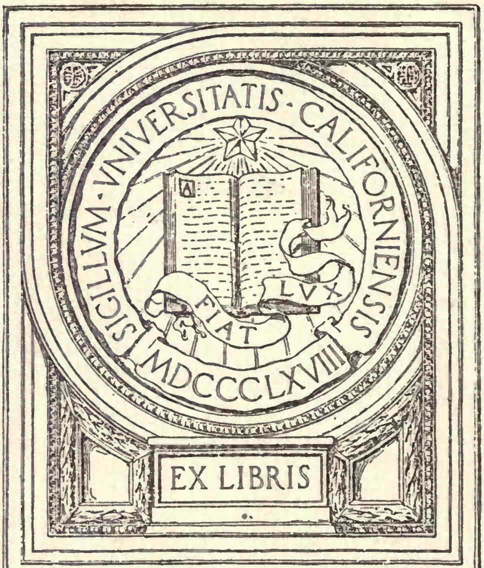
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THEORETICAL ELEMENTS
OF
ELECTRICAL ENGINEERING

THEORETICAL ELEMENTS
OF
ELECTRICAL ENGINEERING

BY
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FOURTH EDITION
THOROUGHLY REVISED AND ENTIRELY RESET

FOURTH IMPRESSION

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PREFACE TO FIRST EDITION

THE first part of the following volume originated from a series of University lectures which I once promised to deliver. This part can, to a certain extent, be considered as an introduction to my work on "Theory and Calculation of Alternating Current Phenomena," leading up very gradually from the ordinary sine wave representation of the alternating current to the graphical representation by polar coördinates, from there to rectangular components of polar vectors, and ultimately to the symbolic representation by the complex quantity. The present work is, however, broader in its scope, in so far as it comprises the fundamental principles not only of alternating, but also of direct currents.

The second part is a series of monographs of the more important electrical apparatus, alternating as well as direct current. It is, in a certain respect, supplementary to "Alternating Current Phenomena." While in the latter work I have presented the general principles of alternating current phenomena, in the present volume I intended to give a specific discussion of the particular features of individual apparatus. In consequence thereof, this part of the book is somewhat less theoretical, and more descriptive, my intention being to present the most important electrical apparatus in all their characteristic features as regard to external and internal structure, action under normal and abnormal conditions, individually and in connection with other apparatus, etc.

I have restricted the work to those apparatus which experience has shown as of practical importance, and give only those theories and methods which an extended experience in the design and operation has shown as of practical utility. I consider this the more desirable as, especially of late years, electrical literature has been haunted by so many theories (for instance of the induction machine) which are incorrect, or too complicated for use, or valueless in practical application. In the class last mentioned are most of the graphical methods, which, while they may give an approximate insight in the inter-relation of phenomena, fail entirely in engineering practice owing to the great difference in the magnitudes of the vectors in the same diagram, and to the synthetic method of graphical representation, which generally require one to start with the quantity which the diagram is intended to determine.

I originally intended to add a chapter on Rectifying Apparatus, as arc light machines and alternating current rectifiers, but had to postpone this, due to the incomplete state of the theory of these apparatus.

The same notation has been used as in the Third Edition of "Alternating Current Phenomena," that is, vector quantities denoted by dotted capitals. The same classification and nomenclature have been used as given by the report of the Standardizing Committee of the American Institute of Electrical Engineers.

CHARLES PROTEUS STEINMETZ.

SCHENECTADY, N. Y., *May* 1*st*, 1901.

PREFACE TO THIRD EDITION

Nearly eight years have elapsed since the appearance of the second edition, during which time the book has been reprinted without change, and a revision, therefore, became greatly desired.

It was gratifying, however, to find that none of the contents of the former edition had to be dropped as superseded or antiquated. However, very much new material had to be added. During these eight years the electrical industry has progressed at least as rapidly as in any previous period, and apparatus and phenomena which at the time of the second edition were of theoretical interest only, or of no interest at all, have now assumed great industrial importance, as for instance the single-phase commutator motor, the control of commutation by commutating poles, etc.

Besides rewriting and enlarging numerous paragraphs throughout the text, a number of new sections and chapters have been added, on alternating-current railway motors, on the control of commutation by commutating poles ("interpoles"), on converter heating and output, on converters with variable ratio of conversion ("split-pole converters"), on three-wire generators and converters, short-circuit currents of alternators, stability and regulation of induction motors, induction generators, etc.

In conformity with the arrangement used in my other books, the paragraphs of the text have been numbered for easier reference and convenience.

When reading the book, or using it as text-book, it is recommended:

After reading the first or general part of the present volume, to read through the first 17 chapters of "Theory and Calculation of Alternating Current Phenomena," omitting, however, the mathematical investigations as far as not absolutely required for the understanding of the text, and then to take up the study of the second part of the present volume, which deals with special apparatus. When reading this second part, it is recommended to parallel its study with the reading of the chapter of "Alternating Current Phenomena" which deals with the same subject in a different manner. In this way a clear insight into the nature and behavior of apparatus will be imparted.

Where time is limited, a large part of the mathematical discussion may be skipped and in that way a general review of the material gained.

Great thanks are due to the technical staff of the McGraw-Hill Book Company, which has spared no effort to produce the third edition in as perfect and systematic a manner as possible, and to the numerous engineers who have greatly assisted me by pointing out typographical and other errors in the previous edition.

CHARLES PROTEUS STEINMETZ.

SCHENECTADY, *September*, 1909.

PREFACE TO THE FOURTH EDITION

With the fourth edition, "Theoretical Elements of Electrical Engineering" has been radically revised and practically rewritten. Since 1897 and 1898, when the first editions of "Alternating Current Phenomena" and "Theoretical Elements" appeared, electrical engineering has enormously expanded and diversified. New material thus had to be added to the successive editions until now it has become utterly impossible to deal with the subject matter adequately within the limited scope of the two books. Therefore in the present edition everything beyond the most fundamental principles of general theory and special apparatus has been withdrawn, to make room for the adequate representation of the theoretical elements of present-day electrical engineering. The same will be done in the new edition of "Alternating Current Phenomena," which is in preparation, and the material, which thus does not find room any more in these two books, together with such additional matters as the development of electrical engineering requires, will be collected in a third volume.

In the present edition, the crank diagram of vector representation, and the symbolic method based on it, which denotes the inductive impedance by $Z = r + jx$, has been adopted in conformity with the decision of the International Electrical Congress of Turin. This crank diagram is somewhat inferior in utility to the polar diagram used in the previous editions, since it is limited to sine waves. I believe it was adopted without sufficient consideration of the relative merits. Nevertheless the advantage of the use of the same vector representation in all elementary text-books on electrical engineering, seems to me to outweigh the advantage of the polar diagram resulting from its ability to represent waves which are not sines, while in advanced electrical engineering both representations will have to remain in use.

CHARLES P. STEINMETZ.

SCHENECTADY, N. Y., *October*, 1915.

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PART I
GENERAL THEORY

1. MAGNETISM AND ELECTRIC CURRENT

1. A magnet pole attracting (or repelling) another magnet pole of equal strength at unit distance with unit force[1] is called a *unit magnet pole.*

The space surrounding a magnet pole is called a *magnetic field of force,* or *magnetic field.*

The magnetic field at unit distance from a unit magnet pole is called a *unit magnetic field,* and is represented by one line of magnetic force (or shortly "one line") per square centimeter, and from a unit magnet pole thus issue a total of 4π lines of magnetic force.

The total number of lines of force issuing from a magnet pole is called its *magnetic flux.*

The magnetic flux Φ of a magnet pole of strength m is,

$$\Phi = 4\pi m.$$

At the distance l from a magnet pole of strength m, and therefore of flux $\Phi = 4\pi m$, assuming a uniform distribution in all directions, the magnetic field has the intensity,

$$H = \frac{\Phi}{4\pi l^2} = \frac{m}{l^2}.$$

since the Φ lines issuing from the pole distribute over the area of a sphere of radius l, that is, the area $4\pi l^2$.

A magnetic field of intensity H exerts upon a magnet pole of strength m the force,

$$mH.$$

Thus two magnet poles of strengths m_1 and m_2, and distance l from each other, exert upon each other the force,

$$\frac{m_1 m_2}{l^2}.$$

[1] That is, at 1 cm. distance with such force as to give to the mass of 1 gram the acceleration of 1 cm. per second.

2. Electric currents produce magnetic fields also; that is, the space surrounding the conductor carrying an electric current is a magnetic field, which appears and disappears and varies with the current producing it, and is indeed an essential part of the phenomenon called an electric current.

Thus an electric current represents a *magnetomotive force* (m.m.f.).

The magnetic field of a straight conductor, whose return conductor is so far distant as not to affect the field, consists of lines of force surrounding the conductor in concentric circles. The intensity of this magnetic field is directly proportional to the current strength and inversely proportional to the distance from the conductor.

Since the lines of force of the magnetic field produced by an electric current return into themselves, the magnetic field is a *magnetic circuit.* Since an electric current, at least a steady current, can exist only in a closed circuit, electricity flows in an *electric circuit.* The magnetic circuit produced by an electric current surrounds the electric circuit through which the electricity flows, and inversely. That is, the electric circuit and the magnetic circuit are *interlinked* with each other.

Unit current in an electric circuit is the current which produces in a magnetic circuit of unit length the field intensity 4π, that is, produces as many lines of force per square centimeter as issue from a unit magnet pole.

In unit distance from an electric conductor carrying unit current, that is, in a magnetic circuit of length 2π, the field intensity is $\frac{4\pi}{2\pi} = 2$, and in the distance 2 the field intensity is unity; that is, unit current is the current which, in a straight conductor, whose return conductor is so far distant as not to affect its magnetic field, produces field intensity 2 in unit distance from the conductor.

One-tenth of unit current is the practical unit, called *one ampere.*

3. One ampere in an electric circuit or turn, that is, one ampere-turn, thus produces in a magnetic circuit of unit length the field intensity 0.4π, and in a magnetic circuit of length l the field intensity $\frac{0.4\pi}{l}$; and F ampere-turns produce in a magnetic circuit of length l the field intensity:

$$H = \frac{0.4\pi F}{l} \text{ lines of force per sq. cm.}$$

regardless whether the F ampere-turns are due to F amperes in a single turn, or 1 amp. in F turns, or $\frac{F}{n}$ amperes in n turns.

F, that is, the product of amperes and turns, is called *magnetomotive force* (m.m.f.).

The m.m.f. per unit length of magnetic circuit, or ratio,

$$f = \frac{\text{m.m.f.}}{\text{length of magnetic circuit}},$$

is called the *magnetizing force,* or *magnetic gradient.*

Hence, m.m.f. is expressed in *ampere-turns;* magnetizing force in *ampere-turns per centimeter* (or in practice frequently ampere-turns per inch), field intensity in lines of magnetic force per square centimeter.

At the distance l from the conductor of a loop or circuit of F ampere-turns, whose return conductor is so far distant as not to affect the field, assuming the m.m.f. $= F$, since the length of the magnetic circuit $= 2\,\pi l$, we obtain as the magnetizing force,

$$f = \frac{F}{2\,\pi l},$$

and as the field intensity,

$$H = 0.4\,\pi f = \frac{0.2\,F}{l}.$$

4. The magnetic field of an electric circuit consisting of two parallel conductors (or any number of conductors, in a polyphase system), as the two wires of a transmission line, can be considered as the superposition of the separate fields of the conductors (consisting of concentric circles). Thus, if there are I amperes in a circuit consisting of two parallel conductors (conductor and return conductor), at the distance l_1 from the first and l_2 from the second conductor, the respective field intensities are,

$$H_1 = \frac{0.2\,I}{l_1},$$

and

$$H_2 = \frac{0.2\,I}{l_2},$$

and the resultant field intensity, if $\tau =$ angle between the directions of the two fields,

$$H = \sqrt{H_1^2 + H_2^2 + 2\,H_1 H_2 \cos\tau},$$

$$= \frac{0.2\,I}{l_1 l_2}\sqrt{l_1^2 + l_2^2 + 2\,l_1 l_2 \cos\tau}.$$

In the plane of the conductors, where the two fields are in the same or opposite direction, the resultant field intensity is,

$$H = \frac{0.2\,I\,(l_1 \pm l_2)}{l_1 l_2},$$

where the plus sign applies to the space between, the minus sign the space outside of the conductors.

The resultant field of a circuit of parallel conductors consists of excentric circles, interlinked with the conductors, and crowded together in the space between the conductors as shown in Fig. 1 by drawn lines.

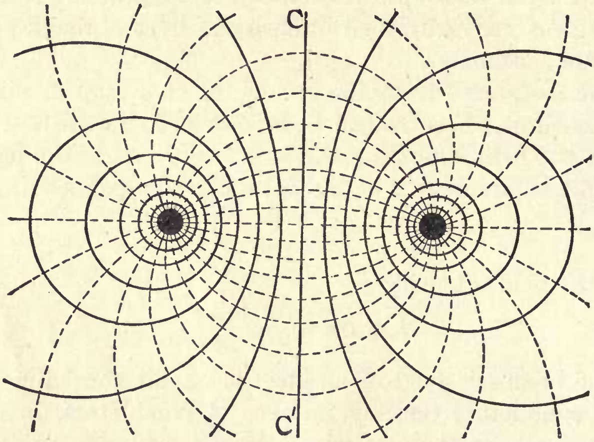

FIG. 1.—Magnetic field of parallel conductors.

The magnetic field in the interior of a spiral (solenoid, helix, coil) carrying an electric current consists of straight lines.

5. If a conductor is coiled in a spiral of l centimeter axial length of spiral, and N turns, thus $n = \frac{N}{l}$ turns per centimeter length of spiral, and I = current, in amperes, in the conductor, the m.m.f. of the spiral is

$$F = IN,$$

and the magnetizing force in the middle of the spiral, assuming the latter of very great length,

$$f = nI = \frac{N}{l}I,$$

thus the field intensity in the middle of the spiral or solenoid,

$$\begin{aligned} H &= 0.4\,\pi f \\ &= 0.4\,\pi n I. \end{aligned}$$

Strictly this is true only in the middle part of a spiral of such length that the m.m.f. consumed by the external or magnetic return circuit of the spiral is negligible compared with the m.m.f. consumed by the magnetic circuit in the interior of the spiral, or in an endless spiral, that is, a spiral whose axis curves back into itself, as a spiral whose axis is curved in a circle.

Magnetomotive force F applies to the total magnetic circuit, or part of the magnetic circuit. It is measured in ampere-turns.

Magnetizing force f is the m.m.f. per unit length of magnetic circuit. It is measured in ampere-turns per centimeter.

Field intensity H is the number of lines of force per square centimeter.

If l = length of the magnetic circuit or a part of the magnetic circuit,

$$F = lf, \qquad f = \frac{F}{l},$$

$$H = 0.4\,\pi f \qquad f = \frac{H}{0.4\,\pi},$$

$$= 1.257\,f \qquad f = 0.796\,H.$$

6. The preceding applies only to magnetic fields in air or other unmagnetic materials.

If the medium in which the magnetic field is established is a "magnetic material," the number of lines of force per square centimeter is different and usually many times greater. (Slightly less in diamagnetic materials.)

The ratio of the number of lines of force in a medium, to the number of lines of force which the same magnetizing force would produce in air (or rather in a vacuum), is called the *permeability* or magnetic conductivity μ of the medium.

The number of lines of force per square centimeter in a magnetic medium is called the *magnetic induction* B. The number of lines of force produced by the same magnetizing force in air, or rather, in the vacuum, is called the *field intensity* H.

In air, magnetic induction B and field intensity H are equal.

As a rule, the magnetizing force in a magnetic circuit is changed by the introduction of the magnetic material, due to the change of distribution of the magnetic flux.

The permeability of air = 1 and is constant.

The permeability of iron and other magnetic materials varies with the magnetizing force between a little above 1 and values beyond 10,000 in soft iron.

The magnetizing force f in a medium of permeability μ produces the field intensity $H = 0.4\,\pi f$ and the magnetic induction $B = 0.4\,\pi\mu f$.

EXAMPLES

7. (1) A pull of 2 grams at 4 cm. radius is required to hold a horizontal bar magnet 12 cm. in length, pivoted at its center, in a position at right angles to the magnetic meridian. What is the intensity of the poles of the magnet, and the number of lines of magnetic force issuing from each pole, if the horizontal intensity of the terrestrial magnetic field $H = 0.2$, and the acceleration of gravity = 980?

The distance between the poles of the bar magnet may be assumed as five-sixths of its length.

Let m = intensity of magnet poles. $l = 5$ is the radius on which the terrestrial magnetism acts.

Thus $2\,mHl = 2\,m$ = torque exerted by the terrestrial magnetism.

2 grams weight $= 2 \times 980 = 1960$ units of force. These at 4 cm. radius give the torque $4 \times 1960 = 7840\,g$ cm.

Hence $2\,m = 7840$.

$m = 3920$ is the strength of each magnet pole and

$\Phi = 4\,\pi m = 49{,}000$, the number of lines of force issuing from each pole.

8. (2) A conductor carrying 100 amp. runs in the direction of the magnetic meridian. What position will a compass needle assume, when held below the conductor at a distance of 50 cm., if the intensity of the terrestrial magnetic field is 0.2?

The intensity of the magnetic field of 100 amp., 50 cm. from the conductor, is $H = \frac{0.2\,I}{l} = 0.2 \times \frac{100}{50} = 0.4$, the direction is at right angles to the conductor, that is, at right angles to the terrestrial magnetic field.

If τ = angle between compass needle and the north pole of the magnetic meridian, l_0 = length of needle, m = intensity of its magnet pole, the torque of the terrestrial magnetism is $Hml_0 \sin\tau = 0.2\,ml_0 \sin\tau$, the torque of the current is

$$Hml_0 \cos\tau = \frac{0.2\,Iml_0 \cos\tau}{l} = 0.4\,ml_0 \cos\tau.$$

In equilibrium, $0.2\,ml_0 \sin\tau = 0.4\,ml_0 \cos\tau$, or $\tan\tau = 2$, $\tau = 63.4°$.

9. (3) What is the total magnetic flux per $l = 1000$ m. length, passing between the conductors of a long distance transmission line carrying I amperes of current, if $l_d = 0.82$ cm. is the diameter of the conductors (No. 0 B. & S.), $l_s = 45$ cm. the spacing or distance between them?

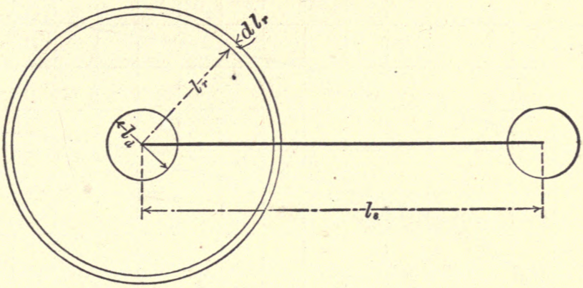

FIG. 2.—Diagram of transmission line for inductance calculation.

At distance l_r from the center of one of the conductors (Fig. 2), the length of the magnetic circuit surrounding this conductor is $2\,\pi l_r$, the m.m.f., I ampere-turns; thus the magnetizing force $f = \frac{I}{2\,\pi l_r}$, and the field intensity $H = 0.4\,\pi f = \frac{0.2\,I}{l_r}$, and the flux in the zone dl_r is $d\Phi = \frac{0.2\,Ildl_r}{l_r}$, and the total flux from the surface of the conductor to the next conductor is,

$$\Phi = \int_{\frac{l_d}{2}}^{l_s} \frac{0.2\,Ildl_r}{l_r} =$$

$$0.2\,Il\Big[\log_\epsilon l_r\Big]_{\frac{l_d}{2}}^{l_s} = 0.2\,Il \log_\epsilon \frac{2l_s}{l_d}.$$

The same flux is produced by the return conductor in the same direction, thus the total flux passing between the transmission wires is,

$$2\,\Phi = 0.4\,Il \log_\epsilon \frac{2l}{l_d}$$

or per 1000 m. $= 10^5$ cm. length,

$$2\,\Phi = 0.4 \times 10^5\,I \log_\epsilon \frac{90}{0.82} = 0.4 \times 10^5 \times 4.70\,I = 0.188 \times 10^6\,I,$$

or 0.188 I megalines or millions of lines per line of 1000 m. of which 0.094 I megalines surround each of the two conductors.

10. (4) In an alternator each pole has to carry 6.4 millions of lines, or 6.4 megalines magnetic flux. How many ampere-turns per pole are required to produce this flux, if the magnetic

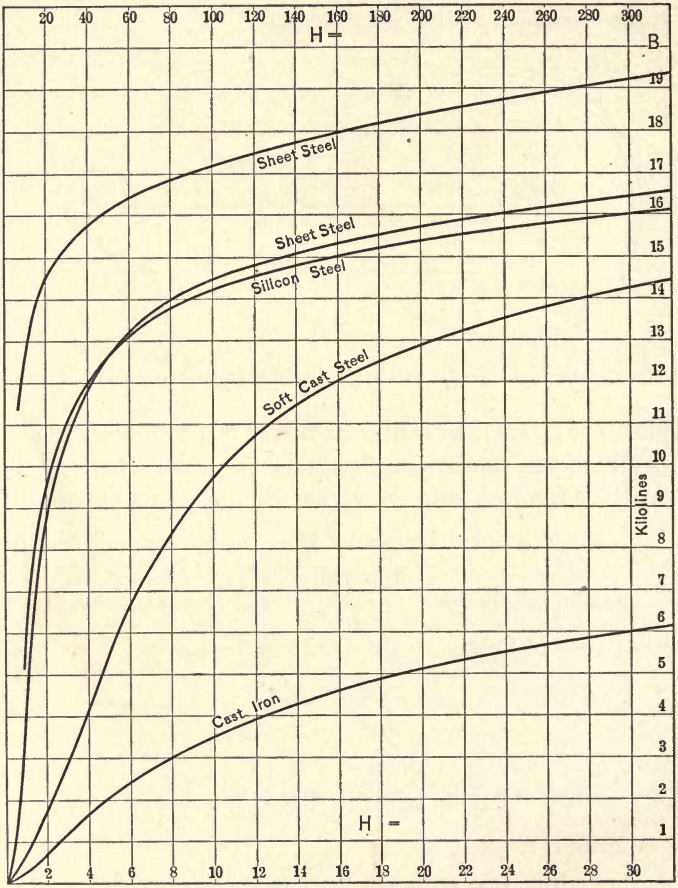

Fig. 3.—Magnetization curves of various irons.

circuit in the armature of laminated iron has the cross section of 930 sq. cm. and the length of 15 cm., the air-gap between stationary field poles and revolving armature is 0.95 cm. in length and 1200 sq. cm. in section, the field pole is 26.3 cm. in length and 1075 sq. cm. in section, and is of laminated iron, and the outside return circuit or yoke has a length per pole of 20 cm. and 2250 sq. cm. section, and is of cast iron?

The magnetic densities are: $B_1 = 6880$ in the armature, $B_2 = 5340$ in the air-gap, $B_3 = 5950$ in the field pole, and $B_4 = 2850$ in the yoke. The permeability of sheet iron is $\mu_1 = 2550$ at $B_1 = 6880$, $\mu_3 = 2380$ at $B_3 = 5950$. The permeability of cast iron is $\mu_4 = 280$ at $B_4 = 2850$. Thus the field intensity $\left(H = \frac{B}{\mu}\right)$ is: $H_1 = 2.7$, $H_2 = 5340$, $H_3 = 2.5$, $H_4 = 10.2$.

The magnetizing force $\left(f = \frac{H}{0.4\pi}\right)$ is, $f_1 = 2.15$, $f_2 = 4250$, $f_3 = 1.99$, $f_4 = 8.13$ ampere-turns per centimeter. Thus the m.m.f. $(F = fl)$ is: $F_1 = 32, F_2 = 4040, F_3 = 52, F_4 = 163$, or the total m.m.f. per pole is

$$F = F_1 + F_2 + F_3 + F_4 = 4290 \text{ ampere-turns.}$$

The permeability μ of magnetic materials varies with the density B, thus tables or curves have to be used for these quantities. Such curves are usually made out for density B and magnetizing force f, so that the magnetizing force f corresponding to the density B can be derived directly from the curve. Such a set of curves is given in Fig. 3.

2. MAGNETISM AND E.M.F.

11. In an electric conductor moving relatively to a magnetic field, an e.m.f. is generated proportional to the rate of cutting of the lines of magnetic force by the conductor.

Unit e.m.f. is the e.m.f. generated in a conductor cutting one line of magnetic force per second.

10^8 times unit e.m.f. is the practical unit, called the *volt.*

Coiling the conductor n fold increases the e.m.f. n fold, by cutting each line of magnetic force n times.

In a closed electric circuit the e.m.f. produces an *electric current.*

The ratio of e.m.f. to electric current produced thereby is called the *resistance* of the electric circuit.

Unit resistance is the resistance of a circuit in which unit e.m.f. produces unit current.

10^9 times unit resistance is the practical unit, called the *ohm.*

The ohm is the resistance of a circuit, in which 1 volt produces 1 amp.

The resistance per unit length and unit section of a conductor is called its resistivity, ρ.

The resistivity ρ is a constant of the material, varying with the temperature.

The resistance r of a conductor of length l, area or section A, and resistivity ρ is $r = \frac{l\rho}{A}$.

12. If the current in the electric circuit changes, starts, or stops, the corresponding change of the magnetic field of the current generates an e.m.f in the conductor carrying the current, which is called the *e.m.f. of self-induction.*

If the e.m.f. in an electric circuit moving relatively to a magnetic field produces a current in the circuit, the magnetic field produced by this current is called its *magnetic reaction.*

The fundamental law of self-induction and magnetic reaction is that these effects take place in such a direction as to oppose their cause (Lentz's law).

Thus the e.m.f. of self-induction during an increase of current is in the opposite direction, during a decrease of current in the same direction as the e.m.f. producing the current.

The magnetic reaction of the current produced in a circuit moving out of a magnetic field is in the same direction, in a circuit moving into a magnetic field in opposite direction to the magnetic field.

Essentially, this law is nothing but a conclusion from the law of conservation of energy.

EXAMPLES

13. (1) An electromagnet is placed so that one pole surrounds the other pole cylindrically as shown in section in Fig. 4, and a copper cylinder revolves between these poles at 3000 rev. per min. What is the e.m.f. generated between the ends of this cylinder, if the magnetic flux of the electromagnet is $\Phi = 25$ megalines?

During each revolution the copper cylinder cuts 25 megalines. It makes 50 rev. per sec. Thus it cuts $50 \times 25 \times 10^6 = 12.5 \times 10^8$ lines of magnetic flux per second. Hence the generated e.m.f. is $E = 12.5$ volts.

Such a machine is called a "unipolar," or more properly a "non-polar" or an "acyclic," generator.

14. (2) The field spools of the 20-pole alternator in Section 1, Example 4, are wound each with 616 turns of wire No. 7 (B. & S.), 0.106 sq. cm. in cross section and 160 cm. mean length of turn. The 20 spools are connected in series. How many amperes and how many volts are required for the excitation of this alternator field, if the resistivity of copper is 1.8×10^{-6} ohms per cm.^{3} [1]

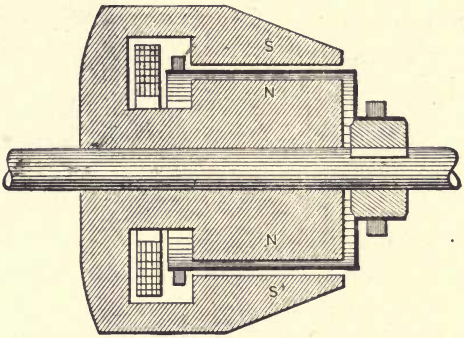

Fig. 4.—Unipolar generator.

Since 616 turns on each field spool are used, and 4280 ampere-turns required, the current is $\frac{4280}{616} = 6.95$ amp.

The resistance of 20 spools of 616 turns of 160 cm. length, 0.106 sq. cm. section, and 1.8×10^{-6} resistivity is,

$$\frac{20 \times 616 \times 160 \times 1.8 \times 10^{-6}}{0.106} = 33.2 \text{ ohms,}$$

and the e.m.f. required, $6.95 \times 33.2 = 230$ volts.

3. GENERATION OF E.M.F.

15. A closed conductor, convolution or turn, revolving in a magnetic field, passes during each revolution through two positions of maximum inclosure of lines of magnetic force A in Fig. 5, and two positions of zero inclosure of lines of magnetic force B in Fig. 5.

[1] cm.^{3} refers to a cube whose side is 1 cm., and should not be confused with cu. cm.

Thus it cuts during each revolution four times the lines of force inclosed in the position of maximum inclosure.

If Φ = the maximum number of lines of force inclosed by the conductor, f = the frequency in revolutions per second or cycles, and n = number of convolutions or turns of the conductor, the lines of force cut per second by the conductor, and thus the *average generated e.m.f.* is,

$$E = 4\, fn\Phi \text{ absolute units,}$$
$$= 4\, fn\Phi\, 10^{-8} \text{ volts.}$$

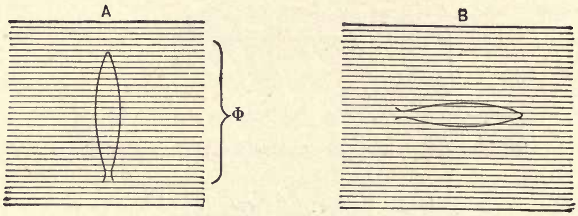

FIG. 5.—Generation of e.m.f.

If f is given in hundreds of cycles, Φ in megalines,

$$E = 4\, fn\Phi \text{ volts.}$$

If a coil revolves with uniform velocity through a uniform magnetic field, the magnetism inclosed by the coil at any instant is,

$$\Phi \cos \tau$$

where Φ = the maximum magnetism inclosed by the coil and τ = angle between coil and its position of maximum inclosure of magnetism.

The e.m.f. generated in the coil, which varies with the rate of cutting or change of $\Phi \cos \tau$, is thus,

$$e = E_0 \sin \tau,$$

where E_0 is the maximum value of e.m.f., which takes place for $\tau = 90°$, or at the position of zero inclosure of magnetic flux since in this position the rate of cutting is greatest.

Since avg. $(\sin \tau) = \frac{2}{\pi}$, the average generated e.m.f. is,

$$E = \frac{2}{\pi} E_0.$$

Since, however, we found above that

$$E = 4\,fn\Phi \text{ is the average generated e.m.f.,}$$

it follows that

$$E_0 = 2\,\pi fn\Phi \text{ is the } \textit{maximum}, \text{ and}$$
$$e = 2\,\pi fn\Phi \sin \tau \text{ the } \textit{instantaneous generated e.m.f.}$$

The interval between like poles forms 360 electrical-space degrees, and in the two-pole model these are identical with the mechanical-space degrees. With uniform rotation, Fig. 6, the space angle, τ, is proportional to time. Time angles are designated by θ, and with uniform rotation $\theta = \tau$, τ being measured in electrical-space degrees.

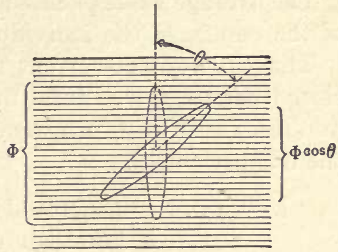

FIG. 6.—Generation of e.m.f. by rotation.

The period of a complete cycle is 360 time degrees, or $2\,\pi$ or $\frac{1}{f}$ seconds. In the two-pole model the period of a cycle is that of one complete revolution, and in a $2\,n_p$-pole machine, $\frac{1}{n_p}$ of that of one revolution.

Thus,

$$\theta = 2\,\pi ft$$
$$e = 2\,\pi fn\Phi \sin 2\,\pi ft.$$

If the time is not counted from the moment of maximum inclosure of magnetic flux, but t_1 = the time at this moment, we have

$$e = 2\,\pi fn\Phi \sin 2\,\pi f\,(t - t_1)$$

or,

$$e = 2\,\pi fn\Phi \sin (\theta - \theta_1),$$

where $\theta_1 = 2\,\pi f t_1$ is the angle at which the position of maximum inclosure of magnetic flux takes place, and is called its phase.

These e.m.fs. are alternating.

If at the moment of reversal of the e.m.f. the connections between the coil and the external circuit are reversed, the e.m.f. in the external circuit is pulsating between zero and E_0, but has the same average value E.

If a number of coils connected in series follow each other successively in their rotation through the magnetic field, as the armature coils of a direct-current machine, and the connections of each coil with the external circuit are reversed at the moment of reversal of its e.m.f., their pulsating e.m.fs. superimposed in the external circuit make a more or less steady or continuous external e.m.f.

The average value of this e.m.f. is the sum of the average values of the e.m.fs. of the individual coils.

Thus in a direct-current machine, if Φ = maximum flux inclosed per turn, n = total number of turns in series from commutator brush to brush, and f = frequency of rotation through the magnetic field.

$E = 4 fn\Phi$ = generated e.m.f. (Φ in megalines, f in hundreds of cycles per second).

This is the *formula of the direct-current generator.*

EXAMPLES

17. (1) A circular wire coil of 200 turns and 40 cm. mean diameter is revolved around a vertical axis. What is the horizontal intensity of the magnetic field of the earth, if at a speed of 900 rev. per min. the average e.m.f generated in the coil is 0.028 volt?

The mean area of the coil is $\frac{40^2 \pi}{4} = 1255$ sq. cm., thus the terrestrial flux inclosed is $1255\,H$, and at 900 rev. per min. or 15 rev. per sec., this flux is cut $4 \times 15 = 60$ times per second by each turn, or $200 \times 60 = 12{,}000$ times by the coil. Thus the total number of lines of magnetic force cut by the conductor per second is $12{,}000 \times 1255\,H = 0.151 \times 10^8\,H$, and the average generated e.m.f. is $0.151\,H$ volts. Since this is = 0.028 volt, $H = 0.186$.

18. (2) In a 550-volt direct-current machine of 8 poles and drum armature, running at 500 rev. per min., the average voltage per commutator segment shall not exceed 11, each armature coil shall contain one turn only, and the number of commutator segments per pole shall be divisible by 3, so as to use the machine as three-phase converter. What is the magnetic flux per field pole?

550 volts at 11 volts per commutator segment gives 50, or as next integer divisible by 3, $n = 51$ segments or turns per pole.

8 poles give 4 cycles per revolution, 500 rev. per min. gives $^{500}/_{60} = 8.33$ rev. per sec. Thus the frequency is $f = 4 \times 8.33 = 33.3$ cycles per second.

The generated e.m.f. is $E = 550$ volts, thus by the formula of direct-current generator,

$$E = 4\,fn\Phi,$$

or,

$$550 = 4 \times 0.333 \times 51\ \Phi,$$

$$\Phi = 8.1 \text{ megalines per pole.}$$

19. (3) What is the e.m.f. generated in a single turn of a 20-pole alternator running at 200 rev. per min., through a magnetic field of 6.4 megalines per pole?

The frequency is $f = \frac{20 \times 200}{2 \times 60} = 33.3$ cycles.

$$e = E_0 \sin \tau,$$

$$E_0 = 2\,\pi fn\Phi,$$

$$\Phi = 6.4,$$

$$n = 1,$$

$$f = 0.333.$$

Thus, $E_0 = 2\,\pi \times 0.333 \times 6.4 = 13.4$ volts maximum, or $e = 13.4 \sin\theta$.

4. POWER AND EFFECTIVE VALUES

20. The power of the continuous e.m.f. E producing continuous current I is $P = EI$.

The e.m.f. consumed by resistance r is $E_1 = Ir$, thus the power consumed by resistance r is $P = I^2r$.

Either $E_1 = E$, then the total power in the circuit is consumed by the resistance, or $E_1 < E$, then only a part of the power is consumed by the resistance, the remainder by some counter e.m.f., $E - E_1$.

If an alternating current $i = I_0 \sin\theta$ passes through a resistance r, the power consumed by the resistance is,

$$i^2r = I_0{}^2r \sin^2\theta = \frac{I_0{}^2r}{2}(1 - \cos 2\,\theta),$$

thus varies with twice the frequency of the current, between zero and $I_0{}^2r$.

The average power consumed by resistance r is,

$$\text{avg. } (i^2r) = \frac{I_0{}^2r}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 r,$$

since avg. (cos) = 0.

Thus the alternating current $i = I_0$ since θ consumes in a resistance r the same effect as a continuous current of intensity

$$I = \frac{I_0}{\sqrt{2}}.$$

The value $I = \frac{I_0}{\sqrt{2}}$ is called the *effective value* of the alternating current $i = I_0 \sin \theta$; since it gives the same effect.

Analogously $E = \frac{E_0}{\sqrt{2}}$ is the effective value of the alternating e.m.f., $e = E_0 \sin \theta$.

Since $E_0 = 2 \pi f n \Phi$, it follows that

$$\begin{aligned} E &= \sqrt{2}\, \pi f n \Phi \\ &= 4.44\, f n \Phi \end{aligned}$$

is the *effective alternating e.m.f.* generated in a coil of turns n rotating at a frequency of f (in hundreds of cycles per second) through a magnetic field of Φ megalines of force.

This is the *formula of the alternating-current generator.*

21. The formula of the direct-current generator,

$$E = 4\, f n \Phi,$$

holds even if the e.m.fs. generated in the individual turns are not sine waves, since it is the average generated e.m.f.

The formula of the alternating-current generator,

$$E = \sqrt{2}\, \pi f n \Phi,$$

does not hold if the waves are not sine waves, since the ratios of average to maximum and of maximum to effective e.m.f. are changed.

If the variation of magnetic flux is not sinusoidal, the effective generated alternating e.m.f. is,

$$E = \gamma \sqrt{2}\, \pi f n \Phi.$$

γ is called the *form factor* of the wave, and depends upon its shape, that is, the distribution of the magnetic flux in the magnetic field.

Frequently *form factor* is defined as the ratio of the effective to the average value. This definition is undesirable since it gives for the sine wave, which is always considered the standard wave, a value differing from one.

EXAMPLES

22. (1) In a star-connected 20-pole three-phase machine, revolving at 33.3 cycles or 200 rev. per min., the magnetic flux per pole is 6.4 megalines. The armature contains one slot per pole and phase, and each slot contains 36 conductors. All these conductors are connected in series. What is the effective e.m.f. per circuit, and what the effective e.m.f. between the terminals of the machine?

Twenty slots of 36 conductors give 720 conductors, or 360 turns in series. Thus the effective e.m.f. is,

$$\begin{aligned} E_1 &= \sqrt{2}\,\pi f n \Phi \\ &= 4.44 \times 0.333 \times 360 \times 6.4 \\ &= 3400 \text{ volts per circuit.} \end{aligned}$$

The e.m.f. between the terminals of a star-connected three-phase machine is the resultant of the e.m.fs. of the two phases, which differ by 60 degrees, and is thus 2 sin 60° $= \sqrt{3}$ times that of one phase, thus,

$$\begin{aligned} E &= E_1 \sqrt{3} \\ &= 5900 \text{ volts effective.} \end{aligned}$$

23. (2) The conductor of the machine has a section of 0.22 sq. cm. and a mean length of 240 cm. per turn. At a resistivity (resistance per unit section and unit length) of copper of $\rho = 1.8 \times 10^{-6}$, what is the e.m.f. consumed in the machine by the resistance, and what the power consumed at 450 kw. output?

450 kw. output is 150,000 watts per phase or circuit, thus the current $I = \frac{150{,}000}{3400} = 44.2$ amperes effective.

The resistance of 360 turns of 240 cm. length, 0.22 sq. cm. section and 1.8×10^{-6} resistivity, is

$$r = \frac{360 \times 240 \times 1.8 \times 10^{-6}}{0.22} = 0.71 \text{ ohms per circuit.}$$

44.2 amp. $\times$ 0.71 ohms gives 31.5 volts per circuit and $(44.2)^2 \times 0.71 = 1400$ watts per circuit, or a total of $3 \times 1400 = 4200$ watts loss.

24. (3) What is the self-inductance per wire of a three-phase line of 14 miles length consisting of three wires No. 0 ($l_d = 0.82$ cm.), 45 cm. apart, transmitting the output of this 450 kw. 5900-volt three-phase machine?

450 kw. at 5900 volts gives 44.2 amp. per line. 44.2 amp. effective gives $44.2\sqrt{2} = 62.5$ amp. maximum.

14 miles = 22,400 m. The magnetic flux produced by I amperes in 1000 m. of a transmission line of 2 wires 45 cm. apart and 0.82 cm. diameter was found in paragraph 1, example 3, as $2\,\Phi = 0.188 \times 10^6 I$, or $\Phi = 0.094 \times 10^6 I$ for each wire.

Thus at 22,300 m. and 62.5 amp. maximum, the flux per wire is

$$\Phi = 22.3 \times 62.5 \times 0.094 \times 10^6 = 131 \text{ megalines.}$$

Hence the generated e.m.f., effective value, at 33.3 cycles is,

$$\begin{aligned} E &= \sqrt{2}\,\pi f \Phi \\ &= 4.44 \times 0.333 \times 131 \\ &= 193 \text{ volts per line;} \end{aligned}$$

the maximum value is,

$$E_0 = E \times \sqrt{2} = 273 \text{ volts per line;}$$

and the instantaneous value,

$$e = E_0 \sin\ (\theta - \theta_1) = 273\ \sin\ (\theta - \theta_1);$$

or, since $\theta = 2\,\pi ft = 210\,t$ we have,

$$e = 273\ \sin\ 210\ (t - t_1).$$

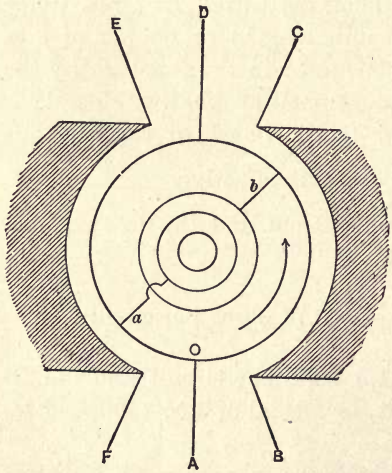

FIG. 7.—Diagram of bipolar generator.

25. (4) What is the form factor (a) of the e.m.f. generated in a single conductor of a direct-current machine having 80 per cent. pole arc and negligible spread of the magnetic flux at the pole corners, and (b) what is the form factor of the voltage between two collector rings connected to diametrical points of the armature of such a machine?

(a) In a conductor during the motion from position A, shown in Fig. 7, to position B, no e.m.f. is generated; from position B to C a constant e.m.f. e is generated, from C to E again no e.m.f., from E to F a constant e.m.f. $-e$, and from F to A again zero e.m.f. The e.m.f. wave thus is as shown in Fig. 8.

The average e.m.f. is

$$e_1 = 0.8\,e;$$

hence, with this average e.m.f., if it were a sine wave, the maximum e.m.f. would be

$$e_2 = \frac{\pi}{2} e_1 = 0.4\,\pi e,$$

and the effective e.m.f. would be

$$e_3 = \frac{e_2}{\sqrt{2}} = \frac{0.4\,\pi e}{\sqrt{2}}.$$

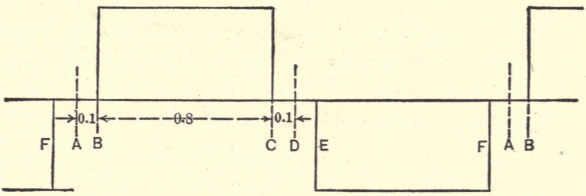

Fig. 8.—E.m.f. of a single conductor, direct-current machine 80 per cent. pole arc.

The actual square of the e.m.f. is e^2 for 80 per cent. and zero for 20 per cent. of the period, and the average or mean square thus is

$$0.8\,e^2,$$

and therefore the actual effective value,

$$e_4 = e\sqrt{0.8}.$$

The form factor γ, or the ratio of the actual effective value e_4 to the effective value e_3 of a sine wave of the same mean value and thus the same magnetic flux, then is

$$\gamma = \frac{e_4}{e_3} = \frac{\sqrt{10}}{\pi}$$
$$= 1.006;$$

that is, practically unity.

(*b*) While the collector leads a, b move from the position F, C, as shown in Fig. 6, to B, E, constant voltage E exists between them, the conductors which leave the field at C being replaced by the conductors entering the field at B. During the motion of the leads a, b from B, E to C, F, the voltage steadily decreases, reverses, and rises again, to $-E$, as the conductors entering the field at E have an e.m.f. opposite to that of the conductors leaving at C. Thus the voltage wave is, as shown by Fig. 9, triangular, with the top cut off for 20 per cent. of the half wave.

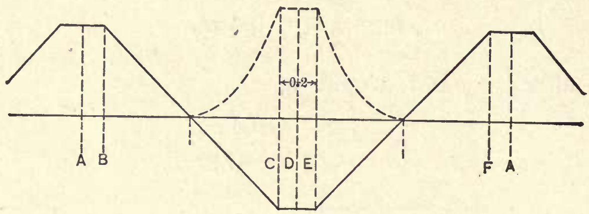

FIG. 9.—E.m.f. between two collector rings connected to diametrical points of the armature of a bipolar machine having 80 per cent. pole arc.

Then the average e.m.f. is

$$e_1 = 0.2\,E + 2 \times \frac{0.4\,E}{2} = 0.6\,E.$$

The maximum value of a sine wave of this average value is

$$e_2 = \frac{\pi}{2}\,e_1 = 0.3\,\pi E,$$

and the effective value corresponding thereto is

$$e_3 = \frac{e_2}{\sqrt{2}} = \frac{0.3\,\pi E}{\sqrt{2}}.$$

The actual voltage square is E^2 for 20 per cent. of the time, and rising on a parabolic curve from 0 to E^2 during 40 per cent. of the time, as shown in dotted lines in Fig. 9.

The area of a parabolic curve is width times one-third of height, or

$$\frac{0.4\,E^2}{3},$$

hence, the mean square of voltage is

$$0.2\,E^2 + 2 \times \frac{0.4\,E^2}{3} = \frac{1.4\,E^2}{3},$$

and the actual effective voltage is

$$e_4 = E\sqrt{\frac{1.4}{3}};$$

hence, the form factor is

$$\gamma = \frac{e_4}{e_3} = \frac{1}{\pi}\sqrt{\frac{280}{27}} = 1.025,$$

or, 2.5 per cent. higher than with a sine wave.

5. SELF-INDUCTANCE AND MUTUAL INDUCTANCE

26. The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in the circuit is called the *inductance* of the circuit.

The number of interlinkages of an electric circuit with the lines of magnetic force of the flux produced by unit current in a second electric circuit is called the *mutual inductance* of the second upon the first circuit. It is equal to the mutual inductance of the first upon the second circuit, as will be seen, and thus is called the mutual inductance between the two circuits.

The number of interlinkages of an electric circuit with the lines of magnetic flux produced by unit current in this circuit and *not* interlinked with a second circuit is called the *self-inductance* of the circuit.

If i = current in a circuit of n turns, Φ = flux produced thereby and interlinked with the circuit, $n\Phi$ is the total number of interlinkages, and $L = \frac{n\Phi}{i}$ the inductance of the circuit.

If Φ is proportional to the current i and the number of turns n,

$$\Phi = \frac{ni}{\mathcal{R}}, \text{ and } L = \frac{n^2}{\mathcal{R}} \text{ the inductance.}$$

$\mathcal{R}$ is called the *reluctance* and ni the m.m.f. of the magnetic circuit.

In magnetic circuits the reluctance $\mathcal{R}$ has a position similar to that of resistance r in electric circuits.

The reluctance $\mathcal{R}$, and therefore the inductance, is not constant in circuits containing magnetic materials, such as iron, etc.

If $\mathcal{R}_1$ is the reluctance of a magnetic circuit interlinked with two electric circuits of n_1 and n_2 turns respectively, the flux produced by unit current in the first circuit and interlinked with the second circuit is $\frac{n_1}{\mathcal{R}_1}$ and the mutual inductance of the first upon the second circuit is $M = \frac{n_1 n_2}{\mathcal{R}_1}$, that is, equal to the mutual inductance of the second circuit upon the first circuit, as stated above.

If no flux leaks between the two circuits, that is, if all flux is interlinked with both circuits, and L_1 = inductance of the first, L_2 = inductance of the second circuit, and M = mutual inductance, then

$$M^2 = L_1 L_2.$$

If flux leaks between the two circuits, then $M^2 < L_1 L_2$.

In this case the total flux produced by the first circuit consists of a part interlinked with the second circuit also, the mutual inductance, and a part passing between the two circuits, that is, interlinked with the first circuit only, its self-inductance.

27. Thus, if L_1 and L_2 are the inductances of the two circuits, $\frac{L_1}{n_1}$ and $\frac{L_2}{n_2}$ is the total flux produced by unit current in the first and second circuit respectively.

Of the flux $\frac{L_1}{n_1}$ a part $\frac{S_1}{n_1}$ is interlinked with the first circuit only, S_1 being its self-inductance or leakage inductance, and a part $\frac{M}{n_2}$ interlinked with the second circuit also, M being the mutual inductance and $\frac{L_1}{n_1} = \frac{S_1}{n_1} + \frac{M}{n_2}$.

Thus, if

L_1 and L_2 = inductance,
S_1 and S_2 = self-inductance,
M = mutual inductance of two circuits of n and n_2 turns respectively, we have

$$\frac{L_1}{n_1} = \frac{S_1}{n_1} + \frac{M}{n_2} \qquad \frac{L_2}{n_2} = \frac{S_2}{n_2} + \frac{M}{n_1},$$

or

$$L_1 = S_1 + \frac{n_1}{n_2} M \qquad L_2 = S_2 + \frac{n_2}{n_1} M,$$

or

$$M^2 = (L_1 - S_1)(L_2 - S_2).$$

The practical unit of inductance is 10^9 times the absolute unit or 10^8 times the number of interlinkages per ampere (since 1 amp. = 0.1 unit current), and is called the *henry* (h); 0.001 of it is called the *milhenry* (mh.).

The number of interlinkages of i amperes in a circuit of

L henry inductance is $iL\ 10^8$ lines of force turns, and thus the e.m.f. generated by a change of current di in time dt is

$$e = -\frac{di}{dt} L\ 10^8 \text{ absolute units}$$

$$= -\frac{di}{dt} L \text{ volts.}$$

A change of current of 1 amp. per second in the circuit of 1 h. inductance generates 1 volt.

EXAMPLES

28. (1) What is the inductance of the field of a 20-pole alternator, if the 20 field spools are connected in series, each spool contains 616 turns, and 6.95 amp. produces 6.4 megalines per pole?

The total number of turns of all 20 spools is $20 \times 616 = 12{,}320$ Each is interlinked with 6.4×10^6 lines, thus the total number of interlinkages at 6.95 amp. is $12{,}320 \times 6.4 \times 10^6 = 78 \times 10^9$.

6.95 amp. = 0.695 absolute units, hence the number of interlinkages per unit current, or the inductance, is

$$\frac{78 \times 10^9}{0.695} = 112 \times 10^9 = 112 \text{ h.}$$

29. (2) What is the mutual inductance between an alternating transmission line and a telephone wire carried for 10 miles below and 1.20 m. distant from the one, 1.50 m. distant from the other conductor of the alternating line; and what is the e.m.f. generated in the telephone wire, if the alternating circuit carries 100 amp. at 60 cycles?

The mutual inductance between the telephone wire and the electric circuit is the magnetic flux produced by unit current in the telephone wire and interlinked with the alternating circuit, that is, that part of the magnetic flux produced by unit current in the telephone wire, which passes between the distances of 1.20 and 1.50 m.

At the distance l_x from the telephone wire the length of magnetic circuit is $2\pi l_x$. The magnetizing force $f = \frac{I}{2\pi l_x}$ if $I =$

current in telephone wire in amperes, and the field intensity $H = 0.4\pi f = \frac{0.2I}{l_x}$, and the flux in the zone dl_x is

$$d\Phi = \frac{0.2Il}{l_x} dl_x.$$

$$l = 10 \text{ miles} = 1610 \times 10^3 \text{ cm.}$$

thus,
$$\Phi = \int_{120}^{150} \frac{0.2Il}{l_x} dl_x$$
$$= 322 \times 10^3 I \log_\epsilon \frac{150}{120} = 72 I \, 10^3;$$

or, $72 I \, 10^3$ interlinkages, hence, for $I = 10$, or one absolute unit,

thus, $M = 72 \times 10^4$ absolute units $= 72 \times 10^{-5}$ h. $= 0.72$ mh.

100 amp. effective or 141.4 amp. maximum or 14.14 absolute units of current in the transmission line produces a maximum flux interlinked with the telephone line of $14.14 \times 0.72 \times 10^{-3} \times 10^9 = 10.2$ megalines. Thus the e.m.f. generated at 60 cycles is

$$E = 4.44 \times 0.6 \times 10.2 = 27.3 \text{ volts effective.}$$

6. SELF-INDUCTANCE OF CONTINUOUS-CURRENT CIRCUITS

30. Self-inductance makes itself felt in continuous-current circuits only in starting and stopping or, in general, when the current changes in value.

Starting of Current. If r = resistance, L = inductance of circuit, E = continuous e.m.f. impressed upon circuit, i = current in circuit at time t after impressing e.m.f. E, and di the increase of current during time moment dt, then the increase of magnetic interlinkages during time dt is

$$Ldi,$$

and the e.m.f. generated thereby is

$$e_1 = -L \frac{di}{dt}.$$

By Lentz's law it is negative, since it is opposite to the impressed e.m.f., its cause.

Thus the e.m.f. acting in this moment upon the circuit is

$$E + e_1 = E - L \frac{di}{dt},$$

and the current is

$$i = \frac{E + e_1}{r} = \frac{E - L\frac{di}{dt}}{r};$$

or, transposing,

$$-\frac{rdt}{L} = \frac{di}{i - \frac{E}{r}},$$

the integral of which is

$$-\frac{rt}{L} = \log_\epsilon \left(i - \frac{E}{r}\right) - \log_\epsilon c,$$

where $-\log_\epsilon c$ = integration constant.

This reduces to

$$i = \frac{E}{r} + c\epsilon^{-\frac{rt}{L}}$$

at $t = 0$, $i = 0$, and thus

$$-\frac{E}{r} = c.$$

Substituting this value, the current is

$$i = \frac{E}{r}\left(1 - \epsilon^{-\frac{rt}{L}}\right),$$

and the e.m.f. of inductance is

$$e_1 = ir - E = -E\epsilon^{-\frac{rt}{L}}.$$

At $t = \infty$,

$$i_0 = \frac{E}{r}, \qquad e_1 = 0.$$

Substituting these values,

$$i = i_0\left(1 - \epsilon^{-\frac{rt}{L}}\right)$$

and

$$e_1 = -ri_0\epsilon^{-\frac{rt}{L}}.$$

The expression $u = \frac{r}{L}$ is called the "*attenuation constant,*" and its reciprocal, $\frac{L}{r}$, the "*time constant of the circuit.*"[1]

[1] The name *time constant* dates back to the early days of telegraphy, where it was applied to the ratio: $\frac{L}{r}$, that is, the reciprocal of the attenuation constant. This quantity which had gradually come into disuse, again became of importance when investigating transient electric phenomena, and in this work it was found more convenient to denote the value $\frac{r}{L}$ as attenuation constant, since this value appears as one term of the more general constant of the electric circuit $\left(\frac{r}{L} + \frac{g}{C}\right)$. (Theory and Calculation of Transient Electric Phenomena and Oscillations, Section IV.)

Substituted in the foregoing equation this gives

$$i = \frac{E}{r}\left(1 - \epsilon^{-ut}\right)$$

and

$$e_1 = - E\epsilon^{-ut}.$$

At $t = \frac{1}{u}$

$$e_1 = - \frac{E}{r} = - 0.368\, E.$$

31. *Stopping of Current.* In a circuit of inductance L and resistance r, let a current $i_0 = \frac{E}{r}$ be produced by the impressed e.m.f. E, and this e.m.f. E be withdrawn and the circuit closed through a resistance r_1.

Let the current be i at the time t after withdrawal of the e.m.f. E and the change of current during time moment dt be di. di is negative, that is, the current decreases.

The decrease of magnetic interlinkages during moment dt is

$$Ldi.$$

Thus the e.m.f. generated thereby is

$$e_1 = - L\frac{di}{dt}.$$

It is negative since di is negative, and e_1 must be positive, that is, in the same direction as E, to maintain the current or oppose the decrease of current, its cause.

Then the current is

$$i = \frac{e_1}{r + r_1} = - \frac{L}{r + r_1}\frac{di}{dt},$$

or, transposing,

$$- \frac{r + r_1}{L} dt = \frac{di}{i},$$

the integral of which is

$$- \frac{r + r_1}{L} t = \log_\epsilon i - \log_\epsilon c,$$

where $- \log_\epsilon c$ = integration constant.

This reduces to $$i = c\epsilon^{-\frac{r + r_1}{L}t},$$

for $$t = 0, \quad i_0 = \frac{E}{r} = c.$$

Substituting this value, the current is

$$i = \frac{E}{r} \epsilon^{-\frac{(r + r_1) t}{L}},$$

and the generated e.m.f. is

$$e_1 = i (r + r_1) = E \frac{r + r_1}{r} \epsilon^{-\frac{(r + r_1) t}{L}}.$$

Substituting $i_0 = \frac{E}{r}$, the current is

$$i = i_0 \epsilon^{-\frac{r + r_1}{L} t},$$

and the generated e.m.f. is

$$e_1 = i_0 (r + r_1) \epsilon^{-\frac{r + r_1}{L} t}.$$

At $t = 0$,

$$e_1 = E \frac{r + r_1}{r};$$

that is, the generated e.m.f. is increased over the previously impressed e.m.f. in the same ratio as the resistance is increased.

When $r_1 = 0$, that is, when in withdrawing the impressed e.m.f. E the circuit is short circuited,

$$i = \frac{E}{r} \epsilon^{-\frac{rt}{L}} = i_0 \epsilon^{-\frac{rt}{L}} \text{ the current, and}$$

$$e_1 = E \epsilon^{-\frac{rt}{L}} = i_0 r \epsilon^{-\frac{rt}{L}} \text{ the generated e.m.f.}$$

In this case, at $t = 0$, $e_1 = E$, that is, the e.m.f. does not rise.

In the case $r_1 = \infty$, that is, if in withdrawing the e.m.f. E the circuit is broken, we have $t = 0$ and $e_1 = \infty$, that is, the e.m.f. rises infinitely.

The greater r_1, the higher is the generated e.m.f. e_1, the faster, however, do e_1 and i decrease.

If $r_1 = r$, we have at $t = 0$,

$$e_{11} = 2 E, \qquad i = i_0,$$

and

$$e_{11} - i_0 r = E;$$

that is, if the external resistance r_1 equals the internal resistance r, at the moment of withdrawal of the e.m.f. E the terminal voltage is E.

The effect at the time t of the e.m.f. of inductance in stopping the current is

$$ie_1 = i_0^2 (r + r_1) \epsilon^{-2\frac{r + r_1}{L}t};$$

thus the total energy of the generated e.m.f.

$$W = \int_0^\infty ie_1 dt$$

$$= i_0^2 (r + r_1) \left[\epsilon^{-2\frac{r + r_1}{L}t} \right]_0^\infty \left(- \frac{L}{2(r + r_1)} \right) = \frac{i_0^2 L}{2};$$

that is, the energy stored as magnetism in a circuit of current i_0 and inductance L is

$$W = \frac{i_0^2 L}{2},$$

which is independent both of the resistance r of the circuit and the resistance r_1 inserted in breaking the circuit. This energy has to be expended in stopping the current.

EXAMPLES

32. (1) In the alternator field in Section 1, Example 4, Section 2, Example 2, and Section 5, Example 1, how long a time after impressing the required e.m.f. $E = 230$ volts will it take for the field to reach (*a*) ½ strength, (*b*) 9/10 strength?

(2) If 500 volts are impressed upon the field of this alternator, and a non-inductive resistance inserted in series so as to give the required exciting current of 6.95 amp., how long after impressing the e.m.f. $E = 500$ volts will it take for the field to reach (*a*) ½ strength, (*b*) 9/10 strength, (*c*) and what is the resistance required in the rheostat?

(3) If 500 volts are impressed upon the field of this alternator without insertion of resistance, how long will it take for the field to reach full strength?

(4) With full field strength, what is the energy stored as magnetism?

(1) The resistance of the alternator field is 33.2 ohms (Section 2, Example 2), the inductance 112 h. (Section 5, Example 1), the impressed e.m.f. is $E = 230$, the final value of current $i_0 = \frac{E}{r} = 6.95$ amp. Thus the current at time t is

$$i = i_0 \left(1 - \epsilon^{-\frac{rt}{L}}\right)$$

$$= 6.95 \, (1 - \epsilon^{-0.296\, t}).$$

(*a*) ½ strength: $i = \frac{i_0}{2}$, hence $(1 - \epsilon^{-0.296\,t}) = 0.5$.

$\epsilon^{-0.296\,t} = 0.5$, $-0.296\,t \log \epsilon = \log 0.5$, $t = \frac{-\log 0.5}{0.296 \log \epsilon}$, and $t = 2.34$ seconds.

(*b*) $\frac{9}{10}$ strength: $i = 0.9\,i_0$, hence $(1 - \epsilon^{-0.296\,t}) = 0.9$, and $t = 7.8$ seconds.

(2) To get $i_0 = 6.95$ amp., with $E = 500$ volts, a resistance $r = \frac{500}{6.95} = 72$ ohms, and thus a rheostat having a resistance of $72 - 33.2 = 38.8$ ohms, is required.

We then have

$$i = i_0 \left(1 \quad \epsilon^{-\frac{rt}{L}}\right)$$
$$= 6.95\,(1 - \epsilon^{-0.643\,t}).$$

(*a*) $i = \frac{i_0}{2}$, after $t = 1.08$ seconds.

(*b*) $i = 0.9\,i_0$, after $t = 3.6$ seconds.

(3) Impressing $E = 500$ volts upon a circuit of $r = 33.2$, $L = 112$, gives

$$i = \frac{E}{r}\left(1 - \epsilon^{-\frac{rt}{L}}\right)$$
$$= 15.1\,(1 - \epsilon^{-0.296\,t}).$$

$i = 6.95$, or full field strength, gives

$$6.95 = 15.1\,(1 - \epsilon^{-0.296\,t}).$$

$$1 - \epsilon^{-0.296\,t} = 0.46$$

and $t = 2.08$ seconds.

(4) The stored energy is

$$\frac{i_0^2 L}{2} = \frac{6.95^2 \times 112}{2} = 2720 \text{ watt-seconds or joules}$$
$$= 2000 \text{ foot-pounds.}$$

(1 joule = 0.736 foot-pounds.)

Thus in case (3), where the field reaches full strength in 2.08 seconds, the average power input is $\frac{2000}{2.08} = 960$ foot pounds per second = 1.75 hp.

In breaking the field circuit of this alternator, 2000 foot-pounds of energy have to be dissipated in the spark, etc.

33. (5) A coil of resistance $r = 0.002$ ohm and inductance $L = 0.005$ mh., carrying current $I = 90$ amp., is short circuited.

(*a*) What is the equation of the current after short circuit?

(*b*) In what time has the current decreased to 0.1. its initial value?

(*a*) $i = I\epsilon^{-\frac{rt}{L}}$

$= 90\ \epsilon^{-400\,t}.$

(*b*) $i = 0.1\ I$, $\epsilon^{-400\,t} = 0.1$, after $t = 0.00576$ second.

(6) When short circuiting the coil in Example 5, an e.m.f. $E = 1$ volt is inserted in the circuit of this coil, in opposite direction to the current.

(*a*) What is equation of the current?

(*b*) After what time does the current become zero?

(*c*) After what time does the current reverse to its initial value in opposite direction?

(*d*) What impressed e.m.f. is required to make the current die out in 1/2000 second?

(*e*) What impressed e.m.f. E is required to reverse the current in 1/1000 second?

(*a*) If e.m.f. $-E$ is inserted, and at time t the current is denoted by i, we have

$$e_1 = -L\frac{di}{dt}, \text{ the generated e.m.f.;}$$

Thus,
$$-E + e_1 = -E - L\frac{di}{dt}, \text{ the total e.m.f.;}$$

and

$$i = \frac{-E + e_1}{r} = -\frac{E}{r} - \frac{L}{r}\frac{di}{dt}, \text{ the current;}$$

Transposing,

$$-\frac{r}{L}dt = \frac{di}{\frac{E}{r} + i},$$

and integrating,

$$-\frac{rt}{L} = \log_\epsilon\left(\frac{E}{r} + i\right) - \log_\epsilon c,$$

where $-\log_\epsilon c =$ integration constant.

At $t = 0$, $i = I$, thus $c = I + \frac{E}{r}$;

Substituting,

$$i = \left(I + \frac{E}{r}\right)\epsilon^{-\frac{rt}{L}} - \frac{E}{r},$$

$$i = 590\ \epsilon^{-400\,t} - 500.$$

(b) $i = 0$, $\epsilon^{-400\,t} = 0.85$, after $t = 0.000405$ second.

(c) $i = -I = -90$, $\epsilon^{-400\,t} = 0.694$, after $t = 0.00091$ second.

(d) If $i = 0$ at $t = 0.0005$, then

$$0 = (90 + 500\,E)\,\epsilon^{-0.2} - 500\,E,$$

$$E = \frac{0.18}{\epsilon^{0.2} - 1} = 0.81 \text{ volt.}$$

(e) If $i = -I = -90$ at $t = 0.001$, then

$$-90 = (90 + 500\,E)\,\epsilon^{-0.4} - 500\,E,$$

$$E = \frac{0.18\,(1 + \epsilon^{-0.4})}{1 - \epsilon^{-0.4}} = 0.91 \text{ volt.}$$

7. INDUCTANCE IN ALTERNATING-CURRENT CIRCUITS

34. An alternating current $i = I_0 \sin 2\pi ft$ or $i = I_0 \sin \theta$ can be represented graphically in rectangular coordinates by a curved line as shown in Fig. 10, with the instantaneous values

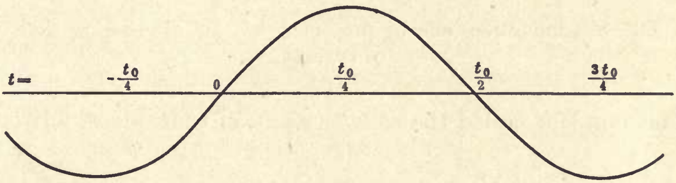

FIG. 10.—Alternating sine wave.

i as ordinates and the time t, or the arc of the angle corresponding to the time, $\theta = 2\pi ft$, as abscissas, counting the time from the zero value of the rising wave as zero point.

If the zero value of current is not chosen as zero point of time, the wave is represented by

$$i = I_0 \sin 2\pi f\,(t - t'),$$

or

$$i = I_0 \sin (\theta - \theta'),$$

where t' and θ' are respectively the time and the corresponding angle at which the current reaches its zero value in the ascendant.

If such a sine wave of alternating current $i = I_0 \sin 2\pi ft$ or $i = I_0 \sin \theta$ passes through a circuit of resistance r and inductance L, the magnetic flux produced by the current and thus its interlinkages with the current, $iL = I_0L \sin \theta$, vary in a wave line similar also to that of the current, as shown in Fig. 11 as Φ. The e.m.f. generated hereby is proportional to the change of iL, and is thus a maximum where iL changes most rapidly, or at its zero point, and zero where iL is a maximum, and according to Lentz's law it is positive during falling and negative during rising current. Thus this generated e.m.f. is a wave following the wave of current by the time $t = \frac{t_0}{4}$, where t_0 is time of one complete period, $= \frac{1}{f}$, or by the time angle $\theta = 90°$.

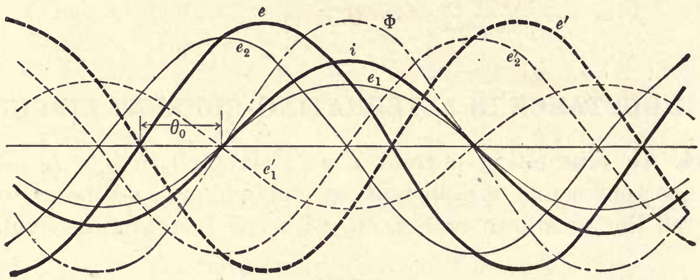

FIG. 11.—Self-induction effects produced by an alternating sine wave of current.

This e.m.f. is called the *counter e.m.f. of inductance.* It is

$$e'_2 = -L\frac{di}{dt}$$
$$= -2\pi f L I_0 \cos 2\pi f t.$$

It is shown in dotted line in Fig. 11 as e'_2.

The quantity $2\pi fL$ is called the *inductive reactance* of the circuit, and denoted by x. It is of the nature of a resistance, and expressed in ohms. If L is given in 10^9 absolute units or henrys, x appears in ohms.

The counter e.m.f. of inductance of the current, $i = I_0 \sin 2\pi ft = I_0 \sin\theta$, of effective value

$$I = \frac{I_0}{\sqrt{2}}, \text{ is}$$

$$e'_2 = -xI_0 \cos 2\pi ft = -xI_0 \cos\theta,$$

having a maximum value of xI_0 and an effective value of

$$E_2 = \frac{xI_0}{\sqrt{2}} = xI;$$

that is, the effective value of the counter e.m.f. of inductance equals the reactance, x, times the effective value of the current, I, and lags 90 time degrees, or a quarter period, behind the current.

35. By the counter e.m.f. of inductance,

$$e'_2 = - xI_0 \cos \theta,$$

which is generated by the change in flux due to the passage of the current $i = I_0 \sin \theta$ through the circuit of reactance x, an equal but opposite e.m.f.

$$e_2 = xI_0 \cos \theta$$

is consumed, and thus has to be impressed upon the circuit. This e.m.f. is called the *e.m.f. consumed by inductance.* It is 90 time degrees, or a quarter period, ahead of the current, and shown in Fig. 11 as a drawn line e_2.

Thus we have to distinguish between counter e.m.f. of inductance 90 time degrees lagging, and e.m.f. consumed by inductance 90 time degrees leading.

These e.m.fs. stand in the same relation as action and reaction in mechanics. They are shown in Fig. 11 as e'_2 and as e_2.

The e.m.f. consumed by the resistance r of the circuit is proportional to the current,

$$e_1 = ri = rI_0 \sin \theta,$$

and in phase therewith, that is, reaches its maximum and its zero value at the same time as the current i, as shown by drawn line e_1 in Fig. 11.

Its effective value is $E_1 = rI$.

The resistance can also be represented by a (fictitious) counter e.m.f.,

$$e'_1 = - rI_0 \sin \theta,$$

opposite in phase to the current, shown as e'_1 in dotted line in Fig. 11.

The counter e.m.f. of resistance and the e.m.f. consumed by resistance have the same relation to each other as the counter e.m.f. of inductance and the e.m.f. consumed by inductance or inductive reactance.

36. If an alternating current $i = I_0 \sin \theta$ of effective value $I = \frac{I_0}{\sqrt{2}}$ exists in a circuit of resistance r and inductance L, that is, of reactance $x = 2 \pi f L$, we have to distinguish:

E.m.f. consumed by resistance, $e_1 = rI_0 \sin \theta$, of effective value $E_1 = rI$, and in phase with the current.

Counter e.m.f. of resistance, $e'_1 = - rI_0 \sin \theta$, of effective value $E_1 = rI$, and in opposition or 180 time degrees displaced from the current.

E.m.f. consumed by reactance, $e_2 = xI_0 \cos \theta$, of effective value $E_2 = xI$, and leading the current by 90 time degrees or a quarter period.

Counter e.m.f. of reactance, $e'_2 = xI_0 \cos \theta$, of effective value $E'_2 = xI$, and lagging 90 time degrees or a quarter period behind the current.

The e.m.fs. consumed by resistance and by reactance are the e.m.fs. which have to be impressed upon the circuit to overcome the counter e.m.fs. of resistance and of reactance.

Thus, the total counter e.m.f. of the circuit is

$$e' = e'_1 + e'_2 = - I_0 (r \sin \theta + x \cos \theta),$$

and the total impressed e.m.f., or e.m.f. consumed by the circuit, is

$$e = e_1 + e_2 = I_0 (r \sin \theta + x \cos \theta).$$

Substituting

$$\frac{x}{r} = \tan \theta_0 \text{ and}$$

$$\sqrt{r^2 + r^2} = z,$$

it follows that

$$x = z \sin \theta_0, \qquad r = z \cos \theta_0,$$

and we have as the total impressed e.m.f.

$$e = zI_0 \sin (\theta + \theta_0),$$

shown by heavy drawn line e in Fig. 11, and total counter e.m.f.

$$e' = - zI_0 \sin (\theta + \theta_0),$$

shown by heavy dotted line e' in Fig. 11, both of effective value

$$e = zI.$$

For $\theta = - \theta_0$, $e = 0$, that is, the zero value of e is ahead of the zero value of current by the time angle θ_0, or the current lags behind the impressed e.m.f. by the angle θ_0.

θ_0 is called the *angle of lag* of the current, and $z = \sqrt{r^2 + x^2}$ the *impedance* of the circuit. e is called the e.m.f. consumed by impedance, e' the counter e.m.f. of impedance.

Since $E_1 = rI$ is the e.m.f. consumed by resistance,
$E_2 = xI$ is the e.m.f. consumed by reactance,
and $E = zI = \sqrt{r^2 + x^2}\, I$ is the e.m.f. consumed by impedance,
we have

$$E = \sqrt{E_1^2 + E_2^2}, \text{ the total e.m.f.}$$

and

$$E_1 = E \cos \theta_0,$$
$$E_2 = E \sin \theta_0, \text{ its components.}$$

The tangent of the angle of lag is

$$\tan \theta_0 = \frac{x}{r} = \frac{2\, \pi f L}{r},$$

and the time constant of the circuit is

$$\frac{L}{r} = \frac{\tan \theta_0}{2\, \pi f}.$$

The total e.m.f., e, impressed upon the circuit consists of two components, one, e_1, in phase with the current, the other one, e_2, in quadrature with the current.

Their effective values are

$$E,\ E \cos \theta_0,\ E \sin \theta_0.$$

EXAMPLES

37. (1) What is the reactance per wire of a transmission line of length l, if l_d = diameter of the wire, l_s = spacing of the wires, and f = frequency?

If I = current, in absolute units, in one wire of the transmission line, the m.m.f. is I; thus the magnetizing force in a zone dl_x at distance l_x from center of wire (Fig. 12) is $f = \frac{I}{2\, \pi l_x}$ and the field intensity in this zone is $H = 4\, \pi f = 2 \frac{I}{l_x}$. Thus the magnetic flux in this zone is

$$d\Phi = H\, l dl_x = \frac{2\, I l dl_x}{l_x};$$

hence, the total magnetic flux between the wire and the return wire is

$$\Phi = \int_{\frac{l_d}{2}}^{l_s} d\Phi = 2\, Il \int_{\frac{l_d}{2}}^{l_s} \frac{dl_x}{l_x} = 2\, Il \log_\epsilon \frac{2\, l_s}{l_d},$$

neglecting the flux inside the transmission wire.

The inductance is

$$L = \frac{\Phi}{I} = 2\,l \log_\epsilon \frac{2\,l_s}{l_d} \text{ absolute units}$$

$$= 2\,l \log_\epsilon \frac{2\,l_s}{l_d}\, 10^{-9} \text{ h.,}$$

and the reactance $x = 2\,\pi f L = 4\,\pi f l \log_\epsilon \frac{2\,l_s}{l_d}$, in absolute units;

or $$x = 4\,\pi f l \log_\epsilon \frac{2\,l_s}{l_d}\, 10^{-9}, \text{ in ohms.}$$

38. (2) The voltage at the receiving end of a 33.3-cycle three-phase transmission line 14 miles in length shall be 5500

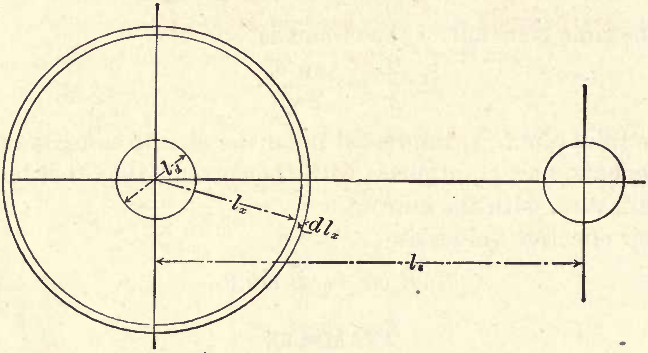

FIG. 12.—Diagram for calculation of inductance between two parallel conductors.

between the lines. The line consists of three wires, No. 0 B. & S. (l_d = 0.82 cm.), 18 in. (45 cm.) apart, of resistivity $\rho = 1.8 \times 10^{-6}$.

(*a*) What is the resistance, the reactance, and the impedance per line, and the voltage consumed thereby at 44 amp.?

(*b*) What is the generator voltage between lines at 44 amp. to a non-inductive load?

(*c*) What is the generator voltage between lines at 44 amp. to a load circuit of 45 degrees lag?

(*d*) What is the generator voltage between lines at 44 amp. to a load circuit of 45 degrees lead?

Here l = 14 miles = $14 \times 1.6 \times 10^5 = 2.23 \times 10^6$ cm.

l_d = 0.82 cm.

Hence the cross section, A = 0.528 sq. cm.

(*a*) Resistance per line, $r = \rho \frac{l}{A} = \frac{1.8 \times 10^{-6} \times 2.23 \times 10^6}{0.528}$ $= 7.60$ ohms.

Reactance per line, $x = 4\pi fl \log_\epsilon \frac{2l_s}{l_d} \times 10^{-9} = 4\pi \times 33.3 \times 2.23 \times 10^6 \times \log_\epsilon 110 \times 10^{-9} = 4.35$ ohms.

The impedance per line, $z = \sqrt{r^2 + x^2} = 8.76$ ohms. Thus if $I = 44$ amp. per line,

the e.m.f. consumed by resistance is $E_1 = rI = 334$ volts,
the e.m.f. consumed by reactance is $E_2 = xI = 192$ volts,
and the e.m.f. consumed by impedance is $E_3 = zI = 385$ volts.

(*b*) 5500 volts between lines at receiving circuit give $\frac{5500}{\sqrt{3}} = 3170$ volts between line and neutral or zero point (Fig. 13), or per line, corresponding to a maximum voltage of $3170\sqrt{2} = 4500$ volts. 44 amp. effective per line gives a maximum value of $44\sqrt{2} = 62$ amp.

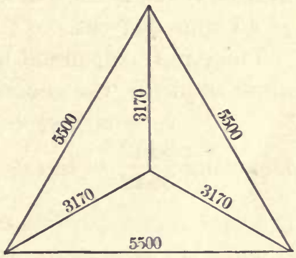

Fig. 13.—Voltage diagram for a three-phase circuit.

Denoting the current by $i = 62 \sin \theta$, the voltage per line at the receiving end with non-inductive load is $e = 4500 \sin \theta$.

The e.m.f. consumed by resistance, in phase with the current, of effective value 334, and maximum value $334\sqrt{2} = 472$, is

$$e_1 = 472 \sin \theta.$$

The e.m.f. consumed by reactance, 90 time degrees ahead of the current, of effective value 192, and maximum value $192\sqrt{2} = 272$, is

$$e_2 = 272 \cos \theta.$$

Thus the total voltage required per line at the generator end of the line is

$$e_0 = e + e_1 + e_2 = (4500 + 472) \sin \theta + 272 \cos \theta$$
$$= 4972 \sin \theta + 272 \cos \theta.$$

Denoting $\frac{272}{4972} = \tan \theta_0$, we have

$$\sin \theta_0 = \frac{\tan \theta_0}{\sqrt{1 + \tan^2 \theta_0}} = \frac{272}{4980}.$$

$$\cos \theta_0 = \frac{1}{\sqrt{1 + \tan^2 \theta_0}} = \frac{4972}{4980}.$$

Hence, $$e_0 = 4980 (\sin \theta \cos \theta_0 + \cos \theta \sin \theta_0)$$
$$= 4980 \sin (\theta + \theta_0).$$

Thus θ_0 is the lag of the current behind the e.m.f. at the generator end of the line, = 3.2 time degrees, and 4980 the maximum voltage per line at the generator end; thus $E_0 = \frac{4980}{\sqrt{2}} = 3520$, the effective voltage per line, and $3520\sqrt{3} = 6100$, the effective voltage between the lines at the generator.

(*c*) If the current

$$i = 62 \sin \theta$$

lags in time 45 degrees behind the e.m.f. at the receiving end of the line, this e.m.f. is expressed by

$$e = 4500 \sin (\theta + 45) = 3170 (\sin \theta + \cos \theta);$$

that is, it leads the current by 45 time degrees, or is zero at $\theta = -45$ time degrees.

The e.m.f. consumed by resistance and by reactance being the same as in (*b*), the generator voltage per line is

$$e_0 = e + e_1 + e_2 = 3642 \sin \theta + 3442 \cos \theta.$$

Denoting $\frac{3442}{3642} = \tan \theta_0$, we have

$$e_0 = 5011 \sin (\theta + \theta_0).$$

Thus θ_0, the angle of lag of the current behind the generator e.m.f., is 43 degrees, and 5011 the maximum voltage; hence 3550 the effective voltage per line, and $3550\sqrt{3} = 6160$ the effective voltage between lines at the generator.

(*d*) If the current $i = 62 \sin \theta$ leads the e.m.f. by 45 degrees, the e.m.f. at the receiving end is

$$e = 4500 \sin (\theta - 45)$$
$$= 3180 (\sin \theta - \cos \theta).$$

Thus at the generator end

$$e_0 = e + e_1 + e_2 = 3652 \sin \theta - 2908 \cos \theta.$$

Denoting $\frac{2908}{3652} = \tan \theta_0$, it is

$$e_0 = 4670 \sin (\theta - \theta_0).$$

Thus θ_0, the time angle of lead at the generator, is 39 degrees, and 4654 the maximum voltage; hence 3290 the effective voltage per line and 5710 the effective voltage between lines at the generator.

8. POWER IN ALTERNATING-CURRENT CIRCUITS

39. The power consumed by alternating current $i = I_0 \sin \theta$, of effective value $I = \frac{I_0}{\sqrt{2}}$, in a circuit of resistance r and reactance $x = 2 \pi f L$, is

$$p = ei,$$

where $e = zI_0 \sin (\theta + \theta_0)$ is the impressed e.m.f., consisting of the components

$$e_1 = rI_0 \sin \theta, \text{ the e.m.f. consumed by resistance}$$

and $$e_2 = xI_0 \cos \theta, \text{ the e.m.f. consumed by reactance.}$$

$z = \sqrt{r^2 + x^2}$ is the impedance and $\tan \theta_0 = \frac{x}{r}$ the phase angle of the circuit; thus the power is

$$\begin{aligned} p &= zI_0^2 \sin \theta \sin (\theta + \theta_0) \\ &= \frac{zI_0^2}{2} (\cos \theta_0 - \cos (2\,\theta + \theta_0)) \\ &= zI^2 (\cos \theta_0 - \cos (2\,\theta + \theta_0)). \end{aligned}$$

Since the average $\cos (2\,\theta + \theta_0) =$ zero, the average power is

$$\begin{aligned} P &= zI^2 \cos \theta_0 \\ &= rI^2 = E_1 I; \end{aligned}$$

that is, the power in the circuit is that consumed by the resistance, and independent of the reactance.

Reactance or self-inductance consumes no power, and the e.m.f. of self-inductance is a *wattless* or *reactive e.m.f.*, while the e.m.f. of resistance is a *power* or *active e.m.f.*

The wattless e.m.f. is in quadrature, the power e.m.f. in phase with the current.

In general, if θ = angle of time-phase displacement between the resultant e.m.f. and the resultant current of the circuit, I = current, E = impressed e.m.f., consisting of two components, one, $E_1 = E \cos \theta$, in phase with the current, the other, $E_2 = E \sin \theta$, in quadrature with the current, the power in the circuit is $IE_1 = IE \cos \theta$, and the e.m.f. in phase with the current $E_1 = E \cos \theta$ is a power e.m.f., the e.m.f. in quadrature with the current $E_2 = E \sin \theta$ a wattless or *reactive* e.m.f.

40. Thus we have to distinguish *power e.m.f.* and *wattless* or *reactive e.m.f.*, or power component of e.m.f., in phase with the current and wattless or reactive component of e.m.f., in quadrature with the current.

Any e.m.f. can be considered as consisting of two components, one, the power component, e_1, in phase with the current, and the other, the reactive component, e_2, in quadrature with the current. The sum of instantaneous values of the two components is the total e.m.f.

$$e = e_1 + e_2.$$

If E, E_1, E_2 are the respective effective values, we have

$$E = \sqrt{E_1^2 + E_2^2}, \text{ since}$$
$$E_1 = E \cos \theta,$$
$$E_2 = E \sin \theta,$$

where θ = phase angle between current and e.m.f.

Analogously, a current I due to an impressed e.m.f. E with a time-phase angle θ can be considered as consisting of two component currents,

$I_1 = I \cos \theta$, the *active* or *power* component of the current, and
$I_2 = I \sin \theta$, the *wattless* or *reactive* component of the current.

The sum of instantaneous values of the power and reactive components of the current equals the instantaneous value of the total current,

$$i_1 + i_2 = i,$$

while their effective values have the relation

$$I = \sqrt{I_1^2 + I_2^2}.$$

Thus an alternating current can be resolved in two components, the power component, in phase with the e.m.f., and the wattless or reactive component, in quadrature with the e.m.f.

An alternating e.m.f. can be resolved in two components: the power component, in phase with the current, and the wattless or reactive component, in quadrature with the current.

The power in the circuit is the current times the e.m.f. times the cosine of the time-phase angle, or is the power component of the current times the total e.m.f., or the power component of the e.m.f. times the total current.

EXAMPLES

41. (1) What is the power received over the transmission line in Section 7, Example 2, the power lost in the line, the power put into the line, and the efficiency of transmission with non-inductive load, with 45-time-degree lagging load and 45-degree leading load?

The power received per line with non-inductive load is $P = EI = 3170 \times 44 = 139$ kw.

With a load of 45 degrees phase displacement, $P = EI \cos 45° = 98$ kw.

The power lost per line $P_1 = I^2R = 44^2 \times 7.6 = 14.7$ kw.

Thus the input into the line $P_0 = P + P_1 = 151.7$ kw. at non-inductive load,

and $\quad = 111.7$ kw. at load of 45 degrees phase displacement.

The efficiency with non-inductive load is

$$\frac{P}{P_0} = 1 - \frac{14.7}{151.7} = 90.3 \text{ per cent.}$$

and with a load of 45 degrees phase displacement is

$$\frac{P}{P_0} = 1 - \frac{14.7}{111.7} = 86.8 \text{ per cent.}$$

The total output is $3\,P = 411$ kw. and 291 kw., respectively.

The total input $3\,P_0 = 451.1$ kw. and 335.1 kw., respectively.

9. VECTOR DIAGRAMS

42. The best way of graphically representing alternating-current phenomena is by a vector diagram. The most frequently used vector diagram is the crank diagram. In this, sine waves of alternating currents, voltages, etc., are represented as projections of a revolving vector on the horizontal. That is, a vector equal in length to the maximum value of the alternating wave is assumed to revolve at uniform speed so as to make one complete revolution per period, and the projections of this revolving vector upon the horizontal then represent the instantaneous values of the wave.

Let, for instance, $\overline{OI}$ represent in length the maximum value of current $i = I \cos (\theta - \theta_0)$. Assume then a vector, $\overline{OI}$, to revolve, left-handed or in positive direction, so that it makes a complete revolution during each cycle or period. If then at a certain moment of time this vector stands in position $\overline{OI_1}$ (Fig. 14), the projection, $\overline{OA_1}$, of $\overline{OI_1}$ on $\overline{OA}$ represents the instantaneous value of the current at this moment. At a later moment $\overline{OI}$ has moved farther, to $\overline{OI_2}$, and the projection, $\overline{OA_2}$, of $\overline{OI_2}$ on $\overline{OA}$ is the instantaneous value. The diagram thus shows the instantaneous condition of the sine waves. Each sine wave

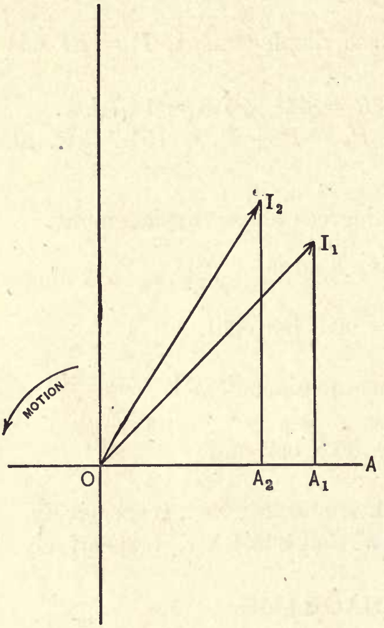

FIG. 14.—Crank diagram showing instantaneous values.

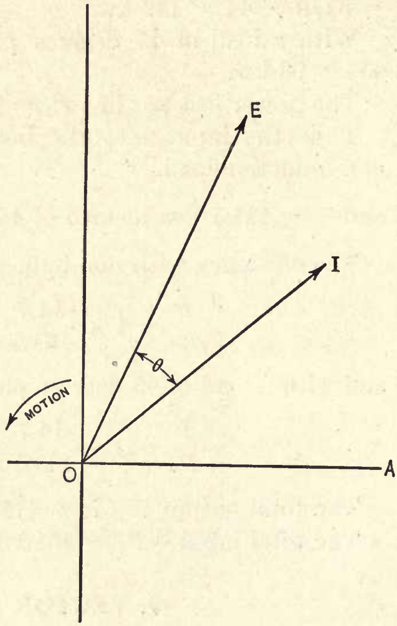

FIG. 15.—Crank diagram of an e.m.f. and current.

reaches the maximum at the moment when its revolving vector, $\overline{OI}$, passes the horizontal, and reaches zero when its revolving vector passes the vertical.

If Fig. 15 represents the crank diagram of a voltage $\overline{OE}$, and a current $\overline{OI}$, and if angle $AOE > AOI$, this means that the current $\overline{OI}$ is behind the voltage $\overline{OE}$, passes during the revolution the zero line or line of maximum intensity, $\overline{OA}$, later than the voltage; that is, the current lags behind the voltage.

In the vector diagram, the first quantity therefore can be put in any position. For instance, the current $\overline{OI}$, in Fig. 15, could be drawn in position $\overline{OI}$, Fig. 16. The voltage then being ahead of the current by angle $EOI = \theta$ would come into the position $\overline{OE}$, Fig. 16.

This vector diagram then shows graphically, by the projections of the vectors on the horizontal, the instantaneous values of the alternating waves at one moment of time. At any other moment

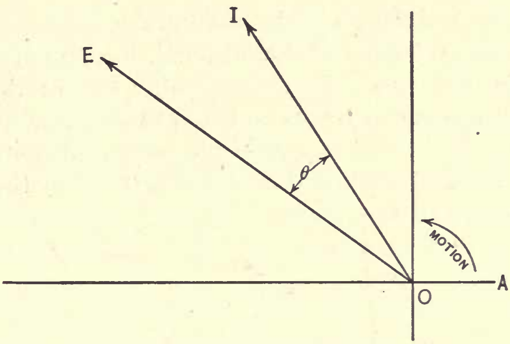

FIG. 16.—Crank diagram.

of time, the instantaneous values would be the projections of the vectors on another radius, corresponding to the other time. The angles between the vector representation are the phase differences between the vectors, and the angles each vector makes with the horizontal may be called its phase. The horizontal then

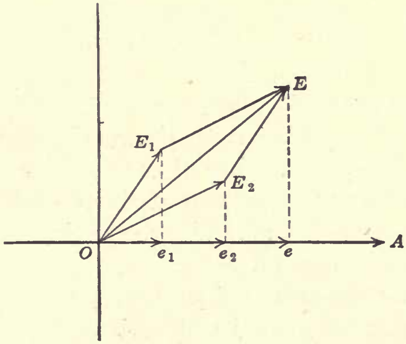

FIG. 17.—Vector diagram of two e.m.f.'s acting in the same circuit.

would be of phase zero. The phase of the first vector may be chosen at random; all other phases are determined thereby.

In this representation, the phase of an alternating wave is given by the time when its maximum value passes the horizontal.

Two voltages, e_1 and e_2, acting in the same circuit, give a resultant voltage e equal to the sum of their instantaneous values. Graphically, voltages e_1 and e_2 are represented in intensity and in phase by two revolving vectors, $\overline{OE_1}$ and $\overline{OE_2}$, Fig. 17. The instantaneous values are the projections $\overline{Oe_1}$, $\overline{Oe_2}$ of $\overline{OE_1}$ and $\overline{OE_2}$ upon the horizontal.

Since the sum of the projections of the sides of a parallelogram is equal to the projection of the diagonal, the sum of the projections $\overline{Oe_1}$ and $\overline{Oe_2}$ equals the projection $\overline{Oe}$ of $\overline{OE}$, the diagonal of the parallelogram with $\overline{OE_1}$ and $\overline{OE_2}$ as sides, and $\overline{OE}$ is thus the resultant e.m.f.; that is, graphically alternating sine waves of voltage, current, etc., are combined and resolved by the parallelogram or polygon of sine waves.

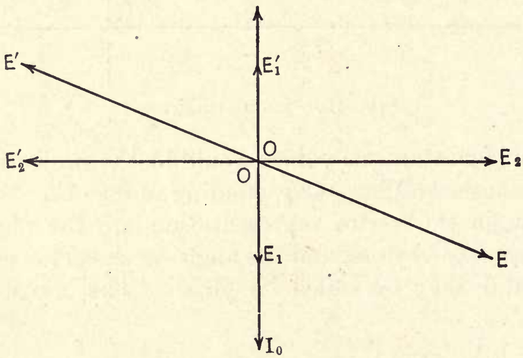

FIG. 18.—Vector diagram.

43. The sine wave of alternating current $i = I_0 \sin \theta$ is represented by a vector equal in length, $\overline{OI_0}$, to the maximum value I_0 of the wave, and located so that at time zero $\theta = 0$, its projection on the horizontal, is zero, and at times $\theta > 0$, but $< \pi$, the projection is positive. Thus this vector $\overline{OI_0}$ is the negative vertical, as shown in Fig. 18.

The voltage consumed by inductance, $e_2 = xI_0 \cos \theta$, is represented by a vector $\overline{OE_2}$ equal in length to xI_0, and located so that at $\theta = 0$, its projection on the horizontal is a maximum. That is, it is the zero vector $\overline{OE_2}$ in Fig. 18.

Analogously, the counter e.m.f. of self-inductance E'_2 is represented by vector $\overline{OE'_2}$ on the negative horizontal of Fig. 18; the voltage consumed by the resistance r, $e_1 = eI_0 \sin \theta$, is represented by vector $\overline{OE_1}$ equal to rI_0, and located on the negative vertical, and the counter e.m.f. of resistance by vector $\overline{OE'_1}$ on the positive vertical.

The counter e.m.f. of impedance:

$$e' = -(rI_0 \sin \theta + xI_0 \cos \theta)$$
$$= -zI_0 \sin (\theta + \theta_0)$$

then is represented graphically as the resultant, by the parallelogram of sine waves of $\overline{OE'_1}$ and $\overline{OE'_2}$, that is, by a vector $\overline{OE'}$, equal in length to zI_0, and of phase $90 + \theta_0$.

The voltage consumed by impedance, or the impressed voltage, is represented by the vector $\overline{OE}$, equal and opposite in direction to the vector $\overline{OE'}$. This vector is the resultant of $\overline{OE_1}$ and $\overline{OE_2}$ and has the phase $\theta_0 - 90$, or $-(90 - \theta_0)$, as shown in Fig. 18.

An alternating wave is thus determined by the length and direction of its vector. The length is the maximum value, intensity or amplitude of the wave; the direction is the phase of its maximum value, usually called the phase of the wave.

44. As phase of the first quantity considered, as in the above instance the current, any direction can be chosen. The further quantities are determined thereby in direction or phase.

The zero vector $\overline{OA}$ is generally chosen for the most frequently used quantity or reference quantity, as for the current, if a number of e.m.fs. are considered in a circuit of the same current, or for the e.m.f., if a number of currents are produced by the same e.m.f., or for the generated e.m.f. in apparatus such as transformers and induction motors, synchronous apparatus, etc.

With the current as zero vector, all horizontal components of e.m.f. are power components, all vertical components are reactive components.

With the e.m.f. as zero vector, all horizontal components of current are power components, all vertical components of current are reactive components.

By measurement from the vector diagram numerical values can hardly ever be derived with sufficient accuracy, since the magnitudes of the different quantities used in the same diagram are usually by far too different, and the vector diagram is therefore useful only as basis for trigonometrical or other calculation, and to give an insight into the mutual relation of the different quantities, and even then great care has to be taken to distinguish between the two equal but opposite vectors, counter e.m.f. and e.m.f. consumed by the counter e.m.f., as explained before.

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45. In a three-phase long-distance transmission line, the voltage between lines at the receiving end shall be 5000 at no load, 5500 at full load of 44 amp. power component, and proportional at intermediary values of the power component of the current; that is, the voltage at the receiving end shall increase proportional to the load. At three-quarters load the current shall be in phase with the e.m.f. at the receiving end. The generator excitation, however, and thus the (nominal) generated

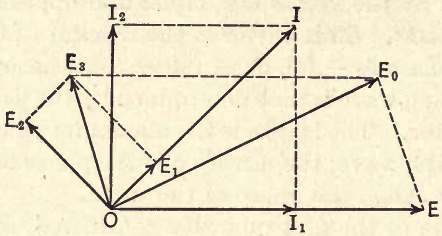

FIG. 19.—Vector diagram of e.m.f. and current in transmission line. Current leading.

e.m.f. of the generator shall be maintained constant at all loads, and the voltage regulation effected by producing lagging or leading currents with a synchronous motor in the receiving circuit. The line has a resistance $r_1 = 7.6$ ohms and a reactance $x_1 = 4.35$ ohms per wire, the generator is star connected, the resistance per circuit being $r_2 = 0.71$, and the (synchronous) reactance is $x_2 = 25$ ohms. What must be the wattless or reactive component of the current, and therefore the total current and its phase relation at no load, one-quarter load, one-half load, three-quarters load, and full load, and what will be the terminal voltage of the generator under these conditions?

The total resistance of the line and generator is $r = r_1 + r_2 = 8.31$ ohms; the total reactance, $x = x_1 + x_2 = 29.35$ ohms.

Let, in the polar diagram, Fig. 19 or 20, $\overline{OE} = E$ represent the voltage at the receiving end of the line, $\overline{OI_1} = I_1$ the power component of the current corresponding to the load, in phase with $\overline{OE}$, and $\overline{OI_2} = I_2$ the reactive component of the current in quadrature with $\overline{OE}$, shown leading in Fig. 19, lagging in Fig. 20.

We then have total current $I = \overline{OI}$.

Thus the e.m.f. consumed by resistance, $\overline{OE_1} = rI$, is in phase with I, the e.m.f. consumed by reactance, $\overline{OE_2} = xI$, is 90 degrees ahead of I, and their resultant is OE_3, the e.m.f. consumed by impedance.

$\overline{OE_3}$ combined with $\overline{OE}$, the receiver voltage, gives the generator voltage $\overline{OE_0}$.

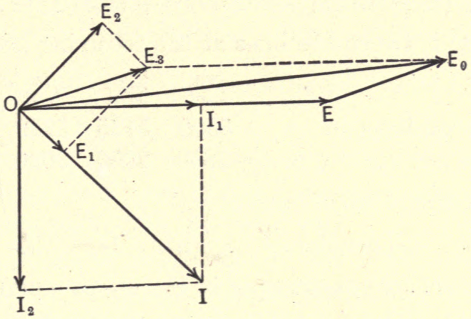

FIG. 20.—Vector diagram of e.m.f. and current in transmission line. Current lagging.

Resolving all e.m.fs. and currents into components in phase and in quadrature with the received voltage E, we have

	Phase component	Quadrature component
Current	I_1	$-I_2$
E.m.f. at receiving end of line, E =	E	0
E.m.f. consumed by resistance, E_1 =	rI_1	$-rI_2$
E.m.f. consumed by reactance, E_2 =	xI_2	$+xI_1$

Thus total e.m.f. or generator voltage,

$$E_0 = E + E_1 + E_2 = \qquad E + rI_1 + xI_2 \qquad xI_1 - rI_2$$

Herein the reactive lagging component of current is assumed as positive, the leading as negative.

The generator e.m.f. thus consists of two components, which give the resultant value

$$E_0 = \sqrt{(E + rI_1 + xI_2)^2 + (xI_1 - rI_2)^2};$$

substituting numerical values, this becomes

$$E_0 = \sqrt{(E + 8.31\,I_1 + 29.35\,I_2)^2 + (29.35\,I_1 - 8.31\,I_2)^2}.$$

At three-quarters load,

$$E = \frac{5375}{\sqrt{3}} = 3090 \text{ volts per circuit,}$$

$I_1 = 33, \qquad I_2 = 0$, thus

$E_0 = \sqrt{(3090 + 8.31 \times 33)^2 + (29.35 \times 33)^2} = 3520$ volts per line or $3520 \times \sqrt{3} = 6100$ volts between lines as (nominal) generated e.m.f. of generator.

Substituting these values, we have

$$3520 = \sqrt{(E + 8.31\,I_1 + 29.35\,I_2)^2 + (8.31\,I_2 - 29.35\,I_1)^2}.$$

The voltage between the lines at the receiving end shall be:

	No load	¼ load	½ load	¾ load	Full load
Voltage between lines,	5000	5125	5250	5375	5500
Thus, voltage per line $\div \sqrt{3}$, $E =$	2880	2950	3020	3090	3160
The power components of current per line, $I_1 =$	0	11	22	33	44

Herefrom we get by substituting in the above equation

	No load	¼ load	½ load	¾ load	Full load
Reactive component of current, $I_2 =$	−21.6	−16.2	−9.2	0	+9.7
hence, the total current, $I = \sqrt{I_1^2 + I_2^2} =$	21.6	19.6	23.9	33.0	45.05
and the power factor, $\frac{I_1}{I} = \cos\theta =$	0	56.0	92.0	100.0	97.7
the lag of the current, $\theta =$	90°	61°	23°	0°	−11.5°

the generator terminal voltage per line is

$$E' = \sqrt{(E + r_1I_1 + x_1I_2)^2 + (x_1I_1 - r_1I_2)^2}$$
$$= \sqrt{(E + 7.6\,I_1 + 4.35\,I_2)^2 + (4.35\,I_1 - 7.6\,I_2)^2}$$

thus:

	No load	¼ load	½ load	¾ load	Full load
Per line, $E' =$	2980	3106	3228	3344	3463
Between lines, $E'\sqrt{3} =$	5200	5400	5600	5800	6000

Therefore at constant excitation the generator voltage rises with the load, and is approximately proportional thereto.

10. HYSTERESIS AND EFFECTIVE RESISTANCE

46. If an alternating current $\overline{OI} = I$, in Fig. 21, exists in a circuit of reactance $x = 2\,\pi fL$ and of negligible resistance, the magnetic flux produced by the current, $\overline{O\Phi} = \Phi$, is in phase with the current, and the e.m.f. generated by this flux, or counter e.m.f. of self-inductance, $\overline{OE'''} = E''' = xI$, lags 90 degrees behind the current. The e.m.f. consumed by self-inductance or impressed e.m.f. $\overline{OE''} = E'' = xI$ is thus 90 degrees ahead of the current.

Inversely, if the e.m.f. $\overline{OE''} = E''$ is impressed upon a circuit of reactance $x = 2\,\pi f L$ and of negligible resistance, the current $\overline{OI} = I = \frac{E''}{x}$ lags 90 degrees behind the impressed e.m.f.

This current is called the *exciting* or magnetizing current of the magnetic circuit, and is wattless.

If the magnetic circuit contains iron or other magnetic material, energy is consumed in the magnetic circuit by a frictional resistance of the material against a change of magnetism, which is called *molecular magnetic friction.*

If the alternating current is the only available source of energy in the magnetic circuit, the expenditure of energy by molecular magnetic friction appears as a lag of the magnetism behind the m.m.f. of the current, that is, as *magnetic hysteresis,* and can be measured thereby.

Magnetic hysteresis is, however, a distinctly different phenomenon from molecular magnetic friction, and can be more or less eliminated, as for instance by mechanical vibration, or can be increased, without changing the molecular magnetic friction.

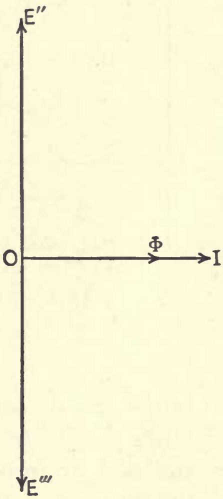

Fig. 21.—Phase relations of magnetizing current, flux and self-inductive e.m.f.

47. In consequence of magnetic hysteresis, if an alternating e.m.f. $\overline{OE''} = E''$ is impressed upon a circuit of negligible resistance, the exciting current, or current producing the magnetism, in this circuit is not a wattless current, or current of 90 degrees lag, as in Fig. 21, but lags less than 90 degrees, by an angle $90 - \alpha$, as shown by $\overline{OI} = I$ in Fig. 22.

Since the magnetism $\overline{O\Phi} = \Phi$ is in quadrature with the e.m.f. E'' due to it, angle α is the phase difference between the magnetism and the m.m.f., or the lead of the m.m.f., that is, the exciting current, before the magnetism. It is called the *angle of hysteretic lead.*

In this case the *exciting current* $\overline{OI} = I$ can be resolved in two components: the *magnetizing current* $\overline{OI_2} = I_2$, in phase with the magnetism $\overline{O\Phi} = \Phi$, that is, in quadrature with the e.m.f. $\overline{OE''} = E''$, and thus wattless, and the *magnetic power component of the current* or the *hysteresis current* $\overline{OI_1} = I_1$, in phase with the e.m.f. $\overline{OE''} = E''$, or in quadrature with the magnetism $\overline{O\Phi} = \Phi$. Magnetizing current and hysteresis current are the two components of the exciting current.

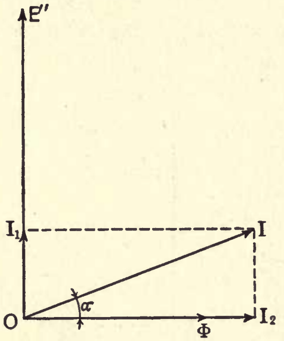

Fig. 22.—Angle of hysteretic lead.

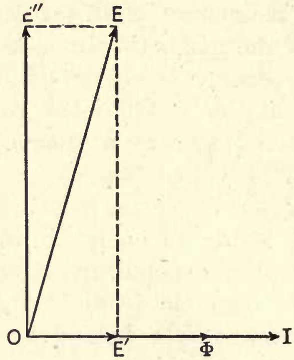

Fig. 23.—Effect of resistance on phase relation of impressed e.m.f. in a hysteresisless circuit.

If the circuit contains besides the reactance $x = 2\,\pi f L$, a resistance r, the e.m.f. $\overline{OE''} = E''$ in the preceding Figs. 21 and 22 is not the impressed e.m.f., but the e.m.f. consumed by self-inductance or reactance, and has to be combined, Figs. 23 and 24, with the e.m.f. consumed by the resistance, $\overline{OE'} = E' = Ir$, to get the impressed e.m.f. $\overline{OE} = E$.

Due to the hysteretic lead α, the lag of the current is less in Figs. 22 and 24, a circuit expending energy in molecular magnetic friction, than in Figs. 21 and 23, a hysteresisless circuit.

As seen in Fig. 24, in a circuit whose ohmic resistance is not negligible, the hysteresis current and the magnetizing current are not in phase and in quadrature respectively with the impressed e.m.f., but with the counter e.m.f. of inductance or e.m.f. consumed by inductance.

Obviously the magnetizing current is not quite wattless, since energy is consumed by this current in the ohmic resistance of the circuit.

Resolving, in Fig. 25, the impressed e.m.f. $\overline{OE} = E$ into two components, $\overline{OE_1} = E_1$ in phase, and $\overline{OE_2} = E_2$ in quadrature with the current $\overline{OI} = I$, the power component of the e.m.f., $\overline{OE_1} = E_1$, is greater than $E' = Ir$, and the reactive component $\overline{OE_2} = E_2$ is less than $E'' = Ix$.

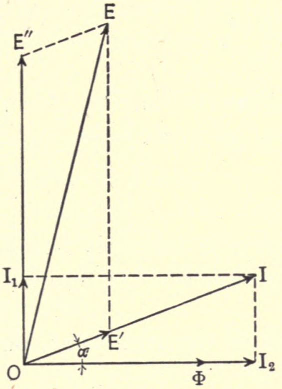

FIG. 24.—Effect of resistance on phase relation of impressed e.m.f. in a circuit having hysteresis.

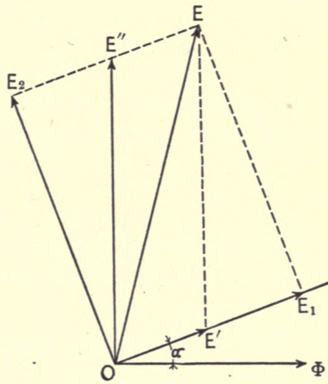

FIG. 25.—Impressed e.m.f. resolved into components in phase and in quadrature with the exciting current.

The value $r' = \frac{E_1}{I} = \frac{\text{power e.m.f.}}{\text{total current}}$ is called the *effective resistance*, and the value $x' = \frac{E_2}{I} = \frac{\text{wattless e.m.f.}}{\text{total current}}$ is called the *apparent* or *effective reactance* of the circuit.

48. Due to the loss of energy by hysteresis (eddy currents, etc.), the effective resistance differs from, and is greater than, the ohmic resistance, and the apparent reactance is less than the true or inductive reactance.

The loss of energy by molecular magnetic friction per cubic centimeter and cycle of magnetism is approximately

$$W = \eta B^{1.6},$$

where B = the magnetic flux density, in lines per sq. cm.

W = energy, in absolute units or ergs per cycle (= 10^{-7} watt-seconds or joules), and η is called the coefficient of hysteresis.

In soft annealed sheet iron or sheet steel and in silicon steel, η varies from 0.60×10^{-3} to 2.5×10^{-3}, and can in average, for good material, be assumed as 1.5×10^{-3}.

The loss of power in the volume, V, at flux density B and frequency f, is thus

$$P = Vf\eta B^{1.6} \times 10^{-7}, \text{ in watts,}$$

and, if I = the exciting current, the hysteretic effective resistance is

$$r'' = \frac{P}{I^2} = Vf\eta\, 10^{-7} \frac{B^{1.6}}{I^2}.$$

If the flux density, B, is proportional to the current, I, substituting for B, and introducing the constant k, we have

$$r'' = \frac{kf}{I^{0.4}},$$

that is, the effective hysteretic resistance is inversely proportional to the 0.4 power of the current, and directly proportional to the frequency.

49. Besides hysteresis, eddy or Foucault currents contribute to the effective resistance.

Since at constant frequency the Foucault currents are proportional to the magnetism producing them, and thus approximately proportional to the current, the loss of power by Foucault currents is proportional to the square of the current, the same as the ohmic loss, that is, the effective resistance due to Foucault currents is approximately constant at constant frequency, while that of hysteresis decreases slowly with the current.

Since the Foucault currents are proportional to the frequency, their effective resistance varies with the square of the frequency, while that of hysteresis varies only proportionally to the frequency.

The total effective resistance of an alternating-current circuit increases with the frequency, but is approximately constant, within a limited range, at constant frequency, decreasing somewhat with the increase of magnetism.

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50. A reactive coil shall give 100 volts e.m.f. of self-inductance at 10 amp. and 60 cycles. The electric circuit consists of 200 turns (No. 8 B. & S.) (= 0.013 sq. in.) of 16 in. mean length of turn. The magnetic circuit has a section of 6 sq. in. and a mean length of 18 in. of iron of hysteresis coefficient $\eta = 2.5 \times 10^{-3}$. An air gap is interposed in the magnetic circuit, of a section of 10 sq. in. (allowing for spread), to get the desired reactance.

How long must the air gap be, and what is the resistance, the reactance, the effective resistance, the effective impedance, and the power-factor of the reactive coil?

The coil contains 200 turns each 16 in. in length and 0.013 sq. in. in cross section. Taking the resistivity of copper as 1.8×10^{-6}, the resistance is

$$r_1 = \frac{1.8 \times 10^{-6} \times 200 \times 16}{0.013 \times 2.54} = 0.175 \text{ ohm},$$

where 2.54 is the factor for converting inches to centimeters. (1 inch = 2.54 cm.)

Writing $E = 100$ volts generated, $f = 60$ cycles per second, and $n = 200$ turns, the maximum magnetic flux is given by $E = 4.44\ fn\Phi$; or, $100 = 4.44 \times 0.6 \times 200\ \Phi$, and $\Phi = 0.188$ megaline.

This gives in an air gap of 10 sq. in. a maximum density $B = 18{,}800$ lines per sq. in., or 2920 lines per sq. cm.

Ten amperes in 200 turns give 2000 ampere-turns effective or $F = 2830$ ampere-turns maximum.

Neglecting the ampere-turns required by the iron part of the magnetic circuit as relatively very small, 2830 ampere-turns have to be consumed by the air gap of density $B = 2920$.

Since
$$B = \frac{4\pi F}{10\, l},$$

the length of the air gap has to be

$$l = \frac{4\pi F}{10\, B} = \frac{4\pi \times 2830}{10 \times 2920} = 1.22 \text{ cm., or } 0.48 \text{ in.}$$

With a cross section of 6 sq. in. and a mean length of 18 in., the volume of the iron is 108 cu. in., or 1770 cu. cm.

The density in the iron, $B_1 = \frac{188{,}000}{6} = 31{,}330$ lines per sq. in., or 4850 lines per sq. cm.

With an hysteresis coefficient $\eta = 2.5 \times 10^{-3}$, and density $B_1 = 4850$, the loss of energy per cycle per cubic centimeter is

$$\begin{aligned} W &= \eta B_1^{1.6} \\ &= 2.5 \times 10^{-3} \times 4850^{1.6} \\ &= 1980 \text{ ergs}, \end{aligned}$$

and the hysteresis loss at $f = 60$ cycles and the volume $V = 1770$ is thus

$$P = 60 \times 1770 \times 1980 \text{ ergs per sec.}$$
$$= 21.0 \text{ watts,}$$

which at 10 amp. represent an effective hysteretic resistance,

$$r_2 = \frac{21.0}{10^2} = 0.21 \text{ ohm.}$$

Hence the total effective resistance of the reactive coil is

$$r = r_1 + r_2 = 0.175 + 0.21 = 0.385 \text{ ohm}$$

the effective reactance is

$$x = \frac{E}{I} = 10 \text{ ohms;}$$

the impedance is

$$z = 10.01 \text{ ohms;}$$

the power-factor is

$$\cos\theta = \frac{r}{z} = 3.8 \text{ per cent.;}$$

the total apparent power of the reactive coil is

$$I^2z = 1001 \text{ volt-amperes,}$$

and the loss of power,

$$I^2r = 38 \text{ watts.}$$

11. CAPACITY AND CONDENSERS

51. The charge of an electric condenser is proportional to the impressed voltage, that is, potential difference at its terminals, and to its capacity.

A condenser is said to have unit capacity if unit current existing for one second produces unit difference of potential at its terminals.

The practical unit of capacity is that of a condenser in which 1 amp. during one second produces 1 volt difference of potential.

The practical unit of capacity equals 10^{-9} absolute units. It is called a farad.

One farad is an extremely large capacity, and therefore one millionth of one farad, called microfarad, mf., is commonly used.

If an alternating e.m.f. is impressed upon a condenser, the charge of the condenser varies proportionally to the e.m.f., and thus there is current to the condenser during rising and from the condenser during decreasing e.m.f., as shown in Fig. 26.

That is, the current consumed by the condenser leads the impressed e.m.f. by 90 time degrees, or a quarter of a period.

Denoting f as frequency and E as effective alternating e.m.f. impressed upon a condenser of C mf. capacity, the condenser is charged and discharged twice during each cycle, and the time of one complete charge or discharge is therefore $\frac{1}{4f}$.

Since $E\sqrt{2}$ is the maximum voltage impressed upon the condenser, an average of $CE\sqrt{2}\ 10^{-6}$ amp. would have to exist during one second to charge the condenser to this voltage, and

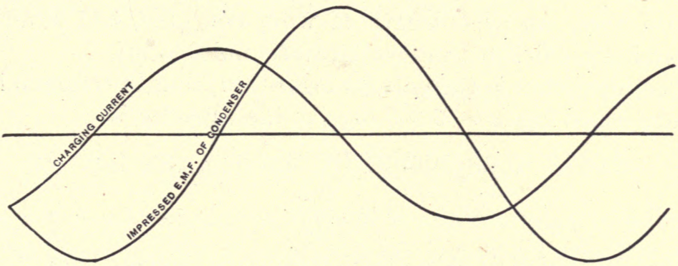

FIG. 26.—Charging current of a condenser across which an alternating e.m.f. is impressed.

to charge it in $\frac{1}{4f}$ seconds an average current of $4fCE\sqrt{2}\ 10^{-6}$ amp. is required.

Since $$\frac{\text{effective current}}{\text{average current}} = \frac{\pi}{2\sqrt{2}},$$

the effective current is $I = 2\pi fCE\ 10^{-6}$; that is, at an impressed e.m.f. of E effective volts and frequency f, a condenser of C mf. capacity consumes a current of

$$I = 2\pi fCE\ 10^{-6} \text{ amp. effective,}$$

which current leads the terminal voltage by 90 degrees or a quarter period.

Transposing, the e.m.f. of the condenser is

$$E = \frac{10^6 I}{2\pi fC} = x_0 I.$$

The value $x_0 = \frac{10^6}{2\pi fC}$ is called the *condensive reactance* of the condenser.

Due to the energy loss in the condenser by dielectric hysteresis, the current leads the e.m.f. by somewhat less than 90 time degrees, and can be resolved into a *wattless charging current* and a *dielectric hysteresis current,* which latter, however, is generally so small as to be negligible, though in underground cables of poor quality, it may reach as high as 50 per cent. or more of the charging or wattless current of the condenser.

52. The capacity of one wire of a transmission line is

$$C = \frac{1.11 \times 10^{-6} \times l}{2 \log_\epsilon \frac{2\,l_s}{l_d}}, \text{ in mf.,}$$

where l_d = diameter of wire, cm.; l_s = distance of wire from return wire, cm.; l = length of wire, cm., and 1.11×10^{-6} = reduction coefficient from electrostatic units to mf.

The logarithm is the natural logarithm; thus in common logarithms, since $\log_\epsilon a = 2.303 \log_{10} a$, the capacity is

$$C = \frac{0.24 \times 10^{-6} \times l}{\log_{10} \frac{2\,l_s}{l_d}}, \text{ in mf.}$$

The derivation of this equation must be omitted here.

The charging current of a line wire is thus

$$I = 2\,\pi f C E\, 10^{-6},$$

where f = the frequency, in cycles per second, E = the difference of potential, effective, between the line and the neutral (E = ½ line voltage in a single-phase, or four-wire quarter-phase system, $\frac{1}{\sqrt{3}}$ line voltage, or Y voltage, in a three-phase system).

EXAMPLES

53. In the transmission line discussed in the examples in 37, 38, 41 and 45, what is the charging current of the line at 6000 volts between lines, at 33.3 cycles? How many volt-amperes does it represent, and what percentage of the full-load current of 44 amp. is it?

The length of the line is, per wire, $l = 2.23 \times 10^6$ cm.

The distance between wires, $l_s = 45$ cm.

The diameter of transmission wire, $l_d = 0.82$ cm.

Thus the capacity, per wire, is

$$C = \frac{0.24 \times 10^{-6}\, l}{\log_{10} \frac{2\,l_s}{l_d}} = 0.26 \text{ mf.}$$

The frequency is $f = 33.3$,
The voltage between lines, 6000.

Thus per line, or between line and neutral point,

$$E = \frac{6000}{\sqrt{3}} = 3460 \text{ volts;}$$

hence, the charging current per line is

$$I_0 = 2\,\pi f C E\, 10^{-6}$$
$$= 0.19 \text{ amp.,}$$

or 0.43 per cent. of full-load current;

that is, negligible in its influence on the transmission voltage.

The volt-ampere input of the transmission is,

$$3\, I_0 E = 2000$$
$$= 2.0 \text{ kv-amp.}$$

12. IMPEDANCE OF TRANSMISSION LINES

54. Let r = resistance; $x = 2\,\pi f L$ = the reactance of a transmission line; E_0 = the alternating e.m.f. impressed upon the line; I = the line current; E = the e.m.f. at receiving end of the line, and θ = the angle of lag of current I behind e.m.f. E.

$\theta < 0$ thus denotes leading, $\theta > 0$ lagging current, and $\theta = 0$ a non-inductive receiver circuit.

The capacity of the transmission line shall be considered as negligible.

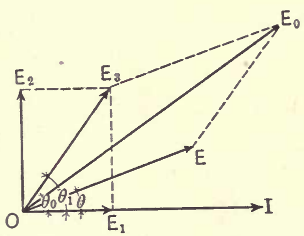

FIG. 27.—Vector diagram of current and e.m.fs. in a transmission line assuming zero capacity.

Assuming the phase of the current $\overline{OI} = I$ as zero in the polar diagram, Fig. 27, the e.m.f. E is represented by vector $\overline{OE}$, ahead of $\overline{OI}$ by angle θ. The e.m.f. consumed by resistance r is $\overline{OE_1} = E_1 = Ir$ in phase with the current, and the e.m.f. consumed by reactance x is $\overline{OE_2} = E_2 = Ix$, 90 time degrees ahead of the current; thus the total e.m.f. consumed by the line, or e.m.f. consumed by impedance, is the resultant $\overline{OE_3}$ of $\overline{OE_1}$ and $\overline{OE_2}$, and is $E_3 = Iz$.

Combining $\overline{OE_3}$ and $\overline{OE}$ gives $\overline{OE_0}$, the e.m.f. impressed upon the line.

Denoting $\tan \theta_1 = \frac{x}{r}$ the time angle of lag of the line impedance, it is, trigonometrically,

$$\overline{OE_0}^2 = \overline{OE}^2 + \overline{EE_0}^2 - 2\,\overline{OE} \times \overline{EE_0} \cos OEE_0.$$

Since

$$\overline{EE_0} = \overline{OE_3} = Iz,$$
$$OEE_0 = 180 - \theta_1 + \theta,$$

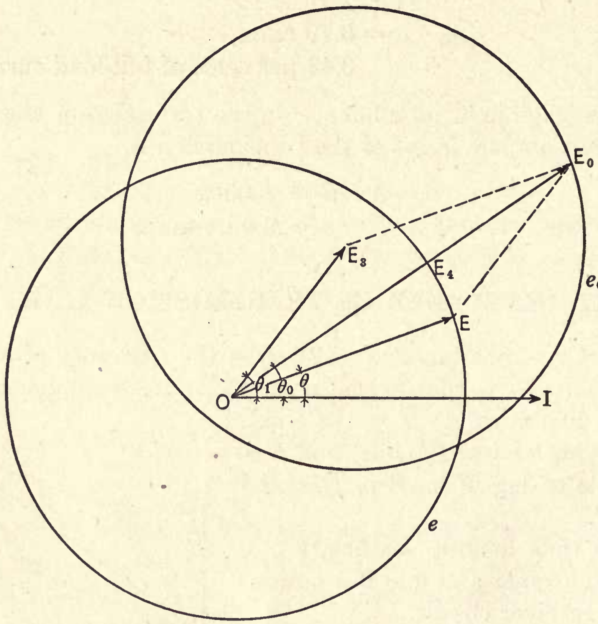

FIG. 28.—Locus of the generator and receiver e.m.fs. in a transmission line with varying load phase angle.

we have

$$E_0^2 = E^2 + I^2z^2 + 2\,EIz \cos (\theta_1 - \theta)$$
$$= (E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2},$$

and

$$E_0 = \sqrt{(E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2}},$$

and the drop of voltage in the line,

$$E_0 - E = \sqrt{(E + Iz)^2 - 4\,EIz \sin^2 \frac{\theta_1 - \theta}{2}} - E.$$

55. That is, the voltage E_0 required at the sending end of a line of resistance r and reactance x, delivering current I at voltage E, and the voltage drop in the line, do not depend upon current and line constants only, but depend also upon the angle of phase displacement of the current delivered over the line.

If $\theta = 0$, that is, non-inductive receiving circuit,

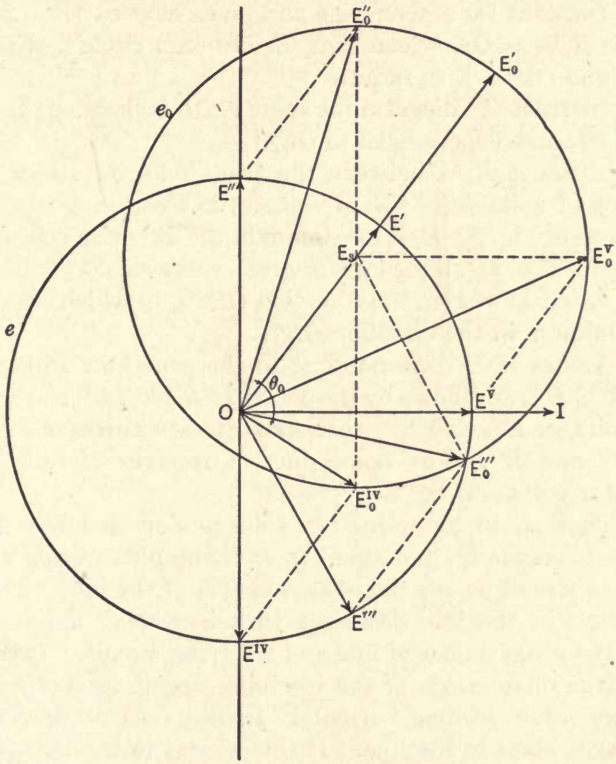

FIG. 29.—Locus of the generator and receiver e.m.fs. in a transmission line with varying load phase angle.

$$E_0 = \sqrt{(E + Iz)^2 - 4\, EIz \sin^2 \frac{\theta_1}{2}};$$

that is, less than $E + Iz$, and thus the line drop is less than Iz.

If $\theta = \theta_1$, E_0 is a maximum, $= E + Iz$, and the line drop is the impedance voltage.

With decreasing θ, E_0 decreases, and becomes $= E$; that is, no drop of voltage takes place in the line at a certain negative value of θ which depends not only on z and θ_1 but on E and I. Beyond this value of θ, E_0 becomes smaller than E; that is, a rise of voltage takes place in the line, due to its reactance. This can be seen best graphically.

Choosing the current vector $\overline{OI}$ as the horizontal axis, for the same e.m.f. E received, but different phase angles θ, all vectors $\overline{OE}$ lie on a circle e with O as center. Fig. 28. Vector $\overline{OE_3}$ is constant for a given line and given current I.

Since $E_3E_0 = \overline{OE}$ = constant, E_0 lies on a circle e_0 with E_3 as center and $\overline{OE} = E$ as radius.

To construct the diagram for angle θ, $\overline{OE}$ is drawn at the angle θ with $\overline{OI}$, and $\overline{EE_0}$ parallel to $\overline{OE_3}$.

The distance E_4E_0 between the two circles on vector $\overline{OE_0}$ is the drop of voltage (or rise of voltage) in the line.

As seen in Fig. 29, E_0 is maximum in the direction $\overline{OE_3}$ as $\overline{OE'_0}$, that is, for $\theta = \theta_0$, and is less for greater as well, $\overline{OE''_0}$, as smaller angles θ. It is $= E$ in the direction $\overline{OE'''_0}$, in which case $\theta < 0$, and minimum in the direction $\overline{OE^{IV}_0}$.

The values of E corresponding to the generator voltages E'_0, E''_0, E'''_0, E^{IV}_0 are shown by the points E' E'' E''' E^{IV} respectively. The voltages E''_0 and E^{IV}_0 correspond to a wattless receiver circuit E'' and E^{IV}. For non-inductive receiver circuit E^{V} the generator voltage is $\overline{OE^{V}_0}$.

56. That is, in an inductive transmission line the drop of voltage is maximum and equal to Iz if the phase angle θ of the receiving circuit equals the phase angle θ_0 of the line. The drop of voltage in the line decreases with increasing difference between the phase angles of line and receiving circuit. It becomes zero if the phase angle of the receiving circuit reaches a certain negative value (leading current). In this case no drop of voltage takes place in the line. If the current in the receiving circuit leads more than this value a rise of voltage takes place in the line. Thus by varying phase angle θ of the receiving circuit the drop of voltage in a transmission line with current I can be made anything between Iz and a certain negative value. Or inversely the same drop of voltage can be produced for different values of the current I by varying the phase angle.

Thus, if means are provided to vary the phase angle of the receiving circuit, by producing lagging and leading currents at will (as can be done by synchronous motors or converters), the voltage at the receiving circuit can be maintained constant within a certain range irrespective of the load and generator voltage.

In Fig. 30 let $\overline{OE} = E$, the receiving voltage; I, the power component of the line current; thus $\overline{OE_3} = E_3 = Iz$, the e.m.f. consumed by the power component of the current in the impedance. This e.m.f. consists of the e.m.f consumed by resistance $\overline{OE_1}$ and the e.m.f. consumed by reactance $\overline{OE_2}$.

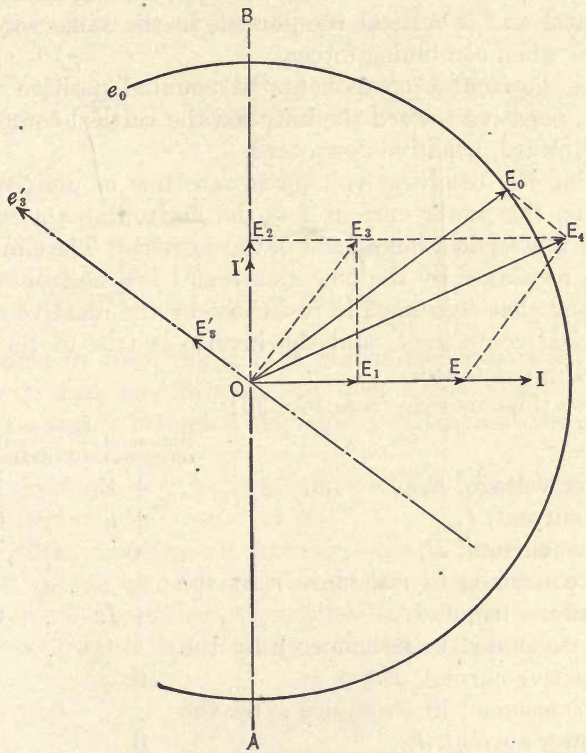

FIG. 30.—Regulation diagram for transmission line.

Reactive components of the current are represented in the diagram in the direction $\overline{OA}$ when lagging and $\overline{OB}$ when leading. The e.m.f. consumed by these reactive components of the current in the impedance is thus in the direction e'_3, perpendicular to $\overline{OE_3}$. Combining $\overline{OE_3}$ and $\overline{OE}$ gives the e.m.f. $\overline{OE_4}$ which would be required for non-inductive load. If E_0 is the generator voltage, E_0 lies on a circle e_0 with $\overline{OE_0}$ as radius. Thus drawing $\overline{E_4E_0}$ parallel to e'_3 gives $\overline{OE_0}$, the generator voltage; $OE'_3 = \overline{E_4E_0}$, the e.m.f. consumed in the impedance by the reactive component of the current; and as proportional thereto, $\overline{OI'} = I'$, the reactive current required to give at generator voltage E_0 and power current I the receiver voltage E. This reactive current I' lags behind E'_3 by less than 90 and more than zero degrees.

57. In calculating numerical values, we can proceed either trigonometrically as in the preceding, or algebraically by resolving all sine waves into two rectangular components; for instance, a horizontal and a vertical component, in the same way as in mechanics when combining forces.

Let the horizontal components be counted positive toward the right, negative toward the left, and the vertical components positive upward, negative downward.

Assuming the receiving voltage as zero line or positive horizontal line, the power current I is the horizontal, the wattless current I' the vertical component of the current. The e.m.f. consumed in resistance by the power current I is a horizontal component, and that consumed in resistance by the reactive current I' a vertical component, and the inverse is true of the e.m.f. consumed in reactance.

We have thus, as seen from Fig. 30:

	Horizontal component	Vertical component
Receiver voltage, E,	$+E$	0
Power current, I,	$+I$	0
Reactive current, I',	0	$\mp I'$
E.m.f. consumed in resistance r by the power current, Ir,	$+Ir$	0
E.m.f. consumed in resistance r by the reactive current, $I'r$,	0	$\mp I'r$
E.m.f. consumed in reactance x by the power current, Ix,	0	$+Ix$
E.m.f. consumed in reactance x by the reactive current, $I'x$,	$\pm I'x$	0
Thus, total e.m.f. required, or impressed e.m.f., E_0,	$E + Ir \pm I'x$	$\mp I'r + Ix$;

hence, combined,

$$E_0 = \sqrt{(E + Ir \pm I'x)^2 + (\mp I'r + Ix)^2};$$

or, expanded,

$$E_0 = \sqrt{E^2 + 2E(Ir \pm I'x) + (I^2 + I'^2)z^2}.$$

From this equation I' can be calculated; that is, the reactive current found which is required to give E_0 and E at energy current I.

The lag of the total current in the receiver circuit behind the receiver voltage is

$$\tan \theta = \frac{I'}{I}.$$

The lead of the generator voltage ahead of the receiver voltage is

$$\tan \theta_1 = \frac{\text{vertical component of } E_0}{\text{horizontal component of } E_0}$$

$$= \frac{\pm I'r - Ix}{E + Ir \pm I'x'}$$

and the lag of the total current behind the generator voltage is

$$\theta_0 = \theta + \theta_1.$$

As seen, by resolving into rectangular components the phase angles are directly determined from these components.

The resistance voltage is the same component as the current to which it refers.

The reactance voltage is a component 90 time degrees ahead of the current.

The same investigation as made here on long-distance transmission applies also to distribution lines, reactive coils, transformers, or any other apparatus containing resistance and reactance inserted in series into an alternating-current circuit.

EXAMPLES

58. (1) An induction motor has 2000 volts impressed upon its terminals; the current and the power-factor, that is, the cosine of the angle of lag, are given as functions of the output in Fig. 31.

The induction motor is supplied over a line of resistance $r = 2.0$ and reactance $x = 4.0$.

(*a*) How must the generator voltage e_0 be varied to maintain constant voltage $e = 2000$ at the motor terminals, and

(*b*) At constant generator voltage $e_0 = 2300$, how will the voltage at the motor terminals vary?

We have

$$e_0 = \sqrt{(e + iz)^2 - 4\, eiz \sin^2 \frac{\theta_1 - \theta}{2}}. \qquad e = 2000.$$

$$z = \sqrt{r^2 + x^2} = 4.472.$$

$$\tan \theta_1 = \frac{x}{r} = 2. \qquad \theta_1 = 63.4°.$$

$$\cos \theta = \text{power-factor}.$$

Taking i from Fig. 31 and substituting, gives (a) the values of e_0 for $e = 2000$, which are recorded in the table, and plotted in Fig. 31.

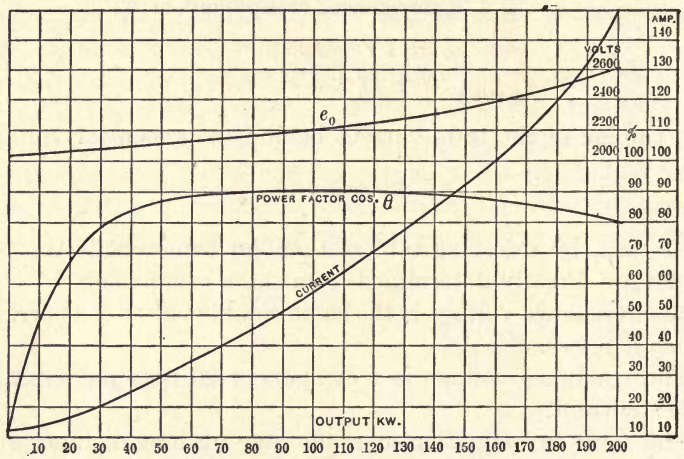

FIG. 31.—Characteristics of induction motor and variation of generator e.m.f. necessary to maintain constant the e.m.f. impressed upon the motor.

(b) At the terminal voltage of the motor $e = 2000$, the current is i, the output P, the generator voltage e_0. Thus at generator voltage $e'_0 = 2300$, the terminal voltage of the motor is

$$e' = \frac{2300}{e_0} e = \frac{2300}{e_0} 2000;$$

the current is

$$i' = \frac{2300}{e_0} i$$

and the power is

$$P' = \left(\frac{2300}{e_0}\right)^2 P.$$

The values of e', i', P' are recorded in the second part of the table under (b) and plotted in Fig. 32.

(a) At $e = 2000$			Thus, e_0	(b) Hence, at $e_0 = 2300$		
Output, P = kw.	Current, i	Lag, θ		Output, P'	Current, i'	Voltage, e'
0	12.0	84.3°	2048	0.	13.45	2240
5	12.6	72.6°	2055	6.25	14.05	2234
10	13.5	62.6°	2060	12.4	15.00	2230
15	14.8	54.6°	2065	18.6	16.4	2220
20	16.3	47.9°	2071	24.4	18.0	2216
30	20.0	37.8°	2084	36.3	22.0	2200
40	25.0	32.8°	2093	48.0	27.5	2198
50	30.0	29.0°	2110	59.5	32.7	2180
69	40.0	26.3°	2146	78.5	42.8	2160
102	60.0	24.5°	2216	110.2	62.6	2080
132	80.0	25.8°	2294	131.0	79.5	1990
160	100.0	28.4°	2382	149.0	96.4	1928
180	120.0	31.8°	2476	156.5	111.5	1860
200	150.0	36.9°	2618	155.0	132.0	1760

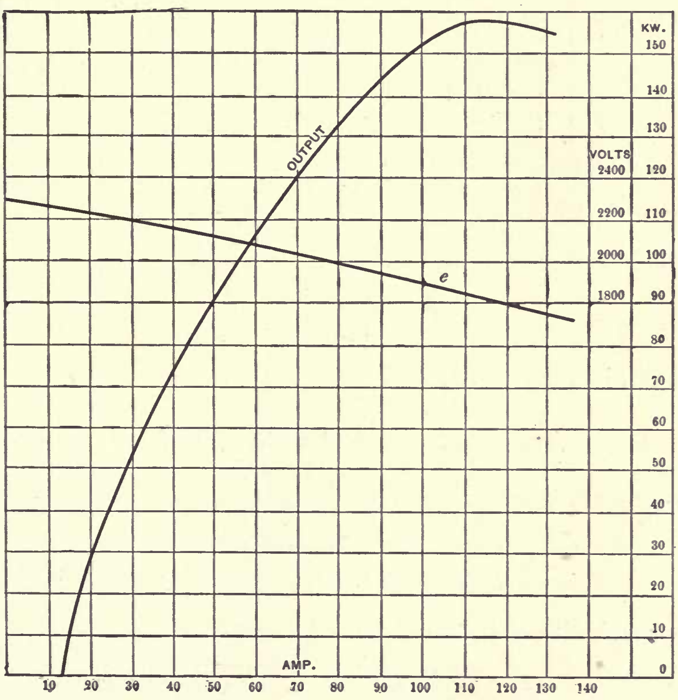

Fig. 32.—Characteristics of induction motor, constant generator e.m.f.

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59. (2) Over a line of resistance $r = 2.0$ and reactance $x = 6.0$ power is supplied to a receiving circuit at a constant voltage of $e = 2000$. How must the voltage at the beginning of the line, or generator voltage, e_0, be varied if at no load the receiving circuit consumes a reactive current of $i_2 = 20$ amp., this reactive current decreases with the increase of load, that is, of power current i_1, becomes $i_2 = 0$ at $i_1 = 50$ amp., and then as leading current increases again at the same rate?

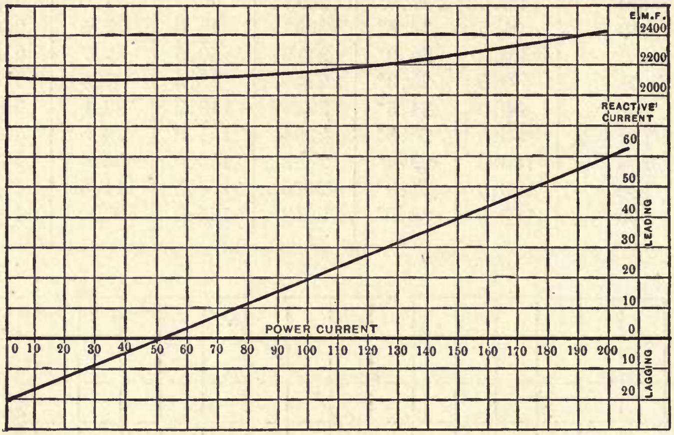


FIG. 33.—Variation of generator e.m.f. necessary to maintain constant receiver voltage if the reactive component of receiver current varies proportional to the change of power component of the current.

The reactive current,

$$i_2 = 20 \text{ at } i_1 = 0,$$
$$i_2 = 0 \text{ at } i_1 = 50,$$

and can be represented by

$$i_2 = \left(1 - \frac{i_1}{50}\right) 20 = 20 - 0.4\, i_1;$$

the general equation of the transmission line is

$$e_0 = \sqrt{(e + i_1 r + i_2 x)^2 + (i_2 r - i_1 x)^2}$$
$$= \sqrt{(2000 + 2\, i_1 + 6\, i_2)^2 + (2\, i_2 - 6\, i_1)^2};$$

hence, substituting the value of i_2,

$$e_0 = \sqrt{(2120 - 0.4\, i_1)^2 + (40 - 6.8\, i_1)^2}$$
$$= \sqrt{4{,}496{,}000 + 46.4\, i_1^2 - 2240\, i_1}.$$

Substituting successive numerical values for i_1 gives the values recorded in the following table and plotted in Fig. 33.

i_1	e_0
0	2120
20	2114
40	2116
60	2126
80	2148
100	2176
120	2213
140	2256
160	2308
180	2365
200	2430

13. ALTERNATING-CURRENT TRANSFORMER

60. The alternating-current transformer consists of one magnetic circuit interlinked with two electric circuits, the primary circuit which receives energy, and the secondary circuit which delivers energy.

Let r_1 = resistance, $x_1 = 2\pi f S_2$ = self-inductive or leakage reactance of secondary circuit,

r_0 = resistance, $x_0 = 2\pi f S_1$ = self-inductive or leakage reactance of primary circuit,

where S_2 and S_1 refer to that magnetic flux which is interlinked with the one but not with the other circuit.

Let a = ratio of $\frac{\text{secondary}}{\text{primary}}$ turns (ratio of transformation).

An alternating e.m.f. E_0 impressed upon the primary electric circuit causes a current, which produces a magnetic flux Φ interlinked with primary and secondary circuits. This flux Φ generates e.m.fs. E_1 and E_i in secondary and in primary circuit, which are to each other as the ratio of turns, thus $E_i = \frac{E_1}{a}$.

Let E = secondary terminal voltage, I_1 = secondary current, θ_1 = lag of current I_1 behind terminal voltage E (where $\theta_1 < 0$ denotes leading current).

Denoting then in Fig. 34 by a vector $\overline{OE} = E$ the secondary terminal voltage, $\overline{OI_1} = I_1$ is the secondary current lagging by the angle $EOI = \theta_1$.

The e.m.f. consumed by the secondary resistance r_1 is $\overline{OE'_1} = E'_1 = I_1 r_1$ in phase with I_1.

The e.m.f. consumed by the secondary reactance x_1 is $\overline{OE''_1} = E''_1 = I_1 x_1$, 90 degrees ahead of I_1. Thus the e.m.f. consumed by the secondary impedance $z_1 = \sqrt{r_1^2 + x_1^2}$ is the resultant of $\overline{OE'_1}$ and $\overline{OE''_1}$, or $\overline{OE'''_1} = E'''_1 = I_1 z_1$.

$\overline{OE'''_1}$ combined with the terminal voltage $\overline{OE} = E$ gives the secondary e.m.f. $\overline{OE_1} = E_1$.

Proportional thereto by the ratio of turns and in phase there-

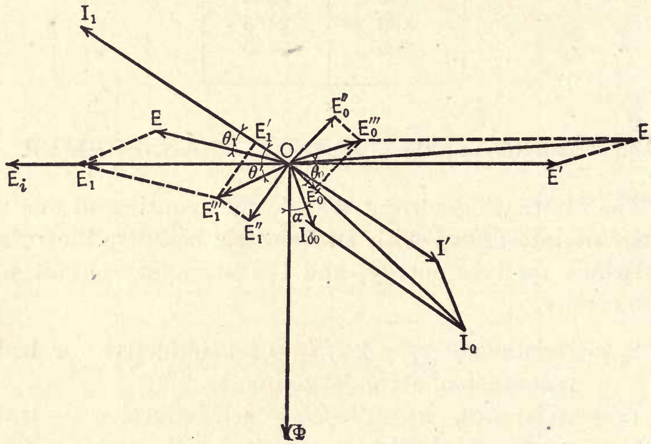


FIG. 34.—Vector diagram of e.m.fs. and currents in a transformer.

with is the e.m.f. generated in the primary $\overline{OE_i} = E_i$ where $E_i = \frac{E_1}{a}$.

To generate e.m.f. E_1 and E_i, the magnetic flux $\overline{O\Phi} = \Phi$ is required, 90 time degrees ahead of $\overline{OE_1}$ and $\overline{OE_i}$. To produce flux Φ the m.m.f. of F ampere-turns is required, as determined from the dimensions of the magnetic circuit, and thus the primary current I_{00}, represented by vector $\overline{OI_{00}}$, leading $\overline{O\Phi}$ by the angle α.

Since the total m.m.f. of the transformer is given by the primary exciting current I_{00}, there must be a component of primary current I', corresponding to the secondary current I_1, which may be called the primary load current, and which is opposite thereto and of the same m.m.f.; that is, of the intensity $I' = aI_1$, thus represented by vector $\overline{OI'} = I' = aI_1$.

$\overline{OI_{00}}$, the primary exciting current, and the primary load current $\overline{OI'}$, or component of primary current corresponding to the secondary current, combined, give the total primary current $\overline{OI_0} = I_0$.

The e.m.f. consumed by resistance in the primary is $\overline{OE'_0} = E'_0 = I_0r_0$ in phase with I_0.

The e.m.f. consumed by the primary reactance is $\overline{OE''_0} = E''_0 = I_0x_0$, 90 degrees ahead of $\overline{OI_0}$.

$\overline{OE'_0}$ and $\overline{OE''_0}$ combined gives $\overline{OE'''_0}$, the e.m.f. consumed by the primary impedance.

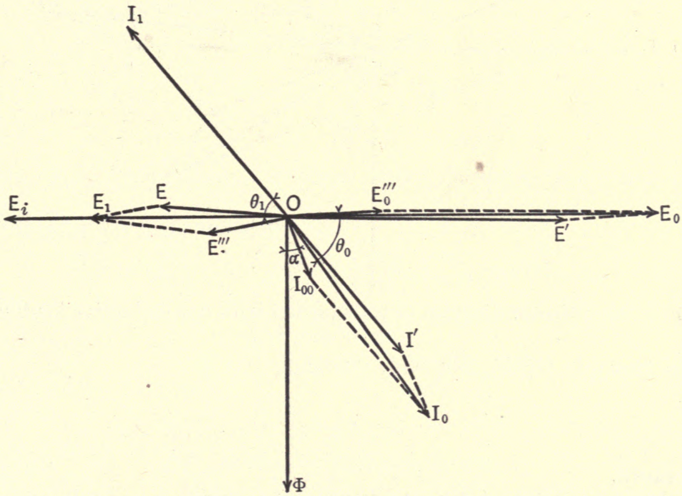


FIG. 35.—Vector diagram of transformer with lagging load current.

Equal and opposite to the primary counter-generated e.m.f. $\overline{OE_i}$ is the component of primary e.m.f., $\overline{OE'}$, consumed thereby.

$\overline{OE'}$ combined with $\overline{OE'''_0}$ gives $\overline{OE_0} = E_0$, the primary impressed e.m.f., and angle $\theta_0 = E_0OI_0$, the phase angle of the primary circuit.

Figs. 35, 36, and 37 give the polar diagrams of $\theta_1 = 45°$ or lagging current, $\theta_1 =$ zero or non-inductive circuit, and $\theta = - 45°$ or leading current.

61. As seen, the primary impressed e.m.f. E_0 required to produce the same secondary terminal voltage E at the same current I_1 is larger with lagging or inductive and smaller with leading current than on a non-inductive secondary circuit; or, inversely, at the same secondary current I_1 the secondary terminal voltage E with lagging current is less and with leading current more than with non-inductive secondary circuit, at the same primary impressed e.m.f. E_0.

The calculation of numerical values is not practicable by measurement from the diagram, since the magnitudes of the different quantities are too different, $E'_1 : E''_1 : E_1 : E_0$ being frequently in the proportion 1 : 10 : 100 : 2000.

Trigonometrically, the calculation is thus:

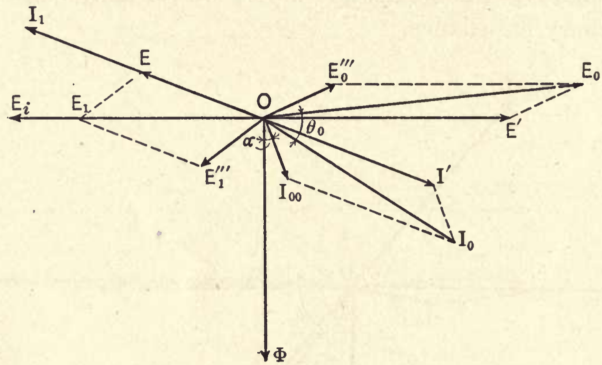


Fig. 36.—Vector diagram of transformer with non-inductive loading.

In triangle OEE_1, Fig. 34, writing

$$\tan \theta' = \frac{x_1}{r_1},$$

we have,

$$\overline{OE_1}^2 = \overline{OE}^2 + \overline{EE_1}^2 - 2\,\overline{OE}\;\overline{EE_1} \cos OEE_1;$$

also,

$$\overline{EE_1} = I_1 z_1$$

$$\sphericalangle OEE_1 = 180 - \theta' + \theta_1,$$

hence,

$$E_1^2 = E^2 + I_1^2 z_1^2 + 2\,EI_1 z_1 \cos(\theta' - \theta_1).$$

This gives the secondary e.m.f., E_1, and therefrom the primary counter-generated e.m.f.

$$E_i = \frac{E_1}{a}.$$

In triangle EOE_1 we have

$$\sin E_1OE \div \sin E_1EO = \overline{EE_1} \div \overline{E_1O}$$

thus, writing

$$\measuredangle E_1OE = \theta'',$$

we have

$$\sin \theta'' \div \sin (\theta' - \theta_1) = I_1z \div E_1,$$

wherefrom we get

$$\measuredangle \theta'', \text{ and } \measuredangle E_1OI_1 = \theta = \theta_1 + \theta'',$$

the phase displacement between secondary current and secondary e.m.f.

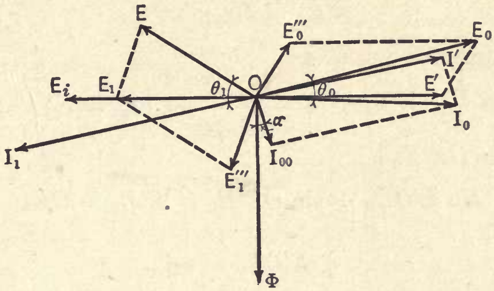


FIG. 37.—Vector diagram of transformer with leading load current.

In triangle $OI_{00}I_0$ we have

$$\overline{OI_0}^2 = \overline{OI_{00}}^2 + \overline{I_{00}I_0}^2 - 2\,\overline{OI_{00}}\,\overline{I_{00}I_0} \cos OI_{00}I_0,$$

since

$$\measuredangle E_1O\phi = 90°,$$
$$\measuredangle OI_{00}I_0 = 90 + \theta + \alpha,$$

and

$$\overline{I_{00}I_0} = I' = aI_1,$$
$$\overline{OI_{00}} = I_{00} = \text{exciting current},$$

calculated from the dimensions of the magnetic circuit. Thus the primary current is

$$I_0^2 = I_{00}^2 + a^2I_1^2 + 2\,aI_1I_{00} \sin (\theta + \alpha).$$

In triangle $OI_{00}I_0$ we have

$$\sin I_{00}OI_0 \div \sin OI_{00}I_0 = \overline{I_{00}I_0} \div \overline{OI_0};$$

writing

$$\measuredangle I_{00}OI_0 = \theta''$$

this becomes

$$\sin \theta''_0 \div \sin (\theta + \alpha) = aI_1 \div I_0;$$

therefrom we get θ''_0, and thus

$$\measuredangle E'OI_0 = \theta_2 = 90° - \alpha - \theta''_0.$$

In triangle $OE'E_0$ we have

$$\overline{OE_0}^2 = \overline{OE'}^2 + \overline{E'E_0}^2 - 2\,\overline{OE'}\;\overline{E'E_0}\cos \overline{OE'}\,E_0;$$

writing

$$\tan \theta'_0 = \frac{x_0}{r_0},$$

we have

$$\sphericalangle\, OE'E_0 = 180° - \theta' + \theta_2,$$

$$\overline{OE'} = E_i = \frac{E_1}{a},$$

$$\overline{E'E_0} = I_0 z_0;$$

thus the impressed e.m.f. is

$$E_0^2 = \frac{E_1^2}{a^2} + I_0^2 z_0^2 + \frac{2E_1 I_0 z_0}{a} \cos(\theta_0' - \theta_2).$$

In triangle $OE'E_0$

$$\sin E'OE_0 \div \sin OE'E_0 = \overline{E'E_0} \div OE_0;$$

thus, writing

$$\sphericalangle\, E'OE_0 = \theta_1'',$$

we have

$$\sin \theta''_1 \div \sin(\theta'_0 - \theta_2) = I_0 z_0 \div E_0;$$

herefrom we get $\sphericalangle\, \theta''_1$, and

$$\sphericalangle\, \theta_0 = \theta_2 + \theta''_1,$$

the phase displacement between primary current and impressed e.m.f.

As seen, the trigonometric method of transformer calculation is rather complicated.

62. Somewhat simpler is the algebraic method of resolving into rectangular components.

Considering first the secondary circuit, of current I_1 lagging behind the terminal voltage E by angle θ_1.

The terminal voltage E has the components $E \cos \theta_1$ in phase, $E \sin \theta_1$ in quadrature with and ahead of the current I_1.

The e.m.f. consumed by resistance r_1, $I_1 r_1$, is in phase.

The e.m.f. consumed by reactance x_1, $I_1 x_1$, is in quadrature ahead of I_1.

Thus the secondary e.m.f. has the components

$E \cos \theta_1 + I_1 r_1$ in phase,

$E \sin \theta_1 + I_1 x_1$ in quadrature ahead of the current I_1, and the total value,

$$E_1 = \sqrt{(E \cos \theta_1 + I_1 r_1)^2 + (E \sin \theta_1 + I_1 x_1)^2},$$

and the tangent of the phase angle of the secondary circuit is

$$\tan \theta = \frac{E \sin \theta_1 + I_1 x_1}{E \cos \theta_1 + I_1 r_1}.$$

Resolving all quantities into components in phase and in quadrature with the secondary e.m.f. E_1, or in horizontal and in vertical components, choosing the magnetism or mutual flux as vertical axis, and denoting the direction to the right and upward as positive, to the left and downward as negative, we have

	Horizontal component	Vertical component
Secondary current, I_1,	$-I_1 \cos \theta$	$+I_1 \sin \theta$
Secondary e.m.f., E_1,	$-E_1$	0
Primary counter-generated e.m.f., $E_1 = \frac{E_1}{a}$,	$-\frac{E_1}{a}$	0
Primary e.m.f. consumed thereby, $E' = -E_i$,	$+\frac{E_1}{a}$	0
Primary load current, $I' = -aI_1$,	$+aI_1 \cos \theta$	$-aI_1 \sin \theta$
Magnetic flux, Φ,	0	$-\Phi$
Primary exciting current, I_{00}, consisting of core loss current,	$I_{00} \sin \alpha$	
magnetizing current,		$-I_{00} \cos \alpha$

hence, total primary current, I_0,

Horizontal component	Vertical component
$aI_1 \cos \theta_1 + I_{00} \sin \alpha$	$-(aI_1 \sin \theta_1 + I_{00} \cos \alpha)$

E.m.f. consumed by primary resistance r_0, $E'_0 = I_0 r_0$ in phase with I_0,

Horizontal component	Vertical component
$r_0 a I_1 \cos \theta + r_0 I_{00} \sin \alpha$	$-(r_0 a I_1 \sin \theta + r_0 I_{00} \cos \alpha)$

E.m.f. consumed by primary reactance x_0, $E_0 = I_0 x_0$, 90° ahead of I_0,

Horizontal component	Vertical component
$x_0 a I_1 \sin \theta + x_0 I_{00} \cos \alpha$	$+x_0 a I_1 \cos \theta + x_0 I_{00} \sin \alpha$

E.m.f. consumed by primary generated e.m.f., $E' = \frac{E_1}{a}$ horizontal.

The total primary impressed e.m.f., E_0,

Horizontal component

$$\frac{E_1}{a} + aI_1\,(r_0 \cos\theta + x_0 \sin\theta) + I_{00}\,(r_0 \sin\alpha + x_0 \cos\alpha).$$

Vertical component

$$aI_1\,(r_0 \sin\theta - x_0 \cos\theta) + I_{00}\,(r_0 \cos\alpha - x_0 \sin\alpha),$$

or writing $\qquad \tan\theta'_0 = \frac{x_0}{r_0},$

since

$$\sqrt{r_0^2 + x_0^2} = z_0, \ \sin\theta'_0 = \frac{x_0}{z_0}, \text{ and } \cos\theta'_0 = \frac{r_0}{z_0}.$$

Substituting this value, the horizontal component of E_0 is

$$\frac{E_1}{a} + az_0I_1 \cos(\theta - \theta'_0) + z_0I_{00} \sin(\alpha + \theta'_0);$$

the vertical component of E_0 is

$$az_0I_1 \sin(\theta - \theta'_0) + z_0I_{00} \cos(\alpha + \theta'_0),$$

and, the total primary impressed e.m.f. is

$$E_0 = \sqrt{\left[\frac{E_1}{a} + az_0I_1\cos(\theta-\theta'_0) + z_0I_{00}\sin(\alpha+\theta'_0)\right]^2 + \left[az_0I_1\sin(\theta-\theta'_0) + z_0I_{00}\cos(\alpha+\theta'_0)\right]^2}$$

$$= \frac{E_1}{a}\sqrt{1 + \frac{2a^2z_0I_1}{E_1}\cos(\theta-\theta'_0) + \frac{2az_0I_{00}}{E_1}\sin(\alpha+\theta'_0) + \frac{a^4z_0^2I_1^2}{E_1^2} + \frac{a^2z_0^2I_{00}^2}{E_1^2} + \frac{2a^3z_0^2I_1I_{00}}{E_1^2}\sin(\theta+\alpha)}.$$

Combining the two components, the total primary current is

$$I_0 = \sqrt{(aI_1 \cos\theta + I_{00} \sin\alpha)^2 + (aI_1 \sin\theta + I_{00} \cos\alpha)}$$

$$= aI_1\sqrt{1 + \frac{2I_{00}}{aI_1}\sin(\theta + \alpha) + \frac{I_{00}^2}{a^2I_1^2}}.$$

Since the tangent of the phase angle is the ratio of vertical component to horizontal component, we have, primary e.m.f. phase,

$$\tan\theta' = \frac{az_0I_1 \sin(\theta - \theta'_0) + z_0I_{00} \cos(\alpha + \theta'_0)}{\frac{E_1}{a} + az_0I_1 \cos(\theta - \theta'_0) + z_0I_{00} \sin(\alpha - \theta'_0)}$$

primary current phase,

$$\tan\theta'' = \frac{aI_1 \sin\theta + I_{00} \cos\alpha}{aI_1 \cos\theta + I_{00} \sin\alpha}$$

and lag of primary current behind impressed e.m.f.,

$$\theta_0 = \theta'' - \theta'$$

EXAMPLES

63. (1) In a 20-kw. transformer the ratio of turns is 20 ÷ 1, and 100 volts is produced at the secondary terminals at full load. What is the primary current at full load, and the regulation, that is, the rise of secondary voltage from full load to no load, at constant primary voltage, and what is this primary voltage?

(*a*) at non-inductive secondary load,

(*b*) with 60 degrees time lag in the external secondary circuit,

(*c*) with 60 degrees time lead in the external secondary circuit.

The exciting current is 0.5 amp., the core loss 600 watts, the primary resistance 2 ohms, the primary reactance 5 ohms, the secondary resistance 0.004 ohm, the secondary reactance 0.01 ohm.

Exciting current and core loss may be assumed as constant.

600 watts at 2000 volts gives 0.3 amp. core loss current, hence $\sqrt{0.5^2 - 3^2} = 0.4$ amp. magnetizing current.

We have thus

$$r_0 = 2 \qquad r_1 = 0.004 \qquad I_{00} \cos \alpha = 0.3 \qquad a = 0.05$$
$$x_0 = 5 \qquad x_1 = 0.01 \qquad I_{00} \sin \alpha = 0.4$$
$$I_{00} = 0.5$$

1. Secondary current as horizontal axis:

	Non-inductive, $\theta_1 = 0$		Lag, $\theta_1 = +60°$		Lead, $\theta_1 = -60°$	
	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
Secondary current, I_1..	200	0	200	0	200	0
Secondary terminal voltage, E..........	100	0	50	+86.6	50	−86.6
Resistance voltage, I_1r_1.	0.8	0	0.8	0	0.8	0
Reactance voltage, I_1x_1.	0	+2.0	0	+ 2.0	0	+ 2.0
Secondary e.m.f., E_1...	100.8	+2.0	50.8	+88.6	50.8	−84.6
Secondary e.m.f., total	100.80		102.13		98.68	
tan θ................	+0.0198		+1.745		− 1.665	
θ....................	+1.1°		+60.2°		−59.0°	

2. Magnetic flux as vertical axis:

	Non-inductive, $\theta_1 = 0$		Lag, $\theta_1 = +60°$		Lead, $\theta_1 = -60°$	
	Hor.	Vert.	Hor.	Vert.	Hor.	Vert.
Secondary generated e.m.f., E	−100.80	0	−102.13	0	− 98.68	0
Secondary current, I_1	−200	+ 4	− 99.4	−172.8	−103	−171.4
Primary load current, $I' = -aI_1$	+ 10	+ 0.2	+ 4.97	− 8.64	+ 5.15	+ 8.57
Primary exciting current, I_{00}	0.3	0.4	0.3	− 0.4	0.3	− 0.4
Total primary current, I_0	+ 10.3	− 0.6	+ 5.27	− 9.04	+ 5.45	+ 8.17
Primary resistance, voltage, I_0r_0	20.6	1.2	10.54	− 18.08	10.90	+16.34
Primary reactance, voltage, I_0x_0	3.0	+51.3	45.20	+ 26.35	− 40.85	+27.25
E.m.f. consumed by primary counter e.m.f., $\frac{-E_1}{a}$	2016	0	2042.6	0	1973.6	0
Total primary impressed e.m.f., E^0	2039.6	+50.1	2098.34	+ 8.27	1943.65	+43.59

Hence,

	Non-inductive, $\theta_1 = 0$	Lag, $\theta_1 = +60°$	Lead, $\theta_1 = -60°$
Resultant E_0	2040.1	2098.3	1944.2
Resultant I_0	10.32	10.47	9.82
Phase of E_0	−1.4°	− 0.2°	− 1.2°
Phase of I_0	+3.3°	+59.8°	−56.3°
Primary lag, θ_0	+4.7°	+60.0°	−55.1°
Regulation $\frac{E_0}{2000}$	1.02005	1.04915	0.9721
Drop of voltage, per cent	2.005	4.915	− 2.79
Change of phase, $\theta_0 - \theta_1$	4.7°	0	4.9°

14. RECTANGULAR COORDINATES

64. The vector diagram of sine waves gives the best insight into the mutual relations of alternating currents and e.m.fs.

For numerical calculation from the vector diagram either the trigonometric method or the method of rectangular components is used.

The method of rectangular components, as explained in the above paragraphs, is usually simpler and more convenient than the trigonometric method.

In the method of rectangular components it is desirable to distinguish the two components from each other and from the resultant or total value by their notation.

To distinguish the components from the resultant, small letters are used for the components, capitals for the resultant. Thus in the transformer diagram of Section 13 the secondary current I_1 has the horizontal component $i_1 = - I_1 \cos \theta_1$, and the vertical component $i'_1 = + I_1 \sin \theta_1$.

To distinguish horizontal and vertical components from each other, either different types of letters can be used, or indices, or a prefix or coefficient.

Different types of letters are inconvenient, indices distinguishing the components undesirable, since indices are reserved for distinguishing different e.m.fs., currents, etc., from each other.

Thus the most convenient way is the addition of a prefix or coefficient to one of the components, and as such the letter j is commonly used with the vertical component.

Thus the secondary current in the transformer diagram, Section 13, can be written

$$i_1 + ji_2 = I_1 \cos \theta_1 + jI_1 \sin \theta_1. \tag{1}$$

This method offers the further advantage that the two components can be written side by side, with the plus sign between them, since the addition of the prefix j distinguishes the value ji_2 or $jI_1 \sin \theta_1$ as vertical component from the horizontal component i_1 or $I_1 \cos \theta_1$.

$$I_1 = i_1 + ji_2 \tag{2}$$

thus means that I_1 consists of a horizontal component i_1 and a vertical component i_2, and the plus sign signifies that i_1 and i_2 are combined by the parallelogram of sine waves.

The secondary e.m.f. of the transformer in Section 13, Fig. 34, is written in this manner, $\underset{\cdot}{E}_1 = - e_1$, that is, it has the horizontal component $- e_1$ and no vertical component.

The primary generated e.m.f. is

$$\underset{\cdot}{E}_i = \frac{- e_1}{a}, \tag{3}$$

and the e.m.f. consumed thereby

$$\underset{\cdot}{E}' = + \frac{e_1}{a}. \tag{4}$$

The secondary current is

$$\underset{\cdot}{I}_1 = - i_1 + j i_2, \tag{5}$$

where

$$i_1 = I_1 \cos \theta_1, \quad i_2 = I_1 \sin \theta_1, \tag{6}$$

and the primary load current corresponding thereto is

$$\underset{\cdot}{I}' = - a\underset{\cdot}{I}_1 = a i_1 - j a i_2. \tag{7}$$

The primary exciting current,

$$\underset{\cdot}{I}_{00} = h - jg, \tag{8}$$

where $h = I_{00} \sin \alpha$ is the hysteresis current, $g = I_{00} \cos \alpha$ the reactive magnetizing current.

Thus the total primary current is

$$\underset{\cdot}{I}_0 = \underset{\cdot}{I}' + \underset{\cdot}{I}_{00} = (a i_1 + h) - j (a i_2 + g). \tag{9}$$

The e.m.f. consumed by primary resistance r_0 is

$$r_0 \underset{\cdot}{I}_0 = r_0 (a i_1 + h) - j r_0 (a i_2 + g). \tag{10}$$

The horizontal component of primary current $(a i_1 + h)$ gives as e.m.f. consumed by reactance x_0 a negative vertical component, denoted by $j x_0 (a i_1 + h)$. The vertical component of primary current $j (a i_2 + g)$ gives as e.m.f. consumed by reactance x_0 a positive horizontal component, denoted by $x_0 (a i_2 + g)$.

Thus the total e.m.f. consumed by primary reactance x_0 is

$$x_0 (a i_2 + g) + j x_0 (a i_1 + h), \tag{11}$$

and the total e.m.f. consumed by primary impedance is

$$r_0 (a i_1 + h) + x_0 (a i_2 + g) - j [r_0 (a i_2 + g) - x_0 (a i_1 + h)]. \tag{12}$$

Thus, to get from the current the e.m.f. consumed in reactance x_0 by the horizontal component of current, the coefficient j has to be added; in the vertical component the coefficient $-j$ omitted; or, we can say the reactance is denoted by jx_0 for the horizontal and by $-\frac{x_0}{j}$ for the vertical component of current. In other words, if $I = i - ji'$ is a current, x the reactance of its circuit, the e.m.f. consumed by the reactance is

$$jxi + xi' = xi' + jxi.$$

65. If instead of omitting $-j$ in deriving the reactance e.m.f. for the vertical component of current we would add j also (as done when deriving the reactance e.m f. for the horizontal component of current), we get the reactance e.m.f.

$$jxi - j^2xi',$$

which gives the correct value $jxi + xi'$, if

$$j^2 = -1; \tag{13}$$

that is, we can say, in deriving the e.m.f. consumed by reactance, x, from the current, we multiply the current by jx, and substitute $j^2 = -1$.

By defining, and substituting, $j^2 = -1$, jx can thus be called the reactance in the representation in rectangular coordinates and $r + jx$ the impedance.

The primary impedance voltage of the transformer in the preceding could thus be derived directly by multiplying the current,

$$I_0 = (ai_1 + h) - j(ai_2 + g), \tag{9}$$

by the impedance,

$$Z_0 = r_0 + jx_0,$$

which gives

$$E'_0 = Z_0 I_0 = (r_0 + jx_0)[(ai_1 + h) - j(ai_2 + g)]$$
$$= r_0(ai_1 + h) - jr_0(ai_2 + g) + jx_0(ai_1 + h) - j^2x_0(ai_2 + g),$$

and substituting $j^2 = -1$,

$$E'_0 = [r_0(ai_1 + h) + x_0(ai_2 + g)] - j[r_0(ai_2 + g) - x_0(ai_1 + h)], \tag{14}$$

and the total primary impressed e.m.f. is thus

$$\underset{\cdot}{E}_0 = \underset{\cdot}{E}' + \underset{\cdot}{E}'_0$$

$$= \left[\frac{e_1}{a} + r_0(ai_1 + h) + x_0(ai_2 + g)\right] - j\left[r_0(ai_2 + g) - x_0(ai + h)\right]. \tag{15}$$

66. Such an expression in rectangular coordinates as

$$\underset{\cdot}{I} = i + ji' \tag{16}$$

represents not only the current strength but also its phase.

It means, in Fig. 38, that the total current $\overline{OI}$ has the two rectangular components, the horizontal component $I \cos \theta = i$ and the vertical component $I \sin \theta = i'$.

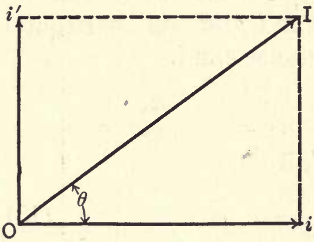


FIG. 38.—Magnitude and phase in rectangular coordinates.

Thus,

$$\tan \theta = \frac{i'}{i}; \tag{17}$$

that is, the tangent function of the phase angle is the vertical component divided by the horizontal component, or the term with prefix j divided by the term without j.

The total current intensity is obviously

$$I = \sqrt{i^2 + i'^2}. \tag{18}$$

The capital letter I in the symbolic expression $\underset{\cdot}{I} = i + ji'$ thus represents more than the I used in the preceding for total current, etc., and gives not only the intensity but also the phase. It is thus necessary to distinguish by the type of the latter the capital letters denoting the resultant current in symbolic expression (that is, giving intensity and phase) from the capital letters giving merely the intensity regardless of phase; that is,

$$\underset{\cdot}{I} = i + ji'$$

denotes a current of intensity

$$I = \sqrt{i^2 + i'^2}$$

and phase

$$\tan \theta = \frac{i'}{i}.$$

In the following, dotted italics will be used for the symbolic expressions and plain italics for the absolute values of alternating waves.

In the same way $z = \sqrt{r^2 + x^2}$ is denoted in symbolic representation of its rectangular components by

$$\dot{Z} = r + jx. \tag{91}$$

When using the symbolic expression of rectangular coordinates it is necessary ultimately to reduce to common expressions.

Thus in the above discussed transformer the symbolic expression of primary impressed e.m.f.

$$\dot{E}_0 = \left[\frac{e_1}{a} + r_0 (ai_1 + h) + x_0(ai_2 + g)\right] - j \left[r_0(ai_2 + g) - x_0(ai_1 + h)\right] \tag{15}$$

means that the primary impressed e.m.f. has the intensity

$$E_0 = \sqrt{\left[\frac{e_1}{a} + r_0 (ai_1 + h) + x_0 (ai_2 + g)\right]^2 + \left[r_0(ai_2 + g) - x_0(ai_1 + h)\right]^2}, \tag{20}$$

and the phase

$$\tan \theta_0 = \frac{r_0 (ai_2 + g) - x_0 (ai_1 + h)}{\frac{e_1}{a} + r_0 (ai_1 + h) + x_0 (ai_2 + g)}. \tag{21}$$

This symbolism of rectangular components is the quickest and simplest method of dealing with alternating-current phenomena, and is in many more complicated cases the only method which can solve the problem at all, and therefore the reader must become fully familiar with this method.

EXAMPLES

67. (1) In a 20-kw. transformer the ratio of turns is 20 : 1, and 100 volts are required at the secondary terminals at full load. What is the primary current, the primary impressed e.m.f., and the primary lag,

(*a*) at non-inductive load, $\theta_1 = 0$;

(*b*) with $\theta_1 = 60$ degrees time lag in the external secondary circuit;

(*c*) with $\theta_1 = -60$ degrees time lead in the external secondary circuit?

	Non-inductive	60° Lag	60° Lead
Secondary current, I_1 =	200	200	200
Secondary impedance voltage, $E' = I_1 Z_1$ =	$0.8 + 2j$	$0.8 + 2j$	$0.8 + 2j$
Secondary terminal voltage, $E = 100 (\cos\theta_1 + j \sin\theta_1)$ =	100	$50 + 86.6j$	$50 - 86.6j$
Thus, secondary counter-generated e.m.f., $E_1 = E + E'_1$ =	$100.8 + 2j$	$50.8 + 88.6j$	$50.8 - 84.6j$
Primary counter-generated e.m.f., $Ei = 20E_1$ =	$2016 + 40j$	$1016 + 1772j$	$1016 - 1692j$
Primary load current' $= I \frac{I_1}{20}$ =	10	10	10
Primary exciting current, at $e = 2000$ volts impressed, I_{00} =	$0.3 - 0.4j$	$0.3 - 0.4j$	$0.3 - 0.4j$
Thus, at primary counter-generated e.m.f. Ei the exciting current is $\frac{E_i I_{00}}{2000}$ =	$\frac{(0.3 - 0.4j)(2016 + 40j)}{2000}$	$\frac{(0.3 - 0.4j)(1016 + 1772j)}{2000}$	$\frac{(0.3 - 0.4j)(1016 - 1692j)}{2000}$
Hence, expanded, I_{00} =	$0.310 - 397j$	$0.507 + 0.063j$	$-0.186 - 0.407j$
Total primary current, $I_0 = I_{00} + I'$	$10.31 - 0.397j$	$10.507 + 0.063j$	$9.814 - 407j$
Primary impedance voltage, $E'_0 = I_0 Z_0$ =	$(2 + 5j)\ 10.31 - 0.397j)$	$(2 + 5j)(10.507 + 0.063j)$	$(2 + 5j)(9.814 - 0.407j)$
Hence, expanded, E'_0 =	$22.6 + 50.76j$	$20.7 + 52.66j$	$21.66 + 48.26j$
Thus, primary impressed e.m.f., $E_0 = Ei + E'_0$ =	$2038.6 + 90.8j$	$1036.7 + 1824.7j$	$1037.7 - 1643.7j$
Hence, primary e.m.f. phase, $\tan \theta^1$ =	$-\frac{90.8}{2038.6}$	$-\frac{1824.7}{1036.7}$	$+\frac{1643.7}{1037.7}$
θ' =	$-2.6°$	$-60.4°$	$+57.7°$
Primary current phase, $\tan \theta''$ =	$+\frac{0.397}{10.31}$	$-\frac{0.063}{10.507}$	$+\frac{0.407}{9.814}$
θ'' =	$+2.2°$	$-0.4°$	$+2.4°$
Primary lag, $\theta_0 = \theta'' - \theta'$ =	$+4.8°$	$+60.0°$	$-55.3°$
And, reduced, primary impressed e.m.f., E_0 =	$\sqrt{2038.6^2 + 90.8^2} = 2041$	$\sqrt{1036.7^2 + 1824.7^2} = 2099$	$\sqrt{1037.7^2 + 1643.7^2} = 1943$
Primary current, I_0	$\sqrt{10.31^2 + 0.397^2} = 10.32$	$\sqrt{10.507^2 + 0.063^2} = 10.51$	$\sqrt{9.814^2 + 0.407^2} = 9.82$

	Non-inductive	60° Lag	60° Lead
We then have	$Z = r = 0.5$	$Z = 0.3 + 0.4j$	$Z = 0.3 - 0.4j$
Secondary current $I_1 = \frac{e}{Z}$	$= \frac{e}{0.5}$	$\frac{e}{0.3 + 0.4j}$	$\frac{e}{0.3 - 0.4j}$
Expanded by the associate term of the denominator, and substitute, $j^2 = -1$, $I_1 =$	$2e$ $= 2e$	$\frac{e(0.3 - 0.4j)}{(0.3 + 0.4j)(0.3 - 0.4j)}$ $= 4e(0.3 - 0.4j)$	$\frac{e(0.3 + 0.4j)}{(0.3 - 0.4j)(0.3 + 0.4j)}$ $= 4e(0.3 + 0.4j)$
Secondary impedance voltage, $E'_1 = I_1 Z_1 =$	$2e(0.004 + 0.01j)$ $= e(0.008 + 0.02j)$	$4e(0.3 - 0.4j)(0.004 + 0.01j)$ $= e(0.0208 + 0.0056j)$	$4e(3 - 4j)(0.004 + 0.01j)$ $= e(-0.0112 + 0.0184j)$
Secondary terminal voltage, $E = e - E'_1 =$	$e(0.992 - 0.02j)$	$e(0.9792 - 0.0056j)$	$e(1.0112 - 0.0184j)$
Or, reduced, $E =$	$e\sqrt{0.992^2 + 0.02^2}$ $= 0.992e$	$e\sqrt{0.9792^2 + 0.0056^2}$ $= 0.9792e$	$e\sqrt{1.011^2 + 0.0184^2}$ $= 1.0114e$
Primary counter-generated e.m.f., $\dot{E}_i =$	$20e$	$20e$	$20e$
Primary load current, $I' = \frac{1}{20} I_1 =$	$0.1e$	$0.2e(0.3 - 0.4j)$	$0.2e(0.3 + 0.4j)$
Primary exciting current, $I_{00} = EY =$	$e(3 - 4j)\,10^{-3}$	$e(3 - 4j)\,10^{-3}$	$e(0.3 - 0.4j)\,10^{2}$
Thus, total primary current $I_0 = I' + I_{00} =$	$e(0.103 - 0.004j)$	$e(0.063 - 0.084j)$	$e(0.063 + 0.076j)$
Primary impedance voltage, $E'_0 = Z_0 I_0 =$	$e(0.103 - 0.004j)(2 + 5j)$	$e(0.063 - 0.084j)\ 2 - 5j)$	$e(0.063 + 0.076j)(2 + 5j)$
Expanded =	$e(0.226 + 505j)$	$e(0.546 + 0.147j)$	$e(-0.254 + 0.467j)$
Thus, primary impressed e.m.f., $E_0 = E_i + E'_0 =$	$e(20.226 + 0.505j)$	$e(20.546 + 0.147j)$	$e(19.746 + 0.467j)$
Or, reduced, $e_0 =$	$e\sqrt{20.226^2 + 0.505^2}$ $= 20.23e$	$e\sqrt{20.564^2 + 0.147^2}$ $= 20.55e$	$e\sqrt{19.746^2 + 0.467^2}$ $= 19.75e$
Or, $e =$	$\frac{e_0}{20.23}$	$\frac{e_0}{20.55}$	$\frac{e_0}{19.75}$
Since $e_0 = 2000$, $e =$	98.85	97.32	101.25
Substituting e gives			
Secondary current, $I_1 =$	197.7	$116.8 - 155.6j$	$121.8 + 162j$
Reduced, $I_1 =$	197.7	194.6	202.5
Secondary terminal voltage, $E_1 =$	$98.1 - 2j$	$95.3 - 0.54j$	$102.4 - 1.86j$
Reduced, $E_1 =$	98.1	95.3	102.4
Primary current, $I_0 =$	$10.18 - 0.004j$	$6.13 - 8.17j$	$6.38 + 7.70j$
Reduced, $I_0 =$	10.18	10.22	10.00

The exciting current is $I'_{00} = 0.3 - 0.4\,j$ amp. at $e = 2000$ volts impressed, or rather, primary counter-generated e.m.f.

The primary impedance, $Z_0 = 2 + 5\,j$ ohms.

The secondary impedance, $Z_1 = 0.004 + 0.01\,j$ ohm.

We have, in symbolic expression, choosing the secondary current I_1 as real axis, the results calculated in tabulated form on page 82.

68. (2) $e_0 = 2000$ volts are impressed upon the primary circuit of a transformer of ratio of turns 20:1. The primary impedance is $Z_0 = 2 + 5\,j$, the secondary impedance, $Z_1 = 0.004 + 0.01\,j$, and the exciting current at $e' = 2000$ volts counter-generated e.m.f. is $I_{00} = 0.3 - 0.4\,j$; thus the exciting admittance, $Y = \frac{I'_{00}}{e'} = (0.15 - 0.2\,j)10^{-3}$.

What is the secondary current and secondary terminal voltage and the primary current if the total impedance of the secondary circuit (internal impedance plus external load) consists of

(*a*) resistance,
$Z = r = 0.5$ – non-inductive circuit.

(*b*) impedance,
$Z = r + jx = 0.3 + 0.4\,j$ – inductive circuit.

(*c*) impedance,
$Z = r + jx = 0.3 - 0.4\,j$ – anti-inductive circuit.

Let e = secondary e.m.f.,

assumed as real axis in symbolic expression, and carrying out the calculation in tabulated form, on page 83.

69. (3) A transmission line of impedance $Z = r + jx = 20 + 50\,j$ ohms feeds a receiving circuit. At the receiving end an apparatus is connected which produces reactive lagging or leading currents at will (synchronous machine); 12,000 volts are impressed upon the line. How much lagging and leading currents respectively must be produced at the receiving end of the line to get 10,000 volts (*a*) at no load, (*b*) at 50 amp. power current as load, (*c*) at 100 amp. power current as load?

Let $e = 10{,}000$ = e.m.f. received at end of line, i_1 = power current, and i_2 = reactive lagging current; then

$$I = i_1 - ji_2 = \text{total line current.}$$

The voltage at the generator end of the line is then

$$\begin{aligned} E_0 &= e + ZI \\ &= e + (r + jx)(i_1 - ji_2) \\ &= (e + ri_1 + xi_2) - j\,(ri_2 - xi_1) \\ &= (10{,}000 + 20\,i_1 + 50\,i_2) - j\,(20\,i_2 - 50\,i_1); \end{aligned}$$

or, reduced,

$$E_0 = \sqrt{(e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2};$$

thus, since $E_0 = 12{,}000$,

$$12{,}000 = \sqrt{(10{,}000 + 20\,i_1 + 50\,i_2)^2 + (20\,i_2 - 50\,i_1)^2}.$$

(*a*) At no load $i_1 = 0$, and

$$12{,}000 = \sqrt{(10{,}000 + 50\,i_2)^2 + 400\,i_2^2};$$

hence,

$$i_2 = +\,39.5 \text{ amp., reactive lagging current, } I = -\,39.5\,j.$$

(*b*) At half load $i_1 = 50$, and

$$12{,}000 = \sqrt{(11{,}000 + 50\,i_2)^2 + (20\,i_2 - 2500)^2};$$

hence,

$$i_2 = +\,16 \text{ amp., lagging current, } I = 50 - 16\,j.$$

(*c*) At full load $i_1 = 100$, and

$$12{,}000 = \sqrt{(12{,}000 + 50\,i_2)^2 + (20\,i - 5000)^2};$$

hence,

$$i_2 = -\,27.13 \text{ amp., leading current, } I = 100 + 27.13\,j.$$

15. LOAD CHARACTERISTIC OF TRANSMISSION LINE

70. The load characteristic of a transmission line is the curve of volts and watts at the receiving end of the line as function of the amperes, and at constant e.m.f. impressed upon the generator end of the line.

Let r = resistance, x = reactance of the line. Its impedance $z = \sqrt{r^2 + x^2}$ can be denoted symbolically by

$$Z = r + jx.$$

Let E_0 = e.m.f. impressed upon the line.

Choosing the e.m.f. at the end of the line as horizontal component in the vector diagram, it can be denoted by $E = e$.

At non-inductive load the line current is in phase with the e.m.f. e, thus denoted by $\dot{I} = i$.

The e.m.f. consumed by the line impedance $Z = r + jx$ is

$$\dot{E}_1 = Z\dot{I} = (r + jx)\, i$$
$$= ri + jx\, i. \quad (1)$$

Thus the impressed voltage,

$$\dot{E}_0 = \dot{E} + \dot{E}_1 = e + ri + jxi. \quad (2)$$

or, reduced,

$$E_0 = \sqrt{(e + ri)^2 + x^2i^2}, \quad (3)$$

and

$$e = \sqrt{E_0^2 - x^2i^2} - ri, \text{ the e.m.f.} \quad (4)$$

$$P = ei = i\sqrt{E_0^2 - x^2i^2} - ri^2, \quad (5)$$

the power received at end of the line.

The curve of e.m.f. e is an arc of an ellipse.

With open circuit $i = 0$, $e = E_0$ and $P = 0$, as is to be expected.

At short circuit, $e = 0$, $0 = \sqrt{E_0^2 - x^2i^2} - ri$, and

$$i = \frac{E_0}{\sqrt{r^2 + x^2}} = \frac{E_0}{z}; \quad (6)$$

that is, the maximum line current which can be established with a non-inductive receiver circuit and negligible line capacity.

71. The condition of maximum power delivered over the line is

$$\frac{dP}{di} = 0; \quad (7)$$

that is,

$$\sqrt{E_0^2 - x^2i^2} + \frac{\frac{1}{2}i(-2x^2i)}{\sqrt{E_0^2 - x^2i^2}} - 2ri = 0;$$

substituting (3):

$$\sqrt{E_0^2 - x^2i^2} = e + ri,$$

and expanding, gives

$$e^2 = (r^2 + x^2)\, i^2 \quad (8)$$
$$= z^2i^2;$$

hence,

$$e = zi, \text{ and } \frac{e}{i} = z. \quad (9)$$

$\frac{e}{i} = r_1$ is the resistance or effective resistance of the receiving circuit; that is, the maximum power is delivered into a non-inductive receiving circuit over an inductive line upon which is impressed a constant e.m.f., if the resistance of the receiving circuit equals the impedance of the line, $r_1 = z$.

In this case the total impedance of the system is

$$Z_0 = Z + r_1 = r + z + jx, \tag{10}$$

or,

$$z_0 = \sqrt{(r + z)^2 + x^2}. \tag{11}$$

Thus the current is

$$\dot{i}_1 = \frac{E_0}{z_0} = \frac{E_0}{\sqrt{(r + z)^2 + x^2}}, \tag{12}$$

and the power transmitted is

$$\begin{aligned} P_1 = i_1^2 r_1 &= \frac{E_0^2 z}{(r + z)^2 + x^2} \\ &= \frac{E_0^2}{2\,(r + z)}; \end{aligned} \tag{13}$$

that is, the maximum power which can be transmitted over a line of resistance r and reactance x is the square of the impressed e.m.f. divided by twice the sum of resistance and impedance of the line.

At $x = 0$, this gives the common formula,

$$P_1 = \frac{E_0^2}{4\,r}. \tag{14}$$

Inductive Load

72. With an inductive receiving circuit of lag angle θ, or power-factor $p = \cos\,\theta$, and inductance factor $q = \sin\,\theta$, at e.m.f. $\dot{E} = e$ at receiving circuit, the current is denoted by

$$\dot{I} = I\,(p - jq); \tag{15}$$

thus the e.m.f. consumed by the line impedance $Z = r + jx$ is

$$\begin{aligned} \dot{E}_1 = Z\dot{I} &= I\,(p - jq)(r + jx) \\ &= I\,[(rp + xq) - j\,(rq - xp)], \end{aligned}$$

and the generator voltage is

$$\begin{aligned} \dot{E}_0 &= \dot{E} + \dot{E}_1 \\ &= [e + I\,(rp + xq)] - jI\,(rq - xp); \end{aligned} \tag{16}$$

or, reduced,

$$E_0 = \sqrt{[e + I(rp + xq)]^2 + I^2(rq - xp)^2}, \tag{17}$$

and

$$e = \sqrt{E_0^2 - I^2(rq - xp)^2} - I(rp + xq). \tag{18}$$

The power received is the e.m.f. times the power component of the current; thus

$$\begin{aligned} P &= eIp \\ &= Ip\sqrt{E_0^2 - I^2(rq - xp)^2} - I^2p(rp + xq). \end{aligned} \tag{19}$$

The curve of e.m.f., e, as function of the current I is again an arc of an ellipse.

At short circuit $e = 0$; thus, substituted,

$$I = \frac{E_0}{z}, \tag{20}$$

the same value as with non-inductive load, as is obvious.

73. The condition of maximum output delivered over the line is

$$\frac{dP}{dI} = 0; \tag{21}$$

that is, differentiated,

$$\sqrt{E_0^2 - I^2(rq - xp)^2} = e + I(rp + xq); \tag{22}$$

substituting and expanding,

$$\begin{aligned} e^2 &= I^2(r^2 + x^2) \\ &= I^2z^2; \\ e &= Iz; \end{aligned}$$

or

$$\frac{e}{I} = z. \tag{23}$$

$z_1 = \frac{e}{I}$ is the impedance of the receiving circuit; that is, the power received in an inductive circuit over an inductive line is a maximum if the impedance of the receiving circuit, z_1, equals the impedance of the line, z.

In this case the impedance of the receiving circuit is

$$Z_1 = z(p + jq), \tag{24}$$

and the total impendance of the system is

$$\begin{aligned} Z_0 &= Z + Z_1 \\ &= r + jx + z(p + jq) \\ &= (r + pz) + j(x + qz). \end{aligned}$$

Thus, the current is

$$I_1 = \frac{E_0}{\sqrt{(r + pz)^2 + (x + qz)^2}}, \tag{25}$$

and the power is

$$P_1 = I_1^2 zp = \frac{E_0^2 zp}{(r + pz)^2 + (x + qz)^2}$$
$$= \frac{E_0^2 p}{2\,(z + rp + xq)}. \tag{26}$$

EXAMPLES

74. (1) 12,000 volts are impressed upon a transmission line of impedance $Z = r + jx = 20 + 50\,j$. How do the voltage

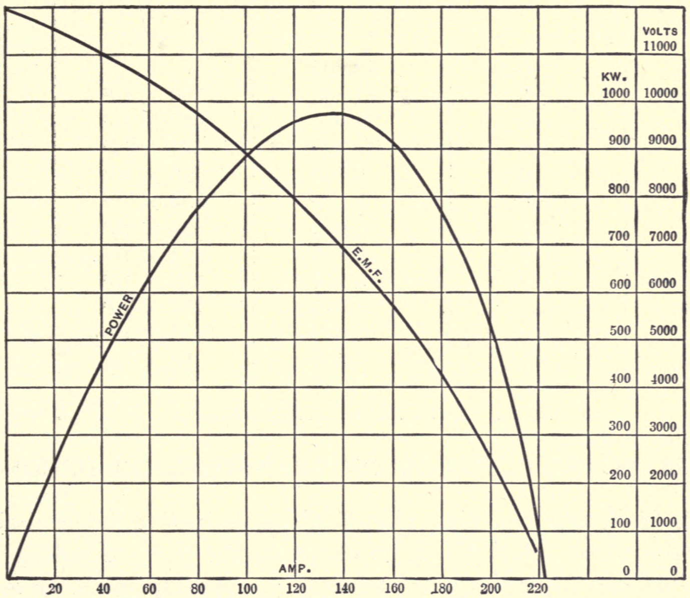


FIG. 39.—Non-reactive load characteristids of a transmission line. Constant impressed e.m.f.

and the output in the receiving circuit vary with the current with non-inductive load?

Let e = voltage at the receiving end of the line, i = current: thus = ei = power received. The voltage impressed upon the line is then

$$E_0 = e + Zi$$
$$= e + ri + jxi;$$

or, reduced,

$$E_0 = \sqrt{(e + ri)^2 + x^2 i^2}.$$

Since $E_0 = 12{,}000$,

$$12{,}000 = \sqrt{(e + ri)^2 + x^2 i^2} = \sqrt{(e + 20\,i)^2 + 2500\,i^2},$$

$$e = \sqrt{12{,}000^2 - x^2 i^2} - ri = \sqrt{12{,}000^2 - 2500\,i^2} - 20\,i.$$

The maximum current for $e = 0$ is

$$0 = \sqrt{12{,}000^2 - 2500\,i^2} - 20\,i;$$

thus,

$$i = 223.$$

Substituting for i gives the values plotted in Fig. 39.

i	e	$p = ei$
0	12,000	0
20	11,500	230×10^3
40	11,000	440×10^3
60	10,400	624×10^3
80	9,700	776×10^3
100	8,900	890×10^3
120	8,000	960×10^3
140	6,940	971×10^3
160	5,750	920×10^3
180	4,340	784×10^3
200	2,630	526×10^3
220	400	88×10^3
223	0	0

16. PHASE CONTROL OF TRANSMISSION LINES

75. If in the receiving circuit of an inductive transmission line the phase relation can be changed, the drop of voltage in the line can be maintained constant at varying loads or even decreased with increasing load; that is, at constant generator voltage the transmission can be compounded for constant voltage at the receiving end, or even over-compounded for a voltage increasing with the load.

1. Compounding of Transmission Lines for Constant Voltage

Let r = resistance, x = reactance of the transmission line, e_0 = voltage impressed upon the beginning of the line, e = voltage received at the end of end line.

Let i = power current in the receiving circuit; that is, $P = ei$ = transmitted power, and i_1 = reactive current produced in the system for controlling the voltage. i_1 shall be considered positive as lagging, negative as leading current.

Then the total current, in symbolic representation, is

$$I = i - ji_1;$$

the line impedance is

$$Z = r + jx,$$

and thus the e.m.f. consumed by the line impedance is

$$\dot{E}_1 = \dot{Z}I = (r + jx)(i - ji_1)$$
$$= ri + jri_1 + jxi - j^2xi_1;$$

and substituting $j^2 = -1$,

$$\dot{E}_1 = (ri + xi_1) - j(ri_1 - xi).$$

Hence the voltage impressed upon the line

$$\dot{E}_0 = e + \dot{E}_1$$
$$= (e + ri + xi_1) - j(ri_1 - xi); \quad (1)$$

or, reduced,

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}. \quad (2)$$

If in this equation e and e_0 are constant, i_1, the reactive component of the current, is given as a function of the power component current i and thus of the load ei.

Hence either e_0 and e can be chosen, or one of the e.m.fs. e_0 or e and the reactive current i_1 corresponding to a given power current i.

76. If $i_1 = 0$ with $i = 0$, and e is assumed as given, $e_0 = e$. Thus,

$$e = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2};$$
$$2e(ri + xi_1) + (r^2 + x^2)(i^2 + i_1^2) = 0.$$

From this equation it follows that

$$i_1 = -\frac{ex \pm \sqrt{e^2x^2 - 2eriz^2 - i^2z^4}}{z^2}. \quad (3)$$

Thus, the reactive current i_1 must be varied by this equation to maintain constant voltage $e = e_0$ irrespective of the load ei.

As seen, in this equation, i_1 must always be negative, that is, the current leading.

i_1 becomes impossible if the term under the square root becomes negative, that is, at the value

$$e^2x^2 - 2\,eriz^2 - i^2z^4 = 0;$$

or,

$$i = \frac{e\,(z - r)}{z^2}. \tag{4}$$

At this point the power transmitted is

$$P = ei = \frac{e^2\,(z - r)}{z^2}. \tag{5}$$

This is the maximum power which can be transmitted without drop of voltage in the line, with a power current $i = \frac{e\,(z - r)}{z^2}$.

The reactive current corresponding hereto, since the square root becomes zero, is

$$i_1 = \frac{ex}{z^2}; \tag{6}$$

thus the ratio of reactive to power current, or the tangent of the phase angle of the receiving circuit, is

$$\tan \theta_1 = \frac{i_1}{i} = - \frac{x}{z - r}. \tag{7}$$

A larger amount of power is transmitted if e_0 is chosen $> e$, a smaller amount of power if $e_0 < e$.

In the latter case i_1 is always leading; in the former case i_1 is lagging at no load, becomes zero at some intermediate load, and leading at higher load.

77. If the line impedance $Z = r + jx$ and the received voltage e is given, and the power current i_0 at which the reactive current shall be zero, the voltage at the generator end of the line is determined hereby from the equation (2):

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2},$$

by substituting $i_1 = 0$, $i = i_0$,

$$e_0 = \sqrt{(e + ri_0)^2 + x^2i_0^2}. \tag{8}$$

Substituting this value in the general equation (2):

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}$$

gives

$$(e + ri_0)^2 + x^2i_0^2 = (e + ri + xi_1)^2 + (ri_1 - xi)^2 \tag{9}$$

as equation between i and i_1.

If at constant generator voltage e_0:
at no load,

$$\left.\begin{array}{l} i = 0, e = e_0, i_1 = i'_0, \\ \text{and at the load,} \\ i = i_0, e = e_0, i_1 = 0 \end{array}\right\} \quad (10)$$

it is, substituted:
no load,

$$e_0 = \sqrt{(e_0 + xi_0)^2 + r^2 i'^2_0}, \quad (11)$$

load i_0,

$$e_0 = \sqrt{(e_0 + ri_0)^2 + x^2 i_0^2}. \quad (12)$$

Thus,

$$(e_0 + xi'_0) + x^2 i'^2_0 = (e_0 + ri_0)^2 + x^2 i_0^2;$$

or, expanded,

$$i'^2_0(r^2 + x^2) + 2\, i'_0\, xe_0 = i_0^2\,(r^2 + x^2) + 2\, i_0 re_0. \quad (13)$$

This equation gives i'_0 as function of i_0, e_0, r, x.

If now the reactive current i_1 varies as linear function of the power current i, as in case of compounding by rotary converter with shunt and series field, it is

$$i_1 = \frac{(i_0 - i)}{i_0}\, i'_0. \quad (14)$$

Substituting this value in the general equation

$$(e_0 + ri_0)^2 + x^2 i_0^2 = (e + ri + xi_1)^2 + (ri_1 - xi)^2$$

gives e as function of i; that is, gives the voltage at the receiving end as function of the load, at constant voltage e_0 at the generating end, and $e = e_0$ for no load,

$$i = 0, i_1 = i'_0,$$

and $e = e_0$ for the load,

$$i = i_0, i_1 = 0.$$

Between $i = 0$ and $i = i_0$, $e > e_0$, and the current is lagging.

Above $i = i_0$, $e < e_0$, and the current is leading.

By the reaction of the variation of e from e_0 upon the receiving apparatus producing reactive current i_1, and by magnetic saturation in the receiving apparatus, the deviation of e from e_0 is reduced, that is, the regulation improved.

2. Over-compounding of Transmission Lines

78. The impressed voltage at the generator end of the line was found in the preceding,

$$e_0 = \sqrt{(e + ri + xi_1)^2 + (ri_1 - xi)^2}. \quad (2)$$

If the voltage at the end of the line e shall rise proportionally to the power current i, then

$$e = e_1 + ai; \tag{15}$$

thus,

$$e_0 = \sqrt{[e_1 + (a + r)\, i + xi_1]^2 + (ri_1 - xi)^2}, \tag{16}$$

and herefrom in the same way as in the preceding we get the characteristic curve of the transmission.

If $e_0 = e_1$, $i_1 = 0$ at no load, and is leading at load. If $e_0 < e_1$, i_1 is always leading, the maximum output is less than before.

If $e_0 > e_1$, i_1 is lagging at no load, becomes zero at some intermediate load, and leading at higher load. The maximum output is greater than at $e_0 = e_1$.

The greater a, the less is the maximum output at the same e_0 and e_1.

The greater e_0, the greater is the maximum output at the same e_1 and a, but the greater at the same time the lagging current (or less the leading current) at no load.

EXAMPLES

79. (1) A constant voltage of e_0 is impressed upon a transmission line of impedance $Z = r + jx = 10 + 20\, j$. The voltage at the receiving end shall be 10,000 at no load as well as at full load of 75 amp. power current. The reactive current in the receiving circuit is raised proportionally to the load, so as to be lagging at no load, zero at full load or 75 amp., and leading beyond this. What voltage e_0 has to be impressed upon the line, and what is the voltage e at the receiving end at ⅓, ⅔, and 1⅓ load?

Let $\dot{I} = i_1 - ji_2$ = current, $\dot{E} = e$ voltage in receiving circuit. The generator voltage is then

$$\begin{aligned} \dot{E}_0 &= e + Z\dot{I} \\ &= e + (r + jx)\,(i_1 - ji_2) \\ &= (e + ri_1 + xi_2) - j\,(ri_2 - xi_1) \\ &= (e + 10\, i_1 + 20\, i_2) - j\,(10\, i_2 - 20\, i_1); \end{aligned}$$

or, reduced,

$$\begin{aligned} e_0^2 &= (e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2; \\ &= (e + 10\, i_1 + 20\, i_2)^2 + (10\, i_2 - 20\, i_1)^2. \end{aligned}$$

When

$$i_1 = 75,\ i_2 = 0,\ e = 10{,}000;$$

substituting these values,

$$e_0^2 = 10{,}750^2 + 1500^2 = 117.81 \times 10^6;$$

hence,

$$e_0 = 10{,}860 \text{ volts is the generator voltage.}$$

When

$$i_1 = 0,\ e = 10{,}000,\ e_0 = 10{,}860, \text{ let } i_2 = i;$$

these values substituted give

$$\begin{aligned} 117.81 \times 10^6 &= (10{,}000 + 20\, i)^2 + 100\, i^2 \\ &= 100 \times 10^6 + 400\, i \times 10^3 + 500\, i^2, \end{aligned}$$

or,

$$i = 44.525 - 1.25\, i^2\, 10^{-3};$$

this equation is best solved by approximation, and then gives

$$p = 42.3 \text{ amp. reactive lagging current at no load.}$$

Since

$$e_0^2 = (e + ri_1 + xi_2)^2 + (ri_2 - xi_1)^2,$$

it follows that

$$e = \sqrt{e_0^2 - (ri_2 - xi_1)^2} - (ri_1 + xi_2);$$

or,

$$e = \sqrt{117.81 + 10^6 - (10\, i_2 - 20\, i_1)^2} - (10\, i_1 + 20\, i_2).$$

Substituting herein the values of i_1 and i_2 gives e.

i_1	i_2	e
0	42.3	10,000
25	28.2	10,038
50	14.1	10,038
75	0	10,000
100	−14.1	9,922
125	−28.2	9,803

80. (2) A constant voltage e_0 is impressed upon a transmission line of impedance $Z = r + jx = 10 + 10\, j$. The voltage at the receiving end shall be 10,000 at no load as well as at full load of 100 amp. power current. At full load the total current shall be in phase with the e.m.f. at the receiving end, and at no load a lagging current of 50 amp. is permitted. How much additional reactance x_0 is to be inserted, what must be the generator voltage e_0, and what will be the voltage e at the receiving end at ½ load and at 1½ load, if the reactive current varies proportionally with the load?

Let x_0 = additional reactance inserted in circuit.

Let $\dot{I} = i_1 - ji_2$ = current.

Then

$$e_0^2 = (e + ri_1 + x_1i_2)^2 + (ri_2 - x_1i_1)^2 = (e + 10\,i_1 + x_1i_2)^2 + (10\,i_2 - x_1i_1)^2,$$

where

$x_1 = x + x_0$ = total reactance of circuit between e and e_0.

At no load,

$$i_1 = 0,\ i_2 = 50,\ e = 10{,}000;$$

thus, substituting,

$$e_0^2 = (10{,}000 + 50\,x_1)^2 + 250{,}000.$$

At full load,

$$i_1 = 100,\ i_2 = 0,\ e = 10{,}000;$$

thus, substituting,

$$e_0^2 = 121 \times 10^6 + 10{,}000\,x_1^2.$$

Combining these gives

$$(10{,}000 + 50\,x_1)^2 + 250{,}000 = 121 \times 10^6 + 10{,}000\,x_1^2;$$

hence,

$$x_1 = 66.5 \pm 40.8$$
$$= 107.3 \text{ or } 25.7;$$

thus

$x_0 = x_1 - x = 97.3$ or 15.7 ohms additional reactance.

Substituting

$$x_1 = 25.7$$

gives

$$e_0^2 = (e + 10\,i_1 + 25.7\,i_2)^2 + (10\,i_2 - 25.7\,i_1)^2,$$

but at full load

$$i_1 = 100,\ i_2 = 0,\ e = 10{,}000,$$

which values substituted give

$$e_0^2 = 121 \times 10^6 + 6.605 \times 10^6 = 127.605 \times 10^6,$$

$e_0 = 11{,}300$, generator voltage.

Since

$$e = \sqrt{e_0^2 - (10\,i_2 - 25.7\,i_1)^2} - (10\,i_1 + 25.7\,i_2),$$

it follows that

$$e = \sqrt{127.605 \times 10^6 - (10\,i_2 - 25.7\,i_1)^2} - (10\,i_1 + 25.7\,i_2).$$

Substituting for i_1 and i_2 gives e.

i_1	i_2	e
0	50	10,000
50	25	10,105
100	0	10,000
150	−25	9,658

81. (3) In a circuit whose voltage e_0 fluctuates by 20 per cent. between 1800 and 2200 volts, a synchronous motor of internal impedance $Z_0 = r_0 + jx_0 = 0.5 + 5j$ is connected through a reactive coil of impedance $Z_1 = r_1 + jx_1 = 0.5 + 10j$ and run light, as compensator (that is, generator of reactive currents). How will the voltage at the synchronous motor terminals e_1, at constant excitation, that is, constant counter e.m.f. $e = 2000$, vary as function of e_0 at no load and at a load of $i = 100$ amp. power current, and what will be the reactive current in the synchronous motor?

Let $\dot{I} = i_1 - ji_2$ = current in receiving circuit of voltage e_1. Of this current $\dot{I}, -ji_2$ is taken by the synchronous motor of counter e.m.f. e, and thus

$$\dot{E}_1 = e - Z_0 j i_2$$
$$= e + x_0 i_2 - j r_0 i_2;$$

or, reduced,

$$e_1^2 = (e + x_0 i_2)^2 + r_0^2 i_2^2.$$

In the supply circuit the voltage is

$$\dot{E}_0 = \dot{E}_1 + IZ_1$$
$$= e + x_0 i_2 - j r_0 i_2 + (i_1 - j i_2)(r_1 + j x_1)$$
$$= [e + r_1 i_1 + (x_0 + x_1) i_2] - j[(r_0 + r_1) i_2 - x_1 i_1];$$

or, reduced,

$$e_0^2 = [e + r_1 i_1 + (x_0 + x_1) i_2]^2 + [(r_0 + r_1) i_2 - x_1 i_1]^2.$$

Substituting in the equations for e_1^2 and e_0^2 the above values of r_0 and x_0: at no load, $i_1 = 0$, we have

$$e_1^2 = (e + 5\, i_2)^2 + 0.25\, i_2^2 \text{ and } e_0^2 = (e + 15\, i_2)^2 + i_2^2;$$

at full load, $i_1 = 100$, we have

$$e_1^2 = (e + 5\, i_2)^2 + 0.25\, i_2^2,$$
$$e_0^2 = (e + 50 + 15\, i_2)^2 + (i_2 - 1000)^2,$$

and at no load, $i_1 = 0$, substituting $e = 2000$, we have

$$e_1^2 = (2000 + 5\,i_2)^2 + 0.25\,i_2^2,$$
$$e_0^2 = (2000 + 15\,i_2)^2 + i_2^2;$$

at full load, $i_1 = 100$, we have

$$e_1^2 = (2000 + 5\,i_2)^2 + 0.25\,i_2^2,$$
$$e_0^2 = (2050 + 15\,i_2)^2 + (i_2 - 1000)^2.$$

Substituting herein e_0 = successively 1800, 1900, 2000, 2100, 2200, gives values of i_2, which, substituted in the equation for e_1^2, give the corresponding values of e_1 as recorded in the following table.

As seen, in the local circuit controlled by the synchronous compensator, and separated by reactance from the main circuit of fluctuating voltage, the fluctuations of voltage appear in a greatly reduced magnitude only, and could be entirely eliminated by varying the excitation of the synchronous compensator.

$e = 2000$				
e_0	No load $i_1 = 0$		Full load $i_1 = 100$	
	i_2	e_1	i_2	e_1
1,800	−13.3	1,937	−39	1,810
1,900	− 6.7	1,965	−30.1	1,850
2,000	0	2,000	−22	1,885
2,100	+ 6.7	2,035	−13.5	1,935
2,200	+13.3	2,074	− 6.5	1,970

17. IMPEDANCE AND ADMITTANCE

82. In direct-current circuits the most important law is Ohm's law,

$$i = \frac{e}{r}, \quad \text{or } e = ir, \quad \text{or } r = \frac{e}{i},$$

where e is the e.m.f. impressed upon resistance r to produce current i therein.

Since in alternating-current circuits a current i through a resistance r may produce additional e.m.fs. therein, when applying Ohm's law, $i = \frac{e}{r}$ to alternating-current circuits, e is the total e.m.f. resulting from the impressed e.m.f. and all e.m.fs. produced by the current i in the circuit.

Such counter e.m.fs. may be due to inductance, as self-inductance, or mutual inductance, to capacity, chemical polarization, etc.

The counter e.m.f. of self-induction, or e.m.f. generated by the magnetic field produced by the alternating current i, is represented by a quantity of the same dimensions as resistance, and measured in ohms: reactance x. The e.m.f. consumed by reactance x is in quadrature with the current, that consumed by resistance r in phase with the current.

Reactance and resistance combined give the impedance,

$$z = \sqrt{r^2 + x^2};$$

or, in symbolic or vector representation,

$$Z = r + jx.$$

In general in an alternating-current circuit of current i, the e.m.f. e can be resolved in two components, a power component e_1 in phase with the current, and a wattless or reactive component e_2 in quadrature with the current.

The quantity

$$\frac{e_1}{i} = \frac{\text{power e.m.f., or e.m.f. in phase with the current}}{\text{current}} = r_1$$

is called the *effective resistance*.

The quantity

$$\frac{e_2}{i} = \frac{\text{reactive e.m.f., or e.m.f. in quadrature with the current}}{\text{current}} = x_1$$

is called the *effective reactance* of the circuit.

And the quantity

$$z_1 = \sqrt{r_1^2 + x^2}$$

or, in symbolic representation,

$$Z_1 = r_1 + jx_1$$

is the impedance of the circuit.

If power is consumed in the circuit only by the ohmic resistance r, and counter e.m.f. produced only by self-inductance, the effective resistance r_1 is the true or ohmic resistance r, and the effective reactance x_1 is the true or inductive reactance x.

By means of the terms effective resistance, effective reactance, and impedance, Ohm's law can be expressed in alternating-current circuits in the form

$$i = \frac{e}{z_1} = \frac{e}{\sqrt{r_1^2 + x_1^2}}; \tag{1}$$

or,
$$e = iz_1 = i\sqrt{r_1^2 + x_1^2}; \tag{2}$$

or,
$$z_1 = \sqrt{r_1^2 + x_1^2} = \frac{e}{i}; \tag{3}$$

or, in symbolic or vector representation,

$$\dot{I} = \frac{\dot{E}}{Z_1} = \frac{\dot{E}}{r_1 + jx}; \tag{4}$$

or,
$$\dot{E} = \dot{I}Z_1 = \dot{I}(r_1 + jx_1); \tag{5}$$

or,
$$Z_1 = r_1 + jx_1 = \frac{\dot{E}}{\dot{I}}. \tag{6}$$

In this latter form Ohm's law expresses not only the intensity but also the phase relation of the quantities; thus

$$e_1 = ir_1 = \text{power component of e.m.f.,}$$
$$e_2 = ix_1 = \text{reactive component of e.m.f.}$$

83. Instead of the term impedance $z = \frac{e}{i}$ with its components, the resistance and reactance, its reciprocal can be introduced.

$$\frac{i}{e} = \frac{1}{z},$$

which is called the *admittance.*

The components of the admittance are called the *conductance* and the *susceptance.*

Resolving the current i into a power component i_1 in phase with the e.m.f. and a wattless component i_2 in quadrature with the e.m.f., the quantity

$$\frac{i_1}{e} = \frac{\text{power current, or current in phase with e.m.f.}}{\text{e.m.f.}} = g$$

is called the *conductance.*

The quantity

$$\frac{i_2}{e} = \frac{\text{reactive current, or current in quadrature with e.m.f.}}{\text{e.m.f.}} = b$$

is called the *susceptance* of the circuit.

The conductance represents the current in phase with the e.m.f., or power current, the susceptance the current in quadrature with the e.m.f., or reactive current.

Conductance g and susceptance b combined give the admittance

$$y = \sqrt{g^2 + b^2}; \tag{7}$$

or, in symbolic or vector representation,

$$Y = g - jb. \tag{8}$$

Thus Ohm's law can also be written in the form

$$i = ey = e\sqrt{g^2 + b^2}; \tag{9}$$

or,

$$e = \frac{i}{y} = \frac{i}{\sqrt{g^2 + b^2}}; \tag{10}$$

or,

$$y = \sqrt{g^2 + b^2} = \frac{i}{e}; \tag{11}$$

or, in symbolic or vector representation,

$$\dot{I} = \dot{E}Y = \dot{E}(g - jb); \tag{12}$$

or,

$$\dot{E} = \frac{\dot{I}}{Y} = \frac{\dot{I}}{g - jb}; \tag{13}$$

or,

$$Y = g - jb = \frac{\dot{I}}{\dot{E}}. \tag{14}$$

and $i_1 = eg$ = power component of current,
$i_2 = eb$ = reactive component of current.

84. According to circumstances, sometimes the use of the terms impedance, resistance, reactance, sometimes the use of the terms admittance, conductance, susceptance, is more convenient.

Since, in a number of series-connected circuits, the total e.m.f., in symbolic representation, is the sum of the individual e.m.fs., it follows that in a number of series-connected circuits the total impedance, in symbolic expression, is the sum of the impedances of the individual circuits connected in series.

Since, in a number of parallel-connected circuits, the total current, in symbolic representation, is the sum of the individual currents, it follows that in a number of parallel-connected circuits the total admittance, in symbolic expression, is the sum of the admittances of the individual circuits connected in parallel.

Thus in series connection the use of the term impedance, in parallel connection the use of the term admittance, is generally more convenient.

Since in symbolic representation

$$Y = \frac{1}{Z}; \tag{15}$$

or,

$$ZY = 1; \tag{16}$$

that is,

$$(r + jx)(g - jb) = 1; \tag{17}$$

it follows that

$$(rg + xb) - j\,(rb - xg) = 1;$$

that is

$$rg + zb = 1,$$
$$rb - xg = 0.$$

Thus,

$$r = \frac{g}{g^2 + b^2} = \frac{g}{y^2}, \tag{18}$$

$$x = \frac{b}{g^2 + b^2} = \frac{b}{y^2}, \tag{19}$$

$$g = \frac{r}{r^2 + x^2} = \frac{r}{z^2} \tag{20}$$

$$b = \frac{r}{r^2 + x^2} = \frac{x}{z^2}, \tag{21}$$

or, in absolute values,

$$y = \frac{1}{z}, \tag{22}$$

$$zy = 1, \tag{23}$$

$$(r^2 + x^2)(g^2 + b^2) = 1. \tag{24}$$

Thereby the admittance with its components, the conductance and susceptance, can be calculated from the impedance and its components, the resistance and reactance, and inversely.

If $x = 0$, $z = r$ and $g = \frac{1}{r}$, that is, g is the reciprocal of the resistance in a non-inductive circuit; not so, however, in an inductive circuit.

EXAMPLES

85. (1) In a quarter-phase induction motor having an impressed e.m.f. $e = 110$ volts per phase, the current is $I_0 = i_1 - ji_2 = 100 - 100\,j$ at standstill, the torque $= D_0$.

The two phases are connected in series in a single-phase circuit of e.m.f. $e = 220$, and one phase shunted by a condenser of 1 ohm capacity reactance.

What is the starting torque D of the motor under these conditions, compared with D_0, the torque on a quarter-phase circuit, and what the relative torque per volt-ampere input, if the torque is proportional to the product of the e.m.fs. impressed upon the two circuits and the sine of the angle of phase displacement between them?

In the quarter-phase motor the torque is

$$D_0 = ae^2 = 12{,}100\ a,$$

where a is a constant. The volt-ampere input is

$$Q_0 = 2\,e\sqrt{i_1^2 + i_2^2} = 31{,}200;$$

hence, the "apparent torque efficiency," or torque per volt-ampere input,

$$\eta_0 = \frac{D_0}{Q_0} = 0.388\ a.$$

The admittance per motor circuit is

$$Y = \frac{I}{e} = 0.91 - 0.91\,j,$$

the impedance is

$$Z = \frac{e}{I} = \frac{110}{100 - 100j} = \frac{110\,(100 + 100j)}{(100 - 100j)\,(100 + 100j)} = 0.55 + 0.55\,j.$$

the admittance of the condenser is

$$Y_0 = j;$$

thus, the joint admittance of the circuit shunted by the condenser is

$$\begin{aligned} Y_1 &= Y + Y_0 = 0.91 - 0.91\,j + j \\ &= 0.91 + 0.09\,j; \end{aligned}$$

its impedance is

$$Z_1 = \frac{1}{Y_1} = \frac{1}{0.91 + 0.09\,j} = \frac{0.91 - 0.09\,j}{0.91^2 + 0.09^2} = 1.09 - 0.11\,j,$$

and the total impedance of the two circuits in series is

$$\begin{aligned} Z_2 &= Z + Z_1 \\ &= 0.55 + 0.55\,j + 1.09 - 0.11\,j \\ &= 1.64 + 0.44\,j. \end{aligned}$$

Hence, the current, at impressed e.m.f. $e = 220$,

$$\begin{aligned} I = i_1 - ji_2 &= \frac{e}{Z_2} = \frac{220}{1.64 + 0.44\,j} = \frac{220\,(1.64 - 0.44\,j)}{1.64^2 + 0.44^2} \\ &= 125 - 33.5\,j; \end{aligned}$$

or, reduced,

$$\begin{aligned} I &= \sqrt{125^2 + 33.5^2} \\ &= 129.4 \text{ amp.} \end{aligned}$$

Thus, the volt-ampere input,

$$\begin{aligned} Q &= eI = 220 \times 129.4 \\ &= 28{,}470. \end{aligned}$$

The e.m.fs. acting upon the two motor circuits respectively are

$$\underset{\cdot}{E}_1 = \underset{\cdot}{I}Z_1 = (125 - 33.5\,j)\,(1.09 - 0.11\,j) = 132.8 - 50.4\,j$$

and

$$\underset{\cdot}{E}' = \underset{\cdot}{I}Z = (125 - 33.5\,j)(0.55 + 0.55\,j) = 87.2 + 50.4\,j.$$

Thus, the tangents of their phase angles are

$$\tan \theta_1 = + \frac{50.4}{132.8} = + 0.30; \text{ hence, } \theta_1 = + 21°;$$

$$\tan \theta' = - \frac{50.4}{87.2} = - 0.579; \text{ hence, } \theta' = - 30°;$$

and the phase difference,

$$\theta = \theta_1 - \theta' = 51°.$$

The absolute values of these e.m.fs. are

$$e_1 = \sqrt{132.8 + 50.4^2} = 141.5$$

and

$$e' = \sqrt{87.2^2 - 50.4^2} = 100.7;$$

thus, the torque is

$$\begin{aligned} D &= ae_1e' \sin \theta \\ &= 11{,}100\,a; \end{aligned}$$

and the apparent torque efficiency is

$$\eta_t = \frac{D}{Q}\,\frac{11{,}100\,a}{28{,}470} = 0.39\,a.$$

Hence, comparing this with the quarter-phase motor, the relative torque is

$$\frac{D}{D_0} = \frac{11{,}100\,a}{12{,}100\,a} = 0.92,$$

and the relative torque per volt-ampere, or relative apparent torque efficiency, is

$$\frac{\eta_t}{\eta_0} = \frac{0.39\,a}{0.388\,a} = 1.005.$$

86. (2) At constant field excitation, corresponding to a nominal generated e.m.f. $e_0 = 12{,}000$, a generator of synchronous impedance $Z_0 = r_0 + jx_0 = 0.6 + 60\,j$ feeds over a transmission line of impedance $Z_1 = r_1 + jx_1 = 12 + 18j$, and of capacity susceptance 0.003, a non-inductive receiving circuit. How will the voltage at the receiving end, e, and the voltage at the generator terminals, e_1, vary with the load if the line capacity is represented by a condenser shunted across the middle of the line?

Let $\dot{I} = i =$ current in receiving circuit, in phase with the e.m.f., $\dot{E} = e$.

The voltage in the middle of the line is

$$\dot{E}_2 = \dot{E} + \frac{Z_1}{2}\dot{I}$$
$$= e + 6\,i + 9\,ij.$$

The capacity susceptance of the line is, in symbolic expression, $Y = 0.003\,j$; thus the charging current is

$$\dot{I}_2 = \dot{E}_2 Y = 0.003\,j\,(e + 6\,i + 9\,ij)$$
$$= 0.027\,i + j\,(0.003\,e + 0.018\,i),$$

and the total current is

$$\dot{I}_1 = \dot{I} + \dot{I}_2 = 0.973\,i + j\,(0.003\,e + 0.018\,i).$$

Thus, the voltage at the generator end of the line is

$$\dot{E}_1 = \dot{E}_2 + \frac{Z_1}{2}\dot{I}_1$$
$$= e + 6\,i + 9\,ij + (6 + 9\,j)[0.973\,i + j\,(0.003\,e + 0.018\,i)]$$
$$= (0.973\,e + 11.68\,i) + j\,(17.87\,i + 0.018\,e),$$

and the nominal generated e.m.f. of the generator is

$$\dot{E}_0 = \dot{E}_1 + Z_0\dot{I}_1$$
$$= (0.973\,e + 11.68\,i) + j\,(17.87\,i + 0.018\,e) + (0.6 + 60\,j)[0.973\,i + j\,(0.003\,e + 0.018\,i)]$$
$$= (0.793\,e + 11.18\,i) + j\,(76.26\,i + 0.02\,e);$$

or, reduced, and $e_0 = 12{,}000$ substituted,

$$e_0^2 = 144 \times 10^6 = (0.793\,e + 11.18\,i)^2 + (76.26\,i + 0.02\,e)^2;$$

thus,

$$e^2 + 33\,ei + 9450\,i^2 = 229 \times 10^6,$$
$$e = -\,16.5\,i + \sqrt{229 \times 10^6 - 9178\,i^2},$$

and

$$e_1 = \sqrt{(0.973\,e + 11.68\,i)^2 + (17.87\,i + 0.018\,e)^2};$$

at

$$i = 0,\ e = 15{,}133,\ e_1 = 14{,}700;$$

at

$$e = 0,\ i = 155.6,\ e_1 = 3327.$$

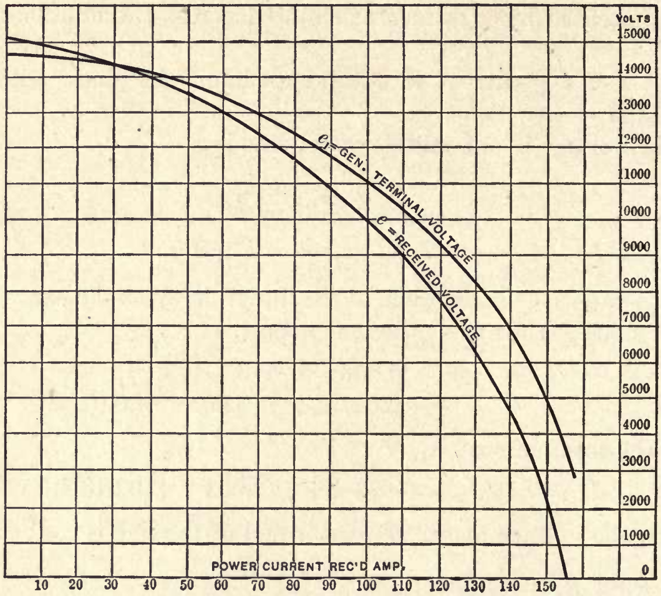


Fig. 40.—Reactive load characteristics of a transmission line fed by synchronous generator with constant field excitation.

Substituting different values for i gives

i	e	e_1	i	e	e_1
0	15,133	14,700	100	10,050	11,100
25	14,488	14,400	125	7,188	8,800
50	13,525	13,800	150	2,325	4,840
75	12,063	12,730	155.6	0	3,327

which values are plotted in Fig. 40.

18. EQUIVALENT SINE WAVES

87. In the preceding chapters, alternating waves have been assumed and considered as sine waves.

The general alternating wave is, however, never completely, frequently not even approximately, a sine wave.

A sine wave having the same effective value, that is, the same square root of mean squares of instantaneous values, as a general alternating wave, is called its corresponding "equivalent sine wave." It represents the same effect as the general wave.

With two alternating waves of different shapes, the phase relation or angle of lag is indefinite. Their equivalent sine waves, however, have a definite phase relation, that which gives the same effect as the general wave, that is, the same mean (ei).

Hence if e = e.m.f. and i = current of a general alternating wave, their equivalent sine waves are defined by

$$e_0 = \sqrt{\text{mean } (e^2)},$$
$$i_0 = \sqrt{\text{mean } (i^2)};$$

and the power is

$$p_0 = e_0 i_0 \cos e_0 i_0 = \text{mean } (ei);$$

thus,

$$\cos e_0 i_0 = \frac{\text{mean } (ei)}{\sqrt{\text{mean } (e^2)}\sqrt{\text{mean } (i^2)}}.$$

Since by definition the equivalent sine waves of the general alternating waves have the same effective value or intensity and the same power or effect, it follows that in regard to intensity and effect the general alternating waves can be represented by their equivalent sine waves.

Considering in the preceding the alternating currents as equivalent sine waves representing general alternating waves, the investigation becomes applicable to any alternating circuit irrespective of the wave shape.

The use of the terms reactance, impedance, etc., implies that a wave is a sine wave or represented by an equivalent sine wave.

Practically all measuring instruments of alternating waves (with exception of instantaneous methods) as ammeters, voltmeters, wattmeters, etc., give not general alternating waves but their corresponding equivalent sine waves.

EXAMPLES

88. In a 25-cycle alternating-current transformer, at 1000 volts primary impressed e.m.f., of a wave shape as shown in

TABLE I

(1) Degrees	(2) e	(3) e^2	(4) $e\frac{1000}{37.22} = e_0$	(5) Σe_0	(6) $\Sigma e_0 - 7324 = B'$	(7) $B' \frac{15,000}{7324} = B$	(8) From hysteresis cycle, f	(9) $50 f = F$	(10) $\frac{F}{500} = i$	(11) i^2	(12) $p = e_0$
0	0	0	0	0	−7,324	−15,000	−20	−1,000	−2.00	4.00	0
10	1	1	27	27	−7,296	−14,950	−19.5	− 975	−1.95	3.80	− 53
20	3.5	12	94	121	−7,203	−14,800	−18	− 900	−1.80	3.24	− 169
30	8	64	216	337	−6,987	−14,300	−14.3	− 715	−1.43	2.04	− 308
40	14	196	377	714	−6,610	−13,550	−10	− 500	−1.00	1.00	− 377
50	22	484	591	1,305	−6,019	−12,350	− 5.5	− 275	−0.55	0.30	− 325
60	31	961	835	2,140	−5,184	−10,600	− 2.3	− 115	−0.23	0.05	+ 191
70	41	1,681	1,100	3,240	−4,084	− 8,370	− 0.2	− 10	−0.02	0.00	− 22
80	50	2,500	1,345	4,585	−2,739	− 5,600	+ 1.0	+ 50	+0.10	0.01	+ 134
90	55	3,025	1,480	6,065	−1,259	− 2,580	+ 1.9	+ 95	+0.19	0.04	+ 281
100	57	3,249	1,535	7,600	+ 276	+ 570	+ 2.6	+ 130	+0.26	0.07	+ 398
110	58	3,364	1,560	9,160	+1,836	+ 3,550	+ 3.3	+ 165	+0.33	0.11	+ 514
120	58	3,364	1,560	10,720	+3,396	+ 6,970	+ 4.5	+ 225	+0.45	0.20	+ 700
130	56	3,136	1,508	12,228	+4,904	+10,050	+ 6.6	+ 330	+0.66	0.44	+ 995
140	43	1,849	1,155	13,383	+6,059	+12,400	+10.0	+ 500	+1.00	1.00	+1,155
150	29	841	780	14,163	+6,839	+14,000	+14.2	+ 710	+1.42	2.02	+1,108
160	14	196	377	14,540	+7,216	+14,800	+18.8	+ 940	+1.88	3.53	+ 710
170	4	16	108	14,648	+7,324	+15,000	+20.0	+1,000	+2.00	4.09	+ 216
180	0	0	0	14,648	+7,324	+15,000					
		Σ = 24,939								Σ = 25.85	Σ = 4,766

$$\text{mean } e^2 = \frac{24,939}{18} = 1385.5$$

$$e' = \text{eff. } e = \sqrt{1385.5} = 37.22.$$

$$\text{mean } i^2 = \frac{25.85}{18} = 1.436$$

$$i' = \text{eff. } i = \sqrt{1.436} = 1.198$$

$$p' = \text{mean } p = \frac{4766}{18} = 264.8$$

Fig. 41 and Table I, the number of primary turns is 500, the length of the magnetic circuit 50 cm., and its section shall be chosen so as to give a maximum density $B = 15{,}000$.

At this density the hysteretic cycle is as shown in Fig. 42 and Table II.

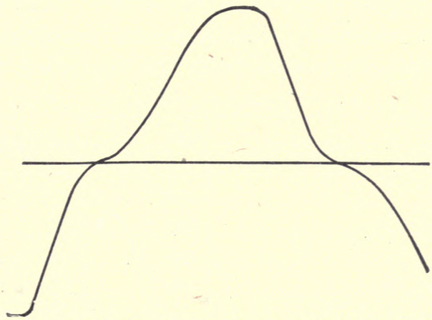


FIG. 41.—Wave-shape of e.m.f. in example 88.

What is the shape of current wave, and what the equivalent sine waves of e.m.f., magnetism, and current?

The calculation is carried out in attached table.

TABLE II

f	*B*
0	± 8,000
2	+ 10,400 − 2,500
4	+ 11,700 + 5,800
6	+ 12,400 + 9,300
8	+ 13,000 + 11,200
10	+ 13,500 + 12,400
12	+ 13,900 + 13,200
14	+ 14,200 + 13,800
16	+ 14,500 + 14,300
18	+ 14,800 + 14,700
20	+ 15,000

In column (1) are given the degrees, in column (2) the relative values of instantaneous e.m.fs., e corresponding thereto, as taken from Fig. 41.

Column (3) gives the squares of e. Their sum is 24,939; thus the mean square, $\frac{24{,}939}{18} = 1385.5$, and the effective value,

$$e' = \sqrt{1385.5} = 37.22.$$

Since the effective value of impressed e.m.f. is = 1000, the instantaneous values are $e_0 = e\frac{1000}{37.22}$ as given in column (4).

Since the e.m.f. e_0 is proportional to the rate of change of magnetic flux, that is, to the differential coefficient of B, B is proportional to the integral of the e.m.f., that is, to Σe_0 plus an integration constant. Σe_0 is given in column (5), and the integration constant follows from the condition that B at 180°

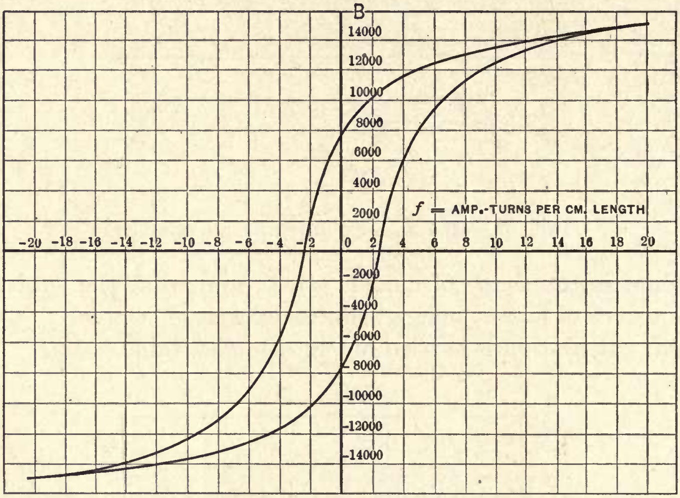


FIG. 42.—Hysteretic cycle in example 88.

must be equal, but opposite in sign, to B at 0°. The integration constant is, therefore,

$$-\tfrac{1}{2}\sum_0^{180} e_0 = -\frac{14,648}{2} = -7324,$$

and by subtracting 7324 from the values in column (5) the values of B' of column (6) are found as the relative instantaneous values of magnetic flux density.

Since the maximum magnetic flux density is 15,000 the instantaneous values are $B = B'\frac{15,000}{7324}$, plotted in column (7).

From the hysteresis cycle in Fig. 42 are taken the values of magnetizing force f, corresponding to magnetic flux density B. They are recorded in column (8), and in column (9) the instantaneous values of m.m.f. $F = lf$, where $l = 50$ = length of magnetic circuit.

$i = \frac{F}{n}$, where $n = 500$ = number of turns of the electric circuit, gives thus the exciting current in column (10).

Column (11) gives the squares of the exciting current, i^2. Their sum is 25.85; thus the mean square, $\frac{25.85}{18} = 1.436$, and the effective value of exciting current, $i' = \sqrt{1.436} = 1.198$ amp.

Column (12) gives the instantaneous values of power, $p = ie_0$. Their sum is 4766; thus the mean power, $p' = \frac{4766}{18} = 264.8$.

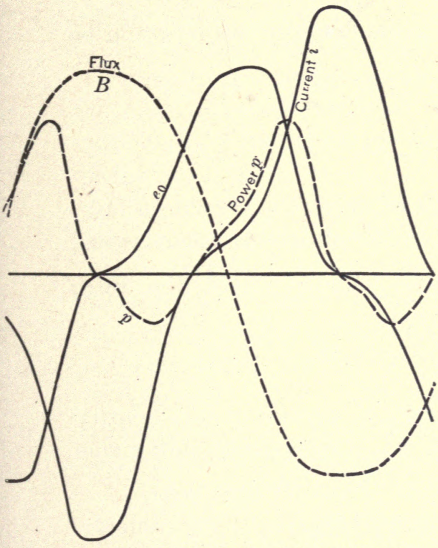


FIG. 43.—Waves of exciting current. Power and flux density corresponding to e.m.f. in Fig. 41 and hysteretic cycle in Fig. 42.

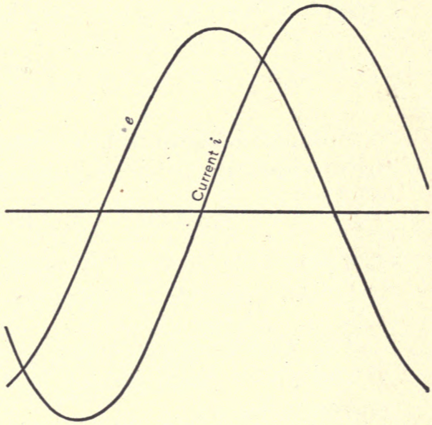


FIG. 44.—Corresponding sine waves for e.m.f. and exciting current in Fig. 43.

Since $$p' = i'e'_0 \cos \theta,$$

where e'_0 and i' are the equivalent sine waves of e.m.f. and of current respectively, and θ their phase displacement, substituting these numerical values of p', e', and i', we have

$$264.8 = 1000 \times 1.198 \cos \theta.$$

hence,

$$\cos \theta = 0.221,$$
$$\theta = 77.2°,$$

and the angle of hysteretic advance of phase,

$$\alpha = 90° - \theta = 12.8°.$$

The hysteresis current is then

$$i' \cos \theta = 0.265,$$

and the magnetizing current,

$$i' \sin \theta = 1.165.$$

Adding the instantaneous values of e.m.f. e_0 in column (4) gives 14,648; thus the mean value, $\frac{14,648}{18} = 813.8$. Since the effective value is 1000, the mean value of a sine wave would be $1000 \frac{2\sqrt{2}}{\pi} = 904$; hence the form factor is

$$\gamma = \frac{904}{813.8} = 1.11.$$

Adding the instantaneous values of current i in column (10), irrespective of their sign, gives 17.17; thus the mean value, $\frac{17.17}{18} = 0.954$. Since the effective value $= 1.198$, the form factor is

$$\gamma = \frac{1.198}{0.954} \frac{2\sqrt{2}}{\pi} = 1.12.$$

The instantaneous values of e.m.f. e_0, current i, flux density B and power p are plotted in Fig. 43, their corresponding sine waves in Fig. 44.

19. FIELDS OF FORCE

89. When an electric current flows through a conductor, power is consumed and heat produced inside of the conductor. In the space outside and surrounding the conductor, a change has taken place also, and this space is not neutral and inert any more, but if we try to move a solid mass of metal rapidly through it, the motion is resisted, and heat produced in the metal by induced currents. Materials of high permeability, as iron filings, brought into this space arrange themselves in chains; a magnetic needle is moved and places itself in a definite direction. Due to the passage of the current in the conductor, there are therefore in the spaces outside of the conductor—where the current does not flow—forces exerted, and this space then is not neutral space, but has become a *field of force*, and the cause of the field, in this case the electric current in the conductor, is its "*motive force*." As in this case the actions exerted in the field of force are magnetic, the space surrounding a conductor traversed by a current is a *field of magnetic force*, and the current in the conductor is the *magnetomotive force.*

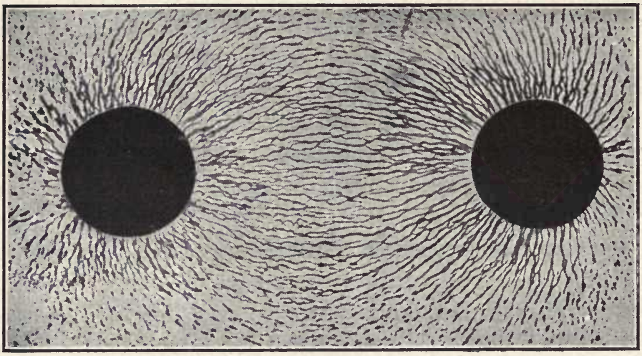
In the space surrounding a ponderable mass, as our earth, forces are exerted on other masses—which cause the stone to fall toward the earth, and water to run down hill—and this space thus is a *field of gravitational force*, the earth the *gravimotive force.*

In the space surrounding conductors having a high potential difference, we observe a *field of dielectric force*, that is, electrostatic or dielectric forces are exerted, and the potential difference between the conductors is the *electromotive force* of the dielectric field.

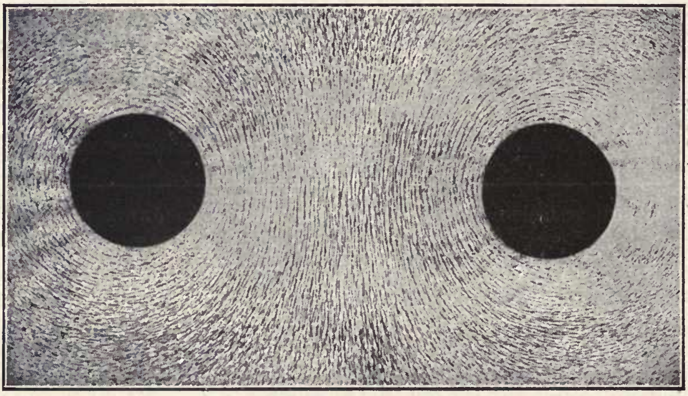
The force exerted by the earth as *gravimotive force*, on any mass in the gravitational field of the earth, causes the mass to move with increasing rapidity. The direction of motion then shows the direction in which the force acts, that is, the direction of the gravitational field. The force g, which the field exerts on unit mass, that is, the acceleration of the mass, measures the intensity of the field: in the gravitational field of the earth 981 cm g sec. The force acting upon a mass m, then, is: $F = gm$, and is called the *weight* of the mass.

In the same manner, in the magnetic field of a current as magnetomotive force, the intensity H of the magnetic field is measured by the force F which the field exerts on a magnetic mass or pole strength m: $F = Hm$; the intensity K of the dielectric field of a potential difference as electromotive force is measured by the force F exerted upon an electric pole strength e: $F = Ke$; the direction of the force represents the direction of the field of force.

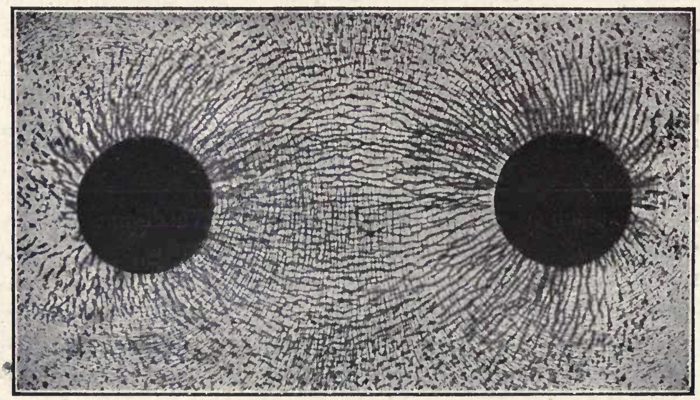
90. This conception of the field of force is one of the most important and fundamental ones of all sciences and applied sciences: a condition of space, brought about by some exciting cause or motive force, whereby the space is not neutral any more, but capable of exerting forces on anything susceptible to these forces: mechanical forces on masses in a gravitational field, magnetic forces on magnetic materials in a magnetic field,



A.—A photograph of a mica-filing map of the dielectric lines of force between two cylinders.



B.—A photograph of an iron-filing map of the magnetic lines of force about two cylinders.



C.—A photographic superposition of A and B representing the magnetic and dielectric fields of the space surrounding two conductors which are carrying energy.

FIG. 45.

dielectric forces on dielectrics in a dielectric field, etc. The field of force then is characterized by having, at any point, a definite direction—the direction in which the force acts—and a definite intensity, to which the forces are proportional.

Such fields of force can be graphically represented by lines showing the direction in which the force acts: the lines of force and, at right angles thereto, the equipotential lines or surfaces, as the direction in which no force acts. Thus the lines of gravitational force of the earth are the verticals, the equipotential surfaces, or level surfaces, are the horizontals. Such pictures of a field of force also illustrate the intensity: where the lines of force and therefore the equipotential lines come closer together, the field is more intense, that is, the forces greater.

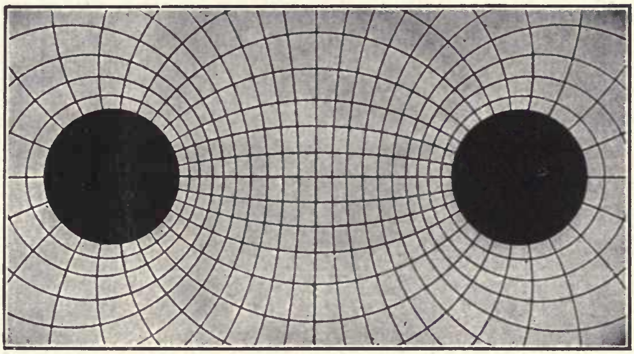


Fig. 46.—A mathematical plot of fields shown in C.

Magnetic fields may be demonstrated by iron filings brought into the field; dielectric fields by particles of a material of high specific capacity, such as mica. Fig. 45 shows the dielectric field of a pair of parallel conductors, the magnetic field between these conductors, and their combination. Fig. 46 shows the same as calculated.

As further illustration, Fig. 47 shows, from observation, half of the dielectric field between a rod with circular disc, as one terminal, passing symmetrically through the center of a cylinder placed in a circular hole in a plate as other terminal: the lines of force pass from terminal to terminal; the equipotential surfaces intersect at right angles (A 10,292).

91. In electrical engineering we have to deal with the electrical quantities: voltage, current, resistance, etc.; the magnetic quantities: magnetic flux, field intensity, permeability, etc.; and the dielectric quantities: dielectric flux, field intensity, permittivity, etc.

The electric current is the *magnetomotive force F* which produces the magnetic field, acting upon space. It is expressed in amperes, or rather in ampere-turns, and thus is an electrical quantity, its

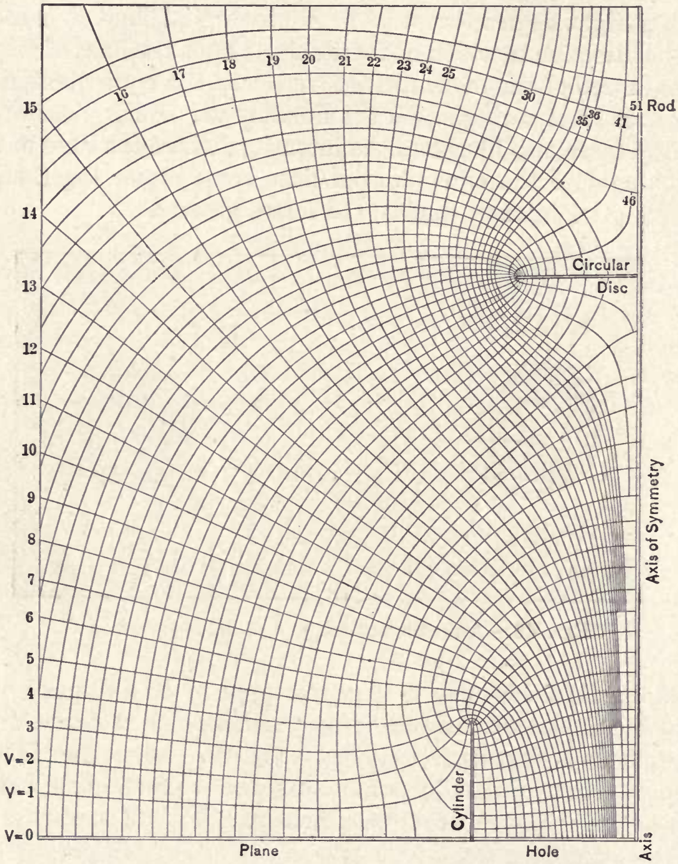


FIG. 47.—Observed dielectric field.

unit being determined by the unit of current, as the ampere-turn equal to 10^{-1} absolute units.

The magnetomotive force per unit length of the magnetic circuit then is the *magnetizing force* or *magnetic gradient f*, in ampere-turns per centimeter, hence still an electrical quantity.

Proportional thereto, and of the same dimension, is the *magnetic field intensity* H. It differs from the magnetic gradient merely by a numerical factor 4π; $H = 4\pi f\, 10^{-1}$. Magnetic field intensity is a magnetic quantity, and its unit defined by the magnetic forces exerted in the field, thus different from the unit of magnetic gradient, which is determined by the unit of electric current; hence the factor 4π. The factor 10^{-1} merely reduces from amperes to absolute unit.

If then μ is the magnetic conductivity of the material in the magnetic field, called its *permeability*, $B = \mu H$ is the magnetic *flux density*, and the total magnetic flux Φ is given by the density B times the area or section of the flux.

Or, passing directly from the magnetomotive force F to the magnetic flux, by the conception of the magnetic circuit: $\Phi = \frac{F}{R}$, where R is the magnetic resistance, or *reluctance* of the magnetic circuit.

R is an electric quantity, and does not contain the 4π.

In the dielectric field, the potential difference e is the electromotive force expressed in volts. The electromotive force per unit length of the dielectric circuit is the *electrifying force* or *voltage gradient* or *dielectric gradient* g, expressed in volts per centimeter. This is still an electric quantity.

Proportional thereto by a numerical factor is the dielectric quantity: *dielectric field intensity* $K = \frac{g}{4\pi v^2}$, and if k is the dielectric conductivity of the medium in the dielectric field, called *specific capacity* or *permittivity*, the *dielectric flux density* is $D = kK$, and the total *dielectric flux* Ψ is flux density times area.

Here again, at the transition from the electric quantity "gradient" to the dielectric quantity "field intensity," a numerical factor $4\pi v^2$ enters, the one quantity being based on the volt as unit, the other on unit force action. v is the velocity of light, 3×10^{10}, and the factor v^2 the result of the convention of assuming the permittivity of empty space as unity.

It is now easy to remember, where in the electromagnetic system of units the factor 4π enters: it is at the transition from the electrical quantities to the magnetic or dielectric quantities, from *gradient* to *field intensity*.

92. The dielectric field and the magnetic field are analogous, and to magnetic flux, magnetic field intensity, permeability, as used in dealing with magnetic circuits, correspond the terms dielectric flux, dielectric field intensity, permittivity, as used in dealing with the electrostatic fields of high potential apparatus, as transmission insulators, transformer bushings, etc. The foremost difference is that in the magnetic field, a line of force must always return into itself in a closed circuit, while in the electrostatic or dielectric field, a line of force may terminate in a conductor. The terminals of the lines of electrostatic flux, Ψ at the conductor, then are represented by the conception of a *quantity of electricity* or *electric charge*, Q, being located on the conductor. Thus, at the terminal of the line of unit dielectric flux, unit electric quantity is located on the conductor.

Dielectric flux Ψ and electric quantity or charge Q thus are identical, and merely different conceptions of the dielectric circuit:

$$Q = \Psi.$$

In using the conception of electric quantity Q, we consider only the terminals of the lines of dielectric flux, that is, deal merely with the effect of the dielectric flux on the electric circuit which produced it. This conception is in many cases more convenient, but it necessarily fails, when the distribution of the dielectric flux in the dielectric field is of importance, such as is the case when dealing with high dielectric field intensities, approaching the possibility of disruptive effects in the field of force, or when dealing with the effect produced by the introduction of materials of different permittivity into the dielectric field. Therefore, with the increasing importance of the dielectric field in engineering, the conception of electric quantity, or charge, is gradually being replaced by the conception of the dielectric flux and the dielectric field, analogous to the magnetic field, which has replaced the previous conception of "magnetic poles."

20. NOMENCLATURE

93. The following nomenclature and symbols of the quantities most frequently used in electrical engineering appears most satisfactory, and is therefore recommended. It is in agreement with the Standardization Rules of the A. I. E. E., but as far as possible standard letters have been used, and script letters avoided as impracticable or at least inconvenient in writing and still more in typewriting. Therefore F has been chosen for m.m.f., and dielectric field intensity changed to K. Also, a few symbols not contained in the Standardization Rules had to be added.

TABLE OF SYMBOLS

Symbol	Name	Unit	Character
E, e.....	Voltage Potential difference Electromotive force	Volt	Electrical
I, i......	Current	Ampere	Electrical
R, r.....	Resistance	Ohm	Electrical
x........	Reactance	Ohm	Electrical
Z, z......	Impedance	Ohm	Electrical
g........	Conductance	Mho	Electrical
b........	Susceptance	Mho	Electrical
Y, y.....	Admittance	Mho	Electrical
ρ........	Resistivity	Ohm-centimeter	Electrical
γ........	Conductivity	Mho-centimeter	Electrical
Φ.......	Magnetic flux	Line; kiloline; megaline	Magnetic
B.......	Magnetic density	Lines per cm.²; kilolines per cm.²	Magnetic
H.......	Magnetic field intensity	Lines per cm.²	Magnetic
μ........	Permeability (magnetic conductivity)	....	Magnetic
f........	Magnetic gradient Magnetizing force	Ampere-turns per centimeter.	Electrical
F........	Magnetomotive force	Ampere-turns	Electrical
R.......	Reluctance (magnetic resistance)	....	Electrical
L........	Inductance	Henry; milhenry	Magnetic
M.......	Mutual inductance	Henry; milhenry	Magnetic
S........	Self-inductance Leakage inductance	Henry; milhenry	Magnetic
Ψ, Q.....	Dielectric flux Electric quantity or charge	Lines of dielectric force Coulombs	Dielectric
D.......	Dielectric density	Dielectric lines per cm.²	Dielectric
K.......	Dielectric field intensity	Coulombs per cm.²	Dielectric
k........	Permittivity Specific capacity	....	Dielectric

TABLE OF SYMBOLS. *Continued*

Symbol	Name	Unit	Character
g	Dielectric gradient Voltage gradient Electrifying force	Volts per centimeter	Electrical
C	Capacity	Farad; microfarad	Dielectric
P, p	Power, effect	Watt; kilowatt	General
W, w....	Energy, work	Joule; kilojoule	General
T, ϑ	Temperature	Degrees Centigrade	General
t	Time	Seconds	General
θ, ϕ, β ...	Time angle	Degrees or radians	General
α, τ	Space angle	Degrees or radians	General
f	Frequency	Cycles per second	General

PART II

SPECIAL APPARATUS

INTRODUCTION

1. By the direction of the energy transmitted, electric machines have been divided into generators and motors. By the character of the electric power they have been distinguished as direct-current and as alternating-current apparatus.

With the advance of electrical engineering, however, these subdivisions have become unsatisfactory and insufficient.

The division into generators and motors is not based on any characteristic feature of the apparatus, and is thus not rational. Practically any electric generator can be used as motor, and conversely, and frequently one and the same machine is used for either purpose. Where a difference is made in the construction, it is either only quantitative, as, for instance, in synchronous motors a higher armature reaction is often used than in synchronous generators, or it is in minor features, as direct-current motors usually have only one field winding, either shunt or series, while in generators frequently a compound field is employed. Furthermore, apparatus have been introduced which are neither motors nor generators, as the synchronous machine producing wattless lagging or leading current, etc., and the different types of converters.

The subdivision into direct-current and alternating-current apparatus is unsatisfactory, since it includes in the same class apparatus of entirely different character, as the induction motor and the alternating-current generator, or the constant-potential commutating machine and the rectifying arc light machine.

Thus the following classification, based on the characteristic features of the apparatus, as adopted by the A. I. E. E. Standardizing Committee, is used in the following discussion. It refers only to the apparatus transforming between electric and electric and between electric and mechanical power.

1st. *Commutating machines,* consisting of a magnetic field and a closed-coil armature, connected with a multi-segmental commutator.

2d. *Synchronous machines,* consisting of a undirectional magnetic field and an armature revolving relatively to the magnetic field at a velocity synchronous with the frequency of the alternating-current circuit connected thereto.

3d. *Rectifying apparatus,* that is, apparatus reversing the direction of an alternating current synchronously with the frequency.

4th. *Induction machines,* consisting of an alternating magnetic circuit or circuits interlinked with two electric circuits or sets of circuits moving with regard to each other.

5th. *Stationary induction apparatus,* consisting of a magnetic circuit interlinked with one or more electric circuits.

6th. *Electrostatic and electrolytic apparatus* as condensers and polarization cells.

Apparatus changing from one to a different form of electric energy have been defined as:

A. *Transformers,* when using magnetism, and as

B. *Converters,* when using mechanical momentum as intermediary form of energy.

The transformers as a rule are stationary, the converters rotary apparatus. Motor-generators transforming from electrical over mechanical to electric power by two separate machines, and dynamotors, in which these two machines are combined in the same structure, are not included under converters.

2. (1) *Direct-current commutating machines* as *generators* are usually built to produce constant potential for railway, incandescent lighting, and general distribution. As *motors* commutating machines give approximately constant speed—shunt motors —or large starting torque—series motors.

When inserted in series in a circuit, and controlled so as to give an e.m.f. varying with the conditions of load on the system, these machines are "*boosters,*" and are generators when raising the voltage, and motors when lowering it.

Commutating machines may be used as *direct-current converters* by transforming power from one side to the other side of a three-wire system.

Alternating-current commutating machines are used as motors of series characteristic for railway and other varying speed service, or with shunt characteristic for constant speed and adjustable speed work, especially where high starting torque efficiency is required. They usually are of single-phase type.

(2) While in commutating machines the magnetic field is

almost always stationary and the armature rotating, *synchronous machines* were built with stationary field and revolving armature, or with stationary armature and revolving field, or as inductor machines with stationary armature and stationary field winding but revolving magnetic circuit. Generally now the revolving field type is used.

By the number and character of the alternating circuits connected to them they are single-phase or polyphase machines. As *generators* they comprise practically all single-phase and polyphase alternating-current generators; as motors a very important class of apparatus, the *synchronous motors,* which are usually preferred for large powers, especially where frequent starting and considerable starting torque are not needed. Synchronous machines may be used as *compensators* or *synchronous condensers,* to produce wattless current, leading by over-excitation, lagging by under-excitation, or may be used as *phase converters* by operating a polyphase synchronous motor by one pair of terminals from a single-phase circuit. The most important class of converters, however, are the synchronous commutating machines, to which, therefore, a special chapter will be devoted in the following.

Inserted in series to another synchronous machine or synchronous converter, and rigidly connected thereto, synchronous machines are also occasionally used as boosters.

Synchronous commutating machines contain a unidirectional magnetic field and a closed circuit armature connected simultaneously to a segmental direct-current commutator and by collector rings to an alternating circuit, generally a polyphase system. Thus these machines can either receive alternating and yield direct-current power as synchronous converters or simply "*converters,*" or receive direct and yield alternating-current power as *inverted converters,* or driven by mechanical power yield alternating and direct current as *double-current generators.* Or they can combine motor and generator action with their converter action. Thus a combination is a synchronous converter supplying a certain amount of mechanical power as a synchronous motor. Usually, they convert from three-phase or single-phase alternating to direct-current power.

(3) *Rectifying machines* are apparatus which by a synchronously revolving rectifying commutator send the successive half waves of an alternating single-phase or polyphase circuit in the same direction into the receiving circuit. The most impor-

tant class of such apparatus were the *open-coil arc light machines*. They have been practically superseded by the mercury arc rectifier.

(4) *Induction machines* are generally used as *motors,* polyphase or single-phase. In this case they run at practically constant speed, slowing down slightly with increasing load. As *generators* the frequency of the e.m.f. supplied by them differs from and is lower than the frequency of rotation, but their operation depends upon the phase relation of the external circuit. As *phase converters,* induction machines can be used in the same manner as synchronous machines. Another occasional use besides as motors is, however, as *frequency converters,* by changing from an impressed primary polyphase system to a secondary polyphase system of different frequency. In this case, when lowering the frequency, mechanical energy is also produced; when raising the frequency, mechanical energy is consumed.

(5) The most important *stationary induction apparatus* is the *transformer,* consisting of two electric circuits interlinked with the same magnetic circuit. When using the same or part of the same electric circuit for primary and secondary, the transformer is called an *auto-transformer* or *compensator.* When inserted in series into a circuit, and arranged to vary the e.m.f., the transformer is called *potential regulator* or booster. The variation of secondary e.m.f. may be secured by varying the relative number of primary and secondary turns, or by varying the mutual inductance between primary and secondary circuit, either electrically or magnetically. The stationary induction apparatus with one electric circuit are used for producing wattless lagging currents, as *reactors, reactive* or *choke coils.*

(6) *Condensers* and *polarization cells* produce wattless leading currents, the latter, however, usually at a low efficiency, while the efficiency of the condenser is extremely high, frequently above 99 per cent.; that is, the loss of power is less than 1 per cent. of the apparent volt-ampere input.

Unipolar, or, more correctly, *non-polar* or *acyclic machines* are apparatus in which a conductor cuts a continuous magnetic field at a uniform rate. They have not become of industrial importance.

Regarding apparatus transforming between electric energy and forms of energy differing from electric or mechanical energy: The transformation between electrical and chemical energy is represented by the primary and secondary battery and the electrolytic cell; the transformation between electrical and heat energy by the thermopile and the electric heater or electric furnace; the transformation between electrical and light energy by the incandescent and arc lamps.

In the following will be given a general discussion of the characteristics of the most frequently used and therefore most important classes of apparatus.

A further discussion and calculation of these apparatus is given in "Theory and Calculation of Alternating Current Phenomena," while a discussion of those characteristics and modifications of these apparatus, which, though important, are less frequently met, and a discussion of the numerous less common types of apparatus, which could not be included in the following, is given in "Theory and Calculation of Electrical Apparatus."

Some important features, as the nature of the reactance of apparatus, mechanical magnetic forces, wave shape distortions caused by some features of design, in apparatus, etc., are discussed in "Theory and Calculation of Electric Circuits."

A. SYNCHRONOUS MACHINES

I. General

3. The most important class of alternating-current apparatus consists of the synchronous machines. They comprise the alternating-current generators, single-phase and polyphase, the synchronous motors, the phase compensators, the phase converters, the phase balancers, the synchronous boosters and the exciters of induction generators, that is, synchronous machines producing wattless lagging or leading currents, and the converters. Since the latter combine features of the commutating machines with those of the synchronous machines they will be considered separately.

In the synchronous machines the terminal voltage and the generated e.m.f. are in synchronism with, that is, of the same frequency as, the speed of rotation.

These machines consist of an armature, in which e.m.f. is generated by the rotation relatively to a magnetic field, and a continuous magnetic field, excited either by direct current, or by the reaction of displaced phase armature currents, or by permanent magnetism.

The formula for the e.m.f. generated in synchronous machines, commonly called alternators, is

$$E = \sqrt{2}\,\pi f n \Phi = 4.44\, f n \Phi,$$

where n is the number of armature turns in series interlinked with the magnetic flux Φ (in megalines per pole), f the frequency of rotation (in hundreds of cycles per second), E the e.m.f. generated in the armature turns.

This formula assumes a sine wave of e.m.f. If the e.m.f. wave differs from sine shape, the e.m.f. is

$$E = 4.44\, \gamma f n \Phi,$$

where γ = form factor of the wave, or $\frac{2\sqrt{2}}{\pi}$ times ratio of effective to mean value of wave, that is, the ratio of the effective value of the generated e.m.f. to that of a sine wave generated by the same magnetic flux at the same frequency.

The form factor γ depends upon the wave shape of the generated e.m.f. The wave shape of e.m.f. generated in a single conductor on the armature surface is identical with that of the distribution of magnetic flux at the armature surface and will be discussed more fully in the chapter on commutating machines. The wave of total e.m.f. is the sum of the waves of e.m.f. in the individual conductors, added in their proper phase relation, as corresponding to their relative positions on the armature surface.

4. In a Y or star-connected three-phase machine, if E_0 = e.m.f. per circuit, or Y or star e.m.f., $E = E_0 \sqrt{3}$ is the e.m.f. between terminals or Δ (delta) or ring e.m.f., since two e.m.fs. displaced by 60 degrees are connected in series between terminals ($\sqrt{3} = 2 \cos 30°$).

In a Δ-connected three-phase machine, the e.m.f. per circuit is the e.m.f. between the terminals, or Δ e.m.f.

In a Y-connected three-phase machine, the current per circuit is the current issuing from each terminal, or the line current, or Y current.

In a Δ-connected three-phase machine, if I_0 = current per circuit, or Δ current, the current issuing from each terminal, or the line or Y current, is

$$I = I_0 \sqrt{3}.$$

Thus in a three-phase system, Δ current and e.m.f., and Y current and e.m.f. (or ring and start current and e.m.f. respectively), are to be distinguished. They stand in the proportion $1 \div \sqrt{3}$.

As a rule, when speaking of current and of e.m.f. in a three-phase system, under current the Y current or current per line, and under e.m.f. the Δ e.m.f. or e.m.f. between lines is understood.

5. While the voltage wave of a single conductor has the same shape as the distribution of the magnetic flux at the armature circumference and so may differ considerably from a sine, that is, contain pronounced higher harmonics, the terminal voltage is the resultant of the waves of many conductors, and, especially with a distributed armature winding, shows the higher harmonics in a much reduced degree; that is, the resultant is nearer sine shape, and some harmonics may be entirely eliminated in the terminal voltage wave, though they may appear in the voltage wave of a single conductor. Thus, for instance, in a three-phase Y-connected machine, the voltage per circuit, or Y voltage, may contain a third harmonic and multiples thereof, while in the voltage between the terminals this third harmonic is eliminated. The voltage between the terminals is the resultant of two Y voltages, displaced from each other by 60 degrees. Sixty degrees for the fundamental, however, is $3 \times 60° = 180°$, or opposition for the third harmonic; that is, the third harmonics in those two Y voltages, which combine to the delta or terminal voltage, are opposite, and so neutralize each other.

Even in a single turn, harmonics existing in the magnetic field and thus in the single conductor can be eliminated by fractional pitch. Thus, if the pitch of the armature turn is not 180 degrees, but less by $\frac{1}{n}$, the e.m.fs. generated in the two conductors of a single turn are not exactly in phase, but differ by $\frac{1}{n}$ of a half wave for the fundamental, and thus a whole half wave for the nth harmonic, so that their nth harmonics are in opposition and thus cancel. Fractional pitch winding of a "pitch deficiency" of $\frac{1}{n}$ thus eliminates the nth harmonic; for instance, with 80 per cent. pitch, the fifth harmonic cannot exist.

In this manner higher harmonics of the e.m.f. wave can be reduced or entirely eliminated, though in general, with a distributed winding, the wave shape is sufficiently close to sine shape without special precaution being taken in the design.

II. Electromotive Forces

6. In a synchronous machine we have to distinguish between terminal voltage E, real generated e.m.f. E_1, virtual generated e.m.f. E_2, and nominal generated e.m.f. E_0.

The real generated e.m.f. E_1 is the e.m.f. generated in the alternator armature turns by the resultant magnetic flux, or magnetic flux interlinked with them, that is, by the magnetic flux passing through the armature core. It is equal to the terminal voltage plus the e.m.f. consumed by the resistance of the armature, these two e.m.fs. being taken in their proper phase relation; thus

$$\dot{E}_1 = \dot{E} + \dot{I}r,$$

where I = current in armature, r = effective resistance.

The virtual generated e.m.f. E_2 is the e.m.f. which would be generated by the flux produced by the field poles, or flux corresponding to the resultant m.m.f., that is, the resultant of the m.m.fs. of field excitation and of armature reaction. Since the magnetic flux produced by the armature, or flux of armature self-inductance, combines with the field flux to the resultant flux, the flux produced by the field poles does not pass through the armature completely, and the virtual e.m.f. and the real generated e.m.f. differ from each other by the e.m.f. of armature self-inductance; but the virtual generated e.m.f., as well as the e.m.f. generated in the armature by self-inductance, have no real and independent existence, but are merely fictitious components of the real or resultant generated e.m.f. E_1.

The virtual generated e.m.f. is

$$\dot{E}_2 = \dot{E}_1 + j\dot{I}x,$$

where x is the self-inductive armature reactance, and the e.m.f consumed by self-inductance Ix is to be combined with the real generated e.m.f. E_1 in the proper phase relation.

7. The nominal generated e.m.f. E_0 is the e.m.f. which would be generated by the field excitation if there were neither self-inductance nor armature reaction, and the saturation were the same as corresponds to the real generated e.m.f. It thus does not correspond to any magnetic flux, and has no existence at all, but is merely a fictitious quantity, which, however, is very useful for the investigation of alternators by allowing the combination of armature reaction and self-inductance into a single effect by a (fictitious) self-inductance or synchronous reactance x_0. The nominal generated e.m.f. would be the terminal voltage with open circuit and load excitation if the saturation curve were a straight line.

The synchronous reactance x_0 is thus a quantity combining armature reaction and self-inductance of the alternator. It is the only quantity which can easily be determined by experiment by running the alternator on short circuit with excited field. If in this case I_0 = current, P_0 = loss of power in the armature coils, E_0 = e.m.f. corresponding to the field excitation at open circuit, $\frac{E_0}{I_0} = z_0$ is the synchronous impedance, $\frac{P_0}{I_0^2} = r_0$ is the effective resistance (ohmic resistance plus load losses), and $x_0 = \sqrt{z_0^2 - r_0^2}$ the synchronous reactance.

In this feature lies the importance of the term "nominal generated e.m.f." E_0,

$$\dot{E}_0 = \dot{E}_1 + j\dot{I}x_0, = \dot{E} + (r + jx)\,\dot{I}$$

the terms being combined in their proper phase relation. In a polyphase machine, these considerations apply to each of the machine circuits individually.

III. Armature Reaction

8. The magnetic flux in the field of an alternator under load is produced by the resultant m.m.f. of the field exciting current and of the armature current. It depends upon the phase relation of the armature current. The e.m.f. generated by the field exciting current or the nominal generated e.m.f. reaches a maximum when the armature coil faces the position midway between

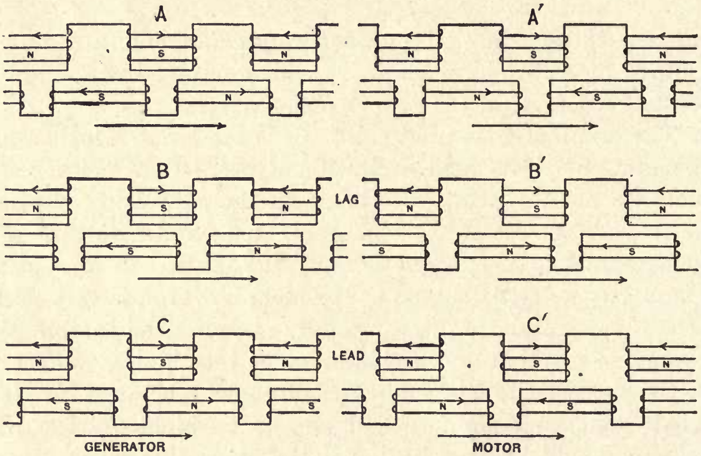


Fig. 48.—Model for study of armature reaction. Armature coils in position of maximum current.

the field poles, as shown in Fig. 48, *A* and *A′*. Thus, if the armature current is in phase with the nominal generated e.m.f., it reaches its maximum in the same position *A*, *A′* of armature coil as the nominal generated e.m.f., and thus magnetizes the preceding, demagnetizes the following magnet pole (in the direction of rotation) in an alternating-current generator *A*; magnetizes the following and demagnetizes the preceding magnet pole in a synchronous motor *A′* (since in a generator the rotation is against, in a synchronous motor with the magnetic attractions and repulsions between field and armature). In this case the armature current neither magnetizes nor demagnetizes the field as a whole, but magnetizes the one side, demagnetizes the other side of each field pole, and thus merely distorts the magnetic field.

9. If the armature current lags behind the nominal generated e.m.f., it reaches its maximum in a position where the armature coil already faces the next magnetic pole, as shown in Fig. 48, B and B', and thus demagnetizes the field in a generator B, magnetizes it in a synchronous motor B'.

If the armature current leads the nominal generated e.m.f., it reaches its maximum in an earlier position, while the armature coil still partly faces the preceding magnet pole, as shown in Fig. 48, C and C', and thus magnetizes the field in a generator, Fig. 48, C, and demagnetizes it in a synchronous motor C'.

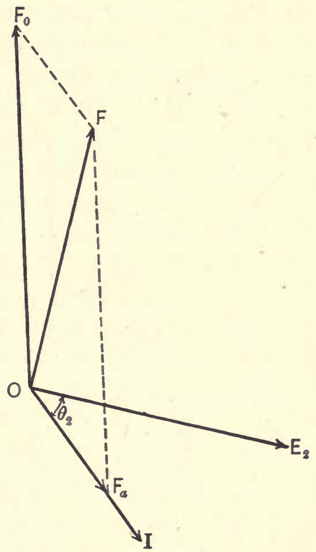


Fig. 49.—Diagram of m.m.fs. in loaded generator.

With non-inductive load, or with the current in phase with the terminal voltage of an alternating-current generator, the current lags behind the nominal generated e.m.f., due to armature reaction and self-inductance, and thus partly demagnetizes; that is, the voltage is lower under load than at no load with the same field excitation. In other words, lagging current demagnetizes and leading current magnetizes the field of an alternating-current generator, while the opposite is the case with a synchronous motor.

10. In Fig. 49 let $\overline{OF} = F$ = resultant m.m.f. of field excitation and armature current (the m.m.f. of the field excitation being alternating with regard to the armature coil, due to its rotation) and θ_2 the lag of the current I behind the virtual e.m.f. E_2 generated by the resultant m.m.f.

The virtual e.m.f. E_2 lags in time 90 degrees behind the resultant flux of $\overline{OF}$, and is thus represented by $\overline{OE_2}$ in Fig. 47, and the m.m.f. of the armature current F_a by $\overline{OF_a}$, lagging by angle θ_2 behind $\overline{OE_2}$. The resultant m.m.f. $\overline{OF}$ is the diagonal of the parallelogram with the component m.m.fs. $\overline{OF_a}$ = armature m.m.f. and $\overline{OF_0}$ = total impressed m.m.f. or field excitation, as sides, and from this construction $\overline{OF_0}$ is found. $\overline{OF_0}$ is thus the position of the field pole with regard to the armature. It is trigonometrically,

$$F_0 = \sqrt{F^2 + F_a{}^2 + 2FF_a \sin \theta_2}.$$

If I = current per armature turn in amperes effective, n = number of turns per pole in a single-phase alternator, the armature reaction is $F_a = nI$ ampere-turns effective, and is pulsating between zero and $nI\sqrt{2}$.

In a quarter-phase alternator with n turns per pole and phase in series and I amperes effective per turn, the armature reaction per phase is nI ampere-turns effective and $nI\sqrt{2}$ ampere-turns maximum. The two phases magnetize in quadrature, in phase and in space. Thus, at the time t, corresponding to angle θ after the maximum of the first phase, the m.m.f. in the direction by angle θ behind the direction of the magnetization of the first phase is $nI\sqrt{2}\cos^2\theta$. The m.m.f. of the second phase is $nI\sqrt{2}\sin^2\theta$; thus the total m.m.f. or the armature reaction $F_a = nI\sqrt{2}$, and is constant in intensity, but revolves synchronously with regard to the armature; that is, it is stationary with regard to the field.

In a three-phaser of n turns in series per pole and phase and I amperes effective per turn, the m.m.f. of each phase is $nI\sqrt{2}$ ampere-turns maximum; thus at angle θ in positon and angle θ in time behind the maximum of one phase;

The m.m.f. of this phase is

$$nI\sqrt{2}\cos^2\theta.$$

The m.m.f. of the second phase is

$$nI\sqrt{2}\cos^2(\theta + 120) = nI\sqrt{2}(-0.5\cos\theta - 0.5\sqrt{3}\sin\theta)^2.$$

The m.m.f. of the third phase is

$$nI\sqrt{2}\cos^2(\theta + 240) = nI\sqrt{2}(-0.5\cos\theta + 0.5\sqrt{3}\sin\theta)^2.$$

Thus the total m.m.f. or armature reaction,

$$F_a = nI\sqrt{2}(\cos^2\theta + 0.25\cos^2\theta + 0.75\sin^2\theta + 0.25\cos^2\theta + 0.75\sin^2\theta) = 1.5\, nI\sqrt{2},$$

constant in intensity, but revolving synchronously with regard to the armature, that is, stationary with regard to the field. These values of armature reaction correspond strictly only to the case where all conductors of the same phase are massed together in one slot. If the conductors of each phase are distributed over a greater part of the armature surface, the values of armature reaction have to be multiplied by the average cosine of the total angle of spread of each phase.

11. The single-phase machine thus differs from the polyphase machines: in the latter, on balanced load, the armature reaction is constant, while in the single-phase machine the armature reaction and thereby the resultant m.m.f. of field and armature is pulsating. The pulsation of the resultant m.m.f. of the single-phase machine causes a pulsation of its magnetic field under load, of double frequency, which generates a third harmonic of e.m.f. in the armature conductors. In machines of high armature reaction, as steam-turbine-driven single-phase alternators, the pulsation of the magnetic field may be sufficient to cause serious energy losses and heating by eddy currents, and thus has to be checked. This is usually done by a squirrel-cage induction machine winding in the field pole faces, or by short-circuited conductors laid in the pole faces in electrical space quadrature to the field coils. In these conductors, secondary currents of double frequency are produced which equalize the resultant m.m.f. of the machine.

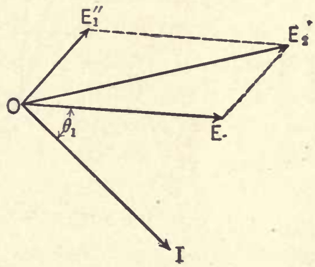


FIG. 50.—Diagram of e.m.fs. in loaded generator.

IV. Self-inductance

12. The effect of self-inductance is similar to that of armature reaction, and depends upon the phase relation in the same manner.

If E_1 = real generated voltage, θ_1 = lag of current behind generated voltage E_1, the magnetic flux produced by the armature current I is in phase with the current, and thus the counter e.m.f. of self-inductance is in quadrature behind the current, and therefore the e.m.f. consumed by self-inductance is in quadrature ahead of the current. Thus in Fig. 50, denoting $\overline{OE}_1 = E_1$ the generated e.m.f., the current is $\overline{OI} = I$, lagging θ_1 behind $\overline{OE}_1$, the e.m.f. consumed by self-inductance $\overline{OE}''_1$, is 90 degrees ahead of the current, and the virtual generated e.m.f. E_2, is the resultant of $\overline{OE}_1$ and $\overline{OE}''_1$. As seen, the diagram of e.m.fs. of self-inductance is similar to the diagram of m.m.fs. of armature reaction.

9

13. From this diagram we get the effect of load and phase relation upon the e.m.f. of an alternating-current generator.

Let E = terminal voltage per machine circuit,
I = current per machine circuit,
and θ = lag of the current behind the terminal voltage.

Let r = resistance,
x = reactance of the alternator armature.

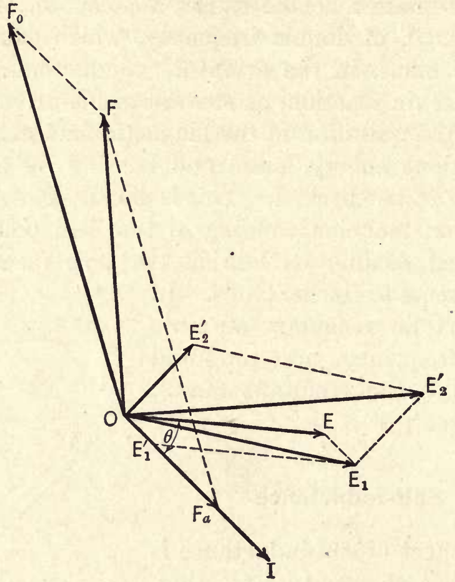


Fig. 51.—Diagram showing combined effect of armature reaction and armature self-inductance.

Then, in the vector diagram, Fig. 51,

$\overline{OE} = E$, the terminal voltage, assumed as zero vector.
$\overline{OI} = I$, the current, lagging by the angle $EOI = \theta$.

The e.m.f. consumed by resistance is $\overline{OE'}_1 = Ir$ in phase with $\overline{OI}$.

The e.m.f. consumed by reactance is $\overline{OE'}_2 = Ix$, 90 degrees ahead of $\overline{OI}$.

The real generated e.m.f. is found by combining $\overline{OE}$ and $\overline{OE'}_1$ to

$$\overline{OE}_1 = E_1.$$

The virtual generated e.m.f. is $\overline{OE}_1$ and $\overline{OE'}_2$ combined to

$$\overline{OE}_2 = E_2.$$

The m.m.f. required to produce this e.m.f. E_2 is $\overline{OF} = F$,

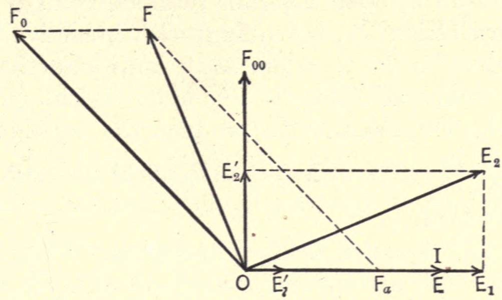


FIG. 52.—Diagram of generator e.m.fs. and m.m.fs. for non-reactive load.

90 deg. ahead of OE_2. It is the resultant of the armature m.m.f. or armature reaction and of the impressed m.m.f. or field excitation. The armature m.m.f. is in phase with the current I, and is nI in a single-phase machine, $n\sqrt{2}\,I$ in a quarter-phase machine, $1.5\sqrt{2}\,nI$ in a three-phase machine, if n = number of armature turns per pole and phase. The m.m.f. of armature reaction is represented in the diagram by $\overline{OF}_a$ of F_a in phase with $\overline{OI}$, and the impressed m.m.f. or field excitation $\overline{OF}_0 = F_0$ is the side of a parallelogram with $\overline{OF}$ as diagonal and $\overline{OF}_a$ as other side; or, the m.m.f. consumed by armature reaction is represented by $\overline{OF'}_a = F_a$ in opposition to $\overline{OI}$. Combining $\overline{OF'}_a$ and $\overline{OF}$ gives $\overline{OF}_0 = F_0$, the field excitation.

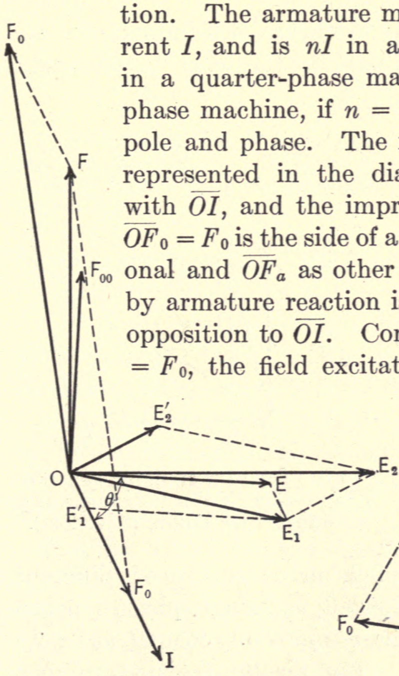


FIG. 53.—Diagram of generator. e.m.fs. and m.m.fs. for lagging reactive load. Power-factor 0.50.

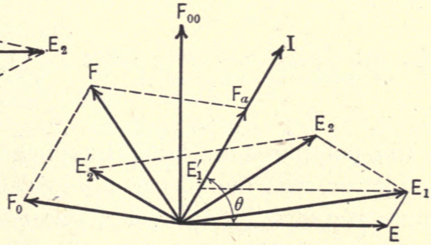


FIG. 54.—Diagram of generator e.m.fs. and m.m.fs. for leading reactive load. Power-factor 0.50.

In Figs. 52, 53, 54 are drawn the diagrams for θ = zero or non-inductive load, $\theta = 60$ degrees, or 60 degrees lag (inductive load of power-factor 0.50), and $\theta = -60$ deg., or 60 deg. lead (anti-inductive load of power-factor 0.50).

Thus it is seen that with the same terminal voltage E a much higher field excitation, F_0, is required with inductive load than with non-inductive load, while with anti-inductive load a much lower field excitation is required. With open circuit the field excitation required to produce the terminal voltage E would be $\frac{E}{E_0} F = F_{00}$, or less than the field excitation F_0 with non-inductive load.

Inversely, with constant field excitation, the voltage of an alternator drops with non-inductive load, drops much more with inductive load, and drops less, or even rises, with anti-inductive load.

V. Synchronous Reactance

14. In general, both effects, armature self-inductance and armature reaction, can be combined by the term "synchronous reactance."

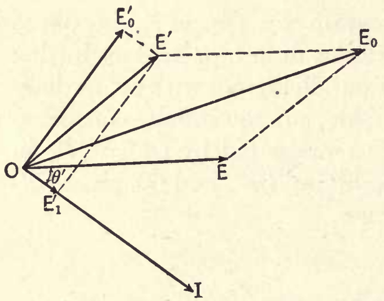


FIG. 55.—Diagram showing effect of synchronous reactance.

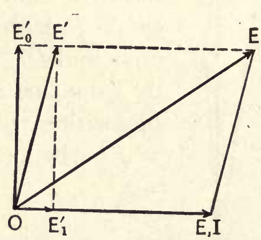


FIG. 56.—Diagram of generator e.m.fs. showing affect of synchronous reactance with non-reactive load.

In a polyphase machine, the synchronous reactance is different, and lower, with one phase only loaded, as "single-phase synchronous reactance," than with all phases uniformly loaded, as "polyphase synchronous reactance." The resultant armature reaction of all phases of the polyphase machine is higher than that with the same current in one phase only, and so also the self-inductive flux, as resultant flux of several phases, and thus represents a higher synchronous reactance.

Let r = effective resistance,
x_0 = synchronous reactance of armature, as discussed in Section II.

Let E = terminal voltage,
I = current,
θ = angle of lag of the current behind the terminal voltage.

It is in vector diagram, Fig. 55.

$OE = E$ = terminal voltage assumed as zero vector. $\overline{OI}$ =

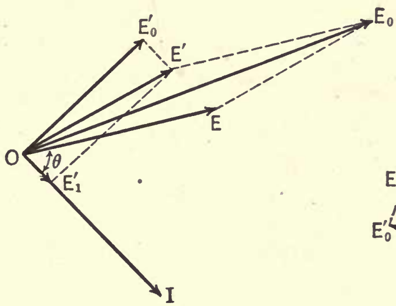


FIG. 57.—Diagram of generator e.m.fs. showing effect of synchronous reactance with lagging reactive load. $\theta = 60$ degrees.

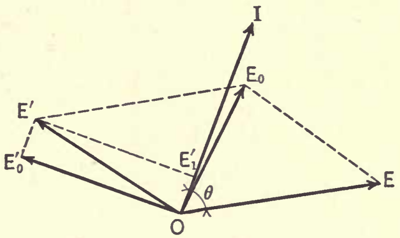


FIG. 58.—Diagram of generator e.m.fs. Showing effect of synchronous reactance with leading reactive load $\theta = -60$ degrees.

I = current lagging by the angle $EOI = \theta$ behind the terminal voltage.

$OE'_1 = Ir$ is the e.m.f. consumed by resistance, in phase with $\overline{OI}$, and $\overline{OE'_0} = Ix_0$ the e.m.f. consumed by the synchronous reactance, 90 degrees ahead of the current $\overline{OI}$.

$\overline{OE'_1}$ and $\overline{OE'_0}$ combined give $\overline{OE'} = E'$ the e.m.f. consumed by the synchronous impedance.

Combining $\overline{OE'_1}, \overline{OE'_0}, \overline{OE}$ gives the nominal generated e.m.f. $\overline{OE_0} = E_0$, corresponding to the field excitation F_0.

In Figs. 56, 57, 58, are shown the diagrams for $\theta = 0$ or non-inductive load, $\theta = 60$ degrees lag or inductive load, and $\theta = -60$ degrees or anti-inductive load.

Resolving all e.m.fs. into components in phase and in quadrature with the current, or into power and reactive components, in symbolic expression we have:

the terminal voltage $\dot{E} = E \cos \theta + jE \sin \theta$;

the e.m.f. consumed by resistance, $E'_1 = ir$;

the e.m.f. consumed by synchronous reactance, $\dot{E}'_0 = + jix_0$,

and the nominal generated e.m.f.,

$$\dot{E}_0 = \dot{E} + \dot{E}'_1 + \dot{E}'_0 = (E \cos \theta + ir) + j\,(E \sin \theta + ix_0);$$

or, since

$$\cos \theta = p = \text{power-factor of the load} \left(= \frac{\text{power current}}{\text{total current}} \right)$$

and

$$q = \sqrt{1 - p^2} = \sin \theta = \text{inductance factor of the load} \left(= \frac{\text{wattless current}}{\text{total current}} \right),$$

it is

$$\dot{E}_0 = (Ep + ir) + j\,(Eq + ix_0),$$

or, in absolute values,

$$E_0 = \sqrt{(Ep + ir)^2 + (Eq + ix_0)^2};$$

hence,

$$E = \sqrt{E_0^2 - i^2\,(x_0 p - rq)^2} - i\,(rp + x_0 q).$$

The power delivered by the alternator into the external circuit is

$$P = iEp;$$

that is, the current times the power component of the terminal voltage.

The electric power produced in the alternator armature is

$$P_0 = i\,(Ep + ir);$$

that is, the current times the power component of the nominal generated e.m.f., or, what is the same thing, the current times the power component of the real generated e.m.f.

VI. Characteristic Curves of Alternating-current Generator

15. In Fig. 59 are shown, at constant terminal voltage E, the values of nominal generated e.m.f. E_0, and thus of field excitation F_0, with the current I as abscissas and for the three conditions,

1. Non-inductive load, $p = 1, q = 0$.
2. Inductive load of $\theta = 60$ degrees lag, $p = 0.5, q = 0.866$.
3. Anti-inductive load of $-\theta = 60$ degrees lead, $p = 0.5$, $q = -0.866$.

The values $r = 0.1$, $x_0 = 5$, $E = 1000$, are assumed. These curves are called the *compounding curves of the synchronous generator.*

In Fig. 60 are shown, at constant nominal generated e.m.f. E_0, that is, at constant field excitation F_0, the values of terminal vol-

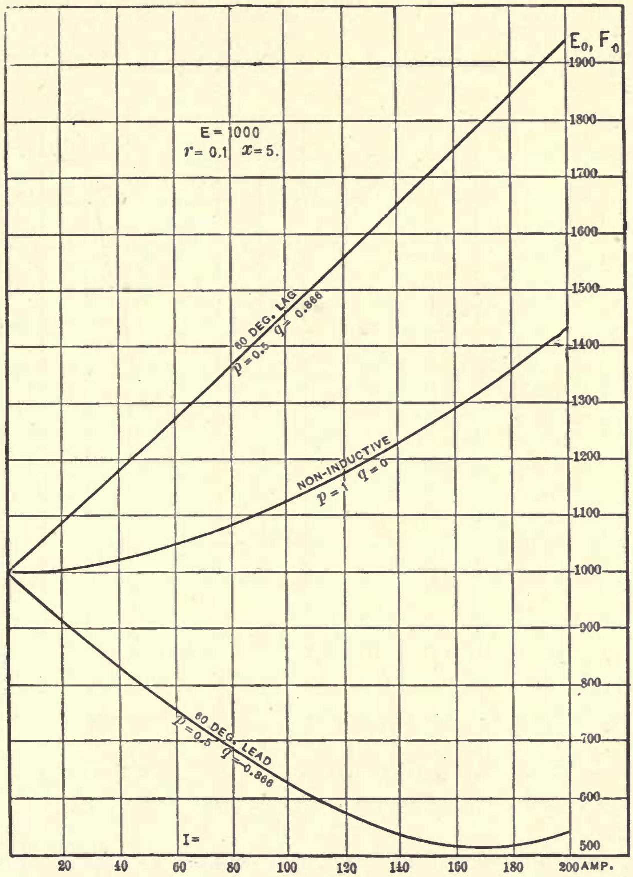


FIG. 59.—Synchronous generator compounding curves.

tage E with the current I as abscissas and for the same resistance and synchronous reactance $r = 0.1, x_0 = 5$, for the three different conditions,

1. Non-inductive load, $p = 1$, $q = 0$, $E_0 = 1127$.
2. Inductive load of 60 degrees lag,

$$p = 0.5, q = 0.866, E_0 = 1458.$$

3. Anti-inductive load of 60 degrees lead,

$$p = 0.5, q = -0.866, E_0 = 628.$$

The values of E_0 (and thus of F_0) are assumed so as to give $E = 1000$ at $I = 100$. These curves are called the *regulation curves* of the alternator, or the *field characteristics* of the synchronous generator.

In Fig. 61 are shown the *load curves* of the machine, with the

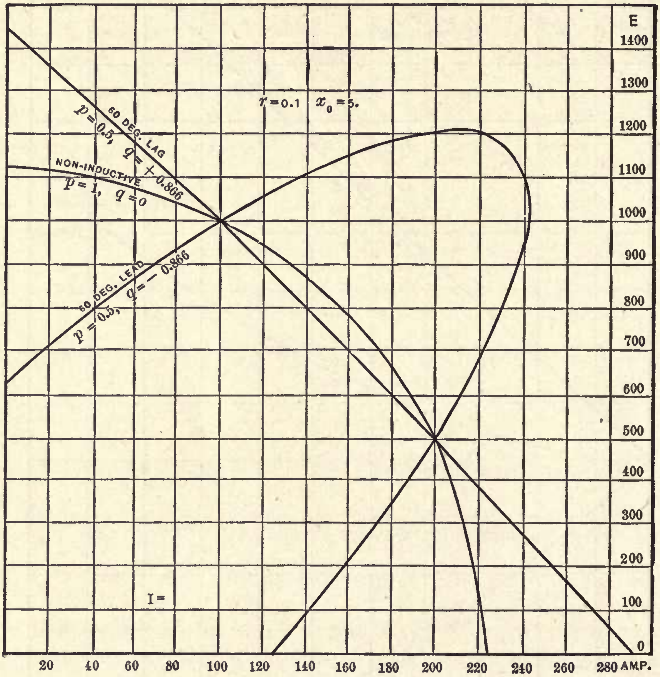


Fig. 60.—Synchronous generator regulation curves.

current I as abscissas and the watts output as ordinates corresponding to the same three conditions as Fig. 60. From the field characteristics of the alternator are derived the open-circuit voltage of 1127 at full non-inductive load excitation, which is 1.127 times full-load voltage; the short-circuit current 225 at full non-inductive load excitation, which is 2.25 times full-load current; and the maximum output, 124 kw., at full non-inductive load excitation, which is 1.24 times rated output, at 775 volts and 160 amp. It depends upon the point on the field characteristic at which the alternator works, whether it tends to regulate for, that is, maintains, constant voltage, or constant current, or constant power, approximately.

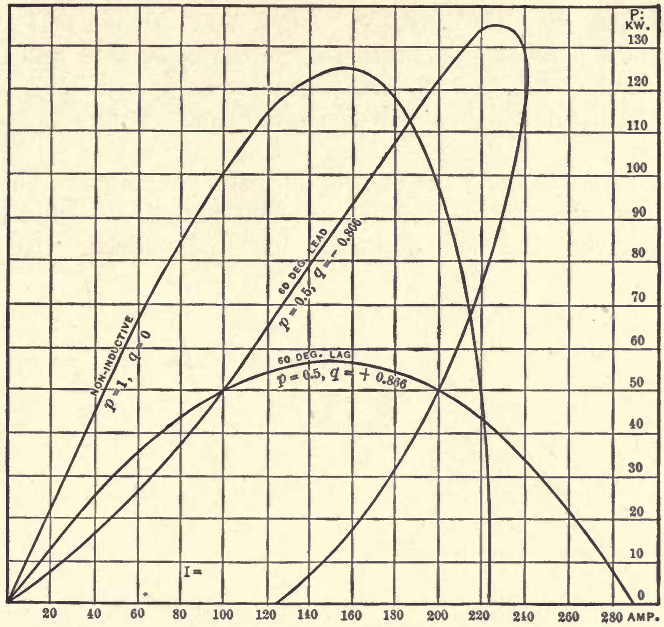


FIG. 61.—Synchronous generator load curves.

VII. Synchronous Motor

16. As seen in the preceding, in an alternating-current generator the field excitation required for a given terminal voltage and current depends upon the phase relation of the external circuit or the load. Inversely, in a synchronous motor the phase relation of the current into the armature at a given terminal voltage depends upon the field excitation and the load.

Thus, if E = terminal voltage or impressed e.m.f., I = current, θ = lag of current behind impressed e.m.f. in a synchronous motor of resistance r and synchronous reactance x_0, the polar diagram is as follows, Fig. 62.

$\overline{OE} = E$ is the terminal voltage assumed as zero vector. The current $\overline{OI} = I$ lags by the angle $EOI = \theta$.

The e.m.f. consumed by resistance is $\overline{OE'}_1 = Ir$. The e.m.f. consumed by synchronous reactance, $\overline{OE'}_0 = Ix_0$. Thus, combining $\overline{OE'}_1$ and $\overline{OE'}_0$ gives $\overline{OE'}$, the e.m.f. consumed by the synchronous impedance. The e.m.f. consumed by the synchronous impedance $\overline{OE'}$ and the e.m.f. consumed by the nominal generated or counter e.m.f. of the synchronous motor $\overline{OE}_0$, combined, give the impressed e.m.f. $\overline{OE}$. Hence $\overline{OE}_0$ is one side of a parallelogram, with $\overline{OE'}$ as the other side, and $\overline{OE}$ as diagonal. $\overline{OE}_{00}$ (not shown), equal and opposite $\overline{OE}_0$, would thus be the nominal counter-generated e.m.f. of the synchronous motor.

In Figs. 63 to 65 are shown the polar diagrams of the synchronous motor for $\theta = 0$ deg., $\theta = 60$ deg., $\theta = -60$ deg. It is seen that the field excitation has to be higher with lead-

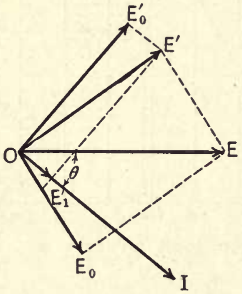


FIG. 62.—Vector diagram of synchronous motor.

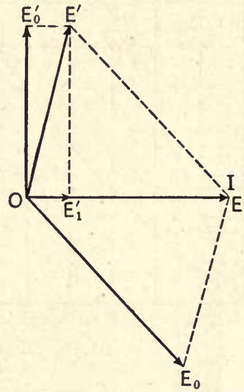


FIG. 63.—Vector diagram of synchronous motor. $\theta = 0$

ing and lower with lagging current in a synchronous motor, while the opposite is the case in an alternating-current generator.

In symbolic representation, by resolving all e.m.fs. into power components in phase with the current and wattless components in quadrature with the current i, we have:

the terminal voltage, $\dot{E} = E \cos \theta + jE \sin \theta = Ep + jEq$;

the e.m.f. consumed by resistance, $\dot{E}'_1 = ir$,

and the e.m.f. consumed by synchronous reactance, $\dot{E}'_0 = + jix_0$.

Thus the e.m.f. consumed by the nominal counter-generated e.m.f. is

$$\dot{E}_0 = \dot{E} - \dot{E}'_1 - \dot{E}'_0 = (E \cos \theta - ir) + j(E \sin \theta - ix_0)$$
$$= (Ep - ir) + j(Eq - ix_0);$$

or, in absolute values,

$$E_0 = \sqrt{(E\cos\theta - ir)^2 + (E\sin\theta - ix_0)^2}$$
$$= \sqrt{(Ep - ir)^2 + (Eq - ix_0)^2};$$

hence,

$$E = i\,(rp + x_0q) \pm \sqrt{E_0^2 - i^2\,(x_0p - rq)^2}.$$

The power consumed by the synchronous motor is

$$P = iEp;$$

that is, the current times the power component of the impressed e.m.f.

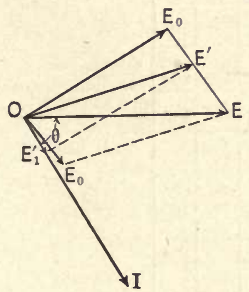


FIG. 64.—Vector diagram of synchronous motor. $\theta = 60$ deg.

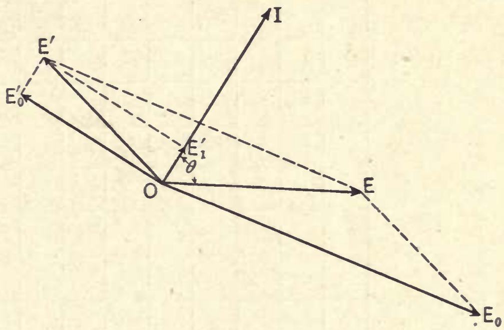


FIG. 65.—Vector diagram of synchronous motor. $\theta = -60$ degrees.

The mechanical power delivered by the synchronous motor armature is

$$P_0 = i\,(Ep - ir);$$

that is, the current times the power component of the nominal counter-generated e.m.f. Obviously to get the available mechanical power, the power consumed by mechanical friction and by molecular magnetic friction or hysteresis, and the power of field excitation, have to be subtracted from this value P_0.

VIII. Characteristic Curves of Synchronous Motor

17. In Fig. 66 are shown, at constant impressed e.m.f. E, the nominal counter-generated e.m.f. E_0 and thus the field excitation F_0 required,

1. At no phase displacement, $\theta = 0$, or for the condition of minimum input;

2. For $\theta = +60$, or 60 deg. lag: $p = 0.5$, $q = +0.866$, and
3. For $\theta = -60$, or 60 deg. lead: $p = 0.5$, $q = -0.866$,

with the current I as abscissas, the constants being

$$r = 0.1, \; x_0 = 5, \text{ and } E = 1000.$$

These curves are called the *compounding curves of the synchronous motors.*

In Fig. 67 are shown, with the power output $P_1 = i\,(Ep - ir)$ $-$ (iron loss and friction) as abscissas, and the same constants

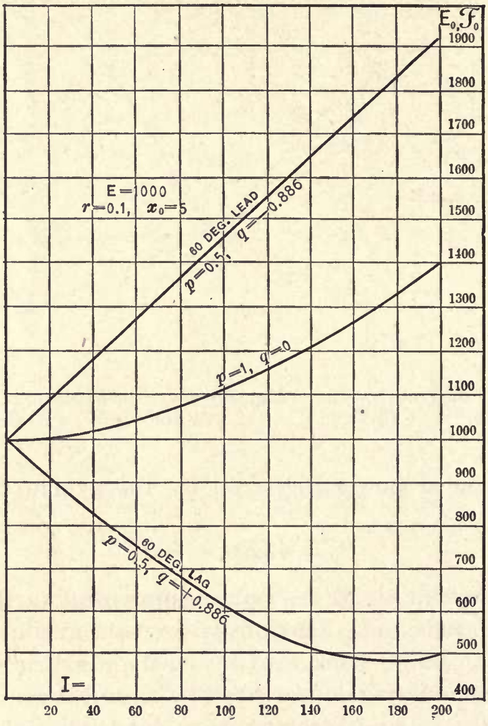


FIG. 66.—Synchronous motor compounding curves.

$r = 0.1$, $x_0 = 5$, $E = 1000$, and constant field excitation F_0; that is, constant nominal counter-generated e.m.f. $E_0 = 1109$ (corresponding to $p = 1$, $q = 0$ at $I = 100$), the values of current I and power-factor p. As iron loss is assumed 3000 watts, as friction 2000 watts. Such curves are called *load characteristics of the synchronous motor.*

18. In Fig. 68 are shown, with constant power output $= P_0$,

$i(Ep - ir)$, and the same constants, $r = 0.1, x_0 = 5, E = 1000$, and with the nominal counter-generated voltage E_0, that is, field excitation F_0, as abscissas, the values of current I for the four conditions,

$P_0 = 5$ kw., or $P_1 = 0$, or no load,
$P_0 = 50$ kw., or $P_1 = 45$ kw., or half load,
$P_0 = 95$ kw., or $P_1 = 90$ kw., or full load,
$P_0 = 140$ kw., or $P_1 = 135$ kw., or 150 per cent. of load.

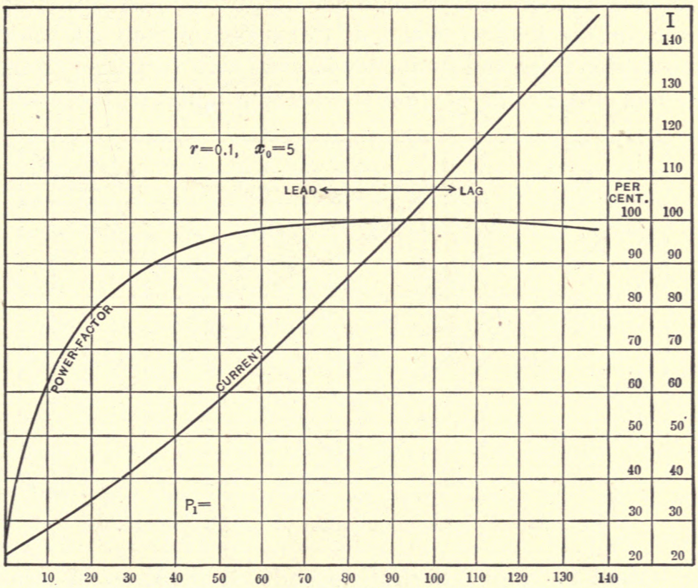


FIG. 67.—Synchronous motor load characteristics.

Such curves are called *phase characteristics of the synchronous motor.*

We have

$$P_0 = iEp - i^2r.$$

Hence,

$$p = \frac{P_0 + i^2r}{iE}, \quad q = \sqrt{1 - p^2}.$$

$$E_0 = \sqrt{(Ep - ir)^2 + (Eq - ix_0)^2}.$$

Similar phase characteristics exist also for the synchronous generator, but are of less interest. It is seen that on each of the four-phase characteristics a certain field excitation gives minimum current, a lesser excitation gives lagging current, a greater excitation leading current. The higher the synchronous reactance x_0, and thus the armature reaction of the synchronous motor, the flatter are the phase characteristics; that is, the less sensitive is the synchronous motor for a change of field excitation or of impressed e.m.f. Thus a relatively high armature reaction is desirable in a synchronous motor to secure stability, that is, independence of minor fluctuations of impressed voltage or of field excitation.

19. The theoretical maximum output of the synchronous motor, or the load at which it drops out of step, at constant impressed voltage and frequency is, even with very high armature reaction, usually far beyond the heating limits of the machine.

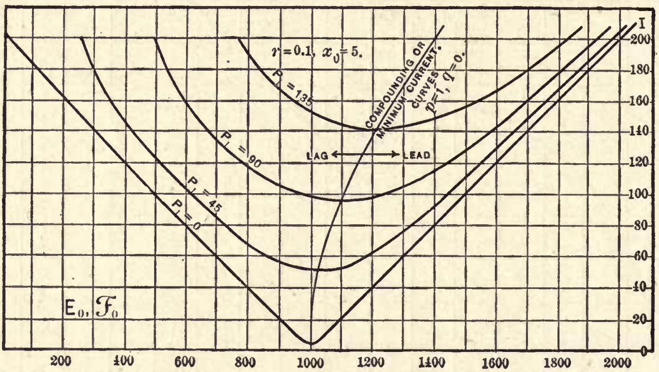


FIG. 66.—Synchronous motor phase characteristics.

The actual maximum output depends on the drop of terminal voltage due to the increase of current, and on the steadiness or uniformity of the impressed frequency, thus upon the individual conditions of operation, but is as a rule far above full load.

Hence, by varying the field excitation of the synchronous motor the current can be made leading or lagging at will, and the synchronous motor thus offers the simplest means of producing out of phase or wattless currents for controlling the voltage in transmission lines, compensating for wattless currents of induction motors, etc. Synchronous machines used merely for supplying wattless currents, that is, synchronous motors or generators running light, with over-excited or under-excited field, are called synchronous condensers. They are used as exciters for induction generators, as compensators for the reactive lagging currents of induction motors, for voltage control of transmission lines, etc. Sometimes they are called "rotary condensers" or "dynamic condensers" when used only for producing leading currents.

IX. Magnetic Characteristic or Saturation Curve

20. The dependence of the generated e.m.f., or terminal voltage at open circuit, upon the field excitation is called the *magnetic characteristic,* or *saturation curve,* of the synchronous

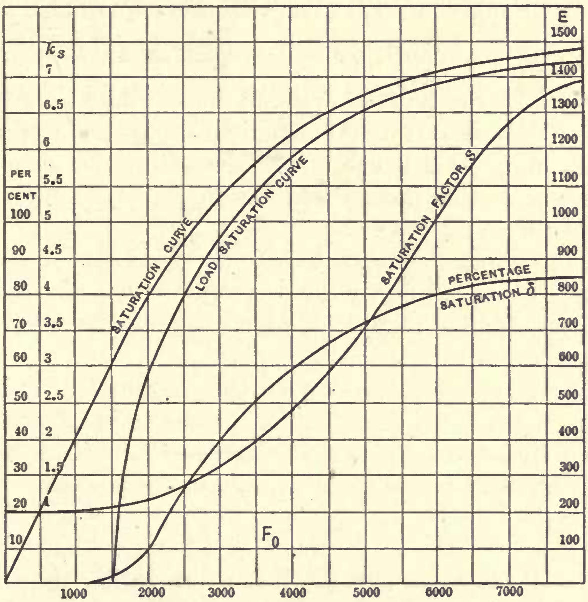


FIG. 69.—Synchronous generator magnetic characteristics.

machine. It has the same general shape as the curve of magnetic flux density, consisting of a straight part below saturation, a bend or knee, and a saturated part beyond the knee. Generally the change from the unsaturated to the over-saturated portion of the curve is more gradual; thus the knee is less pronounced in the magnetic characteristic of the synchronous machines, since the different parts of the magnetic circuit approach saturation successively.

The dependence of the terminal voltage upon the field excitation, at constant full-load current through the amature into a non-inductive circuit, is called the *load saturation curve* of the synchronous machine. It is a curve approximately parallel to the no-load saturation curve, but starting at a definite value of field excitation for zero terminal voltage, the field excitation required to maintain full-load current through the armature against its synchronous impedance.

The ratio $$\frac{dF}{F} \div \frac{dE}{E}$$

is called the *saturation factor* s of the machine. It gives the ratio of the proportional change of field excitation required for a change of voltage. The quantity $\delta = 1 - \frac{1}{s}$ is called the *percentage saturation* of the machine, as it shows the approach to saturation.

In Fig. 69 is shown the magnetic characteristic or no-load saturation curve of a synchronous generator, the load saturation curve and the no-load saturation factor, assuming $E = 1000$, $I = 100$ as full-load values.

In the preceding the characteristic curves of synchronous machines were discussed under the assumption that the saturation curve is a straight line; that is, the synchronous machines working below saturation.

21. The effect of saturation on the characteristic curves of synchronous machines is as follows: The compounding curve is impaired by saturation; that is, a greater change of field excitation is required with changes of load. Under load the magnetic density in the armature corresponds to the true generated e.m.f. E_1, the magnetic density in the field to the virtual generated e.m.f. E_2. Both, especially the latter, are higher than the no-load e.m.f. or terminal voltage E of the generator, and thus a greater increase of field excitation is required in the presence of saturation than in the absence thereof. In addition thereto, due to the counter m.m.f. of the armature current, the magnetic stray field, that is, that magnetic flux which leaks from field pole to field pole through the air, increases under load, especially with inductive load where the armature m.m.f. directly opposes the field, and thus a still further increase of density is required in the field magnetic circuit under load. In consequence thereof, at high saturation the load saturation curve differs more from the no-load saturation curve than corresponds to the synchronous impedance of the machine.

The regulation becomes better by saturation; that is, the increase of voltage from full load to no load at constant field excitation is reduced, the voltage being limited by saturation. Owing to the greater difference of field excitation between no load and full load in the case of magnetic saturation, the improvement in regulation is somewhat reduced.

X. Efficiency and Losses

22. Besides the above described curves the efficiency curves are of interest. The efficiency of alternators and synchronous motors is usually so high that a direct determination by measuring the mechanical power and the electric power is less reliable than

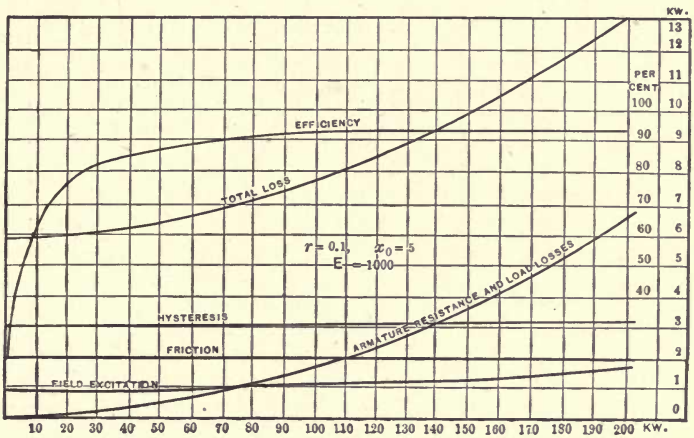


Fig. 70.—Synchronous generator, efficiency and losses.

the method of adding the losses, and the latter is therefore commonly used.

The losses consist of the following: the resistance loss in the armature; the resistance loss in the field circuit; the hysteresis and eddy current losses in the magnetic circuit; the friction and windage losses, and eventually load losses, that is, losses due to eddy currents and hysteresis produced by the load current in the armature.

The resistance loss in the armature is proportional to the square of the current, I.

The resistance loss in the field circuit is proportional to the square of the field excitation current, that is, the square of the nominal generated or counter-generated e.m.f., E_0.

10

The hysteresis loss is proportional to the 1.6th power of the real generated e.m.f., $E_1 = E \pm Ir$.

The eddy current loss is usually proportional to the square of the generated e.m.f., E_1.

The friction and windage loss is assumed as constant.

The load losses vary more or less proportionally to the square of the current in the armature, and should be small with proper design. They can often be represented by an "effective" armature resistance.

Assuming in the preceding example a friction loss of 2000 watts; an iron loss of 3000 watts, at the generated e.m.f. $E_1 = 1000$; a

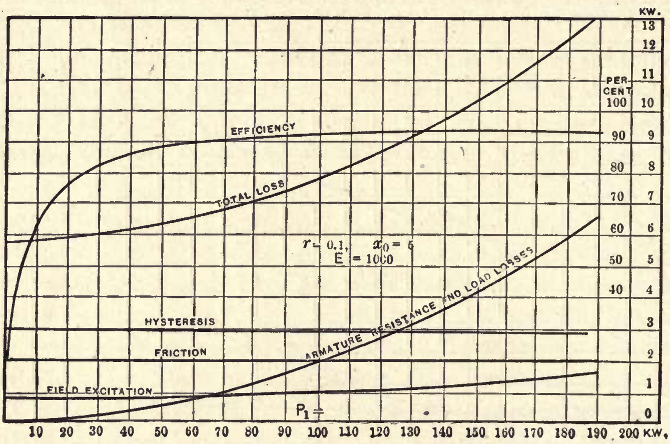


FIG. 71.—Synchronous motor efficiency and losses.

resistance loss in the field circuit of 800 watts, at $E_0 = 1000$, and a load loss at full load of 600 watts.

The loss curves and efficiency curves are plotted in Fig. 70 for the generator, with the current output at non-inductive load or $\theta = 0$ as abscissas, and in Fig. 71 for the synchronous motor, with the mechanical power output as abscissas.

XI. Unbalancing of Polyphase Synchronous Machines

23. The preceding discussion applies to polyphase as well as single-phase machines. In polyphase machines the nominal generated e.m.fs. or nominal counter-generated e.m.fs. are necessarily the same in all phases (or bear a constant relation to each other). Thus in a polyphase generator, if the current or the phase relation of the current is different in the different branches, the terminal voltage must become different also, more or less. This is called the unbalancing of the polyphase generator. It is due to different load or load of different inductance factor in the different branches.

Inversely, in a polyphase synchronous motor, if the terminal voltages of the different branches are unequal, due to an unbalancing of the polyphase circuit, the synchronous motor takes more current or lagging current from the branch of higher voltage, and thereby reduces its voltage, and takes less current or leading current[1] from the branch of lower voltage, or even returns current into this branch, and thus raises its voltage. Hence a synchronous motor tends to restore the balance of an unbalanced polyphase system; that is, it reduces the unbalancing of a polyphase circuit caused by an unequal distribution or unequal phase relation of the load on the different branches. To a less degree the induction motor possesses the same property.

XII. Starting of Synchronous Motors

24. In starting, an essential difference exists between the single-phase and the polyphase synchronous motor, in so far as the former is not self-starting but has to be brought to complete synchronism, or in step with the generator, by external means before it can develop torque, while the polyphase synchronous motor starts from rest and runs up to synchronism with more or less torque.

In starting, the field excitation of the polyphase synchronous motor should be zero or very low.

The starting torque is due to the magnetic attraction of the armature currents upon the remanent magnetism left in the field poles by the currents of the preceding phase, and to the eddy currents produced therein.

Let Fig. 72 represent the magnetic circuit of a polyphase synchronous motor. The m.m.f. of the polyphase armature currents acting upon the successive projections or teeth of the armature, 1, 2, 3, etc., reaches a maximum in them successively; that is, the armature is the seat of a m.m.f. rotating synchronously in the direction of the arrow A. The magnetism in the

[1] Since with lower impressed voltage the current is leading, with higher impressed voltage lagging, in a synchronous motor.

face of the field pole opposite to the armature projections lags behind the m.m.f., due to hysteresis and eddy currents, and thus is still remanent, while the m.m.f. of the projection 1 decreases, and is attracted by the rising m.m.f. of projection 2, etc., or, in other words, while the maximum m.m.f. in the armature has a position *a*, the maximum magnetism in the field-pole face still has the position *b*, and is thus attracted toward *a*, causing the field to revolve in the direction of the arrow *A* (or with a stationary field, the armature to revolve in the opposite direction *B*).

Lamination of the field poles reduces the starting torque caused by eddy currents in the field poles, but increases that caused by remanent magnetism or hysteresis, due to the higher permeability of the field poles. Thus the torque per volt-ampere input is approximately the same in either case, but with laminated

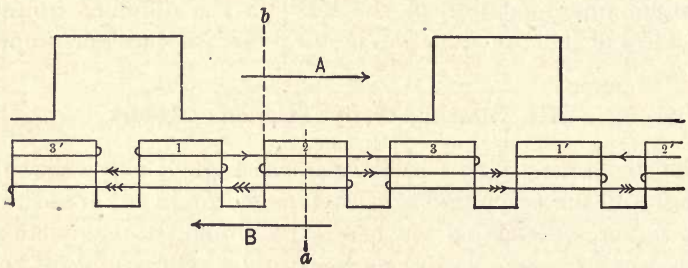


FIG. 72.—Magnetic circuit of a polyphase synchronous motor.

poles the impressed voltage required in starting is higher and the current lower than with solid field poles. In either case, at full impressed e.m.f. the starting current of a synchronous motor is large, since in the absence of a counter e.m.f. the total impressed e.m.f. has to be consumed by the impedance of the armature circuit. Since the starting torque of the synchronous motor is due to the magnetic flux produced by the alternating armature currents, or the armature reaction, synchronous motors of high armature reaction are superior in starting torque.

Very frequently in synchronous motors a squirrel-cage winding is used in the field pole faces, to give powerful starting torque by the induced currents therein, on the induction motor principle. Such squirrel-cage winding should have fairly high resistance to start well from rest, but low resistance to give powerful synchronizing, that is, to pull its load promptly into synchronism.

XIII. Parallel Operation

25. Any alternator can be operated in parallel, or synchronized with any other alternator. A single-phase machine can be synchronized with one phase of a polyphase machine, or a quarter-phase machine operated in parallel with a three-phase machine by synchronizing one phase of the former with one phase of the latter. Since alternators in parallel must be in step with each other and have the same terminal voltage, the condition of satisfactory parallel operation is that the frequency of the machines is identically the same, and the field excitation such as would give the same terminal voltage. If this is not the case, there will be cross currents between the alternators in a local circuit; that is, the alternators are not without current at no load, and their currents under load are not of the same phase and proportional to their respective capacities. The cross currents between alternators when operated in parallel can be wattless currents or power currents.

If the frequencies of two alternators are identically the same, but the field excitation not such as would give equal terminal voltage when operated in parallel, there is a local current between the two machines which is wattless and leading or magnetizing in the machine of lower field excitation, lagging or demagnetizing in the machine of higher field excitation. At load this wattless current is superimposed upon the currents from the machines into the external circuit. In consequence thereof the current in the machine of higher field excitation is lagging behind the current in the external circuit, the current in the machine of lower field excitation leads the current in the external circuit. The currents in the two machines are thus out of phase with each other, and their sum larger than the joint current, or current in the external circuit. Since it is the armature reaction of leading or lagging current which makes up the difference between the impressed field excitation and the field excitation required to give equal terminal voltage, it follows that the lower the armature reaction, that is, the closer the regulation of the machines, the more sensitive they are for inequalities or variations of field excitation. Thus, too low armature reaction is undesirable for parallel operation.

With identical machines the changes in field excitation required for changes of load must be the same. With machines of different compounding curves the changes of field excitation for varying load must be different, and such as correspond to their respective compounding curves, if wattless currents shall be avoided. With machines of reasonable armature reaction the wattless cross currents are small even with relatively great inequality of field excitation. Machines of high armature reaction have been operated in parallel under circumstances where one machine was entirely without field excitation, while the other carried twice its normal field excitation, with wattless currents, however, of the same magnitude as full-load current.

XIV. Division of Load in Parallel Operation

26. Much more important than equality of terminal voltage before synchronizing is equality of frequency. Inequality of frequency, or rather a tendency to inequality of frequency (since by necessity the machines hold each other in step or at equal frequency), causes cross currents which transfer energy from the machine whose driving power tends to accelerate to the machine whose driving power tends to slow down, and thus relieves the latter by increasing the load on the former. Thus these cross currents are power currents, and cause at no load or light load the one machine to drive the other as synchronous motor, while under load the result is that the machines do not share the load in proportion to their respective capacities.

The speed of the prime mover, as steam engine or turbine, changes with the load. The frequency of alternators driven thereby must be the same when in parallel. Thus their respective loads are such as to give the same speed of the prime mover (or rather a speed corresponding to the same frequency). Hence the division of load between alternators connected to independent prime movers depends almost exclusively upon the speed regulation of the prime movers. To make alternators divide the load in proportion to their capacities, the speed regulation of their prime movers must be the same; that is, the engines or turbines must drop in speed from no load to full load by the same percentage and in the same manner.

If the regulation of the prime movers is not the same, the load is not divided proportionally between the alternators, but the alternator connected to the prime mover of closer speed regulation takes more than its share of the load under heavy loads, and less under light loads. Thus, too close speed regulation of prime movers is not desirable in parallel operation of alternators.

XV. Fluctuating Cross Currents in Parallel Operation

27. In alternators operated from independent prime movers, it is not sufficient that the average frequency corresponding to the average speed of the prime movers be the same, but still more important that the frequency be the same at any instant, that is, that the frequency (and thus the speed of the prime mover) be constant. In rotary prime movers, as turbines or electric motors, this is usually the case; but with reciprocating machines, as steam engines, the torque and thus the speed of rotation rises and falls periodically during each revolution, with the frequency of the engine impulses. The alternator connected with the engine will thus not have uniform frequency, but a frequency which pulsates, that is, rises and falls. The amplitude of this pulsation depends upon the design of the engine, the momentum of its fly-wheel, and the action of the engine governor.

If two alternators directly connected to equal steam engines are synchronized so that the moments of maximum frequency coincide, there will be no energy cross currents between the machines, but the frequency of the whole system rises and falls periodically. In this case the engines are said to be synchronized. The parallel operation of the alternators is satisfactory in this case provided that the pulsations of engine speeds are of the same size and duration; but apparatus requiring constant frequency, as synchronous motors and rotary converters, when operated from such a system, will give a reduced maximum output, due to periodic cross currents between the generators of fluctuating frequency and the synchronous motors of constant frequency, and in an extreme case the voltage of the whole system will be caused to fluctuate periodically. Even with small fluctuations of engine speed the unsteadiness of current due to this source is noticeable in synchronous motors and synchronous converters.

If the alternators happen to be synchronized in such a position that the moment of maximum speed of the one coincides with the moment of minimum speed of the other, alternately the one and then the other alternator will run ahead, and thus there will be a pulsating power cross current between the alternators, transferring power from the leading to the lagging machine, that is, alternately from the one to the other, and inversely, with the frequency of the engine impulses. These pulsating cross currents are the most undesirable, since they tend to make the voltage fluctuate and to tear the alternators out of synchronism with each other, especially when the conditions are favorable to a cumulative increase of this effect by what may be called mechanical resonance (hunting) of the engine governors, etc. They depend upon the synchronous impedance of the alternators and upon their phase difference, that is, the number of poles and the fluctuation of speed, and are specially objectionable when operating synchronous apparatus in the system.

28. Thus, for instance, if two 80-pole alternators are directly connected to single-cylinder engines of 1 per cent. speed variation per revolution, twice during each revolution the speed will rise, and fall twice; and consequently the speed of each alternator will be above average speed during a quarter revolution. Since the maximum speed is 1/2 per cent. above average, the mean speed during the quarter revolution of high speed is 1/4 per cent. above average speed, and by passing over 20 poles the armature of the machine will during this time run ahead of its mean position by 1/4 per cent. of 20 or 1/20 pole, that is, 180/20 = 9 electrical space degrees. If the armature of the other alternator at this moment is behind its average position by 9 electrical space degrees, the phase displacement between the alternator e.m.fs. is 18 electrical time degrees; that is, the alternator e.m.fs. are represented by $\overline{OE_1}$ and $\overline{OE_2}$ in Fig. 71, and when running in parallel the e.m.f. $\overline{OE'} = \overline{E_1E_2}$ is short circuited through the synchronous impedance of the two alternators.

Since $E' = \overline{OE_1} = 2\,E_1 \sin 9$ deg. the maximum cross current is

$$I' = \frac{E_1 \sin 9 \text{ deg.}}{z_0} = \frac{0.156\,E_1}{z_0} = 0.156\,I_0,$$

where $I_0 = \dfrac{E_1}{z_0}$ = short-circuit current of the alternator at full-load excitation. Thus, if the short-circuit current of the alternator is only twice full-load current, the cross current is 31.2 per cent. of full-load current. If the short-circuit current is 6 times full-load current, the cross current is 93.6 per cent. of full-load current or practically equal to full-load current. Thus the smaller the armature reaction, or the better the regulation, the larger are the pulsating cross currents between the alternators, due to the inequality of the rate of rotation of the prime movers. Hence for satisfactory parallel operation of alternators connected to steam engines, a certain amount of armature reaction is desirable and very close regulation undesirable.

By the transfer of energy between the machines the pulsations of frequency, and thus the cross currents, are reduced somewhat. Very high armature reaction is objectionable also, since it reduces the synchronizing power, that is, the tendency of the machines to hold each other in step, by reducing the energy transfer between the machines. As seen herefrom, the problem of parallel operation of alternators is almost entirely a problem of the regulation of their prime movers, especially steam engines.

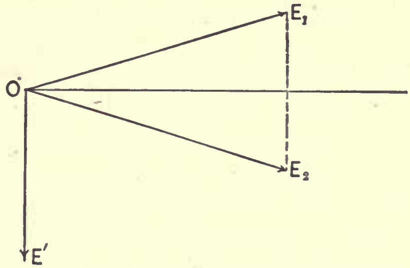


Fig. 73.—Phase displacement between alternators to be synchronized.

With alternators driven by gas engines, the problem of parallel operation is made more difficult by the more jerky nature of the gas-engine impulse. In such machines, therefore, squirrel-cage windings in the field-pole faces are commonly used, to assist synchronizing by the currents induced in this short-circuited winding, on the principle of the induction machine.

From Fig. 73 it is seen that the e.m.f. $\overline{OE'}$ or $\overline{E_1E_2}$, which causes the cross current between two alternators in parallel connection, if their e.m.fs. $\overline{OE_1}$ and $\overline{OE_2}$ are out of phase, is approximately in quadrature with the e.m.fs. $\overline{OE_1}$ and $\overline{OE_2}$ of the machines, if these latter two e.m.fs. are equal to each other. The cross current between the machines lags behind the e.m.f. producing it, $\overline{OE'}$, by the angle ω, where $\tan \omega = \frac{x_0}{r_0}$, and $x_0 =$ reactance, $r_0 =$ effective resistance of alternator armature. The energy component of this cross current, or component in phase with $\overline{OE'}$, is thus in quadrature with the machine voltages $\overline{OE_1}$ and $\overline{OE_2}$, that is, transfers no power between them. The power transfer or equalization of load between the two machines takes place by the wattless or reactive component of cross current, that is, the component which is in quadrature with OE', and thus in phase with one and in opposition with the other of the machine e.m.fs. $\overline{OE_1}$ and $\overline{OE_2}$.

29. Hence, machines without reactance would have no synchronizing power, or could not be operated in parallel. The theoretical maximum synchronizing power exists if the reactance equals the resistance: $x_0 = r_0$. This condition, however, cannot be realized, and if realized would give a dangerously high synchronizing power and cross current. In practice, x_0 is always very much greater than r_0, and the cross current thus practically in quadrature with $\overline{OE'}$, that is, in phase (or opposition) with the machine voltages, and is consequently an energy-transfer current.

If, however, alternators are operated in parallel over a circuit of appreciable resistance, as two stations at some distance from each other are synchronized, especially if the resistance between the stations is non-inductive, as underground cables, with alternators of very low reactance, as turbo alternators, the synchronizing power may be insufficient. In this case, reactance has to be inserted between the stations, to lag the cross current and thereby make it a power-transferring or synchronizing current.

If, however, the machine voltages $\overline{OE_1}$ and $\overline{OE_2}$ are different in value but approximately in phase with each other, the voltage causing cross currents, $\overline{E_1E_2}$, is in phase with the machine voltages and the cross currents thus in quadrature with the machine voltages $\overline{OE_1}$ and $\overline{OE_2}$, and hence do not transfer energy, but are wattless. In one machine the cross current is a lagging or demagnetizing, and in the other a leading or magnetizing, current.

Hence two kinds of cross currents may exist in parallel operation of alternators—currents transferring power between the machines, due to phase displacement between their e.m.fs., and wattless currents transferring magnetization between the machines, due to a difference of their induced e.m.fs.

In compound-wound alternators, that is, alternators in which the field excitation is increased with the load by means of a series field excited by the rectified alternating current, it is almost, but not quite, as necessary as in direct-current machines, when operating in parallel, to connect all the series fields in parallel by equalizers of negligible resistance, for the same reason—to insure proper division of current between machines.

XVI. Higher Frequency Cross Currents between Synchronous Machines

30. If several synchronous machines of different wave shapes are connected into the same circuit, cross currents exist between the machines of frequencies which are odd multiples of the circuit frequency, that is, higher harmonics thereof. The machines may be two or more generators, in the same or in different stations, of wave shapes containing higher harmonics of different order, intensity or phase, or synchronous motors or converters of wave shapes different from that of the system to which they are connected.

The intensity of these cross currents is the difference of the corresponding harmonics of the machines divided by the impedance between the machines. This impedance includes the self-inductive reactance of the machine armatures. The reactance obviously is that at the frequency of the harmonic, that is, if x = reactance at fundamental frequency, it is nx for the nth harmonic.

In most cases these cross currents are very small and negligible. With machines of distributed armature winding, the intensity of the harmonic is low, that is, the voltage nearly a sine wave, and with machines of massed armature winding, as unitooth alternators, the reactance is high. These cross currents thus usually are noticeable only at no load, and when adjusting the field excitation of the machines for minimum current. Thus in a synchronous motor or converter, at no load, the minimum current, reached by adjusting the field, while small compared with full-load current, may be several times larger than the minimum point of the "V" curve in Fig. 68, that is, the value of the energy current supplying the losses in the machine.

It is only in the parallel operation of very large high-speed machines (steam turbine driven alternators) of high armature reaction and very low armature self-induction that such high-frequency cross currents may require consideration, and even then only in three-phase Y-connected generators with grounded neutral, as cross currents between the neutrals of the machines. In a three-phase machine, the voltage between the terminals, or delta voltage, contains no third harmonic or its multiple, as the third harmonics of the Y voltage neutralize in the delta voltage, and such a machine, with a terminal voltage of almost sine shape, may contain a considerable third harmonic in the Y voltage. As the three Y voltages of the three-phase system are 120 degrees apart in phase, their third harmonics are 3×120 deg. = 360 deg. apart, or in phase with each other, from the main terminals to the neutral, and by connecting the neutrals of two three-phase machines of different third harmonics with each other, as by grounding the neutrals, a cross current flows between the machines over the neutral, which may reach very high values. Even in machines of the same wave shape, such a triple frequency current appears between the machines over the neutral, when by a difference in field excitation a difference in the phase of the third harmonic is produced. It therefore is often undesirable to ground or connect together, without any resistance, the neutrals of three-phase machines, but in systems of grounded neutral either the neutral should be grounded through separate resistances or grounded only in one machine.

XVII. Short-circuit Currents of Alternators

31. The short-circuit current of an alternator at full-load excitation usually is from two to five times full-load current, and even less in very large high-speed steam turbine alternators. It is

$$I_0 = \frac{E_0}{z_0},$$

where E_0 = nominal generated e.m.f., z_0 = synchronous impedance of alternator, representing the combined effect of armature reaction and armature self-inductance.

In the first moment after short circuiting, however, the current frequently is many times larger than the permanent short-circuit current, that is,

$$I_1 = \frac{E_0}{z},$$

where z = self-inductive impedance of the alternator.

That is, in the first moment after short circuiting the polyphase alternator the armature current is limited only by the armature self-inductance, and not by the armature reaction, and some appreciable time—occasionally several seconds—elapses before the armature reaction becomes effective.

At short circuit, the magnetic field flux is greatly reduced by the demagnetizing action of the armature current, and the generated e.m.f. thereby reduced from the nominal value E_0 to the virtual value E_2; the latter is consumed by the armature self-inductive impedance z, or self-inductive reactance, which is practically the same in most cases.

The armature self-inductance is instantaneous, since the magnetic field rises simultaneously with the armature current which produces it; armature reaction, however, requires an appreciable time to reduce the magnetic flux from the open-circuit value to the much lower short-circuit value, since the magnetic field flux is surrounded by the field exciting coils, which act as a short-circuited secondary opposing a rapid change of field flux; that is, in the moment when the short-circuit current starts it begins to demagnetize the field, and the magnetic field flux therefore begins to decrease; in decreasing, however, it generates an e.m.f. in the field coils, which opposes the change of field flux, that is, increases the field current so as momentarily to maintain the full field flux against the demagnetizing action of the armature reaction. In the first moment the armature current thus rises to the value given by the e.m.f. generated by the full field flux, while the field current rises, frequently to many times its normal value (hence, if circuit breakers are in the field circuit, they may open the circuit). Gradually the field flux decreases, and with it decrease the field current and the armature current to their normal values, at a rate depending on the resistance and the inductance of the field-exciting circuit. The decrease in value of the field flux will be the more rapid the higher the resistance of the field circuit, the slower the higher the inductance, that is, the greater the magnetic flux of the machine. Thus, the momentary short-circuit current of the machine can be made to decrease somewhat more rapidly by increasing the resistance of the field circuit, that is, wasting exciting power in the field rheostat.

In the very first moment the short-circuit current waves are unsymmetrical, as they must simultaneously start from zero in all phases and gradually approach their symmetrical appearance, *i.e.*, in a three-phase machine three currents displaced by 120 degrees. Hereby the field current is made pulsating, with normal or synchronous frequency, that is, with the same frequency as the armature current. This full frequency pulsation gradually dies out and the field current becomes constant with a polyphase short circuit, while with a single-phase short circuit it remains pulsating with double frequency, due to the pulsating armature reaction. In a polyphase short circuit this full frequency pulsation due to the unsymmetrical starting of the currents is independent of the point of the wave at which the short circuit starts, since the resultant asymmetry of all the polyphase currents is the same regardless of the point of the wave at which the circuit is closed. In a single-phase short circuit, however, the full frequency pulsation depends on the point of the wave at which the circuit is closed, and is absent if the circuit is closed at that moment at which the short-circuit current would pass through zero.

The momentary short-circuit current of an alternator thus represents one of the few cases in which armature self-inductance and armature reaction do not act in the same manner, and the synchronous reactance can be split into two components, thus, $x_0 = x + x'$, where x = self-inductive reactance, which is due to a true self-inductance, and x' = effective reactance of armature reaction, which is not instantaneous.

32. In machines of high self-inductance and low armature reaction, as high frequency alternators, this momentary increase of short-circuit current over its normal value is negligible, and moderate in machines in which armature reaction and self-inductance are of the same magnitude, as large modern multipolar low-speed alternators. In large high-speed alternators of high armature reaction and low self-inductance, as steam turbine alternators, the momentary short-circuit current may exceed the permanent value ten or more times. With such large currents magnetic saturation of the self-inductive armature circuit still further reduces the reactance x, that is, increases the current, and in such cases the mechanical shock on the generator becomes so enormous that it is necessary to reduce the momentary short-circuit current by inserting self-inductance, that is, reactance coils into the generator leads, or by specifically designing the alternator for high armature reactance, or by both.

In view of the excessive momentary short-circuit current, it may be desirable that automatic circuit breakers on such systems have a time limit, so as to keep the circuit closed until the short-circuit current has somewhat decreased.

33. In single-phase machines, and in polyphase machines in case of a short circuit on one phase only, the armature reaction is pulsating, and the field current in the first moment after the short circuit therefore pulsates, with double frequency, and remains pulsating even after the permanent condition has been reached. The double frequency pulsation of the field current in case of a single-phase short circuit generates in the armature a third harmonic of e.m.f. The short-circuit current wave becomes greatly distorted thereby, showing the saw-tooth shape characteristics of the third harmonic, and in a polyphase machine on single-phase short circuit, in the phase in quadrature with the short-circuited phase, a very high voltage appears, which is greatly

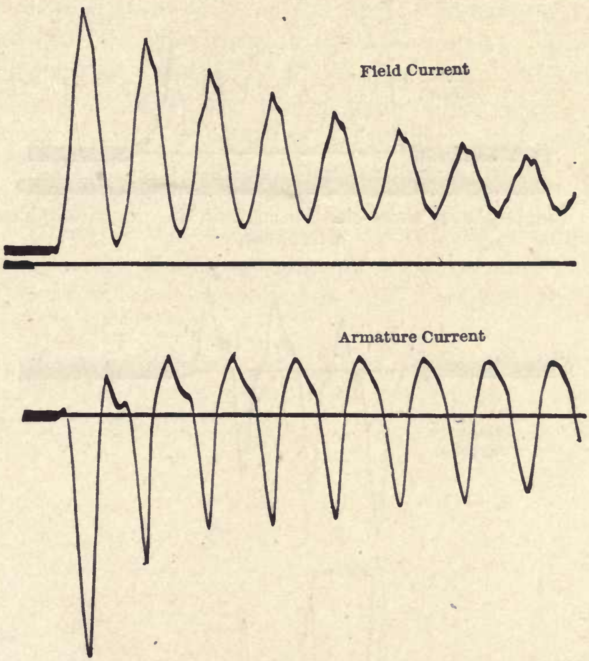


FIG. 74.—Three-phase short-circuit current in a turbo-alternator.

distorted by the third harmonic and may reach several times the value of the open-circuit voltage. Thus, with a single-phase short circuit on a polyphase system, destructive voltages may appear in the open-circuited phase, of saw-tooth wave shape.

Upon this double frequency pulsation of the field current during a single-phase short circuit the transient full frequency pulsation resulting from the unsymmetrical start of the armature current is superimposed and thus causes a difference in the intensity of successive waves of the double frequency pulsation, which gradually disappears with the dying out of the transient full frequency pulsation, and depends upon the point of the wave at which the short circuit is closed, and thus is absent, and the

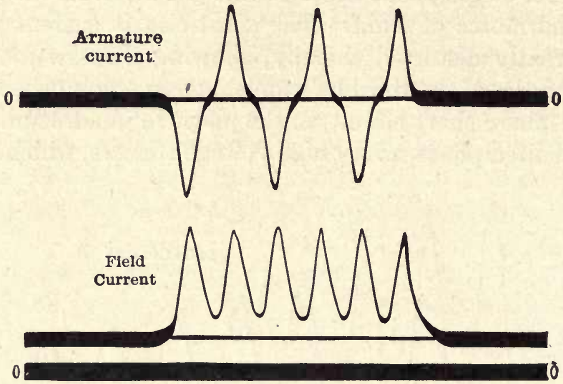


FIG. 75.—Single-phase short-circuit current in a three-phase turbo-alternator.

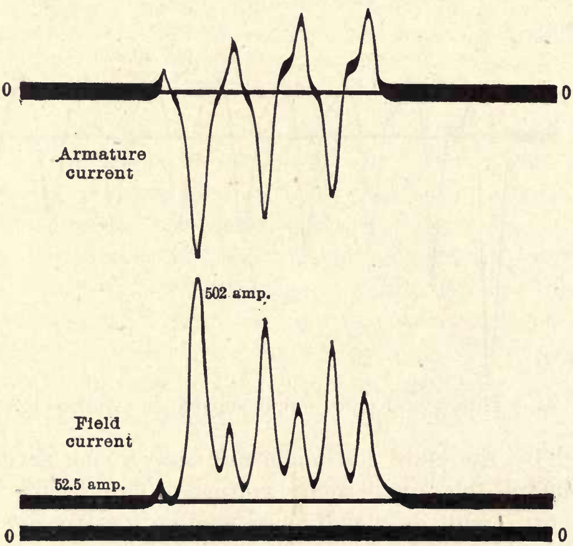


FIG. 76.—Single-phase short-circuit current in a three-phase turbo-alternator.

double frequency pulsation symmetrical, if the circuit is closed at the moment when the short-circuit current should be zero.

34. As illustration is shown, in Fig. 74, the oscillogram of one phase of the three-phase short circuit of a three-phase turbo-alternator, giving the unsymmetrical start of the armature currents and the full frequency pulsation of the field current.

In Fig. 75 is shown a single-phase short circuit of the same machine, in which the circuit is closed at the zero value of the current; the current wave therefore is symmetrical, and the field current shows only the double frequency pulsation due to the single-phase armature reaction.

In Fig. 76 is shown another single-phase short circuit, in which the armature current wave starts unsymmetrical, thus giving a transient full frequency term in the field current. Thus in the double frequency pulsation of the field current at first large and small waves alternate, but the successive waves gradually become equal with the dying out of the full frequency term.

In Figs. 75 and 76 the oscillogram is cut off by the opening of the circuit breaker.

For further discussion, and the theoretical investigation of momentary short-circuit currents, see "Theory and Calculation of Transient Electric Phenomena and Oscillations," Part I, Chapters XI and XII.

For further discussion of the terms reactance, armature reaction and field excitation and their relation, see "Theory and Calculation of Electric Circuits."

B. DIRECT-CURRENT COMMUTATING MACHINES

I. General

35. Commutating machines are characterized by the combination of a continuously excited magnet field with a closed-circuit armature connected to a segmental commutator. According to their use, they can be divided into direct-current generators which transform mechanical power into electric power, direct-current motors which transform electric power into mechanical power, and direct-current converters which transform electric power into a different form of electric power. Since the most important class of the latter are the synchronous converters, which combine features of the synchronous machines with those of the commutating machines, they shall be treated in a separate chapter.

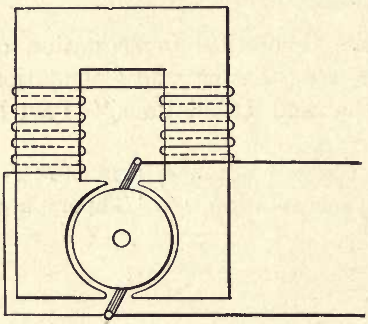


FIG. 77.—Shunt machine.

By the excitation of their magnet fields, commutating machines are divided into magneto machines, in which the field consists of permanent magnets; separately excited machines; shunt machines, in which the field is excited by an electric circuit shunted across the machine terminals, and thus receives a small branch current at full machine voltage, as shown diagrammatically in Fig. 77; series machines, in which the electric field circuit is connected in series with the armature, and thus receives the full machine current at low voltage (Fig. 78); and compound machines, excited by a combination of shunt and series field (Fig. 79). In compound machines the two windings can magnetize either in the same direction (cumulative compounding) or in opposite directions (differential compounding). Differential compounding has been used for constant-speed motors. Magneto machines are used only for very small sizes.

36. By the number of poles commutating machines are divided into bipolar and multipolar machines. Bipolar machines are mainly used in small sizes. By the construction of the armature, commutating machines are divided into smooth-core machines and iron-clad or "toothed" armature machines. In the smooth-core machine the armature winding is arranged on the surface of a laminated iron core. In the iron-clad machine the armature winding is sunk into slots. The iron-clad type has the advantage of greater mechanical strength, but the disadvantage of higher self-inductance in commutation, and thus requires high-resistance, carbon or graphite, commutator brushes. The iron-clad type has the advantage of lesser magnetic stray field, due

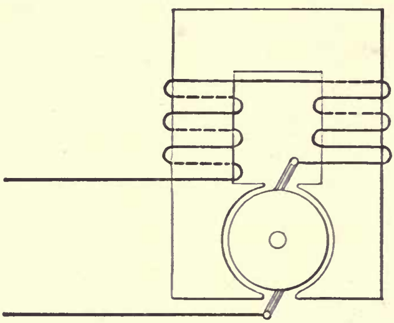


Fig. 78.—Series machine.

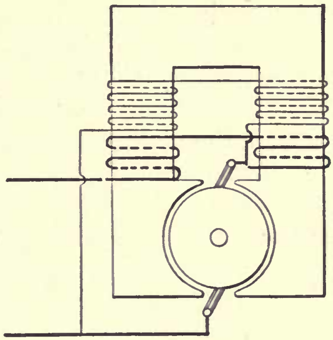


Fig. 79.—Compound machine.

to the shorter gap between field pole and armature iron, and of lesser magnet distortion under load, and thus can with carbon brushes be operated with constant position of brushes at all loads. In consequence thereof, for large multipolar machines the iron-clad type of armature is best adapted; the smooth-core type is hardly ever used nowadays.

Either of these types can be drum wound or ring wound. The drum winding has the advantage of lesser self-inductance and lesser distortion of the magnetic field, and is generally less difficult to construct and thus mostly preferred. By the armature winding, commutating machines are divided into multiple-wound and series-wound machines. The difference between multiple and series armature winding, and their modifications, can best be shown by diagram.

II. Armature Winding

37. Fig. 80 shows a six-pole multiple ring winding, and Fig. 81 a six-polar multiple drum winding. As seen, the armature coils are connected progressively all around the armature in closed circuit, and the connections between adjacent armature coils lead to the commutator. Such an armature winding has as many circuits in multiple, and requires as many sets of commutator brushes, as poles. Thirty-six coils are shown in Figs. 80 and 81, connected to 36 commutator segments, and the two sides of each coil distinguished by drawn and dotted lines. In a drum-wound machine, usually the one side of all coils forms the upper and the other side the lower layer of the armature winding.

Fig. 82 shows a six-pole series drum winding with 36 slots and 36 commutator segments. In the series winding the circuit passes from one armature coil, not to the next adjacent armature coil as in the multiple winding, but first through all the armature coils having the same relative position with regard to the magnet poles of the same polarity, and then to the armature coil next adjacent to the first coil. That is, all armature coils having the same or approximately the same relative position to poles of equal polarity form one set of integral coils. Thus the series winding has only two circuits in multiple, and requires two sets of brushes only, but can be operated also with as many sets of brushes as poles, or any intermediate number of sets of brushes. In Fig. 82, a series winding in which the number of armature coils is divisible by the number of poles, the commutator segments have to be cross connected. Therefore this form of series winding is hardly ever used. The usual form of series winding is the winding shown by Fig. 83. This figure shows a six-polar armature having 35 coils and 35 commutator segments. In consequence thereof the armature coils under corresponding poles which are connected in series are slightly displaced from each other, so that after passing around all corresponding poles the winding leads symmetrically into the coil adjacent to the first armature coil. Hereby the necessity of commutator cross connections is avoided, and the winding is perfectly symmetrical. With this form of series winding, which is mostly used, the number of armature coils must be chosen to follow certain rules. Generally the number of coils is one less or one more than a multiple of the number of poles.

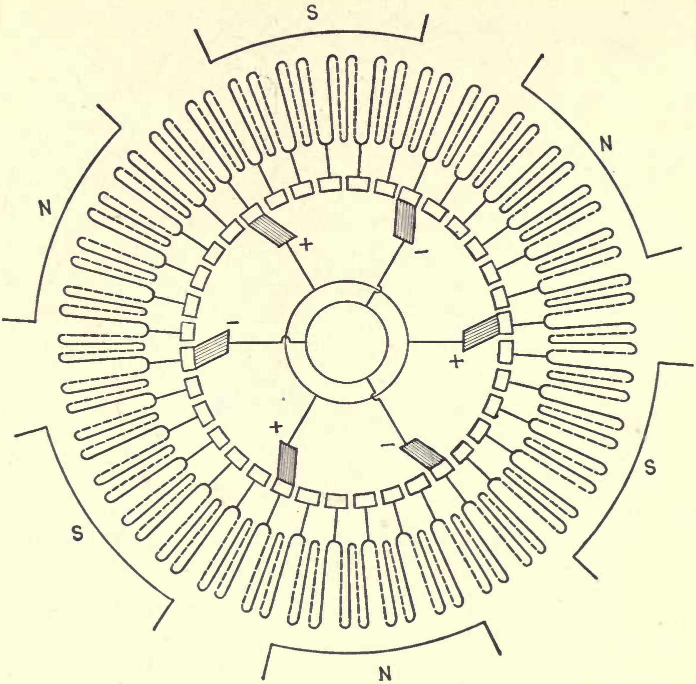


FIG. 80.—Multiple ring armature winding.

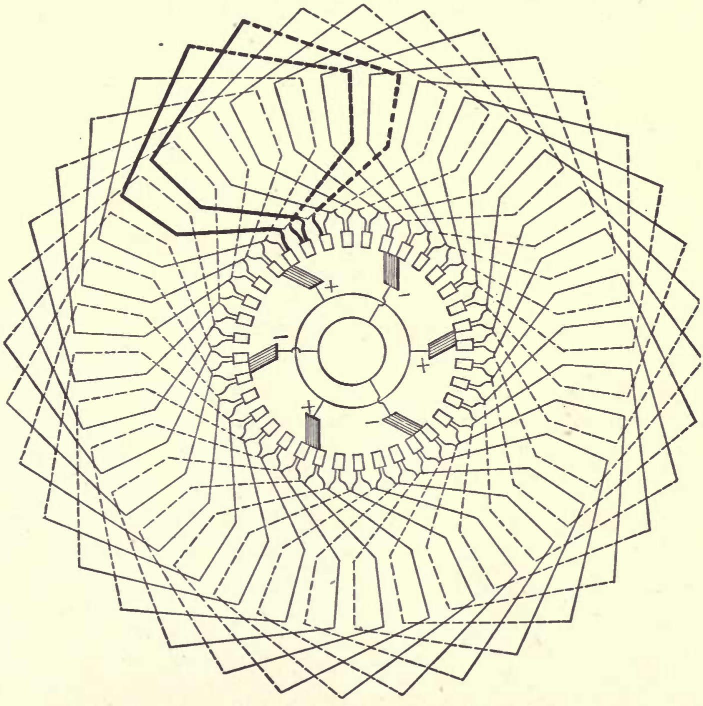


FIG. 81.—Multiple drum full pitch winding.

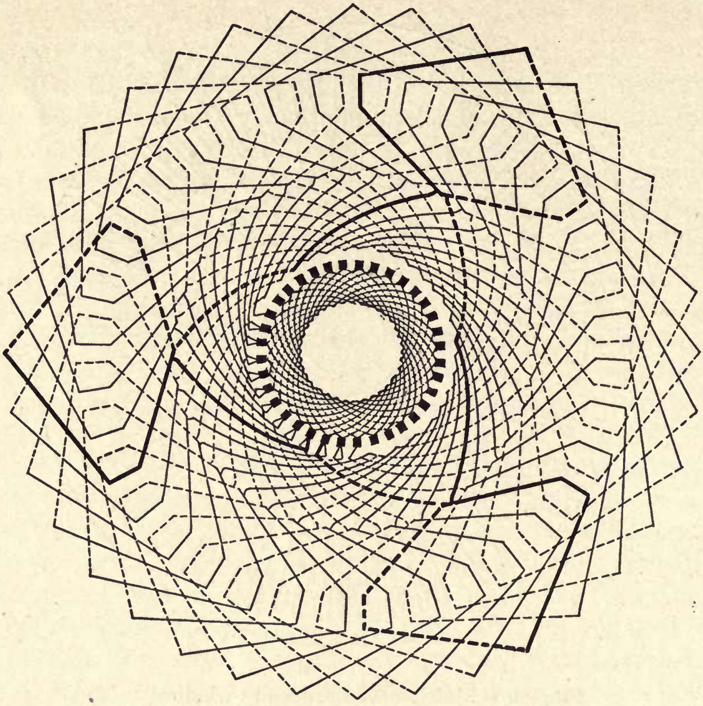


FIG. 82.—Series drum winding with cross-connected commutator.

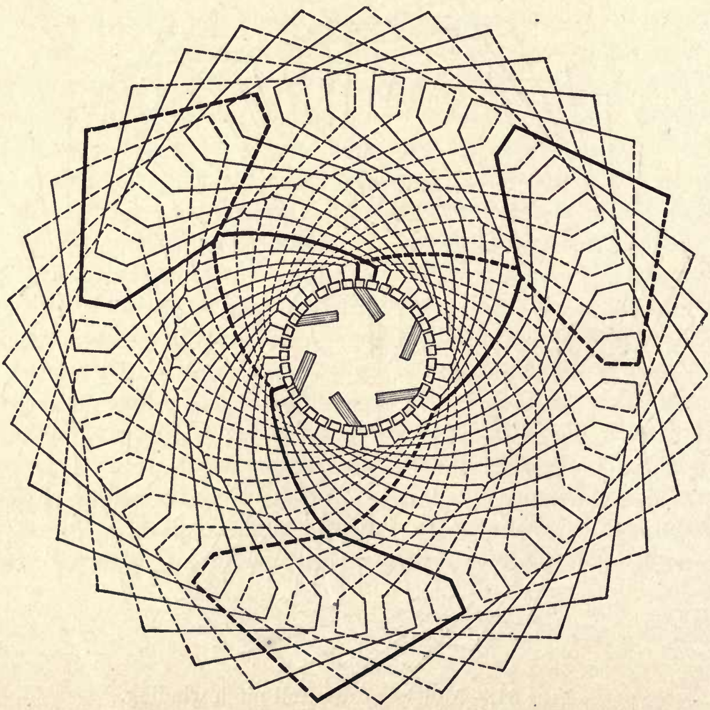


FIG. 83.—Series drum winding.

All these windings are closed-circuit windings; that is, starting at any point, and following the armature conductor, the circuit returns into itself after passing all e.m.fs. twice in opposite direction (thereby avoiding short circuit). An instance of an open-coil winding is shown in Fig. 84, a series-connected three-phase star winding similar to that used in the Thomson-Houston arc machine. Such open-coil windings, however, cannot be used in commutating machines. They are generally preferred in synchronous and in induction machines.

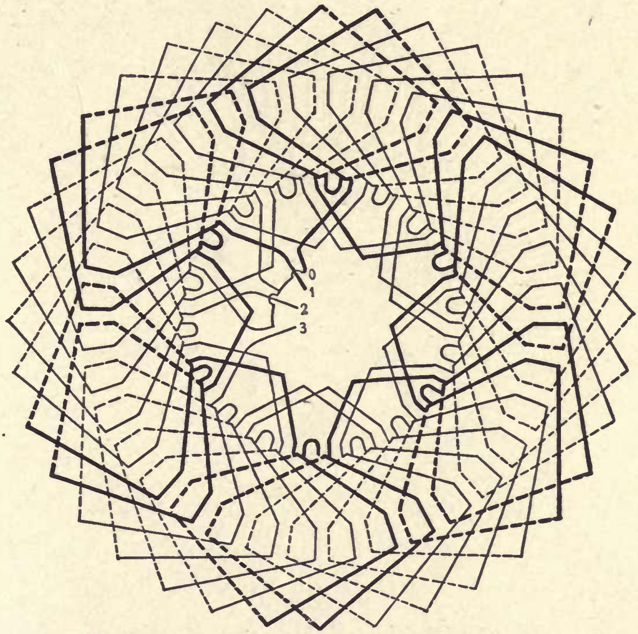


Fig. 84.—Open-circuit three-phase series drum winding.

38. By leaving space between adjacent coils of these windings a second winding can be laid in between. The second winding can either be entirely independent from the first winding, that is, each of the two windings closed upon itself, or after passing through the first winding the circuit enters the second winding, and after passing through the second winding it reenters the first winding. In the first case the winding is called a double spiral winding (or multiple spiral winding if more than two windings are used), in the latter case a double reentrant winding (or multiple reentrant winding). In the double spiral winding the number of coils must be even; in the double reentrant winding, odd.

Multiple spiral and multiple reentrant windings can be either multiple or series wound; that is, each spiral can consist either of a multiple or of a series winding. Fig. 85 shows a double spiral multiple ring winding, Fig. 86 a double spiral multiple drum winding, Fig. 87 a double reentrant multiple drum winding. As seen in the double spiral or double reentrant multiple winding, twice as many circuits as poles are in multiple. Thus such

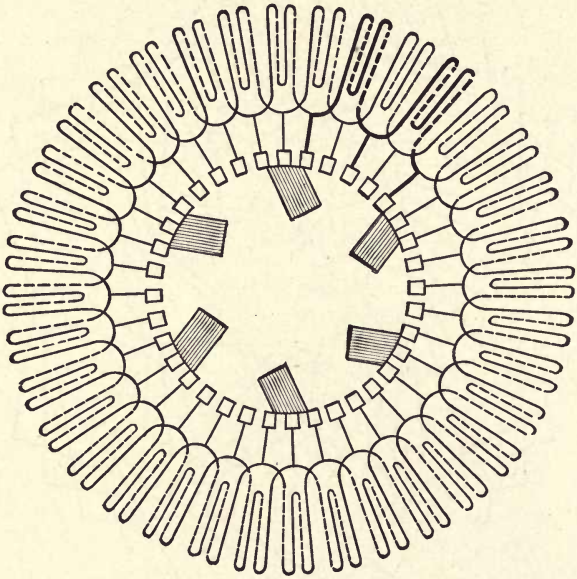


FIG. 85.—Multiple double spiral ring winding.

windings are mostly used for large low-voltage machines, but as very few large direct-current generators are built nowadays, and alternating-current generation with synchronous converters usually preferred, and as multiple spiral or reentrant windings are inconvenient in synchronous converters, their use has greatly decreased.

39. A distinction is frequently made between lap winding and wave winding. These are, however, not different types; but the wave winding is merely a constructive modification of the series drum winding with single-turn coil, as seen by comparing

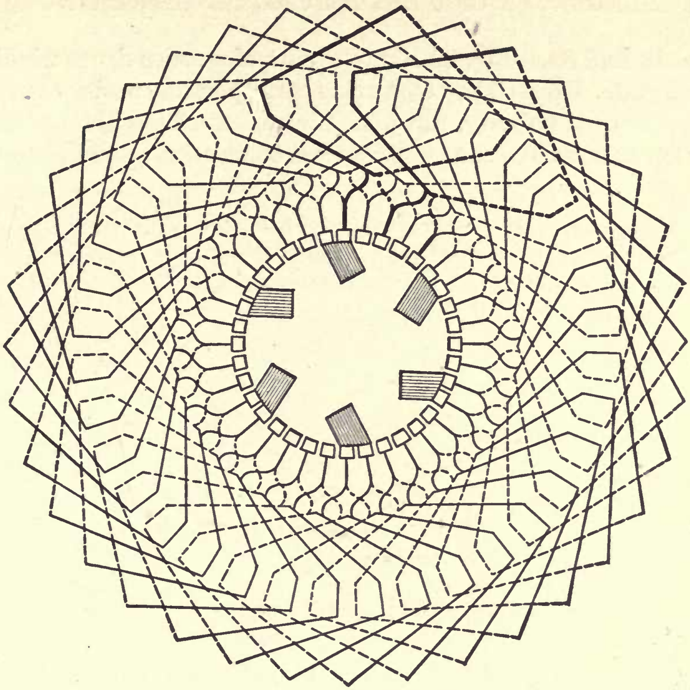


FIG. 86.—Multiple double spiral full pitch winding.

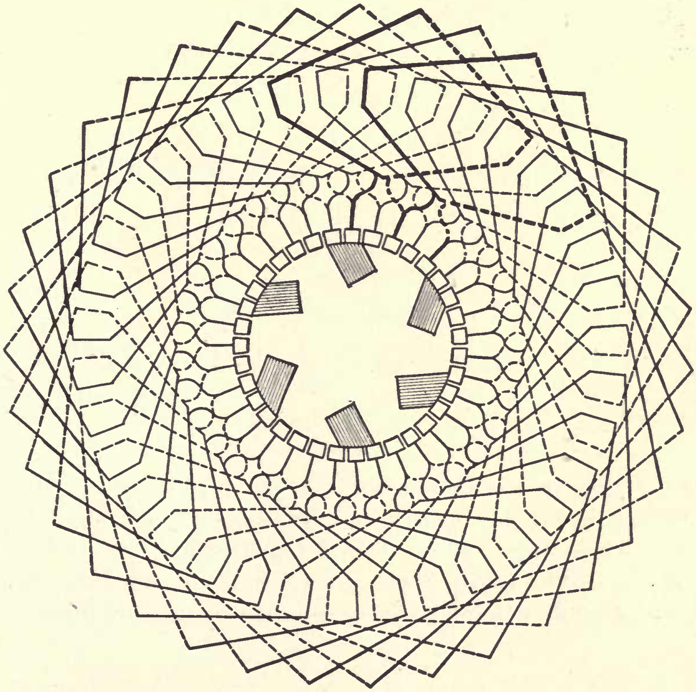


FIG. 87.—Multiple double re-entrant drum full pitch winding.

Figs. 88 and 89. Fig. 88 shows a part of a series drum winding developed. Coils C_1 and C_2, having corresponding positions under poles of equal polarity, are joined in series. Thus the end connection *ab* of coil C_1 connects by cross connection *bc* and *cd* to the

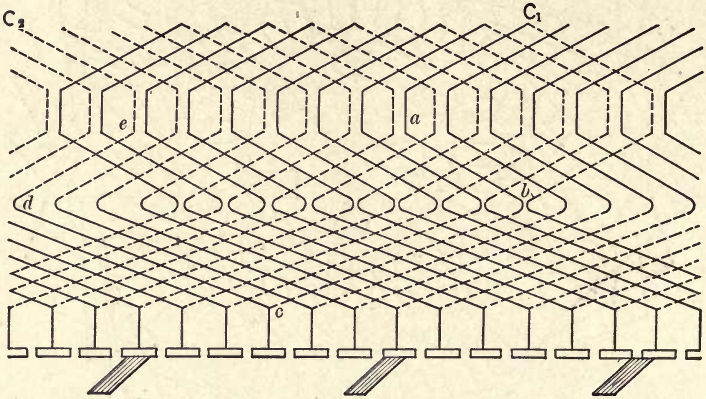


FIG. 88.—Series lap winding.

end connection *de* of coil C_2. If the armature coils consist of a single turn only, as in Fig. 86, and thus are open at *b* and *d*, the end connection and the cross connection can be combined by passing from *a* in coil C_1 directly to *c* and from *c* directly to *e* in

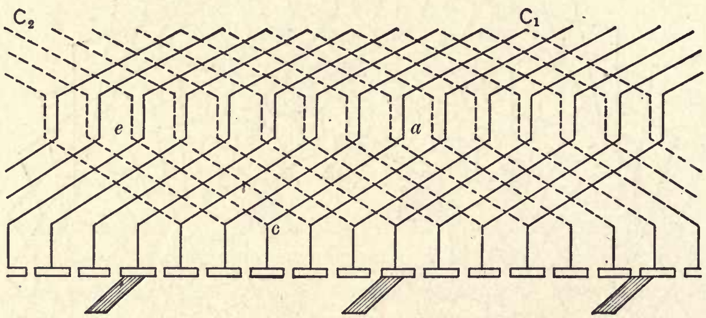


FIG. 89.—Wave winding.

coil C_2; that is, the circuit *abcde* is replaced by *ace*. This has the effect that the coils are apparently open at one side.

Such a winding has been called a wave winding. Only series windings with a single turn per coil can be arranged as wave windings, while windings with several turns per coil must necessarily be lap or coil windings. In Fig. 90 is shown a series drum winding with 35 coils and commutator segments, and a single turn per coil arranged as wave winding. This winding may be compared with the 35-coil series drum winding in Fig. 83.

40. Drum winding can be divided into full-pitch and fractional-pitch windings. In the full-pitch winding the spread of the coil covers the pitch of one pole; that is, each coil covers

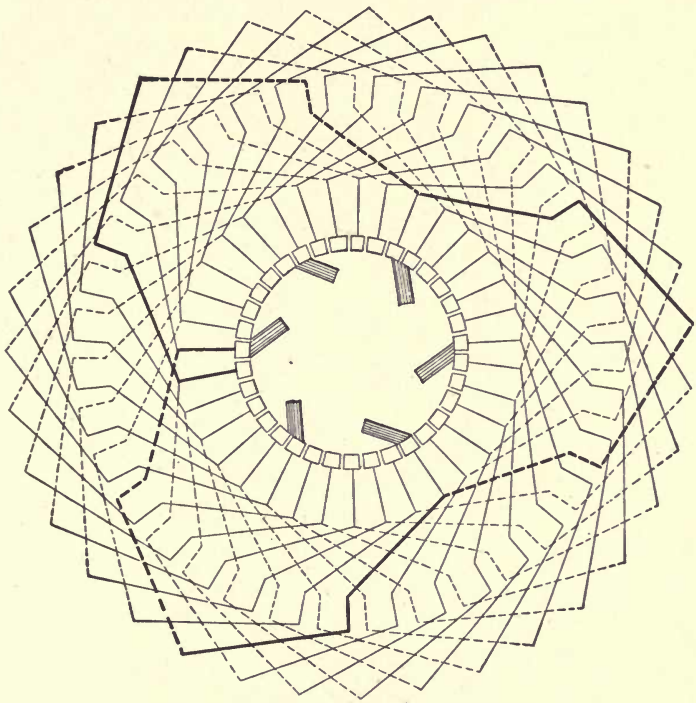


FIG. 90.—Series drum wave winding.

one-sixth of the armature circumference in a six-pole machine, etc. In a fractional-pitch winding it covers less or more.

Series drum windings without cross-connected commutator in which thus the number of coils is not divisible by the number of poles are necessarily always slightly fractional pitch; but generally the expression "fractional-pitch winding" is used only for windings in which the coil covers one or several teeth less than correspond to the pole pitch. Thus the multiple drum winding in Fig. 81 would be a fractional-pitch winding if the coils spread over only four or five teeth instead of over six. As five-sixths fractional-pitch multiple drum winding it is shown in Fig. 91.

Fractional-pitch windings have the advantage of shorter end connections and less self-inductance in commutation, since commutation of corresponding coils under different poles does not take place in the same, but in different, slots, and the flux of self-inductance in commutation is thus more subdivided. Fig. 91 shows the multiple drum winding of Fig. 81 as a frac-

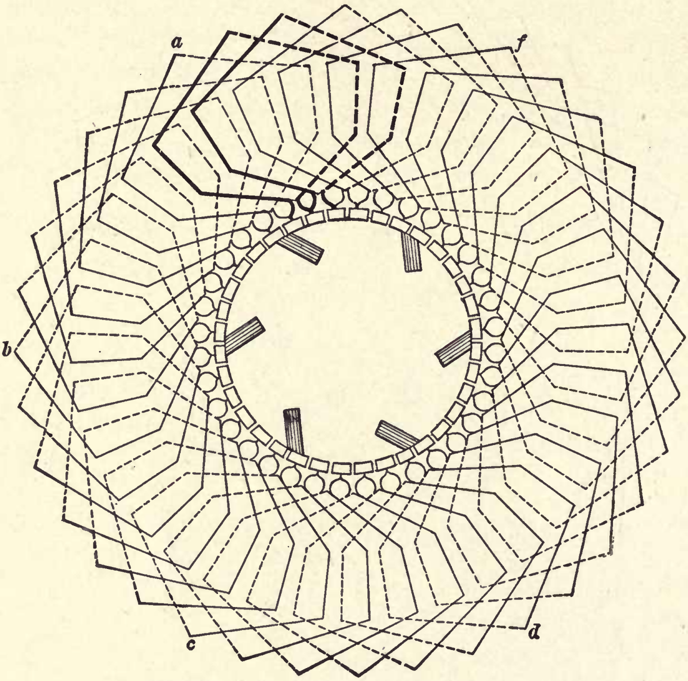


FIG. 91.—Multiple drum five-sixth fractional pitch winding.

tional-pitch winding with five teeth spread, or five-sixths pitch. During commutation the coils *a b c d e f* commutate simultaneously. In Fig. 81 these coils lie by twos in the same slots, in Fig. 91 they lie in separate slots. Thus, in the former case the flux of self-inductance interlinked with the commutated coil is due to two coils; that is, twice that in the latter case. Fractional-pitch windings, however, have the disadvantage of reducing the width of the neutral zone, or zone without generated e.m.f. between the poles, in which commutation takes place, since the one side of the coil enters or leaves the field before the other. Therefore, in commutating machines it is seldom that a pitch is used that falls short of full pitch by more than one or two teeth, while in induction and synchronous machines occasionally as low a pitch as 50 per cent. is used, and two-thirds pitch is frequently employed.

For special purposes, as in single-phase commutator motors fractional-pitch windings are sometimes used.

41. Series windings find their foremost application in machines with small currents, or small machines in which it is desirable to have as few circuits as possible in multiple, and in machines in which it is desirable to use only two sets of brushes, as in smaller railway motors. In multipolar machines with many sets of brushes a series winding is liable to give selective commutation; that is, the current does not divide evenly between the sets of brushes of equal polarity.

Multiple windings are used for machines of large currents, thus generally for large machines, and in large low-voltage machines the still greater subdivision of circuits afforded by the multiple-spiral and the multiple-reentrant winding is resorted to.

To resume, then, armature windings can be subdivided into

(*a*) Ring and drum windings.

(*b*) Closed-circuit and open-circuit windings. Only the former can be used for commutating machines.

(*c*) Multiple and series windings.

(*d*) Single-spiral, multiple-spiral, and multiple-reentrant windings. Either of these can be multiple or series windings.

(*e*) Full-pitch and fractional-pitch windings.

III. Generated E.M.FS.

42. The formula for the generation of e.m.f. in a direct-current machine, as discussed in the preceding, is

$$e = 4fn\Phi,$$

where e = generated e.m.f., f = frequency = number of pairs of poles × hundreds of rev. per sec., n = number of turns in series between brushes, and Φ = magnetic flux passing through the armature per pole, in megalines.

In ring-wound machines, Φ is one-half the flux per field pole, since the flux divides in the armature into two circuits, and each armature turn incloses only half the flux per field pole. In ring-wound armatures, however, each armature turn has only one conductor lying on the armature surface, or face conductor, while in a drum-wound machine each turn has two face conductors. Thus, with the same number of face conductors—that is, the same armature surface—the same frequency, and the same flux per field pole, the same e.m.f. is generated in the ring-wound as in the drum-wound armature.

The number of turns in series between brushes, n, is one-half the total number of armature turns in a series-wound armature, $\frac{1}{p}$ the total number of armature turns in a single-spiral multiple-wound armature with p poles. It is one-half as many in a double-spiral or double-reentrant, one-third as many in a triple-spiral winding, etc.

By this formula, from frequency, series turns, and magnetic flux the e.m.f. is found, or inversely, from generated e.m.f., frequency, and series turns the magnetic flux per field pole is calculated:

$$\Phi = \frac{e}{4fn}.$$

From magnetic flux, and section and lengths of the different parts of the magnetic circuit, the densities and the ampere-turns required to produce these densities are derived, and as the sum of the ampere-turns required by the different parts of the magnetic circuit, the total ampere-turns excitation per field pole is found, which is required for generating the desired e.m.f.

Since the formula for the generation of direct-current e.m.f is independent of the distribution of the magnetic flux, or its wave shape, the total magnetic flux, and thus the ampere-turns required therefor, are independent also of the distribution of magnetic flux at the armature surface. The latter is of importance, however, regarding armature reaction and commutation.

IV. Distribution of Magnetic Flux

43. The distribution of magnetic flux in the air gap or at the armature surface can be calculated approximately by assuming the density at any point of the armature surface as proportional to the m.m.f. acting thereon, and inversely proportional to the nearest distance from a field pole. Thus, if F_0 = ampere-turns acting upon the air gap between armature and field pole, l_a = length of air gap, from iron to iron, the density under the magnet pole, that is, in the range BC of Fig. 90, is

$$B_0 = \frac{4\,\pi F_0}{10\,l_a}.$$

At a point having the distance l_x from the end of the field pole on the armature surface, the distance from the next field pole is $l_d = \sqrt{l_a^2 + l_x^2}$, and the density thus, approximately,

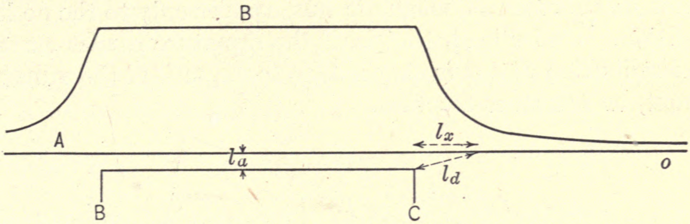


FIG. 92.—Distribution of mganetic flux under a single pole.

$$B_0 = \frac{4\,\pi F_0}{10\sqrt{l_a^2 + l_x^2}}.$$

Herefrom the distribution of magnetic flux is calculated and plotted in Fig. 92, for a single pole BC, along the armature surface A, for the length of air gap $l_a = 1$, and such a m.m.f. as to

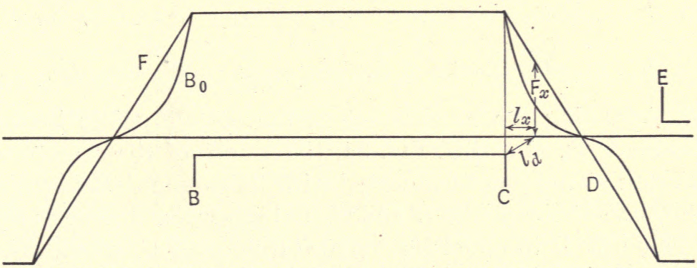


FIG. 93.—Distribution of magnetic force and flux at no load.

give $B_0 = 8000$ under the field pole; that is, for $f_0 = 6400$ or $H_0 = 8000$.

Around the surface of the direct-current machine armature, alternate poles follow each other. Thus the m.m.f. is constant only under each field pole, but decreases in the space between the field poles, from C to E in Fig. 93, from full value at C to full value in opposite direction at E. The point D midway between C and E, at which the m.m.f. of the field equals zero, is called the "neutral." The distribution of m.m.f. of field excitation is thus given by the line F in Fig. 91. The distribution of magnetic flux as shown in Fig. 91 by B_0 is derived by the formula

$$B = \frac{4\,\pi F}{10\,l_d}$$

where

$$l_d = \sqrt{l_a^2 + l_x^2}.$$

This distribution of magnetic flux applies only to the no-load condition. Under load, that is, if the armature carries current, the distribution of flux is changed by the m.m.f. of the armature current, or armature reaction.

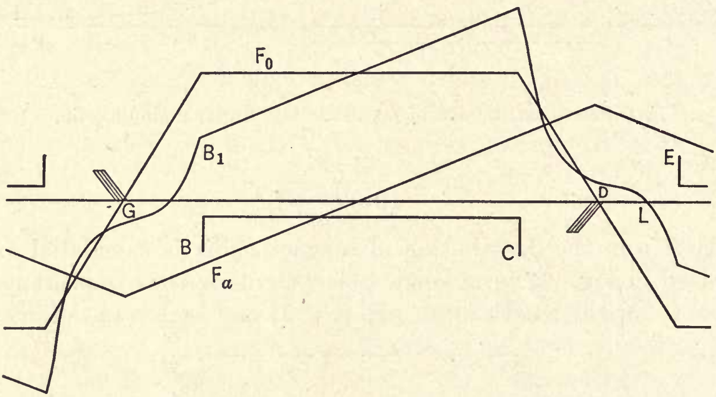


FIG. 94.—Distribution of flux with current in the armature.

44. Assuming the brushes set at the middle points between adjacent poles, D and G, Fig. 94, the m.m.f. of the armature is maximum at the point connected with the commutator brushes, in this case at the points D and G, and gradually decreases from full value at D to equal but opposite value at G, as shown by the line F_a in Fig. 91, while the line F_0 gives the field m.m.f. or impressed m.m.f.

If n = number of turns in series between brushes per pole, i = current per turn, the armature reaction is $F_a = ni$ ampere-turns. Adding F_a and F_0 gives the resultant m.m.f. F, and therefrom the magnetic distribution:

$$B = \frac{4\,\pi F}{10\,l_d}.$$

The latter is shown as line B_1 in Fig. 94.

With the brushes set midway between adjacent field poles, the armature m.m.f. is additive on one side and subtractive on the other side of the center of the field pole. Thus the magnetic intensity is increased on one side and decreased on the other. The total m.m.f., however, and thus, neglecting saturation, the total flux entering the armature, are not changed. Thus, armature reaction, with the brushes midway between adjacent field poles, acts distorting upon the field, but neither magnetizes nor demagnetizes, if the field is below saturation.

The distortion of the magnetic field takes place by the armature ampere-turns beneath the pole, or from B to C. Thus, if τ = pole arc, that is, the angle covered by pole face (two poles or one complete period being denoted by 360 degrees), the distorting ampere-turns of the armature reaction are $\frac{\tau F_a}{180}$.

As seen, in the assumed instance, Fig. 94, where $F_a = \frac{3 F_0}{4}$, the m.m.f. at the two opposite pole corners, and thus the magnetic densities, stand in the proportion 1 to 3. As seen, the generated e.m.f. is not changed by the armature reaction, with the brushes set midway between the field poles, except by the small amount corresponding to the flux entering beyond D and G, that is, shifted beyond the position of brushes. At D, however, the flux still enters the armature, depending in intensity upon the armature reaction; and thus with considerable armature reaction the brushes when set at this point are liable to spark by short-circuiting an active e.m.f. Therefore, under load, the brushes are shifted toward the following pole, that is, toward the direction in which the zero point of magnetic flux has been shifted by the armature reaction.

45. In Fig. 95, the brushes are assumed as shifted to the corner of the next pole, E respectively B. In consequence thereof, the subtractive range of the armature m.m.f. is larger than the additive, and the resultant m.m.f. $F = F_0 + F_a$ is decreased; that is, with shifted brushes the armature reaction demagnetizes the field. The demagnetizing armature ampere-turns are $PM = \frac{GB}{GM} F_a$. That is, if τ_1 = angle of shift of brushes or angle of lead (= GB in Fig. 95), assuming the pitch of two poles = 360 degrees, the demagnetizing component of armature reaction is $\frac{2 \tau_1 F_a}{180}$; the distorting component is $\frac{\tau F_a}{180}$, where τ = pole arc.

Thus, with shifted brushes the field excitation has to be increased under load to maintain the same total resultant m.m.f., that is, the same total flux and generated e.m.f. Hence, in Fig. 95 the field excitation F_0 has been assumed by $\frac{2\tau_1 F_a}{180} = \frac{F}{3}$ larger than in the previous figures, and the magnetic distribution B_1 plotted for these values.

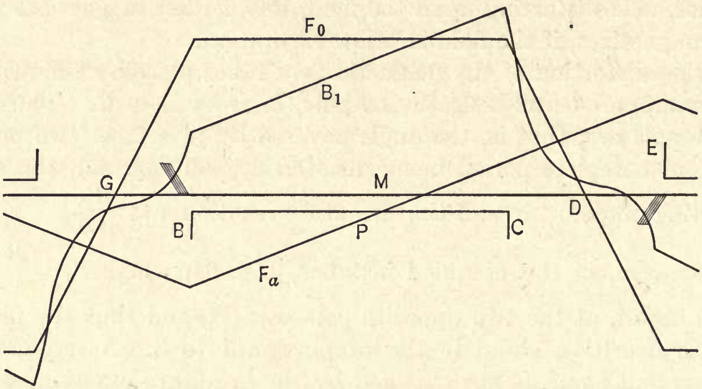


Fig. 95.—Distribution of flux with current in the armature and brushes shifted from the magnetic neutral.

V. Effect of Saturation on Magnetic Distribution

46. The preceding discussion of Figs. 92 to 95 omits the effect of saturation. That is, the assumption is made that the magnetic materials near the air gap, as pole face and armature teeth, are so far below saturation that at the demagnetized pole corner the magnetic density decreases, at the strengthened pole corner increases, proportionally to the m.m.f.

The distribution of m.m.f. obviously is not affected by saturation, but the distribution of magnetic flux is greatly changed thereby. To investigate the effect of saturation, in Figs. 96 to 99 the assumption has been made that the air gap is reduced to one-half its previous value, $l_a = 0.5$, thus consuming only one-half as many ampere-turns, and the other half of the ampere-turns are consumed by saturation of the armature teeth. The length of armature teeth is assumed as 3.2, and the space filled by the teeth is assumed as consisting of one-third of iron and two-thirds of non-magnetic material (armature slots, ventilating ducts, insulation between laminations, etc.).

In Figs. 96, 97, 98, 99, curves are plotted corresponding to those in Figs. 92, 93, 94, and 95. As seen, the spread of magnetic flux at the pole corners is greatly increased, but farther away from the field poles the magnetic distribution is not changed.

47. The magnetizing, or rather demagnetizing, effect of the load with shifted brushes is not changed. The distorting effect

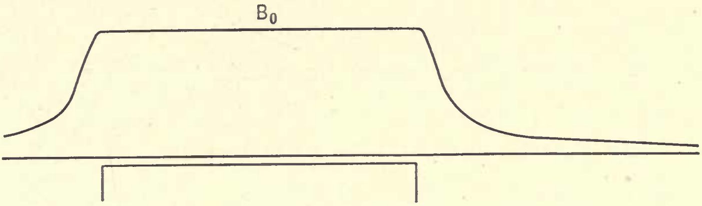


FIG. 96.—Flux distribution under a single pole.

of the load is, however, very greatly decreased, to a small percentage of its previous value, and the magnetic field under the field pole is very nearly uniform under load.

The reason is: Even a very large increase of m.m.f. does not much increase the density, the ampere-turns being consumed by saturation of the iron, and even with a large decrease of m.m.f. the density is not decreased much, since with a small decrease

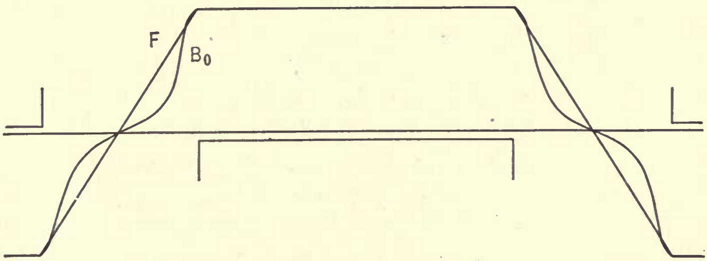


FIG. 97.—Distribution of flux and m.m.f. at no load.

of density the ampere-turns consumed by the saturation of the iron become available for the air gap.

Thus, while in Fig. 95 the densities at the center and the two pole corners of the field pole are 8000, 12,000, and 4000, with the saturated structure in Fig. 99 they are 8000, 9040, and 6550.

At or near the theoretical neutral, however, the saturation has no effect.

That is, saturation of the armature teeth affords a means of reducing the distortion of the magnetic field, or the shifting of flux at the pole corners, and is thus advantageous for machines which shall operate over a wide range of load with fixed position of brushes, if the brushes are shifted near to the next following pole corner.

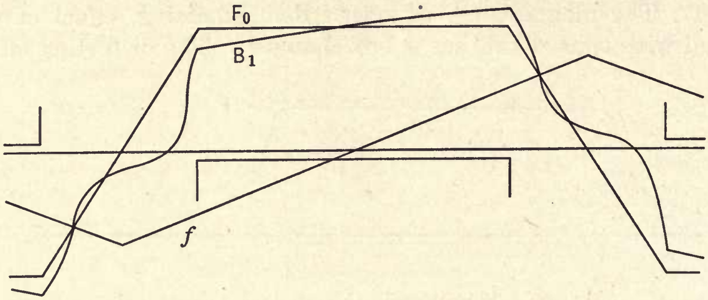


FIG. 98.—Distribution of flux and m.m.f. at load, with Brushes at neutral.

It offers no direct advantage, however, for machines commutating with the brushes midway between the field poles, as converters.

An effect similar to saturation in the armature teeth is produced

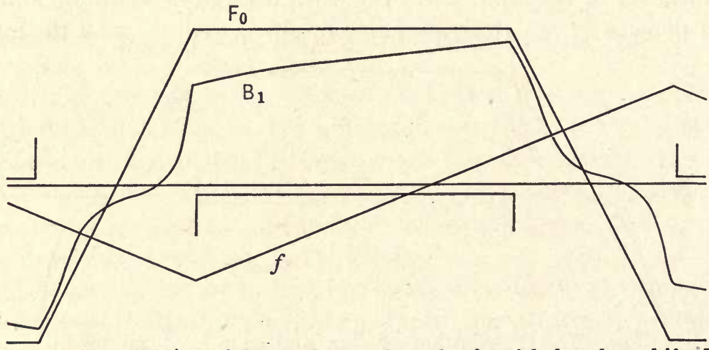


FIG. 99.—Distribution of flux and m.m.f. at load, with brushes shifted to next pole corner.

by saturation of the field pole face, or more particularly, saturation of the pole corners of the field.

VI. Effect of Commutating Poles

48. With the commutator brushes of a generator set midway between the field poles, as in Fig. 94, the m.m.f. of armature reaction produces a magnetic field at the brushes. The e.m.f. generated by the rotation of the armature through this field opposes the reversal of the current in the short-circuited armature coil under the brush, and thus impairs commutation. If therefore the commutation constants of the machines are not abnormally good—high field strength, low armature reaction, low self-inductance and frequency of commutation—the machine does not commutate satisfactorily under load, with the brushes midway between the field poles, and the brushes have to be shifted to the edge of the next field poles, as shown in Fig. 95, until the fringe of the magnetic flux of the field poles reverses the armature reaction and so generates an e.m.f. in the armature coil, which reverses the current and thus acts as commutating flux. The commutating e.m.f. and therefore the commutating flux should be proportional to the current which is to be reversed, that is, to the load. The magnetic flux of the field pole of a shunt or compound machine, however, decreases with increasing load at the pole corners toward which the brushes are shifted, by the demagnetizing action of the armature reaction, and the shift of brushes therefore has to be increased with the load, from nothing at no load. At overload, the pole corners toward which the brushes are shifted may become so far weakened that even under the pole not sufficient reversing e.m.f. is generated, and satisfactory commutation ceases, that is, the sparking limit is reached.

In general, however, varying the brush shift with the load is not permissible, and with rapidly fluctuating load not feasible, and therefore the brushes are set permanently at a mean shift. In this case, however, instead of increasing proportionally with the load, the commutating field is maximum at no load, and gradually decreases with increase of load, and is correct only at one particular load. At constant shift of the brushes, the commutation of the constant potential machine, direct-current generator or motor, is best at a certain load, and usually becomes poorer at lighter or heavier loads, and ultimately becomes bad by inductive sparks due to insufficient commutating flux. In machines in which very good commutating constants cannot be secured, as in large high-speed machines (steam turbine driven direct-current generators), this may lead to bad sparking or even flashing over at sudden overloads as well as when throwing off full load.

49. This has led to the development of the *commutating pole*, also called *interpole*, that is, a narrow magnetic pole located between the main poles at the point of the armature surface, at which commutation occurs, and excited so as to produce a commutating flux proportional to the load, and thus giving the required commutating field at all loads. Such machines then give no inductive sparking, but regarding commutation are limited in overload capacity only by the current density under the brush.

Such commutating poles are excited by series coils, that is, coils connected in series with the armature and having a number of effective turns higher than the number of effective series turns per armature pole, so that at the position of the brushes the

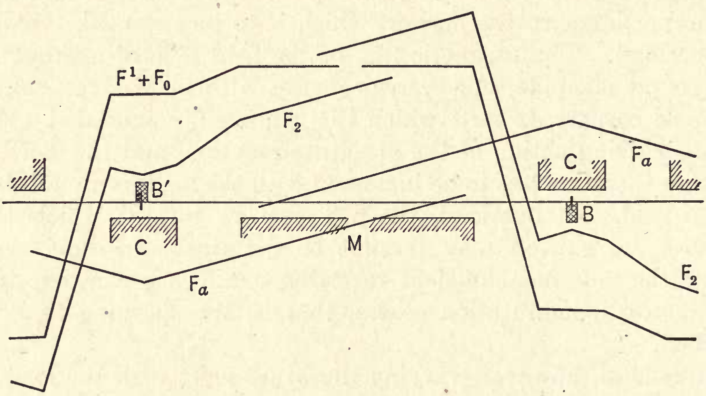


FIG. 100.—Magnetic force distribution with commutating pole.

m.m.f. of the commutating pole overpowers and reverses the m.m.f. of the armature, and produces a commutating m.m.f. equal to the product of the armature current and difference of commutating turns and armature turns, and thereby produces a commutating flux proportional to the load, as long as the magnetic flux in the commutating poles does not reach too high magnetic saturation.

In Fig. 100 is shown the distribution of m.m.f. around the circumference of the armature, and in Fig. 101 the distribution of magnetic flux calculated in the manner as described in paragraphs 46 and 47. *M* represents the main poles, *C* the commutating poles. A main field excitation F_0 is assumed of 10,000 ampere-turns per pole, and an armature reaction F_a of 6000 ampere-turns per pole. Choosing then 8000 ampere-turns per commutating pole F', leaves 2000 ampere-turns as resultant commutating m.m.f. at full load, half as much at half load, etc. The resultant m.m.f. of the main field F_0, the armature F_a, and the commutating pole F' is represented in Fig. 100 by F_2, and the flux produced by it is shown in Fig. 101. As seen, with the commutator brushes midway between the field poles, that is, in the center of the commutating pole, a commutating flux proportional to the armature current enters the armature at the brush B and B', and is cut by the revolving armature during commutation.

The use of the commutating pole or interpole thus permits controlling the commutation, with fixed brush position midway between the field poles, and commutating poles therefore are

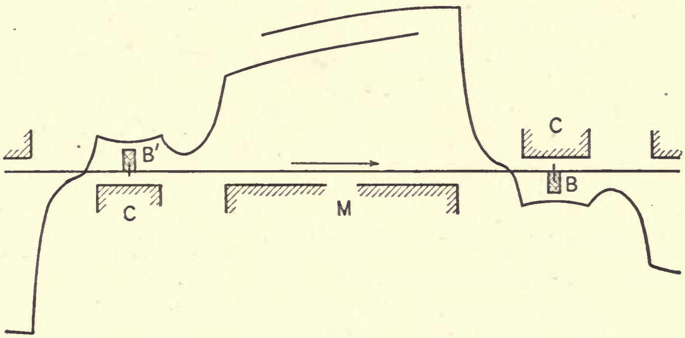


FIG. 101.—Magnetic flux distribution with commutating pole.

extensively used in larger machines, especially of the high-speed type.

The commutating pole makes the commutation independent of the main field strength, and therefore permits the machines to operate with equally good commutation over a wide voltage range, and at low voltage, that is, low field strength, as required for instance in boosters, etc.

50. With multiple-wound armatures, at least one commutating pole for every pair of main poles is required, while with a series-wound armature a single commutating pole would be sufficient for all the sets of armature brushes, if of sufficient strength. In general, however, as many commutating poles as main poles are used.

With the position of the brushes at the neutral, as is the case when using commutating poles, the armature reaction has no demagnetizing component, and the only drop of voltage at load is that due to the armature resistance drop and the distortion of the main field, which at saturation produces a decrease of the total flux, as shown in Fig. 98.

As is seen in Fig. 101, the magnetic flux of the commutating pole is not symmetrical, but the spread of flux is greater at the side of the main pole of the same polarity. As result thereof, the total magnetic flux is slightly increased by the commutating poles; that is, the two halves of the commutating flux on the two sides of the brush do not quite neutralize, and the commutating flux thus exerts a slight compounding action, that is, tends to raise the voltage. This can be still further increased by shifting the brushes slightly back and thus giving a magnetizing component of armature reaction. This can be done without affecting commutation as long as the brushes still remain under the commutating pole. In this manner a compounding or even a slight over-compounding can be produced without a series winding on the main field poles, or, inversely, by shifting the brushes slightly forward, a demagnetizing component of armature reaction can be introduced. Furthermore, the current induced in the short-circuited armature coil by the commutating field is magnetizing, that induced by the magnetic field of armature reaction, demagnetizing.

In operating machines with commutating poles in multiple, care therefore must be taken not to have the compounding action of the commutating poles interfere with the distribution of load; for this purpose an equalizer connection may be used between the commutating pole windings of the different machines, and the commutating windings treated in the same way as series coils on the main poles, that is, equalized between the different machines to insure division of load.

51. The advantage of the commutating pole over the shift of brushes to the edge of the next field pole, in constant potential machines—shunt or compound wound—thus is that the commutating flux of the former has the right intensity at all loads, while that of the latter is right only at one particular load, too high below, too low above that load. In series-wound machines, that is, machines in which the main field is excited in series with the armature, and thus varies in strength with the armature current, armature reaction and field excitation are always proportional to each other, and the distribution of magnetic flux at the armature circumference therefore always has the same shape, and its intensity is proportional to the current, except as far as saturation limits it. As the result thereof, shifting the brushes to the edge of the field poles, as in Fig. 95, brings them in a field which is proportional to the armature current and thus has the proper intensity as a commutating field. Therefore with series-wound machines commutating poles are not necessary for good commutation, but the shifting of the brushes gives the same result. However, in cases where the direction of rotation frequently reverses, as in railway motors, the direction of the shift of brushes has to be reversed with the reversal of rotation. In railway motors this cannot be done without objectionable complication, therefore the brushes have to be set midway, and the use of the magnetic flux at the edge of the next pole, as commutating flux, is not feasible. In this case a commutating pole is used, to give, without mechanical shifting of the brushes, the same effect which a brush shift would give. Therefore in railway motors, especially when wound for high voltage, as 1200 to 2400 volts, a commutating pole is sometimes used. This commutating pole, having a series winding just like the main pole, changes proportionally with the main pole. When reversing the direction of rotation, however, the armature and the commutating poles are reversed, while the main poles remain unchanged, or the main poles are reversed, while the armature and the commutating poles remain unchanged; that is, the separate commutating pole becomes necessary because during the reversal of rotation it has to be treated differently from the main pole.

52. The commutating pole counteracts the armature reaction only at the place of commutation, but not elsewhere, and the field distribution resulting from the armature reaction thus is not eliminated by the commutating pole, except locally. Thus in machines having very low field excitation, and relatively high armature reaction, as alternating-current commutating machines, adjustable speed motors of wide speed range at the high-speed position, boosters near zero voltage, etc., the load losses resulting from excessive field distortion, the tendency to instability of speed, and the liability of flashing at the commutator at sudden changes of load are not eliminated by the commutating pole, but a more complete neutralization of the armature reaction is necessary.

Such is given by a compensating winding. This is a distributed winding, located in the field pole faces closely adjacent to the armature, as shown in Fig. 102. It is connected in series but opposition to the armature winding, and of the same number of effective turns as the armature. By such a compensating winding, the armature reaction is completely eliminated, and with it magnetic distortion, load losses, etc.

By giving the compensating winding some more ampere-turns than the armature, over-compensation is produced, giving a magnetic cross flux under load, opposite to that of armature reaction, that is, a commutating flux. Very commonly in such compensated machines merely the ampere-turns of the compensating winding in the slots at the commutating zone are increased, so that the compensating winding all around the armature exactly neutralizes the armature reaction, except at the commutating zone, where it over-compensates and thus gives a local commutating flux. Such machines, when properly designed, are characterized by absence of load losses, stability at all speeds, instant recovery at sudden load changes, and absence of sparking at commutator even at momentary overloads of several hundred per cent.

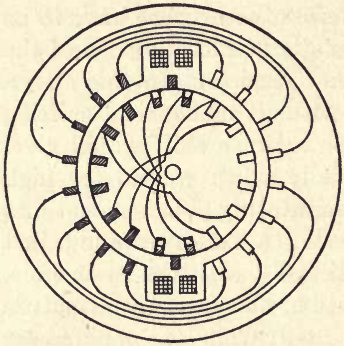


FIG. 102.—Compensated commutating machine with fractional pitch armature winding.

VII. Effect of Slots on Magnetic Flux

53. With slotted armatures the pole face density opposite the armature slots is less than that opposite the armature teeth, due to the greater distance of the air path in the former case. Thus, with the passage of the armature slots across the field pole a local pulsation of the magnetic flux in the pole face is produced, which, while harmless with laminated field pole faces, generates eddy currents in solid pole pieces. The frequency of this pulsation is extremely high, and thus the energy loss due to eddy currents in the pole faces may be considerable, even with pulsations of small amplitude. If S = peripheral speed of the armature in centimeters per second, l_p = pitch of armature slot (that is, width of one slot and one tooth at armature surface), the frequency is $f_1 = \frac{S}{l_p}$. Or, if f = frequency of machine, n = number of armature slots per pair of poles, $f_1 = nf$.

For instance, $f = 33.3$, $n = 51$, thus $f_1 = 1700$.

Under the assumption, width of slots equals width of teeth = 2 × width of air gap, the distribution of magnetic flux at the pole face is plotted in Fig. 103.

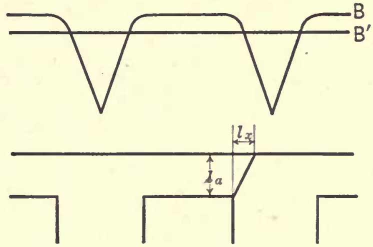


Fig. 103.—Effect of slots on flux distribution.

The drop of density opposite each slot consists of two curved branches equal to those in Fig. 92, that is, calculated by

$$B = \frac{F}{\sqrt{l_a^2 + l_x^2}}.$$

The average flux is 7525; that is, by cutting half the armature surface away by slots of a width equal to twice the length of air gap, the total flux under the field pole is reduced only in the proportion 8000 to 7525, or about 6 per cent.

The flux B pulsating between 8000 and 5700 is equivalent to a uniform flux $B_1 = 7525$ superposed with an alternating flux

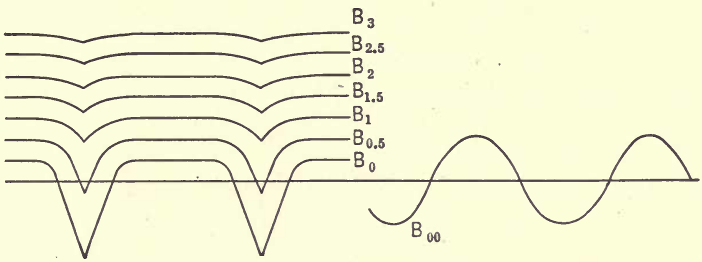


Fig. 104.—Effect of slots on flux distribution.

B_0, shown in Fig. 104, with a maximum of 475 and a minimum of 1825. This alternating flux B_0 can, as regards production of eddy currents, be replaced by the equivalent sine wave B_{00}, that is, a sine wave having the same effective value (or square root of mean square). The effective value is 718.

The pulsation of magnetic flux farther in the interior of the field-pole face can be approximated by drawing curves equidistant from B_0. Thus the curves $B_{0.5}$, B_1, $B_{1.5}$, B_2, $B_{2.5}$, and B_3 are drawn equidistant from B_0 in the relative distances 0.5, 1, 1.5, 2, 2.5, and 3 (where $l_a = 1$ is the length of air gap). They give the effective values:

B_0	$B_{0.5}$	B_1	$B_{1.5}$	B_2	$B_{2.5}$	B_3
718	373	184	119	91	69	57

That is, the pulsation of magnetic flux rapidly disappears toward the interior of the magnet pole, and still more rapidly the energy loss by eddy currents, which is proportional to the square of the magnetic density.

54. In calculating the effect of eddy currents, the magnetizing effect of eddy currents may be neglected (which tends to reduce the pulsation of magnetism); this gives the upper limit of loss

Let B = effective density of the alternating magnetic flux,
S = peripheral speed of armature in centimeters per second, and
l = length of pole face along armature.

The e.m.f. generated in the pole face is then

$$e = SlB \times 10^{-8},$$

and the current in a strip of thickness Δl and 1 cm. width,

$$\Delta i = \frac{e\Delta l}{\rho l} = \frac{SlB\Delta l\, 10^{-8}}{\rho l} = \frac{SB\Delta l\, 10^{-8}}{\rho},$$

where

$$\rho = \text{resistivity of the material.}$$

Thus the effect of eddy currents in this strip is

$$\Delta p = e\Delta i = \frac{S^2 l B^2 \Delta l\, 10^{-16}}{\rho},$$

or per cubic centimeter,

$$p = \frac{S^2 B^2\, 10^{-16}}{\rho};$$

that is, proportional to the square of the effective value of magnetic pulsation, the square of peripheral speed, and inversely proportional to the resistivity.

Thus, assuming for instance,

$$S = 2000,$$
$$\rho = 20 \times 10^{-6}, \text{ for cast steel,}$$
$$\rho = 100 \times 10^{-6}, \text{ for cast iron,}$$

we have in the above example,

At distance from pole face	B	p	
		Cast steel	Cast iron
0	718	10.3	2.06
$\frac{la}{2}$	373	2.78	0.56
la	184	0.677	0.135
$\frac{3la}{2}$	119	0.283	0.057
$2la$	91	0.166	0.033
$\frac{5la}{2}$	69	0.095	0.019
$3la$	57	0.065	0.013

VIII. Armature Reaction

55. At no load, that is, with no current in the armature circuit, the magnetic field of the commutating machine is symmetrical with regard to the field poles.

Thus the density at the armature surface is zero at the point or in the range midway between adjacent field poles. This point, or range, is called the "neutral" point or "neutral" range of the commutating machine.

Under load the armature current represents a m.m.f. acting in the direction from commutator brush to commutator brush of opposite polarity, that is, in quadrature with the field m.m.f. if the brushes stand midway between the field poles; or shifted against the quadrature position by the same angle by which the commutator brushes are shifted, which angle is called the angle of lead.

If n = turns in series between brushes per pole, and i = current per turn, the m.m.f. of the armature is $F_a = ni$ per pole. Or, if n_0 = total number of turns on the armature, n_c = number of turns or circuits in multiple, $2n_p$ = number of poles, and i_0 = total armature current, the m.m.f. of the armature per pole is $F_a = \frac{n_0 i_0}{2 n_p n_c}$. This m.m.f. is called the armature reaction of the continuous-current machine.

Since the armature turns are distributed over the total pitch of pole, that is, a space of the armature surface representing 180 deg., the resultant armature reaction is found by multiplying

F_a with the average $\cos \left\{ \begin{matrix} +90 \\ -90 \end{matrix} \right. = \frac{2}{\pi}$, and is thus

$$F_{a_0} = \frac{2 F_a}{\pi} = \frac{2 ni}{\pi}.$$

When comparing the armature reaction of commutating machines with other types of machines, as synchronous machines etc., the resultant armature reaction $F_{a_0} = \frac{2 F_a}{\pi}$ has to be used. In discussing commutating machines proper, however, the value $F_a = ni$ is usually considered as the armature reaction.

56. The armature reaction of the commutating machine has a distorting and a magnetizing or demagnetizing action upon the magnetic field. The armature ampere-turns beneath the field poles have a distorting action as discussed under "Magnetic Distribution" in the preceding paragraphs. The armature ampere-turns between the field poles have no effect upon the resultant field if the brushes stand at the neutral; but if the brushes are shifted, the armature ampere-turns inclosed by twice the angle of lead of the brushes have a demagnetizing action.

Thus, if τ = pole arc as fraction of pole pitch, τ_1 = shift of brushes as fraction of pole pitch, F_a the m.m.f. of armature reaction, and F_0 the m.m.f. of field excitation per pole, the demagnetizing component of armature reaction is $\tau_1 F_a$, the distorting component of armature reaction is τF_a, and the magnetic density at the strengthened pole corner thus corresponds to the m.m.f. $F_0 + \frac{\tau F_a}{2}$ at the weakened field corner to the m.m.f. $F_0 - \frac{\tau F_a}{2}$.

IX. Saturation Curves

57. As *saturation curve* or *magnetic characteristic* of the commutating machine is understood the curve giving the generated voltage, or terminal voltage at open circuit and normal speed, as function of the ampere-turns per pole field excitation.

Such curves are of the shape shown in Fig. 105 as A. Owing to the remanent magnetism or hysteresis of the iron part of the magnetic circuit, the saturation curve taken with decreasing field excitation usually does not coincide with that taken with increasing field excitation, but is higher, and by gradually first increasing the field excitation from zero to maximum and then decreasing again, the looped curve in Fig. 106 is derived, giving as average saturation curve the curve shown in Fig. 105 as A and as central curve in Fig. 106.

Direct-current generators are usually operated at a point of the saturation curve above the bend, that is, at a point where the terminal voltage increases considerably less than proportionally to the field excitation. This is necessary in self-exciting direct-current generators to secure stability.

The ratio

$$\frac{\text{increase of field excitation}}{\text{total field excitation}} \div \frac{\text{corresponding increase of voltage}}{\text{total voltage}},$$

that is,

$$\frac{dF_0}{F_0} \div \frac{de}{e},$$

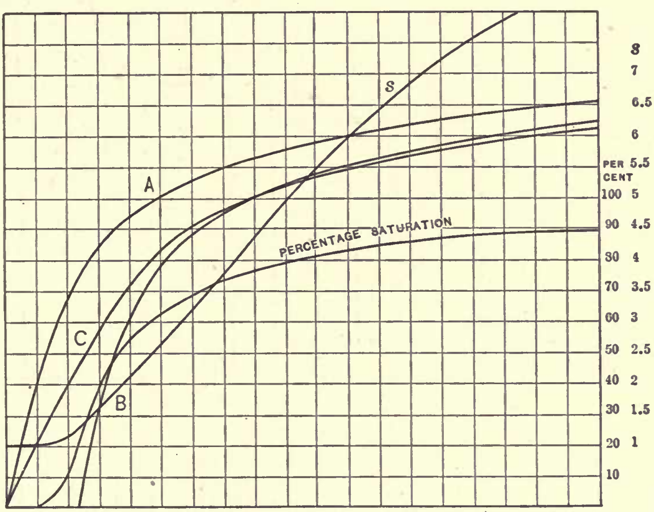


FIG. 105.—Saturation characteristics.

is called saturation factor s, and is plotted in Fig. 105. It is the ratio of a small percentage increase in field excitation to a corresponding percentage increase in voltage thereby produced. The quantity $1 - \frac{1}{s}$ is called the percentage saturation of the machine, as it shows the approach of the machine field to magnetic saturation.

58. Of considerable importance also are curves giving the terminal voltage as function of the field excitation at load. Such curves are called *load saturation curves*, and can be *constant* *current load saturation curve*, that is, terminal voltage as function of field ampere-turns at constant full-load current through the armature, and *constant resistance load saturation curve*, that is, terminal voltage as function of field ampere-turns if the machine circuit is closed through a constant resistance giving full-load current at full-load terminal voltage.

A constant current load saturation curve is shown as B, and a constant resistance load saturation curve as C in Fig. 105.

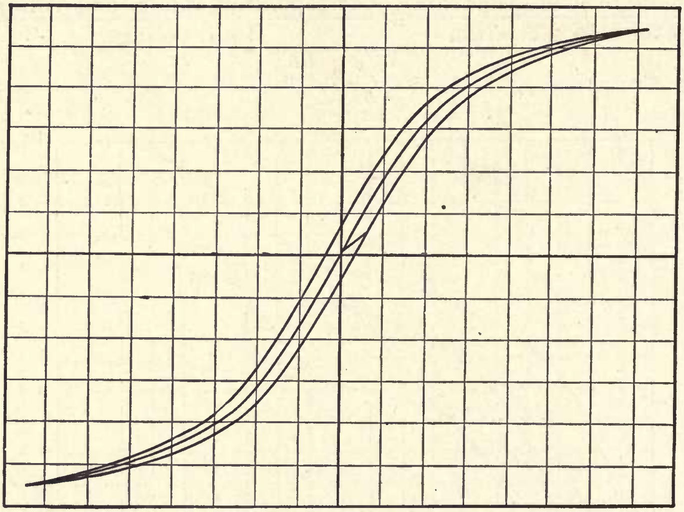


FIG. 106.—Saturation curves.

X. Compounding

59. In the direct-current generator the field excitation required to maintain constant terminal voltage has to be increased with the load. A curve giving the field excitation in ampere-turns per pole, as function of the load in amperes, at constant terminal voltage, is called the *compounding curve* of the machine.

The increase of field excitation required with load is due to:

1. The internal resistance of the machine, which consumes e.m.f. proportional to the current, so that the generated e.m.f., and thus the field m.m.f. corresponding thereto, has to be greater under load. If p = resistance drop in the machine as fraction of terminal voltage, $= \frac{ir}{e}$, the generated e.m.f. at load has to be $e(1 + p)$, and if F_0 = no-load field excitation, and s = satu-

ration coefficient, the field excitation required to produce the e.m.f. $e\,(1 + p)$ is $F_0\,(1 + sp)$; thus an additional excitation of spF_0 is required at load, due to the armature resistance.

2. The demagnetizing effect of the ampere-turns armature reaction of the angle of shift of brushes τ_1 requires an increase of field excitation by $\tau_1 F_a$. (Section VII.)

3. The distorting effect of armature reaction does not change the total m.m.f. producing the magnetic flux. If, however, magnetic saturation is reached or approached in a part of the magnetic circuit adjoining the air gap, the increase of magnetic density at the strengthened pole corner is less than the decrease at the weakened pole corner, and thus the total magnetic flux with the same total m.m.f. is reduced, and to produce the same total magnetic flux an increased total m.m.f., that is, increase of field excitation, is required. This increase depends upon the saturation of the magnetic circuit adjacent to the armature conductors.

4. The magnetic stray field of the machine, that is, that part of the magnetic flux which passes from field pole to field pole without entering the armature, usually increases with the load. This stray field is proportional to the difference of magnetic potential between field poles; that is, at no load it is proportional to the ampere-turns m.m.f. consumed in air gap, armature teeth, and armature core. Under load, with the same generated e.m.f., that is, the same magnetic flux passing through the armature core, the difference of magnetic potential between adjacent field poles is increased by the counter m.m.f. of the armature and by saturation. Since this magnetic stray flux passes through field poles and yoke, the magnetic density therein is increased and the field excitation correspondingly, especially if the magnetic density in field poles and yoke is near saturation. This increase of field strength required by the increase of density in the external magnetic circuit, due to the increase of magnetic stray field, depends upon the shape of the magnetic circuit, the armature reaction, and the saturation of the external magnetic circuit.

Curves giving, with the amperes output as abscissas, the ampere-turns per pole field excitation required to increase the voltage proportionally to the current are called *over-compounding* curves. In the increase of field excitation required for over-compounding, the effects of magnetic saturation are still more marked.

XI. Characteristic Curves

60. The *field characteristic* or *regulation curve*, that is, curve giving the terminal voltage as function of the current output at constant field excitation, is of less importance in commutating machines than in synchronous machines, since commutating machines are usually not operated with separate and constant excitation, and the use of the series field affords a convenient means of changing the field excitation proportionally to the load. The curve giving the terminal voltage as function of current output, in a compound-wound machine, at constant resistance in the shunt field, and constant adjustment of the series field, is, however, of importance as regulation curve of the direct-current generator. This curve would be a straight line except for the effect of saturation, etc., as discussed above.

XII. Efficiency and Losses

61. The losses in a commutating machine which have to be considered when deriving the efficiency by adding the individual losses are:

1. Loss in the resistance of the armature, the commutator leads, brush contacts and brushes, in the shunt field and the series field with their rheostats.

2. Hysteresis and eddy currents in the iron at a voltage equal to the terminal voltage, plus resistance drop in a generator, or minus resistance drop in a motor.

3. Eddy currents in the armature conductors when large and not protected, and in pole faces when solid and the air gap is small.

4. Friction of bearings, of brushes on the commutator, and windage.

5. Load losses, due to the increase of hysteresis and of eddy currents under load, caused by the change of the magnetic distribution, as local increase of magnetic density and of stray field.

The friction of the brushes and the loss in the contact resistance of the brushes are frequently quite considerable, especially with low-voltage machines.

Constant or approximately constant losses are: friction of bearings and of commutator brushes, and windage; hysteresis and eddy current losses; and shunt field excitation. Losses increasing with the load, and proportional or approximately proportional to the square of the current: armature resistance losses; series field resistance losses; brush contact resistance losses; and the so-called "load losses."

XIII. Commutation

62. The most important problem connected with commutating machines is that of commutation.

Fig. 107 represents diagrammatically a commutating machine.

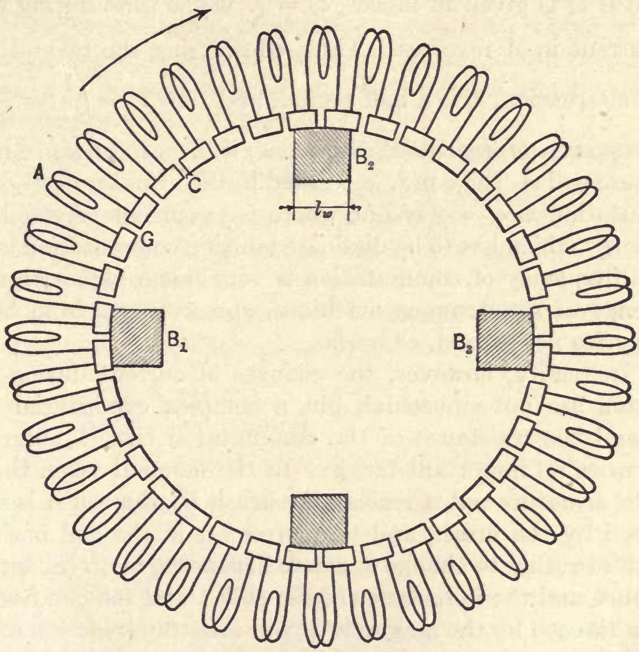


FIG. 107.—Diagram for the study of commutation.

The e.m.f. generated in an armature coil A is zero with this coil at or near the position of the commutator brush B_1. It rises and reaches a maximum about midway between two adjacent sets of brushes, B_1 and B_2, at C, and then decreases again, reaching zero at or about B_2, and then repeats the same change in opposite direction. The current in armature coil A, however, is constant during the motion of the coil from B_1 to B_2. While the coil A passes the brush B_2, however, the current in the coil A reverses, and then remains constant again in opposite direction during the motion from B_2 to B_3. Thus, while the armature coils of a commutating machine are the seat of a system of polyphase e.m.fs. having as many phases as coils, the current in these coils is constant, reversing successively.

63. The reversal of current in coil A takes place while the gap G between the two adjacent commutator segments between which the coil A is connected passes the brush B_2. Thus, if l_w = width of brushes, S = peripheral speed of commutator per second in the same measure in which l_w is given, as in inches per second if l_w is given in inches, $t_0 = \frac{l_w}{S}$ is the time during which the current in A reverses. Thus, considering the reversal as a single alternation, t_0 is a half period, and thus $f_0 = \frac{1}{2\,t_0} = \frac{S}{2\,l_w}$ is the *frequency of commutation;* hence, if L = inductance of the armature coil A, the e.m.f. generated in the armature coil during commutation is $e_0 = 2\pi f_0 L i_0$, where i_0 = current reversed, and the energy which has to be dissipated during commutation is $i_0^2 L$.

The frequency of commutation is very much higher than the frequency of synchronous machines, and averages from 300 to 1000 cycles per second, or more.

64. In reality, however, the changes of current during commutation are not sinusoidal, but a complex exponential function, and the resistance of the commutated circuit enters the problem as an important factor. In the moment when the gap G of the armature coil A reaches the brush B_2, the coil A is short-circuited by the brush, and the current i_0 in the coil begins to die out, or rather to change at a rate depending upon the internal resistance and the inductance of the coil A and the e.m.f. generated in the coil by the magnetic flux of armature reaction and by the field magnetic flux. The higher the internal resistance the faster is the change of current, and the higher the inductance the slower the current changes. Thus two cases have to be distinguished.

1. No magnetic flux enters the armature at the position of the brushes, that is, no e.m.f. is generated in the armature coil under commutation, except that of its own self-inductance. In this case the commutation is entirely determined by the inductance and resistance of the armature coil A, and is called *resistance commutation.*

2. Commutation takes place in an active magnetic field; that

is, in the armature during commutation an e.m.f. is generated by its rotation through a magnetic field. This magnetic field may be the magnetic field of armature reaction, or the reverse magnetic field of a commutating pole, or the fringe of the main field of the machine, into which the brushes are shifted. In this case the commutation depends upon the inductance and the resistance of the armature coil and the e.m.f. generated therein by the main magnetic field, and if this magnetic field is a commutating field, is called *voltage commutation.*

In either case the resistance of the brushes and their contact may either be negligible, as usually the case with copper brushes, or it may be of the same or a higher magnitude than the internal resistance of the armature coil A. The latter is usually the case with carbon or graphite brushes.

In the former case the resistance of the short-circuit of armature coil A under commutation is approximately constant; in the latter case it varies from infinity in the moment of beginning commutation down to minimum, and then up again to infinity at the end of commutation.

65. (a) Negligible resistance of brush and brush contact.

This is more or less approximately the case with copper brushes.

Let i_0 = current,

L = inductance,

r = resistance of armature coil,

$t_0 = \frac{l_w}{S}$ = time of commutation,

and $-e$ = e.m.f. generated in the armature coil by its rotation through the magnetic field, where e is negative for the magnetic field of armature reaction and positive for the commutating field.

Denoting the current in the coil A at time t after beginning of commutation by i, the e.m.f. of self-inductance is

$$e_1 = -L\frac{di}{dt}.$$

Thus the total e.m.f. acting in coil A,

$$-e + e_1 = -e - L\frac{di}{dt},$$

and the current is

$$i = \frac{-e + e_1}{r} = -\frac{e}{r} - \frac{L}{r}\frac{di}{dt}.$$

Transposing, this expression becomes

$$-\frac{rdt}{L} = \frac{di}{\frac{e}{r} + i},$$

the integral of which is

$$-\frac{rt}{L} = \log_\epsilon \left(\frac{e}{r} + i\right) - \log_\epsilon c,$$

where $\log_\epsilon c$ = integration constant.

Since at $t = 0$, $i = i_0$, we have

$$\log_\epsilon c = \log \left(\frac{e}{r} + i_0\right),$$

therefore

$$c = \left(\frac{e}{r} + i_0\right), \text{ and } i = \left(\frac{e}{r} + i_0\right) \epsilon^{-\frac{r}{L}t} - \frac{e}{r},$$

and, at the end of commutation, or, $t = t_0$,

$$i_1 = \left(\frac{e}{r} + i_0\right) \epsilon^{-\frac{r}{L}t_0} - \frac{e}{r}.$$

For perfect commutation,

$$i_1 = -i_0;$$

that is, the current at the end of commutation must have reversed and reached its full value in opposite direction.

Substituting in this last equation the value i_1 from the preceding equation, and transforming, we have

$$\epsilon^{-\frac{r}{L}t_0} = \frac{\frac{e}{r} - i_0}{\frac{e}{r} + i_0}$$

taking the logarithms of both terms,

$$\frac{r}{L}t_0 = \log_\epsilon \frac{\frac{e}{r} + i_0}{\frac{e}{r} - i_0};$$

and, solving the exponential equation for e, we obtain

$$e = ri_0 \frac{1 + \epsilon^{-\frac{r}{L}t_0}}{1 - \epsilon^{-\frac{r}{L}t_0}}.$$

It is evident that the inequality $e > i_0 r$ must be true, otherwise perfect commutation is not possible.

If

$$e = 0,$$

we have

$$i = i_0 \epsilon^{-\frac{r}{L} t_0};$$

that is, the current never reverses, but merely dies out more or less, and in the moment when the gap G of the armature coil leaves the brush B the current therein has to rise suddenly to full intensity in opposite direction. This being impossible, due to the inductance of the coil, the current forms an arc from the brush across the commutator surface for a length of time depending upon the inductance of the armature coil.

Therefore, with low-resistance brushes, resistance commutation is not permissible except with machines of extremely low arma-

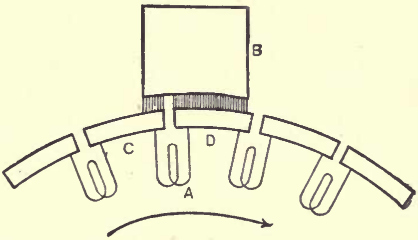


FIG. 108.—Brush commutating coil A.

ture inductance, that is, armature inductance so low that the magnetic energy $\frac{i_0^2 L}{2}$, which appears as "spark" in this case, is harmless.

Voltage commutation is feasible with low-resistance brushes, but requires a commutating e.m.f. e proportional to current i_0; that is, requires shifting of brushes proportionally to the load, or a commutating pole.

In the preceding, the e.m.f. e has been assumed constant during the commutation. In reality it varies somewhat, usually increasing with the approach of the commutated coil to a denser field. It is not possible to consider this variation in general, and e is thus to be considered the average value during commutation.

66. (*b*) High-resistance brush contact.

Fig. 108 represents a brush B commutating armature coil A.

Let r_0 = contact resistance of the brush, that is, resistance from the brush to the commutator surface over the total bearing surface of the brushes. The resistance of the commutated circuit is thus internal resistance of the armature coil r plus the resistance from C to B plus the resistance from B to D.

Thus, if t_0 = time of commutation, at the time t after the beginning of the commutation, the resistance from C to B is $\frac{t_0 r_0}{t}$ and from B to D is $\frac{t_0 r_0}{t_0 - t}$; thus, the total resistance of the commutated coil is

$$R = r + \frac{t_0 r_0}{t} + \frac{t_0 r_0}{t_0 - t} = r + \frac{t_0^2 r_0}{t\,(t_0 - t)}.$$

If i_0 = current in coil A before commutation, the total current into the armature from brush B is $2\,i_0$. Thus, if i = current in commutated coil, the current from B to $D = i_0 + i$, the current from B to $C = i_0 - i$.

Hence, the difference of potential from D to C is

$$\frac{t_0 r_0}{t_0 - t}\,(i_0 + i) - \frac{t_0 r_0}{t}\,(i_0 - i).$$

The e.m.f. acting in coil A is

$$-\,e - \frac{Ldi}{dt},$$

and herefrom the difference of potential from D to C is

$$-\,e - L\frac{di}{dt} - ir;$$

hence,

$$-\,e - L\frac{di}{dt} - ir = \frac{t_0 r_0}{t_0 - t}\,(i_0 + i) - \frac{t_0 r_0}{t}\,(i_0 - i);$$

or, transposing,

$$\frac{Ldi}{dt} + e + ir + \frac{t_0 r_0 i_0\,(2\,t - t_0)}{t\,(t_0 - t)} + \frac{t_0^2 r_0 i}{t\,(t_0 - t)} = 0.$$

$$L\frac{di}{dt} + e + i\left(r + \frac{r_0 t_0^2}{t\,(t_0 - t)}\right) + \frac{r_0 t_0 i_0\,(2\,t - t_0)}{t\,(t_0 - t)} = 0.$$

The further solution of this general problem becomes difficult, but even without integrating this differential equation a number of important conclusions can be derived.

Obviously the commutation is correct and thus sparkless, if the current entering over the brush shifts from segment to segment in direct proportion to the motion of the gap between adjacent segments across the brush, that is, if the current density is uniform all over the contact surface of the brush. This means that the current i in the short-circuited coil varies from $+ i_0$ to $- i_0$ as a linear function of the time. In this case it can be represented by

$$i = i_0 \frac{t_0 - 2t}{t_0};$$

thus,

$$\frac{di}{dt} = - \frac{2 i_0}{t_0}.$$

Substituting this value in the general differential equation gives, after some transformation,

$$\frac{e}{i_0} t_0 + r (t_0 - 2 t) - 2 L = 0;$$

or,

$$e = i_0 \left\{ \frac{2 L}{t_0} - r \left(1 - 2 \frac{t}{t_0}\right) \right\},$$

which gives at the beginning of commutation, $t = 0$,

$$e_1 = i_0 \left(\frac{2L}{t_0} - r\right);$$

at the end of commutation, $t = t_0$,

$$e_2 = i_0 \left(\frac{2 L}{t_0} + r\right);$$

that is, even with high-resistance brushes, for perfect commutation, voltage commutation is necessary, and the e.m.f. e impressed upon the commutated coil must increase during commutation from e_1 to e_2, by the above equation. This e.m.f. is proportional to the current i_0, but is independent of the brush resistance r_0.

Resistance Commutation

67. Herefrom it follows that resistance commutation cannot be perfect, but that at the contact with the segment that leaves the brush the current density must be higher than the average. Let g = ratio of actual current density at the moment of leaving the brush to average current density of brush contact, and considering only the end of commutation, as the most important moment, we have

$$i = i_0 \frac{(2g-1)t_0 - 2gt}{t_0}.$$

For $t = t_0 - t^1$ this gives

$$i = -i_0 + 2g\frac{t^1}{t_0} i_0,$$

while uniform current density would require

$$i = -i_0 + 2\frac{t^1}{t_0} i_0.$$

The general differential equation of resistance commutation, $e = 0$, is

$$L\frac{di}{dt} + i\left(r + \frac{r_0 t_0^2}{t(t_0 - t)}\right) + \frac{r_0 t_0 i_0 (2t - t_0)}{t(t_0 - t)} = 0.$$

Substituting in this equation the value of i from the foregoing equation, expanding and cancelling $t_0 - t$, we obtain

$$2r_0 t_0^2 (g - 1) + rtt_0 (2g - 1) - 2grt^2 - 2gLt = 0;$$

hence,

$$g = \frac{t_0(2r_0 t_0 + rt)}{2(r_0 t_0^2 + rtt_0 - rt^2 - Lt)},$$

and for $t = t_0$,

$$g = \frac{t_0(2r_0 + r)}{2(r_0 t_0 - L)} = 1 + \frac{L + \frac{r}{2}t_0}{r_0 t_0 - L};$$

that is, g is always > 1.

The smaller L and the larger r_0, the smaller is g; that is, the nearer it is to 1, the condition of perfect commutation, and the better is the commutation.

Sparkless commutation is impossible for very large values of g, that is, when L approaches $r_0 t_0$, or when r_0 is not much larger than $\frac{L}{t_0}$. For this reason, in machines in which L cannot be made small, r is sometimes made large by inserting resistors in the leads between the armature and the commutator, so-called "resistance" or "preventive" leads as used in alternating-current commutator motors.

XIV. Types of Commutating Machines

68. By the methods of excitation, commutating machines are subdivided into magneto, separately excited, shunt, series, and compound machines. Magneto machines and separately excited machines are very similar in their characteristics. In either, the field excitation is of constant, or approximately constant, impressed m.m.f. Magneto machines, however, are little used, except for very small sizes.

By the direction of energy transformation, commutating machines are subdivided into generators and motors.

Of foremost importance in discussing the different types of machines is the saturation curve or magnetic characteristic; that is, a curve relating terminal voltage at constant speed to ampere-turns per pole field excitation, at open circuit. Such a curve is shown as *A* in Figs. 109 and 110. It has the same

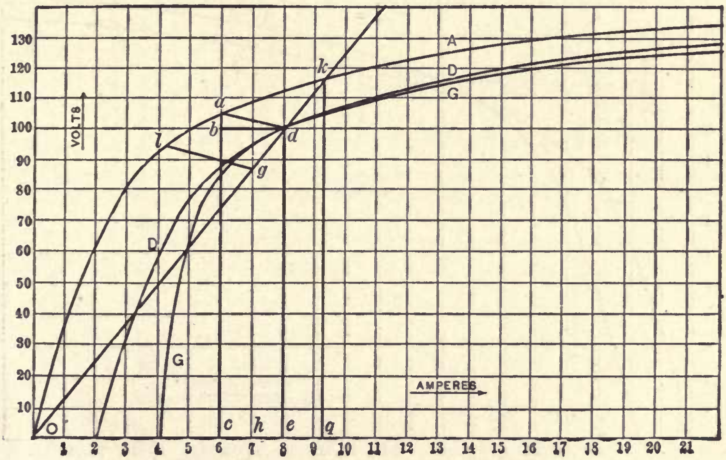


FIG. 109.—Generator saturation curves.

general shape as the magnetic flux density curve, except that the knee or bend is less sharp, due to the different parts of the magnetic circuit saturation successively.

Thus, in order to generate voltage *ac* the field excitation *oc* is required. Subtracting from *ac* in a generator, Fig. 109, or adding in a motor, Fig. 110, the value $ab = ir$, the voltage consumed by the resistance of the armature, commutator, etc., gives the terminal voltage *bc* at current *i*, and adding to *oc* the value $ce = bd = iq$ = armature reaction, or rather field excitation required to overcome the armature reaction, gives the field excitation *oe* required to produce the terminal voltage *de* at current i. The armature reaction iq, corresponding to current i, is calculated as discussed before, and q may be called the *coefficient of armature reaction.*

69. Such a curve, D, shown in Fig. 109 for a generator, and in Fig. 110 for a motor, and giving the terminal voltage de at current i, corresponding to the field excitation oe, is called a *load saturation curve.* Its points are respectively distant from the corresponding points of the no-load saturation curve A a constant distance equal to ad, measured parallel thereto.

Curves D are plotted under the assumption that the armature reaction is constant. Frequently, however, at lower voltage the

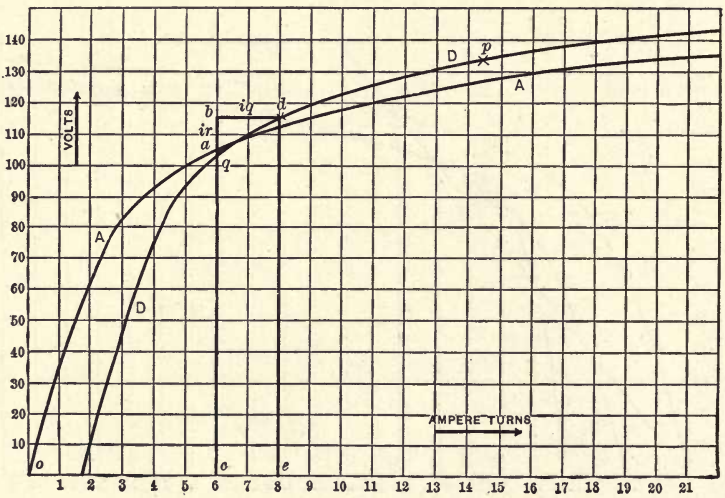


FIG. 110.—Motor saturation curves.

armature reaction, or rather the increase of excitation required to overcome the armature reaction iq, increases, since with voltage commutation at lower voltage, and thus weaker field strength, the brushes have to be shifted more to secure sparkless commutation, and thus the demagnetizing effect of the angle of lead increases. At higher voltage iq usually increases also, due to increase of magnetic saturation under load, caused by the increased stray field. Thus, the load saturation curve of the continuous-current generator more or less deviates from the theoretical shape D toward a shape shown as G.

A. Generators

Separately Excited and Magneto Generator

70. In a separately excited or magneto machine, that is, a machine with constant field excitation F_0, a *demagnetization*

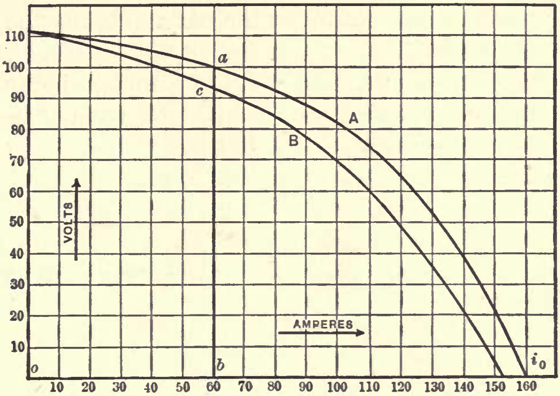


FIG. 111.—Separately excited or magneto-generator demagnetization curve and load characteristic with constant shift of brushes.

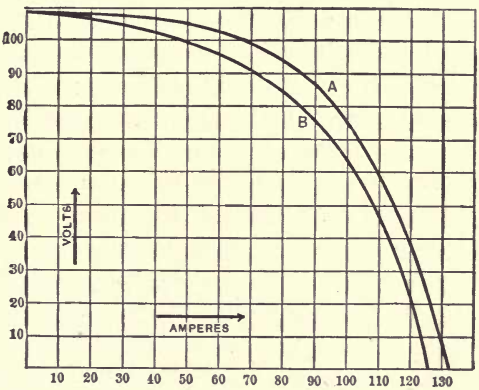


FIG. 112.—Separately excited or magneto-generator demagnetization curve and load characteristic with variable shift of brushes.

curve can be plotted from the magnetization or saturation curve A in Fig. 109. At current i, the resultant m.m.f. of the machine is $F_0 - iq$, and the generated voltage corresponds thereto by the saturation curve A in Fig. 110. Thus, in Fig. 111 a demagnetization curve A is plotted with the current $ob = i$ as abscissas and the generated e.m.f. *ab* as ordinates, under the assumption of constant coefficient of armature reaction q, that is, corresponding to curve D in Fig. 109. This curve becomes zero at the current i_0, which makes $i_0 q = F_0$. Subtracting from curve A in Fig. 111 the drop of voltage in the armature and commutator resistance, $ac = ir$, gives the external characteristic B of the machine as generator, or the curve relating the terminal voltage to the current.

In Fig. 112 the same curves are shown under the assumption that the armature reaction varies with the voltage in the way as represented by curve G in Fig. 109.

In a separately excited or magneto motor at constant speed the external characteristic would lie as much above the demagnetization curve A as it lies below in a generator in Fig. 111, and at constant voltage the speed would vary inversely proportional hereto.

Shunt Generator

71. The external or load characteristic of the shunt generator is plotted in Fig. 113 with the current as abscissas and the terminal voltage as ordinates, as A for constant coefficient of armature reaction, and as B for a coefficient of armature reaction varying with the voltage in the way as shown in G, Fig. 109. The construction of these curves is as follows:

In Fig. 109, *og* is the straight line giving the field excitation *oh* as function of the terminal voltage *hg* (the former obviously being proportional to the latter in the shunt machine). The open-circuit or no-load voltage of the machine is then *kq*.

Drawing *gl* parallel to *da* (assuming constant coefficient of armature reaction, or parallel to the hypothenuse of the triangle *iq*, *ir* at voltage *og*, when assuming variable armature reaction), then the current which gives voltage *gh* is proportional to *gl*, that is, $i : i_0 = gl : da$, where i_0 is the current at the voltage *de*.

As seen from Fig. 113, a maximum value of current exists which is less if the brushes are shifted than at constant position of brushes.

From the load characteristic of the shunt generator the resistance characteristic is plotted in Fig. 114; that is, the dependence of the terminal voltage upon the external resistance $R = \frac{\text{terminal voltage}}{\text{current}}$. Curve A in Fig. 114 corresponds to constant, curve B to varying armature reaction. It is seen that at a certain definite resistance the voltage becomes zero, and for lower resistance the machine cannot generate but loses its excitation.

The variation of the terminal voltage of the shunt generator with the speed at constant field resistance is shown in Fig. 115, at no load as A, and at constant current i as B. These curves are derived from the preceding ones. They show that below a certain speed, which is much higher at load than at no load, the

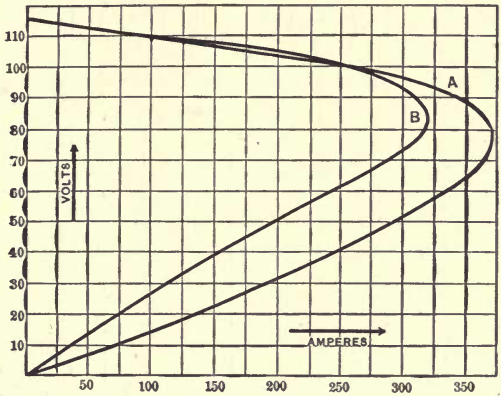


FIG. 113.—Shunt generator load characteristic.

machine cannot generate. The lower part of curve B is unstable and cannot be realized.

Series Generator

72. In the series generator the field excitation is proportional to the current i, and the saturation curve A in Fig. 116 can thus be plotted with the current i as abscissas. Subtracting $ab = ir$, the resistance drop, from the voltage, and adding $bd = iq$, the armature reaction, gives a load saturation curve or external characteristic B of the series generator. The terminal voltage is zero at no load or open circuit, increases with the load, reaches a maximum value at a certain current, and then decreases again and reaches zero at a certain maximum current, the current of short circuit.

Curve B is plotted with constant coefficient of armature reaction q. Assuming the brushes to be shifted with the load and

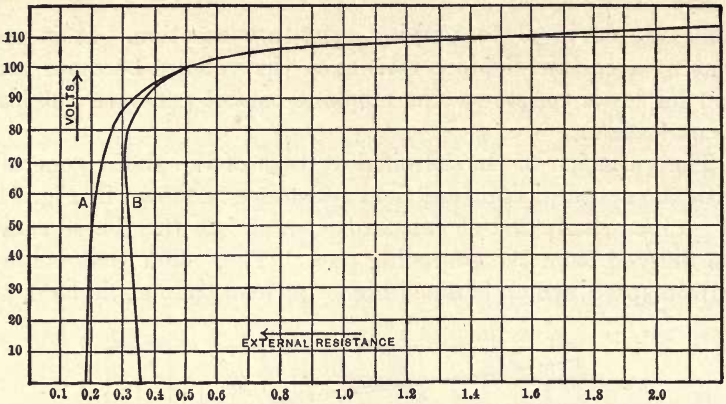


FIG. 114.—Shunt generator resistance characteristic.

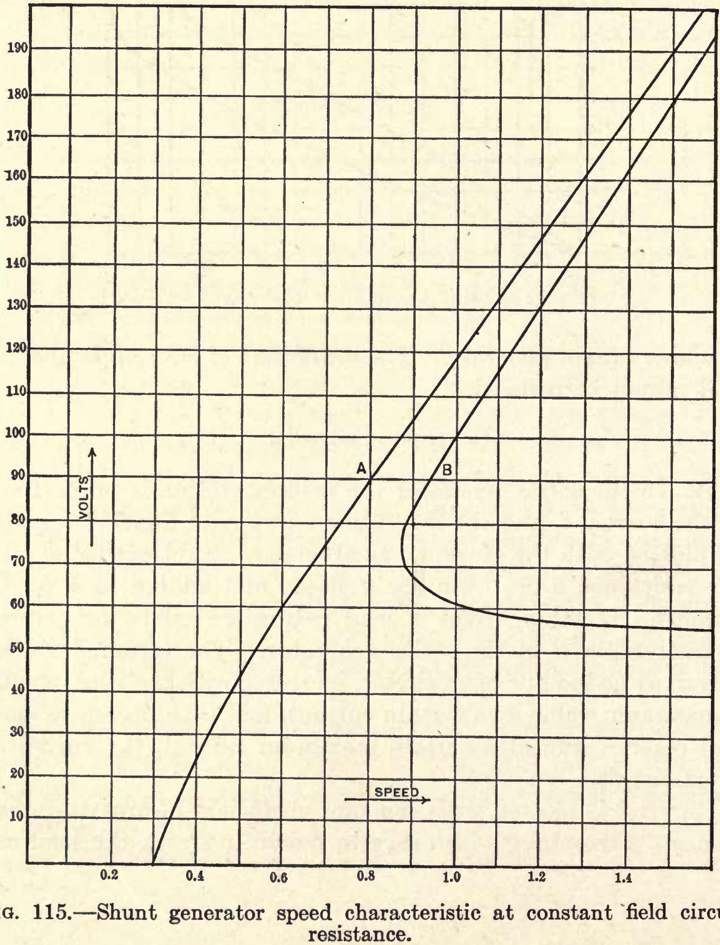


FIG. 115.—Shunt generator speed characteristic at constant field circuit resistance.

proportionally to the load, gives curves *C*, *D*, and *E*, which are higher at light load, but fall off faster at high load. A still further shift of brushes near the maximum current value even overturns the curve as shown in *F*. Curves *E* and *F* correspond to a very great shift of brushes, and an armature demagnetizing effect of the same magnitude as the field excitation, as realized in arc-light machines, in which the last part of the curve is used to secure inherent regulation for constant current.

The resistance characteristic, that is, the dependence of the current and of the terminal voltage of the series generator upon

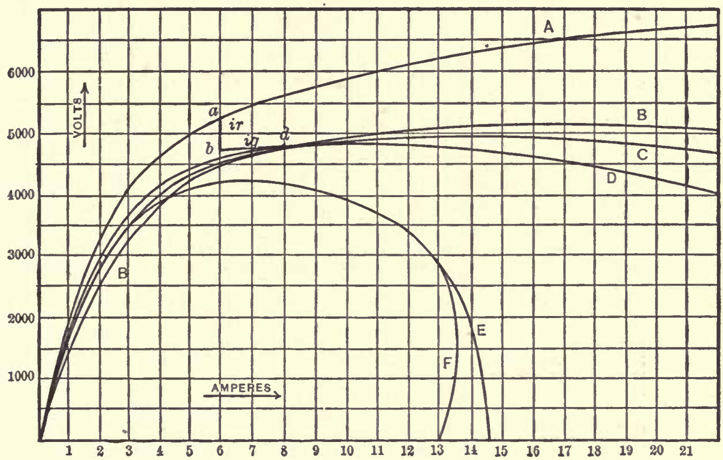


FIG. 116.—Series generator saturation curve and load characteristic.

the external resistance, is constructed from Fig. 116 and plotted in Fig. 117.

B_1 and B_2 in Fig. 117 are terminal volts and amperes corresponding to curve *B* in Fig. 116, E_1, E_2, and F_2 volts and amperes corresponding to curves *E* and *F* in Fig. 116.

Above a certain external resistance the series generator loses its excitation, while the shunt generator loses its excitation below a certain external resistance.

Compound Generator

73. The saturation curve or magnetic characteristic *A*, and the load saturation curves *D* and *G* of the compound generator, are shown in Fig. 118 with the ampere-turns of the shunt field

14

as abscissas. *A* is the same curve as in Fig. 109, while *D* and *G* in Fig. 118 are the corresponding curves of Fig. 109 shifted to

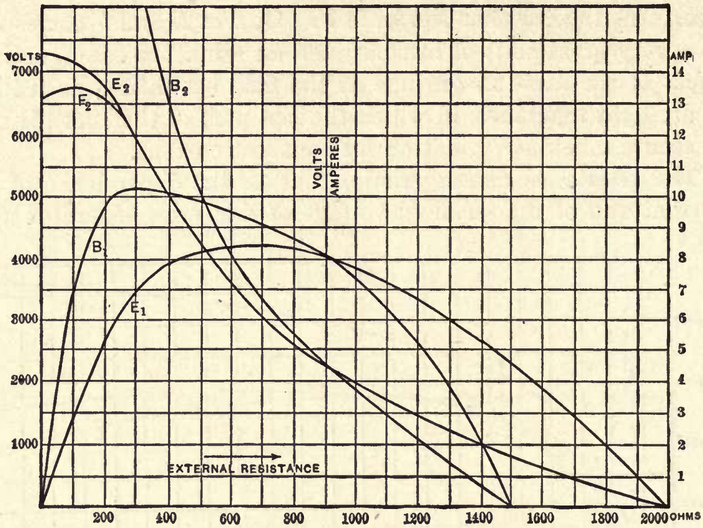


FIG. 117.—Series generator resistance characteristic.

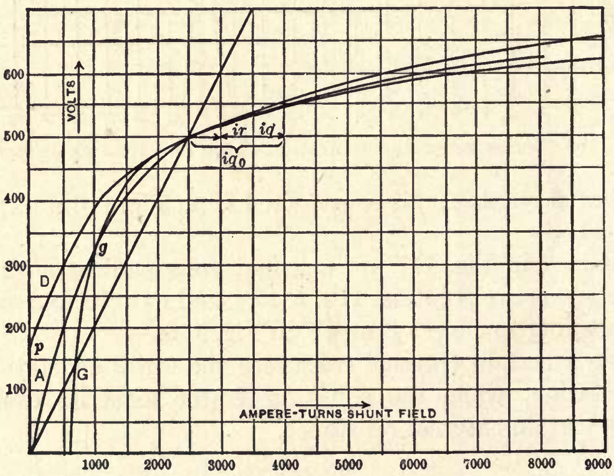


FIG. 118.—Compound generator saturation curve.

the left by the distance iq_0, the m.m.f. of ampere-turns of the series field.

At constant position of brushes the compound generator, when adjusted for the same voltage at no load and at full load, under-compounds at higher and over-compounds at lower voltage, and even at open circuit of the shunt field gives still a voltage *op* as series generator. When shifting the brushes under load, at lower voltage a second point *g* is reached where the machine compounds correctly, and below this point the machine under-compounds and loses its excitation when the shunt field decreases below a certain value; that is, it does not excite itself as series generator.

B. Motors

Shunt Motor

74. Three speed characteristics of the shunt motor at constant impressed e.m.f. *e* are shown in Fig. 116 as *A*, *P*, *Q*, corresponding to the points *d*, *p*, *q* of the motor load saturation curve, Fig. 110. Their derivation is as follows: At constant impressed

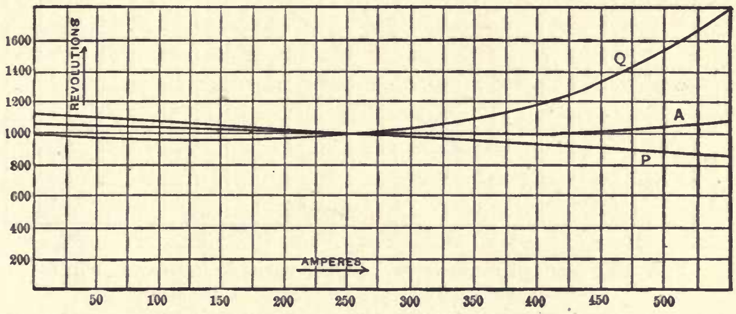


Fig. 119.—Shunt motor speed curves, constant impressed e.m.f.

e.m.f. *e* the field excitation is constant and equals F_0, and at current *i* the generated e.m.f. must be $e - ir$. The resultant field excitation is $F_0 - iq$, and corresponding hereto at constant speed the generated e.m.f. taken from saturation curve *A* in Fig. 110 is e_1. Since it must be $e - ir$, the speed is changed in the proportion $\frac{e - ir}{e_1}$.

At a certain voltage the speed is very nearly constant, the demagnetizing effect of armature reaction counteracting the effect of armature resistance. At higher voltage the speed falls, at lower voltage it rises with increasing current.

In Fig. 120 is shown the speed characteristic of the shunt motor as function of the impressed voltage at constant output, that is, constant product, current times generated e.m.f. If i = current and P = constant output, the generated e.m.f. must be approximately $e_1 = \frac{P}{i}$, and thus the terminal voltage $e = e_1 + ir$. Proportional hereto is the field excitation F_0. The resultant m.m.f. of the field is thus $F = F_0 - iq$, and corresponding thereto from curve A in Fig. 111 is derived the e.m.f. e_0 which would be generated at constant speed by the m.m.f. F.

Since, however, the generated e.m.f. must be e_1, the speed is changed in the proportion $\frac{e_1}{e_0}$.

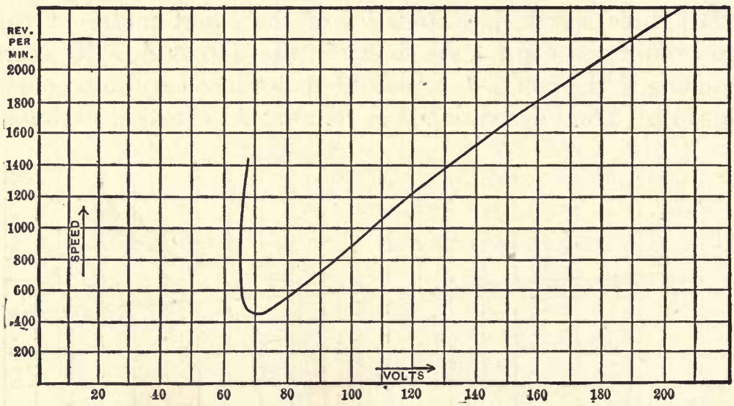


FIG. 120.—Shunt motor speed curve, variable impressed e.m.f.

The speed rises with increasing and falls with decreasing impressed e.m.f. Still further decreasing the impressed e.m.f., the speed reaches a minimum and then increases again, but the conditions become unstable.

Series Motor

75. The speed characteristic of the series motor is shown in Fig. 121 at constant impressed e.m.f. e. A is the saturation curve of the series machine, with the current as abscissas and at constant speed. At current i, the generated e.m.f. must be $e - ir$, and the speed is thus $\frac{e - ir}{e_1}$ times that, for which curve A is plotted, where e_1 = e.m.f. taken from saturation curve A.

This speed curve corresponds to a constant position of brushes midway between the field poles, as generally used in railway motors and other series motors. If the brushes have a constant shift or are shifted proportionally to the load, instead of the saturation curve *A* in Fig. 121 a curve is to be used corresponding to the position of brushes, that is, derived by adding to the abscissas of *A* the values *iq*, the demagnetizing effect of armature reaction.

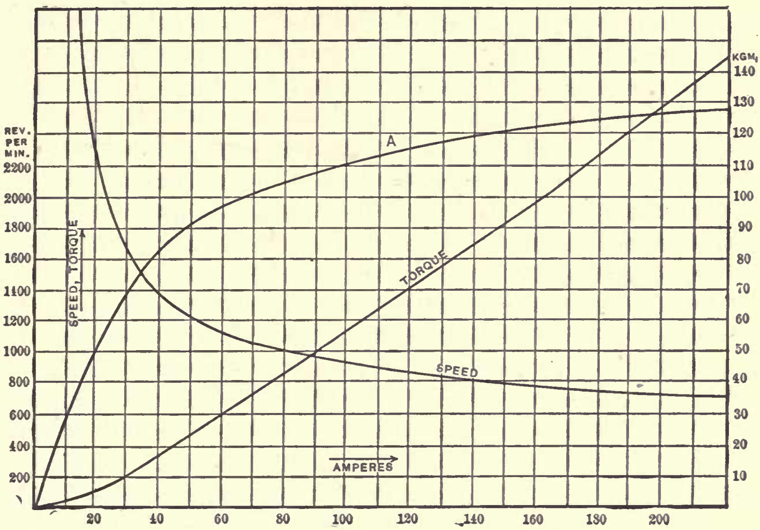


FIG. 121.—Series motor speed curve.

The torque of the series motor is shown also in Fig. 121, derived as proportional to $A \times i$, that is, current $\times$ magnetic flux.

Compound Motors

76. Compound motors can be built with cumulative compounding and with differential compounding.

Cumulative compounding is used to a considerable extent, as in elevator motors, etc., to secure economy of current in starting and at high loads at the sacrifice of speed regulation; that is, a compound motor with cumulative series field stands in its speed and torque characteristic intermediate between the shunt motor and the series motor.

Differential compounding is used to secure constancy of speed with varying load, but to a small extent only, since the speed regulation of a shunt motor can be made sufficiently close, as was shown in the preceding.

Conclusion

77. The preceding discussion of commutating machine types can obviously be only very general, showing the main characteristics of the curves, while the individual curves can be modified to a considerable extent by suitable design of the different parts of the machine when required to derive certain results, as, for instance, to extend the constant-current part of the series generator; or to derive a wide range of voltage at stability, that is, beyond the bend of the saturation curve in the shunt generator; or to utilize the range of the shunt generator load characteristic at the maximum current point for constant-current regulation; or to secure constancy of speed in a shunt motor at varying impressed e.m.f., etc.

The use of the commutating machine as direct-current converter has been omitted from the preceding discussion. By means of one or more alternating-current compensators or autotransformers, connected to the armature by collector rings, the commutating machine can be used to double or halve the voltage, or convert from one side of a three-wire system to the other side and, in general, to supply a three-wire Edison system from a single generator. Since, however, the direct-current converter and three-wire generator exhibit many features similar to those of the synchronous converter, as regards the absence of armature reaction, the reduced armature heating, etc., they will be discussed as an appendix to the synchronous converter.

XV. APPENDIX

ALTERNATING-CURRENT COMMUTATOR MOTOR

78. Since in the series motor and in the shunt motor the direction of the rotation remains the same at a reversal of the impressed voltage, these motors can be operated by an alternating voltage, as alternating-current motors, by making such changes in the materials, proportioning and design, as the alternating nature of the current requires.

In the alternating-current commutator motor, the field structure as well as the armature must be laminated, since the magnetic flux is alternating.

The alternation of the field flux induces an e.m.f. of self induction in the field winding. In the shunt motor, this causes the field exciting current and with it the magnetic field flux to lag and thereby to be out of phase with the armature current which, to represent work, must essentially be an energy current, and thereby reduces output and efficiency and hence requires some method of compensation, as capacity in series with the field winding or excitation of the field from a quadrature phase of voltage. In the series motor the self-inductance of the field causes the main current to lag behind the impressed voltage and thereby lowers the power-factor of the motor. Thus, to get good power-factor, the field self-inductance must be made low, that is, the field as weak and the armature as strong as possible. With such a strong armature, and weak field, the commutating pole is not sufficient to control magnetic distortion by the armature reaction, and complete compensation by a distributed compensating winding, as Fig. 102, page 190, is required.

79. When in the position of commutation the armature coil is short-circuited by the commutator brush, it encloses the full field flux and thus for a moment no e.m.f. is induced in the armature coil by its rotation through the field flux, and in the continuous current machine the coil is without voltage except whatever voltage may be intentionally produced by the commutating flux. In the alternating-current motor, however, the field flux induces voltage also in the armature coil by its alternation, and this voltage is a maximum in the position of commutation, and when short-circuited by the commutator brush tends to produce an excessive current and cause sparking. No position exists on the commutator of the alternating-current motor where the armature coil does not contain an induced e.m.f., but in the position midway between the brushes the e.m.f. induced by the rotation through the magnetic field is a maximum; in the position of commutation the e.m.f. induced by the alternation of the field flux is a maximum. To overcome the destructive sparking caused by the short circuit of the latter e.m.f. by the commutator brush is the problem of making a successful alternating-current commutator:

1. Inducing an opposite e.m.f. by a commutating field. As

the e.m.f. induced by the alternation of the main field is in quadrature with the main field, and the e.m.f. induced by the rotation through the commutating field is in phase with it, the commutating field must be in quadrature with the main field. By properly proportioning this commutating field, as in the series repulsion motor, completely sparkless commutation can be produced at speed. However, at standstill and low speeds this method fails, as the voltage induced by the rotation through the commutating field becomes zero at standstill.

2. Reducing the short-circuit current by high resistance leads between commutator and armature coil. This only mitigates the trouble, but due to the voltage drop in the lead resistance tends to increase sparking at speed. Also, the excessive concentration of heat in the commutating leads in the moment of starting tends to destroy them if the motor does not quickly start.

3. Narrow brushes, to reduce the duration of short circuit.

4. Low impressed frequency, so as to give low values to the induced e.m.f. This is the cause of the desire for abnormally low frequencies, as 15 and even 8 cycles, in alternating-current railway electrification.

5. Low magnetic flux per pole. This is the reason why alternating-current commutator motors of large power usually have such a large number of poles.

These very severe limitations of the design of alternating-current commutating motors are the reason why such motors have found only limited application, except in smaller sizes.

80. Alternating-current motors are usually single-phase, since the possibility of commutation control makes the single-phase easier than a polyphase design. In the single-phase motor, the magnetic field flux is constant in direction, and the direction in quadrature to the main field flux thus is available for producing a suitable commutating flux. In the polyphase motor, however, the magnetic flux rotates, assuming successively all directions, and thus no commutating flux can be used. For this reason, designs of polyphase commutator motors have been made in which the different (2 and 3) phases are kept separate, and spaces left between them for accommodating commutating fluxes.

81. Alternating-current commutator motors are used:

1. In railroading, for securing the advantage of the higher

economy of high voltage alternating-current transmission and distribution. For railroading generally the series motor type is used, either the plain compensated series motor, or inductive modifications thereof, as the repulsion motor etc. In the repulsion motor the armature, instead of being connected in series with field and compensating winding, is closed on itself and thus traversed by a secondary current induced by the compensating winding as primary that is, the armature is connected inductively in series.

2. As constant-speed motor where considerable starting torque is required, as for elevators, hoists, etc., and in general as self-starting single-phase motors. For this purpose, combinations of repulsion and induction type or of series and induction type are used.

3. As adjustable speed, alternating-current motor of single-phase and of polyphase type. The synchronous motor and the induction motor both are constant and fixed speed, the former synchronous, the latter near synchronous. Operating the induction motor materially below synchronism, by armature resistance, is inefficient and gives a speed which varies with the load. By changing the number of poles, or by concatenation, multi-speed induction motors can be produced. The gradual speed adjustment, as given by field control of direct-current motors, requires, however, a commutator on the alternating-current motor. If into the secondary of the induction motor an e.m.f. is introduced, the speed of the motor can be varied by varying the introduced e.m.f.; and lowered, if this e.m.f. is in opposition; raised beyond synchronism, if this e.m.f. is in the same direction as the e.m.f. induced in the motor secondary. As, however, the e.m.f. induced in the induction motor secondary is of the frequency of slip, the speed controlling e.m.f. must either be supplied through the commutator or derived from a low frequency commutating machine as source.

4. For power-factor compensation. In an inductive circuit, the current lags behind the voltage or, what is the same, the voltage leads the current, and the power-factor thus can be raised by compensation either by introducing a leading current, as from condenser or overexcited synchronous motor, or by introducing a lagging voltage. In the commutating machines, the voltage induced in the armature by its rotation is in phase with the field magnetism, and by lagging the field exciting current,

the commutating machines thus can be made to give a lagging voltage, that is, to compensate for low power-factor due to lagging current. Thus, by inserting such a commutating machine into the secondary of an induction machine, the latter can be made to give unity power-factor or even leading current.

Such phase compensation is frequently used in alternating-current commutator motors to get good power-factor. Thus in the series motor, by shunting the field by a non-inductive resistance, and thereby lagging the field exciting component of the current and with it the field flux and the voltage induced in the armature by its rotation, behind the main current, the series motor can at higher speeds be made to give unity power-factor. At low speeds, such complete compensation is not possible, as the compensating voltage is proportional to the speed.

C. SYNCHRONOUS CONVERTERS

I. General

82. For long-distance transmission, and to a certain extent also for distribution, alternating currents, either polyphase or single-phase, are extensively used. For many applications, however, as especially for electrolytic work, direct currents are required, and are usually preferred also for electrical railroading and for low-tension distribution on the Edison three-wire system. Thus, where power is derived from an alternating system, transforming devices are required to convert from alternating to direct current. This can be done either by a direct-current generator driven by an alternating synchronous or induction motor, or by a single machine consuming alternating and producing direct current in one and the same armature. Such a machine is called a converter, and combines, to a certain extent, the features of a direct-current generator and an alternating synchronous motor, differing, however, from either in other features.

Since in the converter the alternating and the direct current are in the same armature conductors, their e.m.fs. stand in a definite relation to each other, which is such that in practically all cases step-down transformers are necessary to generate the required alternating voltage.

Comparing thus the converter with the combination of synchronous or induction motor and direct-current generator, the converter requires step-down transformers; the synchronous motor, if the alternating line voltage is considerably above 10,000 volts, generally requires step-down transformers also; with voltages of 1000 to 10,000 volts, however, usually the synchronous motor and frequently the induction motor can be wound directly for the line voltage and stationary transformers saved. Thus on the one side we have two machines with or sometimes without stationarytransformers, on the other side a single machine with transformers.

Regarding the reliability of operation and first cost, obviously a single machine is preferable.

Regarding efficiency, it is sufficient to compare the converter with the synchronous-motor-direct-current-generator set, since the induction motor is usually less efficient than the synchronous motor. The efficiency of stationary transformers of large size varies from 97 per cent. to 98 per cent., with an average of 97.5 per cent. That of converters or of synchronous motors varies between 91 per cent. and 95 per cent., with 93 per cent. as average, and that of the direct-current generator between 90 per cent. and 94 per cent., with 92 per cent. as average. Thus the converter with its step-down transformers will give an average efficiency of 90.7 per cent., a direct-current generator driven by synchronous motor with step-down transformers an efficiency of 83.4 per cent., without step-down transformers an efficiency of 85.6 per cent. Hence the converter is more efficient, and therefore is almost always preferred.

Mechanically the converter has the advantage that no transfer of mechanical energy takes place, since the torque consumed by the generation of the direct current and the torque produced by the alternating current are applied at the same armature conductors, while in a direct-current generator driven by a synchronous motor the power has to be transmitted mechanically through the shaft.

II. Ratio of e.m.fs. and of Currents

83. In its structure the synchronous converter consists of a closed-circuit armature, revolving in a direct-current excited field, and connected to a segmental commutator as well as to collector rings. Structurally it thus differs from a direct-current machine by the addition of the collector rings, from certain (now very little used) forms of synchronous machines by the addition of the segmental commutator.

In consequence hereof, regarding types of armature windings and of field windings, etc., the same rule applies to the converter as to all commutating machines, except that in the converter the total number of armature coils with a series-wound armature, and the number of armature coils per pair of poles with a multiple-wound armature, must be divisible by the number of phases, and that multiple spiral and reentrant windings are difficult to apply.

Regarding the wave shape of the alternating counter-generated e.m.f., similar considerations apply as for a synchronous machine with closed-circuit armature; that is, the generated e.m.f. usually approximates a sine wave, due to the multi-tooth distributed winding.

Thus, in the following, only those features will be discussed in which the synchronous converter differs from the commutating machines and synchronous machines treated in the preceding chapters.

Fig. 122 represents diagrammatically the commutator of a direct-current machine with the armature coils A connected to adjacent commutator bars. The brushes are B_1B_2, and the field poles F_1F_2.

If now two oppositely located points a_1a_2 of the commutator are connected with two collector rings D_1D_2, it is obvious that

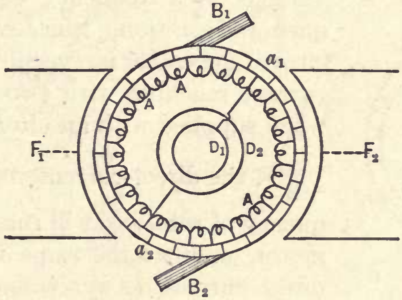


FIG. 122.—Single-phase converter commutator.

the e.m.f. between these points a_1a_2, and thus between the collector rings D_1D_2, will be a maximum in the moment when the points a_1a_2 coincide with the brushes B_1B_2, and is in this moment equal to the direct voltage E of this machine. While the points a_1a_2 move away from this position, the difference of potential between a_1 and a_2 decreases and becomes zero in the moment where a_1a_2 coincide with the direction of the field poles F_1F_2. In this moment the difference in potential between a_1 and a_2 reverses and then increases again, reaching equality with E, but in opposite direction, when a_1 and a_2 coincide with the brushes B_2 and B_1; that is, between the collector rings D_1 and D_2 an alternating voltage is produced whose maximum value equals the direct-current electromotive force E, and which makes a complete period for every revolution of the machine (in a bipolar converter, or p periods per revolution in a machine of $2p$ poles).

Hence, this alternating e.m.f. is

$$e = E \sin 2\pi ft,$$

where f = frequency of rotation, E = e.m.f. between brushes of the machine; thus, the effective value of the alternating e.m.f. is

$$E_1 = \frac{E}{\sqrt{2}}.$$

84. That is, a direct-current machine produces between two collector rings connected with two opposite points of the commutator an alternating e.m.f. of $\frac{1}{\sqrt{2}} \times$ the direct-current voltage, at a frequency equal to the frequency of rotation. Since every alternating-current generator is reversible, such a direct-current machine with two collector rings, when supplied with an alternating e.m.f. of $\frac{1}{\sqrt{2}} \times$ the direct-current voltage at the frequency of rotation, will run as synchronous motor, or if at the same time generating direct current, as synchronous converter.

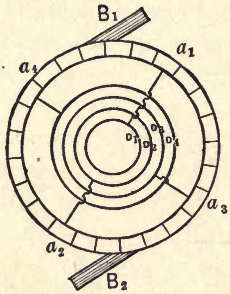


Fig. 123.—Four-phase converter commutator.

Since, neglecting losses and phase displacement, the output of the direct-current side must be equal to the input of the alternating-current side, and the alternating voltage in the single-phase converter is $\frac{1}{\sqrt{2}} \times E$, the alternating current must be $= \sqrt{2} \times I$, where I = direct-current output.

If now the commutator is connected to a further pair of collector rings, D_3D_4 (Fig. 123), at the points a_3 and a_4 midway between a_1 and a_2, it is obvious that between D_3 and D_4 an alternating voltage of the same frequency and intensity will be produced as between D_1 and D_2, but in quadrature therewith, since at the moment where a_3 and a_4 coincide with the brushes B_1B_2 and thus receive the maximum difference of potential, a_1 and a_2 are at zero points of potential.

Thus connecting four equidistant points a_1, a_2, a_3, a_4 of the direct-current generator to four collector rings D_1, D_2, D_3, D_4, gives a four-phase converter of the e.m.f.

$$E_1 = \frac{1}{\sqrt{2}} E \text{ per phase.}$$

The current per phase is (neglecting losses and phase displacement)

$$I_1 = \frac{I}{\sqrt{2}},$$

since the alternating power, $2\, E_1 I_1$, must equal the direct-current power, EI.

Connecting three equidistant points of the commutator to three collector rings as in Fig. 124 gives a three-phase converter.

85. In Fig. 125 the three e.m.fs. between the three collector rings and the neutral point of the three-phase system (or Y voltages) are represented by the vectors $\overline{OE_1}$, $\overline{OE_2}$, $\overline{OE_3}$, thus

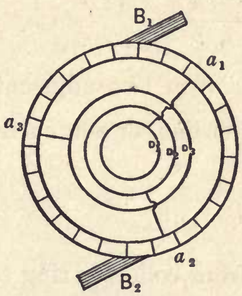


FIG. 124.—Three-phase synchronous converter.

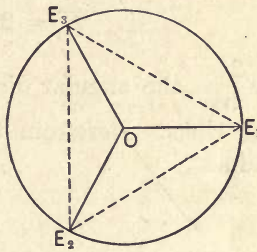


FIG. 125.—E.m.f. diagram of three-phase converter.

the e.m.f. between the collector rings or the delta voltages by vectors $\overline{E_1E_2}$, $\overline{E_2E_3}$, and $\overline{E_3E_1}$. The e.m.f. $\overline{OE_1}$ is, however, nothing but half the e.m.f. E_1 in Fig. 122, of the single-phase converter, that is, $= \frac{E}{2\sqrt{2}}$. Hence the Y voltage, or voltage between collector ring and neutral point or center of the three-phase voltage triangle, is

$$E_1 = \frac{E}{2\sqrt{2}} = 0.354\, E.$$

and thus the delta voltage is

$$E' = E_1 \sqrt{3} = \frac{E\sqrt{3}}{2\sqrt{2}} = 0.612\, E.$$

Since the total three-phase power $3\,I_1E_1$ equals the total continuous-current power IE, it is

$$I_1 = \frac{IE}{3\,E_1} = \frac{2\sqrt{2}}{3}\,I = 0.943\,I.$$

In general, in an n-phase converter, or converter in which n equidistant points of the commutator (in a bipolar machine, or n equidistant points per pair of poles in a multipolar machine with multiple-wound armature) are connected to n collector rings, the voltage between any collector ring and the common neutral, or star voltage, is

$$E_1 = \frac{E}{2\sqrt{2}};$$

consequently the voltage between two adjacent collector rings, or ring voltage, is

$$E' = 2\,E_1 \sin\frac{\pi}{n} = \frac{E \sin\frac{\pi}{n}}{\sqrt{2}},$$

since $\frac{2\pi}{n}$ is the angular displacement between two adjacent collector rings. Herefrom the current per line, or star current, is found as

$$I_1 = \frac{2\sqrt{2}\,I}{n},$$

and the current from line to line, or from collector ring to adjacent collector ring, or ring current, is

$$I' = \frac{\sqrt{2}\,I}{n \sin\frac{\pi}{n}}.$$

86. As seen in the preceding, in the single-phase converter consisting of a closed-circuit armature tapped at two equidistant points to the two collector rings, the alternating voltage is $\frac{1}{\sqrt{2}}$ times the direct-current voltage, and the alternating current $\sqrt{2}$ times the direct current. While such an arrangement of the single-phase converter is the simplest, requiring only two collector rings, it is undesirable, especially for larger machines, on account of the great total and especially local I^2r heating in the armature conductors, as will be shown in the following, and due to the waste of e.m.f., since in the circuit from collector ring to collector ring the e.m.fs. generated in the coils next to the leads are wholly or almost wholly opposite to each other.

The arrangement which I have called the *two-circuit single-phase converter,* and which is diagrammatically shown in Fig. 126, is therefore preferable. The step-down transformer T contains two independent secondary coils A and B, of which one, A, feeds into the armature over conductor rings D_1D_2 and leads a_1a_2, the other, B, over collector rings D_3D_4 and leads a_3a_4, so that the two circuits a_1a_2 and a_3a_4 are in phase with each other, and each spreads over 120 deg. arc instead of 180 deg. arc as in the single-circuit single-phase converter.

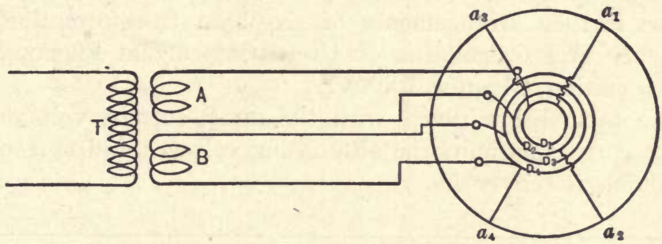


FIG. 126.—Two-circuit single-phase converter.

In consequence thereof, in the two-circuit single-phase converter the alternating counter-generated e.m.f. bears to the continuous-current e.m.f. the same relation as in the three-phase converter, that is,

$$E_1 = \frac{\sqrt{3}}{2\sqrt{2}} E = 0.612\ E,$$

and from the equality of alternating- and direct-current power,

$$2\ I_1E_1 = IE,$$

it follows that each of the two single-phase supply currents is

$$I' = \frac{\sqrt{2}}{\sqrt{3}} I = 0.817\ I.$$

It is seen that in this arrangement one-third of the armature, from a_1 to a_3 and from a_2 to a_4, carries the direct current only, the other two-thirds, from a_1 to a_2 and from a_3 to a_4, the differential current.

A six-phase converter is usually fed from a three-phase system by three transformers or one three-phase transformer. These transformers can either have each one secondary coil only of twice the star or Y voltage, $= \frac{E}{\sqrt{2}}$, which connects with its two terminals two collector rings leading to two opposite points of the armature, or each of the step-down transformers contains two independent secondary coils, and each of the two sets of secondary coils is connected in three-phase delta or Y, but the one set of coils reversed with regard to each other, thus giving two three-phase systems which join to a six-phase system.

The different transformer connections then are distinguished as "diametrical," "double delta" and "double Y."

For further arrangements of six-phase transformation, see "Theory and Calculation of Alternating-current Phenomena," fourth edition, Chapter XXXVI.

The table below gives, with the direct-current voltage and direct current as unit, the alternating voltages and currents of the different converters.

	Direct current	Single-circuit single-phase	Two-circuit single-phase	Three-phase	Four-phase	Six-phase	Twelve-phase	n-phase
Volts between collector ring and neutral point.		$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$	$\frac{1}{2\sqrt{2}}$ $=0.354$
Volts between adjacent collector rings........	1.0	$\frac{1}{\sqrt{2}}$ $=0.707$	$\frac{\sqrt{3}}{2\sqrt{2}}$ $=0.612$	$\frac{\sqrt{3}}{2\sqrt{2}}$ $=0.612$	$\frac{1}{2}=0.5$	$\frac{1}{2\sqrt{2}}$ $=0.354$	0.183	$\frac{\sin\frac{\pi}{n}}{\sqrt{2}}$
Amperes per line......	1.0	$\sqrt{2}$ $=1.414$	$\frac{\sqrt{2}}{\sqrt{3}}$ $=0.817$	$\frac{2\sqrt{2}}{3}$ $=0.943$	$\frac{1}{\sqrt{2}}$ $=0.707$	$\frac{\sqrt{2}}{3}$ $=0.472$	0.236	$\frac{2\sqrt{2}}{n}$
Amperes between adjacent lines..........		$\sqrt{2}$ $=1.414$	$\frac{\sqrt{2}}{\sqrt{3}}$ $=0.817$	$\frac{2\sqrt{2}}{3\sqrt{3}}$ $=0.545$	$\frac{1}{2}=0.5$	$\frac{\sqrt{2}}{3}$ $=0.472$	0.455	$\left\{\begin{matrix}\frac{\sqrt{2}}{n}\times \\ \sin\frac{\pi}{n}\end{matrix}\right.$

These currents give only the power component of alternating current corresponding to the direct-current output. Added thereto is the current required to supply the losses in the machine, that is, to rotate it, and the wattless component if a phase displacement is produced in the converter.

III. Variation of the Ratio of Electromotive Forces

87. The preceding ratios of e.m.fs. apply strictly only to the generated e.m.fs. and that under the assumption of a sine wave of alternating generated e.m.f.

The latter is usually a sufficiently close approximation, since the armature of the converter is a multi-tooth structure, that is, contains a distributed winding.

The ratio between the difference of potential at the commutator brushes and that at the collector rings of the converter usually differs somewhat from the theoretical ratio, due to the e.m.f. consumed in the converter armature, and in machines converting from alternating to continuous current, also due to the shape of the impressed wave.

When converting from alternating to direct current, under load the difference of potential at the commutator brushes is less than the generated direct e.m.f., and the counter-generated alternating e.m.f. less than the impressed, due to the voltage consumed by the armature resistance.

If the current in the converter is in phase with the impressed e.m.f., armature self-inductance has little effect, but reduces the counter-generated alternating e.m.f. below the impressed with a lagging and raises it with a leading current, in the same way as in a synchronous motor.

Thus in general the ratio of voltages varies somewhat with the load and with the phase relation, and with constant impressed alternating e.m.f. the difference of potential at the commutator brushes decreases with increasing load, decreases with decreasing excitation (lag), and increases with increasing excitation (lead).

When converting from direct to alternating current the reverse is the case.

The direct-current voltage stands in definite proportion only to the maximum value of the alternating voltage (being equal to twice the maximum star voltage), but to the effective value (or value read by voltmeter) only in so far as the latter depends upon the former, being $= \frac{1}{\sqrt{2}}$ maximum value with a sine wave.

Thus with an impressed wave of e.m.f. giving a different ratio of maximum to effective value, the ratio between direct and alternating voltage is changed in the same proportion as the ratio of maximum to effective; thus, for instance, with a flat-topped wave of impressed e.m.f., the maximum value of alternating impressed e.m.f., and thus the direct voltage depending thereupon, are lower than with a sine wave of the same effective value, while with a peaked wave of impressed e.m.f. they are higher, by as much as 10 per cent. in extreme cases.

In determining the wave shape of impressed e.m.f. at the converter terminals, not only the wave of generator e.m.f., but also that of the converter counter e.m.f., may be instrumental. Thus, with a converter connected directly to a generating system of very large capacity, the impressed e.m.f. wave will be practically identical with the generator wave, while at the terminals of a converter connected to the generator over long lines with reactive coils or inductive regulators interposed, the wave of impressed e.m.f. may be so far modified by that of the counter e.m.f. of the converter as to resemble the latter much more than the generator wave, and thereby the ratio of conversion may be quite different from that corresponding to the generator wave.

Furthermore, for instance, in three-phase converters fed by ring or delta connected transformers, the star e.m.f. at the converter terminals, which determines the direct voltage, may differ from the star e.m.f. impressed by the generator, by containing different third and ninth harmonics, which cancel when compounding the star voltages to the delta voltage, and give identical delta voltages, as required.

Hence, the ratios of e.m.fs. given in Section II have to be corrected by the drop of voltage in the armature, and have to be multiplied by a factor which is $\sqrt{2}$ times the ratio of effective to maximum value of impressed wave of star e.m.f. ($\sqrt{2}$ being the ratio of maximum to effective of the sine wave on which the ratios in Section II were based), that is, by a "form factor" of the e.m.f. wave.

With an impressed wave differing from the sine shape, there is a current of higher frequency, but generally of negligible magnitude, through the converter armature, due to the difference between impressed and counter e.m.f. wave.

IV. Armature Current and Heating

88. The current in the armature conductors of a converter is the difference between the alternating-current input and the direct-current output.

In Fig. 127, a_1, a_2 are two adjacent leads connected with the collector rings D_1, D_2 in an n-phase converter. The alternating e.m.f. between a_1 and a_2, and thus the power component of the alternating current in the armature section between a_1 and a_2, will reach a maximum when this section is midway between the brushes B_1 and B_2, as shown in Fig. 127.

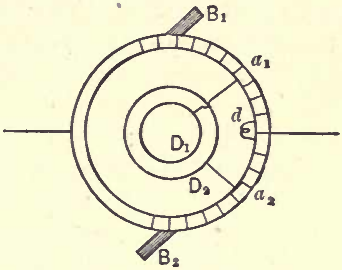


FIG. 127.—Diagram for study of armature heating in synchronous converters.

The direct current in every armature coil reverses at the moment when the coil passes under brush B_1 or B_2, and is thus a rectangular alternating current as shown in Fig. 128 as I. At the moment when the power component of the alternating current is a maximum, an armature coil d midway between two adjacent alternating leads a_1 and a_2 is midway between the brushes B_1 and B_2, as in Fig. 127, and is thus in the middle of its rectangular continuous-current wave, and consequently in this coil the power component of the alternating current and the rectangular direct current are in phase with each other, but opposite, as

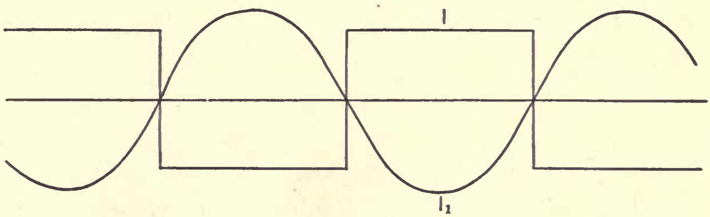


FIG. 128.—Direct current and alternating current in armature coil d, Fig. 127.

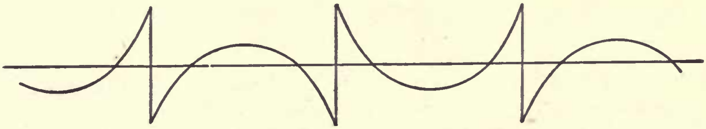


FIG. 129.—Resultant current in coil d, Fig. 127.

shown in Fig. 128 as I_1 and I, and the actual current is their difference, as shown in Fig. 129.

In successive armature coils the direct current reverses successively; that is, the rectangular currents in successive arma-

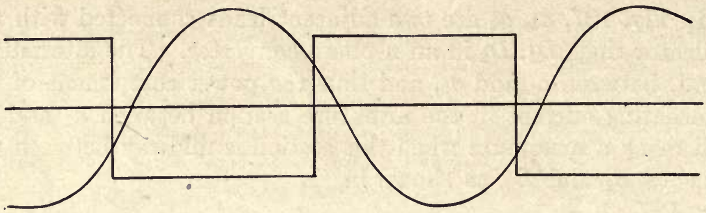


Fig. 130.—Alternating current and direct current in coil between d and a_1 or a_2, Fig. 127.

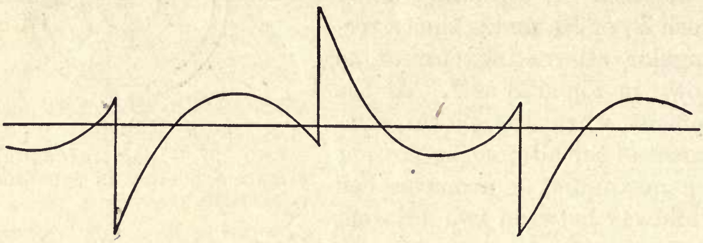


Fig. 131.—Resultant of currents given in Fig. 130.

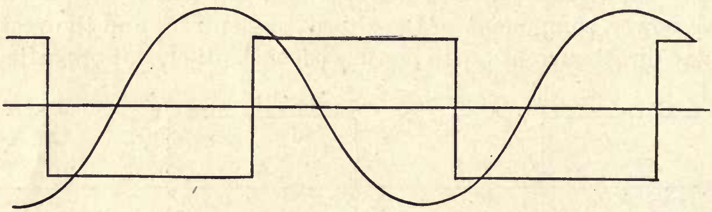


Fig. 132.—Alternating current and direct current in coil between d and a_1 or a_2, Fig. 127.

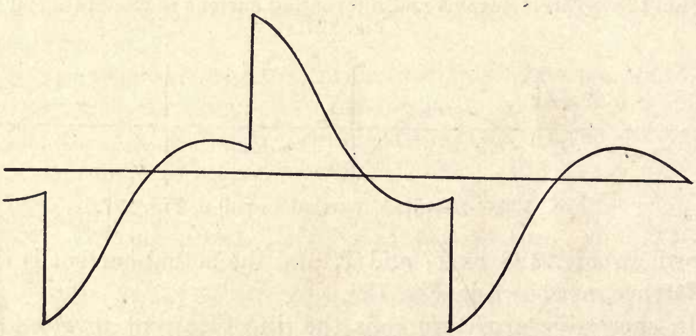


Fig. 133.—Resultant of currents shown in 132.

ture coils are successively displaced in phase from each other; and since the alternating current is the same in the whole section a_1 a_2, and in phase with the rectangular current in the coil d, it becomes more and more out of phase with the rectangular current when passing from coil d toward a_1 or a_2, as shown in Figs. 130 to 133, until the maximum phase displacement between alternating and rectangular current is reached at the alternating leads a_1 and a_2, and is equal to $\frac{\pi}{n}$.

89. Thus, if E = direct voltage, and I = direct current, in an armature coil displaced by angle τ from the position d, midway between two adjacent leads of the n-phase converter, the direct current is $\frac{I}{2}$ for the half period from 0 to π, and the alternating current is

$$\sqrt{2}\, I' \sin(\theta - \tau),$$

where

$$I' = \frac{I\sqrt{2}}{n \sin \frac{\pi}{n}}$$

is the effective value of the alternating current. Thus, the actual current in this armature coil is

$$\begin{aligned} i_0 &= \sqrt{2}\, I' \sin(\theta - \tau) - \frac{I}{2} \\ &= \frac{I}{2}\left\{\frac{4 \sin(\theta - \tau)}{n \sin \frac{\pi}{n}} - 1\right\}. \end{aligned}$$

In a double-current generator, instead of the minus sign, a plus sign would connect the alternating and the direct current in the parenthesis.

The effective value of the resultant converter current thus is:

$$\begin{aligned} I_0 = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_0^2 d\theta} &= \frac{I}{2}\sqrt{\frac{1}{\pi}\int_0^{\pi}\left\{\frac{4 \sin(\theta - \tau)}{n \sin \frac{\pi}{n}} - 1\right\}^2 d\theta} \\ &= \frac{I}{2}\sqrt{\frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \tau}{n\pi \sin \frac{\pi}{n}}}. \end{aligned}$$

Since $\frac{I}{2}$ is the current in the armature coil of a direct-current generator of the same output, we have

$$\gamma_\tau = \left[\frac{I_0}{\frac{I}{2}}\right]^2 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \tau}{n\pi \sin \frac{\pi}{n}},$$

the ratio of the power loss in the armature coil resistance of the converter to that of the direct-current generator of the same output, and thus the ratio of coil heating.

This ratio is a maximum at the position of the alternating leads, $\tau = \frac{\pi}{n}$, and is

$$\gamma_m = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16 \cos \frac{\pi}{n}}{n\pi \sin \frac{\pi}{n}}.$$

It is a minimum for a coil midway between adjacent alternating leads, $\tau = 0$, and is

$$\gamma_0 = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{8}{n\pi \sin \frac{\pi}{n}}.$$

Integrating over τ from 0 (coil d) to $\frac{\pi}{n}$, that is, over the whole phase or section $a_1\, a_2$, we have

$$\Gamma = \frac{n}{\pi}\int_0^{\frac{\pi}{n}} \gamma_\tau d\tau = \frac{8}{n^2 \sin^2 \frac{\pi}{n}} + 1 - \frac{16}{\pi^2},$$

the ratio of the total power loss in the armature resistance of an n-phase converter to that of the same machine as direct-current generator at the same output, or the relative armature heating.

Thus, to get the same loss in the armature conductors, and consequently the same heating of the armature, the current in the converter, and thus its output, can be increased in the proportion $\frac{1}{\sqrt{\Gamma}}$ over that of the direct-current generator.

The calculation for the two-circuit single-phase converter is somewhat different, since in this in one-third of the armature the I^2r loss is that of the direct-current output, and only in the other two-thirds—or an arc $\frac{2\pi}{3}$ —is there alternating current.

Thus in an armature coil displaced by angle τ from the center of this latter section the resultant current is

$$i_0 = \sqrt{2}\, I' \sin(\theta - \tau) - \frac{I}{2}$$
$$= \frac{I}{2}\left\{\frac{4}{\sqrt{3}} \sin(\theta - \tau) - 1\right\},$$

giving the effective value

$$I_0 = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_0^2 d\theta} = \frac{I}{2}\sqrt{\frac{11}{3} - \frac{16}{\pi\sqrt{3}}\cos\tau};$$

thus, the relative heating is

$$\gamma_\tau = \left(\frac{I_0}{\frac{I}{2}}\right)^2 = \frac{11}{3} - \frac{16}{\pi\sqrt{3}}\cos\tau,$$

with the minimum value at $\tau = 0$, it is

$$\gamma_0 = \frac{11}{3} - \frac{16}{\pi\sqrt{3}} = 0.70,$$

and with the maximum value at $\tau = \frac{\pi}{3}$ it is

$$\gamma_m = \frac{11}{3} - \frac{8}{\pi\sqrt{3}} = 2.18;$$

the average current heating in two-thirds of the armature is

$$\Gamma_1 = \frac{3}{\pi}\int^{\frac{\pi}{3}} \gamma_\tau\, d_\tau = \frac{11}{3} - \frac{48}{\pi^2\sqrt{3}} \sin\frac{\pi}{3}$$
$$= \frac{11}{3} - \frac{24}{\pi^2} = 1.236;$$

in the remaining third of the armature, $\Gamma_2 = 1$, thus the average is

$$\Gamma = \frac{2\Gamma_1 + \Gamma_2}{3}$$
$$= 1.151,$$

and therefore the rating is

$$\frac{1}{\sqrt{\Gamma}} = 0.93.$$

By substituting for n, in the general equations of current heating and rating based thereon, numerical values, we get the following table:

Type	Direct-current generator	Single-circuit single-phase	Two-circuit single-phase	Three-phase	Four-phase	Six-phase	Twelve-phase	∞-phase
n	......	2	2	3	4	6	12	
γ_0	1.00	0.45	0.70	0.225	0.20	0.19	0.187	0.187
γ_m	1.00	3.00	2.18	1.20	0.73	0.42	0.24	0.187
Γ	1.00	1.37	1.157	0.555	0.37	0.26	0.20	0.187
Rating (by mean arm. heating)	1.00	0.85	0.93	1.34	1.64	1.96	2.24	2.31

As seen, in the two-circuit single-phase converter the armature heating is less, and more uniformly distributed, than in the single-circuit single-phase converter.

90. A very great gain is made in the output by changing from three-phase to six-phase, but relatively little by still further increasing the number of phases.

In these values, the small power component of current supplying the losses in the converter has been neglected.

These values apply only to the case where the alternating current is in phase with the supply voltage, that is, for unity power-factor of supply. If, however, the current lags, or leads, by the time angle θ, then the alternating current and direct current are not in opposition in the armature coil d midway between adjacent leads, Fig. 127, and the resultant current is a minimum and of the shape shown in Fig. 128, at a point of the armature winding displaced from mid position d by angle $\tau = \theta$. At the leads the displacement between alternating current and direct current then is not $\frac{\pi}{n}$, but $\frac{\pi}{n} + \theta$ at the one, $\frac{\pi}{n} - \theta$ at the other lead, and thus at the other side of the same lead. The resultant current is thus increased at the one, decreased at the other lead, and the heating changed accordingly. For instance, in a quarter-phase converter at zero phase displacement, the resultant current at the lead would be as shown in Fig. 134, $\frac{\pi}{n} = 45$ deg., while at 30 deg. lag the resultant currents in the two coils adjacent to the commutator lead are displaced respectively by $\frac{\pi}{n} + \theta = 75$ deg. and by $\frac{\pi}{n} - \theta = 15$ deg., and so of very different shape, as shown by Figs. 135 and 136, giving very different local heating. Phase displacement thus increases the heating at the one, decreases it at the other side of each commutator lead.

Let again,

I = direct current per commutator brush.

The effective value of the alternating power current in the armature winding, or ring current, corresponding thereto, is

$$I' = \frac{I\sqrt{2}}{n \sin \frac{\pi}{n}}.$$

Let pI' = total power current, allowing for the losses of power in the converter; qI' = reactive current in the converter, assumed as positive when lagging, as negative when leading, and sI' = total current, where $s = \sqrt{p^2 + q^2}$ is the ratio of total current to the load current, that is, power current corresponding to the direct-current output, and $\frac{q}{p} = \tan \theta$ is the time lag of the supply current; p is a quantity slightly larger than 1, by the losses in the converter, or slightly smaller than 1 in an inverted converter.

The actual current in an armature coil displaced in position by angle τ from the middle position d between the adjacent collector leads, then, is

$$\begin{aligned} i_0 &= \sqrt{2}\, I' \{p \sin(\beta - \tau) - q \cos(\beta - \tau)\} - \frac{I}{2} \\ &= \frac{I}{2}\left\{\frac{4}{n \sin \frac{\pi}{n}} [p \sin(\beta - \tau) - q \cos(\beta - \tau)] - 1\right\}, \end{aligned}$$

and, therefore, its effective value is

$$\begin{aligned} I_0 &= \sqrt{\frac{1}{\pi}\int_0^\pi i_0^2 d\beta} \\ &= \frac{I}{2}\sqrt{1 + \frac{8(p^2 + q^2)}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16(p \cos \tau + q \sin \tau)}{\pi n \sin \frac{\pi}{n}}} \\ &= \frac{I}{2}\sqrt{1 + \frac{8 s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16 s \cos(\tau - \theta)}{\pi n \sin \frac{\pi}{n}}}, \end{aligned}$$

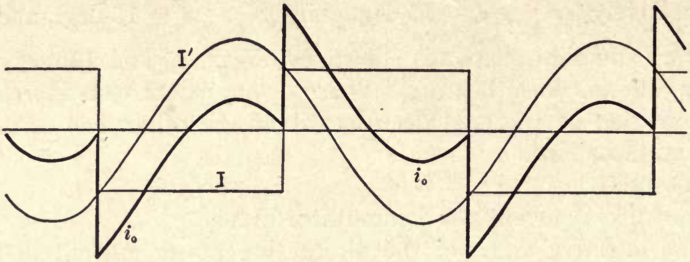


FIG. 134.—Quarter-phase converter unity power-factor, armature current at collector lead.

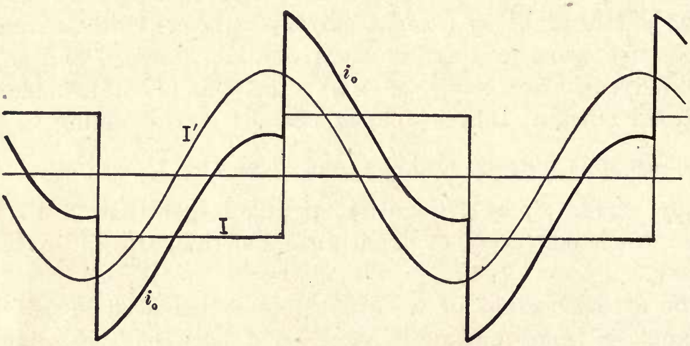


FIG. 135.—Quarter-phase converter phase displacement 30 degrees, armature current at collector lead.

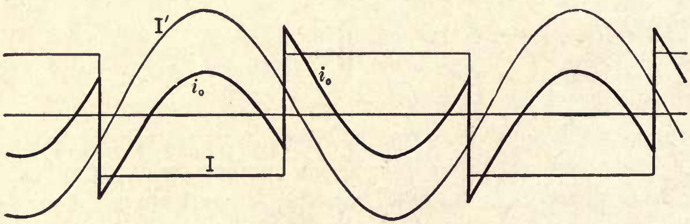


FIG. 136.—Quarter-phase converter phase displacement 30 degrees, armature current at collector lead.

and herefrom the relative heating in an armature coil displaced by angle τ from the middle between adjacent commutator leads:

$$\gamma_\tau = 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,s \cos(\tau - \theta)}{\pi n \sin \frac{\pi}{n}};$$

this gives at the leads, or for $\tau = \mp \frac{\pi}{n}$,

$$\gamma_m = 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,s \cos\left(\frac{\pi}{n} + \theta\right)}{\pi n \sin \frac{\pi}{n}},$$

$$\gamma_m' = 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,s \cos\left(\frac{\pi}{n} - \theta\right)}{\pi n \sin \frac{\pi}{n}},$$

Averaging from $-\frac{\pi}{n}$ to $+\frac{\pi}{n}$ gives the mean current-heating of the converter armature.

$$\Gamma = \frac{2\,n}{\pi} \int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} \gamma_\tau d\tau$$

$$= 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{8\,s\left[\sin\left(\frac{\pi}{n} + \theta\right) + \sin\left(\frac{\pi}{n} - \theta\right)\right]}{n^2 \sin \frac{\pi}{n}}$$

$$= 1 + \frac{8\,s^2}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,s \cos\theta}{\pi^2}$$

$$= 1 + \frac{8\,(p^2 + q^2)}{n^2 \sin^2 \frac{\pi}{n}} - \frac{16\,p}{\pi^2}.$$

91. This gives for

Three-phase, $n = 3$:

$$\gamma_\tau = 1 + 1.185\,s^2 - 1.955\,s \cos(\tau - \theta),$$
$$\gamma_m = 1 + 1.185\,s^2 - 1.955\,s \cos(60 \pm \theta),$$
$$\Gamma = 1 + 1.185\,s^2 - 1.620\,p.$$

Quarter-phase, $n = 4$:

$$\gamma_\tau = 1 + s^2 - 1.795\,s \cos(\tau - \theta),$$
$$\gamma_m = 1 + s^2 - 1.795\,s \cos(45 \pm \theta),$$
$$\Gamma = 1 + s^2 - 1.620\,p.$$

Six-phase, $n = 6$:

$$\gamma_\tau = 1 + 0.889\, s^2 - 1.695\, s \cos(\tau - \theta),$$
$$\gamma_m = 1 + 0.889\, s^2 - 1.695\, s \cos(30 + \theta),$$
$$\Gamma = 1 + 0.889\, s^2 - 1.62\, p,$$

∞-phase, $n = \infty$:

$$\gamma_\tau = \gamma_m = \Gamma = 1 + 0.810\, s^2 - 1.62\, s \cos\theta$$
$$= 1 + 0.810\, s^2 - 1.62\, p.$$

Choosing $p = 1.04$, that is, assuming 4 per cent. loss in friction and windage, core loss and field excitation—the i^2r loss of the armature is not included in p, as it is represented by a drop of direct-current voltage below that corresponding to the alternating voltage, and not by an increase of the alternating current over that corresponding to the direct current—we get, for different phase angles from $\theta = 0$ deg. to $\theta = 60$ deg., the values given below:

	$\theta =$ 0	10	20	30	40	50	60
$s = \dfrac{p}{\cos\theta}$ =	1.04	1.056	1.108	1.20	1.36	1.62	2.08
$q = s \sin\theta = \dfrac{\text{react. cur.}}{\text{power cur.}}$ =	0	0.184	0.379	0.60	0.876	1.24	1.80
$\tan\theta$ =	0	0.176	0.364	0.577	0.839	1.192	1.732
Three-phase:							
γ_m =	1.26	1.62	2.08	2.70	3.65	5.19	8.16
γ_m' =	1.26	1.00	0.80	0.68	0.70	0.99	2.06
Γ =	0.60	0.64	0.77	1.02	1.51	2.43	4.45
Quarter-phase:							
γ_m =	0.76	1.02	1.39	1.88	2.64	3.87	6.30
γ_m' =	0.76	0.55	0.43	0.38	0.42	0.73	1.71
Γ =	— 0.40	0.43	0.54	0.75	1.16	1.94	3.64
Six-phase:							
γ_m =	0.44	0.62	0.88	1.27	1.86	2.85	4.85
γ_m' =	0.44	0.31	0.24	0.25	0.38	0.75	1.79
Γ =	0.28	0.31	0.41	0.60	0.97	1.65	3.17
∞-phase:							
$\gamma_m = \gamma_m' = \Gamma$ =	0.20	0.22	0.32	0.49	0.82	1.45	2.82

92. The values are shown graphically in Figs. 137 and 138, with $\tan \theta = \frac{\text{reactive current}}{\text{energy current}}$ as abscissas, and γ_m as ordinates in Fig. 137, Γ as ordinates in Fig. 138.

As seen, with increasing phase displacement, irrespectively whether lag or lead, the average as well as the maximum armature heating very greatly increases. This shows the necessity of keeping the power-factor near unity at full load and overload, and when applied to phase control of the voltage by converter, means that the shunt field of the converter should be adjusted so as to give a considerable lagging current at no load, so that the

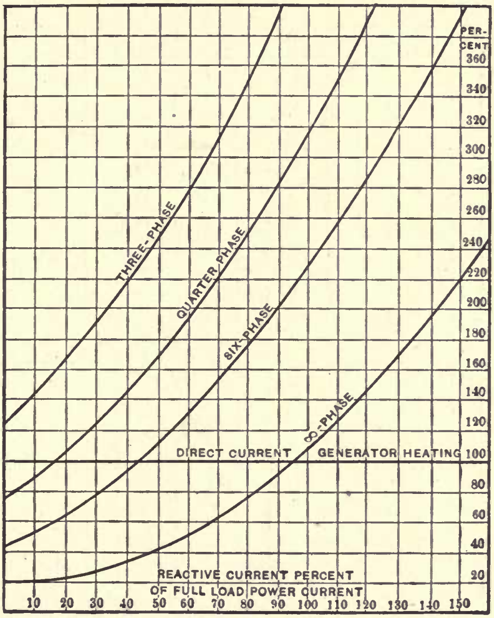


FIG. 137.—Maximum I^2r heating in converter armature coil expressed in per cent. of direct-current generator I^2r heating.

current comes into phase with the voltage at about full load. It therefore is very objectionable in this case to adjust the converter for minimum current at no load, as occasionally done by ignorant engineers, since such wrong adjustment would give considerable leading current at load, and therewith unnecessary armature heating.

It must be considered, however, that above values are referred to the direct-current output, and with increase of phase angle the alternating-current input, at the same output, increases, and the heating increases with the square of the current. Thus at 60 deg. lag or lead, the power-factor is 0.5, and the alternating-current input thus twice as great as at unity power-factor, corresponding to four times the heating. It is interesting therefore to refer the armature heating to the alternating-current input, that is, compare the heating of the converter with that of a synchronous motor of the same alternating-current input. This is given by

$$\Gamma_1 = \frac{\Gamma}{s^2}$$

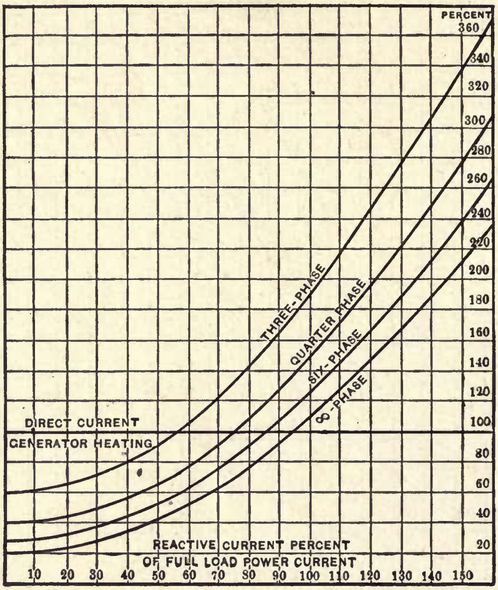


FIG. 138.—Average I^2r heating in converter armature expressed in per cent. of direct-current generator I^2r heating.

and, for $p = 1.04$, gives the following values:

$\theta =$	0	10	20	30	40	50	60
$\tan\theta =$	0	0.176	0.364	0.577	0.839	1.192	14.32
Three-phase:							
$\Gamma_1 =$	0.555	0.57	0.63	0.71	0.82	0.93	1.03
Quarter-phase:							
$\Gamma_1 =$	0.37	0.385	0.44	0.52	0.63	0.74	0.84

Six-phase:							
$\Gamma_1 =$	0.26	0.28	0.335	0.42	0.52	0.63	0.73
∞-phase:							
$\Gamma_1 =$	0.185	0.197	0.26	0.34	0.44	0.55	0.65

It is seen that, compared with the total alternating-current input, the armature heating increases much less with increasing phase displacement, and is almost always much lower than the heating of the same machine at the same input and phase angle, when running a synchronous motor, as shown in Fig. 139.

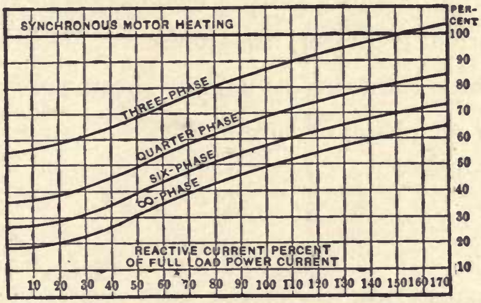


Fig. 139.—Average I^2r heating in converter armature expressed in per cent. of synchronous motor I^2r heating at the same power-factor.

V. Armature Reaction

93. The armature reaction of the polyphase converter is the resultant of the armature reactions of the machine as direct-current generator and as synchronous motor. If the commutator brushes are set at right angles to the field poles or without lead or lag, as is usually done in converters, the direct-current armature reaction consists in a polarization in quadrature behind the field magnetism. The armature reaction due to the power component of the alternating current in a synchronous motor consists of a polarization in quadrature ahead of the field magnetism, which is opposite to the armature reaction as direct-current generator.

Let m = total number of turns on the bipolar armature or per pair of poles of an n-phase converter, I = direct current, then the number of turns in series between the brushes $= \frac{m}{2}$, hence the total armature ampere-turns, or polarization, $= \frac{mI}{2}$. Since, however, these ampere-turns are not unidirectional, but distributed over the whole surface of the armature, their resultant is

$$F = \frac{mI}{2} \text{ avg. cos} \Big|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

and, since

$$\text{avg. cos} \Big|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \frac{2}{\pi},$$

we have $F = \frac{mI}{\pi}$ = direct-current polarization of the converter (or direct-current generator) armature.

In an n-phase converter the number of turns per phase $= \frac{m}{n}$. The current per phase, or current between two adjacent leads (ring current), is

$$I' = \frac{\sqrt{2}\, I}{n \sin \frac{\pi}{n}};$$

hence, the ampere-turn per phase,

$$\frac{mI'}{n} = \frac{\sqrt{2}\, mI}{n^2 \sin \frac{\pi}{n}}.$$

These ampere-turns are distributed over $\frac{1}{n}$ of the circumference of the armature, and their resultant is thus

$$F_1 = \frac{mI'}{n} \text{ avg. cos} \Big|_{-\frac{\pi}{n}}^{+\frac{\pi}{n}},$$

and, since

$$\text{avg. cos} \Big|_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} = \frac{n}{\pi} \sin \frac{\pi}{n},$$

we have

$$F_1 = \frac{\sqrt{2}\, mI}{\pi n} = \text{resultant polarization},$$

in effective ampere-turns of one phase of the converter.

The resultant m.m.f. of n equal m.m.fs. of effective value of F_1, thus maximum value of $F_1 \sqrt{2}$, acting under equal angles

$\frac{2\pi}{n}$, and displaced in phase from each other by $\frac{1}{n}$ of a period, or phase angle $\frac{2\pi}{n}$, is found thus:

Let $F_i = F_1\sqrt{2}\sin\left(\theta - \frac{2i\pi}{n}\right)$ = one of the m.m.fs. of phase angle $\theta = \frac{2i\pi}{n}$, where $i = 0, 1, 2 \ldots n - 1$, acting in the direction $\tau = \frac{2i\pi}{n}$; that is, the zero point of one of the m.m.fs. F_1 is taken as zero point of time θ, and the direction of this m.m.f. as zero point of direction τ.

The resultant m.m.f. in any direction τ is thus

$$F = \sum_1^n F_i \cos\left(\tau + \frac{2i\pi}{n}\right)$$

$$= F_1\sqrt{2}\sum_1^n \sin\left(\theta - \frac{2i\pi}{n}\right)\cos\left(\tau - \frac{2i\pi}{n}\right)$$

$$= \frac{F_1\sqrt{2}}{2}\sum_1^n \left\{\sin\left(\theta + \tau - \frac{4i\pi}{n}\right) + \sin(\theta - \tau)\right\}$$

$$= \frac{F_1\sqrt{2}}{2}\left\{\sum_1^n \sin\left(\theta + \tau - \frac{4i\pi}{n}\right) + n\sin(\theta - \tau)\right\},$$

and, since

$$\sum_1^n \sin\left(\theta + \tau - \frac{4i\pi}{n}\right) = 0,$$

we have

$$F = \frac{nF_1\sqrt{2}}{2}\sin(\theta - \tau);$$

that is, the resultant m.m.f. in any direction τ has the phase

$$\theta = \tau,$$

and the intensity,

$$F' = \frac{nF_1\sqrt{2}}{2},$$

thus revolves in space with uniform velocity and constant intensity, in synchronism with the frequency of the alternating current.

Since in the converter,

$$F_1 = \frac{\sqrt{2}\, mI}{\pi n},$$

we have

$$F' = \frac{mI}{\pi},$$

the resultant m.m.f. of the power component of the alternating current in the n-phase converter.

This m.m.f. revolves synchronously in the armature of the converter; and since the armature rotates at synchronism, the resultant m.m.f. stands still in space, or, with regard to the field poles, in opposition to the direct-current polarization. Since it is equal thereto, it follows that the resultant armature reactions of the direct current and of the corresponding power component of the alternating current in the synchronous converter are equal and opposite, thus neutralize each other, and the resultant armature polarization equals zero. The same is obviously the case in an inverted converter, that is, a machine changing from direct to alternating current.

94. The conditions in a single-phase converter are different, however. At the moment when the alternating current = 0, the full direct-current reaction exists. At the moment when the alternating current is a maximum, the reaction is the difference between that of the alternating and of the direct current; and since the maximum alternating current in the single-phase converter equals twice the direct current, at this moment the resultant armature reaction is equal but opposite to the direct-current reaction.

Hence, the armature reaction oscillates with twice the frequency of the alternating current, and with full intensity, and since it is in quadrature with the field excitation, tends to shift the magnetic flux rapidly across the field poles, and thereby tends to cause sparking and power losses. This oscillating reaction is, however, reduced by the damping effect of the magnetic field structure. It is somewhat less in the two-circuit single-phase converter.

Since in consequence hereof the commutation of the single-phase converter is not as good as that of the polyphase converter, in the former usually voltage commutation has to be resorted to; that is, a commutating pole used, or the brushes shifted from the position midway between the field poles; and in the latter case the continuous-current ampere-turns inclosed by twice the angle of lead of the brushes act as a demagnetizing armature reaction, and require a corresponding increase of the field excitation under load.

While the absence of armature reaction eliminates the need of a commutating pole to counteract the sparking due to the reverse field of armature reaction, nevertheless, commutating poles are very often used in converters, to control the high self-induction of commutation, which economical design requires in such machines. Such commutating poles contain only the ampere turn required to produce the commutating flux, thus less than in generators.

95. Since the resultant main armature reactions neutralize each other in the polyphase converter, there remain only—

1. The armature reaction due to the small power component of current required to rotate the machine, that is, to cover the internal losses of power, which is in quadrature with the field excitation or distorting, but of negligible magnitude.

2. The armature reaction due to the wattless component of alternating current where such exists.

3. An effect of oscillating nature, which may be called a higher harmonic of armature reaction.

The direct current, as rectangular alternating current in the armature, changes in phase from coil to coil, while the alternating current is the same in a whole section of the armature between adjacent leads.

Thus while the resultant reactions neutralize, a local effect remains which in its relation to the magnetic field oscillates with a period equal to the time of motion of the armature through the angle between adjacent alternating leads; that is, double frequency in a single-phase converter (in which it is equal in magnitude to the direct-current reaction, and is the oscillating armature reaction discussed above), sextuple frequency in a three-phase converter, and quadruple frequency in a four-phase converter.

The amplitude of this oscillation in a polyphase converter is small, and its influence upon the magnetic field is usually negligible, due to the damping effect of the field spools, which act like a short-circuited winding for an oscillation of magnetism.

A polyphase converter on unbalanced circuit can be considered as a combination of a balanced polyphase and a single-phase converter; and since even single-phase converters operate quite satisfactorily, the effect of unbalanced circuits on the polyphase converter is comparatively small, within reasonable limits.

Since the armature reaction of the direct current and of the alternating current in the converter neutralize each other, no change of field excitation is required in the converter with changes of load.

Furthermore, while in a direct-current generator the armature reaction at given field strength is limited by the distortion of the field caused thereby, this limitation does not exist in a converter; and a much greater armature reaction can be safely used in converters than in direct-current generators, the distortion being absent in the former.

The practical limit of overload capacity of a converter is usually far higher than in a direct-current generator, since the armature heating is relatively small, and since the distortion of field, which causes sparking on the commutator under overloads in a direct-current generator, is absent in a converter.

The theoretical limit of overload—that is, the overload at which the converter as synchronous motor drops out of step and comes to a standstill—is usually far beyond reach at steady frequency and constant impressed alternating voltage, while on an alternating circuit of pulsating frequency or drooping voltage it obviously depends upon the amplitude and period of the pulsation of frequency or on the drop of voltage.

VI. Reactive Currents and Compounding

96. Since the polarization due to the power component of the alternating current as synchronous motor is in quadrature ahead of the field magnetization, the polarization or magnetizing effect of the lagging component of alternating current is in phase, that of the leading component of alternating current in oppositon to the field magnetization; that is, in the converter no magnetic distortion exists, and no armature reaction at all if the current is in phase with the impressed e.m.f., while the armature reaction is demagnetizing with a leading and magnetizing with a lagging current.

Thus if the alternating current is lagging, the field excitation at the same impressed e.m.f. has to be lower, and if the alternating current is leading, the field excitation has to be higher, than required with the alternating current in phase with the e.m.f. Inversely, by raising the field excitation a leading current, or by lowering it a lagging current, can be produced in a converter (and in a synchronous motor).

Since the alternating current can be made magnetizing or demagnetizing according to the field excitation, at constant impressed alternating voltage, the field excitation of the converter can be varied through a wide range without noticeably affecting the voltage at the commutator brushes; and in converters of high armature reaction and relatively weak field, full load and overload can be carried by the machine without any field excitation whatever, that is, by exciting the field by armature reaction by the lagging alternating current. Such converters without field excitation, or reaction converters, must always run with more or less lagging current, that is, give the same reaction on the line as induction motors, which, as known, are far more objectionable than synchronous motors in their reaction on the alternating system, and therefore they are no longer used.

Conversely, however, at constant impressed alternating voltage the direct-current voltage of a converter cannot be varied by varying the field excitation (except by the very small amount due to the change of the ratio of conversion), but a change of field excitation merely produces wattless currents, lagging or magnetizing with a decrease, leading or demagnetizing with an increase of field excitation. Thus to vary the continuous-current voltage of a converter usually the impressed alternating voltage has to be varied. This can be done either by potential regulator or compensator, that is, transformers of variable ratio of transformation, or by a synchronous machine of the same number of poles as the converter, on the same shaft and connected in series ("synchronous booster") or by the effect of wattless currents on self-inductance. The latter method is especially suited for converters, due to their ability of producing wattless currents by change of field excitation.

The e.m.f. of self-inductance lags 90 deg. behind the current; thus, if the current is lagging 90 deg. behind the impressed e.m.f., the e.m.f. of self-inductance is 180 deg. behind, or in opposition to, the impressed e.m.f., and thus reduces it. If the current is 90 deg. ahead of the e.m.f., the e.m.f. of self-inductance is in phase with the impressed e.m.f., thus adds itself thereto and raises it. Therefore, if self-inductance is inserted into the lines between converter and constant-potential generator, and a wattless lagging current is produced by the converter by a decrease of its field excitation, the e.m.f. of self-inductance of this lagging current in the line lowers the alternating impressed voltage at the converter and thus its direct-current voltage; and if a wattless leading current is produced by the converter by an increase of its field excitation, the e.m.f. of self-inductance of this leading current raises the impressed alternating voltage at the converter and thus its direct-current voltage.

97. In this manner, by self-inductance in the lines leading to the converter, its voltage can be varied by a change of field excitation, or conversely its voltage maintained constant at constant generator voltage or even constant generator excitation, with increasing load and thus increasing resistance drop in the line; or the voltage can even be increased with increasing load, that is, the system over-compounded.

The change of field excitation of the converter with changes of load can be made automatic by the combination of shunt and series field, and in this manner a converter can be compounded or even over-compounded similarly to a direct-current generator. While the effect is the same, the action, however, is different; and the compounding takes place not in the machine as with a direct-current generator, but in the alternating lines leading to the machine, in which self-inductance becomes essential.

As the reactance of the transmission line is rarely sufficient to give phase control over a wide range without excessive reactive currents, it is customary, especially at 25 cycles, to insert reactive coils into the leads between the converter and its step-down transformers, in those cases in which automatic phase control by converter series fields is desired, as in power transmission for suburban and interurban railways, etc., or to specially design the step-down transformers for high internal reactance. Usually these reactive coils are designed to give at full-load current a reactance voltage equal to about 15 per cent. of the converter supply voltage, and therefore capable of taking care of about 10 per cent. line drop at good power-factors.

VII. Variable Ratio Converters ("Split Pole" Converters)

98. With a sine wave of alternating voltage, and the commutator brushes set at the magnetic neutral, that is, at right angles to the resultant magnetic flux, the direct voltage of a converter is constant at constant impressed alternating voltage. It equals the maximum value of the alternating voltage between two diametrically opposite points of the commutator, or "diametrical voltage," and the diametrical voltage is twice the voltage between alternating lead and neutral, or star or Y voltage of the polyphase system.

A change of the direct voltage, at constant impressed alternating voltage, can be produced—

Either by changing the position angle between the commutator brushes and the resultant magnetic flux, so that the direct voltage between the brushes is not the maximum diametrical alternating voltage but only a part thereof,

Or by changing the maximum diametrical alternating voltage, at constant effective impressed voltage, by wave-shape distortion by the superposition of higher harmonics.

In the former case, only a reduction of the direct voltage below the normal value can be produced, while in the latter case an increase as well as a reduction can be produced, an increase if the higher harmonics are in phase, and a reduction if the higher harmonics are in opposition to the fundamental wave of the diametrical or Y voltage.

Both methods are combined in the so-called "Regulating Pole Converter" or "Split Pole Converter," which is used to supply, from constant alternating voltage supply, direct voltage varying sometimes over a range of $\pm$ 20 per cent.

In this type of converter, the field pole is divided into sections, usually two, a smaller one, the regulating pole, and a larger one, the main pole. By varying the excitation of the regulating pole from maximum in one direction, to maximum in the opposite direction, the direction of the resultant magnetic field flux, and the effective width of the field pole, and with the latter the wave shape, are varied. To keep the wave shape variation local in the converter, so as not to reflect it into the primary supply circuit, the proper transformer connection must be used. This is Y primary with preferably Δ or double delta (for three-phase and for six-phase) or Y and double Y or diametrical in the secondary.

VIII. Starting

99. The polyphase converter is self-starting from rest; that is, when connected across the polyphase circuit it starts, accelerates, and runs up to complete synchronism. The e.m.f. between the commutator brushes is alternating in starting, with the frequency of slip below synchronism. Thus a direct-current voltmeter or incandescent lamps connected across the commutator brushes indicate by their beats the approach of the converter to synchronism. When starting, the field circuit of the converter has to be opened or at least greatly weakened. The starting of the polyphase converter is largely a hysteresis effect and entirely so in machines with laminated field poles, while in machines with solid magnet poles or with a short-circuited winding (squirrel-cage) in the field poles, secondary currents in the latter contribute to the starting torque, but at the same time reduce the magnetic starting flux by their demagnetizing effect. The torque is produced by the attraction between the alternating currents of the successive phases upon the remanent magnetism and secondary currents produced by the preceding phase. It is necessarily comparatively weak, and from full-load to twice full-load current at from one-third to one-half of full voltage is required to start from rest without load. Usually, low-voltage taps on the transformers are used to give the lower starting voltage.

While an induction motor can never reach exact synchronism, but must even at no load slip slightly to produce the friction torque, the converter or synchronous motor reaches exact synchronism, due to the difference of the magnetic reluctance in the direction of the field poles and in the direction in electrical quadrature thereto; that is, the field structure acts like a shuttle armature and the polar projections catch with the rotating magnet poles in the armature, in a similar way as an induction motor armature with a single short-circuited coil (synchronous induction motor, reaction machine) drops into step. Obviously, the single-phase converter is not self-starting.

At the moment of starting, the field circuit of the converter is in the position of a secondary to the armature circuit as primary; and since in general the number of field turns is very much larger than the number of armature turns, excessive e.m.fs. may be generated in the field circuit, reaching frequently 4000 to 6000 volts, which have to be taken care of by some means, as by breaking the field circuit into sections, or protecting against excessive voltages by a squirrel-cage starting winding in the pole faces. As soon as synchronism is reached, which usually takes from a few seconds to a minute or more, and is seen by the appearance of continuous voltage at the commutator brushes, the field circuit is closed and the load put on the converter. Obviously, while starting, the direct-current side of the converter must be open-circuited, since the e.m.f. between commutator brushes is alternating until synchronism is reached.

When starting from the alternating side, the converter can drop into synchronism at either polarity; but its polarity can be reversed by strongly exciting the field in the right direction by some outside source, as another converter, etc., or by momentarily opening the circuit and thereby letting the converter slip one pole.

Since when starting from the alternating side the converter requires a very large and, at the same time, lagging current, it is occasionally preferable to start it from the direct-current side as direct-current motor. This can be done when connected to storage battery or direct-current generator. When feeding into a direct-current system together with other converters or converter stations, all but the first converter can be started from the continuous current side by means of rheostats inserted into the armature circuit.

To avoid the necessity of synchronizing the converter, by phase lamps, with the alternating system in case of starting by direct current (which operation may be difficult where the direct voltage fluctuates, owing to heavy fluctuations of load, as railway systems), it is frequently preferable to run the converter up to or beyond synchronism by direct current, then cut off from the direct current, open the field circuit and connect it to the alternating system, thus bringing it into step by alternating current.

If starting from the alternating side is to be avoided, and direct current not always available, as when starting the first converter, a small induction motor (of less poles than the converter) is used as starting motor.

Converters usually are started from the alternating side.

IX. Inverted Converters

100. Converters may be used to change either from alternating to direct current or as inverted converters from direct to alternating current. While the former use is by far the more frequent, sometimes inverted converters are desirable. Thus in low-tension direct-current systems outlying districts have been supplied by converting from direct to alternating, transmitting as alternating, and then reconverting to direct current. Or in a station containing direct-current generators for short-distance supply and alternators for long-distance supply, the converter may be used as the connecting link to shift the load from the direct to the alternating generators, or inversely, and thus be operated either way according to the distribution of load on the system. Or inverted operation may be used in emergencies to produce alternating current.

When converting from alternating to direct current, the speed of the converter is rigidly fixed by the frequency, and cannot be varied by its field excitation, the variation of the latter merely changing the phase relation of the alternating current. When converting, however, from direct to alternating current as the only source of alternating current, that is, not running in multiple with engine- or turbine-driven alternating-current generators, the speed of the converter as direct-current motor depends upon the field strength; thus it increases with decreasing and decreases with increasing field strength. As alternating-current generator, however, the field strength depends upon the intensity and phase relation of the alternating current, lagging current reducing the field strength and thus increasing speed and frequency, and leading current increasing the field strength and thus decreasing speed and frequency.

Thus, if a load of lagging current is put on an inverted converter, as, for instance, by starting an induction motor or another converter thereby from the alternating side, the demagnetizing effect of the alternating current reduces the field strength and causes the converter to increase in speed and frequency. An increase of frequency, however, may increase the lag of the current, and thus its demagnetizing effect, and thereby still further increase the speed, so that the acceleration may become so rapid as to be beyond control by the field rheostat and endanger the machine. Hence inverted converters have to be carefully watched, especially when starting other converters from them, and some absolutely positive device is necessary to cut the inverted converter off the circuit entirely as soon as its speed exceeds the danger limit. The relatively safest arrangement is separate excitation of the inverted converter by an exciter mechanically driven thereby, since an increase of speed increases the exciter voltage at a still higher rate, and thereby the excitation of the converter, and thus tends to check its speed.

This danger of racing does not exist if the inverted converter operates in parallel with alternating generators, provided that the latter and their prime movers are of such size that they cannot be carried away in speed by the converter. In an inverted converter running in parallel with alternators the speed is not changed by the field excitation, but a change of the latter merely changes the phase relation of the alternating current supplied by the converter; that is, the converter receives power from the direct-current system, and supplies power into the alternating-current system but at the same time receives wattless current from the alternating system, lagging at under-excitation, leading at over-excitation, and can in the same way as an ordinary converter or synchronous motor be used to compensate for wattless currents in other parts of the alternating system, or to regulate the voltage by phase control.

X. Frequency

101. While converters can be designed for any frequency, the use of high frequency, as 60 cycles, imposes more severe limitations on the design, especially that of the commutator, as to make the high-frequency converter inferior to the low-frequency or 25-cycle converter.

The commutator surface moves the distance from brush to next brush, or the commutator pitch, during one-half cycle, that is, $1/50$ second with a 25-cycle, $1/120$ second with a 60-cycle converter. The peripheral speed of the commutator, however, is limited by mechanical, electrical, and thermal considerations—centrifugal forces, loss of power by brush friction, and heating caused thereby. The limitation of peripheral speed limits the commutator pitch. Within this pitch must be included as many commutator segments as necessary to take care of the voltage from brush to brush, and these segments must have a width sufficient for mechanical strength. With the smaller pitch required for high frequency, this may become impossible, and the limits of conservative design thus may have to be exceeded.

In a converter, due to the absence of armature reaction and field distortion, a higher voltage per commutator segment can be allowed than in a direct-current generator. Assuming 17 volts as limit of conservative design would give for a 600-volt converter 36 segments from brush to brush. Allowing 0.2 inch for segment and insulation, as minimum conservative value, 37 segments give a pitch of 7.4 inches. Estimating 5000 feet per minute as conservative limit of commutator speed gives 83.3 feet or 1000 inches peripheral speed per second, and with 7.4 inches pitch this gives 136 half cycles, or 68 cycles, as limit of the frequency, permitting conservative commutator design.

At 60 cycles higher voltage per segment, narrower segments and higher commutator speeds thus are necessary than at 25 cycles, and the 60-cycle converter, though still within conservative limits, does not permit as conservative commutator design, especially at higher voltage, as a low-frequency converter, and a lower self-inductance of commutation thus must be aimed at than permissible in a 25-cycle converter, the more so as the frequency of commutation (half the number of commutator segments per pole times frequency of rotation) necessarily is higher in the 60-cycle converter.

Somewhat similar considerations also apply to the armature construction: the peripheral speed of the armature, even if chosen higher for the 60-cycle converter, limits the pitch per pole at the armature circumference, and thereby the ampere conductors per pole and thus the armature reaction, the more so as shallower slots are necessary. The 60-cycle converter cannot be built with anything like the same armature reaction as is feasible at lower frequency. On the armature reaction, however, very largely depends the stability of a synchronous motor or converter, and machines of low armature reaction tend far more to surging and pulsation of current and voltage than machines of high armature reaction.

The 60-cycle converter therefore cannot be made quite as stable and capable of taking care of violent fluctuations of load and of excessive overloads as 25-cycle converters can, and in this respect the lower-frequency machine is preferable, though under reasonably favorable conditions regarding variations of load, variations of supply voltage, and overload 60-cycle converters give excellent service.

It is this inherent inferiority of the 60-cycle converter which has largely been instrumental in introducing 25 cycles as the frequency of electric power generation and distribution.

At 25 cycles, converters are used on railway load—the most fluctuating and therefore most severe service—built for 1200 volts, and even still much higher voltages are available.

XI. Double-current Generators

102. Similar in appearance to the converter, which changes from alternating to direct current, and to the inverted converter, which changes from direct to alternating current, is the double-current generator; that is, a machine driven by mechanical power and producing direct current as well as alternating current from the same armature, which is connected to commutator and collector rings in the same way as in the converter. Obviously the use of the double-current generator is limited to those sizes and speeds at which a good direct-current generator can be built with the same number of poles as a good alternator, that is, low-frequency machines of large output and relatively high speed; while high-frequency low-speed double-current generators are undesirable.

The essential difference between double-current generator and converter is, however, that in the former the direct current and the alternating current are not in opposition as in the latter, but in the same direction, and the resultant armature polarization thus the sum of the armature polarization of the direct current and of the alternating current.

Since at the same output and the same field strength the armature polarization of the direct current and that of the alternating current are the same, it follows that the resultant armature polarization of the double-current generator is proportional to the load regardless of the proportion in which this load is distributed between the alternating- and direct-current sides. The heating of the armature due to its resistance depends upon the sum of the two currents, that is, upon the total load on the machine. Hence, the output of the double-current generator is limited by the current heating of the armature and by the field distortion due to the armature reaction, in the same way as in a direct-current generator or alternator, and is consequently much less than that of a converter.

In double-current generators, owing to the existence of armature reaction and consequent field distortion, the commutator brushes are more or less shifted against the neutral, and the direction of the continuous-current armature polarization is thus shifted against the neutral by the same angle as the brushes. The direction of the alternating-current armature polarization, however, is shifted against the neutral by the angle of phase displacement of the alternating current. In consequence thereof, the reactions upon the field of the two parts of the armature polarization, that due to the continuous current and that due to the alternating current, are usually different. The reaction on the field of the direct-current load can be overcome by a series field. The reaction on the field of the alternating-current load when feeding converters can be compensated for by a change of phase relation, by means of a series field on the converter, with self-inductance in the alternating lines, or reactive coils at the converters.

Thus, a double-current generator feeding on the alternating side converters can be considered as a direct-current generator in which a part of the commutator, with a corresponding part of the series field, is separated from the generator and located at a distance, connected by alternating leads to the generator. Obviously, automatic compounding of a double-current generator is feasible only if the phase relation of the alternating current changes from lag at no load to lead at load, in the same way as produced by a compounded converter. Otherwise, rheostatic control of the generator is necessary. This is, for instance, the case if the voltage of the double-current generator has to be varied to suit the conditions of its direct-current load, and the voltage of the converter at the end of the alternating lines varied to suit the conditions of load at the receiving end, independent of the voltage at the double-current generator, by means of alternating potential regulators or compensators.

Compared with the direct-current generator, the field of the double-current generator must be such as to give a much greater stability of voltage, owing to the strong demagnetizing effect which may be exerted by lagging currents on the alternating side, and may cause the machine to lose its excitation altogether. For this reason it is frequently preferable to excite double-current generators separately. With the general adoption of large three-phase steam-turbine units for electric power generation, the use of inverted converter and double-current generator has greatly decreased.

XII. Conclusion

103. Of the types of machines, converter, inverted converter, and double-current generator, sundry combinations can be dedevised with each other and with synchronous motors, alternators, direct-current motors and generators. Thus, for instance, a converter can be used to supply a certain amount of mechanical power as synchronous motor. In this case the alternating current is increased beyond the value corresponding to the direct current by the amount of current giving the mechanical power, and the armature reactions do not neutralize each other, but the reaction of the alternating current exceeds that of the direct current by the amount corresponding to the mechanical load. In the same way the current heating of the armature is increased. An inverted converter can also be used to supply some mechanical power. Either arrangement, however, while quite feasible, has the disadvantage of interfering with automatic control of voltage by compounding.

Double-current generators can be used to supply more power into the alternating circuit than is given by their prime mover, by receiving power from the direct-current side. In this case a part of the alternating power is generated from mechanical power, and the other converted from direct-current power, and the machine combines the features of an alternator with those of an inverted converter. Conversely, when supplying direct-current power and receiving mechanical power from the prime mover and electric power from the alternating system, the double-current generator combines the features of a direct-current generator and a converter. In either case the armature reaction, etc., are the sum of those corresponding to the two types of machines combined.

104. A combination of the converter with the direct-current generator is represented by the so-called "*motor converter*," which consists of the concatenation of a commutating machine with an induction machine.

If the secondary of an induction machine is connected to a second induction or synchronous machine on the same shaft, and of the same number of poles, the combination runs at half synchronous speed, and the first induction machine as frequency converter supplies half of its power as electric power of half frequency to the second machine, and changes the other half as motor into mechanical power, driving the second machine as generator. (Or, if the two machines have different number of poles, or are connected to run at different speeds, the division of power is at a different but constant ratio). Using thus a double-current generator as second machine, it receives half of its power mechanically, by the induction machine as motor, and the other half electrically, by the induction machine as frequency converter. Such a machine, then, is intermediate between a converter and a direct-current generator, having an armature reaction equal to half that of a direct-current generator.

Such motor converters have been recommended for high-frequency systems, as their commutating component is of half frequency, and thus affords a better commutator design than a high-frequency converter. They are necessarily much larger than standard converters, but are smaller than motor generator sets, as half the power is converted in either machine. One advantage of this type of machine for phase control is that it requires no additional reactive coils, as the induction machine affords sufficient reactance.

The use of the converter to change from alternating to alternating of a different phase, as, for instance, when using a quarter-phase converter to receive power by one pair of its collector rings from a single-phase circuit and supplying from its other pair of collector rings the other phase of a quarter-phase system, or a three-phase converter on a single-phase system supplying the third wire of a three-phase system from its third collector ring, is outside the scope of this treatise, and is, moreover, of very little importance, since induction or synchronous motors are superior in this respect.

Appendix

XIII. Direct-current Converter

105. If n equidistant pairs of diametrically opposite points of a commutating machine armature are connected to the ends of n compensators or autotransformers, that is, electric circuits interlinked with a magnetic circuit, and the centers of these autotransformers connected with each other to a neutral point as shown diagrammatically in Fig. 140 for $n = 3$, this neutral is equidistant in potential from the two sets of commutator brushes, and such a machine can be used as continuous current converter, to transform in the ratio of potentials 1 : 2 or 2 : 1 or 1 : 1, in the latter case transforming power from one side of a three-wire system to the other side.

Obviously either the n autotransformers can be stationary and connected to the armature by $2n$ collector rings, or the autotransformers rotated with the armature and their common neutral connected to the external circuit by one collector ring.

The distribution of potential and of current in such a direct-current converter is shown in Fig. 141 for $n = 2$, that is, two autotransformers in quadrature.

With the voltage $2e$ between the outside conductors of the

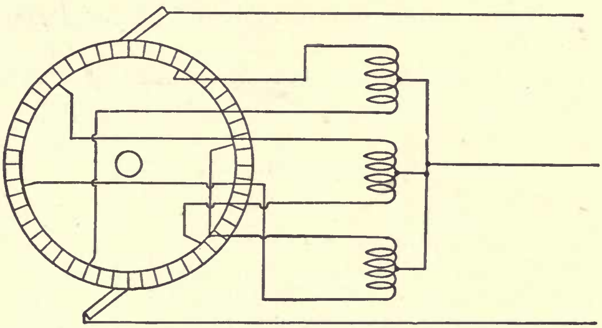


FIG. 140.—Diagram of direct-current converter.

system, the voltage between the neutral and outside conductor is $\pm e$, that on each of the $2n$ autotransformer sections is

$$e \sin\left(\theta - \theta_0 - \frac{\pi k}{n}\right), \qquad k = 0, 1, 2 \ . \ . \ . \ 2n - 1.$$

Neglecting losses in the converter and the autotransformer, the currents in the two sets of commutator brushes are equal and of the same direction, that is, both outgoing or both incoming, and opposite to the current in the neutral; that is, two equal currents i enter the commutator brushes and issue as current $2i$ from the neutral, or inversely.

From the law of conservation of energy it follows that the current $2i$ entering from the neutral divides in $2n$ equal and constant branches of direct current, $\frac{i}{n}$, in the $2n$ autotransformer sections, and hence enters the armature, to issue as current i from each of the commutator brushes.

In reality the current in each autotransformer section is

$$\frac{i}{n} + i_0 \sqrt{2} \cos\left(\theta - \theta_0 - \frac{\pi k}{n} + \alpha\right),$$

where i_0 is the exciting current of the magnetic circuit of the autotransformer, and α the angle of hysteretic advance of phase. At the commutator the current on the motor side is larger than the current on the generator side, by the amount required to cover the losses of power in converter and autotransformer.

In Fig. 141 the positive side of the system is generator, the negative side motor. This machine can be considered as receiving the current i at the voltage e from the negative side of the system, and transforming it into current i at voltage e on the

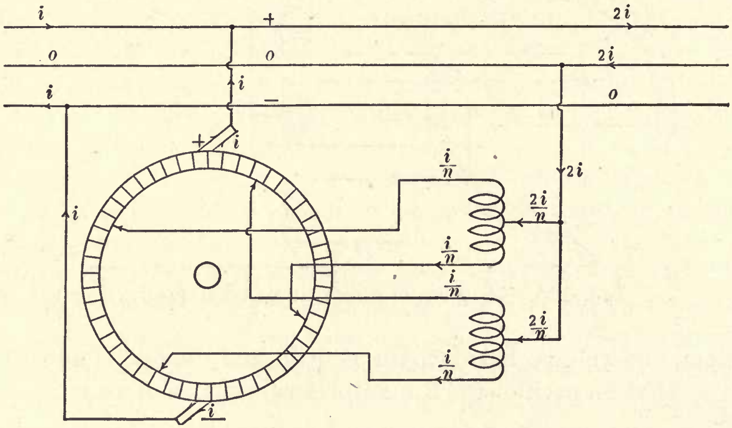


FIG. 141.—Distribution e.m.f. and current in direct-current converter.

positive side of the system, or it can be considered as receiving current i at voltage $2e$ from the system, and transforming it into current $2i$ at the voltage e on the positive side of the system, or of receiving current $2i$ at voltage e from the negative side, and returning current i at voltage $2e$. In either case the direct-current converter produces a difference of power of $2ie$ between the two sides of the three-wire system.

The armature reaction of the currents from the generator side of the converter is equal but opposite to the armature reaction of the corresponding currents entering the motor side, and the motor and generator armature reactions thus neutralize each other, as in the synchronous converter; that is, the resultant armature reaction of the continuous-current converter is practically zero, or the only remaining armature reaction is that corresponding to the relatively small current required to rotate the machine, that is, to supply the internal losses in the same. The armature reaction of the current supplying the electric power transformed into mechanical power obviously also remains, if the machine is used simultaneously as motor, as for driving a booster connected into the system to produce a difference between the voltages of the two sides, or the armature reaction of the currents generated from mechanical power if the machine is driven as generator.

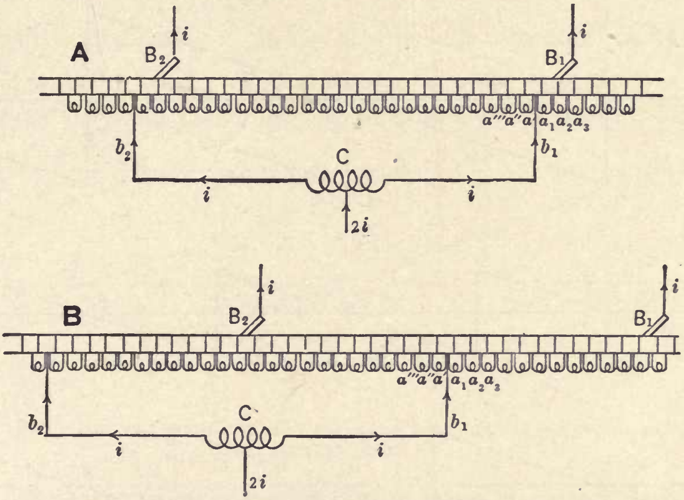


FIG. 142.—Development of a direct-current converter.

106. While the currents in the armature coils are more or less sine waves in the alternator, rectangular reversed currents in the direct-current generator or motor, and distorted triple-frequency currents in the synchronous converter, the currents in the armature coils of the direct-current converter are approximately triangular double-frequency waves.

Let Fig. 142 represent a development of a direct-current converter with brushes B_1 and B_2, and C one autotransformer receiving current $2i$ from the neutral. Consider first an armature coil a_1 adjacent and behind (in the direction of rotation) an autotransformer lead b_1. In the moment when autotransformer leads b_1 b_2 coincide with the brushes B_1 B_2 the current i directly enters the brushes and coil a_1 is without current. In the next moment (Fig. 142*A*) the total current i from b_1 passes coil a_1 to brush B_1, while there is yet practically no current from b_1 over coils a' a'', etc., to brush B_2. But with the forward motion of the armature less and less of the current from b_1 passes through a_1 a_2, etc., to brush B_1 and more over a' a'', etc., to brush B_2, until in the position of a_1 midway between b_1 and b_2 (Fig. 142*B*), one-half of the current from b_1 passes a_1 a_2, etc., to B_1, the other half a' a'', etc., to B_2. With the further rotation the current in a_1 grows less and becomes zero when b_1 coincides with B_2, or half a cycle after its coincidence with B_1. That is, the current in

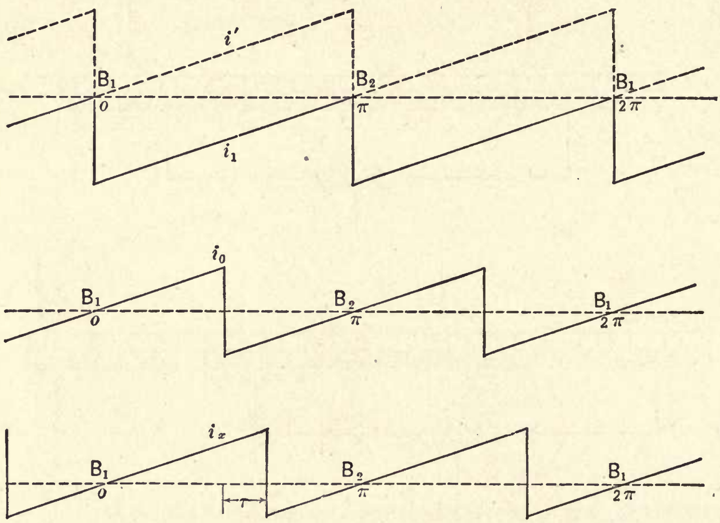


FIG. 143.—Current in the various coils of a direct-current converter.

coil a_1 approximately has the triangular form shown as i_1 in Fig. 143, changing twice per period from 0 to i. It is shown negative, since it is against the direction of rotation of the armature. In the same way we see that the current in the coil a', adjacent ahead of the lead b_1, has a shape shown as i' in Fig. 143. The current in coil a_0 midway between two commutator leads has the form i_0, and in general the current in any armature coil a_x, distant by angle τ from the midway position a_0, has the form i_x, Fig. 143.

All the currents become zero at the moment when the autotransformer leads b_1 b_2 coincide with the brushes B_1 B_2, and change by i at the moment when their respective coils pass a commutator brush. Thus the lines A and A' in Fig. 144 with zero values at B_1 B_2, the position of brushes, represent the currents in the individual armature coils. The current changes from A to A' at the moment $\theta = \tau$ when the respective armature coil passes the brush, twice per period. Due to the inductance of the armature coils, which opposes the change of current, the current waves are not perfectly triangular, but differ somewhat therefrom.

With n autotransformers, each autotransformer lead carries the current $\frac{i}{n}$, which passes through the armature coils as triangular current, changing by $\frac{i}{n}$ in the moment the armature coil passes a commutator brush. This current passes the zero value in the moment the autotransformer lead coincides with a brush. Thus,

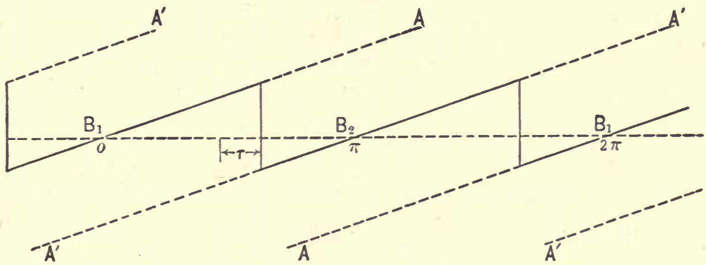


Fig. 144.—Current in individual coils of a direct-current converter with one compensator.

the differents current of n autotransformers which are superposed in an armature coil a_x have the shape shown in Fig. 199 for $n = 3$. That is, each autotransformer gives a set of slanting lines $A_1A'_1$, $A_2A'_2$, $A_3A'_3$, and all the branch currents i_1, i_2, i_3, superposed, give a resultant current i_x, which changes by i in the moment the coil passes the brush. i_x varies between the extreme values $\frac{i}{2}(2p - 1)$ and $\frac{i}{2}(2p + 1)$, if the armature coil is displaced from the midway position between two adjacent autotransformer leads by angle τ, and $p = \frac{\tau}{\pi}$. p varies between $-\frac{1}{2n}$ and $+\frac{1}{2n}$.

Thus the current in an armature coil in position $p = \frac{\tau}{\pi}$ can be denoted in the range from p to $1 + p$, or τ to $\pi + \tau$, by

$$i_x = \frac{i}{2}(2x - 1),$$

where

$$x = \frac{\theta}{\pi}.$$

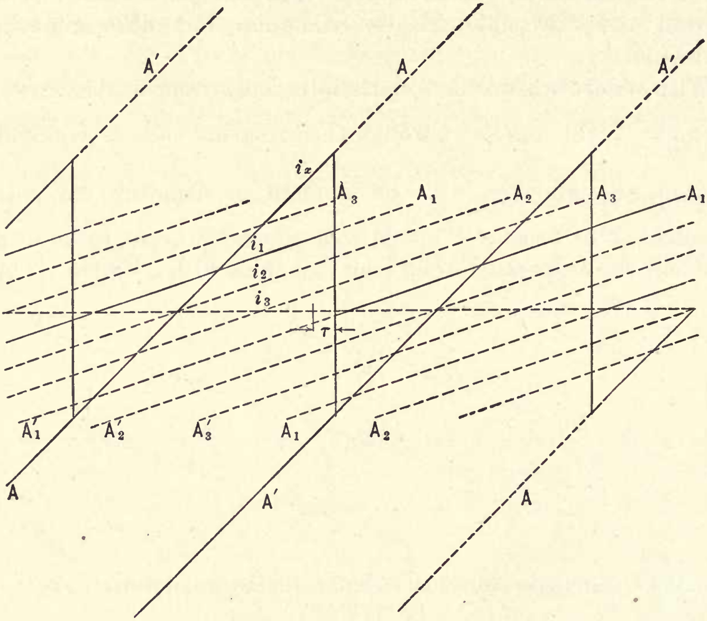


FIG. 145.—Current in a single coil of a direct-current converter with three compensators.

The effective value of this current is

$$I = \sqrt{\int_p^{p+1} i_x^2 dx}$$
$$= \frac{i}{2}\sqrt{1/3 + 4p^2}.$$

Since in the same machine as direct-current generator at voltage $2e$ and current i, the current per armature coil is $\frac{i}{2}$, the ratio of current is

$$\frac{I}{\frac{i}{2}} = \sqrt{1/3 + 4p^2}$$

and thus the relative I^2r loss or the heat developed in the armature coil,

$$\gamma = \left(\frac{I}{\frac{i}{2}}\right)^2 = 1/3 + 4\,p^2,$$

with a minimum,

$$p = 0,\ \gamma_0 = 1/3,$$

and a maximum,

$$p = \frac{1}{2\,n}.$$

$$\gamma_m = \frac{1}{3} + \frac{1}{n^2} = \frac{3 + n^2}{3\,n^2}.$$

The mean heating or I^2r of the armature is found by integrating over γ from

$$p = -\frac{1}{2\,n} \text{ to } p = +\frac{1}{2\,n},$$

as

$$\Gamma = n\int_{-\frac{1}{2n}}^{+\frac{1}{2n}} \gamma dp$$

$$= \frac{1}{3} + \frac{1}{3\,n^2} = \frac{1 + n^2}{3\,n^2}.$$

This gives the following table, for the direct-current converter, of minimum current heating, γ_0, in the coil midway between

DIRECT-CURRENT CONVERTER I^2r RATING

No. of compensators, n =	d. c. gen.	1	2	3	4	n	∞
Minimum current heating $p = 0$, γ_0 =	1	1/3	1/3	1/3	1/3	1/3	1/3
Maximum current heating, $p = +\frac{1}{n}$, γm =	1	4/3	7/12	4/9	19/48	$1/3 + \frac{1}{n^2}$	1/3
Mean current heating, Γ =	1	2/3	5/12	10/27	17/48	$1/3 + \frac{1}{3\,n^2}$	1/3
Rating, $\frac{1}{\sqrt{\Gamma}}$ =	1	1.225	1.549	1.643	1.681	$\sqrt{\frac{3\,n^2}{1 + n^2}}$	1.732

adjacent commutator leads, maximum current heating, γ_m, in the coil adjacent to the commutator lead, mean current heating, Γ, and rating as based on mean current heating in the armature, $\frac{1}{\sqrt{\Gamma}}$:

As seen, the output of the direct-current converter is greater than that of the same machine as generator. Using more than three autotransformers offers very little advantage, and the difference between three and two autotransformers is comparatively small, also, but the difference between two and one autotransformer, especially regarding the local armature heating, is considerable, so that for most practical purposes a two-autotransformer converter would be preferable.

The number of autotransformers used in the direct-current converter has a similar effect regarding current distribution, heating, etc., as the number of phases in the synchronous converter.

Obviously these relative outputs given in above table refer to the armature heating only. Regarding commutation, the total current at the brushes is the same in the converter as in the generator, the only advantage of the former being the better commutation due to the absence of armature reaction.

The limit of output set by armature reaction and corresponding field excitation in a motor or generator obviously does not exist at all in a converter. It follows herefrom that a direct-current motor or generator does not give the most advantageous direct-current converter, but that in the direct-current converter just as in the synchronous converter, it is preferable to proportion the parts differently in accordance with above discussion, as, for instance, to use less conductor section, a greater number of conductors in series per pole, etc.

XIV. Three-wire Generator and Converter

107. A machine based upon the principle of the direct-current converter is frequently used to supply a three-wire direct-current distribution system (Edison system). This machine may be a single generator or synchronous converter, which is designed for the voltage between the outside conductors of the circuit (the positive and the negative conductor), 220 to 280 volts, while the middle conductor of the system, or neutral conductor, is connected to the generator by autotransformer and collector rings, or, in the case of a synchronous converter, is connected to the neutral of the step-up transformers, and the latter thus used as autotransformers.

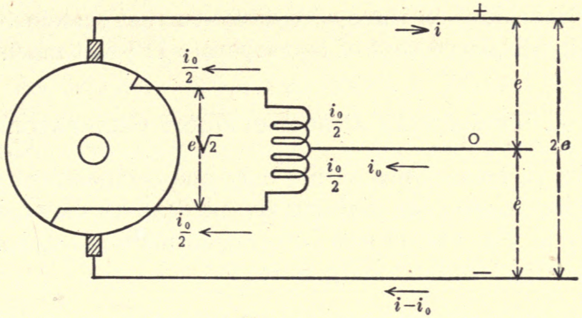


FIG. 146.—Three-wire machine with single autotransformer.

A three-wire generator thus is a combination of a direct-current generator and a direct-current converter, and a three-wire converter is a combination of a synchronous converter and a direct-current converter. Such a three-wire machine has the advantage over two separate machines, connected to the two

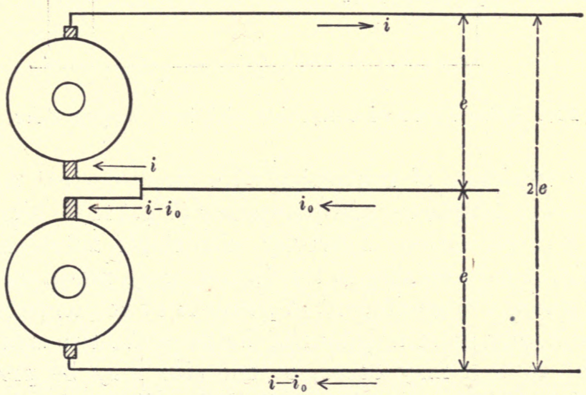


FIG. 147.—Three-wire system with two machines.

sides of the three-wire direct-current system, of combining two smaller machines into one of twice the size, and thus higher space- and operation-economy and lower cost, and has the further advantage that only half as large current is commutated as by the use of two separate machines; that is, the positive brush of the machine on the negative and the negative brush of the machine on the positive side of the system are saved, as seen by the diagrammatic sketch of the machine in Fig. 146 and the two separate two-wire machines in Fig. 147. The use of three-wire 220-volt machines on three-wire direct-current systems thus has practically displaced that of two separate 110-volt machines.

A. Three-wire Direct-current Generator

108. In such machines, either only one compensator or autotransformer is used for deriving the neutral, as shown diagrammatically in Fig. 146, or two autotransformers in quadrature, as shown in Fig. 148, but rarely more.

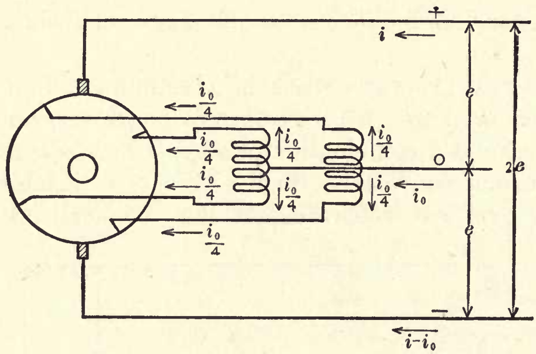


Fig. 148.—Three-wire machine with two compensators.

As the efficiency of conversion of a direct-current converter with two autotransformers in quadrature (Fig. 148) is higher than that of a direct-current converter with single autotransformer (Fig. 146), it is preferable to use two (or even more) autotransformers where a large amount of power is to be converted, that is, where a very great unbalancing between the two sides of the three-wire system may occur, or one side may be practically unloaded while the other is overloaded. Where, however, the load is fairly distributed between the two sides of the system, that is, the neutral current (which is the difference between the currents on the two sides of the system) is small and so only a small part of the generator power is converted from one side to the other, and the efficiency of this conversion thus of negligible influence on the heating and the output of the machine, a single autotransformer is preferable because of its simplicity. In three-wire distribution systems the latter is practically always the case, that is, the load fairly balanced and the neutral current small.

The size of the autotransformers depends upon the amount of unbalanced power, that is, the maximum difference between the load on the two sides of the three-wire system, and thus equals the product of neutral current i_0 and voltage e between neutral and outside conductor; that is, in the three-wire system of voltage e per circuit, voltage $2e$ between the outside conductors, and maximum current i in the outside conductors, the generator power rating is

$$p = 2\,ei.$$

Let now i_0 = maximum unbalanced current in the neutral—usually not exceeding 10 to 20 per cent. of i—and using a single autotransformer, connected diametrically across the armature, Fig. 146, the maximum of the alternating voltage which it receives is $2e$, and its effective voltage therefore $e\sqrt{2}$. As the neutral current i_0 divides when entering the autotransformer, the current in the compensating winding is $\frac{i_0}{2}$ (neglecting the small exciting current), and the volt-ampere capacity of the autotransformer thus is

$$p_0 = \frac{ei_0}{\sqrt{2}},$$

and

$$\frac{p_0}{p} = \frac{1}{2\sqrt{2}}\frac{i_0}{i}$$

$$= 0.354\,\frac{i_0}{i}.$$

Even with the neutral current equal to the current in the outside conductor, or the one side of the system fully loaded, the other not loaded, the autotransformer thus would have only 35.4 per cent. of the volt-ampere capacity of the generator, and as an autotransformer of ratio 1 ÷ 1 is half the size of a transformer of the same volt-ampere capacity, in this case the autotransformer has, approximately, the size of a transformer of 17.7 per cent. of the size of the generator.

With the maximum unbalancing of 20 per cent., or $\frac{i_0}{i} = 0.2$, the autotransformer thus has 7 per cent. of the volt-ampere capacity of the generator, or the size of a transformer of only 3.5 per cent. of the generator capacity, that is, is very small, and this method is therefore the most convenient for deriving the neutral of a three-wire distribution system.

When using n autotransformers, obviously each has $\frac{1}{n}$ of the size which a single autotransformer would have.

The disadvantage of the three-wire generator over two separate generators is that a three-wire generator can only divide the voltage in two equal parts, that is, the two sides of the system have the same voltage at the generator. The use of two separate generators, however, permits the production of a higher voltage on one side of the system than on the other, and thus takes care of the greater line drop on the more evenly loaded side. Even in the case, however, where a voltage difference between the two sides of the system is desired for controlling feeder drops, it can more economically be given by a separate booster in the neutral, as such a booster would require only a capacity equal to the neutral current times half the desired voltage difference between the two sides, and with 20 per cent. neutral current and 10 per cent. voltage difference between the two sides, thus would have only 1 per cent. of the size of the generator.

B. Three-wire Converter

109. In a converter feeding a three-wire direct-current system the neutral can be derived by connection to the transformer neutral. Even in this case, however, frequently a separate autotransformer is used, connected across a pair of collector rings of the converter, since, as seen above, with the moderate unbalancing usually existing, such a compensator is very small.

When connecting the direct-current neutral to the transformer neutral it is necessary to use such a connection that the transformer can operate as autotransformer, that is, that the direct current in each transformer divides into two branches of equal m.m.f., otherwise the direct-current produces a unidirectional magnetization in the transformer, which superimposed upon the magnetic cycle raises the magnetic induction beyond saturation, and thus causes excessive exciting current and heating, except when very small.

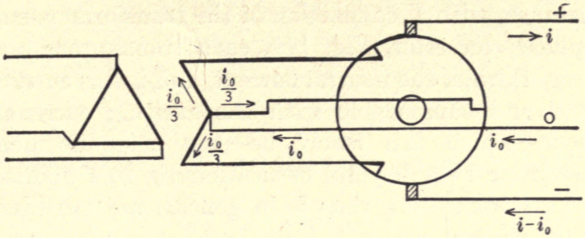


FIG. 149.—Neutral of Y-connected transformers connected to neutral of three-wire system supplied from a three-phase converter.

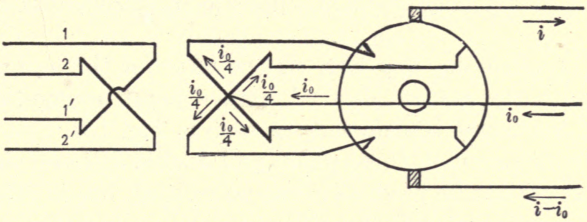


FIG. 150.—Quarter-phase converter with transformer neutral connected to direct-current neutral.

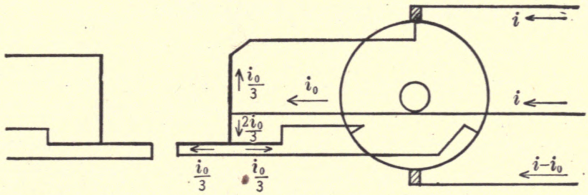


FIG. 151.—Three-phase converter with neutral of the T-connected transformers as direct-current neutral.

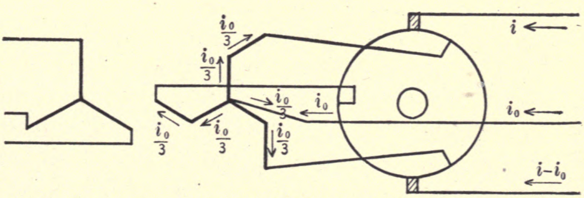


FIG. 152.—Three-phase converter with transformer neutral connected to direct-current neutral.

For instance, with Y connection of the transformers supplying a three-phase converter, Fig. 149, each transformer secondary receives one-third of the neutral current, and if this current is not very small and comparable with the exciting current of the transformer—which can rarely be—the magnetic density in the transformer rises beyond saturation by this unidirectional m.m.f. This connection thus is in general not permissible for deriving the neutral.

In a quarter-phase converter, as shown in Fig. 150, the transformer neutral can be used as direct-current neutral, since in each transformer the direct current divides into two equal branches, which magnetize in opposite direction, and so neutralize.

The T connection, Fig. 151, can be used for three-phase converters with the neutral derived from a point at one-third the height of the teaser transformer, as seen in Fig. 151.

Delta connection on three-phase and double delta on six-phase converters cannot be used, as it has no neutral, but in this case a separate compensator is required.

The diagrammatical connections of transformers can, however, be used on six-phase converters, and the connection shown in Fig. 152, which has two coils on each transformer, connected to different phases, on three-phase converters.

D. ALTERNATING-CURRENT TRANSFORMER

I. General

110. The alternating-current transformer consists of a magnetic circuit interlinked with two electric circuits, the primary, which receives power, and the secondary, which gives out power.

Since the same magnetic flux interlinks primary and secondary turns, the same voltage is induced in every turn of the electric circuits, and the e.m.fs. induced in the primary and in the secondary winding therefore have the ratio of turns:

$$\frac{e'_1}{e'_2} = \frac{n_1}{n_2} = a.$$

This ratio is called the ratio of transformation.

The ratio of transformation of a transformer is the ratio of turns of primary and secondary windings.

In addition to the induced e.m.fs. e'_1 and e'_2, resistance r and reactance x consume voltage in primary and secondary windings. The voltage consumed by the resistance represents waste of power; the voltage consumed by reactance is wattless, but causes lag of current, that is, lowers the power factor; while the induced voltages give the power transfer from primary to secondary. Efficiency therefore requires to make the former voltages as small as possible, and the induced voltages as near to the terminal voltages as possible. Therefore, in first approximation, the ratio of the terminal voltages e_1 and e_2 is the ratio of transformation:

$$\frac{e_1}{e_2} = a.$$

As, approximately, the power output of the secondary equals the power input into the primary, it is:

$$e_2 i_2 = e_1 i_1$$

hence,

$$\frac{i_1}{i_2} = \frac{1}{a},$$

that is, the transformer changes from voltage e_1 and current i_1 to voltage $e_2 = \frac{e_1}{a}$ and current $i_2 = ai_1$.

In general either of the two transformer circuits may be used as primary or as secondary, and by their use transformers thus are distinguished as step-down transformers, if the primary voltage is higher than the secondary, and step-up transformers, if the secondary voltage is higher. Instead of the expression "primary" and "secondary," constructively it therefore is preferable to speak of "high voltage winding" and "low voltage winding."

111. The foremost use of the transformer therefore is for changing of the voltage:

From the medium high primary distribution voltage (2300) to the low secondary consumer voltage (110, 220).

From the high transmission (30 to 150 kilovolts) to the primary distribution voltage (2300) or the voltage required by synchronous motor, synchronous converter, etc.

From the low or medium high generator voltage to the high transmission voltage.

Other occasional uses of transformers are:

To electrically tie systems together, so as to permit exchange of power between them, and synchronous operation. In this case, depending on the distribution of the load in the system, either transformer winding may be primary or secondary.

To break up electrically a very large system, so that a ground in one part does not ground the entire system. In this case, the transformer ratio usually is $1 \div 1$.

In all these cases, the transformers are "constant potential transformers," that is, primary and thus secondary voltage are constant or approximately so.

Transformers supplied with constant current in the primary give practically constant current in the secondary, at a primary voltage varying with the secondary voltage. Such transformers are used in constant-current circuits, for supplying meters in high voltage circuits, etc.

Further uses of transformers are for operating instruments, switches, etc., in high voltage systems. In this case, the transformers may be potential transformers—connected across the constant voltage circuit, or current transformers—connected in series into the circuit, for the supply of meters, the operation of overload circuit breakers, etc.

Where not expressly stated otherwise, in general a constant potential transformer is understood.

II. Excitation

112. The primary current i_1 is not strictly proportional to the secondary current, i_2 by the ratio of transformation,

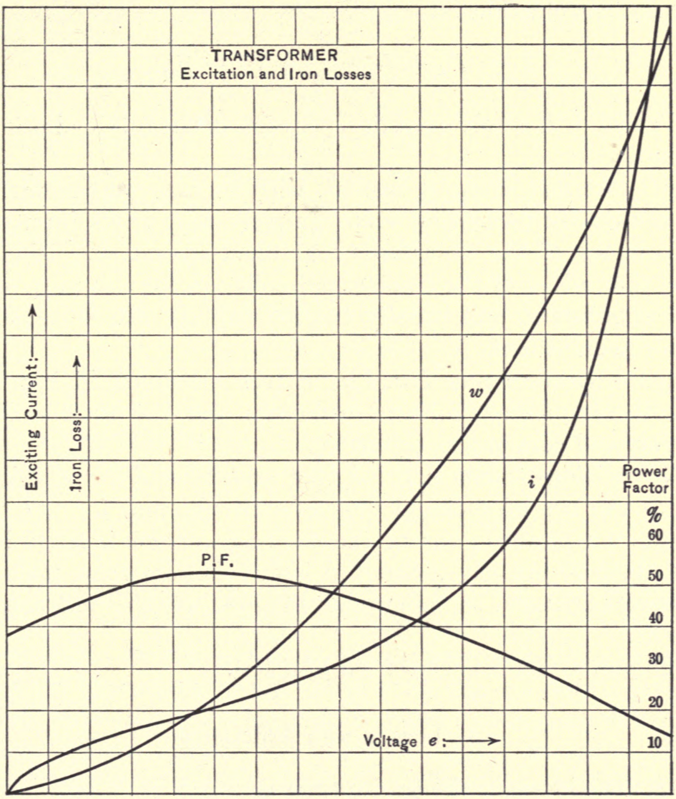


FIG. 153.—Excitation and core loss of transformer.

and does not become zero at no load or open circuit, but a small and lagging current i_0 remains at no load, which is called the exciting current. It produces the magnetic flux and supplies the losses in the iron, so-called "core loss." Its reactive component, i_m, is called the magnetizing current, and is usually greatly distorted in wave shape, while the energy component,

i_h, does not much differ from a sine wave, and is the hysteresis energy current:

$$\dot{I}_0 = i_h - j i_m.$$

Under load, the primary current then consists of two components: the load current I'_2 which is the transformed secondary current $\dot{I}'_2 = \frac{\dot{I}_2}{a_1}$, and the exciting, current $\dot{I}_0$. The total primary current thus is:

$$\dot{I}_1 = \dot{I}'_2 + \dot{I}_0 = \frac{\dot{I}_2}{a} + (i_h - j i_m).$$

In general, I_0 rarely exceeds 5 per cent. of the full-load primary current.

Core loss and exciting current, with its two components, are determined by measuring volts, amperes and watts input into the primary of the transformer at open secondary. It is obvious that either of the transformer coils can for this purpose be used as primary, and usually the low voltage coil is employed as more convenient.

Such excitation and core-loss curves are given in Fig. 153, with the impressed volts as abscissæ, and the total exciting current, and core loss as ordinates.

The exciting current is usually not proportional to the voltage, due to the use of a closed magnetic circuit, and for the same reason, the power-factor of the exciting current is fairly high, from 40 to 60 per cent., except at high voltages, where magnetic saturation causes an abnormal increase of the magnetizing current.

The power-factor is shown on Fig. 153.

III. Losses and Efficiency

113. The losses in the transformer are

(*a*) The core loss, comprising the loss by hysteresis and eddy currents in the iron. This depends on the maximum magnetic flux, and thus on the induced voltage:

$$e'_1 = 2\,\pi f n_1\, \Phi\, 10^{-8}$$

and as the induced voltage is practically equal to the impressed voltage e_1, at constant impressed voltage, the core loss is practically constant, and is often assumed as constant, that is, the core loss is a constant or no-load loss, and is supplied by the exciting current i_0.

(*b*) The i^2r losses in the primary and secondary coils. These are load losses, increasing with the square of the load.

(*c*) Spurious load losses, as eddy currents in the conductors and other metal parts. With proper design these should be negligible.

(*d*) In very high voltage transformers, electrostatic losses in the insulation appear. These usually are small in large well-designed transformers.

In large transformers, the total i^2r loss may be less than 1 per cent., and so also the core loss, resulting in efficiencies of over 98 per cent.

As instance are shown, in Figs. 154 and 155, the loss curves and the efficiency curves of two transformers, of the respective constants, at full load of 20 kw.

	I. Low core-loss type, Fig. 154	II. Low i^2r loss type, Fig. 155
Exciting current	4 per cent.	4 per cent.
Primary resistance loss	1 per cent.	0.5 per cent.
Secondary resistance loss	1 per cent.	0.5 per cent.
Core loss	1 per cent.	2 per cent.

For convenience, exciting current and losses are frequently given in per cent. of the full-load output of the transformer.

The curves correspond to non-inductive load. The core loss comprises hysteresis, which varies with the 1.6 power of the induced voltage and eddies proportional to the square of induced voltage. Hence, within the narrow range of variation of the induced voltage between no load and full load of a constant potential transformer, the core loss can be approximated as proportional to the 1.7 power of the induced voltage. The induced voltage at non-inductive load equals impressed voltage minus primary *ir*, when neglecting the inductive drop, which is permissible at non-inductive load. As the induced voltage thus decreases proportional to primary *ir*, the core loss decreases proportional to 1.7 times the primary *ir*. Thus, with the primary i^2r equal to 1 per cent. at full load, the induced voltage has decreased 1 per cent. and the core loss 1.7 per cent. at full load, and correspondingly at other loads.

As seen, I and II have the same full-load efficiency, but II is more efficient at overload, I at partial load.

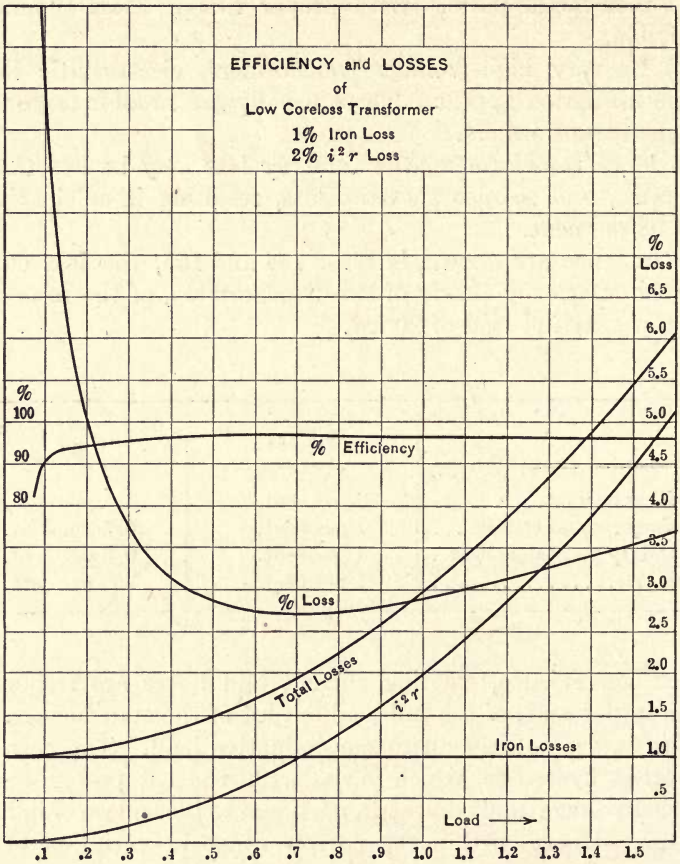


FIG. 154.—Efficiency and losses of low core loss transformer.

114. In transformers for lighting and general distribution (usually with 2300 volt primary and 2 × 115 volt secondary) the transformer is generally heavily loaded only for a short time during the day, partly loaded for a moderate time, and practically unloaded for most of the time. Thus load curves of such a transformer would be:

A. Lighting and power	B. Lighting only
2 hours at 1¼ load.	2 hours at 1¼ load.
2 hours at ¾ load.	2 hours at ¾ load.
6 hours at ½ load.	20 hours at $\frac{1}{20}$ load.
14 hours at $\frac{1}{20}$ load.	

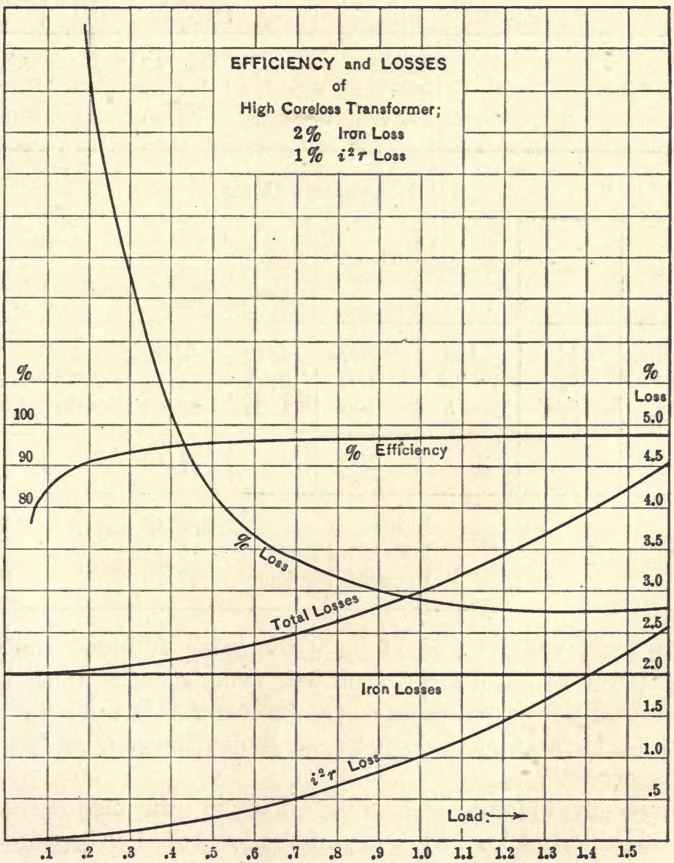


FIG. 155.—Efficiency and losses of low i^1r loss transformer.

This gives for the two types of transformers:

A. Lighting and Power

Time	Load	= Per cent.	Time × load	I		II	
				Losses	Time × losses	Losses	Time × losses
2 hr.	1¼	125	250	4.10	8.20	3.54	7.08
2 hr.	¾	75	150	2.11	4.22	2.55	5.10
6 hr.	½	50	300	1.50	9.00	2.25	13.50
14 hr.	1/20	5	70	1.00	14.00	2.00	28.00
		Σ =	770		35.42		53.68
		Input			805.42		823.68
		Per cent. loss			4.41		6.51
		Per cent. efficiency			95.59		93.49

B. Lighting Only

Time	Load	= Per cent.	Time × load	I		II	
				Losses	Time × losses	Losses	Time × losses
2 hr.	1¼	125	250	4.10	8.20	3.54	7.08
2 hr.	¾	75	150	2.11	4.22	2.55	5.10
20 hr.	1/20	5	100	1.00	20.00	2.00	40.00
		Σ =	500		32.42		52.18
		Input			532.42		552.18
		Per cent. loss			6.11		9.45
		Per cent. efficiency			93.89		90.55

As seen, while I and II have the same full-load efficiency, 97.1 per cent., I, the low core-loss type, gives a much higher all-day efficiency, the more so the shorter the time of heavy load, that is, is far preferable for general distribution, as "lighting transformer."

Inversely, in large power transformers in transmission systems, the high partial load efficiency of the low core-loss type is of less importance, as such transformers are usually not run at partial load, but with a decrease of load on the system, transformers and generators are cut out and the remaining ones kept loaded. Of importance, however, is low i^2r loss. Under emergency conditions requiring overloading of some transformer, the increased loss is all in the copper, and the less therefore the i^2r, the less is the danger of destruction by overheating in a case of a temporary overload. Thus the low i^2r loss type of transformer is preferable for large power units.

IV. Regulation

115. As primary and secondary winding of the transformer cannot occupy the same space, and in addition some insulation—more or less depending on the voltage—must be between them, there is thus a space between primary and secondary through which the primary current can send magnetic flux which does not interlink with the secondary winding, but is a self-inductive or leakage flux and in the same manner the secondary current sends self-inductive or leakage flux through the space between primary and secondary winding. These fluxes give rise to the self-inductive or leakage reactances x_1 and x_2 of the transformer.

Or in other words, two paths exist for magnetic flux in the transformer: the path surrounding primary and secondary coils, through which flows the mutual magnetic flux of the transformer, which is the useful flux, that is, the flux which transfers the power from primary to secondary circuit; and the space between primary and secondary winding through which the self-inductive or leakage flux passes, that is, the flux interlinked with one winding only, but not the other one. The latter flux thus does not transmit power, but consumes reactive voltage and thereby produces a voltage drop and a lag of the current behind the voltage, that is, is in general objectionable.

The mutual magnetic flux passes through a closed magnetic circuit, with the (vector) difference between primary and secondary current, that is, the exciting current $\dot{I}_0 = \dot{I}_1 - \frac{\dot{I}_2}{a}$ as m.m.f. The self-inductive flux passes through an open magnetic circuit of high reluctance, the narrow space between primary and secondary windings, but it is due to the full m.m.f. of primary or secondary current and, therefore, in spite of the high reluctance of the leakage flux path due to the high m.m.f. (20 times as great as that of the mutual flux at 5 per cent. exciting current), this flux and the reactance voltages caused by it are appreciable, usually between 2 per cent. and 8 per cent. in modern transformers.

The distribution of the leakage flux between primary and secondary winding, that is, between primary reactance x_1 and secondary x_2, is to some extent arbitrary (see discussion in "Theory and Calculation of Electric Circuits"), and the methods of test give only the sum of the primary and the secondary reactance, the latter reduced to the primary by the ratio of transformation: $x_1 + a^2x_2$.

116. The total reactance of primary and secondary, and also

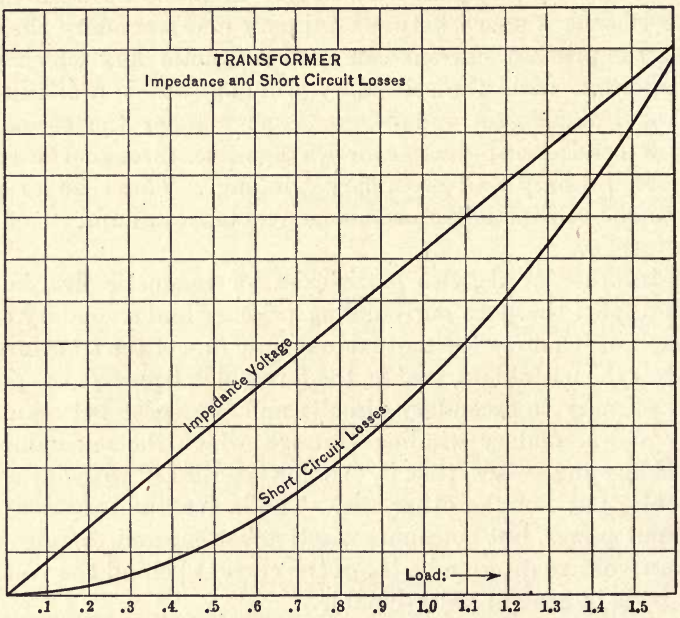


Fig. 156.—Impedance and short circuit losses of transformer.

the total (effective) resistance of primary and secondary winding are measured by impressing voltage on the primary coil, with the secondary winding short-circuited, and measuring volts, amperes and watts.

In this test the voltage usually is impressed upon the high voltage winding, as the impedance voltage is only a small part of the operating voltage of the transformer.

Such "impedance curves" and "short-circuit loss curves" for the transformers in Figs. 154 and 155 are shown in Fig. 156. If the short-circuit loss is greater than the sum of primary and secondary i^2r losses, the difference represents load losses caused by eddy currents in the conductors, etc.

The reactance of the transformer is often given as percentage. Six per cent. reactance thus means that the primary ix, as per cent. of the primary impressed voltage, plus the secondary ix as per cent. of the secondary voltage, is 6 per cent. Or:

$$\frac{i_1x_1 + ai_2x_2}{e_1} = 0.06.$$

Especially since x_1 and x_2 cannot be separated experimentally, but the impedance test gives the sum of primary reactance x_1, and secondary reactance x_2 reduced to the primary by the ratio of transformation a, that is a^2x_2:

$$x_1 + a^2x_2$$

this is permissible.

The foremost effects of the leakage reactance of the transformer are, to affect the voltage regulation, and to determine the short-circuit current and the mechanical forces resulting from it.

117. The exciting current, being a small and practically constant component of the primary current, does not affect the regulation of the transformer appreciably, and thus can be neglected in the calculation of the regulation curve. If this is done, the secondary quantities can be reduced to the primary by the ratio of transformation (or inversely), that is, by multiplying all secondary voltages and dividing all secondary currents by a, and multiplying all secondary impedances by a^2, or inversely when reducing from primary to secondary.[1]

Or, primary and secondary impedances can be given in per cent., that is, the primary ir and ix in per cent. of the primary voltage, the secondary ir and ix in per cent. of the secondary voltage, and in this case, primary and secondary quantities can be directly added. This usually is the most convenient way, at least for approximate calculation.

Thus in the transformer shown in Fig. 154, let

$\xi = 0.02$ be the total reactance (2 per cent.), at full non-inductive load.

[1] As the transformation ratio of the voltage is a, that of the current is $\frac{1}{a}$, the transformation ratio of the impedances (resistance and reactance), is a^2, as impedance $= \frac{\text{volts}}{\text{amperes}}$.

$\rho = 0.02$ is the total resistance, primary and secondary combined.

At the percentage p of the non-inductive load, the voltage consumed by reactance is $p\xi = 0.02\, p$ and in quadrature with the current and thus with the voltage at non-inductive load, hence subtracts by $\sqrt{\text{difference of squares}}$:

$$\sqrt{1 - p^2\xi^2}$$

while the voltage consumed by the resistance is $p\rho = 0.02\, p$ and in phase with the voltage, hence directly subtracts, leaving:

$$\sqrt{1 - p^2\xi^2} - p\rho = \sqrt{1 - 0.0004\, p^2} - 0.02\, p$$

as the voltage at percentage p of load, given as per cent. of the open-circuit or no-load voltage. The voltage drop at fractional load p, as fraction of full-load voltage, that is, the regulation of the transformer at non-inductive load, then is

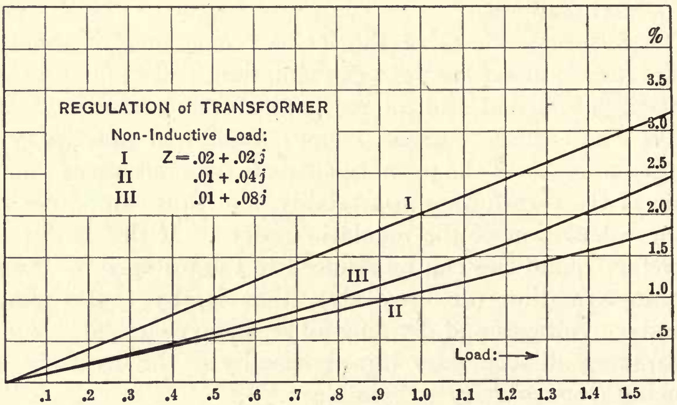


FIG. 157.—Regulation curve of transformer: non-inductive load.

$$R = 1 - \sqrt{1 - p^2\xi^2} + p\rho = 1 - \sqrt{1 - 0.0004\, p^2} + 0.02\, p$$

or, resolved by the binomial, and dropping the higher terms:

$$R = p\rho + \frac{1}{2}\, p^2\xi^2 = 0.02\, p + 0.0002\, p^2$$

$$= p\left(\rho + \frac{1}{2}\, p\xi^2\right) = 0.02\, p\,(1 + 0.01\, p)$$

As curves I, II, III in Fig. 157 are shown the regulation curves of three transformers:

I: 2 per cent. resistance and 2 per cent. reactance.
II: 1 per cent. resistance and 4 per cent. reactance.
III: 1 per cent. resistance and 8 per cent. reactance.

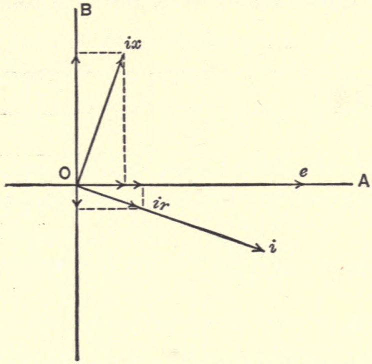


FIG. 158.—Vector diagram of transformer regulation.

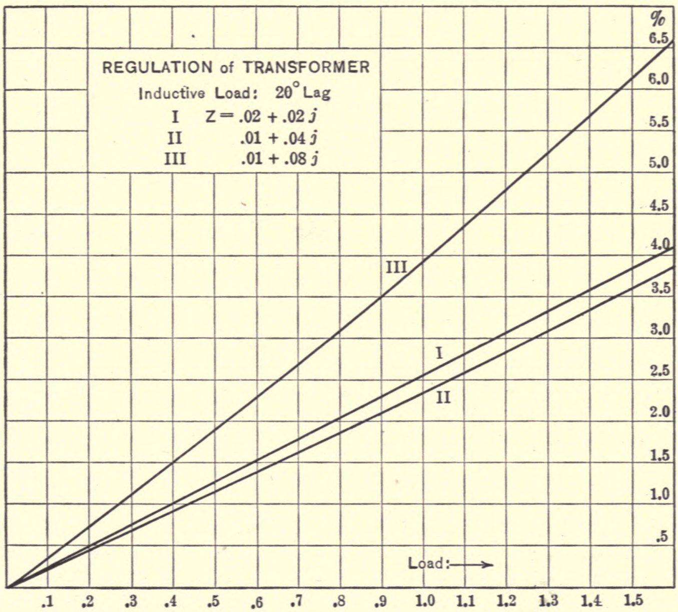


FIG. 159.—Regulation of transformer, moderately inductive load.

Calculated respectively by the equations given at end of next paragraph.

118. At inductive load of power-factor cos ω, that is, the lag of the current behind the voltage by angle ω, the regulation curve is derived from the vector diagram Fig. 158. The *ir* voltage is in phase with the current, the *ix* voltage 90 deg. ahead of the current. Resolving both of these voltages into components in phase and in quadrature with the terminal voltage, gives (Fig. 158):

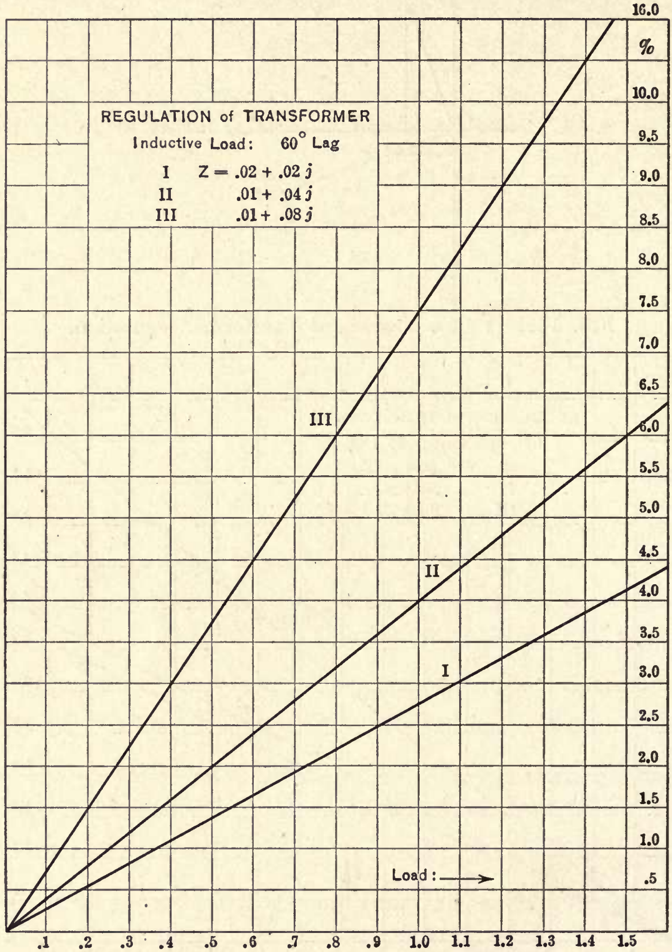


FIG. 160.—Regulation of transformer, highly inductive load.

ir cos ω and *ix* sin ω in phase with *e*,

ix cos ω and −*ir* sin ω in quadrature with *e*.

The former thus directly subtract, and the latter subtract by $\sqrt{\text{difference of squares}}$, thus giving as resultant voltage:

$$\sqrt{e^2 - (ix \cos \omega - ir \sin \omega)^2} - (ir \cos \omega + ix \sin \omega)$$

or, since ir at full load as fraction of e is ρ, and ix as fraction of e is ξ; at the fraction p of the load: $ir = p\rho$, $ix = p\xi$, the resultant voltage is:

$$\sqrt{1 - p^2(\xi \cos \omega - \rho \sin \omega)^2} - p\,(\rho \cos \omega + \xi \sin \omega)$$

and the regulation of the transformer, at inductive load of angle of lag ω, thus is

$$R = 1 - \sqrt{1 - p^2(\xi \cos \omega - \rho \sin \omega)^2} + p\,(\rho \cos \omega + \xi \sin \omega).$$

Resolving again the square root by the binomial, and arranging, gives, by dropping out terms of higher order:

$$R - p\,(\rho \cos \omega + \xi \sin \omega) + \frac{p^2}{2}(\xi \cos \omega - \rho \sin \omega)^2$$

In Figs. 159 and 160 are shown, for the angles of lag $\omega = 20°$ (moderately inductive load, 94 per cent. power-factor), and $\omega = 60°$ (highly inductive load, 50 per cent. power-factor). the regulation of the same three transformers as in Fig. 157, calculated respectively from the expression:

Regulation of Transformers

Per cent. resistance, ρ =	0.02	0.01	0.01
Per cent. reactance, ξ =	0.02	0.04	0.08
∡ ω of lag:	Curve I	Curve II	Curve III
Fig. 157, 0° R =	$0.02\,p\,(1 + 0.01\,p)$	$0.01p\,(1 + 0.08\,p)$	$0.01\,p\,(1 + 0.32\,p)$
Fig. 159, 20° R =	$0.0256\,p\,(1 + 0.0027\,p)$	$0.0231\,p\,(1 + 0.025\,p)$	$0.0368\,p\,(1 + 0.07\,p)$
Fig. 160, 60° R =	$0.0273\,p\,(1 + 0.001\,p)$	$0.0396\,p\,(1 + 0.0016\,p)$	$0.0743\,p\,(1 + 0.0066\,p)$

p	Non-inductive			20° lag.			60° lag.		
	I	II	III	I	II	III	I	II	III
0.2	0.40	0.20	0.21	0.51	0.46	0.75	0.55	0.79	1.49
0.4	0.80	0.41	0.45	1.02	0.93	1.52	1.09	1.58	2.99
0.6	1.20	0.63	0.71	1.54	1.40	2.30	1.64	2.38	4.48
0.8	1.61	0.85	1.00	2.05	1.88	3.13	2.18	3.17	5.98
1.0	2.02	1.08	1.32	2.56	2.37	3.94	2.73	3.96	7.48
1.2	2.43	1.31	1.66	3.08	2.85	4.79	3.28	4.76	9.00
1.4	2.84	1.56	2.03	3.59	3.34	5.67	3.83	5.56	10.50
1.6	3.25	1.80	2.42	4.10	3.83	6.56	4.38	6.35	12.00

119. As seen, at non-inductive load, Fig. 157, the reactance of the transformer, even if fairly high, has practically no effect, but the resistance controls the regulation.

At moderately inductive load reactance as well as resistance affect the regulation; doubling the reactance while halving the resistance, gives practically the same regulation.

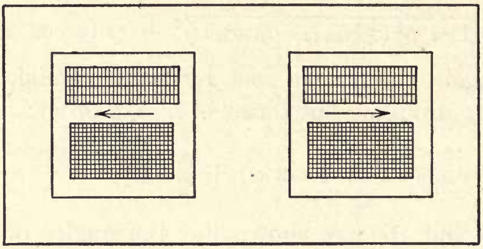


FIG. 161.—High reactance transformer construction.

At highly inductive load the reactance of the transformer begins to predominate over the resistance in affecting the regulation.

Thus, where close regulation is required, as in lighting and general distribution transformers, low reactance is of importance. This is given by reducing the section of the leakage path—that is, bringing primary and secondary windings as close together

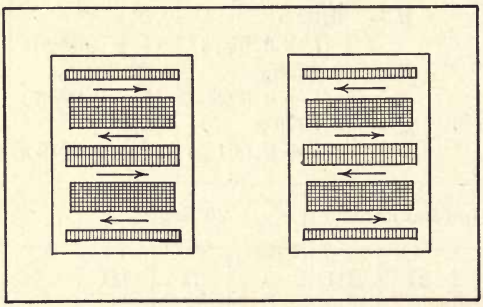


FIG. 162.—Low reactance transformer construction.

as possible—and by reducing the m.m.f. which produces the leakage flux, by subdividing primary and secondary winding into a number of coils and intermixing these coils, so that the leakage flux of each path is due to a small part of the total m.m.f. of primary or secondary only, as shown in Figs. 161 and 162. In Fig. 162 the m.m.f. of each of the four leakage paths is due to one-fourth of the m.m.f. as in Fig. 161, and the leakage flux density thus reduced to one-fourth of what it is in Fig. 161. As furthermore the section of each leakage flux in Fig. 162 is materially less than in Fig. 161, due to the lesser thickness of the coils, it follows that in Fig. 162 the leakage flux interlinked with each turn of each winding, and thus the reactance of the transformer, is materially less than one-quarter of what it is in Fig. 161.

The regulation of the transformer at anti-inductive load, that is, for leading secondary current, obviously is given by the same equation as that for lagging current, by merely substituting $-\omega$ for ω.

V. Short-circuit Current

120. If a short circuit occurs at the secondary terminals of a transformer, and the power supply at the primary is sufficient to maintain the primary terminal voltage, the primary and secondary currents of the transformer are limited by its impedance only. Thus, if

$$\dot{\zeta} = \rho + j\xi$$

is the impedance voltage, as fraction of full-load voltage, the short-circuit current of the transformer is

$$\frac{1}{\dot{\zeta}} = \frac{1}{\sqrt{\rho^2 + \xi^2}}$$

of the full-load current, thus usually is very large. In the three instances illustrated in Figs. 157, 159 and 160, with

$$\begin{aligned} \dot{\zeta} &= 0.02 + 0.02\,j, & \text{hence} \quad \zeta &= 0.028 \\ & 0.01 + 0.04\,j & & 0.04 \\ & 0.01 + 0.08\,j & & 0.08 \end{aligned}$$

the short-circuit current thus is 36, 25 and 12.5 times full-load current, respectively.

As seen, with the exception of very low reactance transformers, it is essentially the reactance which determines the total impedance and thus the short-circuit current.

121. Primary current and secondary current in the transformer, being opposite in phase, repel each other. This repulsion is proportional to the product of primary and secondary current, thus, since primary and secondary current are (approximately) proportional to each other, the repulsion is proportional to the square of the current. The repulsion is small at full load, but in low-reactance transformers, with short-circuit currents from forty to fifty times full-load current, the mechanical forces have increased 1600 to 2500 fold, and then, with large power transformers, reach formidable values, amounting to many hundred tons, and then it is economically difficult to build transformers with the coils supported so rigidly as to stand such forces. Thus far very few generating systems exist of such large size as to be capable of maintaining full voltage at the primary terminals of a large transformer at secondary short circuit, but their number is increasing, and thus the necessity of limiting the short-circuit current of large power transformers to a mechanically safe value is becoming increasingly important. This means a construction providing for considerable internal reactance. As the regulation of large power transformers is of no serious importance, the desirability of low reactance, which exists in the small lighting and general distribution transformers, does not exist in large power transformers, and modern practice tends toward the use of internal reactance of 4 to 8 per cent., to secure reasonable mechanical safety.

VI. Heating and Ventilation

122. As the transformer is a stationary apparatus, it does not have the advantage of dissipating the heat produced by the internal losses, by the natural ventilation of the air currents produced by the centrifugal forces in rotating apparatus, and it is therefore fortunate that the transformer is the most efficient apparatus (except perhaps the electrostatic condenser) and thus has to dissipate less heat than any other apparatus of the same output. Thus in smaller transformers radiation and the natural convection from the surface are often sufficient to keep the temperature within safe limits.

Smaller distribution transformers usually are installed outdoors, on poles, and then require protection by enclosure in an iron case or tank. This still further reduces the heat radiation, and therefore such transformer cases are now almost always filled with oil, the oil serving to carry the heat from the transformer iron and windings to the case. Incidentally, the oil filling also protects the transformer from the failure of insulation by condensation of moisture during the variation of atmospheric temperature and humidity.

In larger oil-cooled transformers, the tank is made corrugated, even with large double corrugations, to give a very large external surface to dissipate the heat.

Much more effectively, however, the heat can be carried away by mechanical ventilation, and size and cost of the transformer thereby materially reduced. Therefore practically all larger transformers have forced ventilation. Various methods of forced ventilation are:

(*a*) Oil circulation. The warm oil is pumped from the top of the transformer tank, through some cooling device. Often also a drying device to take out any trace of moisture—and then fed back into the bottom of the tank.

(*b*) Water circulation. Cooling water is pumped through a system of pipes located under the oil at the top of the transformer tank. This is the most common design of large transformers.

(*c*) Air blast. Coils and iron are subdivided by ventilating ducts, and a low-pressure air blast forced through the ventilating ducts. This is the cleanest method, as no oil is used. However, it is limited to low and moderate voltages—up to about 33,000; at higher voltage, the mechanical and chemical action of corona appearing at the coils reduces their life, and the oil becomes necessary for insulation.

Numerous modifications of the various types have been built and are in use, as water-cooled oil transformers with natural circulation of the water through outside radiating pipes, etc.

VII. Types of Transformers

123. As the transformer consists of a magnetic circuit interlinked with two electric circuits, two constructive arrangements are possible: The electric circuits may be inside, and surrounded by the magnetic circuit as shell, shell-type transformer; or the magnetic circuit may be arranged inside, as core, and surrounded by the electric circuits, core-type transformer.

In their simplest form, Fig. 163 shows diagrammatically the core-type transformer, with the iron Fe as inside circular core, built up of laminations or of iron wire, and the windings Cu outside; Fig. 164 shows diagrammatically the shell-type transformer, with the copper windings inside, as Cu, and the iron shell Fe wound around it, of iron wire, etc. However, the circular form 163 is used to a limited extent only, in small transformers, autotransformers and reactances, and the form 164 practically never used, and in the constructive modification from these diagrammatic types, it is often difficult to decide to which type to assign the transformer.

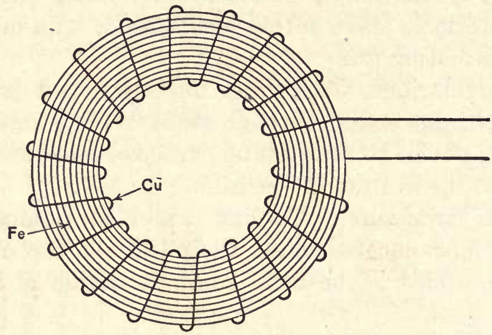


FIG. 163.—Diagram of core type transformer.

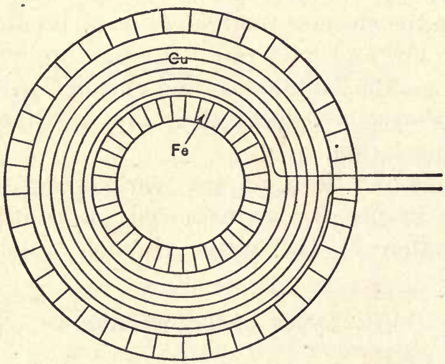


FIG. 166.—Diagram of shell type transformer.

The typical shell-type transformer of today is shown in section in Fig. 165, with the magnetic circuit Fe, and the high voltage windings *P* and low-voltage windings *S* intermixed with each other.

Core-type transformers are shown in section in Figs. 166 and 167, the former with one, the latter with two cores, and with two different coil arrangements, the intermixed and the concentric.

For the transformation of three-phase circuits, three separate single-phase transformers may be used, and their primaries and

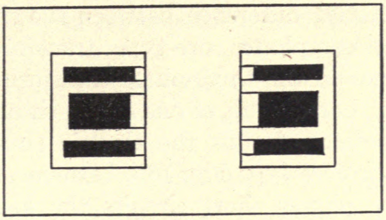


FIG. 165.—Shell type transformer.

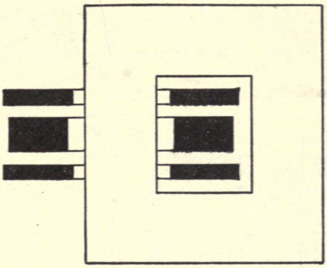


FIG. 166.—Single-coil core type transformer.

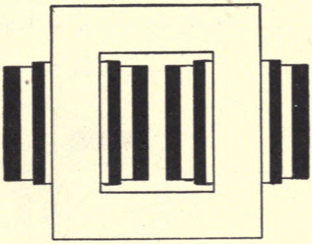


FIG. 167.—Two coil core type transformer.

FIG. 168.—Shell type three-phase transformer diagram.

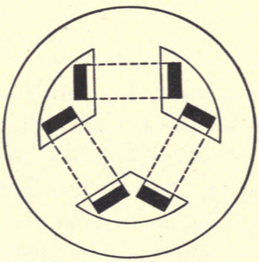


FIG. 169.—Core type three-phase transformer diagram.

secondaries then connected in ring or delta connection or in star or Y connection, giving the four arrangements:

$$\Delta\Delta, \quad \Delta Y, \quad Y\Delta, \quad YY.$$

Or two transformers may be used, arranged in T connection or in open Δ connection, as further discussed under three-phase systems. Or a three-phase transformer may be used. Diagrammatically, the three-phase transformer can be represented by Fig. 168, shell type, and Fig. 169, core type.

124. While in its magnetic and electrical characteristics there is no essential difference between the single-phase shell-type and the single-phase core-type transformer, there is a material difference in the three-phase transformer. In the shell type, Fig. 168, a short circuit of one of the three phases does not affect the magnetic and thus the electric circuit of the other two phases, in the core type Fig. 169, however, a short circuit of one of the three phases short circuits the magnetic return of the other two phases, and so acts as a partial electrical short circuit of these two other phases.

In shell-type transformers, Fig. 168, a triple harmonic of flux can exist, but not in the core type, Fig. 169. In the three-

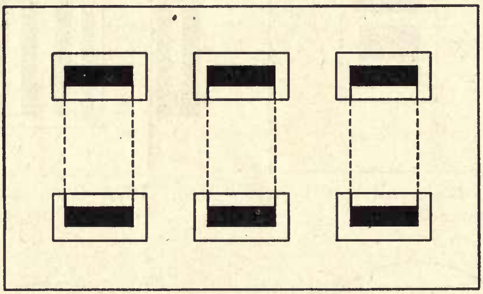


FIG. 170.—Shell type three-phase transformer.

phase system, the three voltages, currents, etc., are displaced in phase from each other by 120°. Their third harmonics therefore are displaced in phase from each other by 3 × 120°, that is, by 360°, or in other words, are in phase with each other. In Fig. 169, such triple frequency fluxes in the three cores would have no magnetic return, except by leakage through the air, that is, cannot exist, except in negligible intensity, and therefore the core type of three-phase transformer cannot give any serious triple frequency voltage. In the shell type Fig. 168, however, the three triple frequency fluxes, being in phase with each other, produce a triple frequency single-phase flux through a closed magnetic circuit. Where the circuit conditions and connections are such as to give a triple harmonic—as with *YY* connection—the shell-type three-phase transformer may produce triple frequency voltages, resulting from the triple frequency flux, and under unfavorable conditions, as when connecting to a system of high capacity—which intensifies these voltages—this may lead to destructive voltages, and YY connections with shell-type three-phase transformers thus lead to serious high voltage dangers.

125. The usual shell-type construction of three-phase transformers is shown in section in Fig. 170, the core type in Fig. 171.

In Fig. 170 economy requires that the middle phase is connected in opposite direction to the outside phases, so that the iron between the successive phases, at 1, 2 and 2, 3, carries the sum of two of the three-phase fluxes, which, as the fluxes are 120 deg. apart, equals one of the fluxes. If the middle phase were not reversed, 1, 2 and 2, 3 would carry the difference of

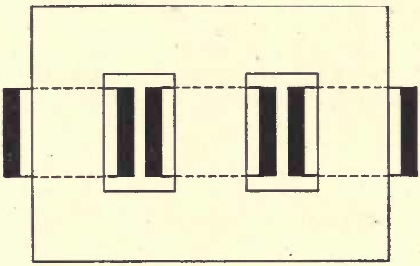


FIG. 171.—Core type three-phase transformer.

two fluxes 120 deg. apart, and this difference is $\sqrt{3}$ times each flux, thus would give a much higher loss.

In Fig. 171 usually the exciting current of the middle phase is somewhat less than that of the outside phase, since the magnetic reluctance of the middle phase is slightly lower.

VIII. Autotransformer

126. If in a transformer a part of the secondary winding is used as primary, or inversely, the transformer is an autotransformer, sometimes also called compensator.

Thus let in a transformer Fig. 172 primary current, voltage and turns be respectively i_1, e_1, n_1, and secondary current, voltage and turns be i_2, e_2, n_2, thus the ratio of transformation $a = \frac{n_1}{n_2}$. Assuming $n_1 > n_2$, then in any n_2 of the n_1 primary turns, the same voltage is induced as in the n_2 secondary turns, and we could thus use any n_2 primary turns as secondary turns, provided we make them of sufficient copper section to carry the secondary current.

The n_2 turns in Fig. 173 thus are in common to primary and secondary circuit. As primary and secondary current are (approximately) opposite in phase, the current in the common turns of Fig. 173 is (approximately, that is, neglecting exciting current) the difference between secondary and primary current, $i_2 - i_1$, thus less than the secondary current i_2, and as the result, the common turns in Fig. 173 may be made of less copper section than the secondary turns in Fig. 172, while the number of primary turns is reduced by n_2. Thus an autotransformer requires less copper, that is, is smaller and cheaper than a transformer of the same output.

127. In the transformer Fig. 172, the size is determined by the number of turns and turn sections, that is, by $e_1 \times i_1 + e_2 \times i_2$

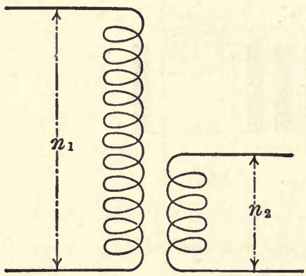


FIG. 172.—Diagram of transformer.

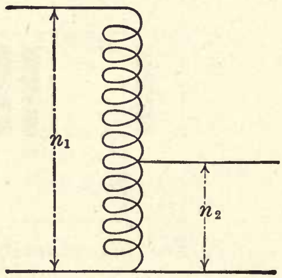


FIG. 173.—Diagram of autotransformer.

(the turns being proportional to the voltage, the turn section to the current, the same magnetic flux assumed). But since $e_1 = ae_2$ and $i_1 = \frac{i_2}{a}$, $e_1 i_1 = e_2 i_2$, and the size of the transformer Fig. 172 thus is proportional to 2 $e_2 i_2$, that is, to 2 P, or twice the output.

In the autotransformer Fig. 173, the n_2 common turns are traversed by the difference of secondary and primary current, at secondary voltage, and the size of this common part of the winding thus is: $e_2 (i_2 - i_1)$. The remaining part of the winding, of $n_1 - n_2$ turns, that is, of voltage $e_1 - e_2$, is traversed by the primary current i_1, hence of size $i_1 (e_1 - e_2)$, and the total size of the autotransformer thus is:

$$e_2 (i_2 - i_1) + i_1 (e_1 - e_2)$$

but, substituting again for i_1 and e_1, gives as the size of the autotransformer:

$$e_2\left(i_2 - \frac{i_2}{a}\right) + \frac{i_2}{a}(ae_2 - e_2) = 2\, e_2 i_2 \left(1 - \frac{1}{a}\right)$$
$$= 2\, P\left(1 - \frac{1}{a}\right)$$

hence, the ratio of size of autotransformer and of transformer of the same output, is:

$$\gamma = \frac{\text{autotransformer}}{\text{transformer}} = 1 - \frac{1}{a}.$$

If the ratio $a = 2$, as transforming between 115 and 230 volts, $\gamma = 1/2$, that is, the autotransformer has half the size of the transformer, or, more correctly stated, the autotransformer is as large as a transformer of half the output.

If the ratio $a = 1.1$, as raising (or lowering) the voltage 10 per cent. by autotransformer, this autotransformer has the size $\gamma = 0.1$ that is, is as small as a transformer of one-tenth the output.

If the ratio $a = 10$, as transforming between 2300 and 230, $\gamma = 0.9$, that is, the autotransformer is only 10 per cent. smaller than the transformer.

The saving in size—and therewith in efficiency and cost—by the use of the autotransformer thus is the greater, the lower the transformation ratio a, but becomes negligible at high transformation ratios. Thus autotransformers are very economical for use in moderate voltage transformation, as a voltage change by 10 or 20 per cent., or even for doubling the voltage, or dividing it in two, but not for high voltage ratios.

128. The most serious disadvantage of the autotransformer obviously is that it electrically interconnects primary and secondary circuit and thereby puts the voltage of the higher voltage circuit onto the lower voltage circuit. Thus, when using autotransformers, the insulation of the low voltage circuit and the high potential tests of all the apparatus used in the low voltage circuit must be those of the high voltage circuit. Furthermore, a ground in one of the two circuits of an autotransformer also is a ground on the other circuit, while with a transformer, a ground on the secondary does not ground the primary, and inversely. With low voltages, as 115 ÷ 230 volt transformation, this is usually of no importance. It would be a serious objection when attempting the use of autotransformers between 2300 and 230 volts. For instance, a ground at the off side of the high-voltage winding, at A in Fig. 173, would put the entire secondary winding 2300 to 2070 volts above ground, and thus the secondary circuit would kill anybody who touches it while standing on the ground.

Any transformer of voltage e_1 and e_2 and currents i_1, i_2 can be converted into an autotransformer, by connecting primary and secondary in series, of voltages $e_1 + e_2$ and e_2 and currents i_1 and $i_2 + i_1$. And inversely, any autotransformer, by disconnecting the two sections of the coil, would give (provided that the insulation is sufficient) a transformer of $(e_1 - e_2) \times i_1$ primary, and $e_2 \times (i_2 - i_1)$ secondary circuit.

The regulation of an autotransformer is better, and the efficiency higher, than that of the same structure as transformer, and the per cent. reactance lower, that is the short-circuit current higher in the autotransformer than in the same structure as transformer. Very often it is difficult to build autotransformers with sufficiently high internal reactance, to make them safe under momentary short circuit as autotransformers, while they may be perfectly safe as transformers, where the reactance is higher. This is a serious objection to the use of autotransformers in high-power systems.

IX. Reactors

(Reactive Coils, Reactances)

129. The reactor consists of one electric circuit interlinked with a magnetic circuit, and its purpose is, not to transform power, but to produce wattless or reactive power, that is, lagging current, or what amounts to the same, leading voltage. While therefore theoretically we cannot speak of an "efficiency" of a reactor, since there is no power output, nevertheless in the industry the expression "efficiency of a reactive coil" is generally used, and generally understood, in the conventional definition:

$$\text{Efficiency} = 1 - \frac{\text{loss}}{\text{input}}$$

and the input is given in total volt-amperes, the loss in energy volt-amperes, that is, watts. The efficiency then is 1 − power-factor.

The transformer at open circuit is a reactor, but a very poor one, as its power-factor is high, that is, the efficiency low.

In the transformer, the exciting ampere-turns are the (vector) difference between primary and secondary ampere-turns, are wasted, and therefore made as low as possible, by using a closed magnetic circuit. In the reactor, no secondary circuit exists, but the exciting ampere-turns are the purpose of the device, thus should be as large as possible. That is, to convert a transformer into a reactor, the reluctance of the magnetic circuit must be increased so as to make the exciting ampere-turns equal to the total full-load ampere-turns of the structure as transformer. This is done by inserting an air gap into the magnetic circuit. Such a gap may be either a single gap, or a number of smaller air gaps, or one or a number of slots cutting almost through the magnetic circuit, but leaving narrow bridges,

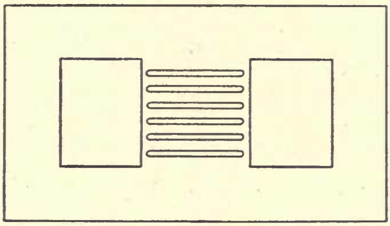


FIG. 174.—Bridged air-gap reactor.

as shown in Fig. 174. This latter offers the advantage of a better mechanical structure, less liability to noise and to magnetic leakage, but when used in series in high voltage circuits, may lead to voltage peaks at the moment of current reversal, which may endanger the insulation. The use of a number of small air gaps instead of one large one distributes the magnetic leakage and thus gives less liability to eddy currents in the conductors.

130. A transformer of output $P = e_2 i_2$ has a size of winding space of $e_2 i_2 + e_1 i_1 = 2\, e_2 i_2$, that is (with the air gap inserted into the magnetic circuit), gives a reactor of the capacity $ei = 2\, P$. That is, a reactor has the size of a transformer of half its output.

Reactors are frequently used in series to apparatus, and the voltage consumed by the reactance then varies with the current, and is, due to the air gap, proportional to the current up to the value where the iron part of the reactance begins to saturate, as shown by the characteristic curve of a reactance, Fig. 175, the "volt-ampere characteristic." Then the voltage increases less than proportional to the current, or inversely, the current increases out of proportion to the voltage, that is, the reactance decreases and wave-shape distortion occurs. Reactances thus must be designed so that at the highest currents (or voltages), at which they may be called upon to develop their reactance, their magnetic circuit is still below saturation.

Industrially, reactors are often denoted in per cent. Thus for

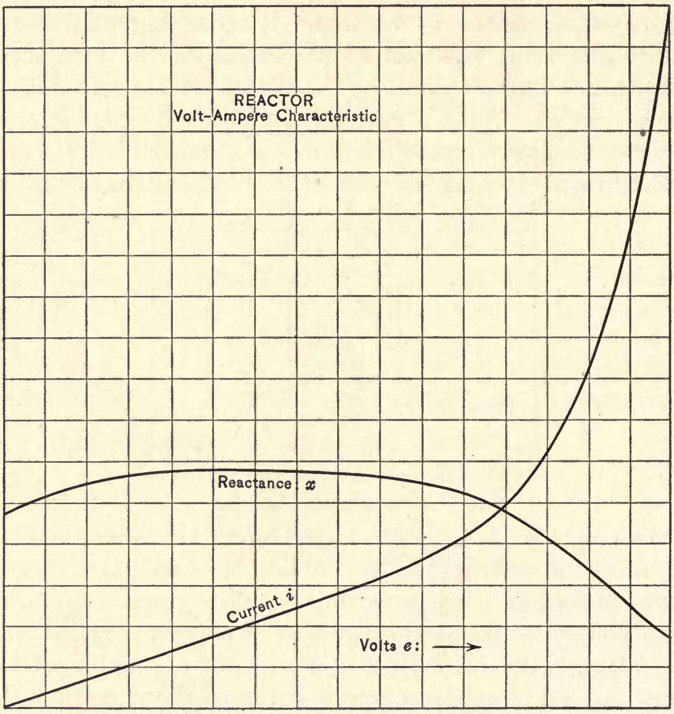


FIG. 175.—Volt-ampere characteristic of reactor.

phase control in synchronous converter circuits, 15 per cent. reactances are used. This means, at full-load current, the voltage consumed by these reactances is 15 per cent. of the circuit voltage.

131. With the increasing size and increasing voltage of modern central stations and the use of high-speed turbo-alternators capable of momentarily giving very high short-circuit currents, the amount of power, which can be developed momentarily by a short circuit in the system near the generating station, has reached such destructive values, that a limitation of this power has become necessary, and as economy of operation forbids sectionalizing the system into a number of smaller units, this has led to the extensive use of power-limiting reactances, in the generator leads, in the bus bars, tie feeders and even the power feeders. Such reactances are used of 2 to 8 per cent., and in bus bars even up to 25 per cent., and in case of a local short circuit, limit the current which can flow. Thus a 4 per cent. reactance would at a short circuit just beyond the reactance limit the current to $\frac{1}{4 \text{ per cent.}}$ = 25 times the normal, etc. But to do so, the reactance must still be there at twenty-five times its rated current, that is, when absorbing full circuit voltage instead of its normal 4 per cent. thereof. If then iron is used in the magnetic circuit of such a reactance, the density must be so low, that at twenty-five times this density (or at 12.5 times with an 8 per cent. reactance, etc.), it does not yet saturate. When limited to such very low magnetic densities in the iron, the mass of iron becomes so enormous, that it becomes more economical to use an air circuit throughout.

Reactances, which must retain their reactance, that is, must not saturate at many times their normal current, such as power limiting reactances, thus are built without iron in the magnetic circuit.

E. INDUCTION MACHINES

I. General

132. The direction of rotation of a direct-current motor, whether shunt- or series-wound, is independent of the direction of the current supplied thereto; that is, when reversing the current in a direct-current motor the direction of rotation remains the same. Thus theoretically any continuous-current motor should operate also with alternating currents. Obviously in this case not only the armature but also the magnetic field of the motor must be thoroughly laminated to exclude eddy currents, and care taken that the currents in the field and armature circuits reverse simultaneously. Obviously the simplest way of fulfilling the latter condition is to connect the field and armature circuits in series as alternating-current series motor. Such motors are used to a considerable extent, but, like the shunt motor, have the disadvantage of a commutator carrying alternating currents.

The shunt motor on an alternating-current circuit has the objection that in the armature winding the current should be power current, thus in phase with the e.m.f., while in the field winding the current is lagging nearly 90 deg., as magnetizing current. Thus field and armature would be out of phase with each other. To overcome this objection either there is inserted in series with the field circuit a condenser of such capacity as to bring the current back into phase with the voltage, or the field may be excited from a separate e.m.f. differing 90 deg. in phase from that supplied to the armature. The former arrangement has the disadvantage of requiring almost perfect constancy of frequency, and therefore is not practicable. In the latter arrangement the armature winding of the motor is fed by one, the field winding by the other phase of a quarter-phase system, and thus the current in the armature brought approximately into phase with the magnetic flux of the field.

Such an arrangement obviously loads the two phases of the system unsymmetrically, the one with the armature power current, the other with the lagging field current. To balance the system two such motors may be used simultaneously and combined in one structure, the one receiving power current from the first, magnetizing current from the second phase, the second motor receiving magnetizing current from the first and power current from the second phase.

The objection that the use of the commutator is complicated and greatly limits the design to avoid serious sparking can be entirely overcome by utilizing the alternating feature of the current; that is, instead of leading the current into the armature by commutator and brushes, producing it therein by electromagnetic induction, by closing the armature conductors upon themselves and surrounding the armature by a primary coil at right angles to the field exciting coil.

Such motors have been built, consisting of two structures each containing a magnetizing circuit acted upon by one phase and a primary power circuit acting upon a closed-circuit armature as secondary and excited by the other phase of a quarter-phase system (Stanley motor).

Going still a step further, the two structures can be combined into one by having each of the two coils fulfill the double function of magnetizing the field and producing currents in the secondary which are acted upon by the magnetization produced by the other phase.

Obviously, instead of two phases in quadrature any number of phases can be used.

This leads us by gradual steps of development from the continuous-current shunt motor to the alternating-current polyphase induction motor.

In its general behavior the alternating-current induction motor is therefore analogous to the continuous-current shunt motor. Like the shunt motor, it operates at approximately constant magnetic density. It runs at fairly constant speed, slowing down gradually with increasing load. The main difference is that in the induction motor the current in the armature does not pass through a system of brushes, as in the continuous-current shunt motor, but is produced in the armature as the short-circuited secondary of a transformer; and in consequence thereof the primary circuit of the induction motor fulfills the double function of an exciting circuit corresponding to the field circuit of the continuous-current machine and a primary circuit producing a secondary current in the secondary by electromagnetic induction.

133. Since in the secondary of the induction motor the currents are produced by induction from the primary impressed currents, the induction motor in its electromagnetic features is essentially a transformer; that is, it consists of a magnetic circuit or magnetic circuits interlinked with two electric circuits or sets of circuits, the primary and the secondary circuits. The difference between transformer and induction motor is that in the former the secondary is fixed regarding the primary, and the electric energy in the secondary is made use of, while in the latter the secondary is movable regarding the primary, and the mechanical force acting between primary and secondary is used. In consequence thereof the frequency of the currents in the secondary of the induction motor differs from, and as a rule is very much lower than, that of the currents impressed upon the primary, and thus the ratio of e.m.fs. generated in primary and in secondary is not the ratio of their respective turns, but is the ratio of the product of turns and frequency.

Taking due consideration of this difference of frequency between primary and secondary, the theoretical investigation of the induction motor corresponds to that of the stationary transformer. The transformer feature of the induction motor predominates to such an extent that in theoretical investigation the induction motor is best treated as a transformer, and the electrical output of the transformer corresponds to the mechanical output of the induction motor.

The secondary of the motor consists of two or more circuits displaced in phase from eaeh other so as to offer a closed secondary to the primary circuits, irrespective of the relative motion. The primary consists of one or several circuits.

In consequence of the relative motion of the primary and secondary, the magnetic circuit of the induction motor must be arranged so that the secondary while revolving does not leave the magnetic field of force. That means, the magnetic field of force must be of constant intensity in all directions, or, in other words, the component of magnetic flux in any direction in space be of the same or approximately the same intensity but differing in phase. Such a magnetic field can either be considered as the superposition of two magnetic fields of equal intensity in quadrature in time and space, or it can be represented theoretically by a revolving magnetic flux of constant intensity, or rotating field, or simply treated as alternating magnetic flux of the same intensity in every direction.

134. The operation of the induction motor thus can also be considered as due to the action of a rotating magnetic field upon a system of short-circuited conductors. In the motor field or primary, usually the stator, by a system of polyphase impressed e.m.fs. or by the combination of a single-phase impressed e.m.f. and the reaction of the currents produced in the secondary, a rotating magnetic field is produced. This rotating field produces currents in the short-circuited armature or secondary winding, usually the rotor, and by its action on these currents drags along the secondary conductors, and thus speeds up the armature and tends to bring it up to synchronism, that is, to the same speed as the rotating field, at which speed the secondary currents would disappear by the armature conductors moving together with the rotating field, and thus cutting no lines of force. The secondary therefore slips in speed behind the speed of the rotating field by as much as is required to produce the secondary currents and give the torque necessary to carry the load. The slip of the induction motor thus increases with increase of load, and is approximately proportional thereto. Inversely, if the secondary is driven at a higher speed than that of the rotating field, the field drags the armature conductors back, that is, consumes mechanical torque, and the machine then acts as a brake or induction generator.

In the polyphase induction motor this magnetic field is produced by a number of electric circuits relatively displaced in space, and excited by currents having the same displacement in phase as the exciting coils have in space.

In the single-phase motor one of the two superimposed magnetic quadrature fields is excited by the primary electric circuit, the other by the secondary currents carried into quadrature position by the rotation of the secondary. In either case, at or near synchronism the magnetic fields are practically identical.

The transformer feature being predominant, in theoretical investigations of induction motors it is generally preferable to start therefrom.

The characteristics of the transformer are independent of the ratio of transformation, other things being equal; that is, doubling the number of turns for instance, and at the same time reducing their cross section to one-half, leaves the efficiency, regulation, etc., of the transformer unchanged. In the same way, in the induction motor it is unessential what the ratio of primary to secondary turns is, or, in other words, the secondary circuit can be wound for any suitable number of turns, provided the same total copper cross section is used. In consequence hereof the secondary circuit is mostly wound with one or two bars per slot, to get maximum amount of copper, that is, minimum resistance of secondary.

The general characteristics of the induction motor being independent of the ratio of turns, it is for theoretical considerations simpler to assume the secondary motor circuits reduced to the same number of turns and phases as the primary, or of the ratio of transformation 1 to 1, by multiplying all secondary currents and dividing all secondary e.m.fs. by the ratio of turns, multiplying all secondary impedances and dividing all secondary admittances by the square of the ratio of turns, etc.

Thus in the following under secondary current, e.m.f., impedance, etc., shall always be understood their values reduced to the primary, or corresponding to a ratio of turns 1 to 1, and the same number of secondary as primary phases, although in practice a ratio 1 to 1 will hardly ever be used, as not fulfilling the condition of uniform effective reluctance desirable in the starting of the induction motor.

II. Polyphase Induction Motor

1. Introduction

135. The typical induction motor is the polyphase motor. By gradual development from the direct-current shunt motor we arrive at the polyphase induction motor.

The magnetic field of any induction motor, whether supplied by polyphase, monocyclic, or single-phase e.m.f., is at normal condition of operation, that is, near synchronism, a polyphase field. Thus to a certain extent all induction motors can be called polyphase machines. When supplied with a polyphase system of e.m.fs. the internal reactions of the induction motor are simplest and only those of a transformer with moving secondary, while in the single-phase induction motor at the same time a phase transformation occurs, the second or magnetizing phase being produced from the impressed phase of e.m.f. by the rotation of the motor, which carries the secondary currents into quadrature position with the primary current.

The polyphase induction motor of the three-phase or quarter-phase type is the one most commonly used, while single-phase motors have found a more limited application only, and especially for smaller powers.

Thus in the following more particularly the polyphase induction machine shall be treated, and the single-phase type discussed only in so far as it differs from the typical polyphase machine.

2. Calculation

136. In the polyphase induction motor,

Let

$Y = g - jb$ = primary exciting admittance, or admittance of the primary circuit with open secondary circuit;

that is,

ge = magnetic power current,

be = wattless magnetizing current,

where e = counter-generated e.m.f. of the motor;

$Z_0 = r_0 + jx_0$ = primary self-inductive impedance, and

$Z_1 = r_1 + jx_1$ = secondary self-inductive impedance, reduced to the primary by the ratio of turns.[1]

All these quantities refer to one primary circuit and one corresponding secondary circuit. Thus in a three-phase induction motor the total power, etc., is three times that of one circuit, in the quarter-phase motor with three-phase armature $1\frac{1}{2}$ of the three secondary circuits are to be considered as corresponding to each of the two primary circuits, etc.

Let e = primary counter-generated e.m.f., or e.m.f. generated in the primary circuit by the flux interlinked with primary and secondary (mutual induction); s = slip, with the primary frequency as unit; that is, $s = 0$ denoting synchronous rotation, $s = 1$ standstill of the motor.

We then have

$1 - s$ = speed of the motor secondary as fraction of synchronous speed,

sf = frequency of the secondary currents,

where

f = frequency impressed upon the primary;

[1] The self-inductive reactance refers to that flux which surrounds one of the electric circuits only, without being interlinked with the other circuits.

hence,

se = e.m.f. generated in the secondary.

The actual impedance of the secondary circuit at the frequency sf is

$$Z_1^s = r_1 + jsx_1;$$

hence, the secondary current is

$$\dot{I}_1 = \frac{se}{Z_1^s} = \frac{se}{r_1 + jsx_1} = e\left(\frac{sr_1}{r_1^2 + s^2x_1^2} - j\frac{s^2x_1}{r_1^2 + s^2x_1^2}\right) = e(a_1 - ja_2),$$

where

$$a_1 = \frac{sr_1}{r_1^2 + s^2x_1^2}, \qquad a_2 = \frac{s^2x_1}{r_1^2 + s^2x_1^2};$$

the primary exciting current is

$$\dot{I}_{00} = eY = e[g - jb],$$

and the total primary current is

$$\dot{I}_0 = e[(a_1 + g) - j(a_2 + b)] = e(b_1 - jb_2),$$

where

$$b_1 = a_1 + g, \qquad b_2 = a_2 + b.$$

The e.m.f. consumed in the primary circuit by the impedance Z_0 is $\dot{I}_0 Z_0$, the counter-generated e.m.f. is e, hence, the primary terminal voltage is

$$\dot{E}_0 = e + \dot{I}_0 Z_0 = e[1 + (b_1 - jb_2)(r_0 + jx_0)] = e(c_1 - jc_2),$$

where

$$c_1 = 1 + r_0b_1 + x_0b_2 \text{ and } c_2 = r_0b_2 - x_0b_1.$$

Eliminating complex quantities, we have

$$E_0 = e\sqrt{c_1^2 + c_2^2},$$

hence, the counter-generated e.m.f. of motor,

$$e = \frac{E_0}{\sqrt{c_1^2 + c_2^2}},$$

where

E_0 = impressed e.m.f., absolute value.

Substituting this value in the equations of $\dot{I}_1$, $\dot{I}_{00}$, $\dot{I}_0$, etc, gives the complex expressions of currents and e.m.fs., and eliminating the imaginary quantities we have the primary current,

$$I_0 = e\sqrt{b_1^2 + b_2^2}, \text{ etc.}$$

The torque of the polyphase induction motor (or any other motor or generator) is proportional to the product of the mutual magnetic flux and the component of ampere-turns of the secondary, which is in phase with the magnetic flux in time, but in quadrature therewith in direction or space. Since the generated e.m.f. is proportional to the mutual magnetic flux and the number of turns, but in quadrature thereto in time, the torque of the induction motor is proportional also to the product of the generated e.m.f. and the component of secondary current in quadrature therewith in time and in space.

Since $\dot{I}_1 = e\,(a_1 - ja_2)$ is the secondary current corresponding to the generated e.m.f. e, the secondary current in the quadrature position thereto in space, that is, corresponding to the e.m.f. je, is

$$j\dot{I}_1 = e\,(a_2 + ja_1),$$

and a_1e is the component of this current in quadrature in time with the e.m.f. e.

Thus the torque is proportional to $e \times a_1e$, or

$$D = e^2a_1$$
$$= \frac{e^2r_1s}{r_1^2 + s^2x_1^2} = \frac{E_0^2r_1s}{(c_1^2 + c_2^2)\,(r_1^2 + s^2x_1^2)}.$$

This value D is in its dimension a *power*, and it is the power which the torque of the motor would develop at synchronous speed.

137. In induction motors, and in general motors which have a definite limiting speed, it is preferable to give the torque in the form of the power developed at the limiting speed, in this case synchronism, as "synchronous watts," since thereby it is made independent of the individual conditions of the motor, as its number of poles, frequency, etc., and made comparable with the power input, etc. It is obvious that when given in synchronous watts, the maximum possible value of torque which could be reached, if there were no losses in the motor, equals the power input. Thus, in an induction motor with 9000 watts power input, a torque of 7000 synchronous watts means that 7/9 of the maximum theoretically possible torque is realized, while the statement, "a torque of 30 pounds at 1-foot radius," would be meaningless without knowing the number of poles and the frequency. Thus, the denotation of the torque in synchronous watts is the most general, and preferably used in induction motors.

Since the theoretical maximum possible torque equals the power input, the ratio

$$\frac{\text{torque in synchronous watts output}}{\text{power input}},$$

that is,

$$\frac{\text{actual torque}}{\text{maximum possible torque}},$$

is called the *torque efficiency* of the motor, analogous to the power efficiency or

$$\frac{\text{power output}}{\text{power input}};$$

that is,

$$\frac{\text{power output}}{\text{maximum possible power output}}.$$

Analogously

$$\frac{\text{torque in synchronous watts}}{\text{volt-amperes input}}$$

is called the *apparent torque efficiency.*

The definitions of these quantities, which are of importance in judging induction motors, are thus:

The "*efficiency*" or "*power efficiency*" is the ratio of the true mechanical output of the motor to the output which it would give at the same power input if there were no internal losses in the motor.

The "*apparent efficiency*" or "*apparent power efficiency*" is the ratio of the mechanical output of the motor to the output which it would give at the same volt-ampere input if there were neither internal losses nor phase displacement in the motor.

The "*torque efficiency*" is the ratio of the torque of the motor to the torque which it would give at the same power input if there were no internal losses in the motor.

The "*apparent torque efficiency*" is the ratio of the torque of the motor to the torque which it would give at the same volt-ampere input if there were neither internal losses nor phase displacement in the motor.

The torque efficiencies are of special interest in starting where the power efficiencies are necessarily zero, but it nevertheless is of importance to find how much torque per watt or per volt-ampere input is given by the motor.

Since $D = e^2a_1$ is the power developed by the motor torque at synchronism, the power developed at the speed of $(1 - s) \times$ synchronism, or the actual power output of the motor, is

$$P = (1 - s) D$$
$$= e^2a_1 (1 - s)$$
$$= \frac{e^2r_1s (1 - s)}{r_1^2 + s^2x_1^2}.$$

The output P includes friction, windage, etc.; thus, the net mechanical output is P − friction, etc. Since, however, friction, etc., depend upon the mechanical construction of the individual motor and its use, it cannot be included in a general formula. P is thus the mechanical output, and D the torque developed at the armature conductors.

The primary current

$$\dot{I}_0 = e (b_1 - jb_2)$$

has the quadrature components eb_1 and eb_2.

The primary impressed e.m.f.

$$\dot{E}_0 = e (c_1 - jc_2)$$

has the quadrature components ec_1 and ec_2.

Since the components eb_1 and ec_2, and eb_2 and ec_1, respectively, are in quadrature with each other, and thus represent no power, the power input of the primary circuit is

$$eb_1 \times ec_1 + eb_2 \times ec_2,$$

or

$$P_0 = e^2 (b_1c_1 + b_2c_2).$$

The volt-amperes or apparent input is obviously,

$$P_a = I_0E_0$$
$$= e^2 \sqrt{(b_1^2 + b_2^2) (c_1^2 + c_2^2)}.$$

138. These equations can be greatly simplified by neglecting the exciting current of the motors, and approximate values of current, torque, power, etc., derived thereby, which are sufficiently accurate for preliminary investigations of the motor at speeds sufficiently below synchronism to make the total motor current large compared with the exciting current.

In this case the *primary current* equals the secondary current, that is,

$$\dot{I}_0 = \dot{I}_1 = \frac{se}{Z^s} = e\,(a_1 - ja_2),$$

where

$$a_1 = \frac{sr_1}{r_1^2 + s^2x_1^2}, \text{ etc.,}$$

and

$$\begin{aligned} \dot{E}_0 &= e + Z_0\dot{I}_0 \\ &= e\left\{1 + \frac{sZ_0}{Z_1^s}\right\} = \frac{e\,(Z_1^s + sZ_0)}{Z_1^s} \\ &= e\,\frac{(r_1 + sr_0) + js\,(x_1 + x_0)}{r_1 + jsx_1}, \end{aligned}$$

and, in absolute values,

$$e_0 = e\,\frac{\sqrt{(r_1 + sr_0)^2 + s^2\,(x_1 + x_0^2)}}{\sqrt{r_1^2 + s^2x_1^2}};$$

hence,

$$e = \frac{e_0\sqrt{r_1^2 + s^2x_1^2}}{\sqrt{(r_1 + sr_0)^2 + s^2\,(x_1 + x_0)^2}},$$

and the *torque*, in synchronous watts, is

$$D = e^2a_1 = \frac{se^2r_1}{r_1^2 + s^2x_1^2};$$

hence, substituting for e,

$$D = \frac{se_0^2r_1}{(r_1 + sr_0)^2 + s^2\,(x_1 + x_0)^2},$$

and the *power* is

$$P = \frac{s\,(1 - s)\,e_0^2r_1}{(r_1 + sr_0)^2 + s^2\,(x_1 + x_0)^2}.$$

If the additional resistance r is inserted into the armature circuit, and the total armature resistance thus becomes $r_1 + r$, instead of r_1, substituting $(r_1 + r)$ in above equations we have

$$D = \frac{se_0^2\,(r_1 + r)}{(r_1 + r + sr_0)^2 + s^2\,(x_1 + x_0)^2}$$

and

$$P = \frac{s\,(1 - s)\,e_0^2\,(r_1 + r)}{(r_1 + r + sr_0)^2 + s^2\,(x_1 + x)^2}, \text{ etc.}$$

Neglecting also the primary self-inductive impedance, $Z_0 =$

$r_0 + jx_0$, which sometimes can be done as first approximation, especially at large values of r, these equations become

$$D = \frac{se_0^2 (r_1 + r)}{(r_1 + r)^2 + s^2 x_1^2},$$

$$P = s \frac{(1 - s)\, e_0^2 (r_1 + r)}{(r_1 + r)^2 + s^2 x_1^2}, \text{ etc.}$$

139. Since the counter-generated e.m.f. e (and thus the impressed e.m.f. E_0) enters in the equation of current, magnetism, etc., as a simple factor, in the equations of torque, power input and output, and volt-ampere input as square, and cancels in the equation of efficiency, power-factor, etc., it follows that the current, magnetic flux, etc., of an induction motor are proportional to the impressed e.m.f., the torque, power output, power input, and volt-ampere input are proportional to the square of the impressed e.m.f., and the torque- and power efficiencies and the power-factor are independent of the impressed voltage.

In reality, however, a slight decrease of efficiency and power-factor occurs at higher impressed voltages, due to the increase of resistance caused by the increasing temperature of the motor and due to the approach to magnetic saturation, and a slight decrease of efficiency occurs at lower voltages when including in the efficiency the loss of power by friction, since this is independent of the output and thus at lower voltage, that is, lesser output, a larger percentage of the output, so that the efficiencies and the power-factor can be considered as independent of the impressed voltage, and the torque and power proportional to the square thereof only approximately, but sufficiently close for many purposes.

3. Load and Speed Curves

140. The calculation of the induction motor characteristics is most conveniently carried out in tabulated form by means of above-given equations as follows:

Let $Z_0 = r_0 + jx_0 = 0.1 + 0.3\, j$ = primary self-inductive impedance.

$Z_1 = r_1 + jx_1 = 0.1 + 0.3\, j$ = secondary self-inductive impedance reduced to primary.

$Y = g - jb = 0.01 - 0.1\, j$ = primary exciting admittance.

E_0 = 110 volts = primary impressed e.m.f.

It is then, per phase,

s	$m = r_1^2 + s^2x_1^2$	$a_1 = \frac{sr_1}{m}$	$a_2 = \frac{s^2x_1}{m}$	$b_1 = a_1 + g$	$b_2 = a_2 + b$	$c_1 = 1 + r_0b_1 + x_0b_2$	$c_2 = r_0b_2 - x_0b_1$	$c = \sqrt{c_1^2 + c_2^2}$	$e = \frac{E_0}{c}$	$u = \sqrt{b_1^2 + b_2^2}$	$I = eu$
0	0.0100	0	0	0.01	0.10	1.031	+0.007	1.031	106.6	0.1010	10.8
0.01	0.0100	0.100	0.003	0.11	0.103	1.042	−0.023	1.042	105.7	0.1507	15.9
0.02	0.0100	0.200	0.012	0.21	0.112	1.055	−0.052	1.056	104.3	0.238	24.8
0.05	0.0102	0.490	0.073	0.50	0.173	1.102	−0.133	1.110	99.2	0.522	51.8
0.1	0.0109	0.920	0.276	0.93	0.376	1.206	−0.241	1.230	89.5	1.003	89.7
0.15	0.0120	1.25	0.563	1.26	0.663	1.325	−0.308	1.360	80.9	1.424	115
0.2	0.0136	1.47	0.883	1.48	0.983	1.443	−0.354	1.485	74.2	1.777	132
0.3	0.0181	1.66	1.49	1.67	1.50	1.617	−0.351	1.654	66.6	2.245	149
0.5	0.0325	1.54	2.31	1.55	2.41	1.878	−0.224	1.891	58.2	2.865	167
1.0	0.1000	1.00	3.00	1.01	3.10	2.031	+0.007	2.031	54.1	3.261	176

s	e^2	$D = e^2a_1$	$P = (1 - s) D$	$P_a = E_0I$	$p = b_1c_1 + b_2c_2$	$P_0 = e^2p$	eff. $= \frac{P}{P_0}$	app. eff. $= \frac{P}{P_a}$	pow. fac. $= \frac{P_0}{P_a}$
0	11,360	0	0	1.19	0.011	0.125	0	0	10.5
0.01	11,170	1.117	1.106	1.75	0.112	1.249	88.5	63.2	71.5
0.02	10,880	2.176	2.133	2.73	0.216	2.350	91.0	78.3	86.2
0.05	9,840	4.82	4.58	5.70	0.528	5.20	88.3	80.5	91.3
0.1	8,010	7.38	6.64	9.87	1.030	8.25	80.7	67.3	83.5
0.15	6,540	8.20	6.97	12.65	1.466	9.60	72.5	55.0	76.0
0.2	5,510	8.10	6.48	14.52	1.782	9.80	66.0	44.6	67.5
0.3	4,440	7.36	5.15	16.4	2.154	9.55	53.8	31.5	58.3
0.5	3,390	5.23	2.61	18.4	2.370	8.04	32.3	14.2	43.8
1.0	2,930	2.93	0	19.4	2.072	6.08	0	0	31.3

Diagrammatically it is most instructive in judging about an induction motor to plot from the preceding calculation—

1st. The *load curves*, that is, with the load or power output as abscissas, the values of speed (as a fraction of synchronism), of current input, power-factor, efficiency, apparent efficiency, and torque.

2d. The *speed curves*, that is, with the speed, as a fraction of synchronism, as abscissas, the values of torque, current input, power-factor, torque efficiency, and apparent torque efficiency.

The load curves are most instructive for the range of speed near synchronism, that is, the normal operating conditions of the motor, while the speed curves characterize the behavior of the motor at any speed.

In Fig. 176 are plotted the load curves, and in Fig 177 the speed curves of a typical polyphase induction motor of moderate size, having the following constants: $e_0 = 110$; $Y = 0.01 - 0.1\,j$; $Z_1 = 0.1 + 0.3\,j$, and $Z_0 = 0.1 + 0.3\,j$.

As sample of a poor motor of high resistance and high admittance or exciting current are plotted in Fig. 178 the load curves of a motor having the following constants: $e_0 = 110$; $Y = 0.04$

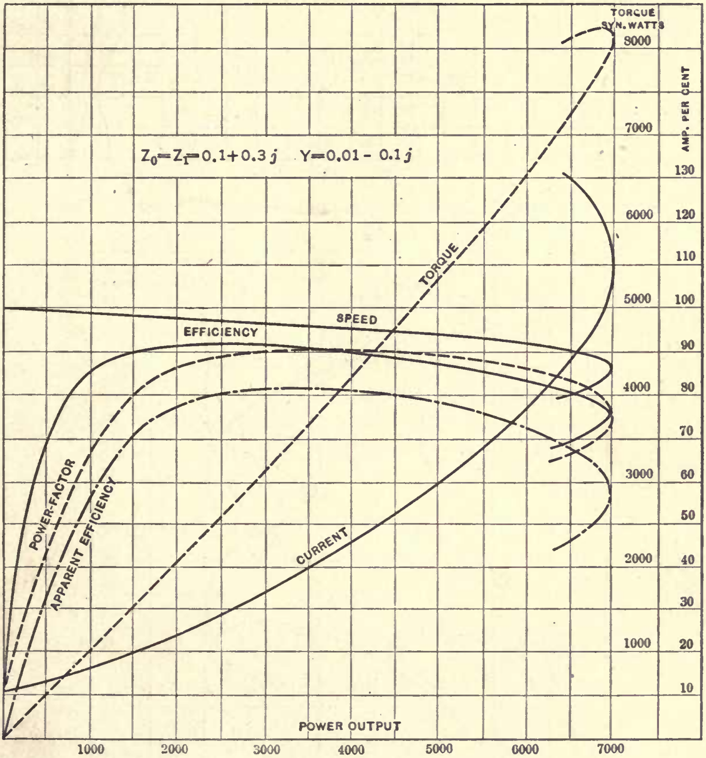


FIG. 176.—Induction motor load curves.

$- 0.4\,j$; $Z_1 = 0.3 + 0.3\,j$, and $Z_0 = 0.3 + 0.3\,j$, showing the overturn of the power-factor curve frequently met in poor motors.

141. The shape of the characteristic motor curves depends entirely on the three complex constants, Y, Z_1, and Z_0, but is essentially independent of the impressed voltage.

Thus a change of the admittance Y has no effect on the characteristic curves, provided that the impedances Z_1 and Z_0 are changed inversely proportional thereto, such a change merely representing the effect of a change of impressed voltage. A

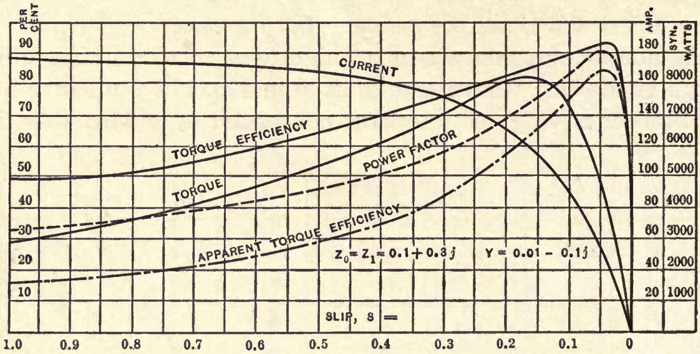


FIG. 177.—Induction motor speed curves.

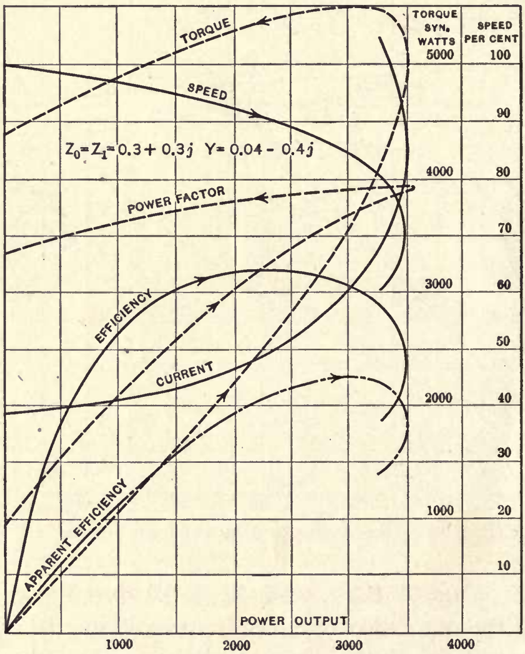


FIG. 178.—Load curves of poor induction motor.

moderate change of one of the impedances has relatively little effect on the motor characteristics, provided that the other impedance changes so that the sum $Z_1 + Z_0$ remains constant, and thus the motor can be characterized by its total internal impedance, that is,

$$Z = Z_1 + Z_0;$$

or $$r + jx = (r_1 + r_0) + j(x_1 + x_0).$$

Thus the characteristic behavior of the induction motor depends upon two complex imaginary constants, Y and Z, or four real constants, g, b, r, x, the same terms which characterize the stationary alternating-current transformer on non-inductive load.

Instead of conductance g, susceptance b, resistance r, and reactance x, as characteristic constants may be chosen: the absolute exciting admittance $y = \sqrt{g^2 + b^2}$; the absolute self-inductive impedance $z = \sqrt{r^2 + x^2}$; the power-factor of admittance $\beta = g/y$, and the power-factor of impedance $\alpha = r/z$.

142. If the admittance y is reduced n-fold and the impedance z increased n-fold, with the e.m.f. $\sqrt{n}E_0$ impressed upon the motor, the speed, torque, power input and output, volt-ampere input and excitation, power-factor, efficiencies, etc., of the motor, that is, all its characteristic features, remain the same, as seen from above given equations, and since a change of impressed e.m.f. does not change the characteristics, it follows that a change of admittance and of impedance does not change the characteristics of the motor provided the product $\gamma = yz$ remains the same.

Thus the induction motor is characterized by three constants only:

The product of exciting admittance and self-inductive impedance $\gamma = yz$, which may be called the characteristic constant of the motor.

The power-factor of exciting admittance $\beta = \frac{g}{y}$.

The power-factor of self-inductive impedance $\alpha = \frac{r}{z}$.

All these three quantities are absolute numbers.

The physical meaning of the characteristic constant or the product of the exciting admittance and impedance is the following:

If I_{00} = exciting current and I_{10} = starting current, we have, approximately,

$$y = \frac{I_{00}}{E_0},$$

$$z = \frac{E_0}{I_{10}},$$

$$\gamma = yz = \frac{I_{00}}{I_{10}}.$$

The characteristic constant of the induction motor $\gamma = yz$ is the ratio of exciting current to starting current or current at standstill.

At given impressed e.m.f., the exciting current I_{00} is inversely proportional to the mutual inductance of primary and secondary circuit. The starting current I_{10} is inversely proportional to the sum of the self-inductance of primary and secondary circuit.

Thus the characteristic constant $\gamma = yz$ is approximately the ratio of total self-inductance to mutual inductance of the motor circuits; that is, the ratio of the flux interlinked with only one circuit, primary or secondary, to the flux interlinked with both circuits, primary and secondary, or the ratio of the waste or leakage flux to the useful flux. The importance of this quantity is evident.

4. Effect of Armature Resistance and Starting

143. The secondary or armature resistance r_1 enters the equation of secondary current thus:

$$\dot{I}_1 = \frac{se}{r_1 + jsx_1} = e\left(\frac{sr_1}{r_1^2 + s^2x_1^2} - j\frac{s^2x_1}{r_1^2 + s^2x_1^2}\right) = e\,(a_1 - ja_2),$$

and the further equations only indirectly in so far as r_1 is contained in a_1 and a_2.

Increasing the armature resistance n-fold, to nr_1, we get at an n-fold increased slip ns,

$$\dot{I}_1 = \frac{nse}{nr_1 + jnsx_1} = \frac{se}{r_1 + jsx};$$

that is, the same value, and thus the same values for e, I_0, D, P_0, P_a, while the power is decreased from $P = (1 - s)\,D$ to $P = (1 - ns)\,D$, and the efficiency and apparent efficiency are correspondingly reduced. The power-factor is not changed; hence, an increase of armature resistance r_1 produces a proportional increase of slip s, and thereby corresponding decrease of power output, efficiency and apparent efficiency, but does not change the torque, power imput, current, power-factor, and the torque efficiencies.

Thus the insertion of resistance in the armature or secondary of the induction motor offers a means of reducing the speed corresponding to a given torque, and thereby the desired torque can be produced at any speed below that corresponding to short-circuited armature or secondary without changing the input or current.

Hence, given the speed curve of a short-circuited motor, the speed curve with resistance inserted in the armature can be derived therefrom directly by increasing the slip in proportion to the increased resistance.

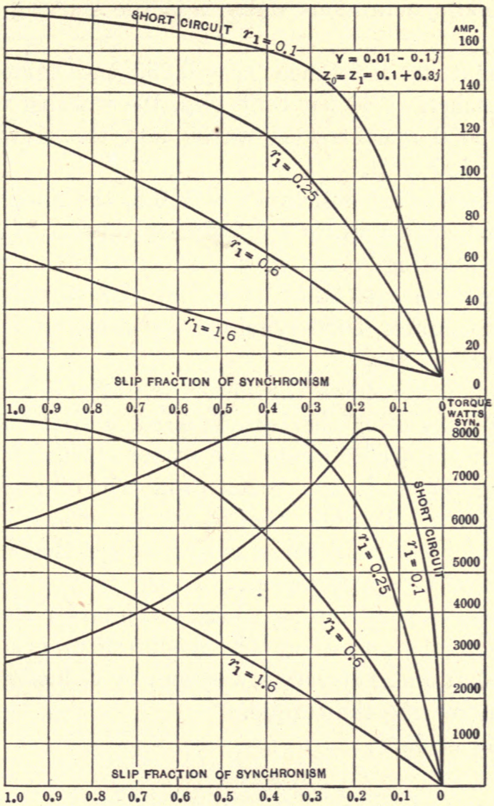


FIG. 179.—Induction motor speed-torque and -current curves.

This is done in Fig. 179, in which are shown the speed curves of the motor Figs. 176 and 177, between standstill and synchronism, for—

Short-circuited armature, $r_1 = 0.1$ (same as Fig. 177).

0.15 ohm additional resistance per circuit inserted in armature, $r_1 = 0.25$, that is, 2.5 times increased slip.

0.5 ohm additional resistance inserted in the armature, $r_1 = 0.6$, that is, 6 times increased slip.

1.5 ohm additional resistance inserted in the armature, $r_1 = 1.6$, that is, 16 times increased slip.

The corresponding current curves are shown on the same sheet.

With short-circuited secondary the maximum torque of 8250 synchronous watts is reached at 16 per cent. slip. The starting torque is 2950 synchronous watts, and the starting current 176 amp.

With armature resistance $r_1 = 0.25$, the same maximum torque is reached at 40 per cent. slip, the starting torque is increased to 6050 synchronous watts, and the starting current decreased to 160 amp.

With the secondary resistance $r_1 = 0.6$, the maximum torque of 8250 synchronous watts approximately takes place in starting, and the starting current is decreased to 124 amp.

With armature resistance $r_1 = 1.6$, the starting torque is below the maximum, 5620 synchronous watts, and the starting current is only 64 amp.

In the two latter cases the lower or unstable branch of the torque curve has altogether disappeared, and the motor speed is stable over the whole range; the motor starts with the maximum torque which it can reach, and with increasing speed, torque and current decrease; that is, the motor has the characteristic of the direct-current series motor, except that its maximum speed is limited by synchronism.

144. It follows herefrom that high secondary resistance, while very objectionable in running near synchronism, is advantageous in starting or running at very low speed, by reducing the current input and increasing the torque.

In starting we have

$$s = 1.$$

Substituting this value in the equations of subsection 2 gives the starting torque, starting current, etc., of the polyphase induction motor.

In Fig. 180 are shown for the motor in Figs. 176, 177 and 179 the values of starting torque, current, power-factor, torque efficiency, and apparent torque efficiency for various values of the secondary motor resistance, from $r_1 = 0.1$, the internal resistance of the motor, or $R = 0$ additional resistance to $r_1 = 5.1$

or $R = 5$ ohms additional resistance. The best values of torque efficiency are found beyond the maximum torque point.

The same Fig. 180 also shows the torque with resistance inserted into the primary circuit.

The insertion of reactance, either in the primary or in the secondary, is just as unsatisfactory as the insertion of resistance in the primary circuit.

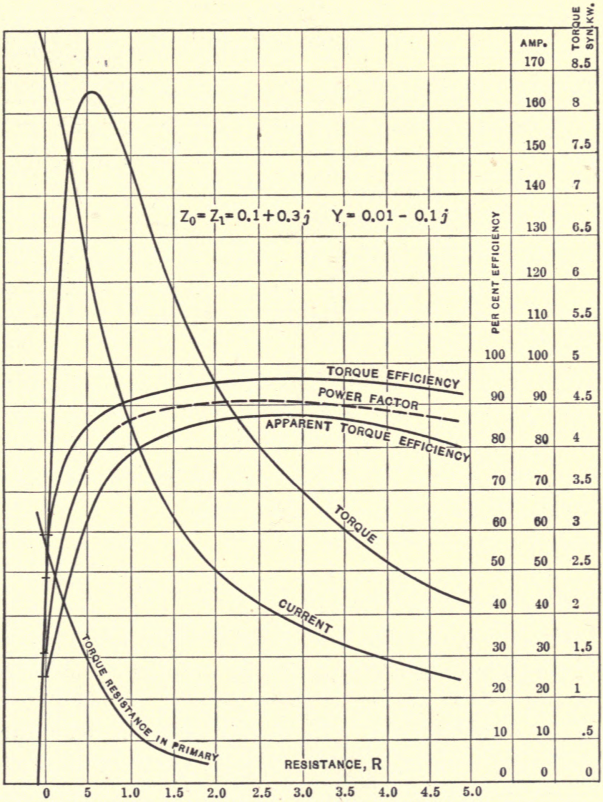


FIG. 180.—Induction motor starting torque with resistance in the secondary.

Capacity inserted in the secondary very greatly increases the torque within the narrow range of capacity corresponding to resonance with the internal reactance of the motor, and the torque which can be produced in this way is far in excess of the maximum torque of the motor when running or when starting with resistance in the secondary.

But even at its best value, the torque efficiency available with capacity in the secondary is below that available with resistance.

For further discussion of the polyphase inductance motor, see "Theory and Calculation of Alternating-current Phenomena."

III. Single-phase Induction Motor

1. Introduction

145. In the polyphase motor a number of secondary coils displaced in position from each other are acted upon by a number of primary coils displaced in position and excited by e.m.fs. displaced in phase from each other by the same angle as the displacement of position of the coils.

In the single-phase induction motor a system of secondary circuits is acted upon by one primary coil (or system of primary coils connected in series or in parallel) excited by a single alternating current.

A number of secondary circuits displaced in position must be used so as to offer to the primary circuit a short-circuited secondary in any position of the armature. If only one secondary coil is used, the motor is a synchronous induction motor and belongs to the class of reaction machines.

A single-phase induction motor will not start from rest, but when started in either direction will accelerate with increasing torque and approach synchronism.

When running at or very near synchronism, the magnetic field of the single-phase induction motor is practically identical with that of a polyphase motor, that is, can be represented by the theory of the rotating field. Thus, in a turn wound under angle τ to the primary winding of the single-phase induction motor, at synchronism an e.m.f. is generated equal to that generated in a turn of the primary winding, but differing therefrom by angle $\theta = \tau$ in time phase.

In a polyphase motor the magnetic flux in any direction is due to the resultant m.m.f. of primary and of secondary currents, in the same way as in a transformer. The same is the case in the direction of the axis of the exciting coil of the single-phase induction motor. In the direction at right angles to the axis of the exciting coil, however, the magnetic flux is due to the m.m.f. of the secondary currents alone, no primary e.m.f. acting in this direction.

Consequently, in the polyphase motor running synchronously, that is, doing no work whatever, the secondary becomes currentless, and the primary current is the exciting current of the motor only. In the single-phase induction motor, even when running light, the secondary still carries the exciting current of the magnetic flux in quadrature with the axis of the primary exciting coil. Since this flux has essentially the same intensity as the flux in the direction of the axis of the primary exciting coil, the current in the armature of the single-phase induction motor running light, and therefore also the primary current corresponding thereto, has the same m.m.f., that is, the same intensity, as the primary exciting current, and the total primary current of the single-phase induction motor running light is thus twice the exciting current, that is, it is the exciting current of the main magnetic flux plus the current producing in the secondary the exciting current of the cross magnetic flux. In reality it is slightly less, especially in small motors, due to the drop of voltage in the self-inductive impedance and the drop of quadrature magnetic flux below the impressed primary magnetic flux caused thereby. In the secondary at synchronism this secondary exciting current is a current of twice the primary frequency; at any other speed it is of a frequency equal to speed (in cycles) plus synchronism.

Thus, if in a quarter-phase motor running light one phase is open-circuited, the current in the other phase doubles. If in the three-phase motor two phases are open-circuited, the current in the third phase trebles, since the resultant m.m.f. of a three-phase machine is 1.5 times that of one phase. In consequence thereof, the total volt-ampere input of the motor remains the same and at the same magnetic density, or the same impressed e.m.f., all induction motors, single-phase as well as polyphase, consume approximately the same volt-ampere input, and the same power input for excitation, and give the same distribution of magnetic flux.

146. Since the maximum output of a single-phase motor at the same impressed e.m.f. is considerably less than that of a polyphase motor, it follows therefrom that the relative exciting current in the single-phase motor must be larger.

The cause of this cross magnetization in the single-phase induction motor near synchronism is that the secondary armature currents lag 90 deg. behind the magnetism, and are carried by the synchronous rotation 90 deg. in space before reaching their maximum, thus giving the same magnetic effect as a quarter-phase e.m.f. impressed upon the primary system in quadrature position with the main coil. Hence they can be eliminated by impressing a magnetizing quadrature e.m.f. upon an auxiliary motor circuit, as is done in the monocyclic motor.

Below synchronism, the secondary currents are carried less than 90 deg., and thus the cross magnetization due to them is correspondingly reduced, and becomes zero at standstill.

The torque is proportional to the power component of the armature currents times the intensity of magnetic flux in quadrature position thereto.

In the single-phase induction motor, the armature power currents $I'_1 = ea_1$ can exist only coaxially with the primary coil, since this is the only position in which corresponding primary currents can exist. The magnetic flux in quadrature position is proportional to the component of e carried in quadrature, or approximately to $(1 - s)\, e$, and the torque is thus

$$D = (1 - s)\, eI' = (1 - s)\, e^2 a_1,$$

thus decreases much faster with decreasing speed, and becomes zero at standstill. The power is then

$$P = (1 - s)^2\, eI' = (1 - s)^2\, e^2 a_1.$$

Since in the single-phase motor only one primary circuit but a multiplicity of secondary circuits exist, all secondary circuits are to be considered as corresponding to the same primary circuit, and thus the joint impedance of all secondary circuits must be used as the secondary impedance, at least at or near synchronism. Thus, if the armature has a quarter-phase winding of impedance Z_1 per circuit, the resultant secondary impedance is $\frac{Z_1}{2}$; if it contains a three-phase winding of impedance Z_1 per circuit, the resultant secondary impedance is $\frac{Z_1}{3}$.

In consequence hereof the resultant secondary impedance of a single-phase motor is less in comparison with the primary impedance than in the polyphase motor. Since the drop of speed under load depends upon the secondary resistance, in the single-phase induction motor the drop in speed at load is generally less than in the polyphase motor; that is, the single-phase induction motor has a greater constancy of speed than the polyphase induction motor, but just as the polyphase induction motor, it can never reach complete synchronism, but slips below synchronism, approximately in proportion to the speed.

The further calculation of the single-phase induction motor is identical with that of the polyphase induction motor, as given in the previous chapter.

Often no special motors are used for single-phase circuits, but polyphase motors adapted thereto. An induction motor with only one primary winding could not be started by a phase-splitting device, and would necessarily be started by external means. A polyphase motor, as for instance a three-phase motor operating single-phase, by having two of its terminals connected to the single-phase mains, is just as satisfactory a single-phase motor as one built with only one primary winding. The only difference is that in the latter case a part of the circumference of the primary structure is left without winding, while in the polyphase motor this part contains windings also, which, however, are not used, or are not effective when running as single-phase motor, but are necessary when starting by means of displaced e.m.fs. Thus, in a three-phase motor operating from single-phase mains, in starting, the third terminal is connected to a phase-displacing device, giving to the motor the cross magnetization in quadrature to the axis of the primary coil, which at speed is produced by the rotation of the secondary currents, and which is necessary for producing the torque by its action upon the secondary power currents.

Thus the investigation of the single-phase induction motor resolves itself into the investigation of the polyphase motor operating on single-phase circuits.

2. Load and Speed Curves

147. Comparing thus a three-phase motor of exciting admittance per circuit $Y = g - jb$ and self-inductive impedances $Z_0 = r_0 + jx_0$ and $Z_1 = r_1 + jx_1$ per circuit with the same motor operating as single-phase motor from one pair of terminals, the single-phase exciting admittance is $Y' = 3\,Y$ (so as to give the same volt-amperes excitation $3\,eY$), the primary self-inductive impedance is the same, $Z_0 = r_0 + jx_0$; the secondary self-inductive impedance single-phase, however, is only $Z'_1 = \frac{Z_1}{3}$, since all three secondary circuits correspond to the same primary circuit, and thus the total impedance single-phase is $Z' = Z_0 + \frac{Z_1}{3}$, while that of the three-phase motor is $Z = Z_0 + Z_1$.

Assuming approximately $Z_0 = Z_1$, we have

$$Z' = \frac{2Z}{3}.$$

Thus, in absolute value,

$$y' = 3y,$$
$$z' = \tfrac{2}{3} z, \text{ and}$$
$$\gamma' = 2\gamma;$$

that is, the characteristic constant of a motor running single-phase is twice what it is running three-phase, or polyphase in

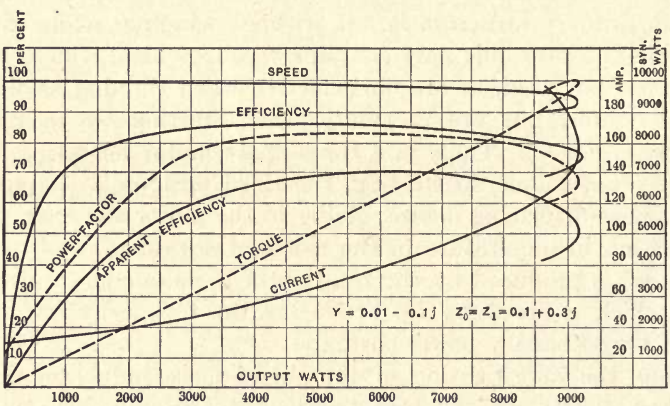


FIG. 181.—Three-phase induction motor on single-phase circuit, load curves.

general; hence, the ratio of exciting current to current at standstill, or of waste flux to useful flux, is doubled by changing from polyphase to single-phase.

This explains the inferiority of the single-phase motor compared with the polyphase motor.

As a rule, an average polyphase motor makes a poor single-phase motor, and a good single-phase motor must be an excellent polyphase motor.

As instances are shown in Figs. 181 and 182 the load curves and speed curves of the three-phase motor of which the curves of one circuit are given in Figs. 176 and 177, having the following constants:

Three-phase	$e_0 = 110$	Single-phase
$Y = 0.01 - 0.1j,$		$Y = 0.03 - 0.3j,$
$Z_0 = 0.1 + 0.3j,$		$Z_0 = 0.1 + 0.3j,$
$Z_1 = 0.1 + 0.3j,$		$Z_1 = 0.033 + 0.1j,$
Thus, $\gamma = 6.36.$		Thus, $\gamma = 12.72.$

It is of interest to compare Fig. 181 with Fig. 176 and to note the lesser drop of speed (due to the relatively lower secondary

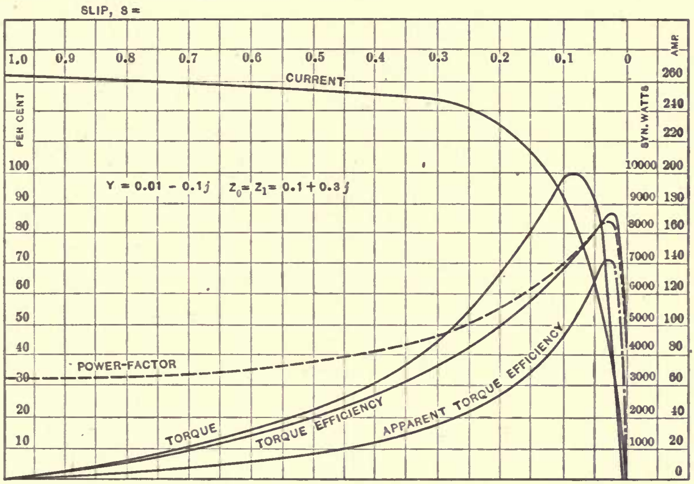


Fig. 182.—Three-phase induction motor on single-phase circuit, speed curves.

resistance) and lower power-factor and efficiencies, especially at light load. The maximum output is reduced from 3 × 7000 = 21,000 in the three-phase motor to 9100 watts in the single-phase motor.

Since, however, the internal losses are less in the single-phase motor, it can be operated at from 25 to 30 per cent. higher magnetic density than the same motor polyphase, and in this case its output is from two-thirds to three-quarters that of the polyphase motor.

148. The preceding discussion of the single-phase induction motor is approximate, and correct only at or near synchronism, where the magnetic field is practically a uniformly rotating field of constant intensity, that is, the quadrature flux produced by the armature magnetization equal to the main magnetic flux produced by the impressed e.m.f.

If an accurate calculation of the motor at intermediate speed and at standstill is required, the changes of effective exciting admittance and of secondary impedance, due to the decrease of the quadrature flux, have to be considered.

At synchronism the total exciting admittance gives the m.m.f. of main flux and auxiliary flux, while at standstill the quadrature flux has disappeared or decreased to that given by the starting device, and thus the total exciting admittance has de-

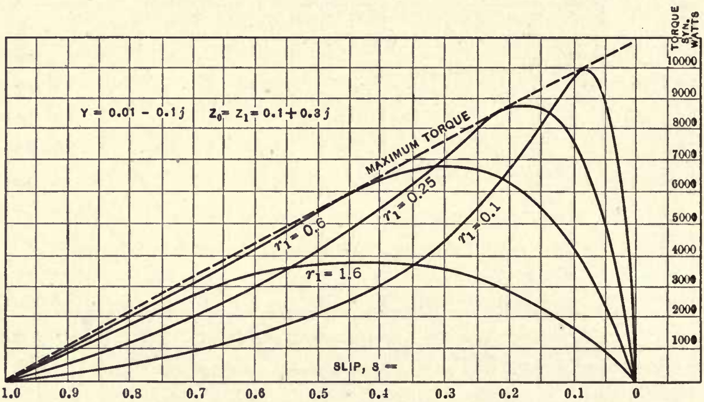


FIG. 183.—Three-phase induction motor on single-phase circuit, torque curves.

creased to one-half of its synchronous value, or one-half plus the exciting admittance of the starting flux.

The effective secondary impedance at synchronism is the joint impedance of all secondary circuits; at standstill, however, only the joint impedance of the projections of the secondary coils on the direction of the main flux, that is, twice as large as at synchronism. In other words, from standstill to synchronism the effective secondary impedance gradually decreases to one-half its standstill value at synchronism.

For fuller discussion hereof the reader must be referred to my second paper on the Single-phase Induction Motor, Transactions A. I. E. E., 1900, page 37.

The torque in Fig. 182 obviously slopes toward zero at standstill. The effect of resistance inserted in the secondary of the single-phase motor is similar to that in the polyphase motor in so far as an increase of resistance lowers the speed at which the maximum torque takes place. While, however, in the polyphase motor the maximum torque remains the same, and merely shifts toward lower speed with the increase of resistance, in the single-phase motor the maximum torque decreases proportionally to the speed at which the maximum torque point occurs, due to the factor $(1 - s)$ entering the equation of the torque,

$$D = e^2 a_1 (1 - s).$$

Thus, in Fig. 183 are given the values of torque of the single-phase motor for the same conditions and the same motor of which the speed curves polyphase are given in Fig. 179.

The maximum value of torque which can be reached at any speed lies on the tangent drawn from the origin onto the torque curve for $r_1 = 0.1$ or short-circuited secondary. At low speeds the torque of the single-phase motor is greatly increased by the insertion of secondary resistance, just as in the polyphase motor.

3. Starting Devices of Single-phase Motors

149. At standstill, the single-phase induction motor has no starting torque, since the line of polarization due to the secondary currents coincides with the axis of magnetic flux impressed by the primary circuit. Only when revolving is torque produced, due to the axis of secondary polarization being shifted by the rotation, against the axis of magnetism, until at or near synchronism it is in quadrature therewith, and the magnetic disposition thus the same as that of the polyphase induction motor.

Leaving out of consideration starting by mechanical means and starting by converting the motor into a series or shunt motor, that is, by passing the alternating current by means of commutator and brushes through both elements of the motor, the following methods of starting single-phase motors are left:

1st. Shifting of the axis of armature or secondary polarization against the axis of generating magnetism.

2d. Shifting the axis of magnetism, that is, producing a magnetic flux displaced in position from the flux producing the armature currents.

The first method requires a secondary system which is unsymmetrical in regard to the primary, and thus, since the secondary is movable, requires means of changing the secondary circuit, such as commutator brushes short-circuiting secondary coils in the position of effective torque, and open-circuiting them in the position of opposing torque.

Thus this method leads to the repulsion motor, which is a commutator motor also.

With the commutatorless induction motor, or motor with permanently closed armature circuits, all starting devices consist in establishing an auxiliary magnetic flux in phase with the secondary currents in time, and in quadrature with the line of secondary polarization in space. They consist in producing a component of magnetic flux in quadrature in space with the primary magnetic flux producing the secondary currents, and in phase with the latter, that is, in time quadrature with the primary magnetic flux.

Thus, if

F_p = polarization due to the secondary currents,

Φ_a = auxiliary magnetic flux,

θ = phase displacement in time between Φ_a and Φ_p,

and

τ = phase displacement in space between Φ_a and F_p,

the torque is

$$D = F_p \Phi_a \sin \tau \cos \theta.$$

In general the starting torque, apparent torque efficiency, etc., of the single-phase induction motor with any of these devices are given in per cent. of the corresponding values of the same motor with polyphase magnetic flux, that is, with a magnetic system consisting of two equal magnetic fluxes in quadrature in time and space.

150. The infinite variety of arrangements proposed for starting single-phase induction motors can be grouped into three classes.

1. *Phase-splitting Devices.* The primary system is composed of two or more circuits displaced from each other in position, and combined with impedances of different inductance factors so as to produce a phase displacement between them.

When using two motor circuits, they can either be connected in series between the single-phase mains, and shunted with impedances of different inductance factors, as, for instance, a condensance and an inductance, or they can be connected in shunt between the single-phase mains but in series with impedances of different inductance factors. Obviously the impedances used for displacing the phase of the exciting coils can either be external or internal, as represented by high-resistance winding in one coil of the motor, etc.

In this class belongs the use of the transformer as a phase-splitting device by inserting a transformer primary in series with one motor circuit in the main line and connecting the other motor circuit to the secondary of the transformer, or by feeding one of the motor circuits directly from the mains and the other from the secondary of a transformer connected across the mains with its primary. In either case it is, respectively, the internal impedance, or internal admittance, of the transformer which is combined with one of the motor circuits for displacing its phase, and thus this arrangement becomes most effective by using transformers of high internal impedance or admittance as constant power transformers or open magnetic circuit transformers.

2. *Inductive Devices.* The motor is excited by the combination of two or more circuits which are in inductive relation to each other. This mutual induction between the motor circuits can take place either outside of the motor in a separate phase-splitting device or in the motor proper.

In the first case the simplest form is the divided circuit whose branches are inductively related to each other by passing around the same magnetic circuit external to the motor.

In the second case the simplest form is the combination of a primary exciting coil and a short-circuited secondary coil on the primary member of the motor, or a secondary coil closed by an impedance.

In this class belong the shading coil and the accelerating coil.

3. *Monocyclic Starting Devices.* An essentially wattless e.m.f. of displaced phase is produced outside of the motor, and used to energize a cross magnetic circuit of the motor, either directly by a special teaser coil on the motor, or indirectly by combining this wattless e.m.f. with the main e.m.f. and thereby deriving a system of e.m.fs. of approximately three-phase or any other relation. In this case the primary system of the motor is supplied essentially by a polyphase system of e.m.fs. with a single-phase flow of energy, a system which I have called "monocyclic."

The wattless quadrature e.m.f. is generally produced by connecting two impedances of different inductance factors in series between the single-phase mains, and joining the connection between the two impedances to the third terminal of a three-phase induction motor, which is connected with its other two terminals to the single-phase lines, as shown diagrammatically in Fig. 184, for a conductance a and an inductive susceptance $-ja$.

This starting device, when using an inductance and a condensance of proper size, can be made to give an apparent starting torque efficiency superior to that of the polyphase induction motor. Usually a resistance and an inductance are used, which, though not giving the same starting torque efficiency as available by the use of a condensance, have the advantage of greater simplicity and cheapness. After starting, the impedances are disconnected.

For a complete discussion and theoretical investigation of the

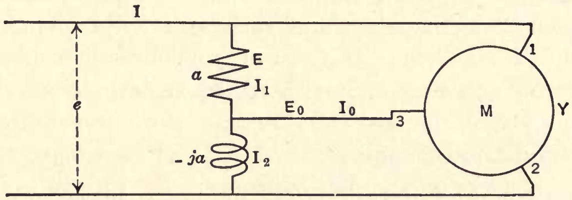


FIG. 184.—Connections for starting single-phase motor.

different starting devices, the reader must be referred to the paper on the single-phase induction motor, American Institute of Electrical Engineers' Transactions, February, 1898."

151. The use of the resistance-inductance, or monocyclic, starting device with three-phase wound induction motor will be discussed somewhat more explicitly as the only method not using condensers which has found extensive commercial application. It gives relatively the best starting torque and torque efficiencies.

In Fig. 184, M represents a three-phase induction motor of which two terminals, 1 and 2, are connected to single-phase mains and the terminal 3 to the common connection of a conductance $a\left(\text{that is, a resistance } \frac{1}{a}\right)$ and an equal susceptance $-ja$ $\left(\text{thus a reactance } + \frac{j}{a}\right)$ connected in series across the mains.

Let $Y = g - jb$ = total admittance of motor between terminals 1 and 2 while at rest. We then have $\frac{4}{3} Y$ = total admittance from terminal 3 to terminals 1 and 2, regardless of whether the motor is delta- or Y-wound.

If e = e.m.f. in the single-phase mains and $\dot{E}$ = difference of potential across conductance a of the starting device, then we have the current in a as $\dot{I}_1 = \dot{E}a,$
and the e.m.f. across $-ja$ as $e - \dot{E};$
thus, the current in $-ja$ is

$$\dot{I}_2 = -ja\,(e - \dot{E}),$$

and the current in the cross magnetizing motor circuit from 3 to 1, 2 is

$$\dot{I}_0 = \dot{I}_1 - \dot{I}_2 = \dot{E}a + ja\,(e - \dot{E}).$$

The e.m.f. $\dot{E}_0$ of the cross magnetizing circuit is, as may be seen from the diagram of e.m.fs., which form a triangle with $e, \dot{E}$ and $e - \dot{E}$ as sides,

$$\dot{E}_0 = \dot{E} - (e - \dot{E}) = 2\,\dot{E} - e,$$

and since

$$\dot{I}_0 = \tfrac{4}{3}\,Y\dot{E}_0,$$

we have

$$\dot{E}a + ja\,(e - \dot{E}) = \tfrac{4}{3}\,Y\,(2\dot{E} - e).$$

This expression solved for $\dot{E}$ becomes

$$\dot{E} = +\,e\,\frac{3\,ja + 4\,Y}{3\,a + 3\,ja - 8\,Y},$$

which from the foregoing value of $\dot{E}_0$ gives

$$\dot{E}_0 = -\,\frac{3\,ea\,(j + 1)}{3a - 3\,ja - 8\,Y};$$

or, substituting

$$Y = g - jb,$$

expanding, and multiplying both numerator and denominator by

$$(3\,a - 8\,g) + j\,(3\,a - 8\,b),$$

gives
$$\dot{E}_0 = ea\,\frac{\tfrac{8}{3}\,(g - b) - j\left(2a - \dfrac{8\,(g + b)}{3}\right)}{(a - \tfrac{8}{3}\,g)^2 + (a - \tfrac{8}{3}\,b)^2},$$

and the imaginary component thereof, or e.m.f. in quadrature to e in time and in space, is

$$\dot{E}_0{}^j = jea\,\frac{2\,a - \tfrac{8}{3}\,(g + b)}{(a - \tfrac{8}{3}\,g)^2 + (a - \tfrac{8}{3}\,b)^2}.$$

In the same motor on a three-phase circuit this quadrature e.m.f. is the altitude of the equilateral triangle with e as sides, thus $= - je\frac{\sqrt{3}}{2}$, and since the starting torque of the motor is proportional to this quadrature e.m.f., the relative starting torque of the monocyclic starting device, or the ratio of starting torque of the motor with monocyclic starting device to that of the same motor on three-phase circuit, is

$$D' = j\frac{E_0^j}{\frac{e\sqrt{3}}{2}} = \frac{2a}{\sqrt{3}}\frac{2a - 8/3(g - b)}{(a - 8/3\, g)^2 + (a - 8/3\, b)^2}.$$

A starting device which has been extensively used is the condenser in the tertiary circuit. In its usual form it can be considered as a modification of the monocyclic starting device, by using a condensance as the one impedance and making the other impedance infinite, that is, omitting it. It thus comprises a three-phase induction motor, in which two terminals are connected to the single-phase supply and the third terminal and one of the main terminals to a condenser. Usually the condenser is left in circuit after starting, and made of such size that its leading current compensates for the lagging magnetizing current of the motor, and the motor thus gives approximately unity power-factor.

For further discussion of this subject the reader is referred to the paper on "Single-phase Induction Motors," mentioned above, and to the "Theory and Calculation of Alternating-current Phenomena" and "Theory and Calculation of Electrical Apparatus."

4. Acceleration with Starting Device

152. The torque of the single-phase induction motor (without a starting device) is proportional to the product of main flux, or magnetic flux produced by the primary impressed e.m.f., and the speed. Thus it is the same as in the polyphase motor at or very near synchronism, but falls off with decreasing speed and becomes zero at standstill.

To produce a starting torque, a device has to be used to impress an auxiliary magnetic flux upon the motor, in quadrature with the main flux in time and in space, and the starting torque is proportional to this auxiliary or quadrature flux. During acceleration or at intermediate speed the torque of the motor is the resultant of the main torque, or torque produced by the primary main flux, and the auxiliary torque produced by the auxiliary quadrature or starting flux. In general, this resultant torque is not the sum of main and auxiliary torque, but often less, due to the interaction between the motor and the starting device.

Most starting devices depend more or less upon the total admittance of the motor and its power-factor. With increasing speed, however, the total admittance of the motor decreases and its power-factor increases, and an auxiliary torque device suited for the admittance of the motor at standstill will not be suited for the changed admittance at speed.

The currents produced in the secondary by the main or primary magnetic flux are carried by the rotation of the motor more or less into quadrature position, and thus produce the quadrature flux giving the main torque as discussed before.

This quadrature component of the main flux generates an e.m.f. in the auxiliary circuit of the starting device, and thus changes the distribution of currents and e.m.fs. in the starting device. The circuits of the starting device then contain, besides the motor admittance and external admittance, an active counter e.m.f., changing with the speed. Inversely, the currents produced by the counter e.m.f. of the motor in the auxiliary circuit react upon the counter e.m.f., that is, upon the quadrature component or main flux, and change it.

Thus during acceleration we have to consider—

1. The effect of the change of total motor admittance and its power-factor upon the starting device.

2. The effect of the counter e.m.f. of the motor upon the starting device and the effect of the starting device upon the counter e.m.f. of the motor.

1. The total motor admittance and its power-factor change very much during acceleration in motors with short-circuited low-resistance secondary. In such motors the admittance at rest is very large and its power-factor low, and with increasing speed the admittance decreases and its power-factor increases greatly. In motors with short-circuited high-resistance secondary the admittance also decreases greatly during acceleration, but its power-factor changes less, being already high at standstill. Thus the starting device will be affected less. Such motors, however, are inefficient at speed. In motors with variable secondary resistance the admittance and its power-factor

can be maintained constant during acceleration by decreasing the resistance of the secondary circuit in correspondence with the increasing counter e.m.f. Hence, in such motors the starting device is not thrown out of adjustment by the changing admittance during acceleration.

In the phase-splitting devices, and still more in the inductive devices, the starting torque depends upon the internal or motor admittance, and is thus essentially affected by the change of admittance during acceleration, and by the appearance of a counter e.m.f. during acceleration, which throws the starting device out of its proper adjustment, so that in some cases while a considerable torque exists at standstill, this torque becomes zero and then reverses at some intermediate speed, and the motor, while starting with fair torque, is not able to run up to speed with the starting device in circuit. Especially is this the case where capacity is used in the starting device. With the monocyclic starting this effect is small in any case and absent when a condenser is used in the tertiary circuit, and therefore the latter may advantageously be left in the circuit at speed.

IV. Induction Generator

1. Introduction

153. In the range of slip from $s = 0$ to $s = 1$, that is, from synchronism to standstill, torque, power output, and power input of the induction machine are positive, and the machine thus acts as a motor, as discussed before.

Substituting, however, in the equations in paragraph 1 for s values > 1, corresponding to backward rotation of the machine, the power input remains positive, the torque also remains positive, that is, in the same direction as for $s < 1$; but since the speed $(1 - s)$ becomes negative or in opposite direction, the power output is negative, that is, the torque in opposite direction to the speed. In this case the machine consumes electrical energy in its primary and mechanical energy by a torque opposing the rotation, thus acting as brake.

The total power, electrical as well as mechanical, is consumed by internal losses of the motor. Since, however, with large slip in a low-resistance motor the torque and power are small, the braking power of the induction machine at backward rotation is, as a rule, not considerable, excepting when using high resistance in the armature circuit.

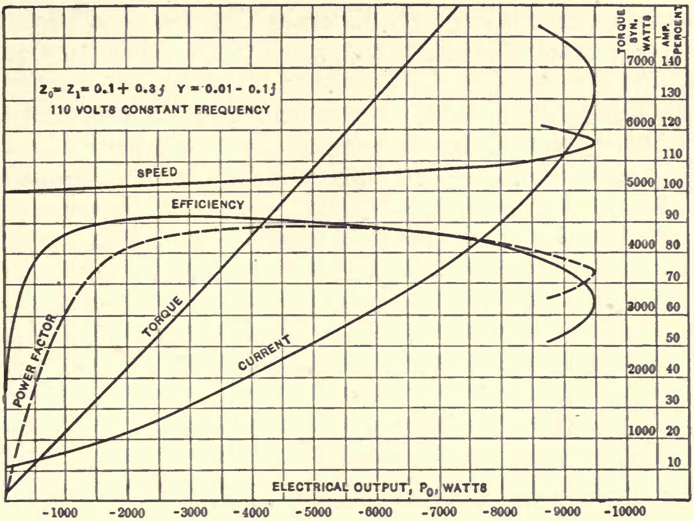


FIG. 185.—Induction generator load curves.

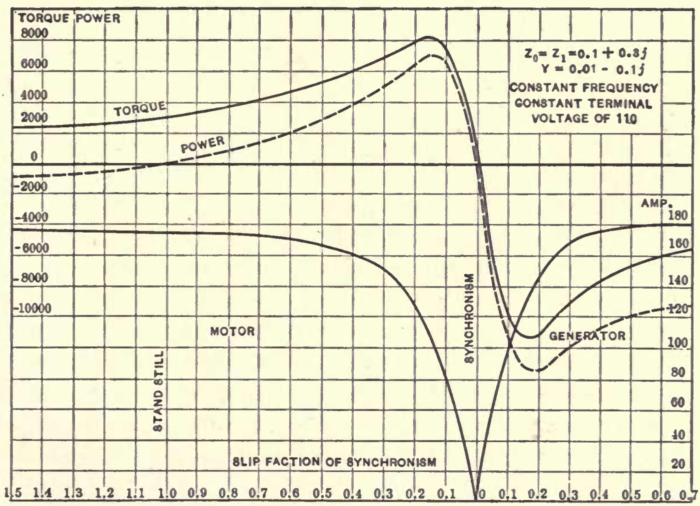


FIG. 186.—Induction machine speed curves.

Substituting for s negative values, corresponding to a speed above synchronism, torque and power output and power input become negative, and a load curve can be plotted for the induction generator which is very similar, but the negative counterpart of the induction motor load curve. It is for the machine shown as motor in Fig. 176 given as Fig. 185, while Fig. 186 gives the complete speed curve of this machine from $s = 1.5$ to $s = -1$.

The generator part of the curve, for $s < 0$, is of the same character as the motor part, $s > 0$, but the maximum torque and maximum output of the machine as generator are greater than as motor.

Thus an induction motor when speeded up above synchronism acts as a powerful brake by returning energy into the lines, and the maximum braking effort and also the maximum electric power returned by the machine will be greater than the maximum motor torque or output.

2. Constant-speed Induction or Asynchronous Generator

154. The curves in Fig. 185 are calculated at constant frequency f, and thus to vary the output of the machine as generator the speed has to be increased. This condition may be realized in case of induction generators running in parallel with synchronous generators under conditions where it is desirable that the former should take as much load as its driving power permits; as, for instance, if the induction generator is driven by a water power while the synchronous generator is driven by a steam engine. In this case the control of speed would be effected on the synchronous generator, and the induction generator be without speed-controlling devices, running up beyond synchronous speed as much as required to consume the power supplied to it.

Conversely, however, if an induction machine is driven at constant speed and connected to a suitable circuit as load, the frequency given by the machine will not be synchronous with the speed, or constant at all loads, but decreases with increasing load from practically synchronism at no load, and thus for the induction generator at constant speed a load curve can be constructed as shown in Fig. 187, giving the decrease of frequency with increasing load in the same manner as the speed of the induction motor at constant frequency decreases with the load. In the calculation of these induction generator curves for constant speed the change of frequency with the load has obviously to be considered, that is, in the equations the reactance x_0 has to be replaced by the reactance $x_0(1 - s)$, otherwise the equations remain the same.

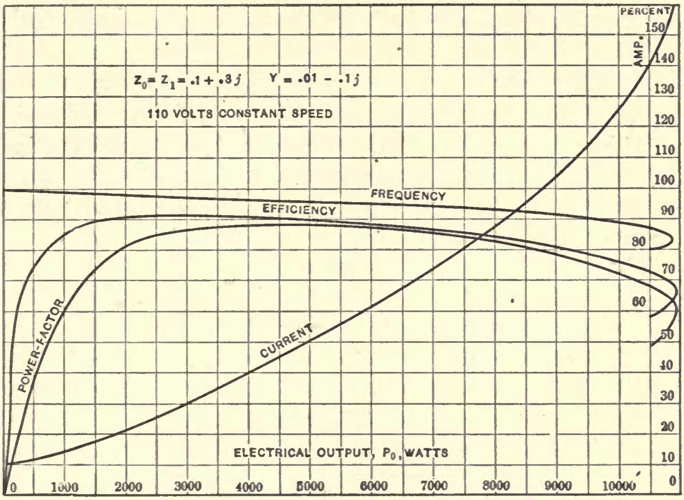


FIG. 187.—Induction generator load curves.

3. Power-factor of Induction Generator

155. The induction generator differs essentially from a synchronous alternator (that is, a machine in which an armature revolves relatively through a constant or continuous magnetic field) by having a power-factor requiring leading current; that is, in the synchronous alternator the phase relation between current and terminal voltage depends entirely upon the external circuit, and according to the nature of the circuit connected to the synchronous alternator the current can lag or lead the terminal voltage or be in phase therewith. In the induction or asynchronous generator, however, the current must lead the terminal voltage by the angle corresponding to the load and voltage of the machine, or, in other words, the phase relation between current and voltage in the external circuit must be such as required by the induction generator at that particular load.

Induction generators can operate only on circuits with leading current or circuits of negative effective reactance.

In Fig. 188 are given for the constant-speed induction generator in Fig. 230 as function of the impedance of the external circuit $z = \frac{e_0}{i_0}$ as abscissas (where e_0 = terminal voltage, i_0 = current in external circuit), the leading power-factor $p = \cos\theta$ required in the load, the inductance factor $q = \sin\theta$, and the frequency.

Hence, when connected to a circuit of impedance z this induction generator can operate only if the power-factor of its circuit is p; and if this is the case the voltage is indefinite, that is, the circuit unstable, even neglecting the impossibility of securing exact equality of the power-factor of the external circuit with that of the induction generator.

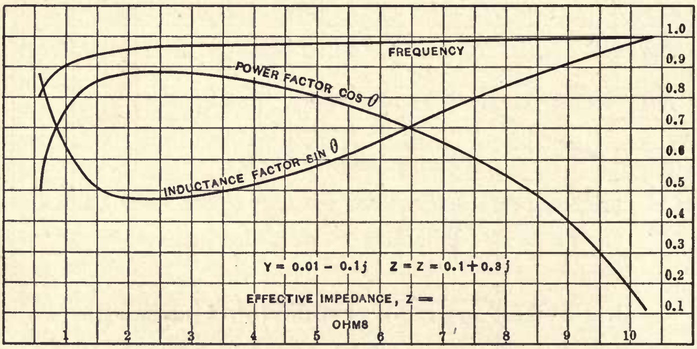


Fig. 188.—Three-phase induction generator power factor and inductance factor of external circuit.

Two possibilities thus exist with such an induction generator circuit.

1st. The power-factor of the external circuit is constant and independent of the voltage, as when the external circuit consists of resistances, inductances, and capacities.

In this case if the power-factor of the external circuit is higher than that of the induction generator, that is, the leading current less, the induction generator fails to excite and generate. If the power-factor of the external circuit is lower than that of the induction generator, the latter excites and its voltage rises until by saturation of its magnetic circuit and the consequent increase of exciting admittance, that is, decrease of internal power-factor, its power-factor has fallen to equality with that of the external circuit.

In this respect the induction generator acts like the direct-current shunt generator, and gives load characteristics very similar to those of the direct-current shunt generator as discussed in B; that is, it becomes stable only at saturation, but

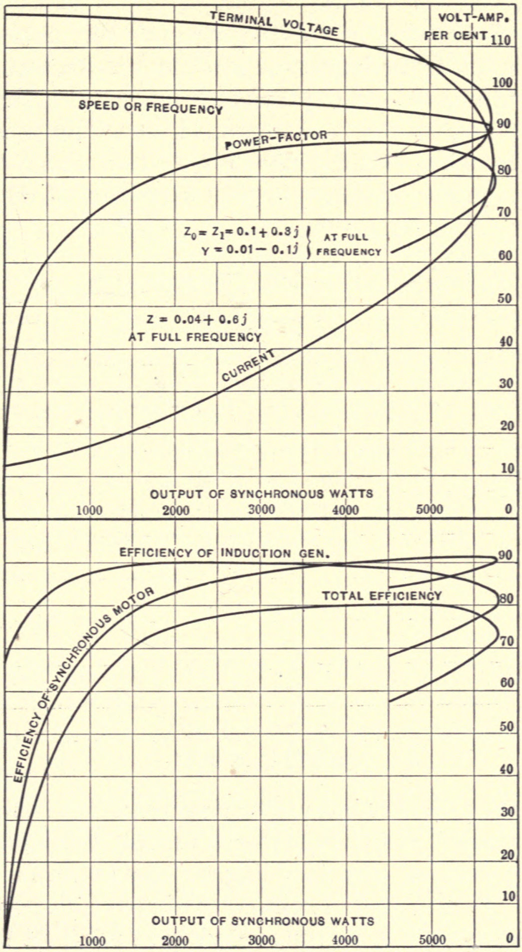


FIG. 189.—Induction generator and synchronous motor load curves.

loses its excitation and thus drops its load as soon as the voltage falls below saturation.

Since, however, the field of the induction generator is alternating, it is usually not feasible to run at saturation, due to excessive hysteresis losses, except for very low frequencies.

2d. The power-factor of the external circuit depends upon the voltage impressed upon it.

This, for instance, is the case if the circuit consists of a synchronous motor or contains synchronous motors or synchronous converters.

In the synchronous motor the current is in phase with the impressed e.m.f. if the impressed e.m.f. equals the counter e.m.f. of the motor plus the internal loss of voltage. It is leading if the impressed e.m.f. is less, and lagging if the impressed e.m.f. is more. Thus when connecting an induction generator with a synchronous motor, at constant field excitation of the latter the

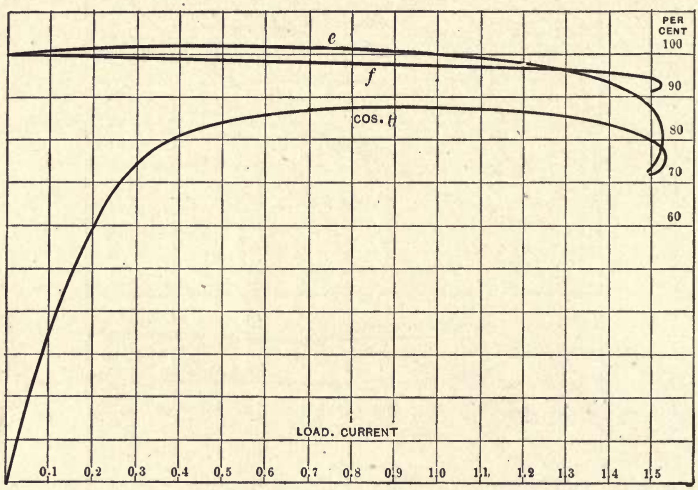


FIG. 190.—Induction generator and synchronous converter, phase control, no line impedance.

voltage of the induction generator rises until it is as much below the counter e.m.f. of the synchronous motor as required to give the leading current corresponding to the power-factor of the generator. Thus a system consisting of a constant-speed induction generator and a synchronous motor at constant field excitation is absolutely stable. At constant field excitation of the synchronous motor, at no load the synchronous motor runs practically at synchronism with the induction generator, with a terminal voltage slightly below the counter e.m.f. of the synchronous motor. With increase of load the frequency and thus the speed of the synchronous motor drops, due to the slip of frequency in the induction generator, and the voltage drops, due to the increase of leading current required and the decrease of counter e.m.f. caused by the decrease of frequency.

By increasing the field excitation of the synchronous motor with increase of load, obviously the voltage of the generator can be maintained constant, or even increased with the load.

When running from an induction generator, a synchronous motor gives a load curve very similar to the load curve of an induction motor running from a synchronous generator; that is, a magnetizing current at no load and a speed gradually decreasing with the increase of load up to a maximum output point, at which the speed curve bends sharply down, the current curve upward, and the motor drops out of step.

The current, however, in the case of the synchronous motor operated from an induction generator is leading, while it is lagging in an induction motor operated from a synchronous generator. In either case it demagnetizes the synchronous machine and magnetizes the induction machine, that is, the synchronous machine supplies magnetization to the induction machine.

In Fig. 189 is shown the load curve of a synchronous motor operated from the induction generator in Fig. 187.

In Fig. 190 is shown the load curve of an over-compounded synchronous converter operated from an induction generator, the over-compounding being such as to give approximately constant terminal voltage *e*.

156. Obviously when operating a self-exciting synchronous converter from an induction generator the system is unstable also; if both machines are below magnetic saturation, since in this case in both machines the generated e.m.f. is proportional to the field excitation and the field excitation proportional to the voltage; that is, with an unsaturated induction generator the synchronous converter operated therefrom must have its magnetic field excited to a density above the bend of the saturation curve.

Since the induction generator requires for its operation a circuit with leading current varying with the load in the manner determined by the internal constants of the motor, to make an induction or asynchronous generator suitable for operation on a general alternating-current circuit, it is necessary to have a synchronous machine as exciter in the circuit consuming leading current, that is, supplying the required lagging or magnetizing current to the induction generator; and in this case the voltage of the system is controlled by the field excitation of the synchronous machine, that is, its counter e.m.f. Either a synchronous motor of suitable size running light can be used herefor as exciter of the induction generator, or the exciting current of the induction generator may be derived from synchronous motors or converters in the same system, or from synchronous alternating-current generators operated in parallel with the induction generator, in which latter case, however, these currents can be said to come from the synchronous alternator as lagging currents. Electrostatic condensers, as an underground cable system, may also be used for excitation, but in this case besides the condensers a synchronous machine or other means is required to secure stability.

The induction machine may thus be considered as *consuming* a lagging reactive magnetizing current at all speeds, and *consuming* a power current below synchronism, as motor, *supplying* a power current (that is, consuming a negative power current) above synchronism, as generator.

Therefore, induction generators are best suited for circuits which normally carry leading currents, as synchronous motor and synchronous converter circuits, but less suitable for circuits with lagging currents, since in the latter case an additional synchronous machine is required, giving all the lagging currents of the system plus the induction generator exciting current.

Obviously, when running induction generators in parallel with a synchronous alternator no synchronizing is required, but the induction generator takes a load corresponding to the excess of its speed over synchronism, or conversely, if the driving power behind the induction generator is limited, no speed regulation is required, but the induction generator runs at a speed exceeding synchronism by the amount required to consume the driving power.

The foregoing consideration obviously applies to the polyphase induction generator as well as to the single-phase induction generator, the latter, however, requiring a larger exciter in consequence of its lower power-factor. Therefore, even in a single-phase induction generator, preferably polyphase excitation is used, that is, the induction machine and its synchronous exciter wound as polyphase machines, but the load connected to one phase only of the induction machine. The curves shown in the preceding apply to the machine as polyphase generator.

The effect of resistance in the secondary is essentially the same in the induction generator as in the induction motor. An increase of resistance increases the slip, that is, requires an increase of speed at the same torque, current, and output, and thus correspondingly lowers the efficiency.

Induction generators have been proposed and used to some extent for high-speed prime movers, as steam turbines, since their squirrel-cage rotor appears mechanically better suited for very high speeds than the revolving field of the synchronous generator.

The foremost use of induction generators will probably be for collecting small water powers in one large system, due to the far greater simplicity, reliability, and cheapness of a small induction generator station feeding into a big system compared with a small synchronous generator station. The induction generator station requires only the hydraulic turbine, the induction machine, and the step-up transformer, but does not even require a turbine governor, and so needs practically no attention, as the control of voltage, speed, and frequency takes place by a synchronous generator or motor main station, which collects the power while the individual induction generator stations feed into the system as much power as the available water happens to supply.

The synchronous induction motor, comprising a single-phase or polyphase primary and a single-phase secondary, tends to drop into synchronism and then operates essentially as reaction machine. A number of types of synchronous induction generators have been devised, either with commutator for excitation or without commutator and with excitation by low-frequency synchronous or commutating machine, in the armature, or by high-frequency excitation. For particulars regarding these very interesting machines, see "Theory and Calculation of Alternating-current Phenomena."

V. Induction Booster

157. In the induction machine, at a given slip s, current and terminal voltage are proportional to each other and of constant phase relation, and their ratio is a constant. Thus when connected in an alternating-current circuit, whether in shunt or in series, and held at a speed giving a constant and definite slip s, either positive or negative, the induction machine acts like a constant impedance.

The apparent impedance and its components, the apparent resistance and apparent reactance represented by the induction machine, vary with the slip. At synchronism apparent impedance, resistance, and reactance are a maximum. They decrease with increasing positive slip. With increasing negative slip the apparent impedance and reactance decrease also, the apparent

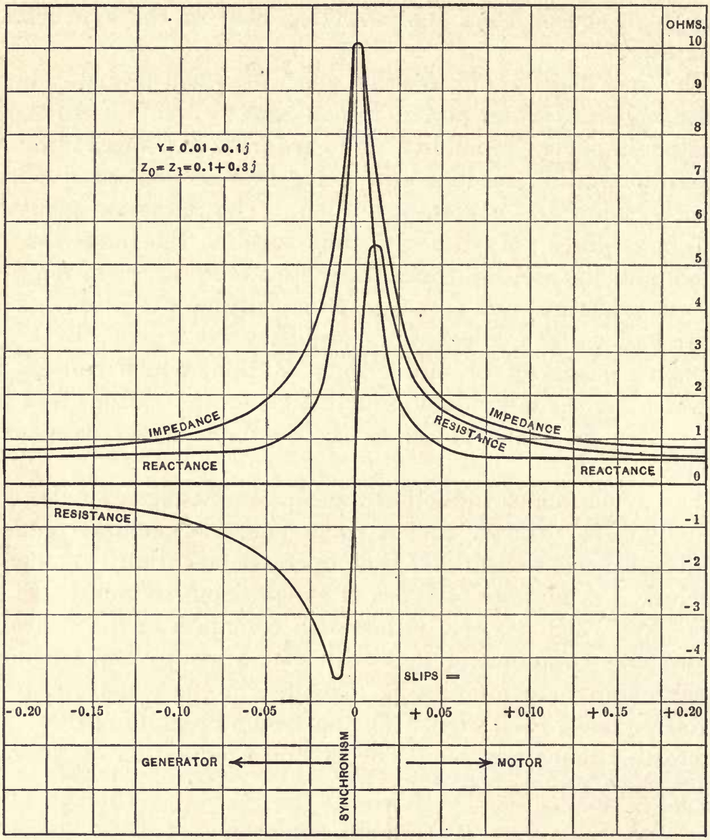


FIG. 191.—Effective impedance of three-phase induction machine.

resistance decreases to zero and then increases again in negative direction as shown in Fig. 191, which gives the apparent impedance, resistance, and reactance of the machine shown in Figs. 176 and 177, etc., with the speed as abscissas.

The cause is that the power current is in opposition to the terminal voltage above synchronism, and thereby the induction machine behaves as an impedance of negative resistance, that is, adding a power e.m.f. into the circuit proportional to the current.

As may be seen herefrom, the induction machine when inserted in series in an alternating-current circuit can be used as a booster, that is, as an apparatus to generate and insert in the circuit an e.m.f. proportional to the current, and the amount of the boosting effect can be varied by varying the speed, that is, the slip at which the induction machine is revolving. Above synchronism the induction machine boosts, that is, raises the voltage; below synchronism it lowers the voltage; in either case also adding an out-of-phase e.m.f. due to its reactance. The greater the slip, either positive or negative, the less is the apparent resistance, positive or negative, of the induction machine.

The effect of resistance inserted in the secondary of the induction booster is similar to that in the other applications of the induction machine; that is, it increases the slip required for a certain value of apparent resistance, thereby lowering the efficiency of the apparatus, but at the same time making it less dependent upon minor variations of speed; that is, requires a lesser constancy of slip, and thus of speed and frequency, to give a steady boosting effect.

VI. Phase Converter

158. It may be seen from the preceding that the induction machine can operate equally well as motor, below synchronism, and as generator, above synchronism.

In the single-phase induction machine the motor or generator action occurs in one primary circuit only, but in the direction in quadrature to the primary circuit there is a mere magnetizing current either in the secondary, in the single-phase motor proper, or in an auxiliary field-circuit, in the monocyclic motor.

The motor and generator action can occur, however, simultaneously in the same machine, some of the primary circuits acting as motor, others as generator circuits. Thus, if one of the two circuits of a quarter-phase induction machine is connected to a single-phase system, in the second circuit an e.m.f. is generated in quadrature with and equal to the generated e.m.f. in the first circuit; and this e.m.f. can thus be utilized to produce currents which, with currents taken from the primary single-phase mains, give a quarter-phase system. Or, in a three-phase motor connected with two of its terminals to a single-phase system, from the third terminal an e.m.f. can be derived which, with the single-phase system feeding the induction machine, combines to a three-phase system. The induction machine in this application represents a phase converter.

The phase converter obviously combines the features of a single-phase induction motor with those of a double transformer, transformation occurring from the primary or motor circuit to the secondary or armature, and from the secondary to the tertiary or generator circuit.

Thus, in a quarter-phase motor connected to single-phase mains with one of its circuits, if

$Y = g - jb$ = primary polyphase exciting admittance,

$Z_0 = r_0 + jx_0$ = self-inductive impedance per primary or tertiary circuit,

$Z_1 = r_1 + jx_1$ = resultant single-phase self-inductive impedance of secondary circuits.

Let

e = e.m.f. generated by the mutual flux and

$Z = r + jx$ = impedance of the external circuit supplied by the phase converter as generator of second phase.

We then have

$I = \frac{e}{Z + Z_0}$ = current of second phase produced by phase converter,

$E = IZ = \frac{eZ}{Z + Z_0} = \frac{e}{1 + \frac{Z_0}{Z}}$ = terminal voltage at generator circuit of phase converter.

The current in the secondary of the phase converter is then

$$I_1 = I + I' + I'',$$

where

I = load current $= \frac{e}{Z + Z_0}$,

$I' = eY$ = exciting current of quadrature magnetic flux,

$I' = \frac{es}{r_1 + jsx_1}$ = current required to revolve the machine.

and the primary current is

$$I_0 - I_1 + I',$$

where

$I' = eY$ = exciting current of main magnetic flux.

From these currents the e.m.fs. are derived in a similar manner as in the induction motor or generator.

Due to the internal losses in the phase converter, the e.m.fs. of the two circuits, the motor and generator circuits, are practically in quadrature with each other and equal only at no load, but shift out of phase and become more unequal with increase of load, the unbalancing depending upon the constants of the phase converter.

An interesting application of the phase converter is made in single-phase induction motor railroading. In this, the phase converter is connected in series to the induction motor which drives the car. This avoids the increase of unbalancing of the phases with increase of load, and makes it possible by properly connected series transformers to maintain perfect phase and voltage balance on the driving motor. Usually, a quarter-phase phase converter and quarter-phase induction motor is used, and the motor phase of the phase converter is connected in series to one of the phases of the motor into the single-phase supply circuit, while the generator phase of the phase convertor feeds the other phase of the driving motor.

It is obvious that the induction machine is used as phase converter only to change single-phase to polyphase, since a change from one polyphase system to another polyphase system can be effected by stationary transformers. A change from single-phase to polyphase, however, requires a storage of energy, since the power arrives as single-phase pulsating, and leaves as steady polyphase flow, and the momentum of the revolving phase converter secondary stores and returns the energy.

With increasing load on the generator circuit of the phase converter its slip increases, but less than with the same load as mechanical output from the machine as induction motor.

An application of the phase converter is made in single-phase motors by closing the tertiary or generator circuit by a condenser of suitable capacity, thereby generating the exciting current of the motor in the tertiary circuit.

The primary circuit is thereby relieved of the exciting current of the motor, the efficiency essentially increased, and the power-factor of the single-phase motor with condenser in tertiary circuit becomes practically unity over the whole range of load. At the same time, since the condenser current is derived by double transformation in the multitooth structure of the induction machine, which has a practically uniform magnetic field, irrespective of the shape of the primary impressed e.m.f. wave, the application of the condenser becomes feasible irrespective of the wave shape of the generator.

Usually the tertiary circuit in this case is arranged on an angle of 60 deg. with the primary circuit, and in starting a powerful torque is thereby developed, with a torque efficiency superior to any other single-phase motor starting device, and when combined with inductive reactance in a second tertiary circuit, the apparent starting torque efficiency can be made even to exceed that of the polyphase induction motor (see page 336).

For further discussion hereof, see A. I. E. E. Transactions, 1900, page 37.

VII. Frequency Converter or General Alternating-current Transformer

159. The e.m.fs. generated in the secondary of the induction machine are of the frequency of slip, that is, synchronism minus speed, thus of lower frequency than the impressed e.m.f. in the range from standstill to double synchronism; of higher frequency outside of this range.

Thus, by opening the secondary circuits of the induction machine and connecting them to an external or consumer's circuit, the induction machine can be used to transform from one frequency to another, as frequency converter.

It lowers the frequency with the secondary running at a speed between standstill and double synchronism, and raises the frequency with the secondary either driven backward or above double synchronism.

Obviously, the frequency converter can at the same time change the e.m.f. by using a suitable number of primary and secondary turns, and can change the phases of the system by having a secondary wound for a different number of phases from the primary, as, for instance, convert from three phase 6000 volts 25 cycles to quarter phase 2500 volts 62.5 cycles.

Thus, a frequency converter can be called a "general alternating-current transformer."

For its theoretical discussion and calculation, see "Theory and Calculation of Alternating-current Phenomena."

The action and the equations of the general alternating-current transformer are essentially those of the stationary alternating-current transformer, except that the ratio of secondary to primary generated e.m.f. is not the ratio of turns but the ratio of the product of turns and frequency, while the ratio of secondary current and primary load current (that is, total primary current minus primary exciting current) is the inverse ratio of turns.

The ratio of the products of generated e.m.f. and current, that is, the ratio of electric power generated in the secondary to electric power consumed in the primary (less excitation), is thus not unity but is the ratio of secondary to primary frequency.

Hence, when lowering the frequency with the secondary revolving at a speed between standstill and synchronism, the secondary output is less than the primary input, and the difference is transformed into mechanical work; that is, the machine acts at the same time as induction motor, and when used in this manner is usually connected to a synchronous or induction generator feeding preferably into the secondary circuit (to avoid double transformation of its output) or to a synchronous converter, which transforms the mechanical power of the frequency converter into electrical power.

When raising the frequency by backward rotation, the secondary output is greater than the primary input (or rather the electric power generated in the secondary greater than the primary power consumed by the generated e.m.f.), and the difference is to be supplied by mechanical power by driving the frequency changer backward by synchronous or induction motor, preferably connected to the primary circuit, or by any other motor device.

Above synchronism the ratio of secondary output to primary input becomes negative; that is, the induction machine generates power in the primary as well as in the secondary, the primary power at the impressed frequency, the secondary power at the frequency of slip, and thus requires mechanical driving power.

The secondary power and frequency are less than the primary below double synchronism, more above double synchronism, and are equal at double synchronism, so that at double synchronism the primary and secondary may be connected in multiple or in series and the machine is then a double synchronous alternator further discussed in the "Theory and Calculation of Electrical Apparatus."

As far as its transformer action is concerned, the frequency converter is an open magnetic circuit transformer, that is, a transformer of relatively high magnetizing current. It combines therewith, however, the action of an induction motor or generator. Excluding the case of over-synchronous rotation, it is approximately (that is, neglecting internal losses) electrical input ÷ electrical output ÷ mechanical output = primary frequency ÷ secondary frequency ÷ speed or primary minus secondary frequency; that is, the mechanical output is negative when increasing the frequency by backward rotation.

Such frequency converters are to a certain extent in commercial use, and have the advantage over the motor-generator plant of requiring an amount of apparatus equal only to the output, while the motor-generator set requires machinery equal to twice the output.

An application of the frequency converter when lowering the frequency is made in concatenation or tandem control of induction machines, as described in the next section. In this case the first motor, or all the motors except the last of the series are in reality frequency converters.

VIII. Concatenation of Induction Motors

160. In the secondary of the induction motor an e.m.f. is generated of the frequency of slip. Thus connecting the secondary circuit of the induction motor to the primary of a second induction motor, the latter is fed by a frequency equal to the slip of the first motor, and reaches its synchronism at the frequency of slip of the first motor, the first motor then acting as frequency converter for the second motor.

If, then, two equal induction motors are rigidly connected together and thus caused to revolve at the same speed, the speed of the second motor, which is the slip s of the first motor at no load, equals the speed of the first motor: $s = 1 - s$, and thus $s = 0.5$. That is, a pair of induction motors connected this way in tandem or in concatenation, that is, "chain connection," as commonly called, or in cascade, as called abroad, tends to approach $s = 0.5$, or half synchronism, at no load, slipping below this speed under load; that is, concatenation of two motors reduces their synchronous speed to one-half, and thus offers as means to operate induction motors at one-half speed.

In general, if a number of induction machines are connected in tandem, that is, the secondary of each motor feeding the primary of the next motor, the secondary of this last motor being short-circuited, the sum of the speeds of all motors tends toward synchronism, and with all motors connected together so as to revolve at the same speed the system operates at $\frac{1}{n}$ synchronous speed, when n = number of motors. If the two induction motors on the same shaft have a different number of poles, they synchronize at some other speed below synchronism, or if connected differentially, they synchronize at some speed above synchronism.

Assuming the ratio of turns of primary and secondary as 1:1, with two equal induction motors in concatenation at standstill, the frequency and the e.m.f. impressed upon the second motor, neglecting the drop of e.m.f. in the internal impedance of the first motor, equal those of the first motor. With increasing speed, the frequency and the e.m.f. impressed upon the second motor decrease proportionally to each other, and thus the magnetic flux and the magnetic density in the second motor, and its exciting current, remain constant and equal to those of the first motor, neglecting internal losses; that is, when connected in concatenation the magnetic density, current input, etc., and thus the torque developed by the second motor, are approximately equal to those of the first motor, being less because of the internal losses in the first motor.

Hence, the motors in concatenation share the work in approximately equal portions, and the second motor utilizes the power which without the use of a second motor at less than one-half synchronous speed would have to be wasted in the secondary resistance; that is, theoretically concatenation doubles the torque and output for a given current, or power input into the motor system. In reality the gain is somewhat less, due to the second motor not being quite equal to a non-inductive resistance for the secondary of the first motor, and due to the drop of voltage in the internal impedance of the first motor, etc.

At one-half synchronism, that is, the limiting speed of the concatenated couple, the current input in the first motor equals its exciting current plus the transformed exciting current of the second motor, that is, equals twice the exciting current.

161. Hence, comparing the concatenated couple with a single motor, the primary exciting admittance is doubled. The total impedance, primary plus secondary, is that of both motors, that is, doubled also, and the characteristic constant of the concatenated couple is thus four times that of a single motor, but the speed reduced to one-half.

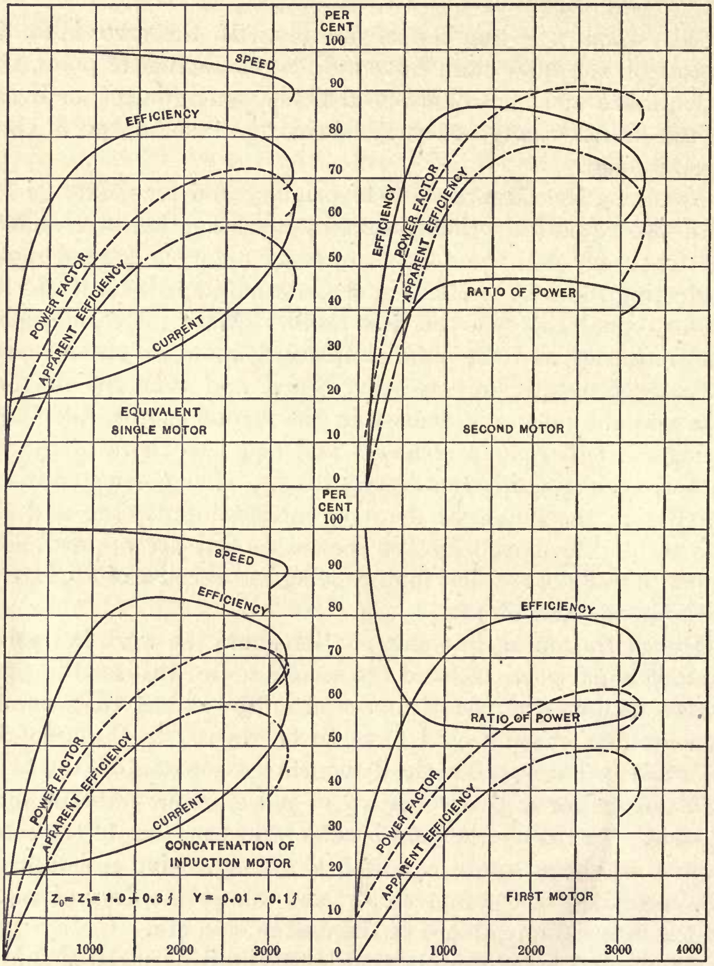


FIG. 192.—Comparison of concatenated motors with a single motor of double the number of poles.

Comparing the concatenated couple with a single motor rewound for twice the number of poles, that is, one-half speed also, such rewinding does not change the self-inductive impedance, but quadruples the exciting admittance, since one-half as many turns per pole have to produce the same flux in one-half the pole arc, that is, with twice the density. Thus the characteristic constant is increased fourfold also. It follows herefrom that the characteristic constant of the concatenated couple is that of one motor rewound for twice the number of poles.

The slip under load, however, is less in the concatenated couple than in the motor with twice the number of poles, being due to only one-quarter the internal impedance, the secondary impedance of the second motor only, and thus the efficiency is slightly higher.

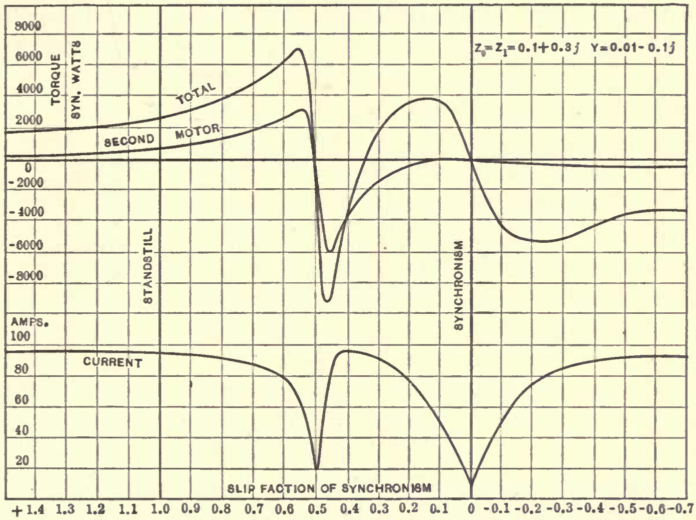


FIG. 193.—Concatenation of induction motors, speed curves.

Two motors coupled in concatenation are in the range from standstill to one-half synchronism approximately equivalent to one motor of twice the admittance, three times the primary impedance, and the same secondary impedance as each of the two motors, or more nearly 2.8 times the primary and 1.2 times the secondary impedance of one motor. Such motor is called the *equivalent motor*.

162. The calculation of the characteristic curve of the concatenated motor system is similar to, but more complex than, that of the single motor. Starting from the generated e.m.f. e of the second motor, reduced to full frequency, we work up to the impressed e.m.f. of the first motor e_0, by taking due consideration of the proper frequencies of the different circuits. Herefor the reader must be referred to "Theory and Calculation of Electrical Apparatus."

The load curves of the pair of three-phase motors of the same constants as the motor in Figs. 176 and 177 are given in Fig. 192, the complete speed curve in Fig. 193.

Fig. 192 shows the load curve of the total couple, of the two individual motors, and of the equivalent motor.

As seen from the speed curve, the torque from standstill to one-half synchronism has the same shape as the torque curve of a single motor between standstill and synchronism. At one-half synchronism the torque reverses and becomes negative. It reverses again at about two-thirds synchronism, and is positive

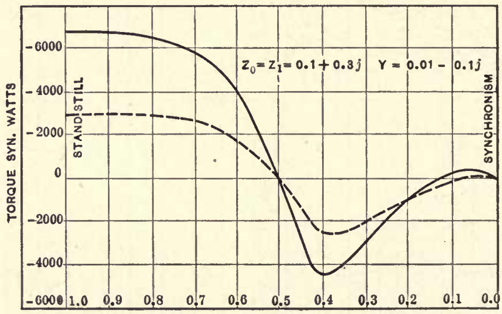


FIG. 194.—Concatenation of induction motors speed curve with resistance in the secondary circuit.

between about two-thirds synchronism and synchronism, zero at synchronism, and negative beyond synchronism.

Thus, with a concatenated couple, two ranges of positive torque and power as induction motor exist, one from standstill to half synchronism, the other from about two-thirds synchronism to synchronism.

In the ranges from one-half synchronism to about two-thirds synchronism, and beyond synchronism, the torque is negative, that is, the couple acts as generator.

The insertion of resistance in the secondary of the second motor has in the range from standstill to half synchronism the same effect as in a single induction motor, that is, shifts the maximum torque point toward lower speed without changing its value. Beyond half synchronism, however, resistance in the secondary lengthens the generator part of the curve, and makes the second motor part of the curve more or less disappear, as seen in Fig. 194, which gives the speed curves of the same motor as Fig. 193, with resistance in circuit in the secondary of the second motor.

The main advantages of concatenation are obviously the ability of operating at two different speeds, the increased torque and power efficiency below half speed, and the generator or braking action between half speed and synchronism, and such concatenation is therefore used to some extent in three-phase railway motor equipments, while for stationary motors usually a change of the number of poles by reconnecting the primary winding through a suitable switch is preferred where several speeds are desired, as it requires only one motor.

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