Mtg 3: Thu, 7 Jan 10

- norm $\|\cdot\|_\infty$ inf. norm

$\|\cdot\|_2$ 2-norm

vectors

functions (gen. from norms for vectors.)

Ref: A. p. 10

$\underline{v} \in \mathbb{R}^n$

underbar ≡ matrix (tensor)

$\{\underline{e}_i\} = \{\underline{e}_1, \ldots, \underline{e}_n\}$ , $\underline{e}_i \in \mathbb{R}^n$

orthonormal basis

$\underline{v} = v_i \underline{e}_i \Rightarrow \{v_i\} = \begin{Bmatrix} v_1 \\ \vdots \\ v_n \end{Bmatrix}$ $n \times 1$ $n \times 1$

$\|\underline{v}\|_2 = \left[ \sum_{i=1}^{n} (v_i)^2 \right]^{1/2} = (\underline{v} \cdot \underline{v})^{1/2}$

Summation conv. on repeated indices (Einstein sum. conv.)

$\underline{v} = \sum_i v_i \underline{e}_i$

$$\|\underline{v}\|_2 = (v_i v_i)^{1/2}$$

$$\|\underline{v}\|_\infty = \max_i |v_i|$$

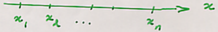
If $\|\underline{v}\|_\infty \to 0$ then $v_i \to 0$ $\forall\, i = 1, \dots n$

Gen. to funcs:

$$(f, g) = \langle f, g \rangle = \int_a^b f(x)\, g(x)\, dx$$

$$\|f\|_2 = [\langle f, f \rangle]^{1/2}$$

$$\|f\|_\infty = \max_x |f(x)|$$




$f(x_1)\ f(x_2)\ \dots\ f(x_n)$

$g(x_1)\ g(x_2)\ \dots\ g(x_n)$

$$\|f\|_\infty \to 0 \Rightarrow f(x) \to 0 \quad \forall x \in [a, b]$$

$\vec{i} = \underline{e}_1$, $\vec{j} = \underline{e}_2$ } $\hat{\underline{v}} = 2\,\underline{e}_1 + 3\,\underline{e}_2$

$\underline{v} = v_1\,\underline{e}_1 + v_2\,\underline{e}_2$

$$\|\underline{v}\|_\infty = \max_i |v_i| = \max(v_1, v_2)$$

$$\|\hat{\underline{v}}\|_\infty = 3$$

$$\|f - f_n\|_\infty = \max_x |f(x) - f_n(x)|$$

$$\neq \underbrace{\max_x f(x)}_{f(\hat{x})} - \underbrace{\max_x |f_n(x)|}_{f_n(\tilde{x})}$$

(matlab, scilab)
octave

HW:

$f(x) = \sin x$

$g(x) = \sin\left(x - \frac{\pi}{2}\right) = -\cos x$

Plot $f, g$ $x \in [0, \pi]$

Find $\|f\|_\infty$, $\|g\|_\infty$, $\|f-g\|_\infty$

## p. 2-4 IMVT cont'd

(3) p. 2-4: $m \leq \underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{\substack{\shortparallel \\ \zeta}} \leq M$

$\zeta$ zeta

Interm. Value Thm: $\exists\, \xi \in [a,b]$ st $f(\xi) = \zeta$

$$\Rightarrow f(\xi) = \frac{1}{b-a}\int_a^b f(x)\,dx$$