BABINET PRINCIPLE AND PARTICLE SIZE DETERMINATION

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Abstract:

Determination of the average diameter of micro particles such as lycopodium particles by scattering is well known classical experiment in optics. The size of lycopodium particles can be determined by scattering of sodium light. With advent of laser light (which is highly monochromatic), this experiment can be easily done. With availability of semiconductor laser, many optics experiments can be done elegantly with cost effectiveness. In this paper an experiment on the determination of the average size of lycopodium particle, with necessary theory, is presented.

Introduction:

After the advent of laser, the experiments on interference, diffraction and scattering of light have gained tremendous importance in physics curriculum. Nowadays semiconductor lasers are available at a relatively low cost. Some classical experiments such as average size of lycopodium particles, diffraction at a slit, double slit, grating, etc can be done with simplicity and elegance. Some of these experiments are included in the list of physics practical in graduate physics programs and engineering programs of colleges affiliated to reputed universities. But lycopodium particles, strewn on a glass substrate, form *a random* opaque micro objects. When the Babinet's principle is invoked this diffraction pattern is similar to the diffraction pattern due to a collection of *random circular apertures.* So the diffraction pattern is Airy's pattern. The maxima and minima circular rings *are not* equispaced. The Airy's pattern is intimately related to the resolving power of imaging system with circular geometry.

Babinet's principle:

Complementary screens are those in which the transparent portions of one are the opaque portions of the other. Let the amplitude of light after propagation through a distance be $u(p)$ when a coherent beam is intercepted by a screen with an aperture in it. When two complementary screens are superimposed the result *is not* a full-opaque screen but a full wave-front *as if no screen were present*. According to Babinet's principle, complementary screens produce the same diffraction pattern. If $u_1(p)$ and $u_2(p)$ are the amplitudes at a point p on a screen at a distance from the aperture then their sum is the amplitude $u(p)$ of the entire wave-front:

$$u_1(p)+u_2(p)=u(p). \tag{1}$$

If we set the amplitude of a uniform plane wave-front as unity, we write eq. (1) as

$$u_2(p)=1-u_1(p)=1+(-u_1(p)). \tag{2}$$

As $u_1(p)$ is binary and takes 1 for clear area and 0 for opaque area, $u_2(p)$ is also binary and takes 1 for opaque area of $u_1(p)$ and 0 for the transparent area of $u_1(p)$. $u_1(p)$ and $u_2(p)$ interchange their transparent and opaque areas. Hence $u_2(p)$ and $u_1(p)$ are complementary to each other. As an example a circular aperture and a circular opaque disk of the same diameter are complementary (see figs.1 and 2).

It is well known that Fraunhofer diffraction is a two dimensional Fourier transform of the transmittance of aperture[1, 4,7,13].

$$U(f_x, f_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(x,y)\, e^{-2\pi i (f_x x + f_y y)} dxdy \ . \tag{3}$$

On the Fraunhofer plane, f_x *and* f_y are spatial frequencies with respect to the screen/aperture/obstacle plane. They are respectively given by

$$f_x = \frac{X}{\lambda Z} \ and \ f_y = \frac{Y}{\lambda Z} \tag{4}$$

where (X, Y) are the *actual* co-ordinates on the Fraunhofer plane which is at a distance of Z from the aperture plane. λ is the wavelength of coherent light incident on the aperture/screen/obstacle. (x, y) are the co-ordinates of a point p on the screen. Taking the Fourier transform of eq. (1) and noting that the Fourier transform of the amplitude $u(x, y)$ of the entire wave-front is a delta function $\delta(0)$, we get

$$U_1(f_x, f_y) + U_2(f_x, f_y) = \delta(0) . \tag{5}$$

Use is made of the fact that Fourier transform is linear.

On the Fraunhofer plane, the bright spot at the centre of symmetry is due to the plane wave propagating along the axis. This is referred to as 'd.c.' component. (zero spatial frequency). This is conspicuous when an opaque object is causing diffraction[2]. The bright 'Poisson spot' on Fraunhofer plane due to a circular disk can be interpreted as a delta function. It is a ***'caustic'*** point for a circular disk. C.V. Raman investigated this for an elliptic disk and found this caustic curve to be the evolute of an ellipse[2]. It is an interesting example of boundary diffraction wave.

The amplitude on the Fraunhofer plane, other than the origin, is zero as delta function has nil (in practice very small) spread. Thus the total amplitudes of the sum of complementary screens is zero on the Fraunhofer plane! For points other than the centre on the diffraction plane, eq. (5) satisfies

$$U_1(f_x, f_y) = -U_2(f_x, f_y) \ . \qquad (f_x \neq 0 \, and f_y \neq 0).$$

In Fraunhofer diffraction, $u(p)$ is of constant amplitude as a plane wave-front is incident on the object/aperture. There is over all phase of difference of π between the amplitudes which is not important as in intensity it does not manifest.

Thus

$$\left|U_1(f_x,f_y)\right|^2=\left|-U_2(f_x,f_y)\right|^2 \; or \; I_1=I_2. \tag{6}$$

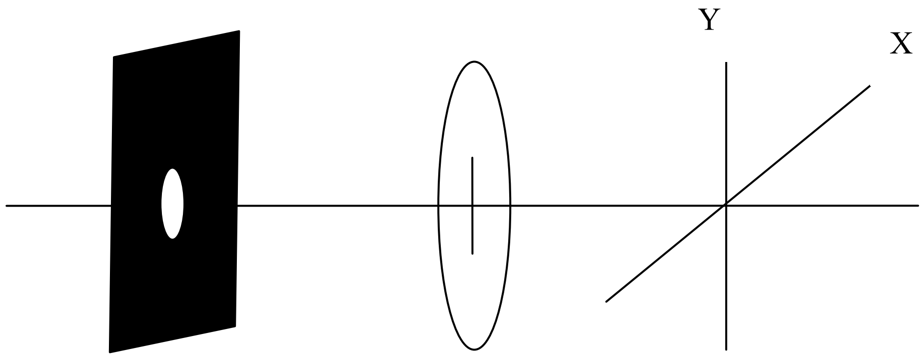

Figure. 1 Geometry for observing Fraunhofer diffraction

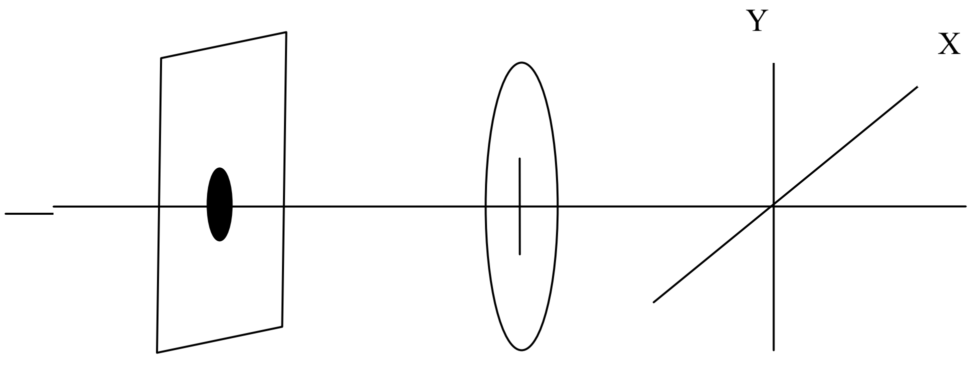

Figure.2 Geometry for observing Fraunhofer diffraction of complementary screen

Therefore the intensity pattern due to either of the complementary screen is the same except at the origin. This is an interesting result. This is the principle behind the experiment on the determination of average size of lycopodium particles.

The Airy's pattern:

A random collection of (opaque) lycopodium particles produces the same diffraction pattern as the random collection of circular apertures of very small diameter comparable to the wavelength of light[7, 9]. The Fraunhofer diffraction pattern due to a large number of random *circular apertures* is the well-known Air's pattern. The diffraction pattern of a circular aperture is Airy's pattern[4, 5, 6]. The diffraction pattern of a random collection of N identical circular apertures is also Airy pattern with N times the brightness due to one aperture. Thus the Fraunhofer diffraction pattern due to lycopodium particles is also an Airy's pattern only (see Fig.2).

The normalized Fraunhofer diffraction amplitude due to a circular aperture of diameter d is given by

$$U(f_x, f_y) = U(\rho) = \left(\frac{2 J_1(\pi\rho\, d)}{\pi\rho\, d} \right) \tag{7}$$

where $J_1(q)$ is the Bessel function or order 1 and argument q.

$$\rho = \sqrt{f_x^{\,2} + f_y^{\,2}} = \frac{\sqrt{X^2 + Y^2}}{\lambda Z} = \frac{R}{\lambda Z}.$$

R is the radial distance from the centre of the Fraunhofer plane. The amplitude and intensity of Airy's patern are shown in fig.3. Airy's disc extends up to the first diffraction minimum and then secondary maxima and minima are seen as circular bright and dark rings respectively. *The widths of the secondary maxima and minima are not equal.* The angular diameters of rings are ***not proportional to*** integer. The radii R_n of dark rings are given by[5, 11]

$$\frac{1.22\lambda Z}{d}, \frac{2.233\lambda Z}{d}, \frac{3.238\lambda Z}{d} \ldots$$

and the radii of the *centres* of bright rings are given by

$$\frac{1.635\lambda Z}{d}, \frac{2.679\lambda Z}{d}, \ldots.$$

Thus it is evident that the radii are not equi-spaced. In a good dark room, two or three dark rings can be seen clearly. The first dark ring is conspicuous as more than 90% of the intensity of the diffraction pattern is contained in the Airy's disk.

The diameter d of the circular aperture or equivalently average diameter of lycopodium particles is given by

$$d = 1.22\lambda\left(\frac{Z}{R_1}\right) = 2.233\lambda\left(\frac{Z}{R_2}\right) = 3.238\lambda\left(\frac{Z}{R_3}\right)....$$

Z is the distance of the screen from the glass plate on which the lycopodium powder is strewn. R_n are the radii of the successive dark rings. The radius of the first dark ring is the radius of the Airy's disc. R_n is *not* integral multiple of $\dfrac{\lambda Z}{R_1}$.

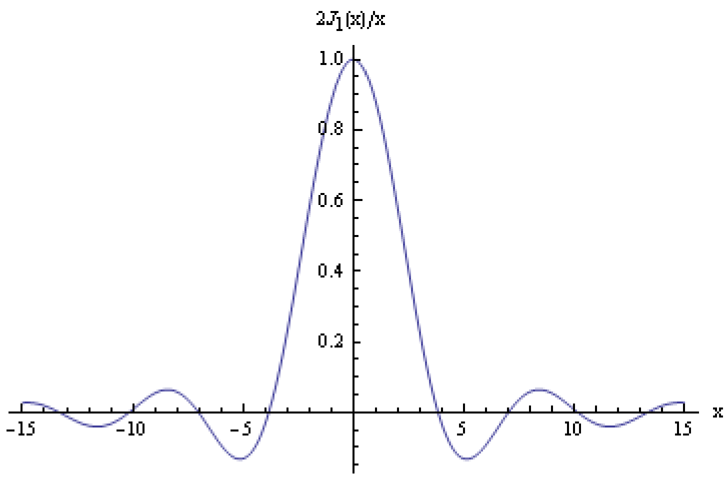

Figure. 3 Airy's function for amplitude

x	Intensity
1.22π	Minimum
2.233π	Maximum
3.238π	Minimum

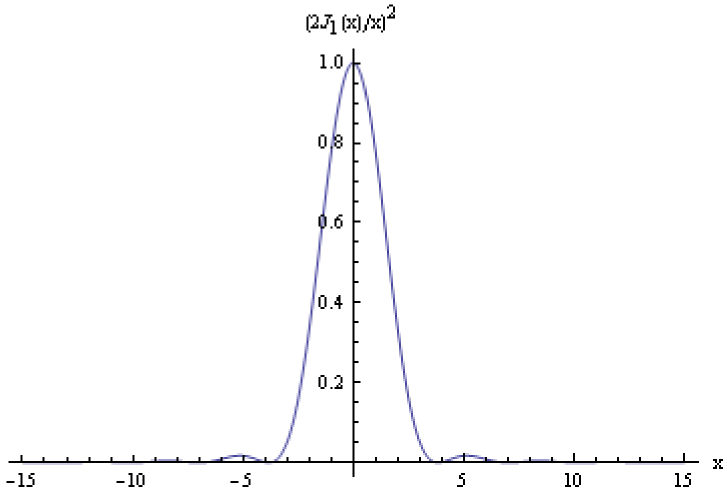

Figure. 4 Modulus square of Airy's function: Intensity Distribution

The Airy pattern plays a key role in resolution of optical instruments with imaging optics having circular symmetry. So the factor 1.22 for the first dark ring can not be ignored [15].

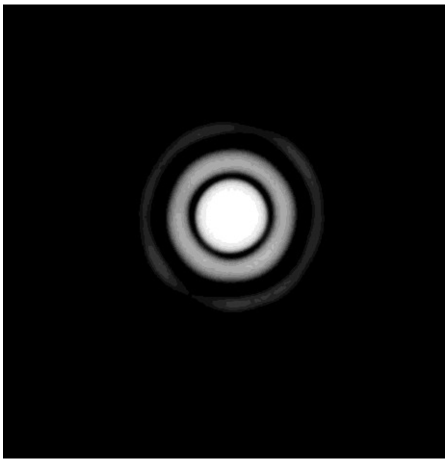

Figure. 5 Intensity distribution of a Circular aperture

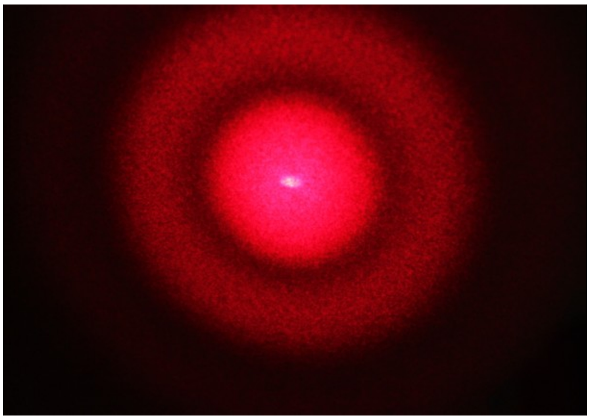

Figure. 6Fraunhofer diffraction of micro particles

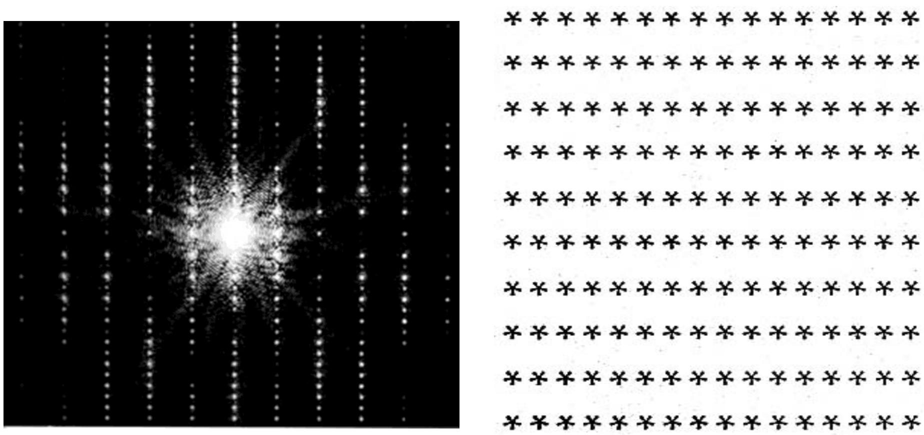

Figure. 7 Aperture matrix and its Diffraction pattern

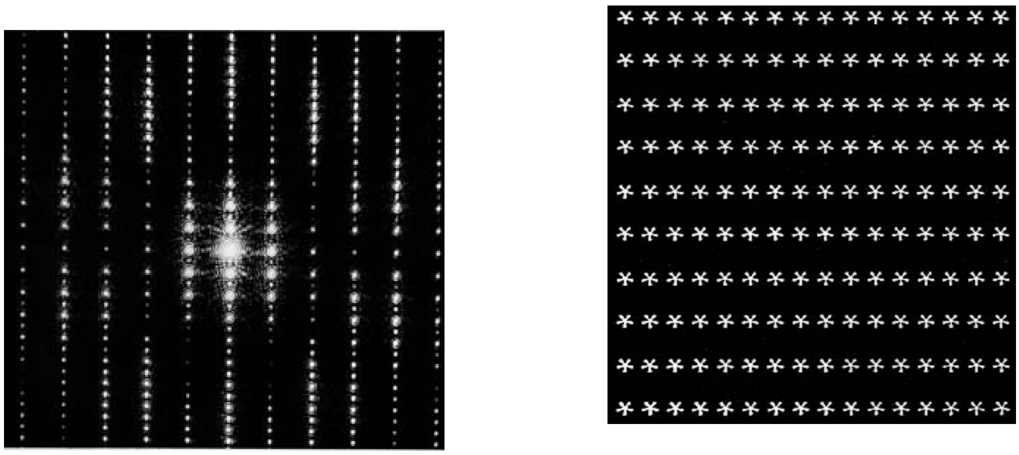

Figure. 8 Complimentary Aperture matrix and its Diffraction pattern

Conclusions

Students at entry level to engineering program are from the higher secondary level. They may not be familiar with Fraunhofer diffraction. Even if a detailed theory of diffraction at a slit, grating, circular aperture are outside the scope of the book, the presence of the factor 1.22 has to be emphasized to distinguish the diffraction at a circular aperture from those of diffractions at a slit and a grating. Earlier practical physics manuals provided a brief description of theory for every experiment[14]. Since we give importance to develop scientific temper among students, we have to be perfect in giving formulae and theory in support of the experiments.

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