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**THESIS**

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**AN ANTI-AIR WARFARE STUDY  
FOR A SMALL SIZE NAVY**

by

**Adrianos M. Poulos**

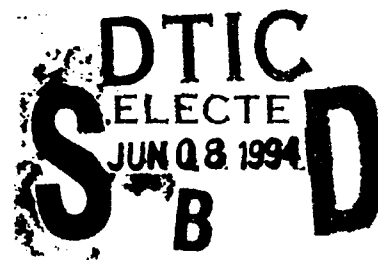
**March 1994**

Thesis Advisor:

**Wayne P. Hughes, Jr.**

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**AN ANTI-AIR WARFARE STUDY FOR A SMALL SIZE NAVY**

by

**Adrianos M Poulos**  
Lieutenant, Hellenic Navy  
B.S., Hellenic Naval Academy, 1981

Submitted in partial fulfillment  
of the requirements for the degree

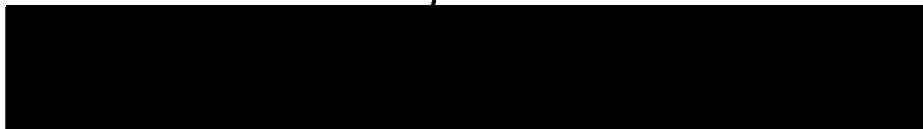
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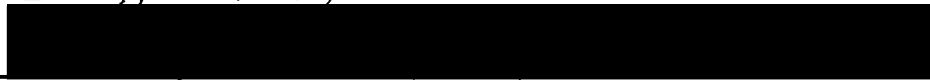
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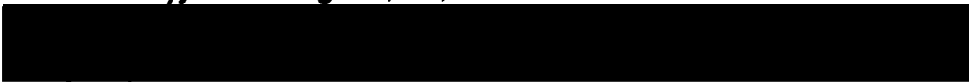


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Department of Operations Research

## ABSTRACT

This thesis is a study of the defensive power of a medium size Naval force subject to air-to-surface missile attack. It evaluates the attrition to an escorted amphibious force and its escorts under different tactical situations for a variety of defense parameters. Using attrition as the measure of effectiveness, it draws conclusions useful to a small Navy regarding its (AAW) defenses. The study models the force-on-force process of aircraft versus warships in discrete time steps, or "salvos." The degradation of the force is expressed in number of ships. This study extends and deepens work by W.Hughes and Lt. E.Hatzopoulos (H.N) model, incorporating new features to analyze AAW principles and concepts.

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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

## TABLE OF CONTENTS

I. INTRODUCTION .....	1
A. BACKGROUND .....	1
B. SEA BATTLE MODELS .....	2
C. THESIS GOAL AND SCOPE .....	4
II. NAVAL COMBAT THEORY .....	7
A. INTRODUCTION .....	7
B. COMBAT THEORY .....	7
1. COMBAT UNIT .....	7
2. COMBAT FORCE .....	7
3. FORCE STRENGTH .....	7
4. OFFENSIVE POWER .....	8
5. COMBAT POWER .....	8
6. FIRE POWER .....	8
7. STAYING POWER .....	9
8. DEFENSIVE POWER .....	10
A. HARDKILL COUNTERACTION .....	10
B. SOFTKILL COUNTERACTION .....	10
(1) Seduction .....	10

(2) Evasion .....	10
(3) Distraction .....	11
9. ALERTNESS (T) .....	11
10. COMBAT WORK .....	11
11. SCOUTING EFFECTIVENESS (S) .....	11
12. UNCERTAINTY IN NAVAL COMBAT .....	12
C. LANCHESTER'S MODEL .....	13
1. SQUARE LAW (AIMED FIRE) .....	13
2. LINEAR LAW (UNAIMED FIRE) .....	15
III. TWO NAVAL MODELS .....	18
A. THE BASIC SALVO EQUATION .....	18
1. ASSUMPTIONS .....	18
2. FORCE-ON-FORCE EQUATIONS .....	19
3. MODEL-BASED CONCLUSIONS .....	20
B. A MODERN NAVAL COMBAT MODEL .....	21
IV. DEFENSIVE POWER IN ANTI-AIR WARFARE .....	24
A. INTRODUCTION .....	24
B. AIR DEFENSE THEORY .....	25



1. DENIAL OF INFORMATION TO THE ENEMY .....	26
2. EARLY WARNING .....	26
3. ATTACK AT THE SOURCE .....	26
4. LAYERED DEFENSE. ....	27
C. MODEL DEVELOPMENT .....	27
1. AREA DEFENSE .....	27
A. FULL ALERTNESS OF THE TOTAL FORCE .....	27
B. ONLY PARTIAL ALERTNESS OF SOME SHIPS .....	28
2. POINT DEFENSE. ....	29
3. SECTOR DEFENSE .....	31
A. MISSILES ARE EQUALLY DISTRIBUTED AMONG ALL .....	32
SECTORS .....	
B. MISSILES ARE CONCENTRATED IN PARTICULAR .....	33
SECTORS .....	
V. APPLICATION OF THE MODEL .....	34
A. INTRODUCTION .....	34
B. STUDY OF DIFFERENT SCENARIOS .....	36
C. ANOTHER USE OF THE MODEL .....	60

<b>VI. CONCLUSIONS AND RECOMMENDATIONS</b> .....	<b>65</b>
<b>A. MAIN CONCLUSIONS</b> .....	<b>65</b>
<b>B. GENERAL CONCLUSIONS</b> .....	<b>67</b>
<b>C. RECOMMENDATION</b> .....	<b>67</b>
<b>APPENDIX A</b> .....	<b>68</b>
<b>A. PROGRAM LISTING</b> .....	<b>68</b>
<b>LIST OF REFERENCES</b> .....	<b>73</b>
<b>INITIAL DISTRIBUTION LIST</b> .....	<b>74</b>

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I would like to dedicate this thesis to my parents for raising me in such a way as to become what I am right now, as well as to my wife Constantina for her support and understanding during all this period, including her help in editing this thesis. To conclude with, I would like to thank the Hellenic Navy for giving me this unique opportunity to be a student at the Naval Postgraduate School.

## **EXECUTIVE SUMMARY**

In the modern naval warfare theater, more and more complicated weapons are being developed and each country has to spend larger and larger amounts of money in order to retain the status of high enough force levels. Many times this situation results from geopolitical reasons, even though a country could spend this money in other areas, like health or education .

Although the more developed and financially independent countries conduct their own research to meet their needs for weapons and ships, the less financially independent ones are, most of the time, obliged to buy these products on the free market. The above mentioned event has two main disadvantages for the less financially independent countries. First is the fact that the weapons/ships do not always meet their real needs 100 percent of the time. Secondly, because of gradual replacement, naval forces are rarely homogeneous.

Therefore the subsequent tactical question is posed : given a number of different warships and weapons, what should be the best allocation under different naval operations in order to maximize the defensive power of the total force?

From the different kinds of naval operations we chose the anti-air warfare environment for our study because we consider aircraft as one of the biggest threats for a naval force.

As a basis for our study we used the salvo model . That model, developed by Capt. W.P. Hughes U.S. Navy, deals with naval missiles and models the attrition as a force-on-force process described in discrete time steps, or "salvos." The general concept of that model is that the losses of units are caused by an enemy "salvo." Loss depends on the offensive power of the attacker, the defensive power of the defender and the defender's units ability to accept missile hits without being operationally placed out of action.

Lt E. Hatzopoulos (H.N) later incorporated in this model the effect of human factors, e.g., scouting and alertness.

First we had to expand on this model in order to study the anti-air warfare principles and concepts. Next we chose the most interesting tactical situations and tried to cover air-defense in all its phases, mainly from the defender's point of view. Only missile attacks were studied but the model can easily be applied for aircraft with bombs.

In order to study the defensive power analytically, we fixed the offensive power of the attacking force at a realistic level and changed the formations of the defender as well as different elements of its defensive power.

For detailed study we chose the following factors by which a small Navy may increase its defensive power:

- The presence of friendly aircraft as combat air patrol units.
- Point defense installations as a last chance for a ship to defend itself.

Defense in depth:

- To raise as many "defense walls" as we can in the path of any missile.
- To compel the enemy's air force to attack particularly valuable units from a greater distance.

## **I. INTRODUCTION**

### **A. BACKGROUND**

The outcome of a battle is final. A military commander cannot fight an engagement again, regardless of the outcome. For military men, unlike other professionals, the only way to implement their profession is during the war. So it would be rhetorical to say that the biggest problem in preparing to fight the next war is that no one has any current experience at doing it!

Few countries have the experience of operating modern weapon systems under the conditions of a full scale conflict, and no one has the assurance of knowing for certain how the potential enemy may choose to employ its forces against him. So it is that for centuries military men have been devising ways to imitate or simulate realistic war situations for analytical study.

As technology improves war increasingly involves a large number of large scale and very complex systems, where people and machines act and interact toward the accomplishment of some military or policy goal. So the greater the complexity of a future war, the more important combat models become. Here, by the term "model" we mean a set of logical and quantitative relationships which represent a particular system.

By manipulating and changing the opposing systems, we can see how the model reacts and so infer how the systems would react. These kinds of models are called "mathematical models". Combat models aim to help decision makers improve tactics or force composition by examining the effectiveness of new tactics, or by different combinations of existing weapon systems. As Sir M.C.Kendall succinctly stated, "Models are for thinking with" [Ref. .1: p.11].

Another purpose of validated combat models is to aid in analyzing data from historical battles. By analyzing such data we can understand the commander's way of thinking during the battle and see if his decisions were correct or not. We can also see if the commander would have persisted in his decisions or changed his tactical plans had he used this model.

A third purpose of a combat model is that of a planning aid or tactical decision aid.

## **B. SEA BATTLE MODELS**

In naval warfare we can distinguish two broad categories of models :

- Interactive war game models with which we can test the skills and capabilities of two or more opposing forces engaged in an unstructured "freeplay" crisis or combat situations.
- Analytical models which are used to evaluate the relative capabilities of alternative forces and weapon systems over a wide range of highly structured tactical situations. [Ref .1: p .145]

While war games and fleet exercises are used mainly for training and naval battle planning, the analytical and simulation models are used primarily as decision aids in tactical development, weapons system procurement and force



planning. Mathematical models have been used extensively to provide the analytical framework in which weapon system capabilities, or alternative courses of tactical action and their potential consequences, are evaluated. The question which arises is how complex an analytical sea battle model should be. The level of complexity of the analysis must be consistent with the decision under consideration. But generally three important qualities of sea battle models are simplicity, transparency and flexibility. [Ref. 1: p.149]

The main types of analytical models are the following:

- Phenomenological models
- Single mission models
- Tactical engagement models
- Campaign models

In this thesis an analytical single mission model is used. Such a model is exemplified by multiple systems (such as aircraft, area SAMs, point defense missiles, guns, active and passive EW systems and decoys ) engaging an incoming raid of ASMs either simultaneously or sequentially. Usually single-mission evaluation models consist of several layers of engagement envelopes. Each envelope decreases the probability of missiles, aircraft or submarines to engage a target as they penetrate from one layer to another. Four crucial effects which are important at this level of aggregation are (1) the effects of weapon system saturation due to raid size and coordination, (2) the effects of raid geometry over the battle space, (3) the cumulative effect of attrition during

the engagement and (4) the effect of command and control on coordination between defensive systems. If the engagement involves a large number of units that interact in a relatively short time span, these effects frequently dominate the results. [Ref.1 p.155]

### **C. THESIS GOAL AND SCOPE**

The way that most of the small and less financially independent countries buy their weapon systems is "off the shelf" meaning that often these weapons do not meet all their needs. So for a small country, the question posed is this: given a number of dissimilar types of warships and weapons, what is the best mix for different naval operations in order to have minimum force losses ?

The goal of this thesis is to try to answer the above question for an anti-air warfare. Since 1912, when Lt. Kaberis of the Greek Army was the first man in the world to use aircraft for military operations by dropping hand grenades on enemy troops, aircraft have evolved to be the major factor of every operation. This is the reason for choosing the air-warfare environment to study. By developing a reasonable anti-air warfare model we will examine the following issues:

- Advantages and disadvantages of layered defense in an air raid.
- Is the point defense vital for some high value unit (HVU) or is the protection from the escorts sufficient?

Secondly, we will show that the same model can be used for the following purposes:

- As a useful tactical planning tool. In this case we can have an estimation for optimal bomber strategy (i.e., how many bombers do we need if we want to cause a certain amount of damage to a given target force? Or, on the contrary, what is the number of escorts in order to protect a HVU from a given number of attackers?)
- As a procurement programming tool (i.e., what is the trade off between AAW ship and point defense escort, or between AAW ship and adding point defense to the landing force?)

Thirdly, we will critique the technical characteristics of the model and its limitations.

The thesis will consist of the following steps. In Chapter II we present the relevant points of naval combat theory and introduce appropriate terminology for concepts that are developed later.

In Chapter III we introduce the concept of the salvo model based on Capt. Hughes' theory about naval warfare. We also describe a modern naval warfare as it was developed by E. Hatzopoulos. The latter is based on the salvo model, but corrects some of its weaknesses by incorporating human factors. The modern naval warfare model will be the basis for the development of our model.

In Chapter IV we present the main principles and doctrines of air defense theory in naval warfare. After that we develop an anti-air warfare combat model, incorporating those principles. We try to cover air defense in all her phases mainly from the defender's point of view. Only missiles are considered in developing the model, but the model may easily be adapted for aircraft with bombs.

Chapter V is devoted to applying the model. We apply it to study several tactical plans analytically. In order to study the defensive power of some force under the above plan we fix the offensive power of the attacking force and change the formations of the defender and different elements of its defensive power.

In Chapter VI we summarize our conclusions about the consistency of the model and also answer the various questions posed above.

Finally, in Appendix A, we give the computer code (MATLAB) used to study the model. The potential user should know that, depending on the specific case studied, some minor modifications will probably be needed.

## **II. NAVAL COMBAT THEORY**

### **A. INTRODUCTION**

This chapter is a brief discussion of combat theory and terminology. It will help the reader understand the concepts and the model that will be developed in the following chapters. The ideas that we present are only those necessary for the reader to become familiar with naval combat and to understand a naval combat model. The basic concepts are drawn from W. Hughes work, " The Value of Warship Attributes in Missile Combat ." [Ref. 7]

### **B. COMBAT THEORY**

#### **1. Combat unit**

A combat unit is ship or aircraft which is capable of delivering firepower.

#### **2. Combat force**

A combat force is a group of combat units that operate and fight together.

#### **3. Force strength**

Force strength is the number of units in an homogeneous combat force on sides A or B, designated A or B respectively. If we have an heterogeneous force then the force strength is the weighted sum of the individual unit values measured against a standard unit, e.g., a Knox class frigate.

#### **4. Offensive power**

This term is a general way of expressing firepower, fighting power, striking power or combat power as appropriate to the circumstances.

#### **5. Combat power**

Combat power is a measure of the actual capability of the force to achieve results in a combat, achieved per unit of time. So it is an observable phenomenon in a battle applicable in a particular environment against a particular enemy and measured by attrition to this enemy. When the forces are activated by a commander, combat power is produced from their firepower.

#### **6. Fire power**

For a combat unit, fire power is the number of accurate shots fired by it per period of time for continuous fire, or per tightly-spaced pulse for salvo fire. For a force, fire power is the number of accurate shots fired by all units per period of time for continuous fire, or per tightly-spaced pulse for salvo fire.

Fire power can be seen as a function of the number of a force's elements on the one hand, and the type of forces and rate of their activity on the other. If  $P$  denotes fire power,  $m$  the number of elements in a force and  $u$  the rate of the force's activities, then the fundamental equation of combat power is given by

$$P=F(m,u)$$

where  $F$  is called the command function.

In this study combat power of a unit or force, is damage imposed, while fire power is the total number of shots delivered by a unit or a force.

## **7. Staying power**

Staying power is the ability of a warship to survive and continue fighting after a hit has been taken by this warship. It is measured by the number of hits that the warship can absorb before being too disabled to continue fighting.

In his master's thesis, T. Beall derived the following expression for measuring the staying power of a ship:

$$SP = 0.070 \times (\text{full load displacement})^{1/3} \quad (2.1)$$

where full load displacement is a characteristic of a particular ship [Ref. 5: p 108]. In this formula staying power is measured in 1000-pound bomb equivalence (TPBE) (equivalent to the explosive power of a 1000-pound bomb in W.W.II, or equal to the explosive power of 660 pounds of TNT).

In order to work with missiles we have to convert the staying power from hits of TPBE to hits by missiles. From the different air-to-surface missiles with different weights of warheads we choose the Harpoon and the Penguin with 507 lb. and 264 lb. warhead weights correspondingly. We then multiply their weights by a factor of 1.3, for the contribution of their kinetic energy and fuel in their destructive power, for an equivalent value of 659 and 343 lb. as their warhead weight. From that we see that the Harpoon has a destructive power equivalent of one TPBE, while the Penguin has a destructive power of half TPBE. So in order to put a ship out of action (OOA) we need as many Harpoon hits as 1000 lb. bomb hits, while we need to double number for Penguins. For convenience in our computations we use the Harpoon as our nominal missile.

In reality, staying power it is not only a function of displacement but also of other design attributes of the warship. For example, the buoyancy design of the ship compartmentation or the use of special materials which reduce the hazard of burning, are some of these factors. For simplicity we'll limit our analysis to the formula derived above in order to compute the staying power.

## **8. Defensive power**

Defensive power can be thought of as the composite of all defensive actions which reduce susceptibility to hits by the enemy. So it can be measured by the number of enemy shots that a target unit can destroy before being hit. Defensive power comprises hardkill and softkill counteractions.

### ***a. Hardkill counteraction***

This term refers to *weapon fire by the target to destroy enemy shots.*

### ***b. Softkill counteraction***

#### **(1) Seduction**

The process of causing accurate shots to miss the target when counterfire has failed, e.g., by seduction chaff.

#### **(2) Evasion**

A process of maneuver to cause good shots to miss the target.



### **(3) Distraction**

A process of causing accurate shots to miss the target before counterfire<sup>1</sup> has its effect. For our purpose we'll consider as defensive power only hardkill counteraction and softkill from seduction.

### **9. Alertness ( $\tau$ )**

By that term we mean the extent to which a target ship fails to take defensive actions up to its designed combat potential due to unreadiness. In such a case the enemy may "surprise" its opponent. By convention, alertness affects only the defensive power of a ship and not its staying or offensive power. It appears normally as a multiplier of defensive power with values between 0 and 1.

### **10. Combat work**

This term is the number of ships put out of action (OOA) by a salvo or within a single period of continuous fire. It may also be the accumulated units put OOA after a series of salvo exchanges.

### **11. Scouting effectiveness ( $\sigma$ )**

By scouting we mean the activity of a warship or a force to collect all the important information about the enemy needed to attack effectively. So we can think of scouting effectiveness as a degradation of offensive power due to incomplete targeting information about the enemy.

---

<sup>1</sup>The fire as a reaction to the enemy's fire

Scouting effectiveness is the difference between the number of accurate shots delivered with perfect knowledge of enemy composition, location, intentions and plans and the number of accurate shots delivered with the existing information. It normally is a multiplier of the offensive power with values between 0 and 1.

## **12. Uncertainty in Naval Combat**

Combat modeling can't be compared with deterministic or other kinds of analytical models. The main reason is the existence of uncertainty. The importance of uncertainty has been adopted by the Military Conflict Institute (TMCI) as one of the six basic axioms to understand and then model a combat situation [Ref. 1]. By the uncertainty we mean all those factors which affect all phases of a combat, especially its outcome. A clear example of uncertainty is that one never knows the exact composition of the enemy (personnel, equipment, intentions, and so forth). Another source of uncertainty is doubt about the exact state of the friendly forces during combat.

And even if we do have perfect information about the enemy and our own force, we still cannot predict the outcome with certainty. The reason is due to the human factors involved in the whole process. In conclusion the outcome of a battle will not be deterministic. A combat leader, of course, can make predictions both by judgment and combat models based on the designed combat potential of both sides. But he cannot trust entirely the outcome of any model. History has

shown us cases where, although the advantage of one side was clearly great, at the last moment some unpredictable factor exploited by the other side overturned the final result.

### C. LANCHESTER'S MODEL

Systems of ordinary differential equations (ODE) have long been used to model various biological phenomena. When military populations are involved, one popular ODE is referred to as the Lanchester system, in honor of F. W. Lanchester, who applied ODE systems to populations of fighter planes in W.W.I. In a Lanchester system, the positive terms are generally due to reinforcement, since most military systems (tanks, ships, unaccompanied men, etc. ) have rates of increase often unrelated to populations currently in the field. In this study we disregard reinforcement and only examine attrition from enemy fire.

#### 1. Square Law (Aimed fire)

The differential equations

$$\frac{dx}{dt} = -ay, a > 0$$

(2.2)

$$\frac{dy}{dt} = -bx, b > 0$$

represent a situation where attrition to each side is proportional to the number of units remaining on the other. In the equations above X and Y are two forces whose force levels are x and y respectively. The numbers a and b are the attrition rate coefficients for the X and Y forces, respectively. It is consistent with this model that each unit has a fixed rate of fire, with each shot having a certain probability of eliminating the opposing unit at which it is aimed (but no other unit), hence the " aimed fire " description.

Equations (2.2) can be solved explicitly for x and y as functions of time, but it is simpler to eliminate time by dividing the second equation by the first to obtain y as a function of x. The result is:

$$\frac{dx}{dy} = \frac{ay}{bx} \quad (2.3)$$

which has the solution

$$b(y_0^2 - y^2) = a(x_0^2 - x^2) \quad (2.4)$$

valid as long as x and y are both non negative. The values  $x_0$  and  $y_0$  are the initial force levels at time  $t=0$  when the engagement begins if it is presumed that the battle will proceed until one side or the other is reduced to 0. If  $x_f$  or  $y_f$  is the final number of survivors on the other side, then, according to (2.4),

$$y_f = 0 \quad \text{and} \quad x_f = \sqrt{x_0^2 - \frac{b}{a}y_0^2} \quad \text{if} \quad by_0^2 \leq ax_0^2 \quad (2.5)$$

$$x_f = 0 \quad \text{and} \quad y_f = \sqrt{y_0^2 - \frac{a}{b}x_0^2} \quad \text{if} \quad by_0^2 \geq ax_0^2$$

Since the outcome in a square law battle is determined by a comparison of  $by_0^2$  with  $ax_0^2$ , these quantities are referred to as "fighting strengths." It is important to note that the fighting strength is proportional to the lethality coefficient and to the square of the initial number of force units. This heavy dependence on force numbers is reasonable if one considers that there are two distinct reasons for introducing a new unit into a square law battle:

- The new unit fires at the enemy, and
- The new unit dilutes the enemy's fire against the units already in battle.

One consequence of these advantages of force numbers is that an attacking force can make up for an inferior lethality coefficient by having superiority in numbers. Note also that the doubling of a force level gives it a four-fold increase in fighting strength, whereas a doubling of the lethality coefficient provides for only a two-fold advantage.

## 2. Linear law (Unaimed fire)

If X's fire is merely directed into Y's operating area, rather than being aimed at a specific Y unit, then the attrition rate for Y will be proportional to both y and x. In the simplest situation, each x unit fires at a constant rate r and each

shot eliminates all y units within some fractional portion  $\frac{p}{A}$  of the total area A over which the y units are uniformly distributed. Here p is the area of lethality of a single firing from an x unit. Thus,  $\frac{dy}{dt} = -(rx)(\frac{p}{A}y)$ . Identifying  $rp/A$  as a, and the corresponding parameter for the other side as b, we are led to the equations for Lanchester's linear law:

$$\frac{dx}{dt} = -bxy \tag{2.6}$$

$$\frac{dy}{dt} = -axy$$

By eliminating time and proceeding as before, we obtain

$$a(x_0 - x) = b(y_0 - y) \quad \text{for} \quad x, y > 0 \tag{2.7}$$

and also

$$y_f = 0 \quad \text{and} \quad x_f = x_0 - \frac{b}{a}y_0 \quad \text{if} \quad ax_0 \geq by_0 \quad \text{and} \tag{2.8}$$

$$x_f = 0 \quad \text{and} \quad y_f = y_0 - \frac{a}{b}x_0 \quad \text{if} \quad ax_0 \leq by_0$$

In this situation the fighting strength is now  $ax_0$  or  $by_0$ ; that is, the product of the lethality coefficient and initial force level. There is no "dilution effect" for the linear law (since fire is unaimed) and consequently there is a lesser influence of force numbers on fighting strength capabilities than in the square law.

The main purpose for presenting the Lanchester models was to show that they are inadequate for describing modern naval combat. They fail to take into account such factors as scouting, the staying power of a unit, or the effects when a force is not homogeneous and consists of different kinds of ships. But perhaps most important these models do not take into account the effects of pulse weapons characterized by instantaneous delivery of substantial combat power.

### III. TWO NAVAL MODELS

#### A. THE BASIC SALVO EQUATION

This equation was developed by Hughes in his book *Fleet Tactics : Theory and Practice* to show the tactical consequences if a warship had the combat power to destroy more than one similar warship with a single salvo. Most of the concepts presented here are from Chapter VII of Hughes' work [Ref. 2 ].

##### 1. Assumptions

- The striking power of the attacker is the number of accurate (good) shots launched
- Good shots are spread equally over all targets. A uniform distribution is not necessarily the best distribution. If each target's defense extracts an equal number of accurate shots, the whole strike may be defeated, whereas an uneven distribution concentrated against only some targets would put at least those targets out of action. In our study we'll examine cases in which shots are unequally distributed, as some of the targets in specific tactical situations have more "value" than the others.
- Counterfire by the target force eliminates with no "leakage" all good shots until the force defenses are saturated, after which all good shots are hits. Mathematically, a subtractive process best describes the effect of counterfire.
- Weapon range is "sufficient" on both sides. In other words, neither side has a weapon range and scouting advantage such that it can detect, track and target the other safely outside the range of the enemy's weapons.



## 2. Force-on-force Equations

The equations which gives us the combat work achieved by a single salvo at any time step from force A or B respectively is:

$$\Delta B = \frac{a \cdot A - b_3 \cdot B}{b_1}$$

and

(3.1)

$$\Delta A = \frac{b \cdot B - a_3 \cdot A}{a_1}$$

where in the first equation:

$\Delta A$  = the number of ships lost in force B by force A's salvo

$a$  = the offensive power of a single unit in force A

$b_3$  = the defensive power of a single unit in force B

$b_1$  = the staying power of a single unit in force B

$a \cdot A$  = the total offensive power of force A

$b_3 \cdot B$  = the total defensive power of force B.

The corresponding terminology holds for the second equation too. The combat power of a salvo is measured in hits that damage the target force, and is the numerator of the equations above. Combat power achieves combat work in hits. When divided by the number of hits a target can take before it is out of action, work on the enemy is measured in ships GOA (out of action).

### 3. Model-based Conclusions

- As missile combat is force-on-force, we may need to examine the fraction of each force that can be put OOA by a salvo. This is expressed by the equations :

$$\frac{\Delta A}{A} = \frac{b \bullet B - a_3 \bullet A}{a_1 \bullet A} \tag{3.2}$$

$$\frac{\Delta B}{B} = \frac{a \bullet A - b_3 \bullet B}{b_1 \bullet B}$$

If we want to have the comparative effectiveness of the two sides we have to divide one equation by the other to obtain a Fractional Exchange Ratio (FER) :

$$\text{FER} = \frac{\Delta B/B}{\Delta A/A} = \frac{(aA - b_3B)(a_1A)}{(bB - a_3A)(b_1B)} \tag{3.3}$$

When FER > 1 then A will have forces remaining when B is out of action, and when FER < 1 then B will have forces remaining.

- "Excess" offensive and defensive power in the form of overkill now have a significant effect on the results.
- The Fractional Exchange Ratio is unreliable when overkill exists.
- From the FER equation we can infer that for B to achieve parity in FER when A is twice as numerous as B, then each B unit must have twice the striking power, twice the defensive power, and twice the staying power of each A unit. This advantage of numerical superiority relative to the other attributes seems to hold over many, if not all, situations.

In his master's thesis, Hatzopoulos developed a naval combat model using the basic Salvo equations. As today's sea battles are based primarily on missiles, his model represents a missile combat. However he proceeded further by incorporating in his model the effects of human factors. So the effects of scouting , training, morale, leadership, and alertness are incorporated in the model.

These terms, although enriching the analytical potential and flexibility of the salvo model, confuse and complicate our basic understanding of the interrelations because they increase the number of parameters.

The assumptions that Hatzopoulos used for the human factors are:

- Scouting effectiveness,  $\sigma_A$  or  $\sigma_B$  , takes values between 0 and 1 and measures the extent to which striking power is diminished due to less than perfect targeting and distribution of fire against the target force.
- Similarly, defender alertness, or readiness  $\tau_A$  or  $\tau_B$  , takes values between 0 and 1 and measures the extent to which counterfire is diminished due to less than perfect readiness or fire control designation to destroy the missiles of an enemy attack.

There are also multipliers which represent the factors seduction chaffs, distraction chaffs and training but as we will not deal with them, we omit them. For our purpose we will assume that each crew has its highest level of training. We will also take into consideration the effect of the seduction chaffs during the development of our model and will compute the contribution of seduction chaffs in the defense of a unit.

A "group" denotes a subdivision of a force and consists of several units that operate together. In order to compute the aggregate staying power of a group we have to sum the staying power of each unit which belong to that group. The staying power of group k in the Blue force is given by:

$$SP_{kb} = \sum_{j \in k} SP_{jkb} \quad \forall k, \quad (3.4)$$

where  $SP_{jkb}$  denotes the staying power of unit j in group k of the Blue force. With the new terms mentioned above, the embellished force-on-force equation which gives the aggregate percentage loss of group k of the (defending) Blue force (which also represents the destroyed staying power of group k) is given by the formula:

$$LOSS_{jkb} = \frac{\sigma_r H_b \sum_j M_{j'k'r} \tau_b \sum_j N_{jkb}}{SP_{kb}} \quad (3.5)$$

where:

H = the probability of striking an undefended target for each missile. In

other words ,H represents the firing accuracy given for each type of missile and depends on the distance to the target.

$M_{j'k'r}$  = the theoretical  <sup>$\epsilon$</sup>  number of missiles that unit j' of group k' in the Red force can fire in a single salvo.

$N_{jkb}$  = The number of missiles a defender (j platform in k group in the Blue (force) can shoot down per salvo (the best he can do).

$\sigma_r$  = Scouting function of the attacking Red force.

$\tau_b$  = Alertness modifier of the defending Blue force.

The equation above was derived in Chapter IV of [Ref. 4]. The summation symbols in the numerator of Eq.(3.5) have the following meaning: the first summation symbol is used to sum the missiles from all the platforms belonging to group k' in the Red force that fire missiles. Thus not all of the platforms or units in group k' necessarily have the ability to fire. This may happen in some particular situation, for example where some ships do not fire due to the formation or other reason.

## **IV. DEFENSIVE POWER IN ANTI-AIR WARFARE**

### **A. INTRODUCTION**

Defensive power has been defined as the number of ASCMs which would hit the attacking unit or force that will be destroyed or averted by a defending unit or force. Our primary goal is to show how a Commander can improve the defensive power of his force in an anti-air warfare situation.

Before developing the model which will help us study defensive power and reach some tactical conclusions about the formations and dispositions of ships, we must introduce the concepts and principles of our model.

We already know that today's sea battles and sea control are based primarily on missiles, the most important of naval weapons. Both surface and anti-air warfare are based on missile attacks and in both cases the ways that a force defends itself against a salvo attack are pretty similar. We believe that the same model, with small changes, can be used for surface warfare as well, and that the tactical conclusions can be easily generalized and used for that situation.

By the term air defense we mean the defense against air-to- surface missiles (ASMs) which have been launched from aircrafts. We are not going to examine explicitly the case of a force defense against aircraft attack with bombs.

## **B. AIR DEFENSE THEORY**

The objective of air defense at sea is to preserve, in the face of airborne attack, the effectiveness of surface units so that they can carry out their assigned missions. We note that the anti-air warfare environment has changed drastically since the second World War. Aircraft and airborne vehicles were about to become much faster with the improvement of jet engines, rocket motors and their variants. The extremely rapid advance of technology in the fields of radar, gyro-controlled stabilization and inertial navigation systems made it possible for weapons and their carriers to be much more precisely controllable, accurate and autonomous. Are naval surface units to become useless after all? Not if a nation wants to dominate at sea. As Themistocles of ancient Greece once said: "The wooden walls<sup>1</sup> "will save the town", meaning that sea power is the most important thing for a maritime nation. Sea power is the ability to use the sea.

Moreover, to secure the use of surface and sea transportation, fighting units on the surface are required. As a result, the provision of air defense for surface units has been a major preoccupation for marine powers worldwide. The same technology that sharpened the threat is used as the means for countering it. Let us now examine in detail the main principles of air defense.

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<sup>1</sup>Wooden walls: Referred to the wooden triremes

### **1. Denial of information to the enemy**

To deny information to the enemy has several advantages. It may present him with fewer targets or even no targets at all, or it may take him longer to piece together such information and therefore delay his attack. In our model the factor which represents the level of information of the attacking force is  $I_s$ , the scouting function for the attacking force.

### **2. Early warning**

It is obvious that air attack incidents at sea tend to be very fast-moving and sudden. So the alertness of the defending force plays a great role in the combat result. In our model  $A$  is the alertness modifier for the blue force. We cannot always assume that all the units have the same alertness, so we'll examine two different cases.

### **3. Attack at the source**

Destroying the platform that poses an air threat is the best way of dealing with it. First, it eliminates the threat from that unit. Second it eliminates another attack from that unit and reduces the opponent's trained manpower. However, for small navies their destruction is not an easy achievement. Since destroying the attacking aircraft, before they approach and fire their weapons, is connected with the presence of CAP units, we cannot expect that a naval force without its own aircraft will always have that possibility.



#### 4. Layered defense.

Air defense at sea is "layered" in the sense that we "defend" all the way from the starting point of the threat up to its intended point of impact. Attack at source is part of that process, the outer layer we could say. Although we may have four layers - outer air battle, area defense, point defense, close-in weapon systems and countermeasure defense - we will combine the last two layers into a single one for reasons of convenience.

### C. MODEL DEVELOPMENT

The three cases which we now study from the defensive power point of view are: area defense, point defense, and sector defense.

#### 1. Area defense

This case follows the CAP's battle. Attacking aircraft which survive the CAP shoot their missiles against the defending force. Here we have to make two new assumptions. First, that only ships with area defense weapon systems will shoot; and second that they will shoot against the incoming missiles without knowing which one of them is going to hit some ship. We examine two subcases:

##### *a. Full alertness of the total force*

In this case the total number of missiles which penetrates the area defense layer is given by the equation :

$$T = [a \cdot \sigma_a \cdot A - \tau_b \cdot \sum_i^{B1} b_{3i} \cdot H_{1b}] \cdot H_{1R} \quad (4.1)$$

where :

$a$  = number of missiles fired by each aircraft,

$A$  = total number of aircraft,

$\sigma_\alpha$  = scouting factor for the attacking red force,

$\tau_\beta$  = alertness factor of the defending force,

$b_{3i}$  = number of missiles that the  $i^{\text{th}}$  ship with area defense weapon systems can fire before saturation.

$H_{1b}$  = probability that a SAM will hit the target,

$H_{1R}$  = probability that a ASCM will hit a ship given that it has not been shot down.

***b. Only partial alertness of some ships***

In this case each ship has a different degree of alertness. But the computed result will be the same as above if the average alertness of the force is equal to some given alertness. If, for example, in the previous case we had a total alertness of 0.5 and now suppose that we have two ships each with alertness 0.2 and 0.8, then the result will be the same as for an average alertness of 0.5, just as it was previously. In that case we may also assume that the attacker has the advantage of surprise, which in our model can be shown by decreasing the probability  $H_{1b}$  of shooting down a missile.

Now the number of missiles which penetrates the area defense layer is given from the equation :

$$T = [a \cdot \sigma_a \cdot A - \sum_i^{B1} \tau_i \cdot b_{3i} \cdot H_{1b}] \cdot H_{1R} \quad (4.2)$$

which incorporates an alertness factor  $t_i$  for each ship  $i$  having an area defense system.

## 2. Point defense.

This situation is more complicated than the previous one because it occurs in the final stages of the battle and the result gives the loss of the total force. In some forces there could be more than one group, and we might become interested in the losses in each of them. As a consequence we must know the distribution of the ASCMs missiles towards each ship or group. Will the missiles be equally distributed among the force? Is it more likely that some of them home on some specific group (e.g. a CV or an amphibious group)?

The ASCM distribution determines whether each ship can use both hard and soft kill weapons simultaneously and each missile can be shot down either from the one or from the other. Even though an ASCM in the point defense layer may have to face the hard kill weapons at first, and then the soft kill weapons (or vice versa, because of the very short time period between these two actions) we model defensive power as if a ship uses its hard and soft kill weapons simultaneously.

Therefore, we can compute the probability  $H_{2R}$  that a ship will destroy an ASCM due to its hard or its soft kill weapon system as follows. Let us say  $P_h$  is

the probability that a hard kill weapon may destroy an ASCM and that  $P_s$  is the probability that a soft kill weapon may destroy the same ASCM. Thus, the probability that an ASCM will not be destroyed by a hard kill weapon is  $(1-P_h)$  and the corresponding probability for a soft kill weapon is  $(1-P_s)$ . As a result the probability that the ship will not destroy the ASCM is  $(1-P_h)(1-P_s)$  yielding the following result:

$$P[\text{a ship destroy an ASCM by any hard or soft kill weapon}] = 1 - (1-P_h)(1-P_s) = H_{2b}$$

In the point defense case the defender sees the missiles which penetrate the point defense layer to home in on him (ship or group) and defends against them accordingly. So, the total loss of group k of the defending force is given by the equation:

$$LOSS_{kb} = \frac{T' \cdot H_{2R} \cdot \sum_i^{B_k} b_{3i} \cdot H_{2b}}{\sum_i^{B_k} SP_{ib}} \quad (4.3)$$

where :

$T'$  = The number of missiles which will home in on some group. These missiles are a portion of the total number  $T$  which have penetrated to the point defense layer.

$H_{2R}$  = The probability that a missile if it penetrates the point defense layer, will hit its target. That probability now is bigger than in the

case of the area defense because the missile now is closer and can "see" more clear, since its seeker has a bigger aspect of the target.

$H_{2b}$  = The same as defined previously.

$SP_i$  = Staying power of  $i^{\text{th}}$  ship on the  $k$  group.

The summation in the nominator gives the total number of missiles that can be shot down by group  $k$ . On the other hand, the summation in the denominator gives the aggregate staying power of group  $k$ . Here it is assumed that the missiles which home on any group  $k$ , are uniformly distributed over all units of that group.

### **3. Sector defense**

In this case, we assume that the total area containing the unit/units we want to protect is divided into a number of equal sectors. At least one ship is assigned to each sector and it is responsible for every missile entering that sector. The main reason for using sectors in air defense is that better coordination and engagement with the attacking missiles can occur. However, these benefits are diminished when using area defense for the following reasons:

- In a small naval force the number of ships with area defense weapon systems is not big enough to cover very many sectors.
- Due to the distance from the force in which area defense engagement takes place, we can easily assume that the total engagement area covers one or two sectors at most. Since any weapon system is limited in the number of missiles it can engage in a given amount of time, then the area defense system of one ship can be saturated more easily. Of course that can easily happen in point defense, if missiles are not equally distributed among

sectors. For these reasons we believe that in the area defense case we have nothing to gain if sector defense is applied.

In point defense we may distinguish two cases: the missiles are equally distributed among the sectors, or they are not. In the first case, we diminish the chance of bad coordination, which may result in overengagement of some missiles. So here, we expect that more ships will engage and more missiles will be shot down. The opposite result happens if the attackers from the area defense concentrate their attack on a few sectors. The weapon systems of ships in those sectors can be saturated more easily.

To avoid that situation, when we know the axis of the threat, we have to divide the total area into a number of unequal sectors, concentrating more ships towards the threat axis. In order to obtain the total number of ships lost, each sector has to be examined separately and then the results summed over all sectors. From the tactical situation the number of sectors and the number of ships in each one of them has to be decided in advance. We assume that each ship shoots only at ASCMs in its sector.

**a. Missiles are equally distributed among all sectors**

If T is the number of penetrators to the point defense area and X is the number of sectors, then each sector should expect  $\frac{T}{X}$  missiles. As a result, for sector i the loss will be given by the formula:

$$Loss_i = \frac{\frac{T}{X} \cdot H_{2R} \cdot \tau_b \cdot \sum_i^{B1} b_{3i} \cdot H_{2b}}{\sum_i^{B1} SP_{ib}} \quad (4.4)$$

where :

$T/X$  = The number of missiles that will home in on sector  $i$ ,

$H_{2R}$  = The same as before,

$\sigma_{\beta}$  = The same as before.

$B_i$  = The number of ships that are assigned to sector  $i$  (If in the same sector there are ships from more than one group then we have to find the losses for each one assuming that the missiles are equally distributed among the ships in the same sector.)

$\tau_b, b_{3i}, H_{2b}, SP_i$  = As before.

***b. Missiles are concentrated in particular sectors***

In this situation it is more likely that the weapon system of ship/ships in those particular sectors will be easily saturated resulting in some missiles reaching their target. Therefore we have to consider different tactical situations and examine each possible outcome. We should reexamine the loss in each sector in order to find the total loss, by using the formula (4.4). The only difference is that  $T/X$  no longer holds. Now the number of missiles is  $T \cdot Y_i$ , where  $Y_i$  is the fraction assumed to attack in the  $i^{\text{th}}$  sector.

## V. APPLICATION OF THE MODEL

### A. INTRODUCTION

In order to use the model the user must determine the following parameters (In parenthesis we give the symbol we use in the program to designate that parameter):

- The numbers of units which compose each force. For our purpose, the red force, or the attackers, is the number of aircraft (A1) which carry ASCMs. The blue force is an amphibious force composed of a number of escorts, DD or frigates, with either area (SC1) or point (SC2) defense weapon systems, but not both. This means that the ships with area defense systems are vulnerable to the missiles' attack after their penetration into the point defense layer. This also results from the assumption that a ship with a point defense weapon system can defend only itself. In addition, there is a number of landing ships (LF) with point defense only (in some scenarios).
- The number of missiles (a) each aircraft can carry.
- The scouting function (SA) for the red force.
- The launch reliability (RL) for each ASCM.
- The probability (H1R) that an ASCM will hit a ship given it has not been shot down, when it is in the area defense layer.
- The probability (H2R) that an ASCM will hit its target, when it is in the point defense layer. For both probabilities we use Figure 8 from Ref. 6, p 34. which gives the hit probabilities as a function of range.
- The maximum number of missiles (B3) in each salvo that each ship with area defense can theoretically engage before it is saturated.
- The probability (H1B) that an engaged missile will be shot down from an area defense weapon system. We consider a shoot-shoot-look engagement



as a doctrine. Therefore, if we assume the single shot probability is approximately 0.7, then the probability of hitting any incoming missile is about 0.91 [Ref. 8, p 34].

- The maximum number of missiles each ship with point defense can engage due to its hard or soft kill weapon systems (B32 for the escorts and B33 for the landing ships).
- The probability that an engaged missile will be shot down by a point defense system for each escort (H2B) and for each landing ship (H3B) Here, for any escort, if we may as well assume that a single kill probability for a hard kill weapon system is approximately 0.67 and for any soft kill weapon system the same probability is approximately 0.65. Thus the probability of defeating a missile due to hard or soft kill systems is about 0.88. The corresponding probability for any landing ship is assumed to be about 0.70.
- The number of CAP's (CAP) as a part of the blue force (if there are any).
- The average number of attackers that each CAP can splash down (AV).
- The displacement for each ship of each type (DISPSC1, DISPSC2, DISPLF) in order to compute the aggregate staying power for each force.

The results from the model are the expected OOA fraction for each type of force DBSC1, DBSC2, or DBLF.

By choosing different values for each of the above parameters we could create a large number of different scenarios. However, as we are primarily interested in AAW engagement in littoral environment (which is carried out by a small naval force) we are able to reduce the number of cases. A computer program was written in MATLAB for the model and implemented on a PC. The code of that program is given in APPENDIX A. The data for each scenario are input one by one from the keyboard by the user. However, if the user so desires

he or she can modify the code and read the data directly from a file.

## **B. STUDY OF DIFFERENT SCENARIOS**

### **SCENARIO 1 (Figure 1)**

In this first scenario the naval force has CAP for support. The missiles which penetrate the point defense area have the same probability of homing on in an escort or a landing ship. Landing ships have no point defense weapon systems. In this and all the following scenarios we assume an initial missile inventory for the enemy of 90 ASCMs. The formation of the opposing forces is shown in Figure 1. The values of the various parameters discussed before are given in Table I.

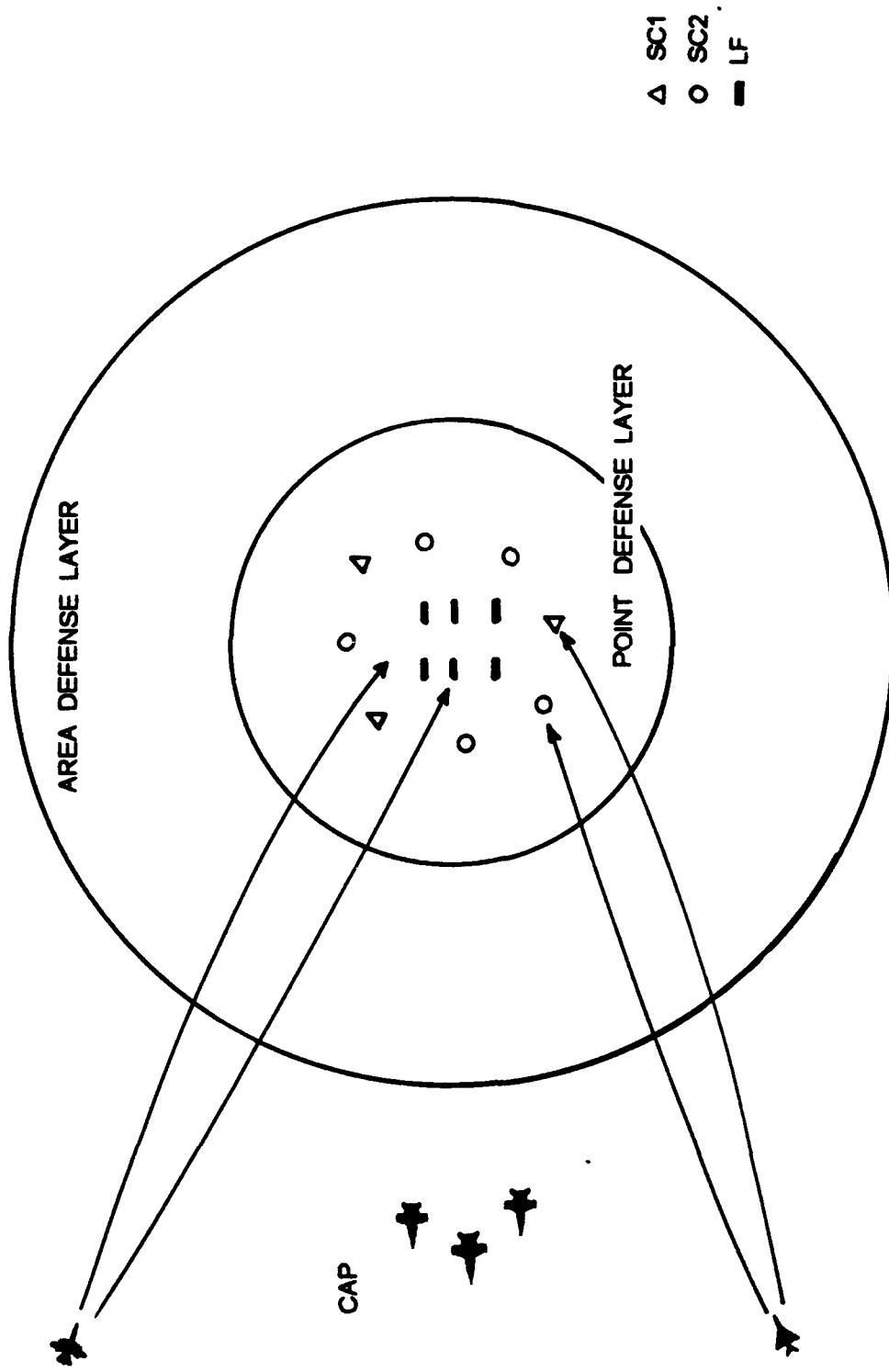


Figure 1. Formation of Opposing Forces in Scenario 1

**TABLE I. CHARACTERISTICS OF RED AND BLUE FORCES IN SCENARIO 1**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	3
SC2	5	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	5
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	0
H2B	0.88	H3B	0
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn		

Results

Number of aircraft that survived after the outer air battle : 10

Initial numbers of missiles launched : 15.66

Penetrators to point defense area : 9.03

DBSC1 = 0.5823 (1.74 ships)

DBSC2 = 0

DBLF = 0.5684 (3.2 ships)

So, in this scenario from a potential number of 30 missiles, 15.66 were actually launched and 6.63 were splashed down during the area defense battle.

The area defense group had a loss of 58%, the point defense had no loss and the landing group had a 56% loss. For the initial inventory of 90 missiles for the red force we see that the loss ratio is less than one for the area defense and the landing group. That means that with the first attack and with consumption of the 1/3 of the initial inventory, the red force destroyed more than 1/3 of the area defense and the landing groups.

Here we would like to note a deficiency of our model: It does not take into account the case of leakers (which means that some missiles have a chance to reach their target even though the target may engage all the incoming missiles). Let us take the SC2 group. Each ship can engage 2 missiles with a probability of 0.88 for each of them. In accordance with our scenario, each SC2 ship may accept a number of 0.645 missiles which may be a leaker with a probability of  $1 - 0.88 = 0.12$ . So the expected number of missiles for an SC2 ship is  $0.645 \times 0.12 = 0.077$ . Since 1.03 is the staying power of that type of ship, the expected loss of fighting capacity for the same ship is  $0.077 / 1.03 = 0.074$ .

**SCENARIO 2 (Figure 2)**

In this scenario we have no CAP, so no outer air- battle. In addition the landing ships now have a hard kill weapon system for point defense. The parameters are presented in Table II. See also Figure 2.

**TABLE II. CHARACTERISTICS OF RED AND BLUE FORCES IN SCENARIO 2**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	3
SC2	5	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	0	CAP	0
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	2
H2B	0.88	H3B	0.7
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn		

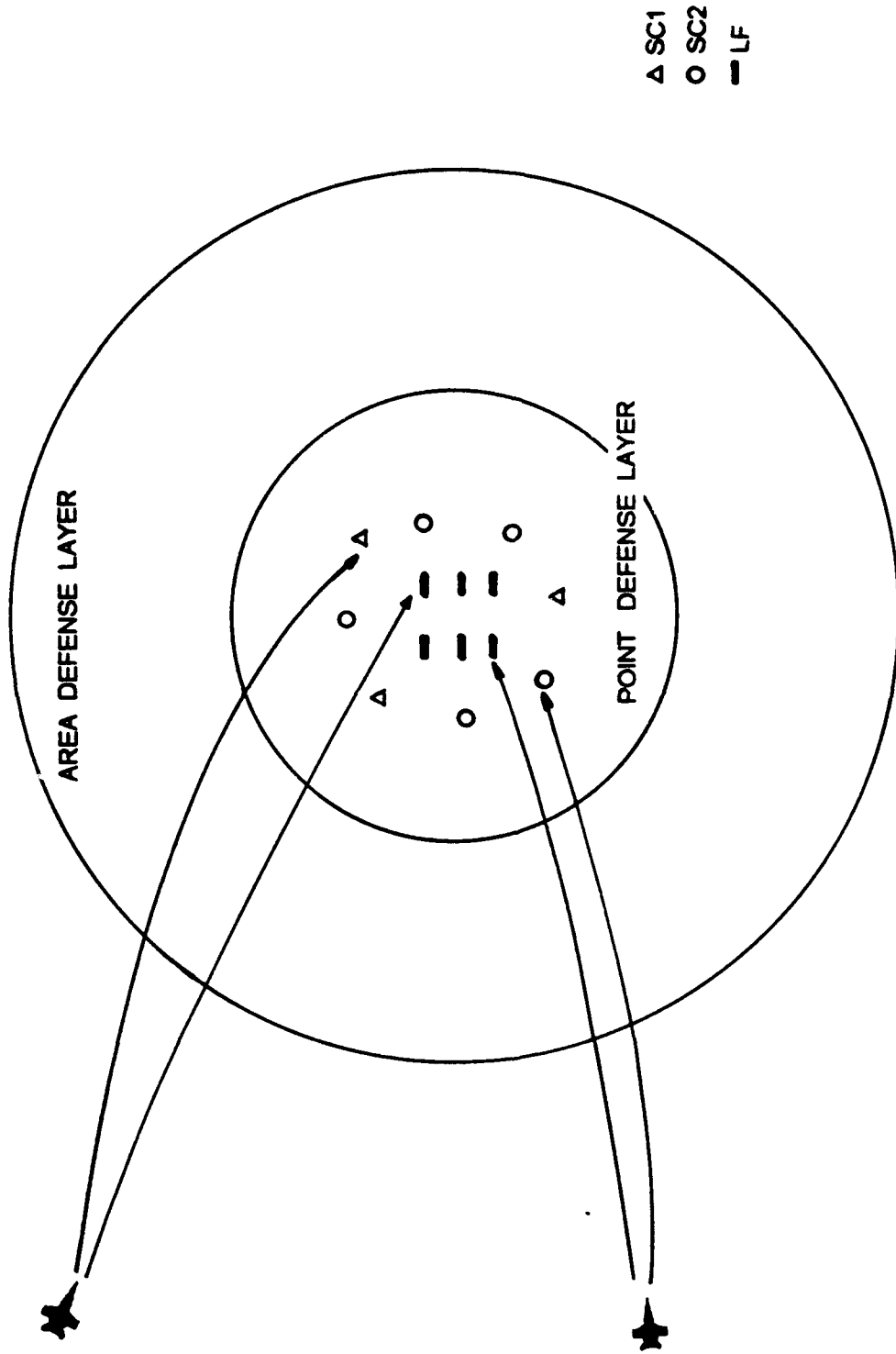


Figure 2. Formation of Opposing Forces in Scenario 2

## Results

Number of aircraft that launch missiles : 15

Penetrators to area defense layer : 23.49

Penetrators to point defense area : 15.30

DBSC1 = 0.9861 (2.95 ships)

DBSC2 = 0

DBLF = 0

We can see that in this scenario we have no CAP support so all the attacking aircraft will try to launch their missiles and finally, due to the scouting factor, 23.49 missiles enter the area defense layer. The number of penetrators to the point defense layer is 15.30. The area defense group now suffers a loss of 98%, which is double that of the loss in the previous scenario. This means that the red force during its first attack destroyed almost all of the area defense ships. On the other hand, even though there wasn't any CAP support, the landing force didn't suffer any loss because the landing ships had point defense and the point defense of the SC2 escorts and the landing ships is strong enough to defeat an even distribution of 15.30 incoming missiles (1.1 ASCMs per ship in the formation).

If we are further interested to see what the expected loss due to leakers would be we may perform the same calculations as for Scenario 1.



The results are that the expected number of missiles for any SC2 ship is  $1.1 \times 0.12 = 0.13$  and the expected loss is  $0.13/1.03 = 0.126$ . The expected number of missiles for any LF ship would  $1.1 \times 0.3 = 0.33$  and the expected loss is  $0.33/1.13 = 0.28$ .

**SCENARIO 3 (Figure 3)**

In this scenario we have CAP and the landing force has point defense, but the ASCMs are unequally distributed among the forces. We assume that the missiles are twice as likely to home on the landing ships as on in the escorts. Within each force each ship has the same probability of accepting a hit.

**TABLE III. CHARACTERISTICS OF RED AND BLUE FORCES IN SCENARIO 3**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	3
SC2	5	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	5
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	2
H2B	0.88	H3B	0.7
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn	Factor of preference LF:SC	2:1

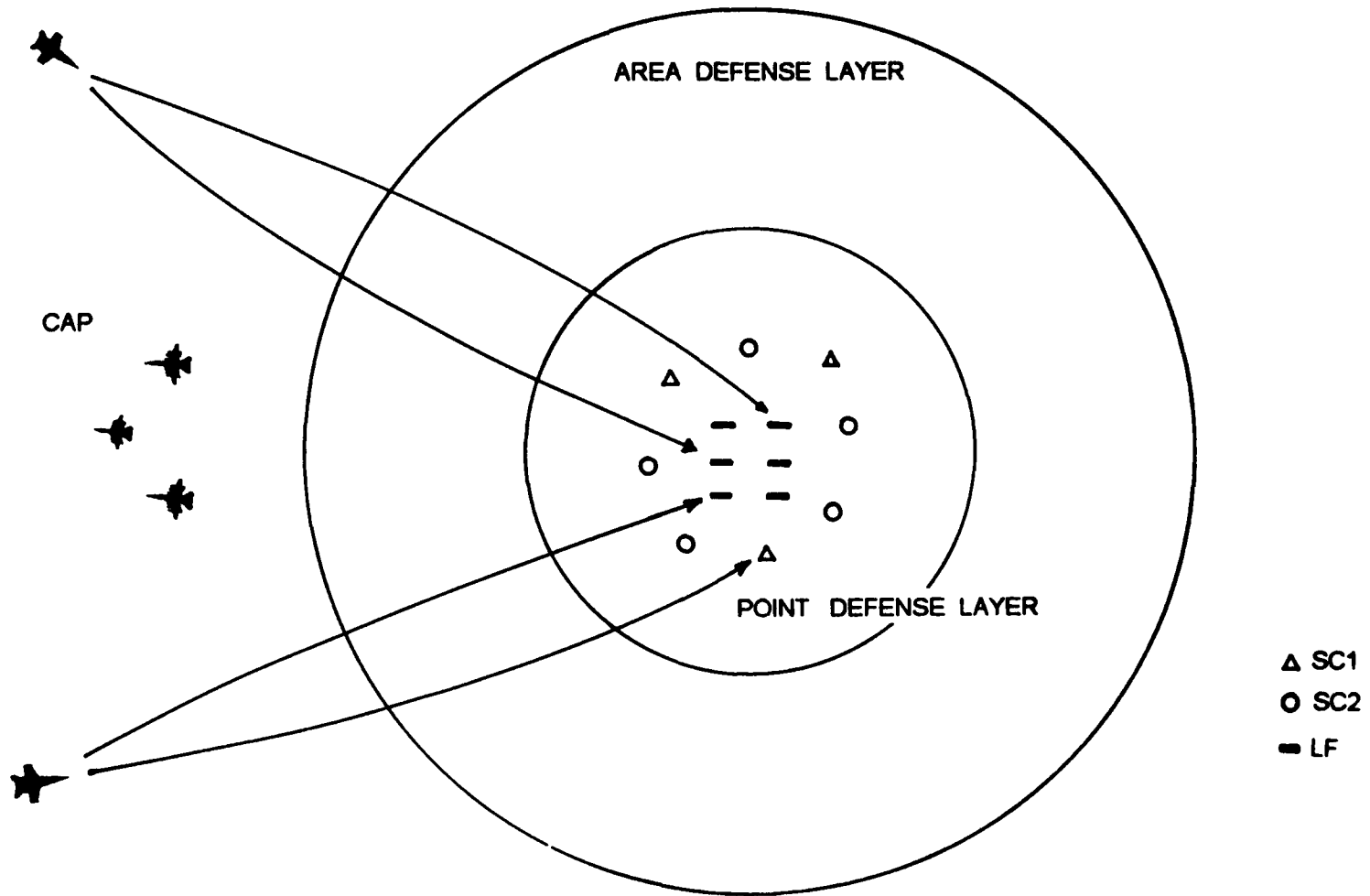


Figure 3. Formation of Opposing Forces in Scenario 3

## Results

Number of aircraft that survived after the outer air battle : 10

Penetrators to area defense layer : 15.66

Penetrators to point defense area : 9.03

DBSC1 = 0.34 (1.02 ships)

DBSC2 = 0

DBLF = 0

We see is that even though the number of missiles which home in on the landing force is larger, no ship of that force is suffering a hit. The number of ASCMs directed at SC1 is fewer, so the percentage of loss in the ships with area defense systems is almost half compared to that in the previous scenario. However the ratio is  $0.3/0.34=0.88$  which means that the rate of reduction of red missiles is still less than the reduction of the area defense ships.

## SCENARIO 4 (Figure 4)

This scenario is based on scenario 3, but now all the missiles are assumed to home in on the landing force. This scenario can occur when the escorts are far enough away from the landing ships so that when the seeker is activated it "sees" only the landing force.

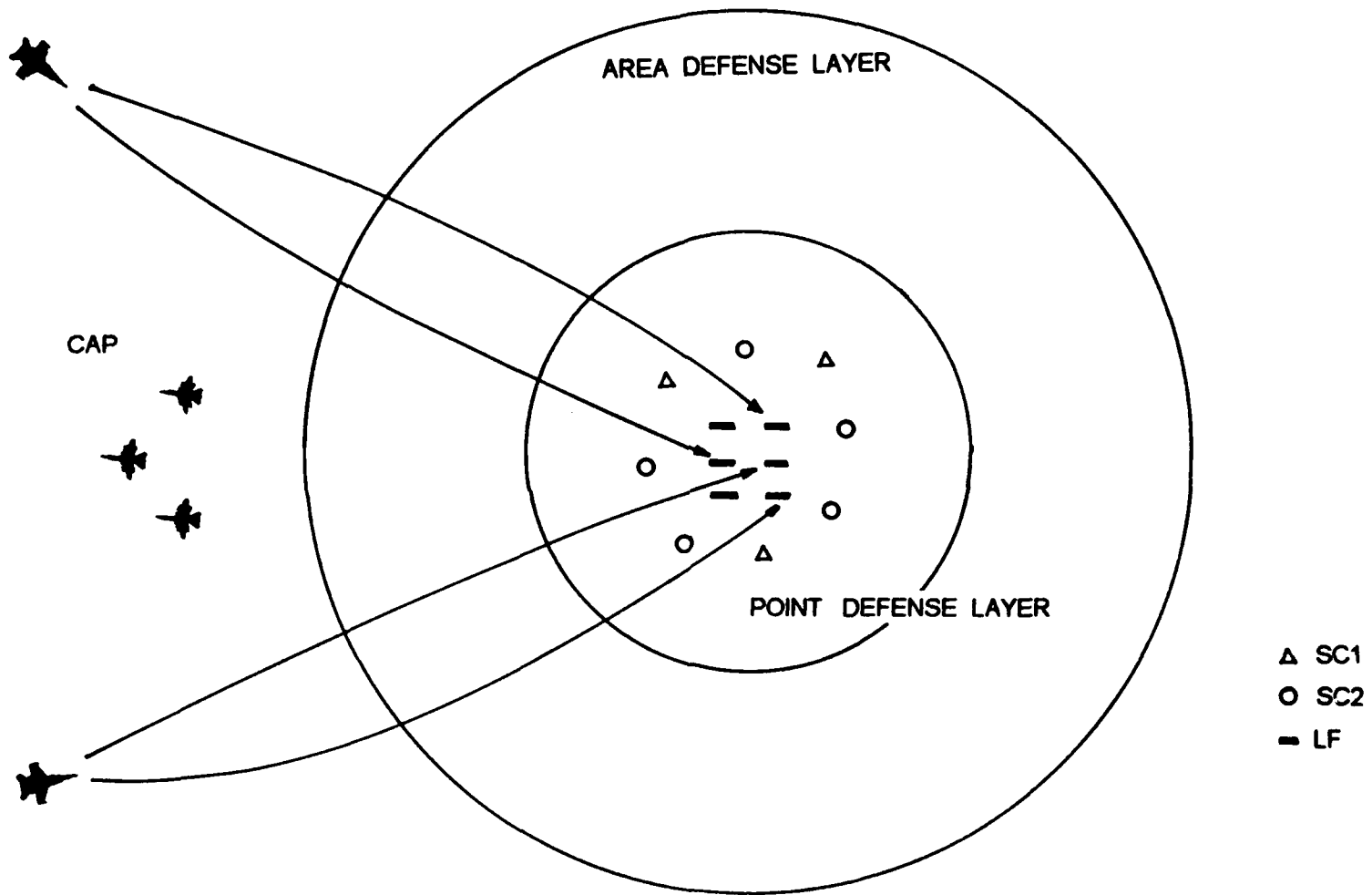


Figure 4. Formation of Opposing Forces in Scenario 4

**TABLE IV. CHARACTERISTICS OF RED AND BLUE FORCE IN SCENARIO 4**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	3
SC2	5	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	5
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	2
H2B	0.88	H3B	0.7
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn		

**Results**

Number of aircraft that survived after the outer air battle : 10

Penetrators to area defense layer : 15.66

Penetrators to point defense area : 9.03

DBSC1 = 0

DBSC2 = 0

DBLF = 0.1196 (0.7 ships)

With our assumption of an inventory of 90 missiles we see that for the landing force the loss ratio = 2.57 which is greater than one. This means that

diminishing of the red missiles is larger than the one of the landing force. What we observe, is that even though the initial homing of the red missiles was directed towards the landing force only, the losses of the latter are not severe. In addition to the above the role which the point defense systems play to defend the landing force becomes clear. If we compare the results with Scenario 1 we see that the landing force without point defense suffers a greater loss, even though the attack is with less missiles.

#### SCENARIO 5

This scenario is based on Scenario 1, with the additional assumption that the landing force has no point defense weapon systems. Here we'll compare the result in the landing force loss if we start with four ships with area defense weapon system and gradually replace each one of them by a ship with only point defense. The total number of escorts remains fixed. In order to have more information we'll assume that the enemy reattacks.

Here we assume that the enemy has already spent his missiles in his first attack. The second attack consists of a new airwing and the number of the aircraft in the second wave is the same as in the initial one. First, we compute the forces remaining from the first attack to obtain the staying power remaining in each group, the remaining defending power, and the number of ships remaining in each group. Then we compute results of the 2nd attack.

By using the percentage loss equation [Ref. 4:p 70] for each group we can find the remaining values of the parameters after the 1st attack. So if have any group k we have the equations

$$SP_k(\text{new}) = SP_k(\text{old}) (1-\text{LOSS}_k) \quad \forall k \text{ under attack} \quad (5.1)$$

$$\text{New number of units in group } k = \text{Old number of units} (1-\text{LOSS}_k) \quad \forall k \quad (5.2)$$

Also the loss percentage can be applied to the defensive ability of any group (i.e.the maximum number of missiles it can engage). If we let B denotethe ability of defense, then the new ability after attack is given by:

$$B(\text{new}) = B(\text{old}) (1-\text{LOSS}_k) \quad \forall k \quad (5.3)$$

**TABLE V. CHARACTERISTICS OF BLUE AND RED FORCES IN SCENARIO 5**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	4 down to 0
SC2	4 up to 8	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	5
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	0
H2B	0.88	H3B	0
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn		

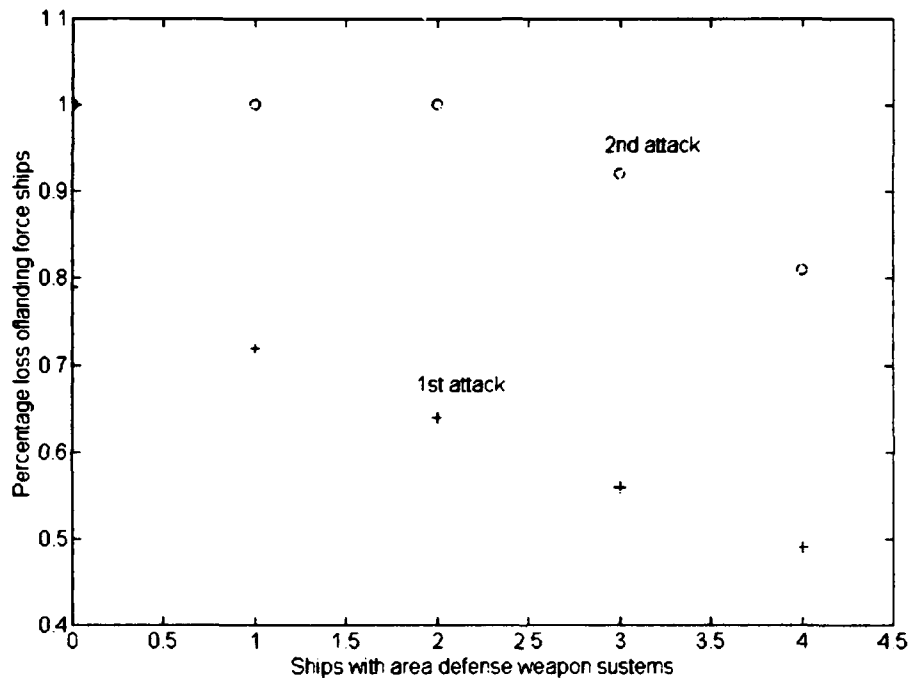
**Results**

In this case the final number of attacking aircraft and launched missiles is the same in all the subcases. However the number of missiles which finally home in on the landing force changes.

The results are shown in Figure 5. We see that the percentage of loss in the landing force diminishes as the number of ships with area defense systems increases, even though the number of ships with point defense systems is decreased by the same number.



In the first three cases we see also that there is a total destruction of the landing force after the second attack. Let us examine in more detail the situation of four escorts with area defense and four with point defense compared to all eight escorts having point defense only.



**Figure 5. Losses of Landing Force vs Number of SC1 Ships**

With four area defense ships, the number of ships remaining in the landing force is  $6 \times (1-0.5)=3$  ships after the first attack. After the second attack, the remaining number is  $3 \times (1-0.82)=0.54$  ships. On the other hand area defense ships  $6 \times (1-0.78)=1.32$  ships after the first attack. Thus we see that in the case

with four escorts having area defense, after the second attack the landing force has only one ship less than it had after the first attack no area defense.

From that we think the importance of the layered defense is clear.

#### SCENARIO 6 (Figure 6)

In this scenario we place three area defense ships far enough away from the landing force towards the threat axis so that the attacking aircraft must either enter in the area defense layer, or launch their missiles from a longer distance. As we have assumed that each area defense ship has no point defense systems we place beside each SC1 ship an SC2 ship for protection. In that case we create three pairs consisting of one SC1 and one SC2 ship for mutual defense (42/22 scenario). This means the two ships are close enough together (i.e., 300 yds) so that any missile which homes in on any SC1 ship enters into the point defense area of the corresponding SC2 ship and, hopefully, becomes engaged.

The total force consists of the following groups. Three pairs of SC1/SC2 ships (SC') with mutual defense and aggregate displacement of  $4000 + 3500 = 7500$  tons for each. Two SC2 ships (SC'') and six landing ships. Since the new groups are far apart from each other, the corresponding hit probabilities of an incoming missile is different for each group.

We examine two cases with two waves of attack for each. In the first case we assume the attacking force tries to destroy the picket pairs on the first attack and then concentrates the second attack the landing force. In the second case we assume the attacking force concentrates on the landing force in both attacks

while ignoring the pickets. In each attack we assume the same number of aircraft participate. The formation of the opposing forces is shown in Figure 6. The model parameter values for both forces are shown in Table VI.

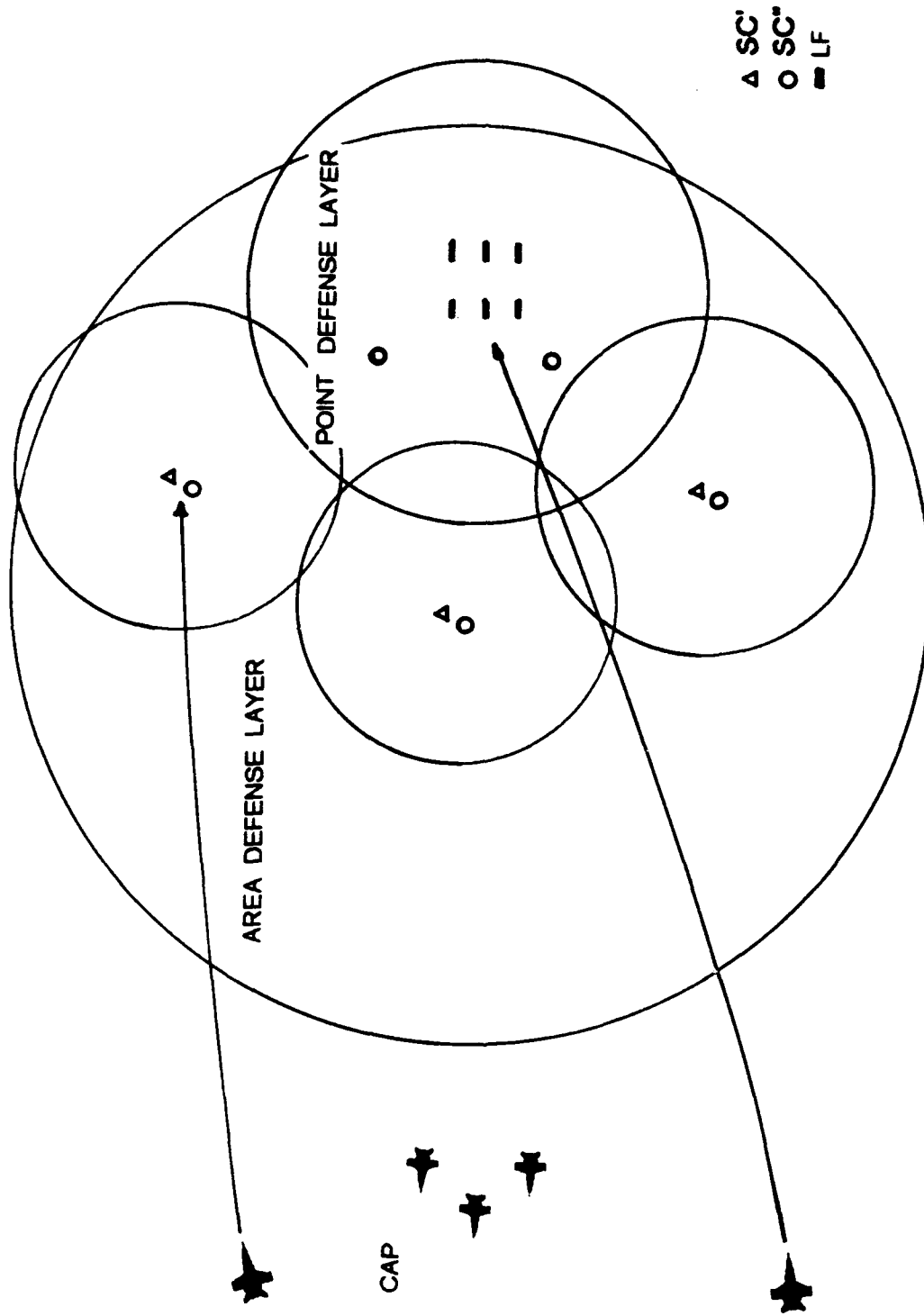


Figure 6. Formation of Opposing Forces in Scenario 6

**TABLE VI. CHARACTERISTICS OF RED AND BLUE FORCES IN SCENARIO 6**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC'	3
SC''	2	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	5
H1R	0.8 Case1 Attack1 0.5 Case1 Attack2 0.5 Case2 Attack2 0.5 Case2 Attack2	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	2
H2B	0.88	H3B	0.7
DISPSC1	7500 tn	DISPSC2	3500 tn
DISPLF	4300 tn		

Results

**TABLE VII. RESULTS OF SCENARIO 6**

Case 1		Case 2	
Attack1	Attack 2	Attack1	Attack 2
DBSC'=0.91 (5.46 ships)	DBSC'=0	DBSC'=0	DBSC'=0
DBSC''=0	DBSC''=0	DBSC''=0	DBSC''=0
DBLF=0	DBLF=0	DBLF=0	DBLF=0

The first thing to observe is that, independently of the way of the attack, neither the group of SC'' nor that of the landing force suffer any damage.

Even in the second attack of case 1, where the SC' group has been almost eliminated from the first attack, the final result for the other two groups is the same. The main reason for this is that we forced the attacking aircraft to attack from a greater distance, reducing the ASCM probability of hit.

With pickets in place such that the enemy aircraft cannot end-around them we put the enemy force in a dilemma of which group to choose first for the attack. So, if he has time, he must try a second or even a third attack in order to have some positive results.

#### SCENARIO 7 (Figure 7)

In this scenario we examine a sector defense. The main assumptions we make for this scenario are the following:

- The area around the HVU (i.e., the landing force) is divided into a number of sectors equal to the number of SC1 ships.
- In each sector we assign an equal number of ships which are responsible for the early detection and engagement of any threat in their sector.
- No ship engages any incoming missile not in its sector.
- It is assumed that the missiles in any sector are equally distributed among the SC of that sector and ships of the landing force.
- Aircraft all attack in one sector.

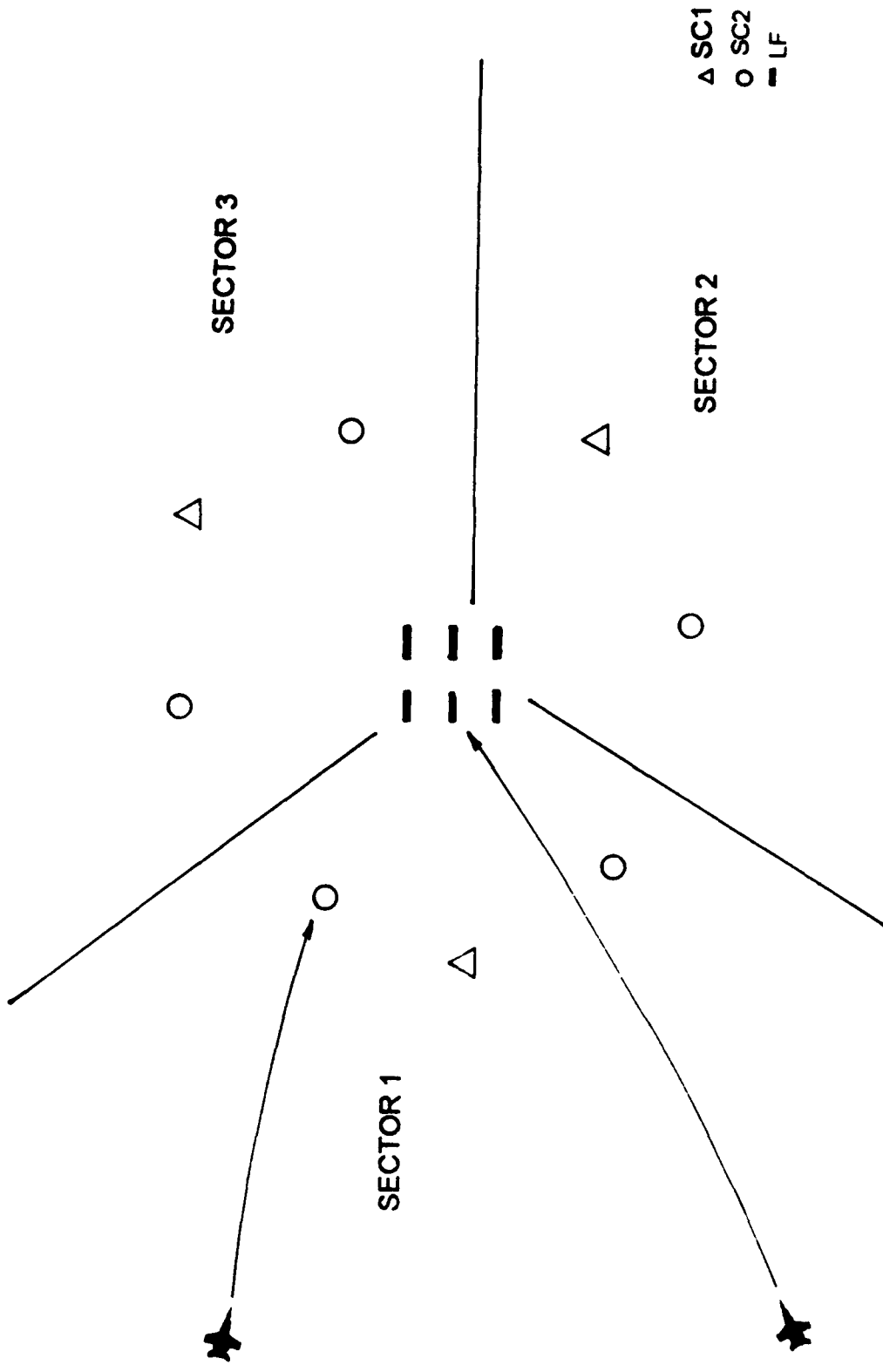


Figure 7. Formation of Opposing Forces in Scenario 7

**TABLE VIII. CHARACTERISTICS OF RED AND BLUE FORCES IN SCENARIO 7**

<u>Factor</u>	<u>Value</u>	<u>Factor</u>	<u>Value</u>
A1	15	SC1	3
SC2	5	LF	6
$t_b$	0.8 for area defen. 1.0 for point defen.	a	2
SA	0.9	RL	0.87
AV	1	CAP	0
H1R	0.8	H2R	1.0
B3	2	H1B	0.91
B32	2	B33	0
H2B	0.88	H3B	0
DISPSC1	4000 tn	DISPSC2	3500 tn
DISPLF	4300 tn	SECTORS	3

Results

**TABLE IX. RESULTS OF SCENARIO 7**

SECTOR	1	SECTOR	2	SECTOR	3
DBSC1	0	DBSC1	1	DBSC1	0
DBSC2	0	DBSC2	0	DBSC2	0
DBLF=1					

Due to the assumption that the missiles home in only on the ships of the sector through which they approach or those in the landing force, we see from the



first attack that all ships of the landing force are OOA . This did not happen in any of the previous cases. (Of course, the fact that we did not allow any point defense to the LF force also contributes to this result).

The other SC2 ships cannot take part in the point defense battle due to our assumption that the missiles attack only in one sector, so they do not face any danger. In order to show the difference between the sector and any other kind of defense, we now allow the other two SC1 ships to take part in the area defense battle. The results for the same sector are :

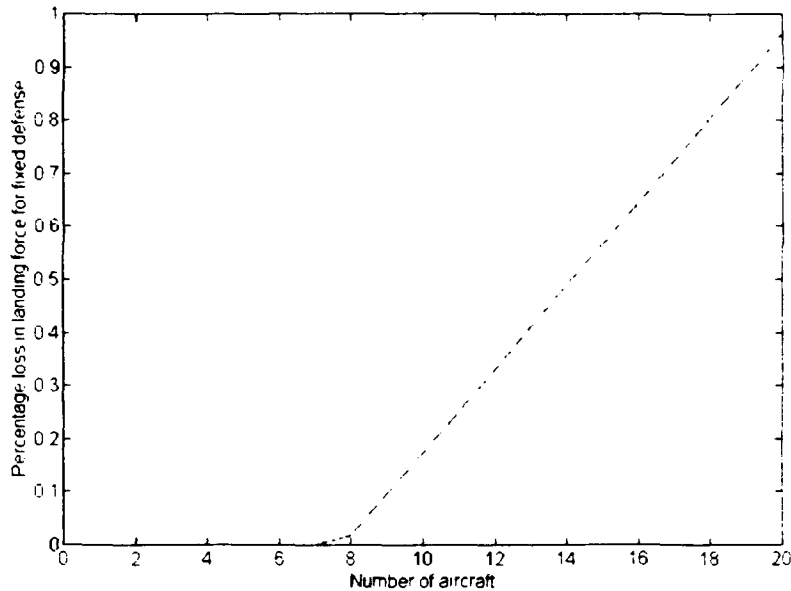
$$DBSC1 = 0.90, DBSC2 = 0, DBLF = 0.88 (5.2 \text{ ships}).$$

The results of Table IX correspond to the total number of ships .

We see that even though the other groups do not suffer any severe loss in numbers, the damage to the landing force is unacceptable. This results from the landing force being in the center of the sectors, so it can receive hits through any of them.

### C. ANOTHER USE OF THE MODEL

Finally, for each scenario, we may construct a graph where the x-axis represents the number of attacking aircraft and the y-axis represents the percentage of loss for a defined group (e.g., the landing force). The construction of such a graph assumes an unchanging defense. Figure 8 displays the graph which corresponds to Scenario 1. This graph can be used by both the attacking aircraft and the defending ships.

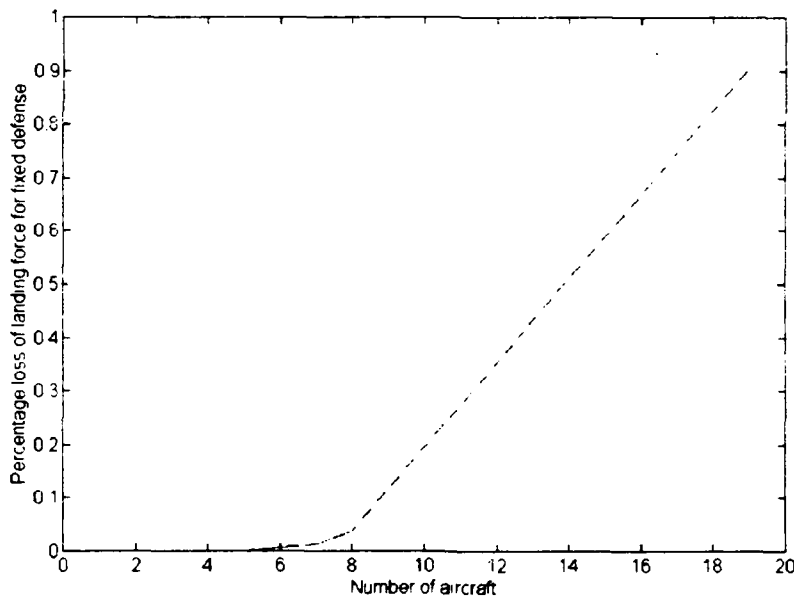


**Figure 8. Losses of Landing Force vs Number of Aircraft**

For a given target the attacking air force may determine the minimum number of aircraft (threshold) in order to have positive results. Therefore, for Scenario 1, we start to have positive results only after eighth aircraft. This means

that for up to eight aircraft the defensive power is strong enough to prevent any hits except for leakers. But after that point, which is its saturation point, we have a linear increase in percentage loss with the number of attacking aircraft.

If we would like to include the effect of potential leakers, then on the previous graph we should have two different lines. The first would be from zero up to the saturation point with a small slope, and the other would be from the saturation point up to the end with the same slope as in the previous graph but in a higher position. The corresponding graph for Scenario 1 showing the leakers, is displayed in Figure 9.



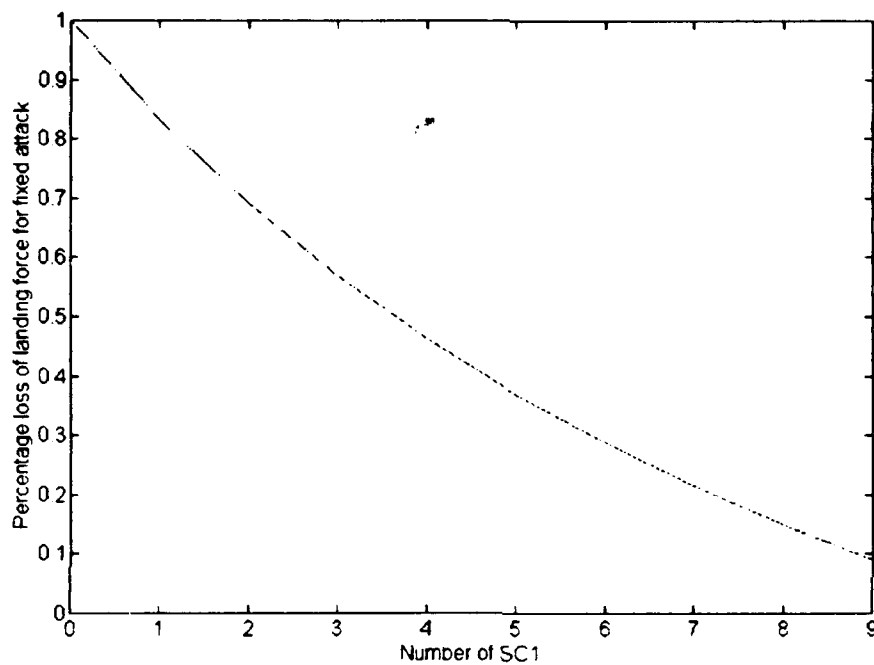
**Figure 9. Losses in Landing Force Including the Leakers**

Up to the first five aircraft there is no positive damage because they are eliminated by the CAP. From five up to eighth aircraft, some positive damage occurs due to the leakers. As, in accordance with the assumptions of Scenario 1, the landing force has no point defense, so the possible leakers come only from the area defense battle. As the maximum number of leakers from the area defense is only 0.34 missiles the net difference in the losses of the LF is not large. This situation can be seen if we compare the lines after the eighth aircraft from both graphs.

In Table X ,(column 2), the saturation point expressed in number of aircraft is given for the different scenarios. As we can see Scenario 2 gives the best protection to landing force. From the same graph we can also determine the number of aircraft which are needed in order to reach a desired level of damage to enemy force .

On the other hand, given the number of attacking aircraft the naval force may determine the required number of escorts in order to eliminate losses up to some preassigned level. In other words, for an expected number of attacking aircraft we can determine the minimum number of surface combatants (with given defensive power) that are needed to escort (or to procure) for a desired level of protection to our HVU. Or in the same way, the kind and numbers of weapons systems required to arm a given number of escorts in order to have the same results.

In Figure 10 we can see the number of SC1 which are needed to keep the damage to the landing force less than 0.1 in Scenario 1, assuming that the five SC2 ships already are in the screen.



**Figure 10.** Required Number of SC1 Ships to Have Less than 0.1 of the LF OOA.

In Table X, (column. 3), the contribution of SC1 ships needed to keep the level of damages in the LF less than 0.1 is shown for different scenarios.

**TABLE X. SATURATION POINTS AND CONTRIBUTION OF SC**

<b>SCENARIO</b>	<b>Saturation point (# AIRCRAFTS)</b>	<b>Contrib. of SC1 ( losses &lt;= .1)</b>
1	8	9
2	18	0.8
3	18	0
4	14	3
7	1	14

## **VI. CONCLUSIONS AND RECOMMENDATIONS**

The principal aim of this thesis was to examine several tactical situations in a possible AAW battle in order to draw some useful conclusions. The entire study was done from the perspective of the defensive power of a Naval force. For this purpose we developed a model which deals with AAW battles. The main characteristic of this model is that the attrition of the Naval force is instantaneous and incurred through the application of pulses or "salvos." Therefore, we assume that the missiles reach their targets simultaneously, and sequential attacks happen in discrete time steps. After each step we compute the outcome of the battle.

Although the values of the parameters we used are not the real ones (e.g. number and displacement of specific ships), they represent those of a small modern Navy and the threat that it faces.

### **A. MAIN CONCLUSIONS**

The main conclusions regarding the scenario are the following :

- The presence of CAP in the battle is a crucial point. CAP represent the long hand of any Naval force. They not only contribute to diminishing the final number of missiles that will home on the Naval force, but also the number of aircraft which can reattack. Of course the latter can also be achieved from the Naval force, but not if the aircraft decide to do a stand - off attack.
- It is vital for each ship to have point defense as a last chance to defend itself. As can be seen from the results in Scenarios 1 and 2, the losses to

the landing force are much less when that the landing force has point defense systems.

- Ships that have no point defense systems should be placed under the protection of a ship with point defense. This is obvious if we compare the results in SC1 force (Scenario 2) where 1.09 missiles homed on each ship (total loss of the force :0.98) with the results in SC' force (Scenario 6) where 3.01 missiles homed on each pair (total loss of the force : 0.91).
- We should try to raise as many "defense walls" as possible in the path of any missile. The value of layered defense is clear from the results of Scenario 5.
- We should force the enemy's air force to attack from the longest possible distance away from the HVU. But how far away is that? This is a difficult question to answer because it highly depends on the geometry of the battle field. For example in littoral warfare, attacks from low flying aircraft are very common. These aircraft are very difficult to detect from long distances, so that a layer of defense in a very long distance situation may be unattainable.
- Sector defense increases the limitations of a Naval force, as was obvious in Scenario 7, where on the first attack we had a total loss of the landing force. Ships should not be prevented from engaging a target unless a friendly unit is directly downrange (on the ASCM bearing).
- A small Navy which, usually purchases ships and weapon systems " off the shelf," has to overcome its limitations through the choice of tactics and dispositions. Therefore it has (1) to study and exploit the circumstances of a possible future theater, (2) to cover the units without point defense under the umbrella of units with point defense, and (3) to use CAP coverage.
- Other crucial factors, through which a small Navy may minimize its limitations, are the fields of early warning and overall command and control. In littoral warfare where reaction times are small, alertness of the force should be high enough to prevent surprise by the attacker. High level in command and control should result in the best allocation of the weapon systems to targets and as a result the best overall air defense.
- Based on the available information for the enemy the model can calculate the saturation point and avoid it by strengthening the force (or avoiding the battle).



- The model can find the contribution of each type of escort (e.g. area defense) ships to the protection of the HVU for any actual operation expected. Then, for some desired level of protection it will be known if more escorts are required or that some of them are available for other missions.

## **B. GENERAL CONCLUSIONS**

- Another use of the model is as a planning tool. We may define the minimum number of ships that we need, in order to keep our losses to an acceptable level, considering the estimated strength of enemy attack. We may also use the model as a decision/programming tool. If, for example, we have to decide between purchasing a new area defense ship or spending the same amount of money for arming all landing ships with point defense weapon systems, which purchase will have the higher level of protection of the landing force? The model can be used from the attacking aircraft viewpoint as a tool for defining its bomber strategy for fixed defense.
- The Fractional Exchange Ratio (FER) did not work well because our forces were nonhomogeneous.
- A weakness of the model is that it doesn't take under consideration possible leakers. As no weapon system has a 100% probability of shooting down a missile, there will always be some small portion of leakers which will affect the results. However, if leaker rate can be estimated, the model can be embellished to reflect the effect they will have.
- Another limitation of the model is that weapon range is not explicit. The inputs and outputs are mean values. Nevertheless, despite its simplicity, the model has a large number of parameters.

## **C. RECOMMENDATION**

A recommendation for future research is to validate the model through historical or wargaming data in different tactical scenarios.

## APPENDIX A

### A. PROGRAM LISTING

```
%PROGRAM "AIR DEFENSE BATTLE"
clear;
clg;
disp('-----THREAT INPUTS-----')
A1=input('How many aircraft attack? ')
SA=input(' Give me the scouting factor SA for the attacking aircraft. ')
a=input(' How many missiles launch each aircraft? ')
RL=input(' What is the launch reliability for each missile? ')
%*****
disp(' ')
disp('-----INPUTS FOR THE AREA DEFENSE-----')
disp(' ')
SC1=input(' How many escorts with area defense W.S there are? ')
disp(' Give the probability H1R that an ASM will hit a ship. ')
H1R=input(' given that it has not been shot down. ')
disp(' ')
disp(' H1B is the probability that each SC1 ship will ')
disp(' shoot down a missile ')
disp(' ')
H1B=[];
for k=1 : SC1
    H1B(k)=input(' Give the probability H1B for each SC1 ship. ')
    fprintf('The prob. H1B for ship %1.1f is %1.1f ',k,H1B(k) )
    disp(' ')
    pause
end
disp(' ')
disp(' B3 is the number of missiles that each SC1 ship can shoot down. ')
disp(' ')
B3=[];
for j=1 : SC1
    B3(j)=input('Give the number of missiles for each SC1 ship. ')
    fprintf('The ship %1.1f can shoot down %1.1f missiles ' ,j, B3(j) )
    disp(' ')
    pause
end
```

```

end
disp(' ')
disp(' ')
disp(' Is the alertness unique for all the ships? If yes press 0 ')
AL=input(' else press any other positive number up to 9. ')
disp(' ')
%*****
disp('-----INPUTS FOR THE POINT DEFENSE-----')
disp(' ')
SC2=input(' How many escorts with point defense W.S there are? ')
Lf=input(' How many landing ships? ')
for i=1 : SC1
    disp(' Give me the displacement for each escort with area defense')
    DISPSC1(i)=input(' weapon system. ')
end
for i1=1 : SC2
    disp(' Give me the displacement for each escort with point defense')
    DISPSC2(i1)=input(' weapon system. ')
end
for l=1 : LF
    DISPLF(l)=input(' Give me the displacement for each landing ship. ')
end
SP1=0.070.*(DISPSC1.^0.333); %Here we compute the staying power
SP2=0.070.*(DISPSC2.^0.333); %Here we compute the staying power
SP3=0.070.*(DISPLF.^0.333); %Here we compute the staying power
pause
disp(' Distribution of missiles ')
disp(' ')
disp(' If the penetrating ASM"s are equally distributed among the')
disp(' the total force press 1.If they are unequally distributed press')
disp(' 2, and if all are homing towards the landing force press any')
X=input('other number up to 9. ')
disp(' ')
disp(' Give me the probability H2R that an ASM will hit a ship ')
H2R=input(' given that it has penetrated the area defense layer. ')
disp(' ')
disp(' H2B is the probability that each SC2 ship will ')
disp(' shoot down a missile due to her hard or soft kill weapons. ')
disp(' ')
H2B=[];
for kk=1 : SC2
    H2B(kk)=input(' Give the probability H2B for each SC2 ship. ')
    fprintf('The prob. H2B for ship %1.1f is %1.1f ',kk,H2B(kk) )
    disp(' ')
end

```

```

        pause
    end
    disp(' ')
    disp('B32 is the number of missiles that each SC2 ship can shoot down. ')
    disp(' ')
    B32=[ ];
    for jj=1 : SC2
        B32(jj)=input('Give the number of missiles for each SC2 ship. ')
        fprintf('The ship %1.1f can shoot down %1.1f missiles ',jj, B32(jj) )
        disp(' ')
        pause
    end
    disp(' ')
    disp(' H3B is the probability that each LF ship will ')
    disp(' shoot down a missile due to her hard or soft kill weapons. ')
    disp(' ')
    H3B=[ ];
    for l=1 : LF
        H3B(l)=input(' Give the probability H3B for each LF ship. ')
        fprintf('The prob. H3B for ship %1.1f is %1.1f ',k,H3B(l) )
        disp(' ')
        pause
    end
    disp(' ')
    disp(' B33 is the number of missiles that each LF ship can shoot down. ')
    disp(' ')
    B33=[ ];
    for jl=1 : LF
        B33(jl)=input('Give the number of missiles for each LF ship. ')
        fprintf('The ship %1.1f can shoot down %1.1f missiles ',jl, B33(jl) )
        disp(' ')
        pause
    end
    disp(' ')
    disp(' ')
    %*****
    disp('-----INPUTS ABOUT THE CAPS-----')
    CAP=input(' How many CAPS there are? ')
    if CAP > 0
        disp('-----OUTER AIR BATTLE-----')
        disp(' ')
        AV=input(' How many aircraft can each CAP splash in the average? ')
        A=A1-CAP*AV;
    else
        disp('-----THERE IS NO OUTER AIR BATTLE-----')
    end

```

```

    A=A1;
end
disp(' ')
disp(' The number of A/F that will try to launch missiles')
fprintf(' are %2.1f ', A)
disp(' ')
%*****
disp('-----AREA DEFENSE BATTLE-----')
if AL == 0
    t1=input(' Give me the alertness of the force. ')
    B=H1B.*B3;
    M=(a*SA*A*RL - t1*sum(B))*H1R;
else
    t=[ ];
    for i=1 : SC1
        t(i)=input('Give the alertness of each SC1 ship. ')
        fprintf('The alertness of ship %1.1f is %1.1f ' i, t(j) )
        pause
    end
    B=(H1B.*B3).*t;
    M=(a*SA*A - sum(B))*H1R;
end
disp(' The total number of missiles which will penetrate the point')
fprintf(' defense layer is %2.2f ',M)
disp(' ')
%*****
disp('-----POINT DEFENSE BATTLE-----')
t1=input(' What is the alertness now? ')
if X==1
    Ratio=1/(SC1+SC2+LF);
    M1=M*Ratio;
    DBSC1=(M1*SC1*H2R)/sum(SP1)
    if DBSC1 < 0
        DBSC1=0
    else
        DBSC1=DBSC1
    end
    B1= H2B.*B32;
    DBSC2=(M1*SC2*H2R - t1*sum(B1))/sum(SP2)
    if DBSC2 < 0
        DBSC2=0
    else
        DBSC2=DBSC2
    end
end

```

```

B2= H3B.*B33;
DBLF=(M1*LF*H2R - t1*sum(B2))/sum(SP3)
    if DBLF < 0
        DBLF=0
    else
        DBLF=DBLF
    end
elseif X==2
    disp(' Give me the factor by which the LF ships are more ')
    p=input('preferable. ')
    den=p+1;
    M1=M*(1/den);
    Ratio= 1/(SC1+SC2)
    M1=M1*Ratio;
    DBSC1=(M1*SC1*H2R)/sum(SP1)
        if DBSC1 < 0
            DBSC1=0
        else
            DBSC1=DBSC1
        end
    B1= H2B.*B32;
    DBSC2=(M1*SC2*H2R - t1*sum(B1))/sum(SP2)
        if DBSC2 < 0
            DBSC2=0
        else
            DBSC2=DBSC2
        end
    M2=M(p/den);
    B2= H3B.*B33;
    DBLF=(M2*H2R - t1*sum(B2))/sum(SP3)
        if DBLF < 0
            DBLF=0
        else
            DBLF=DBLF
        end
else
    M2=M;
    B2=H3B.*B33;
    DBLF=(M2*H2R - t1*sum(B2))/sum(SP3)
        if DBLF < 0
            DBLF=0
        else
            DBLF=DBLF
        end
end

```

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