# LABORATORY ASTRONOMY



WILLSON

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BY

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# TABLE OF CONTENTS

## CHAPTER I

### THE SUN'S DIURNAL MOTION



## CHAPTER II

### THE MOON'S PATH AMONG THE STARS

## CHAPTER III

### THE DIURNAL MOTION OF THE STARS



## CHAPTER IV

### THE COMPLETE SPHERE OF THE HEAVENS

## CHAPTER V

### MOTION OF THE MOON AND SUN AMONG THE STARS



## CHAPTER VI

### MERIDIAN OBSERVATIONS

# PREFATORY NOTE

The following pages are the beginning of an attempt to formulate a course of observation in preparation for the Laboratory Examination in Astronomy for admission to Harvard University.

The first two chapters had been written out nearly in their present form as part of a proposed text-book for that purpose. The later portions, though less fully developed, are still in a form that may prove useful to exceptional students and certainly to teachers.

In order to test the demand for such a presentation of the subject, it seemed worth while to print as much as is here given, for circulation among those who are interested in this method of teaching Astronomy, although the whole is not in suitable shape for use as a text-book.

Chapters VII and VIII treat of the Nautical Almanac and the solution of problems by means of the celestial globe.

These first eight chapters cover the subjects which are required of all candidates offering Astronomy for admission to Harvard University. The succeeding chapters will treat of those subjects from which selection may be made by each student in accordance with his tastes and opportunities.

The method and apparatus here described have been used since 1897 in the summer course in Astronomy for teachers at Harvard, and in part in a few high schools. All the apparatus may be obtained from at least one of the firms supplying laboratory apparatus.

Some copies of this edition have been interleaved in the binding for the convenience of teachers who may wish to make use of it in its present form, and in the hope of drawing suggestions from those interested.

# LABORATORY ASTRONOMY

## Part I

## CHAPTER I

### THE DIURNAL MOTION OF THE SUN

The most obvious and important astronomical phenomenon that men observe is the succession of day and night, and the motion of the sun which causes this succession is naturally the first object of astronomical study. Every one knows that the sun rises in the east and sets in the west, but very many educated people know little more of the course of the sun than this. The first task of the beginner in astronomy should be to observe, as carefully as possible, the motion of the sun for a day. What is to be observed then? A little thought shows that it can only be the direction in which we have to look to see it at different times; that is, toward what point of the compass — how far above the ground. All astronomical observation, indeed, comes down ultimately to this — the direction in which we see things. The strong light of the sun enables us to make use of a very simple method depending on the principle that the shadow of a body lies in the same straight line with the body and the source of light.

**Path of the Shadow of a Pin-head.** — If we place a pin upright on a horizontal plane in the sunlight and mark the position of the shadow of its head at any time, we thus fix the position of the sun at that time, since it is in the prolongation of the line drawn from the shadow to the pin-head. In order to carry out systematic observations by this method in such a form that the results may be easily discussed, it will be convenient to have the following apparatus: (1) A firm *table* in such a position as to receive sunlight for as long a period as possible. It is better that it should be in the open air, in which case it may be made by driving small posts into

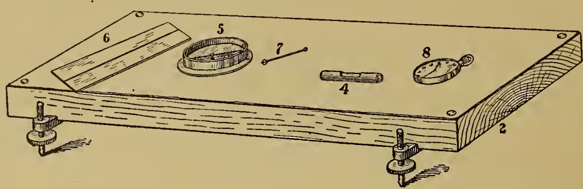


Fig. 1

the ground and securely fastening a stout plank about 18 inches square as a top. (2) A *board*, 18 inches long and 8 inches broad, furnished with leveling screws and smoothly covered with white paper fastened down by (3) *thumb tacks.* (4) A *level* for leveling the board. (5) A *compass.* (6) A *glass plate,* 6 inches long and 2 inches broad, along the median line of which a straight black line is drawn. (7) A *pin,* 5 cm. long, with a spherical head and an accurately turned base for setting it vertical. (8) A *timepiece.*

Draw a straight pencil line across the center of the paper as

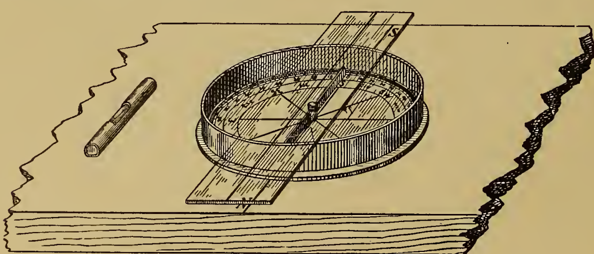

Fig. 2

nearly as possible perpendicular to the length of the board. Place the board upon the table and level approximately. Put the compass on the middle of the pencil line and put the glass plate on the compass with its central line over the center of the needle; turn the plate till its median line is parallel to the pencil line (Fig. 2), and swing the whole board horizontally, till the needle is parallel to the two lines, which are then said to be in the magnetic meridian. Press the leveling screws firmly into the table, and thus make dents by which the board may at any future time be placed in the same position without the renewed use of the compass. Level the board

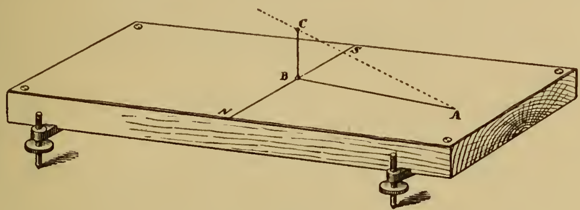


Fig. 3

carefully, placing the level first east and west, then north and south. Place the pin in the pencil line, — in the center if the observation is made between March 20 and September 20, but near the southern edge of the board at any other time of the year, — pressing it firmly down till the base is close to the paper, so that the pin is perpendicular to the paper. Mark with a hard pencil the estimated center of the shadow of the pin-head, *A* (Fig. 3), noting the time by the watch to the nearest minute, affix a number or letter, and affix the same number to the recorded time of the observation in the note-book. It is a good plan to use pencil for notes made while

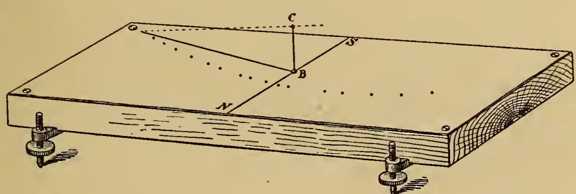


Fig. 4

observing, and ink for computations or notes added afterward in discussing them. Repeat at hourly, or better half-hourly, intervals, thus fixing a set of points (Fig. 4), through which a continuous curve may be drawn showing the path of the shadow for several hours. The same observation should be repeated two weeks later.

## ALTITUDE AND BEARING

By the foregoing process we obtain a diagram on which is shown the position of the pin point, a magnetic meridian line through this point, and a series of numbered points showing the position of the shadow of the pin-head at different times; the height of the pin is known and also the fact that its head was in the same vertical line with its point.

In the discussion of these results, it will be convenient to proceed as follows:

Remove the pin and draw with a hard pencil a fine line, *AB* (Fig. 5), through the pinhole and the point marked at the first observation. This line is called a line of bearing, and the angle which

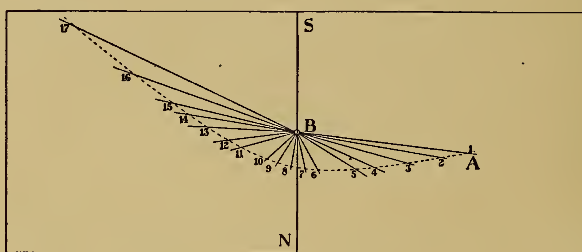


FIG. 5

it makes with the magnetic meridian is called the magnetic bearing of the line. This angle, which may be directly measured on the diagram by a protractor, fixes the position of the vertical plane which contains the observed point and passes also through the center of the pin-head and the sun. If this point bears N.W. from the pin, the sun evidently bears S.E.

Imagine a line, *AC* (Fig. 3), connecting the observed point with the sun's center and passing also through the center of the pin-head. The position of the sun *in* the vertical plane is evidently fixed by this line. The angle between the line of bearing and this line, *BAC*, is called the altitude of the sun; it measures, by the ordinary convention of solid geometry, the angle between the sun's direction and the plane of the horizon.

To determine this angle, lay off the line $B'C'$ (Fig. 6), equal in length to the pin, 5 cm., draw a perpendicular through $B'$; and by means of a pair of compasses or scale laid between the two points $A$ and $B$ (Fig. 5), lay off the line $A'B'$ on the perpendicular, draw $A'C'$, and measure the angle $B'A'C'$ by a protractor. We now have the bearing and altitude of the sun at the time of the first observation, the bearing of the sun from the pin being opposite to that of the point from the pin. In this manner the altitude and bearing are determined for each observed point upon the path of the shadow, and noted against the corresponding time, in the note-book (to avoid confusion, it is convenient to make a separate figure for the morning and afternoon observations, as shown in Fig. 6). We have thus obtained a series of values which will enable us to study more easily the path of the sun upon the concave of the sky.

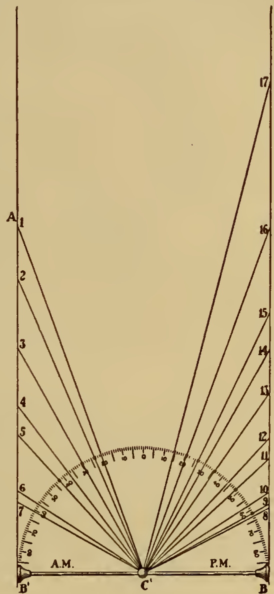


Fig. 6

**Plotting the Sun's Path on a Spherical Surface.** — Probably the most evident method of accomplishing this object would be to

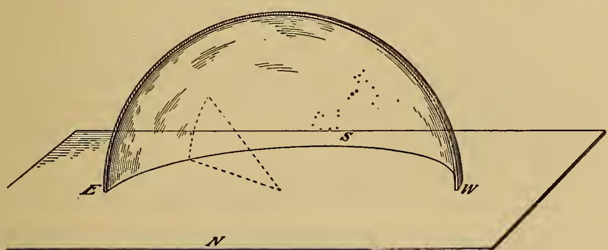


Fig. 7

construct a small concave portion of a sphere, as in the accompanying figure, which suggests how the position of the sun might be referred to the inside of a glass shell.

But the hollow surface offers difficulty in construction and manipulation, and it requires but little stretch of the imagination to pass to the convex surface as follows. The glass shell, as seen from the other side, would appear thus:

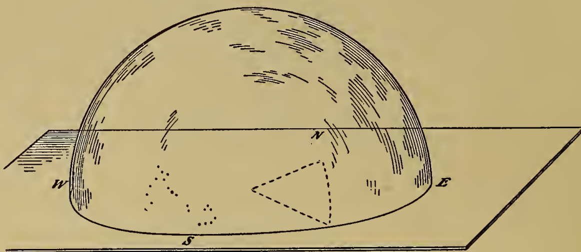


FIG. 8

and we can more readily get at it to measure it, and moreover can more easily recognize the properties of the lines which we shall come to draw upon it, since we are used to looking upon spheres from the outside rather than from the inside, except in the case of the celestial sphere.

On both Figs. 7 and 8 is shown a group of dots which have nearly the configuration of a group of stars conspicuous in the southern heavens in midsummer and called the constellation of Scorpio. It is evident that the constellation has the same shape in both cases, except that in Fig. 8 it is turned right for left or semi-inverted, as is the image of an object seen in a mirror. This property obviously belongs to all figures drawn on the concave surface as seen from the center, when they are looked at from the outside directly toward the center.

So also the diurnal motion of the sun, which as we see it from the center is from left to right, would be from right to left as viewed from the outside of such a surface. This latter is so slight an inconvenience that it is customary to represent the motions of the heavenly bodies in the sky upon an opaque globe, and to determine the angles which these bodies describe about the center, by measuring the corresponding arcs upon the convex surface.

**Plotting on a Hemisphere.** — The apparatus required for plotting the sun's path consists of: a *hemisphere, a,* 4½ inches in diameter; a *circular protractor, b,* a *quadrantal protractor, c,* of 2¼ inches

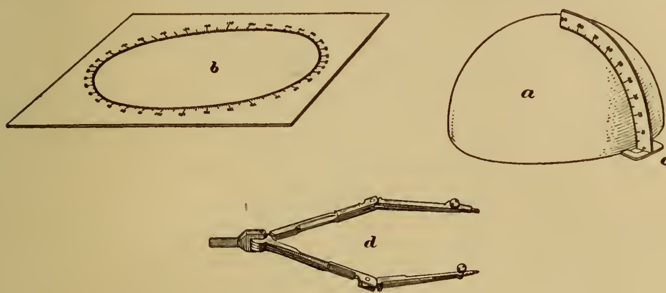


FIG. 9

radius, and a *pair of compasses, d,* whose legs may be bent and one of which carries a hard pencil point.

Determine by trial with the compasses the center of the base of the hemisphere, and mark two diameters by drawing straight lines upon the base at right angles through the center. Prolong these by marks about ⅛ inch in length upon the convex surface. Place the

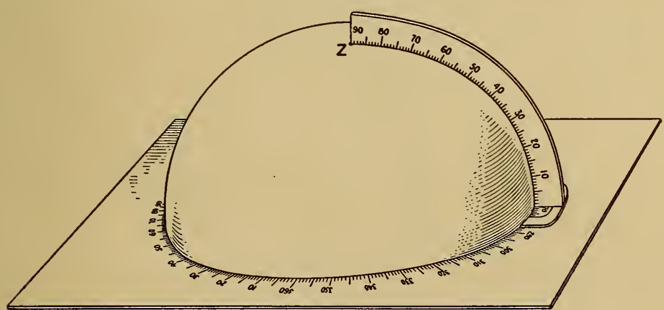


FIG. 10

hemisphere exactly central upon the circular protractor, by bringing the marked ends of one of the diameters upon those divisions of the protractor which are numbered 0° and 180°, and the other on the divisions numbered 90° and 270°. Determine and mark the highest point of the hemisphere by placing the quadrant with its base upon the circular protractor, and its arc closely against the sphere, and marking the end of the scale (Fig. 10). Repeat this with the arc in four positions, 90° apart on the base. The points thus determined should coincide; if they do not, estimate and mark the center of the four points thus obtained. This point represents the highest point of the dome of the heavens — the point directly overhead, called the zenith, and the zero and 180° points on the base protractor may be taken as representing the south and north points respectively of the magnetic meridian.

**The Sun's Path a Circle.** — To plot the altitude and bearing of the first observation, place the foot of the quadrant or altitude arc close against the sphere, the foot of its graduated face on the degree of the protractor which corresponds to the bearing. Mark a fine point on the sphere at that degree of the altitude arc corresponding to the altitude at the first observation. This point fixes the direction in which the sun would have been seen from the center of the hemisphere at the time of observation if the zero line had been truly in the magnetic meridian. Proceed in the same manner with the other observations of bearing and altitude, and thus obtain

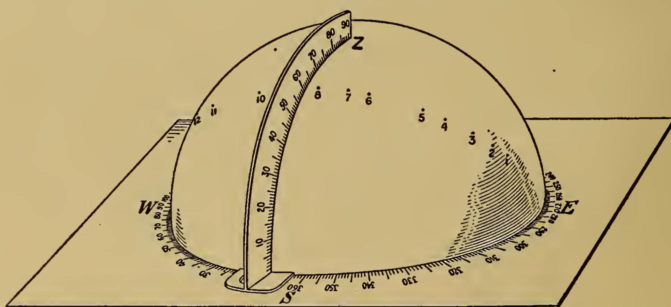


Fig. 11

a series of points (Fig. 11), through which may be drawn a continuous line representing the sun's path upon that day.

It will appear at once that the arcs between the successive points are of nearly equal length if the times of observation were equidistant, and otherwise are proportional to the intervals of time between the corresponding observations — a property which does not at all belong to the shadow curve from which the points are derived. We thus have a noteworthy simplification in referring our observations to the sphere. It will also appear that a sheet of

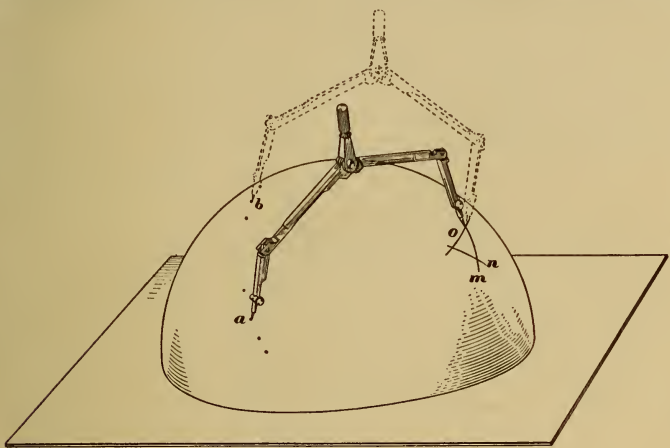


FIG. 12

stiff paper or cardboard may be held edgewise between the hemisphere and the eye, so as to cover all the points; that is, they all lie in the same plane. This fact shows that the sun's path is a circle on the sphere. It is shown by the principles of solid geometry that all sections of the sphere by a plane are circles. If the plane of the circle passes through the center, it is the largest possible, its radius being equal to that of the sphere; it is then called a great circle. Near the 20th of March and 22d of September it will be found that the path of the shadow is nearly a straight line on the diagram, and that the path of the sun is nearly a great circle; that is, the plane of this circle passes nearly through the center of the sphere. In general, the shadow path is a curve, with its concave side toward the pin in summer and its convex side toward it in winter, while the path on the sphere is a small circle, that is, its plane does not pass through the center of the sphere.

**Determining the Pole of the Circle.** — It is proved by solid geometry that all points of any circle on the sphere are equidistant from two points on the sphere, called the poles of the circle. It is important to determine the pole of the sun's diurnal path.

Estimate as closely as possible the position on the sphere of a point which is at the same distance from all the observed points of the sun's path and open the compasses to nearly this distance. For a closer approximation to the position of the pole, place the steel point of the compasses at the point on the hemisphere corresponding to the first observation, *a*, and with the other (pencil) point draw a short arc, *m* (Fig. 12), near the estimated pole. Draw the arc *n* from the point of the last observation, *c*, and join these two arcs by a third drawn from an observed point, *b*, as near as possible to the middle of the path; the pole of the sun's diurnal circle will lie nearly on the great circle drawn from *b* to the middle point *o* of the arc last drawn. Place the steel point at *o*, and the pencil point at *b*, and try the distance of the pencil point from the sun's path at either extremity. If the pencil point lies above (or below) the path at both extremities, the compasses must be opened (or closed) slightly and the assumed pole shifted directly away from (or toward) the middle of the path.

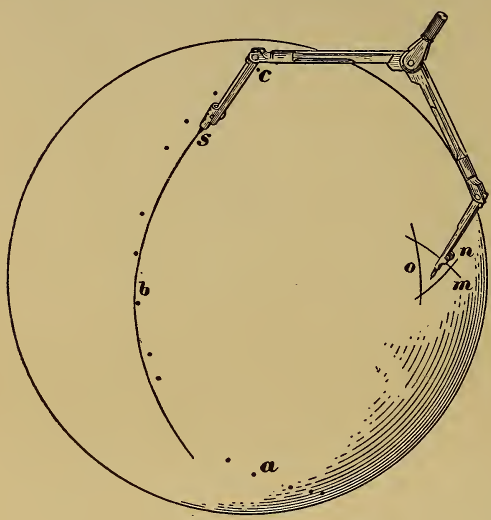

FIG. 13

The proper opening of the compasses is thus quickly determined as well as a close approximation to the position of the pole. Place the steel point at this new position, *p*, the pencil point at *b*, and again test the extreme points. If the west end of the path is below the pencil point (Fig. 13), the latter should be brought directly down to the path by shifting the steel point on the sphere in the plane of the compass legs, that is, along the great circle from *p* to *s*.

From the point thus found a circle can be described with the compasses so as to pass approximately through all the observed points; that is, this point is the pole of the sun's path, and when it is fixed as exactly as possible a circle is to be drawn from horizon to horizon which will represent the sun's path from the point of sunrise to that of sunset, and passing very nearly through all the observed points. The bearing of the points of sunrise and sunset may then be read off on the horizontal circle.

## THE MERIDIAN

The pole as thus determined marks a very interesting and important point in the heavens. We will draw a great circle through the zenith and the pole. To do this, place the altitude arc against the sphere, as if to measure the altitude of the pole; and

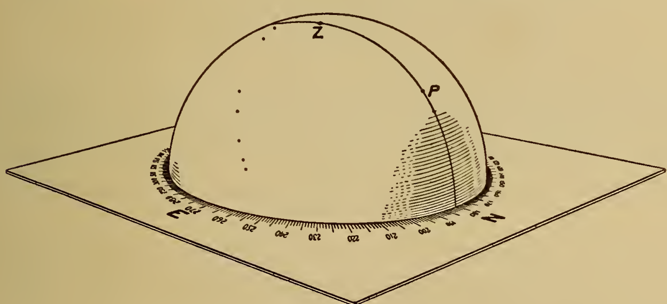


FIG. 14

using it as a guide, draw the northern quadrant of the vertical circle through the zenith and the pole. Note the bearing of this vertical circle. Place the altitude arc at the opposite bearing, and draw another or southern quadrant of the same great circle till it meets the south horizon. This great circle (Fig. 14) is called the meridian of the place of observation, and its plane is called the plane of the meridian of the place of observation, — sometimes the true meridian, to distinguish it from the magnetic meridian.

The line in which it cuts the base of the hemisphere represents the meridian line or true meridian line, just as the line first drawn represents the line of the magnetic meridian. If the observations are made in the United States, near a line drawn from Detroit to Savannah, it will be found that the true meridian coincides very nearly with the magnetic meridian. East of the line joining these cities, the north end of the magnet points to the west of the true meridian by the amounts given in the following table:

| | |
|---|---|
| 21° | at the extreme N.E. boundary of Maine. |
| 15 | at Portland. |
| 10 | at Albany and New Haven. |
| 5 | at Washington and Buffalo. |

While on the west the declination, as it is called, is to the east of the true meridian.

| | |
|---|---|
| 5° | at St. Louis and New Orleans. |
| 10 | at Omaha and El Paso. |
| 15 | at Deadwood and Los Angeles. |
| 20 | at Helena, Montana, and C. Blanco. |
| 23 | at the extreme N.W. boundary of the United States. |

By drawing these lines on the map, as in Fig. 15, it is easy to estimate the declinations at intermediate points within one or two degrees, — at the present time west declinations in the United States are increasing and east declinations decreasing by about 1° in fifteen years.

A great circle perpendicular to the meridian may be drawn by placing the altitude protractor at readings 90° and 270° from the meridian reading and drawing arcs to the zenith in each case. This circle is the prime vertical, and intersects the horizon in the east and west points; thus all the cardinal points are fixed by the meridian determined from our plotting of the sun's path.

**Azimuth.** — Place the hemisphere upon the circular protractor in such a position that the line of the *true meridian* on the hemisphere coincides with the zero line of the protractor.

Place the altitude arc so as to measure the altitude at any part of the sun's path west of the meridian (Fig. 16). The reading of the foot of the arc will give the angle between the true meridian and the vertical plane containing the sun at that point of its diurnal circle. This angle is its *true* bearing and differs from its magnetic

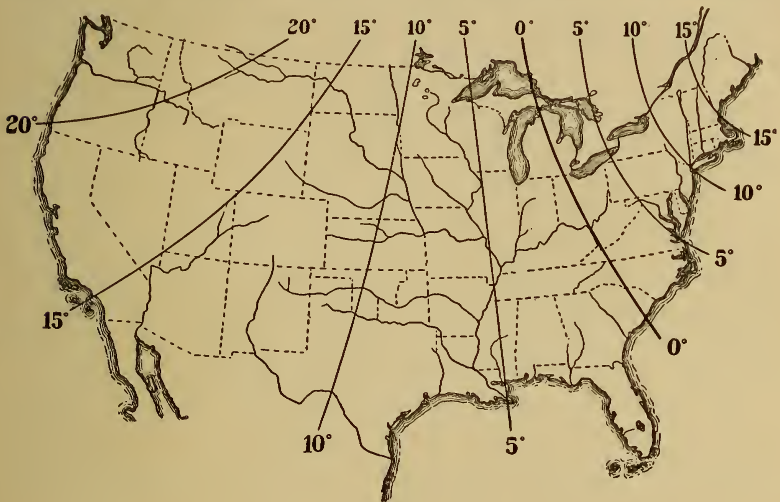


FIG. 15

bearing by the declination of the compass, being evidently less than the magnetic bearing, if the declination is west of north. It is also called the azimuth of the sun's vertical circle, or, briefly, of the sun.

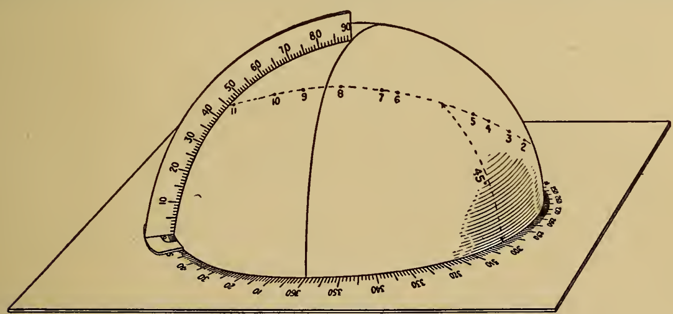

FIG. 16

Formerly azimuth was usually reckoned from north through the west or east, to 180° at the south point. It is now customary to measure it from south through west up to 360°, so that the azimuth of a body when east of the meridian lies between 180° and 360°. The present method is more convenient because the given angle fixes the position of the vertical circle without the addition of the letters E. and W. It is worthy of notice that with this notation the azimuth of the sun as seen in northern latitudes outside of the tropics always increases with the time; and indeed this is true of most of the bodies we shall have occasion to observe.

Now place the altitude quadrant so that its foot is at a point on the circular protractor where the reading is 360° minus the azimuth of the point just measured; the sun at this point of its path is just as far east of the meridian as it was west of the meridian at the point last considered, and it will be found that the altitude of the two points is the same. On the path shown in Fig. 16 the altitude is 45° at the points whose azimuths are 60° and 300° (60 E. of S.).

This fact, that equal altitudes of the sun correspond to equal azimuths east and west of the true meridian, is an important one, and will presently be made use of to enable us to determine the position of the true meridian with a greater degree of precision.

## THE EQUINOCTIAL

We shall find it convenient to draw upon the hemisphere another line, which plays an important rôle in astronomy, the great circle 90° from the pole. Placing the steel point of the compasses at the zenith, open the legs until the pencil point just comes to the horizon plane where the spherical surface meets it, so that if it were revolved about the zenith, the pencil point would move in the horizon. The compass points now span an arc of 90° upon the hemisphere. Place the steel point at the pole, and draw as much of a great circle as can be described on the sphere above the horizon. This will be just one-half of the great circle, and will cut the horizon in the east and west points. The new circle is called the equinoctial or celestial equator (Fig. 17).

We have seen that the path of the sun over the dome of the heavens appears to be a small circle described from east to west about a fixed point in the dome as a pole. The ancient explanation of this fact was that the sun is fixed in a transparent spherical shell of immense size revolving daily about an axis, the earth being a plane in the center of unknown extent, but whose known regions are so small compared to the shell that from points even widely separated on the earth the appearance is the same; just as the

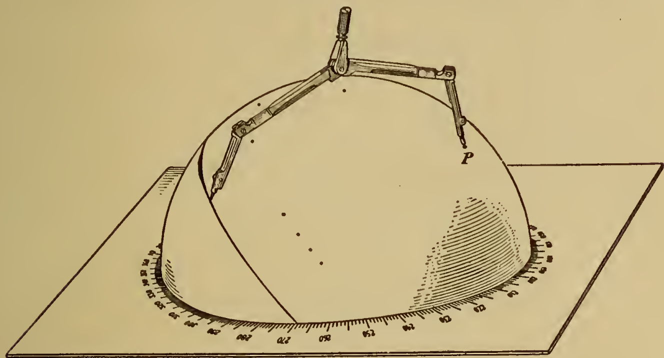


FIG. 17

apparent direction and motion of the sun would be practically the same on our hemisphere to a microscopic observer at the center, and to another anywhere within one-hundredth of an inch of the center. When observations were made, however, at points some hundreds of miles apart on the same meridian, very perceptible differences were found, whose nature will be understood from a comparison of the hemisphere (Fig. 18 *a*), plotted from

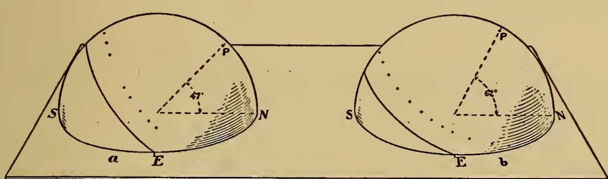


FIG. 18

observations made Aug. 8, 1897, at a point in Canada, not far from Quebec, with a second hemisphere (Fig. 18 *b*), on which is shown the path of the sun on the same date derived from observation of the shadow of a pin-head at Polfos in Norway. It appears on comparison that the distance of the pole above the north horizon is considerably greater in the latter, while the equator is just as much nearer the southern horizon; the sun is at the same distance from the equator in each case. This fact cannot be explained on the supposition that the horizon planes of the two places are the same, for in that case we should have the spherical shell which contains the sun revolving at the same time about two different fixed axes, which is impossible. It is not, however, improbable that the earth's surface should be curved, if we can admit as a possibility that the direction of gravity, which is perpendicular to a horizontal plane, may be different at different places. That the earth's surface in the east and west direction is curved, we know; for men have traversed it from east to west and returned to the starting point, so that we have good reason to believe that its surface is everywhere curved. Long before this conclusive proof was obtained, however, the globular form of the earth was inferred on good grounds.

It was early suggested (regarding the fact that, if the sun is fixed in a shell, that shell is of enormous size as compared with the earth) that it is inherently more probable that the apparent motion of the sun is due to a rotation of the spherical earth about an axis passing through the earth's center and the poles of the sun's circle. This argument is greatly strengthened when we investigate the apparent motion of the stars in connection with their size and distance, and it is now beyond a doubt that this is the true explanation of the apparent diurnal motion of the sun.

## LATITUDE EQUALS ELEVATION OF THE POLE

This subject is treated in all text-books on descriptive astronomy, and it is pointed out that the pole of the sun's path is the point where the line of the earth's axis of rotation cuts the sky, and the equinoctial or celestial equator is the great circle in which the plane of the earth's equator cuts the sky. The fact is proved also that the elevation of the pole above the horizon at any place is equal to the latitude of the place.

This angle, as measured on the hemisphere shown in Fig. 18 *a*, is 47°, and on the hemisphere of Fig. 18 *b* is 62°. The latitudes of

Quebec and Polfos as determined by more accurate measures are 46° 50′ and 61° 57′.

It is easy to see that the arc of the meridian from the zenith to the equinoctial is also equal to the latitude, while the arc from the south point of the horizon to the equator and that from the zenith to the pole are each equal to 90° minus the latitude, or, as it is usually called, the co-latitude.

It will be well here, as in all our measurements, to form some idea of the accuracy of our results. As one degree on our hemisphere is quite exactly equal to $1^{mm}$, a quantity easily measured by ordinary means, it is not difficult with ordinary care to determine the pole of the sun's path so closely that no observed point lies more than a degree from the path. The pole is then fixed within one degree unless the length of the path is very short; usually if the path is more than 90° in length the pole may be placed within less than a degree of its true place and the latitude measured with an error of less than one degree.

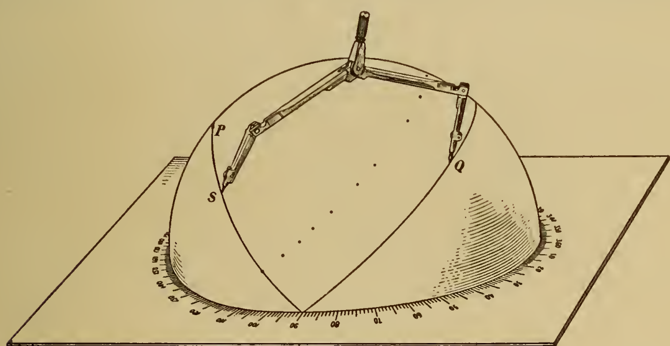


Fig. 19

## HOUR-ANGLE OF THE SUN

Open the dividers as before (see p. 14) so as to draw a great circle. Place the steel point upon the place of the sun, *S*, on its diurnal circle at the time of the last observation in the afternoon (Fig. 19), and with the pencil point strike a small arc cutting the equator at *Q*.

Place the steel point where this arc cuts the equator, and draw a great circle which will pass through the sun's place and the pole; notice that it also cuts the equator at right angles. Such a circle is called an hour-circle. It is the intersection of the surface of the sphere with a plane that passes through the poles and the place of the sun. The number of degrees in the arc of the equator, included between the meridian and the hour-circle which passes through the sun, is called the hour-angle of the sun. By the ordinary convention of solid geometry it measures the wedge angle between the plane of the hour-circle and the plane of the meridian. If a book be placed with its back in the line from the pole to the center of the sphere, and with its title-page to the west, and the western cover opened till it is in the plane of the hour-circle, while the title-page is in the plane of the meridian, the wedge angle between the title-page and the cover will be the hour-angle and will be measured by the arc of the equator indicated above. It is reckoned as increasing from the meridian towards the west in the direction in which the cover is opened. If the hour-circle of the first morning observation is determined in the same way, the hour-angle measured in the opposite direction from the meridian is sometimes called the hour-angle east of the meridian; but more commonly by astronomers this value is subtracted from 360°, and the angle thus obtained is called the hour-angle, this being more convenient because the hour-angle of the sun thus measured constantly increases with the time as the sun pursues its course; being 0° at noon, 180° at midnight, 360° at the next noon, etc.

## UNIFORM INCREASE OF HOUR-ANGLE

Let us now examine more carefully the truth of the surmise previously made, that the arc of the sun's path between two successive observations is proportional to the interval of time between the observations. Draw the hour-circles of the sun at each point of observation (Fig. 20); measure the arc on the equator between the first and the last hour-circles; divide by the number of minutes between the two times. This will give the average increase of hour-angle per minute. Multiply this increase by the difference in minutes of each of the observed times from the time of the first observation, and compare with the progressive increase of the hour-angle as measured off on the equator by means of the graduated quadrant. They will be found to be nearly the same in each case. It is thus shown that the hour-angle of the sun increases uniformly with the time. The rate is nearly a quarter of a degree per minute, since 360° are described in 24 hours. Notice that when the hour-angle is zero, the actual time by the watch is not very far from 12 o'clock (in extreme cases it may be 45 minutes, if the clock is keeping standard time), and that if the hour-angle in degrees (west of the meridian) is divided by 15, the number of hours differs from the

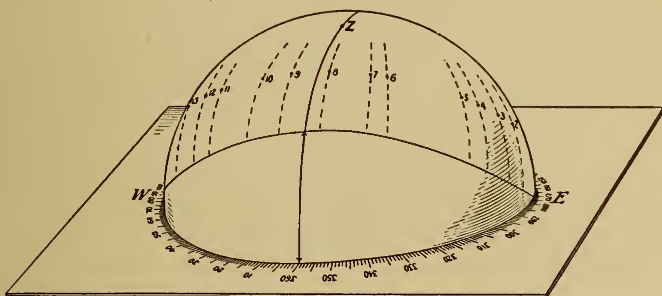


FIG. 20

watch time just as much as the time of meridian passage differs from 12 hours. In fact, the hour-angle of the sun measures what is called apparent solar time, *i.e.*, when H.A. = 15°, it is 1 o'clock; H.A. = 75°, it is 5 o'clock; H.A. = 150°, 10 o'clock, etc.; those angles east of the meridian lying between 180° and 360°, *i.e.*, between $12^h$ and $24^h$, so that 12 hours must be subtracted to give the correct hours by the ordinary clock, which divides the day into two periods of 24 hours each; for instance, if H.A. = 270°, it is $18^h$ past noon or 6 A.M. of the next day. Astronomical clocks usually show the hours continuously from 0 to 24, thus avoiding the necessity of using A.M. and P.M. to discriminate the period from noon to midnight and from midnight to noon.

## DECLINATION OF THE SUN

The distance of the sun's path from the celestial equator, measured along the arc of an hour-circle, is called its declination, and will be found appreciably the same at all points. It requires more delicate observation than ours to find that it changes during the few hours covered by our observation. If, however, the observation be repeated after an interval, say, of two weeks at any time except for a month before or after the 20th of June or December, it will be found that although the sun at the second observation describes a circle, this circle is not in the same position with regard to the equator — that its declination has changed (between March 13 and 27, for instance, by about $5^\circ.5$). The inference to be drawn is that even during the period of our observation the sun's path is not exactly parallel to the equator, although our observations are not delicate enough to show that fact.

It is true in general, as in this case, that the first rude measurements applied to the heavenly bodies give results which when tested by those covering a longer time, or made with more delicate instruments, are found to require correction.

## CHAPTER II

### THE MOON'S PATH AMONG THE STARS

NEXT to the diurnal motion of the sun the most conspicuous phenomenon is the similar motion of the stars and the moon; this will form the subject of a future chapter.

The study of the moon, however, discloses a new and interesting motion of that body. It partakes indeed of the daily motion of the heavenly bodies from east to west, but it moves less rapidly, requiring nearly 25 hours to complete its circuit instead of 24, as do the sun and stars, and returning to the meridian therefore about an hour later on each successive night.

In consequence of this motion it continually changes its place with reference to the stars, moving toward the east among them so rapidly that the observation of a few hours is sufficient to show the fact. At the same time its declination changes like that of the sun, but much more rapidly.

We should begin early to study this motion, and it will be found interesting to continue it at least for some months at the same time that other observations are in progress — a very few minutes each evening will give in the course of time valuable results.

### POSITION BY ALIGNMENT WITH STARS

The first method to be used consists in noting the moon's place with reference to neighboring stars at different times. Some sort of star map is necessary upon which the places of the moon may be laid down so that its path among the stars may be studied. As the configurations that offer themselves at different times are of great variety, it will be well to give a few examples of actual observations of the moon's place by this method.

Dec. 12, 1899, at $12^h\ 0^m$ P.M., the moon was seen to be near three unknown stars, making with them the following configuration, which was noted on a slip of paper as shown in Fig. 21. The relative size of the stars is indicated by the size of the dots. (The *original* papers on which the observations are made should be carefully preserved; indeed, this should always be the practice in all observations.)


FIG. 21

At the same time, for purposes of identification, it was noted that the group of stars formed, with Capella and the brightest star in Orion, both of which were known to the observer, a nearly equilateral triangle. It was also noted that the moon was about 6° from the farthest star, this being estimated by comparison with the known distance between the "pointers" in the "Dipper" (about 5°). With these data it was easily found by the map that these stars were the brightest stars in Aries, and the moon was plotted in its proper place on the map (page 24).

December 13, at $5^h$ $35^m$ P.M., the moon was $\frac{1}{4}°$ (half its diameter) below (south of) a line drawn from Aldebaran (identified by its position with reference to Capella and Orion and by the letter V of stars in which it lies, the Hyades) to the faintest of the three reference stars of December 12. It was also about $\frac{3}{4}°$ west of a line between two unknown stars identified later as Algol (equidistant from Capella and Aldebaran) and $\gamma$ Ceti (at first supposed on reference to the map to be $\alpha$ Ceti, but afterward correctly identified by comparing the map with the heavens). The original observation is given below (Fig. 22) of about one-half the size of the drawing, all except the underscored names being in pencil. The underscored names are in ink and made after the stars were identified. This is a useful practice when additions are made to an original, so that subsequent work may not be given the appearance of notes made at the time of observation. It is well to give on the sketch map several stars in the neighborhood of those used for alignment, to facilitate identification.

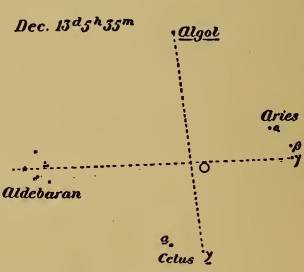


FIG. 22

The alignment was tested by holding a straight stick at arm's length parallel to the line joining the stars.

December 14, $6^h$ $30^m$ P.M. Moon on a line from Algol through the Pleiades (known) about $2\frac{1}{2}°$ (5 diameters of moon) beyond the latter, which were very faint in the strong moonlight. *No figure.*

December 15, $5^h$ $10^m$ P.M. Moon in a line between Capella and Aldebaran. Line from Pleiades to moon bisects line from Aldebaran to $\beta$ Tauri (identified by relation to Aldebaran and Capella).

$9^h$ $25^m$ P.M. Moon in line from $\beta$ Aurigæ to Aldebaran (Fig. 23).

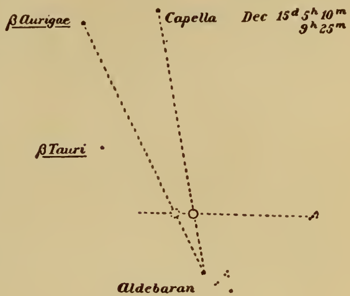


FIG. 23

(NOTE. — Henceforth details of identification are omitted.)

December 16, $7^h$ $40^m$ P.M. Moon almost totally eclipsed $2\frac{1}{4}°$ east of line from $\beta$ Aurigæ to $\gamma$ Orionis; same distance from $\beta$ Tauri as $\zeta$ Tauri (revised estimate about $\frac{1}{2}°$ nearer $\beta$ Tauri than is $\zeta$ Tauri) (Fig. 24).


FIG. 24

December 18, $10^h$ $30^m$ P.M. Observation snatched between clouds. Moon's western edge tangent to line from $\alpha$ Geminorum to Procyon and about 1° north of center of that line.

In the sketch maps above no great accuracy is attempted in placing the stars, but in the final plotting on the map the directions of the notes are carefully followed. The plotting should be done as soon as possible after the observation is made, for even a hasty comparison with the map will often show that stars have been misidentified or that there is some obvious error in the notes, which may be rectified at once if there is an opportunity to repeat the observation. Such a case occurs in the observations of December 13 recorded above, where $\gamma$ Ceti was mistaken for $\alpha$.

## PLOTTING POSITIONS OF THE MOON ON A STAR MAP

Figure 25 shows the positions of the moon plotted from the foregoing observations, together with the lines of construction from which they were determined.

A drawing should be made of the shape of the illuminated portion of the moon at each observation, and the direction among the stars

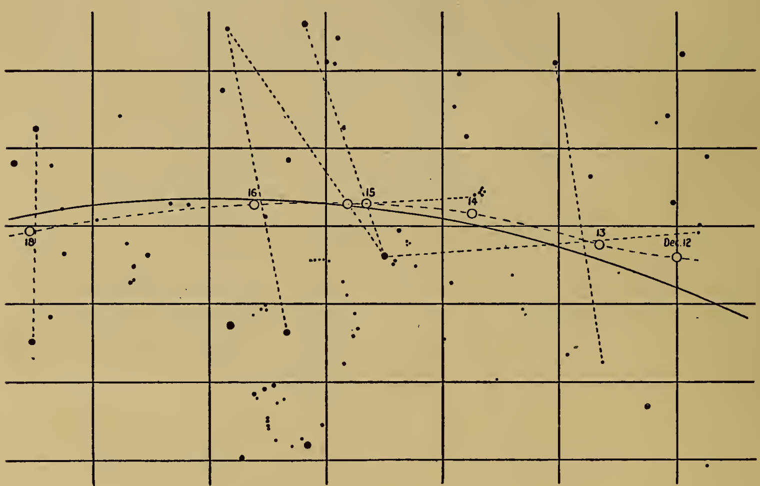


FIG. 25

of the line joining the points of the horns (cusps) for future study of the cause of the moon's changes of phase.

If the star map accompanying this book is used, the identification of the stars consists in determining which of the dots represents the star of reference; the name may be determined by reference to the list; thus the two stars near the line XXIV on the upper portion of the map are "$\alpha$ Andromedæ $0^h\ 5^m + 29°$" and "$\gamma$ Pegasi $0^h\ 8^m + 14°$." The meaning which attaches to these numbers is given in Chapter III. It is a good plan to keep a copy of the map on which to note the names for reference as the stars are learned; most of the conspicuous ones will soon be remembered as they are used.

## THE MOON'S PLACE FIXED BY ITS DISTANCE FROM NEIGHBORING STARS

One month's observation by this method will show that the moon's path is at all points near to the curved line drawn on the map, which is called the ecliptic and which is explained on page 70. To establish more accurately its relations to this line it will be advisable in the later months to adopt a more accurate means of observation, although when the moon is very near a bright star, its position may be quite accurately fixed by the means that we have indicated; and if it chances to pass in front of a bright star and produce an occultation, the moon's position is very accurately fixed indeed, as accurately as by any method. But such opportunities are rare, and for continuous accurate observation we should have a means of measuring the distance of the moon from stars that are at a considerable distance from it. An instrument sufficiently accurate for our purpose is the cross-staff described below. It should be mentioned that, on account of the distortion of the map, the place of the moon is usually more accurately given by distances from the comparison stars than by alignment. The sextant may be used instead of the cross-staff, but is less convenient and also more accurate than is necessary.

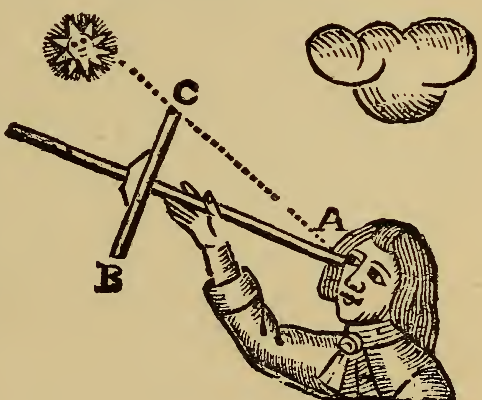

FIG. 26

**The Cross-staff.** — The cross-staff (Fig. 26) consists of a straight graduated rod upon which slides a "transversal" or "cross" perpendicular to the rod; one end of the staff is placed at the eye and the "cross" is moved to such a place that it just fills the angle from one object to another; its length is then the chord of an arc equal to the angle between the objects as seen from that end of the staff at which the eye is placed. The figure, which is taken from an old book on navigation, illustrates the use of this instrument for measuring the sun's altitude above the sea horizon; the rod in the position shown indicates that the sun's altitude is about 40°.

Obviously a given position of the cross corresponds to a definite angle at the end of the rod, and the rod may be graduated to give this angle directly by inspection, or a table may be constructed by which the angle corresponding to any division of the rod may be found; such a table is given on page 27. For our purpose an instrument of convenient dimensions is made by using a cross 20 cm. in length, sliding on a rod divided into millimeters (Fig. 27); this may be used for measuring angles up to 30°, which is enough for our

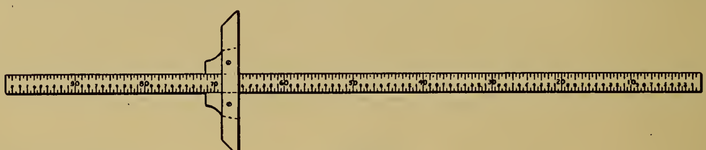

Fig. 27

purpose. The smallest angle that can be measured is about 12°, which corresponds to a chord of $\frac{1}{5}$ of the radius; but by making a part of the cross only 10 cm. long, as shown in the figure, we may measure angles from 6° upwards, and for smaller angles may use the thickness of the cross, which is 5 cm., and thus measure angles as small as 3°; the longer cross will not give good results above 30°, as a slight variation of the eye from the exact end of the rod makes a perceptible difference in the value of the angles greater than 30°.

**Measures with the Cross-staff.** — As an example of the use of the cross-staff, the following observations are given: They were made with a staff about 3 feet in length, graduated by marking the point for each degree at the proper distance in millimeters from the eye end of the staff, as given by Table II on page 27. After the points were marked a straight line was drawn through each entirely across the rod, using the cross itself as a ruler; graduations were thus made on one side for use with the 20 cm. cross, on the other for the

Table I — Angle subtended by Crosses

| Distance from Eye | Length of Cross | | | Distance from Eye | Length of Cross | | |
|---|---|---|---|---|---|---|---|
| | 20 cm. | 10 cm. | 5 cm. | | 20 cm. | 10 cm. | 5 cm. |
| 100cm | 11°.4 | 5°.7 | 2°.9 | 62cm | 18°.3 | 9°.2 | 4°.6 |
| 99 | 11 .5 | 5 .8 | 2 .9 | 61 | 18 .6 | 9 .4 | 4 .7 |
| 98 | 11 .6 | 5 .8 | 2 .9 | 60 | 18 .9 | 9 .5 | 4 .8 |
| 97 | 11 .8 | 5 .9 | 3 .0 | 59 | 19 .2 | 9 .7 | 4 .9 |
| 96 | 11 .9 | 6 .0 | 3 .0 | 58 | 19 .6 | 9 .9 | 4 .9 |
| 95 | 12 .0 | 6 .0 | 3 .0 | 57 | 19 .9 | 10 .0 | 5 .0 |
| 94 | 12 .1 | 6 .1 | 3 .0 | 56 | 20 .2 | 10 .2 | 5 .0 |
| 93 | 12 .3 | 6 .2 | 3 .1 | 55 | 20 .6 | 10 .4 | 5 .2 |
| 92 | 12 .4 | 6 .2 | 3 .1 | 54 | 21 .0 | 10 .6 | 5 .3 |
| 91 | 12 .5 | 6 .3 | 3 .1 | 53 | 21 .4 | 10 .8 | 5 .4 |
| 90 | 12 .7 | 6 .4 | 3 .2 | 52 | 21 .8 | 11 .0 | 5 .5 |
| 89 | 12 .8 | 6 .4 | 3 .2 | 51 | 22 .2 | 11 .2 | 5 .6 |
| 88 | 13 .0 | 6 .5 | 3 .3 | 50 | 22 .6 | 11 .4 | 5 .7 |
| 87 | 13 .1 | 6 .6 | 3 .3 | 49 | 23 .1 | 11 .6 | 5 .8 |
| 86 | 13 .3 | 6 .7 | 3 .3 | 48 | 23 .5 | 11 .9 | 6 .0 |
| 85 | 13 .4 | 6 .7 | 3 .4 | 47 | 24 .0 | 12 .1 | 6 .1 |
| 84 | 13 .6 | 6 .8 | 3 .4 | 46 | 24 .5 | 12 .4 | 6 .2 |
| 83 | 13 .7 | 6 .9 | 3 .5 | 45 | 25 .1 | 12 .7 | 6 .4 |
| 82 | 13 .9 | 7 .0 | 3 .5 | 44 | 25 .6 | 13 .0 | 6 .5 |
| 81 | 14 .1 | 7 .1 | 3 .5 | 43 | 26 .2 | 13 .3 | 6 .7 |
| 80 | 14 .3 | 7 .2 | 3 .6 | 42 | 26 .8 | 13 .6 | 6 .8 |
| 79 | 14 .4 | 7 .2 | 3 .6 | 41 | 27 .4 | 13 .9 | 7 .0 |
| 78 | 14 .6 | 7 .3 | 3 .7 | 40 | 28 .1 | 14 .3 | 7 .2 |
| 77 | 14 .8 | 7 .4 | 3 .7 | 39 | 28 .8 | 14 .6 | 7 .3 |
| 76 | 15 .0 | 7 .5 | 3 .8 | 38 | 29 .5 | 15 .0 | 7 .5 |
| 75 | 15 .2 | 7 .6 | 3 .8 | 37 | 30 .2 | 15 .4 | 7 .7 |
| 74 | 15 .4 | 7 .7 | 3 .9 | 36 | 31 .0 | 15 .8 | 7 .9 |
| 73 | 15 .6 | 7 .8 | 3 .9 | 35 | 31 .9 | 16 .3 | 8 .2 |
| 72 | 15 .8 | 7 .9 | 4 .0 | 34 | 32 .8 | 16 .7 | 8 .4 |
| 71 | 16 .0 | 8 .1 | 4 .0 | 33 | 33 .7 | 17 .2 | 8 .7 |
| 70 | 16 .3 | 8 .2 | 4 .1 | 32 | 34 .7 | 17 .7 | 8 .9 |
| 69 | 16 .5 | 8 .3 | 4 .2 | 31 | 35 .8 | 18 .3 | 9 .2 |
| 68 | 16 .7 | 8 .4 | 4 .2 | 30 | 36 .9 | 18 .9 | 9 .5 |
| 67 | 17 .0 | 8 .5 | 4 .3 | 29 | 38 .1 | 19 .6 | 9 .9 |
| 66 | 17 .2 | 8 .7 | 4 .3 | 28 | 39 .3 | 20 .2 | 10 .2 |
| 65 | 17 .5 | 8 .8 | 4 .4 | 27 | 40 .6 | 21 .0 | 10 .6 |
| 64 | 17 .7 | 8 .9 | 4 .5 | 26 | 42 .1 | 21 .8 | 11 .0 |
| 63 | 18 .0 | 9 .1 | 4 .5 | 25 | 43 .6 | 22 .6 | 11 .4 |

Table II

| Angle subtended by 20 cm. Cross | |
|---|---|
| 12° | 951mm |
| 13 | 878 |
| 14 | 814 |
| 15 | 760 |
| 16 | 711 |
| 17 | 669 |
| 18 | 631 |
| 19 | 598 |
| 20 | 567 |
| 21 | 540 |
| 22 | 514 |
| 23 | 491 |
| 24 | 470 |
| 25 | 451 |
| 26 | 433 |
| 27 | 416 |
| 28 | 401 |
| 29 | 387 |
| 30 | 373 |
| 31 | 361 |
| 32 | 349 |
| 33 | 338 |
| 34 | 327 |
| 35 | 317 |
| 36 | 308 |
| 37 | 299 |
| 38 | 290 |
| 39 | 282 |
| 40 | 275 |

10 cm. cross, and on one edge for the thickness of the cross. By means of these graduations the angle subtended by the cross in any position is read directly from the scale, quarters or thirds of a degree being estimated and recorded in minutes of arc.

The observations are:

| | | |
|---|---|---|
| 1900. January 2. $5^h\ 15^m$. | | |
| Moon to | $\epsilon$ Pegasi, | 35° 45′ |
| " " | Altair, | 26 30 |
| " " | Fomalhaut, | 41 40 |
| January 3. $6^h\ 0^m$. | | |
| Moon to | $\epsilon$ Pegasi, | 23° 30′ |
| " " | Altair, | 29 20 |
| " " | $\beta$ Aquarii, | 8 20 |
| January 4. $5^h\ 20^m$. | | |
| Moon to | $\epsilon$ Pegasi, | 17° 40′ |
| " " | $\beta$ Aquarii, | 8 30 |
| " " | $\delta$ Capricorni, | 9 45 |
| January 6. $5^h\ 50^m$. | | |
| Moon to | $\gamma$ Pegasi, | 12° 0′ |
| " " | $\alpha$ Pegasi, | 16 40 |
| " " | $\epsilon$ Pegasi, | 33 30 |
| January 7. $5^h\ 45^m$. | | |
| Moon to | $\gamma$ Pegasi, | 9° 40′ |
| " " | $\beta$ Arietis, | 19 45 |
| " " | $\alpha$ Andromedæ, | 21 15 |
| " " | $\beta$ Ceti, | 27 30 |
| January 8. $6^h\ 0^m$. | | |
| Moon to | $\alpha$ Arietis, | 11° 0′ |
| " " | $\gamma$ Pegasi, | 21 30 |
| January 9. $10^h\ 0^m$. | | |
| Moon to | $\alpha$ Arietis, | 9° 45′ |
| " " | Alcyone, | 16 0 |
| " " | $\alpha$ Ceti, | 15 30 |

To represent these observations on the star map, open the compasses until the distance of the pencil point from the steel point is equal to the measured distance — making use for this purpose of the scale of degrees in the margin, and then with the steel point carefully centered on the comparison star, strike a short arc with the pencil point near the estimated position of the moon; the intersection of any two of these arcs fixes the position of the moon. If the different stars give different points, those nearest the moon may

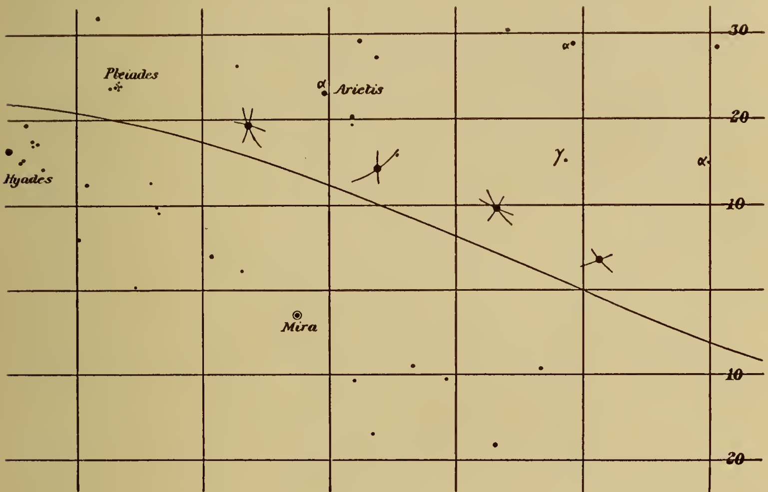


FIG. 28

be assumed to give results nearer the truth. Fig. 28 shows the positions of the moon January 6 to January 9 as plotted from the above measures.

**Length of the Month.** — If it happens that one of the positions observed in the second month falls between the places obtained on two successive days of the first month, or *vice versa*, a determination of the moon's sidereal period may be made by interpolation. Thus, on plotting the observation of December 12 (p. 22), which places the moon between the two observations on January $8^d\ 6^h\ 0^m$ and January $9^d\ 10^h\ 0^m$, its distance from the former is $6°.0$ and from the latter $10°.0$, while the interval is $28^h$; the moon's place on December 12 at $12^h\ 0^m$ is therefore the same as on January 8 at $6^h + \frac{6}{16} \times 28^h$, or January $8^d\ 16^h.5$, that is, January 9 at $4^h\ 30^m$ A.M., and the interval between these two times is $27^d\ 4^h\ 30^m$, which is the time required for the moon to make a complete circuit among the stars or the length of the sidereal month. This is a fairly close approximation; the observation of December 12 having been made under favorable circumstances, the configuration being well defined and the stars near, so that the position on that date by alignment is nearly as accurate as those determined by the measures on January 8 and 9.

After three months the moon comes nearly to the same position at about the same time in the evening, so that it is convenient to determine its period without interpolation by observing the time when the moon comes into the same star line as at the previous observation; moreover, the interval being three months, an error of an hour in the observed interval causes an error of only $20^m$ in the length of the month.

## THE MOON'S NODE

When a sufficiently large number of observations have been plotted to give a general idea of the moon's path among the stars, a smooth curve is to be drawn as nearly as possible through all the points and this curve should be compared with the ecliptic, as shown on the map. Its greatest distance from the ecliptic and the place where it crosses the ecliptic — the position of the node — should be estimated with all possible precision. For this purpose, only the more accurate positions obtained by the cross-staff should be used.

After a few observations of alignment are made, the student will desire to use the more accurate method at once, but it is better to have at least one month's observation by the first method (even if the cross-staff is also used) for comparison with later observations by alignment for the purpose of determining the length of the month, as suggested above, without any instrumental aid whatever.

The records of the positions of the node should be preserved by the teacher for comparison from year to year to show the motion of this point along the ecliptic. The node, as determined by the observations above given, was nearly at the point where the ecliptic crosses the line from $\gamma$ Orionis to Capella. Observations made in November, 1897, by the method of Chapter IV, gave its place on the ecliptic at a point where the latter intersects a line drawn through Castor and Pollux, thus indicating a motion of about 40° in the interval.

Observations made with the cross-staff are sufficiently accurate to show that the motion of the moon is not uniform, but as the distortion of the map complicates the treatment of this subject, we shall defer its consideration until the method of Chapter V has been introduced.

It will be well, however, as soon as measures with the cross-staff are begun, to devote a few minutes each evening to measures of the moon's diameter with an instrument measuring to $10''$, such as a good sextant; or, better, a telescope provided with a micrometer, in order to show the variations of the moon's apparent size at different parts of its orbit. The relative distances of the moon from the earth as inferred from these measures should be compared with the variations of her angular motion as read off from the chart; although on account of the distortion referred to above, it will not be possible to show more than the fact that when the moon is nearest, her angular motion about the earth is greatest, and *vice versa.*

The sextant or micrometer may henceforward be used also for observations of the sun's diameter, which should be measured as often as once a week for a considerable period.

When the moon's diameter is measured, a rough estimate of her altitude should be made in order to make the correction for augmentation in a future more accurate discussion of the measures for determining the eccentricity of her orbit.

## DETERMINING THE ERRORS OF THE CROSS-STAFF

Observations with the cross-staff are most easily made just before the end of twilight or in full moonlight, so that the cross may be seen dark against a dimly lighted background. When used for measuring the distance of stars in full darkness, it is convenient to have a light so placed behind the observer that, while invisible to him, it shall dimly illuminate the arms of the cross.

As the angles which are determined by the cross-staff, especially if large, are affected by the observer's habit of placing the eye too near to or too far from the end of the staff, it is a good plan to measure certain known distances and thus determine a set of corrections to be applied, if necessary, to all measures made with that instrument.

The following table gives the distances between certain stars always conveniently placed for observation in the United States, together with the results of measures made upon them with a cross-staff held in the hands without support, and indicates fairly the accuracy which may be obtained with this instrument. The back of the observer was toward the window of a well-lighted room, and the cross was plainly visible by this illumination.

| Stars | True Distance | Measured Distances | Mean | Correction |
|---|---|---|---|---|
| α Ursæ Majoris to β Ursæ Majoris | 5°.4 | 5°.8 — — | 5°.8 | – 0°.4 |
| α " " " γ " " | 10 .0 | 10 .5 10°.6 10°.6 | 10 .6 | – 0 .6 |
| α " " " ε " " | 15 .2 | 15 .6 15 .5 15 .7 | 15 .6 | – 0 .4 |
| β " " " ζ " " | 19 .9 | 20 .0 20 .3 20 .2 | 20 .2 | – 0 .3 |
| α " " " η " " | 25 .7 | 26 .6 26 .0 26 .1 | 26 .2 | – 0 .5 |
| α " " " Polaris | 28 .5 | 29 .0 29 .2 28 .9 | 29 .0 | – 0 .5 |
| β " " " " | 33 .9 | 35 .1 34 .7 34 .4 | 34 .7 | – 0 .8 |
| η " " " " | 41 .2 | 42 .2 42 .0 42 .0 | 42 .1 | – 0 .9 |

The measured distances are about one-half degree too large, and if a correction of this amount is applied to all angles measured by this instrument up to 30°, the corrected values will seldom be so much as half a degree in error, and the mean of three readings will probably be correct within a quarter of a degree.

## CHAPTER III

### THE DIURNAL MOTION OF THE STARS

As the observations of the moon require but a few minutes each evening, observations may be made on the same nights upon the stars. The first object is to obtain the diurnal paths of some of the brighter stars, and as they cast no shadow we must have recourse to a new method of observation to determine their positions in the sky at hourly intervals.

A simple apparatus for this purpose is represented in Fig. 29. A paper circle is fastened to the leveling board used in the sun

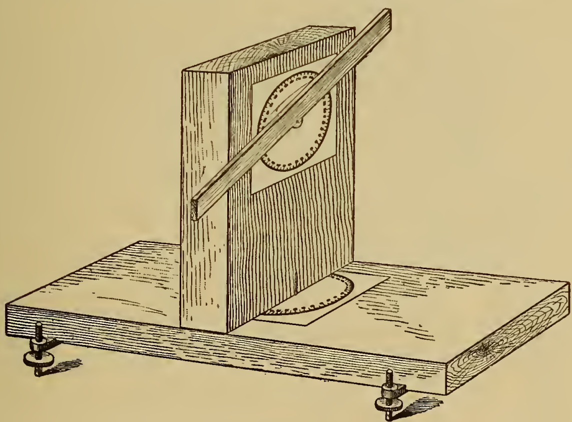

FIG. 29

observations so that the zero of its graduation lies as nearly as possible in the meridian, and a pin with its head removed is placed upright through the center of the circle.

A carefully squared rectangular block about 10 inches by 8 inches by 2 inches is placed against the pin so that the angle which its face makes with the meridian may be read off upon the horizontal circle. A second paper circle is attached to the face of the block with the zero of its graduations parallel to the lower edge; a light ruler is fastened to the block by a pin through the center of its circle; the ruler may be pointed at any star by moving the block about a vertical axis till its plane passes through the star, and then moving the ruler in the vertical plane till it points at the star; a lantern is necessary for reading the circles and for illumination of the block and ruler in full darkness; it should be so shaded that its direct light may not fall on the observer's eye. Sights attached to the ruler make the observation slightly more accurate, but also rather more difficult, and without them the ruler may be pointed within half a degree, which is about as closely as the angles can be determined by the circles.

## THE ALTAZIMUTH

An inexpensive form of instrument for measuring altitude and azimuth is shown in Fig. 30. Here the ruler provided with sights $A$, $B$ is movable about $d$, the center of the semicircle $E$. This semicircle is movable about an axis perpendicular to the horizontal circle $F$, and its position on that circle is read off by the pointer $g$, which reads zero when the plane of $E$ is in the meridian. The circle $F$ is mounted on a tripod provided with leveling screws. If the circle is so placed that the pointer reads zero when the sight-bar is in the magnetic meridian, then its reading when the sights are pointed at any star will give the magnetic bearing of the star. It will, however, be more convenient to adjust the instrument so that the pointer reads zero when the sight-bar is in the true meridian.

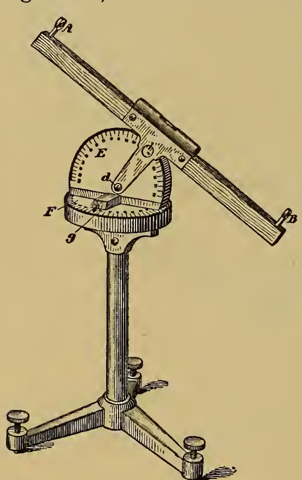

Fig. 30

To insure the verticality of the standard a level is attached to the sight-bar, and by the leveling screws the instrument must be adjusted so that the circle $E$ may be revolved without causing the level bubble to move. (See page 36.)

A more convenient and not very expensive instrument is the altazimuth or universal instrument shown in Fig. 31, which contains some additional parts by the use of which it may be converted into an equatorial instrument. (See page 45.) It consists of a horizontal plate carrying a pointer and revolving on an upright axis which passes through the center of a horizontal circle graduated continuously from 0° to 360°. The plate carries a frame supporting the axis of a graduated circle; this axis is perpendicular to the upright axis, and the circle is graduated from 0° to 90° in opposite directions. Attached to the circle is a telescope whose optical axis is in the plane of the circle. The circle is read by a pointer which is fixed to the frame carrying its axis and reads 0° when the optical axis of the telescope is perpendicular to the upright axis. A level is attached to the telescope so that the bubble is in the center of its tube when the telescope is horizontal. In what follows, all these adjustments are supposed to be properly made by the maker.

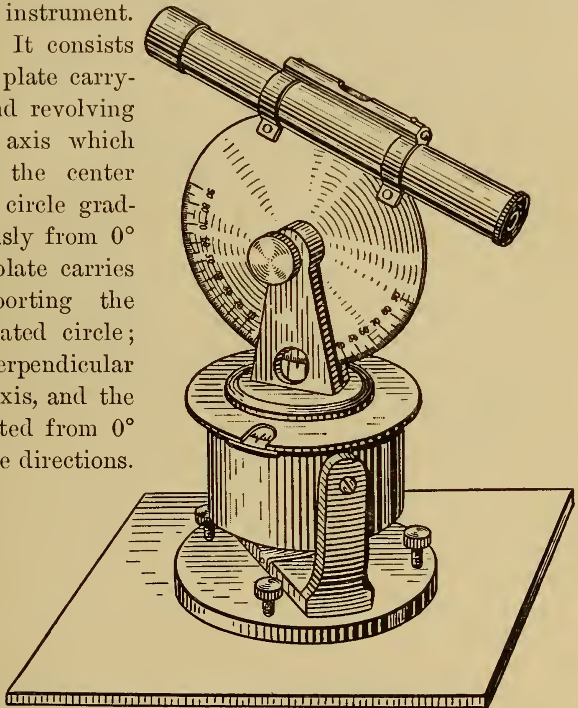

Fig. 31

## ADJUSTMENT OF THE ALTAZIMUTH

If the altazimuth is so adjusted that the upright axis is exactly vertical, and if we know the reading of the horizontal circle when the vertical circle lies in the meridian, we may determine the position of a heavenly body at any time by pointing the telescope upon it and reading the two circles. The difference between the reading of the horizontal circle and its meridian reading is the azimuth, and the reading of the vertical circle is the altitude of the body. Before proceeding to the observation of stars, it will be well to repeat our observations on the sun, using this instrument, and making them in such a manner that we may at the same time get a very exact determination of the meridian reading by the method suggested on page 14.

Place the instrument upon the table used for the sun observation; bring the reading of each circle to 0°; and turn the whole instrument in a horizontal plane until the telescope points approximately south, using the meridian determination obtained from the shadow observations. One leveling screw will then be nearly in the meridian of the center of the instrument, while the two others will lie in an east and west line. Bring the level bubble to the middle of its tube by turning the north leveling screw; then set the telescope pointing east; and "set" the level by turning the east and west screws in opposite directions. Be careful to turn them equally; this can be done by taking one leveling screw between the

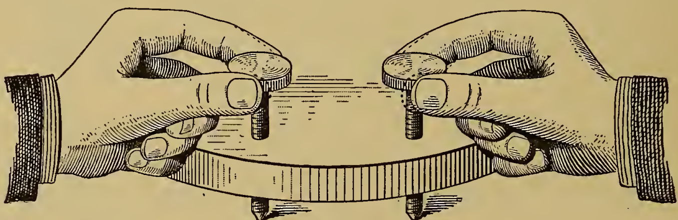

FIG. 32

finger and thumb of each hand, holding them firmly, and turning them in opposite directions by moving the elbows to or from the body by the same amount. Turn the telescope north, and the bubble should remain in place; if it does not, adjust the north screw. The instrument is very easily and quickly adjusted by this method. The upright axis is vertical when the telescope can be turned about it into any position without displacing the bubble.

**Determination of the Meridian and Time of Apparent Noon.** — After completing the adjustment of the instrument, the reading of the circle

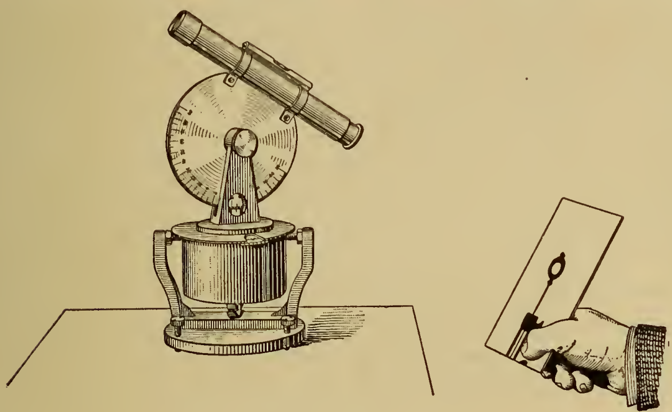

FIG. 33

when the telescope is in the meridian is determined as follows: Point the telescope upon the sun approximately. Place a sheet of paper or a card behind it, and turn the telescope about the vertical axis until the shadow of the vertical circle is reduced to its smallest dimensions and appears as a broad straight line. By moving the telescope about the horizontal axis, bring the shadow of the tube to the form of a circle; in this circle will appear a blurred disk of light. Draw the card about 10 inches back from the eyepiece, and pull out the latter nearly $\frac{1}{8}$ of an inch from its position when focused on distant objects and the disk of light becomes nearly sharp; complete the focusing of this image of the sun by moving the card to or from the eyepiece. The distance of the card and the drawing out of the eyepiece should be such that the sun's image shall be about $\frac{1}{2}$ to $\frac{3}{4}$ of an inch in diameter. Now move the telescope until the image is centered in the shadow of the telescope tube, note the time, and read both circles; this observation fixes the altitude and azimuth of the sun. For determining the meridian it is not necessary that the time should be noted, but it will be convenient to use these observations for a repetition of the determination of the sun's path, determining the altitudes and azimuths by this more accurate method.

This observation should be made at least as early as 9 A.M. Now increase the reading of the vertical circle to the next exact number of degrees, and follow the sun by moving the telescope about the vertical axis. After a few minutes the sun will be again centered by this process. Note the time, and read the horizontal circle. Increase the reading of the vertical circle again by one degree to make another observation, and so on for half an hour. Observations may be made at one-half degree intervals of altitude, but those upon exact divisions will evidently be more accurate. If circumstances admit, observations may be made, during the period of two hours before and after noon, for the purpose of plotting the sun's path; but, owing to the slow change of altitude in that time, the corresponding azimuths are not well determined, and they will be nearly useless for placing the instrument in the meridian.

Some time in the afternoon, as the descending sun approaches the altitude last observed in the forenoon, set the vertical circle upon the reading corresponding to that observation, and repeat the series in inverse order; that is, decrease the readings of altitude by one degree each time, and note the time and the reading of the horizontal circle when the sun is in the axis of the telescope at each successive altitude.

Since equal altitudes correspond to equal azimuths (see page 14), east and west of the meridian, the difference of the horizontal readings is twice the azimuth at either of the two corresponding observations (360° must be added to the western reading, if, as will generally be the case, the 0° point lies between the two readings). Therefore, one-half this difference added to the lesser or subtracted from the greater reading gives the meridian reading. The same value is more easily found by taking half the sum of the two readings. In the same way one-half the interval of time between the two observations added to the time of the first reading gives the watch time of the sun's meridian passage, or apparent noon, as it is called.

Each pair of observations gives the value of the meridian reading and of the watch time of apparent noon; their accordance will give an idea of the accuracy of the observations.

The following observations of the sun were made March 8, 1900, with an instrument similar to that shown in Fig. 33.

| | Time | Altitude | Horizontal Circle | | Time | Altitude | Horizontal Circle |
|---|---|---|---|---|---|---|---|
| 1 | $8^h\ 54^m\ 37^s$ | 27°.5 | 307°.6 | 9 | $2^h\ 32^m\ 10^s$ | 31°.0 | 47°.7 |
| 2 | 8 58 10 | 28 .0 | 308 .4 | 10 | 2 36 30 | 30 .5 | 48 .7 |
| 3 | 9 1 42 | 28 .5 | 309 .2 | 11 | 2 39 45 | 30 .0 | 49 .45 |
| 4 | 9 4 51 | 29 .0 | 310 .0 | 12 | 2 43 27 | 29 .5 | 50 .35 |
| 5 | 9 9 5 | 29 :5 | 310 .9 | 13 | 2 47 0 | 29 .0 | 51 .15 |
| 6 | 9 12 20 | 30 .0 | 311 .8 | 14 | 2 50 17 | 28 .5 | 51 .95 |
| 7 | 9 15 35 | 30 .5 | 312 .6 | 15 | 2 54 7 | 28 .0 | 52 .85 |
| 8 | 9 19 37 | 31 .0 | 313 .45 | 16 | 2 57 33 | 27 .5 | 53 .6 |

The 1st and 16th of these observations give for the meridian reading $\frac{1}{2}[307.6 + (53.60 + 360)] = 360°.60$, and for the corresponding watch time $\frac{1}{2}[8\ 54\ 37 + (2\ 57\ 33 + 12^h)] = 11^h\ 56^m\ 5^s$.

Taking the corresponding A.M. and P.M. observations in this manner, we find for the eight pairs of observations above the following values.

| Altitude | Meridian Reading | Watch Time of Noon |
|---|---|---|
| 27°.5 | 360°.6 | $11^h\ 56^m\ 5.0^s$ |
| 28 .0 | 360 .625 | 56 8.5 |
| 28 .5 | 360 .575 | 55 59.5 |
| 29 .0 | 360 .575 | 55 55.5 |
| 29 .5 | 360 .625 | 56 16.0 |
| 30 .0 | 360 .625 | 56 2.5 |
| 30 .5 | 360 .65 | 56 2.5 |
| 31 .0 | 360 .575 | 55 53.5 |
| mean | 360.61 | 11 56 2.9 |

The agreement of these results is closer than will usually be obtained, the observations being made by a skilled observer and the angles carefully read by means of a pocket lens, which in many cases enabled readings to be made to 0°.05; any reading such as that of the 8th observation, where the value was estimated to lie between two tenths, being recorded as lying halfway between them. This practice adds little to the accuracy if several observations are made, and is not to be recommended to beginners.

## MERIDIAN MARK

It will be convenient to fix a meridian mark for future use. This may be done by fixing the telescope at the meridian reading, turning it down to the horizontal position, and placing some object (as a stake) at as great a distance as possible, so that it may mark the line of the axis of the telescope when in the meridian. A mark on a fence or building will serve if at a greater distance than 50 feet, though a still greater distance is desirable. For setting the telescope upon the mark, it is convenient to have two wires crossing in the center of the field of view, but the setting may be made within 0°.1 without this aid. Having established such a mark, set the horizontal circle at 0°, and move the whole base of the instrument until the telescope points upon the meridian mark. Level carefully; then set the telescope again, if the operation of leveling has caused it to move from the meridian mark; level again, and by repeating this process adjust the instrument so that it is level and that the telescope is in the meridian. Then press hard on the leveling screws, and make dents by which the instrument can be brought into the same position at any future time.

After the A.M. and P.M. observations recorded above, the telescope was pointed upon a meridian mark established by observations made with the shadow of a pin, and the reading of the horizontal circle was 359°.8. The mark was then shifted about a foot toward the west, and the telescope again pointed upon it. As the reading of the circle was then 360°.6, it may be assumed that the mark was now very nearly in the meridian.

If circumstances are such that no point of reference in the meridian is available, it will be necessary, after determining the meridian readings by the sun, to set the telescope upon some well-defined object in or near the horizontal plane and read the circle. The difference between this reading and the meridian reading will be the azimuth of the object. Set the pointer of the horizontal circle to this value, and set the telescope upon the reference mark by moving the whole base as before. If the pointer of the circle is now brought to 0°, the telescope will evidently be in the meridian; and the position is to be fixed by making dents with the leveling screws as before.

## CHOICE OF STARS

We are now ready to begin observations of the stars.

The most familiar group of stars in the heavens is, no doubt, that part of the Great Bear which is variously called the Dipper, Charles's Wain, or the Plough.

At the beginning of October, at 8 o'clock in the evening, an observer anywhere in the United States will see the Dipper at an altitude between 10° and 30° above the N.W. horizon. Set the telescope upon that star which is nearest the north point of the horizon; read both circles to determine its altitude and azimuth, and note the time. Even if the telescope is provided with cross-hairs, the illumination of the light of the sky will not be sufficient to render them visible; but sufficient accuracy in pointing is obtained by placing the star at the estimated center of the field. Observe in succession the altitude and azimuth of the other six stars forming the Dipper, noting the time in each case.

Using the Dipper as a starting point, we will now identify and observe a few other stars.* The total length of the Dipper is about 25°. Following approximately a line drawn joining the last two stars of the handle of the Dipper, at a distance of about 30°, we come to a bright star of a strong red color, much the brightest in that portion of the heavens; this is Arcturus. Observe its altitude and azimuth, and note the time as before. Almost directly overhead, too high to be conveniently observed at this time, is a brilliant white star, Vega ($\alpha$ Lyræ). A little east of south from Vega, at an altitude of about 60°, is a group of three stars forming a line about 5° in length. The central and brightest star of the three is Altair ($\alpha$ Aquilæ), and its position should be observed.

**Diurnal Paths of the Stars.** — Proceed in this way for about an hour, observing also, if time permits, the group of five stars whose middle is at azimuth 220° and altitude 35°. This is the constellation of Cassiopeia. Another interesting asterism will be found — supposing that by this time it is 9 o'clock — at azimuth 270° and altitude 45°, consisting of four stars of about equal magnitude,

* Many of the latest text-books on astronomy contain small star maps which are valuable aids in the identification of the less conspicuous groups.

placed at the corners of a quadrilateral whose sides are about 15° in length, and forming what is called the Square of Pegasus.

It is convenient as an aid in identification to note in each case the magnitude of the star observed. As a rough standard of comparison, it may be remembered that the six bright stars of the Dipper are of about the second magnitude; that at the junction of the handle and bowl is of the fourth. The three stars in Aquila are of the first, third, and fourth magnitudes. Vega and Arcturus are each larger than an average first magnitude star. The brightest stars in the constellation Cassiopeia and in the Square of Pegasus range from the second to the third magnitude.

The little quadrilateral of fourth magnitude stars about 15° east of Altair and known as Delphinus, or vulgarly as Job's Coffin, may be observed.

At the expiration of an hour, set again upon the Dipper stars and repeat the series, going through the same list in the same order. Arcturus will have sunk so low in a couple of hours as to be beyond the reach of observation, even if the place of observation affords a clear view of the horizon. Vega, however, will be less difficult to observe, and may be now added to the list. We should not omit to make an observation of the pole star, which, as its name indicates, may be found near the pole and can be easily found, since the azimuth of the pole is 180°, and its altitude is equal to the latitude of the place.

From the observed values of altitude and azimuth plot the successive places on the hemisphere exactly as in the case of the sun, and thus represent upon the hemisphere the paths of a number of stars in various parts of the heavens. It will be found that these paths are all circles of various dimensions, and that the circles are all parallel to the equator, as determined from the sun observations, that is, they have the same pole as the diurnal circles of the sun.

At this stage it is a good plan to devote some attention to the representation of the various results as shown on the hemisphere, by means of figures on a plane surface, that is, to make careful freehand drawings of the hemisphere and the circles which have now been drawn upon it as seen from various points of view. This is an important aid to the understanding of the diagrams by which it is necessary to explain the statement and solution of astronomical problems; with this purpose in view the drawings should be lettered and the definitions of the various points and lines written under them.

## ROTATION OF THE SPHERE AS A WHOLE

So far the result of our observations is to show that the heavenly bodies appear to move as they would if they were all attached in some way to the same spherical shell surrounding the earth, and were carried about by a common revolution, as if the shell rotated on a fixed axis, passing through the point of observation. The sun may be conceived as carried by the same shell, but observations at different dates show that its place on the shell must slowly change, since its declination changes slightly from day to day.

If these observations on the stars are repeated ten days or one hundred days later, we shall find that the declinations determined from them are the same; that is, the declinations of the diurnal paths of the stars do not change like that of the sun. It will appear also that, as in the case of the sun, equal arcs of the diurnal circle and consequently equal hour-angles are described in equal times. It follows from this, of course, that stars nearer the pole will appear to move more slowly, since they describe paths which are shorter when measured in degrees of a great circle, as may be shown by measuring the diurnal circles on the hemisphere by a flexible millimeter scale, 1 mm. being equal to 1° of a great circle on our hemisphere.

If the field of view of our telescope is 5°, a star on the equinoctial will be carried across its center by the diurnal motion in 20 minutes, while a star at a declination of 60° will remain in the field for twice that time, since its diurnal circle is only half as large as the equinoctial and an angular motion of 10° of its diurnal circle is only 5° of great circle. Since the declinations of the stars do not change, it is unnecessary to make our observations of the stars on the same night; or, rather, observations made on different nights may be plotted as if made on the same night. We may thus obtain extensions of the diurnal circles by working early on one evening and at later hours of the night on following occasions.

## POSITIONS FIXED BY HOUR-ANGLE AND DECLINATION; THE EQUATORIAL

It is evident that we have, in the hour-angles and declinations of the stars, another system of coördinates on the celestial sphere by means of which their position may be fixed. The altitude and azimuth refer the position of the star to the meridian and to the horizon; while the hour-angle and declination refer its position to the meridian and the equator. We have hitherto found it more convenient to deal with the first set of coördinates, but it is often desirable to determine the hour-angle and declination of a body by direct observation, and this may be done by means of an instrument similar to the altazimuth but with the upright axis pointed to the pole of the heavens, so that the horizontal circle lies in the plane of the equator. With this instrument the angles read off on the circle which is directly attached to the telescope measure distances along the hour-circle, perpendicular to the equator, *i.e.*, declinations, while an angle read off on the other circle measures the angle between the meridian and the hour-circle of the star at which the telescope points, and is therefore the star's hour-angle. The two circles are therefore appropriately called the declination circle and the hour-circle of the instrument. As these terms are used with another meaning as applied to circles on the celestial sphere, it would seem that there might be confusion from their use in this sense, but in practice it is never doubtful whether "circle" means the graduated circle of an instrument or a geometrical circle on the surface of the sphere.

It is here supposed that the instrument has been so adjusted that both circles read 0° when the telescope is in the plane of the meridian and points at the equator. An instrument so mounted is called an equatorial instrument. Our altazimuth is adapted to this purpose by constructing the base so that it may be revolved about a horizontal axis perpendicular to the plane in which the altitude circle lies when the azimuth circle reads 0°. If, then, it has been placed in the meridian by the observation of equal altitudes as before described, it may be inclined about this latter axis through an angle equal to the complement of the latitude, and thus brought into the proper position for observing declination and hour-angle directly. An instrument so constructed is called a "universal" equatorial. To adjust the universal equatorial so that the axis points to the pole, adjust it as an altazimuth with both circles reading 0° and level it with the telescope in the meridian pointing south. Depress the telescope till the reading of the vertical circle equals the co-latitude. Tip the whole instrument so as to incline the vertical axis toward the north till the bubble plays and the telescope is horizontal; to do this the vertical axis must have been tipped back through an angle equal to the co-latitude, and it will be in proper adjustment directed toward a point in the meridian whose altitude is equal to the latitude. (Fig. 34 shows the instrument adjusted for latitude 45°.)

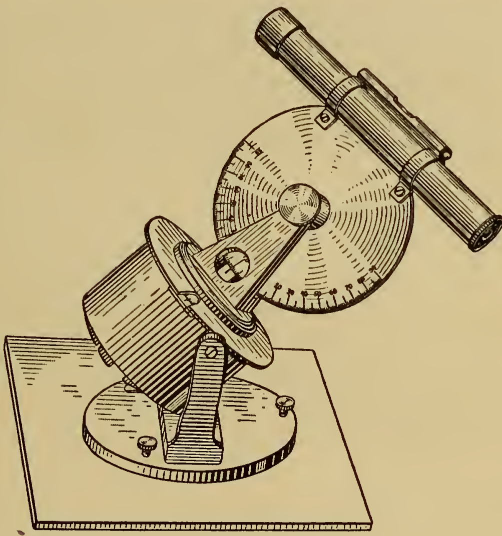

Fig. 34

A notch should be cut in the iron arc at the bottom of the counterpoise, into which the spring-catch may slip when the adjustment is correct, so that the instrument may be quickly changed from one position to the other. If the notch is not quite correctly placed, the final adjustment may be made by a slight motion of the north leveling screw to bring the level exactly into the horizontal position, the vertical circle having been set to the co-latitude for this purpose.

The proper adjustment of the altazimuth is simpler, since it depends only on the use of the level, while to place an equatorial instrument in position we must know the latitude as well. On comparing the two systems of coördinates, it is clear that, while the altitude and azimuth both change continuously, but not uniformly with the time, the hour-angle changes uniformly with the time, and the declination remains the same. One advantage of the latter system of coördinates is that in repeating our observations on the same star after the lapse of an hour, we need only set the declination circle to the previously observed declination, and set the hour-circle at a reading obtained by adding to the former setting the elapsed time in hours reduced to degrees by multiplying by 15; we shall then pick up the star without difficulty. This is an important aid in identifying stars, which has no counterpart in the use of the altazimuth, and we shall henceforth use this method of observation in preference to the other.

# CHAPTER IV

## THE COMPLETE SPHERE OF THE HEAVENS

The study of the motions of the sun, moon, and stars has thus far led to the conclusion that their courses above the plane of the horizon can be perfectly represented by assuming the daily rotation from east to west of a sphere to which they are attached, or a rotation of the earth itself from west to east about an axis lying in the meridian and inclined to the horizon at an angle equal to the latitude of the place of observation, while the sun moves slowly to and from the equator, and the moon, like the sun, changes its declination continually, and has also a motion toward the east on the sphere at a rate of about 13° in each 24 hours. The combination of the two motions of the moon causes it to describe a path which will be more fully discussed later. We shall now begin to observe the sun, to see if its motion among the stars resembles that of the moon in having an east and west component in addition to its motion in declination.

The motion of the moon can be directly referred to the stars, since both are visible at the same time, although the illumination of the dust of our atmosphere, by strong moonlight, cuts us off from the use of the smaller stars, which cannot be seen except when contrasted with a perfectly dark background.

The illumination produced by the sun, however, is so strong that it completely blots out even the brightest stars, so that we cannot apply either of the methods that we have employed in observing the moon.

We are only able to see the stars, of course, when they are above the plane of the horizon, but it is natural to suppose that they continue the same course below the horizon from their points of setting to those of their rising. This inference is confirmed by the fact that some of the bright stars which set within a few degrees of the north point of the horizon, and which we infer complete their course below the horizon, may be seen actually to do so by an observer at a point on the earth some degrees farther north, from which they may be observed throughout the whole of their courses. In the case of the sun, the following facts lead to the same conclusion. Immediately after sunset a twilight glow is seen in the west whose intensity is greatest at the point where the sun has just set. This glow appears to pass along the horizon towards the north, and its point of greatest intensity is observed to be directly over the position which the sun would occupy in the continuation of its path below the horizon, on the assumption that it continues to move uniformly in that path. In high latitudes this change of position in the twilight arch can be followed completely around from the point of sunset to the point of sunrise, the highest point being due north at midnight. It is impossible not to believe that the sun is actually there, though concealed from our sight by the intervening earth. (Of course, too, it is now generally known that in very high latitudes the sun at midsummer is visible throughout its diurnal course.) As the sun sinks farther, the light of the sky decreases, the brighter stars begin to appear, and it is clearly impossible to resist the conclusion that they have been in position during the daylight, but simply blotted out by the overwhelming light of the sun.

## OBSERVATIONS WITH THE EQUATORIAL

When we have fixed the idea that the heavenly sphere revolves as a whole, carrying with it in a certain sense all the bodies that we observe, the next step is to devise some means of locating the different bodies in their proper relative positions on the sphere. For this purpose the equatorial instrument furnishes us with an admirable means of observation. The relative position of two stars is completely fixed when we know the position of their parallels of declination and their hour-circles, since the place of each star is at the intersection of these two circles.

Since an observation with the equatorial gives directly the declination and hour-angle of a star, the method of fixing the relative position of two stars, *A* and *B*, is as follows:

Point the telescope at *A*, and read the circles; then set on *B*, and read the circles; then again on $A$, and read the circles, taking care that the interval between the first and second observations shall be as nearly as possible equal to the interval between the second and third. Obviously the mean of the two readings of the hour-circle at the pointings upon $A$ gives the hour-angle of $A$ at the time when $B$ was observed, since the star's hour-angle changes uniformly. The difference between this mean and the reading of the hour-circle when the pointing was made upon $B$ is, therefore, the difference between the hour-angles of the stars at the time of that observation; and this fixes the relative position of their hour-circles, since this difference is the arc of the equator included between them; their declinations are given by the readings of the declination circles, and thus the relative position of the two stars is completely known.

As an illustration of this method, we may take the following example:

With the telescope pointed at $A$, the readings of the hour-circle and declination circle were 68°.2 and 15°.1, respectively. The telescope was then pointed at $B$, and the circles read 85°.9, 28°.1, and finally upon $A$, the readings being 69°.1, 15°.1; the intervals were nearly the same, as will usually be the case, unless there is some difficulty in finding the second star. Of course the first star can be *re*-found by the readings at the first observation; indeed, if the intervals are plainly unequal, a repetition of the observation may always be made at equal intervals by setting the circles for each star so that no time is lost in finding.

From the above observations we infer that when the hour-angle of $B$ was 85°.9, that of $A$ was 68°.65; and, therefore, that the hour-circles of the two stars cut the equator at points 17°.25 apart; the hour-circle of $B$ being to the west of that of $A$, so that $B$ comes to the meridian earlier, or "precedes" $A$.

It may be noted that the observations apparently occupied a little less than 4 minutes, since in the whole interval the hour-angle of $A$ changed by 0°.9.

## USE OF A CLOCK WITH THE EQUATORIAL

If the intervals between the observations are not exactly equal, it will still be easy to fix the hour-angle of *A* at the time of the observation on *B* if the *ratio* of the intervals is known; if, for instance, the first observation of *A* gives an hour-angle of 25°.3, and the later observation an hour-angle of 26°.3, while the intervals are $1^m$ between the first and second observations, and $3^m$ between the second and third, the hour-angle of *A* at the second observation was obviously 25°.3 + 0°.25. We may thus obtain by "interpolation" the hour-angle of *A* at any known fraction of the interval. Plainly it is an advantage to note the time of each observation for this purpose, as in the following observations, which were made Feb. 5, 1900, for the purpose of determining the relative positions of the stars forming the Square of Pegasus.

| Star | Watch Time | Decl. Circle | Hour-Circle |
|---|---|---|---|
| 1 γ Pegasi | $7^h\ 14^m\ 0^s$ | + 15°.2 | 66°.3 |
| 2 α Pegasi | 15 0 | + 15 .2 | 83 .6 |
| 3 β Pegasi | 16 15 | + 28 .1 | 84 .1 |
| 4 α Andromedæ | 17 10 | + 29 .0 | 68 .3 |
| 5 γ Pegasi | 18 30 | + 15 .2 | 67 .6 |
| 6 γ Pegasi | 21 30 | + 15 .1 | (69 .2) |
| 7 α Andromedæ | 22 30 | + 29 .1 | 69 .6 |
| 8 β Pegasi | 23 30 | + 28 .1 | 85 .9 |
| 9 α Pegasi | 24 20 | + 15 .3 | 86 .0 |
| 10 γ Pegasi | 25 30 | + 15 .1 | 69 .1 |
| 11 γ Pegasi | 27 30 | + 15 .1 | 69 .6 |

The observations here follow each other rapidly. They were made by an experienced observer, and the arrangement of the stars is such that, after setting γ Pegasi, α Pegasi is brought into the field by moving the telescope about the hour-axis only; we pass to β Pegasi by motion around the declination axis only, to α Andromedæ by motion about the hour-axis, and back to γ Pegasi by rotation about the declination axis; so that the stars are found more quickly than if both axes must be altered in position at each change; in observations 6 to 10 the series is observed in reversed order.

If the instrument was correctly adjusted, the declination of the four stars was as follows: $\gamma$ Pegasi $+15^\circ.14$, $\alpha$ Pegasi $15^\circ.25$, $\beta$ Pegasi $28^\circ.1$, $\alpha$ Andromedæ $29^\circ.05$, each being determined as the mean of all the observations made upon the star.

The first advantage of the recorded times is to show that the reading of the hour-circle in 6 was an error, probably for $68^\circ.2$, as we see by comparison with the other values of the hour-angle of $\gamma$ Pegasi, which increase uniformly about $1^\circ$ in each 4 minutes. It will be better, however, to reject the observation entirely, as it is not necessary to use it for the first set of observations 1 to 5, which we will now discuss.

By interpolation between 1 and 5 we find that the hour-angle of $\gamma$ Pegasi at $7^h\,15^m\,0^s$ was $\frac{2}{9}$ of $1^\circ.3$ greater than $66^\circ.3$, or $66^\circ.59$; at $7^h\,16^m\,15^s$ it was $\frac{1}{2}$ of $1^\circ.3$ greater than $66^\circ.3$, or $66^\circ.95$; and at $7^h\,17^m\,10^s$ it was $\frac{80}{270}$ of $1^\circ.3$ less than $67^\circ.6$, or $67^\circ.21$. As the hour-angles of the other stars were observed at these times, we can at once find the differences of their hour-angles from that of $\gamma$ Pegasi, which are as follows: $\alpha$ Pegasi, $17^\circ.01$; $\beta$ Pegasi, $17^\circ.15$; $\alpha$ Andromedæ, $1^\circ.09$. All the hour-angles are greater than those of $\gamma$ Pegasi, so that all the stars precede $\gamma$ Pegasi. By using all the observations we may presumably obtain more accurate results, and it will be well, as in all cases when a considerable number of observations must be dealt with, to arrange the reductions in a more systematic manner.

In the table on the following page the difference of hour-angle is obtained by subtracting the observed hour-angle in each case from the hour-angle of $\gamma$ Pegasi, so that its value is negative, if, as in the results given above, the stars precede $\gamma$ Pegasi, and positive if they follow it. An observation of Venus, made on the same occasion, is added to the list, and an additional observation of $\alpha$ Pegasi is included; the erroneous observation of $\gamma$ Pegasi at $7^h\,21^m\,30^s$ is excluded.

The values of the hour-angle of $\gamma$ Pegasi at the successive times, as given in column 6, are computed from the following considerations, the proof of which is left to the student. If a quantity changes uniformly, and its values at several different times are known, the mean of these values is the same as the value which

| Star | Time | Decl. | H.A. | H.A. of γ Peg. | Star follows γ Peg. |
|---|---|---|---|---|---|
| 1 Venus | $7^h\ 12^m\ 0^s$ | + 4°.0 | 75°.5 | 65°.86 | − 9°.64 |
| 2 α Peg. | 13 0 | + 15 .1 | 83 .1 | 66 .10 | − 17 .00 |
| 3 γ Peg. | 14 0 | + 15 .2 | 66 .3 | 66 .35 | + 0 .05 |
| 4 α Peg. | 15 0 | + 15 .2 | 83 .6 | 66 .59 | − 17 .01 |
| 5 β Peg. | 16 15 | + 28 .1 | 84 .1 | 66 .89 | − 17 .21 |
| 6 α Androm. | 17 10 | + 29 .0 | 68 .3 | 67 .12 | − 1 .18 |
| 7 γ Peg. | 18 30 | + 15 .2 | 67 .6 | 67 .44 | − .16 |
| 8 α Androm. | 22 30 | + 29 .1 | 69 .6 | 68 .44 | − 1 .16 |
| 9 β Peg. | 23 30 | + 28 .1 | 85 .9 | 68 .88 | − 17 .22 |
| 10 α Peg. | 24 30 | + 15 .3 | 86 .0 | 68 .93 | − 17 .07 |
| 11 γ Peg. | 25 30 | + 15 .1 | 69 .1 | 69 .17 | |
| 12 γ Peg. | 27 30 | + 15 .1 | 69 .6 | 69 .65 | + 0 .05 |

the quantity has at the mean of the times. Using this principle, we find the hour-angle of γ Pegasi at $7^h\ 21^m\ 22^s$ was 68°.15.

Between observations 3 and 12 it changed 3°.3 in $13\frac{1}{2}^m$, or 0°.244 per minute. Assuming this rate of change, it is easy, though laborious, to compute the hour-angle at any one of the given times; for example, at $7^h\ 12^m\ 0^s$ the hour-angle was 68°.15 − ($9\frac{22}{60}$ times 0°.244), or 65°.86. Labor will be saved by making a table of the values at the even minutes by successive additions of 0°.244, from which the values at the observed times are rapidly interpolated. The sixth column contains the number of degrees by which the hour-circle of the star follows that of γ Pegasi. The mean values for each star obtained from this column are as follows.

| Star | Decl. | Diff. H.A. |
|---|---|---|
| γ Pegasi . . . . . . . . . . | + 15°.15 | 0°.00 |
| Venus . . . . . . . . . . . | − 4 .0 | − 9 .64 |
| α Pegasi . . . . . . . . . . | + 15 .20 | − 17 .03 |
| β Pegasi . . . . . . . . . . | + 28 .10 | − 17 .22 |
| α Androm. . . . . . . . . . | + 29 .05 | − 1 .17 |

The true values of the declinations of these stars as determined by many years of observations are for γ Pegasi 14°.63, α Pegasi 14°.67, β Pegasi 27°.55, α Andromedæ 28°.53. The values from our observations are 15°.15, 15°.20, 28°.10, 29°.05, so that the latter require corrections of $-$ 0°.52, $-$ 0°.53, $-$ 0°.55, and $-$ 0°.52, respectively. This is due to a faulty adjustment of the instrument, but the error from this cause evidently affects all the observations by nearly the same amount, 0°.53, so that the relative positions are given quite accurately; our observations placing the whole constellation about $\frac{1}{2}$° too far north.

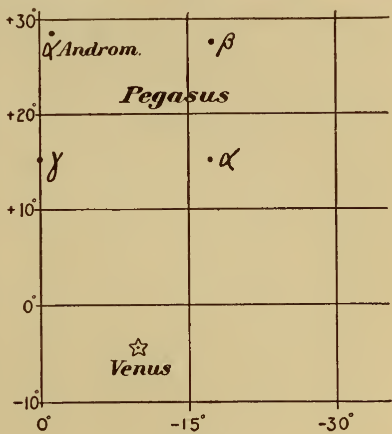

FIG. 35

Since Venus is in the near neighborhood of γ Pegasi, we may assume that the observations of that planet are subject to the same corrections, that she preceded γ Pegasi by 9°.64, and that her true declination was $-$ 4°.0 $-$ 0°.53, or $-$ 4°.53. The correction is applied algebraically with the same sign as to the other stars, since it must be so applied as to make the true place farther south than the observed place.

The places of the Square of Pegasus and the planet Venus, as seen in the sky Feb. 5, 1900, are shown in Fig. 35.

Before plotting the stars on the hemisphere from the above data, it must be prepared by drawing upon it in their proper positions the meridian, zenith, pole, and equator. Draw the hour-circle of γ Pegasi (see Fig. 19, p. 17) at the proper hour-angle from the meridian, to give its position at the time of the last observation, which may be determined by making it intersect the equator at the proper point 69°.6 west of the meridian, and place the star upon it at a distance from the equator equal to the observed declination, 15°.14. The hour-angle of α Pegasi should be drawn in the same manner to cut the equator at 86°.66 from the meridian, and the star placed upon it at the observed declination, 15°.20. Of course on the scale of so small a hemisphere the nearest half degree is sufficiently accurate. Remember that the configurations on the hemisphere and on the map are semi-inverted.

## CLOCK REGULATED TO SHOW THE HOUR-ANGLE OF THE FUNDAMENTAL STAR

The method of calculating the hour-angles of $\gamma$ Pegasi in the last example shows that if the reading of the watch can be relied upon, the observations of that star need only be made at the beginning and at the end of the period of observation, the hour-angle at any time being determined by its uniform increase; or even from a single observation at the beginning of the period, since at the time of observation of any star the hour-angle of $\gamma$ Pegasi can be inferred from that at its first observation by adding the number of degrees which it would have described in the time elapsed, obtained by multiplying the number of hours by 15, or, what gives the same results, dividing the minutes by 4. Moreover, if the rate of the watch is such that it completes its 24 hours in the time in which the stars complete their daily revolution, and if its hands are so set as to read 12 hours when $\gamma$ Pegasi is on the meridian, the difference of hour-angle at any time will be equal to the reading taken directly from the hands of the watch reduced as above to degrees, for when the star is on the meridian and its hour-angle therefore zero, the watch marks $0^h\ 0^m\ 0^s$. Four minutes later by the watch the hour-angle of the star has increased by the diurnal revolution to 1°; in four minutes more to 2°; when the watch indicates 1 hour, the star's hour-angle has increased to 15°, and so on, till 24 hours have elapsed, when the star will again be on the meridian and the cycle recommences.

The rate of an ordinary watch is sufficiently near to that of the stars to allow of its use for this purpose for periods of an hour without causing any error in our observations.

In the use of this method we may regard the observation of the fundamental or zero star as a means of finding out whether the clock is set to the right time: thus, in the following set of observations the first observation gives the hour-angle of $\gamma$ Pegasi 67°.6 at $7^h\ 15^m\ 10^s$, but as 67°.6 equals $4^h\ 30^m\ 24^s$, we may regard the clock as $2^h\ 44^m\ 46^s$ fast; and by applying this correction to all the observed times, may write down at once under the title "corrected time" what the readings would have been if the clock had been set to show 0 hours, when the hour-angle of $\gamma$ Pegasi was 0°. Dividing these by 15 we have the hour-angle in degrees given in column 4.

The following observations were undertaken for determining the configuration of the stars in Orion and its neighborhood, Feb. 6, 1900.

| Star | Obs. Time | Corrected Time | H.A. of $\gamma$ Peg. | Observed H.A. of Star | Decl. | Follows $\gamma$ Peg. |
|---|---|---|---|---|---|---|
| $\gamma$ Pegasi | $7^h\ 15^m\ 10^s$ | $4^h\ 30^m\ 24^s$ | 67°.6 | 67°.6 | + 15°.5 | |
| a | 7 20 0 | 4 35 14 | 68 .8 | 348 .5 | − 1 .4 | 80°.3 |
| b | 7 22 0 | 4 37 14 | 69 .3 | 349 .95 | − 0 .6 | 79 .35 |
| c | 7 23 30 | 4 38 44 | 69 .7 | 348 .1 | − 2 .2 | 81 .8 |
| d | 7 25 20 | 4 40 34 | 70 .1 | 344 .8 | + 7 .1 | 85 .3 |
| e | 7 27 10 | 4 42 24 | 70 .6 | 353 .0 | + 6 .1 | 77 .6 |
| f | 7 28 45 | 4 43 59 | 71 .0 | 347 .5 | − 9 .9 | 83 .5 |
| g | 7 30 20 | 4 45 34 | 71 .4 | 356 .4 | − 8 .2 | 75 .0 |
| h | 7 32 0 | 4 47 14 | 71 .8 | 351 .6 | − 5 .5 | 80 .2 |
| i | 7 34 0 | 4 47 14 | 72 .3 | 334 .7 | − 16 .9 | 97 .6 |
| j | 7 35 45 | 4 50 59 | 72 .7 | 321 .3 | + 5 .05 | 111 .4 |
| a | 7 37 45 | 4 52 59 | 73 .2 | 352 .9 | − 1 .4 | 80 .3 |
| $\gamma$ Pegasi | 7 39 50 | 4 55 4 | 73 .8 | 73 .9 | + 15 .4 | |
| Moon | 7 42 0 | 4 57 14 | 74 .3 | 27 .6 | + 20 .4 | 46 .7 |

The results of columns 6 and 7 enable us to map the constellation as in Fig. 36.

One or two constellations may be plotted in this manner both on the map, which shows the constellation as seen in the sky, and on

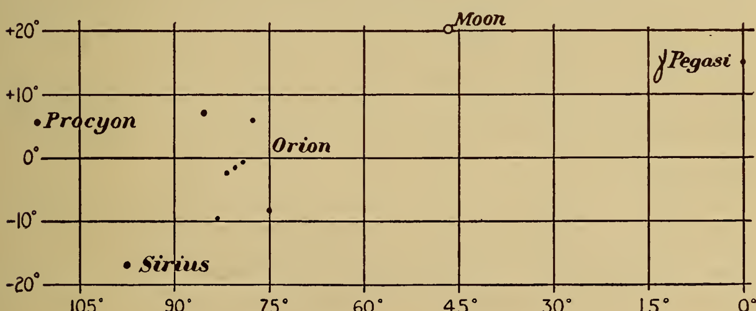

Fig. 36.

the hemisphere, where it is semi-inverted. It will be advisable, however, before much work has been done in this way, to introduce a slight modification.

## THE VERNAL EQUINOX — RIGHT ASCENSION

The precession of the equinoxes causes a change in the position of the equator, which slowly changes the declinations of all the stars. For this reason it is found more convenient to select, instead of $\gamma$ Pegasi as a zero star, the point upon the equator at which the sun crosses it from south to north about March 21 of each year. This point, which is called the vernal equinox, is not fixed, but its motion, due to precession, is simpler than that of any star which might be selected as a zero point; it precedes the hour-circle of $\gamma$ Pegasi at present by about 8 minutes of time, or 2° of arc, and it was because of this proximity that we first selected that star.

Instead, therefore, of adjusting our clock so that it reads $0^h\ 0^m\ 0^s$ when $\gamma$ Pegasi is on the meridian, we set it to that time when the vernal equinox is in that plane; its readings then give the hour-angle of the vernal equinox, and the difference between the hour-angles of that point and of the star may be directly obtained from our observations. The distance by which a star follows the vernal equinox is called its right ascension; more carefully defined, it is the arc of the equator intercepted between the hour-circle of the star and the hour-circle of the vernal equinox (which measures the wedge angle between the planes of these circles); it is also the angle between the tangents drawn to these two circles where they intersect at the pole. Since any star which is east of the vernal equinox follows it, the right ascensions of different stars increase toward the east, that is, toward the left in the sky as we face south, but toward the right on the solid hemisphere as we look down from the outside upon its southern face.

Hereafter we shall fix the positions of the stars by their right ascensions and declinations. We may make use of the observations already reduced with very little additional labor. Since $\gamma$ Pegasi follows the vernal equinox by 2°, we need only add that amount to the quantities given in column 7 on page 55 to know the right ascension of the different stars. If we learn later that on February 6 the right ascension of $\gamma$ Pegasi was more exactly $0^h\ 8^m\ 5^s.64$, we may further correct by adding $5^s$, or even $5^s.64$, if the accuracy of the observations warrants it. The method of determining the exact position of the zero star with reference to the vernal equinox is given in Chapter VI.

Formerly right ascensions were measured altogether in degrees, but owing to the modern use of clocks, it has long been customary to give them in hours; for this reason the hour-circle of instruments mounted as equatorials is graduated to read hours and minutes directly. Since our universal equatorial is intended to serve also as an altazimuth, its circles are both graduated to degrees.

## SIDEREAL TIME

In the last section right ascension has been defined as the angle between the hour-circle passing through a star and the great circle passing through the pole and the vernal equinox. The latter circle is called the equinoctial colure. We have also suggested the use of a clock set to read $0^h\ 0^m\ 0^s$ at the time when the vernal equinox is on the meridian; so that the hour-angle of the vernal equinox at any time will be given directly by the reading of the face of the watch in hours, minutes, and seconds, from which the angle in degrees is computed by dividing by 15. A clock set in this manner, and running at such a rate that it completes 24 hours in the time that the star completes its revolution from any given hour-angle to the same hour-angle again, is said to keep sidereal time. We shall find later that a clock so regulated gains about 4 minutes a day on a clock keeping mean time, thus gaining 24 hours on an ordinary clock in the course of a year, and agreeing evidently with a clock keeping apparent time, as defined on page 19, at that time when the sun is at the vernal equinox and crosses the meridian at the same time with the latter.

Let us suppose now that the vernal equinox has passed the meridian by one hour, then its hour-angle is $1^h$, or $15°$; and our sidereal clock indicates exactly $1^h\ 0^m\ 0^s$. Any star which is at this time on the meridian, that is, whose hour-angle is $0°$, must therefore follow the vernal equinox by $1^h$, or 15°, while at the same instant the time by our sidereal clock is $1^h\ 0^m\ 0^s$. By our definition of right ascension, since the star follows the vernal equinox by $1^h$, its right ascension is $1^h$; in this case, therefore, the right ascension of the star in hours, minutes, and seconds has the same value as the time given by the hands of the clock. In the same way, if the vernal equinox has passed the meridian so far that its hour-angle is $2^h\ 15^m$, the face of the clock will show $2^h\ 15^m$; and any star then upon the meridian follows the vernal equinox by $2^h\ 15^m$. The same relation holds here; namely, that the right ascension of the star is equal to the time by the sidereal clock when the star is upon the meridian. This might have been given as a definition of the term "right ascension"; and, indeed, so closely are the two connected in the mind of the practical astronomer that if the right ascension of a star is given, he at once thinks of this number as representing the time of its meridian passage.

## RIGHT ASCENSION PLUS HOUR-ANGLE EQUALS SIDEREAL TIME

We may here give an explanation of a general principle of very frequent application, and of which this is simply a particular case. Suppose the vernal equinox, represented by the symbol ♈ (Fig. 37), to have passed the meridian by $5^h\ 10^m$. Then a star, *S*, whose right ascension is $2^h\ 15^m$, since it follows the vernal equinox by that amount, will have passed the meridian by $2^h\ 55^m$; and its hour-angle will be $2^h\ 55^m$. The arc of the equator between the meridian and the vernal equinox may be considered as made up of two parts: the right ascension of the star, which is measured by the arc eastward from the vernal equinox to the hour-circle of the star, and the hour-angle of the star, which extends from the meridian westward to the hour-circle of the star. Since this is true of any star, or,

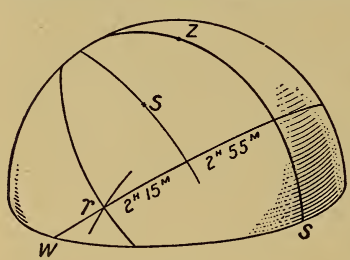


FIG. 37

indeed, of any heavenly body, we may make the following general statement: The right ascension of any body plus its hour-angle at any instant will be equal to the sidereal time at that instant; or, as it is sometimes written: R.A. + H.A. = Sidereal Time. If the body is a point on the meridian, its H.A. = zero; hence the R.A. of a star on the meridian, or briefly, R.A. of the meridian = Sidereal Time, as we have before shown.

From this relation we may most simply determine the right ascension of any heavenly body by observing its hour-angle with the equatorial instrument, and at the same time noting the sidereal time, since R.A. = Sidereal Time − H.A. It is by this method that we shall now proceed to make a somewhat extended catalogue of stars from which we may plot their positions upon the globe.

We will here notice some of the important uses to which this principle may be put. If by any other means the right ascension of a body is known, we may find its hour-angle at any given sidereal time by the equation, Sidereal Time − R.A. = H.A. This gives us an easy way to point upon any object whose right ascension and declination are known, if we have a clock keeping sidereal time; and this is the usual way in which the astronomer finds the objects which he wishes to observe, since they are generally so faint that they cannot be seen by the naked eye. For example, to point the telescope at the great nebula in Orion, whose right ascension is $5^h\ 28^m$, and declination 6° S., we first set the declination circle to −6°, and if the sidereal time is $7^h\ 30^m$ we set the hour-circle to $2^h\ 2^m$, then the telescope will be pointed upon the star. If the sidereal time is $4^h\ 30^m$, in which case the star evidently has not reached the meridian by nearly an hour, we must add 24 hours to the sidereal time; then the expression, H.A. = Sidereal Time − R.A. will become H.A. = $28^h\ 30^m - 5^h\ 28^m$, or $23^h\ 2^m$, the hour-angle being reckoned, as before stated, from $0^h$ up to $24^h$. If then the hour-circle is brought to the reading $345\frac{1}{2}° = 15° \times 23\frac{2}{60}$, we shall find the star in the field.

## THE CLOCK CORRECTION

The same principle enables us to set our clock correctly to sidereal time by observing the hour-angle of *any* star whose right ascension is known. For example, the right ascension of Sirius being $6^h\ 40^m$, or 100°, and its hour-angle being observed to be 330°, or $22^h$, the sidereal time is R.A. + H.A., that is, 430°, or, subtracting 360°, is 70°, corresponding to $4^h\ 40^m$; and a clock may be set to agree; or, by subtracting the time which it then indicates, we determine a correction to be applied to its reading to give the true sidereal time. If, for instance, at the observation above, the clock time is $4^h\ 41^m\ 10^s$, the clock correction is $-1^m\ 10^s$. In this case the clock is $1^m\ 10^s$ fast, the time which it indicates is greater than the true time, and its "error" is said to be $+1^m\ 10^s$. On the other hand, when the clock is slow the correction to true time is positive, while the "error" is negative.

It is customary to observe this distinction between the terms "error" and "correction"; the former is the amount by which the observed value of a quantity exceeds its true value, while the correction is the quantity which must be added to the observed to obtain the true value. They are thus numerically equal but of opposite sign.

The error of the declination circle determined by the observations of page 53 was $+0°.53$, while the correction was $-0°.53$.

For the constantly occurring "clock correction," we shall use the symbol $\Delta t$, the value of which is positive if the clock is slow and negative if it is fast.

If, as is often desirable, we wish to observe a body of known right ascension upon the meridian, we have only to observe it when the time by the sidereal clock is equal to its right ascension.

We may of course find the right ascension of the moon by a direct comparison with the neighboring stars, just as we have determined the difference in right ascension of $\alpha$ Pegasi, from that of $\gamma$ Pegasi, for the brighter stars can be easily observed at the same time as the moon; but no star is so bright that it can be readily observed by our small instrument when the sun is above the horizon,* and we have therefore no means of making a direct comparison between

* See, however, page 70.

a star and the sun. But by means of our clock and our new method of observation this becomes easy; and the sun is to be added to the list of bodies whose right ascension we are to observe regularly. It is only necessary that we should be provided with a clock which keeps correct sidereal time. (See page 67.)

We have already spoken of the means of setting the clock; now a few words as to how the regularity of its rate may be determined. It is only necessary to observe the watch time at which any star is at a given hour-angle on successive nights. If the rate of the clock is such that the interval between the observations is greater than 24 hours, the watch is gaining; if the amount is less than half a minute a day, the watch may be assumed for our purposes to be keeping correct sidereal time, its actual error at any time being checked, as before described, by the observation of the hour-angle of bodies of known right ascension.

## LIST OF STARS

Our first care will be to observe a number of bright stars not very far from the equator which will serve for setting the clock or determining its error, selecting them so that several shall always be above the horizon and may at any time be used for this purpose. Several of those already observed will be found in the list given on the following page, which contains the approximate places of a number of conspicuous stars.

By repeated comparisons of these stars with each other and with $\gamma$ Pegasi, their right ascensions may easily be fixed within $30^s$, and they may then be used for determining the clock error at any time when they are visible. The observations of each evening should be reduced as soon as possible and maps made of the various constellations similar to those of Figs. 35 and 36; it is, however, impossible to represent any large portion of the sphere satisfactorily on a plane surface, and, in order to have a proper idea of the relative positions of the various constellations, we must plot our results on a globe — a proceeding still more necessary when we come to study the motion of the sun and moon among the stars by the method of the following chapter.

A globe 6 inches in diameter is sufficiently large for our purpose; it should be so mounted that it may be turned about its axis on a firm support, and upon it should be traced 24 hour-circles 15° apart, and small circles (parallels of declination) parallel to the equator and 10° apart; its surface should be smooth and white, and of such a texture as to take a lead-pencil mark easily, but permit of erasure.

## TIME STARS

| Star | Mag. | R.A. | $\delta$ | Star | Mag. | R.A. | $\delta$ |
|---|---|---|---|---|---|---|---|
| $\gamma$ Pegasi | 3 | $0^h.1$ | + 15° | Denebola | 2 | $11^h.7$ | + 15° |
| $\beta$ Ceti | 2 | 0 .6 | − 19 | $\delta^2$ Corvi | 3 | 12 .4 | − 16 |
| $\beta$ Andromedæ | 2 | 1 .1 | + 35 | Spica | 1 | 13 .3 | − 11 |
| $\alpha$ Arietis | 2 | 2 .0 | + 23 | Arcturus | 1 | 14 .2 | + 20 |
| $\alpha$ Ceti | 2½ | 3 .0 | + 4 | $\alpha^2$ Libræ | 3 | 14 .8 | + 16 |
| Alcyone | 3 | 3 .7 | + 24 | $\alpha$ Serpentis | 3 | 15 .7 | + 7 |
| Aldebaran | 1 | 4 .5 | + 16 | Antares | 1 | 16 .4 | − 26 |
| Capella | 1 | 5 .2 | + 45 | $\alpha$ Ophiuchi | 2 | 17 .5 | + 13 |
| Rigel | 1 | 5 .2 | − 8 | $\gamma^2$ Sagittarii | 3 | 18 .0 | − 30 |
| $\epsilon$ Orionis | 2 | 5 .5 | − 1 | Vega | 1 | 18 .6 | + 39 |
| Betelgeuze | 1 | 5 .8 | + 7 | Altair | 1 | 19 .8 | + 9 |
| Sirius | 1 | 6 .7 | − 17 | $\alpha^2$ Capricorni | 4 | 20 .2 | − 13 |
| Castor | 2 | 7 .5 | + 32 | $\alpha$ Delphini | 4 | 20 .6 | + 16 |
| Procyon | 1 | 7 .6 | + 5 | $\epsilon$ Pegasi | 2½ | 21 .7 | + 9 |
| Pollux | 1 | 7 .7 | + 28 | $\alpha$ Aquarii | 3 | 22 .0 | − 1 |
| $\alpha$ Hydræ | 2 | 9 .4 | − 8 | $\alpha$ Pegasi | 2½ | 23 .0 | + 15 |
| Regulus | 1 | 10 .1 | + 12 | | | | |

The number attached to the Greek letter indicates that the star to be observed is the following of two neighboring stars.

## CHAPTER V

### MOTION OF THE MOON AND SUN AMONG THE STARS

For plotting the stars on the globe in their proper places, as given by their right ascensions and declinations, it is convenient to have the equator graduated into spaces of $10^m$ each; this may be done by laying the edge of a piece of paper along the equator, and marking off the points of intersection of the equator with two consecutive hour-circles; laying the paper upon a flat surface, bisect the space between the two lines with the dividers, and trisect each of these spaces by trial, testing the equality of the spacing by the dividers; this may be satisfactorily done by two or three trials, and the short scale thus obtained may be easily transferred to the arcs on the equator between each two hour-circles. It may be found convenient to bisect each of the spaces on the scale, thus dividing the equator into spaces of $5^m$ each.

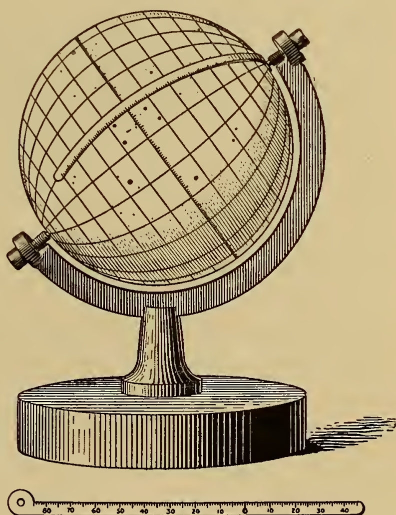

FIG. 38

A strip of parchment or parchment paper about 8 inches long and $\frac{1}{4}$ inch wide, of the shape shown in Fig. 38, and graduated to degrees, completes the apparatus necessary for plotting. The hole being placed over the axis of the globe, the graduated edge of the strip may be made to coincide with the hour-circle of any star by causing it to intersect the equator at a point corresponding to the star's right ascension, taking care that the edge lies in a great circle of the sphere; the graduated edge gives at once the proper declination for plotting the star upon its hour-circle, and the point may be marked with a well-sharpened, hard lead pencil; the latter should be carefully kept, and used for purposes of plotting only. With this simple apparatus the stars may be rapidly and accurately placed upon the globe.

An attempt should be made to represent the magnitudes of the stars by the size of the dots which indicate their places.

## THE MOON'S PATH ON THE SPHERE

The moon should be placed on the list of objects for regular observation, the observations being made in precisely the same manner as those of the stars, and its place should be plotted upon the globe at each observation and marked by a number, giving the date of the month. This method of fixing the moon's place is much more accurate than those made use of in Chapter II, and, as the places are plotted upon a globe, we may study to better advantage those peculiarities of her motion which are masked by the distortion of the map referred to in Chapter II.

The position of the node may now be fixed with such a degree of accuracy that its regression is shown by the observations of two or three months, if some care is taken to observe as nearly as possible at the same altitude in the successive months, so that the corrections for parallax may be nearly the same; indeed, a very few months will force upon the notice of the observer the fact that the moon's path does not lie in one plane, just as observations a few days apart show that the sun's diurnal path is not really a small circle lying in one plane.

We also study the variable motion of the moon by applying dividers between the successive plotted places and then placing the dividers against the parchment scale to measure the distance in degrees traversed in the plane of the orbit. The scale must lie along an hour-circle so as to conform to the curvature of the sphere.

The average rate being about 13° a day, the points on the orbit should be determined as nearly as possible at which the motion is greater and less than this amount, and the point of most rapid motion fixed as closely as possible; this point is most simply fixed by its distance in degrees from the ascending node of the moon's orbit. Since the latter point, however, is continually changing, it is customary to reckon the so-called "longitude in the orbit" of the point by measuring from the vernal equinox along the ecliptic to the node, and adding the angle measured along the orbit from the node to the point.

The variations of the moon's angular diameter and the point of the orbit where the diameter is greatest should be compared with

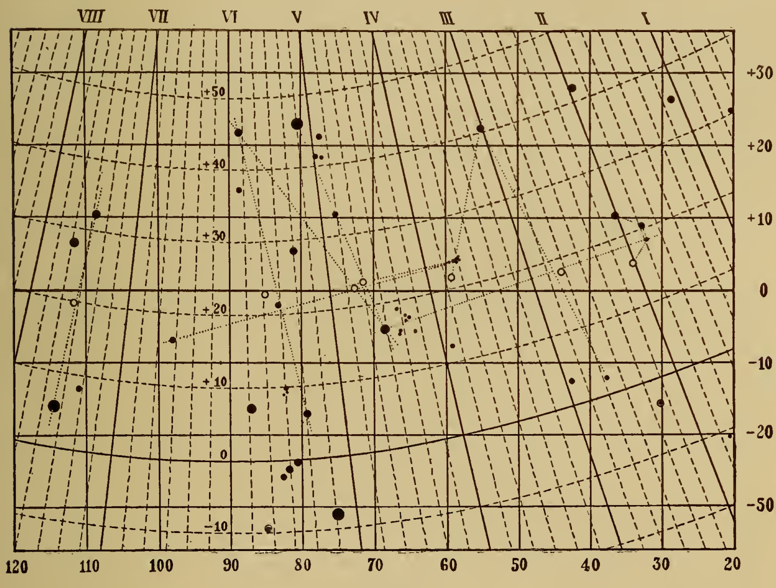


FIG. 39

the results obtained from the investigation of the angular velocity in the orbit, since we thus gain some knowledge of the moon's relative distances from us at different points of its orbit, and of the relation between its distance and its rate of motion about the earth.

The scale of the 6-inch globe is too small to do justice to the accuracy of our observations, which are accurate to a quarter or a tenth of a degree, and it will be interesting to plot these observations on a map constructed on a larger scale, and on a plan which reduces the distortion to very small limits in the region of the ecliptic; such a map is shown in Fig. 39 on a reduced scale; the ecliptic is here taken as a straight horizontal line, as the equator is in the star map previously used; the latitude, or angular distance of a point from the ecliptic measured on a great circle perpendicular to the latter, serves as the coördinate corresponding to the declination on our former map, while right ascension is replaced by longitude, or distance along the ecliptic measured from the vernal equinox up to 360°. The same map will serve also for plotting the paths of the planets in our later study.

For convenience in plotting, the parallels of declination and the hour-circles are printed in broken lines upon the map. The observations of the moon shown in the figure are those of December, 1899, already plotted on the map of Fig. 25.

## THE SUN'S PLACE AMONG THE STARS

By means of the equatorial we may also determine the place of the sun among the stars, although the method of direct comparison with stars we have used in the case of the moon is not applicable, since the stars are not visible when the sun is above the horizon; the most obvious method which is capable of any degree of accuracy involves the use of a clock regulated to sidereal time.

To determine the place of the sun, point upon it with the equatorial about two hours before sunset; note the time, and read the circles; as soon as possible after sunset observe a star in the same manner, with the instrument as near as may be to its position at the sun observation. It is evident that if the circumstances were fortunately such that the telescope did not have to be moved between the observations, the difference in right ascension of the sun and the star would be the difference in time noted by the sidereal clock, while the declinations of the sun and star would be the same. The nearer the star is to the position in which the sun was observed, the less will be the errors arising from imperfect adjustment and orientation of the instrument; while the shorter the interval between the observations, the smaller will be the error due to the uncertainty in the rate of the clock. As the condition of not moving the telescope can seldom be fulfilled, however, we must treat the observation as follows:

Let R.A., H.A., $t$, and $\Delta t$ be the right ascension, hour-angle, clock time, and clock correction at the time of the star observation, and R.A.$'$, H.A.$'$, $t'$, and $\Delta t$, the corresponding quantities at the sun observation. The equation

$$\text{R.A.} + \text{H.A.} = \text{Sidereal Time} = t + \Delta t$$

determines the value of $\Delta t$, which substituted in the equation

$$\text{Sidereal Time} = t' + \Delta t = \text{R.A.}' + \text{H.A.}'$$

determines the value of R.A.$'$, the sun's right ascension at the moment of observation.

The value of $\Delta t$, as determined from the first equation, will be negative if the clock is fast, and positive if the clock is slow; and it must always be applied to the observed time with the proper sign. The declination of the sun is, of course, given directly by the reading of the declination circle.

The following example illustrates the method:

March 30, 1899, an observation of the sun with an equatorial telescope, and a clock keeping sidereal time, gave the following values:

Observed time $= 5^h\ 36^m\ 26^s$; H.A. $= 75°.7 = 5^h\ 2^m\ 48^s$; $\delta = +4°.1$. About an hour after sunset an observation of $\alpha$ Ceti was made in nearly the same position of the instrument, which gave the following values:

Observed time $= 7^h\ 53^m\ 43^s$; H.A. $= 74°.1 = 4^h\ 56^m\ 24^s$; $\delta = +4°.2$. This latter gives, from the known right ascension of $\alpha$ Ceti,

$$2^h\ 57^m\ 0^s + 4^h\ 56^m\ 24^s = \text{Sidereal Time} = 7^h.53^m\ 43^s + \Delta t,$$

and hence $\Delta t = -19^s$; and, applying the same equation to the sun observation,

$$\text{Sun's R.A.} + 5^h\ 2^m\ 48^s = 5^h\ 36^m\ 26^s - 19^s = 5^h\ 36^m\ 7^s;$$

hence the sun's right ascension at the time of the first observation was $0^h\ 33^m\ 19^s$. This is liable to an error equal to the uncertainty of the circle readings, which may be at least one-twentieth of a degree, or $12^s$ of time, and to an error equal to the uncertainty of the gain or loss of the clock during the interval of $2\frac{1}{2}$ hours between the two observations, probably five or ten seconds of time. We may assume that the errors arising from defective adjustment of the instrument were the same for both objects, and may be neglected, since the position of the instrument was very nearly the same for both observations.

## DIFFERENTIAL OBSERVATIONS

The declination of $\alpha$ Ceti, as read from the circles, was $+4°.2$, while its known declination was $+3°.7$. The correction necessary to reduce the circle reading to the true value is, therefore, $-0°.5$, and, applying this quantity to the reading on the sun, we have for the true value of the sun's declination $+4°.1 - 0°.5 = +3°.6$. It is worthy of note that the correction is about the same as that determined from the observations discussed on page 53, which were made with the same instrument in nearly the same adjustment, but from a different place of observation. These results indicate an inherent defect in the instrument, which is at least in great part neutralized by the method of observation. It is a very important thing, even with the most delicate instruments, to avail ourselves of methods which accomplish this object, and surprisingly good work may be done with poor instruments by paying proper attention to the details of observation for this purpose.

Methods by which an unknown body is thus compared with a known body under circumstances as nearly alike as possible are called "differential methods."

## INDIRECT COMPARISON OF THE SUN WITH STARS

It is often possible to determine the difference of right ascension of the sun and some well-known star by using the moon as an intermediary, determining the difference of right ascension of the sun and moon during the daytime and comparing the moon and a star as soon as possible after sunset, the motion of the moon during the interval being allowed for. The irregularity of the moon's motion may, however, introduce a greater error than that arising from uncertainty in the rate of the clock. A better method is offered on those not infrequent occasions when the planet Venus is at its greatest brilliancy, when it may be easily observed in full daylight; the motion of Venus in the interval is much smaller and more nearly uniform, and, therefore, more accurately determined; and by this method the interval between the observations connecting the sun with Venus and Venus with the star may be reduced to a very few minutes, or even seconds, so that the error due to the clock may be regarded as negligible.

The following observations illustrate the method.

| 1900 | | Watch Time | H.A. | δ |
|---|---|---|---|---|
| April 19.3. | Procyon . . . . . . . | $8^h\ 17^m\ 45^s$ | 15°.4 | + 5°.5 |
| | Venus . . . . . . . . | 8 19 33 | 56 .1 | + 26 .1 |
| | Procyon . . . . . . . | 8 21 45 | 16 .4 | + 5 .55 |
| | Venus . . . . . . . . | 8 23 0 | 57 .0 | + 26 .05 |
| | Procyon . . . . . . . | 8 24 53 | 17 .2 | + 5 .30 |
| April 20.0. | Sun . . . . . . . . . . | 1 28 45 | 358 .5 | + 11 .6 |
| | Venus . . . . . . . . | 1 31 35 | 313 .2 | + 25 .35 |
| | Sun . . . . . . . . . . | 1 36 10 | 0 .15 | + 11 .6 |
| | Venus . . . . . . . . | 1 38 33 | 315 .0 | + 25 .3 |
| | Sun . . . . . . . . . . | 1 41 21 | 1 .4 | + 11 .6 |
| April 20.3. | Procyon . . . . . . . | 9 31 27 | 33 .25 | + 5 .45 |
| | Venus . . . . . . . . | 9 32 30 | 72 .9 | + 25 .9 |
| | Procyon . . . . . . . | 9 33 28 | 33 .9 | + 5 .4 |
| | Venus . . . . . . . . | 9 35 0 | 73 .3 | + 25 .9 |
| | Procyon . . . . . . . | 9 36 0 | 34 .25 | + 5 .4 |

The observations April 19.3, that is, April 19 about 7 P.M., give for the hour-angle of Venus 56°.55 at the watch time $8^h\ 21^m\ 17^s$, and for that of Procyon 16°.33 at $8^h\ 21^m\ 28^s$; hence at $8^h\ 21^m$ Procyon followed Venus 40°.22.

In the same way we find that April 20.3 Procyon followed Venus 39°.3, the change of the right ascension of Venus being 0°.92 in 25.2 hours. A simple interpolation shows that April 20.0 Procyon followed Venus 39°.59, and the observations at that time show that Venus followed the sun 45°.92, so that Procyon followed the sun $39°.59 + 45°.92 = 85°.51$, and the difference of right ascension between Procyon and the sun at noon on April 20 was, therefore, $5^h\ 42^m\ 2^s$.

## ADVANTAGES OF THE EQUATORIAL INSTRUMENT

Observation with the equatorial we shall find especially useful in getting exact positions of the moon, since it is available at any time when the moon is above the horizon, and after sunset we can always find some bright star sufficiently near to afford a fairly accurate value of its place.

It is often inconvenient to observe the moon by the more accurate method which is described in Chapter VI, that of meridian observations, which is confined to a short interval of one or two minutes each day, and is often interfered with by clouds passing at the critical moment, although nine-tenths of the whole day may be suitable for observations made out of the meridian. Moreover, until the moon is several days old, it is too faint for observation at its meridian passage. It is, therefore, upon the equatorial that we shall mainly rely for the determination of the moon's motion, as well as for many observations of the planets out of the meridian.

Although it is far more convenient to find the right ascension and declination of the sun by the method of the following chapter, at least a few positions should be found by observations with the equatorial and plotted on the globe. The result will be to show that the path of the sun is very exactly a great circle fixed on the sphere or so nearly fixed that some years of observation with the most refined instruments are necessary to detect any change in its position among the stars, although a much shorter time even would serve to show the slow change of its intersection with the celestial equator due to precession.

This great circle is called the ecliptic, and its position is shown on the map which we have used for plotting our first moon observation.

Three months will give a sufficient arc of this circle to enable us to determine with some accuracy its position with respect to the equator, its inclination to the latter, and their points of intersection;

if possible, observations should, however, be continued throughout the year which the sun requires to complete its circuit, so that the variability of its motion may be observed, most of the work, however, being done with the meridian circle.

The sun's diameter should occasionally be measured to determine the points at which it is nearest to and farthest from the earth.

# CHAPTER VI

## MERIDIAN OBSERVATIONS

We have now arrived at a point where we can see what are the desirable conditions for making observations as accurately as possible of the position of a heavenly body. To adjust the equatorial instrument so that its axis lies in the meridian and at the proper inclination, and to keep it so adjusted, is a matter of some difficulty. In the last chapter we have shown how, by observing an unknown body in a certain fixed position of the instrument, and later a body whose right ascension and declination are known in as nearly as possible the same position of the instrument, we lessen the effect of the instrumental errors. We made our observation of the sun shortly before sunset, so that the interval between this observation and that of the comparison star should be as short as possible. If, however, the rate of the clock can be relied upon, there is no reason why the observation should not be made when the sun is on the meridian, the interval of time required to connect it with stars in that case being not necessarily more than eight or nine hours in the most extreme case; and the comparative ease with which an instrument may be constructed so that it shall be at all observations exactly in the meridian, and the possibility of constructing very accurate time-pieces, has determined the use of such instruments for all the more precise observations in astronomy, such as fix the positions of the fundamental stars and the vernal equinox on the celestial sphere.

The equatorial instrument may be used for this purpose by clamping it in such a position that the reading of the hour-circle is 0°, in which case the declination axis is horizontal east and west, and when the telescope is moved about its axis it always lies in the plane of the meridian. If, with the instrument so adjusted, we observe the sun at the time of its meridian passage, we may find its declination by reading the declination circle, and its right ascension by noting the interval which elapses before the meridian transit of some known star after nightfall, free from any error involved in reading the hour-circle. As before, a star should be chosen at nearly the same declination, so that the interval of time may be very nearly equal to the difference in right ascension between the sun and the star, even if the instrument is not very exactly in the meridian. Observation of several different stars will enable us to determine whether the instrument actually does describe the plane of the meridian as it is rotated about the horizontal axis (see Chapter VIII); and by the observation of stars near the pole, as described on page 81, we may determine whether the declination circle reads exactly 0° when the telescope points to the equator, as should be the case.

## THE MERIDIAN CIRCLE

An instrument which is to be used in this manner, however, is not usually so constructed that it can be pointed at any point in the heavens. Thus, it is unnecessary that it should consist of so many moving parts as the equatorial instrument, and steadiness, strength, and ease of manipulation are very much increased by constructing it as shown in Fig. 40, which represents a very small instrument built on the plan of the meridian circle of the fixed observatory. The strong horizontal axis revolves in two Y's, which are set in strong supports in an east and west line. The axis is enlarged towards the center, and through the center passes at right angles the telescope tube. The axis carries at one end a graduated circle

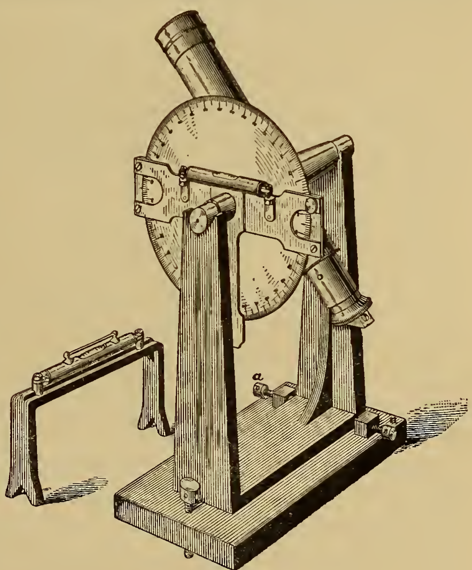


FIG. 40

perpendicular to the axis of rotation. If the axis of the telescope is perpendicular to the axis of rotation, and if the latter is adjusted horizontally east and west, the telescope may be brought into any position of the meridian plane, but must always be directed to some point of the latter. A pointer attached to the support marks the zero of the vertical circle when the telescope points to the zenith, and if the telescope be pointed to a star at the time of its meridian passage, the angle as read off on the circle is the zenith distance of the star; while the time of the star's meridian passage by a clock giving true sidereal time is its right ascension. If the latitude of the place of observation is known, the star's declination is determined by the fact that the zenith distance plus the declination of any body equals the latitude (see page 81). At first the latitude may be used as determined by the sun observation of Chapter I, or from a good map showing the place of observation, but ultimately its value should be determined with the meridian circle itself.

## LEVEL ADJUSTMENT

We will now proceed to show how to make the necessary adjustments for placing the telescope so that it may move in the plane of the meridian.

Place the instrument on its pier and bring the Y's as nearly as possible into an east and west line. If the pier is the same that has been used in the previous work, this may be done by bringing the telescope into the meridian which has been determined by the method of equal altitudes.

The axis must first be brought into a horizontal line, making use for this purpose of the striding level (Fig. 41), which is a necessary auxiliary of this instrument. This is a glass tube nearly but not quite cylindrical, ground inside to such a shape that a plane passing through its axis, $CD$, cuts the wall in an arc, $AB$, of a circle whose center is at $O$. In this tube is hermetically sealed a very mobile liquid in sufficient quantity nearly but not quite to fill it — the space remaining, called the "bubble," always occupying the top of the tube. When $CD$ is horizontal, the bubble rests in the middle of the tube with its ends, of course, at equal distances from the middle; the tube is graduated so that this distance may be measured, the numbering of the graduations usually increasing in both directions from the center of the tube. If the radius of the arc is 14.3 feet, a length of 3 inches of this arc will be equal to about 1°, since the arc of 1° in any circle is about $\frac{1}{57.3}$ of the radius; 1 inch of the arc will then be about 20′, and 0.05 inch 1′. These are about the actual values for the level used with the instrument

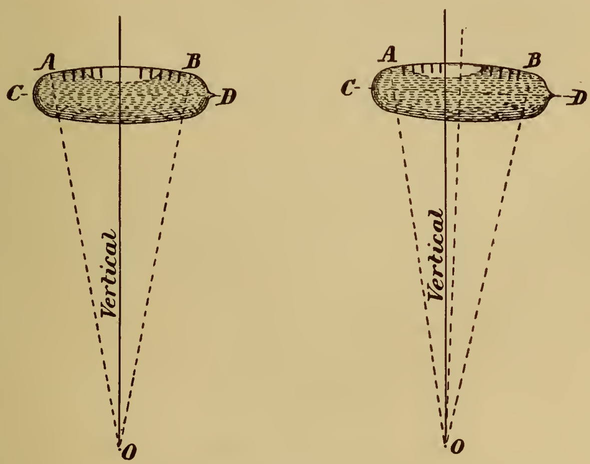


FIG. 41

shown in Fig. 32, the scale divisions being about $\frac{1}{20}$ of an inch apart and therefore corresponding to an arc of 1′.

If the line $CD$ is inclined at an angle of 1′ to the horizontal line by raising the end $A$, the center of curvature will be displaced toward the left, and the level will have the same inclination as if the whole tube had been turned to the right about the point $O$ through an angle of 1′; and the highest point of the arc, which is always directly above $O$, is now $\frac{1}{20}$ of an inch from the middle toward $A$. Since the bubble always rests at the highest point of the arc, it follows that its ends will each be moved toward $A$ by one division; if, for instance, the readings of the ends are 5 and 5 when $CD$ is horizontal, they will be 6 and 4 when $CD$ is inclined by 1′, and evidently 7 and 3 when $CD$ is inclined 2′, etc., the inclination in minutes of arc being one-half the difference of the readings of the ends of the bubble, or $\frac{A-B}{2}$ if $A$ and $B$ represent the readings of the ends of the bubble in each case. If the reading of $B$ is greater, the end $A$ is depressed by one-half the difference of the readings; and the above expression applies to both cases if we agree that it shall always denote the elevation of $A$, a negative value of $\frac{A-B}{2}$ indicating depression of $A$.

## REVERSAL OF THE LEVEL

The level tube is attached to a frame (Fig. 40) resting on two stiff legs terminating in Y's, which are of the same shape and size as those in which the axis of the meridian circle rests, the axis of the level tube being adjusted as nearly as possible parallel to the line joining the Y's. It is difficult to insure this condition, but if it is not exactly fulfilled, the horizontality of the axis may still be determined by placing the level on the axis, and determining the value $\frac{A-B}{2}$, and then turning it end for end, and again reading the value; for if the end $A$ is high by the same amount in each case, the axis is obviously horizontal, and the measured angle of inclination is due to the fact that the leg of the level adjacent to $A$ is longer than the other leg. The practical rule is to read the west and east ends in each position. If these readings are $W_1\ E_1\ W_2\ E_2$, $\frac{W_1-E_1}{2}$ is the elevation of the west end according to the first observation, and $\frac{W_2-E_2}{2}$ at the second. If the leg which is west at the first observation is too long, the first observation gives a value for the elevation of the west end too great, and the second a value too small *by the same amount*; and the average of the two values $\frac{W_1-E_1}{2}$ and $\frac{W_2-E_2}{2}$ gives the true value of the inclination of the axis.

It is usual to write this $\frac{(W_1 + W_2) - (E_1 + E_2)}{4}$ and to record the observations in the following form:

$$\begin{array}{cc} W_1 & E_1 \\ W_2 & E_2 \\ \hline W_1 + W_2 & E_1 + E_2 \end{array}$$

Subtract the second sum from the first and divide by 4. This gives a positive value if the west end is high, and the axis may be made horizontal by turning the leveling screw so as to make the level bubble move through the proper number of divisions. The level should be again determined in the same way, and the axis is level when

$$(W_1 + W_2) - (E_1 + E_2) = 0.$$

The following record of level observation made Feb. 26.3, 1900, conforms to the above scheme:

$$\begin{array}{lcc} & W & E \\ & 1\tfrac{1}{2} & 2\tfrac{1}{2} \\ & 2 & 2 \\ \cline{2-3} & 3\tfrac{1}{2} & 4\tfrac{1}{2} \\ -1 & & \\ \multicolumn{3}{l}{-\tfrac{1}{4} \text{ division} = 15''} \end{array}$$

The west end being too low, the screw was turned so as to raise it enough to move the bubble $\frac{1}{4}$ division toward the west, the level remaining on the axis during the adjustment and watched as the screw was turned; the readings were then as follows:

$$\begin{array}{ccc} 2\tfrac{1}{4} & & 1\tfrac{3}{4} \\ 1\tfrac{3}{4} & & 2\tfrac{1}{4} \\ \hline 4 & & 4 \\ & 0 & \end{array}$$

And the axis was truly level, since $(W_1 + W_2) - (E_1 + E_2) = 0$.

## COLLIMATION ADJUSTMENT

The line of collimation of the telescope is the line drawn from the center of the lens to the wires that cross in the center of the field. When the telescope is "pointed" or "set" upon a star, the image of the star falls upon the point where these wires cross, and when the instrument is correctly adjusted the line of collimation is perpendicular to the axis of rotation, so that the line of collimation cuts the celestial sphere in a great circle as the telescope turns upon its axis.

To make this adjustment, point the telescope exactly upon any well-defined distant point, — the meridian mark will, of course, be chosen if it has been located, — then remove the axis from its Y's and replace it after turning it end for end; if the telescope is still set on the mark in the second position, the adjustment is correct; otherwise move the wire halfway toward the mark by means of the screws *a*, *a* (Fig. 40). Set again upon the mark by moving the screws in the eyepiece tube; reverse the axis again, and thus continue until the telescope points exactly upon the mark in both positions of the axis.

If the adjustments for level and collimation are properly made, the intersection of the wires in the center of the field of view will appear to describe a vertical circle, that is, a great circle through the zenith, as the instrument is turned on its axis. The final adjustment consists in bringing this circle to coincide with the meridian, but for this we must have recourse to observations of stars.

## AZIMUTH ADJUSTMENT

The simplest method is to observe the time of transit by a sidereal clock of a circumpolar star at its upper transit, and again, 12 hours later, at its transit below the pole; if the interval is exactly 12 hours, the adjustment is correct; if the interval is less than 12 hours, the telescope evidently points west of the pole, and the west end of the rotation axis must be moved toward the north. This is done by the screws *a*, *a* (Fig. 40), the fraction of a turn being noted;

the observation is repeated upon the following night, and by comparing the change which has been produced by moving the screws, the further alteration required is readily estimated. On Feb. 26, 1900, the lower transit of $\epsilon$ Ursæ Majoris was observed at $4^h\ 58^m\ 12^s$, and the upper transit at $16^h\ 53^m\ 32^s$; the times were taken by a sidereal clock and have been corrected for its error; the interval being $11^h\ 55^m\ 40^s$, it is evident that the telescope pointed to the east of the meridian, the arc of the star's diurnal path between the lower and upper transits lying to the east of the meridian and being less than $12^h$ by $4^m\ 20^s$ or $260^s$.

To correct the error, the west end of the axis was moved toward the south by turning the adjusting screws through one-quarter of a turn. On the following day the observations were repeated as follows:

Feb. 27.25, lower transit $4^h\ 54^m\ 45^s$; Feb. 27.75, upper transit $16^h\ 54^m\ 28^s$; the eastern arc was still too small, but the error had been reduced to $17^s$, and required a further correction of $\frac{17}{260}$ of a quarter turn of the screws, which were therefore turned through about 6° in the same direction as before, and the instrument was thus brought very closely into the meridian.

This method can only be used with small instruments when the night is more than 12 hours long; but it is the only independent method; it requires that the rate of the clock shall be known between the two observations, and it requires observations at inconvenient times. A more convenient method is always used in practice, but requires an accurate knowledge of the right ascensions of a considerable number of stars in the neighborhood of the pole.

It has been stated that it is often inconvenient to observe the moon when on the meridian, but with this exception all the fundamental observations of astronomy are now made with meridian instruments on account of the simplicity and permanence of the necessary adjustments. A body observed on the meridian is also at its greatest altitude and least affected by atmospheric disturbances, which often interfere with the observation of bodies near the horizon.

## DETERMINATION OF DECLINATIONS WITH THE MERIDIAN CIRCLE

The circle of the meridian instrument may be used to determine the declination of a star in two ways, of which that now described is perhaps the most obvious, but also the least convenient.

If the reading of the circle is known when the telescope is pointed at the pole, the angle through which the telescope must be moved to point upon any star, that is, the polar distance of the star, is the difference between this value and the circle reading when the telescope is pointed at the star; this angle is 90°—the star's declination; if the star is on the equator, the angle is 90°; and if the star is south of the equator, the angle is greater than 90° by an amount equal to the declination of the star; if we consider the declination a negative quantity for a star south of the equator, the value $90° - \delta$ represents the polar distance in all cases.

To determine the reading of the "polar point" we may set the telescope upon a circumpolar star at its "upper culmination" and read the circle, and again, 12 hours later, set on the same star at its "lower culmination," the mean of the two readings is the reading of the polar point. The effect of refraction may be neglected with our small instruments without causing an error of $\frac{1}{40}$ of a degree at any place in the United States if we restrict ourselves to stars within 10° of the pole, or the circle readings may be corrected by a refraction table. Immediately after making this determination it is advisable to make a setting on the meridian mark and note the reading; this point may thereafter be used as a reference point from which the reading of the polar point may be at any time determined if the meridian mark has not in the mean time changed its position.

Better still, the observation of the polar point may be combined with a determination of the circle reading when the telescope points at the zenith, by one of the methods to be described later; the difference of the readings in this case is obviously equal to the co-latitude, and such an observation constitutes an "absolute determination of the latitude," that is, a determination made without reference to observations made at any other place. When the latitude has once been satisfactorily determined, the observations of the declinations of stars can be made to depend upon determinations of the zenith point by means of the fact that for a body on the meridian

$$\text{Declination} = \text{Latitude} - \text{Zenith Distance},$$

latitude and declinations being reckoned positive northward from the plane of the equator, and zenith distance positive southward from the zenith. The proof of this relation is left to the student as well as the interpretation of the result when the observation is made at the transit below the pole.

At the time of observing the transits of $\epsilon$ Ursæ Majoris described on page 79 the following readings of the circle were made when the star was in the center of the field. Each of these observations consists of two readings: one of the index $A$ on the south end of a horizontal bar fixed to the supports of the axis, and the other of an index $B$ at the other extremity of the bar, as nearly as possible half a circumference from $A$. An angle given by the mean of two readings made in this manner is free from the "error of eccentricity," which affects readings by a single index in case the center of the graduated circle does not exactly coincide with the axis about which it is turned between the two observations.

| Date | $A$ | $B$ | Mean |
|---|---|---|---|
| February 26.25 . . . . | 55°.45 | 55°.35 | 55°.40 |
| 26.75 . . . . | 39 .95 | 39 .85 | 39 .90 |
| 27.25 . . . . | 55 .45 | 55 .35 | 55 .40 |
| 27.75 . . . . | 39 .95 | 39 .85 | 39 .90 |

Hence the reading when the instrument was pointed at the pole was $\dfrac{55°.40 + 39°.90}{2} = 47°.65.$

Evidently the polar distance of the star was $\dfrac{55°.40 - 39°.90}{2} = 7°.75,$ and its declination 82°.25; and we have thus obtained an "independent" or "absolute" determination of the declination of $\epsilon$ Ursæ Majoris; that is, a determination independent of the work of other observers, and only dependent on the accuracy of our circle and of our observations.

The circle was known to be adjusted so that the reading of the zenith was very exactly zero, hence the latitude of the place of observation was 42°.35. The exact agreement of these observations indicates that the magnifying power of the telescope was such that it could be set more accurately than the circle could be read, and not that the results are reliable to a hundredth of a degree.

For convenience in recovering the zenith reading, in case the adjustment of the circle should be disturbed, the zenith distance of a meridian mark was measured repeatedly, the result showing that its polar distance was 137°.47, and this was used to check the polar reading in later observations upon stars when it was impossible to get observations of the same star above and below the pole.

Another method of making absolute determinations of the latitude with the meridian circle is to observe the zenith distance of the sun at the solstices; the means of these values being the zenith distance of the equator, which is equal to the latitude. This observation, however, is subject to considerable uncertainty on account of the difference in atmospheric conditions at the summer and winter solstice, and to great inconvenience on account of the lapse of time; it is, however, of course, the means upon which we must rely for the accurate determination of the obliquity of the ecliptic, one of the fundamental quantities of astronomy.

For the use that we shall make of the meridian circle, it will probably be most convenient to make a careful determination of the polar distance of the meridian mark, and use this habitually as a point for reference.

## PROGRAM OF WORK WITH THE MERIDIAN CIRCLE

Work with the meridian circle should at first consist of reobservation of all the stars which have been previously observed with the equatorial, except those which are west of the meridian after nightfall and cannot be observed for six months. Attention should be given to gathering a list of stars within 15° or 20° of the pole for the purpose of quickly setting the instrument in the meridian by the method of page 120. The sun should be observed at least once a week and its place plotted on the globe, and many stars in the neighborhood of the moon's path to form a basis for finding the moon's place by differential observations, of course, also the moon itself, the planets and a comet, if any of sufficient brightness appears. In this way, by observing a few stars each night, a great amount of material may be stored for future use.

Especial attention should be given to getting a good number of observations of stars near the equator, so that fairly accurate values of their differences of right ascension may be obtained, and at the first opportunity the absolute right ascension of one of their number must be determined in order that thus the places of all may be known. The results may be best recorded by making a list of their right ascensions referred to an assumed vernal equinox. Thus, the observations discussed on page 52 show that $\alpha$ Pegasi precedes $\gamma$ Pegasi by $17°.03 = 1^h\ 8^m\ 7^s$, or, in other words, follows it by $22^h\ 51^m\ 53^s$; and if the right ascension of $\gamma$ Pegasi referred to the assumed equinox is $0^h\ 8^m$, that of $\alpha$ Pegasi is $22^h\ 59^m\ 53^s$. If in the course of the year observation shows that the true right ascension of $\gamma$ Pegasi is $0^h\ 8^m\ 5^s$, it is evident that the true value for $\alpha$ Pegasi is $22^h\ 59^m\ 58^s$, and that the right ascension of all stars referred to the assumed equinox by comparison with $\gamma$ Pegasi must be increased by $5^s$.

## DETERMINATION OF THE EQUINOX

An opportunity for observing the absolute right ascension of the zero star, which is often called a "determination of the equinox," occurs about the middle of March and September.

If the course is begun in September, it will be well to make this determination with the help of more experienced observers, even before the nature and object of the measures are understood.

The observation consists in determining the difference of right ascension of some star from the sun at the instant when the latter crosses the equator, for at that time it is either at the vernal or autumnal equinox, and its right ascension is in the one case 0 hours and in the other 12 hours.

If a meridian observation of the sun's altitude shows that the sun is exactly on the equator at meridian passage, and the time of transit is noted by a sidereal clock, and as soon as it is sufficiently dark the transit of a star is observed, the difference of the times is the absolute right ascension of the star if the observation is made at the vernal equinox, or equals the right ascension of the star minus $12^h$ if the observation is made at the autumnal equinox.

Inasmuch as the meridian of the observer will rarely be that one on which the sun happens to be as it crosses the equator, we must make observations on the day before and the day after the equinox, thus getting the difference of right ascension of the star from the sun at noon on both days. The declination of the sun being also measured at these two times, a simple interpolation gives the time at which the sun crossed the equator, and this time being known, another simple interpolation between the differences of right ascension at the two noons gives the difference of right ascension of the sun and star at the time when the sun was at the equinox, which is the star's absolute right ascension.

The first interpolation assumes that the sun's declination changes uniformly with the time, and the second that its right ascension changes uniformly with the time.

Observations should extend over a period of a week before and a week after the equinox to test the truth of these assumptions.

In observing the sun, a shade of colored or smoked glass may be placed over the eyepiece, or the eyepiece may be drawn out as in the method of observation described on page 37, and the screen held in such a position that the cross-wires are sharply focused upon it. As the image of the sun enters the field it should be adjusted by moving the telescope slightly north or south till the horizontal wire passes through the center of the disk, and as the latter advances, the time should be noted when the preceding and following limbs cross the vertical wire, as well as the time when the vertical wire bisects the disk; at the instant of transit the disk should be neatly divided into four equal divisions, a very small deviation from this condition being quite perceptible to the eye.

## THE AUTUMNAL EQUINOX OF 1899

The following table gives the details of observations taken at the autumnal equinox of 1899 for the purpose of determining the equinox.

The latitude of the place of observation was 42°.5, and the declinations given in the last column are calculated by subtracting the zenith distance in each case from this quantity, as explained on page 81.

| Date | Object | Time of Transit | Zen. Dist. | Decl. | |
|---|---|---|---|---|---|
| Sept. 22 | Sun . . . . . . | $12^h$ $0^m$ $2^s.0$ | S42°.2 | + 0°.3 | |
| | η Serpentis . . | 18 18 22.6 | 45 .4 | − 2 .9 | |
| | λ Sagittarii . . | 18 24 2.4 | 67 .95 | − 25 .45 | |
| | Vega . . . . | 18 35 44.5 | 3 .87 | + 38 .63 | |
| | Altair . . . . | 19 48 7.6 | 33 .98 | + 8 .52 | |
| Sept. 23 | Sun . . . . . . | 12 3 45.1 | 42 .62 | − 0 .12 | |
| | η Serpentis . . | 18 18 20.1 | 45 .4 | − 2 .9 | |
| | λ Sagittarii . . | 18 23 57.3 | 67 .97 | − 25 .47 | |
| | Vega . . . . | 18 35 42.6 | 3 .85 | + 38 .65 | |
| | Altair . . . . | 19 48 1.5 | | | |

The intervals between the observed times of transit of each star on the two different dates range from $23^h$ $59^m$ $53^s.9$ to $23^h$ $59^m$ $58^s.1$, showing that the clock was losing about $4^s$ daily, a quantity so small that for our purpose it may be neglected.

Observations of the sun made on different dates between September 18 and September 23, but not here recorded, showed that its right ascension and declination were changing uniformly at the rate of about $3^m$ $45^s$ and 0°.39 per day. The table above shows that from September 22 to September 23 the rates were $3^m$ $43^s.1$ (or, allowing for clock rate, about $3^m$ $39^s$) and 0°.42 per day, and the latter value we shall use to determine the time of the equinox, as follows:

At noon September 22, or September $22^d.0$, as it is expressed by astronomers, the sun's declination was + 0°.3, and September 23.0

its declination was $-0°.12$. Hence its declination was $0°$ September $22\frac{3}{4}\frac{0}{2}$, or September $22^d.714$. It was at that time, as exactly as our observations can show, at the autumnal equinox, and its right ascension was $12^h\ 0^m\ 0^s$.

Since $\eta$ Serpentis followed it to the meridian $6^h\ 18^m\ 20^s.6$, that quantity is the difference between the right ascension of the star and that of the sun September 22.0. Similarly the difference of right ascension of sun and star September 23.0 was $6^h\ 14^m\ 35^s.0$; that is, it was $3^m\ 45^s.6$ less than at the previous date. Assuming this change to be uniform, the difference of right ascension of sun and star at the moment of the equinox on September $22^d.714$ was $0.714 \times 3^m\ 45^s.6$, or $2^m\ 41^s.1$ less than on September 22.0; that is, it was $6^h\ 15^m\ 39^s.5$, and since the right ascension of the sun September 22.714 was $12^h\ 0^m\ 0^s$, the right ascension of $\eta$ Serpentis was $18^h\ 15^m\ 39^s.5$.

The following table gives the data from which the "absolute right ascensions" of the four stars are thus determined. In the last column are the declinations, which are the means obtained from several observations between September 14 and September 23.

| Star | R.A. of Star minus R.A. of Sun | | | Star's | |
|---|---|---|---|---|---|
| | Sept. 22.0 | Sept. 23.0 | Sept. 22.714 | R.A. | Decl. |
| $\eta$ Serpentis | $6^h\ 18^m\ 20^s.6$ | $6^h\ 14^m\ 35^s.0$ | $6^h\ 15^m\ 39^s.5$ | $18^h\ 15^m\ 39^s.5$ | $-\ 2°.89$ |
| $\lambda$ Sagittarii | 6 24 0 .4 | 6 20 12 .2 | 6 21 17 .4 | 18 21 17 .4 | −25 .48 |
| Vega | 6 35 42 .5 | 6 31 57 .5 | 6 33 1 .8 | 18 33 1 .8 | +38 .65 |
| Altair | 7 48 5 .6 | 7 44 16 .4 | 7 45 21 .9 | 19 45 21 .9 | + 8 .59 |

The measurements upon which the above results depend are of two kinds: observed clock times, which are liable to errors of a very few seconds, so that the *differences* of right ascension may be assumed to be correct within perhaps $4^s$; and measures of the sun's declination, which with the greatest care may be in error at least $0°.05$ on any given date.

It is quite within the bounds of probability, for instance, that the sun's declination was $+0°.25$ on September 22.0 and $-0°.17$

on September 23.0; and recomputing with these values, the date of the equinox was September $22\frac{2.5}{4.2}$, or September $22^d.595$, and the right ascensions of the stars $18^h\ 16^m\ 6^s.4$, $18^h\ 21^m\ 44^s.6$, $18^h\ 33^m\ 28^s.6$, $18^h\ 45^m\ 49^s.2$; that is, the uncertainty of the equinox is 0.12 days and of the right ascensions about $27^s$, although the relative right ascension is altered only by a fraction of a second in each case. It is thus evident that the accuracy of the right ascensions depends chiefly upon the accuracy with which the sun's declination can be measured.

## THE AUTUMNAL EQUINOX OF 1900

In order to increase the accuracy of determination of declination, a new circle reading to minutes of arc was substituted for that used for the observations of the equinox in 1899, and the observations were repeated at the same place in 1900. The weather conditions were unfavorable, so that only the following observations could be made.

| DATE | OBJECT | TIME OF TRANSIT | ZEN. DIST. | DECL. |
|---|---|---|---|---|
| Sept. 22 | Sun . . . . . . . | $11^h\ 58^m\ 44^s.8$ | S42° 11′.5 | + 0° 18′.5 |
| | Vega . . . . . . . | 18 35 27.0 | 3 49.0 | +38 41.0 |
| | Altair . . . . . . | 19 47 49.0 | 33 51.0 | + 8 39.0 |
| Sept. 23 | Sun . . . . . . . | 12 3 1.5 | 42 33.1 | − 0 3.1 |
| | Altair . . . . . . | 19 48 35.0 | 33 54.0 | + 8 36.0 |

From these data, by the same method as before, the date of the equinox is found to be September $22\frac{18.5}{21.6}$, or September 22.8565. If each declination of the sun is accurate to $1'$, the result *may* be in error by $\frac{2}{21.6}$ days, or about .09 day; the actual error is probably less than half this amount, and the concluded right ascensions probably within $10^s$ of the true values.

The observed times of Altair on the two dates show that the clock was gaining $46^s$ daily, since the true sidereal time of transit, being equal to the star's right ascension, is the same on both nights. This rate is so large that it cannot be neglected as in the discussion of the result for 1899.

If the clock correction $\Delta t$ (see page 60) at the time of the sun's transit, September 22, be assumed $0^s$ and the gaining rate $46^s$ per day, or $1^s.916$ per hour, the corrections for Vega and Altair September 22 were $-12^s.6$ and $-14.9$, and for the sun and Altair September 23 were $-45.9$ and $-61^s.0$. The times obtained by applying these corrections are said to be "corrected for rate of the clock to the epoch September 22.0."

In this manner the times, as they would have been observed with a clock having an exact sidereal rate, are found to be :

| | September 22 | September 23 |
|---|---|---|
| Time of transit of the Sun . . . | $11^h$ $58^m$ $44^s.8$ | $12^h$ $2^m$ $15^s.6$ |
| " " " Vega . . . . | 18 35 15.4 | |
| " " " Altair . . . . | 19 47 34.1 | 19 47 34.0 |

Hence Altair followed the sun

| | |
|---|---|
| September 22.0 | $7^h$ $48^m$ $49^s.3$ |
| " 23.0 | 7 45 18.4 |
| " 22.856 | 7 45 48.8 |

and the right ascension of Altair was $19^h$ $45^m$ $48^s.8$ ; since Vega precedes Altair by $1^h$ $22^m$ $18^s.7$, its right ascension was $18^h$ $33^m$ $30^s.1$.

In 1899 the difference of right ascension of the two stars was $1^h$ $22^m$ $20^s.1$, but the right ascensions of 1900 are greater by $28^s.3$ and $26^s.7$ than those of 1899.

If we assume the later determination to be absolutely correct, we must regard the earlier as having placed the equinox farther toward the east among the stars than its true place, so that right ascensions referred to the equinox observed in 1899 are too small. We may say that the observations of 1900 indicate a correction of $-27^s.5$ to the "equinox of our little catalogue of four stars"; that is, a correction of $+27^s.5$ to all their right ascensions as determined in 1899.

Applying these corrections, their right ascensions become for

| | | | |
|---|---|---|---|
| $\eta$ Serpentis | $18^h$ | $16^m$ | $7^s.0$ |
| $\lambda$ Sagittarii | 18 | 21 | 44 .9 |
| Vega | 18 | 33 | 29 .3 |
| Altair | 19 | 45 | 49 .4 |

Since the later observations were made with an instrument giving more accurate values of the declination, it is probable that their results are more nearly correct. The clock rate was neglected in the first observations, and the effects of precession, parallax, and refraction in both series, following out the principle that no corrections will be made until observations shall show their necessity.

The effect of refraction is to delay the autumnal equinox about an hour, and hence to decrease the right ascensions of the stars by about $10^s$. At the vernal equinox, however, refraction hastens the equinox an hour and increases the right ascensions by $10^s$; its effect may be shown by observations at the two equinoxes of the same year and eliminated by their combination. Parallax hastens the autumnal and delays the vernal equinox by about $8^m$, thus affecting right ascensions by a little more than $1^s$, the mean of observations at the two equinoxes being free from error from this source. The effect of precession will be manifest in less than ten years with an instrument like that used in the above observations of 1900.

By comparing the equinox of September 22.714 $\pm$ 0.12, 1899, and September 22.856 $\pm$ .09, 1900, the length of the tropical year is found to be 365°.142, but may lie between 364.93 and 365.35 as far as our observations can surely determine. Since refraction delays the vernal and hastens the autumnal equinox by nearly the same amount (about an hour) in each case, it has no effect upon the length of the year. As the greatest error to be feared with our improved instrument is less than 0.1 day, the length of ten or one hundred years may be determined with less than twice that error, in those periods the length of the year may be determined within 0.02 and .002 day, respectively.

With the best modern instrument used to the greatest advantage, the sun's declination may be determined near the equinox within

$0''.5$, and hence the time of the equinox within $30^s$ and right ascensions within $0^s.08$. A single tropical year may be measured with an error of less than $1^m$.

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We have now explained the methods by which it is possible to fix the places of the sun, moon, and stars at different times and thus to obtain data from which their apparent motions about the earth may be studied and theories formed from which their future places may be predicted. More or less complete accounts of these theories are to be found in all works on descriptive astronomy, and the predictions derived from them are published for three years in advance by several governments for the use of navigators and astronomers. Such a publication is the American Ephemeris and Nautical Almanac, of which it will be convenient to give some account before taking up the motions of the planets.

The apparent motions of the planets are less simple than those of the sun, moon, and stars, which at all times seem to move about the earth as a center with approximately uniform velocities. The planets, it is true, in the long run continually move like the sun and moon around the heavenly sphere toward the east, but their velocities are variable within wide limits and at certain times are even reversed, so that they move in the opposite direction or "retrograde" among the stars.

For this reason a longer period of observation is necessary to determine their motions than can be given by the individual student. We may, however, regard the nautical almanacs of past years as predictions that have been verified, and they stand for us as an accredited set of exceptionally accurate observations from which we may draw material to combine with the results of our own observations.