TAYLOR

SURVEYOR'S HAND BOOK

BY

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DEDICATION

TO GEORGE W. BRACKENRIDGE
SAN ANTONIO, TEXAS.

A Civil Engineer and Patriot
Who for a Quarter of a Century Has Been an Active and Useful
Friend of Higher Education in Texas.

PREFACE.

It has been my object to prepare a book for the use of the surveyor in the field, of convenient size and scope, and one that contains all the essentials for ordinary surveying. It is too much to hope that there are no errors in these pages, in theory or example. The preliminary proof has been examined by twelve experienced surveyors and I am indebted to them for many valuable suggestions.

I am under lasting obligations to my colleagues, R. A. Thompson, Expert Engineer of the Texas Railway Commission; Edward C. H. Bantel, Adjunct-Professor of Civil Engineering; and Stanley P. Finch, Instructor in Civil Engineering of the University of Texas. In addition to this assistance I have been aided by valuable suggestions from F. Lavis and Halbert P. Gillette, and from the following leading surveyors and engineers of Texas: C. F. H. von Blücher, Gustav Schleicher, B. F. Love, and W. D. Twichell.

The thanks of the author are hereby expressed to W. & L. E. Gurley, Keuffel & Esser, Eugene Dietzgen Co., and A. Wissler for many illustrations of instruments.

The traverse table has been omitted, as the ordinary ones are useless for angles not multiples of quarter degrees, and the large ones are books in themselves. As lands become more valuable, the transit survey is demanded where angles are read to the nearest minute, and for such surveys the small traverse tables are of no avail.

Tables I, II, III and IV are taken by permission from Henck's "Field Book," while Table V is from Searles' "Field Engineering."

T. U. TAYLOR.

Austin, Texas, September 1, 1908.

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CHAPTER I.

CHAIN SURVEYING.

1. Gunter's Chain.—This chain was invented in 1620 by Edmund Gunter, an English surveyor, and is now in use in a majority of the older states of America. Previous to its invention, chains of irregular lengths had been in use, but there was no uniform system, and as soon as Gunter's chain was invented it was generally adopted.

This chain is 66 ft. or 792 ins. in length, and is divided into 100 equal parts, called links, each link being 7.92 ins. long. Eighty of these chains make one mile. Also, we know

1 acre = 4,840 sq. yds. = 43,560 sq. ft.

1 sq. chain = 66 × 66 = 4,356 sq. ft.

$$\therefore 1 \text{ acre} = \frac{43{,}560}{4{,}356} \text{ sq. chains}, = 10 \text{ sq chains.}$$

Distances are measured in full chains and decimals. If the distance between two points is 9 full lengths and 83 links, we call the length 9.83 chains, as each link is one-hundredth part of a chain. Each link is composed of three parts—a long wire with looped ends and two rings. These rings can be left open or soldered (brazed). If left open they soon become oval and elongated in the direction of the chain, and the chain thus becomes lengthened. It is therefore best to have all joints brazed, as this makes the ring connections more stable and less liable to stretch. Figure 1 is an illustration of one form of Gunter's chain and the two rings at each joint can be seen in the upper right-hand part of the figure. At each end of the chain there are two brass handles, the measuring length of the chain being from back to back of the handles. These loop handles are attached to the chain by nuts that are intended to be adjustable. When fixed in one position, it is important that they remain stationary till adjusted by the surveyor. The wearing surfaces make it necessary to remove links and this renders the chain inaccurate for fractional parts of a chain. There are in all about 600 wearing surfaces and if each surface is worn one-hundredth part of an inch, the chain will be lengthened 6 ins. and this would produce an error of 1 per cent in the calculation of areas.

The tenth link from the end is marked by a one-point brass tag, the twentieth by a two-point tag, the thirtieth by a three-point, the fortieth by a four-point, and the fiftieth by a round tag, it being the middle of the chain. At the center there is generally a snap link for disconnecting the chain, so that a half-chain can

Fig. 1.

be used for steep hills and rough country. The chain is folded by commencing at the middle and folding two links at each time in the form of a warped surface, making when completed a shape something like an hour-glass. In unfolding the chain, take both handles in one hand and with the other throw the chain from you. With a little practice this can be done so that it will stretch its full half-length when thrown and the whole chain can then be opened out.

2. **Engineer's Chain.**—This chain is made similar to Gunter's chain, is 100 ft. in length from back to back of the handles,

and is tagged every 10 ft. Each link is 1 ft. long and it consists of one long wire and two or three rings whose joints are brazed. This chain is now rarely used in railroad or city surveying where great accuracy is required. The steel tape has almost wholly superseded it where accurate work is desired.

3. **Vara Chain.**—The vara chain is 20 varas long, and each vara is divided into five equal parts. Each vara is marked by a tag with its distance from one end stamped, and the tags are numbered from 1 to 19. The chain is thus divided into 100 equal parts, each part being one-fifth of a vara or one one-hundredth of 20 varas, and is, therefore, 0.2 of a vara. It is necessary to remember this, for in the Gunter and engineer chains the chain itself is the unit of length. If the distance between two points is five full lengths, 16 varas and 2 links, then

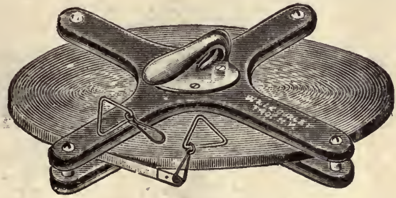


Fig. 2.

the distance is 116.4 varas. In Texas a vara is 33⅓ ins. long by law.

4. **Steel Tapes.**—For precise measurements the steel tape (Fig. 2) is used. It varies in length from 3 ft. to 1,000 ft., and is made of the best steel reasonably flexible. The tape has the advantage of having no wearing surfaces, and is easily folded or looped up like a rope. The width of the tape varies from 3/16 to ½ in., the thickness being about 1/64 in It is marked every 5 ft. from one end and numbered on brass and copper plates bent around the tape from 5 to 95 ft., and every foot is marked by a brass rivet, and each foot from the end is divided into tenths of a foot. The even 5-ft. marks are usually made on a brass plate or sleeve, and the even 10-ft. marks are made on a copper sleeve. In order to assist in identifying the even 10-ft.

marks when the figures have become so worn that they are illegible, rivets are driven through the plate close to the sleeve, one at the 10 and 90-ft. marks, two at the 20 and 80-ft. marks, three at the 30 and 70-ft. marks, and four at the 40 and 60-ft. marks. The rivets are always driven between the sleeve and the 50-ft. mark, so that, by noticing the position of the rivets, it is easy to distinguish the proper point. The 50-ft. mark is marked by two rivets, one on each side of the sleeve.

One of the best forms of steel tapes for railroad or city engineers and surveyors is about ¼ in. wide and has the numbers for the different foot-marks stamped on solder which adheres to the tape. This form of tape has the advantage of not having a shoulder or projection to catch against the reel when the tape is being wound up or run out, or to catch on stones or other rough objects while in use.

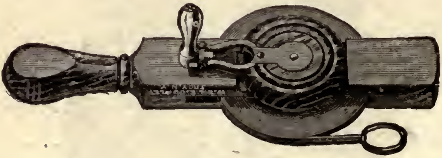


Fig. 3.

5. Standardized Tapes.—For accurate base line measurements a steel tape from 100 to 300 ft. long is used (Fig. 3). Such tapes should be standardized; that is, the absolute length between the marked points under a certain pull at a known temperature should be determined. This is generally done in this country by the United States Coast and Geodetic Survey (Washington, D. C.) for a nominal price. If it is necessary to use any tape unsupported, its correct length when hanging free may be found by direct comparison. Lay the tape on a smooth straight support, give it the proper pull and mark the end points; then, holding one of the ends directly over one of the marks just made, give it a known pull. Drop a plumb line from the other end of the tape and notice the amount by which it differs from the second mark. In this way the correct length of the unsupported tape under any given pull may be determined.

6. **Metallic Tapes.**—The most serviceable tape for ordinary or common use is the metallic tape (Fig. 4), which is a cloth tape manufactured with very fine brass wires interwoven into it. This tape is generally ⅝ in. wide, and is made in lengths of 25, 50 and 100 ft. It is conveniently inclosed in a leather case, and when it is rolled up it can easily be carried in the pocket. For light and irregular work it is much more convenient than the larger steel tapes. It is largely used in building construction, cross-section work, and in railroad engineering, and in many places where its lightness, compactness, and flexibility commend it. It can not be used where accuracy is very im-

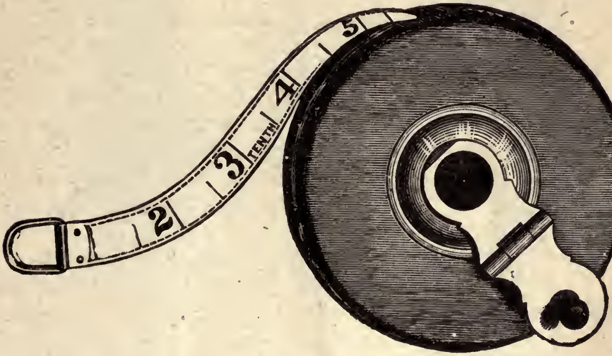


Fig. 4.

portant, for it stretches considerably under pull, but after a short period of use it will be found to have become permanently stretched.

7. **Pins.**—Surveying pins are used to keep tally of the number of chains measured. They are made of pieces of round steel wire 3/16 in. in diameter and about 16 ins. long. One end is pointed and the other is bent to form a ring or handle about 2 ins. in diameter, Fig. 5. Eleven such pins form a set, and they are carried on a key ring, about 6 ins. in diameter, made of the same sized wire. Each pin usually has a small piece

of red flannel tied to its handle so as to make it more easy to be found when used in the field. In railroad chaining stakes are generally used instead of pins and these stakes are driven at every full station (every 100 ft.) and at intermediate points between the stations. For a description of stakes see the chapter on Railroad Surveying.

8. Range Poles.—Range poles are rods of steel, or wooden rods shod in steel, varying from 6 to 10 ft. in length. Alternate foot lengths of the rod are painted red and white to make it more readily distinguishable against any background. They are used by the rear chainman to keep in a straight line when chaining. If the sun is shining and long sights are taken, the bright part of the range pole is seen as the other part is in shadow. To avoid this, a range pole with a flat face is used with the central longitudinal line clearly defined and alternate foot-lengths on each side of this line painted black or red.

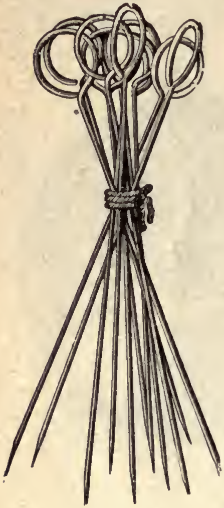
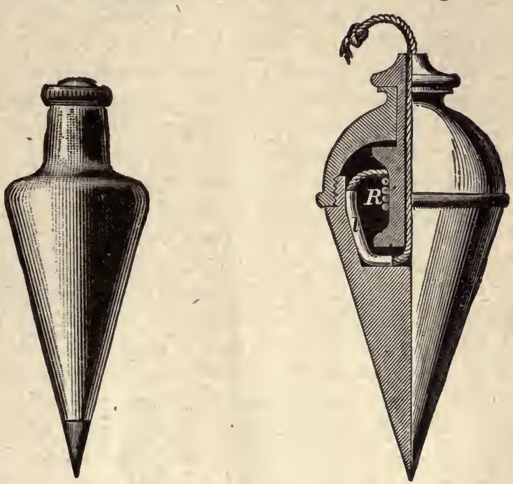


Fig. 5.

9. Plumb-Bob.—In chaining over rough or inclined ground it is often necessary to raise one end of the chain or tape to bring it to a horizontal. To locate a point on the ground directly under such elevated points or ends of chain, a plumb-bob will have to be used. The usual form of a plumb-bob is shown in Fig. 6, (a) and (b), which consists of a conical shaped body rounded into a neck and head in the upper part. The bottom or apex of the cone is usually tipped with a steel point, while the cap-screw at upper end has a hole through its center for the insertion of the cord by which it is suspended. In the ordinary form (a) the cap screw is taken off, the cord is inserted, and a knot tied in the cord to prevent its slipping through the cap-screw when the bob is suspended. Fig. 6 (b) is a special form of plumb-bob which is provided with a spool on the inside by which the cord can be wound up and carried inside the bob instead of being wrapped around the outside as in the ordinary form. This winding is done by turning the cap-screw at the top.

10. Chaining.—A line is measured or chained by two men, called the rear and head chainman. They should start with eleven pins, the rear chainman taking one pin and holding his end of the chain or tape over the intial point, and lines or ranges the head chainman in with the distant flag. The head



(a) (b)

Fig. 6.

chainman sticks a pin at this point and advances to another station, the rear chainman following to the station just left by him. The rear chainman places his end of the chain or tape over this station, and again ranges in the head chainman. The rear chainman must be careful to collect all pins, and when the head chainman calls "Out" he must drop his end of the chain and go to the head chainman, and should hand him 10 pins. The head chainman should count the pins, and if there are not 10 pins the line should be chained over. The number of "Outs" is recorded by each chainman. If we are using the surveyor's chain, and have three "outs," and the head chainman has measured 3 chains and 23 links on the new out, the length of the line is 33.23 chains. The head chainman always starts on a new out with 10 pins, and the head chainman or the rear chainman should never have more than 10 pins in his hand while measuring. The initial point and the end of each out should be carefully marked so that if a mistake is made in a long line the chainmen can return to the last out, and not have to go back to the beginning of the line. The methods of keeping track of the "outs" vary with different surveyors. In chaining long lines, a string tied in the button hole of the coat or shirt, with segments of unequal length, can be used by tying a knot in the long segment for an ordinary "out," and one in the short segment for every ten "outs." Another method is to have the chainmen make tally marks in a note book.

In chaining up a hill which is too steep for one length to be brought horizontal, the head chainman stretches the chain to its full length, and then returns and takes a point on the chain sufficiently near the rear chainman to pull that part horizontal. He marks the point on the ground under the selected point with his pumb-bob, or places the point on the chain immediately on the ground, the rear chainman drops his end of the chain and takes up the point selected by the head chainman and raises it as high as he can over the point as tested by his plumb-bob. The head chainman in the meantime selects another point on the ground in advance and marks that on the ground as before. This process is repeated until the length of the chain is exhausted. This is called "breaking the chain." In "breaking the chain" it is well to take sections of the chain that are multiples of ten.

In measuring down a hill, the process is reversed, so that the rear chainman holds his end on the ground or near it, and the head chainman holds his point over his head as high as he can.

The chain or tape should always be held level, because the horizontal distance between the two fixed points is constant, notwithstanding the fact that changes may be made on the surface of the ground. In the early days surveyors paid no attention to holding their chain level, and there has resulted, in consequence many discrepancies in their surveys, and much litigation. All good surveyors are now very careful in observing this rule. In using the tape in rough countries or thick underbrush, it is a good plan where great accuracy is demanded to attach the handle of the tape by a short loop of strong cord to allow twisting of the tape without breaking.

11. Chaining Over Hills or Across Valleys.—When it is impossible to see one station from the initial station on account of an intervening hill or high timber, a series of range poles is used and a random line marked out so that at least three points can be seen from one station.

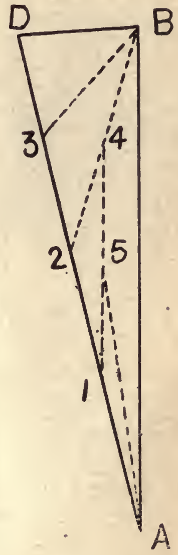


Fig. 7.

Given the two points *A* and *B* (Fig. 7), to set the range poles in line *AB*. We start out from *A* and, guessing at the line, set enough range poles in a random line *AD* so that at least two can be seen from *B*. Then the man at *B* will have the flag pole at 3 set over in the line *B*-2 to the point 4, the man at 4 will have the flag 2 set over in the line 4-1 at 5, the man at 5 will have the flag 1 set over in the line 5-*A*. Then again flag 4 will be set over to some point nearer *AB*, in line between *B* and 5, etc. This process is repeated until all the range poles are in the line *A*-*B*. In the preliminary ranging in the men themselves can act as range poles. Only one man is *absolutely* necessary if he has plenty of range poles, but two can do it with reasonable efficiency.

12. Chain Survey.—When the area of a farm is wanted, or if it is desired to construct a map of same, it may be divided by stations into a system of triangles. All the sides are then measured carefully and a map of the triangulation system can then be made to scale. The buildings and other topographical features, such as roads, fences, etc., can be tied in by measuring from the nearest stations and a sufficient number of points on the building, and map can be completed to scale.

In a recent survey (Fig. 8) the following measurements were made:

$AB = 240$	$DE = 204$	$EC = 340$
$BC = 160$	$EA = 180$	
$CD = 272$	$EB = 300$	

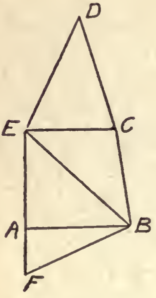


Fig. 8.

In case it were impossible to measure the line *EB*, the area may still be found by a chain survey if use is made of two auxiliary lines *AF* and *BF*, the point *F* being in *AE* produced. By means of these auxiliary lines the triangle *BAF* may be calculated and hence the angle *BAE* becomes known. From this angle and the sides *AB* and *AE* the length of *BE* can be calculated and the area found as before.

PROBLEM 1.—Make a map of the chain survey *ABCDE* to a scale of 1 in. = 50 units.

13. Chain Problems.—(a) To erect a perpendicular to a line at any point:

We know that if the sides of a triangle are 3, 4 and 5, or any multiple of these, the triangle will be right. This is apparent, as the sum of the squares of 3 and 4 equals the square of 5. If, in the triangle *AKB*, Fig. 9, the sides are 18, 24 and 30, it will be a right triangle.

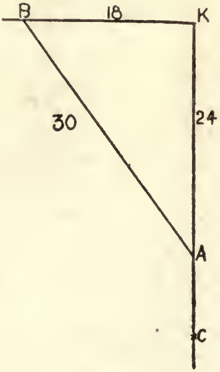


Fig. 9.

The rear chainman holds his end of the chain at *B* in the line *BK* so that the distance *BK* is equal to 18 links; he also holds the end of the seventy-second link at the same point; the head chainman passes the chain around a pin at *K*, which has been firmly driven or pushed into the ground, then takes hold of the forty-second link and stretches the chain so that all parts are taut. A pin is then driven at *A*, which determines the perpendicular *AK*.

In reality there are a great number of ways in which the problem can be solved, for if

$$2n = \text{first side}$$
$$n^2 - 1 = \text{second side}$$
$$n^2 + 1 = \text{third side}$$

the triangle is a right triangle, as $(2n)^2 + (n^2 - 1)^2 = 4n^2 + n^4 - 2n^2 + 1 = n^4 + 2n^2 + 1 = (n^2 + 1)^2$.

Therefore, we can make n equal to any number greater than unity. The following are some of the numbers actually used:

3,	4,	5,
5,	12,	13,
6,	8,	10,
8,	15,	17,
7,	24,	25,
12,	35,	37,
11,	60,	61, etc.

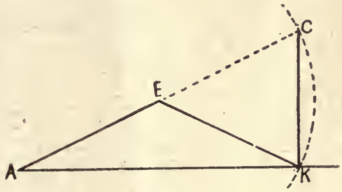


Fig. 10.

(b) Another easy method of erecting a perpendicular to the line *AK* at the point *K* (Fig 10) is to let one of the chainmen hold the end of the chain at *K*, while a second chainman holds the other end of the chain at any point on *AK* so that the chain will be slack. The middle point of the chain is then carried away from the line *AK* until it occupies the position *AEK*. If the end of the chain at *K* is now swung around until it reaches a point *C* in the same straight line as *A* and *E*, the line *CK* will be the perpendicular to *AK* at *K*.

(c) To find the distance across a marsh, river or pond by use of the chain:

Suppose a line that we are chaining reaches a point *A*, Fig. 11, and a river intervenes wider than the length of one chain, and we wish to find the distance *AB*. At the point *A* by the former method we measure the distance to *K* on the perpendicular *AK*, and at the ponit *K* set off the right angle *BKC*, and mark where *KC* produced crosses our original line. Measure AC.

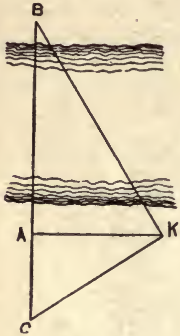


Fig. 11.

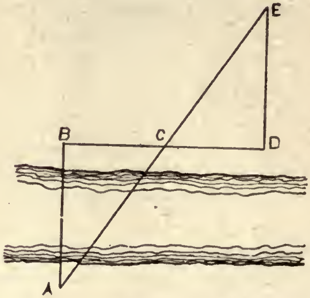


Fig. 12.

In the right triangle *BKC*

$$AK^2 = BA \times AC$$

$$BA = \frac{AK^2}{AC}$$

Caution.—*AK* should be taken at least one-half of *AB*, otherwise *AC* will be so short that a slight error in measuring will produce a large error in the result.

(d) Similar Triangles: To find the distance *AB*, Fig. 12, erect a perpendicular to *AB* at *B* with a chain and prolong it to some point *C*; measure *BC* and set a flag pole at *D* in the line *DC*. Erect a perpendicular, *DE*, to *BD* and have the flagman move along this perpendicular until he is in the line *ACE*.

Set the flag pole firmly in the ground and measure DC and DE.

$$AB : DE :: BC : DC$$

$$AB = \frac{DE \times BC}{DC}$$

14. Correction for Temperature.—Steel tapes are standardized by the Coast Survey by comparison with known standards at Washington, and each standardized tape is marked somewhat as follows: "Length 100 feet at temperature 62° F., pull 12 pounds horizontal."

The average coefficient of linear expansion is 0.0000065 for each degree Fahrenheit, and each unit length.

Let L = Length of tape.
C = Coefficient of tape.
T = Rise in temperature.

Then the increased length of the tape $= LCT$.

Total length of tape $= L + LCT = L(1 + CT)$.

Examples. A 300-ft. tape was standardized at 62° F., pull 12 lbs. A base line was measured when the temperature of the tape was 102° F., find the length of the tape.

Increase $= LCT$.

$= .0000065 \times 300 \times 40 = .078$.

Total length $= 300 + .078 = 300.078$ ft.

15. Stretch of Tape Due to Pull.—It is necessary to subject all tapes to what is called a *standardized pull* for their true lengths. If it takes a 12-lb. pull to make a tape 100 ft. long, any pull greater than this will stretch the tape, and has to be allowed for.

Let P = pull in pounds.
A = cross-section in square inches.

Then the pull per unit area $= \frac{P}{A}$ = unit stress.

If S = total stretch
L = length of tape

Unit stretch $= \frac{S}{L}$

In ordinary pulls the unit stretch varies directly as the unit pull.

Therefore, $\frac{\text{Unit pull}}{\text{Unit stretch}} = \frac{PL}{AS} =$ Constant.

This is Hook's law, which was published in the form *"ut tensio sic vis."* The unit pull divided by the unit stress is constant within the elastic limit and is called "the coefficient of elasticity," and is generaly represented by the letter E. For steel $E = 30{,}000{,}000$ lbs.

EXAMPLE: A bar 1½"x¾"x20' long was subjected to a pull of 18,000 lbs. and produced a stretch of ⅛ in. Find E.

Area = 9/8 sq. in.

$$\text{Unit pull} = \frac{18000}{9/8} = 16{,}000 \text{ lb.}$$

$$\text{Unit stretch} = \frac{1/8}{240} = \frac{1}{1920}$$

$E = 16000$ divided by $1/1920 = 16000 \text{x} 1920 = 30{,}720{,}000.$

EXAMPLE: If a 100-ft. tape was standardized at a pull of 12 lbs., and has a cross-section of .00371 sq. in., find how much it will be stretched by a pull of 26 lbs. if $E = 30{,}000{,}000$. The stretch over the standard length will be due to the extra pull of 14 lbs.

S = total stretch in feet.

$$\text{Unit stretch} = \frac{S}{100}$$

$$\text{Unit pull} = \frac{14}{.00371}$$

$$E = 30{,}000{,}000 = \frac{1400}{.00371S}$$

$$S = \frac{1400'}{30{,}000{,}000 \times .00371} = 0.0125'$$

16. Correction for Sag.—The foregoing corrections for pull and temperature assume that the tape is horizontal, but in field measurements it is never horizontal, although the two ends may be in the same horizontal plane. The tape hangs in a curve, which is practically a parabola, with which a circular curve can coincide almost exactly. The effect is to shorten the chain.

If d = sag
L = length of tape or chain

$$\text{The correction for sag} = \frac{8d^2}{3L}$$

EXAMPLE: A 100-ft. tape, standardized at 62° F. and 12 lbs. pull was used to measure a line when the temperature was 92° F., pull 25 lbs. and sag 0.5 ft. Find the correct length of the tape if the cross-section is 0.003 sq. in.

Correction for temperature = $.0000065 \times 100 \times 30 = 0.0195$
Correction for sag = $8/300 \times 0.25 = 0.0067$

$$\text{Correction for pull} = \frac{13 \times 100 \times 1{,}000}{3 \times 30{,}000{,}000} = 0.1444$$

Length of tape = $300 + 0195 - .0067 + .1444 = 300.1572$

PROBLEM 2.—A 100-ft. tape, cross-section 1/300 sq. in., was standardized at 62° F. and pull 12 lbs. Find the length for a temperature of 96° F., pull 28 lbs. and a sag of 0.5 ft.

PROBLEM 3.—A standardized tape is 100 ft. long between marks at 61° F., and a pull of 11 lbs. Find the length when temperature is 97° F., 20 lbs. pull, and a sag of 0.70 ft., if cross-section is 1/300 sq. in.

17. Erroneous Lengths.—Chains become changed by the breaking of links, the loss of handles, and the wearing of the 600 rubbing surfaces. In the use of the chain two points on the ground, 66 ft. or 100 ft. apart, should be marked, and the chain should be compared with this at frequent intervals. The outer edge of one of the handles is placed over the zero and the 100-ft. mark is marked by a file if the chain is too long. If distances are measured by chains that are too long, we can find the true lengths of the lines by calculation without measurement. If the length of the chain used is $100 + a$, and in the measurement we called it 100 ft., then the length of the line as measured will be too short.

If the extra length of the chain is due to wear or stretch throughout the length, the true length of a line that has been measured with a tape of erroneous length may be found by multiplying the true length of the tape by the number of times it was applied to the ground in measuring the line. After a line 9.864 chains in length had been measured it was found that the chain was really 100.25 ft. long, find the true length of the line. The chain was applied to the line 9.864 times, consequently its true length must be $9.864 \times 100.25 = 988.866$ ft.

However, it might happen that one link of an engineer's chain had been broken and taken out, thus making the chain 99 ft. long. Suppose an engineer's chain was used in measuring a line the length of which was recorded as 628 ft., and it was then discovered that 1 link was out of the 10-ft. section next to the head chainman. What is the true length of the line? Six full lengths were measured $= 6 \times 99 = 594$ ft. If the 28 ft. was measured with the end of the chain next to the rear chainman the true length of the line was 622 ft., but if the 28 ft. was measured with the part of the chain that contained the unknown missing link, then the true length of the line was 621 ft.

Let a = assumed length of chain,
t = true length of chain,
M = measured length of line as measured with chain of erroneous length,
T = true length of line,
n = number of chain lengths in M (whole or fractional.)

Then, $M = na$
$T = nt.$

$$T = M \frac{t}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

18. Erroneous Areas.—If a farm is surveyed with a chain of erroneous length and the area is calculated by use of the erroneous data, we can find the area without rechaining.

Let C = calculated area of farm,
X = true area of farm,
a = assumed length of chain,
t = true length of chain,

Then, na = measured length of side of farm,
nt = true length of same farm.

Now, similar polygons are to each other as the square of their homolgous sides.

$$\therefore X : C :: (nt)^2 : (na)^2.$$

$$\text{or } X = C\left(\frac{t}{a}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

It is well to observe that in the re-calculations for correct length of a line or for correct area of a farm the *assumed* length always appears as denominator of correction ratio. This assumed length is usually an even number, and is generally 20, 66, 100, etc.

EXAMPLE: A line was measured with a chain that was supposed to be 100 ft. long; the length of the line as measured was 986.4 ft. In testing the chain immediately afterwards it was found to be 100.25 ft. long. Assuming that the stretch was proportional throughout, find the length of the line.

$$\text{Correct length of line} = 986.4 \times \frac{100.25}{100} = 988.866$$

PROBLEM 4.—The assumed length of a chain is 100 ft., the calculated area 99.01 acres. The true length of the chain was found after the calculation to be 99 ft. 6 ins. Find the true area.

PROBLEM 5.—A chain used to measure a field was 100 ft. 2 ins. in length, and it was assumed in measuring the farm to be 100 ft. long. If the calculated area, based on the erroneous length of chain, was 11.72 acres, find the true area.

PROBLEM 6.—A farm was surveyed with Gunter's chain and the area was calculated to be 39.6 acres. The chain was tested immediately after the survey was made, and it was found to be 4 ins. too long. Find the true area of the farm.

PROBLEM 7.—If the calculated area was 133⅓ acres and the vara chain was used in chaining which, after the survey, was found to be 3⅓ ins. short, find the true area.

19. Linear Units.—The yard is the primary unit of length in the English measure. The standard yard is the distance between two points at a certain temperature on a bar of platinum kept in London in the office of the Chancellor of Exchequer of Great Britain. A copy of this is kept in Washington, D. C.

An inch is one-thirty-sixth part of a yard, and a foot is one-third part of a yard, or 12 ins.

To convert feet to varas multiply by 0.36.

To convert yards to varas multiply by 1.08.

To convert Gunter's chains to varas multily by 23.76.

To convert poles to varas multiply by 5.94.

To convert meters to varas multily by 1.1811.

20. Units of Land Measure.—

One acre = 4840 square yards,
= 43560 square feet,
= 10 square chains,
= 160 square poles.
= 5645.376 square varas,
= 4046.87 square meters.

One vara = 33⅓ inches.

One yard = 36 inches.

One foot = .36 vara.

One square vara = 1111.1 square inches = $\frac{10,000}{9}$ square ins.

One square yard = 1296 square inches.

A Spanish league was defined as a square, 5,000 varas on a side.

One league = 25,000,000 square varas.
= 4428.203 acres.

One labor = a square of 1000 varas,
= 1,000,000 square varas,
= 177.128 acres,
= 1/25 of a league.

One linear mile = 1900.8. varas.

One meter = 39.37 inches.

One linear mile = 1609.35 meters.

A *labor* was assigned by the Mexican government to settlers for the purposes of agriculture, hence the name; while a *league* was assigned for grazing purposes. In this way a *league* and *labor* became associated.

21. Area of a Triangle.—By geometry we know that the area of a triangle $= \frac{1}{2}(p \times c) = K$, where p represents the altitude CD and c the base of any triangle.

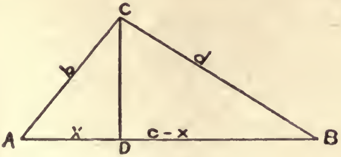


Fig. 13.

In the right triangle ADC, $p^2 = b^2 - x^2$.

In the right triangle BDC, $p^2 = a^2 - (c-x)^2$.

$$b^2 - x^2 = a^2 - a^2 + 2cx - x^2$$

$$x = \frac{b^2 + c^2 - a^2}{2c}$$

But $p^2 = b^2 - \dfrac{(b^2 + c^2 - a^2)^2}{4c^2}$

$$4c^2p^2 = 4b^2c^2 - (b^2 + c^2 - a^2)^2$$
$$= (2bc + b^2 + c^2 - a^2)\,(2bc + a^2 - b^2 - c^2)$$
$$= [(b+c)^2 - a^2) \times (a^2 - (b-c)^2] \div 4c^2$$
$$= (b+c+a)\,(b+c-a)\,(a-b+c)\,(a+b-c)$$

Let $2s = (a+b+c)$

Then $(b+c-a) = 2s - 2a = 2(s-a)$

$(a-b+c) = 2s - 2b = 2(s-b)$

$(a+b-c) = 2s - 2c = 2(s-c)$

$$\therefore\ p^2 = \frac{4s(s-a)(s-b)(s-c)}{c^2}$$

$$p = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{c}$$

Therefore $\frac{1}{2}(pc) = K = \sqrt{s(s-a)(s-b)(s-c)}$ .... (3)

Problem 8.—Calculate the areas of triangles ABE, BEC, and EDC in Fig. 8.

Problem 9.—If the sides of a triangle are 520, 560, and 600 varas, find the area in acres.

PROBLEM 10.—If the sides of a triangle are 13, 20 and 21 chains (66 ft.), find the area in acres.

PROBLEM 11.—If $a=750$ varas, $b=650$ varas, $c=200$ varas, find area in acres.

PROBLEM 12.—If $a=50$ poles. $b=41$ poles, $c=39$ poles, find area in acres.

PROBLEM 13.—If $a=300$ poles, $b=240$ poles, $c=180$ poles, find area in acres.

PROBLEM 14.—If $a=280$ poles, $b=224$ poles, $c=168$ poles, find area in acres.

22. The 57.3 Rule.—Let *EOA* (Fig. 14) be a triangle where the angle x is less than 6°, and the two arms *OA* and *OE* practically equal. If with *O* as a center and *OA* as a radius we describe a circle passing through *E* we have:

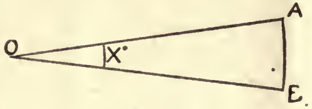


Fig. 14.

$$X^\circ : 360^\circ :: y : 2\pi r$$

where $y = AE$

$$\text{Then } X^\circ = \frac{360}{2\pi} \times \frac{y}{r} = \frac{57.3y}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

That is, the small angle in degrees times the long side is equal to the short side times 57.3.

PROBLEM 15.—A straight roadway 1,320 ft. long has a rise of 21 ft. above the horizontal through the low end. Find its angle of elevation.

23. Applications of the 57.3 Rule.—If the angle *AOE* Fig. 14, equals one-tenth of 57.3°, then we have

$$5^\circ.73\ AOB = \frac{57^\circ.3 \text{ offset}}{\text{Distance}} \quad \therefore \text{Distance} = 10 \times \text{offset}.$$

That is, when the small angle is 5°.73 or 5° 44′, the distance is ten times the small side or offset.

If the angle *EOA* is equal to 0°.573, that is, 34′.38, the long side is one hundred times the offset. Hence $OA = 100 \times AE$.

This is generally expressed by saying that the distance is 100 times the offset. This principle is used in finding the approximate area of a boundary. The angle that *OA* makes with some reference line is measured, and the distance *OA* is found by making the angle equal to 34.38 minutes. The assistant at *A* attaches one end of a tape or chain to the point *A* and then takes *AE* at right angles to *AO* and is sighted in the line *OE* by the distant transitman. When he is located, he reads on the tape the distance *AE* and records it in his note book. The distance from *A* to the instrument man is 100 times this distance *AE*.

PROBLEM 16.—Make a drawing of the following area to a scale of 1 in. equals 100 ft., and find the area in acres, by dividing the boundary into triangles.

Point.	Angle.	Offset.
A	0°	8.50 feet
B	45°	10.00 "
C	75°	9.40 "
D	90°	9.60 "
E	120°	8.60 "
F	150°	7.20 "
G	180°	6.00 "

24. Pacing Survey.—A rough approximate idea of the area of a farm can be obtained by a pacing survey. With a little practice a man can train himself to step off a yard at each stride and in this way a fair approximation can be made to the area of a small farm or parcel of land. In a farm, *ABCDF*, Fig. 15, let *AB* 350 yds.; *BC* 400 yds.; *CD* 90 yds.; *DE* 266.3 yds.; *EF* 250 yds.; *FA* 281.8 yds. Now the area of the farm can be found by dividing the field into triangles or by locating the points, *CDE*, etc., by offsets from some reference line, *AB*. If the land is divided into triangles we pace the distance *BD* 410 yds., *BE* 300 and *AE* 211. This divides the land into four triangles, *BCD*, *BDE*. *BEA* and *AEF*. The area can be calculated by the use of formula (3).

If it is desired to locate the corners by offsets, we adopt some reference line from which to take offsets. This reference line need not be a side of the farm, but can be some line assumed for convenience. However, in the case of Fig. 15, we shall assume *AB* as the reference line. As *ABC* is a right angle, the distance *BC* 400 yds., will locate *C*, and as angle *BCD* is also right, the distance, *CD* 90 yds., will determine the point *D*. Let *DG* be a perpendicular from *D* on line AB. If a perpendicular be dropped from *E* on *AB* cutting *AB* at *H*, where $BH = 240$ and $HE = 180$, the point *E* is determined. The point *F* is similarly located where perpendicular, *MF* 250 and *BM* 480. The areas of the trapezoids, *BCDG, GDEH, HEFM,* and that of the triangle, *AMF*, can be found to be respectively 36,000, 43,500, 51,600, and 16,250 square yards and the area of *ABCDEF* 114,850 sq. yds.

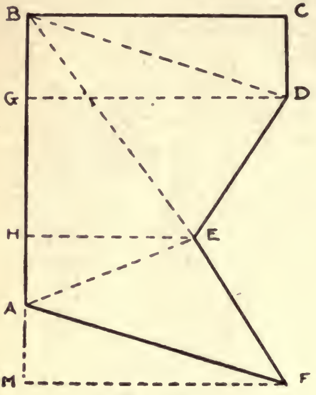


Fig. 15.

Instead of trying to regulate the stride to 1 yd., some prefer to take the usual stride used in walking, counting the number of steps it takes to cover 100 ft. and then estimate the distance. Thus if it takes 40 steps for 100 ft. and there are 114 steps in the length of the line, the number of feet is found by multiplying 114 by 100 and dividing by 40. In this case the line would be about 280 ft., or 93 yds.

25. Location of Houses.—These can be located by *range lines, regular offsets,* or by *intersections.*

Range Lines.—Let *MN*, Fig. 16, be the base line or line nearest any given corner of the house, *FDCE*. Have a range pole set at *A* in line *MN* and in range with *CD,* the side of the building; and another set at *B* in line *MN* and in range with *CE,* another side of the house. Pace the distances from *A* and *B* to end of base line *MN*. On the map locate *AB* on line *MN* and with *AB* as a diameter draw a circle. With *B* as a center and a radius *BC*; cut circle at *C*. Join *BC* and *AC* and produce *BC* in *EC* and *AC* in *CD*. Lay off *CE* and *CD* to scale and locate the rest of the building.

Rectangular Offsets.—Let *CH* and *EK*, Fig. 16. be the perpendiculars from corners of house on base line *MN*. Pace *EK*,

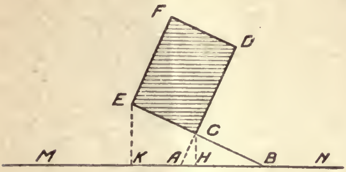


Fig. 16.

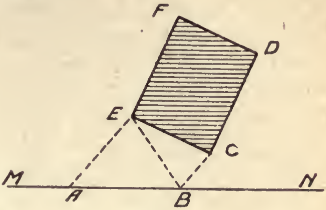


Fig. 17.

CH, and *KH* to end of base line *MN*. The house can thus be located on the map.

By Intersection.—Let *MN*, Fig. 17, be base line and *A* and *B* two points in this line. Pace distances *AB*, *BC*, *BE*, *AE*, *CD*, *EC*, and the distance from *A* or *B* to end of base line. To locate house on map, locate *A* and *B* on map and with *A* as center and *AE* as radius draw arc and with *B* as center and *BE* as radius draw arc cutting first arc at *E*. This locates *E*. Then with *E* as center and *CE* as radius, draw an arc and with *B* as center and *BC* as radius draw arc cutting the other arc at *C*. Draw *CD* perpendicular to *EC* and lay off *CD* to scale, and through *D* and *E* draw *DF* and *EF* parallel to *CE* and *CD* respectively.

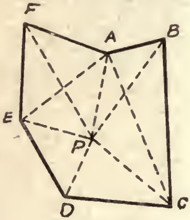


Fig. 18.

26. Survey of Farm by Chain or Pace.—The exact area of a farm *ABCDEF*, Fig. 18, can be found by use of the chain or tape, or an approximate estimate of the area can be found by pacing the sides and diagonals. In Fig. 18 the following lengths of sides were found: *AB* = 170 yds., *BC* = 492 yds., *CD* = 296 yds., *DE* = 272 yds., *EF* = 286 yds., *FA* = 260 yds.

The configuration of the ground was such that the farm could be divided into triangles by running diagonals from the corner *A*. These diagonals, *AC, AD,* and *AE,* were found to be 488, 436, and 322 yds., respectively. The area of *ABC* was found by formula (3) to be 41,000 sq. yds.; that of *ACD,* 63,760; that of *ADE,* 43,680; and that of *AEF,* 35,376, making a total area of the whole farm of 183,816 sq. yds. = 37.98 acres.

To check the foregoing calculation a point *P* was taken on a knoll near the center of the farm and the following distances were paced: *PA* = 196, *PB* = 312, *PC* = 350, *PD* = 240, *PE* = 206, *PF* = 337. The areas calculated by formula (3) are *PAB* = 14,873, *PBC* = 54,577, *PCD* = 34,538, *PDE* = 23,546, *PEF* = 33,504, *PFA* = 25,422, or a total of 184,460 sq. yds. = 38.1 acres. If the distances are all carefully chained instead of paced the two methods should check within one-tenth of an acre.

CHAPTER II.

COMPASS SURVEYING.

27. The Bearing of a Line.—The acute angle that a line makes with the meridian is called its true bearing. If the acute angle is made with that part of the meridian to the north of us it is called north, and if in addition it cuts to the right it is called North $X°$ East, where X equals the acute angle. If the acute angle is made with that part of the meridian to the south of us and cuts to the right it is called $SX°W$. In Fig. 19 the bearing of AB is N 32° E; that of AD, N 54° W; that of EF, S 61° W, while that of EG is S 27° E.

28. Azimuth.—The azimuth of a line is the angle made with the true meridian, and is measured from the south around by the west, north, and east to the south again. If the bearing of a line is S 39° W, the azimuth is 39°; if the bearing is N 39° W, the azimuth is 141°; if the bearing is N 39° E, the azimuth is 219°, and if the bearing is S 39° E, the azimuth is 321°. In some states it is the practice to define "bearing" as the acute angle made by a line with the magnetic meridian (that is, with the needle in its mean position).

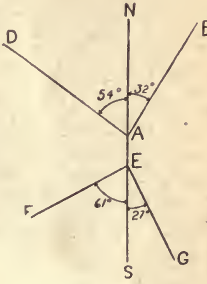


Fig. 19.

29. The Compass.—The essential parts of a surveyor's compass (Fig. 20) are a magnetic needle, a graduated horizontal circle, and a line of sights. These conditions can be fulfilled very crudely or elaborately. It is also convenient to have a declination arc attached to the compass on which we can set off the declination of the needle. A magnetic needle when poised freely will not point towards the North Pole, but will dip towards the north an amount of x degrees. To make it horizontal in the compass it is mounted on an agate pivot and the South end is weighted by having an adjustable brass wire at that end. The accuracy of the compass depends largely upon the activity of the needle, which depends upon the intensity of the magnetic force, which must be kept alive. The pivot upon which the needle is mounted is in the center of a graduated circle which is generally raised to the level of the ends of the needle and is graduated on a silver plate. Inside the compass box we find the letters *E*, *S*, *W*, and *N*. If the compass has no declination arcs the zeros are in the line of sights as determined by the slots in the standards or uprights. The graduated circle' is mounted on a brass plate which has extended arms, to which the uprights are attached by means of mill-head screws. If the arms are not extended the uprights are attached to the graduated circle and fold down over the face when not in use. To set off the declination accurately, each compass should be provided with a declination arc with a vernier attached.

For the purpose of leveling, the compass is provided with two bubble tubes whose axes are at right angles to each other. It is leveled by a

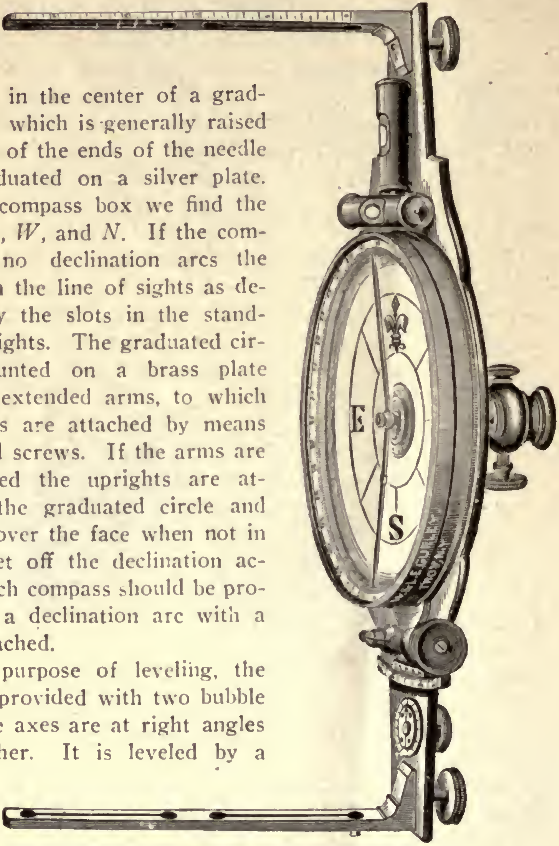


Fig. 20.

ball and socket joint which affords easy and quick methods of setting up. It can be mounted on a Jacob's staff or a tripod, but in most cases county surveyors use the Jacob's staff on account of its ease of transportation. The ball and socket joint is mounted on the Jacob's staff, which has a sharp conical iron shoe. In setting up, the staff is driven into the ground two or three times to get a firm footing so that there will be no vibration. The compass is then set on the staff—leveled—and it is now ready for use. When moving from station to station the compass should always be removed from the staff and carried under the arm, with the needle screwed reasonably tight. In setting up always loosen the ball and socket joint and have the compass almost level and along the line of sights before tightening. If the tripod is used the compass can be taken off in moving from one station to another or it can be left on as with the transit. The tripod gives much more accurate work than the Jacob's staff because you can locate the points more accurately, and it gives a much more stable support.

30. Reading the Bearing.—To read the bearing of a line, set up the instrument over any point on the line, turn the compass so that the arrow in the compass box points in the direction in which you are running the line, and read the north end of the needle. The north end of the needle will lie between two letters, one of which will be *N* or *S*, while the other will be *E* or *W*. If it lies between *N* and *W*, the bearing is northwest; if between *S* and *W*, the bearing is southwest, etc. In sighting always place the eye at the end of the compass box marked *S*.

31. How to Use the Compass.—Set up the tripod with the legs wide apart and firmly pressed into the ground. Place the compass on the brass spindle and then fasten the sights by means of the thumb screws provided for that purpose. This spindle is connected with the head of the tripod by a ball and socket joint, which gives it a limited range of motion. A groove about ⅛ in. wide and about the same depth is cut in the spindle, which engages a pin piercing the socket of the compass body which fits over the spindle and prevents the compass from falling off the tripod. Take hold of the compass with both hands and level it by means of the motion available in the ball-and-socket joint. When both bubbles are in the center of their run—

that is, in the center of the tube—the instrument is level. Do not lower the needle until the compass has been leveled. The compass may now be pointed in any direction by turning it on the spindle axis. In moving the instrument to another point, raise the needle by means of the screw controlling it, remove the compass from the tripod by pulling in the small pin in the socket mentioned above, at the same time lifting the compass from the tripod. Carry the compass under one arm and the tripod in the other hand, or on the other shoulder. If a Jacob staff is used instead of the tripod, the brass spindle connected to the ball-and-socket joint is connected with the staff by a tight fitting joint. When the compass is placed in its box to be stored away the needle should be left free.

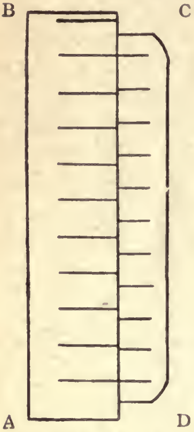


Fig. 21.

During some seasons of the year the compass will be affected by a charge of electricity due to atmospheric conditions. When this is the case one end of the needle will often adhere to the glass plate. If the glass is touched with a damp substance it will relieve this condition and release the needle.

32. **The Vernier.**—The vernier is an auxiliary scale, either straight or circular, designed to read to a certain given part of the finest division on the limb. Thus in the New York rod (Fig. 21) the smallest division that can be read from the rod itself is one one-hundredth of a foot, but the vernier cuts this part into ten parts, so that we can read to one one-thousandth of a foot. In the ordinary transit the finest division is a half degree, but with the aid of the vernier we can read to minutes. If AB is the limb and CD is the vernier scale, let a equal the length of each part of the limb, and b equal the length of each part on the vernier, and n equal the number of parts on the vernier, then $(n-1)$ will be the number of parts on the limb, so arranged

that n parts on the vernier is equal to $n-1$ parts on the limb, consequently $nb = a\,(n-1)$.

If the lowest mark on the vernier agrees with a mark on the limb, then the highest point on the vernier will agree with a mark on the limb, also the second mark on the vernier will not agree by an amount of $a-b$. If the vernier is moved a distance $a-b$, then mark No. 1 on the vernier will agree with a mark on the limb; if moved twice this distance, then mark No. 2 will agree with a mark on the limb; if moved three times this distance, then No. 3 will agree with a mark on the limb. If mark No. 3 on the vernier agrees with a mark on the limb, it means that the zero at the vernier is $3\,(a-b)$ from above the nearest point on the limb.

But $$bn = a(n-1)$$

$$b = \frac{a(n-1)}{n}$$

$$a - b = a - \frac{a(n-1)}{n} = \frac{a}{n}$$

$a-b$ is always one nth of the finest space on the limb and it is called the fineness of reading.

If $n=10$ parts and $a=1/100$ of a foot, then the vernier reads to 1/1,000 of a foot.

If $n=30$ and $a=30'$ then the vernier reads to minutes. This is the case in the ordinary transit; $a=30'$ or ½° and $n=30$, and we can read to minutes.

33. Declination of the Needle.—The magnetic needle at any point when mounted on a pivot and weighted at one end so that it will rest in a horizontal position will make an angle with the true meridian. This angle is called the declination or variation of the needle. In Texas the magnetic meridian cuts to the right of the true meridian passing through a point, and, therefore, the declination is said to be east. In Austin the magnetic meridian makes an angle at the present date of about 8° with the true meridian, or the declination of the needle is said to be 8° east.

The line of zero declination (called the agonic line) now passes near Charleston, S. C.; Asheville, N. C.; Knoxville, Tenn.;

Lima, Ohio; Battle Creek, Mich.; and passes through the remote corner of northeastern Indiana. All sections east of the line have west declinations, while all sections west of this agonic line have east declinations.

The United States Coast and Geodetic Survey determines the magnetic declination at various points in each State at stated intervals; and by this means not only is the declination accurately determined, but its rate of change can be determined by a comparison of the declination for different dates. These results are placed on a map (called the Isogonic Chart) by the Coast Survey. This chart is issued at least every ten years and is of great use to surveyors, as it gives the declination for all parts of the United States with reasonable accuracy. It can be obtained by addressing a letter to the Coast and Geodetic Survey, Washington, D. C.

Fig. 22.

34. **Compass Vernier.**—One form of compass vernier is shown in Fig. 22. This is the usual form of the vernier on the surveyor's compass. The vernier is divided into 30 equal parts and these 30 parts cover 29 parts on the "limb" or graduated circle. The smallest division on the limb is one-half degree or 30 minutes and as the vernier can read to one-thirtieth of the smallest division on the limb, we can read to one-thirtieth of 30 minutes, or to 1 minute.

We further notice that the vernier-zero is nearer the 5th division of the limb, and we find that the 5th division of the vernier to the left of the vernier-zero is opposite or coincides with a division on the limb. Hence the reading for the fractional part is five minutes, which corresponds to this 5th division of the vernier that is opposite a divison of the limb. The whole reading should be 2° 30′ plus 5′ or 2° 35′.

If the zero of the vernier is, as in Fig. 22, nearer the last division between the two zeros than it is to the division beyond the vernier-zero, the fractional part is read on the left half of the vernier. There are 15 divisions in this left half and if the fractional reading is between zero and 15, one division of the left half of the vernier will coincide with one division on the limb, and the number of this division on the vernier is the fractional reading. Thus if the 5th division on the vernier agrees, as in Fig. 22, with a division on the limb, the reading is 5′; if the 9th agrees, the fractional reading is 9′, etc. However, if the vernier reading is greater than 15′, this reading is obtained from the upper part of that half of the vernier that covers a section of the limb reading.

35. To Set Off Declination.—This will be illustrated by an example. Suppose that the declination of the needle is 8° 15′ east. This means that if the needle was allowed to swing freely it would come to rest in a line not pointing to the true north, but in a line that makes 8° 15′ on the east side of the true meridian, or in a line whose bearing is *N.* 8° 15′ *E.* To set off the declination, level the instrument, lower the needle and allow it to come to rest. Turn the compass until the line of sight. through the slots in the standards, coincides in direction with the needle. Clamp the instrument in this position. Since the needle when at rest points N. 8° 15′ E., the line of sight must now be N. 8° 15′ E., or make an angle of 8° 15′ with the true meridian. Then with the instrument clamped, and without disturbing the line of sight, turn the graduated circle in the compass box by means of the milled-head screw until the needle reads N. 8° 15′ E. The vernier scale that marks the declination arc should now read 8° 15′. The final and accurate test is the vernier arc where all declinations should be set off.

36. Changes in Declination.—The declination of all points west of the agonic line has been decreasing, while that to the east of the agonic line has been increasing. In Texas the declination has been decreasing at the rate of about three minutes per year since the time of the first recorded land patents. This steady annual change goes through a large series of years and probably goes through a cycle.

In addition to the annual change there is a daily change. In Texas the needle at about 6 p. m. is in its normal position; at 8 a. m. the north end of the needle swings to the east about two to three minutes, and about 1 p. m. it swings about the same amount to the west of the normal position.

37. Result of Changes.—An old survey was run in 1864 with the correct declination of 10° at the time the survey was made, and a surveyor in 1904, not knowing the present correct declination (which is 8°), sets his compass on the old declination. The bearing of line was N. 42° E.; that is, the line made 42° to the right of the true meridian and 32° with the magnetic meridian. When the correct declination was set off in the compass and the ends of the needle were brought to the zero marks on the graduated circle under the glass top, the line of sights pointed along the true meridian. But since the original survey was made the declination has decreased to 8° and the magnetic meridian has turned 2° to the left of its position in 1864. Now if the surveyor of 1904 had set off the declination of 8° on the declination arc and brought the ends of the needle to agree with the zero marks of the graduated circle, the line of sights would have pointed along the true meridian. But instead he set off a declination of 10°, and when he made the ends of the needle agree with the zero marks the line of sights marked out was 2° to the left of the true meridian. Now if *AB*, Fig. 23, were the original line that made 42° with the true meridian *AN*, and *AM* were the position of the magnetic meridian, the magnetic meridian of 1904 will occupy the position *AM'*, two degrees to

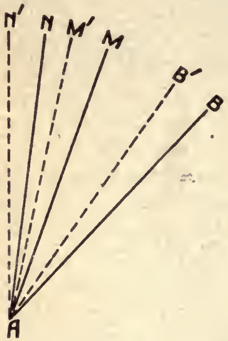


Fig. 23.

the left of *AM*. The surveyor set off 10° on the declination which made the line of sights point to a position *AN′*, which he assumed to be the true meridian, and from this he set off the bearing 42°. As the angle *NAB* is 42°, then the 42° measured from *AN′* will fall to the left of *AB* in some position *AB′*. In all cases in that section west of the agonic line, where the surveyor uses a declination greater than the correct declination, he, in effect, turns the assumed true meridian from which he locates bearings to the left, and all lines thus run will fall to the left of the old lines, if said old lines were surveyed with the correct declination.

34. **Old Lines.**—In surveying old lands the great object is to ascertain a declination which, used with the bearings as obtained from the field notes, will retrace the old lines. This is the prime object. It may be the correct declination for the time and place, and it may not. If two points *A* and *B* can be found on any side of the tract, set the compass at one of these points *A*, run a random line *AC* with an assumed declination and the bearing of the line a distance *AC* equal to the distance *AB*. Measure the distance *BC*, multiply it by 57.3 and divide the product by the length of the line *AB*. The result is the error in degrees in the assumed declination. If the declination is corrected by this error the old bearings will trace out the lines as formerly marked.

39. **Magnetic Bearing.**—In some of the older States the bearing of a line is defined as the angle it makes with the magnetic meridian. The result of this is in all of the States west of the agonic line where the declination has been decreasing for years that the northeast and southwest bearings will be increased over the old bearings by an amount equal to the change in declination, while the northwest and southeast bearings will be decreased by the change in declination since the old line was surveyed. In the country east of the agonic line the reverse of the above is true.

Problem 17.—If the bearing of a line with reference to the magnetic merdian in the States west of the agonic line were N. 72° 18′ E., declination 8° 45′ east at the time of the old survey, find its magnetic bearing at the time when its declination was 7° 24′ east. Find the declination if the magnetic bearing of the line were S. 36° 21′ E.

PROBLEM 18.—The magnetic bearing of a line in a State east of the agonic line was, when the original grant was surveyed in 1806, S. 68° E., with a declination of 25′ west. Find the magnetic bearing in 1896 when the declination had increased to 2° 05′. Find the magnetic bearing if the true bearing was S. 29° 42′ W.

PROBLEM 19.—Find the magnetic bearing in the following:

	True Bearing.	Declination.	Magnetic Bearing.
A............	N 26° 54′	7° 54′ E	....
B............	N 74° 12′ W	7° 54′ E	....
C............	N 33° 28′ W	7° 54′ E	....
D............	S 26° 36′ E	7° 54′ E	....
E............	N 2° 14′ E	8° 17′ E	....
F............	S 87° 14′ E	8° 17′ E	....
G............	N 5° 29′ W	8° 17′ E	....
H............	N 88° 22′ W	8° 17′ E	....

PROBLEM 20.—Find the true bearing for the following courses:

	Magnetic Bearing.	Declination.	True Bearing.
A............	N 3° 14′ W	2° 8′ E	....
B............	S 5° 18′ W	6° 12′ E	....
C............	N 8° 16′ W	3° 16′ W	....
D............	S 74° 26′ W	3° 18′ W	....
E............	N 17° 23′ W	3° 12′ E	....
F............	S 74° 26′ W	4° 02′ W	....
G............	N 17° 23′ W	5° 43′ E	....
H............	N 9° 25′ E	8° 56′ E	....

40. To Find the Declination for Any Special Farm.—To resurvey an old farm or tract of land obtain the field notes from the county clerk's office or from the deeds or grants. These papers should give the declination used in the original survey. This former declination (whether right or wrong) can not be used in a subsequent survey, and it is the surveyor's first duty to ascertain the proper declination to use in his own survey. If he can find one side of the tract marked by corners or trees, he can use these as a basis. If two corners at the end of a line can be found, all he has to do is to set off a declination on the declination arc that will cause the compass when set on the line with the true bearing, to coincide with the line as defined by the trees or corners. However, if the corners can not be seen from each other, the surveyor must select a declination that he thinks will be correct. With this declination he runs a random line with the old bearing the full length of the line, and marks the end of the random line. If the end of the random line does not agree with the corner, he measures the distance between the end of the random line and the true corner. This distance, multiplied by 57.3, and the product divided by the length of the line, will give the correction to be applied to the assumed declination.

41. Local Attraction.—It often happens that ore in the ground, a wire fence, or a railroad track, etc., will pull the needle out of the magnetic meridian. When this is discovered, the only thing to do is to retrace our steps to some point outside the limits of the attraction, set off the correct bearing and locate some point ahead. Then transfer to it, leaving a rear flagman; set up at the point located, and sight on the rear flagman, and then prolong the line by locating the head flagman, transfer and backsight, thus locating another point. This method will not apply when the whole line is within the field of attraction. We then have recourse to the transit and locate the line by internal angles. If the whole farm were within the field of attraction, it would all have to be surveyed with the transit by measuring internal angles.

42. Witnessing a Line or Corner.—All corners should have witness trees or some natural object to establish the corner, even though the stake disappears, thus: Begin at a stake from which a pecan tree 10 ins. in diameter marked *"K"* bears S. 32° E. 84 varas. To find the corner all we have to do is to find the witness tree, set the compass at it on the reverse bearing and chain off the distance. As a check it is well to have the witness line at the corner intersect at a large angle. The line is witnessed or marked by line trees. All trees that can be reached with the arm either way by a man standing on the line should be marked with three hacks on the side next to the line, but these hacks should not cut into the flesh of the tree. It is often the case that the line passes through a tree; such trees are marked on both sides with a hack, blaze, hack. These trees are called "fore and aft" trees. If trees are scattered some surveyors hack trees that are more than three feet from the line.

43. Typical Field Notes.—(From Deed Book 185, p. 235, Travis County, Texas.) Beginning at a stake, a corner to H. P. Sims and R. D. Rone, from which a hackberry marked *"X"* bears N. 61° 30′ W. 10 varas; thence N. 3° W. 216 varas to a stone; thence N. 16° W. 255.6 varas to a stone from which a live oak marked *"A"* bears N. 87° W. 31 varas; thence S. 28° 30′ W. 263.5 varas to a stone on side of hill; thence S. 5° 16′ W. 205.6 varas to a stone, from which a pecan 12 ins. in diameter marked *T* bears S. 63° W. 18 varas; thence S. 85° 15′ E. 227.3 varas to the beginning, containing — acres more or less.

44. Compass Adjustments.—There are in all six adjustments of the compass that should be made.

First. The axis of revolution should be perpendicular to the plane of the plate. This is done by the maker and if the adjustment becomes deranged, the instrument should be sent to the maker or some instrument house that has facilities for making such repairs or adjustments.

Second. The plane of the plate bubbles should be parallel to the plane of the plate. If the first adjustment has been made, level the compass and then turn it through 180°. If the bubble remains in the center of its run, no adjustment is necessary. However, if the bubble does not stay in the middle of its run after the compass has been turned 180°, correct half the apparent error by the screws at the end of the bubble tube. Repeat the operation till the bubble remains in the middle of its run when the compass is turned 180°.

Third. If the needle is bent, its ends will not always read the same, but if the pivot is in the center, the difference of the readings of the ends will be constant. To straighten the needle, set one end at zero and read the other end. This reading will indicate the way the needle must be bent. Repeated trials will be necessary before the needle can be made straight.

If the difference of the readings is not constant, it shows that the pivot is also bent. Read the ends of the needle in any position and then turn the needle by hand till the north end is in the position formerly occupied by the south end. Read the south end of the needle and note the difference of this reading and the first reading of the north end. The needle can then be bent till the north end when swinging free will bisect the space between the first reading of the north end and the second reading of the south end.

Fourth. If the pivot is bent out of its central position, the ends of the needle will not have the same readings, and the difference of the readings will be variable. After the needle is straightened, turn the compass till the difference of the end readings is the greatest. Remove the needle and bend the pivot towards the middle of the larger arc that was between the ends. Repeat till the difference of the end readings is zero.

Fifth. The plane of the sights can be made normal to the place of bubble tubes by leveling the compass and by sighting on some plumb line. If the slot-sight does not agree with the plumb line, the base of sight must be filed till a plumb line can be seen throughout the sights.

Sixth. The diameter through the zero graduations should be made to coincide with the line of sights. This is an adjustment that is always made by reputable makers and the surveyor is rarely called upon to test his compass for this. A very fine wire stretched through the sights and over the compass box will indicate clearly whether the line of sight agrees with the zero lines.

Bibliography.—"Davies' Surveying." By the late Charles Davies. This book has for several decades been one of the standards for the school and camp. and its full discussion of the usual problems of land surveying, together with the traverse and trigonometric tables, makes it a valuable assistant to the surveyor or a guide for the student. In addition to Davies', the works of the late J. B. Johnson, Wm. G. Raymond, Breed and Hosmer, etc., which are described at the end of the chapter on Transit Surveying, contain valuable data and suggestions for the compass surveyor.

CHAPTER III.

TRANSIT SURVEYING.

45. The Transit.—The essential parts of a transit (Fig. 24) are, mathematically, a line of sight and a graduated horizontal circle for reading horizontal angles. Mechanically, the essential parts are the telescope, the horizontal axis, the circular plates, the spindle, leveling head, tripod, and plumb-bob. The line of sight is determined and defined by the telescope mounted on the horizontal axis, the graduated circle by a horizontal circular plate upon which the degrees and fractions of degrees are marked. The telescope is rigidly attached at right angles to two horizontal arms whose axes are in the same straight line, and whose outer ends rest in the standards. These standards consist of two diverging legs rigidly attached to the horizontal plate. Two small levels at right angles to each other are attached to the horizontal plate, and by means of these the plates can be brought to an absolute horizontal. Two verniers (V' and V'), Fig. 25, are attached to the plate with their zeros 180° apart and are provided with a glass cover for protection. These verniers are turned so as to fit the outer graduated circle called the limb. By pulling out the small clip S the whole upper part including the limb B can be taken off the head. The upper part of the transit, including telescope, plate, horizontal axis, standards, and verniers, is called the alidade and is supported on a spindle and can be turned on a vertical axis normal to the vernier plate. However, the limb B and alidade can be clamped tight together by a clamp DF operated by the milled-head screw, which is seen in the faint outline on the right of Fig. 25. When clamped the alidade and limb B can be turned around the interior spindle H by unclamping the lower clamp screw (not shown in Fig. 25 but which can be seen in Fig. 24). The transit is provided with a level head as in the Y-level, which has four leveling screws for bringing the limb B into a horizontal plane. The tripod is generally made of light, tough, straight grained wood, the upper ends of the legs being connected by

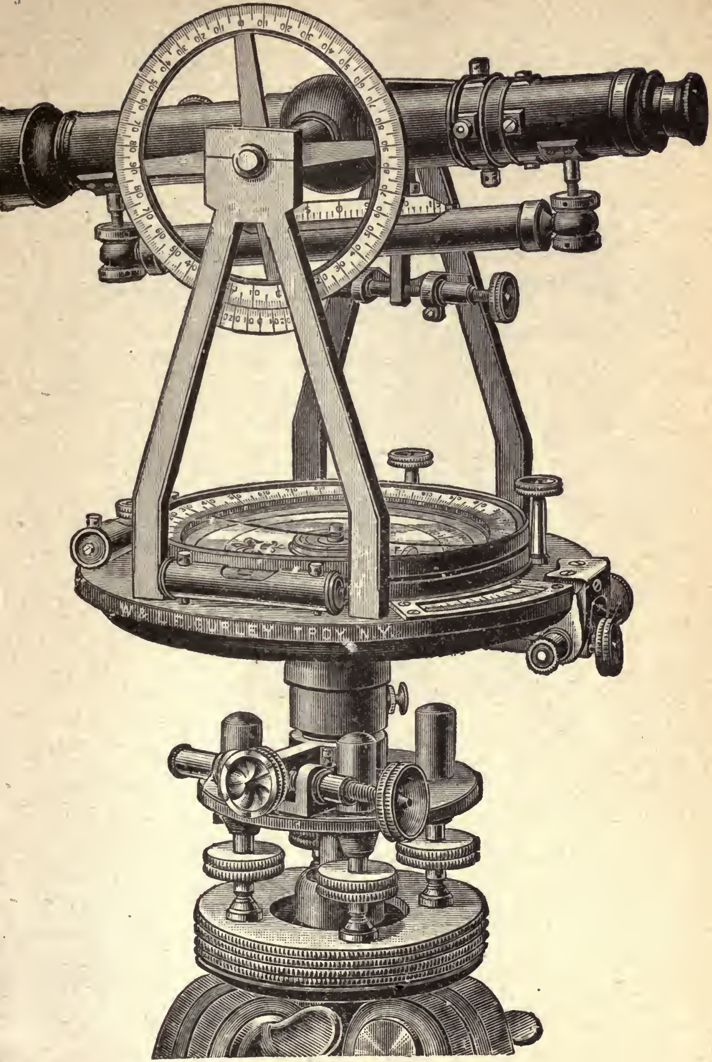


Fig. 21.

pin-joints to the leveling head, while the lower ends are shod with metal shoes. The plumb-bob is one of the mechanical essentials of the transit as the instrument cannot be set over a point below without it.

46. Compass Attachment.—Attached to and supported by the upper horizontal plate is a complete compass box, including graduated circle, needle, pivot, a declination arc inside the box and under the needle. The declination can be set off

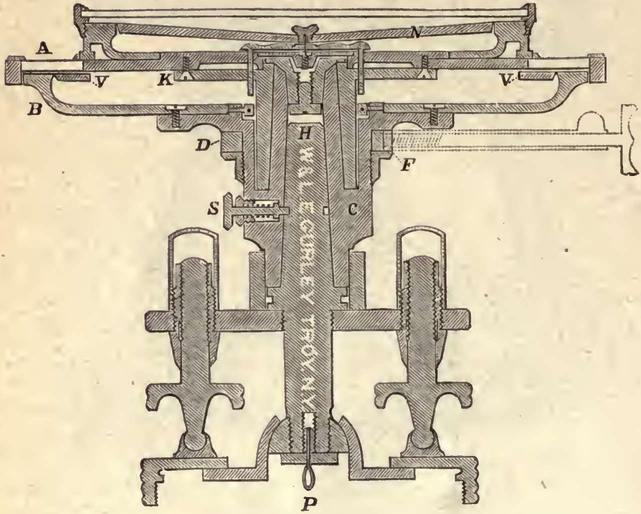


Fig. 25.

and the bearings read as in the compass, and the telescope simply helps to make the line of sight more exact. However, it has the disadvantage of having its line of sight confined to a single line, which a leaf, blade of grass, etc., can interrupt, while in the compass the line of sight is confined to a vertical plane passing through the slots and a slight interruption to the line of sight can be obviated by moving the eye.

47. Vertical Circle.—For the purpose of reading angles of elevation a vertical circle is now generally attached to the end of the horizontal axis and is provided with a tangent screw and a vernier reading to minutes. It is not an essential part of the transit. To bring the line of sight to a horizontal a bubble tube is attached to the telescope whose axis is made parallel to the line of sight of the telescope.

48. Shifting Center.—The modern transits are furnished with a shifting center. The lower part of the spindle to which the loop *P* is attached works in a ball and socket joint which is extended into circular, brim-like plate under the plate on which the leveling screws rest. If these are loosened so that the upper part of the transit can be moved, the point *P* can be moved a short distance in any direction. This is called the shifting center.

49. The Reticule.— The line of sight in the telescope is defined by two cross-wires at right angles to each other, cemented into depressions in a metal ring, Fig. 26. This ring is inside the telescope and is controlled and operated by four capstan screws which can be seen in the view of the telescope of the level or transit. The whole arrangement is called the reticule and it is susceptible to slight motions for the purpose of adjusting the line of sight of the telescope. The reticule is moved by loosening one capstan screw and by tightening the opposite one.

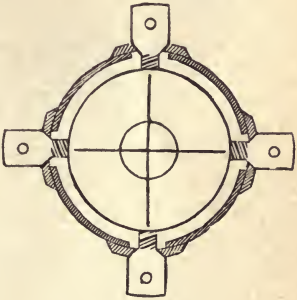


Fig. 26.

50. Setting Up the Transit.—Set up the tripod with the legs widely apart and firmly pressed into the ground; take the transit out of the box by taking hold of the limb and lifting the entire weight with one hand, simply using the other as a guide Never grasp the transit by the telescope to lift it out, as such lifting springs the horizontal axis and otherwise injures the bearings. Set the transit on the tripod, turn it till the threads catch, revolve the telescope vertically and take hold of two legs of the tripod and straighten it up until all the legs are together, and then place the tripod across the shoulder and carry it to the place where the observations are to be made. When it is desired to set the tripod over a point, place the legs wide apart, and move them so that the plumb-bob will be practically over the point. Level up the instrument, and if the plumb-bob is not over the point loosen the leveling screws until the center can be shifted, then move the center until plumb-bob comes over the point below and relevel. If there is not sufficient play in the shifting center to move the plumb-bob over the point the tripod will have to be moved in the direction necessary; then proceed as before.

51. Motions.—If the lower clamp screw be clamped, and the upper loosened, the alidade can be turned on the vertical axis, and it will be noticed that the vernier plate moves with the alidade, and that the limb or graduated circle is stationary. This movement is called the upper motion. If the upper clamp screw be tightened and the lower one loosened, that part of the instrument above the leveling head can be turned around one of the spindles. This movement is called the lower motion.

52. Use of the Transit.—After the transit has been set up over a point, make the zero of the vernier agree with the zero of the limb. Unclamp the upper motion and bring the two zeros as near together as possible; then clamp the upper motion and bring the zeros into exact coincidence by means of the tangent screw controlling the upper clamp. After the zeros have been brought together, loosen the lower motion clamp, take hold of the limb with both hands and turn the telescope till it points towards the object on which we wish to observe. The telescope can be brought approximately into the required direction by sighting over the telescope at the object and turning the instrument until the telescope points towards the point. The cross-wires are brought into the field of view by turning the screw that operates the eye-piece. The large milled-head screw on side of the telescope is then turned till the observed object is seen distinctly and clearly through the telescope. The tangent screws can then be turned till the vertical wire bisects the object.

53. The Transit as a Compass.—If it is desired to use the transit as a compass in regular surveying work, or to use the needle as a check on other work, the milled-head screw shown on the outside of the left leg of the front standard in Fig. 24 is loosened, and the milled-head screw that controls the declination arc, seen between the rear standards, is turned until the proper declination is set off by the vernier inside the compass box. These screws are then clamped and the transit will then read angles with the true meridian. The needle is turned loose by means of the milled-head screw shown above the plate on the right of Fig. 24.

54. Transit Surveying.—If the transit is not used as a compass, we must read the azimuth of each course or line instead of the bearing. As this azimuth is read from the south

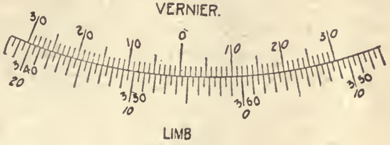


Fig. 27.

point around by the west, north and east, and on to the south again, we can have with a transit reading to minutes, an azimuth of 359° 59′, which could be a bearing of S. 0° 01′ E. These azimuths are read with reference to the true meridian and it is necessary to locate this very accurately if the absolute azimuth is desired. However, if it is only an accurate expression for the area of the farm, a meridian can be assumed for the first course, and then carried around the farm by locating this meridian from each course.

55. Transit Vernier.—The transit vernier is a double vernier (Fig. 27) and has 30 divisions on each side of its zero. Each half of the vernier covers 29 parts or divisions on the limb. The smallest division on the limb is a half-degree, or thirty minutes, and hence the vernier can read to one-thirtieth of a half degree or to one minute. The angle may be measured from the right or left, and the one we use depends upon the special problem under consideration. If read from the right we see that the zero of the vernier is between 5° 30′ and 6° 00′. The reading is 5° 30′ plus the vernier reading. As the reading of the transit is from the right, use the left half of the vernier. On examination, we find that the 14th division of the vernier agrees with a division mark on the limb. The vernier reading is therefore 14′. The whole angle reading is therefore 5° 44′.

If the angle is to be read from the left, use the vernier on the right. The zero of the vernier lies between 354° and 354° 30′. The 16th division of the vernier on the right agrees with a division on the limb and the vernier reading is therefore 16′, and the whole angle reading is 354° 16′.

56. Example.—If a farm is surveyed with the transit, the field notes would be as follows:

Course.	Azimuth.	Distance.
AB..................	203° 30′	255.72 varas
BC..................	248° 00′	182.10 varas
CD..................	3° 47′	329.42 varas
DA..................	84° 15′	249.92 varas

It will be observed that the shape and dimensions of the farm would not have been changed in the slightest if the first course *AB* had been taken at 202° instead of 203° 30′. It simply would have amounted to a turning of all meridians in a clockwise direction and the azimuths would have been as follows: 202°, 246° 30′, 5° 17′, 82° 45′. Then, if it is desired to obtain the area accurately, we can assume a meridian, and it is not necessary that this be the true meridian, but when this meridian is once assumed, the azimuth of all the courses must be with reference to it.

57. Reference Lines.—The line to which the azimuth is referred can be assumed in any desired direction, and one of the sides is often taken as this reference line if only the area is required. Thus, in the example, if *AB* is assumed as the reference line, the azimuths with respect to this line are 180°, 224° 30′, 340° 17′, 60° 45′. In calculating the area the bearing can be taken with respect to the reference line. If AB were our reference line, the field notes would be as follows:

Course.	Bearing.	Distance.
AB..................	North	255.72 varas
BC..................	N 44° 30′ E	182.10 varas
CD..................	S 19° 43′ E	329.42 varas
DA..................	N 60° 45′ W	249.92 varas

PROBLEM 21.—In a farm $ABCDE$, $AB = 19.90$ chains, $BC = 9.03$ chains, $CD = 9.77$ chains, $DE = 5.67$ chains, $EA = 13.24$ chains; $A = 89° 12'$, $B = 73° 37'$, $C = 139° 08'$, $D = 163° 40'$, $E = 74° 24'$.

If the azimuth of $AB = 180°$, find the azimuth and the bearings of the other lines.

58. Repeating Method.—It is often desired to find the angle more accurately than it can be read by a single reading of the verniers. If we have a transit reading by vernier to one minute, we can find any angle ABC to any desired fineness by the repeating method. Thus, if the one transit verniers read to one minute, we can find the angle to ten seconds by repeating the observation six times. The process is as follows:

Telescope normal:

1. Set transit on point B, level up and set cross-wires on point A and read both verniers.
2. Unclamp upper motion and deflect to C, clamp upper motion and read both verniers.
3. Unclamp lower motion, deflect to A and clamp.
4. Unclamp upper motion and deflect to C and clamp.
5. Unclamp lower motion, deflect to A, and set thereon.
6. Unclamp upper motion, deflect to C, set cross-wires thereon and read the angle as given by both verniers. This result is three times the angle ABC, etc., etc., etc.

The process can be carried on till there have been five, ten, or twenty deflections by upper motion from A to C, thus measuring the angle five, ten, or twenty times. Both verniers should be read in every case and the average taken. Usually the angle is read a given number of times, as above, with the telescope normal beginning right (or left) station A, and then read the same number of times with the telescope reversed, beginning on the left (or right) station *C*.

Example.	Vernier *A*.	Vernier *B*.
1......................	31° 42′	31° 42′
3......................	95° 07′	95° 07′
5......................	158° 31′	158° 32′

The average of the five readings gives 158° 31′ 30″, or an angle of 31° 42′ 18″.

59. To Adjust the Plate Levels.—The axis of the plate levels should be at right angles to the vertical axis or the axis of revolution. Set the transit up on the tripod, level it by the plate levels as near as possible, bring one of the level tubes parallel to a pair of leveling screws, and bring the center of the bubble exactly to the center of its run. Then turn the alidade 180° on its vertical axis, and if the bubble remains in the center of its tube, it is in adjustment. If not, lower the high end of the tube or raise the low end by means of the small capstan screws at the end of the tube a sufficient amount to correct half of the displacement of the bubble. Correct the remainder by means of the leveling screws and repeat as a check on your work. Usually it takes several trials to make this adjustment.

60. Line of Sight Adjustment.—To make the line of sight perpendicular to the horizontal axis, set up the instrument on some plane nearly level, bring the plate bubbles to the center of their run, and locate a point about 100 to 200 ft. from the instrument; turn the instrument on its horizontal axis and locate another point the same distance from the instrument, but in an opposite direction; revolve the alidade and bring the vertical wire in coincidence with the point first located; then turn the telescope on its horizontal axis and locate another point near the second point located in the intersection of the cross-wires. If this point last located coincides with the second point located, the line of sights is perpendicular to the horizontal axis. If it is not, correct one-fourth of the displacement and mark this point, and proceed as before.

Let *AB*, Fig. 28, be the position of the horizontal axis when the point 1 is located, and let the line of sights make an angle of $10x = a$ with the perpendicular xy to the axis. Revolve the telescope on the horizontal axis and locate the point 2. Now the angle $102 = 180° - 2a$. Turn the alidade around the vertical axis till the line of sights intersect 1. As 02 has been turned through the angle 102 or $180° - 2a$, AB has been turned through the same angle and occupies the position $A'B'$, where $AO'A = 180° - 2a$, or $A'OB = 2a$. The perpendicular has moved to the position of $x'y'$ where $xOx' = 180° - 2a$ or $x'Oy = 2a$. Let

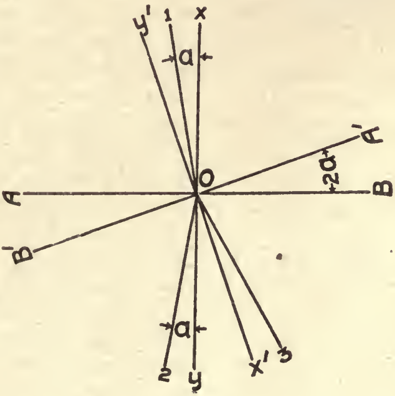


Fig. 28.

telescope point to 1, then revolve it on its horizontal axis and locate a point 3 in the line of sights near 2. The angle $103 = 180° - 2a$. Therefore, the angle $203 = 4a$; hence, all we have to do is to bring the line of sights into coincidence with $x'y'$. Divide the angle 203 into four parts and make $30x' = a$, one-fourth of the angle 302. We can do this by setting a point x' at one-fourth of the distance from 3 to 2, and as 02 and 03 are several hundred times 2-3, this is as accurately as we can measure the angle $30x'$ equal to one-fourth of 203. Now, keep the axis clamped in the position $A'B'$, and move the vertical wire by capstan screws till it coincides with the point x'. Repeat whole work until it checks.

61. Peg Adjustment.—The axis of the bubble tube may be made parallel to the line of sights by the peg adjustment. Drive two pegs or stakes in the ground about 200 ft. apart, whose difference of level is less than 4 ft. Set the transit near peg *A*, level the instrument, and turn the telescope so that the eye end is over the peg, while the bubble is in the center of its run; measure the height of the center of the eye-piece above the peg and call this distance *h*. Have an assistant hold the rod on top of peg *B* and measure from where the line of sights cuts the rod to the top of peg *B* and call this *r*. Transfer the transit to peg *B* and set up as before, measuring the height of the center of the eye-piece from the top of peg *B* and call this distance *h'*. Have the rod placed on top of peg *A* and measure the distance from the line of sights to the top of peg *A* and call this distance *r'*.

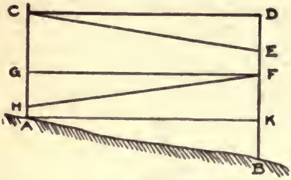


Fig. 29.

In Fig. 29, *AK*, *CD* and *FG* are the horizontal lines as determined by the bubble tube. Suppose the line of sights *CE* cuts below the horizontal line an amount of $DE = e$; when the transit is transferred to *B* it will again cut below when the telescope is sighted to *A* an amount $HG = e$.

Let $AC = h$, $BE = r$, $BF = h'$, $AH = r'$. Then the true difference of level of *A* and $B = BK = BD - AC = r + e - h$.

Also $BK = BF - AG = h' - (r' + e)$

Therefore $r + e - h = h' - r' - e$

Therefore $e = \frac{1}{2}[(h + h') - (r + r')]$.

Rule: The double error is equal to the sum of the instrument heights minus the sum of the rod heights.

62. Location of Meridian by Polaris.—Table I gives the times when Polaris and the mean sun are on the meridian together. For 1907 the "epoch" is 14.1 This means that the mean sun and Polaris are on the meridian together April 14, one-tenth of a day after the beginning of April 14—that is, 2.4 hours after the beginning of April 14. This would make the "epoch" occur on April 14 at 2:24 A. M. For 1909 the "epoch" is 13.8, or April 13, 7:12 P. M. The "epoch," then, is the time or date when Polaris and the mean sun are on a meridian at the same time.

Table I—Epochs equal date in April when Mean Sun and Polaris are on a Meridian together:

Year.	Epoch.	Year.	Epoch.	Year.	Epoch.	Year.	Epoch.	Year.	Epoch.
1907	14.1	1912	13.9	1917	14.6	1922	15.3	1927	15.9
1908	13.5	1913	14.2	1918	15.0	1923	15.6	1928	15.3
1909	13.8	1914	14.6	1919	15.3	1924	15.0	1929	15.6
1910	14.2	1915	14.9	1920	14.7	1925	15.3	1930	15.9
1911	14.5	1916	14.3	1921	15.0	1926	15.6	1931	16.2

If Polaris and the mean sun are on a meridian together, the mean sun will reach the meridian 4 minutes later than Polaris on next day.

The hour angle of the star will be more than that of the sun by 3.94 multiplied by the number of days after the epoch.

EXAMPLE: Find the position of the star (t) in its orbit at 9 P. M. May 6, 1907. The "epoch" for 1907 is on April 14 at 2:24 A. M. The number of days from 2:24 A. M., April 14, to 9 P. M., May 6, is 22.775. Hence Polaris will be $22.775 \times 3.94 = 89.73$ min. ahead of the sun. At 9 P. M. the sun is 9 hrs. past the meridian of the observer, hence Polaris will be 9 hrs. plus 89.73 min. or 10 hrs. 29.73 min. past the meridian. By using this time (t) in Table II we can find the angle Polaris makes at that time with the true meridian.

Table II.—$t =$ local mean time + 3.94 (date—epoch).

Hours. t	Angle, a.	Lat. cor., b.	t
0	0′	—74′	24
1	25′	—72′	23
2	49′	—64′	22
3	69′	—52′	21
4	84′	—37′	20
5	93′	—19′	19
6	96′	0	18
7	92′	+19′	17
8	82′	+36′	16
9	67′	+51′	15
10	47′	+63′	14
11	24′	+70′	13
12	0′	+72′	12

Table III.—Azimuth Coefficients.

	Coefficients $=K$.			
Lat.	1900.	1910.	1920.	1930.
20°	.82	.78	.75	.72
30°	.88	.85	.81	.77
40°	1.00	.96	.92	.87
50°	1.10	1.14	1.09	1.04

Table IV.—Lat. correction Coefficient.

Year.	Coefficient, Q.
1900	1.00
1910	.96
1920	.92
1930	.87

EXAMPLE: Find the angle that Polaris makes with the true meridian at 9 P. M. May 6, 1907, in latitude 30. The time interval from epoch to date was 22.775 days and the increase in time was 1 hr. and 29.77 mins. The value of t was found to be 10 hrs. and 29.73 mins. or 10.50 hrs. (which is near enough for our purposes). Looking in Table II under t for 10.5 hours we find that we have to interpolate between 47′ and 24′, hence the angle is 35.5′. This must be multiplied by the azimuth coefficients. For 1900, lat. 30, the coefficient, Table III, is 0.88, and for 1900 it is 0.85. For one year the decrease is 0.003, and for seven years it is 0.021. The coefficient is therefore 0.86. The angle or azimuth with the north meridian $=35.5 \times .86 = 30.6'$ west. The observed altitude of the star was 29° 8′. The latitude coefficient for 1907 lies between 1.00 and 0.96 and an interpolation gives .972. From Table II, lat. cor. (b) is 66.5′. Hence the correction for the altitude will $= .972 \times 66.5 = 64.64'$. The latitude $= 29°\ 8' + 64.64' = 30°\ 12.64'$.

PROBLEM 22.—Find the angle that Polaris makes with the true meridian 9 P. M. June 12, 1907, in latitude of 33°. Answer $= 20'$ east.

PROBLEM 23.—Given latitude of place $= 36°$, find the angle Polaris makes with meridian on November 6, 1909, 10 P. M. Answer $= 9'.2$ east.

PROBLEM 24.—An observation was made on Polaris at 9:30 P. M. July 22, 1908, in latitude 30°. Find the angle made with the meridian. Answer $= 71'$ east.

63. Circumpolar Stars.—A meridian can be located with sufficient accuracy for ordinary surveying by observations on the North Star (known as Polaris), which is about one and one-fifth degrees from the true North Pole, and if we would observe it for a whole day it would appear to describe a circle about the North Pole in a direction contrary to the motion of the hands of a clock, *i. e.*, contra-clockwise. On account of the invisibility of the true North Pole this motion can be best observed by selecting some star in the Dipper. If we could note exactly when one of the stars of the Dipper is directly above Polaris and could follow its motion throughout the balance of the night, the next day and part of the next night, we would observe that the star would again reach a point directly above Polaris four minutes earlier than it did on the preceding day. If we observed it directly above Polaris at 10 P. M. on one night, the next night it would be at the same position at 56 mins. after 9 o'clock. Thus, each of these stars gains four minutes each day (exactly, 3.945 minutes). In one year it would gain 24 hours and would, therefore, make one more revolution than the earth makes on its axis. All stars that make an apparent complete revolution about the North Pole are called circumpolar stars, and any of them could be used for the location of a meridian when the selected star is directly above or below Polaris.

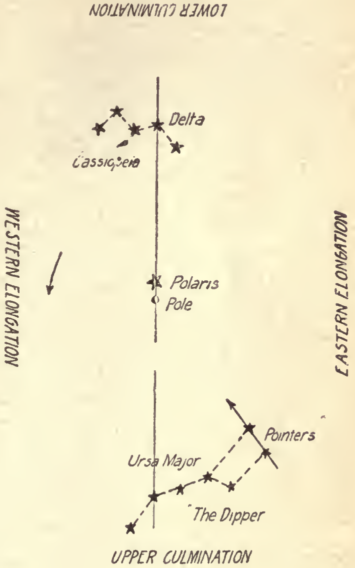


Fig. 30.

There are two groups of stars (called constellations) situated opposite to each other with respect to Polaris and the North Pole that afford favorable opportunity for the location of the meridian by surveyors. These constellations are those of the Great Bear (the Dipper) and of Cassiopeia (the Chair). By a glance at the outlines of these constellations, Fig. 30, it will be seen that the dotted lines outline the shape of a dipper and chair, respectively, hence the names. It must be remembered that Polaris is always opposite the Dipper with respect to the pole, and that it is on the same side as the Chair.

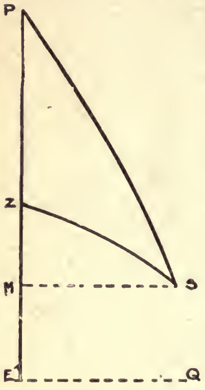


Fig. 31.

When a star is directly above the pole it is said to be at its *upper culmination,* and when directly below, at its *lower culmination.* When at the eastern point of its orbit, it is said to be at its *eastern elongation,* and when at its western point, at its *western elongation.*

64. **Location of Meridian.**—A line passing through the second star (Zeta) in the handle of the Dipper and the third in the back of the Chair (Delta Cassiopeia) passes through Polaris and the North Pole. When Zeta of the Dipper or Delta Cassiopeia is directly above or below Polaris, Polaris is on the meridian, and is at its *upper* or *lower culmination.* If the Dipper is above, Polaris is below the pole, and *vice versa.* But when the star is at either culmination, its horizontal motion is more rapid than at any other point in its path, and a slight error in time affects the result. When the star is at either elongation, the direction of its motion is vertical, and a slight error in time does not have such decided influence on the azimuth.

65. **PZS Triangle.**—The North Pole (P), the Zenith (Z) and the Sun (S) form a spherical triangle *PZS*, Fig. 31, where if

l = latitude of observer
t = hour angle
a = azimuth of sun
d = declination of sun
h = altitude of sun.

We have:

PZ = co-latitude $= 90 - l$;
PS = co-declination $= 90 - d$;
ZS = co-altitude $= 90 - h$;
ZPS = hour angle of sun $= t$;
SZM = azimuth $360 - a$.

66. Formulas.—The usual problem is to locate a meridian at a certain place whose latitude and longitude are known. Drop a perpendicular from S on the meridian PZ of the observer, cutting it at M, and let $ME = N$ where EQ is the celestial equator, or earth's equator extended to the heavens.

$$\text{Now, } ZE = \text{latitude} = l.$$

$$\therefore ZM = l - N.$$

By the application of Napier's Tangent Law, we have, in the right triangle PSM:

$$\cos t = \tan PM \cot PS$$

$$\cos t = \tan (90 - N) \cot (90 - d)$$

$$= \cot N \tan d$$

$$\therefore \tan N = \frac{\tan d}{\cos t} \qquad (5)$$

In the right triangle, SZM

$$\sin ZM = \tan MS \cot a \text{ or } \tan MS = \sin ZM \tan a.$$

In the right triangle MPS,

$$\sin PM = \tan MS \cot t \text{ or } \tan MS = \sin PM \tan t$$

Equating the two values of $\tan MS$, we get:

$$\tan a \sin ZM = \sin PM \tan t$$

$$\tan a = \frac{\sin PM \tan t}{\sin ZM}$$

But

$$PM = 90 - N, \quad ZM = l - N$$

$$\therefore \tan a = \frac{\cos N \tan t}{\sin (l - N)} \qquad (6)$$

67. Observation on Sun.—The best time of day to make an observation for azimuth on the sun is from 8 to 10 A. M. and from 3 to 5 P. M. Before an observation is made it is necessary to have mean local time and if a chronometer is not available, two watches should be set to agree with Western Union time. Thirty minutes before the observation is to be made the transit should be set over the station, the verniers should be brought to zero, and the transit be pointed to some definite terrestrial point, as a church spire. The transit should then be turned by upper motion to point approximately at the sun, and as soon as the sun comes into the field of view of the telescope, the observer should clamp the upper motion and call "angle," when two men read the angles as given by the two opposite verniers. At the signal "angle" the timekeepers, of which there should be at least two, get ready to observe the time. As the disc of the sun approaches the vertical wire, the observer calls, "Get ready," and just as the edge of the sun's disc coincides with the vertical wire he calls "time" and immediately moves the vertical wire by aid of the tangent screw till the opposite edge of the sun's disc coincides with vertical wire, when he calls "time" again. The time interval between the two calls of "time" should not be over six seconds. The timekeepers have noted both the hours, minutes and seconds at each call of "time," and the angle readers read both angles and record same. The data taken in the field therefore consist of reading the spire-station-sun angle for both discs of sun, and the times corresponding to these. The average of each is taken as the angle and time of the sun's center The local mean time is reduced to apparent time, and this to degrees, which gives the hour angle.

The declination of the sun is found for the given time and N is found from Formula 5, and the substitution of values of N, t and l in Formula 6 will give the angle a.

The second method of finding the angle a consists in measuring the altitude of the sun at the time of observation. To do this, the disc of the sun is brought to tangency with the vertical wire and on its left, so that the lower edge of disc coincides with the horizontal wire. If we regard the cross-wires as axes, the sun would be in the second quadrant and tangent to both axes at the first observation. In this position we record time, the spire-station-sun angle, and the vertical angle. The disc is then brought into the fourth quadrant, so that it touches the two axes, when the same data are observed as before. The average of these is taken as the spire-station-sun angle, the angle of elevation, and the time of observation. Then the angle is corrected for refraction and this gives us the complement of *ZS* of the triangle *PZS*. The three sides of the triangle *PZS* are thus known whence the angle *PZS* can be calculated.

Let $s = \frac{1}{2}(PZ + ZS + PS)$.

$$\text{Then } \sin \tfrac{1}{2} PZS = \sqrt{\frac{\sin (s-PZ) \sin (s-ZS)}{\sin PZ \sin ZS}}$$

68. Refraction.—The effect of refraction is to raise all bodies and make them appear higher than their true positions. Thus the sun can be seen wholly above the horizon, when in reality no part of it is above. If *R* represents the amount of refraction in seconds of arc and *h* is the altitude of the sun, we have:

$$R = 58'' \tan h$$

Table V.—Table of Refractions:

Elevation.	Refraction.	Elevation.	Refraction.	Elevation.	Refraction.
5°	9′ 52″	16	3′ 20″	35	1′ 23″
10°	5′ 19″	17	3′ 08″	40	1′ 09″
11°	4′ 51″	18	2′ 58″	45	0′ 58″
12°	4′ 28″	19	2′ 48″	50	0′ 49″
13°	4′ 07″	20	2′ 39″	60	0′ 34″
14°	3′ 50″	25	2′ 04″	70	0′ 21″
15°	3′ 34″	30	1′ 41″	80	0′ 10″

69. Solar Attachment.—There are various forms of solar attachments, but we shall here describe only two. Fig. 32 shows a diagonal prism, which "consists of a prism attached to the cap of the eye-piece, by which the object is presented to the eye when placed at right angles to the telescope. When the telescope is directed to the sun the slide or darkener containing the colored glass is moved over the opening. The circular plate with which the prism is connected is made to turn in the cap, so that when it is substituted for the ordinary cap of the eye-piece the opening of the prism can be easily adjusted to the position of the eye. Observations can be taken with the prism up to an angle of 60° of elevation."

The other form of solar attachment consists of a second telescope, generally smaller in size, attached to the regular telescope of the transit. The second telescope is provided with colored glass to enable the observer to see the sun with distinctness and definition. Fig. 33 illustrates a common form of this solar attachment which is provided with telescope level and tangent screws for horizontal and vertical motions. The line of sight of the solar telescope can be made parallel to that of the transit by bringing both bubble tubes to the middle of their run, while the telescopes are pointed at a vertical line some 200 ft. away. This line should be marked on a white sheet of paper tacked to the side of a house on the same level practically with the telescopes. Draw two heavy horizontal lines on this sheet of paper at a distance apart equal to the distance between the axes of the telescopes. Bring the cross wires of the transit telescope on the lower of these lines, and if the lines of sights are parallel the line of sight of the solar telescope will intersect the upper horizontal line. If it does not, adjust its reticule till the line of sight as defined by the cross-wires intersect the upper line. Check till perfect agreement is secured. An error of 1-16 in. in the distance between the axes in 200 ft. would produce an error in the parallel alignment of the lines of sight of only 5″. A longer base would reduce the error. If the base is 507 ft. and the error in distance between axes is 1-30 in., the lines of sight will make an angle of 1″.

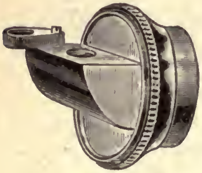


Fig. 32.

To eliminate light errors in latitude and as a check on the work, observations can be taken in the forenoon and afternoon at about the same time from the meridian passage of the sun. In each set of observations the transit is set on a terrestrial mark, the altitude of the sun, the angle mark-station-sun, and the times are taken and recorded. The angle

PZS is calculated and the azimuth of the line from station to mark can be found by addition or subtraction.

70. Meridian Without Calculation.—If the meridian is to be located directly by observation, some solar attachment like that of Fig. 33 is necessary. To locate a meridian by this method we proceed as follows:

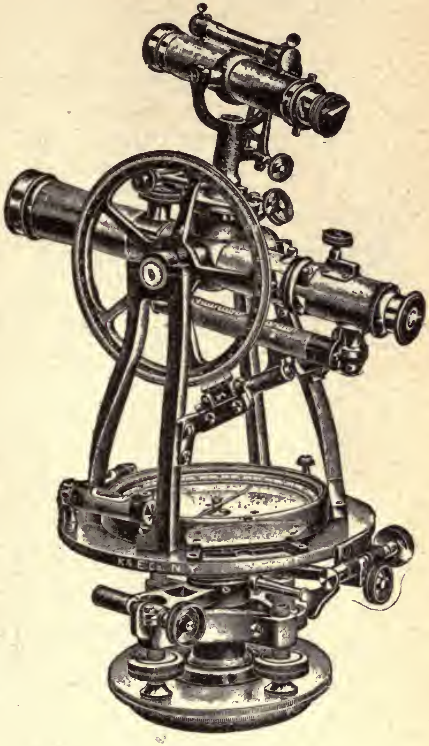


Fig. 33.

1. Make the usual five adjustments for the transit, three for the ordinary transit and two for the solar attachment.

2. Bring the line of sight of solar telescope into the vertical plane of the line of sight of the transit telescope.

3. If declination of sun is south (north) depress (elevate) the transit telescope an amount equal to the declination corrected for refraction, then bring the solar telescope to a horizontal position by means of its bubble tube. The lines of sights of the telescope will now include an angle equal to the corrected declination.

4. Elevate the transit telescope till the vertical arc reads the co-latitude of the place.

5. Revolve both telescopes on their vertical axes till the image of the sun is bisected by the vertical wire of the solar telescope. When this bisection is secured the line of sight of the transit telescope will be in the plane of the meridian and will locate it.

71. Example:—On April 15th, 1907, the following observations were made on the sun at the magnetic station, Austin, Texas (latitude 30° 17′, longitude 97° 44′ 02″):

Disc of Sun.	W. U. Time.	Mark, Station, Sun—Angle.
Right..........	9h. 59m. 57s.	75° 8′
Left............	10h. 0m. 03s.	75° 40′
Average.........	10h. 0m. 0s.	75° 24′

W. U. Time (90 meridian) = 10h 0m 0s

Correction = 30 56

Local Mean Time = 9h 29m 04s

Time from Greenwich mean noon to Austin.

Mean noon = 6h 30m 56s.

Time interval from Greenwich noon to obs. = 4h.

Declination at Greenwich mean noon = 9° 27′ 2″ .90 N.

Hourly increase = 53″ .96.

Total increase = 3′ 35″ .84.

Declination at time of observation = 9° 30′ 38″ .74.

Equation of time at Greenwich, mean noon = 0m 17.15s.

Hourly decrease = 0.626s.

Total decrease = 2.504s.

E. T. at time of obs. = 0m 14 .65s.

Apparent time of obs. = 9h 28m 49.35s = 9.480375h.

t = hour angle SPZ = 2.519625h = 37° 47′ 40″.

$$Tan\ N = \frac{tan\ d}{cos\ t}$$

$Log\ tan\ d = 9.224108$

$log\ cos\ t = 9.897745$

$log\ tan\ N = 9.326363$

$N = 11° 58′ 12″$

$l - N = 18° 18′ 48″$

$$tan\ a = \frac{cos\ N\ tan\ t}{sin\ (l-N)}$$

$log\ cos\ N = 9.990453$
$log\ tan\ t = 9.889594$
$co\text{-}log\ sin\ (l-N) = .502775.$

$log\ tan\ a = 10.382822$
$a = 67°\ 30'\ 8''$
Azimuth of sun $= 292°\ 29'\ 52''$
Azimuth of mark $= 75°\ 24' - 67°\ 30'\ 8''$
$= 7°\ 53'\ 52''$

72. Example:—The following data were taken at a station where latitude $= 29°\ 8'\ .1$ and longitude $= 97°\ 23'$ W.

No.	Sun.	Alt. of Sun.	Angle— Mrk, Sta., Sun.	W. U. Time = 90 M.
1	$\overline{0}\vert$	31° 53′ 48″	5° 56′ 24″	8h. 53.0m. A. M.
2	$\overline{0}\vert$	32° 22′ 00″	5° 33′ 12″	8h. 55.5m. "
3	$\vert\underline{0}$	32°. 26′ 12″	4° 05′ 24″	8h. 58.0m. "
4	$\vert\underline{0}$	32° 55′ 30″	3° 39′ 12″	9h. 00.5m. "
Mean		32° 24′ 22.8″	4° 48′ 33″	8h. 56.75m. "

Declination of sun at Greenwich, mean noon = 2m. 19.3s.
Hourly increase = 58″.4.
Time interval from G. noon to Observation = 2h 56.8m.
Total increase in Declination = 2′.9.
Declination at time of Observation = 2° 17′.2s.
Observed altitude of Sun = 32° 24′.38.
Correction for refraction and parallax = — 1′.3.
True altitude of Sun = 32° 23′.1.
In the *PZS* triangle we have,

$PZ = 60°\ 51'\ 54'' =$ co-lat.
$PS = 92°\ 17'\ 12'' =$ co-dec.
$ZS = 57°\ 36'\ 54'' =$ co-alt.
$\therefore\ 2s = 210°\ 46'\ 00''.$
$s = 105°\ 23'$
$s - codec. = 13°\ 5'\ 48''$

$$(\text{Cos } \tfrac{1}{2} PZS)^2 = \frac{\sin s \quad \sin (s\text{—}codec)}{\sin co\text{-}alt \quad \sin co\text{-}lat}$$

Log sin s = 9.984155
Log sin (s-codec) = 9.355249
cologsin co-lat = 0.073417
colog sin co-lat = 0.058749
2 *log cos* $\frac{1}{2}$ *PZS* = 19.471570
log cos $\frac{1}{2}$ *PZS* = 9.735785.
∴ $\frac{1}{2}$ *PZS* = 57° 1′ 40″
PZS = 114° 3′ 20″

Azimuth of sun at time of obs.	= 294° 3′ 20″
Angle Mk-Sta-Sun	= 4° 48′ 33″
Azimuth of Mark	= 298° 51′ 53″

Bibliography.—"Theory and Practice of Surveying." By J. B. Johnson. This is one of the best, most practical, and comprehensive books upon higher surveying. It includes a discussion of the engineering instruments in their use in ordinary and higher surveying, leveling, topographic, hydrographic, railroad, and earthwork surveying.

"The Principles and Practice of Surveying." By Breed and Hosmer. 526 pages. This is a rather full treatment on the use, care, and adjustments of instruments, land surveying, traverse lines, meridians and latitude, city surveying, mine surveying, plotting, specimen note books and computations.

"Plane Surveying." By Wm. G. Raymond. 485 pages. This is a full discussion of the construction and use of the engineering field instruments, methods of land, city, hydrographic, etc., surveying, and an ample treatment of the slide rule (an unusual feature of a work on surveying), and an excellent set of tables.

"Surveying Manual." By W. D. Pence and Milo S. Ketchum. 252 pages. This is one of the most valuable hand-books or field manuals now in print. While it is modest in size, it covers in a satisfactory way the usual problems confronting the surveyor and engineer. A distinguishing feature is the sample pages of note books executed in freehand lettering.

CHAPTER IV.

CALCULATION OF AREAS.

73. Latitude and Departure of a Course.—Given a course *AB*, Fig. 34, and a meridian through one end of the course, and a perpendicular *B*2 from the other end upon the meridian. Then *A*-2 is called the latitude of the course, and 2-*B* the departure. The latitude of *BC* is *B*-6 or 3-2. All the latitudes that go north are called plus and all those that go south are called minus. Thus in the figure the latitudes of *AB* and *DA* are plus, while those of *BC* and *CD* are minus. The sum of the plus latitudes

$$A2 + A4 = 2\text{-}4.$$

The sum of the minus latitudes

$$B6 + 5D = 2\text{-}4.$$

The algebraic sum of all the latitudes is equal to zero.

All east departures are plus and all west departures are minus. Thus the departure of *AB* and *BC* are plus, while the departures of *CD* and *DA* are minus. The sum of the plus departures $2B + 6C = 3C$, while the sum of the minus or west departures $5C + D4 = C3$.

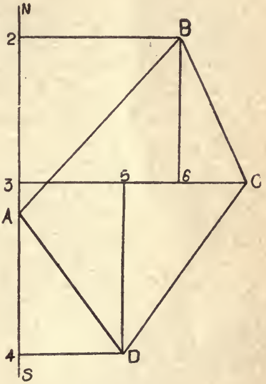


Fig. 34.

The algebraic sum of all the departures is equal to zero.

In the triangle *A*2*B*, let the length $AB = l$, and the angle $BA2 = B$ (called the "bearing").

But $A2 = AB$ cosine $BA2$, that is,

Latitude = length × cosine of bearing..

$$\therefore \; L = l \text{ cosine } B \qquad (7)$$

Also, $B2 = AB \times$ *sine of*.$BA2$, that is,

Departure = length × sine of bearing.

$$\therefore \; D = l \text{ sine } B \qquad (8)$$

Squaring 7 and 8, and adding, we get,

$L^2 + D^2 = l^2 (Cos^2 B + Sin^2 B)$.

But $Sin^2 B + Cos^2 B = 1$,

$\therefore\ L^2 + D^2 = l^2 \quad \therefore\ l = \sqrt{L^2 + D^2}$ .... (9)

Dividing 8 by 7, we get,

$$\text{Tangent } B = \frac{\text{Departure}}{\text{Latitude}} \quad \text{.........................(10)}$$

Example:—The field notes of a farm are given in the following table:

Course.	Bearing.	Distance.
AB	N27°37′E	48.6 chains
BC	S67°14′E	65.4 chains
CD	S38°28′W	52.6 chains
DA	N65°15′W	55.0 chains

To find the latitudes and the departures it is convenient to proceed by finding the natural sines and cosines of all the bearings, and arranging them under the latitudes and departures as follows:

Latitudes.			Departures.		
Cosine.	Distance.	Latitudes.	Sine.	Distance.	Departures.
.88674	48.6	43.10	.46226	48.6	22.47
.38698	65.4	—25.31	.92209	65.4	60.30
.78297	52.6	—41.18	.62206	52.6	—32.72
.41866	55.0	23.03	.90814	55.0	—49.95

The latitudes are found by multiplying the cosine by the distance, and the departures by multiplying the sine by the distance.

74. Traverse Tables.—To facilitate calculation in the office, tables have been prepared by which the latitude and departure can be obtained without arithmetical calculation. Thus for any angle under 45° and for all distances from 1 to 100 the latitude and departures are calculated and tabulated. Thus for an angle of 10° we find:

Sin 10° = .17365

Cos 10° = .98481

Then for any distance x we have

Departure = .17365 x;

Latitude = .98481 x.

Now, if we give to x values from 1 to 10, the following results:

Dist.	10 Deg. Lat.	10 Deg. Dep.	11 Deg. Lat.	11 Deg. Dep.	12 Deg. Lat.	12 Deg. Dep.
1	.9848	.17	.98	.19	.98	.21
2	1.97	.35	1.96	.38	1.96	.42
3	2.95	.52	2.94	.57	2.93	.62
4	3.94	.69	3.93	.76	3.91	.83
5	4.92	.87	4.91	.95	4.89	1.04
6	5.91	1.04	5.89	1.14	5.87	1.25
7	6.89	1.22	6.87	1.34	6.85	1.46
8	7.88	1.39	7.85	1.53	7.83	1.66
9	8.86	1 56	8.83	1.72	8.80	1.87
10	9.85	1.74	9.82	1.91	9.78	2.08

In the same way the latitudes and departures can be calculated for all distance desired and for angles as minute as space will allow. Some works on surveying have traverse tables for all distances from 1 up to 100 and for all angles 15′ apart from zero to 90°.

75. Example:—If the distance is 56.8 chains and the bearing is N. 10° E. we divide up the number into 50, 6, and .8 and find the latitude and departure of each separately and add the results. We look for the latitude and departure of 5 and multiply the result by 10 to get the lat. and dep. for 50. If bearing is 10°, we have for 5 chains,

Lat. = 4.92
Dep. = .87
Hence, we have,

For 50, lat. = 4.92 × 10	= 49.20
" 6, lat. = 5.91	= 5.91
" .8, lat. = .788	= .79
Lat. for 56.8 chains	= 55.90
Dep. for 50 = .87 × 10	= 8.70
" " 6 =	= 1.04
" " .8 =	= .14
Total dep. for 56.8 chains	= 9.88

As an exercise, find the latitude and departure for bearing of 12° and a distance of 37.48 chains.

76. Error of Closure.—In surveying parties the surveyor is usually the only skilled man in the party. The chainmen are usually picked up in the locality and are not supposed to be trained in this work. It is assumed in balancing the survey that the errors are due to the chaining and that the surveyor reads the bearings correctly. If in balancing the error is greater than 1 in 500 the farm should be resurveyed. The error in latitude or departure is the amount that the algebraic sum of the latitudes or departures lacks of being zero. The error of closure is found by squaring the error in latitude and the error in departure and taking the square root of their sum and dividing this result by the perimeter of the farm. This is simply dividing the distance you miss the beginning corner by the length of the perimeter of the farm.

Find the latitudes and departures for the following courses:

Course.	Bearing.	Distance, Yards.	Latitudes.	Departures.
AB	N23°30′E	255.72	234.49	101.96
BC	N68°E	182.1	68.22	168.84
CD	S3°47′W	329.42	—328.67	—21.74
DA	N84°15′W	249.92	25.04	—248.66

Thus in the example the error in latitude is —.92 and the error in departure is + .40, that is, we went north 327.75 and south + 328.67, which leaves us + .92 south of *A*. We went east 270.80 and west 270.40, which leaves us + .40 west of *A* at some point *A′*.

But $AA' = \sqrt{(.92)^2 + (.40)^2}$.

And the error of closure $= \dfrac{\sqrt{(.92)^2 + (.40)^2}}{1017.16} = \dfrac{1}{1014}$

77. Balancing a Survey.—Theoretically the algebraic sum of the latitudes is equal to zero, and the same is true of the departures. But in actual survey work these sums never are equal to zero, owing to unavoidable errors. These errors must be distributed in proportion to the length of the courses. We see that the error in departure is .40, which must be distributed among the courses in proportion to their lengths.

The total distance around the farm (the perimeter) is 1017.16 varas, and the total error in departures is .40 and that for latitudes is .92. The error of any course is to the total error as the length of any course is to the perimeter.

If the compass was used in making the survey this rule for balancing should be followed even if some of the courses are due north-south, or due east-west. The compass cannot define the angle accurately and there is as much probability of error in angle in a due north course as there is in a course whose bearing is N. 26° E. Again, in some of the older states the magnetic bearings are read and a course that is north at the present time could make one degree with the magnetic meridian twenty years hence. If the practice of distributing the errors in departure (or latitude) among those courses that have departure be followed in the calculation of the first survey, the above method would have to be followed in the last survey. Thus the same surveyor would get different results for the area of the farm. The usual rule should be followed in all cases for a compass survey.

Therefore, the error for any course =

$$\frac{\text{total error}}{\text{perimeter}} \times \text{length of course.}$$

Corrections for Latitude of

$$AB = \frac{.92}{1017.16} \times 255.72 = .23$$

$$BC = \frac{.92}{1017.16} \times 182.1 = .16$$

$$CD = \frac{.92}{1017.16} \times 320.42 = .30$$

$$DA = \frac{.92}{1017.16} \times 249.92 = .23$$

Total for Latitude = .92

Corrections for Departure of

$$AB = \frac{.40}{1017.16} \times 255.72 = .10$$

$$BC = \frac{.40}{1017.16} \times 181.1 = .07$$

$$CD = \frac{.40}{1017.16} \times 329.42 = .13$$

$$DA = \frac{.40}{1017.16} \times 249.92 = .10$$

Total for Departure = .40

These are arranged in the following table:

Course	Corrections. Lat.	Dep.	Cor. Lat.	Cor. Dep.
AB	.23	.10	234.72	101.86
BC......	.16	.07	68.38	168.77
CD.....	.30	.13	—328.37	—21.87
DA.....	.23	.10	25.27	—248.76

The sum of the uncorrected plus latitudes is 327.75, and that of the minus latitudes is 328.67; all the plus latitudes must be increased by their corrections, and the minus must be decreased by their corrections. If these corrections are applied properly we will get the numbers in the column "Cor. Lat.," which means corrected latitudes. The sum of the plus departures is 270.80 and the sum of the minus departures is 270.40; the sum of the plus departures is greater by .40; therefore the minus departures must be increased and the plus departures decreased. The column headed "Cor. Dep." gives the corrected departures.

78. The Double Meridian Distance. — The reference meridian generally passes through the most westerly corner of the land. The perpendicular from the mid point of the course upon this meridian is called the meridian distance. The meridian distance of *MN*, Fig. 35, is *xy* where *x* is the midpoint of *MN*. But if *M*3, *N*4 and *O*5 are perpendicular to the meridian, $M3 + N4 = 2xy$, or double the meridian distance, and is called the *D. M. D.* That is, the *DMD* of any course is equal to the sum of the two perpendiculars from its ends upon the reference meridian.

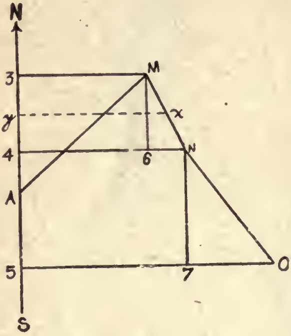


Fig. 35.

$$\text{The } DMD \text{ of } NO = N4 + O5$$
$$= N4 + O7 + N6 + M3$$
$$= (N4 + M3) + N6 + O6$$

That is, the *DMD* of any course is equal to the *DMD* of the preceding course, plus the departure of the preceding course plus the departure of the course itself. The *DMD's* of the first and last courses are always equal to their own departures.

A sketch of the farm whose latitudes and departures were balanced in Art. 77 shows that *A* is the most westerly corner, and it will be convenient to take our reference meridian through this corner. Then the Double Meridian Distance of AB = Departure of AB = 101.86.

The *D. M. D.* of $BC = 101.86 + 101.86 + 168.77 = 372.49$.

The *D. M. D.* of $CD = 372.49 + 168.77 - 21.87 = 519.39$.

The *D. M. D.* of $DA = 519.39 - 21.87 - 248.76 = 248.76$.

The last result proves the correctness of our arithmetical work, as the *DMD* of the last course should equal the departure of that course.

If the course *AB* does not happen to be in the first line of the table of notes, the *DMD's* can be calculated with reference to the most westerly corner without rearranging the table.

79. Area of a Farm.—If we drop perpendiculars from the ends of the courses upon the meridian *NS*, Fig. 36, we form trapezoids, or triangles. If we survey around the farm clockwise, all the areas determined by the courses and perpendiculars that have plus latitudes will be outside the farm, while those that have minus latitudes will include part of the farm and part of the area between the farm and the reference meridian. The algebraic *sum* of the "minus areas" and "plus areas" is equal to the area of the farm.

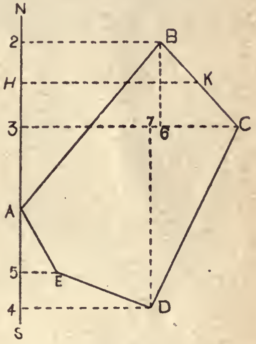


Fig. 36.

The double area of $AB2 = A2 \times B2 = Lat. \times DMD$.

Double area $2B\ C3 = 2\ 3 \times (B2 + C3) = Lat. \times DMD$.

Double area $3C\ D4 = 3\ 4 \times (C3 + D4) = Lat. \times DMD$.

Double area $4D\ E5 = 5\ 4 (D4 + E5) = Lat. \times DMD$.

Double area $5EA = 5A\ (5E) = Lat. \times DMD$.

The areas of $2B3C$ and $3CD4$ have minus latitudes (2 3 and 3 4) and these areas are therefore called "minus areas," and they not only include the whole farm but also the areas between the farm and the reference meridian. The areas $AB2$, $4DE5$, and $5EA$ have plus latitudes, and are called "plus areas." If we add the "plus areas" to the "minus areas," there is left the area of the farm, *A B C D E*.

Course	Bearing	Dist	Lat	Dep	Cor Lat	Cor Dep	D.M.D	Plus Area	Minus Area
AB	N38E	26ch	20.49	16.01	20.40	16.15	16.15	329.4600	
BC	S42E	34ch	-25.27	22.75	-25.39	22.93	55.23		1402.2897
CD	S51W	20ch	- 2.59	-15.54	-12.66	-15.43	62.73		794.1618
DA	N72W	25ch	7.73	-23.78	7.65	-23.65	23.65	180.9225	
			28.22	-39.32				510.3825	2196.4515
			-27.86	38.76					510.3825
			36	- .36					

2 × Area = 1686.0690
Area in Sq Ch = 843.0345
Area in Acres = 84.303

Errors.

Lat Cor	Dep Cor
$C_1 = \frac{36}{103} \times 26 = .09$	$C_1 = \frac{36}{103} \times 26 = .14$
$C_2 =$ " $\times 34 = .12$	$C_2 =$ " $\times 34 = .18$
$C_3 =$ " $\times 20 = .07$	$C_3 =$ " $\times 20 = .11$
$C_4 =$ " $\times 25 = .08$	$C_4 =$ " $\times 25 = .13$
Total = 36	Total = .36

Fig. 37.

80. Area Table.—Placing the D M D's in the table and multiplying each by its corresponding latitude, we find the areas as given in the following table. Dividing the area in square yards by 4840 gives the area in acres:

Course.	Bearing.	Dist.	Lat.	Corrections.		
				Dep.	Lat.	Dep
AB	N 23 30 E	255.72	234.49	101.96	.23	.10
BC	N 68 E	182.1	68.22	168.84	.16	.07
CD	S 3 47 W	329.42	—328.67	—21.74	.30	.13
DA	N 84 15 W	249.92	25.04	—248.66	.23	.10

Cor. Lat.	Cor. Dep.	D. M. D.	Plus Areas.	Minus Areas.
234.72	101.86	101.86	23,905.5792	....
68.38	168.77	372.49	28,479.0662	....
—328.37	—21.87	519.39	....	170,552.0943
25.27	—248.76	248.76	6,286.5652	....
			55,671.2106	170,552.0943
				55,671.5652
			Double area =	114,880.8837

∴ Area = 57,440.44195 sq. yds. = 23.7357 acres.

The standard form of calculation of errors and areas is shown in Fig. 37.

Problem 25.—William James Farm.

Course.	Bearing.	Distance.
AB	S84°45′E	19.73 chains
BC	S21°E	15.85 chains
CD	S78°W	19.53 chains
DA	N16°W	21.51 chains

Area = 35.01575 acres.

Problem 26.—Cambria Farm.

Course.	Bearing.	Distance.
AB	S 41 E	100 poles
BC	S 29 W	41 poles
CD	N 69 W	99 poles
DA	N 31 30 E	90 poles

Area = 39.357 acres.

Problem 27.—Oran Farm.

Course.	Bearing.	Distance.
AB	N5°16′E	205.6 varas
BC	N28°30′W	263.5 varas
CD	S16°E	255.6 varas
DE	S3°15′E	210.6 varas
EA	N84°15 W	227.7 varas

Area = 11.958 acres.

Problem 28.—Diego Blanco Farm.

Course.	Bearing.	Distance.
AB	N56°E	8.18 chains
BC	S16°E	5.39 chains
CD	S59°30′W	7.49 chains
DA	N17°10′W	4.87 chains

Area —3.97879 acres.

PROBLEM 29.—Bowie Blanca Farm.

Course.	Bearing.	Distance.
AB	N56°E	540.0 feet
BC	S16°E	356.0 feet
CD	S59°30′W	524.0 feet
DA	N17°10′W	321.2 feet

Area = 3.9786 acres.

PROBLEM 30.—Bantello Farm.

Course.	Bearing.	Distance.
AB	N33°E	14 chains
BC	S67°E	18 chains
CD	S38°40′W	19 chains
DA	N48°W	16 chains

Area = 27.5937 acres.

PROBLEM 31.—Leon Brooks Farm.

Course.	Bearing.	Distance.
AB	N5°16′E	205.6 varas
BC	S17°10′E	115.6 varas
CD	S56°30′E	207.7 varas
DA	N85°15′W	227.3 varas

Area = 2.80 acres.

PROBLEM 32.—Francis Estell Farm.

Course.	Bearing.	Distance.
AB	N5°E	750.0 feet
BC	S70°15′E	300.0 feet
CD	S16°E	356.8 feet
DA	S56°W	540.0 feet

Area = ... acres.

PROBLEM 33.—Juan Viego Farm.

Course.	Bearing.	Distance.
AB	N59°30′E	7.94 chains
BC	N78°E	4.88 chains
CD	S23°30′W	10.77 chains
DA	N56°30′W	8.74 chains

Area = ... acres.

PROBLEM 34.—John Bruce Farm.

Course.	Bearing.	Distance.
AB	N87°E	376.0 varas
BC	S59°17′W	260.0 varas
CD	S84°45′W	117.3 varas
DA	N16°W	128.4 varas

Area = ... acres.

PROBLEM 35.—H. Yandell Farm.

Course.	Bearing.	Distance.
AB	N59°17′E	260.0 varas
BC	S9°15′W	151.6 varas
CD	S68°W	182.1 varas
DA	N21°W	94.3 varas

Area = ... acres.

PROBLEM 36.—Sidney Dean Farm.

Course.	Bearing.	Distance.
AB	N67°W	11.52 chains
BC	S59°03′W	18.30 chains
CD	S49°E	14.30 chains
DA	N47°30′E	21.17 chains

Area = 23.531 acres.

PROBLEM 38.—George Pierce Farm.

Course.	Bearing.	Distance.
AB	N26°45′E	60 feet
BC	N11°42′E	364 feet
CD	N40°05′E	1038 feet
DE	S41°42′E	1650 feet
EF	S40°15′W	341 feet
FG	S20°15′W	322 feet
GA	N69°34′W	1640 feet

PROBLEM 38.—Old Perry Farm.

Course.	Bearing.	Distance.
AB	S73°45′E	21.60 chains
BC	S53°30′E	22.72 chains
CD	S47°12′W	6.95 chains
DE	S55°00′E	8.38 chains
EF	S48°00′W	5.55 chains
FG	N63°57′W	3.12 chains
GH	N53°25′W	41.60 chains
HA	N3°30′W	6.36 chains

Area = 48.15785 acres.

PROBLEM 39.

Course.	Bearing.	Distance.
AB	N57°42′E	17.47 chains
BC	S30°E	18.47 chains
CD	S38°40′W	2.27 chains
DE	S23°W	1.88 chains
EF	S60°E	13.05 chains
FA	N30°57′W	20.21 chains

Area = 34.2404 acres.

PROBLEM 40.

Course.	Bearing.	Distance.
AB	N3°53′E	7.70 chains
BC	S82°8′E	39.05 chains
CD	S83°42′E	14.39 chains
DE	S56°9′W	14.26 chains
EA	N80°3′W	42.30 chains

Area = 40.604 acres.

PROBLEM 41.

Course.	Bearing.	Distance.
AB	N60°05′E	19.90 chains
BC	S13°32′W	9.03 chains
CD	S27°20′W	9.77 chains
DE	S43°40′W	5.67 chains
EA	N30°43′W	13.24 chains

Area = 16.3432 acres.

81. Courses of No Latitude or Departure.—If a survey is made with the transit, the sum of the interior angles of the polygon should equal two right angles taken as many times as the polygon has sides less two. The error should not amount to more than three minutes, unless the number of sides is large. In a transit survey there can be very little error in the angular measurements and all errors in latitude and departure are largely due to errors in chaining. If a transit line is due north it is presumed that it is in the true meridian and therefore has no departure. Similarly if the course is due east it has no latitude, and if the angles check within three minutes (3′), the errors must be distributed on the assumption that they were due to the chaining. The practice is to distribute the errors in latitude (departure) among those courses that have latitude or departure. Thus no north-south course would receive a correction for departure as its original departure and also its balanced departure is zero. Similarly a due east-west course receives no correction for latitude. Hence if a course is north (east) its length is omitted in the perimeter of the field in calculating the errors in departure (latitude). The following rules are used in balancing:

Rule No. 1.—Distribute all errors in latitude (departure) in proportion to the length of the courses. If any course is north *(east) its length is omitted from the perimeter of the field. Error in latitude (departure) for any course is to the whole error in latitude (departure) as each course is to the corrected perimeter.*

Rule No. 2.—The error in latitude (departure) in any course is to the whole error in latitude as the latitude of the course is to the sum of all the latitudes.

The transit is rapidly becoming the surveyor's instrument, as there is greater demand for accuracy with the advanced price of land. The needle is inaccurate at best and when we consider the effect of barbed wire fences, telephone and telegraph wires, local attraction and other similar influences that render the needle unstable, its efficiency as an instrument of precision is rendered doubtful in the extreme.

Rule No. 2 is by far the most logical in transit surveys and should be used in balancing, and it has the advantage that it is automatic in that it finds no error in departure for north-south courses or in latitude for east-west courses.

82. Example:—In the following survey the errors were distributed in proportion to the length of those courses that have latitude or departure:

Course.	Bearing.	Dist.	Lat.	Dep.	Corrections. Lat.	Corrections. Dep.	Cor. Lat.	Cor. Dep.
AB	N47°E	40.57	27.67	29.67	.03	.01	27.64	29.66
BC	East	34.59	00.	34.59	.00	.00	.....	34.59
CD	S32°E	27.10	—22.98	14.36	.02	.00	—23.00	14.36
DE	South	22.01	—22.01	.00	.01	.00	22.02	.....
EA	N77°30′W	80.51	17.43	—78.60	.05	.01	17.38	—78.61

D. M. D.	Plus Area.	Minus Area.
29.66	819.8024	....
93.91	....	....
142.86	....	3,285.7800
157.22	....	3,461.9844
78.61	1366.2418	....
	2186.0442	6,747.7644
		2,184.0442

Double area = 4,561.7202

Area = 2,280.8601 sq. chains = 228.08601 acres.

If the errors are distributed in proportion to the latitudes and departures, the result is as follows:

Corrections. Lat.	Corrections. Dep.	Cor. Lat.	Cor. Dep.	D. M. D		
.03	.00	27.64	29.67	29.67	820.0788	
.00	.00	0.00	34.59	93.93		
.03	.00	—23.01	14.36	142.88		3287.0688
.03	.00	—23.04	0.00	157.24		3465.5696
.02	.02	17.41	—78.62	78.62	1368.7742	
					2188.8530	6753.2374
						2188.8530

Double area = 4564.3844
∴ area = 2282.1922
= 228.22 acres

PROBLEM 42.

Course.	Bearing.	Distance.
AB......................	N36°9′E	20.0 chains
BC......................	East	8.0 chains
CD......................	South	28.0 chains
DA......................	N59°2′W	23.3 chains

Area = 34.3779 acres.

PROBLEM 43.

Course.	Bearing.	Distance.
AB......................	N39°30′E	10 chains
BC......................	East	11 chains
CD......................	South	17 chains
DA......................	N61°W	20 chains

Area = 19.158 acres.

PROBLEM 44.—Find the area of the following: Beginning at a stake in road 762.5 feet west from Chisholm's southwest corner; thence N. 0° 30′ E. 661 feet; thence up branch S. 81 W. 117 feet, S. 22 W. 124 feet, S. 8 W. 87, S. 70° 30′ W. 162 feet, then S. 27° 30′ W. 153, S. 31° 30′ E. 62 feet, S. 34° W. 94 feet, E. 304 feet, S. 5° W. 129 feet to middle of said road; thence E. along said road 116 feet to beginning.

PROBLEM 45.

Course.	Bearing.	Distance.
AB......................	N39°E	20
BC......................	East	8
CD......................	South	28
DA......................	N60°W	23

83. Area by Co-ordinates.—If the co-ordinates of each corner of the farm are given with reference to two axes OX and OY, we can find the area by dropping perpendiculars from each corner on either axis, as OX, Fig. 38.

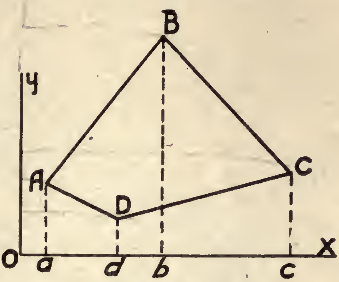


Fig. 38.

Let Oa, Ob, Oc and $Od = x_a, x_b, x_c$ and x_d, repectively; and Aa, Bb, Cc and $Dd = y_a, y_b, y_c$ and y_d.

Now

$$\text{area } aABb = ab\,\frac{(Aa + Bb)}{2}$$

$$\text{area } bBCc = bc\,\frac{(Bb + Cc)}{2}$$

$$\text{area } cCDd = cd\,\frac{(Cc + Dd)}{2}$$

$$\text{area } dDAa = da\,\frac{(Dd + Aa)}{2}$$

$$\text{area of farm} = aABb + bBCc - cCDd - dDAa = (x_b - x_a)\frac{(y_a + y_b)}{2} + (x_c - x_b)\frac{(y_b + y_c)}{2} - (x_c - x_d)\frac{(y_c + y_d)}{2} - (x_d - x_a)\frac{(y_d + y_a)}{2}$$

$\therefore$ Double area $= x_a\,(y_d - y_b) + x_b\,(y_a - y_c) + x_c\,(y_b - y_d) + x_d\,(y_c - y_a)$.

Similarly

Double area $= y_a\,(x_d - x_b) + y_b\,(x_a - x_c) + y_c\,(x_b - x_d) + y_d\,(x_c - x_a)$.

This can be crystallized into the following rule: *To find the double area, multiply each abscissa (ordinate) by the difference of the adjacent ordinates (abscissas) taken in order.*

EXAMPLE.—Find the area of the farm whose co-ordinates are (2, 6), (6, 10), (12, 8), (4, 2).

X.	Y.	Diff. of X's.	Area.	Diff. of Y's.	Area.
2	6	2	12	8	16
6	10	10	100	2	12
12	8	—2	—16	—8	—96
4	2	—10	—20	—2	—8
Area			76.0		76.0

PROBLEM 46.—Find the area by both methods of the farm whose co-ordinates are (2, 4), (4, 8), (12, 12), (16, 4), (10, 0). Answer 96.

PROBLEM 47.—Find area of polygon whose co-ordinates are (0, 0), (0, 12), (10, 9), (18, 14), (22, 13), (9, 0).

84. Traversing.—When it is desired to find the bearing and distance of one point from another, a survey is run from the initial point to the final, making as many straight courses as desired. The latitudes and departures of these courses are calculated, and the closing course is a lost course whose bearing and length are desired and can be found by formulas 8 and 9.

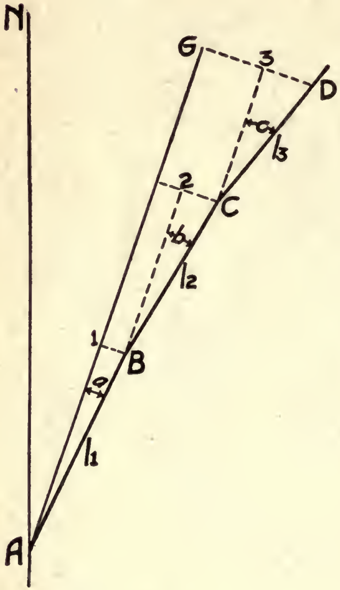


Fig. 39.

85. Example.—Find the bearing and length of *AD* in the following:

Course.	Bearing.	Distance	Latitude.	Departure.
AB	N31°E	20 chains	17.14	10.30
BC	N33°E	24 chains	20.13	13.07
CD	N36°E	26 chains	21.03	15.28
DA	....	....	.....	.....

The tangent of the bearing $= \frac{38.65}{58.30} = .66295.$

Therefore the bearing = N 33° 32′ E

$$\text{Length} = \sqrt{(58.20)^2 + (38.65)^2} = 69.86.$$

86. Approximate Traversing.—Where the bearings of the different courses of a traverse do not differ by more than 6° the bearing can be found by an application of the 57.3 rule. Let *ABCD*, Fig. 39, be a traverse, and let the bearings be as in the preceding example. Take a reference line and let *a, b,* and *c* be the angles that *AB, BC,* and *CD* make with this line *AG*.

$$1B = \frac{al_1}{57.3}$$

$$2C = \frac{bl_2}{57.3}$$

$$3D = \frac{cl_3}{57.3}$$

Let x = angle that *AD* makes with reference line *AG*.

$$DG = \frac{x}{57.3} \times AD$$

But $AD = AB + BC + CD$, nearly

$$= l_1 + l_2 + l_3$$

$$\frac{x}{57.3}(l_1 + l_2 + l_3) = \frac{al_1 + bl_2 + cl_3}{57.3}$$

$$x = \frac{al_1 + bl_2 + cl_3}{l_1 + l_2 + l_3}$$

$$DG = \frac{al_1 + bl_2 + cl_3}{57.3}$$

If B = bearing of the reference line and we add $B(l_1 + l_2 + l_3)$ to each side, we get:

$$B(l_1 + l_2 + l_3) + x(l_1 + l_2 + l_3) = al_1 + bl_2 + cl_3 + Bl_1 + Bl_2 + Bl_3$$

$$\therefore (B + X) = \frac{l_1(B+a) + l_2(B+b) + l_3(B+c)}{l_1 + l_2 + l_3} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

That is, multiplying each bearing by its length, and dividing the sum of the results by the sum of the lengths of the courses gives the bearing required.

Let $a = 32°$, $b = 33°$, $c = 36°$, $AB = 20$, $BC = 24$, $CD = 26$, find bearing of *AD*.

$$(B + X) = \frac{31 \times 20 + 33 \times 24 + 36 \times 26}{70} = 33°.54 = 33°32'.4.$$

PROBLEM 48.—Find the approximate bearing of *AD* from the following notes:

Course.	Bearing.	Distance.
AB....................	S28E	20 chains
BC....................	S32E	18 chains
CD....................	S30E	22 chains
DA....................	....	....

87. Irregular Boundaries.—It often happens that a creek or river is the boundary of a tract of land and the land follows the meanders of the river. Thus the field notes of a certain farm, Fig. 40, are as follows:

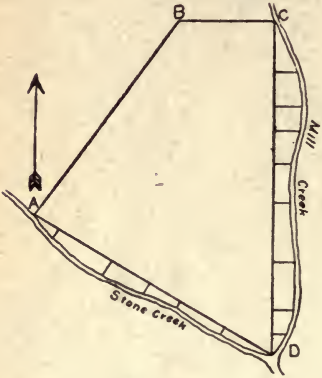


Fig. 40.

Beginning at a pecan tree marked *X* on Stone Creek, thence N. 36° 9′ E. to a stone in the prairie 29 chains; thence E. 8 chains to a cottonwood marked *H* on the west bank of Mill Creek; thence with the meanders of Mill Creek to the junction of Stone Creek; thence up Stone Creek to the beginning.

The following offsets were taken:

	CD			*DA*	
Dist.	Offset.	Area.	Dist.	Offset.	Area.
00 chains	00. chains	.00 acres	00. chains	00. chains	.00 acres
4 chains	2.0 chains	.4 acres	5. chains	2.3 chains	.575 acres
7 chains	2.5 chains	.675 acres	9. chains	2.5 chains	.960 acres
9 chains	2.2 chains	.47 acres	14. chains	2.1 chains	1.15 acres
12 chains	1.0 chains	.48 acres	17. chains	1.8 chains	.32 acres
15 chains	1.4 chains	.36 acres	19. chains	1.4 chains	.07 acres
20 chains	1.8 chains	.80 acres	20. chains	.0 chains	.05 acres
24 chains	2.0 chains	.76 acres	21. chains	1.0 chains	.09 acres
26 chains	1.7 chains	.37 acres	22. chains	.8 chains	.09 acres
28 chains	0.0 chains	.17 acres	23.3 chains	.0 chains	.052 acres
		4.485 acres			3.852 acres

Area of farm *ABCD*	=34.3779 acres
Area of offsets from *C* to *D*	= 4.1250 acres
Area of offsets from *D* to *A*	= 3.852 acres
Total area of farm with offsets	=42.7149 acres

The land lines run up to the bank if the stream is navigable.

PROBLEM 49.—The following offsets were taken where *R* and *L* refer to right and left of the line being surveyed. Find the total area of farm if bounded by straight sides *AB* and *BC* and the meanders of the streams to which offsets were taken from points along *CD* and *DA*.

Length along *CD*	Offsets	Length along *DA*	Offsets
0	0	0	0
3	.6 R	3	.4 L
5	.8 R	5	.6 L
7	.7 R	7	.8 L
8	.3 R	10	.4 L
9	0.0	12	0.0
11	.3 L	14	.3 R
13	.5 L	16	.5 R
15	.4 L	18	.4 R
17	0.0	20	0.0

88. Discrepancies.—It often happens that a survey is found where little care was exercised in the original survey when the grant or patent was taken up. If there are errors in the field notes of the original grant and there are no natural objects to which reference was made, it is very difficult, if not impossible, to re-establish the old lines. But if natural objects were referred to in the original field notes, and these objects can be found and identified, the re-establishment of the old survey is possible and, sometimes, comparatively easy. Corners are often defined or witnessed by natural objects, while the distances in the field notes do not agree with such witness objects. In such cases the natural objects control and the corners must be located as called for by the natural object irrespective of the length of the lines in the notes. If a line begins at a well known tree and runs with a certain bearing to the middle of a certain stream, and thence with the meanders of the same, etc., the line must go to the center of the stream, although the distance of the line may fall short or exceed that called for in the recorded field notes.

PROBLEM 50.—The area was calculated to be 39.357 acres. Find the area of the farm if the line *DA* was a random line from which offsets were taken to a small creek on the left of *DA*, and completely outside the farm as given in problem 26. The following are the field notes for the offsets taken along *DA*:

Dist. from *D*	Offsets to left
0	0
16	8
28	12
40	6
48	12
68	4
90	0

Area = 3.55 acres. If this area is added to the area of problem 26 we get for the whole area 42.907 acres, which is the area of the farm shown in the plot in Fig. 105.

CHAPTER V.
DIVISION OF LAND.

89. Division of Triangle.—There are two cases which generally occur in practice. The first is to draw a line parallel to one side of a triangle to cut off a certain fraction of the whole area, or to divide the triangle into two parts whose areas shall have a certain ratio, while the second is to draw a line from one of the vertices of the triangle to divide it in a given ratio.

First Case: Given the triangle ABC, Fig. 41, the length of whose sides is known. The area of the triangle can be found from Formula 3. It is required to draw a line PQ parallel to BC so that

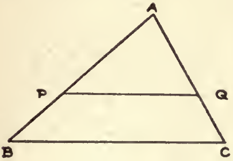


Fig. 41.

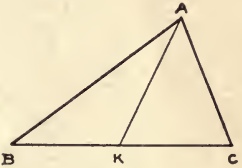


Fig. 42.

$$APQ : ABC :: m : n.$$

Let $AP = x$, and $AQ = y$. Then,

$APQ : ABC :: AP^2 : AB^2$. $\therefore$ $APQ : ABC :: x^2 : c^2$.

$$\therefore \quad x^2 : c^2 :: m : n. \qquad \therefore \quad x = c\sqrt{\frac{m}{n}}$$

In same way, $y = o\sqrt{\frac{m}{n}}$

Example: Given $a = 300$, $b = 240$, $c = 180$. Find a line PQ that will cut off 4/9 of the triangle ABC. $x = 240\sqrt{4/9} = 240 \times 2/3 = 160$. $y = 180 \times 2/3 = 120$.

Second Case: Given the triangle ABC, Fig. 42, to draw a line AK, so that AK will cut off the triangle AKB equal to m/n of the triangle ABC. The triangles ABK and ABC have the same altitude, and are therefore to each other as their bases. Hence,

$ABK : ABC :: m : n.$ But $ABK : ABC :: BK : BC$

$\therefore BK : BC :: m : n.$ $\qquad BK = BC \times m/n$

EXAMPLE: Find BK in the foregoing example when BAK is three-fifths of the triangle ABC. $BK = 3/5 \times 300 = 180.$

PROBLEM 51.—Given $a = 340$, $b = 272$, $c = 204$. Find the area of ABC and AP and AQ when PQ is parallel to BC and the triangle APQ is two-thirds of ABC.

90. Division Line Through Internal Point.—It may be possible that it is desired that the dividing line shall pass through some point inside the triangle and divide the triangle in a cer-

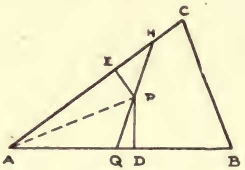


Fig. 43.

tain ratio. Let P be the internal point in the triangle ABC, Fig. 43, and let it be required to pass a line, HPQ, through P that will make the triangle AHQ have the ratio of m to n to the triangle ABC. The point P is known, and the perpendiculars PD and PE are known, or can be calculated. Let the area of the triangle ABC be represented by K, and $PD = p$, $PE = q$, $AQ = x$, and $AH = y$. We have,

Area $APQ = \frac{1}{2}\ PD \times AQ = \frac{1}{2}\ px$

Area $APH = \frac{1}{2}\ PE \times AH = \frac{1}{2}\ qy.$

Area APQ + area APH = area AHQ =

$\frac{1}{2}(px + qy) = m/n\ K$ ....(12)

Also, we have,

Area $AHQ = \frac{1}{2}\ AQ \times AH$ *sin.* $A = \frac{1}{2}\ xy$ *sin.* $A.$

Area $ABC = \quad AB \times AC$ *sin.* $A = \frac{1}{2}\ bc$ *sin.* $A.$

But Area $AHQ = m/n$ area ABC

$$\therefore \tfrac{1}{2}\, xy \sin. A = \frac{m}{2n} bc \sin A$$

$$\therefore xy = m/n\, bc \qquad (13)$$

Thus we have two equations in x and y, and these can be found and laid off on the sides AB and AC.

Example: Given $AB = 420$, $AC = 400$, $BC = 260$, $PD = 100$, $PE = 60$. Find x (AQ) and y (AH), when triangle AHQ is four-tenths of ABC.

By calculation we find area $ABC = 50{,}400$.

Then we have,

$$50x + 30y = 4/10\ 50{,}400 = 20{,}160.$$

$$xy = 4/10 \times 420 \times 400 = 67{,}200.$$

Solving for x and y, we get,

$$x = 219.57 \text{ or } 183.63;$$

$$y = 306.05 \text{ or } 365.75.$$

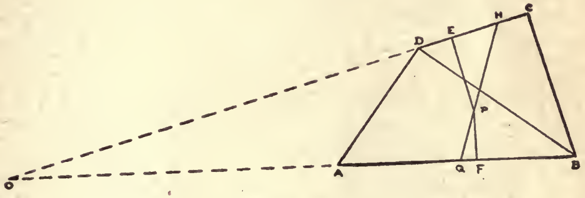


Fig. 44.

Problem 51.—In the triangle, find x and y if the line HQ is to pass through P and bisect the triangle ABC. Answer, $x = 366.47$, $y = 229.21$.

91. Division of Quadrilateral.—Given a quadrilateral $ABCD$, Fig. 44. Required to find a line HQ through an internal point P that will make $ADHQ$ equal to m/n of $ABCD$. Let S = area of $ADHQ$ and K = area $ABCD$. The point P is located by perpendiculars, PE and PF, on two sides of the quadrilateral. Produce two opposite sides AB and CD to intersect in some point O. Let $PF = p$, $PE = q$. The sides and angles of the quadrilateral $ABCD$ are known, and from these the sides and area of *OAD* can be calculated. Adding area of *OAD* to *ADHQ* will give the required area of *OHQ*, and adding the area of *OAD* to the area of *ABCD* will give the area of *OBC*. Find the ratio of *OHQ* to *OBC*. The problem is then reduced to that of finding a line through *P*, dividing the triangle *OBC* into the ratio of *m* to *n*. The solution comes under the case of dividing a triangle by a line through an internal point. After the areas of *AOD*, *OBC* and *OHQ* are found we have, where, $OA=a$, $OD=b$, $PE=q$, $PF=p$, $AQ=x$, $DH=y$,

$$\tfrac{1}{2}p(a+x)+\tfrac{1}{2}q(b+y)=\text{area } OHQ,$$

$$(a+x)\ (b+y)=m/n\ OB\times OC.$$

From these two equations, the values of x and y can be calculated. In the same way we can find the line passing through an internal point in a pentagonal field, dividing the field in a certain ratio.

Problem 52.—If $AB=300$, $BC=192$, $CD=144$, $AD=180$, $DB=240$, $PE=96$ and $PF=60$, find the values of x $(=AQ)$ and y $(=DH)$ when the area *ADHQ* is seven-twelfths of *ABCD*.

92. General Solution.—There are many problems in land dividing that can be solved by special methods, and there are often short operations that can be applied at once. In the majority of cases the line of division is not required to pass through an internal point. Where some certain point is given as the point of beginning of the division line, this point is generally at a corner of the field or on one side at a given distance from a corner. In such cases it is desired to find the bearing and length of the dividing line, and this problem is treated in a general way in the following articles. However, no attempt is made to solve problems of division in regard to the regular geometrical figures, as such solutions are rather simple and offer no difficulties to the student.

We have seen that the sum of the northings and the sum of the southings for a complete survey must each equal zero. Thus, we have two conditions to fulfill and mathematically this gives us two equations. If we let l_1, l_2, l_3, etc., represent the lengths, and B_1, B_2, B_3, etc., represent the bearings of the different courses, we must have:

$$l_1 \, Cos \, B_1 + l_2 \, Cos \, B_2 + l_3 \, Cos \, B_3 \text{ etc.} = 0. \ldots\ldots (14)$$

$$l_1 \, Sin \, B_1 + l_2 \, Sin \, B_2 + l_3 \, Sin \, B_3 \text{ etc} = 0. \ldots\ldots (15)$$

Theoretically, if we know all the parts except two we can find these two unknown parts from equations 14 and 15. The lost or unknown parts can be:

Case I. Bearing and length of one course.

Case II. Length of two courses.

Case III. Length of one course and bearing of another.

Case IV. Bearing of two courses.

93. Case I.—If the bearing and length of one course is unknown, the latitudes and departures of the known courses are first found. The algebraic sum of these must be the latitudes and departure of the unknown course with the signs changed.

If we let L and D be the latitude and departure of the unknown course, respectively, then the length of the course

$$= \sqrt{L^2 + D^2}$$

And the tangent of the bearing$= \dfrac{D}{L}$

Example: Find the lost parts in the following:

Course.	Bearing.	Dist.	Lat.	Dep.
AB............	N62°7′E	9.24	4.32	8.17
BC............	S36°5′E	7.62	—6.16	4.49
CD............	(S45°29′W)	(10.10)	(—7.08)	(—7.20)
DA............	N31°28′W	10.46	8.92	—5.46

$$L = 4.32 + 8.92 - 6.16 = 7.08$$

$$D = 8.17 + 4.49 - 5.46 = 7.20$$

$$\text{Length } CD = \sqrt{(7.08)^2 + (7.20)^2} = 10.10$$

$$\text{Tangent bearing} = \frac{7.20}{7.08} = 1.1070$$

$$\therefore \text{Bearing} = S45°29'W$$

Problem 54.—Find the lost parts in the following:

Course.	Bearing.	Distance.
AB.............................	N46°22′E	38 chains
BC.............................	....	....
CD.............................	S42°W	42 chains
DA.............................	N29°W	54 chains

94. Case II.—If two lengths are unknown we first find the latitudes and departures of the known courses.

Let x and y be the unknown lengths and M and N be the bearings of these courses, respectively. Then from equations 14 and 15 we have:

$$x \,Cos\, M + y \,Cos\, N + L = 0$$
$$x \,Sin\, M + y \,Sin\, N + D = 0$$

Multiply the first equation by Sin N and the second by Cos N and we have:

$$x \,Cos\, M \,Sin\, N + y \,Cos\, N \,Sin\, N + L \,Sin\, N = 0$$
$$x \,Sin\, M \,Cos\, N + y \,Cos\, N \,Sin\, N + D \,Cos\, N = 0$$

Subtracting and transposing, we get:

$$x\,(Sin\, M \,Cos\, N - Cos\, M \,Sin\, N) = L \,Sin\, N - D \,Cos\, N$$
$$x \,Sin\,(M-N) = L \,Sin\, N - D \,Cos\, N$$
$$x = \frac{L \,Sin\, N - D \,Cos\, N}{Sin\,(M-N)}$$

Example.—Find the lost parts in the following survey:

Course.	Bearing.	Dist.	Lat.	Dept.
AB ...	N47°2′E	31.30	21.33	22.90
BC ...	S57°4′E	21.10	—11.47	17.71
CD ...	S60°W	x	—x Cos 60°	—x Sin 60°
DA ...	N40°W	y	y Cos 40°	—y Sin 40°

From formulas (14) and (15), we get,

$$-x \,Cos\, 60° + y \,Cos\, 40 + 9.86 = 0$$
$$-x \,Sin\, 60 - y \,Sin\, 40 + 40.61 = 0$$

Multiplying the first equation by Sin 40° and the second by Cos 40° we have:

$$-x \,Cos\, 60° \,Sin\, 40° + y \,Sin\, 40° \,Cos\, 40° + 9.86 \,Sin\, 40° = 0$$
$$-x \,Sin\, 60° \,Cos\, 40° - y \,Sin\, 40° \,Cos\, 40° + 40.61 \,Cos\, 40° = 0$$

Transposing and changing signs we have:

$$x \,Cos\, 60 \,Sin\, 40° - y \,Sin\, 40° \,Cos\, 40° = 9.86 \,Sin\, 40$$
$$x \,Sin\, 60° \,Cos\, 40° + y \,Sin\, 40° \,Cos\, 40° = 40.61 \,Cos\, 40$$

Adding:

$$x\,(Sin\, 60° \,Cos\, 40° + Cos\, 60° \,Sin\, 40°) = 40.61 \,Cos\, 40° + 9.86 \,Sin\, 40$$
$$x \,Sin\, 100° = 40.61 \,Cos\, 40° + 9.86 \,Sin\, 40°$$
$$x = \frac{40.61 \,Cos\, 40° + 9.86 \,Sin\, 40°}{Sin\, 100°}$$
$$x = \frac{4061 \times .76604 + 9.86 \times .64279}{.98481} = 38.024$$

If we multiply the first equation by Sin 60° and the second by Cos 60° we get:

$$x\ Cos\ 60°\ Sin\ 60° - y\ Sin\ 60°\ Cos\ 40° = 9.86\ Sin\ 60°$$
$$x\ Cos\ 60°\ Sin\ 60° + y\ Cos\ 60°\ Sin\ 40° = 40.61\ Cos\ 60°$$

Subtracting and changing the signs we have:

$$y\ (Sin\ 60°\ Cos\ 40° + Cos\ 60°\ Sin\ 40°) = 40.61\ Cos\ 60° - 9.86\ Sin\ 60°$$
$$y\ Sin\ 100° = 40.61\ Cos\ 60° - 9.86\ Sin\ 60°$$
$$y = \frac{40.61\ Cos\ 60° - 9.86\ Sin\ 60°}{Sin\ 100°}$$
$$y = \frac{40.61 \times .5 - 9.86 \times .86603}{.98481} = 11.94733$$

PROBLEM 55.—Find the lost parts.

Course.	Bearing.	Distance.
AB....................................	N5°E	8.68
BC....................................	S17°E	x
CD....................................	S56°E	y
DA....................................	N85°W	9.58

$x = 4.687,\ y = 8.937$

95. Case III.—The length of one course and the bearing of another lost.

Find the unknown parts in the following example:

Course.	Bearing.	Distance	Latitude.	Departure.
AB	N36°E	12 chains	9.708	7.054
BC	X°	8 chains	— 8 Cos X°	8 Sin X°
CD	S20°E	11 chains	—10.337	3.762
DE	S75°W	y chains	—y Cos 75°	—y sin 75°
EA	N30°W	10 chains	8.660	—5.000

In all cases it is better to make a graphical solution in order to find the direction letters of the bearing. Lay off *AB*. Fig. 45, N. 36° E., equal to 12 chains to some scale, and *EA* *S.* 30° E. 10 chains. *C* will be somewhere on the circumference of a circle whose center is *B* and whose radius is 8 chains, while *D* will be somewhere on *ED*, where *ED* is drawn with

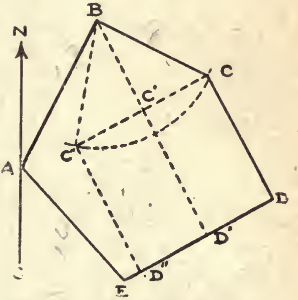


Fig. 45.

a bearing of N. 75° E. Through B draw BD' S. 20° E., and lay off $C'D'$ from D' equal and parallel to CD. Through C' draw CC' parallel to ED and cutting the circle at C and C'' and through C and C'' draw CD and $C''D''$ parallel to BD'. There are two solutions, $ABCDE$ being one and $ABC''D''E$ being the other. From the figure we see that the bearing of BC is southeast, and that of BC'' is southwest. Filling out the table for the southeast bearing and adding the latitude and departures, we get:

$$8 \; Cos\; X° + y\; Cos\; 75° = +8.031$$
$$8 \; Sin\; X° - y\; Sin\; 75° = -5.816$$

Multiplying the first equation by Sin 75° and the second by Cos 75°, we have:

$$8 \; Cos\; X° \; Sin\; 75° + y\; Cos\; 75° \; Sin\; 75° = 8.031 \; Sin\; 75°$$
$$8 \; Sin\; X° \; Cos\; 75° - y\; Cos\; 75° \; Sin\; 75° = -5.816 \; Cos\; 75°$$

Adding, we have:

$$8 \; (Sin\; X° \; Cos\; 75° + Cos\; X° \; Sin\; 75°) =$$
$$8.031 \; Sin\; 75° - 5.816 \; Cos\; 75°$$

$$Sin\; (X° + 75°) = \frac{8.031 \; Sin\; 75° - 5.816 \; cos\; 75°}{8} = .78151$$

$$X° + 75° = 128°36' \text{ or } 51°24'$$
$$X° = 53°36' \text{ or } 23°36'$$

To eliminate $X°$, we have:

$$Cos\; X° = \frac{8.031 - y\; Cos\; 75°}{8}$$

$$Sin\; X° = \frac{y\; Sin\; 75° - 5.816}{8}$$

Squaring and adding, we have:

$$64 = 98.322817 - y\; (16.062 \; Cos\; 75° + 11.632 \; Sin\; 75°) + y^2$$
$$\therefore y^2 - y\; (11.632 \; Sin\; 75° + 16.062 \; Cos\; 75°) = -34.322817$$
$$y^2 - 15.342864 \; y = -34.322817.$$

Completing the square, we have:

$$y = 12.685, \text{ or } 2.705.$$

PROBLEM 55.—Find the bearing X and the distance y in the preceding examples, when the course BC bears southwest.

Answer. Bearing, 23°36. Length, 2.704.

96. Case IV.—Two bearings unknown.

Let X and Y be the unknown bearings, a and b the lengths of these courses, and L and D be the latitude difference and the departure difference of these courses, respectively; then

$$a \, Cos \, X^\circ + b \, Cos \, Y^\circ = L$$
$$a \, Sin \, X^\circ + b \, Sin \, Y^\circ = D$$

Then $Cos \, Y^\circ = \frac{L - a \, Cos \, X^\circ}{b}$ and $Sin \, Y^\circ = \frac{D - a \, Sin \, X}{b}$

Squaring and adding, we have:

$$Sin^2 \, Y^\circ + Cos^2 \, Y^\circ = \frac{(L - a \, Cos \, X^\circ)^2}{b^2} + \frac{(D - a \, Sin \, X^\circ)^2}{b^2}$$

Let $2T = a^2 + L^2 + D^2 - b^2$

But $Cos^2 \, X = 1 - Sin^2 \, X$

Therefore $1 - Sin^2 \, X = \left(\frac{T - aD \, Sin \, X}{aL}\right)^2$

Then $a^2L^2 - T^2 = a^2 (L^2 + D^2) \, Sin^2 X - 2aDT \, Sin \, X$

From this quadratic in $Sin \, X$ two values of $Sin \, X$ will be found and there will be two solutions possible.

97. Example.—Find the unknown parts in the following example:

Course.	Bearing.	Distance.	Latitude.	Departure.
AB.....	N24°E	26 chains	23.752	10.575
BC.....	Sx°E	28 chains	—28 Cos X	28 Sin X
CD.....	S38°E	24 chains	—18.912	14.776
DE.....	Sy°W	36 chains	—36 Cos Y	—36 Sin Y
EA.....	N44°W	18 chains	12.948	—12.504

To find the direction letters draw *AB*, Fig. 46, N. 24° E., and *EA* S. 44° E., move *CD* from its true position to some position *C′D′* parallel and equal to itself where *C′* coincides with *B*. *C* has been moved 28 chains, because the length of *BC* is 28 chains. Now, *D* is 28 chains from *D′*, but *D* is also 36 chains from *E*, hence with *D′* as a center and a radius of 28 chains describe an arc, and with *E* as a center and 36 chains as a radius describe an arc cutting the first arc at *D*. Draw *DC* N. 38° W. 24 chains. Draw *BC* and *DE*. Thus, we see that *BC* bears southeast and that *DE* bears southwest. Putting the direction letters in the table and filling out the latitude and departure columns, we have for our equations:

$$28\ Cos\ X + 36\ Cos\ Y = 17.788$$
$$28 Sin\ X - 36\ Sin\ Y = -12.847$$

Then $Cos\ Y = \dfrac{4.447 - 7\ Cos\ X}{9}$ and $Sin\ Y = \dfrac{3.212 + 7\ Sin\ X}{9}$

$$81 = 30.092753 - 62.258\ Cos\ X + 44.968\ Sin\ X + 49$$
$$4.447\ Cos\ X = 3.212\ Sin\ X - .136232$$
$$Cos\ X = .72228\ Sin\ X - .030635$$
$$1 - Sin^2\ X = .009285 + .04425409\ Sin\ X + .5216884\ Sin^2\ X$$
$$Sin^2\ X + .02908\ Sin\ X = .65655$$
$$Sin\ X = 82498$$
$$X = 50°35'11''$$

PROBLEM 56.—Fnd the lost parts.

Course.	Bearing.	Distance.
AB..............................	N31°E	14 chains
BC..............................	N62°E	20 chains
CD..............................	x	27 chains
DE..............................	S38°W	23 chains
EA..............................	y	24 chains

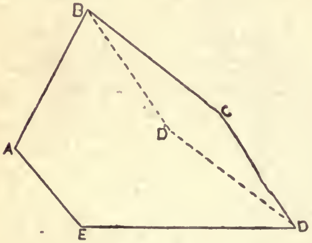


Fig. 46.

98. Dividing Land.—It often becomes necessary to divide farms among the different owners. A certain number of acres is sold from one part of a farm, and it becomes necessary to know the boundaries of the part cut off from the original survey. The partition is generally made in two ways, either by a line starting at a certain point cutting off the required number of acres, or by a line that has a certain bearing. The following examples will serve to illustrate the methods.

99. Example.—Find the bearing and length of a line *AP* that will cut off 40 acres from the farm *ABCD*, as given below in Fig. 47.

Course.	Bearing.	Distance.	Lat.	Dep.	Corrections. Lat.	Corrections. Dep.
AB.....	N47°2′E	31.0 chains	21.13	22.68	.13	.03
BC.....	S57°4′E	21.0 chains	— 11.42	17.63	.08	.02
CD.....	S28°42′W	40.0 chains	— 35.09	— 19.21	.16	.04
DA.....	N40°27′W	32.7 chains	24.88	— 21.22	.13	.03
			— 46.51	— 40.43		
			+ 46.01	40.31		
			— .50	— .12		

Corrected Latitude.	Corrected Departure.	D. M. D.	Area. +	Area. —
21.26	22.71	22.71	482.8146	....
— 11.34	17.65	63.07	....	715.2138
— 34.93	— 19.17	61.55	....	2149.9415
25.01	—21.19	21.19	529.9619	....
			1012.7765	2865.1553
				1012.7765

Double area = 1852.3788 sq. ch.
Area = 92.61894 acres.

Join the starting point *A* of division with the corner *C* nearest the final end of the required course. Find the area of the part thus cut off as follows:

Course.	Cor. Lat.	Cor. Dep.	D. M. D.	Area.
AB...............	21.26	22.71	22.71	482.8146
BC...............	— 11.34	17.65	63.07	— 715.2138
CA...............	— 9.92	— 40.36	40.36	— 400.4712

Area = 31.64352 acres.

As the area of the triangle *ABC* is only 31.64 acres, the line *AP* that makes area *ABCP* equal to 40 acres must cut the side *CD*, hence *P* lies on side *CD*.

Length $CA = \sqrt{(9.92)^2 + (40.36)^2} = 41.561$

Tan. bearing of $CA = \frac{40.36}{9.92} = 4.06855$

$\therefore$ Bearing of $CA = 76°11'28''$

Angle $ACP = 47°29'28''$

Now area $ACP = 400 - 316.4352 = 83.5648$ sq. chains.

But area $ACP = \frac{1}{2}\ CA,\ CP \sin.\ ACP$

$$\therefore CP = \frac{2 \text{ area } ACP}{CA \sin.\ ACP} = \frac{1671296}{41.561 \times .73717} = 5.455 \text{ chains}$$

The latitude and departure of *CP* bear the same ratio to the corrected latitude and departure of *CD* that the length *CP* does to *CD*.

$$\therefore Lat.\ CP = 4.785\ \ Dep.\ CP = 2.62$$

To find the length and bearing of *PA*, complete the table of *ABCP*.

Course.	Latitude.	Departure.	D. M. D.	Area.
AB..............	21.26	22.71	22.71	482.8146
BC..............	—11.34	17.65	63.07	—715.2138
CP..............	—4.785	— 2.62	78.10	—374.4085
PA..............	—5.135	—37.74	37.74	—193.7949

Double area = 800.6026 square chains.

Area *ABCP* = 40.03 acres.

PROBLEM 57.—In the example in Article 99, find the bearing and length of a line *AP* that will cut off an area *ABP* equal to nine acres.

PROBLEM 58.—Find the bearing and length of a line *DK* in the preceding problem that will make area *ADK* equal to six acres.

100. Example.—Find the length of a line that bears N. 52° E. and cuts off 51 acres on the northwest side of the farm *ABCD* above.

Draw a line *CP*, Fig. 47, through *C* that bears N. 52° E., and find the length *CP* and *AP*.

Applying equations (14) and (15) we get:

$$x\ cos.\ 40°27' - y\ cos.\ 52 = -9.92 \qquad (A)$$

$$x\ sin.\ 40°27' + y\ sin.\ 52 = 40.36 \qquad (B)$$

Eliminating y

$$x = \frac{40.36\ cos.\ 52 - 9.92\ sin.\ 52}{sin.\ 92°27'} = 17.046$$

Similarly,

$$y = \frac{40.36\ cos.\ 40°27' + 9.92\ sin.\ 40°27'}{sin.\ 92°27'} = 37.193$$

Find the area of *A B C P*, as follows:

Course.	Latitude.	Departure.	D. M. D.	Area.
AB..............	21.26	22.71	22.71	482.8146
BC..............	−11.34	17.65	63.07	−715.2138
CP..............	−22.89	−29.30	51.42	−1177.0038
PA..............	+12.97	−11.06	11.06	143.4482

Area $ABCP = 63.29724$ acres.

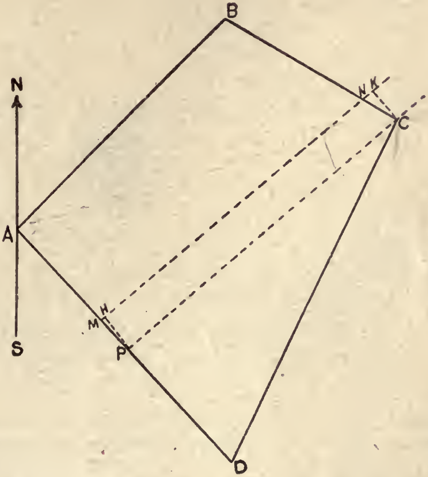


Fig. 47.

The line *CP* cuts off 12.29724 acres in excess. Let the line *MN*, parallel to *CP*, cut off the required area. Hence the area *MNCP* is 122.9724 square chains. From *C* and *P* drop perpendiculars on *MN*, cutting it at *K* and *H*.

Angle $MPH = 2°27'$; angle $NCK = 19°4'$

Let $z =$ altitude of trapezoid $MNCP = PH = CK$

Now,

$$MNCP = HKCP - NCK + MPH$$

$$\therefore\ 122.9724 = 37.193\,z - \frac{z^2}{2}\,tan.\ 19°4' + \frac{z^2}{2}\,tan.\ 2°27'$$

$$\frac{z^2}{2}\,(tan.\ 19°4' - tan.\ 2°27') - 37.193\,z = -122.9724$$

$$.15305z^2 - 37.193z = -122.972x$$

$$z^2 - 243.012z = -803.48$$

$\therefore z = 3.353$ chains

$NC = 3.353 \div cos.\ 19°4' = 3.548$

$PM = 3.353 \div cos.\ 2°27' = 3.356$

$NK = 3.353\ tan.\ 19°4' = 1.16.$

$MH = 3.353\ tan.\ 2°27' = .14$

The field notes of the 51 acres will read as follows:

Course.	Bearing.	Distance.
AB..................................	N47°2′E	31.0 chains
BN..................................	S57°4′E	17.452 chains
NM..................................	S52°W	36.173 chains
MA..................................	N40°27′W	13.690 chains

PROBLEM 57.—Find the bearing and length of a line that starts from mid-point of *CD* and bisects farm, Fig. 47.

PROBLEM 58.—Find bearing and length of a line that starts on *AD* 15 chains from *A* and cuts off 50 acres from west side of farm, Fig. 47.

PROBLEM 59.—In the example of Fig. 47 find the bearing and length of a line that starts at a point *H* on *AB* 15 chains from *D* and bisects farm.

PROBLEM 60.—Find bearing and length of a line *DP* in example of Fig. 47 that cuts off 3 acres on left of dividing line.

PROBLEM 61.—Find the length of line *PQ* that bears N 45° W. and bisects farm in example of Art. 100.

BIBLIOGRAPHY.—The works of the late J. B. Johnson and the late Charles Davies, which have already been described, have sections that deal with the problems of land dividing.

"Plane Surveying," by Daniel Carhart, gives not only a treatment of the land division, but also of the theory and use of instruments and methods of surveying, calculation, earthwork, etc., tables "A Treatise on Surveying," Part I, by the late W.

M. Gillespie, restricts its discussion to land surveying and direct leveling, and under the subject of land division it gives a great number of problems for the division of land, illustrated by figures and examples.

CHAPTER VI.

LEVELING.

101. The Y Level.—The essential parts of a Y level, Fig. 48, are the bubble tube and the line of sight. The latter is determined by the telescope and should be parallel to the axis of the bubble tube. The telescope rests in two *Y*-shaped supports called *Y's* or Wyes, which are attached to a horizontal bar. The lower part of the wye is formed into a threaded bolt that passes through a hole in the end of the horizontal bar. Two capstan nuts are attached to each wye, one above and one below the bar. By turning these capstan screws the wye can be raised or lowered at pleasure. Small, hard, steel pins, about 1-16 in. in diameter, are used for operating the capstan screws. The horizontal bar is attached by a screw-joint to a vertical axis turned in the form of a frustrum of a cone. The telescope with the wyes, horizontal bar, and socket can be removed from the level-head. The level-head consists of a horizontal brass plate enlarged into a ball and socket joint in the center and into a rim with screw threads on the circumference; the former is to provide an adjusting motion for the leveling screws, and the latter for attachments to the tripod head. Above the brass plate, which is attached to the tripod, is another plate provided with four vertical, cylindrical screws, into which the four leveling screws rest in small seats with ball and socket joints, and are operated by milled-head screws. A longitudinal cross-section of the level and telescope is shown in Fig. 49.

102. The Telescope.—The telescope, Fig. 49, consists of an eye-piece, an objective, and a tube to hold them in place. The eye-piece is a very small microscope of a very short length, one end of which is very near the cross wires. In the erecting telescope it consists of four lenses: the eye lens, the field lens, the amplifying lens, and the image lens, arranged in order from the eye. The objective consists of a special tube sliding in the

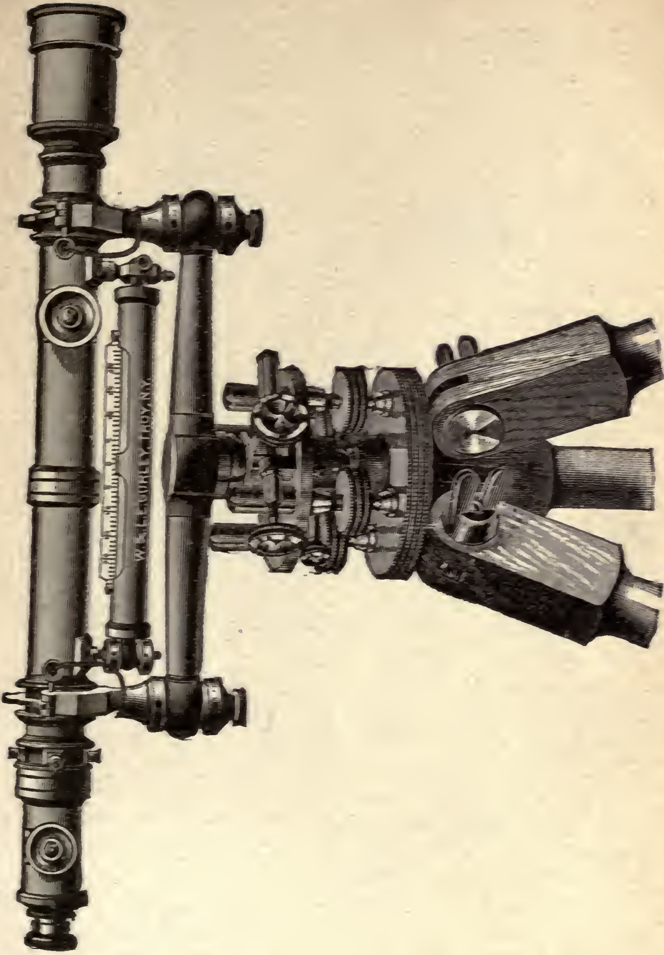


Fig. 48.

main barrel of the telescope with a double lens in the outer end. The objective is held true to its place by two collars inside the main tube. The lens has a long focal length and draws the image to the plane of the cross wires. If this lens were a double convex lens it would neither bring the rays to an exact focus nor make them colorless. Hold a double convex lens so that its central plane is perpendicular to the rays of the sun and hold a sheet of paper back of the lens and move it to and fro to find the focus. If the paper is held between the focus and the lens the edge of the bright circle will be colored red. Move the paper beyond the focus and we find the edge colored blue. In any lens all parallel rays of sunlight, having equal wave lengths, are brought to a focus at a fixed distance behind the lens, called the focal length, or the principal focal distance.

If the lens in the end of the objective were single, the rays of sunlight would not be brought to a focus, but the object would be fringed with colors; that is, the single lens makes the rays planatic (wandering) and chromatic (colored). To make the rays aplanatic and achromatic the object glass is made of two lenses, Fig. 50, a double convex and a plano-concave; the former of crown glass and the latter of flint glass. The refractive indices of these kinds of glass supplement each other and the rays are brought to a focus and are colorless.

The eye-piece is moved by means of milled-head screws attached to a rack and pinion movement, or by a spiral slot into which a pin works. In the first case the eye-piece is moved by the milled-head screws until the cross wires come into view; in the latter case the eye-piece itself is moved backward and forward in the telescope by turning it. The cross wires are attached to a brass ring, called the reticule, which is controlled by small capstan screws outside the telescope.

The tripod is a three-legged support connected to a plate to which the level-head is screwed when the instrument is in use. The legs are made of hard, straight-grained wood, and shod with hard, steel conical shoes.

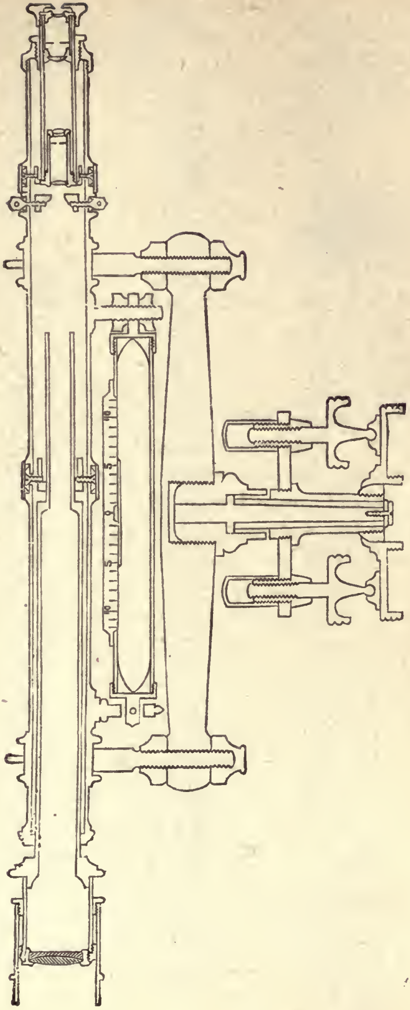


Fig. 49.

103. Setting Up the Instrument.—Set the tripod with legs well spraddled, and then place the level on the tripod, screwing the level-head into the tripod cap. Bring the telescope parallel to two opposite leveling screws; turn the screws both out or both in, making the left thumb move in the direction that the bubble is to shift. After the bubble reaches the center of its turn, turn the telescope over the other pair of opposite screws and repeat the left-thumb process. Repeat and check both on second leveling.

104. Rods.—Leveling rods used by engineers are divided into feet, tenths of a foot, and hundredths of a foot, and have a vernier attachment, which enables the rod to be read to the thousandth part of a foot.

The Philadelphia rod, Fig. 51, is usually 7½ ft. long, and is made in two pieces, which may be effectively extended to a length of 12 ft. This rod has the foot division lines marked by red figures; the even tenths of a foot are marked by black figures; and each alternate hundredth of a foot is painted black half way across the rod on a white background. This enables the rod to be read to the nearest hundredth of a foot from a distance through the telescope by the levelman. When the rod is extended, a continuous graduation to 12 ft. is visible. This rod is provided with a target, a circular piece of metal about 4 ins. in diameter, alternate graduations of which are painted red and white. The target slides along the rod, and its exact distance from the end of the rod may be read by means of a hole in the center. A vernier attached to the target enables the rodman to read to the thousandth part of a foot. This rod is intended for quick work and hard service. It should be made of the best wood, brass trimmings and varnished to resist water.

The New York rod, Fig. 52, is similar to the Philadelphia rod, but it is lighter and much more care is taken in its graduation. The rod can not be read directly from the instrument. It is intended for precise leveling.

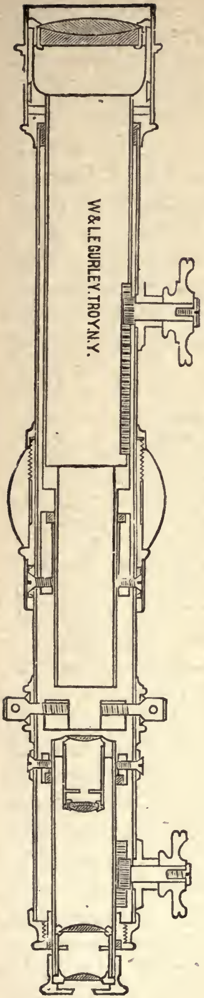


Fig. 50.

Fig. 51.

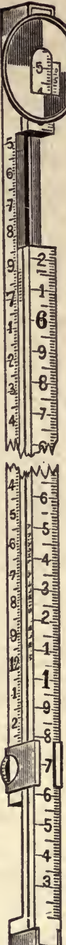


Fig. 52.

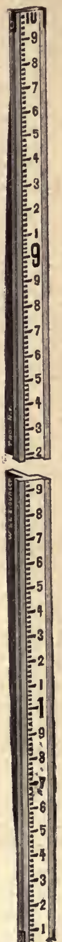


Fig. 53.

Figure 53 shows a form of self-reading rod that can be used when it is desired to read the rod directly from the instrument. Its graduations are similar to those of the Philadelphia rod, but it is somewhat lighter. The Philadelphia rod can be used as a self-reading rod and it is often convenient to use it as such.

105. Theory of Leveling.—When an engineer's level has been set up, and the bubble brought to the center of the bubble-tube, the line of sights is horizontal. The elevation of this horizontal line can be found by noticing how much it strikes above some point whose elevation is known, and adding this distance to the known elevation of the reference point or datum. Having determined the elevation of the horizontal line, the

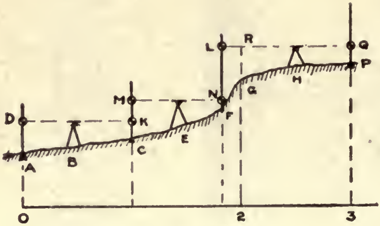


Fig. 54.

elevation of any other point may be easily found by noticing now much the horizontal line is above the point in question, and subtracting this amount from the elevation of the line of sights. The term "height of instrument" is given to the elevation of the horizontal line of sights.

Suppose the elevation of some point *A*, Fig. 54, has been determined and is 100 ft. above some plane known as the datum plane, the elevation of which is called zero. It is desired to find the elevation of some point *P*. Set the instrument at *B* and get the rod reading *AD*, which is 8.46 ft. Adding 8.46 ft. to 100 ft. gives 108.46 ft. as the height of the horizontal line of sights, so we say that the height of the instrument (H. I.) is 108.46 ft. Sight next on point *C* and read the distance *CK* on the rod, which is 2.05 ft.; subtracting this from the H. I., 108.46 ft., gives us 106.41 ft., the elevation of the point *C*. The elevation of the point *C* having been found, the point *C* may be used to find the elevation of another point in the same way that the elevation of the point *A* was used to find *C*. Thus, set the instrument at *E* and read the rod on *C* and let $CM = 6.58$ ft. Then the new H. I. $= 106.41 + 6.58 = 112.99$.

It often happens that the line of sights strikes the ground in front of a regular station as at *G*. If this occurs, hold the rod on some intermediate point as *F*, and take a rod reading. It is necessary in such cases to select a point that is firm and hard. A smooth stone, firmly imbedded in the soil, makes an excellent point for such purposes. Suppose the rod reading on such a turning point was $NF = 1.29$. The elevation of $F = 112.99 - 1.29 = 111.70$. Then set the instrument at some point *H*, level up, and take the rod reading again on *F* (back sight), where $LF = 11.42$. The height of instrument (H. I.) $= 111.70 + 11.42 = 123.12$. The rod reading *GR* on the regular station $G = 6.48$ and the elevation is 116.64, while the rod reading *PQ* on point *P* is 3.32 and the elevation of *P* is 119.80.

It will be well to bear in mind that a back sight is a rod reading taken on a point whose elevation is known, and that a fore sight is a rod reading taken on a point whose elevation is unknown. Always add the back sights to the elevation of the point to get the height of instrument; and subtract fore sights from the height of instrument to get the elevation of the point on which the fore sight was taken. The H. I. is always in the line above the fore sight, and the H. I. will not be changed till the instrument is moved to a new position.

The starting point *A*, the elevation of which has been previously determined, is called the Bench Mark, abbreviated B. M. Intermediate points, such as *C* and *F*, are called Turning Points, T. P. Whenever possible rounded stones, solidly imbedded in the earth and almost covered, are the best T. P.'s.

The following is a convenient arrangement of column headings for level notes:

Station.	B. S.	H. I.	F. S.	El.	Remarks.
0	8.46	108.46	....	100.00	
1	6.58	112.99	2.05	106.41	
+80	11.42	123.12	1.29	111.70	
2	....	......	6.48	116.64	
3	....	......	3.32	119.80	

Figure 55 illustrates a typical level notebook.

PROBLEM 62.—Fill out the column for H. I. and El. in the table below:

Sta.	B. S.	H. I.	F. S.	El.
26	3.26	....	....	76.42
27	...	....	7.42	....
28	1.08	....	11.84	....
29	...	....	5.21	....
30	...	....	9.68	....
+72	1.24	....	11.94	....
31	...	....	4.46	....
32	...	....	8.92	....
33	...	....	11.52	....

PROBLEM 63.—If there is a B. S. on station 27 of 3.22, fill out a table for the remaining H. I.'s and El.'s.

106. Bench Marks.—The relative elevation of any number of points near each other or widely separated may be found by comparing their heights above the datum plane. The datum most extensively used is mean sea level and its elevation is said to be zero. A bench mark is a point, the elevation of which has been carefully and accurately measured, marked and checked, and which may be used as a starting point for any leveling that may be contemplated in its immediate vicinity. The best form of bench mark is a copper bolt firmly imbedded in masonry, which is not likely to settle. The United States Geodetic Survey has established a great many such bench marks throughout the country. The elevation of each of these should be carefully marked either on the head of the bolt or on a copper plate attached to the masonry. In running a line of levels across country excellent and lasting bench marks can be made by chopping away a portion of a large root of a large tree until the part remaining is in the form of a low, broad-based pyramid, and then driving a nail or spike into the vertex.

Station	B.S.	H.I.	F.S.	Elev.		Notes Level Rod — Baird E. Strayer W Strayer
B.M.	8.43	17048		162.05		On top of nail on root of elm tree
T.P. 0+00			1.83	168.65		On tie a P.C.
0+50			1.05	169.43		
T.P.	9.35	179.83	0.00	170.48		Top of Rail
1+00			10.00	169.83		
1+50			4.30	175.53		
2+00			1.50	178.33		
2+42			0.55	179.28		Top of Hub a P.T
3+00			3.90	175.93		
4+00			6.40	173.43		
5+00			6.45	173.38		
T.P	4.23	177.82	6.24	173.59		Top of Sta 5+00
B.M			2.76	175.06		On N.E. Cor. of Stone Foundation
6+00			4.75	173.07		
7+00			4.60	173.22		
8+00			4.65	173.17		
9+00			4.27	173.55		
10+00			3.10	174.72		Top of Tie

Fig. 55.

107. Profiles.—A profile is a drawing that shows the rise and fall of the ground on which the line was surveyed. The surveyed line may be straight, curved, or broken.

To make a profile elevations of points on the line at short regular intervals must be found, as well as the points where there is a sudden change in the surface.

Profiles are usually drawn to a horizontal scale of 1″ = 400′, and a vertical scale of 1″ = 20′. Paper properly divided into squares by horizontal and vertical lines can be purchased by the roll or sheet.

108.—Crosswire Adjustment.—To make the intersection of the cross wires intersect in the axis of the telescope or the line of collimation, set up the instrument, level, and bring the cross wires into view by turning the telescope to clear sky. Focus the objective on some wall, and then have an assistant mark a spot on the wall at the intersection of the cross wires with a soft pencil; loosen the clips or loops that control the telescope, note that it still points to the spot on the wall, then turn the telescope in the wyes with the right hand until the bubble tube is on top. If the cross wires still intersect on the spot the instrument is in adjustment; if it intersects above or below, loosen the small capstan screws that control the wire ring and turn them so that the cross wire will be moved back one-half of the displacement. Bring it back to the spot by the leveling screws, and check by repeating the process.

To correct the vertical wires turn the telescope so that the bubble is to the right or left of the instrument and in the same horizontal plane, and bring the cross wires on the spot by the leveling screws, then turn the telescope on its horizontal axis 180°, and if there is any displacement correct one-half by the capstan screws that control the vertical wire and the other half by the leveling screws. Check by repeating the process.

109. Bubble-Tube Adjustment.—To make the axis of the bubble tube parallel to the line of collimation, loosen the clips and level accurately, then take the telescope in the hand and turn it end for end in the wyes. If the bubble remains in the center of the tube it is in adjustment, but if it does not, raise or lower one end of the bubble tube by means of the small capstan screws to correct one-half of the displacement. The rest is corrected by the leveling screws. Repeat until it checks.

Level accurately, revolve the telescope slowly in the wyes and watch the bubble. If it has a tendency to move towards one of the ends, the bubble tube will have to be moved horizontally by the small horizontal capstan screws at one end of the tube. In some instruments the screws at one end of the bubble tube are to raise it vertically, while the screws at the other end move it horizontally.

110.—Adjustment of Wyes.—To make the axis of the bubble tube and the line of collimation perpendicular to the vertical axis, level accurately over a pair of screws and then turn the telescope 180°. If there is any displacement of the bubble raise or lower the wyes by the capstan screws at the end of the horizontal bar and correct one-half of the displacement. Repeat the process until it checks. As a general check, repeat all the adjustments.

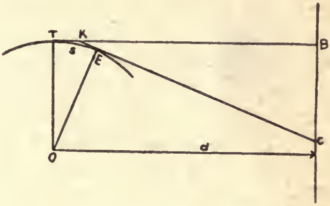


Fig. 56.

111.—The Radius of the Bubble-Tube.—Let *TB*, Fig. 56, the tangent to the interior of the bubble tube cut the rod at *B*, distant *d* from the level, say, 100 ft. or over, turn the leveling screws until the bubble travels a space $s = n$ divisions to some point *E*. The tangent at *E* intersects the rod at some point *C*; take the difference in the readings of *B* and *C*, which gives us *BC* (*r*) in feet, measure the distance *s* the bubble travels, *TE*, in inches and reduce to feet. The two tangents *TB* and *EC* are perpendicular to the radii consequently.

$$\text{Angle } TOE = \text{Angle } BKC = \theta$$

As the angle θ in the sectors is very small we have

$$TO : TE :: KB : BC$$

or

$$R : s :: d : r$$

$$R = \frac{ds}{r} \quad \text{.............................. (16)}$$

Where $R = TO$, the radius of the bubble tube.

Now, KB is not exactly equal to TB, but when TB is 100 ft., KB will be something like 99 ft. 11 ins., so they are practically equal.

To find the angular value of one space on the bubble tube, note how many spaces n the bubble travels in the first operation. In TOE we have

$$\theta = \frac{57.3 \times s}{R}$$

By division

$$\frac{\theta}{n} = \frac{57.3 \times s}{nR}$$

After finding one angular division of the bubble tube, or better the angle subtended between two special marks, we can use the level for measuring distances across swamps, rivers, etc. Thus, bring the end of the bubble to one of the end marks and locate the flag on the level, and have the rod read, shift the bubble until the end reaches the other mark and read the rod again, take the difference in the rod readings and call this r, the angular division of the shift is O; then in the triangle BKC we have

$$KB = \frac{57.3 \times r}{\theta} \quad \text{.............................. (17)}$$

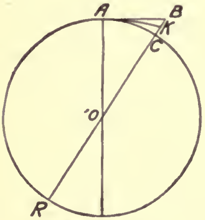


Fig. 57.

PROBLEM 64.—An 18-in. Gurley level gave the following results: Distance (d) $= 100$ ft., rod reading $(r) = 0.071$ ft., shift of bubble $= 0.7$ in., corresponding to seven divisions on the bubble tube scale. Find radius of bubble tube and the angle subtended by one division of the scale.

PROBLEM 65.—If one angular division of the bubble tube scale subtends an angle of 21″ at center of bubble tube circle, find the distance when difference of rod readings was 1.28 ft., when the bubble was shifted five divisions.

112. Curvature of Earth.—Let AB, Fig. 57, be a horizontal line of sight, *ACR* the surface of the earth. Let distance $AB=D$, $BC=c$ and radius of earth r.

In right triangle OAB,

$$\overline{OB}^2=\overline{OA}^2+\overline{AB}^2$$
$$(r+c)^2=r^2+D^2$$
$$2cr=D^2-c^2$$

Now the term c^2 is very small in comparison with D^2 and can be omitted without sensible error.

$$\therefore\ c=\frac{D^2}{2r},\ nearly$$

If we wish the correction in feet while D is in miles, we get

$$c=\frac{5280\times5280D^2}{2\times3926\times5280}=.66D^2=\frac{2}{3}D^2 \dots\dots\dots\dots (18)$$

If $D=1$ mile, $c=2/3$ of 1 ft.$=8$ ins.

If $D=2$ miles, $c=32$ ins.

If $D=3$ miles, $c=72$ ins.

Effect of Refraction.—Refraction has a tendency to make all bodies near the horizon appear higher than their natural positions. Thus if in Fig. 57 the level is at A, the line of sight will be the curved line AK, the radius of which is about seven times the radius of the earth. In formula for curvature, r becomes $7r$.

$$\therefore\ BK=\frac{D^2}{2(7r)}=\frac{D^2}{14r}$$

If c is in feet while D is in miles,

$$BK=\frac{D^2\times5280\times5280}{14\times3926\times5280}=\frac{2D^2}{21}\dots\dots\dots\dots (19)$$

If $D=1$ mile, $BD=2/21$ ft.$=1.14$ ins.

If $D=¼$ mile, $BK=0.07$ in.

If $D=3.25$ miles, $BK=1$ ft., i. e., under ordinary conditions of atmosphere all points 3¼ miles from the observer appear 1 ft. higher than their natural positions.

113.—Vertical Curves.—If two grades meet at a summit B, Fig. 58, it becomes necessary to round off this summit by uniting the two grades by a curve tangent to each. The simplest vertical curve that can be adopted for this purpose is a common parabola that touches the grade lines at A and C where the horizontal distance $AK = KE$, and $AM = MC$. Hence BM is a diameter of the parabola of which BA and BC are tangents. Then

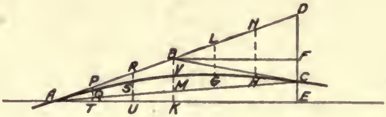


Fig. 58.

$$PQ : BV = \overline{AT}^2 : \overline{AK}^2 \quad \therefore \quad PQ = BV \times \frac{\overline{AT}^2}{\overline{AK}^2}$$

$$PQ : DC = \overline{AT}^2 : \overline{AE}^2 \quad \therefore \quad PQ = DC \times \frac{\overline{AT}^2}{\overline{AE}^2}$$

Let $g =$ grade of AB, rise per station, $g' =$ grade of BC, fall per station, and $n =$ number of stations in AB and BC. Now $BK = ng$.

Draw BF parallel to horizontal line AE

$$\therefore BK = FE = DF = ng.$$

But $FC = ng'$, $\therefore$ $DC = DF + FC = ng + ng' = n\,(n + g')$.

$$\text{But } PQ = DC \times \frac{\overline{AT}^2}{\overline{AE}^2}$$

Now, $AE = 2n$, and if AT one station $= 1$

$$PQ = n(g + g')\,\frac{1}{4n^2} = \frac{(g+g')}{4n}$$

PQ is the change of grade for the first station. Let this change $= a$

$$\therefore a = \frac{g+g'}{4n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$$\text{Change for 2nd station} = \frac{4(g+g')}{4n}$$

$$\text{Change for 3rd station} = \frac{9(g+g')}{4n}$$

Example: Given $g=1.0$, $g'=0.8$, $n=3$, elevation of $B=$ 76.8′, find the elevation of different points on the curve.

$$a=\frac{1}{4\times 3}(1.0+.8)=\frac{1.8}{12}=.15$$

Elevation of *R*, *P*, and *A* are 75.8, 74.8, 73.8 respectively, and the decrease in grade (or elevation) to bring road-bed to curve at points *P*, and *R*, and *B* are .15, 4 × .15, 9 × .15 or .15, .60, 1.35.

Hence the elevations of points on the curve are 73.8 (74.8 — .15), (75.8 — .60) (76.8 — 1.35) or 73.8, 74.65, 75.20, 75.45.

Station.	Original Grade.	Change of Grade.	Grade on Curve.
$A=56$	73.8	.00	73.80
57	74.8	.15	74.65
58	75.8	.60	75.20
$B=59$	76.8	1.35	75.45
60	76.0	.60	75.40
61	75.2	.15	75.05
$C=62$	74.4	.00	74.40

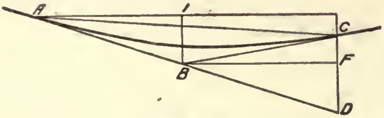


Fig. 59.

PROBLEM 66.—If two grades at a summit are 1.4 and —1.0 and elevation of summit is 94.6, find elevation of points on curve if $n=3$.

114. Curve in Sag.—If the curve occurs at a sag the same formulas will apply in finding the change for each station, but we must remember that the tangents are below the curve and that all elevations must be increased instead of diminished. Thus in Fig. 59, if grade of $AB=-.7$, and of $BC=.5$, elevation of $B=54.8$, and $n=3$, we have

$$a=\frac{g+g'}{4n}=\frac{.7+.5}{4\times 3}=.10$$

Then we have the results as follows:

Station.	Original Grade.	Change of Grade.	Grade on Curve.
$A=22$	56.9	.00	56.9
23	56.2	.10	56.3
24	55.5	.40	55.9
$B=25$	54.8	.90	55.7
26	55.3	.40	55.7
27	55.8	.10	55 9
$C=28$	56.3	.00	56.3

115. Vertical Circular Curves.—If two tangents AB and BL meet at summit B, Fig. 60, a circular curve can be used to unite the two grades. Let O be the center of circular curve. Now g = grade of AB or the amount of rise of AB per station, or 100 ft. If the distance is measured in stations, g is the tangent of the angle the first line, AB, makes with the horizontal. In the rt-triangle AOB, angle AOB equals half of grade angle DBL. $AB=OA\tan AOB \therefore T=R\tan \frac{1}{2} DBL$, where $AB=T$, and $OA=R$.

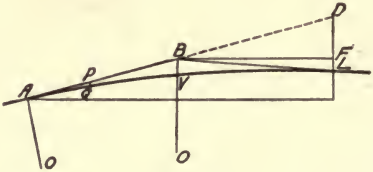


Fig. 60.

The angle DBL is very small, usually less than 4°. $\therefore$ we can write:

$$\text{Tan } AOB=\tan \tfrac{1}{2}DBL=\frac{1}{2}(g+g')$$

$$\therefore T=R\frac{(g+g')}{2} \qquad \therefore R=\frac{2T}{g+g'}$$

By geometry, $\overline{AP}^2=PQ(2R+PQ)=2R \times PQ+\overline{PQ}^2$

Now $\overline{PQ}^2$ is so small in comparison with $2R \times PQ$ that it can be omitted. $\therefore \overline{AP}^2=2R \times PQ$ or $PQ=\dfrac{\overline{AP}^2}{2R}=\dfrac{\overline{AP}^2(g+g')}{4T}$

But $PQ=a$, and $AP=y$

Now $T=AB=n$, number of stations in AB

$$\therefore a=\frac{(g+g')}{4n}\times y^2$$

The last formula is the same one we found for parabolic curve. The curve is really so flat that it can be regarded as a circle or parabola without error.

CHAPTER VII.

TOPOGRAPHIC SURVEY.

116. Topographic Survey.—A compass or transit survey will locate points with reference to each other in a horizontal plane. In other words such surveys show the geographic location of points with respect to each other, but they do not show how such points are situated in elevation with respect to each other. A topographic survey will give not only the relative position of points with respect to their geographic positions, but will also give their elevation vertically. A glance at the map will show the positions of the different objects in the geographic relations, but certain other data must be placed on these maps to indicate the configuration of the terrain.

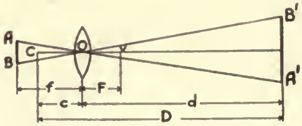


Fig. 61.

117.—Topographic Methods.—There are four general methods of making a topographic survey: (1) By transit and level; (2) by stadia; (3) by plane table; (4) by hand level. The first method is costly, laborious, and slow. With the exercise of care, however, it is the most accurate method, but its cost and the labor required render its use almost prohibitive except for small tracts. The third method is coarse but rapid, and for large areas is by far the most practicable. It is sufficiently accurate for geologic purposes, and a survey by this method is a valuable adjunct to a more detailed survey by either of the other methods. It is useless to discuss here the methods of making a topographic survey by the transit and level, as the use of these instruments is fully discussed in the chapters devoted to their consideration. We shall in this chapter consider the stadia method only.

118. Stadia Formulas.—The two stadia wires are placed in the reticule of the telescope of the transit above and below the horizontal cross-wire and parallel thereto. If these wires be represented by A and B in Fig. 61, and lines be drawn from A and B through the optical center O of the objective, these lines will cut the stadia rod at A' and B'. The lines AA' and BB' are called secondary axes. If we let i represent AB and r represent $A'B'$, then from the similar triangles, OAB and $OA'B'$, we have, $i:r::f:d$.

But by the law of lenses,

$$\frac{1}{f}+\frac{1}{d}=\frac{1}{F}, \quad \dots\dots\dots\dots\dots\dots \quad (22)$$

where F is the "principal focal distance." If parallel rays of light impinge on a lens they will be brought to a focus at some point V, which is called the "Principal Focus" of the lens, and the distance OV is called the principal focal distance. This distance can be found for any given lens by holding the lens so that the central plane of the lens will be perpendicular to the sun's rays. The rays of sunlight will be brought to a focus, which can be found by moving a white sheet of paper parallel to the central plane of the lens. If the sheet of paper is beyond the focus from the lens the circular disc of light will be fringed with blue, while if between the focus and lens it will be fringed with red or yellow. When the sheet of paper is at the focus the rays of light will be concentrated into a very small circular disc of intense light. To find F for the object glass of the telescope, point the telescope to the clear sky and focus on the cross-wires, and then measure from reticule to center of object glass.

From the first of the above equations, we have,

$$\frac{1}{f}=\frac{r}{id}$$

and from the second,

$$\frac{1}{f}=\frac{d-F}{Fd}$$

Equating and reducing, we get,

$$d = \frac{F}{i} r + F.$$

Now the "principal focal distance," F, is fixed for any lens and i (the distance between the stadia wires on the reticule) can be so adjusted that the ratio of F to i will be made any value desired. From the last equation we have,

$$d - F = \frac{F}{i} \times r.$$

But $(d - F)$ is the distance from the "principal focus" V to the stadia rod, and as $F \div i$ is constant, we see that, in reality the distance from the principal focus to the stadia rod varies *directly* as the intercept r on the stadia rod.

If we wish to obtain the distance D from the center of instrument to the rod, we have,

$$D = d + c = \frac{F}{i} r + F + c, \quad \ldots\ldots\ldots\ldots (23)$$

where c is the horizontal distance from center of objective to plumb-bob.

In the majority of transits the distance $F + c$ varies from .80 to 1.25 and 1.00 can be assumed as a fair average without sensible error.

119. Wire Interval.—To fix the stadia wires in a transit we must first find F, and then decide on some distance from the rod to the principal focus, say 400 ft. After this has been done we focus on the rod, then measure the principal focal distance from the lens of the objective, which establishes the principal focus in the line of sight, and from this distance we measure the 400 ft. and set up the rod exactly at the end of this 400 ft. Or we can measure from objective to the rod 400 ft. plus the principal focal distance. We now adjust the stadia wires so that while one of them (the lower) reads 2.00 the upper will read 6.00, the difference being 4.00. Then,

$$400 = \frac{F}{i} 4 \quad \therefore \quad \frac{F}{i} = 100.$$

If the wires are fixed, find F, c, D and r for a given reading, then

$$i = Fr \div (D - F - c).$$

120.—Inclined Sights.—If the line of sights OC is inclined to the horizon at an angle v, as in Fig. 62, we shall for the purpose of mapping have to find the horizontal distance OE and the vertical distance CE. The rod AB is always held vertically. The lines of sight as determined by the stadia wires are OA and OB. Draw $A'B'$ perpendicular to OC, the line of sight as determined by the cross-wires, and let $A'B' = r'$. The angle $BCB' = v$ and the angles at A' and B' differ so slightly from a right angle that for all practical purposes we can assume them equal to 90.

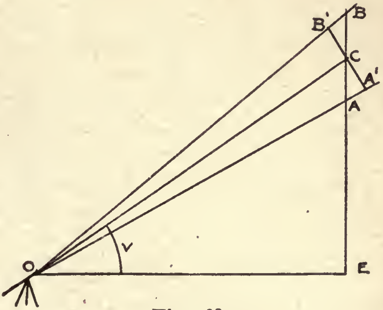


Fig. 62.

$$\therefore\ B'C = BC\ \cos.\ v.$$
$$2B'C = 2BC\ \cos.\ v.$$
or $r' = r\ \cos.\ v.$

$$\text{But } OC = \frac{F}{i} r' + F + c$$
$$= \frac{F}{i} r \cos.\ v + (F + c)$$

$$\text{Then } D = OE = OC \cos.\ v = \frac{F}{i} r \cos.^2 v + (F + c) \cos.\ v.$$

$$H = CE = OC \sin.\ v = \frac{F}{i} r \sin.\ v \cos.\ v + (F + c) \sin.\ v$$

$$=\frac{F}{2i} r \sin. 2v + (F+c) \sin. v$$

$$Let \frac{F}{i} r = K$$

$$\therefore D = K \cos.^2 v + (F+c) \cos. v$$
$$H = \tfrac{1}{2} K \sin. 2v + (F+c) \sin. v$$

Now the last terms in the formulas for D and H are insignificant in comparison with the first term and unless refined accuracy is required these terms can be omitted.

If $F + c = 1.00$, and if $F \div i = 100$, $r = 5.40$, and $v = 6° \ 25'$, we have, $D = 540 \times .9875 + (1 \times .9937) = 534.24$.

$$H = \tfrac{1}{2} \times 540 \times .222 + (1 \times .1118) = 60.05.$$

If the last terms are omitted we have $D = 533.25$ and $H = 59.94$, the errors being 1 in 538 and 1 in 537 respectively. For ordinary maps one-fiftieth of an inch is about as fine as we can indicate on the drawing paper. Thus, if we adopt a scale of 1 in. equals 10 ft., or one-tenth of an inch to the foot, the distance (D) above will be represented by a line 53.4 ins. But if we adopt a scale of 1 in. equal to 100 ft., which is the usual scale in railway topography, we would have. $D = 5.34$ ins. and the error committed by the omission of the last term in the formula for distance would be one hundredth part of an inch.

121. Stadia Rod.—The essentials of a good stadia rod are that it should be clearly, accurately and distinctly graduated and that the graduations should be sufficiently clear to be read to the extreme limits of its longest range. There are many special rods on the market, each possessing special merits in the opinion of the designer, but the Philadelphia rod can be used while the marks are new and clear cut. Fig. 63 shows one form of stadia rod that is extensively used. It is 3.5 ins. wide, ¾ in. thick in the body where the graduations are placed, and ⅞ in. thick on the edges. The rod is made of straight grained wood, is 12 ft. long over all and is hinged in the middle so that it can be folded for convenient transport. The raised flanges (¼x1/16 in.) afford excellent and effective protection to the graduations. The foot marks are indicated in red figures, 1.25 to 0.75 in., while the tenths are indicated by black figures, 0.75 in. high by 0.5 in. width. The space is divided in alternate black and white strips one-hundredth of a foot in width. Each red figure is opposite a black strip 2.5 ins. long, and the figure refers to the *top* edge of the strip and indicates its distance from the bottom of the rod. In the same way each black figure is opposite a black strip of same width but only 1.25 in. in length, the black figures indicating the distance in length of a foot of the top of its strip from the top of the strip through the red figure below. The space between the black figures (the top through the black lines) is divided into ten equal spaces alternately painted black, while the white background forms another strip of the same width. If the wire reads between the red 3 and 4, between the black 6 and 7, and is at the top of the third black strip, the reading is 3.66. It is well to remember that the top of the short black strips (about ¾ in. long) indicate even hundredths, i. e., .02, .04, .06, etc., while the bottom of the black strip indicates the odd hundredths. These remarks apply (except as to lengths of the black strips) to the Philadelphia rod, which for distances under 600 ft. forms an excellent stadia rod.

122. Field Work.— When it is desired to make a topographic survey of a certain district by the stadia method, certain base lines or lines of reference are adopted as a basis to tie into. If the district has been surveyed by triangulation, the triangulation stations form the points from which the survey proceeds. The transit is set up over one of these triangulation stations and sighted to another station of the triangulation survey. The azimuth of this line has been previously determined and the transit can be adjusted by upper

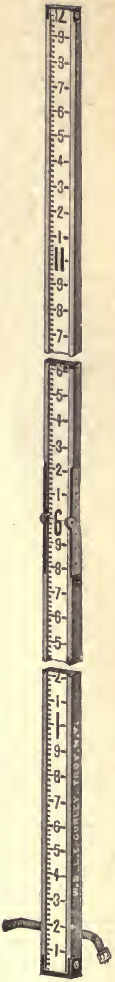


Fig. 63.

motion so that the zeros of the verniers point north and south. When the transit has been set and adjusted so that the zeros will mark out the true meridian, the instrument man can send his rod man to certain strategic points in the terrain. The distance, azimuth and angle of elevation must be read and recorded.

To obtain the distance the lowest stadia wire is brought preferably on some even foot-mark, as the 1 or 2, and the upper wire is then read 7.42. The difference is 5.42 and the distance by stadia 542 ft. To obtain the angle of elevation, the middle cross-wire must be brought on the mark on the rod that indicates the

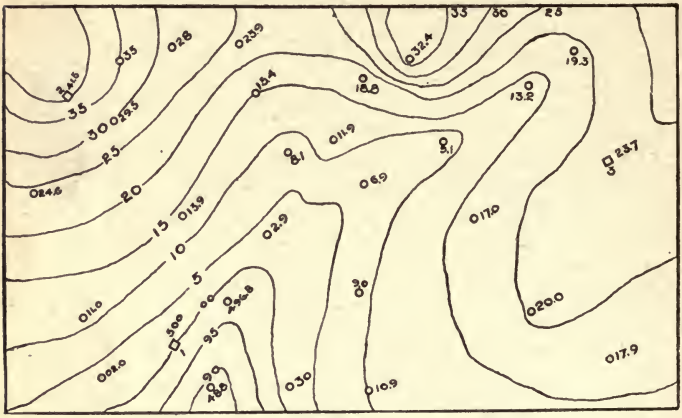


Fig. 64.

height of the center of the horizontal axis of the telescope. It is necessary for the transit man at every set up to take the height of the telescope above the surface under the plumb-bob. The azimuth is read from the south by west, north, east and on to south again.

The primary triangulation stations are indicated by the symbol Δ, while the stadia stations are marked [⊡] with a number following to define it, as [⊡] 3, [⊡] 7, etc. If there has been no triangulation survey the topographic survey proceeds from the same local point to which the stadia stations are connected or "tied in." Other points are variously described in the "object" column as "house," "tree," "cor. fence." If a reading is taken simply for a contour point it is marked C. P.

Smith, Instrument.
Henry, Recorder.
Fox, Rod.

Oct. 14, 1907.

Object.	Azimuth.	Distance. Ft.	Vert. Angle.	Diff. of El.	El.
At [•] 1 Ht. of Inst. = 5′.1			Elevation = 500′.00		
CP	229° 15′	99	—1—52′	—3′.2	496′.8
"	219° 12′	206	0—48′	2′.9	502′.9
"	210° 00′	332	1—24′	8′.1	508′.1
"	228° 45′	370	1— 4′	6′.9	506′.9
"	218° 5′	387	1—46′	11′.9	511′.9
"	254° 10′	281	1—50′	9′.0	509′.0
"	283° 30′	294	2— 8′	10′.9	510′.9
"	290° 8′	181	0—56′	3′.0	503′.0
"	320° 15′	81	—8—38′	—12′.0	488′.0
"	64° 38′	163	0—42′	2′.0	502′.0
[•] 2	157° 17′	401	5—58′	41′.46	.....
[•] 3	246° 51′	754	1—47′	23′.45	.....
At [•] 2 Ht. of Inst. = 4′.8	Mean = 41′.52			541′.52	
[•] 1	387° 17′	401	—5—59′	—41′.58	.....
CP	229 12′	96	—3—54′	—6′.5	535′.0
"	244° 30′	171	—4—32′	—13′.5	528′.0
"	252° 30′	264	—3—50′	—17′.6	523′.9
"	266° 00′	445	—2—56′	—22′.7	518′.8
"	269° 38′	280	—5—22′	—26′.1	515′.4
"	297° 18′	78	—8—58′	—12′.0	529′.5
"	18° 5′	150	—6—30′	—16′.9	524′.6
"	316° 15′	250	—6—22′	—27′.6	513′.9
"	356° 10′	331	—5—18′	—30′.5	511′.0
At [•] 3 Ht. of Inst. = 5′.0	Mean = 23′.69			523′.69	
[•] 1	66° 51′	755	—1—49′	—23′.93	.....
CP	66° 30′	227	—1—44′	—6′.9	516′.8
"	26° 47′	250	—0—38′	—2′.8	520′.9
"	00° 35′	294	—1— 8′	—5′.8	517′.9
"	97° 8′	250	—3—22′	—14′.6	509′.1
"	133° 20′	163	—3—42′	—10′.5	513′.2
"	162° 40′	175	—1—26′	—4′.4	519′.3
"	117° 5′	331	1—30′	8′.7	532′.4

123. Reduction Methods.—The formula for finding the elevation of a point above the instrument,

$$H = \text{inclined distance} \times \sin. v.$$

When v is less than 6°, we can find H readily by the application of the 57.3 rule. But to save time several labor-saving devices have been invented. Two of these make use of the principle of

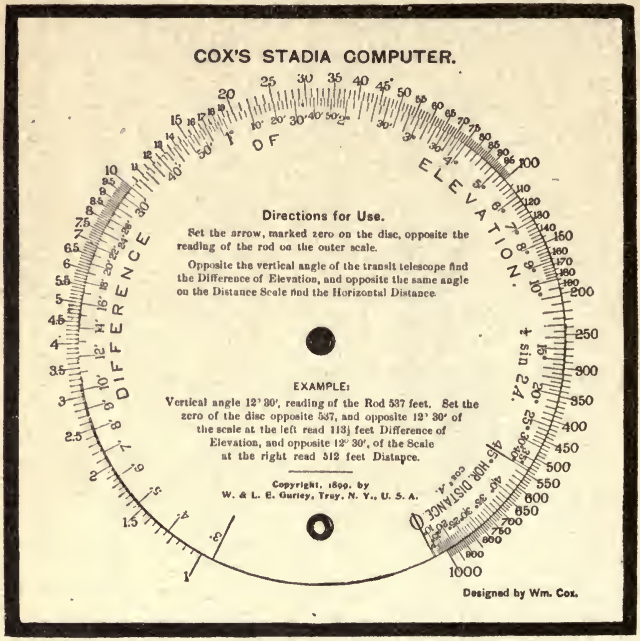


Fig. 65.

the slide rule, Colby's Slide Rule, which can be obtained from the leading dealers in drawing supplies and mathematical instruments, and Cox's "Stadia Computer," manufactured by W. & L. E. Gurley, Troy, N. Y. This "Stadia Computer," Fig. 65, is simply a circular slide rule about 15 ins. in effective length. It consists of a mounted card board, 6¼x6¼ ins., upon which scale is laid off the logarithm of numbers from 1 to 1,000 on the circumference of a circle 5 ins. in diameter. Mounted on this scale is a circular disc concentric with the 5-in. circle on the limb, on which is laid off the logarithm of the sines of angles from 3′ up to 45°. To find the difference of elevation for any distance and angle of elevation, turn the moving disc till the zero of the disc is opposite the required distance. Hold the disc in this position and opposite the given angle of the disc read the number on the limb. This is the required difference in height. The horizontal distance is read opposite the angle in the space marked "Hor. Distance."

EXAMPLE: Given distance = 480, angle of elevation = 5° 10′, find the difference of elevation. Turn the disc till the zero is opposite 480 on the limb and then opposite 5° 10′ on the disc read 43 ft. The whole computer can be carried in the coat pocket and its convenient size makes it a very effective calculator. No correction for horizontal distance is necessary for this angle of elevation.

124. Colby's Slide Rule.—Colby's Slide Rule as shown in Fig. 66 consists of a base piece of trapezoidal cross section on which is laid off the logarithm of the numbers representing the distance read by the stadia, and a sliding runner on which is laid off the angles of elevation to 18° 30′. On the sliding runner is a mark labeled "same unit index," which can be seen on the right on the runner above the space between the numbers 3 and 4. To find the vertical distance between the instrument and rod, set the mark under "same unit index" to agree with the distance read by the stadia, and then opposite the angle of elevation on the slide read the vertical distance on the log scale below.

Fig. 66.

EXAMPLE: Given distance 600 and angle of elevation 3° 10′, to find the difference of elevation. Set index on slide opposite 600 on log scale, and opposite 3° 10′ on the slide, read 33.1 on log scale, which is the difference of elevation.

125. Usual Approximations.—The cosine of all angles less than 18° is greater than 0.95 and we may assume $F+c=1$ and $(F+c)\ cos\ v=.95$. Now, if the horizontal distances are to be read to the nearest tenth of a foot, we can assume $(F+c)$ $cos\ v=1$. The following approximations may be made:

(1) If the last term $=1$ and $D=K$, in the formula,

$D=K\ cos.^2 v+(F+c)\ cos. v$, we have

$D=K\ cos.^2 v+1$

or $K=K\ cos.^2 v+1$

$$Cos^2 v = 1 - \frac{1}{K}.$$

Now if $K=200$,

$$200=200\ cos.^2 v+1 \therefore v=4°04'$$

If $K=700$, $v=2°10'$

Thus, if the angle of elevation is 2° 10′ and the inclined distance 700, we can omit the last term and make the horizontal distance equal to the inclined. The two approximations or assumptions balance each other. Check:

$$D=700\cdot cos^2\ 2°\ 10'+cos\ 2°\ 10'$$
$$=700 \times .9986+.9993$$
$$=699.02+.9993=700.02$$

For an agle of elevation of 2° 10′ and a distance of less than 700 (say, 500) we have

$$D=500 \times .9986+.9993=500.3$$

For all distances less than 700 and a given angle of 2° 10′ the horizontal distance D will be greater than K, but the error is less than 1 foot. For all distances above 700 the horizontal distance (D) is less than K, but the error is less than one foot when K is les than 1,400′. The following table gives the values of v for certain distances when $D=K$:

K	Angle *v*	*K*	Angle *v*
100	5° 44′	700	2° 10′
200	4° 04′	800	2° 02′
300	3° 20′	900	1° 55′
400	2° 52′	1,000	1° 49′
500	2° 34′	1,100	1° 44′
600	2° 20′	1,200	1° 40′

(2) When D is 1′ less than K, i. e., for error of 1 ft. when the last term $= 1'$. we have,

$$D = K - 1;$$

$$D = K\ cos.^2 v + 1;$$

$$\text{or } K - 1 = K\ cos.^2 v + 1,$$

$$\therefore cos.^2 v = 1 - \frac{2}{K}$$

Solving for the different values of K, we can fill out the following table:

K	Angle *v*	*K*	Angle *v*
100	8° 08′	700	3° 04′
200	5° 44′	800	2° 52′
300	4° 41′	900	2° 42′
400	4° 03′	1,000	2° 34′
500	3° 38′	1,100	2° 27′
600	3° 20′	1,200	2° 20′

For any angle given in table and distance less than the corresponding value of K, the error in D will be less than 1 ft.

(3) When last term $= 1'$ and there is a total error of 1 per cent in horizontal distance, we have $D = .99K$,

$$D = K\ cos.^2 v + 1$$

$$\text{or } .99K = K\ cos.^2 v + 1 \quad \therefore \quad cos.^2 v = .99 - \frac{1}{K}$$

This formula gives the following:

K	Angle *v*	*K*	Angle *v*
100	8° 08′	700	6° 08′
200	7° 02′	800	6° 05′
300	6° 38′	900	6° 03′
400	6° 25′	1,000	6° 01′
500	6° 17′	1,100	6° 00′
600	6° 12′	1,200	5° 59′

To find D from table, subtract 1 per cent.

EXAMPLE: If $K=800$, we get $D=800-8=792$.

(4) If the last term be omitted and there is an error of 1 per cent, i. e., if there is a total error of 1 per cent minus 1 ft., or if $D=.99K+1$, we get,

$$D=K\ cos.^2 v+1$$
$$\text{But } D=.99K+1$$
$$.99K+1=K\ cos.^2 v+1$$
$$\therefore cos.^2 v=.99 \therefore v=5^\circ\ 44'$$

That is, if the angle of elevation be 5° 44′, the horizontal distance *(D)* will be less than the inclined *(K)* by 1 per cent of *K* less 1′ or

$$\therefore \text{Error} = \frac{K}{100} - 1$$
$$D = K - \text{Error.}$$

126. Topography by Hand-Level.—The hand level can be used economically to obtain the data for a topographic map of any small area. A base line should be adopted from which the survey proceeds, and lines perpendicular to this base line should be drawn at known intervals. Thus, if in Fig. 67, *ABCD* represents a section of area, adopt a base line *PQ* and at points *P*, 1, 2, 3, and *Q* locate lines normal to *PQ*. These lines should be marked out by stakes so they can be easily followed. In order to leave all elevations positive, assume some datum below the lowest point and refer the elevations of all points to this datum. Begin at some point as *P* and find the elevation of points along this line. The notes should be kept so the height of any point will appear as the numerator of a fraction, while its distance out from base line will appear as the denominator. The height of the eye should first be determined and rod readings should be taken at a sufficient number of points to determine the configuration of the landscape. The

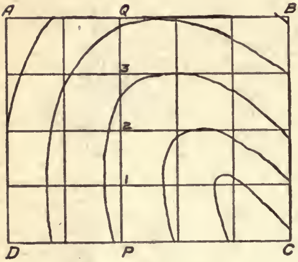


Fig. 67.

bench mark should be located somewhere below the point C, and from this the levelman makes his observation on the rod held on some point in line DC. The difference of the rod reading and height of eye will give the elevation of the point of rod above the observer. Thus, if

$h =$ height of eye of observer,

$r =$ rod reading, then,

$h - r =$ elevation of rodman above observer.

If $h - r$ is negative, the rodman is below the observer.

The following notes were taken on a hand-level survey of a rectangular area:

Line	Left of PQ			Base Line	Right of PQ				
DC..	$\frac{33}{200}$	$\frac{28}{100}$		$\frac{24}{0}$	$\frac{19}{100}$	$\frac{20}{150}$	$\frac{15}{200}$	$\frac{11}{250}$	$\frac{14}{300}$
1....	$\frac{34}{200}$	$\frac{29}{100}$	$\frac{25}{50}$	$\frac{23}{0}$	$\frac{18}{100}$	$\frac{15}{150}$	$\frac{14}{200}$	$\frac{16}{250}$	$\frac{20}{300}$
2....	$\frac{35}{200}$	$\frac{29}{100}$	$\frac{26}{50}$	$\frac{24}{0}$	$\frac{22}{100}$	$\frac{21}{150}$	$\frac{22}{200}$	$\frac{23}{250}$	$\frac{25}{300}$
3....	$\frac{36}{200}$	$\frac{31}{100}$	$\frac{27}{50}$	$\frac{26}{0}$	$\frac{25}{100}$	$\frac{26}{150}$	$\frac{27}{200}$	$\frac{28}{250}$	$\frac{30}{300}$
Q....	$\frac{37}{200}$	$\frac{33}{100}$	$\frac{31}{50}$	$\frac{29}{0}$	$\frac{30}{100}$	$\frac{31}{150}$	$\frac{32}{200}$	$\frac{33}{250}$	$\frac{35}{300}$

BIBLIOGRAPHY.—"A Manual of Topographic Methods," by Henry Gannett. This work is published by the United States Geological Survey and its title indicates its scope, as it deals only with the theory of topography, but gives also the illustrated methods as practiced by the engineers of the Survey, the most expert topographers in the world.

"Topographic Surveying," by Herbert M. Wilson, 910 pages. Fully illustrated, having 18 engraved colored plates, 181 halftone plates and many smaller figures. In addition to the excellent illustrations of the best executed topography, the field instruments and other equipments for field parties are described and the methods explained.

"Elevation and Stadia Tables," by Arthur P. Davis. These tables are for use in reducing inclined sights to the horizontal and for finding the difference of elevation of observer and points.

CHAPTER VIII.

RAILROAD SURVEYING.

127. Railroad Surveying.—By railroad surveying is meant the use of transit and level in selecting and locating the center lines of the track. The location of the straight sections of the track is a matter easily accomplished, but it becomes necessary to unite two straight sections of track that intersect at a definite angle. That a train may pass gently from one straight line to another, making an angle with the first, the two must be connected with each other by an intermediate curve to which each straight line is tangent. On account of the ease of location circular curves are universally used to connect two straight sections of track whose directions are not the same. These straight portions may be joined by a curve of either great or small radius, depending upon the character of the ground. The magnitude of the curve is defined by the size of the angle that a 100-ft. chord subtends at the center of the circle. Thus, in a 4° curve the 100-ft. chord subtends an angle of 4° at the center of the circle. In a 3° curve, 3° at the center, etc.

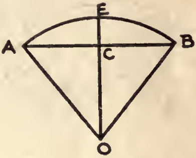


Fig. 68.

128. Degree Formula.—In Fig. 68 let AEB be a circular arc with O as center, and let $AB = 100$ ft. and angle $AOB = D$. Then, if OC is perpendicular to AB,

$$AC = CB = 50 \text{ ft. and } AOC = BOC = \tfrac{1}{2} D$$

Now, $$Sin.\ AOC = \frac{AC}{AO}$$

$$\therefore Sin.\ \tfrac{1}{2}D = \frac{50}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

129. General Formula.—In any curve AKB, Fig. 69, let $AB =$ chord c; $AP =$ tangent T, $AO =$ radius R, $FK =$ mid. ordinate M, $PK =$ External E, $I =$ angle of intersection $GPB = AOB$.

In the right triangle AOP

$$Tan.\ AOP = \frac{AP}{AO}$$

$$\therefore Tan.\ \tfrac{1}{2}I = \frac{T}{R}$$

$$\therefore T = R\ Tan.\ \tfrac{1}{2}I \ldots\ldots\ldots\ldots (25)$$

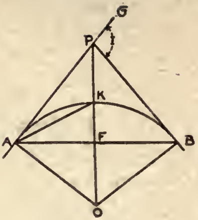


Fig. 69.

In rt. triangle AFO,

$$sin.\ AOF = \frac{AF}{AO}$$

$$\therefore sin.\ \tfrac{1}{2}I = \frac{c}{2R}$$

$$\therefore c = 2R\ sin.\ \tfrac{1}{2}I \ldots\ldots\ldots\ldots (26)$$

In rt. triangle AFK,

$$Tan.\ FAK = \frac{FK}{AF},$$

$$\therefore Tan.\ \tfrac{1}{4}I = \frac{M}{\tfrac{1}{2}c}$$

$$\therefore M = \tfrac{1}{2}c\ tan.\ \tfrac{1}{4}I \ldots\ldots\ldots\ldots (27)$$

In the triangle AKP,

$$\frac{PK}{sin.\ PAK} = \frac{AP}{sin.\ PKA}$$

$$\therefore PK = AP\frac{sin.\ PAK}{sin.\ PKA}$$

$$\therefore E = T\frac{sin.\ \tfrac{1}{4}I}{cos.\ \tfrac{1}{4}I} = T\ tan.\ \tfrac{1}{4}I \ldots (28)$$

If I is known and it is desired to pass a curve through some point on the bisector PO, we measure the distance $PK = E$, and from formula (28) calculate T. Then find R from (25) and D from (24).

130. To Lay Out Curve.—Let QA, Fig. 70, be a straight line or tangent from which a curve turns off at A. The point A where the curve begins is called the "Point of Curve" or P. C., while the point B, where we pass from the curve to the new tangent is called the "Point of Tangent," or P. T. To lay out curve, set up the transit over the station at A, level up and back sight on a tack point in tangent line AQ. Revolve the telescope and turn off the angle of deflection, which is half the degree of curve. The rear chainman holds end of the chain (the zero of chain or tape) on the tack point at A, and the head chainman swings his end of the chain around until the transitman catches the flag pole in field of view. The flag pole is brought accurately to coincide with the line of sight and when the head chainman has the chain or tape straight, a peg is driven at the point l, which is a point on the curve. The chainmen now advance until the rear chainman reaches point 1, the transitman, in the meantime, having set the deflection angle again. The rear chainman holds the end of chain or tape on

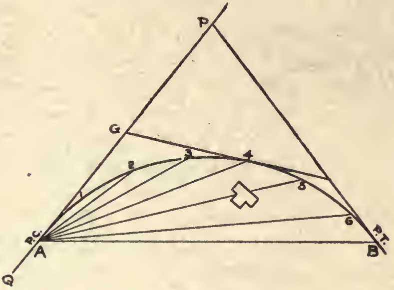


Fig. 70.

point 1, while the head chainman is ranged in the line of sight $A2$. When the chain is straight and the flag pole is in the line of sight, a peg is driven at this point 2. In the same way the other full station points on the curve are located.

Example.—Given $D=2° 30'$ and $I=15° 54'$

$$\text{Now, } R=\frac{50}{sin. \frac{1}{2} D}=\frac{50}{sin. 1° 15'}=2292'.0$$

$$\text{Length of curve}=\frac{15° 54' \times 100}{2° 30'}=636 \text{ feet.}$$

The total angle to deflect will be ½ I or 7° 57′. The angle of deflection is 1° 15′ and there will be six full deflections of 1° 15′ each, making 7° 30′, and a partial deflection of 27′, corresponding to a chord of 36 ft. The usual curve is so flat that the angle of deflection for fractions of 100 ft. is proportional to the length of chord. Thus, if the deflection angle for 100 ft. is 1° 15′, then the deflection for 36 ft. should be $.36 \times 1° 15' =$ 27′, which checks the result found by subtraction.

131. Obstacles.—It often happens that some object will interfere with our line of sight and we cannot locate all the stations from the P. C. Suppose that there were a house or some other obstruction interfering with the line of sight from the P. C. to station 5. In this case the transit must be transferred to station 4, where it is set up, leveled and a back sight taken on the rear flag at *A*, the P. C. Now, if *G*4 is a tangent to the curve at 4, the angle $G4A = GA4$. Hence, if we turn the telescope through an angle equal to the angle *GA*4, the amount deflected from the tangent *AP*, the line of sight will define the tangent 4*G*. Set the transit at 4, level up, bring the verniers to zero, reverse the telescope and set on *A*. Plunge the telescope and set the vernier to read 6° 15′, and the line of sight will define the line 45. This is more fully explained and exemplified in Article 139.

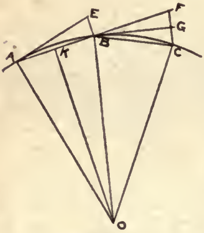


Fig. 71.

132. Location by Offsets.—Let *ABC*, Fig. 71, be a circular curve when $AB = BC = C$, and where $OA = OB = R$. Through *B* draw *BE* parallel to *OA* to cut the tangent *AE* at *E*. Draw *OK* perpendicular to *AB*. Then the triangles *OAK* and *ABE* are similar.

$$\therefore EB : AB = AK : AO.$$

Now, *EB* is called the offset from the tangent to curve or simply tangent offset.

Let $EB = d$

$$\therefore d : C = \frac{1}{2} C : R$$

$$d = \frac{C^2}{2R} \qquad (29)$$

If $AB = C = 100$,

$$d = \frac{5000}{R}$$

Let CF be drawn parallel to OB, to cut chord AB produced at F, and let BG be the tangent at B, cutting CF at G. Then triangle $BCG = BGF$.

$$\text{But } BCG = ABE. \quad \therefore CG = BE,$$
$$\text{But } \quad CF = 2 \times CG = 2 \times BE = 2\ d.$$
$$\therefore \text{ chord offset } CF = \frac{C^2}{R}.$$
$$\text{If } C = 100,$$
$$\text{chord offset} = \frac{10,000}{R}.$$

The formula for the chord offset may be written

$$d = \frac{C^2}{R} = \frac{10,000}{5730} D = 1.75D. \ldots\ldots\ldots\ldots (30)$$

Thus, for a 1° curve the chord offset is 1.75, and that for any other curve can be found by multiplying 1.75 by the degree of the curve.

133. Middle Ordinate.—In Fig. 69 we have by Geometry,

$$KF\ (2R - KF) = AF \times FB.$$
$$\therefore M\ (2R - M) = \frac{1}{2} C \times \frac{1}{2} C = \frac{C^2}{4}$$
$$\therefore 2RM - M^2 = \frac{C^2}{4}$$

Now M^2 is small in comparison with R, and in all practical cases can be omitted.

$$\therefore 2RM = \frac{C^2}{4}$$
$$\therefore M = \frac{C^2}{8R} \ldots\ldots\ldots\ldots\ldots (31)$$

134. Approximate Formulas.—We have established the formula,

$$sin. \frac{1}{2} D = \frac{50}{R}.$$

Now if D is no larger than 8° we can substitute the circular measure of the angle for its sine, that is

$$sin. \frac{1}{2} D = \frac{1}{2} \frac{D^\circ}{57.2965}$$
$$\therefore \frac{D}{2 \text{x} 57.2965} = \frac{50}{R}$$
$$\therefore D = \frac{5729.65}{R}$$

This is usually written,

$$D = \frac{5730}{R}$$

$$\therefore R = \frac{5730}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

Now if $D=1$, $R=5730$ ft. We have the general formula,

$$T = R \tan. \tfrac{1}{2}I = \frac{5730}{D} \tan. \tfrac{1}{2}I.$$

$$C = 2R \sin. \tfrac{1}{2}I = 2\,\frac{5730}{D} \sin. \tfrac{1}{2}I.$$

Let I remain fixed and T_1 and C_1 be the tangent and chord for 1-degree curve. Then,

$$T_1 = 5730 \tan. \tfrac{1}{2}I$$

$$C_1 = 2 \times 5730 \sin. \tfrac{1}{2}I$$

$$\therefore T = \frac{T_1}{D}$$

$$C = \frac{C_1}{D}$$

Again, we have,

$$M = \tfrac{1}{2}C \tan. \tfrac{1}{4}I = \frac{5730}{D} \tan. \tfrac{1}{4}I \sin. \tfrac{1}{2}I.$$

$$E = T \tan. \tfrac{1}{4}I = \frac{5730}{D} \tan. \tfrac{1}{2}I \tan. \tfrac{1}{4}I.$$

For a 1° curve these become,

$$M_1 = 5730 \tan. \tfrac{1}{4}I \sin. \tfrac{1}{2}I.$$

$$E_1 = 5730 \tan. \tfrac{1}{2}I \tan. \tfrac{1}{4}I.$$

$$M = \frac{M_1}{D}, \qquad E = \frac{E_1}{D}.$$

Then for all curves for a fixed I, we have,

$D \times T = T_1 =$ a constant,

$D \times C = C_1 =$ a constant,

$D \times M = M_1 =$ a constant,

$D \times E = E_1 =$ a constant.

135. Reduction Tables.—The value of the tangent T_1, the long chord C_1, the mid-ordinate M_1, and the external E_1 have been calculated for a 1-degree curve, corresponding to value of I from 0 to 117°, for intervals of two minutes. To obtain the values of T, C, M, or E, it is only necessary to look for these for a 1-degree curve for the proper I, and then to divide by the value of D.

EXAMPLE: Find T, C, M, and E, for a 4° curve when $I = 21°$. For a 1-degree curve, we get

$$T_1 = 1062.0,\ C_1 = 2088.5,\ M_1 = 95.95,\ E_1 = 97.58.$$

$$\therefore T = \tfrac{1}{4} \times 1062 = 265.50,$$
$$C = \tfrac{1}{4} \times 2088.5 = 522.125,$$
$$M = \tfrac{1}{4} \times 95.95 = 23.988,$$
$$E = \tfrac{1}{4} \times 97.58 = 24.395.$$

136. Metric Curves.—In Mexico and the South American countries a chain or tape of 20 meters is used instead of the 100-ft. tape that is used in the United States. The degree of the curve is the angle at the center of the circle subtended by a chord of 20 meters. Thus, in Fig. 72 if $AB = 20$ meters, and $AOB = D°$, the number of degrees in the angle D gives the degree of curve.

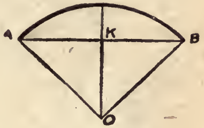


Fig. 72.

$$Sin.\ AOK = \frac{AK}{OA},$$

$$Sin\ \tfrac{1}{2} D = \frac{10}{R}.$$

If D = one degree, we have,

$$Sin\ 30' = \frac{10}{R}. \quad \text{But } sine\ 30' = \frac{1}{2 \times 57.3}$$

$$\therefore \frac{1}{114.6} = \frac{10}{R}. \qquad \therefore R = 1146 \text{ meters.}$$

Now, the radius of a 1-degree curve for the foot system (prevailing in the United States) is 5730 ft. $= 5 \times 1146$.

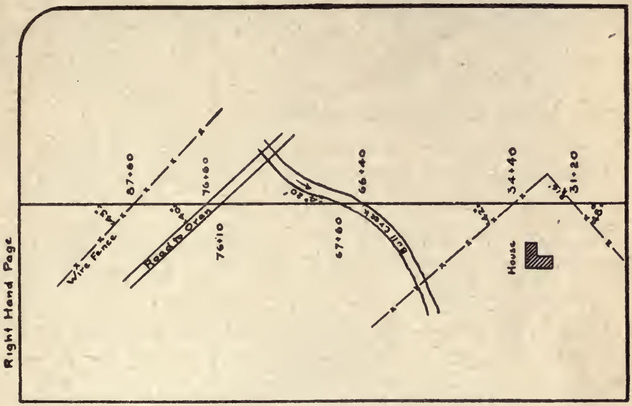
In the same way all the functions of a 1-degree metric curve are one-fifth of the corresponding functions of a 1-degree curve of the foot system. Thus, if $I = 12°$, $T = 602.2'$, $E = 31.56'$, $C = 1197.9'$, for a 1-degree foot curve. Then $T = 120.4$ meters, $E = 6.3$ meters, $C = 239.6$ meters, which were obtained by dividing the former values of T, E and C for the foot curve by 5.

Again, if we have $I = 14° \ 30'$, and wish to find T, E, and C for a 3° metric curve, we can find T, E, and C from the usual tables for the foot curve and divide the results by five times the degree of curvature for the metric system. Thus, for $I = 14° \ 30'$ we have for a 1-degree curve $T = 728.87$, $E = 46.18$, $C = 1446.2$. Then for a metric curve of 3° we divide these values of T, E and $L. C.$ by $3 \times 5 = 15$, as follows:

$$T = \tfrac{1}{15}(728.87) = 48.59 \text{ meters},$$
$$E = \tfrac{1}{15}(46.18) = 3.08 \text{ meters},$$
$$C = \tfrac{1}{15}(1446.2) = 96.41 \text{ meters}.$$

137. Preliminary Survey.—The first instrumental survey on a projected railway line is called the preliminary survey and consists in running a traverse line, staking the line out by means of pegs or stakes, which are driven at the hundred-foot marks, or "stations," as they are called, or at fractional parts thereof. When the survey is finished these stakes mark out a polygonal traverse or survey. There may be two or more preliminary surveys between the same termini, and a comparison of these as to cost of construction, revenue to be derived from probable traffic, and operating expenses will decide the most advantageous route. Fig. 73 is a double page illustration of the form of notes used in the field in preliminary survey.

138. Location Survey.—When one of the preliminary surveys or routes has been adopted, the center line of the proposed track is then located. The different tangents must be connected by curves and the whole line must be surveyed by transit, running in the curves and driving new stakes or changing the position of the old ones. As the curve is shorter than the sum of the two tangents, the first P. T. will be less in distance from the beginning, that is, all stakes after the first P. C. will be moved forward. Those on the tangents (from P. C. to P. I. and from P. I. to P. T.) will be moved over to the curve and all those on the part of tangent from the P. T. to the next P. C. ahead will be moved forward so that the number of each stake will give its distance from the beginning as measured along the proposed center of track. Thus, if the angle of intersection $I = 16° \ 00'$ and we unite the two tangents by a 4° curve, the value of $T_1 = 805.2$, and for a 4° curve $T = 201.3$. Now, if the distance from the beginning to P. I. was 3346 ft., i. e., the P. I. was at station 33 + 46, the P. C. will be located at (3346 — 201.3)



Left Hand Page

Sta.	Dist.	Def.	Calc. Course	Mag. Course	
⊙94		8°48'L.			
	1700		N39°27'E.	N39°30'E.	
⊙77		7°30'L.			
△68					
	1100		N46°57'E.	N47°0'E.	
⊙66		9°54'R.			
	1200		N37°03'E.	N37°0'E.	
⊙54		12°36'R.			
△47					
	2300		N24°27'E.	N24°30'E.	
⊙31		7°18'L.			
	1400		N31°45'E.	N31°45'E.	

Fig. 73.

3144.7, that is, at station 31 + 44.7. The P. T. will be located an equal distance from the P. I. or at 3547.3, according to the preliminary survey. Now, length of curve = 16 ÷ 4 = 400 ft.

Then, according to the location survey, the P. T. will be located at $3144.7 + 400 = 3544.7$, or 2.6 ft. nearer the beginning by the curve route than by the P. I. route. Station 36 was 52.7 ft. from this P. T. according to the preliminary survey, but by the location chaining, the point, instead of being 3,600 ft. from the beginning, will be at 3597.4 ft., and hence the station stake 36 will be taken up and moved forward 2.6 feet, so that it will really be 3600 ft. from the beginning.

PROBLEM 67.—The P. I. in the preliminary was 2614 ft. and $I = 24°$. Find the positions of the P. C. and P. T. for a 3° curve.

PROBLEM 68.—The second P. I. in the previous problem was 9654 ft. Find the position of P. C. in the location survey for a 3° curve if $I = 18°$.

139. Field Book.—It is important that the note book or field book should be neat and accurate and should show all the necessary data for the location of a curve and how it is connected to the tangent points, where it begins and where it ends. The supreme test of note taking and note keeping is that *ANY* engineer can understand fully and accurately exactly what the data mean. Fig. 74 is an illustration of both pages (left and right) of a location survey notebook where a curve has been run in to connect two intersecting tangents. The angle of intersection of the tangents $I = 12° 54'$, and the tangents are united by a 2° 30'. The length of tangent for a 1-degree curve for $I = 12° 54'$ is 647.8 and for a 2° 30' curve the length of tangent $= 647.8 \div 2.5 = 259.1$. This length of tangent can be calculated from the following formula:

$$T = R \tan \tfrac{1}{2} I = \frac{50 \tan \frac{1}{2} I}{\sin \frac{1}{2} D} = \frac{50 \tan 6° 27'}{\sin 1° 15'} = 259.1.$$

The curve is to begin at station 64 + 13.3 and the transit is set up at this point (the P. C.), the verniers brought to zero, and a back sight taken on the last hub. The next station in advance of the P. C. to locate is 65, which is (6500 — 6413.3) 86.7 ft. from the P. C. For a full 100 ft. the deflection is half the degree of curve or 1° 15', and for 86.7 it is $86.7 \div 100$ of

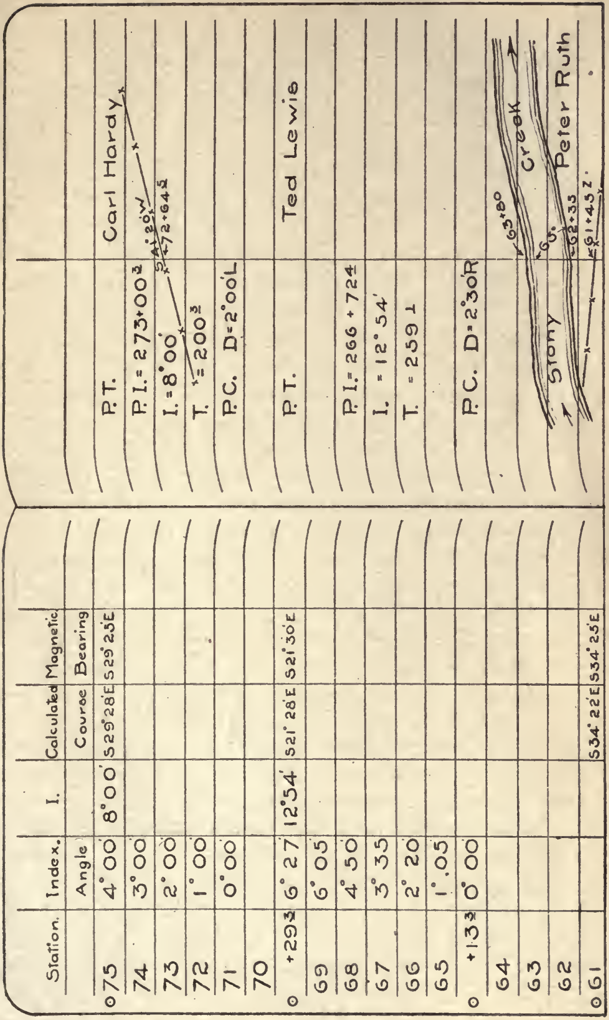

Station.
Index. Angle
I.
Calculated Course
Magnetic Bearing
75 4°00' 8°00' S29°28'E S29°25'E
74 3°00'
73 2°00'
72 1°00'
71 0°00'
70
+29³ 6°27' 12°54' S21°28'E S21°30'E
69 6°05
68 4°50
67 3°35
66 2°20
65 1°05
+13³ 0°00
64
63
62
61 S34°22'E S34°25'E
P.T. Carl Hardy
P.I.= 273+00³
I.=8°00'
T. = 200³
P.C. D=2°00'L
P.T. Ted Lewis
P.I.= 266 + 72⁴
I. = 12°54'
T. = 259.1
P.C. D=2°30'R
S41°20'W
72+64.5
63+80
63
62+33
61+45.7
Stony Creek
Peter Ruth

Fig. 74.

1° 15′. Hence the deflection $= .867 \times 75' = 1°\ 05'$, and this should be recorded opposite the station 65 that it locates. For the full stations 66, 67, 68 and 69 the record in the "index" column should be 2° 20′, 3° 35′, 4° 50′ and 6° 05′, respectively, which are obtained by adding 1° 15′ to the record of the last full station in the index column. Now, the length of the curve $= 100 \times 12°\ 54' \div 2°\ 30' = 516$ ft. Adding this 516 to the 64 + 13.3 (the station number of the P. C.), we get 69 + 29.3, which is the station number for the P. T. The deflection angle for the $29.3 = .293 \times 75' = 22'$, which is added to the index of the last full station, 6° 05′, gives an index of 6° 27′. Now, the reading of 6° 27′ on the P. T. should be half of *I*, that is, if we double the index for the P. T., we should get the value of *I*, or $2(6°\ 27') = 12°\ 54'$, which affords an easy and effective check.

It may happen that in running the curve the transit has to be moved from the P. C. to some station as 67, the index of which is 3° 35′. Now, after setting up the transit over 67, we can back sight on ANY station, provided we set the vernier to read the index of the station *sighted at*. Thus, if we backsight on 65, with telescope reversed, the vernier must read 1° 05′ (on the correct side of the vernier). Then to locate station 68, all we have to do is to revolve the telescope and set the vernier at 4° 50′, the index of the station *sighted at*, and have the stake driven at this point. However, if we should set up the instrument at 67 and backsight on 64 + 13.3 (P. C.) with telescope reversed, we must set the vernier at 0° 00′, the index of the P. C., and then to locate 68 we again make the vernier read 4° 50′, the index of the station *sighted at*. Thus, wherever we set up the transit on the curve, the back sight on any station must read the index opposite the station sighted at, and to locate any other station ahead, revolve the telescope and set the vernier to read the index for that station.

140. Transit Party.—The transit party in the field should consist of transitman, rear chainman, head chainman, rear flagman, stakeman, and axmen. The transitman has charge of the party and should provide himself with the transit, tripod, plumb-bob, reading glass, notebook and pencil.

The rear chainman should have charge of chain or tape and be responsible for it. The head chainman should provide and take care of the flag or range pole. The stakeman provides bag of stakes, keel for marking same, ax or hatchet for driving stakes, and tacks for hub-points. The rear flagman has the silent duty of remaining ever in readiness to be called upon to give a sight at a signal or call from the transit man, and the axmen should have good 4.5-lb. axes to clear the way. It is poor economy to be restricted in the number of men that are to do the clearing.

141. **Stakes.**—The stakeman should provide a sufficient number of stakes for each day's supply at least. The stakes vary in size (Fig. 75), but sawed stakes are 2x1 ins. by 18 ins. in length, while "hubs" should be 2x2 ins. by 18 ins. in length. The flat shaped stake is used to facilitate marking, as the broad surface offers sufficient space for the number of station and the letter indicating the line to be written or printed on the stake. The figures or letters are printed with keel (red chalk), which can be secured from dealers in drawing supplies or from local hardware dealers.

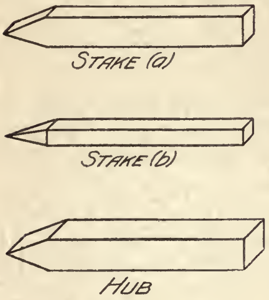


Fig. 75.

142. **Hubs.**—At every angle point or transit station a "hub" is located. This consists of a stake (Fig. 76), 2x2 ins., driven flush with the surface of the ground. A tack is driven in the top of the hub, where the range pole or flag rested in the line of sight. After the tack is driven partly in the hub it should be checked by the transitman so that any error in location can be corrected before it is driven too far to be withdrawn. After it has been checked, it is driven flush with the surface of the hub. About 1 ft. to the left of the hub a "guard" stake is driven with the number of the station marked on it. This guard stake is inclined towards the hub and is left project-

ing from the ground several inches, as shown in Fig. 76. The number of the station of the hub should be marked on the guard with a good system of letters. These figures should be printed with red keel, and in no case should they be written with a rough figure or letter. With care and a little practice the stakesman can soon learn how to mark these in a standard and systematic way.

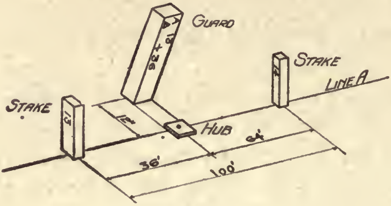


Fig. 76.

143. Hand-Level.—This instrument, Fig. 77, is about 6 ins. long and has a level tube or vial on top. Across one half of the clear glass at object end a horizontal line is drawn. The image of the bubble tube can be seen on half of the glass at object end of tube, as it is reflected by a prism. The ends of the tube are closed with plane glass and a semi-circular convex lens at end of eye-piece or eye-tube magnifies level bubble and

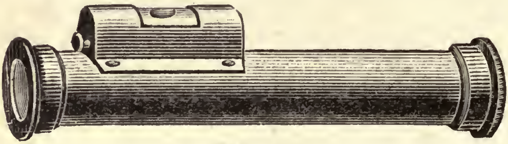


Fig. 77.

the cross wire. The cross wire is fastened to a framework under the level tube and adjusted to its place by the screw shown on end of level case.

To use the level, hold it with the hands so that the eye-end is next the eye, then move it until it is approximately horizontal. The image of the bubble can then be seen on half of the object-end glass. When the bubble appears on the horizontal mark or wire, the line of sight is horizontal. To use the hand-level it is necessary to know the height of your eye. Sight through the hand-level and bring the bubble on the horizontal wire and note the point on the ground indicated by the line of sight. Unless unusual refinement is necessary in taking topography, the hand-level will subserve all necessary requirements and it is an economical, efficient and expeditious instrument for this purpose. In railroad surveying the line of survey affords a base line from which all transverse measurements can be made. The topographer determines the height of his eye when standing in his usual attitude and then taking a position on the line of survey, *ABCD* (Fig. 78), he selects a direction at right angles to the line of survey. Bringing the level to its horizontal position and noting where the line of sight strikes the earth at point 1, he paces the distance from line to point (48 ft., say). At point 1 he notes that the next line of sight strikes ground at point 3, etc. This process is continued until the territory 200 ft. on each side of the line is covered. If the height of the eye is 5.2 ft., then each point of intersection of horizontal line of sight with ground is 5.2 ft. higher than the position of the observer. The elevation of the observer's position is known, or can be ascertained from the levelman's notes, and hence the elevation of each point located can be determined by adding or subtracting height of eye.

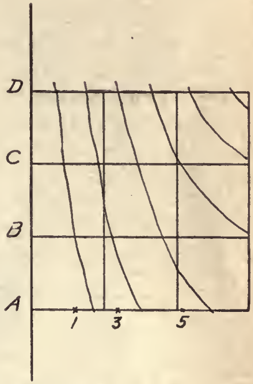


Fig. 78.

On the lower side of the line it is well to have a rodman provided with a rod, graduated to half-feet, at least 12 ft. in length. If it is desired to have all contour points, the uniform height of eye above or below the adjacent points in any one normal line, the topographer can have his rodman walk away from the base line in a normal direction till the rod reads double the eye height. If other points are located, the rod is read by the hand level and the reading recorded. The topographer advances to the rodman's position and sends him on further out to locate other points. If no rodman is used the topographer can pace the distance in the normal direction to some point which he guesses is about the eye-height below his position. If the line of sight from his point strikes below the surface in the normal line, he must go toward the base line till the line of sight strikes the point on the base line. The distance from the base line is found by subtracting or adding the distance between the final location and the assumed point to the distance from base line to assumed point. With a little practice a topographer will soon be able to select a point within a foot or so of the correct point.

144. Slope Stakes in Excavation.—In excavation in earthwork the cross section is defined by the roadbed *AB*, Fig. 79, and the side slopes *AE* and *BC*. The amount of slope of *BC* is determined by the ratio of *BG* to *CG*, and is designated by s. $\therefore s = BG \div CG = tan.BCG$. $\therefore BG = s.CG = sh_1$, where h_1 = height of point *C* above roadbed $AB = CG$, and $h_2 = EF$ height of *E* above roadbed *AB*.

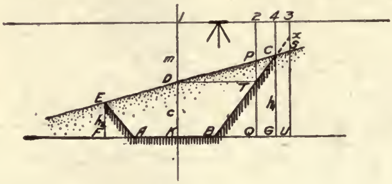


Fig. 79.

$$2b = AB \text{ width of roadbed.}$$

Now
$$BG = sh_1,$$
$$KG = b + sh_1$$
$$KF = b + sh_2$$

$\therefore$ Distance out of stake point C = half width of roadbed plus slope times height of point above roadbed.

The center cut $DK = c$ is already known before any attempt is made to set the slope stakes. The level is set up in some convenient position and a rod reading taken on the station at D. Let 1234 represent the horizontal line of sight,

$$m = D1 = \text{rod reading on station D},$$
$$\therefore H = DK + D1 = c + m.$$

Now, to locate the slope stake at C at a horizontal distance d from D, try some point as P and find the rod reading $P2 = r$.

$$\text{Then } 2Q - P2 = H - r.$$

Now, $H - r$ is the "surface height" of the trial point above roadbed AB.

Calculated distance out $d_c = b + s\,(H - r)$. But
Measured distance out $d_m = b + s\,QT$.

Hence, we see that when the trial point is too near the center the measured distance out is less than the calculated distance out. Try some point S.

$$d_m = \text{measured distance out} = b + s\,UX.$$
$$d_c = \text{calculated distance out} = b + s\,US.$$

$\therefore$ The measured distance out is greater than the calculated distance out when the trial point is too far out, and *vice versa*.

Hence, if

$d_m > d_c$, come in;
$d_m < d_c$, go further out.

Rule: If the measured distance out to the trial point is greater than the calculated distance out, come in, and vice versa.

Slope-Stakes in Level Sections.—If DT, the surface of the ground, is horizontal, then $DK = TQ$. In this case the point T will be the stake point.

Its distance out, $KQ = KB + BQ = b + s\,TQ = b + s\,DK = b + sc$.

Thus, in level sections the distance out is found by multiplying the center cut by the slope and adding the half width of roadbed.

EXAMPLE: If center cut $= 14.6$ and slope $s = 3:2$ and width of roadbed $= 18$ feet, then

$$\text{Distance out} = 9 + \frac{3}{2}\,14.6 = 30.9.$$

Field Methods.—In the field, if the ground is inclined, the usual practice is first to find the distance out on the assumption that the ground is level. This simply serves as a guide and useful help. If the ground slopes, the distance out on the upper side of the center of roadway is always greater than the distance out in a level section, if the ground slopes uniformly. While on the lower side the distance out of the slope stake is less than the level d. o.

EXAMPLE. Given $s = \frac{3}{2}$; $2b = 18$; $c = 14$. The level was set up and the rod reading on the center was 7.2. For a level section, the distance out $= 9 + \frac{3}{2} \times 14 = 30$. On the upper side the trial point was selected at 32 ft. from the center where the rodreading was 5.4 ft.

$$H = 14 + 7.2 = 21.2$$

$$H - r = 21.2 - 5.4 = 15.8$$

$$\text{Calculated d. o.} = 9 + \frac{3}{2} \text{ x } 15.8 = 32.7$$

Now, the calculated d. o. is greater than the true, hence the trial point is too near center. Try a point 34 ft. out when rod reads 5.0.

$$H - r = 21.2 - 5.0 = 16.2$$

$$\text{Calculated d. o.} = 8 + \frac{3}{2} \text{ x } 16.2 = 32.3 \text{ ft.}$$

The calculated d. o. is less than true d. o.

The second trial point is too *far* out. Try point 33, where rod reads 5.2.

$$H - r = 21.2 - 5.2 = 16.0$$

$$\text{Calculated d. o.} = 9 + \frac{3}{2} \text{ x } 16 = 33.0$$

This location is correct.

On the lower side the distance out must be less than 30, the d. o. for a level section. Try a point 29 ft. out, where rod reads $r = 8.4$.

$$H - r = 21.2 - 8.4 = 12.8$$

$$\text{Calculated d. o.} = 9 + \frac{3}{2} \times 12.8 = 28.2$$

Hence, the trial point was too far out. Try a point at 28.4, where rod reading = 8.3.

$$H - r = 21.2 - 8.3 = 12.9$$

$$\text{Calculated d. o.} = 9 + \frac{3}{2} \times 12.9 = 28.35$$

The location is sufficiently accurate.

PROBLEM 69. Center cut = 16.6, $2b = 18'$, $s = 3 \div 2$.

Rod reading on center = 6.2. A trial point was taken at 35, where a rod reading was 5.0. Is the trial point too far out or in? Answer:

If the trial point was at 39.0 and the rod reading was 4.9, is it too far out or in? Answer:

If the point was 36 and the rod reading 4.8, how is it? Answer:

PROBLEM 70.—In the following table:

c = center cut,
m = rod reading on center,
d_m = true distance out of trial point,
r = rod reading on trial point,
d_c = calculated distance out.

Find the results as to accuracy of location point.

Number	c	m	s	b	d_m	r	d_c	Result
A	12.8	6.6	3/2	9	30.2	5.2	..	..
B	12.8	6.6	3/2	9	32.0	4.8	..	..
C	12.8	6.6	3/2	9	30.6	5.0	..	..
D	12.8	6.6	3/2	9	26.0	7.6	..	..
E	12.8	6.6	3/2	9	27.0	8.0	..	..
F	12.8	6.6	3/2	9	26.4	7.8	..	..
G	8.6	5.4	1/1	9	18.0	6.4	..	..
H	8.6	5.4	1/1	9	19.5	4.0	..	..
I	8.6	5.4	1/1	9	17.0	4.2	..	..
J	8 6	5.4	1/1	9	18.9	4.1	..	..
K	11.4	4.8	2/1	9	35.6	3.6	..	..
L	11.4	4.8	2/1	9	33.8	3.8	..	..

145. Slope Stakes in Embankment.—In embankments the road bed *AB* is usually for single track roads 14 ft. wide and the slope varies from 1:1 to 2:1. However, on levees the slope is as flat as 5:1.

The roadbed is *AB*, Fig. 80, and the side slopes *BC* and *AE*. Slope stakes must be set at the foot of the slopes at *E* and *C*. The center fill $KD = c$ is known and it is required to locate these slope stakes. The level is set up. 1, 2, 4, 3 is the horizontal line of sight, the rod reading (m) on the center is $D1$. The height of instrument $(H. I.)$ above roadbed *AB* is *KI*.

Now, $BG = s \times CG$ and $AB = 2b$, $DI = m$

$$H.I. = KI = m - c$$

Distance out $= KG = KB + BG = b + s.CG = b + sh$

The rod reading (r) on C is $4C$,

But $4C = 4G + GC$

$\therefore r = H.I. + h$

or $h = r - H\ I = r - m + c$

Distance out $= b + s\,(r - m + c)$

Suppose we try a point P that is too close to the center.

Rod reading $(r) = P2 = r$

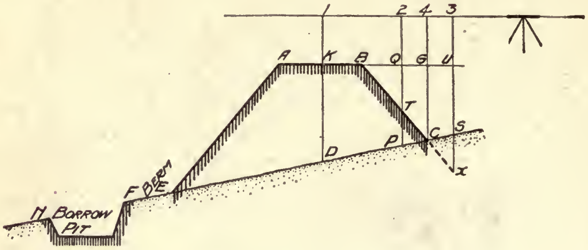


Fig. 80.

d_c = calculated distance out $= b + s(r - m + c) = b + s.PQ$.

But d_m = true distance out $= KQ = KB + BQ = b + sQT$.

Thus d_c is *greater* than d_m.

Hence the calculated distance out is too great and the trial point is too near the center.

Try a point S where rod reading $= 3S$.

$$d_c = b + s(r - m + c) = b + s.US$$

$$\text{But } d_m = KU = b + s.UX$$

$$\therefore d_c \text{ is } less \text{ than } d_m.$$

Hence the calculated distance out is less than the true distance out, or the trial point is too far out. Thus we see that the same rule applies to fills that applies to cuts.

In deep fills the line of sight 1 2 4 3 may be below AB and the height of instrument ($H.\ I.$) will be negative. In this case

$H.\ I. = c - m$

Distance out $= b + s\ (c - m + r)$

Example.—Given $2b = 14$: $s = 3/2$; center fill $= 14.8$ ft.; rod reading on center $= 5.4$. If the ground is level the distance out $= 7 + 3/2\ (14.8) = 29.2$ ft. On the lower side the distance out will be greater than this, while it will be less on upper side.

Try a point 31 out where the rod reading $= 7.20$.

$$d_c = 7 + \tfrac{3}{2}(14.8 - 5.4 + 7.2) = 31.9.$$

∴ Point was too far out.

Try a point 32 ft. out where $r = 7.3$.

$d = 7 + 3/2\ (14.8 - 5.4 + 7.3) = 32.05.$

The location is sufficiently accurate for practical or ordinary requirements.

PROBLEM 71.—In the following table determine the results of the trials, i. e., whether trial point is too far, too near, or correct:

Number	c	m	s	b	d_m	r	d_c	Results
A	17.8	5.2	3/2	7	36	7.4	..	..
B	17.8	5.2	3/2	7	27.2	7.5	..	..
C	17.8	5.2	3/2	7	38	7.6	..	..
D	14.4	4.8	1/1	7	22.0	5.6	..	..
E	14.4	4.8	1/1	7	20.5	5.5	..	..
F	14.4	4.8	1/1	7	20.7	5.	..	..
G	9.2	4.6	2/1	7	23.0	3.4	..	..

146. Berms.—It is often necessary to excavate the earth near the foot of the slope of the embankment to secure enough dirt to make the embankment. When such is the case it is necessary to leave a strip of unbroken original surface at least 4 ft. in width between the borrow pit and the foot of slope to afford a break for earth that washes down or off the slope. Thus in Fig. 80 FE is the berm, a strip of undisturbed natural earth, between the embankment $CBAE$ and the borrow pit NF.

In cuts it is often of the utmost importance to have an undisturbed natural surface on each side of the cut. To do this it is necessary to prevent the deposition of any excavated material within 6 ft. of the edge of the side slope. If the loose earth is piled near the edge of the slope, heavy rains will wash it down the slope into the cut.

Bibliography.—"Railroad Location Surveys and Estimates," by F. Lavis. Published by the Myron C. Clark Publishing Co. This book is a complete epitome of actual field engineering and includes a history of the preliminary survey from the organization of the party to the completion of the line. No better description can be applied to this work than to say that its theme is to tell and show "how to do things." In many respects it covers a territory heretofore not traversed, and is replete with valuable suggestions gained by experience as a field engineer.

"Field Manual for Railroad Engineers." By James C. Nagle. Published by John Wiley & Sons, 403 pp. One of the leading field books of the country, containing full directions, suggestions, tables for the solution of the usual problems met with in field operations in preliminary and location surveys. A full set of tables of trigonometric functions, of a 1° curve, transition curve, coordinates, squares and cube roots.

"Railroad Curves and Earthwork." By C. Frank Allen. Published by Spon & Chamberlain. 490 pp. Contains discussion of the usual railroad curves including the transition curve, rather full treatment of slope stakes and earthwork problems, with diagrams to facilitate the calculation for earth work; field and office tables.

"The Field Engineer." By W. F. Shunk. Published by D. Van Nostrand Company. 329 pp. This work treats of the problems of preliminary and location surveys, many illustrative examples, the essentials of slope stake setting, and the usual tables necessary for an engineer in the field.

"Field Engineering." By Wm. H. Searles. This has been for years one of the standard manuals for field and office engineers, and it covers the problems of railway surveying, location and construction. The book is fully illustrated and has many valuable tables to shorten the labor of calculation.

CHAPTER IX.

EARTHWORK.

147. Prismoidal Formula.—Let Fig. 81 represent a solid bounded by two parallel planes and whose side faces are triangles. Draw the mid-section 12345678 and join any point P in this mid-section with $ABCDEFGH$, 1, 2, 3, 4, 5, 6, 7, and 8. This divides the solid into three kinds or types of pyramids. The first class has P for a vertex and $ABCD$ for a base; the second has P for a vertex and $EFGH$ for a base, while the third class has P for a vertex and for bases the side face triangles, as P—EDC.

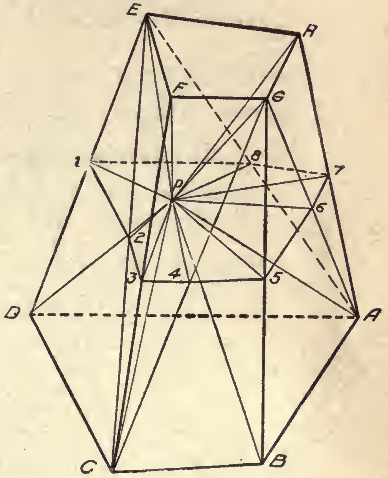


Fig. 81.

Let B_1 = area $ABCD$

B_2 = area $EFGH$

h = perpendicular distance between parallel planes $ABCD$ and $EFGH$.

1. Volume $P—ABCD = \frac{1}{3}\ ABCD \times \frac{1}{2}h = \frac{1}{6}h\ B_1$
2. Volume $P—EFGH = \frac{1}{3}\ EFGH \times \frac{1}{2}h = \frac{1}{6}h\ B_2$
3. To find the volume of the pyramids of the third class, consider $P — EDC$ as a type of the third class. The pyramids $P — E12$ and $P — EDC$ have the same vertex P and bases in the same plane EDC. Hence they are to each other as their bases.

$\therefore P-EDC : P-E12 :: EDC : E12.$

As 1 and 2 are the mid-points of the sides ED and EC, $EDC = 4 \times E12.$

$\therefore P-EDC = 4 \times P-E12.$

But the volume of the Pyramid $P-E12 = \frac{1}{3} \times Area\, P12 \times \frac{1}{2}h = \frac{h}{6} \times P12.$

$$\therefore \text{Volume } P-EDC = 4 \times \frac{h}{6} \times P12 = \frac{4h}{6} \times P12.$$

$$\text{Similarly, Volume } P-EFC = \frac{4h}{6} \times P23.$$

$\therefore$ Total volume of pyramids of third class=

$$= \frac{4h}{6}(P12+P23+P34+P45+P56+P67+P78+P18) = \frac{4h}{6}M,$$

where M=area of mid-section *12345678.*

Adding the volumes of the three types we get for total volume $V = Volume\ of\ solid = \frac{h}{6}(B_1+4M+B_2)$ . . . . (33)

148. Railroad Excavation.—In railroad earthwork, cross-sections at right angles to the center line of track are taken every 100 ft. Slope stakes are set and data obtained for calculating the volume to be excavated between the two sections 100 ft. apart. Such a solid is bounded by a plane roadbed, two parallel end areas, whose planes are perpendicular to the planes of the side slopes, while the upper surface is terminated by planes that are either triangular areas or that can be divided into triangles by drawing the diagonals as $D'C$. The prismoidal formula applies to such a solid. Fig. 82 represents the part of the excavation on one side of the central plane of roadbed. $BKK'B'$ represents half of the roadbed between cross-sections $DKBC$ and $D'K'B'C'$. To find the volume of the excavation by the prismoidal formula given above, it is necessary to find the areas of the ends or bases and of the mid-section.

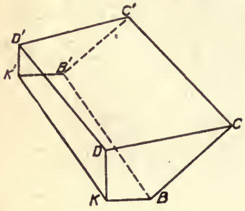


Fig. 82.

149. Level Sections.—Where the intersection of the cross-section plane with the surface of the earth is horizontal, the section is said to be level, or a one-level section.

In Fig. 83 $AB=2b$, $DK=c$, $CG=EF$. Now, $BG=sCG$
$sCG=sc=AF$

$\therefore FG=2b+2sc=EC$

Area $EABC=\frac{1}{2}(EC+AB)DK$

$$=\tfrac{1}{2}(2b+2sc+2b)c,$$

$$=c(2b+sc)=2bc+sc^2 \qquad (34)$$

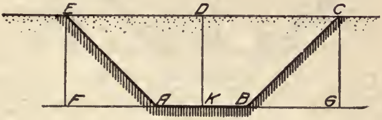


Fig. 83.

Example: Given $2b=18'$, $c=8.4$, $s=3/2$, find area of section.

Area $=2bc+sc^2=18\times 8.4+3/2\times(8.4)^2=257.04$ sq. ft.

150. Two Level Sections.—When the surface of the ground slopes uniformly transverse to the roadway, two points established on the surface will be sufficient to determine the cross-section.

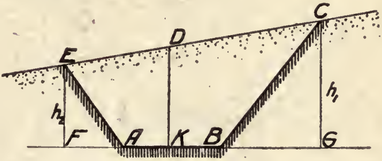


Fig. 84.

Let $h_1=CG$, and $h_2=EF$, $BG=sCG=sh_1$, $AF=sh_2$
$FG=2b+sh_1+sh_2$

Then area $ABCE=ECGF-BCG-AEF$

$$=\tfrac{1}{2}(h_1+h_2)\ (2b+sh_1+sh_2)-\tfrac{1}{2}sh_2-\tfrac{1}{2}sh_1^2$$

$$=b\ (h_1+h_2)+sh_1h_2 \qquad (35)$$

The center cut is used only in locating the slope stakes at C and E, but is not used in the calculation of the area.

EXAMPLE: Given $2b = 18$, $s = 3/2$, $h_1 = 8.4$, $h_2 = 6.6$.

Area of section $= 9(8.4 + 6.6) + 3/2(8.4 \times 6.6) = 135 + 83.16 = 218.16$ sq. ft.

151. Three Level Sections.—By far the most common and usual section is the one where the two side heights and the center cut are used in calculating the area.

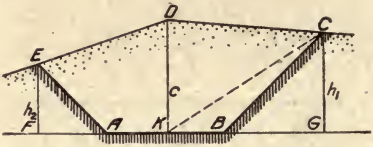


Fig. 85.

As usual, $CG = h_1$, $EF = h_2$, $KG = d_1$, $KF = d_2$, $DK = c$, $AB = 2b$, $BG = sh_1$, $FA = sh_2$, $KG = b + sh_1$, $KF + q = sh_2$.

Area $DKBC = DKC + CKB = \frac{1}{2}cd_1 + \frac{1}{2}bh_1$.

In the same way, $DKAE = \frac{1}{2}cd_2 + \frac{1}{2}bh_2$.

Total area $= c/2(d_1 + d_2) + b/2(h_1 + h_2)$ . . . . (36)

Thus, in the three-level section, the double area is equal to the center cut multiplied by the sum of the distances out, plus the half roadbed multiplied by the sum of the side heights.

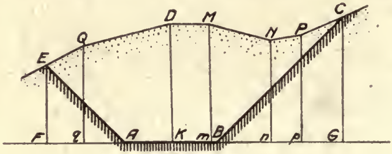


Fig. 86.

152. Irregular Sections.—When the surface of the ground is very irregular, rod readings must be taken at every change in slope of surface. Thus, in Fig. 86 rod readings must be taken at seven different places, and this section would be called a seven-level section. In the field we would locate N by measuring its distance out Kn, and by its elevation Nn above AB the roadbed. Thus, for any point on the surface, we have its co-ordinates, i. e., distance above AB (roadbed) and the distance from K (center of roadbed) to foot of perpendicular. To find the area of the section, we find first the area on the right of the central plane DK and then on the left.

$$BKDMNPC = KDMm + mMNn + nNPp + pPCG - BCG$$

Let c, h_m, h_n, h_p, h_1 be the heights of D, M, N, P, C above AB and d_m, d_n, d_p and d_1 equal the distance out of M, N, etc.

$$\text{Area } KDMm = \tfrac{1}{2}(c + h_m)\, d_m$$
$$\text{Area } mMNn = \tfrac{1}{2}(h_m + h_n)(d_n - d_m)$$
$$\text{Area } nNPp = \tfrac{1}{2}(h_n + h_p)(d_p - d_n)$$
$$\text{Area } pPCG = \tfrac{1}{2}(h_p + h_1)(d_1 - d_p)$$
$$\text{Area } BCG = \tfrac{1}{2}h_1 (d_1 - b)$$

Expanding and simplifying, we have,

$$\text{Double Area } BKDMNPC = cd_m + h_m d_n + h_n d_p + h_p d_1 + bh_1 - h_1 d_p - h_p d_n - h_n d_m \qquad (37)$$

153. Rules.—The notes in the field book are written as follows:

Center	Side				
$\frac{c}{o}$	$\frac{h_m}{d_m}$	$\frac{h_n}{d_n}$	$\frac{h_p}{d_p}$	$\frac{h_1}{d_1}$	$\frac{o}{b}$

Now, the point B is a corner of the polygon whose area we wish. In the table of notes we write each cut as a numerator of a fraction with the distance out of the point as denominator. To complete the notation for each point we can write the notes as follows:

$$\frac{c}{o},\ \frac{h_m}{d_m},\ \frac{h_n}{d_n},\ \frac{h_p}{d_p},\ \frac{h_1}{d_1},\ \frac{o}{b}$$

By an inspection of the formula for the area in connection with the Figure 86, we observe that each positive term consists of each cut or height (numerator) multiplied by the next denominator to the right (left), and that each negative term consists of the numerator multiplied by the denominator to the left (right). This gives the following usual

Rule: To obtain the area of the cross-section:

1. For positive terms, begin at center and multiply each numerator by the next outward denominator.

2. For negative terms, begin at ends and multiply each numerator by the next denominator towards center cut.

3. Take half the algebraic sum of the positive and negative terms for the area of the cross-section.

The data should be arranged as in Figure 87.

If we begin at the center and multiply each numerator by the denominator with which it is connected by the solid arrow and sum the results we get the positive terms, and if we multiply each numerator by the denominator with which it is connected by the dotted arrow, we get the negative terms. Half the algebraic sum of the positive and negative terms gives the area of the cross-section. Thus, from Fig. 87:

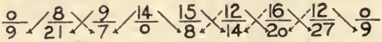


Fig. 87.

Double area on right $= 14 \times 8 + 15 \times 14 + 12 \times 20 + 16 \times 27 + 12 \times 9 - 12 \times 20 - 16 \times 14 - 8 \times 12 = 542$ sq. ft.

Double area on left $= 14 \times 7 + 9 \times 21 + 8 \times 9 - 8 \times 7 = 303$ sq. ft.

Double area of section $= 542 + 303 = 845$ sq. ft.

Second Rule: The double area can be found by arranging the data as in Fig. 87 and by multiplying the sum of two adjacent numerators by the difference of their denominators and by taking the algebraic sum of the products, treating the two extremes as negative.

Thus,

Double area $= -8 \times 12 + 17 \times 14 + 23 \times 7 + 29 \times 8 + 27 \times 6 + 28 \times 6 + 28 \times 7 - 12 \times 18 = 845$ sq. ft.

154. Side Hill Cuts.—It often happens that the railroad runs along the side of a hill and that part of the roadbed will be in cut and part in fill. The elevation of the roadbed is known and the center cut or center fill, as the case may be, is also known. Thus, if *EC*, Fig. 88, is the surface of the earth and *AB* the roadbed, part of the cross-section will be in cut and part in fill. The cut *DK* at the center is known and the slope stake at *C* is located as usual. The point *P* (cross-section grade-point) is located by the levelman and the distance KP measured. Below the point P, grade-point, the ground shown may be roughened or cut into steps, as shown in figure, to prevent slipping during wet weather.

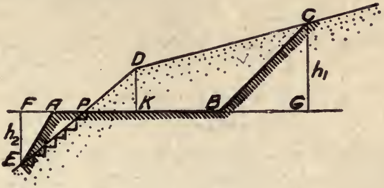


Fig. 88.

Let $BP = a$, then area $PBCD = PDK + DKBC = \frac{c}{2}(a-b) + \frac{b}{2}(c+h_1) - \frac{1}{2}sh_1^2$

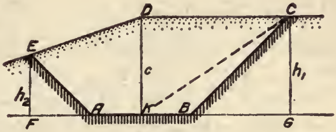


Fig. 89.

Area $PAE = 1/2 EF \times AP = 1/2\ (2b - a)\ h_2$.

Example: Given $2b = 18$, $s = 3/2$ on both sides and $DK = 2'$, $h_1 = 8'$, $h_2 = -4$.

The distances out are 21 on upper side, and 15 on the lower. The grade-point is found 3′ to left of center.

$$\text{Area in cut} = \frac{21}{2}\ (2+8) + \tfrac{1}{2}(2+3) - 48 = 60 \text{ sq. ft.}$$

$$\text{Area in fill} = 1/2\ 6 \times 4 = 12 \text{ sq. ft.}$$

Problem 72.—If $BK = 8'$, $DK = 2'$, $KA = 7'$, $PK = 3'$, slope in cut $= 1:1$, slope in fill $= 3:2$, find area in cut and fill if $h_1 = 8$, $h_2 = -4$.

155. Average End Areas.—In practice, the volume is calculated by the average end area formula.

Fig. 89 represents a form of a three-level section. The central plane DK divides the solid of excavation into two parts that can be treated separately. Let the center and side cuts at one station be c and h_1 and those at the next station 100′ away be c_1 and h_1' and let both sections be three-level sections, as in the figure.

Let B_1 = area $DKBC$

B_2 = area at the next station corresponding to $DKBC$,

We have,

$$B_1 = \tfrac{1}{2}(d_1c_1 + bh_1)$$
$$B_2 = \tfrac{1}{2}(d_1'\,c_1' + bh_1')$$

Now, if the solid is bounded by plane faces, we have center cut, side height, and distance out at mid-section.

$$\tfrac{1}{2}(c_1 + c_1'),\ \tfrac{1}{2}(h_1 + h_1'),\ \tfrac{1}{2}(d_1 + d_1')$$
$$8M = (d_1 + d_1')\ \ (c_1 + c_1') + 2b(h_1 + h_1')$$

But V = true volume. $= \frac{100}{6}(B + 4M + B)$ ......(33)

$$= \frac{100}{12}(2d_1c_1 + 2d_1'c_1' + d_1c_1' + d_1'c_1 + 3bh_1 + 3bh_1')$$

The average end areas $= \frac{1}{2}(B_1 + B_2)$ ....(37)

Let $V_e = \frac{100}{2}(B_1 + B_2) = \frac{100}{12}(3d_1c_1 + 3d_1'c_1' + 3bh_1 + 3bh_1')$

156. Error of Average-End Area Formula.—The average end area formula generally gives an excess of volume. Let E be the excess in volume by end-area formula.

$\therefore E = V_e - V = \frac{100}{12}[(c_1 - c_1')\,(d_1 - d_1')]$.........(38)

In the majority of cases, $c_1 - c_1'$, and $d_1 - d_1'$ have the same sign $\therefore$ excess is positive, that is, there is really an excess. But in passing over a saddle, c_1 can be greater than c_1' and d_1 less than d_1'. In such cases the excess is negative—that is, the volume calculated by the average-end-area formula is smaller than the true volume.

By common consent among engineers, contractors and surveyors, practically all volumes in railway practice are calculated by the average-end-area *(AEA)* formula. In fact, it is highly probable that for the real earth solid, the *AEA* formula gives results as near the actual cubic contents as the true prismoidal formula.

157. Examples.—The stations 1, 2, 3, etc., in the following table are 100 ft. apart. The numerators in each case show the depths of cuts and the denominators the distances out at the different points. Width of roadbed = 18′.0, slope = 3/2.

Station	Cut or Fill: Left	c	Right	Areas	Cubic Yards
1.......	$\frac{5.4}{17.1}$	7.8	$\frac{9.2}{22.8}$	221.31	
					1003.3
2.......	$\frac{7.4}{20.1}$	9.8	$\frac{12.2}{27.3}$	320.46	
					1381.2
3.......	$\frac{8.2}{21.3}$ $\frac{10.2}{10.0}$	12.0	$\frac{13.8}{11.0}$ $\frac{14.2}{30.3}$	425.4	
					1812.7
4.......	$\frac{12.2}{27.3}$	13.8	$\frac{15.0}{12.0}$ $\frac{16.4}{33.6}$	553.47	
					2331.2
5.......	$\frac{14.4}{30.6}$	16.6	$\frac{18.4}{36.6}$	705.4	

In calculating the areas (as at Station 3) we arrange the data as follows:

$$\frac{0}{9} \quad \frac{8.2}{21.3} \quad \frac{10.2}{10.0} \quad \frac{12.0}{0} \quad \frac{13.8}{11.0} \quad \frac{14.2}{30.3} \quad \frac{0}{9}$$

and for positive terms work from the center outward, multiplying each numerator by the next denominator ahead as we pass out from center, and for negative terms multiplying each numerator by the next denominator towards the center.

Calculation:

Area on right $= \frac{1}{2}[12.0\times 11.0 + 13.8\times 30.3 + 14.2\times 9.0 - 14.2\times 11.00] = 260.9$

Area on left $= \frac{1}{2}[12\times 10 + 10.2\times 21.3 + 8.2\times 9 - 8.2\times 10] = 164.53$

Check calculation:

Area on right $= \frac{1}{2}[25.8\times 11.0 + 28\times 19.3 - 14.2\times 21.3] = 260.9$ sq. ft.

Area on left $= \frac{1}{2}[22.2\times 10 + 18.4\times 11.3 - 8.2\times 12.3] = 164.53$ sq. ft.

Total area $= 260.9 + 164.53 = 425.4$

Area at Sta. 4 = $\frac{1}{2}$[13.8 × 12 + 15 × 33.6 + 16.4 × 9 + 13.8 × 27.3 + 12.2 × 9 — 12.0 × 16.4] = 553.47.

Area at Sta. 5 = $\frac{1}{2}$[16.6 × 67.2 + 32.8 × 9] = 705.4.

Volume 1–2 = $\frac{100}{54}$ [221.31 + 320.46] = 1003.3 cubic yds.,

Volume 2–3 = $\frac{100}{54}$ [320.46 + 425.4] = 1381.2 cubic yds.,

Volume 3–4 = $\frac{100}{54}$ [425.4 + 553.47] = 1812.7 cubic yds.,

Volume 4–5 = $\frac{100}{54}$ [553.47 + 705.4] = 2331.2 cubic yds.

Total volume 1–5 = 6528.4 cubic yards.

PROBLEM 73.—Find the areas, volumes and total volume from the following field notes:

Station.	Cut or Fill.					Areas	Cubic Yards
	Left		c	Right			
6.......		$\frac{15.8}{32.7}$	18.4	$\frac{20.2}{39.3}$		......	......
7.......		$\frac{14.8}{31.2}$	16.9	$\frac{18.8}{37.2}$		......	......
8.......	$\frac{12.8}{28.2}$	$\frac{14.7}{13.0}$	15.0	$\frac{16.3}{12.0}$	$\frac{17.4}{35.1}$	......	......
9.......		$\frac{11.4}{26.1}$	13.6	$\frac{14.4}{11.0}$	$\frac{16.2}{33.3}$	......	......

Total volume = 7533.7 cubic yards.

158. Preliminary Estimates.—In comparing preliminary surveys of several lines, it is necessary that we know the number of cubic yards of excavation required on each line. The preliminary profile will give the cut or fill at the different stations, and if we assume that the cross-section is level we can obtain a close approximation to the true areas and hence to the volumes without going to extra expense of setting slope stakes to determine the true cross-section.

From article 149 the area of *B* of a level cross-section is given by

$$B = 2bc + sc^2$$

Where $2b$ = width of roadbed, c = center cut, s = slope
If $2b = 18'$, $s = 3:2$, then $B = 18c + 1.5c^2$.

Now, if we make $c = 1, 2, 3, 4, 5$, etc., we get areas of 19.5, 42, 67.5, 96, etc.

It is assumed that any of these areas is the average of the two sections, 50 ft. on each side of it.

$$\text{But Volume} = \frac{100B}{27} \text{ cubic yards}$$

Making B equal to the areas above, we get the volumes in cubic yards to be 72, 156, 250, 356, 472 cubic yards, etc. In the same way we can find the volumes for any width of roadbed and any slope. The usual widths are 12, 14, 16, etc.

Table V. gives the volumes in cubic yards, slopes 1:4, 1:2, 1:1, 3:2, 2:1 and 3:1 and for the various widths.

Example: If $2b = 18$, $s = 1:1$, and it is desired to find the volume in cubic yards from stations 5 to stations 10, where the center cuts are 6; 8, 10, 12, 11, we look in the table headed "Slopes 3:2" under "base" and opposite 6 we find 600, opposite 8, 889, etc. These are read from Table V. and recorded as below:

Station.	Center Cut.	Volume.
5	6	600
6	8	889
7	10	1,222
8	12	1,600
9	11	1,406
10	9	1,050

Sum of volumes = 6,767 cu. yds.

From this we must subtract half the end volumes, or 825.

Volume between Sta. 5 and Sta. 10 = 5942 cu. yds.

Problem 74.—If the center cuts at Stations 17, 18, 19, 20 and 21 are 12, 14, 15, 16, 15, find the number of cubic yards between Stations 17 and 21 for level sections by use of Table V.

159. Earthwork Note-Book.—The preliminary estimate of the amount of earthwork is for a basis of comparison with other preliminary lines, but the final estimate is based on the actual notes taken in the field in setting the slope stakes. The level notebook, as commonly used, has a left-hand page ruled into six columns, as shown in Fig. 90. The grade column (marked "Gr."

Left hand page. Roadbed in Cut = 18 Feet; " " Fill = 14 "						Right hand page. Slope in Cut = 1 : 1.; " " Fill = 3 : 2.					
Sta.	B.S.	H.I.	F.S.	Elev.	Gr.	C				Areas	Vol. in Cu. Yd.
21	1.22	97.68		96.46	86.2	$\frac{9}{18}$	$\frac{+10.8}{0}$		$\frac{12}{21}$	295.4	
22			5.4	92.3	85.4	$\frac{7.2}{16.2}$	$\frac{+6.9}{0}$		$\frac{10.8}{19.2}$	200.4	518.1
23			7.2	90.5	84.6	$\frac{5.4}{14.4}$	$\frac{+5.9}{0}$		$\frac{7.4}{16.4}$	148.5	646.1
24			9.5	88.2	83.8	$\frac{3.2}{12.2}$	$\frac{+4.4}{0}$		$\frac{5.8}{14.8}$	99.9	460.0
25	1.86	88.30	11.24	86.44	83.0	$\frac{2.4}{11.4}$	$\frac{+3.4}{0}$		$\frac{4.4}{13.4}$	72.8	319.8
26			3.7	84.6	82.2	$\frac{1.2}{10.2}$	$\frac{+2.4}{0}$		$\frac{3.2}{12.2}$	46.7	221.3
27			5.5	82.8	81.4	$\frac{0.5}{9.5}$	$\frac{+1.4}{0}$		$\frac{2.0}{11.0}$	23.6	133.9
28	2.14	82.26	8.18	80.12	80.6	$\frac{-1.4}{9.1}$	$\frac{-0.3}{0}$	$\frac{0}{3}$	$\frac{0.8}{9.8}$	−7.9 | +2.4	51.9
29			3.20	79.1	79.8	$\frac{-1.8}{9.7}$	$\frac{-0.7}{0}$		$\frac{0.0}{9.0}$	−12.8	−38.3
30			5.0	77.3	79.0	$\frac{-2.4}{10.6}$	$\frac{-1.7}{0}$		$\frac{-0.8}{8.2}$	−27.2	−74.1
31			7.2	75.1	78.2	$\frac{-4.0}{13.0}$	$\frac{-3.1}{0}$		$\frac{-1.4}{9.1}$	−53.2	−148.9
32			9.3	73.0	77.4	$\frac{-5.6}{15.4}$	$\frac{-4.4}{0}$		$\frac{-2.8}{11.2}$	−87.9	−261.3

Fig. 90.

in Fig. 90) is filled in from the profile or established rise per 100 feet. The right-hand page of the notebook is ruled into spaces one-tenth to one-fourth inch square. A central line divides the right-hand page into two halves and these can be utilized for the earthwork notes. The difference between the "Elev." and "Gr." is record as the center cut under "C." with a zero for the denominator and the left and right-side cuts are written on the left and right, respectively, with the "distance out" as the denominator. The areas are calculated in square feet and recorded under the heading "Areas" and the volumes are found by the mean-end-area formula, that is, by multiplying the average of the two end areas by 100 to obtain the cubic feet, and by dividing this by 27 to obtain the cubic yards. In passing from cut to fill the usual practice has been followed, averaging the plus area (cuts) between Stations 27 and 28 to obtain the amount of cut or plus volume. In the same way the average of the negative areas between Stations 28 and 29 has been taken for the amount of fill between these two stations. Thus, the amount of cut between Stations 27 and 28 = $\frac{100}{2}(25.6+2.4) \div 27 = 51.9$ cubic yards. The number of cubic yards of fill between stations 28 and 29 = $\frac{100}{2}(7.9 + 12.8) \div 27 = 38.3$ cubic yards.

If a grade-point occurs between two stations and the intersection of cut and fill is approximately normal to line of survey, that part in cut is treated as a wedge whose volume is equal to the half area in cut at last station in cut multiplied by the distance of grade-point from said station. In the same way the part in fill is treated as another wedge whose volume is found the same way.

160. Special Case.—Where there is a rather sudden change from cut to fill a special solution is required to obtain the exact quantity of earth in cut and fill. Let *AB*, Fig. 91, be the contour between the excavation and embankment, *EB* = width of roadbed in cut, and *CH* = width of roadbed in fill. Locate the points *A, B, C, D* on the ground where the plane of the roadbed intersects the surface of the ground. Take level notes on the cross-section *FEB* and *CHG* and measure the distances AE and DH. Then the volumes of $A-FEB$ and $D-CHG$ are treated as pyramids whose bases are FEB and CHG and altitudes AE and DH, respectively. The volume between the sections FMB and the next full station is found by multiplying the average of the end areas by the distance between FMB and the full station and by dividing the product by 27. Thus, if the contour AB occurs between Stations 54 and 55 and M is 42 ft. from Station 54 and the area in cut at Station 54 is 286.8 sq. ft., area $FMB = 73.8$; then the volume between sta. 54 and $FMB = ½\ (73.8 + 286.8) \times 42 \div 27 = 280.5$ c. y.

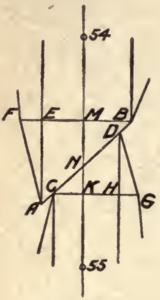


Fig. 91.

EXAMPLE: Given $EB = 18'$, $CH = 14'$, slope in cut = 1:1, slope in fill = 3.2, cut at $F = 6.00'$, cut at $M = 3.2$, fill at $G = 4.8'$, fill at $K = 2.2'$, $AE = 20.0'$, $DH = 15.6$. Then area of $FMB = 73.8$ sq. ft. Volume A $-FEB = ⅓ \times 73.8 \times 20. \div 27 = 18.2$ cu. yds. Area $CKHG = 49.50$ sq. ft. Volume $D-CKHG = ⅓ \times 49.50 \times 15.6 \div 27 = 9.53$ cu. yds.

Now distance $EA = 20$ and $DH = 15.6$, hence $MN = 10.0$, $KN = 7.88$. Therefore, distance from K to station $54 = 42 + 10 + 7.8 = 59.8$. Hence, distance from K to station $55 = 100 - 59.8 = 40.2$ ft. Find number cubic yards in fill between K and 55 if area in fill at $55 = 222.2$ sq. ft.

161. Borrow Pits.—When the excavations will not fill the cuts or embankments, or when the haul is too far for economy, it becomes necessary to obtain earth from the areas adjacent to or near the embankment. Such places are called "borrow pits," and when it is desired to ascertain the amount of earth excavated the area is first divided into rectangular sections about 10x10 ft. With some local point as bench mark or datum, the elevation of each corner of rectangles is determined with reference to the bench mark. After the excavating is finished the points are re-located in the pit and the new elevation of each point again determined with reference to the datum. The difference of the two elevations of any point will be the depth of excavation of that point. The volume taken out of any rectangle will be found by drawing the diagonal (as 13) in the 1 2 3 4.

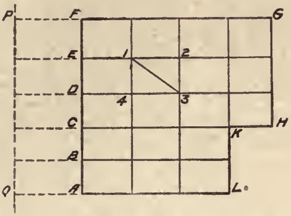


Fig. 92.

Then let $A =$ area 1234,

$h_1 =$ depth of cut at 1,
$h_2 =$ depth of cut at 2,
$h_3 =$ depth of cut at 3,
$h_4 =$ depth of cut at 4.

$$\text{Now, volume } 1\ 2\ 3 = \frac{A}{6}(h_1 + h_2 + h_3)$$

$$\text{volume } 1\ 3\ 4 = \frac{A}{6}(h_1 + h_3 + h_4)$$

$$\therefore \text{Total volume } 1\ 2\ 3\ 4 = \frac{A}{6}(h_2 + h_4 + 2h_1 + 2h_3) \ldots . \quad (39)$$

Rule: Multiply one-sixth the area of rectangle by twice the sum of the two heights at ends of diagonal plus the sum of the other two heights. Ordinarily, the volume can be found with sufficient exactness by taking the average of the four cuts and multiplying this by the area, or,

$$\text{Volume} = \frac{A}{4}(h_1 + h_2 + h_3 + h_4), \text{ nearly}$$

In order to re-establish the points 1, 2, 3, etc., after the excavation has been made it is necessary to establish some base line like *PQ*, that will not be disturbed by the plows or teams and tie every point to this line by rectangular co-ordinates. Thus, to re-establish the point 3, its perpendicular distance from a certain point on *PQ* must be known.

162. End of Fill.—When a fill has a gap in it for a trestle, the dirt is often allowed to spill obliquely beyond the end of the dirt on the track grade. Thus, if AB, Fig. 93, is width of roadbed and is the termination of dirt roadbed, the dirt is allowed to fall down the slope to the irregular line $CDEFG$, where GH and CK are the intersection of the side slopes with the ground surface. The depths of G and C below the plane of roadbed are known from the slope stake notes. It is sufficient to treat the volume whose base is $ABDF$ as a wedge whose base is AMB and whose edge is DEF, and the volumes CBD and AFG as quarter cones whose bases are BCD and GAF, and whose altitudes are the heights of B and A, respectively.

The bases BCD and AFG can be treated as quarter-ellipses.

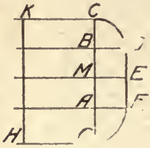


Fig. 93.

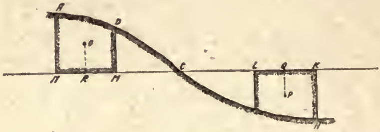


Fig. 94.

Hence, the area $BCD = \frac{1}{4}\ BD \times BC\pi$ and area $AFG = \frac{1}{4}\ AG \times AF\pi$.

EXAMPLE: Given $AB = 14'$; heights of A, M, and $B = 8'$, $7.6'$ and $6.8'$, respectively, and $AG = 12'$, $BC = 10.2$, $BD = 14'$, $AF = 14'$.

Cross-section of wedge $= 14\ (8 + 2 \times 7.6 + 6.8) = 105$.

$$\text{Volume of wedge} = \frac{105}{2} \times \frac{14}{27} = 27.2 \text{ c. y.}$$

$$\text{Volume of } CBD = \tfrac{1}{3} \times \frac{10.2 \times 14\pi}{4} \times \frac{6.8}{27} = 9.4 \text{ c. y.}$$

$$\text{Volume of } AFG = \tfrac{1}{3} \times \frac{12 \times 14\pi}{4} \times \frac{8}{27} = 13.0 \text{ c. y.}$$

Total volume $GCDF = 49.6$ cu. yds.

163. Overhaul.—In contracts for earth work the price per cubic yard is based upon the condition that for this price no material should be transported further than a certain distance (called the "free haul"), and that extra pay should be allowed for all material carried further than this. In Fig. 94 ML = free haul, $ABMN$ represents the excavated material and $LDHK$ represents the material deposited in embankment. If O and P represent the centers of gravity of the volumes $ABMN$ and $LDHK$, the distance RQ is the total haul and the excess of this over the free haul is the overhaul.

$$\therefore \text{Overhaul} = RM + LQ$$

To find the centers of gravity O and P, multiply each elementary mass by its distance from some point C and divide the sum of such products by the sum of the elementary masses. However, it is sufficient in practice to find a point that divides each mass into two equal parts and use these as the centers of gravity.

164. **Shrinkage.**—From a varied mass of data, H. P. Gillette, in his book on "Earthwork and Its Cost," has compiled the deduced principles:

1. Taking extreme cases, earth swells when first loosened with a shovel, so that after loosening it occupies 1 1-7 to 1½ times as much space as it did before loosening. In other words, loose earth is 14 to 50 per cent more bulky than natural bank earth.

2. As an average, we may say that clean sand and gravel swell 1-7, or 14 to 15%; loam, loamy sand or gravel swell 1-5, or 20%; dense clay and dense mixtures of gravel and clay, ⅓ to ½, or 33 to 50%, ordinarily about 35%; while unusually dense gravel and clay banks swell 50%.

3. That this loose earth is compacted by several means: (a) the puddling action of water, (b) the pounding of hoofs and wheels, (c) the jarring and compressive action of artificial rolling.

4. If the puddling action of rains is the only factor, a loose mass of earth will shrink slowly back to its original volume, but an embankment of loose earth will, at the end of a year, be still about 1-12, or 8%, greater than the cut it came from.

5. If the embankment is made with small one-horse carts, or wheel scrapers, at the end of the work it will occupy 5 to

10% less space than the cut from which the earth was taken, and in subsequent years will shrink about 2% more, often less than 2%.

6. If the embankment is made with wagons or dump carts, and made rapidly in dry weather without water, it will shrink about 3% to 10% in the year following the completion of the work, and very little in subsequent years.

7. The height of the embankment appears to have little effect on its subsequent shrinkage.

8. By the proper mixing of clay or loam and gravel, followed by sprinkling and rolling in thin layers, a bank can be made weighing 1¾ times as much as loose earth, or 133 lbs. per cu. ft.

9. The bottom lands of certain river valleys and banks of cemented gravel or hardpan are more than ordinarily dense and will occupy more space in the fill than in the cut unless rolled.

Earthwork is paid for by the cubic yard, usually measured "in place," that is, in the natural bank, cut, or pit before loosening; but there is no good reason why it should not be measured in the fill or embankment, and it often is so measured where it is very difficult to measure the borrow pits. In either case the specifications should distinctly state how the measurements are to be made. Sand or gravel for mortar and concrete are usually paid for by the load in the wagon.

Bibliography.—"Railway and Earthwork Tables." By C. L. Crandall. It is sufficient to say that this book bears out its title, where the tables are arranged by which we can read the volume for railroad cuts and fills for any of the usual data given in the field notebooks for cross-sectioning.

"Railway Earthwork." Parts I and II. By the late A. M. Wellington. Part I discusses the volumes of the various solids in railway earthwork, while Part II consists of a series of diagrams from which the volume corresponding to the field notes can be read at once.

"Railroad Curves and Earthwork" (with Tables). By C. Frank Allen. In the section on earthwork the theory and use of graphical diagrams are treated and the methods of using these diagrams to obtain the volumes are illustrated by many examples.

"Primoidal Formulas and Earthwork." By T. U. Taylor. The history of the different formulas that apply to the earthwork solid and their application to railway cuts and fills are given. A chapter is devoted to the two-term formula wherein it is shown that there is an indefinite number of two-term formulas that give the exact volume of the prismoid; that if we take the average of two sections, these sections must be 21.14 feet from each end of the solid 100 ft. in length.

"Manual of Road Making." By W. M. Gillespie. Contained in appendix some 40 pages upon the subject of earthwork, in which, in addition to the treatment of the ordinary cases, he showed that the prismoidal formulas applied to give the exact volume of the earthwork solid when the upper surface was a warped surface.

"Earthwork and Its Cost." By H. P. Gillette. 244 pages. This work has taken up and considered actual examples, giving date, size of contract, conditions under which constructed, kind of earth, how handled, etc. The author has winnowed from many a contract the essentials as to shrinkage, classification, loosening, cost when carried by wheel barrows, wagons, buck and drag scrapers, wheel scrapers, by elevating grader, steam shovels, cars, etc.

"Rock Excavation. Methods and Cost." By H. P. Gillette. 375 pages. Its title abundantly indicates its scope. Its estimates of cost are from concrete examples where actual conditions are given.

"Handbook of Cost Data." By H. P. Gillette. One of the most valuable books for the engineer that has appeared in many years, and it comes nearer filling a long existing void than any book before the engineering public. It includes a great deal of the material in the two books mentioned above and much additional matter. It deals directly from the ground with such questions as cost of earth and rock excavation, roads and pavements, stone masonry, concrete construction, water works, sewers, piling, trestling, erecting buildings, steam and electric railways, bridge erection, railway and topographic surveys and miscellaneous structures. This book should be a valuable *Vade Mecum* for any engineer who has to deal with the cost of structures.

CHAPTER X.

CITY SURVEYING.

165. The City Engineer.—The most important factor and vital unit in all city surveying is the city engineer. A careless engineer means a careless, loose, inaccurate, conflicting and litigous survey. The city engineer is the supreme court and all the lower courts with respect to the accuracy of city surveying. As the city engineer, so is the survey. The engineer should be the first instrument of precision selected, and it is supreme folly to have a standardized steel tape and a highly sensitive transit in the hands of a careless operator. We apply corrections for sag, temperature and pull to our tape-line measurements, but these are mockeries if the engineer can be sagged from his true course, or if he allows a "pull" to draw him from the straight line. The accurate, just, and fearless performance of his duty should be his platform. To this end should he be born, for this cause came he into the world, and he should bear witness to the truth.

The surveying demanded of a city engineer does not involve any principles, operations, or intricacies that may not be easily overcome by any person who understands thoroughly the use of the ordinary instruments and theory of surveying heretofore described, but as land is much more valuable in cities than in the country it follows that the measurement of city property must be made much more carefully than the survey of a farm. The accuracy of the survey should increase with the value of the property. Small errors that may be neglected now may involve perplexing difficulties in years to come. It is always wise and safe to be considered a little too fine-haired rather than a little too careless.

166. Objects of Survey.—The prime object in a city survey is to establish the points and boundaries of city property with absolute accuracy. To do this, it is necessary to establish certain reference lines or points which will remain permanently fixed and which, like a reference library, are of easy access and of undisputed authenticity. Property is valuable, and to prevent litigation it is necessary to have all property lines authoritatively established beyond the shadow of a doubt. Chains with their many hundred wearing surfaces are unfit for such work, and as it lacks accuracy the compass can not be used. As the ordinary transit measures to the nearest minute and as an angle of 1′ is subtended by an arc of 18 ins. at a distance of one mile its use should be precluded where accurate work is demanded. The primary object of a city survey should be the accurate location of all property lines in accordance with recorded notes or maps, and complete provision for the rapid, convenient and accurate re-establishment of these at any time. The most accurate instruments and greatest care should be used.

167. Monuments.—It is of fundamental importance that lasting monuments be established to which all city lines, points and buildings can be referred. Eternal monuments is the price of accurate work in city surveying. While engineers and surveyors are liable to rail at and descant sneeringly at the loose methods pursued in making the original land surveys, many of such land surveys are monuments of accuracy when compared with the surveys of many of our cities. In fact, although our original land surveys were loosely made, all transfers of property have been based on such surveys. These surveys have many monuments in the shape of trees to stand as silent witnesses to be called upon. The land at least had an original survey, while the original part of a majority of our cities has expanded without the semblance of an original survey. It is worthy of remark that more care and accuracy are displayed in surveying the "additions" and "out-lots" than obtained in the original survey of the nest-egg of the town.

But whether or not monuments were established in the original survey of the town, it is of the utmost and urgent importance that they be established at the earliest possible moment. In some cities a very loose habit has prevailed of using old buildings for reference points. Such a practice should be condemned as a make-shift, for with the enhanced value of property, such buildings are liable to be razed to make room for modern structures.

168. **Additions.**—The map of every "addition" or projected town should when filed in the county clerk's office show clearly the location of all monuments and no map should be admitted to record that does not give these data. Not only should such a map show the location of such monuments, but a full description of such monuments should be made a matter of record. Such requirements should not be a matter of custom, ethics, or taste of the surveyor, but should be a matter of law; and there is no more reason for a law authorizing the employment of skilled surveyors to locate state lands and file a complete set of field notes for the same than there is for a similar law requiring every city to have a similar map or set of notes filed and made a matter of record. These notes should be so clear and include such a number of sketches that they may be readily understood by any person of average intelligence; and such notes should be capable of only one interpretation. Litigation has always fed fat on loose and inaccurate surveying and an unmonumented city.

Monuments should be set and established by the original surveyor. He it is that made the surveys with respect to such monuments and it is his duty to finish his survey. It can be truthfully said, "An unmonumented city has no survey." There is a certain respect paid to the County Surveyor and his work should command respect. So it should be with the work of the city engineer, but while our laws provide for "witness trees," "fore and aft trees," for land surveying, there are in many states no adequate laws for enforcing or establishing imperishable witnesses to the city lines in a city survey.

169. **Kinds of Monuments.**—Monuments should be constructed of permanent material and the special kind will be decided by the question of economy. The materials most commonly used are stone, concrete, wood, and iron rods or pipes. If a stone is used it should be imbedded in the ground with its upper part well underneath the surface, so that the big end will be down and so that it will rest solidly in its bed and have no tendency to change its position. A small hole from ½ to 1 in.

in diameter should be drilled in the upper surface of the stone to a depth of 6 to 8 ins. Into this hole a copper bolt should be inserted and melted lead or babbit metal run around it to hold it securely in position. The upper end of the bolt should be flush with the surface and two normal diametral lines should be marked across the bolt, their intersection forming the reference point over which the plumb-bob of the transit is suspended, or a flag pole set when other points are to be established.

A concrete block, Fig. 95, can be constructed as a monument and it has many advantages over the stone monument, as it can be formed into any desired shape. For economy, the concrete monument should be built in the form of the frustrum of a cone or pyramid, and its upper surface should be kept well below the surface of the street. The copper bolt can be imbedded in the concrete before it hardens and it can be located in any desired position in the concrete.

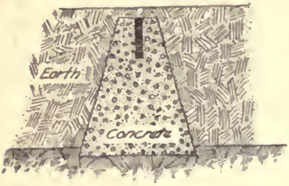


Fig. 95.

If wood is used, the most durable available wood should be selected. The important monuments should be at least 6x6 ins. by 4 ft. in length and should be imbedded on hard soil or preferably on a flat rock or a concrete mixture. Cedar is an excellent material, while osage orange (boisdarc) has no superior. The young mountain locust, 10 ins. in diameter, is the most durable in the east. while mesquite would be practically the only locally available wood of the southwest.

An iron rod or pipe is often driven with maul or sledge for a monument, but these do not make very satisfactory monuments and are not to be recommended, but it must be said that they are infinitely better than none at all and greatly superior to a small wooden stake. Wooden stakes are very easily disturbed or destroyed and unless they are immediately replaced by other monuments of a more permanent character the work will be wasted.

If the street is already graded and paved the monument should be set with its top below the foundation of the pavement and should be protected and made easily accessible by means of an iron jacket and cover plate such as are provided for the valves of the city water supply.

If the street is neither graded nor paved, some thought should be given to the probable final street level and the monument should be located to conform therewith if possible.

It is the duty of the city engineer to establish suitable permanent monuments wherever needed, to indicate the same clearly and correctly on the proper maps, to deposit in the office a complete set of all field notes, to leave his work in such a state that it may all be intelligible and useful to his successor.

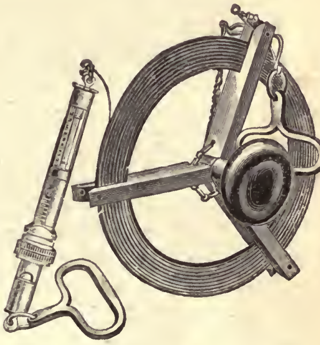


Fig. 96.

170. Location of Monuments.—These should, if possible, be located in the center lines of cross streets and should be on high points. They should be of easy access; a few well located monuments are more valuable than many to which ready reference cannot be made. The fundamental requisites of good monuments are that their location is known and that their distance and azimuth are matters of record.

Sometimes it is impracticable to set monuments in the center of the street. When this is the case, they should be placed as near the center as convenient, but they should always be referenced in to the four corners of the street.

Wherever the monuments are located, the four corners of the streets should be marked by sub-monuments whose distances from the main monument are recorded.

171. Tapes.—It is useless to have an excellent system of monuments unless this excellence prevails throughout the whole organization of the city survey. All lines should be measured with standardized steel tapes. The material of the tape should be of the best steel and its own individual constants should be determined. It should be sent to the U. S. Coast and Geodetic Survey, Washington, D. C., to be standardized. It is there compared with an absolute standard, its coefficient of expansion ascertained, its pull and temperature for standard length determined. These data are returned with the tape and in all important measurements should be used and corrections should be made for temperature, pull, sag, and grade. But accurate work can not be performed with accurate instruments unless accurate methods are used. In chaining, if the street is graded uniformly and the tape can be made straight, the correction for sag would thus be eliminated. If in addition to this, the standardizing pull be applied, the only correction remaining would be that for temperature and grade, and if the street is horizontal, the only correction to be applied would be that due to temperature.

172. Transit.—After the monuments have been located with accuracy and the exact point of these monuments marked by the intersection of lines on the copper bolt head, it becomes necessary to use the most accurate and refined instruments in the prosecution of the further surveying of the city. As the ordinary transit reading to one minute of arc would produce an error of 18 ins. in one mile, its unfitness for accurate city surveying is at once seen. It is useless to locate monuments accurately and to use an accurate standardized tape in connection with a transit that has such possibilities of error as the ordinary engineer's transit. For this reason a special transit (Fig. 97) is constructed with minuter graduations. The same reason that precludes the use of the engineer's transit in refined city work, of course, would exclude the surveyor's compass to a greater degree. In the modern transit constructed for accurate city surveying, the needle and the needle box are omitted and the standards are constructed in one U-shaped piece that gives greater rigidity of bearing to the horizontal axis that supports the telescope, and consequently greater accuracy. The horizontal circle is much larger and the graduations can be made as small as ten seconds of arc. The

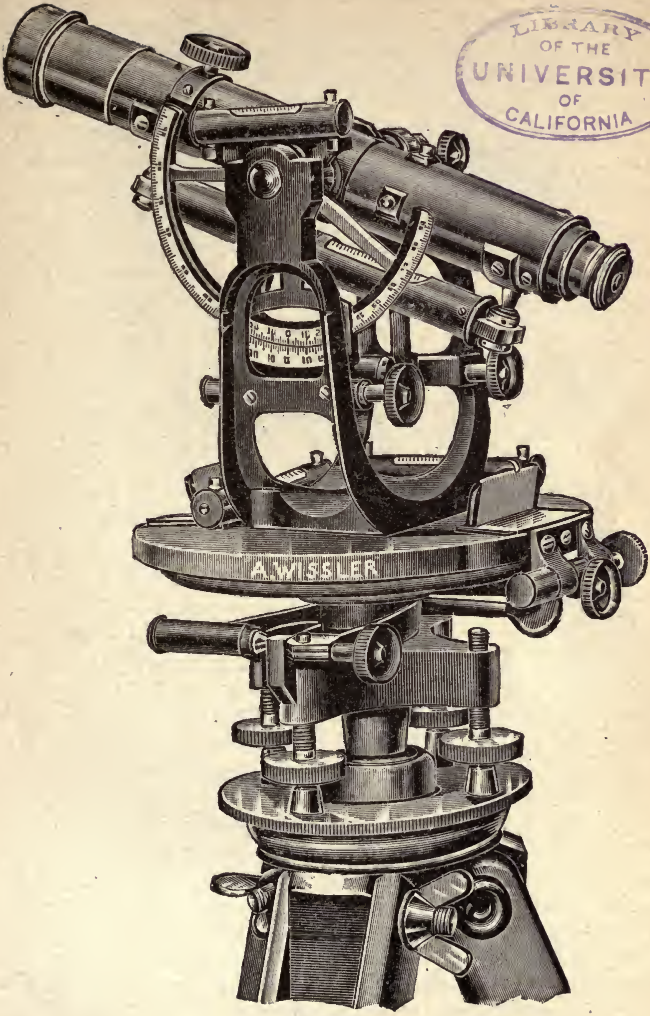


Fig. 97.

horizontal circle is protected from view by a cover plate except where the slot is made for the reading by the verniers. The verniers are read by special reading glasses, which are often attached to the instrument itself. Whatever the fineness of the reading may be, whether it reads to 10″ or 20″, we can by the repeating method read the angle five times and thus reduce the fineness of the reading to one-fifth of that given by the verniers. Thus if the transit is graduated to 30″, we can by repeating the observation five times get a reading of 6″, and if it reads to 10″, we can by the repetition of five times get a reading to 2″. In the length of one mile a reading of 2″ would mean about a half an inch error.

The transit can be provided with stadia wires and complete vertical circle and a heavy tripod. The complete vertical circle and stadia wires are auxiliaries that are added for the purpose of making topographic survey. The transits fulfilling these requirements cost from $300 to $700 and if it is desired the stadia wires and vertical circle can be omitted.

Fig. 98.

173. Datum.—There should be established in every city bench marks to which all elevations should be referred. In the majority of cases, the elevation of the bench marks can be referred to the sea level or mean low tide. In many cities the U. S. Coast and Geodetic Survey has bench marks with reference to sea level that have been established by a system of precise levels run and checked from the coast to the interior. These are by far the most reliable and accurate bench marks that can be obtained. The U. S. Geological Survey has also a chain of bench marks established in certain sections of the country. The bench marks established by these two surveys are often copper bolts set vertically in the cap stone of bridge piers, or horizontal bolts set inside of stone buildings. Another form is a circular disc, Fig. 98, from the center of which a bolt 3 ins. long projects at right angles to the surface of the disc. Two diametrical lines normal to each other are marked across the face of the disc and the elevation is stamped on the horizontal line of the disc. A bed or setting is cut out of the stone for the disc and in the center of this bed a hole is drilled to receive the bolt. The bolt is then leaded into the stone.

174. General Maps.—There is generally a small scale map made of the whole city, but this shows few engineering features and except in the case of small cities it can not show the dimensions of lots and the field notes for the location of monuments. In addition to the map of the whole city there should be a map of certain sections to a scale sufficiently large to show all lengths of all lines and angles made by intersecting lines. It is the practice in many cities to have block maps containing from one to four blocks with the position of all monuments marked with distance from street corners and angles made by such tie lines. These maps should show the center line of street, angles of intersection of center lines, and the location of monuments on street corners.

The map should contain the following data:

1. Length of all lines.
2. Angles made by intersecting lines.
3. The exact position of all monuments.
4. The number of each block and lot.
5. The names of all streets and streams.
6. Water pipes and fire plugs.
7. Sewer pipes.
8. The true meridian.
9. Width of streets.
10. The position of adjoining property lines.
11. A complete title to map.
12. The scale.

175. Water-Pipe Map.—If the city owns the water-works and sewerage systems, it should possess an up-to-date, accurate and distinct map of both the water-pipe lines and the sewer-pipe lines. If the city is small and pipe connections are not intricate nor numerous, one map will suffice for both systems, by adopting a different legend for the two systems. A water-pipe map should show clearly the position of all mains, valves, connections, fire hydrants, size of pipe, and all side connections. Such a map usually pays for itself many times over and it is a very loose city government that does not keep such a map. Without a pipe-line map all extensions and repairs have to be made somewhat upon the temporary makeshift basis. In some cases, the city authorities depend upon the memory of a day laborer to locate sub-mains, and these often have to spend hours in search of the pipe, all of which time could be saved by an accurate map. If a private company owns the water-works, an accurate map is part of its equipment because it is simply a part of good business to have such a map. However, there often seems to be some fatality about municipal ownership in regard to proper records. The city records, covering expenditures of millions of dollars for public improvements are often thrown aside or dumped into boxes, or cases that cannot be used for any other purpose. The proper keeping of engineering data is a weak spot of municipal ownership, an indictment that cannot obtain in the same degree against private ownership.

When city streets are improved by paving, it is of the utmost importance in making repairs or connections to know the exact distance of the main or sub-main from the sidewalk or property lines, as it is a matter of economy in time and renders the tearing up of a large area of paving unnecessary.

176. City Blocks.—The size and shape of city blocks vary in different sections of the country and, in fact, in different sections of the same city. It is difficult to set any limits, but the regular rectangular blocks vary in length from 400 to 900 ft. With a width of street of 80 ft. there will be 5⅓ to 11 blocks to the mile, and of course if the streets are narrower there would be from 6 to 12 to the mile, etc.

177. Rectangular Blocks.—In ordinary cases, a rectangular block consists of two rectangular sections with an alley between. Thus if *ABFG*, Fig. 99, is a rectangular block, there are two sections, *ABCD* and *EFGH*, with an alley *DCEH*. If the length of the block is 300 ft. and if each section contains five lots, these should be 60 ft. wide. The length of the lot is 125 ft. and width of alley 16 ft., the block being 266 ft. wide.

Each lot is described (1) by its number, (2) by the number of the block, (3) by the sub-division or addition, (4) by the name of the city, county, and state. Thus we should write:

"Lot number (3) three in Block thirty-nine (39), Borden Addition, in City of Austin, Travis County, Texas." This description is sufficient if an official map of this "Borden Addition" is on record in the city or county clerk's office, showing all dimensions of such lot. However, if it is desired to insert the metes and bounds, this can be done as follows:

"Lot number three (3) in block thirty-nine (39), Borden Addition, in the City of Austin, County of Travis, State of Texas, and bounded as follows: Beginning at the northeast corner of lot number two (2) in said block, addition and city, one hundred and twenty (120) feet from the northwest corner of said block, thence S 9° W, one hundred and twenty-five (125) feet with the east line of lot number two (2), to a corner on the alley, thence S 81° E sixty (60) feet to the SW corner of lot number four (4); thence N 9° E, with west line of lot number four (4) one hundred and twenty-five (125) feet to a point on the north side of block, the northwest corner of lot number four (4), thence N 81° W with the north line of said block and with the south line of Adams St., sixty (60) feet to the beginning."

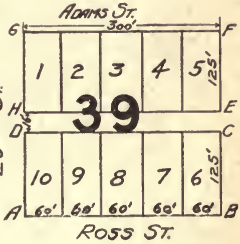


Fig. 99.

178. Rectangular Lots.—The size of lots runs the scale from the narrow business property lot 25 ft. in width to that of the broad frontage, merging into the suburban property defined by the acre and metes and bounds. The lots in the regular residence section vary from 40 to 100 ft. in frontage, but there is infinite variety to the special dimensions and the foregoing figure are approximate only.

In regard to the depth of the regular rectangular residence lot, it can be said that the depths are approximately double the frontage, varying from 90 to 200 ft. unless some irregular boundary, stream or hill intervenes to modify the general plan by which the lots are laid off.

179. Irregular Blocks and Lots.—It often happens that the topography, old roads or streams force the engineer to make a block of irregular shape, the flat-iron, horse-shoe, triangular or oval. In such a case no rules can be laid down for cutting such a block up into lots, and the engineer can have only one guide, and that is to make each lot wide enough for the buildings of that locality (business or residence) and of the ordinary depth.

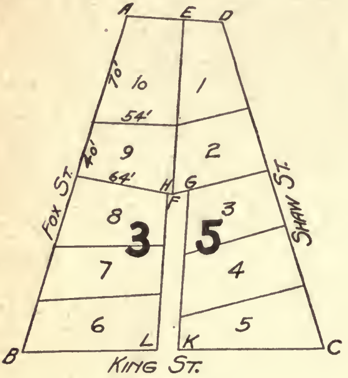


Fig. 100.

If *ABCD*, Fig. 100, represents the apex block between two converging streets it is often difficult to divide this up into lots to the best advantage. The simplest method is to run the lot side lines perpendicular to the street line. This is shown by the side lines of lots 1, 2, 3, 4, and 5, all of which lines are perpendicular to the street line on Shaw St. However, it may happen that for some substantial reason the lot lines are parallel to the alley or some other line. Again the lines may be drawn according to no system whatever. In the latter case, the opposite sides of the lot will not be parallel, and it will be necessary to describe each lot by the metes and bounds. In addition to this the corners should be marked by some permanent marks, as galvanized pipe, stones or concrete blocks.

In the flatiron form of blocks, as in Fig. 100, a dead-end alley can be provided for at the big end of the block, and this can extend as far as the line of lots will permit. A lot in an irregular shaped block should have a rather full description. Thus lot 9 should be described as follows: "Lot number nine (9) in Block thirty-five (35), Division A, in the City of Austin, County of Travis, State of Texas, which is bounded as follows: beginning at an iron pipe in line of Fox Street 70 ft. from north-west corner of said block 35, thence along Fox Street S 6° W 40 ft. to corner of lot number 8, thence S 87° E 64 ft. to a copper bolt in a stone which is a corner to lots number 2 and 9 of said block, thence north 46 ft. to a stone corner to lots 3, 9, and 10, thence S 87° 15′ W 54 ft. to the beginning."

180. Private Notes.—The careful engineer will mark the length of all lines, the angles made by the boundary lines of lots, give the full number of lot, the name of "addition," and all other data necessary to define clearly and distinctly the lot so that another engineer, years later, will have no trouble in tracing the steps of the former. Every modern engineer experiences a genuine appreciation of the original engineer, when he finds that the recorded map shows clearly all distances and angles, and the modern does not hesitate to commend the former when map dimensions, when applied to the field, are found to be true. Too many engineers are stingy with their data when it comes to putting it on the map. The question often arises as to how much data should be placed on the map, and this can be answered by saying that sufficient data should be placed on the map to enable another engineer to go upon the ground and re-locate any lot without doubt or shadow of turning. Until this condition is fulfilled the map is incomplete; the claim of the engineer that his notes are private cannot be set up or maintained. The city engineer "is a public officer and should keep complete records of all work done in his official capacity during his incumbency. If he walks out of his office and retains notes the lack of which would embarrass his successor, he is practically a thief." (Ernest McCullough.)

181. Prescriptive Rights.—Owners in new and sparsely settled additions are often permitted to locate their own lots, and in doing so they get the side lines of the lots shifted a few feet. A fence is usually erected on the lot lines erroneously located and this fence stands as the visible mark of the lot lines for many years. The adjacent lots are not improved and the result is that the owner of the improved lot, although his fence lines are wrongly located and though there may be an excess in his frontage, has been in peaceable possession, undisturbed for a sufficient time to constitute a prescriptive right. This gives him the right of possession and when the owners of adjacent lots want the amount their deeds call for, they find the prescriptive right set up as a bar to moving fence lines. The result is that legal mills have to be set to grinding with no assurance of the quality of the grist.

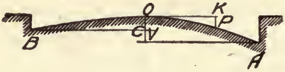


Fig. 101.

Where the prescriptive right obtains it is of the highest importance to property owners to see that their lots are located properly and accurately by an official engineer, and that permanent corners are established.

182. Cross-Section of Streets.—After the blocks and lots have been laid off and accurately marked, it then often falls within the province of the surveyor or engineer to establish the form of cross-section of the street. This cross-section is usually a curve having a certain rise or crown, depending on the material out of which the surface of the street is constructed. If the street is paved with vitrified brick the crown should be from ⅛ to ⅜ in. per foot of half width. Thus for a street width of 96 ft. between side walks there should be a crown of 6 to 18 ins., preferably the latter. If the side walks are at different elevations, local conditions may demand that the cross-section shall consist of two curves tangent to each other at the crown or crest and that the amount of their descent shall be different.

Thus in Fig. 101 the cross-section can be formed by the two curves OA with a fall of OV and OB with a fall of OC.

Let VA = distance from curb to crest = b; $OV = v$; $OK = x$; $PK = y$, fall from O to P.

Then if curve OA is a parabola,

$$PK = \frac{OV}{VA^2} OK^2,$$

$$\text{Or } y = \frac{v}{b^2} x^2 \dots\dots\dots\dots (40)$$

If $b = 48$ feet, and $v = 18'' = 1.5'$, $y = \frac{1.5}{48 \times 48} x^2 = \frac{x^2}{1536}$.

If y equal fall in inches and x = distance in feet, $y = \frac{x^2}{128}$

By making $x = 0$, 4, 8, etc., the falls at these distances are found below.

x	y	x	y	x	y
0	.00	16	2.00	36	10.125
4	.125	24	4.5	40	12.5
8	.5	32	8.00	48	18.0

Formula (40) is a general formula and will apply to any conditions, and does not assume that the crest O is in the center of street.

Circular Curve. — Some engineers prefer to treat the curve OA as a circle and specify the amount of curvature by the radius of the circle.

Let VA = half of chord of circular arc OA; v = rise = OV. As the arc is very flat, KP can be treated as a secant from P to circle.

Then if R = radius of circle,

$$OK^2 = KP\,(2R + PK), \text{ or,}$$

$$x^2 = y\,(2R + y) = 2Ry + y^2.$$

The last term is so small in comparison with the first that it can be omitted.

$$\therefore y = \frac{x^2}{2R}, \text{ or } R = \frac{x^2}{2y}.$$

If the crown is ⅜ or ⅛ in. per horizontal foot, then $R = 192$ ft., or 576 ft. respectively.

183. City Engineering Records.—There are three different kinds of records that should be kept by the City Engineer:

I Field Note-Books.

II Detail maps.

III Orders, letters of correspondence, bids, prices, contracts, specifications, results of tests, etc.

184. Field Note-Books.—For simplicity one kind of style of book that is applicable to all kinds of surveying should be adopted and used exclusively. It should have stiff covers, should be leather bound, and be as large as the average coat pocket will accommodate. If the left hand page is ruled with horizontal blue lines ¼ in. apart and the page divided by vertical red lines into six columns, the right page being divided into small squares by horizontal and vertical blue lines, with a vertical red line in the center of the page, the book will be found to answer admirably for all-round work. In this book, level notes, transit notes, notes on earth-work, sewer-pipe, water-pipe, triangulation, land surveying, etc., can be recorded with clearness and neatness. The measurements can all be placed on the left page, while sketches can be placed on the right page to an approximate scale.

Proper provision should be made for storing or filing all the note-books, preferably in a fire-proof vault. The books should be numbered consecutively and arranged in order on the shelves, and the Chief Engineer should require every note-book to be put in its proper place on the shelves or in the vault over night. Books should be assigned to certain classes of work rather than to particular assistants or transit men. For example, all miscellaneous work relating to property lines should be kept in one book, all work relating to grades of streets in another, etc. Each new book should be immediately given a number, the class of work for which it is intended being plainly lettered on the outside of the cover, thus: "Street Grades and Profiles, 5th, 6th and 7th Wards," and the first half dozen pages should be left blank for an index to its contents. Every new piece of work should be indexed in the book, and also in the general index of all the note-books kept in the office. The Chief Engineer should see that each assistant enters his notes in the proper book so neatly, completely and correctly that at the end of any day's work the book may be handed to any other assistant who would be able to continue the work without the least possible duplication or loss of time.

Each assistant should be required to carry with him the proper note-book, and to make in it the original notes of the work. If this is done, the field-book may be presented as evidence in case of law suits, but it could not be presented as evidence had the notes been copied in it from other books or from scraps of paper.

Note-books should not be permitted to litter the draughting tables or desks of the office. When not in use they should be in their proper places on the shelves, or in cases.

Each member of the office staff should be impressed with the fact that surveys are expensive and that the data contained in these note-books are valuable. Books should not be carelessly thrown about, but on the contrary should be carefully preserved and everything should be done to make the records readily available for future reference.

185. Detail Maps.—In addition to the large wall map of the city there should be smaller maps to a larger scale, showing all essential details of lines, angles, monuments, distances, etc. The wall map may be divided into sections by lines at right angles to each other, or by streets and streams into sections corresponding to the smaller maps. This enables the detailed map of any section of the city to be found with the least loss of time and trouble. On these detail sheets the water, gas, sewer, and steam mains, telephone conduits, etc., should be represented by different colored inks or by specially dotted lines. If there are many of these pipe lines, it may be necessary to have several copies of each sheet, one devoted exclusively to water service (called the water-pipe map), one to sewerage, etc.

Such maps should be made on the best quality of mounted egg-shell paper and should be service maps on which every change in pipe lines should be noted immediately. If it is considered necessary to have records of conditions at different dates—i. e., on the first of January each year—tracings of these service sheets may be made, dated and filed.

An excellent plan for standard sizes for drawings is to adopt the full sheet, half sheet, quarter sheet, and eighth sheet plan, and the dimensions of these can be for full sheets, 24x36 ins.; for half sheets, 24x18 ins.; for quarter sheets, 12x18 ins.; and for eighth sheets, 12x9 ins. Each sheet should be trimmed ½ to 1 in. outside the border except on the left, where a double margin should be left for binding purposes.

However, it is useless to have or to demand accurate city maps and drawings and not at the same time provide safe and secure repositories for such records. Substantial cases should be constructed with a set of drawers (say 40x27 ins. inside dimensions) for the full size drawings. For the half size drawings the 40 by 27 drawer can be divided by a thin partition across the middle, dividing it into two compartments about 27x19¾ ins. Another set can be provided for the quarter size drawings where the 40x27-in. drawer has two divisions or partitions at right angles to each other; and in a similar way the eighth size drawings can be provided for. The drawers should be numbered consecutively and if divided into compartments for fractional sizes each compartment should be given a letter and the drawings in it numbered in a special place on the drawing in addition to the general number that it must bear. Thus the drawing should be labeled "Drawer 26 D, Sheet 14," in one corner, while the general number 76 will indicate that it is the 76th drawing made by the city. The legend "Drawer 26 D, sheet 14," indicates that it is to be replaced in drawer 26 in compartment D, between sheets 13 and 15.

In addition, a systematic record should be kept showing clearly what each numbered drawing refers to in the general series. An alphabetical list should be made of these drawings, where the leading word in title or location will indicate the character of the drawing. Better than this, however, is a card catalogue where every map is cross-indexed in such a manner that it may be readily found. The card catalogue has many advantages over the book catalogue, in that references can be made with greater dispatch, and corrections and new insertions can be made without disturbing the other records.

186. Orders, Bids, Etc.—It is doubtful whether it is necessary to mention the necessity of keeping a record of all correspondence, orders, etc., as this is the usual practice of every good business man, and every engineer should be a good business man, as far as the city is concerned at least.

Contracts and specifications are important documents in connection with large undertakings or important works, and these should be kept in a fire proof safe, to which only the trusted members of the staff have access. Specifications, results of tests, and other data on miscellaneous matters should be indexed and may be filed in a manner similar to that for drawings.

Bibliography.—"Theory and Practice of Surveying." By the late J. B. Johnson. This book has long been a standard work for the surveyor and engineer. Its chapter on City Surveying was prepared by William Bouton, City Engineer of St. Louis, Mo., and gives the conditions necessary for high grade, accurate city surveying.

"Principles and Practice of Surveying." By Breed and Hosmer. An excellent book for the city engineer, containing full directions, discussions, and illustrations of many problems that confront the city engineer.

"Engineering Work in Towns and Cities." By Ernest McCullough. While the author disclaims any intention of writing for city engineers of cities over 10,000 population, the limit should have been placed at 50,000 instead of 10,000. The book is a history of city surveying. With gloves off it deals with the qualifications necessary for the position of city engineer, the compensation he should receive, the problems he has to solve, the difficulties he has to meet, how to keep city records, the necessary theory and principles for the various duties of the position, including the location of monuments, roads, walks, pavements, sanitation, drainage, sewerage, water supply, concrete, contracts and specifications, office system, city engineer's records and field work. It ranks as possibly the best book before the public for the use of the city engineer and especially for that city engineer who wishes to learn the best methods.

CHAPTER XI.

PLOTTING AND LETTERING.

187. Plots.—After a farm is surveyed a line map of the farm or land should be made to some convenient scale, for the purpose of showing the shape of the farm or body of land, its connections with adjoining properties, and its location with respect to natural objects. Such a plot should contain the following data:

1. Boundary lines.
2. Bearing and distance printed on each line.
3. All corners described, as "a hickory 1 ft. diam., marked H," "a stone."
4. Names of adjoining property owners.
5. Meridian, or north and south line.
6. Owner's name printed inside plot.
7. Number of acres printed under owner's name.
8. Complete set of field notes printed below plot.

Fig. 105 illustrates the plot, description and style of letters.

There are various methods used in making a plot from the field notes. These are generally known as the protractor, the tangent, the sine, or the co-ordinate method.

188. Protractor Method.—A protractor, Fig. 102, is a semicircle of horn, celluloid, German silver, etc., graduated to half degrees. A diameter line is marked at one end 0° and at the other end 180°. A bearing is laid off by placing the center of the protractor over the point and the diameter along the meridian and the protractor to the right or left of the meridian as indicated by the last letter of the bearing; that is, east for the right and west for the left. A point is made on the circumference of the protractor at the point of the correct bearing, the protractor is moved and this point joined by a line to the beginning line or point. The length of the course is then laid off on this line to the scale of the map. Through the point thus located another meridian is located and the bearing is laid off as before.

189. Latitude and Departure Method.—Begin at some point *A* as in Fig. 103, and lay off the latitude *AB* due north and south from *A*, and through the point thus located draw an east and west line and lay off the departure on this line, and join the point thus located to *A*. Lay off the latitude of next course on line through *C*, and through the point thus located draw another east and west line and lay off the departure on this line, thus locating the point *D*. Proceed as above until all the points are located.

190. The Tangent Method.—To lay off a line making a given angle with a given line at a given point *A*, Fig. 103. by the tangent method, lay off *AB* equal to ten parts on some scale, and at *B* erect a perdicular to the given line, and on this perpendicular lay off *CB* equal to ten times the natural tangent of the angle desired; join *C* to *A*. Thus, to lay off an angle of 29° 41′, we find from the table that the natural tangent of 29° 41′ = .5700. Make *AB* equal to ten parts and lay off *CB* equal to 5.7 parts, thus locating *C*; then join *C* to *A* and you have the angle required.

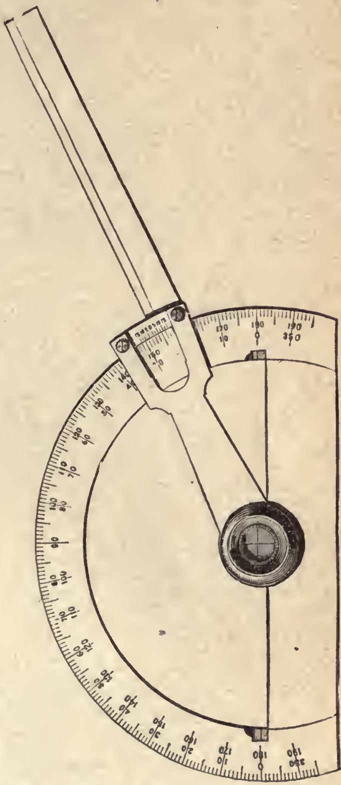


Fig. 102.

191. The Sine Method.—To lay off a given angle at a given point by the natural sine method, take a radius equal to ten parts and with the given point as a center describe a circle. On a perpendicular to the given line at the given point lay off $A3$, Fig. 104, equal to ten times the natural sine of the angle required. Through 3 draw a line parallel to the given line cutting the circumference of the circle at B, join B to A and BAN is the angle required. Example: To construct an angle of 33° 22′ we find that the natural sine of the angle 33° 22′ is .5500. After describing the circle whose radius is ten parts, lay off $A3$ equal to 5.5 parts, and draw $B3$ parallel to the line AN and join B, where it cuts the circumference of the circle, to A, and BAN then will be an angle of 33° 22′.

192. Co-ordinate Method.—Plotting can be done by the Co-ordinate Method: Determine the co-ordinates of each point

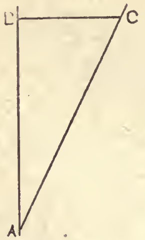


Fig. 103.

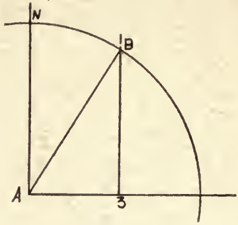
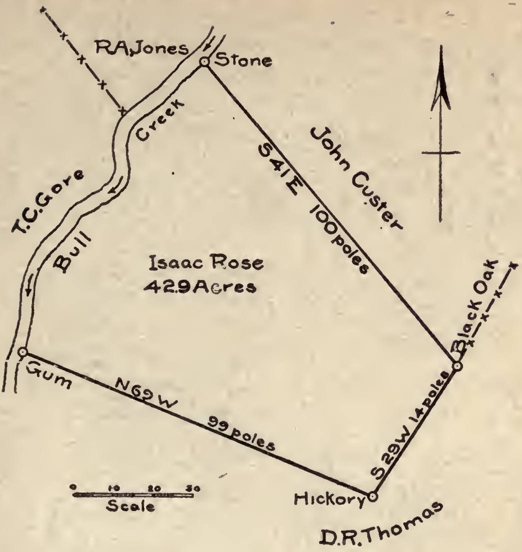


Fig. 104.

with respect to axes (through the initial point, if convenient) and plot from the axes each time. This method will avoid carrying forward any error, as each corner of the survey is found by returning to the original axes. The Y ordinate of any point is equal to the sum (algebraic) of the latitude of the previous points and its own latitude. The X ordinate is equal to the sum of the previous departures plus its own. Using this table of corrected latitudes and departures insures the closing of the plot. This is most accurate method for any large plot, as previous to plotting the sheet can be checked off in squares accurately, say 1,000 ft. on each side, and table of ordinates computed, etc.



Beginning at a stone on Bull Creek a corner to R.A.Jones and John Custer thence with Custer's line S.41°E. 100 poles to a black oak a corner to John Custer and D.R.Thomas,- thence with Thomas' line S 29°W, 41 poles to a hickory a corner to D.R.Thomas;- thence with Thomas' line N.69°W. 99 poles to a gum on Bull Creek a corner to D.R.Thomas and T.C.Gore;- thence up the creek with the meanders of the same to the point of beginning;- containing 429 acres.

Fig. 105.

193. Correcting the Plot.—For the very same reasons that the latitudes and departures very rarely balance, the plot when completed to scale will very rarely close by an amount equal to AA', Fig. 106. In balancing we really shift each corner in the direction of AA', a distance proportional to its length from the beginning corner. To some scale lay off on a straight line the length of the courses $ABCDA'$, and on a line at right angles to this line lay off AA' and through the points B, C, and D draw parallels to AA'.

Through B, C, and D on the plot draw lines parallel to AA' and on these lines lay off distances equal to the amount of correction, locating the points B', C' and D' in the direction that A' has to be moved to close. Then connect these points and close the plot.

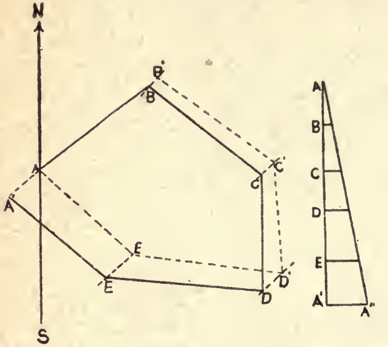


Fig. 106.

194. Lettering.—Every surveyor or engineer should learn some one system of free-hand letters, similar to that in Fig. 107, or some other standard system. Many conclude before trial that they can not letter well, or even make a decent letter. While there is no royal road to good lettering, it is possible for every surveyor or engineer, not afflicted with palsy or extreme nervousness, to learn and execute a good, plain system of letters. But it requires care and implicit obedience to rules. Eternal vigilance and constant practice are required till a system of letters is once learned. After an experience of over twenty years in teaching, it can be asserted that the special books on lettering are far superior to the ordinary alphabets printed as an appendix to works on surveying. If the young engineer will get "Lettering," by

C. W. Reinhardt, published by D. Van Nostrand Company, New York, and will follow instructions faithfully, he can, without doubt, become a good letterer. There is no necessity for fancy letters in a drawing, as neatness, legibility, and clearness are the fundamental requisites. One of the most effective systems of lettering is shown in Fig. 107. Guide lines should always be drawn before the lettering is commenced and the student should adhere strictly to rules.

Bibliography.—"Lettering." By Chas. W. Reinhardt. Published by D. Van Nostrand & Co. This book explains in a clear

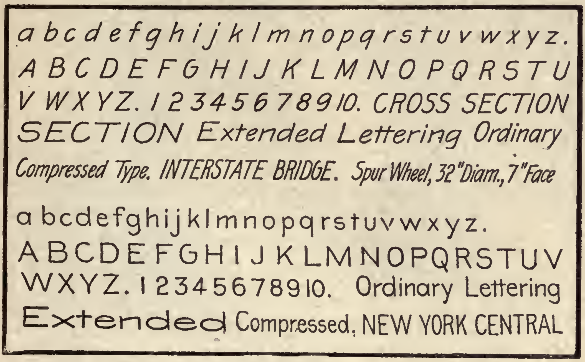


Fig. 107.

and concise manner the system of letters devised by the author and shows by concrete examples how each letter should be formed and how constructed. In addition to this a well selected set of examples of title, heading, and detail lettering is given.

"Mechanical Drawing." By F. E. Giesecke. Part I. Published by Eugene Dietzgen Company. This book has grown out of the necessities of the office and class room and gives an excellent system of free-hand letters for detail work and full instructions are given for the construction of each letter. This book meets all the demands that a learner of lettering can make.

CHAPTER XII.

GOVERNMENT SURVEYING.

195. Radii of Parallels.—Government lands are bounded by meridians and parallels of latitude. If *AB*, Fig. 108, is part of a parallel of latitude, its latitude is the arc *BQ* or the angle *BOQ,* which we will call *L*. Let the radius of the earth be *R* and the radius of the parallel be *r* or *BH*. Then in the right triangle *OBH,*

$$\frac{BH}{OB} = \text{Cos } OBH$$

That is,

$$\frac{r}{R} = \text{Cos L}$$

Or $r = R$ Cos L . . . .

196. Angular Convergence of Meridians.—The two meridians *PA* and *PB,* Fig. 108, at the points *A* and *B* have the direction *AK* and *BK,* respectively, the tangents to these meridians. The amount of convergence is the angle that they lack of being parallel; that is, the angle *AKB* or their angle of intersection. Let Θ = the difference of longitude of *A* and *B* = angle *AHB* = *EOQ.* In the sector *AHB* we have:

$$AB = \frac{\Theta}{57.3} \times BH$$

In *AKB* we have:

$$AB = \frac{X}{57.3} \times BK, \text{ where } X = AKB.$$

Consequently:

$$\frac{X}{57.3} \times BK = \frac{\Theta}{57.3} \times BH$$

$$X = \Theta \times \frac{BH}{BK}$$

$$X = \Theta \; sin. \; L \; \ldots\ldots\ldots\ldots\ldots\ldots \quad (41)$$

197. Linear Convergence.—In the two similar sectors ABH and EOQ we have:

$$AB : EQ :: BH : OQ$$

$$AB = EQ \times \frac{BH}{OQ}$$

$$= EQ \times \frac{r}{R}$$

$$= EQ \, Cos \, L$$

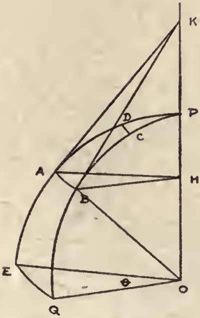


Fig. 108.

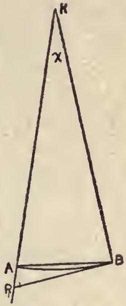


Fig. 109.

If DC is a part of a parallel between the same meridians in latitude L' we have:

$$DC = EQ \, Cos \, L'$$

Let $c = Convergence = AB - DC$

$$= EQ \, Cos \, L - EQ \, Cos \, L'$$

$$= EQ \, (Cos \, L - Cos \, L')$$

But $\dfrac{DC}{AB} = \dfrac{Cos \, L'}{Cos \, L}$

Therefore:

$$c = AB - DC = \frac{AB \, (Cos \, L - Cos \, L')}{Cos \, L} \quad \ldots\ldots (42)$$

Generally we do not know the difference of longitude of A and B, but know the length of AB in miles, and it is necessary to find θ from the data given. The length of one degree on the equator is 69.16 miles.

If D = length of AB in miles, then

$$AB \text{ in degrees} = \frac{D}{69.16 \ Cos\ L}$$

$$\text{But } X = \theta \ Sin\ L$$

$$\text{Therefore } X = \frac{D\ Sin\ L}{69.16\ Cos\ L}$$

$$X^\circ = \frac{D}{69.16} tan\ L$$

$$X'' = \frac{3600}{69.16} D \times tan\ L = 52.05\ D\ tan\ L$$

198. Off-Sets.—If we set the transit at B, Fig. 109, and set the zero on the meridian and turn off a right angle from this meridian, this last line will cut to the left of A. Draw the sector AKB as in the figure and make the angle KBR equal to 90°. The amount the line BR misses A is called the off-set.

The angle ABR = *one-half* X.

$$\text{But } AR = \frac{ABR}{57.3} \times AB$$

If AB=D, we have:

$$AR = Off\text{-}set = \frac{X}{57.3 \times 2} \times D$$

$$\text{But } X = \theta \ Sin\ L = \frac{D}{69.16} tan\ L$$

$$\text{Therefore,} \quad AR = \frac{\frac{1}{2}D}{69.16} tan\ L \times \frac{D}{57.3}$$

$$AR = \frac{D^2}{69.16 \times 57.3 \times 2} tan\ L$$

This is the off-set in miles. If D is in miles and we wish the off-set in feet, we have:

$$\text{Off-set} = \frac{D^2}{69.16 \times 57.3 \times 2} tan\ L \times 5280$$

$$\text{Therefore off-set} = \frac{5280}{69.16 \times 57.3 \times 2} tan\ L \times D^2$$

$$= .66618\ tan\ L\ D^2$$

$$= \tfrac{2}{3} D^2\ tan\ L, \text{ nearly} \ldots\ldots\ldots\ldots (43)$$

199. Running Parallels.—It is impossible to run out the parallel of latitude with the transit directly. We can locate the secant BA or the tangent BR, and then take off-sets to the curve of latitudes at different points, which are generally one-half mile apart. There are two methods of locating points on the parallel of latitude, the *secant method* and the *tangent method.*

200. Tangent Method.—Set up the transit at B and sight along the meridian BK. Then turn off an angle KBR equal to 90°. The line of sight will now locate the line BR, which is tangent to the latitude curve. To obtain the off-sets from this tangent line to the curve at any point on BR, let $d =$ distance from the point to B. Then we have from formula (43):

$$Off\text{-}set = \tfrac{2}{3}\, d^2 \tan. L.$$

After the full distance has been measured, the point R is located. To locate the point A, set up the transit at R, and sight along the line BR, and then turn off an angle of $90° - X°$. The line of sight will now locate the meridian RAK, and if we measure the distance RA along this line an amount equal to the off-set, it will locate the point A on the parallel of latitude passing through B.

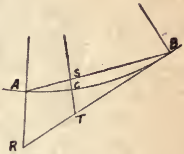


Fig. 110.

201. Secant Method.—Set up the transit at B, as before, and sight along the meridian BK; then turn off an angle of $90° - \frac{1}{2}X$. The line of sight will now locate the secant line BA, which can be run out to the distance BA. To locate points on the parallel of latitude for either method, off-sets must be taken from the tangent or secant.

202. Intermediate Off-Sets.—To find the off-sets at any intermediate point between B and A, let $d =$ distance BT or BS, Fig. 110. The point C on the curve can be located by the off-set TC from the tangent or the off-set SC from the secant. The angle $SBT = \frac{1}{2}$ X°.

$$\therefore \text{ Secant-tangent off-set } ST = \tfrac{1}{2} X° \, BT \div 57.3.$$

$$But\ X = \frac{d}{69.16} \tan. L.$$

$$ST = \tfrac{1}{2}\frac{D}{69.16} \tan L\ d \div 57.3 = \frac{Dd}{2 \times 69.16 \times 57.3} \tan L.$$

If ST is in feet and D and d are in miles, then

Secant-tangent off-set, $ST = \frac{2}{3} Dd \tan. L$.

To find the off-set from the tangent BR to the curve, we have,

$$BT^2 = TC\,(2CK + TC) = 2CK \times TC + TC^2.$$

The last term is so small in comparison with the others that it can be omitted.

$$BT^2 = 2\,TC \times CK = 2\,TC \times BK \;\therefore\; TC = \frac{BT^2}{2\,BK}$$

But $BK = R\ Cot.\ L$ and $BT = d$, then,

$$TC = \frac{d^2}{2\,R\ Cot.\ L} = \frac{d^2}{2R} \tan L.$$ If the offset is in feet and d in miles we have,

Offset $TC = \frac{2}{3}\, d^2 \tan.\ L$.

The secant-curve off-set can be found by subtracting the tangent-curve off-set from the secant-tangent off-set.

$$\therefore SC = \tfrac{2}{3} Dd \tan. L. - \tfrac{2}{3} d^2 \tan. L = 2/3\ d\ (D - d) \tan. L.$$

The secant-curve off-set is equal to two-thirds of the tangent of the latitude multiplied by the segments into which S divides AB.

203. Example.—If a line AB is six miles in length and is a parallel of latitude where $L = 45°$ the different off-sets for each mile can be found as follows:

A. *Tangent-curve off-set* $= \frac{2}{3} d^2 \tan. L. = \frac{2}{3} d^2 \tan 45 = \frac{2}{3} d^2$.

B. *Secant-tangent off-set* $= \frac{2}{3} Dd \tan. L = \frac{2}{3} Dd \tan 45 = \frac{2}{3}$ Dd.

C. *Secant-curve off-set* $= \frac{2}{3} d\ (D - d) \tan. 45 = \frac{2}{3} d\ (D - d)$.

Distance d.	Secant-tangent.	Tangent-curve.	Secant-curve.
	Off-sets.		
1	4	.667	3.333
2	8	2.667	5.333
3	12	6.000	6.000
4	16	10.667	5.333
5	20	16.667	3.333
6	24	24.000	0.000

PROBLEM 75.—Fill out a similar table when latitude = 36°.

204. Reference Meridians and Standard Parallels.—In those states where public lands were surveyed by government surveyors, meridians were located very accurately at certain intervals and parallels of latitude were also accurately located at certain distances apart. As an example the two meridians *BC* and *AD,* Fig. 111, called "reference meridians," were located 24 miles apart, and the "Standard Parallels," *AB* and *CD,* were also located 24 miles apart. This makes a spherical trapezoid whose sides are nearly 24 miles each. The six-mile points on these sides are marked and joined by meridians and parallels, thus dividing the area into smaller trapezoids, with sides 6 miles each way approximately. These trapezoids are called "Townships."

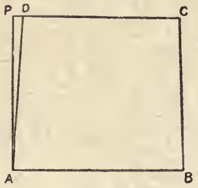


Fig. 111.

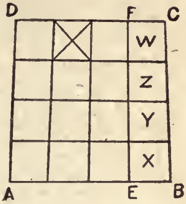


Fig. 112.

The south base of a trapezoid is 24 miles on a standard parallel and the next standard parallel is 24 miles to the north. If the latitude of the south base is 40°, find the amount of convergence.

To find L' the latitude of the north base we have:

One degree = 69.16 miles.

$$24 \text{ miles} = \frac{24}{69.16} \text{ degrees}$$

$$L' = 40° + \left(\frac{24}{69.16}\right)^{\circ} = 40°20'\,49''$$

$$c = \text{convergence} = \frac{24\,[\text{Cos } 40° - \text{Cos } (40°\,20'\,49'')]}{\text{Cos } 40°}$$

$$= \frac{24(.76604 - .76549)}{.76604} = .12219 \text{ miles}$$

PROBLEM 76.—A trapezoid is 24 miles each way, and the latitude of the mid-parallel is 46° 30′. Find the amount of convergence.

PROBLEM 77.—Find the convergence of a trapezoid with 6-mile base and 24 miles north and south, if the latitude of the south base is 36°.

205. Ranges.—In each State or Territory a principal meridian was located as *BC*, Fig. 112. It received a name due to some locality, as the Fayetteville or Butte meridian. Also a principal parallel is located as *AB*. The country is then divided into townships on either side of these axes and they serve as coordinates in locating the townships. Thus in the figure all ranges are west and north. Any row of townships running

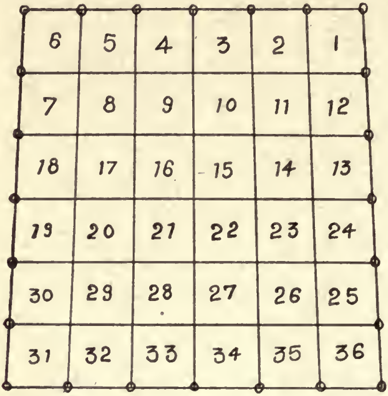


Fig. 113.

north and south is called a Range, while that running east and west is called a Tier. Each township is defined as in Range 1, 2, 3, or 4, Tier 1, 2, 3, or 4, as the case may be, numbering from the Principal Meridian and Principal Parallel. Thus the township crossed will be Range 3 west, Tier 4 north.

206. Townships.—The trapezoid in Fig. 112, 24 miles each way, was surveyed between reference meridians and standard parallels. If the six-mile points on the north and south lines are marked the spherical trapezoid would be divided into approximate squares six miles each way, called townships.

Each township is divided into 36 approximate squares, about one mile on each side, called sections. The sections in each township are numbered as shown in Fig. 113. Section number 1 is in the northeast corner of the township, while number 36 is in the southeast corner.

207. Dividing Up a Township.—All township lines on the south base, on the standard parallels, are full 6 miles, as are all township lines on the meridians. In the range of townships *EBCF*, Fig. 112, there would really be only one east and west line that was fully 6 miles long, as all the others are reduced by the convergence. In dividing the first township *X* into sections we mark off full miles on the south base *EB*, 80 chains each, and also full miles on the north and south lines *BC* and *EF*. If we made the north and south division lines true meridians the sections would decrease materially in size as we proceeded north. To counteract this and keep them approximately 1 mile each way we make the south base of each section bordering on the township lines 80 chains as far as possible. On each east and west township line we commence at the meridian on the east side of the township and measure off 5 full miles, marking the corners; thus out of 144 sections in a Range we would have 21 sections with a full mile for the south base instead of 6 sections if it were divided by true meridians on the mile points of the standard parallel *AB*. The amount of convergence of the townships *X*, *Y*, *Z*, and *W* will be practically the same if they have equal south bases. On the outlines of the townships the corners are marked with stones or posts as indicated by the small circles in Fig. 113. On the township lines the full mile points are all established and marked by corners. In making the survey of sub-division, we begin on the south base of the township at corner to sections 35 and 36, and then run the line between sections 35 and 36 so that it will be parallel to the east line of 36. In the same way all the north-south lines are run parallel to the east line of the township except for sections from 1 to 6 inclusive. From the corner of 1, 2, 11, and 12 the line between 1 and 2 is run directly to the established corner on the north base of the township. The lines between sections 2 and 3, 3 and 4, 4 and 5, and 5 and 6 are run in a similar way. On all north-south lines five full miles are measured from the south base of township, setting a post or stone at the end of each full mile for a section corner. The east-west lines join the corners on the north-south lines. A random line is first run from the section corners to the eastward and if it does not hit the corner, the correction is

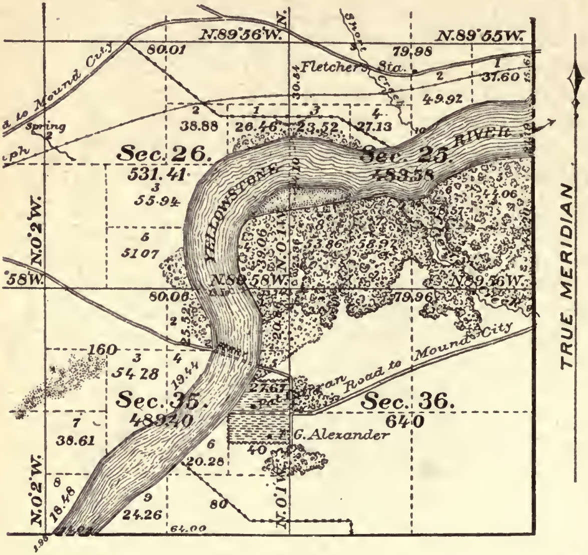


Fig. 114.

made and the true line run. The east-west lines of sections 31, 30, 19, 18, 7, and 6 receive practically all the effect of convergence of the township; and, if these sections are divided into quarter sections, the shortage in length is thrown into the west halves.

The township is subdivided, Fig. 114, as follows:

	Beginning at corner 1-2-35-36 on the south base, thence N1′W between sections 35 and 36.
4.50	Wire fence, bears E. and W.
20	Scattering cottonwood bears east and west. F. G. Alexander's house bears N28°W.
29.30	Leave cottonwood timber bears east and west. Enter road bears north.
30	Southeast corner Alexander's field. Thence along west side of road.
39.50	Cross roads. Bears east to Mound City. Bears north to Link City.
40	Quarter section corner point falls in the road.
57.50	Enter dense cottonwood timber; bears N54°E.
80	Set locust post 4″x4″ — 2′ in the ground for corner sections 25, 26, 35, and 36.

	Thence S89° 57′E on a random line between sections 25 and 36.
40	Set temporary quarter section corner post.
79.96	Intersect east line of township 3 links north of corner of sections 25, 30, 31, and 36, which is a sandstone 5″x8″ set 5″ above the ground, marked and witnessed.
	Thence N89°56′W on a true line between sections 25 and 36 over level bottom land.
18.60	Cherry Creek, 12 links wide, clear water, 1 ft. deep, gentle current, sandy bottom, course northwest.
20.50	Heavy timber, bears north and south.
32.50	Leave heavy timber bearing north and south.
39.98	Deposit a quart of charcoal 12 ins. in the ground as a quarter section corner. Dig pits 18x18x12 ins. east and west 4 ft. and raised a mound of earth 3½ ft. base by 1½ ft. high over the deposit.
46.50	Enter heavy timber bears north and south.
76	Leave heavy timber, enter scattering timber bears N25°E.
79.96	Corner sections 25, 26, 35, and 36.

	Thence N1′W between sections 25 and 26.
25.36	Right bank of Yellowstone river. Set locust post 4″x4″ — 24″ in the ground for meander corner for sections 25 and 26, marked *MC* on north side. Entered shallow water 1 to 2 ft. deep.
26	Across shallow channel 64 links wide to sand bar.
32.12	To right bank of main channel, course east.
40	Quarter section corner falls in the river.
49.46	Left bank of Yellowstone river, 12 ft. high, deposited a marked stone 12 ins. in the ground.
55.70	Wire fence bears east and west.
62.80	Telegraph line bears east and west.
80	Set cedar post for corner sections 23, 24, 25, and 26.

	Thence S89°56′E on a random line.
40	Set temporary quarter section corner.
79.98	Intersect east line of township 3 links north of section corners 25, 24, 30, and 19; which is a sandstone 5x9 ins.—4 ins. above ground marked and witnessed. Thence back N89°55′W on a true line between sections 25 and 24.
39.99	Set a cedar post 3 ft. by 3 ins. square with a marked stone 24 ins. in the ground for a quarter section corner.
58.00	Short creek, 3 links wide.
79.98	Cor. of secs. 23, 24, 25, and 26.

The survey progresses in this way till we reach the corner of sections 1, 2, 11, and 12, when we continue as follows:

	Beginning at corner 1, 2, 11, 12. Thence N1′W on a random line between sections 1 and 2.
40	Set temporary quarter section corner.
79.77	Intersect north line of township at corner of sections 1, 2, 35, and 36, which is a limestone 6″x6″ — 5″ above ground, marked and witnessed. Thence S1′E on a true line between sections 1 and 2.
39.77	Set marked stone for quarter section corner.

In the next Range of sections we begin at corner on south base 2, 3, 34, and 35, and proceed as before. In this case, after the surveyor has located the corner 2, 3, 10, 11 he runs a random line N. 2′ W. between sections 2 and 3 and misses the corner of sections 2, 3, 34, and 35, five links to the west, and thence runs due south on a true line between sections 2 and 3.

Bibliography.—"A Manual of Land Surveying." By F. Hodgman. 374 pages. A very valuable book for the surveyor or field engineer in surveying the public lands. A unique and very important feature is a digest of the legal decisions by the different State and Federal courts in regard to U. S. Lands, surveys, conflicts, etc.

"A Manual of Surveying Instructions." Prepared under direction of the Commissioner of the General Land Office of the United States, Washington, D. C. It contains full and minute directions for the execution of surveys in the field in conformity to the laws of the United States.

CHAPTER XIII.

TRIGONOMETRIC FORMULAS.

208. Formulas for Right Triangle.—In the right triangle *ABC*, Fig. 115, where *C* is the right angle, and *a, b,* and *c* are the sides, we have the following expressions for the different trigonometric functions:

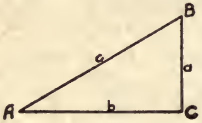


Fig. 115.

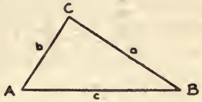


Fig. 116.

$$\sin A = \frac{a}{c}; \qquad \csc A = \frac{c}{a};$$

$$\cos A = \frac{b}{c}; \qquad \sec A = \frac{c}{b};$$

$$\tan A = \frac{a}{b}; \qquad \cot A = \frac{b}{a};$$

Also,

$$\sin A = \frac{1}{\csc A}; \qquad \cos A = \frac{1}{\sec A}; \qquad \tan A = \frac{1}{\cot A}$$

The following relations are sometimes useful:

$$\sin^2 A + \cos^2 A = 1;$$

$$1 + \tan^2 A = \sec^2 A;$$

$$1 + \cot^2 A = \csc^2 A;$$

209. Solutions for Right Triangle.—There are four general cases that can occur, according to the data given, which may be—

I. The hypotenuse and one leg;

II. The two legs;

III. The hypotenuse and one of the acute angles;

IV. A leg and an acute angle.

The data given, the data required, and the solutions are given in the following tabular statement. It is assumed that if angle B is known, A is also known.

Given	Required	Solutions
a, c	b, A, B	$\sin A=\frac{a}{c}$; $b=c\cos A$; $B=90-A$.
a, b	A, c, B	$\tan A=\frac{a}{b}$; $c=\frac{a}{\sin A}$; $B=90-A$.
c, A	a, b, B	$a=c\sin A$; $b=c\cos A$; $B=90-A$.
A, a	c, b, B	$c=\frac{a}{\sin A}$; $b=\frac{a}{\tan A}$; $B=90-A$.

210. Oblique Triangle.—In the general triangle ABC, Fig. 116, three parts, one of which must be a side, have to be given to find the other three. There are four general cases according to the data given. Thus we may have:

I. Two angles and the included side;
II. Two sides and the included angle;
III. Three sides;
IV. Two sides and an angle opposite one of them.

The given parts, the required parts, and the formulas for solution are given in the following table:

Given	Required	Formulas for Solutions
A, C, b	B, c, a	$B=180-(A+C)$, $c=\frac{b\sin C}{\sin B}$; $a=\frac{b\sin A}{\sin B}$.
b, c, A	B, C, a	$B+C=180-A$; $\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\tan\frac{1}{2}(B+C)$ $a=\frac{b\sin A}{\sin B}$
a, b, c	A, B, C	$\sin\frac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{bc}}$; $\cos\frac{1}{2}A=\sqrt{\frac{s(s-a)}{bc}}$; $\tan\frac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$. Check: $A+B+C=180$
$a, b\ A$	B, C, c	$\sin B=\frac{b\sin A}{a}$; $C=180-(A+B)$

$$c = \frac{a \sin C}{\sin A}.$$

Case IV is sometimes ambiguous. We may have the following conditions and results:

If A is obtuse, and $a > b$ there is one solution;

If A is acute and $a =$ or $> b$, there is one solution;

If A is acute and $a < b$ and $a > b \sin A$, there are two solutions;

If A is acute and $a < b$ and $a = b \sin A$, there is one solution;

If A is acute and $a < b$ and $a < b \sin A$, there is no solution.

211. Right Spherical Triangle.—If ABC is a right spherical triangle where $C = 90°$, and the hypotenuse (c), and the two acute angles (A and B) are treated as co-parts, the five parts of the triangle in order are a, b, $90 - A$, $90 - c$, and $90 - B$. To these five parts the following laws (discovered by Napier) apply:

Tangent Law: The sine of any part is equal to the product of the Tangents of the Adjacent parts.

Cosine Law: The sine of any part is equal to the product of the Cosines of the Opposite parts.

The right angle C is not counted or regarded as a part and a and b are regarded as adjacent parts as no significant part comes between them. For any one part the two adjacent parts are those next to it, while the opposite parts are the other two, or parts once removed from the special part under consideration. Thus for $90 - A$, the adjacent parts are b, and $90 - c$, while the opposite parts are a and $90 - B$.

By the application of Napier's laws we can solve any spherical triangle where the three given parts are two sides and an angle or two angles and a side.

212. Oblique Spherical Triangle.—If three sides of a spherical triangle ABC, are given, let

$$2s = a + b + c, \text{ and we have,}$$

$$\sin \tfrac{1}{2}A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}$$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}$$

If the three angles are given, pass to the polar triangle and solve, and then pass back.

CHAPTER XIV.

TABLES OF

LOGARITHMS OF NUMBERS.

LOGARITHMIC SINES, COSINES, TANGENTS, AN
COTANGENTS.

NATURAL SINES AND COSINES.

NATURAL TANGENTS AND COTANGENTS.

CUBIC YARDS PER 100 FT. FOR VARIOUS SLOPES.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272
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160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4252	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198
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1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9279	9472	9666	9860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
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No	0	1	2	3	4	5	6	7	8	9	Diff.
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
9	460898	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4190	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9828	9959	520090	520221	520353	520484	520615	520745	520876	521007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	1351	128
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	[illegible]	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
1	580925	581039	1153	1267	1381	1495	1608	722	1836	1950	114
2	2063	2177	2291	2404	2518	2631	2745	358	2972	3085	114
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	13
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	13
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	13
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	.12
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036	108
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
6	9486	9586	9686	9785	9885	9984	640084	640183	640283	640382	99
7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
7	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
No.	0	1	2	3	4	5	6	7	8	9	Diff

No.	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1624	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
No	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68

No. 0 1 2 3 4 5 6 7 8 9 Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
640	806180	806218	806316	806384	806451	806519	806587	806655	806723	806790	68
1	6858	69 6	6994	7061	7129	7197	7264	7332	7400	7467	68
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
9	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	960042	960090	960138	960185	960233	960280	960328	960376	960423	48
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9364	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE II.

LOGARITHMIC SINES, COSINES, TANGENTS AND COTANGENTS.

NOTE.

The table here given extends to minutes only. The usual method of extending such a table to seconds, by proportional parts of the difference between two consecutive logarithms, is accurate enough for most purposes, especially if the angle is not very small. When the angle is very small, and great accuracy is required, the following method may be used for sines, tangents, and cotangents.

I. Suppose it were required to find the logarithmic sine of 5′ 24″. By the ordinary method, we should have

$$\begin{aligned} \text{log. sin. } 5' &= 7.162696 \\ \text{diff. for } 24'' &= 31673 \\ \hline \text{log. sin. } 5'\,24'' &= 7.194369 \end{aligned}$$

The more accurate method is founded on the proposition in Trigonometry, that the sines or tangents of very small angles are proportional to the angles themselves. In the present case, therefore, we have $\sin. 5' : \sin. 5'\,24'' = 5' : 5'\,24'' = 300'' : 324''$. Hence $\sin. 5'\,24'' = \frac{324 \sin. 5'}{300}$, or $\log. \sin. 5'\,24'' = \log. \sin. 5' + \log. 324 - \log. 300$. The difference for 24″ will, therefore, be the difference between the logarithm of 324 and the logarithm of 300. The operation will stand thus:—

$$\begin{aligned} \text{log. } 324 &= 2.510545 \\ \text{log. } 300 &= 2.477121 \\ \hline \text{diff. for } 24'' &= 33424 \\ \text{log. sin. } 5' &= 7.162696 \\ \hline \text{log. sin. } 5'\,24'' &= 7.196120 \end{aligned}$$

Comparing this value with that given in tables that extend to seconds, we find it exact even to the last figure.

II. Given $\log. \sin. A = 7.004438$ to find A. The sine next less than this in the table is $\sin. 3' = 6.940847$. Now we have $\sin. 3' : \sin. A = 3 : A$. Therefore, $A = \frac{3 \sin. A}{\sin. 3'}$, or $\log. A = \log. 3 +$

log. sin. A − log. sin. 3′. Hence it appears, that, to find the logarithm of A in minutes, we must add to the logarithm of 3 the difference between log. sin. A and log. sin. 3′.

$$\begin{array}{lr}
\text{log. sin. } A = & 7.004438 \\
\text{log. sin. } 3' = & \underline{6.940847} \\
 & 63591 \\
\text{log. } 3 \quad = & \underline{0.477121} \\
A = 3.473 & 0.540712
\end{array}$$

or $A = 3'\ 28.38''$. By the common method we should have found $A = 3'\ 30.54''$.

The same method applies to tangents and cotangents, except that in the case of cotangents the differences are to be subtracted.

*** The radius of this table is unity, and the characteristics 9, 8, 7, and 6 stand respectively for −1, −2, −3, and −4.

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M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	Inf. neg.		0.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	3.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.85	.235244	58
3	.940847	2082.31	.000000	.00	.940847	2082.31	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	2.934214	56
5	.162696	1319.69	.000000	.00	.162696	1319.69	.837304	55
6	.241877	1115.78	9.999999	.00	.241878	1115.78	.758122	54
7	.308824	966.53	999999	.00	.308825	966.54	.691175	53
8	.366816	852.54	.999999	.01	.366817	852.55	.633183	52
9	.417968	762.62	.999999	.01	.417970	762.63	.582030	51
10	7.463726	689.88	9.999998	.01	7.463727	689.88	2.536273	50
11	.505118	629.81	.999998	.01	.505120	629.81	.494880	49
12	.542906	579.37	.999997	.01	.542909	579.37	.457091	48
13	.577668	536.41	.999997	.01	.577672	536.42	.422328	47
14	.609853	499.38	.999996	.01	.609857	499.39	.390143	46
15	.639816	467.14	.999996	.01	.639820	467.15	.360180	45
16	.667845	438.81	999995	.01	.667849	438.82	.332151	44
17	.694173	413.72	.999995	.01	.694179	413.73	.305821	43
18	.718997	391.35	.999994	.01	.719003	391.36	.280997	42
19	.742478	371.27	999993	.01	.742484	371.28	.257516	41
20	7.764754	353.15	9.999993	.01	7.764761	353.16	2.235239	40
21	.785943	336.72	999992	.01	.785951	336.73	.214049	39
22	.806146	321.75	.999991	.01	.806155	321.76	.193845	38
23	.825451	308.05	.999990	.01	.825460	308.07	.174540	37
24	.843934	295.47	999989	.02	.843944	295.49	.156056	36
25	.861662	283.88	.999989	.02	.861674	283.90	.138326	35
26	.878695	273.17	999988	.02	.878708	273.18	.121292	34
27	.895085	263.23	999987	02	.895099	263.25	.104901	33
28	.910879	253.99	999986	.02	.910894	254.01	.089106	32
29	.926119	245.38	999985	.02	.926134	245.40	.073866	31
30	7.940842	237.33	9.999983	.02	7.940858	237.35	2.059142	30
31	.955082	229.80	999982	.02	.955100	229.82	.044900	29
32	.968870	222.73	999981	.02	.968889	222.75	.031111	28
33	.982233	216.08	.999980	.02	.982253	216.10	.017747	27
34	.995198	209.81	.999979	.02	.995219	209.83	.004781	26
35	8.007787	203.90	.999977	.02	8.007809	203.92	1.992191	25
36	.020021	198.31	.999976	.02	.020044	198.33	.979956	24
37	.031919	193.02	.999975	.02	.031945	193.05	.968055	23
38	.043501	188.01	.999973	.02	.043527	188.03	.956473	22
39	.054781	183.25	.999972	.02	.054809	183.27	.945191	21
40	8.065776	178.72	9.999971	.02	8.065806	178.75	1.934194	20
41	.076500	174.42	.999969	.03	.076531	174.44	.923469	19
42	.086965	170.31	999968	.03	.086997	170.34	.913003	18
43	.097183	166.39	.999966	.03	.097217	166.42	.902783	17
44	.107167	162.65	.999964	.03	.107203	162.68	.892797	16
45	.116926	159.08	.999963	.03	.116963	159.11	.883037	15
46	.126471	155.66	.999961	.03	.126510	155.69	.873490	14
47	.135810	152.38	.999959	.03	.135851	152.41	.864149	13
48	.144953	149.24	.999958	.03	144996	149.27	.855004	12
49	.153907	146.22	.999956	.03	.153952	146.25	.846048	11
50	8.162681	143.33	9.999954	.03	8.162727	143.36	1.837273	10
51	.171280	140.54	.999952	.03	.171328	140.57	.828672	9
52	.179713	137.86	.999950	.03	.179763	137 90	.820237	8
53	.187985	135.29	.999948	.03	.188036	135.32	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.84	.803844	6
55	.204070	130.41	.999944	.03	.204126	130.44	.795874	5
56	.211895	128.10	.999942	.03	.211953	128.14	.788047	4
57	.219581	125.87	.999940	.04	.219641	125.91	.780359	3
58	.227134	123.72	.999938	.04	.227195	123.76	.772805	2
59	.234557	121.64	.999936	.04	.234621	121.68	.765379	1
60	.241855		.999934		.241921		.758079	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

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M.	Sine	D. 1".	Cosine.	D 1".	Tang.	D. 1".	Cotang.	M.
0	8.241855	119.63	9.999934	.04	8.241921	119.67	1.758079	60
1	.249033	117.69	.999932	.04	.249102	117.72	.750898	59
2	.256094	115.80	.999929	.04	.256165	115.84	.743835	58
3	.263042	113.98	.999927	.04	.263115	114.02	.736885	57
4	.269881	112.21	.999925	.04	.269956	112.25	.730044	56
5	.276614	110.60	.999922	.04	.276691	110.54	.723309	55
6	.283243	108.83	.999920	.04	.283323	108.87	.716677	54
7	.289773	107.22	.999918	.04	.289856	107.26	.710144	53
8	.296207	105.66	.999915	.04	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.04	.302634	104.18	.697366	51
10	8.308794	102.66	9.999910	.04	8.308884	102.70	1.691116	50
11	.314954	101.22	.999907	.04	.315046	101.26	.684954	49
12	.321027	99.82	.999905	.04	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.06	.327114	98.51	672886	47
14	.332924	97.14	.999899	.05	.333025	97.19	666975	46
15	.338753	95.86	.999897	.05	.338856	95.90	.661144	45
16	.344504	94.60	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.38	.999891	.05	.350289	93.43	.649711	43
18	.355783	92.19	.999888	.05	.355895	92.24	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	8.366777	89.90	9.999882	.05	8.366895	89.95	1.633105	40
21	.372171	88.80	.999879	.05	.372292	88.85	.627708	39
22	.377499	87.72	.999876	.05	.377622	87.77	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.64	.999870	.05	.388092	85.70	.611908	36
25	.393101	84.64	.999867	.05	.393234	84.69	.606766	35
26	.398179	83.66	.999864	.05	.398315	83.71	.601685	34
27	.403199	82.71	.999861	.05	.403338	82.76	.596662	33
28	.408161	81.77	.999858	.05	.408304	81.82	.591696	32
29	.413068	80.86	.999854	.05	.413213	80.91	.586787	31
30	8.417919	79.96	9.999851	.06	8.418068	80.02	1.581932	30
31	.422717	79.09	.999848	.06	.422869	79.14	.577131	29
32	.427462	78.23	.999844	.06	.427618	78.29	.572382	28
33	.432156	77.40	.999841	.06	.432315	77.45	.567685	27
34	.436800	76.58	.999838	.06	.436962	76.63	.563038	26
35	.441394	75.77	.999834	.06	.441560	75.83	558440	25
36	.445941	74.99	.999831	.06	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.06	.450613	74.28	.549387	23
38	.454893	73.47	.999824	.06	.455070	73.53	.544930	22
39	.459301	72.73	.999820	.06	.459481	72.79	.540519	21
40	8.463665	72.00	9.999816	.06	8.463849	72.06	1.536151	20
41	.467985	71.29	.999813	.06	.468172	71.35	.531828	19
42	.472263	70.60	.999809	.06	.472454	70.66	.527546	18
43	.476498	69.91	.999805	.06	.476693	69.98	.523307	17
44	.480693	69.24	.999801	.06	.480892	69.31	.519108	16
45	.484848	68.59	.999797	.06	.485050	68.65	.514950	15
46	.488963	67.94	.999794	.07	.489170	68.01	.510830	14
47	.493040	67.31	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.69	.999786	.07	.497293	66.76	.502707	12
49	.501080	66.08	.999782	.07	.501298	66.15	.498702	11
50	8.505045	65.48	9.999778	.07	8.505267	65.55	1.494733	10
51	.508974	64.89	.999774	.07	.509200	64.96	.490800	9
52	.512867	64.32	.999769	.07	.513098	64.39	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.19	.999761	.07	.520790	63.26	.479210	6
55	.524343	62.65	.999757	.07	.524586	62.72	.475414	5
56	.528102	62.11	.999753	.07	.528349	62.18	.471651	4
57	.531828	61.58	.999748	.07	.532080	61.65	.467920	3
58	.535523	61.06	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.07	.539447	60.62	.460553	1
60	.542819		.999735		.543084		.456916	0
M.	Cosine.	D. 1'.	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

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M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.542819	60.04	9.999735	.07	8.543084	60.12	1.456916	60
1	.546422	59.55	.999731	.07	.546691	59.62	.453309	59
2	.549995	59.06	.999726	.08	.550268	59.14	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.66	.446183	57
4	.557054	58.11	.999717	.08	.557336	58.19	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.73	.439172	55
6	.563999	57.19	.999708	.08	.564291	57.27	.435709	54
7	.567431	56.74	.999704	.08	.567727	56.82	.432273	53
8	570836	56.30	999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	8.577566	55.44	9.999689	.08	8.577877	55.52	1.422123	50
11	.580892	55.02	.999685	.08	.581208	55.10	.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.19	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.79	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.39	.999665	.08	.594283	53.47	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.61	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.86	.999645	.09	.606978	51.94	.393022	41
20	8.609734	51.49	9.999640	.09	8.610094	51.58	1.389906	40
21	.612823	51.12	.999635	.09	.613189	51.21	.386811	39
22	.615891	50.77	.999629	.09	.616262	50.85	.383738	38
23	.618937	50.41	.999624	.09	.619313	50.50	.380687	37
24	.621962	50.06	.999619	.09	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.09	.625352	49.81	.374648	35
26	.627948	49.38	.999608	.09	.628340	49.47	.371660	34
27	.630911	49.04	.999603	.09	.631308	49.13	.368692	33
28	.633854	48.71	.999597	.09	.634256	48.80	.365744	32
29	.636776	48.39	.999592	.09	.637184	48.48	.362816	31
30	8.639680	48.06	9.999586	.09	8.640093	48.16	1.359907	30
31	.642563	47.75	.999581	.09	.642982	47.84	.357018	29
32	.645428	47.43	.999575	.09	.645853	47.53	.354147	28
33	.648274	47.12	.999570	.09	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.09	.651537	46.91	.348463	26
35	.653911	46.52	.999558	.10	.654352	46.61	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.31	.342851	24
37	.659475	45.93	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	8.667689	45.07	9.999529	.10	8.668160	45.16	1.331840	20
41	.670393	44.79	.999524	.10	.670870	44.88	.329130	19
42	.673080	44.51	.999518	.10	.673563	44.61	.326437	18
43	.675751	44.24	.999512	.10	.676239	44.34	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.10	.681544	43.80	.318456	15
46	.683665	43.44	.999493	.10	.684172	43.54	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	8.693998	42.42	9.999469	.10	8.694529	42.52	1.305471	10
51	.696543	42.17	.999463	.11	.697081	42.28	.302919	9
52	.699073	41.93	.999456	.11	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.11	.702139	41.79	.297861	7
54	.704090	41.44	.999443	.11	.704646	41.55	.295354	6
55	.706577	41.21	.999437	.11	.707140	41.32	.292860	5
56	.709049	40.97	.999431	.11	.709618	41.08	.290382	4
57	.711507	40.74	.999424	.11	.712083	40.85	.287917	3
58	.713952	40.51	.999418	.11	.714534	40.62	.285466	2
59	.716383	40.29	.999411	.11	.716972	40.40	.283028	1
60	.718800		.999404		.719396		.280604	0
M.	Cosine.	D. 1″.	Sine	D. 1″.	Cotang.	D. 1″.	Tang.	M.

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M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	8.718800	40.06	9.999404	.11	8.719396	40.17	1.280604	60
1	.721204	39.84	.999398	.11	.721806	39.95	.278194	59
2	.723595	39.62	.999391	.11	.724204	39.74	.275796	58
3	.725972	39.41	.999384	.11	.726588	39.52	.273412	57
4	.728337	39.19	.999378	.11	.728959	39.31	.271041	56
5	.730688	38.98	.999371	.11	.731317	39.10	.268683	55
6	.733027	38.77	.999364	.11	.733663	38.89	.266337	54
7	.735354	38.57	.999357	.11	.735996	38.68	.264004	53
8	.737667	38.36	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.16	999343	.12	.740626	38.27	.259374	51
10	8.742259	37.96	9.999336	.12	8.742922	38.07	1.257078	50
11	.744536	37.76	.999329	.12	.745207	37.88	.254793	49
12	.746802	37.56	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.49	.250260	47
14	.751297	37.17	.999308	.12	.751989	37.29	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.61	.999287	.12	.758668	36.73	.241332	43
18	.760151	36.42	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.24	999272	.12	.763065	36.36	.236935	41
20	8.764511	36.06	9.999265	.12	8.765246	36.18	1.234754	40
21	.766675	35.88	.999257	.12	.767417	36.00	.232583	39
22	.768828	35.70	.999250	.12	.769578	35.83	.230422	38
23	.770970	35.53	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.35	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.18	.999227	.13	.775995	35.31	.224005	35
26	.777333	35.01	999220	.13	.778114	35.14	.221886	34
27	.779434	34.84	999212	.13	.780222	34.97	.219778	33
28	.781524	34.67	999205	.13	.782320	34.80	.217680	32
29	.783605	34.51	999197	.13	.784408	34.64	.215592	31
30	8.785675	34.34	9.999189	.13	8.786486	34.47	1.213514	30
31	.787736	34.18	999181	.13	.788554	34.31	.211446	29
32	.789787	34.02	999174	.13	.790613	34.15	.209387	28
33	.791828	33.86	.999166	.13	.792662	33.99	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.54	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.39	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.23	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.08	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	8.805852	32.78	9.999110	.14	8.806742	32.92	1.193258	20
41	.807819	32.63	.999102	.14	.808717	32.77	.191283	19
42	.809777	32.49	.999094	.14	.810683	32.62	.189317	18
43	.811726	32.34	.999086	.14	.812641	32.48	.187359	17
44	.813667	32.20	.999077	.14	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.14	.816529	32.19	.183471	15
46	.817522	31.91	.999061	.14	.818461	32.05	.181539	14
47	.819436	31.77	.999053	.14	.820384	31.91	.179616	13
48	.821343	31.63	.999044	.14	.822298	31.77	.177702	12
49	.823240	31.49	.999036	.14	.824205	31.63	.175795	11
50	8.825130	31.36	9.999027	.14	8.826103	31.50	1.173897	10
51	.827011	31.22	.999019	.14	.827992	31.36	.172008	9
52	.828884	31.08	.999010	.14	.829874	31.23	.170126	8
53	.830749	30.95	.999002	.14	831748	31.09	.168252	7
54	.832607	30.82	.998993	.14	.833613	30.96	.166387	6
55	.834456	30.69	.998984	.14	.835471	30.83	.164529	5
56	.836297	30.56	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.57	.160837	3
58	.839956	30.30	.998958	.15	.840998	30.45	.159002	2
59	.841774	30.17	.998950	.15	.842825	30.32	.157175	1
60	.843585		.998941		.844644		.155356	0
M.	Cosine.	D. 1"	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.843585	30.05	9.998941	.15	8.844644	30.20	1.155356	60
1	.845387	29.92	.998932	.15	.846455	30.07	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.68	.998914	.15	.850057	29.83	.149943	57
4	.850751	29.55	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.31	.998887	.15	.855403	29.46	.144597	54
7	.856049	29.19	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.96	.998860	.15	.860686	29.11	.139314	51
10	8.861283	28.84	9.998851	.15	8.862433	29.00	1.137567	50
11	.863014	28.73	.998841	.15	.864173	28.88	.135827	49
12	.864738	28.61	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.16	.867632	28.66	.132368	47
14	.868165	28.39	.998813	.16	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.16	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.16	.872770	28.32	.127230	44
17	.873255	28.06	.998785	.16	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.16	.876162	28.11	.123838	42
19	.876615	27.84	.998766	.16	.877849	28.00	.122151	41
20	8.878285	27.73	9.998757	.16	8.879529	27.89	1.120471	40
21	.879949	27.63	.998747	.16	.881202	27.79	.118798	39
22	.881607	27.52	.998738	.16	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.16	.884530	27.58	.115470	37
24	.884903	27.31	.998718	.16	.886185	27.47	.113815	36
25	.886542	27.21	.998708	.16	.887833	27.37	.112167	35
26	.888174	27.11	.998699	.16	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.16	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.16	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	8.894643	26.70	9.998659	.17	8.895984	26.87	1.104016	30
31	.896246	26.60	.998649	.17	.897596	26.77	.102404	29
32	.897842	26.51	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.41	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.31	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.39	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.29	.094430	24
37	.905736	26.03	.998589	.17	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.84	.998568	.17	.910285	26.01	.089715	21
40	8.910404	25.75	9.998558	.17	8.911846	25.92	1.088154	20
41	.911949	25.66	.998548	.17	.913401	25.83	.086599	19
42	.913488	25.56	.998537	.17	.914951	25.74	.085049	18
43	.915022	25.47	.998527	.17	.916495	25.65	.083505	17
44	.916550	25.38	.998516	.17	.918034	25.56	.081966	16
45	.918073	25.29	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.21	.998495	.18	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.29	.077381	13
48	.922610	25.03	.998474	.18	.924136	25.21	.075864	12
49	.924112	24.94	.998464	.18	.925649	25.12	.074351	11
50	8.925609	24.86	9.998453	.18	8.927156	25.04	1.072844	10
51	.927100	24.77	.998442	.18	.928658	24.95	.071342	9
52	.928587	24.69	.998431	.18	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.19	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.11	.998355	.18	.940494	24.29	.059506	1
60	.940296		.998344		.941952		.058048	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M

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M	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	8.940296	24.03	9.998344	.18	8.941952	24.21	1.058048	60
1	.941738	23.95	.998333	.19	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.19	.944852	24.05	.055148	58
3	.944606	23.79	.998311	.19	.946295	23.97	.053705	57
4	.946034	23.71	.998300	.19	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.19	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.19	.950597	23.74	.049403	54
7	.950287	23.48	.998266	.19	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.19	.953441	23.59	.046559	52
9	.953100	23.32	.998243	.19	.954856	23.51	.045144	51
10	8.954499	23.25	9.998232	.19	8.956267	23.44	1.043733	50
11	.955894	23.17	.998220	.19	.957674	23.36	.042326	49
12	.957284	23.10	.998209	.19	.959075	23.29	.040925	48
13	.958670	23.02	.998197	.19	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.19	.961866	23.14	.038134	46
15	.961429	22.88	.998174	.19	.963255	23.07	.036745	45
16	.962801	22.81	.998163	.19	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.93	.033981	43
18	.965534	22.66	.998139	.20	.967394	22.86	.032606	42
19	.966893	22.59	.998128	.20	.968766	22.79	.031234	41
20	8.968249	22.52	9.998116	.20	8.970133	22.72	1.029867	40
21	.969600	22.45	.998104	.20	.971496	22.65	.028504	39
22	.970947	22.38	.998092	.20	.972855	22.58	.027145	38
23	.972289	22.31	.998080	.20	.974209	22.51	.025791	37
24	.973628	22.24	.998068	.20	.975560	22.44	.024440	36
25	.974962	22.17	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.24	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	8.981573	21.83	9.997996	.20	8.983577	22.04	1.016423	30
31	.982883	21.77	.997984	.20	.984899	21.97	.015101	29
32	.984189	21.70	.997972	.20	.986217	21.91	.013783	28
33	.985491	21.64	.997959	.20	.987532	21.84	.012468	27
34	.986789	21.57	.997947	.21	.988842	21.78	.011158	26
35	.988083	21.51	.997935	.21	.990149	21.71	.009851	25
36	.989374	21.44	.997922	.21	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.21	.992750	21.59	.007250	23
38	.991943	21.31	.997897	.21	.994045	21.52	.005955	22
39	.993222	21.25	.997885	.21	.995337	21.46	.004663	21
40	8.994497	21.19	9.997872	21	8.996624	21.40	1.003376	20
41	.995768	21.12	.997860	.21	.997908	21.34	.002092	19
42	.997036	21.06	.997847	.21	.999188	21.27	.000812	18
43	.998299	21.00	.997835	.21	9.000465	21.21	0.999535	17
44	.999560	20.94	.997822	.21	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.21	.003007	21.09	.996993	15
46	.002069	20.82	.997797	.21	.004272	21.03	.995728	14
47	.003318	20.76	.997784	.21	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.21	.006792	20.91	.993208	12
49	.005805	20.64	.997758	.21	.008047	20.85	.991953	11
50	9.007044	20.58	9.997745	.22	9.009298	20.80	0.990702	10
51	.008278	20.52	.997732	.22	.010546	20.74	.989454	9
52	.009510	20.46	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.40	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.35	.997693	.22	.014268	20.56	.985732	6
55	.013182	20.29	.997680	.22	.015502	20.51	.984498	5
56	.014400	20.23	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.17	.997654	.22	.017959	20.39	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.34	.980817	2
59	.018031	20.06	.997628	.22	.020403	20.28	.979597	1
60	.019235		.997614		.021620		.978380	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

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M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.019235	20.00	9.997614	.22	9.021620	20.23	0.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.89	.997588	.22	.024044	20.12	.975956	58
3	.022825	19.84	.997574	.22	.025251	20.06	.974749	57
4	.024016	19.78	.997561	.22	.026455	20.01	.973545	56
5	.025203	19.73	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.67	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.23	.030046	19.85	.969954	53
8	.028744	19.57	.997507	23	.031237	19.79	.968763	52
9	.029918	19.51	.997493	23	.032425	19.74	.967575	51
10	9.031089	19.46	9.997480	23	9.033609	19.69	0.966391	50
11	032257	19.41	.997466	23	.034791	19.64	.965209	49
12	.033421	19.36	.997452	.23	.035969	19.58	.964031	48
13	.034582	19.30	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.38	.959349	44
17	.039197	19.10	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	9.042625	18.95	9.997341	.23	9.045284	19.18	0.954716	40
21	.043762	18.90	.997327	.23	.046434	19.13	.953566	39
22	.044895	18.85	.997313	.24	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.24	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.24	.049869	18.98	.950131	36
25	.048279	18.70	.997271	.24	051008	18.93	.948992	35
26	.049400	18.65	.997257	.24	.052144	18.89	.947856	34
27	.050519	18.60	.997242	.24	.053277	18.84	.946723	33
28	.051635	18.55	.997228	.24	.054407	18.79	.945593	32
29	.052749	18.50	.997214	.24	.055535	18.74	.944465	31
30	9.053859	18.46	9.997199	.24	9.056659	18.70	0.943341	30
31	.054966	18.41	.997185	.24	.057781	18.65	.942219	29
32	.056071	18.36	.997170	.24	.058900	18.60	.941100	28
33	.057172	18.31	.997156	.24	.060016	18.56	.939984	27
34	.058271	18.27	.997141	.24	.061130	18.51	.938870	26
35	.059367	18.22	.997127	.24	.062240	18.46	.937760	25
36	.060460	18.17	.997112	.24	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.24	.064453	18.37	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.33	.934444	22
39	.063724	18.04	.997068	.25	.066655	18.28	.933345	21
40	9.064806	17.99	9.997053	.25	9.067752	18.24	0.932248	20
41	.065885	17.95	.997039	.25	.068846	18.19	.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.86	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.81	.996994	.25	.072113	18.06	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.72	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.68	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.64	.996934	.25	.076432	17.89	.923568	12
49	.074424	17.59	.996919	.25	.077505	17.84	.922495	11
50	9.075480	17.55	9.996904	.25	9.078576	17.80	0.921424	10
51	.076533	17.51	.996889	.25	.079644	17.76	.920356	9
52	.077583	17.46	.996874	.25	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.26	.082833	17.63	.917167	6
55	.080719	17.34	.996828	.26	.083891	17.59	.916109	5
56	.081759	17.29	.996812	.26	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.26	.086000	17.51	.914000	3
58	.083832	17.21	.996782	.26	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.26	.088098	17.43	.911902	1
60	.085894		.996751		.089144		.910856	0
M.	Cosine.	D. 1".	Sine	D. 1".	Cotang.	D. 1".	Tang.	M

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.085894	17.13	9.996751	.26	9.089144	17.39	0.910856	60
1	.086922	17.09	.996735	.26	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.26	.091228	17.31	.908772	58
3	.088970	17.00	.996704	.26	.092266	17.27	.907734	57
4	.089990	16.96	.996688	.26	.093302	17.23	.906698	56
5	.091008	16.92	.996673	.26	.094336	17.19	.905664	55
6	.092024	16.88	.996657	.26	.095367	17.15	.904633	54
7	.093037	16.84	.996641	.26	096395	17.11	.903605	53
8	.094047	16.80	.996625	.26	.097422	17.07	.902578	52
9	.095056	16.76	.996610	.26	.098446	17.03	.901554	51
10	9.096062	16.73	9.996594	.27	9.099468	16.99	0.900532	50
11	.097065	16.69	996578	.27	.100487	16.95	.899513	49
12	.098066	16.65	996562	.27	.101504	16.91	.898496	48
13	.099065	16.61	996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	996530	.27	.103532	16.84	.896468	46
15	.101056	16.53	996514	.27	.104542	16.80	.895458	45
16	.102048	16.49	996498	.27	105550	16.76	.894450	44
17	.103037	16.46	996482	.27	.106556	16.72	.893444	43
18	.104025	16.42	996465	.27	.107559	16.69	.892441	42
19	.105010	16.38	.996449	.27	.108560	16.65	.891440	41
20	9.105992	16.34	9.996433	.27	9.109559	16.61	0.890441	40
21	106973	16.30	.996417	.27	.110556	16.58	.889444	39
22	.107951	16.27	.996400	.27	.111551	16.54	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.19	996368	.27	.113533	16.47	.886467	36
25	.110873	16.16	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	996335	.28	.115507	16.39	.884493	34
27	.112809	16.08	996318	.28	.116491	16.36	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.32	.882528	32
29	.114737	16.01	.996285	.28	.118452	16.29	.881548	31
30	9.115698	15.98	9.996269	.28	9.119429	16.25	0.880571	30
31	.116656	15.94	.996252	.28	.120404	16.22	.879596	29
32	.117613	15.90	.996235	.28	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.11	.876683	26
35	.120469	15.80	.996185	.28	124284	16.08	.875716	25
36	.121417	15.76	.996168	.28	125249	16.04	.874751	24
37	.122362	15.73	.996151	.28	126211	16.01	.873789	23
38	.123306	15.69	.996134	.28	127172	15.98	.872828	22
39	.124248	15.66	.996117	.28	.128130	15.94	.871870	21
40	9.125187	15.62	9.996100	.28	9.129087	15.91	0.870913	20
41	.126125	15.59	996083	.28	.130041	15.87	.869959	19
42	.127060	15.56	996066	.28	.130994	15.84	.869006	18
43	.127993	15.52	.996049	.29	.131944	15.81	.868056	17
44	.128925	15.49	.996032	.29	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.29	.133839	15.74	.866161	15
46	.130781	15.42	.995998	.29	.134784	15.71	.865216	14
47	.131706	15.39	.995980	.29	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.29	.136667	15.64	.863333	12
49	.133551	15.32	.995946	.29	.137605	15.61	.862395	11
50	9.134470	15.29	9.995928	.29	9.138542	15.58	0.861458	10
51	.135387	15.26	.995911	.29	139476	15.55	.860524	9
52	.136303	15.22	.995894	.29	140409	15.51	.859591	8
53	.137216	15.19	.995876	.29	141340	15.48	.858660	7
54	.138128	15.16	.995859	.29	.142269	15.45	.857731	6
55	.139037	15.13	995841	.29	.143196	15.42	.856804	5
56	.139944	15.09	995823	.29	.144121	15.39	.855879	4
57	.140850	15.06	995806	.29	145044	15.36	.854956	3
58	141754	15.03	995788	.29	.145966	15.32	.854034	2
59	142655	15.00	995771	.30	.146885	15.29	.853115	1
60	.143555		.995753		.147803		.852197	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine	D. 1″.	Cosine.	D. 1″	Tang.	D. 1″.	Cotang	M.
0	9 143555	14.97	9.995753	.30	9.147803	15.26	0.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.87	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.84	995681	.30	.151454	15.14	.848546	56
5	.148026	14.81	.995664	.30	152363	15.11	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.75	.995628	.30	154174	15.05	.845826	53
8	.150686	14.72	.995610	.30	155077	15.02	.844923	52
9	.151569	14.69	.995591	.30	.155978	14.99	.844022	51
10	9.152451	14.66	9.995573	.30	9.156877	14.96	0.843123	50
11	.153330	14.63	.995555	.30	.157775	14.93	.842225	49
12	154208	14.60	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.54	.995501	.30	.160457	14.84	.839543	46
15	.156830	14.51	.995482	31	.161347	14.81	.838653	45
16	.157700	14.48	.995464	.31	.162236	14.78	.837764	44
17	.158569	14.45	.995446	.31	.163123	14.75	.836877	43
18	.159435	14.42	.995427	.31	.164008	14.73	835992	42
19	.160301	14.39	.995409	.31	.164892	14.70	.835108	41
20	9.161164	14.36	9.995390	.31	9.165774	14.67	0.834226	40
21	.162025	14.33	.995372	.31	.166654	14.64	.833346	39
22	.162885	14.30	.995353	31	.167532	14.61	832468	38
23	.163743	14.27	.995334	.31	.168409	14.58	.831591	37
24	.164600	14.24	.995316	.31	.169284	14.56	.830716	36
25	.165454	14.22	.995297	.31	.170157	14.53	.829843	35
26	.166307	14.19	.995278	.31	.171029	14.50	.828971	34
27	.167159	14.16	.995260	.31	.171899	14.47	.828101	33
28	168008	14.13	.995241	.31	.172767	14.44	.827233	32
29	.168856	14.10	.995222	.31	.173634	14.42	.826366	31
30	9.169702	14.07	9.995203	.31	9.174499	14.39	0.825501	30
31	.170547	14.05	.995184	.32	.175362	14.36	.824688	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	13.99	.995146	.32	.177084	14.31	.822916	27
34	.173070	13.96	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.94	.995108	.32	.178799	14.25	.821201	25
36	.174744	13.91	.995089	.32	.179655	14.23	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	176411	13.85	.995051	.32	.181360	14.17	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.15	.817789	21
40	9.178072	13.80	9.995013	.32	9.183059	14.12	0.816941	20
41	.178900	13.77	.994993	.32	.183907	14.09	.816093	19
42	.179726	13.75	.994974	.32	.184752	14.07	.815248	18
43	.180551	13.72	.994955	.32	.185597	14.04	.814403	17
44	.181374	13.69	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.32	.187280	13.99	.812720	15
46	.183016	13.64	.994896	.33	.188120	13.97	811880	14
47	.183834	13.61	.994877	.33	.188958	13.94	.811042	13
48	.184651	13.59	.994857	.33	.189794	13.91	.810206	12
49	.185466	13.56	.994838	.33	.190629	13.89	.809371	11
50	9.186280	13.54	9.994818	.33	9.191462	13.86	0.808538	10
51	.187092	13.51	.994798	.33	.192294	13.84	.807706	9
52	.187903	13.48	.994779	.33	.193124	13.81	.806876	8
53	.188712	13.46	.994759	.33	.193953	13.79	.806047	7
54	.189519	13.43	.994739	.33	.194780	13.76	.805220	6
55	.190325	13.41	.994720	.33	.195606	13.74	.804394	5
56	.191130	13.38	994700	.33	.196430	13.71	.803570	4
57	.191933	13.36	.994680	.33	.197253	13.69	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.66	.801926	2
59	.193534	13.31	.994640	.33	.198894	13.64	.801106	1
60	.194332		.994620		199713		.800287	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.194332	13.28	9.994620	.33	9.199713	13.62	0.800287	60
1	.195129	13.26	.994600	.33	.200529	13.59	.799471	59
2	.195925	13.23	.994580	.34	.201345	13.57	.798655	58
3	.196719	13.21	.994560	.34	.202159	13.54	.797841	57
4	.197511	13.18	.994540	.34	.202971	13.52	.797029	56
5	.198302	13.16	.994519	.34	.203782	13.49	.796218	55
6	.199091	13.13	.994499	.34	.204592	13.47	.795408	54
7	.199879	13.1	.994479	.34	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.34	.206207	13.42	.793793	52
9	.201451	13.06	.994438	.34	.207013	13.40	.792987	51
10	9.202234	13.04	9.994418	.34	9.207817	13.38	0.792183	50
11	.203017	13.01	.994398	.34	.208619	13.35	.791381	49
12	.203797	12.99	.994377	.34	.209420	13.33	.790580	48
13	.204577	12.96	.994357	.34	.210220	13.31	.789780	47
14	.205354	12.94	.994336	.34	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.34	.211815	13.26	.788185	45
16	.206906	12.89	.994295	.34	.212611	13.24	.787389	44
17	.207679	12.87	.994274	.34	.213405	13.21	.786595	43
18	.208452	12.85	.994254	.35	.214198	13.19	.785802	42
19	.209222	12.82	.994233	.36	.214989	13.17	.785011	41
20	9.209992	12.80	9.994212	.35	9.215780	13.15	0.784220	40
21	.210760	12.78	.994191	.35	.216568	13.12	.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.08	.781858	37
24	.213055	12.71	.994129	.35	.218926	13.06	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.66	.994087	.35	.220492	13.01	.779508	34
27	.215338	12.64	.994066	.35	.221272	12.99	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.59	.994024	.35	.222830	12.95	.777170	31
30	9.217609	12.57	9.994003	.35	9.223607	12.92	0.776393	30
31	.218363	12.55	.993982	.35	.224382	12.90	.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.86	.774071	27
34	.220618	12.48	.993918	.36	.226700	12.84	.773300	26
35	.221367	12.46	.993897	.36	.227471	12.82	.772529	25
36	.222115	12.44	.993875	.36	.228239	12.79	.771761	24
37	.222861	12.42	.993854	.36	.229007	12.77	.770993	23
38	.223606	12.39	.993832	.36	.229773	12.75	.770227	22
39	.224349	12.37	.993811	.36	.230539	12.73	.769461	21
40	9.225092	12.35	9.993789	.36	9.231302	12.71	0.768698	20
41	.225833	12.33	.993768	.36	.232065	12.69	.767935	19
42	.226573	12.31	.993746	.36	.232826	12.67	.767174	18
43	.227311	12.29	.993725	.36	.233586	12.65	.766414	17
44	.228048	12.26	.993703	.36	.234345	12.63	.765655	16
45	.228784	12.24	.993681	.36	.235103	12.60	.764897	15
46	.229518	12.22	.993660	.36	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.36	.236614	12.56	.763386	13
48	.230984	12.18	.993616	.36	.237368	12.54	.762632	12
49	.231715	12.16	.993594	.36	.238120	12.52	.761880	11
50	9.232444	12.14	9.993572	.37	9.238872	12.50	0.761128	10
51	.233172	12.12	.993550	.37	.239622	12.48	.760378	9
52	.233899	12.10	.993528	.37	.240371	12.46	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.44	.758882	7
54	.235349	12.05	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.01	.993440	.37	.243354	12.38	.756646	4
57	.237515	11.99	.993418	.37	.244097	12.36	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.34	.755161	2
59	.238953	11.95	.993374	.37	.245579	12.32	.754421	1
60	.239670		.993351		.246319		.753681	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″	Tang.	M.

10° 169°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang	M.
0	9.239670	11.93	9.993351	.37	9.246319	12.30	0.753681	60
1	.240386	11.91	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.89	.993307	.37	.247794	12.26	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.24	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.22	.750736	56
5	.243237	11.83	993240	.37	.249998	12.20	.750002	55
6	.243947	11.81	.993217	.38	.250730	12.18	.749270	54
7	.244656	11.79	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.75	.993149	.38	.252920	12.13	.747080	51
10	9.246775	11.73	9.993127	.38	9.253648	12.11	0.746352	50
11	.247478	11.71	.993104	.38	.254374	12.09	.745626	49
12	.248181	11.69	.993081	.38	.255100	12.07	.744900	48
13	.248883	11.67	.993059	38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.01	.742731	45
16	.250980	11.61	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.59	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.58	.992944	.38	.259429	11.96	.740571	42
19	.253067	11.56	.992921	.38	.260146	11.94	.739854	41
20	9.253761	11.54	9.992898	.38	9.260863	11.92	0.739137	40
21	.254453	11.52	.992875	.38	.261578	11.90	.738422	39
22	.255144	11.50	.992852	.39	.262292	11.89	.737708	38
23	.255834	11.48	.992829	39	.263005	11.87	.736995	37
24	.256523	11.46	.992806	.39	.263717	11.85	.736283	36
25	.257211	11.44	.992783	39	.264428	11.83	.735572	35
26	.257898	11.42	.992759	39	.265138	11.81	.734862	34
27	.258583	11.41	.992736	39	.265847	11.79	.734153	33
28	.259268	11.39	.992713	39	.266555	11.78	.733445	32
29	.259951	11.37	.992690	39	.267261	11.76	.732739	31
30	9.260633	11.35	9.992666	39	9.267967	11.74	0.732033	30
31	.261314	11.33	.992643	39	.268671	11.72	.731329	29
32	.261994	11.31	.992619	39	.269375	11.70	730625	28
33	.262673	11.30	.992596	39	.270077	11.69	.729923	27
34	.263351	11.28	.992572	.39	.270779	11.67	.729221	26
35	.264027	11.26	.992549	.39	.271479	11.65	.728521	25
36	.264703	11.24	.992525	.39	.272178	11.64	.727822	24
37	.265377	11.22	.992501	.39	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.19	.992454	.40	.274269	11.58	.725731	21
40	9.267395	11.17	9.992430	.40	9.274964	11.57	0.725036	20
41	.268065	11.15	.992406	.40	.275658	11.55	.724342	19
42	.268734	11.13	.992382	.40	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.51	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.06	.992287	.40	.279113	11.46	.720887	14
47	.272064	11.05	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.40	.280488	11.43	.719512	12
49	.273388	11.01	.992214	.40	.281174	11.41	.718826	11
50	9.274049	10.99	9.992190	.40	9.281858	11.40	0.718142	10
51	.274708	10.98	.992166	.40	.282542	11.38	.717458	9
52	.275367	10.96	.992142	.40	.283225	11.36	.716775	8
53	.276025	10.94	.992118	.41	.283907	11.35	.716093	7
54	.276681	10.92	.992093	.41	.284588	11.33	.715412	6
55	.277337	10.91	.992069	.41	.285268	11.31	.714732	5
56	.277991	10.89	.992044	.41	.285947	11.30	.714053	4
57	.278645	10.87	.992020	.41	.286624	11.28	.713376	3
58	.279297	10.86	.991996	.41	.287301	11.26	.712699	2
59	.279948	10.84	.991971	.41	.287977	11.25	.712023	1
60	.280599		.991947		.288652		.711348	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

100° 79°

11° 168°

M.	Sine	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.280599	10.82	9.991947	.41	9.288652	11.23	0.711348	60
1	.281248	10.81	.991922	.41	.289326	11.22	.710674	59
2	.281897	10.79	.991897	.41	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.41	.290671	11.18	.709329	57
4	.283190	10.76	991848	.41	.291342	11.17	.708658	56
5	.283836	10.74	991823	.41	.292013	11.15	.707987	55
6	.284480	10.72	.991799	.41	.292682	11.14	.707318	54
7	.285124	10.71	.991774	.41	.293350	11.12	.706650	53
8	.285766	10.69	.991749	.41	.294017	11.11	.705983	52
9	.286408	10.67	.991724	42	.294684	11.09	.705316	51
10	9.287048	10.66	9.991699	.42	9.295349	11.07	0.704651	50
11	.287688	10.64	.991674	.42	.296013	11.06	.703987	49
12	.288326	10.63	.991649	.42	.296677	11.04	.703323	48
13	.288964	10.61	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.59	.991599	42	.298001	11.01	.701999	46
15	.290236	10.58	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.56	.991549	.42	.299322	10.98	.700678	44
17	.291504	10.55	.991524	.42	.299980	10.97	.700020	43
18	.292137	10.53	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.51	.991473	.42	.301295	10.93	.698705	41
20	9.293399	10.50	9.991448	.42	9.301951	10.92	0.698049	40
21	.294029	10.48	.991422	.42	.302607	10.90	697393	39
22	.294658	10.47	.991397	.42	.303261	10.89	.696739	38
23	.295286	10.45	.991372	.42	.303914	10.87	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.86	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.84	.694782	35
26	.297164	10.40	.991295	.43	.305869	10.83	.694131	34
27	.297788	10.39	.991270	.43	.306519	10.81	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.36	.991218	.43	.307816	10.78	.692184	31
30	9.299655	10.34	9.991193	.43	9.308463	10.77	0.691537	30
31	.300276	10.33	.991167	.43	.309109	10.76	.690891	29
32	.300895	10.31	.991141	.43	.309754	10.74	.690246	28
33	.301514	10.30	.991115	.43	.310399	10.73	.689601	27
34	.302132	10.28	.991090	.43	.311042	10.71	.688958	26
35	.302748	10.26	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.22	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.64	.685753	21
40	9.305819	10.19	9.990934	.44	9.314885	10.62	0.685115	20
41	.306430	10.17	.990908	.44	.315523	10.61	.684477	19
42	.307041	10.16	.990882	.44	.316159	10.60	.683841	18
43	.307650	10.14	.990855	.44	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.44	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.44	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.44	.318697	10.54	.681303	14
47	.310080	10.09	.990750	.44	.319330	10.53	.680670	13
48	.310685	10.07	.990724	.44	.319961	10.51	.680039	12
49	.311289	10.06	.990697	.44	.320592	10.50	.679408	11
50	9.311893	10.04	9.990671	.44	9.321222	10.48	0.678778	10
51	.312495	10.03	.990645	.44	.321851	10.47	.678149	9
52	.313097	10.01	.990618	.44	.322479	10.46	.677521	8
53	.313698	10.00	.990591	.44	.323106	10.44	.676894	7
54	.314297	9.98	.990565	.44	.323733	10.43	.676267	6
55	.314897	9.97	.990538	.44	.324358	10.41	.675642	5
56	.315495	9.96	.990511	.45	.324983	10.40	.675017	4
57	.316092	9.94	.990485	.45	.325607	10.39	.674393	3
58	.316689	9.93	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.91	.990431	.45	326853	10.36	.673147	1
60	.317879		.990404		.327475		.672525	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

101° 78°

M.	Sine	D. 1″	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.317879	9.90	9.990404	.45	9.327475	10.35	0.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.86	.990324	.45	.329334	10.31	.670666	57
4	.320249	9.84	.990297	.45	.329953	10.29	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.81	.990243	.45	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.79	.990188	.45	.332418	10.24	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.23	.666967	51
10	9.323780	9.76	9.990134	.45	9.333646	10.21	0.666354	50
11	.324366	9.75	.990107	.45	.334259	10.20	.665741	49
12	.324950	9.73	.990079	.46	.334871	10.19	.665129	48
13	.325534	9.72	.990052	.46	.335482	10.17	.664518	47
14	.326117	9.70	990025	.46	.336093	10.16	.663907	46
15	.326700	9.69	.989997	.46	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.46	.337311	10.14	.662689	44
17	.327862	9.66	.989942	.46	.337919	10.12	.662081	43
18	.328442	9.65	.989915	.46	.338527	10.11	.661473	42
19	.329021	9.64	.989887	.46	.339133	10.10	.660867	41
20	9.329599	9.62	9.989860	.46	9.339739	10.08	0.660261	40
21	.330176	9.61	.989832	.46	.340344	10.07	.659656	39
22	.330753	9.60	.989804	.46	.340948	10.06	.659052	38
23	.331329	9.58	.989777	.46	.341552	10.05	.658448	37
24	.331903	9.57	.989749	.46	.342155	10.03	.657845	36
25	.332478	9.56	.989721	.46	.342757	10.02	.657243	35
26	.333051	9.54	.989693	.46	343358	10.01	.656642	34
27	.333624	9.53	.989665	.47	343958	10.00	.656042	33
28	.334195	9.52	.989637	.47	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	9.335337	9.49	9.989582	.47	9.345755	9.96	0.654245	30
31	.335906	9.48	989553	.47	.346353	9.95	.653647	29
32	.336475	9.46	989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	989497	.47	.347545	9.92	.652455	27
34	.337610	9.44	989469	.47	.348141	9.91	.651859	26
35	.338176	9.43	989441	.47	.348735	9.90	.651265	25
36	.338742	9.41	989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	989385	.47	.349922	9.87	.650078	23
38	339871	9.39	989356	.47	.350514	9.86	.649486	22
39	.340434	9.37	989328	.47	.351106	9.85	.648894	21
40	9.340996	9.36	9.989300	.47	9.351697	9.84	0.648303	20
41	.341558	9.35	.989271	.47	.352287	9.82	.647713	19
42	.342119	9.34	.989243	.47	.352876	9.81	.647124	18
43	.342679	9.32	989214	.48	.353465	9.80	.646535	17
44	.343239	9.31	989186	.48	.354053	9.79	.645947	16
45	.343797	9.30	989157	.48	.354640	9.78	.645360	15
46	.344355	9.29	.989128	.48	.355227	9.76	.644773	14
47	.344912	9.27	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.26	.989071	.48	.356398	9.74	643602	12
49	.346024	9.25	.989042	.48	.356982	9.73	.643018	11
50	9.346579	9.24	9.989014	.48	9.357566	9.72	0.642434	10
51	.347134	9.22	.988985	.48	.358149	9.70	.641851	9
52	.347687	9.21	.988956	.48	.358731	9.69	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.68	.640687	7
54	.348792	9.19	.988898	.48	.359893	9.67	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.66	.639526	5
56	.349893	9.16	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.14	.988782	.49	.362210	9.62	.637790	2
59	.351540	9.13	988753	.49	.362787	9.61	.637213	1
60	.352088		.988724		363364		.636636	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.352088	9.11	9.988724	.49	9.363364	9.60	0.636636	60
1	.352635	9.10	.988695	.49	.363940	9.59	.636060	59
2	.353181	9.09	.988666	.49	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.49	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.49	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.49	.366237	9.54	.633763	55
6	.355358	9.04	.988548	.49	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.49	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.49	.367953	9.51	.632047	52
9	.356984	9.01	.988460	.49	.368524	9.50	.631476	51
10	9.357524	8.99	9.988430	.49	9.369094	9.49	0.630906	50
11	.358064	8.98	.988401	.49	.369663	9.48	.630337	49
12	.358603	8.97	.988371	.49	.370232	9.47	.629768	48
13	.359141	8.96	.988342	.50	.370799	9.45	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.44	.628633	46
15	.360215	8.94	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.50	.372499	9.42	.627501	44
17	.361287	8.91	.988223	.50	.373064	9.41	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.89	.988163	.50	.374193	9.39	.625807	41
20	9.362889	8.88	9.988133	.50	9.374756	9.38	0.625244	40
21	.363422	8.87	.988103	.50	.375319	9.37	.624681	39
22	.363954	8.86	.988073	.50	.375881	9.36	.624119	38
23	.364485	8.84	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.81	.987953	.50	.378122	9.31	621878	34
27	.366604	8.80	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.79	987892	.50	.379239	9.29	.620761	32
29	.367659	8.78	987862	.51	.379797	9.28	.620203	31
30	9.368185	8.76	9.987832	.51	9.380354	9.27	0.619646	30
31	.368711	8.75	.987801	.51	.380910	9.26	.619090	29
32	369236	8.74	987771	.51	.381466	9.25	.618534	28
33	369761	8.73	.987740	.51	.382020	9.24	.617980	27
34	.370285	8.72	.987710	.51	.382575	9.23	.617425	26
35	.370808	8.71	.987679	.51	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.51	.383682	9.21	.616318	24
37	.371852	8.69	.987618	.51	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.51	.384786	9.19	.615214	22
39	.372894	8.66	.987557	.51	.385337	9.18	.614663	21
40	9.373414	8.65	9.987526	.51	9.385888	9.17	0.614112	20
41	.373933	8.64	.987496	.51	.386438	9.16	.613562	19
42	.374452	8.63	.987465	.51	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.51	.387536	9.14	.612464	17
44	.375487	8.61	.987403	.51	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.11	.611369	15
46	.376519	8.59	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.58	.987310	.52	.389724	9.09	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.56	.987248	.52	.390815	9.07	.609185	11
50	9.378577	8.55	9.987217	.52	9.391360	9.06	0.608640	10
51	.379089	8.53	.987186	.52	.391903	9.05	.608097	9
52	.379601	8.52	.987155	.52	.392447	9.04	.607553	8
53	.380113	8.51	.987124	.52	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.02	.606469	6
55	.381134	8.49	.987061	.52	.394073	9.01	.605927	5
56	.381643	8.48	.987030	.52	.394614	9.00	.605386	4
57	.382152	8.47	.986998	.52	.395154	8.99	.604846	3
58	.382661	8.46	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.52	.396233	8.97	.603767	1
60	.383675		.986904		.396771		.603229	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang	M.
0	9.383675	8.44	9.986904	.53	9.396771	8.96	0.603229	60
1	.384182	8.43	.986873	.53	.397309	8.96	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.41	.986809	.53	.398383	8.94	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.39	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.53	399990	8.91	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.36	.986651	.53	.401058	8.89	.598942	52
9	.388210	8.35	.986619	53	.401591	8.88	.598409	51
10	9.388711	8.34	9.986587	.53	9.402124	8.87	0.597876	50
11	.389211	8.33	.986555	.53	.402656	8.86	.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.31	.986491	.53	.403718	8.84	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.83	.595751	46
15	.391206	8.29	.986427	.54	.404778	8.82	.595222	45
16	.391703	8.28	.986395	.54	.405308	8.81	.594692	44
17	.392199	8.27	.986363	.54	.405836	8.80	.594164	43
18	.392695	8.26	.986331	.54	.406364	8.79	.593636	42
19	.393191	8.25	.986299	.54	.406892	8.78	.593108	41
20	9.393685	8.24	9.986266	.54	9.407419	8.77	0.592581	40
21	.394179	8.23	.986234	.54	.407945	8.76	.592055	39
22	.394673	8.22	.986202	.54	.408471	8.75	.591529	38
23	.395166	8.21	.986169	.54	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.54	.409521	8.74	.590479	36
25	.396150	8.19	.986104	.54	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.54	.410569	8.72	.589431	34
27	.397132	8.17	.986039	.54	.411092	8.71	.588908	33
28	.397621	8.16	.986007	.54	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.54	.412137	8.69	.587863	31
30	9.398600	8.14	9.985942	.54	9.412658	8.68	0.587342	30
31	.399088	8.13	.985909	.55	.413179	8.67	.586821	29
32	.399575	8.12	.985876	.55	.413699	8.66	.586301	28
33	.400062	8.11	.985843	.55	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.09	.985778	.55	.415257	8.64	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.06	.985679	.55	.416810	8.61	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	9.403455	8.04	9.985613	.55	9.417842	8.59	0.582158	20
41	.403938	8.03	.985580	.55	.418358	8.58	.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.01	.985514	.55	.419387	8.56	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.56	.580099	16
45	.405862	7.99	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.56	.420927	8.54	.579073	14
47	.406820	7.97	.985381	.56	421440	8.53	.578560	13
48	.407299	7.96	.985347	.56	.421952	8.52	.578048	12
49	.407777	7.96	.985314	.56	.422463	8.51	.577537	11
50	9.408254	7.95	9.985280	.56	9.422974	8.50	0.577026	10
51	.408731	7.94	.985247	.56	.423484	8.49	.576516	9
52	.409207	7.93	.985213	.56	.423993	8.49	.576007	8
53	.409682	7.92	.985180	.56	.424503	8.48	.575497	7
54	.410157	7.91	.985146	.56	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.56	.425519	8.46	.574481	5
56	.411106	7.89	.985079	.56	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.56	.426534	8.44	.573466	3
58	.412052	7.87	.985011	.56	.427041	8.43	.572959	2
59	.412524	7.86	.984978	.56	.427547	8.43	.572453	1
60	.412996		.984944		.428052		.571948	0
M.	Cosine.	D. 1″	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

15° **164°**

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.412996	7.85	9.984944	.56	9.428052	8.42	0.571948	60
1	.413467	7.84	.984910	.57	428558	8.41	.571442	59
2	.413938	7.84	.984876	.57	429062	8.40	.570938	58
3	414408	7.83	984842	.57	.429566	8.39	.570434	57
4	.414878	7.82	984808	.57	430070	8.38	.569930	56
5	.415347	7.81	984774	.57	.430573	8.38	.569427	55
6	.415815	7.80	984740	.57	.431075	8.37	.568925	54
7	.416283	7.79	984706	.57	.431577	8.36	.568423	53
8	.416751	7.78	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.77	984638	.57	.432580	8.34	.567420	51
10	9.417684	7.76	9.984603	.57	9.433080	8.33	0.566920	50
11	.418150	7.75	984569	.57	433580	8.33	.566420	49
12	.418615	7.75	.984535	.57	.434080	8.32	.565920	48
13	.419079	7.74	.984500	.57	.434579	8.31	.565421	47
14	.419544	7.73	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.57	.435576	8.29	.564424	45
16	.420470	7.71	.984397	.58	.436073	8.28	.563927	44
17	420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.69	.984328	.58	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.26	.562437	41
20	9.422318	7.67	9.984259	.58	9.438059	8.25	0.561941	40
21	422778	7.67	.984224	.58	438554	8.24	.561446	39
22	423238	7.66	.984190	.58	439048	8.24	.560952	38
23	423697	7.65	.984155	.58	.439543	8.23	560457	37
24	424156	7.64	.984120	.58	.440036	8.22	.559964	36
25	424615	7.63	.984085	.58	440529	8.21	.559471	35
26	425073	7.62	.984050	.58	441022	8.20	.558978	34
27	425530	7.61	.984015	.58	.441514	8.20	.558486	33
28	425987	7.61	.983981	.58	442006	8.19	.557994	32
29	426443	7.60	.983946	.58	442497	8.18	.557503	31
30	9.426899	7.59	9.983911	.58	9.442988	8.17	0.557012	30
31	427354	7.58	.983875	.58	443479	8.16	.556521	29
32	427809	7.57	.983840	.59	443968	8.16	.556032	28
33	428263	7.56	.983805	.59	444458	8.15	.555542	27
34	428717	7.55	.983770	.59	.444947	8.14	.555053	26
35	429170	7.55	.983735	.59	.445435	8.13	.554565	25
36	429623	7.53	.983700	.59	.445923	8.13	.554077	24
37	430075	7.52	983664	.59	446411	8.12	.553589	23
38	430527	7.52	.983629	.59	.446898	8.11	.553102	22
39	430978	7.51	983594	.59	.447384	8.10	.552616	21
40	9.431429	7.50	9.983558	.59	9.447870	8.09	0.552130	20
41	.431879	7.49	983523	.59	448356	8.09	.551644	19
42	432329	7.49	.983487	.59	448841	8.08	.551159	18
43	432778	7.48	983452	.59	449326	8.07	.550674	17
44	433226	7.47	.983416	.59	449810	8.06	.550190	16
45	433675	7.46	983381	.59	450294	8.06	.549706	15
46	434122	7.45	983345	.59	450777	8.05	.549223	14
47	434569	7.44	983309	.60	451260	8.04	.548740	13
48	435016	7.44	983273	.60	451743	8.03	.548257	12
49	435462	7.43	.983238	.60	452225	8.03	.547775	11
50	9.435908	7.42	9.983202	.60	9.452706	8.02	0.547294	10
51	436352	7.41	.983166	.60	.453187	8.01	.546813	9
52	436798	7.40	983130	.60	.453668	8.00	.546332	8
53	437242	7.40	983094	.60	.454148	8.00	.545852	7
54	437686	7.39	.983058	.60	454628	7.99	.545372	6
55	438129	7.38	.983022	.60	455107	7.98	.544893	5
56	438572	7.37	.982986	.60	455586	7.97	.544414	4
57	.439014	7.36	.982950	.60	456064	7.97	.543936	3
58	.439456	7.36	.982914	.60	456542	7.96	.543458	2
59	.439897	7.35	.982878	.60	457019	7.95	.542981	1
60	.440338		.982842		457496		.542504	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

105° **74°**

16° **163°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.440338	7.34	9.982842	.60	9.457496	7.94	0.542504	60
1	.440778	7.33	.982805	.60	.457973	7.94	.542027	59
2	.441218	7.32	.982769	.61	.458449	7.93	.541551	58
3	.441658	7.31	.982733	.61	.458925	7.92	.541075	57
4	.442096	7.31	.982696	.61	.459400	7.91	.540600	56
5	.442535	7.30	.982660	.61	.459875	7.91	.540125	55
6	.442973	7.29	.982624	.61	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.61	.460823	7.89	.539177	53
8	.443847	7.27	.982551	.61	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.61	.461770	7.88	.538230	51
10	9.444720	7.26	9.982477	.61	9.462242	7.87	0.537758	50
11	.445155	7.25	.982441	.61	.462715	7.86	.537285	49
12	.445590	7.24	.982404	.61	.463186	7.86	.536814	48
13	.446025	7.24	.982367	.61	.463658	7.85	.536342	47
14	.446459	7.23	.982331	.61	.464128	7.84	.535872	46
15	.446893	7.22	.982294	.61	.464599	7.83	.535401	45
16	.447326	7.21	.982257	.61	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.81	.533992	42
19	.448623	7.19	.982146	.62	.466477	7.81	.533523	41
20	9.449054	7.18	9.982109	.62	9.466945	7.80	0.533055	40
21	.449485	7.17	.982072	.62	.467413	7.79	.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.16	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.14	.981924	.62	.469280	7.76	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.76	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.62	.470676	7.74	.529324	32
29	.452915	7.11	.981774	.62	.471141	7.74	.528859	31
30	9.453342	7.10	9.981737	.62	9.471605	7.73	0.528395	30
31	.453768	7.10	.981700	.62	.472069	7.72	.527931	29
32	.454194	7.09	.981662	.63	.472532	7.71	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.71	.527005	27
34	.455044	7.07	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.63	.473919	7.69	.526081	25
36	.455893	7.06	.981512	.63	.474381	7.69	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.04	.981436	.63	.475303	7.67	.524697	22
39	.457162	7.04	.981399	.63	.475763	7.67	.524237	21
40	9.457584	7.03	9.981361	.63	9.476223	7.66	0.523777	20
41	.458006	7.02	.981323	.63	.476683	7.65	.523317	19
42	.458427	7.01	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.01	.981247	.63	.477601	7.64	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	6.99	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.64	.479432	7.61	.520568	13
48	.460946	6.97	.981057	.64	.479889	7.61	.520111	12
49	.461364	6.96	.981019	.64	.480345	7.60	.519655	11
50	9.461782	6.96	9.980981	.64	9.480801	7.59	0.519199	10
51	.462199	6.95	.980942	.64	.481257	7.59	.518743	9
52	.462616	6.94	.980904	.64	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.64	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.64	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.64	.483075	7.56	.516925	5
56	.464279	6.91	.980750	.64	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.64	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.64	.484435	7.54	.515565	2
59	.465522	6.89	.980635	.64	.484887	7.53	.515113	1
60	.465935		.980596		.485339		.514661	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.465935	6.88	9.980596	.64	9.485339	7.53	0.514661	60
1	466348	6.88	.980558	.64	.485791	7.52	.514209	59
2	466761	6.87	.980519	.65	.486242	7.51	.513758	58
3	.467173	6.86	.980480	.65	.486693	7.51	.513307	57
4	467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.84	.980364	.65	.488043	7.49	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	9.470046	6.81	9.980208	.65	9.489838	7.46	0.510162	50
11	.470455	6.81	.980169	.65	.490286	7.46	.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.79	.980091	.65	.491180	7.44	.508820	47
14	.471679	6.78	.980052	.65	.491627	7.44	.508373	46
15	.472086	6.78	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.76	.979934	.66	.492965	7.42	.507035	43
18	.473304	6.76	.979895	.66	.493410	7.41	.506590	42
19	473710	6.75	.979855	.66	.493854	7.41	.506146	41
20	9.474115	6.74	9.979816	.66	9.494299	7.40	0.505701	40
21	.474519	6.74	.979776	.66	.494743	7.39	.505257	39
22	474923	6.73	.979737	.66	.495186	7.39	.504814	38
23	475327	6.72	.979697	.66	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.66	.496073	7.38	.503927	36
25	476133	6.71	.979618	.66	.496515	7.37	.503485	35
26	476536	6.70	.979579	.66	.496957	7.36	.503043	34
27	.476938	6.69	.979539	.66	497399	7.36	.502601	33
28	.477340	6.69	.979499	.66	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.66	.498282	7.34	.501718	31
30	9.478142	6.67	9.979420	66	9.498722	7.34	0.501278	30
31	.478542	6.67	.979380	.66	499163	7.33	.500837	29
32	.478942	6.66	.979340	.67	.499603	7.33	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.31	.499519	26
35	.480140	6.64	.979220	.67	.500920	7.31	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.63	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.67	.502235	7.29	.497765	22
39	.481731	6.61	.979059	.67	502672	7.28	.497328	21
40	9.482128	6.61	9.979019	.67	9.503109	7.28	0.496891	20
41	.482525	6.60	.978979	.67	503546	7.27	.496454	19
42	.482921	6.59	.978939	.67	503982	7.27	.496018	18
43	.483316	6.59	.978898	.67	.504418	7.26	.495582	17
44	.483712	6.58	.978858	.67	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.24	.494276	14
47	.484895	6.56	.978737	.68	.506159	7.24	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.68	.507027	7.23	.492973	11
50	9.486075	6.54	9.978615	.68	9.507460	7.22	0 492540	10
51	.486467	6.54	.978574	.68	.507893	7.21	.492107	9
52	.486860	6.53	.978533	.68	.508326	7.21	.491674	8
53	.487251	6.52	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	68	.509191	7.20	.490809	6
55	.488034	6.51	.978411	.68	.509622	7.19	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.19	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	.489982		.978206		.511776		.488224	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

18° **161°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.489982	6.48	9.978206	.68	9.511776	7.16	0.488224	60
1	.490371	6.47	.978165	.69	.512206	7.16	.487794	59
2	.490759	6.46	.978124	.69	.512635	7.15	.487365	58
3	.491147	6.46	.978083	.69	.513064	7.14	486936	57
4	.491535	6.45	.978042	.69	513493	7.14	186507	56
5	.491922	6.45	.978001	.69	.513921	7.13	.486079	55
6	.492308	6.44	.977959	.69	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.69	.514777	7.12	485223	53
8	.493081	6.43	.977877	.69	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.69	.515631	7.11	.484369	51
10	9.493851	6.41	9.977794	.69	9.516057	7.10	0.483943	50
11	.494236	6.41	.977752	.69	.516484	7.10	.483516	49
12	.494621	6.40	.977711	.69	.516910	7.09	.483090	48
13	.495005	6.39	.977669	.69	517335	7.09	.482665	47
14	.495388	6.39	.977628	.69	.517761	7.08	.482239	46
15	.495772	6.38	.977586	.69	.518186	7.08	.481814	45
16	.496154	6.38	.977544	.70	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.36	.977461	.70	.519458	7.06	.480542	42
19	.497301	6.36	.977419	.70	.519882	7.05	.480118	41
20	9.497682	6.35	9.977377	.70	9.520305	7.05	0.479695	40
21	.498064	6.34	.977335	.70	.520728	7.04	.479272	39
22	.498444	6.34	.977293	.70	.521151	7.04	.478849	38
23	.498825	6.33	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.31	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.31	.977083	.70	.523259	7.01	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.01	.476320	32
29	.501099	6.30	.976999	.70	.524100	7.00	.475900	31
30	9.501476	6.29	9.976957	.70	9.524520	6.99	0.475480	30
31	.501854	6.28	.976914	.71	.524940	6.99	.475060	29
32	.502231	6.28	.976872	.71	.525359	6.98	.474641	28
33	.502607	6.27	.976830	.71	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.71	526197	6.97	.473803	26
35	.503360	6.26	.976745	.71	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.71	.527033	6.96	472967	24
37	.504110	6.25	.976660	.71	.527451	6.96	472549	23
38	.504485	6.24	.976617	.71	.527868	6.95	.472132	22
39	.504860	6.24	.976574	.71	.528285	6.95	.471716	21
40	9.505234	6.23	9.976532	.71	9.528702	6.94	0.471298	20
41	.505608	6.22	.976489	.71	.529119	6.94	.470881	19
42	.505981	6.22	.976446	.71	.529535	6.93	.470465	18
43	.506354	6.21	.976404	.71	.529951	6.93	.470049	17
44	.506727	6.21	.976361	.71	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.91	.469219	15
46	.507471	6.19	.976275	.72	.531196	6.91	.468804	14
47	.507843	6.19	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	467975	12
49	.508585	6.18	.976146	.72	.532439	6.89	.467561	11
50	9.508956	6.17	9.976103	.72	9.532853	6.89	0.467147	10
51	.509326	6.16	.976060	.72	.533266	6.88	.466734	9
52	.509696	6.16	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.72	.534092	6.87	465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.14	.975887	.72	.534916	6.86	.465084	5
56	.511172	6.14	.975844	.72	.535328	6.86	.464672	4
57	.511540	6.13	.975800	.72	.535739	6.85	464261	3
58	.511907	6.12	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.72	536561	6.84	.463439	1
60	.512642		.975670		.536972		.463028	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

108° **71°**

M	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.512642	6.11	9.975670	.73	9.536972	6.84	0.463028	60
1	.513009	6.11	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.09	.975539	.73	.538202	6.82	.461798	57
4	.514107	6.09	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.81	.460980	55
6	.514837	6.08	.975408	.73	.539429	6.81	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.06	.975277	.73	.540653	6.79	.459347	51
10	9.516294	6.05	9.975233	.73	9.541061	6.79	0.458939	50
11	.516657	6.05	.975189	.73	.541468	6.78	.458532	49
12	.517020	6.04	.975145	.73	.541875	6.78	.458125	48
13	.517382	6.04	.975101	.73	.542281	6.77	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.03	.975013	.74	.543094	6.76	.456906	45
16	.518468	6.02	.974969	.74	.543499	6.76	.456501	44
17	.518829	6.02	.974925	.74	.543905	6.75	.456095	43
18	.519190	6.01	.974880	.74	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.74	.544715	6.74	.455285	41
20	9.519911	6.00	9.974792	.74	9.545119	6.74	0.454881	40
21	.520271	5.99	.974748	.74	.545524	6.73	.454476	39
22	.520631	5.99	.974703	.74	.545928	6.73	.454072	38
23	.520990	5.98	.974659	.74	.546331	6.72	.453669	37
24	.521349	5.98	.974614	.74	.546735	6.72	.453265	36
25	.521707	5.97	.974570	.74	.547138	6.71	.452862	35
26	.522066	5.97	.974525	.74	.547540	6.71	.452460	34
27	.522424	5.96	.974481	.74	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.74	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.75	.548747	6.69	.451253	31
30	9.523495	5.94	9.974347	.75	9.549149	6.69	0.450851	30
31	.523852	5.94	.974302	.75	.549550	6.68	.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.67	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.67	.448847	25
36	.525630	5.91	.974077	.75	.551552	6.66	.448448	24
37	.525984	5.90	.974032	.75	.551952	6.66	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.89	.973942	.75	.552750	6.65	.447250	21
40	9.527046	5.89	9.973897	.75	9.553149	6.64	0.446851	20
41	.527400	5.88	.973852	.75	.553548	6.64	.446452	19
42	.527753	5.88	.973807	.75	.553946	6.63	.446054	18
43	.528105	5.87	.973761	.75	.554344	6.63	.445656	17
44	.528458	5.87	.973716	.76	.554741	6.62	.445259	16
45	.528810	5.86	.973671	.76	.555139	6.62	.444861	15
46	.529161	5.86	.973625	.76	.555536	6.61	.444464	14
47	.529513	5.85	.973580	.76	.555933	6.61	.444067	13
48	.529864	5.85	.973535	.76	.556329	6.60	.443671	12
49	.530215	5.84	.973489	.76	.556725	6.60	.443275	11
50	9.530565	5.83	9.973444	.76	9.557121	6.59	0.442879	10
51	.530915	5.83	.973398	.76	.557517	6.59	.442483	9
52	.531265	5.82	.973352	.76	.557913	6.59	.442087	8
53	.531614	5.82	.973307	.76	.558308	6.58	.441692	7
54	.531963	5.81	.973261	.76	.558703	6.58	.441297	6
55	.532312	5.81	.973215	.76	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.76	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.76	.559885	6.56	.440115	3
58	.533357	5.79	.973078	.77	.560279	6.56	.439721	2
59	.533704	5.79	.973032	.77	.560673	6.55	.439327	1
60	.534052		.972986		.561066		.438934	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

20° **159°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.534052	5.78	9.972986	.77	9.561066	6.55	0.438934	60
1	.534399	5.78	.972940	.77	.561459	6.54	.438541	59
2	.534745	5.77	.972894	.77	.561851	6.54	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.54	.437756	57
4	.535438	5.76	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.76	.972755	.77	.563028	6.53	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.75	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.74	.972617	.77	.564202	6.51	.435798	52
9	.537163	5.74	.972570	.77	.564593	6.51	.435407	51
10	9.537507	5.73	9.972524	.77	9.564983	6.50	0.435017	50
11	.537851	5.73	.972478	.77	.565373	6.50	.434627	49
12	.538194	5.72	.972431	.78	.565763	6.50	.434237	48
13	.538538	5.71	.972385	.78	.566153	6.49	.433847	47
14	.538880	5.71	.972338	.78	.566542	6.49	.433458	46
15	.539223	5.70	.972291	.78	.566932	6.48	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.69	.972198	.78	.567709	6.47	.432291	43
18	.540249	5.69	.972151	.78	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.46	.431514	41
20	9.540931	5.68	9.972058	.78	9.568873	6.46	0.431127	40
21	.541272	5.67	.972011	.78	.569261	6.46	.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.66	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.66	.971870	.78	.570422	6.44	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.44	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.64	.971729	.79	.571581	6.43	.428419	33
28	.543649	5.64	.971682	.79	.571967	6.43	.428033	32
29	.543987	5.63	.971635	.79	.572352	6.42	.427648	31
30	9.544325	5.63	9.971588	.79	9.572738	6.42	0.427262	30
31	.544663	5.62	.971540	.79	.573123	6.41	.426877	29
32	.545000	5.62	.971493	.79	.573507	6.41	.426493	28
33	.545338	5.61	.971446	.79	.573892	6.40	.426108	27
34	.545674	5.61	.971398	.79	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.79	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.79	.575044	6.39	.424956	24
37	.546683	5.59	.971256	.79	.575427	6.39	.424573	23
38	.547019	5.59	.971208	.79	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.79	.576193	6.38	.423807	21
40	9.547689	5.58	9.971113	.79	9.576576	6.37	0.423424	20
41	.548024	5.57	.971066	.80	.576959	6.37	.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.56	.970970	.80	.577723	6.36	.422277	17
44	.549027	5.56	.970922	.80	.578104	6.36	.421896	16
45	.549360	5.55	.970874	.80	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5 55	.970779	.80	.579248	6.34	.420752	13
48	.550359	5.54	.970731	.80	.579629	6.34	.420371	12
49	.550692	5.54	.970683	.80	.580009	6.34	.419991	11
50	9.551024	5.53	9.970635	.80	9.580389	6.33	0.419611	10
51	.551356	5.53	.970586	.80	.580769	6.33	.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5 52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.51	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.51	.970394	.81	.582286	6.31	.417714	5
56	.553010	5.50	.970345	.81	.582665	6.31	.417335	4
57	.553341	5.50	.970297	.81	.583044	6.30	.416956	3
58	.553670	5.49	.970249	.81	.583422	6.30	.416578	2
59	.554000	5.49	.970200	.81	.583800	6.30	.416200	1
60	.554329		.970152		.584177		.415823	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

110° **69°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.554329	5.48	9.970152	.81	9.584177	6.29	0.415823	60
1	.554658	5.48	.970103	.81	.584555	6.29	.415445	59
2	.554987	5.47	.970055	.81	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.81	.585309	6.28	.414691	57
4	.555643	5.46	.969957	81	.585686	6.28	.414314	56
5	.555971	5.46	.969909	.81	.586062	6.27	.413938	55
6	.556299	5.45	.969860	.81	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.81	.586815	6.26	.413185	53
8	.556953	5.44	.969762	.81	.587190	6.26	.412810	52
9	.557280	5.44	.969714	.81	.587566	6.26	.412434	51
10	9 557606	5.44	9.969665	.82	9.587941	6.25	0.412059	50
11	.557932	5.43	.969616	.82	.588316	6.25	.411684	49
12	.558258	5.43	.969567	.82	.588691	6.24	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.24	.410934	47
14	.558909	5.42	.969469	82	.589440	6.24	.410560	46
15	.559234	5.41	.969420	.82	.589814	6.23	.410186	45
16	.559558	5.41	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.39	.969223	.82	.591308	6.22	.408692	41
20	9.560855	5.39	9.969173	.82	9.591681	6.21	0.408319	40
21	.561178	5.38	.969124	.82	.592054	6.21	.407946	39
22	.561501	5.38	.969075	.82	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.20	.406829	36
25	.562468	5.37	.968926	.83	.593542	6.19	.406458	35
26	.562790	5.36	.968877	.83	.593914	6.19	.406086	34
27	.563112	5.36	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.35	.968777	.83	.594656	6.18	.405344	32
29	.563755	5.35	.968728	83	.595027	6.18	.404973	31
30	9.564075	5.34	9.968678	.83	9.595398	6.17	0.404602	30
31	.564396	5.34	.968628	83	.595768	6.17	.404232	29
32	.564716	5.33	.968578	.83	.596138	6.16	.403862	28
33	.565036	5.33	.968528	.83	.596508	6.16	.403492	27
34	.565356	5.32	.968479	.83	.596878	6.16	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.31	.968329	.83	.597985	6.15	.402015	23
38	.566632	5.31	.968278	.84	.598354	6.14	.401646	22
39	.566951	5.30	.968228	.84	.598722	6.14	.401278	21
40	9.567269	5.30	9.968178	84	9.599091	6.13	0.400909	20
41	.567587	5.29	.968128	.84	.599459	6 13	.400541	19
42	.567904	5.29	.968078	.84	.599827	6.13	.400173	18
43	.568222	5.28	.968027	.84	.600194	6.12	.399806	17
44	.568539	5.28	.967977	.84	.600562	6.12	.399438	16
45	.568856	5.28	.967927	.84	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.84	.601296	6.11	.398704	14
47	.569488	5.27	.967826	.84	.601663	6.11	.398337	13
48	.569804	5.26	.967775	.84	.602029	6.10	.397971	12
49	.570120	5.26	.967725	.84	.602395	6.10	.397605	11
50	9.570435	5.25	9.967674	.84	9.602761	6.10	0.397239	10
51	.570751	5.25	.967624	.84	.603127	6.09	.396873	9
52	.571066	5.24	.967573	.85	.603493	6.09	.396507	8
53	.571380	5.24	.967522	.85	.603858	6.09	.396142	7
54	.571695	5.24	.967471	.85	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.23	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.22	.967319	.85	.605317	6.07	.394683	3
58	572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.21	.967217	.85	.606046	6.06	.393954	1
60	.573575		.967166		.606410		.393590	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.573575	5.21	9.967166	.85	9.606410	6.06	0.393590	60
1	.573888	5.20	.967115	.85	.606773	6.06	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.85	.607500	6.05	.392500	57
4	.574824	5.19	.966961	.85	.607863	6.05	.392137	56
5	.575136	5.19	.966910	.85	.608225	6.04	.391775	55
6	.575447	5.18	.966859	.86	.608588	6.04	.391412	54
7	.575758	5.18	.966808	.86	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.86	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.86	.609674	6.03	.390326	51
10	9.576689	5.17	9.966653	.86	9.610036	6.02	0.389964	50
11	.576999	5.16	.966602	.86	.610397	6.02	.389603	49
12	.577309	5.16	.966550	.86	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.86	.611120	6.01	.388880	47
14	.577927	5.15	.966447	.86	.611480	6.01	.388520	46
15	.578236	5.14	.966395	.86	.611841	6.01	.388159	45
16	.578545	5.14	.966344	.86	.612201	6.00	.387799	44
17	.578853	5.14	.966292	.86	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.86	.612921	6.00	.387079	42
19	.579470	5.13	.966188	.86	.613281	5.99	.386719	41
20	9.579777	5.12	9.966136	.87	9.613641	5.99	0.386359	40
21	.580085	5.12	.966085	.87	.614000	5.98	.386000	39
22	.580392	5.11	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.11	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.11	.965929	.87	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.09	.965772	.87	.616151	5.96	.383849	33
28	.582229	5.09	.965720	.87	.616509	5.96	.383491	32
29	.582535	5.09	.965668	.87	.616867	5.96	.383133	31
30	9.582840	5.08	9.965615	.87	9.617224	5.95	0.382776	30
31	.583145	5.08	.965563	.87	.617582	5.95	.382418	29
32	.583449	5.07	.965511	.87	.617939	5.95	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.94	.381705	27
34	.584058	5.06	.965406	.88	.618652	5.94	.381348	26
35	.584361	5.06	.965353	.88	.619008	5.94	.380992	25
36	.584665	5.06	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.05	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.05	.965195	.88	.620076	5.93	.379924	22
39	.585574	5.04	.965143	.88	.620432	5.92	.379568	21
40	9.585877	5.04	9.965090	.88	9.620787	5.92	0.379213	20
41	.586179	5.04	.965037	.88	.621142	5.92	.378858	19
42	.586482	5.03	.964984	.88	.621497	5.91	.378503	18
43	.586783	5.03	.964931	.88	.621852	5.91	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.91	.377793	16
45	.587386	5.02	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.01	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.01	.964720	.88	.623269	5.90	.376731	13
48	.588289	5.01	.964666	.89	.623623	5.89	.376377	12
49	.588590	5.00	.964613	.89	.623976	5.89	.376024	11
50	9.588890	5.00	9.964560	.89	9.624330	5.89	0.375670	10
51	.589190	4.99	.964507	.89	.624683	5.88	.375317	9
52	.589489	4.99	.964454	.89	.625036	5.88	.374964	8
53	.589789	4.99	.964400	.89	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.89	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.89	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.89	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.89	.626797	5.86	.373203	3
58	.591282	4.97	.964133	.89	.627149	5.86	.372851	2
59	.591580	4.96	.964080	.89	.627501	5.86	.372499	1
60	.591878		.964026		.627852		.372148	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.591878	4.96	9.964026	.89	9.627852	5.85	0.372148	60
1	.592176	4.95	.963972	.89	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.84	.371095	57
4	.593067	4.94	.963811	.90	.629255	5.84	.370745	56
5	.593363	4.94	.963757	.90	.629606	5.84	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.83	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.82	.368995	51
10	9.594842	4.92	9.963488	.90	9.631355	5.82	0.368645	50
11	.595137	4.91	.963434	.90	.631704	5.82	.368296	49
12	.595432	4.91	.963379	.90	.632053	5.81	.367947	48
13	.595727	4.91	.963325	.90	.632402	5.81	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.81	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.89	.963163	.91	.633447	5.80	.366553	44
17	.596903	4.89	.963108	.91	.633795	5.80	.366205	43
18	.597196	4.89	.963054	.91	.634143	5.79	.365857	42
19	.597490	4.88	.962999	.91	.634490	5.79	.365510	41
20	9.597783	4.88	9.962945	.91	9.634838	5.79	0.365162	40
21	.598075	4.88	.962890	.91	.635185	5.78	.364815	39
22	.598368	4.87	.962836	.91	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.91	.635879	5.78	.364121	37
24	.598952	4.86	.962727	.91	.636226	5.78	.363774	36
25	.599244	4.86	.962672	.91	.636572	5.77	.363428	35
26	.599536	4.86	.962617	.91	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.91	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.91	.637611	5.76	.362389	32
29	.600409	4.84	.962453	.92	.637956	5.76	.362044	31
30	9.600700	4.84	9.962398	.92	9.638302	5.76	0.361698	30
31	.600990	4.84	.962343	.92	.638647	5.75	.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.74	.360318	26
35	.602150	4.82	.962123	.92	.640027	5.74	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.74	.359629	24
37	.602728	4.81	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.81	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.81	.961902	.92	.641404	5.73	.358596	21
40	9.603594	4.80	9.961846	.92	9.641747	5.73	0.358253	20
41	.603882	4.80	.961791	.92	.642091	5.72	.357909	19
42	.604170	4.79	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.79	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.79	.961624	.93	.643120	5.71	.356880	16
45	.605032	4.78	.961569	.93	.643463	5.71	.356537	15
46	.605319	4.78	.961513	.93	.643806	5.71	.356194	14
47	.605606	4.78	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.77	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	9.606465	4.76	9.961290	.93	9.645174	5.69	0.354826	10
51	.606751	4.76	.961235	.93	.645516	5.69	.354484	9
52	.607036	4.76	.961179	.93	.645857	5.69	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.69	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.74	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.74	.960955	.93	.647222	5.68	.352778	4
57	.608461	4.74	.960899	.94	.647562	5.67	.352438	3
58	.608745	4.73	.960843	.94	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.94	.648243	5.67	.351757	1
60	.609313		.960730		.648583		.351417	0
M.	Cosine.	D. 1″.	Sine	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.609313	4.73	9.960730	.94	9.648583	5.67	0.351417	60
1	.609597	4.72	.960674	.94	.648923	5.66	.351077	59
2	.609880	4.72	.960618	.94	.649263	5.66	.350737	58
3	.610164	4.72	.960561	.94	.649602	5.66	.350398	57
4	.610447	4.71	.960505	.94	.649942	5.65	.350058	56
5	.610729	4.71	.960448	.94	.650281	5.65	.349719	55
6	.611012	4.71	.960392	.94	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.94	.650959	5.64	349041	53
8	.611576	4.70	.960279	.94	.651297	5.64	.348703	52
9	.611858	4.69	.960222	.94	.651636	5.64	.348364	51
10	9.612140	4.69	9.960165	.95	9.651974	5.64	0.348026	50
11	.612421	4.69	.960109	.95	.652312	5.63	.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.95	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.66	.959711	.95	.654674	5.61	.345326	42
19	.614665	4.66	.959654	.95	.655011	5.61	.344989	41
20	9.614944	4.65	9 959596	.95	9.655348	5.61	0.344652	40
21	.615223	4.65	.959539	.95	.655684	5.61	.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.64	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.64	.959368	.96	.656692	5.60	.343308	36
25	.616338	4.64	.959310	.96	.657028	5.59	.342972	35
26	.616616	4.63	.959253	.96	.657364	5.59	.342636	34
27	.616894	4.63	.959195	96	.657699	5.59	.342301	33
28	.617172	4.63	.959138	.96	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.96	.658369	5.58	.341631	31
30	9.617727	4.62	9.959023	.96	9.658704	5.58	0.341296	30
31	.618004	4.61	.958965	.96	.659039	5.58	.340961	29
32	.618281	4.61	.958908	.96	.659373	5.57	.340627	28
33	.618558	4.61	.958850	.96	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.96	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.96	.660376	5.56	.339624	25
36	.619386	4.60	.958677	.96	660710	5.56	.339290	24
37	.619662	4.59	.958619	.97	.661043	5.56	.338957	23
38	.619938	4.59	.958561	.97	.661377	5.56	.338623	22
39	.620213	4.59	.958503	.97	.661710	5.55	.338290	21
40	9.620488	4.58	9.958445	.97	9.662043	5.55	0.337957	20
41	.620763	4.58	.958387	.97	.662376	5.55	.337624	19
42	.621038	4.58	.958329	.97	.662709	5.54	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.54	.336958	17
44	.621587	4.57	.958213	.97	.663375	5.54	336625	16
45	.621861	4.57	.958154	.97	.663707	5.54	336293	15
46	.622135	4.56	.958096	.97	664039	5.53	335961	14
47	.622409	4.56	.958038	.97	.664371	5.53	335629	13
48	.622682	4.56	.957979	.97	.664703	5.53	335297	12
49	.622956	4.55	.957921	.97	665035	5.53	334965	11
50	9.623229	4.55	9.957863	.97	9.665366	5.52	0.334634	10
51	.623502	4.54	.957804	.98	.665698	5.52	.334302	9
52	.623774	4.54	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.54	.957687	.98	666360	5.51	.333640	7
54	.624319	4.53	.957628	.98	666691	5.51	.333309	6
55	.624591	4.53	.957570	.98	667021	5.51	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.51	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.50	.332318	3
58	.625406	4.52	.957393	.98	.668013	5.50	.331987	2
59	625677	4.52	.957335	.98	.668313	5.50	.331657	1
60	.625948		.957276		.668673		.331327	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

25° 154°

M	Sine.	D. 1".	Cosine.	D 1".	Tang.	D. 1".	Cotang.	M.
0	9.625948	4.51	9.957276	.98	9.668673	5.50	0.331327	60
1	.626219	4.51	.957217	.98	.669002	5.49	.330998	59
2	.626490	4.51	.957158	.98	.669332	5.49	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.49	.330339	57
4	.627030	4.50	.957040	.99	.669991	5.49	.330009	56
5	.627300	4.50	.956981	.99	.670320	5.48	.329680	55
6	.627570	4.49	.956921	.99	.670649	5.48	.329351	54
7	.627840	4.49	.956862	.99	.670977	5.48	.329023	53
8	.628109	4.49	.956803	.99	.671306	5.47	.328694	52
9	.628378	4.48	.956744	99	.671635	5.47	.328365	51
10	9.628647	4.48	9.956684	.99	9.671963	5.47	0.328037	50
11	.628916	4.48	.956625	.99	.672291	5.47	.327709	49
12	.629185	4.47	.956566	.99	.672619	5.46	.327381	48
13	.629453	4.47	.956506	.99	.672947	5.46	.327053	47
14	.629721	4.47	.956447	.99	.673274	5.46	.326726	46
15	.629989	4.46	.956387	99	.673602	5.46	.326398	45
16	.630257	4.46	.956327	.99	.673929	5.45	.326071	44
17	.630524	4.46	956268	.99	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	1.00	.674911	5.46	.325089	41
20	9.631326	4.45	9.956089	1.00	9.675237	5.44	0.324763	40
21	.631593	4.44	.956029	1.00	.675564	5.44	.324436	39
22	.631859	4.44	.955969	1.00	.675890	5.44	.324110	38
23	.632 25	4.44	.955909	1.00	.676217	5.44	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.43	.955789	1.00	.676869	5 43	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.42	.322480	33
28	.633454	4.42	.955609	1.00	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	321829	31
30	9.633984	4.41	9.955488	1.00	9.678496	5.42	0.321504	30
31	.634249	4 41	.955428	1.01	.678821	5.41	.321179	29
32	.634514	4.41	.955368	1.01	.679146	5.41	.320854	28
33	.634778	4.40	.955307	1.01	.679471	5.41	.320529	27
34	.635042	4.40	.955247	1.01	.679795	5.41	.320205	26
35	.635306	4.40	.955186	1.01	.680120	5.40	.319880	25
36	.635570	4.39	.955126	1.01	.680444	5.40	.319556	24
37	.635834	4.39	.955065	1.01	.680768	5.40	.319232	23
38	.636097	4.39	.955005	1.01	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.01	.681416	5.39	.318584	21
40	9.636623	4.38	9.954883	1.01	9.681740	5.39	0.318260	20
41	.636886	4.38	.954823	1.01	.682063	5.39	.317937	19
42	.637148	4.37	.954762	1.01	.682387	5.39	.317613	18
43	.637411	4.37	.954701	1 01	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.36	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.36	.954518	1.02	.683679	5.38	.316321	14
47	.638458	4.36	.954457	1.02	.684001	5.37	315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	9.639242	4.35	9.954274	1.02	9.684968	5.37	0.315032	10
51	.639503	4.34	.954213	1.02	.685290	5.36	.314710	9
52	.639764	4.34	.954152	1.02	.685612	5.36	.314388	8
53	.640024	4.34	.954090	1.02	.685934	5.36	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.36	.313745	6
55	.640544	4.33	.953968	1.02	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.32	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.03	687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	687861	5.35	.312139	1
60	.641842		.953660		.688182		.311818	0
M	Cosine	D 1"	Sine	D. 1".	Cotang.	D. 1'.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.641842	4.32	9.953660	1.03	9.688182	5.34	0.311818	60
1	.642101	4.31	.953599	1.03	.688502	5.34	.311498	59
2	.642360	4.31	.953537	1.03	.688823	5.34	.311177	58
3	.642618	4.31	.953475	1.03	.689143	5.34	.310857	57
4	.642877	4.30	.953413	1.03	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.30	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.29	.953228	1 03	.690423	5.33	.309577	53
8	.643908	4.29	.953166	1.03	.690742	5.32	.309258	52
9	.644165	4.29	.953104	1.03	.691062	5.32	.308938	51
10	9.644423	4.28	9.953042	1.03	9.691381	5.32	0.308619	50
11	.644680	4.28	.952980	1.04	.691700	5.32	.308300	49
12	.644936	4.28	.952918	1.04	.692019	5.31	.307981	48
13	.645193	4.27	.952855	1.04	.692338	5.31	.307662	47
14	.645450	4.27	.952793	1.04	.692656	5.31	.307344	46
15	.645706	4.27	.952731	1.04	.692975	5.31	.307025	45
16	.645962	4.26	.952669	1.04	.693293	5.30	.306707	44
17	.646218	4.26	.952606	1.04	.693612	5.30	.306388	43
18	.646474	4.26	.952544	1.04	.693930	5.30	.306070	42
19	.646729	4.26	.952481	1.04	.694248	5.30	.305752	41
20	9.646984	4.25	9.952419	1.04	9.694566	5.29	0.305434	40
21	.647240	4.25	.952356	1.04	.694883	5.29	.305117	39
22	.647494	4.25	.952294	1.04	.695201	5.29	.304799	38
23	.647749	4.24	.952231	1.04	.695518	5.29	.304482	37
24	.648004	4.24	.952168	1.05	.695836	5.29	.304164	36
25	.648258	4.24	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	303530	34
27	.648766	4.23	.951980	1.05	.696787	5.28	.303213	33
28	.649020	4.23	.951917	1 05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	9.649527	4.22	9.951791	1.05	9.697736	5.27	0.302264	30
31	.649781	4.22	.951728	1.05	.698053	5.27	.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.21	.951602	1.05	.698685	5.26	.301315	27
34	.650539	4.21	.951539	1.05	.699001	5.26	.300999	26
35	.650792	4.21	.951476	1.05	.699316	5.26	.300684	25
36	.651044	4.20	.951412	1.05	.699632	5.26	.300368	24
37	.651297	4.20	.951349	1.06	.699947	5.26	.300053	23
38	.651549	4.20	.951286	1.06	.700263	5.25	.299737	22
39	.651800	4.19	.951222	1.06	.700578	5.25	.299422	21
40	9.652052	4.19	9.951159	1.06	9.700893	5.25	0.299107	20
41	.652304	4.19	.951096	1.06	.701208	5.25	.298792	19
42	.652555	4.18	.951032	1.06	.701523	5.24	.298477	18
43	.652806	4.18	.950968	1.06	.701837	5.24	.298163	17
44	.653057	4.18	.950905	1.06	.702152	5.24	.297848	16
45	.653308	4.18	.950841	1.06	.702466	5.24	.297534	15
46	.653558	4.17	.950778	1.06	.702781	5.24	.297219	14
47	.653808	4.17	.950714	1.06	.703095	5.23	.296905	13
48	654059	4.17	.950650	1.06	.703409	5.23	.296591	12
49	654309	4.16	.950586	1.06	.703722	5.23	.296278	11
50	9.654558	4.16	9.950522	1.07	9.704036	5.23	0.295964	10
51	.654808	4.16	.950458	1.07	.704350	5.22	.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.22	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.14	.950138	1.07	.705916	5.21	.294084	4
57	.656302	4.14	.950074	1.07	.706228	5.21	.293772	3
58	.656551	4.14	.950010	1.07	.706541	5.21	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.21	.293146	1
60	.657047		.949881		.707166		.292834	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	0.292834	60
1	.657295	4.13	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.12	.949752	1.07	707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.08	.708726	5.19	.291274	55
6	.658531	4.11	.949494	1.08	.709037	5.19	.290963	54
7	.658778	4.11	.949429	1.08	.709349	5.19	.290651	53
8	659025	4.11	.949364	1.08	.709660	5.19	.290340	52
9	.659271	4.10	.949300	1.08	709971	5.18	.290029	51
10	9.659517	4.10	9.949235	1.08	9.710282	5.18	0.289718	50
11	.659763	4.10	.949170	1.08	.710593	5.18	.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.09	.949040	1.08	.711215	5.18	.288785	47
14	.660501	4.09	.948975	1.08	.711525	5.17	.288475	46
15	.660746	4.09	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.09	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.09	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.09	.712766	5.17	.287234	42
19	.661726	4.08	.948650	1.09	.713076	5.16	.286924	41
20	9.661970	4.07	9.948584	1.09	9.713386	5.16	0.286614	40
21	.662214	4.07	.948519	1.09	.713696	5.16	.286304	39
22	.662459	4.07	.948454	1.09	.714005	5.16	.285995	38
23	.662703	4.06	.948388	1.09	.714314	5.15	.285686	37
24	.662946	4.06	.948323	1.09	.714624	5.15	.285376	36
25	.663190	4.06	.948257	1.09	.714933	5.15	.285067	35
26	.663433	4.05	.948192	1.09	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.09	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.09	.715860	5.14	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.14	.283832	31
30	9.664406	4.04	9.947929	1.10	9.716477	5.14	0.283523	30
31	.664648	4.04	.947863	1.10	.716785	5.14	.283215	29
32	.664891	4.04	.947797	1.10	.717093	5.14	.282907	28
33	.635133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.10	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.10	.718017	5.13	.281983	25
36	.665859	4.03	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.02	.947467	1.10	.718633	5.13	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.12	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	9.666824	4.01	9.947269	1.10	9.719555	5.12	0.280445	20
41	.667065	4.01	.947203	1.11	.719862	5.12	.280138	19
42	.667305	4.01	.947136	1.11	.720169	5.11	.279831	18
43	.667546	4.01	.947070	1.11	720476	5.11	.279524	17
44	.667786	4.00	.947004	1.11	.720783	5.11	.279217	16
45	.668027	4.00	.946937	1.11	.721089	5.11	.278911	15
46	.668267	4.00	.946871	1.11	.721396	5.11	.278604	14
47	.668506	3.99	.946804	1.11	.721702	5.10	.278298	13
48	.668746	3.99	.946738	1.11	.722009	5.10	.277991	12
49	.668986	3.99	.946671	1.11	.722315	5.10	.277685	11
50	9.669225	3.99	9.946604	1.11	9.722621	5.10	0.277379	10
51	.669464	3.98	.946538	1.11	.722927	5.10	.277073	9
52	669703	3.98	.946471	1.11	.723232	5.09	.276768	8
53	.669942	3.98	.946404	1.11	.723538	5.09	.276462	7
54	.670181	3.98	.946337	1.12	.723844	5.09	.276156	6
55	.670419	3.97	.946270	1.12	.724149	5.09	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.09	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.96	.946069	1.12	.725065	5.08	.274935	2
59	671372	3.96	.946002	1 12	.725370	5.08	.274630	1
60	.671609		.945935		.725674		.274326	0
M	Cosine.	D. 1″.	Sine.	D 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	0.274326	60
1	.671847	3.96	.945868	1.12	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.12	.726892	5.07	.273108	56
5	.672795	3.94	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.94	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.94	.945464	1.13	.727805	5.06	.272195	53
8	.673505	3.94	.945396	1.13	.728109	5.06	.271891	52
9	.673741	3.93	.945328	1.13	.728412	5.06	.271588	51
10	9.673977	3.93	9.945261	1.13	9.728716	5.06	0.271284	50
11	.674213	3.93	.945193	1.13	.729020	5.06	.270980	49
12	.674448	3.93	.945125	1.13	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.92	.944990	1.13	.729929	5.05	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.05	.269767	45
16	.675390	3.91	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.91	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.91	.944718	1.13	.731141	5.04	.268859	42
19	.676094	3.91	.944650	1.13	.731444	5.04	.268556	41
20	9.676328	3.90	9.944582	1.14	9.731746	5.04	0.268254	40
21	.676562	3.90	.944514	1.14	.732048	5.04	.267952	39
22	.676796	3.90	.944446	1.14	.732351	5.04	.267649	38
23	.677030	3.90	.944377	1.14	.732653	5.03	.267347	37
24	.677264	3.89	.944309	1.14	.732955	5.03	.267045	36
25	.677498	3.89	.944241	1.14	.733257	5.03	.266743	35
26	.677731	3.89	.944172	1.14	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.14	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.14	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.14	.734463	5.02	.265537	31
30	9.678663	3.88	9.943899	1.14	9.734764	5.02	0.265236	30
31	.678895	3.87	.943830	1.14	.735066	5.02	.264934	29
32	.679128	3.87	.943761	1.15	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.01	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.01	.264031	26
35	.679824	3.86	.943555	1.15	.736269	5.01	.263731	25
36	.680056	3.86	.943486	1.15	.736570	5.01	.263430	24
37	.680288	3.86	.943417	1.15	.736870	5.01	.263130	23
38	.680519	3.86	.943348	1.15	.737171	5.01	.262829	22
39	.680750	3.85	.943279	1.15	.737471	5.00	.262529	21
40	9.680982	3.85	9.943210	1.15	9.737771	5.00	0.262229	20
41	.681213	3.85	.943141	1.15	.738071	5.00	.261929	19
42	.681443	3.84	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.84	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.84	.942934	1.15	.738971	4.99	.261029	16
45	682135	3.84	.942864	1.16	.739271	4.99	.260729	15
46	.682365	3.83	.942795	1.16	.739570	4.99	.260430	14
47	.682595	3.83	.942726	1.16	.739870	4.99	.260130	13
48	.682825	3.83	.942656	1.16	.740169	4.99	.259831	12
49	.683055	3.83	.942587	1.16	.740468	4.98	.25953[illegible]	11
50	9.683284	3.82	9.942517	1.16	9.740767	4.98	0.259233	10
51	.683514	3.82	.942448	1.16	.741066	4.98	.258934	9
52	.683743	3.82	.942378	1.16	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.16	.741664	4.98	.258336	7
54	.684201	3.81	.942239	1.16	.741962	4.98	.258038	6
55	.684430	3.81	.942169	1.16	.742261	4.97	.257739	5
56	.684658	3.81	.942099	1.16	.742559	4.97	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	.685571		.941819		.743752		.256248	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	0.256248	60
1	685799	3.79	.941749	1.17	.744050	4.96	.255950	59
2	.686027	3.79	.941679	1.17	.744348	4.96	.255652	58
3	686254	3.79	.941609	1.17	.744645	4.96	.255355	57
4	.686482	3.79	.941539	1.17	.744943	4.96	.255057	56
5	.686709	3.78	.941469	1.17	.745240	4.96	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	687163	3.78	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.17	.746132	4.95	.253868	52
9	.687616	3.77	.941187	1.17	.746429	4.95	.253571	51
10	9.687843	3.77	9.941117	1.18	9.746726	4.95	0.253274	50
11	.688069	3.77	.941046	1.18	.747023	4.95	.252977	49
12	.688295	3.77	.940975	1.18	.747319	4.94	.252681	48
13	.688521	3.76	.940905	1.18	.747616	4.94	.252384	47
14	.688747	3.76	.940834	1.18	.747913	4.94	.252087	46
15	.688972	3.76	.940763	1.18	.748209	4.94	.251791	45
16	.689198	3.76	.940693	1.18	.748505	4.94	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	9.690098	3.75	9.940409	1.18	9.749689	4.93	0.250311	40
21	.690323	3.74	.940338	1.18	.749985	4.93	.250015	39
22	.690548	3.74	.940267	1.19	.750281	4.93	.249719	38
23	.690772	3.74	.940196	1.19	.750576	4.92	.249424	37
24	.690996	3.74	.940125	1.19	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.19	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.19	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.19	.751757	4.92	.248243	33
28	.691892	3.73	.939840	1.19	.752052	4.92	.247948	32
29	.692115	3.72	.939768	1.19	.752347	4.91	.247653	31
30	9.692339	3.72	9.939697	1.19	9.752642	4.91	0.247358	30
31	.692562	3.72	.939625	1.19	.752937	4.91	.247063	29
32	.692785	3.72	.939554	1.19	.753231	4.91	.246769	28
33	.693008	3.71	.939482	1.19	.753526	4.91	.246474	27
34	.693231	3.71	.939410	1.19	.753820	4.91	.246180	26
35	.693453	3.71	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.71	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.20	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	9.694564	3.70	9.938980	1.20	9.755585	4.89	0.244415	20
41	.694786	3.69	.938908	1.20	.755878	4.89	.244122	19
42	.695007	3.69	.938836	1.20	.756172	4.89	.243828	18
43	.695229	3.69	.938763	1.20	.756465	4.89	.243535	17
44	.695450	3.69	.938691	1.20	.756759	4.89	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.89	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.21	.757638	4.88	.242362	13
48	.696334	3.68	.938402	1.21	.757931	4.88	.242069	12
49	.696554	3.67	.938330	1.21	.758224	4.88	.241776	11
50	9.696775	3.67	9.938258	1.21	9.758517	4.88	0.241483	10
51	.696995	3.67	.938185	1.21	.758810	4.88	.241190	9
52	.697215	3.67	.938113	1.21	.759102	4.87	.240898	8
53	.697435	3.66	.938040	1.21	.759395	4.87	.240605	7
54	.697654	3.66	.937967	1.21	.759687	4.87	.240313	6
55	.697874	3.66	.937895	1.21	.759979	4.87	.240021	5
56	.698094	3 66	.937822	1.21	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.21	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.21	.760856	4.86	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.86	.238852	1
60	.698970		.937531		.761439		.238561	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

80° 149°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.698970	3.65	9.937531	1.22	9.761439	4.86	0.238561	60
1	.699189	3.64	.937458	1.22	.761731	4.86	.238269	59
2	.699407	3.64	.937385	1.22	.762023	4.86	.237977	58
3	.699626	3.64	.937312	1.22	.762314	4.86	.237686	57
4	.699844	3.64	.937238	1.22	.762606	4.86	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.63	.936946	1.22	.763770	4.85	.236230	52
9	.700933	3.62	.936872	1.22	.764061	4.85	.235939	51
10	9.701151	3.62	9.936799	1.22	9.764352	4.85	0.235648	50
11	.701368	3.62	.936725	1.23	.764643	4.84	.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.84	.235067	48
13	.701802	3.61	.936578	1.23	.765224	4.84	.234776	47
14	.702019	3.61	.936505	1.23	.765514	4.84	.234486	46
15	.702236	3.61	.936431	1.23	.765805	4.84	.234195	45
16	.702452	3.61	.936357	1.23	.766095	4.84	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	9.703317	3.60	9.936062	1.23	9.767255	4.83	0.232745	40
21	.703533	3.59	.935988	1.23	767545	4.83	.232455	39
22	.703749	3.59	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.59	.935840	1.23	768124	4.82	231876	37
24	.704179	3.59	.935766	1.24	768414	4.82	.231586	36
25	.704395	3.59	.935692	1.24	768703	4.82	.231297	35
26	.704610	3.58	.935618	1.24	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.24	.769281	4.82	.230719	33
28	.705040	3.58	.935469	1.24	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.24	.769860	4.82	.230140	31
30	9.705469	3.57	9.935320	1.24	9.770148	4.81	0.229852	30
31	.705683	3.57	.935246	1.24	.770437	4.81	.229563	29
32	.705898	3.57	.935171	1.24	.770726	4.81	.229274	28
33	.706112	3.57	.935097	1.24	.771015	4.81	.228985	27
34	.706326	3.56	.935022	1.24	.771303	4.81	.228697	26
35	.706539	3.56	.934948	1.24	.771592	4.81	.228408	25
36	.706753	3.56	.934873	1.24	.771880	4.80	.228120	24
37	.706967	3.56	.934798	1.25	.772168	4.80	.227832	23
38	.707180	3.55	.934723	1.25	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	9.707606	3.55	9.934574	1.25	9.773033	4.80	0.226967	20
41	.707819	3.55	.934499	1.25	.773321	4.80	.226679	19
42	.708032	3.54	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.54	.934349	1.25	.773896	4.79	.226104	17
44	.708458	3.54	.934274	1.25	.774184	4.79	.225816	16
45	.708670	3.54	.934199	1.25	.774471	4.79	.225529	15
46	.708882	3.54	.934123	1.25	.774759	4.79	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.79	.224954	13
48	.709306	3.53	.933973	1.26	.775333	4.79	.224667	12
49	.709518	3.53	.933898	1.26	.775621	4.78	.224379	11
50	9.709730	3.53	9.933822	1.26	9.775908	4.78	0.224092	10
51	.709941	3.52	.933747	1.26	.776195	4.78	.223805	9
52	.710153	3.52	.933671	1.26	.776482	4.78	.223518	8
53	.710364	3.52	.933596	1.26	.776768	4.78	.223232	7
54	.710575	3.52	933520	1.26	.777055	4.78	.222945	6
55	.710786	3.51	933445	1.26	.777342	4.77	.222658	5
56	.710997	3.51	933369	1.26	.777628	4.77	.222372	4
57	.711208	3.51	933293	1.26	.777915	4.77	.222085	3
58	.711419	3.51	933217	1.26	.778201	4.77	.221799	2
59	.711629	3.51	.933141	1.26	.778488	4.77	.221512	1
60	.711839		.933066		.778774		.221226	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

120° 59°

31° | 148°

M.	Sine.	D. 1″	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang	M.
0	9.711839	3.50	9.933066	1.27	9.778774	4.77	0.221226	60
1	.712050	3.50	932990	1.27	.779060	4.77	.220940	59
2	.712260	3.50	932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.76	.220368	57
4	.712679	3.49	.932762	1.27	.779918	4.76	.220082	56
5	.712889	3.49	.932685	1.27	.780203	4.76	.219797	55
6	.713098	3.49	.932609	1.27	.780489	4.76	.219511	54
7	.713308	3.49	.932533	1.27	.780775	4.76	.219225	53
8	.713517	3.48	.932457	1.27	.781060	4.76	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.76	.218654	51
10	9.713935	3.48	9.932304	1.27	9.781631	4.75	0.218369	50
11	.714144	3.48	.932228	1.27	.781916	4.75	218084	49
12	.714352	3.48	.932151	1.28	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.47	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.28	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.46	.931768	1.28	.783626	4.74	.216374	43
18	.715602	3.46	.931691	1.28	.783910	4.74	.216090	42
19	.715809	3.46	.931614	1.28	.784195	4.74	.215805	41
20	9.716017	3.46	9.931537	1.28	9 784479	4.74	0.215521	40
21	.716224	3.46	.931460	1.28	784764	4.74	.215236	39
22	.716432	3.45	.931383	1.28	785048	4.74	.214952	38
23	.716639	3.45	931306	1.28	.785332	4.74	.214668	37
24	716846	3.45	.931229	1.29	.785616	4.73	.214384	36
25	.717053	3.45	931152	1.29	785900	4.73	.214100	35
26	717259	3.44	931075	1.29	.786184	4.73	213816	34
27	.717466	3.44	.930998	1.29	.786468	4.73	.213532	33
28	.717673	3.44	.930921	1.29	.786752	4.73	.213248	32
29	.717879	3.44	.930843	1.29	.787036	4.73	.212964	31
30	9.718085	3.43	9.930766	1.29	9.787319	4.73	0.212681	30
31	.718291	3.43	.930688	1.29	.787603	4.72	.212397	29
32	.718497	3.43	.930611	1.29	.787886	4.72	.212114	28
33	.718703	3.43	.930533	1.29	.788170	4.72	.211830	27
34	.718909	3.43	.930456	1.29	.788453	4.72	.211547	26
35	.719114	3.42	.930378	1.29	.788736	4.72	211264	25
36	.719320	3.42	.930300	1.30	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3 42	.930145	1.30	.789585	4.71	.210415	22
39	.719935	3.41	.930067	1.30	.789868	4.71	.210132	21
40	9.720140	3.41	9.929989	1.30	9.790151	4.71	0.209849	20
41	.720345	3.41	.929911	1.30	.790434	4.71	.209566	19
42	.720549	3.41	.929833	1.30	.790716	4.71	.209284	18
43	.720754	3.41	.929755	1.30	.790999	4.71	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.71	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.70	.208437	15
46	.721366	3.40	.929521	1.30	.791846	4.70	.208154	14
47	.721570	3 40	.929442	1.31	.792128	4.70	.207872	13
48	.721774	3.39	.929364	1.31	.792410	4.70	.207590	12
49	.721978	3.39	.929286	1.31	.792692	4.70	.207308	11
50	9.722181	3.39	9.929207	1.31	9.792974	4.70	0.207026	10
51	.722385	3.39	.929129	1.31	.793256	4.70	.206744	9
52	.722588	3.39	.929050	1.31	.793538	4.70	.206462	8
53	.722791	3.38	.928972	1.31	.793819	4.69	.206181	7
54	.722994	3.38	.928893	1.31	.794101	4.69	.205899	6
55	.723197	3.38	.928815	1.31	.794383	4.69	.205617	5
56	.723400	3.38	.928736	1.31	.794664	4.69	.205336	4
57	.723603	3.37	.928657	1.31	794946	4.69	.205054	3
58	.723805	3.37	.928578	1.31	.795227	4.69	.204773	2
59	.724007	3.27	.928499	1.32	.795508	4 69	.204492	1
60	.724210		.928420		.795789		.204211	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang	M

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	0.204211	60
1	.724412	3.37	.928342	1.32	.796070	4 68	.203930	59
2	.724614	3.36	.928263	1.32	.796351	4.68	.203649	58
3	.724816	3.36	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.36	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.36	.928025	1.32	.797194	4.68	.202806	55
6	.725420	3.36	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.32	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	9.726225	3.35	9.927629	1.32	9.798596	4.67	0.201404	50
11	.726426	3.34	.927549	1.33	.798877	4.67	.201123	49
12	.726626	3.34	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.34	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.34	.927310	1.33	.799717	4.67	.200283	46
15	.727228	3.34	.927231	1.33	.799997	4.66	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.66	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.66	.199443	43
18	.727828	3.33	.926991	1.33	.800836	4.66	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.66	.198884	41
20	9.728227	3.33	9.926831	1.33	9.801396	4.66	0.198604	40
21	.728427	3.32	.926751	1.33	.801675	4.66	.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.66	.198045	38
23	728825	3.32	.926591	1.34	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.34	.802513	4.65	.197487	36
25	.729223	3.31	.926431	1.34	.802792	4.65	.197208	35
26	.729422	3.31	.926351	1.34	.803072	4.65	.196928	34
27	.729621	3.31	.926270	1.34	.803351	4.65	.196649	33
28	.729820	3.31	.926190	1.34	.803630	4.65	.196370	32
29	.730018	3.31	.926110	1.34	.803909	4.65	.196091	31
30	9.730217	3.30	9.926029	1.34	9.804187	4.65	0.195813	30
31	.730415	3.30	.925949	1.34	.804466	4.64	.195534	29
32	.730613	3.30	925868	1.34	.804745	4.64	.195255	28
33	.730811	3.30	.925788	1.34	.805023	4.64	.194977	27
34	.731009	3.30	.925707	1.35	.805302	4.64	.194698	26
35	.731206	3.29	.925626	1.35	.805580	4.64	.194420	25
36	.731404	3.29	.925545	1.35	.805859	4.64	.194141	24
37	.731602	3.29	.925465	1.35	.806137	4.64	.193863	23
38	.731799	3.29	.925384	1.35	.806415	4.64	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	9.732193	3.28	9.925222	1.36	9.806971	4.63	0.193029	20
41	.732390	3.28	.925141	1.35	.807249	4.63	.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.28	.924979	1.35	.807805	4.63	.192195	17
44	.732980	3.27	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.63	.191639	15
46	.733373	3.27	.924735	1.36	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.36	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.36	.809193	4.62	.190807	12
49	.733961	3.26	.924491	1.36	.809471	4.62	.190529	11
50	9.734157	3.26	9.924409	1.36	9.809748	4.62	0.190252	10
51	.734353	3.26	.924328	1.36	.810025	4.62	.189975	9
52	.734549	3.26	.924246	1.36	.810302	4.62	.189698	8
53	.734744	3.26	.924164	1.36	.810580	4.62	.189420	7
54	.734939	3.25	.924083	1.36	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.36	.811134	4.61	.188866	5
56	.735330	3.25	.923919	1.36	.811410	4.61	.188590	4
57	.735525	3.25	.923837	1.37	.811687	4.61	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.61	.188036	2
59	.735914	3.24	.923673	1.37	.812241	4.61	187759	1
60	.736109		.923591		.812517		.187483	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang	D. 1"	Tang	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	0.187483	60
1	.736303	3.24	.923509	1.37	.812794	4.61	.187206	59
2	.736498	3.24	.923427	1.37	.813070	4.61	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.61	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.37	.813899	4.60	.186101	55
6	.737274	3.23	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.37	.814452	4.60	.185548	53
8	.737661	3.22	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	9.738048	3.22	9.922768	1.38	9.815280	4.60	0.184720	50
11	.738241	3.22	.922686	1.38	.815555	4.60	.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.59	.184169	48
13	.738627	3.21	.922520	1.38	.816107	4.59	.183893	47
14	.738820	3.21	.922438	1.38	.816382	4.59	.183618	46
15	.739013	3.21	.922355	1.38	.816658	4.59	.183342	45
16	.739206	3.21	.922272	1.38	.816933	4.59	.183067	44
17	.739398	3.21	.922189	1.38	.817209	4.59	.182791	43
18	.739590	3.20	.922106	1.38	.817484	4.59	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.59	.182241	41
20	9.739975	3.20	9.921940	1.39	9.818035	4.59	0.181965	40
21	.740167	3.20	.921857	1.39	.818310	4.58	.181690	39
22	.740359	3.20	.921774	1.39	.818585	4.58	.181415	38
23	.740550	3.19	.921691	1.39	.818860	4.58	.181140	37
24	.740742	3.19	.921607	1.39	.819135	4.58	.180865	36
25	.740934	3.19	.921524	1.39	.819410	4.58	.180590	35
26	.741125	3.19	.921441	1.39	.819684	4.58	.180316	34
27	.741316	3.19	.921357	1.39	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.39	.820234	4.58	.179766	32
29	.741699	3.18	.921190	1.39	.820508	4.58	.179492	31
30	9.741889	3.18	9.921107	1.39	9.820783	4·57	0.179217	30
31	.742080	3.18	.921023	1.39	.821057	4.57	.178943	29
32	.742271	3.18	.920939	1.40	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.17	.920688	1.40	.822154	4.57	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.16	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.16	.920352	1.40	.823251	4.56	.176749	21
40	9.743792	3.16	9.920268	1.40	9.823524	4.56	0.176476	20
41	.743982	3.16	.920184	1.40	.823798	4.56	.176202	19
42	.744171	3.16	.920099	1.40	.824072	4.56	.175928	18
43	.744361	3.15	.920015	1.41	.824345	4.56	.175655	17
44	.744550	3.15	.919931	1.41	.824619	4.56	.175381	16
45	.744739	3.15	.919846	1.41	.824893	4.56	.175107	15
46	.744928	3.15	.919762	1.41	.825166	4.56	.174834	14
47	.745117	3.15	.919677	1.41	.825439	4.56	.174561	13
48	.745306	3.14	.919593	1.41	.825713	4.55	.174287	12
49	.745494	3.14	.919508	1.41	.825986	4.55	.174014	11
50	9.745683	3.14	9.919424	1.41	9.826259	4.55	0.173741	10
51	.745871	3.14	.919339	1.41	.826532	4.55	.173468	9
52	.746060	3.14	.919254	1.41	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.41	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.13	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.54	.171830	3
58	.747187	3.12	.918745	1.42	.828442	4.54	.171558	2
59	.747374	3.12	.918659	1.42	.828715	4.54	.171285	1
60	.747562		.918574		.828987		.171013	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	0.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.54	.170740	59
2	.747936	3.12	.918404	1.42	.829532	4.54	.170468	58
3	.748123	3.11	.918318	1.42	.829805	4.54	.170195	57
4	.748310	3.11	.918233	1.42	.830077	4.54	.169923	56
5	.748497	3.11	.918147	1.43	.830349	4.54	.169651	55
6	.748683	3.11	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.11	.917976	1.43	.830893	4.53	.169107	53
8	.749056	3.10	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	9.749429	3.10	9.917719	1.43	9.831709	4.53	0.168291	50
11	.749615	3.10	.917634	1.43	.831981	4.53	.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.10	.917462	1.43	.832525	4.53	.167475	47
14	.750172	3.09	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.09	.917290	1.43	.833068	4.53	.166932	45
16	.750543	3.09	.917204	1.43	.833339	4.53	.166661	44
17	.750729	3.09	.917118	1.44	.833611	4.52	.166389	43
18	.750914	3.09	.917032	1.44	.833882	4.52	.166118	42
19	.751099	3.08	.916946	1.44	.834154	4.52	.165846	41
20	9.751284	3.08	9.916859	1.44	9.834425	4.52	0.165575	40
21	.751469	3.08	.916773	1.44	.834696	4.52	.165304	39
22	.751654	3.08	.916687	1.44	.834967	4.52	.165033	38
23	.751839	3.08	.916600	1.44	.835238	4.52	.164762	37
24	.752023	3.07	.916514	1.44	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.44	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.44	.836051	4.51	.163949	34
27	.752576	3.07	.916254	1.44	.836322	4.51	.163678	33
28	752760	3.07	.916167	1.45	.836593	4.51	.163407	32
29	.752944	3.06	.916081	1.45	.836864	4.51	.163136	31
30	9.753128	3.06	9.915994	1.45	9.837134	4.51	0.162866	30
31	.753312	3.06	.915907	1.45	.837405	4.51	.162595	29
32	.753495	3.06	.915820	1.45	.837675	4.51	.162325	28
33	.753679	3.06	.915733	1.45	.837946	4.51	.162054	27
34	.753862	3.05	.915646	1.45	.838216	4.51	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.51	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.45	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.50	.160703	22
39	.754778	3.05	.915210	1.46	.839568	4.50	.160432	21
40	9.754960	3.04	9.915123	1.46	9.839838	4.50	0.160162	20
41	.755143	3.04	.915035	1.46	.840108	4.50	.159892	19
42	.755326	3.04	.914948	1.46	.840378	4.50	.159622	18
43	.755508	3.04	.914860	1.46	.840648	4.50	.159352	17
44	.755690	3.04	.914773	1.46	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.46	.841187	4.49	.158813	15
46	.756054	3.03	.914598	1.46	.841457	4.49	.158543	14
47	.756236	3.03	.914510	1.46	.841727	4.49	.158273	13
48	.756418	3.03	.914422	1.46	.841996	4.49	.158004	12
49	.756600	3.03	.914334	1.46	.842266	4.49	.157734	11
50	9.756782	3.02	9.914246	1.47	9.842535	4.49	0.157465	10
51	.756963	3.02	.914158	1.47	.842805	4.49	.157195	9
52	.757144	3.02	.914070	1.47	.843074	4.49	.156926	8
53	.757326	3.02	.913982	1.47	.843343	4 49	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.49	.156388	6
55	.757688	3.02	.913806	1.47	.843882	4.49	.156118	5
56	.757869	3.01	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.01	913630	1.47	.844420	4.48	.155580	3
58	.758230	3.01	.913541	1.47	.844689	4.48	.155311	2
59	.758411	3.01	.913453	1.47	.844958	4.48	.155042	1
60	.758591		.913365		.845227		.154773	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang	D. 1″.	Tang.	M.

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	0.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.48	.154504	59
2	.758952	3.00	.913187	1.48	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.48	.846302	4.48	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	2.99	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.99	.912744	1.48	.847108	4.47	.152892	53
8	.760031	2.99	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.99	.912566	1.48	.847644	4.47	.152356	51
10	9.760390	2.99	9.912477	1.48	9.847913	4.47	0.152087	50
11	.760569	2.99	.912388	1.48	.848181	4.47	.151819	49
12	.760748	2.98	.912299	1.49	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.49	.848717	4.47	.151283	47
14	.761106	2.98	.912121	1.49	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.49	.849254	4.47	.150746	45
16	.761464	2.98	.911942	1.49	.849522	4.47	.150478	44
17	.761642	2.97	.911853	1.49	.849790	4.46	.150210	43
18	.761821	2.97	.911763	1.49	.850057	4.46	.149943	42
19	.761999	2.97	.911674	1.49	.850325	4.46	.149675	41
20	9.762177	2.97	9.911584	1.49	9.850593	4.46	0.149407	40
21	.762356	2.97	.911495	1.49	.850861	4.46	.149139	39
22	.762534	2.97	.911405	1.49	.851129	4.46	.148871	38
23	.762712	2.96	.911315	1.50	.851396	4.46	.148604	37
24	.762889	2.96	.911226	1.50	.851664	4.46	.148336	36
25	.763067	2.96	.911136	1.50	.851931	4.46	.148069	35
26	.763245	2.96	.911046	1.50	.852199	4.46	.147801	34
27	.763422	2.96	.910956	1.50	.852466	4.46	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.46	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	9.763954	2.95	9.910686	1.50	9.853268	4.45	0.146732	30
31	.764131	2.95	.910596	1.50	.853535	4.45	.146465	29
32	.764308	2.95	.910506	1.50	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.51	.854069	4.45	.145931	27
34	.764662	2.94	.910325	1.51	.854336	4.45	.145664	26
35	.764838	2.94	.910235	1.51	.854603	4.45	.145397	25
36	.765015	2.94	.910144	1.51	.854870	4.45	.145130	24
37	.765191	2.94	.910054	1.51	.855137	4.45	.144863	23
38	.765367	2.94	.909963	1.51	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.51	.855671	4.44	.144329	21
40	9.765720	2.93	9.909782	1.51	9.855938	4.44	0.144062	20
41	.765896	2.93	.909691	1.51	.856204	4.44	.143796	19
42	.766072	2.93	.909601	1.51	.856471	4.44	.143529	18
43	.766247	2.93	.909510	1.51	.856737	4.44	.143263	17
44	.766423	2.93	.909419	1.52	.857004	4.44	.142996	16
45	.766598	2.92	.909328	1.52	.857270	4.44	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.44	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.44	.142197	13
48	.767124	2.92	.909055	1.52	.858069	4.44	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.44	.141664	11
50	9.767475	2.91	9.908873	1.52	9.858602	4.44	0.141398	10
51	.767649	2.91	.908781	1.52	.858868	4.43	.141132	9
52	.767824	2.91	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.91	.908599	1.52	.859400	4.43	.140600	7
54	.768173	2.91	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.91	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.90	.908324	1.53	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.43	.139270	2
59	.769045	2.90	.908049	1.53	.860995	4.43	.139005	1
60	.769219		.907958		.861261		.138739	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	0.138739	60
1	.769393	2.90	.907866	1.53	.861527	4.43	.138473	59
2	.769566	2.89	.907774	1.53	.861792	4.43	.138208	58
3	.769740	2.89	.907682	1.53	.862058	4.42	.137942	57
4	.769913	2.89	.907590	1.53	.862323	4.42	.137677	56
5	.770087	2.89	.907498	1.53	.862589	4.42	.137411	55
6	.770260	2.89	.907406	1.54	.862854	4.42	.137146	54
7	.770433	2.88	.907314	1.54	.863119	4.42	.136881	53
8	.770606	2.88	.907222	1.54	.863385	4.42	.136615	52
9	.770779	2.88	.907129	1.54	.863650	4.42	.136350	51
10	9.770952	2.88	9.907037	1.54	9.863915	4.42	0.136085	50
11	.771125	2.88	.906945	1.54	.864180	4.42	.135820	49
12	.771298	2.88	.906852	1.54	.864445	4.42	.135555	48
13	.771470	2.87	.906760	1.54	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.54	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.54	.865240	4.41	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.41	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.41	.134230	43
18	.772331	2.87	.906296	1.55	.866035	4.41	.133965	42
19	.772503	2.86	.906204	1.55	.866300	4.41	.133700	41
20	9.772675	2.86	9.906111	1.55	9.866564	4.41	0.133436	40
21	.772847	2.86	.906018	1.55	.866829	4.41	.133171	39
22	.773018	2.86	.905925	1.55	.867094	4.41	.132906	38
23	.773190	2.86	.905832	1.55	.867358	4.41	.132642	37
24	.773361	2.85	.905739	1.55	.867623	4.41	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.41	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.41	.131848	34
27	.773875	2.85	.905459	1.56	.868416	4.41	.131584	33
28	.774046	2.85	.905366	1.56	.868680	4.40	.131320	32
29	.774217	2.85	.905272	1.56	.868945	4.40	.131055	31
30	9.774388	2.84	9.905179	1.56	9.869209	4.40	0.130791	30
31	.774558	2.84	.905085	1.56	.869473	4.40	.130527	29
32	.774729	2.84	.904992	1.56	.869737	4.40	.130263	28
33	.774899	2.84	.904898	1.56	.870001	4.40	.129999	27
34	.775070	2.84	.904804	1.56	.870265	4.40	.129735	26
35	.775240	2.84	.904711	1.56	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.56	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	9.776090	2.83	9.904241	1.57	9.871849	4.40	0.128151	20
41	.776259	2.83	.904147	1.57	.872112	4.39	.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.39	.127624	18
43	.776598	2.82	.903959	1.57	.872640	4.39	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.39	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.39	.126833	15
46	.777106	2.82	.903676	1.57	.873430	4.39	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.39	.126306	13
48	.777444	2.81	.903487	1.58	.873957	4.39	.126043	12
49	.777613	2.81	.903392	1.58	.874220	4.39	.125780	11
50	9.777781	2.81	9.903298	1.58	9.874484	4.39	0.125516	10
51	.777950	2.81	.903203	1.58	.874747	4.39	.125253	9
52	.778119	2.81	.903108	1.58	.875010	4.39	.124990	8
53	.778287	2.81	.903014	1.58	.875273	4.39	.124727	7
54	.778455	2.80	.902919	1.58	.875537	4.38	.124463	6
55	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
56	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
57	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
58	.779128	2.80	.902539	1.59	.876589	4.38	.123411	2
59	.779295	2.79	.902444	1.59	.876852	4.38	.123148	1
60	.779463		.902349		.877114		.122886	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

37° 142°

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.779463	2.79	9.902349	1.59	9.877114	4.38	0.122886	60
1	.779631	2.79	.902253	1.59	.877377	4.38	.122623	59
2	.779798	2.79	.902158	1.59	.877640	4.38	.122360	58
3	.779966	2.79	.902063	1.59	.877903	4.38	.122097	57
4	.780133	2.79	.901967	1.59	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.59	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.59	.878691	4.38	.121309	54
7	.780634	2.78	.901681	1.59	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.59	.879216	4.37	.120784	52
9	.780968	2.78	.901490	1.60	.879478	4.37	.120522	51
10	9.781134	2.78	9.901394	1.60	9.879741	4.37	0.120259	50
11	.781301	2.77	.901298	1.60	.880003	4.37	.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.37	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	45
16	.782132	2.77	.900818	1.60	881314	4.37	.118686	44
17	.782298	2.76	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.76	.900626	1.60	.881839	4.37	.118161	42
19	.782630	2.76	.900529	1.61	.882101	4.37	.117899	41
20	9.782796	2.76	9.900433	1.61	9.882363	4.37	0.117637	40
21	.782961	2.76	.900337	1.61	.882625	4.37	.117375	39
22	.783127	2.76	.900240	1.61	.882887	4.36	.117113	38
23	.783292	2.75	.900144	1.61	.883148	4.36	.116852	37
24	.783458	2.75	.900047	1.61	.883410	4.36	.116590	36
25	.783623	2.75	.899951	1.61	.883672	4.36	.116328	35
26	.783788	2.75	.899854	1.61	.883934	4.36	.116066	34
27	.783953	2.75	.899757	1.61	.884196	4.36	.115804	33
28	.784118	2.75	.899660	1.61	.884457	4.36	.115543	32
29	.784282	2.74	.899564	1.62	.884719	4.36	.115281	31
30	9.784447	2.74	9.899467	1.62	9.884980	4.36	0.115020	30
31	.784612	2.74	.899370	1.62	.885242	4.36	.114758	29
32	.784776	2.74	.899273	1.62	.885504	4.36	.114496	28
33	.784941	2.74	.899176	1.62	.885765	4.36	.114235	27
34	.785105	2.74	.899078	1.62	.886026	4.36	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.36	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.36	.113451	24
37	.785597	2.73	.898787	1.62	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.62	.887333	4.35	.112667	21
40	9 786089	2.73	9.898494	1.63	9.887594	4.35	0.112406	20
41	.786252	2.73	.898397	1.63	.887855	4.35	.112145	19
42	.786416	2.72	.898299	1.63	.888116	4.35	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.72	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.35	.110839	14
47	.737232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.71	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.71	.897614	1.63	.889943	4.35	.110057	11
50	9.787720	2.71	9.897516	1.64	9.890204	4.35	0.109796	10
51	.787883	2.71	.897418	1.64	.890465	4.35	.109535	9
52	.788045	2.71	.897320	1.64	.890725	4.34	.109275	8
53	.788208	2.71	.897222	1.64	.890986	4.34	.109014	7
54	.788370	2.70	.897123	1.64	891247	4.34	.108753	6
55	.788532	2.70	.897025	1.64	.891507	4.34	.108493	5
56	788694	2.70	.896926	1.64	891768	4.34	.108232	4
57	.788856	2.70	.896828	1.64	892028	4.34	.107972	3
58	789018	2.70	.896729	1.64	.892289	4.34	.107711	2
59	.789180	2.70	.896631	1.64	.892549	4.34	.107451	1
60	.789342		.896532		.892810		.107190	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang	D. 1".	Tang.	M.

127° 52°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.789342	2.69	9.896532	1.65	9.892810	4.34	0.107190	60
1	.789504	2.69	.896433	1.65	.893070	4.34	.106930	59
2	.789665	2.69	.896335	1.65	.893331	4.34	.106669	58
3	.789827	2.69	.896236	1.65	.893591	4.34	.106409	57
4	.789988	2.69	.896137	1.65	.893851	4.34	.106149	56
5	.790149	2.69	.896038	1.65	.894111	4.34	.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.34	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.34	.105368	53
8	.790632	2.68	.895741	1.65	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	9.790954	2.68	9.895542	1.66	9.895412	4.33	0.104588	50
11	.791115	2.68	.895443	1.66	.895672	4.33	.104328	49
12	.791275	2.67	.895343	1.66	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.66	.896192	4.33	.103808	47
14	.791596	2.67	.895145	1.66	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.66	.896712	4.33	.103288	45
16	.791917	2.67	.894945	1.66	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.66	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.66	.897491	4.33	.102509	42
19	.792397	2.66	.894646	1.66	.897751	4.33	.102249	41
20	9.792557	2.66	9.894546	1.67	9.898010	4.33	0.101990	40
21	.792716	2.66	.894446	1.67	.898270	4.33	.101730	39
22	.792876	2.66	.894346	1.67	.898530	4.33	.101470	38
23	.793035	2.66	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.66	.894146	1.67	.899049	4.33	.100951	36
25	.793354	2.65	.894046	1.67	.899308	4.33	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.67	.899827	4.32	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.67	.900346	4.32	.099654	31
30	9.794150	2.65	9.893544	1.68	9.900605	4.32	0.099395	30
31	.794308	2.64	.893414	1.68	.900864	4.32	.099136	29
32	.794467	2.64	.893343	1.68	.901124	4.32	.098876	28
33	.794626	2.64	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.64	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.64	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.64	.892940	1.68	.902160	4.32	.097840	24
37	.795259	2.64	.892839	1.68	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.68	.902938	4.32	.097062	21
40	9.795733	2.63	9.892536	1.69	9.903197	4.32	0.096803	20
41	.795891	2.63	.892435	1.69	.903456	4.32	.096544	19
42	.796049	2.63	.892334	1.69	.903714	4.31	.096286	18
43	.796206	2.63	.892233	1.69	903973	4.31	.096027	17
44	.796364	2.62	.892132	1.69	.904232	4.31	.095768	16
45	.796521	2.62	.892030	1.69	.904491	4.31	.095509	15
46	.796679	2.62	.891929	1.69	.904750	4.31	.095250	14
47	.796836	2.62	.891827	1.69	.905008	4.31	.094992	13
48	.796993	2.62	.891726	1.69	.905267	4.31	.094733	12
49	.797150	2.61	.891624	1.69	.905526	4.31	.094474	11
50	9.797307	2.61	9.891523	1.70	9.905785	4.31	0.094215	10
51	.797464	2.61	.891421	1.70	.906043	4.31	.093957	9
52	.797621	2.61	.891319	1.70	.906302	4.31	.093698	8
53	.797777	2.61	.891217	1.70	.906560	4.31	.093440	7
54	.797934	2.61	.891115	1.70	.906819	4.31	.093181	6
55	.798091	2.61	.891013	1.70	.907077	4.31	.092923	5
56	.798247	2.61	.890911	1.70	.907336	4.31	.092664	4
57	.798403	2.60	.890809	1.70	.907594	4.31	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.31	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.31	.091889	1
60	.798872		.890503		.908369		.091631	0
M.	Cosine.	D. 1	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

39° **140°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.798872	2.60	9.890503	1.71	9.908369	4.30	0.091631	60
1	.799028	2.60	.890400	1.71	.908628	4.30	.091372	59
2	.799184	2.60	.890298	1.71	.908886	4.30	.091114	58
3	.799339	2.59	.890195	1.71	.909144	4.30	.090856	57
4	.799495	2.59	.890093	1.71	.909402	4.30	.090598	56
5	.799651	2.59	.889990	1.71	.909660	4.30	.090340	55
6	.799806	2.59	.889888	1.71	.909918	4.30	.090082	54
7	.799962	2.59	.889785	1.71	.910177	4.30	.089823	53
8	.800117	2.59	.889682	1.71	.910435	4.30	.089565	52
9	.800272	2.59	.889579	1.71	.910693	4.30	.089307	51
10	9.800427	2.58	9.889477	1.72	9.910951	4.30	0.089049	50
11	.800582	2.58	.889374	1.72	.911209	4.30	.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.72	.911725	4.30	.088275	47
14	.801047	2.58	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.57	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.72	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.30	.086986	42
19	.801819	2.57	.888548	1.72	.913271	4.30	.086729	41
20	9.801973	2.57	9.888444	1.73	9.913529	4.29	0.086471	40
21	.802128	2.57	.888341	1.73	.913787	4.29	.086213	39
22	.802282	2.57	.888237	1.73	.914044	4.29	.085956	38
23	.802436	2.56	.888134	1.73	.914302	4.29	.085698	37
24	.802589	2.56	.888030	1.73	.914560	4.29	.085440	36
25	.802743	2.56	.887926	1.73	.914817	4.29	.085183	35
26	.802897	2.56	.887822	1.73	.915075	4.29	.084925	34
27	.803050	2.56	.887718	1.73	.915332	4.29	.084668	33
28	.803204	2.56	.887614	1.73	.915590	4.29	.084410	32
29	.803357	2.55	.887510	1.74	.915847	4.29	.084153	31
30	9.803511	2.55	9.887406	1.74	9.916104	4.29	0.083896	30
31	.803664	2.55	.887302	1.74	.916362	4.29	.083638	29
32	.803817	2.55	.887198	1.74	.916619	4.29	.083381	28
33	.803970	2.55	.887093	1.74	.916877	4.29	.083123	27
34	.804123	2.55	.886989	1.74	.917134	4.29	.082866	26
35	.804276	2.55	.886885	1.74	.917391	4.29	.082609	25
36	.804428	2.54	.886780	1.74	.917648	4.29	.082352	24
37	.804581	2.54	.886676	1.74	.917906	4.29	.082094	23
38	.804734	2.54	.886571	1.74	.918163	4.29	.081837	22
39	.804886	2.54	.886466	1.75	.918420	4.29	.081580	21
40	9.805039	2.54	9.886362	1.75	9.918677	4.28	0.081323	20
41	.805191	2.54	.886257	1.75	.918934	4.28	.081066	19
42	.805343	2.54	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.53	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.75	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.76	.920990	4.28	.079010	11
50	9.806557	2.52	9.885311	1.76	9.921247	4.28	0.078753	10
51	.806709	2.52	.885205	1.76	.921503	4.28	.078497	9
52	.806860	2.52	.885100	1.76	.921760	4.28	.078240	8
53	.807011	2.52	.884994	1.76	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.76	.922274	4.28	.077726	6
55	.807314	2.52	.884783	1.76	.922530	4.28	.077470	5
56	.807465	2.51	.884677	1.76	.922787	4.28	.077213	4
57	.807615	2.51	.884572	1.76	.923044	4.28	.076956	3
58	.807766	2.51	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.51	.884360	1.77	.923557	4.28	.076443	1
60	.808067		.884254		.923814		.076186	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang	D. 1″.	Tang	M.

129° **50°**

40° | 139°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.808067	2.51	9.884254	1.77	9.923814	4.28	0.076186	60
1	.808218	2.51	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.51	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.77	.924583	4.27	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.77	.925352	4.27	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.27	.074135	52
9	.809419	2.50	.883297	1.78	.926122	4.27	.073878	51
10	9.809569	2.49	9.883191	1.78	9.926378	4.27	0.073622	50
11	.809718	2.49	.883084	1.78	.926634	4.27	.073366	49
12	.809868	2.49	.882977	1.78	.926890	4.27	.073110	48
13	.810017	2.49	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.49	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.49	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.79	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.79	.928427	4.27	.071573	42
19	.810912	2.48	.882229	1.79	.928684	4.27	.071316	41
20	9.811061	2.48	9.882121	1.79	9.928940	4.27	0.071060	40
21	.811210	2.48	.882014	1.79	.929196	4.27	.070804	39
22	.811358	2.48	.881907	1.79	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.79	.929708	4.27	.070292	37
24	.811655	2.47	.881692	1.79	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.79	.930220	4.27	.069780	35
26	.811952	2.47	.881477	1.79	.930475	4.26	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.26	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.26	.069013	32
29	.812396	2.47	.881153	1.80	.931243	4.26	.068757	31
30	9.812544	2.46	9.881046	1.80	9.931499	4.26	0.068501	30
31	.812692	2.46	.880938	1.80	.931755	4.26	.068245	29
32	.812840	2.46	.880830	1.80	.932010	4.26	.067990	28
33	.812988	2.46	.880722	1.80	.932266	4.26	.067734	27
34	.813135	2.46	.880613	1.80	.932522	4.26	.067478	26
35	.813283	2.46	.880505	1.80	.932778	4.26	.067222	25
36	.813430	2.46	.880397	1.81	.933033	4.26	.066967	24
37	.813578	2.45	.880289	1.81	.933289	4.26	.066711	23
38	.813725	2.45	.880180	1.81	.933545	4.26	.066155	22
39	.813872	2.45	.880072	1.81	.933800	4.26	.066200	21
40	9.814019	2.45	9.879963	1.81	9.934056	4.26	0.065944	20
41	.814166	2.45	.879855	1.81	.934311	4.26	.065689	19
42	.814313	2.45	.879746	1.81	.934567	4.26	.065433	18
43	.814460	2.45	.879637	1.81	.934822	4.26	.065178	17
44	.814607	2.44	.879529	1.81	.935078	4.26	.064922	16
45	.814753	2.44	.879420	1.81	.935333	4.26	.064667	15
46	.814900	2.44	.879311	1.82	.935589	4.26	.064411	14
47	.815046	2.44	.879202	1.82	.935844	4.26	.064156	13
48	.815193	2.44	.879093	1.82	.936100	4.26	.063900	12
49	.815339	2.44	.878984	1.82	.936355	4.26	.063645	11
50	9.815485	2.44	9.878875	1.82	9.936611	4.26	0.063389	10
51	.815632	2.43	.878766	1.82	.936866	4.26	.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.26	.062879	8
53	.815924	2.43	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.82	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.25	.061858	4
57	.816507	2.43	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.42	.877999	1.83	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	.816943		.877780		.939163		.060837	0
M.	Cosine	D. 1″.	Sine	D. 1″.	Cotang.	D. 1″	Tang.	M

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	0.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.42	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.42	.877340	1.84	.940183	4.25	.059817	56
5	.817668	2.41	.877230	1.84	.940439	4.25	.059561	55
6	.817813	2.41	.877120	1.84	.940694	4.25	.059306	54
7	.817958	2.41	.877010	1.84	.940949	4.25	.059051	53
8	.818103	2.41	.876899	1.84	.941204	4.25	.058796	52
9	.818247	2.41	.876789	1.84	.941459	4.25	.058541	51
10	9.818392	2.41	9.876678	1.84	9.941713	4.25	0.058287	50
11	.818536	2.41	.876568	1.84	.941968	4.25	.058032	49
12	.818681	2.40	.876457	1.84	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.84	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.85	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.25	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.39	.875682	1.85	.944007	4.25	.055993	41
20	9.819832	2.39	9.875571	1.85	9.944262	4.25	0.055738	40
21	.819976	2.39	.875459	1.85	.944517	4.25	.055483	39
22	.820120	2.39	.875348	1.85	.944771	4.24	.055229	38
23	.820263	2.39	.875237	1.86	.945026	4.24	.054974	37
24	.820406	2.39	.875126	1.86	.945281	4.24	.054719	36
25	.820550	2.39	.875014	1.86	.945535	4.24	.054465	35
26	.820693	2.38	.874903	1.86	.945790	4.24	.054210	34
27	.820836	2.38	.874791	1.86	.946045	4.24	.053955	33
28	.820979	2.38	.874680	1.86	.946299	4.24	.053701	32
29	.821122	2.38	.874568	1.86	.946554	4.24	.053446	31
30	9.821265	2.38	9.874456	1.86	9.946808	4.24	0.053192	30
31	.821407	2.38	.874344	1.86	.947063	4.24	.052937	29
32	.821550	2.38	.874232	1.87	.947318	4.24	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.24	.052428	27
34	.821835	2.37	.874009	1.87	.947827	4.24	.052173	26
35	.821977	2.37	.873896	1.87	.948081	4.24	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.24	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.24	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.24	.051156	22
39	.822546	2.37	.873448	1.87	.949099	4.24	.050901	21
40	9.822688	2.37	9.873335	1.87	9.949353	4.24	0.050647	20
41	.822830	2.36	.873223	1.88	.949608	4.24	.050392	19
42	.822972	2.36	.873110	1.88	.949862	4.24	.050138	18
43	.823114	2.36	.872998	1.88	.950116	4.24	.049884	17
44	.823255	2.36	.872885	1.88	.950371	4.24	.049629	16
45	.823397	2.36	.872772	1.88	.950625	4.24	.049375	15
46	.823539	2.36	.872659	1.88	.950879	4.24	.049121	14
47	.823680	2.36	.872547	1.88	.951133	4.24	.048867	13
48	.823821	2.35	.872434	1.88	.951388	4.24	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.24	.048358	11
50	9.824104	2.35	9.872208	1.89	9.951896	4.24	0.048104	10
51	.824245	2.35	.872095	1.89	.952150	4.24	.047850	9
52	.824386	2.35	.871981	1.89	.952405	4.24	.047595	8
53	.824527	2.35	.871868	1.89	.952659	4.24	.047341	7
54	.824668	2.35	.871755	1.89	.952913	4.24	.047087	6
55	.824808	2.34	.871641	1.89	.953167	4.24	.046833	5
56	.824949	2.34	.871528	1.89	.953421	4.24	.046579	4
57	.825090	2.34	.871414	1.89	.953675	4.23	.046325	3
58	.825230	2.34	.871301	1.89	.953929	4.23	.046071	2
59	.825371	2.34	.871187	1.90	.954183	4.23	.045817	1
60	.825511		.871073		.954437		.045563	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	0.045563	60
1	.825651	2.34	.870960	1.90	.954691	4.23	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.23	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.90	.956215	4.23	.043785	53
8	.826631	2.33	.870161	1.91	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.91	.956723	4.23	.043277	51
10	9.826910	2.32	9.869933	1.91	9.956977	4.23	0.043023	50
11	.827049	2.32	.869818	1.91	.957231	4.23	.042769	49
12	.827189	2.32	.869704	1.91	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.91	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.91	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.91	.958247	4.23	.041753	45
16	.827745	2.32	.869245	1.91	.958500	4.23	.041500	44
17	.827884	2.31	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.31	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.31	.868900	1.92	.959262	4.23	.040738	41
20	9.828301	2.31	9.868785	1.92	9.959516	4.23	0.040484	40
21	.828439	2.31	.868670	1.92	.959769	4.23	.040231	39
22	.828578	2.31	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.31	.868440	1.92	.960277	4.23	.039723	37
24	.828855	2.31	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.92	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.93	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.23	.038708	33
28	.829407	2.30	.867862	1.93	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.23	.038201	31
30	9.829683	2.30	9.867631	1.93	9.962052	4.23	0.037948	30
31	.829821	2.30	.867515	1.93	.962306	4.23	.037694	29
32	.829959	2.29	.867399	1.93	.962560	4.23	.037440	28
33	.830097	2.29	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.29	.867167	1.93	.963067	4.23	.036933	26
35	.830372	2.29	.867051	1.94	.963320	4.23	.036680	25
36	.830509	2.29	.866935	1.94	.963574	4.23	.036426	24
37	.830646	2.29	.866819	1.94	.963828	4.23	.036172	23
38	.830784	2.29	.866703	1.94	.964081	4.23	.035919	22
39	.830921	2.29	.866586	1.94	.964335	4.23	.035665	21
40	9.831058	2.28	9.866470	1.94	9.964588	4.22	0.035412	20
41	.831195	2.28	.866353	1.94	.964842	4.22	.035158	19
42	.831332	2.28	.866237	1.94	.965095	4.22	.034905	18
43	.831469	2.28	.866120	1.94	.965349	4.22	.034651	17
44	.831606	2.28	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.22	.034145	15
46	.831879	2.28	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.27	.865653	1.95	.966362	4.22	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.27	.865419	1.95	.966869	4.22	.033131	11
50	9.832425	2.27	9.865302	1.95	9.967123	4.22	0.032877	10
51	.832561	2.27	.865185	1.95	.967376	4.22	.032624	9
52	.832697	2.27	.865068	1.95	.967629	4.22	.032371	8
53	.832833	2.27	.864950	1.96	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.96	.968136	4.22	.031864	6
55	.833105	2.26	.864716	1.96	.968389	4.22	.031611	5
56	.833241	2.26	.864598	1.96	.968643	4.22	.031357	4
57	.833377	2.26	.864481	1.96	.968896	4.22	.031104	3
58	.833512	2.26	.864363	1.96	.969149	4.22	.030851	2
59	.833648	2.26	.864245	1.96	.969403	4.22	.030597	1
60	.833783		.864127		.969656		.030344	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	0.030344	60
1	.833919	2.26	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.22	.029838	58
3	.834189	2.25	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.97	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.22	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.25	.863183	1.97	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	9.835134	2.24	9.862946	1.98	9.972188	4.22	0.027812	50
11	.835269	2.24	.862827	1.98	.972441	4.22	.027559	49
12	.835403	2.24	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.24	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.24	.862471	1.98	.973201	4.22	.026799	46
15	.835807	2.24	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.24	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.99	.974466	4.22	.025534	41
20	9.836477	2.23	9.861758	1.99	9.974720	4.22	0.025280	40
21	.836611	2.23	.861638	1.99	.974973	4.22	.025027	39
22	.836745	2.23	.861519	1.99	.975226	4.22	.024774	38
23	.836878	2.23	.861400	1.99	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.99	.975732	4.22	.024268	36
25	.837146	2.22	.861161	1.99	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.99	.976238	4.22	.023762	34
27	.837412	2.22	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	9.837812	2.22	9.860562	2.00	9.977250	4.22	0.022750	30
31	.837945	2.22	.860442	2.00	.977503	4.22	.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.21	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.21	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.21	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.21	.859842	2.01	.978768	4.22	.021232	24
37	.838742	2.21	.859721	2.01	.979021	4.22	.020979	23
38	.838875	2.21	.859601	2.01	.979274	4.22	.020726	22
39	.839007	2.21	.859480	2.01	.979527	4.22	.020473	21
40	9.839140	2.21	9.859360	2.01	9.979780	4.22	0.020220	20
41	.839272	2.20	.859239	2.01	.980033	4.22	.019967	19
42	.839404	2.20	.859119	2.01	.980286	4.22	.019714	18
43	.839536	2.20	.858998	2.01	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.21	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.21	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.21	.018450	13
48	.840196	2.19	.858393	2.02	.981803	4.21	.018197	12
49	.840328	2.19	.858272	2.02	.982056	4.21	.017944	11
50	9.840459	2.19	9.858151	2.02	9.982309	4.21	0.017691	10
51	.840591	2.19	.858029	2.02	.982562	4.21	.017438	9
52	.840722	2.19	.857908	2.02	.982814	4.21	.017186	8
53	.840854	2.19	.857786	2.03	.983067	4.21	.016933	7
54	.840985	2.19	.857665	2.03	.983320	4.21	.016680	6
55	.841116	2.19	.857543	2.03	.983573	4.21	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.21	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.21	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.21	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4,21	.015416	1
60	.841771		.856934		.984837		.015163	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	0.015163	60
1	.841902	2.18	.856812	2.04	.985090	4.21	.014910	59
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657	58
3	.842163	2.18	.856568	2.04	.985596	4.21	.014404	57
4	.842294	2.17	.856446	2.04	.985848	4.21	.014152	56
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899	55
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646	54
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393	53
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140	52
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888	51
10	9.843076	2.17	9.855711	2.05	9.987365	4.21	0.012635	50
11	.843206	2.17	.855588	2.05	.987618	4.21	.012382	49
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129	48
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877	47
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624	46
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371	45
16	.843855	2.16	.854973	2.05	.988882	4.21	.011118	44
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866	43
18	.844114	2.16	.854727	2.06	.989387	4.21	.010613	42
19	.844243	2.16	.854603	2.06	.989640	4.21	.010360	41
20	9.844372	2.15	9.854480	2.06	9.989893	4.21	0.010107	40
21	.844502	2.15	.854356	2.06	.990145	4.21	.009855	39
22	.844631	2.15	.854233	2.06	.990398	4.21	.009602	38
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349	37
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097	36
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844	35
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591	34
27	.845276	2.15	.853614	2.07	991662	4.21	.008338	33
28	.845405	2.14	.853490	2.07	.991914	4.21	.008086	32
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833	31
30	9.845662	2.14	9.853242	2.07	9.992420	4.21	0.007580	30
31	.845790	2.14	.853118	2.07	.992672	4.21	.007328	29
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075	28
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822	27
34	.846175	2.14	.852745	2.07	.993431	4.21	.006569	26
35	.846304	2.14	.852620	2.08	.993683	4.21	.006317	25
36	.846132	2.13	.852496	2.08	.993936	4.21	.006064	24
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306	21
40	9.846944	2.13	9.851997	2.08	9.994947	4.21	0.005053	20
41	.847071	2.13	.851872	2.08	.995199	4.21	.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548	18
43	.847327	2.13	.851622	2.09	.995705	4.21	.004295	17
44	.847454	2.12	.851497	2.09	.995957	4.21	.004043	16
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790	15
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537	14
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285	13
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032	12
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779	11
50	9.848218	2.12	9.850745	2.09	9.997473	4.21	0.002527	10
51	.848345	2.12	.850619	2.10	.997726	4.21	.002274	9
52	.848472	2.11	.850493	2.10	.997979	4.21	.002021	8
53	.848599	2.11	.850368	2.10	.998231	4.21	.001769	7
54	.848726	2.11	.850242	2.10	.998484	4.21	.001516	6
55	.848852	2.11	.850116	2.10	.998737	4.21	.001263	5
56	.848979	2.11	.849990	2.10	.998989	4.21	.001011	4
57	.849106	2.11	.849864	2.10	.999242	4.21	.000758	3
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505	2
59	.849359	2.11	.849611	2.11	.999747	4.21	.000253	1
60	.849485		.849485		0.000000		.000000	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

TABLE III.

NATURAL SINES AND COSINES.

	0°		1°		2°		3°		4°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
M.	Cosin.	Sine	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	89°		88°		87°		86°		85°		

M.	5° Sine.	5° Cosin.	6° Sine.	6° Cosin.	7° Sine.	7° Cosin.	8° Sine.	8° Cosin.	9° Sine.	9° Cosin.	M.
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin	Sine.	M.
	84°		83°		82°		81°		80°		

M.	10° Sine.	10° Cosin	11° Sine.	11° Cosin.	12° Sine.	12° Cosin.	13° Sine.	13° Cosin.	14° Sine.	14° Cosin.	M.
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	79°		78°		77°		76°		75°		

	15°		16°		17°		18°		19°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96.90	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
M	Sine.	Cosin	Sine.	Cosin	Sine.	Cosin.	Sine.	Cosin	Sine.	Cosin	M
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	69°		68°		67°		66°		65°		

M.	25° Sine.	25° Cosin.	26° Sine.	26° Cosin.	27° Sine.	27° Cosin.	28° Sine.	28° Cosin.	29° Sine.	29° Cosin.	M.
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86127	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86113	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	63135	77550	51
10	.57596	.81748	59014	.80730	.60414	.79688	.61795	.78622	63158	.77531	50
11	.57619	.81731	59037	.80713	60437	.79671	.61818	.78604	63180	.77513	49
12	57643	81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	57738	.81647	59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	59201	.80593	.60599	.79547	.61978	.78478	63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	63361	.77366	41
20	.57833	.81580	59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	59272	.80541	.60668	.79494	.62046	.78424	63406	.77329	39
22	57881	.81546	59295	.80524	.60691	.79477	.62069	.78405	63428	.77310	38
23	57904	.81530	59318	.80507	.60714	.79459	.62092	.78387	63451	.77292	37
24	57928	.81513	59342	.80489	.60738	.79441	.62115	.78369	63473	.77273	36
25	57952	.81496	59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	63518	.77236	34
27	.57999	.81462	59412	.80438	.60807	.79388	.62183	.78315	63540	.77218	33
28	58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	63563	.77199	32
29	.58047	.81428	59459	.80403	.60853	.79353	.62229	.78279	63585	.77181	31
30	.58070	.81412	59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	59506	.80368	.60899	.79318	.62274	.78243	63630	.77144	29
32	.58118	.81378	59529	.80351	.60922	.79300	.62297	.78225	63653	.77125	28
33	.58141	.81361	59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	59576	.80316	60968	.79264	.62342	.78188	63698	.77088	26
35	.58189	.81327	59599	.80299	.60991	.79247	.62365	.78170	63720	.77070	25
36	.58212	.81310	59622	.80282	.61015	.79229	.62388	.78152	63742	.77051	24
37	.58236	.81293	59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	59669	.80247	.61061	.79193	.62433	.78116	63787	.77014	22
39	.58283	.81259	59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	62683	.77916	.64033	.76810	11
50	.58543	.81072	59949	.80038	.61337	.78980	.62706	.77897	64056	.76791	10
51	.58567	.81055	59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	59995	.80003	.61383	.78944	.62751	.77861	64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	64145	.76717	6
55	58661	.80987	60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	60089	.79934	.61474	.78873	.62842	.77788	64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	64212	.76661	3
58	.58731	.80936	60135	.79899	61520	.78837	.62887	.77751	64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	54°		53°		52°		51°		50°		

	40°		41°		42°		43°		44°		
M.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	M.
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
M.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	Cosin.	Sine.	M.
	49°		48°		47°		46°		45°		

TABLE IV.

NATURAL TANGENTS AND COTANGENTS.

	0°		1°		2°		3°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	01891	52.8821	.03638	27 4899	05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	05503	18.1708	51
10	.00291	313.774	.02036	49.1039	.03783	26.4316	05533	8.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	05620	17.7934	47
14	.00407	215.552	.02153	46.4489	.03900	25.6418	05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3676	.05854	17.0837	39
22	.00640	156.259	02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15 6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	04774	20 9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15 2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	06817	14.6686	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14 4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	89°		88°		87°		86°		

	4°		5°		6°		7°		
M.	Tang	Cotang	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14436	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	10657	9.38307	12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	10952	9.13093	12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	12751	7.84242	44
17	07490	13.3515	.09247	10.8139	.11011	9.08211	12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	12869	7.77035	40
21	07607	13.1461	.09365	10.6783	.11128	8.98598	12899	7.75254	39
22	.07636	13.0958	09394	10.6450	.11158	8.96227	12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	09629	10.3854	.11394	8.77689	13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	11423	8.75425	13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	13224	7 56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	13343	7.49465	24
37	.08075	12.3838	09834	10.1683	.11600	8.62078	13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	08661	11.5461	.10422	9.59490	12190	8.20352	13965	7.16071	3
58	08690	11.5072	.10452	9.56791	12219	8.18370	13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	12249	8.16398	.14024	7.13042	1
60	.08749	11 4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	85°		84°		83°		82°		

	8°		9°		10°		11°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	14054	7.11537	.15838	6.31375	.17633	5.67128	19438	5.14455	60
1	.14084	7.10038	15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	19740	5.06584	50
11	.14381	6.95385	16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	19801	5.05037	48
13	.14440	6.92525	16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	16346	6.11779	.18143	5.51176	19952	5.01210	43
18	.14588	6.85475	16376	6.10664	.18173	5.50264	19982	5.00451	42
19	.14618	6.84082	16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	14648	6.82694	16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	16465	6.07340	18263	5.47548	20073	4.98188	39
22	14707	6.79936	16495	6.06240	18293	5.46648	.20103	4.97438	38
23	14737	6.78564	16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	16555	6.04051	.18353	5.44857	20164	4.95945	36
25	14796	6.75838	16585	6.02962	.18384	5.43966	20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	20224	4.94460	34
27	14856	6.73133	16645	6.00797	.18444	5.42192	20254	4.93721	33
28	14886	6.71789	16674	5.99720	.18474	5.41309	20285	4.92984	32
29	.14915	6.70450	16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	19046	5.25048	.20861	4.79370	13
48	15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	17513	5.71013	19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	17543	5.70037	19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	81°		80°		79°		78°		

	12°		13°		14°		15°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	21377	4.67786	23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	21499	4.65138	23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	21529	4.64480	23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	21560	4.63825	23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	21590	4.63171	23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	21621	4.62518	23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	21651	4.61868	23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	21682	4.61219	23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	21712	4.60572	23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	21743	4.59927	23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	21773	4.59283	23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	21804	4.58641	23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	21834	4.58001	23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	21864	4.57363	23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	21895	4.56726	23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	21925	4.56091	23762	4.20842	.25614	3.90417	27482	3.63874	38
23	21956	4.55458	23793	4.20298	.25645	3.89945	27513	3.63461	37
24	21986	4.54826	23823	4.19756	.25676	3.89474	27545	3.63048	36
25	22017	4.54196	.23854	4.19215	.25707	3.89004	27576	3.62636	35
26	22047	4.53568	.23885	4.18675	.25738	3.88536	27607	3.62224	34
27	.22078	4.52941	23916	4.18137	.25769	3.88068	27638	3.61814	33
28	22108	4.52316	23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	22231	4.49832	.24069	4.15465	.25924	3.85745	27795	3.59775	28
33	22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	22292	4.48600	24131	4.14405	.25986	3.84824	27858	3.58966	26
35	22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	22353	4.47374	24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	22383	4.46764	24223	4.12825	.26079	3.83449	27952	3.57758	23
38	22414	4.46155	24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	22444	4.45548	24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	22475	4.44942	24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	22505	4.44338	24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	22536	4.43735	24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	22567	4.43134	24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	22597	4.42534	24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	22658	4.41340	24501	4.08152	.26359	3.79378	28234	3.54179	14
47	22689	4.40745	24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	22719	4.40152	24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	22781	4.38969	24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	22842	4.37793	24686	4.05092	.26546	3.76709	28423	3.51829	8
53	22872	4.37207	24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	22903	4.36623	24747	4.04081	26608	3.75828	.28486	3.51053	6
55	22934	4.36040	24778	4.03578	26639	3.75388	.28517	3.50666	5
56	22964	4.35459	24809	4.03076	26670	3.74950	.28549	3.50279	4
57	22995	4.34879	24840	4.02574	26701	3.74512	.28580	3.49894	3
58	23026	4.34300	24871	4.02074	26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	24902	4.01576	26764	3.73640	28643	3.49125	1
60	23087	4.33148	24933	4.01078	26795	3.73205	.28675	3.48741	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	77°		76°		75°		74°		

	16°		17°		18°		19°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	73°		72°		71°		70°		

	20°		21°		22°		23°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	42791	2.33693	50
11	.36760	2.72036	38754	2.58038	.40775	2.45246	42826	2.33505	49
12	.36793	2.71792	38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	36925	2.70819	38921	2.56928	.40945	2.44230	42998	2.32570	44
17	.36958	2.70577	38955	2.56707	.40979	2.44027	43032	2.32383	43
18	.36991	2.70335	38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	37024	2.70094	39022	2.56266	.41047	2.43623	43101	2.32012	41
20	37057	2.69853	39055	2.56046	.41081	2.43422	43136	2.31826	40
21	.37090	2.69612	39089	2.55827	.41115	2.43220	43170	2.31641	39
22	37123	2.69371	39122	2.55608	.41149	2.43019	43205	2.31456	38
23	37157	2.69131	39156	2.55389	41183	2.42819	43239	2.31271	37
24	37190	2.68892	.39190	2.55170	.41217	2.42618	43274	2.31086	36
25	37223	2.68653	39223	2.54952	.41251	2.42418	43308	2.30902	35
26	37256	2.68414	39257	2.54734	.41285	2.42218	43343	2.30718	34
27	37289	2.68175	39290	2.54516	.41319	2.42019	43378	2.30534	33
28	37322	2.67937	39324	2.54299	.41353	2.41819	43412	2.30351	32
29	37355	2.67700	39357	2.54082	.41387	2.41620	43447	2.30167	31
30	37388	2.67462	39391	2.53865	.41421	2.41421	43481	2.29984	30
31	37422	2.67225	39425	2.53648	.41455	2.41223	43516	2.29801	29
32	37455	2.66989	39458	2.53432	.41490	2.41025	43550	2.29619	28
33	37488	2.66752	39492	2.53217	.41524	2.40827	43585	2.29437	27
34	37521	2.66516	39526	2.53001	.41558	2.40629	43620	2.29254	26
35	37554	2.66281	39559	2.52786	.41592	2.40432	43654	2.29073	25
36	37588	2.66046	39593	2.52571	41626	2.40235	43689	2.28891	24
37	37621	2.65811	39626	2.52357	.41660	2.40038	43724	2.28710	23
38	37654	2.65576	39660	2.52142	41694	2.39841	43758	2.28528	22
39	37687	2.65342	39694	2.51929	.41728	2.39645	43793	2.28348	21
40	37720	2.65109	39727	2.51715	41763	2.39449	43828	2.28167	20
41	.37754	2.64875	39761	2.51502	41797	2.39253	43862	2.27987	19
42	37787	2.64642	39795	2.51289	41831	2.39058	43897	2.27806	18
43	.37820	2.64410	39829	2.51076	41865	2.38863	43932	2.27626	17
44	37853	2.64177	39862	2.50864	.41899	2.38668	43966	2.27447	16
45	.37887	2.63945	39896	2.50652	.41933	2.38473	44001	2.27267	15
46	37920	2.63714	39930	2.50440	41968	2.38279	44036	2.27088	14
47	37953	2.63483	39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	37986	2.63252	39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	40065	2.49597	42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	40098	2.49386	42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	40132	2.49177	42173	2.37118	44244	2.26018	8
53	38153	2.62103	40166	2.48967	42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	40200	2.48758	42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	40267	2.48340	42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	69°		68°		67°		66°		

	24°		25°		26°		27°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang	Cotang.	M.
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	51503	1.94162	45
16	45082	2.21819	47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	45117	2.21647	47234	2.11711	.49387	2.02483	51577	1.93885	43
18	45152	2.21475	47270	2.11552	.49423	2.02335	51614	1.93746	42
19	45187	2.21304	47305	2.11392	.49459	2.02187	51651	1.93608	41
20	45222	2.21132	47341	2.11233	.49495	2.02039	51688	1.93470	40
21	45257	2.20961	47377	2.11075	.49532	2.01891	51724	1.93332	39
22	45292	2.20790	47412	2.10916	.49568	2.01743	51761	1.93195	38
23	45327	2.20619	.47448	2.10758	.49604	2.01596	51798	1.93057	37
24	45362	2.20449	47483	2.10600	.49640	2.01449	51835	1.92920	36
25	45397	2.20278	47519	2.10442	.49677	2.01302	51872	1.92782	35
26	45432	2.20108	47555	2.10284	49713	2.01155	51909	1.92645	34
27	45467	2.19938	47590	2.10126	.49749	2.01008	51946	1.92508	33
28	45502	2.19769	47626	2.09969	49786	2.00862	51983	1.92371	32
29	45538	2.19599	47662	2.09811	.49822	2.00715	52020	1.92235	31
30	45573	2.19430	47698	2.09654	.49858	2.00569	52057	1.92098	30
31	45608	2.19261	47733	2.09498	.49894	2.00423	52094	1.91962	29
32	45643	2.19092	47769	2.09341	.49931	2.00277	52131	1.91826	28
33	45678	2.18923	47805	2.09184	.49967	2.00131	52168	1.91690	27
34	45713	2.18755	47840	2.09028	.50004	1.99986	52205	1.91554	26
35	45748	2.18587	.47876	2.08872	50040	1.99841	52242	1.91418	25
36	45784	2.18419	47912	2.08716	50076	1.99695	52279	1.91282	24
37	45819	2.18251	47948	2.08560	.50113	1.99550	52316	1.91147	23
38	45854	2.18084	47984	2.08405	.50149	1.99406	52353	1.91012	22
39	45889	2.17916	48019	2.08250	.50185	1.99261	52390	1.90876	21
40	45924	2.17749	48055	2.08094	.50222	1.99116	52427	1.90741	20
41	45960	2.17582	48091	2.07939	.50258	1.98972	52464	1.90607	19
42	45995	2.17416	.48127	2.07785	50295	1.98828	52501	1.90472	18
43	46030	2.17249	.48163	2.07630	.50331	1.98684	52538	1.90337	17
44	46065	2.17083	.48198	2.07476	.50368	1.98540	52575	1.90203	16
45	46101	2.16917	.48234	2.07321	50404	1.98396	52613	1.90069	15
46	46136	2.16751	.48270	2.07167	50441	1.98253	52650	1.89935	14
47	46171	2.16585	.48306	2.07014	.50477	1.98110	52687	1.89801	13
48	46206	2.16420	.48342	2.06860	.50514	1.97966	52724	1.89667	12
49	46242	2.16255	.48378	2.06706	.50550	1.97823	52761	1.89533	11
50	46277	2.16090	.48414	2.06553	.50587	1.97681	52798	1.89400	10
51	46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	46348	2.15760	.48486	2.06247	.50660	1.97395	52873	1.89133	8
53	46383	2.15596	.48521	2.06094	50696	1.97253	.52910	1.89000	7
54	46418	2.15432	48557	2.05942	50733	1.97111	.52947	1.88867	6
55	46454	2.15268	48593	2.05790	.50769	1.96969	52985	1.88734	5
56	46489	2.15104	48629	2.05637	50806	1.96827	53022	1.88602	4
57	46525	2.14940	48665	2.05485	50843	1.96685	53059	1.88469	3
58	46560	2.14777	48701	2.05333	50879	1.96544	53096	1.88337	2
59	46595	2.14614	48737	2.05182	50916	1.96402	53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	65°		64°		63°		62°		

	28°		29°		30°		31°		
M	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang	M.
0	.53171	1.88073	55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	53283	1.87677	55545	1.80034	.57851	1.72857	60205	1.66099	57
4	53320	1.87546	55583	1.79911	.57890	1.72741	60245	1.65990	56
5	.53358	1.87415	55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	55697	1.79542	.58007	1.72393	60364	1.65663	53
8	.53470	1.87021	55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	55850	1.79051	.58162	1.71932	60522	1.65228	49
12	.53620	1.86499	55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	53657	1.86369	55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	53694	1.86239	55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	53769	1.85979	56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	53807	1.85850	56079	1.78319	.58396	1.71244	60761	1.64579	43
18	53844	1.85720	56117	1.78198	.58435	1.71129	60801	1.64471	42
19	53882	1.85591	56156	1.78077	.58474	1.71015	60841	1.64363	41
20	53920	1.85462	56194	1.77955	.58513	1.70901	60881	1.64256	40
21	53957	1.85333	56232	1.77834	.58552	1.70787	60921	1.64148	39
22	53995	1.85204	56270	1.77713	.58591	1.70673	60960	1.64041	38
23	54032	1.85075	56309	1.77592	.58631	1.70560	61000	1.63934	37
24	54070	1.84946	56347	1.77471	.58670	1.70446	61040	1.63826	36
25	54107	1.84818	56385	1.77351	.58709	1.70332	61080	1.63719	35
26	54145	1.84689	56424	1.77230	.58748	1.70219	61120	1.63612	34
27	54183	1.84561	56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	54220	1.84433	56501	1.76990	.58826	1.69992	61200	1.63398	32
29	54258	1.84305	56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	54635	1.83033	56923	1.75675	.59258	1.68754	61641	1.62230	21
40	.54673	1.82906	56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	54748	1.82654	57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	59730	1.67419	62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	59770	1.67309	62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	61°		60°		59°		58°		

	32°		33°		34°		35°		
M	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	71725	1.39421	21
40	.64117	1.55966	.66608	1 50133	.69157	1.44598	71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1 49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1 49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	57°		56°		55°		54°		

	36°		37°		38°		39°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1 31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	46
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73096	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	,80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.76264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	53°		52°		51°		50°		

	40°		41°		42°		43°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	93633	1 06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1 14363	.90569	1.10414	93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	85912	1.16398	.88992	1.12369	.92170	1.08496	95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M
	49°		48°		47°		46°		

	44°				44°				44°		
M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M.	M.	Cotang.	Tang.	M.
	45°				45°				45°		

TABLE V.

CUBIC YARDS PER 100 FEET. SLOPES ¼ : 1; ½ : 1; 1 : 1; 1½ : 1; 2 : 1; 3 : 1.

Depth	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	Base 26	Base 28
1	45	53	60	68	82	90	97	105
2	93	107	122	137	167	181	196	211
3	142	163	186	208	253	275	297	319
4	193	222	252	281	341	370	400	430
5	245	282	319	356	431	468	505	542
6	300	344	389	433	522	567	611	656
7	356	408	460	512	616	668	719	771
8	415	474	533	593	711	770	830	889
9	475	542	608	675	808	875	942	1008
10	537	611	685	759	907	981	1056	1130
11	601	682	764	845	1008	1090	1171	1253
12	667	756	844	933	1111	1200	1289	1378
13	734	831	926	1023	1216	1312	1408	1505
14	804	907	1010	1115	1322	1426	1530	1633
15	875	986	1096	1208	1431	1542	1653	1764
16	948	1067	1184	1304	1541	1659	1778	1896
17	1023	1149	1274	1401	1653	1779	1905	2031
18	1100	1233	1366	1500	1767	1900	2033	2167
19	1179	1319	1460	1601	1882	2023	2164	2305
20	1259	1407	1555	1704	2000	2148	2296	2444
21	1342	1497	1653	1808	2119	2275	2431	2586
22	1426	1589	1752	1915	2241	2404	2567	2730
23	1512	1682	1853	2023	2364	2534	2705	2875
24	1600	1778	1955	2133	2489	2667	2844	3022
25	1690	1875	2060	2245	2616	2801	2986	3171
26	1781	1974	2166	2359	2744	2937	3130	3322
27	1875	2075	2274	2475	2875	3075	3275	3475
28	1970	2178	2384	2593	3007	3215	3422	3630
29	2068	2282	2496	2712	3142	3356	3571	3786
30	2167	2389	2610	2833	3278	3500	3722	3944
31	2268	2497	2726	2956	3416	3645	3875	4105
32	2370	2607	2844	3081	3556	3793	4030	4267
33	2475	2719	2964	3208	3697	3942	4186	4431
34	2581	2833	3085	3337	3841	4093	4344	4596
35	2690	2949	3208	3468	3986	4245	4505	4764
36	2800	3067	3333	3600	4133	4400	4667	4933
37	2912	3186	3460	3734	4282	4556	4831	5105
38	3026	3307	3589	3870	4433	4715	4996	5278
39	3142	3431	3719	4008	4586	4875	5164	5453
40	3259	3556	3852	4148	4741	5037	5333	5630
41	3379	3682	3986	4290	4897	5201	5505	5808
42	3500	3811	4122	4433	5056	5367	5678	5989
43	3623	3942	4260	4579	5216	5534	5853	6171
44	3748	4074	4400	4726	5378	5704	6030	6356
45	3875	4208	4541	4875	5542	5875	6208	6542
46	4004	4344	4684	5026	5707	6048	6389	6730
47	4134	4482	4830	5179	5875	6223	6571	6919
48	4267	4622	4978	5333	6044	6400	6756	7111
49	4401	4764	5127	5490	6216	6579	6942	7305
50	4537	4907	5278	5648	6389	6759	7130	7500
51	4675	5053	5430	5808	6564	6942	7319	7697
52	4815	5200	5584	5970	6741	7126	7511	7896
53	4956	5349	5741	6134	6919	7312	7705	8097
54	5100	5500	5900	6300	7100	7500	7900	8300
55	5245	5653	6060	6468	7282	7690	8097	8505
56	5393	5807	6222	6637	7467	7881	8296	8711
57	5542	5964	6386	6808	7653	8075	8497	8919
58	5693	6122	6552	6981	7841	8270	8700	9130
59	5845	6282	6719	7156	8031	8468	8905	9342
60	6000	6444	6889	7333	8222	8667	9111	9556

Depth	Base 12	Base 14	Base 16	Base 18	Base 22	Base 24	Base 26	Base 28
1	46	54	61	69	83	91	98	106
2	96	111	126	141	170	185	200	215
3	150	172	194	217	261	283	306	328
4	207	237	267	296	356	385	415	444
5	269	306	343	380	454	491	528	565
6	333	378	422	467	556	600	644	689
7	402	454	506	557	661	713	765	317
8	474	533	593	652	770	830	889	948
9	550	617	683	750	883	950	1017	1083
10	630	704	778	852	1000	1074	1148	1222
11	713	794	876	957	1120	1202	1283	1365
12	800	889	978	1067	1244	1333	1422	1511
13	891	987	1083	1180	1372	1469	1565	1661
14	985	1089	1193	1296	1504	1607	1711	1815
15	1083	1194	1306	1417	1639	1750	1861	1972
16	1185	1304	1422	1541	1779	1896	2015	2133
17	1291	1417	1543	1669	1920	2046	2172	2298
18	1400	1533	1667	1800	2067	2200	2333	2467
19	1513	1654	1794	1935	2217	2357	2498	2639
20	1630	1778	1926	2074	2370	2519	2667	2815
21	1750	1906	2061	2217	2528	2683	2839	2994
22	1874	2037	2200	2363	2689	2852	3015	3178
23	2002	2172	2343	2513	2854	3024	3194	3365
24	2133	2311	2489	2667	3022	3200	3378	3556
25	2269	2454	2639	2824	3194	3380	3565	3750
26	2407	2600	2793	2985	3370	3563	3756	3948
27	2550	2750	2950	3150	3550	3750	3950	4151
28	2696	2904	3111	3319	3733	3941	4148	4356
29	2846	3061	3276	3491	3920	4135	4350	4565
30	3000	3222	3444	3667	4111	4333	4556	4778
31	3157	3387	3617	3846	4306	4535	4765	4994
32	3319	3556	3793	4030	4504	4741	4978	5215
33	3483	3728	3972	4217	4706	4950	5194	5439
34	3652	3904	4156	4407	4911	5163	5415	5667
35	3824	4083	4343	4602	5120	5380	5639	5898
36	4000	4267	4533	4800	5333	5600	5867	6133
37	4180	4454	4728	5002	5550	5824	6098	6372
38	4363	4644	4926	5207	5770	6052	6333	6615
39	4550	4839	5128	5417	5994	6283	6572	6861
40	4741	5037	5333	5630	6222	6519	6815	7111
41	4935	5239	5543	5846	6454	6757	7061	7365
42	5133	5444	5756	6067	6689	7000	7311	7622
43	5335	5654	5972	6291	6928	7246	7565	7883
44	5541	5867	6193	6519	7170	7496	7822	8148
45	5750	6083	6417	6750	7417	7750	8083	8417
46	5963	6304	6644	6985	7667	8007	8348	8689
47	6180	6528	6876	7224	7920	8269	8617	8965
48	6400	6756	7111	7467	8178	8533	8889	9244
49	6624	6987	7350	7713	8439	8802	9165	9528
50	6852	7222	7593	7963	8704	9074	9444	9815
51	7083	7461	7839	8217	8972	9350	9728	10106
52	7319	7704	8089	8474	9244	9630	10015	10400
53	7557	7950	8343	8735	9520	9913	10306	10698
54	7800	8200	8600	9000	9800	10200	10600	11000
55	8046	8454	8861	9269	10083	10491	10898	11306
56	8296	8711	9126	9541	10370	10785	11200	11615
57	8550	8972	9394	9817	10661	11083	11506	11928
58	8807	9237	9667	10096	10956	11385	11815	12244
59	9069	9506	9943	10380	11254	11691	12128	12565
60	9333	9778	10222	10667	11556	12000	12444	12889

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	48	56	63	70	78	107	115	122
2	104	119	133	148	163	222	237	252
3	167	189	211	233	256	344	367	389
4	237	267	296	326	356	474	504	533
5	315	352	389	426	463	611	648	685
6	400	444	489	533	578	756	800	844
7	493	544	596	648	700	907	959	1011
8	593	652	711	770	830	1067	1126	1185
9	700	767	833	900	967	1233	1300	1367
10	815	889	963	1037	1111	1407	1481	1556
11	937	1019	1100	1181	1263	1589	1670	1752
12	1067	1156	1244	1333	1422	1778	1867	1956
13	1204	1300	1396	1493	1589	1974	2070	2167
14	1348	1452	1556	1659	1763	2178	2281	2385
15	1500	1611	1722	1833	1944	2389	2500	2611
16	1659	1778	1896	2015	2133	2607	2726	2844
17	1826	1952	2078	2204	2330	2823	2959	3085
18	2000	2133	2267	2400	2533	3067	3200	3333
19	2181	2322	2463	2604	2744	3307	3448	3589
20	2370	2519	2667	2815	2963	3556	3704	3852
21	2567	2722	2878	3033	3189	3811	3967	4122
22	2770	2933	3096	3259	3422	4074	4237	4444
23	2981	3152	3322	3493	3663	4344	4515	4685
24	3200	3378	3556	3733	3911	4622	4800	4978
25	3426	3611	3796	3981	4167	4907	5093	5278
26	3659	3852	4044	4237	4430	5200	5393	5585
27	3900	4100	4300	4500	4700	5500	5700	5900
28	4148	4356	4563	4770	4978	5807	6015	6222
29	4404	4619	4833	5048	5263	6122	6337	6552
30	4667	4889	5111	5333	5556	6444	6667	6889
31	4937	5167	5396	5626	5856	6774	7004	7233
32	5215	5452	5689	5926	6163	7111	7348	7585
33	5500	5744	5989	6233	6478	7456	7700	7944
34	5793	6044	6296	6548	6800	7807	8059	8311
35	6093	6352	6611	6870	7130	8167	8426	8685
36	6400	6667	6933	7200	7467	8533	8800	9067
37	6715	6989	7263	7537	7811	8907	9181	9456
38	7037	7319	7600	7881	8163	9289	9570	9852
39	7367	7656	7944	8233	8522	9678	9967	10256
40	7704	8000	8296	8593	8889	10074	10370	10667
41	8048	8352	8656	8959	9263	10478	10781	11085
42	8400	8711	9022	9333	9644	10889	11200	11511
43	8759	9078	9396	9715	10033	11307	11626	11944
44	9126	9452	9778	10104	10430	11733	12059	12385
45	9500	9833	10167	10500	10833	12167	12500	12833
46	9881	10222	10563	10904	11244	12607	12948	13289
47	10270	10619	10967	11315	11663	13056	13404	13752
48	10667	11022	11378	11733	12089	13511	13867	14222
49	11070	11433	11796	12159	12522	13974	14337	14700
50	11481	11852	12222	12593	12963	14444	14815	15185
51	11900	12278	12656	13033	13411	14922	15300	15678
52	12326	12711	13096	13481	13867	15407	15793	16178
53	12759	13152	13544	13937	14330	15900	16293	16685
54	13200	13600	14000	14400	14800	16400	16800	17200
55	13648	14056	14463	14870	15278	16907	17315	17722
56	14104	14519	14933	15348	15763	17422	17837	18252
57	14567	14989	15411	15833	16256	17944	18367	18789
58	15037	15467	15896	16326	16756	18474	18904	19333
59	15515	15952	16389	16826	17263	19011	19448	19885
60	16000	16444	16889	17333	17778	19556	20000	20444

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	50	57	65	72	80	109	117	124
2	111	126	141	156	170	230	244	259
3	183	206	228	250	272	361	383	406
4	267	296	326	356	385	504	533	563
5	361	398	435	472	509	657	694	731
6	467	511	556	600	644	822	867	911
7	583	635	687	739	791	998	1050	1102
8	711	770	830	889	948	1185	1244	1304
9	850	917	983	1050	1116	1388	1450	1517
10	1000	1074	1148	1222	1296	1593	1667	1741
11	1161	1243	1324	1406	1487	1813	1894	1976
12	1333	1422	1511	1600	1689	2044	2133	2222
13	1517	1613	1709	1806	1902	2287	2383	2480
14	1711	1815	1919	2022	2126	2541	2644	2748
15	1917	2028	2139	2250	2361	2806	2917	3028
16	2133	2252	2370	2489	2607	3081	3200	3319
17	2361	2487	2613	2739	2865	3369	3494	3620
18	2600	2733	2867	3000	3133	3667	3800	3933
19	2850	2991	3131	3272	3413	3976	4117	4257
20	3111	3259	3407	3556	3704	4296	4444	4592
21	3383	3539	3694	3850	4005	4628	4783	4939
22	3667	3830	3993	4156	4318	4970	5133	5296
23	3961	4131	4302	4472	4642	5324	5494	5665
24	4267	4444	4622	4800	4978	5689	5867	6044
25	4583	4769	4954	5139	5324	6065	6250	6435
26	4911	5104	5296	5489	5681	6452	6644	6837
27	5250	5450	5650	5850	6050	6850	7050	7250
28	5600	5807	6015	6222	6430	7259	7467	7674
29	5961	6176	6391	6606	6820	7680	7894	8109
30	6333	6556	6778	7000	7222	8111	8333	8555
31	6717	6946	7176	7406	7635	8554	8783	9013
32	7111	7348	7585	7822	8059	9007	9244	9482
33	7517	7761	8006	8250	8494	9472	9717	9962
34	7933	8185	8437	8689	8941	9948	10200	10452
35	8361	8620	8880	9139	9398	10435	10694	10954
36	8800	9067	9333	9600	9867	10933	11200	11467
37	9250	9524	9798	10072	10346	11443	11717	11991
38	9711	9993	10274	10556	10837	11963	12244	12526
39	10183	10472	10761	11050	11339	12494	12783	13072
40	10667	10963	11259	11556	11852	13037	13333	13630
41	11161	11465	11769	12072	12376	13591	13894	14198
42	11667	11978	12289	12600	12911	14156	14467	14778
43	12183	12502	12820	13139	13457	14731	15050	15369
44	12711	13037	13363	13689	14015	15319	15644	15970
45	13250	13583	13917	14250	14583	15917	16250	16583
46	13800	14141	14481	14822	15163	16526	16867	17207
47	14361	14709	15057	15406	15754	17146	17494	17843
48	14933	15289	15644	16000	16356	17778	18133	18489
49	15517	15880	16243	16606	16968	18420	18783	19146
50	16111	16481	16852	17222	17592	19074	19444	19815
51	16717	17094	17472	17850	18228	19739	20117	20494
52	17333	17719	18104	18489	18874	20415	20800	21185
53	17961	18354	18746	19139	19531	21102	21494	21887
54	18600	19000	19400	19800	20200	21800	22200	22600
55	19250	19657	20065	20472	20880	22509	22917	23324
56	19911	20326	20741	21156	21570	23230	23644	24059
57	20583	21006	21428	21850	22272	23961	24383	24805
58	21267	21696	22126	22556	22985	24704	25133	25563
59	21961	22398	22835	23272	23709	25457	25894	26332
60	22667	23111	23556	24000	24444	26222	26667	27111

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	52	59	67	74	81	111	119	126
2	119	133	143	163	178	237	252	267
3	200	222	244	267	289	378	400	422
4	296	326	356	385	415	533	563	593
5	407	444	481	519	556	704	741	778
6	533	578	622	667	711	889	933	978
7	674	726	778	830	881	1089	1141	1193
8	830	889	948	1007	1067	1304	1363	1422
9	1000	1067	1133	1200	1267	1533	1600	1667
10	1185	1259	1333	1407	1481	1778	1852	1926
11	1385	1467	1548	1630	1711	2037	2119	2200
12	1600	1689	1778	1867	1956	2311	2400	2489
13	1830	1926	2022	2119	2215	2600	2696	2793
14	2074	2178	2281	2385	2489	2904	3007	3111
15	2333	2444	2556	2667	2778	3222	3333	3444
16	2607	2726	2844	2963	3081	3556	3674	3793
17	2896	3022	3148	3274	3400	3904	4030	4156
18	3200	3333	3467	3600	3733	4267	4400	4533
19	3519	3659	3800	3941	4081	4644	4785	4926
20	3852	4000	4148	4296	4444	5037	5185	5333
21	4200	4356	4511	4667	4822	5444	5600	5756
22	4563	4730	4889	5052	5215	5867	6030	6193
23	4941	5111	5281	5452	5622	6304	6474	6644
24	5333	5511	5689	5867	6044	6756	6933	7111
25	5741	5926	6111	6296	6481	7222	7407	7593
26	6163	6356	6548	6741	6933	7704	7896	8089
27	6600	6800	7000	7200	7400	8200	8400	8600
28	7052	7259	7467	7674	7881	8711	8919	9126
29	7519	7733	7948	8163	8378	9237	9452	9667
30	8000	8222	8444	8667	8889	9778	10000	10222
31	8496	8726	8956	9185	9415	10333	10563	10793
32	9007	9244	9481	9719	9956	10904	11141	11378
33	9533	9778	10022	10267	10511	11489	11733	11978
34	10074	10326	10578	10830	11081	12089	12341	12593
35	10330	10889	11148	11407	11667	12704	12963	13222
36	11200	11467	11733	12000	12267	13333	13600	13867
37	11785	12059	12333	12607	12881	13978	14252	14526
38	12385	12667	12948	13230	13511	14637	14919	15200
39	13006	13289	13578	13867	14156	15311	15600	15889
40	13630	13926	14222	14519	14815	16000	16296	16593
41	14274	14578	14881	15185	15489	16704	17007	17311
42	14933	15244	15556	15867	16178	17422	17733	18044
43	15607	15926	16224	16563	16881	18156	18474	18793
44	16296	16622	16948	17274	17600	18904	19230	19556
45	17000	17333	17667	18000	18333	19667	20000	20333
46	17719	18059	18400	18741	19081	20444	20785	21126
47	18452	18800	19148	19496	19844	21237	21585	21933
48	19200	19556	19911	20267	20622	22044	22400	22756
49	19963	20326	20689	21052	21415	22867	23230	23593
50	20741	20711	21481	21852	22222	23704	24074	24444
51	21[illegible]33	21911	22289	22667	23044	24556	24933	25311
52	22341	22726	23111	23496	23881	25422	25807	26193
53	23163	23556	23948	24341	24733	26304	26696	27089
54	24000	24400	24800	25200	25600	27200	27600	28000
55	24852	25259	25667	26074	26481	28111	28519	28926
56	25719	26133	26548	26963	27378	29037	29452	29867
57	2[illegible]600	27022	27444	27867	28289	29978	30400	30822
58	27496	[illegible]926	28356	28785	29215	30933	31363	31793
59	28407	28844	29281	29719	30156	31904	32341	32778
60	29333	29778	30222	30667	31111	32889	33333	33778

Depth	Base 12	Base 14	Base 16	Base 18	Base 20	Base 28	Base 30	Base 32
1	56	63	70	78	85	115	122	130
2	133	148	163	178	193	252	267	281
3	233	256	278	300	322	411	433	456
4	356	385	415	444	474	593	622	652
5	500	537	574	611	648	796	833	870
6	667	711	756	800	844	1022	1067	1111
7	856	907	959	1011	1063	1270	1322	1374
8	1067	1126	1185	1244	1304	1541	1600	1659
9	1300	1367	1433	1500	1567	1833	1900	1967
10	1556	1630	1704	1778	1852	2148	2222	2296
11	1833	1915	1996	2078	2159	2485	2567	2648
12	2133	2222	2311	2400	2489	2844	2933	3022
13	2456	2552	2648	2744	2841	3226	3322	3419
14	2800	2904	3007	3111	3215	3630	3733	3837
15	3167	3278	3389	3500	3611	4056	4167	4278
16	3556	3674	3793	3911	4030	4504	4622	4741
17	3967	4093	4219	4344	4470	4974	5100	5226
18	4400	4533	4667	4800	4933	5467	5600	5733
19	4856	4996	5137	5278	5419	5981	6122	6263
20	5333	5481	5630	5778	5926	6519	6667	6815
21	5833	5989	6144	6300	6456	7078	7233	7389
22	6356	6519	6681	6844	7007	7659	7822	7985
23	6900	7070	7241	7411	7581	8263	8433	8504
24	7467	7644	7822	8000	8178	8889	9067	9144
25	8056	8241	8426	8611	8796	9537	9722	9807
26	8667	8859	9052	9244	9437	10207	10400	10593
27	9300	9500	9700	9900	10100	10900	11100	11300
28	9956	10163	10370	10578	10785	11615	11822	12030
29	10633	10848	11063	11278	11493	12352	12567	12781
30	11333	11556	11778	12000	12222	13111	13333	13556
31	12056	12285	12515	12744	12974	13893	14122	14352
32	12800	13037	13274	13511	13748	14696	14933	15170
33	13567	13811	14056	14300	14544	15522	15767	16011
34	14356	14607	14859	15111	15363	16370	16622	16874
35	15167	15426	15685	15944	16204	17241	17500	17759
36	16000	16267	16533	16800	17067	18133	18400	18667
37	16856	17130	17404	17678	17952	19048	19322	19596
38	17733	18015	18296	18578	18859	19985	20267	20548
39	18633	18922	19211	19500	19789	20944	21233	21522
40	19556	19852	20148	20444	20741	21926	22222	22516
41	20500	20804	21107	21411	21715	22930	23233	23537
42	21467	21778	22089	22400	22711	23956	24267	24578
43	22456	22774	23093	23411	23730	25004	25322	25641
44	23467	23793	24119	24444	24770	26074	26400	26726
45	24500	24833	25167	25500	25833	27167	27500	27833
46	25556	25896	26237	26578	26919	28281	28622	28963
47	26633	26981	27330	27678	28026	29419	29767	30115
48	27733	28089	28444	28800	29156	30578	30938	31289
49	28856	29219	29581	29944	30307	31759	32122	32485
50	30000	30370	30741	31111	31481	32963	33333	33704
51	31167	31544	31922	32300	32678	34189	34567	34944
52	32356	32741	33126	33511	33896	35437	35822	36207
53	33567	33959	34352	34744	35137	36707	37100	37493
54	34800	35200	35600	36000	36400	38000	38400	38800
55	36056	36463	36870	37278	37685	39315	39722	40130
56	37333	37748	38163	38578	38993	40652	41067	41481
57	38633	39056	39478	39900	40322	42011	42433	42856
58	39956	40385	40815	41244	41674	43393	43822	44252
59	41300	41737	42174	42611	43048	44796	45233	45670
60	42667	43111	43556	44000	44444	46222	46667	47111

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