Kawamoto.

Electrical Ignition.

ELECTRICAL IGNITION

BY

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B. S. University of Illinois, 1916

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1 9 1 6

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1916.

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Tane Kawamoto

ENTITLED Electrical Ignition

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Electrical Engineering

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Recommendation concurred in:*

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*Required for doctor's degree but not for master's.

C O N T E N T S

NOMENCLATURE

E Constant electromotive force

e Transient electromotive force

e_i Transient electromotive force, induced

e_t Transient electromotive force, armature terminals

I Constant current

i Transient current

i_1 Transient current, primary coil

i_2 Transient current, secondary coil

i_c Transient current, condenser

i_a Transient current, armature

Q Constant quantity of electricity

q Transient quantity of electricity

R Resistance

R_1 Resistance, primary coil

R_2 Resistance, secondary coil

R_a Resistance, armature

r_s Resistance, spark gap

r_c Resistance, interrupter's contacts

R_S Resistance, secondary circuit, total

R_p Resistance, primary circuit, total

L Coefficient of self-induction

L_1 Coefficient of self-induction, primary coil

L_2 Coefficient of self-induction, secondary coil

M Coefficient of mutual-induction

C Capacity

X Inductive reactance

X_1 Inductive reactance, primary coil $= 2\pi f L_1$

X_2 Inductive reactance, secondary coil $= 2\pi f L_2$

X_a Armature ohmic Reactance

X_m Mutual-inductive reactance

X_c Capacity reactance

X_p Equivalent reactance of quantity $n(X_1 + X_a)$

Z Equivalent impedance of quantity $\sqrt{X_p^2 + R_p^2}$

N_1 Number of turns of the primary coil

N_2 Number of turns of the secondary coil

N_a Number of turns of the armature windings

w Angular velocity, equivalent to $2\pi f$

t Time

θ Angle, equivalent to $w t$

γ Angle, constant

f Frequency

π Ratio of the circumference of a circle to its diameter

n Number, 1, 2, 3, etc.

ϵ Base of naperian logarithms

j Imaginary quantity of $\sqrt{-1}$

l Length

S Speed, revolutions per minute

A, B, a, b, , , constants

D "Symbolic operator" (see any text on differential equations)

ELECTRICAL IGNITION

Introduction

One of the most important details of gas engine operation has been the development of a suitable means of ignition.

Ignition, with reference to the internal combustion engine, is the igniting of the explosive charge within the engine cylinder at the proper time to deliver the force of the expanding gases to the piston. Good ignition can be obtained by igniting the charge at the proper position of the piston with uniformity at varying speeds. It would seem at first thought to be a simple matter to construct a reliable machine to accomplish this, but the description of the different improvements given in the next chapter will show how much time and thought have been necessary to develop the present ignition systems.

At the present time, the most used and the most successful form of ignition is by electric spark, which is known as the jump spark ignition. This has proven to be the most satisfactory in the large majority of internal combustion engines and in automibles it is used exclusively. It is by far the most reliable and flexible method in use. There are two general systems of producing the jump spark, namely; battery and magneto systems.

The principles involved in these two systems are to produce a transient voltage across the spark plug points

which is sufficiently high to break down the dielectric medium between the plug points, producing spark discharges across them. Hence the successful operation of these two systems are entirely dependent upon the transient electric phenomena.

The object of this thesis is to study these transient phenomena experimentally and verify the results if possible by means of mathematical solutions.

Chapter I

Method of Ignition

The first successful form of ignition was obtained by means of an open flame which was drawn into the cylinder at the proper time. Flame ignition, however, is uncertain, difficult of application, not economical, and therefore has been abandoned on modern engines.

The next form of ignition used was the hot tube, in which a closed tube connecting with the engine cylinder was first heated to a red heat by means of an external flame. The gas is injected into the cylinder, compressed. This charge of gas under compression becomes explosive and will ignite from the hot tube. The heat of the burning gas is sufficient to keep the tube hot. This form of ignition is satisfactory in small engines especially with the constant speed stationary types, but is hardly sufficient to ignite a large volume of gas such as is admitted to a large engine, and is poorly adapted to timing variable speed engines.

One of the simplest forms of igniters is that used by the Deisel Engine Company. In this engine the air is compressed to a very high pressure and the temperature is then sufficient to ignite the entering charge of gas, which is delivered to the cylinder at a pressure slightly higher than the compression pressure. This then requires no special ignition apparatus, and the time of ignition is controlled by the time of admission to the cylinder.

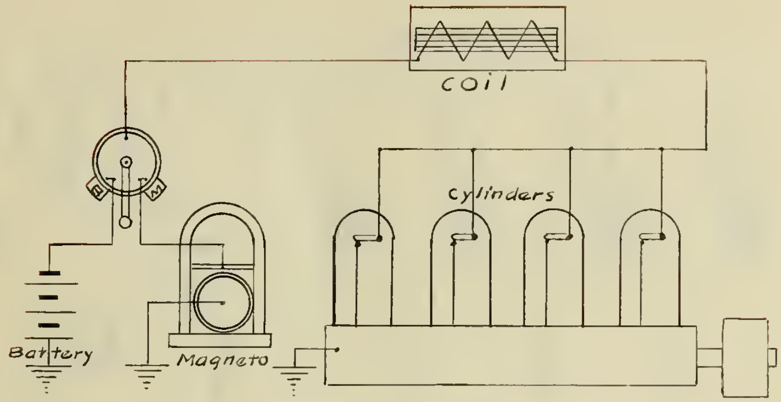

Fig. 1 "Make and Break".

"Make and break" was the first type of electrical ignition. The equipment consists of a coil of insulated wire wound around a soft iron wire core; a contact device inside the cylinder, operated by a cam mechanism, to close and open the circuit connected to the storage batteries or dry cells. The operation of a make and break ignition system is as follows:

When the contact is closed a current is established in the coil, when the contact is opened, the sudden break in the circuit causes an inductive kick, forming an arc between the opening contact thus igniting the charge. There was much trouble experienced with this system as it is difficult to keep the contacts clean.

"Jump spark ignition" was the next development. With this system the contact was brought outside the cylinder, eliminating the burned and dirty contacts, and in place a spark plug was used. The equipment used consists of one coil for each cylinder with a primary and secondary winding, a

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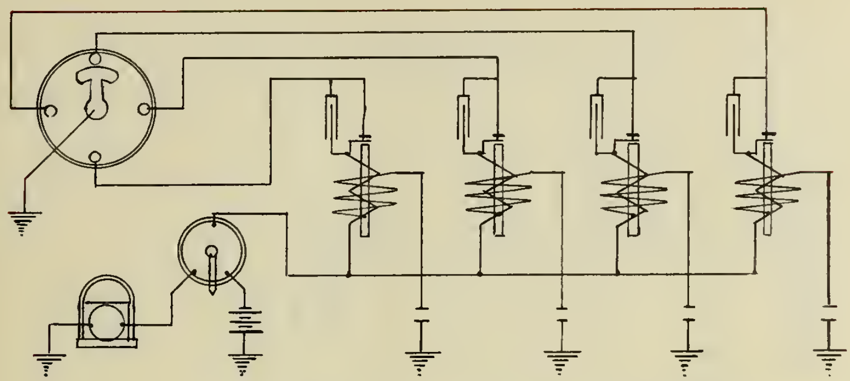

Fig. 2 "Jump Spark System"

vibrator being connected in series with the primary circuit of this coil, shunted with a condenser to make the interruption more effective, a low tension distributor with as many points as cylinders, a spark plug for each cylinder and a source of energy, either batteries or magneto.

Similar to the "make and break", after establishing a current in the primary coil, the primary circuit is suddenly broken and this change induces a high transient voltage in the secondary winding which in turn produces a spark across the plug points. It should be noted that this is the first system to use a transformer and a distributor timing mechanism for ignition purposes. Undoubtedly a great improvement was made in this system. Still the ignition was not satisfactory especially with a high speed engine as it was found impossible to make the different vibrators equally sensitive. This would cause an uneven running of the engine.

"A master vibrator", which controlled all the coils from one vibrator came next. It gave a uniformity in the

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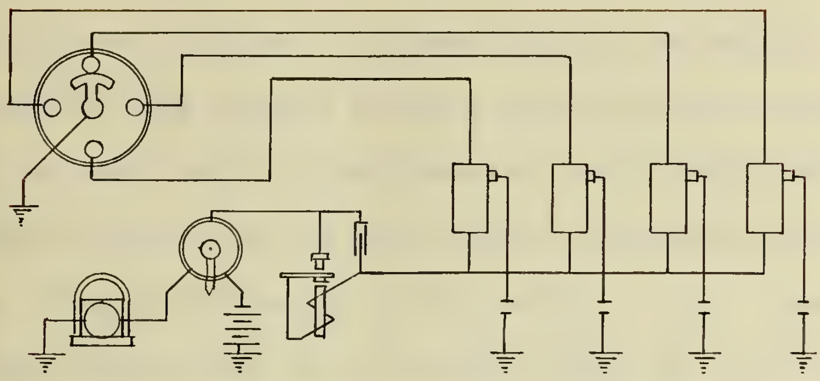

Fig 3 "Master Vibrator".

time of firing and the engine would operate very smoothly when the proper adjustment of this one vibrator was obtained.

The single spark system of ignition made use of an interrupter instead of the master vibrator. This was placed

Fig. 4 One type of Interrupter.

at one end of the magneto and a cam on the same shaft opened the interrupter contact, the same as did the master vibrator, except that it was opened without any lag and occurred at exactly the same angle regardless of the engine speed. The interrupter could also be adjusted for advance and retard position. The low-tension distributor at this stage practically disappeared, and in its place came the high-tension distributor, which eliminated all but one coil. Also at this stage all direct current types of magnetos were discarded and the alternating current type was used, arranged so that the time of break in the interrupter occurred at the peak of the e.m.f. wave. This system of ignition is one that is most commonly employed in all recent internal combustion engines especially with the high power, high speed engines, such as those used in automibles, aeroplanes, and motor boats.

The scope of this paper is to present the operating characteristics of this system which may however be subdivided into three general classifications, namely:

1. Battery distributor ignition
2. Low-tension magneto ignition
3. High-tension magneto ignition

Each of these three methods will be fully described and discussed separately in the chapters III and V.

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Chapter II

Apparatus Employed

The principal apparatus used throughout the entire experiment was an oscillograph. Since the experimental part of the subject is the study of electrical transients that occur at different stages of the operation of ignition units, ~~and~~ this can best be done by making photographic records of the transient currents or voltages with an oscillograph. The oscillograph in question was made by the General Electric Company, the type of which is Duddel's. As the construction, manipulation, and working principles of an oscillograph is well known to Electrical Engineers, they will purposely be omitted in the paper. However, it must be pointed out that the data obtained from the oscillograph is not accurate enough for precision work even with a very accurate calibration, but is fairly consistent for the purposes, such as the object of this paper, which requires the results within three to five percent errors. This error is due to the fact that the width of the curves themselves may be several percent of their magnitudes, in some cases they are as high as fifty percent.

The figure of merit or sensitivity of the oscillograph galvanometer elements were calibrated with direct current from a storage battery, with the ordinary ammeters and voltmeters of the Weston type.

The magnetoes that were tested in the course of

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Fig. 6 Bosch.

Fig. 5 Dixie.

Fig 7 Splitdorf.

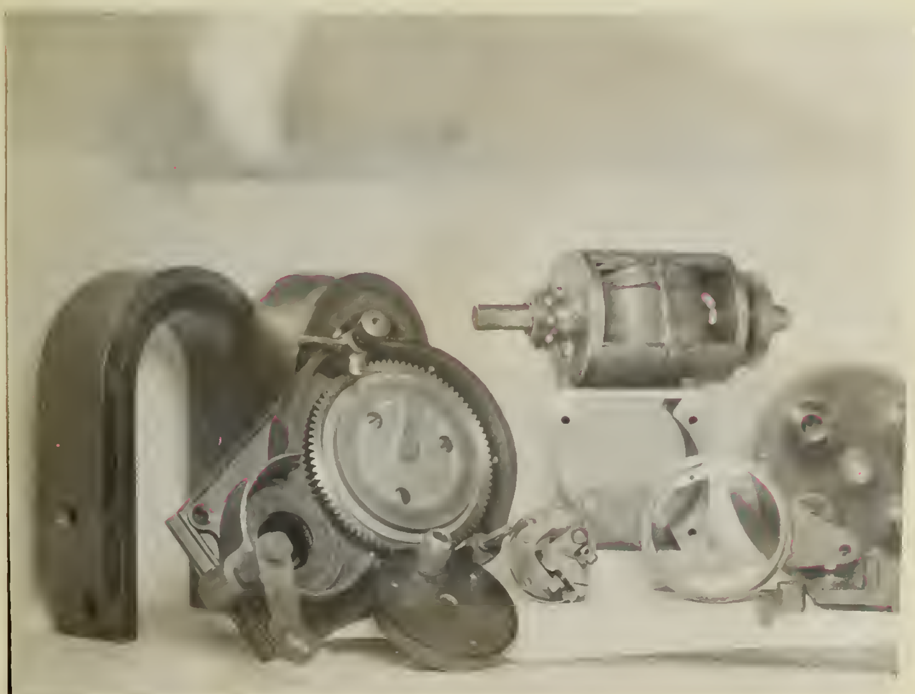

Fig. 8 G. E.

Fig. 9 Kingston.

Fig. 10 National.

these experiments were as follows;

Dixie high-tension

Splitdorf low-tension, Model T

Bosch high tension, Type N U - 4

G. E. low-tension

Kingston low-tension, Model L

National low-tension, Model C - 4

A large percent of the following data relates to the first two types including the battery distributor system of the second machine. The description of the principal types will be given separately in the following chapters.

The motor used to drive the magnetoe was a direct current shunt motor rated 4 H.P., 110 volts, 1275 R.P.M. It will be noted that the capacity of the motor is many times larger than the necessary power to drive a magneto. This was purposely chosen in order to keep a constant speed for any given set of the tests. The variation of the motor speed was obtained by armature rheostat method and the field excitation was kept as stiff as possible so that the disturbance of speed due to the armature reaction was a minimum.

A definite wave point closing switch the function, of which was to close the relay switch at any predetermined phase angle, that is any desired point on the alternating electro-motive force wave, was devised. The complete analysis of the working principles, construction, and manipulation of this special device, which was originated by the

author, will be found in Appendix "B". The transients of the alternating current depend upon the phase angle at which the circuit is closed or opened. If the switch is closed or opened at random the curve thus obtained is of little or no use for the analytical purposes. There is, however, a similar device developed by Mr. Bagley in 1913 at University of Illinois which serves the same purpose, but for the author's use the switch was somewhat inconvenient, as it required too much power to operate. Further the contact point made by means of a roller draws arcs due to poor contact when the switch is closed, which in turn produces an irregularity in the curve at the most important part of the transient.

A rotating contact maker which is a part of the above switch was made use of for another purpose by connecting it directly to the shaft of the magneto and impressing and e.m.f. across the two stationery brushes attached to the contact maker, the curcuit of which contains an oscillograph element in series with a resistance. Thus when the magneto is running, at a certain definite position of the armature with reference to the field magnet, the contact point on the rotating disc and one of the brushes close the circuit, the other brush being at rest on a slip ring. This starts an impulse current that flows through the oscillograph element making a record on the photographic film at this particular instant. If we use another or two more elements for some characteristic waves, such as current or e.m.f., this impulse kick serves

as a reference point for any number of films so long as the position of brush is not changed. The reference point due to this device will be seen in the Figs. 20 to 23.

The determinations of the constants such as the numerical values of resistances, capacity, and self inductances were made by the wheatstone bridge methods. The inductance of the armature winding was determined by the same method - Anderson modification of the Maxwell's method - for several different positions of the armature over a complete revolution.(See graph 5).

Chapter III

Battery Distributor System

The battery distributor ignition consists of three or more storage batteries as a source of energy, high-tension distributor, interrupter, and a transformer, the diagram of connections of which are shown in Fig. 11.

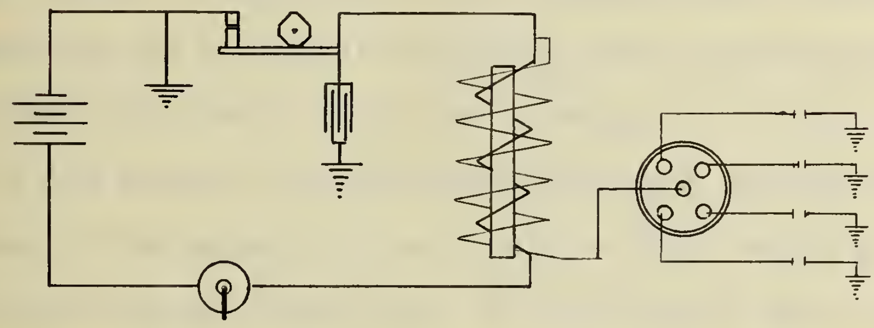

Fig 11 Battery Distributor System.

This system is usually used in combination with a low tension magneto ignition. A switch is provided to shift the source of power from battery to magneto or vice verse. This method is very satisfactory as an igniter especially with a low speed engine. However, it is somewhat unsatisfactory with high speed engines, the reason for which will be explained later in connection with the mathematic formula. Hence this system is ideal ignition for the starting or cranking speed. Thus after the engine is once started the source of energy is transferred from the battery to magneto which is more suitable means of ignition for higher engine speed.

The operating principles of the battery ignition system may briefly be presented here before taking up the

analytical part of the principles involved.

On rotating the engine shaft the interrupter moves to a position where the primary or the battery circuit is closed and a current is established in the primary coil. This current produces a flux proportional to the product of current and magnetic permeability in the transformer core. For the sake of simplicity, let us assume that the magnetic permeability of a core is constant, then the flux produced is directly proportional to the strength of current and the part of the energy thus withdrawn from the battery is stored in form of the magnetic flux equals to $\frac{L\ I^2}{2}$ where L is the coefficient of self-induction of the primary coil and I is the final value of the current, and the rest of the energy is discipated in the form of heat.

But as the engine is still further rotated the primary circuit is interrupted; the current and therefore flux rapidly fall to zero. This sudden change of flux induces a high voltage in the secondary coil which in turn causes a spark to jump across one of the plugs to the engine frame and thence through the frame to the ground connection of the secondary coil. This process is repeated as the engine rotates, excepting that on each successive break the distributor has moved so as to connect the secondary with the proper engine cylinder.

The theoretical explanation of these transient currents, hence the useful transient voltage which tends to produce spark discharges can best be made by means of the

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mathematical formulae. The process of deriving these formulae will be found in Appendix "A".

From the equation (3), the growth of primary current from the instant the interrupter closes the circuit,

$$i_1 = \frac{E}{R_1}\left(1 - \epsilon^{-\frac{R_1}{L_1}t}\right) \quad \text{-----------------} (3)$$

which shows that an infinite time is required for the current to reach the steady value $\frac{E}{R_1}$. But in circuits met with in practice i_1 will be very nearly equal to $\frac{E}{R_1}$ in a comparatively short interval of time since the term $\epsilon^{-\frac{R_1}{L_1}t}$ converges very rapidly. It may also be seen that the rate of growth of the current depends upon the time constant $\frac{R}{L}$. Therefore, it is evident that with a given coil there must be allowed a certain length of time for the current to build up to some value which is sufficiently high to produce sparks across the plug when the circuit is interrupted. Hence if this interruption comes too soon, as in case of high engine speeds, the current in the primary coil has had no time to attain the required value to magnetize the core. The result is the production of a much weaker spark, which may or may not be satisfactory as an igniter.

However, if a ballast or external rheostat be provided so that the total primary resistance is increased, the time necessary to build up the primary current to any pre-assigned value may be varied within a reasonable range. Such a device is found in the Westinghouse vertical ignition unit.

Table I gives the calculated and observed values of

Table I

Growth of the primary current

Splitdorf - Battery circuit

E = 6.2 *volts*
R = 1.48 *ohms*
L = .0099 *henry*

---calculated-----				observed	
t	$\epsilon^{-\frac{R}{L}t}$	$1-\epsilon^{-\frac{R}{L}t}$	i_1	i_1	Diff.
0	1	0	0	0	0
.001	.861	.139	.581	.523	-.058
.002	.74	.26	1.09	1.024	-.066
.003	.638	.362	1.52	1.5	-.02
.004	.549	.451	1.89	1.8	-.09
.005	.472	.528	2.21	2.2	-.01
.006	.407	.593	2.48	2.605	.125
,007	.35	.65	2.72	2.75	.03
.008	.301	.699	2.92	2.99	.07
.009	.259	.741	3.1	3.24	.14
.010	.223	.777	3.25	3.37	.12
.011	.192	.808	3.38	3.52	.14
.012	.165	.835	3.5	3.64	.14
.013	.142	.858	3.59	3.735	.145
.014	.122	.878	3.68	3.81	.13
.015	.105	.895	3.75	3.87	.12
.016	.091	.909	3.82	3.92	.1
.017	.078	.922	3.86	3.97	.09
.018	.067	.933	3.9	4.02	.12
.019	.058	.942	3.94	4.06	.12
.02	.05	.95	3.97	4.08	.11
.023	.032	.968	4.05	4.1	.05
.025	.024	.976	4.08	4.125	.045
.03	.011	.989	4.14	4.18	.04
.044	0.	1.	4.18	4.18	0.

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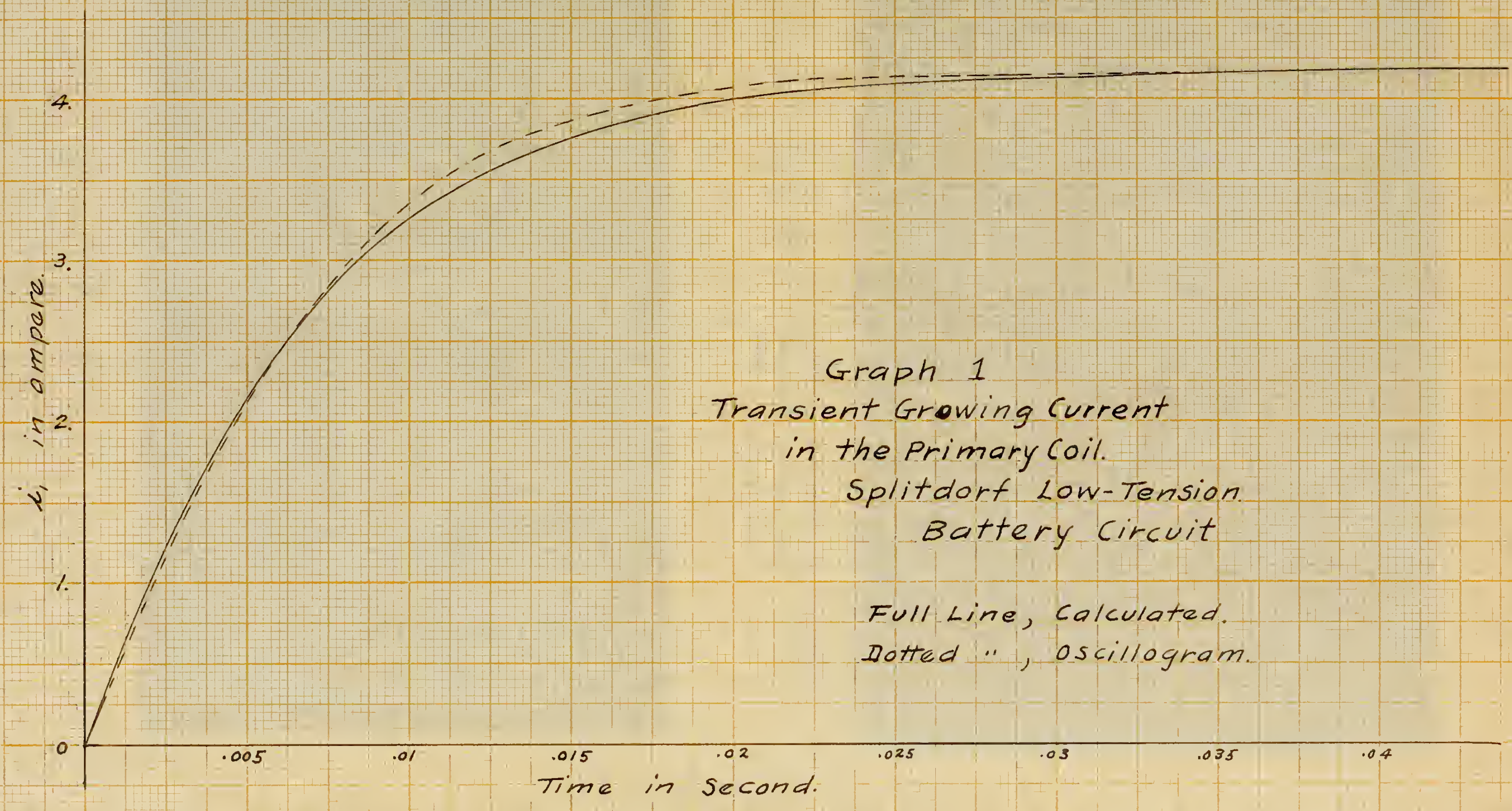

Graph 1
Transient Growing Current
in the Primary Coil.
Splitdorf Low-Tension
Battery Circuit
Full Line, Calculated.
Dotted " , Oscillogram.
i, in ampere.
0
1.
2.
3.
4.
.005
.01
.015
.02
.025
.03
.035
.04
Time in Second.

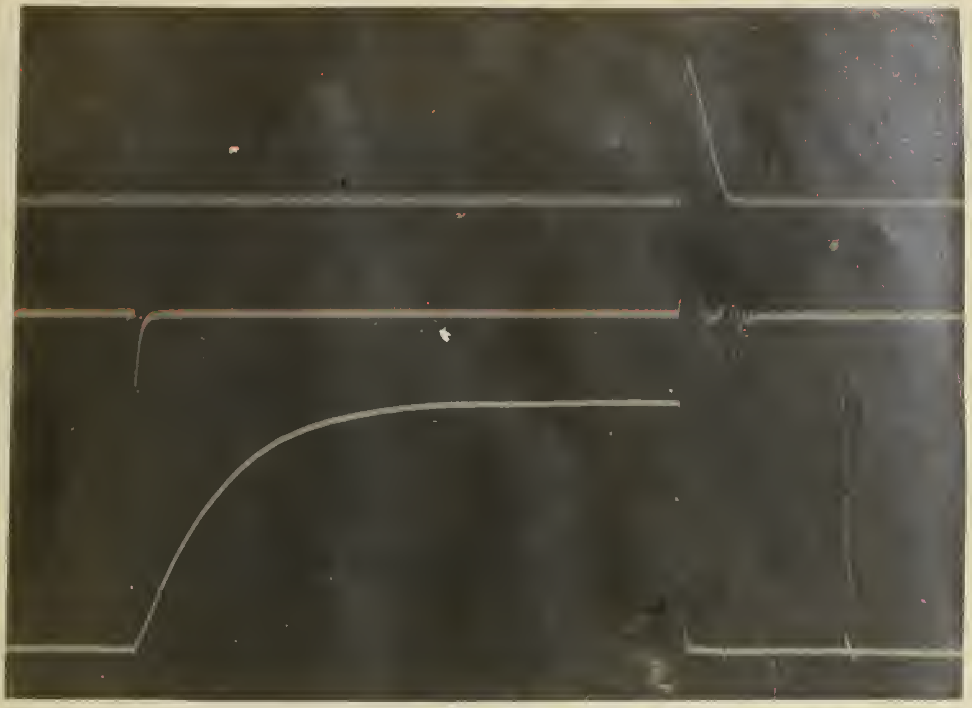


Fig. 12 — First: Secondary current. Second: Condenser " . Third: Primary " .

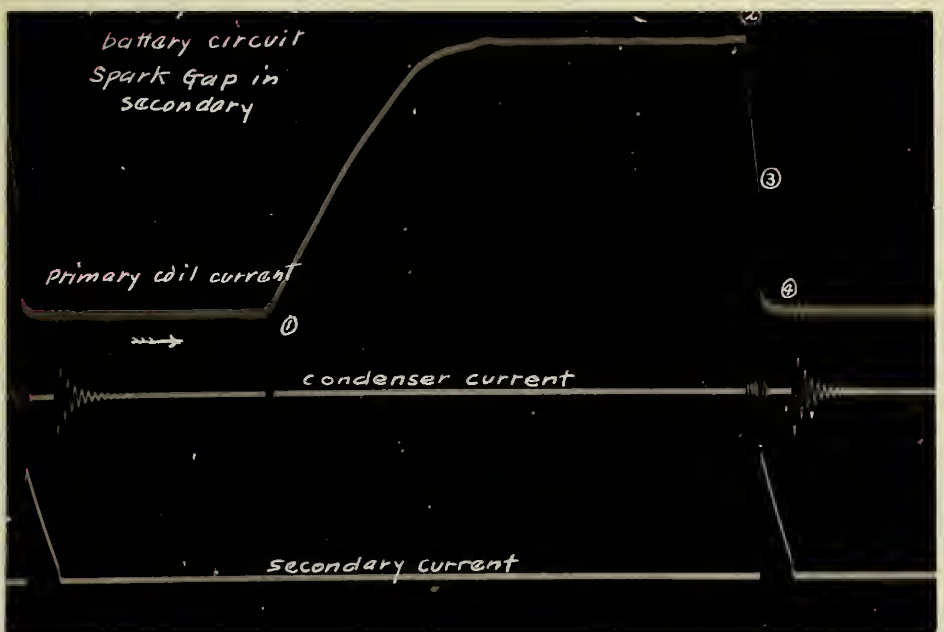


Fig. 13

growing current of a Splitdorf low-tension magneto when used on battery circuit and these two curves are shown in Graph I.

Fig. 12 is an oscillograph from which the observed value of growing current was obtained.

The decaying process of the primary current and the current (spark) in the secondary circuit should follow either of two sets of the equations,

$$i_1 = \frac{R_s M I + L_2 E}{R_s M + L_2 R_1} \frac{\epsilon^{-\alpha t}}{\cos\theta} \cos(bt+\theta) + \frac{\sqrt{a^2+b^2}}{\cos\theta} \epsilon^{-\alpha t} \sin bt \quad \cdots (25)$$

and

$$i_2 = \frac{n_1}{n_2} \frac{L_2}{R_s M} \left[R_1 i_1 + \left(\frac{R_s M I + L_2 E}{C(R_s M + L_2 R_1)} \cdot \frac{1}{b} - E \tan\theta \right) \epsilon^{-\alpha t} \sin bt - E \epsilon^{-\alpha t} \cos bt \right] (26)$$

or

$$i_1 = \frac{R_s M I + L_2 E}{R_s M + L_2 R_1} \left[\frac{\alpha}{\alpha-\beta} \epsilon^{-\alpha t} - \frac{\beta}{\alpha-\beta} \epsilon^{-\beta t} \right] - \frac{\alpha\beta C E}{\alpha-\beta} \left[\epsilon^{-\alpha t} - \epsilon^{-\beta t} \right] - (21)$$

and

$$i_2 = \frac{n_1}{n_2} \frac{L_2}{R_s M} \left[R_1 i_1 - \frac{1}{\alpha-\beta} \left(\frac{R_s M I + L_2 E}{C(R_s M + L_2 R_1)} - E \right) \left(\epsilon^{-\alpha t} - \epsilon^{-\beta t} \right) \right] \quad \cdots (22)$$

The first set being an oscillatory and the second a logarithmic form the Fig. 12 - Splitdorf - shows the former while the Fig. 13 - G.E. - indicates the latter case.

It should be noted in these two oscillograms that the dying away of the secondary spark current is very sharp. It is entirely due to the variation of spark gap resistance. For if the resistance of secondary circuit is constant the decaying of secondary current follows exactly as equations (23) or (27). Fig. 14 shows the shape of induced current in secondary circuit with constant resistance of 7500 ohms across the gap - Splitdorf.

Thus the apparent resistance of the spark gap is a function of the current density. We have the relation be-

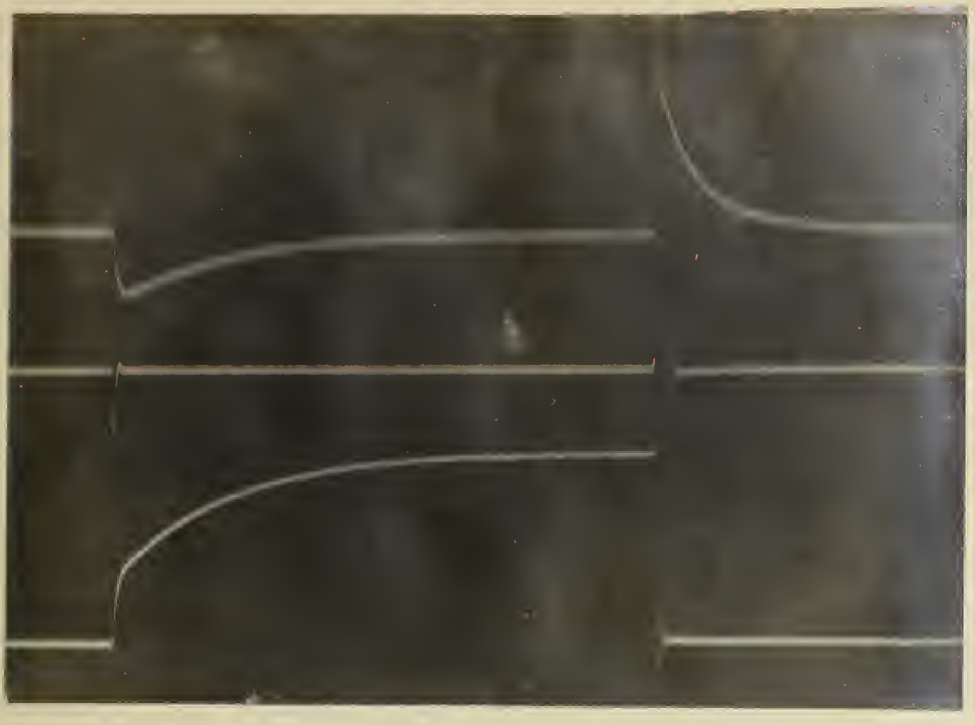


Fig. 14 First, Secondary current.
Second, Condenser " .
Third, Primary " .

tween the e.m.f. e across the plug points and spark current i, which may be expressed in an equation:

$$e = e_o + \frac{K}{\sqrt{i_2}} \quad \text{-------------------} \quad (A)$$

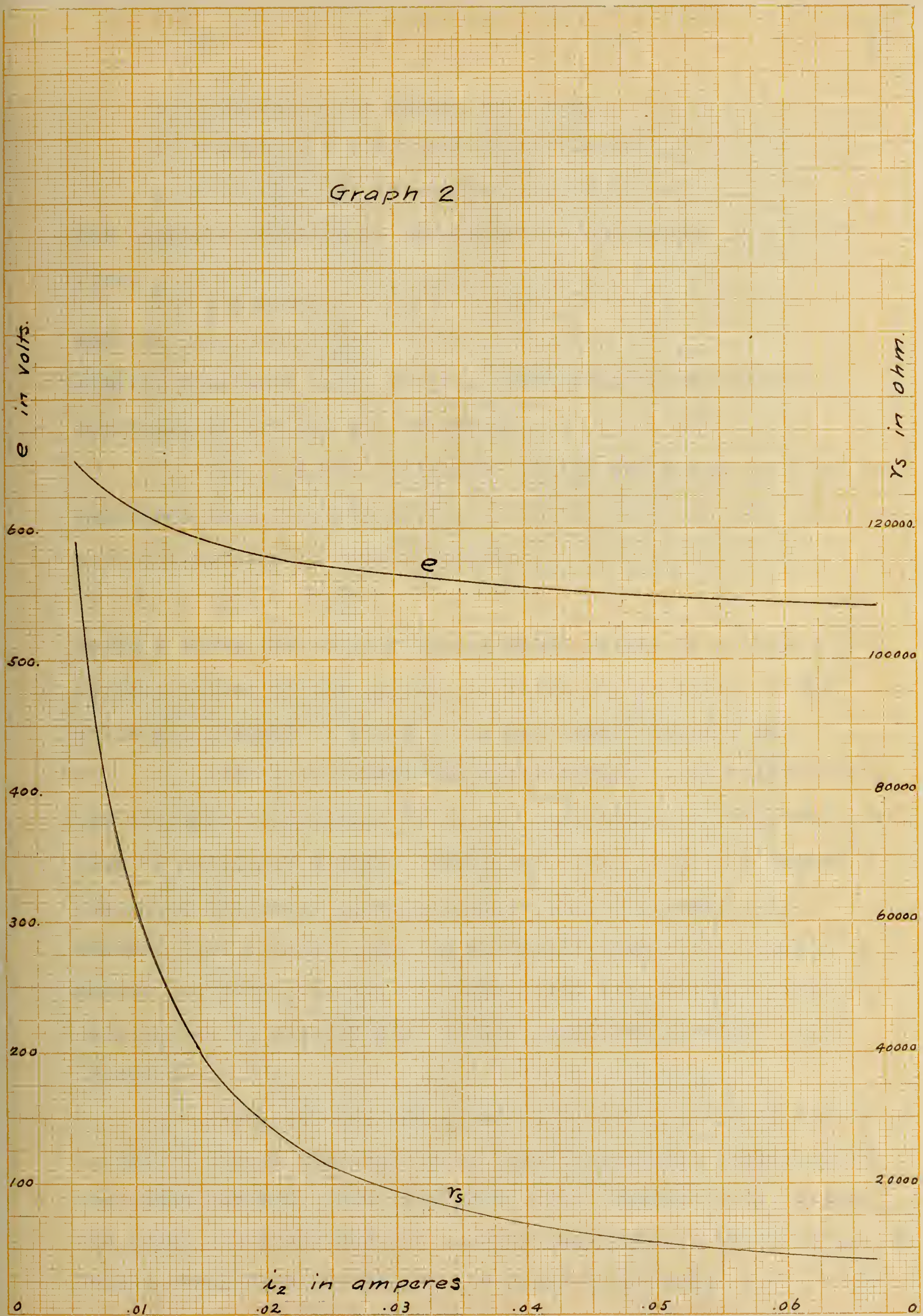
where e_o is voltage drop across the gap independent of the strength of current flowing through the medium (gaseous mixture in our case) K being a function of the spark gap length and is constant for any set of plugs which have definite shape and length of the gap.

The approximate values of e_o and K for a few known substances may give more definite explanation of the equation (A):

e_o = 13 volts for mercury vapor

= 16 volts for zinc and cadium arc

= 30 volts for magnetite arc


Graph 2
e in volts.
r_s in ohm.
e
r_s
600.
500.
400.
300.
200
100
0
120000.
100000
80000
60000
40000
20000
0
.01
.02
.03
.04
.05
.06
i_2 in amperes
U. OF I. S. S. FORM 3

e_o = 36 volts for carbon arc, etc.

K = 48.5 (l + .125) for magnetite arc

= 51 (l - .8) for carbon arc, etc.

the complete equation of the magnetite and carbon arcs therefore,

$$e = 30 + 48.5 \frac{l + .125}{\sqrt{i_2}}$$

and $$e = 36 + 51. \frac{l - .8}{\sqrt{i_2}}$$

similar constants exist for the spark plug points as the terminals of arc and gas as medium.

The apparent resistance of the spark gap may also be expressed in a function of i_2. From the equation (A), the resistance across the plug points is

$$r_s = \frac{1}{i_2}\left(e_o + \frac{K}{\sqrt{i_2}}\right) \quad \text{- - - - - - - - - - - - - - - - - - - -} \quad (B)$$

Graph 2 shows the roughly approximated values of e and r_s for various values of the current in which e_o is taken as 500 volts and K equal to 11.25. The gap length being 1 mm.

From graph 2 and the oscillograph it is evident that the increase of apparent spark gap resistance is so great, the flow of secondary current ceases abruptly. And this sudden change of the spark current induces in the primary coil a further disturbance producing an oscillatory current which is damped out very quickly. The magnified view of this damped oscillation is clearly seen in the condenser current. Figs. 12 and 13.

Figs. 15, 16, and 17 are the oscillograms of similar transient currents of Splitdorf low-tension unit operating at the speeds of 800, 1400, and 2000 R.P.M. respectively. Higher the speed the spark current becomes less uniform which is due to the early interruption of the growing current.

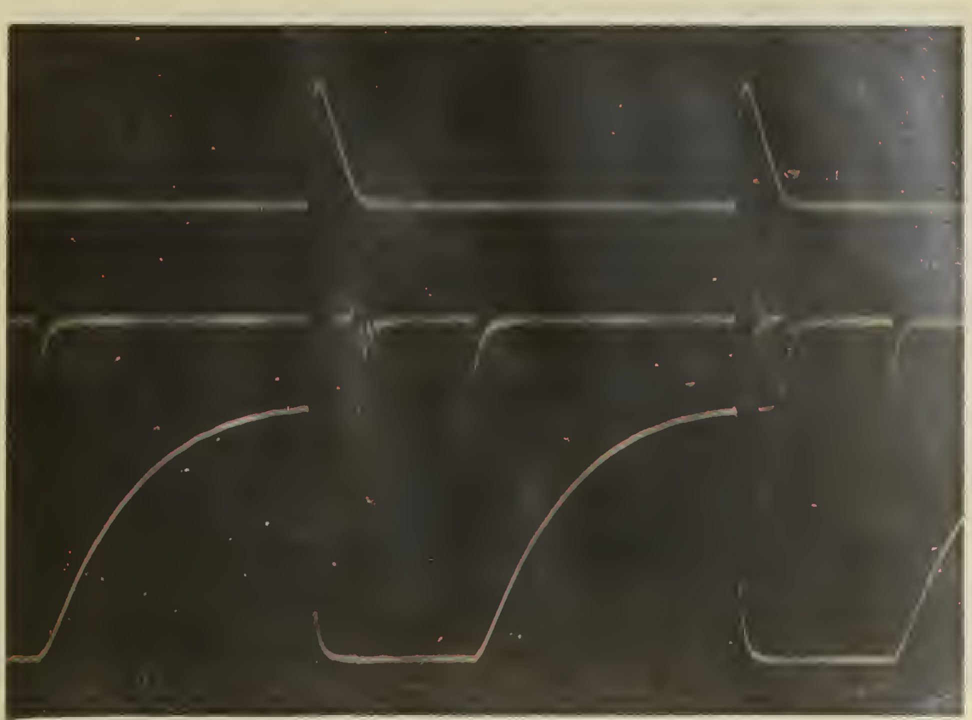


Fig. 15

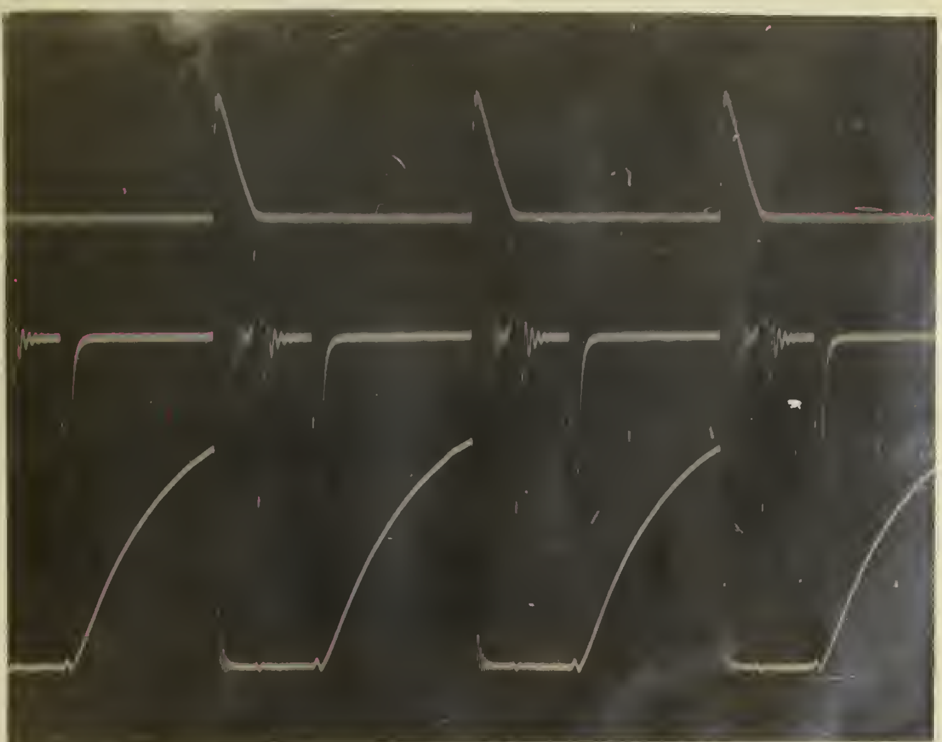


Fig. 16

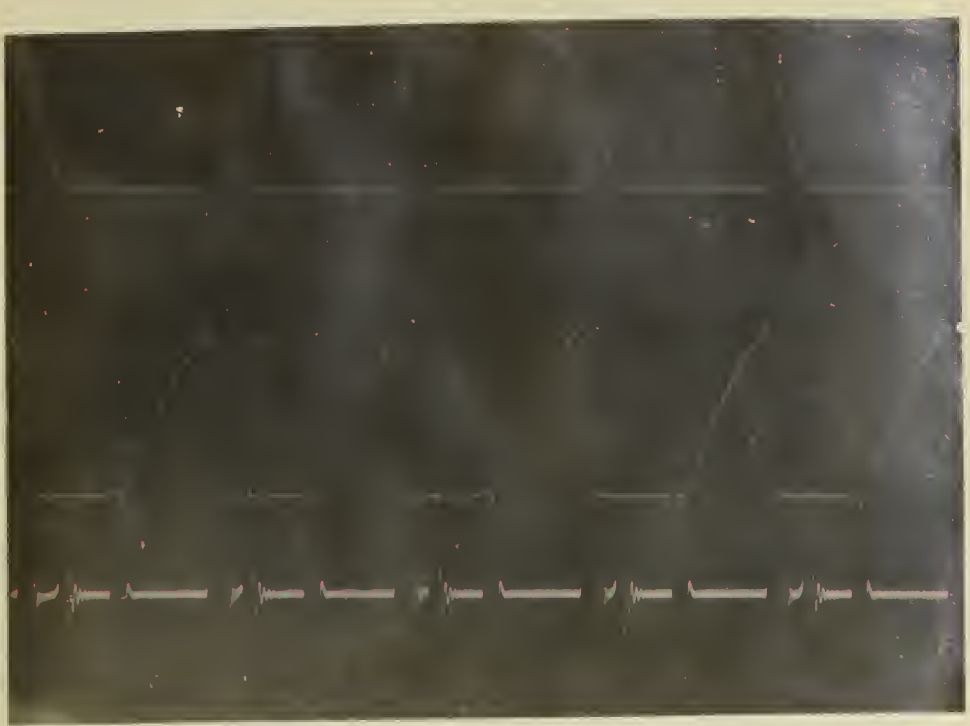


Fig. 17

Chapter IV

Open and short circuit characteristics of the Magneto Generator

The characteristics of a magneto generator may be divided into open circuit characteristic, short circuit characteristic, and the combination of above two constitutes operating or performance characteristic.

The open circuit characteristic of any magneto includes flux distribution in the air gap and the e.m.f. generated. The flux distribution wave of the machine of this type is naturally a flat top consequently the e.m.f. wave is highly peaked form. Fig. 20, 21, and 22 are oscillograms of open circuit e.m.f. waves of Splitdorf low-tension, Dixie high-tension, and Bosch high-tension respectively. Analysis of Splitdorf low-tension e.m.f. wave is given in the tables 2-a to 2-d which shows e.m.f. wave contains appreciable harmonics as high as 11th. The subsequent analyses show that 33rd harmonic has the maximum value of nearly .03 volts that is somewhere near one-third percent of the fundamental wave.

The graph 3 shows approximate wave forms of e.m.f. and flux distribution at no load with the proper phase angle displaced. The flux distribution wave is obtained from the following;

we have $$e = -N_a 10^{-8} \frac{d\phi}{dt} = -2\pi f 10^{-8} \frac{d\phi}{d\theta}$$

or
$$\phi = -\frac{10^8}{2\pi f N_a} \int e\, d\theta = -K \int e\, d\theta$$

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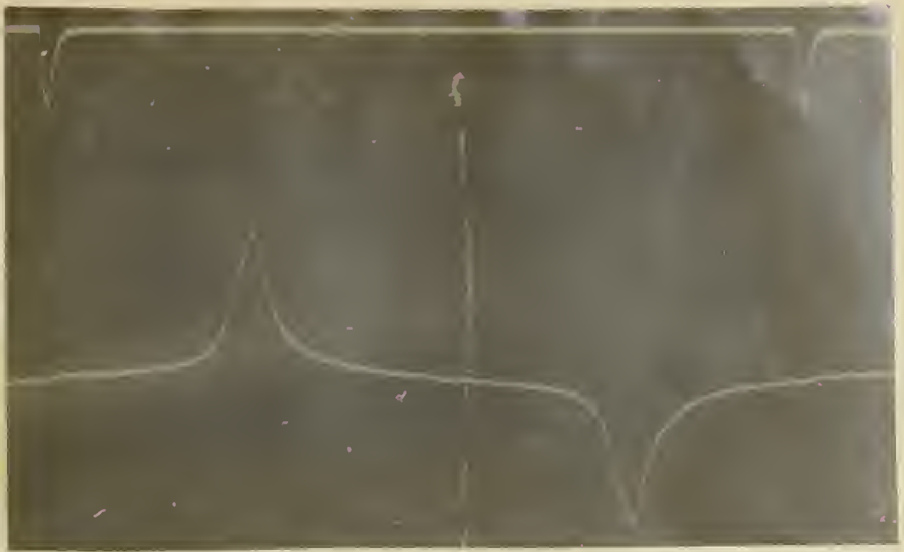


Fig. 20

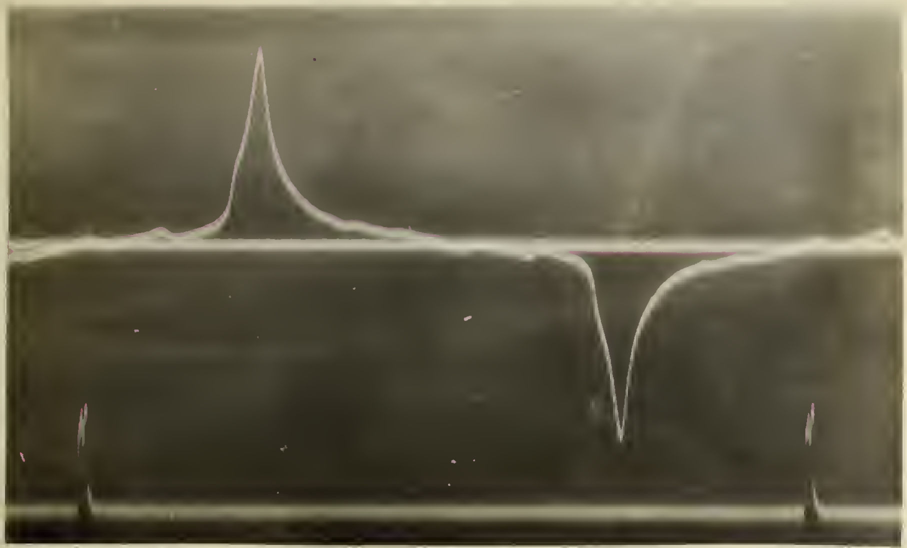


Fig. 21

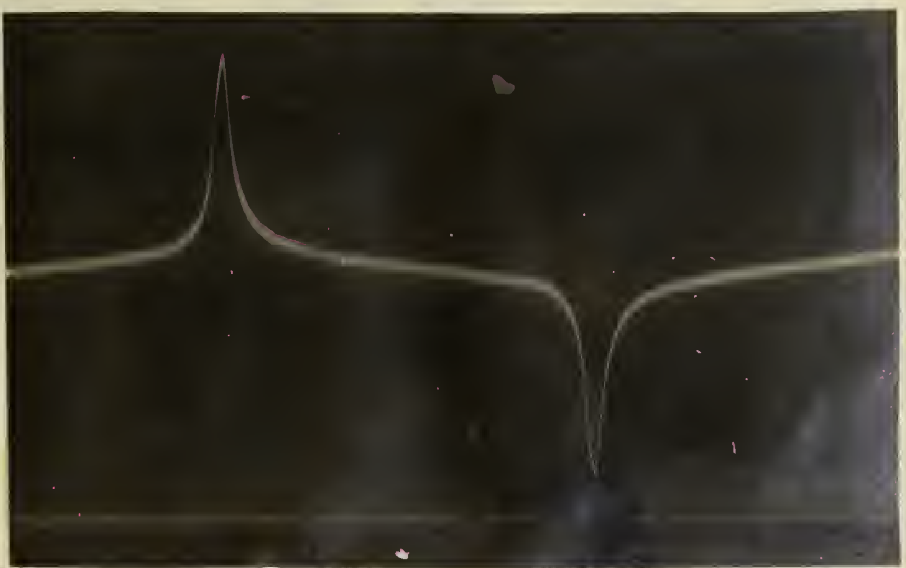


Fig. 22

Table 2 - a

Wave Analysis of the Open Circuit Voltage

Splitdorf Low Tension Magneto

$\theta°$	e	$e\,\sin\theta$	$e\,\sin 3\theta$	$e\,\sin 5\theta$	$e\,\sin 7\theta$	$e\,\sin 9\theta$	$e\,\sin 11\theta$
0	0	0	0	0	0	0	0
5	.158	.014	.0409	.0669	.0906	.1115	.1294
10	.317	.055	.1585	.2425	.2977	.317	.298
15	.475	.123	.3359	.459	.459	.336	.123
20	.633	.216	.548	.623	.407	0	-.407
25	.792	.335	.765	.6475	.0688	-.56	-.788
30	.95	.475	.95	.475	-.475	-.95	-.475
35	1.109	.636	1.07	.0965	-1.003	-.783	.469
40	1.267	.816	1.097	-.434	-1.247	0	1.247
45	1.505	1.063	1.063	-1.063	-1.063	1.064	1.063
50	1.742	1.335	.871	-1.635	-.303	1.742	-.303
55	1.98	1.62	.513	-1.971	+.838	1.4	-1.79
60	2.376	2.05	0	-2.058	+2.058	0	-2.055
65	3.168	2.87	-.627	-1.82	+3.06	-2.238	-.256
70	5.069	4.76	-2.0345	-0.882	+3.88	-5.069	3.88
75	6.336	6.12	-4.325	+1.64	+1.64	-4.475	6.12
80	10.692	10.52	-9.27	+6.87	-3.65	0	3.542
85	20.59	20.5	-19.9	+18.62	-16.82	14.53	-11.81
90	25.74	25.74	-25.74	+25.74	-25.74	25.74	-25.74
Σ		+79.248	-54.4842	+45.6174	-37.6019	+31.1685	-26.7526

Table 2 - b

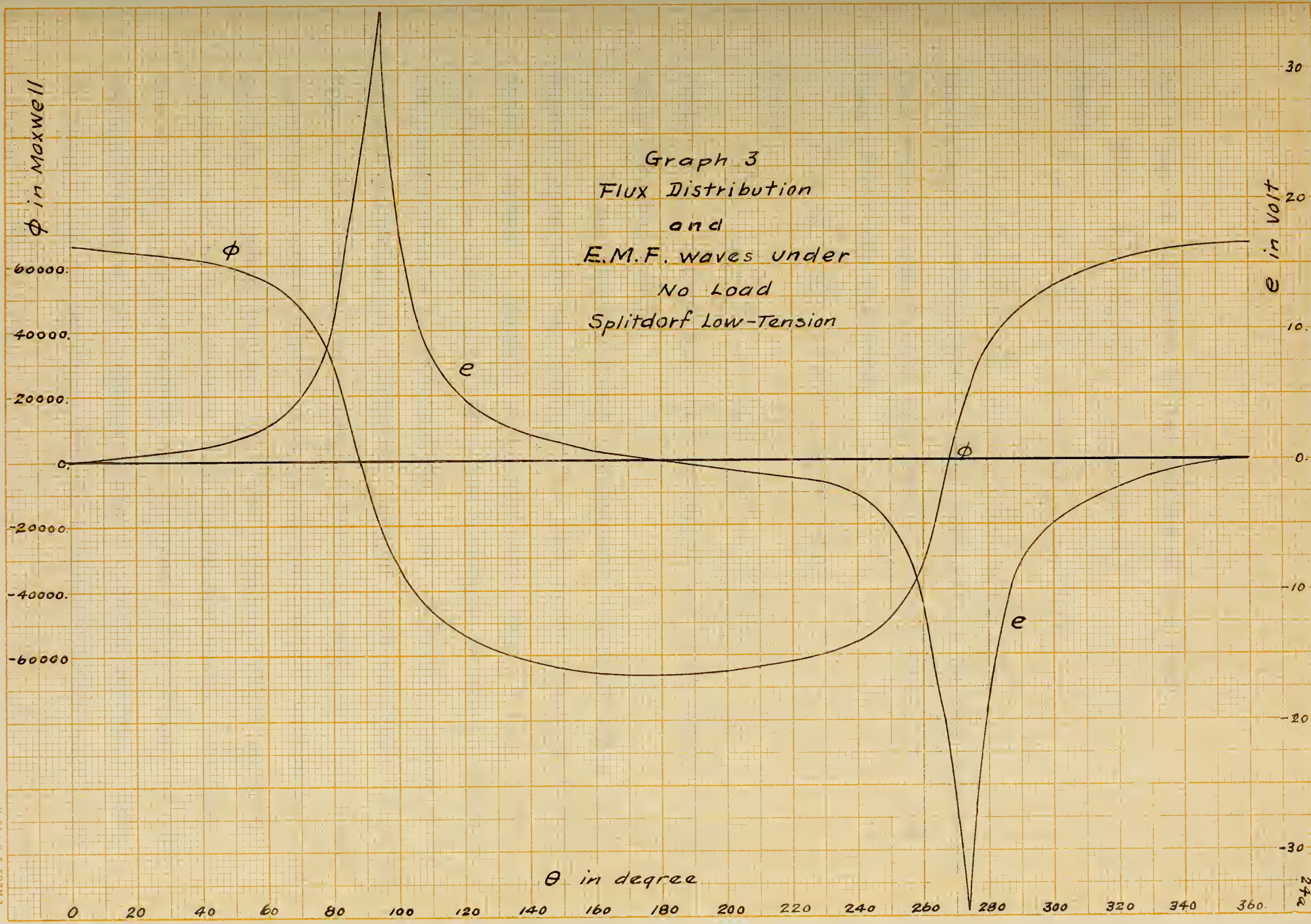
θ°	e	$e\,\mathrm{Sin}\,\theta$	$e\,\mathrm{Sin}\,3\theta$	$e\,\mathrm{Sin}\,5\theta$	$e\,\mathrm{Sin}\,7\theta$	$e\,\mathrm{Sin}\,9\theta$	$e\,\mathrm{Sin}\,11\theta$
95	32.472	32.38	-31.4	29.4	-26.6	23.9	-18.62
100	17.424	17.15	-15.08	11.2	-5.96	0	5.96
105	11.088	10.7	-7.83	2.87	2.87	-7.84	10.7
110	8.158	7.66	-3.83	-1.418	6.25	-8.158	6.25
115	6.415	5.81	-1.66	-3.68	6.38	-4.53	-.558
120	4.91	4.25	0	-4.25	4.25	0	-4.25
125	3.96	3.24	1.025	-3.94	1.675	2.795	-3.59
130	3.247	2.485	1.6235	-3.03	-.565	3.247	-.565
135	2.772	1.955	1.956	-1.956	-1.956	1.957	1.96
140	1.98	1.272	1.713	-.677	-1.95	0	1.95
145	1.663	.955	1.607	.145	-1.51	-1.175	.704
150	1.426	.713	1.426	.713	-.713	-1.426	-.713
155	1.104	.47	1.07	.907	.0965	-.784	-1.1
160	.871	.282	.754	.848	.56	0	-.56
165	.713	.184	.504	.688	.688	.504	.1845
170	.473	.0825	.2365	.363	.445	.473	.444
175	.238	.0207	.0617	.1007	.1365	.168	.195
180	0	0	0	0	0	0	0
Σ		89.6092	-47.8233	28.2837	-15.903	9.131	-1.6085
Grand total		168.8572	-102.3075	73.9011	-53.505	40.2965	-28.3611
Divided by 18		9.381	-5.684	4.100	-2.972	2.238	-1.575

Table 2 - c

θ^o	e	$e\cos\theta$	$e\cos 3\theta$	$e\cos 5\theta$	$e\cos 7\theta$	$e\cos 9\theta$	$e\cos 11\theta$	$e\cos 13\theta$
0	0	0	0	0	0	0	0	0
5	.158	.1575	.1524	.143	.1294	.1115	.0907	.062
10	.317	.312	.2741	.204	.1336	0	-.1085	-.2038
15	.475	.469	.335	.123	-.123	-.336	-.459	-.458
20	.633	.595	.3165	-.11	-.485	-.633	-.485	-.11
25	.792	.722	.205	-.454	-.789	-.56	+.0688	.648
30	.95	.822	0	-.822	-.822	0	.823	.823
35	1.109	.908	-.287	-1.104	-.47	.783	1.005	-.0965
40	1.267	.972	-.486	-1.19	.2208	1.267	.2205	-1.19
45	1.505	1.151	-1.062	-1.064	1.064	1.064	-1.064	-1.063
50	1.742	1.12	-1.507	-.596	1.715	0	-1.715	.597
55	1.98	1.135	-1.91	.1599	1.795	-1.4	-.837	1.97
60	2.376	1.187	-2.376	1.188	1.188	-2.376	1.188	1.188
65	3.168	1.341	-3.06	2.59	-.255	-2.238	3.16	-1.82
70	5.069	1.73	-4.382	4.99	-3.26	0	3.26	-4.99
75	6.336	1.64	-4.47	6.12	-6.12	4.475	-1.643	-1.64
80	10.692	1.86	-5.346	8.19	-10.05	10.692	-10.02	8.2
85	20.59	1.78	-5.325	8.72	-11.8	14.53	-16.83	18.62
90	25.74	0	0	0	0	0	0	0
Σ		17.8015	-28.928	27.0879	-27.9582	25.2375	-23.3455	20.5367

Table 2 - d

θ°	e	$e\cos\theta$	$e\cos3\theta$	$e\cos5\theta$	$e\cos7\theta$	$e\cos9\theta$	$e\cos11\theta$	$e\cos13\theta$
95	32.472	-2.81	8.42	-13.75	18.6	-23.9	26.6	-29.4
100	17.424	-2.98	8.712	-13.36	16.39	-17.424	16.37	-13.35
105	11.088	-2.77	7.84	-10.7	10.7	-7.84	2.87	2.87
110	8.158	-2.62	7.06	-8.04	5.25	0	-5.25	8.04
115	6.415	-2.46	6.2	-5.25	.57	4.53	-6.39	3.68
120	4.91	-2.125	4.91	-2.455	-2.455	4.91	-2.455	-2.455
125	3.96	-1.858	3.824	-.345	-3.59	2.795	1.675	-3.94
130	3.247	-1.595	2.81	1.11	-3.2	0	3.2	-1.11
135	2.772	-1.38	1.955	1.955	-1.955	-1.957	1.96	+1.955
140	1.98	-.975	.99	1.866	-.3442	-1.98	-.245	1.86
145	1.663	-.782	.4315	1.655	.705	-1.175	-1.507	.1445
150	1.426	-.618	0	1.235	1.235	0	-1.234	-1.235
155	1.109	-.426	-.287	.636	1.103	.784	-.0965	.908
160	.871	-.265	-.4355	.1515	.668	.871	.667	.1515
165	.713	-.1776	-.5035	-.1845	.1845	.504	.688	.688
170	.473	-.0812	-.4095	-.304	-.1995	0.	.162	.304
175	.238	-.0206	-.23	-.216	-.195	-.168	-.1365	-.1008
180	0	0	0	0	0	0	0	0
Σ		-23.7434	51.287	-46.002	43.4668	-40.05	34.6232	-32.8068
Grand total		-5.9419	22.359	-18.9141	15.5082	-14.8125	11.2777	-12.2701
Divided by 18		-.33	1.242	-1.05	.862	-.822	.627	-.681


Graph 3
Flux Distribution
and
E.M.F. waves under
No Load
Splitdorf Low-Tension
φ in Maxwell
e in Volt
θ in degree
φ
e
60000.
40000.
20000.
0.
-20000.
-40000.
-60000
30
20
10.
0.
-10
-20
-30
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360.

But e may be expressed by the series of the form

$$e = \sum_1^\infty \left[E_n \sin n\theta + E_n' \cos n\theta \right]$$

hence

$$\phi = -K \int \sum_1^\infty \left[E_n \sin n\theta + E_n \cos n\theta \right] d\theta$$

$$= -K \left[\sum_1^\infty \left[-\frac{E_n}{n} \cos n\theta + \frac{E_n'}{n} \sin n\theta \right] \right] + K_1$$

the integration constant K_1 must necessarily be zero since the flux wave is symmetrical with respect to neutral line. The flux wave takes the form,

$$\phi = \sum_1^\infty \left[\Phi_n \cos n\theta - \Phi_n' \sin n\theta \right]$$

where

$$\Phi_n = \frac{10^8 E_n}{2\pi f N_a n} \quad \text{and} \quad \Phi_n' = \frac{10^8 E_n'}{2\pi f N_a n}$$

Tables 3 a - c show the calculation of the flux distribution wave from the e.m.f.

The short circuit (permanent) characteristic of a magneto generator is similarly to that of an ordinary alternator, except a magneto is so designed that its full load condition is short circuiting the armature terminals through the interruptor's breaker points. The transient sudden short circuit phenomena of a magneto, however, are entirely different from those observed in a power generating machine. In an alternator, a sudden short circuit current often reaches as high as fifty to seventy-five times the permanent short circuit current, while in a magneto even under the worst condition, the current does not exceed one hundred fifty percent of the maximum value of the permanent short circuit current. This is mainly due to the fact that in a magneto there exists no

Table 3 - a

Calculations of Flux Distribution at No Load

from Open Circuit E.M.F. Wave

$$K = \frac{10^6}{N_a} = 7800.$$

n	E_n	E_n/n	Φ_n/K	E_n'	E_n'/n	Φ_n'/K
1	9.381	9.381	9.96	-.33	-.33	-.351
3	-5.684	-1.89	-2.01	1.242	.415	.442
5	4.100	.82	.872	-1.05	-.21	-.223
7	-2.972	-.425	.452	.862	.123	.131
9	2.238	.249	.264	-.822	-.091	-.097
11	-1.575	-.143	-.152	.627	.057	.06
13				-.681	-.052	-.055

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[illegible]	[illegible]	[illegible]				[illegible]

Table 3 - b

θ°	0	30	60	90	120	150	180
Φ_1	77700	77700	77700	77700	77700	77700	77700
cos θ	1.	.866	.5	0	-.5	-.866	-1.
ϕ_1	77700	67200	38850	0	-38850	-67200	-77700
Φ_3	-16400	-16400	-16400	-16400	-16400	-16400	-16400
cos 3θ	1.	0	-1.	0	1.	0	-1.
ϕ_3	-15700	0	15700	0	-15700	0	15700
Φ_5	6800	6800	6800	6800	6800	6800	6800
cos 5θ	1.	-.866	.5	0	-.5	.806	-1.
ϕ_5	6800	-5890	3400	0	-3400	5890	-6800
Φ_7	-3520	-3520	-3520	-3520	-3520	-3520	-3520
cos 7θ	1.	-.866	.5	0	-.5	.866	-1.
ϕ_7	-3520	3050	-1760	0	1760	-3050	+3520
Φ_9	2060	2060	2060	2060	2060	2060	2060
cos 9θ	1.	0	.5	0	1.	-.866	-1.
ϕ_9	2060	0	1030	0	2060	-1780	2060
Φ_{11}	-1180	-1180	-1180	-1180	-1180	-1180	-1180
cos 11θ	1.	.866	.5	0	-.5	-.866	-1.
ϕ_{11}	-1180	-1020	-590	0	590	.020	1180
$\Sigma\phi_n$	66160	63340	56630	0	-53540	-65120	-66160

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Table 3 - c

θ°	0	30	60	90	120	150	180
Φ_1'	-2740	-2740	-2740	-2740	-2740	-2740	-2740
sin θ	0	.5	.866	1.	.866	.5	0
ϕ_1'	0	-1370	-2380	-2740	-2380	-1370	0
Φ_3'	3450	3450	3450	3450	3450	3450	3450
sin 3θ	0	1.	0	-1.	0	1.	0
ϕ_3'	0	3450	0	-3450	0	3450	0
Φ_5'	-1740	-1740	-1740	-1740	-1740	-1740	-1740
sin 5θ	0	.5	-.866	1.	-.866	.5	0
ϕ_5'	0	-870	1510	-1740	1510	-870	0
Φ_7'	1020	1020	1020	1020	1020	1020	1020
sin 7θ	0	-.5	.866	-1.	.866	-.5	0
ϕ_7'	0	.510	885	-1020	885	-510	0
Φ_9'	-755	-755	-755	-755	-755	-755	-755
sin 9θ	0	-1.	0	1.	0	-1.	0
ϕ_9'	0	755	0	-755	0	755	0
Φ_{11}'	470	470	470	470	470	470	470
sin 11θ	0	-.5	-.866	-1.	-.866	-.5	0
ϕ_{11}'	0	-235	-407	-470	-407	-235	0
Φ_{13}'	-428	-428	-428	-428	-428	-428	-428
sin 13θ	0	.5	.866	1.	866	.5	0
ϕ_{13}'	0	-214	-372	-428	-372	-214	0
$\Sigma \phi_n'$	0	1006	-764	-10603	-764	1006	0
ϕ	66160	62334	57394	10603	-52780	-66126	-66160

appreciable transient flux which is one of the most prominent factors in case of an alternator under the similar conditions, also partly due to comparatively high reactance of the armature.

The flux distribution and generating e.m.f. waves under load are somewhat distorted from those taken under no load. Unfortunately there is no known method to determine these waves correctly so far as the author's knowledge goes, since the wave form of flux distribution, therefore that of e.m.f. is far from a sinusoidal.

In spite of this fact the relations that exist among the flux, e.m.f. and current may be expressed in an equation,

$$e = -10^{-8} N_a \frac{d(\phi + .04\pi N_a \mu i)}{dt}$$

where $0.4\pi N_a ui$ is ~~opposing~~ flux due to the current in the armature. We know the instantaneous value of ϕ and the constant Na, but we do not know the value of u which is some function of the armature current. So if it is assumed that the value of magnetic permeability u is constant and the magnitude of which is unity, then it is possible to calculate approximately the wave form of the e.m.f. under short circuit conditions from the following equation, provided the instantaneous values of the load current are known.

Substituting θ for $2\pi ft$ and changing the differentials $d(\phi + 0.4\pi N\mu i)$ and $d\theta$ to increments $\Delta(\phi + 0.4\pi N\mu i)$ and $\Delta\theta$, we have

$$e = -10^{-8} 2\pi f N_a \frac{\Delta(\phi + 0.4\pi N_a i)}{\Delta\theta}$$

Taking the increment of the angle $\Delta\theta$ as 10° which is equal to

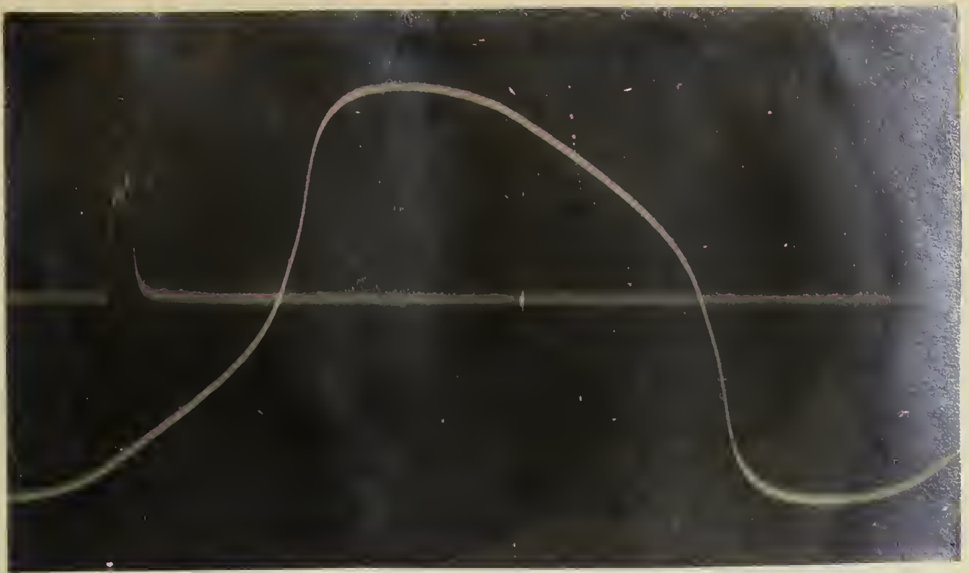


Fig 23 Splidorf

Fig. 24 Dixie

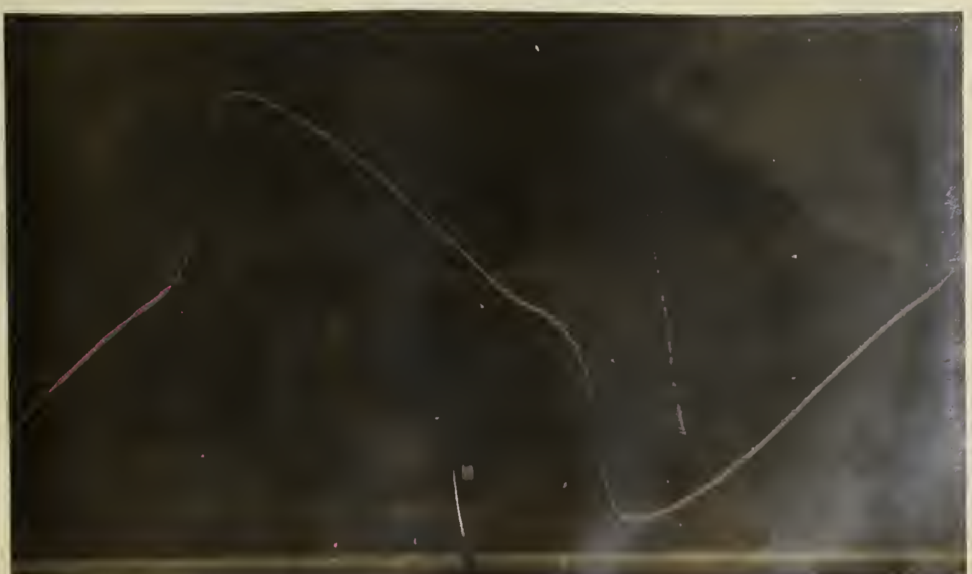


Fig 25 Bosch

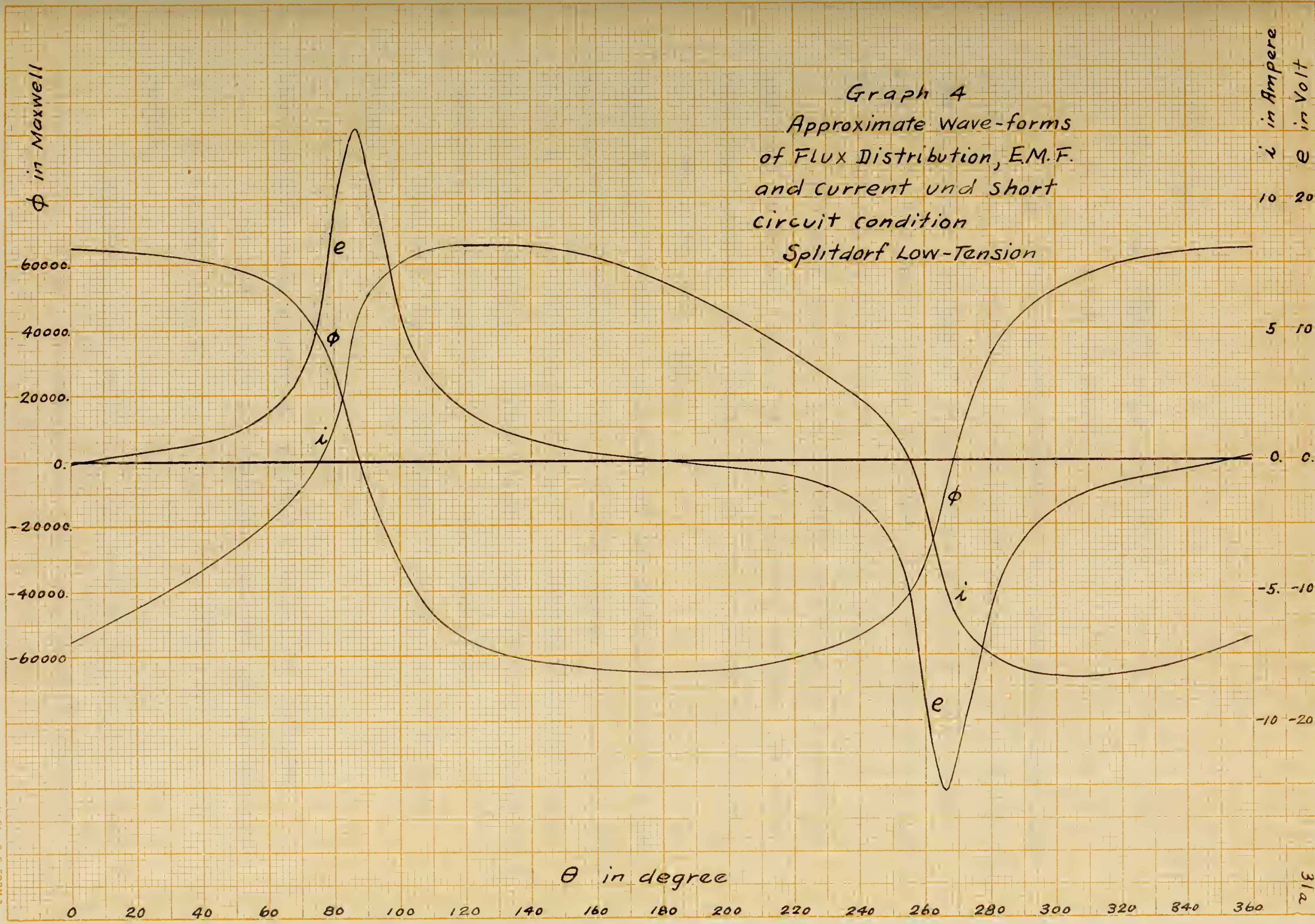
Table 4

Calculations of the Flux Distribution and E.M.F. Waves under Short Circuit Condition

Splitdorf Low Tension Magneto

f = 150 N_a = 120 K = 0.00066

θ°	i	ϕ	$.4\pi N_a i$	$\phi + .4\pi N_a i$	$\Delta(\phi + .4\pi N_a i)$	e
-10.	-7.38	66160.	-1115.	65045.	+76.	-.05
0	-6.87	66160.	-1039.	65121.	-382.	+.25
10	-6.23	65679.	-940.	64739.	-585.	.386
20	-5.54	64990.	-836.	64154.	-1340.	.885
30	-4.9	63554.	-740.	62814.	-1807.	1.23
40	-4.15	61634.	-627.	61007.	-2515.	1.66
50	-3.32	58994.	-502.	58492.	-3373.	2.22
60	-2.49	55495.	-376.	55119.	-7465.	4.92
70	-1.13	47825.	-171.	47654.	-16051.	10.6
80	1.51	31375.	228.	31603.	-38283.	25.2
90	6.14	-7606.	926.	-6680.	-26162.	17.2
100	7.56	-34000.	1158.	-32842.	-12389.	8.2
110	8.12	-46356.	1225.	-45231.	-7300.	4.82
120	8.25	-53776.	1245.	-52531.	-4913.	3.24
130	8.3	-58696.	1252.	-57444.	-3010.	1.98
140	8.23	-61696.	1242.	-60454.	-2182.	1.44
150	8.07	-63856.	1220.	-62636.	-1460.	.965
160	7.79	-65271.	1175.	-64096.	-949.	.625
170	7.38	-66160.	1115.	-65045.	-76.	.05
180	6.87	-66160.	1039.	-65121.	+382.	-.25
190	6.23	-65679	940	-64739		


Graph 4
Approximate Wave-forms
of Flux Distribution, E.M.F.
and Current und Short
circuit condition
Splitdorf Low-Tension
φ in Maxwell
i in Ampere
e in Volt
θ in degree
60000.
40000.
20000.
0.
-20000.
-40000.
-60000
10 20
5 10
0. 0.
-5. -10.
-10 -20
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
e
φ
i
φ
i
e

0.175, then,

$$e = -10^{-8}\frac{2\pi f N_a}{.175}\Delta(\phi + 0.4\pi N_a i)$$

$$= -K\ \Delta(\phi + 0.4\pi N_a i)$$

Fig. 23 is an oscillogram of the permanent short circuit current of Splitdorf low-tension magneto, from which the instantaneous values of current were measured. The phase angle of said current wave is found from the reference point which was set at zero degree of the open circuit e.m.f. wave.

The wave form of e.m.f. under the similar condition may also be approximated by still another method. We have a relation between the induced e.m.f. and current as follows,

$$e = i R_a + X_a \frac{di}{d\theta}$$

where X_a is the ohmic reactance of the armature which is not by any means a constant, but a careful measurement shows that it is a function of the position of armature with reference to the field magnets. Graph 5 shows the values of armature reactance at various position of the armature.

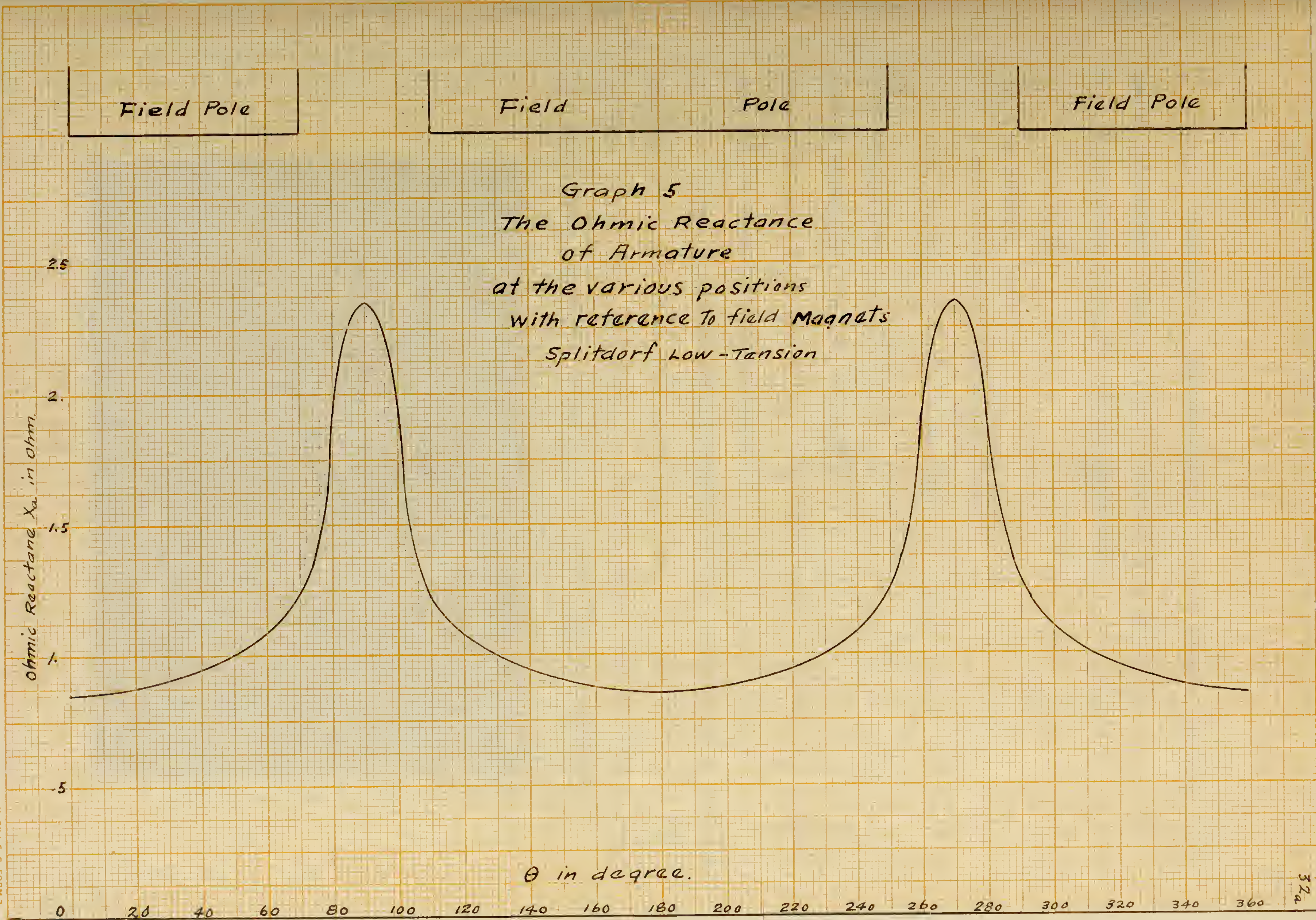
Reducing the last equation similarly as the first method, we get,

$$e = i R_a + \frac{X_a}{\Delta\theta}\Delta i$$

the instantaneous values of e could easily be computed.

In both of these methods the smaller the increments Δi and $\Delta\theta$ taken in the calculation the more accurate the result.

A similar relation holds when the primary coil of the transformer is in the armature circuit, also when there is



Field Pole
Field
Pole
Field Pole
Graph 5
The Ohmic Reactance
of Armature
at the various positions
with reference to field Magnets
Splitdorf Low-Tension
Ohmic Reactane Xa in ohm
2.5
2.
1.5
1.
.5
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
θ in degree.

a current flowing in the secondary of the same. In the first case, secondary coil is open circuited, we have,

$$e = -2\pi f N_a 10^{-8} \frac{d(\phi + .4\pi N_a \mu i_1)}{d\theta}$$

$$X_p \frac{di_1}{d\theta} + i_1 R_p$$

In the second case

$$e = -2\pi f N_a 10^{-8} \frac{d(\phi + .4\pi N_a \mu i)}{d\theta}$$

$$= X_p \frac{di_1}{d\theta} + i_1 R_p + X_m \frac{di_2}{d\theta}$$

For the primary circuit and

$$o = X_s \frac{di_2}{d\theta} + i_2 R_s + X_m \frac{di_1}{d\theta}$$

For the secondary circuit. The solutions of these equations are to be found in appendix A.

Fig. 26 is an oscillogram of the primary coil and condenser currents with the secondary open. Fig. 27 shows the effect of mutual inductance upon the wave form of the primary current when there is a current flowing in the secondary.

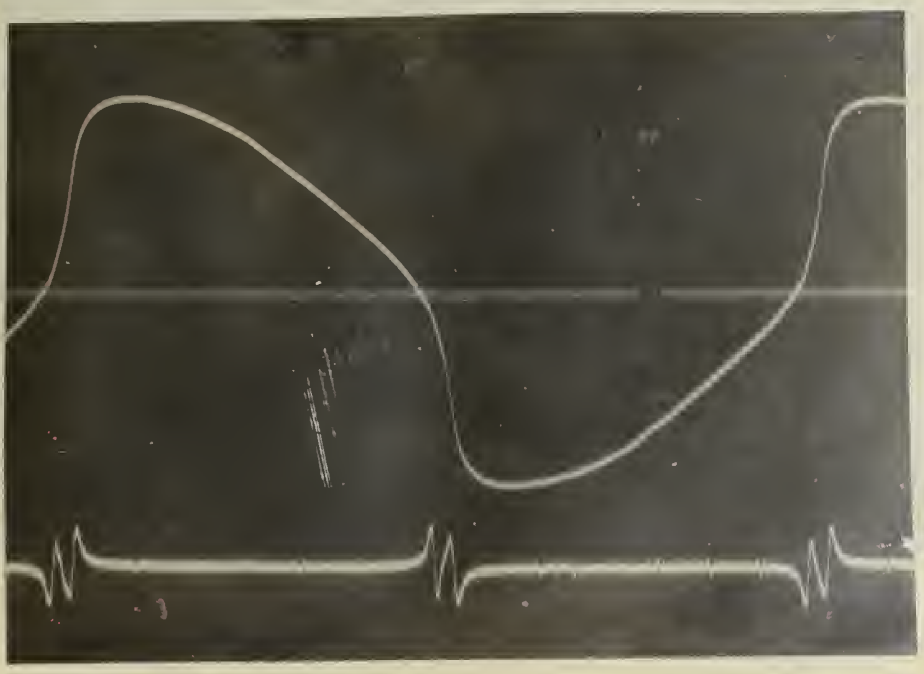


Fig. 26

It has already been mentioned in the previous paragraph that the transient (sudden short circuit current) in a magneto is quite different from that of an alternator, and this transient current follows approximately as in an equation,

$$i_{s.c.} = A\,\epsilon^{-\frac{R_a}{X_a}\theta} + i_{s.c.}(permanent)$$

where A is constant and equal to the instantaneous value of the permanent short circuit current when $\theta = \theta_1$ at which the switch is closed. The method of determining this constant is found in appendix A.

Fig. 28 and 29 show the sudden short circuit currents of the Splitdorf low-tension magneto with $\theta = 0°$ and $90°$ respectively.

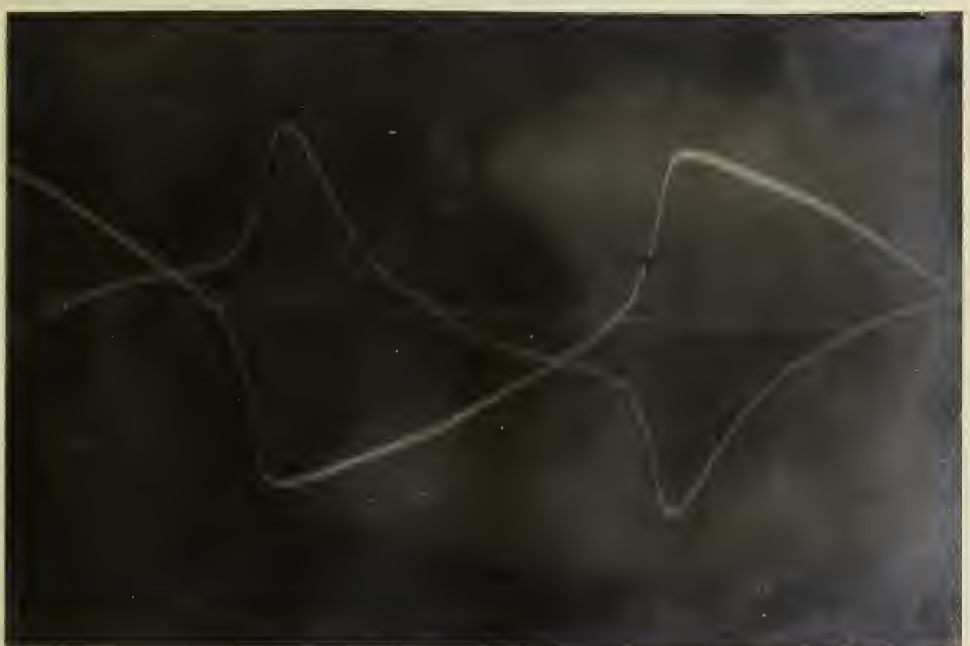


Fig. 27

Table 5

Transient Sudden Short Circuit Current

Switch is closed at $\theta = 0^\circ$

$R_a = .5$ A = 6.87

	---Calculated----						observed	
θ°	i	X_a	$\frac{R_a}{X_a}$	$\frac{R_a}{X_a}\theta$	$A\epsilon^{-\frac{R_a}{X_a}\theta}$	i_o #	i_o	Diff.
0	-6.87	.86	.581	0	6.87	0	0	0
20	-5.54	.895	.558	.195	5.65	.11	.18	-.07
40	-4.15	.96	.521	.364	4.77	.62	.42	.2
60	-2.49	1.065	.46	.483	4.23	1.74	1.21	.53
80	-1.51	1.88	.266	.372	4.74	3.23	3.49	-.26
100	7.56	1.58	.316	.554	3.94	10.5	10.15	.35
120	8.25	1.085	.46	.966	2.6	10.85	10.4	.81
140	8.23	.96	.521	1.275	1.91	10.14	10.	.14
160	7.79	.895	.558	1.565	1.44	9.23	9.35	.12
180	6.87	.86	.581	1.83	1.1	7.97	8.2	.23
200	5.54	.895	.558	1.95	.98	6.52	6.87	.35
220	4.15	.96	.521	2.	.93	5.08	5.28	.2
240	2.49	1.085	.46	1.93	1.	3.49	3.8	.31
260	-1.51	1.58	.316	1.44	1.68	0.17	0.7	.53
280	-7.56	1.88	.266	1.30	1.87	-5.71	-5.7	.01
300	-8.25	1.085	.46	2.41	.62	-7.63	-6.98	.65
320	-8.23	.96	.521	2.92	.37	-7.86	-7.1	.76
340	-7.79	.895	.558	3.32	.25	-7.54	-6.65	.89
360	-6.87	.86	.581	3.66	.18	-6.69	-6.1	.68

Sudden short circuit current = i_o

Permanent short circuit current = i

Graph 6 shows the transient currents calculated from the equation, and from the oscillogram. The tabulation of the calculated value of current is given in Table 5.

Fig. 28 $\theta_1 = 0°$

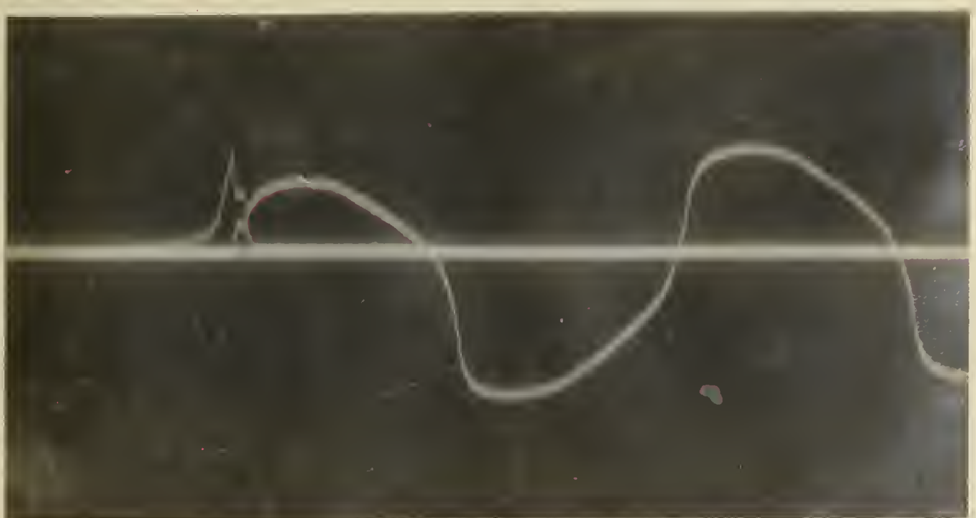
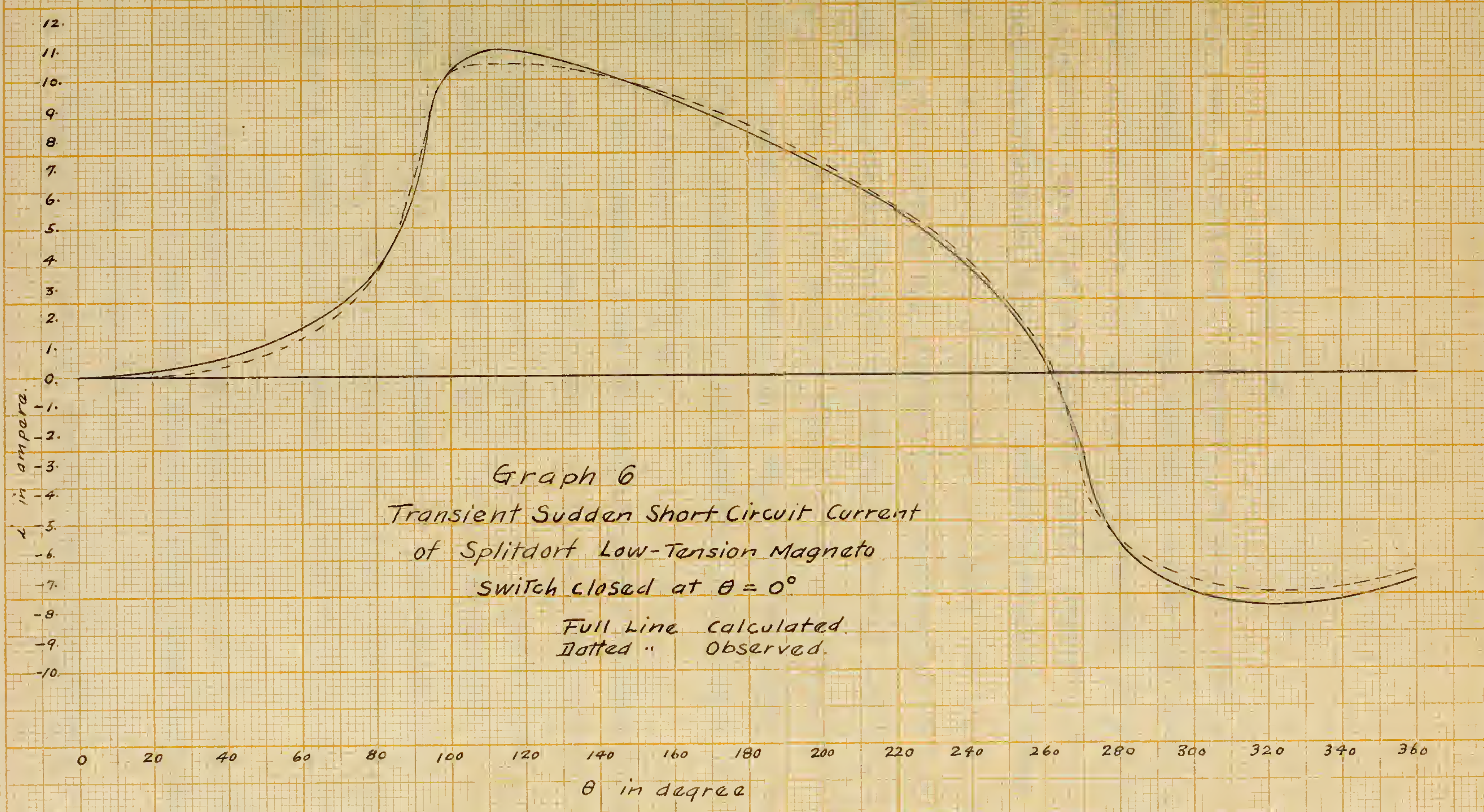


Fig. 29 $\theta_1 = 90°$



12.
11.
10.
9.
8.
7.
6.
5.
4
3.
2.
1.
0.
-1.
-2.
-3.
-4.
-5.
-6.
-7.
-8.
-9.
-10.
i in ampere.
Graph 6
Transient Sudden Short Circuit Current
of Splitdorf Low-Tension Magneto
Switch closed at θ = 0°
Full Line calculated
Dotted " Observed
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
θ in degree

Chapter V

Low and High-tension Magneto Ignition Systems

Though there are many different makes and kinds of magnetos on the market for the ignition purposes, we may, however, classify them into two general types, namely, low-tension and high-tension magnetos . Strictly speaking, there exists no definite boundary line between these two types, as the power generated by any magneto must necessarily be low-tension. The classification therefore is merely a conventional one. A low-tension magneto signifies a unit equipped with a separate transformer, while in a high-tension magneto the armature itself serves as the transformer, the primary of which is the armature winding.

At the present time we have two types of armatures which are used in all recent makes of ignition magnetos , namely, shuttle and inductor types. It is, however, hard to determine which is the better type, as the advantages of one are offset by those of the other and the working principles of these two types are identical. The advantages and disadvantages are confined to the design and construction of the machine.

The connections of ignition magnetos generally consist of two different methods; closed circuit type and open circuit type. Figs. 30 and 31.

Usually a low-tension magneto is connected by the

[illegible]

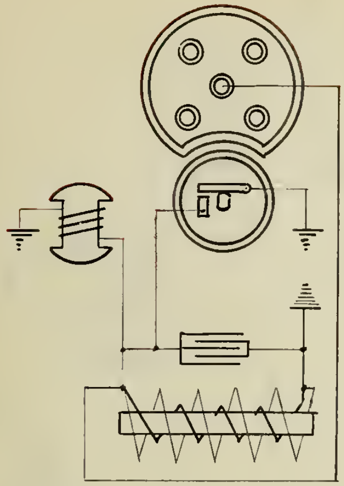


Fig. 30
Closed Circuit Type.

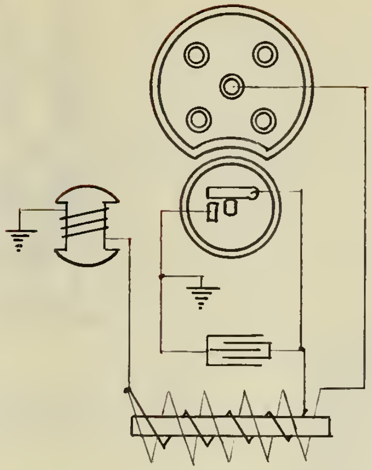


Fig 31
Open Circuit Type.

first method, while a high-tension magneto is connected by the latter method. A closed circuit type magneto requires less abrupt interruption of the armature current, because the armature has a very low impedance circuit whether the interrupter is open or closed hence the interrupter points do not need constant care. But in the case of the open circuit type magneto the abrupter the interruption the better the ignition, hence the interrupter must be given close attention in order to operate satisfactorily.

As to the performance characteristics of magnetos, the author confined his studies to two typical types of magnetos rather than to investigate loosely many varieties. The chosen types were Splitdorf low-tension and Dixie high-tension magnetos. These two represent low and high-tension magnetos, which have shuttel and inductor types of armatures. They have closed and open circuit type connections and therefore these

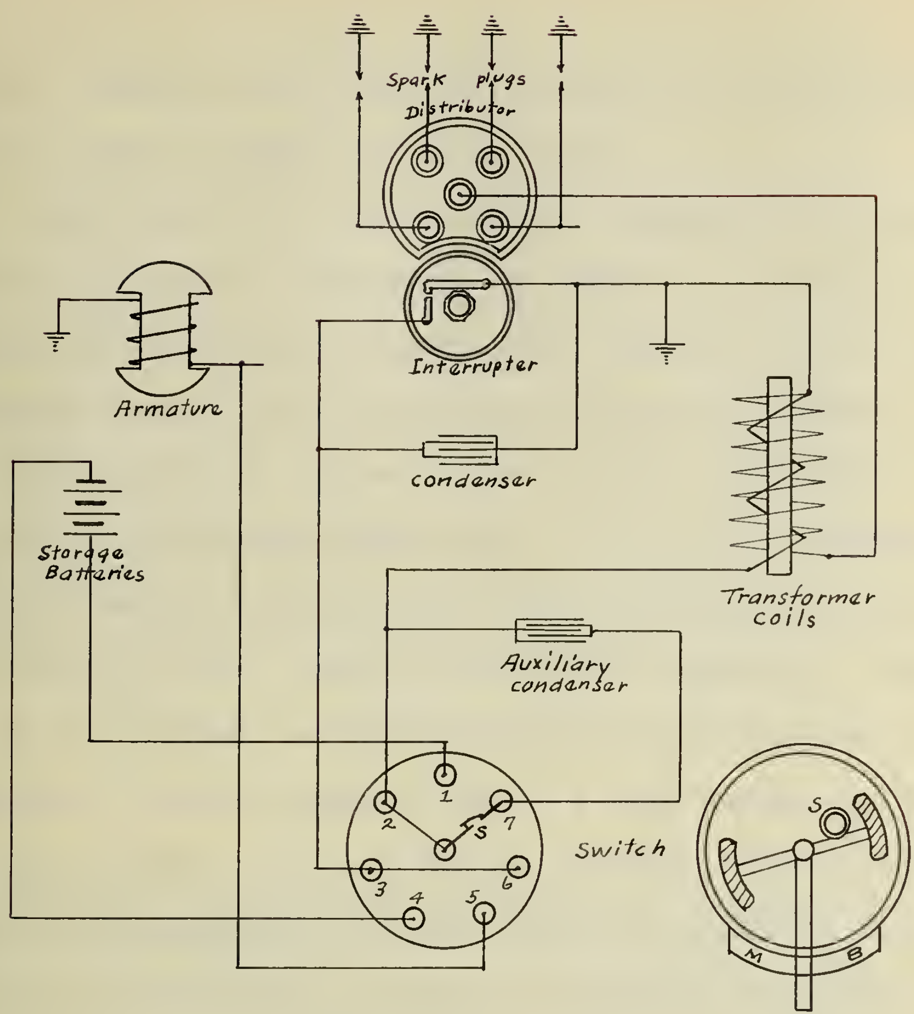


Fig 32

For Battery circuit connection, the switch short circuits contact points 1-7 and 3-4.

For Magneto circuit connection, the switch short circuits contact points 5-6 and 2-3

A push button "S" is provided, which when pressed down, opens the primary circuit and inserts auxiliary condenser in series with the primary coil and battery. Thus growth and decay of the primary current is produced independently.

magnetos represent the characteristics of all the different makes and kinds in use at present time.

The actual interior coiling diagram of the Splitdorf low-tension magneto is as shown in Fig. 32, in which the switch is closed on the battery circuit, the current flow through the primary coil of the transformers from the storage battery if the interrupter is closed which in turn establishs a magnetic flux in the transformer core. Now if the armature shaft is further rotated the cam opens the interrupter and the current in the coil will decay through the condenser. This sudden decay of the current and hence magnetic flux induces an e.m.f. sufficiently high to cause a spark to jump across one of the spark plug points. For the magneto circuit this unit is connected as a closed circuit type, that is, when the interrupter is closed the terminals of armature are short circuited through the interrupter and a very feeble current is flowing through the primary coil. If the interrupter opens the breaker points then there will be a sudden rise of the current in the primary coil which produces a similar transient voltage in the secondary as in case of the battery circuit.

For convenience, let us call the period during which the interrupter is closed "the first transient", and the period beginning from the instant the interrupter opens "the second transient".

Figs. 33, 34, and 35 are the oscillograms of the transient current in the various parts of the circuits, the

Fig. 33 Top curve, Armature Current.
Bottom " , Spark current.

Fig 34 Top curve, Primary Coil current.
Middle " , Condenser current.
Bottom " , Reference $\theta_1 = 0°$
Point indicated by kick.

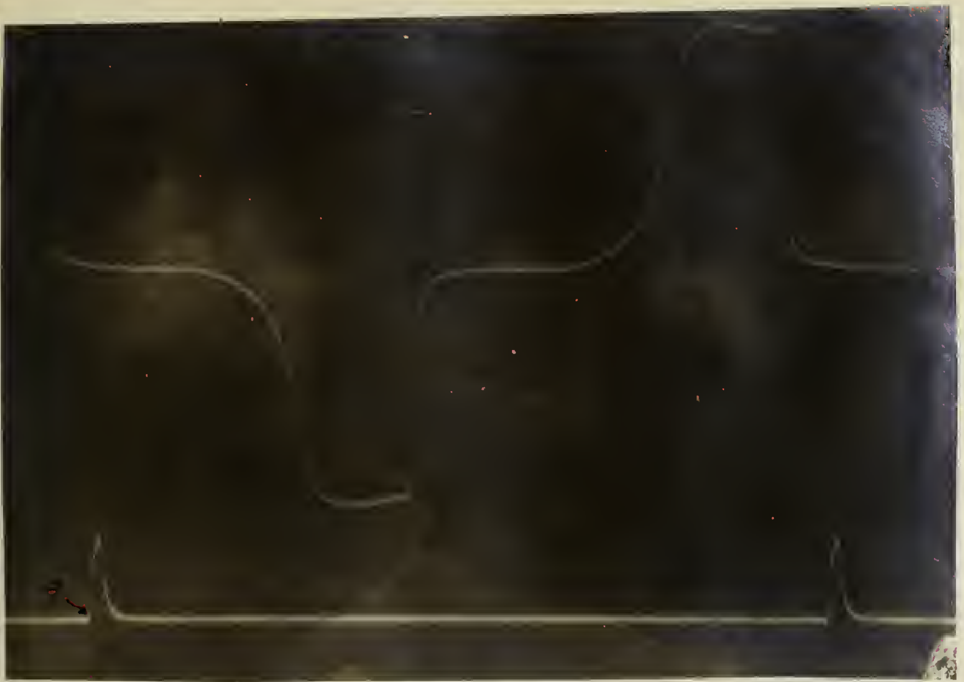


Fig. 35 Top Curve, Interrupter Current
Bottom " Reference $\theta_1 = 0°$

explanation of which is given under each picture. As to the mathematical expressions for these transient currents see appendix A under low tension magneto.

The high-tension magnetos are the most recent development in magneto design, and they are in general connected as the open circuited type. Winding the secondary coil on the magneto armature is an improvement over the low-tension magneto in several ways, - first, it eliminates the primary winding and core, therefore eliminating the losses in these parts; second, the generation of energy in the secondary winding itself by cutting the lines of force in the magnetic field add to the efficiency; and third, a high-tension magneto includes all the equipment necessary for ignition purposes and requires much smaller space for installation, etc.

The high-tension magneto armatures like those of low-tension magnetos, may be either a shuttle type or an inductor alternator type. Figs. 36 and 37 show these two types of armatures, the former being shuttle type high-tension magneto armature (Bosch), and the latter inductor type (Dixie). Note the slip rings on the Bosch which serve as the distributor.

The useful magnetic flux through the core upon which the armature is wound - Dixie - at the various positions of the armature are shown in photographic plates by means of the fine iron filing. The poles shown may be regarded as two terminals of the armature core. The deflection of the flux in all pictures is due to the leakage in the magnetic field of

Fig. 36

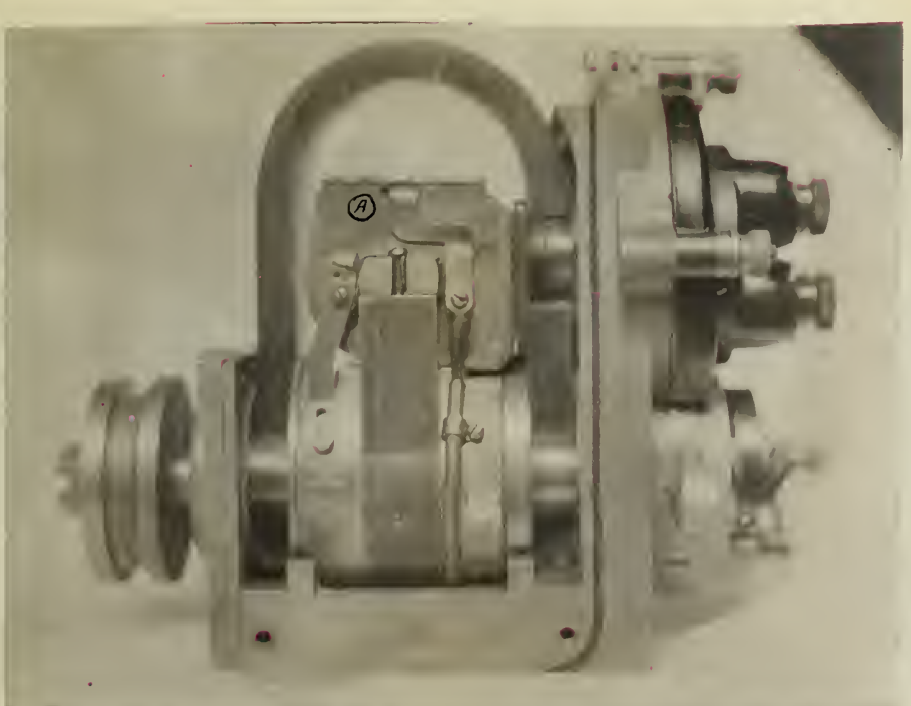


Fig. 37 Ⓐ Armature

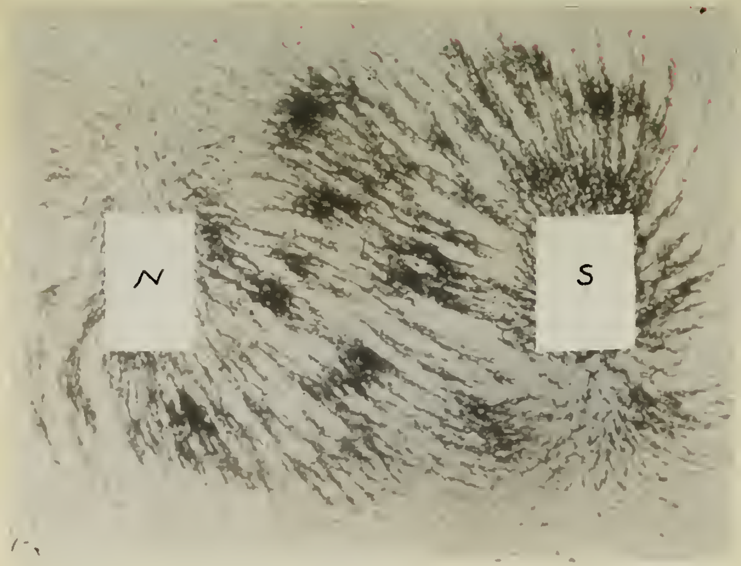


Fig. 38 0°

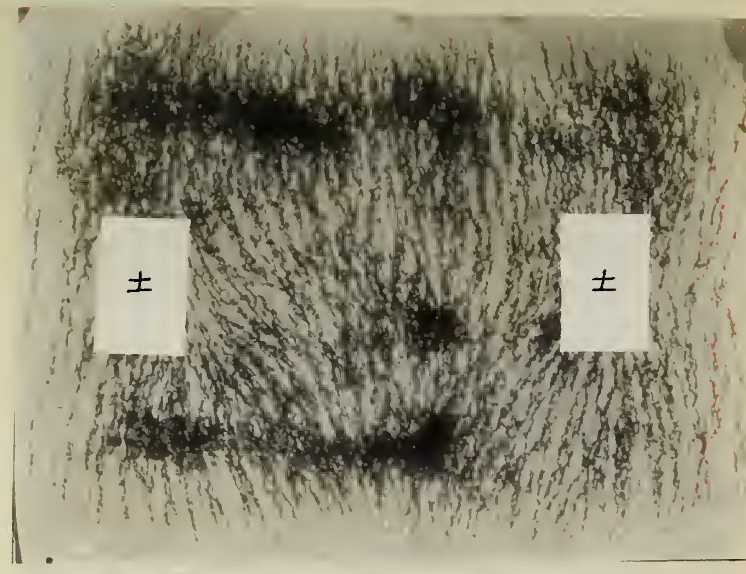


Fig. 40 90°

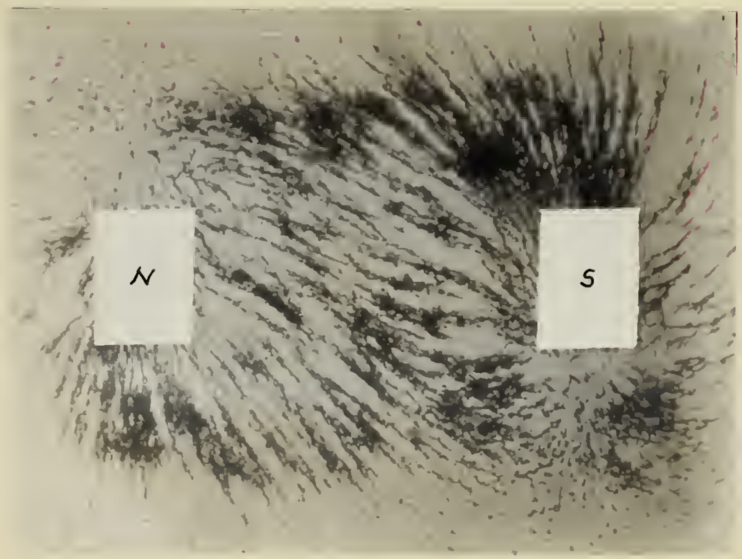


Fig. 39 45°

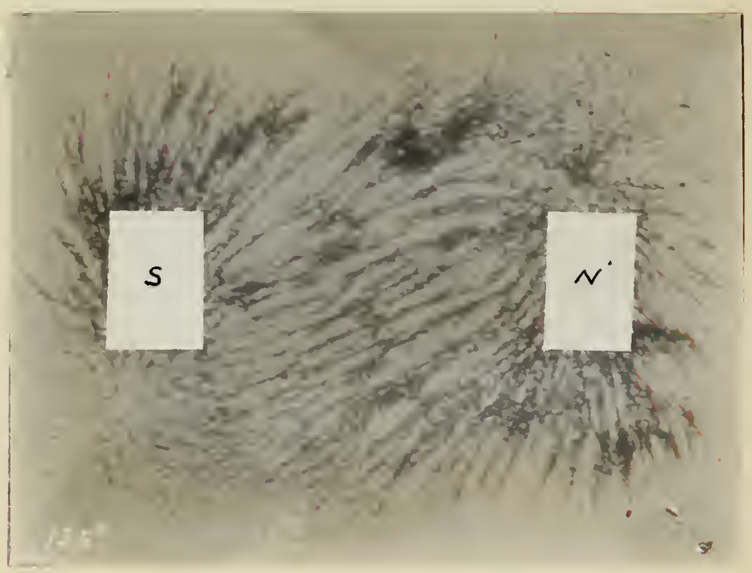


Fig. 41 135°

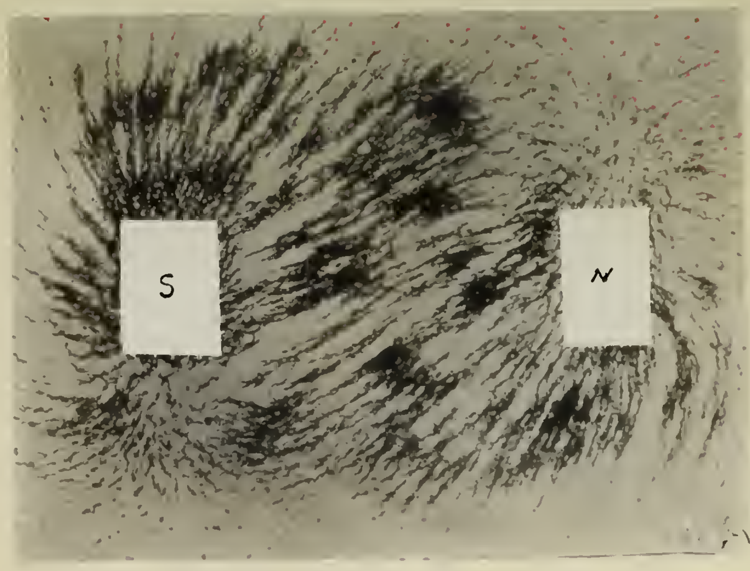


Fig. 42 180°

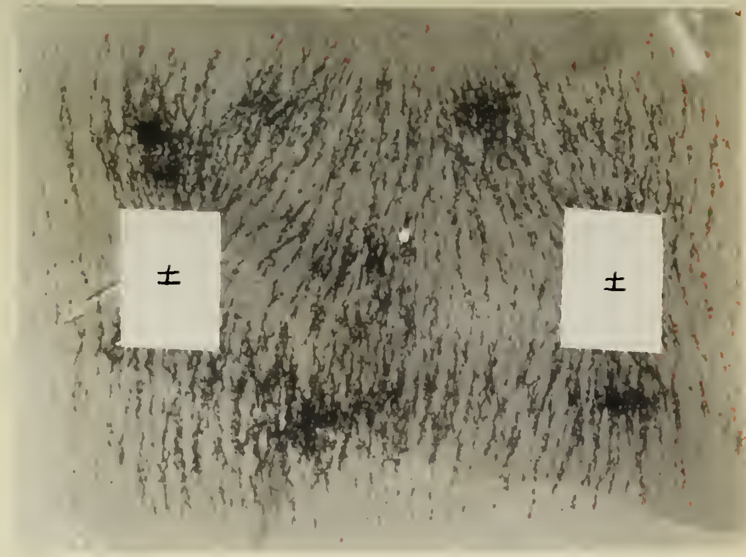


Fig 44 270°

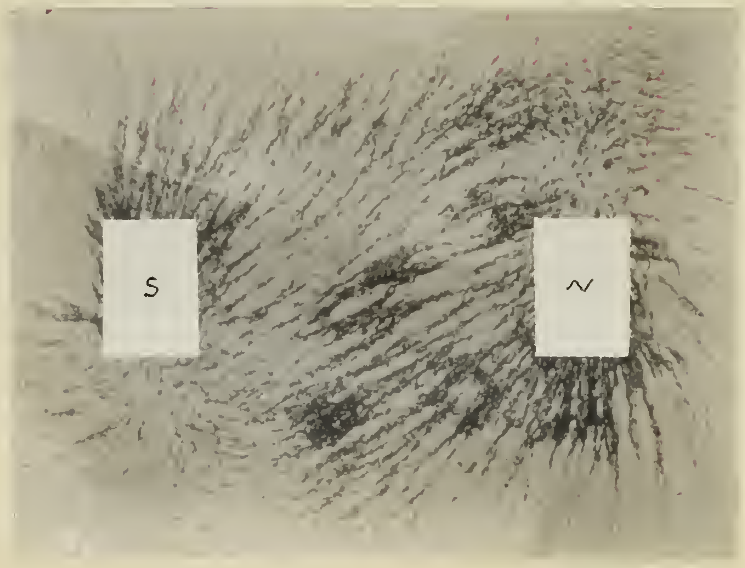


Fig. 43 225°

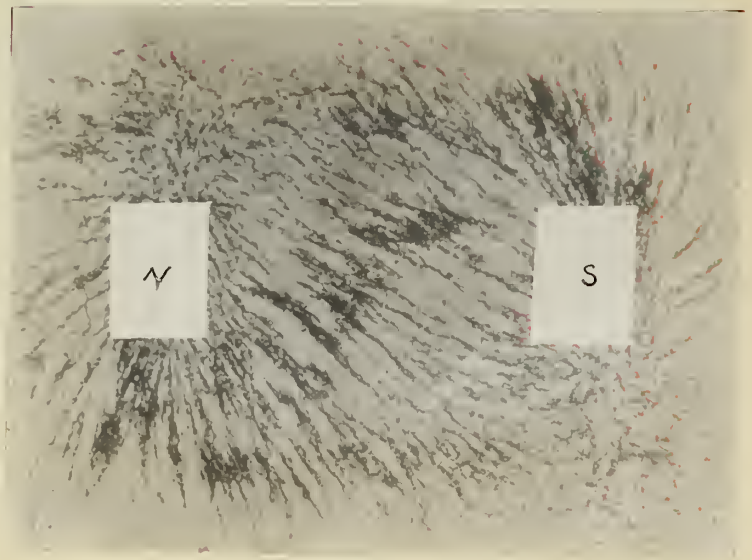


Fig 45 315°

the magnets, the north and south poles of which are located above and below the pictures.

The wiring diagram of the Dixie high-tension is as shown in Fig. 46.

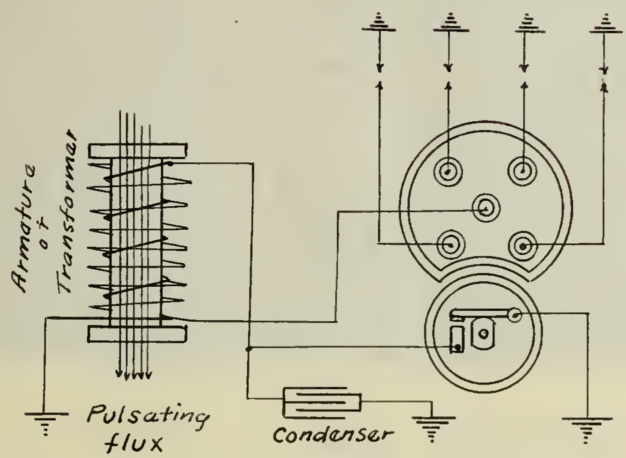


Fig. 46

The operation of this unit is as follows: Beginning from the first transient a short circuit current is established in the armature or primary coil which is suddenly interrupted at the instant the interrupter opens, this abrupt change of the current in the primary induces a transient voltage in secondary which is its self generating an e.m.f. induced in it by the alternating flux. The sum of these two e.m.fs causes the spark to jump across the gap. Fig . 47 is the oscillogram taken for the primary coil current, and secondary spark current with reference point by the contact maker device.

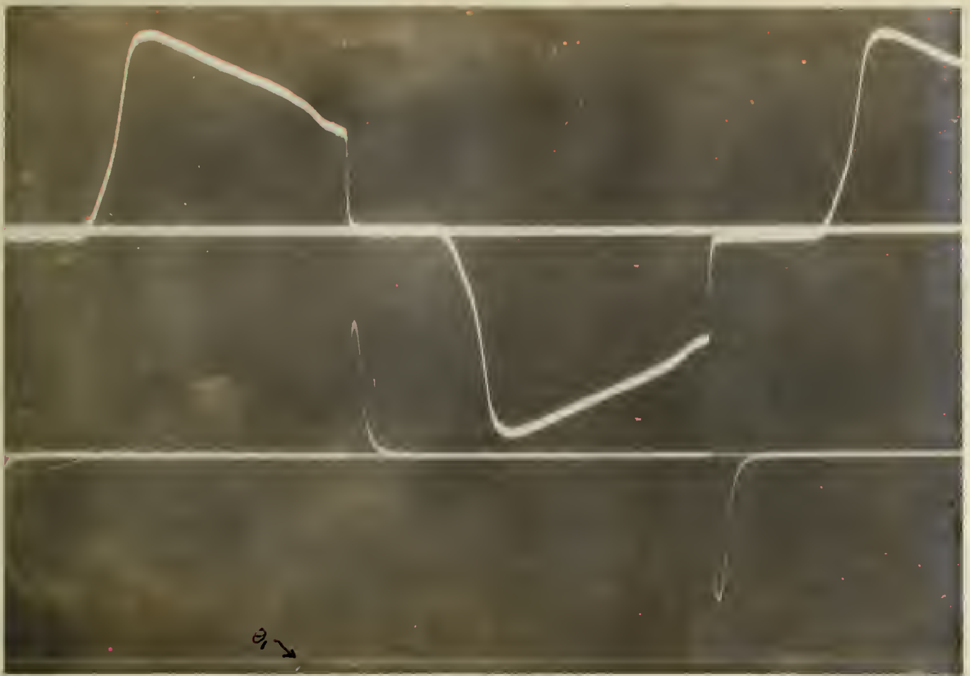


Fig. 47

Top, Armature current.
Middle Spark current.
Bottom Referance point

APPENDIX A

The Mathematical Equations of Transient Currents.

Preliminary Note.

In deriving the mathematical equations of various transients, the following conventions are used

a The starting moment of all transients is chosen as the zero time

b All quantities of the secondary circuit are expressed in the equivalent terms of the primary circuit.

Further assumed that the magnetic permeability of transformer core is constant. Also there exists no leakage flux. These lead to the following assumptions:

a The numerical values of inductances, therefor reactances of the transformer coils are constant.

b Energy loss due to hysteresis is nill.

c Mutual inductance of two coils is perfect.

or $M = \sqrt{L_1 L_2}$ and $X_m = \sqrt{X_1 X_2}$

I Storage Battery system.

For "the first transient", the e.m.f. equation of such circuit may be written by applying Kirchhoff's Law,

$$E = i_1 R_1 + L_1 \frac{di_1}{dt} \quad \text{------------------------} \quad (1)$$

Rearranging the terms,

$$\frac{di_1}{dt} + \frac{R_1}{L_1} i_1 = \frac{E}{L_1}$$

Solving for i_1 by integration,

$$i_1 = \frac{E}{R_1} + K \epsilon^{-\frac{R_1}{L_1} t} \quad \text{------------------} \quad (2)$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

where K is constant of integration and can be determined from initial condition, $i_1 = 0$ when $t = 0$,

or $0 = \frac{E}{R_1} + K$ which gives

$$K = -\frac{E}{R_1}$$

Substituting above for K in (2), we have

$$i_1 = \frac{E}{R_1}\left[1 - \epsilon^{-\frac{R_1}{L_1}t}\right] \qquad (3)$$

Thus the primary current grows from zero to

$$i_1 = \frac{E}{R_1}\left[1 - \epsilon^{-\frac{R_1}{L_1}t_1}\right] = I$$

at this instant, i.e., t_1 seconds after it started, the interrupter breaks the circuit and the decay of i_1 or "the second transient" period begins.

It is evident that the second transient involves two circuits, primary and secondary of the transformer, since the decay of i_1 induces an e.m.f. sufficiently high enough to produce a spark current i_2 in the secondary circuit. The e.m.f. equations of the second transient are,

$$i_1 R_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} + \frac{1}{C}\int i_1\,dt = E \qquad (4)$$

$$i_2 R_s + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0 \qquad (5)$$

Differentiating (4) and (5),

$$R_1\frac{di_1}{dt} + L_1\frac{d^2 i_1}{dt^2} + M\frac{d^2 i_2}{dt^2} + \frac{1}{C}i_1 = 0 \qquad (6)$$

$$R_s\frac{di_2}{dt} + L_2\frac{d^2 i_2}{dt^2} + M\frac{d^2 i_1}{dt^2} = 0 \qquad (7)$$

From (6),

$$\frac{d^2 i_2}{dt^2} = -\frac{1}{MC}i_1 - \frac{R_1}{M}\frac{di_1}{dt} - \frac{L_1}{M}\frac{d^2 i_1}{dt^2} \qquad (8)$$

Differentiating again,

$$\frac{d^3 i_2}{dt^3} = -\frac{1}{MC}\frac{di_1}{dt} - \frac{R_1}{M}\frac{d^2 i_1}{dt^2} - \frac{L_1}{M}\frac{d^3 i_1}{dt^3} \qquad (9)$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Defferentiating (7),

$$L_2 \frac{d^3 i_2}{dt^3} + R_s \frac{d^2 i_2}{dt^2} + M \frac{d^3 i_1}{dt^3} = 0 \text{ ------------- } (10)$$

Substituting (8) and (9) in the equation (10),

$$-\left[\frac{L_2}{CM}\frac{di_1}{dt} + \frac{R_1 L_2}{M}\frac{d^2 i_1}{dt^2} + \frac{L_1 L_2}{M}\frac{d^3 i_1}{dt^3}\right] + M\frac{d^3 i_1}{dt^3}$$

$$-\left[\frac{R_s}{CM} i_1 + \frac{R_1 R_s}{M}\frac{di_1}{dt} + \frac{L_1 R_s}{M}\frac{d^2 i_1}{dt^2}\right] = 0$$

which involves only i_1 and t as variables, rearranging the terms,

$$(L_1 L_2 - M^2)\frac{d^3 i_1}{dt^3} + (R_s L_1 + R_1 L_2)\frac{d^2 i_1}{dt^2}$$

$$+ (R_1 R_s + \frac{L_2}{C})\frac{di_1}{dt} + \frac{R_s}{C} i_1 = 0$$

But the coefficient of the first term is zero according to the assumption. The last equation becomes,

$$\frac{d^2 i_1}{dt^2} + A\frac{di_1}{dt} + B i_1 = 0 \text{ ---------------- } (11)$$

Where

$$A = \frac{R_1 R_s + \frac{L_2}{C}}{L_1 R_s + L_2 R_1} \quad \text{and} \quad B = \frac{R_s}{C(L_1 R_s + L_2 R_1)}$$

The general solution of the equation (11) is

$$i_1 = K_1 \epsilon^{-\frac{A+\sqrt{A^2-4B}}{2}t} + K_2 \epsilon^{-\frac{A-\sqrt{A^2-4B}}{2}t} \text{ ------ } (12)$$

But $\sqrt{A^2 - 4B} = \frac{1}{C(L_1 R_s + L_2 R_1)}\sqrt{(R_1 R_s C - L_2)^2 - 4R_s^2 L_1 C}$

From which we may say that (12) represents a general equation for three different solutions, namely :

case 1 $\quad (R_1 R_s C - L_2)^2 - 4L_1 C R_s^2 > 0$

case 2 $\quad (R_1 R_s C - L_2)^2 - 4L_1 C R_s^2 = 0$

case 3 $\quad (R_1 R_s C - L_2)^2 - 4L_1 C R_s^2 < 0$

Again from (4) and (5), we get

$$\frac{di_2}{dt} = -\left[\frac{L_1}{M}\frac{di_1}{dt} + \frac{R_1}{M} i_1 + \frac{1}{MC}\int i_1 dt\right] + \frac{E}{M}$$

[illegible]

$$\frac{di_2}{dt} = -\frac{R_s}{L_2} i_2 - \frac{M}{L_2}\frac{di_1}{dt}$$

Eliminating $\frac{di_2}{dt}$ from these two and solving for i_2, we have

$$i_2 = \frac{L_1L_2 - M^2}{R_sM}\frac{di_1}{dt} + \frac{R_1L_2}{R_sM} i_1 + \frac{L_2}{CR_sM}\int i_1 dt - \frac{L_2E}{R_sM}$$

The first term on the right side again vanishes, or

$$i_2 = \frac{R_1L_2}{R_sM} i_1 + \frac{L_2}{CR_sM}\int i_1 dt - \frac{L_2E}{R_sM} \text{ ---------- } (13)$$

The equation (13), like (12), represent a general solution for the secondary current, however, it depend upon the particular solution of the primary current. It is, however, necessary to reduce integral term into an ordinary algebraic expression. Integrating (12),

$$\int i_1 dt = -\frac{K_1}{\alpha}\epsilon^{-\alpha t} - \frac{K_2}{\beta}\epsilon^{-\beta t} + K_3 \text{ ------ } (14)$$

where $\alpha = \frac{A + \sqrt{A^2 - 4B}}{2}$ and $\beta = \frac{A - \sqrt{A^2 - 4B}}{2}$

and K_3 is a constant of integration which must be determined before proceeding further. The condition of the circuit is such that

$$\int i\, dt = 0 \quad \text{when} \quad t = 0$$

that is, the quantity of electricity stored in the condenser during the first transient is nill since the terminals of condenser was short circuited, the equation (14) takes the form for $t = 0$

$$0 = -\frac{K_1}{\alpha} - \frac{K_2}{\beta} + K_3 \quad \text{or} \quad K_3 = \frac{K_1}{\alpha} + \frac{K_2}{\beta}$$

$$\therefore \int i_1 dt = \frac{K_1}{\alpha}\left[1 - \epsilon^{-\alpha t}\right] + \frac{K_2}{\beta}\left[1 - \epsilon^{-\beta t}\right] \text{ ------- } (15)$$

Putting (12) and (15) in (13),

$$i_2 = \frac{L_2}{R_sM}\Big[R_1(K_1\epsilon^{-\alpha t} + K_2\epsilon^{-\beta t}) - E$$

$$+ \frac{K_1}{C\alpha}(1 - \epsilon^{-\alpha t}) + \frac{K_2}{C\beta}(1 - \epsilon^{-\beta t})\Big] \text{ --------- } (16)$$

The constants K_1 and K_2 can be determined from two equations (12) and (16). The initial conditions of the circuits are as follow:

For $t = 0$ $\quad i_1 = I - i_2$

and $\quad t = \infty$ $\quad i_2 = 0$

(12) and (16) for $t = 0$ become

$$i_{1\,t=0} = K_1 + K_2 \quad \text{-------------------- (17)}$$

$$i_{2\,t=0} = \frac{L_2}{R_s M}\left[R_1(K_1 + K_2) - E\right] \quad \text{-------------- (18)}$$

$$\therefore \quad K_1 + K_2 = \frac{R_s M I + L_2 E}{R_s M + L_2 R_1} \quad \text{---------------- (19)}$$

and from the second condition,

$$\frac{K_1}{\alpha} + \frac{K_2}{\beta} = cE \quad \text{----------------------- (20)}$$

Solving simultaneously (19) and (20) for K_1 and K_2,

$$K_2 = -\frac{\beta}{\alpha-\beta}\,\frac{R_s M I + L_2 E}{R_s M + L_2 R_1} + \frac{\alpha\beta}{\alpha-\beta}\,cE$$

$$K_1 = \frac{\alpha}{\alpha-\beta}\,\frac{R_s M I + L_2 E}{R_s M + L_2 R_1} - \frac{\alpha\beta}{\alpha-\beta}\,cE$$

Substituting these in (12) and (16),

$$i_1 = \frac{R_s M I + L_2 E}{R_s M + L_2 R_1}\left[\frac{\alpha}{\alpha-\beta}\,\epsilon^{-\alpha t} - \frac{\beta}{\alpha-\beta}\,\epsilon^{-\beta t}\right]$$

$$- \frac{\alpha\beta c E}{\alpha-\beta}\left[\epsilon^{-\alpha t} - \epsilon^{-\beta t}\right] \quad \text{------------- (21)}$$

$$i_2 = \frac{L_2}{R_s M}\left[R_1 i_1 - E + \frac{1}{\alpha-\beta}\,\frac{RMI + L_2 E}{c(R_s M + L_2 R_1)}\left(\epsilon^{-\beta t} - \epsilon^{-\alpha t}\right)\right.$$

$$\left. - \frac{\beta E}{\alpha-\beta}\left(1 - \epsilon^{-\alpha t}\right) + \frac{\alpha E}{\alpha-\beta}\left(1 - \epsilon^{-\beta t}\right)\right] \quad \text{----- (22)}$$

The expression thus found for i_2 is an equivalent term of the primary circuit, the actual value of which may be obtained by multiplying (22) with the ratio of transformation n_1/n_2, or

$$\text{Actual secondary current } \bar{i}_2 = \frac{n_1}{n_2}\,i_2 \quad \text{------ (23)}$$

[illegible]

Coming back to consider three cases of the particular solution of the equation (21) for the further davalopemant. It is evident from the inaqualities that the exponents α and β are real for the case (1) while those in case (3) are complex numbers. In the first case the equations of the currents i_1 and i_2 can be used in their present form. Since the functions are logarithmic, this is called the logarithmic form. In the third case trigonometric functions appear; therefore this is called the trigonometric or oscillatory form. The second case, howaver belong to neithar form marking the transition between the case (1) and (3) and called the critical case. Furthermore for the case (2) the equations (21) and (22) fail, and a modification of these equations is necessary. This will be omitted, for this case is an extremly rare in the practical circuits. The reduction of the equations (21) and (22) will be made for the oscillatory case.

Let $\alpha = a + jb$ and $\beta = a - jb$

where $a = \frac{A}{2}$ and $b = \sqrt{B - \frac{A^2}{4}}$

then (21) assumes the form

$$i_1 = \frac{R_5 MI + L_2 E}{R_5 M + L_2 R_1}\left[\frac{a+jb}{2jb}\epsilon^{-(a+jb)t} - \frac{a-jb}{2jb}\epsilon^{-(a-jb)t}\right]$$

$$-(a^2+b^2)CE\left[\frac{\epsilon^{-(a+jb)t} - \epsilon^{-(a-jb)t}}{2jb}\right]$$

$$= \frac{R_5 MI + L_2 E}{R_5 M + L_2 R_1}\frac{\epsilon^{-at}}{b}\left[b\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2} - a\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2j}\right]$$

$$+(a^2+b^2)CE\frac{\epsilon^{-at}}{b}\left[\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2j}\right]$$

$$= \frac{R_5 MI + L_2 E}{R_5 M + L_2 R_1}\frac{\epsilon^{-at}}{b}\left[b\cos bt - a\sin bt\right]$$

$$+(a^2+b^2)CE\frac{\epsilon^{-at}}{b}\sin bt \quad \cdots\cdots\cdots\cdots (24)$$

Let $\theta = \tan^{-1}\left[\frac{a}{b}\right]$ $\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$ and $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

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Putting these in (24)

$$i_1 = \frac{R_s M I + L_2 E}{R_s M + L_2 R_1} \epsilon^{-at} \frac{\sqrt{b^2+a^2}}{b} \left] \frac{b}{\sqrt{a^2+b^2}} \cos bt - \frac{a}{\sqrt{a^2+b^2}} \sin bt \right]$$

$$+ (a^2+b^2) C E \frac{\epsilon^{-at}}{b} \sin bt$$

$$= \frac{R_s M I + L_2 E}{R_s M + L_2 R_1} \epsilon^{-at} \sec\theta \cos(bt+\theta)$$

$$+ (a^2+b^2) C E \frac{\epsilon^{-at}}{b} \sin bt \text{ - - - - - - - - - - - - - - - } (25)$$

Similarly

$$i_2 = \frac{L_2}{R_s M}\left[R_1 i_1 - E + \frac{R_s M I + L_2 E}{C(R_s M + L_2 R_1)} \frac{\epsilon^{-at}}{b} \left(\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2j} \right) \right.$$

$$\left. - E \frac{a-jb}{2jb}\left(1 - \epsilon^{-(a+jb)t}\right) + E \frac{a+jb}{2jb}\left(1 - \epsilon^{-(a-jb)t}\right) \right]$$

$$= \frac{L_2}{R_s M}\left[R_1 i_1 - E + \frac{R_s M I + L_2 E}{C(R_s M + L_2 R_1)} \frac{\epsilon^{-at}}{b} \sin bt \right.$$

$$- E \frac{a}{b} \epsilon^{-at} \left(\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2j} \right)$$

$$\left. + E \left\{ 1 - \epsilon^{-at} \left(\frac{\epsilon^{jbt} + \epsilon^{-jbt}}{2} \right) \right\} \right]$$

$$= \frac{L_2}{R_s M}\left[R_1 i_1 - E + \frac{R_s M I + L_2 E}{C(R_s M + L_2 R_1)} \frac{\epsilon^{-at}}{b} \sin bt \right.$$

$$\left. - E \tan\theta\, \epsilon^{-at} \sin bt + E(1 - \epsilon^{-at} \cos bt) \right] \cdot (26)$$

Again the true value of the secondary current is

$$\bar{i}_2 = \frac{n_1}{n_2} i_2 \text{ - } (27)$$

It has already been discussed in the previous chapter that the spark resistance r_s is a function of the density of the spark current that jump across the plug points, and less than a certain value of this current density the increase of resistance is so great that the passage of any current become prohibitive, its flow will cease abruptly. This sudden change of the secondary current induce, for momentary, an e.m.f. on the primary coil. The e.m.f. thus

[illegible]

causes another disturbunce in the primary circuit. This last transient involves primary circuit only, and its e.m.f. equation can be written as follow:

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{c}\int i_1 dt = E + e \quad \text{-----------} (28)$$

where $e = L_2 \frac{di_2}{dt}$. Since this has no theoretical value whatever, its value must be determined from the experimental data. Differentiating (28),

$$L_1 \frac{d^2 i_1}{dt^2} + R_1 \frac{di_1}{dt} + \frac{1}{c} i_1 = \frac{de}{dt} \quad \text{-----------} (29)$$

The solution of (29) is

$$i_1 = K_1 \epsilon^{-(\alpha+\beta)t} + K_2 \epsilon^{-(\alpha-\beta)t}$$

$$+ \left(C \frac{de}{dt} + C^2 R_1 \frac{d^2 e}{dt} + \text{---------} \right) \text{----} (30)$$

Since the value of C is an order of 10^{-7}, the terms within the bracket, which converge with the power of C, could be neglected, or

$$i_1 = K_1 \epsilon^{-(\alpha+\beta)t} + K_2 \epsilon^{-(\alpha-\beta)t} \quad \text{-----------} (31)$$

where $\alpha = \frac{R_1}{2L_1}$ and $\beta = \sqrt{\frac{R_1^2}{4L_1^2} - \frac{1}{LC}}$

K_1 and K_2 are constants of the integration which can be evaluated from the following conditions

$i_1 = 0$ when $t = 0$

and $\frac{1}{c}\int i_1 dt = E$ " $t = 0$

Theoretically i_1 is not zero when t is zero, but its magnitude is so small that may be neglected. Defferentiating (31) and multiplied by L_1,

$$L_1 \frac{di_1}{dt} = -(\alpha+\beta) K_1 \epsilon^{-(\alpha+\beta)t} - (\alpha-\beta) K_2 \epsilon^{-(\alpha-\beta)t} \quad \text{----} (32)$$

But from (28)

$$L_1 \frac{di_1}{dt} = E + e - i_1 R_1 - \frac{1}{c}\int i_1 dt$$

Equating the right members of above two equation and applying the condition of circuit for $t = 0$

$$-e = L_1\left[(\alpha+\beta)K_1 + (\alpha-\beta)K_2\right] \qquad (33)$$

Also from (31)

$$K_1 + K_2 = 0 \qquad (34)$$

Solving for K_1 and K_2 from (33) and (34), we get

$$K_1 = -\frac{e}{2\beta L_1} \qquad \text{and} \qquad K_2 = \frac{e}{2\beta L_1}$$

$$i_1 = -\frac{e}{2\beta L_1}\epsilon^{-\alpha t}\left[\epsilon^{+\beta t} - \epsilon^{-\beta t}\right] \qquad (35)$$

$$= -\frac{e}{\beta L_1}\epsilon^{-\alpha t}\sinh\beta t \qquad (36)$$

Again the equation (35) leads to three distinct solutions according to the values of constants R_1, L_1, and C. It is however, the characteristics of this equation is the same as that of the equation (12); and (36) is one of this particular solutions, i.e. logarithmic case expressed in a hyperbolic form. For the oscillatory case, let $\beta = jb$ or $b = \sqrt{\frac{1}{L_1C} - \frac{R_1^2}{4L_1^2}}$, then (35) becomes

$$i_1 = -\frac{e}{bL_1}\epsilon^{-\alpha t}\left[\frac{\epsilon^{jbt} - \epsilon^{-jbt}}{2j}\right] \qquad (37)$$

$$= -\frac{e}{bL_1}\epsilon^{-\alpha t}\sin bt \qquad (38)$$

The condenser is charged with the quantity Q equal to CE when the primary current has entirely died away and stored energy in the plates of condenser in the form of an electric charge. The energy thus stored will be discharged through the breaker's points in the form of current when the interrupter closed the primary circuit. Therefore the e.m.f. equation of this transient will be

$$i_c r_c + \frac{q}{c} = 0 \qquad (39)$$

But i_c = condenser's discharging current $= \frac{dq}{dt}$

$$\therefore \quad \frac{dq}{dt} + \frac{q}{r_c C} = 0 \qquad (40)$$

or

$$q = K\epsilon^{-\frac{1}{r_c C}t} \qquad (41)$$

Since $q = Q = CE$ when $t = 0$, $K = CE$

or $q = CE\epsilon^{-\frac{1}{r_c C}t}$

and $i_c = \frac{dq}{dt} = \frac{E}{r_c}\epsilon^{-\frac{1}{r_c C}t}$ ------------------ (42)

The transient charging current of the condenser during the decay of primary current is, of course, exactly the same as the primary coil current which has been derived — the equation (25).

II The General Equation of the Transient Current in an Alternating Current circuit with a Complex E.M.F. Wave.

An alternating current or electromotive force is rarely an exact sine wave, nevertheless it is always a periodic function of the time. This is especially true in the magneto generators of all descriptions. The generating e.m.f. wave of this particular generator is far from the sinusoidal, but is highly peaked wave. The mathematical treatment of such a distorted wave can best be carried by expressing it in an infinite trigonometric series, or Fouriers' series.

Since an e.m.f. generated by any alternator has no constant term nor contains even harmonics, the expression of an complex e.m.f. wave is the form

$$e = E_1 \sin\theta + E_3 \sin 3\theta + E_5 \sin 5\theta + \text{-----}$$

$$+ E_m \sin m\theta + \text{---------------}$$

$$+ E_1' \cos\theta + E_3' \cos 3\theta + E_5' \cos 5\theta + \text{------}$$

$$+ E_m' \cos m\theta + \text{--------} \qquad (43)$$

or

$$= \sum_{n=1}^{n=\infty} E_n \sin n\theta + \sum_{n=1}^{n=\infty} E_n' \cos n\theta \text{ ---------- } (44)$$

Suppose we impress an e.m.f. expressed by the equation (43) upon the circuit of any natures; such as one contains resistance, inductive reactance, and condensive reactance; the current that flows in the circuit could be expressed in terms of e.m.f. and constants of the circuit. The general equation of current may be derived from the e.m.f. equation of such circuit,

$$X \frac{di}{d\theta} + Ri + X_c \int i \, d\theta = \sum E_n \sin n\theta + \sum E_n' \cos n\theta \quad (45)$$

Differentiating and dividing all terms by X,

$$\frac{d^2 i}{dt^2} + \frac{R}{X}\frac{di}{dt} + \frac{X_c}{X} i = \sum \frac{nE_n}{X} \cos n\theta - \sum \frac{nE_n'}{X} \sin n\theta \quad (46)$$

The solution of (46) composed of two terms, i.e. the complimental solution or transient term, and the particular solution or parmanent term. The parmanent term can be obtained by using the symbolic operator "D" which is equal to $\frac{d}{d\theta}$

$$i_p = \frac{1}{D^2 + \frac{R}{X} D + \frac{X_c}{X}} \left[\sum \frac{nE_n}{X} \cos n\theta - \sum \frac{nE_n'}{X} \sin n\theta \right] \text{----} (47)$$

$$= \sum \frac{\left[RE_n + (nX - \frac{X_c}{n}) E_n' \right]}{R^2 + (nX - \frac{X_c}{n})^2} \sin n\theta$$

$$+ \sum \frac{\left[RE_n' - (nX - \frac{X_c}{n}) E_n \right]}{R^2 + (nX - \frac{X_c}{X})^2} \cos n\theta \text{ ----} (48)$$

$$= \sum I_n \sin n\theta + \sum I_n' \cos n\theta \text{ ---------------} (49)$$

The transient term of the equation (46) is

$$i_t = A \epsilon^{-\left[\frac{R}{2X} + \sqrt{\frac{R^2}{4X^2} - \frac{X_c}{X}}\right]\theta} - B \epsilon^{-\left[\frac{R}{2X} - \sqrt{\frac{R^2}{4X^2} - \frac{X_c}{X}}\right]\theta} \quad (50)$$

where A and B are constants and to be determined from the initial condition.

The total current then the sum of permanent and transient terms

$$i = i_p + i_t \text{ ---------------------------} (51)$$

The equation (51) can be applied to a circuit

[illegible]

contains resistance and inductive reactance but no capacity, or with a circuit that has resistance only. For the first case (51) reduced to

$$i = A' \epsilon^{-\frac{R}{X}\theta} + \sum \left[\frac{RE_n + nXE_n'}{R^2 + n^2X^2} \sin n\theta \right]$$

$$+ \sum \left[\frac{RE_n' - nXE_n}{R^2 + n^2X^2} \cos n\theta \right] \text{ --------- } (52)$$

For the second case

$$i = \sum \frac{E}{R} \sin n\theta + \sum \frac{E}{R} \cos n\theta \text{ ---------- } (53)$$

III Low-Tension Magneto System.

The short circuit armature current for "the first transient" is approximately from the equation (52)

$$i_1 = A_1 \epsilon^{-\frac{R_a}{X_a}\theta} + \sum \left[\frac{R_aE_n + nX_aE_n'}{R_a^2 + n^2X_a^2} \sin n\theta \right]$$

$$+ \sum \left[\frac{R_aE_n' - nX_aE_n'}{R_a^2 + n^2X_a^2} \cos n\theta \right] \text{ ---------- } (54)$$

The value of A may be determined from the condition

$i_1 = I$ some predetermined value,

when $\theta = \theta_1$, or (54) becomes

$$I = A_1 \epsilon^{-\frac{R_a}{X_a}\theta_1} + \sum \left[\frac{R_aE_n + nX_aE_n'}{R_a^2 + n^2X_a^2} \sin n\theta_1 \right]$$

$$+ \sum \left[\frac{R_aE_n' - nX_aE_n'}{R_a^2 + n^2X_a^2} \cos n\theta_1 \right]$$

$$= A_1 \epsilon^{-\frac{R_a}{X_a}\theta_1} + i_{p\,\theta=\theta_1} \text{ ---------------- } (55)$$

$$\therefore \quad A = \epsilon^{\frac{R_a}{X_a}\theta_1} \left[I - i_{p\,\theta=\theta_1} \right] \text{ ------------------ } (56)$$

Thus the armature is short circuited through the interrupter during the period of the first transient in which the position of armature, respect to that of the field poles, moves from θ_1 to some other phase angle

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say θ_2 when the second transient begins. The e.m.f. equation of this transient involves primary and secondary circuits interlinked with a magnetic circuit, or

$$R_p i_1 + X_p \frac{di_1}{dt} + X_m \frac{di_2}{dt} = e \quad \text{------------} (57)$$

$$R_s i_2 + X_s \frac{di_2}{dt} + X_m \frac{di_1}{dt} = 0 \quad \text{------------} (58)$$

Solving these two equations simultaneously for i_1 and i_2 in a manner exactly the same as that of the equations (4) and (5), we get

$$i_1 = A_2 \epsilon^{-\frac{Y}{W}\theta} + \frac{e}{R_p} - \frac{X_p}{R_p^2}\Big[\frac{de}{d\theta} - \frac{W}{Y}\frac{d^2e}{d\theta^2} + \frac{W^2}{Y^2}\frac{d^3e}{d\theta^3} - \cdots\cdots - (-1)^m\Big(\frac{Y}{W}\Big)^{m-1}\frac{d^me}{d\theta^m} + \cdots\cdots\Big] \cdots (59)$$

where $Y = R_p R_s$ and $W = R_p X_s + R_s X_p$

$$i_2 = -\frac{1}{R_s}\sqrt{\frac{X_s}{X_p}}\Big[e - R_p i_1\Big] \quad \text{------------} (60)$$

and

$$\bar{i}_2 = -\frac{1}{R_s}\sqrt{\frac{X_s}{X_p}}\Big[e - R_p i_1\Big]\frac{n_1}{n_2} \quad \text{------------} (61)$$

The constant A_2 can be determined from the condition

$$i_1 = I' - i_2 \qquad \text{when} \quad \theta = \theta_2$$

and has the value

$$A_2 = \epsilon^{\frac{Y}{W}\theta_2}\Bigg[\frac{R_s I'}{R_s + R_p\sqrt{\frac{X_s}{X_p}}} + \Bigg(\frac{\sqrt{\frac{X_s}{X_p}}}{R_s + R_p\sqrt{\frac{X_s}{X_p}}} - \frac{1}{R_p}\Bigg)e + \frac{X_p}{R_p^2}\Big(\frac{de}{d\theta} - \frac{W}{Y}\frac{d^2e}{d\theta^2} + \cdots\cdots - (-1)^m\Big(\frac{Y}{W}\Big)^{m-1}\frac{d^me}{d\theta^m} + \cdots\Big)\Bigg] (62)$$

substituting this value in (59) for A_2, the complete expression for the primary current is derived.

IV High-Tension Magneto System.

The first transient of a high-tension magneto is the same as that of a low-tension magneto, except the initial condition of the armature current. Because

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of the fact that a high tension magneto is connected in an open circuit type. Hence the initial current condition is zero instead of I for zero time

$$\therefore \quad i_1' = A_1 \epsilon^{-\frac{R_p}{X_p}\theta} + \Sigma\left[\frac{R_p E_n + n X_p E_n'}{R_p^2 + n^2 X_p^2} \sin n\theta\right]$$

$$+ \Sigma\left[\frac{R_p E_n' - n X_p E_n}{R_p^2 + n^2 X_p^2} \cos n\theta\right] \quad \text{-------} \quad (63)$$

in which

$$A_1 = -\epsilon^{\frac{R_p}{X_p}\theta_1} \, i_{p\,\theta=\theta_1} \quad \text{-----------------} \quad (64)$$

The second transient of a high-tension magneto, however, slightly differs from that of a low-tension magneto.

$$i_1 R_p + X_p \frac{di'}{d\theta} + X_m \frac{di_2}{dt} + X_c \int i_1 d\theta = e \quad \text{---------} \quad (65)$$

$$i_2 R_S + X_S \frac{di_2}{dt} + X_m \frac{di_1}{dt} = e \quad \text{-----------------} \quad (66)$$

Solving these two equations by usual method, we have

$$i_1' = K_1 \epsilon^{-\alpha\theta} + K_2 \epsilon^{-\beta\theta} + \frac{1}{X_c}\frac{de}{d\theta}$$

$$+ \frac{X_S - X_m - R_S(X_S X_p + R_p R_S)}{X_c R_S} \frac{d^2 e}{d\theta^2} + \frac{\cdots\cdots}{X_c^2 R_S^2} \frac{d^3 e}{d\theta^3}$$

$$+ \text{-----------}$$

Those terms after second member of right side contains X_c or its higher power as denominator, the numerical value of which is an order of 10^7. Therefore these terms can be neglected

$$\therefore \quad i_1' = K_1 \epsilon^{-\alpha_1\theta} + K_2 \epsilon^{-\beta_1\theta} \quad \text{-----------------} \quad (67)$$

Similarly

$$i_2' = K_3 \epsilon^{-\alpha_2\theta} + K_4 \epsilon^{-\beta_2\theta} \quad \text{-----------------} \quad (68)$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2, K_1, \text{----} K_4$ are the functions of the constants. The determination of which repeat the process that already has been carried out in the previous equations, this will be omitted.

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Appendix B

The Definite Wave - Point Relay Switch

In all forms of the electrical transient phrnomena in an alternating current circuit, the magnitude of the transient always depends upon the phase angle at which the circuit is closed or opened. By closing or opening such circuit with an ordinary switch at random gives results which are of little or no use for an analytical purpose.

The function of the definite wave-point relay switch is to close or open an alternating current circuit at any predetermined instant, i.e., at any phase angle from the neutral point of the electro motive-force wave, in connection with an oscillograph.

The essential part of the apparatus is a rotating contact-maker which makes a momentary contact once per revolution. Referring to the figure A, two ebonite discs K are rigidly mounted on a shaft which is directly coupled to the shaft of generator. To the circumference of one disc is fastened a thin conducting strip forming a slip ring. On the other disc a point conductor is attached on the surface of the rim. These two conductors on the discs are connected internally. A stud which carries a movable brush F is mounted loose upon the fixed shaft. The movable brush F is resting on the disc carrying a point contact, while a fixed brush E is resting permanently upon the slip ring. The relative po-

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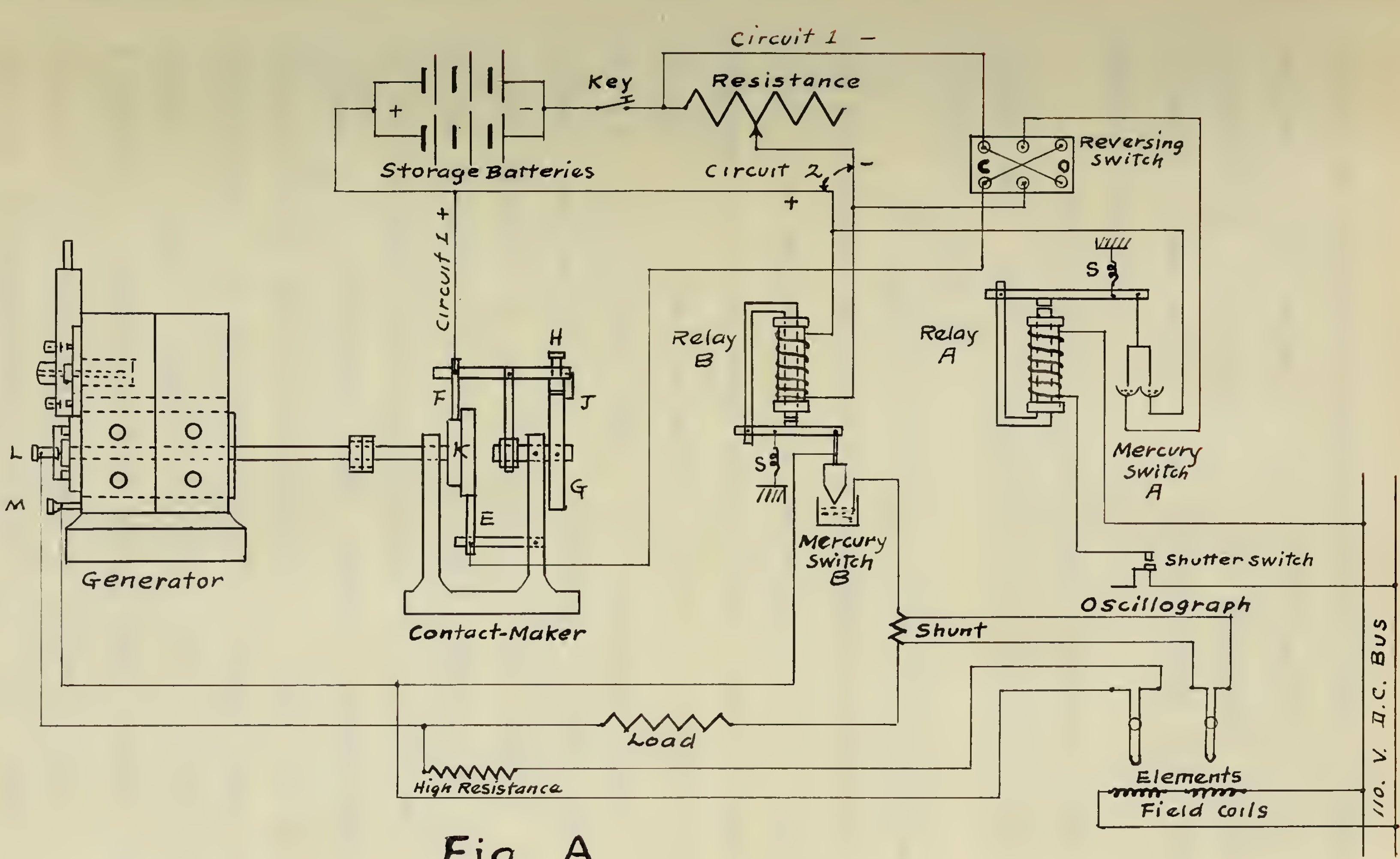

Fig. A

E	Fixed Brush Resting on the Slipe Ring.
F	Movable Brush " " " Contact-Making Disc.
G	Graduated Circular Scale.
H	Set Screw to Hold F in any Desired Position.
J	Point Indicator of the scale.
K	Rotating Discs. S Spring

sition of the movable brush therefore that of armature of the generator with respect to the field magnets of generator is indicated by the pointer J on the graduated circular scale G. By means of set screw H the movable brush may be set in any desired position.

Beside this contact-maker, there are two sensitive relays used, each accompanied with a mercury switch arranged as shown in Fig. A.

The operation of this device is as follows; By closing the simple switch at the storage batteries the core of relay B is magnetized, the resistance in the circuit is so arranged that the lever arm is barely held up against the pull of spring S, thus the mercury switch B is kept open. The circuit 1, on the other hand, from the same source of power through the contact-maker, reversing switch, mercury switch A, and finally to the terminals of the relay B. The polarity of the circuit 1 and that of 2 are opposing at the relay coil. If the circuit which operates shutter of the oscillograph is closed by an extraneous mean, current in the relay A comes to close mercury switch A and remain so until the shutter current is interrupted. In the mean time the contact-maker completes the circuit 1, and an impulse current flows through the relay coil B against a steady current, that already established from the circuit 2. Since the resistance in the circuit 1 is much smaller in comparison with that of 2, this impulse current is sufficient to cause demagnetization of the core of relay which

in turn releases the lever arm. The mercury switch B thus closes the armature circuit through a load impeadance. The transient current and e.m.f. take place for this particular phase angle and are recorded on the photographic film.

The use of this device for the transients which due to opening the switch, the manipulation of apparatus is exactly the same, except that the reversing switch is closed on the opposite side marked "O" and adjust relay B in such a way that the core is magnetized to a strength such that it can hold the lever arm against the tension but not sufficiently strong enough to pull up the lever arm. Now if the circuit 1 is completed by relay switch A and contact-maker the impulse current, for momentary, flow through the coil in the same direction with the steady current from the circuit 2. The magnetism in the core due to sum of these two currents is capable of pulling the lever arm up and hold it in that position by the relay coil even after the circuit 1 is broken.

The resistances of the circuit 1 and 2 and the voltage of batteries must be kept constant for a set of films to be taken including a calibration film which definitely determines the position of movable brush. Since there exists a time lag from the instant the shutter circuit is closed till the mercury switch B opens or closes and the constancy of this time lag depends entirely upon the constancy of the circuits condition.

The following oscillograms show the transient short circuit currents of a magneto generator running at the speed of 1200 R.P.M. The switch was set to close the circuit at 0°, 30°, 60°, 75°, and 90° respectively. ~~It is~~ However, the actual measurement shows a slight error each picture, the maximum of which is one and eight tenths electrical degree or one four thousandth of ~~the~~ a second.

It is possible, so far as the author's opinion is concerned, to make this apparatus accurate within an error of two electrical degrees at the frequency of 60 cycles or nearly one ten thousandth of the second, if a slight improvement is given to the relays.

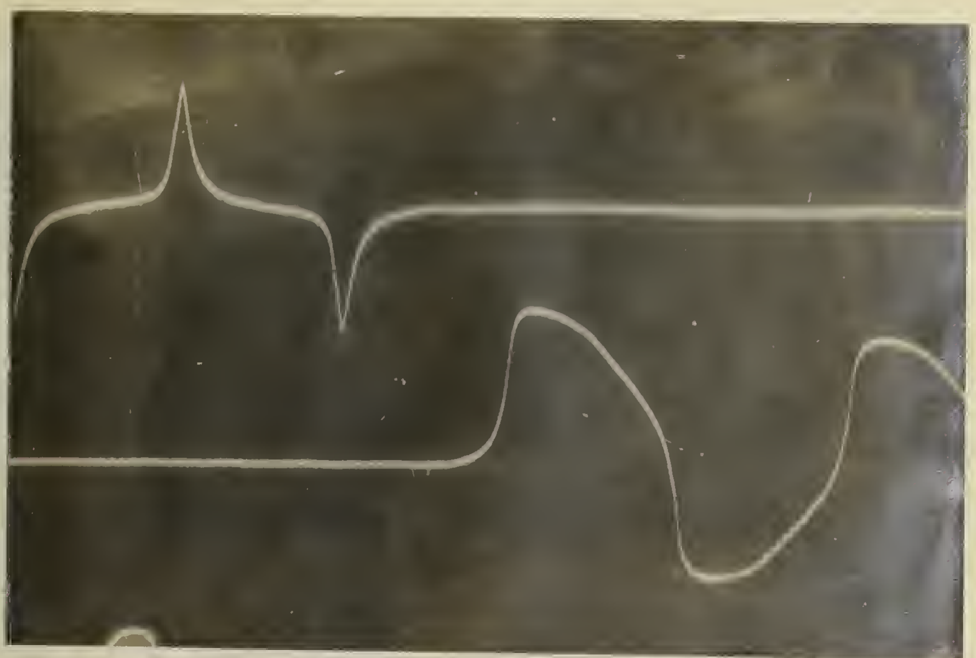

Fig B Swich set for 0°
Calibration 0°

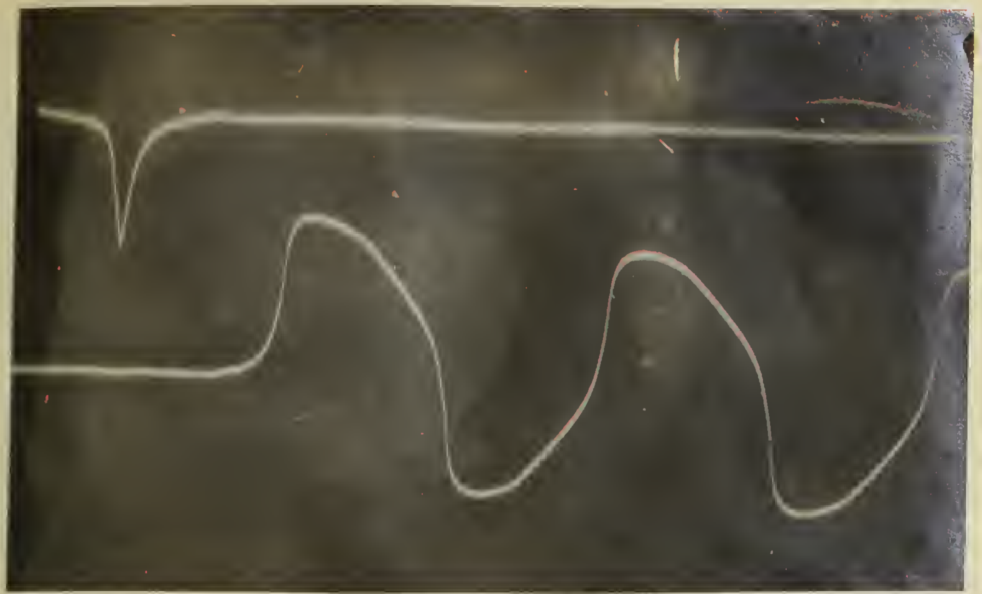

Fig. C Switch set for 30°
Calibration 28.2°

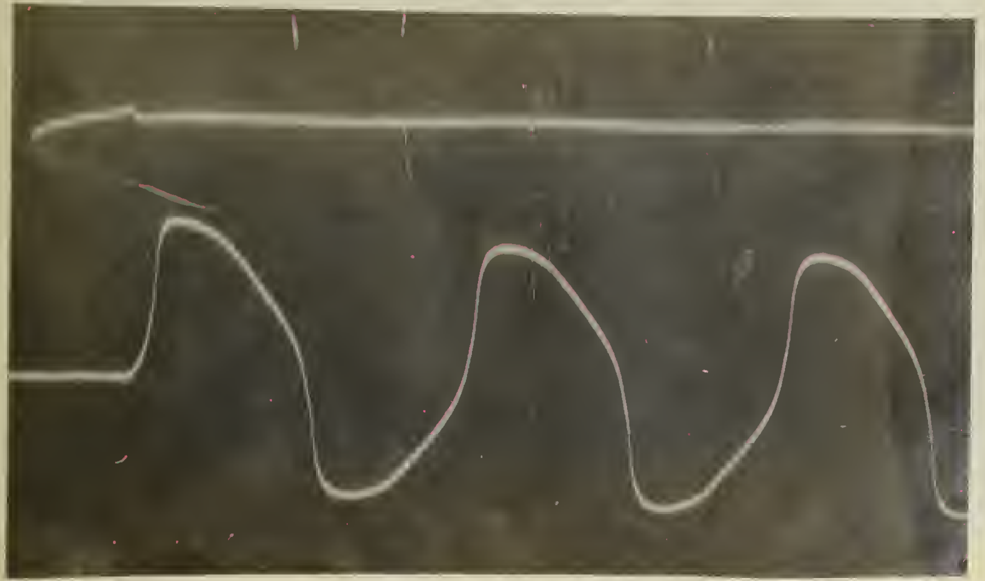

Fig. D Switch set for 60°
Calibration 61.1°

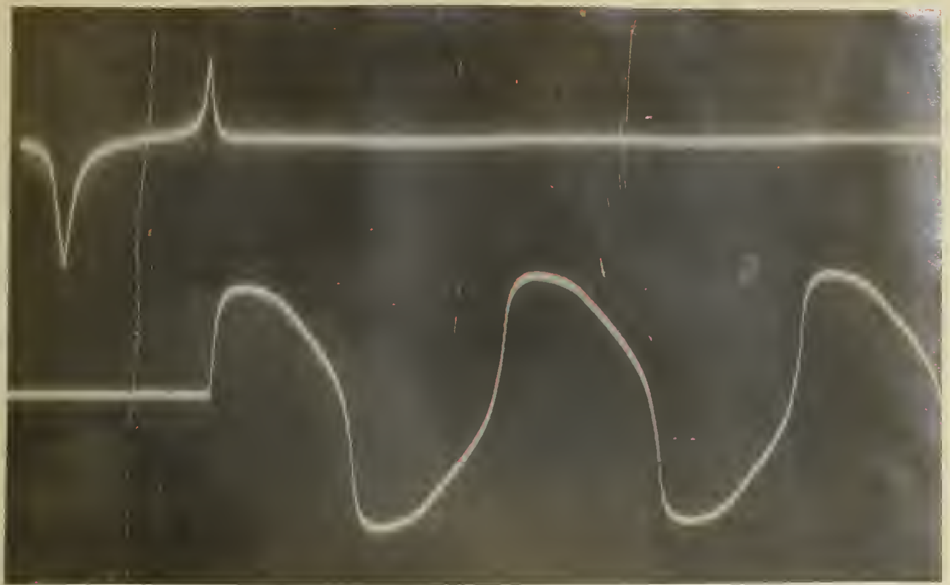

Fig. E Switch set for 90°
Calibration 88.8°

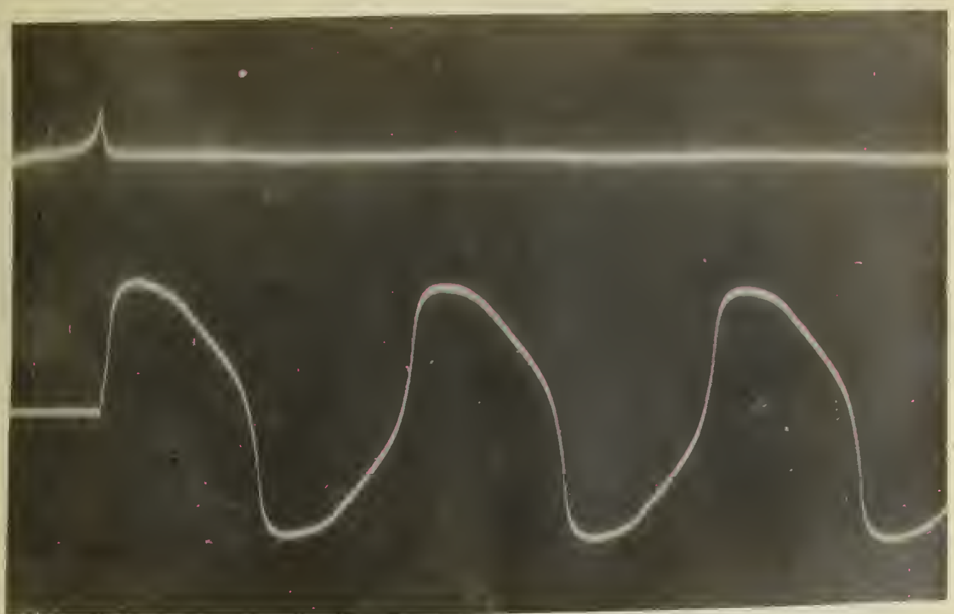

Fig. F Switch set for 75°
Calibration 76.4°

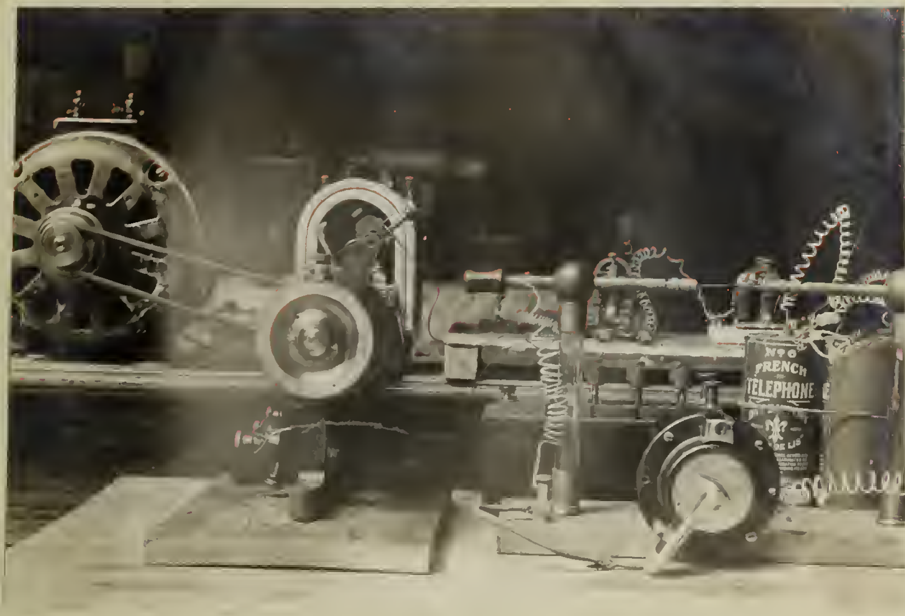

Fig. G The Definite wave point switch — contact maker and Relays — Set up for a short circuit test of a magneto

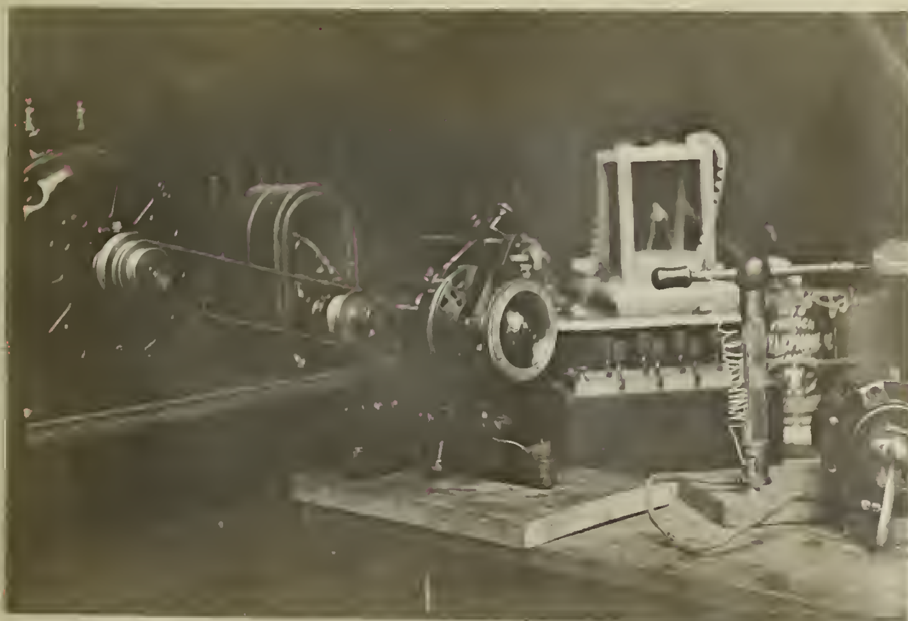

Fig. H Another View of the Definite wave point switch connected to a magneto.