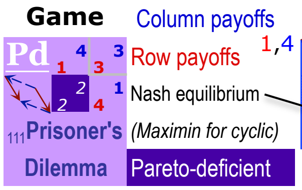
2 X 2 Games

1. Visualizing the Adjacent Possible in the Topology of Two-player, Two-strategy Games



Pareto-optimal outcomes in **bold** font

Adjacent games are neighbors by payoff swaps

1↔2 swaps form tiles of 4 games

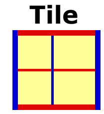
2↔3 swaps join tiles into 4 layers

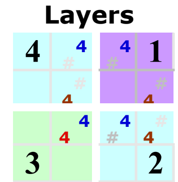
3↔4 swaps link layers

Layers differ by alignment of 4s

Each layer is a torus, table is a torus

Layers scrolled to center Prisoner's Dilemma





Families	Harmonious
1.**Win-win** 4,4	Stag Hunt
2.**Biased** 4,3	Battle
Self-serving	Altruistic
3.Second Best 3,3	
4.**Unfair** 4,2	Winner Loser
5.**PD Family**	Tragic 3,2
Pd 2,2	Alibi 3,2
6.**Cyclic**	

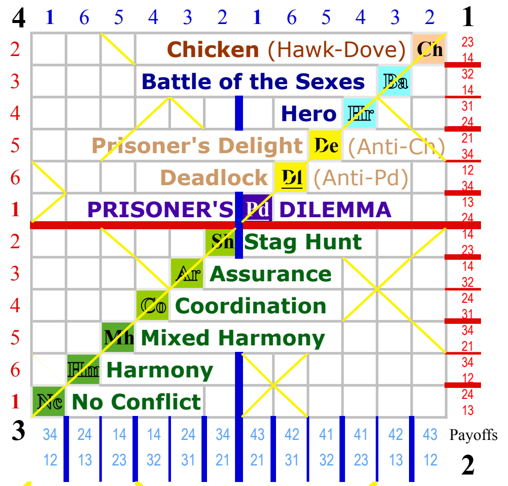
Game Numbers: Layer: Row: Column. Symmetric across SW-NE diagonal. Swapping Row and Column positions swaps indices, and Layer for 2&4

To find a game: Make ordinal 4<3<2<1. Put column with Row's 4 right, row with Column's 4 up. Find layer by alignment of 4s; then intersection of Row&Column payoffs.

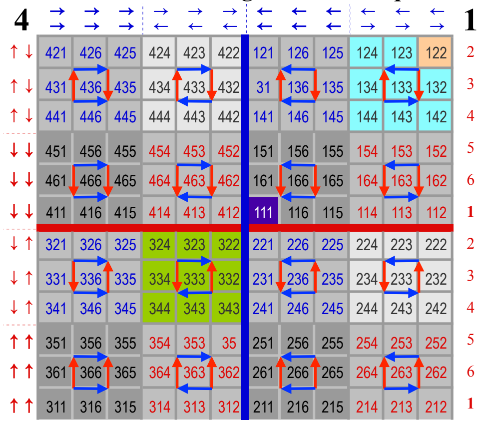
see Robinson & Goforth 2005 *The Topology of the 2x2 Games: A New Periodic Table* www.cs.laurentian.ca/dgoforth/home.html www.BryanBruns.com/2x2chart.html

2. Structures in the Topology of 2X2 Games

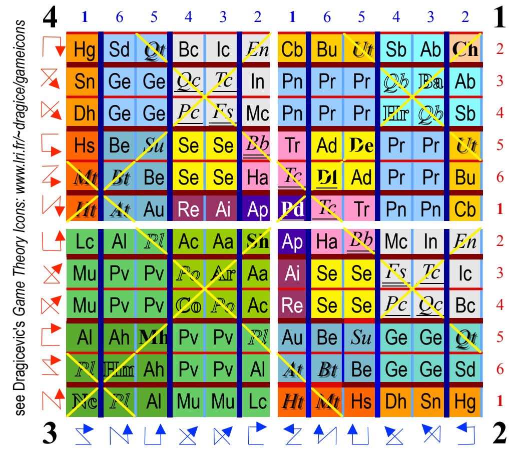
a. Twelve Symmetric Games on the Diagonal



b. Dominant Strategies & Nash Equilibria



c. 4 Layers, 12 Payoff Patterns, 144 Games



4 games per tile, 36 games and 9 tiles per layer

66 asymmetric pairs: 66 + 12 = 78 "unique" games

d. High swaps (3↔4) Link Tiles Across Layers

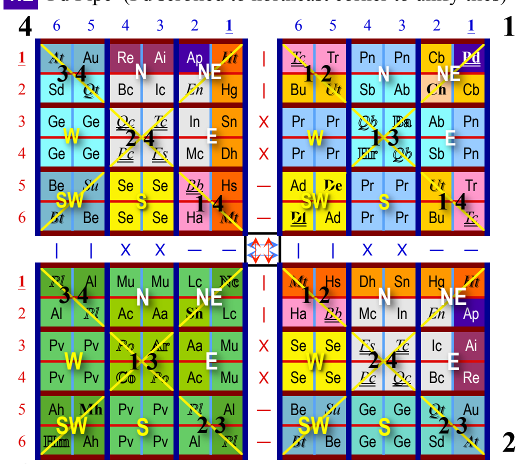
connecting equivalently-located tiles on different layers

6 Hotspots double-link 2 tiles

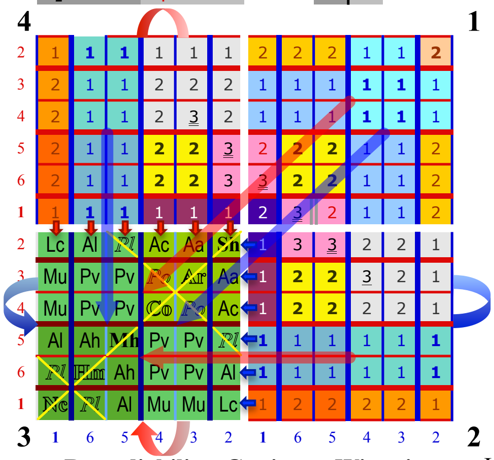
6 Pipes link 4 tiles on 4 layers

13 Coord.-Battle Hotspot

NE Pd Pipe (Pd scrolled to northeast corner to unify tiles)



3 High swaps realign 4s; switch row or column in tile



e. Remediability: Getting to Win-win

Swaps to reach win-win 1= single 3↔4 swap

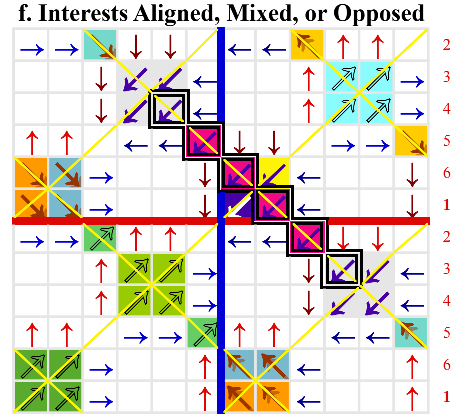
Paths with 2 or 3 steps include 2↔3 and 1↔2 swaps

Arrows show vectors for slices of games

\# **Bold** = each player has a pathway

Pareto-efficient paths: each swap step leaves player with same or better-ranked outcome

f. Interests Aligned, Mixed, or Opposed



2 Externalities and inducement correspondences

Jekyll-Hyde Type ++ Pure Cooperation

±

Pure Conflict - - Jekyll-Hyde Type

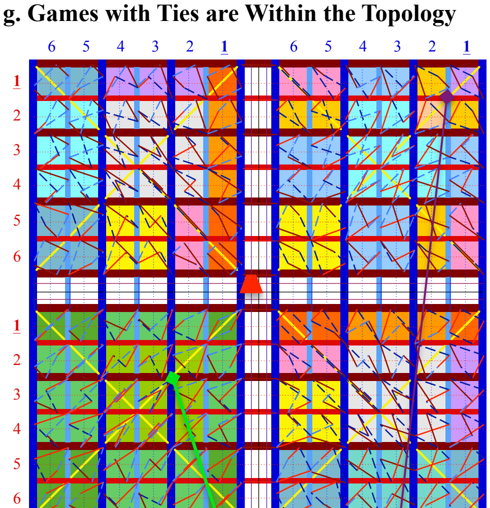
Fixed Rank-sum - - ± + +

(Zero Sum) see Schelling 1963 *The Strategy of Conflict*

2 Greenberg 1990 *The Theory of Social Situations*

Robinson & Goforth 2005 *The Topology of 2x2 Games*

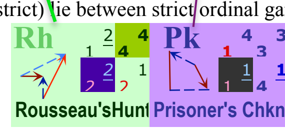
g. Games with Ties are Within the Topology



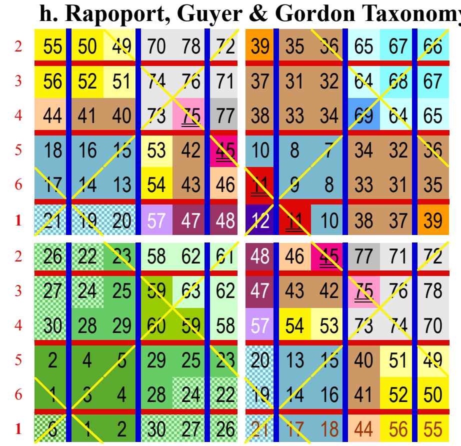
Strict Nonstrict: Half-swaps 2=3 1=2

Ordinal games at grid intersections (graph nodes/vertices)

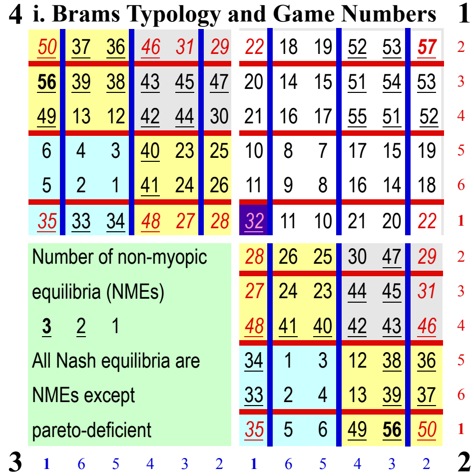
Games with ties (non-strict) lie between strict ordinal games, (Robinson et al. 2007) as do all normalized 2x2 games (1-4 scale)



h. Rapoport, Guyer & Gordon Taxonomy



i. Brams Typology and Game Numbers



	Stable	Weakly Stable	Unstable	
No Conflict N EP	D_2o D_1o D_0o	D_2c D_1c	D_0c	E=Natural outcome is Equilibrium
Mixed Motive M EP	D_2o D_1o	D_2l D_1t D_1f D_0f	D_1tf D_1ctf	2 1 0 Dominant strategies
M Ep	D_2o pareto-deficient	D_1f	D_1tf	no pressures
M e	no D_0	2 equilibria	e_2o e_2c	competitive pressure
M e	dominance D_0	No equilibria	e_0o e_0c	threat-vulnerable
Complete Opposition Z E	D_2o D_1o	e	eD_0c	force-vulnerable

see Rapoport et al.1976 *2x2 Games;* Robinson & Goforth 2003

	Strongly cyclic
Not cyclic	Moderately cyclic
No conflict	Weakly cyclic

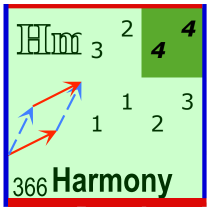
see Brams 1994 *Theory of Moves;*

Difficult see Brams & Kilgour 2008

www.BryanBruns.com/2x2chart.htm

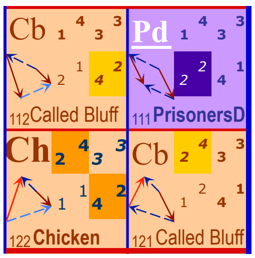
A Brief Overview of the Topology of 2x2 Ordinal Games

The topology of 2x2 games (Robinson and Goforth 2005) elegantly displays the relationships between strict ordinal 2x2 games, where two players each have two strategies and four differently ranked preferences for the outcomes. *Swaps* in the two lowest payoffs (1↔2) form *tiles* of four games closest to each other in the payoff space. Swaps in middle payoffs (2↔3) and additional low swaps form *layers*. The four layers differ by the alignment of 4s. Each layer is a torus. Scrolling Prisoner's Dilemma (Game 111) to the center helps visualize the structure of the topology.



Harmony. *Row or Column's choice benefits the other, and leads to win-win*

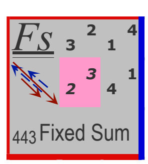
The 12 symmetric games, where players face identical payoffs, form a diagonal axis from southwest to northeast. Row payoffs are the same across rows, and column payoffs the same down columns, so the 66 asymmetric games on either side of this axis of symmetry combine payoffs from different symmetric games, for a total of 12 + 66 = 78 "unique" games.

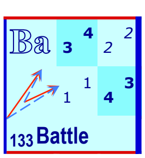


Prisoner's Dilemma Tile*: Swaps in lowest two payoffs transform Prisoner's Dilemma into Chicken.*
Prisoner's Dilemma*: If both prisoners both keep silent, they each get a light sentence, their second-best outcome (3,3). If only one confesses, he goes free and the other gets a long sentence. But the dominant strategy for each is to confess (defect), ending up with both getting a medium-length sentence, the second-worst outcome (2,2).*
Chicken*: If both play a Hawk strategy, both die (1,1). If both play Dove, both get second-best (3,3). Two Hawk-Dove Nash Equilibria yield very unequal outcomes (4,2 and 2,4).*

Three rows in each layer have dominant strategies for Row (with higher payoffs whatever Column does), as do three columns for Column. Based on dominant strategies, three-fourths of games have a single Nash Equilibrium, (a pair strategies that are best replies to each other). *Battles of the Sexes* and *Stag Hunts*, including asymmetric variants, have no dominant strategies and two equilibria. *Cyclic* games have no dominant strategies and (for ordinal payoffs) no equilibria.

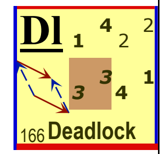
Fixed Rank-sum*. Ordinal equivalent of Zero-sum; gains for one are matched by losses for the other. Cyclic, since one player always has an incentive to move. Maximin strategies avoid the worst payoff.*



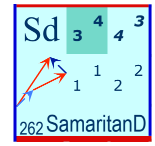


Battle of the Sexes*. A couple prefers doing something together, but each has a different first choice*

Prisoner's Dilemma Family games have a Pareto-superior outcome that both players would prefer to the Nash Equilibrium. The family can extend to include *Tragic* games, also with a poor equilibrium but lacking a better alternative. In *Second-Best* games such as Prisoner's Delight and Deadlock, both players can achieve their second-ranked preference (3,3). *Biased* games with high but unequal (4-3) equilibria form the largest payoff family. In *Altruistic* games, the largest subfamily, a player with a dominant strategy gets their second-ranked payoff. In *Selfish* games, the dominant strategy gets the best of a biased equilibrium. *Unfair* games have highly unequal (4-2) equilibria.



Deadlock*.Incentives lead to second-best*



Samaritan's Dilemma*. Column could help himself, without Row's aid. Row prefers that both invest but prefers to help whatever Row does, while Column most prefers to be helped and do less.*

Swaps in high payoffs (3↔4) link layers. In six *hotspots*, swaps for Row or Column connect the same two tiles, double-linking two layers, as in the Layer 1-3 Hotspot where Battles of the Sexes turn into Coordination games. In six *pipes*, high swaps for Row or Column link to different tiles, weaving four tiles on four layers. High swaps, one for each player, convert Prisoner's Dilemma into an Asymmetric Prisoner's Dilemma (412 or 221) and then into the ordinal Stag Hunt (322). Most games, except fixed sum, can be converted to win-win through one or two swaps.

Most games have mixed interests: a strategy may help or hurt other player. Games of pure cooperation or conflict are less frequent, and fixed-sum (zero-sum) games even rarer. In *Jekyll-Hyde Type* games, such as Blackmail (216/461), incentives make one player kind and the other cruel.

Games with ties lie between the strict ordinal games, linked by *half-swaps* that make (or break) ties in preferences. For games with interval-scale or real payoffs, normalized versions can be mapped into the topology. Game numbers uniquely identify similar and related games, like scientific names for species, and so could aid comparative and cumulative research.

Rousseau's Stag Hunt*. A hunter could choose to cooperate in the risky hunt for a stag, or could catch a hare regardless of what the other hunter does. This game with ties lies between Assurance (333) and the strict Stag Hunt (322).*