# Erratum: Neutrinoless double-$\beta$ decay in effective field theory: The light-Majorana neutrino-exchange mechanism [Phys. Rev. C 97, 065501 (2018)]

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Equation (44) describes the amplitude for the process $\pi^-(q) \to \pi^+(q)e^-e^-$ with on-shell pions $q^2 = m_\pi^2$ and massless electrons with zero four-momentum and not as written in the text above Eq. (44) for $\pi^-\pi^- \to e^-e^-$.

We now consider the more general situation $\pi^-(p_a)\pi^-(p_b) \to e^-(p_1)e^-(p_2)$ with on-shell pions $p_a^2 = p_b^2 = m_\pi^2$ and massless electrons $p_1^2 = p_2^2 = 0$ and introduce the Mandelstam variables $s = (p_a + p_b)^2,\ t = (p_a - p_1)^2,\ u = (p_a - p_2)^2$. The amplitude $T_{\pi^-\pi^-\to e^-e^-}$ can be written as

$$T_{\pi^-\pi^-\to e^-e^-} = T_{\rm lept} 2F_\pi^2 \mathcal{S}_{\pi\pi} + 4G_F^2 V_{ud}^2 m_{\beta\beta} \bar{u}_L(p_{e1})\sigma_{\mu\nu} C\bar{u}_L^T(p_{e2})\mathcal{A}_{\pi\pi}^{\mu\nu}, \tag{1}$$

where $T_{\rm lept} = 4G_F^2 V_{ud}^2 m_{\beta\beta}\bar{u}_L(p_{e1})C\bar{u}_L^T(p_{e2})$. The antisymmetric leptonic structure $\mathcal{A}_{\pi\pi}$ vanishes if $p_1^\mu = p_2^\mu$. Note that we capture a subset of N$^2$LO corrections by normalizing the amplitude in terms of the physical pion decay constant $F_\pi$ rather than $F_0$.

For the amplitude $\mathcal{S}_{\pi\pi}$, we find

$$\mathcal{S}_{\pi\pi} = -\left[\frac{1}{4}\left(\frac{1}{t} + \frac{1}{u}\right)(s - 2m_\pi^2) + \frac{1}{(4\pi F_\pi)^2}\left(\mathcal{V}_{\pi\pi} + \frac{s - 2m_\pi^2}{2}\frac{5}{6}g_\nu^{\pi\pi}(\mu)\right)\right], \tag{2}$$

where the loop contribution is given by

$$\begin{aligned}\mathcal{V}_{\pi\pi} = {}& \frac{3(s - 2m_\pi^2)}{2}\ln\frac{\mu^2}{m_\pi^2} - \frac{(2m_\pi^2 - s)^2 \ln^2\left(-\frac{1+\sqrt{1-\frac{4m_\pi^2}{s}}}{1-\sqrt{1-\frac{4m_\pi^2}{s}}}\right)}{4s}\\ &- \frac{(m_\pi^2 - t)\ln\left(1 - \frac{t}{m_\pi^2}\right)\left[m_\pi^4 + 6m_\pi^2 t + t(-s + t)\right]}{4t^2}\\ &- \frac{(m_\pi^2 - u)\ln\left(1 - \frac{u}{m_\pi^2}\right)\left[m_\pi^4 + 6m_\pi^2 u + u(-s + u)\right]}{4u^2}\\ &- \frac{6m_\pi^4(t + u) + 132m_\pi^2 tu + tu[-45s + 12(t + u)]}{24tu}.\end{aligned} \tag{3}$$

At threshold, that is, for $s = 4m_\pi^2,\ t = -m_\pi^2,\ u = -m_\pi^2$, we obtain

$$\mathcal{S}_{\pi\pi} = 1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\ln\frac{\mu^2}{m_\pi^2} + \frac{7}{2} + \frac{\pi^2}{4} + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right). \tag{4}$$

At the kinematic point, $s = 0,\ t = m_\pi^2,\ u = m_\pi^2,\ q^2 = m_\pi^2$, which corresponds to the kinematics $\pi^-(q) \to \pi^+(q)e^-(0)e^-(0)$, we recover Eq. (44),

$$\mathcal{S}_{\pi\pi} = 1 + \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(3\ln\frac{\mu^2}{m_\pi^2} + 6 + \frac{5}{6}g_\nu^{\pi\pi}(\mu)\right). \tag{5}$$

In this Erratum we specified the correct kinematics for which the amplitude in Eq. (44) of the paper applies. In addition, Eqs. (2) and (3) extend Eq. (44) to a completely general kinematics, which can be useful for the chiral extrapolation of lepton-number-violating amplitudes computed in lattice QCD to the physical point.