A REPAIRABLE ITEM INVENTORY SYSTEM USING DYNAMIC PROGRAMMING AND MARKOV PROCESSES

JAMES ALFRED BIGGINS

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THESIS

A REPAIRABLE ITEM INVENTORY SYSTEM USING DYNAMIC PROGRAMMING AND MARKOV PROCESSES

by

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A Repairable Item Inventory System Using Dynamic Programming and Markov Processes

by

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ABSTRACT

This thesis developed an analytical model of a repairable item inventory system. The system consisted of a depot that repaired failed units according to some repair policy and stocked ready-for-issue units in support of a finite number of customers. A least-cost repair policy and stock level was determined by use of a computer program which is included as an appendix.

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I. BACKGROUND

This thesis studied the problem of inventory management of repair parts that have high unit cost and few customers.

The importance of a model such as this lies in the fact that, although these parts represent a very low percentage of items managed by the Navy, they account for a large percentage of the dollar investment in inventory.

Because of the high dollar value, it costs less to repair failed units than to buy new ones. Thus, it pays to establish a repair facility to renew failed items.

This thesis attempted to model this repair process to determine how this type of system might operate at minimum cost.

II. INTRODUCTION

A repairable item inventory system consists of three elements: users or customers, repair facilities, and stocking activities.

When a part fails, the customer forwards that unit to the repair facility and requests a new unit from the stocking activity. Upon receipt of the failed unit, the repair facility refurbishes it. The number requiring repair at any one time is called the "repairable stock". The third element, the stocking activity, stores the parts received from the repair facility and issues them on request from the end user. The number of ready-for-issue parts in the stocking activity is called the safety stock.

This is the repairable item inventory system. One should note that the system as defined is closed. That is, every failed unit received by the repair facility can be repaired. Thus, after the inventory system is in operation, there is no reason to order new units to replace those already in the system.

In order to operate the repairable item inventory system optimally, two basic questions must be answered.

1. How many failed units should be in the repair facility before one should repair and how many should one repair when the decision to repair is made.

2. How much safety stock should be carried by this stocking activity so that orders from end-users might be promptly and economically filled.

When one has made these two decisions one can compute a cost per cycle to operate under these conditions. The goal of course is to make these decisions so that the system operates at minimum cost.

The answer to the first question, when and how much to repair, was determined using a technique developed by Howard in his Dynamic Programming and Markov Processes. It is, in part, the purpose of this thesis, to look into the feasibility of utilizing such a method on this problem.

The answer to the second question, how much to stock, can be found only after solving the repair problem.

Since the problem is naturally divided into answering question (1) and then question (2), Section III will treat the problem in this sequence. In Section IV, there will be a sample solution and a discussion of the computer program. And, finally, the methods used will be criticized, and conclusions will be made in Section V.

III. MODEL

A. GENERAL

A model such as the one that was described in the introduction requires that a decision about the repair policy be made at specified time periods. The point in time at which this decision is made is called the review time. The time between reviews is held constant and because of this the model is called a periodic review model. The length of time between reviews is defined as the average time needed to repair a failed unit. This is called the repair cycle. By defining the length of the repair cycle in this way, the model with its associated assumptions will better fit the real-world situation. This will be seen more clearly later.

Graphically, the repairable item inventory system operates in the following manner during a repair cycle.

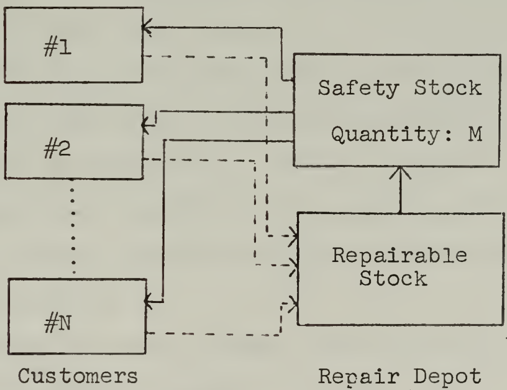


Figure 1.

REPAIRABLE ITEM INVENTORY SYSTEM

Thus, during the repair cycle, customers' repairable units fail and set in motion the dynamics of Figure 1. In this illustration there are N customers and M units of safety stock.

Consider the effects of this M and N upon the quantity of repairable stock. The maximum number that can be in need of repair is N+M, while the minimum number is zero. Zero in repair implies that each customer has a working unit and that there are M units in a ready-for-issue condition in stock. At the other end of the range N+M units in repair implies that each customer lacks this part and that there are zero units of safety stock available. This is the worst condition in which the system is allowed to function. The amount of repairable stock, a number between zero and N+M is defined as the "state" of the system. Thus, there are N+M states.

The depot, of course, would not need to repair all failed items on hand. The alternatives available are to repair from zero up to the total amount awaiting repair. For example, if there were 5 units of repairable stock, the depot could make six decisions: repair 0, repair 1, ..., repair 5. Define this decision as the repair policy for that state. Define a set of decisions, one for every state, as the "repair policy".

The preceding has been a verbal description of a repairable item inventory system. The next few sections will translate this into mathematical terms.

1. <u>The Probability Matrix</u>

Let S_n be the amount of repairable stock at the beginning of the n^{th} repair cycle.

R_n be the amount of repairable stock actually repaired during the n^{th} repair cycle.

D_n be the amount demanded during the n^{th} repair cycle.

Thus $S_n = S_{n-1} - R_{n-1} + D_{n-1}$. This describes a Markov chain. The distinctive element in a Markov chain is that the probability law of the future of a process, once it is in a given state, depends only on the state and not on how the process arrived in that state.

Using the recursive definition of S_n above, a matrix can be formed that gives the probability of having any amount of repairable stock on hand, given the amount at the beginning of the previous repair cycle. This movement from one state to another is called a transition.

Thus, if the present state is i, and the alternative chosen is to repair k, then the next state of the system will be j, if and only if j+k-i items fail and hence are demanded during the cycle.

Assumptions can be made concerning the number and rate of these demands. A typical one might be that the demands are distributed Poisson with mean λ. This assumption, although not necessary to the model, will be used in the remainder of the thesis.

If $p_k(i,j)$ denotes the probability that the current state is i, k items are repaired and the next states is j then $p_k(i,j)$ is given by:

$$
(1)\quad p_k(i,j) = \begin{cases} 0 & \text{if} \quad i-k < 0 \\ & \quad\quad j+k-i < 0 \\ & \quad\quad j+k-i > N \\ & \quad\quad j+k-i > N-i+M \ \& \ i > M \\ \dfrac{\lambda^{j+k-i} e^{-\lambda}}{(j+k-i)!} & \text{otherwise as } i=0,\dots,N+M \\ & \quad\quad k=0,\dots,i \\ & \quad\quad j=0,\dots,N+M \end{cases}
$$

The restrictions upon $p_k(i,j)$ when $p_k(i,j)=0$ in the formula above can be explained in the following way:

a. $i-k < 0$. The probability p(.) equals zero when one wants to repair more than is in repairable stock. The decision is not a valid one.

b. $j+k-i < 0$. This situation can occur only if there is negative demand, which is clearly impossible.

c. $j+k-i > N$. This can occur only if there are more demands than customers can generate.

d. $j+k-i > N-i$ M and $i > M$.

If $i > M$ then the number of demands in the cycle can be no greater than N-(i-M) since i-M customers will not have items. Thus those cases in which j+k-i exceed N-i+M must be assigned zero probability.

Suppose $\bar{A}$ is a vector that was defined earlier as the repair policy, that is, a repair decision for every possible state. The transpose of the (N+M+1) vector $\bar{A}$ can be denoted by:

$$\bar{A}' = (a_o, a_1, \ldots, a_i, \ldots a_{N+M}),$$

where a_i is the amount to be repaired, if the repair cycle starts in the i^{th} state.

For any given repair policy, a probability matrix describing this passage from state to state can be defined as:

$$\text{(2)} \qquad \bar{P}(\bar{A}) = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ N+M \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 0 & 1 & 2 & & & & & & & & N+M \end{array} \\ \left[\begin{array}{ccccccccccc} P & P & P & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & P & P & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & P & P & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & & & & & & & \\ \vdots & & & & & & & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & P \end{array} \right] \end{array}$$

Where the P's denote those probabilities which are positive

Because of the number of alternative repair policies available for a given starting state, there are (N+M+1)! possible probability state matrices. By defining the matrices in this way, three additional assumptions are made: 1) even after a decision to repair items in the repairable stock is made, those items will not be available until the next period. In other words, demands can only be filled from those items that are ready for use at the beginning of the repair period. 2) Parts that have failed during a repair period are delivered to the repair facility prior to the end of the period regardless of when they fail. 3) Shipments from the repair facility are received by the customers at the end of the repair cycle.

2. <u>Complete Ergodicity</u>

To use the techniques discussed in this thesis, the states of the probability matrices described in the previous section must not just be Markov chains, but ergodic Markov chains. That is, the one-step probability transition matrices which result from repair policies must be completely ergodic. By this is meant that after a large number of transitions, the state of the system is independent of the initial state. For a matrix $\bar{P}(\bar{A})$ to exhibit this quality, it is sufficient to show that it has one recurrent chain.

Consider first a repair policy in which the alternative is "do not repair" regardless of the state of the system. The one-step probability matrix would be as follows:

$$
\begin{array}{c}
\\ 0 \\ 1 \\ 2 \\ \vdots \\ \\ \\ \\
\end{array}
\begin{array}{c}
\begin{array}{cccccccccc}
0 & 1 & 2 & & N & N+1 & & & & N+M
\end{array} \\
\left[\begin{array}{cccccccccc}
P_o & P_o & P_o & \cdots & P_o & 0 & \cdots & & & 0 \\
0 & P_o & P_o & \cdots & P_o & P_o & 0 & \cdots & & 0 \\
0 & 0 & P_o & \cdots & P_o & P_o & P_o & \cdot & & 0 \\
\vdots & & & \ddots & & & & & & \vdots \\
\vdots & & & & \ddots & & & & & \vdots \\
\vdots & & & & & \ddots & & & & \cdot \, 0 \\
0 & \cdots & & & & & & & 0 & P_o
\end{array}\right]
\end{array}
$$

Figure 2.

Repair Policy: "Do Not Repair"

The symbol P_o denotes those probabilities which are positive under the repair decision "do not repair." From

the figure one can see that eventually the system will end up in state N+M, independent of the initial state, and remain there.

To get a better feel for this property in conjunction with a repairable item inventory system, let the matrix shown in Figure 2. change slightly. Consider the probability transition matrix which results from changing just one of the alternatives: repair k items if the state is i.

$$
\begin{array}{c|cccccccc|}
 & 0 & 1 & & N & & & & N+M \\
\hline
0 & P_o & P_o & \cdots & P_o & 0 & \cdots & & 0 \\
1 & 0 & P_o & \cdots & P_o & P_o & 0 & \cdot & 0 \\
\vdots & \vdots & & \ddots & & & & & \vdots \\
i & \cdot & P_k & P_k & P_k & P_k & & & 0 \\
\vdots & \vdots & & & & \ddots & & & \vdots \\
N+M & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & P_o \\
\hline
\end{array}
$$

Figure 3.

Repair Policy: "Repair k if in states i; otherwise do not repair any"

The decision to repair k items shifts the associated row of the matrix k units to the left. Thus, a decision to repair is equivalent to allowing the system to shift back to some lower state with positive probability. This must be kept in mind when considering the next question.

Is there any repair policy that would make this probability transition matrix non-ergodic? Consider the following repair policy: If in states 0 through N+M-1, repair all; if in state N+M, repair none.

$$
\begin{array}{c|ccccccccc|}
 & 0 & 1 & 2 & & & N & & & N+M \\
\hline
0 & P_o & P_o & P_o & \cdots & & P_o & 0 & \cdot & 0 \\
1 & P_1 & P_1 & P_1 & \cdots & & P_1 & 0 & \cdot & 0 \\
2 & P_2 & P_2 & P_2 & \cdots & & P_2 & 0 & \cdot & 0 \\
\vdots & \vdots & & & & & & & & \\
M & P_m & P_m & P_m & \cdots & & P_m & 0 & \cdot & 0 \\
M+1 & P_{m+1} & P_{m+1} & P_{m+1} & \cdots & P_{m+1} & 0 & 0 & \cdot & 0 \\
\vdots & \vdots & \ddots & & & & & & & \\
N+M-1 & & & & & & & & & 0 \\
N+M & 0 & 0 & 0 & \cdots & & & 0 & & P_{n+m}
\end{array}
$$

Figure 4.

Non-ergodic Repair Policy

In this example, if, after a long period of time one found himself in state N+M, one would know he had started in N+M. Thus, this repair policy produces a probability transition matrix that is non-ergodic.

A non-ergodic repair policy could arise in three different instances in a problem of this nature. Each of these will be considered separately along with the method for handling that situation.

a. A non-ergodic repair policy might exist in the set of possible repair policies, but might not be utilized in any way while determining the optimal repair policy. The method developed in the following sections requires that only the repair policies considered be ergodic, not all possible repair policies.

b. A non-ergodic repair policy could exist and could be in the set of repair policies considered in deriving an optimal policy that is completely ergodic. In this case, there is a method developed by Howard in his Dynamic Programming and Markov Processes that can solve this type of problem.

c. Finally, a non-ergodic repair policy might be optimal. But, if a non-ergodic policy were optimal, one would actually have two separate repairable item inventory problems. Thus, they could be separated and treated as such.

From the example that had to be used to show that a non-ergodic repair policy might exist, one can see that it is an unusual case. So, with the solutions offered for these cases, no generality will be lost in considering all repair policies completely ergodic.

3. Costs

Up until now the thesis has described the repairable item inventory problem as a series of transitions, going from one amount of repairable stock to another, depending upon the repair policy. But, from this, nothing can be said

about the relative merit of one repair policy over another. This is a function of costs.

For instance, safety stock serves as an alternative to repairing a failed unit. Having an item available in safety stock allows one to delay repair of the failed unit and still offer immediate delivery. Since both of these alternatives offer delivery at once, the choice of one method of operation over the other is dependent upon costs.

There is also the alternative between immediate delivery and delayed delivery. But, here again, one can attach a cost for immediate delivery and delayed delivery. Thus, there is a set of trade-offs and each has an associated cost. It must only be determined which repair policy and stock level will allow this system to operate at minimum cost.

For this repairable item model, charges will be made at the end of each repair cycle. At that time one will determine what has happened during the previous cycle and what is on hand at the moment and then make appropriate changes. The following costs will be considered while doing this:

a. Stock Level Costs
b. Set-up and Repair Costs
c. Additional Holding Costs
d. Backorder Costs

a. Stock Level Costs

This cost includes the rental or depreciation of a warehouse. The maintenance, taxes, and insurance are

also considered in this charge. These costs are considered fixed after the size of the operation has been determined.

It is assumed, in this model, that if it costs \$K in fixed costs for a stocking policy of M units, it will cost \$2K to have a stocking policy of 2M.

But, after the stocking policy has been set, the cost, be it \$K or \$2K, will be charged no matter how much safety stock is actually on hand at the end of the repair cycle.

The stock level M will initially be assumed to be given and the optimal repair policy will be determined for that given level. In this instance, the stock level cost is constant and independent of repair policy. So it need not be used in this computation.

Later in the thesis the optimal choice of M will be considered; and, at that time, the stock level cost will enter the calculations.

b. Additional Holding Cost

At times the depot might have an amount in excess of the fixed stock level M on hand. This comes about when the amount demanded exceeds the safety stock. An additional amount is charged for each unit in the depot in excess of this safety stock to cover the cost of such things as additional warehouse space and overtime required to handle these items.

So, if S is the number in repair at the end of the period; h is the holding cost per unit

Then

(3) $$\text{Cost}_{\text{hold}} = \begin{cases} 0 & S \leq M \\ h(S-M) & S > M \end{cases}$$

c. Set-up and Repair Costs

When the decision is made to repair some failed units, two costs are incurred: a set-up cost and a repair cost.

The set-up cost is a fixed cost that is levied every time the decision is made to repair and is independent of the quantity repaired. It is considered the cost to tool up.

The second cost is the charge that is made for repairing each individual unit. This cost includes the labor and materials required to repair it.

So let C = the set-up cost
r = per unit repair cost
x = quantity repaired

Then

(4) $$\text{Cost}_{\text{rep}} = \begin{cases} C + rx & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

d. Backorder Costs

This is a cost charged when the quantity demanded exceeds available safety stock. It is a penalty for loss of goodwill or sales. In the Navy's case, it might be a penalty levied for lack of readiness.

In this model the penalty cost can only be levied if repairable stock exceeds M. If it is less than M there will be at least one unit of safety stock available.

So if the number in repair at the end of the repair period is denoted by S, and P is the backorder cost per unit, then

$$(5) \qquad Cost_{back} = \begin{cases} 0 & S \leq M \\ P(S-M) & S > M \end{cases}$$

The total variable cost per transition would then be the sum of the repair cost, the holding cost, and the backorder cost.

B. DETERMINATION OF THE OPTIMAL REPAIR POLICY FOR A GIVEN STOCK LEVEL

With the model, the transition probability matrix, and the costs defined as above, the minimum cost repair policy for a given stock level can be developed.

1. The Problem as a Markov Chain

Let $\bar{b}(n)$ be a vector whose i^{th} element is the probability that the state of the system will be in i after n repair cycles. $\bar{P}(\bar{A})$, as has been defined before, is the one-step transition probability matrix for a given repair policy $\bar{A}$.

If one knows the state vector $\bar{b}(0)$ and the Matrix $\bar{P}(\bar{A})$ then the state of the system after one repair cycle is

$$(6) \qquad \bar{b}(1)=\bar{b}(0)\bar{P}(\bar{A})$$

If one wishes to know the state vector $\bar{b}(n+1)$ with $\bar{b}(0)$ known then

(7) $$\bar{b}(n+1) = \bar{b}(n)\, \bar{P}(\bar{A})$$
$$= \bar{b}(n-1)\left[\bar{P}(\bar{A})\right]^2 = \bar{b}(n-2)\left[\bar{P}(\bar{A})\right]^3 = \ldots$$
(8) $$= \bar{b}(0)\left[\bar{P}(\bar{A})\right]^n$$

Now, if $\bar{P}(\bar{A})$ is ergodic, it can be shown that steady state probabilities exist and can be determined by solving the following equations:

(9) $$\bar{b} = \bar{b}\bar{P}(\bar{A})$$

(10) $$\bar{1}'\bar{b} = 1$$

where the elements of $\bar{b}$ are the steady-state probabilities of having a given quantity in repair.

Suppose then that repair policies vector $\bar{A}$ consists of $a_0, a_1, \ldots, a_i, \ldots a_{m+n}$ and suppose further that there are i units of repairable stock. The expected cost for the period would be

(11) $$K_{a_i}(i) = \sum_{j=0}^{N+M} p_{a_i}(i,j) c_{a_i}(i,j)$$

where $p_{a_i}(i,j)$ is the probability of ending in state j, having started in i, using the repair policy, repair a_i.

$C_{a_i}(i,j)$ is the cost associated with $p_{a_i}(i,j)$

Probabilities $p_{a_i}(i,j)$ and costs $C_{a_i}(i,j)$ are known and, so, expected cost $K_{a_i}(i)$ can be easily computed.

Since the expected cost of being in each state is known and the long-run probabilities of being in each state are known (Equations (9) and (10)), the long-run expected

cost per period can be determined and would be:

$$(12) \qquad K_A = \sum_{i=0}^{N+M} \bar{b}\, \bar{K}_{a_i}(i)$$

One could look at each possible repair policy, $\bar{A}$, and compute this K_A. Then the problem of determining a minimum cost policy for a fixed stock level M could be solved by comparing each cost, K_A, and choosing the minimum value. However, this would involve determining a cost for a large number of repair policies.

For example, for three customers and a two unit stock level, the number of repair policies that must be compared is 6! or 720. Thus, even for a very small size problem, the number of repair policies to be examined is quite large. For a larger problem the number of repair policies becomes so large as to prohibit even the use of a computer to solve the problem in the manner described above.

Thus the method is feasible but computationally not practical. The equations developed in this section, though, will be used later.

An alternative procedure for solving the problem in a manner which requires less computation is necessary.

2. <u>The Problem as a Dynamic Program</u>

To gain an understanding of this alternative approach it is best to look initially at a finite time horizon.

Consider a repairable item system over the next L-periods. Let there be n cycles remaining until the time

interval terminates. Define $a_i(n)$ as the repair policy that will be utilized if the system has i units in repair and n cycles remaining. When i is specified for each n, the repair policy over the life of the system will be specified. That repair policy which minimizes the total expected cost over the entire time period would be optimal.

Define $V_i(n)$ as the minimum total expected cost of maintaining the system with n cycles remaining, having started with i in repair (assuming an optimal policy has been followed up to this point).

$V_i(n)$ can be computed by first looking at the minimum cost policy if the system had one cycle remaining. If at the start of this period, one had i in repair, then

$$(13) \qquad V_i(1) = \underset{k}{\text{MIN}} \sum_{j=0}^{N+M} p_K(i,j)c_k(i,j)$$

With $V_i(1)$ known one could look at $V_i(2)$.

$$(14) \qquad V_i(2) = \underset{k}{\text{MIN}} \sum_{j=0}^{N+M} \begin{bmatrix} \text{Prob. of} \\ \text{Transition} \\ \text{To state } j \\ \text{repairing } k \end{bmatrix} \begin{bmatrix} \text{Cost of} \\ \text{Transition} \\ \text{to state } j \end{bmatrix} + \begin{bmatrix} \text{Minimum expected} \\ \text{cost for week 1} \\ \text{if in state } j \end{bmatrix}$$

$$(15) \qquad V_i(2) = \underset{k}{\text{MIN}} \sum_{j=0}^{N+M} p_k(i,j) \left[c_k(i,j)+V_j(1)\right]$$

In this manner one could work backward and determine an optimal policy for each cycle.

A recursive relation arises out of this approach.

$$V_i(n) = \underset{k}{\text{MIN}} \sum_{j=0}^{N+M} p_K(i,j) \left[c_k(i,j) + V_j(n-1) \right] \tag{16}$$

If termination of this repairable item system were imminent, this approach would be valid and efficient. If one did not know the number of cycles until termination, but only that it was a long time into the future, this method seems less effective.

Consider, then, what happens to $V_i(n)$, defined above, when n gets large. To accomplish this goal, generating functions will be used.

3. <u>Asymtotic Form of Recursive Relation</u>

The generating function of an arbitrary function defined on the non-negative integers is defined by

$$f(z) = \sum_{n=0}^{\infty} f(n) \, z^n \tag{17}$$

Some general properties of generating functions are the following:

General Properties of Generating Functions	
Function	Generating Function
$f(n)$	$f(z)$
$f(n-1)$	$zf(z)$
$f(n+1)$	$z^{-1}[f(z)-f(0)]$

TABLE I

During the following development, generating functions of vectors will be used. A vector generating function is simply a vector whose elements are generating functions.

To develop the asymptotic form of $V_i(n)$ (Equation 16) through generating functions, one must also develop the asymptotic expression for $\bar{b}(n)$ (Equation 7).

Transforming Equation 7 using the above table produces:

$$z^{-1}\left[\bar{b}(z) - \bar{b}(0)\right] = \bar{b}(z)\,\bar{P}\,(\bar{A}) \tag{18}$$

With some manipulation, the above equation becomes:

$$\bar{b}(z) = \bar{b}(0)\,(\bar{I} - z\bar{P}(\bar{A}))^{-1} \tag{19}$$

Of interest in this equation, for use a little later in the section, is the inverse transform of the following matrix from Equation (19)

$$\bar{H}(z) = (\bar{I} - z\,\bar{P}(\bar{A}))^{-1} \tag{20}$$

Consider $(\bar{I}-z\bar{P}(\bar{A}))^{-1}$. By the adjoint method of evaluating an inverse, one can write:

$$\bar{H}(z) = \frac{\text{Adjoint } (\bar{I}-z\bar{P}(\bar{A}))}{(\bar{I} - z\bar{P}(\bar{A}))} \tag{21}$$

Since z=1 is a characteristic root of every Markov matrix, $\left|\bar{I}-z\bar{P}(\bar{A})\right|$ can be written as:

$$(1-z)\ \ (r_o + r_1 z + \ldots + r_{n-1} z^{n-1}) \tag{22}$$

Thus by partial fraction expansion of Equation 21 one can show that $\bar{H}(z) = \frac{\bar{S}}{1-z} + \bar{G}(z)$

where $\bar{G}(z)$ is a rational function of z

Since from Table I, the inverse transform for $\bar{S}/(1-z)$ is $\bar{S}$, the inverse transform of $\bar{H}(z)$ would be

$$(24) \qquad \bar{H}(n) = \bar{S} + \bar{G}(n)$$

H(n) is simple $\left[\bar{P}(\bar{A})\right]^n$ as can be seen by Equation (8) and Equation (19). Since $\bar{P}(\bar{A})$ is a completely ergodic matrix, it must be true that $\bar{G}(n) \rightarrow 0$ as $n \rightarrow 0$ and $\bar{S}$ is the steady-state probability matrix.

So, each row of $\bar{S}$ is the row vector $\bar{b}$.

With this information at hand, an equation for $\bar{V}(n)$ can be developed. Suppose now that the repair policy has been given. The subscript, $\bar{A}$, describing this policy will be dropped from the development since it adds no information. Multiplying through by p(i,j) Equation (16) becomes:

$$(25) \qquad V_i(n+1) = \sum_{j=0}^{N+M} p(i,j)c(i,j) + \sum_{j=0}^{N+M} p(i,j)V_i(n)$$

$$i=1,\ldots,N+M$$

Combining Equation (25) with the notation of Equation (11) this becomes

$$(26) \qquad V_i(n+1)=K(i) + \sum_{j=0}^{N+M} p(i,j)V_j(n) \quad i=1,\ldots,N+M$$

Switching to matrix form this can be written:

(27) $$\bar{V}(n+1) = \bar{K}_b + \bar{P}\bar{V}(n)$$

where $\bar{K}_b$ is a column vector of the K(i)'s.

Through the use of transforms in Table I and the fact that the transform of f(n) = 1 is 1/(1-z), the matrix of generating functions becomes

(28) $$z^{-1}\left[\bar{V}(z) - \bar{V}(0)\right] = 1/(1-z)\bar{K}_b + \bar{P}\,\bar{V}(z)$$

With a little manipulation this becomes

(29) $$\bar{V}(z) = \frac{z}{1-z}(\bar{I}-z\,\bar{P})^{-1}\bar{K}_b + (\bar{I}-z\bar{P})^{-1}\bar{V}(0)$$

Substituting Equation (24) into Equation (29) and collecting terms, one gets:

(30) $$\bar{V}(z) = \frac{z}{(1-z)^2}\bar{S}\bar{K}_b + \frac{z}{1-z}\,\bar{G}(z) + \frac{1}{1-z}\,\bar{S}\bar{V}(0) + \bar{G}(z)\bar{V}(0)$$

Since the inverse transform of $z/(1-z)^2$ is f(n)=n, then the first term is n $\bar{S}\bar{K}_b$. By using partial fraction expansion and dropping terms that tend to zero as n becomes large (the asymptotic form is only of interest), the inverse transform of the second term is $\bar{G}(1)\bar{K}_b$. The third term has an inverse transform of $\bar{S}\bar{V}(0)$, since the inverse transform of 1/(1-z) is f(n) = 1. Finally, since the geometric terms of $\bar{G}(z)\ \bar{V}(0)$ approach zero as n gets large, this term can be ignored. Thus the asymptotic inverse transform of $\bar{V}(z)$ is:

(31) $$\bar{V}(n) = n\bar{S}\bar{K}_b + \bar{G}(1)\,\bar{K}_b + \bar{S}\bar{V}\,(0)$$

Since the matrix $\bar{S}$ is made up of N+M $\bar{b}$ vectors, one can write:

(32) $$\bar{K} = \bar{S}\,\bar{K}_b$$

where each element of $\bar{K}$ is the constant $\bar{K}_A$ per Equation (12).

So $\bar{V}(n)$ becomes

(33) $$\bar{V}(n) = n\bar{K} + \bar{G}(1)\,\bar{K}_b + \bar{S}\,\bar{V}(0)$$

What has been derived here is an equation for long-run cost as a function of N. Looking at the equation in this manner, $\bar{G}(1)\bar{K}_b+\bar{S}\bar{V}(0)$ will be defined as $\bar{V}$, the intercept. The asymptotic long-run cost equation finally becomes:

(34) $$\bar{V}(n) = n\bar{K} + \bar{V}$$

4. Review and Collection of Information

To quickly review, one initially wanted to find a method of determining the minimum cost repair policy. To do this, it was decided first to determine the minimum cost repair policy given a certain stock level.

Two approaches were used and combined: the Markov chain and Dynamic Programming approach. In the former, an equation for the cost per cycle can be obtained but it is shown that it is computationally impractical to solve a problem in this manner. In the latter approach, a simple

asymptotic form of total expected cost after n repair cycles is found. This leads to a simplified method of solution.

Suppose the repair system is operating under a certain policy. Then, by breaking the asymptotic form of the matrix $\bar{V}(n)$ into its elements one gets;

(35) $$V_i(n) = nK + V_i \quad i=0,\dots,N+M$$

From equation (26) one has:

(36) $$V_i(n) = K(i) + \sum_{j=0}^{N+M} p(i,j)V_j(n-1) \quad i=0,\dots,N+M$$

Now combining these two equations,

(37) $$nK+V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)V_j(n-1)$$

$$nK+V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)(nK+V_j)$$

and since $\sum_{j=0}^{N+M} p(i,j) = 1$

(38) $$K+V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)V_j \quad j=0,\dots, N+M$$

Equation (38) produces N+M+1 simultaneous equations with N+M+2 unknowns. That is, there are N+M+1 V_i's and the K as unknowns.

But, it one of the V_i's are set equal to zero, one can solve these equations. This causes all of the V_i's to be transformed by a constant amount, but does not affect the K. To show this, let:

(39) $$V_j' = V_j + a$$

Then

(40)

$$K+V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)\, V'_j$$

$$K+V_i+a = K(i) + \sum_{j=0}^{N+M} p(i,j) \left[V_j+a \right]$$

$$K+V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)\, V_j$$

Thus, scaling down the V's by setting one of them equal to zero does not affect the computation of the cost K of the system.

The development of these formulas assumes an arbitrary repair policy and thus computes the cost per repair cycle for this policy. Equation (12) was an equally easy way to solve this identical problem. The advantage to the present way, though, is that it presents a method of starting with an arbitrary repair policy and rapidly converging to the optimal repair policy that the previous method did not provide.

5. Algorithm

Equation 16 showed that if we were in repair cycle n the least cost policy for cycle n+1 in the ith state would be:

(41) $$\underset{a_i}{\text{MIN}} \left[K_{a_i} + \sum_{j=0}^{N+M} p_{a_i}(i,j)\, V_j(n) \right]$$

Since in the long run

(42) $$V_j(n) = nK + V_j$$

and

(43) $$\sum_{j=0}^{N+M} p\ (i,j) = 1$$

we can minimize cost over possible decisions in the i^{th} state by choosing the decision:

(44) $$\underset{a_i}{\text{MIN}} \left[K_{a_i}(1) + \sum_{j=0}^{N+M} p_{a_i}(i,j)\ V_j \right]$$

Using Equation (44) in conjunction with Equation (38) produces an interactive scheme that converges to the optimal policy. This will be proved in the next section. Assuming this, though, the algorithm would operate as follows:

ALGORITHM

A. For an arbitrary set of decisions, or for decisions generated from Part B, solve the set of simultaneous equations (setting one of the V_j's = 0).

(38) $$K + V_i = K(i) + \sum_{j=0}^{N+M} p(i,j)\ V_j \quad i=0,\ldots,N+M$$

B. Using the V_j's obtained in the above equations, compute over all i's and all a_i's

(44) $$K_{a_i}\ (i) + \sum_{j=0}^{N+M} p_{a_i}(i,j)\ V_j$$

For each state choose that repair alternative for which the equation above is minimum. Go to part A.

Each iteration will decrease the value of K. Upon reaching the optimal policy the same decision will recur. Thus, the system will have converged to the optimal repair policy and the variable cost for a given stock level.

Now, it only remains to prove the point, and a practical method of solving a repair problem of this nature will have been found.

6. Proof of Convergence

Suppose we have solved the system of simultaneous equations using a set of repair decisions A. Suppose further that in using part B of the algorithm, a repair policy B is generated. If it can be shown that $K_B \leq K_A$, convergence by this iterative scheme will be proven.

Since policy B produces a lower cost than the policy A from part B of the algorithm, we know that

$$(45) \qquad K_B(i) + \sum_{j=0}^{N+M} p_B(i,j)\, V_j^A \leq K_A(i) + \sum_{j=0}^{N+M} p_A(i,j) V_j^A \qquad i=0,\ldots,N+M$$

Let

$$(46) \qquad C_i = K_A(i) + \sum_{j=0}^{N+M} p_A(i,j) V_j^A - \sum_{j=0}^{N+M} p_B(i,j) V_j^A - K_B(i)$$

Thus C_i is greater than or equal to zero.

From part A of the algorithm we know

$$(47) \qquad K_B + V_i^B = K_B(i) + \sum_{j=0}^{N+M} p_B(i,j)\, V_j^B$$

(48) $$K^A + V_i^A = K_A(i) + \sum_{j=0}^{N+M} p_A(i,j)\, V_j^A$$

Subtracting the above two equations results in

(49) $$K^A - K^B + V_i^A - V_i^B = K_A(i) - K_B(i) + \sum_{j=0}^{N+M} \left[p_A(i,j)V_j^B - p_B(i,j) \right.$$

$$= C_i - \sum_{j=0}^{N+M} p_A(i,j)V_j^A + \sum_{j=0}^{N+M} p_B(i,j)V_j^A$$

$$+ \sum_{j=0}^{N+M} p_A(i,j)V_j^B - \sum_{j=0}^{N+M} p_B(i,j)V_j^B$$

$$= C_i + \sum_{j=0}^{N+M} p_A(i,j) \left[V_j^B - V_j^A \right]$$

Let

(50) $$\Delta(K) = K^A - K^B$$

$$\Delta V(i) = V_i^A - V_i^B$$

So the above equation by substitution becomes:

(51) $$\Delta(K) + \Delta(V_i) = C_i + \sum_{j=0}^{N+M} p_B(i,j)\ \Delta(V_j)$$

Equation (51) is identical to Equation (38) except that the terms are differences rather than absolute quantities.

Thus by analogy the solution to ΔK is

$$\Delta(K) = \sum_{j=0}^{N+M} b_i^{\,B}\, C_i \tag{52}$$

where b_i is the i^{th} element of $\bar{b}$ from Equation 9. Since all the elements of $\bar{b}$ are greater than or equal to zero, and $C_i \geqslant 0$ by definition, $\Delta K \geqslant 0$. So K^B is less than K^A and the algorithm converges.

So for a given stock level a method for determining the optimal repair policy and the variable cost in a few iterations has been found; its value increases as N+M+1, increases. One must now develop a method of determining the optimal stock level.

7. Computation of the Total Cost

In paragraph III.A.3.a., it was pointed out that there is a cost associated with the stock level that was fixed while determining repair policy. This fixed cost (FC) is added to the variable cost (VC) for a total cost for a given stock level.

$$TC = FC + VC$$

There remains to determine the stock level with its repair policy that gives minimum cost.

C. DETERMINATION OF AN OPTIMAL STOCK LEVEL

In determining an optimal stock level, the range of stock level can be any number between zero and infinity. One cannot, therefore, simply compute the optimal repair

policy for each stock level and then choose that stock level that generates the minimum total cost per cycle.

The total cost is a function of stock level and cannot be written without considerable computation. So, to examine the nature of the cost function, it will help to analyze the fixed costs and variable costs associated with each M separately. Fixed cost is a linear function of M so, it causes no particular problem. Variable cost though requires a little more analysis.

1. Analysis of Variable Cost

a. Set-up Cost.

As the stock level increases there will be fewer occasions for which one will repair failed items while minimizing cost. Thus the set-up cost per cycle approaches zero as stock level approaches infinity.

b. Backorder cost.

The greater the stock level the less frequently the system will enter a backorder state. So, again, the backorder cost per cycle will tend to approach zero as the stock level approaches infinity.

c. Repair Costs.

The cost to repair the failed items is independent of the stock level. In the long run, each item which fails will be repaired. Since the expected number of failures per cycle is λ, the expected repair cost per cycle must be λ times the cost of repairing the item.

Adding the three costs mentioned above, one can see that, as stock level increases to infinity the variable cost per cycle approaches the average repair cost per cycle.

2. <u>Analysis of the Total Cost</u>

The graph of the total cost functions, can be plotted as is shown below:

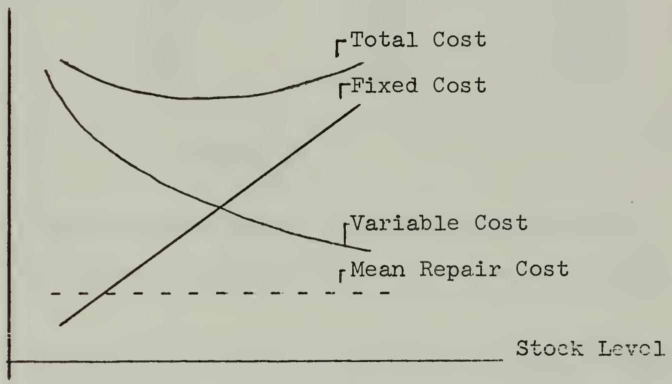


Figure 5.

GRAPH OF THE TOTAL COST FUNCTION

As an example of this total cost function the following parameters were used to solve a problem of this nature:

Number of customers	10
Mean Cyclic Demand	2
Set-Up Cost	\$ 3.
Repair Cost	\$ 3.
Backorder Cost	\$ 4.
Additional Carrying Cost	\$ 1.
Per Unit Fixed Carrying Cost	\$ 1.

Stock Level	Total Variable Cost	Total Fixed Cost	Total Cost
1	14.24	1.00	15.24
2	11.29	2.00	13.27
3	9.45	3.00	12.45
4	8.52	4.00	12.52
5	7.90	5.00	12.90
6	7.52	6.00	13.52
7	7.27	7.00	14.27
8	7.09	8.00	15.07
9	6.93	9.00	15.93
10	6.82	10.00	16.82

TABLE II

Here, the variable cost is approaching $6.00 while fixed cost is a linear function of stock level with 1 as the slope.

The heuristic arguments presented here indicate that the total costs function is convex. The importance of this lies in the fact that if there is a stock level let us say a, such that TC_a is the total cost of this stock level and if

$$TC_a \leqslant TC_{a+1}$$

and

$$TC_a \leqslant TC_{a-1}$$

then TC_a is the minimum total cost repair system and there is no $TC_{(\cdot)}$ that has a lower cost in the range.

$$0 \leqslant MA < \infty$$

IV. AN EXAMPLE

In the example initially worked by the author, the number of customers was small and as a result, all possible repair policies as discussed in Section III.A were studied. But, even with all these alternatives, each state eventually converged to one of two repair policies; repair none or repair all. This seemed reasonable since, with a set-up cost, if one repaired any, it would be the least-cost policy to repair all failed units.

So, in expanding the size of the program, all other possible repair policies were deleted. Poisson demands were also initially assumed in the thesis but as the mean number of demands per cycle increased the calculation of the Poisson probability became computationally unfeasible. To circumvent the difficulty the following theorem was used:

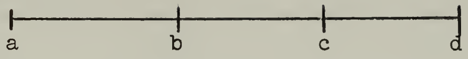
Theorem: A random variable K that is distributed Poisson with a mean, m, approaches the Normal distribution with mean, m, and variance m^2 as m gets large.

$$\lim_{m \to \infty} \left[\frac{e^{-m} m^k}{k!} - \frac{1}{\sqrt{2\pi}} \int_{k-m-\frac{1}{2}}^{k-m+\frac{1}{2}} e^{-\frac{1}{2}y^2} \, dy \right] = 0$$

For m reasonably large, this theorem was used to approximate the Poisson probabilities in the computations discussed in this section.

Golden section search was utilized to find the optimal stock level. Briefly this method of search operates in the

following manner:



Let $[a,b]$ be the range of search. Let b and c be points picked in some efficient manner (golden section search). Then:

1. compute costs for b and c.
2. if the cost at b $<$ the cost at c, search new interval a,d using the same method.
3. if the cost at b $>$ the cost at c, search new interval c,b using the same method.

A method such as this will allow one to get as close as one wants to the optimal or converge to the optimal, depending on the method of choosing the points b and c at each stage.

A computer program has been written and can be found in the back of the thesis. The program utilized the following data:

Number of customers	75
Mean cyclic demand	15
Set-up cost	$ 20.
Repair Cost	$ 3.
Backorder cost	$ 3.
Additional carrying costs	$ 2.
Per unit fixed carrying cost	$ 2.
Stock level range	0-24

Using this data the computer routine picked the stock levels in the sequence shown from left to right and generated the following repair policies:

TABLE III

Stock Level	9 Iteration 1	9 Iteration 2	14 Iteration 1	14 Iteration 2	18 Iteration 1	18 Iteration 2	20 Iteration 1	20 Iteration 2	21 Iteration 1	21 Iteration 2	22 Iteration 1	22 Iteration 2	23 Iteration 1	23 Iteration 2	22 Iteration 1	22 Iteration 2
# in Repair	R*	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	6	0	6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	7	0	7	0	7	0	7	0	0	0	0	0	0	0	0
8	0	8	0	8	0	8	0	8	0	8	0	8	0	8	0	8
9	0	9	0	9	0	9	0	9	0	9	0	9	0	9	0	9
10	0	10	0	10	0	10	0	10	0	10	0	10	0	10	0	10
11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11
12	0	12	0	12	0	12	0	12	0	12	0	12	0	12	0	12
13	13	13	0	13	0	13	0	13	0	13	0	13	0	13	0	13
14	14	14	0	14	0	14	0	14	0	14	0	14	0	14	0	14
15	15	15	0	15	0	15	0	15	0	15	0	15	0	15	0	15
16	16	16	0	16	0	16	0	16	0	16	0	16	0	16	0	16
17	17	17	17	17	0	17	0	17	0	17	0	17	0	17	0	17
18	18	18	18	18	0	18	0	18	0	18	0	18	0	18	0	18
19	19	19	19	19	0	19	0	19	0	19	0	19	0	19	0	19
20	20	20	20	20	0	20	0	20	0	20	0	20	0	20	0	20
21	21	21	21	21	21	21	0	21	0	21	0	21	0	21	0	21
22	22	22	22	22	22	22	0	22	0	22	0	22	0	22	0	22
23	23	23	23	23	23	23	0	23	0	23	0	23	0	23	0	23
24	24	24	24	24	24	24	24	24	0	24	0	24	0	24	0	24
25	25	25	25	25	25	25	25	25	25	25	0	25	0	25	0	25
26	26	26	26	26	26	26	26	26	26	26	0	26	0	26	0	26
27	27	27	27	27	27	27	27	27	27	27	27	27	0	27	27	27
28 and over: repair total amount always.																
Total Cost	148.37		141.47		138.43		137.75		137.61		137.60		137.72		137.60	

* R is the quantity to be repaired

The program converged on the optimal policy of maintaining a stock level of 22. In addition it made the following repair decision: if, at the beginning of the repair cycle, the number in repair is greater than or equal to 8, one should repair all the failed units. Otherwise, do not repair.

This program used 236K bits of computer space and 6 min.30 second of computer time.

V. CRITIQUE AND CONCLUSIONS

A. SUMMARY

Howard's computation technique was used in determining the optimal repair policy and the variable cost for a given stock level. The algorithm associated with this can be found in Section III.B.5. The total cost for this stock level was then computed by adding its fixed cost to the variable cost.

It was then argued that the cost of operating the repair system was a convex function of the stock level. Thus, any of the many unimodal search methods could be used. With this information at hand, the following algorithm could be used.

Minimum Cost Algorithm
1. Select two stock levels over a general range using a unimodal search method.
2. Compute the total cost(s) using the repair policy algorithm and compare. Select range associated with that stock level that has minimum total cost.
3. Choose new stock level to compare with present optimum stock level.
4. If there is no stock level better, stop. Otherwise, go to step 2.

B. CRITIQUE

Although this algorithm did locate the optimal repair policy and stock level, it presented some computational problems. In addition to this, there is some question as to the usefulness of the results because of the assumptions that were made.

1. Assumptions

The model envisaged a cyclic repair system in which all transactions took place within that cycle. At the same time, probability distributions were assumed that were not consistent with this assumption. For instance, a Poisson distribution does not prevent a part from failing a couple of seconds prior to the end of the cycle. But, it is assumed that, if this happened, the failed item would be delivered to the repair activity prior to termination of the cycle.

It was further assumed that all work started in a repair cycle would be completed in that cycle. This implied that everything required to repair the failed item would be on hand at all times. This particular assumption might be good or bad depending upon what is required to repair the item.

It was also assumed that every failed item received could be repaired. This is very unrealistic. But, it is felt that this assumption could be relaxed by allowing for some attrition, although no suggestions are to be offered at this time.

There are two properties of this model, though, that make it particularly useful. First of all, the model assumes that the repair periods are cyclic. This is the way most workloads are formed and work is accomplished today. Secondly, the model is independent of the demand distribution used. So, even some empirical demand distribution could be used as easily as the Poisson.

2. Computational Problems

The two major weaknesses of the model are program size and computer run-time. In the example in Section IV, it was noted that 236K bits of computer space were required for a problem that contained only 75 customers and a range of possible stock levels of 0-24. 236K is considered a very large program, while 75 customers is a very small problem. The large storage space requirement is due to the matrix size required to hold all the state probabilities for the associated decisions. In the example in Section IV, the matrix was 100x100x2 which uses 80,000 bits of computer space by itself. A single computer could not handle a problem with 1000 customers which is not an unrealistic repairable item problem.

The second major weakness was the running time of the program. The small example took 6½ minutes. This lengthy running time was due to the fact that a probability of demand must be computed for each state and each repair alternative. And then, having done this, one was required to solve a series of systems of simultaneous equations to determine the optimal repair policy.

3. Parameter Sensitivity

The parameters or costs were not tested in the thesis. This would normally be done to see how repair policies and stock levels were affected by changes in the set-up cost, and the unit costs of carrying and repairing failed items. It was felt, though, that the computational problem mentioned in the previous section required a solution prior to testing parameters.

C. ALTERNATIVES

There are ways to avoid some of the problems. The suggested alternative ways given here deal with either reducing the number of states or reducing the number of alternatives.

1. Reduction of Storage Area

Information could be generated as required for each iteration rather than storing it. But, by doing this, computation time would be increased significantly. For example, even in determining a repair policy for a given stock level, the entire state space and cost space would have to be computed at each iteration. And, as was pointed out earlier, computer running time was already quite high. Thus, there does not seem to be a practical means of reducing storage space utilized while using the same approach developed in the thesis.

2. Compression of State Space

A study of the results of Section IV led to two possible alternatives for reducing state space. Both alternatives required the following facts to be noted from Table III in Section IV.

Let x be the amount of repairable stock on hand; let b be the smallest x such that the repair decision is: repair all failed items. Then, if, at the beginning of a repair cycle, x is greater than b, the decision will be to repair all failed units.

Secondly, note that the "active" state space is quite small. By "active" is meant those states or repair decisions that actually change during cost reduction routines. The size of the active state space seems to be a function more of cyclic demand and stock size than of the number of customers. The following two alternatives are offered:

a. Reduce the Size of the Problem:

Let M ≡ the number in stock
Amean ≡ the cyclic demand
N ≡ number of customers

Define the state space, the number in repair, in the following way. Let the state space = 0,1,...,AMEAN+M,K where K is any possible state greater than M + Amean. The bulk of the states of a larger problem are combined into state K. A major difficulty in solving this problem is the determination of a cost associated with the state K. Although no specific solution will be offered in this paper, it is felt that this can be solved utilizing expected cost rather than actual cost.

b. Reduce the Number of Alternatives for Certain States

Define M, Amean and N in the same way, use the following general rule. For the states 0,...M+Amean, one can make two decisions: repair none or repair all. For all states K greater than M+Amean, repair all. This would reduce storage space while retaining the structure of the problem as it is.

D. EXTENSION-MULTI-ITEM SYSTEM

A discussion of a repairable item inventory system would not be complete without at least a cursory look at a repairable item system that is multi-item. Since this was not the main thrust of this thesis, a solution to this problem will only be suggested.

1. No Resource Constraint

With the absence of constraints on resources, the multi-item problem reduces trivally to a set of individual repairable-item problems. The minimum cost repair policy and stock level can be determined for each item individually.

2. Resources Constraint

The more common repairable item systems would have a combination of two constraints: a budget constraint and a workload constraint. Suppose minimizing the probability of a stock-out is the measure of effectiveness. To work with this measure, stock-out cost must be deleted from the cost function and the problem must be reformulated in the following manner:

$$\text{MIN} \sum_{i=1}^{L} a_i \; P(S/O)_i$$

$$\text{ST} \sum_{i=1}^{L} C_i(P_i) \leq D$$

$$\sum_{i=1}^{L} E_i x_i \leq F$$

Where L = number of customers

$P(S/O)_i$ = probability for the i^{th} item being out of stock

a_i = weighting factor for item i

$C_i(P_i)$ = cost of the repair policy associated with item i

E_i = work units required to repair item i

x_i = total number of the i^{th} item to be repaired

D = total budget constraint

F = workload constraint

With the time required to compute the optimal repair policy and stock level even for one item, solving this type of minimization problem is not amenable to solution at this time.

E. CONCLUSIONS

Three major conclusions can be drawn from this paper.

1. The total cost of maintaining a part in a repairable item system is a convex function of stock level.

2. The optimal repair policy will be of the following form:

Let x be the number of failed items in repair. Let b be the state of the smallest quantity in repair for which all will be repaired. Then the following decision rule will be used:

$$\text{If } x < b, \text{ do not repair}$$

$$x \geq b, \text{ repair all.}$$

This is similar to the (s,S) policy in inventory theory.

3. The dynamic programming-Markov process approach to the repairable inventory system leads to a minimum cost solution.

```
C*****************************************************************C
C                                                                 C
C  DEFINITION OF VARIABLES                                        C
C                                                                 C
C*****************************************************************C
C                                                                 C
C    A1              LOWER BOUND OF GOLDEN SECTION                C
C                    SEARCH                                       C
C                                                                 C
C    A2              UPPER BOUND OF GOLDEN SECTION                C
C                    SEARCH                                       C
C                                                                 C
C    AMEAN           MEAN DEMAND DURING REPAIR                    C
C                    PERIOD                                       C
C                                                                 C
C    B(I)            EXPECTED IMMEDIATE COST USED                 C
C                    IN A GIVEN COMPUTATION                       C
C                                                                 C
C    BLL(I)          PROBABILITIES USED IN A                      C
C                    GIVEN COMPUTATION                            C
C                                                                 C
C    CFIX            FIXED PER UNIT COST OF                       C
C                    CARRYING A UNIT IN STOCK                     C
C                                                                 C
C    COST(J,K)       COST ASSOCIATED WITH                         C
C                    PROB(I,J,K).  THIS IS                        C
C                    INDEPENDENT OF STARTING                      C
C                    STATE                                        C
C                                                                 C
C    CPOL            COST ASSOCIATED WITH STOCK                   C
C                    LEVEL AND COST OF BACKORDER                  C
C                                                                 C
C    CREP            REPAIR COST PER ITEM                         C
C                                                                 C
C    CSET            FIXED COST OF SETTING UP                     C
C                    MACHINES FOR REPAIR                          C
C                                                                 C
C    CXLFT           COST IF STOCK LEVEL WERE AT LEFT             C
C                    SEARCH POINT                                 C
C                                                                 C
C    CXRT            COST IF STOCK LEVEL WERE AT RIGHT            C
C                    SEARCH POINT                                 C
C                                                                 C
C    DENOM           TRUNCATION FACTOR FOR                        C
C                    POISSON DISTRIBUTION                         C
C                                                                 C
C    DPOL            TOTAL CPOL                                   C
C                                                                 C
C    DREP            TOTAL CREP                                   C
C                                                                 C
C    DSET            TOTAL CSET                                   C
C                                                                 C
C    GELG            SUBROUTINE THAT SOLVES A                     C
C                    SYSTEM OF SIMULTANEOUS                       C
C                    EQUATIONS                                    C
C                                                                 C
C    H               RANGE OF GOLDEN SECTION SEARCH               C
C                                                                 C
C    LDEM            SYSTEM DEMAND DURING REPAIR                  C
C                    CYCLE                                        C
C                                                                 C
C    LEND            QUANTITY UNREPAIRED AT                       C
C                    END OF REPAIR CYCLE                          C
C                                                                 C
C    LREP            QUANTITY BEING REPAIRED                      C
C                                                                 C
C                    QUANTITY AVAILABLE FOR REPAIR                C
C                    AT START OF REPAIR CYCLE                     C
C                                                                 C
C    LSTART          NUMBER IN REPAIR AT BEGINNING                C
C                    OF THE REPAIR CYCLE                          C
C                                                                 C
```

```
C     MA              STOCK LEVEL                               C
C                                                               C
C     MD(I)           BEST REPAIR POLICY VECTOR                 C
C                                                               C
C     NCCUNT          NUMBER                                    C
C                     OF IMPROVEMENT ROUTINES                   C
C                     INITIATED                                 C
C                                                               C
C     NCUST           NUMBER OF CUSTOMERS                       C
C                                                               C
C     ND(I)           BEST ALTERNATIVE REPAIR                   C
C                     POLICY VECTOR                             C
C                                                               C
C     NUMBER          TOTAL NUMBER OF STATES                    C
C                     IN SYSTEM                                 C
C                                                               C
C     NYESNO          DETERMINES WHETHER OR NOT                 C
C                     PRESENT SET OF ALTERNATIVES               C
C                     MATCH PREVIOUS SET                        C
C                                                               C
C     POISS(X,Y)      SUBROUTINE THAT COMPUTES                  C
C                     POISSON PROBABILITIES OF                  C
C                     DEMAND                                    C
C                                                               C
C     PRCB(I,J,K)     PROBABILITY  J ITEMS ARE                  C
C                     IN REPAIR AFTER ONE PERIOD                C
C                     GIVEN THAT I ITEMS NEEDED                 C
C                     REPAIR AT BEGINNING CF                    C
C                     PERIOD AND K ITEMS WERE                   C
C                     REPAIRED DURING PERIOD                    C
C                                                               C
C     QUE(I,K)        IMMEDIATE EXPECTED COST                   C
C                     DUE TO HAVING I ITEMS                     C
C                     AND REPAIRING K
C                                                               C
C     R               GOLDEN SECTION SEARCH CONSTANT            C
C                                                               C
C     XLFT            LEFTHAND POINT OF SEARCH                  C
C                                                               C
C     XRT             RIGHTHAND POINT OF SEARCH                 C
C                                                               C
C***************************************************************C
C                                                               C
C   CECLARATICN OF VARIABLES                                    C
C                                                               C
C***************************************************************C
C                                                               C
      DIMENSION PRCB(100,100,2),COST(100,100),
     1QUE(100,2),BLL(10000),B(100),ND(100),MD(100)
      NCOUNT = 0
      NCUST=75
      AMEAN = 15.0
      CSET=20.00
      CREP =3.0
      CPOL = 5.0
      CFIX=2.00
      LMNOP=0
      R=.5*(SQRT(5.0)-1.0)
C                                                               C
C***************************************************************C
C                                                               C
C   INITIALIZE FIRST TWO POINTS OF GOLDEN                       C
C   SECTION SEARCH                                              C
C                                                               C
C***************************************************************C
C                                                               C
      A1=C.0
      A2=24.0
      H=A2-A1
      XLFT=A1+(R**2)*H
      XRT=A1+R*H
      MA= IFIX(XLFT)
```

```
      WRITE(6,0003) MA
 0003 FORMAT('1','THE NEW SEARCH VALUE IS',I4///)
      GO TO 0015
 0007 MA = IFIX(XRT)
      WRITE(6,0008) MA
 0008 FORMAT('1','THE NEW SEARCH VALUE IS',I4///)
C                                                                       C
C***********************************************************************C
C                                                                       C
C   START SEARCH OF OPTIMAL REPAIR POLICY                               C
C   FOR A GIVEN STOCK LEVEL                                             C
C                                                                       C
C***********************************************************************C
C                                                                       C
 0015 NCOUNT = 0
      NUMBER = NCUST + MA + 1
      DO 10 I = 1,21
      MD(I) = 0
      ND(I) = 0
      B(I) = 0.0
      DO 14 J= 1,21
      NNNN = (I-1)*NUMBER + J
      BLL(NNNN) = 0.0
      COST(I,J) = 0.0
      DO 20 K=1,2
      QUE(J,K) = 0.0
      PROB(I,J,K) = 0.0
 0020 CONTINUE
 0014 CONTINUE
 0010 CONTINUE
C                                                                       C
C***********************************************************************C
C                                                                       C
C   COMPUTATION OF PROBABILITY MATRICES                                 C
C                                                                       C
C***********************************************************************C
C                                                                       C
      DO  1200 I = 1,NUMBER
      LSTART = I-1
      DO 1100 K =1,2
      IF(K.GT.I) GO TO 1100
      LREP=LSTART
      IF(K.EQ.1) LREP=0
      DENOM = 0.0
      DO 500 J = 1,NUMBER
      LEND = J-1
C
C
      IF((LSTART-LREP).GT.LEND) GO TO 400
      LDEM = LEND - LSTART + LREP
      IF(LDEM.GT.NCUST) GO TO 400
      IF((LSTART.GT.MA).AND.(LDEM.GT.(NCUST-LSTART+MA)))
     1GO TO 400
      D = 44.0/7.0
      BB = FLOAT(LDEM)
      PROB(I,J,K)=EXP(-(((BB-AMEAN)/AMEAN)**2)/2.0)/
     1SQRT(D)*AMEAN
      DENOM = DENOM + PROB(I,J,K)
      GO TO 500
  400 PROB(I,J,K)= 0.0
  500 CONTINUE
      DO 1000 J =1,NUMBER
      A = PROB(I,J,K)/DENOM
 1000 PROB(I,J,K) = A
 1100 CONTINUE
 1200 CONTINUE
C                                                                       C
C***********************************************************************C
C                                                                       C
C   COMPUTATION OF COST MATRICES                                        C
C                                                                       C
C***********************************************************************C
```

```
C                                                              C
      DO 1500 J = 1,NUMBER
      LEND = J-1
      DO 1400 K = 1,NUMBER
      LREP = K-1
      DSET = CSET
      IF((K.EQ.1)) DSET = 0.0
      DREP = CREP*FLOAT(LREP)
      KA = 0
      IF(LEND.GT.MA) KA = LEND - MA
      DPOL = CPOL*FLOAT(KA)
 1400 COST(J,K) = DSET + DREP + DPOL
 1500 CONTINUE
C                                                              C
C**************************************************************C
C                                                              C
C   COMPUTATION OF IMMEDIATE EXPECTED COST                     C
C                                                              C
C**************************************************************C
C                                                              C
      DO 1600 I = 1,NUMBER
      DO 1550 K=1,2
 1550 QUE(I,K) = 0.0
 1600 CONTINUE
C
C
      DO 1800 I = 1,NUMBER
      DO 1700 K=1,2
      IF(K.GT.I) GO TO 1700
      LL=I
      IF(K.EQ.1) LL=1
      DO 1650 J=1,NUMBER
 1650 QUE(I,K)=QUE(I,K) + PROB(I,J,K)*COST(J,LL)
 1700 CONTINUE
 1800 CONTINUE
C                                                              C
C**************************************************************C
C                                                              C
C   CHOOSE ALTERNATIVE THAT GIVES MINIMUM                      C
C   IMMEDIATE EXPECTED COST                                    C
C                                                              C
C**************************************************************C
C                                                              C
      DO 2100 I=1,NUMBER
      IF(I.EQ.1) GO TO 1900
      LB = 1
      IF(QUE(I,1).GT.QUE(I,2)) LB=2
      B(I) = QUE(I,LB)
      MD(I) = LB
      GO TO 2000
 1900 LB=1
      MD(I) =1
      B(I)= QUE(I,LB)
 2000 DO 2050 J = 1,NUMBER
      NNNN=I+(J-1)*NUMBER
 2050 BLL(NNNN)=PROB(I,J,LB)
 2100 CONTINUE
C                                                              C
C**************************************************************C
C                                                              C
C   WRITE INITIAL REPAIR POLICY                                C
C                                                              C
C**************************************************************C
C                                                              C
      WRITE(6,2200)
 2200 FORMAT(' ','INITIAL REPAIR POLICY'//)
      WRITE(6,2225)
 2225 FORMAT(' ','NUMBER IN REPAIR',10X,'REPAIR'/)
      DO 2275 I=1,NUMBER
      J=I-1
      JJ=0
      IF(MD(I).EQ.2) JJ=I-1
```

```
      WRITE(6,2240) J,JJ
 2240 FORMAT(' ',4X,I4,18X,I4)
 2275 CONTINUE
      WRITE(6,2280)
 2280 FORMAT(///)
C                                                              C
C**************************************************************C
C                                                              C
C   ORGANIZE PROBLEM AS SYSTEM OF SIMULTANEOUS                 C
C   EQUATIONS                                                  C
C                                                              C
C**************************************************************C
C                                                              C
 2300 MMMM= NUMBER**2
      DO 2320 I = 1,MMMM
      Y=(-1)*BLL(I)
 2320 BLL(I) = Y
      DO 2400 I = 1,NUMBER
 2400 BLL(I) = 1.0
      J= 1
      DO 2500 I = 2,NUMBER
      J= J+1
      K= (I-1)*NUMBER + J
      Y= BLL(K) + 1.0
 2500 BLL(K) = Y
      CALL GELG (B,BLL,NUMBER,1,.0005,NIER)
      B(1) = 0.0
      DO 3500 I =1,NUMBER
      DO 3400 K = 1,2
      IF(K.GT.I) GO TO 3400
      Y = 0.0
      DO 3300 J = 1,NUMBER
      Y = Y+ PROB(I,J,K)*B(J)
 3300 CONTINUE
      Y = Y + QUE(I,K)
      IF(K.EQ.1) E=Y
      IF(Y.GT.E) GO TO 3400
      ND(I) = K
      E = Y
 3400 CONTINUE
 3500 CONTINUE
      NYESNO = 0
      DO 3600 I = 1,NUMBER
 3600 IF (MD(I).NE.ND(I)) NYESNO = 1
      IF(NYESNO.EQ.0) GO TO 5000
C                                                              C
C**************************************************************C
C                                                              C
C   WRITE PRESENT REPAIR POLICY                                C
C                                                              C
C**************************************************************C
C                                                              C
      NCOUNT = NCOUNT + 1
      WRITE (6,2950) NCOUNT
 2950 FORMAT(' ','POLICY IMPROVEMENT ROUTINE NUMBER  ',
     1I3/)
      WRITE(6,3630)
 3630 FORMAT(' ','NUMBER IN REPAIR',10X,'REPAIR'/)
      DO 3625 I=1,NUMBER
      J=I-1
      JJ=0
      IF(ND(I).EQ.2) JJ=I-1
      WRITE(6,3635) J,JJ
 3635 FORMAT(' ',4X,I4,18X,I4)
 3625 CONTINUE
      WRITE(6,3640)
 3640 FORMAT(///)
      DO 3650 I = 1,NUMBER
 3650 MD(I) = ND(I)
C                                                              C
C**************************************************************C
C                                                              C
```

```
C   CHOOSE PROBABILITY MATRIX AND ASSOCIATED                    C
C   IMMEDIATE EXPECTED COSTS FOR NEXT ITERATION                 C
C                                                               C
C***************************************************************C
C                                                               C
      DO 4000 I = 1,NUMBER
      NN = ND(I)
      B(I) = QUE(I,NN)
      DO 3800 J = 1,NUMBER
      NNNN = I + (J-1)*NUMBER
 3800 BLL(NNNN) = PROB(I,J,NN)
 4000 CONTINUE
      GO TO 2300
C                                                               C
C***************************************************************C
C                                                               C
C   COMPUTE THE STEADY STATE PROBABILITIES                      C
C                                                               C
C***************************************************************C
C                                                               C
 5000 DO 5100 I =1,NUMBER
      NN = ND(I)
      DO 5050 J = 1,NUMBER
      NNNN = J + (I-1)*NUMBER
 5050 BLL(NNNN) = PROB(I,J,NN)
 5100 CONTINUE
C
C
      J = 0
      DO 5200 I = 1,NUMBER
      B(I) = 0.0
      J = J+1
      K =(I-1)*NUMBER + J
      A=BLL(K)-1.0
      BLL(K) = A
      KKKK= (I-1)*NUMBER + 1
      BLL(KKKK) = 1.0
 5200 CONTINUE
C
C
      B(1) = 1.0
C
C
      CALL GELG (B,BLL,NUMBER,1,.0005,NIER)
C                                                               C
C***************************************************************C
C                                                               C
C   NOW COMPUTE EXPECTED PERIODIC COST FOR                      C
C   GIVEN INVENTORY                                             C
C                                                               C
C***************************************************************C
C                                                               C
      A = 0.0
      DO 5300 I =1,NUMBER
      NN = ND(I)
 5300 A = A + B(I)*QUE(I,NN)
C
C
      WRITE (6,5400) A
 5400 FORMAT(' ','THE TOTAL VARIABLE COST IS  ',F7.2)
      IF(MA.EQ.IFIX(XLFT)) CXLFT = A+FLOAT(MA*2)
      IF(MA.EQ.IFIX(XRT)) CXRT = A + FLOAT(MA*2)
      IF(MA.EQ.IFIX(XLFT)) WRITE(6,8100) CXLFT
      IF(MA.EQ.IFIX(XRT)) WRITE(6,8100) CXRT
 8100 FORMAT(' ','THE TOTAL COST OF SYSTEM IS ',F7.2)
      LMNOP= LMNOP +1
      IF(LMNOP.EQ.1) GO TO 0007
C                                                               C
C***************************************************************C
C                                                               C
C   COMPARE COSTS USING GOLDEN SECTION SEARCH                   C
C                                                               C
```

```
C***********************************************************C
C                                                           C
      IF(CXLFT.LT.CXRT) GO TO 8500
      A1=XLFT
      H=A2-A1
      IF(H.LT.1.0) GO TO 9000
      XLFT = XRT
      CXLFT =CXRT
      XRT=A1+R*H
      MA= IFIX(XRT)
      WRITE(6,8300) MA
 8300 FORMAT('1','THE NEW SEARCH VALUE IS ',I4///)
      GO TO 0015
 8500 A2=XRT
      H=A2-A1
      IF(H.LT.1.0) GO TO 9000
      XRT = XLFT
      CXRT  = CXLFT
      XLFT=A1+(R**2)*H
      MA = IFIX(XLFT)
      WRITE(6,8600) MA
 8600 FORMAT('1','THE NEW SEARCH VALUE IS ',I4///)
      GO TO 0015
 9000 STOP
      END
```

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13. ABSTRACT

This thesis developed an analytical model of a repairable item inventory system. The system consisted of a depot that repaired failed units according to some repair policy and stocked ready-for-issue units in support of a finite number of customers. A least-cost repair policy and stock level was determined by use of a computer program which is included as an appendix.

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