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BULLETIN NO. 67

REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS

BY

ARTHUR N. TALBOT

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 67 MARCH, 1913

REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS

BY ARTHUR N. TALBOT, PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING AND IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

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REINFORCED CONCRETE WALL FOOTINGS AND COLUMN FOOTINGS.

I. Introduction.

1. *Preliminary.*—Footings form an important element in the design of masonry structures. The two forms of footing most commonly used may be named the wall footing and the column footing, the former projecting laterally on the two sides of a longitudinal wall and the latter extending in four directions from the base of a column or pedestal block. It is usually assumed in the design of foundations that the earth conditions are such as to make the upward pressure on the footing uniform over its surface. Wide differences exist in methods of designing, due to differences in the assumptions made with reference to the structural action of the footing. It is not strange that these differences exist, since little or no experimental data are available which apply directly to the conditions of footings. Relatively short and deep beams and slabs under heavy uniform loads, with the supporting pressure largely concentrated at the center of the structure, may not be expected to give the same results as have been obtained in tests with the more slender beams and slabs and with the methods of support and of application of load which have generally been used in tests. With the present extensive application of reinforced concrete to footings, especially in connection with tall buildings carrying very heavy column loads, a more definite knowledge of the structural action of footings has come to be of importance. It is appreciated that the tests herein described are applicable only to a limited field, but they are offered as a contribution on a subject in which little experimentation has been done.

It may seem strange, considering the wide variations in practice, that few failures of footings have been publicly reported. It must be remembered, however, that these structures are out of sight, buried deep in the earth without opportunity for inspection. A failure in a footing may effect a change in the distribution of the load over the bed of the footing, resulting only in increased settlement. Possibly many instances of undue settlement of buildings may be due to failure in the footings. Possibly, in other cases, the earth at the center of the footing may be able to take the increased load under the conditions of side restraint developed. It is also probable that many footings have been made unduly strong.

2. *Acknowledgment.*—The investigations were made in the Laboratory of Applied Mechanics of the University of Illinois as a part of the work of the University of Illinois Engineering Experiment Station. Direct supervision of the work of making both the wall footings and the column footings was given by Mr. D. A. Abrams, Associate in the Engineering Experiment Station. He and Mr. W. A. Slater, First Assistant in the Engineering Experiment Station, directed and assisted in the tests of column footings. The tests of the wall footings were made under the direction of members of the laboratory staff. Acknowledgment is also made to Mr. Slater and to Mr. H. F. Gonnerman, Assistant in the Engineering Experiment Station, for assistance in the preparation of this bulletin. The investigation was undertaken at the suggestion of Dr. N. C. Ricker, Professor of Architecture. Assistance in the work of testing was given by senior students in architectural engineering and civil engineering who used the results in their theses. The following participated in the tests: Wall footings—series of 1908, Herbert Amery Brand, Horace Leland Bushnell, Arch. Eng'g, '08. Wall footings—series of 1909, Charles Emery Bressler, Jr., Nels Reuben Hjort, Civil Eng'g, '09. Wall footings—series of 1911, Charles Aloysius Petry, William Henry Ruskamp, Civil Eng'g, '11, Thomas James Giboney, Civil Eng'g, '12. Column footings—series of 1909, Norman Haden Hill, Edward Forde Zahrobsky, Arch. Eng'g, '09. Column footings—series of 1910, Charles Harris, James Verney Richards, Arch. Eng'g, '10. Column footings—series of 1911, Edward Raylor Kent, Earle Robinson Math, Arch. Eng'g, '11. Column footings—series of 1912, William Howard Farnum, Cyrus Edmund Palmer, Arch. Eng'g, '12.

3. *Scope of Bulletin.*—The tests of 114 wall footings and 83 column footings are described in the bulletin. The wall footings were 12 in. wide, generally 5 ft. in length and 12 in. in depth or 10 in. to the center of the reinforcing bars, with a 12x12x12-in. stem in the middle to represent the wall through which the test load was applied. The wall footing rested on a bed of springs arranged in such a way as to approximate conditions of uniform upward pressure on the bottom surface of the footing. A variety in method of reinforcement was employed to throw light on the development of tensile stress in the steel and on the resistance to bond, diagonal tension, and shear. Tests of brick footings, unreinforced concrete footings, and footings having I-beams encased in concrete were included in the investigation of wall footings. The column footings were 5 ft. square and generally 12 in. thick or 10 in. to the

center of the reinforcing bars, and had a 12x12x12-in. pier built over the middle through which the load was applied. The column footings also were tested on a bed of springs which gave conditions approximating those of uniform upward pressure. Variety was given to the amount and method of reinforcement and to other conditions with a view of determining the structural action with respect to tension, bond, diagonal tension, and shear, and to give information which would bear upon methods of calculation of stresses. It is thought that these are the first experimental tests on column footings, and probably the first on wall footings on a bed of springs. Analyses are given of the stresses in wall footings and column footings and methods of calculation are discussed and compared with the results of the tests.

4. *General Theory.*—In wall footings and pier footings the weight or load is applied vertically through the wall or base block or pier, and the upward bearing pressure of the soil (which may also be called the load, since its amount and distribution determine the stresses) supports this weight from below. The usual assumption on which design of footings is based is that the soil pressure is uniform over the bed of the footing. Before uniformity of pressure on the footing will obtain, the footing must bend to the amount and form which would be caused by a uniformly distributed load. The assumption of uniform pressure is warranted if the earth layer is an elastic compressible soil of considerable thickness and of not too high a modulus of compressibility, as under these conditions the amount of bend of the projection of the footing is slight in comparison with the amount of compression of the earth. Also, in soft soils which flow laterally, as in a so-called floating foundation, the settlement and changes in the soil will produce conditions approximating uniform pressure. Where the bed is rock the pressure will be transmitted more nearly directly from the wall or pier to the rock, and as the projections of the footing have little opportunity for being bent upward this portion of the footing may be expected to take only a small part of the load. This lack of uniformity of distribution of pressure is more likely to be present with reinforced footings than with the less flexible unreinforced footings which would carry the same load.

The principles of beam action are, in general, applicable to wall footings, but not so fully to column footings, which partake more of the nature of slabs. The formulas for calculating stresses in reinforced concrete beams have been treated in Bulletin No. 4, "Tests of Reinforced Concrete Beams: Series of 1905," and in Bulletin No. 29, "Tests of

Reinforced Concrete Beams: Resistance to Web Stresses." The principal formulas for beam action in rectangular beams reinforced for tension only, as used in this bulletin, will be repeated here.

The resisting moment of the reinforced concrete beam is (see Bulletin No. 29, page 6)

$$M = Afd' = Afjd \qquad (13)$$

where A is the area of cross section of longitudinal reinforcement, d is the distance from the compression face to the center of the longitudinal reinforcement, d' is the distance from the center of the reinforcement to the center of gravity of compressive stresses, j is the ratio of d' to d (which, for the beams of this bulletin, may be considered to vary from .82 to .92), and f is the tensile stress per unit of area in the metal reinforcement.

The formula for the maximum vertical shearing unit-stress in the concrete in any vertical section is

$$v = \frac{V}{jbd} = \frac{V}{bd'} \qquad (18)$$

where V is the total vertical shear at the given section (equivalent to the resultant of vertical forces on one side of the section considered), and b is the breadth of the beam. This formula neglects any horizontal tensile stresses in the concrete.

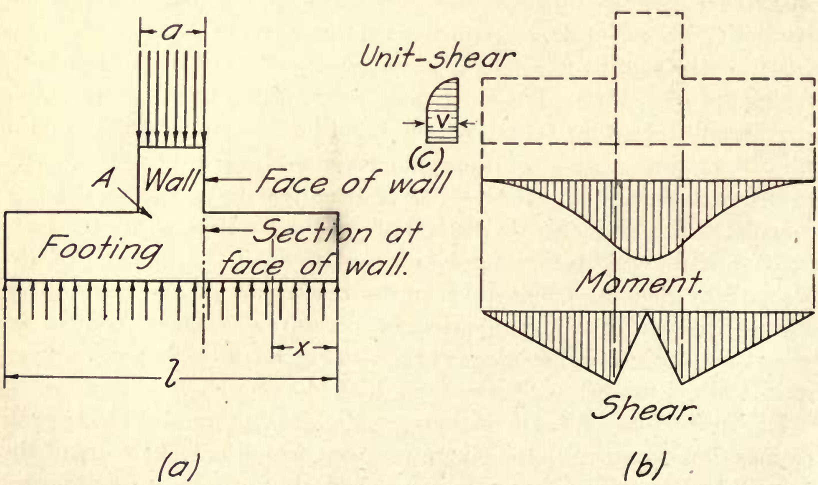

FIG. 1. (*a*) DISTRIBUTION OF LOAD AND PRESSURE IN WALL FOOTING. (*b*) MOMENT CURVE AND SHEAR CURVE. (*c*) DISTRIBUTION OF SHEARING STRESS OVER VERTICAL SECTION.

The formula for bond unit-stress in horizontal reinforcing bars is

$$u=\frac{V}{mod'}\qquad\qquad\qquad(17)$$

where o is the periphery of one longitudinal reinforcing bar, m is the number of bars, and the other symbols are as used before. This formula neglects any horizontal tensile stresses in the concrete.

These formulas were derived for certain assumed conditions in the beam. Since it is convenient to use them as a means of comparison for conditions other than those assumed, as, for example, when the bars are bent up at the end, the values obtained from these formulas will sometimes be referred to as nominal vertical shearing stresses and nominal bond stresses.

The value of the maximum diagonal tensile unit-stress in any section when tensile stresses exist is

$$t=\frac{1}{2}s+\sqrt{\frac{1}{4}s^2+v^2}\qquad\qquad\qquad(19)$$

where s is the horizontal tensile unit-stress existing in the concrete and v is the horizontal or vertical shearing unit-stress. The direction and amount of this maximum diagonal tensile stress will vary with the relative values of s and v. In general, it may be said that in the ordinary reinforced concrete beam the value of t probably varies from one to two times v. This applies to the parts where tensile stresses exist in the concrete. Where the tensile strength of the concrete has been exceeded, it is customary to use the same formula.

It is evident that the value of the diagonal tension is generally indeterminate. No working formulas are available. For this reason it is the practice, now becoming nearly universal, in beams without web reinforcement to calculate the value of the vertical shearing unit-stress v, and to use this as the measure or means of comparison of the diagonal tensile stress developed in the beam; with the understanding, of course, that the actual diagonal tension is considerably greater than the vertical shearing stress. It has been found that the value of v developed in beams will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam.

5. *Analysis of Wall Footings.*—Fig. 1(a) shows a wall footing and a typical set of external forces acting upon the footing. In the discussion, the stem or pier above will be called the wall and the remainder the footing proper. The projecting portion of the footing will be called the projection.

The bending moment at a section of the footing x distant from the end (calling w the uniform upward pressure per lineal foot of length of footing for a given width of section) is

$$M=\frac{1}{2}wx^2 \qquad (22)$$

For a section at the face of the wall, the bending moment will be

$$M=\frac{1}{8}w(l-a)^2 \qquad (23)$$

where l is the extreme dimension of the footing and a is the thickness of wall. For a section through the middle of the wall, assuming the load to be distributed uniformly over the wall, the bending moment will be

$$M=\frac{1}{8}w\,(l^2-la) \qquad (24)$$

The variation of the bending moment along the footing is shown in Fig. 1(b).

Although the maximum bending moment is shown by the above analysis to be at the section which passes through the middle of the wall, the resisting moment of that section will be far greater than that of a section of the projection of the footing in those cases where the wall and footing are poured at the same time or where they are well bonded together. Even with a weak bond the horizontal shearing stress at the junction of wall and footing will, in footings of the ordinary proportions, be so small that the combined section may be expected to act together. Besides, the pressure from the wall, instead of being distributed as shown, will be concentrated to some extent on the footing near the faces of the wall, as at A, Fig. 1(a), and this will act to reduce differences of moments. Altogether, it may be expected that the section at the face of the wall will be the critical section for bending moment and resisting moment and that equation (23) will express the value of the critical bending moment as closely as may be determined by ordinary analysis.

Fig. 1(b) shows also the variation in the external vertical shear V over the length of the footing for uniform loading. The theory of beams gives a distribution of the intensity of the vertical shearing stresses throughout the vertical section which is represented in Fig. 1(c) and is more fully discussed in Bulletin No. 29, page 9. Due to a concentration of pressure near the face of the footing (as at A, Fig. 1(a) and to the transmission of pressure diagonally therefrom in a manner which is analogous to arch action (as is also to be found in short simple beams), it may be expected that at vertical sections near the wall the vertical shearing stresses will be greater in the compression portion of the vertical section and less below the neutral axis than is given by the beam analysis of Bulletin No. 29. This modification of the distribution of the vertical shearing stresses may be expected to reduce the amount of the diagonal tension stress de-

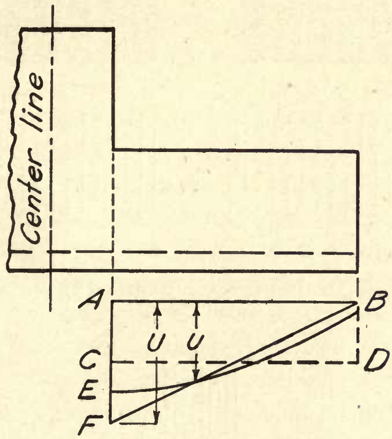

Fig. 2. Distribution of Bond Stress along the Reinforcement in a Wall Footing.

veloped near the wall, and the position of the critical section for diagonal tension failure may be expected to be away from the face of the wall. The values of the vertical shearing stresses given in this bulletin as a means of comparing or measuring the resistance to diagonal tension in the wall footing tests are based upon a section distant d from the wall (a section which is shown to give reasonable values), and the vertical shear V at this section is used in equation (18). A comparison with the values at a section at the face of the wall will also be made.

The bond stress between the surface of the horizontal reinforcement and the concrete will also be affected by variations from true beam action. By equation (17), page 8, the bond stress is a maximum at the face of the wall as represented by the line AF in Fig. 2, and decreases uniformly toward the end of the beam, as shown by ordinates to the line FB, becoming zero at B. Due to the deformations accompanying the stretching of the steel under the wall and to the relative deformations necessary to develop bond between the steel and the concrete, as well as to variation from true beam action, the bond stress will not follow the ordinates to the straight line. It seems probable

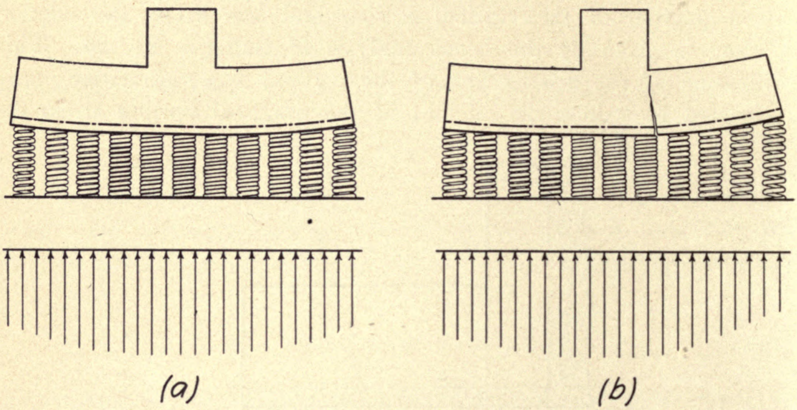

FIG. 3. EFFECT OF DEFLECTION OF WALL FOOTING UPON DISTRIBUTION OF LOAD BY SPRINGS.

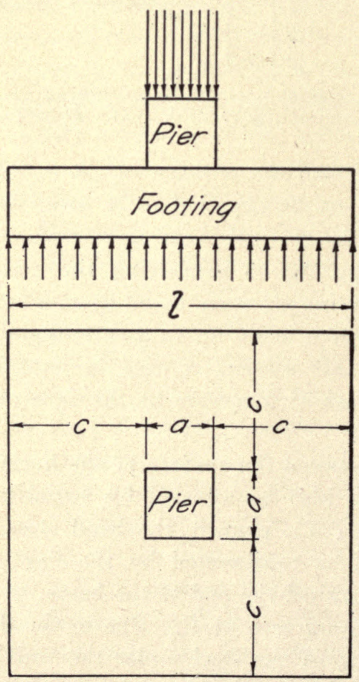

FIG. 4. DISTRIBUTION OF LOAD AND PRESSURE IN COLUMN FOOTING.

that the bond stresses developed are less at the face of the wall and greater at points farther out from the wall than is indicated by the analysis. It would seem that the bond stress will be expressed by some such line as the curved line EB, of the figure. This distribution is different still from the uniform bond stress indicated by the dotted line, which is based upon length of embedment and total amount of surface, a method assumed by some in such calculations. The distribution of the load may also affect the bond stresses. However, although the true bond stress at a section at a face of the wall may be expected to be less than that given by the ordinary beam analysis, in the absence of a better method it seems best to use equation (17), page 8, for the calculation of bond stresses.

When footings are tested on a bed of springs, the deflection produced in the beam results in compressing the springs at the middle of the footing more than at the ends, and hence the pressure will not be uniformly distributed along the length of the footing. If the compression of the springs at points along the length of the footing is known, and also the deflection of the footing at these points, the distribution of the load may be determined and the resulting bending moment calculated. Fig. 3 illustrates the effect of the deflection of the footing upon the distribution of the load. The bending moment so calculated will be somewhat less than that based on uniform distribution of the load, and the amount of the resulting tensile stress, bond stress, and vertical shearing stress will be less. The amount of the difference will depend upon the stiffness of the springs and the deflection of the footing under load, but within so-called critical loads it will not be large. Of course, in designing footings, our knowledge of the distribution of the pressure by the soil is too imperfect to consider the effect of deflection upon distribution of pressure.

In testing on a bed of springs, the load may not be symmetrically applied, and one end of the footing may receive more load than the other. The stresses in the end in which the springs receive the greater compression will, of course, be larger than values based on uniform distribution of load.

6. *Analysis of Column Footings.*—Fig. 4 represents a column footing of the form used in the tests. The stem representing the bottom of a column or a pedestal block will be termed the pier, and its lateral faces the faces of the pier. The load will be considered as applied uniformly over the top of the pier and the upward pressure as uniformly distributed over the lower surface of the footing. It is seen that the

footing may be considered to be a cantilever slab (rather than cantilever beams) supported at the top over a central area and loaded uniformly by an upward load, and that as the projecting portion of the footing deflects upward its surface will be bowl-shaped, in reality a double-curved surface. The determination of the distribution of the stresses over the various parts of the column footing is a much more difficult problem than is presented in wall footings.

Various methods of calculating the strength of column footings have been proposed. In some cases the offsets have been considered as cantilever beams having the full width of the footing and the full load on this area is considered to be taken by this beam, the critical section being at AB, Fig. 5. Although the load at the corners is counted twice,

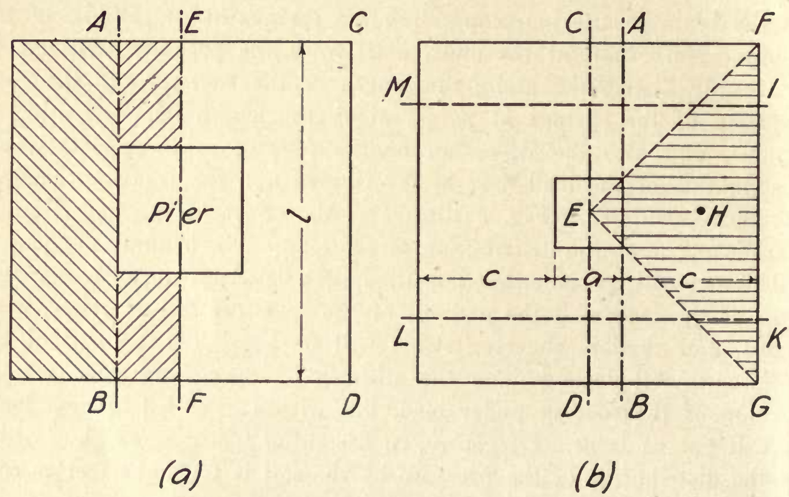

Fig. 5. Loaded Areas Assumed in Designing Column Footings.

the error is not great when the offset is small in comparison with the dimensions of the pier. If the dangerous section is considered to pass through the center of the footing, EF, Fig. 5 (a), a greater discrepancy exists. A common method of design is to consider that one-fourth of the total load is applied on the triangle EFG, Fig. 5 (b), and having found the center of pressure (as H) of the part of the load at one side of AB or CD (according to which is used as the dangerous section), to calculate the bending moment as the product of this amount of load by the distance from this center of pressure to AB or CD, the dangerous or critical section.

When the bending moment has been obtained by one of these methods, it is considered to be resisted by a beam IKLM, Fig. 5 (b) of width somewhat greater than the width of the pier, say, the width plus once the depth of the footing, according to the views of the designer. That is to say, the reinforcement in this assumed width is considered to develop stresses which altogether are sufficient in a beam of the depth of the footing to withstand the calculated bending moment. If the cross section of the steel lying within the assumed width is A, the resisting moment will be $M=Afjd$. The steel lying outside the dotted lines is considered to carry load to the beam formed by the reinforcement which lies at right angles to these lines, just as the steel parallel to and near FG carries load to the beam IKLM, and in this method of design no account is taken of it in the main beam. Whether the spacing of the outer bars should be the same as that of the interior or be greater is then left as a matter of judgment. In the determination of both bending moment and resisting moment, then, the practice of engineers varies considerably.

A rational analysis of the stresses would involve a determinate expression for the deflection of the footing at every point of the cantilever slab and also for the radius of curvature in each direction. A full treatment would include a consideration of the effect produced by having stresses act at right angles to each other and of the action of other combined stresses. However, it may be expected that this effect will not be large, as in reinforced concrete footings the presence of stresses in two directions affects principally the amount of the compressive stresses and the compressive stresses will not be the controlling element of strength in footings as ordinarily designed. It is easily seen that an analytical treatment of a cantilever slab of this kind which approached completeness would be very complicated. This and the uncertainty involved in the assumptions made at some steps of the analysis renders the correctness of the results of the mathematical work of such an analysis quite problematical. In view of the complexity of the problem and the uncertainty of portions of the work, it seems futile to attempt to derive thoroughly rational formulas for stresses in column footings. This being so, it seems best to utilize approximate solutions based on other considerations.

A study of the phenomena of the flexure of the column footing may be helpful in judging of the division of the load in the production of bending moment in the two directions and of the development of stress in the different parts. It is apparent that the stresses will be proportional to the deformations developed and that the deformations at any point will depend upon the curvature at that point. It will be recalled that in the analysis of beams in mechanics of materials the stresses developed are found to be inversely proportional to the radius of curvature or directly proportional to the curvature. At corresponding points on similar sections the curvature and hence the stress may be considered to vary somewhat as the change of deflection at these points. With these considerations in mind, we may be able to judge of the effect of the varying curvature in different parts of the footing.

Fig. 6 represents in a general way the form which the footing under load may be expected to take along various sections. The full lines in the lower figure represent the deflections or flexure curves of vertical

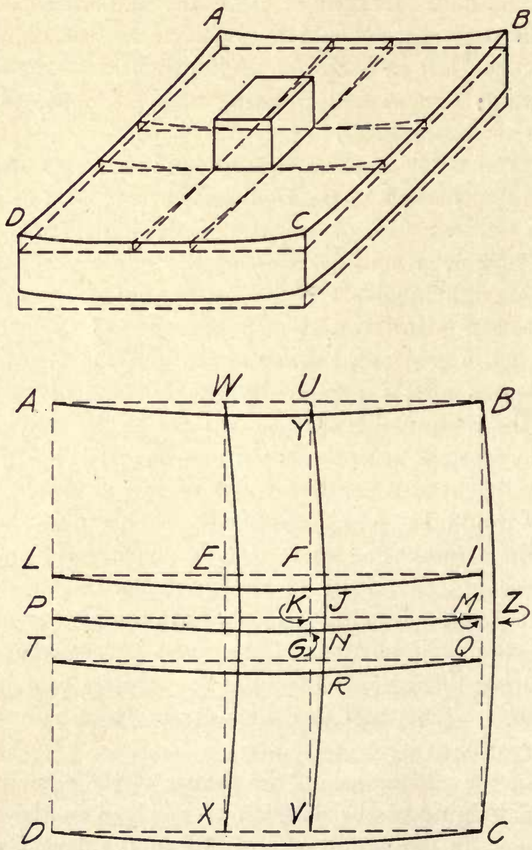

FIG. 6. FORM TAKEN BY COLUMN FOOTINGS UNDER LOAD.

sections taken along the dotted lines. The vertical rise at a corner B will be the sum of the deflection at M (KG) and the deflection of the lateral face BC (MZ). The three sections through the faces of the pier and the center of the pier which give the flexure curves LJI, PNM, and TRQ, may be considered to have nearly the same stress. The section at a lateral face of the footing will give a flexure curve AYB which will generally have less deflection and less flexural curvature at Y (and hence less stress) than is to be found in the section at the face of the pier which gives the curve LJI. The amount of the difference between these two curves will depend upon the relation of the cantilever span to thickness of pier and to amount and distribution of reinforcement. For sections between AB and LI the flexure curves and the conditions of curvature will range between those of the limiting curves. If we knew the flexure curves in all parts of the footing we should be able to get at the distribution of stresses.

If, with two-way reinforcement, we consider the load or pressure on the footing to be carried by two beams or sets of beams running parallel to the sides of the footing, the proportion of load or pressure taken by each beam from any elementary area may be considered to depend in some way upon the relative deflection of the beams in the two directions. In Fig. 7 (a), for convenience of description, consider the top of the

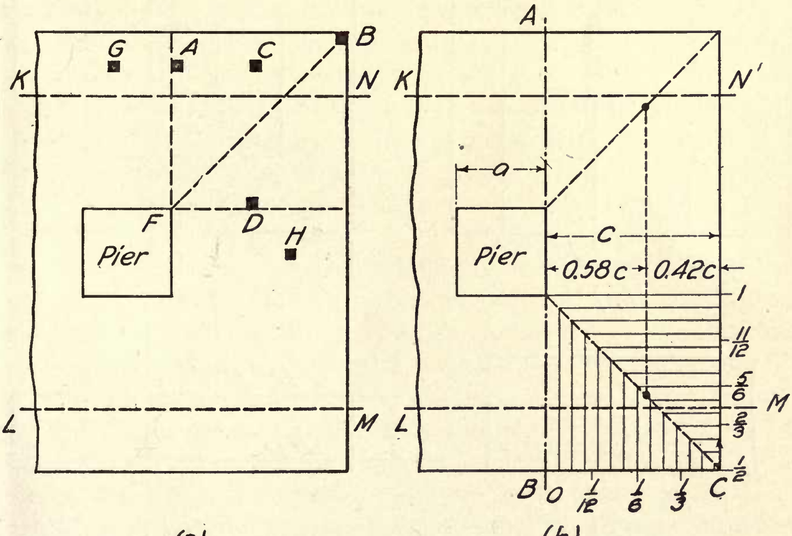

FIG. 7. DIAGRAM SHOWING ASSUMED DISTRIBUTION OF LOAD IN PRODUCING BENDING MOMENT.

diagram north, and that the footing is formed of a beam running in the east and west direction, and of another running in the north and south direction. For an element at A the deflection laterally from the north and south beam will be very slight and the total load on this element may, without much error, be considered to produce bending in the beam running in the north and south direction. For the corner area B part of the load may be considered as producing moment in the beam which runs in the north and south direction and part in the beam which runs laterally (east and west). For an element at C the amount of deflection of the footing from C to A will be much less than that from C to D; it seems evident that the proportion of load at C producing moment in the north-and-south beam is much greater than that acting on the east-and-west beam. Similarly, at D a greater proportion acts on the east-and-west beam than on the north-and-south beam. Along the diagonal line BF we may consider that half of the load acts on each beam. At G all acts on the north-and-south beam; at H none of it.

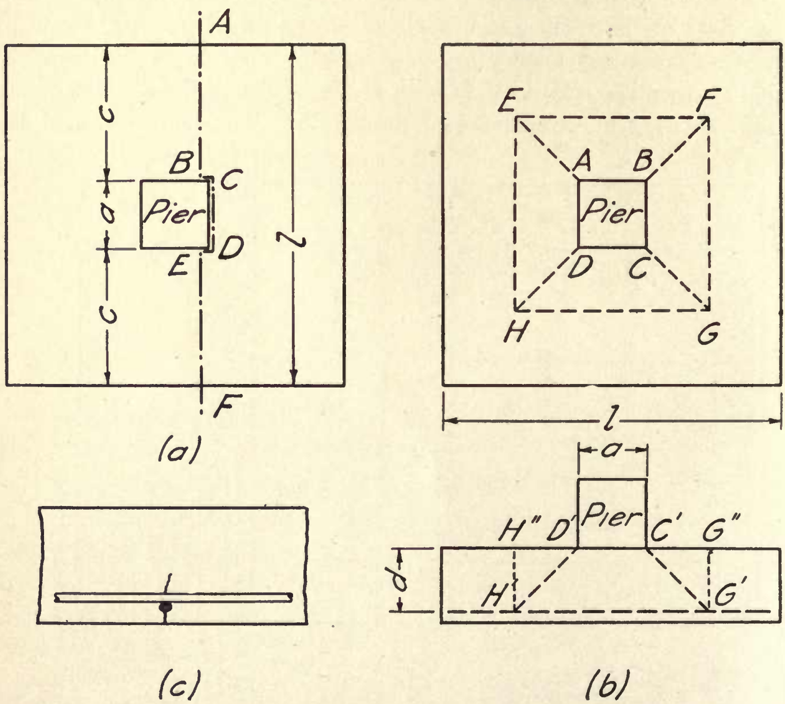

Fig. 8. Critical or Dangerous Section for Resisting Moment and Position of Section for Calculating Vertical Shearing Stresses.

After making a study of the flexure curves obtained on a number of the column footings tested, the fractions given on the diagram in Fig. 7 (b) were taken as roughly representing the proportion of the unit-load at the points indicated which acts upon the east-and-west beam to produce bending moment and curvature. For the variation of the proportions along the lines of the diagram between the limits noted a curvilinear relation was assumed, and a process of approximate integration was applied to the load division problem. Of the part above the diagonal line, approximately two-thirds of the load or pressure upon the triangle was found to go to produce bending moment in the east-and-west beam, and of the part below the diagonal line approximately one-third, the remainder in both cases going to produce bending moment in the beam in the north-and-south direction; and of course altogether one-half of the load on the corner square must be considered to produce bending moment on each beam. By the calculation, under the assumed division of load, the center of pressure of the various parts of the load tributary to the north-and-south beam from a corner square was found to be 0.58 c from a line through the face of the pier. That is to say, this analysis results in considering that the pressure on the corner square affects the bending moment of the north-and-south beam the same as if one-half of the load of this corner square were placed at a point distant 0.58 of the width of the square from a line through the face of the pier, see Fig. 7 (b). As the method of assuming the division of load will not warrant refinement of calculation it seems well to adopt the more convenient and more conservative value of 0.6 c for the position of the center of pressure, and this value will be used in the calculations in this bulletin. It may be added, also, that other methods of attacking the problem locate the center of pressure not far from the position here chosen.

The location of the critical or dangerous section for which the bending moment is to be found is also of importance. For footings made in such a way that the pier and footing are bonded together sufficiently not to permit failure by horizontal shear between them, as were all the column footings described in this bulletin, a section at the face of the pier CD, Fig. 8 (a), will be the critical section for the part of the beam immediately in front of the pier. For the part of the footing on either side of this, the critical section possibly may be somewhat back of the face of the pier. From some of the tests which were made it would seem that a combination section made up of three sections, one coinciding with the face of the pier and the other two slightly back of this, as shown by AB, CD, and EF in Fig. 8 (a), might represent the critical section. However, after making a study of all the tests, it is concluded that a section through the face of the pier is fairly representative of the tests, and this section will be used in the calculations in this bulletin. For very broad footings a section somewhat back of the pier may properly be assumed. The formula for the critical bending moment may then be expressed as follows:

$$M=\left[\frac{1}{8}a(l-a)^2+\frac{3}{40}(l-a)^3\right]w$$

or

$$M=\left(\frac{1}{2}ac^2+0.6c^3\right)w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

where a is one dimension of the square pier, l one dimension of the square footing, and c is the dimension of the offset of the footing, see Fig. 8 (a).

The bending moment thus obtained goes to produce curvature across the section and may be said to be resisted by the entire section, but the stresses may be expected to be different in different parts of the section, being a maximum under the pier and having the least stress at the edge

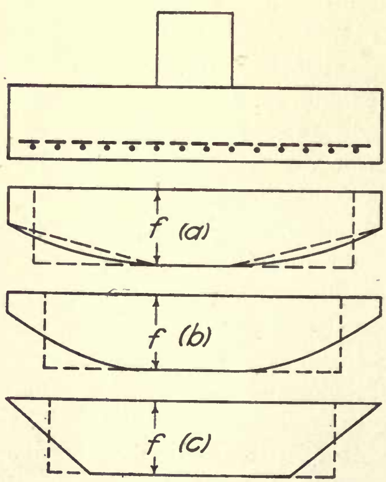

FIG. 9. VARIATION OF STRESS AMONG REINFORCING BARS.

of the section. The range in stresses may be illustrated by Fig. 9, where the stress in the reinforcing bars at right angles to the section considered is represented by the ordinates in the diagram. The stress of the bars lying under the pier may be considered to be uniform and represented by f. The bars in the projection of the footing which lie near the pier will be stressed nearly as high. The stress in a rod near the edge of the footing will be less, say from $0.25f$ to $0.75f$, depending upon the proportions of the footing and the distribution of the reinforcement. Between the pier and the edge of the footing the stress in the bars will vary by some law, probably a curvilinear relation. The total resisting moment developed in the full width of beam may be made up by using the stresses in the several bars. We may obtain this resisting moment in terms of the *maximum* stress f, by finding the equivalent proportion of bars which when stressed to the maximum stress f will give the same total resisting moment as is developed by all the bars with their varying stresses. If the bars are uniformly spaced, this is the same as taking the bars within a rectangle which will give the same area as is included by the curved line. For the dimensions of footing and pier used in the tests, if the minimum stress be $0.25f$ and a curvilinear variation be assumed, then 80% of the bars stressed to the maximum stress f will produce a resisting moment equivalent to that due to the assumed distribution of stress. If the stress at the edge be $0.50f$ and a curvilinear relation be used, the resisting moment will be equivalent to the use of 87% of the bars; if a rectilinear relation from the pier to the edge of the footing be used, 80% of the bars would give the equivalent resisting moment. As an extreme assumption, if the stress at the edge of the footing be $0.75f$ the use of 93% of the steel will give an equivalent resisting moment. In footings with short thick projections the stress in a bar near the edge will be nearly as great as in a bar under the pier, while in broad thin footings the stress in a bar at the edge of the pier will be considerably less than the maximum.

In connection with this dicussion, it seems well to point out that the ordinary assumption of beams superimposed in two directions presumes that outside the pier and out on the projections bars must act to give lateral stiffness and that these bars have a function as carrying bars to what may be considered the main beam, so that the value of the stress in these outer bars for the purpose in hand must be taken as auxiliary rather than as directly tributary to the main beams. It is uncertain to what extent this action must be considered in determining resisting moments of the section of the footing. If the distribution of stress across the section were known, it would seem that the stress in all the rods should be used in calculating the resisting moment of the section.

The preceding discussion assumes a uniform spacing of bars. If the bars are spaced more closely toward the middle the same methods may be employed and the probable distribution of stress across the section determined. If provision is to be made for lateral stiffness or carrying-stress, a further estimate must be made. If the bars are bunched near the edge of the footings the assumptions would have to be modified.

Another view may be had by assuming two equivalent main beams at right angles across the footing which resist the bending moment already obtained. The width of beam assumed as the equivalent width will be that width for which the calculated stress will agree with the actual stress in the most stressed bar, when only the reinforcing bars within the equivalent width are used in the calculation of resisting moment. It is plain that this width is greater than the width of the pier and less than the full width of the footing. It is evident that the equivalent width will vary with the size of the pier, the thickness of the footing, the dimension of the projecting portion, and the amount and distribution of reinforcement. An expression for the equivalent width of beam for use in calculations, even though empirical and not altogether general, will be useful.

A study of the observations and results of the tests of the footings made in the laboratory indicates that the bars for some distance on either side of the pier have nearly the same stress as those under the pier. As a working basis applicable when the spacing of the bars is uniform or does not vary far from this, the conclusion was reached that the resisting moment of the footing in each of the two directions may be based upon the amount of steel in a width of beam equal to the width of pier plus twice the depth of the footing to the reinforcement, plus one-half the remainder of the width of footing and that the use of this amount of steel will determine the maximum steel stress. Expressed as a formula the equivalent beam width then is

$$b=a+2d+\frac{1}{2}(l-a-2d) \qquad \dots\dots\dots\dots\dots\dots (25)$$

where l is the width of the footing. If the width given by the first two terms of the second member is greater than the width of the footing, then the width of the beam may be taken as the full width of the footing.

It may be thought that the concrete along the edges of the footing will of its own strength be sufficient to carry the loads laterally without reinforcement, but the deformation due to flexure along these edges may be much greater than concrete will stand and reinforcement near the edges serves a useful and necessary purpose, especially in distributing the deformations of the concrete preventing the concentration of elongation at single cracks.

The resisting moment, of course, will be

$$M = Afjd \quad \text{(13)}$$

where A is the area of the reinforcement in the given direction for the equivalent width of beam above specified, f is the unit-stress in the most stressed reinforcing bars, and the other symbols are as given on page 8. If the relative stress in the individual bars across the section is known or assumed, $M = \Sigma Afjd$ will express the total resisting moment developed over the section, f here being a variable denoting the unit-stress in the individual bar and A the area of one bar.

The bond stresses may be based upon the shear at the section at the face of the pier. For this the external vertical shear will be the amount of load used in determining the critical bending moment. At the face of the pier this shear is

$$V = \frac{1}{4}(l^2 - a^2)w = (ac + c^2)w \quad \text{(29)}$$

The expression for bond stress will be taken to be

$$u = \frac{V}{mojd} \quad \text{(17)}$$

where m is the number of reinforcing bars included within the equivalent width of beam as used in calculating the maximum tensile stress.

The calculated bond stress is greater at this section than it is towards the end of the bar, and hence the bond stress is considerably greater than the average bond stress found by considering that the total stress in the steel at the given section is taken off in bond over the surface of the bar between this section and the end of the bar. The same variation of bond stress from middle to end does not hold for the bars near the edge of the footing, and in these the concentration of bond stress is probably considerably greater towards the end of the bar. Where bars are bent up towards their ends the bond stress is also increased in parts of the bar. It is also apparent that the method of calculating bond stress will not apply when the bars are placed in exterior bands without reinforcement under the pier.

In measuring the resistance to diagonal tension failure we may follow the practice used in beams, and for comparison of resistance to diagonal tension we may use the vertical shearing stress developed. Because the diagonal tension failures in footings tested gave fractures at an angle of about 45° with the vertical, the frustum running from the faces of the pier and reaching the reinforcement at a distance d (the depth from surface to center of reinforcement) from a section through a face of the pier, it seems reasonable to take as the critical section a vertical section enveloping the base of the frustum indicated by EFGH, Fig. 8(b). This position gives results in agreement with those found for wall footings, and by analogy with the reasoning used in wall footings it may be expected that this is the section which has the distribution of shear giving maximum diagonal tensile stresses. In order to be in agreement with the other formulas for vertical shearing stresses, jd will be used in the formula for shearing stress, thus using the maximum unit-stress of the section instead of the average stress. The external vertical shear V may be considered to be that part of the load on the footing outside of the sections considered. The following formula expresses the amount of the vertical shear by this assumption.

$$V=\left[l^2-(a+2d)^2\right]w \qquad (30)$$

The expression for the critical vertical shearing stress becomes

$$v=\frac{V}{4(a+2d)jd} \qquad (31)$$

It will be borne in mind that these values of the vertical shearing stress will be used as a measure of the tendency to produce diagonal tension failure. The shearing stress at sections around the pier (punching shear) may be considered to be that given by the expression $\frac{(l^2-a^2)\,w}{4ajd}$, and the working stresses for punching shear applied.

II. Materials, Test Pieces, and Method of Testing

7. *Materials.*—The materials used in making the test footings were similar to those used in the reinforced concrete beams described in Bulletin No. 29. The stone and sand were bought in the open market. The Universal portland cement was furnished by the manufacturers. The Chicago AA portland cement and the Lehigh portland cement were bought in the open market. The mild steel rods used for the reinforcement were furnished by the Illinois Steel Company. The corrugated bars were supplied by the Corrugated Bar Company.

Stone. The stone was a good quality of crushed limestone from Kankakee, Illinois, ordered screened through a 1-in. screen and over a

¼-in. screen. It contained from 45% to 50% voids and weighed from 80 to 83 lb. per cu. ft. The mechanical analyses made agree very closely with those given on page 21 of Bulletin No. 29.

Sand. The sand was of good quality, sharp, well graded, and generally clean. It weighed 100 to 105 lb. per cu. ft. and contained about 28% voids. The mechanical analyses for that used in the series of 1908 is the same as for the 1908 sand given on page 21 of Bulletin No. 29, while that for the series of 1909 is nearly the same as that for the 1907 sand given on the same page. The sand used in 1910, 1911, and 1912 had the same general characteristics.

Cement. Tests of the three brands of cement are given in Table 2, Table 1 gives analyses of fineness.

Concrete. Men skilled in mixing concrete and making test pieces were employed in the work. The foreman, a contractor for small con-

TABLE 1.

Mechanical Analysis of Cement

Sieve No.	Per cent passing					
	1908		1909		1911	1912
	Universal	Chicago AA	Universal	Chicago AA	Universal	Universal
75	99.4	98.2	98.8	97.5	98.9	....
100	98.3	94.9	96.3	92.8	96.5	97.2
200	89.9	80.0	81.3	74.7	82.5	81.8

crete work, has had the making of test pieces in the laboratory for the past seven years. Care was taken in measuring, mixing, and tamping to secure as uniform a concrete as is possible under working conditions. In 1908 and 1909 all materials were proportioned by loose volume, and weights were taken as a check on the measurement. In the later years, the method of measurement of the sand and stone was the same, but 95 lb. of cement was taken to be a cubic foot. The latter method of proportioning gives a somewhat richer concrete than when the cement is measured loose. Except in 1912, when all the concrete was machine mixed, the mixing was done with shovels by hand. The sand and cement were first mixed dry; the stone, which had previously been thoroughly moistened, was added, and the mass then turned until of a uniform appearance. Water was then added in such proportion as to give a fairly wet mixture. The mass was again turned until thoroughly mixed.

Steel. The steel reinforcing bars consisted of plain round rods and deformed bars. The round rods were open-hearth mild steel. The deformed bars were square and round corrugated bars, types B and C.

TABLE 2.

Tensile Strength of Cement

These tests were made with standard Ottawa sand. Each value is the average of 5 briquettes.

Ref. No.	Ultimate Strength, lb. per sq. in.							
	Chicago AA Cement				Universal Cement			
	Age 7 days		Age 28 days		Age 7 days		Age 28 days	
	Neat	1–3	Neat	1–3	Neat	1–3	Neat	1–3
SERIES OF 1908.								
1...........	559	145	707	247	563	244	764	319
2...........	732	192	857	318	809	248	885	336
3...........	665	175	779	266	728	232	776	285
4...........	811	227	833	307	699	242	754	292
5...........	666	182	792	284	702	229	763	315
6...........	693	191	781	283	...	295*	...	366*
7...........	719	206	767	303	...	...	...	...
	...	248*	...	335*				
Average...	692	188	788	287	700	239	788	309
SERIES OF 1909.								
1...........	742	205	783	270	617	160	853	278
2...........	716	232	807	306	595	179	772	280
		288*		331*				
3...........	725	176	768	254	607	197	732	281
Average...	728	204	786	277	606	179	786	280
SERIES OF 1910.								
1...........	...	...	...	...	629	219	678	328
2...........	...	...	...	...	613	217	649	315
3...........	...	...	...	...	670	190	697	297
Average...	...	...	...	...	637	209	675	313
SERIES OF 1911.								
1...........	719†	248†	805†	329†	589	198	674	278
						265*		323*
2...........	...	...	...	...	684	227	709	283
3...........	...	...	...	...	653	240	731	319
4...........	...	...	...	...	662	214	696	282
Average...	...	...	...	...	647	220	702	290
SERIES OF 1912.								
1...........	...	...	...	...	585	239	685	315
2...........	...	...	...	...	577	225	694	297
3...........	...	...	...	...	691	242	715	306
4...........	...	...	...	...	617	231	792	326
5...........	...	...	...	...	588	246	672	333
6...........	...	...	...	...	612	253	758	323
7...........	...	...	...	...	698	287	884	372
Average...	...	...	...	...	624	246	743	325

*Briquettes made with the same sand as was used in concrete. Not included in average.
† Lehigh Portland cement.

Test pieces were cut from the reinforcing bars. Table 3 gives the average of the results of tension tests for the steel used in the different years.

8. *Wall Footings.*—All the concrete wall footings were 12 in. wide. The stem of the footing (the "wall") was in all cases 12x12x12 in. The length of the footing was 5 ft., except that four of the series of

TABLE 3.

Tension Tests of Reinforcing Bars

Nominal Size inches	Description	Stress at Yield Point lb. per sq. in.	Number of Tests	Maximum Variation from Average per cent
		1908 TESTS		
½	Plain round	41 500	17	7.4
½	Cor. square	50 400	22	12.7
¾	Plain round	41 000	2	1.2
5	I-beam	......	..	....
		1909 TESTS		
¼	Cor. square	33 000	1	....
⅓	Cor. square	31 200	10	10.3
⅜	Plain round	42 600	8	6.2
½	Plain round	41 200	16	13.4
½	Cor. square	46 800	8	8.5
9/16	Cor. round	53 500	16	6.0
⅝	Plain round	37 300	9	9.9
¾	Plain round	38 800	8	1.7
¾	Cor. square	50 300	6	6.1
No. 7*	Wire mesh	124 000†	6	8.1
No. 11*	Wire mesh	114 000†	6	1.3
		1910 TESTS		
½	Plain round	34 000	2	5.0
½	Cor. square	52 700	3	9.7
9/16	Cor. round	53 500	16	6.0
⅝	Plain round	37 100	2	1.4
¾	Plain round	41 700	4	3.6
		1911 TESTS		
⅜	Plain round	41 200	11	8.8
⅜	Cor. round	44 000	11	10.4
½	Plain round	39 500	10	3.9
½	Cor. square	51 500	3	5.9
⅝	Plain round	40 100	8	2.4
		1912 TESTS		
⅜	Plain round	48 100	14	5.2
⅝	Plain round	35 900	8	2.2
⅝	Cor. round	52 200	10	6.9

*Birmingham or Stubs' gauge.
†Ultimate strength—No yield point could be detected.

1908 were 6 ft. 8 in. long, two of the series of 1909 were 7 ft. long, three of the unreinforced footings of the series of 1911 were 7 ft. long and three were 3 ft. long. The depth was, in most cases, 10 in. to the center of steel. In the test pieces made in 1908 and in 1909, the depth over all was 11 in., in 1911, 12 in. over all. In the series of 1908 one reinforced concrete footing was 6 in. to the steel instead of 10 in., and two of those reinforced with I-beams had other depths. In the series of 1908 two footings had their upper surfaces sloped from 11 in. to 5½ in. at the end of the projection and two were stepped as shown in Fig. 14, page 40. In the greater number of the footings the reinforcing bars were carried straight throughout their length. In some the bars were bent up with easy curves to points near the upper surface of the footing. In a few U-shaped stirrups were used, passing around and

TABLE 4.

Compression Tests of 6-in. Cubes
Series of 1908, 1909, 1910, 1911, and 1912.

No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Maximum Load lb. per sq. in.
1301	1–3–6	64	1315	1535	1–2½–5	121	2510	1718	1–1–2	546	4640
1302	1–3–6	65	1137	1536	1–2½–5	94	2768				
1303	1–3–6	60	1073	1541	1–2½–5	109	2438	1721	1–2–4	640	2980
				1542	1–2½–5	79	2272	1722	1–2–4	606	2890
1306	1–1½–3	64	3970	1551	1–2½–5	121	2290	1723	1–2–4	368	2910
1307	1–1½–3	64	2107	1552	1–2½–5	101	2857	1725	1–2–4	606	2840
1308	1–1½–3	62	1807	1553	1–2½–5	120	2835	1726	1–2–4	368	2910
				1554	1–2½–5	103	2242	1727	1–2–4	641	2810
1311	1–3–6	63	2210	1561	1–2½–5	114	2550	1728	1–2–4	606	2840
1312	1–3–6	64	1610	1562	1–2½–5	87	2187	1729	1–2–4	373	3225
1313	1–3–6	64	2097	1563	1–2½–5	113	2462	1731	1–2–4	641	2810
1314	1–3–6	62	1420	1564	1–2½–5	83	2322	1732	1–2–4	436	2590
1315	1–3–6	62	1372	1631	1–2½–5	63	1173	1733	1–2–4	360	2570
1316	1–3–6	61	1433	1632	1–2½–5	60	1412	1741	1–2–4	629	3040
1317	1–3–6	62	1203	1633	1–2½–5	63	1173	1742	1–2–4	436	2590
1318	1–3–6	61	1433	1634	1–2½–5	66	1288	1743	1–2–4	360	2570
1319	1–3–6	60	1263	1635	1–2½–5	64	814	1744	1–2–4	629	3040
1322	1–3–6	60	1507	1636	1–2½–5	69	1666	1745	1–2–4	590	2705
1325	1–3–6	64	1910	1641	1–2½–5	69	1538	1746	1–2–4	359	2890
1326	1–3–6	60	1553	1642	1–2½–5	60	1412	1747	1–2–4	628	2870
1341	1–3–6	60	1383	1645	1–2½–5	61	1663	1748	1–2–4	590	2705
1342	1–3–6	64	1457	1646	1–2½–5	59	1235	1749	1–2–4	359	2890
1351	1–3–6	59	1283	1651	1–2½–5	68	1797	1751	1–2–4	634	2685
1352	1–3–6	60	1403	1652	1–2½–5	62	1350	1752	1–2–4	585	2725
1361	1–3–6	58	2103	1655	1–2½–5	61	1663	1753	1–2–4	354	3030
1362	1–3–6	59	1215	1661	1–2½–5	73	1757	1754	1–2–4	640	2980
1371	1–3–6	62	1211	1662	1–2½–5	75	1692	1755	1–2–4	585	2725
1372	1–3–6	59	1290	1665	1–2½–5	73	1842	1756	1–2–4	354	3030
1375	1–3–6	62	1228	1666	1–2½–5	55	1319	1757	1–2–4	640	2980
1376	1–3–6	59	960	1671	1–2½–5	59	1235	1758	1–2–4	591	3410
				1672	1–2½–5	75	1530	1759	1–2–4	354	3030
1411	1–2½–5	92	1395	1673	1–2½–5	63	1735	1761	1–2–4	634	2685
1412	1–2½–5	75	1523	1674	1–2½–5	60	1465	1762	1–2–4	591	3410
1414	1–2½–5	77	1533	1675	1–2½–5	69	1538	1763	1–2–4	354	3030
1415	1–2½–5	88	2028	1676	1–2½–5	55	1319	1806	1–2–4	70	2094
1416	1–2½–5	64	1307	1681	1–2½–5	63	1735	1807	1–2–4	73	1831
1417	1–2½–5	73	1343	1682	1–2½–5	62	1350	1808	1–2–4	64	1610
1418	1–2½–5	64	1292	1685	1–2½–5	57	1293	1809	1–2–4	66	2645
1421	1–2½–5	72	1877	1687	1–2½–5	67	1573	1810	1–2–4	63	1562
1422	1–2½–5	61	864	1688	1–2½–5	75	1692	1811	1–2–4	62	2203
1425	1–2½–5	94	1717	1692	1–2½–5	60	1465	1812	1–2–4	135	3123
1426	1–2½–5	60	1468	1693	1–2½–5	57	1293	1813	1–2–4	118	3197
1429	1–2½–5	82	1622	1694	1–2½–5	67	1573	1814	1–2–4	77	2450
1431	1–2½–5	100	1718					1815	1–2–4	62	2223
1432	1–2½–5	64	1385	1701	1–3–6	638	2640	1816	1–2–4	61	1877
1435	1–2½–5	104	1245	1702	1–3–6	298	1862	1817	1–2–4	66	2577
1436	1–2½–5	70	1151	1702a	1–3–6	422	2550	1818	1–2–4	61	2030
1437	1–2½–5	84	1543					1819	1–2–4	73	2772
1439	1–2½–5	100	1605	1703	1–2–4	520	3845	1820	1–2–4	69	1526
1447	1–2½–5	78	1482	1704	1–2–4	628	2870	1821	1–2–4	86	1678
1448	1–2½–5	58	1583	1704a	1–2–4	367	4170	1822	1–2–4	80	1692
1449	1–2½–5	92	1720					1823	1–2–4	61	2640
1451	1–2½–5	67	1418	1705	1–1–2	586	4816	1831	1–2–4	71	2620
1501	1–2½–5	112	2910	1706	1–1–2	431	4403	1832	1–2–4	108	3100
1502	1–2½–5	104	2939	1706a	1–1–2	546	4640	1833	1–2–4	67	2500
1503	1–2½–5	112	2307					1834	1–2–4	115	3240
1504	1–2½–5	109	1761	1707	1–2–4	640	2980	1835	1–2–4	67	2085
1505	1–2½–5	128	3618	1708	1–2–4	430	3090	1836	1–2–4	110	3410
1506	1–2½–5	116	2260	1708a	1–2–4	367	4170	1837	1–2–4	74	2625
1507	1–2½–5	121	3180	1710	1–2–4	430	3090	1838	1–2–4	109	3710
1508	1–2½–5	109	1988	1710a	1–2–4	367	4170	1839	1–2–4	73	2785
1515	1–2½–5	127	2980	1712	1–2–4	680	2175	1840	1–2–4	98	3235
1516	1–2½–5	118	2679	1713	1–2–4	606	2890	1841	1–2–4	116	2755
1522	1–2½–5	94	2935	1714	1–2–4	373	3225	1842	1–2–4	105	3050
1526	1–2½–5	88	1917					1843	1–2–4	116	3580
1531	1–2½–5	120	3193	1716	1–1–2	678	3870	1844	1–2–4	102	3630
1532	1–2½–5	114	2637	1717	1–1–2	431	4403				

TABLE 5.

Flexure Tests of 6 in. x 8 in. x 36 in. Control Beams. Series of 1908, 1909, 1910, 1911, and 1912.

No.	Kind of Concrete.	Age at Test days.	Modulus of Rupture, lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Modulus of Rupture, lb. per sq. in.	No.	Kind of Concrete.	Age at Test days.	Modulus of Rupture, lb. per sq. in.
1302	1–3–6	65	249	1516	1–2½–5	99	385	1708a	1–2–4	597	432
1303	1–3–6	71	167	1521	1–2½–5	107	357	1709	1–2–4	82	214
				1522	1–2½–5	91	296	1710a	1–2–4	597	432
1306	1–1½–3	63	437	1525	1–2½–5	107	294	1712	1–2–4	101	266
1307	1–1½–3	64	355	1526	1–2½–5	93	287	1713	1–2–4	36	226
1308	1–1½–3	71	385	1531	1–2½–5	93	342	1714	1–2–4	597	291
				1532	1–2½–5	96	440				
1312	1–3–6	64	140	1535	1–2½–5	105	377	1716	1–1–2	100	402
1313	1–3–6	64	286	1536	1–2½–5	94	230	1718	1–1–2	597	656
1314	1–3–6	64	258	1541	1–2½–5	104	387				
1315	1–3–6	63	151	1542	1–2½–5	85	397	1721	1–2–4	89	220
1316	1–3–6	60	194	1551	1–2½–5	88	363	1722	1–2–4	36	226
1317	1–3–6	71	175	1552	1–2½–5	96	368	1724	1–2–4	82	214
1318	1–3–6	60	194	1553	1–2½–5	87	365	1725	1–2–4	55	169
1319	1–3–6	59	284	1554	1–2½–5	99	362	1728	1–2–4	55	169
1321	1–3–6	63	379	1561	1–2½–5	81	375	1729	1–2–4	597	291
1322	1–3–6	59	309	1562	1–2½–5	84	408	1732	1–2–4	113	384
1325	1–3–6	67	289	1563	1–2½–5	80	306	1742	1–2–4	113	384
1326	1–3–6	77	210	1564	1–2½–5	89	313	1745	1–2–4	84	367
1341	1–3–6	69	237	1631	1–2½–5	56	231	1747	1–2–4	58	231
1342	1–3–6	60	219	1632	1–2½–5	65	283	1748	1–2–4	84	367
1351	1–3–6	60	182	1633	1–2½–5	56	231	1751	1–2–4	64	265
1352	1–3–6	56	228	1634	1–2½–5	65	267	1752	1–2–4	78	350
1361	1–3–6	60	296	1635	1–2½–5	63	301	1754	1–2–4	63	272
1362	1–3–6	60	191	1636	1–2½–5	67	323	1755	1–2–4	78	350
1371	1–3–6	63	258	1641	1–2½–5	81	274	1757	1–2–4	63	272
1372	1–3–6	60	179	1642	1–2½–5	65	283	1758	1–2–4	84	335
1375	1–3–6	63	250	1645	1–2½–5	72	341	1761	1–2–4	64	265
1376	1–3–6	60	174	1646	1–2½–5	63	252	1762	1–2–4	84	335
				1651	1–2½–5	61	260	1806	1–2–4	98	260
1411	1–2½–5	58	282	1652	1–2½–5	80	269	1807	1–2–4	88	346
1412	1–2½–5	74	151	1655	1–2½–5	72	341	1808	1–2–4	106	289
1414	1–2½–5	82	224	1661	1–2½–5	63	306	1809	1–2–4	67	362
1415	1–2½–5	78	295	1662	1–2½–5	86	325	1810	1–2–4	68	215
1416	1–2½–5	69	214	1665	1–2½–5	63	306	1811	1–2–4	74	358
1417	1–2½–5	71	322	1666	1–2½–5	65	255	1812	1–2–4	100	243
1418	1–2½–5	69	264	1671	1–2½–5	63	252	1813	1–2–4	93	224
1421	1–2½–5	70	287	1672	1–2½–5	86	315	1814	1–2–4	78	339
1422	1–2½–5	64	183	1673	1–2½–5	62	290	1815	1–2–4	74	432
1425	1–2½–5	87	301	1674	1–2½–5	60	227	1816	1–2–4	56	259
1426	1–2½–5	64	274	1675	1–2½–5	81	274	1817	1–2–4	67	415
1429	1–2½–5	75	287	1676	1–2½–5	65	255	1818	1–2–4	63	246
1431	1–2½–5	105	283	1681	1–2½–5	62	290	1819	1–2–4	74	405
1435	1–2½–5	109	287	1682	1–2½–5	80	269	1820	1–2–4	69	217
1436	1–2½–5	68	171	1685	1–2½–5	57	290	1822	1–2–4	95	277
1437	1–2½–5	83	265	1687	1–2½–5	66	288	1823	1–2–4	612	341
1439	1–2½–5	67	240	1688	1–2½–5	86	325	1831	1–2–4	301	401
1447	1–2½–5	72	294	1692	1–2½–5	60	227	1832	1–2–4	280	394
1448	1–2½–5	58	281	1693	1–2½–5	57	290	1833	1–2–4	298	331
1449	1–2½–5	98	294	1694	1–2½–5	66	288	1834	1–2–4	287	310
1451	1–2½–5	60	240					1835	1–2–4	293	303
1501	1–2½–5	79	309	1701	1–3–6	205	230	1836	1–2–4	281	405
1502	1–2½–5	87	423	1702a	1–3–6	597	281	1837	1–2–4	295	354
1503	1–2½–5	79	373					1839	1–2–4	303	358
1504	1–2½–5	109	425	1703	1–2–4	205	415	1840	1–2–4	267	453
1505	1–2½–5	95	376	1704	1–2–4	58	231	1841	1–2–4	287	356
1506	1–2½–5	99	272	1704a	1–2–4	597	432	1842	1–2–4	277	312
1507	1–2½–5	105	409	1706	1–2–4	668	383	1843	1–2–4	288	310
1508	1–2½–5	93	363	1706a	1–2–4	597	656	1844	1–2–4	274	387
1515	1–2½–5	95	371	1707	1–2–4	89	220				

under the longitudinal reinforcing bars. The reinforcing bars when straight were 4 ft. 11 in. and 4 ft. 11½ in. in length, except that in 1909 a length of 4 ft. 6 in. was used. The reinforcement of the footings is described in Tables 9 to 11, and the position of the bars is shown in Fig. 14. Except where otherwise noted, the wall and footing were poured at the same time. In the footings made in 1909 and 1911 four ¼-in. bars were placed vertically in the corners of the wall, extending down into the footing, to prevent displacement of the wall in handling.

The brick footings were 5 ft. long and about 12 in. wide. The wall was 12 in. in thickness. The depth, offsets, and number of courses are shown in Fig. 13.

9. *Column Footings.*—The general form of a column footing is shown in Fig. 4, page 12. The column, or pier as it will be called, was 12x12x12 in. The footings were 5 ft. square. The depth over all varied from 6 in. to 18 in. One footing (No. 1451) had a sloping upper surface, the depth being 7 in. less at the edges than at the face of the pier. The dimension given in the table for the position of the reinforcement is the distance from the upper surface of the footing to the center of the two layers of bars, or to the center of the four layers in the case of four-way reinforcement. Unless otherwise noted the reinforcing bars were straight throughout their length. In the series of 1909 the reinforcing bars were generally 4 ft. 6 in. long; in 1910, 4 ft. 10 in. long; in 1911 and 1912, about 4 ft. 11½ in. long. In a few cases in 1909 the reinforcing rods were 9 in. shorter, and alternate bars were run within 12 in. of one face of the footing, the other bars going to within the same distance from the opposite face. For the footings made in 1909 the depth over all was in most cases 11 in., it being in a few 11½ in. and 12 in. For the footings made in 1910, 1911, and 1912, the depth over all was 12 in., except for the shallower footings. The general make-up of the footings and the disposition of the reinforcing bars is given in Tables 14 to 18, and in the diagrams. Eyes U-formed of steel rods were embedded in the footings at two points; hooks were inserted in these eyes when the footings were lifted and moved.

10. *Making and Storing Footings.*—The footings were built in wooden side forms directly on the concrete floor of the mixing room with a strip of building paper beneath the forms. The forms were generally removed after 7 days. In the work of the first two years the wall footings were left on the floor of the mixing room until the test, when they were removed to the Materials Testing Laboratory, but in the later years they were piled one above the other for storage. The column footings were piled one above the other for storage; they were tested in the mixing room. The specimens were wet down with water from

a hose at frequent intervals for some time after making. The temperature of the mixing room ranged from 50° to 70° F. and was somewhat irregular, so that the average temperature for the hardening for different specimens varied. As noted under Article 29, "Phenomena of Tests of Column Footings," part of the column footings were not removed from the place of making until just before the test, and the difference in moisture conditions probably affected the rate of hardening.

11. *Minor Test Pieces.*—In Tables 4 and 5 are given the results of compression tests of 6-in. cubes and of flexure tests of 6x8x36-in. plain concrete control beams. These minor test pieces were made from the same batch of concrete as the corresponding footings and serve to give an estimate of the strength and quality of the concrete used. The control beams were tested with a 3-ft. span and one-third-point loading upon a wooden base, so arranged as to insure a good distribution of the loads and pressures across the width of the beam.

12. *Testing Wall Footings.*—The wall footings were tested in the 200 000-lb. Olsen testing machine of the Laboratory of Applied Mechanics, except that in the tests made in 1912 the 600 000-lb. Riehle testing machine was used. A nest of springs was placed on the bed of the machine. These springs were "car springs," one set being 2¾x-7x½-in. springs and the other set 3x12x9/16-in. springs. The first size closed 1.7 in. with a load of 1 700 lb., and the second size about 3 in. with a load of 2 000 lb. A calibration of a considerable number of these springs showed close uniformity among them, and their shortening was directly proportional to the load. The springs were held in place in setting the footing by ¼x3x12-in. plates with a dowel fastened on the under side which extended down into the opening of the spring. The springs were spaced to suit the load expected, and were most commonly 3 in. center to center, both lengthwise and crosswise of the footing. The 7-ft. footings of the series of 1907 had the sets of springs spaced 4 in. apart in the lengthwise direction. A view of a wall footing in the testing machine is given in Fig. 10. The footing rested directly on the spacing plates. On top of the stem or wall an iron plate was bedded in plaster of paris. On this plate a spherical bearing block was centered with respect to the wall and adjusted to the head of the machine and the load was applied directly to this bearing block, or the load was centered by using a rod across the plate to act as a pivot.

As the load was applied by the testing machine, the springs compressed and the ends of the footing deflected somewhat. Vertical measurements were taken from the bed of the machine to marks on

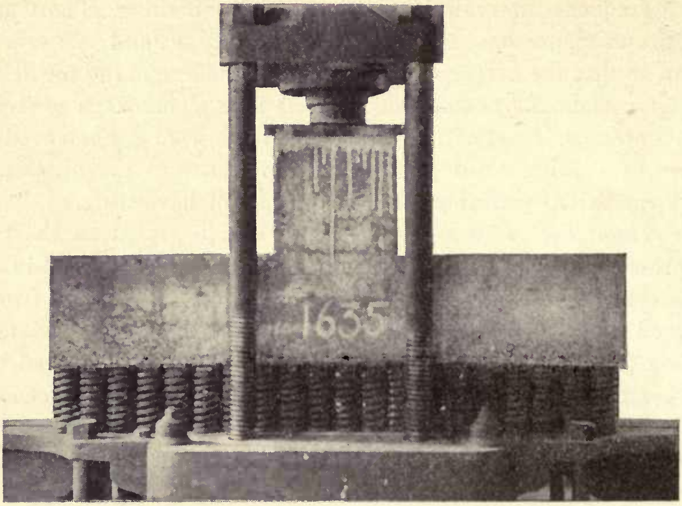

FIG. 10. VIEW OF WALL FOOTING IN TESTING MACHINE.

both sides of the footing at five points along its length. These measurements were taken at a load of 2 000 lb., and at the various loads applied thereafter. A careful watch was made for cracks and their appearance and growth was recorded. After failure the footing was broken up and examined. Measurement of slip of bar and of deformations in the bar were made in some cases, as is described elsewhere.

It was sometimes difficult to keep the test specimen in its place at high loads, as the bed of springs canted or otherwise got out of place. In those failures in which the footing separated into pieces, parts were thrown violently from the testing machine.

13. *Testing Column Footings.*—The column footings were tested in a machine built especially for the purpose. Fig. 11 (a) gives a view of the apparatus which was used in the tests in 1910, 1911, and 1912. A bed was formed by placing 10-in. I-beams side by side, the edges of the flanges touching. On this rested a bed of car springs on which the test footing was placed. Transversely under the bed of I-beams and near their ends were two 12-in. x 55-lb. I-beams 6 ft. long which took the load from above. Under these I-beams were two cast-iron blocks through which eight rods passed to similar cast-iron blocks on the upper part of the machine. The two hydraulic jacks by which the load was applied were placed between these blocks and a 24-in. I-beam. This I-beam transmitted the load through blocks to the top

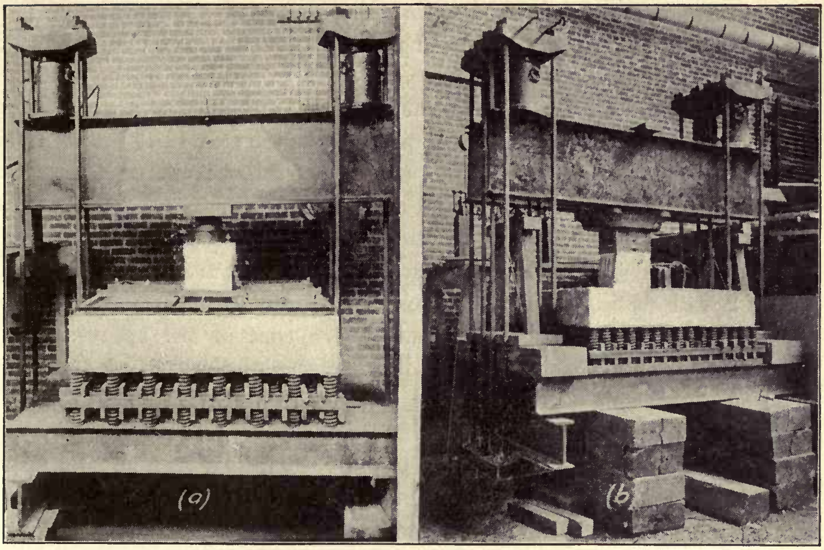

FIG. 11. TESTING MACHINES FOR COLUMN FOOTINGS.

of the pier. One of these blocks was an adjustable spherical bearing block. The lower or base block was bedded to the top of the pier with plaster of paris. The whole of the machine rested on a timber foundation. The pumps which operated the hydraulic jacks were placed on a platform near by, a gage being connected with each pump.

In the 1909 tests four jacks were used and two 24-in. I-beams were placed across the top with four sets of vertical rods running from their ends. A heavy steel block carried the load from the two I-beams. In some of the 1909 tests no adjustable bearing blocks were used, the bearing plates consisting of flat plates only and the adjustment being made by the jacks. This apparatus is shown in Fig. 11 (b). On account of the low loads on the jacks the results with this machine are less satisfactory than with the 1910 machine.

The springs used were the 3x12x9/16-in. helical car springs used in tests of wall footings, ground to a length of 12 in. In the 1910 tests there were generally 113 springs used, though for the heavy loads the number was increased to 225. In the 1909 tests seven footings were tested with 225 springs. The amount of shortening with this number of springs was so small at the lower loads as not to give a good distribution of the load over the bottom of the footing. A bed composed of 113 springs would compress about 1 in. under a load of 75 000 lb.

The springs completely closed with a shortening of about 3 in., so that with some deflection in the footing not more than 200 000 lb. could generally be carried by 113 springs.

In the operation of the tests the load was applied in varying increments. It was kept as nearly as possible equally divided between the jacks. In some cases, due to imperfect bearing, the springs tilted to one side and a release of load and a readjustment was necessary. The load was taken from the gage indication, and was corrected by means of a calibration graph which had been prepared from the calibration of the jacks in a testing machine. Many of the tests were continued beyond the critical load and in some cases the rupture of the specimen was followed by a violent throwing of large pieces of the footing from the machine.

Measurements of the compression of the springs were made at the corners of the footings. In the 1910 and 1911 tests a frame made up of 1¼x1¼-in. angles was supported at three points on the upper surface of the footing close to the pier. Measurements taken from this frame at numerous points on the upper surface of the footing by means of Ames test dials enabled the deflection of the footing at these points to be determined. In the 1909 tests threads were stretched along two opposite lateral faces of the footing and the deflection at the faces obtained by means of mirror-and-scale apparatus. A yoke clamped to the sides of the pier gave a basis of measurements for the transverse deflections.

Observations were taken of cracks after they became visible on the lateral faces. Due to the form of construction of the machine and the presence of the nest of springs, no observations could be made on the bottom surface of the footing during the progress of the test. The footings were examined after being taken from the machine, but it must be borne in mind that the cracks formed and fractures obtained indicated conditions that may have been brought into existence after the critical load was applied.

III. Experimental Data and Discussion.

A. Wall Footings.

14. *Tables.*—Table 6 gives values of j used in the calculations in connection with equations (13), (17) and (18). Tables 7, 9, 10 and 11 give descriptions of the wall footing test pieces, the results of the tests, and calculated quantities. These quantities are calculated by the formulas and methods given on pages 8 to 10. In all the calcula-

tions, a uniform distribution of load is assumed. This is done because in some cases the measurements of deflection and of compression of springs are not available, or are available only at loads below the critical load considered, and their use would put some tests on a different basis from the others. In the stiffer footings and for the lower loads the error in assuming that the distribution is uniform is slight. For a

TABLE 6.

VALUES OF j USED IN CALCULATIONS.

Reinforcement per cent	1–2½–5 and 1–3–6 concrete	1–1–2 and 1–2–4 concrete
0.20	.92	.93
0.30	.90	.915
0.40	.89	.90
0.50	.88	.895
0.60	.87	.89
0.70	.865	.88
0.80	.86	.875
1.00	.85	.86
1.20	.84	.85
1.50	.825	.84

very few cases the calculations will give perhaps 7% greater tensile stress than the real distribution, the same excess in bond stress, and perhaps as much in the vertical shearing stresses at the section used. Generally the error is well within this limit. Of course, after the yield point of the reinforcement is reached, or failure by bond occurs, the effect of uneven distribution of pressure is far greater than that outlined above.

It should be noted that there was some difficulty and an element of uncertainty in determining the amount of the critical load, especially when readings were not taken close together. Generally the critical load was taken at the marked increase in deflection, though dropping of load, marked increase in deformation in steel, and other features were considered when the information was available. The additional load beyond the critical load carried by most footings, will be useful as a safeguard, but is not available for design purposes.

In these tables the tensile stresses and bond stresses are calculated for a section at the face of the wall, the vertical shearing stresses for a section distant d from the face of the wall. The reason for using this particular section for the vertical shearing stresses is discussed under Article 25, "Vertical Shearing Stresses and Diagonal Tension Failures." Table 12 gives values of the vertical shearing stresses for wall footings

TABLE 7.

Tests of Wall Footings without Reinforcement.

Footing No.	Year made	Kind of Concrete	Cement		Length feet	Age at Test days	Load at Failure pounds	Modulus of Rupture lb. per sq. in.	Control Beams		6-in. Cubes.	
			Kind	Per cent					Modulus of Rupture lb. per sq. in.	Age days	Maximum Load lb. per sq. in.	Age days
1301	1908	1–3–6	U	8.5	5	64	14 600	289	...	..	1 315	64
1302	1908	1–3–6	AA	9.9	5	63	12 200	242	249	65	1 137	65
1303	1908	1–3–6	AA	10.1	5	61	12 000	238	167	71	1 073	60
1306	1908	1–1½–3	U	18.7	5	64	23 050	457	437	63	3 970	64
1307	1908	1–1½–3	AA	20.2	5	61	22 300	442	355	64	2 107	64
1308	1908	1–1½–3	AA	19.4	5	61	20 000	396	385	71	1 807	62
1701	1911	1–3–6	U	12.0	5	86	20 300	339	230	205	2 660	519
1702	1911	1–3–6	U	12.2	5	355	15 000	250	...	...	1 862	298
1702a	1911	1–3–6	L	11.8	5	295	23 300	389	...	...	2 630	360
1703	1911	1–2–4	U	15.5	5	86	28 100	469	415	205	3 845	520
1704	1911	1–2–4	U	17.2	5	89	15 400	257	231	58	.....	...
1704a	1911	1–2–4	L	17.1	5	296	22 700	379	...	...	4 170	367
1705	1911	1–1–2	U	36.5	5	100	27 900	464	...	...	4 920	363
1706	1911	1–1–2	U	34.3	5	355	26 500	442	...	...	4 403	431
1706a	1911	1–1–2	L	35.0	5	304	24 600	410	...	...	3 680	367
1707	1911	1–2–4	U	17.4	3	88	49 600	344	220	89	.....	...
1708	1911	1–2–4	U	17.1	3	374	35 100	244	...	...	3 090	430
1708a	1911	1–2–4	L	17.1	3	304	56 700	394	...	...	4 170	367
1709	1911	1–2–4	U	17.2	7	83	11 800	316	214	82	.....	...
1710	1911	1–2–4	U	17.1	7	375	11 900	319	...	...	3 090	430
1710a	1911	1–2–4	L	17.1	7	329	16 400	439	...	...	4 170	367

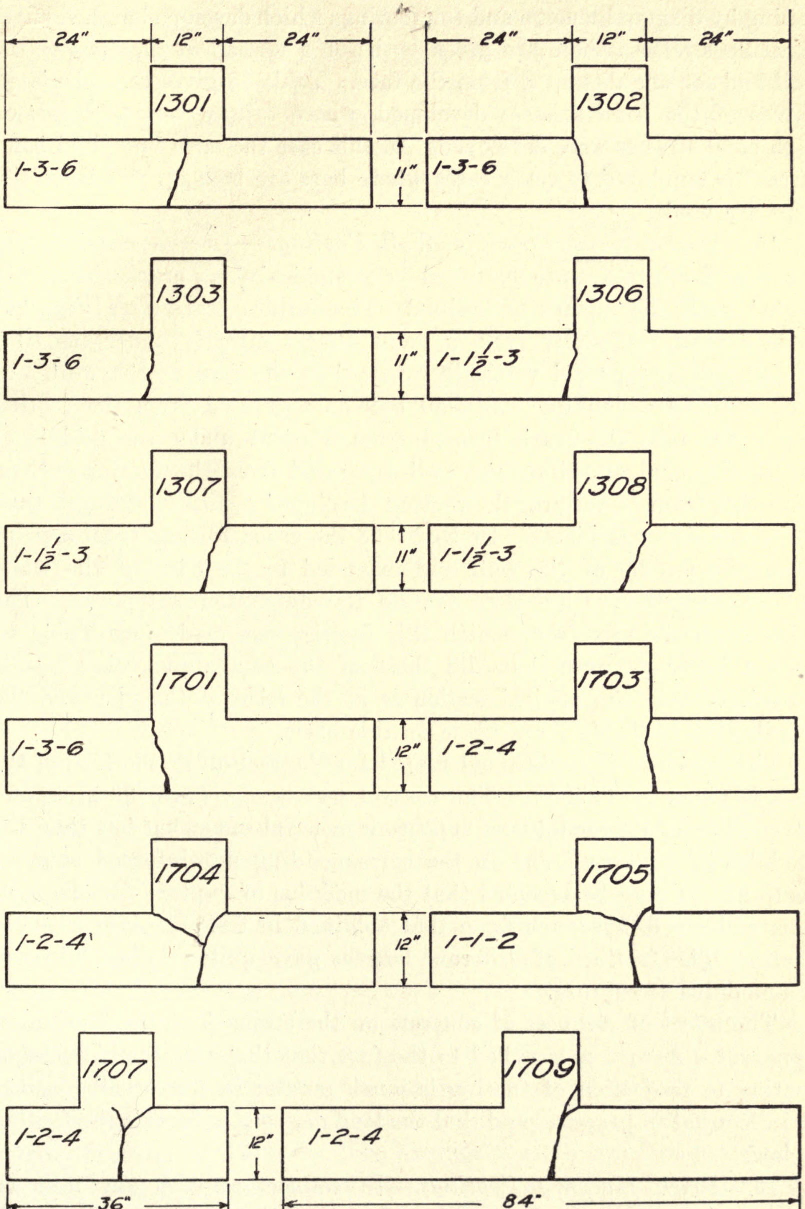

FIG. 12. UNREINFORCED CONCRETE WALL FOOTINGS.

failing by diagonal tension and for footings which developed high vertical shearing stresses; these are given both for a section at the face of the wall and for one distant d from the face. Table 13 gives the calculated values of the bond stresses developed, where failure was by bond, or high bond stresses were developed. In this case the term "nominal bond stress" is employed to cover cases where bars are bent up or where stirrups are used.

15. *Unreinforced Concrete Wall Footings.*—In the unreinforced concrete footings failure occurred very suddenly, no cracks being observed until the instant of failure. The failure crack (see Fig. 12) formed most frequently from a point almost directly under the edge of the wall and passed vertically upward to the face of the wall. In No. 1301, in which the wall had become separated from the footing before the test, the crack formed from a point under the middle of the footing and at failure the wall separated from the footing. Even with this form of failure, the footing developed a higher strength than its companion test pieces. In No. 1308 the crack formed from a point below the middle of the wall and extended to the edge of the wall; but its strength was less than that of its companion test pieces. The cubes from the mix from which this footing was made (see Table 4) gave a lower strength than did those of the companion cubes. It is considered that the critical section is at the edge of the pier and the calculations in Table 7 are made on this basis.

The modulus of rupture calculated for the section at the face of the wall is given in Table 7. The control test beams (6x8, 36-in. span) gave values of the modulus of rupture in general somewhat less than the modulus of rupture found in the corresponding unreinforced concrete footings. It may be expected that the modulus of rupture for the footings will not differ much from that obtained in tests of plain concrete beams. The footings of different lengths gave quite similar values of the modulus of rupture.

The effect of richness of concrete on the strength of the footings is apparent. Attention is called to the fact that the ratio of thickness of footing to projection of footing is much greater in these wall footings than is usual in practice, and that the load per square foot on the footing is low.

16. *Brick Masonry Footings.*—In connection with the tests of brick and terra cotta columns, described in Bulletin No. 27, four wall footings of brick masonry were built, and the results of the tests of these brick footings will be recorded here. The form and dimensions of these

footings are shown in Fig. 13. All the footings were well laid up in 1-3 portland cement mortar, joints being broken in workmanlike manner, and the workmanship was the same as that described in Bulletin No. 27 for the brick columns. Two grades of brick were used, shale build-

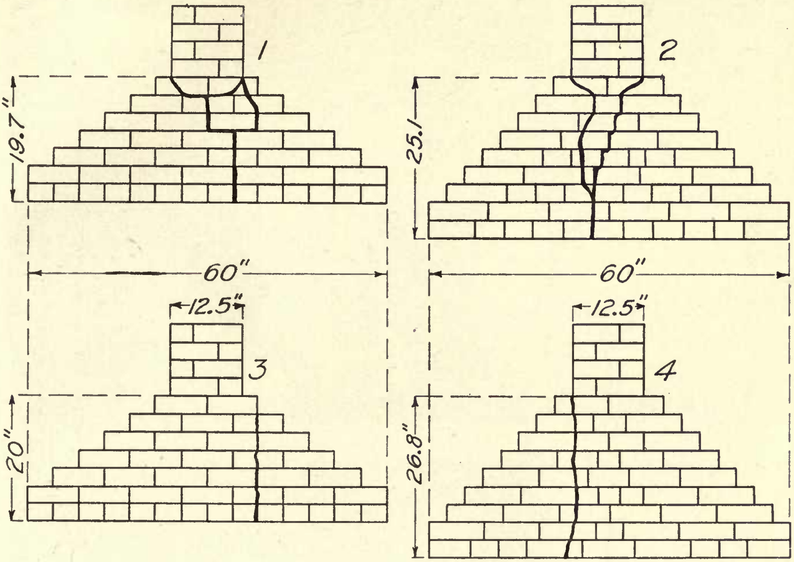

FIG. 13. BRICK WALL FOOTINGS.

ing brick and underburned clay brick, as described in that bulletin. The tests were made in the same manner as the tests of the concrete footings.

The footings failed suddenly at the maximum applied load. The section of failure was usually at the edge of the wall or stem of the footing. The failure was along vertical joints and through the bricks. In the footings made with hard building brick the line of fracture was irregular, a second break running to the other face of the wall. In the footings made with soft brick there was but one line of failure.

The modulus of rupture calculated for the section at which failure occurred is given in Table 8. The average for the two footings made with the hard building brick is 281 lb. per sq. in., and for the two made with soft brick 76 lb. per sq. in. Table 8 also gives the results of tests made on brick beams built at the same time and in the same way. In No. 1 and No. 3 the courses were not well laid as to joints,

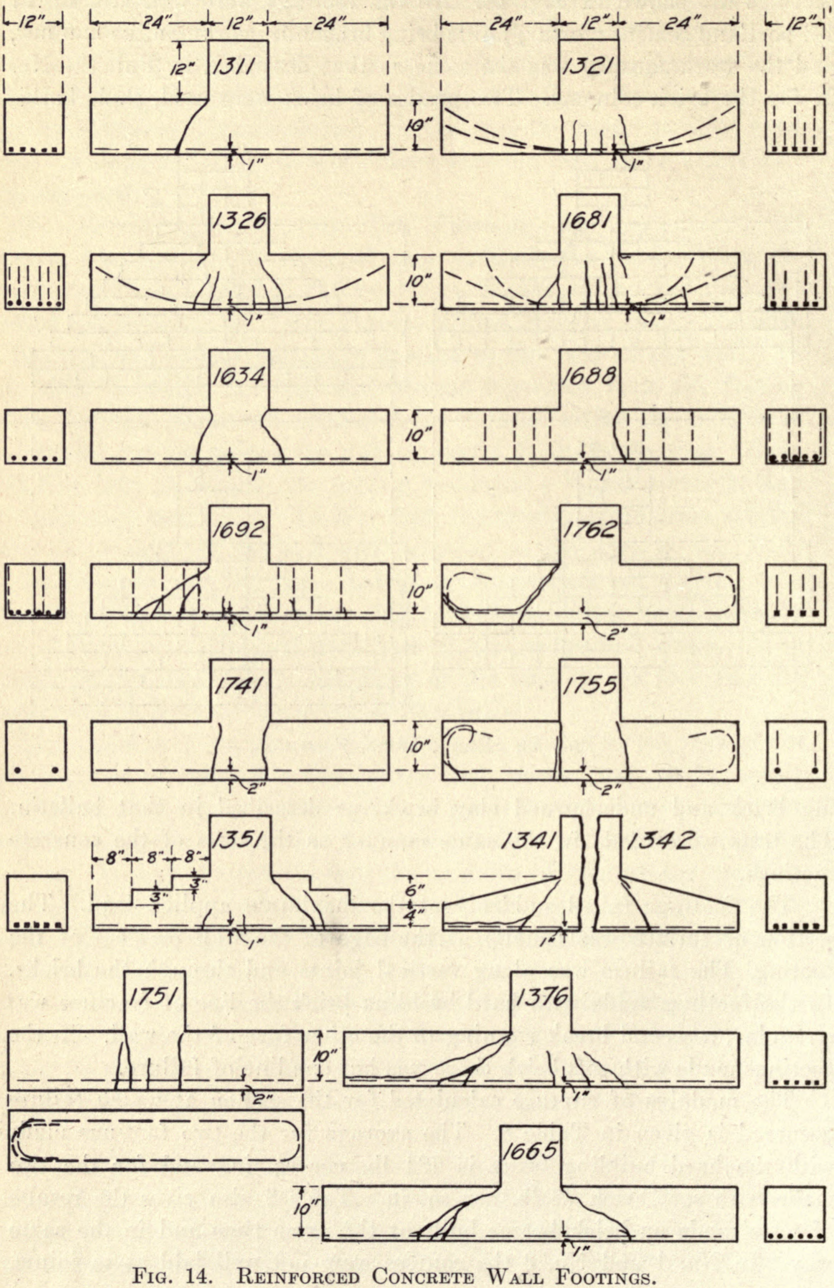

FIG. 14. REINFORCED CONCRETE WALL FOOTINGS.

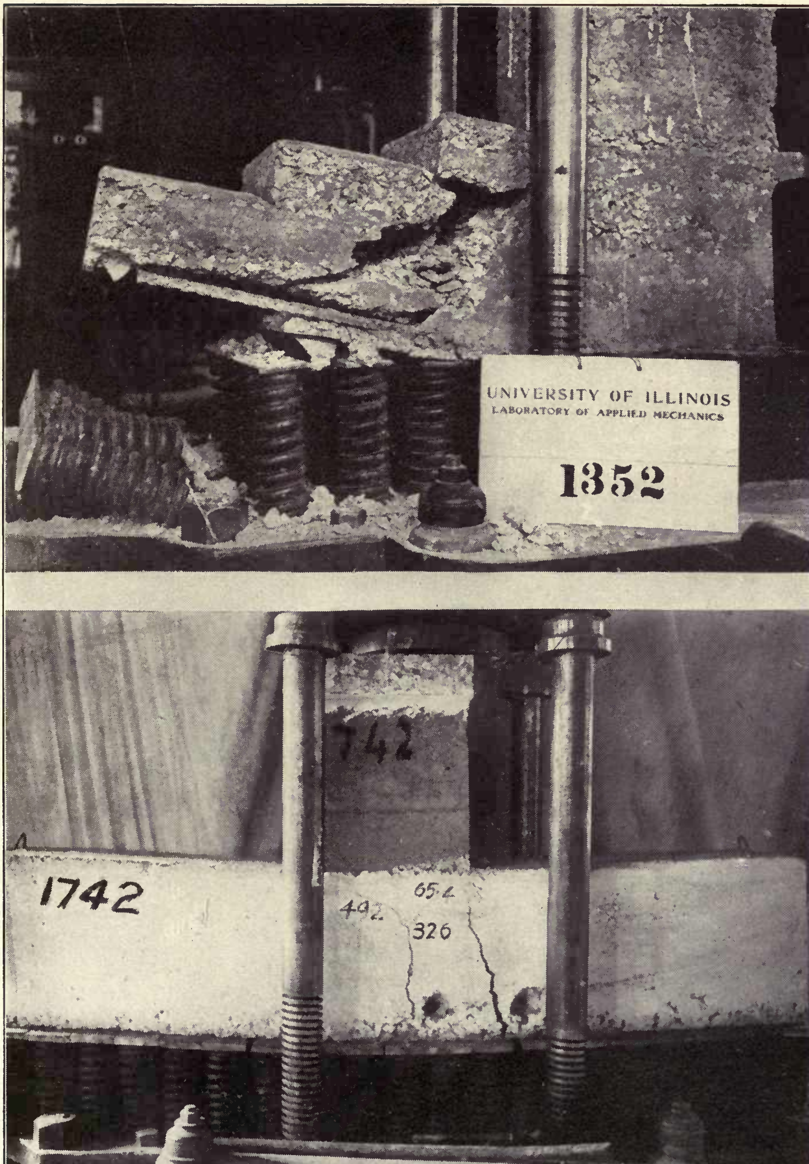

FIG. 15. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

FIG. 16. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

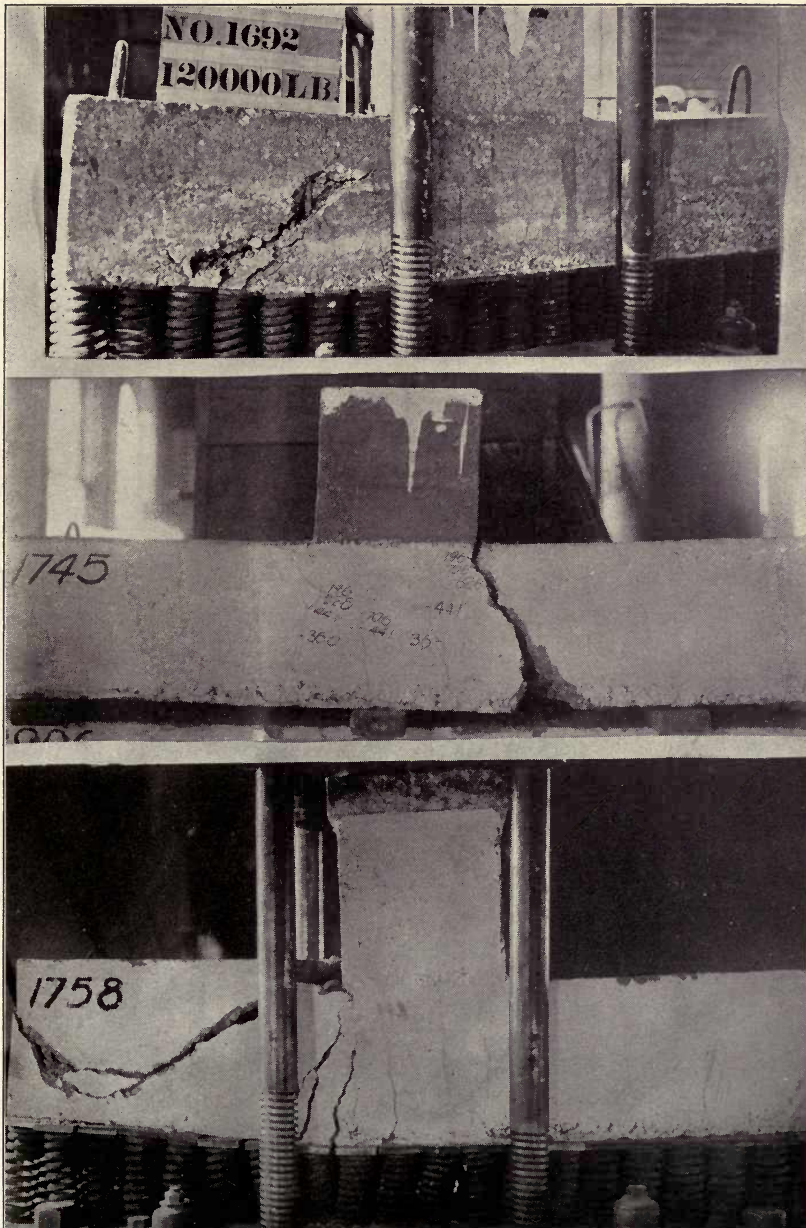

FIG. 17. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

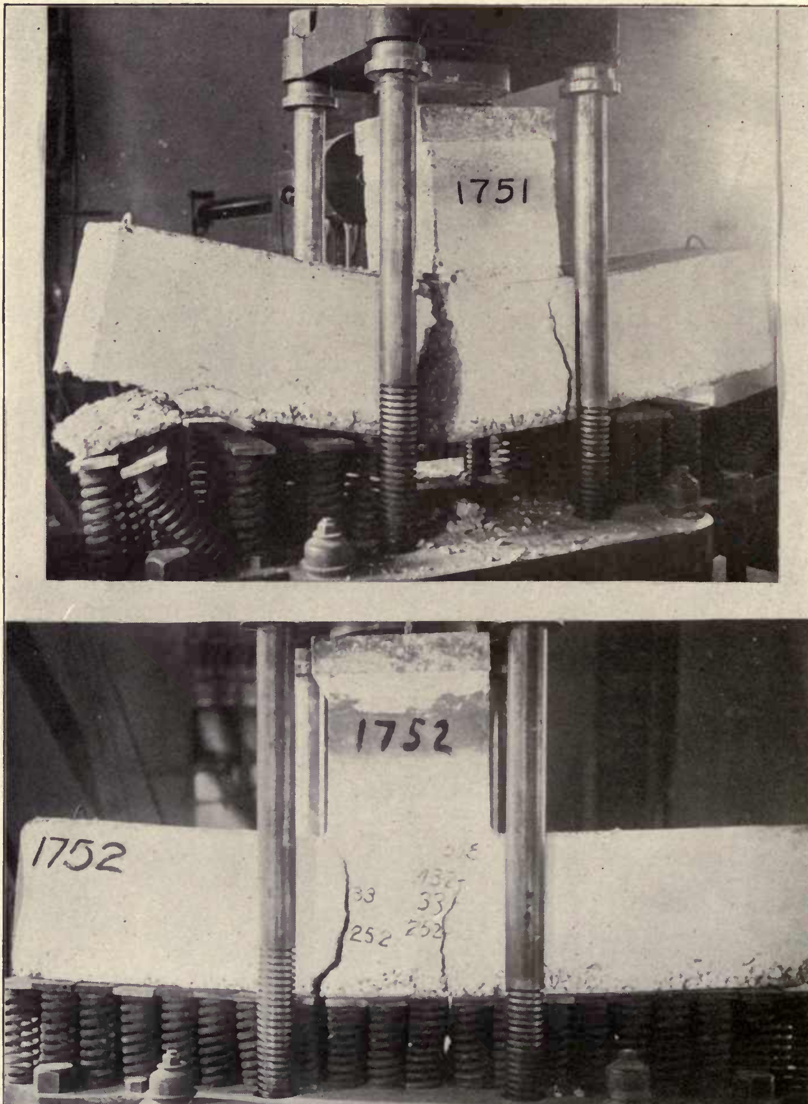

FIG. 18. VIEWS OF REINFORCED CONCRETE WALL FOOTINGS AFTER TEST.

TABLE 8.

TESTS OF BRICK BEAMS AND FOOTINGS.

1–3 cement mortar used. Beams tested with a 54-in. span.

Specimen	Age days	Length feet	Breadth inches	Depth inches	Material	Maximum Applied Load pounds	Modulus of Rupture, lb. per sq. in.	Remarks
Beam No. 1	76	5	12.5	13.9	Shale building brick	10 450	235	Courses not carefully laid as regards joints.
Beam No. 2	76	5	12.75	13.6	Shale building brick	20 900	478	Courses carefully laid.
Beam No. 3	76	5	13.0	14.75	Underburned clay brick	2 960	57	Courses not carefully laid.
Beam No. 4	74	5	13.1	14.50	Underburned clay brick	4 310	85	Courses carefully laid.
Footing No. 1	76	5	12.5	19.7	Shale building brick	50 000	292	
Footing No. 2	77	5	12.5	25.1	Shale building brick	75 200	270	
Footing No. 3	74	5	12.5	20.0	Underburned clay brick	14 800	85*	Depth at line of fracture 18 in.
Footing No. 4	76	5	12.5	26.8	Underburned clay brick	21 000	66	

*Calculations made for section at line of fracture.

but in No. 2 and No. 4 more care was given to the position of the cross joints with respect to the adjacent bricks. It will be seen that the modulus of rupture for the brick beams is not far from that for the brick footings, but it is also apparent that the method of laying will greatly affect the strength of the structure. As the modulus of rupture of the shale brick was found to average 1 670 lb. per sq. in., and of the soft clay brick 481 lb. per sq. in. (see page 11, Bulletin No. 27), and the footings broke across the brick, it appears that with the excellent mortar used the strength of the footing is dependent upon the strength of the brick. However, it should be noted that the modulus of rupture of the footings is far below that of the brick, and the strength of the footing is greatly affected by the joints, and probably also by the thickness of the brick.

17. *Phenomena of Tests of Reinforced Concrete Wall Footings.*—In the tests, as the load was increased the springs forming the bed compressed, the amount of compression or shortening being proportional to the load applied. Although the deflection of the ends of the footing modified somewhat the distribution of the load over the bottom of the footing, this deflection was so slight, below what may be called the critical load on the beam, as compared with the total shortening of the springs, that the distribution up to the point of failure was not far from uni-

form. In those cases in which the deflection became considerable, as after the longitudinal reinforcement was stressed well beyond the yield point or the reinforcing bars had slipped considerably, an appreciably smaller load came at the ends than the middle. In some cases before the test was discontinued, the load was finally carried principally by the springs immediately under the wall or stem of the footing, due to the large deflection of the ends or to the closing up of the springs, and the applied load was not representative of the load taken in flexure. The point of failure or critical load was determined from the point of marked change in the deflection curve. By reason of the lack of definiteness of this point, the load at failure is somewhat uncertain in some of the tests, as has already been discussed. In a few of the tests the load seemed not to have been centrally applied on the wall or stem, and the springs were compressed much more at one end of the footing than at the other. As was to be expected, these footings failed at lower total loads than their companion test pieces. In the calculations, for simplicity the loads have been used as symmetrically applied, but it must be understood that in these cases the load on one projection of the footing was larger than the normal proportion of the total load.

As in ordinary beam tests, in the reinforced footings cracks formed in the concrete generally at loads well below the load which produced failure. In some cases these were tension cracks and in other cases diagonal tension cracks, while in some the cracks were evidently caused by slip of bar. The failures were usually slow, especially in the case of the tension failures and in some of the bond failures. With slow failures and in cases where deflection of end of footing became large, the load could finally be applied in an amount considerably above the load which may be considered to be the failure load. In diagonal tension failures, the failure was usually sudden and violent, often part of the footing being thrown off the weighing table of the machine. It will be appreciated that the amount of energy stored in the compressed springs was very large, and the sudden release of this energy resulted in a violent displacement of the specimen.

As high percentages of reinforcement were not used, no failures by compression were found. In two specimens the concrete in the stem or wall proved to be very poor or the wall was poorly made, and the wall failed before the full strength of the footing was developed. These tests have not been included in the tables.

The following are brief notes of the tests. The location of the cracks is shown in Figs. 14 and 19. Heavy lines indicate the crack

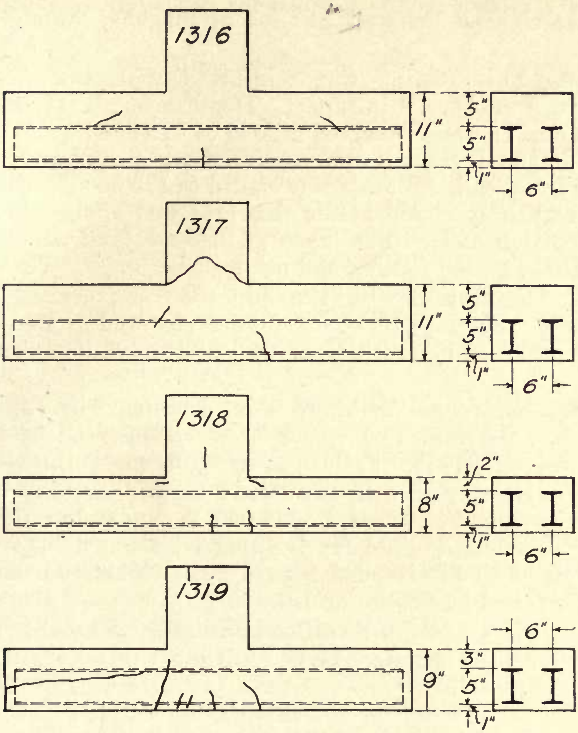

FIG. 19. WALL FOOTINGS REINFORCED WITH I-BEAMS.

along which failure took place. Reference may be made to Tables 8, 9, 10 and 11, page 41 and 55-58.

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No. 1311. At a load of 100 000 lb. springs became closed under the load and load was released. No sign of failure. Reloaded the following day, an additional number of springs being used. Slight cracks (diagonal) noted at load of 115 000 lb. Failed by diagonal tension at a load of 129 000 lb. See Fig. 14.

No. 1312. At load of 60 000 lb. small tension cracks were noted. At 80 000 lb. diagonal tension crack was noted. Failed by diagonal tension at load of 96 000 lb.

No. 1313. Failed at 95 000 lb. by diagonal tension.

No. 1314. At load of 78 500 lb. small tension crack was noted. At 84 000 lb. tension crack noted under left face of wall. Failure occurred by bond at 111 300 lb.

No. 1315. Fine diagonal crack noted at load of 59 000 lb. about 5 in. to the left of the wall. At 65 000 lb. another diagonal crack noted a little to the right of the wall. Failed by diagonal tension at load of 89 500 lb.

No. 1316. This footing was reinforced with two 5-in. x 9¾-lb. I-beams, 4 ft. 9 in. long, 6 in. apart. Depth over all, 11 in. (See Fig. 19.) Small tension crack observed at load of 115 000 lb. At 130 000 lb. two small diagonal cracks were observed at either end of footing about 12 in. from wall. Footing was loaded to 140 000 lb., but as middle springs were entirely closed before this load was applied less than the normal proportion of load was taken at the ends, and the test was discontinued, although the footing had not failed.

No. 1317. Reinforced with two 5-in. I-beams; arrangement same as in No. 1316. (See Fig. 19.) First crack observed at load of 115 000 lb. Failure occurred by crushing of wall at 134 400 lb. Concrete split lengthwise.

No. 1318. Reinforced with two 5-in. I-beams 6 in. apart. Depth over all 8 in. (See Fig. 19.) Tension crack observed at 105 000 lb. Maximum load 120 000 lb. Failure occurred by tension in steel.

No. 1319. Reinforced with two 5-in. I-beams. (See Fig. 19.) Wall crooked. First crack observed at 117 000 lb. under left face of wall. At 120 000 lb. a longitudinal crack appeared about 6 in. from top of footing at left end and extended toward wall. Maximum load 140 600 lb. Probably failed by flexure of I-beam.

No. 1321. Bars bent up to different heights. Failed by tension in steel at 128 000 lb. (See Fig. 14.) Continued to take load, the ends finally deflecting 0.4 in.

No. 1322. Reinforcement the same as No. 1321. Tension crack observed at load of 74 000 lb. At 80 000 lb. a diagonal crack was noted a little to right of wall. Numerous tension cracks appeared up to the maximum load of 130 000 lb. Tension failure.

No. 1325. Bars bent up to different heights. At 81 000 lb. first crack appeared under right face of wall. Load at failure 100 000 lb. Failure occurred by tension in steel.

No. 1326. Bars bent up to different heights. (See Fig. 14.) At load of 67 000 lb. a tension crack was noted near center and a crack near right edge of wall somewhat inclined to the vertical. At 75 000 lb. another inclined crack was noted to left of wall. At 80 000 lb. the inclined cracks were growing rapidly. At 105 000 lb. tension cracks were noted under right and left faces of wall. Footing failed by tension in steel.

No. 1341. Sloped footing. (See Fig. 14.) At 69 000 lb. a diagonal crack appeared about 8 in. to left of wall. Failed by diagonal tension at 80 000 lb.

No. 1342. Sloped footing. Poor concrete. At 70 000 lb. diagonal crack noted 18 in. from right end. Failure occurred suddenly at 80 300 lb. by diagonal tension.

No. 1351. Stepped footing. (See Fig. 14.) Small tension cracks noted at load of 86 300 lb. At 90 000 lb. a diagonal crack was noted about 12 in. from right end. Failure occurred suddenly by diagonal tension at 107 300 lb.

No. 1352 Stepped footing. (See Fig. 15). At 80 000 lb. a diagonal crack was noted. At 85 800 lb. a diagonal crack was noted a few inches left of wall. Failure occurred suddenly by diagonal tension at 86 800 lb.

No. 1362. At load of 70 000 lb. small crack was observed a little to right of wall. At 80 000 lb. tension crack noted under left face of wall. At 90 000 lb. a tension crack was noted near right face of wall and steel passed its yield point. Load continued to be taken and final rupture occurred suddenly by diagonal tension at load of 94 100 lb.

No. 1371. Footing 6 ft. 8 in. long. Small diagonal cracks noted near right and left faces of wall at 52 700 lb. Tension crack noted toward the middle of footing at load of 68 000 lb. Failure by tension at 69 300 lb., followed by diagonal tension.

No. 1372. Footing 6 ft. 8 in. long. Small inclined cracks noted near right and left faces of wall at load of 50 200 lb. At 67 000 lb. tension cracks appeared near center of footing. Failure by tension at 67 000 lb., followed by diagonal tension.

No. 1375. Footing 6 ft. 8 in. long, reinforcement, 5 ½-in. corrugated bars. At 60 000 lb. small inclined cracks noted a little to right of wall. At 69 000 lb. another small inclined crack noted near left face of wall. Failure occurred suddenly by diagonal tension at load of 75 500 lb.

No. 1376. Footing 6 ft. 8 in. long, similar to No. 1375. (See Fig. 14.) Diagonal cracks were noted at 40 000 lb. at both ends of footing near wall. At 60 000 lb. a diagonal crack was noted at right extending toward wall. At 67 000 lb. small tension crack near center of footing. Failure occurred by diagonal tension at load of 67 000 lb.

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No. 1631. Low percentage of reinforcement. At a load of 40 000 lb. a crack appeared 6 in. to the left of the wall and extended diagonally upward toward the wall. At this load a crack was also noted at 2½ in. inside the right edge of wall extending vertically 6 in. As the load was increased the cracks grew and at 55 000 lb. the yield point of the steel had been passed. With continued application of the load, the test piece deflected considerably at the ends, allowing much of the load to be taken directly under the wall. Failure by tension.

No. 1632. At 55 000 lb. vertical cracks were noted under the edges of the wall. Tension failure at 60 000 lb.

No. 1633. At 43 000 lb. a vertical crack was noted 2½ in. to left of the wall. Failure occurred slowly by tension and bond between steel and concrete at 78 000 lb.

No. 1634. (See Fig. 14.) At 40 000 lb. cracks were noted 2½ in. from each edge of wall extending almost vertically. Failure occurred slowly by slipping of bars (bond) at load of 73 000 lb.

No. 1635. In this test no cracks were noted before failure, which occurred suddenly at a load of 55 000 lb. by bond.

No. 1636. Diagonal cracks appeared at a load of 80 000 lb. 7 in. on each side of and extending toward wall. The footing failed slowly at load of 89 500 lb., probably by bond, although crack was inclined. Tension failure in steel probably imminent.

No. 1641. At 60 000 lb. a diagonal crack appeared 7 in. to the left of the wall. Failure occurred suddenly at 92 000 lb. by bond, the suddenness of failure possibly being occasioned by diagonal tension weakness. An examination of the end of the specimen showed that the bars had slipped.

No. 1642. At 60 000 lb. a diagonal crack was noted 5 in. to the left of the wall. Failure occurred slowly by bond at 80 000 lb.

No. 1645. At 60 000 lb. a crack was noted 2 in. to the left of the wall and inclined slightly toward it. The footing failed suddenly at 80 000 lb., the end of the footing being thrown off the machine. Failure probably by bond.

No. 1646. At 60 000 lb. a crack appeared on each side of the wall 8 in. from it. Failure occurred at 100 000 lb. by bond.

No. 1651. Failure occurred violently at 80 000 lb. by diagonal tension. The test piece became tipped so that there was more load on the left end than on the right end.

No. 1652. At 60 000 lb. a diagonal crack was noted 6 in. to right of wall. Failure occurred violently at 116 000 lb. by diagonal tension, the right end of the specimen being thrown off the machine.

No. 1655. At 60 000 lb. a diagonal crack was noted 8 in. to the left of the wall. Failure occurred suddenly by diagonal tension at 72 000 lb., the left end of the specimen being thrown off the machine. The specimen tipped so that there was more load on the left end.

No. 1656. At 60 000 lb. a diagonal crack was noted 6 in. to the left of the wall. The specimen failed violently at 108 000 lb. by diagonal tension.

No. 1661. At 80 000 lb. a diagonal crack appeared on each side of the wall and 8 in. from it. At 141 000 lb. failure occurred violently by diagonal tension and stripping of bars.

No. 1662. At 80 000 lb. a diagonal crack was noted 8½ in. to the left of the wall. At 114 000 lb. failure occurred suddenly by diagonal tension. The footing tipped slightly so there was more load on the left end.

No. 1665. Footing 7 ft. long. (See Fig. 14.) At 60 000 lb. cracks were noted on each side of the wall 15 in. from it. Failure occurred suddenly at 84 500 lb. by diagonal tension. There was more load on left end.

No. 1666. Footing 7 ft. long. Two cracks appeared at 55 000 lb., one 8 in. to the left of the wall and the other 2 in. to the right. The specimen failed violently at 94 000 lb. by diagonal tension along crack which ran from point on bottom 10 in. from wall, the concrete below reinforcement stripping off from bars.

No. 1671. One bar straight, 4 bent up at two points. At 70 000 lb. a vertical crack was noted under left edge of wall. Beyond 80 000 lb. the crack had opened wide and the load was very unevenly distributed because of the large deflection at the ends. At 100 000 lb. the crack was ¼ in. wide. 80 000 lb. considered as the critical load. Failure probably by tension followed by bond.

No. 1672. One bar straight, 4 bent up at two points. At 60 000 lb. a crack was noted 2 in. to the right of the wall and inclined slightly toward it. Vertical cracks also appeared under the left edge of the wall and 2 in. from it. Beyond 80 000 lb. these cracks opened up and much of the load was transmitted from wall direct to springs below. At 100-000 lb. the cracks extended to the top of the footing and were ¼ in. wide at the bottom. Failure probably by tension and bond.

No. 1673. One bar straight, 4 bent up at two points. This footing was first loaded up to 80 000 lb. on 3-in. x 12-in. springs placed 3 in. center to center. At this load the springs bent and the specimen was thrown out of position. The first cracks had appeared at 60 000 lb. one on each side of the wall and about 1 in. from it. The springs were reset and a load of 98 000 lb. applied when the footing again swung out of position. The specimen was then placed on the 2¾ x 7-in. springs, as in the other tests, and loaded. The notes are indefinite, and the critical load is not known. After being taken from the machine the cracks were ¼ in wide at the bottom. It seems that failure probably occurred at a load greater than 98 000 lb. by tension or bond.

No. 1674. One bar straight, four bent up at two points. At 60 000 lb. the first crack was noted just under the right edge of the wall and nearly vertical. The diagonal tension crack 8 in. to the left of the wall opened slowly. The footing failed at 99 500 lb., probably by a combination of bond and diagonal tension. Final failure was sudden. Upon examination it was found that the turned-up bars had slipped ½ in.

No. 1675. Two bars straight, four bent up at two points. At 80 000 lb. two cracks appeared on right 2 in. and 12 in. from wall. Failure was slow and occurred at 135 000 lb., slipping of the bars permitting diagonal tension crack to be formed. Bars found to have slipped at right end.

No. 1676. Two bars straight, four bent up at two points. At 75 000 lb. the first crack was noted 4 in. to the left of the wall. Failure occurred slowly at 99 000 lb., probably by bond, the crack beginning 7 in. to the left of the wall. The bars were found to have slipped slightly.

No. 1681. Six corrugated bars, two straight, four bent up. (See Fig. 14.) First cracks noted at 80 000 lb. Load was applied up to 125 000 lb., six vertical cracks opening under the wall and one a little to the left of it. As the springs were practically closed the load was removed. The specimen was then tested as a simple beam on supports 4 ft. 4 in. apart.

By this method of loading the footing failed at 60 000 lb., evidently by tension, although in this test one of the inner bars slipped.

No. 1682. Six corrugated bars, two straight, four bent up. At 80 000 lb. the first cracks appeared, one 3 in. inside left edge of wall and one 4 in. outside, extending diagonally toward edge of wall. Beyond 140 000 lb. the action of the springs was untrustworthy as the middle springs had entirely closed. No failure.

No. 1685. Four round stirrups near wall at each end, one stirrup exposed to view on face of footing. At 60 000 lb. a diagonal crack was noted 3½ in. to the right of the wall. Failure occurred slowly at 61 500 lb. by bond.

No. 1686. Four round stirrups near wall at each end. At 60 000 lb. a vertical crack appeared 1 in. inside right edge of wall. Footing failed suddenly at 82 000 lb. by bond. Examination of bars showed that they had slipped ⅜ in.

No. 1687. Four corrugated stirrups near wall at each end. At 40 000 lb. a vertical crack appeared 2 in. to the left of wall. Failure occurred slowly at 80 000 lb. by slip of bars. The inner stirrup also slipped.

No. 1688. Four corrugated stirrups near wall at each end. (See Fig. 14.) At 60 000 lb. a crack appeared 1½ in. to the right of the wall, extending 1 in. inside of wall near top at failure. Failure occurred slowly by bond at 108 000 lb.

No. 1692. Four round stirrups near wall at each end. (See Fig. 14 and 17.) At 61 000 lb. a vertical crack was noted 1 in. to left of wall. The footing failed slowly at 120 000 lb. by diagonal tension.

No. 1693. Four corrugated stirrups near wall at each end. At 80 000 lb. a crack was noted under the left edge of wall, extending inward slightly but almost vertically. Failure occurred suddenly at 106 600 lb. by diagonal tension between two stirrups.

No. 1694. Four corrugated stirrups near wall at each end. Two cracks were noted at 60 000 lb. 2 in. and 5 in. to left of wall and joining 5½ in. above base. Failure occurred suddenly at 113 000 lb. at a crack 9 in. to the left of the wall by diagonal tension. The bars and inner stirrups were found to have slipped.

Series of 1911.

No. 1712. At a load of 78 500 lb. small cracks appeared on both sides of footing near right face of wall extending toward edge of wall. The critical load was 85 000 lb. Failure occurred by tension or bond.

No. 1713. At a load of 40 000 lb. a vertical crack appeared under right face of wall and at a load of 47 000 lb. a similar crack appeared under left face of wall. A small tension crack was noted near center of footing at 65 000 lb. At a load of 71 700 lb. a diagonal crack appeared 16 in. from right end and at a load of 119 800 lb. another diagonal crack appeared 16 in. from left end. Critical load was 85 000 lb., the steel passing its yield point. The load was increased to 125 300 lb. when complete failure occurred suddenly along the diagonal crack which appeared at 119 800 lb. On account of the greater load taken by the springs near the middle after the yield point of the steel was passed, the increase in load does not represent the increase in bending moment.

No. 1714. Deformation in steel measured. At a load of 100 000 lb. cracks under the left and right faces of wall had extended vertically about 10 in. At the critical load of 100 000 lb. the measured stress in the steel was 42 000 lb. per sq. in. Failure was by tension, though under continued loading the bars finally slipped considerably.

No. 1716. 1-1-2 concrete. At a load of 22 800 lb. a crack appeared directly under the right face of wall. At a load of 86 400 lb. a number of fine cracks, which extended diagonally upward to the left, appeared just to the left. The critical load was 90 000 lb. At a load of 106 300 lb. a vertical crack was noted near the center of the footing. The deflection of the springs became very unequal, and at a load of 139 000 lb. the springs at the center of the footing had completely closed.

No. 1717. (See Fig. 16.) 1-1-2 concrete. At a load of 35 600 lb. cracks appeared under right and left faces of wall and 2 in. to right of center of the footing. At a load of 60 100 lb. a diagonal crack was noted 2½ in. to right of wall. Critical load 85 000 lb. At a load of 97 900 lb. the concrete appeared to be failing in compression at the upper surface of footing near wall and the cracks opened appreciably. At a load of 131 400 lb. the springs at the center had completely closed.

No. 1718. 1-1-2 concrete. Deformation in steel measured. At the critical load of 100 000 lb. the measured tension in the steel was 42 000 lb. per sq. in. At a load of 122 800 lb. the steel deformations had exceeded the range of the extensometer. Failure occurred by tension in steel.

No. 1721. Wall made one day later than footing. At a load of 48 100 lb. cracks appeared on both sides of footing about 2 in. inside right face of wall. Critical load was 60 000 lb. At a load of 63 800 lb. cracks appeared 18½ in. from left end and 20 in. from right end. At a load of 72 000 lb. the cracks first noted opened perceptibly.

No. 1722. Wall made one day later than footing. At a load of 34 700 lb. cracks appeared 2 in. and 3½ in. to right of left face of wall. The critical load was 60 000 lb. As load was increased to 70 200 lb. other cracks were noted under both faces of wall. The load then fell off and cracks opened. Failure occurred by tension in steel.

No. 1723. Deformation in steel measured. Wall made one day later than footing. At the critical load of 60 000 lb. the measured stress was above 40 000 lb. per sq. in. Failure occurred slowly by tension in steel.

No. 1724. Wall made one day later than footing. Building paper placed between wall and footing. At a load of 32 300 lb. a vertical crack was noted 4 in. to the left of center of footing. At a load of 40 000 lb. a vertical crack appeared under right face of wall. At a load of 48 000 lb. a crack appeared 21 in. from left end of footing and extended toward the wall. The critical load was 55 000 lb., failure occurring by tension in the steel. At a load of 62 000 lb. the crack under right face of wall opened appreciably.

No. 1725. (See Fig. 16.) Wall made one day later than footing. Building paper placed between wall and footing. At a load of 26 500 lb. the first crack appeared 3 in. inside left face of wall. At a load of 33 700 lb. a vertical crack appeared directly under the right face of wall. At a load of 40 300 lb. a crack was noted 1 in. to left of left face of the wall. Critical load 55 000 lb. At a load of 62 300 lb. the load was being taken slowly and cracks were opening. Failure occurred by tension in the steel.

No. 1726. Deformation in steel measured. At the critical load of 60 000 lb. the measured tension in the steel was 42 000 lb. per sq. in.

No. 1727. At a load of 32 000 lb. cracks appeared under right and left faces of pier. At a load of 40 500 lb. a small crack appeared near center of footing extending vertically 5 in. At a load of 49 500 lb. cracks appeared 20 in. from right end and 17½ in. from left end. The critical load was 55 000 lb. The cracks opened considerably at a load of 62 300 lb. Failure occurred by tension in the steel.

No. 1728. (See Fig. 16.) First crack appeared at a load of 24 000 lb. 4 in. to left of left face of wall and extended vertically. At a load of 30 600 lb. a crack appeared 2 in. to left of left face of wall. At a load of 36 900 lb. a crack appeared 2½ in. to right of wall. The critical load was 55 000 lb.

No. 1729. Deformation in steel measured. At a load of 40 000 lb. a small crack was noted near center of footing. Measurement of the deformation in steel at the critical load of 60 000 lb. indicated that the yield point had been passed.

No. 1731. Reinforced with six ½-in. corrugated square bars bent up in a manner similar to that shown in Fig. 14 for No. 1681. At a load of 35 800 lb. the first crack was noted 18½ in. from the left end of the footing extending upward toward the wall. At a load of 61 600 lb. a crack appeared under the left face of the wall. The critical load was 145 000 lb. Capacity of springs was reached at a load of 155 600 lb. Failure occurred by tension in the steel.

No. 1733. Reinforcement the same as No. 1731. Deformation in steel measured. At a critical load of 160 000 lb. the measured tension in steel was 58 000 lb. per sq. in. At 110 000 lb. load the wall crushed.

No. 1741. (See Fig. 14 and 22.) Deflection of the spings was not measured. Extensometer dials were attached to the reinforcing rods to measure slip. The curves in which slip is plotted against load (Fig. 23) show that at the face of the wall movement of steel relative to concrete began between loads of 6 000 lb. and 20 000 lb. At a load of 35 000 lb. the slip at this position was .001 in. The critical load was 50 000 lb. and at this load a slip of .0001 in. was observed at the end of one bar. The slip at this point progressed with the load, reaching .0007 in.

at a load of 65 000 lb. At a load of 68 900 lb. complete failure occurred by sudden slipping of the bars at the end where slip had previously been observed. If the curve above referred to, of slip at the end, is produced to the load of 68 900 lb. it indicates a slip of about .001 in., which pull-out tests indicate as the critical amount of slip. The last measurement taken (that at 65 000 lb. load) showed that the slip at the ends of all the other bars was very small, not over .0002 in. Failure was primarily by tension in the steel. At a load of 60 000 lb. crushing of concrete was observed at the intersection of the wall and the footing.

No. 1742. (See Fig. 15.) Deformation in steel was measured on one side of footing under face of wall. First crack appeared at a load of 32 600 lb. Critical load 53 000 lb. Measured stress at this load, over the gage length used, was 37 000 lb. per sq. in. Failure by tension.

No. 1743. The measured deformation of the bars shows tension failure. Critical load was 52 000 lb.

No. 1744. Extensometer dials attached to rods to measure the slip. Slip under face of wall began at a load of about 25 000 lb. At the critical load of 70 000 lb. bars at right end had slipped about .004 in. and about 3/8 in. at the maximum load of 84 000 lb. Bond failure.

No. 1745. (See Fig. 17.) At a load of 36 000 lb. first crack was noted 1 in. inside the right face of wall and extended vertically 5 in. At a load of 44 100 lb. cracks appeared at the center of the footing, 19½ in. from the left end and 20 in. from the right end. Critical load 70 000 lb. At a load of 79 600 lb. the cracks opened appreciably and at a load of 98 200 lb. the middle springs had closed. Bond failure.

No. 1746. Deformation in steel measured. At the critical load of 75 000 lb. the measured steel stress, over the gage length used, was 40 000 lb. per sq. in. Failure occurred by tension in steel.

No. 1747. Reinforcement two ½-in. square corrugated bars. At a load of 19 500 lb. first crack was noted 25 in. from right end. At a load of 25 000 lb. cracks appeared under left face of wall. At a load of 55 300 lb. the cracks were widening but not extending very much. Critical load 60 000 lb. At a load of 82 000 lb. the rods had slipped at the left end. Tension failure.

No. 1748. Reinforcement same as 1747. First crack was noted at a load of 21 800 lb. under right face of wall. As the load increased cracks appeared under left face of wall and about 8 in. to the right of the wall. Critical load 50 000 lb. Tension failure.

No. 1749. Reinforcement same as 1747. Critical load 60 000 lb. Tension failure.

No. 1751. (See Fig. 14 and 18.) At a load of 32 600 lb. a vertical crack was noted at the center of the footing. At a load of 40 400 lb. vertical cracks appeared under the right and left faces of the wall. Critical load 50 000 lb. At a load of 57 800 lb. a crack appeared 1½ in. to left of wall, and at this load the cracks opened considerably. Concrete split off bottom of footing at the hooked end of the bars. Tension failure.

No. 1752. (See Fig. 18.) Reinforcement same as in 1751. At a load of 25 200 lb. cracks appeared under the left face of the wall and 9 in. to right of the center of footing. Critical load 46 000 lb. At 50 800 lb. load the cracks opened perceptibly. Bars did not slip at the ends. Tension failure.

No. 1753. Reinforcement same as in 1751. Measured stress in steel at critical load of 58 000 lb. was 42 000 lb. per sq. in. Tension failure.

No. 1754. Reinforcement two ⅝-in. plain round bars curved up at ends to within 2 in. of top and back 10 in. First crack appeared at center of footing at a load of 17 500 lb. At a load of 24 300 lb. cracks appeared 3 in. to left of wall and under the right face of wall. Critical load 40 000 lb. At 40 200 lb. the latter crack opened appreciably and after 47 500 lb. the load was taken on much slower than before. Tension failure.

No. 1755. (See Fig. 14.) Reinforcement same as in No. 1754. Deformation in steel measured. At a load of 29 800 lb. a crack appeared under right face of wall. At a load of 39 800 lb. a crack appeared under left face of wall. At a load of 47 400 lb. the cracks had not opened very much. Critical load 47 500 lb. Measured steel stress 40 000 lb. At a load of 56 300 lb. the cracks opened rapidly. Tension failure.

No. 1756. Reinforcement same as in No. 1754. Critical load was 60 000 lb. Tension failure.

No. 1757. Reinforcement four ⅝-in. round rods curved up at ends to within 2 in. of top and back 10 in. First crack appeared at a load of 42 000 lb. at center of footing. At a load of 50 000 lb. a crack appeared 1½ in. to the left of the wall. At a load of 75 300 lb. a crack was noted 4 in. to right of wall. Critical load 92 000 lb. At 105 000 lb. the cracks opened appreciably and load was taken more slowly. Springs at the center closed at a load of 112 900 lb. Tension failure.

No. 1758. (See Fig. 17.) Reinforcement same as in 1757. First crack appeared at a load of 32 500 lb. under the right face of wall. As the load increased several cracks appeared under the wall and at a load of 82 200 lb. the cracks had not opened much. Critical load 90 000 lb. Load was increased up to 121 000 lb. when the footing failed suddenly by diagonal tension. Initial failure was by tension in steel.

No. 1759. Reinforcement same as in 1757. Deformation in steel measured. Critical load 90 000 lb. Failure occurred by tension followed by diagonal tension.

No. 1761. Reinforcement four ½-in. square corrugated bars curved up at ends to within 2 in. of top and back 10 in. First crack appeared under the right face of wall at a load of 27 700 lb. At a load of 35 000 lb. a crack appeared 2 in. to right of center and at a load of 44 100 lb. a crack appeared under the left face of the wall. At a load of 61 000 lb. a diagonal crack appeared 16 in. from the north end extending upward about 5 in. At a load of 69 500 lb. a fine diagonal crack appeared 7½ in. to left of wall. Critical load was 98 000 lb. At a load of 107 200 lb.

crack near center opened appreciably and at a load of 115 200 lb. failure occurred suddenly by diagonal tension. Initial failure due to tension.

No. 1762. Reinforcement same as in 1761. (See Fig. 14.) First crack appeared at a load of 40 200 lb. at center. At a load of 67 400 lb. a diagonal crack appeared 16 in. from left end of footing and extended toward wall. Critical load was 100 000 lb. At 112 000 lb. the load dropped off but cracks did not open perceptibly. At a load of 123-000 lb. crushing of concrete was observed. At a load of 132 700 lb. failure occurred suddenly by diagonal tension. Initial failure due to tension.

No. 1763. Reinforcement same as in 1761. Deformation of steel measured. Critical load 125 000 lb. Measured stress 65 000 lb. per sq. in. Failure occurred by tension in steel. At a load of 147 000 lb. the concrete split off above the bars.

18. *Reinforced Concrete Wall Footings: Bars Straight.*—In the series of 1908 (see Table 9) ten of the reinforced concrete footings having the longitudinal reinforcement straight throughout the length of the footing gave diagonal tension failures. One gave a bond failure and four failed by tension. In the series of 1909 (see Table 10), there were nine diagonal tension failures, seven bond failures, and two tension failures. In the series of 1911 (see Table 11), there were two bond failures and twenty-two tension failures. In many cases diagonal tension failures occurred at loads which gave high tensile stresses in the reinforcement. With a high percentage of reinforcement, diagonal tension failures were frequent.

19. *Reinforced Concrete Wall Footings: Bars Bent Up.*—The four footings with longitudinal reinforcement bent up (as shown in Fig. 14), of the series of 1908, No. 1325, No. 1326, No. 1321, and No. 1322, failed by tension in the reinforcement at calculated stresses generally somewhat above the yield point of the steel and at very high values of the bond and shearing stresses. There is some uncertainty in the manner of failure in some of the series of 1909, but it seems that all of the failures were by bond, although in No. 1671 and No. 1672 the calculated stress in the steel was above the yield point of the material and the cracks opened up somewhat and although in several cases the failures were complicated by diagonal cracks which ordinarily might be considered to be diagonal tension cracks, No. 1681 and No. 1682, reinforced with deformed bars, did not fail under the highest load applied, and No. 1681 when tested as a simple beam on supports 4 ft. 4 in. apart failed by tension and one of the bars bent up next to the wall was found to have slipped. In the series of 1911, footings No. 1731 and No. 1733 reinforced with deformed bars (two bars straight, two

curved upward, and two bent up, in a manner similar to that shown in Fig. 14 for No. 1681), gave high resistance to diagonal tension and developed the elastic strength of the steel. The other footings of this series having the bars bent up did not have the bending at such a point as to have any material effect upon resistance to diagonal tension.

The effect of bending up bars (anchorage of bars is not referred to here) was to increase the resistance to diagonal tension, higher values of vertical shearing stresses being obtained than in footings of similar reinforcement and proportions in which the bars were laid straight and failure was by diagonal tension. An increase in the tendency to failure by slip of bars was also apparent. The amount of bond stress developed will be discussed under Art. 26, "Bond."

20. *Reinforced Concrete Wall Footings with Stirrups.*—Of the series of 1909, all the footings having stirrups failed by bond or diagonal tension. Not considering the two footings in which the stirrups were exposed to view, the footings having deformed bars gave higher loads and developed higher bond stress, vertical shearing stress, and tensile stress than those having plain bars. The failures of the footings reinforced with plain rods were definitely bond failures, and the footings reinforced with deformed bars gave diagonal tension failures, though in No. 1694 both longitudinal bars and stirrups were found to have slipped. The footings having deformed bar stirrups gave somewhat higher loads than those having stirrups made of plain rods. As the stirrups were made without end anchorage it was expected that slip might occur, the purpose here being (as in former tests) to determine at what loads slipping occurred. It seems to be apparent from these tests that there was concentration of bond stress at the stirrup points and that bond failures are more likely to occur when this web reinforcement is used.

21. *Stepped and Sloped Wall Footings.*—(See Table 9, page 55). All the failures in the stepped and sloped footings were by diagonal tension. Fig. 15 is a view of a stepped footing after failure. The loads at failure were generally less than for the same amount and kind of reinforcement in the other forms of footings, but the calculated vertical shearing stresses in the section a distance d from the face of the wall were as large as in the other footings having similar reinforcement. It should be noted that the depth of the footing at the section considered was used in these calculations.

22. *Wall Footings Reinforced with I-beams.*—(See Table 9, page 55, and Fig. 19). The footings reinforced with two 5-in. x 9.75 lb.

TABLE 9.

Tests of Reinforced Concrete Wall Footings. Series of 1908.

All footings of 1–3–6 concrete, hand-mixed. Depth to center of steel 10 in. Length of footing 5 ft. Stresses are given in lb. per sq. in. Chicago AA Portland cement.

Footing No.	Age days	Longitudinal Reinforcement			Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance d	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1313	61	6 ½-in. round	0.98	Bars straight	95 000	45 500	217	475	Diagonal tension
1315	60	do.	0.98	do.	89 500	42 900	205	447	do.
1362	58	do.	0.98	do.	90 000	43 200	206	449	Tension followed by diag. tension
1371*	61	do.	0.98	do.	69 300	50 000	205	367	do.
1372*	58	do.	0.98	do.	67 000	48 400	197	355	do.
1375*	61	5 ½-in. cor. square	1.04	do.	75 500	60 500	222	378	Diagonal tension
1376*	58	do.	1.04	do.	67 000	53 600	197	335	do.
1311	61	do.	1.04	do.	129 000	58 200	295	607	do.
1312	61	do.	1.04	do.	96 000	43 300	219	451	do.
1314	61	4 ¾-in. round	1.47	do.	111 300	36 700	263	575	Bond
1321	62	5 ½-in. cor. square	1.04	Bars bent up to different heights	128 000	57 800	292	602	Tension
1322	57	do.	1.04		130 000	58 700	298	611	"
1325	63	6 ½-in. round	0.98	do.	100 000	48 000	229	500	"
1326	57	do.	0.98	Bars bent up to same height	105 000	50 400	240	525	"
1341†	59	5 ½-in. cor. square	1.04	Bars straight	80 000	36 100	250	376	Diagonal tension
1342†	61	do.	1.04	do.	80 300	36 200	251	378	do.
1351‡	61	do.	1.04	do.	107 300	48 500	362	505	do.
1352‡	61	do.	1.04	do.	86 800	39 200	292	408	do.
1316	60	2 5-in. I-Beams	....	See Fig. 19	130 000	....	...	...	No failure at this load.
1317	61	do.	....	do.	134 000	.....	...	...	Wall crushed
1318	60	do.	....	do.	120 000	.....	...	...	Tension
1319	58	do.	....	do.	140 600	.....	...	...	"

* Length 6 ft. 8 in. † Sloped footing. ‡ Stepped footing.

TABLE 10.

Tests of Reinforced Concrete Wall Footings. Series of 1909.

All footings 5 ft. long and of 1–2½–5 hand-mixed concrete unless otherwise noted. Chicago AA Portland cement. Depth to center of steel 10 in. Stresses are given in lb. per sq. in.

Footing No.	Age days	Longitudinal Reinforcement			Load at Failure pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1631	63	6 ⅜-in. round	0.55	Bars straight	55 000	46 800	122	355	Tension in steel
1632	64	do.	0.55	do.	60 000	49 900	133	388	do.
1633	63	5 ½-in. round	0.82	do.	78 000	44 400	176	467	Tension and bond
1634	63	do.	0.82	do.	73 000	41 600	165	436	Bond
1635	57	6 ½-in. round	0.98	do.	55 000	26 400	126	274	"
1636	64	do.	0.98	do.	89 500	41 800	204	446	Probably bond
1641	70	5 ⅝-in. round	1.28	do.	92 000	34 200	213	445	Bond and possibly diagonal tension
1642	64	do.	1.28	do.	80 000	29 800	185	387	Bond
1645	59	6 ⅝-in. round	1.53	do.	80 000	25 300	189	318	Probably bond
1646	76	do.	1.53	do.	100 000	31 700	236	412	Bond
1651	65	4 ½-in. cor. square	0.84	do.	80 000	44 700	181	465	Diagonal tension
1652	73	do.	0.84	do.	116 000	64 800	262	675	do.
1655	59	5 ½-in. cor. square	1.04	do.	72 000	31 800	165	339	do.
1656	67	do.	1.04	do.	108 000	47 800	247	509	do.
1661	70	6 ½-in. cor. square	1.25	do.	141 000	53 600	327	560	do.
1662	69	do.	1.25	do.	114 000	43 400	264	453	do.
1665*	62	do.	1.25	do.	84 500	51 800	260	360	do.
1666*	58	6 9/16-in. cor. round	1.24	do.	94 000	57 900	289	453	do.
1671	76	5 ½-in. round	0.82	1 bar straight,	80 000	45 500	181	474	Probably tension
1672	66	do.	0.82	4 bars bent up at different points	80 000	45 500	181	474	do.

*Length 7 ft.

TABLE 10 (CONTINUED).

Footing No.	Age days	Longitudinal Reinforcement			Load at Failure pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
		Description	Per cent	Disposition					
1673	60	5 ⅝-in. round	1.28	1 bar straight, 4 bars bent up at different points	98 000‡	36 400	227	475	Uncertain
1674	60	do.	1.28		99 500	37 000	230	482	Bond and diag. tension
1675	69	6 ⅝-in. round	1.53	2 bars straight, 4 bars bent up at different points	135 000	42 600	318	555	Bond
1676	58	do.	1.53		99 000	31 200	234	400	"
1681	60	6 ½-in. cor. square	1.25	do.	125 000	47 600	289	496	Did not fail at this load; see notes of test.
1682	73	do.	1.25	do.	140 000	53 300	324	555	do.
1685§	61	5 ⅝-in. round	1.25	Bars straight	61 500	22 900	142	298	Bond
1686§	67	do.	1.25	do.	82 000	30 500	190	396	"
1687†	64	5 ⅝-in. round	1.28	Bars straight	80 000	29 800	185	387	Bond, inner stirrup also slipped
1688†	69	do.	1.28	do.	108 000	40 200	250	522	Bond
1692§	60	6 $\frac{9}{16}$-in. cor. round	1.24	do.	120 000	46 000	278	540	Diagonal tension
1693†	61	6 ½-in. cor. square	1.25	do.	106 600	40 800	247	423	do.
1694†	64	do.	1.25	do.	113 000	43 000	262	449	do.

§ ½-in. round **stirrups.** † **½-in.** sq. cor. stirrups. ‡ Critical load may be beyond 98 000 lb.

TABLE 11.

Tests of Reinforced Concrete Wall Footings. Series of 1911.

All footings of 1–2–4, hand-mixed concrete unless otherwise noted. Depth to center of steel 10 in. Length of footing 5 ft. Stresses given in lb. per sq. in.

Footing No.	Age days	Cement	Longitudinal Reinforcement			Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
			Description	Per cent	Disposition					
1712	73	U	6 ½-in. round	0.98	Bars straight	85 000	40 200	192	418	Tension or bond
1713	98	U	do.	0.98	do.	85 000	40 200	192	418	Tension
1714	339	L	do.	0.98	do.	100 000	47 200	226	493	Tension
1716*	72	U	6 ½-in. round	0.98	Bars straight	90 000	42 500	203	443	Tension
1717*	94	U	do.	0.98	do.	85 000	40 200	192	418	"
1718*	338	L	do.	0.98	do.	100 000	47 200	226	493	"
1721†	64	U	6 ⅜-in. round	0.55	Bars straight	60 000	48 800	131	382	Tension
1722†	99	U	do.	0.55	do.	60 000	48 800	131	382	"
1723†	347	L	do.	0.55	do.	60 000	48 800	131	382	"
1724‡	67	U	6 ⅜-in. round	0.55	Bars straight	55 000	44 800	120	350	Tension
1725‡	88	U	do.	0.55	do.	55 000	44 800	120	350	"
1726‡	345	L	do.	0.55	do.	60 000	48 800	131	382	"
1727	66	U	6 ⅜-in. round	0.55	Bars straight	55 000	44 800	120	350	Tension
1728	88	U	do.	0.55	do.	55 000	44 800	120	350	"
1729	345	L	do.	0.55	do.	60 000	48 800	131	382	"
1731	83	U	6 ½-in. cor. sq.	1.25	Two bars straight, two curved, and two bent up.	145 000	54 500	331	568	Tension
1733	359	L	do.	1.25		160 000	60 300	365	628	"
1741	61	U	2 ⅝-in. round	0.52	Bars straight	50 000	43 800	109	571	Tension followed by bond
1742	101	U	do.	0.52	do.	53 000	46 400	116	605	Tension
1743	346	L	do.	0.52	do.	52 000	45 500	114	595	"
1744	73	U	3 ⅝-in. round	0.77	Bars straight	70 000	41 800	156	544	Bond
1745	73	U	do.	0.77	do.	70 000	41 800	156	544	"
1746	358	L	do.	0.77	do.	75 000	44 800	167	584	Tension

*1–1–2 concrete. †Wall made one day after footing. ‡Wall made one day after footing, building paper placed between wall and footing.

TABLE 11 (CONTINUED).

Footing No.	Age days	Cement	Longitudinal Reinforcement			Critical Load pounds	Stress in Longitudinal Reinforcement	Nominal Vertical Shearing Stress at Distance *d*	Nominal Bond Stress	Manner of Failure
			Description	Per cent	Disposition					
1747	65	U	2 ½-in. cor. sq.	0.42	Bars straight	60 000	64 000	129	666	Tension
1748	73	U	do.	0.42	do.	50 000	53 300	108	555	"
1749	359	L	do.	0.42	do.	60 000	64 000	129	666	"
1751	77	U	2 ⅝-in. round	0.52	Reinforcement of one horizontal bar looped at one end and bent in at other	50 000	43 800	109	571	Tension
1752	69	U	do.	0.52		46 000	40 400	100	525	"
1753	336	L	do.	0.52		58 000	50 600	127	663	"
1754	67	U	2 ⅝-in. round	0.52	Curved up to within 2 in. of top and back 10 in.	40 000	35 000	88	457	Tension
1755	70	U	do.	0.52		47 500	41 700	104	542	"
1756	340	L	do.	0.52		60 000	52 300	131	685	"
1757	69	U	4 ⅝-in. round	1.04	Curved up to within 2 in. of top and back 10 in.	92 000	41 800	208	544	Tension
1758	76	U	do.	1.04		90 000	40 750	204	535	"
1759	354	L	do.	1.04		90 000	40 750	204	535	"
1761	74	U	4 ½-in. cor. sq.	0.85	Curved up to within 2 in. of top and back 10 in.	98 000	54 000	219	562	Tension
1762	76	U	do.	0.85		100 000	55 100	224	574	"
1763	353	L	do.	0.85		125 000	68 800	279	720	"

I-beams carried high loads. One did not fail under a load of 130 000 lb., and in another the stem crushed under a load of 134 000 lb. without sign of failure in the footing. In No. 1318 the failure was a tension failure in the lower flange of the I-beams and in No. 1319 the failure was coincidently by tension of flange of I-beam and by bond, the steel slipping or splitting from the concrete. The calculated tensile stress in the steel at a section at the face of the wall using the lower flange of the I-beam as the tension area of the steel and considering the combination to act as a reinforced concrete beam, was somewhat higher than the yield point of mild steel. The total vertical shear was very high. The loads carried by these footings were among the highest in the experimental wall footing tests. Of course, the amount of steel in the I-beams was much larger than in the footings reinforced with longitudinal rods. The load carried was about double what would be carried by counting I-beams alone to take the full bending moment at a section at the face of the wall, using 35 000 lb. per sq. in. as the value of the modulus of rupture of the I-beams.

23. *Effect of Pouring Wall Separately from Footing.*—In construction it is generally necessary from the standpoint of convenience to pour the wall after the footing has taken its set. To determine whether this method of construction has an effect upon the choice of section which should be taken as the critical section in design, in a number of cases the wall or stem of the footing was built 24 hours after the footing had been finished. In three footings, No. 1724, 1725, and 1726, a layer of building paper was placed over the footing and the wall was constructed upon this. Two pieces of wire 0.1 in. in diameter passed from footing to wall to resist breakage in handling. The conditions were such as to make the bond very slight. Three footings, No. 1727, 1728, and 1729, were constructed monolithically under otherwise similar conditions. There was no marked difference in the loads carried, the method of failure, or the phenomena of tests for footings constructed under these different conditions, all giving tension failures at the face of the wall. By calculation the horizontal shearing stress at the face of the wall may be shown to be less than the probable coefficient of friction. These tests corroborate the view that the critical section for design and calculation may properly be taken at the face of the wall.

24. *Tension Failures and Tensile Stresses.*—In the footings which gave tension failures, the vertical cracks which had formed enlarged, similarly to the action in ordinary beam tests, and at the critical load the cracks opened and a marked increase in the end deflection occurred.

Beyond the critical load the footing usually took an increase of load, the ends bending up so that the distribution was no longer uniform, and generally the ultimate failure was slow. The failure crack was at or near the section at the face of the wall. The stress in the steel for this section, as calculated by the method given on page 8, was in general somewhat larger than the yield point of the steel determined by tests on coupons taken from the same bars.

The calculated value of the tensile stress in the reinforcement for the beams tested at an age of nearly a year is in some cases considerably higher than the yield point of the steel and higher than the calculated stress in the companion test pieces which were tested at an age of about 60 days and which are given as failing by tension. Part of the difference may be due to the use of the same values of *jd* in the older beams as in the others. As has already been stated, it was in many cases difficult to determine the manner of failure, as the phenomena of tension failure and bond failure have points in common, and it is possible that some of the footings reported as failing by tension in reality failed by bond.

In a number of footings of the series of 1911 measurements of the deformation of the steel were made by inserting an extensometer of the Berry type in gage holes drilled in the reinforcing bars at the side of the footing. Fig. 20 gives the results of some of the measurements, the deformations being translated into equivalent stresses. Generally, one gage line (usually 6 in. in length) was placed so that it was bisected by the plane of the face of the wall (marked B in the figure). Gage line A (when used) was bisected by the center line of the footing, gage line C (when used) was adjacent to B and nearer the end of the footing. As the stress varies from point to point, the instrument reading will give the average stress over the gage length and not the maximum stress. Especially may the average stress over gage line B be less than the stress at the section at the face of the pier. The measured deformation at the center gage line was found to be generally somewhat greater than that on gage line B. Evidently little bond stress is developed over the thickness of the wall. The amount of the measured stress was generally lower than the calculated values given in the tables, but the difference perhaps was not more than that due to the effect of the smaller deformation toward the outer gage point and the greater stiffness found in the older footings.

To determine whether the reinforcing bars had been stressed beyond their yield point, several test pieces were afterward broken up and the

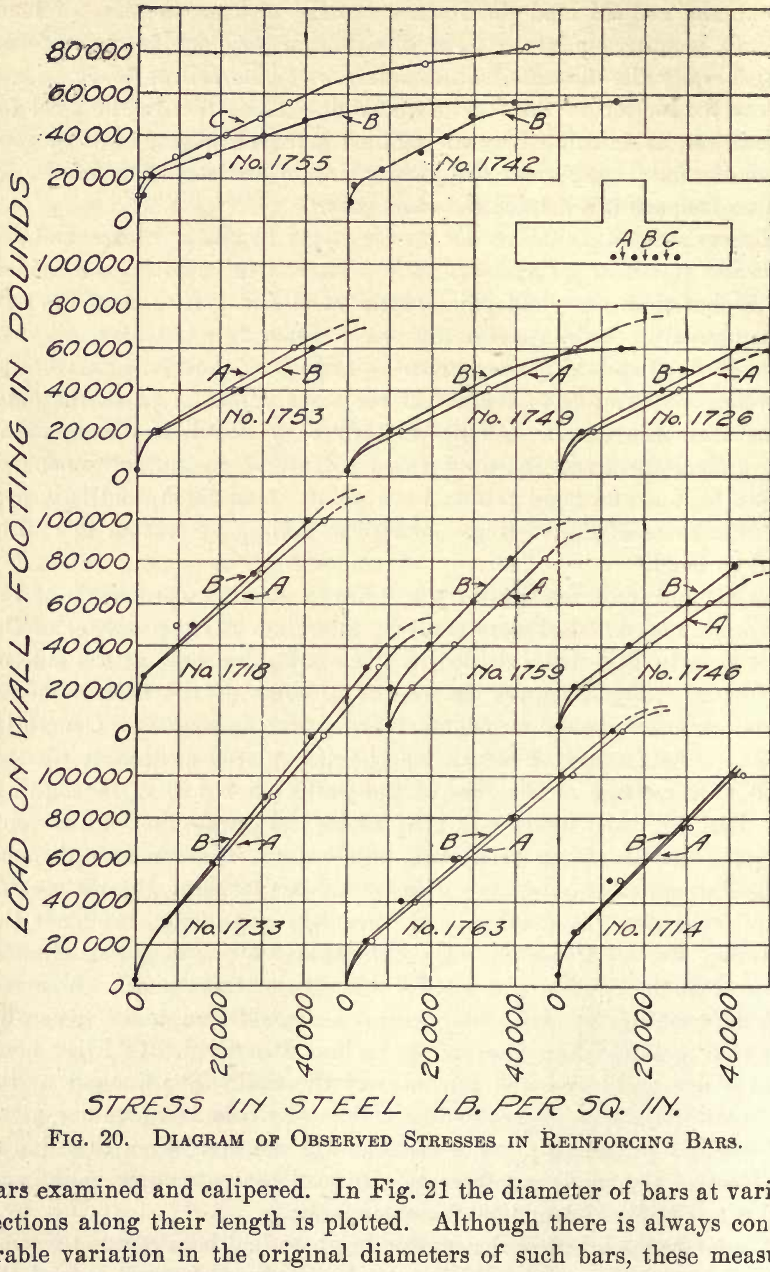


FIG. 20. DIAGRAM OF OBSERVED STRESSES IN REINFORCING BARS.

bars examined and calipered. In Fig. 21 the diameter of bars at various sections along their length is plotted. Although there is always considerable variation in the original diameters of such bars, these measurements were useful in helping to determine the method of failure.

The fact that the tension failures occurred at the section at the face of the wall, together with the approximate agreement of the calculated stress with the observed deformation and with the yield point of the material, justifies the use of the section at the face of the wall as the critical section in calculations of bending moment and of tensile stress in the reinforcing bars. This is apparent with footings of different richness of concrete, different percentages of reinforcement, and different grades of steel.

25. *Vertical Shearing Stresses and Diagonal Tension Failures.*—As was noted under Article 4, "General Theory," the diagonal tension stresses developed in reinforced concrete beams may be expected to be roughly proportional to the vertical shearing stresses, though the diagonal tension may vary from one to two times the vertical shearing stress. As was stated on page 9, the value of the vertical shearing stress has come to be used as a convenient means of measuring the resistance to diagonal tension, although, of course, it is not the numerical equivalent of the stress. There is, however, some question as to what section should be taken as the critical section in short cantilever

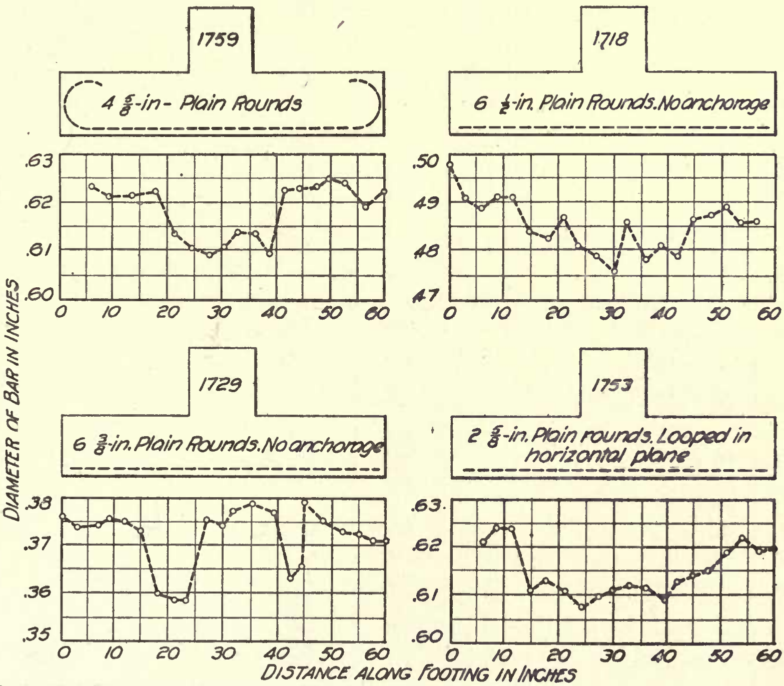


Fig. 21. Diagram Showing Diameter along Bars after Test of Footing.

TABLE 12.

Values of Vertical Shearing Stress.

Footing No.	Reinforcement		Nominal Vertical Shearing Stress lb. per sq. in.		Manner of Failure
	Disposition	Per cent	At face of pier	Distance d from face of pier	
1311	Cor. bars straight	1.04	506	295	Diagonal tension
1312	do.	1.04	376	219	do.
1313	Round bars straight	0.98	372	217	do.
1315	do.	0.98	351	205	do.
1371*	do.	0.98	289	205	Tension followed by diagonal tension.
1372*	do.	0.98	279	197	do.
1375*	Cor. bars straight	1.04	314	222	Diagonal tension
1376*	do.	1.04	279	197	do.
1651	do.	0.84	310	181	do.
1652	do.	0.84	450	262	do.
1655	do.	1.04	282	165	do.
1656	do.	1.04	424	247	do.
1661	do.	1.25	560	327	do.
1662	do.	1.25	452	264	do.
1665§	do.	1.25	359	260	do.
1666§	do.	1.25	400	289	do.
1314	Round bars straight	1.47	450	263	Bond
1634	do.	0.82	283	165	"
1635	do.	0.98	216	126	"
1636	do.	0.98	351	204	Probably bond
1641	do.	1.28	365	213	Bond and possibly diagonal tension.
1642	do.	1.28	317	185	Bond
1645	do.	1.53	323	189	Probably bond
1646	do.	1.53	404	236	Probably diagonal tension and bond
1341	Cor. bars straight	1.04	314	244	(Sloped footing) Diag. tens.
1342	do.	1.04	315	245	do.
1351	do.	1.04	421	351	(Stepped footing) Diag. tens.
1352	do.	1.04	340	284	do.
1673	Round bars, 1 straight, 4 bent up at different points	1.28	388	227	Uncertain
1674	do.	1.28	394	230	Bond and diagonal tension
1675	Round bars, 2 straight, 4 bent up at different points	1.53	545	318	Bond
1676	do.	1.53	400	234	"
1685	Round bars straight with plain round stirrups	1.25	244	142	"
1686		1.25	325	190	"
1687	Round bars straight with cor. stirrups	1.28	317	185	Bond
1688		1.28	429	250	"
1692	Cor. bars straight with plain round stirrups	1.24	476	278	Diagonal tension
1693	Cor. bars straight with cor. stirrups	1.25	423	247	do.
1694		1.25	449	262	do.
1712	Round bars straight	0.98	329	192	Tension and bond
1713	do.	0.98	329	192	Tension
1714	do.	0.98	387	226	Tension
1716	do.	0.98	348	203	Tension
1717	do.	0.98	329	192	"
1718	do.	0.98	387	226	"

*Length 6 ft. 8 in. §Length 7 ft.

TABLE 12 (CONTINUED).

Footing No.	Reinforcement		Nominal Vertical Shearing Stress lb. per sq. in.		Manner of Failure
	Disposition	Per cent	At face of pier	Distance d from face of pier	
1731	Cor. bars, 4 bent up at different points	1.25	567	331	Tension
1733		1.25	628	365	"
1757	Round bars curved up at ends	1.04	356	208	Tension
1758		1.04	349	204	Tension
1759	do.	1.04	349	204	Tension
1761	Cor. bars curved up at ends	0.85	374	219	Tension
1762		0.85	382	224	"
1763	do.	0.85	480	279	Tension

beams supported and loaded as were these footings. It has been stated on page 10 that the application of the load on the wall (Fig. 1, page 8) and the uniform support of the footing along its bed may be expected to cause a different distribution of shear throughout the vertical section at the face of the wall than is usually assumed in normal beam action, and it seems probable that the vertical shear is more largely taken by the compression area and that relatively less of it is borne in this section below the neutral axis. As a result, the diagonal tension may be expected to be less at this section than would be the case with normal beam action. Tests of beams show that diagonal tension failures start at the reinforcing bars some distance from the support, even if the total vertical shear is much greater at the support. It may then be expected that the critical section for diagonal tension will be some distance from the face of the wall. In the tests of wall footings the diagonal crack is generally formed at the level of the reinforcing bars at a point distant from the face of the wall about equal to the vertical distance d from surface of footing to center of reinforcing bar. Without knowing exactly what point to select we may use a section through this point tentatively as the critical section (that is, a vertical section at a distance from the face of the wall equal to the depth of the footing down to the center of the steel) and compare the vertical shearing stresses obtained in footings having a variety of proportions of depth to length.

In Table 12 are given calculated values of the vertical shear for a section at the face of the wall and for a section distant d from it for those footings in which diagonal tension failures were found, and also for others in which high vertical shearing stresses were developed. It will be seen that the values of the vertical shear at the new section are much more consistent among themselves in the diagonal tension failures than is the case with the section next to the face of the wall. The values of the shearing stress for this section are generally greater in the footings which failed by diagonal tension than the values found for other forms of failure. The fact that in the tests diagonal cracks generally formed first at a point somewhat near this section also favors its use. It would seem to be safe practice, where the vertical shearing stress is to be used as the measure of the resistance of the footing to diagonal tension failure, to consider the section distant d from the face of the wall as the critical section.

It should be noted that the values of the vertical shearing stress at this so-called critical section are larger than those which have been found in beam tests. This, probably, is due partly to the fact that short beams give higher resistance to diagonal tension (possibly on account of less deflection and on account of less frequent tension cracks in the concrete), as has been shown in Bulletin No. 29, and possibly partly to not taking the critical section far enough from the face of the wall.

26. *Bond Stresses.*—The analysis given in Article 5, "Analysis of Wall Footings," indicates that in wall footings with a uniformly distributed load the bond stress is greatest at the face of the wall and decreases uniformly to the end of the projection, if ordinary beam action is to govern. The distortion of the concrete at the wall, necessary to develop the tensile stress in the bar at this point, or a slip of the bar to produce the same effect, and the formation of tension cracks in the concrete (which take the place of much of the general deformation of the concrete), and other considerations which detract from true beam action, lead us to expect that equation (17) may not express the actual bond stress developed. It is even possible in the case of short bars that after slip of bar begins at the face of the wall the bond stress may for a time be fairly uniform along the bar and thus its intensity at the face of the wall be, say, only half of that given by this equation. However, for simplicity and because slip is very undesirable, the bond stress u given in the tables has been calculated on the basis of equation (17) for a section at the face of the wall. Although the ordinary analysis does not hold for stepped and sloped footings nor where the longitudinal reinforcing bars are bent up at the ends, for the sake of comparison equation (17) has been used for these also. It is realized that the bond stress so calculated will not be the true bond stress.

In failures by bond a crack, vertical or nearly vertical, formed at a section near the face of the wall and opened somewhat as the test progressed. Generally only one crack of this kind formed, though sometimes one formed at the second face. If the bar slipped at its end the crack opened widely. In some cases the bar slipped ¼ in. or more. In all the cases in which instruments were used for detecting the movements of the bar, motion was detected first at a section at the face of the wall and later at the end of the bar. The load for which at a section at the face of a wall a movement of the bar with respect to the concrete was first detected, corresponded with the load at which the concrete would be expected to fail in tension; and it seems that this early slip is intimately connected with the formation of tension cracks in the concrete and that it is more or less local. The amount of the movement was affected by the position of the crack with reference to the location of the instrument. The development of local slip will affect the distribution of the tensile stress in the bar and also will increase the bond stress at other points. In some cases the passing of the yield point of the steel and a considerable slip of the bar came at loads close together and it was difficult to tell which developed first. In other cases the cause of failure is uncertain, but the statements given in the tables were decided upon from a study of the notes of the tests, the position and growth of the cracks, the instrument readings when taken, and an examination of the test piece after failure including the calipering of the bars.

Calculated values of bond stresses are given in Tables 9, 10, and 11. Table 13 repeats these values for footings failing by bond or developing high bond stresses. It will be seen that in footings having ⅜-in. bars failure by slip did not occur. There were a few bond failures with ½-in. bars, and there were a number of bond failures with ⅝-in. bars. In the footings with plain straight rods it will be noted that the values of the bond stress, as calculated by the method used, range somewhat higher than the values of bond resistance which have been found in bond tests of plain rods. In footings with straight corrugated bars, like No. 1747, 1748, and 1749, high bond stresses were developed, and no failure of a footing reinforced in this manner is attributable to bond, though in No. 1694, at the end of the test, bars were found to have slipped. In some cases the concrete in front of the corrugations was found to have the appearance of being powdered, and slight movements were detected.

In three of the footings, measurement of slip of bars was made at different points along the length of the bar. A graduated dial carrying a pointer was attached to a reinforcing bar. A silk-covered wire, weighted at its free end to keep it taut, was wrapped around the shaft which carried the pointer and attached at its other end to the concrete

TABLE 13.

Values of Bond Stress.

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Bond Stress lb. per sq. in.	Manner of Failure
1314	Round bars, straight	1.47	575	Bond
1633	do.	0.82	467	Tension and bond
1634	do.	0.82	436	Bond
1635	do.	0.98	274	"
1636	do.	0.98	446	Probably bond
1641	do.	1.28	445	Bond and possibly diagonal tension
1642	do.	1.28	387	Bond
1645	do.	1.53	318	Probably bond
1646	do.	1.53	412	Bond
1311	Cor. bars, straight	1.04	607	Diagonal tension
1312	do.	1.04	451	do.
1313	Round bars, straight	0.98	475	do.
1315	do.	0.98	447	do.
1371*	do.	0.98	367	Tension followed by diag. tension
1372*	do.	0.98	355	do.
1375*	Cor. bars, straight	1.04	378	Diagonal tension
1376*	do.	1.04	335	do.
1651	do.	0.84	465	do.
1652	do.	0.84	675	do.
1655	do.	1.04	339	do.
1656	do.	1.04	509	do.
1661	do.	1.25	560	do.
1662	do.	1.25	453	do.
1665§	do.	1.25	360	do.
1666§	do.	1.25	453	do.
1362	Round bars straight	0.98	449	Tension followed by diag. tension
1631	do.	0.55	355	Tension
1632	do.	0.55	388	Tension
1673	Round bars, 1 straight, 4 bent up two points	1.28	475	Uncertain
1674		1.28	482	Bond and diagonal tension
1675	do.	1.53	555	Bond
1676	do.	1.53	406	"
1321	Cor. bars, bent up at two points	1.04	602	Tension
1322		1.04	611	"
1325	Round bars bent up at two points.	0.98	500	"
1326	Round bars bent up at one point.	0.98	525	"
1671	Round bars, one straight 4 bent up at two points	0.82	474	Probably tension
1672		0.82	474	do.
1681	Cor. bars, two straight, 4 bent up at two points	1.25	496	No failure at maximum applied load. See notes of test.
1682		1.25	555	
1341	Cor. bars straight	1.04	376	(Sloped footing) Diagonal tension
1342	do.	1.04	378	do.
1351	do.	1.04	505	(Stepped footing) Diagonal tension
1352	do.	1.04	408	do.
1685	Round bars, straight with plain round stirrups	1.25	298	Bond
1686		1.25	396	"
1687	Round bars, straight with cor. stirrups	1.28	387	"
1688		1.28	522	"
1692	Cor. bars, straight with cor. stirrups	1.24	540	Diagonal tension
1693		1.25	423	do.
1694	do.	1.25	449	do.
1712	Round bars straight	0.98	418	Tension and bond

*Length 6 ft. 8 in. §Length 7 ft.

TABLE 13 (CONTINUED).

Footing No.	Reinforcement: Disposition	Reinforcement: Per cent	Nominal Bond Stress lb. per sq. in.	Manner of Failure
1714	Round bars straight	0.98	493	Tension
1716	do.	0.98	443	Tension
1718	do.	0.98	493	"
1731	Cor. bars, 2 straight and 4 bent at two points	1.25	568	"
1733		1.25	628	"
1741	Round bars straight	0.52	571	Tension followed by bond
1742	do.	0.52	605	Tension
1743	do.	0.52	595	"
1744	do.	0.77	544	Bond
1745	do.	0.77	544	"
1746	do.	0.77	584	Tension
1747	Cor. bars straight	0.42	666	Tension
1748	do.	0.42	555	"
1749	do.	0.42	666	"
1751	Round bars looped at ends	0.52	571	"
1752	do.	0.52	525	"
1753	do.	0.52	663	"
1754	Round bars curved up and back at ends	0.52	457	Tension
1755		0.52	542	"
1756	do.	0.52	685	"
1757	do.	1.04	544	"
1758	do.	1.04	535	"
1759	do.	1.04	535	"
1761	Cor. bars, curved up and back at ends	0.85	562	Tension
1762		0.85	574	"
1763	do.	0.85	720	"

adjacent to the bar. The slip measuring apparatus is shown in Fig. 22. Fig. 23 gives the results for No. 1741 and No. 1744, the location of the points of measurement being shown in the plan at the right of the figure. Distance to the right of the zero line represents movement of the bar toward the center of the footing relatively to the concrete. In No. 1741 the measured movement at the face of the wall for a load of 35 000 lb. was 0.001 in. At a load of 65 000 lb. the end slip was nearly 0.001 in. At a load of 68 900 lb. complete failure occurred by sudden slipping of the bars at the end where slip had previously been observed, but there were indications of previous critical failure by tension in the steel. In No. 1744 at a load of 70 000 lb. the ends of the bars had slipped about 0.004 in., and this slip rapidly increased under a slightly larger load. The measurements go to show that there is a complicated relation between the slip of bars and the formation of tension cracks in the concrete. Tests have since been made to determine the relation between bond and slip movement, and the subject will be more fully treated in a forthcoming bulletin.

Of the footings with anchored bars, attention may be called to the

FIG. 22. INSTRUMENTS IN PLACE TO MEASURE SLIP OF BARS.

ones with horizontally looped bars, No. 1751, 1752, and 1753, which developed high strength and gave tension failures. The footings having bars bent or hooked back in a long curved bend, No. 1754, 1755, 1756, 1757, 1758, and 1759, did not show failure in bond, but for some reason No. 1754 did not carry a high load.

B. COLUMN FOOTINGS

27. *Tables.*—Table 14 gives data of the unreinforced concrete column footings, results of the tests, and calculated values of the modulus of rupture. Tables 15, 16, 17, and 18 give data of the reinforcement of the column footings for the series of 1909, 1910, 1911 and 1912, the results of the tests and the calculated stresses. The stresses were calculated by the methods outlined on pages 20 to 24. The values of j used are those given in Table 6. In the calculation of tensile stress and bond stress in footings of 10-in. depth, except where otherwise noted, the area of steel in an equivalent beam width of $\frac{46}{60}$ of the width of footing was used.

28. *Unreinforced Concrete Column Footings.*—The concrete footings without reinforcement generally failed suddenly and without warning at the maximum load applied. In some, as in No. 1506, the maxi-

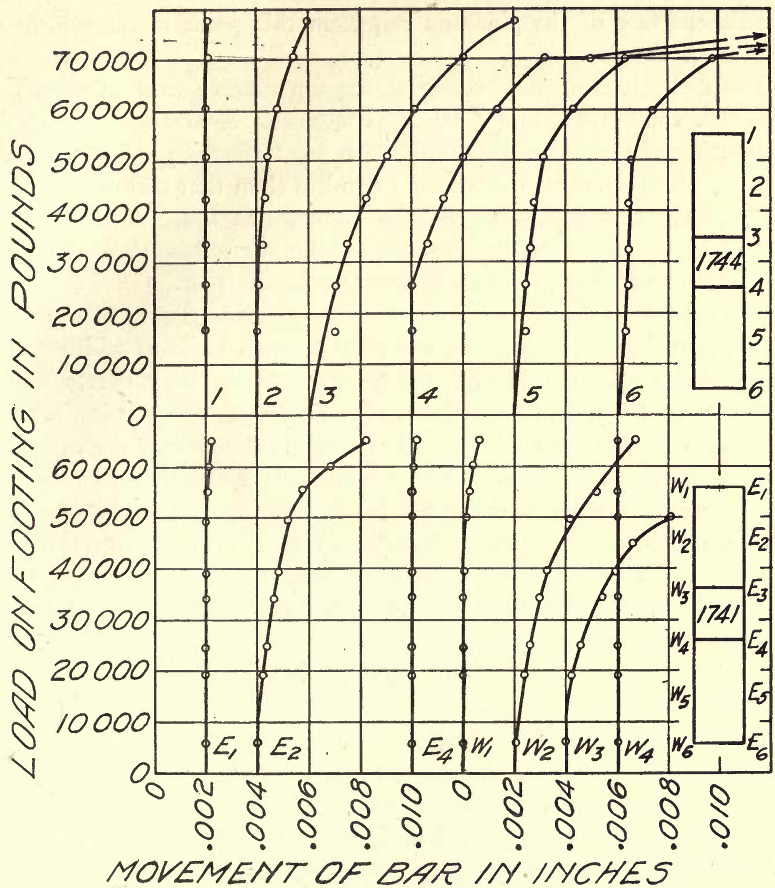


Fig. 23. Diagram Showing Slip of Bars.

mum load was held some little time before failure occurred. In the thicker footings, having at the higher loads much energy stored in the springs, the failure was violent, heavy pieces being thrown to one side and the testing machine giving the appearance of a wreck. In every case, failure was by tension and the footings broke into two or more pieces. The fracture generally occurred in vertical planes, except as it became inclined toward the edge of pier. Fig. 24 shows the position of the lines of fracture. The fractures on the side faces are not shown but the cracks on those faces were vertical in all cases. Along the top surface of the footings the fracture coincided with one face of the pier and in one of the failures cracks jogged back somewhat along an adjacent face of the pier and ran from this point to the middle of the side of the footing.

The deflections of the footing at the top surface were measured in No. 1505, 1506, 1507, and 1508 in an effort to determine roughly the distribution of stresses. The results are not sufficiently definite for the purpose. The curvature at an edge was less than that through the face of the pier. It would seem to follow from this that the stresses developed in the concrete at the outside edge were less than those developed in a parallel direction at the face of the pier.

The moduli of rupture for the footings given in Table 14 were calculated by using a resisting moment based upon the full width of the footing, that is by considering the fiber stress in the concrete at the bottom of the footing to be uniform over the length of a section passing through the face of the wall, instead of taking into account the variation in stress across the section. The method of calculating the bending moment was the same as that used in the reinforced footings. Variations in concrete and in the actual distribution of load over the footing masked any effect due to the variation in proportion of depth to projection. It should be noted that the values of the modulus of rupture in the table are smaller than the modulus of rupture found in the control beams, averaging perhaps two-thirds as great if some allowance be made for the greater age of the control beams. The projection in an unreinforced footing usually is relatively short, and it may be more convenient in designing to use the full width of section, but it appears that the working stress used should be based on a modulus of rupture smaller than that found by beam tests. The probability of variation in tensile strength of concrete also must be taken into account in choosing the working stresses for unreinforced footings.

29. *Phenomena of Tests of Reinforced Concrete Column Footings.*—In the tests of the reinforced concrete column footings, as the load was applied the springs forming the bed compressed. The deflection in these footings was so slight and the consequent difference in the amount of shortening in the springs was so small that in the calculations the load was considered to be uniformly distributed over the footing up to the point of failure. In cases where the failure was by tension or by slip of bars, there followed bending up of the edges of the footing which modified this distribution of the load as soon as failure became evident. Three forms of failure may be distinguished: (a) tension in reinforcement; (b) bond between steel and concrete; and (c) diagonal tension or shear.

TABLE 14.

Data of Unreinforced Concrete Column Footings.
Series of 1910.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Universal Portland cement. Weight of cement averaged 12.6 per cent of weight of aggregate.

Footing No.	Age days	Depth inches	Load at Failure pounds	Modulus of Rupture lb. per sq. in.	Control Beams		6-in. Cubes	
					Modulus of Rupture lb. per sq. in.	Age days	Maximum Load lb. per sq. in.	Age days
1501	77	6	30 000	272	309	79	2910	112
1502	74	6	28 000	254	423	87	2939	104
1503	77	8	49 000	250	373	79	2307	112
1504	73	8	31 000	158	425	109	1761	109
1505	86	12	86 000	195	376	95	3618	128
1506	77	12	67 000	152	272	99	2260	116
1507	75	18	238 000	240	409	105	3180	121
1508	73	18	190 000	191	363	93	1988	109

(a) In the failures by tension in the steel the cracks which had appeared at the bottom or on the lateral faces of the footings near the middle portion of the length opened at the maximum load, and the maximum load was maintained for some time under steady pumping of the jacks, the edges of the footing meanwhile deflecting considerably.

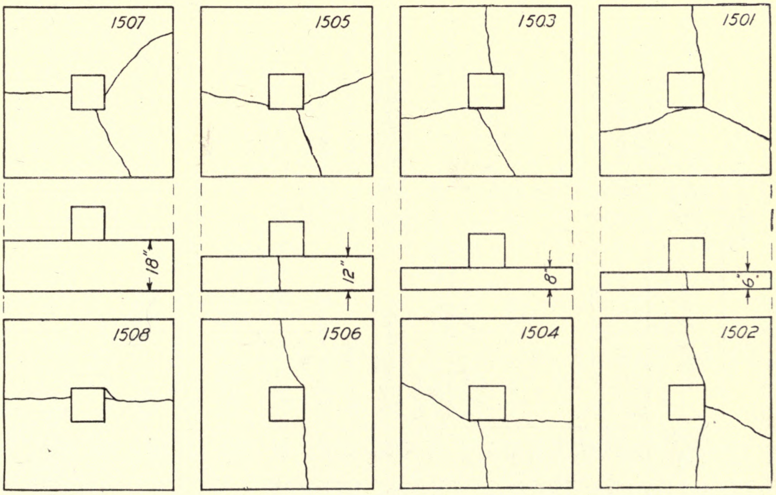


Fig. 24. Unreinforced Concrete Column Footings.

TABLE 1[illegible].

REINFORCED CONCRETE COLUMN FOOTINGS. SERIES OF 1909.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Chicago AA Portland cement. Total depth 11 in., depth to center of steel 10 in., unless otherwise noted.

Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Calculated Stresses lb. per sq. in.: Bond Stress	Calculated Stresses lb. per sq. in.: Vertical Shearing Stress	Manner of Failure	Remarks
1411*	81	15 ⅝-in. round	0.28	Bars spaced 4 in. c. to c. in each of two directions.	112 000	31 900	220	69	Tension	
1412	76	do.	0.28		160 000	45 600	314	99	"	
1413*	82	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	144 000	29 200	269	90	Bond	
1414	71	do.	0.39		192 000	38 900	358	120	"	
1415	80	do.	0.39	do.	160 000	32 400	299	100	"	Bars cut 3 ft. 9 in. long and ends staggered.
1416	68	do.	0.39	do.	128 000	25 900	239	81	"	
1417*	81	12 ¾-in. round	0.88	do.	160 000	14 900	206	104	"	Load eccentric.
1418	61	do.	0.88	do.	176 000	16 400	226	114	"	do. Total depth, 12 in.
1421*	84	1 ¾-in., 2 ⅝-in., 4 ½-in. and 4 ⅜-in. round.	0.38	do.	136 000	24 300	...	85	"	
1422	57	3 ¾, 4 ½, and 4 ⅜-in. round	0.42	do.	160 000	25 200	...	100	Not determined	Load eccentric. Total depth, 11½ in.
1425*	79	12 ½-in. round	0.39	Bars spaced from 3 to 7 in. in each of two directions	160 000	29 900	275	100	Probably tension	Load eccentric
1426	65	do.	0.39		160 000	29 900	275	100	do.	do.
1429	77	Wire mesh	0.13	Two double layers	128 000	53 700	...	77	....	
1431†	76	15 ⅓-in. cor. sq.	0.28	Bars spaced 4 in. c. to c. in each of two directions	156 000	44 200	271	97	Probably tension	
1432†	71	do.	0.28		136 000	38 500	236	84	do.	
1435	77	5 ¾-in., 2 ½-in., 2 ⅓-in. and 2 ¼-in. cor. sq.	0.61	Bars spaced 5 in. c. to c. in each of two directions	208 000	22 000	...	134	Diagonal tension	Total depth, 11½ in.
1436	64	5 ¾-in., 4 ½-in., 2 ¼-in. cor. sq. bars	0.67	do.	176 000	17 500	...	113	do.	Total depth. 12 in.
1437	82	3 ½-in., 4 ⅓-in., 4 ¼-in. cor. sq. bars	0.24	do.	128 000	34 800	...	79	Tension	
1439*	82	1 ¾-in., 4 ½-in., 4 ⅓-in. cor. sq. bars	0.33	Bars spaced 6 in. c. to c. in each of two directions	160 000	32 600	...	98	"	Load eccentric
1447	78	½-in. round	0.31	6 bars parallel to each side	208 000	42 100	...	130	Diagonal tension	Total depth, 12 in.
1448	63	do.	0.31	5 bars parallel to each diag.	176 000	35 600	...	110	do.	do.
1449	75	½ and ⅝-in. round	0.27	5 ½-in bars 5 in. c. to c. parallel to each side, 3 ⅝-in. bars 7 in. c. to. c. parallel to each diagonal	192 000	33 400	...	122	Tension	do.
1451	63	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	88 000	17 800	131	59	Bond	Sloped. 3 in. to steel at edges

* Tested with a flat bearing plate. † Mild steel.

TABLE 16.

Reinforced Concrete Column Footings. Series of 1910.

All footings 5 ft. square. 1–2½–5 concrete, hand-mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in. unless otherwise noted.

Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Calculated Stresses lb. per sq. in.: Bond Stress	Calculated Stresses lb. per sq. in.: Vertical Shearing Stress	Manner of Failure	Remarks
1515	81	12 ½-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	185 000	37 600	347	117	Tension	
1516	80	do.	0.39		170 000	34 600	317	107	Tension followed by bond	
1521*	78	do.	0.58	do.	116 000	38 300	352	158	Tension	Total depth, 8 in.
1522†	71	do.	0.56	do.	122 000	38 700	356	156	Bond with possible tension on west side	do.
1525‡	79	do.	0.78	do.	63 000	29 700	273	144	Bond	Total depth, 6 in.
1526‡	71	do.	0.78	do.	85 000	40 000	368	194	Bond; tension probably imminent	do.
1531	81	18 ½-in. round	0.59	Bars spaced 3¼ in. c. to c. in each of two directions	280 000	38 700	357	180	Diagonal tension	
1532	79	do.	0.59		252 000	34 800	321	162	do.	
1535	85	8 ¾-in. round	0.59	Bars spaced 8 in. c. to c. in each of two directions	194 000	20 600	372	125	Bond	
1536	70	12 ⅝-in. round	0.61	Bars spaced 5 in. c. to c. in each of two directions	182 000	24 200	279	117	"	
1541‡	78	8 ¾-in. round	1.18	Bars spaced 8 in. c. to c. in each of two directions	52 000	16 700	231	122	Bond, lack of concrete below bars	Total depth, 6 in.
1542‡	61	12 ⅝-in. round	1.23	Bars spaced 5 in. c. to c. in each of two directions	95 000	29 400	340	222	Bond	Total depth, 6½ in.
1551	82	10 9/16-in. cor. round	0.42	Bars spaced 6 in. c. to c. in each of two directions	225 000	43 400	448	142	Diagonal tension	
1552	72	10 ½-in. cor. square	0.42		236 000	45 100	415	149	do.	
1553	81	15 9/16-in. cor. round	0.62	Bars spaced 4 in. c. to c. in each of two directions	327 000	43 000	445	211	do.	
1554	75	15 ½-in. cor. square	0.62		288 000	37 800	346	185	do.	
1561	77	½-in. round	0.59	Bars spaced 7 in. c. to c. uniformly in four directions	240 000	33 300		154	Probably combination of bond and tension	
1562	64	do.	0.59		210 000	29 100		135		
1563	77	do.	0.59	Bars spaced 7 in. c. to c. in four directions, 3 short diag. bars at corners omitted.	174 000	24 100		112	Probably bond	
1564	64	do.	0.59		210 000	29 100		135	do.	

* 6¾ in. to center of steel. † 7 in. to center of steel. ‡ 5 in. to center of steel.

TABLE 17.

Reinforced Concrete Column Footings. Series of 1911.

All footings 5 ft. square. 1–2–4 concrete, hand mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in. unless otherwise noted.

Footing No.	Age at Test days	Reinforcement			Load at Failure pounds	Calculated Stresses lb. per sq. in.			Manner of Failure
		Description	Per cent	Disposition		Tensile Stress in Steel	Bond Stress	Vertical Shearing Stress	
1806	62	22 3/8-in. round	0.41	Spaced 2 3/4 in. c. to c. in each of two directions	179 000	34 800	241	111	Probably bond
1807	62	do.	0.41		210 000	40 800	283	130	Tension
1808	62	27 3/8-in. round	0.50	Bars spaced 2 1/4 in. c. to c. in each of two directions	198 000	31 600	218	124	Diagonal tension
1809	62	do.	0.50		236 000	37 700	260	148	Tension followed by diagonal tension
1810	59	33 3/8-in. round	0.61	Bars spaced 1 13/16 in. c. to c. in each of two directions	219 000	28 700	198	138	Diagonal tension
1811	60	do.	0.61		261 000	34 200	236	164	do.
1812	78	12 1/2-in. round	0.39	Bars spaced 5 in. c. to c. in each of two directions	171 000	34 300	315	106	Bond
1813	65	do.	0.39		121 000	24 300	223	75	"
1814	62	22 1/2-in. round	0.72	Bars spaced 2 3/4 in. c. to c. in each of two directions.	301 000	33 700	310	192	"
1815	60	do.	0.72		294 000	33 000	303	187	Diagonal tension and bond
1816	65	8 5/8-in. round	0.41	Bars spaced 7 1/2 in. c. to c. in each of two directions.	132 000	25 400	293	82	Bond
1817	59	do.	0.41		159 000	30 600	353	99	"
1818	64	21 3/8-in. cor. round	0.39	Bars spaced 2 7/8-in. c. to c. in each of two directions.	198 000	40 400	279	123	Diagonal tension
1819	60	do.	0.39		261 000	53 300	367	162	Tension followed by diagonal tension.
1820	64	10 1/2-in. cor. square	0.42	Bars spaced 6 in. c. to c. in each of two directions.	179 000	33 800	313	111	Diagonal tension
1821	30	do.	0.42		159 000	30 000	278	99	do.
1822	60	22 3/8-in. round	0.41	Bars spaced 2 3/16 in. c. to c. in each of two directions No rods under pier	198 000	38 500*	266*	123	Tension
1823	60	do.	0.41		210 000	40 800*	282*	131	"

* 46/60 of steel considered as effective.

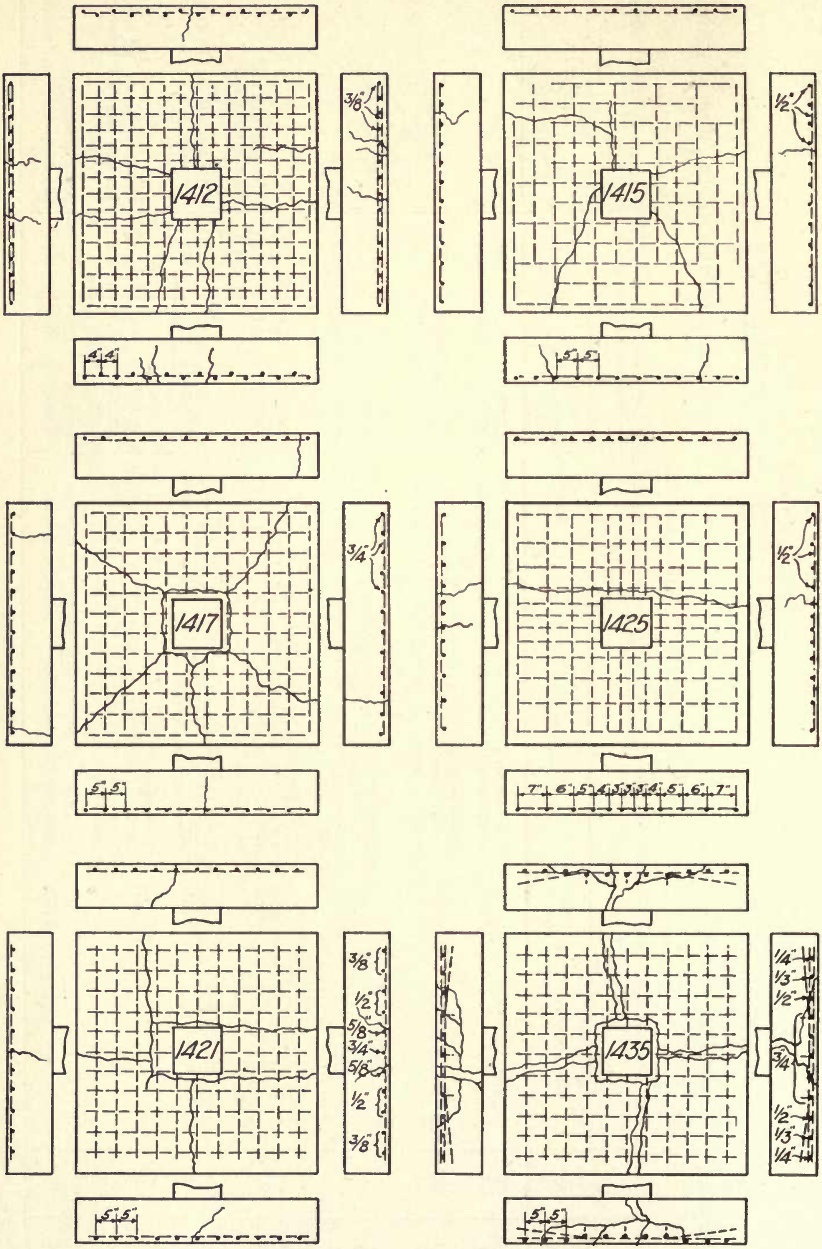
TABLE 18.

Reinforced Concrete Column Footings. Series of 1912.

All footings 5 ft. square. 1–2–4 concrete, machine-mixed. Universal Portland cement. Total depth 12 in., depth to center of steel 10 in.

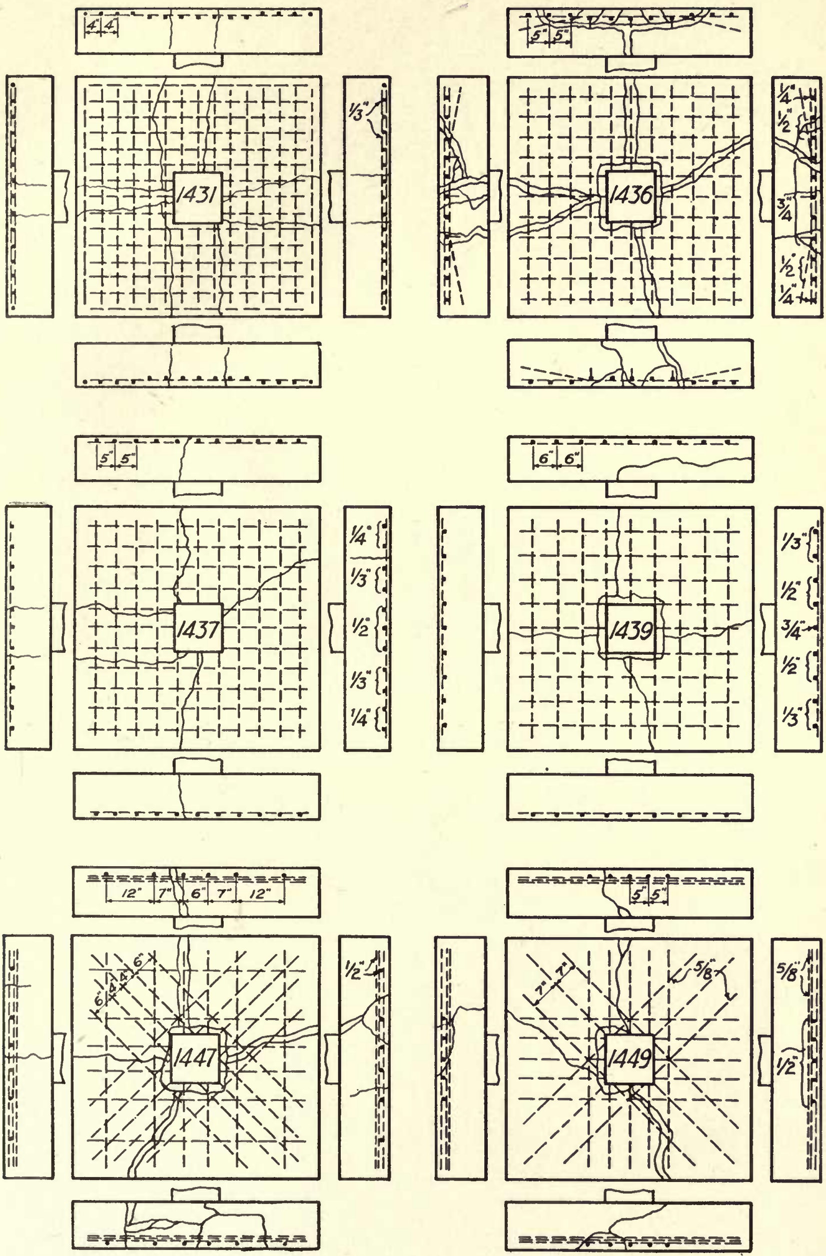
Footing No.	Age at Test days	Reinforcement: Description	Reinforcement: Per cent	Reinforcement: Disposition	Load at Failure pounds	Calculated Stresses lb. per sq. in.: Tensile Stress in Steel	Calculated Stresses lb. per sq. in.: Bond Stress	Calculated Stresses lb. per sq. in.: Vertical Shearing Stress	Manner of Failure
1831	69	8 ⅝-in. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions	161 000	31 000	357	100	Bond
1832	73	do.	0.41		192 000	37 100	425	119	"
1833	66	22 ⅜-in. round	0.41	Bars spaced 1⅛ in. c. to c. in each of two directions. No bars nearer center of footing than 18 in.	113 000	22 000*	152*	71	"
1834	71	do.	0.41		152 000	29 800*	206*	95	"
1835	76	do.	0.41	Bars spaced 1⅝ in. c. to c. in each of two directions. No bars nearer center of footing than 12 in.	183 000	35 800*	248*	114	"
1836	66	do.	0.41		212 000	41 200*	285*	132	"
1837	77	do.	0.41	Bars spaced 1⅛ in. c. to c. in each of two directions. No bars farther from center of footing than 12 in.	192 000	28 700	198	119	"
1838	69	do.	0.41		247 000	36 900	254	154	Tension followed by sudden bond failure.
1839	69	8 ⅝-in. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions. Bent from two long bars.	192 000	37 100	425	119	Tension followed by diagonal tension
1840	65	do.	0.41		201 000	38 800	445	125	
1841	71	do.	0.41	Bars spaced 7½ in. c. to c. in each of two directions. Curved up at ends to within 2 in. of top and back 8 in.	186 000	35 900	412	116	Tension followed by bond
1842	70	do.	0.41		203 000	39 200	450	126	Tension followed by diagonal tension
1843	71	8 ⅝-in. cor. round	0.41	Bars spaced 7½ in. c. to c. in each of two directions.	227 000	43 800	507	141	Diagonal tension
1844	67	do.	0.41		269 000	52 000	596	167	Bond followed by diagonal tension

* $\frac{43}{60}$ of steel considered as effective.



The Drawings of No. 1412, 1415 and 1421 show the Cracks as they appeared on the Bottom Surface of the Footing.

FIG. 25. REINFORCED CONCRETE COLUMN FOOTINGS.



The Drawings of No. 1431 and 1437 show the Cracks as they appeared on the Bottom Surface of the Footing.

Fig. 26. Reinforced Concrete Column Footings.

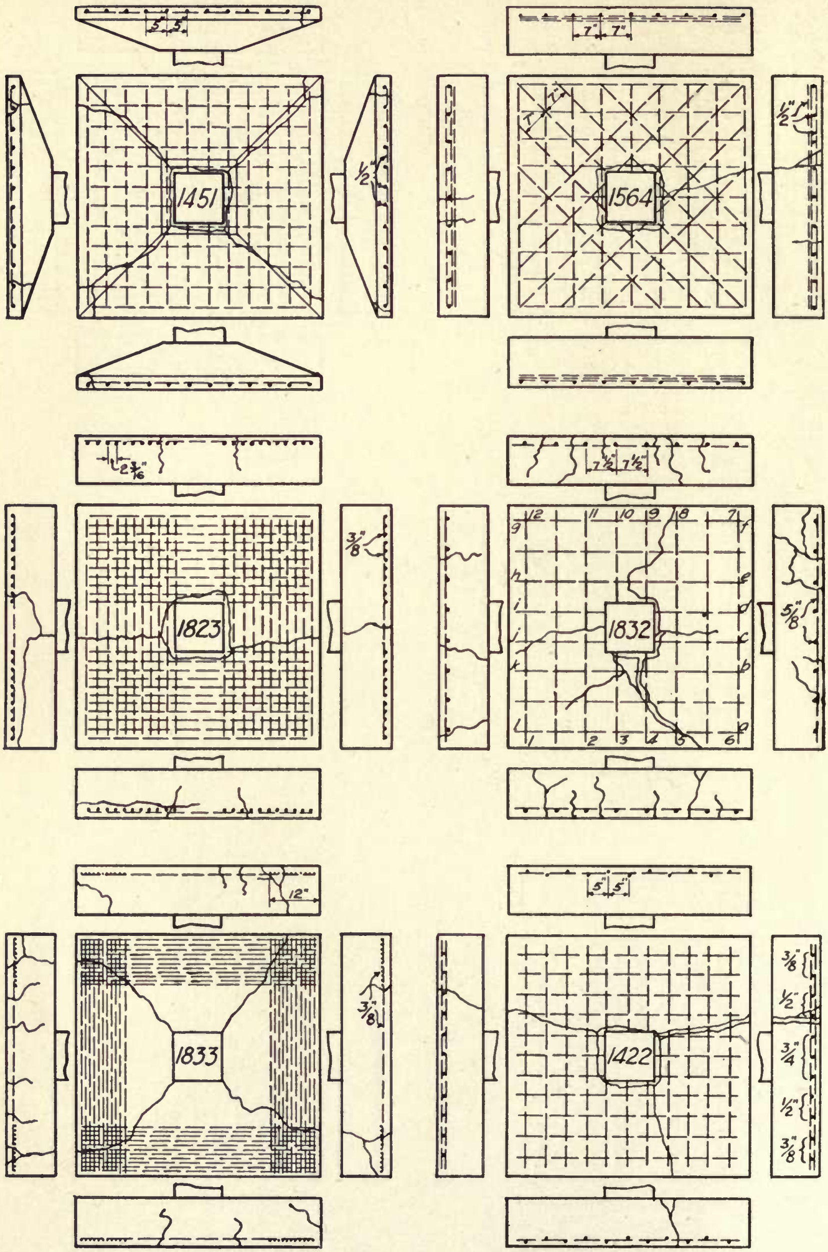


Fig. 27. Reinforced Concrete Column Footings.

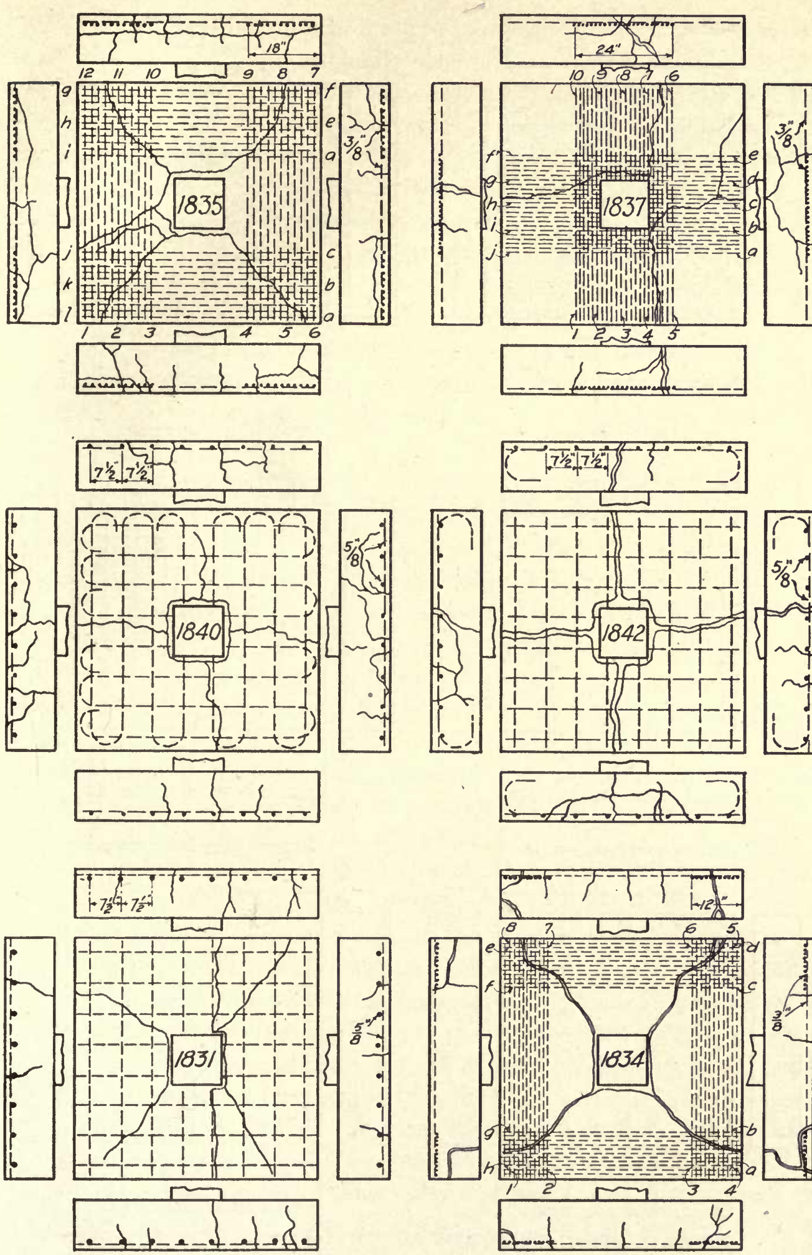


FIG. 28. REINFORCED CONCRETE COLUMN FOOTINGS.

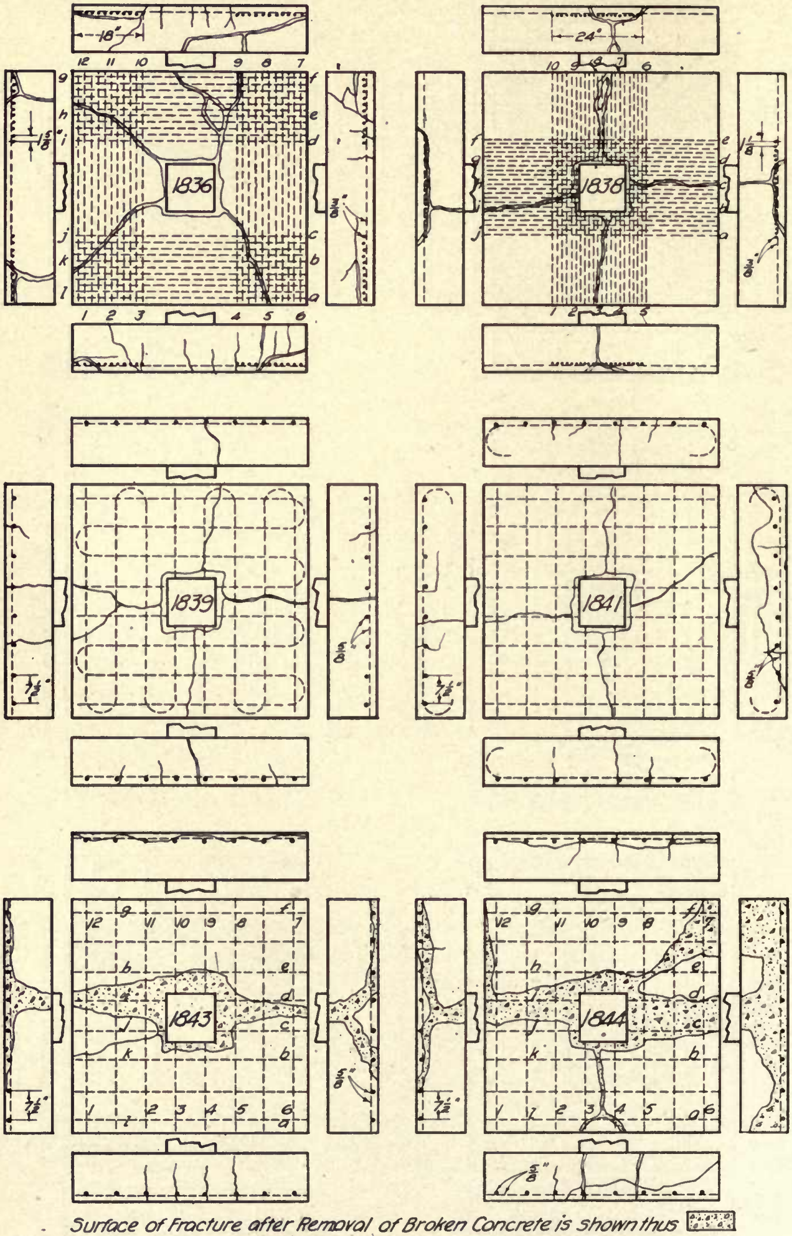


FIG. 29. REINFORCED CONCRETE COLUMN FOOTINGS.

When the operation of the jacks was continued after the maximum load was reached, the pier finally sheared or punched through the footing (see No. 1838, Fig. 31), the angle of the shearing face generally being steeper than 45° with the vertical. This shearing took place after considerable deflection and when the projection of the footing was taking only a fraction of its share of the load. After the phenomena just noted the cracks on the upper surface of the footing extended from the corner of the pier to a point on the lateral face of the footing opposite the face of the pier (see No. 1412, Fig. 25, page 78). In a few cases the line of fracture ran to a point near the middle of the pier. It should be borne in mind that the shearing and formation of cracks referred to occurred after the yield point of the reinforcement had been reached. In some footings examined the reinforcing rods were found to be necked.

(b) In the bond failures the failure was gradual, cracks forming near the ends of the lateral faces and finally opening up considerably, while the tension hair cracks which had opened in the middle of the lateral faces finally closed. (See No. 1415, Fig. 25, and No. 1417, Fig. 25). In general, the large cracks formed in the bond failures ran diagonally inward from near the corners of the footing. In many cases the bars were seen to have slipped at the ends. In cases where reinforcement was placed close to the lower surface of the footing, the tension cracks which formed by beam action in a direction parallel to the lower reinforcing bar, loosened the bond of these bars and caused failure at lower loads than might otherwise be expected. The manner of this loosening is apparent from Fig. 8 (c), page 18. Several of the footings of the series of 1909 suffered from this cause. In a number of cases, when the test was continued farther than the maximum load, an ultimate failure by punching through the footing in the manner noted under "Tension Failures" was found. The measurements of slip of end of bar taken in the tests of 1912 and discussed under "Bond Failures" give information on the first slip of bar.

(c) In the form of failure termed diagonal tension or shear failure the footing below the pier suddenly separated from the outer portion of the pier, leaving a mass in the form of a frustum of a pyramid below the pier, the reinforcing bars being stripped from a part or all of the remainder of the footing. The outer portion of the footing was generally intact, except as to tension cracks which had formed as usual. The face of the fracture was about 45° with the vertical. No. 1843, Fig. 31, failed in this way. These failures are similar to the failures in ordinary reinforced concrete beams known as diagonal tension or shear failures in suddenness, in direction of the fractured face, and in the stripping of the reinforcing bars. Since these failures are in the interior the formation of diagonal tension cracks could not be observed. In all probability the failure is due to diagonal tension stresses, the cracks forming first at or near the reinforcing bars. For the highest loads obtained incipient compression failure was observed near the junction of pier and upper face of the footing.

The following are brief notes of tests. The location of cracks is shown in Fig. 25-29. The heavy lines on the lateral faces of the pier indicate cracks along which failure took place. Reference may be made to Table 15-18. It should be remembered that when first observed the cracks noted were hair cracks, which continued to be very fine cracks until they opened when the steel reached its yield point or bond resistance was overcome. As the faces of the piers varied in roughness and as they were not whitewashed, the load when a crack was first observed may be expected to be different in one footing from that in a companion test piece. The footings marked "Stored in place of making" were left on the floor of the mixing room in the place they were made until just before the test. In this position they were subject to considerable dampness during the time of seasoning, as the floor was frequently wet from the work. It seems evident that the concrete in this condition hardened more slowly and did not attain the same strength in the storage period as did those footings which were stored above the floor.

Series of 1909.

No. 1411. Tested with flat bearing plate. Shortening of springs at north end nearly double that at south end. Very uneven distribution of load. Tension crack formed at 96 000 lb. Failure at 112 000 lb. by gradual opening of tension cracks.

No. 1412. Load was applied through a spherical-seated bearing block. At 120 000 lb. tension crack appeared on east face directly in line with north face of pier. At 144 000 lb. a second tension crack appeared on the east face directly in line with the south face of the pier. Footing failed slowly at 160 000 lb., tension cracks opening. (See Fig. 25.)

No. 1413. This was the first footing tested and much of the work was experimental, and many of the springs were without bearing at the beginning of the test. A flat bearing block was used and the load was not uniformly distributed. Possibly some error in data. No cracks noted until directly before failure. The main cracks ran from points on the lateral faces near the corners of the footing towards points under the pier. Slip of bars was observed. Bond failure.

No. 1414. First crack observed at 120 000 lb. in middle of west lateral face. A second crack formed at 136 000 lb. and closed up at the failure of the footing. Failed at 192 000 lb. by bond, new cracks forming on the bottom from near the corners.

No. 1415. (See Fig. 25.) Short rods extended alternately to within 3 in. and 12 in. of faces of footing. Southeast corner carried less load than remainder. Tension cracks which formed finally closed up. Failure at 160 000 lb. by bond, cracks forming and opening on lateral face near corners.

No. 1416. Tension crack at 120 000 lb. on two opposite faces. Gradual failure at 128 000 lb. Manner of failure not definitely known, probably bond. The use of shorter rods may have caused concentration of bond stresses.

No. 1417. (See Fig. 25 and 30.) No cracks were noted until the maximum load of 160 000 lb. was applied, when cracks formed gradually near the corners. Bond failure. Reinforcing bars were very close to surface, especially on east side.

No. 1418. No cracks were noted until maximum load of 176 000 lb. was applied, when cracks formed slowly on the lateral faces near the corners and failure was gradual. The slipping of reinforcing bars was very noticeable. Cracks formed in horizontal plane of rods and the concrete below split off.

No. 1421. (See Fig. 25 and 30.) Flat bearing plate used. Reinforced with rods of varying size. At 128 000 lb. tension cracks appeared on three faces. These cracks followed the lines of the reinforcing bars and reduced the effectiveness of the bond resistance. Method of failure not definitely known, probably bond.

No. 1422. (See Fig. 27.) Footing stored in place of making. Load not uniformly distributed. No cracks appeared until sudden failure occurred at 160 000 lb. Method of failure not determined.

No. 1425. (See Fig. 25.) Flat bearing plate. Reinforcing rods with varying spacing across the footing. Load not uniformly distributed, the north side taking a greater load. At 112 000 lb. cracks were seen on two faces. Maximum load 160 000 lb. Probably tension failure.

No. 1426. Tension crack at 112 000 lb. on east face and at 144 000 lb. on south face. Failure gradual at 160 000 lb. Seemingly tension failure, perhaps beginning at edge where rods were spaced far apart.

No. 1429. Reinforced with wire mesh. First crack at 96 000 lb. on east and west faces. Broke suddenly at 128 000 lb. Tension failure.

No. 1431. (See Fig. 26.) Reinforced with mild steel corrugated bars. Two cracks on east face in line with north-and-south face of pier at 112 000 lb. and one on west face and one on south face at 128 000 lb. Cracks grew as load was increased and at the maximum load of 156 000 lb. failure occurred. Tension failure.

No. 1432. Reinforced with mild steel corrugated bars. Crack at 128 000 lb. and others appeared later. Principal cracks opened. Gradual failure at 136 000 lb. Tension failure.

No. 1435. (See Fig. 25.) Reinforced with corrugated bars of varying sizes bent up somewhat at ends. At 136 000 lb. tension crack appeared in west face in line with the south face of pier, at 144 000 lb. on east face opposite north face of pier. Failed at 208 000 lb. by diagonal tension, the angle of the faces of fracture being about 45° on all four sides. The lower layer of concrete to the top of reinforcement fell away and outer portion of footing broke into four pieces. One ¼-in. bar near north face was found broken near the center of its length.

No. 1436. (See Fig. 26.) Reinforced as No. 1435. Stored in place of making. At 160 000 lb. tension crack appeared on east and west faces. At 176 000 lb. failed suddenly much as No. 1435.

No. 1437. (See Fig. 26.) Light reinforcement of corrugated bars. At 104 000 lb. tension cracks appeared on east and west faces on line with north face of pier; at 112 000 lb. on north and south faces. Failed by tension at 128 000 lb.

No. 1439. (See Fig. 26.) Flat bearing plate. At 136 000 lb. tension cracks appeared at center of length of east face and at 144 000 lb. near center of west and north faces. Failed at 160 000 lb. by tension in steel. Pier finally sheared through.

No. 1447. (See Fig. 26.) Reinforced with rods in four directions. At 144 000 lb. two cracks appeared on west face, one directly in center and one 12 in. from corner, the latter crack closing before final failure. At 160 000 lb. crack in north face in line with east face of pier and at 167 000 lb. on south face about center. Sudden failure at 208 000 lb. Diagonal tension failure.

No. 1448. Footing stored in place of making. At 144 000 lb. crack appeared on east face, one on the south face and two on the west face. Three were near center and one 8 in. from corner, the last nearly closing up before final rupture. Sudden failure at 176 000 lb. by diagonal tension.

No. 1449. (See Fig. 26.) At 96 000 lb. crack appeared at center of length of east face and on west face in line with north face of pier. At 160 000 lb. at center of south face and at 192 000 lb. at center of north face. Tension failure at 192 000 lb.

No. 1451. (See Fig. 27.) Sloped footing. Stored in place of making. At 80 000 lb. crack appeared on west face 9 in. from corner. Failed at 88 000 lb., evidently by bond. It would seem that in this form of footing bond stresses would be more concentrated towards the ends of bars than in footings of rectangular cross section.

Series of 1910.

No. 1515. Tension cracks appeared at the middle of the two lateral faces at a load of 120 000 lb. At 166 000 lb. tension cracks appeared at the middle of the other two lateral faces. At 185 000 lb. gradual failure by tension occurred, the tension cracks opening up. With a continuation of the test there was a punching through the footing by the pier.

No. 1516. First crack (tension) at 102 000 lb. Instruments removed at 138 000 lb. Load released at 143 000 lb. to adjust the testing machine. Load again brought to 138 000 lb. and then released to adjust upper nuts of testing machine. Load again applied and released because of dangerous leaning of the springs. The test was continued the following day. The footing held the maximum load at 170 000 lb. for several minutes under steady pumping of the jacks, while the tension cracks on the sides slowly opened. Failure was by tension in the reinforcement. With continued operation of the test, rods in the lower layer slipped, cracks having formed along the lower surface beneath them. The pier finally sheared or punched through along diagonal planes.

No. 1521. Seven-inch depth to center of reinforcement. First cracks (tension) at 52 000 lb. When the load of 116 000 lb. was reached the tension cracks widened perceptibly and the load fell off at once to 112 000 lb. Tension failure along these cracks at this load. The pier finally punched through as shown in diagram.

No. 1522. Seven-inch depth. First crack (tension) at 85 000 lb. Instruments removed. At maximum load of 122 000 lb. cracks slowly opened. Failure was very gradual. Examination of footing after failure showed that rods had slipped at northeast corner and at south edge. Examination of three rods along west side showed no indication of slip. In general the slip of bars was accompanied by cracks in the concrete immediately underneath and in the direction of the bar. Rods which were calipered showed no indication of reduction of section.

No. 1525. Five-inch depth. First cracks (tension) at 45 000 lb. Failure at 65 000 lb. by gradual opening of cracks on face near corners and final appearance of diagonal cracks on top face. General indications of failure by bond. In the final punching through by the pier, the faces of the fractures were nearly vertical. Examination after failure showed that most of the rods had slipped, many of them at both ends. The bottom bars showed no reduction of section.

No. 1526. Five-inch depth. First crack (tension) at 38 000 lb. Instruments removed at 49 000 lb. At 85 000 lb. cracks on lateral faces near corners gradually opened. Bond failure, diagonal cracks finally reaching the top surface. After failure all the rods in this footing were examined and all but two found to have slipped at one or both ends. These two were in the bottom layer and next to the south edge. They were gaged with a micrometer caliper and found not to have necked.

No. 1531. Ten-inch depth. First crack (tension) at 85 000 lb. Springs leaned considerably and load was released, machine adjusted, and load reapplied. At 177 000 lb. the springs again needed adjustment and the load was released. Four weeks later additional springs were placed on the bed and a second test made. At 280 000 lb. concrete above the base of the pier began to show signs of compression failure. The load fell off slowly and the footing finally failed suddenly by shearing through from the edge of the pier at the top of the footing to a line about as shown in Fig. 8(b) at the bottom, and the reinforcement together with the concrete layer below it was stripped off. The tension cracks which had formed closed up. Diagonal tension failure. Where concrete remained on rods examination was made after failure and no rods could be found to have slipped. The angle of fracture with the horizontal was about 45°.

No. 1532. First crack (tension) at 138 000 lb. Instruments removed at 198 000 lb. At 242 000 lb. the springs had closed up. The load was then released and one week later with additional springs placed on the bed the footing was loaded to failure with 252 000 lb. Failure was sudden by the pier shearing through. The fracture made an angle of about 45° with the horizontal. Of the few rods still encased in concrete when examined after failure none had slipped. As to the rest nothing could be determined.

No. 1535. First crack (tension) at 158 000 lb. Gradual failure at 194 000 lb. Failure was accompanied by opening of cracks near corners on lateral faces and by closing of tension hair cracks which had appeared near the center of the lateral faces earlier in the test. Bond failure. After failure all the rods were examined and all but two were found to have slipped at one or both ends and it is possible that even they had slipped at one end. These two were next to the north and east edges and in the top and bottom layers respectively.

No. 1536. Very gradual failure at 182 000 lb. Indications of failure by slipping of reinforcement. All bars which were found to have slipped were in the upper layer. Under the ends of bars found to have slipped cracks were found running in the direction of the bar. No such cracks were found at the ends of bars which had not slipped.

No. 1541. Five-inch depth. First crack (tension) at 45 000 lb. Instruments removed at 52 000 lb. Load fell off slowly while instruments were being removed and load could not be raised above 52 000 lb. by further pumping. This maximum load was held under steady pumping for about two minutes while footing deflected visibly. The failure was gradual, the bars slipping ½ in. to ¾ in. As the load was released some of the concrete below the bars dropped off. It seems probable that rods were placed too close to lower surface and that tension cracks reduced bond resistance.

No. 1542. Five-inch depth. First cracks (tension) appeared at 67 000 lb. on two opposite faces. Instruments removed at 85 000 lb. Gradual failure at 95 000 lb. Crack on lateral face near corners formed and opened on application of maximum load. Bond failure.

No. 1551. (See Fig. 30.) Ten-inch depth. Reinforced with round corrugated bars. Tension cracks formed at 102 000 lb. Instru-

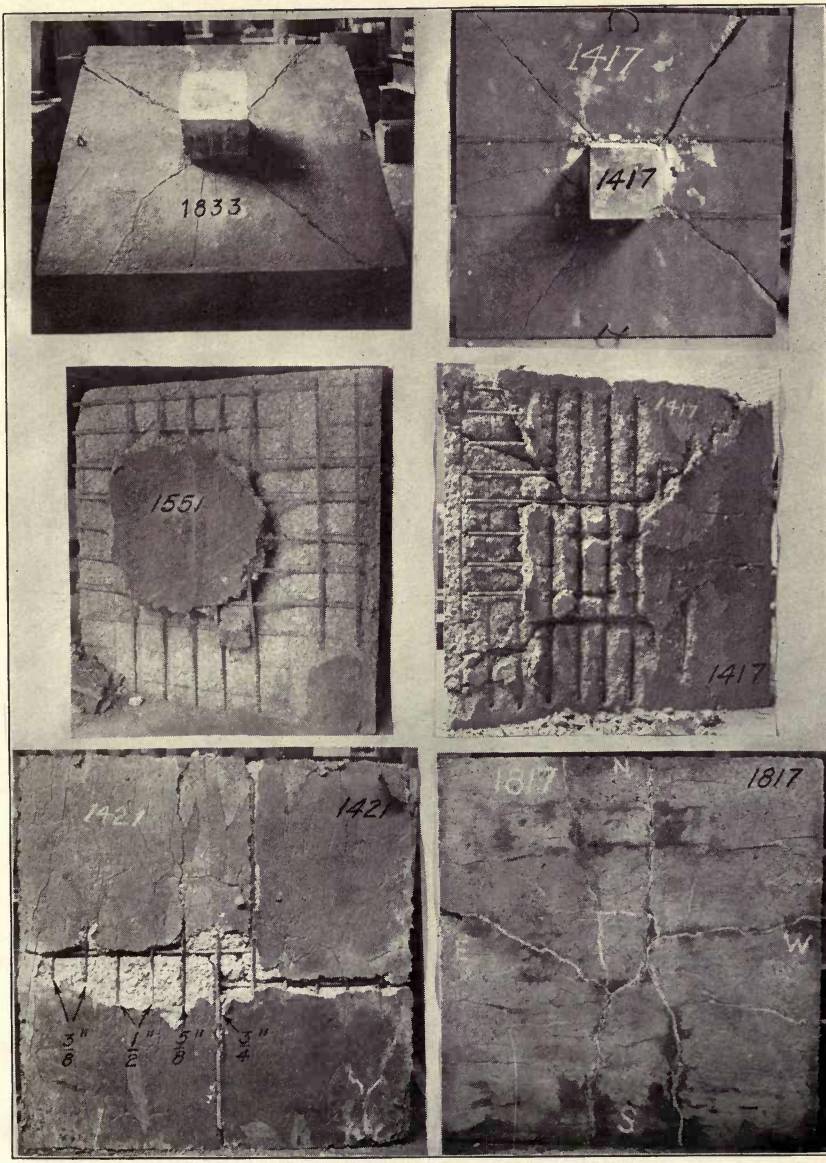


FIG. 30. VIEWS SHOWING COLUMN FOOTINGS AFTER FAILURE.

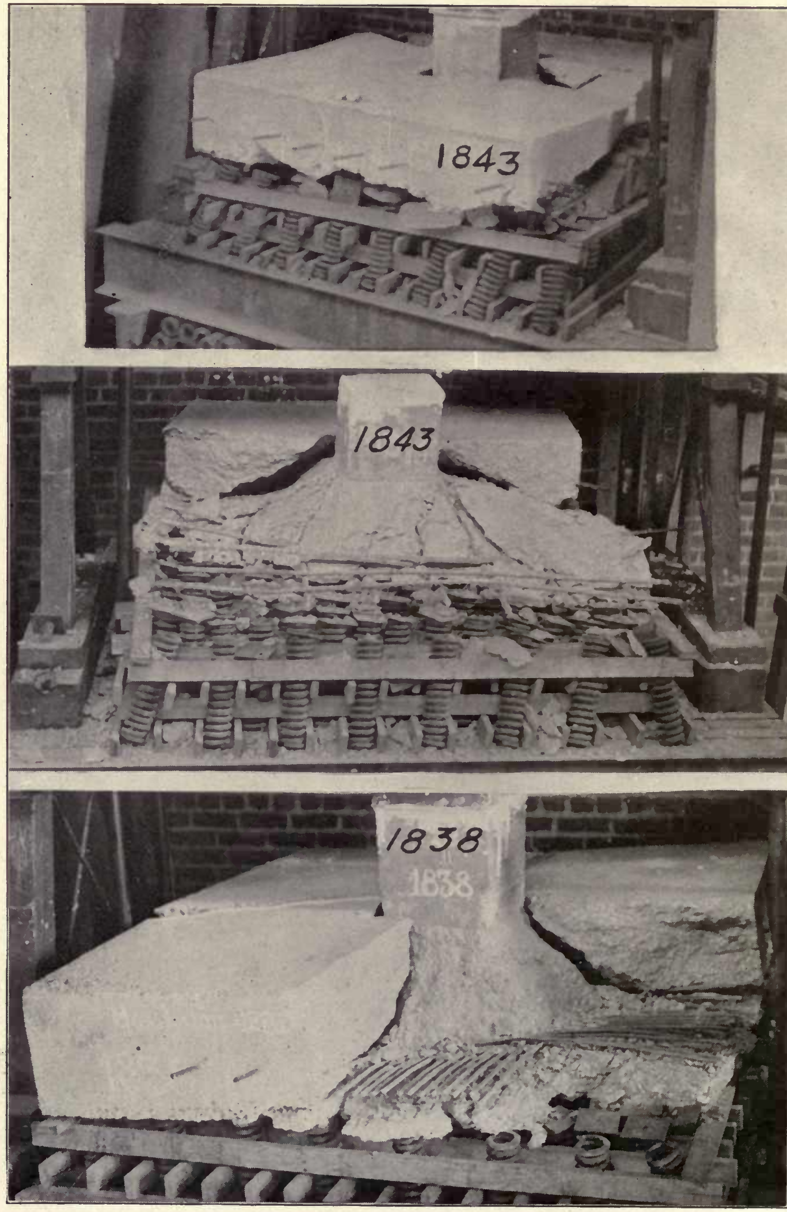


FIG. 31. VIEWS SHOWING COLUMN FOOTINGS AFTER FAILURE.

ments removed at 102 000 lb. At 218 000 lb. tension cracks formed at middle of third lateral face. At 225 000 lb. failure by diagonal tension, as shown in Fig. 30, the reinforcing bars stripping off.

No. 1552. Reinforced with square corrugated bars. First crack (tension) at 102 000 lb. on three lateral faces. Instruments removed at 177 000 lb., and at 198 000 lb. load was released, footing removed, additional springs put in, and the test was continued. Failure at 236 000 lb., sudden and violent and similar to No. 1551. Diagonal tension. The reinforcing stripped off and the footing broke across as a plain concrete footing.

No. 1553. Reinforced with 0.6% of corrugated bars in each of two directions. At 218 000 lb. the springs had closed and the test was discontinued and the footing removed from the machine. A month later with additional springs on the bed of the machine the test was completed. At 327 000 lb. the footing suddenly failed by diagonal tension. Highest load of any test.

No. 1554. First crack (tension) at 120 000 lb. Final failure by diagonal tension at 288 000 lb. Angle of face of fracture about 45° with the vertical.

No. 1561. Reinforcement laid in four directions. First crack (tension) at 138 000 lb. Load released twice to adjust machine. Failure gradual at 240 000 lb., the tension cracks on the four faces opening. Examination after failure showed that of the rods in the upper layer those close to the edges of the pier had slipped at the ends while those passing under the center of the pier and those close to the edges of the footing had not slipped. Some of the diagonal rods had slipped.

No. 1562. Reinforcement in four directions. First crack (tension) at 120 000 lb. Instruments removed at 198 000 lb. Failed at 210 000 lb. by gradual opening of tension cracks on east and west faces, followed after a slight falling off of load by a sudden shearing around pier. Of the several rods on each side examined after failure many showed conclusively that they had not slipped and the remainder were in such condition that nothing about slip could be determined.

No. 1563. Reinforced in four directions. First crack (tension) at 120 000 lb. Failure at 174 000 lb. by gradual opening of cracks on east and west faces, followed finally by shearing.

No. 1564. (See Fig. 27.) First crack (tension) at 138 000 lb. Failure at 210 000 lb. by gradual opening up of tension cracks on two opposite faces followed by shearing around pier. After failure it was found that of the rods in the bottom layer those in the north half only had slipped at the west end. Of the north and south rods (third layer from bottom) those in about the middle third had slipped at the south end. Of the diagonal rods in the second layer from the bottom those passing under the edges of the pier had slipped while those passing directly under the center of the pier had not slipped.

SERIES OF 1911.

No. 1806. At a load of 138 000 lb. fine cracks were noted on all faces of footing over reinforcing bars, extending upward about 5 in. At a load of 159 000 lb. the cracks opened somewhat. As the load was increased, these cracks opened very little and at a load of 179 000 lb. failure occurred gradually by bond.

No. 1807. At a load of 159 000 lb. the cracks which had formed on the north face and the east face were opening and extending. Another crack was noted on west face of footig 6 in. north of pier. As the load was increased the cracks lengthened and became more prominent and at a load of 198 000 lb. they were gradually opening up. Failure occurred slowly at a load of 210 000 lb., probably by tension.

No. 1808. At a load of 120 000 lb. a vertical crack 4 in. high was noted on north face of footing in line with the west face of the pier. At a load of 138 000 lb. cracks were noted at the center of the north, east, and west faces 5, 7, and 6 in. high, respectively. A crack 6 in. high was also noted on west side in line with south face of pier. At a load of 198 000 lb. the pier began to fail and the load was released, a new pier set in place, and the footing again loaded. Failure occurred at a load of 198 000 lb. by diagonal tension. The cube tests show that the concrete in this footing was not quite up to standard.

No. 1809. At a load of 219 000 lb. the cracks on the north face were gradually opening up and failure was imminent. Maximum load was 236 000 lb. Failure occurred by tension followed by diagonal tension. Pier punched through footing.

No. 1810. At a load of 138 000 lb. fine vertical cracks were noted on north face of footing in line with east face of pier, on west face in line with north face of pier 6 in. high, and on east face of footing at center 6 in. high. At a load of 158 000 lb. the cracks became more prominent. At a load of 178 000 lb. cracks were noted on north face 2 in. west of center 6 in. high, on west face at center and in line with north face of pier, and on south face in line with east and west faces of pier 6 in. high. At a load of 198 000 lb. cracks were noted on north face 2 in. east of center 6 in. high and on west face 6 in. south of pier 6 in. high. Failure occurred at a load of 219 000 lb. by diagonal tension.

No. 1811. At a load of 219 000 lb. cracks previously observed were extending but no new ones were noted. At a load of 261 000 lb. the cracks began to widen, one of the rods in the upper layer slipped at its west end and failure occurred suddenly by diagonal tension.

No. 1812. At a load of 159 000 lb. all the cracks were prominent. Failure occurred at a load of 171 000 lb. by bond. Pier finally punched through the footing. Examination afterward showed that middle third of both layers of bars had slipped ¼ in. at their end.

No. 1813. At a load of 121 000 lb. the cracks were opening and extending. It was difficult to maintain the load. Load was removed and the under side of footing examined and it was found that the bars had slipped. Bond failure.

No. 1814. At a load of 219 000 lb. the cracks were not opening very fast. As the load was increased to 260 000 lb. the cracks became more prominent but there were no signs of failure. At a load of 301 000 lb., the ends of bars were found to be slipping at holes cut into the concrete, and the cracks at reinforcing bars opened up gradually. Failure occurred suddenly at this load by bond and with great violence.

No. 1815. Cracks formed at reinforcing bars. At a load of 282 000 lb. the cracks had widened very little. Fine hair cracks were noted around some of the bars. Failure occurred at a load of 294 000 lb. by diagonal tension and bond.

No. 1816. As the load was increased to 138 000 lb. the cracks on the north face of footing opened up considerably and the rods slipped. Failure occurred by bond. Bars showed slip at three faces.

No. 1817. (See Fig. 30.) At a load of 159 000 lb. the bar at center running in a north and south direction had slipped and a crack at this bar had opened up considerably. Some of the bars at the west face of footing showed indications of slip. The load was released and cracks closed up very little. The footing was again loaded to 159 000 lb., the cracks opened up considerably, and failure occurred by bond. A number of the rods were observed to have slipped.

No. 1818. Reinforced with 3/8-in. round corrugated bars. Failure occurred at a load of 198 000 lb. by diagonal tension.

No. 1819. Reinforced with 3/8-in. round corrugated bars. At a load of 198 000 lb. a prominent crack was noted on north face 13 in. to west of pier 5 in. high. At this load the cracks on west side were opening up. As the load was increased the cracks opened gradually and failure occurred suddenly at a load of 261 000 lb. Failure due to tension followed by diagonal tension.

No. 1820. Reinforced with 1/2-in. square corrugated bars. As the load was increased small cracks were noted at the reinforcing bars on north, east, and west faces. Failure occurred at a load of 179 000 lb. by diagonal tension.

No. 1821. Reinforced with 1/2-in. square corrugated bars. Tested at an age of 30 days. At a load of 159 000 lb. failure occurred by diagonal tension. There was no indication that the bars had slipped. After failure the vertical cracks on sides of footing had practically closed. A prominent horizontal crack was visible at plane of bars.

No. 1822. Reinforced with 3/8-in. round rods. No rods under the pier. At a load of 102 000 lb. first crack was noted on north face of footing 6 in. east of center 8 in. high. At a load of 138 000 lb. a vertical crack was noted on west face 8 in. north of center. At a load of 158 000 lb. other cracks noted in the middle fourth of length on east, west, and north faces. Failure occurred gradually at a load of 198 000 lb. by tension in steel. Examination of bars showed that they had necked.

No. 1823. (See Fig. 27.) Reinforced in same manner as No. 1822. As the load was applied up to 159 000 lb. vertical cracks were noted in middle third of length on the north, east, and west faces of footing. At a load of 178 000 lb. the cracks opened considerably and a new crack was noted on east face 8 in. south of pier 6 in. high. At a load of 198 000 lb. the cracks opened up gradually and at a load of 210 000 lb. failure occurred. Tension failure. Examination of the bars showed necking, as is discussed on page 62.

SERIES OF 1912.

No. 1831. Cracks on north and south faces were higher than those in east and west faces, the lower layer of bars running north and south. At 153 000 lb., during the process of reading, the load fell off slightly. The maximum load was 161 000 lb. At this load the cracks on the north face had opened about ¼ in. while those on the east and west faces had not opened appreciably. Failure was gradual and probably by slipping of north and south rods.

No. 1832. (See Fig. 27.) Cracks were as shown in sketch. Graphs showing slip of bars indicate the critical point for slipping to have occurred at about 140 000 lb. At 184 000 lb. measurements showed that the rods continued to slip after the increase of load had been discontinued. At 192 000 lb., the maximum load, the rods on all faces could be seen to be slipping. Failure was by bond.

No. 1833. (See Fig. 27 and 30.) Cracks were as shown in sketch. At a load of 92 000 lb. the pier failed, no cracks having been previously noted. A 12-inch cube about 4 years old was put in its place, embedded in plaster of paris, and the test continued. Failure took place violently at a load of 113 000 lb. about 20 seconds after pumping of the jacks had been stopped. The main cracks after failure were found where none had been observed during the test. Examination after failure showed that the bars lying near the north face of the footing (bottom layer) had slipped about ½ in. and ¾ in. at their ends. No other slipping was apparent.

No. 1834. Cracks were as shown in sketch. Careful search for cracks was made at lower loads but none were found. Failure was sudden at 153 000 lb., developing an entirely new set of cracks. Those previously observed did not open appreciably. At this load most of the bars slipped from ½ in. to ⅞ in.

No. 1835. (See Fig. 28.) The cracks were as shown in the sketch. The first crack occurred under rod number 3, the point at which the measurements detected slip first. Failure was gradual and took place at a load of 184 000 lb. The cracks which formed at failure had not been observed previously. Rods slipped from ¼ in. to ½ in.

No. 1836. At a load slightly less than 211 000 lb. an accident to the apparatus interrupted the test. The measurements for slip showed about the same characteristics on the second test as on the first. Before the second test, the cracks observed in the first test had nearly closed. Failure was probably by bond.

No. 1837. (See Fig. 28 and 32.) At a load of 162 000 lb. the pier of this footing failed. After a 12-in. cube had been put in its place the test was continued. Cracks formed in the first test closed before the second test. Failure was by slipping of bars.

No. 1838. (See Fig. 31.) No cracks appeared except the four cracks shown in the sketch. These could not be detected at a load of 74 000 lb. but were found at 92 000. At 240 000 lb. the cracks had opened about 1/4 in. Failure was by tension followed by the slipping of all the rods at the maximum load of 247 000 lb. The slab portion of the footing broke into four separate pieces, each breaking from the pier at an angle of about 45° with the vertical as shown in Fig. 31.

No. 1839. First cracks were seen on north and south faces at the ends of the lower layer of bars. These cracks opened rapidly near maximum load and final failure came suddenly. Failure by tension followed by diagonal tension.

No. 1840. (See Fig. 28.) Just before failure the cracks at the middle of the faces had opened to a width of about 1/8 in. and a very large deflection of the center of the footing was apparent. At the time that the pier punched through the footing, failure seemed to be occurring also by crushing of the pier. Failure was by tension followed by diagonal tension.

No. 1841. (See Fig. 29.) At failure the first observed cracks had become about 1/16 in. wide. An examination after failure showed that although the bars were bent backward in vertical planes the ends of the four center bars in the top layer had slipped from 1/16 in. to 3/8 in. Of the bottom layer only the two center bars had slipped. The proximity of large vertical cracks probably is largely responsible for the slipping. The main slip apparently occurred at the time of the collapse of the footing. Measurement with caliper showed some indication of necking.

No. 1842. (See Fig. 28.) First cracks were seen at the ends of the upper layer of bars. Failure occurred at 203 000 lb. after this load had been held under continuous pumping about 2 minutes. Examination of bars with caliper indicated some necking. Failure probably by tension followed by diagonal tension. The pier punched through, the angle of fracture being about 60° with the vertical.

No. 1843. (See Fig. 31.) This footing was reinforced with eight 5/8 in. round corrugated bars in each layer. The first slip as indicated by the measurements was in a rod of the bottom layer which ended in the crack first observed. This rod passed under the pier very near one edge.

No. 1844. The reinforcement was the same as in No. 1843. The first cracks appeared simultaneously on the north and south faces at a load of 93 000 lb. The measurements indicate that the first slip occurred in the two rods ending in these two cracks and at the same end as that at which the cracks were noted. Slipping of the bars was visible to the eye at the maximum load and before complete failure.

One crack opened 1/16 in. at a load of 252 000 lb. and ⅛ in. at the maximum load. Failure was by bond followed by diagonal tension, though there are indications that the slip of part of the bars threw greater stress on other bars and that these bars may have been stressed beyond the yield point.

30. *Reinforced Concrete Column Footings: Tension Failures.*—As stated on page 73, the tension failures were marked by an appreciable opening of tension cracks at the lateral faces of the footing. The maximum load was generally maintained for some time under a steady pumping of the jacks, the edges of the footing meantime deflecting upward. In many cases these tension cracks appeared at a point on the lateral face of the footings in line with a face of the pier, and these cracks were found to extend entirely along the lower surface of the footing, passing through points immediately below the face of the pier referred to. In other cases the cracks were nearer the middle of the length of the lateral face, and either extended directly across the bottom surface or offsetted somewhat toward or directly over the face of the pier. As the crack extended upward, it sometimes became directed towards the junction of the face of the pier and upper surface of the footing, or it made a square turn at the corner. It was difficult to find the condition of these cracks in the interior, and it is evident that the cracks seen on the upper surface were the results of conditions which obtained after the maximum load was reached. No. 1412, Fig. 25, and No. 1431, Fig. 26, may be referred to as illustrations of the formation and direction of these cracks. Of course, it seems probable, as the bending up was greater along a middle or central section of the pier than along a section near a lateral face of the footing, that the tension cracks formed first in the interior and also that the bars in the interior reached their yield point before this stress was reached by bars at the lateral face. Possibly after the yield point was reached in the interior there was an adjustment of the stresses through the bars and more was taken by the rods near the lateral faces. The general phenomena of failure indicate that the resisting moment developed must have been greatest at a section passing through the face of the pier or else at a combination section through the part of the footing just below the face of the pier and across the remainder of the footing just a little back of this face, as shown in Fig. 8(a), page 18.

For failures by tension in reinforcement the loads carried were, in a general way, proportional to the amount of reinforcement, though in some cases the weaker footing of two companion test pieces failed at a load below what might be expected, due, no doubt, to some part of the footing receiving a larger proportion of the load than the remainder and for which the assumption of uniform distribution of load gives incorrect results. For footings with the heavy reinforcement, sufficient load was not carried to develop the yield-point strength of the reinforcement but instead failure was by bond or diagonal tension. It is evident that the amount of reinforcement which may be made effective is limited by the resistance to diagonal tension and bond stress which may be developed, and that bond and diagonal tension strength must be considered in the design of footings.

In footings with depths of 5 in., 7 in., and 10 in., the complication of bond and tension failures prevents the drawing of final conclusions, but there is nothing to indicate a difference in action for footings of different thicknesses or different relative lengths of projection.

31. *Reinforced Concrete Column Footings: Bond Failures.*—As outlined on page 83, the failures by bond were generally gradual failures, cracks first becoming visible on the lateral faces near the corners of those footings in which the reinforcement was spaced over the entire footing and the tension cracks in the middle of the lateral faces finally closing or closing when the load was released. It seems probable that cracks had formed somewhat earlier in the interior nearer the pier, the central bars slipping, and that after this slip of the central bars greater stress would be given to the outer bars, probably at points nearer their ends, and the bond stress at the ends of these outer bars would increase and finally slip would occur there. The result was a failure crack in a diagonal direction. After these cracks opened the bars were found to have slipped.

Values of the bond stress developed in the footings which failed by bond and in others developing high bond stresses, calculated by the method described on page 23, are given in Table 19. They are fairly consistent and are somewhat lower than values of bond stress derived from simple pulling tests. It must be borne in mind that the method of calculation is empirical and that the analysis does not apply to the arrangement of bars in exterior bands. Low values are explained by the nearness of the bar to the surface in some of the footings and by the formation of tension cracks across one set of bars, which cracks extended longitudinally along the other set of reinforcing bars and acted to loosen the bond, see Fig. 8 (c). The footings reinforced with ¾-in. round rods are especially noticeable in this respect.

Although the bond stresses developed in footings reinforced with

TABLE 19.

Values of Bond Stress Developed in Column Footings.

In the calculations, the bars within a width of 46 inches were considered, except as otherwise noted.

Footing No.	Reinforcement		Calculated Bond Stress lb. per sq. in.	Manner of Failure
	Kind	Per cent		
1413	Plain round bars	0.39	269	Bond
1414	do.	0.39	358	"
1415	do.	0.39	299	Bond, (bars staggered)
1416	do.	0.39	239	do.
1417	do.	0.88	206	Bond
1418	do.	0.88	226	"
1451	do.	0.39	131	Bond, (sloped footing)
1522*	do.	0.56	356	Bond with possible tension
1525§	do.	0.78	273	Bond
1526§	do.	0.78	368	Bond; tension probably imminent
1535	do.	0.59	372	Bond
1536	do.	0.61	279	"
1541§	do.	1.18	231	Bond; lack of concrete below bars
1542§	do.	1.23	340	Bond
1812	do.	0.39	315	"
1813	do.	0.39	223	"
1814	do.	0.72	310	"
1816	do.	0.41	293	"
1817	do.	0.41	353	"
1831	do.	0.41	357	"
1832	do.	0.41	425	"
1833‖	do.	0.41	152	"
1834‖	do.	0.41	206	"
1835‖	do.	0.41	248	"
1836‖	do.	0.41	285	"
1837	do.	0.41	198	"
1412	Plain round bars	0.28	314	Tension
1515	do.	0.39	347	"
1516	do.	0.39	317	Tension followed by bond
1521*a*	do.	0.58	352	Tension
1531	do.	0.59	357	Diagonal tension
1532	do.	0.59	321	do.
1806	do.	0.41	241	Probably bond
1807	do.	0.41	283	Tension
1815	do.	0.72	303	Diagonal tension and bond
1822‖	do.	0.41	266	Tension
1823‖	do.	0.41	282	"
1838	do.	0.41	254	Tension followed by sudden bond failure
1839†	do.	0.41	425	Tension followed by diagonal tension
1840†	do.	0.41	445	do.
1841‡	do.	0.41	412	do.
1842‡	do.	0.41	450	Tension
1552	Cor. square bars	0.42	415	Diagonal tension
1554	do.	0.62	346	do.
1820	do.	0.42	313	do.
1551	Cor. round bars	0.42	448	Diagonal tension
1553	do.	0.62	445	do.
1818	do.	0.39	279	do.
1819	do.	0.39	367	Tension followed by diagonal tension
1843	do.	0.41	507	Diagonal tension
1844	do.	0.41	596	Bond followed by diagonal tension

* Depth 7 in. to center of steel. § Depth 5 in. to center of steel. † Continuous bar looped. ‡ Bars curved up at ends. ‖ Reinforcement placed in exterior bands; $\frac{46}{60}$ of steel used in calculations. *a* Depth 6¾ in. to center of steel.

deformed bars were in some cases above those developed with plain bars, only one bond failure was found (No. 1844), the calculated bond stress being 596 lb. per sq. in.

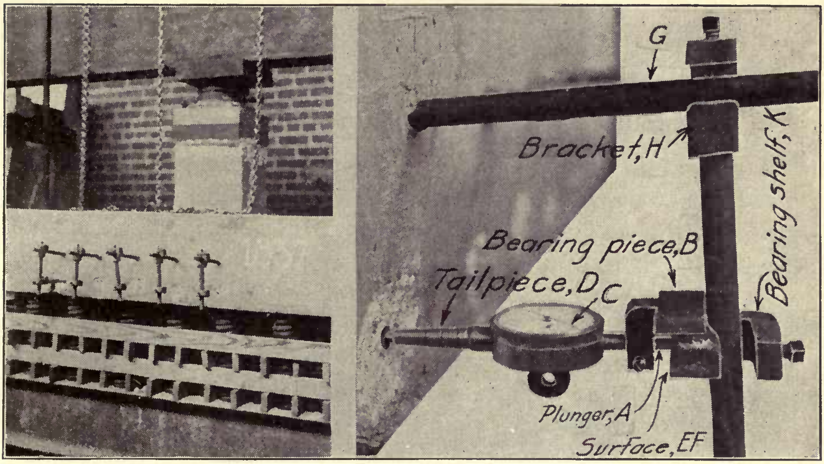


FIG. 32. APPARATUS FOR DETERMINING END SLIP OF BARS.

In order to determine definitely when first slip occurs at the end of the bars and whether bond is likely to be a primary cause of failure, a device was used in some of the 1912 tests by means of which an end movement of the bar as small as 0.0001 in. was measured with considerable certainty. This movement was determined by measuring the change in distance between the end of the reinforcing bar and a point in line with its axis and about 5½ in. from the face of the footing. The apparatus is shown in Fig. 32. The measuring instrument consists of an Ames gage micrometer equipped with a pointed tail piece D and a bearing piece B. Movement of the pointer C indicates change in distance between the point of the tail piece and the end of the micrometer plunger A. In operation the point of the tail piece is inserted in a small hole drilled in the end of the reinforcing bar, and the end of the plunger is brought to bear at a definite point upon the surface EF which has a fixed position relative to the face of the footing. The position of the surface EF is maintained by means of the auxiliary rod G, the cast-iron bracket H, and the bearing shelf K. The auxiliary rod G is embedded a short distance in the concrete at the time of pouring the concrete and the other parts are put in place at the time of the test. In order that the end of the plunger shall always have contact with the same point of the surface EF, two conical contact points attached to the under side of the bearing piece serve to insure that the bearing of the plunger of the micrometer shall always be in the same horizontal element of the bearing surface while the engagement of one of these contact points in a groove of the bearing shelf insures that the bearing shall always be in the same vertical element. The bearing surface EF is sufficiently curved to insure pressure against the plunger spring while the instrument is being seated. Hence accuracy of results does not depend upon the plunger being forced into position by the stiffness of the spring. In the view at the left of Fig. 32, means of measuring slip on five bars are shown. The micrometer, a movable instrument, is shown in place against one of these bars.

By means of this instrument measurements of slip were taken on footings No. 1832, 1834, 1835, 1836, 1837, 1838, 1843, and 1844. It was found that with careful handling of the instrument results could be obtained which among themselves appear fairly consistent. Fig. 33, 34, and 35 give representative results showing the slip at the end of the bar. The position of the bars on which measurements were taken is shown by the letters and numbers on the diagrams of column footings given in Fig. 25-29. In a few instances there appears to be a progressive movement of the bars in the wrong direction for slip, and this indicates the possibility that some warping of the face of the footing may have been mistaken for movement of the bars. It seems unlikely that this would be of much importance in the results, since the slip was usually quite pronounced after it began, and since later observations usually strengthened the conclusion that slip actually began at the point where the slip curve makes a sharp bend to the right.

The tests of 1909, 1910, and 1911 had indicated that bond stress in column footings is an important consideration and that the formation of tension cracks in the footing must go along with a loosening of the bars which are parallel to these cracks, thus hastening bond failure. In the 1912 tests in which slip was measured, a careful record of cracks was kept, and these tests show an intimate relation between the formation of cracks on the lateral face of the footing and the slipping of the bars. Table 20 records the position of the cracks at the face of the footing and the position of the bars which give end slip, together with the corresponding loads. The position of the bar may be identified by reference to Figs. 27, 28, and 29, pages 80 to 82. In seven out of the eight footings in which observations were taken with the instrument the measurements showed slip. Of these seven, first slip at the end of bar occurred in four footings in bars which end at points where the first crack was detected. In one of the remaining three, first slip occurred where the second crack was detected. In the other two foot-

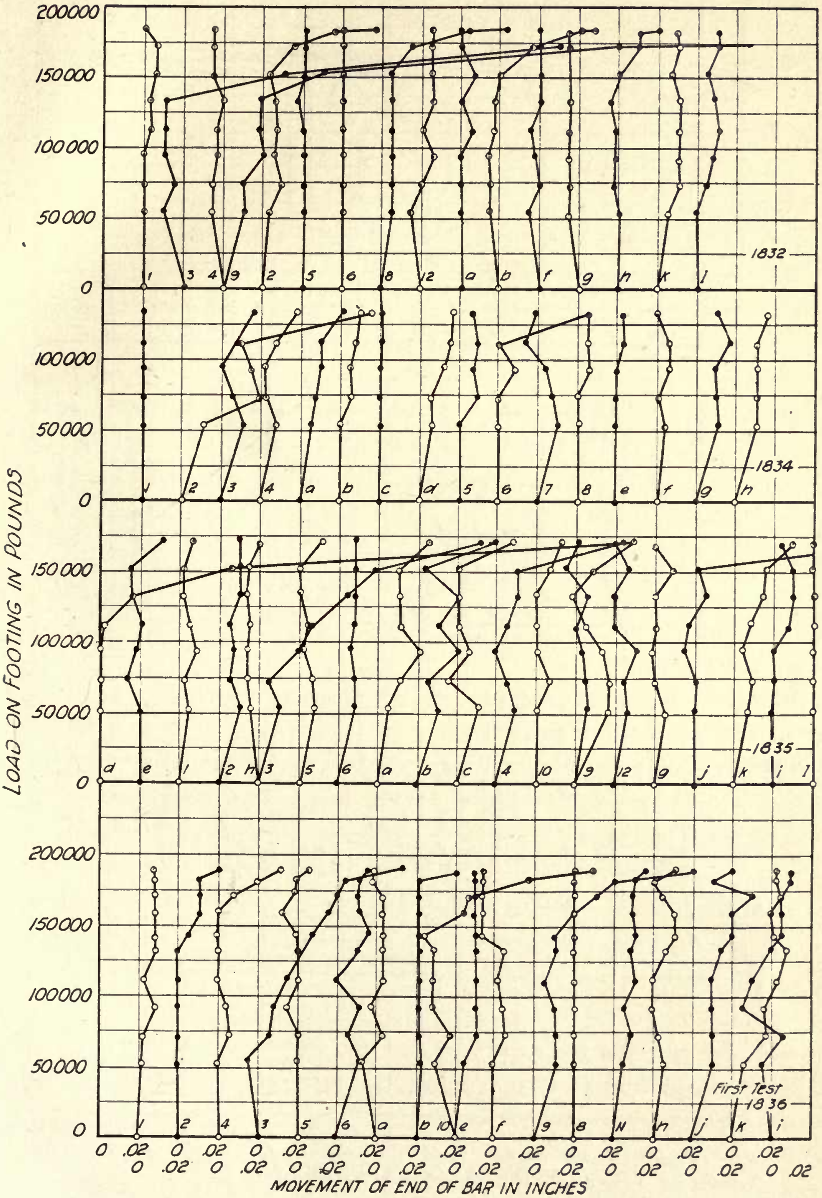


FIG. 33. DIAGRAM SHOWING END SLIP OF BARS.

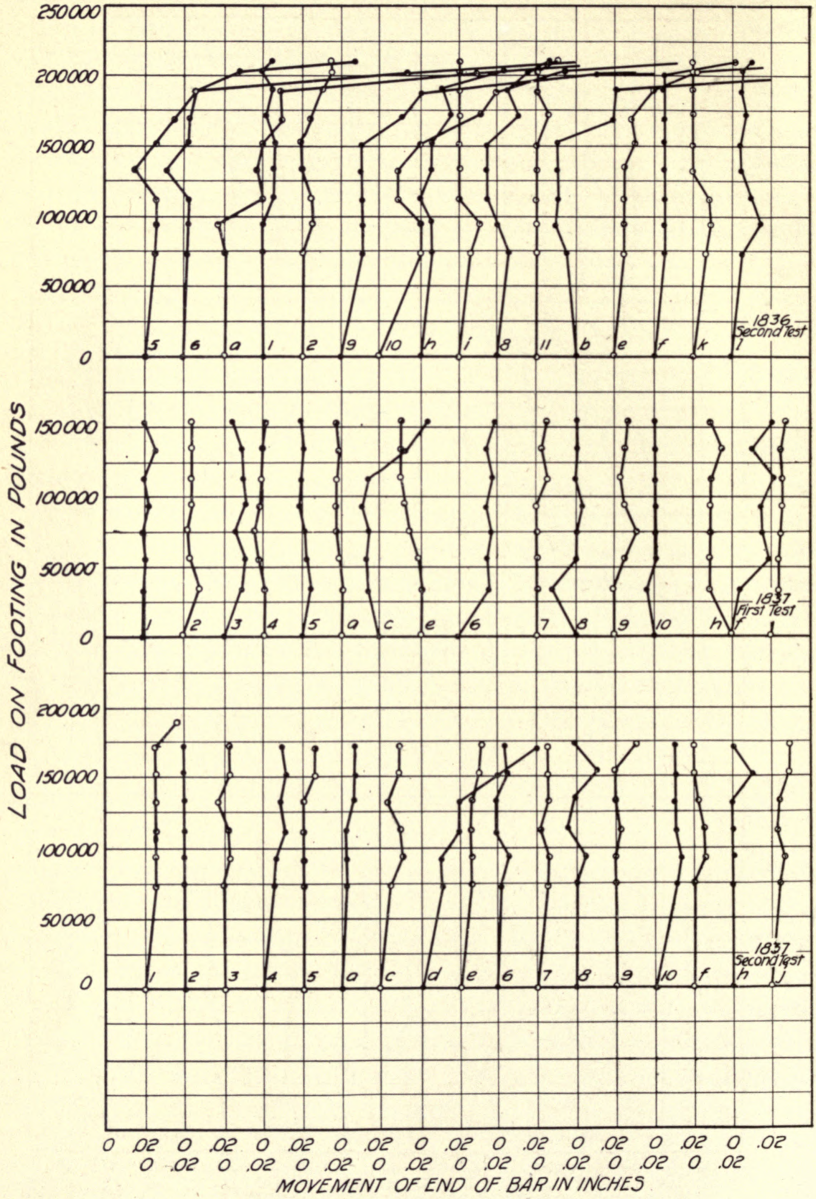


FIG. 34. DIAGRAM SHOWING END SLIP OF BARS.

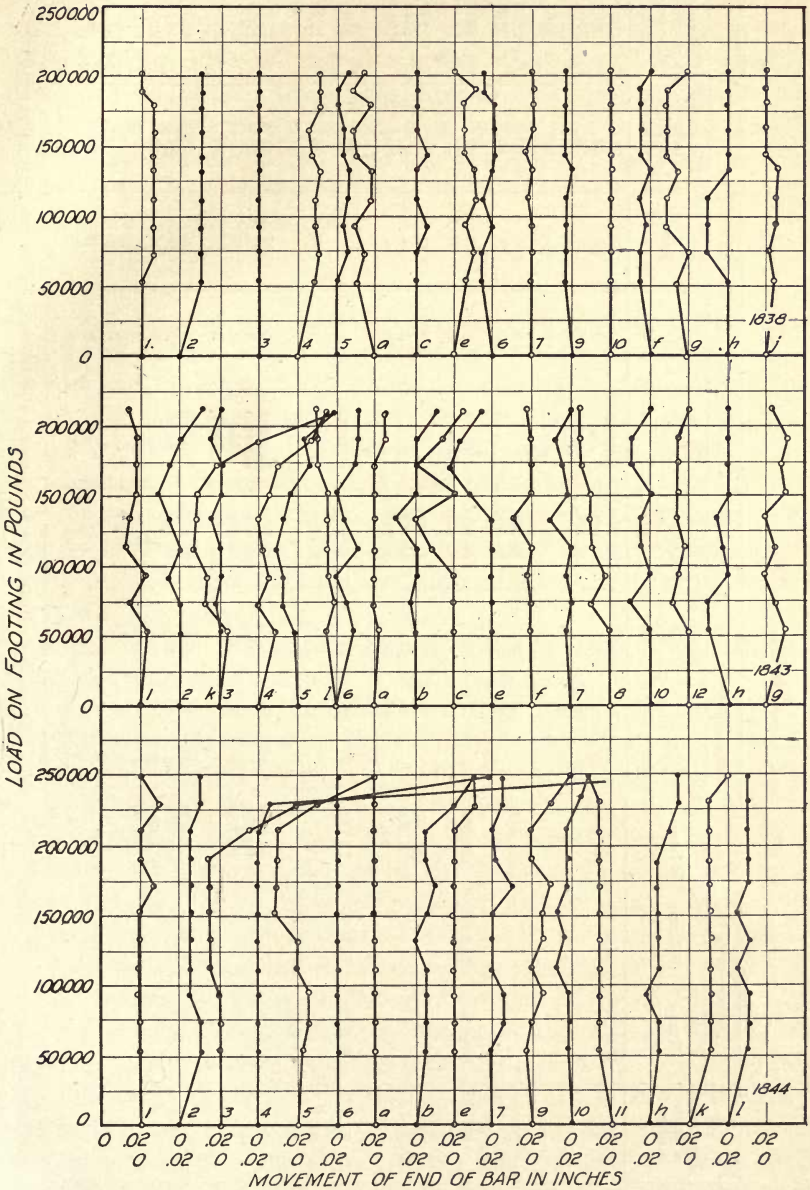


FIG. 35. DIAGRAM SHOWING END SLIP OF BARS.

TABLE 20.

Location of Cracks and Slips at Faces of Column Footings

No.	Diameter of Bars inches	Direction of: Top Layer	Direction of: Bottom Layer	First Cracks: Load pounds	First Cracks: No. of Bar	First Cracks: Face	First Cracks: Distance from Left Edge inches	End Slip: Load pounds	End Slip: Layer	End Slip: No. of Bar	End Slip: Face	End Slip: Distance from Left Edge inches	Calculated Bond Stress lb. per sq. in.
1832	⅝	E & W	N & S	93 000	3	N	26	134 000	Bottom	3	N	26	295
				114 000	9	S	34	134 000	do.	9	S	34	295
				do.	10	S	34	152 000	do.	2	S	18	335
				do.	b	W		do.	Top	b	W	18	335
								do.	do.	8	N	18	335
1834	⅜	......	......	114 000		N	40	113 000	......	2	N	10	252*
						N	18	113 000	......	6	S	10	252*
1835	⅜	N & S	E & W	114 000	3	N	20	74 000	Top	3	N	20	100*
								95 000	Bottom	d	W	20	128*
1836	⅜	......	......	114 000		N	28	54 000	......	3	N	20	72*
				132 000		S	24	144 000	......	9	S	20	194*
				do.		S	36	144 000	......	10	S	40	194*
				do.	3	N	18						
1837†	⅜	N & S	E & W	73 000	d	W	24	112 000	Bottom	c	W	30	116
1837‡	⅜	......	......	......	..	..	....	120 000	Bottom	d§	W	24	124
1838	⅜	N & S	E & W	......	..	..	....	‖	......	....	...	....	....
1843	⅝ *a*	E & W	N & S	53 000	e	W	41¼	113 000	Bottom	5	N	41¼	252
				93 000	3	N	26¼	133 000	Bottom	4	N	33¼	295
				93 000	9	S	26¼	152 000	do.	3	N	26¼	337
								152 000	do.	2	N	18¾	337
1844	⅝ *a*	E & W	N & S	93 000	3	N	26¼	190 000	Bottom	3	N	26¼	420
				93 000	9	S	26¼	211 000	do.	9	S	26¼	467
				113 000	5	N	41¼	do.	do.	5	N	41¼	do.
				do.	4	N	33¼	do.	do.	4	N	33¼	do.
				do.	10	S	3¾	do.	do.	10	S	33¼	do.
								do.	do.	7	S	3¾	do.
								do.	Top	b	W	18¾	do.
								do.	do.	e	W	41¼	do.

* Bars placed in bands near edges of footing. † First test. ‡ Second test. § Readings at *d* not taken in first test. First slip occurred here in second test.
‖ Readings showed no slip until failure. *a* Corrugated round bars. All others plain round.

ings there seemed to be some relation between the formation of these cracks and the slipping of bars but the connection was not so close. In footing No. 1844 the first five points where slip occurred were coincident with the first five cracks detected. It may be expected that the loosening effect would begin as soon as any load is applied, and slip of bars was observed in two cases before any cracks had been detected. In all other cases a crack was found at a load lower than that at which slip occurred at the same point.

In all but one instance the bar in which first slip occurred was a bar which was located nearer the edge of the pier than any other on which measurement was taken. This was true whether the bars were spaced over the whole footing, grouped in the space between the edge of the pier and the edge of the footing, or confined to single bands somewhat wider than the pier. It may be noted then that the bar which showed the most marked tendency to slip lies in the vertical section for which the stress in the bars at right angles to those under consideration appears to be a maximum. Evidently stress in one system of bars tends to reduce the bond resistance of the bars at right angles, and the results are in keeping with the assumption that the critical section is at the face of the pier.

Attention should be called to the fact that the method of calculation of bond stress is not applicable to footings in which the reinforcement is placed in exterior bands, as is indicated by the very low values for No. 1834, 1835, and 1836. Footing No. 1837, in which the reinforcement was placed in central bands, was loaded twice, the first time to a load of 162 000 lb. and the second time to failure which occurred at 192 000 lb. On the first loading bar c gave indications of slip at 110 000 lb., and on second loading it showed no slip but bar d gave indications of slip at 120 000 lb. No other bar gave slip measurement. A crack had formed along bar d at a load of 73 000 lb. In the companion footing, No. 1838, no slip was observed until failure at a load of 247 000 lb.

32. *Reinforced Concrete Column Footings: Diagonal Tension Failures.*—The four faces of fracture found in the failures here named diagonal tension failures extended from the pier at the top of the footing at an angle of about 45° with the vertical to the bottom surface of the footing, forming a frustum of a square pyramid having the corners or edges rounded off somewhat. As the diagonal tension cracks would begin at or above the longitudinal reinforcement it seems a reasonable procedure to take as a measure of the diagonal tension stress the vertical shearing stress obtained by using the vertical sections located at a

TABLE 21.

VALUES OF VERTICAL SHEARING STRESS DEVELOPED IN COLUMN FOOTINGS

Footing No.	Reinforcement		Calculated Vertical Shearing Stress lb. per sq. in.	Manner of Failure
	Kind	Per cent		
1447	Plain round bars	0.31	130	Diagonal tension
1448	do.	0.31	110	do.
1531	do.	0.59	180	do.
1532	do.	0.59	162	do.
1808	do.	0.50	124	do.
1810	do.	0.50	148	Tension followed by diagonal tension
1811	do.	0.61	164	Diagonal tension
1435	Cor. square bars	0.61	134	do.
1436	do.	0.67	113	do.
1552	do.	0.42	149	do.
1554	do.	0.62	185	do.
1820	do.	0.42	111	do.
1821	do.	0.42	99	do.
1551	Cor. round bars	0.42	142	do.
1553	do.	0.62	211	do.
1818	do.	0.39	123	do.
1843	do.	0.41	141	do.
1521*	Plain round bars	0.58	158	Tension
1522†	do.	0.56	156	Bond and possible tension
1525‡	do.	0.78	144	Bond
1526‡	do.	0.78	194	Bond; tension probably imminent
1542‡	do.	1.23	222	Bond
1807	do.	0.41	130	Tension
1809	do.	0.50	148	Tension followed by diagonal tension
1814	do.	0.72	192	Bond
1815	do.	0.72	187	Diagonal tension and bond
1823	do.	0.41	131	Tension
1836	do.	0.41	132	Bond
1838	do.	0.41	154	Tension followed by sudden bond failure
1839	do.	0.41	119	Tension followed by diagonal tension
1840	do.	0.41	125	do.
1841	do.	0.41	116	do.
1842	do.	0.41	126	Tension
1819	Cor. round bars	0.39	162	Tension followed by diagonal tension
1844	do.	0.41	167	Bond followed by diagonal tension

*6¾ in. to center of steel. †7 in. to center of steel. ‡5 in. to center of steel.

distance from the face of the pier equal to the depth of the steel reinforcement from the upper surface of the footing and to use as a length of section the four sides of the square base thus formed. The external vertical shear at this section would be the amount of load or upward pressure on the footing outside of this square base. The procedure in getting a measure of the diagonal tension is analogous to that used in ordinary beams, and the position of the section is analogous to that used in wall footings described on page 63. The formula for the vertical shearing stress already given on page 24 is

$$v=\frac{V}{4(a+2d)jd}.$$

Values of the vertical shearing stress thus calculated are given in Table 21. The values found seem to be fairly consistent with the results obtained in beam tests. Higher values are noticeable with the larger percentage of reinforcement. This perhaps is explainable by the greater stiffness given by the larger reinforcement, as has been noted in the results for diagonal tension in ordinary beams. (See Bulletin No. 29.) The values obtained with the deformed bars were not greatly different from those with the plain rounds.

Attention should be called to the probability that the method here used of placing the critical section for diagonal tension may not be applicable in the case of stepped and built-up footings.

33. *Disposition of Reinforcing Bars.*—A variety of arrangement of reinforcing bars was used. In the two-way reinforcement the usual disposition of bars was to space uniformly across the full width of the footing, but in some cases a closer spacing was made across the middle portion and the remaining bars were spaced farther apart. Footings were also made with the bars spaced uniformly over a width somewhat greater than the width of the pier and no bars outside of this, thus making what may be considered as two central beams. In other footings the bars were placed in bands at the outer edge of the footing with no bars in the interior. In one set bars of a shorter length were used, and these were staggered in such a way that alternate bars ended near the face of the footing. Four-way reinforcement was also tried as shown in Fig. 26 and 27.

Uniform spacing of bars was used in the effort to determine the proportion of the reinforcement which may be considered to be effective in resisting the calculated bending moment or the amount to be used in the calculations to determine the stress in the most stressed bars. Judging from the calculated stresses in the footings of this kind which failed by tension in the steel, for footings of the proportions tested, about three-fourths of the steel is effective in resisting the calculated bending moment, or rather the stress in the highest stressed bars is the same as if three-fourths of the steel bars, equally stressed, made up the resisting steel.

The footings which had the reinforcement placed in the form of two central beams carried high loads. In No. 1837 the bond stress was the critical stress and in No. 1838 (which was made of unusually strong concrete, the test cubes giving an average strength of 3710 lb. per sq. in. at an age of 109 days) the yield point of the steel was exceeded and this was followed immediately by a bond failure. It seems probable that the bars were stressed nearly equally. It is obvious that with the central beam construction the corners outside the main reinforcement must have the strength to carry the loads coming on this portion. In the case of No. 1838 it would seem that the tensile deformations in the concrete in the corner squares must have been very high. In fact, from another point of view, it does not seem probable that the steel in the bands could have reached its yield point at a section through the face of the pier without the concrete in the unreinforced space near it being stretched far beyond the ordinary limit of deformation of plain concrete. It seems probable that cracks would form in this space and that these cracks would preclude the development of the resistance needed in the corner square. The action in this portion of the footing warrants further study.

The footings in which the reinforcement was placed in outer bands and which had no reinforcement under the piers were made with a view of getting light on general footing action; it was not expected that this would be an effective way of placing reinforcement. It will be noted that No. 1822 and 1823 carried as high a load as the two footings with evenly spaced reinforcement made the same year (1911), No. 1806 and 1807. Similarly, in the series of 1912, No. 1835 and 1836 carried as high loads as No. 1831 and 1832, but No. 1833 and 1834 carried smaller loads. Of course the conditions for bond resistance were worse for the $\frac{5}{8}$-in. bars. The calculations for stresses given in Tables 17 and 18 for the footings with outer bands were made by using the same proportion of the bars as effective as was used for the footings with evenly spaced reinforcement ($\frac{46}{60}$), merely as a means of comparison and for want of any definite method of calculation which would be applicable to this kind of spacing, and the calculated bond stresses for the different dispositions of bars may not be comparable. It is seen that this method of calculation does not deal with the bending moment about a diagonal of the footing which for reinforcement in exterior bands may become an important consideration. The tests do not give information on the relation between the stresses in the different bars of any band. The tests bring out two points of interest: (1) the loads carried with this disposition of the reinforcing bars are large in comparison with what might be judged from the ordinary analyses and discussions which have appeared in engineering literature; and (2) there is seemingly a greater tendency to failure by bond when the bars are placed in bands near the edge of the footing, as is shown by the results in the bond failures in No. 1833, 1834, 1835 and 1836. The latter condition may result from a concentration of bond stress near the end of the bars. It goes to show the difficulties connected with the calculation of bond stress in slabs. The location of the bars in which slip was first detected has already been discussed under the head of "Bond Failures."

A form of footing in which short bars are placed with their ends staggered, as shown in Fig. 25, is in line with designs which have been used in practice. This arrangement of bars is defended on the ground that there is the full amount of steel at the critical section and that there is no need of carrying all the bars to the face of the footing. In No. 1415 and 1416 one end of the bar extended to within 3 in. of the face of the footing while the other end was 12 in. from the face, the next bar alternating in position with this. As was to be expected this arrangement gave less bond resistance, and the footings failed by bond at lower loads than those in which the bars were made full length. It is hardly necessary to make the comment that this form of construction is not good practice, especially when the dimensions are such that resistance to bond stresses forms an essential part of the strength of the structure.

In the footings with the reinforcement placed in four directions (four-way reinforcement), the total weight of steel in the diagonal direction in the 1909 tests was made about the same as that in the other directions. In the calculations for No. 1447, 1448, and 1449, for want of a better method, the bending moment has been computed by the usual methods of the bulletin and one-half of this bending moment has been considered to be taken by one set of rectangular reinforcement. In the 1910 footings a larger amount of steel was used. Of these, No. 1561 carried a very high load. The significance of the results is obscured by the variety of manner of failure (bond, diagonal tension, and tension) and by variations in the quality of concrete, and a comparison with two-way reinforcement on the basis of load carried would not be of value. This type of distribution of reinforcement should receive further attention, and tests may well involve the measurement of deformation in the reinforcing bars.

The footings having the reinforcing bars looped in a horizontal plane (No. 1839 and No. 1840) developed high calculated bond stresses. Those having the reinforcing bars bent upward and backward in vertical planes (No. 1841 and No. 1842) also failed by tension, but it will be seen that the opening of vertical cracks will tend to reduce the effect of this kind of anchorage.

IV. Summary

34. *Wall Footings.*—The tests of wall footings cover a variety of reinforcement. The method used to secure a distributed upward pressure introduced difficulties in testing. It also made it difficult to determine the load which should be taken as the critical load, and the loads which have been so specified may not always be the true critical load. The use of the bed of springs on the whole proved very satisfactory and is probably the best available arrangement for tests of the number and range used. The tests bring out phenomena which might not be apparent from analytical considerations alone or which might not be accepted without physical verification. Variations in concrete add to the complications encountered in analyzing such a series of tests. The tables and diagrams and discussions present information and data of the tests in a detailed way. The following statements summarize in a general way some of the points which are brought out by the tests and which have a bearing upon the principles and methods of design:

1. Wall footings under load follow the general laws of flexure. The section for maximum moment, the critical section for calculation of vertical shearing stress for use in judging of resistance to diagonal tension, and the method of calculating bond stress received experimental consideration.

2. The values of the modulus of rupture found in the unreinforced concrete footings are not far from the values of modulus of rupture obtained in simple beam tests such as the control beams. Increasing the richness of the mixture gives the added strength which tests of simple beams would lead us to expect. Variations in the tensile strength of concrete are to be expected, and considerable variation was found in the moduli of rupture of the test pieces, the variation being augmented by differences incident to the method of testing. The tests on footings of different lengths, undertaken to determine whether the section at the face of the wall should be used for the critical section, do not disclose any marked differences in modulus of rupture.

3. The results of the tests and the measurements of deformation of the reinforcement indicate that the critical section may be considered to be at the face of the wall and that the calculated tensile stress in the bars at this section is probably somewhat above the maximum tensile stress developed. Whether the maximum compressive stress may properly be calculated in the same way was not determined. It may be expected that high compressive stresses exist at the intersection of

wall and projection. Indications of high compression and of incipient compression failure were found at the intersection of the wall and footing at loads above the critical loads.

Test pieces in which the wall was poured after the footing had taken its set, gave results which indicate that a section at the face of wall may properly be used in calculations of moments even when the wall is to be poured separately from the footing.

4. The calculations for bond stress, based upon the total external vertical shear at the section at the face of the wall and calculated by equation (17), evidently give stresses higher than the existing stresses. This is shown by the fact that the values calculated in this way are higher than those found in pull-out tests and beam tests. A study of the analytical conditions existing at this section tends to confirm the statement. However, as bond resistance is so important a strength element in a short cantilever beam, this method of calculation and the use of the working value of bond stress ordinarily assumed in design seems only reasonably conservative and may be recommended for general practice. Attention may properly be called to the importance of making calculation of bond stress in wall footings and other beams in which the length is short relatively to the depth. The advantage of using relatively small bars in such cases is also apparent.

Anchorage of bars by bending upward and back in a long curve or by looping the bar in a horizontal plane was found to add materially to bond resistance.

5. The tests indicate that the vertical shearing stresses developed at the face of the wall, calculated by the usual method, are higher than the vertical shearing stress which is found to exist in simple beams with concentrated loading when diagonal tension failures are developed. It was found that diagonal tension failures start at a point some distance away from the section at the face of the wall. This observation and certain analytical considerations such as the probable greater proportion of shear taken in the compression area at sections near the face of the wall show that, in calculating the vertical shearing stress which shall be used as a basis for judging the resistance to diagonal tension, a section some distance from the face of the wall should be used. The tests and the discussion indicate that a section d distant from the face of the wall (d being the distance from center of reinforcing bar to top of footing) may properly be used as the critical section for calculating the vertical shearing stress for this purpose, and that at this section the ordinarily accepted working stress may properly be used for calculating resistance to diagonal tension failure.

6. The bending up of bars at several points along the length of the projection gave added resistance against diagonal tension failure. Vertical stirrups also added to the resistance against diagonal tension failure but were not especially effective. Neither method of web reinforcement would be very convenient in construction. Generally speaking, it will be best to try to design the footing so that the vertical shearing stresses will be within the limit of the working stress permitted in beams without web reinforcement, and thus avoid the use of web reinforcement. In large important footings, when diagonal tension is a critical element, it would seem that some kind of unit frame with well-formed web reinforcement would be preferable to placing stirrups or to bending up bars at the necessary intervals. In stepped and sloping footings attention should be called to the larger diagonal tension and bond stresses developed. The increase in these stresses over those found in footings of uniform depth may be sufficient to decide against the use of stepped and sloping footings.

7. The footings having I-beams embedded in the concrete carried high loads, perhaps corresponding to the yield-point tensile strength of the lower flange of the I-beams and more than double what would be carried by naked I-beams. The weight of the I-beams, of course, was greater than that of the reinforcing bars used in the reinforced concrete wall footings.

35. *Column Footings.*—The requirement of uniform load and the presence of double-curved flexure complicate an investigation of column footings. In this investigation methods of testing were developed. As these are presumably the first tests on column footings, the phenomena of the tests and data of their action will be of interest to designers, especially in the directions in which tests have brought out weaknesses not always recognized and usually not guarded against. The results contribute data toward the settlement of methods of calculating of both the bending moment and the resisting moment for square footings, and the principles may with care be extended to other forms. The results may not easily be summarized, but the following statements are intended to cover the principal matters brought out in the tests:

1. A square column footing under load may be expected to take a bowl-shaped form. In slabs subject to bending in two directions, the stress in a fiber can not differ from that in an adjoining fiber at the same level without setting up longitudinal shear; and as there is considerable resistance to variation from equality of stress in adjoining fibers, it may be expected that in stiff thick pieces (as are footings

of ordinary design, where the thickness is large in comparison with the length of the projection) the deformations and consequent stresses will be distributed over the width of a cross section and that considerable stress will be developed even in the fibers at the edge of the footing.

2. For footings having projections of ordinary dimensions, the critical section for the bending moment for one direction (which in two-way reinforced concrete footings is to be resisted by one set of bars) may be taken to be at a vertical section passing through the face of the pier. In calculating this moment, all the upward load on the rectangle lying between a face of the pier and the edge of the footing is considered to act at a center of pressure located at a point half-way out from the pier, and half of the upward load on the two corner squares is considered to act at a center of pressure located at a point six-tenths of the width of the projection from the given section. By equating this bending moment and the resisting moment which is available at the given section, the maximum tensile stress in the concrete or in the reinforcing bars may be calculated.

3. As is usually the case when plain concrete is used in flexure, the unreinforced footings show considerable variation in results. The variations were such as not to permit a method of determining the effective width of resisting section to be established or to obtain a formula for resisting moment. Based upon the full section of the footing, the moduli of rupture obtained were considerably less than the moduli of rupture of control beams made with the same concrete.

4. In reinforced concrete column footings, resistance to non-uniformity of stress in adjoining bars will be given by bond and by longitudinal shear in the concrete, and the amount of variation from uniformity of stress in the various bars will depend upon the spacing of the bars as well as upon the relative dimensions of the footing. With two-way reinforcement evenly spaced over the footing, it seems that the tensile stress is approximately the same in bars lying within a space somewhat greater than the width of the pier and that there is also considerable stress in the bars which lie near the edges of the footing. For intermediate bars stresses intermediate in amount will be developed. For footings having two-way reinforcement spaced uniformly over the footing, the method proposed for determining the maximum tensile stress in the reinforcing bars, is to use in the calculation of resisting moment at a section at the face of the pier the area of all the bars which lie within a width of footing equal to the width of pier plus twice the thickness of footing, plus half the remaining distance on each

side to the edge of the footing. This method gives results in keeping with the results of tests. When the spacing through the middle of the width of the footing is closer, or even when the bars are concentrated in the middle portion, the same method may be applied without serious error. Enough reinforcement should be placed in the outer portion to prevent the concentration of tension cracks in the concrete and to provide for other distribution stress.

5. The method proposed for calculating maximum bond stress in column footings having two-way reinforcement evenly spaced, or spaced as noted in the preceding paragraph, is to use the ordinary bond stress formula, and to consider the circumference of all the bars which were used in the calculation of tensile stress, and to take for the external shear that amount of upward pressure or load which was used in the calculation of the bending moment at the given section.

An important conclusion of the tests is that bond resistance is one of the most important features of strength of column footings, and probably much more important than has been appreciated by the average designer. The calculations of bond stress in footings of ordinary dimensions where large reinforcing bars are used show that the bond stress may be the governing element of strength. The tests show that in multiple-way reinforcement a special phenomenon affects the problem and that lower bond resistance may be found in footings than in beams. Longitudinal cracks form under and along the reinforcing bar due to the stretch in the reinforcing bars which extend in another direction, and these cracks act to reduce the bond resistance. The development of these cracks along the reinforcing bars must be expected in service under high tensile stresses, and low working bond stresses should be selected. An advantage will be found in placing under the bars a thickness of concrete of two inches, or better three inches, for footings of the size ordinarily used in buildings.

Difficulty may be found in providing the necessary bond resistance, and this points to an advantage in the use of bars of small size, even if they must be closely spaced. Generally speaking, bars of ¾-in. size or smaller will be found to serve the purpose of footings of usual dimensions. The use of large bars, because of ease in placing, leads to the construction of footings which are insecure in bond resistance. In the tests the column footings which were reinforced with deformed bars developed high bond resistance. Curving the bar upward and backward at the end increased the bond resistance, but this form is awkward in construction. Reinforcement formed by bending long bars in a series of horizontal loops covering the whole footing gave a footing with high bond resistance.

6. As a means of measuring resistance to diagonal tension failure, the vertical shearing stress calculated by using the vertical sections formed upon the square which lies at a distance from the face of the pier equal to the depth of the footing was used. This calculation gives values of the shearing stress, for the footings which failed by diagonal tension, which agree fairly closely with the values which have been obtained in tests of simple beams. The formula used in this calculation is $v=\frac{V}{bjd}$, where V is the total vertical shear at this section taken to be equal to the upward pressure on the area of the footing outside of the section considered, b is the total distance around the four sides of the section, and jd is the distance from the center of reinforcing bars to the center of the compressive stresses. This stress is somewhat larger than the average vertical shear over the section which is sometimes used. The working stress now frequently specified for this purpose in the design of beams, 40 lb. per sq. in., for 1-2-4 concrete, may be applied to the design of footings.

The punching shear may be calculated for the vertical sections which inclose the pier footing, although it may be expected that shear failure may not be produced exactly on this section. The value now generally accepted for punching shear, 120 lb. per sq. in. for 1-2-4 concrete, may be used for the working stress in this case.

7. No failures of concrete in compression were observed, and none would be expected with the low percentages of reinforcement used. The compressive stresses in the pier of the footing were in some cases very high and in a few instances the pier failed and was replaced by a cube of concrete. In frequent cases there were signs of distress near the intersection of pier and footing where there is an abrupt change in direction of surfaces and where the combined stresses are very high.

8. In stepped footings, the abrupt change in the value of the arm of the resisting moment at the point where the depth of footing changes may be expected to produce a correspondingly abrupt increase of stress in the reinforcing bars. Where the step is large in comparison with the projection, the bond stress must become abnormally large. It is evident that the distribution of bond stress is quite different from that in a footing of uniform thickness. The sloped footing also gives a distribution of stress which is different from that in a footing of uniform thickness. However, for footings of uniform thickness the bond stress

is a maximum at the section at the face of the pier; in a sloped footing the bond stress at the section at the face of the pier would be less accordingly than in a footing of uniform thickness, and a moderate slope may be found to distribute the bond stress more uniformly throughout the length of the bar. This is not of advantage if the full embedment of the bar is effective in resisting any pull due to bond.

9. The use of short bars placed with their ends staggered increases the tendency to fail by bond and cannot be considered as acceptable practice in footings of ordinary proportions. In footings in which the projection is short in comparison with the depth the objection is very great.

10. Footings having reinforcement placed in the direction of the diagonals as well as parallel to the sides (four-way reinforcement) gave good tests. The significance of the results is so obscured by the variety of manner of failure (bond, diagonal tension, and perhaps tension) and by variations in the quality of the concrete, that a comparison with two-way reinforcement on the basis of loads carried would not be of value. This type of distribution of reinforcement should be included in further tests. Measurements of deformation in the bars are needed to determine the division of stress among the four sets of bars.

36. *Concluding Remarks.*—The tests of wall footings and column footings leave uncertainty in some parts of the problem and there are gaps in other parts. The recent development of the portable extensometer or strain gage and the skill and experience which have been gained in its use in recent tests have opened opportunities for obtaining information on the stresses developed in such test pieces which were not available when the series of tests was undertaken. It is suggested that some of the remaining unsolved problems may most readily be attacked by measurement of deformations in the steel and concrete, and that further investigation may best be carried on by constructing a form of apparatus which will permit such measurements to be taken under the conditions of uniform loading.

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