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UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF MINES HELIUM ACTIVITY HELIUM RESEARCH CENTER

INTERNAL REPORT

DESIGN OF INTERCHANGER FOR DETERMINING THE RATIO OF C MIXED TO

C UNMIXED IN THE HELIUM-NITROGEN SYSTEM

BY

Robert E. Barieau

BRANCH Fundamental Research

PROJECT NO. 4329

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HELIUM RESEARCH CENTER

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Estimation of heat leak and weight of insulation

Robert E. Barieau

of nitrogen, and nixture . . .

Heat transfer

Fundamental Research Branch

the interchanger

Project 4329

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BUREAU OF LAND MANAGEMENT LIBRARY BLDG. 50, DENVER FEDERAL CENTER P.O. BOX 25047 DENVER, COLORADO 80225



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I/ Project Leaders Thermodynamics, Helium Leaventh Genter, Bureau et Hines, Amerillo, Texas

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DESIGN OF INTERCHANGER FOR DETERMINING THE RATIO OF C MIXED TO C P

UNMIXED IN THE HELIUM-NITROGEN SYSTEM

By

Robert E. Barieau $\frac{1}{}$

ABSTRACT

In this report the design parameters for an interchanger to be used in determining the ratio of C mixed to C unmixed in a heliumnitrogen system have been calculated. It was found that an interchanger constructed of 3/16 inch 0.D. hard-drawn copper tubing would be satisfactory for a design pressure of 5000 psia. An arrangement of 5 helium tubes, 19 nitrogen tubes, and 31 mixture tubes will provide the required flow rate with sufficiently low pressure drops through the interchanger (maximum $\Delta p = 1.7$ psi). The interchanger will provide sufficient heat transfer surface with a length of 10.5 feet.

Since it is assumed that thermocouples for temperature measurement will be placed on the copper tube-wall exterior instead of in the flowing gas stream, calculations were made to determine the additional interchanger surface needed to provide accurate indication

Work on manuscript completed October 1964.

<u>1</u>/ Project Leader, Thermodynamics, Helium Research Center, Bureau of Mines, Amarillo, Texas

DESIGN OF INTERCHANCES FOR LETERMINING THE RATIO OF C

Robert S. Barisse

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I/ Project Leader, Thermodynemics, Heline Kessarch Conter, Durean II. Minne, Amarillo, Texas

Work on manuscript completed Occober 198

of the final equilibrium gas temperature. It was found that less than one foot of additional length on each end of the interchanger would provide such temperature readings within 0.01° F.

Weight of solder required in fabrication has been calculated and heat leak and insulation requirements have been estimated.

INTRODUCTION

As part of our experimental determinations of the enthalpy of helium-nitrogen mixtures, we will determine the ratio of the heat capacity of the mixture to that of the same components unmixed. This method has been described in a Report of Invention by Robert E. Barieau, entitled "The Adiabatic Mixing Flow Calorimeter and Double Heat Exchanger Method for the Determination of the Enthalpies of Mixtures", dated August 9, 1963. The invention referred to consists of the following: Unmixed streams of helium and nitrogen are brought to the same temperature by means of a high temperature heat sink. They are then cooled by passing through a heat exchanger, pass to a low-temperature heat sink and are cooled to its temperature, pass through a second heat exchanger, where they are warmed, pass to the high temperature heat sink where they are brought to its temperature, then pass to an adiabatic calorimeter where the gases are mixed and the delta T of mixing is determined. The mixed stream then goes to the high temperature heat sink, is brought to its temperature, then passes to the second exchanger, where it is cooled by giving up heat

of the final equilibrium gas temperature. It was found that fees than one foot of additional length on each and of the intercharger would provide such temperature readings within 0.01° F.

Motght of solder required in fabrication has been calcolated

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They are then cooled by passing through a near evelonger, pass to a

to the unmixed streams. The mixed stream then passes to the low temperature heat sink where it is brought to its temperature, then passes through the first heat exchanger where it is warmed by the unmixed streams.

This report gives the calculated design characteristics of the interchangers mentioned in the above description.

DESCRIPTION OF INTERCHANGER

This interchanger will be constructed of 3/16-inch O.D. hard drawn copper tubing with 0.042-inch wall thickness. There will be a total of 55 parallel tubes, each 10.5 feet long, for a total length of 577.5 feet. The total weight of copper tubing will be 40.85 pounds. The tubes will be soft-soldered into a close-packed cylindrical array. Because the tubing is to be hard drawn, and we desire a safe working pressure of 5000 psia, it will not be possible to use ordinary tin-lead soft solder as this would anneal the copper tubing. Therefore, a bismuth, lead, tin, cadmium, low melting alloy-melting below 200°F, will be used to solder the copper tubes together. The weight of low melting alloy will be 6.8 pounds. When packed into a close-packed cylindrical array the outside diameter of the tubing will be 1.46 inches. The tubing will be surrounded by Linde CS-5 insulation, 0.52 inch thick. The outside diameter of the insulation will be 2.5 inches. The weight of insulation will be 2.6 pounds. The insulation will be surrounded by a vacuum tight tube, as it is

to the omminad stiteans. The mined stream then passes to the low temperators heat sink where it is brought to its temperature, then passes through the first heat exchanger where it is warmed by the unmixed streams.

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DESCRIPTION, OF LNERGUARDING

Therefore; a bismuth, lead, tin, cadmium, low melting alloy-melting

necessary to evacuate the insulation to about 1 micron (10⁻³mm) of mercury.

The helium gas will be carried by five of the parallel copper tubes. The nitrogen gas will be carried by 19 of the parallel copper tubes and the mixture stream of helium and nitrogen will be carried by the remaining 31 parallel copper tubes. The total weight of the interchanger exclusive of the outside tube will be 50.25 pounds.

SUMMARY OF CALCULATED OPERATING PERFORMANCE OF INTERCHANGER

A maximum of 20 standard cubic feet per minute of nitrogen will be available from the Amarillo Helium Plant at a pressure of 600 psia. A maximum of 20 standard cubic feet per minute of grade-A helium will be available from the Amarillo Helium Plant at a pressure of 2500 psia. After mixing, the mixture will be returned to the Amarillo Helium Plant, at essentially atmospheric pressure, for recycle purification.

We have calculated the operating performance of this interchanger for mixture compositions of 25-75, 50-50, and 75-25 heliumnitrogen mixtures, and also for the pure components. The calculations were made for a pressure of 500 psia. The inlet temperature of the pure helium and nitrogen streams, at the warm end of the interchanger, was chosen as 68° F (20° C) and the inlet temperature of the helium-nitrogen mixture, at the cold end of the interchanger matessary to avacuate the insulation to about 1 micron (10 'as) of mercury.

The helium gas will be carried by the of the parallel copper tubes. The attropse gas will be carried by 19 of the parallel topper tubes and the mixture seream of belium and oltremon will-be carried by the remaining 11 parallel copper tubes. The total weight of the intercharger exclusive of the colorer tubes will be 50.25 pounds.

SUMAILY OF CALCULATED OTTER DELTASTING PERCENDER OF TVERTALLER

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-8

was chosen as $32^{\circ} F (0^{\circ} C)$.

The calculated pressure drops and temperatures of the streams are given in table 1.

It is seen that the ΔT at the warm end of the interchanger is less than 4°F. Also, the pressure is so low that pressure drop will affect the heat capacity to less than 0.1 percent.

It is seen from table 1 that the smallest rate of heat transfer is for pure helium, amounting to 506 Btu hr.⁻¹. Using Linde CS-5 insulation, I have calculated the heat leak to be 0.50 Btu hr.⁻¹, or of the order of a 0.1 percent of the heat transferred.

The details of the various calculations follow.

SAFE WORKING PRESSURE OF 3/16 INCH O.D. COPPER TUBING WITH 0.042 INCH WALL

If t is the wall thickness in inches and R is the inside radius in inches, then

t = 0.042 inch. 2R = 3/16 - 2 × 0.042 2R = .1875 - .084 = 0.1035 R = 0.05175 inch t/R = 0.8116.

Since t/R > 1/2, it is necessary, in calculating working pressures, to use the formulas applicable to thick wall cylindrical shells.

was chosen as 32°F (0°C)

The calculated pressure drops and temperatures of the streams are given in table 1.

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The details of the warthus delculations follow

SAFE WORKING PURSSERS OF 1/10 DOTE 0.01. LOPPER. TUBING WITH 0. 042 1800 PALL

If t is the wall thickness in inches and R is the inside railies to inches, then

t = 0.042 inch
22 = 3/10 - 2 × 0.042
23 = .1075 - .084 = 0.1035
R = 0.05175 inch
c/R = 0.8116.

Since t/R > 1/2, it is mecessary, in calculating merking pres-

	TABL	E 1	Calculated p streams u	ressure drops nder various	s and tempo relative	eratures of flow rates.	inlet and ou P = 500 psi	<u>tlet</u>
S	tream	Gas	SCFM	Io	$\mathbf{T}^{\mathbf{L}}$	$T_1^L - T_2^L$	Q,Btu hr ⁻¹	∆p,psi
	1 2 3	He He	20 20 	35.1065 32.0000	68.0000 64.8935	3.1065	- 506 + 506	1.669 0.069*
	*Lam	inar Fl	ow					
	1 2	He He-N ₂	20 80/3	35.5505 32.0000	68.0000 64.7527	3.2473	- 500 + 748	1.669
	3	N ₂	20/3	35.3765	68.0000		- 248	0.098
	1 2	He He-N ₂	20 40	35.8399 32.0000	68.0000 64.3398	3.6602	- 495 + 1224	1.669 0.642
	3	N ₂	20	36.0805	68.0000		- 729	0.664
	1 2	He He-N ₂	20/3 80/3	35.4078 32.0000	68.0000 64.3430	3.6570	- 167 + 898	0.283 0.397
	3	N ₂	20	36.0102	68.0000		- 731	0.664
	1							
	2	N ₂	20	32.0000	64.1402	3.8598	+ 734	0.282
	3	N ₂	20	35.8598	68.0000		- 734	0.664

			2051 0000 ,	, 25.			
					20 40 20		
						"2 He He-N ₂	

ISIN 1. - Galculated pressure drops and temperatures of inter and outle streams under various relative flow rungs. 8 - 500 orts.

.51

These formulas are (1)

(1) ASME Boiler and Pressure Vessel Code. Section VIII. Unfired Pressure Vessels, pages 9 and 121.

(1) Circumferential stress

$$t = R(Z^{1/2} - 1); Z = \frac{SE + P}{SE - P}$$

(2) Longitudinal stress

$$t = R(Z^{1/2} - 1); Z = \frac{P}{SE} + 1$$

where t = wall thickness, inches

P = allowable working pressure, psi
R = inside radius of shell, inches
S = maximum allowable stress value, psi
E = joint efficiency
(E = 1 for tubes)

with t/R = 0.8116,

$$z^{1/2} = 1.8116$$

Z = 3.2819

The maximum allowable working pressure is also given in Laole 1.

These formulas are (1)

() AND soffee and Pressure Vessel Code: Sofelse VIII. Untited

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(1) Gircunterential atrian

(2) Long Live Land Line (2)

deres f - will finderes, inches P - silonable inching pressure, psi R - inclus redine of shall, inches R - psint officience

ATTA A REAL AND A REAL

SILE & SILS

1.85. e E

$$Z = \frac{1 + P/S}{1 - P/S}$$

$$Z - Z P/S = 1 + P/S$$

$$P/S = \frac{Z - 1}{Z + 1} = \frac{2.2819}{4.2819}$$

$$P/S = 0.5329$$

For longitudinal stress, we have

$$P/S = Z - 1 = 2.2819$$

Therefore, the determining expression is P/S = 0.5329. Copper tubing comes in the following conditions: annealed, light drawn, and hard drawn. Below 100° F, the maximum allowable stress values for these conditions are given in table 2.

TABLE 2. - Allowable stress and safe working pressure of copper tubing

Annealed 6,000 3,200 Light Drawn 9,000 4,800 Hard Drawn 11,300 6,000	Condition	S psi	P = .5329 S psi
Annealed6,0003,200Light Drawn9,0004,800Hard Drawn11,3006,000			
Hard Drawn 11,300 6,000	Annealed Light Drawn	6,000 9,000	3,200 4,800
	Hard Drawn	11,300	6,000

The maximum allowable working pressure is also given in table 2. . For 4500 psi operation, the copper tubing must be either light or hard

and for circumforential stress

$$\frac{1}{2} = \frac{1 + \frac{p}{2}}{1 - \frac{p}{3}}$$

For longitudinal stress, we have

١

5/8 H 7 - 2 H 5/8

Therefore, the determining expression is 2/2 = 0.2122; Copper tubing comes in the following conditions: somealed, light drawn, and hard drawn. Relive 100°F, the maximum allowable stress values for these conditions are given in table 2.

The maximum allowable working pressure is also given in table in the second state of hard

5.7

drawn. For 5000 psi operation the copper tubing must be hard drawn. We have decided to specify that the tubing be hard drawn. We will then plan on a top working pressure of 4500 psi and we will set our relief valves at 5000 psi. The ASME Boiler and Pressure Vessel Code (2) indicates that the maximum allowable stress in copper tubing

(2) Reference 1, page 90.

is the same at 400° F for annealed, soft, and hard drawn copper tubing. This means that heating hard drawn copper tubing to 400° will anneal it. As the melting point of the common lead-tin solders are around 400° F, this means we cannot use ordinary solder without annealing the hard drawn copper tubing. At 250° F, the maximum allowable stress is different for annealed, light, and hard drawn copper tubing.

To be on the safe side, we have decided to specify that the alloy to be used to solder the copper tubing, in a close-packed cylindrical array, must melt below 200°F. Table 3 gives the composition of some eutectic fusible alloys.

	M .	Т.					
Item	°F	°C	Bi	РЪ	Sn	Cd	Other
A B C D	117 . 136 158 197	46.8 58. 70 91.5	44.70 49.00 50.00 51.60	22.60 18.00 26.70 40.20	8.30 12.00 13.30	5.30 10.00 8.20	19.10 In 21.00 In

TABLE 3. - Composition and melting temperatures of eutectic fusible alloys

rray. For this packing, the filling factor

drawni For 5000 pei operation the copper tubing must be have drawn. We will We have decided to apacify that the toning he have drawn. We will then plan on a top sorting presents of 000 pei and we will set our relief valves at 5000 pei. The hold bailer and Presente Versel Code (2) indicates that the maimum allowible stress in copper tubin

(2) Reference L, page 90.

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To be on the safe wide, we have doulded to specify that the alloy to be used to solder the copper libits. In a clume-packed sylledrical array, must male below 100 F. Yanke 3 gives the composition of some subootic factble alloys.

111.12 			117 136 158 197	

3. - Comments of a second and the local second a second

WEIGHT OF COPPER TUBING IN INTERCHANGER

We have a total of 55 parallel, 3/16-inch O.D., 0.042-inch wall, copper tubes each 10.5 feet long. This is a total of 577.5 feet.

3/16'' = 0.1875 inch

 $.1875 - 2 \times .042 = 0.1035$

area or

$$\pi/4 \times (.1875)^2 = 0.0276116$$

$$\pi/4 \times (.1035)^2 = 0.0084134$$

The cross-sectional area occupied by copper in a single tube is

= 0.0276116 - 0.0084134 = 0.0191982 square inches

Volume =
$$\frac{0.0191982 \times 577.5}{144}$$

= 0.076993 cu. ft.
= 133.044 cu. in.
= 2180.2 cm³
Weight copper = 2180.2 × 8.5
= 18,531.7 grams
= 40.85 lbs.

APPROXIMATE OUTSIDE DIAMETER OF PACKED INTERCHANGER TUBING

It is planned to soft solder the interchanger tubes in a closepacked cylindrical array. For this packing, the filling factor is

WEIGHT OF COPPEN TUNING IN INTROLOUS WEIGHT

We have a ratal of 55 paralets. 3/16-10ch 0.0., 0.002-10ch wall, copper tubes each 10.5 feet long. This is a rotal of 577.5

feel

3/16", - 0.1875 Mach

The cross-sections! area consided by suppor in a single fore 1

- 0.0276116 - 0.0024134

APPROXIMATE OUTSIDE DIAMETER OF PACKED INTRODUCTION TO THE

It is planned to act acider the interchanger tubes in a store packed cylindrical array. For this packing, the rilling factor is

given by

$$f = \frac{\pi}{2\sqrt{3}} = 0.9069.$$

The area taken up per tube is $\frac{\pi D^2}{4} = \frac{\pi \times (.1875)^2}{4} = 0.027612$ sq in.

There are a total of 5 + 19 + 31 = 55 tubes, making a total area of

$$55 \times .027612 = 1.51866$$
 sq. in.

The interchanger will then take up an area of

$$\frac{1.51866}{.9069} = 1.6746 \text{ sq. in.}$$

This area is enclosed in a circle with a diameter of

D = 1.4602 inches.

ESTIMATION OF WEIGHT OF LOW MELTING SOLDER IN INTERCHANGER

The cross-sectional area of the interchanger is 1.6746 square inches. The actual area taken up by the copper tubes is 1.51866 square inches. The difference, 0.15594, is the area available to be filled with solder.

Volume of solder = $\frac{0.15594 \times 10.5}{144}$

= 0.01137 cubic feet

given by

The sees callen up per suite is the market and a suite

There are a total of 2 + 12 + 31 - 35 tabes, making a coral

TO BUTE

35 X .D37612 = 1.51866 Hq. 1A

the interchanger will they take up an area of

111 . pz 0278.1 - 84816.1

This area is enclosed in a circle ofth a diameter of

D = 1 A602 Inchas

AS CARACTERS OF MERCHINE OF LOS NELTING SQLIPE IN INTROCESS BY

The cause-sectional area of the interchanger is from a sphare inches. The actual area taken up by for court takes in 1.51mm square inches. The difference, 0.15592, is the area outlinks of the

Velame of solder - velaminy

JOAL STORE TELLO

= 19.647 cubic inches = 321.96 cm³

and with a specific gravity of 9.58 for solder, we have a total weight of

 $321.96 \times 9.58 = 3084$ grams

or

 $\frac{3084}{453.6}$ = 6.80 lbs of solder.

ESTIMATION OF HEAT LEAK AND WEIGHT OF INSULATION

(3) with an apparent thermal conductivity of

(3) Riede, P. M., and D. I-J. Wang, Characteristics and Applications of Some Superinsulation. Advances in Cryogenic Engineering,
 v. 5, K. D. Timmerhaus, Editor. Plenum Press, Inc., New York, 1960, pp. 209-215.

 $k = 22 \times 10^{-5}$ Btu hr⁻¹ ft⁻² (deg F)⁻¹ ft and a density of 11.0 pounds per cubic foot. In order to obtain this low value, it is necessary to evacuate the insulation. A vacuum of 1 micron of mercury is sufficient.

We will calculate the heat leak for a thickness of 0.5199 inch. The outside diameter of the insulation is then 19.647 cubic inches
 321.90 cm³

-

and with a specific gravity of 9.58 for molder, we have a total weight of

221.96 x 9.58 = 3086 grouns

10 14 = 6,80 1bs or solder.

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(2) Riede, I. M., and D. I.J. Mang, Characteristics and Applications of Some Superinsulation. Advances in Cryogenic Englasering, 7. 5. K. D. Timmerinaus, Editor. Plenum Press, Inc., See Tork, 1980, pp. 209-215.

 $k = 21 \times 10^{-5}$ Bin M⁻¹ ft⁻² (deg F)⁻¹ Ft and a density of 11.0 prounds per cubit four. In order to obtain this low value, 15 is necessary to evacuate the insulation. A variant of 1 ploton of mercury is suith-

We will calculate the heat loak for a thickness of Grievy inch.

$$1.4602 + 2 \times .5199 = 2.500 \text{ inches}$$

$$A_{i} = \frac{\pi \times 1.4602 \times 10.5}{12} = 4.01393$$

$$A_{o} = \frac{\pi \times 2.5 \times 10.5}{12} = 6.87223$$

$$A_{o} - A_{i} = 2.8583; A_{o}/A_{i} = 1.712095$$

$$\log A_{o}/A_{i} = 0.23353; \ln A_{o}/A_{i} = 0.53772$$

$$A_{av} = \frac{2.8583}{0.53772} = 5.3156$$
$$Q = \frac{k \Delta T A_{av}}{T}$$

We will take the average
$$\Delta T$$
 as being 18° F. Then

$$Q = \frac{22 \times 10^{-5} \times 18 \times 5.3156 \times 12}{0.5199}$$

$$Q = 0.486 \text{ Btu hr}^{-1}$$

With 20 scfm of He and N_2 and 40 scfm of He- N_2 mixture going through the interchanger, the heat exchanged is of the order of 1200 Btu hr⁻¹, so the above heat leak is less than 0.1% of the heat exchanged.

The cross-sectional area taken up by the insulation is given by

$$\pi/4$$
 (2.5² - 1.4602²) = 3.2341 sq. in.

With 20 sets of He and H and H actual to actual 10 He-H attained actual and a through the intercharger, the hear excharged is of the crise of 1200 Bes he⁻¹, so the atom (heat heat is the time that the heat the charged

The dross-saucturer area taken up by the previation is given by

14 (2.1" - 1.4002") . " J. 2341 Mg. 43
The volume of the insulation is

$$= \frac{3.2341 \times 10.5}{144} = 0.2358 \text{ cu ft.}$$

Then the weight of the insulation is

$$0.2358 \times 11.0 = 2.59$$
 lbs.

DETAILS OF CALCULATION OF TEMPERATURES IN INTERCHANGER

We have calculated the operating characteristics of this interchanger under the following operating conditions.

Pressure = 500 psia

Temperature of the entering helium and nitrogen streams = 20° C = 68° F.

Temperature of the entering mixture stream = $0^{\circ}C = 32^{\circ}F$.

Let stream 1 be the helium stream.

Let stream 2 be the mixture stream.

Let stream 3 be the nitrogen stream.

The fundamental equation for heat transfer is given by (4)

and a is the length of interalmenter

(4) Walker, W. H., W. K. Lewis, W. H. McAdams and E. R. Gilliland, Principles of Chemical Engineering, McGraw-Hill Book Company, Inc. New York and London, 3rd ed. 1937, p. 107. The volume of the Leasthritic is

Then the weight of the inuplanting is

DETAILS OF CALCULATION OF TEMPT'S TONES. IN INTERCASSO

We have galcolated the operatory margersticitud of this inter-

Terreratine of the suluring belive and ritores street

(a) "Walker, W. H., W. Er Lewis, W. U. Mindama and E. W. Giffflamm, If the total of Channes I had inverter, Hofern Hill Each Company, Inc. New Three and Confere 3rd ad 1912, p. 107.

$$\frac{dQ}{dA} = h (T_{Cu} - T) = \pm WC_{p} \frac{dT}{dA}$$
(1)

where

 $\frac{dQ}{dA}$ is the heat transferred to the gas per unit area of heat transfer surface per unit time in Btu hr⁻¹ ft⁻², h is the individual heat transfer coefficient for heat transferred from the metal surface to the gas in Btu hr⁻¹ ft⁻² (deg F)⁻¹.

T_{Cu} is the temperature of the copper tube surface in deg F. T is the temperature of the gas in deg F.

W is the mass flow rate of the gas in lbs hr^{-1} .

 C_{p} is the heat capacity of the gas in Btu $1b^{-1}$ (deg F)⁻¹.

The + sign in front of W is to be taken when the gas is flowing in the direction of dA positive and the - sign is to be taken when dA positive is in the opposite direction of gas flow.

Now $A = \underline{A} \times and$

 $dA = \underline{A} dx$ where \underline{A} is the area available for heat transfer per unit length of interchanger and x is the length of interchanger.

In our interchanger, streams 1 and 3 are flowing in the same direction and stream 2 is flowing in the opposite direction through the interchanger. We then have for the three streams

$$\frac{dQ_1}{dx} = A_1 h_1 (T_{Cu_1} - T_1) = -W_1 C_{p_1} \frac{dT_1}{dx}$$
(2)

$$\frac{dQ}{dA} = h \left(T_{QU} - T \right) = \pm R T_{V} \frac{dT}{dA}$$
(3)

Where

 $\frac{dQ}{dA}$ is the least transferred to the gas per whit area of beat transfer surface per unit time in Ruchr⁻¹ ft⁻², h is the individual heat transfer coefficient for heat transferred from the metal surface to the are to Bea br⁻¹ ft⁻² (dec F)⁻¹.

Tou to the complexature of the copper rule surrace in dog T.

W in the mass flow rate of the gas in the hr

to the the heat canadity of the sam in Stu 16" (deg F)".

The + sign in front of W is to be taken when the was is flowing in the direction of dA positive and the - sign is to be taken when dA nestrive is in the opposite direction of gas flow.

DEL X A = A Mai

dA = A da whate A is the area available for hant transfer per unit length of interchanger and x is the length of interchanger.

In our interchanger, streams 1 and 1 iro lineire in the second direction and stream 2 is finwing in the opposite direction require to the interchanger. We then have for the three streams

$$\frac{10}{20} = \frac{1}{2} \frac{10}{2} \left(\frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2} \right) = \frac{10}{2} \frac{10}{2} \frac{10}{2} = \frac{10}{20}$$

$$\frac{dQ_2}{dx} = \underline{A}_2 h_2 (T_{Cu_2} - T_2) = W_2 C_{p_2} \frac{dT_2}{dx}$$
(3)

$$\frac{\mathrm{d}Q_3}{\mathrm{d}x} = \underline{A}_3 h_3 \left(\mathbf{T}_{\mathrm{Cu}_3} - \mathbf{T}_3 \right) = -W_3 C_{\mathrm{p}_3} \frac{\mathrm{d}\mathbf{T}_3}{\mathrm{d}x}$$
(4)

The length of interchanger increases in the direction of the mixture flow, stream 2. We neglect the thermal lag through the copper tube surface. Thus, we assume that

$$T_{Cu_1} = T_{Cu_2} = T_{Cu_3} = T_{Cu}$$
 (5)

Then

$$(T_{Cu} - T_{1}) = -\frac{W_{1}C_{p_{1}}}{\underline{A}_{1}h_{1}} \frac{dT_{1}}{dx}$$
 (6)

or

$$(T_{Cu} - T_1) = -a_1 T_1'$$
 (7)

where

$$a_{1} = \frac{\overset{w_{1}}{} \overset{p_{1}}{}}{\underline{A}_{1} \overset{h_{1}}{} \overset{h_{1}}{}}$$
(8)

and

$$T_1' = \frac{dT_1}{dx}$$
(9)

Similarly

$$(T_{Cu} - T_2) = a_2 T_2'$$
 (10)



$$(4) \qquad \frac{1}{26} \int_{0}^{1} \frac{1}{2} \frac{1$$

The length of inforthanger increases in the direction of the copies mixture Elow, stream 2. We negledt the thermal ing through the copies tube aveface. Thos, we argume that

Then

.

$$\frac{1.16}{\pi b} \frac{1^{q_1}}{1^{d_1} \Delta} = = (1^{q_1} - 1^{q_1})$$

20

prede

$$\frac{q}{1^{d}1^{\Delta}} = 1^{\Delta}$$

Similarly

$$(T_{C_{1}} - T_{2}) = a_{2}T_{2}$$

$$(T_{Cu} - T_3) = -a_3 T_3'$$
 (11)

and eliminating T_{Cu}, we have

$$(T_1 - T_2) = a_1 T_1' + a_2 T_2'$$
 (12)

$$(T_3 - T_2) = a_2 T_2' + a_3 T_3'$$
(13)

Assuming there is no heat leak into the exchanger, we have

$$\frac{\mathrm{d}Q_1}{\mathrm{d}x} + \frac{\mathrm{d}Q_2}{\mathrm{d}x} + \frac{\mathrm{d}Q_3}{\mathrm{d}x} = 0 \tag{14}$$

Therefore, we can write

$$W_{1}C_{p_{1}}T_{1}' + W_{3}C_{p_{3}}T_{3}' = W_{2}C_{p_{2}}T_{2}'$$
(15)

or

$$b_1 T'_1 + b_3 T'_3 = b_2 T'_2$$
 (16)

where

$$p = WC$$
(17)

Equations (12), (13), and (16) are the differential equations that fix the solution of our heat transfer problem. The solutions

and elimination I real have

$$\frac{(21)}{2} = \frac{1}{2} \frac{1}{2}$$

$$(z_3 - z_2) = a_2 z_2^2 + a_3 z_3^2$$
(23)

Assuming there is no heat leak into the exchanger, we have

$$(M) = \frac{2^{0}h}{xb} + \frac{2^{0}h}{xb} + \frac{10h}{xb}$$

Therefore, we cam write.

(01) <u><u><u></u></u></u>

where

100

Equargence (12); (11), and (16) are the differential equations that fix the solution of our heat sumpter problem. The solutions

are (5)

(5) Barieau, Robert E. Design Equations for Multistream Interchangers. Memorandum Report No. 33, Helium Research Center, Bureau of Mines, January 1964, 29 pp.

$$\Gamma_{1} = \alpha_{2} + \left[\frac{1 + a_{2}r_{2}}{1 - a_{1}r_{2}}\right]\beta_{2}e^{r_{2}x} + \left[\frac{1 + a_{2}r_{3}}{1 - a_{1}r_{3}}\right]\gamma_{2}e^{r_{3}x}$$
(18)

$$T_{2} = \alpha_{2} + \beta_{2} e^{r_{2}x} + \gamma_{2} e^{r_{3}x}$$
(19)

$$T_{3} = \alpha_{2} + \left[\frac{1 + a_{2}r_{2}}{1 - a_{3}r_{2}}\right] \beta_{2}e^{r_{2}x} + \left[\frac{1 + a_{2}r_{3}}{1 - a_{3}r_{3}}\right] \gamma_{2}e^{r_{3}x}$$
(20)

where r_2 and r_3 are the roots of the equation

$$(a_{1}a_{3}b_{2} + a_{2}a_{3}b_{1} + a_{1}a_{2}b_{3})r^{2}$$

- $[b_{2}(a_{1} + a_{3}) + b_{1}(a_{2} - a_{3}) + b_{3}(a_{2} - a_{1})]r = 0$ (21)
+ $b_{2} - b_{1} - b_{3}$

The constants α_2 , β_2 , and γ_2 are fixed by the boundary conditions of the problem. The boundary conditions are fixed by the temperatures of the streams when they flow into the interchanger. By our convention, stream 2 (mixture) flows in at x = 0 and streams 1

(5) Barieau, Robart E Danigh Equations for Multiafreen Erect, changers. Monoraydum Kepert No. 33, Hellum Research Contast, Bureau of Mices, Jansory 1961, 29 pp.

$$T_2 = a_2 + 8_2 a_2^2 + v_2 a_3^2$$
 (19)

$$(22) = \alpha_2 + \left[\frac{1}{1} + \frac{1}{2}\right] = \alpha_2^{2} + \frac{1}{2} + \frac{1}{2}$$

where ry and ry are the roots of the equation

$$\left[a_{1}a_{2}b_{2}+a_{2}a_{3}b_{1}+a_{1}a_{2}b_{3}b_{1}^{2}\right] = 0 \quad (23)$$

The constants 22.23 and 21 are liked by the housdary condtions of the problem. The houseary conditions are liked by the temperatures of the streams when they lise into the interchanger. Mo our convention, stream 2 (misture) flows to at s = 0 and streams i

22

(helium) and 3 (nitrogen) flow in at x = L, where L is the length of interchanger. Then we have the three boundary conditions

$$T_{1}^{L} = \alpha_{2} + \left[\frac{1 + a_{2}r_{2}}{1 - a_{1}r_{2}}\right] \beta_{2}e^{r_{2}L} + \left[\frac{1 + a_{2}r_{3}}{1 - a_{1}r_{3}}\right] \gamma_{2}e^{r_{3}L}$$
(22)

$$T_2^{\circ} = \alpha_2 + \beta_2 + \gamma_2$$
 (23)

assumed,

$$\mathbf{T}_{3}^{\mathrm{L}} = \alpha_{2} \left[\frac{1 + a_{2}r_{2}}{1 - a_{3}r_{2}} \right] \beta_{2} e^{r_{2}L} + \left[\frac{1 + a_{2}r_{3}}{1 - a_{3}r_{3}} \right] \gamma_{2} e^{r_{3}L}$$
(24)

These equations are to be used in the following way: From the assumed flow conditions, a_1 , a_2 , a_3 , b_1 , b_2 , and b_3 are calculated. These values are then substituted in equation (21) and r_2 and r_3 are determined as the roots of equation (21). r_2 and r_3 are then substituted in equations (22), (23), and (24), and these equations are solved for α_2 , β_2 , and γ_2 . Then these values for α_2 , β_2 , and γ_2 are substituted in equations (18), (19), and (20) and T_1^0 , T_3^0 , and T_2^L are calculated. We can then calculate the ΔT 's at the ends of the interchanger.

Sample calculation for 20 scfm of helium, 20 scfm of nitrogen, and 40 scfm of helium-nitrogen mixture

In this section, I give a sample calculation for 20 scfm of helium, 20 scfm of nitrogen and 40 scfm of a 50-50 helium-nitrogen mixture. Calculations were also made for 20 scfm of pure helium (helium) and 3 (nitrogen) flow in at x = L, where L is the longth of interchanger. Then we have the three boundary conditions

$$(55) \quad 4^{2} = 2^{2} \left[\frac{1}{2} + \frac{1}{2} +$$

$$T_2^2 = \alpha_2 + \theta_2 + \gamma_2 \tag{23}$$

$$r_{3}^{L} = a_{2} \left[\frac{1}{1} + \frac{a_{3} r_{2}}{a_{3} r_{2}} \right] s_{2}^{a} r_{2}^{b} + \left[\frac{1}{1} + \frac{a_{3} r_{3}}{a_{3} r_{3}} \right] r_{2}^{a} r_{3}^{b} \qquad (24)$$

These equations are to be used in the following way: From the assumed flow conditions, a_1 , a_2 , a_3 , b_1 , b_2 , and b_3 are calculated. These values are then substituted in equation (21) and c_2 and c_3 are determined as the roots of equation (21). c_2 and c_3 are then substituted in equations (22), (23), and (24), and these equations are solved for a_2 , b_2 , and v_3 . These values for a_2 , b_3 , and v_3 and v_4 are the substituted in equations (18), (19), and (20) and c_1^2 , c_3^2 , and v_3^2 are calculated to ends of these equations (16), (19), and (20) and c_1^2 , c_3^2 , and c_3^2 are character the ends of the function of the ends of the ends of the function of the ends of the function of the ends of the function of the ends o

Sample calculation for 20 acts of helium. 20 stim of

In this section, I give a sample calculation for 20 scin of helium, 20 scin of nitregen and 40 scin of a 50-30 helium-mitrupon mixture. Calculations were also made for 20 scin of pure helium

only; 20 scfm of pure nitrogen only; 20 scfm of helium, 20/3 scfm nitrogen and 80/3 scfm of a 75-25 helium-nitrogen mixture; and for 20/3 scfm helium, 20 scfm nitrogen, and 80/3 scfm of a 25-75 heliumnitrogen mixture. Calculations, other than for the 50-50 mix, are summarized in the tables.

In my calculations, I have taken C = $1.23960 \text{ Btu } 1b^{-1}(\text{deg F})^{-1}$ for helium, and C = 0.26274 for nitrogen. For the mixture, I have P_3 assumed, at 500 psia,

$$\frac{W_2^C P_2}{W_1^C P_1 + W_3^C P_3} = 1 - 0.04y(1-y)$$
(25)

where y is the mole fraction of helium in the mixture.

Then for the 50-50 mix

 $\frac{W_2^{C} p_2}{W_1^{C} p_1 + W_3^{C} p_3} = 0.99$ (26)

I take a standard cubic foot as the amount of gas, at 1 atmosphere and 70° F, in a cubic foot

$$1 \text{ cu. ft.} = 28,316.8 \text{ cm}^3$$
 (27)

The molal volume of a standard cubic foot of gas at 70° F and 1 atmosphere is

$$= 22,414.6 \times \frac{294.26}{273.15} = 24,146.9 \text{ cm}^3$$
(28)

only: 20 seim of pure nitrogen only: 20 seim of belium attune, 2013 seim of a 75-25 belium attrogen mixture; and for 20/3 seim belium, 20 seim bil:rogen, and 80/3 seim of a 25-75 helium attrogen mixtore. Galculetions, other than for the 50-50 mix, are

In my calculations, I have taken C = 1.23960 Bto 1b (deg I) for halium, and C = 0.26274 for nitrogen. For the misture, I have

$$\frac{W_2^{C} P_2}{W_1^{C} P_1} = 1 - 0.04\gamma(1-\gamma)$$
(25)

where y is the mole fraction of halium in the mateure.

$$\frac{W_2 C_{P_2}}{W_1 C_{P_1} + W_3 C_{P_3}} = 0.99$$
(26)

I take a standard cubic foot as the amount of gas, at I stands

The molal volume of a standard cubic foot of gas at 70 F and

1 atmosphere is

$$1 \text{ SCF} = \frac{28,316.8}{24,146.9} = 1.17269 \text{ g moles.}$$

$$M. W. He = 4.0028$$

$$M. W. N_2 = 28.016$$

$$1 \text{ SCFM} = \frac{1.17269 \times 60 \times M.W.}{453.592} \text{ lbs hr}^{-1}$$

$$1 \text{ SCFM} = 0.155120 \times M. W. \text{ lbs hr}^{-1}$$

$$20 \text{ SCFM (He)} = 12.4184 \text{ lbs hr}^{-1} = W_1 \qquad (29)$$

$$20 \text{ SCFM (N}_2) = 86.9170 \text{ lbs hr}^{-1} = W_3 \qquad (30)$$

Then

$$b_1 = W_1 C_{p_1} = 15.3938 \text{ Btu } \text{hr}^{-1} (\text{deg F})^{-1}$$
 (32)

$$b_3 = W_3 C_{p_3} = 22.8366 \text{ Btu hr}^{-1} (\text{deg F})^{-1}$$
 (33)

$$W_2 C_{p_2} = b_2 = 0.99 (b_1 + b_3) = 37.8481 \text{ Btu hr}^{-1} (\text{deg F})^{-1}$$
 (34)

$$C_{p_2} = \frac{37.8481}{99.3354} = 0.381013 \text{ Btu } 1b^{-1} \text{ (deg F)}^{-1}$$
 (35)

Table 4 gives data on the various flow rates for the different calculations.

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Table à gives dats un dhe varides liuw rates for the d'rierent

St	ream		Gas	SCFM W,	lbs. hr ⁻¹ C _p ,1	Btu lb ⁻¹ (deg F) ⁻¹			
	1 2 3		He He(100)	20 20 0	12.4184 12.4184	1.23960 1.23960			
	1 2 3		He He-N ₂ (75-25) N ₂	20 80/3 20/3	12.4184 41.3907 28.9723	1.23960 0.551658 0.26274			
	1 2 3		He He-N ₂ (50-50) N ₂	20 40 20	12.4184 99.3354 86.9170	1.23960 0.381013 0.26274			
	1 2 3		He He-N ₂ (25-75) N ₂	20/3 80/3 20	4.1395 91.0565 86.9170	1.23960 0.304845 0.26274			
	1 2 3		 N ₂ (100) N ₂	0 20 20	86.9170 86.9170	0.26274 0.26274			

TABLE 4. - Flow rates and heat capacities for various assumed flow conditions

n is heat capacity in Stu 1h (day F)

Malker, Lewis, Mohdams, and Gilliand (7) say that for the

(7) Reference 4, p. 113

common games equation (36) can be simplified to give the dimensional ecuation

(37)

		20 - 0 - 0	He He (100)	
		03 6 \ 08 6 \ 08 6 \ 05		
Darets . ; escape . o, . milos . o	A. 1395 0500 86,9170			
 0.26274 0.26274				

TABLE 4. - Plew rates and heat care this for various as inter

5.6

Heat Transfer Coefficients for Gases in Circular Tubes

The heat transfer coefficient, for a gas under turbulent flow, inside a circular tube is given by $(\underline{6})$

(6) Reference 4, p. 112.

$$h = 0.023 \frac{k}{D} \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{C \mu}{k}\right)^{1-n}$$
(36)

where

h is the heat transfer coefficient in Btu $hr^{-1}ft^{-2} (deg F)^{-1}$ k is the thermal conductivity in Btu $hr^{-1} ft^{-2} (deg F)^{-1} ft$. D is the inside diameter of the tube in feet G is the mass velocity in 1bs $hr^{-1} (sq. ft. of cross-section)^{-1}$ μ is the viscosity in 1bs $hr^{-1} ft^{-1} = 2.42 \times centipoises$ Cp is heat capacity in Bt μ 1b⁻¹ (deg F)⁻¹ Walker, Lewis, McAdams, and Gilliland (7) say that for the

(7) Reference 4, p. 113.

common gases equation (36) can be simplified to give the dimensional equation

h = 0.0144 Cp
$$\frac{G^{0.8}}{D^{0.2}}$$
 (37)

Hear Transfor Controlement 1. 19 1920

The heat tracetor coefficient, for a sus order rathered line to the sus order rathered to be the tast of the subsect of the su

(6) Reference - 1 LL

where

Part and a second of second

common gases equalion (36) can be simplified to give the dimensional

$$u = 0.0100 \, \text{Gr}_{0}^{-2.8}$$

(18)

However, Giauque (8) obtained the expression

(8) Giauque, W. F. Liquid Oxygen Trailer Unit, Design, Construction, and Operation. Final Report to National Defense Research Committee. Report OSRD No. 4141, July 1944, p. V-11, 539 pp.

h = 0.0120 Cp
$$\frac{G^{0.8}}{D^{0.2}}$$
 (38)

for high pressure air, inside straight circular tubes. We have accepted equation (38) for calculating all heat transfer coefficients.

$$G = \frac{W4}{n\pi D^2}$$
(39)

where n is the number of parallel tubes and

$$G^{0.8} = \frac{4^{0.8} W^{0.8}}{\frac{0.8}{n^{0.8} D^{1.6}}}$$
(40)

h = 0.0120 Cp
$$\frac{4^{0.8}}{\pi^{0.8}} \left(\frac{W}{n}\right)^{0.8} \frac{1}{D^{1.8}}$$
 (41)

$$\frac{4^{0.8}}{10.8} = 1.213188 \tag{42}$$

Thus

h = 0.0145583
$$\left(\frac{W}{n}\right)^{0.8} \frac{Cp}{D^{1.8}}$$
 (43)

so that

Nowewer, Giaunice (B) opination the expression

(8) Giacque, W. F. Liquid Heyezzi Frailer Unit. Beatyr, Construction, and Operation. Winal Report to Barianal Deredse Research Conmittee. Report OSED-No. 4061. July 1964. p. V-11. 535 pp.

$$b = 0.0120 - C_{\rm P} \frac{G^{0.3}}{g^{0.2}}$$

for high pressure air, inside straight circular rules. We have

.

where n is the temper of parailal tubes and

so that

$$\frac{1}{8.1_{B}} \left(\frac{1}{n}\right) \frac{8.0}{8.0_{A}} \left(\frac{1}{n}\right) \frac{8.0}{8.0_{A}} \left(\frac{1}{n}\right) \frac{1}{8.0_{A}} \left(\frac{1}{n}\right) \frac{1}{1} \frac{1}{1}$$

(53) HULELS I - 8.0

= Infi

$$n = 0.0143981 \left(\frac{w}{u}\right)^{1.6} \frac{2u}{u^{1.6}}$$

83.

29

For 3/16" O.D. tubing with 0.042" wall

$$I.D. = .1875 - .084 = 0.1035$$
 inch (44)

$$D = \frac{0.1035}{12} = 0.008625 \text{ ft}. \tag{45}$$

$$\frac{1}{D} = 115.942029 \text{ ft}^{-1}$$
 (46)

we have

$$\frac{1}{D^{1.8}} = 5195.575$$
 (47)

Then

h = 75.6387 (
$$\frac{W}{n}$$
) Cp (48)

For helium,
$$n_1 = 5$$

For nitrogen, $n_3 = 19$
For mixture, $n_2 = 31$

$$n_1^{0.8} = 3.623898$$
 $Cp_1 = 1.2396$

$$n_2^{0.8} = 15.598734$$

$$n_3^{0.8} = 10.543939 \text{ Cp}_3 = 0.26274$$

Then we have for the helium at P = 500 psia,

$$h_1 = 25.8732 W_1^{0.8}$$
(49)

For 3/16" C.D. subing with 0,042" ball

$$\frac{1}{\alpha} = \frac{1}{\alpha} = \frac{1}{\alpha}$$

mon1

Then we have for the fullym as F = 500 pals,

and for the mixture at P = 500 psia,

$$h_2 = 4.84903 \text{ Cp}_2 \text{ W}_2^{0.8}$$
 (50)

and for the nitrogen at P = 500 psia,

$$h_3 = 1.884809 W_3^{0.8}$$
(51)

With $W_1 = 12.4184$, $W_2 = 99.3354$, $W_3 = 86.9170$, and $Cp_2 = 0.381013$, we have

1 5 194-1345 194,9938 970.673 0,585280

$$W_1^{0.8} = 7.5033035$$
 (52)

$$N_2^{0.8} = 39.598910$$
 (53)

$$W_3^{0.8} = 35.586378$$
 (54)

Finally, we have

$$h_1 = 194.1345 \text{ Btu } \text{hr}^{-1} \text{ ft}^{-2} \text{ (deg F)}^{-1}$$
 (55)

$$h_2 = 73.1607 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{deg F})^{-1}$$
 (56)

$$h_3 = 67.0735 \text{ Btu } \text{hr}^{-1} \text{ ft}^{-2} (\text{deg F})^{-1}.$$
 (57)

Now

1

A

$$a = \frac{WCp}{\underline{A}h} = \frac{WCp}{n\pi Dh}$$

$$\pi D = \pi \times 0.008625 = 0.0270962366$$

and for the mixture at F = 500 pain,

and for the nitroyen at 2 = 500 pains

With W₁ = 12.4184, W₂ = 99.3354, W₃ = 80.9170, and Gp₂ = 0.361013,

svad aw

$$I_1^{0.8} = 7.5030035$$
 (52)

Finally, we have

$$h_2 = 73.1607$$
 Stu ht⁻¹ ft⁻² (deg N)⁻¹ (56)
b. = 57.0735 Stu ht⁻¹ ft⁻² (deg S)⁻¹ (57)

NoW

•

1

mB = n x 0.008625 = 0.0276962366

39.

		а	$=\frac{36.905494}{nh}$	WCp					
Stream	n n	h	WCp = b	nh	a				
1 2 3	5 31 19	194.1345 73.1607 67.0735	15.3938 37.8481 22.8366	970.673 2267.982 1274.397	0.585280 0.615879 0.661329				
The	e various	values of h	n, b, and a are	summarized in	table 5. We				
the	en have	$a_1 = 0.5$ $a_2 = 0.6$	585280	$b_1 = 15.393$ $b_2 = 37.848$	38				
		$a_2 = 0.6$ $a_3 = 0.6$	561329	$b_3 = 22.836$	56				
		20 20	$a_1 a_3 b_2 = 14.6$	495854					
		é	$a_2 a_3 b_1 = 6.26$	987385					
		é	$a_1 a_2 b_3 = 8.23$	171877					
	$a_1 a_3 b_2 + a_2 a_3 b_1 + a_1 a_2 b_3 = 29.15117802$ (58)								
	$b_2(a_1 + a_3) = 47.18178209$								
$b_1 (a_2 - a_3) = -0.69964821$									
	$b_3(a_2 - a_1) = 0.69877712$								
	$b_2(a_1 + a_3) + b_1(a_2 - a_3) + b_3(a_2 - a_1) = 47.18091100$ (59)								

	15.3038 37.2451 22.8356	194.1345 73,2607 67.0735	

The varinus values of h. b. ond a are summarized in table 5. My

than have

a, a, b, e, b4.6493254

 $a_2^{a_3}b_1 = 6.26987385$

a. 4.23171877

a, a, b, + 2, a, b, + 4, a, b, * 29.1117802

52 (41 + 53) = (7.18173203 13 (41 + 53) = -0.504556821

signeed - (1 - 2m) 14

 $b_2(a_1 + a_3) + b_1(a_2 - a_3) + b_2(a_2 - a_1) + 47.10001100 (39)$

Stream	SCFM	h,Btu hr ⁻¹ ft ⁻² (deg	F) ⁻¹ b	a
1 2 3	20 20 0	194.1345 45.1013	15.3938 15.3938	0.585280 0.406337
1	20	194.1345	15.3938	0.585280
2	80/3	52.5832	22.8335	0.516957
3	20/3	27.8518	7.6122	0.530878
1	20	194.1345	15.3938	0.585280
2	40	73.1607	37.8481	0.615879
3	20	67.0735	22.8366	0.661329
1	20/3	80.6136	5.1313	0.469829
2	80/3	54.5988	27.7581	0.605252
3	20	67.0735	22.8366	0.661329
1 2 3	0 20 20	45.3383 67.0735	22.8366 22.8366	0.599647 0.661329

TABLE 5. - Various values of h, b, and a for the various flow rates

SLE 5. - Mouschur for y

 1
 20
 r²
 1.6149473224 x
 0.00110402853174 = 0

 1
 20
 r²
 1.823590300
 x
 0.00110402853174 = 0

 1
 20/3
 r²
 1.9184507114 r
 0.001293713692 = 0

 1
 20
 r²
 1.9184507114 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.9184503184 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.9184503184 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.918453384 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.918453884 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.918453884 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.918453884 r
 0.01293713692 = 0

 1
 20/3
 r²
 1.91855388 r
 0.001293713692 = 0

 1
 20/3
 r²
 1.91855388 r
 0.001293713692 = 0

 1
 20/3
 r²
 1.918553888 r
 0.001293713692 = 0

 1
 20
 r²
 1.918573388 r
 0.001293713692 = 0

</tabularray>

	· 1.101 ch		
		· 12	
	1		

TABLE 5. - Various values of h. D. and a for the variants for eater

$$b_2 - b_1 - b_3 = -0.3823$$

Substituting in equation (21), we have

$$29.15117802 r^{2} - 47.180911r - 0.3823 = 0$$

or

$$r^{2} - 1.6184907164 r - 0.013114392829 = 0$$
 (60)

The roots of this equation are

$$r_2 = 1.626553404$$
 (61)

$$r_3 = -0.0080626880 \tag{62}$$

The equations for r and the roots for the various calculations carried out are given in table 6.

T.	ABLE	6.	- Equation	for	r	and roots for the various calculations
						carried out
Stre	am		SCFM			1 + A.T.
1 2 3			20 80/3 20/3	r ² r ₂ r ₃	1 11 11	1.8169475284 r - 0.012662853174 = 0 1.823890300 -0.0069427715
1 2 3			20 40 20	r ² r ₂ r ₃		1.6184907164 r - 0.013114392829 = 0 1.626553404 -0.0080626880
1 2 3			20/3 80/3 20	r ² r ₂ r ₃	1 11 11	1.991757261 r - 0.01221713692 = 0 1.997872335 -0.006115074

Substituting in squation (21), os have

30

The roots of this equation are

The squartons for t and the roots for the various calculations carted out are given in table b.

			63	

8.8

Then

 $a_1r_2 = 0.95198917629; 1 - a_1r_2 = 0.04801082371$

 $a_2r_2 = 1.0017600839; 1 + a_2r_2 = 2.0017600839$

 $a_{3}r_{2} = 1.07568693611; 1 - a_{3}r_{2} = -0.07568693611$

$$\frac{1 + a_2 r_2}{1 - a_1 r_2} = 41.69393335$$

$$\frac{1 + a_2 r_2}{1 - a_3 r_2} = -26.44789427$$

 $a_1r_3 = -0.004718930033; 1 - a_1r_3 = 1.00471893003$

 $a_2r_3 = -0.004965640223; 1 + a_2r_3 = 0.995034359777$

$$a_3r_3 = -0.005332089392; 1 - a_3r_3 = 1.005332089392$$

$$\frac{1 + a_2 r_3}{1 - a_1 r_3} = 0.99036091593; \quad \frac{1 + a_2 r_3}{1 - a_3 r_3} = 0.98975688796$$

Substituting in equations (18), (19), and (20), we have

$$T_{1} = \alpha_{2} + 41.69393335 \beta_{2} e^{r_{2}x} + 0.99036091593 \gamma_{2} e^{r_{3}x}$$
(63)

$$T_{2} = \alpha_{2} + \beta_{2} e^{r_{2}x} + \gamma_{2} e^{r_{3}x}$$
(64)

$$\frac{1+a_2r_3}{1-a_1r_3} = 0.990360913931 \frac{1+a_2r_3}{1-a_3r_3} = 0.99973688796$$

Substituting in equations (18), (19), and (20), we have

)

$$T_{3} = \alpha_{2} - 26.44789427 \beta_{2} e^{r_{2}x} + 0.98975688796 \gamma_{2} e^{r_{3}x}$$
(65)

With the length of the interchanger being 10.50 feet, we have

$$r_2L = 17.078810742; \frac{r_2L}{2.303} = 7.4172333$$
 (66)

$$r_{3}L = -0.084658224; \frac{r_{3}L}{2.303} = -0.0367666$$
 (67)

$$[10] \frac{r_2^L}{2.303} = e^r 2^L = 2.6135650 \times 10^7$$
(68)

$$[10] \frac{-r_3^{L}}{2.303} = e^{-r_3^{L}} = 1.0883450$$
(69)

$$e^{r_3 L} = 0.91882629129$$
 (70)

We then have the three boundary condition equations,

$$68 = \alpha_2 + 41.69393335 \times 2.613565 \times 10^7 \beta_2$$
(71)

+ 0.99036091593 × 0.91882629129
$$\gamma_2$$

32 = $\alpha_2 + \beta_2 + \gamma_2$ (72)

$$68 = \alpha_2 - 26.44789427 \times 2.613565 \times 10^7 \beta_2$$
(73)

or

f

$$\delta 8 = \alpha_2 + 1.0896980492 \times 10^9 \beta_2 + 0.9099696474 \gamma_2 \quad (74)$$

$$\mathbf{T}_{3} = \alpha_{2} - 26.4a789427 \beta_{2}e^{c_{2}a_{1}} + 0.9897968796v_{2}e^{c_{3}a_{1}}$$
(65)

With the length of the interchanger being 10.50 feet, we have

$$r_2 t = 17.078810342; \frac{r_2 t}{2.303} = 7.4172333 (00)$$

We then have the three biordary condition equations

62.1

18 - a2 + 1.08983986692 × 10⁹ 62 + 0.9099696678 22 (74)
$$32 = \alpha_2 + \beta_2 + \gamma_2 \tag{75}$$

$$68 = \alpha_2 - 0.6912329079 \times 10^9 \beta_2 + 0.90941465069 \gamma_2$$
(76)

Subtracting equation (76) from (74), we have

$$0 = 1.780930957 \times 10^{9} \beta_{2} + 0.0005549967 \gamma_{2}$$
(77)

and subtracting equation (75) from (74), we have

$$36 = 1.0896980482 \times 10^9 \beta_2 - 0.0900303526 \gamma_2$$
(78)

From equation (77)

1

$$0 = \beta_2 + 3.116329118 \times 10^{-13} \gamma_2 \tag{79}$$

and from equation (78)

$$33.036674761 \times 10^{-9} = \beta_2 - 826.19541041 \times 10^{-13} \gamma_2$$
(80)

Subtracting equations (79) and (80)

$$33.036674761 \times 10^{-9} = -829.3117395 \times 10^{-13} \gamma_2$$
(81)

$$Y_2 = -398.36256$$
 (82)

$$\beta_2 = 1.241429 \times 10^{-10}$$
(83)

and substantiate equilibrius (23) from (7011 as here

Seven squarton (77)

(83) mukneups move here

Subtraction equations (74) and (80)

(SR) CERCI. ROC. - - -

and

Thereford

$$\alpha_2 = 430.36256$$

Then we have

$$\Gamma_1 = 430.3626 + 5.1760 \times 10^{-9} e^{r_2 x} - 394.5227 e^{r_3 x}$$
 (84)

$$T_2 = 430.3626 + 1.2414 \times 10^{-10} e^{r_2 x} - 398.3626 e^{r_3 x}$$
 (85)

$$T_3 = 430.3626 - 3.2833 \times 10^{-9} e^{r_2 x} - 394.2821 e^{r_3 x}$$
 (86)

Equations (84), (85), and (86) give the temperatures of the streams through the interchanger. When x = L = 10.50 feet, we have

$$T_1^L = 430.3626 + 0.1353 - 362.4978 = 68.0000^\circ F$$
 (87)

$$T_2^L = 430.3626 + 0.0032 - 366.0260 = 64.3398^\circ F$$
 (88)

$$T_3^L = 430.3626 - 0.0858 - 362.2768 = 68.0000^\circ F$$
 (89)

At the cold end of the interchanger, where x = 0, we have

$$T_1^{\circ} = 430.3626 - 394.5227 = 35.8399^{\circ} F$$
 (90)

$$T_2^{\circ} = 430.3626 - 398.3626 = 32.0000^{\circ} F$$
 (91)

$$T_3^{\circ} = 430.3626 - 394.2821 = 36.0805^{\circ} F$$
 (92)

bra

THEN WW MENT

$$T_2 = 430.3626 \pm 1.2414 \times 10^{-10} c^2 c^2 - 398.3626 c^2 3^2$$
 (85)

Equations (84), (85), and (80) give the temperatures of the

At the cold and of the interchanger, where a a up we have

Therefore,

$$T_1^L - T_2^L = T_3^L - T_2^L = 3.660^\circ F$$
 (93)

or

$$T_1^L - T_2^L = 2.034^\circ C$$

or the mixed stream of He and N₂ will be warmed to 2.03° C of the temperature of the entering pure helium and nitrogen. The helium stream will be cooled to 35.840° F or to 3.840° F of the temperature of the entering cold mixture stream.

While the helium and nitrogen streams start out at the same temperature at the warm end of the interchanger, when they come out of the interchanger at the cold end, the helium stream is 0.241° F colder than the nitrogen stream. The problem of allowing or correcting for this temperature difference will be considered later. The temperatures of the various streams as a function of interchanger length are given in table 7. The temperature of the copper tube wall surface as a function of interchanger length is also given.

Table 8 gives the temperatures calculated at the hot and cold end of the interchanger from the equations given in table 7. The last column in table 8 gives the temperature difference between the streams at the warm end of the interchanger. This temperature difference would be zero in an infinite interchanger.

Table 9 gives the rate of heat transfer in the interchanger.

- Therefore,

or the minut stream of its and N₂ will be varmed to 2.02°C of the temperature of the extering pure heltus and altroint. The helium stream will be cooked to 35.840°F or to 3.840°F of the bespect-

While the letter as of the set each of the interchanger, which they tone temperature at the warm and of the interchanger, which they tone out of the interchators at the cold wod, the inclume stream is 0.251° 2 coulder theo the attropic atteam. The periodem of allowing or secretable for this temperature difference will be considered later. The conjectators of the various streams as a function of the interchanger length are given in table 7. The temperature of the copper teme vali surface as a function of interchanger length is also given.

Table 8 giver the temperators releviated at the her and cold and of the teterchanger from the equiviant table 7. The last column to table 5 gives restangerators difference bethen the streams at the varm and of the incoronanger This camperators difference would be serv in a millette interchanger.

Table 9 gives the rate of home transfer in the interchanger

Stream	SCFM	
1	20	$T_1 = 35.1065 + 3.13272x$
2	20	$T_2 = 32.0000 + 3.13272x$
		$T_{Cu} = 33.2729 + 3.13272x$
		-10 rev rev
1	20	$T_1 = 497.8375 - 2.4925 \times 10^{-10} e^{12x} - 462.2870e^{13x}$
2	80/3	$T_2 = 497.8375 + 0.0866 \times 10^{-10} e^{r_2 x} - 465.8375 e^{r_3 x}$
3	20/3	$T_3 = 497.8375 + 5.3003 \times 10^{-10} e^{r_2 x} - 462.4610 e^{r_3 x}$
		$T_{c} = 497.8375 + 0.1682 \times 10^{-10} e^{r_2 x} - 464.1655 e^{r_3 x}$
1	20	$T_1 = 430.3626 + 51.760 \times 10^{-10} e^{r_2 x} - 394.5227 e^{r_3 x}$
2 -	40	$T_2 = 430.3626 + 1.2414 \times 10^{-10} e^{r_2 x} - 398.3626 e^{r_3 x}$
3	20	$T_3 = 430.3626 - 32.833 \times 10^{-10} e^{r_2 x} - 394.2821 e^{r_3 x}$
		$T_{Cu} = 430.3626 + 2.4850 \times 10^{-10} e^{r_2 x} - 396.3844 e^{r_3 x}$
1	20/3	$T = 551.85215 + 36.777 \times 10^{-11} r^{2x} - 516.44433 r^{3x}$
1	20/3	$\frac{1}{1} = 551.85215 + 1.021 \times 10^{-11} r^{2x} = 510.85215 r^{3x}$
2	80/3	$\frac{1}{2} = 551.65215 + 1.021 \times 10^{-11} r_{2}x = 515.65215e^{-13}x$
3	20	$T_3 = 551.85215 - 7.023 \times 10 = 2 - 515.84199e^{-11}$
		$T_{Cu} = 551.85215 + 2.256 \times 10^{-1} e^{-2t} - 517.92809e^{-5t}$
2	20	$T_2 = 32.0000 + 3.06097x$
3	20	$T_2 = 35.8598 + 3.06097x$
		$T_{Cu} = 33.8355 + 3.06097x$

TABLE 7. - <u>Temperature of streams and tube wall surface as a function</u> of interchanger length

	SCEN	
The assistance + Julianass		
	6.440	
	. US-	
T. = 35.6598 + 3.060972.		

Stream	SCFM	Т	$\mathbf{T}^{\mathbf{L}}$	$T^{L} - T^{\circ}$	$T_1^L - T_2^L$
1 2 Cu	20 20	35.1065 32.0000 33.2729	68.0000 64.8935 66.1665	32.8935 32.8935	3.1065
1 2 3 Cu	20 80/3 20/3	35.5505 32.0000 35.3765 33.6720	68.0000 64.7527 68.0000 66.3089	32.4495 32.7527 32.6235	3.2473
1 2 3 Cu	20 40 20	35.8399 32.0000 36.0805 33.9782	68.0000 64.3398 68.0000 66.1607	32.1601 32.3398 31.9195	3.6602
1 2 3 Cu	20/3 80/3 20	35.4078 32.0000 36.0102 33.9241	68.0000 64.3430 68.0000 66.1633	32.5922 32.3430 31.9898	3.6570
2 3 Cu	20 20	32.0000 35.8598 33.8355	64.1402 68.0000 65.9757	32.1402 32.1402	3.8598

TABLE 8. - Temperatures at hot and cold end of interchanger

	35.1005 ,32.0700 33.2729	
	25.9000 35.9000 35.9005	

In 8 - Temperatures at hat and cold and of interciventy

Die -

T.	ABLE 9.	- <u>Calculation</u>	of	heat transfe	rre	<u>d</u>		
Str	eam	SCFM	TL	- T ⁰		b		Q, Btu hr ⁻¹
1 2		20 20	32 32	.8935 .8935	15 15	.3938 .3938		-506.356 +506.356
1 2 3		20 80/3 20/3	32 32 32	.4495 .7527 .6235	15 22 7	.3938 .8335 .6122		-499.521 +747.859 -248.337
							Σ	= +0.001
1 2 3		20 40 20	32 32 31	.1601 .3398 .9195	15 37 22	.3938 .8481 .8366		-495.066 +1224.000 -728.933
							Σ	= +0.001
1 2 3		20/3 80/3 20	32 32 31	.5922 .3430 .9898	5 27 22	.1313 .7581 .8366		-167.240 +897.780 -730.538
							Σ	= +0.002
2 3		20 20	32 32	.1402 .1402	22 22	.8366 .8366		+733.973 -733.973

where B is the inside discourse in feet and I is the loogth of

Dubu In LeoL

= n x 0.008625 x 10.54

L, # 0 28031 ft

	d			
			20 20	
	15.3938 22.8335 7.6122	32.4495 32.7527 32.6235		
		32,1601 32,3398 51,9195		
+753.973				

e.

TABLE 9. - Calcolation of heat transferred

•

CALCULATION OF TEMPERATURE DROP THROUGH COPPER TUBE WALL

The equations used in calculating the performance of the interchanger have been derived on the assumption that the temperature drop through the copper wall is small and can be neglected. It is important that this temperature difference be calculated to see if it is permissible to neglect it.

From table 9, we see that for the 50-50 mix experiment,

$$Q_1 = -495.066 \text{ Btu/hr}$$

and since for the helium stream there are five tubes, we have

$$\frac{Q_1}{n_1} = -99.013 \text{ Btu/hr tube}.$$

The area inside a tube is

$$A_i = \pi D_i L$$

where D is the inside diameter in feet and L is the length of tube in feet

 $A_{i} = \pi \times 0.008625 \times 10.50$ $A_{i} = 0.28451 \text{ ft}^{2}$

CALCULATEON OF TENNESSATURE DROT DATONIN COLLER TOPS WILL

The equations used in calculating the performance of the interchanger have seen derives on the an articler that the trapets ture drop through the coppet walk is small set cen be hereinsted is to tapertant that this teapertate difference to calculated to

From table 9, we see that for the 50-10 mix experiment,

and state for the hullow stream there are two public, we have

The ered thilds a tube is

where if is the maids disserts in feat and b is the langth of take in feat

$$A_{\rm r} = \pm \times 0.008625 \times 10.50$$

A. = 0.23151 m

$$A_{o} = \pi D_{o} L$$

= $\frac{\pi \times .1875}{12} \times 10.50$

we have

$$A_{o} = 0.51542 \text{ ft}^{2}$$
$$A_{av} = \frac{A_{o} - A_{i}}{\ln A_{o}/A_{i}}$$

$$A_{0} - A_{1} = 0.23091$$

 $A_0/A_i = 1.811606$

 $\log_{10} A_{0}/A_{i} = 0.25806$

$$\ln A_{i} = 0.59421$$

and chus, for

$$A_{av} = 0.38860$$

$$k \land \Delta T$$

$$Q = \frac{\frac{1}{av} \frac{av}{Cu}}{L}$$

$$L = \frac{.042}{12} = 0.0035 \text{ ft.}$$

$$\Delta T_{Cu} = \frac{QL}{kA_{av}}$$

 $q = \frac{k A_{av} \alpha \alpha_{out}}{L}$

 $L = \frac{100L}{12} = 0.0033$ it.

The - The

$$\Delta T_{Cu} = Q \times \frac{0.0035}{222 \times 0.38860}$$

$$\Delta T_{Cu} = Q \times 0.0000406$$

Then for the helium, with $\frac{Q_1}{n_1} = -99.013$ Btu hr⁻¹ tube⁻¹,

we have

$$\Delta T_{Cu_1} = 0.0040^{\circ} F$$

Table 9 indicates that for the nitrogen stream

$$Q_3 = -728.933$$
 Btu hr⁻¹

and with 19 parallel tubes for the nitrogen stream, we have

$$\frac{Q_3}{n_3} = -38.365 \text{ Btu hr}^{-1} \text{ tube}^{-1}$$

and thus, for the nitrogen stream

 $\Delta T_{Cu_3} = 38.365 \times .0000406$

$$\Delta T_{Cu_3} = 0.0016^{\circ} F$$

Table 9 indicates that for the 50-50 mixture stream

$$Q_2 = 1224.000$$
 Btu hr⁻¹

Then for the balling, with
$$\frac{Q_1}{\alpha_1} = -99.013$$
 Bec he const

avail av

Table 9 indicates that for the mittingen strough

and with 19 parallal tubes for the nitroget stream, -t lave

and thus, for the plirogen stream

Table 9 indicates that for the '30-50 mixture bireas

6 = 1224.000 Btu hr

and with 31 parallel tubes for the mixture stream, we have

$$\frac{Q_2}{n_2} = 39.484 \text{ Btu hr}^{-1} \text{ tube}^{-1}$$

Thus, for the mixture stream

$$\Delta T_{Cu_2} = 39.484 \times 0.0000406$$

$$\Delta T_{Cu_2} = 0.0016^{\circ} F$$

With the ΔT in the interchanger between the streams being cooled and heated being $3.7^{\circ}F$, we see that the ΔT_{Cu} through the copper tube wall surface is completely negligible.

DETAILS OF CALCULATION OF PRESSURE DROPS THROUGH THE INTERCHANGER

If we neglect kinetic energy and gravitational potential energy effects, the pressure drop due to friction for circular tubes can be expressed by the so-called Fanning equation (9)

(9) Reference 1, p. 77.

$$\frac{\mathrm{d}\mathbf{F}}{\mathrm{d}\mathbf{N}} = -\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}\mathbf{N}}\frac{1}{\rho} = 2\mathrm{f}\frac{\mathbf{V}^2}{\mathrm{g}\mathbf{D}}$$
(94)

and with 31 parallel tunes for the mixture stream, we have

Thus, for the mixture attain

With the AT in the interchanger between the streams being cooled and heated being 3.7°F, we are that the AT_{Cu} through the copper tube wall surface is completely negligible.

DETAILS OF CALCULATION OF PRESSIRE INOPE THEOREM THE INTERCEMENTS If we neglect kinetic evergy and gravitational potential every effects, the pressure drap due to Priction for strular takes can be expressed by the so-called Familio eduation (9)

(9) Reference L, p. 37

$$\frac{d\pi}{dm} = -\frac{d\pi}{dM}\frac{1}{\mu} = 2\pi\frac{v^2}{RR}$$
 (we

where p is the pressure in pounds per square foot

 $\boldsymbol{\rho}$ is the density in pounds per cubic foot

N is the length in feet

f is the dimensionless friction factor

V is the average linear velocity in feet per second

g is the acceleration due to gravity

g = 32.17405 ft sec⁻²

D is the inside diameter of the circular pipe in feet.

For gas flow, we replace ρ by $\frac{1}{v}$, where v is the specific volume in cubic feet per pound. v is then replaced by v = $\frac{\text{RTZ}}{\text{Mp}}$, where R is the universal gas constant in ft-lb/lb-mole deg F.

R = 1545.393 ft-lb/lb-mole deg F

T is the absolute temperature in degrees Rankine

Z is the dimensionless compressibility factor

M is the molecular weight of the gas

p is the pressure in pounds per square foot

$$V = \frac{W}{n\rho S}$$
(95)

where w is the total mass rate of flow in 1bs per second

n is the number of parallel tubes

S is the cross-sectional area at right angles to the tube in square feet per tube

where p is the pressure in pounds pay square in

o is the density in provide per cubic foot

W is the length in fee

f is the dimensioniess frighten fautor

V is the sverage lithest velocity in fact per success

g is the acceleration due to gravity

s = 32.17405 fc sec

Is the inside discussiv of the circular pipe in feet.

For gas flow, we replace a by $\frac{1}{2}$, where v is the specific volume in cubic feat per pound. v is then replaced by v = $\frac{372}{Mp}$, where k is the universal gas constant in ft-1b/1b-mole deg f.

R = 1345.393 ft-10/15-mole deg f

I is the absolute temperature in degrees Wankins

Z is the dimensionless compressibility factor

H is the molecular weight of the gas

p is the pressure in pounds per square foot

<u>v</u> = 1 200

n is the meder of samilari money

E is the cross-seccional area at right augies to the tabs in square feet par tabs

In our interchanger .

$$S = \frac{\pi D^2}{4}$$
(96)

$$v^{2} = \frac{w^{2}}{n^{2}\rho^{2}s^{2}}$$
(97)

$$s^2 = \frac{\pi^2 p^4}{16}$$
(98)

$$V^{2} = \frac{16}{\pi^{2} D^{4} \rho^{2} n^{2}}$$
(99)

Substituting in equation (94), we have

$$\frac{1}{f} \frac{dp}{dN} = \frac{32 w^2 \rho}{\pi^2 p^4 \rho^2 n^2 g D}$$
$$= \frac{32 w^2 v}{\pi^2 g p^5 n^2}$$
(100)

and substituting for v, we have

$$-\frac{1}{f}\frac{dp}{dN} = \frac{32RTZw^2}{\pi^2 gMpD^5n^2}$$
(101)

$$\frac{32R}{\pi g} = \frac{32 \times 1545.393}{(3.14159)^2 \times 32.17405} = 155.734263$$
(102)

Then for gas flow in general,

$$-\frac{1}{f}\frac{dp}{dN} = \frac{155.734263 \text{ TZw}^2}{\text{MpD}^5 n^2}$$
(103)

47

(78)

(64)

 $y^2 = \frac{16}{r^2 0^2 p^2 n^2}$

Substituting in equation (94), we have

<u>w.v</u> 05_2

and substituting for v. se have

<u>328</u> <u>32. χ 1965. 39.</u> - 1231 Παλάδα (162) π²g (3.16159)⁻ x 32.17607

Then for gas fims in general.

$$-\frac{1}{6}\frac{d_{D}}{dR} = \frac{155.7352653.12w^{2}}{800^{-2}}$$
(103)

In our interchanger,

$$D = \frac{3/16 - 2 \times .042}{12}$$

$$D = 0.008625 \text{ ft.} = 0.8625 \times 10^{-2} \text{ ft.}$$

$$D^5 = 0.477304 \times 10^{-10}$$

Then

$$-\frac{1}{f}\frac{dp}{dN} = \frac{326.279 \times 10^{10} \text{ TZw}^2}{\text{Mpn}^2}$$
(104)

As our lowest operating pressure will be 500 psia, we will take

$$p = 500 \times 144 = 72000 \text{ psfta},$$

and we will take

$$T = 283.15 \times 1.8 = 509.67^{\circ} R$$

so that

$$\frac{326.279 \times 10^{10} \times 509.67}{72000} = 2.30965 \times 10^{10}$$

Thus,

$$-\frac{1}{f}\frac{dp}{dN} = \frac{2.30965 \times 10^{10} \text{ Zw}^2}{\text{Mn}^2}$$
(105)

For stream 3, the nitrogen stream, Z_3 at 500 psia = 0.99149

$$M_3 = 28.016$$
 and $n_3 = 19$

In our interchanger,

Then

As our lowest overating pressure will be 500 pais, we will take

p = 500 × 144 = 72000 parts,

and we will take

so that

, BIRLIT

$$-\frac{1}{2}\frac{dp}{dN} = \frac{2.30965 \times 10^{10} \text{ cm}^2}{\text{Mm}^2}$$
(105)

For stream 3, the nitrogen stream, 2, at 500 pais = 0.99149

$$M_3 = 28.016 \text{ and } n_3 = 19$$

Thus

$$n_3^2 = 361$$

$$-\frac{1}{f_3}\frac{dp_3}{dN} = \frac{2.30965 \times 10^{10} \times 0.99149 w_3^2}{28.016 \times 361}$$
(106)

$$-\frac{1}{f_3}\frac{dp_3}{dN} = 2.26423 \times 10^6 w_3^2$$
(107)

For stream 1, the helium stream

$$Z_1 = 1.01639$$
 $M_1 = 4.0028$

$$n_1 = 5$$
 $n_1^2 = 25$

so that

$$-\frac{1}{f_1}\frac{dp_1}{dN} = \frac{2.30965 \times 10^{10} \times 1.01639 w_1^2}{4.0028 \times 25}$$
(108)

$$\frac{1}{f_1} \frac{dp_1}{dN} = 2.34586 \times 10^8 w_1^2$$
(109)

For stream 2, the mixture stream, with 50-50 mix, we will take $Z_2 = 1.0000$

$$M_2 = \frac{28.016 + 4.0028}{2} = 16.0094$$

$$n_2 = 31$$
 $n_2^2 = 961$

For stream 1, the holion stream

$$x_1 = 1.01039$$
 $M_1 = 0.0029$

ag that

$$\frac{1}{5} \frac{dp_1}{dx} = \frac{2.30965 \times 10^{10} \times 1.01539 \text{ yr}}{5.0028 \times 25}$$

$$\frac{1}{2} = \frac{1}{2} \frac{1}{2} \frac{1}{2} = \frac{1}{2} \frac{1}{2}$$

or stress 2, the pixture stream, with 50-50 min, we will take

$$n_2 = 31$$
 $n_2^2 = 361$

Thus, $-\frac{1}{f_2} \frac{dp_2}{dN} = \frac{2.30965 \times 10^{10} w_2^2}{16.0094 \times 961}$ (110) $-\frac{1}{f_2} \frac{dp_2}{dN} = 1.50123 \times 10^6 w_2^2$ (111)

In order to apply equations (107), (109), and (111), it is necessary to read the friction factor, f, from a graph given on page 78, of reference 1. In order to do this, it is necessary to know the Reynold's number, Re,

$$Re = \frac{4 w}{\pi D \mu n}$$
(112)

where w is the mass flow rate in 1bs per sec. and D is the I.D. of the tube in ft.

 μ is the viscosity in 1b/sec ft. $\mu = 0.0672$ times the viscosity in poises n is the number of parallel tubes. $D = 0.8625 \times 10^{-2}$ ft. $\frac{4}{\pi D} = 147.622$

Therefore,

$$Re = \frac{147.622 \text{ w}}{\mu n}$$
(113)

TIME .

In order to apply equations (107), (107), and (111), tr is necessary to read the relation factor is from a gravit given on page 78, of reference i. In order to do this, it is necessary to know the Reynold's restorer, Se.

where w is the mass flow rate in the per sec. and D is the L.D. of the rube in fr.

$$\mu \text{ Is the veccetty in lower the
$$\mu = 0.0672 \text{ times are viscousis; in poisses
$$\pi \text{ is the memory of parallel radius.}$$

$$D = 0.8625 \times 10^{-2} \text{ re}.$$$$$$

Therefore,

I have taken the viscosity of helium and nitrogen from Richardson, Gordon, Cooper, and Walker (10) as being

(10) Richardson, H. P., J. L. Gordon, J. L. Cooper and J. D. Walker. Thermophysical Properties of Selected Gases Below 300°K. Helium Research Center Internal Report No. 34, July 1963.

$$M_{He} = 192 \text{ micropoises at 500 psia and } 10^{\circ} \text{ C}$$

 $M_{N_2} = 177 \text{ micropoises at 500 psia and } 10^{\circ} \text{ C}$
 $\mu_{He} = 192 \times 10^{-6} \times 0.0672 = 1.29 \times 10^{-5}$
 $\mu_{N_2} = 177 \times 10^{-6} \times 0.0672 = 1.19 \times 10^{-5}$
 $\mu_{2} = \mu_{(He - N_2)} = 1.24 \times 10^{-5}$

For the 50-50 mixture, I take the mean of the pure component values. Then with $n_1 = 5$,

$$\operatorname{Re}_{1} = \frac{147.622 \text{ w}_{1}}{1.29 \times 10^{-5} \times 5}$$

$$Re_1 = 2.2887 \times 10^6 w_1 \tag{114}$$

E have taken the viscosity of helium and nitrogen itom Richardson, Gordon, Cooper, and Walker (10) as helts

(10) Richardson, R. P., J. L. Gordon, J. L. Gaoper and J. D. Walker. Thermophysical Properties of Selected Gases Balow 300° i. Heline Research Center Internal Record No. 34, July 1963.

$$T_{m_{e}} = 192 \text{ micropolass at 500 pais and 10°C
 $T_{m_{e}} = 177 \text{ micropolass at 500 pais and 10°C
 $T_{m_{e}} = 152 \times 10^{-6} \times 0.0672 = 1.29 \times 10^{-5}$
 $T_{m_{e}} = 177 \times 10^{-6} \times 0.0672 = 1.19 \times 10^{-5}$
 $T_{m_{e}} = 1.24 \times 10^{-5} \times 0.0672 = 1.24 \times 10^{-5}$$$$

For the 50-50 mixture, I take the mean of the pure component values

•

(111)

With $n_2 = 31$,

	$Re_{2} = \frac{147.622 w_{2}}{5}$
	$1.24 \times 10^{-5} \times 31$
	$Re_2 = 3.8403 \times 10^5 w_2$

R

and with $n_3 = 19$,

$$e_3 = \frac{147.622 w_3}{1.19 \times 10^{-5} \times 19}$$

$$Re_3 = 6.5291 \times 10^5 w_3 \tag{116}$$

I summarize pressure drop calculations for various flow rates in tables 10 and 11 of this report.

ESTIMATION OF ADDITIONAL INTERCHANGER TUBING NEEDED TO HAVE THE COPPER TUBE WALL INDICATE THE FINAL EQUILIBRIUM TEMPERATURE OF THE HELIUM-NITROGEN MIXTURE STREAM TO WITHIN 0.01°F

At the warm end of the interchanger, the copper tube wall surface will be at a temperature between that of the entering gases and the exiting mixture gas stream. We ask the question, "How much additional length of tubing is required to have the temperature of the tube wall within 0.01° F of the mixed gas temperature?"

In order to answer this question, it is necessary to allow for heat conduction along the tube wall. If x represents the variable length of the interchanger, we assume that the temperature of the tube

(115)

$$\frac{147,622}{2} = \frac{147,622}{1.24 \times 10^{-5} \times 31}$$

and with n. = 19.

I summarias pressure drop calculations for various flow rates in cables 10 and 11 of this report.

ESTIMATION OF ADDITIONAL INTEROMANDES IDDING NEEDED TO HAVE DE COFFER TUBE WALL INDICATE YES FIRAL SQUILCENTER TRACTER OF THE RELIUN-NITROGES MINTINE STREAM TO VIINTE O FILE

At the warm and of the interchanger, the copper tube wall surface will be at a temperature between that of the cutering gases and the exiting mixture gas stream. We suk the question, "How much additional length of tubing is required to have the temperature of the cube wall within 0.01° F of the mixed gas temperature?"

In order to answer this question, it is necessary to allow for heat conduction along the tube wall. If x represents the variable length of the interchanger, we assume that the temperature of the inbu

Stream	SCFM	w, lbs sec ⁻¹	Re	f	- <u>dp</u> dN
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	20	3.44956×10^{-3}	1,273	0.013	0.944
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	80/3	11.49742 × 10^{-3}	4,328	0.0098	3.112
3	20/3	8.04786 × 10^{-3}	5,255	0.0092	1.349
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	40	27.59316 × 10^{-3}	10,597	0.0077	8.801
3	20	24.14361 × 10^{-3}	15,764	0.0069	9.107
1	20/3	$\begin{array}{r} 1.14986 \times 10^{-3} \\ 25.29347 \times 10^{-3} \\ 24.14361 \times 10^{-3} \end{array}$	2,632	0.0125	3.877
2	80/3		• 9,913	0.0078	5.448
3	20		15,764	0.0069	9.107
23-	20	24.14361×10^{-3}	9,661	0,0078	3.867
	20	24.14361×10^{-3}	15,764	0.0069	9.107

TABLE 10. - Pressure drop calculations for various flow rates

TABLE 11. - Pressure drop calculations for various flow rates, N = 10.5 ft

Stream	SCFM	Δp , psf.	∆p, psi
1 2	20	240.3	1.669
	20	9.91	0.069*
*Laminar Flo	W		
1	20	240.3	1.669
2	80/3	32.68	0.227
3	20/3	14.16	0.098
1	20	240.3	1.669
2	40	92.41	0.642
3	20	95.62	0.664
1	20/3	40.71	0.283
2	80/3	57.20	0.397
3	20	95.62	0.664
2	20	40.61	0.282
3	20	95.62	

. 22.69				
			20 40 20	
		I 14980 × 10-3 25.29347 × 10-3 24.14301 × 10-3		
		C-01 x 10001.40 . C-01 x 10001.40 .		

WRIE 10. - Freesure drum critevist (son for Various fice cate)

TABLE 11. - Environe. Aron calculations for verious flow cares,
wall is only a function of x. We thus neglect radial temperature gradients. In a differential length of interchanger, we have

$$\frac{dQ_2}{dx} = n_2 \pi D_2 h_2 (T_{Cu} - T_2) = W_2 C_{p_2} \frac{dT_2}{dx}$$
(117)

$$T_{Cu} - T_2 = a_2 T_2'$$
 (118)

where

$$a_{2} = \frac{W_{2}^{C} p_{2}}{n_{2} \pi D_{2}^{h} p_{2}}$$
(119)

The heat flowing into the differential length, dx, is given by

$$Q_{in} = -kA \frac{dT_{Cu}}{dx}$$
(120)

where k is the thermal conductivity of copper and A is the crosssectional area available for heat conduction. The heat flowing out of this differential length is given by

$$Q_{out} = Q_{in} + \frac{dQ_{in}}{dx} dx$$
 (121)

$$= - kA \left(\frac{dT_{Cu}}{dx} + \frac{d^2T_{Cu}}{dx^2} dx \right)$$
(122)

.

The heat transferred to the gas is given by

well is only a fonction of x. We thus neglere redial than white

$$T_{0_{1}} = T_{2} = s_{2}T_{3}^{2}$$
 (113)

3.211114

The heat flowing into the differential length, dz, is given by

where k is the thermal communicativity of compary and A is the creassectional area available for heat conduction. The heat flowing out of this differential leasth is given by

$$q_{out} = q_{in} + \frac{dq_{in}}{d\pi} d\pi$$
 (121)

$$= -kA \left(\frac{d^{T}Cu}{dx} + \frac{a^{T}T}{dx^{2}} dx \right)$$
 (122)

The heat rransferred to the ges is given by

$$= \frac{dQ}{dx} dx = kA \frac{d^2 T_{Cu}}{dx^2} dx$$
(123)

or

$$\frac{dQ}{dx} = kA \frac{d^2 T_{Cu}}{dx^2}$$
(124)

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = \frac{\mathrm{d}Q_2}{\mathrm{d}x} \tag{125}$$

and

$$kA \frac{d^{2}T_{Cu}}{dx^{2}} = \frac{dQ_{2}}{dx} = W_{2}C_{p_{2}}\frac{dT_{2}}{dx}$$
(126)

or

$$kAT''_{Cu} = b_2 T_2'$$
 (127)

We try, as solutions to equations (118) and (127),

$$\Gamma_2 = \alpha_2 e^{rx}$$
(128)

$$T_{Cu} = \alpha_{Cu} e^{rx}$$
(129)

$$T_2' = \alpha_2 r e^{rx}$$
(130)

$$x_{1} \frac{10^{2} k_{1}}{2x_{2}} = x_{1} \frac{10^{2} k_{1}}{2}$$

$$\frac{dQ}{dx} = \frac{d}{dx}$$

MAT Cu = b2T2 (127

We try, as solutions to squations (118) and (127),

$$T_2^{t} = \pi_2^{w_{\rm H}^{\rm TX}}$$
 (130)

$$T_{Cu} - T_2 = (\alpha_{Cu} - \alpha_2)e^{rx}$$
 (131)

$$T''_{Cu} = \alpha_{Cu} r^2 e^{rx}$$
(132)

Substituting in equation (118), we have

$$(\alpha_{\rm Cu} - \alpha_2) e^{\rm rx} = a_2 \alpha_2 {\rm r} e^{\rm rx}$$
(133)

and for this equation to be true, it is necessary and sufficient that

 $\alpha_{\rm Cu} = \alpha_2 \ (1 + a_2 r) \tag{134}$

Substituting in equation (127), we have

$$kA \alpha_{Cu} r^2 e^{rx} = b_2 \alpha_2 r e^{rx}$$
(135)

and for this equation to be true, it is necessary and sufficient that

$$r\left[kA \alpha_{Cu}r - b_2\alpha_2\right] = 0$$
(136)

Substituting from equation (134), we have

$$r\left[kAr (1 + a_2r) - b_2\right] = 0.$$
 (137)

This is a cubic equation with

$$r_1 = 0.$$
 (138)

$$(12.1)$$
 $z_2 = (a_{(0)} - a_2)e^{-1}$ (12.1)

Substituting is equation (123), we have

and for this equation to be true. It is necessary and solficiant that

$$z_{GU} = v_{2} (1 + u_{1}r)$$
 (134)

Substituting in equation (127), we have

and for this equation to be true, it is modentary and sufficient that

$$r\left[xA \alpha_{CM}^{2} - b_{2}^{2}a_{2}\right] = 0 \qquad (136)$$

Substituting from aquation (134), we have

$$r[kAr(1 + a_2r) - b_2] = 0.$$
 (137)

This is a cubic equation with

*

The other two roots are given by

$$kAa_2r^2 + kAr - b_2 = 0.$$
 (139)

There are 31 parallel tubes carrying the mixed stream; therefore, the cross-sectional area for heat conduction is

$$A = \frac{31 \times 0.019198}{144} = 0.00413296148 \text{ ft}^2$$

For copper

$$k = 222 Btu hr^{-1} ft^{-2} (deg F)^{-1} ft$$

$$kA = 0.917517$$

For the 50-50 mixture stream,

$$a_2 = 0.615879$$

$$b_2 = 37.8481$$

$$kAa_2 = 0.5650794524$$

Substituting in equation (139), we have

$$0.5650794524r^2 + 0.917517r - 37.8481 = 0$$
(140)

The other two reate are given by

There are it peralle; token carcying the mixed atraem; therefore,

For couput

112 - W.SIFSET

For the 50-30 minture stream.

.

kka. = 0.5650794524

Subscituting in equation (139), we have

or

$$r^{2} + 1.623695564r - 66.9783689 = 0$$
(141)

The roots of this equation are

$$r_2 = -9.036047785 \tag{142}$$

$$r_3 = +7.412352225$$
 (143)

Only negative or zero roots will fit our boundary conditions, so we have

$$T_{Cu} = \alpha_{Cu} + \beta_{Cu} e^{r_2 x}$$
(144)

$$T_2 = \alpha_{Cu} + \beta_2 e^{r_2 x}$$
(145)

But from equation (134)

$$\beta_{Cu} = \beta_2 \ (1 + a_2 r_2) \tag{146}$$

$$a_2 = 0.615879$$

Floral Ly.

$$r_2 = -9.036047785$$

~

 $a_2r_2 = -5.565112074$

$$1 + a_2 r_2 = -4.565112074$$

D = CHINE as = seatcher 1 + "

The roots of this equation are

SPO - 9,0300019- - 21

(14) - +7.012358225

Only constitue or sare roots will fit our boundary conditions,

avail an o's

Nuc from equation (134)

300 - R2 (L+ A2 R2) (LA

T2 = -9.036047785

"2"2 - -5.56511207A

1 + m2rg = -0.565112076

so that

$$\Gamma_{Cu} = \alpha_{Cu} - 4.565112074\beta_2 e^{r_2 x}$$
(147)

$$\Gamma_2 = \alpha_{Cu} + \beta_2 e^{\Gamma_2 x}$$
(148)

When x = 0,

$$T_2^o = 64.3398^\circ F$$

 $T_{C11}^o = 66.1607^\circ F$

 $T_{Cu}^{o} - T_{2}^{o} = 1.8209 = -5.565112074\beta_{2}$

 $\beta_2 = -0.3271991607$

 $-4.565112074\beta_2 = 1.493700839$

 $\alpha_{\rm Cu} = 64.3398 + 0.3272$

 $\alpha_{\rm Cu} = 64.6670^{\circ} \, {\rm F}.$

Finally,

"Nos many add in

$$T_{Cu} = 64.6670 + 1.4937 e^{r_2 x}$$
 (149)

$$\Gamma_2 = 64.6670 - 0.32720 e^{r_2 x}$$
(150)

and the

. (147)

(841)

When a - 0,

To a bliastic F

757-97 - 97 - 3⁻¹

A. CD07'99 - "21

Te. - - 1.6209 - - 3.3631207462

B. = -0.3271991607

ecanotces.i = sections.e-

a__ = 64.6610° F.

Finally,

T. - 64.6670 + 1.4937 . - 28

T. - 01.0070 - 0.32720 - 72

(150)

Table 12 lists the various constants needed to solve the differential equations. I have listed the pure helium and pure nitrogen streams for the low temperature end of the interchanger as the solutions are of the same form.

Table 13 gives the equations for r and the various negative roots resulting therefrom.

Table 14 gives the temperatures of the gas streams and the copper tube wall surface as a function of interchanger length.

Table 15 gives the additional length of interchanger tubing needed at the warm end of the interchanger to reduce the temperature of the copper tube wall to within 0.01°F of its final value.

It is clear from table 15 that if we isolate the 31 tubes of the mixture stream from the helium and nitrogen streams and allow a foot additional length, we can be confident that the tube wall surface will measure the temperature of the gas to within 0.01°F.

ESTIMATION OF ADDITIONAL INTERCHANGER TUBING NEEDED TO HAVE THE COPPER TUBE WALL INDICATE THE FINAL EQUILIBRIUM TEMPERATURE OF THE PURE HELIUM AND NITROGEN STREAMS TO WITHIN 0.01° F

When streams 1 and 3 come out of the interchanger at the cold end, they will not be at exactly the same temperature. Also, the copper tube wall surface will be at a temperature between streams 1 and 3 and the mixture stream, stream 2. We then ask the question, "How much additional length of tubing is required to have the temperature of the tube wall within 0.01° F of the final equilibrium Table 12 lists the various constants meeded to solve the differential equations. I have listed the pure holium and pure nitrogen streams for the low temperature and of the interchanger as the solutions are of the same form.

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ESTIMATION OF ADDITIONAL INTERCHANCER TUBING NEEDED TO HAVE THE COPPER TUBE WALL INDICATE THE FINAL BOUILIBRIUM TEMPERATURE OF THE FURE HELIUM AND NITROGEN STREAMS TO WITHIN 0.01°F

When screams 1 and 3 come out of the interchanger at the cold and, they will not be at exactly the same temperature. Also, the copper tube wall surface will be at a temperature between streams 1 and 3 and the mixture stream, stream 2. We than ask the question, "How much additional length of tubing is required to have the tem-

TABL	E 12 Constants	for the equa	tion kAa2r ²	$+ kAr - b_2 =$	0	
ream	Gas	SCFM	T°,°F	а	b	kA
2 Cu	He	20	64.8935 66.1665	0.406337	15.3938	0.917517
2 Cu	He-N ₂ (75-25)	80/3	64.7527 66.3089	0.516957	22.8335	0.917517
2 Cu	He-N ₂ (50-50)	40	64.3398 66.1607	0.615879	37.8481	0.917517
2 Cu	He-N ₂ (25-75)	80/3	64.3430 66.1633	0.605252	27.7581	0.917517
2 Cu	N ₂	20	64.1402 65.9757	0.599647	22.8366	0.917517
1 Cu	Не	20	35.1065 33.2729	0.585280	15.3938	0.710336
3 Cu	N ₂	20	35.8598 33.8355	0.661329	22.8366	0.710336

	5 - 26 +			
			205	
		64.7527 66.3089		
			20	

And the second second

	TABLE	E 13 <u>Equation</u>	s for r	and <u>ca</u>	roots for the various calculations arried out
Str	eam	Gas	SCFM		
	2	Не	20	r ²	+ 2.4610114266r - 41.290044434 = 0 $r_2 = -7.772996735$
	2	He-N ₂ (75-25)	80/3	r ²	+ $1.934396865r - 48.13976288 = 0$ $r_2 = -7.97257035$
	2	He-N ₂ (50-50)	40	r ²	+ 1.623695564r - 66.97836887 = 0 $r_2 = -9.036047785$
	2	He-N ₂ (25-75)	80/3	r ²	+ 1.652204371r - 49.98496394 = 0 $r_2 = -7.94420648$
	2	N ₂	20	r ²	+ 1.6676477994r - 41.5070301 = 0 $r_2 = -7.33015305$
	1	Не	20	r ²	+ $1.7085839255r - 37.026983333 = 0$ $r_2 = -6.998948025$
	3	N ₂	20	r ²	+ 1.5121066823r - 48.612734622 = 0 $r_2 = -7.769208925$

$$T = \alpha_{Cu} + \frac{1}{1 + a_2 r_2} \beta_{Cu} e^{r_2 x}$$

$T_{Cu} = \alpha_{Cu} + \beta$	Cu e ^{r2x}	

			r.	
		6\/08		
			N2	

r. = -7.769208923

$$c = \alpha_{C_{11}} + \frac{1}{1 + \alpha_2 \tau_2} \, \Pi_{C_{11}} \, \alpha^{\pi} 2^{\pi}$$

	Las. The measurement of the		i of interchanger ren	SCII
Stream	Gas	SCFM		
2 Cu	He	20	$T_2 = 65.2965 - 0.403$ $T_{Cu} = 65.2965 + 0.83$	30e ^r 2 ^x 700e ^{r2x}
2 Cu	He-N ₂ (75-25)	80/3	$T_2 = 65.1303 - 0.372$ $T_{Cu} = 65.1303 + 1.12$	76e ^{r2x} 786e ^{r2x}
2 Cu	He-N ₂ (50-50)	40	$T_2 = 64.6670 - 0.327$ $T_{Cu} = 64.6670 + 1.49$	72e ^{r2x} 937e ^{r2x}
2 . Cu	He-N ₂ (25-75)	80/3	$T_2 = 64.7216 - 0.378$ $T_{Cu} = 64.7216 + 1.44$	36e ^r 2 ^x +17e ^{r2x}
2 Cu	N ₂	20	$T_2 = 64.5578 - 0.417$ $T_{Cu} = 64.5578 + 1.41$	76e ^{r2x} 179e ^{r2x}
TABLE 1	5 <u>Additional ler</u> <u>end of inter</u> <u>copper_tube</u>	ngth of inter rchanger, to e wall to wit <u>value</u>	changer needed, at the reduce the temperatur the temperatur the first fi	he warm re of nal
Stream	Gas		SCFM	L, ft.
2 2 2 2 2	He He-N ₂ (75-2 He-N ₂ (50-5 He-N ₂ (25-7 N ₂	25) 50) 75)	20 80/3 40 80/3 20	0.5745 0.5982 0.5541 0.6257 0.6759

TABLE	14.	-	Equations	for	ter	nperature	of	the	gas	and	the	copper	tube
			Wa	11 a	as a	a function	n of	int	terch	nange	er le	ength	

•

	,	,	
T _{Cu} = 64.5670 + 1.4937e ^{12x}			
		He-N2(25-75)	
T _{Cu} = 64.5578 + 1.4179e ²²⁸			

temperature of streams 1 and 3?"

Again, to answer this question, we must allow for heat conduction along the tube wall. Neglecting radial temperature gradients in the copper, we have as our differential equations, as before

$$T_{Cu} - T_{1} = a_{1} T_{1}'$$
 (151)

$$T_{Cu} - T_3 = a_3 T_3'$$
 (152)

$$kAT''_{Cu} = b_1T'_1 + b_3T'_3$$
(153)

As solutions to equations (151), (152), and (153), we will try

$$T_1 = \alpha_1 e^{rx}$$
(154)

$$T_{Cu} = \alpha_2 e^{rx}$$
(155)

$$T_3 = \alpha_3 e^{rx}$$
(156)

Then

$$T'_{1} = \alpha_{1} r e^{rx}$$
 (157)

$$T'_{Cu} = \alpha_2 r e^{rx}$$
(158)

$$T'_{3} = \alpha_{3} r e^{rx}$$
(159)

Camparature of streams 1 and 37"

Again, to answer this question, we must allow for heat conduction along the tube wall. Registring radial temperature gredients in the copper, we have as our differential equations, as before

$$(121) = a_1 T_1 = a_1 T_1$$
 (121)

$$T_{Cu} = T_3 = \frac{4}{3}T_3^T$$
 (152)

$$(LEI) = b_1 T_1 + b_3 T_3$$
(133)

As solutions to equetions (151), (152), and (153), we will try

$$\Gamma_{\rm Gu} = \sigma_2 e^{TN} \tag{155}$$

nenI

(157)

"to " t

L's = agree

$$T''_{Cu} = \alpha_2 r^2 e^{rx}$$
(160)

(

Substituting in equation (151), we have

$$(\alpha_2 - \alpha_1) e^{rx} = a\alpha_1 r e^{rx}$$
(161)

and in order for equation (161) to be true for all x, it is necessary and sufficient that

$$\alpha_2 = \alpha_1 \ (1 + a_1 r) \tag{162}$$

Substituting in equation (152), we have

$$(\alpha_2 - \alpha_3) e^{rx} = a_3 \alpha_3 r e^{rx}$$
(163)

and in order for equation (163) to be true for all x, it is necessary and sufficient that

$$\alpha_2 = \alpha_3 (1 + a_3 r) \tag{164}$$

Substituting in equation (153), we have

$$kA \alpha_2 r^2 e^{rx} = (b_1 \alpha_1 + b_3 \alpha_3) r e^{rx}$$
(165)

and in order for equation (165) to be true for all x, it is necessary and sufficient that

$$kA\alpha_2 r^2 = (b_1 \alpha_1 + b_3 \alpha_3) r$$
 (166)

Substituting in squation (151), we have

and in order for equation (101) to be true for all x, it is necessary and sufficient that

$$a_2 = a_1 (1 + a_1 c)$$
 (162)

Subscituting in equation (152), we have

~

and in order for equation (163) to be true for all x. it is necessary and sufficient that

/

Substituting in equation (153), we have

$$kA \alpha_2 r^2 e^{rx} = (b_1 \alpha_1 + b_3 \alpha_3) r e^{rx}$$
(105)

and in order for equation (165) to be true for all x, it is necessary and sufficient that

$$\log_2 x^2 = (b_1 \sigma_1 + b_3 \sigma_3) \pi$$
 (166)

Substituting for α_1 from equation (162) and for α_3 from equation (164), we have

$$kA\alpha_2 r^2 = \left(\frac{b_1}{1+a_1 r} + \frac{b_3}{1+a_3 r}\right)\alpha_2 r \qquad (167)$$

and equation (167) will be true for all x, if,

$$r\left[kAr \left(1 + a_{1}r\right) \left(1 + a_{3}r\right) - b_{1} \left(1 + a_{3}r\right) - b_{3}\left(1 + a_{1}r\right)\right] = 0 \quad (168)$$

Equation (168) is a fourth degree equation in r. There are four, roots to this equation. Let these roots be r_1 , r_2 , r_3 , and r_4 . Obviously, r = 0 is one of the roots. Let $r_1 = 0$. Then the other three roots are given by the roots of the cubic equation:

kAr
$$(1 + a_1 r) (1 + a_3 r) - b_1 (1 + a_3 r) - b_3 (1 + a_1 r) = 0$$
 (169)

$$kAa_1a_3r^3 + kA(a_1 + a_3)r^2 + (kA - b_1a_3 - b_3a_1)r - b_1 - b_3 = 0.$$
 (170)

The cross-sectional area available for heat conduction, along the tube for a 3/16-inch 0.D., 0.042-inch wall, per tube, is

$$\frac{\pi \left[(0.D.)^2 - (I.D.)^2 \right]}{4}$$

$$O.D. = 3/16'' = 0.1875''$$

Substituting for of from equation (162) and for of from equation (164), we have

$$(167) = \frac{1}{1 + a_1 r} + \frac{1}{1 + a_3 r} a_3 r$$

and equation (107) will be true for all x, 1f.

$$\pi \left[bAr \left(1 + a_{1}r \right) \left(1 + a_{3}r \right) - b_{1} \left(1 + a_{3}r \right) - b_{3}(1 + a_{1}r) \right] = 0 \quad (168)$$

Equation (168) is a fourth degree equation in r. There are four, roots to this equation. Let these roots be $r_1, r_2, r_3, and r_4$. Obviously, r = 0 is one of the roots. Let $r_1 = 0$. Then the other three roots are given by the roots of the cubic equations

$$bar (1 + a_1r) (1 + a_3r) - b_1 (1 + a_3r) - b_3 (1 + a_1r) = 0 \quad (169)$$

$$(Aa_1a_3x^3 + kA (a_1 + a_3)x^2 + (kA - b_1a_3 - b_3a_1)x - b_1 - b_3 = 0.$$
 (170)

The cross-sectional area available for heat conduction, alon the tuba for a 3/16-inch 0.D., 0.042-inch wall, per tube, is

0.D. = 3/16" + 0.1875"

$$I.D. = 0.1875 - 2 \times .042'' = .1035''$$

$$\frac{(0.D.)^2}{4} = 0.0087890625 \text{ in}^2$$

$$\frac{(I.D.)^2}{4} = 0.0026780625 \text{ in}^2$$

$$\frac{(0.D.)^2 - (I.D.)^2}{4} = 0.0061110000$$

$$\pi \left[(0.D.)^2 - (I.D.)^2 \right] = 0.0191982727 \text{ in}^2$$

For stream 1 there will be five tubes and for stream 3 there will be 19 tubes, making a total of 24 tubes available for heat conduction. Total available cross-section area available for heat conduction is then

$$A = \frac{24 \times .0191982727}{144} = 0.0031997121 \text{ ft}^2$$

For the thermal conductivity of copper, we have

$$k = 222 Btu hr^{-1} ft^{-2} (deg F)^{-1} ft.$$

Then

$$kA = 0.710336$$

For stream 1 there will be five tubes and for stream 3 there []] be 19 tubes, making a fotal of 24 tubes available for heat co

duction. Total available cross-section area available for heat con-

$$L = 24 \times .0191982717 = 0.0031997131 fc^2$$

For the chernal conductivity of capper, we have

Then

$$kA = 0.710336$$

with
$$a_1 = 0.585280$$
 $b_1 = 15.3938$

$$a_3 = 0.661329$$
 $b_3 = 22.8366$

$$a_1a_3 = 0.3870626371$$
 $b_1a_3 = 10.18036636$

$$a_1 + a_3 = 1.246609$$
 $b_3 a_1 = 13.36580525$

 $kAa_1a_3 = 0.2749445254$

$$kA (a_1 + a_3) = 0.8855112506$$

 $kA - b_1 a_3 - b_3 a_1 = -22.83583561$

$$-b_1 - b_3 = -38.2304$$

Then equation (170) becomes

 $0.2749445254 r^{3} + 0.8855112506 r^{2} - 22.83583561 r - 38.2304 = 0 \quad (171)$

We divide by 0.2749445254, and we have

 r^{3} + 3.220690609 r^{2} - 83.05615679r - 139.0476859 = 0. (172)

The three roots of this equation are:

$$r_2 = -1.623455805 \tag{173}$$

$$r_3 = -10.087696295$$
 (174)

Then equation (170) becomes

 $0.2749445254 r^{3} + 0.8855112506 r^{2} - 22.83583561 r - 38.2364 = 0 \quad (171)$

We divide by 0,27494452254, and we have

³ + 3.220690609r² - 83.05613679r - 139.0476659 = 0. (172)

The three roots of this equation are:

 $r_q = -10.087696295$

$$r_{1} = +8.490461495 \tag{175}$$

The solutions to our differential equations (151), (152), and (153) are

$$T_{1} = \alpha_{1} e^{r_{1}x} + \beta_{1} e^{r_{2}x} + \gamma_{1} e^{r_{3}x} + \xi_{1} e^{r_{4}x}$$
(176)

$$T_{Cu} = \alpha_2 e^{r_1 x} + \beta_2 e^{r_2 x} + \gamma_2 e^{r_3 x} + \xi_2 e^{r_4 x}$$
(177)

$$T_{3} = \alpha_{3} e^{r_{1}x} + \beta_{3} e^{r_{2}x} + \gamma_{3} e^{r_{3}x} + \xi_{3} e^{r_{4}x}$$
(178)

For an interchanger of infinite length,

$$T_{1}^{\infty} = T_{3}^{\infty} = T_{Cu}^{\infty}.$$
 (179)

This boundary condition indicates that

$$\xi_1 = \xi_2 = \xi_3 = 0.$$
 (180)

Also with $r_1 = 0$,

$$\alpha_1 = \alpha_2 = \alpha_3 \tag{181}$$

Equation (162) leads to

$$\beta_2 = \beta_1 (1 + a_1 r_2)$$
(182)

and

$$\gamma_2 = \gamma_1 (1 + a_1 r_3)$$
 (183)

$$f_{Gu} = \alpha_2 e^{r_1 r_1} + \beta_2 e^{r_2 r_2} + \gamma_2 e^{r_3 r_1} + \xi_2 e^{r_4 r_1}$$
(177)

$$I_{3} = \alpha_{3} e^{x_{1}x} + \beta_{3} e^{x_{2}x} + \gamma_{3} e^{x_{3}x} + \xi_{3} e^{z_{6}x}$$
(178)

For an interchanger of infinite longth,

$$T_1 = T_3 = T_{Gu}$$
 (179)

This boundary condition indicates that

$$\xi_1 = \xi_2 = \xi_3 = 0.$$
 (180)

Also with r, = 0.

$$x_1 = \alpha_2 = \alpha_3$$
 (181)

Equation (152) leads to

-

$$B_2 = B_1 (1 + a_1 r_2) \tag{182}$$

bus

$$v_2 = v_1 (1 + a_1 r_3)$$
 (183)

while equation (164) leads to

$$\beta_2 = \beta_3 (1 + a_3 r_2)$$
(184)

and

$$V_2 = Y_3 (1 + a_3 r_3)$$
 (185)

With $a_1 = 0.585280$ and $a_3 = 0.661329$, we have

$$a_1r_2 = -0.95017621355;$$
 1 + $a_1r_2 = 0.049782378645$

$$a_1r_3 = -5.904126890;$$
 $1 + a_1r_3 = -4.904126890$

$$a_{3}r_{2} = -1.07363840407;$$
 $1 + a_{3}r_{2} = -0.07363840407$

$$a_3r_3 = -6.671286106;$$
 1 + $a_3r_3 = -5.671286106$

Then

$$\beta_{1} = 20.070734708\beta_{2}$$

$$\beta_{3} = -13.57987062\beta_{2}$$

$$\gamma_{1} = -0.2039098952\gamma_{2}$$

$$\gamma_{3} = -0.1763268474\gamma_{2}$$

Then our solutions for equations (176), (177), and (178) become $T_{1} = \alpha_{2} + 20.070734708\beta_{2}e^{r_{2}x} - 0.2039098952\gamma_{2}e^{r_{3}x}$ (186)

while equation (164) leads to

-

$$B_2 = B_3 (1 + a_2 \pi_2)$$
 (184)

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bras

With a. = 0.585280 and a. = 0.001329, we have

Then

$$B_1 = 20.070734708B_2$$

$$B_3 = -13.57987062B_2$$

$$Y_1 = -0.2039098952Y_2$$

$$Y_3 = -0.1763268474Y_3$$

Then our solutions for equations (176), (177), and (178) become $T_1 = \alpha_2 + 20.0707347088_2 e^{T_2 X} - 0.2039098952 v_2 e^{T_3 X}$ (186)

3

$$T_{Cu} = \alpha_2 + \beta_2 e^{r_2 x} + \gamma_2 e^{r_3 x}$$
(187)

$$r_3 = \alpha_2 - 13.57987062\beta_2 e^{r_2 x} - 0.1763268474 \gamma_2 e^{r_3 x}$$
(188)

When x = 0,

$$T_1^\circ = 35.8399^\circ F$$
, $T_3^\circ = 36.0805^\circ F$, $T_{Cu}^\circ = 33.9782^\circ F$

so that

$$35.8399 = \alpha_2 + 20.070734708\beta_2 - 0.2039098952\gamma_2$$
(189)

$$33.9782 = \alpha_2 + \beta_2 + \gamma_2 \tag{190}$$

$$36.0805 = \alpha_2 - 13.57987062\beta_2 - 0.1763268474\gamma_2$$
(191)

or

$$1.8617 = 19.070734708\beta_2 - 1.203909895\gamma_2$$
(192)

$$2.1023 = -14.57987062\beta_2 - 1.1763268474\gamma_2$$
(193)

$$0.09762078013 = \beta_2 - 0.06312865831\gamma_2 \tag{194}$$

$$0.1441919517 = -\beta_2 - 0.08068156969\gamma_2$$
(195)

$$0.2418127318 = -0.1438102280\gamma_2$$
(196)

,

$$1.8617 = 19.0707347083_2 - 1.203903693_{2}$$

$$2.1023 = -14.579870628_2 - 1.1763208674_{2}$$

$$0.09762078013 = 8_2 - 0.06312865831_{2}$$

$$0.09762078013 = 8_2 - 0.06312865831_{2}$$

$$0.1441919517 = -8_2 - 0.08068156969_{2}$$

$$0.1441919517 = -8_2 - 0.1838102280_{2}$$

$$(195)$$

.

$$35.8399 = \alpha_2 + 20.0707347088_2 - 0.20390989529_2 \quad (139)$$
$$\gamma_2 = -1.6814710272 \tag{197}$$

$$\beta_2 = 0.097620780127 - .1061490099216$$

$$\beta_2 = -0.008528229792 \tag{198}$$

$$\alpha_2 = 33.9782 + 1.681471027 + 0.008528229792$$

$$\alpha_2 = 35.66820^{\circ} \,\mathrm{F} \tag{199}$$

Then,

$$\beta_1 = -0.1711678377$$

 $\beta_3 = +0.1158122572$
 $\gamma_1 = +0.3428685809$

$$\gamma_2 = +0.2964884852$$

We then have as our solutions

$$T_1 = 35.66820 - 0.17117e^{r_2x} + 0.34287e^{r_3x}$$
 (200)

$$T_{Cu} = 35.66820 - 0.00853e^{r_2 x} - 1.68147e^{r_3 x}$$
 (201)

$$T_3 = 35.66820 + 0.11581e^{r_2x} + 0.29649e^{r_3x}$$
 (202)

with

 $r_2 = -1.6234558$

$$Y_2 = -1.6814710272$$
 (197)

, nadī

$$\beta_1 = -0.1711678377$$

$$\beta_3 = +0.1158122572$$

$$\gamma_1 = +0.5428685809$$

$$\gamma_3 = +0.2964884832$$

We then have as our solutions

•

$$\Gamma_{e} = 35.66820 + 0.11581e^{\Gamma_{2}\pi} + 0.29649e^{\Gamma_{3}\pi}$$
 (202)

£2 = −1.6234558

WIED

and

$$r_{2} = -10.087696$$

Solving equation (201) for x, when T_{Cu} is within 0.01° F of its final value, we obtain

$$x = 0.5506$$
 feet.

Then we have

$$(T_1)_{x=0.5506} = 35.5995^{\circ} F$$
 (203)

$$(T_{C1})_{x=0} 5506 = 35.6582^{\circ} F$$
 (204)

$$(T_3)_{x=0} 5506 = 35.7167^{\circ} F$$
 (205)

This calculation indicates that just about 6-1/2 inches of interchanger length are needed to bring the copper tube surface between the nitrogen and helium streams to within 0.01° F of its final value. The streams at this point are about 0.12° F apart in temperature. Thus, we do not have to worry over the fact that streams 1 and 3 will be at different temperatures when they come out of the interchanger.

In table 16, I list the various constants for additional calculations, carried out in this section.

In table 17, I list the equations for r and the roots for the calculations carried out in this section.

T. + -10 05769

Solving equation (201) for x, when Tou is within 0.01'F of its final value, we obtain

x = 0.5506 feet.

Than we have

This calculation indicates that just about 6-1/2 inches of interchanger langth are needed to bring the copper tube surface between the nitrogen and halium streams to within 0.01°F of its final value. The streams at this point are about 0.12°F spart in temperature. Thus, we do not have to worry over the fact that streams 1 and 3 will be at different temperatures when they come out of the interchanger.

In table 15, J list the various constants for additional calculations, carried out in this section.

In table 17, I list the equations for r and the roots for the calculations carried out in this section.

In table 18, I give the equations for the temperatures of the gases and the copper tube wall as a function of interchanger length.

In table 19, I give the additional length of interchanger needed, at the cold end of the interchanger, to reduce the temperature of the copper tube wall to within 0.01° F of its final value.

Table 19 shows that if we allow about a foot of additional tubing length, we will measure a temperature of the tube wall which will be well within 0.01° F of the equilibrated temperature of streams 1 and 3.

· 0.0164555900492" + 0.05990/20181

In table 18. I give the equations for the temperatures of the gauges and the dopper tube wall as a function of interchanger length. In table 19. I give the additional length of interchanger needed, at the cold and of the interchanger, to reduce the temperature of the convert tube wall to within 0.01° r of its final value.

•

Table 19 shows that if we allow about a foot of additional tubing langth, we will measure a temperature of the tube wall which will be well within 0.01°F of the equilibrated temperature of streams 1 and 3.

. .

TABLE	16 <u>Con</u>	stants for	the equati	on		
	kAa	$1^{a}3^{r}$ + kA	$(a_1 + a_3)r^2$	+ (kA - $b_1 a_3$	- b ₃ a ₁)r - b ₁	$-b_3 = 0$
Stream	Gas	SCFM	T°,°F	a	Ъ	kA
1 3 Cu	He N ₂	20 20/3	35.5505 35.3765 33.6720	0.585280 0.530878	15.3938 7.6122	0.710336
1 3 Cu	He N ₂	20 20	35.8399 36.0805 33.9782	0.585280 0.661329	15.3938 22.8366	0.710336
1 3 Cu	He N ₂	20/3 20	35.4078 36.0102 33.9241	0.469829 0.661329	5.1313 22.8366	0.710336
TABLE	17 <u>Equ</u>	<u>ations for</u>	<u>r and root</u> <u>carrie</u>	<u>s for the varia</u> d out	ous calculati	ons
Stream	Gas		SCFM			
1 3	He N ₂		20 20/3	0.01852035 - r - 1.93	843r ³ + 0.066 0493154 = 0	52986652r ²
				$r_2 = -1.82$ $r_3 = -8.50$	1673673 1313605	
1 3	He N ₂		20 20	0.01204004 - r - 1.67	$662r^3 + 0.038$ 41406207 = 0	77726508r ²
				$r_2 = -1.62$	3455805	
				$r_3 = -10.0$	87696295	
1 3	He N ₂		20/3 20	0.01645559 -r-2.0852	$049r^3 + 0.059$ 21127 = 0	907246r ²
				$r_2 = -1.97$	8175358	
				$r_3 = -8.87$	7854985	

TABLE 16. - Constants for the equation

	Enlo - May +			
		35.5505 35.2765 33.6720	20 20/3	

 $k_{Aa_1a_3}r^3 + k_A(a_1 + a_3)r^2 + (k_A - b_1a_3 - b_3a_1)r - b_1 + b_3 - b_3a_1)r - b_1 + b_3a_1 +$

Encileuiations		
	sarried out	

	20 20/3		
$\begin{array}{r} 0.01204004662r^{2} + 0.03877726508r^{2} \\ - r - 1.6741406207 = 0 \\ r_{2} = -1.623455805 \end{array}$, He N	

TABLE 18.	- Equation	ns for temper wall as a fu	ratures of the gases and the copper tube unction of interchanger length
Stream	Gas	SCFM	
1	Не	20	$T_1 = 34.6589 + 0.4476e^{rx}$
Cu			$T_{Cu} = 34.6589 - 1.38598e^{rx}$
			(r = -6.998948025)
1	He	20	$T_1 = 35.11513 + 0.07360e^{r_2x} + 0.36177e^{r_3x}$
3	N ₂	20/3	$T_3 = 35.11513 - 0.14802e^{r_2x} + 0.40939e^{r_3x}$
Cu			$T_{Cu} = 35.11513 - 0.00487e^{r_2x} - 1.43826e^{r_3x}$
1	Не	20	$T_1 = 35.6682 - 0.17117e^{r_2x} + 0.34287e^{r_3x}$
3	N ₂	20	$T_3 = 35.6682 + 0.11581e^{r_2x} + 0.29649e^{r_3x}$
Cu			$T_{Cu} = 35.6682 - 0.00853e^{r_2x} - 1.68147e^{r_3x}$
1	He	20/3	$T_1 = 35.54255 - 0.63108e^{r_2x} + 0.49633e^{r_3x}$
3	N ₂	20	$T_3 = 35.54255 + 0.14454e^{r_2x} + 0.32311e^{r_3x}$
Cu	-		$T_{Cu} = 35.54255 - 0.04455e^{r_2x} - 1.57390e^{r_3x}$
3	N ₂	20	$T_3 = 35.4658 + 0.3940e^{rx}$
Cu	2		$T_{Cu} = 35.4658 - 1.63031e^{rx}$
			(r = -7.769208925)

	TANGET TANGET TANGET TANGET TANGET TANGET		
	T _{Cu} = 34.6589 - 1.38598e ^{TK}	,	
	x = -6.998948025		
ADRTe ^F D ^R			
		05	
1			

TABLE 18. - Equations for temperatures of the made and the copper table vall as a function of inverchanger length

	wall to within	0.01° F of its fina	al value
Stream	Gas	SCFM	L, ft.
1	Не	20	0.7046
1 3	He N ₂	20 20/3	0.6052
1 3	He N ₂	20 20	0.5506
1 3	He N ₂	20/3 20	0,8157
3	N ₂	20	0.6557

TABLE	19.	-	Addit	ional	leng	th	of	inte	ercha	angei	r,	need	ed	at	the	cold	end	of
			the	inter	chang	ger	, to	red	luce	the	te	mper	atu	ire	of	copper	c tu	be
				<u>7</u>	wall	to	wit	hin	0.01	L°F (of	its	fir	nal	val	ue		

20	, He	
	AR SK	

TABLE 19. - Additional length of interchanger, needed at the cold and of the interchanger, to reduce the temperature of copper tube wall to within 0.01° F of its final value



