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HELIUM RESEARCH CENTER

INTERNAL REPORT

DESIGN OF INTERCHANGER FOR DETERMINING THE RATIO OF C_p MIXED TO C_p UNMIXED IN THE HELIUM-NITROGEN SYSTEM

BY

Robert E. Barieau

BRANCH Fundamental Research

PROJECT NO. 4329

DATE October 1964

HD
9660
.H43
M56
no.58

AMARILLO, TEXAS

#915080018 ID 88069876

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DESIGN OF INTERCHANGER FOR DETERMINING THE RATIO OF C_p MIXED TO C_p UNMIXED IN THE HELIUM-NITROGEN SYSTEM

By

Robert E. Barieau[1]

ABSTRACT

In this report the design parameters for an interchanger to be used in determining the ratio of C_p mixed to C_p unmixed in a helium-nitrogen system have been calculated. It was found that an interchanger constructed of 3/16 inch O.D. hard-drawn copper tubing would be satisfactory for a design pressure of 5000 psia. An arrangement of 5 helium tubes, 19 nitrogen tubes, and 31 mixture tubes will provide the required flow rate with sufficiently low pressure drops through the interchanger (maximum Δp = 1.7 psi). The interchanger will provide sufficient heat transfer surface with a length of 10.5 feet.

Since it is assumed that thermocouples for temperature measurement will be placed on the copper tube-wall exterior instead of in the flowing gas stream, calculations were made to determine the additional interchanger surface needed to provide accurate indication

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1/ Project Leader, Thermodynamics, Helium Research Center, Bureau of Mines, Amarillo, Texas

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Work on manuscript completed October 1964.

of the final equilibrium gas temperature. It was found that less than one foot of additional length on each end of the interchanger would provide such temperature readings within 0.01°F.

Weight of solder required in fabrication has been calculated and heat leak and insulation requirements have been estimated.

INTRODUCTION

As part of our experimental determinations of the enthalpy of helium-nitrogen mixtures, we will determine the ratio of the heat capacity of the mixture to that of the same components unmixed. This method has been described in a Report of Invention by Robert E. Barieau, entitled "The Adiabatic Mixing Flow Calorimeter and Double Heat Exchanger Method for the Determination of the Enthalpies of Mixtures", dated August 9, 1963. The invention referred to consists of the following: Unmixed streams of helium and nitrogen are brought to the same temperature by means of a high temperature heat sink. They are then cooled by passing through a heat exchanger, pass to a low-temperature heat sink and are cooled to its temperature, pass through a second heat exchanger, where they are warmed, pass to the high temperature heat sink where they are brought to its temperature, then pass to an adiabatic calorimeter where the gases are mixed and the delta T of mixing is determined. The mixed stream then goes to the high temperature heat sink, is brought to its temperature, then passes to the second exchanger, where it is cooled by giving up heat of the final equilibrium gas temperature. It was found that less than one foot of additional length on each end of the interchanger would provide such temperature readings within 0.01°F.

Weight of solder required in fabrication has been calculated and heat leak and insulation requirements have been estimated.

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This report gives the calculated design characteristics of the interchangers mentioned in the above description.

DESCRIPTION OF INTERCHANGER

This interchanger will be constructed of 3/16-inch O.D. hard drawn copper tubing with 0.042-inch wall thickness. There will be a total of 55 parallel tubes, each 10.5 feet long, for a total length of 577.5 feet. The total weight of copper tubing will be 40.85 pounds. The tubes will be soft-soldered into a close-packed cylindrical array. Because the tubing is to be hard drawn, and we desire a safe working pressure of 5000 psia, it will not be possible to use ordinary tin-lead soft solder as this would anneal the copper tubing. Therefore, a bismuth, lead, tin, cadmium, low melting alloy-melting below 200°F, will be used to solder the copper tubes together. The weight of low melting alloy will be 6.8 pounds. When packed into a close-packed cylindrical array the outside diameter of the tubing will be 1.46 inches. The tubing will be surrounded by Linde CS-5 insulation, 0.52 inch thick. The outside diameter of the insulation will be 2.5 inches. The weight of insulation will be 2.6 pounds. The insulation will be surrounded by a vacuum tight tube, as it is to the unmixed streams. The mixed stream then passes to the low temperature heat sink where it is brought to its temperature, then passes through the first heat exchanger where it is warmed by the unmixed streams.

This report gives the calculated design characteristics of the interchangers mentioned in the above description.

DESCRIPTION OF INTERCHANGER.

This interchanger will be constructed of 3/16-inch O.D. hard drawn copper tubing with 0.032-inch wall thickness. There will be a total of 55 parallel tubes, each 10.5 feet long, for a total length of 577.5 feet. The total weight of copper tubing will be 40.85 pounds. The tubes will be soft-soldered into a close packed cylindrical array. Because the tubing is to be hard drawn, and we desire a safe working pressure of 5000 psia, it will not be possible to use ordinary tin-lead soft solder as this would anneal the copper tubing. Therefore; a bismuth, lead, tin, cadmium, low melting alloy-melting below 200°F, will be used to solder the copper tubes together. The weight of low melting alloy will be 6.8 pounds. When packed into a close-packed cylindrical array the outside diameter of the tubing will be 1.46 inches. The tubing will be surrounded by Linde CS-5 insulation, 0.52 inch thick. The outside diameter of the insulation will be 2.5 inches. The weight of insulation will be 2.6 pounds. The insulation will be surrounded by a vacuum tight tube, as it is necessary to evacuate the insulation to about 1 micron (10^{-3}mm) of mercury.

The helium gas will be carried by five of the parallel copper tubes. The nitrogen gas will be carried by 19 of the parallel copper tubes and the mixture stream of helium and nitrogen will be carried by the remaining 31 parallel copper tubes. The total weight of the interchanger exclusive of the outside tube will be 50.25 pounds.

SUMMARY OF CALCULATED OPERATING PERFORMANCE OF INTERCHANGER

A maximum of 20 standard cubic feet per minute of nitrogen will be available from the Amarillo Helium Plant at a pressure of 600 psia. A maximum of 20 standard cubic feet per minute of grade-A helium will be available from the Amarillo Helium Plant at a pressure of 2500 psia. After mixing, the mixture will be returned to the Amarillo Helium Plant, at essentially atmospheric pressure, for recycle purification.

We have calculated the operating performance of this interchanger for mixture compositions of 25-75, 50-50, and 75-25 helium-nitrogen mixtures, and also for the pure components. The calculations were made for a pressure of 500 psia. The inlet temperature of the pure helium and nitrogen streams, at the warm end of the interchanger, was chosen as 68°F (20°C) and the inlet temperature of the helium-nitrogen mixture, at the cold end of the interchanger necessary to evacuate the insulation to about 1 micron (10^{-3} mm) of mercury.

The helium gas will be carried by five of the parallel copper tubes. The nitrogen gas will be carried by 19 of the parallel copper tubes and the mixture stream of helium and nitrogen will be carried by the remaining 11 parallel copper tubes. The total weight of the interchanger exclusive of the outside tube will be 50.25 pounds.

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The calculated pressure drops and temperatures of the streams are given in table 1.

It is seen that the ΔT at the warm end of the interchanger is less than 4°F. Also, the pressure is so low that pressure drop will affect the heat capacity to less than 0.1 percent.

It is seen from table 1 that the smallest rate of heat transfer is for pure helium, amounting to 506 Btu $hr.^{-1}$. Using Linde CS-5 insulation, I have calculated the heat leak to be 0.50 Btu $hr.^{-1}$, or of the order of a 0.1 percent of the heat transferred.

The details of the various calculations follow.

SAFE WORKING PRESSURE OF 3/16 INCH O.D. COPPER TUBING WITH 0.042 INCH WALL

If t is the wall thickness in inches and R is the inside radius in inches, then

$$t = 0.042 \text{ inch.}$$

$$2R = 3/16 - 2 \times 0.042$$

$$2R = .1875 - .084 = 0.1035$$

$$R = 0.05175 \text{ inch}$$

$$t/R = 0.8116.$$

Since $t/R > 1/2$, it is necessary, in calculating working pressures, to use the formulas applicable to thick wall cylindrical shells.

TABLE 1. - Calculated pressure drops and temperatures of inlet and outlet streams under various relative flow rates. P = 500 psia

Stream	Gas	SCFM	T^o	T^L	$T_1^L - T_2^L$	Q, Btu hr^{-1}	Δp, psi
1	He	20	35.1065	68.0000	3.1065	- 506	1.669
2	He	20	32.0000	64.8935		+ 506	0.069*
3	--	--					
*Laminar Flow							
1	He	20	35.5505	68.0000	3.2473	- 500	1.669
2	He-N_2	80/3	32.0000	64.7527		+ 748	0.227
3	N_2	20/3	35.3765	68.0000		- 248	0.098
1	He	20	35.8399	68.0000	3.6602	- 495	1.669
2	He-N_2	40	32.0000	64.3398		+ 1224	0.642
3	N_2	20	36.0805	68.0000		- 729	0.664
1	He	20/3	35.4078	68.0000	3.6570	- 167	0.283
2	He-N_2	80/3	32.0000	64.3430		+ 898	0.397
3	N_2	20	36.0102	68.0000		- 731	0.664
1	--	--					
2	N_2	20	32.0000	64.1402	3.8598	+ 734	0.282
3	N_2	20	35.8598	68.0000		- 734	0.664

These formulas are (1) .

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(1) ASME Boiler and Pressure Vessel Code. Section VIII. Unfired Pressure Vessels, pages 9 and 121.

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(1) Circumferential stress

$$t = R(Z^{1/2} - 1);\ Z = \frac{SE + P}{SE - P}$$

(2) Longitudinal stress

$$t = R(Z^{1/2} - 1);\ Z = \frac{P}{SE} + 1$$

where t = wall thickness, inches

P = allowable working pressure, psi

R = inside radius of shell, inches

S = maximum allowable stress value, psi

E = joint efficiency

(E = 1 for tubes)

with t/R = 0.8116,

$$Z^{1/2} = 1.8116$$

$$Z = 3.2819$$

and for circumferential stress

$$Z = \frac{1 + P/S}{1 - P/S}$$

$$Z - Z\,P/S = 1 + P/S$$

$$P/S = \frac{Z - 1}{Z + 1} = \frac{2.2819}{4.2819}$$

$$P/S = 0.5329$$

For longitudinal stress, we have

$$P/S = Z - 1 = 2.2819$$

Therefore, the determining expression is P/S = 0.5329. Copper tubing comes in the following conditions: annealed, light drawn, and hard drawn. Below 100° F, the maximum allowable stress values for these conditions are given in table 2.

TABLE 2. - Allowable stress and safe working pressure of copper tubing

Condition	S psi	P = .5329 S psi
Annealed	6,000	3,200
Light Drawn	9,000	4,800
Hard Drawn	11,300	6,000

The maximum allowable working pressure is also given in table 2. For 4500 psi operation, the copper tubing must be either light or hard

drawn. For 5000 psi operation the copper tubing must be hard drawn. We have decided to specify that the tubing be hard drawn. We will then plan on a top working pressure of 4500 psi and we will set our relief valves at 5000 psi. The ASME Boiler and Pressure Vessel Code (2) indicates that the maximum allowable stress in copper tubing

(2) Reference 1, page 90.

is the same at 400° F for annealed, soft, and hard drawn copper tubing. This means that heating hard drawn copper tubing to 400° will anneal it. As the melting point of the common lead-tin solders are around 400° F, this means we cannot use ordinary solder without annealing the hard drawn copper tubing. At 250° F, the maximum allowable stress is different for annealed, light, and hard drawn copper tubing.

To be on the safe side, we have decided to specify that the alloy to be used to solder the copper tubing, in a close-packed cylindrical array, must melt below 200° F. Table 3 gives the composition of some eutectic fusible alloys.

TABLE 3. - Composition and melting temperatures of eutectic fusible alloys

	M.T.		Composition				
Item	°F	°C	Bi	Pb	Sn	Cd	Other
A	117	46.8	44.70	22.60	8.30	5.30	19.10 In
B	136	58.	49.00	18.00	12.00	----	21.00 In
C	158	70	50.00	26.70	13.30	10.00	----
D	197	91.5	51.60	40.20	----	8.20	----

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D	197	91.5	51.60	40.20	----	8.20	----

WEIGHT OF COPPER TUBING IN INTERCHANGER

We have a total of 55 parallel, 3/16-inch O.D., 0.042-inch wall, copper tubes each 10.5 feet long. This is a total of 577.5 feet.

$$3/16'' = 0.1875 \text{ inch}$$

$$.1875 - 2 \times .042 = 0.1035$$

$$\pi/4 \times (.1875)^2 = 0.0276116$$

$$\pi/4 \times (.1035)^2 = 0.0084134$$

The cross-sectional area occupied by copper in a single tube is

$$= 0.0276116 - 0.0084134$$

$$= 0.0191982 \text{ square inches}$$

$$\text{Volume} = \frac{0.0191982 \times 577.5}{144}$$

$$= 0.076993 \text{ cu. ft.}$$

$$= 133.044 \text{ cu. in.}$$

$$= 2180.2 \text{ cm}^3$$

$$\text{Weight copper} = 2180.2 \times 8.5$$

$$= 18{,}531.7 \text{ grams}$$

$$= 40.85 \text{ lbs.}$$

APPROXIMATE OUTSIDE DIAMETER OF PACKED INTERCHANGER TUBING

It is planned to soft solder the interchanger tubes in a close-packed cylindrical array. For this packing, the filling factor is

given by

$$f = \frac{\pi}{2\sqrt{3}} = 0.9069.$$

The area taken up per tube is $\frac{\pi D^2}{4} = \frac{\pi \times (.1875)^2}{4} = 0.027612$ sq in.

There are a total of 5 + 19 + 31 = 55 tubes, making a total area of

$$55 \times .027612 = 1.51866 \text{ sq. in.}$$

The interchanger will then take up an area of

$$\frac{1.51866}{.9069} = 1.6746 \text{ sq. in.}$$

This area is enclosed in a circle with a diameter of

$$D = 1.4602 \text{ inches.}$$

ESTIMATION OF WEIGHT OF LOW MELTING SOLDER IN INTERCHANGER

The cross-sectional area of the interchanger is 1.6746 square inches. The actual area taken up by the copper tubes is 1.51866 square inches. The difference, 0.15594, is the area available to be filled with solder.

$$\text{Volume of solder} = \frac{0.15594 \times 10.5}{144}$$

$$= 0.01137 \text{ cubic feet}$$

$$= 19.647 \text{ cubic inches}$$

$$= 321.96 \text{ cm}^3$$

and with a specific gravity of 9.58 for solder, we have a total weight of

$$321.96 \times 9.58 = 3084 \text{ grams}$$

or

$$\frac{3084}{453.6} = 6.80 \text{ lbs of solder.}$$

ESTIMATION OF HEAT LEAK AND WEIGHT OF INSULATION

Linde insulation CS-5 is a mixture of copper flakes and Santocel (3) with an apparent thermal conductivity of

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(3) Riede, P. M., and D. I-J. Wang, Characteristics and Applications of Some Superinsulation. Advances in Cryogenic Engineering, v. 5, K. D. Timmerhaus, Editor. Plenum Press, Inc., New York, 1960, pp. 209-215.

$k = 22 \times 10^{-5}$ Btu hr^{-1} ft^{-2}(deg F)$^{-1}$ ft and a density of 11.0 pounds per cubic foot. In order to obtain this low value, it is necessary to evacuate the insulation. A vacuum of 1 micron of mercury is sufficient.

We will calculate the heat leak for a thickness of 0.5199 inch. The outside diameter of the insulation is then

$$1.4602 + 2 \times .5199 = 2.500 \text{ inches}$$

$$A_i = \frac{\pi \times 1.4602 \times 10.5}{12} = 4.01393$$

$$A_o = \frac{\pi \times 2.5 \times 10.5}{12} = 6.87223$$

$$A_o - A_i = 2.8583; \; A_o/A_i = 1.712095$$

$$\log A_o/A_i = 0.23353; \; \ln A_o/A_i = 0.53772$$

$$A_{av} = \frac{2.8583}{0.53772} = 5.3156$$

$$Q = \frac{k \, \Delta T \, A_{av}}{L}$$

We will take the average ΔT as being 18°F. Then

$$Q = \frac{22 \times 10^{-5} \times 18 \times 5.3156 \times 12}{0.5199}$$

$$Q = 0.486 \text{ Btu hr}^{-1}$$

With 20 scfm of He and N_2 and 40 scfm of He-N_2 mixture going through the interchanger, the heat exchanged is of the order of 1200 Btu hr^{-1}, so the above heat leak is less than 0.1% of the heat exchanged.

The cross-sectional area taken up by the insulation is given by

$$\pi/4 \, (2.5^2 - 1.4602^2) = 3.2341 \text{ sq. in.}$$

The volume of the insulation is

$$= \frac{3.2341 \times 10.5}{144} = 0.2358 \text{ cu ft.}$$

Then the weight of the insulation is

$$0.2358 \times 11.0 = 2.59 \text{ lbs.}$$

DETAILS OF CALCULATION OF TEMPERATURES IN INTERCHANGER

We have calculated the operating characteristics of this interchanger under the following operating conditions.

Pressure = 500 psia

Temperature of the entering helium and nitrogen streams $= 20^\circ C = 68^\circ F$.

Temperature of the entering mixture stream $= 0^\circ C = 32^\circ F$.

Let stream 1 be the helium stream.

Let stream 2 be the mixture stream.

Let stream 3 be the nitrogen stream.

The fundamental equation for heat transfer is given by (4)

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(4) Walker, W. H., W. K. Lewis, W. H. McAdams and E. R. Gilliland, Principles of Chemical Engineering, McGraw-Hill Book Company, Inc. New York and London, 3rd ed. 1937, p. 107.

$$\frac{dQ}{dA} = h\,(T_{Cu} - T) = \pm\, WC_p \frac{dT}{dA} \tag{1}$$

where

$\frac{dQ}{dA}$ is the heat transferred to the gas per unit area of heat transfer surface per unit time in Btu hr^{-1} ft^{-2}, h is the individual heat transfer coefficient for heat transferred from the metal surface to the gas in Btu hr^{-1} ft^{-2} (deg F)$^{-1}$.

T_{Cu} is the temperature of the copper tube surface in deg F.

T is the temperature of the gas in deg F.

W is the mass flow rate of the gas in lbs hr^{-1}.

C_p is the heat capacity of the gas in Btu lb^{-1} (deg F)$^{-1}$.

The + sign in front of W is to be taken when the gas is flowing in the direction of dA positive and the - sign is to be taken when dA positive is in the opposite direction of gas flow.

Now $A = \underline{A}\, x$ and

$dA = \underline{A}\, dx$ where $\underline{A}$ is the area available for heat transfer per unit length of interchanger and x is the length of interchanger.

In our interchanger, streams 1 and 3 are flowing in the same direction and stream 2 is flowing in the opposite direction through the interchanger. We then have for the three streams

$$\frac{dQ_1}{dx} = \underline{A}_1 h_1 (T_{Cu_1} - T_1) = -W_1 C_{p_1} \frac{dT_1}{dx} \tag{2}$$

$$\frac{dQ_2}{dx} = \underline{A}_2 h_2 (T_{Cu_2} - T_2) = W_2 C_{p_2} \frac{dT_2}{dx} \tag{3}$$

$$\frac{dQ_3}{dx} = \underline{A}_3 h_3 (T_{Cu_3} - T_3) = -W_3 C_{p_3} \frac{dT_3}{dx} \tag{4}$$

The length of interchanger increases in the direction of the mixture flow, stream 2. We neglect the thermal lag through the copper tube surface. Thus, we assume that

$$T_{Cu_1} = T_{Cu_2} = T_{Cu_3} = T_{Cu} \tag{5}$$

Then

$$(T_{Cu} - T_1) = - \frac{W_1 C_{p_1}}{\underline{A}_1 h_1} \frac{dT_1}{dx} \tag{6}$$

or

$$(T_{Cu} - T_1) = -a_1 T_1' \tag{7}$$

where

$$a_1 = \frac{W_1 C_{p_1}}{\underline{A}_1 h_1} \tag{8}$$

and

$$T_1' = \frac{dT_1}{dx} \tag{9}$$

Similarly

$$(T_{Cu} - T_2) = a_2 T_2' \tag{10}$$

$$(T_{Cu} - T_3) = - a_3 T_3' \tag{11}$$

and eliminating T_{Cu}, we have

$$(T_1 - T_2) = a_1 T_1' + a_2 T_2' \tag{12}$$

$$(T_3 - T_2) = a_2 T_2' + a_3 T_3' \tag{13}$$

Assuming there is no heat leak into the exchanger, we have

$$\frac{dQ_1}{dx} + \frac{dQ_2}{dx} + \frac{dQ_3}{dx} = 0 \tag{14}$$

Therefore, we can write

$$W_1 C_{p_1} T_1' + W_3 C_{p_3} T_3' = W_2 C_{p_2} T_2' \tag{15}$$

or

$$b_1 T_1' + b_3 T_3' = b_2 T_2' \tag{16}$$

where

$$b = WC_p \tag{17}$$

Equations (12), (13), and (16) are the differential equations that fix the solution of our heat transfer problem. The solutions

are (5)

(5) Barieau, Robert E. Design Equations for Multistream Interchangers. Memorandum Report No. 33, Helium Research Center, Bureau of Mines, January 1964, 29 pp.

$$T_1 = \alpha_2 + \left[\frac{1 + a_2 r_2}{1 - a_1 r_2}\right] \beta_2 e^{r_2 x} + \left[\frac{1 + a_2 r_3}{1 - a_1 r_3}\right] \gamma_2 e^{r_3 x} \qquad (18)$$

$$T_2 = \alpha_2 + \beta_2 e^{r_2 x} + \gamma_2 e^{r_3 x} \qquad (19)$$

$$T_3 = \alpha_2 + \left[\frac{1 + a_2 r_2}{1 - a_3 r_2}\right] \beta_2 e^{r_2 x} + \left[\frac{1 + a_2 r_3}{1 - a_3 r_3}\right] \gamma_2 e^{r_3 x} \qquad (20)$$

where r_2 and r_3 are the roots of the equation

$$\left[\begin{array}{l} (a_1 a_3 b_2 + a_2 a_3 b_1 + a_1 a_2 b_3) r^2 \\ - \left[b_2(a_1 + a_3) + b_1(a_2 - a_3) + b_3(a_2 - a_1)\right] r \\ + b_2 - b_1 - b_3 \end{array}\right] = 0 \qquad (21)$$

The constants α_2, β_2, and γ_2 are fixed by the boundary conditions of the problem. The boundary conditions are fixed by the temperatures of the streams when they flow into the interchanger. By our convention, stream 2 (mixture) flows in at x = 0 and streams 1

(helium) and 3 (nitrogen) flow in at x = L, where L is the length of interchanger. Then we have the three boundary conditions

$$T_1^L = \alpha_2 + \left[\frac{1 + a_2 r_2}{1 - a_1 r_2}\right] \beta_2 e^{r_2 L} + \left[\frac{1 + a_2 r_3}{1 - a_1 r_3}\right] \gamma_2 e^{r_3 L} \qquad (22)$$

$$T_2^o = \alpha_2 + \beta_2 + \gamma_2 \qquad (23)$$

$$T_3^L = \alpha_2 \left[\frac{1 + a_2 r_2}{1 - a_3 r_2}\right] \beta_2 e^{r_2 L} + \left[\frac{1 + a_2 r_3}{1 - a_3 r_3}\right] \gamma_2 e^{r_3 L} \qquad (24)$$

These equations are to be used in the following way: From the assumed flow conditions, a_1, a_2, a_3, b_1, b_2, and b_3 are calculated. These values are then substituted in equation (21) and r_2 and r_3 are determined as the roots of equation (21). r_2 and r_3 are then substituted in equations (22), (23), and (24), and these equations are solved for α_2, β_2, and γ_2. Then these values for α_2, β_2, and γ_2 are substituted in equations (18), (19), and (20) and T_1^o, T_3^o, and T_2^L are calculated. We can then calculate the ΔT's at the ends of the interchanger.

<u>Sample calculation for 20 scfm of helium, 20 scfm of nitrogen, and 40 scfm of helium-nitrogen mixture</u>

In this section, I give a sample calculation for 20 scfm of helium, 20 scfm of nitrogen and 40 scfm of a 50-50 helium-nitrogen mixture. Calculations were also made for 20 scfm of pure helium

only; 20 scfm of pure nitrogen only; 20 scfm of helium, 20/3 scfm nitrogen and 80/3 scfm of a 75-25 helium-nitrogen mixture; and for 20/3 scfm helium, 20 scfm nitrogen, and 80/3 scfm of a 25-75 helium-nitrogen mixture. Calculations, other than for the 50-50 mix, are summarized in the tables.

In my calculations, I have taken $C_{p_1} = 1.23960$ Btu $lb^{-1}(deg\ F)^{-1}$ for helium, and $C_{p_3} = 0.26274$ for nitrogen. For the mixture, I have assumed, at 500 psia,

$$\frac{W_2 C_{p_2}}{W_1 C_{p_1} + W_3 C_{p_3}} = 1 - 0.04y(1-y) \qquad (25)$$

where y is the mole fraction of helium in the mixture.

Then for the 50-50 mix

$$\frac{W_2 C_{p_2}}{W_1 C_{p_1} + W_3 C_{p_3}} = 0.99 \qquad (26)$$

I take a standard cubic foot as the amount of gas, at 1 atmosphere and 70° F, in a cubic foot

$$1 \text{ cu. ft.} = 28{,}316.8 \text{ cm}^3 \qquad (27)$$

The molal volume of a standard cubic foot of gas at 70° F and 1 atmosphere is

$$= 22{,}414.6 \times \frac{294.26}{273.15} = 24{,}146.9 \text{ cm}^3 \qquad (28)$$

$$1 \text{ SCF} = \frac{28,316.8}{24,146.9} = 1.17269 \text{ g moles.}$$

$$\text{M. W. He} = 4.0028$$

$$\text{M. W. } N_2 = 28.016$$

$$1 \text{ SCFM} = \frac{1.17269 \times 60 \times \text{M.W.}}{453.592} \text{ lbs hr}^{-1}$$

$$1 \text{ SCFM} = 0.155120 \times \text{M. W. lbs hr}^{-1}$$

$$20 \text{ SCFM (He)} = 12.4184 \text{ lbs hr}^{-1} = W_1 \quad (29)$$

$$20 \text{ SCFM } (N_2) = 86.9170 \text{ lbs hr}^{-1} = W_3 \quad (30)$$

$$40 \text{ SCFM (50-50, He - } N_2) = 99.3354 \text{ lbs hr}^{-1} = W_2 \quad (31)$$

Then

$$b_1 = W_1 C_{p_1} = 15.3938 \text{ Btu hr}^{-1} (\text{deg F})^{-1} \quad (32)$$

$$b_3 = W_3 C_{p_3} = 22.8366 \text{ Btu hr}^{-1} (\text{deg F})^{-1} \quad (33)$$

$$W_2 C_{p_2} = b_2 = 0.99 (b_1 + b_3) = 37.8481 \text{ Btu hr}^{-1} (\text{deg F})^{-1} \quad (34)$$

$$C_{p_2} = \frac{37.8481}{99.3354} = 0.381013 \text{ Btu lb}^{-1} (\text{deg F})^{-1} \quad (35)$$

Table 4 gives data on the various flow rates for the different calculations.

TABLE 4. - Flow rates and heat capacities for various assumed flow conditions

Stream	Gas	SCFM	W, lbs. hr^{-1}	C_p, Btu lb^{-1} $(deg\ F)^{-1}$
1	He	20	12.4184	1.23960
2	He(100)	20	12.4184	1.23960
3	--	0		
1	He	20	12.4184	1.23960
2	He-N_2(75-25)	80/3	41.3907	0.551658
3	N_2	20/3	28.9723	0.26274
1	He	20	12.4184	1.23960
2	He-N_2(50-50)	40	99.3354	0.381013
3	N_2	20	86.9170	0.26274
1	He	20/3	4.1395	1.23960
2	He-N_2(25-75)	80/3	91.0565	0.304845
3	N_2	20	86.9170	0.26274
1	--	0		
2	N_2(100)	20	86.9170	0.26274
3	N_2	20	86.9170	0.26274

Heat Transfer Coefficients for Gases in Circular Tubes

The heat transfer coefficient, for a gas under turbulent flow, inside a circular tube is given by (6)

(6) Reference 4, p. 112.

$$h = 0.023 \frac{k}{D} \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{C_p \mu}{k}\right)^{1-n} \qquad (36)$$

where

h is the heat transfer coefficient in Btu $hr^{-1} ft^{-2} (deg\ F)^{-1}$

k is the thermal conductivity in Btu $hr^{-1} ft^{-2} (deg\ F)^{-1}$ ft.

D is the inside diameter of the tube in feet

G is the mass velocity in lbs hr^{-1} (sq. ft. of cross-section)$^{-1}$

μ is the viscosity in lbs $hr^{-1} ft^{-1}$ = 2.42 X centipoises

Cp is heat capacity in Btu $lb^{-1} (deg\ F)^{-1}$

Walker, Lewis, McAdams, and Gilliland (7) say that for the

(7) Reference 4, p. 113.

common gases equation (36) can be simplified to give the dimensional equation

$$h = 0.0144\ Cp \frac{G^{0.8}}{D^{0.2}} \qquad (37)$$

However, Giauque (8) obtained the expression

(8) Giauque, W. F. Liquid Oxygen Trailer Unit, Design, Construction, and Operation. Final Report to National Defense Research Committee. Report OSRD No. 4141, July 1944, p. V-11, 539 pp.

$$h = 0.0120\ Cp\ \frac{G^{0.8}}{D^{0.2}} \tag{38}$$

for high pressure air, inside straight circular tubes. We have accepted equation (38) for calculating all heat transfer coefficients.

$$G = \frac{W4}{n\pi D^2} \tag{39}$$

where n is the number of parallel tubes and

$$G^{0.8} = \frac{4^{0.8}\ W^{0.8}}{n^{0.8}\pi^{0.8}\ D^{1.6}} \tag{40}$$

so that

$$h = 0.0120\ Cp\ \frac{4^{0.8}}{\pi^{0.8}}\ (\frac{W}{n})^{0.8}\ \frac{1}{D^{1.8}} \tag{41}$$

$$\frac{4^{0.8}}{\pi^{0.8}} = 1.213188 \tag{42}$$

Thus

$$h = 0.0145583\ (\frac{W}{n})^{0.8}\ \frac{Cp}{D^{1.8}} \tag{43}$$

For 3/16" O.D. tubing with 0.042" wall

$$\text{I.D.} = .1875 - .084 = 0.1035 \text{ inch} \tag{44}$$

$$D = \frac{0.1035}{12} = 0.008625 \text{ ft.} \tag{45}$$

$$\frac{1}{D} = 115.942029 \text{ ft}^{-1} \tag{46}$$

$$\frac{1}{D^{1.8}} = 5195.575 \tag{47}$$

Then

$$h = 75.6387 \left(\frac{W}{n} \right)^{0.8} Cp \tag{48}$$

For helium, $n_1 = 5$

For nitrogen, $n_3 = 19$

For mixture, $n_2 = 31$

$$n_1^{0.8} = 3.623898 \quad Cp_1 = 1.2396$$

$$n_2^{0.8} = 15.598734$$

$$n_3^{0.8} = 10.543939 \quad Cp_3 = 0.26274$$

Then we have for the helium at P = 500 psia,

$$h_1 = 25.8732 \; W_1^{0.8} \tag{49}$$

and for the mixture at P = 500 psia,

$$h_2 = 4.84903\ Cp_2\ W_2^{0.8} \tag{50}$$

and for the nitrogen at P = 500 psia,

$$h_3 = 1.884809\ W_3^{0.8} \tag{51}$$

With $W_1 = 12.4184$, $W_2 = 99.3354$, $W_3 = 86.9170$, and $Cp_2 = 0.381013$, we have

$$W_1^{0.8} = 7.5033035 \tag{52}$$

$$W_2^{0.8} = 39.598910 \tag{53}$$

$$W_3^{0.8} = 35.586378 \tag{54}$$

Finally, we have

$$h_1 = 194.1345 \text{ Btu hr}^{-1} \text{ ft}^{-2} \text{ (deg F)}^{-1} \tag{55}$$

$$h_2 = 73.1607 \text{ Btu hr}^{-1} \text{ ft}^{-2} \text{ (deg F)}^{-1} \tag{56}$$

$$h_3 = 67.0735 \text{ Btu hr}^{-1} \text{ ft}^{-2} \text{ (deg F)}^{-1}. \tag{57}$$

Now

$$a = \frac{WCp}{Ah} = \frac{WCp}{n\pi Dh}$$

$$\pi D = \pi \times 0.008625 = 0.0270962366$$

$$a = \frac{36.905494\ WCp}{nh}$$

Stream	n	h	WCp = b	nh	a
1	5	194.1345	15.3938	970.673	0.585280
2	31	73.1607	37.8481	2267.982	0.615879
3	19	67.0735	22.8366	1274.397	0.661329

The various values of h, b, and a are summarized in table 5. We then have

$a_1 = 0.585280 \qquad b_1 = 15.3938$

$a_2 = 0.615879 \qquad b_2 = 37.8481$

$a_3 = 0.661329 \qquad b_3 = 22.8366$

$$a_1a_3b_2 = 14.6495854$$

$$a_2a_3b_1 = 6.26987385$$

$$a_1a_2b_3 = 8.23171877$$

$$a_1a_3b_2 + a_2a_3b_1 + a_1a_2b_3 = 29.15117802 \qquad (58)$$

$$b_2\ (a_1 + a_3) = 47.18178209$$

$$b_1\ (a_2 - a_3) = -0.69964821$$

$$b_3\ (a_2 - a_1) = 0.69877712$$

$$b_2(a_1 + a_3) + b_1\ (a_2 - a_3) + b_3\ (a_2 - a_1) = 47.18091100 \qquad (59)$$

TABLE 5. - Various values of h, b, and a for the various flow rates

Stream	SCFM	h, Btu $hr^{-1}ft^{-2}(deg\ F)^{-1}$	b	a
1	20	194.1345	15.3938	0.585280
2	20	45.1013	15.3938	0.406337
3	0			
1	20	194.1345	15.3938	0.585280
2	80/3	52.5832	22.8335	0.516957
3	20/3	27.8518	7.6122	0.530878
1	20	194.1345	15.3938	0.585280
2	40	73.1607	37.8481	0.615879
3	20	67.0735	22.8366	0.661329
1	20/3	80.6136	5.1313	0.469829
2	80/3	54.5988	27.7581	0.605252
3	20	67.0735	22.8366	0.661329
1	0			
2	20	45.3383	22.8366	0.599647
3	20	67.0735	22.8366	0.661329

$$b_2 - b_1 - b_3 = -0.3823$$

Substituting in equation (21), we have

$$29.15117802\ r^2 - 47.180911r - 0.3823 = 0$$

or

$$r^2 - 1.6184907164\ r - 0.013114392829 = 0 \qquad (60)$$

The roots of this equation are

$$r_2 = 1.626553404 \qquad (61)$$

$$r_3 = -0.0080626880 \qquad (62)$$

The equations for r and the roots for the various calculations carried out are given in table 6.

TABLE 6. - Equation for r and roots for the various calculations carried out

Stream	SCFM	
1	20	$r^2 - 1.8169475284\ r - 0.012662853174 = 0$
2	80/3	$r_2 = 1.823890300$
3	20/3	$r_3 = -0.0069427715$
1	20	$r^2 - 1.6184907164\ r - 0.013114392829 = 0$
2	40	$r_2 = 1.626553404$
3	20	$r_3 = -0.0080626880$
1	20/3	$r^2 - 1.991757261\ r - 0.01221713692 = 0$
2	80/3	$r_2 = 1.997872335$
3	20	$r_3 = -0.006115074$

Then

$a_1r_2 = 0.95198917629; \quad 1 - a_1r_2 = 0.04801082371$

$a_2r_2 = 1.0017600839; \quad 1 + a_2r_2 = 2.0017600839$

$a_3r_2 = 1.07568693611; \quad 1 - a_3r_2 = -0.07568693611$

$$\frac{1 + a_2r_2}{1 - a_1r_2} = 41.69393335$$

$$\frac{1 + a_2r_2}{1 - a_3r_2} = - 26.44789427$$

$a_1r_3 = -0.004718930033; \quad 1 - a_1r_3 = 1.00471893003$

$a_2r_3 = -0.004965640223; \quad 1 + a_2r_3 = 0.995034359777$

$a_3r_3 = -0.005332089392; \quad 1 - a_3r_3 = 1.005332089392$

$$\frac{1 + a_2r_3}{1 - a_1r_3} = 0.99036091593; \quad \frac{1 + a_2r_3}{1 - a_3r_3} = 0.98975688796$$

Substituting in equations (18), (19), and (20), we have

$$T_1 = \alpha_2 + 41.69393335\, \beta_2 e^{r_2x} + 0.99036091593 \gamma_2 e^{r_3x} \quad (63)$$

$$T_2 = \alpha_2 + \beta_2 e^{r_2x} + \gamma_2 e^{r_3x} \quad (64)$$

$$T_3 = \alpha_2 - 26.44789427\ \beta_2 e^{r_2 x} + 0.98975688796 \gamma_2 e^{r_3 x} \quad (65)$$

With the length of the interchanger being 10.50 feet, we have

$$r_2 L = 17.078810742; \quad \frac{r_2 L}{2.303} = 7.4172333 \quad (66)$$

$$r_3 L = -0.084658224; \quad \frac{r_3 L}{2.303} = -0.0367666 \quad (67)$$

$$[10]^{\frac{r_2 L}{2.303}} = e^{r_2 L} = 2.6135650 \times 10^7 \quad (68)$$

$$[10]^{\frac{-r_3 L}{2.303}} = e^{-r_3 L} = 1.0883450 \quad (69)$$

$$e^{r_3 L} = 0.91882629129 \quad (70)$$

We then have the three boundary condition equations,

$$68 = \alpha_2 + 41.69393335 \times 2.613565 \times 10^7\ \beta_2 + 0.99036091593 \times 0.91882629129\ \gamma_2 \quad (71)$$

$$32 = \alpha_2 + \beta_2 + \gamma_2 \quad (72)$$

$$68 = \alpha_2 - 26.44789427 \times 2.613565 \times 10^7\ \beta_2 + 0.98975688796 \times 0.91882629129\ \gamma_2 \quad (73)$$

or

$$68 = \alpha_2 + 1.0896980492 \times 10^9\ \beta_2 + 0.9099696474\ \gamma_2 \quad (74)$$

$$32 = \alpha_2 + \beta_2 + \gamma_2 \tag{75}$$

$$68 = \alpha_2 - 0.6912329079 \times 10^9 \beta_2 + 0.90941465069 \gamma_2 \tag{76}$$

Subtracting equation (76) from (74), we have

$$0 = 1.780930957 \times 10^9 \beta_2 + 0.0005549967 \gamma_2 \tag{77}$$

and subtracting equation (75) from (74), we have

$$36 = 1.0896980482 \times 10^9 \beta_2 - 0.0900303526 \gamma_2 \tag{78}$$

From equation (77)

$$0 = \beta_2 + 3.116329118 \times 10^{-13} \gamma_2 \tag{79}$$

and from equation (78)

$$33.036674761 \times 10^{-9} = \beta_2 - 826.19541041 \times 10^{-13} \gamma_2 \tag{80}$$

Subtracting equations (79) and (80)

$$33.036674761 \times 10^{-9} = -829.3117395 \times 10^{-13} \gamma_2 \tag{81}$$

$$\gamma_2 = -398.36256 \tag{82}$$

$$\beta_2 = 1.241429 \times 10^{-10} \tag{83}$$

and

$$\alpha_2 = 430.36256$$

Then we have

$$T_1 = 430.3626 + 5.1760 \times 10^{-9} e^{r_2 x} - 394.5227\ e^{r_3 x} \qquad (84)$$

$$T_2 = 430.3626 + 1.2414 \times 10^{-10} e^{r_2 x} - 398.3626\ e^{r_3 x} \qquad (85)$$

$$T_3 = 430.3626 - 3.2833 \times 10^{-9} e^{r_2 x} - 394.2821\ e^{r_3 x} \qquad (86)$$

Equations (84), (85), and (86) give the temperatures of the streams through the interchanger. When x = L = 10.50 feet, we have

$$T_1^L = 430.3626 + 0.1353 - 362.4978 = 68.0000^\circ F \qquad (87)$$

$$T_2^L = 430.3626 + 0.0032 - 366.0260 = 64.3398^\circ F \qquad (88)$$

$$T_3^L = 430.3626 - 0.0858 - 362.2768 = 68.0000^\circ F \qquad (89)$$

At the cold end of the interchanger, where x = 0, we have

$$T_1^o = 430.3626 - 394.5227 = 35.8399^\circ F \qquad (90)$$

$$T_2^o = 430.3626 - 398.3626 = 32.0000^\circ F \qquad (91)$$

$$T_3^o = 430.3626 - 394.2821 = 36.0805^\circ F \qquad (92)$$

Therefore,

$$T_1^L - T_2^L = T_3^L - T_2^L = 3.660^\circ F \qquad (93)$$

or

$$T_1^L - T_2^L = 2.034^\circ C$$

or the mixed stream of He and N_2 will be warmed to $2.03^\circ C$ of the temperature of the entering pure helium and nitrogen. The helium stream will be cooled to $35.840^\circ F$ or to $3.840^\circ F$ of the temperature of the entering cold mixture stream.

While the helium and nitrogen streams start out at the same temperature at the warm end of the interchanger, when they come out of the interchanger at the cold end, the helium stream is $0.241^\circ F$ colder than the nitrogen stream. The problem of allowing or correcting for this temperature difference will be considered later. The temperatures of the various streams as a function of interchanger length are given in table 7. The temperature of the copper tube wall surface as a function of interchanger length is also given.

Table 8 gives the temperatures calculated at the hot and cold end of the interchanger from the equations given in table 7. The last column in table 8 gives the temperature difference between the streams at the warm end of the interchanger. This temperature difference would be zero in an infinite interchanger.

Table 9 gives the rate of heat transfer in the interchanger.

TABLE 7. - Temperature of streams and tube wall surface as a function of interchanger length

Stream	SCFM	
1	20	$T_1 = 35.1065 + 3.13272x$
2	20	$T_2 = 32.0000 + 3.13272x$
		$T_{Cu} = 33.2729 + 3.13272x$
1	20	$T_1 = 497.8375 - 2.4925 \times 10^{-10} e^{r_2 x} - 462.2870 e^{r_3 x}$
2	80/3	$T_2 = 497.8375 + 0.0866 \times 10^{-10} e^{r_2 x} - 465.8375 e^{r_3 x}$
3	20/3	$T_3 = 497.8375 + 5.3003 \times 10^{-10} e^{r_2 x} - 462.4610 e^{r_3 x}$
		$T_{Cu} = 497.8375 + 0.1682 \times 10^{-10} e^{r_2 x} - 464.1655 e^{r_3 x}$
1	20	$T_1 = 430.3626 + 51.760 \times 10^{-10} e^{r_2 x} - 394.5227 e^{r_3 x}$
2	40	$T_2 = 430.3626 + 1.2414 \times 10^{-10} e^{r_2 x} - 398.3626 e^{r_3 x}$
3	20	$T_3 = 430.3626 - 32.833 \times 10^{-10} e^{r_2 x} - 394.2821 e^{r_3 x}$
		$T_{Cu} = 430.3626 + 2.4850 \times 10^{-10} e^{r_2 x} - 396.3844 e^{r_3 x}$
1	20/3	$T_1 = 551.85215 + 36.777 \times 10^{-11} e^{r_2 x} - 516.44433 e^{r_3 x}$
2	80/3	$T_2 = 551.85215 + 1.021 \times 10^{-11} e^{r_2 x} - 519.85215 e^{r_3 x}$
3	20	$T_3 = 551.85215 - 7.023 \times 10^{-11} e^{r_2 x} - 515.84199 e^{r_3 x}$
		$T_{Cu} = 551.85215 + 2.256 \times 10^{-11} e^{r_2 x} - 517.92809 e^{r_3 x}$
2	20	$T_2 = 32.0000 + 3.06097x$
3	20	$T_3 = 35.8598 + 3.06097x$
		$T_{Cu} = 33.8355 + 3.06097x$

TABLE 8. - Temperatures at hot and cold end of interchanger

Stream	SCFM	T^{o}	T^{L}	$T^{L} - T^{o}$	$T_1^{L} - T_2^{L}$
1	20	35.1065	68.0000	32.8935	3.1065
2	20	32.0000	64.8935	32.8935	
Cu		33.2729	66.1665		
1	20	35.5505	68.0000	32.4495	3.2473
2	80/3	32.0000	64.7527	32.7527	
3	20/3	35.3765	68.0000	32.6235	
Cu		33.6720	66.3089		
1	20	35.8399	68.0000	32.1601	3.6602
2	40	32.0000	64.3398	32.3398	
3	20	36.0805	68.0000	31.9195	
Cu		33.9782	66.1607		
1	20/3	35.4078	68.0000	32.5922	3.6570
2	80/3	32.0000	64.3430	32.3430	
3	20	36.0102	68.0000	31.9898	
Cu		33.9241	66.1633		
2	20	32.0000	64.1402	32.1402	3.8598
3	20	35.8598	68.0000	32.1402	
Cu		33.8355	65.9757		

TABLE 9. - Calculation of heat transferred

Stream	SCFM	$T^L - T^o$	b	Q, Btu hr^{-1}
1	20	32.8935	15.3938	-506.356
2	20	32.8935	15.3938	+506.356
1	20	32.4495	15.3938	-499.521
2	80/3	32.7527	22.8335	+747.859
3	20/3	32.6235	7.6122	-248.337
				Σ = +0.001
1	20	32.1601	15.3938	-495.066
2	40	32.3398	37.8481	+1224.000
3	20	31.9195	22.8366	-728.933
				Σ = +0.001
1	20/3	32.5922	5.1313	-167.240
2	80/3	32.3430	27.7581	+897.780
3	20	31.9898	22.8366	-730.538
				Σ = +0.002
2	20	32.1402	22.8366	+733.973
3	20	32.1402	22.8366	-733.973

CALCULATION OF TEMPERATURE DROP THROUGH COPPER TUBE WALL

The equations used in calculating the performance of the interchanger have been derived on the assumption that the temperature drop through the copper wall is small and can be neglected. It is important that this temperature difference be calculated to see if it is permissible to neglect it.

From table 9, we see that for the 50-50 mix experiment,

$$Q_1 = -495.066 \text{ Btu/hr}$$

and since for the helium stream there are five tubes, we have

$$\frac{Q_1}{n_1} = -\ 99.013 \text{ Btu/hr tube.}$$

The area inside a tube is

$$A_i = \pi D_i L$$

where D is the inside diameter in feet and L is the length of tube in feet

$$A_i = \pi \times 0.008625 \times 10.50$$

$$A_i = 0.28451 \text{ ft}^2$$

The area outside a tube is

$$A_o = \pi D_o L$$

$$= \frac{\pi \times .1875}{12} \times 10.50$$

$$A_o = 0.51542 \text{ ft}^2$$

$$A_{av} = \frac{A_o - A_i}{\ln A_o/A_i}$$

$$A_o - A_i = 0.23091$$

$$A_o/A_i = 1.811606$$

$$\log_{10} A_o/A_i = 0.25806$$

$$\ln A_o/A_i = 0.59421$$

$$A_{av} = 0.38860$$

$$Q = \frac{k A_{av} \Delta T_{Cu}}{L}$$

$$L = \frac{.042}{12} = 0.0035 \text{ ft.}$$

$$\Delta T_{Cu} = \frac{QL}{kA_{av}}$$

$$\Delta T_{Cu} = Q \times \frac{0.0035}{222 \times 0.38860}$$

$$\Delta T_{Cu} = Q \times 0.0000406$$

Then for the helium, with $\frac{Q_1}{n_1} = -99.013$ Btu hr^{-1} tube^{-1}, we have

$$\Delta T_{Cu_1} = 0.0040^{\circ} F$$

Table 9 indicates that for the nitrogen stream

$$Q_3 = -728.933 \text{ Btu hr}^{-1}$$

and with 19 parallel tubes for the nitrogen stream, we have

$$\frac{Q_3}{n_3} = -38.365 \text{ Btu hr}^{-1} \text{ tube}^{-1}$$

and thus, for the nitrogen stream

$$\Delta T_{Cu_3} = 38.365 \times .0000406$$

$$\Delta T_{Cu_3} = 0.0016^{\circ} F$$

Table 9 indicates that for the 50-50 mixture stream

$$Q_2 = 1224.000 \text{ Btu hr}^{-1}.$$

and with 31 parallel tubes for the mixture stream, we have

$$\frac{Q_2}{n_2} = 39.484 \text{ Btu hr}^{-1} \text{ tube}^{-1}$$

Thus, for the mixture stream

$$\Delta T_{Cu_2} = 39.484 \times 0.0000406$$

$$\Delta T_{Cu_2} = 0.0016^\circ F$$

With the ΔT in the interchanger between the streams being cooled and heated being $3.7^\circ F$, we see that the ΔT_{Cu} through the copper tube wall surface is completely negligible.

DETAILS OF CALCULATION OF PRESSURE DROPS THROUGH THE INTERCHANGER

If we neglect kinetic energy and gravitational potential energy effects, the pressure drop due to friction for circular tubes can be expressed by the so-called Fanning equation (9)

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(9) Reference 1, p. 77.

$$\frac{dF}{dN} = -\frac{dp}{dN}\frac{1}{\rho} = 2f\frac{V^2}{gD} \qquad (94)$$

where p is the pressure in pounds per square foot

ρ is the density in pounds per cubic foot

N is the length in feet

f is the dimensionless friction factor

V is the average linear velocity in feet per second

g is the acceleration due to gravity

$g = 32.17405 \text{ ft sec}^{-2}$

D is the inside diameter of the circular pipe in feet.

For gas flow, we replace ρ by $\frac{1}{v}$, where v is the specific volume in cubic feet per pound. v is then replaced by $v = \frac{RTZ}{Mp}$, where R is the universal gas constant in ft-lb/lb-mole deg F.

R = 1545.393 ft-lb/lb-mole deg F

T is the absolute temperature in degrees Rankine

Z is the dimensionless compressibility factor

M is the molecular weight of the gas

p is the pressure in pounds per square foot

$$V = \frac{w}{n\rho S} \tag{95}$$

where w is the total mass rate of flow in lbs per second

n is the number of parallel tubes

S is the cross-sectional area at right angles to the tube in square feet per tube

$$S = \frac{\pi D^2}{4} \tag{96}$$

$$V^2 = \frac{w^2}{n^2 \rho^2 S^2} \tag{97}$$

$$S^2 = \frac{\pi^2 D^4}{16} \tag{98}$$

$$V^2 = \frac{16 \quad w^2}{\pi^2 D^4 \rho^2 n^2} \tag{99}$$

Substituting in equation (94), we have

$$- \frac{1}{f} \frac{dp}{dN} = \frac{32 \, w^2 \rho}{\pi^2 D^4 \rho^2 n^2 gD}$$

$$= \frac{32 \, w^2 v}{\pi^2 g D^5 n^2} \tag{100}$$

and substituting for v, we have

$$- \frac{1}{f} \frac{dp}{dN} = \frac{32 RTZw^2}{\pi^2 gMpD^5 n^2} \tag{101}$$

$$\frac{32R}{\pi^2 g} = \frac{32 \times 1545.393}{(3.14159)^2 \times 32.17405} = 155.734263 \tag{102}$$

Then for gas flow in general,

$$- \frac{1}{f} \frac{dp}{dN} = \frac{155.734263 \; TZw^2}{MpD^5 n^2} \tag{103}$$

In our interchanger,

$$D = \frac{3/16 - 2 \times .042}{12}$$

$$D = 0.008625 \text{ ft.} = 0.8625 \times 10^{-2} \text{ft.}$$

$$D^5 = 0.477304 \times 10^{-10}$$

Then

$$-\frac{1}{f}\frac{dp}{dN} = \frac{326.279 \times 10^{10} \ TZw^2}{Mpn^2} \tag{104}$$

As our lowest operating pressure will be 500 psia, we will take

$$p = 500 \times 144 = 72000 \text{ psfta},$$

and we will take

$$T = 283.15 \times 1.8 = 509.67^\circ R$$

so that

$$\frac{326.279 \times 10^{10} \times 509.67}{72000} = 2.30965 \times 10^{10}$$

Thus,

$$-\frac{1}{f}\frac{dp}{dN} = \frac{2.30965 \times 10^{10} \ Zw^2}{Mn^2} \tag{105}$$

For stream 3, the nitrogen stream, Z_3 at 500 psia = 0.99149

$$M_3 = 28.016 \text{ and } n_3 = 19$$

$$n_3^2 = 361$$

$$-\frac{1}{f_3}\frac{dp_3}{dN} = \frac{2.30965 \times 10^{10} \times 0.99149\, w_3^2}{28.016 \times 361} \qquad (106)$$

$$-\frac{1}{f_3}\frac{dp_3}{dN} = 2.26423 \times 10^{6}\, w_3^2 \qquad (107)$$

For stream 1, the helium stream

$$Z_1 = 1.01639 \qquad M_1 = 4.0028$$

$$n_1 = 5 \qquad n_1^2 = 25$$

so that

$$-\frac{1}{f_1}\frac{dp_1}{dN} = \frac{2.30965 \times 10^{10} \times 1.01639\, w_1^2}{4.0028 \times 25} \qquad (108)$$

$$-\frac{1}{f_1}\frac{dp_1}{dN} = 2.34586 \times 10^{8}\, w_1^2 \qquad (109)$$

For stream 2, the mixture stream, with 50-50 mix, we will take

$$Z_2 = 1.0000$$

$$M_2 = \frac{28.016 + 4.0028}{2} = 16.0094$$

$$n_2 = 31 \qquad n_2^2 = 961$$

Thus,

$$-\frac{1}{f_2}\frac{dp_2}{dN} = \frac{2.30965 \times 10^{10} w_2^2}{16.0094 \times 961} \tag{110}$$

$$-\frac{1}{f_2}\frac{dp_2}{dN} = 1.50123 \times 10^6 w_2^2 \tag{111}$$

In order to apply equations (107), (109), and (111), it is necessary to read the friction factor, f, from a graph given on page 78, of reference 1. In order to do this, it is necessary to know the Reynold's number, Re,

$$Re = \frac{4 w}{\pi D \mu n} \tag{112}$$

where w is the mass flow rate in lbs per sec. and D is the I.D. of the tube in ft.

μ is the viscosity in lb/sec ft.

$\mu = 0.0672$ times the viscosity in poises

n is the number of parallel tubes.

$D = 0.8625 \times 10^{-2}$ ft.

$\frac{4}{\pi D} = 147.622$

Therefore,

$$Re = \frac{147.622 w}{\mu n} \tag{113}$$

I have taken the viscosity of helium and nitrogen from Richardson, Gordon, Cooper, and Walker (10) as being

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(10) Richardson, H. P., J. L. Gordon, J. L. Cooper and J. D. Walker. Thermophysical Properties of Selected Gases Below 300°K. Helium Research Center Internal Report No. 34, July 1963.

$$\eta_{He} = 192 \text{ micropoises at } 500 \text{ psia and } 10^{\circ}C$$

$$\eta_{N_2} = 177 \text{ micropoises at } 500 \text{ psia and } 10^{\circ}C$$

$$\mu_{He} = 192 \times 10^{-6} \times 0.0672 = 1.29 \times 10^{-5}$$

$$\mu_{N_2} = 177 \times 10^{-6} \times 0.0672 = 1.19 \times 10^{-5}$$

$$\mu_2 = \mu_{(He - N_2)} = 1.24 \times 10^{-5}$$

For the 50-50 mixture, I take the mean of the pure component values.

Then with $n_1 = 5$,

$$Re_1 = \frac{147.622 \, w_1}{1.29 \times 10^{-5} \times 5}$$

$$Re_1 = 2.2887 \times 10^{6} \, w_1 \tag{114}$$

With $n_2 = 31$,

$$Re_2 = \frac{147.622\ w_2}{1.24 \times 10^{-5} \times 31}$$

$$Re_2 = 3.8403 \times 10^5\ w_2 \qquad (115)$$

and with $n_3 = 19$,

$$Re_3 = \frac{147.622\ w_3}{1.19 \times 10^{-5} \times 19}$$

$$Re_3 = 6.5291 \times 10^5 w_3 \qquad (116)$$

I summarize pressure drop calculations for various flow rates in tables 10 and 11 of this report.

ESTIMATION OF ADDITIONAL INTERCHANGER TUBING NEEDED TO HAVE THE COPPER TUBE WALL INDICATE THE FINAL EQUILIBRIUM TEMPERATURE OF THE HELIUM-NITROGEN MIXTURE STREAM TO WITHIN 0.01°F

At the warm end of the interchanger, the copper tube wall surface will be at a temperature between that of the entering gases and the exiting mixture gas stream. We ask the question, "How much additional length of tubing is required to have the temperature of the tube wall within 0.01°F of the mixed gas temperature?"

In order to answer this question, it is necessary to allow for heat conduction along the tube wall. If x represents the variable length of the interchanger, we assume that the temperature of the tube

TABLE 10. - Pressure drop calculations for various flow rates

Stream	SCFM	w, lbs sec^{-1}	Re	f	$-\frac{dp}{dN}$
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	20	3.44956×10^{-3}	1,273	0.013	0.944
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	80/3	11.49742×10^{-3}	4,328	0.0098	3.112
3	20/3	8.04786×10^{-3}	5,255	0.0092	1.349
1	20	3.44956×10^{-3}	7,895	0.0082	22.89
2	40	27.59316×10^{-3}	10,597	0.0077	8.801
3	20	24.14361×10^{-3}	15,764	0.0069	9.107
1	20/3	1.14986×10^{-3}	2,632	0.0125	3.877
2	80/3	25.29347×10^{-3}	9,913	0.0078	5.448
3	20	24.14361×10^{-3}	15,764	0.0069	9.107
2	20	24.14361×10^{-3}	9,661	0.0078	3.867
3	20	24.14361×10^{-3}	15,764	0.0069	9.107

TABLE 11. - Pressure drop calculations for various flow rates, N = 10.5 ft

Stream	SCFM	Δp, psf.	Δp, psi
1	20	240.3	1.669
2	20	9.91	0.069*
*Laminar Flow			
1	20	240.3	1.669
2	80/3	32.68	0.227
3	20/3	14.16	0.098
1	20	240.3	1.669
2	40	92.41	0.642
3	20	95.62	0.664
1	20/3	40.71	0.283
2	80/3	57.20	0.397
3	20	95.62	0.664
2	20	40.61	0.282
3	20	95.62	0.664

wall is only a function of x. We thus neglect radial temperature gradients. In a differential length of interchanger, we have

$$\frac{dQ_2}{dx} = n_2 \pi D_2 h_2 (T_{Cu} - T_2) = W_2 C_{p_2} \frac{dT_2}{dx} \qquad (117)$$

$$T_{Cu} - T_2 = a_2 T_2' \qquad (118)$$

where

$$a_2 = \frac{W_2 C_{p_2}}{n_2 \pi D_2 h_2} \qquad (119)$$

The heat flowing into the differential length, dx, is given by

$$Q_{in} = - kA \frac{dT_{Cu}}{dx} \qquad (120)$$

where k is the thermal conductivity of copper and A is the cross-sectional area available for heat conduction. The heat flowing out of this differential length is given by

$$Q_{out} = Q_{in} + \frac{dQ_{in}}{dx} dx \qquad (121)$$

$$= - kA \left(\frac{dT_{Cu}}{dx} + \frac{d^2 T_{Cu}}{dx^2} dx \right) \qquad (122)$$

The heat transferred to the gas is given by

$$Q_{in} - Q_{out}$$

$$= \frac{dQ}{dx} dx = kA \frac{d^2T_{Cu}}{dx^2} dx \tag{123}$$

or

$$\frac{dQ}{dx} = kA \frac{d^2T_{Cu}}{dx^2} \tag{124}$$

$$\frac{dQ}{dx} = \frac{dQ_2}{dx} \tag{125}$$

and

$$kA \frac{d^2T_{Cu}}{dx^2} = \frac{dQ_2}{dx} = W_2C_{p_2} \frac{dT_2}{dx} \tag{126}$$

or

$$kAT''_{Cu} = b_2T'_2 \tag{127}$$

We try, as solutions to equations (118) and (127),

$$T_2 = \alpha_2e^{rx} \tag{128}$$

$$T_{Cu} = \alpha_{Cu}e^{rx} \tag{129}$$

$$T'_2 = \alpha_2re^{rx} \tag{130}$$

$$T_{Cu} - T_2 = (\alpha_{Cu} - \alpha_2)e^{rx} \tag{131}$$

$$T''_{Cu} = \alpha_{Cu} r^2 e^{rx} \tag{132}$$

Substituting in equation (118), we have

$$(\alpha_{Cu} - \alpha_2)e^{rx} = a_2\alpha_2 re^{rx} \tag{133}$$

and for this equation to be true, it is necessary and sufficient that

$$\alpha_{Cu} = \alpha_2 \ (1 + a_2 r) \tag{134}$$

Substituting in equation (127), we have

$$kA \ \alpha_{Cu} r^2 e^{rx} = b_2\alpha_2 re^{rx} \tag{135}$$

and for this equation to be true, it is necessary and sufficient that

$$r\left[kA \ \alpha_{Cu} r \ - \ b_2\alpha_2\right] \ = \ 0 \tag{136}$$

Substituting from equation (134), we have

$$r\left[kAr \ (1 + a_2 r) - b_2\right] = 0. \tag{137}$$

This is a cubic equation with

$$r_1 \ = \ 0. \tag{138}$$

The other two roots are given by

$$kAa_2r^2 + kAr - b_2 = 0. \qquad (139)$$

There are 31 parallel tubes carrying the mixed stream; therefore, the cross-sectional area for heat conduction is

$$A = \frac{31 \times 0.019198}{144} = 0.00413296148 \text{ ft}^2$$

For copper

$$k = 222 \text{ Btu hr}^{-1} \text{ ft}^{-2}(\text{deg F})^{-1}\text{ft}$$

$$kA = 0.917517$$

For the 50-50 mixture stream,

$$a_2 = 0.615879$$

$$b_2 = 37.8481$$

$$kAa_2 = 0.5650794524$$

Substituting in equation (139), we have

$$0.5650794524r^2 + 0.917517r - 37.8481 = 0 \qquad (140)$$

or

$$r^2 + 1.623695564r - 66.9783689 = 0 \quad (141)$$

The roots of this equation are

$$r_2 = -9.036047785 \quad (142)$$

$$r_3 = +7.412352225 \quad (143)$$

Only negative or zero roots will fit our boundary conditions, so we have

$$T_{Cu} = \alpha_{Cu} + \beta_{Cu} e^{r_2 x} \quad (144)$$

$$T_2 = \alpha_{Cu} + \beta_2 e^{r_2 x} \quad (145)$$

But from equation (134)

$$\beta_{Cu} = \beta_2 \ (1 + a_2 r_2) \quad (146)$$

$$a_2 = 0.615879$$

$$r_2 = -9.036047785$$

$$a_2 r_2 = -5.565112074$$

$$1 + a_2 r_2 = -4.565112074$$

so that

$$T_{Cu} = \alpha_{Cu} - 4.565112074\beta_2 e^{r_2 x} \quad (147)$$

$$T_2 = \alpha_{Cu} + \beta_2 e^{r_2 x} \quad (148)$$

When x = 0,

$$T_2^o = 64.3398^o F$$

$$T_{Cu}^o = 66.1607^o F$$

$$T_{Cu}^o - T_2^o = 1.8209 = -5.565112074\beta_2$$

$$\beta_2 = -0.3271991607$$

$$-4.565112074\beta_2 = 1.493700839$$

$$\alpha_{Cu} = 64.3398 + 0.3272$$

$$\alpha_{Cu} = 64.6670^o F.$$

Finally,

$$T_{Cu} = 64.6670 + 1.4937\ e^{r_2 x} \quad (149)$$

$$T_2 = 64.6670 - 0.32720\ e^{r_2 x} \quad (150)$$

Table 12 lists the various constants needed to solve the differential equations. I have listed the pure helium and pure nitrogen streams for the low temperature end of the interchanger as the solutions are of the same form.

Table 13 gives the equations for r and the various negative roots resulting therefrom.

Table 14 gives the temperatures of the gas streams and the copper tube wall surface as a function of interchanger length.

Table 15 gives the additional length of interchanger tubing needed at the warm end of the interchanger to reduce the temperature of the copper tube wall to within $0.01^{\circ}F$ of its final value.

It is clear from table 15 that if we isolate the 31 tubes of the mixture stream from the helium and nitrogen streams and allow a foot additional length, we can be confident that the tube wall surface will measure the temperature of the gas to within $0.01^{\circ}F$.

ESTIMATION OF ADDITIONAL INTERCHANGER TUBING NEEDED TO HAVE THE COPPER TUBE WALL INDICATE THE FINAL EQUILIBRIUM TEMPERATURE OF THE PURE HELIUM AND NITROGEN STREAMS TO WITHIN $0.01^{\circ}F$

When streams 1 and 3 come out of the interchanger at the cold end, they will not be at exactly the same temperature. Also, the copper tube wall surface will be at a temperature between streams 1 and 3 and the mixture stream, stream 2. We then ask the question, "How much additional length of tubing is required to have the temperature of the tube wall within $0.01^{\circ}F$ of the final equilibrium

TABLE 12. - Constants for the equation $kAa_2r^2 + kAr - b_2 = 0$

ream	Gas	SCFM	T°, °F	a	b	kA
2 Cu	He	20	64.8935 66.1665	0.406337	15.3938	0.917517
2 Cu	He-N_2(75-25)	80/3	64.7527 66.3089	0.516957	22.8335	0.917517
2 Cu	He-N_2(50-50)	40	64.3398 66.1607	0.615879	37.8481	0.917517
2 Cu	He-N_2(25-75)	80/3	64.3430 66.1633	0.605252	27.7581	0.917517
2 Cu	N_2	20	64.1402 65.9757	0.599647	22.8366	0.917517
1 Cu	He	20	35.1065 33.2729	0.585280	15.3938	0.710336
3 Cu	N_2	20	35.8598 33.8355	0.661329	22.8366	0.710336

TABLE 13. - Equations for r and roots for the various calculations carried out

Stream	Gas	SCFM	
2	He	20	$r^2 + 2.4610114266r - 41.290044434 = 0$ $r_2 = -7.772996735$
2	$He\text{-}N_2$(75-25)	80/3	$r^2 + 1.934396865r - 48.13976288 = 0$ $r_2 = -7.97257035$
2	$He\text{-}N_2$(50-50)	40	$r^2 + 1.623695564r - 66.97836887 = 0$ $r_2 = -9.036047785$
2	$He\text{-}N_2$(25-75)	80/3	$r^2 + 1.652204371r - 49.98496394 = 0$ $r_2 = -7.94420648$
2	N_2	20	$r^2 + 1.6676477994r - 41.5070301 = 0$ $r_2 = -7.33015305$
1	He	20	$r^2 + 1.7085839255r - 37.026983333 = 0$ $r_2 = -6.998948025$
3	N_2	20	$r^2 + 1.5121066823r - 48.612734622 = 0$ $r_2 = -7.769208925$

$$T = \alpha_{Cu} + \frac{1}{1 + a_2 r_2} \beta_{Cu} e^{r_2 x}$$

$$T_{Cu} = \alpha_{Cu} + \beta_{Cu} e^{r_2 x}$$

TABLE 14. - Equations for temperature of the gas and the copper tube wall as a function of interchanger length

Stream	Gas	SCFM	
2	He	20	$T_2 = 65.2965 - 0.4030e^{r_2 x}$
Cu			$T_{Cu} = 65.2965 + 0.8700e^{r_2 x}$
2	$He-N_2$(75-25)	80/3	$T_2 = 65.1303 - 0.3776e^{r_2 x}$
Cu			$T_{Cu} = 65.1303 + 1.1786e^{r_2 x}$
2	$He-N_2$(50-50)	40	$T_2 = 64.6670 - 0.3272e^{r_2 x}$
Cu			$T_{Cu} = 64.6670 + 1.4937e^{r_2 x}$
2	$He-N_2$(25-75)	80/3	$T_2 = 64.7216 - 0.3786e^{r_2 x}$
Cu			$T_{Cu} = 64.7216 + 1.4417e^{r_2 x}$
2	N_2	20	$T_2 = 64.5578 - 0.4176e^{r_2 x}$
Cu			$T_{Cu} = 64.5578 + 1.4179e^{r_2 x}$

TABLE 15. - Additional length of interchanger needed, at the warm end of interchanger, to reduce the temperature of copper tube wall to within 0.01° F of its final value

Stream	Gas	SCFM	L, ft.
2	He	20	0.5745
2	$He-N_2$(75-25)	80/3	0.5982
2	$He-N_2$(50-50)	40	0.5541
2	$He-N_2$(25-75)	80/3	0.6257
2	N_2	20	0.6759

temperature of streams 1 and 3?"

Again, to answer this question, we must allow for heat conduction along the tube wall. Neglecting radial temperature gradients in the copper, we have as our differential equations, as before

$$T_{Cu} - T_1 = a_1 T_1' \tag{151}$$

$$T_{Cu} - T_3 = a_3 T_3' \tag{152}$$

$$kAT''_{Cu} = b_1 T_1' + b_3 T_3' \tag{153}$$

As solutions to equations (151), (152), and (153), we will try

$$T_1 = \alpha_1 e^{rx} \tag{154}$$

$$T_{Cu} = \alpha_2 e^{rx} \tag{155}$$

$$T_3 = \alpha_3 e^{rx} \tag{156}$$

Then

$$T_1' = \alpha_1 r e^{rx} \tag{157}$$

$$T_{Cu}' = \alpha_2 r e^{rx} \tag{158}$$

$$T_3' = \alpha_3 r e^{rx} \tag{159}$$

$$T''_{Cu} = \alpha_2 r^2 e^{rx} \qquad (160)$$

Substituting in equation (151), we have

$$(\alpha_2 - \alpha_1)\ e^{rx} = a\alpha_1 re^{rx} \qquad (161)$$

and in order for equation (161) to be true for all x, it is necessary and sufficient that

$$\alpha_2 = \alpha_1\ (1 + a_1 r) \qquad (162)$$

Substituting in equation (152), we have

$$(\alpha_2 - \alpha_3)\ e^{rx} = a_3\alpha_3 re^{rx} \qquad (163)$$

and in order for equation (163) to be true for all x, it is necessary and sufficient that

$$\alpha_2 = \alpha_3\ (1 + a_3 r) \qquad (164)$$

Substituting in equation (153), we have

$$kA\ \alpha_2 r^2 e^{rx} = (b_1\alpha_1 + b_3\alpha_3) re^{rx} \qquad (165)$$

and in order for equation (165) to be true for all x, it is necessary and sufficient that

$$kA\alpha_2 r^2 = (b_1\alpha_1 + b_3\alpha_3) r \qquad (166)$$

Substituting for α_1 from equation (162) and for α_3 from equation (164), we have

$$kA\alpha_2 r^2 = \left(\frac{b_1}{1 + a_1 r} + \frac{b_3}{1 + a_3 r}\right)\alpha_2 r \qquad (167)$$

and equation (167) will be true for all x, if,

$$r\left[kAr\ (1 + a_1 r)\ (1 + a_3 r) - b_1\ (1 + a_3 r) - b_3 (1 + a_1 r)\right] = 0 \qquad (168)$$

Equation (168) is a fourth degree equation in r. There are four roots to this equation. Let these roots be r_1, r_2, r_3, and r_4. Obviously, r = 0 is one of the roots. Let $r_1 = 0$. Then the other three roots are given by the roots of the cubic equation:

$$kAr\ (1 + a_1 r)\ (1 + a_3 r) - b_1\ (1 + a_3 r) - b_3\ (1 + a_1 r) = 0 \qquad (169)$$

or

$$kAa_1 a_3 r^3 + kA\ (a_1 + a_3) r^2 + (kA - b_1 a_3 - b_3 a_1) r - b_1 - b_3 = 0. \qquad (170)$$

The cross-sectional area available for heat conduction, along the tube for a 3/16-inch O.D., 0.042-inch wall, per tube, is

$$\frac{\pi\left[(O.D.)^2 - (I.D.)^2\right]}{4}$$

$$O.D. = 3/16'' = 0.1875''$$

$$\text{I.D.} = 0.1875 - 2 \times .042'' = .1035''$$

$$\frac{(\text{O.D.})^2}{4} = 0.0087890625 \text{ in}^2$$

$$\frac{(\text{I.D.})^2}{4} = 0.0026780625 \text{ in}^2$$

$$\frac{(\text{O.D.})^2 - (\text{I.D.})^2}{4} = 0.0061110000$$

$$\frac{\pi\left[(\text{O.D.})^2 - (\text{I.D.})^2\right]}{4} = 0.0191982727 \text{ in}^2$$

For stream 1 there will be five tubes and for stream 3 there will be 19 tubes, making a total of 24 tubes available for heat conduction. Total available cross-section area available for heat conduction is then

$$A = 24 \times \frac{.0191982727}{144} = 0.0031997121 \text{ ft}^2$$

For the thermal conductivity of copper, we have

$$k = 222 \text{ Btu hr}^{-1} \text{ ft}^{-2} (\text{deg F})^{-1} \text{ft.}$$

Then

$$kA = 0.710336$$

with $a_1 = 0.585280$ $b_1 = 15.3938$

$a_3 = 0.661329$ $b_3 = 22.8366$

$a_1a_3 = 0.3870626371$ $b_1a_3 = 10.18036636$

$a_1 + a_3 = 1.246609$ $b_3a_1 = 13.36580525$

$$kAa_1a_3 = 0.2749445254$$

$$kA\ (a_1 + a_3) = 0.8855112506$$

$$kA - b_1a_3 - b_3a_1 = -\ 22.83583561$$

$$-b_1 - b_3 = -38.2304$$

Then equation (170) becomes

$$0.2749445254\ r^3 + 0.8855112506\ r^2 - 22.83583561\ r - 38.2304 = 0 \quad (171)$$

We divide by 0.2749445254, and we have

$$r^3 + 3.220690609r^2 - 83.05615679r - 139.0476859 = 0. \quad (172)$$

The three roots of this equation are:

$$r_2 = -1.623455805 \quad (173)$$

$$r_3 = -10.087696295 \quad (174)$$

$$r_4 = +8.490461495 \tag{175}$$

The solutions to our differential equations (151), (152), and (153) are

$$T_1 = \alpha_1 e^{r_1 x} + \beta_1 e^{r_2 x} + \gamma_1 e^{r_3 x} + \xi_1 e^{r_4 x} \tag{176}$$

$$T_{Cu} = \alpha_2 e^{r_1 x} + \beta_2 e^{r_2 x} + \gamma_2 e^{r_3 x} + \xi_2 e^{r_4 x} \tag{177}$$

$$T_3 = \alpha_3 e^{r_1 x} + \beta_3 e^{r_2 x} + \gamma_3 e^{r_3 x} + \xi_3 e^{r_4 x} \tag{178}$$

For an interchanger of infinite length,

$$T_1^{\infty} = T_3^{\infty} = T_{Cu}^{\infty}. \tag{179}$$

This boundary condition indicates that

$$\xi_1 = \xi_2 = \xi_3 = 0. \tag{180}$$

Also with $r_1 = 0$,

$$\alpha_1 = \alpha_2 = \alpha_3 \tag{181}$$

Equation (162) leads to

$$\beta_2 = \beta_1 (1 + a_1 r_2) \tag{182}$$

and

$$\gamma_2 = \gamma_1 (1 + a_1 r_3) \tag{183}$$

while equation (164) leads to

$$\beta_2 = \beta_3 (1 + a_3 r_2) \tag{184}$$

and

$$\gamma_2 = \gamma_3 (1 + a_3 r_3) \tag{185}$$

With $a_1 = 0.585280$ and $a_3 = 0.661329$, we have

$$a_1 r_2 = -0.95017621355; \quad 1 + a_1 r_2 = 0.049782378645$$

$$a_1 r_3 = -5.904126890; \quad 1 + a_1 r_3 = -4.904126890$$

$$a_3 r_2 = -1.07363840407; \quad 1 + a_3 r_2 = -0.07363840407$$

$$a_3 r_3 = -6.671286106; \quad 1 + a_3 r_3 = -5.671286106$$

Then

$$\beta_1 = 20.0707347088\beta_2$$

$$\beta_3 = -13.57987062\beta_2$$

$$\gamma_1 = -0.2039098952\gamma_2$$

$$\gamma_3 = -0.1763268474\gamma_2$$

Then our solutions for equations (176), (177), and (178) become

$$T_1 = \alpha_2 + 20.0707347088\beta_2 e^{r_2 x} - 0.2039098952\gamma_2 e^{r_3 x} \tag{186}$$

$$T_{Cu} = \alpha_2 + \beta_2 e^{r_2 x} + \gamma_2 e^{r_3 x} \tag{187}$$

$$T_3 = \alpha_2 - 13.579870628\beta_2 e^{r_2 x} - 0.1763268474\gamma_2 e^{r_3 x} \tag{188}$$

When x = 0,

$$T_1^\circ = 35.8399^\circ F,\ T_3^\circ = 36.0805^\circ F,\ T_{Cu}^\circ = 33.9782^\circ F$$

so that

$$35.8399 = \alpha_2 + 20.070734708\beta_2 - 0.2039098952\gamma_2 \tag{189}$$

$$33.9782 = \alpha_2 + \beta_2 + \gamma_2 \tag{190}$$

$$36.0805 = \alpha_2 - 13.579870628\beta_2 - 0.1763268474\gamma_2 \tag{191}$$

or

$$1.8617 = 19.070734708\beta_2 - 1.203909895\gamma_2 \tag{192}$$

$$2.1023 = -14.579870628\beta_2 - 1.1763268474\gamma_2 \tag{193}$$

$$0.09762078013 = \beta_2 - 0.06312865831\gamma_2 \tag{194}$$

$$0.1441919517 = -\beta_2 - 0.08068156969\gamma_2 \tag{195}$$

$$0.2418127318 = -0.1438102280\gamma_2 \tag{196}$$

$$\gamma_2 = -1.6814710272 \tag{197}$$

$$\beta_2 = 0.097620780127 - .1061490099216$$

$$\beta_2 = -0.008528229792 \tag{198}$$

$$\alpha_2 = 33.9782 + 1.681471027 + 0.008528229792$$

$$\alpha_2 = 35.66820^\circ F \tag{199}$$

Then,

$$\beta_1 = -0.1711678377$$

$$\beta_3 = +0.1158122572$$

$$\gamma_1 = +0.3428685809$$

$$\gamma_3 = +0.2964884852$$

We then have as our solutions

$$T_1 = 35.66820 - 0.17117e^{r_2 x} + 0.34287e^{r_3 x} \tag{200}$$

$$T_{Cu} = 35.66820 - 0.00853e^{r_2 x} - 1.68147e^{r_3 x} \tag{201}$$

$$T_3 = 35.66820 + 0.11581e^{r_2 x} + 0.29649e^{r_3 x} \tag{202}$$

with

$$r_2 = -1.6234558$$

and

$$r_3 = -10.087696$$

Solving equation (201) for x, when T_{Cu} is within $0.01^\circ F$ of its final value, we obtain

$$x = 0.5506 \text{ feet.}$$

Then we have

$$(T_1)_{x=0.5506} = 35.5995^\circ F \quad (203)$$

$$(T_{Cu})_{x=0.5506} = 35.6582^\circ F \quad (204)$$

$$(T_3)_{x=0.5506} = 35.7167^\circ F \quad (205)$$

This calculation indicates that just about 6-1/2 inches of interchanger length are needed to bring the copper tube surface between the nitrogen and helium streams to within $0.01^\circ F$ of its final value. The streams at this point are about $0.12^\circ F$ apart in temperature. Thus, we do not have to worry over the fact that streams 1 and 3 will be at different temperatures when they come out of the interchanger.

In table 16, I list the various constants for additional calculations, carried out in this section.

In table 17, I list the equations for r and the roots for the calculations carried out in this section.

In table 18, I give the equations for the temperatures of the gases and the copper tube wall as a function of interchanger length.

In table 19, I give the additional length of interchanger needed, at the cold end of the interchanger, to reduce the temperature of the copper tube wall to within 0.01°F of its final value.

Table 19 shows that if we allow about a foot of additional tubing length, we will measure a temperature of the tube wall which will be well within 0.01°F of the equilibrated temperature of streams 1 and 3.

TABLE 16. - Constants for the equation

$$kAa_1a_3r^3 + kA(a_1 + a_3)r^2 + (kA - b_1a_3 - b_3a_1)r - b_1 - b_3 = 0$$

Stream	Gas	SCFM	T°, °F	a	b	kA
1	He	20	35.5505	0.585280	15.3938	0.710336
3	N_2	20/3	35.3765	0.530878	7.6122	
Cu			33.6720			
1	He	20	35.8399	0.585280	15.3938	0.710336
3	N_2	20	36.0805	0.661329	22.8366	
Cu			33.9782			
1	He	20/3	35.4078	0.469829	5.1313	0.710336
3	N_2	20	36.0102	0.661329	22.8366	
Cu			33.9241			

TABLE 17. - Equations for r and roots for the various calculations carried out

Stream	Gas	SCFM	
1	He	20	$0.01852035843r^3 + 0.06652986652r^2$
3	N_2	20/3	$- r - 1.930493154 = 0$
			$r_2 = -1.821673673$
			$r_3 = -8.501313605$
1	He	20	$0.01204004662r^3 + 0.03877726508r^2$
3	N_2	20	$- r - 1.6741406207 = 0$
			$r_2 = -1.623455805$
			$r_3 = -10.087696295$
1	He	20/3	$0.01645559049r^3 + 0.059907246r^2$
3	N_2	20	$- r - 2.085221127 = 0$
			$r_2 = -1.978175358$
			$r_3 = -8.877854985$

TABLE 18. - Equations for temperatures of the gases and the copper tube wall as a function of interchanger length

Stream	Gas	SCFM	
1	He	20	$T_1 = 34.6589 + 0.4476e^{rx}$
Cu			$T_{Cu} = 34.6589 - 1.38598e^{rx}$
			(r = -6.998948025)
1	He	20	$T_1 = 35.11513 + 0.07360e^{r_2x} + 0.36177e^{r_3x}$
3	N_2	20/3	$T_3 = 35.11513 - 0.14802e^{r_2x} + 0.40939e^{r_3x}$
Cu			$T_{Cu} = 35.11513 - 0.00487e^{r_2x} - 1.43826e^{r_3x}$
1	He	20	$T_1 = 35.6682 - 0.17117e^{r_2x} + 0.34287e^{r_3x}$
3	N_2	20	$T_3 = 35.6682 + 0.11581e^{r_2x} + 0.29649e^{r_3x}$
Cu			$T_{Cu} = 35.6682 - 0.00853e^{r_2x} - 1.68147e^{r_3x}$
1	He	20/3	$T_1 = 35.54255 - 0.63108e^{r_2x} + 0.49633e^{r_3x}$
3	N_2	20	$T_3 = 35.54255 + 0.14454e^{r_2x} + 0.32311e^{r_3x}$
Cu			$T_{Cu} = 35.54255 - 0.04455e^{r_2x} - 1.57390e^{r_3x}$
3	N_2	20	$T_3 = 35.4658 + 0.3940e^{rx}$
Cu			$T_{Cu} = 35.4658 - 1.63031e^{rx}$
			(r = -7.769208925)

TABLE 19. - Additional length of interchanger, needed at the cold end of the interchanger, to reduce the temperature of copper tube wall to within 0.01°F of its final value

Stream	Gas	SCFM	L, ft.
1	He	20	0.7046
1	He	20	0.6052
3	N_2	20/3	
1	He	20	0.5506
3	N_2	20	
1	He	20/3	0.8157
3	N_2	20	
3	N_2	20	0.6557