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INTERNAL REPORT

ELASTIC DISTORTION OF THE HIGH PRESSURE

COMPRESSIBILITY BOMBS AT 30° C

BY

Ted C. Briggs

BRANCH Fundamental Research

PROJECT NO. 4330

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ELASTIC DISTORTION OF THE HIGH PRESSURE COMPRESSIBILITY BOMBS AT 30° C

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ABSTRACT

This report contains equations representing the change of compressibility bomb volume as a function of internal and jacket pressures. Methods to correct for elastic distortion of the bombs are included.

INTRODUCTION

The original Burnett compressibility method (4)[2/] depended upon the supposition that the ratio $\frac{V_1 + V_2}{V_1}$ of the isothermal volumes of two containers is constant. In practice, the isothermal volume of a container is a function of the confined pressure, and of the pressure surrounding the container.

The Thermodynamics Section of the Helium Research Center is using the Burnett method to measure compressibility factors of gases. The compressibility bombs, currently in use, are surrounded by oil

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2/ Underlined numbers in parentheses refer to items in the list of references at the end of this report.

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jackets. Pressure in the oil jackets can be varied at will by use of an oil displacement pump. Volumes of the two compressibility bombs are functions of the internal bomb pressure and pressure in the oil jackets. Internal gas pressures can be measured with a high degree of precision by use of Ruska Instrument Corporation piston gage No. 9274 (2). Jacket pressures are measured with a much lower degree of precision by use of a Heise Bourdon tube gage. Precision of measurement of the internal gas pressure $\leq$ 0.01 psi over the pressure range 30-10,000 psig. Precision of measurement of the jacket pressure is about $\pm$ 5 psi over the pressure range 0-5,000 psi.

Elastic distortion of the PVT bombs results in an error in the experimental compressibility factors unless a correction is made for distortion effects. The purpose of this report is to develop equations to represent the change of bomb volumes V_1 and $V_1 + V_2$ as a function of the internal and jacket pressures, and to present methods to correct for elastic distortion of the bombs.

DESIGNATION OF THE COMPONENT VOLUMES OF V_1 AND V_2

A Burnett apparatus consists of two containers designated as V_1 and V_2. The original method consisted of a series of expansions of gas from V_1 into an evacuated V_2. To develop equations to represent V_1 and $V_1 + V_2$ as a function of the internal and jacket pressures, one must know the dimensions of V_1 and V_2, and the magnitude of the

jacketed and unjacketed portions of V_1 and V_2. For the following derivations notations are assigned to the various volumes.

V_1^o = volume of V_1 at zero internal and jacket pressures

V_2^o = volume of V_2 at zero internal and jacket pressures

$V_{b_1}^o$ = volume of jacketed bomb portion of V_1 at zero internal and jacket pressures

$V_{b_2}^o$ = volume of jacketed bomb portion of V_2 at zero internal and jacket pressures

$V_{t_1}^o$ = volume of unjacketed tubing portion of V_1 at zero internal pressure

$V_{t_2}^o$ = volume of unjacketed tubing portion of V_2 at zero internal pressure

$V_{f_1}^o$ = volume of unjacketed fittings connected to V_1 at zero internal pressure

$V_{f_2}^o$ = volume of unjacketed fittings connected to V_2 at zero internal pressure

CHANGE OF JACKETED BOMB VOLUME AS A FUNCTION OF INTERNAL AND JACKET PRESSURES

The PVT bombs were machined from 303 stainless steel bar stock. Dimensions of one of the jacketed bombs are listed in figure 1. In the following derivations a number of simplifying assumptions are made. Important assumptions are numbered.

Assumption 1: Dimensions of jacketed bomb volumes $V_{b_1}^o$ and $V_{b_2}^o$ are identical.

From the dimensions of figure 1, the jacketed bomb volume is estimated to be 4.649 in^3. Examination of figure 1 reveals that the

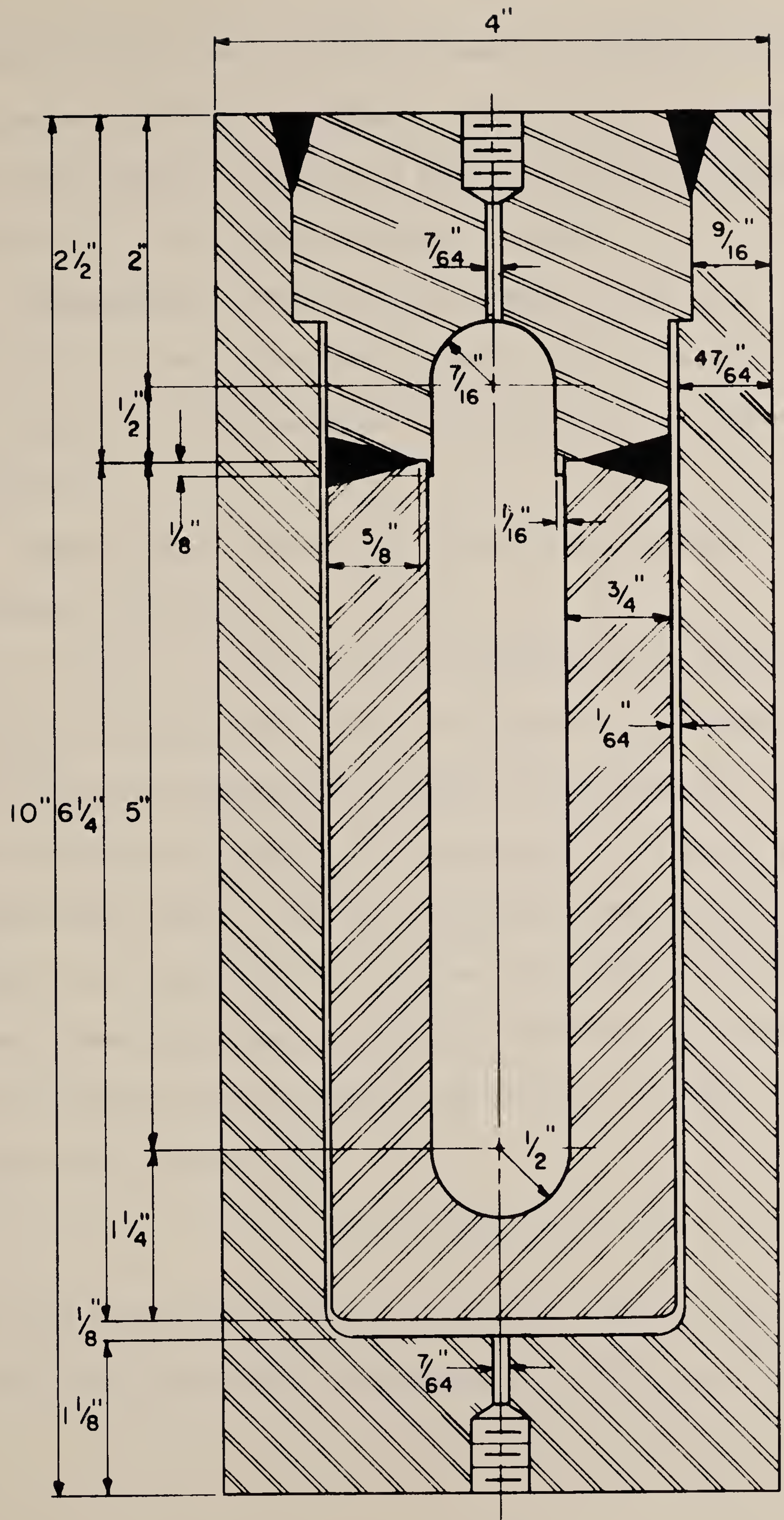

FIGURE 1.-Jacketed High-Pressure Compressibility Bomb

jacketed bomb volume consists of several volumes of various wall thickness and of various shapes. It would be difficult to compute the exact pressure function of each of the various jacketed volumes; therefore, a simplifying assumption is made.

<u>Assumption 2</u>: The jacketed volume consists of a uniform, thick-wall, closed plane end cylinder, with an internal radius of 0.5 inch, a wall thickness of 0.75 inch, and an internal volume of 4.649 in^3.

<u>Assumption 3</u>: Distortion of the internal bombs, tubing, and fittings is elastic.

A number of references are available in the literature pertaining to elastic distortion of cylinders. Timoshenko (<u>8</u>) presents an equation for the change of radius of an open-end cylinder. Love (<u>6</u>) developed an equation for the change of volume of a thick-wall closed-end cylinder. Newitt (<u>7</u>) derived equations for the change of radius and length for both open-end and closed-end thick-wall cylinders. These references are used to calculate the change of volume of the internal bombs, tubing, and fittings. Volume of a cylinder is given by equation (1).

$$V = \pi r^2 L \tag{1}$$

Take the derivative of equation (1) with respect to the radius and length, and replace the derivatives with delta quantities to obtain

equation (2).

$$\Delta V = 2\pi\, r\, L\, \Delta\, r + \pi\, r^2\, \Delta\, L \tag{2}$$

Divide equation (2) by equation (1) to obtain equation (3)

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L} \tag{3}$$

A jacketed cylinder with closed ends is subject to a longitudinal stress due to pressure acting on the ends. The elongation is given by equation (4), references (6) and (7).

$$\frac{\Delta L}{L} = \frac{(1-2\sigma)(a^2 P_i - b^2 P_j)}{E(b^2 - a^2)} \tag{4}$$

Change of internal radius of a thick-wall, closed-end, jacketed cylinder is given by equation (5), reference (7).

$$\frac{\Delta r}{r} = \frac{(1-2\sigma)(a^2 P_i - b^2 P_j)}{E(b^2 - a^2)} + \frac{(1+\sigma)(P_i - P_j)b^2}{E(b^2 - a^2)} \tag{5}$$

Substitute equations (4) and (5) into equation (3) to obtain equation (6).

$$\frac{\Delta V_b}{V_b^o} = \frac{1}{E(b_b^2 - a_b^2)} \left[3(1-2\sigma)(a_b^2 P_i - b_b^2 P_j) + 2(1+\sigma)(P_i - P_j) b_b^2\right] \text{ 3/} \tag{6}$$

3/ The subscript b denotes bomb dimensions.

The terms in the preceding equations are:

V = cylinder volume

L = internal cylinder length

ΔL = change of internal cylinder length due to pressure distortion

$r = a$ = internal cylinder radius

Δr = change of internal cylinder radius due to pressure distortion

b = external cylinder radius

P_i = internal cylinder pressure

P_j = jacket pressure

σ = Poisson's ratio

E = Young's modulus

Equation (6) can be rearranged to give equation (7).

$$\frac{\Delta V_b}{V_b^o} = \frac{3(1-2\sigma)a_b^2 + 2(1+\sigma)b_b^2}{E(b_b^2 - a_b^2)} P_i - \frac{(5-4\sigma)b_b^2}{E(b_b^2 - a_b^2)} P_j \qquad (7)$$

Equation (7) is in the form of equation (8).

$$\frac{\Delta V_b}{V_b^o} = \alpha' P_i - \beta' P_j \qquad (8)$$

where

$$\alpha' = \frac{3(1-2\sigma)a_b^2 + 2(1+\sigma)b_b^2}{E(b_b^2 - a_b^2)} \qquad (9)$$

and

$$\beta' = \frac{(5-4\sigma) b_b^2}{E(b_b^2 - a_b^2)} \qquad (10)$$

Equation (10) can be rearranged to give equation (11).

$$E = \frac{(5-4\sigma) b_b^2}{\beta'(b_b^2 - a_b^2)} \qquad (11)$$

CHANGE OF UNJACKETED TUBING VOLUME AS A FUNCTION OF THE INTERNAL PRESSURE

The valves, fittings, and bombs of V_1 and V_2 are connected by 1/8 inch od, 0.035 inch wall thickness, high pressure, stainless steel tubing. The connecting tubing constitutes a portion of the unjacketed volumes of V_1 and V_2. Figure 2 is an unscaled block diagram showing the components of V_1 and V_2. The total length of tubing in V_2 is 20.5 inches, and the total length of tubing in V_1 is 13.6 inches.

$$V_{t_1}^o = 0.032 \text{ in}^3$$

$$V_{t_2}^o = 0.049 \text{ in}^3$$

The change of tubing radius is calculated from equation (12), reference (7), page 45. Equation (12) gives the change of internal radius of a thick-wall, closed-end, unjacketed cylinder as a function

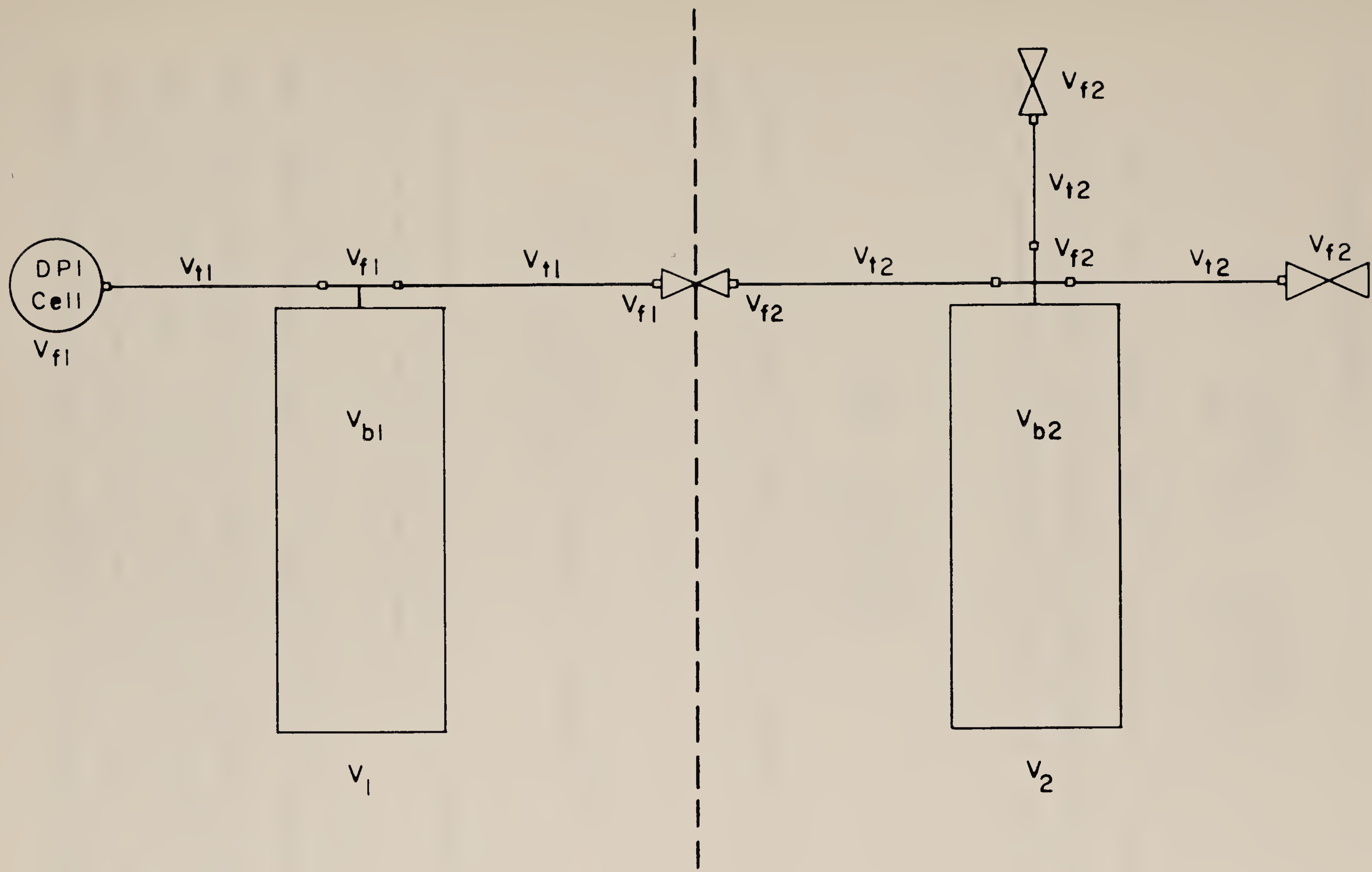

FIGURE 2.-Block Diagram of Volumes V_1 and V_2

of the internal pressure.

$$\frac{\Delta r}{r} = \frac{(1-2\sigma)a^2 P_i}{E(b^2 - a^2)} + \frac{(1+\sigma)b^2 P_i}{E(b^2 - a^2)} \tag{12}$$

The change of tubing length is calculated as if the tubing were a closed-end cylinder. The change of tubing length as a function of internal pressure is calculated from equation (13), reference (7), page 45.

$$\frac{\Delta L}{L} = \frac{(1-2\sigma)a^2 P_i}{E(b^2 - a^2)} \tag{13}$$

Equations (12) and (13) are substituted into equation (3) to obtain equation (14).

$$\frac{\Delta V_t}{V_t^o} = \frac{P_i}{E(b_t^2 - a_t^2)} \left[3(1-2\sigma)a_t^2 + 2(1+\sigma)b_t^2\right] \text{4/} \tag{14}$$

4/ The subscript t denotes tubing dimensions.

The change of tubing volume as a function of the internal pressure was also calculated as if the tubing were an open-end cylinder. In one case, the change of tubing volume was calculated for an open-end cylinder with the assumption that the change of volume is due to a change of radius only; and in another case, with the assumption that

the volume change is due to a change of radius and a decrease in length. The various assumptions give different values for $\frac{\Delta V_t}{V_t^o}$; however, the effect of the different values for $\frac{\Delta V_t}{V_t^o}$ on the change of V_1 and $V_1 + V_2$ with pressure is negligible.

CHANGE OF UNJACKETED FITTING VOLUME AS A FUNCTION OF THE INTERNAL PRESSURE

It would be difficult to estimate the exact pressure function of the fittings connected to V_1 and V_2. The fittings which constitute $V_{f_1}^o$ are the Ruska differential pressure cell (0.038 in^3), a Ruska tee (0.014 in^3), a Ruska valve (0.002 in^3), and two Ruska connectors (0.022 in^3). The fittings which constitute $V_{f_2}^o$ are three Ruska valves (0.006 in^3), three Ruska connectors (0.033 in^3), and a Ruska cross (0.012 in^3).

$$V_{f_1}^o = 0.076 \text{ in}^3$$

$$V_{f_2}^o = 0.051 \text{ in}^3$$

Assumption 4: (a) The change of fitting volume can be computed as if the fittings were a thick-wall, open-end cylinder, with an id of 0.070 inch, and an od of 0.32 inch. (b) The change of length of the hypothetical cylinder is negligible.

The change of fitting volume is calculated from equation (15).

$$\frac{\Delta V_f}{V_f^o} = \frac{2(1-\sigma)a_f^2 + 2(1+\sigma)b_f^2}{E(b_f^2 - a_f^2)} P_i \quad \text{5/} \qquad (15)$$

5/ The subscript f denotes hypothetical fitting dimensions.

The assumed fitting dimensions are a rough estimate of the average fitting dimensions; therefore, calculation of the change of fitting volume with pressure is, at best, approximate.

EXPERIMENTAL DISTORTION COEFFICIENT AND CALCULATION OF YOUNG'S MODULUS

The volume of V_1 as a function of jacket and internal pressures can be written as equation (16), reference (3).

$$V_1 = V_1^o(1 + \alpha P_i - \beta P_j) \qquad (16)$$

Equation (16) can be written as equation (17) when $\Delta V_1 = V_1 - V_1^o$.

$$\frac{\Delta V_1}{V_1^o} = \alpha P_i - \beta P_j \qquad (17)$$

The coefficient β was determined to be $1.5163 \times 10^{-7} \pm 0.0068 \times 10^{-7}$, psi^{-1} at 30° C (3). The change of V_1 due to change of jacket pressure is essentially equal to the change of the jacketed bomb volume

due to change of jacket pressure.

$$\therefore \qquad \beta' = \beta \cdot \frac{V_1^o}{V_{b_1}^o}$$

$$V_{b_1}^o = 4.649 \text{ in}^3$$

$$V_1^o = 4.757 \text{ in}^3$$

$$\beta' = 1.5515 \times 10^{-7}, \text{ psi}^{-1}$$

Young's modulus can be calculated from equation (11).

$$E = \frac{(5 - 4\sigma) b_b^2}{\beta'(b_b^2 - a_b^2)} \tag{11}$$

$$\sigma = 0.305 \quad (\underline{1})$$

$$a_b = 0.5 \text{ in}$$

$$b_b = 1.25 \text{ in}$$

$$E = 29.004 \times 10^6 \text{ psi}$$

CHANGE OF VOLUMES V_1 AND $V_1 + V_2$ AS A FUNCTION OF THE INTERNAL AND JACKET PRESSURES

Equation (7) represents the change of jacketed bomb volume as a function of the internal and jacket pressures.

$$\frac{\Delta V_b}{V_b^o} = \frac{3(1-2\sigma)a_b^2 + 2(1+\sigma)b_b^2}{E(b_b^2 - a_b^2)} P_i - \frac{(5-4\sigma)b_b^2}{E(b_b^2 - a_b^2)} P_j \tag{7}$$

Numerical values for the constants are substituted into equation (7) to obtain equation (18).

$$a_b = 0.5 \text{ in}$$

$$b_b = 1.25 \text{ in}$$

$$\sigma = 0.305$$

$$E = 29.004 \times 10^6 \text{ psi}$$

$$\frac{\Delta V_b}{V_b^o} = 1.1481 \times 10^{-7} P_i - 1.5515 \times 10^{-7} P_j \tag{18}$$

Equation (14) expresses the change of unjacketed tubing volume as a function of the internal pressure.

$$\frac{\Delta V_t}{V_t^o} = \frac{3(1-2\sigma)a_t^2 + 2(1+\sigma)b_t^2}{E(b_t^2 - a_t^2)} P_i \tag{14}$$

<u>Assumption 5</u>: Young's modulus and Poisson's ratio for the bombs, tubing, and fittings are identical.

Numerical values for the constants are substituted into equation (14) to obtain equation (19).

$$a_t = 0.028 \text{ in}$$

$$b_t = 0.063 \text{ in}$$

$$\sigma = 0.305$$

$$E = 29.004 \times 10^{6} \text{ psi}$$

$$\frac{\Delta V_t}{V_t^o} = 1.2206 \times 10^{-7} P_i \tag{19}$$

Equation (15) represents the change of unjacketed fitting volume as a function of the internal pressure.

$$\frac{\Delta V_f}{V_f^o} = \frac{2(1-\sigma)a_f^2 + 2(1+\sigma)b_f^2}{E(b_f^2 - a_f^2)} P_i \tag{15}$$

The constants

$$a_f = 0.035 \text{ in}$$

$$b_f = 0.160 \text{ in}$$

$$\sigma = 0.305$$

$$E = 29.004 \times 10^{6} \text{ psi}$$

are substituted into equation (15) to obtain equation (20).

$$\frac{\Delta V_f}{V_f^o} = 0.9692 \times 10^{-7} P_i \tag{20}$$

$$V_1^o = V_{b_1}^o + V_{t_1}^o + V_{f_1}^o \tag{21}$$

$$V_2^o = V_{b_2}^o + V_{t_2}^o + V_{f_2}^o \tag{22}$$

$$\Delta V_1 = \Delta V_{b_1} + \Delta V_{t_1} + \Delta V_{f_1} \tag{23}$$

$$\Delta V_2 = \Delta V_{b_2} + \Delta V_{t_2} + \Delta V_{f_2} \tag{24}$$

$$\frac{\Delta V_1}{V_1^o} = \frac{\Delta V_{b_1}}{V_1^o} + \frac{\Delta V_{t_1}}{V_1^o} + \frac{\Delta V_{f_1}}{V_1^o} \tag{25}$$

$$\frac{\Delta(V_1 + V_2)}{V_1^o + V_2^o} = \frac{\Delta(V_{b_1} + V_{b_2})}{V_1^o + V_2^o} + \frac{\Delta(V_{t_1} + V_{t_2})}{V_1^o + V_2^o} + \frac{\Delta(V_{f_1} + V_{f_2})}{V_1^o + V_2^o} \tag{26}$$

Volumes estimated from the component dimensions are:

$$V_{b_1}^o = 4.649 \text{ in}^3$$

$$V_{b_2}^o = 4.649 \text{ in}^3$$

$$V_{t_1}^o = 0.032 \text{ in}^3$$

$$V_{t_2}^o = 0.049 \text{ in}^3$$

$$V_{f_1}^o = 0.076 \text{ in}^3$$

$$V_{f_2}^o = 0.051 \text{ in}^3$$

$$V_1^o = 4.757 \text{ in}^3$$

$$V_2^o = 4.749 \text{ in}^3$$

$$V_1^o + V_2^o = 9.506 \text{ in}^3$$

From equation (18) and the dimensions of V_1^o and $V_{b_1}^o$

$$\frac{\Delta V_{b_1}}{V_1^o} = \frac{\Delta V_{b_1}}{V_{b_1}^o} \cdot \frac{V_{b_1}^o}{V_1^o} = 1.1220 \times 10^{-7} P_i - 1.5163 \times 10^{-7} P_j \qquad (27)$$

From equation (19) and the dimensions of V_1^o and $V_{t_1}^o$

$$\frac{\Delta V_{t_1}}{V_1^o} = \frac{\Delta V_{t_1}}{V_{t_1}^o} \cdot \frac{V_{t_1}^o}{V_1^o} = 0.0082 \times 10^{-7} P_i \qquad (28)$$

From equation (20) and the dimensions of V_1^o and $V_{f_1}^o$

$$\frac{\Delta V_{f_1}}{V_1^o} = \frac{\Delta V_{f_1}}{V_{f_1}^o} \cdot \frac{V_{f_1}^o}{V_1^o} = 0.0155 \times 10^{-7} P_i \qquad (29)$$

Equations (27), (28), and (29) are substituted into equation (25) to obtain equation (30).

$$\frac{\Delta V_1}{V_1^o} = 1.1457 \times 10^{-7} P_i - 1.5163 \times 10^{-7} P_j \qquad (30)$$

P_i and P_j are in psi

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

Equation (30) represents the change of volume V_1^o as a function of internal and jacket pressures.

Equation (31) is derived from equations (18), (19), (20), (26), and the component dimensions.

$$\frac{\Delta(V_1 + V_2)}{V_1^o + V_2^o} = 1.1463 \times 10^{-7} P_i - 1.5176 \times 10^{-7} P_j \qquad (31)$$

P_i and P_j are in psi

Equation (31) represents the change of volume $(V_1^o + V_2^o)$ as a function of internal and jacket pressures.

Suppose one wishes to adjust the jacket pressure so that the volume ratio $\left(\frac{V_1^o + V_2^o}{V_1^o}\right)$ will be a constant. When the gas sample is confined in V_1, the jacket pressure should be adjusted to 0.7556 of the internal pressure for ΔV_1 to be zero. When the gas sample is confined in $V_1 + V_2$, the jacket pressure should be adjusted to 0.7553 of the internal pressure for $\Delta(V_1 + V_2)$ to be zero. Adjusting the jacket pressure for each measured internal pressure is one way to compensate for elastic distortion of the PVT bombs.

CORRECTION OF THE VOLUME RATIO FOR ELASTIC DISTORTION

Another method of correcting for elastic distortion of the PVT bombs is the method used by Canfield (5). Canfield's method consists of correcting the volume ratio for elastic distortion.

The volume ratio $\left(\frac{V_1^o + V_2^o}{V_1^o}\right)$ at zero pressure is defined to be N_o. The volume ratio for the r th expansion is given by equation (32).

$$N_r = N_o \frac{\left[1 + \frac{\Delta(V_1 + V_2)}{V_1^o + V_2^o}\right]_{\text{at } P_r}}{\left[1 + \frac{\Delta V_1}{V_1^o}\right]_{\text{at } P_{r-1}}} \qquad (32)$$

P_{r-1} is the pressure before the r th expansion and P_r is the pressure after the r th expansion.

$$r = 1, 2, 3, 4, 5 \ldots$$

Substituting equations (30) and (31) into equation (32) gives equation (33).

$$N_r = N_o \frac{(1 + 1.1463 \times 10^{-7} P_i - 1.5176 \times 10^{-7} P_j)_{\text{at } P_r}}{(1 + 1.1457 \times 10^{-7} P_i - 1.5163 \times 10^{-7} P_j) \text{ at } P_{r-1}} \qquad (33)$$

Let the jacket pressure be zero and equation (33) reduces to equation (34).

$$N_r = N_o \frac{(1 + 1.1463 \times 10^{-7} P_i) \text{ at } P_r}{(1 + 1.1457 \times 10^{-7} P_i) \text{ at } P_{r-1}} \qquad (34)$$

The fundamental equation for calculation of a compressibility factor by the Burnett method is equation (35).

$$Z_r = \frac{Z_o}{P_o} \cdot P_r \cdot N_1 \cdot N_2 \cdot N_3 \cdot\cdot\cdot\cdot\cdot N_r \tag{35}$$

Substitution of equation (34) into equation (35) gives equation (36).

$$Z_r = \frac{Z_o}{P_o} \cdot P_r \cdot N_o^r \frac{(1 + k_1P_1)(1 + k_1P_2)}{(1 + k_2P_o)(1 + k_2P_1)} \cdot\cdot\cdot\cdot \frac{(1 + k_1P_r)}{(1 + k_2P_{r-1})} \tag{36}$$

where

$$k_1 = 1.1463 \times 10^{-7}, \text{ psi}^{-1}, \text{ at } 30° \text{ C}$$

$$k_2 = 1.1457 \times 10^{-7}, \text{ psi}^{-1}, \text{ at } 30° \text{ C}$$

A compressibility factor, obtained by the Burnett method and corrected for elastic distortion of the PVT bombs, can be calculated from equation (36).

The constants k_1 and k_2 are dependent upon the dimensions of V_1^o and V_2^o, Poisson's ratio for V_1 and V_2, and Young's modulus for V_1 and V_2. Young's modulus is temperature dependent; therefore, the constants k_1 and k_2 are temperature dependent.

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