TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

NEW SOUTH WALES,

FOR THE YEAR 1873.

SYDNEY: THOMAS RICHARDS, GOVERNMENT PRINTER.

1874.

[2s. 6d.]

Royal Society of New South Wales.

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FUNDAMENTAL RULES.

Objects of the Society.

1. The object of the Society is to receive at its stated meetings original papers on subjects of Science, Art, Literature, and Philosophy, and especially on such subjects as tend to develop the resources of Australia, and to illustrate its Natural History and Productions.

President.

2. The Governor of New South Wales shall be *ex officio* the President of the Society.

Other Officers.

3. The other Officers of the Society shall consist of two Vice-Presidents, a Treasurer, and two or more Secretaries, who, with six other Members, shall constitute a Council for the management of the affairs of the Society.

Election of Officers.

4. The Vice-Presidents, Treasurer, Secretaries, and the six other Members of Council, shall be elected annually at a General Meeting in the month of May.

Vacancies during the Year.

5. Any vacancies occurring in the Council of Management, during the year, may be filled up by the Council.

Fees.

6. The entrance money paid by members on their admission shall be One Guinea; and the annual subscription shall be One Guinea, payable in advance.

The sum of Ten Pounds may be paid at any time as a composition for the ordinary annual payment for life.

Honorary Members.

7. The Honorary Members of the Society shall be persons who have been eminent benefactors to this or to some other of the Australian Colonies, or distinguished patrons and promoters of the objects of the Society. Every person proposed as an Honorary Member must be recommended by the Council and elected by the Society. Honorary Members shall be exempted from payment of fees and contributions, they may attend the meetings of the Society, and they shall be furnished with copies of transactions and proceedings, published by the Society, but they shall have no right to hold office, to vote, or otherwise interfere in the business of the Society.

Confirmation of By-laws.

8. By-laws proposed by the Council of Management shall not be binding until ratified by a General Meeting.

Alteration of Fundamental Rules.

9. No alteration of or addition to the Fundamental Rules of the Society shall be made, unless carried at two successive General Meetings.

BY-LAWS.

Ordinary Meetings.

1. An Ordinary Meeting of the Royal Society, to be convened by public advertisement, shall take place at 8 p.m. on the first Wednesday in every month, during the last eight months of the year. These Meetings will be open for the reception of contributions, and the discussion of subjects of every kind, if brought forward in conformity with the Fundamental Rules and By-laws of the Society.

Council Meetings.

2. Meetings of the Council of Management shall take place on the last Wednesday in every month, and on such other days as the Council may determine.

Contributions to the Society.

3. Contributions to the Society, of whatever character, must be sent to one of the Secretaries, to be laid before the Council of Management. It will be the duty of the Council to arrange, for promulgation and discussion at an Ordinary Meeting, such communications as are suitable for that purpose, as well as to dispose of the whole in the manner best adapted to promote the objects of the Society.

Ordinary Members.

4. Candidates for admission as Ordinary Members to be proposed and seconded at one of the stated meetings of the Society. The vote on their admission to take place, by ballot, at the next subsequent meeting; the assent of the majority of the Members voting at the latter meeting being requisite for the admission of the Candidate.

New Members to be notified of their Election.

5. Every Member shall receive due notification of his election, together with a copy of the Fundamental Rules and By-laws of the Society.

Introduction of New Members to the Society.

6. Every candidate duly elected as Member should, on his first attendance at a meeting of the Society, be introduced to the Chair, by his proposer or seconder, or by some person acting on their behalf.

Annual Subscriptions when due.

7. Annual subscriptions shall become due on the first of May for the year then commencing. The entrance fee and first year's subscription of a new Member shall become due on the day of his election.

Members whose Subscriptions are not paid to enjoy no privileges.

8. Members will not be entitled to attend the meetings, or to enjoy any of the privileges of the Society, until their entrance fee and subscription for the year have been paid.

Subscriptions in arrears.

9. Members who have not paid their subscriptions for the current year shall be informed of the fact by the Treasurer. If, thirty days after such intimation, any are still indebted, their names will be formally laid before the Society at the first Ordinary Meeting. At the next Ordinary Meeting, those whose subscriptions are still due will be considered to have resigned.

Expulsion of Members.

10. A majority of Members present at any Ordinary Meeting shall have power to expel an obnoxious Member from the Society, provided that a resolution to that effect has been moved and seconded at the previous Ordinary Meeting, and that due notice of the same has been sent in writing to the Member in question, within a week after the meeting at which such resolution has been brought forward.

Admission of Visitors.

11. Every Ordinary Member shall have the privilege of admitting one friend as a Visitor to an Ordinary Meeting of the Society, on the following conditions :—

1. That the name and residence of the Visitor, together with the name of the Member introducing him, be entered in a book at the time.
2. That the Visitor does not permanently reside within ten miles of Sydney, and,
3. That he shall not have attended two Meetings of the Society in the current year.

The Council shall have power to introduce Visitors, irrespective of the above restrictions.

Management of Funds.

12. The funds of the Society shall be lodged at a Bank, named by the Council of Management. Claims against the Society, when approved by the Council, shall be paid by the Treasurer.

Audit of Accounts.

13. Two Auditors shall be appointed annually, at an Ordinary Meeting, to audit the Treasurer's Accounts. The Accounts as audited to be laid before the Annual Meeting in May.

LIST OF MEMBERS

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Royal Society of New South Wales.

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Stephens, W. J., Darlinghurst Road.
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Tunks, William, M.P., North Shore.
Trebeck, P. N., George-street.
Tarleton, Rev. Waldyre W., Bourke.

Ward, R. D., North Shore.
Watt, Charles, Parramatta.
Walker, P. B., Telegraph Office, George-street.
Wallis, William, Moncur Lodge, Potts' Point.
Weigall, A. B., Head Master, Sydney Grammar School.
Williams, J. P., New Pitt-street.
Wood, Harrie, Department of Lands.
Wright, Horatio G. A., Wynyard Square.

CORRIGENDA.

Page.	Line.	
11	13	*dele* "most"
18	10	*dele* "1 ton"
28	Note, for "95" read "45"	

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ANNIVERSARY ADDRESS,

Delivered 25th June, 1873, by the Rev W. B. Clarke, *M.A., F.G.S., F.R.G.S., &c., Vice-President.*

Gentlemen of the Royal Society,—

The last, an introductory meeting, held on the 30th May, in which we were interested and instructed by the exhibition of numerous most ingenious and valuable instruments in the service of spectroscopic and electric science, very ably described by Professor Smith, reduced the amount of necessary official work which would otherwise have occupied us this evening. Before I proceed to address you on the various topics which I propose to bring to your notice, I must pause, in order to congratulate you on the improved condition of our finances, and on the absence of such losses by deaths of associates and friends of our Society as it was my painful duty to introduce into the Anniversary Address of 1872.

We have, however, to regret the departure of one who was only this morning laid in his grave—the President of the Legislative Council, Sir Terence Aubrey Murray,—whose tastes were literary and scientific, and whose name will now disappear from the list of our members. To him, as a friend, who took an interest in my own pursuits, with whom I have been officially associated in advancing the Industrial Progress of the Colony, and to whom I owe many personal attentions, I would desire to record this brief tribute "*in memoriam.*"

I must also mention, with the thanks of the Council to the respective authors, the independent valuable contributions to our Transactions mentioned in the following list of papers read during the last session. But, whilst so doing, I must not forget to tender also our united acknowledgments to the Government, for the acceptable aid they have rendered us in undertaking the printing of the volume for 1872, which has relieved our Treasurer of some anxiety, and given us fresh encouragement to persevere in our otherwise unassisted work.

List of Papers read before the Royal Society of New South Wales, during the year 1872.

(1.) The Anniversary Address, by the Rev. W. B. Clarke, M.A., F.G.S.

(2.) " A Suggestion for an improvement in the projection of Maps," by the Rev. Thomas Horton.

(3.) " On Australian Gems," by George Milner Stephen, Esq., F.G.S.

(4.) " Astronomical Notices," by H. C. Russell, Esq., B.A., F.R.A.S.

(5.) " On the Coloured Cluster Stars about Kappa Crucis," by H. C. Russell, Esq., B.A., F.R.A.S.

(6.) " On an improved method of separating Gold from Argentic Chloride, as obtained in Gold-refining by Chloride Gas," by Dr. Leibius, of the Royal Branch Mint.

(7.) " Remarks on the fallacy of a certain method of assaying Antimony Ores given by some Manuals of Assaying," by Dr. Leibius.

(8.) " Remarks on Tin Ore, and what may appear like it," by Dr. Leibius.

(9.) " Statistical Review of the Progress of New South Wales in the last ten years, 1862–1871," by Christopher Rolleston, Esq., Auditor-General.

(10.) " On the Deniliquin Meteorite," by Archibald Liversidge, Esq., Reader in Geology, University of Sydney.

Discovery of Australia.

Among the many interesting events that have recently invited the attention of geographers at Home is one which ought to awaken sympathy among ourselves. Certain opinions expressed during a discussion some time ago in a colonial journal have, in my humble judgment, done injustice to the claims to our respect of the great circumnavigator, Captain Cook. Long ago, also, it had been insinuated that Captain Cook was not the discoverer of New South Wales, but that he borrowed the names and places on his chart from a chart of some older navigator. It is frequently the case that merit is denied to those who work for the public, and unworthy judgments are passed by ill-natured critics. On reading Mr. Major's work, published by the Hakluyt Society, I find the author has very properly quoted, in refutation of the unjust surmise, the able and satisfactory defence of Captain Cook's reputation from the jealousies of an adversary, by M. Metz, a Frenchman, published in the year 1805. M. Metz states that Dalrymple never forgave Cook on account of his having been preferred to himself in the command of the "Endeavour," and therefore he endeavoured himself to do Cook an injury, by trying to show that localities named by him were to some extent very much like places named by others in distant parts of the world. The French author shows conclusively that it is perfectly foolish to think that Cook could choose names that would betray plagiarism of the kind. The particular object of my reference to this matter is to lead to an earlier event in the history of Australia, and to point out, from recent researches, to whom the first discovery of it is to be assigned.

The Rev. Julian E. T. Woods, in his carefully compiled history of the discovery and exploration of this great Country, has given an excellent account of "Early Voyages" to it, and stated in a foot-note that he was indebted for many facts and quotations to the work written by Mr. R. H. Major, and published by the Hakluyt Society, in 1859. That work bears the title "Early Voyages to Terra Australis, now called Australia." It consists of an introduction, copies of charts, and various documents. The author, who is a distinguished member of the Society of Antiquaries, as well as a member of the Hakluyt Society, and also one of the Secretaries of the Royal Geographical Society, has for many years occasionally turned his attention to solving the problem—"Who discovered Australia?"

About twelve years since, he had come to the conclusion that the discovery was not made, as generally supposed, by a Dutchman, but by a Portuguese. He has more recently published another paper, in the "Transactions of the Society of Antiquaries," which points to the discoverer in a very different direction. At the time mentioned, he held as authentic that this great land was first made known to Europeans by Manoel Godinho de Eredia, in the year 1601. The supposed fact was founded on a comparison of a MS. map and certain printed documents; and more recently, in the Burgundian Library, at Brussels, there has been found the original report of Eredia, illustrated by charts and other drawings.

In 1809, Mr. Major had pointed out the existence in the British Museum of several French maps, in which the date of the discovery was placed at an earlier period than 1601, the date of the oldest map being 1542.

The names of places on the coast of Australia, called on the charts "*Jave la Grande*," were considered to be written in "*Gallicized Portuguese*," and hence the discovery was given to Portugal.

Another map, of the date of 1531, still more recently has been found, in which Australia is denominated "*Regio Patalis*" by the author, Oronce Finé, the Astronomer of Briançon. Coincident with this is the fact that, on the Portuguese maps of the period, where Australia ought to have been, if discovered by that nation at the time, there is a perfect blank. Mr. Major was, therefore, induced to reconsider the idea of "*Gallicized Portuguese*," and soon came to the conclusion that instead, the names on the old charts of "*Jave la Grande*" were written in the common idiom of the Langue Romane or Provençal—a language then spoken commonly in the south of France. Moreover, it was found that on one of the maps now in the War Office in Paris, bearing date 1555, the name of the author was the renowned Provençal navigator Guillaume le Testu, a native of Grasse. The conclusion from these inquiries is, that the discovery of Australia is due to France, and not to Portugal.

Sir H. C. Rawlinson, President of the Royal Geographical Society, from whose Anniversary Address of 27th May, 1872, I have taken, for brevity's sake, the above particulars, states that Mr. Major's subsequent researches, communicated to him, remove all doubt as to the claim of France to this discovery, and adds,—"We must own that the French are without a rival in the field."

One of the curious points indicated is, that, "Cap de Grâce," on the French maps of "*Jave la Grande*," was named in honor of Le Testu, who describes himself on the chart as a native "*de la ville Françoyse de Grace.*" It is satisfactory to be able to announce the settlement of the disputed claim to the discovery of this continent in favour of the great nation who have become our neighbours, without in the very least infringing on the satisfaction we ought to feel as Englishmen, that there is no well-grounded claim to the discovery of the coast on which we reside but that of the illustrious navigator whose services the members of the Society which we ourselves represent have placed on record, on the tablet erected by them to commemorate the event, at the entrance of Botany Bay. (See Trans. Roy. Soc. N.S.W., 1869, p. 3.)

It is also satisfactory to have the pleasure of repeating this announcement to the Society this evening, in the presence of the Representative of France in this Colony, who is about to be elected a member of our body.

Recent Explorations in Northern Australia and Queensland.

(1.) From these greater events we may now turn to some more recent local discoveries, made in the progress of settlement and development. These are, respectively, on the west and east sides of the Gulf of Carpentaria, in Arnheim's Land, and in the York Peninsula. The former is now familiarly known as the Port Darwin country, a settlement there having been made by South Australia, which now holds it, though widely separated from it. Some of its features are known to us from the notices in the public papers and reports respecting its supposed auriferous character; and long ago I expressed an opinion in print that gold might occur there, judging from the character of the rocks which I had examined.

The whole of the country forming that part of Arnheim's Land appears to be a trough resting on granite as a base, which is flanked by slates supporting sandstones, and Tertiary deposits still younger. The general strike of these formations appears to be N.E. or N.N.E. Melville Island is an outlier of the granitic formation, with littoral deposits of white clay and red sandstone, such as occur from Port Essington round Van Diemen's Gulf to Cape Hotham on its southern border. Granite again occurs S.E. of Port Darwin and on the Alligator River.

A similar strike of the rocks appears to mark the line of country from the Victoria River of Gregory to Caledon, Arnheim, and Melville Bays, at the northern and north-western corner of the Gulf of Carpentaria, where granite again occurs, the slates following the same trend. There is, no doubt, a large area of country in many respects resembling the auriferous regions elsewhere.

From Leichhardt's description of the massive plateau along the Alligator, it would appear to have much in common with the great sandstone tracts in New South Wales. But in absence of any fossil evidence (for the data of that kind collected by Leichhardt he was compelled to abandon), the exact age can hardly be assigned at present. But having received from a more recent explorer of the country to the south-east a trilobite of a Devonian species, and from Leichhardt's statements to myself personally, believing some shells which he found to be upper Palæozoic, it is probable that much of the region may be of similar character to that of Queensland.

Captain (Admiral) P. P. King gives in his book a view of the red sandstone on Prince Regent's River, about 360 geographical miles, on a parallel coast trend to S.W. of Port Darwin, which looks there to be a thick bedded rock. The fossils Leichhardt found on the Roper River were in a baked rock, which occurs in various places in connection with basalt. The transmutations occurring in other districts where copper, iron, and gold exist, are frequent; and these changes are on both sides of the great sandstone plateau. Taking the Victoria River as the southern boundary of the region, we know that Admiral Stokes found fossils which were lost on their way to England; and Fossil Head obtained its name from them. Jukes says the rock in which they must have occurred resembled the Palæozoic sandstone of New South Wales. Stokes says they were "casts of shells not of a recent appearance." Although, in the absence of other evidence, Mr. Daintree may claim the sandstone for his "Desert" rocks of that name, I believe that they are far older. Some of the granite is very large-grained, with lithia mica. The granite passes occasionally into pegmatite and syenite. Much of the sandstone is fine-grained.

After all, however, the chief advantage of a settlement so far to the north-west, although in a climate unsuitable to the labour of Englishmen, holds out prospects of intercourse with India, and therefore, by transmarine communication, with England; and on that account, if on no other, now the overland line is completed, we ought to wish well to the opening up of the whole of that distant territory; especially as we saw in the late Exhibition proofs of the value of exploration, in the pearls which formed part of the exhibition from Western Australia, and which occur also in the great oyster beds to the northward of that Colony. It would be a grand consummation of colonial enterprise if a railway to the new settlement could enable the south-east of Australia to be so connected with the north-west of it. We should then be free from the jealousies that now trouble the settlement of the postal communication with Europe.

It may add to the interest excited by ideas of gold in the north-west of Australia, to mention that, in a letter to Sir Henry Ellis, dated 1st May, 1861, Mr. Major has quoted from a Portuguese author an account of a visit to "the Island of Gold, which lies in the sea on the opposite coast, or coast outside Timor, which is properly called the Southern coast." The pamphlet itself was printed at the Royal Press, in 1807, and bears in its title that it is an account of the Golden Chersonese, or Auriferous Peninsula and Islands, &c. The discoverers were fishermen, who were driven out of their course by a tempest, and found so much gold that their boat would hold no more. Whether the story be true or not, the land indicated is no other than the north coast of Arnheim's Land, and in the territory now occupied by the Port Darwin Settlement.

(2.) I can speak more confidently respecting discoveries in the York Peninsula.

In my Anniversary Address of 1872 I mentioned two expeditions—one undertaken in 1871, the other to be commenced last year.

The first was under the direction of Mr. Hackett, of Lolworth Station, accompanied by Mr. Hann.

In Leichhardt's journal of his expedition to Port Essington, mention is made of a "fine conspicuous range," named Kirchner's Range; and as the geological features were considered by Mr. Daintree and others to be of considerable importance, and as it afterwards proved that it was on the same general strike as that of the schist rocks on the Gilbert River, the search for this range became of interest to those who wished to ascertain whether the metalliferous formations, so important to the southward, extended to the westward of the line traversed by my unfortunate friend Mr. Kennedy. My own impression, as stated many years ago, was that which was afterwards shared by others—viz., that gold would be found in that direction. In my paper, read before this Society, September, 1867 (Trans., vol. 1, p. 52), on the "Auriferous Districts of Northern Queensland," I then stated that I always expected another gold-field about the 144th meridian, on the heads of the Mitchell waters and the Kennedy River, and that it was not unlikely that at the back of the east coast there are patches of auriferous country as far as 13° S."

In this opinion it will be shown I was correct; for although Mr. Hackett's explorations referred to in my last Anniversary Address did not prove it, it was proved by the second expedition, under Mr. W. Hann and Mr. Norman Taylor in 1872, up to the north of 16°, the course of the Kennedy River being still left unexplored.

Mr. Hackett's party (as many others have done) did not clearly recognize the actual position of Kirchner's Range according to Leichhardt's definition of its features, but passed it unobserved, and so they followed down the Lynd River, as, on comparison of Leichhardt's and Mr. Hackett's memoranda, I found to be the case, nearly to the junction of the Mitchell. During this journey they suffered considerable privation, and had to feed their dogs and themselves on fish, which abounded in the river, and were often of great size,—but they found no gold, or, at most, only minute indications of it. From a description of one of the fish, I presume it to have been allied to that shot in the Burdekin by Leichhardt; or, as the account is not very clear, it may have been a Ceratodus. At any rate, there is evidence of abundance of life of that kind in the otherwise almost azoic region through which the river flows.

On their return a second exploration was made to the north-east during which the party crossed to the summit of the coast range. A third journey carried them to a district more northerly, in which they became entangled in a mesh of water-channels, which probably belonged to the heads of the Mitchell.

Subsequently, on the 26th June, 1872, the second expedition, under the leadership of Mr. William Hann, accompanied by Mr. Norman Taylor (geologist), Dr. Tate (botanist), Mr. Warner (surveyor), and Messrs. Stewart and Nation, and Jerry (an aboriginal), commenced a more systematic and organized survey. Leichhardt did not, however (as surmised by Mr. Hann), say anything as to the probability of gold occurring near Kirchner's Range; but the latter gentleman, reasoning from discoveries of a later date, did right in looking for it there, though he only met with a result similar to that of Mr. Hackett's expedition. Both explorers, nevertheless, confirm Leichhardt's statement as to the general features of the country; and Mr. Hann calls Kirchner Range itself "a bold and conspicuous feature, abutting on the river."

The range this time was ascended; and from it, to the north and north-east, as well as from Gregory's Bluff (a precipitous notable feature, not mentioned by Leichhardt), was obtained a view of a large, wild and broken, undulating country, in which gold was, after search, not found. Mr. Hann—not accepting Mr. Taylor's opinion, that that part is not auriferous—still thinks strongly that it is "well worthy of the attention of gold-diggers."

It would seem to me to be of a geological formation too much transmuted, as it mainly consists of mica-schist often very garnetiferous; for so soon as they got into a region in which slate, with quartz ridges, occurred (though quartz drift was common enough before), they struck gold. This was on the Palmer River, one of the new discoveries north of the Mitchell; but in all the country to the south, between it and the Lynd district, watered by the Tate and Walsh Rivers (both new to geographers), no gold was found. The principal formations there besides granite, porphyry, and garnetiferous and mica-schist, with some greenstone and basalt, consisted of sandstone, with occasional patches of limestone, some undoubtedly Devonian, in the vicinity of which occurred Mesozoic deposits, which from the fossils I have examined, are partly Cretaceous, and partly Jurassic.

In the vicinity of the older limestone, not far from the junction of the Walsh with the Mitchell, Mr. Taylor recognized what he considered to be a Carboniferous formation, and a range of that character was marked upon the chart. The junction of the Lynd with the Mitchell was fixed in 16° 23′; that of the Walsh was found to be in 16° 24′ 39″. Conglomerates are common in the area between the rivers, and signs of "coal" were noticed.

In Mr. Taylor's report the existence of "Glossopteris" is also mentioned. This cannot now be verified, as in the long travel the specimens collected were destroyed, and all I could find in the collection submitted to me for examination of this character were a minute portion of coal and a piece or two of shale, which, in defect of evidence, may have belonged to one of the most Mesozoic formations, as these, according to Mr. Taylor, were found occasionally to be bedded *in situ*. It would be very interesting to have determined that our own Carboniferous formation, however reduced in area, exists so far to the north.*

Of other minerals, magnetic iron, agates, and a profusion of garnet of a pale rosy tint, forming vast accumulations in some of the water channels, attracted attention; the former, no doubt, derived from disintegrated amygdaloidal basalt, the latter from decomposed mica-schist. Quartz drift and sand are also marked features.

On the Palmer River which comes in from the east where the Coast Range seems to make a great bend in that direction, but yet a westerly water, in 15° 49′ 14″, gold was first found. Below this the country was on mica-schist, and barren; but above, where the bed-rock was granite or slate, gold was traceable for several miles, generally in or close to the river on both sides.

The slate country extends to the coast between Weary Bay and the Endeavour River, with quartz-ridges, basalt and sandstone. There can be no doubt that the great masses of the latter and the associated conglomerates may be connected with a Carboniferous deposit, unless they belong to Daintree's "Desert sandstone" which, according to the specimens I have examined, his descrip-

* See Appendix A.

tion, and photographs (of which a very interesting series was shown at the late Exhibition,) imitates the features of some of our Hawkesbury rocks in the plateau and ravines of the Blue Mountains.

"Carboniferous" rànges and "fossil shells" were found by Mr. Taylor on the Normanby, a new river coming from the south-east, and joining the Kennedy not far from latitude 14° 36′ south.

Connecting this locality with others of similar character, we may assume a strike of about N.E. or N.N.E. for these sandstone ranges. They seem tó have a resemblance to the Arnheim's Land ranges.

Gold was prospected for on the return journey from Princess Charlotte's Bay in several places, but without result. Considering the physical difficulties presented by the country along the East side of the main Coast Range, and the necessarily imperfect way, in which, under the circumstances, any search would be made, together with the impossibility of examining any reefs that may have existed (and such do exist on the Tate River), this reported want of success is not to be taken as a final determination: and the fact that gold (and of a very good quality) was found on the Palmer River, proves that there are sources yet undeveloped. It is, therefore, to be hoped that the Queensland Government will pursue the investigation, by means less harassing than those to which Mr. W. Hann's zealous and energetic labours and those of his Staff were exposed.

They discovered much good, as well as some bad pasturable country; and in the difficulties of theEastern Coast Range they merely experienced that which Kennedy suffered from in his ascent from Rockingham Bay.

In the course of the journey crocodiles were found to be numerous, and on the Normanby River a number of heads of these animals were found attached to a sapling; and as the blacks were numerous there, we may suppose it to be a trophy. The existence of the "climbing kangaroo" is attested in the diary; and of a snake sixteen feet in length. There is, therefore, probably, a new field for the naturalist in this northern region.

As to the Aborigines, they seemed congregated in some localities, but appeared sparsely distributed; and though some were friendly, others seemed as treacherous as those who offered poor Kennedy as a martyr to Colonial exploration.

I have had an opportunity of inspecting hastily a considerable portion of the fossils collected by Mr. W. Hann and Mr. Norman Taylor, which were kindly submitted to me by the Queensland Minister for Works, on their way to the Agent-General in London. The principal portion consisted of Mesozoic genera and species of lower Cretaceous and Jurassic ages, among which were a Crioceras, several Ammonites, Belemnites, and other shells; and many of these I can identify with the fossils in my own collection, from the more southern and western great Secondary formations of Queensland. That area is indicated by the brown colour on the map I exhibited in this room in 1872, and which I again produce.

Although there is nothing actually new to Queensland Geology in some of these Mesozoic species, yet inasmuch as their existence proves the former extension of the Secondary formations far to the north of previous discoveries, they are entitled to great consideration; and, if I cannot exhibit them to-night (because I have already forwarded them to England), I can at least supplement the evidence by showing you some of the species from other parts of Queensland, which, having been just received back from England, whither I sent them for examination, description and comparison with European species, are to be considered as the typical Australian representatives of various Jurassic and lower Cretaceous animals.

Of the kind assistance I have received in connection with these from geological friends at Home, I have before spoken in various publications; but I have not been able till now to exhibit them as the authenticated species which are named on the labels I have attached to them. It may be interesting to some here present to see how far upwards in the zoological scale we have been able to carry out Australian evidence. Of this, however, I have yet something more to say. I must now add that, with the fossils mentioned from the Mitchell and Walsh Rivers, there were in the collection sent to me by the Hon. W. H. Walsh several very minute fossils ; among them a beautiful foraminiferous shell, and several others to which no clue was given as to actual locality. I do not, at present, know whether any of these were supposed to be of the Carboniferous age ; they exhibit, however, a proof of another formation besides those mentioned already.

Moreover, there was included with the above a very fine centrum of the vertebral bone of some gigantic Saurian reptile, which did not appear to me to be that of an Icthyosaurus. The only illustration of it I can now offer is a photograph kindly made for me at the Museum. These, and some bones (not in good condition) of Diprotodon, and, probably, of a species allied to Wombat, and some small Marsupials—with a few specimens of agate, limestone, basalt, &c.—are gone home to be immortalized in the records of Science, and exhibited to British geologists in the Annexe to the Exhibition building in London. Whatever may be their actual value to theoretical Palæontology, these things furnish an additional testimony to the fact of the wide distribution of the relics of the peculiar mammalian creatures that are now known to have existed from the south to the north of Eastern Australia, and which must have lived very close up to, if not into, the recent epoch to which we ourselves belong.

Queensland Mesozoic Fossils.

The collection which I sent to Mr. Moore, of Bath, F.G.S., who kindly reported on it in his paper on "Australian Mesozoic Geology," read before the Geological Society in 1870, embraced no less than eighty-nine species, of which eight at least are also European. It is from this collection that the specimens on the table have been selected. Many others in my possession in addition to the collection spoken of, have not yet been examined ; but those, and the collection from the Mitchell and Walsh Rivers, will swell the amount of species considerably. As Mr. Moore included in his paper, from his own collections, fifty-six species of Liassic and Oolitic fossils from Western Australia, of which twenty-four are also indigenous to British or other European districts,—and as I have also already numerous uncompared fossils from that distant Colony, received from the Colonial Secretary, the Hon. F. P. Barlee, and the Rev. J. C. Nicolay,—the list of these Secondary fossils, ranging from the lower Cretaceous downwards to the Lias, will doubtless ere long be greatly extended, both by purely Australian species, and by those which have a more cosmopolitan range; the latter, in some degree, linking this continent, as the fossils of the Palæozoic formations also do, to the world north of the Equator.

A more recent service has also been performed by the "Notes on the Geology of Queensland," read before the Geological Society, in April, 1872, by my friend Mr. Daintree, illustrated by the description of fossils by Mr. Etheridge, F.R.S., and Mr. Carruthers, F.R.S. This was published in the Quarterly Journal, in August, 1872. Among the fossils are those I mentioned in my last Address (p. 37) as having been sent home by me to Mr. Daintree, in consequence of his loss by shipwreck. Mr. Etheridge describes thirty-eight new forms, of which ten are common to England and Queensland. Of these, fifteen are Devonian marine species from Gympie: five from the Barwon River, six from the Don River, one from Wealwandangie, and two from Cracow Creek (all Carboniferous). Of Mesozoic fossils the list contains six from the lower Oolite, at Gordon Downs; and, belonging to the Cretaceous system, fifteen species from Maryborough, five from Marathon, two from Hughenden, and one from M'Kinlay's Range.

As Mr. Daintree acknowledges the above contributions made by myself, from Gordon Downs, Cracow Creek, Wealwandangie, and the Don River, it is my duty to transfer the acknowledgment to the gentlemen who, at my request, procured them for me from the respective localities, in order that I might have them described. And here I would earnestly entreat the assistance of all who are interested in the full development of the physical structure and former condition of this continent to follow the example of those friends in the other Colonies who have already assisted me, and to forward me such fossils, whether shells or corals, or whatever they may meet with of fossil organisms, in order that we may, as soon as possible, be able to complete the Palæontological history of *this* Colony, which, in some respects, is yet defective, and in which men of science at home are equally interested with ourselves. Many persons living in the very heart of rich Palæontological treasures are not, perhaps, aware of their scientific value; and if I may be pardoned for the expression I would say they do not recognize the duty they owe to the world in the endeavour to make them known.

Mr. Etheridge, in reference to one of the fossils, has this remark:—"I figure it, like many other forms, to draw the attention of Australian geologists to the Lamellibranchiata of the Palæozoic rocks of Queensland and elsewhere, through the continent, in the hope that search may be made for more perfect specimens."

I cannot forbear quoting from another passage in his most valuable Appendix to Mr. Daintree's paper, respecting an additional series of fossils from Queensland forwarded by Mr. W. Hann:— "Nearly the whole," he says, "are Cretaceous. The localities from which they are derived are Bowen Downs, head of the Thomson River, Tower Hill and Barcoo River beds, near Tambo." "These interesting fossils will receive further elucidation; it was, however, deemed of sufficient interest to thus notice their occurrence, which bears out geographically the distribution of the Cretaceous series previously noticed, and confirms the view taken of the great westerly extension of the Cretaceous rocks through the plains of Queensland, thus, perhaps, accounting for the universally and widely spread level land in the western plains."

In his Appendix the learned Palæontologist has given his view of the "Succession of the Stratified Rocks of Queensland," an enlarged copy of which I also introduce as likely to afford instruction to those who may look for it, but who also may be unable to obtain it otherwise except from a repetition in this way. And this, let it be observed, has been the sole object I have had in view in referring so often to the work done out of the Colonies, in assisting those in them to understand what progress is being made in the knowledge of Australia.

At present we have no discoveries in New South Wales of Cretaceous or of such other Upper Mesozoic deposits as are referred to in the above remarks. But of the Palæozoic rocks in this Colony and Tasmania we have abundant examples, and small amounts of their fossils have been described by Dana, Sowerby, Lonsdale, Salter, Morris, M'Coy, and de Koninck of Belgium: and I am now about to dispatch about 1,000 individuals selected from the Palæozoic strata of this Colony to the latter authority who has undertaken (at my request) to examine, compare, and describe what, I trust, will give us a complete account of the great marine formation in the lower portion of our coal-beds.

Coal.

This allusion very naturally suggests a reference to the great question of our coal supply.

(1.) In England there has been of late immense excitement as to the probability of coal diminishing rapidly and generally, owing to the enormous expenditure of fuel in various ways, and the increase of machinery and manufactures, and also of population; and various calculations have been made as to how long, even at the present rate of consumption, the mines will last, and also as to the possibility of obtaining additional supply from fresh localities below geological formations that have never been pierced for that purpose.

I confess, after all I have read on the subject, that I am not convinced of the necessity for any alarm, so far as natural resources are involved. Economy is, undoubtedly, the best protection against useless risk; and if some system could be adopted by which the fearful waste of fuel in domestic expenditure could be regulated, it would be well.

According to the latest calculations—founded on the completest data—it is computed that, allowing for the present annual rate of increase in consumption, there is sufficient coal in Great Britain, within a limit of depth suitable for working, to last 276 years—allowing that at the end of the first century the consumption might be 415 millions of tons per annum; and, allowing the consumption in that time to reach 274 millions of tons, with a diminishing rate of increase, the coal would last 360 years. As one of the means of great expenditure of coal is in the working of iron, attempts have been successfully made in some districts to reduce the consumption of coal. An instance is given by Mr. Hull, where 1 ton 33 cwt. 1 qr. in the North Lancashire line has been found sufficient for a ton of pig iron, whereas seventy-seven years since the consumption of coal on the Clyde was 9½ tons. Again, it is said, a saving of 25 per cent. has been introduced in ocean steamers; but, according to Mr. Hunt, as quoted by Mr. Hull, a third of the whole quantity raised is consumed in households, amounting to 37,000,000 tons, or about a ton per head of the whole population; but the Coal Committee say that in 1869 it was 14 cwt. per head. It becomes, therefore, no longer a mere theoretical but a practical inquiry, whether the system of heating apartments must not be revised. This, and improvements in scientific inventions of various kinds, will probably postpone the time assumed for the evil hour. Mr. Hull, whose work on the Coal Fields of Great Britain has now gone into a third edition, has embodied the facts just referred to, in an Essay on what he calls "The Coal Famine," *Quarterly Journal of Science*, April, 1873, and points out how the high price of coal at home, so vexatious to the poor, and so injurious to all classes, does not arise so much from the croakers as from the abominable system of strikes, which inevitably ruin the prospects of the miners themselves—a system fostered by the worst enemies of the pitmen, and nourished by immorality and semi-barbarism of life adopted by or forced on that population; so that after all, as in most other cases, immorality and irreligion in a portion of a community have, even in unlooked-for ways, a tendency to bring injury to the whole.

Now, it may be asked how does this affect us at the Antipodes? Perhaps there is an effect even here from what goes on in the collieries at home; but if I mistake not, we even here are at this time suffering from similar causes as to advanced prices of coal, occasioned by strikes. But this is not the question we have now to consider. Our manufactories here are on the increase, and the present rate of consumption of coal in this Colony is also wonderfully increased.

A few weeks ago I had the opportunity of witnessing a state of great activity in the city of Newcastle, and also a proportionate activity in the Illawarra district. There is also a kind of coal-fever breeding among speculators, which may, unless well looked after, give as much trouble to many as the late gold, copper, and tin eruptions have occasioned.

It is quite true, however, that these mineral epidemics cannot really affect the amount of underground wealth that the Providence of God has placed beneath our feet. But what that amount may be is another question.

At present our idea of the extent and thickness of coal in this Colony is not founded altogether on positive proof so much as on scientific deductions. Nevertheless, it may be satisfactory to be assured that so far as such deductions go, when founded on the best data we have (and for the present we can have no others), the probability is great that there is an abundant supply for a long period of time.

I am fond of believing in a Providential ruler of man's destiny, and it appears to me hardly philosophical to suppose that the nation to which we belong, whose tendencies and ingenuities are somehow connected with what coal has done and is doing for it, should be permitted to send off large offshoots of its population into distant regions without everything being already prepared to render them capable of following the instincts of their brethren at home.

This may be fanciful, but the fact is that all the great colonizations by the Anglo-Saxon race have settled where there are the natural supplies which its genius does or will require.

Witness the United States, and British possessions in North America; witness India, witness Tasmania, New Zealand, New South Wales, and Queensland, in which Colonies, if anywhere in Australia, coal is abundantly distributed in proportion to their extent, though not everywhere of the same geological epoch.

All the other Colonies come to us for some of their supplies, and even India, that does not want them, for she has enough for, herself and us too; and China and Japan, where coal abounds come also. The coal fields in China have an area of, at least, 400,000 square miles.

(2.) This seems only a natural process, for, on looking over the tables of a very old friend of my younger days (Mr. R. C. Taylor, who emigrated to, and died in the United States in 1850, having made himself a great name in that country), I find it stated (Statistics of Coal, p. 240) that from 1801 to 1853, the United States imported no less than 2,995,047 tons of British, Colonial, and other foreign coal, which gives an average of 57,597 annual importations of tons at 2,240 lbs. to the ton; and of this, from British Parliamentary returns, from 1831 to 1846, the imports from Great Britain amounted to 687,936 tons. The British exportation is on the increase.

As illustrating the above remarks, let us see first, what is the calculated extent of the American and Indian coal-fields. I take the last estimates by Professor Hitchcock, in his Returns to the United States Government. He says there are eight distinct areas:—

1. New England basin: area, 750 square miles; maximum thickness of coal seams, 23 feet.
2. Pennsylvania: 434 square miles; average thickness of from twenty to twenty-five seams, 70 feet.
3. Apalachian basin: 63,475 square miles, all of bituminous coal.
4. Michigan basin: 6,700 square miles; thickness of seams up to 11 feet.
5. Illinois basin: 51,700 square miles; ten to thirteen seams, 31 to 35 feet aggregate thickness.

6. Missouri basin: more than 100,000 square miles, all considered productive.
7. Texas: 5,000 square miles; coal in places 4 feet thick.
8. Arizona: not fully developed, but known to be rich in numerous localities.

The total amounts to 230,659 square miles, and the whole are of the old Carboniferous era; besides which, there are the Virginia Triassic coal, and the immense amount of California.

In 1861, Mr. Hull stated the produce of the coal-fields of the North American British possessions to have been 2,000,000 tons. In 1871, as I learn from reports received from Canada, 500,000 tons; and in 1872, 800,000 tons of Cape Breton coal, were sold in that dominion. Mr. Hull also calculated that the whole of the area in North America was thirty-eight times larger than that of the British coal-fields, whilst the production of the latter was eight times greater than the American. ("Coal-fields of Great Britain," p. 217.)

In 1867, Mr. Warington Smyth ("Treatise on Coal and Coal-mining," p. 96) rates the annual production of the United States at 18,000,000 tons, but with an annual importation of half a million.

The same writer adds a cautionary remark, justifying what I said before:—"It is reported"—he quotes from "Her Majesty's Secretaries of Embassy and Legation," 1866—"that great looseness seems to exist in the compilation of figures involving large sums, as well as in the returns required to be made by the Companies." He adds this:—"On passing, then, to what is of more weight—the thickness of workable coal—we are constrained to believe, whilst fully realizing the colonial value of the Apalachian, and of the Illinois and Indiana deposits, that the data for the estimation of the contents of the others are not yet satisfactory, and that the progress of exploration in such vast tracts will show many an element for subtraction." (p. 98.)

What may have been the management of our own coal mines up to this date is not generally known; but there can be no harm in making one more quotation from Mr. Smyth's work,—that with which he concludes. He is speaking of Great Britain. "In no other country in Europe is there such a laxity of vital importance to our successors. Under the Inspection Act every colliery is bound to keep up plans on a certain scale; but how partial is the advantage when, at the end of a lease, the documents are subject to be lost or destroyed. And, unless the Government, on behalf of the Nation, insist upon the deposition of duly guaranteed mining plans in a suitable office, and lessors and lessees co-operate in rendering available at a future day those tracts which the exigencies of trade prevent us from turning to present account, we remain open to the charge of an unworthy stewardship of the riches which a bountiful Nature has committed to our care."

The comparative amount of coal produced at present in the world is taken by Professor Leone Levi to be 200,000,000 tons per annum, of which 120,000,000—the Treasury statistics make it in 1871, 117,000,000—are the produce of Great Britain; the other Countries, such as France, Germany, Belgium, and North America, notwithstanding her enormous coal areas, consuming more than they produce. During the last half-century, Great Britain has exported, in a continually increasing decennial proportion, no less than 43,442,000 tons, the export in 1871 amounting to 12,816,000 tons. In 1870, France took 2,074,000; Russia, 805,000; Denmark, 695,000; Italy, 612,000; Egypt, 374,000; Sweden, 370,000; Brazil, 261,000; Norway, 248,000; and Cuba, 222,000 tons; and in addition, other Countries, including America, took, 5,842,000 tons. So that in fact British coal has fed the naval and commercial prosperity of half the world.

It is quite clear, therefore, that when the other coal-bearing countries have their mines in full operation, unless Great Britain can undersell them all, she must lose her prestige. To use the words of the author last cited,—"Cheap iron, coals, and labour, have made England what she is; there is nothing of such prime importance to England as cheap coal. If she is deprived of this, she will lose the very prop to her manufacturing and commercial prosperity. The rise in coal means a rise in steam, and, therefore in the price of all goods produced by its aid. It is the same with iron. That metal is necessary for all kinds of machinery; the price of it, therefore, is an essential element in the cost of machinery and in the value of the goods produced thereby; and as coal and machinery are equally useless without labour, a rise in it must necessitate a rise in produce. A general advance in the price of all English manufactured goods is therefore inevitable, if the advance in the price of coal be maintained. The enhanced value of British goods will offer a new opportunity to increase American manufactures. As soon as England ceases to be able to undersell her competitors, her hold on foreign markets, which is the basis of her commercial supremacy, will diminish."—(*Daily Telegraph.*) We may presume that this reasoning is, in some degree, applicable to this Colony; and that, as respects the world at large, it is for the universal prosperity of all nations to remain at peace.

(3.) It was my intention to offer some special remarks on the Coal Statistics of India, in continuation of those made in my Address in the year 1870; but I am compelled to be brief, though the subject is most extensive.

By the latest document on Mineral Statistics, by Dr. Oldham, Superintendent of the Geological Survey of India (Memoirs, vol. vii.), I find that the quantity of coal raised in all India in the year 1868 was 547,971 tons, this amount being more than double what it was in 1858, and that the Raniganj field produced in 1868 more than six times that raised in 1850. The East Indian Railway alone consumes nearly half the total quantity raised in the country. The number of steam-engines has more than doubled in the Raniganj field in eight years. Other coal-fields are now being wrought, and fresh discoveries are being often made, so that the supply from India itself must rapidly increase. Dr. Oldham says that, in 1868, the Madras Railway used of English coal, imported direct, 1,285 tons, together with a small proportion of Australian coal. The Scinde Railway used in 1867 of coal, coke, and patent fuel imported from Europe, 5,645 tons; and in 1,868, 4,016. And in 1867 and 1868, the great Peninsula line consumed 116,824 tons, all imported. But on the lines connected with Calcutta, it appears that, of 447,644 tons used in the two years mentioned, on the East Indian Railway, only 4,029 were imported coal.

The Eastern Bengal Railway used in 1867, 16,120 tons of Indian coal, and in 1868, 16,330 tons of Indian, and only about 573 tons of English coal, and these chiefly in river steamers. The Calcutta and South-eastern line, and the Delhi and Umballa line used only Indian coal from Raniganj. The Salt Range supplied a portion used between Lahore and Umritsur. These data are sufficient to show that Indian coal bids fair to supplement the fuel imported from England and Australia. And to point out what India is doing with her coal, I quote a passage of a speech delivered in the British Parliament by Mr. Laing, on the 7th March, 1873, on moving for the continuation of the broad railway gauge in India.

The main railway system of British India now comprises 5,000 miles actually opened, inaugurated by Lord Dalhousie, and constructed on the wide gauge of 5 feet 6 inches, by separate Companies, under guarantees, at a tota cost of £90,000,000, or between £16,000 and £17,000 per mile. * * * The construction of those railways has been an enormous advantage to the Indian Empire, where within fifteen years the revenue has been raised from 30 millions to 50 millions sterling per annum; and the aggregate import and export trade from 50 millions to 100 millions sterling—an increase in a great measure attributable to the railways. That lamented statesman, Lord Mayo —one of the most able and popular of the many great Viceroys they have had in India—being impressed with the advantages conferred by the railways on that country, was very anxious for a large extension of the system, and arrived at the conclusion that 10,000 additional miles of railway were urgently required. (*Times*, 8th March.)

Fresh coal-fields in India have been recently explored; but of one of them, the Kurhurbárí field, Mr. Hughes says (Memoirs vii, part 2) that the assays proved its coal was superior to the Raniganj: and that, deducting waste and all impediments to working this field, it will produce 80,000,000 tons; and that, at an annual consumption of 250,000 tons, the field has a life of 300 years. The seams in this area vary from 9 to 32 feet, giving an average of 6 feet each to twenty-two seams; and on the whole an average of 15 feet.

The same explorer describes the Káranpúrá coal-fields, in which some of the seams attain a thickness of from 14 to 21 feet. These fields form a part of the Damuda Valley, and Mr. Hughes gives the areas of all the basins in it thus:—

1. Ránigauj, 1,000 square miles.
2. North Káranpúrá, 472 ditto
3. Bokáro, 220 ditto
4. Jherria, 200 ditto
5. South Káranpúrá, 72 ditto
6. Rámgurh, 40 ditto.

These 2,004 square miles do not, however, make up all, and only a portion of the coal-fields of India. But of this we may take the calculation of Mr. Hughes as correct,—that of the Káranpúrá fields alone there is proved to be a quantity capable of meeting all demands, and that in the northern basin of the name (No. 2 of the list), taking only 250 miles instead of 472 square miles, there are *eight thousand seven hundred and fifty million* tons of coal.

In the south (No. 5) basin, taking only 15 square miles, as the average ascertained thickness of the seams (deducting 20 feet for partings) is fully *seventy* feet, that area will supply 75,000,000 tons. There are eight seams, with a total thickness of 62 feet, and sixteen with a thickness of 159 feet 3 inches.

The thickness of the seams in most of the Indian fields is remarkable. Thus, in the Rániganj beds, nine seams have an aggregate thickness of 120 feet; eleven, amounting to 100 feet; and four, in the lowest Damuda series, attain to 69 feet (W. T. Blandford, in Memoirs, vol. 3, part 1). The last announcement is of a new coal area, west of the Damuda Valley, which supplied coal during the mutiny war of 1857; and which Mr. Hughes thinks will prove of great advantage. It is called the Daltonganj field.

Besides these data as to the amount of coal in India may be mentioned the Jherrhia field, 170 miles from Calcutta, with 465,000,000 of tons; and Bokáro with 1,500,000,000. The upper or Rajmahal Hills turned out 657,827 tons in 1858-9-60; and the Khasi Hills 1,917,000 tons.

No time is left to me for any reference to the great iron beds in various parts of India; but there remain some pressing considerations which will induce me to trespass a little longer on your patience, in regard to our own coal prospects. I may ask, perhaps, first of all,—is not it very probable that the coal trade now carried on with Indian ports will be soon cut short, when the cost of transport from the mines of India will be reduced to an equality with, or to a level, and perhaps below the cost of the importation of Australian coal? Some may suppose that the superiority of the latter will always carry a market. But that superiority is not so apparent. Some of the Indian coal (though much is inferior) rivals the Australian; and some of the Australian is as good, for certain purposes, as British. It is quite true that this year the Admiralty contracts for the Eastern depôts are for Australian, to the extent of 11,000 tons, and not for English coal; and, in the West, the Colonial coals of Cape Breton are to be firstly employed with American instead of British. But this arises from the difference in price when the contracts were made; since then, Australian coal has risen, and the saving calculated on by the Admiralty of £23,000 per annum will have to be diminished by the difference between the old prices and the new of our Australian coal.

Australian coal will, perhaps, before many years, be as little needed in China or India as British will be wanted in the United States.

(4). But in case of necessity, what is the actual known amount of our Australian coal? Whatever may be wanted for ourselves, we are not likely to get any from Victoria; for although upwards of one hundred thousand pounds have been expended by the Government in geological surveys and mining operations*

* See Appendix B.

including coal, from 1851 to 1873, and upwards of four thousand vertical feet of strata have been pierced in the search for coal, yet it is found necessary in this year of grace to send for the New South Wales Examiner of Coal Fields to find it, if possible, for those unquestionably able geologists who have declared over and over again that it cannot exist in any great payable quantity.* For my own part, I do not believe that the gentlemen at the head and on the field staff of the late Geological Survey Department could have been mistaken, though I sincerely hope that my friend the Examiner may receive a crown of Glossopteris and Vertebraria for his pains. The field staff of the survey alluded to would do honor to any scientific body in the world; and we may suppose that all that could be done has been done to find what Nature either never deposited or has removed by denudation. At any rate, we have the confirmation of this view in the opinion long ago expressed by the late experienced Director of the Geological Survey of Victoria.

A question, as the members of this Society well know, has been raised as to the age of our worked New South Wales coal; and, basing his opinion on the existence of certain plants in the beds of the formation, Professor M'Coy holds that the coal now supplying the markets is not of Palæozoic, but of Mesozoic age, and of the Oolitic epoch. My own opinion has been that it is not Mesozoic, but Palæozoic. Finding that, after great search for coal in Victoria, nothing valuable could be discovered, a Commission issued by the Parliament invested three gentlemen (Mr. Clement Hodgkinson, Mr. R. Brough Smyth, and Mr. Thomas Couchman) with the direction to report upon the coal-fields of "the south-eastern part of the Colony." This was a praiseworthy proceeding, and I am sorry to find that it has not been successful. Out of forty-nine seams enumerated, there are but three that attain the thickness of a foot, and of these a foot seam is said to occur at Cape Otway; and at Cape Paterson are two, respectively 4 feet and 3 fect 6 inches thick. Of course, as the Report states, there may be others; but such are

* See Appendix C.

put down as "said" to have occurred—"said" to have been worked—"said" to have been "discovered, but not seen." There is, that is to say, no actual knowledge of them. And this is all we learn from the Report as to the existence of coal of all ages (for "lignite" and "tertiary" are included) under the title—"Number and thickness of coal seams in Victoria."

So far, so good. What Commissioners have seen should be respected; but unhappily the two thick seams spoken of in the last report as occurring at Cape Paterson, they confess they "did not see; the excavations were filled in with rocks and sand, and we had no means of descending the shafts." They add, however, that they "accept the statements made regarding the character and thickness of them as correct. (Report, p. 11.) I do not call in question the correctness of these statements, but I would merely say that, including the three seams pointed out above, the average thickness of the forty-nine seams is just a little under (I would not rob them of a decimal even) four and a half inches *

I had intended to discuss this Report at considerable length, in order to point out some errors in it, and to explain more fully than I can now the history of the controversy which has taken place about the ages of our various Australian Coal-fields. But finding that what I had prepared on the subject would occupy more time than we can spare this evening, I have withdrawn it in order to use it in another way.

I will merely say now, that although within a brief recent period, as explained before, great progress has been made in the exploring of large regions of Mesozoic age to the north of us, and in the finding of coal therein, some of which is younger than our Newcastle coal, yet I adhere to my opinion that the latter is not Mesozoic, and maintain still, that the field in which it occurs rests on Palæozoic rocks having lower seams beneath, which repose on other marine beds (so that they do "interpolate" each other in the series), in this respect exactly resembling the older coal-fields of Queensland; and further, I maintain that both series of coal beds hold the same characteristic fossil plant.

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* See Appendix C (p. 95).

I may therefore add, for Professor M'Coy's information, that the coal seams at Rix's Creek, which he distinguishes as of a different epoch, by a line drawn below "shale with Glossopteris," because he thinks the lower do not hold that plant, do carry Glossopteris; and this fact I had confirmed in writing by the present Examiner of Coal Fields, whom I requested to verify the fact. Rix's Creek is only another example of what may be seen at Stony Creek, at Mount Wingan, at Coyeo, at Murrurundi, and in various other parts of the Hunter River basin. And Mr. Daintree, on my appeal to His Excellency Sir Henry Barkly, went up to Stony Creek, examined the place, confirmed my statements, refuted Professor M'Coy's notions of a reversal of beds, and published his report in No. 100 of the *Melbourne Yeoman*, 29th August, 1863. Such evidence cannot be set aside, though it remains unnoticed by Victorian annalists; and as to Glossopteris there, it was that very plant which astonished the Professor, and led to a most unjustifiable and indefensible misrepresentation of what I said to him when I produced it as proof of a Newcastle plant below the marine beds, and finally, also, to Mr. Daintree's investigations.*

(5.) I pass on now to some further remarks on the prospect of Coal production in this Colony.

It is the fashion to consider a patch of colour on a map indicative of a Carboniferous formation to be tantamount to asserting the existence of beds of coal under the whole area so coloured. This is, however, a wrong conclusion. In a very large portion of our Carboniferous area, no sufficient operations have exhibited the proof that any coal, or how much, exists under the surface. All mineral deposits are found to thicken, grow thin, and sometimes to die out altogether, or to renew their strength again, and there are plenty of proofs of this in New South Wales. Coal beds also change their character, as shown in the occurrence of oil-cannel and other hydro-carbons, passing from or into ordinary coal, and bearing still the prevailing phytological impressions which

* See Colliery Guardian, Feb. 20, 1869.

are common in the shales associated with it. Thus, all the area indicating "Coal" on certain maps recently published must not be supposed to be so full of coal as to justify the search for it in every portion of that area, and other parts may be untinted as coal where coal does exist. But there is, so far as has been proved, coal enough to last for a long period, with proper economy and due attention to the limitation of exports.

The quantity, for instance, actually produced in 1871, from mines now in operation at or near Newcastle, was 790,143 tons, and of this, 565,429 tons were exported; and of these exports 84,916 tons went to Victoria, 39,705 to New Zealand, 1,694 to Queensland, 29,786 to South Australia, 5,974 to Tasmania, and 390 to Western Australia; the United States took 24,814, China 372,800, and India 7,118, &c. The whole was exported to twenty-seven different ports, in nearly 1,000 vessels.

Smelting operations will increase, and, in places where coal cannot be obtained on or near the spot, the cost of working in the interior will be enhanced; and as steam communication is encouraged from place to place along our leagues and leagues of Australian coast, or to foreign ports, the demand on our coal mines will increase the cost. It would be well, therefore, if steps were taken to ascertain not the possible or probable, but certain, existence of coal in such districts as have never yet been practically sounded. There may be a promising coal area, but the thickness of the seams may be insignificant; and to point out this was one chief reason that induced me to enter so minutely into the case of America and India, and to quote the words of Mr. Warington Smyth in relation to what he strikingly denominates "the elements for subtraction."

I have had on my mind another impression to which I must give utterance. There is no doubt that there may be almost insurmountable difficulties in obtaining coal for certain localities; and it is just in such places that all the growing timber has been removed by the axe of the miner, leaving large tracts quite bare where wood and coal are necessary. Many tracts of this description have been traversed by me; and especially in some of the cedar districts is this clearing the effect of wanton wastefulness.

If the time has arrived for a Minister for Mines to be appointed, would it not be well to extend his jurisdiction to Woods and Forests also?

In the year 1863, in conjunction with me, the present Examiner of Coal-fields published a series of sections of the coal-field from Newcastle to Morpeth and Stony Creek into the lower coal seams; and this, on one sheet, was shown by him during the late Exhibition. He also exhibited there a new series of sections, marked A to J consecutively,—showing the actual thickness and depths of the coal-beds in the following localities, viz., Newcastle, Wolgan, Lithgow Valley, Burragorang, at Fitzroy Iron Mines, Kangaroo Creek, Mount Keira, and Coalcliff. The whole of these are in the upper coal-beds, between the first out-crop and the underlying Palæozoic marine beds,—being the seams that the Victorian Commissioners insist on being Oolitic, though one of the Newcastle beds contained fish which the highest icthyological authority in England considered Palæozoic. In the large section the place of this was shown, but in the new sections it has been omitted; but the whole of the beds are shown to hold Glossopteris and Vertebraria, and those underlying the marine beds to hold excellent coal and cannel.

The contrast between this display and the forty-nine Victorian threads of coal is very striking.

Mr. Mackenzie has ventured on a calculation as to the amount of coal in a certain area of country "known to himself," which he considers to occupy "15,419 *square* miles. One of the seams of coal, 8 feet in thickness, under this area, should, he says, after allowing one-third for loss and waste in getting, &c., yield 84,208,298,667 tons, which, at the present production of about 1,000,000 tons per annum in New South Wales, would last about

84,208 years," and, at the present production of Great Britain, "112,000,000 tons per annum, our 8-foot seam would last about 751 years."

This calculation is not, perhaps, strictly accurate, and it may be doubted whether there is any continuous area of the kind as 9,868,160 statute acres, uninterrupted by intrusive rocks.

But regarding this as a hypothetical case, the result may be accepted, under the assured conviction that it falls very far short of the actual quantity of coal in New South Wales, as will some day be proved.

This subject, Gentlemen, has occupied so much of your time as to leave but a small margin for two other topics, which I must not pass over in silence.

Metalliferous Areas.

During the last year there has been a great declension in the marketable value of shares in the metalliferous mines of the Colony. But it is not surprising that disappointment of the kind should have occurred, seeing that it has arisen in most instances without any bearing upon the mines themselves. Stock-jobbing on the part of persons who have had no interest in the mines, but only in their own profit by ventures in the market, does not affect the actual value of the productive capacity of well-ascertained underground deposits. They remain just as they were, and cannot be affected by the trickery or the deception of such as have taken advantage of ignorance or credulity, or the too sanguine expectations of honest but incautious speculators. Far from being discouraged by the results of these mistakes, I look forward to a nearer realization of the words I ventured to use in my last year's Address, viz. :—"When the excitement that is (was then) rife shall have subsided, and we shall have less dread of spelling speculation without the initial letter, there will be found inducements to study the structure of the country more than the market price of its products."

But even the "excitement" had some good effects, for various fresh finds of diamonds, cinnabar, copper, gold, iron, and tin have occurred, and the large area between the Darling and Bogan

Rivers has more and more cast off its obscurity. Copper, iron, and gold have been found in fresh portions of that region; and even within four or five miles of the spot where Mr. Cunningham lost his way in the scrubs, and finally his life, during Sir T. L. Mitchell's Expedition of 1835, a lode of copper has been found, in beds of metamorphic rock which the first explorers did not know existed there. Moreover, several new gold fields have been proclaimed in 1872–3.

(1.) Another result to science and true mining industry had been derived from examinations in the tin country to the northward,—not by half or almost entirely ignorant mining managers, but by skilled and properly educated geological surveyors, qualified to report on all the delicate questions involved.

Thus, in the "Quarterly Journal of the Geological Society of London, for February, 1873" (vol. xxix) we have first a report, read on 6th November, 1872, bearing date 2nd July, from Mr. T. F. Gregory, in a letter to the Queensland Government, and communicated by the Earl of Kimberley to the Society.

In this report Mr. Gregory gives the outlines of the stanniferous region, the principal stream beds and fluviatile flats, and some of the minor lodes or veins in the Queensland part of the area; one of the latter, as he says, traceable, at intervals, for nine or ten miles. Mr. Aplin, whom he quotes, states that he met in that area no other tin ore than the peroxide (cassiterite) which is associated only with an "invariably red granite," the felspar being a pink or red orthoclase, and the mica generally black, but when tin is present *in situ* the mica is white. The granite, he adds, is coarse-grained and readily disintegrating, with bands of granitoid character highly micaceous, traversed by veins and bands of quartz, in which the tin ore abounds, as if they had been local feeders in the courses of drainage. The crystals of tin ore are found along the margin of the quartz veins, though sometimes in the micaceous portions, in which cases the mica is white. "The strike of the bands and the distinct quartz veins is generally N. E. and S.W. No tin floors, as at Elsmore mine, in New South Wales, had been discovered." This is an abstract only of Mr. Aplin's notes.

Mr. Gregory calculated then, that the population of miners and their families would not for three years be less than from 5,000 to 8,000, but would increase.

(2.) The next communication made to the Geological Society, on the same evening, is entitled "Observations on some recent Tin Ore discoveries in New England, New South Wales; by Mr. G. H. F. Ulrich, F.G.S."

The author speaks first of the granite plateau of which Ben Lomond is the summit, the height of which he gives as nearly 4,000 feet above the sea. It is, however, exactly 5,000 feet, as may be seen in my eighth Northern Report to the Government, 7th May, 1853, p. 17. [Parliamentary Papers, 26th May, 1853.]

He says the predominant rocks are granite and basalt, in closing subordinate ranges of slate and sandstone; the basalt having broken through and overflowed the summits, greenstone occurring in the slate. He describes the country as park-like, with the climate of central Europe. He says of Elsmore, twelve miles east of Inverell, that it includes a granite range two miles long, and extending under basalt, micaceous in character, rendered porphyritic by crystals of white orthoclase (sometimes several inches long), with occasionally bluish grey oligoclase; quartz veins traverse the rock up to a foot in thickness, and druses, seams, and crystals of cassiterite stud them. Portions of the veins are micaceous, representing the "greisen" of the Saxon and Bohemian tin ore districts, differing only from the rock at Beechworth, in Victoria, in that the granite there is fine and euritic, and rarely porphyritic.

He considers the veins of softer granite, which are highly micaceous, of more importance, the quartz and felspar being insignificant compared with 75 per cent. of mica.

In these micaceous dykes Cassiterite is distributed in crystals from the size of a pin's head to above that of a pea, and in "nests and branches yielding lumps of mostly pure ore to above 50 lbs in weight; part of the mass of one of these dykes forming a breccia of mica and imperfectly crystallized tin ore, cemented by hydrate of iron." He comes to the conclusion that the granite represents one of what in Saxony and Bohemia are called "stocks or stockworks, but of incomparably greater size and thickness."

He found beryl associated with quartz crystals, in a ferruginous clay in the spoil heaps of a shaft, and the same mineral on tin crystals, in fragile small thin crystals; rock crystal, holding tin ore; arsenical pyrites; wolfram in the granite, disassociated from tin. The wolfram itself is either pitch-black, brown, or hyacinth-red in colour, forming occasionally in twins, as at Schlaggenwald, with twelve-sided prisms and one pyramid.

The drift is rich, and of recent granite detritus, from six inches to twelve inches thick, spread over the range; and there is an older, probably Pliocene Tertiary cemented gravel of water-worn pebbles and quartz (rock crystal and Cairngorm), hard granite and hornstone, capping the top and dipping under basalt. The granite detritus gave from 3 oz. to 2 lbs. of ore to 20 lbs. weight. He says that for 150 miles far into Queensland all the creeks of the granite country have proved to be stanniferous.

At Glen Creek the granite simulates that at Beechworth, in a small patch of 10 chains, in the creek which runs through hard flinty, unfossiliferous slate. Small veins of arsenical and copper pyrites are enclosed in the granite, which gradually passes into slate; the veins of the granite also intruding, without change or interruption.

This flinty metamorphic slate forms the base of the area, but in it are outcroppings of micaceous granite, with large radiating crystals of schorl; and near these protrusions are veins of solid tin ore traversing the slate. The granite is harder than at Elsmore, and is occasionally traversed by an augitic greenstone diabase, rougher at the surface than under a covering of drift or alluvium, and passing occasionally into a variety of serpentine.

Mr. Ulrich concludes with this remark:—"Positive want of water, or too great an expense attached to the bringing of it to the stanniferous localities will, however, I am afraid, be prohibitory of the working of a great number of those recently discovered. Still, the produce of such as can be worked will doubtless, in no long time, sensibly affect the tin-markets of the world; in fact, it seems not unlikely that the production of tin ore in this part of Australia will reach, if not surpass, that of all the old tin-mining countries combined."

These two papers are very important, as giving the settled opinions of persons whose word may be taken. No guarantee is required; but I may add that some of the facts stated were notified by me in 1853, especially the way Wolfram occurred at Dundee, from which locality and for many miles northward on the eastern side of the great granite platform, I have a collection of rocks and ores that enables me to confirm of that side what Messrs. Alpin and Ulrich say of the western side.

(3.) Let me conclude these selections by a reference to another of the officers of the staff of the late Victorian Survey, who is now attached to this Colony, and whose experience will make him of great service in the district between the Murray and the Murrumbidgee, where he is at present employed. I mean Mr. C. S Wilkinson, who some time since published an excellent Report on the Inverell and Cope's Creek country.

Mr. Wilkinson's Report is in close agreement with those before quoted, and with my own explorations in the vicinity of his researches made in the year 1853; but he has, however, made some advances in the development of the structure of the tin country. He points out variations in the composition of the granite, its change from ternary to binary character; and shows how it becomes porphyritic from the admission of orthoclase felspar, just as Mr. Ulrich found it at Elsmore. Radiating crystals of schorl were also observed by him, and smoky quartz in the geodic hollows of the granite.

Greenstone of a somewhat peculiar kind, with epidote, also occurs on Cope's Creek, where it is traversed in places by dykes of entire granite running N.W. These are split up by vertical joints parallel with those in the main granite, striking E. 5° N. In the greenstone occur quartz veins which are believed to be auriferous, as alluvial gold was found hard by. The veins and joints seem to be faulted by successive slips, so as to give the rock a resemblance to a sedimentary deposit. Rex's ground, on Middle Creek, Cope-Hardinge mine near Tiengha, and a still wider area, exhibit the same phenomena. Where the strike of the veins and joints is meridional, no minerals have been found; only in the newer, or E. 5° N. and N.E. direction, tin lodes occur near Cope's Creek.

In the boundary tin mines, veins of quartz in the granite, striking E. 20° N., with a very high angle of dip, carry tin as well as the walls of granite. Felspathic dykes traversing porphyritic granite, E. 15° to E. 20° N., near Sutherland's Water, carry quartz veins with tin. Solid lodes of tin occur also in other localities, in the centre of euritic dykes, going E.N.E., and nearly vertical. Similar cases present themselves to the south-westward, bearing E.N.E. Eleven thin seams of Cassiterite have been found here in a width of 5 feet. Iron pyrites, galena, and copper in a quartz dyke, are found on Darby's Creek in the granite, going E. 20° N.

These examples are sufficient to point out the character of the country and the main features of the lodes.

Loose tin occurs in places of considerable size, rounded sometimes as by water, like the drift quartz with gold, which much resembles the Victorian Tertiary drifts. I have no doubt myself that those drifts, especially when between granite and the overlying basalt, which also occur in the same neighbourhood, as well as the quartz veins in the greenstone, are auriferous, and that much of the loose tin is of the same age as distributed with the other drift. I have before mentioned that I found Cassiterite in various places of the granite region in New England, with gold and gems.* And in a letter not long ago received from Mr. Wilkinson, he gives me a drawing of some curious diamonds from Cope's Creek; and some were brought to me long before from Boro, having exact resemblance to those from Suttor's Bar, on the Macquarie.

* See Trans. Roy. Soc. N.S.W., 1872, p. 67.

Limestone occurs near Elsmore, and not far from Barraba, and elsewhere also to the eastward; and there is a possibility of finding near Elsmore outliers of what may be considered the Carboniferous beds that are in force to the westward.*

(4.) I now desire to express my opinion that the region in which this tin country occurs is Devonian, and that there is nothing that is usually called primary about the granite which twenty years ago I traversed in various directions, and of which I gave a full account to our Government in the reports of my northern explorations. The slates flanking the granites are seen to be older than the latter in many places; and this view is supported by Mr. Wilkinson's opinion of a transmutation of the slates, besides other data which cannot now be mentioned.

Cinnabar.

As an addition to the above discussion in relation to tin, I may notice here that I have received samples of tin from the granite highlands of Tasmania, this year discovered; and also last year I had placed in my hands tin drift from Flinder's Island, in Bass's Strait, where topaz abounds, brought by Mr. Gould, the late head of the Geological Survey there, who is now added to the staff of the Topographical Survey of New South Wales. I have also recently received from a New Zealand friend some excellent samples of cinnabar. It occurs in a swampy locality, not far from the Bay of Islands, and under circumstances that fully justify the opinion I have previously expressed respecting the agency of hot springs in the production of this ore of mercury as it has been found in this Colony and in Queensland. It is a new find, but owes its first discovery to a lady now resident on the North Shore, who noticed it lying exposed on the surface.

The only occurrence of colonial cinnabar in solid siliceous rock that I am acquainted with is at Wide Bay Creek, near Kilkivan, in Queensland. Specimens from both of these new habitats have been placed by me on the table, for comparison of the modes of occurrence.

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* See Appendix D

Conclusion.

In bringing before you, gentlemen, at such a length the various topics touched on to-night, my desire has been to state some of the discoveries that have taken place since the last Anniversary, and to continue the connected series of reviews of matters most interesting to ourselves, which I have considered the fittest way of addressing you in the discursive range allowed on these occasions.

Other subjects of perhaps equal importance have been passed by; and even as it is, I must offer an apology for detaining you so long. It is said, as men grow older they sometimes become more garrulous. Should I ever grow young again, I will try to be more concise, and pack my articles in a smaller compass.

Note.—This Address was illustrated by numerous geological and mineralogical specimens, a coloured map, and sections of Cainozoic, Mesozoic, and Palæozoic formations, as they occur in Queensland.

I have read a second Report to the Surveyor General, from Mr. C. S. Wilkinson, on the Inverell Tin District, which has not yet been published, but which deals more minutely with the subject discussed in the first Report mentioned at p. 36. It is a most valuable document.

APPENDIX.

A. (p. 11.)

Since the text was in print, three or four small pieces of shale were found, which had been brought down by the Expedition and were forwarded to me from Brisbane for inspection; and on one of them is a portion of a frond which has the characters of *Glossopteris*. (19th August, 1873.)

B. (p. 26.)

In alluding to surveys and mining investigations, I had in view all mineral inquiries, including that for coal, because I considered that if the surveyors and searchers for metals had come across any coal deposits or associated strata, in places not included in any special search, we should have heard of them.

No one who has read the Address can doubt that I scrupulously avoided all exaggerations, and kept in view the sensible remark I quoted from Mr. Warington Smyth respecting "elements for subtraction."

It is, however, difficult to find out what precise sums Victoria has expended in her most useful and inimitable surveys and mining operations; nor do I know exactly how much ought to be set apart for coal search alone, seeing that private operations have left but few traces capable of being expressed in figures.

But the particulars which I will now mention will be sufficient to show that no charge of exaggeration can justly be brought against me.

In Mr. Selwyn's return as to total cost of his surveys from 1852 to 1861 (inclusive) [Report of 1861], I find (at page 27) an amount of £32,516 0s. 5d., or at an average of £3,612 *per annum*. This, in the approximate estimate of Mr. Braché (Selwyn's Report, 1863, p. 41) from 1852 to 1861 inclusive, comes up to £41,116 9s. 10d.; and in the introduction to the first volume of the Memoirs of the Geological Society of Italy, according to information derived from Victoria, it is stated that Mr. Selwyn's survey for thirteen years, including salaries, cost 2,500,000 Italian *lire*, which, at the value of 8⅛d. to the *lira*, is £84,635. The introduction is by the President of the Society, Igino Cocchi, and is entitled, "Brevi Cenni sui Principali Instituti e Comitati Geologici e sul R. Comitato Geologico d'Italia." (Firenze, 1871.)

In addition to these estimates is one made by Mr. Braché for Mining Surveys, from 1858 to 1862, which seems to be independent of the Geological Survey, as it occurs in the same report quoted above (from Mr. Selwyn, 24th June, 1863), as to a cost of £24,300.

On searching the "Votes and Proceedings" of the Parliament of Victoria from 1863 to 1872, it is found that the united annual expenditures under the head of "Minister for Mines" for that period come to no less a sum than £197,983 14s. 5d.

Selecting out of these amounts only Mr. Braché's estimates, Mr. Selwyn's own return, and the figures given in the Votes and Proceedings, we have an acknowledged expenditure for Surveys and Mining establishments, from 1852 to 1872 inclusive, not merely £100,000, but of no less a sum than £263,400 4s. 3d. sterling.

Such an example of faith and perseverance is not deserving of censure, but of praise and imitation; and it would be well for us of this Colony if we could quote from among our many speculative operations a similar instance of devotion to the cause of development of natural resources.

As the above data are taken from official sources, they are probably correct, although there may be others which have not yet fallen in my way.

Of this great amount, some was, I presume, expended on "*search for coal*"; but there are items of expenditure for coal only which deserve consideration, and which were incurred either by the Government or by private individuals and Companies.

Thus, in the district of Bellerine and Paywit, including Queenscliff, the Government cost of sinking and boring through 4,688 feet of strata was, up to 1863, £3,557 11s. 3d., but without any practical result.

The Government expenditure on account of the Griffith's Point Company, up to 19th January, 1866, was £888 10s. 6d. They "reached a depth of 822 feet 2 inches, without any coal having been cut."—(Selwyn's Report, 1865, p. 21.)

I do not know the full cost of Government coal search about Cape Patterson, but the Coal Company of that name had, in 1860, expended £3,050.—(Selwyn, May, 1860.) The same author tells us that, "during the last ten or twelve years, probably more than double that amount has been expended in the district, while about 100 tons of coal is all that has been brought to market."—(Selwyn's Catalogue of Victorian Exhibition, 1861, p. 185.)

Mr. Brough Smyth (Official Record, 1872–3) admits that "during the period from 1854 to 1868 many thousands of pounds were expended in boring and sinking shafts in the Cape Patterson Coal Field" and (p. 101–3) up to 31st December, 1871, there had been raised in the Colony 2,033 tons of coal, and 1,992 tons of lignite, whilst at Lal Lal, in 1871, there were raised 995 tons of brown coal, about half of which was saleable.

Mr. H. Levi says (Minutes of Evidence, 23rd April, 1872; Report of Board, p. 28) that "the Victorian Coal Company expended between £12,000 and £14,000 in improving their boat harbour, in boats, shipping coal, and proving the seams, but the impossibility of transporting the coals at any reasonable cost put success out of the question."

At Barrabool Hills, near Geelong, Mr. Thomas spent £8,000 in sinking and boring. This spot was examined by myself in 1856; the depth reached was 600 feet, and all that had been discovered was a 6-inch layer of coal.

The Newton and Chilwell Prospecting Association (as I am informed by a proprietor) expended at least £2,000 in their operations, reaching a depth of 1,150 feet.

The Griffiths Point Company expended, in addition to the Government grant of £888 10s. 6d., the sum of £444 5s. 3d., making for that adventure £1,332 15s. 9d.

Collating these statements, we have the sum of nearly £31,000 sterling, as the cost, up to the times fixed, of the particular localities enumerated. What may have been the expenditure in other places does not appear.

As to some of the researches, I may mention, as a proof that I have not spoken of the Victorian coal simply from what I have read, that the directors of the Newton and Chilwell Association placed in my hands sections of the borings, together with the materials brought up, so that I was enabled to judge for myself.

The Company working on the Bass River also consulted me, and furnished me with plans and charts, &c.; and the same opportunity was afforded me by the Company at work in the Glenelg district, near Coleraine.

I may say here, briefly, in each case I was compelled to state that I recognized nothing approaching to the data that could be furnished by the coal seams of Newcastle or the Illawarra, and that I concluded no such strata would be reached except at very great depths, and then only if not cut out by intrusive or bed rocks of other formations.

In all these beds from Victoria I saw no trace of our distinguished plant *Glossopteris*, nor has it ever been recognized by any geologist or palæontologist in that Colony. But it exists in New South Wales in the Hunter River beds, both below and above the marine fossiliferous beds among the workable coal seams; and also in the sandstone at Muree, abounding in the marine fossils of Lower carboniferous age, in which a *Conularia* and a variety of fossils are seen with the remains of a *Glossopteris* that must have been washed into the sea when the marine beds were being deposited.

The marine fossils of that locality were figured by Professor M'Coy, from the collection sent by me to the late Professor Sedgwick of Cambridge. (Annals of Natural History, vol. xx., 1847, 1st Series. See also 2nd Series, September, 1848, and 3rd Series, August, 1862.)

C. (p. 28.)

THE VICTORIAN COAL-FIELDS.

The following is the first Report of Mr. John Mackenzie, F.G.S., Government Examiner of Coal-fields, New South Wales, on the Coal-fields of the south-eastern district of Victoria :—

To the Honorable Angus Mackay, M.L.A., Minister for Mines.

Sir, Melbourne, July 28, 1873.

In accordance with your request, and with the permission of the Honorable the Minister for Lands, I have visited and examined the coal and strata of The Bass, Griffith's Point, Blue Mountains, Sandy Waterholes, Kilcunda, Cape Paterson, Strzelecki near Anderson's Inlet, Stockyard Creek at Corner Inlet, Traralgon, and Crossover. I have now the honor to submit the following report thereon.

THE BASS.

A shaft has been sunk on the river bank (see A on plan), and some coal said to have been found in it, but owing to its being half full of water, I was unable to see the strata sunk through.

I examined both sides of the river near the shaft, and could observe no trace of coal. Several days' rain prevented my seeing the rock in the bed of the creek, where Mr. Krausé informed me there was a small vein or patch of coal, and I could not hear of any regular seam of coal having been discovered.

GRIFFITH'S POINT.

In this district I examined the cliff sections and position of the old shafts sunk for coal. The carboniferous strata, where the shaft lettered B on plan has been sunk and a small vein of coal found, are lying at an angle of 75°, and no workable seam of coal exists there. It is, in my opinion, only a waste of money to sink or bore further in this locality.

At C on plan the strata dip north-east, and have been bored through to a depth of 850 ft. below the sea level, and no coal seam found. From here to the Sandy Waterholes (see letter D on plan), a distance of about two and a half miles, a constantly ascending series of beds, consisting of conglomerates, sandstones, and shales, with drifted pieces of fossil wood, junks and streaks of coal and carbonaceous matter, are exposed in natural sections in the cliffs, but in which there are no regular seams of coal.

It is, therefore, useless to look for, or to expect to find, any workable seams below those exposed in the cliffs at the Sandy Waterholes and Griffith's Point, as the cliff sections and borings show us that there are none.

SANDY WATERHOLES.

On Mr. Turnbull's land (see letter D on plan) there are seams of coal exposed in natural cliff sections as shown on section I.

These are regular seams of coal extending over a considerable area, and I believe them to be identical in geological position with those commonly called the Rock and Queen veins at Cape Patterson, and that it is here where they first make their appearance above the sea-level again on the coast west of I on plan, near Cape Patterson.

The coal is of very good quality, but it is very much disturbed by faults, and dips at an inclination of 21° towards the north-east.

The only workable portion of the No. 1 seam is 11 in. of good coal (see section No. I) and this is too thin to be of any commercial value.

No. 2 measures 13½ in. of good coal (see section); and if it had been formed with a better roof, and had been lying at a less angle, it might possibly have been worked by holing in the 3 in. of coal lying about 2 ft. below.

KILCUNDA.

I was accompanied in my inspection of this mine by Messrs. Krausé, Watson, and Thomas. Six different measurements taken in the main heading gave 20 in. as the average thickness of the seam of coal at E on plan. At F on plan, about one quarter of a mile south-west of the main heading, it is 2 ft. in thickness.

The dip is about 8 deg. to the north-east, and two faults have been proved, one of 120 ft. and another of 20 ft.

The coal produced is bright, bituminous, and non-caking, and the Coal Board's estimate of 15 in. of good or round coal is, I consider, a very liberal one, and quite as much as it will yield.

The seam extends over a large area, and I believe it to be identical with the one found at the Blue Mountain and Strzelecki Ranges, and on the sea-coast west of the Rock and Queen veins (see letter H). It has a bad roof, is disturbed by faults, and near the latter, as is usually the case, the greater part of the coal is very soft, and has an irregular cleavage, and when exposed to the weather decrepitates. In my opinion it will be impossible to mine it at such a price as will enable the proprietors to compete with the New South Wales or other intercolonial coal in the Melbourne market. The following is a section of this mine:—

	ft.	in.
Yellow sandstone.		
Grey and blue shale	10	0
Coal (average of six measurements)	1	8
Floor—Indurated clay.		

I annex drawings showing how this coal is worked, and the Newcastle coal in New South Wales. (See sketch section, page 11.)

BLUE MOUNTAIN.

At a height of about 310 feet above the sea-level a 17-inch seam of coal (see G on plan) is to be seen outcropping in the side of a creek. It lies at an inclination of 28°, dips north 25° west, has brown sandstones and shales above it, similar to those at Kilcunda, and is, I believe, identical in geological position with the 20-inch coal at that place. I consequently infer that no other thicker seam of coal is likely to be found at a workable depth in this locality. (See general section, No. 2.)

It is too thin to be of any commercial value in such a position. The following is its section:—Alluvial, 2 feet; sandstone, 4 feet; brown shale, 1 foot; good coal, 1 foot 5 inches.

CAPE PATTERSON.

Here I find the coal measures intersected by numerous basaltic dykes and faults, and the dip changing in inclination and direction at very short distances.

The undulating or folding nature of the strata exposes the basset edges of two seams of coal, exceeding 1 foot in thickness, in three different places, at short distances apart. These might make it appear to a casual observer that they were the outcrops of three others, although they are really only the same again appearing at the surface of the ground.

Their measurements are shown in vertical section No. 3.

I believe the Rock and Queen veins are identical in geological position with those before mentioned, exposed in natural cliff sections on Mr. Turnbull's land at the Sandy Waterholes.

The quality of the coal is good, but the faulty nature of the ground, the irregular and constantly changing dip, the thinness of the beds of coal, and distance from a shipping port, prevent its being worked at a profit.

The average of three different measurements of the Rock vein only gives 2 feet 4 inches of coal, which is divided by two bands of clay, &c.; and the average of three measurements of the Queen vein gives 2 feet 2¾ inches of coal, intersected by no less than three bands of shale, &c., although the Rock vein has been called and reported to be a 4-feet coal, and the Queen vein a 3 feet 6 in. seam of coal.

The numerous boreholes put down in this locality have proved beyond doubt that there are no other payable seams of coal likely to be found at a workable depth here.

STRZELECKI (M'CALL AND CO'S LEASE.)

In a creek on these ranges, and at a height of about 660 feet above the level of the sea, a seam of good coal is to be seen exposed. The sandstones and shales lying over it are similar to those at Kilcunda and the Blue Mountain, and I believe it to be the same coal as is found there.

The following is a section of it at L on plan:—Alluvial, 2 feet; shale, 2 feet; coal, 8 inches; indurated clay, 8 inches; coal, 1 foot 3 inches; band, 1 inch; coal, 5 inches; total of coal, 2 feet 4 inches.

The coal and strata are lying nearly horizontal, having only a very slight inclination or dip towards the north-west. This coal has a friable shale roof, which would make it expensive and difficult to work, and as the owners of the lease have never attempted to work it, but are boring below, in hopes of finding a more workable seam, I presume that they, like myself, do not consider that it could be worked to a profit.

On the 28th ultimo, the lessees had bored a distance of 256 feet below the above-mentioned coal, and the borer told me that the strata gone through consisted of sandstones with grey and blue shales, and no coal.

My opinion is that they will have to bore at least 1,200 feet before any other regular seams of coal will be met with, and that they would then intersect those identical with the Rock and Queen veins at Cape Patterson. (See general section, No. 2.)

The rocks now being bored through at this place will probably have junks and pieces of coal in them similar to those seen on the coast between Sandy Waterholes and Kilcunda, and if any should be found, a seam of coal will appear to have been struck, and will no doubt be recorded as such.

STOCKYARD CREEK (HILL'S PROSPECTING LEASE.)

A very thin and inferior coal is outcropping in one place on a creek on this lease, and a thin stratum of bituminous shale in another.

The latter was described to me by those interested in it as a valuable seam of coal. The deposits lie at an angle of 18 deg. to 24 deg., and rest on Silurian rocks, which are to be seen about a quarter of a mile lower down the creek. The following is their measurement. (A sketch is here given of the first deposit, showing a layer of sandstone, followed by inferior coal 9 in. to 1 ft., after which is a stratum of very hard shale. In the case of the second deposit, the strata came in the following order:—Very hard shale 6 ft. in thickness, indurated clay and black bituminous shale 3 in., black bituminous shale 2 in., stone 2 in., black shale.)

How any one having the slightest pretensions to a knowledge of coal-mining could ever look upon these as workable seams of coal I am at a loss to understand; for of all the reported discoveries of coal I have ever seen, here or elsewhere, during the last twenty-five years, I never saw one of less promise.

TRARALGON (N ON PLAN.)

I was accompanied in my examination of the coal discoveries here by Mr. Krausé, Dr. Simmons (one of the Coal Committee), and Mr. Duncan Campbell. At the time of my inspection, and previous to it, there was heavy rain, which made my examination rather difficult.

I looked at the place where the coal had been found, and on proceeding a short distance higher up the creek I saw the same strata and coal exposed in a natural section above the bed of the creek.

There was one layer of coal 2 in. in thickness, and another 10 in. of shale and coal.

This appears to be the north-easterly edge of the western Port and Cape Patterson coal basin, and the shales here contain similar fossil flora.

Thick beds of conglomerates, sandstones, and shales, with no workable coal seams in them, are to be seen rising from under this coal as you ascend the creek, and they rest on Silurian rocks. Therefore, no workable coal seams will be found by boring or sinking below the bed of the creek where the shaft was sunk.

Whether the 300 ft. or 400 ft. of sandstones and shales, &c., in the ranges over where the shaft was sunk contain any workable coal, it is impossible to say for certain without provings being made.

But I think that, if they did contain any thick or workable seams of coal, we should have seen some pieces or trace of it in walking round the ranges. We discovered none.

CROSSOVER (O ON PLAN).

Having given my knee-joint a very severe wrench through a buggy accident, whilst proceeding on my journey here, I was unable to examine all the different outcrops of lignite and brown coal in this locality, as it was impossible to go to two of them without walking several miles through a scrubby country, which I was then unable to do.

I therefore left instructions for specimens and sections to be procured me from the two places I was unable to visit, and engaged two men to further test the nature of the brown coal in the drive I inspected.

After receiving the specimens and measurements, I hope to be able to form an opinion as to the value of these deposits.

To summarize the remarks made in the foregoing Report, I may briefly state that, having given the whole subject my very best consideration, I have arrived at the following conclusions :—

1. That it is useless to expend any further sums of money in searching for payable seams of coal in The Bass, Griffith's Point, Western Port, Cape Patterson, Strzelecki, or Stockyard Creek districts.

2. That the Kilcunda, Blue Mountain, or Strzelecki seam of coal might be sought for and opened out in the ranges east and north of Messrs. M'Call and Co.'s lease at Strzelecki, proving it at intervals of a few miles apart to determine the thickness.

3. That the country might be examined between the above-mentioned ranges and the river Latrobe, or north-easterly and north-westerly edge of the coal basin, to see whether the Rock or Queen veins, or their equivalents, rise to the surface again in this direction, and are of any value. Such an exploration can only be properly and efficiently carried out by your Mining Department, and under a responsible person. The very excellent and valuable geological maps prepared by the late Director-General of the Geological Survey, and the more recent maps published by your present Secretary for Mines—Mr. R. Brough Smyth, F.G.S.—are proofs beyond dispute that you will, by this means, have the work done in the best and most efficient manner possible.

I cannot conclude this Report without expressing my thanks to Mr. Krausé for his valuable assistance in conducting me to many of the places herein referred to, which were very difficult of access; and also for his kind attention to me personally when I met with the accident at Moe; also to Mr. Murray, for his kind attention and assistance at Cape Patterson and the neighbourhood.

[The above Report was laid before the Victorian Parliament Tuesday, 12 August, 1873, and is reprinted here in justice to my own opinion previously expressed.]

The following is a second Report of subsequent date, also laid before the Parliament of Victoria:—

To the Hon. Angus Mackay, M.P., Minister for Mines.

Melbourne, August 19, 1873.

Sir,

I have the honor to submit my Report upon the coal and lignite you desired me to examine in the Crossover, Barrabool Hills, Winchelsea, and Loutit Bay districts.

CROSSOVER—LIGNITE DEPOSIT.

In my previous Report I mentioned that, owing to an accident, I was unable to examine two other alleged discoveries in this locality, and before expressing any opinion I thought it advisable to have measurements and samples sent me from all the different places, the alleged discoveries being represented as of a much superior quality to that being worked in the drive I inspected.

After putting on men for a fortnight to procure specimens, I have only received some from the original drive.

No reference being made to the new ground, I am inclined to think that its productions are either inferior in quality or that no lignite exists therein.

The following is an account of the strata proved in the cutting and drive opened out:—

	ft.	in.
Surface soil	1	0
White clay and grit	2	6
Black clay, with imperfectly carbonized pieces of wood	1	6
Lignite or brown coal	3	4
Brown coal (conchoidal fracture)	1	7
Brown bituminous shale	0	2½
Brown shale	2	10
Yellow clay, with fossil resin	1	10
Brown coal or lignite	4	4
Brown clay	1	0
Yellowish-white plastic clay	0	6
Greenish sandstone.		

The lignite in the drive is of a very fair quality, but is at the present time of no commercial value in such a position, as it could not compete with firewood for house-fire purposes in any of your large cities, and is not suitable for locomotive engines.

BARRABOOL HILLS, GEELONG, AND BELLERINE.

The lithological character of the Barrabool Hills sandstone is the same as that which lies over the 17-inches to 2-feet coal in the Western Port district.

I inspected the 1,200 feet shaft and borehole made by Mrs. Thomas and others at Barrabool Hills, in which 6 inches of coal were said to be found. I also carefully looked over the accounts of the strata sunk and bored through in numerous places in the Bellerine district, where a vertical thickness of over 4,000 feet of strata has been tested, and no workable seam discovered.

I am of opinion that the above-mentioned shafts and boreholes have proved that there are no payable coal seams at a workable depth in either of these districts, or in the intervening country at Drysdale, Geelong, &c.

WORMBETE, NEAR WINCHELSEA.

I was accompanied, in my examination of the coal worked here, by Messrs. Stirling, Krausé, and Moran.

At a height of about 750 feet above the level of the sea, near the head of Wormbete Creek, a 3½-inch to 4-inch layer of coal is to be seen in several places.

The following is a section of it, and the overlying and underlying strata:—

	ft.	in.
Brown sandstones, a great thickness.		
Blue shale, about	12	0
Coal, $3\frac{1}{2}$ inches to	0	4
Blue sandy shale.		
Sandstone.		

The strata and coal dip about $7\frac{1}{2}°$ to the east, 20° south.

No other seam of coal has been seen, either higher up or lower down the creek, although there are excellent natural sections exposed to view in several places here, as well as in other parts of the district; it is therefore useless to expect to find a payable seam of coal in this locality.

The place is one of those described by people inexperienced in such matters as giving "good indications," and money has been uselessly expended in driving in a $3\frac{1}{2}$-inch coal, with the hope of its becoming thicker when further developed. It was also supposed to be a "good indication" of finding a thick and payable coal below, although they could see no coal underneath it in the natural exposed sections of strata lower down the creek, and consequently will find none by sinking or boring.

LOUTIT BAY TO STONY CREEK, ALONG THE COAST.

Excellent natural exposed cliff sections are to be seen here, and they show us that there are considerably more than 1,000 feet of sandstones, with shale very similar to those at Barrabool Hills, and on the Blue Mountain and Strzelecki Ranges, in Western Port, in which there is no coal of any value, the thickest being about 5 inches.

A shaft has been sunk and a borehole put down by the Colac Company to the depth of about 120 feet, on "good indications," and money expended in piercing strata similar to that exposed to view in natural cliff sections adjacent to where the borehole was made.

Thin irregular patches of coal, called "good indications," are to be seen in the cliff sections, and also, at low water, in the rocks outcropping on the beach, the shaft and borehole only going through the same strata as are seen in the cliffs.

STONY CREEK, NEAR AIREY'S INLET.

I went down a shaft 45 feet deep, at the bottom of which a place has been driven about five yards in an inferior bed of lignite 18 inches in thickness. Other shafts have been sunk through the upper portion of the lignite deposit, and in one of them 4 feet of inferior lignite and sandy shale have been cut through.

I was unable to measure a section of the different strata between the upper 4-feet bed of lignite and sandy shale, and the lower 18-inch seam, on account of the main shaft being timbered; but the two beds examined were considered to be the best.

The extent of these lignite deposits is very limited, and they have now been sufficiently tested to show us that their quality is not likely to improve by any further sinking or driving; and they may therefore be regarded as of no commercial value, so long as there is any quantity of cheap firewood to be obtained.

In conclusion, it may be as well for me to state my reasons for recommending, in a previous Report, that any future provings should be carried out under the direction of your department.

They are as follows:—

Because I found that large sums of money had been expended by the Government in supplementing sinkings and borings in different districts, at the recommendation of private individuals, who had their own more or less crude ideas as to "good indications," and where coal was likely to be found.

Thus, some one in the Western Port District, not qualified to give an opinion, but who happens to have lived in a mining township in Wales or Lancashire, imagines that the surface of the ground here reminds him of a spot he knew there where coal was found, and reports it as "good indications," where money should be expended by the Government.

Another—in Cape Otway or Wincheslea—who by chance may have been born in a coal-mining district in Scotland, sees a place which reminds him of his native country, and forthwith proclaims its "good indications," and a consequent appeal is made for Government funds.

A third imagines that if there are a few inches of coal in any rock in his district, a "good indication" exists to justify sinking through it in the expectation of finding a thicker seam below.

I have already pointed out the fallacy of such reasoning.

In another case, a borer or sinker passes through some black shale which reminds him of a similar deposit over a 6- or 10-foot coal in England, and therefore concludes that a like seam will be found under the black shale here.

I have, &c.,

JOHN MACKENZIE, F.G.S.,
Government Examiner of Coal Fields, N.S.W.

D. (p. 38.)

The extension of coal in this direction is rendered probable by the fact that a seam of coal crosses the Macintyre, about nine miles below Inverell, and is doubtless connected with the Warialda and Gragin country.

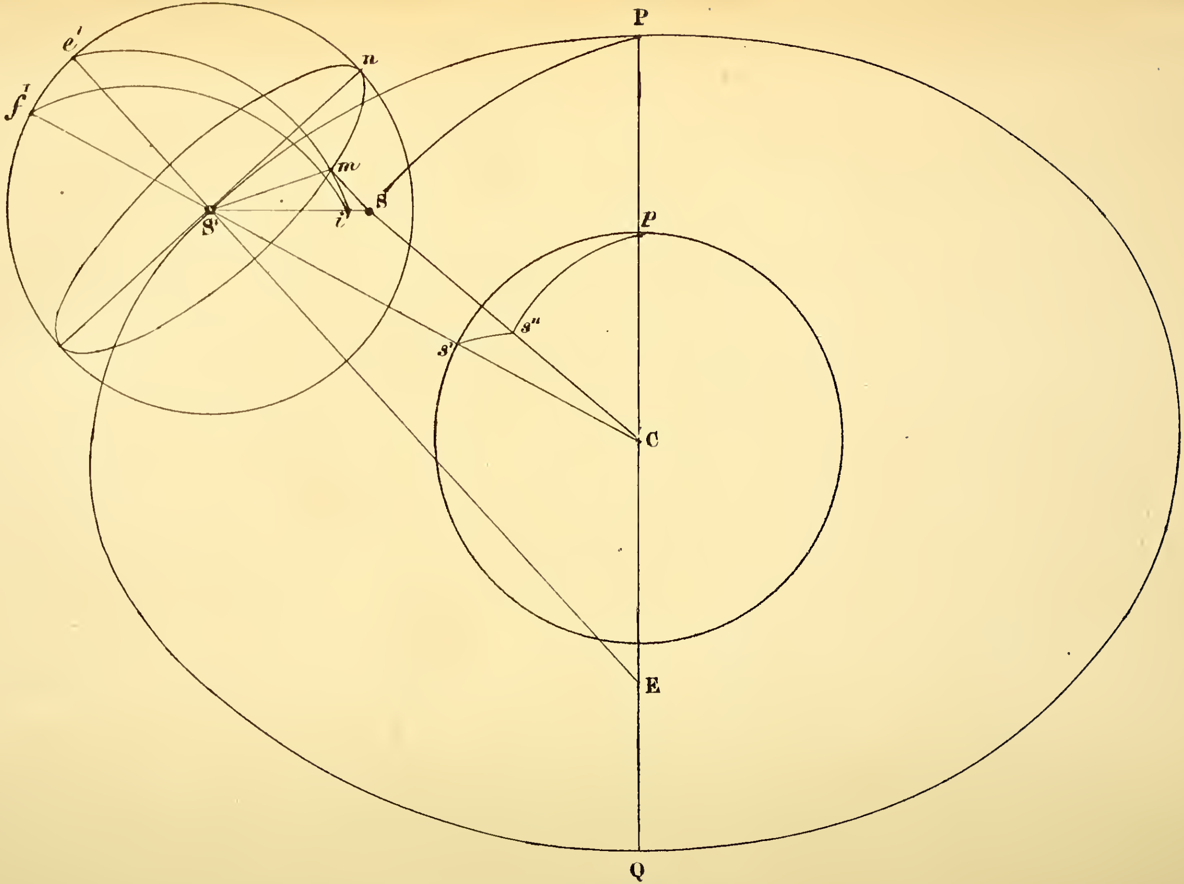

Figure accompanying "Geodesic Investigations".

Read before the Society, July 9th, 1873.

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GEODESIC INVESTIGATIONS.

A NEW and simple method of computing with precision the unknown entities:—latitudes, longitudes, azimuths, circular measure of spheroidal arc, angles between normal planes, and angles between geocentric radii and chord of arc—pertaining to the principal problem in Geodesic Surveying.

By MARTIN GARDINER, *C.E.*, *Member of the Mathematical Society of London.* [*Read before the Society*, 9 *July*, 1873.]

(*See plate.*)

GIVEN the lengths a, b, of the equatorial and polar radii of the earth, the geodesic distance d between two stations S', S'' on its spheroidal surface, the geographic latitude l' of the station S', and the geographic azimuth A' of the other station as taken from the station S', to find:—

1°. The circular measure Σ of the geodesic arc d, the length c of chord of this arc, and the angle which it subtends at the centre of the earth.

2°. The geocentric azimuth $A_{,}$ of the station S'' as if taken from S',—the dihedral angle i' between the two planes of the chord c which contain the normal at S' and the geocentric radius to S' respectively, — and the angle a' which the chord c makes with the geocentric radius to S'.

3°. The difference of longitude ω of the stations, and the geocentric azimuth $A_{,,}$ of the station S' as if taken from S'', — and also the geocentric and geographic latitudes λ'', l'' of the station S''.

4°. The geographic azimuth A'' of the station S' as if observed from the station S'', — the dihedral angle i'' of the two planes which contain the chord c and the normal, and the chord c and geocentric radius to the station S'', respectively.

5°. The angle $\triangle$ between the two planes, one of which contains the chord c and normal at S', and the other—the chord c and the normal at S''. (The two planes may be designated the "normal chordal planes.")

Before proceeding to the question, I consider it necessary (in order to render the investigation satisfactory throughout) to state the following preliminary theorems, for the proof of which the appended notes can be consulted.

1°. If $a_1, a_2, \ldots\ldots a_n$ be any number n of small and consecutively connected arcs forming one "geodesic" or shortest arc d between any two points on the spheroidal surface of the earth, and that $\rho_1, \rho_2, \ldots\ldots \rho_n, \rho_{n+1}$ are the lengths of the radii of curvature taken in order at the first extremities of the series of arcs and at the final extremity of the last of the series; then will the circular measure of the whole geodesic arc d be equal to the sum Σ of the n terms of the series.

$$\frac{2a_1}{\rho_1+\rho_2}, \quad \frac{2a_2}{\rho_2+\rho_3} \ldots\ldots\ldots\ldots \frac{2a_n}{\rho_n+\rho_{n+1}}.$$

And the radius $R=\frac{d}{\Sigma}$ is the mean radius of curvature of d.

2°. If a geodesic arc d connecting any two stations S', S'' on the spheroidal surface of the earth be not more than 60 miles in length, and that ρ', ρ'' are the radii of curvature of the arc at the points S', S'' respectively; then, if on the normal at either station we assume a centre whose distance from the station is equal to $\frac{\rho'+\rho''}{2}$, the circle described from such centre with $\frac{\rho'+\rho''}{2}$ as radius will pass through both stations S', S''.

And $\frac{d}{\rho'+\rho''}$ will be the circular measure of the angle between the chord connecting the stations and the straight line drawn from either station, in its tangent plane, to the foot of the perpendicular from the other station on such tangent plane.

3°. Having the length of a geodesic arc d on the spheroidal surface of the earth, and the geographic latitude l' of one extremity, and also the geographic azimuth of the other extremity as observed from the first,—there are simple, well-known methods of computing the latitude l'' of the second extremity to within one or two seconds of accuracy, and the azimuth of the first as taken from the second to within one second of accuracy.

Let C be the centre of the earth, PQ its polar axis, $PS'Q$, $PS''Q$ the geodetic meridians through the stations S', S'', and let E' be the point of intersection of the normal at S' with the polar axis. Let m be the point in which the line $S''m$ parallel to the normal $S'E'$ pierces the tangent plane to the spheroidal surface of the earth at S' (the plane of the horizon at S').

Conceive a sphere described having S' as centre and $S'\,m'$ as radius ; and let i' be the point in which the chord c connecting the stations pierces the surface of this sphere. Let $e'\ f'$ be the points in which its surface is pierced by the productions of the lines $E'\,S'$, CS' through S'; and let $S'\,n'$ be the trace of the meridian plane $PS'E'$ upon the tangent plane to the earth at S'.

It is evident that — the arcs $e'm'$, $e'n'$ are quadrants ; that arc $n'm'$, or its equivalent, the spherical angle $n'e'm'$ is equal to the given geographic azimuth A' ; that the arc $m'i'$ is equivalent to the angle between the lines $S'S''$, $S'm'$, or to half the circular measure Σ of the arc $S'S''$; that the arc $e'f'$ is the angle v' of the vertical at S' ; and that the angle $e'f'i'$ of the spherical triangle $e'f'i'$ is equivalent to the angle between the planes $CS'f'P$, $CS'f'i'S''$, or to the geocentric azimuth A_1.

Now we can easily compute the angle v' of the vertical at S' from the given latitude l' of the station S'.

And it is well known that by means of close approximate values for l', l'', A', A'', we can obtain extremely accurate values of the circular measure Σ of the arc d and of the length of its chord c. And therefore, in the spherical triangle $e'f'i'$ we may consider as known :—the side $e'f'$ (equal to the angle of the vertical v' as S'),— the side $e'i'$ (equal to $90°+\frac{1}{2}\,\Sigma$),—and the angle $f'e'i'$ (equal $180°—A'$). Hence, from this triangle we can find, by means of Napier's Analogies, the angles $e'f'i'$, $f'i'e'$ which are the respective values of $A_{,}$ and i' ; we can also find the side $f'i'$ which is the measure of the angle between the geocentric radius CS' and the chord c of the arc d.

It may be proper to observe here, that if we were given the geocentric azimuth $A_{,}$ instead of the geographic azimuth A', then we could obtain A', provided we had the arc $e'i'$, and arc $e'f'$: for in the spherical triangle we should have two sides and an angle opposite one of them, from which to find the supplement of the angle between the sides. In fact, a like case occurs when in the sequel we have to obtain A'' from a kindred spherical triangle, and it is for this reason the observation has been made in order that the process of investigation may be the more clearly comprehended.

Now conceive a sphere concentric with the earth, and let p, s', s'', be the points in which it is pierced by the central vectors CP, CS', CS''. It is evident that $s'ps''$ is a spherical triangle in which the side $s's''$ is equivalent to the angle that the geodesic arc d or its chord c subtends at the centre of the earth ; it is also evident that the side ps' is equal to the geocentric co-latitude of the station S', and that the angle $ps's''$ is equal to the geocentric azimuth of the station S'' as if taken from the station S''.

But having l' and a close approximate to l'' we can find the geocentric radius r' to S' and a close approximate to r'' the geocentric radius to S''; and therefore, since we may consider the comparatively short chord c as known accurately, we can find the angle which this chord subtends at the centre of the earth with an extreme degree of approximative accuracy. Hence, in the spherical triangle we may consider the sides $s'p$, $s's''$, and the angle between them as known, and obtain by Napier's Analogies the remaining parts, viz.:—The angle $ps''s'$ which is the geocentric azimuth $A_{''}$ of the station S' as taken from S'',—the angle $s'ps''$ which is the difference of longitude ω of the stations S', S'',—and the side ps'' which is the geocentric latitude λ'' of the station S''. We can also obtain, by a well-known formula, the geographic latitude l'' of the station S'' from its geocentric latitude λ'', and therefore the angle ν'' of the vertical at S'' which is equal to $l''-\lambda''$.

Now, having the angle ν'' of the vertical at S'', the geocentric azimuth $A_{''}$ of the station S' as if taken from S'', and the circular measure Σ of the geodesic arc d, it is evident, from an observation already made, that we can find i'', A'' and a'':—

For if from S' we let fall a perpendicular on the tangent plane at the station S'', and that we conceive a sphere described having S'' as centre and the distance to the foot of the perpendicular as radius, and that i'', e'', f'' are the points in which it is pierced by the chord c, the normal $S''E''$, and radius CS'' of the earth; then it is evident we have on its surface a spherical triangle $e''f''i''$ of kindred kind to the triangle $e'f'i'$ on the sphere having S' as centre, and that one side is $90° + \frac{1}{2}\Sigma$, another side ν'', and the angle opposite the first side supplement to $A_{''}$. Moreover, the three remaining parts are evidently the angle i'' opposite side ν'', the angle A'' between the sides ν'' and $90 + \frac{1}{2}\Sigma$, and the side a'' opposite the angle A''.

FORMULAE ARRANGED FOR CALCULATIONS.

For approximate values of l'', ω, and A''.

$$R_{,} = \frac{a}{(1-e^2 \sin^2 l')^{\frac{1}{2}}}$$ in which e is earth's excentricity.

$$l'' - l' = \left\{ \frac{d \cos A'}{R_{,} \sin 1''} - \frac{d^2 \sin^2 A' \tan l'}{2 R_{,} \sin 1''} \right\} (1 + e^2 \cos^2 l').$$

$$\omega = \frac{d \sin A'}{R_{,} \cos l'' \sin 1''}$$

$$A'' - A' = \omega \frac{\sin \frac{1}{2} (l'' + l')}{\cos \frac{1}{2} (l'' - l')}$$

For λ', ν', and approximate value of λ'' and ν''.

$$\tan \lambda' = \frac{b^2}{a^2} \tan l'$$

$$\tan \lambda'' = \frac{b^2}{a^2} \tan l'' \quad \text{approx.}$$

$$\nu' = l - \lambda'$$

$$\nu'' = l'' - \lambda'' \quad \text{approx.}$$

For the geocentric radii-rectors r', r'' to stations.

$$r' = \left\{ \frac{a^2 \cos l'}{\cos \lambda' \cos \nu'} \right\}^{\frac{1}{2}}$$

$$r'' = \left\{ \frac{a^2 \cos l''}{\cos \lambda'' \cos \nu''} \right\}^{\frac{1}{2}} \quad \text{approx.}$$

For the radii of curvature of the meridians at the stations.

$$R' = \frac{a (1 - e^2)}{(1 - e^2 \sin^2 l')^{\frac{3}{2}}}$$

$$R'' = \frac{a (1 - e^2)}{(1 - e^2 \sin^2 l'')^{\frac{3}{2}}} \quad \text{approx.}$$

For radii of curvature of arcs perpendicular to the meridians at the stations.

$$R_{,} = \frac{a}{(1-e^2 \sin^2 l')^{\frac{1}{2}}}$$

$$R_{,,} = \frac{a}{(1-e^2 \sin^2 l'')^{\frac{1}{2}}} \quad \text{approx.}$$

For the approximate radii of curvature of arc d at its extremities.

$$\rho' = \frac{R' \times R_{\prime}}{R' \sin^2 A' + R_{\prime} \cos^2 A'}$$

$$\rho'' = \frac{R'' \times R_{\prime\prime}}{R'' \sin^2 A'' + R_{\prime\prime} \cos^2 A''} \quad \text{approx.}$$

For the circular measure of arc d; the number of seconds contained in the circular measure of arc d; and the length of the chord of the arc.

$$\Sigma = \frac{2d}{\rho' + \rho''}$$

$$n'' = \Sigma \times 206264''{\cdot}80624$$

$$c = \frac{2d \sin \frac{\Sigma}{2}}{\Sigma}$$

Plane Triangle.

For the angle C which the chord c subtends at the earth's centre.

$$\text{Sin } \tfrac{1}{2} C = \left\{ \frac{(s-r')\ (s-r'')}{r'\ r''} \right\}^{\frac{1}{2}}$$

in which $s = \frac{1}{2} (r' + r'' + c)$

First Spherical Triangle.

For the angles $A_{\prime}$, i', a' respectively.

$$\tan \tfrac{1}{2} (A_{\prime} + i') = \frac{\cos \frac{1}{2} (\gamma' - \nu')}{\cos \frac{1}{2} (\gamma' + \nu')} \cot \tfrac{1}{2} V$$

$$\tan \tfrac{1}{2} (A_{\prime} - i') = \frac{\sin \frac{1}{2} (\gamma' - \nu')}{\sin \frac{1}{2} (\gamma' + \nu')} \cot \tfrac{1}{2} V$$

$$\cot \tfrac{1}{2} a' = \frac{\sin \frac{1}{2} (A_{\prime} + i')}{\sin \frac{1}{2} (A_{\prime} - i')} \tan \tfrac{1}{2} (\gamma' - \nu')$$

in which $\gamma' = 90° + \frac{1}{2} \Sigma$

and $\frac{1}{2} V = 90° - \frac{1}{2} A'$

Second Spherical Triangle.

For ω, $A_{''}$ and λ'' respectively; and also l'' and ν''

$$\tan \tfrac{1}{2} (A_{''} + \omega) = \frac{\cos \frac{1}{2} (\lambda - C_{'})}{\cos \frac{1}{2} (\lambda + C_{'})} \cot \tfrac{1}{2} A_{'}$$

$$\tan \tfrac{1}{2} (A_{''} - \omega) = \frac{\sin \frac{1}{2} (\lambda - C_{'})}{\sin \frac{1}{2} (\lambda + C_{'})} \cot \tfrac{1}{2} A_{'}$$

$$\tan \tfrac{1}{2} \lambda_{''} = \frac{\sin \frac{1}{2} (A_{''} + \omega)}{\sin \frac{1}{2} (A_{''} - \omega)} \tan \tfrac{1}{2} (\lambda - C_{'})$$

$$\lambda'' = 90^\circ - \lambda_{''}$$

$$\tan l'' = \frac{a^2}{b^2} \tan \lambda''$$

$$\nu'' = l'' - \lambda''$$

in which $\lambda = 90^\circ - \lambda'$

Third Spherical Triangle.

For i'', A'', a'', and the angle Δ between the normal planes at the stations, both of which contain the chord c of stations.

$$\sin i'' = \frac{\sin \nu'' \sin A_{''}}{2 \sin \frac{1}{2} \gamma'' \cos \frac{1}{2} \gamma''} = \frac{\sin \nu'' \sin \frac{1}{2} A_{''} \cos \frac{1}{2} A_{''}}{\sin \frac{1}{2} \gamma'' \cos \frac{1}{2} \gamma''}$$

$$\cot \tfrac{1}{2} A'' = \frac{\sin \frac{1}{2} (\gamma'' - \nu'')}{\sin \frac{1}{2} (\gamma'' + \nu'')} \cot \tfrac{1}{2} (A_{''} - i'')$$

$$\cot \tfrac{1}{2} a'' = \frac{\sin \frac{1}{2} (A_{''} + i'')}{\sin \frac{1}{2} (A_{''} - i'')} \tan \tfrac{1}{2} (\gamma'' - \nu'')$$

$$\text{angle } \Delta = i' - i''$$

in which $\gamma'' = 90^\circ + \frac{1}{2} \Sigma$

For testing the correctness of the calculations we have—

$$a' + a'' + C = 180°$$

☞ They are angles of one plane triangle, but obtained from two spherical triangles and one plane triangle.

Example.

Given :—$a = 20992639$; $b = 20852899$; $d = 316800$;
$l' = 48° \, 50' \, 48''{\cdot}590$; $A' = 46° \, 15' \, 36''{\cdot}970$

To find :—Σ , c , C , $A_{,}$, $A_{,,}$, A'' , ω , i , i'' , Δ
λ' , λ'' , l'' , ν' , ν'' .

We find :—$\log a = 7{\cdot}3206164$; $\log b = 7{\cdot}3191664$
$\log a^2 = 14{\cdot}6412329$; $\log b^2 = 14{\cdot}6383328$

$$\log e^2 = \bar{3}{\cdot}82317107$$

To arive at approximate values of l'' , ω , and A'' we have—

$$M = 1 + e^2 \cos^2 l'' \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$\log e^2 = \bar{3} \cdot 823117107$

$\log \cos^2 l' = \bar{1} \cdot 63654980$ ∴ $e^2 \cos^2 l' = 0{\cdot}002882178$

$\bar{3} \cdot 45972087$ $\quad 1 + e^2 \cos^2 l' = 1{\cdot}002882178 = M$

∴ $\log M = 0{\cdot}0012499$

$$k = \log \sin 1'' = \bar{6}{\cdot}6855749 \ldots \quad \ldots \quad \ldots \quad (2)$$

$$R_{,} = \frac{a}{\left\{1 - e^2 \sin^2 l'\right\}^{\frac{1}{2}}} \ldots \quad \ldots \quad \ldots \quad (3)$$

$\log e^2 = \bar{3}{\cdot}8231710$

$\log \sin^2 l' = \bar{1}{\cdot}7535358$ ∴ $e^2 \sin^2 l' = 0{\cdot}0037732$

$\bar{3}{\cdot}5767068$ $\quad 1 - e^2 \sin^2 l' = 0{\cdot}9962268$

$\log a = 7{\cdot}3206164$

$\log (1 - e^2 \sin^2 l')^{\frac{1}{2}} = \bar{1}{\cdot}9991791$ ∴ $\log R_{,} = 7{\cdot}3214373$

∴ $\log R_{,} = 7{\cdot}3214373$

$$l'' - l' = \left(\frac{d \cos A'}{k \cdot R_{,}} - \frac{d^2 \sin^2 A' \tan l'}{2\, k \cdot R_{,}^2} \right) \cdot M \quad \ldots \quad (4)$$

$$\begin{aligned}
\log d &= 5\cdot5007851 \\
\log \cos A' &= \bar{1}\cdot8397190 \\
\log M &= 0\cdot0012499 \\
\text{co} \cdot \log R_{,} &= \bar{8}\cdot6785627 \\
\text{co} \cdot \log k &= 5\cdot3144251 \\
\hline
&\ 3\cdot3347418 \\
\text{cor. num.} &= 2161''\cdot4338
\end{aligned}$$

$$\begin{aligned}
\log d^2 &= 11\cdot0015703 \\
\log \sin^2 A' &= \bar{1}\cdot7176611 \\
\log \tan l' &= 0\cdot0584930 \\
\log M &= 0\cdot0012499 \\
\text{co} \cdot \log R_{,}^2 &= \overline{15}\cdot3571253 \\
\text{co} \cdot \log k &= 5\cdot3144251 \\
\text{co} \cdot \log 2 &= \bar{1}\cdot6989700 \\
\hline
&\ 1\cdot1194947 \\
\text{cor. num.} &= 14''\cdot1089
\end{aligned}$$

$$\begin{aligned}
\therefore\ l'' - l' &= 2161''\cdot4338 - 14''\cdot1089 \\
&= 2147''\cdot3249 \\
&= 0^\circ\ 35'\ 47''\cdot3249 \\
\therefore\ l'' &= 49^\circ\ 26'\ 35''\cdot9149 \text{ approx.}
\end{aligned}$$

$$\omega = \frac{d \sin A'}{R_{,} k \cos l''} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$\begin{aligned}
\log d &= 5\cdot5007851 \\
\log \sin A' &= \bar{1}\cdot8588306 \\
\hline
&\ 5\cdot3596157 \\
&\ 1\cdot8200593 \\
\hline
\therefore \text{ log of seconds in } \omega &= 3\cdot5395564
\end{aligned}$$

$$\begin{aligned}
\log R_{,} &= 7\cdot3214373 \\
\log \cos l'' &= \bar{1}\cdot8130471 \\
\log k &= \bar{6}\cdot6855749 \\
\hline
&\ 1\cdot8200593
\end{aligned}$$

$$\begin{aligned}
\therefore\ \omega &= 3463''\cdot8288 \\
&= 0^\circ\ 57'\ 43''\cdot8288 \text{ approx.}
\end{aligned}$$

$$\text{Supplement of } A'' - A' = \omega \frac{\sin \frac{1}{2}(l'' + l')}{\cos \frac{1}{2}(l'' - l')} \quad \ldots \quad (6)$$

$$\begin{aligned}
\log \omega &= 3\cdot5395564 \\
\log \sin \tfrac{1}{2}(l'' + l') &= \bar{1}\cdot8787333 \\
\hline
&\ 3\cdot4182897 \\
\log \cos \tfrac{1}{2}(l'' - l') &= \bar{1}\cdot9999942 \\
\hline
\therefore\ \log (A'' - A') &= 3\cdot4182955
\end{aligned}$$

$$\begin{aligned}
\therefore\ A'' - A' &= 2619''\cdot9651 \\
&= 0^\circ\ 43'\ 39''\cdot9651 \\
\therefore\ A'' &= 133^\circ\ 0'\ 43''\cdot0649 \text{ approx.}
\end{aligned}$$

With the given values of l', A', a, b, and e,
and the approx. values of l'' and A'', we find (either from tables which are constructed for the purpose, or from well known formulæ) :—

The geocentric latitude	$\lambda' =$	48° $39'$ $25''\cdot8018$	
The angle of the vertical	$\nu' =$	0° 11 $22''\cdot7882$	
The geocentric radius	$r' =$	20,883,240.	
The geocentric radius	$r'' =$	20,882,524.	approx.
The radius of curvature	$\rho' =$	20,933,190.	at station S'.
The radius of curvature	$\rho'' =$	20,935,360.	at station S''. (approx.)

$$\Sigma = \frac{2\,d}{\rho' + \rho''}$$
$$= \frac{633{,}600.}{41{,}868{,}550.}$$
$$= 0\cdot01{,}513{,}308 \text{ circular measure of } d.$$

The number of seconds in Σ is given by—

$$\Sigma'' = \Sigma \times 206264''\cdot80624$$
$$= 3121''\cdot4218$$

$\therefore\ \Sigma = 0^\circ\ 52'\ 1''\cdot4218$ angular measure.
$\frac{1}{2}\ \Sigma = 0^\circ\ 26'\ 0''\cdot711$ nearly.

$$\text{chord C} = \frac{2d \sin, \frac{1}{2}\Sigma}{\Sigma}$$

$\log 2d = 5\cdot8018152$

$\log \sin \frac{1}{2}\Sigma = \bar{3}\cdot8788931$ $\qquad \therefore c = 316796\cdot93$

$3\cdot6807083$ $\qquad = 316797\cdot$ nearly.

$\log \Sigma = \bar{2}\cdot1799273$

$\therefore \log c = 5\cdot5007810$ ☞ The ver sin of d is about 600 feet.

Plane Triangle.

$$\text{Sin}\ \tfrac{1}{2}\ C = \left\{\frac{(s-r').\ (s-r'')}{r'.r''}\right\}^{\frac{1}{2}}$$

$r' = 20883240$
$r'' = 20882524$
$c = 316797$

$42082561 \div 2$

$s = 21041280\cdot5$

$s-r' = 158040\cdot5$
$s-r'' = 158756\cdot5$

$\log (s-r') = 5\cdot1987683$
$\log (s-r'') = 5\cdot2007154$
$\text{co.} \log (r'-r'') = 5\cdot3604191$

$15\cdot7599028 \div 2$

$\therefore \log \sin \frac{1}{2}\ C = 7\cdot8799514$

$\therefore \frac{1}{2}\ C = 0^\circ\ 26'\ 4''\cdot518$
$C = 0^\circ\ 52'\ 9''\cdot036$

First Spherical Triangle.

$\gamma' = 90° \;\; 26' \;\; 0''{\cdot}711 \qquad A' = 46° \;\; 15' \;\; 36''{\cdot}970$, and its supp
$\nu' = 0° \;\; 11' \;\; 22''{\cdot}788 \qquad V = 133° \;\; 44' \;\; 23''{\cdot}030$,

$$\tfrac{1}{2}\, V = 66° \;\; 52' \;\; 11''{\cdot}515$$
$$\tfrac{1}{2}\,(\gamma'-\nu') = 45° \;\; 7' \;\; 18''{\cdot}961$$
$$\tfrac{1}{2}\,(\gamma'+\nu') = 45° \;\; 18' \;\; 41''{\cdot}749$$

$$\tan \tfrac{1}{2}\,(A' + i') = \frac{\cos \tfrac{1}{2}\,(\gamma'-\nu')}{\cos \tfrac{1}{2}\,(\gamma'+\nu')} \cot \tfrac{1}{2}\, V$$

log cot $\frac{1}{2}$ V = 9·6305884
log cos $\frac{1}{2}$ $(\gamma'-\nu')$ = 9·8485587

19·4791471 $\quad \therefore \tfrac{1}{2}\,(A+i') = 23°$ „ $11'$ „ $57''{\cdot}281$
log cos $\frac{1}{2}$ $(\gamma'+\nu')$ = 9·8471102

9·6320369

$$\tan \tfrac{1}{2}\,(A-i') = \frac{\sin \tfrac{1}{2}\,(\gamma'-\nu')}{\sin \tfrac{1}{2}\,(\gamma'+\nu')} \cot \tfrac{1}{2}\, V.$$

log cot $\frac{1}{2}$ V = 9·6305884
log sin $\frac{1}{2}$ $(\gamma'-\nu')$ = 9·8504073

19·4809957 $\quad \therefore \tfrac{1}{2}\,(A_{,}-i') = 23°\; 3'\; 43''{\cdot}972$
log sin $\frac{1}{2}$ $(\gamma'+\nu')$ = 9·8518340

9·6291617 $\quad \therefore A_{,} = 46°\; 15'\; 41''{\cdot}253$
$\therefore i' = 0°\; 8'\; 13''{\cdot}309$

$$\cot \tfrac{1}{2}\, a' = \frac{\sin \tfrac{1}{2}\,(A_{,}+i')}{\sin \tfrac{1}{2}\,(A_{,}-i')} \tan \tfrac{1}{2}\,(\gamma'-\nu')$$

log tan $\frac{1}{2}$ $(\gamma'-\nu')$ = 10·0018485
log sin $\frac{1}{2}$ $(A_{,}+i')$ = 9·5954188

19·5972673 $\quad \therefore \tfrac{1}{2}\, a' = 44°\; 43'\; 3''{\cdot}634$
log sin $\frac{1}{2}$ $(A_{,}-i')$ = 9·5929873

10·0042800 $\quad \therefore a' = 89°\; 26'\; 7''{\cdot}268$

Second Spherical Triangle.

$\tfrac{1}{2}\, A_{,} = 23° \;\; 7' \;\; 50''{\cdot}626$;
$\tfrac{1}{2}\,(\lambda_{,}-C) = 20° \;\; 14' \;\; 12''{\cdot}581$; $\qquad \lambda_{,} = 90°-\lambda' = 41°\; 20'\; 34''{\cdot}198$;
$\tfrac{1}{2}\,(\lambda_{,}+C) = 21° \;\; 6' \;\; 21''{\cdot}617$; $\qquad C = 0° \;\; 52' \;\; 9''{\cdot}036$

$$\tan \tfrac{1}{2}\,(A_{,,}+\omega) = \frac{\cos \tfrac{1}{2}\,(\lambda_{,}-C)}{\cos \tfrac{1}{2}\,(\lambda_{,}+C)} \cot \tfrac{1}{2}\, A_{,}\,.$$

log cot $\frac{1}{2}$ $A_{,}$ = 10·3693990
log cos $\frac{1}{2}$ $(\lambda_{,}-C)$ = 9·9723282

20·3417272 $\quad \therefore \tfrac{1}{2}\,(A_{,,} + \omega) = 66°\; 59'\; 15''{\cdot}017$
log cos $\frac{1}{2}$ $(\lambda_{,}+C)$ = 9·9698424

10·3718848

$$\tan \tfrac{1}{2}(A_{\prime\prime}-\omega)=\frac{\sin \frac{1}{2}(\lambda_{\prime}-C)}{\sin \frac{1}{2}(\lambda_{\prime}+C)} \cot \tfrac{1}{2} A_{\prime}$$

$$\begin{array}{ll}
\log \cot \frac{1}{2} A_{\prime} & = 10 \cdot 3693990 \\
\log \sin \frac{1}{2}(\lambda-C) & = 9 \cdot 5389522 \\
\hline
 & 19 \cdot 9083512 \qquad \therefore \tfrac{1}{2}(A_{\prime\prime}-\omega)=66^{\circ}\ 1'\ 31'' \cdot 338 \\
\log \sin \frac{1}{2}(\lambda_{\prime}+C) & = 9 \cdot 5564166 \\
\hline
 & 10 \cdot 3519346 \qquad \therefore A_{\prime\prime}=133^{\circ}\ 0'\ 46'' \cdot 355
\end{array}$$

And diff. of longitude $\omega=0^{\circ}\ 57'\ 43'' \cdot 679$

$$\tan \tfrac{1}{2} \lambda_{\prime\prime}=\frac{\sin \frac{1}{2}(A_{\prime\prime}+\omega)}{\sin \frac{1}{2}(A_{\prime\prime}-\omega)} \tan \tfrac{1}{2}(\lambda_{\prime}-C).$$

$$\begin{array}{ll}
\log \tan \frac{1}{2}(\lambda_{\prime}-C) & = 9 \cdot 5666240 \\
\log \sin \frac{1}{2}(A_{\prime\prime}+\omega) & = 9 \cdot 9639858 \qquad \therefore \tfrac{1}{2} \lambda_{\prime\prime}=20^{\circ}\ 22'\ 22'' \cdot 574 \\
\hline
 & 19 \cdot 5306098 \qquad \lambda_{\prime\prime}=40^{\circ}\ 44'\ 45'' \cdot 147 \\
\log \sin \frac{1}{2}(A_{\prime\prime}-\omega) & = 9 \cdot 9608157 \\
\hline
 & 9 \cdot 5697941 \qquad \therefore \lambda''=90^{\circ}-\lambda_{\prime\prime}=49^{\circ}\ 15'\ 14'' \cdot 853
\end{array}$$

$$\tan l''=\frac{a^{2}}{b^{2}} \tan \lambda''$$

$$\begin{array}{ll}
\log \tan \lambda'' & = 10 \cdot 0647298 \\
\log a^{2} & = 14 \cdot 6412329 \\
\hline
 & 24 \cdot 7059627 \qquad \therefore l''=49^{\circ}\ 26'\ 35'' \cdot 634 \\
\log b^{2} & = 14 \cdot 6383328 \\
\hline
 & 10 \cdot 0676299
\end{array}$$

$$\therefore \nu''=l''-\lambda''=0^{\circ}\ 11'\ 20'' \cdot 781$$

Third Spherical Triangle.

$$\sin i''=\frac{\sin \nu'' \cdot \sin A_{\prime\prime}}{2 \sin \frac{1}{2} \gamma'' \cos \frac{1}{2} \gamma''}$$

$$\begin{array}{ll}
\log \sin \nu' & = 7 \cdot 5185809 \\
\log \sin A_{\prime\prime} & = 9 \cdot 8640364 \\
\hline
 & 17 \cdot 3826173 \\
 & 9 \cdot 9999875 \\
\hline
\therefore \log \sin i'' & = 7 \cdot 3826298\ ;
\end{array}
\qquad
\begin{array}{ll}
\log 2 & = 0 \cdot 3010300 \\
\log \sin \frac{1}{2} \gamma'' & = 9 \cdot 8511218 \\
\log \cos \frac{1}{2} \gamma'' & = 9 \cdot 8478357 \\
\hline
 & 9 \cdot 9999875 \\
 & \therefore i''=0^{\circ}\ 8'\ 17'' \cdot 800
\end{array}$$

$$\cot \tfrac{1}{2} A''=\frac{\sin \frac{1}{2}(\gamma''-\nu'')}{\sin \frac{1}{2}(\gamma''+\nu'')} \cot \tfrac{1}{2}(A_{\prime\prime}-i'')$$

$$\begin{array}{ll}
\log \cot \frac{1}{2}(A_{\prime\prime}-i'') & = 9 \cdot 6396002 \\
\log \sin \frac{1}{2}(\gamma''-\nu'') & = 9 \cdot 8504094 \\
\hline
 & 19 \cdot 4900096 \qquad \therefore \tfrac{1}{2} A''=66^{\circ}\ 30'\ 21'' \cdot 5625 \\
\log \sin \frac{1}{2}(\gamma''+\nu'') & = 9 \cdot 8518319 \\
\hline
 & 9 \cdot 6381777 \qquad \therefore A''=133^{\circ}\ 0'\ 43'' \cdot 125
\end{array}$$

$$\cot \tfrac{1}{2} a'' = \frac{\sin \tfrac{1}{2} (A_{\prime\prime}+i'')}{\sin \tfrac{1}{2} (A_{\prime\prime}-i'')} \tan \tfrac{1}{2} (\gamma''-\nu'')$$

$$\begin{array}{lrl}
\log \tan \tfrac{1}{2} (\gamma''-\nu'') = & 10{\cdot}0018527 & \\
\log \sin \tfrac{1}{2} (A_{\prime\prime}+i'') = & 9{\cdot}9626464 & \\
\hline
 & 19{\cdot}9644991 & \therefore \tfrac{1}{2} a'' = 44^\circ \quad 50' \quad 51''{\cdot}853 \\
\log \sin \tfrac{1}{2} (A_{\prime\prime}-i'') = & 9{\cdot}9621908 & \\
\hline
 & 10{\cdot}0023083 & \therefore a'' = 89^\circ \quad 41' \quad 43''{\cdot}706
\end{array}$$

$$\begin{aligned}
\Delta &= i'' - i'' = 0^\circ \quad 8' \quad 17''{\cdot}800 - 0^\circ \quad 8' \quad 13''{\cdot}309 \\
&= 0^\circ \quad 0' \quad 4''{\cdot}491 \\
&= \text{The angle between the “\textit{Normal Chordal planes}.”}
\end{aligned}$$

☞ From this it is evident that the normals (at S' and S'') at their nearest points (points $\frac{1}{2}$ $(\rho'+\rho'')$ distant from the respective stations) have their distance asunder denoted by

$$\begin{aligned}
D &= \frac{(\rho'+\rho''-\text{versin } d)}{2} \times \sin 4''{\cdot}491 \\
&= 20933670 \times {\cdot}00002177298 \\
&= 455{\cdot}78 \text{ feet.}
\end{aligned}$$

Test of accuracy of calculations :—

The angle C which the chord subtends at the centre of earth has been obtained from the plane triangle, and the angles a', a'' which the chord makes with the central radii to its extremities have been obtained from the first and third spherical triangles respectively, so that, if the work be correct, the sum of the three must be equal 180°.

$$\begin{array}{rl}
\text{We have } C = & 0^\circ \quad 52' \quad 9''{\cdot}036 \\
a' = & 89^\circ \quad 26' \quad 7''{\cdot}268 \\
a'' = & 89^\circ \quad 41' \quad 43''{\cdot}706 \\
\hline
 & 180^\circ \quad 0' \quad 0''{\cdot}010
\end{array}$$

And this is very satisfactory, considering that the tables used are to seven places of decimals only.

The accuracy of the work may also be tested by “Dalby's Theorem,” which gives results very near to rigorous accuracy :—

$$\tan \tfrac{1}{2} \omega = \frac{\cos \tfrac{1}{2} (l''-l')}{\sin \tfrac{1}{2} (l''+l')} \cot \tfrac{1}{2} (A''+A')$$

$$\begin{array}{lrl}
\log \cot \tfrac{1}{2} (A''+A') = & 7{\cdot}8028362 & \\
\log \cos \tfrac{1}{2} (l''-l') = & 9{\cdot}9999940 & \\
\hline
 & 17{\cdot}8028302 & \therefore \tfrac{1}{2} \omega = 0^\circ \quad 28' \quad 51''{\cdot}857 \\
\log \sin \tfrac{1}{2} (l''+l') = & 9{\cdot}8787330 & \\
\hline
 & 7{\cdot}9240972 & \omega = 0^\circ \quad 57' \quad 43''{\cdot}714
\end{array}$$

which differs only by about $\frac{3}{100}$ of a second from the result already obtained for the difference of longitude of the stations.

Notes.

Assuming the earth to be an oblate spheroid, of which the axis of revolution is the polar diameter, and that we have the values of a, b, its polar and equatorial radii, and those of the given data with all attainable accuracy, I propose to show the assumptions on which the investigations are based, to be compatible with the most rigorous requirements in the actual practice of trigonometrical surveying.

1°. First, then, I may observe that when the geodesic arc d is not over 60 miles in length the approximate values of l'' and A'' derived from the formulæ recommended by Oliver Byrne, C.E., in his treatise on Geodesy in "Bohn's Dictionary of Engineering" (which I have used in preference to all others, as giving the nearest approximations hitherto attainable), are equal in every respect to the absolutely rigorous values in their applications to the determination of Σ and c the circular measure and chord of the geodesic arc d, and that this would be the case even were their differences from the true values of l'' and A'' twice as large as they really are. We can easily prove this when the stations S', S'' are on one meridian, by means of plane analytic geometry; but when, as is generally the case, the stations are not so situated, it is necessary to elucidate the relations which subsist between radii of curvature on the spheroidal surface of the earth. This may perhaps be effected by the following observations, without entering into the extensive calculations necessary in order to afford a rigorous test. Although any two normals to the spheroidal figure of the earth will cut each other only when the stations are either both on one meridian or on a parallel of latitude, it can nevertheless be clearly inferred that $\frac{2d}{\rho' + \rho''}$ is an extremely close approximate to the circular measure Σ of a geodesic arc d not over 60 miles in length, and the more so the greater the difference in longitudes of the stations S', S''. In the case in which d is part of a meridian, the angle made by the normals or radii of curvature at its extremities is at once attainable as the difference of the geographic latitudes of the extremities. But when the geodesic arc d is not a portion of one meridian, we have a means of computing the rigorously correct value of Σ supplied to us by the higher calculus. We can form the equation of the spheroidal surface of the earth referred to rectangular axes, making the centre the origin and the polar diameter the axis of z, and find expressions for the co-ordinates of the stations S', S'' in terms of their differences of latitude and longitude and the equatorial and polar radii. We can also form the differential equations expressing the circular measure $d\Sigma$ between two consecutive normals to any geodesic arc d which is not a meridian (thus assuming the arcs $a_1, a_2, \ldots a_n$, into which we conceive d divided to be infinitely small, of the *first* order); and we can then integrate between the limits or extremities of d for the total circular measure Σ of the arc d. That we shall, by such means, obtain the exact value of Σ is evident; and for the method of integrating the differential equations Salmon's "Geometry of Three Dimensions" can be consulted.

From this and the fact that the approximate expression for Σ holds even in the case in which S' and S'' are on one meridian (in which case the difference of curvature of the extremities of d is a maximum) it should be evident that the greater the difference of longitude of S' and S'' the nearer to absolute accuracy will be the expression $\frac{2d}{\rho'+\rho'}$ for Σ.

However, if we be content with approximations to accuracy equal to those by which the sides of the triangles have been obtained, and that we wish to keep our results in strict conformity with the lengths of such sides, then it must be admitted that the value $\frac{2d}{\rho'+\rho''}$ for Σ is preferable to the absolutely rigorous value were such rigorous value to give results whose differences from those obtained by means of the approximate one could be appreciated in practice: but such is not the case. And for like reasons, it is evident that $\frac{1}{2}\Sigma$ is the proper value for the circular measure of the angle which the chord c of the arc d makes with the tangent plane at either of the stations.

2°. The angles at the centre of the earth C as obtained from the chord c, and central radii r', r'', can be easily proved to be correct to the thousand part of a second, even though r'' may have been computed from a value of l'' differing by $1''$ from the correct value.

3°. The method of solution has a great advantage in supplying a rigorous test to the accuracy of all the calculations, inasmuch as the magnitudes of the three angles of the triangle, whose base is the chord c and opposite vertex the centre of the earth, are found from three different triangles—one of which is the plane triangle itself, and the two others the principal spherical triangles used in obtaining the most important entities. In the example, which I have worked out, it may be seen that the sum of the three angles is about $\frac{1}{100}$ of a second over 180°; but this is owing to the tables of logs being carried out to seven places of decimals only, when tables of ten places of decimals are necessary in order to find results correctly to $\frac{1}{100}$ part of a second. However, the application of the test is sufficient to prove that the computed latitudes, azimuths, angles of normal planes, &c., have been obtained as correctly as the sides of the triangles in the most accurate trigonometrical surveys.

The second test applied to the correctness of the work would be sufficient to show the calculated results to be *close approximates* to rigorous accuracy, had the method of solution itself not furnished an easier and more rigorous test.

I may mention that Dalby's Theorem is retained in "The Account of the Principal Triangulation of Great Britain and Ireland," and submitted as a most important formula, with two proofs of its approximation to truth, and historical notes of a curious character.

It is there stated that, in applying it to the determination of the difference of longitude of Beachy Head and Dunnose, it gave results at variance with those obtained otherwise.

That it would be likely to give such a result in that case is evident—for when the deviations of the plumb-lines at the stations is such as to affect the azimuthal angles separately in a greater degree than it affects the *sum* of the azimuths, the like paradox will present itself, whether the latitudes have been correctly obtained or not. However, Dalby's theorem is a close test to the accuracy of calculations for longitude, made by using correct data.

4°. From the example in which d is assumed as 60 miles, it appears that the $\triangle$ between the "normal chordal planes" is $4''\cdot491$. And assuming one of the stations to be 4,000 feet higher than the other; then since 4,000 multiplied by the sine of $4''\cdot491$ $=\cdot087$ feet, and that this fraction of a foot subtends but a very small fraction of a second, we may consider the traces of the "normal chordal planes" on the earth's surface as one and the same trace. However, this small angle causes the shortest distance of the normals from each other to be $455\cdot78$ feet$=\left\{\frac{1}{2}(\rho'+\rho'')-\text{versin } d\right\}\cdot\sin\triangle$.

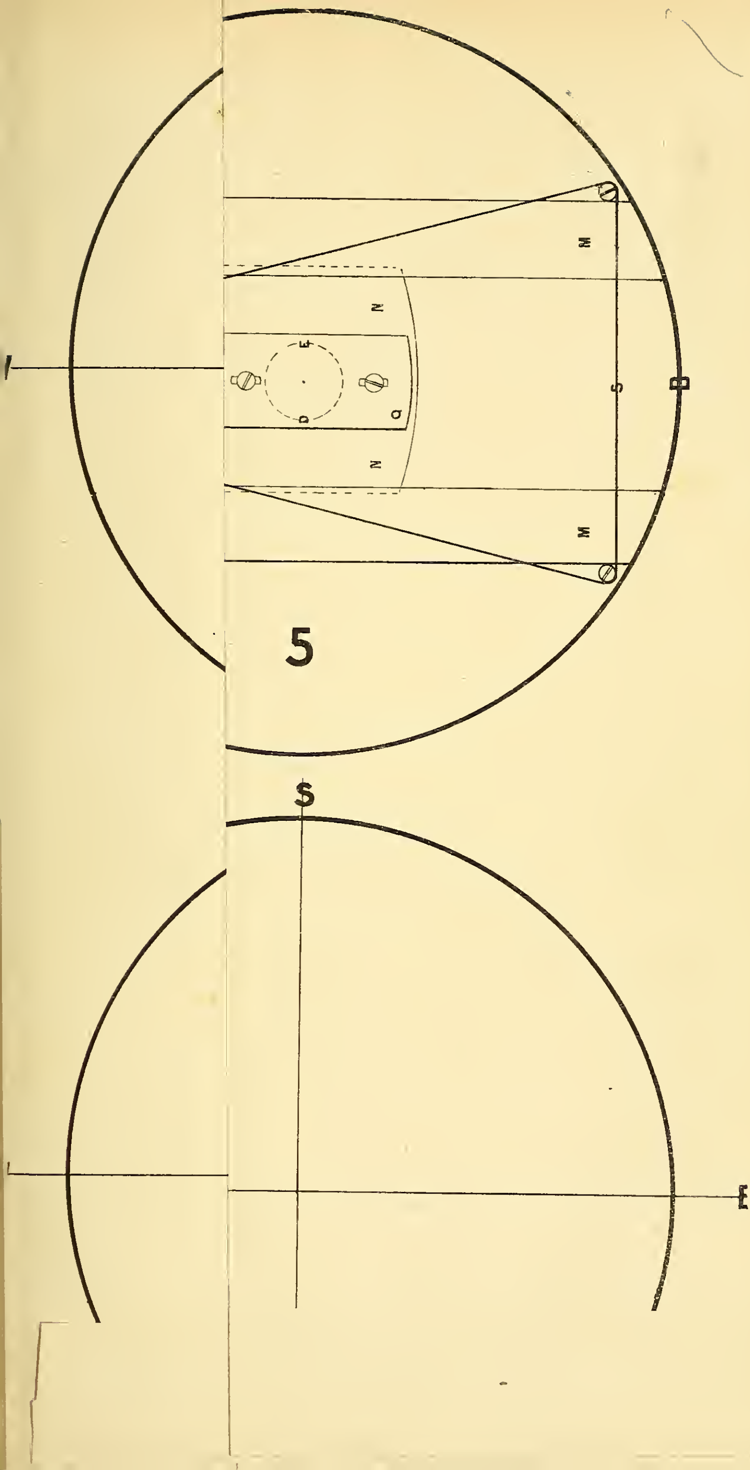
5°. The method of investigation is no doubt of a very elementary character when compared with the analysis usually employed in treating such questions; but, for this reason, it affords a clear view of the various relations of the involved entities, and leads to more elegant formulae (as a natural consequence) which can be worked with great precision. I need scarcely mention that the utmost precision is absolutely necessary: for if the relative latitudes, longitudes and azimuths of the principal stations of a trigonometrical survey be not attainable with at least equal precision to that of the lengths of the sides of the triangles, their positions cannot be geographically defined with accuracy. This will appear evident when we consider that an error of one second in either latitude or longitude would misrepresent the position of a station by something close on a hundred feet,—an amount of error that would not be tolerated in the length of one of the computed sides of the loosest triangulation.

It is well known that the resultant latitudes and longitudes of stations obtained by means of calculation, and by direct astronomical observations are generally discordant:—at some station; differing by hundreds of feet, and at others by less amounts; but scarcely ever agreeing. It has been demonstrated by Pratt, in his treatise "On the Figure of the Earth," that in some instances the error in the length or amplitude of an arc got from astronomical observations is such as to exceed $\frac{3}{10}$ of a mile in about 600 miles. He has computed, by means of the laws of attraction, that at some stations of the Trigonometrical Survey of India, the plumb-line deviates from the vertical or normal due to the spheroidal figure of the earth by as much as 6″, 9″, 10″ and 19″. And by applying the computed corrections to the observed angles, the geographical positions of the stations astronomically obtained were brought more into harmony with the positions as computed from those in more favourable localities for observations. But (notwithstanding laborious calculations), by taking into account only the visible disturbing forces in the vicinity of the stations—such as ranges of mountains and sea coast—there were discrepancies which were accounted for by assuming unequal local attraction in the spheroidal earth; and attributing it to the diversity of density of the matter adjacent to the stations.

Such being the case, it is evident that astronomical observations for latitudes or longitudes should be taken only in favourable localities, in order to get a near approximate to the absolute geographic positions of a few of the principal stations; and that with such as starting points or data, all the other stations should have their positions computed with the greatest accuracy possible.

Admitting that the absolute position of one of the stations is correctly obtained by astronomical observation, it is evident we can choose various strings of connected sides of triangles issuing from such station, and so obtain by alternations of the data, computed expressions for the same desired magnitudes; and then, by means of equations of conditions, eliminate any discrepancies due to imperfect readings of the measured angles between the sides of the triangles. But in carrying out the calculations for any such string of sides, care should be taken to use only the angles which every two of the stations subtend at the intermediate station as actually measured by the altitude and azimuth instrument,—subject merely to corrections due to attractions of mountains, and to defect of attraction of the sea coast,—for otherwise, we could not, by mere addition and subtraction of angles, obtain the geographic azimuths of one of the sides from that of the other at their point of junction, so as to get by calculation the relative positions of all the stations from that of the first.

However, in order to obtain complete sets of equations of conditions, it is necessary that we should have means of computing the azimuths (geocentric and geographic) of disconnected stations; and at some future time, I intend to submit solutions of this problem.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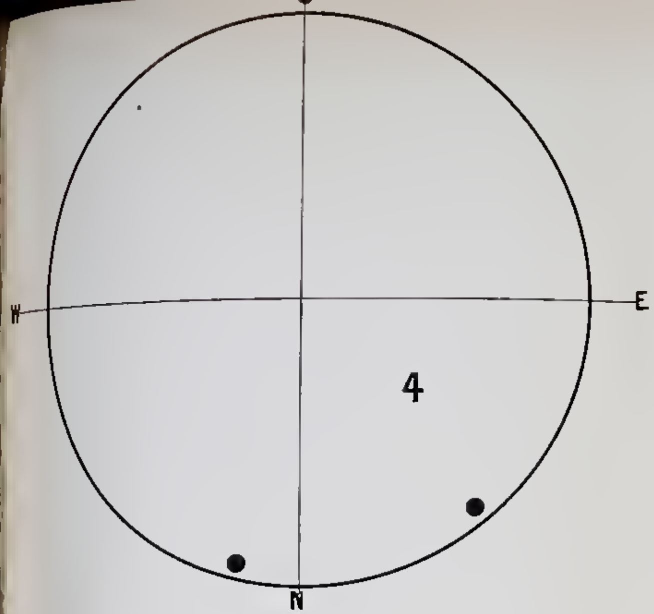
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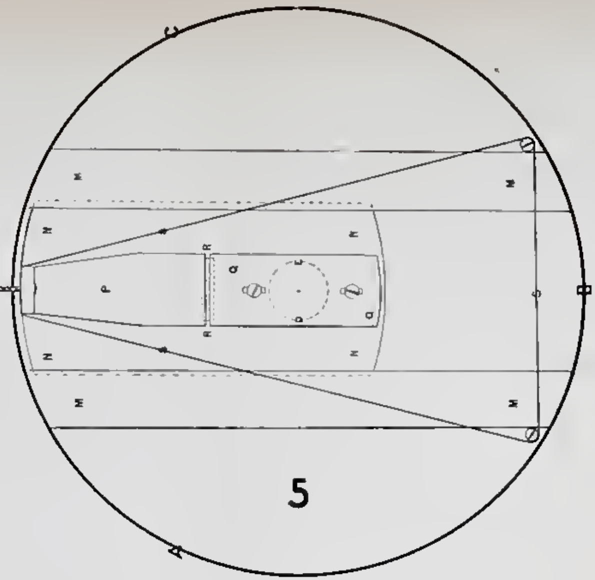
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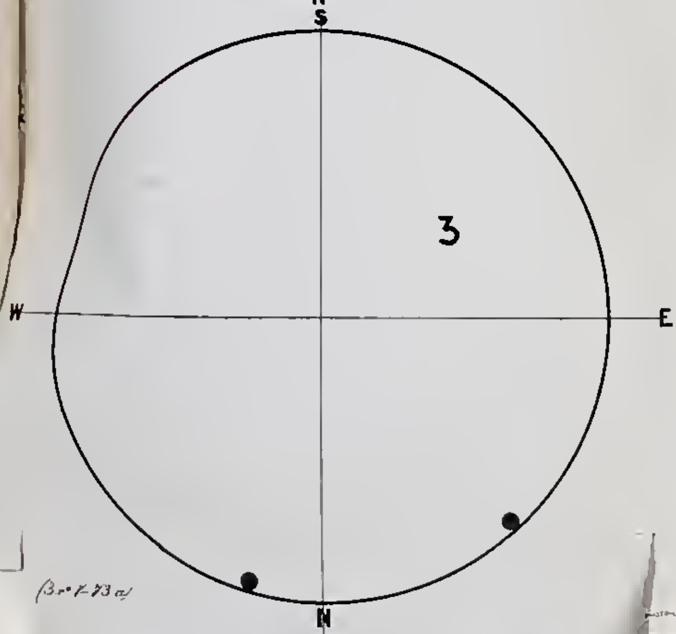
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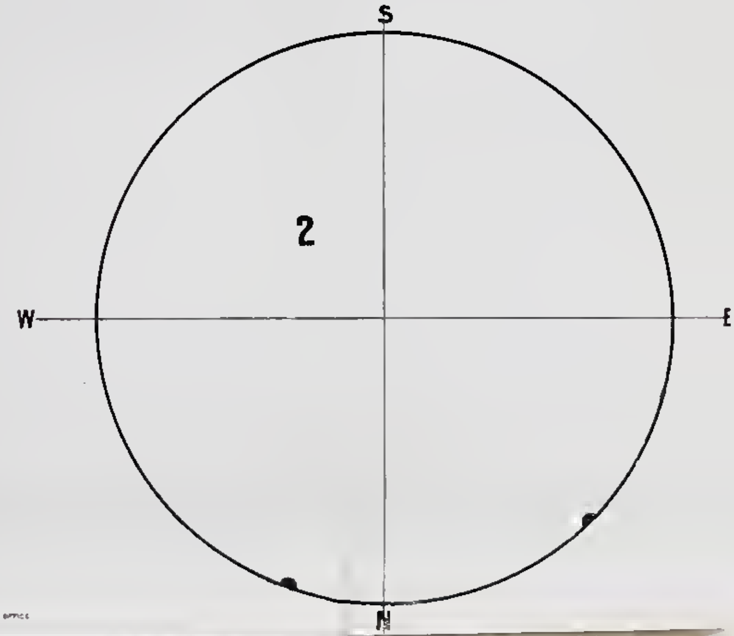
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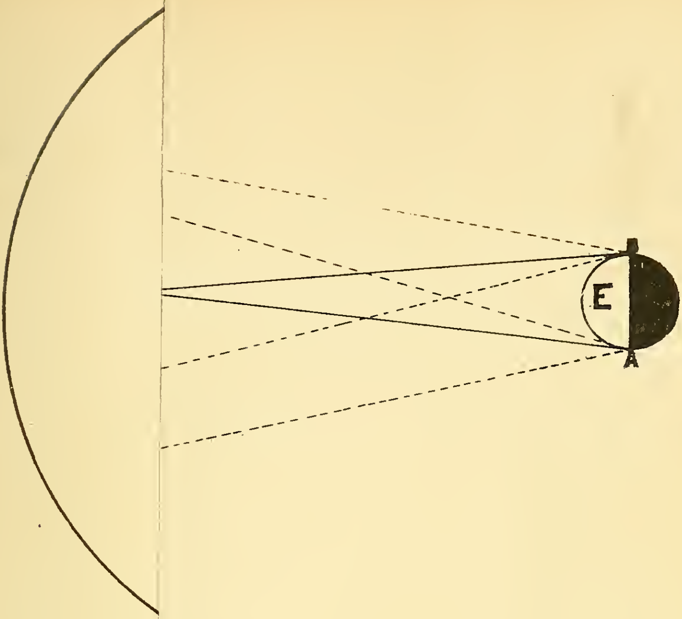
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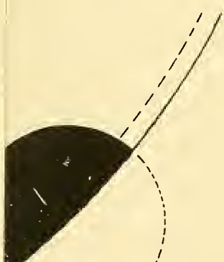

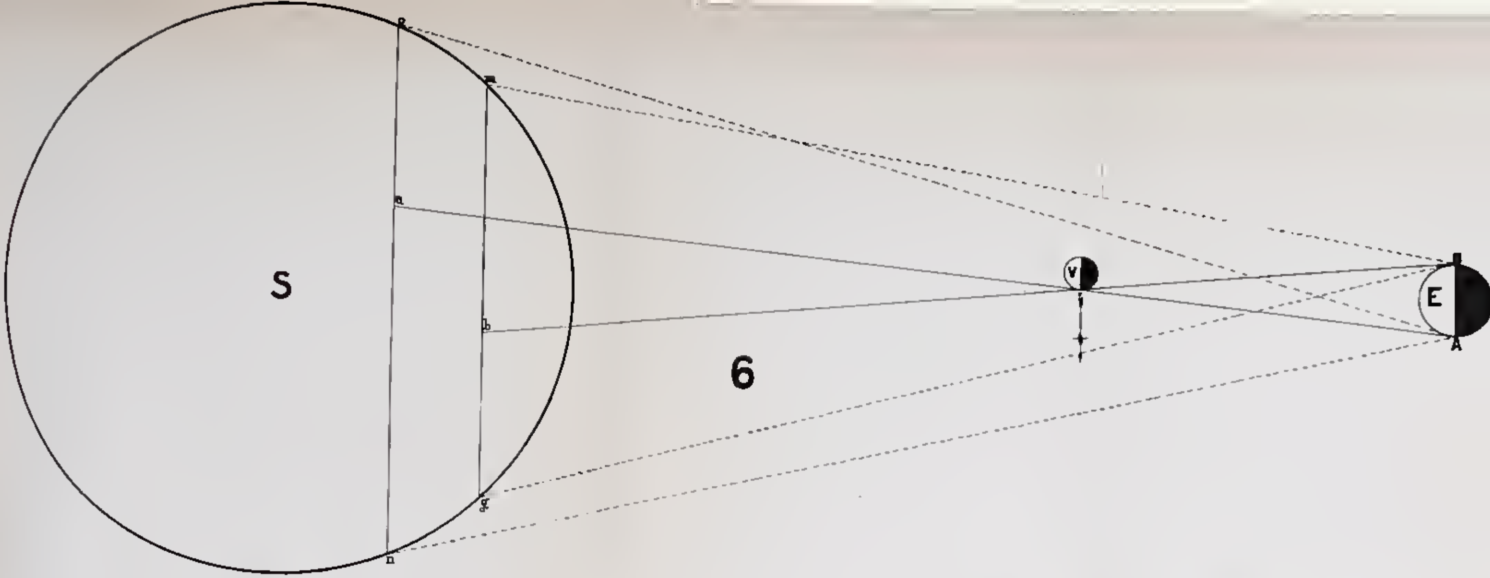

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6. REAL INTERNAL CONTACT

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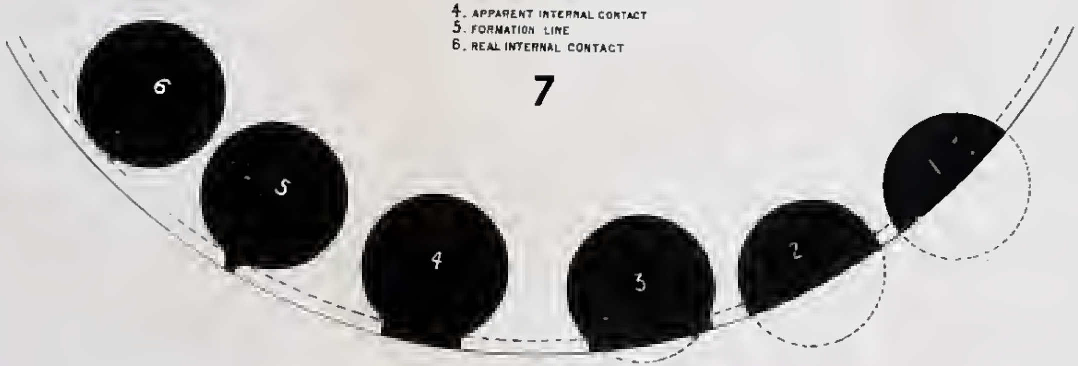

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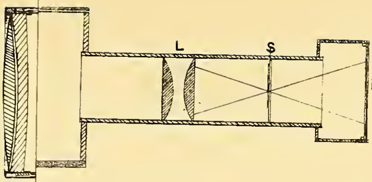

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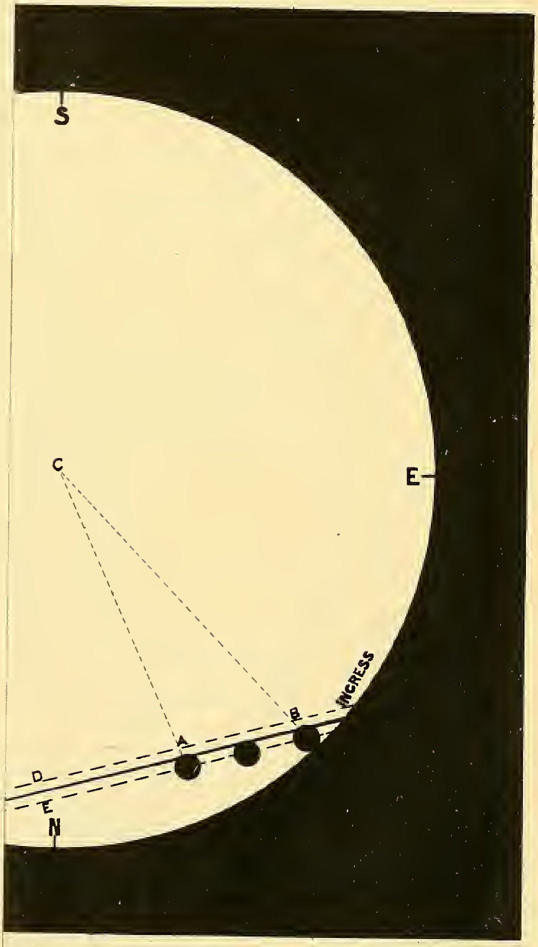

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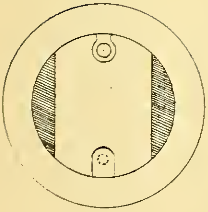

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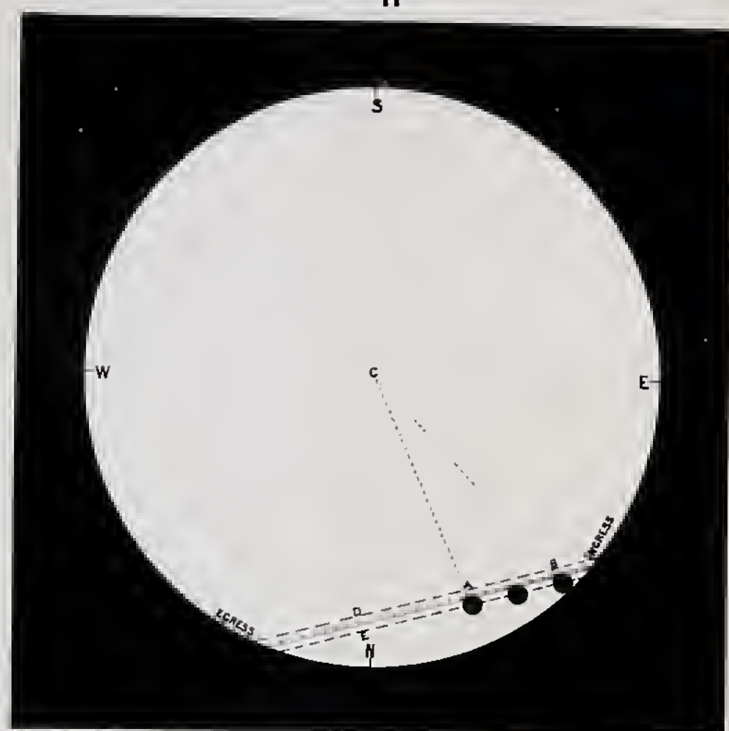

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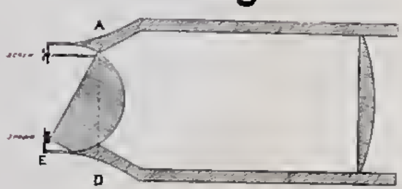

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LOCAL PARTICULARS OF THE TRANSIT OF VENUS IN 1874.

By H. C. Russell, Esq., B.A., Government Astronomer.

[*Read before the Royal Society, September* 3, 1873.]

It is not my intention in the following paper to enter at all into the general question of the Transit of Venus ; so much has been written on this subject, both in regard to the method of observing it and the best stations, that it would seem nothing of further interest can be said. But I propose to give such an account of the methods of observing and the best stations in the Colony as will enable us to make the best use of the circumstances in which we are placed. A few preliminary considerations will, however, enable us to apprehend more easily the methods of observation which are possible, and the difficulties which present themselves to the observer. The Transit will occur on the 9th December, 1874, and the first contact will take place at 11h. 52m. a.m. Sydney mean time. The sun will then be in the zenith of the place whose longitude is 151° 36′ E., and 22° 57′ S. latitude, or near Rockhampton, in Queensland. If we could at that time be at Rockhampton, and, turning to the north, look up at the sun, we should not see Venus on the sun's limb, for the planet would then be a little to the right, or north-east, of our line of sight—to us, of course, invisible, owing to the superior brilliance of the sun ; but if we could at that same moment move to the north-east, and take our station at the point of first contact, we should see Venus just beginning to encroach on the sun's limb. Our point of observation at that time would be on the mail steamer route from San Francisco to Honolulu, and about one-third of the distance from the former city.

If from this point a series of curves were described, increasing the radius each time, they would indicate, at least so far as they were on that part of the earth from which the sun was visible, sections of Venus's shadow cone, or all those parts of the earth from which Venus in transit would be visible ; and the time which this shadow cone will take to include the whole earth is twenty-one minutes, or, from the time that Venus is first visible on the earth's surface until it will be visible all over the earth will be twenty-one minutes, the last point on the earth to see the first contact from will be a place near Prince Edward's Island. Unfortunately, both first and last contact points are on the ocean, and have no convenient places near them for observation. At the time of completed ingress, the transit will be visible from the whole hemisphere of the earth of which Rockhampton is the centre; and, excepting Australia, and on the north-west, where it includes India, China, and part of Russia, it is nearly all water, hence the difficulty of finding stations to fulfil the required conditions. But at the end of the transit the earth will have moved in its diurnal rotation, so that to all the stations on the east and north-east which were well situated to observe the ingress the sun will have set, and to all those on the west he will have risen higher. Many new places will have come within view of the phenomena, and the centre of the hemisphere of the earth from which it will be visible will be no longer Rockhampton, but a point situated 88° 46′ east of Greenwich, and 22° 58′ s. latitude, or not far from Amsterdam Island. This hemisphere includes more than half of Africa, the south-east corner of the Mediterranean, part of the Black Sea, nearly the whole of Russia and Japan, and all the countries to the south of them. Australia and New Zealand are included in its south-east side, and the Southern Continent on the south. At a point not far from South Victoria Land the egress will be first seen, and at a place near Orsk, in Russia, it will be last seen.

At the ingress Australia is not well situated for observation; but at egress, New Zealand and South-east Australia are very favourably situated for observation. It thus appears that Europe and the whole of America will see nothing of the transit, and it will be for those who are favourably situated to make the most of their opportunities for observing phenomena so simple but of such vast importance to science.

There are some facts in reference to the phenomena which will repay a few moments' consideration, and it will greatly simplify the matter if we for the time neglect the earth's motion, and assume that we are on a fixed earth watching the passage of Venus across the sun's disc. In diagram 6 we take E to represent the earth, V Venus, and the large circle S the Sun, and the arrow to indicate the direction in which Venus will appear to move, viz., from east to west.

If two observers are placed at A and B, they will each see Venus like a black spot crossing the sun's disc; but A will of course see it in the line A Va moving in the direction *o n*, and B will see it in the line BVb moving in the line and direction *m g*, and if Venus would only trace the lines on the sun as definite as these we have on paper all difficulty would be over, we could at our leisure make all the measures which would be required, and thus determine the angular distance between the lines *o n* and *m g*, which is all that we require to know in order to find the sun's distance.

This is known as the direct method, and will be one of those used at the coming transit. But since the actual amount of displacement of Venus is very small, all the measures must be made with a faultless micrometer, and each observation really involves two measures which should be made simultaneously, viz., the distance of Venus from the nearest part of the sun's edge, and its angular distance from a meridian or north and south line, on the sun. Both measures require extreme accuracy, and present serious difficulties in actual operation. First, the necessity to make two measures at the same instant; for since Venus is moving in two directions, or at least appearing to do so, any difference of time in the measures would destroy their value. Another great difficulty is the ill-defined edge of the sun, which is constantly varying from atmospheric causes, and is never sharply defined in any part, but liable to distortion or apparent enlargement from various causes.

It is possible, I think, that this difficulty might be overcome by a micrometer made for the purpose, so that a pair of cross wires being made to bisect the sun's limb in one part might be made to follow it all round, and so by finding the average edge, measure from the centre just as the solar photos are put on the plate of the microscope and their average edge determined. But of course such a method would require perfect clock-work.

It will be observed that the direct method assumes an exact knowledge of the dimensions of the earth, and the power of finding exact positions, both of latitude and longitude, so as to make allowance for difference of time in the observation. Now, a century since, the exact determination of longitude was not possible, for the lunar theory was an unsolved problem, and the lunar tables little better than guesses, upon which it was useless to base any attempt to find accurate longitudes; and it was well known that if astronomers had to depend upon that method, there was little chance of a satisfactory result. Hence the illustrious Halley was led to look for some other method, and devised a way out of the difficulty. He saw that the relative lengths of the paths of Venus, as observed from two stations, would give data by which to measure the distance between *o n* and *m g*, or the amount of displacement, without any need of determining accurately the true positions of the observers. All that was requisite was to choose stations so that the difference might be as great as possible, and provide the observers with the means of measuring the time of transit as accurately as possible.

These conditions were easily fulfilled, but still observers went to their work unprepared for the serious and perplexing difficulty they had to meet. No sooner had the transit commenced and the planet encroached more than half its diameter on the sun, than it was found that its figure became distorted on the side next the sun's limb, taking the form first of the letter D, or a dome, then that of a circle with a piece stretched out on one side; then as the ingress went on, becoming like a pear, and finally becoming suddenly round, but some distance within the sun's edge. Now at what time during these phenomena did ingress take place? No one could answer the question, and a very serious uncertainty, amounting to fifteen or twenty seconds, existed in many of the observations. Fortunately the observers recorded what they did see very carefully, and a century afterwards all the difficulty was cleared up by Mr. Stone, who was able, by interpreting the observers' notes, to ascertain when the ingress took place, at least for all the principal places, and he thus cleared up a difficulty which had long been a sore puzzle to astronomers. All the complete observations were found to agree within small quantities, which were not greater than probable errors of observation.

Of course, in the coming transit, with the experience of the past before us, it is hoped that observations made for Halley's method will be much more successful and accurate than those of 1769. One of the essential conditions, however, of Halley's method, viz., that the observer should see both beginning and end of transit, necessarily limits the possible stations for its application. The uncertainty of the weather is also introduced, by which ingress or egress may be lost, and it becomes necessary to spread the stations over as wide a space as possible, to lessen the chances of cloudy and bad weather.

Still, another method of solving the question of the sun's distance by means of a transit of Venus was proposed by the astronomer Delisle. It only requires the observer to see one phase, either ingress or egress, and his observations are at once of value. The principle upon which this method is based is simply this—that if two observers are so situated as to see the ingress or egress of Venus at different times, the difference of time represents a certain space over which Venus has moved. By reference to figure 6 it will be seen that A and B see Venus leave the sun one after the other—that B looking in the direction B *g* sees it leaving when it crosses that line, and that A looking in the direction A *n* sees it leaving some time after B. Now it is evident that Venus moved from the line B *g* to A *n* in the interval; and if A and B pick their station, they may both see Venus leave the sun at the same point, and therefore the lines B *g* and A *n* will meet at the sun and form a triangle of which we have a known base, A B, and the relation between the distances, Sun to Venus and Venus to Earth. Hence we find the linear distance over which Venus moved in the interval. And since we know the time in which the given space was moved, we find the whole length of orbit, and hence its radius, and by proportion the earth's distance.

And, lastly, we have the photographic method, which is really a modification of the first or direct method. Similar stations are required for both; but it is thought that as the photographic offers the best means of obtaining a great number of pictures in which the image of the whole sun with Venus on it will be taken, and the time at which each is photographed recorded instantaneously, that the results thus obtained will be better than those obtained by the micrometer, because the position of Venus is recorded instantly in such a way that the two measures required can be made at leisure; and the whole, or average of the whole edge of the sun used so as to do away with uncertainties depending upon distortion affecting the whole or part of it. Some even go so far as to expect from this method the best results; and there can be no doubt that, if the instrumental difficulties can be overcome, it is the best form of the direct method, and has the great advantage of doing away with all personality or uncertainty depending upon peculiarities of the observer. Very great care, however, will be requisite in the preparation and management of the instrument and the photographic manipulation; and although the results will be free from peculiarities of the observer, they will be affected with instrumental peculiarities, of which distortion is perhaps the most troublesome. By this is meant a defect depending upon the form of the lenses used, which will make the images of equal portions of the sun occupy unequal portions of the photograph, according to the part of the field of the telescope which is used; in other words, a given angular space on the sun will occupy a different space on the photograph if it is at the centre of the field to what it will if near the edge of it. So that if, when one picture is taken, the centre of the sun occupies the centre of the field, and when another is taken it occupies another place, the two pictures will not be strictly comparable unless the actual amount of distortion of every part of the field is known and allowed for. But another great difficulty is to ensure that the exact angle of position of Venus on the sun may be determined. For this purpose some *fixed* object in the telescope must be photographed by its shadow with every picture taken; and lastly, we notice the difficulty of exposing with sufficient rapidity and uniformity the the whole disc of the sun to the photographic plate; for if one part of the sun be exposed longer than another it will extend the sun's edge in that direction, and of course vitiate the result. No thoroughly satisfactory method of doing this has yet been published, and the first plan used seems to be the best. It is that which places a flashing shutter in that point of the telescope where the rays cross, and where of course the least amount of motion makes the exposure. But I must defer the consideration of these points till afterwards.

We have thus rapidly pointed out what is to be observed, and the methods of doing it, and we come now to the consideration of those points which are of local interest. And we notice that Australia is not well situated for observing the commencement of the transit; for since the question is one of parallax, those who are best situated for observing it are those who are nearest to the edge of the illuminated hemisphere of the earth, or whose positions, as seen from the sun, are as far from the centre as possible. Now, as I have pointed out, one part of Australia as seen from the sun will occupy the centre of the earth, and is therefore not well suited for observation: but towards the end, circumstances will have changed, and Australia will have moved so that the south-east part of it will occupy a good position for observing egress. In some respects I think it will be the best; for although we shall not be so well situated in regard to the actual amount of displacement or parallax as New Zealand, we shall have nearly as much, and a far better prospect of getting good observations, both on account of the greater altitude of the sun and the probable state of the weather, for I find that a great deal of rain falls at the meteorological stations of New Zealand in December, and the weather seems very uncertain.

For observing the end of the transit, then, according to Delisle's method, which has been strongly recommended by the Astronomer Royal, we are well situated for observing the accelerated egress; and for Halley's and the direct or photographic method, all parts of Australia are available; and it is my intention to make the observations at each station serve for each of the three methods, by observing the ingress and egress, and filling up the time by taking as many photographs as possible. I communicated this intention to the Astronomer Royal, and have received a letter from him in which he says :—

"No better arrangement for observing the transit of Venus can be made than that which you propose, namely, to observe by eye at the telescope the completed ingress and the beginning of egress (which I fully expect to be the most accurate observation of all, noting carefully the two phenomena, of when Venus *seems* to be just wholly on, and when Venus *is* wholly on, as shown by the breaking of the ligament), and to fill up the time by taking photographs."

The stations which are now provided for are Sydney, Eden, and Blue Mountains—Sydney, because we have greater facility for observation than is possible at a temporary station; Eden, because it is near the best point of Australia, and within telegraphic reach for determining its position; and Blue Mountains as a stand-by, in case the two coast districts are cloudy, and because I find, from recent experiments, that the additional altitude adds much to the probability of good observations. I have asked for money for a fourth station, which, if granted, will provide for one probably on the dry plains about Urana, where there seems to be a singularly clear atmosphere.

Of other stations to be occupied in Australia I at present only know of Melbourne, Adelaide, and Brisbane. But it is probable that Victoria will have two other stations, though this is not yet decided. I have no doubt also that Mr. Tebbutt will observe at Windsor; so that the best part of Australia will have such a number of stations as will give us a fair chance of blue sky somewhere. Now, if a line be supposed to be drawn through Sydney, Mundooran and Palmerston in Northern Australia, all places on that line will see first contact at the same time as Sydney, viz., 12h. 4m. p.m. December 9th. All points to the east of this line will see it earlier, and all points to the west of it will see it later. If a line be drawn through Brisbane, parallel to the line described, it will indicate all places at which first contact will be one minute earlier than Sydney. A similar line through Melbourne will indicate places one minute later than Sydney.

The point on the sun's disc where Venus will first be seen is situated 49 degrees from the north point of the sun towards the east, or on the lower right hand side as we look at the sun, half-way between the lowest (north) point of the sun, and the east side. If an inverting (astronomical) telescope is used, it will be 49 degrees from the upper limb towards the west or following side.

We now come to the particulars of observations; and first as to the instruments. I will describe as briefly as possible some of the principal forms. The great majority of the telescopes which are to be used have an aperture of from 4 to 5 inches (or stopped down to that), and it is decided that a magnifying power of 150 will be best for general purposes. The telescopes to be used for photographic work are for the most part of the same size, but the longer the focal length and the larger the object-glass for this purpose, the better. They all, with the exception of the American form, to be described presently, will be provided with a combination of lenses inserted in the position of the ordinary eye-piece, which will enlarge the picture of the sun to about 4 inches. The enlarging combination may be either an ordinary low power Huygenian eye-piece, a short-focus camera combination, or one made by fixing the object glasses of a binocular with their convex sides together, as indicated at L (figure 8). The latter form I have found very satisfactory. A camera box will also be attached firmly to each of the telescopes holding the plate at P, (figure 8). At S, where the rays cross, the flashing shutter is to be inserted.

The American form of photo-telescope is totally different. It consists of an object-glass of about 5 inches diameter and 40 feet focus, corrected only for photographic or actinic rays. It is to be mounted with its tube horizonal and true north and south, but the object-glass and camera are to be supported on brick or stone pillars. In front of the object-glass and driven by clock-work is a plane mirror arranged so as to throw the image of the sun steadily into the long telescope. Similar means are provided for taking the photos, but of course the enlarging lens is not required, and the shutter used in the other form of telescope is not suitable, and is to be altered, but how is not yet decided.

For all forms except the last it is essential for good work that the telescopes should be mounted on good firm equatorial stands adjusted to meridian and elevation of the pole, and driven by clock-work (I have devised a very simple clock for this purpose, which I cannot now describe, but which I shall be happy to show to any one interested).

For making observations of the time of ingress and egress the instruments first described are sufficient; but for either application of the direct method, other things are essential beside a satisfactory equatorial telescope and movement. For a small error in the adjustment of the polar axis, though of no consequence for observing ingress or egress, is fatal to the value of micrometer or photographic results. And first, the micrometer, which must, in addition to the means of measuring distances to a fraction of a second, have the means of reading angles of position to w thin 10 seconds; it is very desirable, also, that the field should take in the whole diameter of the sun, and the measures be made alternately to the nearest north and south points of the limb of the sun; the sum of every pair of observations should then be equal to the diameter of the sun, which will afford a good test of their accuracy. Moreover, the measures require to be made as nearly as possible in the same part of the field, or have the part specified with each observation and the value of the micrometer determined for all parts of it.

The best method of finding the zero of position is by allowing a star near the Equator to run along the position line or wire and repeat the observation a number of times, reading the position circle every time. A small well-defined sun spot or the edge of the sun may be used, but the result requires a correction for the sun's change in declination. As Venus is in effect moving in two directions, it will be necessary to make the measure of distance and angle simultaneous, or an error will be introduced.

The micrometer may be made of great use in observing ingress or egress, by assisting the judgment as to the time of apparent ingress or egress, in the following way:—As soon as Venus is bisected at ingress, set the micrometer lines to the diameter of Venus, as it appears on the sun's limb, then turn it so that one line becomes a tangent to the limb at the point where the planet enters, and as soon as Venus reaches the other line apparent internal contact will have taken place.

For the photographic method, the following things in addition to those mentioned are essential:—A small motion for adjustment of focus must be provided, either in the mounting of the object glass or camera box; preferably in the former—for the camera box must be so attached to the tube that motion round its axis is impossible. In other words, the angle of position must be absolutely fixed. Another provision must be made by which every plate when under exposure will take exactly the same position with reference to the object glass; and the plan which seems to me best adapted to attain this end is to have the supports for the four corners of the plate attached to the camera box and do away with the ordinary dark slide altogether. Of course this involves having the end of the telescope in the dark room, but this is easily done by making the side next the telescope of dark light-proof cloth, through which the telescope may pass and have the cloth tied round it. If properly done this will not interfere with the clockwork, but will afford great facility of manipulation, and the cloth hanging about the telescope will tend to prevent vibration.

Provision must also be made to have one fine wire, or better two, at right angles to each other, stretched across close to the sensitive plate; in one or both provision must be made so that the wire may be adjusted to the zero of position as in the micrometer. A piece of plate-glass, with a line and scale upon it, may take the place of the wires, and then every picture would not only have the zero line, but also a scale of reference printed upon it, which would be a valuable check on distortion. The care required in determining the true position of the wire or glass may be gathered from the fact that if we take 10″ as the limit of error to be allowed, we shall require to know the position of the end of the wire or line within (about) 1-10000th part of an inch. The methods of adjusting all the lenses will be found in Mr. Rutherford's paper, and I need not add anything to what he says.

I have before alluded to the flashing shutter; its details will be best understood by reference to figure 5, in which A B C is the diaphragm put in the telescope; it has a hole in the centre, d e, half an inch in diameter; M M M M are pieces put on A B C, as guides for the shutter N N N N. The shutter has a hole in it between R R, corresponding to d e, but half of it is covered by the piece p which is fixed to the shutter, and the other half nearly covered by the piece o o, which by means of the slots and screws is adjustable, so that the slit R R may be varied at pleasure; but about 1-50th of an inch is enough to separate p and o o. S S S is a spring (common elastic) which causes the shutter to flash past D E as soon as the string T is cut or burned. If cut, it should be done with scissors, to prevent vibration in the telescope. As the shutter flashes past d e, R R opens every part of it in succession, and exposes the whole sun. In the position in which the shutter is represented, the hole d e is covered, and the shutter must be put into that position before every plate is put in. It is usual to burn the string, but a device such as that in figure 10A will fasten and release it as often as necessary, by just pressing the spring with the fingers at A & B, and it will release the shutter at C. The methods of determining the distortion produced in the photograph will be found in Mr. De la Rue's and Mr. Rutherford's papers. I have only to say that, by photographing a known pair of double stars such as Alpha Centauri in all parts of the field, the distortion would be easier and better determined. I shall erect one of the proposed scales about two miles from the Observatory, which will be open to any one who would like to use it (on application). Lastly, and very important, the exact time of each flash of the shutter must be exactly recorded.

It seems desirable, if possible, to have something which shall form a reliable point of reference during the transit. For this purpose I propose to set up one of De la Rue's scales in such a position as, by simply moving the telescope in right ascension, it may be brought into the field and photographed. If set up truly horizontal, its angle of position could be calculated exactly, and determined by repeated reference from the telescope after star adjustments, and having obtained its true angle of position, it would form a valuable reference line for the true zero of position, and at the same time furnish a photo. of the scale as a check on the distortion which might be changed by an alteration of telescopic adjustments during the progress of the observations.

If means are not at command for making the adjustments for temperature of tube, as suggested by Mr. Rutherford, the tubes may be covered with blankets to prevent change of temperature.

I hope no one will think that, because a telescope is less than 4 inches, it is not therefore to be used. There is no charm in $4\frac{1}{2}$ inches. It is only taken as a size which may be within the reach of a great many observers, and, for the reasons stated, it is desirable (not necessary) that all the telescopes should be as nearly as possible one size.

I had nearly forgotten one important point, viz.,—the method of getting the focus. Mr. Rutherford describes it (and the same plan applies to an ordinary telescope); but if the telescope is a good one, much time may be lost, for in some cases the hundredth part of an inch is sufficient to put the telescope in and out of focus. It will greatly facilitate finding it if the plates are put in with a piece of glass of the same thickness, under one end, *i.e.*, out of collimation. One part of the picture will then be found sharper than another, or perhaps quite sharp, and the amount of motion necessary can be seen and made at once. The moon answers well for this plan, and gives, of course, the true focus of the telescope.

PHENOMENA OF INGRESS AND EGRESS.

In order to understand clearly the nature of the phenomena presented at the ingress and egress of the planet Venus in transit, it is necessary to remember that all bright objects, such as the sun, stars, &c., when seen in a telescope present a fictitious diameter, or, in other words, a bright object always looks larger than it really is. The defect arises from imperfections in the telescope and in part from peculiarities of vision which cause us to estimate the size of a bright object unfairly when it is placed in comparison with a dark one of the same size. Mr. Stone, who has explained this, and made such valuable use of the fact in explaining the last transit of Venus, estimates the enlargement as one-sixtieth of the diameter of Venus, and he strongly urges that observations of ingress and egress should be mainly directed to the determination of the time of these two phenomena which are to be explained presently, known as *doubtful* and *certain* contacts.

The first external contact is not considered important, because in all the best stations for observing the transit the sun is low down when it takes place; and the sun's limb, always an unsteady seething line, is then so much so that it is considered impossible to observe at all satisfactorily when the edge of the sun becomes dented with that of Venus; but if it is seen the time should be carefully stated. As a help in this matter, the Astronomer-Royal suggests that such coloured glasses should be used as will absorb the ends of the spectrum, and leave the green predominant; and he further suggests that, for observing the sun at small altitudes, great advantage may be derived from adding to the eye-lens a small prism of which the base is towards the earth. It must be equal in effect to the dispersion produced by the atmosphere. This is greatest, of course, for objects near the horizon; but in the inverting telescope it is in the upper part of the field, hence the prism is so placed as to have the opposite effect. His method for introducing this prism, which must be variable to meet the variable states of atmosphere, is exceedingly simple and beautiful. The lens of the eye-piece nearest the eye is made movable, so that it may take any position between its normal one A D (figure 9) and A E; where it will be seen EAD is in effect a prism introduced, which is variable at pleasure. As the details of construction of this eye-piece are not given, I have added to the figure just enough to convey to any one wishing to make one an easy method of doing it. First, the eye lens must be taken out, and two flat edges cut to it (figure 10), which represents the lens so cut, then the sockets for the lens must be cut to the same curve as the lens, and lined with soft leather; two flat pieces must then be fixed in the eye-piece so as to touch the flat edges of the glass and keep it from turning round; lastly, a spring must be placed, tending always to press in one part of the lens, and a screw to press in the other part, by which motion in or out may be given to that end of the lens. In use, then, if the atmospheric dispersion become troublesome, it is only necessary to turn on the screw until the prism so formed is equal to the effect required. (The following notes are based on Mr. Stone's paper in Royal Astronomical Society's Notices.)

In figure 7 I have represented Venus in the several stages of ingress and egress, and have shown the apparent edge of the sun by the continuous line and the real edge by the dotted line. I have on purpose grossly exaggerated the difference between these, in order to make the explanation more obvious, and I have moreover assumed that the whole amount is on the sun, for simplicity; but in fact the dark body of Venus is reduced when in transit by the same amount, and from just the same cause as the sun's is increased. Starting then with No. 1, in which Venus is bisected by the sun's apparent edge, we notice that it is not really bisected, and that it will not be bisected until the centre of Venus comes on the dotted line, as in No. 2. Now observe what takes place here. All the light between the continuous and dotted lines is fictitious; and, since the real light is cut off at the dotted line, Venus takes the form represented, that of a dome resting on the sun's edge. In No. 3, the light cut off at the dotted line is not equal to the diameter of Venus; and it appears as if a portion of the edge of the planet were drawn out to the apparent edge of the sun. This is called the formation of the drop. In No. 4 the drop is smaller; and it appears to the observer that if Venus were only round, its outline would be within that of the sun—that is, *apparent internal contact;* but it is only apparent, for it will be seen that a part of Venus still covers a fraction of the sun's limb, hence a drop still remains. This is one of the two phenomena that we are particularly requested to observe with the greatest care, because, owing to the uncertainties of atmosphere, it is very doubtful whether observers will be able to see the real contact satisfactorily. The use of the micrometer may be of great assistance here, as I have before pointed out, for it will enable the observer to estimate more accurately when Venus, if made round, will just meet the apparent edge of the sun. Card 2 should be used for representing, at this stage, the diameter of the drop; it should be estimated as a part of the diameter of the planet, and full notes written on the card, in reference to the observation at the time, expressing the estimated diameter of the drop, the state of the sun particularly, whether steady, boiling, or whatever it may be, also the magnifying power used, whether Venus has a well-defined outline away from the drop, in fact everything required to convey to a stranger *all* the circumstances—atmospheric, telescopic, and personal, then existing.

After apparent internal contact the drop will very rapidly decrease in size, and in the course of fifteen or twenty seconds it will become a line (No. 5), and suddenly break. This is *real internal contact* and the last moment before the breaking of the line should be carefully represented on card 4, and another card should be made to show the position of Venus the moment after the line is broken. This is *certain contact*, the second phenomenon to be observed.

True internal contact simply means, as will be seen by reference to figure 7, that the true limb of Venus has crossed the true limb of the sun, and therefore allows the light to appear all round it; but since the motion of Venus on the sun is very slow, the formation of the line of light is gradual, and will be seen sooner by some persons than by others; and if telescopes of large apertures are used without being stopped down, a considerable difference of time may be made, amounting, perhaps, to ten seconds. But if the sun's limb is unsteady, it becomes almost impossible to catch this phase at all, and in many cases it will probably not be seen; though, when caught, it is by far the most satisfactory observation that can be made. It must be remembered that at real internal contact, or the breaking of the drop, Venus will appear some distance within the outline of the sun; and the apparent amount, as a fraction of the diameter of the planet, should be carefully estimated and recorded on the card, as it will afford the means of ascertaining how the observer caught the phase which was required to be observed.

After this phase has been observed, the direct method must be applied until Venus approaches the other limb of the sun, when the phenomena described at ingress will be reproduced in reverse order. If the formation of the line No. 5, fig 7 can be well observed, it is to be done with all the care of the previous observation, and Venus will arrive at phase 4 in about eighteen seconds afterwards, and the other phases in order—3, 2, 1. Cards should be used at all phases of egress, and with the same care as at ingress, care being taken to mark each card *Egress.* The cards I have represented in figures 2, 3, and 4 are drawn to scale, and represent the sun and Venus seen upon it, each of their relative dimensions: if thought desirable, cards for intermediate phases may be prepared. If these cards are held in the left hand, by taking hold of the side marked W, then the card represents the part of the sun on which Venus will appear to an observer without a telescope at ingress ; for cards at egress Venus is shewn between N and W., but cards will be printed for each position separately.

Time.

The question of time now has to be considered, and it is one of the most important, for without an exact record of the time of each observation the most perfect measures or photos. will be worthless. Where a chronograph can be had it should be used, and means taken not only to mark on the tape the dot which represents the time, but also to write opposite each observation or dot what it represents, so that there may be no possibility of subsequent confusion. In the absence of a chronograph, the observer will note the ticks of the clock or chronometer and record the time accordingly ; but it is to be remembered that the rates of clocks and chronometers must be steady, and that nothing but the transit instrument will determine their errors with the requisite accuracy. Of course, should the observer be on any telegraph line he may get time from the Observatory ; and I shall be only too glad to furnish any aid that may be in my power.

The exact latitude and longitude of the observer must be known, if he obtains good observations : but as that can only be known after the transit, the determination of true position may be left till then.

[*Extracts.*]

Warren De La Rue, Esq.

At the request of the Astronomer Royal, Mr. De La Rue has furnished a paper on the Observation of the Transit of Venus by Photography, from which we extract the following notes :—

"I do not suggest that photographic observations should displace eye observations ; on the contrary, I think that both eye and photographic observations ought to be made. The conditions which the transit of Venus offer for the determination of the relative position of the sun and planet's centres are more advantageous than those presented by solar eclipses ; inasmuch that it is far more easy to measure directly the distance between the centre of the disk of the sun and that of the image of the planet upon it, than to measure the distances between peripheries of the sun and moon ; and in the transits of Venus any error of observation would not affect the final result nearly so much as in solar eclipses. For example, an error of 1″ in the measurements of maximum displacement in 1874 or 1882, would give an error of only 0·185 in the deduced solar parallax. Moreover, it is by no means important to catch exactly the phases of contact, as two photographs obtained at a sufficient interval will serve the purpose. Nor is it in any way essential, as it is with eye observations, that favourable conditions should exist for retarding the period of contact at one station and accelerating it at another, because in the photographic method the length of the cords need not be directly considered in determining the nearest approach to the sun and planet's centres. During the transit, photographs at intervals of two or three minutes may be obtained, and any or all of them useful for comparison with other stations. The epoch of photographic record is determinable with the utmost accuracy, because the time is not more than 1-50th or 1-100th of a second, and the instantaneous slide makes an audible signal which may be recorded or made to record itself on the chronograph, the small fraction of a second between actual exposure and the striking of the slide may be determined and allowed for.* Figure 11 represents the solar disc of the dimensions of the photograph which would be taken with the Kew photo-heliograph at the transit of 1874, and under the same conditions of adjustment as existed during the eclipse observations of 1860. On this occasion 1-thousandth of an inch represented 0·496 second of arc. Figure 11 is direct as given in that instrument. a b two positions of Venus; C, the centre of the sun. The dark line marked ingress egress the true path of Venus, and the dotted lines d and e the possible amount of displacement from parallax. The points N. E. S. and W. are the apparent north, east, south, and west points of the sun.

"In the transit of 1874 the solar disc photographed as above would have a semi-diameter of 1965·8 thousandths of an inch (or a diameter of nearly 4 inches), and Venus a semi-diameter of 63·33 of these units, and the parallax of Venus referred to the sun would be represented by 47·85 of these units; the maximum possible displacement being 95·7 units, or nearly 1-tenth of an inch.

"Means have been devised for ascertaining the distortion of the photograph picture, by which the measures are made satisfactory. By photographing a scale of equal parts placed at one or two miles from the instrument, a picture will be obtained in which different equal portions of the scale would occupy different places and spaces in the photograph; and by comparing these by means of a micrometer, the effect of optical distortion can be obtained to a great nicety. Such a scale of equal parts, if erected at a distance of two miles, would have to be about 110 feet long, in order that its image might be somewhat longer than the diameter of a sun picture. It could be made by fixing two horizontal rails one over the other, so that their outer edges would be distant 3 feet. On these, plates of zinc, 2 feet wide and 3 feet long, would have to be fastened so as to leave an interval of exactly 2 feet between adjacent plates. The plates might be made of the same width by clamping the whole series together and planing their edges in a planing machine. In placing the plates on the rails, one might be used temporarily as a template to govern the interval between a plate previously fixed and one about to be screwed down. Of course care must be taken to make the supporting rails firm and straight, and if possible with a sky background. The focus of the Kew heliograph for an object two miles distant would be about 1-50th of an inch longer than that for parallel rays. This produces a little indistinctness of outline in the picture, but does not prevent measurements being made. Before finally fixing the enlarging lens for sun pictures, sharply defined pictures of the scale should be obtained by adjusting the focus. Fears have been expressed that the collodion in drying becomes distorted. Experiments, however, in 1860–1861, have demonstrated that the shrinkage is only in the direction of the thickness."

* The electric signal may be given at the exact moment of exposure, by placing the contact pieces so that they pass as the opening in the slide passes the centre of the opening through which the image passes.—(Note by H. C. R.)

Mr. Rutherford to American Commissioners.

"I think I can best reply to your letter of the 19th instant, making inquiries as to the application of photography to the transit of Venus, by describing the methods now and for the last three years in use at my observatory for photographing the sun, with results which I have not seen equalled by any other process.

"1st. The object glass is corrected for photographic rays without reference to the effect on the visual image. This correction I have effected in two modes; first, by constructing a double objective of flint and crown glass with such curves as will produce the correction; second, by applying to an ordinary achromatic objective a lens of such curves and density as will produce the required correction. Of the first description is the $11\frac{1}{4}$-inch, now the property of the Argentine Republic; of the second description is the 13-inch objective now mounted in my observatory.

"Without a proper correction the photographic images are without sharpness, and consequently entirely unfit for measurement. (*See note at end.*)

"With an objective of the above description it is not possible to adjust the focus by the eye. This must be done by trial, and there must be a micrometer screw, so that the true focus once found may be recovered at pleasure. I use a galvanized iron tube with three thermometers, from whose reading the focal adjustment is made by the screw for the change in the length of the tube. I find that stars furnish the best means of determining the focus.

"The camera box is arranged so that a plate-holder may be inserted, and a picture taken at the focus of the objective. The size of a picture so taken is about one-tenth of an inch diameter for each foot of focal length, and is too large to permit an instantaneous and uniform exposure. The image of the sun is enlarged by a photographic camera lens of ordinary construction. I use my 13-inch Harrison $\frac{1}{4}$-plate portrait tube. No doubt a better enlarging lens could be devised with a flat field and corrected for photographic rays alone; whereas the best portrait lenses have considerable curvature of field, and profess only to unite the photographic and visual rays; a compromise which sacrifices about one-half of the sharpness of each. After traversing this combination the rays cross on their way to the sensitive plate. The point where the rays cross is so small that a very simple and light snap for instantaneous exposure is possible, and an additional and very important advantage is secured in that the whole image is simultaneously exposed and cut off.

"Directly in front of the sensitive plate is a very fine platina wire, stretched east and west with a simple adjustment, by which it may be made to coincide with the course of a star or sun spot; the shadow of this wire will mark on the photo the zero of position. Of course this supposes the instrument accurately adjusted to the meridian and the elevation of the pole. Behind the plate, with a motion from side to side, is mounted a Ramsden eye-piece, for the purpose of finding the angular value of any part of the plate.

"The adjustments required are:—1. Collimation of objective. The object-glass must be adjusted by the proper screws until the flame of a candle held at a small hole in the middle of the plate-holder, is seen reflected, and superimposed by an eye looking through the blue transparent part of the candle. 2. Collimation of the plate. This is effected by putting in the plate-holder a piece of plate-glass smoked on the side most distant from the object-glass and covered on the other side all but the central quarter of an inch. A cap with a quarter-inch central hole is then put over the objective, and when the image of a candle held before this opening is reflected back through it by the smoked-glass, the plate-holder is in collimation. The enlarging lens must also be put in collimation by proper screws, using the reflection of a candle in the same way as for the object glass.

"The quantity and direction of the inevitable distortion is ascertained in two ways:—1st, By placing in the focus of the objective a plate of plane parallel glass upon which a reticule of lines is engraved, and taking a photograph of this when the focus is exactly adjusted. If then the reticule and the photo are compared, the amount of distortion produced by the enlarging lens will be known. 2. Place in the position of the sensitive plate a plate of parallel glass; upon which lines running north and south (transit lines) are engraved about three seconds of equatorial time apart. Stop down the object glass to about one inch, and clamp the telescope firmly. Then by means of the chronograph take transits of many bright stars over all the lines; observing them with the eyepiece before mentioned. A comparison of these transits will give not only the angular value of the intervals between the lines, but also the amount of distortion produced by the instrument, objective and enlarging lens included."

"A second method of ascertaining the angular value of the parts of the field is to photograph a known object at a known distance, such as a building by day, or two electric lights expressly arranged at night; or a group of stars like the Pleiades, whose distances are accurately known. By a combination of these methods I think the angular value of a given linear space may be accurately known. But with all these precautions it is important that no reliance should be placed, for precision, upon the apparent outline of the sun at any particular part; for the photograph of the sun will have a greater or less diameter, by many seconds, according to the energy of the sun's rays, by a change in aperture of the telescope, by the time of exposure, or the hour of the day at which the picture is taken, or by the sensibility of the chemicals. You may be tempted to say, if this be true, what reliance can be placed upon the results of photography? I answer, that the sun has no defined outlines at any time, even to the eye; but in its best state is an irregular, seething, ever restless object, utterly unfit to be the starting point for measures of precision, and that while the eye is confined in its attempts at measures to some small part of the sun's limb, the photograph can be placed upon the stage of the micrometer, and accurately centred with reference to the average of the whole contour, and thus escape the errors sure to be the result of measures based upon local bisections."

"I would recommend for photographing the transit an achromatic object glass, 5-inch aperture, 70 inches focus, reduced to 60 inches by the front lens before described, and an enlarging combination which makes the sun's image 2 inches diameter at 70 inches from the objective. I would make the camera box part of the tube, and the focal adjustment near the object glass."

PROFESSOR NEWCOMB says:—

"It is probable 200 negatives may be taken at each station, and it is to be hoped that the average error of the whole will not exceed 0·03. It is *desirable* that the constant error peculiar to the station should not exceed a fraction of this, say 0·02; this would require that all the arrangements should be so made that the error in measuring the distances between the centres, which on the average is about 800 seconds, should not exceed 1-40000th of the entire space measured (*i.e.*, one-fiftieth of a second); and in order to make photography useful as compared with other methods, it is *necessary* that the error should not exceed 1-10000th part of the space measured (if the photos are taken 4 inches diameter this becomes 1-7000th part of an inch). To secure corresponding accuracy in measuring the angle of position, it is necessary that the zero line should be correct within 15″, or, in other words, on the same scaled photo, the position of the end of the zero line must be known within about 1-6000th of an inch."

"As to the size of the negative, we may assume that when in the microscope it should seem the same size as the sun seen through a telescope with a magnifying power of 240. To effect this with a microscope magnifying five times, the diameter of the negative must be that formed in the image of a telescope 40 feet long, or $4\frac{1}{2}$ inches; but if the negatives can certainly be made to bear a higher power a smaller image may be advantageous, and if it will bear a power of 10 on the microscope it might be 2 inches (I have seen a negative that would bear a power of 50). In order to take the photographs of the size mentioned we must either have the telescopes 40 feet long, as proposed by Professor Winlock, or magnify the image with an enlarging combination. But the 40-feet telescope will be too long to point direct at the sun, and Professor Winlock suggested it should be fixed horizontally north and south, and the sun reflected into it by a moving mirror in front of the object glass. It seems to offer decided advantages over the other method.

"1. It gives an image free from all distortion, except such as can be accurately determined and allowed for.

"2. Granting the plane surface of the reflector preserved, it affords the most certain means of determining the angular value of any linear space on the picture.

"3. It affords the means of determining the angle of position with great certainty.

"4. The plate-holder may be firmly fixed on a stone pier in the dark room.

In reference to the methods of carrying out these plans, he says, "the reflector of the heliostat may be made of glass, silvered glass, or speculum metal, the requirements being that it shall not change its shape, or become curved by necessary exposure to the sun. To reduce the probability of this, it is best to use a metallic reflector when the exposure being only for about one second in every minute (if photos are taken at the rate of one per minute) or 1-60th of the whole time, the mirror will not change much, and may in the meantime be carefully covered, or, if used for focusing, covered with blue or green glass. The heliostat should be worked by clock-work, free from all vibration produced by the wheels striking together. It is probable the reflector may at all stations be placed within one foot of the object glass.

"The object glass should be five or six inches diameter, forty feet focal length, and corrected not for visual but photographic rays; it must not be supported by the tube, but by a stone or brick pier the same as the plate-holder. The tube is only useful to cut off extraneous light. The object glass should have screw motions to adjust it to the meridian and focal distance.

"For southern stations the heliostat and object glass should be south of the plate-holder.

"Zero Lines.—The pier which supports the plate-holder must be so formed that a plumb line may be suspended close to the sensitive plate and passing through the middle of focal point. The plumb line must be a very fine platinum wire, with a heavy bob. Provision must be made in the support for turning the wire round so that any want of perfect flexibility may be overcome by reversing the wire at each picture. The perfect verticality of this wire must be carefully investigated and secured. A horizontal wire must also be placed crossing the vertical one (without touching it) in the centre of the focal point. Exposing the plate involves some mechanical difficulties. Equal exposures of every part of the picture may be attained by cutting off the sunlight in front of the objective by a narrow slit moving past it, but in this arrangement the image will be elongated by diffraction in a direction at right angles to the slit. To overcome this difficulty it appears desirable to put the *cut-off* or exposing arrangement within one or two feet of the face; but if this is operated in the usual way, viz., by having a narrow slit moving across the rays by the action of a spring, the last part is exposed less than the first because the motion of the slide is accelerated by the spring, and therefore quicker at the end than at the beginning. As a possible method of overcoming this difficulty, for which something must be contrived, he suggests that a thin disc of cardboard, three feet in diameter, and mounted like a glass in an electrical machine, and having on the same axis a grooved wheel six or eight inches in diameter, and connected with the card-board disc by ratchet, the three-feet disc has its axis level and parallel with the optical axis of the tube, and has a slit placed radially in it, which will expose in passing the whole sensitive plate every time the disc is turned; therefore, one exposure will be made during the passage of the slit, and the light will be cut off during the other part of the turn. The exposure is to be made as follows:—A weight is attached to the grooved wheel by a string of such a length that when the slit is near the line at which it begins to make the exposure the weight rests on the ground; the weight is now wound up nearly one turn, or until the slit comes nearly in a line of exposure on the other side, or past it, as it were. Now, when an exposure is wanted, the weight carries on the disc with a continually increasing speed until it touches the ground, when of course its action ceases, and the disc goes on with its own momentum at a speed quite uniform, except the slight retardation due to friction.

"NOTE.—As to definition, this is not quite correct. If a telescope lens of the ordinary kind is a good one, it will be found to have a very sharp actinic focus, from one-half to three-quarters of an inch further from the object glass than the visual focus."

NOTE *added* 15*th October*, 1873.—Since the remarks on enlarging lenses, in page 7, were written, I have found by actual experiment that a better form of enlarging combination is made as follows:—Take two simple *plano-convex* lenses of equal focal length, and place them with the convex sides towards each other, and separated by a distance equal to rather more (one and one-fourth) than the focal length.

The diameter of these lenses should be about one and a half times the diameter of the image of the sun formed by the object glass, and the focal length about three or four inches. The advantages are,—first, that the enlargement is effected without any distortion measurable by ordinary means; and, second, the lenses being simple, affect the actinic rays most, and so correct to a great extent the faults of a telescopic lens applied to photography, and so produce sharper pictures.

H.C.R.

THE BINGERA DIAMOND FIELD.

By Archibald Liversidge, F.C.S., F.G.S., Reader in Geology and Mineralogy of the Sydney University, late University Demonstrator of Chemistry, Cambridge.

[*Read before the Royal Society*, 1st *October*, 1873.]

In the following note I purpose giving a few facts concerning the recently opened diamond workings in the neighbourhood of the town of Bingera. Bingera is situated some 400 miles north of Sydney, on the Horton, or, as it is more popularly termed, the Big River; this River runs into the Gwydir River, the Gwydir in turn losing itself in the Barwon or Darling River.

Being on my way last winter (June, 1873) to visit the tin districts of New England, I turned aside and availed myself of the opportunity to pay a hurried visit to the above diamond workings. The trip was not a satisfactory one, for, owing to the persistent rains and floods, travelling was at times quite impracticable, and at all times done under difficulties; hence, in the limited time at my disposal, all hopes of anything like a thorough geological examination of the spot had, to my great regret, to be relinquished. However, I was enabled to acquire a certain amount of information, which I venture to lay before you this evening, in the hope that it may not prove to be altogether devoid of interest and value.

But, in the first place, I may perhaps be permitted, *en passant*, to preface my remarks upon Bingera by briefly mentioning a few of the facts relating to the other and longer known diamond-bearing localities of Australia, but only so far as they throw light upon the Bingera deposits. For fuller information I must refer you to the Rev. W. B. Clarke's Addresses to the Royal Society of New South Wales in the years 1870 and 1872, and to the very complete account of the Mudgee diamond district, by Mr. Norman Taylor and the late Dr. Thomson, read before this Society in 1870.

Diamonds in Australia.

As early as 1860 the Rev. W. B. Clarke mentions the discovery of diamonds in the Macquarie River, but no information is furnished as to the conditions under which they were found, and it is not stated whether they occurred in the present river bed or in an ancient river drift.

But we have a full account of the geology of the diamond-bearing district detailed in the above-mentioned paper by Messrs. Norman Taylor and Thomson, and from it we shall see that the Mudgee and Bingera districts have many points of resemblance.

The Mudgee diamond workings are distant some 170 miles south of Bingera, on the Cudgegong River, which runs into the Macquarie River, and that again into the Darling River.

Diamonds were first discovered here in 1867 by the gold diggers, who neglected them for some time, but in 1869 they were worked pretty extensively. The localities lie along the river in the form of outliers of an old river drift, at varying distances from the river, and at heights of 40 feet or so above it. These outliers are capped by deposits of basalt, hard and compact, and in some cases columnar. This basalt is regarded by Mr. Taylor as of Post-Pleiocene age, but this has not been determined directly by any fossil evidence.

The great denudation which the district has sustained is at once apparent from the drift, together with its protective covering of basalt, having been cut up into these isolated patches or outliers.

The remains of the drift can still be traced for some 17 miles up the river, and in parts it still retains a thickness of 70 feet.

The patches which were worked, as enumerated in the above-mentioned paper, are as follows:—Jordan's Hill, 40 acres; Two-mile Flat, 70 acres; Rocky Ridge, 40 acres; Horseshoe Bend, 20 acres; Hassall's Hill, 340 acres. Total, 510 acres.

A peculiar deposit of crystalline cinnabar was found in one patch.

In the above localities the drift has invariably been met with in tunnelling under or sinking through the basalt, and in places where the basalt had been denuded away the drift has either disappeared or has been scattered over the neighbourhood.

No diamonds have been been found in the river bed, except in places where the diggers have discharged the drift into the river when washing for gold.

The basalt when not resting on the drift frequently lies upon metamorphic shales, slates, sandstones, or greenstone.

The general formation of the neighbourhood is regarded as Upper Silurian, with overlying outliers of undoubtedly carboniferous age.

The rocks in the vicinity are nearly vertical, with a general strike of N.N.W., and consist of red and yellow coarse and fine grained indurated sandstones; thin white platy argillaceous shales; pink and brown fine-grained sandstone, banded with purple stripes; slates and hard metamorphic schists; hard brecciated conglomerate, containing limestone nodules, flint, and red felspar in a greenish silicious base. And with these occur dykes and ejections of intrusive greenstone.

The rocks are generally devoid of mica. For the most part the Older Pleiocene diamond-bearing drift is coarse and loose, but parts are cemented together into a compact conglomerate by a white cement of a silicious nature, sometimes rendered green by admixture with silicate of iron; in other cases oxides of iron and manganese have been the agglutinating agents. Diamonds were proved to exist in this solid portion by a special experiment of Mr. Taylor's.

The drift is chiefly made up of boulders and pebbles of quartz, jasper, agate, quartzite, flinty slate, shale, sandstone, with abundance of coarse sand, and more or less clay.

The quartz pebbles are white, like vein quartz, but often encrusted with films of iron or manganese oxides.

Many of the boulders and pebbles are remarkable for a most peculiar brilliant silicious polish, which is evidently not due to friction, since the cavities are equally well polished. Silicified wood is common, and coal has been found in the river higher up: also carboniferous fossils, such as Favorites Gothlandica and others.

Amongst the minerals associated with the diamond are the following. This list, we shall see, is almost identical with that furnished by Bingera:—

1. Black vesicular pleonast. This mineral has not yet been found at Bingera.
2. Topaz.
3. Quartz.
4. Corundum.
   a. Sapphire.
   b. Adamantine spar.
   c. Barklyite.
   d. A bluish white variety, characteristic of Mudgee.
   e. Ruby.
   f. Rolled corundum, dirty white and pink.
5. Zircon.
6. Tourmaline.
7. Black titaniferous sand.
8. Black magnetic ironsand.
9. Brookite.
10. Woodtin, rare.
11. Garnets.
12. Iron, from tools.
13. Gold.
14. The Diamond.

The largest found was $5\frac{5}{8}$ carats = roughly 16·2 grains. The average sp. gr. was 3·44; and the average weight 0·23 carat, or nearly one carat grain each. The carat contains 4 carat grains, which are equal to 3·16 grains troy.

The Newer Pleiocene drift afforded a few diamonds, and being derived partly from the older drift, its materials are somewhat similar; but in addition to the gems as enumerated, a few grains of osmiridium have been collected from it.

Diamonds have also been found in Victoria, but in no large quantities, and of but small size, but no report of their geological position appears to have been published.

We will now return to the more immediate subject of this note.

The Bingera Diamond Workings.

The diamond-bearing deposits at present undergoing development are some seven or eight miles, more or less, to the south of Bingera, and are situated in a kind of basin or closed valley amidst the hills; this basin is about four miles long by three wide, and is open to the north.

This, together with the surrounding district, is evidently of Devonian or Carboniferous age, but all attempts to procure fossils in order to verify this have hitherto failed. As before mentioned, the weather was too wet to allow me to make a proper search myself; in fact, it was only with very great difficulty that one could get about at all in the then state of the country. The weather was so thick from the pouring rain and constant mists that but meagre and unsatisfactory glimpses were obtained even of the country's general aspect. Nearly the whole of the basin seems to have been originally more or less covered with drift, parts of it having since been removed by denudation.

Running into the valley are various spurs of basalt, which apparently cover portions of the drift; but at present this is only a conjecture, since the workings have not yet been carried on sufficiently far to show whether this be the case or not, neither by tunnelling under it nor by sinking shafts, but I hope soon to receive information upon this head, for when on the spot I suggested that a shaft should be sunk which will decide the question. Should the drift be proved to pass under the basalt, the known diamond-bearing area will be greatly increased. The probabilities are in favour that it does.

Both the basalt and the drift have undergone much denudation.

The drift is said to be traceable along the course of the river for some (30) thirty miles.

The drift is the forsaken bed of some river, and in all probability that of the Horton.

The rock upon which the diamond drift rests, or the "bed rock" of the minerals, is an argillaceous shale. Outcrops of this are seen in one or two places, but no good section is shown.

In other parts of the ground we see a compact, rather small-grained silicious brecciated conglomerate, strongly agglutinated together by a ferruginous cement; occasionally the pebbles incorporated in this conglomerate are of rather large size.

In one part of the field the junction of the conglomerate with the argillaceous shale is clearly shown in the cutting formed by a small gully.

Both the shale and conglomerate beds appear to have undergone much disturbance; and at this particular spot diamonds are said to be plentiful on the conglomerate but not on the shale. The surface of the shale is here free from drift, but the conglomerate does not appear to be quite free from it. The miners regard the conglomerate as being of itself diamond-bearing, but this has not been put to any absolute proof.

Up to the present all the diamonds have been found within a foot or so of the surface, in fact just at the grass roots. In no case have the workings been carried to greater depths than two or three feet; in some parts examined the drift itself is not thicker than that.

In the former sinkings made by the gold diggers diamonds have occasionally been met with at depths of 60 feet, or even more; but, as the men were working for gold, no great attention was paid to the diamonds, and it is quite likely that they fell in from the surface.

The method employed in the search for the precious stones is very simple:—The drift is stripped off and carted to the puddling-machines, where it undergoes a great diminution in bulk by the removal of the clay and fine sand; the large pebbles are then screened off, and the clean gravel remaining is passed through one of Hunt's diamond-saving machines. But since this apparatus depends upon the principle of separation by difference in specific gravity, it does not perhaps afford the best method which could be devised; it may answer well enough for gold and other bodies of very high specific gravity, but must certainly answer very imperfectly for diamonds, on account of their comparatively low specific gravity, viz., 3·4 to 3·5, which is nearly equalled by most of the accompanying minerals, and exceeded by some.

I should be inclined to recommend the methods employed in Africa and Brazil, since they would probably prove more efficacious.

We may now pass on to consider in more detail the mineralogical nature of the drift, or "wash-dirt" as it is termed by the miners.

From Messrs. Westcott and M'Caw's claims I obtained three different specimens. See samples Nos. 1, 2, and 3.

Wash-dirt No. 1.

This is a pale brown clay, binding together well-rolled pebbles, subangular and angular fragments of variously coloured jasper, red, green, brown, &c. Also black flinty slate, tourmaline, argillaceous sandstone, and shale, &c.

Wash-dirt No. 2.

This is rather darker in colour than No. 1, and the clay is more tenacious, the contained pebbles are of much the same character; the clay has a brecciated structure, and differs in colour in parts, fragments of it being nearly white. On the spot, when freshly dug out, portions of the clay are of a bright green colour, due to the presence of a ferrous silicate, which, by exposure to the air, absorbs oxygen, and passes into the reddish ferric silicate which imparts the red colour to the clay.

Wash-dirt No. 3.

This kind contains a larger proportion of pebbles than either No. 1 or No. 2; it is of a light colour, and much less indurated, being of a sandy nature. This also contains pebbles of argillaceous shale.

Unrolled blocks of the bed rock are met with in all the drift. In all three we find occasional minute crystals of selenite, probably of very recent origin.

During the process of extracting the diamond from the wash-dirt, the material is sized as it passes through the machines; but as it is hardly necessary to consider these sands and gravels separately, it will be as well to consider their constituents merely, irrespective of the size, since they all contain nearly the same minerals, although not in the same proportions; but as the large pebbles and boulders which are removed immediately after the stuff is puddled do differ from the finer parts considerably, we shall take them by themselves.

Pebbles and Boulders.

These consist of masses of red, green, brown, and other coloured jaspers; white quartz, common agate, black flinty slate; fine sandstone, into which manganese and iron oxides have infiltered, leaving dendritic markings between the joints. Many of the pebbles are also coated externally in the same way. Nodules of magnesite and concretions of limonite or brown iron ore, of concentric structure,—some of the magnesite still showing the limonite *in situ*. Rolled masses of hard compact brecciated conglomerate, often containing much manganese in the cement; masses of silicified wood (but this is not very common), cacholong, and greenstone. The rolled masses of sandstone, and especially the argillaceous sandstone, often assume long finger-shaped forms, and are accordingly termed "finger stones" by the diggers. The pebbles are not polished, as at Mudgee.

The list of gems, stones, and other minerals accompanying the diamond, includes the following:—

1. *Tourmaline*, or "jet stone," of the miners, occurs as rolled prisms, usually from a $\frac{1}{4}$ to $\frac{3}{4}$ inch long. They usually retain the trigonal section, but sometimes no trace of crystalline form is left, and they appear merely as more or less rounded black pebbles, often with a pitted surface, totally unlike the usual appearance of tourmaline; the blow-pipe decides their character at once, for they intumesce before it, and in other respects answer to the well-known tests. These "black jet stones" are invariably found with the diamond, and are regarded by the miners as one of the best indications of its presence.

2. *Zircon* occurs in small crystals of red and brown colours, also nearly colourless, but more commonly as rolled pieces of a brown shade. A cleavage plane is usually to be seen.

3. *Sapphire*, generally in small angular pieces and usually of a pale colour; in many the blue tint does not overspread the whole of the fragment. The *Ruby* is present, but very rare. One fragment showed the faces of an acute hexagonal pyramid and basal pinacoid. The lower half of the crystal had been fractured; it was of a red colour, but possessed a purple-coloured central mass. The fragments of sapphire are far less in size than those found at Mudgee and in New England, and far less rolled; the major part often appears to have undergone no rounding at all—thus presenting a broad distinction between the gem sand from the two places. A little corundum is found.

4. *Topaz*, as rounded fragments, and sometimes with rough crystalline outline. They are generally of a dull yellowish colour, colourless and transparent, small in size, and often apparently freely fractured.

5. *Garnet*, in small, rough-looking ill-formed crystals, of a dull red colour.

6. *Spinelle*:—Not very common, generally in small red or pinkish fragments.

7. *Quartz*:—Small prisms, capped with the pyramid, more or less rolled, transparent, and of a pale dirty red, also smoky; also small jasper pebbles, &c., &c. Amongst the jasper pebbles are some of pale mottled tints of yellow, pink, drab, brown, bluish grey, &c.; these are termed "morlops" by the miners, and are regarded by them with much favour, as they say they never find one of them in the dish without diamonds accompanying it. Their

average specific gravity, taken from a large number, is 3·25. As this is nearly the same as that of the diamond, we can readily understand their being found together. Many must be lost in the washing processes. They are oval in form, smooth, and rarely exceed a quarter-inch in length. The miners can give no origin for the name, and it does not appear to be mentioned in any works on mineralogy, &c.

8. *Brookite:*—Small flat fragments, very rare.

9. *Titaniferous iron:*—Rather common.

10. *Magnetic iron ore*, in small grains, showing an octohedral form under the microscope, coated with hidrated sesquioxide of iron, easily removed by the magnet. Gold in small particles was often found attached to the grains of magnetite.

11. *Wood tin:*—Rare, in small rolled particles.

12. *Gold:*—Fine grains and scales, present but in small quantity, and the greater portion attached to the magnetite; hence the magnet was found the most ready means of removing it.

13. *Osmiridium:*—In small brittle plates; rare.

14. *The Diamond:*—As already stated, they are for the most part small in size. Some are clear, colourless, and transparent, while others have a pale straw-yellow tint. One or two dark ones, very small, have been seen; also a greenish one. The sp. gr., as deduced from nineteen specimens, is 3·42 (the Mudgee being 3·44).

In some the crystalline form is well and distinctly shown, but others possess very much rounded faces. Some of the best crystals were those of the triakis-octohedron, the triakis-tetrahedron, the octohedron, the tetrakis-hexahedron (or four-faced cube), and the hexakis-octohedron.

No fractured specimens have been detected, but it is rather common to find them with very much pitted surfaces, and with internal black specks.

One of the Companies, when prospecting the ground, found the drift to yield as follows:—

6 loads yielded	... ...	41	diamonds
$4\frac{1}{2}$,,	... ...	143	,,
6 ,,	... ...	88	,,
6 ,,	... ...	125	,,
6 ,,	... ...	163	,,
6 ,,	... ...	89	,,
Refuse from machines, &c.	...	41	,,
$34\frac{1}{2}$		690	diamonds,

or an average of 20 diamonds per ton of stuff, regarding the load as equal to one ton. The above were obtained by the Gwydir Diamond Mining Company.

The following is an account of the number obtained by Messrs. Westcott and M'Caw from the Eaglehawk claim, up to August 26th 1873:—

400	diamonds, weighing	192 grains
420	" "	199 "
310	" "	153 "
200	" "	109 "
350	" "	150 "
1,680	" "	803 " troy.

And, as examples of the number obtained per load of stuff, the following may be cited:—

5 loads yielded 86 diamonds, weighing 32 grains.

8 loads yielded 68 diamonds, weighing 30 grains.

Up to the present no large diamonds have been found, the largest hitherto met with being one only of 8 grains

1 of 4 grains
6 of 3 "
85 of 2 "
1,587 of less than 2 grains.

No mention is made of the kind of drift from which the above quantities were obtained; they, however, afford an opportunity of roughly estimating the yield.

"It is reported from Bingera, in the *Tamworth News* of September 26, that Mr. Gardiner has obtained 115 diamonds, and that the Gwydir Company are progressing vigorously. The Giant's Knob is rich in gems, the yield averaging about 140 to the machine full, when the dirt is taken from the diamond drift.

"A correspondent of the *Tamworth Examiner*, on the 12th instant, states that there have lately been large finds of diamonds in the district of Bingera. The Gwydir Diamond Company have prospected now twenty-one pieces of land, nineteen of which have proved to be more or less diamond-producing soil, containing Grupiara or alluvial deposit, whose surface shows it to be the unused bed of a stream or river; Burgalhas, small angular fragments of rocks, bestrewing the surface of the ground; Cascalho, fragments of rocks and sand mixed up with clay and forming the bed of a river; and Takoa Carza, which are the above materials cemented together into a conglomerate mass. All the above, however, are known by the generic name of Cascalho. The masses of stones themselves, which rarely exceed a cubic foot in size, contain itacolumute jasper, and perdots and granite. These are the known indications of the whereabouts of diamonds as trusted to and found to be correct both in the East Indies and the Brazils. The nineteen successful prospects of the Gwydir Company have produced each on an average thirty-five diamonds to every six loads (of one ton) of wash-dirt, and they have now by them some 11,000* glistering pebbles, ready to transmit to Amsterdam, Paris, or some other European continental market; and are at present making extensive arrangements for the formation of three more dams and puddling apparatus on other parts of their land where good supplies of water are to be found. He also gives the following as the find of Messrs. M'Caw and Westcott:—Up to the week ending July 12, 100 diamonds; up to the week ending July 19, 113 diamonds; up to the week ending July 26, 119 diamonds —total 322."—*Herald*, August 21, 1873.

The only minerals found at Mudgee which have not yet been discovered at Bingera are cinnabar and vesicular pleonaste. Bingera in turn seems to possess one or two characteristics, such as the magnesite containing the nodules of limonite – these are perfectly spherical at times—and the "morlops" form of jasper. As these are nothing more than small jasper pebbles, careful search would probably prove their presence in most river drifts containing rolled jasper.

From the foregoing we see that the diamond in Australia is associated with sandstones, shales, conglomerates, and trap-rocks; and, perhaps, it would not be amiss just to see if this be the case in other Countries. Thus, in the Brazils, in Bahia, the matrix of the diamond is said undoubtedly to be a tertiary sandstone. Burton,† in his book on the Highlands of Brazil, states that it occurs in itacolumite, a metamorphosed palæozoic rock; but this statement requires confirmation. This, however, is known indisputably,—that they occur in the alluvial drifts of various kinds similar to those of New South Wales.

Diamonds found on the Cuddapah Hills in India are stated to occur in a conglomerate, and between Saugor and Mirzapore in a solid sandstone, and also in a ferruginous conglomerate; and in a gravel at Cuddapore containing pebbles of trap, granite, schist, quartz, jasper, sandstone, and also of the neighbouring limestone; basalt also is found near by,

And at Bangnapilly the diamond is said to have been found in a sandstone, together with corundum and magnetite, as well as in breccia, and the slates there are flinty. The district of Kumarea and Bridgepore is conglomeratic, and associated with sandstone beds. Other diamond-bearing localities of India also are conglomeratic.

In Russia, too, conglomerates seem to be the present receptacle of the diamond; iridium is there associated with it.

Borneo—again here we find it in a conglomerate containing quartz, &c., and associated with gold, platinum, and osmiridium.

* (1,100 Query ?) † See vol. II, p. 144.

Then too, in Africa, they are found in a drift, and usually within a few feet of the surface, from 3 to 9 feet, and rarely down so far as 30 feet.

Here, again, one of the main features of the district is the presence of sandstone, either of Upper Silurian or Devonian age; trap is also present, and a conglomerate or breccia containing boulders of granite, gneiss, mica schist, porphyry, sandstone, jasper, slate, agate, &c.

In conclusion, we are still as much in the dark respecting the origin of the diamond or even its true matrix, for no good proof has yet been offered on this question, as we have seen in nearly all cases it occurs in an ancient river drift, and is usually associated with sandstones, conglomerates, and trap rocks; neither do we know the matrix of the sapphire, zircon, &c., which are usually much rolled, as if they had been borne a great distance. The sapphires from Bingera seem to have undergone but little alteration, and consequently have not travelled far, so that perhaps we may soon light upon their source and that of the diamonds simultaneously. Bingera certainly seems the most hopeful locality to elucidate this point of any at present known.

Before closing this paper, I must express my obligations to Messrs. Westcott and M'Caw, and to Mr. Dougherty of the Gwydir Diamond Co., for their great assistance in procuring and sending me suites of specimens illustrating the various rocks and minerals of the diamond workings, and wish them success in their endeavours to open up this industry, which I hope will prove to be a new source of wealth to New South Wales. And this result appears to be highly probable, since the whole of the above-mentioned valuable finds have been made by the exertions of but a few, perhaps not more than five or six workers.

APPENDIX.

REPORT ON THE DISCOVERY OF DIAMONDS AT BALD HILL, NEAR HILL END.

University of Sydney,
December 5, 1873.

To the Hon. the Minister for Lands.

Sir,

In reply to your request of the 2nd instant, I have the honor to furnish you with the following particulars relating to the mineral specimens from Bald Hill, near Hill End, which accompanied your communication.

Diamonds—Three in number ; the largest of them is in the form of a six-faced octahedron, rather flattened, owing to four of the groups of faces being more highly developed than the remaining four. The faces and edges are rounded somewhat, but this has not been caused by attrition ; diamonds often appear as if water-worn, but in reality this is seldom the case ; the rolled and water-worn appearance is due to the fact that the diamond usually crystallizes with curved faces and rounded edges. It is clear and colourless, and perfectly free from all visible internal flaws ; the surface is likewise free from flaws ; but scattered over some of the faces are a few minute and insignificant triangular markings, but these are quite superficial and will disappear during the ordinary process of cutting. It possesses a specific gravity of 3·58, and weighs 9·6 grains (Troy), *i.e.*, a little over three carats. It is generally calculated that diamonds lose one-half their weight during the process of cutting and polishing ; and their true value cannot be ascertained until this has been done. The diamond next in size possesses the same crystallographic form as the one above mentioned, but is not so much compressed. It has a weight of 4·5 grains (Troy), or nearly one and a half carats. It has a chip on one edge, and contains a speck of foreign matter. It is a straw-colour. The smallest diamond weighs about half a grain ; it has the form of a six-faced tetrahedron, and possesses a high lustre, but is rather off colour.

Accompanying the diamonds were two small specimens of gem sand.

Gem Sand No. 1.

In this the following substances were found to be present :—

I. *Corundum*—When blue this is known as the *sapphire*, and when red as the *ruby*.

- (*a*) *Common Corundum*—Present in small fragments of bluish, greenish, and grey tints.
- (*b*) *Sapphire*—In small particles of a blue colour, some so dark as to appear almost black, and others very light. Some of the fragments still show their crystalline form, viz., a hexagonal pyramid, but most of them do not, and are either much rolled, subangular, or angular in their outline.

The ruby is absent, but probably would have been present had the sample of gem sand been larger.

II. *Zircon*—Plentiful, usually in the form of much rolled pieces. Generally of a brown colour, sometimes red, and at others nearly colourless. The small and nearly colourless crystals possess a very high lustre, almost equal to that of the diamond, so that they might readily, without careful examination, be mistaken for that gem.

III. *Quartz*—Usually as small, well-rolled grains, either colourless, milky, or yellowish. Sometimes as hexagonal prisms, capped with the hexagonal pyramid. Jasper of various colours, such as red, yellow, grey, also occurs, together with black grains of flinty slate.

IV. *Rutile*—In angular fragments, still showing traces of crystallization. Distinguished by its brown colour and metallic lustre, and by the presence of numerous fine striæ on the faces of the prism. It very much resembles tin stone in appearance. In composition it consists of titanic acid.

V. *Brookite* also occurs. This is another form of titanic acid. Rutile crystallizes in striated tetragonal prisms, whilst this crystallizes in tabular forms belonging to the rhombic system. It is present in small quantity, in the form of flat irregular plates, brown or grey in colour.

VI. *Topaz*—Present in small rolled and angular fragments, colourless, and in pale tints of yellow and greenish blue. The latter coloured topaz is often erroneously termed the aquamarine.

VII. *Beryl or Emerald*, doubtful, but one or two very small fragments resembling it.

VIII. *Garnet*—Small, rough, common garnets, of no value.

IX. *Tourmaline*—A few rounded pieces, but none showing the crystalline form, which is that of a three-sided prism.

X. *Gold*—Present in the form of scales.

Gem Sand No. 2.

This consists of larger grains than No. 1, in fact they are small pebbles.

I. *Quartz*—Present principally in the form of jasper, of various colours, red, brown, green, yellowish, &c., &c. ; also variegated. Colourless and yellow quartz pebbles are also found, together with black pebbles of flinty slate.

II. *Corundum*—Present as common corundum, and as the sapphire.

III. *Brookite*—Same as gem sand No. 1, only in larger pieces.

IV. *Topaz*—Clear and colourless ; also tinted.

From the foregoing it will be seen that the Ball Hill gem sands very closely resemble those from Bingera and Mudgee.

None of the gems contained in the parcels submitted to me, with the exception of the diamonds, are of any commercial value, except for grinding and polishing purposes. Still, they are of great value as indications, for where such occur there is every prospect of finding others of larger size and better quality.

An examination of the original "washdirt" or "drift" might yield valuable information, and larger samples of the gem sand will probably be found to contain such minerals as iridium, titaniferous iron, tin stone, magnetite, &c., like the Bingera gem sand.

I may, perhaps, mention that, in 1867, a brilliant of the first water and without flaw, weighing one carat, was worth about £20 ; if weighing one and a half carat, about £45 ; and if two carats, about £80, and so on ; but since that time the prices have probably undergone much change. According to the September number of the British Trade Circular, the prices ruling for Cape diamonds, uncut, are in proportion much lower than the above.

I return the diamonds and gem sand per bearer.

I have the honor to be,
Sir,
Your obedient servant,

ARCHD. LIVERSIDGE.

OUR COAL AND OUR COAL PORTS.

By James Manning, Esq.

[*Read before the Royal Society*, 6*th August*, 1873.]

The interesting subject of our coal export has lately occupied much attention, and has given rise to the publication of various letters in the newspapers, to some of which, namely to those signed "New South Wales" and "Veritas," I was about to reply in the same form; when it was suggested that the subject might be made available for a paper to be read before this Society, with a view to its first inviting discussion here.

The subject of our coal and of our coal ports being of all-absorbing interest at present, I trust I may not be out of place by giving expression to those reflections which have long been rife in my own mind, but which I had not committed to paper until now.

Then, with a hope of rendering some service to this really very important cause, I will venture to lay before you the following remarks and suggestions for what they may be worth.

I quite agree with the writer "New South Wales" that Newcastle should not be the only outlet for our Northern coal. Port Stephens and other places must soon come to our assistance, as the trade gradually grows into magnitude; but I think it would be a great folly to try to induce the Government or the public to believe that the entrance into Lake Macquarie can ever be made available for large shipping. I presume, from my own long experience in bar harbours, that if the sum of £15,000, which is said by "New South Wales" to have been declared to be "enough for removing all obstacles to the navigation of that harbour," was multiplied by ten, it would still be insufficient, and would only lead to ultimate disappointment.

Little or nothing of value can be done to any bar harbour without enormous expenditure. I have always advocated that it is wiser to meet Nature in such positions, by leaving her work alone, and by suiting the vessels to the port, instead of attempting to suit the port to the vessels. Thus, Lake Macquarie may possibly be utilized upon this principle (or at most to render only some slight assistance), by adopting the existing entrance to the neighbouring valuable coal fields, and suit the boats to the place by the adoption of the following measures.

The great facilities presented at Lake Macquarie for utilizing very thick, easily worked, and excellent coal, would seem to warrant the adoption, by an enterprising Company, of a small fleet of the so-called "West Hartley" flat-bottomed vessels, with moveable centreboards—these to trade in and out at high water by aid of twin screw steam launches as tug-boats. Such vessels are made on about 3 to 4 feet draught of water, whilst carrying 150 tons of coal.

With a capital invested in an adequate number of such vessels and with two or three such steam-launches as I describe, a very large and, I suppose, safe trade might be done in thus conducting coals to large ships in the Newcastle bay, Port Stephens, or in Port Jackson. In such case it would be desirable to have one of the little screw tugs in the Newcastle harbour to bring the vessels in and out, and to utilize its steam power to help the discharging of coals.

Possibly some of the Lake Macquarie coal may be shipped from the seaward side by means of jetties having some southern protection from a promontory in same form as at Bulli; but to endeavour to connect Lake Macquarie by rail with Newcastle would avail nothing, as at that port they can give the fullest employment to their steam cranes, staiths, and every yet available means for shipping, by opening out more of their illimitable supplies of coal along the existing Great Northern Railway within easy distance of Newcastle.

So much then for our possible future exports from Lake Macquarie; and now for Newcastle.

I, for one, quite rejoice at the fact of one ship having lately left Newcastle in ballast for California. The captain or agent for that ship ought to be rewarded with a suitable present in the form of a chronometer, to be subscribed for at general expense, for having thus taught our authorities a most valuable lesson, namely, the due appreciation of the value of *time;* and for having adopted the most effective manner of showing the public how wretchedly we are behind the times in our appliances for availing ourselves of Nature's *biggest* gift in our possession in these Colonies.

The public are, however, in some measure to blame for their present difficulties, by reason of the outcry that was made some years ago against the existing Government, for their reputed "lavish expenditure at Newcastle."

With reference to Newcastle, I beg to submit the following suggestions, as being possibly the best means of rapidly increasing our shipment of coals as fast as required; and, by such method, to reduce the existing waste of capital in ships and the waste of labour in idle ships' crews.

My idea is that the Government should at once send to the winds the avowed and too paltry idea of maintaining a sort of Government monopoly of the haulage of coals from the different collieries to Newcastle, and by prompt action encourage all the colliery Companies to expend their own capital in the adoption of private railways to the nearest shipping points on the river above Newcastle, in the same way that the Waratah Company now enjoys the use of its own line, with its own locomotives to and from their own shipping wharf and twin "shoots" at the so-called "Port Waratah" on the river bank, four miles above Newcastle.

The "Wallsend," the "Old Lambton," the "New Lambton," and the "Co-operative" collieries are all obliged to crowd their coal on to Newcastle wharf and to the steam cranes; whereas "Waratah," by some happy political fluke, enjoys the right of having their own railway line, and can convey all their large "output" of coals to the ships at their own wharf at a working expense for the line of only 2¼d. a ton, over a distance of four miles from the coal pits; whilst all the other Companies, close to them, are obliged to use the Government locomotives for the haulage of their coal to Newcastle, at an expense of not less than 10d. for the shortest distance.

Thus it would seem that the Government should either, by themselves, promptly connect the above-named collieries by rail by the present crossing of the Waratah line over the Great Northern line; give ample wharfage and shipping accommodation just below the river boundary frontage of the Waratah Company; or, let the Companies have the right of doing all this themselves. Then to let each Company carry on its own shipping business in the same manner as the "Waratah" does.

But as only vessels of from 500 to 700 tons can go so far up the river, I would suggest (the above important arrangement being first conceded) that it should then become a port regulation that all vessels and steamers under (say) 600 tons, should be compelled to take their coals from the new wharfs on the river, and thus leave the Newcastle wharf and its steam cranes for the sole use of the larger ships, to be supplied as at present from the collieries by the direct Great Northern trunk line; the A. A. Company's coal supplies from their "Borehole" mine being necessarily excluded from this restriction, as that colliery, being to the eastward of Newcastle, is quite differently situated from the others.

If this suggestion should find favour, I foresee great extension of business at Newcastle and its neighbourhood, by the gradual association with every colliery of iron or other smelting works on the river bank, or near their pits, whereby to utilize their own small coal in the same manner that the Waratah Company dispose of over 30,000 tons each year of its small coal to the great Waratah and Moonta Copper Smelting Works; and which works, I may here mention, smelt annually on the river bank at Port Waratah no less than 25,000 tons of poor Moonta copper ore from South Australia, which averages only 6 per cent. of copper, and yet pays well.

I trust that these suggestions for Newcastle may find more favour than the only other plan I have heard of for facilitating the coal shipping trade—namely, by the costly utilization of Bullock Island, between Port Waratah on the river and Newcastle itself. By this plan I think the increased accommodation could be made available in less than a quarter of the time, and for less than a quarter of the expense, than by the proposed adoption of Bullock Island—for the present, at any rate. I may mention here that it is probable that in remoter days, and when Sydney itself may require relief from overgrowing trade, a railway branch line from the Great Northern will cross the Hunter River at Hexham, and go direct to deep water at Salamander Bay (Port Stephens), by an easy line, not exceeding twenty-five miles, and then make that port the great outlet for all or most of the general northern trade, and for coal ships of greater draught than can enter the splendid harbour of Port Jackson.

So much, then, for our northern coal ports. But having headed this paper "Our Coal and our Coal Ports," I must add a few remarks on the subject of our coal fields and coal ports south of Lake Macquarie, herein expressed.

Coming south for the utilization of more of our wonderful coal fields, it is, I think, extremely doubtful whether our fine port at Broken Bay may ever come in for coal shipments. Broken Bay lies too near the inverted apex of the great coal basin on our coast (between Coalcliff and Bulli to the south, Lake Macquarie to the north, and Bowenfels and Nattai to the westward) to sanction the belief that the deep sinking that would be required there could be warranted. In the Old World alone could such a parallel case be profitably availed of.

Farther south, and below Sydney, we come to the north-easterly dip and out-crop of the great coal basin at Coalcliff, Bulli, and Wollongong. The illimitable supplies of coal immediately at those places and on the seaside can only be used by the adoption of steam colliers and small vessels plying to and from Sydney or Melbourne. Such shipping appliances from that direction can never do more than make a small impression on the future demand from all parts of the world.

About forty miles south of Wollongong, at Kangaroo Creek, in the Shoalhaven country, we have one of the finest seams of coal existing anywhere, being 12 feet thick. It is, however, extremely improbable that these coals will be used for export, as the access by the Shoalhaven River, or by Jervis Bay, seems to be impracticable.

Again, and still farther south, and some thirty miles from Jervis Bay, we find the last southern remains of our grand coal measures; where insignificant seams of coal and of oil shales crop out in the Ulladulla country and at the head of the Clyde River. The oil shale on the Clyde, being the oldest and the last evidence of the "wedging," or "pinching out," of the great palæozoic formation, which, being seen here for the last time, seems to point to this position as the last resting place, south, of the true coal measures.

South of where the coal terminates, and along the remainder of our coast to our boundary at Cape Howe, a distance of 100 miles, there is no evidence of the existence of any old carboniferous formation; because below Bateman's Bay and the Clyde River we come to the Moruya granite, which continues to the parallel of Montague Island and the "Dromedary," where passers-by may observe the termination of the granite; as the southern half of that little island—of some four miles in circumference—is composed of granite, whilst its northern half is composed of basaltic rocks.

Between this parallel of Montague Island and Cape Howe we have an almost uninterrupted prevalence of porphyritic and sandstone cliffs, pierced only in one place along the coast, for about one mile in width, by a very extraordinary upheaval or deposit of a tertiary, if not of a still younger formation. This spot exists 16 miles north of Twofold Bay, at a place called "Boonda." Here at 60 feet above the sea level there exists a regular seam of carbonaceous substance, which is fully 7 feet thick, is based by most excellent fireclay, and capped by kaolin and fine sands, and is visible along the whole of this extraordinary formation and section of the cliff, in a perfectly horizontal position with the sea, having a dip of two degrees, or of 1 foot in 30, inland, corresponding in dip to the true coal seams at Wollongong.

This coaly matter is, however, only a *lignite*, which, although containing 65 per cent. of inflammable matter, is without commercial value, and is replete with sulphur and arsenic; and as no fossils (except recent-looking plants) of any kind have been found there, even by the assistance of borings (effected and carefully watched by myself) through white sands and white clays for 120 feet below this otherwise perfect seam of apparent coal, it thus proves that this singular formation is not belonging to the old and valuable carboniferous measures, but seems to resemble very much the lignite deposit found at Lal Lal, near Ballarat, in Victoria.

Leaving this geological *lusus naturæ;* the porphyritic and red sandstone and slaty cliffs continue all the way past Merimbula, Twofold Bay, and Green Cape to Gabo Island, the Genoa River, and Cape Howe, where our boundary with our Victorian neighbours is constituted by a most remarkable formation wholly composed of the most durable of all rocks—namely by the red granite of Syene, which has its exact parallel in the well-known quarries of ancient "Syene," at the first cataract of the Nile, beyond Thebes; the Egyptian differing from ours at Gabo and Cape Howe only by being coarser-grained. From this time-defying rock of Syene came the great monoliths of the ancient Egyptians, in the wholly unworn shapes of Pompey's Pillar and Cleopatra's Needles.

Of such stone is our extreme south boundary composed. It is a rock which, by reason of its being composed of felspar and and hornblende only, is so massive and hard, and is, of all other rocks, the one which is freest from fractures or clefts; and therefore is the only kind of stone from which the Egyptians could have formed and transported their wonderful monuments, which will remain unchanged to the end of time, and are already over 2,000 years old.

Our northern territory towards Queensland carries on coal measures from Port Stephens inland, and in patches all the way to the Clarence and Richmond Rivers, where shipments may be made.

Beyond our northern boundary with Queensland, coal passes through the whole of that Colony in disjointed patches, right on to Gladstone and Rockhampton at least. As yet, however, Queensland cannot avail of its wealth in coal as we do, nor is that grand Colony ever likely to be able to do so, because of the relative positions of the existing coal measures with the peculiar ports of that country. The continuation of the Ipswich Railway to Brisbane, may, however, lead to some important results in the delivery of coals from Ipswich, Goodna, and the Bremer River, if deeper and better seams are found than those yet worked. Some coal may also come down the Toowoomba Railway line from the neighbourhood of Helidon; and, if modifications to the line are made from Warwick downwards, by means of more "loops," it is not improbable that the superior coal from Allora, near Warwick, may be able to bear the freight to Brisbane. Also, I would mention that should the coal that has lately been found near Gladstone be within a payable distance to bring it by rail to the head of the tidal waters at the limestone falls on the Calliope, it might lead to some important results, as it could be brought down from the falls in barges and in vessels of light draught, and be conveyed to ships lying in Port Curtis.

Having dealt generally with the coal measures along our coast, I have yet to impose upon you some remarks upon the all-important consideration of *Port Jackson*, as being the means at command for increasing our coal export to a practically unlimited extent.

The anonymous writer in *Sydney Morning Herald*, with his *nom de plume* of "Veritas," has in his letter made the strong assertion in italics that Newcastle is the only legitimate coal port in the southern hemisphere. These are his words:—"Newcastle physically is so situated that it can fear no other rival (unless Port Stephens), and that it is fallacious and the height of absurdity to imagine that a Government would be so short-sighted and narrow-minded as to try and cripple *the only legitimate coal port in the southern hemisphere.*

To these strong expressions of "Veritas" I beg to take exception, and I fear not to throw down the gauntlet before him, and boldly to assert it as my opinion that Nature points out that the central and incomparably superior harbour of Port Jackson must soon be at the head and front of our foreign coal shipping operations, and for the following reasons:—

Firstly. By consequence of large increase of steam colliers, and perhaps of West Hartley centreboard vessels acting as tenders and coal-feeders to large ships in Port Jackson.

Secondly. By consequence of necessary railway modifications, which must soon take place to facilitate the vast increasing traffic on the Bathurst and Goulburn lines, which will thus open an easy doorway for the admission of that enterprise that will lead to the transport of considerable quantities of coal from the western and south-western portion of our coal basins—namely, from Bowenfels and Wallerawang to the west, and from Nattai and Sutton Forest to the south-west.

Thirdly and chiefly, by a new source of supply—namely, by a *direct* and cheap acquisition of coal from the back of the Bulli country, under thirty-five miles in a straight line, from Sydney, south, over a road that can be made accessible by means of a double line of broad gauge railway along the Illawarra coast road, thence by the "Bottle Forest" country, on to or over a dam at George's River; or even by a railway pontoon or other bridge over the same place and water, and thence straight on to the nearest deep water in Port Jackson, crossing the Great South-western Railway, over the deep cutting between Newtown and Petersham to staiths on North Balmain near Elliott's Chemical Works, directly opposite Cockatoo Island, in the expansive and deep waters of that part of the Parramatta River which is immediately between and close to the two large dry docks and the great engineering and ship-building establishments.

By this route, from the first available coal taken *at the dip*, some two or three miles inland from the sea, and with probably not more than 600 feet of sinking, the coal can be brought direct by easy gradients to three and five fathoms of water, by a total distance not exceeding forty miles.

Such being the case, the haulage at even a half-penny per ton per mile, as the proved working cost of colliery railways, including wear and tear of rails, would make only 1s. 6d. per ton. Thus, under these circumstances it might not be unreasonable to expect that from 5,000 to 10,000 tons a day could arrive daily from that quarter alone, for the supply of some of the foreign ships that would come with confidence to our great coal country, when we can give quickest possible dispatch frem either Newcastle or Sydney.

Apropos to the dam at George's River above noticed, I am quite aware of the objections that will be raised respecting its formation, as being, in my opinion, the best means of making a highway for the transit of the southern coal to Sydney.

It is not in the province of this paper to touch on the subject of any water-supply scheme in this direction, unless it be to show briefly that the erection of an immense dam at George's River should be no stumbling-block to the coal enterprise; and therefore I think it necessary to say here that I could show good cause why such a dam might be made effective in every way, and to yield of itself a compensating interest on whatever capital it might cost, if it was undertaken by a public Company, under liberal concessions from the Government.

Entirely in view of this southern coal enterprise, I believe that the dam might be made highly remunerative in the mode suggested by me in a letter I addressed to the public prints some months ago, even if it were only to give the motory power I pointed out as being available thereby.

For further substantiation of this assertion, I would refer this Society to a recent number of "Dingler's Polytechnical Journal," published this year in Augsburg (which came under my notice only a week or two ago, and which I now lay on the table). The article is written by the celebrated German engineer, G. Delabar, now of St. Gallen, and contains a description of the grand waterworks at Freibourg, in Switzerland, as conceived and carried out solely by the genius of Herr Ritter, an engineer of that town.

On an expenditure of 2,000,000 francs, of privately subscribed capital, this most able man has succeeded in just completing (within four years) works that must be among the greatest wonders of the present age. By a dam of 64,000 cubic mètres, equal to 82,450 cubic yards, made across the River Saane, which flows through a deep gorge, and costing 340,000 francs—equal to £13,600—for the embankment alone, that is built of cement and pebbles. By this means and by the further aid of hydraulic turbines, he has obtained motory action up to the extent of 2,600 to 4,000 horse-power.

Primarily he applies a part of this enormous power by using 300-horse power of the whole to the working of four of Girard's double force pumps, to drive water up 525 feet to a reservoir made on the highest land, at Parolles, near the town, at a distance of about a mile and a half, transmitting each minute, by pipes of 15-inch diameter, 690 gallons of crystal water, for drinking purposes only, and which water is all filtered at the waterworks by the dam, prior to its entering the main.

The filter works alone cost 34,600 francs, £1,380.

Her Ritter, as Engineer and Director of his Company, farms out the rest, or as yet only a part of the rest, of this great water power, to various manufactories which have been recently erected around these works. This he does by means of the new and so-called telodynamic action, transmitted to long distances, up to half an English mile, by endless wire ropes carried through tunnels and along valleys in various directions by ingenious contrivances, and over stone pillars capped with rollers to keep the traction straight—one of which pillars is 60 feet high.

For this he obtains a rent equal to 150 to 200 francs per annum for each horse-power, according to quantity used—thus, if we include the 300-horse power for pumping the drinking water for the town, at same rate of rent as the rest, it will be found that he can obtain an annual rent on the whole 4,000-horse power available, and at only the minimum price of 150 francs for each horse-power, equal to 600,000 francs, or £24,000. But let us now reduce the returns by one-half, or as upon only 2,000-horse power used, and he would still have £12,000, which would be equal to 15 per cent. on the total subscribed capital of two million francs, or £80,000 for the entire undertaking.

The Company afterwards extended their capital, and purchased 1,400 English acres of forest in the immediate neighbourhood, for the sum of 1,400,000 francs, such purchase being made from the town of Freibourg with the consent of the State.

The Company is therefore now called the "Société Générale des Eaux et Forêts," and it retains for its own use no less than than 300-horse power for the effective working of their own large sawmills, with twelve saws and other machines. This power is transmitted to a distance of half a mile by means of the telodynamic cable.

The rest of the motory power is all farmed off to the numerous manufactories started, and which are already seven in number, one of which alone demands from 50 to 150 horse power, and is applied to an immense railway-carriage manufactory that sends its carriages to all parts of the Continent.

This establishment also absorbs one-seventh part of all the timber sawn by the parent Company.

The cable gives the necessary power, in several directions, to the following named industries, namely :—

A large foundry and engineering works, 20-horse power. A paper manufactory. Chemical manure works, 20-horse power. A flour mill. A stone-cutting and stone-polishing works. Pottery works, and gypsum-grinding.

This was all that was in work, or in progress of erection, to the end of last year. The rest of the available power was expected to be soon farmed off.

The farming of this immense motory power is, however, only one source of profit out of many others that Herr Ritter obtains for his flourishing Company by his great engineering design, to wit:

He has obtained fifty years' right to the artificial fisheries produced.

He is erecting ten ice-houses, each of which is to contain 200 waggon-loads of best ice, to be harvested each winter from off the artificial lake, and which ice will be transmitted in summer to all parts of the Continent by rail.

He has obtained all the sewage of the town free of cost. The whole of this he removes in a diluted state, just fitted for irrigation purposes.

These three last-named industries he sub-lets to another branch Company, which is called "Société de Pisciculture, Glaciers et Irrigation." In this last undertaking, as in all others, Herr Ritter has obtained the greatest fame, and has achieved important results by an especial appliance of his own, used here at Freibourg for the first time—namely, by sending off all the diluted sewage to two suburbs, north-west and south-west of the town, by means of *compressed air*, and without the aid of any pumps. The sewage and the water is received into two large reservoirs at each place, from whence it is all utilized, with wonderfully fertilizing results, whilst the town is thereby kept in a state of great purity and healthiness.

Herr Ritter is also working off (in prudent quantities) the forest timber of the large acreage purchased for this purpose. The forest being situated above the dam, and now on the borders of this long artificial lake (measuring 160 acres) he is enabled to float the logs across the lake and down to the dam, from whence he conveys them by a tramroad for half a mile, to the large saw-mills which are annexed to the great railway line. The profits on this timber enterprise alone, and consequent on the cheap motive power at his command, yields no less a return for this Company than thirty francs on every cubic mètre of timber sawn into various forms.

Besides the above industries there are various minor means applied for increasing rental, namely, by large washing and bleaching establishments, and by swimming and skating schools established on this artificial Lake Perolles—added to which the shores of the lake are embellished with pleasure-grounds and tea-gardens, for the healthful recreation of the inhabitants of Freibourg and its increasing number of tourist visitors. The consequence of all this is that Freibourg bids fair to become a place of world-wide fame and of the greatest industrial activity, whilst all the properties around are doubling in value through that prosperity which has been brought about by the genius of one person, who has conceived and carried out all these designs.

Before leaving this interesting subject, I would wish to quote the words of the eminent engineer Delabar, who says of these works :—

"Thus are the wants of the inhabitants of Freibourg thoroughly cared for—with pure drinking water for their houses, and with water power for their industries, given them to such an extent that their great advantages will become the envy of far more important towns. In fact, nothing more noble can be conceived than these water-works, which are perfected for the sole purpose of making water subservient to man's use, for his home, for his fields, and for his industries in life."

Such is the high commendation passed by one great engineer upon the merits of another in the same calling !

And now I take this opportunity of venturing to give the following suggestion to business men, and to house and property holders of every class in and about Sydney—namely, that they should constitute themselves into a Society, that might go forth accredited to Herr Ritter (after the Freibourg manner) as the "Société Générale Australienne des Eaux, Charbons et Irrigation," and that such Society should send him an invitation, under very liberal arrangements, to visit this place for the purpose of inspecting our natural resources, and to advise with such Society as to the best method of utilizing Port Jackson in connection with our coal-fields south and north of Sydney ; how to act for our future and permanent water supply, and how to dispose of and utilize our valuable sewage.

I have thus digressed considerably from our coal and coal port subjects, for the express purpose of showing how, by the development of somewhat parallel resources at our command (less the superior advantage enjoyed by Freibourg of a permanent Alpine stream) we can make a self-supporting, though expensive, railway dam across George's River, whereby to connect the southern coal-fields with Port Jackson.

In our case, and in order to bring about such results, we should seek to raise a head of water inside the dam not exceeding four feet above highest spring tides ; and, this important action being made a success of, we could obtain quite as much or more water-power at George's River, for nine months of the year, in the mode I have before stated, as they now enjoy at Freibourg. Our power would thus have to be applied by all the river waters being brought to fall over a graduated rocky weir, to give direct action on undershot water-wheels, in the same manner as is done at Geneva—where shafting applied to *one* single large water-wheel, driven by the Rhone, gives power enough to pumps to raise water above the highest levels of, and for the supply of all Geneva.

Many will exclaim against the magnitude of, and the risk of failure of making such dam (as I propose), tight enough to obtain only four feet head of water. But what is it after all? The affair would not exceed 350,000 cubic yards of stone and clay work, and should be viewed in the light of a *national* undertaking that would be worth all the risk of failure of making it watertight—when there would be, under the worst circumstances, a certain resultant benefit to Sydney of procuring a direct railway communication with our Southern coal-fields and with all the Illawarra lands.

But admitting the success of such undertaking being made watertight and the inner waters being rendered fresh, then the beneficial consequences would be beyond price. Motory power could be obtained to a very large extent as long as there was inflow from those tributaries of George's River, which drain such an enormous extent of country; and Sydney might have a never-failing supply of water, at a cost that would be less than *nil*, if the existing water reserves were sold for that necessary extension of the city that would be required on the establishment of a railway line and a great coal trade, &c., with Illawarra.

And now, finally, and in anticipation of Sydney's future greatness, I venture to suggest that twenty years more may not pass away before it will be found to be necessary to meet the immensely increasing Hunter River and Northern trade (quite irrespectively of the coal trade at Newcastle), by connecting the Great Northern Railway Line from Maitland with Sydney, by another line that will intersect rich coal-fields, will cross the Hawkesbury by a tubular, a suspension, or by a high-level bridge, and arrive by easy gradients at the North Shore, opposite Cockatoo Island and North Balmain.

Sydney would thus have another prominent source for increasing the future export of coal when it may be required, and thus complete her ability of shipping best house coal from the *North*, best steaming coal from the *South*, excellent and condensed steaming coal from Bowenfels, bright house and gas coal from Wallerawang, and the finest oil shales in the World from the *West*.

But even without this ultimate connection of Port Jackson with the Northern "outcrop" of our immense coal basin, it would follow that if the easy approach to the Southern coal-field by rail to Sydney be brought about, we should render such great support to a large foreign and home trade, and to ships coming with cargoes or in ballast, that, so far from our prominent coal ports —Sydney, Newcastle, and Port Stephens—becoming injuriously "rivals," we should then jointly, and in all probability, barely keep pace with the increased demand for foreign ships, for our large iron and other smelting operations, and for the manufacturing demands which would arise in the neighbourhood of George's River, Cook's River, Newtown, Botany, and the general suburbs of Sydney.

Such conditions being brought about, I would ask, finally—would not the realization of these proposals cause Sydney to become incomparably the greatest emporium of the Southern Hemisphere?

OUR COAL AND COAL PORTS.

BY JAMES MANNING, ESQ.

[*Read before the Royal Society, 3rd September*, 1873.]

I BEG briefly to mention that I had it in view to attach an appendix to my late paper that was read before this Society last month. I had desired to postpone such action until the next meeting in October, partly because I am not prepared at present, and partly because I thought that I might by that time see and hear of all objections that might be raised against the proposal for the development of the Southern Coal Fields, and of the Illawarra lands generally, and that I might gain further information with reference to the proposal.

In my late paper I omitted a few interesting matters, from a fear that it was already too long, and that I might weary the Society by any prolongation.

The newspaper discussions that have followed on the reading of my paper before the Society, give evidence that I need not trouble you again with any replies to objections raised. I have had the pleasure of setting the ball going. I must now leave it with others to play the game out.

But as I may consider the opportunity thus lost of affixing an appendix to my late paper, I will be contented by availing myself of this evening's meeting to say that, in a late visit that I made to Illawarra, I discovered how I could amend the plan, at the Bulli and Wollongong end, in such manner as to enable us to obtain the coal for quite eighteen-pence per ton less than by sinking for it at the dip behind the Bulli Mountain, as I had proposed; that is to say, by taking the line to the east of the coal range, and by this means getting the coal by tunnels at the outcrop.

Hereby I am thoroughly convinced that Nature could not have done more for Sydney and for this Colony than she has done by this great gift, namely, by that of this Southern Coal, aided by the most extraordinary natural facilities of bringing the same from Illawarra and Port Hacking Creek by one of the easiest railroads ever to be made over and along a mountain coast. This route will present no great engineering difficulties, and may not have any heavier gradient than 1 in 45, which will enable an ordinary railway engine to bring probably 500 tons of coal to the deep waters of Port Jackson by one journey and in less than two hours from the back of where the first coal crops out on the Bulli Range, near Stanwell Park, and Coalcliff.

Here I beg also to call attention to a small arrangement of mine, brought about for the express purpose of unveiling to the public gaze by ocular demonstration, as to how great and boundless is the coal at their hand. On the acknowledged principle "that seeing is believing," I arranged for one of the officers of the Illawarra Steam Company's vessels to go to the top of the tower of the new Town Hall (the ascent to which is as yet only by ladder), and to report to me all he could see to the south from that point. He returned and stated that he had had a distinct sight of the whole coast passing Botany Bay, Port Hacking, Coalcliff, Bulli, Mount Keira behind Wollongong, and all the way further south (twenty-five miles) to the high Saddle-back Range behind Kiama, in which I may mention there exists enormous seams of excellent coal.

Thus my sailor friend had a clear gaze at over forty miles of magnificent coal country, and along the whole line of which the coal crops out to the eastward, and may be worked by tunnelling in the same manner as at Bulli, and supplied to the railway beyond Bulli proper by means of self-acting inclines.

I have spoken of forty miles of coal country showing its eastern outcrop, but I may add that in fact the coal seams extend at a workable depth under ground, up to ten miles of the Sydney side of Coalcliff (which is itself about thirty miles south of Sydney), and at which place the outcrop dips into the sea.

Also I would mention, that even beyond the lofty "Saddle-back," south of Kiama, the coal fields extend towards Shoalhaven, but are out of view from the tower of the Town Hall.

From that elevated position this ship's officer could also see the straight direction of the proposed railway line from Wollongong, Bulli, Coalcliff, and along the leading and easy descent of the Bottle Forest and Port Hacking Creek range on to George's River, and finally by its straight course direct on to the deep waters of North Balmain, opposite Cockatoo Island.

After this unveiling, I ask shall we let these vast resources lie dormant? And shall we leave unopened to our Sydney population the delights they would derive from having access to Illawarra by one of the most picturesque and charming railway lines that the world could produce? But a question of far greater importance is, shall we postpone unnecessarily that development which would speedily place Sydney and New South Wales in the position of proud commercial pre-eminence?

APPENDIX.

OUR COAL AND COAL PORTS.

THE seeming favour with which my papers on coal subjects were received by the Royal Society, and by the public, caused me to be anxious, lest by any possibility I might have made wrong or delusive statements in my advocacy of railway communication with the southern coal fields, in order to command the coal trade from those parts in connection with Port Jackson. Such anxiety made me determine to visit the localities, so as to satisfy myself that I had not been holding out a phantom or a shadow for the grasp of public enterprise; and having been to Illawarra by the proposed road for the railway line, I think it is my duty to make a statement of the result of my inspection. This I will therefore do, and am happy to be able to preface my remarks by saying that such inspection has been highly satisfactory, and far exceeds my preconceived ideas of the practicability and value of the proposed project.

I was accompanied by a friend. We left Sydney and passed by Cook's River, and over George's River by the punt. We then continued on our course by the Bottle Forest line that leads to Wollongong, when we rode at a brisk pace over a remarkably easy yet rising gradient; and in the course of 21 miles I took and noted nineteen observations of the aneroid at various defined spots, and at positions which I determined by marking trees and numbering such as I went along. The result of this went to prove that the rise was extremely gradual throughout the whole line of 21 miles, and which reached an elevation of 864 feet for the whole distance, as the approximate and barometrical height.

The result gives only a rise of 42 feet in each mile, or 1 foot in 125; or, practically, the whole of this excellent road can be cantered over from end to end, except where there are a few light depressions and corresponding rises, in the last 3 or 4 miles; but the whole of which can be avoided by diverging the proposed railway line a very little to the eastward and along the top of the eastern slopes of the Port Hacking Creek, which are almost immediately by the Bottle Forest Road. The whole line is perfectly firm, is based with sandstone, and for some miles it is bestrewed with sunken clay ironstone ore, which ore I judged, from its ponderousness in the hand, to be quite rich enough for smelting, when the coal and it can be cheaply brought together.

During the whole of these 21 miles not a single bridge or culvert would be required for a railway line, beyond ordinary small drainage culverts. In other words, this part of the road is so singularly good that to form it into a first-class double railway line would involve no other expense than to shape it, to free it from obstructions and to lay down the permanent way and rails at the least possible expense, when it could be traversed with speed from end to end, and going or coming.

So much then for the first part of the proposed road, which must be traversed by the line from whatever pass of the Illawarra coast mountain the railway might pass from the sea levels to the table-land.* For reasons that will be obvious presently, I do not expect that the Coalcliff proposed line will continue any further along the Bottle Forest Road than to about the turn-off at 21 miles from George's River to Stanwell Park, near Coalcliff, which is distant about 3 to 4 miles from such turnoff.

* The last survey in November proved the fallacy of this remark, by reason of our having found a vastly superior route through the Port Hacking Valley, and by means of tunnelling through the Bulgo range.

But in the event of the railway coming up by Westmacott's Pass behind Bulli, as contemplated by residents in Wollongong, &c., I beg to remark that the remainder of the 9 miles along the Bottle Forest to its junction with the Appin Road on the top of the Bulli Pass is a continuation of a very good road through Madden's Plains, interrupted here and there by small depressions and rises, which may probably be avoided ; but these nine miles, by my measurements, and, as far as the aneroid can be trusted, give an extra elevation of 164 feet, and consequently give an equally greater descent down the mountain than where I shall show the line, in my opinion, ought to go.

Passing from the turn-off of the Bottle Forest and Stanwell Park Roads, we reached Stanwell Park, near the seaside, by a distance of three or four miles, with the last mile ridden down the mountain after dark by a road that baffles description.

The next morning we were mounted on fresh horses, kindly supplied by Mr. Hargraves, and being conducted by the overseer we again ascended the mountain by another pass, known as Mitchell's Appin Pass. We ascended this mountain, close to Stanwell Park House, expressly to see from the tops of the cliffs, at various points, where it might be possible to make a railway line. By this track we intersected a seam of rich clay iron ore at an elevation of 460 feet, and reached the table-land at a place showing only 820 feet, and at a point that was almost in sight of the Bottle Forest Road, and distant about two miles from where we had left it the night before to descend by the direct track to Stanwell Park.

In order that I may be easily understood, I beg to say here that I afterwards found that at or about this very spot it proved, in my opinion, to be by far the best way to make the proposed descent to the lower lands.

Passing along on horseback and on foot from cliff-top to cliff-top, we commanded views in every direction down and over the frightful but magnificent abysses, with the whole of Illawarra before us for fully fifty miles. The view was bounded by the high Saddleback mountain behind Kiama to the south, by the cliffs beyond Sydney and Broken Bay to the north, and by an enormous expanse of ocean to the eastward, with a few white sails showing in the distance, and two pigmy steamers approaching the coast, the one from Sydney to Bulli, and the other by Five Islands, steering its course for Wollongong. Splendid and expansive views were then obtained from point to point over elevations exceeding 1,000 feet. When all seemed hopeless for a roadway to the lower country, we passed on until we again skirted the Bottle Forest Road, and came to the Bulli Pass, where the descent is made by a road which, once seen, can never be forgotten, because of the impressions made by its grand scenery.

Bulli being reached by a descent of about two miles, I saw at a glance from the lowest ground that a railroad could be made here by a long, gradual, and straight descent, which would give a gradient of about 1 in 45, as I had before been informed; and here I think that heavy trains of coal might pass up at a moderately high speed if aided by a second engine.

After resting some time at Bulli we returned by the coast to where we started from in the morning, in order that we might view the cliffs and possible passes, from below upwards, and also from a desire to see if there was any possibility of getting out of this coal country by means of tunnelling through the Bulgo range, which fences in the extreme north of Illawarra, this having been a suggestion which had formed matter for discussion at the Royal Society's last meeting.

I may at once dispose of this tunnelling question by saying that it is quite impracticable, and would be futile if even the first great Bulgo range was tunnelled at the bottom ; because when once through it, and stationed on its north side, the same difficulties would have to be overcome of clearing away out of the broken hills and gullies at the head of Port Hacking Creek on to the level of the Bottle Forest Road before it would be practicable to pass on.

From Bulli to Coal Cliff is nine miles, and along the whole of this distance most of the fine seams of coal crop out to the eastward, in the same manner as they do all along from Bulli to Wollongong and down to Gerringong, over a distance of some extra 40 miles. Therefore, if the egress by rail should be made by Westmacott's Pass, at Bulli, the whole of these upper or more northern nine miles of rich coal country would continue to be valueless, unless enterprise should venture a partial development of those coal fields by attempting to make jetties in open roadsteads, having the most insignificant protection from heavy seas, and with far less shelter than is even afforded by the dangerous position of the Bulli jetty.

Interested parties having met me on our ride up through these nine miles of coal country, they called my attention to the possibility of having the railroad at the extreme northern end of this coal district, to admit of all proprietors being benefited alike, and thus to develop more feeders to the railway. This was exactly what I wished, but which I saw was impracticable by means of the proposed tunnelling. I was also much encouraged in this new idea, because my measurements of elevation proved what I knew must be the case, namely, that the northerly dip of the coal basin in these parts is not confined to the dip of the coal alone, but that the whole geological formation had endured similar depression, and that therefore it must be easier to rise out of this coast abyss the further north we went, provided other natural features warranted the attempt.

Thus, from a suggestion made by others, I carried out the investigation of the special locality, and was much pleased by the result, which convinces me that not only can all the difficulties be overcome, and the egress brought to that comparatively low elevation that I had reached in the morning, on ascending the cliff-tops at the extreme north, but I am also satisfied that the features of the country will admit of such an easy gradient being made that we may have it as light as 1 in 50, which is only 100 feet to the mile. This success would be attained by keeping about 200 feet above the sea all the way from Bulli, so as to escape the gullies and broken ground, and pass on by the flats below the high mountain cliffs which continue for several miles from Bulli; thus by keeping that level of 200 feet at least, the great Coal Cliff itself can be rounded by a natural basement existing on it at that elevation, but which would have to be relieved by cuttings.

This would thus leave only 600 feet more to rise before the table-top would be reached; and this elevation can, in my opinion, be commanded by making a winding circuit in serpentine form around and up two great spurs of the mountain, which, with the east and west bend of the Bulgo Range at the back of Stanwell Park, terminate the northern boundary of the Illawarra country, where the northerly dip of the coal passes down under the sea level.

This course will admit of a circuitous upward route of from four to six miles on to the table-top by a road which could be made perfectly safe, and also to be of the grandest nature in its scenery afforded, and give a gradient that will admit of large coal trains going up at half speed without the aid of a second engine, by taking up full load at twice.

At the same time that I advocate the adoption of this special line, I do not suppose that it should be even nominally adopted in preference to the one by the Bulli Pass without survey; but I do strenuously hope that a sufficient sum of money, say £2,000, be subscribed in Sydney and Illawarra, in the relative proportions of £1,500 from the former and £500 from the latter, for the express purpose of carrying out effective preliminary surveys with accurate cross-sections of both these passes; and that priority in the surveys be given to those difficult engineering parts, before more of the money is spent on the rest of the simple surveys along the whole line.

With reference to the pecuniary aspect of this matter and of its important results, I confirm all I have formerly said, although the altered conditions of the coal line by the first planned and easier approach to the table-land at the back of the Bulli Mountains (where it was proposed to lift the coal from the dip) would have made the railway project *cheaper*, although not half as efficient, as by adopting the amended and more expensive proposed approach to the coal at the outcrop on the easterly side of the range, and which altered proposal presents the further facilities of opening out all the Illawarra lands for their surface as well as for their coal value.

Assuming that the line from Port Jackson to Wollongong only should cost £500,000—with railway dam, or perhaps bridge at George's River included—still I repeat with confidence that perhaps even the very first year of the railway being opened it would reach a return of 5 per cent. on all the capital invested, besides covering working expenses and wear and tear of rails and engines; and all this by reason of the coal trade only, without the aid of the passenger and other produce and goods trade with Illawarra, and without the help of that suburban traffic which would be sure to arise from George's River, whether by the direct route between Canterbury and Cook's River, over the hills and over the valley of Wolli Creek, or by the longer but much more level route by the neighbourhood of the Seven-mile Beach, and on by the Cook's River valley to Petersham and North Balmain, or on to Newtown and Sydney.

Every succeeding year would increase the income of the line very largely, and eventually it would be so prolific in its returns that the excess of the profits would cover a great share, if not all, of the losses that must accrue to the Government by reason of the necessary extension of the new forming railways to the sparse inland population, and by the comparatively small and non-paying carrying trade of those countries.

At 2s. 6d. to 3s. a ton on only 200,000 tons a year to be sent up by the Illawarra proposed line, and which is only at the rate of just a *fifth part* of what Newcastle is now shipping so sluggishly, the interests of money, with working expenses, would be covered. And as the demand would be sure to increase by reason of the facilities that would be offered by direct and prompt deliveries in Sydney, and without that breakage of the coal which is caused by transhipment to the steam colliers, and worth 1s. a ton, so would the annual returns of the Southern railway coal traffic increase with such probable rapidity that the demand and the "out-put" would increase at the rate of at least 100,000 tons a year for many consecutive years, if not perhaps permanently; and 2s. 6d. a ton freight on each extra 100,000 would represent 2½ per cent. more interest on the original cost of the line of £500,000.

Such being the case, I would also venture to say that we should find that the earliest customers for the Illawarra railway would be represented by those Colliery Companies already in existence at Bulli and at Wollongong, and which now do all their irregular business by means of steam colliers. Their properties would become doubled in value, and the general coal trade from thence would probably grow into enormous proportions, whilst Sydney and Illawarra would derive such accession to their population and prosperity that it would be soon shown that there is much more material wealth and healthy industry arising from a coal and iron trade than from all the combined efforts to grow rich by the treacherous production of gold.

OUR COAL AND COAL PORTS.

BY MR. JAMES MANNING.

[*Read before the Royal Society, December* 10, 1873.]

THE very rapidly increasing interest which has been taken in the progressive development of the proposed Illawarra railway, together with the fact that I had been invited by the Government to accompany their engineer, Mr. Stephens, on his exploration of the projected railroad to the Southern coal-fields, must be my excuse for again handling this subject, and coming before this Society with a paper purporting briefly to state what we saw on our journey.

I am aware that a paper of a purely narrative character, such as this purports to be, would be against the rules of this Society for admission, were it not that we may consider that such narrative has arisen entirely from the consequences of the Society's adoption of my former papers on "Our Coal and our Coal Ports," which had for their object the development of the resources of the country. I therefore trust I shall not be considered to be out of order in attaching this paper as an appendix to the former ones.

Preliminarily, I beg to say that before venturing to publish our experiences through your Society, I obtained the consent of the Engineer-in-Chief for Railways, and also that of Mr. Stephens to do so.

I also wish to premise that after a short visit to Illawarra to form my own opinion of how we could get a railroad into that rich country, I was impressed with the belief that there was only one way of getting in and out of Illawarra by rail. This route I described in the appendix to the paper read by me on the 3rd September last. And now that I have returned from a second examination of my first projected line, I have increased good opinion of its efficacy, and would maintain its intrinsic value; but subsequent investigation and researches, consequent on my first action, has led to the discovery of an infinitely better route. With regard therefore to my first projected line, it was to have commenced from Bulli by a gradual acclivity towards the Bottle Forest Road and Sydney, by way of the formidable "coal cliff" which rises perpendicularly from the sea. This difficulty I proposed at first to overcome by rounding the natural ledge on the cliff at a height of 200 feet, and then to have further contoured two large creeks, and two mountain spurs by a supposed available upward gradient of 1 in 50, to bring the line out on the tableland, and then the line was to have passed on by a course along the eastern slopes of the highest part of the Bottle Forest Falls for Port Hacking Creek, to have come out on the Bottle Forest Road, about two miles south of the old station and creek of that name, and to have passed on by the seemingly very easy downward gradients to George's River, &c.

My late visit to the same neighbourhood with Mr. Stephens has confirmed my opinion of the practicability of this line had it been wanted, or should it hereafter be wanted as a tributary to the main trunk line, for getting over the mountains to the westward, and to the coal at the dip. On Mr. Stephens' examination of the "coal cliff" he has found it will be necessary to pierce this mountain range at an elevation of about 275 feet from the sea for either of the lines to which this tunnel of 50 chains would be common to both, so as to soften down the gradients of the new line towards Bulgo and Port Hacking Creek, which are within three miles of Coalcliff. This tunnel, therefore, will be 75 feet higher than my proposed rounding of same of 200 feet, and would be very materially in favour of my after contouring and ascents to the table-land, and where my proposed landing-place proves to be 70 feet lower than I had before estimated; so that I should have gained 145 feet of elevation at starting, and which would now only leave the difference of the height of landing-place 752 feet less 275 feet for the elevation of the tunnel, or only 477 feet in all.

This comparatively easy rise out of the low lands would be quite practicable by an upward gradient of 1 in 50, in the shortest possible distance to give that necessary gradient. Could we have done no better than this we should even thus have secured the only other possible means of ingress and egress across the Illawarra mountains by my first proposed course, at the extreme north end of this sea and land-locked coal country, where the geological dip of the strata trends to the northward by a gradient of about 1 in 120; a dip from the south to the north, which proves the coal seams and all the country to have a fall of 660 feet in a space of about 15 miles in a straight direction from Mount Keira to Coalcliff—a geological fact which renders it impossible ever to get a railway out of Illawarra by any other exit than by the extreme north end, where the northerly line of coast ranges with its continuous downward dip of about 1 in 120 northerly, is formidably intercepted by a coast range (Bulgo), running direct east into the sea, with an elevation there of 540 feet at its lowest part, and terminating bluffly over the sea. My first idea, on the occasion of this obstructing cross range presenting itself, was to tunnel through it so as to make a passage by the downward northerly dip of the surface, and by the falls of the Port Hacking Creek to continue my course to nearly the level of Port Hacking itself. From this idea I was deterred by the confident assertions of two very intelligent gentlemen that the attempt would be utterly futile, and I therefore gave it up; and in my letter to the *Herald* with details of my then expedition I disposed of the suggestion of tunnelling the Bulgo Range as being wholly worthless in its consequences, if attempted.

Happily, however, I was subsequently encouraged to renew the idea by the forcible remarks of the Surveyor General, who had reason to believe, partly through Lord Audley's survey of this creek in 1862, that such line up Port Hacking Creek could, by aid of tunnelling here and there, be made available for the purpose. Mr. Justice Hargrave also wrote to me on the same subject in part, and I then promised to give attention to the suggestions. It is therefore to the intelligence of these gentlemen that much of the subsequent success is owing, and most specially so to the Surveyor General, to whom the honour is due of first drawing attention to the feasibility of obtaining a railway direct up this, our Endeavour rivulet.

On hearing that the petition for a preliminary survey had been so promptly granted, and as I was courteously invited to accompany the engineer, I sought the assistance of a guide in Mr. Blake, who subsequently proved himself most intelligent, and from his knowledge of the country of great service; and I requested him to accompany us up the Port Hacking Creek, on our expedition in search of a railway passage.

I was glad to find that he, a practical man, and well acquainted with most of the country through which we should have to pass, was strongly impressed with the belief that we should find such a passage by following up Port Hacking Creek, and up which he was aware that the tides ascended to more than one-third of its supposed whole length to its northern bend at Bulgo Range, by Mr. Gilbert, Hargraves House, whither it comes by an easterly course from the Bottle Forest Road direction.

With this additional encouragement we started hopefully, on November 18, from North Balmain, where I had previously proposed the coal terminus should be. We were accompanied by Mr. Thomas Mort and Mr. Moriarty to this, the starting-point of our so-called "trial," or more properly, our "reconnaissance-survey," and which survey had necessarily and throughout to be made almost entirely on foot. Our good friends having bid us farewell, and a successful issue to the excursion, we started from an elevation of about twenty-five feet above the sea-level, such rise being necessary for the discharge of coal into large ships from the staithes and coal shoots, and guided almost only by both our aneroids, we kept the uniform level along the near banks of Long Cove, going through every place as it came in our way, and in one place meeting a garden with a fountain, statues, and garden ornaments, which we feared might have to give way to the iron-horse. Within two miles of our starting we crossed the old Parramatta Road, and struck straight for the northern arches of the Petersham viaduct (under which the line would have to pass for its level course on to the staithes or shoots), and we went then by Canterbury, across Cook's River, and in by Cup and Saucer Creek to Tom Ugly's Point at George's River. Along the whole of this course we found a practicable line.

We returned to Canterbury that night. Next morning we went back to the Petersham viaduct, and defined an excellent siding from our line of the day before to turn off into the Petersham station close by—for the use of the passenger and general produce trade of Illawarra.

It was our desire to see if a better line for the coal trade could be obtained by going through the head of the Cook's River and Marrickville country, and then on to Cook's River Dam, and by Seven-Mile Beach levels on to George's River. To this we found a formidable obstruction close to the viaduct, in the form of the "Wardell Hill" and road, which could not be got over without considerable expense, and too sharp in curve from under the arches of the viaduct; added to which there would have been objections by reason of heavy compensations for passing through numerous villa and other properties nearer Sydney, besides this route being longer than by our direct line *viâ* Canterbury. This is, however, yet an undecided matter.

Next day, November 20th, we were joined at Tom Ugly's ferry, at George's River, by Mr. Blake and his son, who took us about three miles in a spring-cart, with a few days' provisions on to Gymmea Bay, in Port Hacking, where a boat awaited us. The intermediate easy elevation and depressions having been noted by Mr. Stephens, we went by the boat across by the north-western arm of Port Hacking into the Port Hacking Creek itself; then by rowing about three miles up the creek against the tide we reached the junction of Kangaroo Creek, under rain, and found shelter for the night under an overhanging rock or cave.

Next morning, Saturday, we started at 4 o'clock, in order to make the most of a falling tide with our boat, and then after passing up with considerable difficulty, from the shallowness of water, we reached the point where we intended to leave the boat. Having breakfasted here, we started again at 9 o'clock, on foot, carrying the least possible amount of baggage, and thus began our difficult march, or rather our floundering through a wilderness. The difficulties of this walking expedition are almost beyond description, through ever constant entanglements of untrodden scrub and vines; over loose leafsoil, logs, stones, and every conceivable obstruction inherent to a thick, wild, and untenanted country; annoyed by bush leeches and ticks, our feet wet through all day, our clothes in ribbons—these and many other troubles are what we had to contend with. Resting from time to time to make a fresh start, we toiled on up the meandering and fine brook.

As we went away from all action of tidal or brackish waters we came into the most gorgeous tropical vegetation, existing more or less on both sides of the creek. The magnificence of this vegetation requires to be seen to have the effect of its beauties conveyed to the mind. The hilly slopes on the sides of the running streams covered with various kinds of palms, fern trees, immense trees of every description, and all in such places having rich green leaves, covered with creepers and with flexible vines varying in all sizes up to that of the wrist, and some hanging down perhaps fifty feet or more; whilst underneath every decaying dead log, and every living root projecting everywhere over-ground, was covered with a perfect coating of thick green moss; all gave a charm to the picture which seemed to be alone a sufficient compensation for the fatigue we had to endure. We pushed on as hard as we could up stream under a lively hope to reach the most southern dwelling of the free selectors, who live on the creek near Bulgo.

The weather was rainy, cold, and unpropitious for the prospect of a night's lodging on damp ground by the side of a burning log. But our energies failed to gain the desired object. The country, if anything, became richer and denser in vegetation—the branch creeks deeper and more numerous, some of which had to be traversed by logs which, fortunately for us, bridged them. As we progressed on and on we found the timber, both live and dead, to be larger and larger; large dead trees had to be constantly stepped over, straddled over, or climbed over, in spite of their being covered with from one to two inches of rich, cold, and wet moss. At 6 o'clock in the evening we agreed that, as we could not have progressed much over a quarter of a mile in the last hour, we had had enough of it for one day, and since 4 o'clock in the morning; so we gave up all hopes of reaching the first house, and searched for lodgings away from the rich and green jungle. Most fortunately we obtained very welcome homes, under the certain prospect of a wet night, by finding two immense "turpentine" trees near each other, which towered a prodigious height over us; whilst the bottoms of these two trees had been hollowed out by decay and by fire in such a manner as to afford us two dry resting-places for the night. A heap of dead cabbage-tree leaves was secured for the bedding of each bedroom; Mr. Stephens and I shared one—Blake and his son took the other. By this happily-found shelter we slept soundly through a wet night, and awoke refreshed the next morning.

Before passing on to even more important subjects after we cleared out of this noble forest, I should call your attention to the splendour and to the intrinsic value of the "turpentine" tree in particular, which abound here in every direction. I will not attempt to give any estimate of the girth or height of these noble trees, but will content myself by pourtraying the dimensions of one of these two trees that we slept under, by informing you that all four of our party took easy shelter from a heavy shower of rain after our breakfast was over—one was sitting down at his ease, whilst the other three were standing up, without its being at all inconvenient to any of us. This valuable timber is very scarce in other parts, and indeed is almost extinct. Its wood defies all action of the sea cobbera, and it is admirably suited for under water or under ground uses, and is quite equal in such virtues to the Jarrah wood of Western Australia. Whilst we were at Bulli we visited the fine jetty of the Mining Company, and there we had pointed out to us a fine specimen of a turpentine pile which had been at the end of the jetty for six years, and had only been recently removed because the top of it had been broken and rendered useless. On examining it, we found it to be perfectly sound from the action of the cobbera beyond the thin external so-called "sap." We were informed that an ironbark or gum pile would not stand in sea water beyond three years without being perforated like a sponge, and rendered entirely useless. This being the case, some idea may be gained of how much additional valuable produce exists in the new line of intended railway for Government or for mercantile purposes.

Here I may venture to hope that as this late *terra incognita* becomes a sort of treasure trove for the public, it would be a pride for the country to preserve the rich tropical jungles on this rivulet, so that they may not become destroyed and be lost to the future tourists who will by-and-bye delight to pass by these magnificent productions of vegetable nature, whilst proceeding along their delightful tour to other scenery of a wholly different character, though surpassingly grand. Also owing to the broken nature of the ground on the Hacking Creek, and the narrowness of the various small flats, there will be nothing that should allure the husbandman to destroy the beautiful brushwood that abounds there, for the sake of any doubtful returns he could obtain.

I may mention that I saw evidence of the proximity of coal, namely, by that regular seam of clay iron ore with same dip as the coal, having tolerably defined and known elevation over the coal, and being, as it were, the capping of the coal measures. I could not possibly see any coal itself, because none exists above the surface here (not even of the 7-feet thick top seam), as it dips under the sea at "Coalcliff," on the south side of the Bulgo Range, which divides Hacking Creek from Illawarra, through which the tunnel should pass, and where the magnificent mountain and marine scenery will be first met in its fullest and stupendous grandeur, overlooking Stanwell Park and a great deal of the Illawarra coast country.

Returning to my narrative of our journey, we reached the first inhabited free selector's house about mid-day on Sunday, and felt much relief in finding any beaten path under our feet once more. The pleasure evinced by the family of the free selector (Hamilton) on hearing of our mission, was shown by their overwhelming hospitality. Our constant aneroid elevations taken along the creek gave us cheering hopes that the total height of the creek would not be at all obstructive to the forming of a railway in such a broken country; whilst at the same time I was satisfied to find that Mr. Stephens (who has had great experience in the Old World, and who partially surveyed the Toowoomba grand mountain line, in Queensland, for eighteen miles, and engineered the whole of it, with its eleven tunnels, being in all two miles and a half) made light of the difficulties that would seem insurmountable to non-professional men; and when we finally worked our way out, the next day, to the Bulgo Range, at the head of the creek, by Mr. Gilbert Hargrave's house, and after Mr. Stephens had taken his views of the hills that he overlooked from the top of the dividing range, then only 200 feet above us, I was extremely pleased to hear him say that, by tunnelling at this spot through Bulgo, out near the head of the Hacking Creek, he would only have 350 feet of greatest height to overcome between Wollongong and Sydney; that if all was equally feasible to the south, it would be a good coal-traffic route, all downward gradients being of a very easy character, whilst probably fifteen miles of the road along Hacking Creek would be at about 1 in 400, or practically a level line but with sufficient downward gradient which might admit of all coal trains and other trains to pass down for fifteen miles entirely by their own gravity, and without need of much break power.

On Sunday afternoon we descended to Stanwell Park House, which is 91 feet above the sea. The next day was devoted to the viewing of Coalcliff, when Mr. Stephens, with practised quickness, determined on the expediency of a 50-chain tunnel at 275 feet above the sea, to pierce this great coal cliff, which had before seemed to be so serious a barrier, and by which he at once showed the saving of half the length of line that would have been used by going round the cliff, as first proposed by me. Having fixed the gradients and height of tunnel by aneroid observation, he proceeded to inspect the contouring of the two creeks and two large spurs, with a view in the first place of connecting the coal cliff with the Bulgo Range for the new line by a gradient that should not exceed 1 in 150, and also with the view of seeing if my first proposed road from the Coalcliff tunnel (which would be common to both) would admit of my stated ascent by contouring same ranges higher up to my proposed landing-place at 752 feet on the tableland. That day, being devoted satisfactorily to this work, we passed on from Coalcliff next day to Bulli and to Wonoona, when Mr. Stephens found that he could not pass a railway line satisfactorily for the three first miles from Coalcliff by a safer gradient than about 1 in 80 or 90, owing to the proximity of the cliffs to the sea, and then to pass on by an almost level line partly on the seaward side of the road, and repass the road at foot of Plunkett's Hill, to take the line as near the foot of the coal-bearing ranges as possible from near Brokersnose Point on to Mount Keira, and then on by Mount Kembla, Dapto, Kiama, Jamberoo, and Shoalhaven.

The direct line to Wollongong would be by a short branch line from Fairy Meadows on by the road to the Wollongong wharf, which would thus connect the Sydney and the Illawarra waters.

We reached Wollongong on Thursday, where we had a very pleasing meeting with several gentlemen who have been most prominent in the railway matter. The result of our discoveries took them all by surprise, and created much pleasurable excitement.

In the afternoon we returned to our previous night's quarters at the Wonoona Inn, going by Mount Keira and Mount Pleasant, where the main trunk line would pass round that promontory of land, and then *viâ* Mount Kembla, &c., to Shoalhaven. From Mount Keira we passed along, on foot, across every fence and other obstruction under the coal-bearing cliffs all the way to the tramroads of the Bellambi coal mine, near Mr. M'Cabe's house —thus sighting the whole of this country with careful aneroid measurements, after viewing it only, by the direct road course to Wollongong, in the morning. The future course for the trunk coal line, and for the branch line to Wollongong wharf, became clearly defined to us, and will be quite practicable, and so adjusted that the former should pass close to the present and future " inclines" of the various coal companies. Next morning we started very early on hired horses to go up Westmacott Bulli Pass, at the summit of which range of grand scenery we commenced a survey, on foot, of every elevation and depression, from the junction of the Appin and Bottle Forest roads, where we found the rise to be 1,178 feet. The elevations continued along the road for 4½ miles more towards Sydney, until we reached the maximum point of 1,250 feet. This survey requiring two days, we descended the mountain to Stanwell Park, by my first proposed descent, after taking the measurements of seven miles of the road. During our survey we had commanding views of the whole of the wild and utterly valueless and unproductive country before us, from off Madden's Plains towards Appin by the one line of ranges, and towards Liverpool by another.

Next morning, very early, we returned up the mountain to where we left off the previous day, and completed the whole of this interesting survey to the ferry at George's River by 4·30 in the afternoon.

Here we crossed this fine river, which I regard in the light of another great national friend, at present in salt-water disguise; but when affiliated with the Illawarra proposed railway, in manner briefly advocated by me in my first paper read before this Society, it should become only second to it in importance to Sydney in its rich consequences.

George's River being passed we found a conveyance which soon took us to Cook's River and Sydney.

And now, finally, and to summarise the result of Mr. Stephens' experienced eye, and of our reconnaissance survey by aneroid measurements, the consequences of the probable adoption of this improved proposed and *double and broad gauge railway line* will be to give an excellent coal traffic line. The gradients throughout the 48 miles to Wollongong will not average more than 1 in 200. Half the route will be at 1 in 400, and the very worst part of the line will not be worse than 1 in 80, and that only for three miles on the Wollongong side of the Bulgo tunnel, which tunnel will be nearly half-way to Sydney; and at this point will be the highest elevation of 350 feet over the sea. From Wollongong to the Bulgo tunnel the gradients will be nearly all upwards, whilst those from Bulgo to George's River will be nearly all downwards—so that the coal trains can come on to Bulgo half-full in number of trucks, and complete their loading, up to 300 to 500 tons probably, according to engine power, on the downward side of the Bulgo tunnel, by the new line being fed by the illimitable supply of coal (some of it within twenty miles of Sydney) that can be easily raised by shafting for miles along the Port Hacking Creek, and by working the coal at the dip with its easterly rise of 1 in 30, and light northerly dip of 1 in 120, barring the consequences of so-called "faults," an extraordinary one, of which such "faults" exists at Coalcliff to the extent of 169 feet, and which has caused quite a bouleversement there of the usual dip of the strata, having produced for the proprietors a highly advantageous north-easterly dip there, for a probably short distance only.

I need not enlarge on the certain commercial results of such an undertaking, whereby we shall be able to encourage the consumption of our coal and perhaps of our iron to a probably, prodigious extent; neither need I dwell on the general consequences that will be caused by the use of and by the development of the surface products all along this hitherto lost country, including all Illawarra proper; nor should we fail to consider that the apparent costliness of these works should be as nothing compared with the great advantages to the public that will accrue from their adoption, and that by making people familiar with the idea of costly works, they at last come to consent to their being entered upon.

But, though last, it may not be the least in the important consequences of the proposed work that this beautiful line of suburban railway will induce us, from time to time, to shake off some of our utilitarian habits, to avail ourselves of that social enjoyment and relaxation from business which such short railway trip to Illawarra will give in an unsurpassable degree.

AUSTRALIAN NATURAL HISTORY.

By Gerard Krefft, F.L.S., &c.,
Curator and Secretary of the Australian Museum.

[*Read before the Royal Society*, 5 *November*, 1873.]

MAMMALS OF AUSTRALIA AND THEIR CLASSIFICATION.

Part I.—Ornithodelphia and Didelphia.

According to geological evidence, the class Mammalia (animals who develop mammary glands for the nourishment of their young) made their appearance on this earth during the Oolitic period.

The fossils obtained, a few lower jaws and teeth, were referred to the sub-class *Didelphia*, comprising at present a single order, the Marsupialia. These are distinguished from the Monodelphia, or Placentalia, by bringing forth their young in a very rudimentary state, nourishing them in a "marsupium," which is either a regular pouch or a simple skinfold, such as our native cats and antechini develop at the time of parturition. The living species are almost entirely confined to Australia, to the neighbouring islands, such as the Solomons, Timor, the Aru Group, to New Guinea, and to Celebes; in America a single genus still lingers, represented by one northern and about thirty southern species.

The extinct genera found in England, in the Stonesfield slate and Purbeck beds, are of small size, about as large as our Antechini or Phascogales, and generally considered to represent the most ancient form of mammalian life hitherto discovered.

According to the theory of evolution, the *Ornithodelphia* (represented in Australia alone by the order *Monotremata*, the duck-bill or Ornithorhynchus, and the spiny ant-eater or Echidna) should have made their appearance first, but fossil remains of them have not yet been found except in the post-pleiocene deposits of Wellington. Some allowance must be made, however, for the incompleteness of our geological or palæontological record, so that during future and more systematic investigation additional evidence may be looked for.

The discovery of fossil remains in Australia extends over a good many years, bones and teeth of mammals of all kinds have been found and shipped Home in large quantities; palæontologists have examined and reported upon them, but owing to a scanty supply of the skeletons of modern marsupials, the classification of these distinguished men has never been as correct as the owners of the fossils had a right to expect.

The errors which have been made are indeed numerous and varied, the most harmless of creatures have been represented as "the fellest of the fell," animals with all the true characters of phalangers have been persistently described as allied to the kangaroos—the peculiar short tarsal bones of harmless kangaroos have been explained to be those of great flesh-eaters. New species have been created for the numerous still living bettongs, wallabies, and rat kangaroos, phalangers, dasyures, and thylacines when found fossil; whilst the peculiar character of the old short-footed kangaroos, with their firmly joined lower mandible, their immovable incisors, and their other marked distinctions, have never engaged the attention of foreign investigators. So little are our living animals understood that anatomists have not yet pointed out the peculiar structure of the kangaroo's molar teeth. I allude especially to their fangs or roots. I doubt very much that many men have made the observation that, as far as the grinders are concerned, the bandicoots, but more particularly the rabbit-rat or peragalea, are near relations of the wombats. Three years ago such teeth were lithographed for our Museum catalogue, but they have not been published. It must have struck observant people that, with the exception of the Monotremes (the platypus and ant-eaters) and the dasyures, all our animals have their hind feet constructed on a peculiar and uniform plan, possessing invariably two small conjoined inner toes, much less in size than the rest of the digits. All, with one exception—the Thylacine—have two bones articulated to the lower part of the pelvis. All marsupials, except the native bear and the dactylopsila, have the angle of the lower jaw bent inwards, and all members of the kangaroo tribe have a wide opening at the base of the lower jaw, below the ascending ramus. In all phalangers, bandicoots, and dasyures, this opening is closed, except in the highly herbivorous native bear and wombat, which sometimes have a small foramen remaining. Such a perforation is also present in the very typical Australian form, the Thylacoleo.

The Teeth.

(*Ornithodelphia or Monotremata.*)

The Monotremata, who must be regarded as the most ancient mammals known, possess either horny teeth, such as the Ornithorhynchus, or none at all, like the Echidna. The development of these animals appears to have taken place from the *Sauropsida* (a combination of the two classes of birds and reptiles), and points in the direction of that curious lizard-bird, the *Archæopteryx*. This creature, with its toothed beak, is perhaps the most important missing link ever discovered, and when Australia is better explored we shall perhaps find fossil remains of mammals more reptile or bird like than the Platypus or the Echidna. The Australian antiquated living representatives of the early mammalian type must have been developed at a period more remote than the Oolite, which preserved the supposed marsupial remains of

Phascolotherium, *Triconodon*, and other forms. Some of our living marsupials resemble the Echidna (in the structure of their skull and in their scanty or curiously-arranged teeth), and in this direction the connection between the two lowest orders of mammals must be looked for. I regret to say that the most diligent search for Platypus remains has not yet been successful, but of the Echidna I found three fragments of the humerus and a femur, the latter almost perfect, and indicating a larger kind.

The teeth of the Monotremata consist in the Platypus of four horny plates without roots or fangs, well adapted for crushing the food. The Echidna is toothless, but at the posterior end of the mandibular symphysis there is a small alveolus in each ramus, which appears to me indicative of a rudimentary tooth, perhaps corresponding to a canine. The Myrmecobius (with its peculiar skull, which resembles that of the Echidna, but is well provided with fifty-two very small teeth) and the little Tarsipes, with its irregular dentition, appear to be the nearest relations to the Monotremes.*

(*Didelphia or Marsupialia.*)

The teeth of the marsupials—the peculiar arrangement of some, and the presence of almost toothless genera—point, as already stated, to a probable development from the Monotremes.

When examining the Echidna, with its long spiny tongue, we can easily imagine a kind of connection between this form, the Myrmecobius and the little honey-sucking Tarsipes. Both marsupials possess a bird-like skull and very weak mandibles, both are covered with comparatively coarse hair, and have few or irregular teeth, not touching each other. One, with a nailless thumb, conjoined inner toes, and only one pair of lower incisors, connects the herbivorous marsupials with the Monotremes ; the other, with many cutting teeth, without conjoined inner toes, with tuberculated grinders and regular canines, appears to diverge towards the marsupial carnivores. With the platypus no such connection can at present be established. We must not forget, however, that all our efforts at elucidation are comparable to looking for a pin in a bale of hay. The dentition of the two

* It is still an open question how the young of these Monotremeous animals are conveyed to the mammary glands. The Echidna has two deep pouches, about the size of the new-born young, without nipples ; and on two occasions young animals have been found inside, but how they get there we are unable to tell. The general belief that the mother uses her lips in the conveyance is untenable when we consider that the animal is destitute of lips, and has nothing but a stiff beak and paws as clumsy as it is possible to imagine. Year after year passes without a solution of this most important question, and the Echidna will probably have disappeared by the time that a more liberal-minded generation produces men who will devote a little time and some money to the investigation of his interesting problem.

little animals just mentioned is of so exceptionable a character that they cannot well be included in the group with more highly developed teeth. For the sake of arrangement, however, the tarsipes is added to the phalangers and the myrmecobius to the dasyures.

Following the clue thus received, we arrive at the dentition of the aberrant phalangers represented by the genus *Thylacoleo.*

This supposed marsupial lion, believed to have been the "fellest of the fell," was, after all, a harmless creature, which is proved by his weak incisors, small canines, and the highly inflected scoop-like angle of the lower jaw. This animal bruised his food with a formidable premolar tooth, whereof one was developed in each ramus above and below. Cuvier's well-known sentence about the molars of a mammal, explaining its character and position in the system, failed in this instance. A much worn large premolar in the Australian Museum, and an upper pair with perfectly flat grinding surface in Professor Owen's possession—a present from Dr. George Bennett—have probably convinced anatomists that the view I took first of the herbivorous habits of this "lion in phalanger hide" was a perfectly correct one. The incisors are simply large editions of the typical phalanger's front teeth, such as may be examined in the native bear, the yellow-bellied flying phalanger, or the northern dactylopsila or striped phalanger. It would be waste of time to describe them in detail; those gentlemen who take sufficient interest in the matter can get a Phascolarctos or "native bear" skull any day, and those who do not care about it will perhaps feel thankful for being spared the infliction of a long description. The Thylacoleo was just three times the size of a native bear, and if this scale is borne in mind the incisive dentition can be reconstructed without trouble by those interested. The great premolar corresponds to the same tooth of the phalangers, and makes its appearance early. The first molar below is, however, the tooth of a carnivore, and corresponds to that of the Sarcophilus, so does the last tooth above. The last molar, like that of all the marsupial carnivores, stands transversely and across the palate. The second or last molar below is a small tubercular tooth, and quite unlike the last and largest trenchant one in the pouched flesh-eaters. The canine above is an enlarged example of the canine of Bettongia rufescens. The canines found vary, and may be those of several species of "pouched lions." They are placed far into the palate, and are more or less covered by simple, single-rooted, and blunt premolars, the crown of which resemble the head of a common wrought nail.

In the true phalangers the upper canines and premolars, Nos. 2 and 3, are generally well developed, the first premolar being lost in early life.

Below, the Thylacoleo has generally two or three small teeth, sometimes on the inner side of the great premolar, which represents the diminutive canine and premolars No. 1 and No. 2. The shape of these small and functionless teeth is not known, as all the specimens of mandibles in collections show only empty sockets.

Fam. Phalangistidæ.

The phalangers proper, whereof the Thylacoleo is an aberrant form, comprise animals the molar dentition of which is very different in the several genera composing the family. They all possess, however, the six incisors above and two below; their canines are always well developed in the upper jaw, and the molars have tapering fangs or roots. The living genera and species are represented by the genus *Cuscus*, a northern form, in many respects resembling the extinct Thylacoleo; the genus *Dactylopsila*, with greatly developed front cutting teeth and small grinders; the common flying phalangers, or sugar squirrels, of the genus *Belideus*, with small and slightly tuberculated molars; the feather-tailed genus, *Acrobata*, with much developed canines, and with grinders reduced to three above and below in each ramus; the phalangers known as "opossums," of the genus Phalangista, with a powerful third premolar turned more or less outwards, to which (and not to the kangaroo rat premolar) the great tooth of Thylacoleo bears a close resemblance, and the ring-tailed phalangers of the genus *Pseudocheirus* which close the phalanger series proper. These animals, generally called ring-tailed 'possums, resemble in their dentition the aberrant Phascolarctos, or native bear; and in the loosely anchylosed and movable mandibles and the scooped out lower incisors, they approach the kangaroos.

The relationship between the two animals, the great ring-tail flying squirrel and Cook's ring-tail phalanger or opossum, is so close that I am often obliged to look and compare skulls of both, where in other cases it is easy enough to feel without looking to which genus a skull or jaw belongs.

The Thylacoleo alone combines in its dentition, and in the form of the mandibles, characteristics which are found scattered about among the whole Phalanger and Bettong tribe.

Sub-Family *Phascolarctodidæ*.

The presence of a second species of the genus Phascolarctos, lately described by me in the Zoological Society's proceedings, makes it necessary to establish a sub-family for their reception. The dentition of this group is a very peculiar one, being chiefly distinguished by the total absence of canine teeth below, and by only one premolar (the third) above and below in each ramus. In this respect the native bears approach the kangaroos on the one hand and the gigantic extinct phalangers on the other.

There is also some relationship with the wombats in the shape of several bones, and in the occasional reduction of the upper incisor teeth to four, or even a single pair. The second and third upper incisors are small, and sometimes either missing or lost at an early age. Many individuals examined by me had only two incisors above in each ramus and two below, a fact which I desire to mention, as it may lead to further investigation. The upper grinders of the native bears are very broad, almost square, and provided with four sharp tubercles, the lower ones are more compressed. The undeveloped premolar of certain large extinct phalangers resembles the molars of the native bears, and young individuals of these again possess bones which bear a great likeness to those of full-grown Diprotodons. To this resemblance I shall refer again farther on.

Sub-Family *Diprotodontidæ*.

The Diprotodons were gigantic animals, with teeth constructed on the phalanger type, that is, six incisors above, and a pair below, without canines, the premolar generally present but often shed at an early age, molar teeth with a two-ridged crown divided by a valley and with rims or talons in front; the enamel either rugged and of a worm-eaten appearance or smooth.

These animals form two groups, the Zygomaturi and the Diprotodons proper, and at present they are not well understood by naturalists.

The chief difference consists in the cutting teeth, but as the mandibles and skulls are seldom found together, and as it cannot be proved when so found that the one really belongs to the other, we have been obliged to accept the additional genus Nototherium for certain loose mandibles. Professor Owen claims the only perfect skull of the genus Zygomaturus ever discovered, which was described by the late Mr. W. S. Macleay as belonging to his genus Nototherium—but this claim, as the lawyers say, has been disallowed. An exhaustive review of all Professor Owen's papers on Australian Fossil Remains has lately been published in the pages of the *Sydney Mail*, and to this I refer for particulars. Our *Zygomaturus* skull retains its incisor teeth, and I possess the fractured portion of the upper jaw of another Zygomaturus, containing the first incisor, the broken off second, and the alveolus of the third. These fragments were discovered by Dr. Creed, near Scone, and formed part of a skull which unfortunately broke to pieces when touched. The first of these teeth is figured on plate No. 2. The principal difference between the two genera is as follows :—

Genus Diprotodon.

First pair of upper front teeth broad, scalpiform or chisel-like, without compressed sides. The following teeth much smaller, right below the first pair, and not in a line with them, not unlike the corresponding ones of the native bear. Lower incisors very large, rounded, and tusk-like.

Genus Zygomaturus.

First upper incisor with compressed sides, like wombat teeth, of equal width throughout, and forming generally one-fourth of the segment of a circle; the next pair iu a line with the first, not pushed beneath them, much smaller, with straight fangs, and not unlike the same teeth iu the *Bettongia campestris*—or kangaroo rat.

Genus Nototherium.

The upper teeth of this genus are unknown; it was founded on certain lower jaws destitute of incisors, but others have since been discovered containing incisive teeth, and these have been added to the geuus, so that a definition thereof, according to Owen, stands at present thus—incisors absent, very small, or sometimes very large, compressed, fusiform, and not rounded or tusklike as in the genus Diprotodon.

The molars vary much in shape, but all appear to have tapering fangs or roots. Premolar very small or absent.

Fam. Phascolomyidæ.

This family comprises the wombats, which retain many of the phalanger characters, but are chiefly distinguished by their peculiar continuously growing teeth. The incisors are two above and below, canines not developed, grinders five in each ramus above and below, the first being a premolar. The crown of very younr wombat molars resembles that of the Diprotodons, but this peculiarity is soon lost when the teeth get into use. Their insertion is in this manner that both series when viewed from in frotn would form figures like this $\begin{matrix})(\\()\end{matrix}$ the upper ones turned outwards, the lower ones inwards. The incisors above are formed like the first pair of the Zygomaturus teeth, whilst the lower ones resemble the Diprodoton tusks—a curious fact, which points to one common progenitor. It is also interesting to notice that the form of the first pair of cutting teeth in the native bear is more like the Diprodoton's upper incisors, whilst the lower teeth are an exact representation of the fusiform Nototherium teeth. Again, the upper ones of the Bettong closely resemble the Zygomaturus incisors, whilst the lower come near the Diprotodon. It is in this manner that our animals are intimately connected with each other, and characters concentrated in a few extinct species are still scattered about among the recent genera, each retaining some peculiarity from the Thylacoleo, Zygomaturus, Nototherium, or Diprotodon.

Fam. Macropodidæ.

This family comprises the kangaroo tribe, and is another branch or offshoot from the great phalanger family, as I shall presently show. Some of the old fossil kangaroos are chiefly distinguished by having the mandibles closely anchylosed, like the wombats; their lower incisors small, and not fit, owing to the firmness of the jaw, to nip the grass as modern kangaroos do. On this account they probably succumbed in the battle of life at an early period, whilst the co-existing smaller and fleeter species, who could move rapidly from place to place, lived on till the present day. The teeth of the kangaroos have always been in number the same as those of the native bear, with this exception, that the upper canine is seldom developed, and that in one group—the kangaroos proper—the grinding teeth are almost lost as quick in front as they came into place behind. Some extinct kangaroos are also distinguished by their thick premolars, but co-existing with these animals were such already as cannot now be separated from the living red kangaroo (*Macropus rufus*), or the black wallaroo (*Macropus robustus*). The teeth differed as much in shape as they do now. Some exhibited simple lobed grinders, with a connecting ridge; others had these lobes strengthened by fangs or buttresses; others again had teeth like the Diprotodons or Zygomaturi, but all invariably had firmly-rooted grinders, whose fangs expanded below, and thus prevented the perfect and functional molar teeth from being easily lost after death. There is nothing so scarce in collections as a perfect fossil kangaroo grinder, and I refer for particulars to the illustrations of our still unpublished fossil remains. The tribe of kangaroos is connected with the phalangers proper, through the ring-tail phalangers (*Pseudocheirus*) and the great flying phalangers or petaurista (*Petaurista*), animals having incisor teeth which above and below resemble those of the *Macropodidæ.* The upper ones, though very small, are of the same shape as the corresponding teeth in the nail-tail wallabies of the genus *Onychogalea.*

The connection with the bandicoots, the last of the herbivorous family of Marsupials, is effected through the Hypsiprimni, or short-tailed rat kangaroo, which approach nearest in form to the genus *Perameles.*

Fam. Peramelidæ.

The bandicoots bridge over the space between the grass and flesh eating tribes. Though they still retain the peculiar hind feet with the two small inner toes, they have developed already ten small cutting teeth above and six below; they also retain their three premolars and four molars through life, and they possess sometimes large canines, though their food remains grass and herbs. Their grinders, studded with sharp tubercles, appear admirably adapted for the insectivorous diet on which they are believed to exist, but a close examination reveals the astonishing fact that these teeth are inserted on the same principle as those of the wombat—in one genus at least—and that they have conical roots with a much smaller pair of fangs on the inner side. In the genus *Peragalea*, the one alluded to, the outward appearance of the grinders is perfectly wombat-like, and though a pair of most powerful canines are developed, still the habits of the creatures are almost entirely those of vegetarians, and excrements examined by me seldom showed remains of insects.

Fam. Dasyuridæ.

The number of teeth in this, the "native cat" family, is in one genus almost as in the bandicoots, with the exception of the upper cutting teeth. The bandicoots have five in each ramus above, or ten in all, and the dasyures only eight. The ordinary dasyures are deficient in one premolar tooth in each ramus above and below, and they approach in the form of the first molar the ancient thylacoleo—the animal with which this discourse was commenced, and which now closes the circle of our marsupial families who, apparently very different, are still closely connected with each other, and are probably developed from some remote mammalian form whereof the platypus is the only living representative.

The Bones of Marsupials.

Having discussed the dentition of the order, it is necessary to say a few words about their bones.

The chief distinguishing characteristic of a marsupial animal's skull is the vacuity of the palate, which is, however, not constant. The second is the inflection of the portion of the mandibles situated below the articulating condyle. The broader this inflection the more peaceful the animal. All highly carnivorous marsupials have this process narrow and sharp, all vegetarians broad and hollow. To give an example: The process is deepest in the living wombats, in the extinct Thylacoleo, in the great kangaroos, and the wallabies; it is less deep in the rat kangaroos and bettongs, in the Diprotodon and Nototherium, and in the native bear. The corresponding character is a functional canine tooth in the upper jaw. It may be argued that the gigantic Phalangers, the Nototherium, Zygomaturus, and Diprotodon did not possess such a tooth; but there are no rules without exceptions, and at earlier stages they may have possessed the tooth in question. We only know one or two perfect skulls of aged individuals, and as the rule holds good as far as recent marsupials are concerned, it may be accepted for them at all events.

The slightly carnivorous bandicoots, and the small phalangers known as flying-squirrels and flying-mice, show a sharp angle like all true flesh-eaters ; and though a bandicoot may live on herbs and roots, he will also kill mice and prove his carnivority whenever an opportunity is offered. The rule laid down by some of the earlier comparative anatomists, that the articulating condyle is below the dental line of the ramus in carnivores does not hold good in all cases, and, in a very exceptional form, the Dactylopsila, which is a fruit-eating phalanger, the condyle is as low as in our greatest carnivores. The dental series in a line with the ascending ramus has been pointed out by me as a carnivorous peculiarity; and this position of the teeth in the Thylacoleo, combined with upper canines and molars of a flesh-eater, have induced me to admit that the Thylacoleo was as carnivorous as other phalangers, but certainly not more so, because the broad expanse of the inflected angle—a proof of non-carnivority—neutralizes the other characteristics. The condyle of the most savage of our flesh-eaters—the Tasmanian devil—has a broad upper surface, and not the spindle or roller shape of the true placental beasts of prey.

The last important evidence of marsupiality in the herbivores is the wide foramen at the base of the ascending ramus. This opening becomes smaller in many of the insectivorous phalangers, though it is very much smaller, sometimes absent, in the native bear and wombat.

All marsupials have arm-bones with a rotating motion, except the pig-footed bandicoots. All except the Thylacine have a pair of marsupial bones attached to the lower portion of the pelvis, and all have the pelvic bones very narrow. All except the bandicoots have five well-developed nailed toes to the fore-foot ; and the whole tribe except the true carnivores has the peculiar arrangement of the hind toes, that is, two conjoined small digits on the inner side of the foot.

The humerus, through often modified, cannot easily be mistaken in the more common members of the tribe. There is always a strong deltoid ridge, and the supra-condylar foramen is almost always present, except in some small Dasyures, and the gigantic fossil herbivorons species, the Diprotodon, for example. That the hand or manus in all marsupials is provided with five digits, except in the Chæropus, or pig-footed bandicoot, has been mentioned already.

The scapula appears to differ in shape considerably at first sight, but closer examination reveals a certain uniformity of structure. I can do no more at present than draw attention to the corresponding form of this bone in the wombats, the thylacine, and the bandicoots.

All marsupials which have the rotating movement of the lower arm-bones possess clavicles—the exception being the bandicoots. The clavicles of the Diprotodons are exactly like those of the wombat.

It is necessary to state here that the shape of the ulna in the Diprotodons resembles that bone of the elephant, the olecranon process being little developed.

The femur of the terrestrial marsupials, who progress by a succession of leaps, is generally slightly bent; in the wombats (and more or less in the phalangers) it is a remarkably straight bone, very short, the shaft flattened (in the Diprotodon), and the distal portion much expanded. The tibia and fibula in the phalanger tribe enjoy much freedom of motion. The kangaroos have these bones closely attached, and the great Diprotodon had so short a tibia and fibula that I could not make up my mind for years to accept fragments of these bones as belonging really to a tibia. There is no doubt about them any longer, and a restored tibia and fibula in the Museum collection, will convince even the most sceptical.

The *os calcis* or heelbone of the Diprotodon resembles that of the wombat and native bear. The digits were probably very small, but I cannot say more about them at present, though we possess bones which may turn out to be those of the toes of a Diprotodon.

The vertebræ of these great animals resembled again those of the Phascolarctos, or native bear, and the wombat—the first, or atlas, consisting of two parts when young, never joining below, not even in adult subjects, just as the atlas of living phalangers remains permanently open below.

The ribs of the Diprotodon were probably thirteen pair, rather broad, and not unlike those of the wombat. The tail was short, and wombat-like also.

The numerous large bones hitherto discovered are in almost every instance a proof of being those of phalangers, either of the wombats or Diprotodon family, and not a single bone or tooth indicates the existence in Australia of a large carnivore—larger than the Tasmanian Thylacine.

I shall now give a list of the animals hitherto discovered in a fossil state, and arrange them in the following order:—

Fam. Phalangistidæ.

To this family belong all the gigantic fossil mammals. The following genera are represented:—

Genus Diprotodon.

With two described species *D. australis* and *D. bennettii*. The last-mentioned animal has lately been been found by Messrs.

King and Bennett, at Gowrie, in the Darling Downs district. The splendid casts now in the Museum were prepared under my direction. These casts and models represent the four legs of the marsupial giant, named in honor of Dr. George Bennett, of this city, who was kind enough to put the material for the restoration of an almost perfect skeleton at my disposal.

There were at least a dozen or more different kinds of Diprotodons, but their description cannot now be entered on.

Genus Zygomaturus.

Two species are at present described, but I possess proof that more existed.

Genus Nototherium.

A numerous tribe, represented by perhaps twenty or more species.

Genus Thylacoleo.

Several kinds of this Phalanger have been proved to exist, full descriptions of which will shortly be given.

Genus Phascolarctos.
Phalangista.
Belideus.

Fossil remains of these three genera have been found.

Fam. Macropodidæ.

This extensive family was represented by numerous species, many of which are still living. All the short-footed animals with firmly joined mandibles are now extinct, and for these the genus *Halmatutherium* (*Krefft*) has been established.

The fossil Bettongs are identical with still living species.

Fam. Phascolomyidæ.

The wombats were also numerous in olden times, and twenty fossil species at least can be demonstrated.

Fam. Peramelidæ.

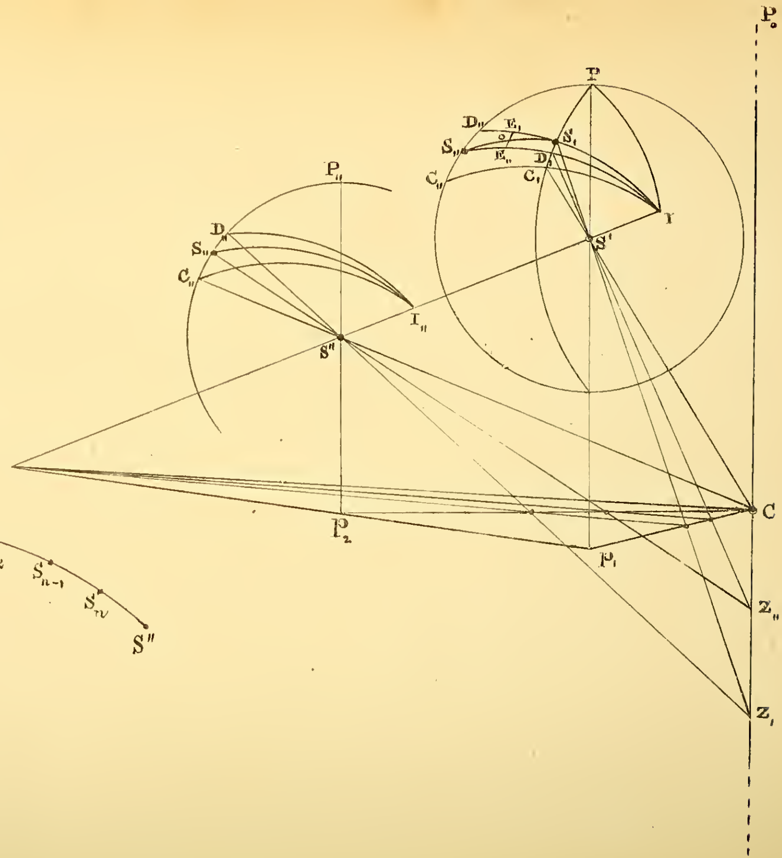
The bandicoots are also plentiful in a fossil state. The Peragalea, or rabbit rat, with its peculiar wombat-like grinders, occurs already, and many of these fossil teeth show a continuous growth, like the teeth of all wombats.

FAM. DASYURIDÆ.

Common native cats and the Thylacine and Sarcophilus, identical with the animals now inhabiting Tasmania, were common in the Wellington District in particular. Elsewhere their remains are very rare. The Thylacine was the largest of our carnivores.

The teeth in the Museum collection have now been all determined, and there is not one which indicates the presence in former times of animals which could not be referred to any one of the genera enumerated in this paper.

This closes the Ornithodelphia and Didelphia.



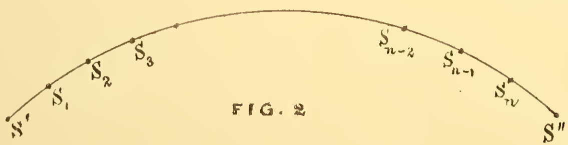


FIG. 2

Geodesic Investigations by M. Gardiner C.E.

GEODESIC INVESTIGATIONS.

CONTAINING numerous new* theorems in Practical Geodesy, and solutions to all the problems relating to the determination of latitudes, azimuths, difference of longitude, length and circular measure of geodesic arc, &c.

By MARTIN GARDINER, *C.E., of the Queen's University, Ireland, Member of the Mathematical Society of London.*

[*Read before the Royal Society*, 5 *November*, 1873.]

(*See plate.*)

LET P_0 be the pole of reference of the spheroidal earth; C the earth's centre; S', S'', any two stations on its surface; $Z_{\prime}$, $Z_{\prime\prime}$, the points in which the normals to the surface at S', S'', cut the polar axis.

The planes $S'S''Z_{\prime}$, $S''S'Z_{\prime\prime}$, shall be referred to as the "normal-chordal" planes. The first of these planes, which contains the normal to the surface at the point S', shall be designated the normal-chordal plane S'; and the other, which contains the normal to the surface at the point S'', shall be designated the normal-chordal plane S''.

And any plane whatever which contains the chord of the geodesic arc $S'S''$ shall be referred to as a chordal plane.

The polar and equatorial radii of the earth being something of about 3,950 miles, and 3,963 miles in length, it is evident that (owing to the great magnitude and small ellipticity of central and normal sections), for arcs of not more than 100 miles in length, we may, with due respect to the most minute accuracy attainable in actual calculations, assume that the normals to the surface at the stations make angles with the chord connecting the stations whose sines are equals. And we may consider the traces of the two normal-chordal planes as equals in length and in circular measure to the true "geodesic" or shortest arc between the stations.

Let s and Σ represent the common length, and common circular measure of either of these plane traces on the spheroidal earth. And let $\frac{1}{2}\ \Sigma'$, $\frac{1}{2}\ \Sigma''$, be the depressions of the chord of $S'S''$ below the tangent planes at S' and S''. Conceive two unit spheres described having S', S'' as centres.

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* All the theorems in the paper are given as *new*, with the exception of (1) and (18); and all the problems, with exception of problems 1 and 5.

Let $C_{,}$, $S_{,}$, I, P, be the points in which the sphere S' is pierced by the productions of the lines CS', $Z_{,}S'$, $S''S'$ through the centre S', and by the line $S'P$ parallel to the polar radius and in the same direction.

Let $C_{,,}$, $S_{,,}$, $I_{,,}$, $P_{,,}$, be the points in which the sphere S'' is pierced by the productions of the lines CS'', $Z_{,,}S''$, by the chord $S'S''$ taken in the direction $S''S'$, and by the line $S_{,,}P_{,,}$ parallel to and in the same direction as the polar radius CP_{0}.

Then, evidently, the points $P_{,}$, $C_{,}$, $S_{,}$, are in the trace, on the unit sphere S', of the earth's meridian plane through S'; and the points $P_{,,}$, $C_{,,}$, $S_{,,}$, are in the trace, on the unit sphere S'', of the earth's meridian plane through the station S''.

We have, evidently, the arc PI = the arc $P_{,,}I_{,,}$; arc $S_{,}I = 90° - \frac{1}{2} \Sigma'$; arc $S_{,,}I_{,,} = 90° + \frac{1}{2} \Sigma''$; angle $PS_{,}I$ = supplement of the geographic azimuth of the station S'' as observed at S′ (which azimuth we shall denote by the letter A'); angle $P_{,,}S_{,,}I$ = the geographic azimuth of the station S' as if taken at S'' (which we shall denote by A''); arcs $PS_{,}$, $P_{,,}S_{,,}$ equals to the geographic co-latitudes of the stations S', S''; arcs $PC_{,}$, $PC_{,,}$, equals to the geocentric co-latitudes of the stations S', S''; angle $PC_{,}I$ = supplement of angle $C_{,}$, which we shall call the geocentric azimuth of S'' as if taken at S' (this angle cannot be directly measured with the instruments); and the angle $P_{,,}C_{,,}I_{,,}$ is the geocentric azimuth $C_{,,}$, which corresponds to the geographic azimuth A''.

☞ In the figure, and all through this paper, the latitude of the station S' is considered *greater*, or not less, than the latitude of the station S''.

Now conceive the sphere S'' moved by direct translation along the chord $S''S'$, carrying its lines and points rigidly fixed, until its centre coincides with the centre S' of the sphere S'. Then it is evident the points $I_{,,}$, $P_{,,}$, will coincide with I, P, and that the points I, $C_{,}$, $C_{,,}$, will lie in one great circle of the sphere $S_{,}$.

It is also evident that the points $PS_{,,}C_{,,}$ lie in the trace of one plane, or in one great circle, and that the spherical angle $S_{,}PS_{,,}$ or $C_{,}PC_{,,}$ represents the difference of longitude of the stations S', S''.

Let $p_{,}$, $p_{,,}$, be the points in which the lines PS', $P_{,,}S''$, pierce the earth's equator. It is evident that the plane angle $p_{,}Cp_{,,}$ is = the difference of longitude of the stations S', S''.

Let $\phi_{,}$, $\phi_{,,}$, represent the angles $Cp_{,}p_{,,}$, $Cp_{,,}p_{,}$, of the plane triangle $p_{,}Cp_{,,}$. It is evident $\phi_{,}$, $\phi_{,,}$, are equivalents to the dihedral angles which the chordal plane $S'S''p_{,}p_{,,}$ parallel to the polar axis, makes with the meridian planes passing through the stations S', S'', respectively.

It is also evident that the spherical angle $S_{,}PI = \phi_{,}$, and that the spherical angle $S_{,,}PI = 180° - \phi_{,,} = \phi_{,} +$ the difference of longitude of the stations S', S''.

The arc PI is the measure of the angle which the chord $S'S''$ makes with the polar axis; and is also the complement of the angle θ which the chord $S'S''$ makes at i with the plane of the equator.

Let $D_{\prime}$, $D_{\prime\prime}$, be the points in which the great circles $IS_{\prime\prime}$, $IS_{\prime}$, cut the great circles $PS_{\prime}C_{\prime}$, $PS_{\prime\prime}C_{\prime\prime}$, respectively.

The arc $S_{\prime}S_{\prime\prime}$ is the measure of the angle between the normals $S'Z_{\prime}$, $S''Z_{\prime\prime}$; the arc $C_{\prime}C_{\prime\prime}$ is the measure of the angle C between the central radii to the stations; the arc $S_{\prime}D_{\prime\prime}$ is the measure of the plane angle $S'Z_{\prime}S''$; and the arc $S_{\prime\prime}D_{\prime}$ is the measure of the plane angle $S'Z_{\prime\prime}S''$.

The arcs $S_{\prime}C_{\prime}$, $S_{\prime\prime}C_{\prime\prime}$, are the measures of "the angles of the vertical" at the respective stations S', S''.

The arc $IC_{\prime}$ and the supplement of the arc $IC_{\prime\prime}$ are the measures of the two interior angles of the triangle formed by the chord of the geodesic arc, and central radii to its extremities.

The small spherical angle $S_{\prime}\,IS_{\prime\prime}$ is evidently equivalent to the dihedral angle $\triangle$ between the two normal-chordal planes.

☞ The interpretation of the points, lines, angles, &c., of the whole figure can present no difficulty to any one, and need not be further elucidated. But, in order to comprehend the investigations it is necessary to master the following notation, which is adhered to throughout as closely as possible:—

NOTATION.

l', l'' represent the geographic latitudes of the stations S', S''.
$l_{\prime}$, $l_{\prime\prime}$ „ „ „ colatitudes „ „ „
λ', λ'' „ „ geocentric latitudes „ „ „
A', A'' „ „ geographic azimuths as already explained.
Σ, represents the circular measure of the arc between S' and S''.
k „ „ chord of the arc.
s „ „ length of the arc.
$k_{\circ}$ „ „ projection $p_{\prime}p_{\prime\prime}$ of the chord k.
θ „ „ angle which the chord k makes with $k_{\circ}$.
$\phi_{\prime}$, $\phi_{\prime\prime}$ represent the angles $Cp_{\prime}p_{\prime\prime}$, $Cp_{\prime\prime}p_{\prime}$ in the plane of the equator.
$\beta_{\prime}$, $\beta_{\prime\prime}$ „ „ angles which the chordal plane parallel to the polar axis makes with the normal-chordal planes S', S''.
Δ represents the angle between the two normal-chordal planes.
Ω „ either of the angles $S_{\prime}S_{\prime\prime}D_{\prime}$, $S_{\prime\prime}S_{\prime}D_{\prime\prime}$.
ν „ the arc $S_{\prime}S_{\prime\prime}$, or angle between the normals at the stations.
$L_{\prime}$, $L_{\prime\prime}$ „ the arcs $PD_{\prime}$, $PD_{\prime\prime}$.
$z_{\prime}$, $z_{\prime\prime}$ „ the arcs $S_{\prime}D_{\prime\prime}$, $S_{\prime\prime}D_{\prime}$.
$D_{\prime}$, $D_{\prime\prime}$ „ the angles $PD_{\prime}S_{\prime\prime}$, $PD_{\prime\prime}S_{\prime}$.
$R_{\prime}$, $R_{\prime\prime}$ „ the lengths of the lines $S'Z_{\prime}$, $S''Z_{\prime\prime}$.
$r_{\prime}$, $r_{\prime\prime}$ „ the lengths of the central radii CS', CS''.
a, b, „ the lengths of the equatorial and polar radii.
e „ the earths eccentricity.
$\frac{1}{2}\Sigma'$ „ the depression of the chord below the tangent plane at S'.
$\frac{1}{2}\Sigma''$ „ „ „ „ „ S''.
$\delta_{\prime}$, $\delta_{\prime\prime}$, „ the arcs $S_{\prime}D_{\prime}$, $S_{\prime\prime}D_{\prime\prime}$.

1 If with I as pole we describe a great circle of the unit sphere S' to cut the arcs $S_{\prime}D_{\prime\prime}$, $S_{\prime\prime}D_{\prime}$, $S_{\prime}S_{\prime\prime}$, in $E_{\prime}$, $E_{\prime\prime}$, O, respectively, it is evident that—

$$\text{arc } S_{\prime}E_{\prime} = \tfrac{1}{2}\Sigma'$$
$$\text{arc } S_{\prime\prime}E_{\prime\prime} = \tfrac{1}{2}\Sigma''$$

Moreover, it is evident that for any pair of mutually visible stations S', S'' on the earth, we have—

$$\Sigma = \tfrac{1}{2}\Sigma' + \tfrac{1}{2}\Sigma''$$

And we may also assume—

$$\cos\tfrac{1}{2}\Sigma' = \cos\tfrac{1}{2}\Sigma'' = \cos\tfrac{1}{2}\Sigma,$$

without appreciable error in calculated results.

This can be easily inferred from the following elucidation:—

We have— $S_{\prime}E_{\prime} = \tfrac{1}{2}\Sigma'$; $S_{\prime}I = \tfrac{1}{2}\pi - \tfrac{1}{2}\Sigma'$;
$S_{\prime\prime}E_{\prime\prime} = \tfrac{1}{2}\Sigma''$; $S_{\prime\prime}I = \tfrac{1}{2}\pi + \tfrac{1}{2}\Sigma''$;

$$\tfrac{1}{2}\Sigma' + \tfrac{1}{2}\Sigma'' = \Sigma;$$
$$\tfrac{1}{2}\Sigma'' > \tfrac{1}{2}\Sigma'.$$

Applying formula (4) given by Serret, on page 158 of his "Traité de Trigonométrie," to the spherical triangle $S_{\prime}IS_{\prime\prime}$, and putting ϵ for the spherical excess of this triangle, we have—

$$\tan\tfrac{1}{2}(\Delta - \epsilon) = \tan\tfrac{1}{2}\Delta.\,\frac{\sin\frac{1}{2}(\frac{1}{2}\Sigma' - \frac{1}{2}\Sigma'')}{\cos\frac{1}{2}(\frac{1}{2}\Sigma' + \frac{1}{2}\Sigma'')}$$

And since we know that the numerator of the fractional portion on the right-hand side of this equation is necessarily negative, $\therefore$ we learn that the angle Δ or $S_{\prime}IS_{\prime\prime}$ is less than ϵ, the spherical excess of the triangle $S_{\prime}IS_{\prime\prime}$. Hence we infer that—

$$\text{angle } IS_{\prime}S_{\prime\prime} + \text{angle } IS_{\prime\prime}S_{\prime} > \pi$$

and $\therefore$ also—

$$\text{angle } S_{\prime}S_{\prime\prime}D_{\prime} > \text{angle } S_{\prime\prime}S_{\prime}D_{\prime\prime}$$

and that—

$$A' + A'' > \text{angle } P_{\prime}S_{\prime}S_{\prime\prime} + \text{angle } PS_{\prime\prime}S_{\prime}, \text{ or } > a' + a''.$$

And we perceive that the sum of the azimuths A', A'' is greater than the sum of the two angles a', a'', of the triangle $S_{\prime}PS_{\prime\prime}$ by the amount which the angle $S_{\prime}S_{\prime\prime}D_{\prime}$ exceeds $S_{\prime\prime}S_{\prime}D_{\prime\prime}$; or, which is the same, by the amount which ϵ, the spherical excess of the triangle $S_{\prime}IS_{\prime\prime}$, exceeds Δ, or angle $S_{\prime}IS_{\prime\prime}$.

The equation—

$$\tan\tfrac{1}{2}(\epsilon - \Delta) = \tan\tfrac{1}{2}\Delta.\,\frac{\sin\frac{1}{2}(\frac{1}{2}\Sigma'' - \frac{1}{2}\Sigma')}{\cos\frac{1}{2}(\frac{1}{2}\Sigma'' + \frac{1}{2}\Sigma')}$$

shows what a very small angle $\epsilon - \Delta$ must be. In actual practice, in which the stations are not over 100 miles asunder, $\epsilon - \Delta$ will always be less than $\frac{1}{1000}$ part of a second.

$$\therefore\quad \frac{\cos\frac{1}{2}\Sigma'}{\cos\frac{1}{2}\Sigma''} = \frac{\sin S_{\prime}S_{\prime\prime}D}{\sin S_{\prime\prime}S_{\prime}D} = 1.$$

2 The following relations are also worthy of particular notice:—

$$D_{\prime} > A'$$
$$A' > A''$$
$$A'' > D_{\prime\prime}$$

Their demonstration may be as follows:—

The triangle $S_{\prime}ID_{\prime}$ is evidently such that,—

$$\text{angle } IS_{\prime}D_{\prime} + \text{angle } ID_{\prime}S_{\prime} < \pi$$

but $\quad \text{angle } PD_{\prime}S_{\prime\prime} + \text{angle } ID_{\prime}S_{\prime} = \pi$

$\therefore \quad \text{angle } PD_{\prime}S_{\prime\prime} > \text{angle } IS_{\prime}D$

or $\quad D_{\prime} > A'$.

The triangle $S_{\prime\prime}ID_{\prime\prime}$ is evidently such that,—

$$\begin{array}{ll} & \text{angle } IS_{\prime\prime}D_{\prime\prime} + \text{angle } ID_{\prime\prime}S_{\prime\prime} > \pi \\ \text{but} & \text{angle } PD_{\prime\prime}S_{\prime} + \text{angle } ID_{\prime\prime}S_{\prime\prime} = \pi \\ \therefore & \text{angle } IS_{\prime\prime}D_{\prime\prime} > \text{angle } PD_{\prime\prime}S_{\prime} \\ \text{or} & A'' > D_{\prime\prime}. \end{array}$$

If we apply Serret's theorem (already mentioned) to the triangles $S_{\prime}ID_{\prime}$, $S_{\prime\prime}ID_{\prime\prime}$, we easily find—

$$\tan \tfrac{1}{2}(D_{\prime}-A') = \tan \tfrac{1}{2}\Delta \cdot \frac{\sin \frac{1}{2}\left\{z_{\prime\prime}-(\frac{1}{2}\Sigma''-\frac{1}{2}\Sigma')\right\}}{\cos \frac{1}{2}\left\{z_{\prime\prime}-(\frac{1}{2}\Sigma''+\frac{1}{2}\Sigma')\right\}}$$

$$\tan \tfrac{1}{2}(A''-D_{\prime\prime}) = \tan \tfrac{1}{2}\Delta \cdot \frac{\sin \frac{1}{2}\left\{z_{\prime}+(\frac{1}{2}\Sigma''-\frac{1}{2}\Sigma')\right\}}{\cos \frac{1}{2}\left\{z_{\prime}-(\frac{1}{2}\Sigma''+\frac{1}{2}\Sigma')\right\}}.$$

intimating that $D_{\prime}-A'$ and $A''-D_{\prime\prime}$ are both positive ; and $\therefore$ that $D_{\prime} > A'$; and $A'' > D_{\prime\prime}$.

And, from these, we have—

$$\frac{\tan \frac{1}{2}(D_{\prime}-A')}{\tan \frac{1}{2}(A''-D_{\prime\prime})} = \frac{\cos \frac{1}{2}\left\{z_{\prime}-(\frac{1}{2}\Sigma''+\frac{1}{2}\Sigma')\right\}}{\cos \frac{1}{2}\left\{z_{\prime\prime}-(\frac{1}{2}\Sigma''+\frac{1}{2}\Sigma')\right\}} \cdot \frac{\sin \frac{1}{2}\left\{z_{\prime\prime}-(\frac{1}{2}\Sigma''-\frac{1}{2}\Sigma')\right\}}{\sin \frac{1}{2}\left\{z_{\prime}+(\frac{1}{2}\Sigma''-\frac{1}{2}\Sigma')\right\}}$$

That A' is greater than A'' is evident from equations (12) given in the sequel.

☞ In the "Account of the Trigonometrical Survey of Great Britain and Ireland" (see page 249 of that work) it is assumed that—

$D_{\prime}$ is greater than A'
and that $D_{\prime\prime}$ is also greater than A''

It is easy to perceive that the formulæ there given to express the differences $D_{\prime}-A'$, $A''-D_{\prime\prime}$ is erroneous, viz. :—

$$D-A' = \tfrac{1}{4}\frac{e^2}{1-e^2} \cdot \cos^2 l' \cdot \sin 2A' \cdot z_{\prime}^2$$

$$D_{\prime\prime}-A'' = \tfrac{1}{4}\frac{e^2}{1-e^2} \cdot \cos^2 l'' \cdot \sin 2A'' \cdot z_{\prime\prime}^2 .$$

For, assuming the difference of longitude of the two stations to be 1°, and $l' = 50°$, it is evident that we can have values for A' and A'' each less than 90°, and $\therefore$ the right-hand members of these equations positive :— thus intimating that $D_{\prime\prime}$ is greater than A'', which we know to be absurd.

If we were to assume $A' = 90°$, then it is evident the above formulæ would intimate that

$$D_{\prime} = A' = 90° ;$$

which is also absurd : for from the point I, which is not the pole of the great circle $PS_{\prime}$, we cannot have two arcs $IS_{\prime}$, $ID_{\prime}$ perpendicular to this circle.

If we suppose the latitudes l', l'' to be equal to each other, then we know that in all such cases

$$D_{\prime} - A' = 0 ; \quad A'' - D_{\prime\prime} = 0 ;$$

but in such cases, the above formulæ would intimate that both $D_{\prime}-A'$ and $A''-D_{\prime\prime}$ have real finite values. (See note to problem 5).

3 Putting a' and a'' to represent the angles $PS_{,}S_{,,}$, $PS_{,,}S_{,}$, of the triangle $S_{,}PS_{,,}$; it is evident, from what has been already shown, that in all cases of mutually visible stations S', S'', on the earth's surface we have, to a very minute accuracy—

$$A' + A'' = a' + a''$$

And since we can, by one of Napier's analogies, express the angle $S_{,}PS_{,,}$ or ω in terms of the sum of a', a'', and the sides $l_{,}$, $l_{,,}$; therefore, by substituting $A' + A''$ instead of $a' + a''$ in such analogy, we have—

$$\tan \tfrac{1}{2}\,\omega = \frac{\cos \frac{1}{2}\,(l_{,,} - l_{,})}{\cos \frac{1}{2}\,(l_{,,} + l_{,})} \cot \tfrac{1}{2}\,(A' + A'');$$

$$\therefore \quad \tan \tfrac{1}{2}\,\omega = \frac{\cos \frac{1}{2}\,(l' - l'')}{\sin \frac{1}{2}\,(l' + l'')} \cot \tfrac{1}{2}\,(A' + A'') \quad \ldots \quad \ldots \quad (1)$$

This important formula is known as Dalby's theorem.

By applying Delambre's analogies to the triangle $S_{,}PS_{,,}$, and assuming $A' + A'' = a' + a''$, we have—

$$\left.\begin{aligned}
\sin \tfrac{1}{2}\,(A' + A'') &= \cos \tfrac{1}{2}\,(l' - l'') \cdot \frac{\cos \frac{1}{2}\,\omega}{\cos \frac{1}{2}\,\nu} \\
\cos \tfrac{1}{2}\,(A' + A'') &= \sin \tfrac{1}{2}\,(l' + l'') \cdot \frac{\sin \frac{1}{2}\,\omega}{\cos \frac{1}{2}\,\nu} \\
\tan \tfrac{1}{2}\,(A' + A'') &= \frac{\cos \frac{1}{2}\,(l' - l'')}{\sin \frac{1}{2}\,(l' + l'')} \cdot \cot \tfrac{1}{2}\,\omega \\
\cot \tfrac{1}{2}\,(A' + A'') &= \frac{\sin \frac{1}{2}\,(l' + l'')}{\cos \frac{1}{2}\,(l' - l'')} \cdot \tan \tfrac{1}{2}\,\omega
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (2)$$

Again, by applying formulæ (1), page 157 of Serret's Trigonometry, to the triangle $S_{,}PS_{,,}$ we have—

$$-\frac{\cos \frac{1}{2}\,(A' + A'' + \omega)}{\cos \frac{1}{2}\,(A' + A'' - \omega)} = \tan\,(45° - \tfrac{1}{2}\,l') \cdot \tan\,(45° - \tfrac{1}{2}\,l'') \quad \ldots \quad (3)$$

$$-\frac{\cos \frac{1}{2}\,(A' + A'' + \omega)}{\cos \frac{1}{2}\,(A' + A'' - \omega)} = \tan \tfrac{1}{2}\,l_{,} \cdot \tan \tfrac{1}{2}\,l_{,,} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

4 From the triangles $S_{,}PI$, $S_{,,}PI$, we have—

$$\left.\begin{aligned}
\sin A' \cos \tfrac{1}{2}\,\Sigma' &= \cos\theta \sin\phi_{,} \\
\sin A'' \cos \tfrac{1}{2}\,\Sigma'' &= \cos\theta \sin\phi_{,,}
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (5)$$

$$\left.\begin{aligned}
\sin A' \cos l' &= \cos\theta \sin\beta_{,} \\
\sin A'' \cos l'' &= \cos\theta \sin\beta_{,,}
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (6)$$

$$\left.\begin{aligned}
\sin\theta &= \sin \tfrac{1}{2}\,\Sigma' \sin l' - \cos \tfrac{1}{2}\,\Sigma' \cos l' \cos A' \\
\sin\theta &= -\sin \tfrac{1}{2}\,\Sigma'' \sin l'' + \cos \tfrac{1}{2}\,\Sigma'' \cos l'' \cos A''
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (7)$$

$$\left.\begin{aligned}
\tan \tfrac{1}{2}\,\Sigma' \cos l' &= \cot\phi_{,} \sin A' - \sin l' \cos A' \\
\tan \tfrac{1}{2}\,\Sigma'' \cos l'' &= \cot\phi_{,,} \sin A'' - \sin l'' \cos A''
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (8)$$

$$\left.\begin{aligned}
\tan l' \cos \tfrac{1}{2}\,\Sigma' &= \cot\beta_{,} \sin A' - \sin \tfrac{1}{2}\,\Sigma' \cos A' \\
\tan l'' \cos \tfrac{1}{2}\,\Sigma'' &= \cot\beta_{,,} \sin A'' - \sin \tfrac{1}{2}\,\Sigma'' \cos A''
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (9)$$

5 From the equations (5) we have—

$$\frac{\sin A'}{\sin A''} \cdot \frac{\cos \frac{1}{2} \Sigma'}{\cos \frac{1}{2} \Sigma''} = \frac{\sin \phi_{\prime}}{\sin \phi_{\prime\prime}}$$

But it is evident that $\dfrac{\sin \phi_{\prime}}{\sin \phi_{\prime\prime}} = \dfrac{R_{\prime\prime} \cos l''}{R_{\prime} \cos l'} = \dfrac{r_{\prime\prime} \cos \lambda''}{r_{\prime} \cos \lambda'}$

And since with respect to any pair of mutually visible stations, we may assume $\dfrac{\cos \frac{1}{2} \Sigma'}{\cos \frac{1}{2} \Sigma''} = 1$; we have—

$$\left.\begin{aligned} \frac{\sin A'}{\sin A''} &= \frac{R_{\prime\prime} \cos l''}{R_{\prime} \cos l'} \\ \frac{\sin A'}{\sin A''} &= \frac{r_{\prime\prime} \cos \lambda''}{r_{\prime} \cos \lambda'} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (10)$$

The rigorously true formulæ, for any two stations on the earth, being—

$$\left.\begin{aligned} \frac{\sin A'}{\sin A''} &= \frac{R_{\prime\prime} \cos l''}{R_{\prime} \cos l'} \cdot \frac{\cos \frac{1}{2} \Sigma''}{\cos \frac{1}{2} \Sigma'} \\ \frac{\sin A'}{\sin A''} &= \frac{r_{\prime\prime} \cos \lambda''}{r_{\prime} \cos \lambda'} \cdot \frac{\cos \frac{1}{2} \Sigma''}{\cos \frac{1}{2} \Sigma'} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (11)$$

From these we easily deduce—

$$\left.\begin{aligned} \frac{\sin^2 A'}{\sin^2 A''} &= \frac{(1-e^2) \tan^2 l' + 1}{(1-e^2) \tan^2 l'' + 1} \cdot \frac{\cos^2 \frac{1}{2} \Sigma''}{\cos^2 \frac{1}{2} \Sigma'} \\ &= \frac{\tan^2 l' + \dfrac{a^2}{b^2}}{\tan^2 l'' + \dfrac{a^2}{b^2}} \cdot \frac{\cos^2 \frac{1}{2} \Sigma''}{\cos^2 \frac{1}{2} \Sigma'} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (12)$$

$$\left.\begin{aligned} \frac{\sin^2 A'}{\sin^2 A''} &= \frac{\dfrac{1}{e^2} \cdot \sec^2 \lambda' - 1}{\dfrac{1}{e^2} \sec^2 \lambda'' - 1} \cdot \frac{\cos^2 \frac{1}{2} \Sigma''}{\cos^2 \frac{1}{2} \Sigma'} \quad \ldots \quad \ldots \quad (13) \\ &= \frac{\dfrac{a^2}{b^2} \tan^2 \lambda' + 1}{\dfrac{a^2}{b^2} \tan^2 \lambda'' + 1} \cdot \frac{\cos^2 \frac{1}{2} \Sigma''}{\cos^2 \frac{1}{2} \Sigma'} \quad \ldots \quad \ldots \quad (13) \end{aligned}\right\}$$

And if we find m' and m'' such, that $\tan^2 m' = (1-e^2) \tan^2 l'$; $\tan^2 m'' = (1-e^2) \tan^2 l''$; then will

$$\frac{\sin A'}{\sin A''} = \frac{\cos m''}{\cos m'} \cdot \frac{\cos \frac{1}{2} \Sigma''}{\cos \frac{1}{2} \Sigma'}$$

If we find an angle χ such, that

$$\tan^2 \chi = \frac{(1-e^2) \tan^2 l' + 1}{(1-e^2) \tan^2 l'' + 1} \cdot \frac{\cos^2 \frac{1}{2} \Sigma''}{\cos^2 \frac{1}{2} \Sigma'}$$

$$\text{then} \quad \frac{\sin A'}{\sin A''} = \frac{\tan \chi}{1}$$

$$\frac{\tan \frac{1}{2} (A'-A'')}{\tan \frac{1}{2} (A'+A'')} = \tan (\chi-45^\circ)$$

$$\therefore \quad \tan \tfrac{1}{2} (A'-A'') = \tan \tfrac{1}{2} (A'+A'') \tan (\chi-45^\circ) \quad \ldots \quad \ldots \quad (14)$$

$$\tan \tfrac{1}{2} (A'-A'') = \frac{\cos \frac{1}{2} (l'-l'')}{\sin \frac{1}{2} (l'+l'')} \cdot (\tan \chi-45^\circ) \cot \tfrac{1}{2}\omega \quad \ldots \quad (15)$$

And since $a'-a'' = A' + \Omega - (A''-\Omega) = A'-A'' + 2\Omega$

$$\therefore \tan\left\{\tfrac{1}{2}(A'-A'') + \Omega\right\} = \frac{\sin\frac{1}{2}(l'-l'')}{\cos\frac{1}{2}(l'+l'')}\cot\tfrac{1}{2}\omega \qquad \dots \quad \dots \quad (16)$$

6 Multiplying both sides of equations (5) by the chord k, and remembering that the projection k_o, of the chord on the plane of the equator is $= k\cos\theta$, we have

$$\left.\begin{aligned} k.\ \sin A' \cos\tfrac{1}{2}\Sigma' &= k_o\,.\,\sin\phi_, \\ k.\ \sin A'' \cos\tfrac{1}{2}\Sigma'' &= k_o\,.\,\sin\phi_{,,} \end{aligned}\right\} \qquad \dots \quad \dots \quad \dots \quad (17)$$

But from the plane triangle $p_, Cp_{,,}$, we have

$$k_o = \frac{R_,\cos l'\sin\omega}{\sin\phi_{,,}} = \frac{R_{,,}\cos l''\sin\omega}{\sin\phi_,}$$

$$\therefore \left.\begin{aligned} k\cdot\sin A' \cos\tfrac{1}{2}\Sigma' &= R_{,,}\cos l''\sin\omega \\ k\cdot\sin A'' \cos\tfrac{1}{2}\Sigma'' &= R_,\cos l'\sin\omega \end{aligned}\right\} \qquad \dots \quad \dots \quad \dots \quad (18)$$

$$\left.\begin{aligned} k\sin A' \cos\tfrac{1}{2}\Sigma' &= r_{,,}\cos\lambda''\sin\omega \\ k\sin A'' \cos\tfrac{1}{2}\Sigma'' &= r_,\cos\lambda'\sin\omega \end{aligned}\right\} \qquad \dots \quad \dots \quad \dots \quad (19)$$

Again $$k = \frac{2s\sin\frac{1}{2}\Sigma}{\Sigma}$$

$$\therefore \frac{2s.\sin A'\sin\frac{1}{2}\Sigma\cos\frac{1}{2}\Sigma'}{\Sigma} = R_{,,}\cos l''\sin\omega$$

$$\frac{2s.\sin A''\sin\frac{1}{2}\Sigma\cos\frac{1}{2}\Sigma''}{\Sigma} = R'\cos l'\sin\omega.$$

And if we assume $\cos\frac{1}{2}\Sigma = \cos\frac{1}{2}\Sigma' = \cos\frac{1}{2}\Sigma''$, these may be written

$$\left.\begin{aligned} s\,.\,\sin A'\,.\,\frac{\sin\Sigma}{\Sigma} &= R_{,,}\cos l''\sin\omega \\ s\,.\,\sin A''\,.\,\frac{\sin\Sigma}{\Sigma} &= R_,\cos l'\sin\omega \end{aligned}\right\} \qquad \dots \quad \dots \quad (20)$$

in which s and Σ are the length and circular measure of the geodesic arc.

7 From the triangles $D_, S_, I$, $D_{,,} S_{,,} I$, we have—

$$\left.\begin{aligned} \cos\tfrac{1}{2}\Sigma'\,.\,\sin A' &= \cos(z_{,,}-\tfrac{1}{2}\Sigma'')\sin D_, \\ \cos\tfrac{1}{2}\Sigma''\,.\,\sin A'' &= \cos(z_,-\tfrac{1}{2}\Sigma')\sin D_{,,} \end{aligned}\right\} \dots \quad \dots \quad \dots \quad (21)$$

And from the triangles $S_,PI$, $S_{,,}PI$, we have—

$$\left.\begin{aligned} \cos l'\sin\omega &= \sin z_,\sin D_{,,} \\ \cos l''\sin\omega &= \sin z_{,,}\sin D_, \end{aligned}\right\} \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (22)$$

From these we easily deduce—

$$\cot z_, = \frac{\sin A''\,.\,\cos\frac{1}{2}\Sigma''}{\cos l'\,.\,\sin\omega\,.\,\cos\frac{1}{2}\Sigma'} - \tan\tfrac{1}{2}\Sigma' \qquad \dots \quad \dots \quad (23)$$

$$\cot z_{,,} = \frac{\sin A'\,.\,\cos\frac{1}{2}\Sigma'}{\cos l''\,.\,\sin\omega\cdot\cos\frac{1}{2}\Sigma''} - \tan\tfrac{1}{2}\Sigma'' \qquad \dots \quad \dots \quad (24)$$

or, with close approximate accuracy for mutually visible stations—

$$\cot z_, = \frac{\sin A''}{\cos l'\sin\omega} - \tan\tfrac{1}{2}\Sigma' \qquad \dots \quad \dots \quad \dots \quad \dots \quad (25)$$

$$\cot z_{,,} = \frac{\sin A'}{\cos l''\,.\,\sin\omega} - \tan\tfrac{1}{2}\Sigma'' \qquad \dots \quad \dots \quad \dots \quad \dots \quad (26)$$

From these we have—

$$\sin z_{,} = \frac{\cos l' \sin \omega}{\left\{(\sin A'' - \tan \tfrac{1}{2} \Sigma' \cos l' \sin \omega)^2 + (\cos l' \sin \omega)^2\right\}^{\frac{1}{2}}} \quad (27)$$

$$\sin z_{,,} = \frac{\cos l'' \sin \omega}{\left\{(\sin A' - \tan \tfrac{1}{2} \Sigma'' \cos l'' \sin \omega)^2 + (\cos l'' \sin \omega)^2\right\}^{\frac{1}{2}}} \quad (28)$$

And from these and equations (22) we have—

$$\sin D_{,,} = \left\{(\sin A'' - \tan \tfrac{1}{2} \Sigma' \cos l' \sin \omega)^2 + (\cos l' \sin \omega)^2\right\}^{\frac{1}{2}} \quad (29)$$

$$\sin D_{,} = \left\{(\sin A' - \tan \tfrac{1}{2} \Sigma'' \cos l'' \sin \omega)^2 + (\cos l'' \sin \omega)^2\right\}^{\frac{1}{2}} \quad (30)$$

8 We can express $z_{,}$, $z_{,,}$, in other useful forms.

In the plane triangle $S'S''Z_{,}$ we know that the side $S'Z_{,}$ is $R_{,}$; that $S'S''$ is k; and that the angle $S''S'Z_{,} = 90° - \frac{1}{2}\Sigma'$.
From it we have—

$$S''Z_{,} = (k^2 + R_{,}^2 - 2.k.R_{,} \sin \tfrac{1}{2} \Sigma')^{\frac{1}{2}}$$

also, $$\sin z_{,} = \frac{k. \cos \frac{1}{2} \Sigma'}{S''Z_{,}}$$

$\therefore$

$$\left.\begin{aligned} \sin z_{,} &= \frac{k \cos \frac{1}{2} \Sigma'}{\left\{k^2 + R_{,}^2 - 2\, k\, R_{,} \sin \frac{1}{2} \Sigma'\right\}^{\frac{1}{2}}} \\ \text{and, since } k &= \frac{2\, s \,.\, \sin \frac{1}{2} \Sigma_{,}}{\Sigma} \\ \sin z_{,} &= \frac{s \,.\, \sin \Sigma}{\left\{(R_{,}\, \Sigma)^2 - 4 s \,.\, \sin^2 \frac{1}{2} \Sigma\, (R_{,}\, \Sigma - s)\right\}^{\frac{1}{2}}} \end{aligned}\right\} \quad (31)$$

Similarly,

$$\left.\begin{aligned} \sin z_{,,} &= \frac{k. \cos \frac{1}{2} \Sigma''}{\left\{(k^2 + R_{,,}^2 - 2.k.R_{,,} \sin \frac{1}{2} \Sigma''\right\}^{\frac{1}{2}}} \\ \sin z_{,,} &= \frac{s \cdot \sin \Sigma}{\left\{(R_{,,}\, \Sigma)^2 - 4 s. \sin^2 \frac{1}{2} \Sigma\, (R_{,,}\, \Sigma - s)\right\}^{\frac{1}{2}}} \end{aligned}\right\} \quad (32)$$

If we put ρ for the mean radius of curvature of the arc s; then $\Sigma = \frac{s'}{\rho}$; and $\sin^2 \frac{1}{2} \Sigma = \frac{1}{4} \left(\frac{s}{\rho}\right)^2$ nearly when s is not more than 1°. Hence, for such small arcs, we have—

$$\frac{\sin z_{,}}{\sin \Sigma} = \frac{1}{\left\{\left(\frac{R_{,}}{\rho}\right)^2 - \left(\frac{s}{\rho}\right)^2 \left(\frac{R_{,} - \rho}{\rho}\right)\right\}^{\frac{1}{2}}} \quad \ldots \quad (33)$$

It is evident the first side is a little greater than $\frac{z_{,}}{\Sigma}$; and we have, with rough approximate accuracy, for arcs of not more than 1°, the relation—

$$z_{,}=\frac{\Sigma}{\left\{\left(\frac{R_{,}}{\rho}\right)^{2}-\Sigma^{2}\left(\frac{R_{,}-\rho}{\rho}\right)\right\}^{\frac{1}{2}}} \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

And ∴ also the rude unreliable approximates—

$$\left.\begin{aligned} z_{,} &= \frac{\rho}{R'}\cdot\Sigma=\frac{s}{R_{,}} \\ z_{,,} &= \frac{\rho}{R_{,,}}\cdot\Sigma=\frac{s}{R_{,,}} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (35)$$

(See foot-note, Problem 5).

We can express the arcs $z_{,}$, $z_{,,}$, in the following manner :—

From the plane triangle $S'S''Z_{,}$, we have—

$$k\cdot\sin(\text{arc } ID_{,,})=R_{,}\cdot\sin z_{,}$$

$$\therefore \qquad \sin(\text{arc } ID_{,,})=\frac{R_{,}\cdot\sin z_{,}}{k}$$

$$=\frac{R_{,}\cos\frac{1}{2}\Sigma'}{(k^{2}+R^{2}_{,}-2.k.R_{,}\sin\frac{1}{2}\Sigma')^{\frac{1}{2}}}$$

$$=\frac{\cos\frac{1}{2}\Sigma'}{\left(1+\frac{k^{2}}{R_{,}^{2}}-\frac{2.k}{R_{,}}\cdot\sin\frac{1}{2}\Sigma'\right)^{\frac{1}{2}}}$$

$$\left.\begin{aligned} &\therefore \qquad z_{,}=\sin^{-1}\left(\frac{\cos^{2}\frac{1}{2}\Sigma'}{1+\frac{k^{2}}{R^{2}_{,}}-\frac{2.k}{R_{,}}\cdot\sin\frac{1}{2}\Sigma'}\right)^{\frac{1}{2}}-\tfrac{1}{2}(\pi-\Sigma')\,; \\ &\text{Similarly,} \\ &z_{,,}=\tfrac{1}{2}(\pi+\Sigma'')-\sin^{-1}\left(\frac{\cos^{2}\frac{1}{2}\Sigma''}{1+\frac{k^{2}}{R_{,,}^{2}}-\frac{2.k}{R_{,,}}.\sin\frac{1}{2}\Sigma''}\right)^{\frac{1}{2}} \end{aligned}\right\} (36)$$

These expressions for $z_{,}$ and $z_{,,}$ are rigorously accurate and very useful.

9 From the triangle $S_{,,}P\,D_{,}$ we have

$$\cos l''\sin A''=\sin L_{,}\sin.\,\text{D}_{,}$$

$$\therefore \qquad \sin \text{L}_{,}=\frac{\cos l''\sin \text{A}''}{\sin D_{,}}$$

$$\sin L_{,}=\frac{\cos l''\sin A''}{\left\{(\sin A'-\tan\frac{1}{2}\Sigma''\cos l''\sin\omega)^{2}+(\cos l''\sin\omega)^{2}\right\}^{\frac{1}{2}}} \quad \ldots \quad (37)$$

Similarly,

$$\sin \text{L}_{,,}=\frac{\cos l'\sin A'}{\left\{(\sin \text{A}''-\tan\frac{1}{2}\Sigma'\cos l'\sin\omega)^{2}+\cos l'\sin\omega)^{2}\right\}^{\frac{1}{2}}} \quad \ldots \quad (38)$$

Again, it is evident $\text{L}_{,}$ is the circular measure of the angle $S'Z_{,,}C$ between the line $\text{S}_{,}\text{Z}_{,,}$ and the polar axis, and that

$$\cot L_{,}=\frac{CZ_{,,}}{Cd_{,}}$$

$$\therefore \qquad \cot L_{,} = \frac{R_{,}\,(1-e^2)\ \sin\ l' + R_{,,}\,e^2 .\ \sin l''}{R_{,}\cos l'} \qquad \ldots \qquad (39)$$

Similarly,

$$\cot L_{,,} = \frac{R_{,,}\,(1-e^2)\ \sin\ l'' + R_{,}\,e^2\ \sin l'}{R_{,,}.\ \cos l''} \qquad \ldots \qquad (40)$$

☞ It is evident that $L_{,}$ and $L_{,,}$ are functions of the latitudes of the stations S' and S''; and (no matter what may be the relative positions of the stations) $L_{,}$ $L_{,,}$ will remain constant magnitudes, provided the latitudes are constant in magnitude. When l' and l'' remain constant in magnitude, then obviously $\frac{\sin A'}{\sin D_{,,}}$ and $\frac{\sin A''}{\sin D_{,}}$ remain constant in magnitude.

The expressions for cot $L_{,}$, cot $L_{,,,}$ in terms of the latitudes only, as given in equations (39) and (40) are of importance. They enable us to find the rigorous values of $\frac{1}{2}\,\Sigma'$, $\frac{1}{2}\,\Sigma''$, &c., when the values of the latitudes l', l'', are known, and that either the difference of longitude or one of the azimuths is also known.

(10) The accuracy of the following most important set of formulæ can be relied upon. The simple investigations, by means of which they have been evolved, are omitted in order to economize space in the "Transactions."

Expressions for the angles of depression of the chord of the geodesic arc.

$$\left.\begin{aligned} \cos\tfrac{1}{2}\,\Sigma' &= \frac{R_{,,}}{k}\cdot\frac{\cos l''}{\sin A'}\cdot\sin\omega \\ \cos\tfrac{1}{2}\,\Sigma'' &= \frac{R_{,}}{k}\cdot\frac{\cos l'}{\sin A''}\cdot\sin\omega \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (41)$$

$$\left.\begin{aligned} \sin\tfrac{1}{2}\,\Sigma' &= \frac{R_{,}R_{,,}\,(\cos l'\cos l''\cos\omega + \frac{b^2}{a^2}\sin l'\sin l'') - a^2}{R_{,}\cdot k} \\ \sin\tfrac{1}{2}\,\Sigma'' &= \frac{R_{,}R_{,,}\,(\cos l'\cos l''\cos\omega + \frac{b^2}{a^2}\sin l'\sin l'') - a^2}{R_{,,}\cdot k} \end{aligned}\right\} \qquad (42)$$

$$\left.\begin{aligned} \tan\tfrac{1}{2}\,\Sigma' &= \frac{R_{,}}{R_{,,}}\cdot\frac{\sin A'}{\cos l''}\cdot\frac{1}{\sin\omega} - \frac{\cot\omega\sin A' + \sin l'\cos A'}{\cos l'} \\ \tan\tfrac{1}{2}\,\Sigma'' &= \frac{R_{,,}}{R_{,}}\cdot\frac{\sin A''}{\cos l'}\cdot\frac{1}{\sin\omega} - \frac{\cot\omega\sin A'' + \sin l''\cos A''}{\cos l''} \end{aligned}\right\} \qquad (43)$$

$$\left.\begin{aligned} \tan\tfrac{1}{2}\,\Sigma' &= \frac{\sin A'}{\cos l''\sin\omega}\cdot\left(\cos l'\cos l''\cos\omega + \frac{b^2}{a^2}\sin l'\sin l'' - \frac{a^2}{R_{,}R_{,,}}\right) \\ \tan\tfrac{1}{2}\,\Sigma'' &= \frac{\sin A''}{\cos l'\sin\omega}\cdot\left(\cos l'\cos l''\cos\omega + \frac{b^2}{a^2}\sin l'\sin l'' - \frac{a^2}{R_{,}R_{,,}}\right) \end{aligned}\right\} \qquad (44)$$

$$\left.\begin{aligned} \tan\tfrac{1}{2}\,\Sigma' &= \frac{\cos l'}{\sin A''}\cdot\frac{R_{,}\sin A'\cos A'' + R_{,,}\sin A''\cos A'}{R_{,}\sin l'' + R_{,,}\sin l'} \\ \tan\tfrac{1}{2}\,\Sigma'' &= \frac{\cos l''}{\sin A'}\cdot\frac{R_{,}\sin A'\cos A'' + R_{,,}\sin A''\cos A'}{R_{,}\sin l'' + R_{,,}\sin l'} \end{aligned}\right\} \qquad \ldots \qquad (45)$$

And from these we immediately obtain the following relations :—

$$\left.\begin{aligned} \frac{\cos\frac{1}{2}\,\Sigma'}{\cos\frac{1}{2}\,\Sigma''} &= \frac{R_{,,}\cos l''\sin A''}{R_{,}\cos l'\sin A'} \\ \frac{\sin\frac{1}{2}\,\Sigma'}{\sin\frac{1}{2}\,\Sigma''} &= \frac{R_{,,}}{R_{,}} \\ \frac{\tan\frac{1}{2}\,\Sigma'}{\tan\frac{1}{2}\,\Sigma''} &= \frac{\cos l'\sin A'}{\cos l''\sin A''} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (46)$$

All these expressions are rigorously accurate for any two stations on the earth, or any ellipsoid of revolution whatever; and from the expressions for either the cosines, sines, or tangents, we can obtain those for all the elementary functions of $\frac{1}{2}\Sigma'$, $\frac{1}{2}\Sigma''$, in terms of the implicated entities.

☞ By equating the values of $\tan\frac{1}{2}\Sigma'$ given in (44) and (45), we can easily express the sine of the difference of longitude of the two stations as a direct and explicit function of the two latitudes and two azimuths,—the resulting formula being rigorously exact for any two stations on any spheroid. This has not been hitherto effected; for Dalby's Theorem is applicable only to mutually visible stations on the earth, and is even then but a close approximate.

☞ By means of the equations (46) we can easily express the squares of the sines, cosines, and tangents of the angles of depression, as rational functions of the latitudes and azimuths only; but the forms so arrived at take the indefinite form $\frac{0}{0}$ when $R_{,} = R_{,,}$ (which is the case on any spheroid when the latitudes of the stations are equal, and always the case on the sphere, no matter how the stations may be situated). They are not adapted for calculations in obtaining the angles of depression, unless the latitudes of the stations differ considerably. However, they are often of use in transformation of equations. They are—

$$\left.\begin{aligned}
(\cos\tfrac{1}{2}\Sigma')^2 &= \frac{(R_{,}\cos l''\sin A'')^2 - (R_{,,}\cos l''\sin A'')^2}{(R_{,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}\\
(\cos\tfrac{1}{2}\Sigma'')^2 &= \frac{(R_{,}\cos l'\sin A')^2 - (R_{,,}\cos l'\sin A')^2}{(R_{,,}\cos l''\sin A'')^2 - (R_{,,}\cos l'\sin A')^2}\\
(\sin\tfrac{1}{2}\Sigma')^2 &= \frac{(R_{,,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}{(R_{,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}\\
(\sin\tfrac{1}{2}\Sigma'')^2 &= \frac{(R_{,,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}{(R_{,,}\cos l''\sin A'')^2 - (R_{,,}\cos l'\sin A')^2}\\
(\tan\tfrac{1}{2}\Sigma')^2 &= \frac{(R_{,,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}{(R_{,}\cos l''\sin A'')^2 - (R_{,,}\cos l''\sin A'')^2}\\
(\tan\tfrac{1}{2}\Sigma'')^2 &= \frac{(R_{,,}\cos l''\sin A'')^2 - (R_{,}\cos l'\sin A')^2}{(R_{,}\cos l'\sin A')^2 - (R_{,,}\cos l'\sin A')^2}
\end{aligned}\right\} \cdots \quad \cdots (47)$$

Expressions for the angle between the normals at the stations.

$$\left.\begin{aligned}
\cos\nu &= \sin l'\sin l'' + \cos l'\cos l''\cos\omega\\
\sin^2\tfrac{1}{2}\nu &= \sin^2\tfrac{1}{2}(l' - l'') + \cos l'\cos l''\sin^2\tfrac{1}{2}\omega
\end{aligned}\right\} \cdots \quad \cdots (48)$$

Expressions for the angle between the normal chordal planes.

$$\left.\begin{aligned}
\sin\Delta &= \tfrac{1}{2}e^2\left(\frac{R_{,}}{R_{,,}}\cdot\frac{\sin A'\sin 2l'}{\cos\frac{1}{2}\Sigma''} - \frac{R_{,,}}{R_{,}}\cdot\frac{\sin A''\sin 2l''}{\cos\frac{1}{2}\Sigma'}\right)\\
\sin\Delta &= e^2.\frac{R_{,}\sin l' - R_{,,}\sin l''}{(R_{,}^2 - R_{,,}^2)^{\frac{1}{2}}}.(\cos^2 l''\sin^2 A'' - \cos^2 l'\sin^2 A')^{\frac{1}{2}}\\
\sin\Delta &= e^2\cdot k\cdot\frac{\sin A'\sin A''}{\sin\omega}.\left(\frac{\sin l'}{R_{,,}} - \frac{\sin l''}{R_{,}}\right)
\end{aligned}\right\} (49)$$

$$\sin\tfrac{1}{2}\Delta = \left\{\frac{\sin\frac{1}{2}(\nu + \frac{1}{2}\Sigma' + \frac{1}{2}\Sigma'').\sin\frac{1}{2}(\nu - \frac{1}{2}\Sigma' - \frac{1}{2}\Sigma'')}{\cos\frac{1}{2}\Sigma'.\cos\frac{1}{2}\Sigma''}\right\}^{\frac{1}{2}} \quad (50)$$

☞ In the "Account of the Principal Triangulation of Great Britain and Ireland," an erroneous expression is given for the angle Δ, viz.:—

$$\Delta = e^2\sin 2A'\cos^2(l' + l'').\tfrac{1}{2}\Sigma$$
$$\Delta = e^2\sin 2A''\cos^2(l' + l'').\tfrac{1}{2}\Sigma$$

That the formula is erroneous is easily seen: for, independent of the oversight committed in assuming $\sin 2A' = \sin 2A''$, it is obvious that when $l' = l''$, we must have $\Delta = 0$, no matter what may be the common value of l' and l''. This is contrary to what the formula intimates in such case. In fact, the expression is not even a rough approximate to the angle Δ. And it may be further observed that the formulæ given in that work, as pertaining to a true spheroidal triangle, are arrived at by making use of the erroneous expression for Δ, so that they too are erroneous.

Expressions for the chord k of the geodesic arc—

$$\left.\begin{aligned} k^2 &= (R_{\prime} \cos l')^2 + (R_{\prime\prime} \cos l'')^2 - 2\, R_{\prime}\, R_{\prime\prime} \cos l' \cos l'' \cos \omega + \left(\frac{b}{a}\right)^4 . \\ &\quad . (R_{\prime} \sin l' - R_{\prime\prime} \sin l'')^2 \\ k &= R_{\prime\prime} . \frac{\cos l''}{\sin A'} . \frac{\sin \omega}{\cos \frac{1}{2}\Sigma'} = R_{\prime} . \frac{\cos l'}{\sin A''} . \frac{\sin \omega}{\cos \frac{1}{2}\Sigma''} \end{aligned}\right\} \quad (51)$$

Expressions for the radius ρ_0 of mean curvature of the arc—

$$\left.\begin{aligned} \rho_0 &= \tfrac{1}{2}k . \frac{\cos\frac{1}{2}\,(\frac{1}{2}\Sigma' - \frac{1}{2}\Sigma'')}{\sin\frac{1}{2}\,(\frac{1}{2}\Sigma' + \frac{1}{2}\Sigma'')} \\ \rho_0 &= \frac{\sin \omega}{\sin \Sigma} . \left(\frac{R_{\prime} R_{\prime\prime} \cos l' \cos l''}{\sin A' \sin A''}\right)^{\frac{1}{2}} \\ &\text{closely approximate only.} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (52)$$

Expressions for the length of the geodesic arc between the stations—

$$\left.\begin{aligned} s &= \tfrac{1}{2}k\,(\tfrac{1}{2}\Sigma' + \tfrac{1}{2}\Sigma'') . \frac{\cos \frac{1}{2}\,(\frac{1}{2}\Sigma' - \frac{1}{2}\Sigma'')}{\sin \frac{1}{2}\,(\frac{1}{4}\Sigma' + \frac{1}{2}\Sigma'')} \\ s &= \frac{\Sigma}{\sin \Sigma} . \left(\frac{R_{\prime} R_{\prime\prime} \cos l'' \cos l'}{\sin A' \sin A''}\right)^{\frac{1}{2}} \sin \omega \\ &\text{closely approximate only.} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (53)$$

☞ The complete expressions for the angles of depression of the chord, the angle between the normals, the angle between the normal-chordal planes, and the length of the chord, cannot fail to be of great practical use in questions of Geodesy. It would be an easy matter to give much simpler formulæ of an approximate kind, such as are usually supplied in treatises on Practical Geodesy, and in *accounts of trigonometrical surveys*; but it may be proper to observe that the application of approximate formulæ should be a matter of necessity; for when extensively used in geodesical surveying (as they are certain to be by persons who wish to avoid extensive calculations) they give a low inaccurate character to the work, and leave it entirely impossible to deduce anything reliable therefrom as to the figure of the earth.

Problem 1.

Given the latitudes l', l'', of two stations S', S'', and their difference of longitude ω; to find the azimuths A', A'', at the stations; the circular measure Σ, the chord k, and length s of the geodesic arc between the stations, &c.

We can find the arcs $PD_{\prime\prime\prime}$, $PD_{\prime}$, or $L_{\prime\prime\prime}$, $L_{\prime}$, from—

$$\cot L_{\prime} = (1-e^2) \tan l' + e^2 . \frac{R_{\prime\prime}}{R_{\prime}} . \frac{\sin l''}{\cos l'} ;$$

$$\cot L_{\prime\prime} = (1-e^2) \tan l'' + e^2 \cdot \frac{R_{\prime}}{R_{\prime\prime}} \cdot \frac{\sin l'}{\cos l''} .$$

* The triangles $S_,PD''$, $S''PD_,$ give the following formulæ from which to obtain the azimuths A', A'', and angles $D_{,,}$, $D_,$;—

$$\tan\tfrac{1}{2}(A' + D_{,,}) = \frac{\cos\frac{1}{2}(L_{,,} - l_,)}{\cos\frac{1}{2}(L_{,,} + l_,)}.\cot\tfrac{1}{2}\omega$$

$$\tan\tfrac{1}{2}(A' - D_{,,}) = \frac{\sin\frac{1}{2}(L_{,,} - l_,)}{\sin\frac{1}{2}(L_{,,} + l_,)}.\cot\tfrac{1}{2}\omega$$

$$\tan\tfrac{1}{2}(D_, + A'') = \frac{\cos\frac{1}{2}(l_{,,} - L_,)}{\cos\frac{1}{2}(l_{,,} + L_,)}.\cot\tfrac{1}{2}\omega$$

$$\tan\tfrac{1}{2}(D_, - A'') = \frac{\sin\frac{1}{2}(l_{,,} - L_,)}{\sin\frac{1}{2}(l_{,,} + L_,)}.\cot\tfrac{1}{2}\omega$$

And since $S_,D_, = L_, - l_,$; and $S_{,,}D_{,,} = l_{,,} - L_{,,}$; we have from the triangles $S_,ID_,$, $S_{,,}ID_{,,}$, the following set from which to find $IS_,$, $ID_,$, $IS_{,,}$, $ID_{,,}$,

$$\tan\tfrac{1}{2}(IS_, + ID_,) = \frac{\sin\frac{1}{2}(D_, + A')}{\sin\frac{1}{2}(D_, - A')}.\tan\tfrac{1}{2}(L_, - l_,)$$

$$\tan\tfrac{1}{2}(IS_, - ID_,) = \frac{\cos\frac{1}{2}(D_, + A')}{\cos\frac{1}{2}(D_, - A')}.\tan\tfrac{1}{2}(L_, - l_,)$$

$$\tan\tfrac{1}{2}(IS_{,,} + ID_{,,}) = \frac{\sin\frac{1}{2}(A'' + D_{,,})}{\sin\frac{1}{2}(A'' - D_{,,})}.\tan\tfrac{1}{2}(l_{,,} - L_{,,})$$

$$\tan\tfrac{1}{2}(IS_{,,} - ID_{,,}) = \frac{\cos\frac{1}{2}(A'' + D_{,,})}{\cos\frac{1}{2}(A'' - D_{,,})}.\tan\tfrac{1}{2}(l_{,,} - L_{,,})$$

Then $\frac{1}{2}\Sigma' = \frac{1}{2}\pi - IS_,$; $\frac{1}{2}\Sigma'' = IS_{,,} - \frac{1}{2}\pi$;

$z_, = ID_{,,} - IS_,$; $z_{,,} = IS_{,,} - ID_,$;

from which to find $\frac{1}{2}\Sigma'$, $\frac{1}{2}\Sigma''$, $z_,$, $z_{,,}$.

$\therefore\ \Sigma = \frac{1}{2}\Sigma' + \frac{1}{2}\Sigma''$ is also known.

To find the chord k and length s of the geodesic arc connecting the stations, we have—

$$k = \frac{R_, \cos l' \sin\omega}{\sin A''.\cos\frac{1}{2}\Sigma''}$$

$$= \frac{R_{,,} \cos l'' \sin\omega}{\sin A'.\cos\frac{1}{2}\Sigma'}$$

$$s = \frac{k\cdot\Sigma}{2\sin\frac{1}{2}\Sigma} = \tfrac{1}{2}k\left(\tfrac{1}{2}\Sigma' + \tfrac{1}{2}\Sigma''\right).\frac{\cos\frac{1}{2}(\frac{1}{2}\Sigma' - \frac{1}{2}\Sigma'')}{\sin\frac{1}{2}(\frac{1}{2}\Sigma' + \frac{1}{2}\Sigma'')}$$

or we can find s at once without finding k, from—

$$s = \frac{R_, \cos l' \sin\omega.\ \Sigma}{2\sin A'' \sin\frac{1}{2}\Sigma.\cos\frac{1}{2}\Sigma''}$$

$$= \frac{R_{,,} \cos l'' \sin\omega.\ \Sigma}{2.\sin A'' \sin\frac{1}{2}\Sigma.\cos\frac{1}{2}\Sigma'}$$

We can find the angle Δ from either of the triangles $S_,D_,I$, $S_{,,}D_{,,}I$, in each of which we know the three sides and the remaining angles.

We can also find the angle ν (the arc $S_,S_{,,}$) between the normals, from the triangles $S_,PS_{,,}$, of which we know the two sides and included angle ω.

☞ This method of solution is rigorously accurate as to the determination of A', A'', $\frac{1}{2}\Sigma'$, $\frac{1}{2}\Sigma''$, no matter how distant the stations may be from each other.

And we must have—

$$\frac{\sin A'}{\sin A''} = \frac{R_{,,} \cos l''}{R_, \cos l'}.\frac{\cos\frac{1}{2}\Sigma''}{\cos\frac{1}{2}\Sigma}$$

* The *direct* expressions for the cotangents of the azimuths (as given in the Account of the Trig. Survey of Great Britain and Ireland) can be at once obtained from the triangles $PS_,D_{,,}$, $PS_{,,}D_,$, by applying the well known formula (Todhunter's Trigonometry, art. 44) and using the above expressions for cot $L_{,,}$, cot $L_,$.

Now, if we put G', G'', for the angles which the *true* geodesic connecting the stations makes with the meridians through S' and S'', we know (see notes at end of this paper) that—

$$\frac{\sin G'}{\sin G''} = \frac{R_{,,} \cos l''}{R_{,} \cos l'}$$

$\therefore \dfrac{\sin G'}{\sin G''} = \dfrac{\sin A'}{\sin A''} \cdot \dfrac{\cos \frac{1}{2}\Sigma'}{\cos \frac{1}{2}\Sigma''}$; and, since $\dfrac{\cos \frac{1}{2}\Sigma'}{\cos \frac{1}{2}\Sigma''}$ is greater than 1, we have $\dfrac{\sin A'}{\sin A''}$ less than $\dfrac{\sin G'}{\sin G''}$.

For any pair of mutually visible stations on the earth's surface, we may regard $\dfrac{\cos \frac{1}{2}\Sigma'}{\cos \frac{1}{2}\Sigma''}$ as equal unity, without sensible error in calculated results; or, which amounts to the same, we may regard $\dfrac{\sin A'}{\sin A''}$ as equal to $\dfrac{\sin G'}{\sin G''}$.

This is evidently equivalent to regarding $A' + A'' = a' + a''$, the sum of the angles at the base of the triangle $S_{,}PS_{,,}$. By actually calculating the angles a', a'', from the two sides and included angle ω of this triangle, we shall find $(A' + A'') - (a' + a'')$ to be inappreciable in the actual practice of the most accurate trigonometrical surveying.

Otherwise,

When the station S', S'', are mutually visible from each other.

To find the azimuths, we have—

$$\tan^2 \psi = \frac{\tan^2 l' + \dfrac{a^2}{b^2}}{\tan^2 l'' + \dfrac{a^2}{b^2}}$$

$$\tan \tfrac{1}{2}(A' + A'') = \frac{\cos \frac{1}{2}(l' - l'')}{\sin \frac{1}{2}(l' + l'')} \cdot \cot \tfrac{1}{2}\omega$$

$$\tan \tfrac{1}{2}(A' - A'') = \tan \tfrac{1}{2}(A' + A''). \tan(\psi - 45^\circ).$$

We have $$\cos \tfrac{1}{2}\nu = \frac{\sin \frac{1}{2}(l' + l'')}{\cos \frac{1}{2}(A' + A'')} \cdot \sin \tfrac{1}{2}\omega$$

$$\tan\left\{\tfrac{1}{2}(A' - A'') + \Omega\right\} = \frac{\sin \frac{1}{2}(l' - l'')}{\cos \frac{1}{2}(l' + l'')} \cdot \cot \tfrac{1}{2}\omega$$

from which to find ν and Ω. Then to find Σ, k, s, Δ, we have—

$$\tan \tfrac{1}{2}\Sigma = \tan \tfrac{1}{2}\nu \,.\, \cos \Omega$$

$$k = \frac{R_{,} \cos l'. \sin \omega}{\sin A'' \cos \frac{1}{2}\Sigma}$$

$$s = \frac{k}{2 \sin \frac{1}{2}\Sigma} \cdot \Sigma$$

$$\tan \tfrac{1}{2}\Delta = \sin \tfrac{1}{2}\Sigma \tan \Omega$$

$$\sin \tfrac{1}{2}\Delta = \sin \tfrac{1}{2}\nu \sin \Omega$$

Problem 2.

Given the azimuths A', A'', taken at the stations S', S'', and the latitude l' of one of the stations S': to find — the latitude l'' of the station S''; the difference of longitude ω of the stations; the circular measure Σ, the chord k, and the length s of the geodesic arc between the stations; the angle ν which the two normals make with each other; the angle Δ between the two normal-chordal planes; &c. ....

To find l'', we have

$$\tan^2 l'' = \frac{\sin^2 A''}{\sin^2 A'} \left(\tan^2 l' + \frac{a^2}{b^2}\right) - \frac{a^2}{b^2}$$

If we use the ratio of the squares of the equatorial and polar radii of the earth as determined by Bessel, the above formula can be written :—

$$\tan^2 l'' = \frac{\sin^2 A''}{\sin^2 A'} (\tan^2 l' + 1{\cdot}00671945) - 1{\cdot}00671945$$

To find the difference of longitude ω, we have—

$$\tan \tfrac{1}{2} \omega = \frac{\cos \frac{1}{2} (l' - l'')}{\sin \frac{1}{2} (l' + l'')} \cdot \cot \tfrac{1}{2} (A' + A'')$$

To find the circular measure Σ, chord k, and length of arc s, we have

$$\tan \tfrac{1}{2} \Sigma = \frac{\cos l' \cos A' + \cos l'' \cos A''}{R_{\prime} \sin l' + R_{\prime\prime} \sin l''} \cdot (R_{\prime} R_{\prime\prime})^{\frac{1}{2}}$$

$$k = \frac{R_{\prime} \cos l' \sin \omega}{\sin A'' \cos \frac{1}{2} \Sigma}$$

$$s = \frac{k \cdot \Sigma}{2 \sin \frac{1}{2} \Sigma}.$$

We can find the circular measure otherwise, by first finding the angle Ω which a plane parallel to the two normals makes with either of the normal-chordal planes, and also the angle ν between the normals. Thus :—

From the triangle $S_{\prime} P S_{\prime\prime}$ we have formula (16)—

$$\tan \left\{ \tfrac{1}{2} (A' - A'') + \Omega \right\} = \frac{\sin \frac{1}{2} (l' - l'')}{\cos \frac{1}{2} (l' + l'')} \cdot \cot \tfrac{1}{2} \omega$$

from which to find Ω.

$$* \cos \tfrac{1}{2} \nu = \frac{\sin \frac{1}{2} (l' + l'')}{\cos \frac{1}{2} (A' + A'')} \cdot \sin \tfrac{1}{2} \omega$$

from which to find ν.

Then the following set—

$$\tan \tfrac{1}{2} \Sigma = \tan \tfrac{1}{2} \nu . \cos \Omega.$$

$$k = \frac{R_{\prime} \cos l' . \sin \omega}{\sin A'' . \cos \frac{1}{2} \Sigma} = \frac{R_{\prime\prime} \cos l'' . \sin \omega}{\sin A' . \cos \frac{1}{2} \Sigma}$$

$$s = \frac{k . \Sigma}{2 . \sin \frac{1}{2} \Sigma}$$

$$\tan \tfrac{1}{2} \Delta = \sin \tfrac{1}{2} \Sigma . \tan \Omega$$

$$\sin \tfrac{1}{2} \Delta = \sin \tfrac{1}{2} \nu . \sin \Omega$$

from which to find Σ, k, s, and Δ.

Problem 3.

Given the latitudes l', l'', and the azimuth A'; to find the azimuth A''. the difference of longitude ω of the stations, and all the other entities determined in the preceding problem.

We can at once find the azimuth A'' from either formulæ (10), (12), which give—

$$\sin A'' = \sin A' \,.\, \frac{R_{,} \cos l'}{R_{,,} \cos l''}$$

$$^{*}\sin^2 A'' = \sin^2 A' \,.\, \frac{\tan^2 l'' + \dfrac{a^2}{b^2}}{\tan^2 l' + \dfrac{a^2}{b^2}}$$

or
$$\sin^2 A'' = \sin^2 A' \,.\, \frac{\tan^2 l'' + 1{\cdot}00671945}{\tan^2 l' + 1{\cdot}00671945} \,.$$

Or we can proceed otherwise. Thus :—

Find the arc $PD_{,,}$ or $L_{,,}$ by means of

$$\cot L_{,,} = (1 - e^2) \tan^2 l'' + e^2 \,.\, \frac{R_{,} \,.\, \sin l'}{R_{,,} \,.\, \cos l''}$$

Then from the triangle $S_{,} \, P \, D_{,,}$ we have—

$$\sin D_{,,} = \frac{\cos l' \,.\, \sin A'}{\sin L_{,,}}$$

$$\cot \tfrac{1}{2} \omega = \frac{\cos \frac{1}{2} (L_{,,} + l_{,})}{\cos \frac{1}{2} (L_{,,} - l_{,})} \,.\, \tan \tfrac{1}{2} (A' + D_{,,}) \,.$$

And to find A''

$$\tan \tfrac{1}{2} (A' + A'') = \frac{\cos \frac{1}{2} (l' - l'')}{\sin \frac{1}{2} (l' + l'')} \,.\, \cot \tfrac{1}{2} \omega \,.$$

The other entities can now be found as in Problem 1.

☞ When we know the latitudes l', l'', and azimuths A', A'', of two mutually visible stations on the earth from actual observations, we can find an approximate value of $\frac{a}{b}$ from the equation.

$$\frac{a^2}{b^2} = \frac{\tan^2 l' - \tan^2 l'' \left(\dfrac{\sin A'}{\sin A''}\right)^2}{\left(\dfrac{\sin A'}{\sin A''}\right)^2 - 1}$$

Using this value of $\frac{a^2}{b^2}$, we should omit each one of the four measured entities l', l'', A', A'', in turn, and compute new values of the same, and the four corresponding values of ω by means of problems 2 and 3.

Then taking the means as the most accurate values of l', l'', and ω, we should employ them as in problem 1, and find the corresponding values for A', A'', $\frac{1}{2}\Sigma'$, $\frac{1}{2}\Sigma''$.

* When either the sine, cosine or tangent of an angle is equated to a known quantity f, we can express the other elementary functions by means of f; and ∴ always find the angle with precision from the tables.—(See Todhunter's Trig., pages 165, 166.)

Then taking the means of the three values of each of the angles A, A'', as the most reliable, we should compute $\frac{a}{b}$ from the more accurate equation.

$$\frac{a^2}{b^2} = \frac{\tan^2 l' - \tan^2 l'' \left(\frac{\sin A' \, . \, \cos \frac{1}{2} \Sigma'}{\sin A'' \, . \, \cos \frac{1}{2} \Sigma''}\right)^2}{\left(\frac{\sin A' \, . \, \cos \frac{1}{2} \Sigma'}{\sin A'' \, . \, \cos \frac{1}{2} \Sigma''}\right)^2 - 1}$$

By selecting suitable pairs of stations it is evident we should be enabled to approximate closely to the value of $\frac{a}{b}$.

Problem 4.

Given the two latitudes l', l'', and the length s and circular measure Σ of the geodesic arc joining the stations: to find the azimuths A', A'', the difference of longitude ω, and all the other entities.

We can find the arcs $S_{,}\,D_{,,,}$, $S_{,,}\,D_{,}$ or $z_{,}$, $z_{,,}$, from —

$$\sin z_{,} = \frac{s . \sin \Sigma}{\left\{(R_{,}\,\Sigma)^2 - 4s . \sin^2 \frac{1}{2} \Sigma\, (R_{,}\,\Sigma - s)\right\}^{\frac{1}{2}}}$$

$$\sin z_{,,} = \frac{s . \sin \Sigma}{\left\{(R_{,,}\,\Sigma)^2 - 4\, s . \sin^2 \frac{1}{2} \Sigma\, (R_{,,}\,\Sigma - s)\right\}^{\frac{1}{2}}}$$

We can find the arcs $P\,D_{,}$, $P\,D_{,,,}$ or $L_{,}$, $L_{,,}$, by means of —

$$\cot L_{,} = \frac{R_{,}\,(1 - e^2) \sin l' + R_{,,}\, e^2 \sin l''}{R_{,} \cos l'}$$

$$\cot L_{,,} = \frac{R_{,,}\,(1 - e^2) \sin l'' + R_{,}\, e^2 \sin l'}{R_{,,} \cos l''}$$

Then from the triangles $S_{,}\,P\,D_{,,}$, $S_{,,}\,P\,D_{,}$, we can at once obtain the two azimuths A', A'', and the difference of longitude ω. We have —

$$\tan \tfrac{1}{2} A' = \left\{\frac{\sin (p - z_{,}) \sin (p - l_{,})}{\sin p \, . \sin (p - L_{,,})}\right\}^{\frac{1}{2}}$$

$$\tan \tfrac{1}{2} A'' = \left\{\frac{\sin (q - z_{,,}) \sin (q - l_{,,})}{\sin q \, . \sin (q - L_{,})}\right\}^{\frac{1}{2}}$$

$$\tan \tfrac{1}{2} \omega = \left\{\frac{\sin (p - L_{,,}) \sin (p - l_{,})}{\sin p \, . \sin (p - z_{,})}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{\sin (q - L_{,}) \sin (q - l_{,,})}{\sin q \, . \sin (q - z_{,,})}\right\}^{\frac{1}{2}}$$

in which $p = \frac{1}{2}(l_{,} + z_{,} + L_{,,})$, and $q = \frac{1}{2}(l_{,,} + z_{,,} + L\ )$

The other entities can be found as in Problem 1.

Problem 5.

Given the latitude l' of one station S', the azimuth A' of the other station S'', as taken at S', and given also the length s and circular measure Σ of the geodesic arc between the stations : to find the latitude l'' of the station S'', the difference of longitude ω of the stations, the azimuth A'' of the station S'' as if taken at S', and all the other entities.

From the triangle $P\,S_{,}I$ we have—

$$\tan \tfrac{1}{2}(\phi_{,} + \beta_{,}) = \frac{\cos \frac{1}{2}(l' - \frac{1}{2}\Sigma)}{\sin \frac{1}{2}(l' + \frac{1}{2}\Sigma)} \,.\, \tan \tfrac{1}{2} A' \ldots \quad \ldots \quad \ldots \quad (a)$$

$$\tan \tfrac{1}{2}(\phi_{,} - \beta_{,}) = \frac{\sin \frac{1}{2}(l' - \frac{1}{2}\Sigma)}{\cos \frac{1}{2}(l' + \frac{1}{2}\Sigma)} \,.\, \tan \tfrac{1}{2} A' \ldots \quad \ldots \quad \ldots \quad (a)$$

from which to find the angles $\phi_{,}$, $\beta_{,}$. Then to find the angle θ that the chord makes with the plane of the equator, we have—

$$\cos \theta = \frac{\sin A' \cos \frac{1}{2}\Sigma}{\sin \phi_{,}}$$

The chord k is given by—

$$k = \frac{2\,s\,\sin \frac{1}{2}\Sigma}{\Sigma} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (b)$$

And its projection $p_{,}\,p_{,,}$ on the plane of the equator can be found from—

$$k_{\circ} = \frac{k \cdot \sin A' \cos \frac{1}{2}\Sigma}{\sin \phi_{,}}$$

Now, in the plane triangle $Cp_{,}\,p_{,,}$, we know that the side $Cp_{,} = R_{,}\cos l'$; that the side $p_{,}p_{,,} = k_{\circ}$; and that the angle $C = \omega$. And it is evident that by finding two subsidiary angles $h_{,}$, $h_{,,}$, such that—

$$\tan h_{,} = R_{,} \cos l' \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (c)$$

$$\tan h_{,,} = \frac{k \,.\, \sin A' \cos \frac{1}{2}\Sigma}{\sin \phi_{,}} \quad \ldots \quad \ldots \quad \ldots \quad (d)$$

we shall have—

$$\tan \tfrac{1}{2}(\phi_{,,} - \omega) = \frac{\sin (h_{,,} - h_{,})}{\sin (h_{,,} + h_{,})} \cdot \cot \tfrac{1}{2}\phi_{,} \ldots \quad (e)$$

$$\tfrac{1}{2}(\phi_{,,} + \omega) = 90^{\circ} - \tfrac{1}{2}\phi_{,} \quad \ldots \quad \ldots \quad \ldots \quad (f)$$

from which to find $\phi_{,,}$ and the difference of longitude ω.

The azimuth A'' is given by—

$$\sin A'' = \sin A' . \frac{\sin \phi_{,,}}{\sin \phi_{,}} \quad \ldots \quad \ldots \quad \ldots \quad (g)$$

And from formula 6 we have—

$$\tan^2 l'' = \frac{\sin^2 A''}{\sin^2 A'}\left(\tan^2 l' + \frac{a^2}{b^2}\right) - \frac{a^2}{b^2} \quad \ldots \quad (h)$$

from which to find l''.

The other entities can be found as already indicated in the solution of Problem 1.

☞ If the given data were l''., A'', Σ, s: then it is evident that formulæ (a) should be written as follows:—

$$\cot \tfrac{1}{2} (\phi_{\prime\prime} - \beta_{\prime\prime}) = \frac{\cos \tfrac{1}{2} (l'' + \tfrac{1}{2} \Sigma)}{\sin \tfrac{1}{2} (l'' - \tfrac{1}{2} \Sigma)} . \cot \tfrac{1}{2} A''$$

$$\cot \tfrac{1}{2} (\phi_{\prime\prime} + \beta_{\prime\prime}) = \frac{\sin \tfrac{1}{2} (l'' + \tfrac{1}{2} \Sigma)}{\cos \tfrac{1}{2} (l'' - \tfrac{1}{2} \Sigma)} . \cot \tfrac{1}{2} A''.$$

Otherwise:—

To find the arc $S_{\prime} D_{\prime\prime}$ or $z_{\prime}$,—

$$\sin. z_{\prime} = \frac{s. \sin \Sigma}{\left\{ (R_{\prime} \Sigma)^2 - 4. s. \sin^2 \tfrac{1}{2} \Sigma (R_{\prime} \Sigma - s) \right\}^{\frac{1}{2}}}$$

The triangle $P S_{\prime} D_{\prime\prime}$ gives—

$$\tan \tfrac{1}{2} (D_{\prime\prime} + \omega) = \frac{\cos \tfrac{1}{2} (l_{\prime} - z_{\prime})}{\cos \tfrac{1}{2} (l_{\prime} + z_{\prime})} . \cot \tfrac{1}{2} A'$$

$$\tan \tfrac{1}{2} (D_{\prime\prime} - \omega) = \frac{\sin \tfrac{1}{2} (l_{\prime} - z_{\prime})}{\sin \tfrac{1}{2} (l_{\prime} + z_{\prime})} . \cot \tfrac{1}{2} A'$$

from which to find $D_{\prime\prime}$ and ω.

Then we have, $\sin A'' = \dfrac{\cos \tfrac{1}{2} (z_{\prime} - \tfrac{1}{2} \Sigma) \sin D_{\prime\prime}}{\cos \tfrac{1}{2} \Sigma}$

from which to find A''.

$$\tan \tfrac{1}{2} l_{\prime\prime} = - \frac{\cos \tfrac{1}{2} (A' + A'' + \omega)}{\cos \tfrac{1}{2} (A' + A'' - \omega)} \cdot \cot \tfrac{1}{2} l_{\prime}$$

from which to find l''.

The various other entities $\tfrac{1}{2} \Sigma'$, $\tfrac{1}{2} \Sigma''$, &c., can now be found as in Problem 1.

☞ In the determination of the difference of longitude ω, as in Chambers's "Practical Mathematics," and in the treatise on "Geodesy" in "Bohn's Dictionary of Engineering," the formula employed is erroneous in principle, and must lead to results more or less incompatible.

The formula is—

$$\omega = \frac{s. \sin A'}{R_{\prime} \cos l'' . \sin 1''}$$

It would be better to find l'', and $R_{\prime\prime}$ approximately (l'' is there found before finding ω), and put—

$$\sin \omega = \frac{s. \sin A' . \sin \Sigma}{R'' \cos l'' . \Sigma}$$

For $\dfrac{\sin A'}{\sin A''} = \dfrac{R_{\prime\prime} \cos l''}{R_{\prime} \cos l'}$ is at variance with the expressions—

$$\omega = \frac{s \sin A'}{R_{\prime} \cos l'' . \sin 1''} = \frac{s . \sin A''}{R_{\prime\prime} \cos l' . \sin 1''} .$$

☞ This problem and problem 1 are the only two contained in this paper which have been solved elsewhere. They have received special attention in the elaborate work published by the Ordnance Department in 1858, entitled—"Account of the Principal Triangulation of Great Britain and Ireland."

* The solution given to this problem in that work, as the one to be preferred when great accuracy is required, is based on defective conceptions and incorrect formulæ.

In order to obtain the difference of longitude it has been the first object to find, in terms of the given data, an expression for the difference between the angle $D_{\prime\prime}$ and the azimuth A''; so as to have the side $PS_{\prime} = l_{\prime}$, the angle $PS_{\prime} D_{\prime\prime} = A'$, and the angle $D_{\prime\prime}$ of the triangle $PS_{\prime} D_{\prime\prime}$.

The general expression given for this difference, which has been denoted by ζ, is, when put in the notation adopted in this paper,

$$\zeta = \tfrac{1}{4}\frac{e^2}{1-e^2}.\ \cos^2 l'.\ \sin 2A'.\ z_{\prime}^2$$

and if the given data were l'', A'', s, Σ, it would be

$$\zeta = \tfrac{1}{4}\frac{e^2}{1-e^2}\ \cos^2 l''.\ \sin 2A''.\ z_{\prime\prime}^2.$$

And it is assumed that *in all cases*, the azimuth is *less* than the angle D, viz., that A' is *less* than $D_{\prime}$, and that A'' is *less* than $D_{\prime\prime}$

Now I have clearly proved (in article 2 of the investigations) that when the latitude l' is greater than the latitude l'',

A' is *less* than $D_{\prime}$

A'' is *greater* than $D_{\prime\prime}$.

So that if the *magnitude* of ζ were even admitted to be correctly determined by the above formula, the fact of its misapplication would still cause greater errors in the results of actual calculations, than could arise from neglecting the small angle ζ altogether, and thus regarding the figure of the earth to be a perfect sphere.

But a little consideration will suffice to show that the formula does not give a correct value for the magnitude of angle ζ. For, assuming the latitude l' of the station S' to be 50°, the difference of longitude $\omega = 1°$, and the azimuth $A' = 90°$, it is obvious that the azimuth A'' must be less than 90°; and $\therefore$ since $2A' = 180°$, one of the values of ζ given by the formula is zero or $= 0$, and the other value a positive quantity.

Now, from the investigations in this paper, it is evident that, by retaining the above-mentioned values of ω and l', we might have the azimuth A' less than 90°, and also the unequal azimuth A'' less than 90°; and that the formula in such case would lead us to infer that the angle ζ should be positive whether $D_{\prime}$ or $D_{\prime\prime}$ may be the angle we wish to find.

It is stated, on page 248 of the above-mentioned work, that the angle ζ is so small as not to amount to $\frac{1}{10}$ of a second even when the geodesic arc between the stations is 100 miles with an average azimuth of 45°. Nevertheless, if the angle $D_{\prime\prime}$ were to be assumed equal to A'', and that we were to compute the co-latitude $l_{\prime\prime}$ from the triangle, we should obtain a value $= L_{\prime\prime}$, leaving a defect $= \delta_{\prime\prime}$. Now $\delta_{\prime\prime}$. sin A'' is very nearly equal the angle Δ between the normal-chordal planes; and for an azimuth A' of about 47°, latitude $l' = 49° 30'$, and arc $s = 60$ miles the angle $\Delta = 4{\cdot}5$ seconds in the example worked out in my last paper. Hence $4''.5 \div \sin 47°$ gives very nearly $6''$ as the error in the latitude l'' if calculated from such data.

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* In the first solution there given, it is assumed that $z_{\prime} = \frac{s}{R_{\prime}}$, $z_{\prime\prime} = \frac{s}{R_{\prime\prime}}$ which is equivalent to assuming that the earth is a perfect sphere. In reality $\frac{s}{R_{\prime}}$ or $\frac{s}{R_{\prime\prime}}$ is always less than Σ; and it may be seen in note (5), at the end of this paper, that in some instances $z_{\prime\prime}$ is greater than Σ. However, that method of solution is tacitly admitted as not suitable when great accuracy is required.

This shows that great caution should be observed in using inexact formulae or expressions, based on no principle or theorem, as if they were approximations based on scientific principles; for although the angle ζ is as small as above stated, yet if it were neglected (as many important small terms are often neglected) it would be equivalent to regarding the earth as a perfect sphere.

Problem 6.

Given the azimuth A', the latitude l'', and the circular measure Σ and length s of the geodesic arc between the stations; to find — the azimuth A'', the latitude l', the difference of longitude ω, and the other entities determined in the solution of problem 1.

To find the difference of longitude ω we have

$$\sin\omega = \frac{s.\sin\Sigma\sin A'}{\Sigma.\, R_{/\!/}\cos l''}$$

To find the arc $S_{/\!/}D_{/\!/}$ or $z_{/\!/}$, we have

$$\sin z_{/\!/} = \frac{s.\sin\Sigma}{\left\{(R_{/\!/}\Sigma)^2 - 4s\sin^2\tfrac{1}{2}\Sigma\,(R_{/\!/}\ \Sigma - s)\right\}^{\frac{1}{2}}}$$

Then in the triangle $S_{/\!/}PD_{/\!/}$ we have the side $PS_{/\!/} = l_{/\!/} = 90^\circ - l''$; the side $S_{/\!/}D_{/} = z_{/\!/}$; and the angle $P = \omega$. Therefore

$$\sin D_{/} = \frac{\sin l_{/\!/}\sin\omega}{\sin z_{/\!/}}\ ;\ \text{it is also} = \frac{\cos\frac{1}{2}\Sigma}{\cos(z_{/\!/} - \frac{1}{2}\Sigma)}\sin A'$$

$$\tan\tfrac{1}{2}A'' = \frac{\sin\frac{1}{2}(l_{/\!/} - z_{/\!/})}{\sin\frac{1}{2}(l_{/\!/} + z_{/\!/})}.\cot(D_{/} - \omega)$$

$$= \frac{\cos\frac{1}{2}(l_{/\!/} - z_{/\!/})}{\cos\frac{1}{2}(l_{/\!/} + z_{/\!/})}.\cot(D_{/} + \omega)$$

from which to find $A_{/\!/}$. And to find l' we have

$$\tan^2 l' = \left(\tan^2 l'' + \frac{a^2}{b^2}\right)\frac{\sin^2 A'}{\sin^2 A''} - \frac{a^2}{b^2}$$

Or we can find it from

$$\tan\tfrac{1}{2}l_{/} = -\frac{\cos\frac{1}{2}(A' + A'' + \omega)}{\cos\frac{1}{2}(A' + A'' - \omega)}.\cot\tfrac{1}{2}l_{/\!/}$$

The other entities can now be found as in problem 1.

Problem 7.

Given the latitude l', the difference of longitude ω, and the length s and circular measure Σ of the geodesic arc between the stations: to find — the azimuths, the latitude l'', and all the other entities determined in the solution of problem 1.

To find $z_{/}$ or arc $S_{/}D_{/\!/}$ we have—

$$\sin z_{/} = \frac{s.\sin\Sigma}{\left\{(R_{/}\ \Sigma)^2 - 4\,s.\sin^2\tfrac{1}{2}\Sigma\,(R_{/}\Sigma - s)\right\}^{\frac{1}{2}}}$$

Then, in the triangle $S, PD_{,,}$ we have the angle $P = \omega$, the side $SD_{,,} = z_{,}$, and the side $PS' = l_{,} = 90°—l'$. Hence, to find the azimuth A' we have—

$$\sin D_{,,} = \frac{\sin l_{,} \sin \omega}{\sin z_{,}}$$

$$\tan \tfrac{1}{2} A' = \frac{\sin \frac{1}{2} (l_{,} - z_{,})}{\sin \frac{1}{2} (l_{,} + z_{,})} \cdot \cot \tfrac{1}{2} (D_{,,} - \omega)$$

To find the azimuth A'' we have—

$$\sin A'' = \frac{\Sigma \cdot \sin \omega \cdot R' \cos l'}{s \cdot \sin \Sigma}$$

And to find the latitude l'', we have either of the following :—

$$\tan^2 l'' = \left(\tan^2 l' + \frac{a^2}{b}\right) \cdot \frac{\sin^2 A''}{\sin^2 A'} - \frac{a^2}{b^2}$$

$$\tan \tfrac{1}{2} l_{,,} = - \frac{\cos \frac{1}{2} (A' + A'' + \omega)}{\cos \frac{1}{2} (A' + A'' - \omega)} \cdot \cot \tfrac{1}{2} l_{,}$$

The other entities can now be found as in problem 1.

Problem 8.

Given the azimuth A', the difference of longitude ω, and the length s and circular measure Σ of the geodesic arc between the stations : to find—the latitudes of the stations, the azimuth A'', and all other entities.

Formula 20 gives us—

$$R_{,,} \cos l'' = \frac{s \cdot \sin \Sigma \sin A'}{\Sigma \sin \omega}$$

$\therefore$ if we find G such that

$$G = \frac{s \sin \Sigma \sin A'}{\Sigma \cdot \sin \omega}$$

We have—

$$\frac{a^2 \cos^2 l''}{1 - e^2 \sin^2 l''} = G^2$$

$$\therefore \qquad \sin l'' = \left\{ \frac{(a + G) \cdot (a - G)}{(a + eG) \cdot (a - eG)} \right\}^{\frac{1}{2}}$$

from which to find the latitude l''.

We can now find the arc $S_{,,}D_{,}$ or $z_{,,}$ as in problem 6, or from—

$$\sin z'' = \frac{\cos l'' \sin \omega}{\left\{ (\sin A' - \tan \frac{1}{2} \Sigma \cos l'' \sin \omega)^2 + (\cos l'' . \sin \omega)^2 \right\}^{\frac{1}{2}}}$$

and the azimuth A'' from formulae—

$$\sin D_{,} = \frac{\sin l_{,,} \sin \omega}{\sin z_{,,}} = \frac{\cos \frac{1}{2} \Sigma}{\cos (z_{,,} - \frac{1}{2} \Sigma)} . \sin A'$$

$$\tan \tfrac{1}{2} A'' = \frac{\sin \frac{1}{2} (l_{,,} - z_{,,})}{\sin \frac{1}{2} (l_{,,} + z_{,,})} \cdot \cot (D_{,} - \omega) \quad .$$

and then the latitude l' from—

$$\tan \tfrac{1}{2} l_{,} = - \frac{\cos \frac{1}{2} (A' + A'' + \omega)}{\cos \frac{1}{2} (A' + A'' - \omega)} \cdot \cot \tfrac{1}{2} l_{,,}$$

The other entities can now be found as in problem 1.

Problem 9.

Given the azimuths A', A'', and the length s and circular measure Σ of the geodesic arc between the stations; to find the latitudes l', l'', the difference of longitude ω, and the other various entities determined in the solution of problem 1.

We have (see formula 36) the arcs $S_{,}D_{,,,}$, $S_{,,}D_{,}$ or sides $z_{,}$ $z_{,,}$ of the triangles $S_{,}PD_{,,,}$, $S_{,,}PD_{,}$ expressed by—

$$z_{,} = \sin^{-1}\left\{\frac{\cos^2 \frac{1}{2}\Sigma}{1 + \frac{k^2}{R_{,}^2} - \frac{2.k}{R_{,}} . \sin\frac{1}{2}\Sigma}\right\}^{\frac{1}{2}} - (90° - \tfrac{1}{2}\Sigma)$$

the arc within the brackets being greater than 90°;

$$z_{,,} = (90° + \tfrac{1}{2}\Sigma) - \sin^{-1}\left\{\frac{\cos^2 \frac{1}{2}\Sigma}{1 + \frac{k^2}{R_{,,}^2} - \frac{2 \cdot k}{R_{,,}} . \sin\frac{1}{2}\Sigma}\right\}^{\frac{1}{2}}$$

the arc within the brackets being less than 90°

We know that $z_{,}$ is a little less than Σ, and that $z_{,,}$ differs little from Σ. But in order to obtain $z_{,}$ and $z_{,,}$ with great accuracy, we require only sufficiently approximate values of $R_{,}$ and $R_{,,}$. These we can find by assuming, for the occasion, that the arc IO (which bisects the angle Δ) cuts the arcs $PS_{,}$, $PS_{,,}$ at angles respectively equal to A' and A''; and that the portion of it intercepted between them is equal to Σ. Then putting $D_{\circ}$, D_{∞} for the points in which arc IO cuts $PS_{,}$ $PS_{,,}$, we have—

$$\tan\tfrac{1}{2}(PD_{\infty} + PD_{\circ}) = \frac{\cos\frac{1}{2}(A' - A'')}{\cos\frac{1}{2}(A' + A'')} . \tan\tfrac{1}{2}\Sigma$$

$$\tan\tfrac{1}{2}(PD_{\infty} - PD_{\circ} = \frac{\sin\frac{1}{2}(A' - A'')}{\sin\frac{1}{2}(A' + A'')} . \tan\tfrac{1}{2}\Sigma$$

from which we find colatitudes $PD_{\circ}$, PD_{∞} that cannot differ in any case from the absolutely rigorous values by more than a few seconds.

Then with approximate values of the latitudes obtained from

$$l' = 90° - PD_{\circ}\,;\; l'' = 90° - PD_{\infty}\,;$$

we can find values for the lengths of the normals $R_{,}$, $R_{,,}$ (terminating in the polar axis), which are equally as efficient in determining extremely correct values of $z_{,}$, $z_{,,}$, as if they were found with absolute accuracy.

Now having found the values of $z_{,}$ and $z_{,,}$, we can find the angles $D_{,}$, $D_{,,}$ by means of formulae (21) which give—

$$\sin D_{,} = \frac{\cos\frac{1}{2}\Sigma}{\cos(z_{,,} - \frac{1}{2}\Sigma)} . \sin A'$$

$$\sin D_{,,} = \frac{\cos\frac{1}{2}\Sigma}{\cos(z_{,} - \frac{1}{2}\Sigma)} . \sin A''$$

Then in the triangles $S_{,}PD_{,,,}$, $S_{,,}PD_{,}$, we have the bases $z_{,}$, $z_{,,}$, and the angles at the bases, so that we can find their common vertical angle ω, and their sides $l_{,}$, $l_{,,}$ which are the complements of the latitudes—

$$\tan\tfrac{1}{2}(L_{,,} + l_{,}) = \frac{\cos\frac{1}{2}(A' - D_{,,})}{\cos\frac{1}{2}(A' + D_{,,})} . \tan\tfrac{1}{2}z_{,}.$$

$$\tan \tfrac{1}{2} (L_{\prime\prime} - l_{\prime}) = \frac{\sin \frac{1}{2} (A' - D_{\prime\prime})}{\sin \frac{1}{2} (A' + D_{\prime\prime})} . \tan \tfrac{1}{2} z_{\prime}.$$

$$\tan \tfrac{1}{2} (l_{\prime\prime} + L_{\prime}) = \frac{\cos \frac{1}{2} (D_{\prime} - A'')}{\cos \frac{1}{2} (D_{\prime} + A'')} . \tan \tfrac{1}{2} z_{\prime\prime}.$$

$$\tan \tfrac{1}{2} (l_{\prime\prime} - L_{\prime}) = \frac{\sin \frac{1}{2} (D_{\prime} - A'')}{\sin \frac{1}{2} (D_{\prime} + A'')} . \tan \tfrac{1}{2} z_{\prime\prime}.$$

$$\tan \tfrac{1}{2} \omega = \frac{\cos \frac{1}{2} (L_{\prime\prime} - l_{\prime})}{\cos \frac{1}{2} (L_{\prime\prime} + l_{\prime})} . \cot \tfrac{1}{2} (A' + D_{\prime\prime})$$

$$\tan \tfrac{1}{2} \omega = \frac{\cos \frac{1}{2} (l_{\prime\prime} - L_{\prime})}{\cos \frac{1}{2} (l_{\prime\prime} + L_{\prime})} . \cot \tfrac{1}{2}(D_{\prime} + A'')$$

From these we can find $l_{\prime}$, $l_{\prime\prime}$, $L_{\prime}$, $L_{\prime\prime}$ and ω with precision.

To obtain the latitudes l', l'', we have

$$l' = 90° - l_{\prime} ; \; l'' = 90° - l_{\prime\prime}.$$

And the other various entities can be found as in problem 1.

☞ We can find l'' and l'' from the two equations—

$$\frac{\tan^2 l' + 1}{\tan^2 l'' + 1} = \left(\frac{\sin z_{\prime\prime} . \sin D_{\prime}}{\sin z_{\prime} \cdot \sin D_{\prime\prime}}\right)^2 \quad \ldots \quad \ldots \quad \text{(a)}$$

$$\frac{\tan^2 l' + \frac{a^2}{b^2}}{\tan^2 l'' + \frac{a^2}{b^2}} = \frac{\sin^2 A'}{\sin^2 A''} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(b)}$$

☞ We can also find the latitudes (independently of $z_{\prime}$, $z_{\prime\prime}$, $D_{\prime}$, $D_{\prime\prime}$) by solving the two following equations :—

$$\tan \tfrac{1}{2} \Sigma \sin l' - \cos A' \cos l' = - \tan \tfrac{1}{2} \Sigma \sin l'' + \cos A'' \cos l'' ;$$

$$\frac{\tan^2 l' + \frac{a^2}{b^2}}{\tan^2 l'' + \frac{a^2}{b^2}} = \frac{\sin^2 A'}{\sin^2 A''} .$$

They may be written in the forms—

$$\frac{\tan^2 l' + 1}{\tan^2 l'' + 1} = \frac{(\tan \frac{1}{2} \Sigma . \tan l' - \cos A')^2}{(\tan \frac{1}{2} \Sigma . \tan l'' - \cos A'')^2} \quad \ldots \quad \ldots \quad (\alpha)$$

$$\frac{\tan^2 l' + \frac{a^2}{b^2}}{\tan^2 l'' + \frac{a^2}{b^2}} = \frac{1 - \cos^2 A'}{1 - \cos^2 A''} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (\beta)$$

It is evident that by having any three of the five entities A', A'', l', l'', Σ, we can find the remaining two from these equations.

Problem 10.

Given the two azimuths A', A'', and the difference of longitude ω of the stations, to find the two latitudes l', l'', and the other entities.

The co-latitudes l', l'', can be easily found from—

$$\frac{\cos\frac{1}{2}(l_{,}-l_{,,})}{\cos\frac{1}{2}(l_{,}+l_{,,})}=\frac{\cot\frac{1}{2}(A'+A'')}{\tan\frac{1}{2}\omega}$$

$$\frac{\cot^2 l_{,}+\frac{a^2}{b^2}}{\cot^2 l_{,,}+\frac{a^2}{b^2}}=\frac{\sin^2 A'}{\sin^2 A''}$$

By putting N to represent the dexter of the first equation we can replace it by the following one, which can be easily derived from it :—

$$\cot^2\tfrac{1}{2}l_{,}\cdot\cot^2\tfrac{1}{2}l_{,,}=\left(\frac{N+1}{N-1}\right)^2=M \quad \ldots \quad \ldots \quad \ldots \quad (a')$$

The second can be put under the form—

$$\frac{\frac{(\cot^2\frac{1}{2}l_{,}-1)^2}{(2\cot\frac{1}{2}l_{,})^2}+\frac{a^2}{b^2}}{\frac{(\cot^2\frac{1}{2}l_{,,}-1)^2}{(2\cot\frac{1}{2}l_{,,})^2}+\frac{a^2}{b^2}}=\frac{\sin^2 A'}{\sin^2 A''} \quad \ldots \quad \ldots \quad \ldots \quad (b')$$

By developing this last equation, and making use of equation (a'), it is easy to put it in the form—

$$\frac{M\cot^2\frac{1}{2}l_{,}+\cot^2\frac{1}{2}l_{,,}-M\left(4\frac{a^2}{b^2}-2\right)}{M\cot^2\frac{1}{2}l_{,,}+\cot^2\frac{1}{2}l_{,}-M\left(4\frac{a^2}{b^2}-2\right)}=\frac{\sin^2 A'}{\sin^2 A''} \quad \ldots \quad \ldots \quad (c)$$

It is evident that the values of $\cot^2\frac{1}{2}l_{,}$ and $\cot^2\frac{1}{2}l_{,,}$ can be easily found from the formula a' and c.

The resulting values can be arrived at by means of the following set of formulæ :—

$$\tan\psi=\frac{\cot\frac{1}{2}(A'+A'')}{\tan\frac{1}{2}\omega} \qquad (d)$$

$$M=\tan^2(\psi+45^\circ) \qquad (e)$$

$$G=M\left(4\frac{a^2}{b^2}-2\right).(\sin^2 A'-\sin^2 A'')\,; \qquad (f)$$

$$U=M\sin^2 A''-\sin^2 A'\,; \qquad (g)$$

$$V=M\sin^2 A'-\sin^2 A''\,; \qquad (h)$$

$$\cot^2\tfrac{1}{2}l_{,}=\frac{G+\sqrt{G^2+4M.U.V}}{2\cdot U} \qquad (i)$$

$$\cot^2\tfrac{1}{2}l_{,,}=\frac{-G+\sqrt{G^2+4M.U.V}}{2\cdot V} \qquad (j)$$

☞ It is evident M is positive and greater than 1; and therefore, since $\sin^2 A'$ is greater than $\sin^2 A''$, it follows that G and V are also positive.

And as $\cot^2 \frac{1}{2} l_{\prime\prime}$ must be positive, it is evident that the radical in equation *(j)* must have the sign $+$ before it, and that U must also be positive. Moreover, from this and the fact that $\cot^2 \frac{1}{2} l_{\prime}$ is greater than $\cot^2 \frac{1}{2} l_{\prime\prime}$ (or from the relation, $\cot^2 \frac{1}{2} l_{\prime} \cdot \cot^2 \frac{1}{2} l_{\prime\prime} = M$) it is evident that the radical in equation (*i*) must have the sign $+$ before it.

Having found $l_{\prime}, l_{\prime\prime}$, from the equations *d, e, f, g, h, i, j*, we have

$$l' = 90^\circ - l_{\prime}$$
$$l'' = 90^\circ - l_{\prime\prime}$$

The various other entities can be found as in problem 1 .

☞ If we were to assume $D_{\prime\prime} = A_{\prime\prime}$, so as to find $l_{\prime\prime}$ from the triangle $S_{\prime}PD''$, the result would be too small (&∴ the latitude l'' too great) by the arc $\delta_{\prime\prime}$, which amounts to about 6 seconds in the case worked out in my last paper. (See the last note to problem 5.)

The equations —

$$\frac{\cos \frac{1}{2}(l_{\prime} - l_{\prime\prime})}{\cos \frac{1}{2}(l_{\prime} + l_{\prime\prime})} = \frac{\cot \frac{1}{2}(A' + A'')}{\tan \frac{1}{2}\omega}$$

$$\frac{\cot^2 l_{\prime} + \frac{a^2}{b^2}}{\cot^2 l_{\prime\prime} + \frac{a^2}{b^2}} = \frac{\sin^2 A'}{\sin^2 A''}$$

can be put under the forms :—

$$\tan \tfrac{1}{2}\omega \,.\, \frac{\cot \frac{1}{2} l_{\prime} \cot \frac{1}{2} l_{\prime\prime} + 1}{\cot \frac{1}{2} l_{\prime} \cot \frac{1}{2} l_{\prime\prime} - 1} = \frac{\cot \frac{1}{2} A'' \cot \frac{1}{2} A' - 1}{\cot \frac{1}{2} A'' + \cot \frac{1}{2} A'}$$

$$\frac{\cot^2 \frac{1}{2} l_{\prime\prime} \left\{ (\cot^2 \frac{1}{2} l_{\prime} - 1)^2 + 4\frac{a^2}{b^2} \,.\cot^2 \frac{1}{2} l_{\prime} \right\}}{\cot^2 \frac{1}{2} l_{\prime} \left\{ (\cot^2 \frac{1}{2} l_{\prime\prime} - 1)^2 + 4\frac{a^2}{b^2} \,.\cot^2 \frac{1}{2} l'' \right\}} = \frac{\cot^2 \frac{1}{2} A' \,(1 + \cot^2 \frac{1}{2} A_{\prime\prime})^2}{\cot^2 \frac{1}{2} A'' \,(1 + \cot^2 \frac{1}{2} A')^2}$$

It is evident that by knowing any three of the five entities l', l'', A', A'', ω, we can, by solving these equations, find the remaining two.*

☞ In determining entities, all correct methods available should be employed, and the means of the results taken in order to ameliorate the discrepancies which inevitably occur, owing to imperfections in the data. Want of space has compelled me to omit many useful improvements and additions of this character in the solutions of the preceding problems.

For the same reason I have omitted expressions for the magnitudes of the various straight lines of the figure, and the angles they make with each other ; and also the relations or formulae derivable from the anharmonic divisions and pencils.

To establish the lengths and positions of great Base-Lines more accurately than by the usual method of triangulating from a short Base measured by means of rods and bars :—

Select three mutually visible stations S_1, S_2, S_3, no two of which shall differ in longitude by less than half of a degree, or be of a less distance asunder than 50 miles. (The greater the difference in longitude, the better as respects the two more distant stations S_1 and S_3).

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* If l', A'. ω be the given entities, we can casily find $L_{\prime\prime\prime}$, $z_{\prime\prime}$, $D_{\prime\prime}$ from the triangle $D_{\prime\prime}P\,S_{\prime}$. And by equating the expressions for cos (PI) in the two triangles $D_{\prime\prime}PI$, $S_{\prime}PI$, we can find the value of $\tan \frac{1}{2} \Sigma'$. Then by means of formula (25) we can find A'' ; and by means of formula (4) or (30) we can find l''. &c., &c.

With first-class instruments, find by means of actual observation (using the method of least squares, &c.) the latitudes of the stations, and their three pairs of azimuths. (It is supposed that the observer knows how to use the instruments, so as to obtain correct readings of angles of altitude and azimuth, no matter what may be the deviations of the plumb-line from the normals at the stations).

Let l', l'' be the latitudes of the stations S^1, S_2.

Let A' be the azimuth of S_2 as taken at S_1; and let A'' be the azimuth of S_1 as taken at S_2.

Then we know the values of the four entities—

$$l',\ l'',\ A',\ A''.$$

Assume each three of these four entities as data, and compute therefrom the obtainable values of the latitudes and azimuths, which (written in order) we may denote by—

$$l'_1,\ l''_1,\ A'_1,\ A''_1.$$

Find also the four obtainable values of the difference of longitude of the stations.

From the observed and computed values of the latitudes and azimuths, and the computed values of the difference of longitude of the stations, find the most probably correct values; and represent them in order by—

$$l'_2,\ l''_2,\ A'_2,\ A''_2,\ \omega_2.$$

Assume each three of these five entities as new data, and compute the *four* values of each entity obtainable therefrom; and find the most probable value of each entity from *all* the known values (observed and computed).

Represent the values so derived by—

$$l'_3,\ l''_3,\ A'_3,\ A''_3,\ \omega_3.$$

Assuming these last as more compatible values of the latitudes, azimuths, and difference of longitude, proceed in like manner, and find a new set of values for the five entities. And represent such values by—

$$l'_4,\ l''_4,\ A'_4,\ A''_4,\ \omega_4.$$

Assuming these last as still more compatible values of the latitudes, azimuths, and difference of longitude, proceed in like manner, and find a new set of values for the five entities.

Represent such values by—

$$l'_5,\ l''_5,\ A'_5,\ A''_5,\ \omega_5.$$

Now, under the circumstances (in which the stations are supposed to be chosen by a competent person, and the observations taken with the greatest care), it will be found that this last set of values of the five entities are equals respectively to the immediately preceding set, or so nearly so as not to differ from them by amounts appreciable in the most rigorous practice.

Omitting the subscript figures, we may write the last obtained values of the latitudes, azimuths, and difference of longitudes, in the adopted notation, as follows:—

$$l'_1,\ l''_1,\ A'_1,\ A''_1,\ \omega_1.$$

Find the angles $\frac{1}{2}\gamma', \frac{1}{2}\gamma''$, of depression of the chord $S_1 S_2$ at the stations S_1, S_2 from formulæ (44), (45) ; and take the means of the values so found as the more compatible values.

Find the length of the chord k_3 connecting the stations S_1, S_2, by means of formulæ (51) ; and take the mean of the values so found as the more compatible.

Find the length s of the geodesic arc between the stations S_1, S_2, by means of formulæ (53).

Proceed with respect to the stations S_2, S_3, in like manner, to find the most compatible values of their latitudes, azimuths, and difference of longitude, the angles of depression at S_1, S_2, the length of the chord, and the length of the geodesic arc. The values of the entities so found may be written in order—

$$l''_2,\ l'''_2,\ A''_2,\ A'''_2,\ \omega_2.\ \tfrac{1}{2}a',\ \tfrac{1}{2}a'',\ k_1.$$

Proceed in like manner with respect to the stations S_3, S_1. The values of the entities so arrived at can be written in order—

$$l'''_3,\ l'_3,\ A'''_3,\ A'_3,\ \omega_3,\ \tfrac{1}{2}\beta',\ \tfrac{1}{2}\beta'',\ k_2.$$

Now we should have $l'_1 = l'_3$; $l''_1 = l''_2$; $l'''_2 = l''_3{}'$. And the stations S_1, S_3, being supposed to be the pair having the greatest difference of longitude, we should have $\omega_3 = \omega_1 + \omega_2$.

If such equalities subsist to within an extremely close degree of absolute accuracy, they are a test of the correctness of the work. But if the differences be appreciable, take the means $\frac{1}{2}(l'_1 + l'_3)$; $\frac{1}{2}(l''_1 + l''_2)$; $\frac{1}{2}(l'''_2 + l'''_3)$ as the most compatible values of the latitudes : and, retaining the rectified azimuths, repeat the whole process, when values of the latitudes, azimuths, and difference of longitude will be at length arrived at which are compatible in every respect, or sufficiently so for the most scrupulous accuracy in actual practice.

We can find the angles A, B, C, of the spheroidal triangle at the respective vertices S_1, S_2, S_3, by mere addition or subtraction of the rectified azimuths.

Now putting K_1, K_2, K_3, to represent the angles between the pairs of chord at the vertices S_1, S_2, S_3, we have—

$$\left.\begin{aligned} \cos K_1 &= \frac{k_2^2 + k_3^2 - k_1^2}{2k_2 k_3} \\ \cos K_2 &= \frac{k_3^2 + k_1^2 - k_2^2}{2k_3 k_1} \\ \cos K_3 &= \frac{k_1^2 + k_2^2 - k_3^2}{2k_1 k_2} \end{aligned}\right\} \quad \dots \quad \dots \quad \dots \quad (a)$$

We have also the following set of formulæ rigorously true for any spheroidal triangle on any spheroid or any analogous triangle on any surface whatever—

$$\left.\begin{aligned} \cos K_1 &= \sin \tfrac{1}{2}\beta'' \sin \tfrac{1}{2}\gamma' + \cos \tfrac{1}{2}\beta'' \cos \tfrac{1}{2}\gamma' \cos A \\ \cos K_2 &= \sin \tfrac{1}{2}\gamma'' \sin \tfrac{1}{2}\alpha' + \cos \tfrac{1}{2}\gamma'' \cos \tfrac{1}{2}\alpha' \cos B \\ \cos K_3 &= \sin \tfrac{1}{2}\alpha'' \sin \tfrac{1}{2}\beta' + \cos \tfrac{1}{2}\alpha'' \cos \tfrac{1}{2}\beta' \cos C \end{aligned}\right\} \quad \ldots \quad \ldots \quad (b)$$

The values of the chordal angles obtained from both sets of equations should be equals if the work has been correctly performed; and we should also have

$$K_1 + K_2 + K_3 = 180^\circ.$$

If these equations or relations subsist to within an extremely close degree of absolute accuracy, they are (in conjunction with the tests already afforded) sufficient to enable any *competent* mathematician to state, with confidence, that the magnitudes of the geodesic arcs, chords, and angles have been found with the greatest accuracy, and that the relative astronomical positions and mutual bearings of the stations have been determined with precision.

It is evident that the differences of the spheroidal and chordal angles of any spheroidal triangle on any spheroid whatever, are rigorously expressed by

$$\left.\begin{aligned} &A - \cos^{-1} (\sin \tfrac{1}{2}\beta'' \sin \tfrac{1}{2}\gamma' + \cos \tfrac{1}{2}\beta'' \cos \tfrac{1}{2}\gamma' \cos A) \\ &B - \cos^{-1} (\sin \tfrac{1}{2}\gamma'' \sin \tfrac{1}{2}\alpha' + \cos \tfrac{1}{2}\gamma'' \cos \tfrac{1}{2}\alpha' \cos B) \\ &C - \cos^{-1} (\sin \tfrac{1}{2}\alpha'' \sin \tfrac{1}{2}\beta' + \cos \tfrac{1}{2}\alpha'' \cos \tfrac{1}{2}\beta' \cos C) \end{aligned}\right\} \quad \ldots \quad (c)$$

The sum of the three expressing the spheroidal excess.

☞ I may here observe that if the process just indicated, with respect to three chosen stations, be applied to any three mutually visible stations forming the vertices of a great primary triangle of a Trigonometrical Survey, it will be found to be an efficient method of exposing the errors due to the use of erroneous formulæ and the misrepresentations of the bearings and positions of stations which may possibly result from permitting the officer who makes the principal astronomical observations to have any control over the necessary amount or the character of the computations; for computations, when rigorously and efficiently performed, are certain to add so considerably to the amount of work required from the chief astronomical observer as to keep him constantly in the field supplying the necessary data, and testing his own and subordinates' work, whenever such is reported to be too incompatible by the computers.

The method of testing the accuracy of a survey by means of a "base of verification" of a few miles in length is thoroughly deceptive when the primitive base and base of verification are nearly in the same latitude; for a compensation of errors is very likely, in such case, to make the computed and measured lengths of short lines to closely agree, even though the usual erroneous and inefficient formulæ be used in the computation. If the bases differ extensively in latitude and longitude, it is probable that, under like circumstances, the measured and computed lengths will differ by some small amount. But, in this case also, the test is thoroughly inadequate as regards the lengths and bearings of the sides of the great primary triangles, and particularly so as regards the astronomical or geodesic positions of their vertices, which is a matter of much more consequence, when looked at from either a practical or scientific point of view.

It is scarcely necessary to add that, by selecting a set of connected triangles extending through the triangulation of any extensive country, we could, by such means, test the general accuracy of the entire work.

NOTES.

(1) The problems solved in this paper are almost all new. Two only (as far as my knowledge extends) have been solved elsewhere or applied in the actual practice of trigonometrical surveys, viz. :—

1°. When the given data consists of the latitude of one station S', the azimuth A', from it to the other station S'', and the length and circular measure of the geodesic arc between the stations.

2°. When the latitudes of the two stations are given, and also the difference of longitude of the stations.

It seems to be the general practice to find methods of determining the unknown latitude, or azimuths, or difference of longitude only, without paying much respect to the various other entities and the relations subsisting amongst them, by means of which errors of observation can be more easily detected and rectified.

I have given new methods of solving the two above-mentioned cases. The other equally important cases, which have been hitherto overlooked, will no doubt attract more attention in the future. The whole set, when fully worked out by the various processes suggested by the figure, constitute the basis of a system of Geodesy in which the Equatorial and Polar Radii of the earth are the only standards required, in order to obtain the accurate lengths and bearings of sides of triangles on its spheroidal surface, and the positions of the stations or vertices.

(2) In the preliminary portion of the paper many useful theorems and formulæ have been evolved.

I would direct particular attention to the direct evolution of Dalby's celebrated theorem ; to the remarkable relation connecting the circular measure Σ of the geodesic arc joining two stations, the angle Δ between the two normal-chordal planes, and the angle ν between the two normals to the surface at the stations, viz. :—that their halves can be represented by the sides of a right-angled spherical triangle.

(3) The errors which have been demonstrated to exist in the investigations and formulæ of the "Account of the Principal Triangulation of Great Britain and Ireland" relate to the most important questions in practical Geodesy. They pertain to formulæ specially recommended as the most exact and reliable, and such as should be employed to test the approximative accuracy of all other methods or formulæ. It must be manifest, from what has been shown in the present paper, that they are not fitted for the purposes intended, and that they lead to results more erroneous than those arrived at by considering the earth to be a perfect sphere. Some persons may think it strange that those errors were not detected long since in the actual work of the English and Indian Trigonometrical Surveys ; but those who have paid special attention to the science of Geodesy will no doubt at once perceive that it was almost impossible to detect errors in that manner, as all imperfections are generally attributed to "the deviation of the plumb-line from *the vertical.*"

That the plumb-line must deviate from the normal or vertical line at some stations is evident ; and calculations based on the laws of attraction and measurements of mountain masses can be made, giving a rough estimate of the amount of such deviation ; but, in order to get a correct estimate of its amount and direction, it is, in my opinion, absolutely necessary to take such observations at each of the principal stations of a triangulation as will enable one to apply *all* the problems in the present paper, *to every two of the stations*, and thus obtain a complete set of entities to which the principles of probabilities can be *properly* applied, instead of confining the calculated results, as is usually done, to what may be given by the application of the first and fifth problems only.

Until the science of Geodesy is thus fully applied, not the least reliance can be placed on the lengths of arcs of meridian determined from triangulations.

4 It is easy to prove that the principal theorems arrived at apply to short arcs on any surface whatever, as well as to the spheroidal surface of the earth, even when such surface is so irregular as to be inexpressible by means of an equation.

We can assume any straight line cutting the normals at the stations S', S'' as polar axis of reference, and any point P in it as a pole; and then assuming any point C in this polar axis as centre of reference, we can take the plane through it perpendicular to the polar axis as the equatorial plane of reference. Thus the figure can be constructed as already indicated in the case in which the surface is a spheroid.

And in this general case, it is evident that—

$$\sin A' \cos \tfrac{1}{2} \Sigma' = \cos \theta \sin \phi_{,} ;$$

$$\sin A'' \cos \tfrac{1}{2} \Sigma'' = \cos \theta \sin \phi_{,,} ;$$

$$\frac{\sin A'}{\sin A''} = \frac{R_{,,} \cos l''}{R_{,} \cos l'} \cdot \frac{\cos \frac{1}{2} \Sigma'}{\cos \frac{1}{2} \Sigma''}$$

and that all the formulæ, not implicating peculiar properties of the spheroid, hold good for the general surface, when the stations S', S'' are so near to each other as to permit us to regard the normals as making angles with the chord whose sines are equals, and the traces of the normal-choral planes as equals in length and circular measure.

If there be three stations S', S'', S''', to be simultaneously considered, the assumable position for the polar axis of reference is generally restricted; as such axis must cut the three normals to the surface drawn at the stations.

If the three normals intersect in one point, any line through this point can be assumed as the polar axis. If two of the normals cut each other, and that neither of them is cut by the third normal, then the polar axis must pass through the point of intersection and lie in the plane of this point and the third normal. If the three normals have no point of intersection, then the polar axis must lie in a ruled surface of the second degree, &c.

And when there are four stations on the surface, then should no two of the four normals lie in one plane, there can be but two transversal lines drawn to cut them, and ∴ but two assumable positions for the polar axis of reference.

However, with respect to all surfaces of revolution (whose normals must all cut the axis of revolution), we can, by giving a more extended signification to some of the involved entities, arrive at general theorems applying to any stations whatever on the surface, of which the theorems already evolved may be regarded as particular cases for short arcs, so stated as to be easier of application when actual results have to be computed.

For instance, we can easily demonstrate the following:—

Theorem. If S', S'', be any two stations on a surface of revolution of any kind, and A', A'', the angles which the *true* "geodesic" joining the stations makes with the traces of the meridian planes through the stations, and that $R_{,}$, $R_{,,}$, are the normals terminating in the axis, and $r_{,}$, $r_{,,}$, the central radii to the stations, then will—

$$\frac{\sin A'}{\sin A''} = \frac{R_{,,} \cdot \cos l''}{R_{,} \cdot \cos l'} = \frac{r_{,,} \cdot \cos \lambda''}{r_{,} \cdot \cos \lambda'} = \frac{Cp_{,,}}{Cp_{,}} .$$

(See Fig. 2.) Conceive the "Geodesic" $S'S''$, to be divided into infinitesimally small parts or elements, $S'S_{,}$, $S_{,}S_2$, S_2S_3, ... S_n S''.

Let $A_{,}$, A_2, A_3, ... A_n, represent the azimuths of the stations S', $S_{,}S_2$,

. . . . . S_{n-1}, if observed from $S_{,}$. S_2, S_3 . . . . S_n ; and let A' represent the azimuth of $S_{,}$ as if observed from S'; and let A'' represent the azimuth of S_n as if observed from S''.

Put R', $R_{,}$, R_2, ... R_n, R'' for the normals (terminating in the axis) at S', $S_{,}$, S_2, ... S_n, S'' ; and r', $r_{,}$, $r_{,,,}$ . . . . . r_n, r'' for the central radii to the stations taken in like order.

Now, from the elements of analytic geometry, we know that the tangent lines to any infinitesimally small arc of the *first* order, which forms part of a "geodesic," have their least distance an infinitesimally small of the *third* order; and that the lengths of these tangents, from the points of contact to their points of least distance from each other, are *equals*. We know also that the plane of every two consecutive elements of any "geodesic" contains the normal at their point of junction; and therefore that sin $A_{,}$, sin A_2, sin A_3, ... sin A_n, are respectively equal to the sines of the azimuths of the stations S_2, S_3, S_4 . . . . . S'' as if observed from $S_{,}$, S_2, ... S_n, which are their supplements. Hence—(as the cosines of infinitesimally small arcs are equals)—

$$\frac{\sin A'}{\sin A_1}=\frac{R_{,}\cos l_{,}}{R'\cos l'};\quad \frac{\sin A_1}{\sin A_2}=\frac{R\cos l_2}{R^1\cos l};\quad \frac{\sin A_2}{\sin A_3}=\frac{R_3\cos l_3}{R_2\cos l_2};\ \ldots\ldots$$

$$\frac{\sin A_{n-1}}{\sin A_n}=\frac{R_n\cos l_n}{R_{n-1}\cos l_{n-1}};\quad \frac{\sin A_n}{\sin A''}=\frac{R''\cos l''}{R_n\cos l_n}$$

$\therefore$ evidently—
$$\frac{\sin A'}{\sin A''}=\frac{R''\cos l''}{R'\cos l'}.$$

And in a similar manner it may be shown that—

$$\frac{\sin A'}{\sin A''}=\frac{r''\cos\lambda''}{r'\cos\lambda'}.$$

$$\frac{\sin A'}{\sin A''}=\frac{\text{perpendicular from } S'' \text{ on the axis}}{\text{perpendicular from } S' \text{ on the axis}}.$$

For a spheroid such as the earth's surface, we can prove, in like manner, that for any two stations whatever—no matter how distant from each other,—

$$\frac{\sin^2 A'}{\sin^2 A''}=\frac{\tan^2 l'+\frac{a^2}{b^2}}{\tan^2 l''+\frac{a^2}{b^2}}$$

in which A', A'', are the angles made by the geodesic joining the stations, with the traces of the meridian planes through the stations. And if the earth's surface were that of a circular cylinder or cone, or of any other known surface of revolution expressible by means of an equation, we could easily find the particular equivalents to substitute for the perpendiculars from the stations to the axis in the above general expression for sin A' ÷ sin A''.

The following theorems (and others of a kindred character) are evident from what has been shown in the few last notes :—

1°. If $\beta_{,}$ be the point in which a geodesic on any surface of revolution is cut perpendicularly by the trace $t_{,}$ of a meridian plane, and that $\beta_{,,}$ is the point in which it is cut obliquely by the trace $t_{,,}$ of another meridian plane: then, putting $p_{,,}$ $p_{,,,}$ to represent the lengths of the perpendiculars from $\beta_{,,}$ $\beta_{,,}$

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upon the axis of revolution, and ϵ the angle which the geodesic makes with the trace $t_{\prime\prime\prime}$,—

$$\sin \epsilon = \frac{p_{\prime}}{p_{\prime\prime}}$$

2°. If the traces of meridian planes cut a geodesic at right angles, then will the perpendiculars from the points of intersection upon the axis of revolution be equal to each other; and the distances of the points of intersection from the poles will be equals.

3°. If the trace of one meridian cut a geodesic on a symmetrical surface of revolution (such as the earth's surface) in two points perpendicularly, then will the line joining the points of intersection pass through the centre of the surface.

4°. If i be the point of intersection of two geodesics on any surface of revolution, and $\beta_{\prime}$, $\beta_{\prime\prime}$, the points on which they are cut perpendicularly by traces of meridian planes; then, if $p_{\prime}$, $p_{\prime\prime}$, g denote the lengths of perpendiculars from $\beta_{\prime}$, $\beta_{\prime\prime}$, i upon the axis, and that ψ represents the angle between the geodesics at their point of intersection—

$$\psi = \sin^{-1}\left(\frac{g}{p_{\prime}}\right) + \sin^{-1}\left(\frac{g}{p_{\prime\prime}}\right).$$

5°. If two geodesics cut each other in two points equidistant from the axis on a symmetrical surface of revolution: then will their angles of intersection at one point be equal to their angles of intersection at the other point.

5 From formulae (21) it is evident that when $A' = A''$, we must have $z_{\prime\prime} = z_{\prime} = \frac{1}{2}\Sigma' + \frac{1}{2}\Sigma''$. It is also evident that when $\sin A' = \sin D_{\prime}$, and that A' is acute, and $D_{\prime}$ obtuse, then will $z_{\prime\prime} = \frac{1}{2}\Sigma' + \frac{1}{2}\Sigma''$. And putting V to represent the value of the angle A', in this case, it is evident that when A' is greater than V, then will $z_{\prime\prime}$ be less than $\frac{1}{2}\Sigma' + \frac{1}{2}\Sigma''$; and when A' is less than V then will $z_{\prime\prime}$ be greater than $\frac{1}{2}\Sigma' + \frac{1}{2}\Sigma''$.

Hence we have the following theorem pertaining to the figure of the earth, or to any surface of revolution whose normals cut the axis in points analogous to the way in which the normals to the earth cut its axis.

Theorem.—If S'' be any fixed point within any convex closed curve on the surface of an oblate spheroid whose adjacent pole is $P_{\circ}$, and that $Z_{\prime\prime}$ is the point in which the normal to the surface at S'' cuts the axis; there are 4 real points S' on this curve, and 4 points only, such that the angle $S''\ Z_{\prime\prime}\ S'$ subtended at $Z_{\prime\prime}$ is equal to the sum of the angles of depression of the chord $S''\ S'$ below the tangent planes at S'' and S'.

1°. The two points in which the curve is cut by the plane N through S'' perpendicular to the axis.

2°. The two other points lying on the same side of the plane N, and such that the azimuth of S' as taken at S'' is acute, and the azimuth of S'' as taken at S' is also acute, but greater than the other, and approaching very nearly to 90° when (as in case of the earth) the ellipticity of the surface is small.

☞ It may perhaps be proper to mention that the circular measure of the arc between the stations, which is given as data in some of the problems of this paper, is supposed to be more accurate than is usually obtained from the mere triangulation calculations of a trig. survey. In a future paper, when considering three stations simultaneously, I will show that the circular measures so obtained must, if used, lead to very loose determinations of latitudes, azimuths, and differences of longitude.

Sydney: Thomas Richards, Government Printer.—1871.