Given $f : \mathbb{R}^+ \to \mathbb{R}$ and $n \in \mathbb{N}$ with $n \geqslant 2$

Define $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} \arctan(x) - \pi & \text{if } x \in [\,0, \infty) \\ x - \frac{\pi}{2} & \text{if } x \in [-(n-1)\frac{\pi}{2}, 0) \\ f(\tan(x + n\frac{\pi}{2})) & \text{if } x \in [-n\frac{\pi}{2}, -(n-1)\frac{\pi}{2}) \end{cases}$$

Then

$$\begin{array}{rcccl} & x & & & \in [\,0, \infty) \\ \Rightarrow & g(x) & = & \arctan(x) - \pi & \in [-\pi, -\frac{\pi}{2}) \\ \Rightarrow & g(g(x)) & = & \arctan(x) - 3\frac{\pi}{2} & \in [-3\frac{\pi}{2}, -\pi) \\ & & \vdots & & \vdots \\ \Rightarrow & g^{n-2}(x) & = & \arctan(x) - (n-1)\frac{\pi}{2} & \in [-(n-1)\frac{\pi}{2}, -(n-2)\frac{\pi}{2}) \\ \Rightarrow & g^{n-1}(x) & = & \arctan(x) - n\frac{\pi}{2} & \in [-n\frac{\pi}{2}, -(n-1)\frac{\pi}{2}) \end{array}$$

Hence

$$\begin{aligned} & g^n(x) \\ = \; & f(\tan(g^{n-1}(x) + n\tfrac{\pi}{2})) \\ = \; & f(\tan(\arctan(x) - n\tfrac{\pi}{2} + n\tfrac{\pi}{2})) \\ = \; & f(\tan(\arctan(x))) \\ = \; & f(x) \end{aligned}$$

That is, g is an nth functional root of f.