APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO DENSITY FIELD DETERMINATION IN TRANSONIC CORNER FLOW

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THESIS

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by

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ABSTRACT

The successful application of holographic interferometry to the study of density fields around opaque bodies in wind tunnel experiments has been reported in the literature. The present report extends this technique to the study of the three-dimensional asymmetric flow fields encountered near the wing-fuselage junction of an aerodynamic model in the transonic flow regime. Finite fringe interferometry has been used to obtain fringe information about a partially transparent wing-body structure. A FORTRAN computer program was utilized to invert the fringe information and produce a plot of the density field around the model. The resulting asymmetric density field and shock wave structure are shown to be an accurate representation of the phenomena encountered in aerodynamic corner flow.

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I. INTRODUCTION

The field of flow measurement has been revolutionized in recent years with the perfection of holography and holographic interferometry techniques. High power Q-switched and dye-switched lasers and sophisticated double-pulsing trigger mechanisms provide exposure times on the order of twenty nanoseconds, thereby "freezing" the flow during the hologram production process. The precision optical quality components and measurement techniques of Mach-Zehnder interferometry have given way to the much less restrictive requirements of holographic interferometry which provide high quality interferograms in three dimensions.

Techniques for the application of holography to interferometry have been reported by Heflinger, et al. [1], and by Brooks [2]. In the determination of the density field around a free jet in the supersonic regime, Matulka [3, 4] expressed the fringe data in a series of orthogonal polynomials and transformed them to polynomials representing density using an inversion technique reported in [5, 6]. The method was extended by Jagota [7, 8] to the determination of the three-dimensional density field around a ten-degree half angle cone in a supersonic wind tunnel. The ability to produce readable holograms in wind tunnel studies using transparent phase objects was verified by Heyer [9]. In the present report the aforementioned techniques

have been combined to study the three-dimensional density field near the wing-fuselage junction of an aerodynamic model in transonic flow. The experiment was conducted at the Naval Ship Research and Development Center, Carderock, Maryland in an eighteen inch transonic blowdown wind tunnel at a Mach number of 0.937, using a semi-transparent model of original design.

Since reasonably small variations in density were anticipated, a finite fringe technique was used in obtaining the interferograms. The horizontal finite fringe field was produced by a vertical translation of a diffusing glass in the scene beam a distance of 0.003 inches between the two exposures of the holographic plate. Fringe data obtained from the interferograms were reduced to density information using a modified form of the inversion computer program used in [7]. A self-testing procedure incorporated in the program verified the resulting density data as an accurate representation of the actual flow around the model.

II. EXPERIMENTAL APPARATUS

A. THE WIND TUNNEL

The investigation was conducted in the Naval Ship Research and Development Center blowdown supersonic wind tunnel. Transonic flow conditions were produced through incorporation of slotted upper and lower tunnel walls. The tunnel is a nominal eighteen inch blow-down-to-vacuum facility with a test section fourteen inches by eighteen inches in cross-section and twenty-nine inches in length with the slotted surfaces installed. Optical quality windows twenty-two inches in diameter in the side walls provided complete viewing of the flow in the test section as the model was rolled through 180 degrees for hologram production. A functional schematic of the wind tunnel is shown in Figure 1.

B. THE MACH NUMBER AND PRESSURE MEASUREMENT PROCEDURE

The Mach number at the test section is determined as a function of total and static pressure measurements and is maintained by carefully controlled butterfly valve settings. Total pressure is determined by recording atmospheric pressure prior to tunnel operation and accounting for the pressure loss between the plenum and the test section during operation. Static pressure is measured directly at a central wall port in the test section. Data recordings

were made on a Beckman Instruments Company 210 Digital Recorder, shown in Figure 2, and were read out on line via a Franklin Strip Tape Printer.

C. THE HOLOGRAPHIC ARRANGEMENT

The holographic arrangement is illustrated in Figure 3 and shown in photographs included as Figures 4, 5, 6, and 7. Two large wooden tables were constructed and linked together with two-by-four beams under the tunnel to form the experimental platform. Thick rubber pads were attached to the table legs to dampen possible floor vibrations. The bulk of the platform provided sufficient stability and vibration damping for the experiment. The monochromatic light source used was a KORAD K-1 pulsed ruby laser operating at a wavelength of 6943 Angstroms, together with a Pockels cell Q-switching device. The resultant effective exposure time was approximately twenty nanoseconds, eliminating the problems due to possible model vibration during hologram exposure. To maintain the laser head and output etalon at a constant temperature of 27.0 degrees Centigrade, a LAUDA constant temperature circulator Model K2R was used.

The reference beam was directed under the wind tunnel by four front surface mirrors, and the beam size was manipulated by means of lenses (Figure 3). The scene beam was routed through the test section to intersect the reference beam on the holographic plate at an angle of approximately fifty degrees. A diffuse glass, mounted on

a precision X-Y translation table in the scene beam, was used to produce light field holograms. Alignment of the Q-switched laser and system optics was accomplished using a continuous wave, low-power helium-neon laser. Reference grids were mounted accurately on the outer surfaces of the tunnel windows using a survey's transit. Details of the model mounting and reference grids are shown in Figure 7. To enable hologram production during daylight hours, the entire tunnel room was blacked out using drop curtains and light shields.

D. THE WIND TUNNEL MODEL

The aerodynamic model used is shown in Figures 8, 9, and 10. The metal portions of the model were stainless steel. The greater part of the modified double wedge platform wing was constructed of optical lucite, as was the portion of the fuselage at the wing root. Detailed model dimensions are shown in Figure 11. The choice of aerodynamic design provided good flow characteristics and a relatively stable lambda-type shock wave on the wing; the largely transparent construction facilitated hologram production through 180 degrees of view.

The model was rotated about its sting mount in the wind tunnel from the zero degree position, wings level, to the 180 degree position, wings level inverted. Alignment for the desired rotation angle was accomplished by manually aligning prescribed lines on the sting mount collar with a scribed mark on the sting support.

III. ANALYTICAL EVALUATION OF THE DENSITY FIELD

A. THE BASIC EQUATION OF INTERFEROMETRY

Interferograms are created when two originally coherent light beams are superimposed and projected on a viewing screen. The two rays will reinforce or annul each other, depending on their relative phase difference at the screen. This phase difference is directly a function of the optical pathlengths traversed by the two waves. Consider a coherent beam which is split and then recombined on a viewing screen. A difference in optical pathlengths of the two component beams may be achieved by causing the beams to traverse through different media prior to recombination, with their physical pathlengths maintained equal. Each component beam will travel at a speed c_0/n where c_0 is the speed of light in a vacuum and n is the index of refraction of the medium traversed. The difference in optical pathlength is then given by

$$L = L\,(n_2 - n_1) = c_0 \Delta t \qquad (1)$$

where Δt is the time difference of travel in the two media. If the optical pathlength is changed by an amount $N\lambda$, where λ is the wavelength of the light source and N is an integer, then the order of interference changes by an amount N. In other words, a shift of N fringes occurs in the interference pattern. The fringe shift may be expressed as

$$g = L/\lambda \qquad (2)$$

where $g =$ fringe shift

$\lambda =$ light source wavelength

$L =$ change in optical pathlength

Substituting equation (1) into equation (2) yields

$$g = \frac{L}{\lambda}(n_2 - n_1) \qquad (3)$$

The index of refraction for a given medium is a function of density. In the case of gases, since the speed of light is very nearly the same as in a vacuum, the index of refraction is well represented by the first two terms of a Taylor series expansion [10]:

$$n = 1 + \beta \frac{\rho}{\rho_s} \qquad (4)$$

where $\beta =$ dimensionless constant related to the Gladstone-Dale constant by $K = \beta/\rho_s$

$\rho_s =$ reference density at 0° C, 760 mm. Hg.

The value of β for air at $\lambda = 5893$ Angstroms (deep red light) is 0.000292; variation with wavelength is very small.

For a fixed difference in the index of refraction between the two component beams the fringe shift relation becomes:

$$g = \beta \frac{L}{\lambda}\left(\frac{\rho_2 - \rho_\infty}{\rho_s}\right) \qquad (5)$$

For variable density in the test section, the net change in optical pathlength is the integrated effect along the beam path, or

$$g = \frac{\beta}{\lambda \rho_s}\int_0^L (\rho - \rho_\infty)\,ds = Q\int_0^L f(x, y, z_c)\,ds \qquad (6)$$

where: $$Q = \frac{\beta \, \rho_\infty}{\lambda \, \rho_s} \tag{7}$$

$$f(x, y, z_c) = \frac{\rho(x, y, z_c)}{\rho_\infty} - 1 \tag{8}$$

z_c = plane of constant z

ds = incremental distance along beam path

Equation (6) is the basic integral equation for the unknown density. With known fringe shift values from an interferogram, the equation is inverted to obtain the density along a beam path.

B. THE INTEGRAL INVERSION

The integral inversion procedure utilized in this investigation was first reported by C. D. Maldonado, et al [5, 6]. It was used subsequently by R. D. Matulka [3] and R. C. Jagota [7] to determine the density variation in an asymmetric free jet and about a cone at angle of attack in supersonic flow, respectively. The procedure involves the representation of the function $f(x, y, z_c)$ of Equation (6) in a complete set of orthogonal functions, with the expansion coefficients evaluated by use of the orthogonality condition between the functions f and g of Equation (6). The set of functions used is orthogonal over the entire (x, y) plane for every z_c and remains an orthogonal set under a rotation of the coordinate system. The coordinate system used for the inversion is shown in Figure 12. It consists of (a) a set of fixed coordinates x, y and (b) a set of moving coordinates

x', y' in which the fringe number function is defined and which rotates with respect to x, y as the viewing angle through the test section is varied.

In operator form, Equation (6) can be represented as

$$g(\xi, y', z_c) = \mathcal{T} f(x, y, z_c) \tag{9}$$

and f is evaluated by inversely transforming the equation to obtain

$$f(x, y, z_c) = \mathcal{T}^{-1} g(\xi, y', z_c) \tag{10}$$

This inversion is achieved by utilizing a pair of orthogonal polynomials $U_{m+2k}^{\pm m}(\alpha x, \alpha y)$ and $H_{m+2k}(\alpha y')$ which are related by the transform relationship

$$\mathcal{T}\left[U_{2k}(\alpha x, \alpha y) e^{-\alpha^2 x'^2}\right] = \frac{e^{\pm i m \xi}}{[k!(m+k)!]^{1/2}} \cdot \frac{1}{2^{m+2k}} \cdot H_{m+2k}(\alpha y') \tag{11}$$

where $H_{m+2k}(\alpha y')$ are Hermite polynomials of order m+2k. The unknown function $f(x, y, z_c)$ is expanded in a set of functions $U_{m+2k}^{\pm m}$ as

$$f(x, y, z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \left\{ C_{m+2k}^{+m}(\alpha) U_{m+2k}^{+m}(\alpha x, \alpha y) + C_{m+2k}^{-m}(\alpha) U_{m+2k}^{-m}(\alpha x, \alpha y) \right\} e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \tag{12}$$

where $\varepsilon_m = 1/2$ for m = 0, $\varepsilon_m = 1$ for m = 1, 2, 3, . . ., and $C_{m+2k}^{\pm m}$ are the unknown coefficients of the expansion. α is an arbitrary scale factor which may be considered the reciprocal of a non-dimensionalizing coefficient.

Utilizing the transform relation of Equation (11), Equation (6) becomes

$$g(\xi, y', z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \left[k!\,(m+k)!\,2^{2(m+2k)}\right]^{1/2}$$
$$\times \left[C_{m+2k}^{+m}(\alpha)\, e^{im\xi} + C_{m+2k}^{-m}(\alpha)\, e^{-im\xi}\right] H_{m+2k}(\alpha)\, e^{-\alpha^2 y'^2} \qquad (13)$$

Equation (13) is subject to the orthogonality condition

$$\int_{-\pi}^{\pi} e^{\pm im\xi} e^{\mp in\xi} d\xi \int_{-\infty}^{+\infty} H_{m+2k}(\alpha y') H_{n+2\ell}(\alpha y')\, e^{-\alpha^2 y'^2} dy'$$
$$= \frac{2\pi^{3/2}}{\alpha} \left[(m+2k)!\,(n+2\ell)!\,2^{m+2k}\, 2^{n+2\ell}\, \delta_{mn}\, \delta_{(m+2k)(n+2\ell)}\right] \qquad (14)$$

where δ is the Kronecker delta function. The solution of Equation (14) applied to Equation (13) yields the series coefficients

$$C_{m+2k}^{\pm m}(\alpha) = \frac{\alpha}{2\pi^{3/2}} \left[\frac{k!\,(m+k)!}{(m+2k)!}\right] \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(\xi, y', z_c)\, H_{m+2k}(\alpha y')\, e^{\mp im\xi} dy'\, d\xi \qquad (15)$$

With the substitution of the coefficients of Equation (15), Equation (7) becomes

$$f(x, y, z_c) = \frac{\alpha}{\pi^{3/2}} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \varepsilon_m \frac{\left[k!\,(m+k)!\,\right]^{1/2}}{(m+2k)!}\, e^{-(\alpha^2 x^2 + \alpha^2 y^2)}$$
$$\times \operatorname{Re}\left[\int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(\xi, y', z_c)\, e^{-im\xi} H_{m+2k}(\alpha y')\, dy'\, d\xi \times U_{m+2k}^{m}(\alpha x, \alpha y)\right] \qquad (16)$$

The functions $U_{m+2k}^{\pm m}$ are defined as

$$U_{m+2k}^{\pm m}(\alpha x, \alpha y) = (-1)^k \alpha \left[\frac{k!\,(\alpha^2 x^2 + \alpha^2 y^2)^m}{\pi (m+k)!}\right]^{1/2} e^{\pm im\phi}\, L_k^m(\alpha^2 x^2 + \alpha^2 y^2) \qquad (17)$$

where $\phi = \tan^{-1}(y/x) - (\pi/2)$ and L_k^m are the associated Laguerre polynomials

$$L_k^m(\alpha^2x^2+\alpha^2y^2) = \sum_{s=0}^{\infty} \frac{(m+k)!}{(k-s)!(m+s)!\, s!}\left[(-1)(\alpha^2x^2+\alpha^2y^2)\right]^s \quad (18)$$

Insertion of Equation (17) into Equation (16) yields

$$f(x,y,z_c) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{\infty}\sum_{k=0}^{\infty} \varepsilon_m \frac{(-1)^k k!}{(m+2k)!}\left(\alpha^2x^2+\alpha^2y^2\right)^{m/2} L_k^m(\alpha^2x^2+\alpha^2y^2)$$
$$\times\left[B_{m+2k}^m(\alpha)\cos(m\phi) + D_{m+2k}^m(\alpha)\sin(m\phi)\right] e^{-(\alpha^2x^2+\alpha^2y^2)} \quad (19)$$

where

$$B_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(\xi, y', z_c)\cos(m\xi)\, H_{m+2k}(\alpha y')\,dy'\,d\xi \quad (20)$$

$$D_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(\xi, y', z_c)\sin(m\xi)\, H_{m+2k}(\alpha y')\,dy'\,d\xi \quad (21)$$

Equations (19), (20), and (21) are the basic equations used to obtain the density distribution from the experimentally determined fringe variations in a completely asymmetric flow field.

C. THE NUMERICAL PROCEDURE

Because the function $g(\xi, y', z_c)$ is an experimentally determined quantity the unknown coefficients $B_{m+2k}^m(\alpha)$ and $D_{m+2k}^m(\alpha)$ in the series representation of $f(x, y, z_c)$ in Equation (19) cannot be calculated analytically. It is therefore necessary to evaluate the double integrals of Equations (20) and (21) numerically. This is accomplished by noting in Figure 12 and Equation (8) that there is an area outside which the density is invariant, namely outside the test

section where the known density is ρ_∞. Since the function $f(x, y, z_c) = 0$ outside this circular domain, the limits of integration of $+\infty$ and $-\infty$ in Equations (20) and (21) can be replaced by finite values. The fringe distribution is then approximated by small increments over the test domain, resulting in the representation of the B and D coefficients as double series:

$$B_{m+2k}^{m}(\alpha)=\sum_{i=1}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, X_i+\Delta X_i)\int_{\xi_j}^{\xi_{j+1}}\cos(m\xi)\,d\xi\int_{X_i}^{X_{i+1}} H_{m+2k}(\alpha x)\,dx \tag{22}$$

and

$$D_{m+2k}^{m}(\alpha)=\sum_{i=1}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, X_i+\Delta X_i)\int_{\xi_j}^{\xi_{j+1}}\sin(m\xi)\,d\xi\int_{X_i}^{X_{i+1}} H_{m+2k}(\alpha x)\,dx \tag{23}$$

Using the derivative formula for Hermite polynomials, Equations (22) and (23) can be manipulated to yield workable series expressions:

$$B_{m+2k}^{m}(\alpha)=\left[\frac{1}{2\alpha m}\cdot\frac{1}{(m+2k+1)}\right]\sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, X_i+\Delta X_i)$$
$$\times\left[\sin(m\xi_{j+1})-\sin(m\xi_j)\right]\left[H_{m+2k+1}(\alpha X_{i+1})-H_{m+2k+1}(\alpha X_i)\right] \tag{24}$$

$$D_{m+2k}^{m}(\alpha)=-\left[\frac{1}{2\alpha m}\cdot\frac{1}{(m+2k+1)}\right]\sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, X_i+\Delta X_i)$$
$$\times\left[\cos(m\xi_{j+1})-\cos(m\xi_j)\right]\left[H_{m+2k+1}(\alpha X_{i+1})-H_{m+2k+1}(\alpha X_i)\right] \tag{25}$$

Since it is impossible to sum over an infinite number of terms, Equation (19) is necessarily expressed as the sum of a finite series:

$$f(x,y,z_c) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{k=0}^{K} \sum_{m=0}^{M} \varepsilon_m (-1)^k \left[\frac{k!}{(m+2k)!}\right] (\alpha^2 x^2 + \alpha^2 y^2)$$

$$\times L_k^m(\alpha^2x^2+\alpha^2y^2)\left[B_{m+2k}^{m}(\alpha)\cos(m\phi) + D_{m+2k}^{m}(\alpha)\sin(m\phi)\right] e^{-(\alpha^2x^2+\alpha^2y^2)} \qquad (26)$$

It has been demonstrated that judicious selection of the parameters $\Delta\xi$, Δx, K, M, and α yields density distributions with very good accuracy [3, 6].

IV. EXPERIMENTAL PROCEDURE

A. LABORATORY TECHNIQUES

In order to visualize the general flow patterns and localize shock or expansion waves about the model a series of standard Schlieren photographs were taken at varying flow Mach numbers. Pictures were produced for roll angles of 0^{o}, 45^{o}, and 90^{o} at Mach numbers from 0.925 to 1.10. A representative series of Schlieren photographs is shown in Figures 13, 14, and 15. Analysis of the Schlieren photographs dictated a flow Mach number of 0.937 for the experimental study; this Mach number yielded uniform upstream flow conditions and located the lambda-type shock wave ideally near the center of the lucite section of the model wing.

The coherence length of the pulsed ruby laser was approximately ten centimeters for the output power utilized. This reduced the normally critical requirement for pathlength equality in the scene and reference beams that must be fulfilled in the classical Mach-Zehnder interferometric approach. Consequently, a length of string proved to be a sufficiently accurate measuring device to maintain the two beam pathlengths within the coherence length of the laser, a requirement for interferogram production. To compensate for the fact that the scene beam traversed approximately five inches of glass tunnel walls and lucite grids which the reference beam did not, the scene beam

was adjusted to be some 2.5 inches shorter than the reference beam. Reference beam pathlength was maintained at approximately 138 inches throughout the experiment.

Holograms produced using the basic holographic setup shown in Figure 3 exhibited clear, well-defined fringe patterns in nearly every instance. In deciding on the final arrangement, several techniques were tested to improve upon fringe pattern definition. Horizontal, vertical and diagonal translations of the diffuser plate in the scene beam were considered, varying from 0.001 inches to 0.005 inches. A vertical translation of 0.003 inches yielded clear horizontal fringes that were quite easily analyzed. The transverse mode selector aperture was varied from 1.0 mm. to 3.0 mm. in increments of 0.5 mm; best lighting of the model resulted with use of a 2.5 mm. aperture. The temperature of the cooling water circulated through the laser head and etalon was varied from 26.0°C. to 28.0°C. in increments of 0.2°C., with 27.0°C. providing the best fringe definition. Finally, a variety of beam splitters and lenses were tested prior to final selection of the best available optics arrangement for the experiment. A 2:1 reference to scene beam strength ratio was found to yield very good holograms.

Two double exposure holograms were taken for each model viewing angle. The first, labeled a double-static exposure, consisted of two exposures in a no-flow condition with a 0.003 inch vertical translation of the diffuser plate between exposures. The fringe

patterns in this hologram provided a measure of the effect of tunnel wall glass, grid plexiglass and model lucite on the subsequent double exposure. The second, or static-dynamic, exposure consisted of a no-flow exposure, a 0.003 inch diffuser translation, and finally an exposure at flow Mach number 0.937. The fringe deviations recorded in the region behind the lucite portion of the model by the double-static hologram were measured and subtracted from the fringe shifts measured in the corresponding static-dynamic hologram.

Holograms were produced on Agfa-Gaevert 8E75 holographic plates, 4 inches by 5 inches in size. As recommended by Collier, et al. [11] the development process included:

1. Five minutes in Kodak D-19 developer
2. Thirty seconds in a flowing water bath
3. Five minutes in standard rapid fixer
4. Thirty seconds in a flowing water bath
5. One and one-half minutes in Kodak Hypo Clearing agent
6. Five minutes in a flowing water bath
7. Five minutes in methanol bath
8. One minute in a flowing water bath
9. Drying

B. PHOTOGRAPHIC TECHNIQUES

Normal reconstructions of the original scene were made by illuminating the holograms with a seven milliwatt continuous wave

helium-neon laser beam at a wavelength of 6328 Angstroms. There was some slight distortion in the reconstructed scene because of the difference in wavelengths of the original scene beam and the reconstruction beam; however, the effect was almost totally negated by shrinkage of the holographic plate emulsion during the development process.

A common technique of image reconstruction was employed, utilizing a conjugate reference beam to reilluminate the exposed hologram, as shown in Figure 16. The resulting scene was recorded on photographic film. Individual points on the photograph are produced by a series of non-parallel rays originating from various source points on the diffuser plate in the scene beam. Using a reilluminating beam of small diameter has the effect of a small aperture at the focal point of the imaging lens, filtering out all but a set of nearly parallel rays, as shown in Figure 17. The real images produced in this manner have a large depth of field, permitting simultaneous projection on the film of front and rear grids, the model and the fringe patterns. The imaging lens was focused as near to the plane of the fringes as possible, producing photographs at various planes of constant z_c.

C. DATA REDUCTION

Photographic interferograms were obtained using the arrangement shown in Figure 16, with the camera viewing screen in the position of the real image. The line of sight in the plane desired was achieved by translating and elevating the hologram until common points on the

front and rear grids were aligned. The Graphic View camera, with the wide angle lens aperture set fully open at f7.8, was adjusted to yield the best focus on the fringe plane. Exposure times of from 1/5 to 3/4 seconds were used to produce workable interferogram photographs on Polaroid Type 55 P/N film.

Fringe shift analysis was accomplished on 8 inch by 10 inch enlargements of the 4 inch by 5 inch film used to record the images. The enlargements were placed face down on a light table, and the fringes, model contours and shock wave were traced on the back surface at the desired cross-sectional plane. Fringe shift values were recorded by measuring the distance between (1) the intersection of the hypothetically undeviated fringe and the cross-sectional plane and (2) the intersection of the deviated fringe with the same plane. Fringe shifts so obtained were corroborated by placing the negative in a photographic enlarger and tracing the lines of interest directly onto graph paper.

The known model fuselage diameter of 1.1 inches was compared with that measured in each individual photograph to yield magnification factors relating projected dimensions to actual dimensions. These factors were then used as corrections to the fringe shift measurements. A grid reference point located on the cross-sectional plane of interest served as the datum for all fringe shift measurements. A base point was located at the intersection of the cross-sectional plane of interest and the body longitudinal axis. Fringe shift measurements were

corrected using this base point as the new datum so that the inversion circle was properly centered on the body axis. The radius of the inversion circle was selected to be nearly equal to the semi-span of the wing. Fringe shift measurements were converted to fringe numbers using the average free stream spacing, while fringe locations were nondimensionalized using the inversion circle diameter. From the data so obtained, the radial variation of fringe number was plotted for each viewing angle. Fringe numbers at 201 equidistant points, as required for input into Mode 3 of the computer program, HOLOFER, were recorded from the resulting curves. Further details concerning this inversion computer program are outlined in Appendix B. A typical reduction of an interferogram to obtain the radial variation of fringe number at a particular cross-sectional plane is detailed in Appendix A.

V. EXPERIMENTAL RESULTS AND DISCUSSION

A pair of double exposure holograms was taken of the model at $11\frac{1}{4}$ degree intervals through a 180 degree field of view. Experimental data from the wind tunnel runs are recorded in Table I. Initial resulting density patterns indicated relatively smooth contours across adjacent intervals; the interval was therefore doubled to $22\frac{1}{2}$ degrees to simplify and speed the analysis. Fringe data were first inserted into the inversion computer program along nine lines of sight in the 180 degree field of view. A numerical comparison of views from 0 degrees to 90 degrees and from 90 degrees to 180 degrees verified to within 0.20 percent the assumption of a single plane of symmetry in the experiment. The fringe data input was then reduced to five lines of sight in a 90 degree field of view, as shown in Figure 18. The resulting output was an inverted density field along nine radial lines spanning a 180 degree field of view, with a mirror image on the opposite side of the plane of symmetry, as shown in Figure 19.

The static-dynamic photographic interferogram for the 0 degree view, along with its corresponding double-static interferogram, is shown in Figure 20. The diffraction effects caused by the presence of the lucite portions of the model are clearly visible in the double-static exposure, where the free stream fringe lines are bent and displaced toward the model axis. This displacement was measured

and subtracted from subsequent measurements made on the static-dynamic exposure, as outlined in Section IV.A. Photographic interferograms of the remainder of the static-dynamic exposures are shown in Figures 21 through 24. Clearly visible and reduceable in nearly all views were (1) the region of uniform subsonic flow, commonly called the free stream condition, (2) the transition from local subsonic to local supersonic flow, and (3) the lambda-type shock wave on the model wing. These characteristics are shown in schematic representation in Figure 25.

Contour plots of the density function, as expressed in Equation (8) of Section III.A., for successive z-planes of analysis are shown in Figures 26 and 27. The cross-sectional plane of analysis for the plot of Figure 26 was located at 186.75 mm. from the model nose along the longitudinal axis. For Figure 27, the plane of interest was 195.25 mm. from the model nose. It is apparent from both contour plots that the model went to a very small angle of attack under the loading forces produced during tunnel operation; this is evidenced by the compression of the contour lines above the model and the corresponding expansion of the contours below the model. Measurements made from photographic interferograms confirm this angle of attack to be, at most, 0.05 degrees. The closed contours above and adjacent to the wing surface in both figures may very well be the result of a vortex originating at the intersection of the wing leading edge and the

fuselage on either side of the model and traveling aft and outward over the wing surface.

A comprehensive quantitative analysis of the shock wave structure was not undertaken, with the exception of estimating the strength of the shock by comparison of fringe line separation immediately ahead of and aft of the shock wave. Fringe line separation measurements on either side of the shock wave were converted first to density information and thence to pressure information, disregarding compressibility effects. An approximate strength value of 0.207 was computed using the accepted definition of $(p_2 - p_1)/p_1$. This corresponds to a local Mach number of 1.08 in the supersonic region just ahead of the shock wave. Qualitative shock wave analysis resulted in the construction of a three-dimensional structural representation as shown in Figure 28, using input information from several interferogram viewing angles. While location of the leading and trailing edges of the lambda-type shock wave was very accurate, interior structure was largely indefinable due to "smearing" and blurring of the fringes transiting the shock wave itself.

As a preliminary step to possible future studies in this field, photographic interferograms were made from holograms produced with the aerodynamic model set at small angles of attack. Orientations included angles of attack of five and ten degrees, with roll angles varying from zero to ninety degrees. Although the holograms themselves were of very good quality, the photographic reproductions

were relatively poor due to the fact that an inferior photographic arrangement had to be used. They were therefore omitted from this report. It was inferred from the holograms, however, that a complete study at angle of attack using the basic procedures followed in the present study would be both totally feasible and rewardingly fruitful.

The original character of the experimental data prevented comparison with published results. Qualitative studies of transonic phenomena are widely available, and the general characteristics of the resulting density field and shock wave structure serve to bear out the schematics based on theoretical and mathematical models. Moreover, the self-testing mode of the inversion computer program, HOLOFER, verified the consistency and reproductibility of the resulting density distributions to within 2.0 percent through proper choice of the input parameters, primarily the slope-matching parameter α. The errors encountered in the final results are due primarily to errors in the fringe data input to the inversion program. The intrinsic presence of laser speckle, the extended pathlengths of the scene and reference beams and the unavoidable beam scattering and diffraction within the lucite model sections, created difficulty in obtaining precisely the slope of the fringe lines behind the lucite sections. Fringe spacing measurements in the free stream flow were conservatively judged accurate to within 0.5 mm. This assumption was quite reasonable since all measurements were effected with a scale graduated at half millimeter intervals. This figure of 0.5 mm.,

combined with the mean free stream fringe spacing of 5.197 mm. for all interferograms, indicated measurement accuracy to within one tenth (0.1) of a fringe. The mean systematic error of the free stream spacing in each view was computed to be a maximum of 3.9 percent. Associated with this systematic error was a random error of 2.1 percent in the measurement of fringe shifts in each view to a conservative accuracy of 0.5 mm. The resulting error for each viewing angle was therefore a maximum of 6.0 percent, found by merely adding the two types of error for each view. The minimum error limit was found by considering the error resulting from the reproduction of the same interferogram view five separate times. Statistically, with 6.0 percent error in each view, the composite error for the repeated view is 2.6 percent. As five different views, or lines of sight, were used for the data between zero and ninety degrees, the final total error in the analysis was therefore in the interval between 2.6 percent and 6.0 percent. To insure contour clarity and guard against overlapping, the maximum error figure of 6.0 percent was used in construction of the plots shown in Figures 26 and 27. In general, the rather large fringe shifts led to a very low mean fringe sensitivity value of 0.1259. This coefficient indicated a resulting density function (Equation (8)) inaccuracy of less than 1.5 percent for a fringe shift measurement misreading of 0.5 mm.

Physical limitations of beam diameter and hologram plate area dictated the choice of an inversion circle diameter somewhat smaller

than the full data circle normally used in the finite fringe procedure. This, in effect, introduced an inconsistency in reference density into the analysis since the density on the selected circle and immediately outside it was not the calculated ρ_{∞} ; the density function was therefore not zero outside the actual inversion region. To alleviate this inconsistency, a new, updated reference density was computed for each cross-sectional plane of analysis by averaging the density values on the selected circle from the first inversion process. The actual reference densities used were ρ = 1.777 mg/cc for the 186.75 mm. plane and ρ = 1.642 mg/cc for the 195.25 mm. plane. This procedure was justified since all density values between the selected circle and the full data circle were constant to within approximately fifteen percent. The updated reference densities were then used to produce the final output density field. The net effect was a scaled, uniform shift toward density function values slightly lower than those computed on the basis of the original reference density.

VI. CONCLUSIONS

The finite fringe procedure for the production of holographic interferograms has been applied successfully to the determination of the three-dimensional density distribution of the flow near the wing-fuselage junction of a partially transparent aerodynamic model in the transonic regime. Density contours accurate to within six percent enabled a thorough analysis of the flow field to be conducted, highlighting flow characteristics and the presence of the shock wave. Subsequent studies of similar models at angle of attack have been shown to be entirely feasible. Procedures used in the experiment also exhibit promise for the direct analysis of duct and inlet flows as well as comprehensive study of shock wave structure.

The inversion computer program, HOLOFER, was found to be adequate in handling a general asymmetric flow field analysis. However, it was considered quite cumbersome and difficult to modify for various experimental situations. Subsequent analysts will find the procedures advocated by Sweeney and Vest [12] for the recording and analyzing of interferograms of considerable interest. In addition, the efforts of Van Houton [13], who utilized the method proposed by Junginger and van Haeringen [14], may prove valuable in reducing computer time significantly.

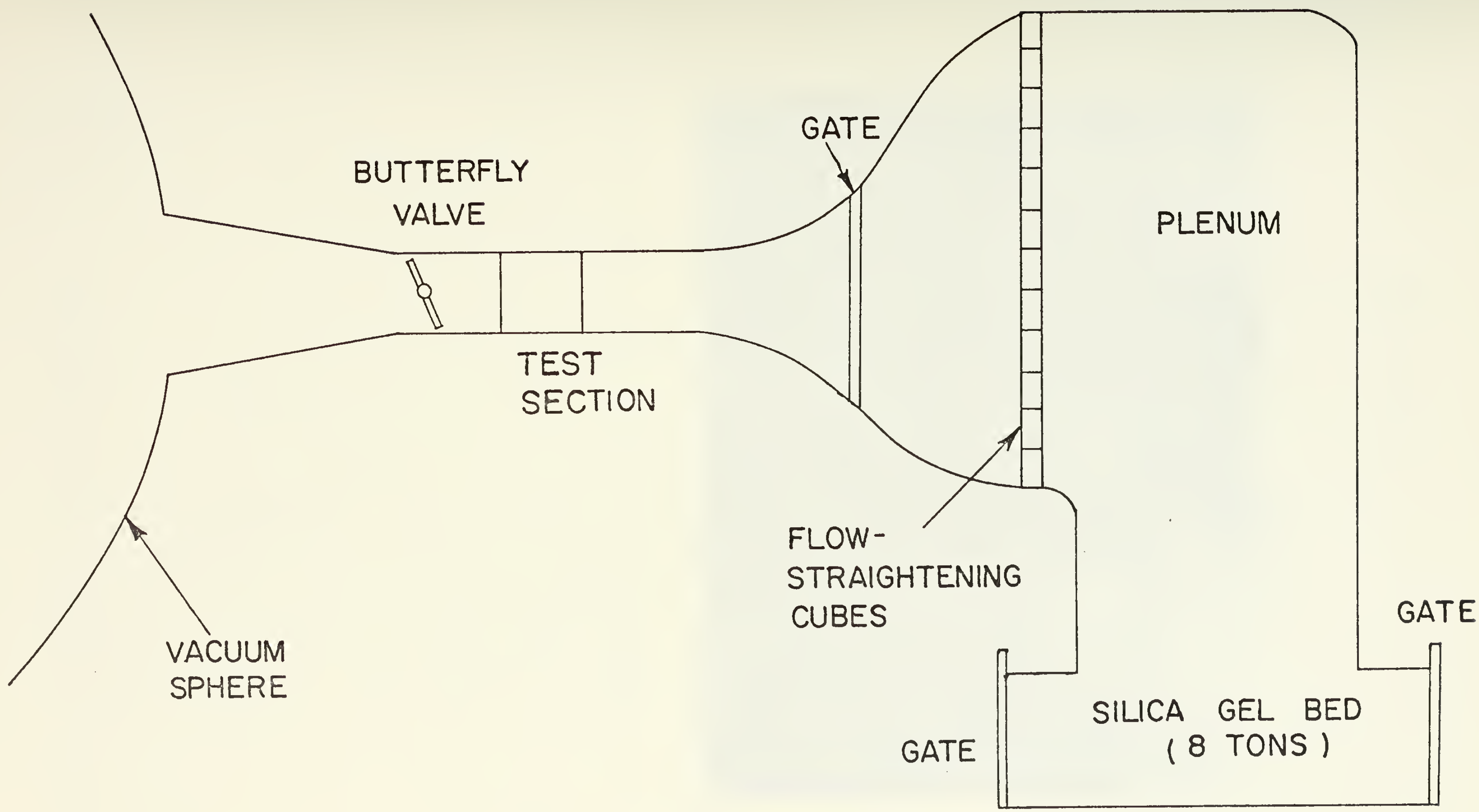


FIGURE I. FUNCTIONAL SCHEMATIC OF NSRDC WIND TUNNEL

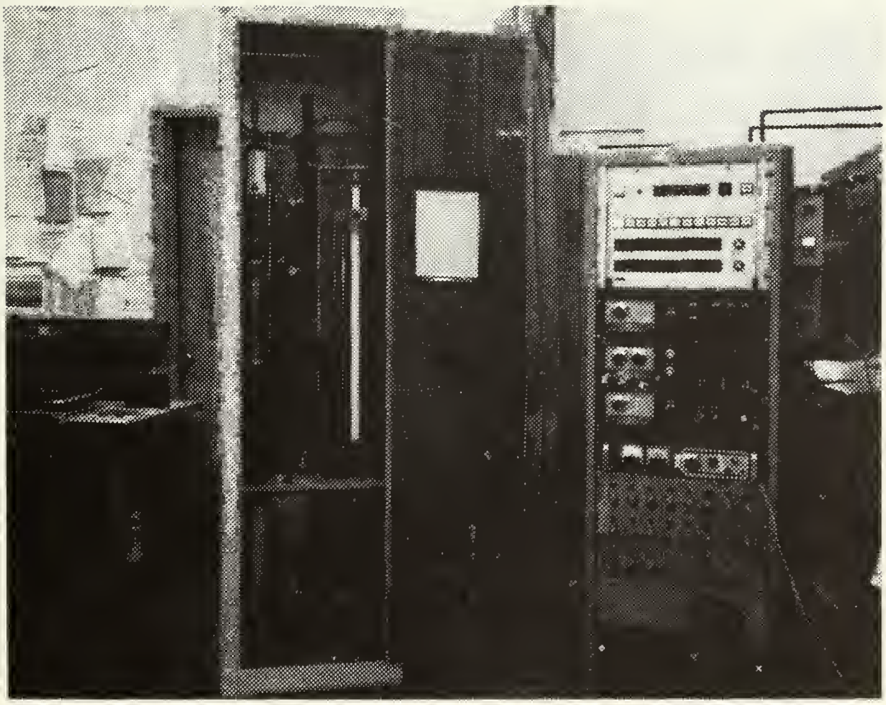


FIGURE 2. BECKMAN 210 DATA RECORDING SYSTEM

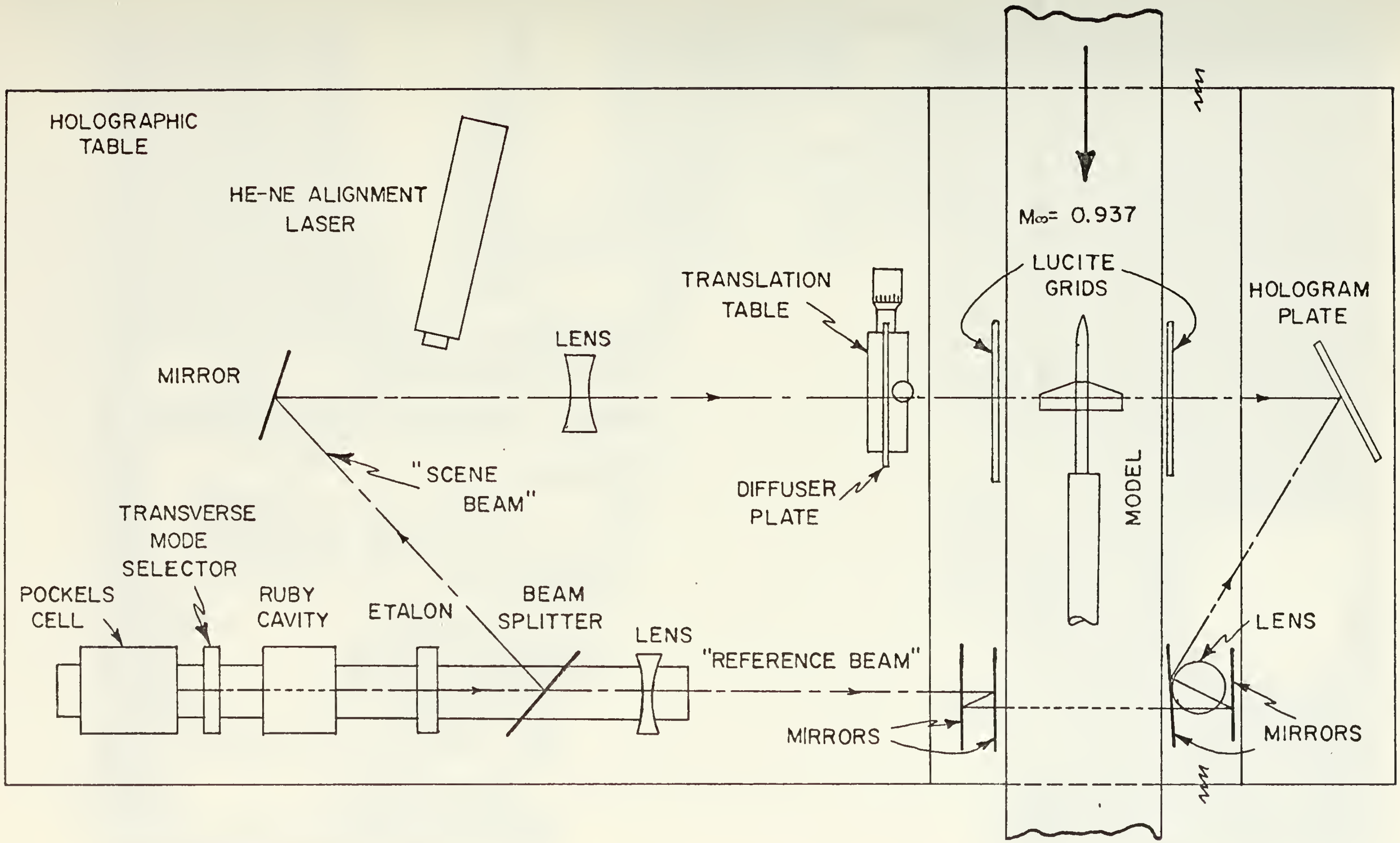


FIGURE 3. SCHEMATIC DRAWING OF THE HOLOGRAPHIC ARRANGEMENT

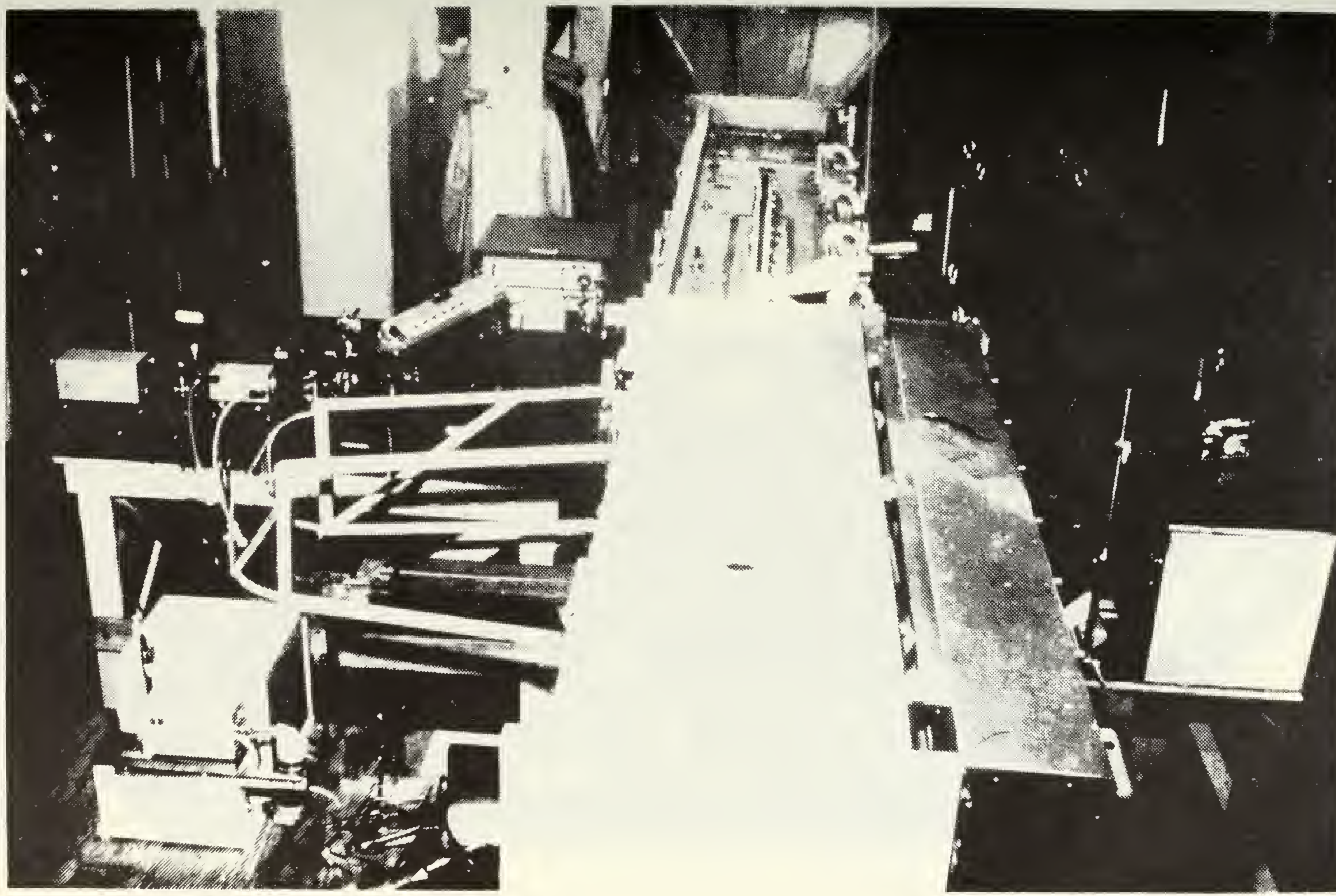


FIGURE 4. OVERHEAD VIEW OF TUNNEL AND ENTIRE HOLOGRAPHIC SYSTEM

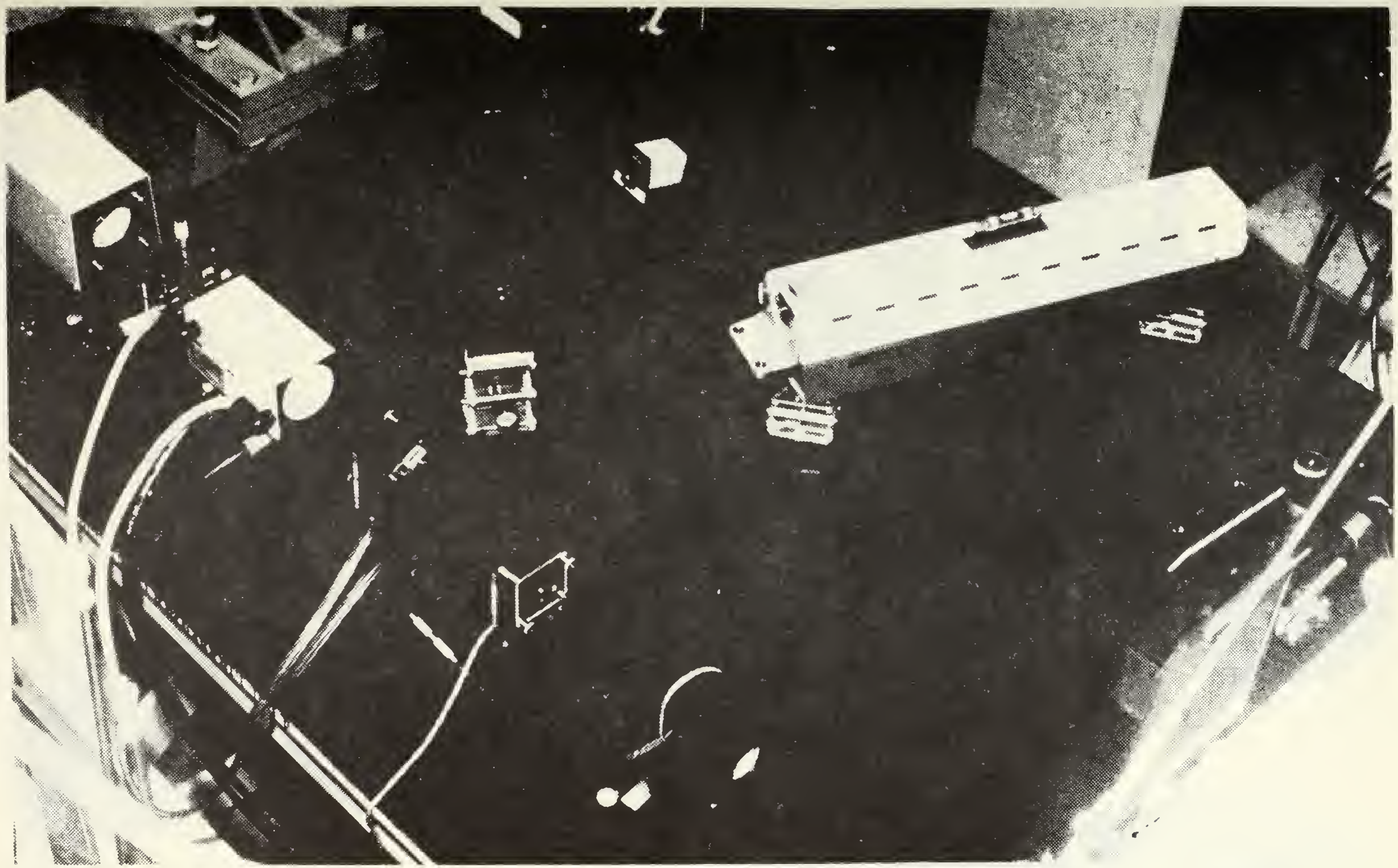


FIGURE 5. OBLIQUE VIEW OF HOLOGRAPHIC TABLE

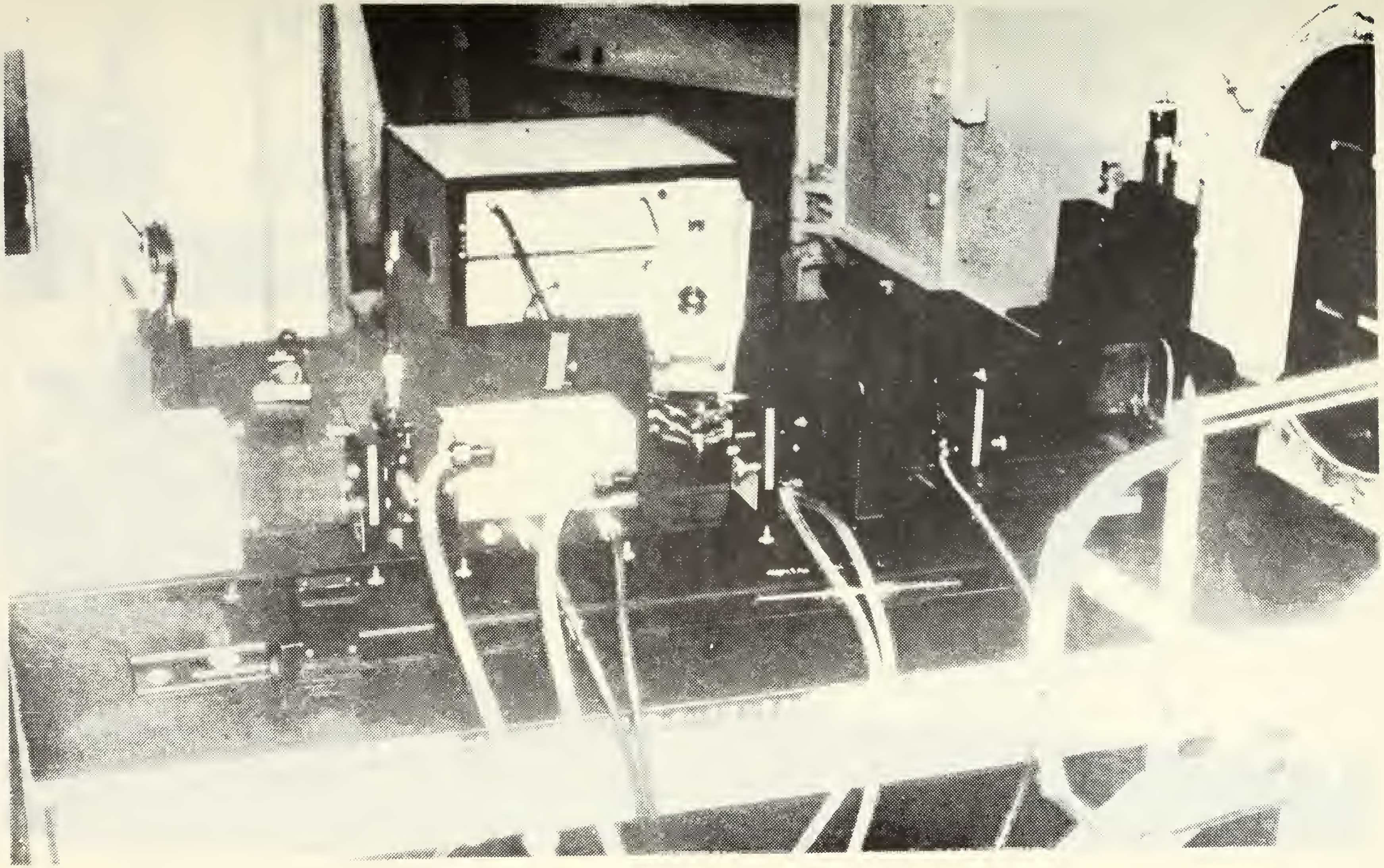


FIGURE 6. OBLIQUE VIEW OF HOLOGRAPHIC TABLE

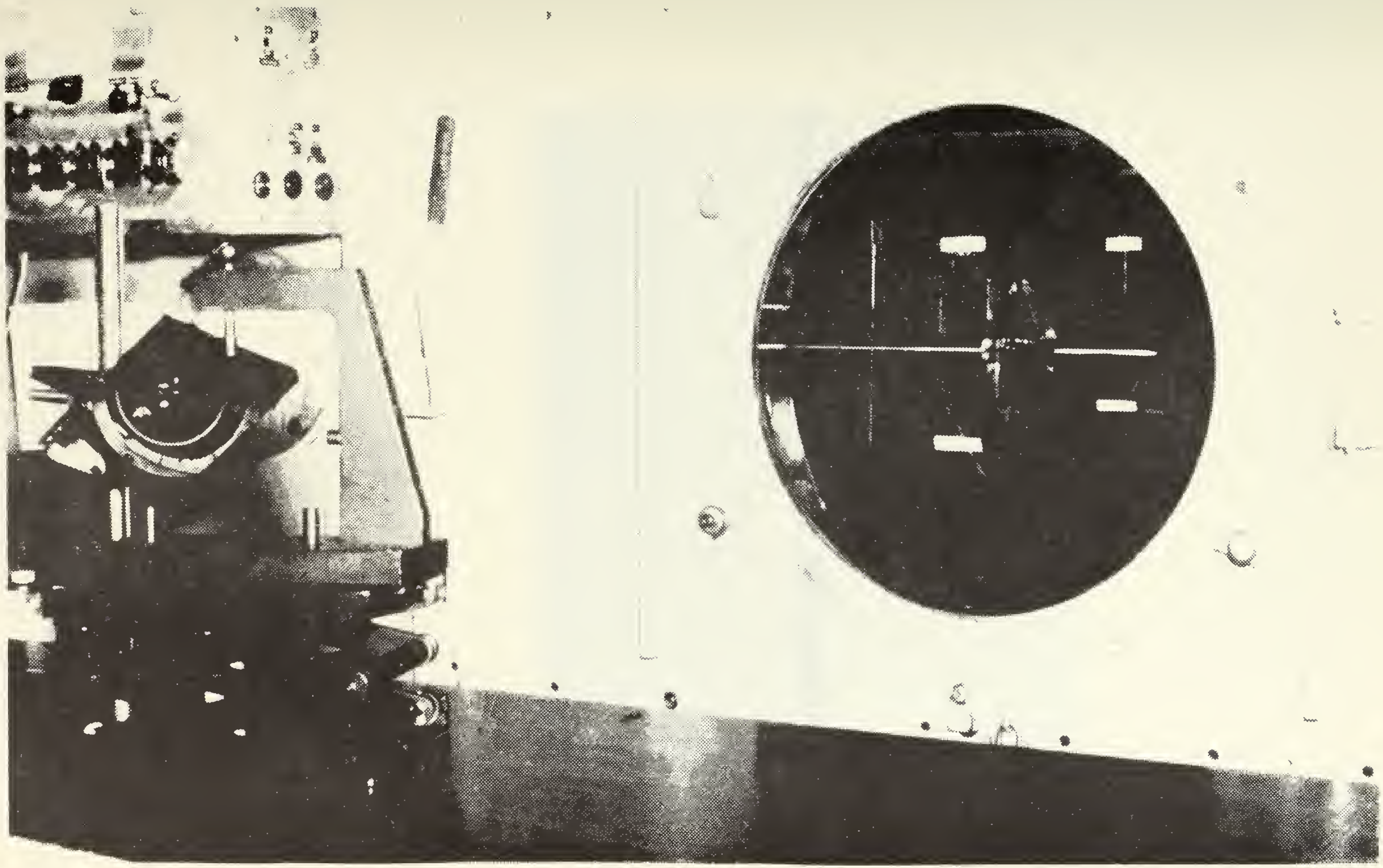


FIGURE 7. DETAIL OF MODEL MOUNTING AND REFERENCE GRIDS

FIGURE 8. AERODYNAMIC TEST MODEL: 0 DEG. ROLL ANGLE

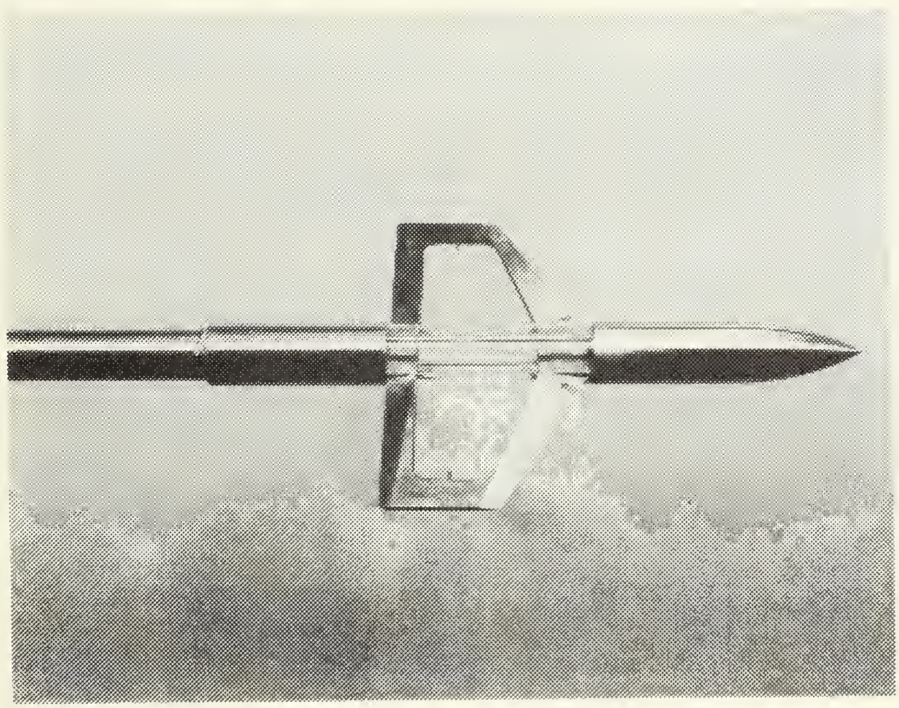


FIGURE 9. AERODYNAMIC TEST MODEL; 45 DEG. ROLL ANGLE

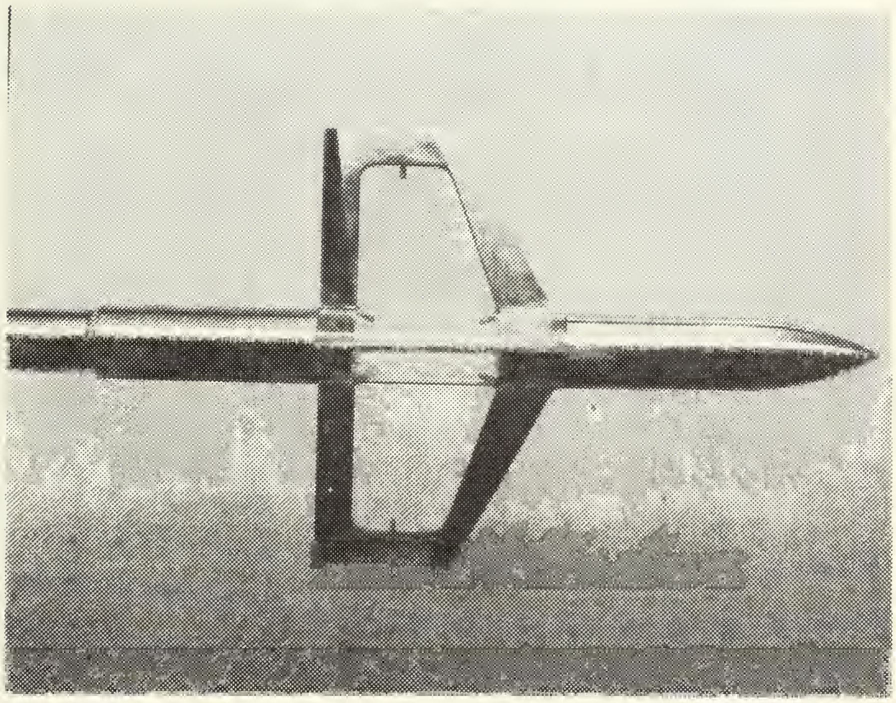


FIGURE 10. AERODYNAMIC TEST MODEL; 90 DEG. ROLL ANGLE

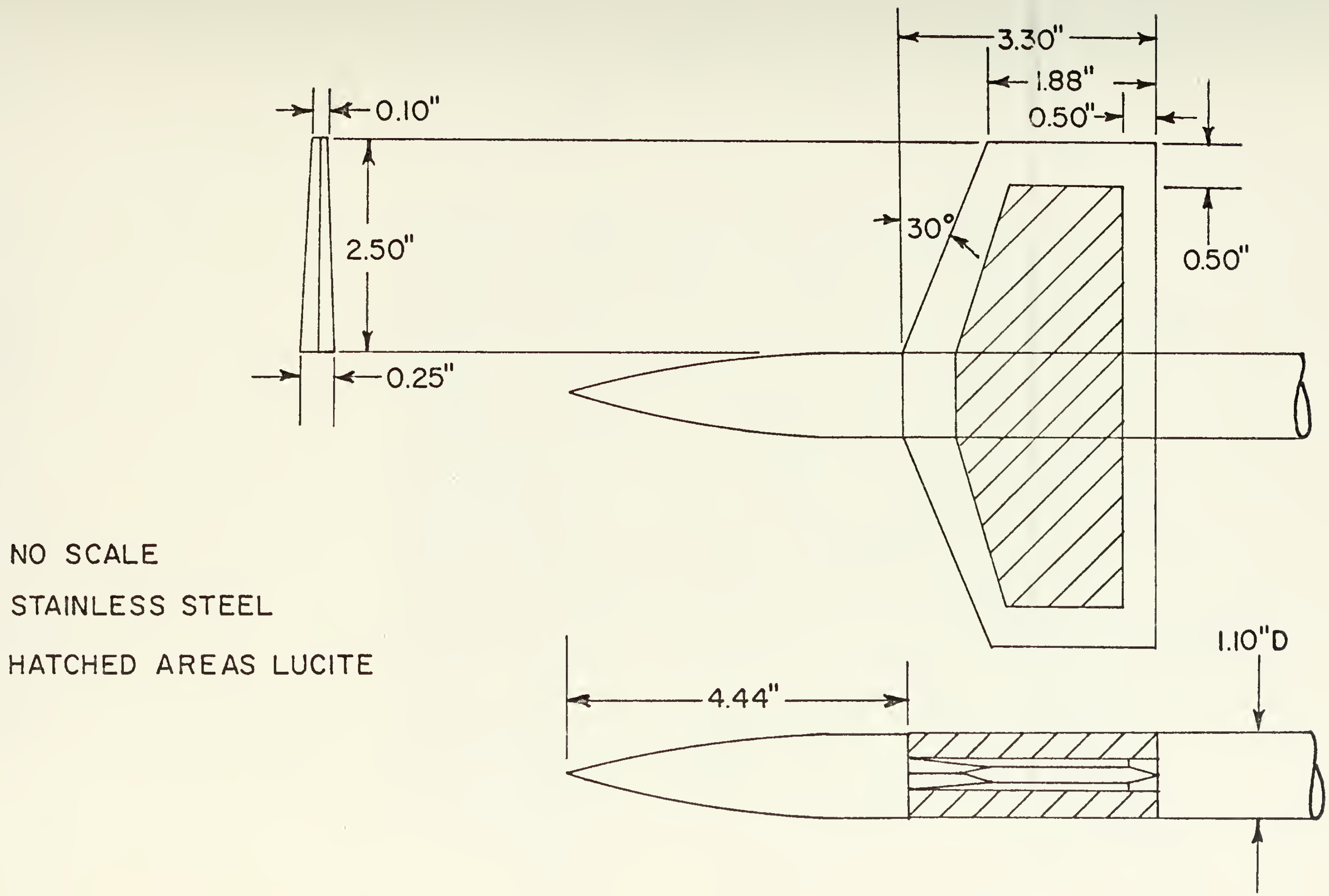


FIGURE 11. DETAILS OF THE AERODYNAMIC TEST MODEL

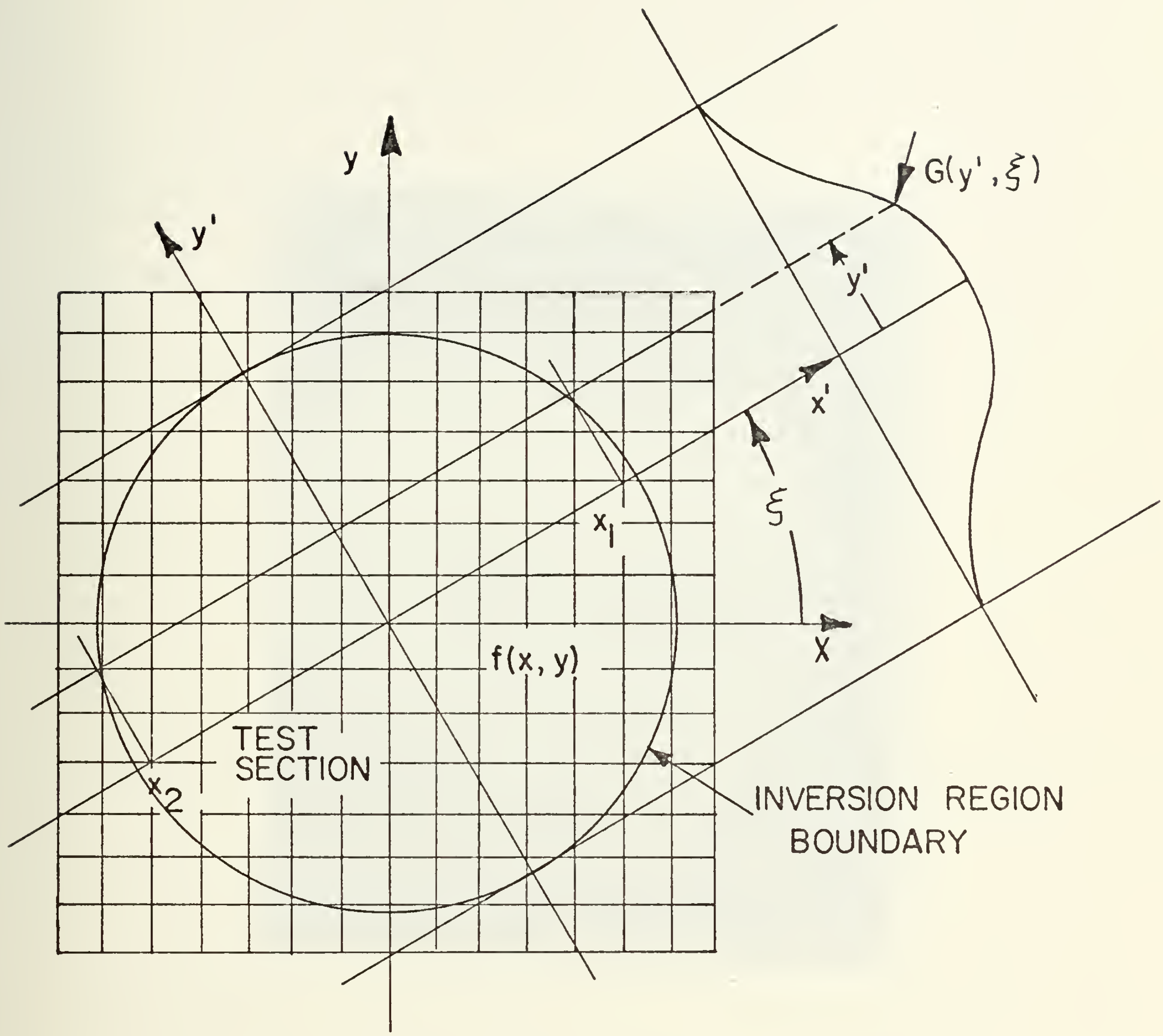


FIGURE 12. CO-ORDINATE SYSTEM USED FOR INVERSION OF FRINGE NUMBER TO DENSITY

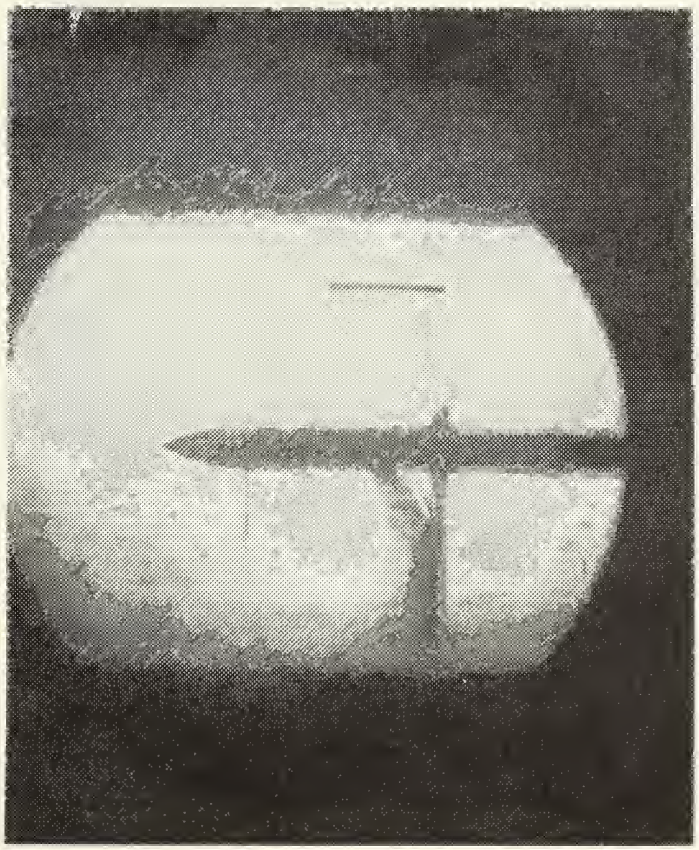


FIGURE 13. SCHLIEREN PHOTOGRAPH; 0 DEG. ROLL ANGLE, 0.967 MACH NUMBER

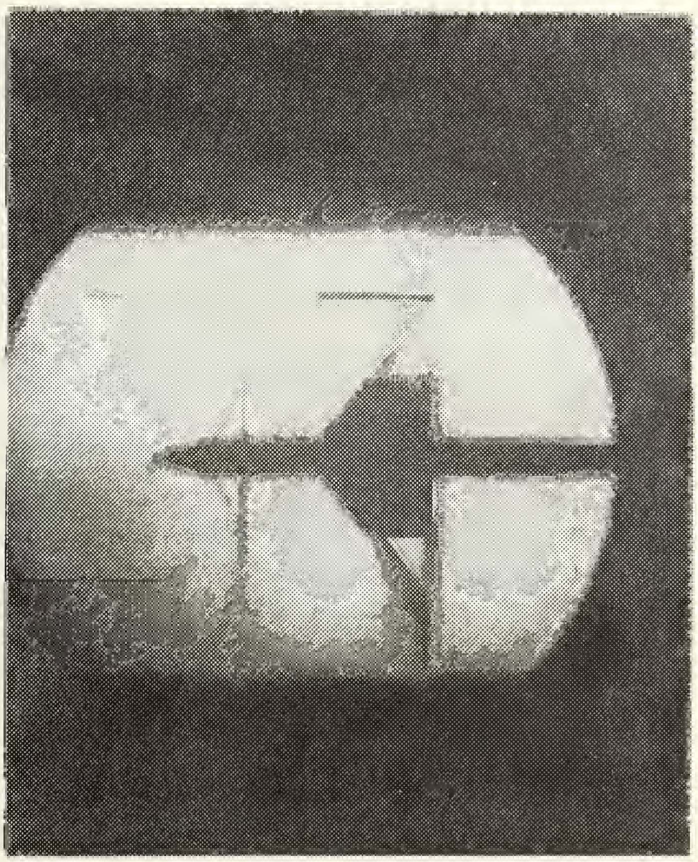


FIGURE 14. SCHLIEREN PHOTOGRAPH; 45 DEG. ROLL ANGLE. 0.967 MACH NUMBER

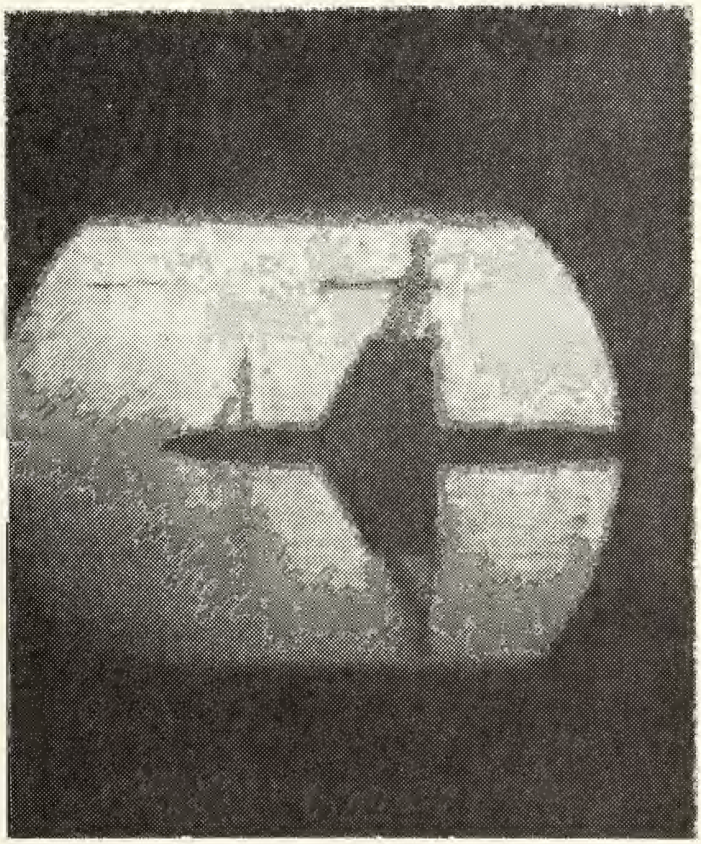


FIGURE 15. SCHLIEREN PHOTOGRAPH; 90 DEG. ROLL ANGLE, 0.967 MACH NUMBER

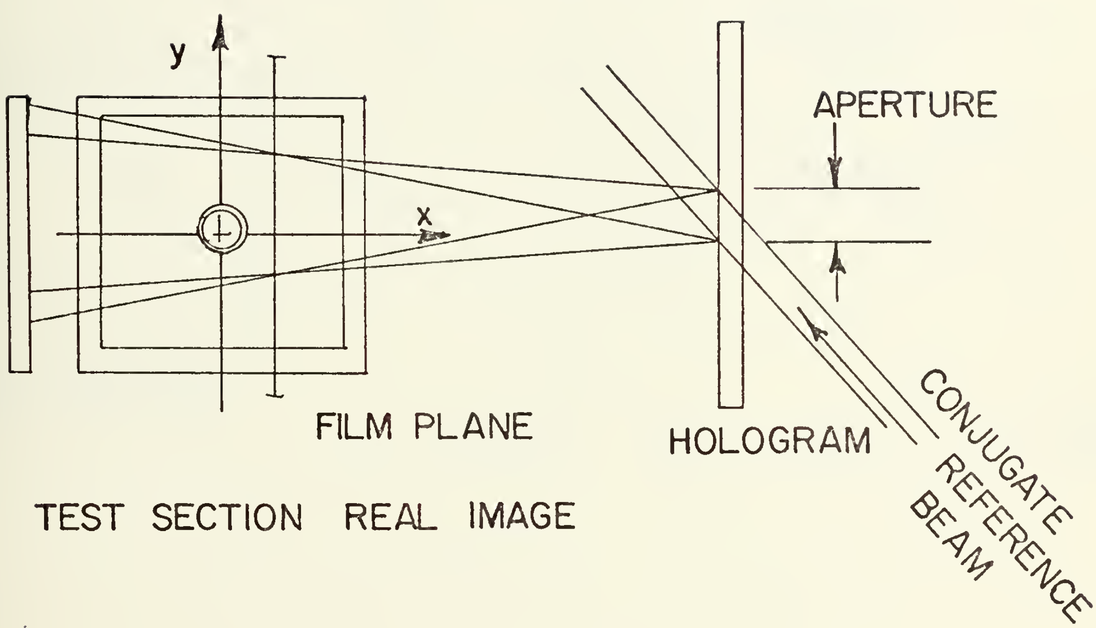


FIGURE 16. LENSLESS PHOTOGRAPHIC TECHNIQUE USING A CONJUGATE REFERENCE BEAM OF SMALL DIAMETER

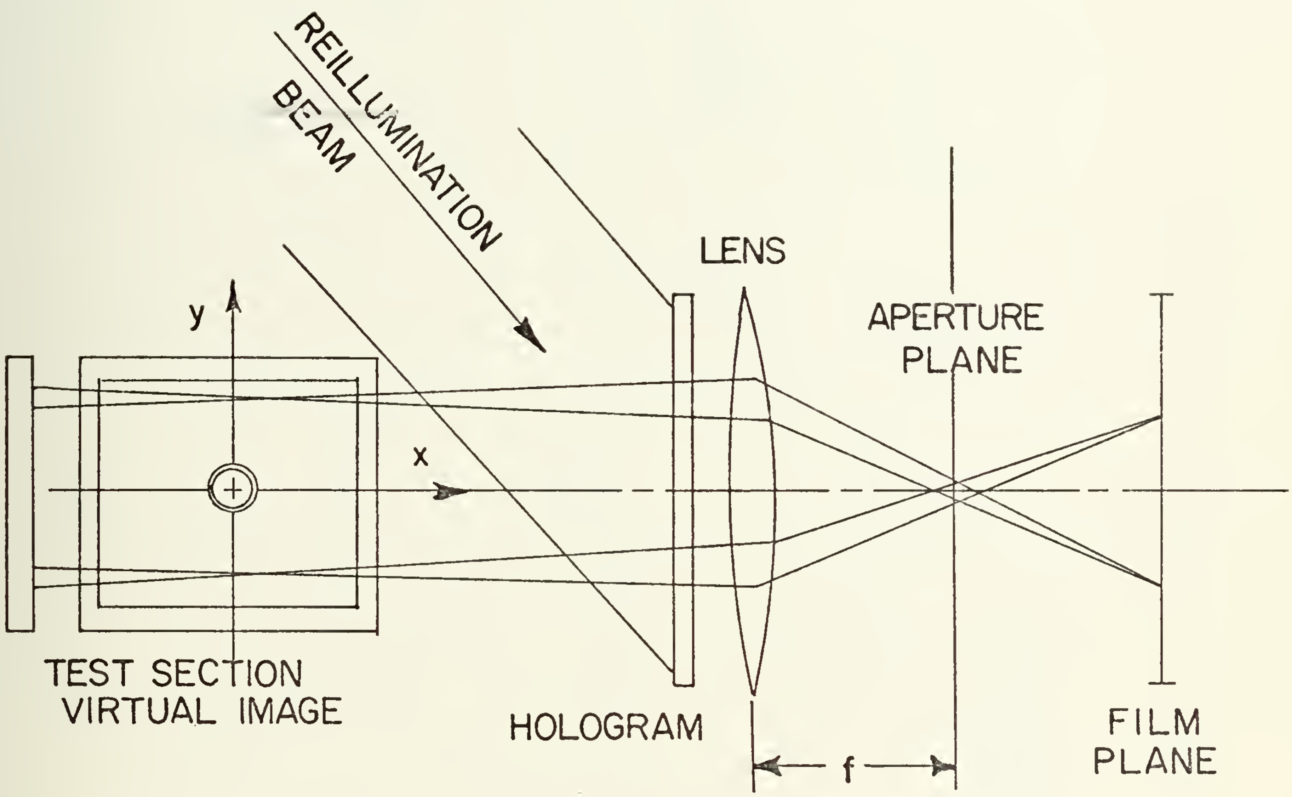


FIGURE 17. SPATIAL FILTERING TECHNIQUE FOR SELECTING PHOTOGRAPH OF CONSTANT ANGLE LINES OF LIGHT

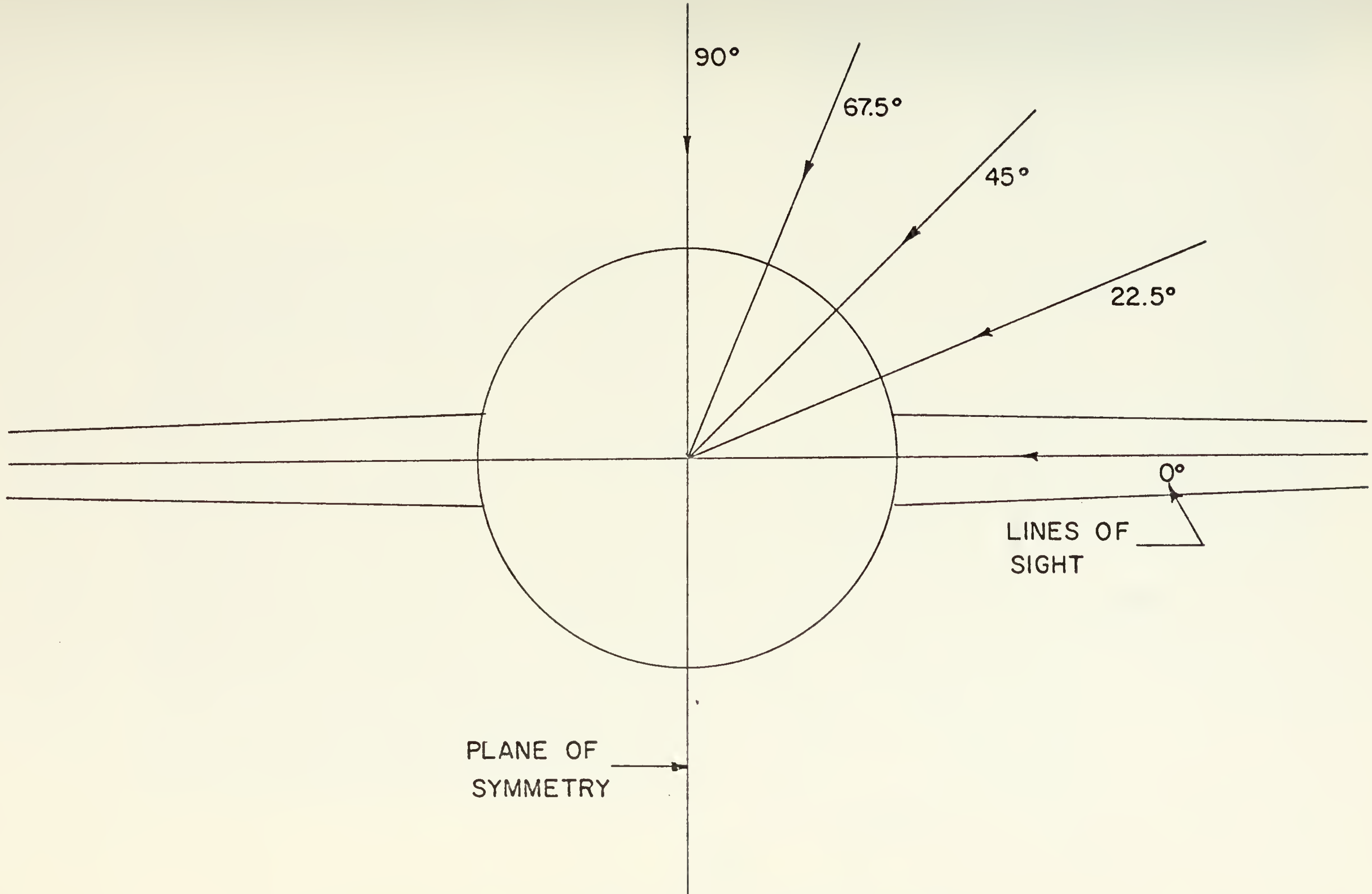


FIGURE 18. FRINGE DATA INPUT INFORMATION

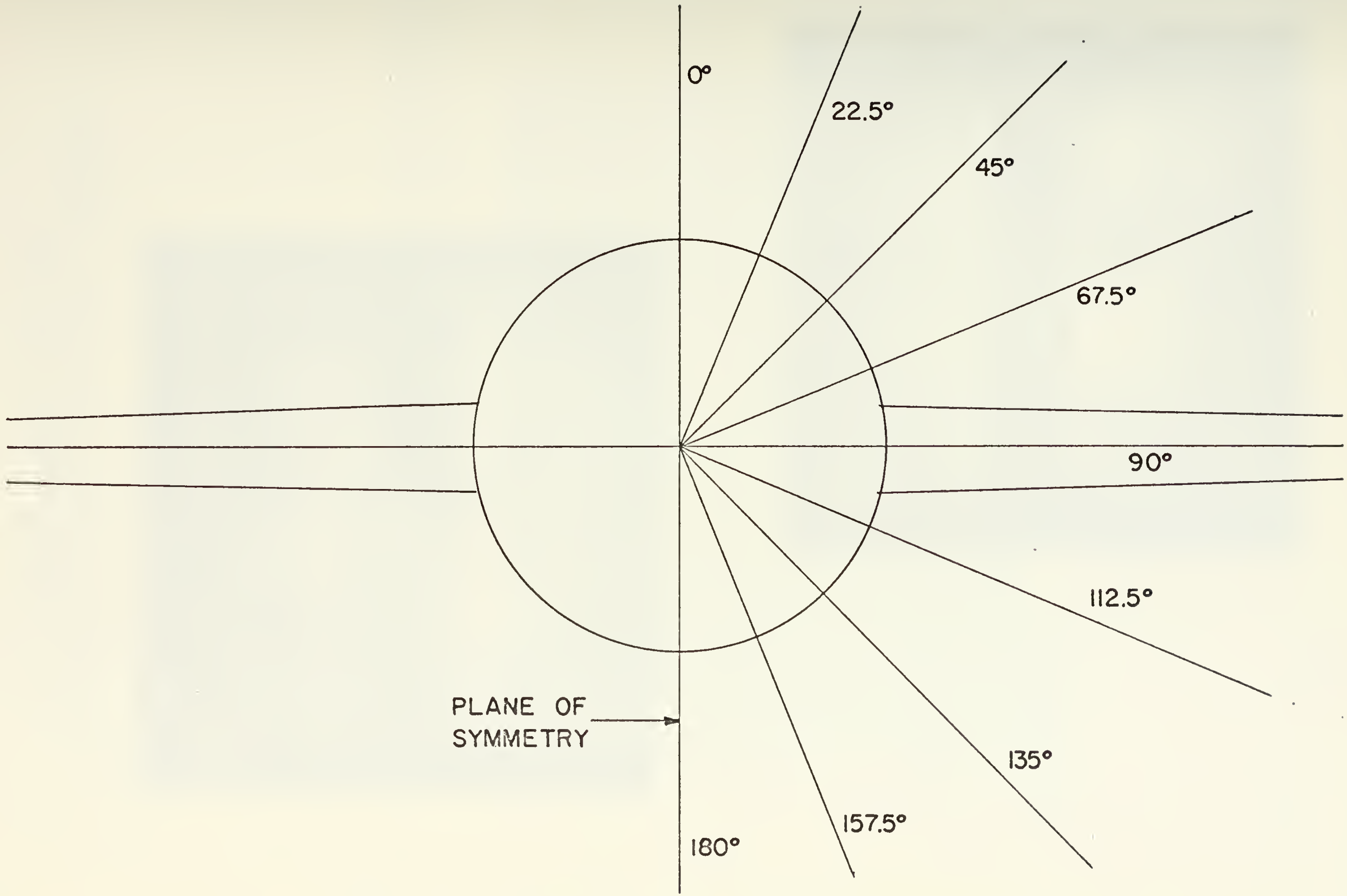
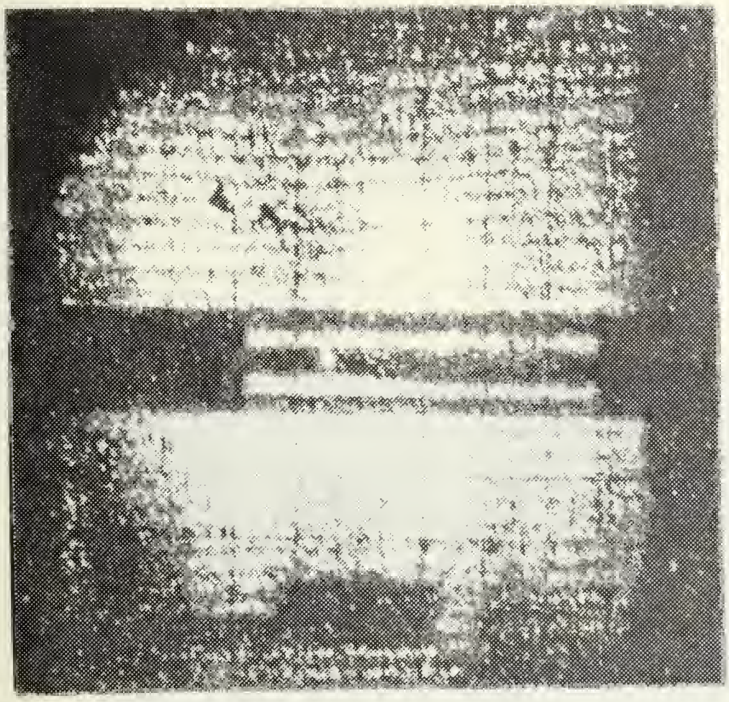


FIGURE 19. DENSITY DATA OUTPUT INFORMATION



DOUBLE-STATIC
INTERFEROGRAM

STATIC-DYNAMIC
INTERFEROGRAM

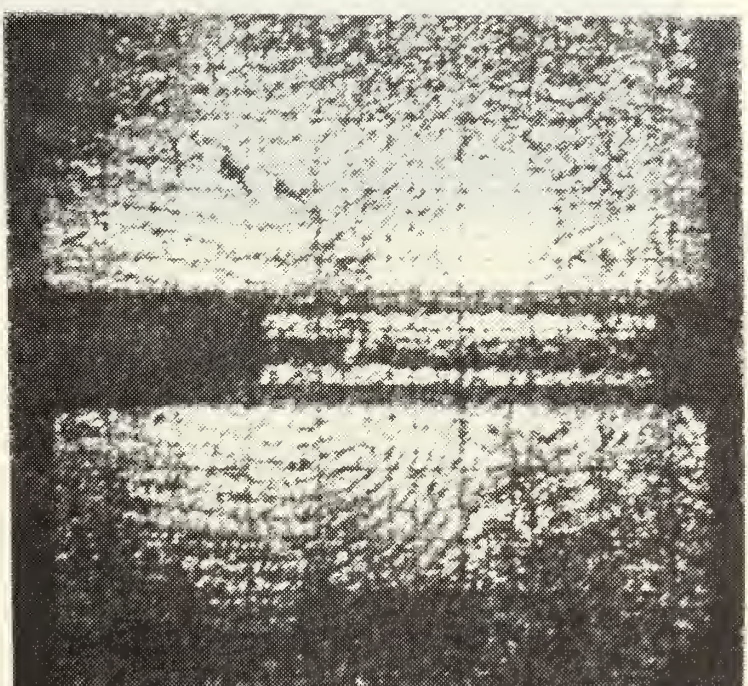


FIGURE 20. PHOTOGRAPHIC INTERFEROGRAMS
FOR 0 DEG. VIEWING ANGLE

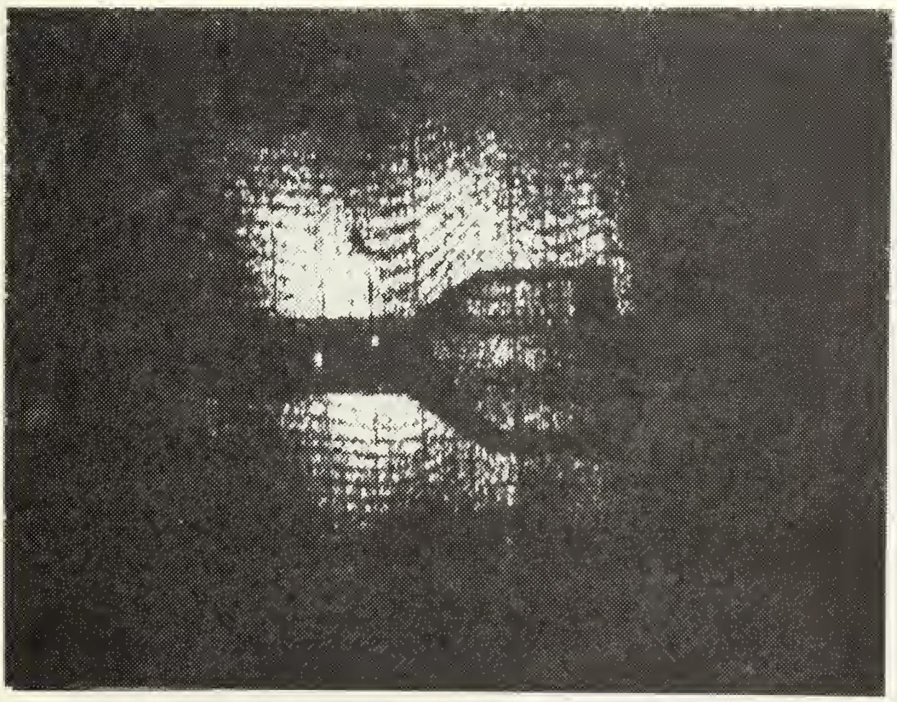


FIGURE 21. STATIC-DYNAMIC INTERFEROGRAM FOR 22½ DEG. VIEWING ANGLE

FIGURE 22. STATIC-DYNAMIC INTERFEROGRAM FOR 45 DEG. VIEWING ANGLE

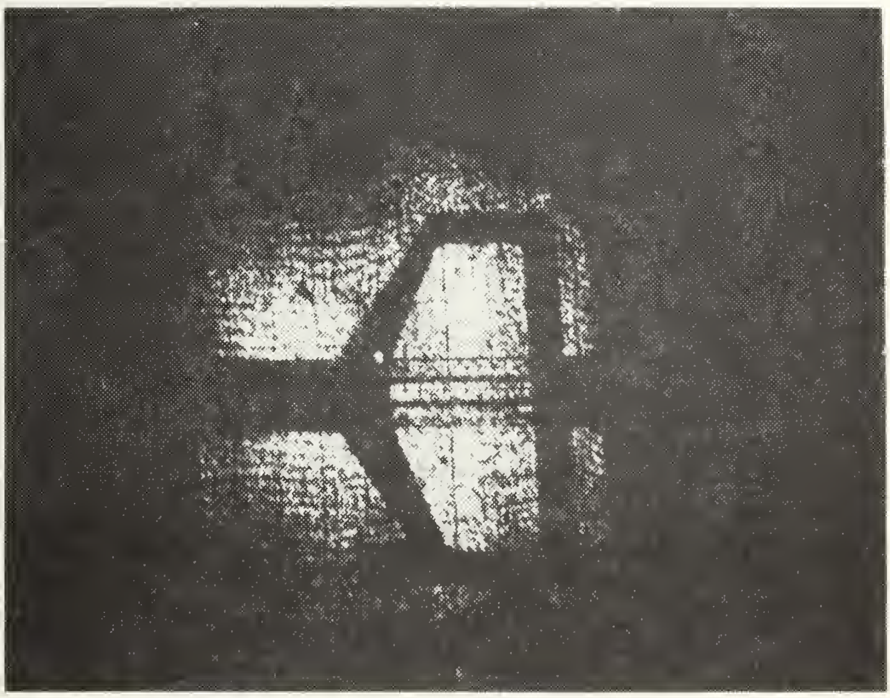


FIGURE 23. STATIC-DYNAMIC INTERFEROGRAM FOR 67½ DEG. VIEWING ANGLE

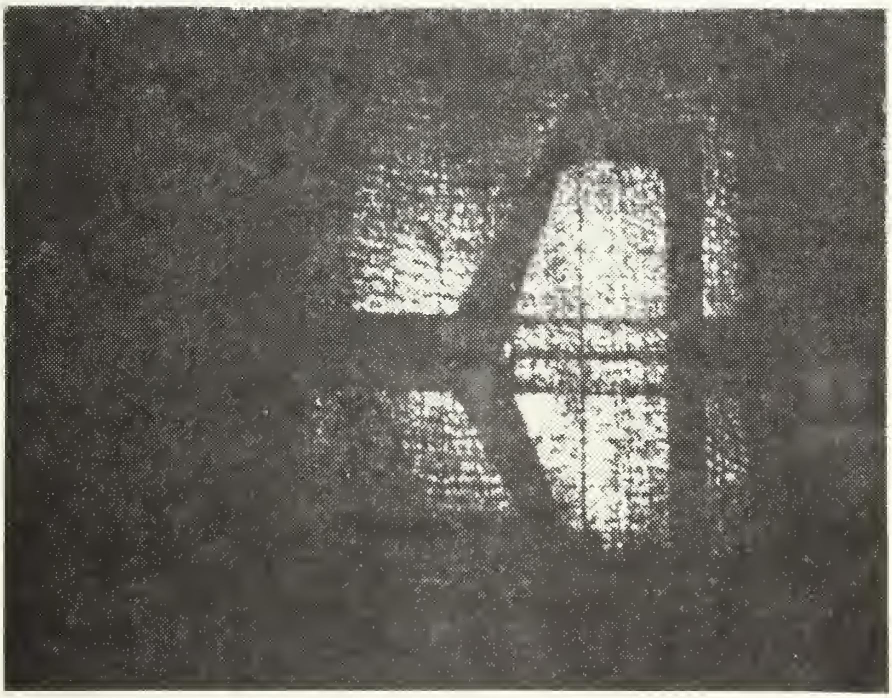


FIGURE 24. STATIC-DYNAMIC INTERFEROGRAM
FOR 90 DEG. VIEWING ANGLE

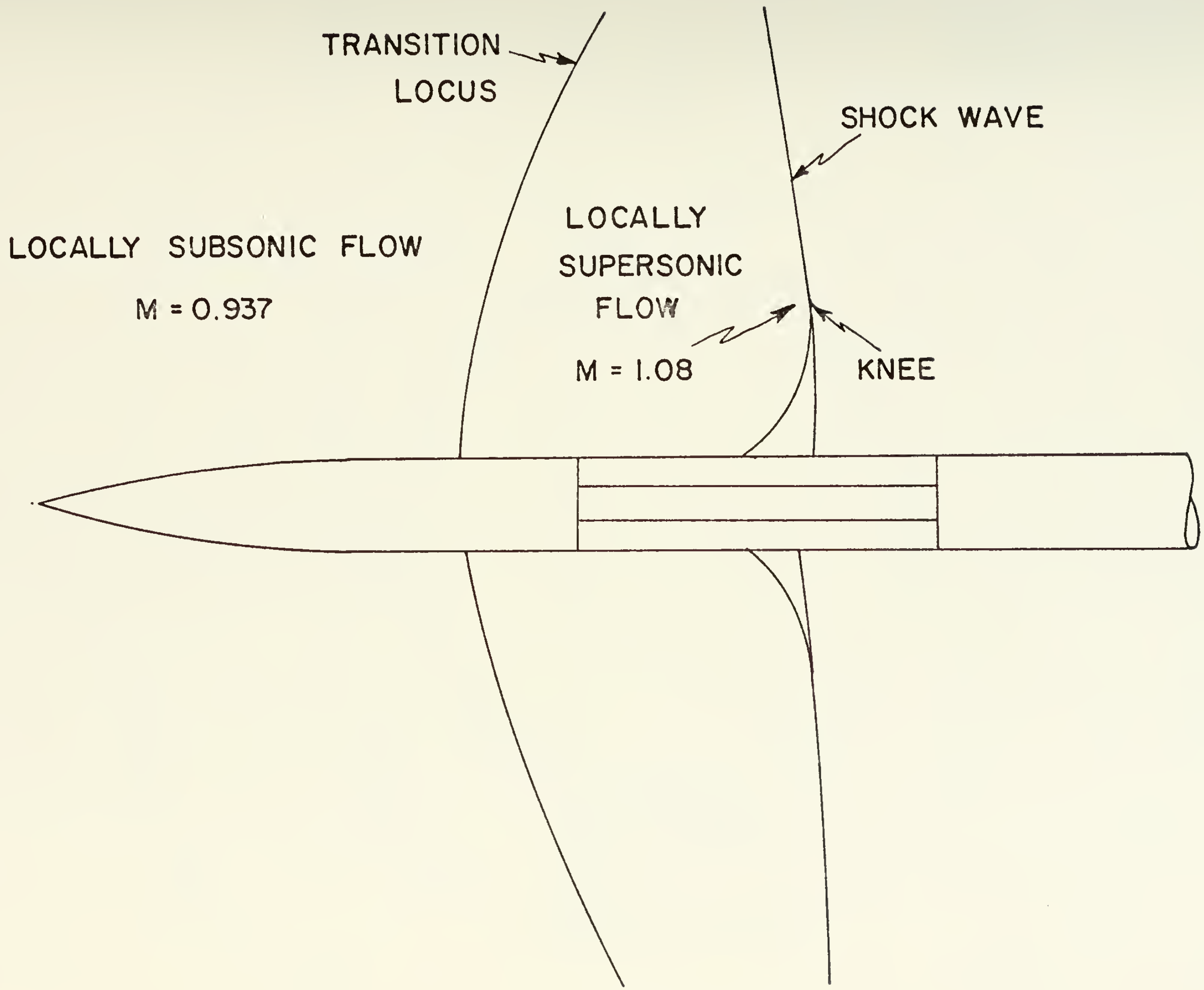


FIGURE 25.
CHARACTERISTIC TRANSONIC FLOW REGIONS; FROM SEVERAL INTERFEROGRAMS

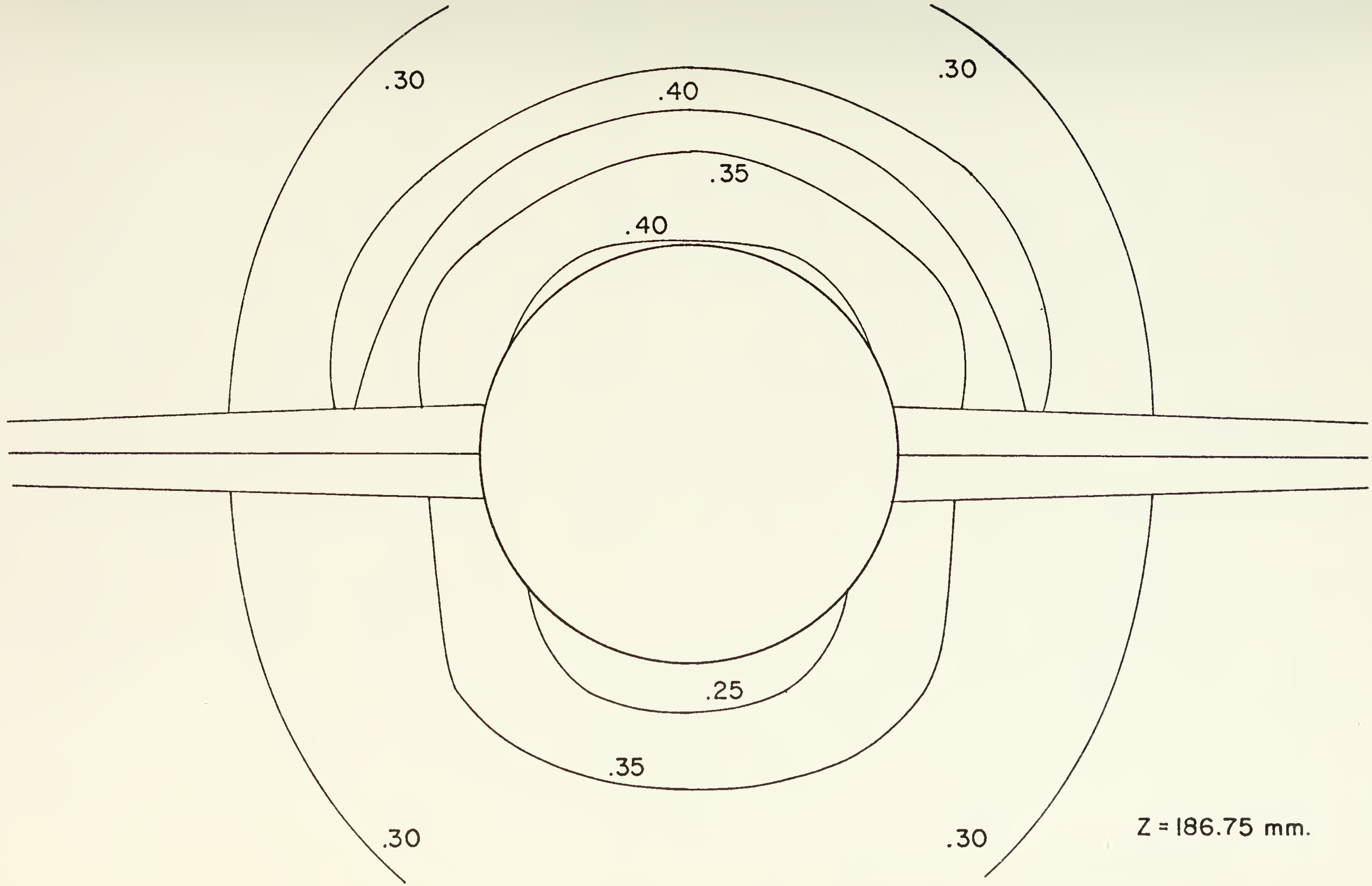


FIGURE 26. CONTOUR PLOT OF DENSITY FUNCTION, (ρ / ρ_∞) - 1, FOR GIVEN CROSS-SECTIONAL PLANE

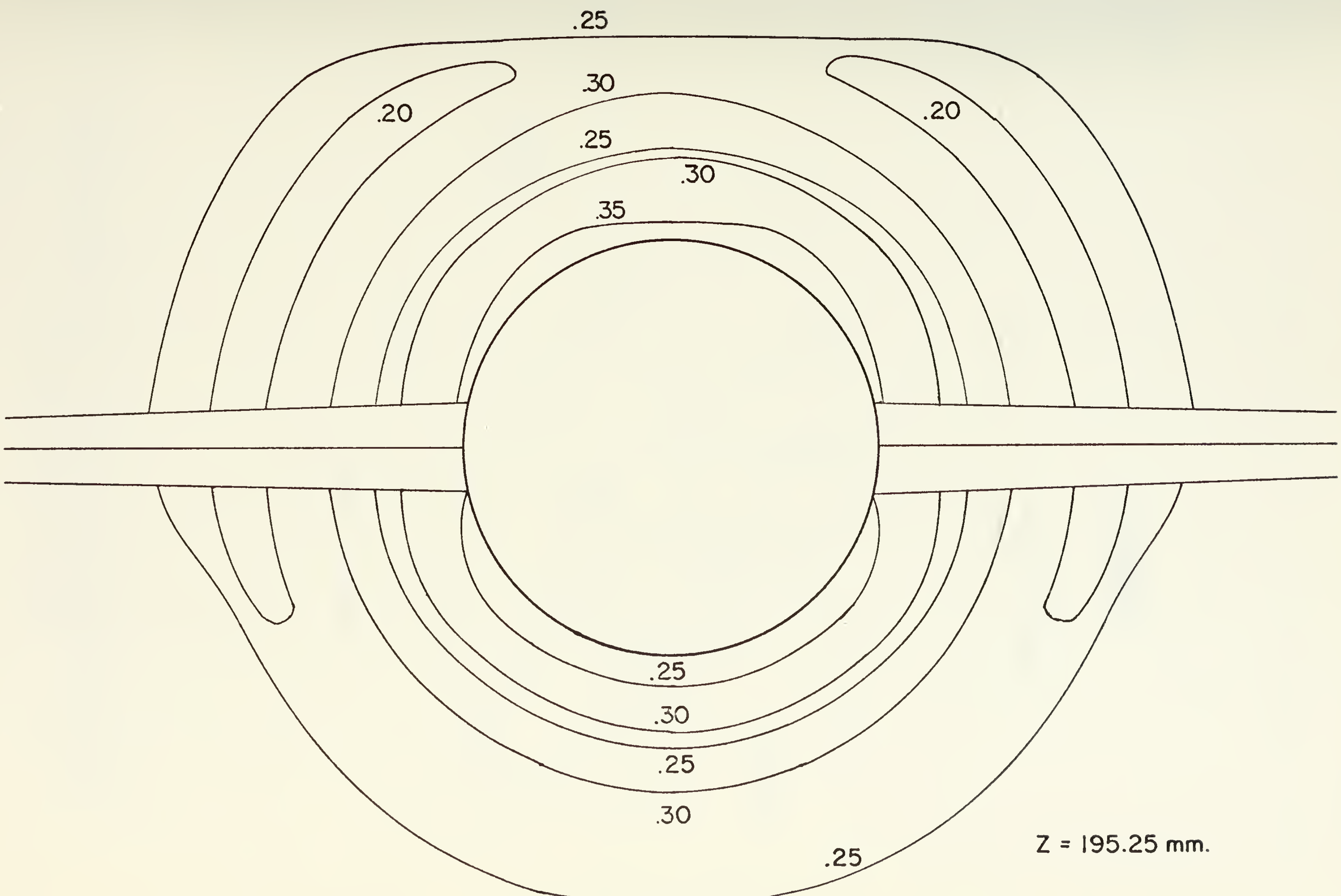


FIGURE 27.
CONTOUR PLOT OF DENSITY FUNCTION, $(\rho / \rho_\infty) - 1$, FOR GIVEN CROSS-SECTIONAL PLANE

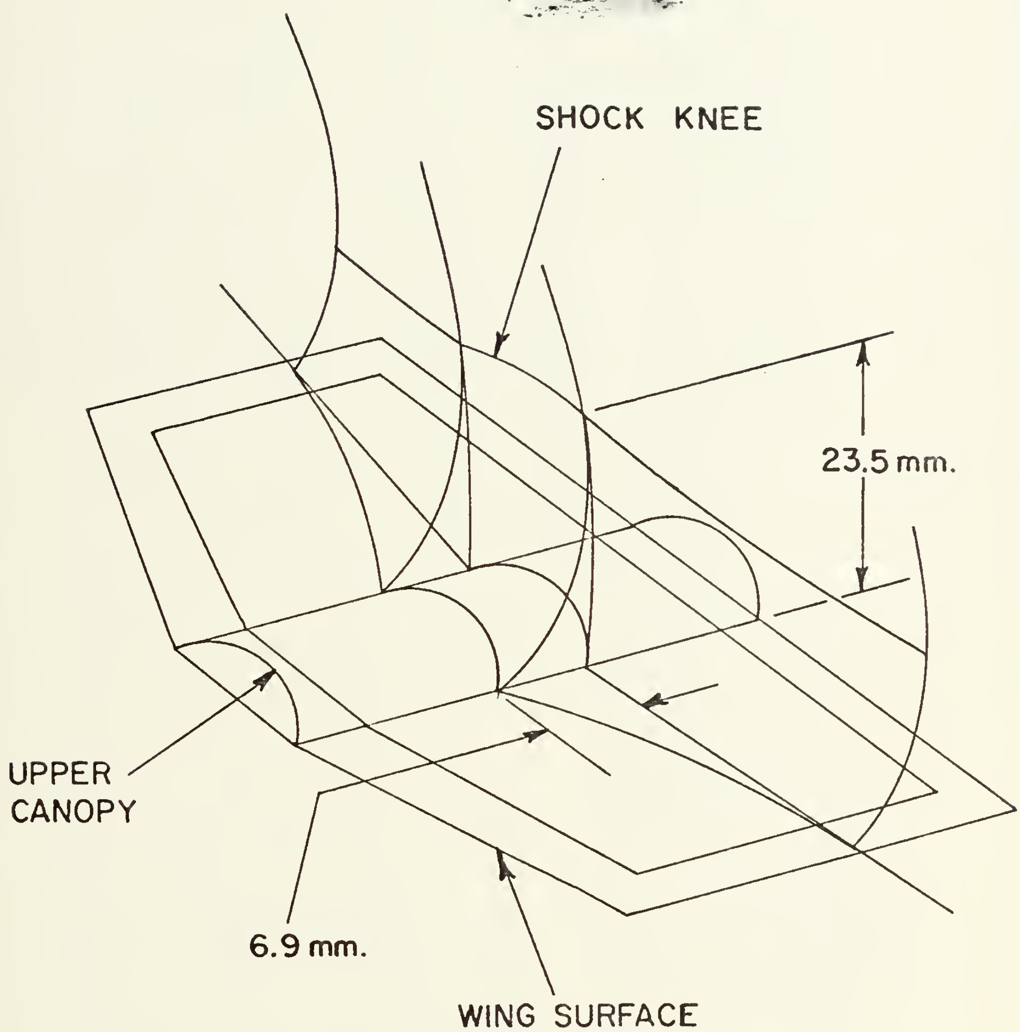


FIGURE 28. THREE DIMENSIONAL SCHEMATIC OF SHOCK WAVE STRUCTURE; CONSTRUCTED USING SEVERAL INTERFEROGRAM VIEWS

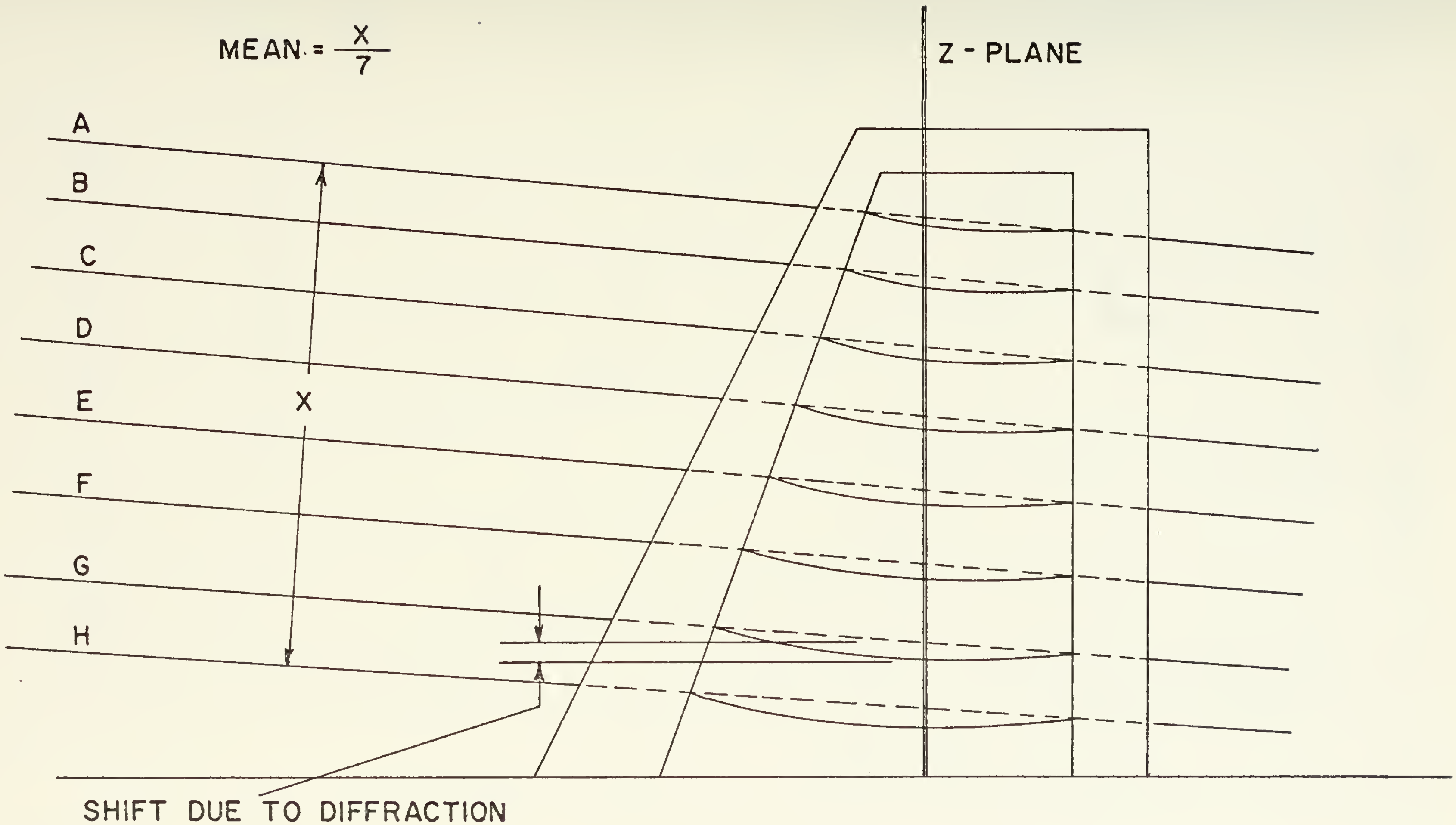


FIGURE 29. DOUBLE-STATIC HOLOGRAM REDUCTION PROCESS; Z = 186.75 mm.

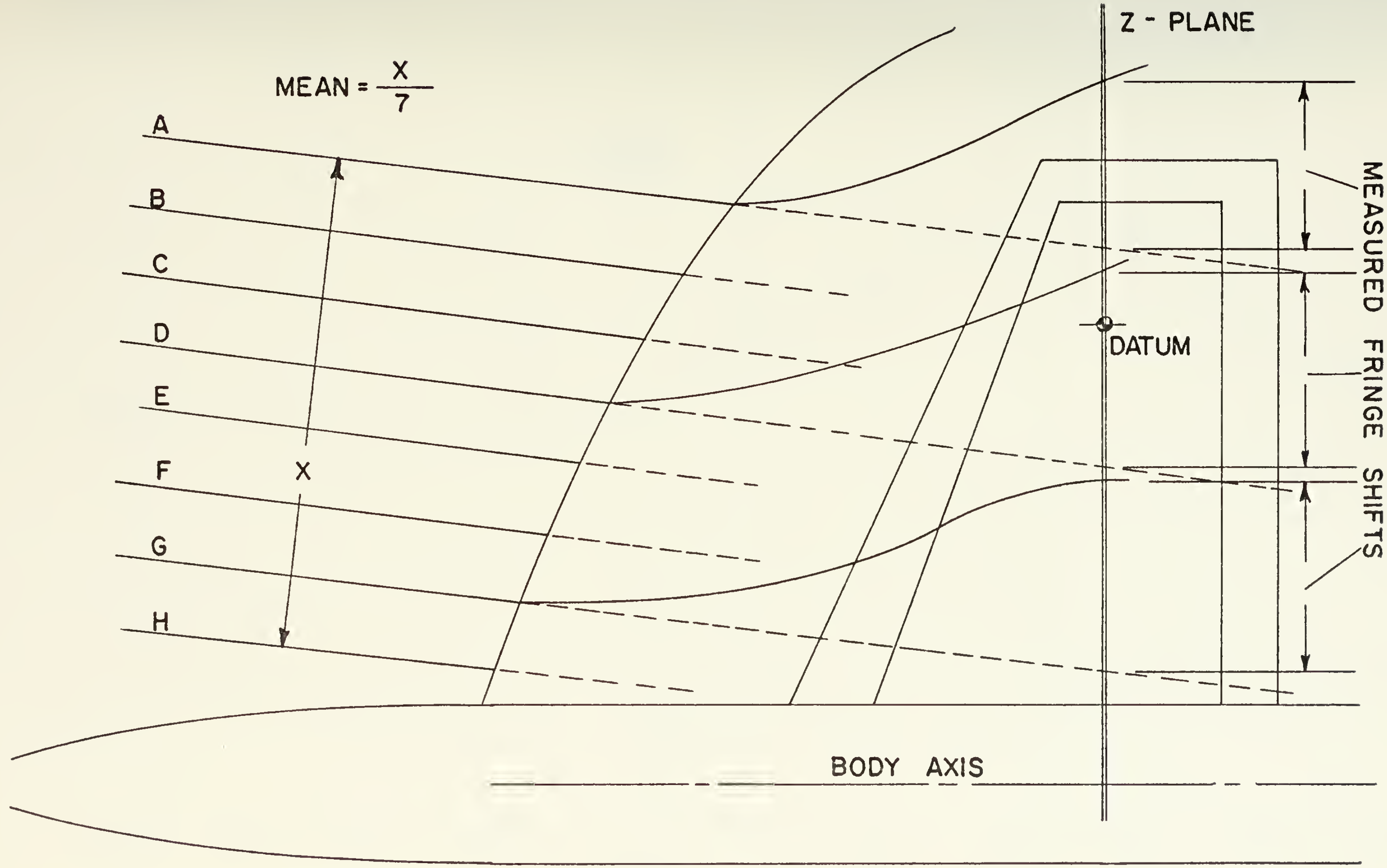


FIGURE 30. STATIC-DYNAMIC HOLOGRAM REDUCTION PROCESS; Z = 186.75 mm.

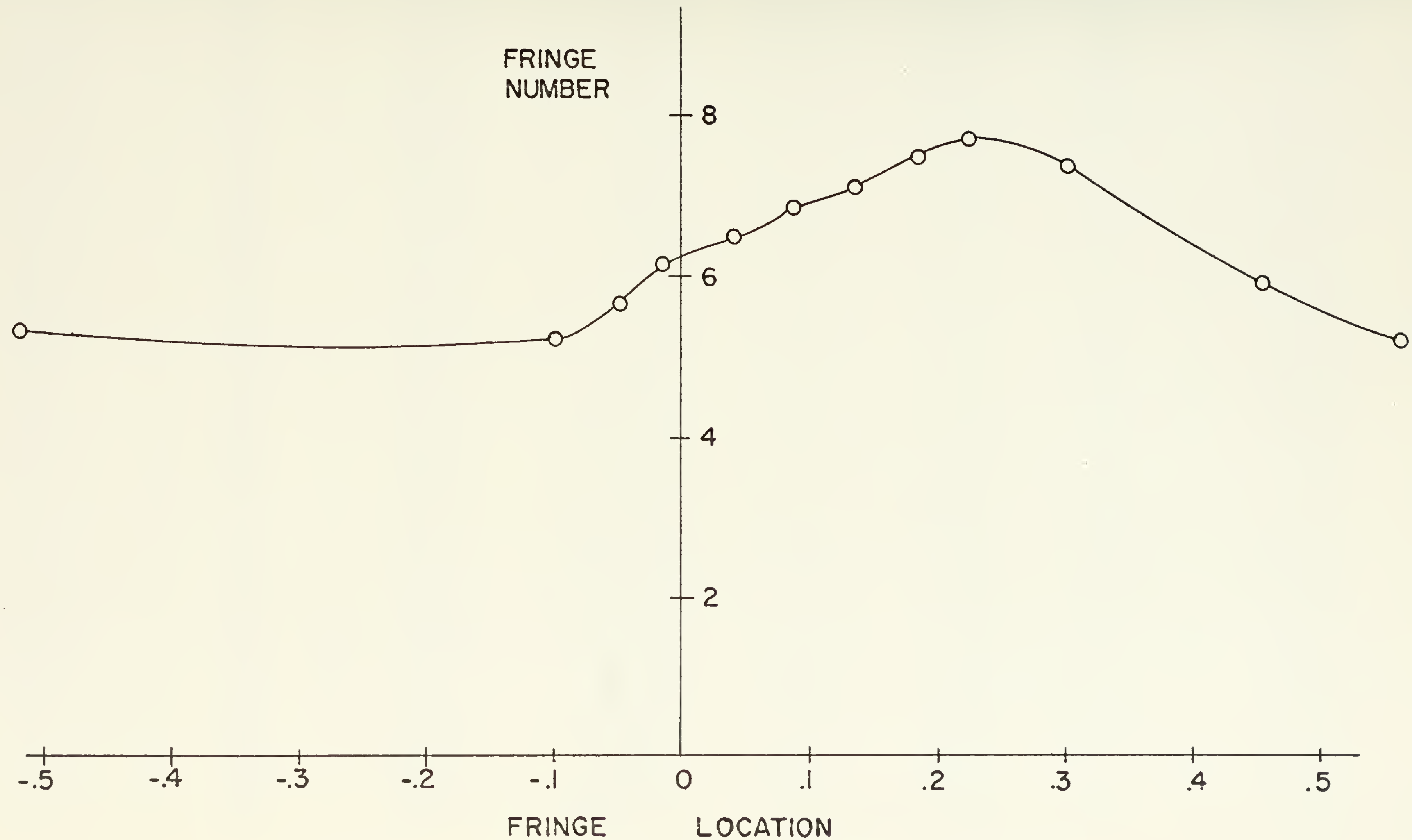


FIGURE 31. RADIAL VARIATION OF FRINGE NUMBER; ZERO DEGREE VIEW, Z = 186.75 mm.

RUN NUMBER	HOLOGRAM NUMBER	DOUBLE EXPOSURE	ROLL ANGLE	P_A (psi)	P_T (psi)	T_T (deg. F)	MACH NUMBER
1	9	S/S	0.00				
2	10	S/D	0.00	14.781	14.181	64.6	.9371
3	23	S/S	11.25				
4	24	S/D	11.25	14.820	14.228	64.2	.9398
5	19	S/S	22.50				
6	20	S/D	22.50	14.784	14.187	64.6	.9361
7	15	S/S	33.75				
8	16	S/D	33.75	14.800	14.206	64.2	.9392
9	11	S/S	45.00				
10	12	S/D	45.00	14.785	14.185	64.6	.9391
11	25	S/S	56.25				
12	26	S/D	56.25	14.830	14.237	64.3	.9361
13	21	S/S	67.50				
14	22	S/D	67.50	14.787	14.195	64.3	.9365
15	17	S/S	78.75				
16	18	S/D	78.75	14.870	14.285	64.8	.9342
17	13	S/S	90.00				
18	14	S/D	90.00	14.787	14.195	64.6	.9402
19	35	S/S	101.25				
20	36	S/D	101.25	14.781	14.274	61.5	.9392
21	27	S/S	112.50				
22	28	S/D	112.50	14.875	14.281	62.5	.9371
23	37	S/S	123.75				
24	38	S/D	123.75	14.878	14.284	61.0	.9369
25	29	S/S	135.00				
26	30	S/D	135.00	14.875	14.278	62.0	.9372
27	39	S/S	146.25				
28	40	S/D	146.25	14.881	14.286	60.3	.9352
29	31	S/S	157.50				
30	32	S/D	157.50	14.875	14.276	61.8	.9350
31	41	S/S	168.75				
32	42	S/D	168.75	14.881	14.287	61.0	.9371
33	33	S/S	180.00				
34	34	S/D	180.00	14.875	14.282	62.0	.9357

TABLE I. EXPERIMENTAL DATA FOR WIND TUNNEL RUNS AT NSRDC

FRINGE I.D.	MEASURED SHIFT	CORRECTED SHIFT	FRINGE NUMBER	DISTANCE FROM DATUM	DEMAG. DISTANCE	DISTANCE FROM AXIS	NONDIMEN. LOCATION
A	25.0	21.475	3.53	-27.0	-25.1	-63.8	-.798
B	29.6	26.075	4.29	-23.2	-21.6	-59.8	-.748
C	30.4	26.875	4.42	-18.5	-17.2	-55.4	-.693
D	31.6	28.075	4.62	-13.0	-12.1	-50.3	-.623
E	35.5	31.975	5.23	- 9.5	- 8.8	-47.0	-.588
F	36.0	32.475	5.34	- 3.6	- 3.4	-41.6	-.520
G	34.0	30.475	5.23	+32.8	+30.5	- 7.7	-.096
H	36.4	32.875	5.64	+37.0	+34.4	- 3.8	-.048
I	40.0	36.475	6.26	+40.0	+37.2	- 1.0	-.013
J	41.5	37.975	6.51	+44.4	+41.3	+ 3.1	+.039
K	43.5	39.975	6.86	+48.5	+45.2	+ 7.0	+.088
L	45.3	41.775	7.17	+52.7	+49.1	+10.9	+.136
M	47.4	43.875	7.53	+57.0	+53.1	+14.9	+.186
N	48.6	45.075	7.73	+60.5	+56.3	+18.1	+.226
O	46.8	43.275	7.42	+67.0	+62.4	+24.2	+.303
P	38.5	34.975	5.99	+80.3	+74.8	+36.6	+.458
Q	34.0	30.475	5.23	+89.6	+83.4	+45.2	+.565

Mean free stream spacing = 5.96 mm.

Diffraction Correction = 3.525 mm.

Magnification factor = 1.074

Body axis location = +38.2 mm

All measurements in millimeters

- = above, + = below

TABLE II.
RECORDED DATA FOR ZERO DEGREE VIEW AT Z = 186.75 mm. FROM BODY NOSE

APPENDIX A

REDUCTION OF AN INTERFEROGRAM TO OBTAIN FRINGE SHIFT DATA

The reduction of data for a cross-sectional plane of interest involved a complete analysis of both doubly exposed holograms, and their corresponding interferograms, for each viewing angle.

For each view, or line of sight, the double-static exposure was first placed face down on a light table. The average fringe spacing was recorded on the back by measuring the perpendicular distance between two sufficiently separated fringe lines in the free stream and dividing by the number of spacings in between. The method is portrayed in Figure 29. Of primary importance was the measurement of the uniform shift of fringe lines due to the beam diffractive quality of the lucite sections of the model; this was taken to be the average distance between hypothetically unaltered fringe lines and the corresponding shifted fringes at their intersection with the z-plane.

The static-dynamic exposure was then placed face down on the light table and the fringe and model contours traced on the back, as shown in Figure 30. Again, the average free stream fringe line spacing was measured and checked against the value from the double-static exposure. A reference point, or datum, was selected at the intersection of a well-defined horizontal grid line and the z-plane of interest. Fringe shifts were computed in the following manner:

1. The distance from datum to a hypothetically undeviated fringe line at its intersection with the z-plane was measured in millimeters.
2. The distance from datum to the actual deviated, or shifted, fringe at its intersection with the z-plane was measured in millimeters. Measured shifts should be made perpendicular to the fringe direction. The present procedure is convenient and only introduces an error of about 1% in the overall level of the density field.
3. The raw fringe shift distance was then the distance in 1. above minus the distance in 2.
4. To correct for diffraction error, the uniform shift measured in the double-static exposure was then subtracted from the distance in 3.

Fringe numbers were calculated by dividing the shift for each fringe by the average free stream spacing for the exposure. Magnification factors were computed for each exposure by comparing photographically recorded model diameter with actual model diameter.

Since the datum location varied slightly from exposure to exposure, it was necessary to reference all measurements to a central point for each plane of analysis. This point was taken to be the intersection of the longitudinal axis of the model fuselage and the cross-sectional plane of interest. The intersections of shifted fringe lines with the z-plane were referenced to this fixed point and demagnified. The resulting distances were then nondimensionalized using the selected inversion circle radius. Table II contains typical data recorded for the zero degree view at the 186.75 mm. plane of interest.

A plot of fringe numbers versus nondimensionalized fringe location was produced for each viewing angle using data obtained from the interferograms. Fringe numbers at 201 equidistant points, as required for input into Mode 3 of the inversion computer program, were recorded from the resulting curves. The curve plotted using the data from Table II is shown in Figure 31.

APPENDIX B

APPLICATION OF COMPUTER PROGRAM "HOLOFER"

The computer program used in this study is designed to invert an array of fringe numbers across a field to obtain the associated density field using a special adaptation of the inversion technique first proposed by C. D. Maldonado [5, 6]. Three different modes of operation are available to the operator, as described below:

(a) Mode 1

This mode provides the basic self-testing capability of the computer program. It can either generate its own input density field using Subroutine FUNCT or read in a density field through Subroutine FREAD. The fringe number array corresponding to the input density data is first generated; this information is then used as the input for the inversion to obtain the original density distribution once again. This self-testing procedure was utilized in the present investigation to determine the best value of the scale factor, α , required to assure accurate density in the region of inversion.

(b) Mode 2

This mode generates the fringe array at regular intervals across the test field from irregularly spaced fringe values read in through Subroutine SHEET. Simulated fringe arrays may be generated

by one of the functions specified in Subroutine FUNCT if NCODE = 1 is specified. Mode 2 has not been utilized in the present investigation.

(c) Mode 3

Mode 3 operation reads in a regularly spaced array of input density values directly utilizing Subroutine READ, which is first called by Subroutine GARRAY. The parameters in the first two cards preceding the input fringe data serve to identify the size, location and symmetry of the fringe field. The following parameters were used in calculating the asymmetric density field in the present experiment:

PARAMETER	INPUT
NOF	Run Number
IMAX	201
JMAX	20
ISYM	1
JSYM	1
IMS	201
JMS	5
Z	0.0, 1.0
XO	0.0
YO	0.0
PHISYM	0.0

Amplifying details and applications of the inversion computer program are found in References [3, 7, 9]. A listing of the program is included in this appendix for reference.

```
C
C     ***************************HOLOVERT*******************************
C
C     HOLOVERT INVERTS THE  FIELD AT AN ARRAY OF POINTS IN A VARIABLE
C     COORDINATE SYSTEM.  IT SURVEYS NPTS POINTS EACH FOR A SET OF NLINS
C     LINES ACROSS THE FIELD.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT
      COMMON /TAB2/ IPT,KPT,LPT,BND,NPTS,NLINS,RHOINF,RLAMDA,BETA
      COMMON /OUT/ XP,THEO,CALC,ERR,RHO,CA,FA
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,NOF,NAF
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
      COMMON /IO/ CMS,IN1,IN2,IN4
      DATA BL,PL,ST,EX,OH,SC,DH,BR/1H ,1H+,1H*,1HX,1HO,1H:,1H-,1H|/
      DIMENSION RB(7),TL(62),RO(101),RA(101)
C-----------------------------------------------------------------------
      DIMENSION G(5151),GA(5151),H(202,5),SCF(73,6),BDA(4000)
      DIMENSION THEO(51,11),CALC(51,11),ERR(51),RHO(51)
      DIMENSION CA(51),FA(51,11),AR(42),XP(51),YP(51)
      CMS=0.
C-----------------------------------------------------------------------
      REWIND 3
      IN1=5
      IN2=5
      IN4=5
      IF (CMS.NE.1.) GO TO 20
      IN1=1
      IN2=2
      IN4=4
      IF (CMS.EQ.1.) REWIND 4
20    READ (IN4,89) (AR(I),I=1,42)
      IMAX=AR(1)
      JMAX=AR(2)*2.
      IF (JMAX.LE.0) JMAX=1
      KLIMIT=AR(3)
      MLIMIT=AR(4)
      KEXTRA=AR(5)
      MEXTRA=AR(6)
      JSYM=AR(14)
      ALPHA=AR(7)
      SIZE=AR(8)
      EPS=AR(9)
      MODE=AR(13)
      DGN=AR(17)
      RHOINF=AR(10)
      RLAMDA=AR(11)*1.E-8
      BETA=AR(12)
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      NPTS=AR(15)                                                       CAL00480
      NLINS=AR(16)                                                      CAL00490
      SD=AR(18)                                                         CAL00500
      PHIZ=AR(19)                                                       CAL00510
      DELPHI=AR(20)                                                     CAL00520
      YPZERO=AR(21)                                                     CAL00530
      YPRNG=AR(22)                                                      CAL00540
      XPZERO=AR(23)                                                     CAL00550
      XPRNG=AR(24)                                                      CAL00560
      NOF=AR(25)                                                        CAL00570
      NAF=AR(26)                                                        CAL00580
      IPT=AR(27)                                                        CAL00590
      KPT=AR(28)                                                        CAL00600
      LPT=AR(29)                                                        CAL00610
      BND=AR(30)                                                        CAL00620
      A=AR(31)                                                          CAL00630
      B=AR(32)                                                          CAL00640
      C=AR(33)                                                          CAL00650
      D=AR(34)                                                          CAL00660
      E=AR(35)                                                          CAL00670
      P=AR(36)                                                          CAL00680
      S=AR(37)                                                          CAL00690
      T=AR(38)                                                          CAL00700
      U=AR(39)                                                          CAL00710
      V=AR(40)                                                          CAL00720
      W=AR(41)                                                          CAL00730
      Q=AR(42)                                                          CAL00740
      WRITE(6,90)
      WRITE(6,98)  (ST,I=1,75)
      WRITE(6,91) (AR(I),I=1,6)
      WRITE(6,92) (AR(I),I=7,12)
      WRITE(6,93) (AR(I),I=13,18)
      WRITE(6,94) (AR(I),I=19,24)
      WRITE(6,95) (AR(I),I=25,30)
      WRITE(6,96) (AR(I),I=31,36)
      WRITE(6,97) (AR(I),I=37,42)
      WRITE(6,98)  (ST,I=1,75)
   90 FORMAT(/////, ' THE INPUT PARAMETERS FOR THIS RUN ARE :',///)
   91 FORMAT (/5X,'*  IMAX   * JMAX/2  * KLIMIT  * MLIMIT  * KEXTRA  *',SET00310
     1' MEXTRA  *'/3X,6F10.0)
   92 FORMAT (/5X,'*  ALPHA  *  SIZE   *  EPS.   * RHO-INF * LAMBDA  *',SET00340
     1'  BETA   *'/3X,2F10.3,F11.6,2X,F8.7,F9.1,F11.6)                  SET
   93 FORMAT (/5X,'*  MODE   *  SYM.   * POINTS  *  LINES  * DIAGNOS *',SET00360
     1' STD.DEV *'/2X,5F10.0,F10.3)
   94 FORMAT (/5X,'* PHIZERO * DELPHI  * YPZERO  * YPRANGE * XPZERO  *',
     1' XPRANGE *'/4X,6F10.3)
   95 FORMAT (/5X,'* TST FUN * ADD FUN * GARRAY  *  GRAPH  * LIN PRT *',SET00400
     1' MAP BND *'/2X,6F10.0)
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 96   FORMAT (/5X,'*    A     *     B     *     C     *     D     *     E     *',SET00420
     1'     P      *'/4X,6F10.3)
 97   FORMAT (/5X,'*    S     *     T     *     U     *     V     *     W     *',SET00440
     1'     Q      *'/4X,6F10.3)
 98   FORMAT (/3X,75A1)
      IF (DGN.GE.4) WRITE (6,89) (AR(I),I=1,42)                            CAL00750
      NNN=2                                                                CAL00760
      IF (MODE.LT.0) NNN=1                                                 CAL00770
      IF (MODE.GT.5) NNN=3                                                 CAL00780
      IF (MODE.GT.5) MODE=MODE-10                                          CAL00790
      NGP=0                                                                CAL00800
      IF (KLIMIT.LT.KEXTRA) KEXTRA=KLIMIT                                  CAL00810
      IF (MLIMIT.LT.MEXTRA) MEXTRA=MLIMIT                                  CAL00820
      IF (IPT.LT.0) NGP=IPT                                                CAL00830
      IF (IPT.LT.0) IPT=-IPT                                               CAL00840
      ISYM=2.1-(FLOAT(JSYM)/2.-FLOAT(JSYM/2))*2                            CAL00850
      IF (JSYM.EQ.0) ISYM=1                                                CAL00860
      IF (JSYM.GT.JMAX) ISYM=2                                             CAL00870
      IF (ISYM.EQ.1) JMAX=((JMAX+1)/2)*2                                   CAL00880
      RJMX=JMAX                                                            CAL00890
      MSYM=JSYM                                                            CAL00900
      IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                            CAL00910
      FCU=ISYM*JSYM*JMAX                                                   CAL00920
      IF ((JSYM.GT.JMAX).OR.(JSYM.EQ.0)) FCU=JMAX                          CAL00930
      QSYM=FCU/RJMX                                                        CAL00940
      IMS=(IMAX+ISYM-1)/ISYM                                               CAL00950
      JMS=JMAX                                                             CAL00960
      IF (ISYM.EQ.1) JMS=(JMAX/2+1)/2                                      CAL00970
      IF (JSYM.EQ.0) JMS=JMAX/2                                            CAL00980
      MODE=ABS(AR(13))                                                     CAL00990
      XO=0.                                                                CAL01000
      YO=0.                                                                CAL01010
      ZD=0.                                                                CAL01020
      PHISYM=0.                                                            CAL01030
      HS=SIZE/2.                                                           CAL01040
      RHOS=1.286                                                           CAL01050
      BOX=RHOINF*BETA/RHOS/RLAMDA                                          CAL01060
      RPTS=NPTS                                                            CAL01070
      XPR=0.                                                               CAL01080
      IF (NPTS.GT.1) XPR=XPRNG/(RPTS-1.)/2.                                CAL01090
      XPM=-XPR                                                             CAL01100
      PIE=3.141592653589793                                                CAL01110
      MONE=1                                                               CAL01120
        WRITE (6,58) IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU             CAL01130
 58     FORMAT (3X,'IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU',/           CAL01140
     1   7I5,2F7.3/)                                                       CAL01150
      NTWO=2                                                               CAL01160
      IX=IMAX+JMAX                                                         CAL01170
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      NF=IN1                                                            CAL01180
      IF ((MODE.EQ.1.).AND.(NOF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)     CAL01190
      IF ((MODE.EQ.1).AND.(NOF.EQ.8)) CALL FREAD (NO,RO,NF,ZD)          CAL01200
      Z=ZD                                                              CAL01210
      IF (DGN.GE.1.) WRITE (6,68)                                       CAL01220
      CALL GARRAY (G,GA,NOF,DGN,MONE,XO,YO,PHISYM)                      CAL01230
      LM=1                                                              CAL01240
      IF ((LPT.EQ.0).AND.(BND.EQ.0)) LM=0                               CAL01250
      IIMX=IMAX+1                                                       CAL01260
      JJMX=JMAX+1                                                       CAL01270
      IJMX=IMAX*JMAX                                                    CAL01280
      NBD=1                                                             CAL01290
      IF (JSYM.EQ.0) NBD=2                                              CAL01300
      KBD=KLIMIT*NBD                                                    CAL01310
      DO 15 IJ=1,IJMX                                                   CAL01320
15    GA(IJ)=0.                                                         CAL01330
      IF (NAF.EQ.0) GO TO 16                                            CAL01340
      NF=IN2                                                            CAL01350
      IF ((NAF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)                      CAL01360
      IF (NAF.EQ.8) CALL FREAD (NA,RA,NF,ZD)                            CAL01370
      MST=MODE                                                          CAL01380
      MODE=1                                                            CAL01390
      IF (DGN.GE.1.) WRITE (6,68)                                       CAL01400
      IF (NAF.NE.0) CALL GARRAY (GA,G,NAF,DGN,NTWO,XO,YO,PHISYM)        CAL01410
      MODE=MST                                                          CAL01420
      DO 6 IJ=1,IJMX                                                    CAL01430
6     G(IJ)=G(IJ)+GA(IJ)                                                CAL01440
16    RLINS=NLINS                                                       CAL01450
      IF (NAF.EQ.8) WRITE (6,88) NA,(RA(L),L=1,NA)                      CAL01460
      IF (NOF.EQ.8) WRITE (6,87) NO,(RO(I),I=1,NO)                      CAL01470
      IF (LM.EQ.0) GO TO 14                                             CAL01480
      RB(1)=-1.                                                         CAL01490
      DO 1 I=2,7                                                        CAL01500
1     RB(I)=RB(I-1)+.5                                                  CAL01510
      TPIE=2.*PIE                                                       CAL01520
      MPIE=-PIE                                                         CAL01530
      DYP=0.                                                            CAL01540
      DXP=0.                                                            CAL01550
      IF (NLINS.GT.1) DYP=YPRNG/(RLINS-1.)                              CAL01560
      IF (NPTS.GT.1) DXP=XPRNG/(RPTS-1.)                                CAL01570
      IF ((DGN.GE.1.).AND.(NNN.EQ.2)) WRITE (6,64)                      CAL01580
      IF(NNN.EQ.2) CALL BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)                 CAL01590
      DO 5 J=1,NLINS                                                    CAL01600
      IF (DGN.GE.1.) WRITE (6,67) J                                     CAL01610
      RJM=J-1                                                           CAL01620
      PHI=PHIZ+DELPHI*RJM                                               CAL01630
      YP(J)=YPZERO+DYP*RJM                                              CAL01640
      PSI=(PHI+90.)*PIE/180.                                            CAL01650
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      TAU=PSI-PHISYM
      IF (LPT.EQ.0) GO TO 9
      IF (LPT.LE.1) WRITE (6,78) (ST,I=1,124)
      IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)
      IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ
      WRITE (6,86)
      WRITE (6,85) Z,PHI,YP(J)
      WRITE (6,76)
      IF (MODE.EQ.1) WRITE (6,83) (RB(I),I=1,7)
      IF (MODE.GT.1) WRITE (6,80) (RB(I),I=1,7)
      WRITE (6,81) (DH,I=1,54),(PL,II=1,13)
9     IC=0
      DO 3 I=1,NPTS
      RIM=I-1
      THEO(I,J)=0.
      CA(I)=0.
      FA(I,J)=0.
      ERR(I)=0.
      CALC(I,J)=0.
      RHO(I)=0.
      XP(I)=XPZERO+DXP*RIM
      XPII=ABS(XP(I))
      IF (XPII.LT.1.E-10) XP(I)=0.
      RS=SQRT(XP(I)**2+YP(J)**2)/HS
      IF (RS.GT.1.) GO TO 13
      THT=ATANM(YP(J),XP(I))
      IF (XPII.EQ.0.) THT=0.
      SIG=TAU-PIE/2.+THT
      IF (SIG.GT.PIE) SIG=SIG-TPIE
      IF (SIG.LT.MPIE) SIG=SIG+TPIE
      SIGI=SIG
      XS=RS*COS(SIG)
      IF (DGN.GE.1.) WRITE (6,44) SIG
44    FORMAT ('   SIG=',E10.3)
      YS=RS*SIN(SIG)
      IF(DGN.GE.5) WRITE (6,57) PHI,DELPHI,PSI,TAU,THT,SIG,SIGI,XS,YS
57      FORMAT ('  ANGLES=',10E10.3)
      RI=I
      F=0.
      IF (DGN.GE.2.) WRITE (6,66) I
      CALL FUNCT (XS,YS,FA(I,J),NAF,DGN,NTWO)
      IF (MODE.EQ.1) CALL FUNCT (XS,YS,F,NOF,DGN,MONE)
      THEO(I,J)=F
      IF (NNN.GE.2) REWIND 3
      IF(NNN.GE.2) CALL FIELD (RS,SIGI,SOLN,NBD,BDA,DGN,KBD)
      IF(NNN.EQ.1) CALL FIELD2 (RS,SIGI,SOLN,G,H,SCF,DGN)
      CA(I)=SOLN/BOX/HS
      CALC(I,J)=CA(I)-FA(I,J)
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      RHO(I)=RHOINF*(CALC(I,J)+1.)                                      CAL02140
      ERR(I)=CA(I)                                                      CAL02150
      IF (MODE.EQ.1) ERR(I)=(CALC(I,J)-THEO(I,J))                       CAL02160
      IF (MODE.GT.1) THEO(I,J)=FA(I,J)                                  CAL02170
      IF (LPT.EQ.0) GO TO 3                                             CAL02180
13    LC=0                                                              CAL02190
      TL(1)=BL                                                          CAL02200
      TTL=0.                                                            CAL02210
      IF ((XP(I).GT.XPM).AND.(XP(I).LT.XPR)) TTL=1.
      IF (IC.EQ.5) IC=0
      IF (IC.EQ.0) TL(1)=PL
      DO 2 L=2,62
      TL(L)=BL                                                          CAL02260
      IF ((I.EQ.1).OR.(TTL.EQ.1.).OR.(I.EQ.NPTS)) TL(L)=PL              CAL02270
      IF (LC.EQ.13) LC=0                                                CAL02280
      IF ((IC.EQ.0).AND.(LC.EQ.0)) TL(L)=PL                             CAL02290
2     LC=LC+1                                                           CAL02300
      TL(2)=PL                                                          CAL02310
      TL(22)=PL                                                         CAL02320
      TL(62)=PL                                                         CAL02330
      IC=IC+1                                                           CAL02340
      RLW=(CA(I)+1.)*20.+2.5                                            CAL02350
      LW=RLW                                                            CAL02360
      IF (LW.GT.62) LW=62                                               CAL02370
      IF (LW.LT.2) LW=2                                                 CAL02380
      TL(LW)=SC                                                         CAL02390
      RLY=(FA(I,J)+1.)*20.+2.5                                          CAL02400
      LY=RLY                                                            CAL02410
      IF (LY.GT.62) LY=62                                               CAL02420
      IF (LY.LT.2) LY=2                                                 CAL02430
      IF (NAF.NE.0) TL(LY)=ST                                           CAL02440
      RLX=(THEO(I,J)+1.)*20.+2.5                                        CAL02450
      LX=RLX                                                            CAL02460
      IF (LX.GT.62) LX=62                                               CAL02470
      IF (LX.LT.2) LX=2                                                 CAL02480
      IF (MODE.EQ.1) TL(LX)=OH                                          CAL02490
      RLZ=(CALC(I,J)+1.)*20.+2.5                                        CAL02500
      LZ=RLZ                                                            CAL02510
      IF (LZ.GT.62) LZ=62                                               CAL02520
      IF (LZ.LT.2) LZ=2                                                 CAL02530
      TL(LZ)=EX                                                         CAL02540
      WRITE (6,82)MOUT,KOUT,INDEX,THEO(I,J),ERR(I),CALC(I,J),RHO(I),    CAL02550
     1XP(I),(TL(L),L=1,62)                                              CAL02560
      IF ((NPTS.LE.20).AND.(I.NE.NPTS)) WRITE (6,79)                    CAL02570
3     CONTINUE                                                          CAL02580
      IF (LPT.NE.0) WRITE (6,81) (DH,I=1,54),(PL,I=1,13)                CAL02590
      TMAX=0.                                                           CAL02600
      TMIN=0.                                                           CAL02610
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      IE=0                                                              CAL02620
      BE=0.                                                             CAL02630
      EB=0.                                                             CAL02640
      DO 4 I=1,NPTS                                                     CAL02650
      TH=THEO(I,J)                                                      CAL02660
      IF (TH.GT.TMAX) TMAX=TH                                           CAL02670
      IF (TH.LT.TMIN) TMIN=TH                                           CAL02680
      ER=ABS(CALC(I,J)-TH)                                              CAL02690
      IF (ER.LE.BE) GO TO 4                                             CAL02700
      BE=ER                                                             CAL02710
      IE=I                                                              CAL02720
4     CONTINUE                                                          CAL02730
      TMM=TMAX-TMIN                                                     CAL02740
      EB=RHOINF*(CALC(IE,J)-THEO(IE,J))                                 CAL02750
      IF (TMM.NE.0.) BE=(CALC(IE,J)-THEO(IE,J))*100./TMM                CAL02760
      IF ((MODE.EQ.1).AND.(LPT.NE.0)) WRITE (6,75) EB,XP(IE),BE         CAL02770
      IF (DELPHI.NE.0.) YP(J)=PHI                                       CAL02780
5     CONTINUE                                                          CAL02790
      IF (BND.EQ.0.) GO TO 14                                           CAL02800
      IF (LPT.EQ.1) WRITE (6,78) (ST,I=1,124)                           CAL02810
      IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)                            CAL02820
      IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ                    CAL02830
      IF (DGN.GE.1.) WRITE (6,63)                                       CAL02840
      CALL MAP (NPTS,NLINS,CALC,NOF,Z,BND)                              CAL02850
14    IF (NAF.EQ.0) GO TO 10                                            CAL02860
      NAO=10*NOF+NAF                                                    CAL02870
      IF ((DGN.GE.1.).AND.(NGP.EQ.-3)) WRITE (6,62)                     CAL02880
      IF (NGP.EQ.-3) CALL GPUNCH (Z,XO,YO,PHISYM,NAO,IMAX,JMAX,G)       CAL02890
      DO 7 IJ=1,IJMX                                                    CAL02900
7     G(IJ)=G(IJ)-GA(IJ)                                                CAL02910
10    IF (IPT.LE.0) GO TO 11                                            CAL02920
      IF ((IPT.EQ.1).OR.(IPT.EQ.3)) WRITE (6,78) (ST,I=1,124)           CAL02930
      IF ((IPT.EQ.2).OR.(IPT.GE.4)) WRITE (6,74) (ST,I=1,95)            CAL02940
      IF ((CMS.EQ.1.).AND.((IPT.EQ.2).OR.(IPT.GE.4))) READ (5,79) ZZ    CAL02950
      CALL GPRINT (G,MONE)                                              CAL02960
      IF (NGP.EQ.-1) CALL GPUNCH (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)       CAL02970
      IF (IPT.EQ.3) WRITE (6,78) (ST,I=1,124)                           CAL02980
      IF (IPT.GE.4) WRITE (6,74) (ST,I=1,95)                            CAL02990
      IF ((CMS.EQ.1.).AND.(IPT.GE.4)) READ (5,79) ZZ                    CAL03000
      IF (IPT.GE.3) CALL GPRINT (GA,NTWO)                               CAL03010
11    IF (KPT.LE.0) GO TO 12                                            CAL03020
      IF ((KPT.EQ.1).OR.(KPT.EQ.3)) WRITE (6,78) (ST,I=1,124)           CAL03030
      IF ((KPT.EQ.2).OR.(KPT.GE.4)) WRITE (6,74) (ST,I=1,95)            CAL03040
      IF ((CMS.EQ.1.).AND.((KPT.EQ.2).OR.(KPT.GE.4))) READ (5,79) ZZ    CAL03050
      IF (DGN.GE.1.) WRITE (6,61)                                       CAL03060
      CALL GPLOT (G,GA,JMS)                                             CAL03070
12    WRITE (6,73) (EX,I=1,124)                                         CAL03080
      AGAIN=ST                                                          CAL03090
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      IF (CMS.NE.1.) READ(5,60) AGAIN                                   CAL03100
      IF (AGAIN.EQ.BL) GO TO 20                                         CAL03110
      WRITE (6,77)                                                      CAL03120
 89   FORMAT (6F12.7)                                                   CAL03130
 88   FORMAT (/'      THE INPUT DATA FOR ADD-ON FUNCTION NO.8 (',I3,    CAL03140
     1' POINTS) WAS:'/7(11F10.3/))                                      CAL03150
 87   FORMAT (/'      THE INPUT DATA FOR THE TEST FUNCTION NO.8, (',I3, CAL03160
     1' POINTS) WAS:'/7(11F10.3/))                                      CAL03170
 86   FORMAT (1H1//'    THE INVERTED CROSS SECTION FOR:')               CAL03180
 85   FORMAT (10X,'Z  =',F8.3,' CM.'/10X,'PHI=',F8.3,' DEGREES'/10X,    CAL03190
     14HY' =,F8.3,' CM.',44X,'0 = ORIGINAL FUNCTION'/                   CAL03200
     270X,'* = ADD-ON FUNCTION')                                        CAL03210
 84   FORMAT (' ADJUST PAGE, HIT SPACE AND RETURN.')                    CAL03220
 83   FORMAT ('   LIMIT  MAX  ORIGINAL      ABS.  COMPUTED (MG/CC)'/'  MCAL03230
     1   K TERM  FUNCTION     ERROR  FUNCTION DENSITY ',6H  X'  ,F4.1,  CAL03240
     26F10.1)                                                           CAL03250
 82   FORMAT (2X,I2,1X,I3,1X,I4,1X,F9.4,2X,F7.3,1X,F9.4,1X,F7.3,        CAL03260
     1F7.3,1X,62A1)                                                     CAL03270
 81   FORMAT (3X,54A1,2X,A1,12(4X,A1))                                  CAL03280
 80   FORMAT ('   LIMIT  MAX',3X,'ADD-ON',6X,'THE',4X,'DENSITY (MG/CC)'/CAL03290
     1'   M   K TERM',2X,'FUNCTION',5X,'SUM',3X,'FUNCTION DENSITY ',    CAL03300
     26H  X'  ,F4.1,6F10.1)                                             CAL03310
 79   FORMAT (1X,F10.3)                                                 CAL03320
 78   FORMAT (1X,124A1)                                                 CAL03330
 77   FORMAT (1X//)                                                     CAL03340
 76   FORMAT (4X,'FUNCTION = (RHO/RHO-INFINITY)-1.0',33X,': = THE'      CAL03350
     1,' INVERTED SUM'/70X,'X = COMPUTED FUNCTION')                     CAL03360
 75   FORMAT ('     LARGEST ERROR: ',F8.6,' GMS/CC;      AT ',3HX'=,F6.3CAL03370
     1,';',4X,F10.2,' PERCENT.'//)                                      CAL03380
 74   FORMAT (1X,47A1,' SET PAGE, HIT SPACE, RETURN ',48A1)             CAL03390
 69   FORMAT ('   CALL FREAD')                                          CAL03400
 68   FORMAT ('   CALL GARRAY')                                         CAL03410
 67   FORMAT ('   LINE',I3,' DO LOOP')                                  CAL03420
 66   FORMAT ('   POINT',I3,' CALL FUNCT')                              CAL03430
 65   FORMAT ('   CALL FIELD')                                          CAL03440
 64   FORMAT ('   CALL BDGEN')                                          CAL03450
 63   FORMAT ('   CALL MAP')                                            CAL03460
 62   FORMAT ('   CALL GPUNCH')                                         CAL03470
 61   FORMAT ('   CALL GPLOT')                                          CAL03480
 60   FORMAT (80A1)                                                     CAL03490
      STOP                                                              CAL03500
      END                                                               CAL03510
                                                                        CAL03520
C000001                                                                 SUB00010
C                                                                       SUB00020
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      SUBROUTINE BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)
C
C   BDGEN EVALUATES THE B AND D COEFFICIENTS FOR ALL M AND K, AND WRITES
C   THE ARRAY LINEARLY ON DISK.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
      DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,6),BDA(KBD)
C   INITIALIZE THE VALUES:
      INDEX=0
      KL2=NBD*KLIMIT
      REWIND 3
      JJMX6=JJMX*6
      IIMX2=(IIMX+1)/2
      PIE=3.14159265358979 3
      RIMAX=IMAX
      KLMP=KLIMIT+1
      DX=2./RIMAX
      RJMAX=JMAX
      DXI=2.*PIE/FCU
C   INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY;  VECTORS:
C   (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED
      DO 1 II=1,IIMX2
      RII=II
      IIM=IIMX-II+1
      H(II,3)=ALPHA*(RII*DX-DX-1.)
      H(IIM,3)=-H(II,3)
      H(II,1)=2.*H(II,3)
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.
      H(IIM,1)=-H(II,1)
      H(IIM,2)=H(II,2)
      H(II,5)=H(II,2)
      H(IIM,5)=H(IIM,2)
      H(II,4)=H(II,1)
1     H(IIM,4)=H(IIM,1)
      SIGN=1.
C   INITIATE THE SIN/COS ARRAY:
      DO 2 J=1,JJMX
      RJM=J-1
      SCF(J,1)=0.
      SCF(J,2)=1.
      SCF(J,3)=SIN(RJM*DXI-PIE/2.)
      SCF(J,4)=COS(RJM*DXI-PIE/2.)
      SCF(J,5)=0.
2     SCF(J,6)=0.
      MS=0
C   COMMENCE THE M LOOP:
```

```
      DO 7 MP=1,MLIMIT                                                  SUB00510
      M=MP-1                                                            SUB00520
      RM=M                                                              SUB00530
      SIGN=-SIGN                                                        SUB00540
      IF (DGN.LE.-4) WRITE (6,88) SCF(1,1),SCF(2,1),SCF(1,2),SCF(2,2)   SUB00550
C     TEST FOR SYMMETRY SKIPS:                                          SUB00560
      IF (MS.EQ.MSYM) MS=0                                              SUB00570
      TOTAL=0.                                                          SUB00580
      MS=MS+1                                                           SUB00590
      IF (MS.NE.1) GO TO 6                                              SUB00600
C     COMMENCE THE K LOOP:                                              SUB00610
      DO 5 KP=1,KLIMIT                                                  SUB00620
      K=KP-1                                                            SUB00630
      PK=KP                                                             SUB00640
      RK=K                                                              SUB00650
      INDEX=INDEX+1                                                     SUB00660
C     CALL THE B & D COEFFICIENTS AND WRITE THEM ON DISK:               SUB00670
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                   SUB00680
      IF(DGN.EQ.3.) WRITE(6,89) M,K,B,D
      IF (DGN.LE.-2) WRITE (6,89) M,K,B,D                               SUB00690
      IF (DGN.LE.-4) WRITE (6,88) H(1,1),H(1,2),H(1,4),H(1,5)           SUB00700
      KK=K*NBD+1                                                        SUB00710
      K2=KP*NBD                                                         SUB00720
      BDA(K2)=D                                                         SUB00730
      BDA(KK)=B                                                         SUB00740
C     GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:SUB00750
      ORDER=M+2*KP+1                                                    SUB00760
      HA=SQRT(PK*(PK+RM))/ORDER                                         SUB00770
      HB=2.*SQRT((PK+1.)*(RM+PK+1.))/(ORDER+1.)/(ORDER+2.)              SUB00780
      DO 5 II=1,IIMX2                                                   SUB00790
      IIM=IIMX-II+1                                                     SUB00800
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                           SUB00810
      H(IIM,1)=SIGN*H(II,1)                                             SUB00820
5     H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                        SUB00830
C     ADVANCE THE SIN/COS ARRAY FOR THE NEXT M:                         SUB00840
      DO 3 J=1,JJMX                                                     SUB00850
      IF (DGN.LE.-5) WRITE (6,87) (SCF(J,NT),NT=1,6)                    SUB00860
87      FORMAT ('  SIN/COS MXI:'8E10.3)                                 SUB00870
      STEMP=SCF(J,1)                                                    SUB00880
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                      SUB00890
3     SCF(J,2)=SCF(J,2)*SCF(J,4)-STEMP*SCF(J,3)                         SUB00900
      DO 4 J=1,JMAX                                                     SUB00910
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                      SUB00920
4     SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                      SUB00930
      WRITE (3) (BDA(I),I=1,KBD)                                        SUB00940
      IF (DGN.LE.-3) WRITE (6,88) (BDA(I),I=1,10)                       SUB00950
      IF (JSYM.GT.JMAX) RETURN                                          SUB00960
6     RM=RM+1                                                           SUB00970
```

```
C     REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:
        DO 7 II=1,IIMX2
        IIM=IIMX-II+1
        H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)
        H(II,1)=H(II,5)*(RM+2.)
        H(IIM,1)=-SIGN*H(II,1)
        H(II,2)=2.*SQRT(RM+1)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))
        H(II,2)=H(II,2)/(RM+2.)/(RM+3.)
        H(II,4)=H(II,1)
7       H(II,5)=H(II,2)
89      FORMAT ('    M=',I4,',    K=',I4,',    B=',E10.4,',    D=',E10.4)
88      FORMAT (2X,10E10.3)
        RETURN
        END
C000002
C


        SUBROUTINE FIELD (RS,SIG,SOLN,NBD,BDA,DGN,KBD)
C
C     FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR
C     POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION
C     EQUATION OF MALDONADO, ET.AL.  FIELD USES THE ARRAY OF  B & D
C     COEFFICIENTS GENERATED IN SUBROUTINE BDGEN.
C
        COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
        COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT
        COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
        DIMENSION BDA(KBD), STK(52), STM(52)
C     INITIALIZE THE VALUES:
        INDEX=0
        MTIMER=0
        KOUT=0
        MOUT=0
        MMAX=0
        KMAX=0
        TOTAL=0.
        JJMX6=JJMX*6
        REWIND 3
        AR=ALPHA*RS
        ARG=AR**2
        EXPON=EXP(-ARG)
        PIE=3.141592653589793
        APP=ALPHA/PIE/PIE
        M=0
        RM=M
        RIMAX=IMAX
        DX=2./RIMAX
```

```
      RJMAX=JMAX                                                        SUB01440
      SIGN=1.                                                           SUB01450
      STK(1)=0.                                                         SUB01460
      STM(1)=0.                                                         SUB01470
      SMS=0.                                                            SUB01480
      CMS=1.                                                            SUB01490
      SMI=SIN(SIG)                                                      SUB01500
      CMI=COS(SIG)                                                      SUB01510
      MEP=MEXTRA+1                                                      SUB01520
      DO 16 MB=1,MEP                                                    SUB01530
16    STM(MB)=0.                                                        SUB01540
      FM=1.                                                             SUB01550
      MS=0                                                              SUB01560
C   COMMENCE THE M LOOP:                                                SUB01570
2     SIGN=-SIGN                                                        SUB01580
      K=0                                                               SUB01590
      RK=K                                                              SUB01600
      RM=M                                                              SUB01610
      ARM=1.                                                            SUB01620
      IF (M.NE.0) ARM=AR**M                                             SUB01630
      KTIMER=0                                                          SUB01640
      KEP=KEXTRA+1                                                      SUB01650
      DO 15 KB=1,KEP                                                    SUB01660
15    STK(KB)=0.                                                        SUB01670
      SIGNK=-1.                                                         SUB01680
C   COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:    SUB01690
      PM=0.                                                             SUB01700
      P=SQRT(1./FM)                                                     SUB01710
      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUB01720
C   TEST FOR SYMMETRY SKIPS:                                            SUB01730
      IF (MS.EQ.MSYM) MS=0                                              SUB01740
      MS=MS+1                                                           SUB01750
      IF (MS.NE.1) GO TO 7                                              SUB01760
C   READ A LINE OF B & D COEFFICIENTS FOR GIVEN M:                      SUB01770
      READ (3) (BDA(I),I=1,KBD)                                         SUB01780
      IF (DGN.LE.-6) WRITE (6,88) (BDA(I),I=1,10)                       SUB01790
C   COMMENCE THE K LOOP:                                                SUB01800
3     INDEX=INDEX+1                                                     SUB01810
      SIGNK=-SIGNK                                                      SUB01820
C   COMPUTE THE M,K SUMMATION TERM:                                     SUB01830
      KK=K*NBD+1                                                        SUB01840
      B=BDA(KK)                                                         SUB01850
      D=0.                                                              SUB01860
      IF (NBD.EQ.2) D=BDA(KK+1)                                         SUB01870
      BRAKET=B                                                          SUB01880
      IF(RM.EQ.0.) GO TO 4                                              SUB01890
      BRAKET=B*CMS+D*SMS                                                SUB01900
4     ADD=SIGNK*BRAKET*P*ARM                                            SUB01910
```

```
      TOTAL=TOTAL+ADD                                                   SUB01920
      IF (DGN.GT.-5) GO TO 5                                            SUB01930
      STOT=TOTAL*EXPON*APP/BOX/SIZE                                     SUB01940
      WRITE (6,89) M,K,STOT,ADD,BRAKET,P,ARM,B,CMS,D,SMS                SUB01950
C   ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES: SUB01960
5     CHECK=ABS(ADD)                                                    SUB01970
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                            SUB01980
C   ADVANCE THE K INDEX:                                                SUB01990
      K=K+1                                                             SUB02000
      RK=K                                                              SUB02010
      DO 10 KA=1,KEXTRA                                                 SUB02020
      KB=KEXTRA-KA+1                                                    SUB02030
10    STK(KB+1)=STK(KB)                                                 SUB02040
      STK(2)=TOTAL                                                      SUB02050
      ORDER=M+2*K+1                                                     SUB02060
C   GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:           SUB02070
      PM=P                                                              SUB02080
      P=PP                                                              SUB02090
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                              SUB02100
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                    SUB02110
C   SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:             SUB02120
      KTIMER=KTIMER+1                                                   SUB02130
      IF (K.GE.KLIMIT) GO TO 6                                          SUB02140
      IF (CHECK.GE.EPS) KTIMER=0                                        SUB02150
      IF (KTIMER.LE.KEXTRA) GO TO 3                                     SUB02160
      GO TO 7                                                           SUB02170
6     KOUT=KOUT+1                                                       SUB02180
      IF (KEXTRA.EQ.0) GO TO 7                                          SUB02190
      TOTAL=0.                                                          SUB02200
      DO 11 KA=1,KEXTRA                                                 SUB02210
11    TOTAL=TOTAL+STK(KA+1)                                             SUB02220
      RKX=KEXTRA                                                        SUB02230
      TOTAL=TOTAL/RKX                                                   SUB02240
C   END OF K LOOP;  ADVANCE M:                                          SUB02250
7     M=M+1                                                             SUB02260
      RM=M                                                              SUB02270
      STP=SMS                                                           SUB02280
      SMS=SMS*CMI+CMS*SMI                                               SUB02290
      CMS=CMS*CMI-STP*SMI                                               SUB02300
      IF (K.GT.KMAX) KMAX=K                                             SUB02310
      FM=FM*RM                                                          SUB02320
      DO 12 MA=1,MEXTRA                                                 SUB02330
      MB=MEXTRA-MA+1                                                    SUB02340
12    STM(MB+1)=STM(MB)                                                 SUB02350
      STM(2)=TOTAL                                                      SUB02360
C   SET M TIMER FOR EXTRA M TERMS:                                      SUB02370
      IF (MS.EQ.1) MTIMER=MTIMER+1                                      SUB02380
      IF (JSYM.GT.JMAX) GO TO 9                                         SUB02390
```

```
      IF (K.GT.KEXTRA) MTIMER=0
      IF (M.GE.MLIMIT) GO TO 13
      IF (MTIMER.LE.MEXTRA) GO TO 2
13    IF (MEXTRA.EQ.0) GO TO 9
      TOTAL=0.
      DO 14 MA=1,MEXTRA
14    TOTAL=TOTAL+STM(MA+1)
      RMX=MEXTRA
      TOTAL=TOTAL/RMX
C   END OF M LOOP;  COMPUTE OUTPUT SOLN.
9     MOUT=M-1
      IF (KOUT.EQ.0) KOUT=KMAX-1
      SOLN=TOTAL*EXPON*APP/2.
89    FORMAT ('   M=',I4,',  K=',I4,',  SUBTOTAL=',9E10.3)
88    FORMAT (2X,10E10.3)
      RETURN
      END
C000003
C


      SUBROUTINE BD (M,K,G,H,SCF,B,D,JJMX6)
C
C
C   BD EVALUATES THE FIRST (B) AND SECOND (D) COEFFICIENTS IN THE
C   INVERSION EQUATION, FOR A PARTICULAR SET OF INDEXES M & K.
C   BD MAKES USE OF THE HERMITE POLYNOMIAL ARRAY GENERATED BY
C   SUBROUTINE FIELD AS M & K ADVANCE.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
      DIMENSION G(IJMX),SCF(JJMX6),H(IIMX)
      PIE=3.14159265358979З
      B=0.
      D=0.
      RM=M
      RK=K
      RJMAX=JMAX
      JJMX4=4*JJMX
      DXI=2.*PIE/FCU
 200  FORMAT(1X,I10,/)
      IF (JSYM.LE.0) GO TO 4
      IF (M.NE.0) GO TO 2
      S=DXI
      DO 1 J=1,JMAX
      DO 1 I=1,IMAX
      II=I+1
      IJ=IMAX*(J-1)+I
```

```
      DH=H(II)-H(I)                                                     SUB02850
1     B=B+G(IJ)*S*DH                                                    SUB02860
      B=B*QSYM/2.                                                       SUB02870
      RETURN                                                            SUB02880
2     DO 3 J=1,JMAX                                                     SUB02890
      JS=J+JJMX4                                                        SUB02900
      S=SCF(JS)/RM                                                      SUB02910
      DO 3 I=1,IMAX                                                     SUB02920
      II=I+1                                                            SUB02930
      IJ=IMAX*(J-1)+I                                                   SUB02940
      DH=H(II)-H(I)                                                     SUB02950
3     B=B+G(IJ)*S*DH                                                    SUB02960
      B=B*QSYM                                                          SUB02970
      RETURN                                                            SUB02980
4     IF (M.NE.0) GO TO 6                                               SUB02990
      S=DXI                                                             SUB03000
      DO 5 J=1,JMAX                                                     SUB03010
      DO 5 I=1,IMAX                                                     SUB03020
      II=I+1                                                            SUB03030
      IJ=IMAX*(J-1)+I                                                   SUB03040
      DH=H(II)-H(I)                                                     SUB03050
5     B=B+G(IJ)*S*DH                                                    SUB03060
      B=B/2.                                                            SUB03070
      RETURN                                                            SUB03080
6     DO 7 J=1,JMAX                                                     SUB03090
      JS=J+JJMX4                                                        SUB03100
      J2=JS+JJMX                                                        SUB03110
      S=SCF(JS)/RM                                                      SUB03120
      C=SCF(J2)/RM                                                      SUB03130
      DO 7 I=1,IMAX                                                     SUB03140
      II=I+1
      IJ=IMAX*(J-1)+I                                                   SUB03160
      DH=H(II)-H(I)                                                     SUB03170
      B=B+G(IJ)*S*DH                                                    SUB03180
66    FORMAT ('  BD:',4I5,10F6.2)                                       SUB03190
7     D=D-G(IJ)*C*DH                                                    SUB03200
      RETURN                                                            SUB03210
      END                                                               SUB03220
C000004                                                                 SUB03230
C                                                                       SUB03240


      SUBROUTINE FIELD2 (RS,SIG,SOLN,G,H,SCF,DGN)                       SUB03250
C                                                                       SUB03260
C    FIELD2 COMPUTES THE SAME INVERSION AS SUBROUTINE FIELD, EXCEPT THAT SUB03270
C    THE COEFFICIENTS B AND D ARE COMPUTED INDIVIDUALLY AS USED BY      SUB03280
C    CALLING BD.  DISK STORAGE IS NOT REQUIRED, BUT COMPUTING TIME IS   SUB03290
C    MUCH GREATER.  FIELD2 IS UTILIZED BY SPECIFYING A NEGATIVE MODE ON SUB03300
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C     THE INPUT PARAMETER.                                                SUB03310
C     FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR     SUB03320
C     POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION SUB03330
C     EQUATION OF MALDONADO, ET AL.  FIELD CALLS SUBROUTINES BD & GARRAY. SUB03340
C                                                                         SUB03350
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z     SUB03360
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT            SUB03370
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                        SUB03380
      DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,6)                             SUB03390
C     INITIALIZE THE VALUES:                                              SUB03400
      INDEX=0                                                             SUB03410
      MTIMER=0                                                            SUB03420
      KOUT=0                                                              SUB03430
      MOUT=0                                                              SUB03440
      MMAX=0                                                              SUB03450
      KMAX=0                                                              SUB03460
      TOTAL=0.                                                            SUB03470
      JJMX6=JJMX*6                                                        SUB03480
      IIMX2=(IIMX+1)/2                                                    SUB03490
      AR=ALPHA*RS                                                         SUB03500
      ARG=AR**2                                                           SUB03510
      EXPON=EXP(-ARG)                                                     SUB03520
      PIE=3.141592653589793                                               SUB03530
      APP=ALPHA/PIE/PIE                                                   SUB03540
      M=0                                                                 SUB03550
      RM=M                                                                SUB03560
      RIMAX=IMAX                                                          SUB03570
      DX=2./RIMAX                                                         SUB03580
      RJMAX=JMAX                                                          SUB03590
      DXI=2.*PIE/FCU                                                      SUB03600
C     INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY;  VECTORS:         SUB03610
C     (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED    SUB03620
      DO 1 II=1,IIMX2                                                     SUB03630
      RII=II                                                              SUB03640
      IIM=IIMX-II+1                                                       SUB03650
      H(II,3)=ALPHA*(RII*DX-DX-1.)                                        SUB03660
      H(IIM,3)=-H(II,3)                                                   SUB03670
      H(II,1)=2.*H(II,3)                                                  SUB03680
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.                                     SUB03690
      H(IIM,1)=-H(II,1)                                                   SUB03700
      H(IIM,2)=H(II,2)                                                    SUB03710
      H(II,5)=H(II,2)                                                     SUB03720
      H(IIM,5)=H(IIM,2)                                                   SUB03730
      H(II,4)=H(II,1)                                                     SUB03740
    1 H(IIM,4)=H(IIM,1)                                                   SUB03750
      SIGN=1.                                                             SUB03760
      FM=1.                                                               SUB03770
C     INITIATE THE SIN/COS ARRAY:                                         SUB03780
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      DO 11 J=1,JJMX                                                    SUB03790
      RJM=J-1                                                           SUB03800
      SCF(J,1)=0.                                                       SUB03810
      SCF(J,2)=1.                                                       SUB03820
      SCF(J,3)=SIN(RJM*DXI-PIE)                                         SUB03830
      SCF(J,4)=COS(RJM*DXI-PIE)                                         SUB03840
      SCF(J,5)=0.                                                       SUB03850
   11 SCF(J,6)=0.                                                       SUB03860
      MS=0                                                              SUB03870
C     COMMENCE THE M LOOP:                                              SUB03880
    2 SIGN=-SIGN                                                        SUB03890
      K=0                                                               SUB03900
      RK=K                                                              SUB03910
      ARM=1.                                                            SUB03920
      IF (M.NE.0) ARM=AR**M                                             SUB03930
      KTIMER=0                                                          SUB03940
      SIGNK=-1.                                                         SUB03950
C     COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:  SUB03960
      PM=0.                                                             SUB03970
      P=SQRT(1./FM)                                                     SUB03980
      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUB03990
C     ADVANCE THE SIN/COS ARRAY FOR NEW M:                              SUB04000
      DO 12 J=1,JJMX                                                    SUB04010
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                      SUB04020
   12 SCF(J,2)=SCF(J,2)*SCF(J,4)-SCF(J,1)*SCF(J,3)                      SUB04030
      DO 13 J=1,JMAX                                                    SUB04040
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                      SUB04050
   13 SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                      SUB04060
C     TEST FOR SYMMETRY SKIPS:                                          SUB04070
      IF (MS.EQ.MSYM) MS=0                                              SUB04080
      TOTAL=0.                                                          SUB04090
      MS=MS+1                                                           SUB04100
      IF (MS.NE.1) GO TO 7                                              SUB04110
      RMS=RM*SIG                                                        SUB04120
      CMS=COS(RMS)                                                      SUB04130
      SMS=SIN(RMS)                                                      SUB04140
C     COMMENCE THE K LOOP:                                              SUB04150
    3 INDEX=INDEX+1                                                     SUB04160
      SIGNK=-SIGNK                                                      SUB04170
C     CALL THE B & D COEFFICIENTS AND COMPUTE THE M,K SUMMATION TERM:   SUB04180
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                   SUB04190
      IF (DGN.LE.-2.) WRITE (6,89) M,K,B,D                              SUB04200
      BRAKET=B                                                          SUB04210
      IF(RM.EQ.0.) GO TO 4                                              SUB04220
      BRAKET=B*CMS+D*SMS                                                SUB04230
    4 ADD=SIGNK*BRAKET*P*ARM                                            SUB04240
      TOTAL=TOTAL+ADD                                                   SUB04250
C     ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES: SUB04260
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      CHECK=ABS(ADD)                                                    SUB04270
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                            SUB04280
C     ADVANCE THE K INDEX:                                              SUB04290
      K=K+1                                                             SUB04300
      RK=K                                                              SUB04310
      ORDER=M+2*K+1                                                     SUB04320
C     GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:         SUB04330
      PM=P                                                              SUB04340
      P=PP                                                              SUB04350
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                              SUB04360
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                    SUB04370
C     GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:SUB04380
      HA=SQRT(RK*(RK+RM))/ORDER                                         SUB04390
      HB=2.*SQRT((RK+1.)*(RM+RK+1.))/(ORDER+1.)/(ORDER+2.)              SUB04400
      DO 5 II=1,IIMX2                                                   SUB04410
      IIM=IIMX-II+1                                                     SUB04420
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                           SUB04430
      H(IIM,1)=SIGN*H(II,1)                                             SUB04440
    5 H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                        SUB04450
C     SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:           SUB04460
      KTIMER=KTIMER+1                                                   SUB04470
      IF (K.GE.KLIMIT) GO TO 6                                          SUB04480
      IF (CHECK.GE.EPS) KTIMER=0                                        SUB04490
      IF (KTIMER.LE.KEXTRA) GO TO 3                                     SUB04500
      GO TO 7                                                           SUB04510
C     END OF K LOOP:  ADVANCE M AND COMPUTE NEW TOTAL:                  SUB04520
    6 KOUT=KOUT+1                                                       SUB04530
    7 M=M+1                                                             SUB04540
      IF (K.GT.KMAX) KMAX=K                                             SUB04550
      RM=M                                                              SUB04560
      FM=FM*RM                                                          SUB04570
C     REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:                      SUB04580
      DO 8 II=1,IIMX2                                                   SUB04590
      IIM=IIMX-II+1                                                     SUB04600
      H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)                                  SUB04610
      H(II,1)=H(II,5)*(RM+2.)                                           SUB04620
      H(IIM,1)=-SIGN*H(II,1)                                            SUB04630
      H(II,2)=2.*SQRT(RM+1.)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))          SUB04640
      H(II,2)=H(II,2)/(RM+2.)/(RM+3.)                                   SUB04650
      H(II,4)=H(II,1)                                                   SUB04660
    8 H(II,5)=H(II,2)                                                   SUB04670
C     SET M TIMER FOR EXTRA M TERMS:                                    SUB04680
      IF (MS.EQ.1) MTIMER=MTIMER+1                                      SUB04690
      IF (JSYM.GT.JMAX) GO TO 9                                         SUB04700
      IF (K.GT.KEXTRA) MTIMER=0                                         SUB04710
      IF (M.GE.MLIMIT) GO TO 9                                          SUB04720
      IF (MTIMER.LE.MEXTRA) GO TO 2                                     SUB04730
C     END OF M LOOP:  COMPUTE OUTPUT SOLN.                              SUB04740
```

```
9       MOUT=M-1                                                        SUB04750
        IF (KOUT.EQ.0) KOUT=KMAX-1                                      SUB04760
        SOLN=TOTAL*EXPON*APP/2.                                         SUB04770
89      FORMAT ('   M=',I4,',   K=',I4,',   B=',E10.4,',   D=',E10.4)   SUB04780
        RETURN                                                          SUB04790
        END                                                             SUB04800
                                                                        SUB04810
                                                                        SUB04820
C000005                                                                 SUB00010
C                                                                       SUB00020
        SUBROUTINE GARRAY (G,GA,NOF,DGN,NUMB,XO,YO,PHISYM)              SUB00030
C                                                                       SUB00040
C     GARRAY FILLS THE DATA ARRAY OVER AN ORTHOGONAL AREA WITH          SUB00050
C     THE REGULAR DATA OBTAINED BY THE METHOD CORRESPONDING TO THE      SUB00060
C     PARTICULAR MODE:                                                  SUB00070
C                                                                       SUB00080
C         MODE 1  -  DATA OBTAINED BY SAMPLING A KNOWN FUNCTION SUPPLIED SUB00090
C                    IN SUBROUTINE FUNCT AND SAMPLED IN SUBROUTINE GOLF. SUB00100
C                                                                       SUB00110
C         MODE 2  -  DATA OBTAINED BY GENERATING A REGULAR ARRAY FROM   SUB00120
C                    IRREGULAR EXPERIMENTAL INPUT DATA READ IN.  CALLS  SUB00130
C                    SUBROUTINE SHEET. (EXPERIMENTAL DATA MAY           SUB00140
C                    BE SIMULATED, SEE 'SHEET')                         SUB00150
C                                                                       SUB00160
C         MODE 3  -  UTILIZES RAW DATA TAKEN AT THE PROPER INTERVAL,    SUB00170
C                    OR PREVIOUSLY GENERATED, AND READ DIRECTLY INTO THE SUB00180
C                    GARRAY.  CALLS SUBROUTINE READ.                    SUB00190
C                                                                       SUB00200
        COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z SUB00210
        COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                    SUB00220
        COMMON /IO/ CMS,IN1,IN2,IN4                                     SUB00230
        DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX)                            SUB00240
        PIE=3.141592653589793                                           SUB00250
        HS=SIZE/2.                                                      SUB00260
        IF (MODE.GT.3) MODE=1                                           SUB00270
        RIMX=IMAX                                                       SUB00280
        RJMX=JMAX                                                       SUB00290
        DELR=SIZE/RIMX                                                  SUB00300
        DELXI=2.*PIE/FCU                                                SUB00310
        IF (MODE.GT.1) GO TO 2                                          SUB00320
        DO 1 J=1,JMS                                                    SUB00330
        RJ=J                                                            SUB00340
        XI=(RJ-.5)*DELXI-PIE                                            SUB00350
        J2=J+2*(JMS-J)                                                  SUB00360
        J3=J+JMAX/2                                                     SUB00370
        J4=J2+JMAX/2                                                    SUB00380
```

```
      DO 1 I=1,IMS                                                      SUB00390
      RI=I                                                              SUB00400
      II=IMAX+1-I                                                       SUB00410
      R=(RI-.5)*DELR-HS                                                 SUB00420
      CALL GOLF (R,XI,GIJ,NOF,DGN,NUMB)                                 SUB00430
      G(I,J)=GIJ                                                        SUB00440
      IF (ISYM.EQ.2) G(II,J)=GIJ                                        SUB00450
      IF (ISYM.EQ.2) GO TO 1                                            SUB00460
      G(II,J3)=GIJ                                                      SUB00470
      IF (JSYM.EQ.0) GO TO 1                                            SUB00480
      G(I,J2)=GIJ                                                       SUB00490
      G(II,J4)=GIJ                                                      SUB00500
1     CONTINUE                                                          SUB00510
      GO TO 4                                                           SUB00520
2     IF (MODE.GT.2) GO TO 3                                            SUB00530
      CALL SHEET (G,GA,XO,YO,PHISYM,NOF)                                SUB00540
      GO TO 4                                                           SUB00550
3     CALL READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)                        SUB00560
4     IF (DGN.GE.2) WRITE (6,39)                                        SUB00570
      RETURN                                                            SUB00580
39    FORMAT ('   GARRAY RETURNS')                                      SUB00590
      END                                                               SUB00600
C000006                                                                 SUB00610
C                                                                       SUB00620

      SUBROUTINE GOLF (R,XI,GIJ,NOF,DGN,NUMB)                           SUB00630
C                                                                       SUB00640
C   GOLF COMPUTES THE FUNCTION G(R,XI) FOR A PARTICULAR LINE OF SIGHT   SUB00650
C   FROM A KNOWN FUNCTION CONTAINED IN SUBROUTINE FUNCT.                SUB00660
C                                                                       SUB00670
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB00680
      ZERO=0.                                                           SUB00690
      LMAX=IMAX*3                                                       SUB00700
      RLMAX=LMAX                                                        SUB00710
      DELXP=SIZE/RLMAX                                                  SUB00720
      SXI=SIN(XI)                                                       SUB00730
      CXI=COS(XI)                                                       SUB00740
      DELXS=DELXP*CXI                                                   SUB00750
      DELYS=DELXP*SXI                                                   SUB00760
      XP=DELXP*.5-SIZE/2.                                               SUB00770
      XS=XP*CXI-R*SXI                                                   SUB00780
      YS=XP*SXI+R*CXI                                                   SUB00790
      GIJ=0.                                                            SUB00800
      DO 1 L=1,LMAX                                                     SUB00810
      RL=L                                                              SUB00820
      CALL FUNCT(XS,YS,F,NOF,DGN,NUMB)                                  SUB00830
      GIJ=GIJ+F                                                         SUB00840
```

```
      XS=XS+DELXS
1     YS=YS+DELYS
      IF (GIJ.NE.0.) GIJ=GIJ*DELXP*BOX
      IF ((SD.EQ.0.).OR.(NUMB.EQ.1)) GO TO 2
      IF (DGN.GE.3) WRITE (6,28) IX
      CALL GAUSS (IX,SD,ZERO,RV)
      GIJ=GIJ+RV
2     IF (DGN.GE.3) WRITE (6,29) R,XI,GIJ
      RETURN
29    FORMAT ('    R=',F8.3,',    XI=',F8.3,',    GIJ=',F8.3)
28    FORMAT ('    GAUSS, IX=',I8)
      END
C000007
C

      SUBROUTINE FUNCT (XS,YS,F,NOF,DGN,NUMB)
C

CP67USERID  1095BOXJ
C
C   FUNCT EVALUATES AS INPUT FUNCTION AT POSITION (X,Y) IN THE TEST
C   SECTION COORDINATE SYSTEM.  NOF IDENTIFIES THE EQUATION USED.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2
      DIMENSION RO(101),RA(101)
      AA=A
      BB=B
      CC=C
      DD=D
      EE=E
      PP=P
      IF (NUMB.LE.1) GO TO 50
      AA=S
      BB=T
      CC=U
      DD=V
      EE=W
      PP=Q
50    PIE=3.141592653589793
      HS=SIZE/2.
      R=SQRT(XS**2+YS**2)/HS
      F=0.
      IF (R.GT.1.) GO TO 11
      IF (NOF.LE.0) GO TO 11
C
C   1.  AXISYMMETRIC GAUSSIAN:
```

```
1       IF (NOF.GT.1) GO TO 2                                           SUB01290
        F=AA*EXP(-1.*(R*HS/BB)**2)                                      SUB01300
        GO TO 11                                                        SUB01310
C                                                                       SUB01320
C    2.  ADJUSTABLE RECTANGULAR STEP FUNCTION:                          SUB01330
2       IF (NOF.GT.2) GO TO 3                                           SUB01340
        F=PP                                                            SUB01350
        IF ((ABS(XS-DD).LE.BB).AND.(ABS(YS-EE).LE.CC)) F=AA             SUB01360
        GO TO 11                                                        SUB01370
C                                                                       SUB01380
C    3.  DISPLACABLE ELLIPTICAL GAUSSIAN:                               SUB01390
3       IF (NOF.GT.3) GO TO 4                                           SUB01400
        F=AA*EXP(-1.*(((XS-DD)/BB)**2+((YS-EE)/CC)**2))                 SUB01410
        GO TO 11                                                        SUB01420
C                                                                       SUB01430
C    4.  CONSTANT:                                                      SUB01440
4       IF (NOF.GT.4) GO TO 5                                           SUB01450
        F=AA                                                            SUB01460
        GO TO 11                                                        SUB01470
C                                                                       SUB01480
C    5.  ADJUSTABLE AND DISPLACABLE ELLIPTIC RAMP FUNCTION:             SUB01490
5       IF (NOF.GT.5) GO TO 6                                           SUB01500
        RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                       SUB01510
        F=0.                                                            SUB01520
        IF (RBC.LT.1.) F=AA*((1.-RBC)**PP)                              SUB01530
        GO TO 11                                                        SUB01540
C                                                                       SUB01550
C    6.  DISPLACABLE ELLIPTIC STEP FUNCTION:                            SUB01560
6       IF (NOF.GT.6) GO TO 7                                           SUB01570
        RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                       SUB01580
        F=0.                                                            SUB01590
        IF (RBC.LT.1.) F=AA                                             SUB01600
        GO TO 11                                                        SUB01610
C                                                                       SUB01620
C    7.  CIRCULAR COSINE-SQUARED FUNCTION OF BB MAXIMA:                 SUB01630
7       IF (NOF.GT.7) GO TO 8                                           SUB01640
        F=AA*COS((2.*BB-1.)*PIE*R/2.)**2                                SUB01650
        GO TO 11                                                        SUB01660
C                                                                       SUB01670
C    8.  NUMERICAL FUNCTION:  REQUIRES AN INPUT ARRAY READ IN BY        SUB01680
C        SUBROUTINE FREAD; N FOLLOWED BY N POINT VALUES. (101 MAX)      SUB01690
C        A CONSTANT VALUE AA IS ADDED TO THE FUNCTION.                  SUB01700
8       IF (NOF.GT.8) GO TO 9                                           SUB01710
        IF (NUMB.LE.1) N=NO                                             SUB01720
        IF (NUMB.GT.1) N=NA                                             SUB01730
        NM=N-1                                                          SUB01740
        NMM=N-2                                                         SUB01750
        RN=N                                                            SUB01760
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      RI=R*(RN-1.)+1.                                                   SUB01770
      IR=INT(RI)                                                        SUB01780
      RIR=FLOAT(IR)                                                     SUB01790
      DI=RI-RIR                                                         SUB01800
      IF (NUMB.LE.1) F=RO(IR)                                           SUB01810
      IF (NUMB.GT.1) F=RA(IR)                                           SUB01820
      IF ((IR.NE.N).AND.(NUMB.LE.1)) F=F+DI*(RO(IR+1)-RO(IR))           SUB01830
      IF ((IR.NE.N).AND.(NUMB.GT.1)) F=F+DI*(RA(IR+1)-RA(IR))           SUB01840
      F=F*AA+BB                                                         SUB01850
      GO TO 11                                                          SUB01860
C                                                                       SUB01870
C   9.  SPECIAL FUNCTION:  MAY BE WRITTEN FOR THE OCCASION AND          SUB01880
C         INSERTED IN SUBROUTINE SPFUN                                  SUB01890
9     IF (NOF.GT.9) GO TO 10                                            SUB01900
      CALL SPFUN (XS,YS,F)                                              SUB01910
      GO TO 11                                                          SUB01920
C                                                                       SUB01930
C   EQUATIONS NO. 10 AND BEYOND ARE SET TO ZERO.                        SUB01940
10    F=0.                                                              SUB01950
C                                                                       SUB01960
C                                                                       SUB01970
11    IF (DGN.GE.4) WRITE  (6,99) XS,YS,F                               SUB01980
99    FORMAT ('    XS=',F8.3,',    YS=',F8.3,',    F=',F8.3)            SUB01990
      RETURN                                                            SUB02000
      END                                                               SUB02010
C000008                                                                 SUB02020
C                                                                       SUB02030


      SUBROUTINE SPFUN (XS,YS,F)                                        SUB02040
C                                                                       SUB02050
C   SPFUN IS A SPECIAL ROUTINE FOR EQ'N NO. 9.  ANY FUNCTION MAY BE     SUB02060
C   ENTERED.                                                            SUB02070
C                                                                       SUB02080
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2         SUB02090
      DIMENSION RO(101),RA(101)                                         SUB02100
      F=0.                                                              SUB02110
      IF ((ABS(XS).LE.B).AND.(ABS(YS).LE.C))  F=A                       SUB02120
      RETURN                                                            SUB02130
      END                                                               SUB02140
C000009                                                                 SUB02150
C                                                                       SUB02160
```

```
      SUBROUTINE SHEET (G,D,XO,YO,PHISYM,NOF)
C
C     SHEET READS IRREGULARLY SPACED VALUES OF THE LINE INTEGRAL, AS
C     OBTAINED FROM HOLOGRAPHIC INTERFEROGRAMS.  THE INTEGRAL LINES MAY BE
C     DEFINED EITHER BY GRID INTERCEPT POSITIONS, OR BY ANGLE AND RADIUS
C     ABOUT THE CENTER OF THE LABORATORY COORDINATE SYSTEM CENTER.  LINES
C     MUST BE ENTERED IN CONSECUTIVE ORDER FROM LOWEST (NEG.) TO HIGHEST
C     (POS.) RADIUS.  DATA MAY BE SIMULATED BY SPECIFYING NCODE=1,
C     FOLLOWED BY APERATURE POSITIONS FOR A FUNCTION NUMBER IN 'SUBFUNCT'.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      COMMON /SYM/ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
      COMMON /IO/ CMS,IN1,IN2,IN4
      DIMENSION G(IMAX,JMAX),D(IMAX,JMAX),XI(303),RR(303)
      DIMENSION XG(303),XD(303),YG(303),YD(303),XY(303)
      NAR=303
      PIE=3.141592653589793
      MPIE=-PIE
      TPIE=2.*PIE
      MPIT=MPIE/2.
      PIET=PIE/2.
C     ZERO THE ARRAYS:
      DO 1 J=1,JMAX
      DO 1 I=1,IMAX
      G(I,J)=0.
1     D(I,J)=0.
      DO 2 I=1,NAR
      XG(I)=0.
      XD(I)=0.
      YG(I)=0.
      YD(I)=0.
      XY(I)=0.
      XI(I)=0.
2     RR(I)=0.
C     READ THE BASIC DATA:
      IF (CMS.EQ.1.) REWIND 1
      READ (IN1,59) NOF,NCODE
      READ (IN1,58) Z,XO,YO,PHISYM,XMX,XMN,YMX,YMN
      READ (IN1,59) JM
      RIMX=IMAX
      DR=SIZE/RIMX
      R=(-DR-SIZE)/2.
      RZO=SQRT(XO**2+YO**2)
      GAM=ATANM(YO,XO)
      TP=3-ISYM
      BT=JSYM
      DAN=PIE*TP/BT
      HS=SIZE/2.
```

```
      MXY=1
      IF((XMX.NE.0.).OR.(XMN.NE.0.).OR.(YMX.NE.0.).OR.(YMN.NE.0.))MXY=0
      IF (MXY.EQ.1) GO TO 3
      XMX=XO+HS
      YMX=YO+HS
      XMN=XO-HS
      YMN=YO-HS
C   COMMENCE THE READ AND FILL LOOP:
3     DO 12 J=1,JM
      READ (IN1,59)IM
      MN=0
      XH=0.
      YH=0.
C   READ THE LINES, DETERMINE CODE, CALCULATE RADIUS & ANGLE FOR CODE 1:
      DO 5 I=1,IM
      IF(NCODE.LE.0)READ(IN1,58) XD(I),YD(I),XG(I),YG(I),D(I,J),RR(I),
     1XI(I),XY(I)
      IF(NCODE.GE.1)CALL SIM(XD(I),YD(I),XG(I),YG(I),RR(I),XI(I),
     1XY(I),XO,YO,PHISYM,XMX,XMN,YMX,YMN,NOF,I,IM)
      IF (XY(I).EQ.3.) GO TO 5
      IF((XD(I).NE.0.).OR.(YD(I).NE.0.)) XY(I)=1.
      IF((XG(I).NE.0.).OR.(YG(I).NE.0.)) XY(I)=1.
      IF(((RR(I).NE.0.).OR.(XI(I).NE.0.)).AND.(XY(I).EQ.0.)) XY(I)=2.
      IF((XY(I).EQ.0.).AND.(D(I,J).NE.0.)) XY(I)=2.
      IF (XY(I).NE.1.) GO TO 4
      DEN=SQRT((XG(I)-XD(I))**2+(YG(I)-YD(I))**2)
      IF (DEN.EQ.0.) XY(I)=4.
      IF (XY(I).EQ.4.) GO TO 4
      RR(I)=((XO-XD(I))*(YG(I)-YD(I))-(XG(I)-XD(I))*(YO-YD(I)))/DEN
      XI(I)=ATANM((YG(I)-YD(I)),(XG(I)-XD(I)))
      XIM=XI(I)
4     IF (XY(I).EQ.2.) XIM=XI(I)
      XIN=XIM
      IF(XY(I).EQ.2.) RR(I)=RR(I)+RZO*SIN(GAM-XI(I))
5     CONTINUE
C   COMPUTE MAX AND MIN ANGLE INDEXES FOR APERATURE POSITION LOCATION:
      DO 6 I=1,IM
      IF((XY(I).NE.1.).OR.(XY(I).NE.2.)) GO TO 6
      IF (XI(I).GT.XIM) XIM=XI(I)
      IF (XI(I).GT.XIM) IMT=I
      IF (XI(I).LT.XIN) XIN=XI(I)
      IF (XI(I).LE.XIN) INT=I
6     CONTINUE
C   DETERMINE APERATURE LOCATION:
      LPR=0
      XID=XI(IMT)-XI(INT)
      IF(ABS(XID).LT..00001) LPR=1
      XIH=(XI(IMT)+XI(INT))/2.
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```
      RRH=10000.                                                        SUB03130
      XH=RRH*COS(XIH)                                                   SUB03140
      YH=RRH*SIN(XIH)                                                   SUB03150
      IF (LPR.EQ.1) GO TO 7                                             SUB03160
      YTX=-RR(IMT)*SIN(XI(IMT))-YO                                      SUB03170
      YTN=-RR(INT)*SIN(XI(INT))-YO                                      SUB03180
      XTX=RR(IMT)*COS(XI(IMT))-XO                                       SUB03190
      XTN=RR(INT)*COS(XI(INT))-XO                                       SUB03200
      UA=TAN(XI(IMT))                                                   SUB03210
      UC=TAN(XI(INT))                                                   SUB03220
      UB=YTX-UA*XTX                                                     SUB03230
      UD=YTN-UC*XTN                                                     SUB03240
      XH=(UD-UB)/(UA-UC)                                                SUB03250
      YH=XH*UA+UB                                                       SUB03260
      RRH=SQRT((XH-XO)**2+(YH-YO)**2)                                   SUB03270
      XIH=ATANM((YH-YO),(XH-XO))                                        SUB03280
    7 CONTINUE                                                          SUB03290
C   FILL THE ANGLE AND RADIUS FOR ANY CODE 3 OR 4 LINES:                SUB03300
      DO 9 I=1,IM                                                       SUB03310
      IF(XY(I).NE.3.) GO TO 8                                           SUB03320
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUB03330
      XI(I)=XIH-ATANM(RR(I),BAS)                                        SUB03340
      GO TO 9                                                           SUB03350
    8 XI(I)= ATANM((YH-YD(I)),(XH-XD(I)))                               SUB03360
      RR(I)=RRH*SIN(XI(I)-XIH)                                          SUB03370
    9 CONTINUE                                                          SUB03380
C   ANGLES AND RADII ARE NOW FILLED FOR ALL POINTS IN THIS LINE.        SUB03390
C   VACATE THE SET OF VECTORS TO BE USED AS TEMPORARY STORAGE:          SUB03400
      DO 10 I=1,IM                                                      SUB03410
      XD(I)=0.                                                          SUB03420
      YD(I)=0.                                                          SUB03430
      XG(I)=0.                                                          SUB03440
      YG(I)=RR(I)                                                       SUB03450
      XY(I)=D(I,J)                                                      SUB03460
      RR(I)=0.                                                          SUB03470
      D(I,J)=0.                                                         SUB03480
   10 XI(I)=0.                                                          SUB03490
C   CONVERT THE LINE TO REGULAR RADII USING INTERPOLATION:              SUB03500
      RR(1)=R+DR                                                        SUB03510
      CALL SPLINE(YG,XY,IM,RR(1),D(1,J))                                SUB03520
      DO 11 I=2,IMAX                                                    SUB03530
      RI=I                                                              SUB03540
      RR(I)=R+DR*RI                                                     SUB03550
   11 CALL SPLINN(YG,XY,IM,RR(I),D(I,J))                                SUB03560
C   GENERATE THE VECTOR OF ANGLES FOR THIS COLUMN AND STORE IN G ARRAY: SUB03570
      DO 12 I=1,IMAX                                                    SUB03580
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUB03590
      G(I,J)=XIH-ATANM(RR(I),BAS)                                       SUB03600
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      YG(I)=0.                                                          SUB03610
      D(I,J)=XY(I)                                                      SUB03620
12    XY(I)=0.                                                          SUB03630
C     COLUMNS ARE NOW ALL REGULARLY FILLED.                             SUB03640
C     NEXT,INTERPOLATE EACH ROW REGULARLY OVER THE ANGLES.              SUB03650
      DO 23 I=1,IMAX                                                    SUB03660
C     EXPAND THE DATA TO 2 SETS TO ESTABLISH SMOOTH INTERPOLATION.      SUB03670
      JM3=3*JM                                                          SUB03680
      II=IMAX+1-I                                                       SUB03690
      IF (JSYM.NE.0) GO TO 14                                           SUB03700
      DO 13 J=1,JMS                                                     SUB03710
      J2=J+JMS                                                          SUB03720
      J3=J2+JMS                                                         SUB03730
      XD(J)=D(I,J)                                                      SUB03740
      XD(J2)=D(II,J)                                                    SUB03750
      XD(J3)=D(I,J)                                                     SUB03760
      XG(J)=G(I,J)-TPIE-PHISYM                                          SUB03770
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUB03780
13    XG(J3)=G(I,J)-PHISYM                                              SUB03790
      GO TO 16                                                          SUB03800
14    DO 15 J=1,JMS                                                     SUB03810
      J1=JMS+1-J                                                        SUB03820
      J2=JMS+J                                                          SUB03830
      J3=JM3+1-J                                                        SUB03840
      XD(J1)=D(I,J)                                                     SUB03850
      XD(J2)=D(II,J)                                                    SUB03860
      XD(J3)=D(II,J)                                                    SUB03870
      XG(J1)=G(I,J)-2.*(G(I,J)-PHISYM)-PIE-PHISYM                       SUB03880
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUB03890
15    XG(J3)=G(II,J)+2.*(DAN+PHISYM-G(II,J))-PIE-PHISYM                 SUB03900
16    CONTINUE                                                          SUB03910
      JM2=2*JMS                                                         SUB03920
      JP=JMS/2                                                          SUB03930
      DO 17 J=1,JM2                                                     SUB03940
      XD(J)=XD(J+JP)                                                    SUB03950
17    XG(J)=XG(J+JP)                                                    SUB03960
      JJS=JM2+1                                                         SUB03970
      DO 18 J=JJS,JM3                                                   SUB03980
      XD(J)=0.                                                          SUB03990
18    XG(J)=0.                                                          SUB04000
C     FIND THE SMALLEST ANGLE                                           SUB04010
      XY(1)=1.                                                          SUB04020
      SA=XG(1)                                                          SUB04030
      DO 19 J=1,JM2                                                     SUB04040
      IF (XG(J).GE.SA) GO TO 19                                         SUB04050
      SA=XG(J)                                                          SUB04060
      XY(1)=J                                                           SUB04070
19    CONTINUE                                                          SUB04080
```

```
C    FIND THA MAX ANGLE IN THE ROW:
       XY(JM2)=JM2
       SB=XG(JM2)
       DO 20 J=1,JM2
       IF (XG(J).LE.SB) GO TO 20
       SB=XG(J)
       XY(JM2)=J
20     CONTINUE
C    DETERMINE THE ORDER OF INCREASING ANGLE IN THE ROW
       SB=XG(JM2)
       JJ=2
21     JSA=XY(JJ-1)
       SA=XG(JJ-1)
       JTS=0
       DO 22 J=1,JM2
       IF (XG(J).LE.SA) GO TO 22
       IF (XG(J).GT.SB) GO TO 22
       SB=XG(J)
       XY(JJ)=J
       JTS=1
22     CONTINUE
       IF (JTS.EQ.0) JM2=JJ
       JJ=JJ+1
       IF (JJ.LE.JM2) GO TO 21
       DO 23 J=1,JM2
       JX=XY(J)
23     YD(J)=XD(JX)
C    INTERPOLATE:
       DXI=2.*PIE/FCU
       XI(J)=DXI/2.-PIE-PHISYM
       CALL SPLINE (XG,YD,JM2,XI(J),G(I,J))
       DO 24 J=2,JMS
       XI(J)=XI(J-1)+DXI
24     CALL SPLINN (XG,YD,JM2,XI(J),G(I,J))
       DO 25 J=1,JMS
       XIJ=XI(J)
       XU=XMX
       IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XU=XMN
       YU=YMN
       IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YU=YMX
       XL=XMN
       IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XL=XMX
       YL=YMX
       IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YL=YMN
       SXIJ=SIN(XIJ)
       CXIJ=COS(XIJ)
       RMN=(XO-XL)*SXIJ-(YO-YL)*CXIJ
       RMX=(XO-XU)*SXIJ-(YO-YU)*CXIJ
```

```
      DO 25 I=1,IMAX                                                    SUB04570
      IF (RR(I).LT.RMN) G(I,J)=0.                                       SUB04580
      IF (RR(I).GT.RMX) G(I,J)=0.                                       SUB04590
25    CONTINUE                                                          SUB04600
C   EXPAND SYMMETRY SECTOR INTO AN ORTHOGONAL INTERVAL.                 SUB04610
      IF (ISYM.EQ.2) GO TO 27                                           SUB04620
      DO 26 J=1,JMS                                                     SUB04630
      J2=JMAX/2+1-J                                                     SUB04640
      J3=JMAX/2+J                                                       SUB04650
      J4=JMAX+1-J                                                       SUB04660
      DO 26 I=1,IMAX                                                    SUB04670
      II=IMAX+1-I                                                       SUB04680
      G(I,J2)=G(I,J)                                                    SUB04690
      G(II,J3)=G(I,J)                                                   SUB04700
26    G(II,J4)=G(I,J)                                                   SUB04710
      RETURN                                                            SUB04720
C   FOR EVEN SYMMETRY, AVERAGE THE GARRAY COLUMNS.                      SUB04730
27    IMS=(2*IMAX+1)/2                                                  SUB04740
      DO 28 J=1,JMAX                                                    SUB04750
      DO 28 I=1,IMS                                                     SUB04760
      II=IMAX+1-I
      GST=(G(I,J)+G(II,J))/2.                                           SUB04780
      G(I,J)=GST                                                        SUB04790
28    G(II,J)=GST                                                       SUB04800
      RETURN                                                            SUB04810
59    FORMAT (5I5)                                                      SUB04820
58    FORMAT (10F7.3)                                                   SUB04830
      END                                                               SUB04840
C000010                                                                 SUB04850
C                                                                       SUB04860


      FUNCTION ATANM(Y,X)                                               SUB04870
C                                                                       SUB04880
C   COMPUTES THE ARCTAN OF Y/X BETWEEN -P1 AND +P1.                     SUB04890
C                                                                       SUB04900
      PIE=3.141592653589793                                             SUB04910
      P12=PIE/2.                                                        SUB04920
      ATANM=SIGN(PI2,Y)                                                 SUB04930
      IF(X.NE.0.) ATANM=ATAN(Y/X)                                       SUB04940
      IF(X.GE.0.) RETURN                                                SUB04950
      IF(Y.GE.0.) ATANM=PIE+ATANM                                       SUB04960
      IF(Y.LT.0.) ATANM=-PIE+ATANM                                      SUB04970
      RETURN                                                            SUB04980
      END                                                               SUB04990
C000011                                                                 SUB05000
C                                                                       SUB05010
```

```
      SUBROUTINE SIM (XD,YD,XG,YG,D,R,XI,XY,XO,YO,PS,XM,XN,YM,YN,I,IM,   SUB05020
     1DG,NF)                                                             SUB05030
C    SIM SIMULATES THE FRINGE NUMBER DATA ONE WOULD OBTAIN FROM THE      SUB05040
C    HOLOGRAPHIC INTERFEROGRAM PROCESS FOR A KNOWN FUNCTION AS           SUB05050
C    CONTAINED IN SUBROUTINE FUNCT.  THE GRID BOX DIMENSIONS MUST        SUB05060
C    EXCEED THE INVERSION CIRCLE SIZE, AND APERATURE POINTS SPECIFIED    SUB05070
C    MUST FALL BETWEEN XI=-40 DEGREES, AND XI=+130 DEGREES.              SUB05080
C                                                                        SUB05090
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALF,SIZ,EPS,MOD,BOX,SD,IX,Z        SUB05100
      COMMON /IO/ CMS,IN1,IN2,IN4                                        SUB05110
      READ (IN1,29) XH,YH                                                SUB05120
      ZER=0.                                                             SUB05130
      RIM=IM                                                             SUB05140
      RI=I                                                               SUB05150
      DX=(XM-XN)/(RIM-1.)                                                SUB05160
      DY=(YM-YN)/(RIM-1.)                                                SUB05170
      X1=XM-(RIM-1.)*DX                                                  SUB05180
      Y1=YM-(RIM-1.)*DY                                                  SUB05190
      XI1=ATANM (YH-Y1,XH-X1)-PS                                         SUB05200
      RRH=SQRT((XH-XO)**2+(YH-YO)**2)                                    SUB05210
      XIO=ATANM(YH-YO,XH-XO)-PS                                          SUB05220
      R1=RRH*SIN(XI1-XIO)                                                SUB05230
      IF ((I.GT.1).AND.(I.LT.IM)) GO TO 1                                SUB05240
      IF (ABS(R1).LT.SIZ/2.) R1=SIGN(SIZ/2.,R1)                          SUB05250
      XY=3.                                                              SUB05260
      GO TO 2                                                            SUB05270
1     CALL GOLF (R,XI,D,NR,ZER,ZER)                                      SUB05280
      XD=XN                                                              SUB05290
      YD=YN                                                              SUB05300
      IF (XH.NE.X1) YD=Y1-(X1-XN)*(YH-Y1)/(XH-X1)                        SUB05310
      IF (YS.GE.YN) GO TO 2                                              SUB05320
      YD=YN                                                              SUB05330
      IF (YH.NE.Y1) XD=X1-(Y1-YN)*(XH-X1)/(YH-Y1)                        SUB05340
2     RETURN                                                             SUB05350
29    FORMAT (10F7.3)                                                    SUB05360
      END                                                                SUB05370
                                                                         SUB05380
C000012                                                                  SUB00010
C                                                                        SUB00020

      SUBROUTINE FREAD (NO,RO,NF,ZZ)                                     SUB00030
C                                                                        SUB00040
C    FREAD READS THE NUMERIC ARRAY WHICH IS USED FOR EQUATION 8 OF       SUB00050
C    SUBROUTINE FUNCT.  FIRST CARD IS NUMBER OF POINTS (N.GE.1),         SUB00060
C    FOLLOWED BY ONE POINT PER CARD.                                     SUB00070
C                                                                        SUB00080
      DIMENSION RO(101)                                                  SUB00090
```

```
      READ (NF,89) NO,ZZ                                                SUB00100
      WRITE(6,90) NO,ZZ
      DO  10  I=1,NO
      READ(NF,88)  RO(I)
      WRITE(6,88)  RO(I)
   10 CONTINUE
89    FORMAT (I5,F9.3)                                                  SUB00120
   88 FORMAT(F8.5)
   90 FORMAT(1X,I5,F9.3)
      RETURN                                                            SUB00140
      END                                                               SUB00150
C000013                                                                 SUB00160
C                                                                       SUB00170


      SUBROUTINE GPRINT (G,NUMB)                                        SUB00180
C                                                                       SUB00190
C   GPRINT PRINTS THE DATA ARRAY 'G'  WHICH WAS INPUT TO                SUB00200
C   THE PROGRAM IN SUBROUTINE GARRAY.                                   SUB00210
C                                                                       SUB00220
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB00230
      DIMENSION G(IJMX)                                                 SUB00240
      DIMENSION X(15)                                                   SUB00250
      DATA HYP,VERT/1H-,1HI/                                            SUB00260
      IF (NUMB.EQ.1) WRITE (6,99) MODE,Z                                SUB00270
      IF (NUMB.EQ.2) WRITE (6,92) Z                                     SUB00280
      JM2=JMAX/2                                                        SUB00290
      RIMAX=IMAX                                                        SUB00300
      RJMAX=JMAX                                                        SUB00310
      DX=SIZE/RIMAX                                                     SUB00320
      DXI=360./RJMAX                                                    SUB00330
C   INTRVL SETS THE NUMBER OF TERMS PRINTED PER LINE. IF IT IS ALTERED, SUB00340
C   ONE MUST ALSO REDIMENSION X AND ALTER FORMATS 98, 97, AND 95.       SUB00350
      INTRVL=15                                                         SUB00360
      IB=1                                                              SUB00370
      IT=IB+INTRVL-1                                                    SUB00380
1     IF (IT.GT.IMAX) IT=IMAX                                           SUB00390
      IBT=IT-IB+1                                                       SUB00400
      WRITE (6,98) (II,II=IB,IT)                                        SUB00410
      DO 2 I=1,IBT                                                      SUB00420
      RI=IB-1+I                                                         SUB00430
2     X(I)=-SIZE/2.+(RI-.5)*DX                                          SUB00440
      LM=7*IBT+1                                                        SUB00450
      WRITE (6,97) (X(I),I=1,IBT)                                       SUB00460
      WRITE (6,96) (HYP,L=1,LM),VERT                                    SUB00470
      JMH=JM2+1                                                         SUB00480
      DO 3 J=JMH,JMAX                                                   SUB00490
      RJ=J                                                              SUB00500
```

```
      XI=-180.+DXI*(RJ-.5)
      IGB=(J-1)*IMAX+IB
      IGT=IGB-IB+IT
3     WRITE (6,95) J,XI,(G(L),L=IGB,IGT)
      WRITE (6,94) (HYP,L=1,LM),VERT
      IB=IB+INTRVL
      ITOLD=IT
      IT=IT+INTRVL
      IF (ITOLD.LT.IMAX) GO TO 1
      WRITE (6,93)
99    FORMAT (1H1//'   THE ARRAY OF INPUT DATA (G), OBTAINED BY GARRAY,
     1MODE ',I1,', FOR Z='F7.3,' CM:')
98    FORMAT (///11X,'I= ',15I7)
97    FORMAT (11X,'X=   ',15F7.3)
96    FORMAT ('   J=      XI= |',115A1)
95    FORMAT (2X,I3,F9.2,'|',15F7.3)
94    FORMAT (14X,'|',115A1)
93    FORMAT (///5X)
92    FORMAT (1H1//'   THE ADD-ON FUNCTION GARRAY FOR Z=',F7.3,' CM:')
      RETURN
      END
C000014
C


      SUBROUTINE GPUNCH (Z,XO,YO,PHS,NOF,IMX,JMX,G)
C
C   GPUNCH PUNCHES OUT THE FIRST NON-SYMMETRIC PORTION OF GARRAY
C   (OR WRITES IT ON FILE 7 IN CMS VERSION)
C
      COMMON /SYM/ ISM,JSM,MSM,FCU,IMS,JMS,QSM
      DIMENSION G(IMX,JMX)
      WRITE (7,39) NOF,IMX,JMX,ISM,JSM,IMS,JMS
      WRITE (7,38) ((G(I,J),I=1,IMS),J=1,JMS)
39    FORMAT(10I5)
38    FORMAT(10F7.3)
      RETURN
      END
C000015
C


      SUBROUTINE READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)
C
C   READS THE NON-SYMMETRIC PORTION OF THE GARRAY AND EXPANDS IT TO AN
C   ORTHOGONAL SET. NOTE, INSURE SUFFICIENT DIMENSIONS IN MAIN PROGRAM.
C
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM
```

```
      COMMON /IO/ CMS,IN1,IN2,IN4                                       SUB00950
      DIMENSION G(IMAX,JMAX)                                            SUB00960
      READ (IN1,39) NOF,IMAX,JMAX,ISYM,JSYM,IMS,JMS                     SUB00970
      READ (IN1,38) Z,XO,YO,PHISYM                                      SUB00980
      READ (IN1,38) ((G(I,J),I=1,IMS),J=1,JMS)                          SUB00990
      WRITE(6,37) NOF,Z,XO,YO,PHISYM,IMAX,JMAX,JSYM                     SUB01000
      RJMX=JMAX                                                         SUB01010
      MSYM=JSYM                                                         SUB01020
      IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                         SUB01030
      FCU=ISYM*JSYM*JMAX                                                SUB01040
      IF (JSYM.GT.JMAX) FCU=JMAX                                        SUB01050
      QSYM=FCU/RJMX                                                     SUB01060
      DO 4 J=1,JMS                                                      SUB01070
      IF(ISYM.EQ.1) GO TO 2                                             SUB01080
      DO 1 I=1,IMS                                                      SUB01090
      II=IMAX+1-I                                                       SUB01100
1     G(II,J)=G(I,J)                                                    SUB01110
      GO TO 4                                                           SUB01120
2     J2=JMAX/2+1-J                                                     SUB01130
      J3=JMAX/2+J                                                       SUB01140
      J4=JMAX+1-J                                                       SUB01150
      DO 3 I=1,IMAX                                                     SUB01160
      II=IMAX+1-I                                                       SUB01170
      G(I,J2)=G(I,J)                                                    SUB01180
      G(II,J3)=G(I,J)                                                   SUB01190
3     G(II,J4)=G(I,J)                                                   SUB01200
4     CONTINUE                                                          SUB01210
39     FORMAT(10I5)                                                     SUB01220
38    FORMAT (10F7.3)                                                   SUB01230
37    FORMAT(///'  MODE 3 READS GARRAY DIRECTLY:   NOF='I4/'     Z='F7.3SUB01240
     1,',   XO=',F7.3,',   YO=',F7.3,',   PHISYM=',F7.3/'    IMAX=',I4,SUB01250
     2',   JMAX=',I4,',   JSYM=',I4///)                                 SUB01260
      RETURN                                                            SUB01270
      END                                                               SUB01280
C000016                                                                 SUB01290
C                                                                       SUB01300

      SUBROUTINE MAP (IM,JM,A,N,Z,BAND)                                 SUB01310
C                                                                       SUB01320
C    MAP CALLS SUBROUTINE MIMPII AND PLOTS A CONTOUR MAP OF THE ARRAY   SUB01330
C                                                                       SUB01340
      DIMENSION A(IM,JM),T(24)                                          SUB01350
      DATA BL/1H /                                                      SUB01360
      DO 1 I=1,24                                                       SUB01370
1     T(I)=BL                                                           SUB01380
      ICON=1                                                            SUB01390
      IF(BAND.LT.0.) ICON=0                                             SUB01400
```

```
      IF(BAND.LT.0.) BAND=-BAND
      AMIN=0.
      IJT=0
      AZ=1.
      BZ=0.
      WRITE(6,49) N,Z
      CALL MTMPII (A,IM,JM,T,BAND,AZ,BZ,AMIN,IJT,ICON)
49    FORMAT (1H1//'  THE FUNCTION SURFACE, TEST NO.'I3,'  Z='F5.3//)
      RETURN
      END
C000017
C


      SUBROUTINE GPLOT (G,GA,JMS)
C
C   GPLOT PRINTS A ROUGH PLOT OF THE LINE INTEGRAL FUNCTIONS IN GARRAY.
C
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z
      CCMMON /TAB/ INDEX(7),JSYM,ISYM
      COMMON /TAB2/ IPT,KPT,LPT,MPT,REST(5)
      DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX),ROW(101)
      DIMENSION A(201),B(101),C(201),D(101)
      JM=101
      DATA BL,PL,ST,DH,EX/1H ,1H+,1H*,1H-,1HX/
      JMS2=JMAX/2+1
      IF(ISYM.EQ.2)JMS2=1
      JMS3=JMS2+JMS-1
      DO 8 J=JMS2,JMS3
      WRITE(6,67) (ST,I=1,120)
      DO 1 I=1,IMAX
      A(I)=G(I,J)
1     C(I)=GA(I,J)
      AS=.5
      BS=.0
      CALL INTERP (A,IMAX,AS,B,JM,BS)
      CALL INTERP (C,IMAX,AS,D,JM,BS)
      WRITE (6,69) J
      BIG=0.
      SMALL=0.
      DO 2 I=1,IMAX
      IF(A(I).GT.BIG) BIG=A(I)
      IF(C(I).GT.BIG) BIG=C(I)
      IF(A(I).LT.SMALL) SMALL=A(I)
2     IF(C(I).LT.SMALL) SMALL=C(I)
      RANGE=BIG-SMALL
      RINK=RANGE/80.
      TOP=BIG+RINK
```

```
      CEN=BIG
      BOT=BIG-RINK
      KC=0
      DO 7 K=1,41
      IC=0
      DO 6 I=1,101
      ROW(I)=BL
      IF((I.EQ.1).OR.(I.EQ.51).OR.(I.EQ.101))ROW(I)=PL
      IF((K.EQ.1).OR.(K.EQ.41)) ROW(I)=PL
      IF((TOP.GE.0.).AND.(BOT.LE.0.)) ROW(I)=DH
      IF(IC.EQ.5) GO TO 3
      GO TO 4
3     IC=0
      IF(KC.EQ.10) ROW(I)=PL
4     IF(KPT.LE.2) GO TO 5
      IF((D(I).LE.TOP).AND.(D(I).GE.BOT)) ROW(I)=ST
5     IF((B(I).LE.TOP).AND.(B(I).GE.BOT)) ROW(I)=EX
6     IC=IC+1
      IF(KC.EQ.5) KC=0
      IF(KC.NE.0) WRITE (6,65) (ROW(I),I=1,101)
      IF(KC.EQ.0) WRITE (6,68) CEN,(ROW(I),I=1,101)
      TOP = TOP-2.*RINK
      CEN=CEN-2.*RINK
      BOT=BOT-2.*RINK
7     KC=KC+1
8     WRITE (6,66) (ST,I=1,120)
69    FORMAT (//'      J=',I3///)
68    FORMAT (1X,F8.3,1X,101A1)
67    FORMAT (1H1,//121A1//)
66    FORMAT (///121A1///)
65    FORMAT (10X,101A1)
      RETURN
      END
C000018
C

      SUBROUTINE INTERP (A,IM,AS,B,JM,BS)
C
C   INTERP CONVERTS A REGULAR VECTOR A OF IM POINTS TO A REGULAR VECTOR
C   B OF JM POINTS.  OS=.5 FOR A VECTOR WITH POINTS DEFINED IN THE
C   CENTER OF THE INTERVAL, AS AND BS ARE THE % OF AN INTERVAL FROM THE
C   EDGE OF THE FIELD TO THE FIRST POINT (.0 OR .5 FOR EDGE OR CENTER
C   DEFINED POINTS)
C
      DIMENSION A(IM),B(JM)
      RIM=IM
      RJM=JM
```

```
      RAT=(RIM-1.+2.*AS)/(RJM-1.+2.*BS)
      DO 2 I=1,JM
      BI=I
      AI=RAT*(BI+BS)-AS
      IA=AI
      F=AI-FLOAT(IA)
      IF((IA.EQ.0).OR.(IA.EQ.JM)) GO TO 1
      B(I)=A(IA)+F*(A(IA+1)-A(IA))
      GO TO 2
1     IF(IA.EQ.0) B(I)=A(1)*(F-AS)/(1.-AS)
      IF(IA.EQ.JM) B(I)=A(JM)*F/(1.-AS)
2     CONTINUE
      RETURN
      END

C000019
C     -------------------------------------------------------------------
C
C        SUBROUTINE SPLINE
C
C        PURPOSE
C           PROVIDES INTERPOLATED VALUE USING "CUBIC SPLINE FITTING"
C
C        USAGE
C           FIRST CALL TO SUBROUTINE:
C           CALL SPLINE(X,Y,M,XINT,YINT)
C
C           SUBSEQUENT CALLS:
C           CALL SPLINN(X,Y,M,XINT,YINT)
C
C        DESCRIPTION OF PARAMETERS
C           X:      MONOTONICALLY INCREASING ABSCISSA ARRAY
C           Y:      ONE-FOR-ONE CORRESPONDING ORDINATE ARRAY
C           M:      NUMBER OF X AND Y VALUES SUPPLIED < OR = 300
C           XINT:   VALUE OF ABSCISSA FOR WHICH CORRESPONDING ORDINATE
C                   IS TO BE INTERPOLATED (OR EXTRAPOLATED)
C           YINT:   INTERPOLATED (OR EXTRAPOLATED) ORDINATE VALUE
C
C
C        REMARKS
C           IF SPECIFIED X FALLS OUTSIDE OF RANGE, AN EXTRAPOLATED
C           VALUE WILL BE SUPPLIED
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           SUBROUTINE SPLICO IS INCLUDED IN SUBROUTINE SPLIN PACKAGE
C
C        MATHEMATICAL METHOD
C           UPON FIRST ENTRY TO SPLIN, A CALL TO SPLICO IS MADE TO
```

```
C             DETERMINE THE COEFFICIENTS TO BE USED IN PERFORMING THE     SPL00340
C             INTERPOLATIONS. SEARCH FOR BRACKETING ABSCISSA VALUES IS    SPL00350
C             ALWAYS MADE FROM THE REFERENCE LAST USED IN INTERPOLATING.  SPL00360
C                                                                         SPL00370
C          REFERENCE                                                      SPL00380
C             PENNINGTON, RALPH H.,"INTRODUCTORY COMPUTER METHODS AND     SPL00390
C             NUMERICAL ANALYSIS", THE MACMILLAN COMPANY, NEW YORK, 1965  SPL00400
C                                                                         SPL00410
C     ------------------------------------------------------------------SPL00420
C                                                                         SPL00430

      SUBROUTINE SPLINE(X,Y,M,XINT,YINT)                                  SPL00440
      DIMENSION X(M),Y(M),C(4,300)                                        SPL00460
      CALL SPLICO(X,Y,M,C)                                                SPL00470
      K=1                                                                 SPL00480
      ENTRY SPLINN(X,Y,M,XINT,YINT)                                       SPL00490
    3 IF(XINT-X(1)) 70,1,2                                                SPL00500
   70 K=1                                                                 SPL00510
      GO TO 7                                                             SPL00520
    1 YINT=Y(1)                                                           SPL00530
      RETURN                                                              SPL00540
    2 IF(XINT-X(K+1))6,4,5                                                SPL00550
    4 YINT=Y(K+1)                                                         SPL00560
      RETURN                                                              SPL00570
    5 K=K+1                                                               SPL00580
      IF(M-K) 71,71,3                                                     SPL00590
   71 K=M-1                                                               SPL00600
      GO TO 7                                                             SPL00610
    6 IF(XINT-X(K))13,12,11                                               SPL00620
   12 YINT=Y(K)                                                           SPL00630
      RETURN                                                              SPL00640
   13 K=K-1                                                               SPL00650
      GO TO 6                                                             SPL00660
    7 PRINT 101,XINT                                                      SPL00670
  101 FORMAT(8H0XINT = E18.9,32H, OUT OF RANGE FOR INTERPOLATION)         SPL00680
   11 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))                 SPL00690
      YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))                SPL00700
      RETURN                                                              SPL00710
      END                                                                 SPL00720

      SUBROUTINE SPLICO(X,Y,M,C)                                          SPL00730
      DIMENSION X(M),Y(M),C(4,300),D(300),P(300),E(300),A(300,3),B(300),SPL00750
     1Z(300)                                                              SPL00760
      MM=M-1                                                              SPL00770
      DO 2 K=1,MM                                                         SPL00780
      D(K)=X(K+1)-X(K)                                                    SPL00790
```

```
      P(K)=D(K)/6.                                                      SPL00800
    2 E(K)=(Y(K+1)-Y(K))/D(K)                                           SPL00810
      DO 3 K=2,MM                                                       SPL00820
    3 B(K)=E(K)-E(K-1)                                                  SPL00830
      A(1,2)=-1.-D(1)/D(2)                                              SPL00840
      A(1,3)=D(1)/D(2)                                                  SPL00850
      A(2,3)=P(2)-P(1)*A(1,3)                                           SPL00860
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)                                 SPL00870
      A(2,3)=A(2,3)/A(2,2)                                              SPL00880
      B(2)=B(2)/A(2,2)                                                  SPL00890
      DO 4 K=3,MM                                                       SPL00900
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)                           SPL00910
      B(K)=B(K)-P(K-1)*B(K-1)                                           SPL00920
      A(K,3)=P(K)/A(K,2)                                                SPL00930
    4 B(K)=B(K)/A(K,2)                                                  SPL00940
      Q=D(M-2)/D(M-1)                                                   SPL00950
      A(M,1)=1.+Q+A(M-2,3)                                              SPL00960
      A(M,2)=-Q-A(M,1)*A(M-1,3)                                         SPL00970
      B(M)=B(M-2)-A(M,1)*B(M-1)                                         SPL00980
      Z(M)=B(M)/A(M,2)                                                  SPL00990
      MN=M-2                                                            SPL01000
      DO 6 I=1,MN                                                       SPL01010
      K=M-I                                                             SPL01020
    6 Z(K)=B(K)-A(K,3)*Z(K+1)                                           SPL01030
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)                                     SPL01040
      DO 7 K=1,MM                                                       SPL01050
      Q=1./(6.*D(K))                                                    SPL01060
      C(1,K)=Z(K)*Q                                                     SPL01070
      C(2,K)=Z(K+1)*Q                                                   SPL01080
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)                                        SPL01090
    7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)                                    SPL01100
      RETURN                                                            SPL01110
      END                                                               SPL01120

C000020
C                                                                       MET00010
C         **************************************************************MET00020
C                                                                       MET00030
C         SUBROUTINE MTMPII                                             MET00040
C                                                                       MET00050
C         PURPOSE                                                       MET00060
C                   MTMPII WILL PRODUCE,ON THE PRINTER, A CONTOUR MAP   MET00070
C                   OF ANY SINGLE PRECISION TWO DIMENSIONAL ARRAY.      MET00080
C                                                                       MET00090
C         USAGE                                                         MET00100
C                   CALL MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)        MET00110
C                                                                       MET00120
```

```
C       DESCRIPTION OF PARAMETERS                                          MET00130
C                                                                          MET00140
C                 Y    - THE ARRAY TO BE CONTOURED. DIMENSIONED Y(N,M)MET00150
C                 N    - NUMBER OF ROWS IN Y.                             MET00160
C                 M    - NUMBER OF COLUMNS IN Y.                          MET00170
C                 T    - ARRAY FOR PLOT TITLE. REAL*4 T(24).              MET00180
C                 BND  - BANDWIDTH FOR THE CONTOURING. IF BND IS ZERO    MET00190
C                        A BANDWIDTH WILL BE CALCULATED AS FOLLOWS        MET00200
C                        BND=(MAX(Y)-MIN(Y))/15.                          MET00210
C                 AZ   - A LINEAR TRANSFORMATION MAYBE PERFORMED ON       MET00220
C                        THE ARRAY Y OF THE FOLLOWING FORM AZ*Y+BZ.       MET00230
C                        IF AZ=0 THEN AZ WILL BE COMPUTED SUCH THAT       MET00240
C                        MAX(|MAX(Y)|,|MIM(Y)|) WILL BE LESS THAN 1,      MET00250
C                        AND BZ WILL BE LEFT AS INPUT.                    MET00260
C                 BZ   - SEE UNDER AZ                                     MET00270
C                 AMIN - THE LEVEL AT WHICH COUTOURING WILL BEGIN. IF    MET00280
C                        AMIN > MIN(Y) THEN AMIN WILL BE CALCULATED       MET00290
C                        TO BE ,WITH REFERENCE AT ZERO, THE NEXT LOWERMET00300
C                        CONTOUR LEVEL FROM MIN(Y) AS DETERMINED BY       MET00310
C                        BND.                                             MET00320
C                 IJT  - IF IJT=0 AMIN WILL BE CALCULATED AS DESCRIBEDMET00330
C                        ABOVE.                                           MET00340
C                 ICON - IF ICON=0 NO CONTOURING WILL BE DONE BUT THE    MET00350
C                        ARRAY Y WILL BE PRINTED IN THE PLOT FORMAT.      MET00360
C                                                                          MET00370
C       REMARKS                                                            MET00380
C                 MTMPII REQUIRES A PRINTER WITH 132 PRINT POSITIONS.    MET00390
C                 IF NECESSARY THE MAP WILL BE SEGMENTED COLUMNWISE.     MET00400
C                 THE ROWS AND COLUMNS ARE NUMBERED ALONG THE EDGES SOMET00410
C                 THAT A SEGMENTED MAP MAYBE EASILY JOINED TOGETHER.     MET00420
C                 ONLY THREE SIGNIFICANT FIGURES WILL BE PRINTED AT      MET00430
C                 EACH POINT. THE POSITION OF THE FIRST SIGNIFICANT      MET00440
C                 DIGIT WILL BE DETERMINED BY MAX(|MAX(Y)|,|MIN(Y)|).    MET00450
C                 THE PLOT WILL BE PRODUCED ON A INCH INCH GRID. IT      MET00460
C                 WILL BE ASSUMED THAT THE SPACING BETWEEN POINTS IN     MET00470
C                 BOTH DIRECTIONS IS THE SAME AND EQUAL FOR ALL POINTSMET00480
C                                                                          MET00490
C       SUBROUTINES REQUIRED                                               MET00500
C                 NONE                                                     MET00510
C                                                                          MET00520
C       METHOD                                                             MET00530
C                 THE CONTOUR LEVELS ARE DETERMINED BY SIMPLE LINEAR     MET00540
C                 INTERPOLATION FROM THE FOUR SURROUNDING PIONTS.        MET00550
C                                                                          MET00560
C       **************************************************************MET00570
C                                                                          MET00580
C                                                                          MET00590
C    MTMPII SUBROUTINE FOR ONE-INCH GRID SPACING                          MET00600
```

```
C     OAKES CODE 5105    15 JAN 69                                       MET00610
C                                                                        MET00620
      SUBROUTINE MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)                MET00630
      REAL*4 IH,KG,ITJZ                                                  MET00640
      DIMENSION A(140),B(140),C(140),D(140),IH(20),Y(N,M),TP(10),TPX(10)MET00650
     Z  ,TPM(10),XMT(10),BTM(10),BTX(10),BT(10),KG(10),T(24)            MET00660
      DIMENSION E(140),F(140),G(140),H(140)                             MET00670
C                                                                        MET00680
C                                                                        MET00690
      DATA DUE/4H    /,EPL/4H+   /,EMI/4H-   /,IH/1H0,1H ,1H1,1H ,1H2,  MET00700
     11H ,1H3,1H ,1H4,1H ,1H5,1H ,1H6,1H ,1H7,1H ,1H8,1H ,1H9,1H /,KG/  MET00710
     21H0,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/,BLK/4H    /              MET00720
C                                                                        MET00730
      YMIN=Y(1,1)                                                        MET00740
      YMAX=Y(1,1)                                                        MET00750
      DO 20 I=1,M                                                        MET00760
      DO 10 J=1,N                                                        MET00770
      YMIN=AMIN1(YMIN,Y(J,I))                                            MET00780
      YMAX=AMAX1(YMAX,Y(J,I))                                            MET00790
   10 CONTINUE                                                           MET00800
   20 CONTINUE                                                           MET00810
      DELY=YMAX-YMIN                                                     MET00820
      IF(BND) 25,25,30                                                   MET00830
   25 BND=DELY/15.0                                                      MET00840
   30 IF (AMIN-YMIN) 31,31,32                                            MET00850
   31 IF (IJT) 33,32,33                                                  MET00860
   32 PD=YMIN/BND                                                        MET00870
      PF=ABS(PD-INT(PD))                                                 MET00880
      IF (YMIN) 2,1,1                                                    MET00890
    1 AMIN=YMIN-PF*BND                                                   MET00900
      GO TO 33                                                           MET00910
    2 AMIN=YMIN-(1.0-PF)*BND                                             MET00920
   33 AHLD=AZ                                                            MET00930
      IF(AZ) 55,35,55                                                    MET00940
   35 SM=AMAX1(ABS(YMIN),ABS(YMAX))                                      MET00950
      NS=0                                                               MET00960
   40 NS=NS+1                                                            MET00970
      SM=10.0*SM                                                         MET00980
      IF(SM-1.0)40,50,45                                                 MET00990
   45 NS=NS-1                                                            MET01000
      SM=SM/10.0                                                         MET01010
      IF(SM-1.0)50,50,45                                                 MET01020
   50 AHLD=10.0**NS                                                      MET01030
   55 HBND=BND/2.0                                                       MET01040
      PRINT 70                                                           MET01050
      PRINT 6,T                                                          MET01060
```

```
    6 FORMAT(5X,24A4,//)                                                  MET01070
      PRINT 57,AHLD,BZ                                                    MET01080
   57 FORMAT(1H0,65HTHE FOLLOWING TRANSFORMATION WAS PERFORMED ON THE INMET01090
     1PUT MATRIX    /5X,1H(,E12.5,8H*Y(I,J)+,E12.5,1H) //2X,73HAND THREEMET01100
     2 DIGITS TO THE RIGHT OF THE DECIMAL POINT ARE PRINTED IN THE MAP )MET01110
C                                                                         MET01120
      PRINT 54,YMAX,YMIN                                                  MET01130
   54 FORMAT(/4X,5HYMAX=,E15.7,5X,5HYMIN=,E15.7)                          MET01140
      IF (ICON)5,58,5                                                     MET01150
    5 PRINT 11,BND                                                        MET01160
   11 FORMAT(2X,17HTHE BAND WIDTH IS,E12.5,6H UNITS //4X,14HCONTOUR LEVEMET01170
     1LS         )                                                        MET01180
      I=0                                                                 MET01190
      YTOP=AMIN                                                           MET01200
      IF(ABS(YMIN-YMAX)-100.0*BND)53,53,58                                MET01210
   53 YB=YTOP                                                             MET01220
      YTOP=YTOP+BND                                                       MET01230
      I=I+1                                                               MET01240
      J=MOD(I,20)                                                         MET01250
      ITJZ=IH(J)                                                          MET01260
      IF(YB-YMAX)59,58,58                                                 MET01270
   59 PRINT 61,YB,YTOP,ITJZ                                               MET01280
   61 FORMAT(/4X,E10.3,4H TO ,E10.3,2H =,1X,A1)                           MET01290
      GO TO 53                                                            MET01300
   58 NCCP=0                                                              MET01310
      NCP=0                                                               MET01320
   60 PRINT 70                                                            MET01330
   70 FORMAT(1H1)                                                         MET01340
      PRINT 6,T                                                           MET01350
      NLINE=0                                                             MET01360
      NCCP=NCP+1                                                          MET01370
      NCP = NCP + 13                                                      MET01380
   73 IF(NCP-M)80,80,75                                                   MET01390
   75 NCP=M                                                               MET01400
   80 CONTINUE                                                            MET01410
      J=-2                                                                MET01420
      NLINE=NLINE+1                                                       MET01430
      LLINE=N-NLINE+1                                                     MET01440
C SET UP HEADING                                                          MET01450
      IF(NCCP-1) 85,85,90                                                 MET01460
   85 J = - 1                                                             MET01470
   90 DO 100 I = 1,135                                                    MET01480
      A(I)=BLK                                                            MET01490
      B(I)=BLK                                                            MET01500
      H(I) = BLK                                                          MET01510
  100 CONTINUE                                                            MET01520
  110 DO 160 L=NCCP,NCP                                                   MET01530
      J = J+8                                                             MET01540
```

```
      KI=L
      IF(KI-100) 130,120,120
  120 LL=KI/100
      A(J)=KG(LL+1)
      KI=KI-100*LL
      GO TO 135
  130 A(J)=KG(1)
  135 J=J+1
      IF(KI-10) 150,140,140
  140 LL=KI/10
      A(J)=KG(LL+1)
      KI=KI-10*LL
      GO TO 155
  150 A(J)=KG(1)
  155 J=J+1
      A(J)=KG(KI+1)
  160 CONTINUE
C SETUP FIRST ROW OF ARRAY
      GO TO 260
  170 NLINE=NLINE+1
      LLINE=N-NLINE+1
      IF(NLINE-N) 180,180,380
  180 DO 190 I=1,135
      A(I)=BLK
      B(I)=BLK
      C(I)=BLK
      D(I)=BLK
      E(I) = BLK
      F(I) = BLK
      G(I) = BLK
      H(I) = BLK
  190 CONTINUE
      IF (ICON)195,260,195
  195 NCY=NCCP-1
      J=4
      IF(NCY)200,200,210
  200 J=5
  210 NCY=NCY+1
      IF(NCY-NCP) 220,220,260
  220 IF(NCY-M) 230,260,260
  230 NLINE = NLINE - 1
      YD1 = Y(NLINE,NCY) - Y(NLINE+1,NCY)
      YD2=Y(NLINE,NCY+1)-Y(NLINE+1,NCY+1)
      TP(1) = Y(NLINE,NCY)-0.125*YD1
      TPX(1)=Y(NLINE,NCY)-0.250*YD1
      TPM(1)=Y(NLINE,NCY)-0.375*YD1
      XMT(1)=Y(NLINE,NCY)-0.500*YD1
      BTM(1)=Y(NLINE,NCY)-0.625*YD1
```

```
      BTX( 1)=Y(NLINE,NCY)-0.750*YD1
      BT  (1)=Y(NLINE,NCY)-0.875*YD1
      TP  (10)=Y(NLINE,NCY+1)-0.125*YD2
      TPX(10)=Y(NLINE,NCY+1)-0.250*YD2
      TPM(10)=Y(NLINE,NCY+1)-0.375*YD2
      XMT(10)=Y(NLINE,NCY+1)-0.500*YD2
      BTM(10)=Y(NLINE,NCY+1)-0.625*YD2
      BTX(10)=Y(NLINE,NCY+1)-0.750*YD2
      BT  (10)=Y(NLINE,NCY+1)-0.875*YD2
      NLINE = NLINE + 1
      D1=0.1 * (TP (10)-TP (1))
      D2=0.1 * (TPX(10)-TPX(1))
      D3=0.1 * (TPM(10)-TPM(1))
      D4=0.1 * (XMT(10)-XMT(1))
      D5=0.1 * (BTM(10)-BTM(1))
      D6=0.1 * (BTX(10)-BTX(1))
      D7=0.1 * (BT (10)-BT (1))
      DO 240 I = 2,9
      TP (I)=TP (I-1)+D1
      TPX(I)=TPX(I-1)+D2
      TPM(I)=TPM(I-1)+D3
      XMT(I)=XMT(I-1)+D4
      BTM(I)=BTM(I-1)+D5
      BTX(I)=BTX(I-1)+D6
      BT (I)=BT (I-1)+D7
  240 CONTINUE
      DO 250 I=1,10
      J=J+1
      I1=MOD(IFIX((TP (I)-AMIN)/BND),20)+1
      I2=MOD(IFIX((TPX(I)-AMIN)/BND),20)+1
      I3=MOD(IFIX((TPM(I)-AMIN)/BND),20)+1
      I4=MOD(IFIX((XMT(I)-AMIN)/BND),20)+1
      I5=MOD(IFIX((BTM(I)-AMIN)/BND),20)+1
      I6=MOD(IFIX((BTX(I)-AMIN)/BND),20)+1
      I7=MOD(IFIX((BT (I)-AMIN)/BND),20)+1
      A(J)=IH(I1)
      B(J)=IH(I2)
      C(J)=IH(I3)
      D(J)=IH(I4)
      E(J)=IH(I5)
      F(J)=IH(I6)
      G(J)=IH(I7)
  250 CONTINUE
      GO TO 210
  260 NCY=NCCP-1
      J=-2
      IF(NCY) 265,265,270
  265 J=-1
```

```
      GO TO 330                                                         MET02510
  270 NCY=NCY+1                                                         MET02520
      IF(NCY-NCP) 280,280,310                                           MET02530
  280 J=J+7                                                             MET02540
      THLD=AHLD*Y(NLINE,NCY)+BZ                                         MET02550
      IF(THLD) 285,290,290                                              MET02560
  285 H(J)=EMI                                                          MET02570
      GO TO 295                                                         MET02580
  290 H(J)=EPL                                                          MET02590
  295 NUM=INT(ABS(THLD-INT(THLD))*1000.0+0.5)                           MET02600
      NDS=100                                                           MET02610
      DO 300 KK=1,3                                                     MET02620
      J=J+1                                                             MET02630
      KI=NUM/NDS                                                        MET02640
      H(J)=KG(KI+1)                                                     MET02650
      NUM=NUM-KI*NDS                                                    MET02660
      NDS=NDS/10                                                        MET02670
  300 CONTINUE                                                          MET02680
      GO TO 270                                                         MET02690
  310 IF(NCP-M) 360,320,320                                             MET02700
  320 IF(J-127)330,330,360                                              MET02710
  330 J=J+3                                                             MET02720
      KI=NLINE                                                          MET02730
      IF(KI-100) 340,335,335                                            MET02740
  335 LL=KI/100                                                         MET02750
      H(J)=KG(LL+1)                                                     MET02760
      KI=KI-100*LL                                                      MET02770
      GO TO 343                                                         MET02780
  340 H(J)=KG(1)                                                        MET02790
  343 J=J+1                                                             MET02800
      IF(KI-10) 350,345,345                                             MET02810
  345 LL=KI/10                                                          MET02820
      H(J)=KG(LL+1)                                                     MET02830
      KI=KI-10*LL                                                       MET02840
      GO TO 355                                                         MET02850
  350 H(J)=KG(1)                                                        MET02860
  355 J=J+1                                                             MET02870
      H(J)=KG(KI+1)                                                     MET02880
      J=J-5                                                             MET02890
      IF(NCY-1) 270,270,360                                             MET02900
  360 IF(NLINE-1)362,362,368                                            MET02910
  362 PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(H(IP2),IP2=1,132)    MET02920
      GO TO 170                                                         MET02930
  368 PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(C(IP2),IP2=1,132),   MET02940
     1(D(IP3),IP3=1,132),(E(IP4),IP4=1,132),(F(IP5),IP5=1,132),         MET02950
     2(G(IP6),IP6=1,132),(H(IP7),IP7=1,132)                             MET02960
  370 FORMAT(132A1)                                                     MET02970
      GO TO 170                                                         MET02980
```

```
  380 DO 390 I=1,135                                                    MET02990
      A(I)=BLK                                                          MET03000
      B(I)=BLK                                                          MET03010
      C(I)=BLK                                                          MET03020
      D(I)=BLK                                                          MET03030
  390 CONTINUE                                                          MET03040
      J=-2                                                              MET03050
      IF(NCCP-1) 395,395,400                                            MET03060
  395 J=-1                                                              MET03070
  400 DO 430 L=NCCP,NCP                                                 MET03080
      J=J+8                                                             MET03090
      KI=L                                                              MET03100
      IF(KI-100) 410,405,405                                            MET03110
  405 LL=KI/100                                                         MET03120
      C(J)=KG(LL+1)                                                     MET03130
      KI=KI-100*LL                                                      MET03140
      GO TO 412                                                         MET03150
  410 C(J)=KG(1)                                                        MET03160
  412 J=J+1                                                             MET03170
      IF(KI-10) 420,415,415                                             MET03180
  415 LL=KI/10
      C(J)=KG(LL+1)                                                     MET03200
      KI=KI-10*LL                                                       MET03210
      GO TO 422                                                         MET03220
  420 C(J)=KG(1)                                                        MET03230
  422 J=J+1                                                             MET03240
      C(J)=KG(KI+1)                                                     MET03250
  430 CONTINUE                                                          MET03260
      PRINT 370, (B(IP1),IP1=1,132),(C(IP2),IP2=1,132)                  MET03270
      IF(NCP-M)60,500,500                                               MET03280
  500 RETURN                                                            MET03290
      END                                                               MET03300

C000021
C                                                                       GAUS  10
C     ..................................................................GAUS  20
C                                                                       GAUS  30
C        SUBROUTINE GAUSS                                               GAUS  40
C                                                                       GAUS  50
C        PURPOSE                                                        GAUS  60
C           COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN  GAUS  70
C           MEAN AND STANDARD DEVIATION                                 GAUS  80
C                                                                       GAUS  90
C        USAGE                                                          GAUS 100
C           CALL GAUSS(IX,S,AM,V)                                       GAUS 110
C                                                                       GAUS 120
C        DESCRIPTION OF PARAMETERS                                      GAUS 130
```

```
C          IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR        GAUS 140
C              LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER        GAUS 150
C              IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM     GAUS 160
C              NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT     GAUS 170
C              ENTRY TO THE SUBROUTINE.                                   GAUS 180
C          S  -THE DESIRED STANDARD DEVIATION OF THE NORMAL               GAUS 190
C              DISTRIBUTION.                                              GAUS 200
C          AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION                GAUS 210
C          V  -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE           GAUS 220
C                                                                         GAUS 230
C       REMARKS                                                           GAUS 240
C          THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC           GAUS 250
C                                                                         GAUS 260
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                     GAUS 270
C          RANDU                                                          GAUS 280
C                                                                         GAUS 290
C       METHOD                                                            GAUS 300
C          USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM        GAUS 310
C          NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN           GAUS 320
C          ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.       GAUS 330
C          THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE      GAUS 340
C          ARE FOUND BY THE POWER RESIDUE METHOD.                         GAUS 350
C                                                                         GAUS 360
C     ..................................................................GAUS 370
C                                                                         GAUS 380
      SUBROUTINE GAUSS(IX,S,AM,V)                                         GAUS 390
      A=0.0                                                               GAUS 400
      DO 50 I=1,12                                                        GAUS 410
      CALL RANDU(IX,IY,Y)                                                 GAUS 420
      IX=IY                                                               GAUS 430
   50 A=A+Y                                                               GAUS 440
      V=(A-6.0)*S+AM                                                      GAUS 450
      RETURN                                                              GAUS 460
      END                                                                 GAUS 470
C000022
C                                                                         RAND  10
C     ..................................................................RAND  20
C                                                                         RAND  30
C       SUBROUTINE RANDU                                                  RAND  40
C                                                                         RAND  50
C       PURPOSE                                                           RAND  60
C          COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN     RAND  70
C          0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND                 RAND  80
C          2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER       RAND  90
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C           AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.
C
C        USAGE
C           CALL RANDU(IX,IY,YFL)
C
C        DESCRIPTION OF PARAMETERS
C           IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER
C                NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY,
C                IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS
C                SUBROUTINE.
C           IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT
C                ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS
C                BETWEEN ZERO AND 2**31
C           YFL- THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT,
C                RANDOM NUMBER IN THE RANGE 0 TO 1.0
C
C        REMARKS
C           THIS SUBROUTINE IS SPECIFIC TO SYSTEM/360 AND WILL PRODUCE
C           2**29 TERMS BEFORE REPEATING.  THE REFERENCE BELOW DISCUSSES
C           SEEDS (65539 HERE), RUN PROBLEMS, AND PROBLEMS CONCERNING
C           RANDOM DIGITS USING THIS GENERATION SCHEME.  MACLAREN AND
C           MARSAGLIA, JACM 12, P. 83-89, DISCUSS CONGRUENTIAL
C           GENERATION METHODS AND TESTS.  THE USE OF TWO GENERATORS OF
C           THE RANDU TYPE, ONE FILLING A TABLE AND ONE PICKING FROM THE
C           TABLE, IS OF BENEFIT IN SOME CASES.  65549 HAS BEEN
C           SUGGESTED AS A SEED WHICH HAS BETTER STATISTICAL PROPERTIES
C           FOR HIGH ORDER BITS OF THE GENERATED DEVIATE.
C           SEEDS SHOULD BE CHOSEN IN ACCORDANCE WITH THE DISCUSSION
C           GIVEN IN THE REFERENCE BELOW.  ALSO, IT SHOULD BE NOTED THAT
C           IF FLOATING POINT RANDOM NUMBERS ARE DESIRED,AS ARE
C           AVAILABLE FROM RANDU, THE RANDOM CHARACTERISTICS OF THE
C           FLOATING POINT DEVIATES ARE MODIFIED AND IN FACT THESE
C           DEVIATES HAVE HIGH PROBABILITY OF HAVING A TRAILING LOW
C           ORDER ZERO BIT IN THEIR FRACTIONAL PART.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011,
C           RANDOM NUMBER GENERATION AND TESTING
C
C     ..................................................................
C
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      SUBROUTINE RANDU(IX,IY,YFL)                                       RAND 540
      IY=IX*65539                                                       RAND 550
      IF(IY)5,6,6                                                       RAND 560
    5 IY=IY+2147483647+1                                                RAND 570
    6 YFL=IY                                                            RAND 580
      YFL=YFL*.4656613E-9                                               RAND 590
      RETURN                                                            RAND 600
      END
```

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13. ABSTRACT

The successful application of holographic interferometry to the study of density fields around opaque bodies in wind tunnel experiments has been reported in the literature. The present report extends this technique to the study of the three-dimensional asymmetric flow fields encountered near the wing-fuselage junction of an aerodynamic model in the transonic flow regime. Finite fringe interferometry has been used to obtain fringe information about a partially transparent wind-body structure. A FORTRAN computer program was utilized to invert the fringe information and produce a plot of the density field around the model. The resulting asymmetric density field and shock wave structure are shown to be an accurate representation of the phenomena encountered in aerodynamic corner flow.

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