A

COMPLETE

ARITHMETIC,

ORAL AND WRITTEN.

PART FIRST.

BY

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PREFACE.

THE aim of the author in the preparation of this work may be stated as follows:

1. To present each subject in arithmetic in such a manner as to lead the pupil by means of preparatory steps and propositions which he is required to examine for himself, to gain clear perceptions of the elements necessary to enable him to grasp as a reality the more complex and complete processes.

2. To present, wherever it can be done, each process objectively, so that the truth under discussion is exhibited to the eye and thus sharply defined in the mind.

3. To give such a systematic drill on oral and written exercises and review and test questions as will fix permanently in the mind the principles and processes of numbers with their applications in practical business.

4. To arrange the pupil's work in arithmetic in such a manner that he will not fail to acquire such a knowledge of principles and facts, and to receive such mental discipline, as will fit him properly for the study of the higher mathematics.

The intelligent and experienced teacher can readily determine by an examination of the work how well the author has succeeded in accomplishing his aim.

Special attention is invited to the method of presentation given in the teacher's edition. This is arranged at the beginning of each subject, just where it is required, and contains definite and full instructions regarding the order in which the subject should be presented, the points that require special attention and illustration, the kind of illustrations that should be used, a method for drill exercise, additional oral exercises where required for the teacher's use, and such other instructions as are necessary to form a complete guide to the teacher in the discussion and presentation of each subject.

The plan adopted of having a separate teacher's edition avoids entirely the injurious course usually pursued of cumbering the pupil's book with hints and suggestions which are intended strictly for the teacher.

Attention is also invited to the Properties of Numbers, Greatest Common Divisor, Fractions, Decimals, Compound Numbers, Business Arithmetic, Ratio and Proportion, Alligation, and Square and Cube Root, with the belief that the treatment will be found new and an improvement upon former methods.

The author acknowledges with pleasure his indebtedness to Prof. D. H. MacVicar, LL.D., Montreal, for valuable aid rendered in the preparation of the work, and to Charles D. McLean, A. M., Principal of the State Normal and Training School, at Brockport, N. Y., for valuable suggestions on several subjects.

M. MacVICAR.

Potsdam, *September*, 1877.

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ARITHMETIC

NOTATION AND NUMERATION.

NUMBERS FROM 1 TO 1000.

Art. 1. Numbers are expressed by means of ten figures.

Figures,	0	1	2	3	4	5	6	7	8	9
Names,	*Naught,*	*One,*	*Two,*	*Three,*	*Four,*	*Five,*	*Six,*	*Seven,*	*Eight,*	*Nine.*

Observe regarding the ten figures:

1. The *naught* is also called *cipher* or *zero,* and when written alone stands for *no number.*

2. The other nine figures are called *digits* or *significant figures,* and each stands for the number written under it.

3. Any number of objects not greater than *nine* is expressed by *one figure.*

Thus, 2 boys, 5 girls, 7 pens, 9 desks, 4 windows.

2. PROP. I.—*Numbers from nine upwards are represented by means of the nine digits and the cipher, by regarding objects as arranged in groups of different orders thus,*

1. We regard *one* more than *nine* objects as a group which we call ***Ten,*** and represent by ***1*** and ***0*** thus,

9 *and* ***1*** = ***1*** *ten, written* ***10.***

Observe, that any digit written, as the ***1*** is in ***10,*** in the second place from the right, represents the *number* of *tens.*

Hence ***20*** means ***2*** *tens,* ***30*** means ***3*** *tens*, and so on.

2. We regard ***10*** *tens* as a *group* of a *higher* order, which we call ***Hundred*** and represent thus, ***100.***

Observe, that any digit written, as the ***1*** is in ***100,*** in the third place from the right, represents the *number* of hundreds.

Hence ***200*** means ***2*** *hundred,* ***300*** means ***3*** hundred, and so on.

3. *Higher Orders* of groups are formed in the same manner as *Tens* and *Hundreds,* by regarding *ten* of any order as *one* of the next higher. The number is also represented by writing each new order one place farther from the right.

Thus *Ten* hundred make ***1*** *Thousand,* and the number is written ***1000.*** Hence ***2000*** means ***2*** *thousand,* ***3000*** means ***3*** *thousand,* and so on.

4. *Observe,* the position which a figure occupies determines the name of the order whose number it expresses.

Thus, in 379, the 9 stands for the number of ***Units,*** or single things; the 7 for the number of ***Tens,*** or groups of *ten single things,* and the 3 stands for the number of ***Hundreds,*** or groups of *ten tens.*

5. *Observe,* also, that when *one or more* orders are wanting in a number, their places are filled by ciphers. Thus, 5 hundred 7 is expressed 507; 8 hundred 3 tens is expressed 830.

Laws of Notation and Numeration.

3. 1. *A figure standing alone, or at the right of one or more figures, expresses the number of* **Units** *or single things.*

2. *A figure in the second place from the right expresses the number of* **Tens** *or groups of ten single things.*

3. *A figure in the third place from the right expresses the number of* **Hundreds** *or groups of ten tens.*

4. *The cipher is used to locate significant figures in their proper positions.*

EXAMPLES FOR PRACTICE.

4. Express in figures the following:

1. Eight; five; seven; four; nine; nine and one.

2. Three tens; nine tens; six tens; two tens; 8 tens; nine tens and one ten.

3. Six hundred; four hundred; 3 hundred; 9 hundred.

4. Ten and three; ten and nine; ten and seven; 2 tens and three; 5 tens and 2.

5. Nine tens and seven; 8 tens and 4; 6 tens and 9; 3 tens and 7; 9 tens and 7.

6. Five tens and nine; 9 tens and 3; 4 tens and 8; 7 tens and 3; 9 tens and 9.

7. Nine tens and one; one hundred two tens; one hundred nine; four hundred seven.

8. Two hundred seven tens and two; 5 hundred 9 tens and 1; 8 hundred 3.

9. Seven hundred 4; 9 hundred 6; 4 hundred eight tens.

10. 4 hundred ten; 8 hundred one; 3 hundred 2.

11. 9 hundred 9 tens; 9 hundred 9; 9 hundred 9 tens and 9.

12. How many *tens,* and how many *units* are 39?

13. How many *hundreds,* and how many *tens* are 750?

14. How many *hundreds,* and how many *units* are 408?

15. How many hundreds, tens, and units are 396?

5. Prop. II.—*The names of numbers are formed by combining the names of the figures used to express the numbers with the names of the orders of groups represented.*

1. ***11*** is ten and one, read ***Eleven***. ***12*** is ten and two, read ***Twelve.*** These two numbers are the only exceptions to the proposition.

2. The names of numbers from *twelve* to *two tens* are formed by changing *ten* into *teen,* and prefixing the name of the digit which expresses how many the number is greater than ten. The name of the digit, when necessary to combine properly with *teen,* is changed, thus:

13 is three and ten, or ***Thir-teen;*** *three* changed to *thir.*
14 is four and ten, or ***Four-teen.***
15 is five and ten, or ***Fif-teen;*** *five* changed to *fif.*
16 is six and ten, or ***Six-teen.***
17 is seven and ten, or ***Seven-teen.***
18 is eight and ten, or ***Eigh-teen;*** *eight* changed to *eigh.*
19 is nine and ten, or ***Nine-teen.***

3. The name of any number of tens from *one ten* to *ten tens* is formed by changing the word *tens* to *ty* and prefixing the name of the digit which expresses the required number of tens, making the necessary changes in the name of the digits to combine properly with *ty,* thus:

20 is two tens, or ***Twen-ty;*** *two* changed to *twen.*
30 is three tens, or ***Thir-ty;*** *three* changed to *thir.*
40 is four tens, or ***For-ty;*** *four* changed to *for.*
50 is five tens, or ***Fif-ty;*** *five* changed to *fif.*
60 is six tens, or ***Six-ty.***
70 is seven tens, or ***Seven-ty.***
80 is eight tens, or ***Eigh-ty;*** *eight* is changed to *eigh.*
90 is nine tens, or ***Nine-ty.***

4. Tens, and ones or units, when written together, are read by uniting the two names in one, thus:

21 is two tens and one, read ***Twenty-one.***

35 is three tens and five, read ***Thirty-five.***

5. Hundreds are read by naming the digit that expresses them. Thus, 400 is read *Four hundred.*

6. Hundreds, tens, and units, when written together, are read by uniting the three names. Thus, 683 is read *Six hundred eighty-three.*

EXAMPLES FOR PRACTICE.

6. Read the following numbers:

1. 17. 13. 19. 12. 18. 14. 11. 15. 10. 16.
2. 30. 70. 40. 80. 60. 90. 24. 54. 84. 93. 99.
3. 500. 200. 700. 410. 820. 390. 605. 903. 509.
4. 783. 625. 936. 473. 897. 369. 704. 990.
5. 888. 111. 273. 909. 990. 999. 777. 222.

Name the orders in the following numbers commencing at the right. Thus, 839 is 9 units, 3 tens, and 8 hundreds.

6. 493. 765. 892. 375. 906. 580. 734. 983. 306.
7. 572. 409. 603. 942. 300. 850. 903. 872.

Read and analyze the following, thus:

8. 37 horses. ANALYSIS.—Thirty-seven horses may be regarded as 3 groups of *ten* horses and 7 single horses, or as 37 single horses.

9. 542 trees. ANALYSIS.—Five hundred forty-two trees may be regarded as 5 groups of *one* hundred trees each, 4 groups of *ten* trees each, and 2 *single* trees; or it may be regarded as 54 groups of *ten* trees each and 2 single trees; or as 542 single trees.

10. 67 tops. 46 rings. 14 boys. 17 men. 75 sheep.
11. 127 tables. 108 beds. 205 chairs. 511 stoves.
12. 696 beetles. 478 bees. 930 flies.
13. 444 robins. 309 hawks. 875 blackbirds.
14. 196 lambs. 480 goats. 555 cows. 909 foxes.
15. 303. 940. 508. 490. 736. 893. 999.

READING LARGE NUMBERS.

7. Prop. III.—*The names of the orders in large numbers are formed by giving a new name to the order in every third place counting* FROM *the* UNITS.

1. We indicate the orders to which new names are applied by inserting a comma at the left of every third figure, counting from the right.

The commas are inserted and the names applied; thus,

Trillions.	*Billions.*	*Millions.*	*Thousands.*	*Units.*
73,	**069,**	**294,**	**780,**	**569**
5th Period.	4th Period.	3d Period.	2d Period.	1st Period.

2. The *commas* separate the number into sets of three figures. Each set is called a ***Period.***

3. The right-hand order in each period has a new name, as shown in the illustration. The figure in this place expresses ***ones*** of the given name.

4. The *second figure* in each period expresses ***tens,*** and the *third* ***hundreds,*** of whatever the first order is called.

For example, the figures in the third period of the above number are 294, and the right-hand order is called millions; hence the period is read, two hundred ninty-four *millions.*

5. The figures in each period are read in the same manner as they would be if there were but one period in the number.

Thus, in the above number, the fifth period is read, *seventy-three trillions,* the fourth is read, *sixty-nine billions.* There being no hundreds expressed in the fourth period nothing is said about hundreds. Each succeeding period is read in the same manner.

6. A large number can be read as easily as a number of three places, when the following names of the first order on the right of the successive periods are fixed in the memory:

PERIODS.	NAMES.	PERIODS.	NAMES.
1st.	Units.	7th.	Quintillions.
2d.	Thousands.	8th.	Sextillions.
3d.	Millions.	9th.	Septillions.
4th.	Billions.	10th.	Octillions.
5th.	Trillions.	11th.	Nonillions.
6th.	Quadrillions.	12th.	Decillions.

From these illustrations, we obtain for reading numbers the following

RULE.—*I. Begin at the right and separate the number, by inserting commas, into periods of three figures each.*

II. Begin at the left and read the hundreds, tens, and ones of each period, giving the name of the ones in each case except in the last period.

EXAMPLES FOR PRACTICE.

8. Point off and read the following numbers:

1. 307. 560. 293. 1348. 4592. 8347. 6241.
2. 84385. 93761. 352634. 893625. 38297634.
3. 1001. 4032. 9306. 8400. 3080. 5906. 3103.
4. 85000. 34006. 59040. 80307. 306205. 340042.
5. 307009. 85004. 230060. 903560. 100001.
6. 8060. 50040. 3040006. 2406007. 5030062.
7. 9000000. 40006003. 60304090. 200006000. 300000804.
8. 800800800800. 3005006004. 407000060060.
9. 806042. 35064. 9003005. 100100100101.
10. 3000050030. 8300400706005. 9000100130004.
11. 97304206590734059034. 3000700000597034006276.

WRITING LARGE NUMBERS.

9. Prop. IV.—*Numbers are written one period at a time and in the order in which the periods are read.*

Observe regarding this proposition:

1. Each period in a number except the one at the left must contain three figures. Hence the places for which significant figures are not given must be filled with ciphers.

Thus, three hundred seven million, four thousand, eighty-two, is written 307,004,082. Observe in this number a significant figure is given only for the *hundreds* and *ones* in the million's period, hence the *ten's* place is filled with a *cipher*. For a like reason the ten's and hundred's place in the thousand's period and the hundred's place in the unit's period are filled with ciphers.

2. When a number is read, a period in which all the orders are wanting is not named. Care must therefore be taken to notice such periods and fill their places in each case with three ciphers.

For example, in the number seven million three hundred four, the thousands period is not named, but when the number is expressed in figures its place is filled with three ciphers; thus, 7,000,304.

Rule.—*Begin at the left and write the figures expressing the hundreds, tens, and ones of each period in their proper order, filling with ciphers all periods or places where no significant figures are given.*

EXAMPLES FOR PRACTICE.

10. Express in figures the following numbers:

1. Two hundred seven. Four hundred fifty. Seven hundred ninety. Three hundred eighty-seven.

2. Three thousand nine. Eight thousand sixty.

3. Eleven hundred. Twenty-five hundred.

4. Ten tens. One hundred tens. Ten tens and three. One hundred tens and fifteen. Eight hundred six tens.

5. Ten thousand. Ten thousand nine. Twenty thousand. Twenty thousand fifty-three.

6. Eleven thousand eleven. One million one.

7. Eighty million eighty thousand eighty.

8. Two thousand three hundred five million.

9. Eight hundred sixty-two tens. Five hundred seven tens. Two thousand sixty-three tens.

10. Forty two thousand million. Seven thousand and six million. Forty-four billion seven.

11. 87 million 1 thousand 2. 907 trillion 4 million 6.

12. 11 billion 108 thousand 39. 1 trillion 1 million 1.

DEFINITIONS.

11. A ***Unit*** is a single thing, or group of single things, regarded as one; as, *one* ox, *one* yard, *one* ten, *one* hundred.

12. ***Units are of two kinds*** — Mathematical and Common. A *mathematical* unit is a single thing which has a fixed value; as, one *yard*, one *quart*, one *hour*, one *ten*. A *common* unit is a single thing which has no fixed value; as, one *house*, one *tree*, one *garden*, one *farm*.

13. A ***Number*** is a unit, or collection of units; as, *one* man, *three* houses, *four*, *six* hundred.

Observe, the *number* is "*the how many*" and is represented by whatever answers the question, How many? Thus in the expression *seven* yards, *seven* represents the number.

14. The ***Unit of a Number*** is one of the things numbered. Thus, the unit of eight bushels is *one bushel*, of five boys is *one boy*, of nine is *one*.

15. A ***Concrete Number*** is a number which is applied to *objects* that are named; as four *chairs*, ten *bells*.

16. An ***Abstract Number*** is a number which is not applied to any named objects ; as *nine, five, thirteen.*

17. ***Like Numbers*** are such as have the same unit. Thus, four *windows* and eleven *windows* are like numbers, eight and ten, three *hundred* and seven *hundred.*

18. ***Unlike Numbers*** are such as have different units. Thus, twelve *yards* and five *days* are unlike numbers, also six *cents* and nine *minutes.*

19. ***Figures*** are characters used to express numbers.

20. The ***Value*** of a figure is the number which it represents.

21. The ***Simple*** or ***Absolute Value*** of a figure is the number it represents when standing alone, as 8.

22. The ***Local*** or ***Representative Value*** of a figure is the number it represents in consequence of the place it occupies. Thus, in 66 the 6 in the second place from the right represents a number ten times as great as the 6 in the first place.

23. ***Notation*** is the method of writing numbers by means of figures or letters.

24. ***Numeration*** is the method of reading numbers which are expressed by figures or letters.

25. A ***Scale*** in Arithmetic is a succession of mathematical units which increase or decrease in value according to a fixed order.

26. A ***Decimal Scale*** is one in which the fixed order of increase or decrease is uniformly ten.

This is the scale used in expressing numbers by figures.

27. ***Arithmetic*** is the Science of Numbers and the Art of Computation.

ROMAN NOTATION.

28. *Characters Used.*—The Roman Notation expresses numbers by seven letters and a dash.

Letters.—	I,	V,	X,	L,	C,	D,	M.
Values.—	One,	Five,	Ten,	Fifty,	One Hundred,	Five Hundred,	One Thousand.

29. *Laws of Roman Notation.*—The above seven letters and the dash are used in accordance with the following laws:

1. *Repeating a letter repeats its value.*

Thus, I denotes one; II, two; III, three; X, ten; XX, two tens, or twenty.

2. *When a letter is placed at the left of one of greater value, the difference of their values is the number expressed.*

Thus, IV denotes four; IX, nine; XL, forty.

3. *When a letter is placed at the right of one of greater value, the sum of their values is the number expressed.*

Thus, VI denotes six; XI, eleven; LX, sixty.

4. *A dash placed over a letter multiplies its value by one thousand.*

Thus, $\overline{\text{X}}$ denotes ten thousand; $\overline{\text{IV}}$, four thousand; $\overline{\text{V}}$, five thousand.

EXERCISE FOR PRACTICE.

30. Express the following numbers by Roman Notation:

1. Two.	5. Eight.	9. Nineteen.	13. Thirty-seven.
2. Five.	6. Ten.	10. Fourteen.	14. Thirty-nine.
3. Three.	7. Thirteen.	11. Twenty.	15. Forty-six.
4. Four.	8. Seventeen.	12. Seventy-four.	16. Forty-four.

17. One hundred twenty-seven. Eight hundred four.
18. One hundred forty-nine. Ninety-five.
19. One thousand. Five thousand. Fifty thousand.

20. Ten thousand. One hundred thousand. Five hundred thousand. Ninety thousand.

21. 1800. 1875. 8065. 7939. 1854. 20365. 85342.

22. Read the following: MIX; MDLXIV; $\overline{X}$; $\overline{D}$; $\overline{MM}$; $\overline{MD}$; $\overline{D}$VII; MDCCCLXXVI; $\overline{ML}$; $\overline{DLX}$.

REVIEW AND TEST QUESTIONS.

31. Study carefully and answer each of the following questions:

1. Define a scale. A decimal scale.

2. How many figures are required to express numbers in the decimal scale, and why?

3. Explain the use of the cipher, and illustrate by examples.

4. State reasons why a scale is necessary in expressing numbers.

5. Explain the use of each of the three elements—*figures, place,* and *comma*—in expressing numbers.

6. What is meant by the *simple* or *absolute* value of figures? What by the *local* or *representative* value?

7. How is the *local value* of a figure affected by changing it from the *first* to the *third* place in a number?

8. How by changing a figure from the second to the fourth? From the fourth to the ninth?

9. Explain how the names of numbers from twelve to twenty are formed. From twenty to nine hundred ninety.

10. What is meant by a *period* of figures?

11. Explain how the name for each order in any period is formed.

12. State the name of the right-hand order in each of the first six periods, commencing with units.

13. State the two things mentioned in (**9**) which must be observed when writing large numbers.

14. Give a rule for reading numbers; also for writing numbers.

ADDITION

PREPARATORY STEPS.

32. Step I.—*To find how many any two groups of objects, each less than ten, will make when united in one group.*

1. This *must* be done in the *first place* by counting, that is, by putting one group with the other *one at a time.*

Thus, to find how many 9 oranges and 7 oranges will make when united in one lot, we put *one* of the 7 oranges with the 9, and know at once that there are 10 oranges together; we then put another of the 7 oranges with the 10, and know that there are 11 oranges together, and so on, until the 7 oranges are put with the 9, making 16 oranges.

2. The ***Addition Table*** consists of the *sums* of the numbers from 1 to 9 inclusive, taken *two at a time.* These sums must at first be found by counting; but when found, they should be fixed in the memory so that they can be given at sight of the figures.

EXERCISES.

Find by counting the sum of.

1. 4 books and 5 books; 6 balls and 3 balls; 8 boys and 4 boys; 7 pencils and 3 pencils.

2. 9 marks and 6 marks; 5 apples and 7 apples; 3 desks and 5 desks; 4 windows and 6 windows.

Use objects, and find by counting the sum of

3. 7 and 3; 9 and 4; 6 and 8; 5 and 5; 9 and 8; 6 and 7.

4. 9 and 7; 8 and 8; 7 and 7; 9 and 9; 4 and 9.

33. STEP II.—*To memorize the Addition Table.*

This is done readily by pursuing the following course:

1. Write on your slate in irregular order the nine digits, with 1 under each figure, thus:

1	3	6	2	4	7	9	5	8
1	1	1	1	1	1	1	1	1

Commence at the left and write the sum under each set just as rapidly as possible. When you have them all written, erase them, and write them again and again, until you can do it just as quickly as you can make the figures.

2. Write 2 under each figure, thus:

1	4	7	3	6	8	2	9	5
2	2	2	2	2	2	2	2	2

Practice in writing the sums in the same manner as you did with 1. Be particular not to use your fingers or other objects with which to count. Think the sums out in your mind without whispering or moving your lips.

When you can write the sums with 2, at sight of the figures, then take 3 and practice in the same way, then 4, then 5, and so on up to 9.

3. When you have practiced in this way on each table by itself, then write on your slate sets of numbers, mixing the tables together, thus:

2	5	8	4	7	4	9	5	9	8
7	3	6	9	8	4	8	7	9	7

Write several sets in this form, leaving room for answers. Practice as before in writing the sums until you can do it at sight of the figures.

4. Practice on giving the sums orally at sight of the figures. Thus, write on your slate sets of numbers as in the *third exercise*, and pronounce *inaudibly* the sums instead of writing them.

5. To practice in giving the sums orally when out of school; write sets of numbers as in *third exercise*, and get some one to pronounce rapidly and repeatedly the pairs of numbers while you pronounce simply the sums.

Thus, the person pronouncing the numbers says *eight, seven;* you answer at once *fifteen.*

34. Step III.—*To find the sum of two or more numbers, each expressed by one figure, by using the Addition Table.*

1. Find the sum of 7, 9, and 8.

Solution.—(1) We know at once from the *memorized results* of the addition table, that the sum of 7 and 9 is 16 or 1 *ten* and 6 *units.*

(2) We add the 8 units to the 6 units of the last result and know in the same manner that the sum of the 8 and 6 is 14, or 1 *ten* and 4 *units.* Uniting this ten with the ten found by adding the 7 and 9, we have 2 *tens* and 4 units, or 24. Hence the sum of 7, 9, and 8 is 24.

2. The process in finding the sum of any column of figures consists in noting the *tens* which the column makes.

Thus, suppose the figures in a column to be 9, 6, 8, 5, and 7. Commencing with 9 we note that 9 and 6 make 1 *ten* and 5. We add the 8 to the 5 and we have another *ten* and 3, making 2 tens and 3. We add the 5 to the 3, making 2 tens and 8. We now add the 7 to the 8 and we have another *ten* and 5, making in all 3 *tens* and 5 units, or 35.

3. Be careful to observe that in practice each new number is added to the excess of the tens *mentally*, and nothing named but results.

For example, in finding the sum of a column consisting of the figures 5, 8, 6, 9, 7 and 8, commencing with 5 the results should be named, thus, *five, thirteen, nineteen, twenty-eight, thirty-five, forty-three.*

4. The numbers to be added are called *Addends*. The result found is called the *Sum* or *Amount*, and the process of finding the sum is called *Addition*. The *Sign* +, read plus, placed between numbers; thus, 8 + 5 + 9, shows that these numbers are to be added. The sign =, read equals, denotes that what is written before it is equal to what is written after it; thus, 5 + 9 = 14, is read 5 plus 9 equals 14.

5. To become expert and accurate in adding, you must practice on columns first of *three* figures, then *four*, then five, until you can give the sums of such columns at sight. You must also at the same time apply this practice on long columns, so as to acquire the habit of holding the *tens* in your mind while you perform the addition.

Examples for this practice can be copied from the following table:

ARITHMETICAL DRILL TABLE NO. 1.

	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.
1.	2	3	4	7	1	3	8	5	4	2
2.	1	3	8	5	2	5	3	8	3	5
3.	3	9	5	6	4	8	6	4	8	6
4.	5	5	7	6	6	8	7	9	5	9
5.	4	8	7	5	8	6	9	6	4	7
6.	6	8	6	9	1	9	5	8	6	9
7.	8	4	8	4	3	7	8	9	5	3
8.	7	9	7	8	5	9	8	6	8	5
9.	9	5	9	7	6	8	9	7	4	9
10.	9	8	5	7	8	9	6	7	8	3
11.	7	5	9	4	7	8	5	6	4	9
12.	9	8	6	8	9	4	7	9	7	8

35. Copy from this table examples as follows:

1. Commence with column **A** opposite **1** and copy three numbers for the first example, then opposite **2** and copy three numbers for the second example, and so on to the bottom of the column. The first six examples copied from column **A** in this way are

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
2	1	3	5	4	6
1	3	5	4	6	8
3	5	4	6	8	7

2. Copy examples with three numbers from each column in the same way, and practice on finding the sums as directed for memorizing the addition table.

3. Copy in the same manner examples with *four* numbers, *five* numbers, and so on up to *ten* numbers.

Continue to practice in this way until you can add rapidly and accurately.

ILLUSTRATION OF PROCESS.

36. Prob. 1.—**To find the sum of two or more numbers, each containing only one order of units, and all the same order.**

Find the sum of

(1.)	(2.)	(3.)
6	60	600
9	90	900
8	80	800
23	230	2300

Explanation.—1. The sum of 8, 9, and 6 is found by forming *groups of ten.* Thus, 8 and 9 make 1 *ten* and 7; 7 and 6 make 1 *ten* and 3; hence, 8, 9, and 6 make 2 *tens* and 3, or 23.

2. The sum of 8, 9, and 6 is the same whether these figures express units, tens or hundreds, etc. Hence, when their sum is found, if they express *units,* as in the first example, the sum is *units*; if they express *tens,* as in the second example, the sum is *tens*; if *hundreds, hundreds,* etc.

SIGHT EXERCISES.

Find the sum of

(1.) 80+9 (2.) 700+50+8 (3.) 900+60+3
(4.) 8000+50+3 (5.) 3600+80+2 (6.) 7006+800
(7.) 60+80 (8.) 90+70 (9.) 500+900
(10.) 7000+6000 (11.) 9000+5000 (12.) 800+300

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
30	200	9000	10000	900	7000
70	600	5000	80000	600	5000
80	700	7000	70000	800	9000

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
40	900	4000	50000	500	6000
20	400	3000	90000	300	3000
50	800	6000	40000	200	8000

37. PROB. 2.—**To find the sum of any two or more numbers.**

Find the sum of 985, 854, and 698.

(1.) ANALYSIS.

$$\begin{aligned} 985 &= 900 + 80 + 5 \\ 854 &= 800 + 50 + 4 \\ 698 &= 600 + 90 + 8 \end{aligned}$$

$$\left.\begin{array}{r} 17 \\ 220 \\ 2300 \end{array}\right\} = 2300 + 220 + 17$$

2537

(2.)

$$\left.\begin{array}{r} 985 \\ 854 \\ 698 \end{array}\right\} \text{Addends}$$

2537 Sum.

EXPLANATION.—1. The orders of units in the numbers to be added are *independent* of each other, and may be separated as shown in the *analysis*.

2. The sum of each order is found by finding the sum of the figures expressing that order (**36**).

3. The sums of the separate orders may be united into one sum, as shown in the analysis; or,

4. By commencing with the units' order, the number of tens found can at once be added to the tens' order; so with the hundreds found by adding the tens' order, etc., and thus the sum may be found in one operation, as shown in (2).

From these illustrations we obtain the following

38. Rule.—*I. Write the numbers to be added in such a manner that units of the same order will stand in the same column.*

II. Add each column separately, commencing with the units.

III. When the sum of any column is expressed by two or more figures, place the right-hand figure under the column, and add the number expressed by the remaining figures to the next column.

IV. Write under the last column its entire sum.

Proof.—*Add the numbers by commencing at the top of the columns. If the results agree the work is probably correct.*

EXERCISES FOR PRACTICE.

39. For practice with abstract numbers, copy from Table No. 1, page 16, examples as follows:

Three Numbers of Three Places.

1. Use any three consecutive columns as **A, B, C.** Commence opposite **1** and copy three numbers for the *first* example, then opposite **2** and copy three more for the *second* example, and so on to the bottom of the table.

The first six examples copied in this way are as follows:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
234	138	395	557	487	686
138	395	557	487	686	848
395	557	487	686	848	797

2. Copy in the same manner examples with three numbers from columns B, C, D; C, D, E; D, E, F; E, F, G; F, G, H; G, H, I; and H, I, J.

Four Numbers of Four Places.

40. 1. Copy as before the numbers from any four consecutive columns, as C, D, E, F. Commence in each case opposite **1** for the first number of the first example, opposite **2** for the first number of the second example, and so on to the bottom of the table.

2. Copy in the same manner examples from A, B, C, D; B, C, D, E; D, E, F, G; E, F, G, H; F, G, H, I; and G, H, I, J.

Numbers of Five Places.

41. Continue the practice by copying numbers of five places, as already directed. Commence with examples of five numbers, then six, then seven, and so on.

ORAL EXAMPLES.

42. 1. A farmer sold 50 bushels of wheat to one man, 30 to another, and 20 to another; how many bushels did he sell?

SOLUTION.—He sold as many bushels as the sum of 50, 30, and 20, which is 100. Hence he sold 100 bushels.

2. Mr. Weston owns 20 acres of land, Mr. Fuller owns 40, and Mr. Easty 60; how many acres do they all own?

3. James Merriam sold a cow for $80 and fifteen sheep for $45; how much did he receive for the cow and sheep?

4. A lady paid $24 for a shawl, $35 for a dress, and $9 for a scarf; how much did she pay for all?

5. A boy bought 3 balls and paid 45 cents for each ball. How much did he pay for the three?

6. J. L. Eaton sold a tub of butter for $37, a cheese for $24, and some beans for $18; how much money did he receive?

7. A tailor sold a coat for $35, a vest for $7, and a hat for $6; how many dollars did he get for all?

8. A lady gave $62 for a watch, $42 for a chain, $2 for a key, and $6 for a case; what did she give for all?

WRITTEN EXAMPLES.

43. 1. A store-keeper paid $375 for coffee, $280 for tea, $564 for sugar, $108 for dried apples, and $198 for spices; what was the amount of the purchases? *Ans.* $1525.

2. A newsboy sold 244 papers in January, 301 in February, 278 in March, and 390 in April; how many papers did he sell in the four months? *Ans.* 1213.

3. In a city containing 4 wards, there are 340 voters in the first ward, 523 in the second, 311 in the third, and 425 in the fourth; how many voters in the city?

4. Simon Esty has a house worth $850, and five more each worth $975; what is the value of the six?

5. What is the distance from the Gulf of St. Lawrence to Lake Michigan, passing up the River St. Lawrence 750 miles, Lake Ontario 180 miles, Niagara River 34 miles, Lake Erie 250 miles, Detroit River 23 miles, Lake and River St. Clair 45 miles, and Lake Huron 260 miles? *Ans.* 1542 miles.

6. In 1870 the population of Albany was 69452, Utica 28798, Syracuse 43081, Rochester 63424, Buffalo 117778; what was the united population of these cities?

7. A man bought a house for $3420; he paid $320 to have it painted, and $40 to have it shingled; for what amount must he sell it in order to gain $250? *Ans.* $4030.

8. A grain dealer paid $1420 for a lot of flour, and $680 for a lot of meal; he gained $342 on the flour and $175 on the meal; how much did he receive for both lots?

9. Bought a horse for $275 and a carriage for $342; sold the horse at an advance of $113 and the carriage at an advance of $65; how much did I get for both? *Ans.* $795.

10. Bought 3 house-lots; the first cost $325, the second $15 more than the first, and the third as much as both the others; what was the cost of the whole? *Ans.* $1330.

UNITED STATES MONEY.

44. The sign $ stands for the word ***dollars.*** Thus, $13 is read 13 *dollars.*

45. The letters ***ct.*** stand for ***cents.*** Thus, 57 ct. is read fifty-seven *cents.*

46. When dollars and cents are written together, the cents are separated from the dollars by a (.). Thus, $42 and 58 ct. are written $42.58.

47. When the number of cents is less than 10, a cipher must occupy the first place at the right of the period. Thus, $8 and 4 ct. are written $8.04.

48. In arranging the numbers for adding, dollars must be placed under dollars and cents under cents, in such a manner that the periods in the numbers stand over each other, thus:

(1.)	(2.)	(3.)
$376.84	$3497.03	$53.70
43.09	69.50	786.
706.40	240.84	9.08

WRITTEN EXAMPLES.

49. Read, arrange, and add the following:

1. $93.48 + $406.30 + $8.07 + $5709.80.
2. $4.75 + $3083.09 + $72.50 + $9.32 + $384.
3. $500 + $93.05 + $364.80 + $47.09.

Express in figures the following:

4. Seventy-five dollars and thirty-eight cents.
5. Nine hundred six dollars and seventy-five cents.
6. Three hundred twelve dollars and nine cents.
7. Eighty-four cents; seven cents; three cents.

8. Find the sum of $305.08, $6.54, and $296.03.

9. A farmer sold a quantity of wheat for $97.75, of barley for $42.06, of oats for $39.50. How much did he receive for the whole? *Ans.* $179.31.

10. Bought a house for $4368.90, furniture for $790.07, carpeting $280.60, and made repairs on the house amounting to $307.05. How much did the whole cost? *Ans.* $5746.62.

11. A man bought a horse for $342.50, a carriage for $185.90, and sold them so as to gain on both $85.50. How much were they sold for? *Ans.* $613.90.

12. A furniture dealer sold a bedroom set for $135.86, a bookcase for $75.09, and 3 rocking-chairs for $5.75 each. How much did he receive for the whole? *Ans.* $228.20.

13. A man is in debt to one man $873.60, to another $500.50, to another $75.08, to another $302.04; how much does he owe in all? *Ans.* $1751.22.

14. James Williams bought a saw-mill for $8394.75, and sold it so as to gain $590.85; for how much did he sell it?

15. A lady after paying $23.85 for a shawl, $25.50 for a dress, $2.40 for gloves, and $4.08 for ribbon, finds she has $14.28 left; how much had she at first? *Ans.* $70.11.

DEFINITIONS.

50. ***Addition*** is the process of uniting two or more numbers into one number.

51. ***Addends*** are the numbers added.

52. The ***Sum*** or ***Amount*** is the number found by addition.

53. The ***Process of Addition*** consists in forming units of the same order into groups of ten, so as to express their amount in terms of a higher order.

54. The ***Sign of Addition*** is +, and is read *plus*. When placed between two or more numbers; thus, 8 + 3 + 6 + 2 + 9, it means that they are to be added.

55. The ***Sign of Equality*** is =, and is read *equals,* or *equal to;* thus, $9 + 4 = 13$ is read, *nine plus four equals thirteen.*

56. Principles.—*I. Only numbers of the same denomination and units of the same order can be added.*

II. The sum is of the same denomination as the addends.

III. The whole is equal to the sum of all the parts.

REVIEW AND TEST QUESTIONS.

57. 1. Define Addition, Addends, and Sum or Amount.

2. Name each step in the process of Addition.

3. Why place the numbers, preparatory to adding, units under units, tens under tens, &c.?

4. Why commence adding with the units' column?

5. What objections to adding the columns in an irregular order? Illustrate by an example.

6. Construct, and explain the use of the addition table.

7. How many combinations in the table, and how found?

8. Explain carrying in addition. What objection to the use of the word?

9. Define counting and illustrate by an example.

10. Write five examples illustrating the general problem of addition, "Given all the parts to find the whole."

11. State the difference between the addition of objects and the addition of numbers.

12. Show how addition is performed by using the addition table.

13. What is meant by the denomination of a number? What by units of the same order?

14. Show by analysis that in adding numbers of two or more places, the orders are treated as independent of each other.

SUBTRACTION

PREPARATORY STEPS.

58. STEP 1.—*To find the difference between two numbers when the smaller is expressed by one figure and the larger does not exceed the smaller by 10.*

1. The *difference* between two numbers is the amount that one number is greater than the other. Thus, 7 is 2 greater than 5; hence 2 is the *difference* between 7 and 5.

2. The greater number is called the *Minuend,* the smaller the *Subtrahend,* and the process of finding the *difference* is called *Subtraction.*

3. Subtraction is an application of the Addition Table. From our knowledge of the parts that make up the numbers in the table, we can tell at once, if a number and one of its parts are given, what the other part is.

Thus, we know that 9 and 6 are equal to 15; hence if 15 be given and the part 9, we can name 6 at once as the other part, or difference between 15 and 9.

EXERCISE FOR PRACTICE.

59. 1. Arrange on your slate in irregular order the numbers from 1 to 10, and write 1 under each number, thus:

1	5	2	8	4	7	3	10	6	9
1	1	1	1	1	1	1	1	1	1

Practice writing and pronouncing the differences between these pairs of numbers in the same manner as you did in addition. See (**33**).

Arrange on your slate in the same way and practice upon each of the following:

2. Numbers from 2 to 11 with 2 written under.
3. Numbers from 3 to 12 with 3 written under.
4. Numbers from 4 to 13 with 4 written under.
5. Numbers from 5 to 14 with 5 written under.
6. Numbers from 6 to 15 with 6 written under.
7. Numbers from 7 to 16 with 7 written under.
8. Numbers from 8 to 17 with 8 written under.
9. Numbers from 9 to 18 with 9 written under.

60. STEP 2.—*To find the difference between two numbers when the smaller number is expressed by one figure and the greater by two, the units' figure of which is less than the smaller number*

Observe carefully the following:

1. Numbers of two figures above 20 can at sight be made into two parts one of which contains 1 *ten* and the *units.* Thus, 74 = 60 + 14; 90 = 80 + 10.

2. To subtract, for example, 9 from 85, we regard the 85 as 70 and 15, and take the 9 from the 15. We know *at sight* that 6 is the difference between 15 and 9. Uniting this 6 to 70 we have 76, the difference between 85 and 9.

3. Find the difference between 73 and 6, 32 and 5, 94 and 9, and explain the process as illustrated in 1 and 2.

EXERCISE FOR PRACTICE.

61. 1. Write on your slate every number from 20 to 99, and make each into two parts one of which will contain 1 *ten* and the *units.*

2. Write on your slate in irregular order all the numbers from 20 to 30, with 4 under each number thus,

20	*25*	*21*	*28*	*24*	*30*	*27*	*22*	*26*	*29*
4	*4*	*4*	*4*	*4*	*4*	*4*	*4*	*4*	*4*

Subtract the 4 from each number and write the difference under. Erase and repeat the work until you can write the difference at sight of the numbers.

Practice in the same manner, on subtracting from the same numbers each number from 1 to 9 inclusive.

3. Write on your slate in the same way numbers from 30 to 40 inclusive; then from 40 to 50; 50 to 60; 60 to 70; 70 to 80; 80 to 90.

Practice on subtracting from each set in order numbers from 1 to 9 inclusive, as directed for numbers from 20 to 30. Continue the practice on each set until you can give the differences at sight.

ORAL EXAMPLES.

62. 1. Frank had 9 apples in his basket, but he ate 2 and gave away 3 more; how many were left?

SOLUTION.—He had as many left as the difference between 9 and the sum of 2 and 3, which difference is 4. Hence he had 4 apples left.

2. Irving picked 16 cherries, and gave 4 to his sister and 5 to his mother; how many had he left?

3. A man bought some bacon for $6, and enough flour to make the whole cost $14; what did the flour cost him?

4. A man who had 20 dollars, spent 5 for a hat and 2 for a pair of rubbers; how many dollars had he left?

5. I bought a saddle for $14 and a bridle for $3, and paid $9; how much was left unpaid?

6. A farmer having 40 turkeys sold 3 to one man and 6 to another; how many had he left?

7. William had 42 chickens, but 3 of them died and a hawk carried away 2 more; how many had he then?

8. A drover bought 21 sheep of one man, 12 of another, and 7 of another, and then sold 9; how many had he left?

9. I gave a cow and 9 dollars in money for a horse worth 97 dollars; how much did I get for the cow?

ILLUSTRATION OF PROCESS.

63. PROB. I.—**To find the difference between two numbers, each containing only one order of units and both the same order.**

Find the difference between:

(1.)	(2.)	(3.)
9	90	900
4	40	400
5	50	500

EXPLANATION.—1. The difference between 9 and 4 is found by making 9 into two parts, one of which is 4, the other 5, the difference.

2. The difference between 9 and 4 is the same, whether these figures express units, tens, or hundreds, etc. Hence, when their difference is found, if they express units, as in the first example, the difference is units; if they express tens, as in the second example, the difference is tens; if hundreds, hundreds, etc.

SIGHT EXERCISES.

Find the difference between the following numbers:

(1.)	(2.)	(3.)	(4.)	(5.)
80	500	900	90	8000
30	300	400	70	5000

(6.)	(7.)	(8.)	(9.)	(10.)
13	130	1300	150	1500
6	60	600	90	900

(11.)	(12.)	(13.)	(14.)	(15.)
160	12000	14000	11000	17000
80	5000	9000	3000	8000

64. Prob. II.—**To find the difference between any two numbers.**

Find the difference between 853 and 495.

ANALYSIS.

Minuend,	853	=	700	+	140	+	13
Subtrahend,	495	=	400	+	90	+	5
Difference,	358	=	300	+	50	+	8

Explanation.—1. The 5 units cannot be taken from the 3 units; hence 1 of the 5 tens is added to the 3 units, making 13, as shown in the analysis, and the 5 units are then taken from 13, leaving 8 units.

2. One ten has been taken from the 5 tens in the minuend, leaving 4 tens. The 9 tens of the subtrahend cannot be taken from the 4 tens that are left. Hence 1 of the 8 hundreds is added to the 4 tens, making 14 tens, or 140, as shown in the analysis. The 9 tens are taken from the 14 tens, leaving 5 tens.

3. One hundred has been taken from the 8 hundreds, leaving 7 hundreds. Hence the difference between 853 and 495 is 358.

From these illustrations we obtain the following

65. Rule.—*I. Write the subtrahend under the minuend, placing units of the same order in the same column.*

II. Begin at the right, and subtract the number of units of each order of the subtrahend from the number of units of the corresponding order of the minuend, and write the result beneath.

III. If the number of units of any order of the subtrahend is greater than the number of units of the corresponding order of the minuend, increase the latter by 10 and subtract; then diminish by 1 the units of the next higher order of the minuend and proceed as before.

Proof.—*Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is probably correct.*

EXAMPLES FOR PRACTICE.

66. Copy examples for practice with abstract numbers from Arithmetical Table No. 1, on page 16, as follows:

Examples with Three Figures.

1. Take the numbers from columns A, B, C. For the first example use the numbers opposite **1** and **2**; for the second those opposite **2** and **3**; then **3** and **4, 4** and **5,** and so on to the bottom of the columns. The first six examples are as follows:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
234	395	557	557	686	848
138	138	395	487	487	686

2. Copy examples in the same manner from columns B, C, D; then C, D, E; D, E, F; E, F, G; F, G, H; G, H, I; and H, I, J.

Examples with Four Figures.

67. For examples with four figures, copy the numbers for the first set from columns A, B, C, D; for the second, from B, C, D, E; the third, C, D, E, F; the fourth, D, E, F, G; the fifth, E, F, G, H; the sixth, F, G, H, I; and the seventh, G, H, I, J.

Examples with Six Figures.

68. For examples with six figures copy the numbers from the columns as follows: first set, A, B, C, D, E, F; second set, B, C, D, E, F, G; third set, C, D, E, F, G, H; fourth set, D, E, F, G, H, I; fifth set, E, F, G, H, I, J.

Let all these examples be worked out of school and between recitations, and brought to class on paper for the correction of the answers.

WRITTEN EXAMPLES.

69. 1. A man deposited $1680 in a bank and afterwards drew out $195; how much was left? *Ans.* $1485.

2. The independence of the United States was declared in 1776; how long after that event is the year 1876?

3. The population of Utica in 1860 was 22529 and in 1870 it was 28798; what was the increase? *Ans.* 6269.

4. The height of Mt. Etna is 10840 feet and that of Mt. Vesuvius 3948 feet; how many feet higher is Etna than Vesuvius? *Ans.* 6892 feet.

5. The sum of two numbers is 8627, and the smaller number is 2687; what is the greater? *Ans.* 5940.

6. The number of pupils attending school in Boston in 1870 was 38944, and of these 35442 attended the public schools; how many in all other schools? *Ans.* 3502 pupils.

7. A man bought four houses, for which he paid $15960; for the first he paid $3186, for the second $2783, and for the third $4789; how much did he pay for the fourth?

SOLUTION.—If the man paid $15960 for the four houses and the sum of $3186 + $2783 + $4789, which is $10758 for three of them, he must have paid for the fourth the difference between $15960 and $10758, which is $5202.

8. A man's salary is $1200 a year, and he has money at interest which brings him $225 more; if his expenses are $875, how much can he save? *Ans.* $550.

9. Warren Newhall deposited $362 in the Wakefield Bank on Monday, $760 on Tuesday, and $882 on Thursday; on Wednesday he drew out $380, on Friday $350, and on Saturday $200; how much remained on deposit at the end of the week? *Ans.* $1074.

10. A has $6185, B has $15181, C has $858 less than A and B together, and D has as much as all the rest; how much has D? *Ans.* $41874.

11. My property is valued at $7096, and I owe a debt of

$600, another of $1247, and another of $429; what am I worth? *Ans.* $4820.

12. A man deposits $1110 in the bank at one time, and $1264 at another; he then draws out $786 at one time, $654 at another, $689 at another; how much still remained in the bank? *Ans.* $245.

13. A merchant paid $4570 for goods; he sold a part of them for $3480, and the rest for $2724; how much did he gain by the transaction? *Ans.* $1634.

14. I bought a farm for $4750, and built a house and barn upon it at a cost of $3475, and then sold the whole for $7090; how much did I lose? *Ans.* $1135.

15. A grain-dealer bought 8756 bushels of grain; he then sold 2368 bushels at one time and 5383 bushels at another; how many bushels had he left? *Ans.* 1005 bushels.

16. Find the difference between $527.03 and $264.39.

$$\begin{array}{r} \$527.03 \\ 264.39 \\ \hline \$262.64 \end{array}$$

Explanation.—Write the subtrahend under the minuend, so that dollars are under dollars and cents under cents. Subtract as if the numbers were abstract and place a period in the result between the second and third figures from the right. The figures on the left of the period express dollars and those on the right cents.

17. I received $352.07, and paid out of this sum to one man $73.12, to another $112.57; how much have I left of the money? *Ans.* $166.38.

18. A lady had $23.37, and paid out of this $7.19 for flour, $3.07 for sugar, $2.05 for butter; how much had she left?

19. A farmer sold $153 worth of wheat, $54.75 of barley, and $29.05 of oats. He paid out of the money received to one man $32.13, to another $109.55; how much had he left?

20. A merchant sold in one day $732.17 of goods. He received in cash $459.58; how much did he sell on credit?

21. Three men are to pay a debt of $4809. The first man pays $1905.38, the second $2001.70; how much has the third to pay? *Ans.* $901.92.

DEFINITIONS.

70. ***Subtraction*** is the process of finding the difference between two numbers.

71. The ***Minuend*** is the greater of two numbers whose difference is to be found.

72. The ***Subtrahend*** is the smaller of two numbers whose difference is to be found.

73. The ***Difference*** or ***Remainder*** is the result obtained by subtraction.

74. The ***Process of Subtraction*** consists in comparing two numbers, and resolving the greater into two parts, one of which is equal to the less and the other to the difference of the numbers.

75. The ***Sign of Subtraction*** is —, and is called *minus*. When placed between two numbers it indicates that their difference is to be found; thus, 14 — 6 is read, 14 minus 6, and means that the difference between 14 and 6 is to be found.

76. ***Parentheses*** ( ) denote that the numbers inclosed between them are to be considered as one number.

77. A ***Vinculum*** $\overline{\quad\quad}$ affects numbers in the same manner as parentheses. Thus, $19 + (13 - 5)$, or $19 + \overline{13 - 5}$ signifies that the difference between 13 and 5 is to be added to 19.

78. Principles.—*I. Only like numbers and units of the same order can be subtracted.*

II. The minuend is the sum of the subtrahend and difference, or the minuend is the whole of which the subtrahend and difference are the parts.

III. An equal increase or decrease of the minuend and subtrahend does not change the difference.

REVIEW AND TEST QUESTIONS.

79. 1. Define the process of subtraction. Illustrate each step by an example.

2. Explain how subtraction should be performed when an order in the subtrahend is greater than the corresponding order in the minuend. Illustrate by an example.

3. Indicate the difference between the subtraction of numbers and the subtraction of objects.

4. When is the result in subtraction a remainder, and when a difference?

5. Show that so far as the process with numbers is concerned the result is always a difference.

6. Prepare four original examples under each of the following problems and explain the method of solution:

PROB. I.—*Given the whole and one of the parts to find the other part.*

PROB. II.—*Given the sum of four numbers and three of them to find the fourth.*

7. Construct a Subtraction Table.

8. Define counting by subtraction.

9. Show that counting by addition, when we add a number larger than *one*, necessarily involves counting by subtraction.

10. What is the difference between the meaning of *denomination* and *orders of units?*

11. State Principle III and illustrate its meaning by an example.

12. Show that the difference between 63 and 9 is the same as the difference between (63 + 10) and (9 + 10).

13. Show that 28 can be subtracted from 92, without analyzing the minuend as in (**64**), by adding 10 to each number.

14. What must be added to each number, to subtract 275 from 829 without analyzing the minuend as in (**64**)?

15. What is meant by *borrowing* and *carrying* in subtraction?

MULTIPLICATION

PREPARATORY STEPS.

80. STEP I.—*To find the sum of 2 times, 3 times, and so on to 12 times, any number expressed by one figure.*

1. The sum of any number of times a given number is found by addition. Thus, 3 times 9 or *three nines* equals $9 + 9 + 9 = 27$.

2. The sign $\times$ stands for the word *times*. Thus, 4×5 is read either 4 times 5 which means $5 + 5 + 5 + 5$, or 5 times 4 which means $4 + 4 + 4 + 4 + 4$.

3. Be particular to notice that the sum of 2 *threes* and 3 *twos* is the same, and so with 3 *fours* and 4 *threes*, 4 *fives* and 5 *fours*, and so on.

This may be shown with marks on your slate, thus:

***3 fours* = *4 threes* = 12.**

= 12.

$4 + 4 + 4 = 3+3+3+3 = 12.$

4. The sum of any number of times a given number is called a *Product*. Thus, 3 times $7 = 7+7+7 = 21$, hence 21 is the *product* of 3 and 7; and 3 and 7 are called *Factors* of 21.

5. The ***Multiplication Table*** consists of the products of numbers from 2 to 12 inclusive. These products are found by addition, and then memorized so that they can be given at sight of their factors.

EXERCISES FOR PRACTICE.

81. Copy on your slate and find by addition the product of each of the following examples, and write it in place of the question mark.

1.	$7 \times 3 = ?$	$5 \times 7 = ?$	$3 \times 3 = ?$
2.	$4 \times 7 = ?$	$8 \times 9 = ?$	$4 \times 9 = ?$
3.	$9 \times 5 = ?$	$4 \times 8 = ?$	$7 \times 8 = ?$
4.	$6 \times 3 = ?$	$5 \times 5 = ?$	$8 \times 2 = ?$
5.	$3 \times 4 = ?$	$8 \times 8 = ?$	$6 \times 9 = ?$

6. Find by addition each of the products in the following Multiplication Table.

MULTIPLICATION TABLE.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

82. STEP II.—*To memorize the Multiplication Table.* Pursue the following course:

1. Write on your slate in two sets and in irregular order

2 times 2 are, 3 times 2 are, and so on, up to 12 times 2 are, thus:

(1.)	(2.)
2 times 2 are	7 times 2 are
5 times 2 are	11 times 2 are
3 times 2 are	9 times 2 are
8 times 2 are	12 times 2 are
4 times 2 are	6 times 2 are
9 times 2 are	10 times 2 are

2. Find, by adding, the *product* of each example and write it after the word "are."

3. Read very carefully the first set several times, then erase the *products* and write them again from memory as you read the example. Continue to erase and write the *products* in this way until they are firmly fixed in your memory.

In every case where you feel the least uncertainty about the correctness of a product, find it again by addition.

4. Practice on the second set in the same way. When you have its products fixed in your memory, then practice on writing at once the products of both sets.

To vary the exercises, erase all the *products*, and give them orally as you repeat the example mentally; thus, Two times two are *four;* Five times two are *ten;* three times two are *six,* and so on.

5. Write on your slate a series of *twos,* and write under them in irregular order the numbers from 2 to 12 inclusive; thus,

2	2	2	2	2	2	2	2	2	2	2
2	6	8	3	9	5	12	7	11	4	10

Write the product under each example as you repeat *mentally* the number of *twos.* Thus, as you say mentally *two twos,* write 4 under the first example; as you say, *six twos,* write 12 under the second example, and so on with each of the other examples. Erase the products and write them again and again, until each product is called up to your mind just as soon as you look at the two numbers.

6. Pursue the same course in memorizing the products of 3's, 4's, 5's, 6's, 7's, 8's, and 9's.

MULTIPLIER ONE FIGURE.

PREPARATORY STEPS.

83. STEP I.—*Find by using the Multiplication Table the product of each of the following:*

Thus, 5 × 7 = 35, 5 tens × 7 = 35 tens, 500 × 7 = 3500.

Find the product of

1. 8 × 6; 8 tens × 6; 8 hundred × 6; 8000 × 6.
2. 9 × 7; 90 × 7; 900 × 7; 9000 × 7.
3. 3 × 5; 30 × 5; 300 × 5; 3000 × 5.
4. 7000 × 4; 500 × 9; 8000 × 4; 4000 × 3.
5. 60000 × 6; 900000 × 9; 5000000 × 7.

84. STEP II.—*The orders in a number are independent of each other; hence, to find any number of times a given number, we multiply each order separately,* thus:

To find 6 times 748, we regard the 748 = 700 + 40 + 8. We know from *memorized results* that 6 times 8 are 48, that 6 times 40 are 240, and that 6 times 700 are 4200. Having taken each of the three parts of 748 6 times, the sum of these products must be 6 times 748. Hence, 48 + 240 + 4200 = 4488 = 6 times 748.

EXERCISE FOR PRACTICE.

Multiply and explain, as shown in Step II, each of the following:

1. 342 × 2.	5. 834 × 3.	9. 437 × 9.
2. 132 × 3.	6. 527 × 6.	10. 685 × 7.
3. 421 × 4.	7. 289 × 5.	11. 392 × 4.
4. 711 × 9.	8. 837 × 8.	12. 759 × 8.

85. The method of finding the *sum* of two or more times a given number by using memorized results is called *Multiplication.* The number taken is called the *Multiplicand,* and the number which denotes how many times the multiplicand is taken is called the *Multiplier.*

ILLUSTRATION OF PROCESS.

86. Prob. I.—**To multiply any number by numbers less than 10.**

How many are 4 times 369?

(1.) ANALYSIS.

$$369 \times 4 = \begin{cases} 9 \times 4 = 36 \\ 60 \times 4 = 240 \\ 300 \times 4 = 1200 \end{cases}$$

$$\underline{\qquad\quad} \\ 1476$$

(2.)

$$\begin{array}{r} 369 \\ 4 \\ \hline 1476 \end{array}$$

Explanation.—1. The 369 is equal to the three parts, 9, 60, and 300.

2. By taking each of these parts four times, the 369 is taken four times. Hence, to find 4 times 369, the 9 is taken 4 times; then the 60; then the 300, as shown in the analysis.

3. Uniting the 36, the 240, and the 1200 in one number, we have 4 times 369. Hence, 1476 is 4 times 369.

4. In practice, no analysis is made of the number. We commence with the units and multiply thus:

(1.) 4 times 9 units are 36 units or 3 tens and 6 units. We write the 6 units in the units' place and reserve the 4 tens to add to the product of the tens.

(2.) 4 times 6 tens are 24 tens, and the 3 tens reserved are 27 tens or 2 hundred and 7 tens. We write the 7 tens in the tens' place, and reserve the 2 hundred to add to the product of the hundreds.

(3.) We proceed in the same manner with hundreds, thousands, etc.

From these illustrations, we obtain the following

87. Rule.—*Begin at the right hand and multiply each order of the multiplicand by the multiplier. Write in the product, in each case, the units of the result, and add the tens to the next higher result.*

EXAMPLES FOR PRACTICE.

Perform the multiplication in the following:

1. 837 × 3.	7. 986 × 4.	13. 579 × 7.
2. 5709 × 8.	8. 7093 × 9.	14. 90703 × 6.
3. 83095 × 2.	9. 50739 × 6.	15. 29073 × 8.
4. 39706 × 5.	10. 79060 × 8.	16. 40309 × 7.
5. 95083 × 6.	11. 79350 × 2.	17. 73290 × 9.
6. 70639 × 8.	12. 60790 × 5.	18. 30940 × 6.

88. Continue the practice with abstract numbers by taking examples from Arithmetical Table No. 1, page 16, in the following order:

Three Figures in the Multiplicand.

1. Use three columns and copy for multiplicands each number in the columns, commencing at the top of the Table.

2. Take as multiplier the figure immediately under the right-hand figure of the multiplicand.

The first six examples taken in this way from columns **A, B, C,** are

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
234	138	395	557	487	686
8	5	7	7	6	8

3. Let examples be copied in this way from columns A, B, C; B, C, D; C, D, E; D, E, F; E, F, G; F, G, H; G, H, I; and H, I, J.

Four Figures in the Multiplicand.

1. Use four columns, and copy the multiplicands and multipliers in the same way as with three figures, taking the multipliers from the first column on the right.

2. Copy from columns A, B, C, D; then B, C, D, E; C, D, E, F; D, E, F, G; E, F, G, H; F, G, H, I.

Six Figures in the Multiplicand.

1. Copy, as already directed, examples from columns A, B, C, D, E, F; then B, C, D, E, F, G; C, D, E, F, G, H; D, E, F, G, H, I; and E, F, G, H, I, J. Take the multipliers from the right-hand column used.

2. Let the examples from each of these sets be worked at your seat between recitations or out of school.

ORAL EXAMPLES.

89. 1. Bought 3 barrels of flour, at $12 a barrel, and a barrel of crackers for $5; how much did the whole cost?

SOLUTION.—The whole cost three times $12, plus $5, which is $41.

2. If it requires 4 yards of cloth to make a coat, and 1 yard to make a vest, how many yards will make 8 of each? 12 of each? 9 of each?

3. Bought 12 chairs at $2 each, a sofa at $45, and 5 tables at $9 each; how much did the whole cost?

4. Gave $7 each to 4 men, paid for 9 yards of cloth at $3 a yard, and for a coat $18; how much money have I spent?

5. At 8 dollars a cord, what will 5 cords of wood cost? 7 cords? 11 cords? 9 cords? 12 cords?

WRITTEN EXAMPLES.

90. 6. How much will 7 acres of land cost, at $285 an acre? *Ans.* $1995.

SOLUTION.—7 acres will cost 7 times $285. 7 times $285 = 7 times $5 + 7 times $80 + 7 times $200 = $1995. Hence, 7 acres cost $1995.

7. What will 647 cords of wood cost at $6 a cord?

8. What will be the cost of building 213 yards of iron fence, at 3 dollars a yard? *Ans.* 639 dollars.

9. There are 5280 feet in a mile; how many feet in 12 miles? *Ans.* 63360 feet.

10. I sold 284 acres of land at 9 dollars an acre; how much money did I receive? *Ans.* $2556.

11. There are 4 farthings in one penny; how many farthings in 379 pennies? *Ans.* 1516 farthings.

12. James Reed went to market with $485; he paid for 20 barrels of flour at $8 a barrel; 16 boxes of soap at $3 a box; and 3 tubs of butter at $12 a tub; how much money did he have left? *Ans.* $241.

13. A merchant bought 9 hogsheads of molasses at $52 a hogshead, and sold the whole for $544; how much did he gain by the transaction? *Ans.* $76.

14. Sold 89 bushels of beans at $2 a bushel, and 7 loads of hay at $19 a load; how much did I receive for both?

MULTIPLIERS 10 AND ABOVE.

PREPARATORY STEPS.

91. STEP I.—*To multiply any number by 10, 100, 1000, and so on.*

1. A figure is multiplied by 10 by moving it one place to the left, by 100 by moving it two places, etc. Thus, 4 expresses *four*, 40 expresses 10 *fours*, 400 expresses 100 *fours*, etc.

2. A cipher placed at the right of a number moves each significant figure in it one place to the left; hence, multiplies it by 10.

Thus, in 372 the 2 is in the first place, the 7 in the second, and the 3 in the third; but in 3720 the 2 is in the second, the 7 in the third, and the 3 in the fourth place; hence, annexing the cipher has removed each figure one place to the left, and consequently multiplied each order in the number by 10.

3. In like manner annexing two ciphers, three ciphers, etc., multiplies a number by 100, 1000, etc., respectively.

92. STEP II.—*To multiply by using the parts of the multiplier.*

1. The multiplier may be made into any desired parts, and the multiplicand taken separately the number of times expressed by each part. The sum of the products thus found is the required product.

Thus, to find 9 times 12 we may take 4 times 12 which are 48, then 5 times 12 which are 60. 4 times 12 plus 5 times 12 are 9 times 12; hence, 48 plus 60, or 108, are 9 times 12.

2. When we multiply by one of the equal parts of the multiplier, we find one of the equal parts of the required product. Hence, by multiplying the part thus found by the number of such parts, we find the required product.

For example, to find 12 times 64 we may proceed thus:

(1.) ANALYSIS.

$$\left.\begin{array}{l} 64 \times 4 = 256 \\ 64 \times 4 = 256 \\ 64 \times 4 = 256 \end{array}\right\} = 3 \text{ times } 256.$$

$$64 \times 12 = 768$$

(2.)

$$\begin{array}{r} 64 \\ 4 \\ \hline 256 \\ 3 \\ \hline 768 \end{array}$$

(1.) Observe, that $12 = 4 + 4 + 4$; hence, 4 is one of the 3 equal parts of 12.

(2.) That 64 is taken 12 times by taking it 4 times + 4 times + 4 times, as shown in the analysis.

(3.) That 4 times 64, or 256, is one of the 3 equal parts of 12 times 64. Hence, multiplying 256 by 3 gives 12 times 64, or 768.

3. In multiplying by 20, 30, and so on up to 90, we invariably multiply by 10 one of the equal parts of these numbers, and then by the number of such parts.

For example, to multiply 43 by 30, we take 10 times 43, or 430, and multiply this product by 3; $430 \times 3 = 1290$, which is 30 times 43.

We multiply in the same manner by 200, 300, etc., 2000, 3000, etc.; multiplying first by 100, 1000, etc., then the product thus found by the number of 100's, 1000's, etc.

ILLUSTRATION OF PROCESS.

93. Prob. II.—**To multiply by a number containing only one order of units.**

1. Multiply 347 by 500.

	(1.) ANALYSIS.	(2.)
First step,	347 × 100 = 34700	347
Second step,	34700 × 5 = 173500	500
		173500

Explanation.—500 is equal to 5 times 100; hence, by taking 347, as in *first step,* 100 times, 5 times this result, or 5 times 34700, as shown in *second step,* will make 500 times 347. Hence 173500 is 500 times 347.

2. In practice we multiply first by the significant figure, and annex to the product as many ciphers as there are ciphers in the multiplier, as shown in (2); hence the following

94. Rule.—*Multiply by the significant figure and annex as many ciphers to the result as there are ciphers in the multiplier.*

95. EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)	(4.)
Multiply	34	256	573	968
By	50	70	90	60

	(5.)	(6.)	(7.)	(8.)
Multiply	3465	8437	2769	4763
By	600	300	800	200

	(9.)	(10.)	(11.)	(12.)
Multiply	70	850	7300	8390
By	40	30	600	900

96. PROB. III.—**To multiply by a number containing two or more orders of units.**

1. Multiply 539 by 374.

(1.) ANALYSIS.

$$539 \times 374 = \begin{cases} 539 \times 4 = 2156 \\ 539 \times 70 = 37730 \\ 539 \times 300 = 161700 \end{cases}$$

$$201586$$

(2.)

539	Multiplicand.
374	Multiplier.
2156	1st partial product.
37730	2d partial product.
161700	3d partial product.
201586	Whole product.

EXPLANATION.—1. The multiplier, 374, is analyzed into the parts 4, 70, and 300, according to (**92**).

2. The multiplicand, 539, is taken first 4 times = 2156 (**86**); then 70 times = 37730 (**93**); then 300 times = 161700 (**93**).

3. 4 times + 70 times + 300 times are equal to 374 times; hence the sum of the partial products, 2156, 37730, and 161700, is equal to 374 times 539 = 201586.

4. Observe, that in practice we arrange the partial products as shown in (2), omitting the ciphers at the right, and placing the first significant figure of each product under the order to which it belongs. Hence the following

97. RULE.—*I. Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

II. Multiply the multiplicand by each significant figure in the multiplier, successively, beginning at the right, and place the right-hand figure of each partial product under the order of the multiplier used. Add the partial products, which will give the product required.

PROOF.—*I. Repeat the work. II. Use the multiplicand as multiplier; if the results are the same the work is probably correct.*

EXAMPLES FOR PRACTICE.

98. Copy examples from Arithmetical Table No. 1, page 16.

Multiplicand five figures, Multiplier three.

1. Take the multiplicands in order, commencing opposite **1**, from columns A, B, C, D, E; B, C, D, E, F; C, D, E, F, G; D, E, F, G, H; and E, F, G, H, I.

2. Take the multipliers in each set from the *three right-hand columns* used for multiplicands, the number immediately under the multiplicand.

Multiplicand six figures, Multiplier five.

1. Take the multiplicands in order from columns A, B, C, D, E, F; B, C, D, E, F, G; C, D. E, F, G, H; and D, E, F, G, H, I.

2. Take the multipliers in each set from the *five right-hand columns* used for multiplicand.

WRITTEN EXAMPLES.

99. 1. A man left $4500 to his wife, $3254 to each of his five daughters, and the remainder of his property, amounting to $3860 to his only son; what was the value of his estate?

2. If you should buy 2682 barrels of flour, at $9 a barrel, and pay $15838 down, how much would you still owe for the flour? *Ans.* $8300.

3. I bought 8 barrels of sugar, at $54 a barrel; 3 barrels of it were spoiled by exposure, but the rest was sold at $72 a barrel; how much did I lose on the sugar? *Ans.* $72.

4. Sold 5 oxen at $75 each, 3 horses at $256 each, a carriage at $325, and a plow for $25; how much did I receive for the whole? *Ans.* $1493.

5. There are 63 gallons in a hogshead; how many gallons in 8290 hogsheads? *Ans.* 522270.

6. If an acre yields 114 bushels of potatoes, how many bushels may be raised on 124 acres? *Ans.* 14136 bushels.

7. If 276 men can do a piece of work in 517 days, in what time could one man do the same work? *Ans.* 142692 days.

8. A man owns 2 orchards, in each of which there are 21 rows of trees, with 213 trees in each row; how many trees do both orchards contain? *Ans.* 8946 trees.

9. If 2 tons of hay, worth $13 a ton, winter one cow, what will be the cost of wintering 348 cows? *Ans.* $9048.

10. I bought 14 cows at 39 dollars each, and 29 oxen at 63 dollars each; how much did I pay for all? *Ans.* $2373.

11. France contains 203736 square miles, and the population averages 176 per square mile; what is the entire population? *Ans.* 35857536.

12. A square mile contains 640 acres; find the cost of 36 square miles at $45 an acre. *Ans.* $1036800.

13. What is the cost of 5 yards of cloth at $2.25 a yard.

SOLUTION.—Since 1 yard costs $2.25, 5 yards will cost 5 times $2.25, which is $11.25.

Observe, that when the multiplicand contains cents, we multiply without regard to the period, and insert a period between the second and third figures of the result. The two figures at the right express the cents in the answer.

14. A farmer sold 57 bushels of beans at $2.36 per bushel, and 285 bushels of wheat at $1.75. How much did he receive for both? *Ans.* $633.27.

15. A fruit merchant bought 295 baskets of peaches at $1.25 a basket; finding that 43 baskets were worthless, he sold the rest at $1.75; how much did he make on the transaction?

16. A drover bought 94 head of cattle at $39 a head and 236 sheep at $3.89 a head. He sold the cattle at a gain of $9 a head and the sheep at a loss of $.75 a head; what was the total amount of the sale, and the gain on the transaction?

17. A merchant bought 472 yards of cloth at $1.25 a yard; 147 were damaged and had to be sold at $.67 a yard. He sold the remainder at $1.58 a yard; did he gain or lose on the transaction, and how much? *Ans.* $21.99 gain.

18. A mechanic employed on a building 78 days received $2.75 a day. His family expenses during the same time were $1.86 a day; how much did he save? *Ans.* $69.42.

19. Bought 167 bushels of wheat at $1.65 a bushel, and 287 bushels of oats at $.37 a bushel. I sold the wheat at a loss of 4 cents on a bushel, and 34 bushels of oats at a gain of 18 cents a bushel, the remainder at a gain of 13 cents. What did I gain on the transaction?

20. A merchant purchased 16 pieces of cloth, each containing 48 yards, at $2.75 a yard. He sold the entire lot at an advance of $.45 per yard. How much did he pay for the cloth, and what was his entire gain?

DEFINITIONS.

100. ***Multiplication*** is the process of taking one number as many times as there are units in another.

101. The ***Multiplicand*** is the number taken, or multiplied.

102. The ***Multiplier*** is the number which denotes how many times the multiplicand is taken.

103. The ***Product*** is the result obtained by multiplication.

104. A ***Partial Product*** is the result obtained by multiplying by one order of units in the multiplier, or by any part of the multiplier.

105. The ***Total*** or ***Whole Product*** is the sum of all the partial products.

106. The ***Process of Multiplication*** consists, *first,* in finding partial products by using the memorized results of the Multiplication Table; *second,* in uniting these partial products by addition into a total product.

107. A ***Factor*** is one of the *equal parts* of a number. Thus, 12 is composed of six 2's, four 3's, three 4's, or two 6's; hence, 2, 3, 4, and 6 are factors of 12.

The multiplicand and multiplier are factors of the product. Thus, 36 × 25 = 925. The product 925 is composed of *twenty-five* 37's, or *thirty-seven* 25's. Hence, both 37 and 25 are equal parts or factors of 925.

108. The ***Sign of Multiplication*** is ×, and is read *times,* or *multiplied by.*

When placed between two numbers, it denotes that either is to be multiplied by the other. Thus, 8 × 6 shows that 8 is to be taken 6 times, or that 6 is to be taken 8 times; hence it may be read either 8 times 6 or 6 times 8.

109. PRINCIPLES.—*I. The multiplicand may be either an abstract or concrete number.*

II. The multiplier is always an abstract number.

III. The product is of the same denomination as the multiplicand.

REVIEW AND TEST QUESTIONS.

110. 1. Define Multiplication, Multiplicand, Multiplier, and Product.

2. What is meant by Partial Product? Illustrate by an example.

3. Define Factor, and illustrate by examples.

4. What are the factors of 6? 14? 15? 9? 20? 24? 25? 27? 32? 10? 30? 50? and 70?

5. Show that the multiplicand and multiplier are factors of the product.

6. What must the denomination of the product always be, and why?

7. Explain the process in each of the following cases and illustrate by examples:

I. To multiply by numbers less than 10.
II. To multiply by 10, 100, 1000, and so on.
III. To multiply by one order of units.
IV. To multiply by two or more orders of units.
V. To multiply by the factors of a number (**92**—2).

8. Give a rule for the third, fourth, and fifth cases.

9. Give a rule for the shortest method of working examples where both the multiplicand and multiplier have one or more ciphers on the right?

10. Show how multiplication may be performed by addition.

11. Explain the construction of the Multiplication Table, and illustrate its use in multiplying.

12. Why may the ciphers be omitted at the right of partial products?

13. Why commence multiplying the units' order in the multiplicand first, then the tens', and so on? Illustrate your answer by an example.

14. Multiply 8795 by 629, multiplying first by the tens, then by the hundreds, and last by the units.

15. Multiply 3572 by 483, commencing with the thousands of the multiplicand and hundreds of the multiplier.

16. Show that *hundreds* multiplied by *hundreds* will give *ten thousands* in the product.

17. Multiplying *thousands* by *thousands*, what order will the product be?

18. Name at sight the *lowest order* which each of the following examples will give in the product:

(1.) 8000 × 3000; 2000000 × 3000; 5000000000 × 7000.

(2.) 40000 × 20000; 7000000 × 4000000.

19. What orders in 3928 can be multiplied by each order in 473, and not have any order in the product less than thousands?

DIVISION

PREPARATORY STEPS.

111. Step I.—*To find how many times a number expressed by one figure is contained in any number not greater than* 9 *times the given number.*

1. This is done by our knowledge of the Multiplication Table. Thus, if asked how many 3's in 15, we can answer at once *five* 3's.

Answer the following questions:

1. How many 2's in 4? In 8? In 12?

2. How many 5's in 15? In 25? In 10? In 30? In 40? In 45? In 20?

3. How many 7's in 14? In 35? In 49? In 63?

4. How many 4's in 12? In 20? In 8? In 28? In 16? In 24? In 36?

5. How many 6's in 8, and what remaining? In 13? In 26? In 37? In 45? In 32?

6. How many 3's in 4, and what remaining? In 7? In 11? In 16? In 14? In 25?

2. The method of finding how many times one number is contained in another by using the Multiplication Table is called *Division.*

3. The number we divide is called the *Dividend*, and the number by which we divide is called the *Divisor*.

4. The number which tells how many times the divisor is contained in the dividend is called the *Quotient*, and what is left of the dividend after the division is performed is called the *Remainder*.

112. The sign ÷ stands for the words, *how many —— in*, or *divided by*. Thus, 9 ÷ 3 is read, *How many 3's in 9*, or *9 divided by 3*.

Express on your slate with the sign ÷ each of the following questions, and then give the answer:

1. How many 3's in 12? In 18? In 27? In 9? In 21?

2. How many 7's in 21? In 35? In 14? In 42? In 63?

3. How many 4's in 16? In 28? In 36? In 12? In 24? In 32? In 9? In 17?

4. How many 9's in 18? In 10? In 27? In 45? In 38? In 72? In 30? In 63? In 67?

Read and give the answer for each of the following:

5. 24 ÷ 8. 40 ÷ 8. 16 ÷ 8. 56 ÷ 8. 16 ÷ 8. 48 ÷ 8. 72 ÷ 8. 9 ÷ 8. 27 ÷ 8. 19 ÷ 8.

6. 18 ÷ 6. 24 ÷ 6. 36 ÷ 6. 48 ÷ 6. 12 ÷ 6. 30 ÷ 6. 42 ÷ 6. 54 ÷ 6. 14 ÷ 6.

7. 27 ÷ 9. 45 ÷ 9. 9 ÷ 9. 36 ÷ 9. 54 ÷ 9. 81 ÷ 9. 63 ÷ 9. 18 ÷ 9. 11 ÷ 9. 30 ÷ 9.

113. STEP II.—*To apply the Multiplication Table in finding at sight how many times a number expressed by one figure is contained in any number not greater than* 9 *times the given number.*

Pursue the following course:

1. Write on your slate in irregular order the products of the Multiplication Table, commencing with the products of 2. Write immediately before, the number whose products you have taken; thus,

2)10 *2)4* *2)14* *2)6* *2)12* *2)16* *2)8*

2. Write under the line from memory the number of 2's in 10, in 4, in 14, etc. When this is done, erase each of these results, and rewrite and erase again and again, until you can give the quotients at sight of the other two numbers.

3. Look at the numbers, and question yourself. Thus, you say mentally, *twos* in *ten,* and you follow with the answer, *five; twos* in four, *two; twos* in fourteen, *seven.*

4. Omit the questions entirely, and pass your eye along the examples and name the results; thus, *five, two, seven,* etc.

EXERCISE FOR PRACTICE.

114. Practice as above directed on each of the following:

1.	3) 6	3) 12	3) 15	3) 9	3) 24	3) 18	3) 21
2.	4) 12	4) 24	4) 32	4) 8	4) 16	4) 28	4) 20
3.	5) 15	5) 25	5) 10	5) 30	5) 20	5) 45	5) 35
4.	6) 12	6) 24	6) 36	6) 18	6) 42	6) 54	6) 30
5.	7) 14	7) 35	7) 49	7) 21	7) 42	7) 56	7) 28
6.	8) 16	8) 40	8) 24	8) 56	8) 32	8) 64	8) 48
7.	9) 27	9) 45	9) 18	9) 54	9) 72	9) 36	9) 63
8.	7) 56	4) 36	8) 48	5) 35	9) 81	6) 54	8) 72
9.	5) 55	9) 99	6) 84	8) 72	4) 36	3) 27	9) 63

DIVISORS FROM 2 TO 12.

PREPARATORY STEPS.

115. STEP I.—*To divide when the quotient is expressed by two or more places, but contains only one order of units.*

1. To do this we regard the dividend as made into equal parts, and we divide one of these equal parts by the given divisor and multiply the quotient thus found by the number of equal parts; thus,

Take for example 60 divided by 3. We know 6 is one of the 10 equal parts of 60. We know also that there are

2 *threes* in 6, and that each 6 in the 60 must contain 2 *threes*. Then as 60 contains 10 times 6, it must contain 10 times 2 threes, or 20 threes. Hence the quotient of 60 divided by 3 is 20.

2. The equal parts of the dividend which we divide may be expressed by two or more figures.

Take, for example, 3500 divided by 7. Here we divide first the 35 by 7. We know that 35 is one of the 100 equal parts of 3500. We know also that there are 5 *sevens* in 35, and that each 35 in 3500 must contain 5 *sevens*. We know, therefore, that as 3500 contains 100 times 35, it must contain 100 times 5 *sevens*, or 500 *sevens*. Hence the quotient of 3500 divided by 7 is 500.

3. When there is only one order in the quotient, it can be given at sight of the dividend and divisor.

Thus, in dividing 2700 by 9, you know at once that there are 3 *nines* in 27, and hence that there are 300 nines in 2700.

116. EXAMPLES FOR PRACTICE.

1. 80 ÷ 2.	8. 4200 ÷ 7.	15. 9600 ÷ 12.
2. 90 ÷ 3.	9. 2500 ÷ 5.	16. 1080 ÷ 9.
3. 60 ÷ 2.	10. 7200 ÷ 9.	17. 64000 ÷ 8.
4. 120 ÷ 4.	11. 3600 ÷ 12.	18. 49000 ÷ 7.
5. 180 ÷ 9.	12. 5400 ÷ 6.	19. 55000 ÷ 11.
6. 350 ÷ 5.	13. 5600 ÷ 8.	20. 63000 ÷ 9.
7. 320 ÷ 8.	14. 4400 ÷ 11.	21. 45000 ÷ 5.

117. STEP II.—*To divide when the quotient contains two or more orders of units.*

Observe carefully the following:

1. Each order of the dividend may contain the divisor an exact number of times. In this case the division of each order is performed independently of the others.

For example, to divide 888 by 2, we may separate the orders thus:

$$888 \div 2 = \left\{ \begin{array}{r} 800 \div 2 = 400 \\ 80 \div 2 = 40 \\ 8 \div 2 = 4 \end{array} \right\} = 444.$$

2. When each order does not contain the divisor an exact number of times, we take the largest part of the dividend which we know does contain it.

Thus, in dividing 92 by 4, we observe at once that 80 is the largest part of the dividend which we know contains 4 an exact number of times. We divide 80, and obtain 20 as the quotient. We have now left of the dividend undivided 1 ten and 2 units, which make 12 units. We know that 12 contains 3 times 4, and we have already found that 80 contains 20 times 4. Hence 80 + 12, or 92, must contain 20 + 3 or 23 times 4.

EXAMPLES FOR PRACTICE.

118. Perform the division in the following examples, and explain each step in the process, as above:

1. 222 ÷ 2.	15. 8888 ÷ 4.	29. 4826 ÷ 2.
2. 444 ÷ 2.	16. 9693 ÷ 3.	30. 6396 ÷ 3.
3. 666 ÷ 3.	17. 684 ÷ 2.	31. 8480 ÷ 4.
4. 52 ÷ 2.	18. 96 ÷ 4.	32. 87 ÷ 3.
5. 84 ÷ 3.	19. 84 ÷ 6.	33. 870 ÷ 3.
6. 960 ÷ 3.	20. 780 ÷ 3.	34. 8700 ÷ 3.
7. 680 ÷ 4.	21. 85 ÷ 5.	35. 9800 ÷ 7.
8. 950 ÷ 5.	22. 940 ÷ 2.	36. 8000 ÷ 5.
9. 980 ÷ 2.	23. 92 ÷ 4.	37. 9000 ÷ 6.
10. 180 ÷ 6.	24. 240 ÷ 8.	38. 4200 ÷ 7.
11. 192 ÷ 6.	25. 272 ÷ 8.	39. 4620 ÷ 7.
12. 272 ÷ 8.	26. 360 ÷ 9.	40. 3600 ÷ 8.
13. 405 ÷ 5.	27. 387 ÷ 9.	41. 4050 ÷ 9.
14. 245 ÷ 7.	28. 260 ÷ 5.	42. 2680 ÷ 4.

ILLUSTRATION OF PROCESS.

119. PROB. I.—**To divide any number by any divisor not greater than 12.**

1. Divide 986 by 4.

ANALYSIS.

	4)986	(200
$4 \times 200 =$	800	40
	186	
$4 \times 40 =$	160	6
	26	$246\frac{2}{4}$
$4 \times 6 =$	24	
	2	

EXPLANATION.—Follow the *analysis* and notice each step in the process; thus,

1. We commence by dividing the higher order of units. We know that 9, the figure expressing hundreds, contains twice the divisor 4, and 1 remaining. Hence 900 contains, according to (**117**—2), 200 times the divisor 4, and 100 remaining. We multiply the divisor 4 by 200, and subtract the product 800 from 986, leaving 186 of the dividend yet to be divided.

2. We know that 18, the number expressed by the two left-hand figures of the undivided dividend, contains 4 times 4, and 2 remaining. Hence 18 tens, or 180, contains, according to (**117**—2), 40 times 4, and 20 remaining. We multiply the divisor 4 by 40, and subtract the product 160 from 186, leaving 26 yet to be divided.

3. We know that 26 contains 6 times 4, and 2 remaining, which is less than the divisor, hence the division is completed.

4. We have now found that there are 200 fours in 800, 40 fours in 160, and 6 fours in 26, and 2 remaining; and we know that $800 + 160 + 26 = 986$. Hence 986 contains $(200 + 40 + 6)$ or 246 fours, and 2 remaining. The remainder is placed over the divisor and written after the quotient; thus, $246\frac{2}{4}$.

EXAMPLES FOR PRACTICE.

120. Solve and explain as above each of the following:

1. $51 \div 3$.	6. $195 \div 4$.	11. $38567 \div 8$.
2. $72 \div 2$.	7. $387 \div 8$.	12. $73046 \div 9$.
3. $96 \div 4$.	8. $932 \div 5$.	13. $50438 \div 2$.
4. $85 \div 5$.	9. $795 \div 7$.	14. $39050 \div 7$.
5. $98 \div 6$.	10. $352 \div 2$.	15. $20807 \div 3$.

SHORT AND LONG DIVISION COMPARED.

121. Compare carefully the following forms of writing the work in division:

(1.)

FORM USED FOR EXPLANATION.

Two steps in the process written.

	4) 986	(200
4 × 200 =	800	40
	186	6
4 × 40 =	160	246
	26	
4 × 6 =	24	

(2.)

LONG DIVISION.

One step written.

4) 986 (246
8
18
16
26
24

(3.)

SHORT DIVISION.

Entirely mental.

4) 986
$246\frac{2}{4}$

Observe carefully the following:

1. The division is performed by a successive division of parts of the dividend.

2. There are three steps in the process: *First*, finding the quotient figures; *Second*, multiplying the divisor by the quotient figures; *Third*, subtracting from the undivided dividend the part that has been divided, to find what remains yet to be divided.

3. In (1) the form for explanation, the numbers used in the *second* and *third* steps of the process are written. This is done to avoid taxing the memory with them, and thus concentrate the whole attention on the *explanation.*

4. In (2), the form called *Long Division,* the numbers used in the *second step* in the process are held in the memory, and those used in the *third step* are only partially written, the ciphers on the right being omitted. This method is always used when the divisor is greater than 12.

5. In (3) the form called *Short Division,* all the numbers used in the process are held in the memory, the quotient only being expressed. This method should invariably be used in practice when the divisor is not greater than 12.

ARITHMETICAL DRILL TABLE NO. 2.

	A.	B.	C.	D.	E.	F.	G.	H.	I.	J.
1.	2	4	3	7	5	8	6	9	4	8
2.	1	6	8	6	9	4	9	7	3	6
3.	4	2	4	6	3	7	5	4	9	3
4.	3	9	2	4	6	3	8	6	7	9
5.	5	7	6	8	4	9	6	2	5	2
6.	7	6	8	3	7	5	3	8	4	8
7.	4	2	4	9	2	6	7	3	8	5
8.	6	3	6	5	8	4	9	5	6	9
9.	8	5	2	7	3	8	4	9	2	7
10.	3	9	4	2	9	3	8	7	9	3
11.	9	4	8	4	7	5	3	4	6	6
12.	5	9	2	8	5	9	6	8	4	9

EXAMPLES FOR PRACTICE.

122. Copy, as follows, examples with *one figure* in the divisor from the above Table, and perform the work in each case by *Short Division.*

Three Figures in the Dividend.

1. Commence opposite **2**, and take the numbers for the dividends from the columns in the same manner and order as was done in multiplication.

2. Take as divisor the figure immediately above the right-hand figure of the dividend.

The first six examples from columns **A, B, C,** are:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
3) 168	8) 424	4) 392	2) 576	6) 768	8) 424

Five Figures in the Dividend.

1. Commencing opposite **2**, take the dividends from columns A, B, C, D, E; B, C, D, E, F; C, D, E, F, G; D, E, F, G, H; E, F, G, H, I; and F, G, H, I, J.

2. Take as divisor the figure immediately above the right-hand figure of the dividend.

ORAL EXAMPLES.

123. 1. A party of eight boys went fishing; they had a boat for every two boys; how many boats had they?

SOLUTION.—They had as many boats as 2 boys are contained times in 8 boys, which is 4. Hence they had 4 boats.

2. Justus earns 6 cents a day; how many days must he work to earn 24 cents?

3. A man buys 54 pounds of sugar; how many weeks will it last, if his family use 9 pounds a week?

4. There are 36 windows on one side of a building, arranged in 6 rows; how many windows in each row?

5. How many ranks of 6 soldiers each will 24 soldiers make? 42 soldiers? 54 soldiers? 72 soldiers?

6. At 8 dollars apiece, how many trunks can be bought for 32 dollars? For 56 dollars? For 88 dollars?

7. When 5 plows cost $40, what is the cost of 3 plows?

SOLUTION.—If 5 plows cost $40, one plow will cost as many dollars as 5 is contained times in 40, which is 8. Hence, one plow costs $8. Three plows will cost 3 times $8, which is $24. Hence, etc.

8. Daniel paid 28 cents for 4 oranges, and Luke bought 7 at the same price; how much did Luke pay for his?

9. If you can earn 56 dollars in 8 weeks, how much can you earn in 6 weeks?

10. When 5 yards of cloth can be bought for 20 dollars, how many yards of the same cloth can be bought for 32 dollars?

11. When 88 dollars will pay for 11 barrels of flour, how many barrels can be bought for 64 dollars?

WRITTEN EXAMPLES.

124. 1. A father left $4925, which he wished to be divided equally among his two sons and three daughters; how much did each one receive? *Ans.* $985.

2. At $3 a cord, how many cords of wood could be bought for $693? For $906? *Answers.* 231; 302.

3. If a man walk at the rate of 4 miles an hour, in how many hours can he walk 840 miles? *Ans.* 210 hours.

4. How many barrels of flour can be made of 588 bushels of wheat, if it takes 4 bushels to make a barrel? *Ans.* 147.

5. A certain laborer saves $6 a month; how many months will it take him to save $726? *Ans.* 121 months.

6. How long will a man be employed in cutting 175 cords of wood, if he cut 7 cords each week? *Ans.* 25 weeks.

7. How many baskets, each of which holds 6 pecks, would be needed to hold 804 pecks of apples? *Ans.* 134.

8. There are 604800 seconds in a week; how many seconds in one day? *Ans.* 86400 seconds.

9. How many revolutions will be made by a wheel 11 feet in circumference, in running one mile, which is equal to 5280 feet? *Ans.* 480.

10. A man distributed $423 among poor persons, giving each $9; how many persons received the money? *Ans.* 47.

11. A furniture dealer expended $413 in purchasing chairs at $7 a dozen; how many dozen did he buy?

12. A farmer sold 184 bushels of wheat at $1.50 per bushel, and expended the amount received in buying sheep at $4 a head; how many sheep did he buy?

13. A merchant expended $534 in purchasing boots at $6 a pair, which he sold at $8 a pair; how much did he gain on the transaction?

14. A grain dealer sold 912 bushels of corn at $.75 a bushel, and expended the amount received in buying flour at $9 a barrel; how many barrels of flour did he purchase?

DIVISORS GREATER THAN 12.

PREPARATORY STEPS.

125. STEP I.—*Examples with one order of units in the quotient, where the quotient figure can be found at once by dividing by the left-hand figure of the divisor;* thus,

Divide 13600 by 34.

```
34)13600(400
   13600
   -----
```

Here observe that 3, the left-hand figure of the divisor, is contained 4 times in 13, the two left-hand figures of the dividend, and that 34 multiplied by 4 equals 136. Hence 34 is contained 4 times in 136, and, according to (**115**), 400 times in 13600.

EXAMPLES FOR PRACTICE.

126. Divide and explain each of the following examples:

1. 1680 ÷ 84.	6. 37100 ÷ 53.	11. 665000 ÷ 95.
2. 2790 ÷ 93.	7. 31500 ÷ 63.	12. 117000 ÷ 39.
3. 3280 ÷ 82.	8. 5800 ÷ 29.	13. 276000 ÷ 46.
4. 3780 ÷ 97.	9. 33500 ÷ 67.	14. 336000 ÷ 56.
5. 6480 ÷ 72.	10. 59200 ÷ 74.	15. 623000 ÷ 89.

127. STEP II.—*Examples with one order in the quotient, where the quotient figure must be found by trial.*

In examples of this kind, we proceed thus:

Divide 1769 by 287.

FIRST TRIAL.

```
287)1769(8
    2296
    ----
```

1. We divide as before by 2, the left-hand figure of the divisor, and find the quotient 8. This course will always give the largest *possible* quotient figure. Multiplying the divisor 287 by 8, we observe at once that the product 2296 is greater than the dividend 1769. Hence 287 is not contained 8 times in 1769.

SECOND TRIAL.

```
287)1769(7
    2009
    ----
```

2. We erase the 8 and 2296 and try 7 as the quotient figure. Multiplying 287 by 7, we observe again that the product 2009 is greater than the dividend 1769. Hence 287 is not contained 7 times in 1769.

THIRD TRIAL.

```
287)1769(6
    1722
    ----
      47
```

3. We erase the 7 and 2009, and try 6 as the quotient figure. Multiplying 287 by 6, we observe that the product 1722 is less than the dividend 1769. Subtracting 1722 from 1769, we have 47 remaining, a number less than the divisor 287. Hence 287 is contained 6 times in 1769 and 47 remains.

EXAMPLES FOR PRACTICE.

128. Find the quotients and remainders in each of the following:

1. 119÷27.	8. 3275÷458.	15. 215400÷359.
2. 236÷36.	9. 4936÷643.	16. 410900÷587.
3. 199÷39.	10. 2758÷582.	17. 638400÷798.
4. 419÷58.	11. 3657÷739.	18. 866700÷963.
5. 248÷38.	12. 1890÷496.	19. 191600÷479.
6. 845÷97.	13. 4760÷ 68.	20. 577500÷825.
7. 665÷74.	14. 2850÷ 39.	21. 474400÷593.

ILLUSTRATION OF PROCESS.

129. Prob. II.—**To divide any number by any given divisor.**

1. Divide 21524 by 59.

```
59)21524(364
   177
   ---
    382
    354
    ---
     284
     236
     ---
      48
```

Explanation.—1. We find how many times the divisor is contained in the fewest of the left-hand figures of the dividend which will contain it.

59 is contained 3 times in 215, with a remainder 38, hence, according to (**115**—1), it is contained 300 times in 21500, with a remainder 3800.

2. We annex the figure in the next lower order of the dividend to the remainder of the previous division, and divide the number thus found by the divisor.

2 tens annexed to 380 tens make 382 tens. 59 is contained 6 times in 382, with a remainder 28, hence, according to (**115**—1), it is contained 60 times in 3820, with a remainder 280.

3. We annex the next lower figure and proceed as before.

4 units annexed to 280 units make 284 units. 59 is contained 4 times in 284, with a remainder of 48, a number smaller than the divisor, hence the division is completed, and we have found that 59 is contained 364 times in 21524, with a remainder 48.

Observe carefully the following analysis of the process in the preceding example:

Multiplying the divisor by the part of the quotient found each step.	Part of dividend divided each step.		Part of divided dividend subtracted from the part undivided.
			21524
59 × 300 =	17700		17700
			3824
59 × 60 =	3540		3540
			284
59 × 4 =	236		236
59 × 364 =	21476	+	48

From these illustrations we obtain the following

130. Rule.—*I. Find how many times the divisor is contained in the least number of orders at the left of the dividend that will contain it, and write the result for the first figure of the quotient.*

II. Multiply the divisor by this quotient figure, and subtract the result from the part of the dividend that was used; to the remainder annex the next lower order of the dividend for a new partial dividend and divide as before. Proceed in this manner with each order of the dividend.

III. If there be at last a remainder, place it after the quotient, with the divisor underneath.

Proof.—*Multiply the divisor by the quotient and add the remainder, if any, to the product. This result will be equal to the dividend, when the division has been performed correctly.*

131. EXAMPLES FOR PRACTICE.

1.	18450 ÷ 90.	17.	13824 ÷ 128.
2.	9225 ÷ 45.	18.	142692 ÷ 517.
3.	28035 ÷ 89.	19.	35904 ÷ 204.
4.	26840 ÷ 61.	20.	1678306 ÷ 313.
5.	255798 ÷ 81.	21.	64109742 ÷ 706.
6.	17472 ÷ 21.	22.	31899868 ÷ 4004.
7.	72144 ÷ 72.	23.	5332114 ÷ 4321.
8.	9590 ÷ 70.	24.	10205721 ÷ 3243.
9.	59644 ÷ 62.	25.	19014604 ÷ 406.
10.	137505 ÷ 309.	26.	7977489 ÷ 923.
11.	467775 ÷ 105.	27.	31907835 ÷ 4005.
12.	1292928 ÷ 312.	28.	203812983 ÷ 5049.
13.	264375 ÷ 705.	29.	61142488 ÷ 4136.
14.	289520 ÷ 517.	30.	406070736 ÷ 8056.
15.	1143723 ÷ 509.	31.	119836687 ÷ 3041.
16.	2750283 ÷ 603.	32.	330445150 ÷ 3145.

132. Additional examples for practice should be taken from Arithmetical Table No. 2, page 58, as follows:

Dividend four figures, Divisor two.

1. Take the dividends in order from columns A, B, C, D; B, C, D, E; C, D, E, F; D, E, F, G; E, F, G, H; F, G, H, I; G, H, I, J.

2. Take as divisors in each set the figures immediately above the dividend, in the two right-hand columns of those used.

Dividend six figures, Divisor three.

1. Take the dividends in order from columns A, B, C, D, E, F; B, C, D, E, F, G; C, D, E, F, G, H; D, E, F, G, H, I; E, F, G, H, I, J.

2. Take the divisors as before from the three right-hand columns of those used for dividend.

WRITTEN EXAMPLES.

133. 1. A hogshead of molasses contains 63 gallons; how many hogsheads in 16002 gallons?

Solution.—As one hogshead contains 63 gallons, 16002 gallons will make as many hogsheads as 63 is contained times in 16002. 16002 ÷ 63 = 254. Hence there are 254 hogsheads in 16002 gallons.

2. A man wishes to carry to market 2623 bushels of potatoes; if he carries 61 bushels at a load, how many loads will they make? *Ans.* 43 loads.

3. An army contractor furnished horses, at $72 each, to the amount of $1131264; how many did he furnish? *Ans.* 15712.

4. A certain township contains 192000 acres; how many square miles in the town, there being 640 acres in a square mile? *Ans.* 300 miles.

5. A man paid $1548 for a farm at the rate of $43 an acre; how many acres did the farm contain? *Ans.* 36 acres.

6. How many acres of land at $100 an acre, can be bought for $26700? *Ans.* 267 acres.

7. A certain product is 43964 and one of the factors is 58; what is the other factor? *Ans.* 758.

8. At what yearly salary will a man earn 20400 dollars in 17 years? *Ans.* $1200.

9. If light travels 192000 miles in a second, in how many soconds will it travel 691200000 miles? *Ans.* 3600.

10. Henry Pendexter divided $47400 into 3 equal parts, one of which he gave to his wife; the rest, after paying a debt of $3280, he divided equally among 4 children; what did each child receive? *Ans.* $7080.

11. A piano maker expended in one year for material $20644, and for labor $4925, paying each week the same amount; what was his weekly expense?

12. A western farmer raised in one year 13475 bushels of wheat; the average yield was 49 bushels per acre; how many acres did he have sown?

DIVISION BY FACTORS.

PREPARATORY STEPS.

134. Step I.—*Any number may be expressed in terms of one of its factors by taking another factor as the* **Unit.** **(14.)**

Thus, $12 = 4 + 4 + 4$; hence, 12 may be expressed as 3 *fours*, the *four* being the *unit* of the number 3.

Write the following numbers:

1. Express 12 as 2's; as 3's; as 4's; as 6's.
2. Express 36 as 3's; as 9's; as 18's; as 12's; as 6's.
3. Express 45 as 5's; as 3's; as 9's; as 15's.
4. Express 42 and 24 each as 6's.
5. Express 45 and 225 each as 9's; as 5's; as 3's.

135. Step II.—*When a number is made into three or more factors, any two or more of them may be regarded as the unit of the number expressed by the remaining factors.*

For example, $24 = 3 \times 4 \times 2$. This may be expressed thus, $24 = 3$ (4 twos). Here the 3 expresses the number of 4 twos; hence, (4 twos) is regarded as the *unit* of the number 3.

Write the following:

1. Express 12 as (3 *twos*); as (2 *twos*); as (2 *threes*).
2. Express 30 as (3 *twos*); as (2 *fives*); as (3 *fives*).
3. Express 42 and 126 each as (2 *sevens*); as (7 *threes*).
4. Express 75, 225, and 375 each as (5 *threes*).
5. Express 66, 198, and 264 each as (11 *threes*) and as (2 elevens).

136. Step III.—*When the same factor is made the* **unit** *of both the dividend and divisor, the division is performed as if the numbers were concrete.*

Thus, $60 \div 12$ may be expressed, 20 threes $\div$ 4 threes, and the division performed in the same manner as in 6 feet $\div$ 3 feet. 4 threes are contained 5 times in 20 threes; hence, 12 is contained 5 times in 60.

The division may be performed in this way when the factors are connected by the sign of multiplication; thus, $60 \div 12 = (20 \times 3) \div (4 \times 3)$. We can regard as before the 3 as the unit of both dividend and divisor, and hence say, 4 *threes* are contained 5 times in 20 threes.

Perform the division in each of the following examples, without performing the multiplication indicated:

1. 25 threes ÷ 5 threes = ?	5. $(64 \times 9) \div (8 \times 9) = ?$
2. 42 eights ÷ 6 eights = ?	6. $(49 \times 13) \div (7 \times 13) = ?$
3. 88 twos ÷ 11 twos = ?	7. $(96 \times 7) \div (12 \times 7) = ?$
4. 108 fives ÷ 9 fives = ?	8. $(78 \times 11) \div (26 \times 11) = ?$

ILLUSTRATION OF PROCESS.

137. Prob. III.—**To divide by using the factors of the divisor.**

Ex. 1. Divide 315 by 35.

```
        5 ) 3 1 5
  7 fives ) 6 3 fives
              9
```

Explanation.—1. The divisor 35 = 7 *fives.*

2. Dividing the 315 by 5, we find that 315 = 63 *fives.* (**138.**)

3. The 63 *fives* contain 9 times 7 *fives;* hence 315 contains 9 times 7 *fives* or 9 times 35.

Ex. 2. Divide 359 by 24.

```
           2 | 3 5 9
      3 twos | 1 7 9 twos        and 1 remaining             =  1
4 (3 twos)   |  5 9 (3 twos)  and 2 twos remaining        =  4
   Quotient,    1 4          and 3 (3 twos) remaining     = 1 8
                                       True remainder,      2 3
```

Explanation.—1. The divisor $24 = 4 \times 3 \times 2 = 4$ (3 *twos*).

2. Dividing 359 by 2, we find that 359 = 179 *twos* and 1 unit remaining.

3. Dividing 179 *twos* by 3 *twos*, we find that 179 *twos* = 59 (3 *twos*) and 2 *twos* remaining.

4. Dividing 59 (3 *twos*) by 4 (3 *twos*), we find that 59 (3 *twos*) contain 4 (3 *twos*) 14 times and 3 (3 *twos*) remaining.

Hence 359, which is equal to 59 (3 *twos*) and 2 *twos* + 1, contains 4 (3 *twos*), or 24, 14 times, and 3 (3 *twos*) + 2 *twos* + 1, or 23, remaining.

From these illustrations we obtain the following

138. Rule.—*I. Resolve the divisor into convenient factors; divide the dividend by one of these factors, the quotient thus obtained by another, and so on until all the factors have been used. The last quotient will be the true quotient.*

II. The true remainder is found by multiplying each remainder, after the first, by all the divisors preceding its own, and finding the sum of these products and the first remainder.

EXERCISE FOR PRACTICE.

139. Examples for practice in dividing by the factors of the divisor:

1.	376 ÷ 100.	10.	19437 ÷ 40.
2.	8975 ÷ 100.	11.	13658 ÷ 42.
3.	76423 ÷ 1000.	12.	27780 ÷ 60.
4.	92768 ÷ 1000.	13.	7169 ÷ 90.
5.	774 ÷ 18.	14.	4947 ÷ 108.
6.	876 ÷ 24.	15.	30683 ÷ 400.
7.	4829 ÷ 28.	16.	75947 ÷ 900.
8.	15836 ÷ 30.	17.	8460 ÷ 180.
9.	7859 ÷ 84.	18.	14025 ÷ 165.

ONE ORDER IN DIVISOR.

PREPARATORY STEPS.

140. Step I.—*To divide by 10, 100, 1000, etc.*

1. Observe the figure in the *second* place in a number denotes *tens*, and this figure, with those to the left of it, express the number of tens. Hence, to find how many tens in a number, we cut off the right-hand figure.

Thus, in 7369 the 6 denotes *tens*, and 736 the number of tens in 7369, hence 7369 ÷ 10 = 736, and 9 remaining.

2. In like manner the figure in the *third* place denotes *hundreds,* the figure in the *fourth* place *thousands,* etc. Hence by cutting off *two* figures at the right, we divide by 100; by cutting off *three,* we divide by 1000, etc. The figures cut off are the remainder.

Give the quotient and remainder of the following at sight:

587 ÷ 10.	8973 ÷ 100.	73265 ÷ 1000.
463 ÷ 100.	50380 ÷ 100.	58207 ÷ 10000.

141. STEP II.—*A number consisting of only one order of units, contains two factors which can be given at sight.*

Thus, 20 = 2 × 10. 400 = 4 × 100. 7000 = 7 × 1000.

Observe that the significant figure of the number, in each case, is one factor and that the other factor is 1 with as many ciphers annexed as there are ciphers at the right of the significant figure.

ILLUSTRATION OF PROCESS.

142. PROB. IV.—**To divide when the divisor consists of only one order of units.**

1. Divide 8736 by 500.

```
5 ) 8 7 | 3 6
    ---------
    1 7 and 236 remaining.
```

EXPLANATION. 1. We divide first by the factor 100. This is done by cutting off 36, the units and tens at the right of the dividend.

2. We divide the quotient, 87 hundreds, by the factor 5, which gives a quotient of 17 and 2 hundred remaining, which added to 36 gives 236, the true remainder.

EXAMPLES FOR PRACTICE.

143. Divide and explain each of the following examples:

1. 852 ÷ 100.	7. 8365 ÷ 1000.	13. 93689 ÷ 800.
2. 593 ÷ 20.	8. 3973 ÷ 700.	14. 79365 ÷ 9000.
3. 762 ÷ 60.	9. 62850 ÷ 4000.	15. 57842 ÷ 3000.
4. 938 ÷ 400.	10. 97462 ÷ 6000.	16. 90000 ÷ 700.
5. 852 ÷ 300.	11. 76352 ÷ 900.	17. 40034 ÷ 900.
6. 983 ÷ 700.	12. 49730 ÷ 800.	18. 20306 ÷ 700.

DEFINITIONS.

144. ***Division*** is the process of finding how many times one number is contained in another.

145. The ***Dividend*** is the number divided.

146. The ***Divisor*** is the number by which the dividend is divided.

147. The ***Quotient*** is the result obtained by division.

148. The ***Remainder*** is the part of the dividend left after the division is performed.

149. A ***Partial Dividend*** is any part of the dividend which is divided in one operation.

150. A ***Partial Quotient*** is any part of the quotient which expresses the number of times the divisor is contained in a partial dividend.

151. The ***Process of Division*** consists, *first*, in finding the partial quotients by means of memorized results; *second*, in multiplying the divisor by the partial quotients to find the partial dividends; *third*, in subtracting the partial dividends from the part of the dividend that remains undivided to find the part yet to be divided.

152. ***Short Division*** is that form of division in which no step of the process is written.

153. ***Long Division*** is that form of division in which the *third* step of the process is written.

154. The ***Sign of Division*** is ÷, and is read *divided by*. When placed between two numbers, it denotes that the number before it is to be divided by the number after it; thus, 28 ÷ 7 is read, 28 divided by 7.

Division is also expressed by placing the dividend above the divisor, with a short horizontal line between them; thus, $\frac{35}{5}$ is read, 35 divided by 5.

155. Principles.—*I. The dividend and divisor must be numbers of the same denomination.*

II. The denomination of the quotient is determined by the nature of the problem solved.

III. The remainder is of the same denomination as the dividend.

REVIEW AND TEST QUESTIONS.

156. 1. Define Division, and illustrate each step in the process by examples.

2. Explain and illustrate by examples Partial Dividend, Partial Quotient, and Remainder.

3. Prepare two examples illustrating each of the following problems:

I. Given all the parts, to find the whole.

II. Given the whole and one of the parts, to find the other part.

III. Given one of the equal parts and the number of parts, to find the whole.

IV. Given the whole and the size of one of the parts, to find the number of parts.

V. Given the whole and the number of equal parts, to find the size of one of the parts.

4. Show that 45 can be expressed as *nines*, as *fives*, as *threes*.

5. What is meant by true remainder, and how found?

6. Explain division by factors. Illustrate by an example.

7. Why cut off as many figures at the right of the dividend as there are ciphers at the right of the divisor? Illustrate by an example.

8. Give a rule for dividing by a number with one or more ciphers at the right. Illustrate the steps in the process by an example.

9. Explain the difference between Long and Short Division, and show that the process in both cases is performed mentally.

10. Illustrate each of the following problems by three examples:

VI. Given the final quotient of a continued division, the true remainder, and the several divisors, to find the dividend.

VII. Given the product of a continued multiplication and the several multipliers, to find the multiplicand.

VIII. Given the sum and the difference of two numbers, to find the numbers.

APPLICATIONS.

157. PROB. I.—**To find the cost when the number of units and the price of one unit is given.**

Ex. 1. What is the cost of 42 yards of silk, at $2.36 a yard?

SOLUTION.—If one yard cost $2.36, 42 yards must cost 42 times $2.36. Hence, $2.36 × 42 = $99.12, is the cost of 42 yards.

Find the cost of the following:

2. 94 stoves, at $36 for each stove. *Ans.* $3384.
3. 436 bushels wheat, at $1.76 a bushel. *Ans.* $767.36.
4. 259 yards of broadcloth, at $2.84 per yard.
5. 84 tons of coal, at $7.84 per ton. *Ans.* $658.56.
6. 2 farms, each containing 139 acres, at $73.75 per acre.
7. 436 bushels of apples, at $1.45 per bushel.
8. 432 yards cloth, at $1.75 per yard.
9. 897 pounds butter, at $.37 per pound.
10. 346 bushels of wheat, at $1.73 a bushel.

158. PROB. II.—**To find the price per unit when the cost and number of units are given.**

Ex. 1. Bought 25 cows for $1175; how much did each cost?

SOLUTION.—Since 25 cows cost $1175, each cow cost as many dollars as 25 is contained times in 1175. Hence, 1175 ÷ 25 = 47, the number of dollars each cow cost.

Find the price of the following:

2. If 42 tons of hay cost \$546; what is the price per ton?

3. Bought 536 yards cloth for \$1608; how much did I pay per yard? *Ans.* \$3.

4. Sold 196 acres land for \$10192; how much did I receive an acre? *Ans.* \$52.

5. Paid \$1029 for 147 barrels flour; what did I pay per barrel? *Ans.* \$7.

6. Received \$980 for 28 weeks' work; what was my wages per week? *Ans.* \$35.

7. The total cost for conducting a certain school for 14 years was \$252000; what was the yearly expense? *Ans.* \$18000.

8. A merchant pays his clerks annually \$3744. How much is this per week? *Ans.* \$72.

159. Prob. III.—**To find the cost when the number of units and the price of two or more units are given.**

1. At \$15 for 3 cords of wood, what is the cost of 39 cords?

Solution 1.—Since 3 cords cost \$15, 39 cords must cost as many times \$15 as 3 is contained times in 39. Hence, *first step*, $39 \div 3 = 13$; *second step*, $\$15 \times 13 = \$195$, the cost of 39 cords.

Solution 2.—Since 3 cords cost \$15, each cord cost as many dollars as 3 is contained times in 15; hence each cord cost \$5, and 39 cords cost 39 times \$5, or \$195. Hence, *first step*, $15 \div 3 = 5$; *second step*, $\$5 \times 29 = \$195$, the cost of 39 cords.

Observe carefully the difference between these two solutions. Let both be used in practice. Take for each example the one by which the division can be most readily performed.

Solve and explain the following:

2. Paid \$28 for 4 barrels of flour; how much, at the same rate, will I pay for 164 barrels? *Ans.* \$1148.

3. If 9 stoves cost \$135, what is the cost at the same rate, of 84 stoves?

4. A farmer paid for 32 sheep \$128; what is the cost at the same rate, of 793? *Ans.* \$3172.

5. A man traveled by railroad 1728 miles in 3 days; how many miles, at the same rate, will he travel in 54 days?

6. A merchant bought 150 yards of cotton for \$18; how much will he pay, at the same rate, for 1350 yards?

7. A book-keeper receives for his service at the rate of \$624 for 13 weeks; what is his yearly salary? *Ans.* \$2496.

160. PROB. IV.—**To find the number of units when the cost and the price per unit are given.**

1. At \$7 a ton, how many tons of coal can be bought for \$658?

SOLUTION.—Since 1 ton can be bought for \$7, there can be as many tons bought for \$658 as \$7 is contained times in \$658. Hence, \$658÷\$7 = 94, the number of tons that can be bought for \$658.

Solve and explain the following:

2. For \$765, how many barrels of pears can be bought at \$9 a barrel? *Ans.* 85.

3. How many horses can be bought for \$9928, at \$136 per horse? *Ans.* 73.

4. A man paid for a farm \$6134, at \$38 per acre; how many acres does the farm contain?

5. A mechanic received at one time from his employer \$357. He was paid at the rate of \$21 a week; how many weeks had he worked? *Ans.* 17.

6. A lumber merchant paid for walnut lumber \$3528, at \$98 a thousand feet; how many thousand feet did he buy?

161. PROB. V.—**To find the number of units that can be purchased for a given sum when the cost of two or more units is given.**

1. When 8 bushels of wheat can be bought for \$12, how many bushels can be bought for \$6348?

SOLUTION.—Since 8 bushels can be bought for \$12, there can be as many times 8 bushels bought for \$6348 as \$12 is contained times in \$6348. Hence, *first step*, \$6348 ÷ \$12 = 529; *second step*, 529 × 8 = 4232, the number of bushels that can be bought for \$6348.

Solve and explain the following:

2. If 28 pounds of sugar cost $4, how many pounds can be bought for $348? *Ans.* 2436 pounds.

3. The cost of 4 boxes of oranges is $12. How many boxes, at the same rate, can be bought for $552?

4. When peaches are sold at $6 for 8 baskets, how many baskets must a man sell to receive $582? *Ans.* 776 baskets.

5. A farmer sold a quantity of butter at $35 a hundred pounds, and received $1715; how many pounds did he sell?

6. A carpenter was paid at the rate of $42 for 12 days, and received $588; how many days was he employed?

7. At $69 for 12 cords of wood, how many cords can be bought for $966? *Ans.* 168 cords.

REVIEW EXAMPLES.

162. 1. I sold 60 pounds of cheese at 12 cents a pound, and laid out the proceeds in coffee at 20 cents a pound; how many pounds of coffee did I buy? *Ans.* 36 pounds.

2. Bought a quantity of wood for $3959, and sold it for $6095, thus gaining $3 on each cord sold; how much wood did I buy? *Ans.* 712 cords.

3. I sold a farm of 244 acres at $48 an acre, and another farm of 160 acres at $72 an acre; how much more did the first farm bring than the second? *Ans.* $192.

4. The expenses of a young lady at school were $75 for tuition, $20 for books, $68 for clothes, $17 for railroad fare, $5 a week for board for 42 weeks, and $36 for other expenses; what was the total expense? *Ans.* $426.

5. I paid $8960 for 8 city lots, and sold them at a loss of $12 on each lot; how much did I receive for 3 of them?

6. I bought 27 acres of land at $41 an acre, and 26 acres at $27 an acre, and sold the whole at $43 an acre; how much did I gain or lose? *Ans.* $470 gain.

7. What is the total cost of 45 acres of land at $17 an

acre, two horses at $132 each, a yoke of oxen for $130, a horse-rake for $65, and a plow for $17? *Ans.* $1241.

8. A grocer bought 14 barrels of apples for $42; how much will he pay at the same rate for 168 barrels? *Ans.* $504.

9. The sum of two numbers is 73, and their difference 47; what are the numbers?

SOLUTION.—The sum 73 is equal to the greater number plus the less, and the less number plus the difference 47, are equal to the greater; hence, if 47 be added to the sum 73, we have twice the greater number. Hence, *first step*, 73 + 47 = 120; *second step*, 120 ÷ 2 = 60, the greater number; *third step*, 60 − 47 = 13, the less number.

10. The sum of two numbers is 8976, and the difference 452; what are the numbers? *Ans.* 4714 and 4262.

11. Two men owed together $3957, one of them owed $235 more than the other; what was each man's debt?

12. A house and lot are worth $7394. The house is valued at $2462 more than the lot; what is each worth?

13. If 15 cows can subsist on a certain quantity of hay for 10 days, how long will the same suffice for 3 cows?

14. A fruit merchant bought 576 barrels of apples at $36 for every 8 barrels, and sold them at $33 for every 6 barrels; how much did he gain on the transaction? *Ans.* $576.

15. A drover bought cattle at $47 a head to the amount of $2961, and sold them for $3528; what was the selling price per head? *Ans.* $56.

16. A farmer sold 62 bushels of wheat for $50, also 14 cords of wood at $5 a cord, 4 tons of hay at $15 a ton, and 2 cows at $30 apiece; he took in payment $145 in money, a coat worth $50, a horse-rake worth $21, and the balance in clover-seed at $4 a bushel; how many bushels of seed did he receive? *Ans.* 6 bushels.

17. A farmer paid $22541 for two farms, and the difference in the cost of the farms was $3471. The price of the farm for which he paid the smaller sum was $64 an acre, and of the other $87 an acre. How many acres in each farm?

PROPERTIES OF NUMBERS.

DEFINITIONS.

163. An ***Integer*** is a number that expresses how many there are in a collection of *whole* things.

Thus, 8 yards, 12 houses, 32 dollars.

164. An ***Exact Divisor*** is a number that will divide another number without a remainder.

Thus, 3 or 5 is an exact divisor of 15.

All numbers with reference to exact divisors are either prime or composite.

165. A ***Prime Number*** is a number that has no exact divisor besides 1 and itself.

Thus, 1, 3, 5, 7, 11, 13, etc., are prime numbers.

166. A ***Composite Number*** is a number that has other exact divisors besides 1 and itself.

Thus, 6 is divisible by either 2 or 3 ; hence is composite.

167. A ***Prime Divisor*** is a prime number used as a divisor.

Thus, in 35 ÷ 7, 7 is a prime divisor.

168. A ***Composite Divisor*** is a composite number used as a divisor.

Thus, in 18 ÷ 6, 6 is a composite divisor.

EXACT DIVISION.

169. The following *tests* of exact division should be carefully studied and fixed in the memory for future use.

PROP. I.—*A divisor of any number is a divisor of any number of times that number.*

Thus, 12 = 3 fours. Hence, 12 × 6 = 3 fours × 6 = 18 fours. But 18 fours are divisible by 4. Hence, 12 × 6, or 72, is divisible by 4.

PROP. II.—*A divisor of each of two or more numbers is a divisor of their sum.*

Thus, 5 is a divisor of 10 and 30; that is, 10 = 2 fives and 30 = 6 fives. Hence, 10 + 30 = 2 fives + 6 fives = 8 fives. But 8 fives are divisible by 5. Hence, 5 is a divisor of the sum of 10 and 30.

PROP. III.—*A divisor of each of two numbers is a divisor of their difference.*

Thus, 3 is a divisor of 27 and 15; that is, 27 = 9 threes and 15 = 5 threes. Hence, 27 — 15 = 9 threes — 5 threes = 4 threes. But 4 threes are divisible by 3. Hence 3 is a divisor of the difference between 27 and 15.

PROP. IV.—*Any number ending with a cipher is divisible by the divisors of* 10, *viz.*, 2 *and* 5.

Thus, 370 = 37 times 10. Hence is divisible by 2 and 5, the divisors of 10, according to Prop. I.

PROP. V.—*Any number is divisible by either of the divisors of* 10, *when its right-hand figure is divisible by the same.*

Thus, 498 = 490 + 8. Each of these parts is divisible by 2. Hence the number 498 is divisible by 2, according to Prop. II.

In the same way it may be shown that 495 is divisible by 5.

Prop. VI.—*Any number ending with two ciphers is divisible by the divisors of* 100, *viz.*, 2, 4, 5, 10, 20, 25, *and* 50.

Thus, 8900 = 89 times 100. Hence is divisible by any of the divisors of 100, according to Prop. I.

Prop. VII.—*Any number is divisible by any one of the divisors of* 100, *when the number expressed by its two right-hand figures is divisible by the same.*

Thus, 4975 = 4900 + 75. Any divisor of 100 is a divisor of 4900 (Prop. VI). Hence, any divisor of 100 which will divide 75 is a divisor of 4975 (Prop. II).

Prop. VIII.—*Any number ending with three ciphers is divisible by the divisors of* 1000, *viz.*, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, 200, 250, *and* 500.

Thus, 83000 = 83 times 1000. Hence is divisible by any of the divisors of 1000, according to Prop. I.

Prop. IX.—*Any number is divisible by any one of the divisors of* 1000, *when the number expressed by its three right-hand figures is divisible by the same.*

Thus, 92625 = 92000 + 625. Any divisor of 1000 is a divisor of 92000 (Prop. VIII). Hence, any divisor of 1000 which will divide 625 is a divisor of 92625 (Prop. II).

Prop. X.—*Any number is divisible by* 9, *if the sum of its digits is divisible by* 9.

This proposition may be shown thus:

(1.) 486 = 400 + 80 + 6.

(2.) 100 = 99 + 1 = 11 nines + 1. Hence, 400 = 44 nines + 4, and is divisible by 9 with a remainder 4.

(3.) 10 = 9 + 1 = 1 nine + 1. Hence, 80 = 8 nines + 8, and is divisible by 9 with a remainder 8.

(4.) From the foregoing it follows that 400 + 80 + 6, or 486, is divisible by 9 with a remainder 4 + 8 + 6, the sum of the digits. Hence, if the sum of the digits is divisible by 9, the number 486 is divisible by 9 (Prop. II).

Prop. XI.—*Any number is divisible by* 3, *if the sum of its digits is divisible by* 3.

This proposition is shown in the same manner as Prop. X; as 3 divides 10, 100, 1000, etc., with a remainder 1 in each case.

Prop. XII.—*Any number is divisible by* 11, *if the difference of the sums of the digits in the odd and even places is zero or is divisible by* 11.

This may be shown thus:

(1.) $4928 = 4000 + 900 + 20 + 8$.

(2.) $1000 = 91$ elevens $- 1$. Hence, $4000 = 364$ elevens $- 4$.

(3.) $100 = 9$ elevens $+ 1$. Hence, $900 = 81$ elevens $+ 9$.

(4.) $10 = 1$ eleven $- 1$. Hence, $20 = 2$ elevens $- 2$.

(5.) From the foregoing it follows that $4928 = 364$ elevens $+ 81$ elevens $+ 2$ elevens $- 4 + 9 - 2 + 8$.

But $-4 + 9 - 2 + 8 = 11$. Hence, $4928 = 364$ elevens $+ 81$ elevens $+ 2$ elevens $+ 1$ eleven $= 448$ elevens, and is therefore divisible by 11.

The same course of reasoning applies where the difference is minus or zero. Hence, etc.

EXAMPLES FOR PRACTICE.

170. Find exact divisors of each of the following numbers by applying the foregoing tests:

1.	470.	12.	9375.	23.	5478.
2.	975.	13.	15264.	24.	3825.
3.	2304.	14.	37128.	25.	8694.
4.	4500.	15.	28475.	26.	3270.
5.	8712.	16.	47000.	27.	3003.
6.	9736.	17.	69392.	28.	8004.
7.	5725.	18.	34604.	29.	7007.
8.	8375.	19.	38745.	30.	1005.
9.	6000.	20.	53658.	31.	9009.
10.	8500.	21.	25839.	32.	3072.
11.	3625.	22.	21762.	33.	8008.

PRIME NUMBERS.

PREPARATORY PROPOSITIONS.

171. Prop. I.—*All even numbers are divisible by 2 and consequently all even numbers, except 2, are composite.*

Hence, in finding the prime numbers, we cancel as composite all even numbers except 2.

Thus, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, 9, ~~10~~, 11, ~~12~~, and so on.

Prop. II.—*Each number in the series of odd numbers is 2 greater than the number immediately preceding it.*

Thus, the numbers left after cancelling the even numbers are

$$\begin{array}{cccccc} 3 & 5 & 7 & 9 & 11 & 13, \text{ and so on.} \\ 3 & \overbrace{3+2} & \overbrace{5+2} & \overbrace{7+2} & \overbrace{9+2} & \overbrace{11+2} \end{array}$$

Prop. III.—*In the series of odd numbers, every* THIRD *number from* 3 *is divisible by* 3, *every* FIFTH *number from* 5 *is divisible by* 5, *and so on with each number in the series.*

This proposition may be shown thus:

According to Prop. II, the series of odd numbers increase by 2's. Hence the *third* number from 3 is found by adding 2 *three* times, thus:

$$\begin{array}{cccc} 3 & 5 & 7 & 9 \\ 3 & \overbrace{3+2} & \overbrace{3+2+2} & \overbrace{3+2+2+2} \end{array}$$

From this it will be seen that 9, the third number from 3, is composed of 3, plus 3 *twos*, and is divisible by 3 (Prop. II); and so with the third number from 9, and so on.

By the same course of reasoning, each fifth number in the series, counting from 5, may be shown to be divisible by 5; and so with any other number in the series; hence the following method of finding the prime numbers.

ILLUSTRATION OF PROCESS.

172. Prob.—**To find all the Prime Numbers from 1 to any given number.**

Find all the prime numbers from 1 to 63.

1	3	5	7	9	11	13
				3		
15	17	19	21	23	25	27
3 5			3 7		5	3 9
29	31	33	35	37	39	41
		3 11	5 7		3 13	
43	45	47	49	51	53	55
	3 5 9 15		7	3 17		5 11
	57		59	61	63	
	3 19				3 7 9 21	

Explanation.—1. Arrange the series of odd numbers in lines, at convenient distances from each other, as shown in illustration.

2. Write 3 under every *third* number from 3, 5 under every *fifth* number from 5, 7 under every *seventh* number from 7, and so on with each of the other numbers.

3. The terms under which the numbers are written are composite, and the numbers written under are their factors, according to Prop. III. All the remaining numbers are prime.

Hence all the prime numbers from 1 to 63 are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61.

EXAMPLES FOR PRACTICE.

173. 1. Find all the prime numbers from 1 to 95.

2. Find all the prime numbers from 42 to 103.

3. Find all the prime numbers from 70 to 130.

4. Find all the prime numbers from 1 to 200.

5. Find all the prime numbers from 200 to 400.

6. Show by an example that every seventh number from seven, in the series of odd numbers, is divisible by seven.

FACTORING.

PREPARATORY STEPS.

174. Step I.—*Find by inspection all the exact divisors of each of the following numbers, and write them in order on your slate,* thus : $6 = 3 \times 2$, $10 = 5 \times 2$.

1.	6	10	14	15	21	22	26
2.	33	34	35	38	39	46	51
3.	55	57	58	62	65	69	74
4.	77	79	82	85	86	87	91
5.	3	11	115	119	123	129	141

Refer to the results on your slate and observe

(1.) Each *prime* exact divisor is called a *prime factor* of the number of which it is a divisor.

(2.) Each number is equal to the product of its *prime* factors.

Step II.—*The same prime factor may enter into a number two or more times.* Thus, $18 = 2 \times 3 \times 3$. Hence the prime factor, 3, enters twice into 18.

Resolve the following numbers into their prime factors, and name how many times each factor enters into a number.

1.	4	8	16	32	64	9	27
2.	18	20	28	40	44	45	50
3.	54	56	75	80	98	100	108
4.	71	25	49	125	121	213	343

DEFINITIONS.

175. A ***Factor*** is one of the *equal parts* of a number, or one of its exact divisors.

Thus, 15 is composed of 3 *fives* or 5 *threes ;* hence, 5 and 3 are factors of 15.

176. A ***Prime Factor*** is a prime number which is a factor of a given number.

Thus, 5 is a prime factor of 30.

177. A ***Composite Factor*** is a composite number which is a factor of a given number.

Thus, 6 is a composite factor of 24.

178. ***Factoring*** is the process of resolving a composite number into its factors.

179. An ***Exponent*** is a small figure placed at the right of a number and a little above, to show how many times the number is used as a factor.

Thus, $3^5 = 3 \times 3 \times 3 \times 3 \times 3$. The 5 at the right of 3 denotes that the 3 is used 5 times as a factor.

180. A ***Common Factor*** is a number that is a factor of each of two or more numbers.

Thus, 3 is a factor of 6, 9, 12, and 15; hence is a common factor.

181. The ***Greatest Common Factor*** is the greatest number that is a factor of each of two or more numbers.

Thus, 4 is the greatest number that is a factor of 8 and also of 12. Hence 4 is the greatest common factor of 8 and 12.

ILLUSTRATION OF PROCESS.

182. Find the prime factors of 462.

```
2 ) 462
  ─────
3 ) 231
  ─────
  7 ) 77
    ────
     11
```

EXPLANATION.—1. We observe that the number 462 is divisible by 2, the smallest prime number. Hence we divide by 2.

2. We observe that the first quotient, 231, is divisible by 3, which is a prime number. Hence we divide by 3.

3. We observe that the second quotient, 77, is divisible by 7, which is a prime number. Hence we divide by 7.

4. The third quotient, 11, is a prime number. Hence the prime factors of 462 are 2, 3, 7, and 11; that is, $462 = 2 \times 3 \times 7 \times 11$.

Any composite number may be factored in the same manner. Hence the following

183. RULE.—*Divide the given number by any prime number that is an exact divisor, and the resulting quotient by another, and so continue the division until the quotient is a prime number. The several divisors and the last quotient are the required prime factors.*

EXAMPLES FOR PRACTICE.

184. Find the prime factors of the following numbers :

1. 210.	12. 16028.	23. 5184.
2. 630.	13. 19175.	24. 9160.
3. 462.	14. 10323.	25. 8030.
4. 1386.	15. 1250.	26. 4165.
5. 1470.	16. 6400.	27. 62500.
6. 8136.	17. 2240.	28. 81000.
7. 3234.	18. 1000.	29. 64000.
8. 4361.	19. 4515.	30. 45500.
9. 30030.	20. 7854.	31. 16875.
10. 11025.	21. 2310.	32. 18590.
11. 14600.	22. 5450.	33. 16380.

CANCELLATION.

PREPARATORY PROPOSITIONS.

185. Study carefully the following propositions :

PROP. I.—*Rejecting a factor from a number divides the number by that factor.*

Thus, $72 = 24 \times 3$. Hence, rejecting the factor 3 from 72, we have 24, the quotient of 72 divided by 3.

PROP. II.—*Dividing both dividend and divisor by the same number does not change the quotient.*

Thus, $60 \div 12 = 20$ threes $\div$ 4 threes $= 5$.

Observe that the unit *three*, in 20 *threes* ÷ 4 *threes*, does not in any way affect the size of the quotient; therefore, it may be rejected and the quotient will not be changed.

Hence, dividing both the dividend 60 and the divisor 12 by 3 does not change the quotient.

ILLUSTRATION OF PROCESS.

186. Ex. 1. Divide 462 by 42.

$$\frac{6\,)\,462}{6\,)\,42} = \frac{77}{7} = 11.$$

EXPLANATION.—We divide both the divisor and dividend by 6. According to Prop. II, the quotient is not changed.

Hence, $77 \div 7 = 462 \div 42 = 11$.

Ex. 2. Divide $65 \times 24 \times 55$ by $39 \times 15 \times 35$.

$$\frac{\overset{\cancel{13}}{\cancel{65}} \times \overset{8}{\cancel{24}} \times \overset{11}{\cancel{55}}}{\underset{3}{\cancel{39}} \times \underset{\cancel{3}}{\cancel{15}} \times \underset{7}{\cancel{35}}} = \frac{8 \times 11}{3 \times 7} = \frac{88}{21} = 4\tfrac{4}{21}.$$

EXPLANATION.—1. We divide any factor in the dividend by any number that will divide a factor in the divisor.

Thus, 65 in the dividend and 15 in the divisor are divided each by 5. In the same manner, 55 and 35, 13 and 39, 24 and 3 are divided.

The remaining factors, 8 and 11, in the dividend are prime to each of the remaining factors in the divisor. Hence, no further division can be performed.

2. We divide the product of 8 and 11, the remaining factors in the dividend, by the product of 3 and 7, the remaining factors in the divisor, and find as a quotient $4\frac{4}{21}$, which, according to (**185**—II), is equal to the quotient of $65 \times 24 \times 55$ divided by $39 \times 15 \times 35$.

All similar cases may be treated in the same manner; hence, the following

187. RULE.—*I. Cancel all the factors that are common to the dividend and divisor.*

II. Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor. The result will be the quotient required.

WRITTEN EXAMPLES.

188. 1. Divide 9009 by 6006. *Ans.* $1\frac{1}{2}$.

2. Divide 8475 by 525. *Ans.* $16\frac{1}{7}$.

3. Divide 3328 by 216. *Ans.* $15\frac{11}{27}$.

4. Divide $8 \times 15 \times 40$ by 10×24. *Ans.* 20.

5. Divide $49 \times 25 \times 12$ by $16 \times 36 \times 5$. *Ans.* $5\frac{5}{48}$.

6. Divide 12500 by 75. *Ans.* $166\frac{2}{3}$.

7. Divide $64 \times 81 \times 25$ by 24×27. *Ans.* 200.

8. Divide $12 \times 49 \times 27$ by 42×14. *Ans.* 27.

9. What is the quotient of 16 times 5 times 4 divided by 8 times 20? *Ans.* 2.

10. Multiply 8 times 66 by 5 times 18, and divide the product by 33 times 72. *Ans.* 20.

11. If 10, 12, 84, and 42 are the factors of the dividend, and 12, 5, 24, and 7 are the factors of the divisor, what is the quotient? *Ans.* 42.

12. How many barrels of flour, at 12 dollars a barrel, are worth as much as 16 cords of wood, at 3 dollars a cord?

13. At 18 dollars a week, how many weeks must a man work to pay 3 debts of 180 dollars each? *Ans.* 30 weeks.

14. When a laborer can buy 36 bushels of potatoes, at 4 shillings a bushel, with the earnings of 24 days, how many shillings does he earn a day? *Ans.* 6 shillings.

15. How many loads of potatoes, each containing 15 bushels, at 42 cents a bushel, will pay for 12 rolls of carpeting, each containing 56 yards, at 75 cents a yard? *Ans.* 80 loads.

16. A man exchanged 75 bushels of onions, at 90 cents a bushel, for a number of boxes of tea, containing 25 pounds each, at 54 cents a pound; how many boxes did he receive?

17. How many pounds of tea, at 72 cents a pound, would pay for 3 hogsheads of sugar, each weighing 1464 pounds, at 15 cents a pound? *Ans.* 915 pounds.

GREATEST COMMON DIVISOR.

PREPARATORY STEPS.

189. STEP I.—*Find by inspection an exact divisor for each of the following sets of numbers:*

1. 3, 9, 15, and 12.	4. 18, 45, 27, and 72.
2. 7, 14, 21, and 35.	5. 36, 84, 108, and 60.
3. 8, 12, 36, and 28.	6. 42, 70, 28, and 112.

STEP II.—*Find by inspection the greatest number that is an exact divisor of each of the following pairs of numbers:*

1. 5, 25.	3. 6, 120.	5. 25, 750.
2. 7, 28.	4. 13, 1300.	6. 45, 9000.

Find in the same manner the greatest exact divisor of the following:

7. 14, 35.	9. 36, 96.	11. 84, 132.
8. 25, 45.	10. 72, 108.	12. 88, 121.

STEP III.—*Express the numbers in each of the foregoing examples in terms of their greatest exact divisor.*

Thus, the greatest exact divisor of 16 and 40 is 8, hence 16 may be expressed as 2 *eights,* and 40 as 5 *eights.*

DEFINITIONS.

190. A ***Common Divisor*** is a number that is an exact divisor of each of two or more numbers.

Thus, 5 is a divisor of 10, 15, and 20.

191. The ***Greatest Common Divisor*** is the greatest number that is an exact divisor of each of two or more numbers.

Thus, 3 is the greatest exact divisor of each of the numbers 6 and 15. Hence 3 is their greatest common divisor.

192. Numbers are ***prime to each other*** when they have no common divisor besides 1; thus, 8, 9, 25.

METHOD BY FACTORING.

PREPARATORY PROPOSITION.

193. Illustrate the following proposition by examples.

The greatest common divisor is the product of the prime factors that are common to all the given numbers ; thus,

$$42 = 7 \times 2 \times 3 = 7 \text{ sixes};$$
$$66 = 11 \times 2 \times 3 = 11 \text{ sixes}.$$

7 and 11 being prime to each other, 6 must be the greatest common divisor of 7 sixes and 11 sixes. But 6 is the product of 2 and 3, the common prime factors; hence the greatest common divisor of 42 and 64 is the product of their common prime factors.

ILLUSTRATION OF PROCESS.

194. Prob. I.—**To find the Greatest Common Divisor of two or more numbers by factoring.**

Find the greatest common divisor of 98, 70, and 154.

(1.)

2)98	2)70	2)154
7)49	7)35	7)77
7	5	11

Or,

(2.)

2)98	70	154
7)49	35	77
7	5	11

$2 \times 7 =$ greatest common divisor.

Explanation.—1. We resolve each of the numbers into their prime factors, as shown in (1) or (2).

2. We observe that 2 and 7 are the only prime factors common to all the numbers. Hence the product of 2 and 7, or 14, according to (**193**). is the greatest common divisor of 98, 70, and 154.

The greatest common divisor of any two or more numbers is found in the same manner ; hence the following

195. Rule.—*Resolve each number into its prime factors, and find the product of the prime factors that are common to all the numbers.*

EXAMPLES FOR PRACTICE.

196. Find the greatest common divisor of

1. 30, 75, 105.	10. 68, 102, 238.
2. 70, 15, 210.	11. 66, 132, 231.
3. 63, 105, 147.	12. 138, 184, 322.
4. 78, 195, 117.	13. 105, 245, 315.
5. 112, 196, 272.	14. 195, 280, 345.
6. 126, 234, 306.	15. 147, 339, 483.
7. 187, 221, 323.	16. 228, 276, 348.
8. 405, 567, 324.	17. 360, 315, 495.
9. 225, 525, 300.	18. 840, 312, 408.

METHOD BY DIVISION.

PREPARATORY PROPOSITIONS.

197. Let the two following propositions be carefully studied and illustrated by other examples, before attempting to find the greatest common divisor by this method.

PROP. I.—*The greatest common divisor of two numbers is the greatest common divisor of the smaller number and their difference.*

Thus, 3 is the greatest common divisor of 15 and 27.

Hence 15 = 5 threes and 27 = 9 threes;
and 9 threes — 5 threes = 4 threes.

But 9 and 5 are prime to each other; hence, 4 and 5 must be prime to each other, for if not, their common divisor will divide their sum, according to (**169**—II), and be a common divisor of 9 and 5.

Therefore, 3 is the greatest common divisor of 5 threes and 4 threes, or of 15 and 12. Hence, the greatest common divisor of two numbers is the greatest common divisor of the smaller number and their difference.

PROP. II.—*The greatest common divisor of two numbers is the greatest common divisor of the smaller number and the remainder after the division of the greater by the less.*

This proposition may be illustrated thus:

$22 - 6 = 16$
$16 - 6 = 10$
$10 - 6 = 4$

1. Subtract 6 from 22, then from the difference, 16, etc., until a remainder less than 6 is obtained.

2. Observe that the number of times 6 has been subtracted is the quotient of 22 divided by 6, and hence that the remainder, 4, is the remainder after the division of 22 by 6.

3. According to Prop. I, the greatest common divisor of 22 and 6 is the greatest common divisor of their difference, 16, and 6. It is also, according to the same Proposition, the greatest common divisor of 10 and 6, and of 4 and 6. But 4 is the remainder after division and 6 the smaller number. Hence the greatest common divisor of 22 and 6 is the greatest common divisor of the *smaller number* and the *remainder* after division.

ILLUSTRATION OF PROCESS.

198. PROB. II.—**To find the Greatest Common Divisor of two or more numbers by continued division.**

Find the greatest common divisor of 28 and 176.

```
28)176(6
   168
   ---
     8)28(3
       24
       --
        4)8(2
          8
          -
          0
```

EXPLANATION.—1. We divide 176 by 28 and find 8 for a remainder; then we divide 28 by 8, and find 4 for a remainder; then we divide 8 by 4, and find 0 for a remainder.

2. According to Prop. II, the greatest common divisor of 28 and 176 is the same as the greatest common divisor of 28 and 8, also of 8 and 4. But 4 is the greatest common divisor of 8 and 4. Hence 4 is the greatest common divisor of 28 and 176.

The greatest common divisor of any two numbers is found in the same manner; hence the following

199. RULE.—*Divide the greater number by the less, then the less number by the remainder, then the last divisor by the last remainder, and so on until nothing remains. The last divisor is the greatest common divisor sought.*

To find the greatest common divisor of three or more numbers by this method we have the following

200. RULE.—*Find the greatest common divisor of two of the numbers, then of the common divisor thus found and a third number, and so on with a fourth, fifth, etc., number.*

ARITHMETICAL DRILL TABLE NO. 3.

201. Table for Oral Exercises in Greatest Common Divisor, and for Oral and Written Exercises in Least Common Multiple.

	A.	B.	C.	D.	E.	F.	G.
1.	4	10	14	6	12	16	8
2.	18	9	15	6	18	24	12
3.	21	27	8	16	24	12	28
4.	20	32	36	15	25	10	30
5.	20	35	40	45	12	24	36
6.	18	42	54	30	48	14	35
7.	21	42	63	49	28	56	24
8.	40	16	32	56	72	48	64
9.	18	45	72	27	63	81	36
10.	54	70	90	60	80	44	66
11.	33	55	99	77	22	88	24
12.	72	48	84	60	96	36	108

ARITHMETICAL DRILL TABLE NO. 4.

202. Table for Written Exercises in Greatest Common Divisor and Least Common Multiple.

	A.	B.	C.	D.	E.	F.
1.	30	36	154	176	88	198
2.	48	210	72	54	84	126
3.	252	396	264	480	220	792
4.	60	120	40	420	175	195
5.	132	264	396	462	594	528
6.	140	105	420	156	315	585
7.	96	280	112	192	336	840
8.	198	315	297	693	567	594
9.	210	350	240	300	720	630
10.	132	220	264	308	660	528
11.	168	480	504	420	252	540
12.	156	312	130	364	273	351

EXAMPLES FOR PRACTICE.

203. Find the greatest common divisor of the following:

1. 195 and 465.
2. 357 and 483.
3. 418 and 330.
4. 455 and 1085.
5. 808, 546, and 124.
6. 335 and 1085.
7. 356 and 808.
8. 195 and 483.
9. 465, 1365, and 215.
10. 546, 4641, and 364.

204. Continue the practice in finding the greatest common divisor of abstract numbers by taking examples from the above Arithmetical Tables. Let all the examples taken from Table No. 3 be worked orally and in sets in the same manner as directed for written exercises.

Examples with Two Numbers.

205. First Set.—Take examples for written exercises from the first line of Table No. 4, thus:

(1.) 30 36
(2.) 36 154
(3.) 154 176
(4.) 176 88
(5.) 88 196

Observe that in each new example, the first number taken in the last example is omitted and a new number added.

Take in the same manner examples from each line in the table.

206. Second Set.—Take examples from the first and second lines thus:

(1.) 30 48
(2.) 36 210
(3.) 154 72
(4.) 176 54
(5.) 88 84
(6.) 198 126

To present other examples, omit the *first* line and use the *second* and *third,* then the *third* and *fourth,* and so on to the bottom of the table.

207. Third Set.—Take the numbers from the *first* and *third* line, then from the *second* and *fourth,* then from the *third* and *fifth,* etc., to the bottom of the card.

Examples with Three Numbers.

208. First Set.—Take examples from each line, thus:

(1.) 30 36 154
(2.) 36 154 176
(3.) 154 176 88
(4.) 176 88 198

209. Second Set.—Take the numbers from the *first, second,* and *third* lines, then from the *second, third,* and *fourth,* and so on to the bottom of the table.

WRITTEN EXAMPLES.

210. 1. Divide the greatest common divisor of 48, 72, 96, and 120 by the greatest common divisor of 21, 30, 39, and 84.

2. I have rooms 12 feet, 15 feet, and 24 feet wide; what is the width of the widest carpeting that will fit any room in my house? *Ans.* 3 feet.

3. In the village of Potsdam, some of the sidewalks are 48 inches wide, some 60 inches, and others 72 inches; what is the widest flagging that can be used in each of these sidewalks without cutting? *Ans.* 12 inches.

4. A man owns 3 village lots of equal depth, the first having a front of 72 feet, the second 144 feet, and the third 108 feet, which he wishes to divide into as many lots as possible having equal fronts; how many feet will each front contain? *Ans.* 36 feet.

5. I have a lot whose sides measure, respectively, 42 feet, 84 feet, 112 feet, and 126 feet; I wish to enclose it with boards having the greatest possible uniform length; what will be the length of each board? *Ans.* 14 feet.

6. A teamster agrees to cart 132 barrels of flour for a merchant on Monday, 84 barrels on Wednesday, and 108 barrels on Friday; what is the largest number he can carry at a load, and yet have the same number in each? *Ans.* 12 barrels.

7. A merchant has three pieces of cloth containing respectively 42, 98, and 84 yards, which he proposes to sell in dress patterns of uniform size. What is the largest number of yards the dress patterns can contain so that there may be nothing left of either piece?

8. If two farms containing each an exact number of acres were purchased for $8132 and $6270 respectively, what is the highest uniform price per acre that could have been paid, and in this case how many acres in each farm?

LEAST COMMON MULTIPLE.

PREPARATORY PROPOSITIONS.

211. Study carefully each of the following propositions:

PROP. I.—*A multiple of a number contains as a factor each prime factor of the number as many times as it enters into the number.*

Thus, 60, which is a multiple of 12, contains 5 times 12, or 5 times $2 \times 2 \times 3$, the prime factors of 12. Hence, each of the prime factors of 12 enters as a factor into 60 as many times as it enters into 12.

PROP. II.—*The least common multiple of two or more given numbers must contain, as a factor, each prime factor in those numbers the greatest number of times that it enters into any one of them.*

Thus, $12 = 2 \times 2 \times 3$, and $9 = 3 \times 3$. The prime factors in 12 and 9 are 2 and 3. A multiple of 12, according to Prop. I, must contain 2 as a factor twice and 3 once. A multiple of 9, according to the same proposition, must contain 3 as a factor twice. Hence a number which is a multiple of both 12 and 9 must contain 2 as a factor twice and 3 twice, which is equal to $2 \times 2 \times 3 \times 3 = 36$. Hence 36 is the least common multiple of 12 and 9.

DEFINITIONS.

212. A ***Multiple*** of a number is a number that is exactly divisible by the given number.

Thus, 24 is divisible by 8; hence, 24 is a multiple of 8.

213. A ***Common Multiple*** of two or more numbers is a number that is exactly divisible by each of them.

Thus, 36 is divisible by each of the numbers 4, 9, and 12; hence, 36 is a common multiple of 4, 9, and 12.

214. The ***Least Common Multiple*** of two or more numbers is the least number that is exactly divisible by each of them.

Thus, 24 is the least number that is divisible by each of the numbers 6 and 8; hence, 24 is the least common multiple of 6 and 8.

METHOD BY FACTORING.

ILLUSTRATION OF PROCESS.

215. Prob. I.—**To find, by factoring, the least common multiple of two or more numbers.**

Find the least common multiple of 18, 24, 15, and 35.

3	18	24	15	35
2	6	8	5	35
5	3	4	5	35
	3	4		7

Explanation.—1. We observe that 3 is a factor of 18, 24, and 15. Dividing these numbers by 3, we write the quotients with 35, in the second line.

2. Observing that 2 is a factor of 6 and 8, we divide as before, and find the third line of numbers. Dividing by 5, we find the fourth line of numbers, which are prime to each other ; hence cannot be further divided.

3. Observe the divisors 3, 2, and 5 are all the factors that are common to any two or more of the given numbers, and the quotients 3, 4, and 7 are the factors that belong each only to one number. Therefore the divisors and quotients together contain each of the prime factors of 18, 24, 15, and 35 as many times as it enters into any one of these numbers. Thus, the divisors 3 and 2, with the quotient 3, are the prime factors of 18; and so with the other numbers.

Hence, according to (**211**—II), the continued product of the divisors 3, 2, and 5, and the quotients 3, 4, and 7, which is equal to 2520, is the least common multiple of 18, 24, 15, and 35.

From this illustration we have the following

216. Rule.—*I. Write the numbers in a line, and divide by any prime factor that is contained in any two or more of them, placing the quotients and the undivided numbers in the line below.*

II. Operate upon the second line of numbers in the same manner, and so on until a line of numbers that are prime to each other is found.

III. Find the continued product of the divisors used and the numbers in the last line ; this will give the least common multiple of the given numbers.

WRITTEN EXAMPLES.

217. 1. What is the least common multiple of the nine digits?

2. What is the least number of cents that can be exactly expended in oranges, whether they cost 4, 5, or 6 cents apiece?

3. What is the smallest quantity of milk that will exactly fill either six-quart, nine-quart, or twelve-quart cans?

4. A can lay 42 rows of shingles on my house in a day, and B can lay 56 rows; what is the least number of rows that will give a number of full days' work to either A or B?

5. What is the smallest sum of money that I can exactly lay out in calves at 14 dollars each, cows at 38 dollars each, or oxen at 57 dollars each? *Ans.* 798 dollars.

6. What is the width of the narrowest street across which stepping-stones either 2, 4, or 9 feet long will exactly reach?

7. Three separate parties are measuring the distance from the town hall in Middleton to the town hall in Danvers; one party uses a chain 33 feet long, another a chain 66 feet long, and the third a chain 50 feet long, marking each chain's length with a stake; at what intervals of space will three stakes be driven at the same place? *Ans.* Every 1650 feet.

METHOD BY GREATEST COMMON DIVISOR.

ILLUSTRATION OF PROCESS.

218. PROB. II.—**To find, by using the greatest common divisor, the least common multiple of two or more numbers.**

Find the least common multiple of 195 and 255.

EXPLANATION.—1. We find the greatest common divisor of 195 and 255, which is 15.

2. The greatest common divisor, 15, according to (**193**), contains all the prime factors that are common to 195 and 255. Dividing each of these numbers by 15, we find the factors that are not common, namely, 13 and 17.

3. The common divisor 15 and the quotient 13 contain all the prime factors of 195, and the common divisor 15 and the quotient 17 contain all the prime factors of 255.

Hence, according to (**211**—II), the continued product of the common divisor 15 and the quotients 13 and 17, which is 3315, is the least common multiple of 195 and 255.

The least common multiple of any two numbers is found in the same manner, hence the following

219. Rule.—*I. Find the greatest common divisor of the two given numbers, and divide each of the numbers by this divisor.*

II. Find the continued product of the greatest common divisor and the quotients; this will give the least common multiple of the two given numbers.

To find the least common multiple of three or more numbers by this method we have the following

220. Rule.—*Find the least common multiple of two of them; then find the least common multiple of the multiple thus found and the third number, and so on with four or more numbers.*

EXAMPLES FOR PRACTICE.

221. Find the least common multiple of

1. 91 and 182.	5. 154 and 231.
2. 110 and 165.	6. 385 and 455.
3. 78 and 195.	7. 364 and 637.
4. 143 and 165.	8. 462 and 546.

222. For further practice take examples with two numbers from Table No. 4, page 93, as directed in (**205**), (**206**), and (**207**); and examples with three numbers from Table No. 3, as directed in (**208**) and (**209**).

Continue to practice with abstract numbers until you can find the least common multiple of two or more numbers accurately and rapidly.

REVIEW AND TEST QUESTIONS.

223. 1. Define Prime Number, Composite Number, and Exact Divisor, and illustrate each by an example.

2. What is meant by an Odd Number? An Even Number?

3. Show that if an even number is divisible by an odd number, the quotient must be even.

4. Name the prime numbers from 1 to 40.

5. Why are all even numbers except 2 composite?

6. State how you would show, in the series of odd numbers, that every fifth number from 5 is divisible by 5.

7. What is a Factor? A Prime Factor?

8. What are the prime factors of 81? Of 64? Of 125?

9. Show that rejecting the same factor from the divisor and dividend does not change the quotient.

10. Explain Cancellation, and illustrate by an example.

11. Give reasons for calling an exact divisor a *measure*.

12. What is a Common Measure? The Greatest Common Measure? Illustrate each answer by an example.

13. Show that the greatest common divisor of 42 and 114 is the greatest common divisor of 42 and the remainder after the division of 114 by 42.

14. Explain the rule for finding the greatest common divisor by factoring; by division.

15. Why must we finally get a common divisor if the greater of two numbers be divided by the less, and the divisor by the remainder, and so on?

16. What is a Multiple? The Least Common Multiple?

17. Explain how the Least Common Multiple of two or more numbers is found by using their greatest common divisor.

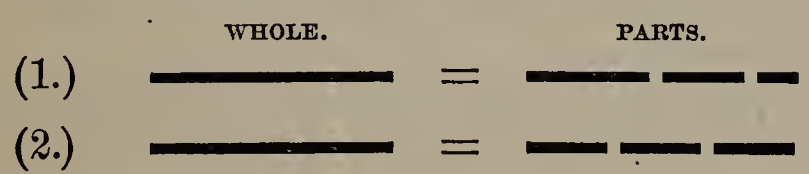
18. Prove that a number is divisible by 9 when the sum of its digits is divisible by 9.

19. Prove that a number is divisible by 11 when the difference of the sums of the digits in the odd and even places is zero.

FRACTIONS

PREPARATORY PROPOSITIONS.

224. Prop. I.—*Any thing regarded as a whole can be divided into unequal or equal parts;* thus,



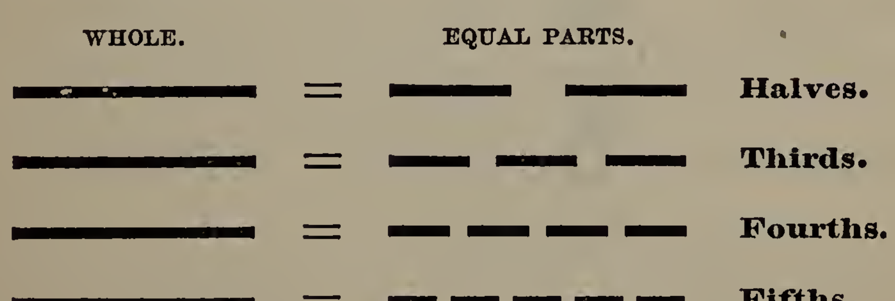
1. Equal parts of a whole are called *Fractions.*

2. Into what kind of parts can a pear be divided? A bushel of wheat? A rope? A garden? Anything?

3. Make $12 into unequal parts in six ways, and into equal parts in five ways.

4. In how many ways can 15 be made into equal parts? Into unequal parts?

Prop. II.—*The same whole can be divided into equal parts of different sizes;* thus,



1. Observe the equal parts are named by using the ordinal corresponding with the number of parts. Thus, when the whole is made into *three* parts, one part is called a *third,* when

into four parts, one part is called a fourth, and so on to any number of parts.

2. When the whole is made into ten equal parts, what is one part called? Into sixteen equal parts? Into twenty-four? Into forty-three?

3. What are the largest equal parts that can be made of a whole? The next largest? The next largest?

4. What is meant by one-half of an apple? One-third? One-fifth?

5. What is meant by two-thirds of a line? Of an hour? Of a day?

6. How would you find the fourth of anything? The seventh? The tenth?

7. Find the third of 6. Of 12. Of 15. Of 24. Of 48.

8. If a whole is made into twelve equal parts, how would you name three parts? Seven parts? Five parts? Nine parts?

9. How many *halves* make a whole? How many *thirds?* How many *sevenths?* How many *tenths?* How many *fifteenths?*

PROP. III.—*Equal parts of a whole, or* ***Fractions,*** *are expressed by two numbers written one over the other, with a line between them;* thus,

NUMERATOR,	4	Shows the number of equal parts in the fraction.
DIVIDING LINE,	—	Shows that 4 and 5 express a fraction.
DENOMINATOR,	5	Shows the number of equal parts in the whole.

Read, *Four-fifths.*

1. Read the following: $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{7}$, $\frac{9}{10}$, $\frac{8}{13}$, $\frac{17}{25}$, $\frac{42}{89}$.

2. What does $\frac{5}{7}$ signify? $\frac{8}{9}$? $\frac{12}{17}$? $\frac{23}{30}$?

3. Express in numbers three-fifths; nine-thirteenths; eleven-fifteenths.

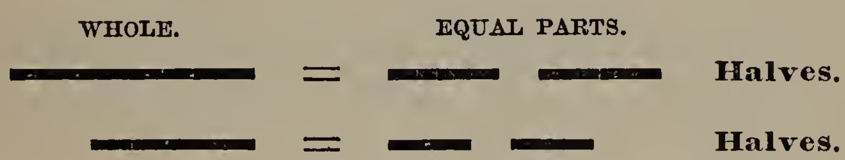
4. What does *Numerator* mean? *Denominator?* Dividing line? *Terms* of a fraction?

5. How is a fraction expressed by numbers?

6. Name the terms of $\frac{4}{5}$, $\frac{9}{13}$, $\frac{15}{23}$, $\frac{3}{11}$, $\frac{42}{97}$, $\frac{124}{263}$.

7. Express in numbers seven-ninths? Nineteen forty-fifths.

Prop. IV.—*The size or value of the same kind of equal parts depends upon the size or value of the whole of which they are parts;* thus,



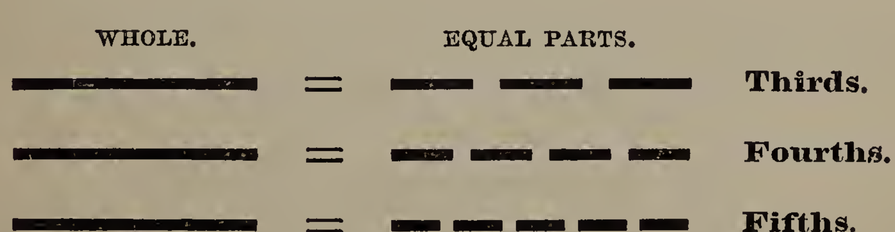
1. The equal parts in the illustration, although halves in both cases, are unequal in size, because the wholes are unequal in size.

2. Which is the larger, the half of $4 or the half of $6?

3. Which is the smaller, the fourth of 12 inches, or the fourth of 20 inches, and why?

4. If offered the half of either of two farms, which would you take, and why?

Prop. V.—*The size or value of the equal parts of a whole diminish as the number of parts increase, or increase as the number of parts diminish;* thus,



1. Which is the greater, one-half or one-third? One-fourth or one-fifth? One-sixth or one-ninth, and why?

2. How much is $\frac{1}{6}$ of $48 smaller than $\frac{1}{8}$ of it?

3. Upon what two things does the value of one-half, one-third, one-fourth, one-fifth, etc., depend? Illustrate your answer by two examples.

DEFINITIONS.

225. A ***Fractional Unit*** is one of the *equal parts* of anything regarded as a whole.

226. A ***Fraction*** is *one* or *more* of the equal parts of anything regarded as a whole.

227. The ***Unit of a Fraction*** is the unit or whole which is considered as divided into equal parts.

228. The ***Numerator*** is the number above the dividing line in the expression of a fraction, and indicates how many *equal parts* are in the *fraction.*

229. The ***Denominator*** is the number below the dividing line in the expression of a fraction, and indicates how many *equal parts* are in the *whole.*

230. The ***Terms*** of a fraction are the numerator and denominator.

231. Taken together, the *terms* of a *fraction* are called a ***Fraction,*** or ***Fractional Number.***

232. Hence, the word ***Fraction*** means one or more of the equal parts of anything, or the *expression* that denotes one or more of the equal parts of anything.

REDUCTION.

PREPARATORY STEPS.

233. Step I.—*A fraction is represented by lines thus:*

$\frac{2}{3}$ = 

Part taken.

Whole.

Observe carefully the following:

1. In $\frac{2}{3}$, the denominator 3 expresses the *whole,* or 3 *thirds,* and the numerator 2 expresses *two* parts of the *same size.*

Hence, 3 equal lines for the denominator and 2 equal lines for the numerator, of the same length as those in the denominator, represent correctly the whole, the parts taken, and the relation of the parts to each other, as expressed by the fraction $\frac{2}{3}$.

2. Represent by lines $\frac{3}{4}$; $\frac{5}{7}$; $\frac{9}{12}$; $\frac{1}{10}$; $\frac{1}{6}$; $\frac{8}{13}$.

Why can the numerator and denominator of a fraction be represented by equal lines?

Step II.—*Show by representing the fraction with lines that one-half is equal to two-fourths;* thus,

$$\frac{1}{2} = \frac{2}{4}$$

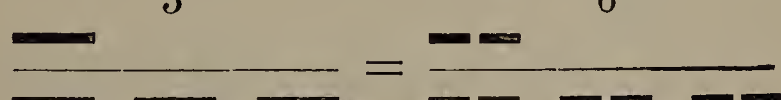
1. By observing the illustration, it will be seen that the value of the numerator and denominator is not changed by making *each part* in *each* into two equal parts. It will also be seen that when this is done the numerator contains 2 parts and the denominator 4. Hence $\frac{1}{2} = \frac{2}{4}$.

2. Show in the same manner that one-half is equal to three-sixths, four-eighths, five-tenths, and so on.

Step III.—*Any fractional unit can, without changing its value, be divided into any desired number of equal parts.*

Study carefully and explain the following illustrations:

1. $$\frac{1}{3} = \frac{2}{6}$$



2. $$\frac{1}{4} = \frac{3}{12}$$

ORAL EXAMPLES.

234. 1. How many tenths in $\frac{1}{5}$ of an orange? How many fifteenths? How many twentieths? etc., and why?

2. How can $\frac{1}{4}$ of a yard be made into sixteenths of a yard?

3. How many twelfths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$? In $\frac{1}{6}$?

4. Make $\frac{1}{7}$ into twenty-firsts, and explain the process.

5. Show by lines that $\frac{1}{5} = \frac{3}{15}$; that $\frac{1}{6} = \frac{4}{24}$; that $\frac{1}{2} = \frac{9}{18}$.

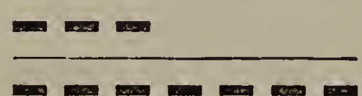
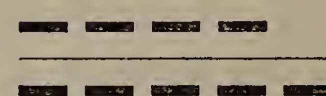
6. Change $\frac{1}{7}$, without altering its value, into a fraction containing 7 equal parts; 10 equal parts; 25 equal parts.

PRINCIPLES OF REDUCTION.

235. Let each of the following principles be illustrated by the pupil with a number of examples:

PRIN. I.—*The numerator and denominator of a fraction represent, each, parts of the same size;* thus,

$\frac{3}{7}$ $\frac{4}{5}$

(1.)  (2.) 

Observe in illustration (1) the denominator 7 represents the whole or 7 *sevenths*, and the numerator 3 represents 3 *sevenths*; in illustration (2), the denominator represents 5 *fifths*, and the numerator 4 *fifths*. Hence the numerator and denominator of a fraction represent parts of the same size.

PRIN. II.—*Multiplying both the terms of a fraction by the same number does not change the value of the fraction;* thus,

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Be particular to observe in the illustration that the *amount* expressed by the 2 in the numerator or the 3 in the denominator of $\frac{2}{3}$ is not changed by making each part into 4 equal parts; therefore $\frac{2}{3}$ and $\frac{8}{12}$ express, each, the same amount of the same whole.

Hence, multiplying the numerator and denominator by the same number means, so far as the real fraction is concerned, *dividing the equal parts in each into as many equal parts as there are units in the number by which they are multiplied.*

PRIN. III.—*Dividing both terms of a fraction by the same number does not change the value of the fraction;* thus,

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

The amount expressed by the 9 in the numerator or the 12 in the denominator of $\frac{9}{12}$ is not changed by putting every 3 parts into *one*, as will be seen from the illustration.

Hence $\frac{9}{12}$ and $\frac{3}{4}$ express each the same amount of the same whole, and dividing the numerator and denominator by the same number means *putting as many parts in each into one as there are units in the number by which they are divided.*

DEFINITIONS.

236. The ***Value*** of a fraction is the amount which it represents.

237. ***Reduction*** is the process of changing the terms of a fraction without altering its value.

238. A fraction is reduced to ***Higher Terms*** when its numerator and denominator are expressed by larger numbers. Thus, $\frac{4}{5} = \frac{8}{10}$.

239. A fraction is reduced to ***Lower Terms*** when its numerator and denominator are expressed by smaller numbers. Thus, $\frac{8}{12} = \frac{4}{6}$.

240. A fraction is expressed in its ***Lowest Terms*** when its numerator and denominator are *prime* to each other.

Thus, in $\frac{4}{9}$, the numerator and denominator 4 and 9 are prime to each other; hence the fraction is expressed in its *lowest terms.*

241. A ***Common Denominator*** is a denominator that belongs to two or more fractions.

242. The ***Least Common Denominator*** of two or more fractions is the least denominator to which they can all be reduced.

243. A ***Proper Fraction*** is one whose numerator is less than the denominator; as $\frac{2}{3}$, $\frac{5}{7}$.

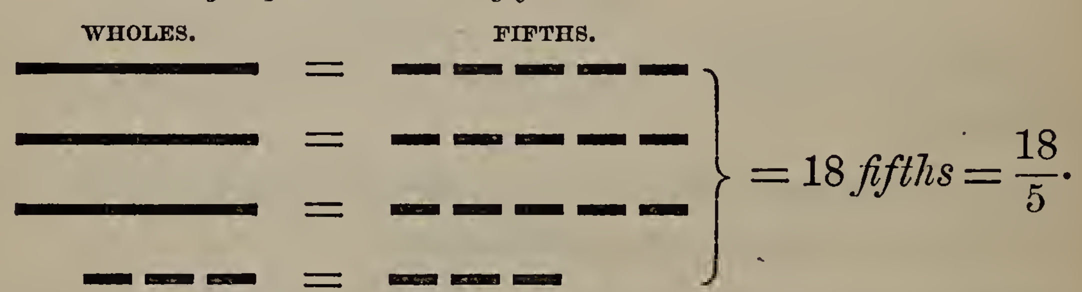
244. An ***Improper Fraction*** is one whose numerator is equal to, or greater than, the denominator; as $\frac{5}{5}$, $\frac{8}{3}$.

245. A ***Mixed Number*** is a number composed of an integer and a fraction; as $4\frac{2}{3}$, $18\frac{5}{7}$.

ILLUSTRATION OF PROCESS.

246. Prob. I.—**To reduce a whole or mixed number to an improper fraction.**

1. Reduce $3\frac{3}{5}$ equal lines to *fifths.*



Explanation.—Each whole line is equal to 5 *fifths,* as shown in the illustration; 3 lines must therefore be equal to 15 *fifths.* 15 *fifths* + 3 *fifths* = 18 fifths. Hence in $3\frac{3}{5}$ lines there are $\frac{18}{5}$ of a line.

From this illustration we have the following:

247. Rule.—*Multiply the whole number by the given denominator, and to the product add the numerator of the given fraction, if any, and write the result over the given denominator.*

EXAMPLES FOR PRACTICE.

248. Reduce orally the following:

1. In 5 pounds of sugar how many *fourths* of a pound?

SOLUTION.—In 1 pound of sugar there are 4 *fourths*; hence, in 5 pounds there are 5 times 4 *fourths*, which is $\frac{20}{4}$ of a pound.

2. In 7 tons of coal how many *ninths* of a ton?
3. How many *tenths* in \$63? In 42 yards? In 17 pounds?
4. Express 20 as *fourths*. As *sevenths*. As *hundredths*.
5. In \$$9\frac{3}{7}$ how many *sevenths* of a dollar?

SOLUTION.—In \$1 there are 7 *sevenths*. In \$9 there must therefore be 9 times 7 *sevenths* or 63 *sevenths*. 63 *sevenths* + 3 *sevenths* are equal to 66 *sevenths*. Hence, in \$$9\frac{3}{7}$ there are $\frac{66}{7}$ of a dollar.

6. In $12\frac{5}{12}$ acres how many *twelfths* of an acre?
7. How many *eighths* in $9\frac{7}{8}$? In $11\frac{5}{8}$? In $7\frac{3}{8}$? In $5\frac{6}{8}$?

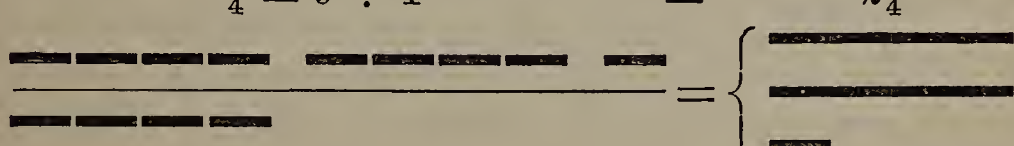
Reduce the following to improper fractions:

8.	$45\frac{3}{5}$.	12.	$340\frac{3}{4}$.	16.	$3\frac{23}{45}$.
9.	$83\frac{4}{7}$.	13.	$187\frac{5}{7}$.	17.	$7\frac{120}{213}$.
10.	$13\frac{11}{12}$.	14.	$462\frac{3}{8}$.	18.	$4\frac{7}{100}$.
11.	$76\frac{5}{8}$.	15.	$463\frac{5}{12}$.	19.	$2\frac{87}{1000}$.

249. PROB. II.—**To reduce an improper fraction to an integer or a mixed number.**

1. Reduce 9 *fourths* of a line to whole lines.

$$\frac{9}{4} = 9 \div 4 \qquad = \qquad 2\frac{1}{4}$$



EXPLANATION.—A whole line is composed of 4 *fourths*. Hence, to make the 9 *fourths* of a line into whole lines, we put every *four* parts into *one*, as shown in the illustration, or divide the 9 by 4, which gives 2 wholes and 1 of the *fourths* remaining. Hence the following

250. RULE.—*Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

251. Reduce and explain orally the following:

1. How many bushels are $\frac{28}{4}$ of a bushel? $\frac{63}{9}$? $\frac{51}{7}$? $\frac{75}{12}$?

2. In \$$\frac{54}{6}$, how many dollars? In $\frac{67}{8}$ of a yard, how many yards? In $\frac{50}{9}$ of a foot, how many feet?

3. How many miles in $\frac{101}{12}$ of a mile? $\frac{58}{9}$? $\frac{75}{8}$?

Reduce to whole or mixed numbers the following:

4.	$\frac{832}{25}$.	9.	$\frac{3682}{100}$.	14.	$\frac{87634}{107}$.
5.	$\frac{936}{43}$.	10.	$\frac{4664}{54}$.	15.	$\frac{50709}{308}$.
6.	$\frac{592}{84}$.	11.	$\frac{21075}{325}$.	16.	$\frac{90057}{1009}$.
7.	$\frac{925}{73}$.	12.	$\frac{3273}{101}$.	17.	$\frac{2065}{32}$.
8.	$\frac{437}{100}$.	13.	$\frac{7356}{500}$.	18.	$\frac{6304}{909}$.

252. PROB. III.—**To reduce a fraction to higher terms.**

1. Reduce $\frac{2}{3}$ of a line to twelfths.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

EXPLANATION.—1. To make a *whole*, which is already in *thirds*, into 12 equal parts, *each third* must be made into *four* equal parts.

2. The numerator of the given fraction expresses 2 *thirds*, and the denominator 3 *thirds*; making each third in both into *four* equal parts (**245**—II), as shown in the illustration, the new numerator and denominator will each contain 4 times as many parts as in the given fraction.

Hence, $\frac{2}{3}$ of a line is reduced to *twelfths* by multiplying both numerator and denominator by 4.

Hence the following rule for reducing a fraction to higher terms:

253. RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply the terms of the given fraction by the quotient.*

EXAMPLES FOR PRACTICE.

254. Reduce and explain orally the following:

1. How would you make *halves* of an apple into *fourths?* Into *sixths?* Into *tenths?* Into *sixteenths?*
2. How many *twelfths* in $\frac{3}{4}$ of a cord of wood?
3. Explain how $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{5}$ can each be reduced to *twentieths.*
4. Show by the use of lines that $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$.
5. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{9}{14}$, and $\frac{13}{21}$ each to *forty-seconds.*
6. In $\frac{3}{7}$ how many *ninety-eighths?*
7. Change $\frac{5}{9}$, $\frac{3}{10}$, $\frac{7}{12}$, $\frac{3}{8}$, and $\frac{13}{30}$ each to 360ths.
8. Reduce $\frac{3}{5}$, $\frac{7}{11}$, $\frac{4}{15}$, $\frac{16}{33}$, and $\frac{24}{55}$ to 165ths.

255. PROB. IV.—**To reduce a fraction to lower terms.**

Reduce $\frac{9}{12}$ of a given line to *fourths.*

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

EXPLANATION.—1. To make into 4 equal parts or *fourths* a whole, which is already in 12 equal parts, or twelfths, every 3 of the 12 parts must be put into one.

2. The numerator of the given fraction expresses 9 *twelfths*, and the denominator 12 *twelfths;* putting every 3 *twelfths* into *one*, in both (**235**—III) as shown in the illustration, the new numerator and denominator will each contain *one-third* as many parts as in the given fraction.

Hence, $\frac{9}{12}$ of a line is reduced to *fourths* by dividing both numerator and denominator by 3.

Hence the following rule for reducing a fraction to its lowest terms:

256. RULE.—*Reject from the terms of the given fraction all their common factors.* Or,

Divide the terms of the given fraction by their greatest common divisor.

EXAMPLES FOR PRACTICE.

257. Reduce and explain orally the following:

1. In $\frac{4}{6}$ of a bushel, how many *thirds* of a bushel?

2. How can *twelfths* of a bushel be made into *fourths* of a bushel? Into *thirds?* Into *halves?*

3. Reduce $\frac{8}{20}$ of a dollar to *fifths* of a dollar.
4. Show by the use of lines that $\frac{12}{36} = \frac{6}{18} = \frac{4}{12} = \frac{3}{9} = \frac{1}{3}$.
5. Reduce $\frac{8}{12}$ to its lowest terms. $\frac{9}{15}$. $\frac{10}{25}$. $\frac{18}{42}$. $\frac{15}{40}$.
6. Express $\frac{16}{24}$ in parts 8 times as great in value.

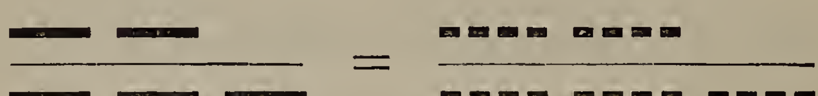
Reduce the following to their lowest terms:

7. $\frac{63}{135}$.	10. $\frac{247}{323}$.	13. $\frac{825}{2709}$.	16. $\frac{936}{2368}$.
8. $\frac{66}{121}$.	11. $\frac{272}{425}$.	14. $\frac{630}{936}$.	17. $\frac{5184}{6912}$.
9. $\frac{246}{342}$.	12. $\frac{330}{726}$.	15. $\frac{324}{612}$.	18. $\frac{3444}{3556}$.

258. PROB. V.—To change fractions to equivalent ones having a common denominator.

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ of a line to fractions having a common denominator.

$$\frac{2}{3} \qquad\qquad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

(1.) 

$$\frac{3}{4} \qquad\qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

(2.) 

EXPLANATION.—1. We find the least common multiple of the denominators 3 and 4, which is 12.

2. We reduce each of the fractions to twelfths (**252**—III), as shown in illustrations (1) and (2).

Hence the following

259. RULE.—*Find the least common multiple of all the denominators for a common denominator; divide* *this by each denominator separately, and multiply the corresponding numerator by the quotient, and write the product over the common denominator.*

EXAMPLES FOR PRACTICE.

260. Reduce and explain orally the following:

1. Reduce $\frac{2}{3}$ and $\frac{1}{2}$ to sixths. $\frac{2}{3}$ and $\frac{3}{4}$ to twelfths.

2. Change $\frac{5}{8}$ and $\frac{5}{6}$ to fractions having the same denominator, and explain each step in the process.

3. Express $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{3}{8}$ as fortieths.

4. What is the least common denominator of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{4}{7}$?

Observe, fractions have a *least* common denominator when their denominators are alike and there is no factor common to all the numerators and the common denominator.

Reduce the following to their least common denominator:

5. $\frac{5}{6}$, $\frac{9}{10}$, and $\frac{14}{15}$.
6. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
7. $\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{7}{10}$.
8. $\frac{5}{14}$, $\frac{13}{21}$, and $\frac{7}{9}$.
9. $\frac{31}{54}$, $\frac{11}{28}$, $\frac{53}{63}$, and $\frac{3}{12}$.
10. $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{9}{14}$, $\frac{3}{28}$, and $\frac{17}{32}$.
11. $\frac{7}{9}$, $\frac{5}{11}$, $\frac{13}{18}$, $\frac{3}{22}$, and $\frac{1}{36}$.
12. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{27}$, $\frac{8}{81}$, $\frac{16}{243}$, and $\frac{31}{729}$.

ADDITION.

PREPARATORY PROPOSITIONS.

261. PROP. I.—*Fractional units of the* SAME KIND, *that are fractions of the same whole, are added in the same manner as integral units.*

Thus, $\frac{2}{5}$ of a yard can be added to $\frac{4}{5}$ of a yard, because they are each fifths of *one yard.* But $\frac{2}{5}$ of a *yard* cannot be added to $\frac{4}{5}$ of a *day.*

Solve orally the following:

1. $\frac{3}{4}+\frac{1}{4}+\frac{5}{4}$.
2. $\frac{5}{9}+\frac{8}{9}+\frac{2}{9}$.
3. $\frac{7}{12}+\frac{5}{12}+\frac{11}{12}$.
4. $\frac{5}{7}+\frac{3}{7}+\frac{6}{7}$.
5. $\frac{13}{15}+\frac{8}{15}+\frac{7}{15}$.
6. $\frac{9}{11}+\frac{12}{11}+\frac{6}{11}$.

7. In $\frac{5}{9}+\frac{7}{9}+\frac{8}{9}$ of a yard, how many yards?

8. How many are $\$\frac{7}{12}+\$\frac{5}{12}+\$\frac{8}{12}+\$\frac{11}{12}+\$\frac{5}{12}$?

9. Find the sum of $\frac{13}{24}+\frac{15}{24}+\frac{7}{24}+\frac{5}{24}+\frac{8}{24}$ miles.

10. Why cannot $\frac{8}{9}$ of a bushel and $\frac{5}{9}$ of a peck be added as now expressed?

PROP. II.—*Fractions expressed in different fractional units must be changed to equivalent fractions having the same fractional unit, before they can be added.*

For example, $\frac{2}{3}$ and $\frac{3}{4}$ of a foot cannot be added until both fractions are expressed in the same fractional unit. Thus, $\frac{2}{3}$ of a foot is equal $\frac{8}{12}$ of a foot, and $\frac{3}{4}$ of a foot is equal $\frac{9}{12}$ of a foot; $\frac{8}{12}+\frac{9}{12}$ of a foot $=\frac{17}{12}$, or $1\frac{5}{12}$ feet. Hence the sum of $\frac{2}{3}+\frac{3}{4}$ of a foot $=1\frac{5}{12}$ feet.

Find orally the sum of the following:

1. $\frac{2}{5}+\frac{3}{10}$.	4. $\frac{7}{10}+\frac{18}{20}$.	7. $\frac{3}{4}+\frac{5}{6}+\frac{7}{12}$.
2. $\frac{2}{3}+\frac{4}{9}$.	5. $\frac{3}{5}+\frac{7}{15}$.	8. $\frac{4}{5}+\frac{8}{15}+\frac{2}{3}$.
3. $\frac{5}{12}+\frac{3}{4}$.	6. $\frac{8}{12}+\frac{5}{24}$.	9. $\frac{7}{8}+\frac{5}{6}+\frac{11}{12}$.

262. PROB. I.—**To find the sum of any two or more given fractions.**

1. Find the sum of $\frac{4}{9}+\frac{5}{6}+\frac{7}{8}$.

$$\left.\begin{array}{l}\frac{4}{9}=\frac{32}{72}\\ \frac{5}{6}=\frac{60}{72}\\ \frac{7}{8}=\frac{63}{72}\end{array}\right\}=\frac{155}{72}=2\frac{11}{24}$$

EXPLANATION.—1. We reduce the fractions to the same fractional unit, by reducing them to their least common denominator, which is 72 (**258**—V).

2. We find the sum of the numerators, 155, and write it over the common denominator, 72, and reduce $\frac{155}{72}$ to $2\frac{11}{24}$

The sum of any number of fractions may be found in the same manner; hence the following

263. RULE.—*I. Change the fractions to equivalent ones having the least common denominator, then add the numerators, write the result over the common* *denominator, and reduce, when possible, to lower terms or to a whole or mixed number.*

II. When there are mixed numbers or integers, add the fractions and integers separately, then add the results.

WRITTEN EXAMPLES.

264. Find the sum of each of the following:

1. $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.
2. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{4}{8}$, $\frac{6}{7}$, and $\frac{11}{14}$.
3. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{6}$, and $\frac{1}{8}$.
4. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{9}{16}$.
5. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
6. $\frac{3}{8}$, $\frac{4}{5}$, and $\frac{1}{2}$.
7. $\frac{1}{4}$, $\frac{3}{5}$, and $\frac{5}{9}$.
8. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.
9. $\frac{3}{9}$, $\frac{8}{48}$, $\frac{15}{16}$, and $\frac{63}{84}$.
10. $\frac{3}{8}$, $\frac{6}{12}$, $\frac{46}{64}$, and $\frac{6}{48}$.
11. $\frac{1}{2}$, $7\frac{1}{4}$, and $8\frac{2}{3}$.
12. $1\frac{2}{3}$, $2\frac{3}{4}$, and $4\frac{1}{2}$.
13. $4\frac{1}{5}$, $2\frac{1}{4}$, $3\frac{1}{3}$, and $6\frac{1}{2}$.
14. $8\frac{4}{15}$, $2\frac{7}{9}$, $3\frac{3}{5}$, and $4\frac{2}{3}$.
15. $1\frac{1}{4}$, $4\frac{1}{3}$, $\frac{7}{11}$, and $1\frac{1}{2}$.
16. $4\frac{5}{8}$, $10\frac{1}{6}$, and $83\frac{33}{48}$.
17. $68\frac{4}{9}$, $28\frac{1}{2}$, $32\frac{3}{7}$, $7\frac{8}{21}$, and $6\frac{125}{126}$.
18. $8\frac{1}{2}$, $25\frac{3}{7}$, 19, and $68\frac{4}{11}$.

19. Herbert Adams has $10\frac{3}{4}$ acres of land in one field, $10\frac{1}{8}$ in another, and $11\frac{3}{16}$ in a third; how many acres has he in the three fields? *Ans.* $32\frac{1}{16}$ acres.

20. A farmer has three tubs of butter, weighing, respectively, $44\frac{1}{6}$ pounds, $56\frac{1}{4}$ pounds, and $78\frac{2}{3}$ pounds; how much butter in the three tubs? *Ans.* $179\frac{1}{12}$ pounds.

21. A carpenter has a board $7\frac{4}{9}$ feet long, another $11\frac{2}{3}$ feet long, and a third $9\frac{5}{6}$ feet long; what is their united length?

22. How many yards in three remnants of cotton cloth, containing, respectively, $2\frac{1}{4}$ yards, $1\frac{1}{9}$ yards, and $2\frac{3}{8}$ yards?

23. Henry earned $3\frac{5}{8}$ dollars, his father gave him $5\frac{1}{12}$ dollars, and his brother gave him $1\frac{1}{12}$ dollars more than his father; how much money did he have in all? *Ans.* $14\frac{7}{8}$ dollars.

24. A man traveled $42\frac{5}{12}$ miles on Monday, $30\frac{2}{7}$ miles on Tuesday, $48\frac{1}{8}$ miles on Wednesday, and $25\frac{3}{4}$ miles on Thursday; how far did he travel during the four days?

25. Three pieces of cloth contain, respectively, $43\frac{2}{3}$, $54\frac{5}{7}$, and $87\frac{3}{8}$ yards; how many yards in all?

SUBTRACTION.

265. Prop. I.—*Fractional units of the* SAME KIND *that are fractions of the same whole are subtracted in the same manner as integral units.*

Thus, 7 *ninths* — 5 *ninths* = 2 *ninths*, or $\frac{7}{9} - \frac{5}{9} = \frac{2}{9}$.

Perform orally the subtraction in the following:

1. $\frac{8}{9} - \frac{3}{9}$.
3. $\frac{15}{14} - \frac{7}{14}$.
5. $\frac{32}{43} - \frac{12}{43}$.
7. $\frac{113}{124} - \frac{7}{124}$.
2. $\frac{12}{17} - \frac{5}{17}$.
4. $\frac{19}{24} - \frac{11}{24}$.
6. $\frac{106}{108} - \frac{40}{108}$.
8. $\frac{87}{100} - \frac{9}{100}$.

Prop. II.—*Fractions expressed in different fractional units must be reduced to the same fractional unit before subtracting.*

Thus, in $\frac{7}{8} - \frac{5}{16}$ we reduce the $\frac{7}{8}$ to *sixteenths*; $\frac{7}{8} = \frac{14}{16}$, and $\frac{14}{16} - \frac{5}{16} = \frac{9}{16}$; hence, $\frac{7}{8} - \frac{5}{16} = \frac{9}{16}$.

Perform orally the subtraction in the following:

1. $\frac{5}{6} - \frac{7}{12}$.
4. $\frac{2}{3} - \frac{5}{18}$.
7. $\frac{7}{8} - \frac{5}{24}$.
2. $\frac{7}{9} - \frac{3}{18}$.
5. $\frac{4}{5} - \frac{7}{20}$.
8. $\frac{5}{6} - \frac{4}{18}$.
3. $\frac{11}{12} - \frac{13}{36}$.
6. $\frac{8}{15} - \frac{11}{30}$.
9. $\frac{13}{21} - \frac{11}{63}$.

266. Prob. I.—**To find the difference of any two given fractions.**

1. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

$$\frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$$

Explanation.—1. We reduce the given fractions to their least common denominator, which is 24.

2. We find the difference of the numerators, 21 and 10, and write it over the common denominator, giving $\frac{11}{24}$, the required difference.

2. Find the difference between $35\frac{2}{3}$ and $16\frac{3}{4}$.

$$\begin{aligned} 35\tfrac{2}{3} &= 35\tfrac{8}{12} \\ 16\tfrac{3}{4} &= 16\tfrac{9}{12} \\ & \overline{18\tfrac{11}{12}} \end{aligned}$$

Explanation.—1. We reduce the $\frac{2}{3}$ and $\frac{3}{4}$ to their least common denominator.

2. $\frac{9}{12}$ cannot be taken from $\frac{8}{12}$; hence, we increase the $\frac{8}{12}$ by $\frac{12}{12}$ or 1, taken from the 35. We now subtract $\frac{9}{12}$ from $\frac{20}{12}$, leaving $\frac{11}{12}$.

3. We subtract 16 from the remaining 34, leaving 18, which united with $\frac{11}{12}$ gives $18\frac{11}{12}$, the required difference.

The difference between any two fractions or mixed numbers may be found in the same manner; hence the following

267. RULE.—*I. Reduce the given fractions to equivalent ones having the least common denominator; then find the difference of the numerators and write it over the common denominator.*

II. When there are mixed numbers, subtract the fraction first, then the integer.

If the fraction in the minuend is smaller than that in the subtrahend, increase it by one from the integral part of the minuend; then subtract.

WRITTEN EXAMPLES.

268. Perform the following subtractions:

1. $\frac{8}{4} - \frac{5}{9}$.	5. $6\frac{3}{8} - 4\frac{1}{2}$.	9. $84\frac{2}{3} - 37\frac{5}{7}$.
2. $\frac{13}{17} - \frac{4}{7}$.	6. $37\frac{4}{15} - 33\frac{5}{24}$.	10. $73\frac{8}{9} - 29\frac{11}{12}$.
3. $7\frac{1}{2} - 4\frac{7}{9}$.	7. $13\frac{5}{12} - 9\frac{7}{13}$.	11. $51\frac{13}{16} - 34\frac{19}{48}$.
4. $9\frac{5}{7} - 6\frac{3}{4}$.	8. $50\frac{1}{16} - 47\frac{1}{24}$.	12. $65\frac{7}{30} - 59\frac{13}{20}$.

13. From a cask of vinegar containing $31\frac{1}{2}$ gallons, $16\frac{3}{8}$ gallons were drawn; how many remained? *Ans.* $15\frac{1}{8}$ gallons.

14. If flour be bought for \$$9\frac{7}{10}$ a barrel, and sold for \$$12\frac{5}{6}$, what is the gain per barrel? *Ans.* $3\frac{2}{15}$ dollars.

15. If a grocer buys $4\frac{2}{3}$ and $6\frac{1}{2}$ barrels of flour, and then sells $1\frac{1}{2}$ and $4\frac{1}{3}$ barrels, how many does he still have?

16. The sum of two numbers is $59\frac{8}{9}$, and the greater is $30\frac{35}{36}$; what is the other number? *Ans.* $28\frac{11}{12}$.

17. A man who contracted to build $45\frac{1}{4}$ miles of railroad, has completed $25\frac{5}{8}$ miles; how many miles has he to build?

18. James found \$$2\frac{3}{4}$, earned \$$1\frac{1}{2}$, and had \$$2\frac{1}{4}$ given him; how much more money had he then than George, who earned \$$6\frac{2}{3}$ and spent \$$4\frac{3}{10}$?

19. I bought two tubs of butter, the tubs and butter together weighing $111\frac{3}{4}$ pounds, and the tubs alone weighing $7\frac{5}{8}$ and 8 pounds respectively; what was the weight of the butter?

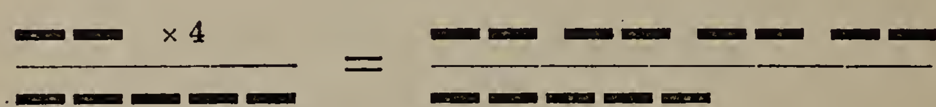
MULTIPLICATION.

PREPARATORY PROPOSITIONS.

269. The following propositions must be mastered perfectly, to understand and explain the process in multiplication and division of fractions.

PROP. I.—*Multiplying the numerator of a fraction, while the denominator remains unchanged, multiplies the fraction;* thus,

$$\frac{2 \times 4}{5} = \frac{8}{5}$$



Observe that since the denominator is not changed, the size of the parts remain the same. Hence the fraction $\frac{2}{5}$ is multiplied by 4, as shown in the illustration, by multiplying the numerator by 4.

PROP. II.—*Dividing the denominator of a fraction while the numerator remains unchanged multiplies the fraction;* thus,

$$\frac{2}{12 \div 4} = \frac{2}{3}$$

(1.) (2.)

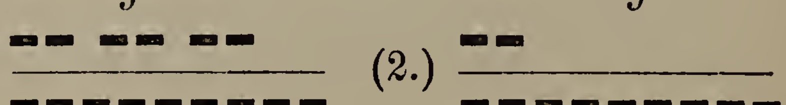
Observe that in (1) the whole is made into 12 equal parts. By putting every 4 of these parts into one, or dividing the denominator by 4, the whole, as shown in (2), is made into 3 equal parts, and each of the 2 parts in the numerator is 4 *times* 1 *twelfth*.

Hence, dividing the denominator of $\frac{2}{12}$ by 4, the number of parts in the numerator remaining the same, multiplies the fraction by 4.

PROP. III.—*Dividing the numerator of a fraction while the denominator remains unchanged divides the fraction;* thus,

$$\frac{6 \div 3}{9} = \frac{2}{9}$$

(1.) $\frac{1}{3}$ of (2.)

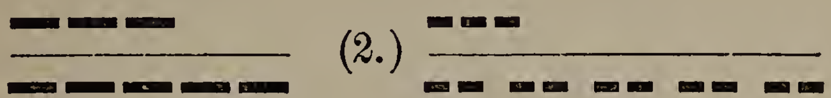


In (1) the numerator 6 expresses the parts taken, and *one-third* of these 6 parts, as shown by comparing (1) and (2), the denominator remaining the same, is one-third of the value of the fraction. Hence, the fraction $\frac{6}{9}$ is divided by 3 by dividing the numerator by 3.

PROP. IV.—*Multiplying the denominator of a fraction while the numerator remains unchanged divides the fraction;* thus,

$$\frac{3}{5} \times 2 = \frac{3}{10}$$

(1.) (2.)



In (1) the whole is made into 5 equal parts; multiplying the denominator by 2, or making each of these 5 parts into 2 equal parts, as shown in (2), the whole is made into 10 equal parts, and the 3 parts in the numerator are one-half the size they were before.

Hence, multiplying the denominator of $\frac{3}{5}$ by 2, the numerator remaining the same, divides the fraction by 2.

EXERCISES.

270. Show by the use of lines or objects that

1. $\frac{4 \times 3}{7} = \frac{12}{7}$.
2. $\frac{5}{18 \div 6} = \frac{5}{3}$.
3. $\frac{14 \div 2}{17} = \frac{7}{17}$.
4. $\frac{4}{5 \times 3} = \frac{4}{15}$.
5. $\frac{3}{12 \div 4} = 1$.
6. $\frac{3 \times 4}{12} = 1$.
7. $\frac{9}{20 \div 5} = 2\frac{1}{4}$.
8. $\frac{15 \div 3}{19} = \frac{5}{19}$.
9. $\frac{3 \times 5}{5} = 3$.
10. $\frac{5}{7}$ is how many times $\frac{5}{7 \times 3}$, and why?
11. Why is $\frac{3}{20 \div 5} = \frac{3 \times 5}{20}$? Explain by lines.
12. $\frac{7}{18}$ is how many times $\frac{7}{18 \div 6}$, and why?
13. Why is $\frac{3}{8}$ greater than $\frac{3}{8+1}$? Explain by lines.

ILLUSTRATION OF PROCESS.

271. PROB. I.—**To multiply a fraction by an integer.**

1. Multiply $\frac{4}{9}$ by 7.

SOLUTION.—1. According to (**269**—I), multiplying the numerator, the denominator remaining the same, multiplies the fraction. Hence, 7 times $\frac{4}{9}$ is equal to $\frac{4 \times 7}{9} = \frac{28}{9} = 3\frac{1}{9}$.

2. According to (**269**—II), a fraction is also multiplied by dividing the denominator. Hence the following

272. RULE.—*Multiply the numerator of the fraction by the given integer, or divide the denominator.*

EXAMPLES FOR PRACTICE.

273. Multiply orally the following, reduce the results to their lowest terms, and explain as above.

1. $\frac{5}{7} \times 3$.	4. $\frac{2}{3} \times 9$.	7. $\frac{7}{8} \times 5$.
2. $\frac{3}{8} \times 12$.	5. $\frac{5}{11} \times 6$.	8. $\frac{4}{9} \times 9$.
3. $\frac{8}{9} \times 4$.	6. $\frac{6}{7} \times 8$.	9. $\frac{13}{14} \times 14$.

Multiply the following and reduce. Cancel when possible.

10. $\frac{37}{94} \times 9$.	13. $\frac{254}{1000} \times 50$.	16. $\frac{800}{1000} \times 90$.
11. $\frac{43}{100} \times 8$.	14. $\frac{376}{843} \times 48$.	17. $\frac{852}{973} \times 75$.
12. $\frac{84}{100} \times 10$.	15. $\frac{500}{1000} \times 100$.	18. $\frac{606}{6006} \times 100$.

274. PROB. II.—**To find any given part of an integer.**

1. Find $\frac{4}{5}$ of \$395.

SOLUTION.—1. We find the $\frac{1}{5}$ of \$395 by dividing it by 5. Hence the *first step*, \$395 ÷ 5 = \$79.

2. Since \$79 is 1 fifth of \$395, four times \$79 will be 4 fifths. Hence the *second step*, \$79 × 4 = \$316.

To avoid fractions until the final result, we multiply by the numerator first, then divide by the denominator; hence the following

275. RULE.—*Divide by the denominator and multiply by the numerator of the fraction, which indicates the part required.*

EXAMPLES FOR PRACTICE.

1. Find $\frac{7}{9}$ of $342; of $1000; of $4860; of $10001.

2. How many are $\frac{9}{10}$ of 8730? of 57? of 835? of 10? of 100?

3. A has $189, B has $\frac{4}{7}$ of A's money, and C has $\frac{7}{12}$ of B's money: what is the sum of their money? *Ans.* $360.

4. A farmer owning 395 acres of land, sold at one time $\frac{2}{9}$ of it, and at another time $\frac{5}{12}$ of it; how many acres does he still own? *Ans.* $142\frac{23}{36}$ acres.

5. A man having $1305 gave $\frac{2}{9}$ of it to A and $\frac{3}{7}$ of what remained to B; how much had he left? *Ans.* $580.

6. A merchant having three pieces of cloth containing respectively 187 yards, 162 yards, and 208 yards, sold $\frac{2}{3}$ of the first piece, $\frac{3}{5}$ of the second; how many yards has he left?

276. Prob. III.—**To find any given part of a fraction,** or *To multiply a fraction by a fraction.*

Find the $\frac{2}{3}$ of $\frac{3}{4}$ of a given line.

FIRST STEP.

$$\frac{1}{3} \text{ of } \frac{3}{4} = \frac{3}{4 \times 3} = \frac{3}{12}$$

$\frac{1}{3}$ of 

SECOND STEP.

$$\frac{3 \times 2}{12} = \frac{6}{12} = \frac{1}{2}$$

Explanation.—According to (**269**—IV), a fraction is divided by multiplying its denominator. Hence we find the $\frac{1}{3}$ of $\frac{3}{4}$ by multiplying the denominator 4 by 3, as shown in *First Step.*

Having found 1 *third* of $\frac{3}{4}$, we find 2 *thirds*, according to (**269**—I), by multiplying the numerator of $\frac{3}{12}$ by 2, as shown in *Second Step.* Hence the following

277. Rule.—*Divide by the denominator and multiply by the numerator of the fraction which indicates the part required.*

EXAMPLES FOR PRACTICE.

278. Solve orally the following, and explain as above:

1. What is $\frac{3}{4}$ of $\frac{5}{8}$? $\frac{2}{7}$ of $\frac{4}{9}$? $\frac{7}{12}$ of $\frac{3}{5}$? $\frac{5}{7}$ of $\frac{3}{8}$?
2. What part of 1 is $\frac{1}{5}$ of $\frac{2}{3}$? $\frac{3}{4}$ of $\frac{1}{2}$? $\frac{5}{7}$ of $\frac{3}{8}$?
3. If a yard of cloth cost $\$\frac{4}{5}$, what is $\frac{3}{4}$ of a yard worth?

Find the value of the following:

4. $\frac{7}{23}$ of $\frac{5}{21}$; $\frac{42}{183}$ of $\frac{3}{7}$; $\frac{63}{108}$ of $\frac{12}{45}$; $\frac{203}{475}$ of $\frac{5}{7}$.
5. $\frac{4}{7}$ of $\frac{43}{73}$; $(\frac{2}{3}+\frac{4}{5}) \times 8$; $(\frac{8}{9}-\frac{3}{4}) \times \frac{5}{7}$; $(\frac{5}{9}$ of $\frac{8}{13}) - \frac{13}{117}$.
6. What is the cost of $\frac{7}{12}$ of a yard of cloth, at $\$\frac{3}{5}$ per yard?
7. A farmer gave $\frac{2}{7}$ of his farm to one son, and $\frac{3}{5}$ of what was left to another; what part of the whole farm did he give the second son? *Ans.* $\frac{3}{7}$.

279. PROB. IV.—**To multiply by a mixed number.**

Multiply 372 by $6\frac{5}{7}$.

$$\begin{array}{r} 372 \\ 6\frac{5}{7} \\ \hline 2232 \\ 265\frac{5}{7} \\ \hline 2497\frac{5}{7} \end{array}$$

EXPLANATION.—1. In multiplying by a mixed number, the multiplicand is taken separately (**92**), as many times as there are units in the multiplier, and such a part of a time as is indicated by the fraction in the multiplier; hence,

2. We multiply 372 by $6\frac{5}{7}$ by multiplying first by 6, which gives the product 2232, and adding to this product $\frac{5}{7}$ of 372, which is $265\frac{5}{7}$ (**274**), giving $2497\frac{5}{7}$, the product of 372 and $6\frac{5}{7}$. Hence the following

280. RULE.—*Multiply first by the integer, then by the fraction, and add the products.*

EXAMPLES FOR PRACTICE.

281. Multiply orally and explain the following:

1. 12 by $5\frac{2}{3}$.	4. 20 by $3\frac{4}{5}$.	7. 80 by $3\frac{3}{8}$.
2. 18 by $2\frac{1}{9}$.	5. 100 by $7\frac{2}{5}$.	8. 36 by $2\frac{7}{9}$.
3. 8 by $9\frac{3}{4}$.	6. 400 by $2\frac{1}{4}$.	9. 24 by $10\frac{3}{4}$.

Perform the multiplication in the following:

10. $39 \times 24\frac{3}{5}$.	12. $425 \times 104\frac{7}{8}$.	14. $1000 \times 73\frac{3}{5}$.
11. $75 \times 12\frac{7}{10}$.	13. $63\frac{11}{12} \times 24$.	15. $8000 \times 9\frac{35}{100}$.

282. PROB. V.—**To multiply when both multiplicand and multiplier are mixed numbers.**

Multiply $86\frac{2}{3}$ by $54\frac{5}{7}$.

$$(1.)\quad 86\tfrac{2}{3} = \tfrac{260}{3}; \qquad 54\tfrac{5}{7} = \tfrac{383}{7}.$$

$$(2.)\quad \tfrac{260}{3} \times \tfrac{383}{7} = \tfrac{99580}{21} = 4741\tfrac{19}{21}.$$

EXPLANATION.—1. We reduce, as shown in (1), both multiplicand and multiplier to improper fractions.

2. We multiply, as shown in (2), the numerators together for the numerator of the product, and the denominators together for the denominators of the product (**275**), then reduce the result to a mixed number. Hence the following

283. RULE.—*I. Reduce the mixed numbers to improper fractions.*

II. Multiply the numerators together for the numerator of the product, and the denominators for the denominators of the product.

III. Reduce the result to a whole or mixed number.

284. The process in multiplication is shortened by cancellation; thus,

Multiply $32\frac{4}{7}$ by $28\frac{7}{12}$.

$$(1.)\quad 32\tfrac{4}{7} = \frac{228}{7}; \qquad 28\tfrac{7}{12} = \frac{343}{12}.$$

$$(2.)\quad \frac{\overset{19}{\cancel{228}}}{\cancel{7}} \times \frac{\overset{49}{\cancel{343}}}{\cancel{12}} = 19 \times 49 = 931.$$

EXPLANATION.—1. We reduce the multiplicand and multiplier to improper fractions, as shown in (1).

2. We indicate the multiplication, and cancel, as shown in (2), the factors common to any numerator and any denominator. Hence the following

285. RULE.—*Indicate all the multiplications, and cancel the factors common to any numerator and any denominator.*

EXERCISE FOR PRACTICE.

286. Multiply the following, canceling common factors:

1. $\frac{36}{84} \times \frac{7}{18}$.
2. $\frac{125}{255} \times \frac{45}{75}$.
3. $\frac{63}{110} \times \frac{44}{81}$.
4. $\frac{39}{45} \times \frac{60}{117}$.
5. $\frac{78}{85} \times \frac{51}{72}$.
6. $\frac{200}{1000} \times \frac{50}{75}$.
7. $\frac{100}{374} \times \frac{26}{75}$.
8. $\frac{360}{535} \times \frac{125}{340}$.
9. $\frac{100}{1000} \times \frac{400}{700}$.

Find the continued product of

10. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{3}{10}$.
11. $\frac{15}{64}$, $\frac{16}{27}$, $\frac{11}{10}$, $\frac{72}{385}$, and $\frac{21}{85}$.
12. $\frac{5}{6}$, $2\frac{3}{4}$, $3\frac{5}{11}$, $5\frac{2}{19}$, and $6\frac{1}{194}$.
13. $\frac{95}{116}$, $\frac{17}{21}$, and $\frac{87}{153}$.
14. $\frac{355}{520}$, $\frac{51}{211}$, and $\frac{35}{85}$.
15. $3\frac{3}{5}$, $4\frac{5}{6}$, and $7\frac{1}{29}$.

16. What is the cost of 12 cords of wood at $\$6\frac{3}{8}$ a cord, and 8 tons of coal at $\$11\frac{7}{8}$ a ton? *Ans.* $171.50.

17. A lady bought 15 yards of silk at $\$2\frac{7}{9}$ a yard, and $7\frac{3}{8}$ yards of lace at $\$3\frac{2}{3}$ a yard; what was the cost of both?

18. The State of Kentucky has an area of 37680 square miles, and the average population to a square mile, in 1870, was $35\frac{3}{10}$; what was the population of the State?

19. What is the cost of $45\frac{7}{10}$ tons of iron, at $\$27\frac{3}{4}$ per ton?

20. At $\$3\frac{2}{5}$ a yard, what is the cost of $15\frac{7}{8}$? Of $32\frac{5}{9}$?

21. A merchant sold $12\frac{2}{3}$ yards of cloth at $\$2\frac{4}{7}$ a yard; $28\frac{5}{8}$ at $\$1\frac{3}{4}$; and $52\frac{7}{8}$ at $\$3\frac{2}{5}$; what did he receive for the whole?

22. Find the product of $\frac{3}{7}$ of $25\frac{4}{9}$, and $\frac{8}{11}$ of $16\frac{3}{10}$.

23. Find the product of $4\frac{5}{6}$ of $\frac{3}{4}$ of 12, and $7\frac{2}{3}$ of 15.

24. Bought 19 pounds of butter at $23\frac{1}{2}$ cents a pound, giving in return $27\frac{5}{6}$ pounds of lard at 15 cents a pound, and the rest in cash; what did I give in cash? *Ans.* 29 cents.

25. A farmer has two fields containing respectively $11\frac{1}{2}$ acres and $21\frac{1}{3}$ acres; how much hay will he take from both fields, at the rate of $1\frac{3}{5}$ tons an acre for the first, and $2\frac{3}{8}$ tons an acre for the second? *Ans.* $69\frac{1}{15}$ tons.

26. Find the value of $(\frac{2}{3} \text{ of } 8) - (\frac{4}{7} \text{ of } 9 - 2\frac{5}{14})$.

27. Find the value of $(\$37\frac{4}{9} - \$13\frac{5}{7}) \times (\frac{2}{5} \text{ of } 8 - 2\frac{3}{4})$.

28. Find the value of $\overline{2\frac{2}{3} + 3\frac{4}{7}} \times \overline{8\frac{4}{5} - 4\frac{1}{2}}$.

DIVISION.

ILLUSTRATION OF PROCESS.

287. PROB. I.—**To divide a fraction by an integer.**

1. Divide $\frac{8}{9}$ by 4.

(1.) $\frac{8}{9} \div 4 = \frac{8 \div 4}{9} = \frac{2}{9}$

(2.) $\frac{8}{9} \div 4 = \frac{8}{9 \times 4} = \frac{8}{36} = \frac{2}{9}$

EXPLANATION.—1. According to (**269**—III), a fraction is divided by dividing the numerator. Hence we divide $\frac{8}{9}$ by 4, as shown in (1), by dividing the numerator 8 by 4.

2. According to (**269**—IV), a fraction is divided by multiplying the denominator. Hence we divide $\frac{8}{9}$ by 4, as shown in (2), by multiplying the denominator by 4, and reducing the result to its lowest terms. Hence the following

288. RULE.—*Divide the numerator, or multiply the denominator, by the given integer.*

EXAMPLES FOR PRACTICE.

289. Divide orally and explain the following. Dividing the numerator in every case where it can be done, in preference to multiplying the denominator.

1. $\frac{12}{13} \div 4$.
2. $\frac{42}{57} \div 7$.
3. $\frac{13}{14} \div 3$.
4. $\frac{5}{9} \div 8$.
5. $\frac{8}{11} \div 4$.
6. $\frac{7}{13} \div 6$.

Perform the division in the following:

7. $\frac{128}{236} \div 32$.
8. $\frac{285}{432} \div 57$.
9. $\frac{225}{236} \div 75$.
10. $\frac{43}{84} \div 25$.
11. $\frac{200}{365} \div 50$.
12. $\frac{300}{432} \div 75$.
13. If 7 yards of cloth cost $\$\frac{5}{8}$, what will 1 yard cost?
14. At $\$\frac{2}{3}$ for 4 boxes of figs, what will 1 box cost?
15. Show that multiplying the denominator of $\frac{8}{9}$ by 4 divides the fraction. Explain by lines.
16. The product of two numbers is $149\frac{3}{5}$, and one of them is 23; what is the other?

17. If a compositor earns $\$45\frac{3}{4}$ in 18 days, how much does he earn in 1 day? In 9 days? In 5 days? In 27 days?

Find the value

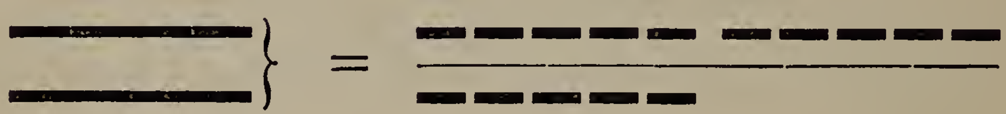
18. Of $(\frac{3}{4}$ of $\frac{5}{9} - \frac{7}{18}) \div 8$.
19. Of $(\frac{7}{10} + \frac{4}{5}) \div 32$.
20. Of $(\frac{2}{3}$ of $5\frac{1}{4} - 2\frac{5}{6}) \div 12$.
21. Of $(\frac{8}{9}$ of $10\frac{4}{6} + \frac{8}{15}) \div 32$.

290. Prob. II.—**To divide by a fraction.**

1. How many times is $\frac{3}{5}$ of a given line contained in twice the same line?

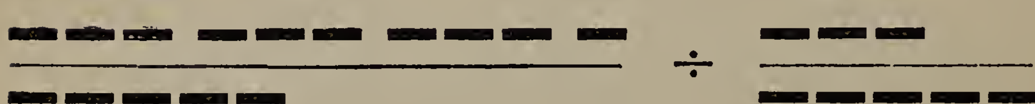
FIRST STEP.

2 lines $= \frac{10}{5}$ of a line.



SECOND STEP.

$\frac{10}{5} \div \frac{3}{5} = \frac{10}{3} = 3\frac{1}{3}$.



$= 3\frac{1}{3}$.

Explanation.—1. We can find how many times one number is contained in another, only when both are of the same denomination (**155**). Hence we first reduce, as shown in *First Step*, the 2 lines to 10 *fifths* of a line; the same *fractional* denomination as the divisor, 3 *fifths.*

2. The 3 fifths in the divisor, as shown in *Second Step,* are contained in the 10 fifths in the dividend 3 times, and 1 part remaining, which makes $\frac{1}{3}$ of a time. Hence 2 equal lines contain $\frac{3}{5}$ of one of them $3\frac{1}{3}$ times.

Observe the following regarding this solution:

(1.) The dividend is reduced to the same fractional denomination as the divisor by multiplying it by the denominator of the divisor; and when reduced, the division is performed by dividing the numerator of the dividend by the numerator of the divisor.

(2.) By inverting the terms of the divisor these two operations are expressed by the sign of multiplication. Thus, $2 \div \frac{3}{5} = 2 \times \frac{5}{3}$, which means that 2 is to be multiplied by 5, and the product divided by 3.

2. How many times is $\frac{1}{2}$ of a given line contained in $\frac{2}{3}$ of it?

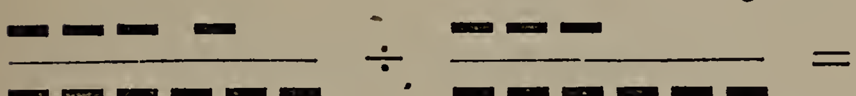
FIRST STEP.

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{1}{2} = \frac{3}{6}$$

SECOND STEP.

$$\frac{4}{6} \div \frac{3}{6} = \frac{4}{3} = 1\tfrac{1}{3}$$



EXPLANATION.—1. We reduce, as shown in *First Step*, the dividend $\frac{2}{3}$ and the divisor $\frac{1}{2}$ both to *sixths* (**155**—1).

2. We divide the $\frac{4}{6}$ by $\frac{3}{6}$ by dividing the numerator of the dividend by the numerator of the divisor. The $\frac{3}{6}$ is contained in $\frac{4}{6}$, as shown in *Second Step*, $1\frac{1}{3}$ times. Hence $\frac{1}{2}$ is contained $1\frac{1}{3}$ times in $\frac{2}{3}$.

291. When dividing by a fraction, we abbreviate the work by *inverting the divisor*, as follows:

1. In reducing the dividend and divisor to the same fractional unit, the product of the denominators is taken as the common denominator, and each numerator is multiplied by the denominator of the other fraction; thus,

$$\frac{5}{7} \div \frac{2}{3} = \frac{5 \times 3}{7 \times 3} \div \frac{2 \times 7}{3 \times 7} = \frac{15}{21} \div \frac{14}{21} = \frac{15}{14} \quad \begin{matrix}\text{Numerator of dividend.}\\ \text{Numerator of divisor.}\end{matrix}$$

2. By inverting the divisor, thus, $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$, the numerators 15 and 14 are found at once, without going through the operation of finding the common denominator. Hence the following

292. RULE.—*Invert the terms of the divisor and proceed as in multiplication.*

EXAMPLES FOR PRACTICE.

293. Solve orally the following and explain as above.

1. $\frac{4}{5} \div \frac{2}{3}$.	4. $12 \div \frac{5}{7}$.	7. $90 \div \frac{10}{11}$.
2. $\frac{5}{7} \div \frac{1}{4}$.	5. $\frac{8}{9} \div \frac{2}{3}$.	8. $\frac{10}{11} \div \frac{5}{9}$.
3. $8 \div \frac{3}{5}$.	6. $9 \div \frac{5}{6}$.	9. $200 \div \frac{100}{123}$.

10. 1 is how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$?

11. At $\$\frac{7}{8}$ a bushel, how many bushels of corn can be bought for $9?

SOLUTION.—As many bushels as $\$\frac{7}{8}$ is contained times in $9. $9 are equal to $\$\frac{72}{8}$, and $\$\frac{7}{8}$ is contained in $\$\frac{72}{8}$ $10\frac{2}{7}$ times. Hence, etc.

12. At $\$\frac{5}{7}$ a yard, how many yards of cloth can be bought for $3? For $10? For $15? For $7? For $25? For $9?

13. For $12 how many pounds of tea can be bought at $\$\frac{3}{4}$ per pound? At $\$\frac{5}{8}$? At $\$\frac{4}{5}$? At $\$\frac{2}{3}$? At $\$\frac{7}{8}$? At $\$\frac{4}{9}$?

14. 5 are how many times $\frac{2}{3}$? $\frac{5}{7}$? $\frac{3}{4}$? $\frac{7}{8}$? $\frac{10}{11}$?

15. If $\frac{4}{9}$ of a ton of coal cost $3, what will 1 ton cost?

SOLUTION.—Since $\frac{4}{9}$ of a ton cost $3, $\frac{1}{9}$ will cost $\frac{1}{4}$ of $3, or $\$\frac{3}{4}$, and 1 ton, or $\frac{9}{9}$, will cost 9 times $\$\frac{3}{4}$, or $\$\frac{27}{4}$, equal to $\$6\frac{3}{4}$. Hence, etc.

Or, 1 ton will cost as many times $3 as $\frac{4}{9}$ of a ton is contained times in 1 ton. 1 ton $\div \frac{4}{9} = \frac{9}{4} = 2\frac{1}{4}$. Hence, 1 ton will cost $2\frac{1}{4}$ times $3, or $\$6\frac{3}{4}$.

16. At $\$\frac{7}{8}$ for $\frac{4}{5}$ of a pound of tea, what is the cost of 1 pound? Of 7 pounds? Of $\frac{9}{10}$ of a pound? Of $\frac{7}{9}$ pounds?

17. If $\frac{7}{8}$ of a cord of wood cost $4, what will 1 cord cost? 4 cords? 11 cords? $\frac{4}{5}$ of a cord? $\frac{9}{10}$ of a cord?

18. How many bushels of wheat can be bought for $8, if $\frac{2}{3}$ of a bushel cost $\$\frac{8}{9}$? If $\frac{3}{5}$ of a bushel cost $\$\frac{9}{10}$?

19. At $\$\frac{3}{5}$ a yard, how much cloth can be bought for $\$\frac{9}{10}$?

SOLUTION.—As many yards as $\$\frac{3}{5}$ is contained times in $\$\frac{9}{10}$. $\$\frac{3}{5}$ equals $\$\frac{6}{10}$, and $\$\frac{6}{10}$ is contained $1\frac{1}{2}$ times in $\$\frac{9}{10}$. Hence, etc.

20. At $\$\frac{5}{12}$ a bushel, how many bushels of potatoes can be bought for $\$\frac{7}{8}$. For $\$\frac{23}{24}$? For $\$\frac{2}{3}$? For $\$\frac{1}{2}$? For $\$\frac{3}{4}$?

21. How many pounds of sugar at $\$\frac{1}{8}$ can be bought for $\$\frac{2}{3}$? For $\$\frac{5}{7}$? For $\$\frac{17}{24}$? For $\$\frac{25}{32}$? For $\$\frac{7}{9}$?

22. If $\frac{9}{10}$ of a yard of cloth can be bought for $\$\frac{7}{12}$, how much will 1 yard cost? 5 yards? $7\frac{2}{3}$ yards?

23. A grocer expended $\frac{2}{5}$ of $480 in purchasing tea at $\$\frac{5}{7}$ per pound, and the balance in purchasing coffee at $\$\frac{3}{8}$ per pound. How many pounds did he buy of each?

Perform the division in the following. Invert the divisor and cancel common factors. (**168.**)

24. $\frac{36}{79} \div \frac{12}{23}$.
25. $\frac{54}{65} \div \frac{18}{39}$.
26. $\frac{75}{88} \div \frac{15}{22}$.
27. $\frac{100}{861} \div \frac{50}{91}$.
28. $324 \div \frac{8}{9}$.
29. $573 \div \frac{7}{12}$.
30. $\frac{85}{734} \div \frac{35}{42}$.
31. $862 \div \frac{34}{57}$.
32. $\frac{100}{365} \div \frac{46}{125}$.
33. $573 \div \frac{25}{36}$.
34. $\frac{43}{86} \div \frac{100}{124}$.
35. $\frac{1000}{2800} \div \frac{100}{380}$.
36. $1000 \div \frac{100}{354}$.
37. $3000 \div \frac{200}{753}$.
38. $\frac{387}{569} \div \frac{23}{85}$.

294. PROB. III.—**To divide when the divisor or dividend is a mixed number, or both.**

1. Divide 48 by $4\frac{2}{3}$.

(1.) $48 \div 4\frac{2}{3} = 48 \div \frac{14}{3}$

(2.) $48 \div \frac{14}{3} = 48 \times \frac{3}{14} = 10\frac{2}{7}$

EXPLANATION.—1. We reduce the divisor $4\frac{2}{3}$ as shown in (1), to the improper fraction $\frac{14}{3}$.

2. We invert the divisor, as shown in (2) according to (**291**), and multiply the 48 by $\frac{3}{14}$ giving $10\frac{2}{7}$ as the quotient of 48, divided by $4\frac{2}{3}$.

2. Divide $8\frac{6}{7}$ by $3\frac{7}{8}$.

(1.) $8\frac{6}{7} \div 3\frac{7}{8} = \frac{62}{7} \div \frac{31}{8}$

(2.) $\frac{62}{7} \div \frac{31}{8} = \frac{\overset{2}{\cancel{62}}}{7} \times \frac{8}{\cancel{31}} = \frac{16}{7} = 2\frac{2}{7}$

EXPLANATION.—1. We reduce the dividend and divisor, as shown in (1), to improper fractions, giving $\frac{62}{7} \div \frac{31}{8}$.

2. We invert the divisor, $\frac{31}{8}$, as shown in (2) according to (**291**), and cancel 31 in the numerator 62 and denominator 31 (**186**), giving $\frac{16}{7}$, or $2\frac{2}{7}$. Hence, $8\frac{6}{7} \div 3\frac{7}{8} = 2\frac{2}{7}$.

From these illustrations we obtain the following

295. RULE.—*Reduce mixed numbers to improper fractions; then invert the divisor and proceed as in multiplication, canceling any factors that are common to any numerator and a denominator.*

WRITTEN EXAMPLES.

296. Perform and explain the division in the following:

1. $2\frac{8}{21} \div 4\frac{1}{6}$.
2. $7\frac{3}{5} \div 2\frac{4}{7}$.
3. $9\frac{7}{12} \div 5\frac{2}{3}$.
4. $89 \div 7\frac{4}{5}$.
5. $43\frac{27}{100} \div \frac{9}{1000}$.
6. $862 \div 42\frac{3}{5}$.
7. $100\frac{1}{10} \div 5\frac{3}{10}$.
8. $36\frac{5}{9} \div 8\frac{3}{10}$.
9. $732 \div 14\frac{2}{5}$.
10. $85\frac{7}{13} \div 23$.
11. $37\frac{7}{10} \div 6\frac{3}{10}$.
12. $7\frac{13}{100} \div 2\frac{3}{1000}$.
13. $1000\frac{1}{10} \div \frac{7}{100}$.
14. $936 \div 5\frac{3}{100}$.
15. $873 \div \frac{13}{27}$.
16. $5\frac{23}{100} \div 2\frac{19}{1000}$.
17. $362 \div \frac{37}{100}$.
18. $46\frac{8}{13} \div \frac{5}{39}$.
19. $\frac{2}{3}$ of $5\frac{1}{2} \div \frac{7}{8}$.
20. $\frac{4}{5}$ of $15\frac{5}{7} \div \frac{3}{4}$.
21. $\frac{7}{9}$ of $28 \div \frac{1}{2}$ of $\frac{4}{5}$.

22. If a bushel of wheat cost $\$1\frac{3}{5}$, how much can be bought for $\$12\frac{5}{9}$? For $\$28\frac{2}{3}$? For $\$273\frac{7}{8}$?

23. At $\$\frac{5}{7}$ for $\frac{3}{70}$ of an acre of land, what is the cost of 1 acre? Of $\frac{3}{10}$ of an acre? Of $5\frac{2}{3}$ of an acre? Of $29\frac{7}{9}$ of an acre?

24. A merchant expended $\$597\frac{3}{4}$ in buying cloth at $\$2\frac{2}{3}$ a yard. He afterwards sold the whole of it at $\$3\frac{2}{9}$ a yard. How much did he gain by the transaction?

COMPLEX FRACTIONS.

297. Certain results are obtained by dividing the numerator and denominator of a fraction by a number that is not an exact divisor of each, which are fractional in form, but are not fractions according to the *definition* of a fraction. These fractional forms are called *Complex Fractions.*

The following examples, which illustrate the three classes of complex fractions, should be carefully studied:

Ex. 1. Show that $\frac{8}{12}$ of a line is equal to $\frac{2\frac{2}{3}}{4}$ of the same line.

$$\frac{8 \div 3}{12 \div 3} = \frac{2\frac{2}{3}}{4}$$

$$\frac{\text{--- --- --}}{\text{--- --- --- ---}} = \frac{\text{--- --- --}}{\text{--- --- --- ---}}$$

EXPLANATION.—1. Dividing the numerator and denominator of $\frac{8}{12}$ by 3 makes every *3 parts* in each into *1 part*, as shown in the illustration, but does not change the value of the fraction (**235**—III).

2. The denominator or whole contains 4 of these parts, and the numerator 2 of them and $\frac{2}{3}$ of one of them, as will be seen by the illustration. Hence, $\frac{8}{12}$ of a line is equal to $\frac{2\frac{2}{3}}{4}$ of the same line.

Ex. 2. Show that $\frac{5}{12}$ of a line is expressed by $\frac{1}{2\frac{2}{5}}$.

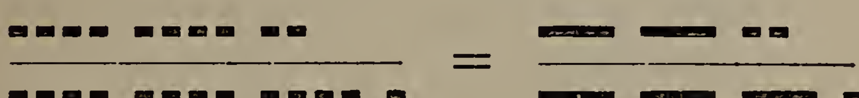
$$\frac{5 \div 5}{12 \div 5} = \frac{1}{2\frac{2}{5}}$$

EXPLANATION.—1. Dividing the numerator and denominator of $\frac{5}{12}$ by 5 makes every *5 parts* in each into *1 part*, as shown in the illustration.

2. The denominator or whole contains 2 of these parts and $\frac{2}{5}$ of one of them, and the numerator contains 1 part, as shown in the illustration. Hence, $\frac{5}{12}$ of a line is represented by the fractional expression $\frac{1}{2\frac{2}{5}}$.

Ex. 3. To show that $\frac{10}{13}$ of a line is expressed by $\frac{2\frac{1}{2}}{3\frac{1}{4}}$.

$$\frac{10 \div 4}{13 \div 4} = \frac{2\frac{1}{2}}{3\frac{1}{4}}$$



EXPLANATION.—1. Dividing the numerator and denominator of $\frac{10}{13}$ by 4 makes every *4 parts* in each into *1 part*, as shown in the illustration.

2. The denominator or whole contains 3 of these parts and $\frac{1}{4}$ of one of them, and the numerator contains 2 of them and $\frac{2}{4}$ or $\frac{1}{2}$ of one of them. Hence, $\frac{10}{13}$ of a line is represented by the fractional expression $\frac{2\frac{1}{2}}{3\frac{1}{4}}$.

From these illustrations we have the following definitions:

298. A ***Complex Fraction*** is an expression in the form of a fraction, having a fraction in its numerator or denominator, or in both; thus, $\frac{\frac{3}{5}}{7}$, $\frac{4}{5\frac{2}{3}}$, $\frac{2\frac{1}{2}}{6\frac{4}{5}}$.

299. A ***Simple Fraction*** is a fraction having a whole number for its numerator and for its denominator.

PROBLEMS IN COMPLEX FRACTIONS.

300. Prob. I.—**To reduce a complex fraction to a simple fraction.**

Reduce $\frac{4\frac{2}{3}}{7\frac{3}{4}}$ to a simple fraction.

$$\frac{4\frac{2}{3}}{7\frac{3}{4}} = \frac{4\frac{2}{3} \times 12}{7\frac{3}{4} \times 12} = \frac{56}{93}$$

Explanation.—1. We find the least common multiple of the denominators of the partial fractions $\frac{2}{3}$ and $\frac{3}{4}$, which is 12.

2. Multiplying both terms of the complex fraction by 12 (**235**—II), which is divisible by the denominators of the partial fractions, $\frac{2}{3}$ and $\frac{3}{4}$, reduces each term to a whole number. $4\frac{2}{3} \times 12 = 56$; $7\frac{3}{4} \times 12 = 93$. Therefore $\frac{4\frac{2}{3}}{7\frac{3}{4}}$ is equal to the simple fraction $\frac{56}{93}$. Hence the following

301. Rule.—*Multiply both terms of the complex fraction by the least common multiple of all the denominators of the partial fractions.*

302. The three classes of complex fractions are forms of expressing three cases of division; thus,

(1.) $\frac{5\frac{2}{3}}{7} = 5\frac{2}{3} \div 7.$ A mixed number divided by an integer.

(2.) $\frac{32}{9\frac{5}{6}} = 32 \div 9\frac{5}{6}.$ An integer divided by a mixed number.

(3.) $\frac{8\frac{3}{5}}{2\frac{2}{3}} = 8\frac{3}{5} \div 2\frac{2}{3}.$ A mixed number divided by a mixed number.

Hence, when we reduce a complex fraction to a simple fraction, as directed (**301**), we in fact reduce the dividend and divisor to a common denominator, and reject the denominator by indicating the division of the numerator of the dividend by the numerator of the divisor; thus, (1.) $\dfrac{5\frac{3}{4}}{2\frac{2}{3}} = \dfrac{5\frac{3}{4} \times 12}{2\frac{2}{3} \times 12} = \dfrac{69}{32}$, according to (**300**).

(2.) $5\frac{3}{4} \div 2\frac{2}{3} = \frac{23}{4} \div \frac{8}{3}$, and $\frac{23}{4} \div \frac{8}{3} = \frac{69}{12} \div \frac{32}{12} = \frac{69}{32}$, the same result as obtained by the method of multiplying by the least common multiple of the denominators of the partial fractions.

EXAMPLES FOR PRACTICE.

303. Reduce to simple fractions, and explain as above:

1. $\dfrac{8\frac{2}{3}}{9\frac{3}{5}}$.
2. $\dfrac{13\frac{4}{7}}{16\frac{3}{4}}$.
3. $\dfrac{15\frac{3}{10}}{23\frac{43}{100}}$.
4. $\dfrac{13\frac{5}{12}}{4\frac{5}{16}}$.
5. $\dfrac{32\frac{7}{8}}{54\frac{11}{12}}$.
6. $\dfrac{10\frac{3}{100}}{5\frac{7}{10}}$.

When the numerator or denominator contains two or more terms connected by a sign, perform the operation indicated by the sign first, then reduce to a simple fraction.

Reduce the following to simple fractions:

7. $\dfrac{3\frac{1}{2} + 2\frac{4}{5}}{8\frac{2}{3} + 5\frac{3}{4}}$.
8. $\dfrac{4\frac{7}{8} - 2\frac{1}{4}}{6\frac{3}{7} - 1\frac{3}{5}}$.
9. $\dfrac{(8\frac{4}{5} - \frac{3}{4}) \times \frac{2}{5}}{(5 \times \frac{7}{8}) + (\frac{4}{5} \text{ of } \frac{2}{3})}$.
10. $\dfrac{(\frac{3}{7} \text{ of } \frac{2}{3}) - \frac{3}{14}}{(2\frac{3}{5} \text{ of } 2) - \frac{7}{10}}$.
11. $\dfrac{(\frac{3}{10} \text{ of } 9) + (\frac{4}{5} \text{ of } 2)}{\frac{3}{8} \text{ of } 5}$.
12. $\dfrac{5\frac{3}{100} - \frac{7}{25}}{1000}$.

304. Prob. II.—**To reduce a fraction to any given denominator.**

1. *Examples where the denominator of the required fraction is a factor of the denominator of the given fraction.*

Reduce $\frac{17}{24}$ to a fraction whose denominator is 8.

$$\frac{17}{24} = \frac{17 \div 3}{24 \div 3} = \frac{5\frac{2}{3}}{8}$$

Explanation.—We observe that 8, the denominator of the required fraction, is a factor of 24, the denominator of the given fraction. Hence, dividing both terms of $\frac{17}{24}$ by 3, the other factor of 24, the fraction is reduced (**235**—III) to $\dfrac{5\frac{2}{3}}{8}$, a fraction whose denominator is 8.

2. *Examples where the denominator of the required fraction is not a factor of the denominator of the given fraction.*

Reduce $\frac{8}{13}$ to a fraction whose denominator is 10.

$$(1.)\quad \frac{8}{13} = \frac{8 \times 10}{13 \times 10} = \frac{80}{130}$$

$$(2.)\quad \frac{80}{130} = \frac{80 \div 13}{130 \div 13} = \frac{6\frac{2}{13}}{10}$$

EXPLANATION.—1. We introduce the given denominator 10 as a factor into the denominator of $\frac{8}{13}$ by multiplying, as shown in (1) both terms of the fraction by 10 (**235**—II).

2. The denominator 130 now contains the factors 13 and 10. Hence, dividing both terms of the fraction $\frac{80}{130}$ by 13 (**235**—III), as shown in (2), the result is $\frac{6\frac{2}{13}}{10}$, a fraction whose denominator is 10.

From these examples we obtain the following:

305. RULE.—*Multiply both terms of the fraction by the given denominator, and then divide them by the denominator of the fraction.*

Observe that when the given denominator is a factor or multiple of the denominator of the fraction, it is not necessary to multiply by it, as will be seen in the first example above.

EXAMPLES FOR PRACTICE.

306. 1. Reduce $\frac{6}{9}$, $\frac{8}{12}$, $\frac{13}{15}$, $\frac{19}{24}$, $\frac{32}{36}$, and $\frac{40}{48}$ each to *thirds.*

2. How many sevenths in $\frac{17}{21}$? In $\frac{39}{42}$? In $\frac{61}{63}$?

3. Reduce $\frac{2}{3}$, $\frac{5}{9}$, $\frac{3}{5}$, $\frac{7}{12}$, $\frac{3}{8}$, and $\frac{5}{6}$ each to *sevenths.*

4. In $\frac{4}{5}\frac{3}{4}$ how many twentieths? How many ninths?

5. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{10}$, $\frac{4}{5}$, and $\frac{9}{20}$ to hundredths.

6. How many tenths in $\frac{2}{3}$? In $\frac{7}{8}$? In $\frac{5}{12}$? In $\frac{37}{39}$?

7. Express as hundredths $\frac{5}{7}$, $\frac{8}{9}$, $\frac{7}{12}$, $\frac{27}{45}$, and $\frac{327}{462}$.

8. How many thousandths in $\frac{5}{8}$? In $\frac{19}{25}$? In $\frac{59}{125}$? In $\frac{87}{93}$?

9. Reduce to hundredths $\frac{273}{456}$; $\frac{43}{264}$; $\frac{390}{507}$; $\frac{406}{703}$.

10. How many hundredths in $1\frac{2}{3}$? In $4\frac{1}{2}$? In $7\frac{3}{4}$? In $9\frac{4}{5}$? In 17? In $\frac{4}{5}$? In $\frac{7}{10}$? In $2\frac{3}{10}$?

11. Reduce to hundredths $\frac{2\frac{3}{5}}{7}$; $\frac{4}{5\frac{2}{3}}$; $\frac{9\frac{4}{7}}{8}$; $\frac{7}{3\frac{2}{9}}$.

REVIEW EXAMPLES.

307. 1. How many *thirtieths* in $\frac{5}{6}$, and why? In $\frac{2}{5}$?

2. Reduce $\frac{3}{7}$, $\frac{5}{14}$, $\frac{3}{4\frac{2}{3}}$, $\frac{9}{56}$, and $\frac{27}{84}$ each to twenty-eighths.

3. Reduce to a common denominator $\frac{5}{6\frac{2}{3}}$, $\frac{3\frac{1}{2}}{8}$, and $\frac{7}{32}$.

4. State the reason why $\frac{5}{9 \div 4} = \frac{5 \times 4}{9}$ (**269**).

5. Reduce $\frac{4}{5}$ to a fraction whose numerator is 12; is 20; is 2; is 3; is 7 (**235**).

6. Reduce to a common numerator $\frac{3}{5}$ and $\frac{5}{7}$ (**252**).

7. Find the sum of $\frac{8}{9}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{11}{18}$.

8. Find the value of $(\frac{5}{7} \text{ of } \frac{4}{15} - \frac{1}{14}) \div \left(\frac{3}{4} + \frac{2}{3\frac{1}{2}}\right)$.

9. If $\frac{3}{8}$ of an estate is worth \$3460, what is $\frac{4}{7}$ of it worth?

10. \$4 is what part of \$8? Of \$12? Of \$32? Of \$48?

Write the solution of this example, with *reason* for each step.

11. If a man can do a piece of work in 150 days, what part of it can he do in 5 days? In 15 days? In 25 days? In $7\frac{1}{2}$ days? In $3\frac{3}{4}$ days? In $12\frac{1}{2}$ days?

12. A's farm contains 120 acres and B's 280; what part of B's farm is A's? *Ans.* $\frac{3}{7}$.

13. 42 is $\frac{6}{7}$ of what number?

Write the solution of this example, with *reason* for each step.

14. \$897 is $\frac{5}{9}$ of how many dollars?

15. $\frac{4}{7}$ of 76 tons of coal is $\frac{8}{10}$ of how many tons?

16. A piece of cloth containing 73 yards is $\frac{3}{5}$ of another piece. How many yards in the latter?

17. Bought a horse for \$286, and sold him for $\frac{7}{9}$ of what he cost; how much did I lose?

18. 84 is $\frac{5}{12}$ of 8 times what number?

Write the solution of this example, with *reason* for each step.

19. A has $694 in a bank, which is $\frac{4}{5}$ of 3 times the amount B has in the same bank; what is B's money?

20. Two men are $86\frac{3}{4}$ miles apart; when they meet, one has traveled $8\frac{7}{8}$ miles more than the other; how far has each traveled?

21. If $\frac{7}{12}$ of a farm is valued at $$4732\frac{5}{8}$, what is the value of the whole farm?

22. The less of two numbers is $432\frac{5}{9}$, and their difference $123\frac{7}{12}$. Find the greater number.

23. A man owning $\frac{4}{9}$ of a saw-mill, sold $\frac{3}{5}$ of his share for $2800; what was the value of the mill?

24. What number diminished by $\frac{5}{7}$ and $\frac{2}{9}$ of itself leaves a remainder of 32? *Ans.* 504.

25. I put $\frac{4}{9}$ of my money in the bank and gave $\frac{3}{5}$ of what I had left to a friend, and had still remaining $400. How much had I at first? *Ans.* $1800.

26. Sold 342 bushels of wheat at $$1\frac{5}{7}$ a bushel, and expended the amount received in buying wood at $$4\frac{3}{4}$ a cord. How many cords of wood did I purchase? *Ans.* $123\frac{3}{7}$ cords.

27. If $\frac{2}{3}$ of 4 pounds of tea cost $$2\frac{1}{2}$, how many pounds of tea can be bought for $$7\frac{4}{5}$? For $$12\frac{8}{9}$? For $$\frac{7}{12}$?

28. If 5 be added to both terms of the fraction $\frac{3}{7}$, how much will its value be changed, and why?

29. I exchanged $47\frac{3}{8}$ bushels of corn, at $$\frac{5}{7}$ per bushel, for $24\frac{2}{3}$ bushels of wheat; how much did the wheat cost a bushel?

30. A can do a piece of work in 5 days, B can do the same work in 7 days; in what time can both together do it?

31. Bought $\frac{2}{7}$ of $84\frac{4}{5}$ acres of land for $\frac{4}{9}$ of $$3584\frac{7}{8}$; what was the price per acre?

32. A boy while fishing lost $\frac{2}{3}$ of his line; he then added 8 feet, which was $\frac{4}{5}$ of what he lost; what was the length of the line at first? *Ans.* 15 feet.

33. A merchant bought a quantity of cloth for $$2849\frac{4}{5}$, and sold it for $\frac{7}{10}$ of what it cost him, thereby losing $$\frac{5}{7}$ a yard. How many yards did he purchase, and at what price per yard?

34. A tailor having $276\frac{2}{3}$ yards of cloth, sold $\frac{2}{9}$ of it at one time and $\frac{3}{7}$ at another; what is the value of the remainder at \$3 a yard?

35. A man sold $\frac{5}{12}$ of his farm at one time, $\frac{2}{7}$ at another, and the remainder for \$180 at \$45 an acre; how many acres were there in the farm?

36. A merchant owning $\frac{12}{25}$ of a ship, sells $\frac{1}{3}$ of his share to B, and $\frac{3}{8}$ of the remainder to C for $\$600\frac{3}{4}$; what is the value of the ship?

REVIEW AND TEST QUESTIONS.

308. 1. Define Fractional Unit, Numerator, Denominator, Improper Fraction, Reduction, Lowest Terms, Simple Fraction, Common Denominator, and Complex Fraction.

2. What is meant by the *unit* of a fraction? Illustrate by an example.

3. When may $\frac{1}{3}$ be greater than $\frac{1}{2}$? $\frac{1}{6}$ than $\frac{1}{4}$?

4. State the three principles of Reduction of Fractions, and illustrate each by lines.

5. Illustrate with lines or objects each of the following propositions:

I. To diminish the numerator, the denominator remaining the same, diminishes the value of the fraction.

II. To increase the denominator, the numerator remaining the same, diminishes the value of the fraction.

III. To increase the numerator, the denominator remaining the same, increases the value of the fraction.

IV. To diminish the denominator, the numerator remaining the same, increases the value of the fraction.

6. What is meant by the Least Common Denominator?

7. When the denominators of the given fractions are prime to each other, how is the Least Common Denominator found, and why?

8. State the five problems in reduction of fractions, and illustrate each by the use of lines or objects.

9. Show that multiplying the denominator of a fraction by any number divides the fraction by that number (**269**).

10. Show by the use of lines or objects the truth of the following:

(1.) $\frac{1}{3}$ of 2 equals $\frac{2}{3}$ of 1.
(2.) $\frac{3}{5}$ of 1 equals $\frac{1}{5}$ of 3.
(3.) $\frac{1}{6}$ of 5 equals $\frac{5}{6}$ of 1.
(4.) $\frac{4}{7}$ of 9 equals 4 times $\frac{1}{7}$ of 9.

11. To give to another person $\frac{3}{5}$ of 14 silver dollars, how many of the dollar-pieces must you change, and what is the largest denomination of change you can use?

12. Show by the use of objects that the quotient of 1 divided by a fraction is the given fraction inverted.

13. Why is it impossible to perform the operation in $\frac{4}{5} + \frac{2}{3}$, or in $\frac{5}{9} - \frac{3}{7}$, without reducing the fractions to a common denominator?

14. Why do we invert the divisor when dividing by a fraction? Illustrate your answer by an example.

15. What objection to calling $\frac{5}{7\frac{2}{3}}$ a fraction (**226**)?

16. State, and illustrate with lines or objects, each of the three classes of *so-called* Complex Fractions.

17. Which is the greater fraction, $\frac{8}{9}$ or $\frac{37}{40}$, and how much?

18. To compare the value of two or more fractions, what must be done with them, and why?

19. Compare $\frac{3\frac{1}{2}}{7}$ and $\frac{4}{8}$; $\frac{2\frac{4}{5}}{9}$ and $\frac{3}{10}$; $\frac{5\frac{2}{3}}{7\frac{4}{5}}$ and $\frac{2\frac{3}{5}}{3\frac{1}{2}}$, and show in each case which is the greater fraction, and how much?

20. State the rule for working each of the following examples:

(1.) $3\frac{2}{3} + 4\frac{5}{7} + 8\frac{2}{9}$.
(2.) $(7\frac{3}{4} + 5\frac{1}{2}) - (8 - 2\frac{1}{3})$.
(3.) $5 \times \frac{4}{9}$ of $\frac{7}{8}$ of 27.
(4.) $8\frac{3}{7} \times 5\frac{2}{3}$.
(5.) $\frac{7}{9} \times \frac{5}{8}$. Explain by objects.
(6.) $\frac{5}{12} \div \frac{7}{9}$. Explain by objects.

21. Illustrate by an example the application of Cancellation in multiplication and division of fractions.

DECIMAL FRACTIONS

DEFINITIONS.

309. A ***unit*** is separated into ***decimal parts*** when it is divided into *tenths;* thus,

UNIT. = DECIMAL PARTS.

310. A ***Decimal Fractional Unit*** is one of the *decimal parts* of anything.

311. By making a *whole* or *unit* into decimal parts, and one of these parts into decimal parts, and so on, we obtain a series of distinct orders of *decimal fractional units,* each $\frac{1}{10}$ of the preceding, having as denominators, respectively, 10, 100, 1000, and so on.

Thus, separating a whole into decimal parts, we have, according to (**246**), $1 = \frac{10}{10}$; making $\frac{1}{10}$ into decimal parts, we have, according to (**252**), $\frac{1}{10} = \frac{10}{100}$; in the same manner, $\frac{1}{100} = \frac{10}{1000}$, $\frac{1}{1000} = \frac{10}{10000}$, and so on. Hence, in the series of fractional units, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and so on, each unit is *one-tenth* of the preceding unit.

312. A ***Decimal Fraction*** is a fraction whose denominator is 10, 100, 1000, etc., or 1 with any number of ciphers annexed. Thus, $\frac{3}{10}$, $\frac{7}{100}$, $\frac{43}{1000}$, are decimal fractions.

313. The ***Decimal Sign*** (.), called the *decimal point,* is used to express a decimal fraction without writing the denominator, and to distinguish it from an integer.

NOTATION AND NUMERATION.

314. Prop. I.—*A decimal fraction is expressed without writing the denominator by using the decimal point, and placing the numerator at the right of the period.*

Thus, $\frac{7}{10}$ is expressed .7; $\frac{35}{100}$ is expressed .35.

Observe that the number of figures at the right of the period is always the same as the number of ciphers in the denominator; hence, the denominator is indicated, although not written.

Thus, in .54 there are two figures at the right of the period; hence we know that the denominator contains two ciphers and that $.54 = \frac{54}{100}$.

Express the following decimal fractions without writing the denominators:

1. $\frac{9}{10}$.	4. $\frac{25}{100}$.	7. $\frac{375}{1000}$.	10. $\frac{53}{100}$.
2. $\frac{5}{10}$.	5. $\frac{83}{100}$.	8. $\frac{632}{1000}$.	11. $\frac{705}{1000}$.
3. $\frac{1}{10}$.	6. $\frac{97}{100}$.	9. $\frac{486}{1000}$.	12. $\frac{3086}{10000}$.

315. A ***decimal fraction*** expressed without writing the denominator is called simply a ***Decimal.***

Thus, we speak of .79 as the *decimal* seventy-nine, yet we mean the *decimal fraction* seventy-nine hundredths.

316. Prop. II.—*Ciphers at the left of significant figures do not increase or diminish the number expressed by these figures.*

Thus, 0034 is thirty-four, the same as if written 34 without the two ciphers.

From this it will be seen that the number of figures in the numerator of decimal fractions can, without changing the fraction, be made equal to the number of ciphers in the denominator by writing ciphers at its left; thus, $\frac{7}{1000} = \frac{007}{1000}$.

Hence, $\frac{7}{1000}$ is expressed by using the decimal point and two ciphers thus, .007.

Observe, that while the *number* of parts in the numerator is not changed by *prefixing* the *two ciphers*, yet the 7 is moved to the third place on the right of the decimal point. Hence, according to (**314**), the denominator is indicated.

317. Prop. III.—*When the fraction in a mixed number is expressed decimally, it is written after the integer, with the decimal point between them.*

Thus, 57 and .09 are written 57.09; 8 and .0034 are written 8.0034.

Express as *one number* each of the following:

1. 9 and .7.	4. 43 and .1.	7. 703 and .02.
2. 32 and .04.	5. 7 and .03.	8. 560 and .008.
3. 73 and .507.	6. 3 and .064.	9. 900 and .062.

From these illustrations we obtain the following rule for writing decimals:

318. Rule.—*Write the numerator of the given decimal fraction. Make the number of figures written equal to the number of ciphers in the denominator by prefixing ciphers. Place at the left the decimal point.*

EXERCISE FOR PRACTICE.

319. Express the following decimal fractions without writing the denominator:

1. $\frac{5}{100}$.	7. $\frac{342}{1000}$.	13. $\frac{705}{100000}$.
2. $\frac{9}{100}$.	8. $\frac{43}{1000}$.	14. $\frac{3}{100000}$.
3. $\frac{38}{100}$.	9. $\frac{5}{1000}$.	15. $\frac{1001}{100000}$.
4. $\frac{6}{100}$.	10. $\frac{76}{10000}$.	16. $\frac{38604}{1000000}$.
5. $\frac{3}{100}$.	11. $\frac{8}{10000}$.	17. $\frac{7}{1000000}$.
6. $\frac{9}{100}$.	12. $\frac{306}{10000}$.	18. $\frac{501}{1000000}$.

19. Three hundred seven hundred-thousandths.
20. Nine thousand thirty-four millionths.
21. Seventy-five ten-millionths.
22. Eight thousand sixty-three hundred-millionths.

Express the following by writing the denominator; thus, $.032 = \frac{32}{1000}$.

23. .073.
24. .0026.
25. .36093.
26. .0000302.
27. .005062.
28. .000009.
29. .00904.
30. .063.
31. .00005.

320. Prop. IV.—*Every figure in the numerator of a decimal fraction represents a distinct order of decimal units.*

Thus, $\frac{537}{1000}$ is equal $\frac{500}{1000} + \frac{30}{1000} + \frac{7}{1000}$. But, according to (**255**), $\frac{500}{1000} = \frac{5}{10}$, and $\frac{30}{1000} = \frac{3}{100}$. Hence, 5, 3, and 7 each represent a distinct order of decimal fractional units, and $\frac{537}{1000}$, or .537 may be read 5 *tenths* 3 *hundredths* and 7 *thousandths.*

Analyze the following; thus, $.0709 = \frac{7}{100} + \frac{9}{10000}$.

1. .036.
2. .907.
3. .0025.
4. .01007.
5. .00063.
6. .04095.
7. .0300601.
8. .0003092.
9. .0070405.

321. Prop. V.—*A decimal is read correctly by reading it as if it were an integer and giving the name of the right-hand order.*

Thus, $.975 = \frac{900}{1000} + \frac{70}{1000} + \frac{5}{1000}$. Hence is read, *nine hundred seventy-five thousandths.*

1. Observe that when there are ciphers at the left of the decimal, according to (**316**), they are not regarded in reading the number; thus, .062 is read sixty-two *thousandths.*

2. The name of the lowest order is found, according to (**314**), by prefixing 1 to as many ciphers as there are figures in the decimal. For example, in .00209 there are five figures; hence the denominator is 1 with five ciphers; thus, 100000, read hundred-thousandths.

From these illustrations we obtain the following

322. Rule.—*Read the decimal as a whole number; then pronounce the name of the lowest or right-hand order.*

Read the following:

1. .003.
2. .00502.
3. .3097.
4. .06409.
5. .00006.
6. .30009.
7. .8306.
8. .00007.
9. .0052.
10. .000304.
11. .030972.
12. .000053.
13. .010302.
14. .0070409.
15. .00000503.
16. .00100049.
17. .0000007.
18. .00030659.

19. What is the denominator of .000309? How is it found? What is the numerator, and how read?

20. What effect have the ciphers in .0083?

21. Write a rule for expressing a *decimal* by writing its denominator.

323. The relation of the orders of units in an integer and decimal will be seen from the following table:

DECIMAL NUMERATION TABLE.

Billions.	Hundred-millions.	Ten-millions.	**Millions.**	Hundred-thousands.	Ten-thousands.	**Thousands.**	Hundreds.	Tens.	**Units.**	DECIMAL POINT.	Tenths.	Hundredths.	**Thousandths.**	Ten-thousandths.	Hundred-thousandths.	**Millionths.**	Ten-millionths.	Hundred-millionths.	**Billionths.**
5	5	5	5	5	5	5	5	5	5	.	5	5	5	5	5	5	5	5	5

ORDERS OF INTEGERS. ORDERS OF DECIMAL FRACTIONS.

Observe carefully the following:

1. The ***Unit*** is the standard in both cases. The integral orders are *multiples* of ***one unit,*** and the decimal orders are *decimal fractions* of ***one unit.***

2. Figures that are equally distant from the units' place on the right or left, have corresponding names; thus, *tenths* correspond to *tens, hundredths* to *hundreds,* and so on.

3. In reading an integer and decimal together, "*and*" should not be used anywhere but between the integer and fraction.

Thus, 9582.643 should be read, nine thousand five hundred eighty-two *and* six hundred forty-three thousandths.

4. *Dimes, cents,* and *mills* being respectively *tenths, hundredths,* and *thousandths* of a dollar, are written as a decimal. Thus, \$.347 is 3 *dimes,* 4 *cents,* and 7 *mills.* In reading dimes, cents, and mills, the dimes are read as cents. Thus, \$62.538 is read, 62 dollars, 53 cents, 8 mills.

EXAMPLES FOR PRACTICE.

324. Read the following:

1. \$384.57.	7. 10010.30102.	13. 7.00007.
2. \$700.903.	8. 90307.00806.	14. 10.1.
3. \$302.08.	9. 6001.0001.	15. 100.0001.
4. \$90.097.	10. 50000.000005.	16. 35.00035.
5. \$100.01.	11. 3070.00101.	17. 8.30005.
6. \$7.809.	12. \$9005.009.	18. 10.000001.

Express the following without writing the denominator.

19. $\$39\frac{5}{100}$.	22. $307\frac{6}{1000000}$.	25. $10001\frac{1}{1000}$.
20. $10\frac{1001}{10000}$.	23. $80\frac{32}{10000}$.	26. $7030\frac{201}{10000}$.
21. $73\frac{21}{10000}$.	24. $9\frac{704}{10000000}$.	27. $100\frac{1}{1000000}$.

28. Write with figures: Seventy-three thousandths; four hundred five millionths; eight ten-thousandths.

29. Three thousand nine hundred-millionths; ninety-one millionths; six hundred four thousand three billionths.

30. Eighty-four *and* seven ten-thousandths; nine thousand six *and* five hundred seven ten-millionths; six *and* three millionths.

31. Four thousand thirty-seven *and* nine hundred seven billionths; one million one *and* one thousand one ten-millionths.

REDUCTION.

PREPARATORY PROPOSITIONS.

The following preparatory propositions should be *very carefully* studied.

325. PROP. I.—*Annexing a cipher or multiplying a number by 10 introduces into the number the two prime factors 2 and 5.*

Thus, 10 being equal 2×5, 7×10 or $70=7\times(2\times5)$. Hence a number must contain 2 and 5 as a factor at least as many times as there are ciphers annexed.

326. PROP. II.—*A fraction in its lowest terms, whose denominator contains no other prime factors than 2 or 5, can be reduced to a simple decimal.*

Observe that every cipher annexed to the numerator and denominator makes each divisible once by 2 and 5 (**325**). Hence, if the denominator of the given fraction contains no other factors except 2 and 5, by annexing ciphers the numerator can be made divisible by the denominator, and the fraction reduced to a *decimal*.

Thus, $\frac{7}{8}=\frac{7000}{8000}$ (**235**—II). Dividing both terms of the fraction by 8 (**235**—III), we have $\frac{7000}{8000}=\frac{875}{1000}=.875$.

Reduce to decimals and explain as above:

1. $\frac{1}{2}$.	4. $\frac{7}{20}$.	7. $\frac{3}{5}$.	10. $\frac{17}{50}$.	13. $\frac{49}{320}$.
2. $\frac{3}{4}$.	5. $\frac{11}{40}$.	8. $\frac{7}{25}$.	11. $\frac{5}{16}$.	14. $\frac{147}{250}$.
3. $\frac{5}{8}$.	6. $\frac{9}{80}$.	9. $\frac{13}{125}$.	12. $\frac{23}{64}$.	15. $\frac{15}{64}$.

16. How many ciphers must be annexed to the numerator and denominator of $\frac{3}{4}$ to reduce it to a decimal?

17. Reduce $\frac{7}{8}$ to a decimal, and explain why the decimal must contain three places.

18. If reduced to a decimal, how many decimal places will $\frac{2}{5}$ make? Will $\frac{9}{25}$ make? Will $\frac{5}{16}$ make, and why?

327. Prop. III.—*A fraction in its lowest terms, whose denominator contains any other prime factors than 2 or 5 can be reduced only to a complex decimal.*

Observe that in this case annexing ciphers to the numerator and denominator, which (**325**) introduces only the factors 2 and 5, cannot make the numerator divisible by the given denominator, which contains other prime factors than 2 or 5.

Hence, a fraction will remain in the numerator, after dividing the numerator and denominator by the denominator of the given fraction, however far the division may be carried.

Thus, $\frac{11}{21} = \frac{11000}{21000}$ (**235**—II). Dividing both numerator and denominator by 21, we have $\frac{11000}{21000} = \frac{523\frac{17}{21}}{1000} = .523\frac{17}{21}$, a complex decimal.

Reduce and explain the following:

1. How many tenths in $\frac{2}{3}$? In $\frac{5}{7}$? In $\frac{4}{9}$? In $\frac{3}{4}$? In $\frac{5}{12}$?
2. Reduce to hundredths $\frac{8}{9}$; $\frac{6}{7}$; $\frac{5}{11}$; $\frac{7}{13}$; $\frac{8}{27}$; $\frac{20}{23}$.
3. How many thousandths in $\frac{1}{3}$? In $\frac{4}{9}$? In $\frac{4}{7}$? In $\frac{3}{43}$?

328. Prop. IV.—*The same set of figures must recur indefinitely in the same order in a complex decimal which cannot be reduced to a simple decimal.*

Thus, $\frac{7}{11} = \frac{70000}{110000} = \frac{6363\frac{7}{11}}{10000} = .6363\frac{7}{11}$.

Observe carefully the following:

1. In any division, the number of different remainders that can occur is 1 less than the number of units in the divisor.

Thus, if 5 is the divisor, 4 must be the greatest remainder we can have, and 4, 3, 2, and 1 are the only possible different remainders; hence, if the division is continued, any one of these remainders *may recur*.

2. Since in dividing the numerator by the denominator of the given fraction, each partial dividend is formed by annexing a cipher to the remainder of the previous division, when a

remainder recurs the partial dividend must again be the same as was used when this remainder occurred before; hence the same remainders and quotient figures must recur in the same order as at first.

3. If we stop the division at any point where the given numerator recurs as a remainder, we have the same fraction remaining in the numerator of the decimal as the fraction from which the decimal is derived.

Thus, $$\frac{7}{11}=\frac{700}{1100}=\frac{63\frac{7}{11}}{1000}=.63\tfrac{7}{11};$$

or $$\frac{7}{11}=\frac{70000}{110000}=\frac{6363\frac{7}{11}}{10000}=.6363\tfrac{7}{11}, \text{ and so on.}$$

329. PROP. V.—*The value of a fraction which can only be reduced to a complex decimal is expressed, nearly, as a simple decimal, by rejecting the fraction from the numerator.*

Thus, $\frac{3}{11}=\frac{27\frac{3}{11}}{100}$ (**327**). Rejecting the $\frac{3}{11}$ from the numerator, we have $\frac{27}{100}$, a simple fraction, which is only $\frac{3}{11}$ of $\frac{1}{100}$ smaller than the given fraction $\frac{3}{11}$ or $\frac{27\frac{3}{11}}{100}$.

Observe the following:

1. By taking a sufficient number of places in the decimal, the true value of a complex decimal can be expressed so nearly that what is rejected is of no consequence.

Thus, $\frac{3}{11}=\frac{27272727\frac{3}{11}}{100000000}$; rejecting the $\frac{3}{11}$ from the numerator, we have $\frac{27272727}{100000000}$, or .27272727, a simple decimal, which is only $\frac{3}{11}$ of 1 hundred-millionths smaller than the given fraction.

2. The *approximate value* of a complex decimal which is expressed by rejecting the given fraction from its numerator is called a *Circulating Decimal,* because the same figure or set of figures constantly recur.

330. Prop. VI.—*Diminishing the numerator and denominator by the same fractional part of each does not change the value of a fraction.*

Be particular to master the following, as the reduction of circulating decimals to common fractions depends upon this proposition.

1. The truth of the proposition may be shown thus:

$$\frac{9}{12}=\frac{9-\frac{1}{3}\text{ of }9}{12-\frac{1}{3}\text{ of }12}=\frac{9-3}{12-4}=\frac{6}{8}=\frac{3}{4}.$$

Observe that to diminish the numerator and denominator each by $\frac{1}{3}$ of itself is the same as multiplying each by $\frac{2}{3}$. But to multiply each by $\frac{2}{3}$, we multiply each by 2 (**235**—II), and then divide each by 3 (**235**—III), which does not change the value of the fraction; hence the truth of the proposition.

2. From this proposition it follows that the value of a fraction is not changed by subtracting 1 from the denominator and the fraction itself from the numerator.

Thus, $\frac{3}{5}=\frac{3-\frac{3}{5}}{5-1}=\frac{2\frac{2}{5}}{4}$. Observe that 1 is the $\frac{1}{5}$ of the denominator 5, and $\frac{3}{5}$ is $\frac{1}{5}$ of the numerator 3; hence, the numerator and denominator being each diminished by the same fractional part, the value of the fraction is not changed.

DEFINITIONS.

331. A ***Simple Decimal*** is a decimal whose numerator is a whole number; thus, $\frac{93}{100}$ or .93.

Simple decimals are also called *Finite Decimals*.

332. A ***Complex Decimal*** is a decimal whose numerator is a mixed number; as $\frac{26\frac{2}{3}}{100}$ or $.26\frac{2}{3}$.

There are two classes of *complex decimals :*

1. Those whose value can be expressed as a simple decimal (**326**), as $.23\frac{1}{2} = .235$; $.32\frac{3}{4} = .3275$.

2. Those whose value cannot be expressed as a simple decimal (**327**), as $.53\frac{1}{3} = .53333$ and so on, leaving, however far we may carry the decimal places, $\frac{1}{3}$ of 1 of the lowest order unexpressed. See (**328**).

333. A ***Circulating Decimal*** is an *approximate value* for a complex decimal which *cannot be reduced* to a simple decimal.

Thus, $.\dot{6}6\dot{6}$ is an approximate value for $.666\frac{2}{3}$ (**329**).

334. A ***Repetend*** is the figure or set of figures that are repeated in a circulating decimal.

335. A ***Circulating Decimal is expressed*** by writing the repetend once. When the repetend consists of one figure, a point is placed over it; when of more than one figure, points are placed over the first and last figures; thus, .333 and so on, and .592592 + are written $.\dot{3}$ and $.\dot{5}9\dot{2}$.

336. A ***Pure Circulating Decimal*** is one which commences with a repetend, as $.\dot{8}$ or $.\dot{3}9\dot{4}$.

337. A ***Mixed Circulating Decimal*** is one in which the repetend is preceded by one or more decimal places, called the *finite part* of the decimal, as $.7\dot{3}$ or $.004\dot{7}2\dot{5}$, in which .7 or .004 is the finite part.

ILLUSTRATION OF PROCESS.

338. PROB. I.—**To reduce a common fraction to a decimal.**

Reduce $\frac{3}{8}$ to a decimal.

$$\frac{3}{8} = \frac{3000}{8000} = \frac{375}{1000} = .375$$

EXPLANATION.—1. We annex the same number of ciphers to both terms of the fraction (**235**—Prin. II), and divide the resulting terms by 8, the significant figure in the denominator which must give a decimal denominator. Hence, $\frac{3}{8}$ expressed decimally is .375.

2. In case annexing ciphers does not make the numerator divisible (**327**) by the significant figures in the denominator, the number of places in the decimal can be extended indefinitely.

In practice, we abbreviate the work by annexing the ciphers to the numerator only, and dividing by the denominator of the given fraction, pointing off as many decimal places in the result as there were ciphers annexed. Hence the following

339. RULE.—*I. Annex ciphers to the numerator and divide by the denominator.*

II. Point off as many places in the result as there are ciphers annexed.

EXERCISE FOR PRACTICE.

340. Reduce to simple decimals:

1. $\frac{7}{16}$.	3. $\frac{27}{32}$.	5. $\frac{89}{125}$.	7. $\frac{313}{625}$.
2. $\frac{39}{65}$.	4. $\frac{9}{25}$.	6. $\frac{37}{80}$.	8. $\frac{17}{320}$.

Reduce to a complex decimal of four decimal places:

9. $\frac{5}{14}$.	11. $\frac{13}{70}$.	13. $\frac{3}{85}$.	15. $\frac{37}{193}$.
10. $\frac{11}{26}$.	12. $\frac{23}{65}$.	14. $\frac{8}{91}$.	16. $\frac{43}{251}$.

Find the repetend or approximate value of the following:

17. $\frac{3}{7}$.	20. $\frac{11}{13}$.	23. $\frac{19}{22}$.	26. $8\frac{5}{7}$.
18. $\frac{13}{21}$.	21. $\frac{22}{39}$.	24. $\frac{20}{51}$.	27. $24\frac{9}{28}$.
19. $\frac{15}{28}$.	22. $\frac{17}{52}$.	25. $\frac{24}{55}$.	28. $32\frac{53}{130}$.

341. PROB. II.—**To reduce a simple decimal to a common fraction.**

Reduce .35 to a common fraction.

$$.35 = \frac{35}{100} = \frac{7}{20}$$

EXPLANATION.—We write the decimal with the denominator, and reduce the fraction (**255**) to its lowest terms; hence the following

342. RULE.—*Express the decimal by writing the denominator, then reduce the fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

343. Reduce to common fractions in their lowest terms:

1. .215.	4. .0054.	7. .00096.	10. .0625.
2. .840.	5. .0125.	8. .008025.	11. .00512.
3. .750.	6. .0064.	9. .00075.	12. .00832.

344. PROB. III.—**To find the true value of a pure circulating decimal.**

Find the true value of $.\dot{7}\dot{2}$.

$$.\dot{7}\dot{2} = \frac{\dot{7}\dot{2}}{100} = \frac{72}{100-1} = \frac{72}{99} = \frac{8}{11}$$

EXPLANATION.—In taking $.\dot{7}\dot{2}$ as the *approximate value* of a given fraction, we have subtracted the given fraction from its own numerator, as shown in (**329**—V). Hence, to find the true value of $\frac{\dot{7}\dot{2}}{100}$, we must, according to (**330**—VI, 2), subtract 1 from the denominator 100, which makes the denominator as many 9's as there are places in the repetend; hence the following

345. RULE.—*Write the figures in the repetend for the numerator of the fraction, and as many 9's as there are places in the repetend for the denominator, and reduce the fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

346. Find the true value of

1. $\dot{3}\dot{6}$.	4. $\dot{3}7\dot{2}$.	7. $\dot{1}8\dot{9}$.	10. $\dot{5}36\dot{8}$.
2. $\dot{7}\dot{8}$.	5. $\dot{8}5\dot{6}$.	8. $\dot{3}2\dot{4}$.	11. $\dot{2}71\dot{8}$.
3. $\dot{5}\dot{4}$.	6. $\dot{1}3\dot{5}$.	9. $\dot{8}3\dot{6}$.	12. $\dot{8}16\dot{3}$.

Find the true value as improper fractions of

13. $37.\dot{8}\dot{1}$.	16. $89.\dot{5}\dot{4}$.	19. $63.\dot{2}74\dot{5}$.
14. $9.\dot{1}0\dot{8}$.	17. $53.\dot{3}2\dot{4}$.	20. $29.\dot{1}88\dot{1}$.
15. $3.\dot{5}0\dot{4}$.	18. $23.\dot{7}5\dot{8}$.	21. $6.\dot{0}3\dot{6}$.

347. PROB. IV.—**To find the true value of a mixed circulating decimal.**

Find the true value of $.3\dot{1}\dot{8}$.

$$(1)\quad .3\dot{1}\dot{8} = .3\tfrac{18}{99} = \frac{3\frac{18}{99}}{10} = \frac{315}{990} = \frac{7}{22}$$

EXPLANATION.—1. We find, according to (**344**), the true value of the repetend $.0\dot{1}\dot{8}$, which is $.0\frac{18}{99}$. Annexing this to the .3, the finite part, we have $.3\frac{18}{99}$, the true value of $.3\dot{1}\dot{8}$ in the form of a *complex decimal.*

2. We reduce the complex decimal $.3\frac{18}{99}$, or $\frac{3\frac{18}{99}}{10}$, to a simple fraction by multiplying, according to (**300**), both terms of the fraction by 99, giving $\frac{3\frac{18}{99}}{10} = \frac{315}{990} = \frac{7}{22}$. Hence the true value of $.3\dot{1}\dot{8}$ is $\frac{7}{22}$.

$$(2)\quad \begin{array}{rl} .318 & \text{Given decimal.} \\ \underline{3} & \text{Finite part.} \\ 315 & \frac{315}{990} = \frac{7}{22}. \end{array}$$

ABBREVIATED SOLUTION.—Observe that in simplifying $\frac{3\frac{18}{99}}{10}$, we multiplied both terms by 99. Instead of multiplying the 3 by 99, we may multiply by 100 and subtract 3 from the product. Hence we add the 18 to 300, and subtract 3 from the result, which gives us the true numerator. Hence the following

348. RULE.—*I. Find the true value of the repetend, annex it to the finite part, and reduce the complex decimal thus formed to a simple fraction.*

To abbreviate the work:

II. From the given decimal subtract the finite part for a numerator, and for a denominator write as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part.

EXAMPLES FOR PRACTICE.

349. Find the true value of

1. $.7\dot{1}\dot{2}$.
2. $.9\dot{5}\dot{9}$.
3. $.4\dot{8}\dot{6}$.
4. $.04\dot{3}2\dot{8}$.
5. $.006\dot{4}\dot{1}$.
6. $.032\dot{8}\dot{7}$.
7. $.000035\dot{7}3\dot{9}$.
8. $.00830\dot{2}68\dot{5}$.
9. $.02073\dot{4}82\dot{7}$.

Find the true value, in the form of an improper fraction, of

10. $5.3\dot{2}\dot{8}$.	12. $2.4\dot{3}$.	14. $12.2\dot{2}\dot{7}$.
11. $9.7\dot{5}\dot{2}$.	13. $7.8\dot{6}$.	15. $5.3\dot{9}$.

ADDITION.

PREPARATORY PROPOSITION.

350. *Any two or more decimals can be reduced to a common denominator by annexing ciphers.*

Thus, $.7 = \frac{7}{10}$, and, according to (**235**—II), $\frac{7}{10} = \frac{70}{100} = \frac{700}{1000} = \frac{7000}{10000}$, and so on; therefore, $.7 = .70 = .700 = .7000$, Hence any two or more decimals can be changed at once to the same decimal denominator by annexing ciphers.

ILLUSTRATION OF PROCESS

351. Find the sum of 34.8, 6.037, and 27.62.

(1.)	(2.)
34.800	34.8
6.037	6.037
27.620	27.62
68.457	68.457

EXPLANATION.—1. We arrange the numbers so that units of the same order stand in the same column.

2. We reduce the decimals to a common denominator, as shown in (1), by annexing ciphers.

3. We add as in integers, placing the decimal point before the tenths in the sum.

In practice, the ciphers are omitted, as shown in (2), but the decimals are regarded as reduced to a common denominator.

Thus the 3 *hundredths* in the second number and the 2 *hundredths* in the third, when added are written, as shown in (2), as 50 thousandths; in the same manner, the 8 *tenths* and 6 *tenths* make 1400 *thousandths*, or 1 unit and 400 thousandths. The 1 unit is added to the units and the 4 written in the *tenths' place* as 400 *thousandths*.

From this it will be seen that the addition of decimals is subject to the same laws (**261**—I and II) and rule (**263**) as other fractions.

EXAMPLES FOR PRACTICE.

352. Find the sum of the following, and explain as above:

1. 38.9, 7.05, 59.82, 365.007, 93.096, and 8.504.
2. 9.07, 36.009, 84.9, 5.0036, 23.608, and .375.
3. $42.08, $9.70, $89.57, $396.02, and $.89.
4. .039, 73.5, .0407, 2.602, and 29.8.
5. 395.3, 4.0701, 9.96, and 83.0897.
6. 8.0093, .805, .03409, 7.69, and .0839.
7. $.87, $32.05, $9, $75.09, $.67, and $3.43.
8. .80003, 3.09, 13.36, 97.005, and .9999.

SUBTRACTION.

353. Find the difference between 83.7 and 45.392.

```
83.700
45.392
------
38.308
```

Explanation.—1. We arrange the numbers so that units of the same order stand in the same column.

2. We reduce the decimals, or regard them as reduced to a common denominator, and then subtract as in whole numbers.

The reason of this course is the same as given in addition. The ciphers are also usually omitted.

EXAMPLES FOR PRACTICE.

354. Subtract and explain the following:

1. 834.9 — 52.47.
2. 39.073 — 7.0285.
3. $67.09 — $29.83.
4. $95.02 — $78.37.
5. 83.003 — 45.879.
6. 379.000001 — 4.0396.
7. 54.5 — 37.00397.
8. 96.03 — 89.09005.
9. .09 — .0005903.
10. .7 — .099909.

11. A man paid out of \$3432.95 the following sums; \$342.06, \$593.738, \$729.039, \$1362.43, \$296.085, \$37.507. How much has he left? *Ans.* \$72.091.

12. In a mass of metal there are 183.741 pounds; $\frac{1}{3}$ of it is iron, 25.305 pounds are copper, and 3.0009 pounds are silver, and the balance lead. How much lead is there in the mass?

13. A druggist sold 74.52 pounds of a costly drug. He sold in March $10\frac{3}{5}$ pounds, in April 25.125, in May $21\frac{3}{8}$, and the balance in June. How many pounds did he sell in June?

Find the decimal value of

14. $(3\frac{1}{5} - 2\frac{3}{8}) + (7\frac{7}{25} - 1\frac{5}{8}) - (9.23 - 8.302)$.

15. $(\$85\frac{3}{4} - \$37\frac{3}{8}) + (\frac{2}{3} \text{ of } \$184.20 - \$\frac{5}{8})$.

16. $\$859.085 - (\$128\frac{3}{8} + \$\frac{7}{16}) + \$7\frac{3}{4}$.

MULTIPLICATION.

355. Multiply 3.27 by 8.3.

(1.) $$3.27 \times 8.3 = \frac{327}{100} \times \frac{83}{10}$$

(2.) $$\frac{327}{100} \times \frac{83}{10} = \frac{27141}{1000} = 27.141$$

Explanation.—1. Observe that 3.27 and 8.3 are mixed numbers; hence, according to (**282**), they are reduced before being multiplied to improper fractions, as shown in (1).

2. According to (**276**), $\frac{327}{100} \times \frac{83}{10}$, as shown in (2), equals 27.141. Hence 27.141 is the product of 3.27 and 8.3.

The work is abbreviated thus:

(3.)
```
  3.27
   8.3
 -----
   981
  2616
 -----
27.141
```

We observe, as shown in (2), that the product of 3.27 and 8.3 must contain as many decimal places as there are decimal places in both numbers. Hence we multiply the numbers as if integers, as shown in (3), and point off in the product as many decimal places as there are decimal places in both numbers. Hence the following

356. RULE.—*Multiply as in integers, and from the right of the product point off as many figures for decimals as there are decimal places in the multiplicand and multiplier.*

EXAMPLES FOR PRACTICE.

357. Multiply and explain the following:

1. 7.3×4.9.
2. $13.4 \times .37$.
3. 35.08×6.2.
4. $\$97.03 \times 42$.
5. $\$83.65 \times .7$.
6. 34.0007×8.43.
7. $73.406 \times .903$.
8. $.4903 \times .06$.
9. $.935 \times .008$.
10. $5.04 \times .072$.
11. $.009 \times .008$.
12. $.0007 \times .036$.
13. $.0405 \times .09$.
14. $.00101 \times .001$.
15. $.307 \times .005$.

Multiply and express the product decimally:

16. $\$35\frac{3}{5}$ by $9\frac{2}{3}$.
17. $3\frac{7}{9}$ by $6\frac{2}{5}$.
18. $\$.05\frac{3}{4}$ by $18\frac{1}{2}$.
19. $7\frac{3}{4}$ thousandths by $\frac{4}{5}$.
20. $12\frac{1}{8}$ by $3\frac{4}{5}$ hundredths.
21. $9\frac{7}{8}$ tenths by $.0003\frac{2}{5}$.

22. What is the value of 325.17 pounds of iron at $.023 per pound? *Ans.* 7.47891 dollars.

23. What would 12.34 acres of land cost at $43.21 per acre?

24. A merchant sold 86.43 tons of coal at $9.23 a ton, thereby gaining $112.12; what was the cost of the coal?

25. A French gramme is equal to 15.432 English grains; how many grains are $14\frac{4}{9}$ grammes equal to?

26. A metre is equal to 39.3708 inches; how many inches are there in 1.325 metres?

27. A merchant uses a yardstick which is .00538 of a yard too short; how many yards will he thus gain in selling 438 yards measured by this yardstick?

Find the value of the following:

28. $\$240.09 \times (2.3\frac{4}{5} - \frac{3}{4} \text{ of } 1\frac{2}{3})$.
29. $(\$375\frac{7}{8} - \$87.093) \times (\frac{4}{7} \text{ of } 36 - \frac{2}{3} \text{ of } 3\frac{1}{2})$.
30. $(\overline{\frac{5}{9} \text{ of } 12\frac{3}{5}} - \overline{.903\frac{3}{4} + 1.00\frac{5}{8}}) \times 375$.

31. A dealer in wood and hay bought 2395 tons of hay at \$14.75 a ton, and $2387\frac{1}{2}$ cords of wood at \$4.50 a cord; how much did he pay for all? *Ans.* \$46070.

32. Bought 18 books at \$1.37$\frac{1}{2}$ each, and sold them at a gain of .50$\frac{3}{4}$ cents each; what did I receive for the whole?

33. A boy went to a grocery with a \$10 bill, and bought $3\frac{1}{2}$ pounds of tea at \$.90 a pound, 7 pounds of flour at \$.07 a pound, and 4 pounds of butter at \$.35 a pound; how much change did he return to his father? *Ans.* \$4.96.

34. What would 15280 feet of lumber cost, at \$2.37$\frac{1}{2}$ for each 100 feet? *Ans.* \$362.90.

DIVISION.

PREPARATORY PROPOSITIONS.

358. PROP. I.—*When the divisor is greater than the dividend, the quotient expresses the part the dividend is of the divisor.*

Thus, $4 \div 6 = \frac{4}{6} = \frac{2}{3}$. The quotient $\frac{2}{3}$ expresses the part the 4 is of 6.

1. Observe that the process in examples of this kind consists in reducing the fraction formed by placing the divisor over the dividend to its lowest terms. Thus, $32 \div 56 = \frac{32}{56}$, which reduced to its lowest terms gives $\frac{4}{7}$.

2. In case the result is to be expressed decimally, the process then consists in reducing to a decimal, according to (**338**), the fraction formed by placing the dividend over the divisor. Thus, $5 \div 8 = \frac{5}{8}$, reduced to a decimal equals .625.

Divide the following, and express the quotient decimally. Explain the process in each case as above.

1. 3÷4.	4. 13÷40.	7. 5÷7.	10. 3÷20.
2. 7÷20.	5. 15÷32.	8. 8÷11.	11. 7÷88.
3. 5÷8.	6. 9÷80.	9. 5÷6.	12. 4÷13.

359. Prop. II.—*The fraction remaining after the division of one integer by another expresses the part the* REMAINDER *is of the divisor.*

Thus, $42 \div 11 = 3\frac{9}{11}$. The divisor 11 is contained 3 times in 42 and 9 left, which is 9 parts or $\frac{9}{11}$ of the divisor 11. Hence we say that the divisor 11 is contained $3\frac{9}{11}$ times in 42. We express the $\frac{9}{11}$ decimally by reducing it according to (**338**). Hence, $3\frac{9}{11} = 3.\dot{8}\dot{1}$.

Divide the following and express the remainder decimally, carrying the decimal to four places:

1. $324 \div 7$.	4. $89 \div 103$.	7. $5374 \div 183$.
2. $473 \div 23$.	5. $65 \div 17$.	8. $3000 \div 547$.
3. $783 \div 97$.	6. $37 \div 43$.	9. $1000 \div 101$.

360. Prop. III.—*Division is possible only when the dividend and divisor are both of the same denomination* (**155**—I).

For example, $\frac{3}{10} \div \frac{7}{100}$, or $.3 \div .07$ is impossible until the dividend and divisor are reduced to the same fractional denomination; thus, $.3 \div .07 = .30 \div .07 = 4\frac{2}{7} = 4.\dot{2}8571\dot{4}$.

ILLUSTRATION OF PROCESS.

361. Ex. 1. Divide .6 by .64.

(1.) $.6 \div .64 = .60 \div .64$

(2.) $60 \div 64 = \frac{60}{64} = .9375$

Explanation.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit or denomination (**290**).

2. We divide, according to (**290**), as shown in (2), the numerator 60 by the numerator 64, which gives $\frac{60}{64}$. Reducing $\frac{60}{64}$ to a decimal (**338**), we have $.6 \div .64 = .9375$.

Ex. 2. Divide .63 by .0022.

(1.) $.63 \div .0022 = .6300 \div .0022$

(2.) $6300 \div 22 = 286\frac{4}{11} = 386.\dot{3}\dot{6}$

EXPLANATION.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit by annexing ciphers to the dividend (**350**).

2. We divide, according to (**290**), as shown in (2), the numerator 6300 by the numerator 22, giving as a quotient $286\frac{4}{11}$.

3. We reduce, according to (**338**), the $\frac{4}{11}$ in the quotient to a decimal, giving the repetend $.\dot{3}\dot{6}$. Hence, $.63 \div .0022 = 286.\dot{3}\dot{6}$.

Ex. 3. Divide 16.821 by 2.7.

(1.) $16.821 \div 2.7 = 16.821 \div 2.700$

(2.) $16.821 \div 2.700 = \frac{16821}{1000} \div \frac{2700}{1000}$

(3.)
```
27|00 ) 168|21 ( 6.23
        162
        ---
         62
         54
         --
          81
          81
          --
```

EXPLANATION.—1. We reduce, as shown in (1), the dividend and divisor to the same decimal unit by annexing ciphers to the divisor (**350**).

2. The dividend and divisor each express thousandths as shown in (2). Hence we reject the denominators and divide as in integers (**290**).

3. Since there are ciphers at the right of the divisor, they may be cut off by cutting off the same number of figures at the right of the dividend (**142**). Dividing by 27, we find that it is contained 6 times in 168, with 6 remaining.

4. The 6 remaining, with the two figures cut off, make a remainder of 621 or $\frac{621}{2700}$. This is reduced to a decimal by dividing both terms by 27. Hence, as shown in (3), we continue dividing by 27 by taking down the two figures cut off.

The work is abbreviated thus:

We reduce the dividend and divisor to the same decimal unit by cutting off from the right of the dividend the figures that express lower decimal units than the divisor. We then divide as shown in (3), prefixing the *remainder* to the figures cut off and reducing the result to a decimal.

From these illustrations we obtain the following

362. Rule.—*Reduce the dividend and divisor to the same decimal unit; divide as in integers and reduce the fractional remainder in the quotient, if any, to a decimal.*

EXAMPLES FOR PRACTICE.

363. In the following examples carry the answer in each case to four decimal places:

1. Divide 53.28 by 3.12; by 7.3; by 9.034.
2. Divide $27\frac{5}{8}$ by 4.03; by .72; by $2.3\frac{3}{4}$.
3. Divide \$725.42 by \$.37; by \$3.08; by $\$.95\frac{3}{4}$.
4. Divide \$.93 by \$.847; $\$73.09\frac{4}{5}$ by $\$.75\frac{1}{2}$; $\$.37\frac{1}{2}$ by \$.74.

What is the value of

5. $\$75.83 \div \$100$.
6. $(\frac{3}{4} \text{ of } .73) \div .09$.
7. $734\frac{7}{8} \div 4.5\frac{3}{4}$.
8. $\$10000 \div \$.07$.
9. $8.345 \div 2.0007$.
10. $(8\frac{4}{5} + 12.07) \div (15.03 - \frac{7}{8})$.
11. $(\$354.07 - \frac{5}{7} \text{ of } \$10.84) \div \frac{2}{3} \text{ of } \$7.08$.
12. $(\overline{\frac{5}{9} \div .03} \times 64) \div (\frac{7}{8} \text{ of } \frac{2}{3} \text{ of } 12\frac{3}{5})$.
13. $(\$3.05\frac{2}{3} \div \frac{3}{7}) - (\frac{4}{5} \text{ of } \$1.08 \div \frac{2}{3})$.
14. $(\frac{5}{9} \text{ of } \$324.18 - \$\frac{7}{8}) \div \$2.0005$.
15. At \$2.32, how many yards of cloth can be bought for \$373.84?
16. The product of two numbers is 375.04 and one of them is 73.009; what is the other? *Ans.* 5.1369+.
17. How much tea can be bought for \$134.84, if $23\frac{3}{5}$ pounds cost \$17.70? *Ans.* 179.7866+ pounds.
18. A merchant received \$173.25, \$32.19, and \$89.13. He expended the whole in buying silk $\$1.37\frac{1}{2}$. How many yards of silk did he buy?
19. A farmer sold $132\frac{4}{5}$ bushels wheat at \$1.35 per bushel, and 184 bushels corn at $\$.73\frac{1}{2}$ per bushel. He bought coal with the amount received, at \$9.54 a ton. How many tons did he buy?

20. What decimal part of a farm worth \$3965 can be bought for \$1498.77? *Ans.* .378.

21. What is the value of $27\frac{5}{8}$ acres of land when .57 of an acre is worth \$48?

22. A merchant lost .47 of his capital, and had to use .13 more for family expenses, and had still remaining \$5380. What was his original capital? *Ans.* \$13450.

REVIEW EXAMPLES.

364. Answers involving decimals, unless otherwise stated, are carried to four decimal places.

What is the cost

1. Of $.7\frac{1}{2}$ of a pound of tea, if 7 pounds cost \$6.95.
2. Of 4.5 acres of land, if 100 acres cost \$7385.
3. Of $9\frac{3}{4}$ cords of wood, at \$12.60 for 2.8 cords.
4. Of 5384 feet lumber, at \$5.75 per 100 feet.
5. Of 13.25 yards of cloth, if 3.75 yards cost $\$9.93\frac{3}{4}$.
6. Of 31460 bricks, at \$8.95 per 1000 bricks.
7. Of $158\frac{1}{2}$ pounds butter, if 9.54 pounds cost \$3.239.

Reduce each of the following examples to decimals:

8. $\frac{11}{12}$.
9. $\frac{8}{9}$.
10. $\dfrac{2\frac{1}{2}}{5}$.
11. $\dfrac{3\frac{2}{3}}{7\frac{1}{2}}$.
12. $\frac{3}{7}$ of $1\frac{4}{5}$.
13. $5\frac{8}{11} - 5\frac{2}{9}$.
14. $\dfrac{(3\frac{2}{5}+\frac{1}{2})\times\frac{4}{5}}{8}$.
15. $\dfrac{\frac{2}{7}\text{ of }.3}{8\frac{3}{5} - 4.3}$.
16. $(1\frac{1}{3}+\frac{7}{8})$ of $\frac{4}{5}$.
17. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{23}$.
18. $\dfrac{(\frac{5}{9}-\frac{4}{15})\times 2\frac{3}{4}}{25}$.
19. $\dfrac{\frac{5}{7}\text{ of }(4\frac{2}{5}-\frac{1}{4})}{7.5\frac{1}{2}}$.

20. Four loads of hay weighed respectively 2583.07, $3007\frac{3}{7}$, $2567\frac{5}{8}$, and $3074\frac{11}{16}$ pounds; what was the total weight?

21. Seven car-loads of coal, each containing 13.75 tons, were sold at \$8.53 per ton. How much was received for the whole?

22. At \$1.75 per 100, what is the cost of 5384 oranges?

23. What is the cost of carrying 893850 pounds of corn from Chicago to New York, at \$.35$\frac{5}{8}$ per 100 pounds?

24. If freight from St. Louis to New York is \$.39$\frac{4}{5}$ per 100 pounds, what is the cost of transporting 3 boxes of goods, weighing respectively 783$\frac{2}{3}$, 325$\frac{3}{5}$, and 286$\frac{7}{8}$ pounds?

25. A piece of broadcloth cost \$195.38$\frac{1}{4}$, at \$3.27 per yard. How many yards does it contain?

26. A person having \$1142.49$\frac{3}{4}$ wishes to buy an equal number of bushels of wheat, corn, and oats; the wheat at \$1.37, the corn at \$.87$\frac{1}{2}$, and the oats at \$.35$\frac{3}{4}$. How many bushels of each can he buy?

27. Expended \$460.80 in purchasing silk, .3 of it at \$2.25 per yard, $\frac{1}{3}$ of it at \$1.86 per yard, and the balance at \$3.45 per yard. How many yards did I buy of each quality of silk?

28. What is the value of $\left(\frac{\frac{2}{3} \text{ of } 7 - 2\frac{3}{8}}{1\frac{3}{4} + \frac{5}{12}}\right) \div \frac{.36}{.48}$.

29. A produce dealer exchanged 48$\frac{3}{5}$ bushels oats at 39$\frac{3}{4}$ cts. per bushel, and 13$\frac{1}{2}$ barrels of apples at \$3.85 a barrel, for butter at 37$\frac{1}{2}$ cts. a pound; how many pounds of butter did he receive?

30. A grain merchant bought 1830 bushels of wheat at \$1.25 a bushel, 570 bushels corn at 73$\frac{1}{2}$ cts. a bushel, and 468 bushels oats at 35$\frac{3}{4}$ cts. a bushel. He sold the wheat at an advance of 17$\frac{1}{2}$ cts. a bushel, the corn at an advance of 9$\frac{3}{4}$ cts. a bushel, and the oats at a loss of 3 cts. a bushel. How much did he pay for the entire quantity, and what was his gain on the transaction?

31. A fruit merchant expended \$523.60 in purchasing apples at \$3.85 a barrel, which he afterwards sold at an advance of \$1.07 per barrel; what was his gain on the sale?

32. The cost of constructing a certain road was \$5050.50. There were 35 men employed upon it 78 days, and each man received the same amount per day; how much was the daily wages?

REVIEW AND TEST QUESTIONS.

365. 1. Define Decimal Unit, Decimal Fraction, Repetend, Circulating Decimal, Mixed Circulating Decimal, Finite Decimal, and Complex Decimal.

2. In how many ways may $\frac{3}{5}$ be expressed as a decimal fraction, and why?

3. What effect have ciphers written at the left of an integer? At the left of a decimal, and why in each case (**316**)?

4. Show that each figure in the numerator of a decimal represents a distinct order of decimal units (**320**).

5. How are *integral orders* and *decimal orders* each related to the *units* (**323**)? Illustrate your answer by lines or objects.

6. Why in reading a decimal is the lowest order the only one named? Illustrate by examples (**321**).

7. Give reasons for not regarding the ciphers at the left in reading the numerator of the decimal .000403.

8. Reduce $\frac{7}{8}$ to a decimal, and give a reason for each step in the process.

9. When expressed decimally, how many places must $\frac{13}{125}$ give, and why? How many must $\frac{5}{32}$ give, and why?

10. Illustrate by an example the reason why $\frac{17}{21}$ cannot be expressed as a *simple decimal* (**327**).

11. State what fractions can and what fractions cannot be expressed as simple decimals (**326** and **327**). Illustrate by examples.

12. In reducing $\frac{5}{7}$ to a complex decimal, why must the numerator 5 recur as a remainder (**328**—1 and 2)?

13. Show that, according to (**235**—II and III), the value of $\frac{15}{45}$ will not be changed if we diminish the numerator and denominator each by $\frac{2}{5}$ of itself.

14. Show that multiplying 9 by $1\frac{2}{3}$ increases the 9 by $\frac{2}{3}$ of itself.

15. Multiplying the numerator and denominator of $\frac{14}{35}$ each by $1\frac{3}{7}$ produces what change in the fraction, and why?

16. Show that in diminishing the *numerator* of $\frac{4}{5}$ by $\frac{4}{5}$ and the *denominator* by 1 we diminish each by the same part of itself.

17. In taking $.\dot{3}$ as the value of $\frac{1}{3}$, what fraction has been rejected from the numerator? What must be rejected from the denominator to make $.\dot{3} = \frac{1}{3}$, and why?

18. Show that the true value of $.\dot{8}\dot{1}$ is $\frac{81}{99}$. Give a reason for each step.

19. Explain the process of reducing a mixed circulating decimal to a fraction. Give a reason for each step.

20. How much is .33333 less than $\frac{1}{3}$, and why?

21. How much is .571428 less than $\frac{4}{7}$, and why?

22. Find the sum of .73, .0049, .089, 6.58, and 9.08703, and explain each step in the process (**261**—I and II).

23. If *tenths* are multiplied by *hundredths*, how many decimal places will there be in the product, and why (**355**)?

24. Show that a number is multiplied by 10 by moving the decimal point one place to the right; by 100 by moving it two places; by 1000 three places, and so on.

25. State a rule for *pointing off* the decimal places in the product of two decimals. Illustrate by an example, and give reasons for your rule.

26. Multiply 385.23 by .742, multiplying *first* by the 4 *hundredths*, then by the 7 *tenths*, and *last* by the 2 *thousandths.*

27. Why is the quotient of an integer divided by a *proper fraction* greater than the dividend?

28. Show that a number is divided by 10 by moving the decimal point one place to the left; by 100 by moving it two places; by 1000, three places; by 10000, four places, and so on.

29. Divide 4.9 by 1.305, and give a reason for each step in the process. Carry the decimal to three places.

30. Give a rule for division of decimals.

DENOMINATE NUMBERS

DEFINITIONS.

366. A ***Related Unit*** is a unit which has an invariable relation to one or more other units.

Thus, 1 foot = 12 inches, or $\frac{1}{3}$ of a yard; hence, 1 foot has an invariable relation to the units *inch* and *yard*, and is therefore a *related unit.*

367. A ***Denominate Number*** is a concrete number (**15**) whose unit (**14**) is a *related unit.*

Thus, 17 yards is a denominate number, because its unit, *yard*, has an invariable relation to the units *foot* and *inch*, 1 yard making always 3 feet or 36 inches.

368. A ***Denominate Fraction*** is a fraction of a *related unit.*

Thus, $\frac{3}{4}$ of a yard is a denominate fraction.

369. The ***Orders*** of related units are called ***Denominations.***

Thus, *yards, feet*, and *inches* are denominations of length; *dollars, dimes*, and *cents* are denominations of money.

370. A ***Compound Number*** consists of several numbers expressing *related denominations*, written together in the order of the relation of their units, and read as one number.

Thus, 23 yd. 2 ft. 9 in. is a compound number.

371. A ***Standard Unit*** is a unit established by law or custom, from which other units of the same kind are derived.

Thus, the standard unit of measures of extension is the yard. By dividing the yard into 3 equal parts, we obtain the unit *foot;* into 36 equal parts, we obtain the unit *inch;* multiplying it by $5\frac{1}{2}$, we obtain the unit *rod,* and so on.

372. Related units may be classified into *six kinds:*

1. Extension.	3. Weight.	5. Angles or Arcs.
2. Capacity.	4. Time.	6. Money or Value.

373. ***Reduction of Denominate Numbers*** is the process of changing their denomination without altering their value.

UNITS OF WEIGHT.

374. The ***Troy*** pound of the mint is the ***Standard Unit*** of weight.

TROY WEIGHT.

TABLE OF UNITS.

24 *gr.*	=	1 *pwt.*
20 *pwt.*	=	1 *oz.*
12 *oz.*	=	1 *lb.*
3.2 *gr.*	=	1 *carat.*

1. ***Denominations.*** — Grains (*gr.*), Pennyweights (*pwt.*), Ounces (*oz.*), Pounds (*lb.*), and Carats.

2. ***Equivalents.***—1 lb. = 12 oz. = 240 pwt. = 5760 gr.

3. ***Use.***—Used in weighing gold, silver, and precious stones, and in philosophical experiments.

AVOIRDUPOIS WEIGHT.

TABLE OF UNITS.

16 *oz.*	=	1 *lb.*
100 *lb.*	=	1 *cwt.*
20 *cwt.*	=	1 *T.*

1. ***Denominations.*** — Ounces (*oz.*), pounds (*lb*), hundredweights (*cwt.*), tons (*T.*).

2. ***Equivalents.***—1 Ton = 20 cwt. = 2000 lb. = 32000 oz.

3. ***Use.***—Used in weighing groceries, drugs at wholesale, and all coarse and heavy articles.

4. In the United States Custom House, and in wholesale transactions in coal and iron, 1 quarter = 28 lbs., 1 cwt. = 112 lb., 1 T. = 2240 lb. This is usually called the *Long Ton* table.

APOTHECARIES' WEIGHT.

TABLE OF UNITS.

20 *gr.* = 1 *sc.* or ℈.
3 ℈ = 1 *dr.* or ʒ.
8 ʒ = 1 *oz.* or ℥
12 *oz.* = 1 *lb.*

1. ***Denominations.*** — Grains (*gr.*), Scruples (℈), Drams (ʒ), Ounces (℥), Pounds (*lb.*)

2. ***Equivalents.*** — *lb.* 1 = ℥ 12 = ʒ 96 = ℈ 288 = gr. 5760.

3. ***Use.***—Used in medical prescriptions.

4. Medical prescriptions are usually written in Roman notation. The number is written after the symbol, and the final "i" is always written j. Thus, ℥ vij is 7 ounces.

Comparative Table of Units of Weigh

		TROY.		AVOIRDUPOIS.		APOTHECARIES.
1 pound	=	5760 grains	=	7000 grains	=	5760 grains.
1 ounce	=	480 "	=	437.5 "	=	480 "

Table of Avoirdupois Pounds in a Bushel, as Established by Law in the States named.

	Cal.	Conn.	Del.	Ill.	Ind.	Ia.	Ky.	La.	Me.	Mass.	Mich.	Minn.	Mo.	N. H.	N. J.	N. Y.	O.	Or.	Penna.	Vt.	Wash. T.	Wis.	N. C.
Wheat..............	60	56	60	60	60	60	60	60		60	60	60	60		60	60	60	60	60	60	60	60	60
Indian Corn........	52	56	56	52	56	50	56	56		56	56	56	52		56	58	56	56	56	56	56	56	54
Oats...	32	28		32	32	35	33⅓	32	30	30	32	32	35	30	30	32	32	34	32	32	36	32	
Barley.............	50			48	48	48	48	32		46	48	48	48		48	48	48	46	47	46	45	48	48
Buckwheat	40	45		40	50	52	52			46	42	42	52		50	48		42	48	46	42	42	50
Rye................	54	56		54	56	56	56	32		56	56	56	56		56	56	56	56	56	56	56	56	
Clover Seed........				60	60	60	60				60	60	60		64	60	60	60			60	60	
Timothy Seed......				45	45	45	45						45			44						46	

Peas, Beans, and Potatoes are usually weighed 60 lb. to the bushel.

Table of Special Denominations.

100 lb. of Grain or Flour	= 1 Cental.		200 lb. Pork or Beef	= 1 Barrel.
100 " of Dry Fish	= 1 Quintal.		196 " Flour	= 1 Barrel.
100 " of Nails	= 1 Keg.		240 " Lime	= 1 Cask.

280 lb. salt at N. Y. Salt Works = 1 Barrel.

PROBLEMS ON RELATED UNITS.

375. PROB. I.—**To reduce a denominate or a compound number to a lower denomination.**

Reduce 23 lb. 7 oz. 9 pwt. to pennyweights.

```
 23 lb. 7 oz. 9 pwt.
 12
 283 oz.
  20
5669 pwt.
```

SOLUTION.—1. Since 12 oz. make 1 lb., in any number of pounds there are 12 times as many ounces as pounds. Hence we multiply the 23 lb. by 12, and add the 7 oz., giving 283 oz.

2. Again, since 20 pwt. make 1 oz., in any number of ounces there are 20 times as many pennyweights as ounces. Hence we multiply the 283 oz. by 20, and add the 9 pwt., giving 5669 pwt.

376. RULE.—*I. Multiply the number of the highest denomination given, by the number of units of the next lower denomination that make 1 of the higher, and to the product add the number given of the lower denomination.*

II. Proceed in this manner with each successive denomination obtained, until the number is reduced to the required denomination.

EXAMPLES FOR PRACTICE.

377. Reduce and explain orally the following:

1. How many drams in 2 lb. 7 oz.? In 5 lb. 10 oz. 4 dr.?
2. How many grains in 5 dr. 2 sc.? In 1 oz. 4 dr. 1 sc.?
3. How many pennyweights in 2 lb. 6 oz.? In 4 oz. 7 pwt.?
4. How many pounds in 4 T. 132 lb.? In 7 T. 19 lb.?

Reduce

5. 13 T. 17 cwt. to pounds.
6. 19 lb. 7 oz. 5 dr. to drams.
7. 3 lb. 9 ℥ 5 ʒ to grains.
8. 17 lb. 11 ℥ 2 ℈ to grains.
9. 27 lb. 5 oz. 17 pwt. to grains.
10. 173 T. 5 cwt. 47 lb. to pounds.

11. 23 lb. 11 oz. 9 pwt. to pennyweights.
12. 87 T. 13 cwt. 93 lb. to pounds.
13. In 37 lb. 8 oz. 15 pwt. 19 gr. how many grains?
14. Reduce 184 T. to hundredweights; to pounds; to ounces.
15. How many grains in 1 pound Apothecaries' weight? In 1 pound Troy weight? In 1 pound Avoirdupois weight?
16. Reduce 164 lb. 17 pwt. to pennyweights.

378. Prob. II.—**To reduce a denominate number to a compound or a higher denominate number.**

Reduce 7487 sc. to a compound number.

```
3 ) 7 4 8 7 sc.
8 ) 2 4 9 5 dr. + 2 sc.
1 2 ) 3 1 1 oz. + 7 dr.
        2 5 lb. 11 oz.
```

Solution.—Since 3 sc. make 1 dr., 7487 sc. must make as many drams as 3 is contained times in 7487, or 2495 dr. + 2 sc.

2. Since 8 dr. make 1 oz., 2495 dr. must make as many ounces as 8 is contained times in 2495, or 311 oz. + 7 dr.

3. Since 12 oz. make 1 lb., 311 oz. must make as many pounds as 12 is contained times in 311, or 25 lb. + 11 oz. Hence, 7487 sc. are equal to the compound number 25 lb. 11 oz. 7 dr. 2 sc. Hence the following

379. Rule.—*I. Divide the given number by the number of units of the given denomination that make one of the next higher denomination.*

II. In the same manner divide this quotient and each successive quotient, omitting the remainders, until the denomination required is reached. The last quotient, with the remainders annexed, is the required result.

EXAMPLES FOR PRACTICE.

380. Reduce and explain orally:

1. How many pounds Troy in 24 oz.? In 30 oz.? In 79 oz.? In 400 pwt.? In 1200 pwt.? In 114 oz.?
2. How many hundredweights in 1384 lb.? In 1975 lb.?
3. How many tons in 4800 lb.? In 7259 lb.?

4. In 240 sc. how many drams? How many ounces?

Reduce

5. 38964 gr. to pounds Troy.
6. 876445 lbs. to tons.
7. ʒ 97634 to pounds.
8. 503640 oz. to tons.
9. 279647 gr. to pounds Apoth.
10. 3468 cwt. to tons.
11. 8597 pwt. to pounds.
12. 93875 gr. to drams.
13. 873604 oz. to tons.
14. 534278 gr. to pounds Troy.

15. In 93645 gr. how many pounds Troy? How many Avoirdupois? How many Apothecaries'?

16. In 9 lb. Troy how many pounds Avoirdupois?

17. Reduce 57 lb. 13 oz. Avoir. to Troy weight.

18. Reduce 14 lb. 7 oz. Avoir. to Apothecary weight.

19. Reduce 5 lb. 10 oz. 17 pwt. to Apothecary weight.

381. Prob. III.—**To reduce a denominate fraction or decimal to integers of lower denominations.**

Reduce $\frac{5}{7}$ of a ton to lower denominations.

(1.) $\frac{5}{7}$ T. $= \frac{5}{7}$ of 20 cwt. $= \frac{5}{7} \times 20 = 14$ cwt. $+ \frac{2}{7}$ cwt.
(2.) $\frac{2}{7}$ cwt. $= \frac{2}{7}$ of 100 lb. $= \frac{2}{7} \times 100 = 28$ lb. $+ \frac{4}{7}$ lb.
(3.) $\frac{4}{7}$ lb. $= \frac{4}{7}$ of 16 oz. $= \frac{4}{7} \times 16 = 9\frac{1}{7}$ oz.

Solution.—Since 20 cwt. is equal 1 T., $\frac{5}{7}$ of 20 cwt., or $14\frac{2}{7}$ cwt., equals $\frac{5}{7}$ of 1 T. Hence, to reduce the $\frac{5}{7}$ of a ton to hundredweights, we take $\frac{5}{7}$ of 20 cwt., or multiply, as shown in (1), the $\frac{5}{7}$ by 20, the number of hundredweights in a ton.

In the same manner we reduce the $\frac{2}{7}$ cwt. remaining to pounds, as shown in (2), and the $\frac{4}{7}$ lb. remaining to ounces, as shown in (3). Hence the following

382. Rule.—*I. Multiply the given fraction or decimal by the number of units in the next lower denomination, and reduce the result to a mixed number, if any.*

II. Proceed in the same manner with the fractional part of each successive product.

III. The integral parts of the several products, with the fraction, if any, in the last product, arranged in proper order, is the required result.

EXAMPLES FOR PRACTICE.

383. Find the value in lower denominations:

1. Of $\frac{5}{9}$ of a ton.
2. Of $\frac{7}{10}$ of a dram.
3. Of $\frac{8}{11}$ of a pound Troy.
4. Of $\frac{5}{7}$ of a pound Apoth.
5. Of .6 of a pound Avoir.
6. Of .85 of a ton.
7. Of .94 of a dram.
8. Of .73 of an ounce Troy.
9. Of $\frac{7}{12}$ of a hundredweight.
10. Of $\frac{5}{8}$ of a quarter.
11. Of $7\frac{4}{9}$ pounds Troy.
12. Of 3.7 hundredweights.
13. Of 5.94 pounds Apoth.
14. Of $13\frac{5}{7}$ tons.
15. Of .7364 of a pound Troy.
16. Of .9356 of a ton.
17. Reduce .84 of a hundredweight to Troy weight.
18. In $\frac{3}{8}$ of a pound Avoir. how much Troy weight?
19. How much will $\frac{5}{7}$ of a cwt. make expressed in Troy weight? Expressed in Apothecary weight?

384. PROB. IV.—**To reduce a denominate fraction or decimal of a lower to a fraction or decimal of a higher denomination.**

Reduce $\frac{3}{5}$ of a dram to a fraction of a pound.

$$(1.)\quad \tfrac{3}{5}\text{ dr.} = \tfrac{1}{8}\text{ oz.} \times \tfrac{3}{5} = \tfrac{3}{40}\text{ oz.}$$
$$(2.)\quad \tfrac{3}{40}\text{ oz.} = \tfrac{1}{12}\text{ lb.} \times \tfrac{3}{40} = \tfrac{1}{160}\text{ lb.}$$

SOLUTION.—1. Since 8 drams = 1 ounce, 1 dram is equal $\frac{1}{8}$ of an oz., and $\frac{3}{5}$ of a dram is equal $\frac{3}{5}$ of $\frac{1}{8}$ oz. Hence, as shown in (1), $\frac{3}{5}$ dr. = $\frac{3}{40}$ oz.

2. Since 12 ounces = 1 pound, 1 ounce is equal $\frac{1}{12}$ of a pound, and, as shown in (2), $\frac{3}{40}$ of an ounce is equal $\frac{3}{40}$ of $\frac{1}{12}$ lb., or $\frac{1}{160}$ lb. Hence, $\frac{3}{5}$ dr. = $\frac{1}{160}$ lb. Hence the following

385. RULE.—*I. Find the part which a unit of the given denomination is of a unit of the next higher denomination, and multiply this fraction by the given fraction or decimal.*

II. Proceed in the same manner with the result, and each successive result, until reduced to the denomination required. Reduce the result to its lowest terms or to a decimal.

EXAMPLES FOR PRACTICE

386. Reduce and explain orally:

1. $\frac{2}{3}$ dr. to a fraction of a pound.
2. $\frac{4}{5}$ sc. to a fraction of a pound.
3. .7 oz. to a fraction of a pound Troy.
4. .8 lb. to a fraction of a ton.
5. .5 pwt. to a fraction of a pound.
6. $\frac{7}{8}$ lb. to a fraction of a hundredweight.
7. Reduce $\frac{5}{9}$ pwt., $\frac{7}{12}$ gr., $\frac{18}{23}$ oz., .32 oz., .74 pwt., and .64 gr. each to the fraction of a pound Troy.
8. Reduce $\frac{6}{7}$ of a scruple to a fraction of a pound.

$$\frac{\overset{2}{\cancel{6}}}{7}\times\frac{1}{\cancel{3}}\times\frac{1}{\underset{4}{\cancel{8}}}\times\frac{1}{12}=\frac{1}{336}\text{ lb.}$$

Explanation.—Since we divide the given fraction by the numbers in the ascending scale successively (**384**) between the given and the required denomination, we may arrange them as shown in the margin, and cancel.

9. Reduce .3 oz., .84 lb., and $\frac{4}{7}$ cwt. each to the fraction of a ton.
10. What fraction of a pound is $\frac{5}{9}$ of a dram? .8 of a sc.?
11. Reduce to a fraction of a pound Troy .42 gr.; .96 pwt.
12. $\frac{3}{4}$ of an ounce is what fraction of 1 lb.? of 1 cwt.?
13. Reduce to the fraction of a ton $\frac{5}{8}$ cwt.; .9 cwt.; $\frac{4}{5}$ lb.; .34 cwt.; .86 lb.; .16 oz.; $\frac{3}{4}$ oz.

387. Prob. V.—**To reduce a compound number to a fraction of a higher denomination.**

Reduce ℥4 ʒ6 ℈2 to a fraction of 1 pound.

(1.) ℥4 ʒ6 ℈2 = ℈116; lb. 1 = ℈288.

(2.) $\frac{116}{288}=\frac{29}{72}$; hence, ℥4 ʒ6 ℈2 = lb. $\frac{29}{72}$.

Solution.—1. Two numbers can be compared only when they are the same denomination. Hence we reduce, as shown in (1), the ℥4 ʒ6 ℈2 and the lb. 1 to scruples, the lowest denomination mentioned in either number.

2. ℥4 ʒ6 ℈2 being equal ℈116, and lb. 1 being equal ℈288, ℥4 ʒ6 ℈2 is the same part of lb. 1 as ℈116 is of ℈288, which is $\frac{116}{288}$, or $\frac{29}{72}$. Hence ℥4 ʒ6 ℈2 = lb. $\frac{29}{72}$, or $.00402\dot{7}$.

388. RULE.—*Reduce the given number to its lowest denomination for the numerator of the required fraction, and a unit of the required denomination to the same denomination for the denominator, and reduce the fraction to its lowest terms or to a decimal.*

EXAMPLES FOR PRACTICE.

389. 1. Reduce 5 cwt. 4 lb. to a fraction of one ton.

2. What fraction of a pound Troy are 7 oz. 12 pwt. 8 gr.?

3. What fraction of a hundredweight are 64 lb. 12 oz.?

Reduce to the fraction of a pound:

4. 10 oz. 8 pwt. 16 gr.
5. 9 oz. 5 dr. 2 sc.
6. 6 dr. 1 sc. 18 gr.
7. 5 oz. 4 dr. 2 sc. 20 gr.
8. 4 oz. 18 pwt. 20 gr.
9. 11 oz. 19 pwt. 23 gr.

10. Reduce to the fraction of a ton 8 cwt. 64 lb.

11. What part of 6 lb. Troy are 2 lb. 8 oz. 16 pwt.?

12. What part of 4 cwt. are 1 cwt. 25 lb.? 2 cwt. 50 lb.?

13. Reduce 8 oz. 16 pwt. to the decimal of 6 pounds.

14. Reduce 8 cwt. 60 lb. to the decimal of 1 ton; of 8 tons.

15. Reduce 8 cwt. 3 qr. 16 lb. to the decimal of a ton.

```
25 ) 16.00 lb.
 4 )  3.64 qr.
20 )  8.91 cwt.
      .4455 T.
```

ABBREVIATED SOLUTION.—Since the 16 pounds are reduced to a decimal of a quarter by reducing $\frac{16}{25}$ to a decimal, we annex two ciphers to the 16, as shown in the margin, and divide by 25, giving .64 qr.

To this result we prefix the 3 quarters, giving 3.64 qr., which is equivalent to $\frac{3.64}{4}$ hundredweights; hence we divide by 4, as shown in the margin, giving .91 cwt.

To the result we again prefix the 8 cwt., giving 8.91, which is equivalent to $\frac{8.91}{20}$ of a ton, equal .4455 T. Hence, 8 cwt. 3 qr. 16 lb. = .4455 T.

16. Reduce 8 oz. 6 dr. 2 sc. to the decimal of a pound.
17. What decimal of 24 lb. Troy is 2 lb. 8 oz. 16 pwt. ?
18. 9 oz. 16 pwt. 12 gr. are what decimal of a pound ?
19. Reduce 12 cwt. 2 qr. 18 lb. to the decimal of a ton.
20. What decimal of a pound are ℥ 9 ʒ 5 ℈ 2 gr. 18. ?
21. Reduce 11 oz. 16 pwt. 20 gr. to the decimal of a pound.
22. Reduce 7 lb. 5 oz. Avoir. to a decimal of 12 lb. 5 oz. 3 pwt. Troy.
23. 1 lb. 9 oz. 8 pwt. is what part of 3 lb. Apoth. weight. ?

390. PROB. VI.—**To find the sum of two or more denominate or compound numbers, or of two or more denominate fractions.**

1. Find the sum of 7 cwt. 84 lb. 14 oz., 5 cwt. 97 lb. 8 oz., and 2 cwt. 9 lb. 15 oz.

cwt.	lb.	oz.
7	84	14
5	97	8
2	9	15
15	92	5

SOLUTION.—1. We write numbers of the same denomination under each other, as shown in the margin.

2. We add as in Simple Numbers, commencing with the lowest denomination. Thus, 15, 8, and 14 ounces make 37 ounces, equal 2 lb. 5 oz. We write the 5 oz. under the ounces and add the 2 lb. to the pounds.

We proceed in the same manner with each denomination until the entire sum, 15 cwt. 92 lb. 5 oz., is found.

2. Find the sum of $\frac{4}{9}$ lb., $\frac{4}{5}$ dr., and $\frac{3}{4}$ sc.

	oz.	dr.	sc.	gr.
$\frac{4}{9}$ lb. =	5	2	2	0
$\frac{4}{5}$ dr. =			2	8
$\frac{3}{4}$ sc. =				15
	5	3	2	3

SOLUTION.—1. According to (**261**), only fractional units of the same kind and of the same whole can be added; hence we reduce $\frac{4}{9}$ lb., $\frac{4}{5}$ dr., and $\frac{3}{4}$ sc. to integers of lower denominations (**381**), and then add the results, as shown in the margin. Or,

2. The given fractions may be reduced to fractions of the same denomination (**384**), and the results added according to (**261**), and the value of the sum expressed in integers of lower denominations according to (**381**).

EXAMPLES FOR PRACTICE.

391. 1. Add 13 T. 18 cwt. 2 qr. 19 lb. 13 oz., 15 cwt. 3 qr. 20 lb., 32 T. 19 cwt., 17 T. 15 cwt. 14 oz., and 8 T. 12 cwt. 13 lb. 15 oz.

2. Add 13 lb. 7 oz. 5 dr. 2 sc., 9 oz. 7 dr. 15 gr., 7 lb. 9 oz. 7 dr., 11 oz. 5 dr. 2 sc. 19 gr., and 9 lb. 10 oz. 6 dr. 1 sc. 18 gr.

3. Find the sum of $\frac{5}{9}$ lb., $\frac{3}{4}$ pwt., $\frac{2}{3}$ oz., and $\frac{5}{7}$ lb.

4. Find the sum of .7 oz., .5 pwt., .75 lb., .45 oz., .9 pwt.

5. What is the sum of $4\frac{4}{5}$ cwt., $39\frac{7}{8}$ lb., and $14\frac{3}{4}$ oz.

6. Find the sum of 3.75 T., 7.9 cwt., and $19\frac{5}{8}$ lb.

7. Find the sum of 13.45 lb., 8.75 oz., and 3.7 dr.

8. A trader bought three lots of sugar, the first containing 10 cwt. 3 qr. 17 lb., the second 11 cwt. 3 qr. 27 lb., and the third 26 cwt. 2 qr. 12 lb.; how much did he buy?

9. A wholesale produce dealer bought 3 T. 9 cwt. 15 lb. 4 oz. of butter during the spring, 1 T. 12 cwt. 18 lb. 6 oz. during the summer, 2 T. 7 cwt. 10 lb. 6 oz. during the autumn, and 3 T. 2 cwt. 98 lb. 15 oz. during the winter; how much did he buy during the entire year?

10. A manufacturing company bought 4 bars of silver, weighing respectively 11 lb. 8 oz. 10 pwt. 23 gr.; 10 lb. 8 oz. 15 pwt. 10 gr.; 9 lb. 11 oz. 9 pwt. 11 gr.; and 4 lb. 9 oz. 16 pwt. 22 gr.; what was their united weight?

11. If a druggist sells in prescriptions lb. 4 ℥4 ʒ6 ℈2 of a certain drug in January, lb. 8 ℥7 ʒ7 ℈2 in February, lb. 10 ℥10 ʒ2 ℈1 in March, lb. 9 ℥1 ʒ2 ℈6 in April, and lb. 7 ℥9 ʒ3 ℈2 in May, how much was sold during the five months? *Ans.* lb. 40 ℥10 ℈1.

12. What is the sum of 8.7 lb. 3.34 oz. and $4\frac{5}{6}$ pwt.?

13. Find the sum of .8 cwt. and .5 oz.

14. A grocer sold 4 lots of tea containing respectively 7 cwt. 39 lb. 13 oz., 5 cwt. 84 lb. 15 oz., 13 cwt. 93 lb. 7 oz., and 7 cwt. 74 lb. 11 oz.; what was the entire weight of the tea sold?

392. PROB. VII.—**To find the difference between any two denominate or compound numbers, or denominate fractions.**

Find the difference between 27 lb. 7 oz. 15 pwt. and 13 lb. 9 oz. 18 pwt.

lb.	oz.	pwt.
27	7	15
13	9	18
13	9	17

SOLUTION.—1. We write numbers of the same denomination under each other.

2. We subtract as in simple numbers. When the number of any denomination of the subtrahend cannot be taken from the number of the same denomination in the minuend, we add as in simple numbers (**65**—III) one from the next higher denomination. Thus, 18 pwt. cannot be taken from 15 pwt.; we add 1 of the 7 oz. to the 15 pwt., making 35 pwt. 18 pwt. from 35 pwt. leaves 17 pwt., which we write under the pennyweights.

We proceed in the same manner with each denomination until the entire difference, 13 lb. 9 oz. 17 pwt., is found.

To subtract denominate fractions we reduce as directed in addition, and then subtract.

EXAMPLES FOR PRACTICE.

393. Find the difference between

1. 25 T. 16 cwt. 2 qr. 19 lb. and 13 T. 18 cwt. 22 lb. 13 oz.
2. lb. 13 ℥ 7 ʒ 5 ℈ 1 15 gr. and lb. 7 ℥ 9 ʒ 6 ℈ 2 12 gr.
3. 29 lb. 1 oz. 13 pwt. and 17 lb. 8 oz. 19 pwt. 12 gr.
4. $4\frac{3}{5}$ lb. and $\frac{5}{8}$ oz.
5. $1\frac{4}{5}$ T. and $2\frac{3}{8}$ cwt.
6. $\frac{7}{12}$ lb. and $\frac{5}{6}$ dr.
7. $5\frac{7}{8}$ lb. and $2\frac{5}{8}$ pwt.
8. $3\frac{5}{8}$ cwt. and $7\frac{3}{4}$ lb.
9. 9.7 oz. and 5.3 pwt.
10. 8.36 T. and 19.75 cwt.
11. 7.5 lb. and 4.75 sc.

12. A druggist had 13 lb. 4 oz. 5 dr. of a certain medicine, and sold at one time 3 lb. 7 oz. 6 dr. 2 sc., at another 5 lb. 9 oz. 4 dr. 1 sc. 10 gr. How much has he left?

13. Out of a stack of hay containing 16 T. 9 cwt. 1 qr. 12 lb. three loads were sold containing, respectively, 3 T. 4 cwt., 2 T. 19 cwt. 2 qr. 9 lb., and 3 T. 13 cwt. 14 lb. How much hay is left in the stack?

394. Prob. VIII.—**To multiply a denominate or compound number by an abstract number.**

Multiply 18 cwt. 74 lb. 9 oz. by 6.

	18 cwt.	74 lb.	9 oz.
			6
5 T.	12 cwt.	47 lb.	6 oz.

Solution.—We multiply as in simple numbers, commencing with the lowest denomination. Thus, 6 times 9 oz. equals 54 oz. We reduce the 54 oz. to pounds (**378**), equal 3 lb. 6 oz. We write the 6 oz. under the ounces, and add the 3 lb. to the product of the pounds.

We proceed in this manner with each denomination until the entire product, 5 T. 12 cwt. 47 lb. 6 oz. is found.

EXAMPLES FOR PRACTICE.

395. 1. Multiply 3 lb. 9 oz. 12 pwt. 17 gr. by 4; by 7.

2. Multiply 7 cwt. 2 qr. 18 lb. 5 oz. by 9; by 12; by 63.

3. Each of 8 loads of hay contained 2 T. 7 cwt. 19 lb.; what is the weight of the whole?

4. A grocer sold 12 firkins of butter, each containing 63 lb. $13\frac{3}{4}$ oz. How much did they all contain?

5. A druggist bought 25 boxes of a certain medicine, each box containing 2 lb. 5 oz. 7 dr. 1 sc. 19 gr.; what was the weight of the whole?

Multiply and reduce to a compound number:

6. $8\frac{5}{9}$ lb. by 14.
7. 13 lb. $7\frac{4}{5}$ oz. by 8.
8. 9.56 cwt. by 7.
9. 10.95 lb. by 5.
10. $2\frac{7}{8}$ lb. by 9.
11. 6.84 T. by .9.
12. 2.13 dr. by .4.
13. 7.63 cwt. by .34.
14. $\frac{5}{8}$ T. by $\frac{7}{9}$.
15. $\frac{4}{5}$ lb. by .7.
16. $\frac{7}{12}$ cwt. by $\frac{6}{7}$.
17. .9 pwt. by .9.

18. If a load of coal by the *long ton* weigh 2 T. 8 cwt. 3 qr. 11 lb., what will be the weight of 32 loads?

19. A drayman delivered on board of a boat 12 loads of coal, each containing 3 T. 7 cwt. 16 lb. How much coal did he put on board?

396. Prob. IX.—**To divide a denominate or compound number by any abstract number.**

Divide 29 lb. 7 oz. 2 dr. by 7.

	lb.	oz.	dr.
7)	29	7	2
	4	2	6

Solution.—1. The object of the division is to find $\frac{1}{7}$ of the compound number. This is done by finding the $\frac{1}{7}$ of each denomination separately. Hence the process is the same as in finding one of the equal parts of a concrete number.

Thus, the $\frac{1}{7}$ of 29 lb. is 4 lb. and 1 lb. remaining. We write the 4 lb. in the quotient, and reduce the 1 lb. to ounces, which added to 7 oz. make 19 oz. We now find the $\frac{1}{7}$ of the 19 oz., and proceed as before until the entire quotient, 4 lb. 2 oz. 6 dr. is found.

EXAMPLES FOR PRACTICE.

1. Divide 9 T. 15 cwt. 3 qr. 18 lb. by 2; by 5; by 8; by 12.

2. If 29 lb. 7 oz. 16 pwt. are made into 7 equal parts, how much will there be in each part?

3. A druggist made 12 powders of ℥1 ʒ5 of a certain medicine; what was the weight of each powder?

4. Divide lb. 3 ℥7 ʒ4 ℈2 by 4; by 6; by 12; by 32.

5. The aggregate weight of 43 equal sacks of coffee is 2 T. 7 cwt. 2 qr. 12 lb.; what is the weight of each sack?

6. Divide 5 T. 14 cwt. 2 qr. 8 lb. by 3 cwt. 1 qr. 12 lb.

Reduce both the dividend and divisor to the same denomination, and divide as in simple numbers.

7. Divide 2 lb. 5 oz. 2 pwt. 7 gr. by 1 oz. 3 pwt. 7 gr.

8. How many boxes, each containing 96 lb., will hold 1 T. 13 cwt. 2 qr. 10 lb.? *Ans.* 35.

9. Divide lb. 6 ℥9 ʒ7 ℈2 by ʒ7 ℈2 gr. 10.

10. Divide .98 lb. by .46 pwt.; $\frac{4}{9}$ of a ton by $\frac{5}{6}$ of a pound.

11. A druggist purchased 154 equal bottles of a certain medicine, containing in the aggregate lb. 34 ℥2 ʒ5 ℈1; how much did each bottle contain?

12. Divide lb. 75 by ʒ.58; .08 T. by .6 qr.

UNITS OF LENGTH.

397. A *yard* is the ***Standard Unit*** in *linear, surface,* and *solid* measure.

LINEAR MEASURE.

TABLE OF UNITS.

12 *in.*	=	1 *ft.*
3 *ft.*	=	1 *yd.*
$5\frac{1}{2}$ *yd.*	=	1 *rd.*
320 *rd.*	=	1 *mi.*

1. ***Denominations.***—Inches (in.), Feet (ft.), Yards (yd.), Rods (rd.), Miles (mi.).

2. ***Equivalents.***—1 mi. = 320 rd. = 5280 ft. = 63360 in.

3. ***Use.***—Used in measuring lines and distances.

4. In measuring cloth the yard is divided into *halves, fourths, eighths,* and *sixteenths.* In estimating duties in the Custom House, it is divided into *tenths* and *hundredths.*

Table of Special Denominations.

60 Geographic *or* 69.1$\dot{6}$ Statute Miles	= 1 Degree	of Latitude on a Meridian or of Longitude on the Equator.
360 Degrees	= the Circumference of the Earth.	
1.1$\dot{6}$ Statute Miles	= 1 Geog. Mi.	*Used to measure distances at sea.*
3 Geographical Miles	= 1 League.	
6 Feet	= 1 Fathom.	*Used to measure depths at sea.*
4 Inches	= 1 Hand.	*Used to measure the height of horses at the shoulder.*

SURVEYORS' LINEAR MEASURE.

TABLE OF UNITS.

7.92 *in.*	=	1 *l.*
25 *l.*	=	1 *rd.*
4 *rd.*	=	1 *ch.*
80 *ch.*	=	1 *mi.*

1. ***Denominations.***—Link (l.), Rod (rd.), Chain (ch.), Mile (mi.).

2. ***Equivalents.***—1 mi. = 80 ch. = 320 rd. = 8000 l.

3. ***Use.***—Used in measuring roads and boundaries of land.

4. The ***Unit*** of measure is the *Gunter's Chain,* which contains 100 links, equal 4 rods or 66 feet.

EXAMPLES FOR PRACTICE.

398. Reduce and explain the following:

1. 7 rods to inches.
2. 3 miles to yards.
3. 38465 yd. to miles.
4. 84 rods to links.
5. $\frac{3}{7}$ of a rd. to inches.
6. $\frac{5}{9}$ of a ch. to links.
7. 15 degrees to statute miles.
8. 3.76 geographical miles to statute miles.
9. 12 rd. 4 yd. 2 ft. to inches.
10. 210 geographical miles to statute miles.
11. 2 mi. 5 ch. 3 rd. to links.
12. .73 of a mile to a compound number.
13. .85 of a yd. to a decimal of a mile.
14. 7 yd. 2 ft. to a decimal of 3 rd.
15. Find the sum of $\frac{7}{9}$ of a mi., .85 of a ch., and 3 ch. 2 rd.
16. Find the difference between 3 mi. 5 ch. 2 rd. 13 l., and $\frac{5}{12}$ of (1 mi. 7 ch. 3 rd. 18 l.)
17. The four sides of a tract of land measure respectively 3 mi. 5 ch. 2 rd., 2 mi. 7 ch. 3 rd. 13 l., 3 mi. 17 l., and 2 mi. 2 rd.; what is the distance round it?
18. On a railroad 132 mi. 234 rd. 4 yd. 2 ft. long, there are 18 stations at equal distances from each other. How far are the stations apart, there being a station at each end of the road?
19. A ship moving due north sailed 15.7 degrees. How far did she sail in statute miles?
20. A ship sailing on the equator moved 45 leagues. How many degrees is she from the place of starting, and what is the distance in statute miles?
21. $\frac{5}{6}$ of a rod is what part of 3 chains?
22. 32 fathoms are what decimal of a mile?
23. In 125 geog. miles how many statute miles?
24. 3 ft. are what decimal of 3 rods?
25. 1 link is what decimal of 1 foot?

UNITS OF SURFACE.

399. A *square yard* is the ***Standard Unit*** of *surface measure.*

400. A ***Surface*** has two dimensions—*length* and *breadth.*

401. A ***Square*** is a *plane surface* bounded by four equal lines, and having four right angles.

402. A ***Rectangle*** is any *plane surface* having four sides and four right angles.

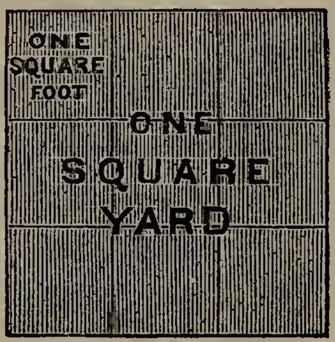
403. The ***Unit of Measure*** for surfaces is usually a square, each side of which is one unit of a known length.

Thus, in 14 sq. ft., the *unit* of *measure* is a *square foot.*

404. The ***Area*** of a rectangle is the *surface* included within its boundaries, and is expressed by the number of times it contains a given *unit* of *measure.*

3 feet long.

3 feet long.



3 sq. ft. × 3 = 9 sq. ft.

Thus, since a square yard is a surface, each side of which is 3 feet long, it can be divided into 3 rows of square feet, as shown in the diagram, each row containing 3 square feet. Hence, if 1 sq. ft. is taken as the ***Unit of Measure,*** the area of a square yard is 3 sq. ft. × 3 = 9 sq. ft.

The area of any rectangle is found in the same manner ; hence the following

405. RULE. *Find the product of the numbers denoting the length and breadth, expressed in the lowest denomination named in either ; the result is the area, which can be reduced to any required denomination.*

To find either *dimension* of a rectangle.

406. RULE. *Divide the number expressing the area, by the given dimension ; the quotient is the other.*

SQUARE MEASURE.

TABLE OF UNITS.

144 *sq. in.*	=	1 *sq. ft.*
9 *sq. ft.*	=	1 *sq. yd.*
$30\frac{1}{4}$ *sq. yd.*	=	1 *sq. rd.*, or *P.*
160 *sq. rd.*	=	1 *A.*
640 *A.*	=	1 *sq. mi.*

1. ***Denominations.***—Square Inch (sq. in.), Square Yard (sq. yd.), Square Rod (sq. rd.), Acre (A.), Square Mile (sq. mi.).

2. ***Equivalents.***—1 sq. mi. = 640 A=102400 sq. rd.=3097600 sq. yd.=27878400 sq. ft. =4014489600 sq. in.

3. ***Use.***—Used in computing areas or surfaces.

4. Glazing and stone-cutting are estimated by the *square foot;* plastering, paving, painting, etc., by the *square foot* or *square yard;* roofing, flooring, etc., generally by the *square* of 100 *square feet.*

5. In laying shingles, *one thousand*, averaging 4 inches wide, and laid 5 inches to the weather, are estimated to cover a square.

SURVEYORS' SQUARE MEASURE.

TABLE OF UNITS.

625 *sq. l.*	=	1 *P.*
16 *P.*	=	1 *sq. ch.*
10 *sq. ch.*	=	1 *A.*
640 *A.*	=	1 *sq. mi.*
36 *sq. mi.*	=	1 *Tp.*

1. ***Denominations.***—Square Link (sq. l.), Poles (P.), Square Chain (sq. ch.), Acres (A.), Square Mile (sq. mi.), Township (Tp.).

2. ***Equivalents.***—1 Tp. = 36 sq. mi. = 23040 A. = 230400 sq. ch. = 3686400 P. = 2304000000 sq. l.

3. ***Use.***—Used in computing the area of land.

4. The ***Unit*** of land measure is the *acre.* The measurement of a tract of land is usually recorded in *square miles, acres,* and *hundredths* of an acre.

EXAMPLES FOR PRACTICE.

407. Reduce and explain the following:

1. 3 acres to sq. ft.
2. 5 sq. mi. to sq. yards.
3. .83 of an A. to sq. yards.
4. 3 sq. mi. to sq. chains.
5. .007 mi. to sq. links.
6. .08 of an A. to sq. links.

7. 25 sq. yd. to a decimal of an acre.
8. 14 P. to a decimal of a sq. mi.
9. $\frac{4}{9}$ of a sq. mi. to a compound number.
10. .0005 of an A. to sq. feet.
11. $\frac{5}{7}$ of a Tp. to a compound number.
12. .0008 of a sq. mi. to a compound number.

Find the sum of

13. $\frac{3}{7}$ of an A., $\frac{5}{8}$ of a sq. rd., and 3 A. 158 sq. rd. 25 sq. yd.

14. $\frac{7}{8}$ of (1 Tp. 18 sq. mi. 584 A.), and $\frac{5}{12}$ of (378 A. 9 sq. ch. 12 P.)

15. Find the difference between ($\frac{3}{4}$ of 6 sq. mi. + $\frac{5}{8}$ of an A.), and ($\frac{2}{3}$ of an A. + $\frac{3}{5}$ of a pole).

16. Subtract 1 sq. l. from 1 acre; from 1 township.

17. A tract of land containing 984 A. 7 sq. ch. 12 P. was divided into 7 equal farms; what was the size of each farm?

What is the area of rectangles of the following dimensions:

18. 15 yd. by 12 yd?
19. $16\frac{2}{3}$ yards square?
20. $9\frac{3}{4}$ yd. by $18\frac{1}{2}$ yd?
21. 7.5 ch. by 3 ch. 8 l?
22. 4 yd. 2 ft. 4 in. by $3\frac{4}{5}$ yd?
23. 3.4 yd. by $9\frac{3}{4}$ yd?

24. How many square feet of lumber required for the floors of a house containing 2 rooms 15 ft. by 19 ft., 5 rooms 14 ft. by 16 ft., and 3 rooms 12 ft. by 15 ft.? *Ans.* 2230 sq. ft.

25. How many boards 12 ft. long and 4 in. wide required to floor a room which is 48 ft. by 32 ft.? *Ans.* 384 boards.

26. How many yards of carpeting 2 ft. 3 in. wide will be required for 3 rooms 18 ft. by 24 ft., and 4 rooms 12 ft. by 16 ft. 6 in.? *Ans.* $309\frac{1}{3}$ yds.

27. Find the cost of carpeting a house containing rooms as follows: 4 rooms 15 ft. by 19 ft. 6 in., carpet $\frac{3}{4}$ yd. wide at \$1.26 per yard; 2 rooms 18 ft. by 25 ft., carpet $\frac{7}{8}$ yd. wide at \$2.45 per yard; and 5 rooms 12 ft. 8 in. by 16 ft., carpet 1 yd. wide at \$1.08. *Ans.* \$620.

28. Find the cost of glazing 10 windows, each 9 ft. 10 in. by 5 ft. 3 in., at $.94 a square foot. *Ans.*

29. How many tiles 10 inches square will lay a floor 32 ft. 5 in. by 23 ft. 6 in.? *Ans.* 1096.98.

30. How many flag-stones, 3 ft. 5 in. by 2 ft. 6 in. will be required to cover a court 125 ft. by 82 ft., and what will be the cost of flagging the court at $1.87 a square yard?

31. Find the cost of lathing and plastering a house at $.52 per square yard, containing the following rooms, no allowance being made for doors, windows, and baseboard; 3 rooms 14 ft. by 18 ft., and 2 rooms 12 ft. by 15 ft., height of ceiling 11 ft.; 4 rooms 12 ft. by 16 ft., and 2 rooms 12 ft. by 14½ ft., height of ceiling 9 ft. 6 in. *Ans.*

32. The ridge of the roof of a building is 44 ft. long, and the distance from each eave to the ridge is 19 ft. 3 in. How many shingles 4 in. wide, laid 5½ in. to the weather will be required to roof the building, the first row being double?

33. What will be the cost of papering a room 20 ft. by 32, height of ceiling 12 ft., with rolls of paper 8 yards long 18 inches wide, at $1.63 per roll, deducting 132 sq. ft. for doors, windows, and baseboard?

UNITS OF VOLUME.

408. A ***Solid*** or ***Volume*** has three dimensions—*length, breadth,* and *thickness.*

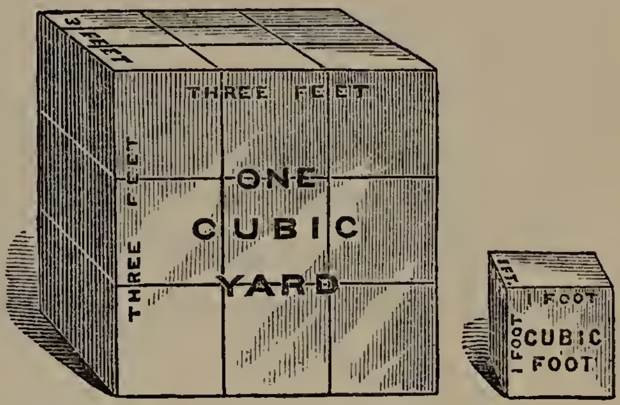
409. A ***Rectangular Solid*** is a body bounded by six *rectangles* called *faces.*

410. A ***Cube*** is a *rectangular solid,* bounded by six equal squares.

411. The ***Unit*** of ***Measure*** is a cube whose edge is a unit of some known length.

412. The ***Volume,*** or ***Solid Contents*** of a body is expressed by the number of times it contains a given *unit* of *measure.* For example, the contents of a *cubic yard* is expressed as 27 *cubic feet.*

Thus, since each face of a cubic yard contains 9 square feet, if a section 1 ft. thick is taken it must contain 3 times 3 cu. ft., or 9 cu. ft., as shown in the diagram.



And since the cubic yard is 3 feet thick, it must contain 3 sections, each containing 9 cu. ft., which is 27 cu. ft.

Hence, the *volume* or *contents* of a cubic yard expressed in cubic feet, is found by taking the product of the numbers denoting its 3 dimensions in feet.

The *contents* of any rectangular solid is found in the same manner; hence the following

413. RULE.—*Find the product of the numbers denoting the three dimensions expressed in the lowest denomination named. This result is the* VOLUME, *and can be reduced to any required denomination.*

To find a required dimension.

414. RULE.—*Divide the volume by the product of the numbers denoting the other two dimensions.*

The volume, before division, must be reduced to a cubic unit corresponding with the square unit of the product of the two dimensions.

CUBIC MEASURE.

TABLE OF UNITS.

1728 *cu. in.* = 1 *cu. ft.*
27 *cu. ft.* = 1 *cu. yd.*

1. ***Denominations.***—Cubic Inch (cu. in.), Cubic Foot (cu. ft.), Cubic Yard (cu. yd.).

2. ***Equivalents.***—1 cu. yd. = 27 cu. ft. = 46656 cu. in.

3. ***Use.***—Used in computing the volume or contents of solids.

EXAMPLES FOR PRACTICE.

415. Reduce and explain the following:

1. 97 cu. ft. to cu. in.
2. .09 of a cu. yd. to cu. ft.
3. 4 cu. yd. 394 cu. ft. to cu. in.
4. .0007 of a cu. yd. to cu. in.
5. $\frac{3}{5}$ of a cu. ft. to a decimal of a cu. yd.
6. .8 of a cu. ft. to a decimal of a cu. yd.
7. Find the sum of $\frac{2}{3}$ of a cu. yd. and .625 of a cu. ft.
8. Find the difference between $\frac{3}{8}$ of a cu. yd. and .75 of a cu. ft.

Find the *contents* of rectangular solids of the following dimensions:

9. A solid 24 ft. long by 1 ft. 6 in. by 2 ft. 9 in.
10. A cube whose edge is 3 yd. 2 ft. 8 in.
11. A solid 7 ft. 9 in. long by 3 ft. 4 in. by 4 ft. 6 in.
12. A solid 12 yd. 1 ft. 9 in. long by 2 yd. 2 ft. by 2 ft. 8 in.
13. How many cubic feet in a stick of timber 38 ft. long by 2 ft. 3 in. by 1 ft. 9 in?
14. A cistern 9 ft. square contains 1092 cu. ft.; what is its depth?
15. A stick of square timber contains 189 cu. ft., 2 of its dimensions are 1 ft. 9 in. and 2 ft. 3 in.; what is the other?
16. A bin contains 326$\frac{1}{4}$ cu. ft.; 2 of its dimensions are 9 ft. 8 in. and 7 ft. 6 in.: what is the other?
17. How many cubic feet of air in a room 74 ft. 9 in. long, 52 ft. 10 in. wide, and 23 ft. 6 in. high?
18. How many cubic yards of earth in an embankment 283 ft. by 42 ft. 8 in. by 18 ft. 6 in.?
19. A vat is 7 ft. 2 in. by 4 ft. 9 in. by 3 ft. 4 in. How many cubic feet does it contain?
20. In digging a cellar 48 ft. 6 in. by 39 ft. 8 in., and 8 ft. 4 in. deep, how many cubic yards of earth must be removed?

21. What will be the cost of the following bill of square timber, at \$.33$\frac{1}{3}$ per cubic foot:

(1.) 3 pieces 13 ft. by 9 in. by 7 in.?
(2.) 8 pieces 15 ft. 6 in. by 10 in. by 8 in.?
(3.) 4 pieces 23 ft. by 8 in. by 9 in.?
(4.) 6 pieces 36 ft. by 1 ft. 6 in. by 1 ft.?
(5.) 9 pieces 18 ft. 9 in. by 1 ft. 3 in. by 9 in.?
(6.) 12 pieces 15 ft. by 7$\frac{1}{2}$ in. by 9$\frac{1}{4}$ in.?

22. How many perches in a wall 37 ft. long, 23 ft. 6 in. high, and 2 ft. 6 in. thick?

Table of Units for Measuring Wood and Stone.

16 *cu. ft.*	= 1 Cord Foot (*cd. ft.*)	Used for measuring both wood and stone.
8 *cd. ft.* or 128 *cu. ft.*	= 1 Cord (*cd.*)	
24$\frac{3}{4}$ *cu. ft.*	= 1 perch (*pch.*) of stone or masonry.	

1 *cu. yd.* of earth is called a load.

1. The materials for masonry are usually estimated by the *cord* or *perch,* the work by the *perch* and *cubic foot,* also by the *square foot* and square yard.

2. In estimating the mason work in a building, each wall is measured on the outside and no allowance is ordinarily made for doors, windows, and cornices, unless specified in contract. In estimating the material, the doors, windows, and cornices are deducted.

3. Brickwork is usually estimated by the *thousand bricks.* The size of a brick varies thus: North River bricks are 8 in. × 3$\frac{1}{2}$ × 2$\frac{1}{4}$, Philadelphia and Baltimore bricks are 8$\frac{1}{4}$ in. × 4$\frac{1}{8}$ × 2$\frac{3}{8}$, Milwaukee bricks 8$\frac{1}{2}$ in. × 4$\frac{1}{8}$ × 2$\frac{3}{8}$, and Maine bricks 7$\frac{1}{2}$ in. × 3$\frac{3}{8}$ × 2$\frac{3}{8}$.

4. *Excavations* and *embankments* are estimated by the cubic yard.

EXAMPLES FOR PRACTICE.

416. Reduce and explain the following:

1. 64 pch. to cu. feet.
2. 42 cords to cd. feet.
3. 36 cords to cu. feet.
4. .84 pch. to cu. feet.

5. $\frac{5}{7}$ of a cd. to cu. feet.

6. .85 of a cord to a decimal of 3 cu. yd.

7. $\frac{5}{9}$ of a cord to a decimal of a cu. yd.

8. $\frac{4}{5}$ of a cu. ft. to a decimal of a pch.

9. .73 of a cu. ft. to a decimal of a cd.

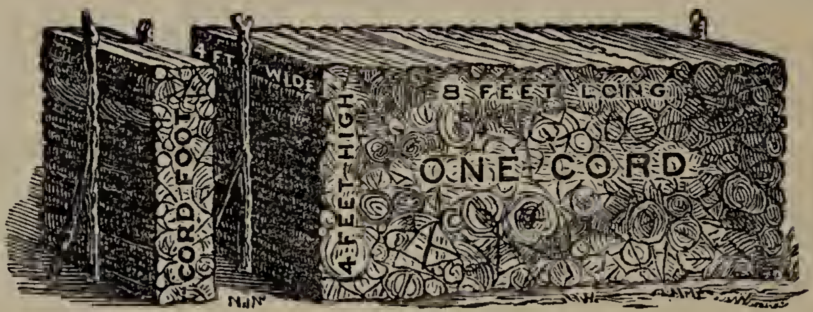
10. $\frac{2}{3}$ of 8 cd. to a decimal of 12 cd.

11. Find the sum of $\frac{3}{4}$ pch., $\frac{5}{8}$ cd., and 11 cd. ft. 38 cu. ft.

12. How many North River bricks make 1 cubic foot laid without any mortar?

13. A pile of wood containing 84 cd. 7 cd. ft. 12 cu. ft. was made into 5 equal piles; what was the size of each?

14. How many cords in a pile of wood 196 ft. long, 7 ft. 6 in. high, and 8 ft. wide?



A ***Cord*** is a pile of wood, stone, etc., 8 ft. long, 4 ft. wide, and 4 ft. high.

A ***Cord Foot*** is 1 ft. long, 4 ft. wide, and 4 ft. high, or $\frac{1}{8}$ of cord, as shown in the cut.

15. What is the cost of a pile of stone 28 ft. long, 9 ft. wide, and 7 ft. high, at $3.85 per cord?

16. A load of wood containing 1 cord is 3 ft. 9 in. high and 4 ft. wide; what is its length?

17. How many perches of masonry will 18 cd. 5 cd. ft. of stone make, allowing 22 cu. ft. of stone for 1 perch of wall?

18. How many cords of stone will be required to enclose with a wall built without mortar a lot 28 rods long and 17 rods wide, the wall being 5 ft. high and 2 ft. 9 in. thick?

19. How many Philadelphia bricks in a cubic foot of wall $13\frac{1}{2}$ in. wide, laid in courses of mortar $\frac{1}{4}$ of an inch thick?

SOLUTION.—Since the mortar is $\frac{1}{4}$ of an inch thick, each brick in the wall is increased $\frac{1}{4}$ of an inch in *length* and in *thickness.* Hence the *length* occupied by a Philadelphia brick *in the wall* is $8\frac{1}{4}$ in. + $\frac{1}{4}$ in. = $8\frac{1}{2}$ in., and the *thickness* is $2\frac{3}{8}$ in. + $\frac{1}{4}$ in. = $2\frac{5}{8}$ in.

Again, since the wall is $13\frac{1}{2}$ in. wide, 3 bricks are placed side by side, and $13\frac{1}{2}$ in. ÷ 3 = $4\frac{1}{2}$ in. the *width* occupied by a brick *in the wall.*

Hence, the volume occupied *in the wall* by a Philadelphia brick, with the given width of wall and thickness of mortar, is $8\frac{1}{2}$ in. × $2\frac{5}{8}$ × $4\frac{1}{2}$ = 100.40625 cu. in. And since 1728 cu. in. equals 1 cu. ft., 1728÷100.40625 = 17.21 + the number of Philadelphia bricks in 1 cu. ft. of wall.

From this solution we obtain the following rule for finding the number of bricks of a given size in a given wall:

417. RULE.—*Find the number of bricks of the given size in a cubic foot of the given wall. Multiply this number by the number of cubic feet in the wall.*

20. How many Maine bricks will be required to build a house 54 feet long, 32 feet wide, and 25 feet high, the brick being laid in mortar $\frac{1}{4}$ of an inch thick, the wall being 11 in. wide, and 258 cu. ft. being allowed for doors and windows?

21. How many perches of stone laid dry will build a wall 9 ft. 6 in. high, 384 ft. long, and 2 ft. 9 in. thick?

22. What will it cost to remove an embankment 325 ft. long, 25 ft. wide, and 12.8 ft. high, at 58 cts. per cubic yard?

23. What is the cost of building a wall 89 ft. long, 28 ft. high, $19\frac{1}{2}$ in. wide, with brick $8\frac{1}{2}$ in. long, $4\frac{1}{2}$ in. wide, $2\frac{3}{8}$ in. thick, laid in mortar $\frac{1}{4}$ in. thick, at $12.85 a 1000 bricks laid in the wall?

24. What will be the cost of a pile of wood 114 yd. 2 ft. long, 4 ft. wide, and 6 ft. 8 in. high, at $4.50 per cord?

25. What will be the cost of removing the earth from the cellar of a house 48 ft. 9 in. by 32 ft., the cellar to be 9 ft. deep, at $.57 per load?

BOARD MEASURE.

TABLE OF UNITS.

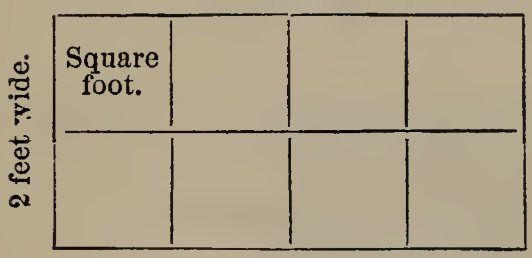
12 B. in. = 1 B. ft.
12 B. ft. = 1 cu. ft.

418. A ***Board Foot*** is 1 ft. long, 1 ft. wide, and 1 in. thick. Hence, 12 *board feet* equals 1 cu. ft.

419. A ***Board Inch*** is 1 ft. long, 1 in. wide, and 1 in. thick, or $\frac{1}{12}$ of a *board foot.* Hence, 12 *board inches* equals 1 *board foot.*

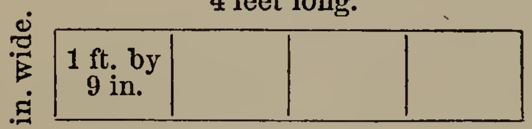
Observe carefully the following:

(1.)
4 feet long.



4 × 2 = 8 sq. ft. or 8 B. ft.

(2.)
4 feet long.



4 × 9=36 B. in.; 36 B. in.÷12=3 B. ft.

1. Diagram 1 represents a board where both dimensions are feet. Hence the product of the two dimensions gives the square feet in surface (**405**), or the number of board feet when the lumber is not more than 1 inch thick.

2. Diagram (2) represents a board where one dimension is feet and the other inches. It is evident (**418**) that a board 1 foot long, 1 inch thick, and any number of inches wide, contains as many *board inches* as there are inches in the width. Hence the number of square feet or *board feet* in a board 1 inch thick is equal to the length in feet multiplied by the width in inches divided by 12, the number of *board inches* in a *board foot.*

3. In case the lumber is more than 1 inch thick, the number of board feet is equal to the number of square feet in the surface multiplied by the thickness.

EXAMPLES FOR PRACTICE.

420. Find the contents of boards measuring

1. 24 ft. by 13 in.
2. 28 ft. by 15 in.
3. 18 ft. by 16 in.
4. 9 ft. by 32 in.
5. 13 ft. by 26 in.
6. 17 ft. by 30 in.
7. 5 ft. by 18 in.
8. 34 ft. by 15 in.
9. 25 ft. by 14 in.

Find the contents of boards measuring

10. 15 ft. by 1 ft. 3 in.
11. 27 ft. by 1 ft. 6 in.
12. 19 ft. by 2 ft. 4 in.
13. 23 ft. by 1 ft. 5 in.

14. Find the contents of a board 18 ft. long and 9 in. wide.

15. How many board feet in a stick of square timber 48 ft. long, 9 inches by 14 inches.

16. Find the length of a stick of timber 8 in. by 10 in., which will contain 20 cu. ft.

OPERATION.—$(1728 \times 20) \div (8 \times 10) = 432$; $432 \div 12 = 36$ ft., the length.

17. A piece of timber is 10 in. by 12 in. What length of it will contain 26 cubic feet?

Find the cost of the following:

18. Of 234 boards 14 ft. long 8 in. wide, at $3.25 per hundred.

19. Of 5 sticks of timber 27 ft. long, 9 in. by 14 in., at $1.75 per hundred feet board measure.

20. Of 84 plank 20 ft. long, 11 in. wide, 3 in. thick, at $1.84 per hundred feet board measure.

UNITS OF CAPACITY.

421. The ***Standard Units*** of capacity are the *Gallon* for Liquid, and the *Bushel* for Dry Measure.

LIQUID MEASURE.

TABLE OF UNITS.

4 *gi.*	=	1 *pt.*
2 *pt.*	=	1 *qt.*
4 *qt.*	=	1 *gal.*
$31\frac{1}{2}$ *gal.*	=	1 *bbl.*

1. ***Denominations.***—Gills (gi.), Pints (pt.), Quarts (qt.), Gallons (gal.), Barrels (bbl.)

2. ***Equivalents.***—1 gal. = 4 qt. = 8 pt. = 32 gi.

3. ***Use.***—Used in measuring liquids.

4. The capacity of cisterns, vats, etc., is usually estimated by considering a barrel $31\frac{1}{2}$ gal.; but barrels are made of various sizes, from 30 to 56 gallons. The *hogshead, butt, tierce, pipe,* and *tun* are names of casks, and have usually their capacity in gallons marked upon them.

APOTHECARIES' FLUID MEASURE.

TABLE OF UNITS.

♏ 60	=	*f* ʒ 1
f ʒ 8	=	*f* ℥ 1
f ℥ 16	=	*O.* 1
O. 8	=	*Cong.* 1

1. ***Denominations.***—Minims or drops (♏), Fluid Drachm (f ʒ), Fluid Ounce (f ℥), Pint (O., for *octarius*, the Latin for one-eighth or pint), Gallon (Cong., for *congius*, the Latin for gallon).

2. ***Equivalents.*** — Cong. 1 = O. 8 = f ℥ 128 = f ʒ 1024 = ♏ 61440.

3. ***Use.***—Used in prescribing and compounding liquid medicine.

4. The symbols precede the numbers, as in Apothecaries' Weight, as shown in the table of units.

DRY MEASURE.

TABLE OF UNITS.

2 *pt.*	=	1 *qt.*
8 *qt.*	=	1 *pk.*
4 *pk.*	=	1 *bu.*

1. ***Denominations.***—Pints (pt.), Quarts (qt.), Pecks (pk.), Bushels (bu.).

2. ***Equivalents.***—1 bu. = 4 pk. = 32 qt. = 64 pt.

3. ***Use.***—Used in measuring grain, roots, fruits, salt, etc.

4. *Heaped measure,* in which the bushel is heaped in the form of a cone, is used in measuring potatoes, corn in the ear, coarse vegetables, large fruits, etc. *Stricken measure* is used in measuring grains, seeds, and small fruits.

EXAMPLES FOR PRACTICE.

422. Solve and explain orally the following:

1. How many gills in 4 qt.? In 2 gal.? In 7 qt.? In 3 qt. 1 pt.? In 3 gal. 3 qt.?

2. How many pints in 2 bu.? In 3 pk. 5 qt.? In 1 bu. 2 pk. 7 qt.?

3. What is the sum of O. 5 f ℥ 12 f ʒ 7 and f ℥ 8 f ʒ 3 ♏ 15?

4. Multiply 3 pk. 5 qt. 1 pt. by 3; by 5; by 10; by 7; by 12.

Reduce

5. ♏ 8465 to gallons.
6. 23649 pt. to bushels.
7. 93584 pt. to barrels.
8. f ʒ 7649 to gallons.
9. 57364 gi. to barrels.
10. 93654 pt. to bushels.

11. 3 qt. 1 pt. to a decimal of a gallon.

12. $\frac{3}{4}$ of 5 qt. 1 pt. to a decimal of 2 bushels.

13. f ℥ 7 ♏ 15 to a decimal of Cong. 3.

14. A merchant bought 5860 bushels wheat in Ohio at \$1.25, and sold the whole in Connecticut at the same price. How much did he gain on the transaction?

15. A grocer bought 12 firkins of butter, each containing 73 lb. 13 oz., at 36 cts. a pound; 7 bu. 3 pk. clover seed, at \$1.15 a peck; and 5 loads of potatoes, each load containing 43 bu. 3 pk., at \$.32 a bushel. How much was the cost?

Comparative Table of Units of Capacity.

	CUBIC IN. IN ONE GALLON.	CUBIC IN. IN ONE QUART.	CUBIC IN. IN ONE PINT.
Liquid Measure	231	$57\frac{3}{4}$	$28\frac{7}{8}$
Dry Measure ($\frac{1}{2}$ pk.)	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$

1. The ***Standard Bushel*** of United States contains 2150.42 cu. in. and the ***Imperial Bushel*** of Great Britain contains 2216.192 cu. in.

2. An English Quarter contains 8 imp. bu. or $8\frac{1}{4}$ U. S. bu. A *quarter* of 8 U. S. bu., or 480 lb., is used in shipping grain from New York.

3. A ***Register Ton*** is 100 cu. ft.; used in measuring the internal capacity or tonnage of a vessel. A ***Shipping Ton*** is 40 cu. ft. in the U. S. and 42 cu. ft. in England.

4. A cubic foot of pure water weighs 1000 oz. or $62\frac{1}{2}$ lb. Avoir.

EXAMPLES FOR PRACTICE.

423. 1. How many U. S. bushels in a bin of wheat 6 ft. long, 5 ft. 6 in. wide, and 4 ft. 9 in. deep? *Ans.*

How many cubic feet in a space that holds

2. 1000 U. S. bushels?
3. 1000 imp. bushels?
4. 120 bbl. water?
5. 240 English quarters?
6. 18 T. 16 cwt. of pure water?
7. 804 bu. 3 pk. U. S. bu.?

8. A cistern containing 5300 gal. of water is 10 ft. square. How deep is it? *Ans.* 7.085+.

9. How many ounces in gold are equal in weight to 9 pounds 14 ounces of iron?

UNITS OF TIME.

424. The *mean solar day* is the ***Standard Unit*** of time.

TABLE OF UNITS.

60 *sec.*	=	1 *min.*
60 *min.*	=	1 *hr.*
24 *hr.*	=	1 *da.*
7 *da.*	=	1 *wk.*
365 *da.*	=	1 *common yr.*
366 *da.*	=	1 *leap yr.*
100 *yr.*	=	1 *cen.*

1. ***Denominations.***—Seconds (sec.), Minutes (min.), Hours (hr.), Days (da.), Weeks (wk.), Months (mo.), Years (yr.), Centuries (cen.).

2. There are 12 Calendar Months in a year; of these, April, June, September, and November, have 30 da. each. All the other months except February have 31 da. each. February, in *common years*, has 28 da., in *leap years* it has 29 da.

3. In computing interest, 30 days are usually considered *one month.* For business purposes the day begins and ends at 12 o'clock midnight.

425. The reason for *common* and *leap years* will be seen from the following:

The *true year* is the time the earth takes to go *once around the sun,* which is 365 days, 5 hours, 48 minutes and 49.7 seconds. Taking 365 days as a *common year,* the time lost in the calendar in 4 years will lack only 44 minutes and 41.2 seconds of 1 day. Hence we add 1 day to February every fourth year, making the year 366 days, or ***Leap Year.*** This correction is 44 min. 41.2 sec. more than should be added, amounting in 100 years to 18 hr. 37 min. 10 sec.; hence at the end of 100 years we omit adding a day, thus losing again 5 hr. 22 min. 50 sec., which we again correct by adding a day at the end of 400 years; hence the following rule for finding *leap year:*

426. Rule.—*Every year, except centennial years, exactly divisible by 4, is a leap year. Every centennial year exactly divisible by 400 is also a leap year.*

This will render the calendar correct to within one day for 4000 years.

427. Prob. 10.—**To find the interval of time between two dates.**

How many yr., mo., da. and hr. from 6 o'clock P. M., July 19, 1862, to 6 o'clock A. M., April 9, 1876.

yr.	*mo.*	*da.*	*hr.*
1876	4	9	7
1862	7	19	18
13	8	19	13

SOLUTION.—1. Since the latter date denotes the greater period of time, it is the minuend, and the earlier date, the subtrahend.

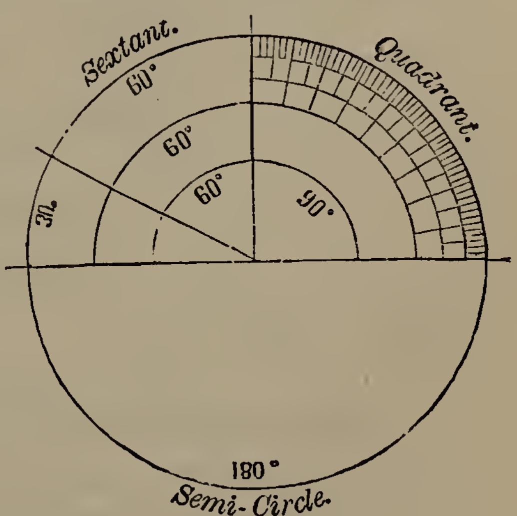
2. Since each year commences with January, and each day with 12 o'clock midnight, 7 o'clock A. M., April 9, 1876, is the 7th hour of the 9th day of the fourth month of 1876; and 6 o'clock P. M., July 19, 1862, is the 18th hour of the 19th day of the 7th month of 1862. Hence the minuend and subtrahend are written as shown in the margin.

3. Considering 24 hours 1 day, 30 days 1 month, and 12 months 1 year, the subtraction is performed as in compound numbers (**392**), and 13 yr. 8 mo. 19 da. 13 hr. is the interval of time between the given dates.

Find the interval of time between the following dates:

1. March 14, 1776, and August 3, 1875.
2. 5 A. M. May 19, 1854, and 7 P. M. Sept. 3, 1876.
3. 10 P. M. October 3, 1812, and 8 A. M. April 17, 1879.
4. 7 P. M. November 25, 1754, and 2 A. M. May 13, 1873.
5. The American revolution began April 19th, 1775, and ended Jan. 20, 1783. How long did it continue?
6. Washington died Dec. 14th, 1799, at the age of 67 yr. 9 mo. 22 da. At what date was he born?

CIRCULAR MEASURE.



428. A ***Circle*** is a plane figure bounded by a curved line, all points of which are equally distant from a point within called the centre.

429. A ***Circumference*** is the line that bounds a circle.

430. A ***Degree*** is one of the 360 equal parts into which the circumference of a circle is supposed to be divided.

431. The degree is the ***Standard Unit*** of circular measure.

TABLE OF UNITS.

60″	=	1′
60′	=	1°
30°	=	1 *S.*
12 *S.*	=	1 *Cir.*
360°	=	1 *Cir.*

1. ***Denominations.***—Seconds (″), Minutes (′), Degrees (°), Signs (S.), Circle (Cir.).

2. *One-half* of a *circumference*, or 180°, as shown by the figure in the margin, is called a *Semi-circumference; One-fourth*, or 90°, a *Quadrant; One-sixth*, or 60°, a *Sextant;* and *One-twelfth*, or 30°, a *Sign.*

3. The length of a degree varies with the size of the circle, as will be seen by examining the foregoing diagram.

4. A degree of latitude or a degree of longitude on the Equator is 69.16 statute miles. A *minute* on the earth's circumference is a *geographical* or *nautical* mile.

SPECIAL UNITS.

Table for Paper.

24 Sheets	=	1 Quire.
20 Quires	=	1 Ream.
2 Reams	=	1 Bundle.
5 Bundles	=	1 Bale.

Table for Counting.

12 Things	=	1 Dozen (doz.)
12 Dozen	=	1 Gross (gro.)
12 Gross	=	1 Great Gross (G. Gro.)
20 Things	=	1 Score (Sc.)

EXAMPLES FOR PRACTICE.

432. Reduce and explain the following:

1. 5° 27′ 43″ to seconds.
2. 1 cir. 5 s. to minutes.
3. 3 s. 17° 9′ to seconds.
4. 7° 4′ to a fraction of a sign.
5. 9° 12′ to a decimal of a circle.
6. .83 of a cir. to a compound number.
7. What part of a circumference are 60°? 90°? 180°?
8. Reduce $\frac{4}{5}$ of a quadrant to a compound number.
9. How many sextants in 120°? In 150°? In 165°? In 248°? In 295°?
10. In 5 cir. 7 s. 17°, how many sextants and what left?
11. How many degrees, minutes, etc., in $\frac{5}{7}$ of a quadrant?
12. America was discovered Oct. 14, 1492. What interval of time between the discovery and the Centennial Exposition, July 4, 1876?

13. Washington was born Feb. 22, 1732, and Napoleon Aug. 15, 1769. How much older was Washington than Napoleon?

14. How many dozen in $7\frac{5}{6}$ gross? In $13\frac{3}{4}$ gro.?

15. How many dozen in $8\frac{3}{5}$ great gross? In $15\frac{5}{6}$?

16. How many dozen in $17\frac{4}{5}$ scores? In $196\frac{7}{10}$? In $84\frac{3}{4}$?

17. Reduce 13 bundles 1 ream 15 quires of paper to sheets.

18. 136 sheets are what decimal of 1 bundle? Of 17 quires?

UNITS OF MONEY.

UNITED STATES MONEY.

433. The *dollar* is the ***Standard Unit*** of United States money.

TABLE OF UNITS.

10 *m.*	=	1 *ct.*
10 *ct.*	=	1 *d.*
10 *d.*	=	$1.
$10	=	1 *E.*

1. ***Denominations.***—Mills (m.), Cents (ct.), Dimes (d.), Dollars ($), Eagles (E.).

2. The United States *coin*, as fixed by the "New Coinage Act" of 1873, is as follows: ***Gold,*** the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar, and one-dollar; ***Silver,*** the trade-dollar, half-dollar, quarter-dollar, and ten-cent; ***Nickel,*** the five-cent and three-cent; ***Bronze,*** one-cent.

3. ***Composition of Coins.***—*Gold coin* contains .9 pure gold and .1 silver and copper. *Silver coin* contains .9 pure silver and .1 pure copper. *Nickel coin* contains .25 nickel and .75 copper. *Bronze coin* contains .95 copper and .05 zinc and tin.

4. The *Trade-dollar* weighs 420 grains and is designed for commercial purposes solely.

CANADA MONEY.

434. 1. ***Denominations.***—*Mills, Cents,* and *Dollars.* These have the same nominal value as in United States Money.

2. The *Coin* of the Dominion of Canada is as follows: ***Gold,*** the coins in use are the sovereign and half-sovereign; ***Silver,*** the fifty-cent, twenty-five cent, ten-cent, and five-cent pieces; ***Bronze,*** the one-cent piece.

ENGLISH MONEY.

435. The *pound sterling* is the ***Standard Unit*** of English money. It is equal to $4.8665 United States money.

TABLE OF UNITS.

4 *far.*	=	1 *d.*
12 *d.*	=	1 *s.*
20 *s.*	=	1 *Sov.* or £1.
2 *s.*	=	1 *fl.*
5 *s.*	=	1 *cr.*

1. ***Denominations.***—Farthings (far.), Pennies (d.), Shillings (s.), Sovereign (sov.), Pound (£), Florin (fl.), Crown (cr.).

2. The Coins in general use in Great Britain are as follows: ***Gold,*** sovereign and half-sovereign; ***Silver,*** crown, half-crown, florin, shilling, six-penny, and three-penny; ***Copper,*** penny, half-penny, and farthing.

FRENCH MONEY.

436. The *silver franc* is the ***Standard Unit*** of French money. It is equal to $.193 United States money.

TABLE OF UNITS.

10 *m.*	=	1 *ct.*
10 *ct.*	=	1 *dc.*
10 *dc.*	=	1 *fr.*

1. ***Denominations.***—Millimes (m.), Centimes (ct.), Decimes (dc.), Francs (fr.).

2. ***Equivalents.***—1 fr. = 10 dc. = 100 ct. = 1000 m.

3. The Coin of France is as follows: ***Gold,*** 100, 40, 20, 10, and 5 francs; ***Silver,*** 5, 2, and 1 franc, and 50 and 25 centimes; ***Bronze,*** 10, 5, 2, and 1 centime pieces.

GERMAN MONEY.

437. The *mark* is the ***Standard Unit*** of the *New German Empire.* It is equal to 23.85 cents United States money, and is divided into 100 equal parts, one of which is called a ***Pfennig.***

1. The *Coins* of the *New Empire* are as follows: ***Gold,*** 20, 10, and 5 marks; ***Silver,*** 2 and 1 mark; ***Nickel,*** 10 and 5 pfennig.

2. The coins most frequently referred to in the United States are the silver Thaler, equal 74.6 cents, and the silver Groschen, equal $2\frac{1}{2}$ cts.

EXAMPLES FOR PRACTICE.

438. Reduce and explain the following:

1. £2 17s. to farthings.
2. 83745 mills to dollars.
3. $\frac{3}{4}$s. to a decimal of a £.
4. $34 to mills.
5. .7d to a decimal of a £.
6. .9s. to a decimal of £3.
7. $\frac{5}{8}$ of a £ to a compound number.
8. £.84 to a compound number.
9. How many pounds sterling in $8340 of American gold?
10. In 2368 francs how many dollars U. S. gold?
11. Remitted to England $436 gold to pay a debt. How much is the debt in English money?
12. Received from Germany 43864 marks. How much is the amount in American money?
13. £240 17s. is how much in United States money? In German money? In French money?
14. Reduce 7 marks to a decimal of $4.
15. Reduce 12 francs to a decimal of $5.
16. Exchanged $125 for French money. How much French money did I receive?

THE METRIC SYSTEM.

Decimal Related Units.

439. The ***Metric System*** of *Related Units* is formed according to the *decimal scale.*

440. The ***Meter,*** which is 39.37079 inches long, or nearly *one ten-millionth* of the distance on the earth's surface from the equator to the pole, is the *base* of the system.

441. The ***Primary*** or ***Principal Units*** of the system are the *Meter,* the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram.* All other units are multiples and sub-multiples of these.

442. The *names* of ***Multiple Units*** or higher denominations are formed by prefixing to the names of the *primary units* the Greek numerals *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myria* (10000).

443. The names of ***Sub-multiple Units***, or lower denominations, are formed by prefixing to the names of the *primary units.* the Latin numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), and *Milli* ($\frac{1}{1000}$).

UNITS OF LENGTH.

444. The ***Meter*** is the *principal unit* of *length.*

TABLE OF UNITS.

10 Millimeters,	*mm.*	=	1 Centimeter	=	.3937079 in.
10 Centimeters,	*cm.*	=	1 Decimeter	=	3.937079 in.
10 Decimeters,	*dm.*	=	1 ***Meter***	=	39.37079 in.
10 METERS,	*M.*	=	1 Dekameter	=	32.808992 ft.
10 Dekameters,	*Dm.*	=	1 Hectometer	=	19.927817 rd.
10 Hectometers,	*Hm.*	=	1 Kilometer	=	.6213824 mi
10 Kilometers,	*Km.*	=	1 Myriameter (*Mm.*)	=	6.213824 mi.

The *meter* is used in place of one yard in measuring cloth and short distances. Long distances are usually measured by the *kilometer.*

UNITS OF SURFACE.

445. The ***Square Meter*** is the *principal unit* of surfaces.

TABLE OF UNITS.

100 Sq. Millimeters,	*sq. mm.*	=	1 Sq. Centimeter	=	.155 + sq. in.
100 Sq. Centimeters,	*sq. cm.*	=	1 Sq. Decimeter	=	15.5 + sq. in.
100 Sq. Decimeters,	*sq. dm.*	=	1 ***Sq. Meter*** (*Sq. M.*)	=	1.196 + sq. yd.

446. The ***Are***, a square whose side is 10 meters, is the *principal unit* for measuring land.

TABLE OF UNITS.

100 Centiares,	*ca.*	=	1 ***Are***	=	119.6034 sq. yd.
100 Ares,	*A.*	=	1 Hectare (*Ha.*)	=	2.47114 acres.

UNITS OF VOLUME.

447. The ***Cubic Meter*** is the *principal unit* for measuring ordinary solids, as embankments, etc.

TABLE OF UNITS.

1000 Cu. Millimeters	*cu. mm.*	=	1 Cu. Centimeter	=	.061 cu. in.
1000 Cu. Centimeters,	*cu. cm.*	=	1 Cu. Decimeter	=	61.026 cu. in.
1000 Cu. Decimeters,	*cu. dm.*	=	1 ***Cu. Meter***	=	35.316 cu. ft.

448. The ***Stere,*** or *Cubic Meter*, is the *principal unit* for measuring wood.

TABLE OF UNITS.

10 Decisteres,	*dst.*	= 1 ***Stere***		=	35.316+ cu. ft.
10 STERES,	*St.*	= 1 Dekastere	(*Dst.*	=	13.079+ cu. yd.

UNITS OF CAPACITY.

449. The ***Liter*** is the *principal unit* both of Liquid and Dry Measure. It is equal to a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter*.

TABLE OF UNITS.

10 Milliliters,	*ml.*	= 1 Centiliter	=	.6102 cu. in.	=	.338 fl. oz.
10 Centiliters,	*cl.*	= 1 Deciliter	=	6.1022 " "	=	.845 gill.
10 Deciliters,	*dl.*	= 1 ***Liter***	=	.908 qt.	=	1.0567 qt.
10 LITERS,	*L.*	= 1 Dekaliter	=	9.08 "	=	2.6417 gal.
10 Dekaliters,	*Dl.*	= 1 Hectoliter	=	2.8372+ bu.	=	26.417 "
10 Hectoliters,	*Hl.*	= 1 Kiloliter	=	28.372+ "	=	264.17 "
10 Kiloliters,	*Kl.*	= 1 Myrialiter	=	283.72+ "	=	2641.7 "

The Hectoliter is used in measuring large quantities in both liquid and dry measure.

UNITS OF WEIGHT.

450. The ***Gram*** is the *principal unit* of *weight*, and is equal to the weight of a cube of distilled water whose edge is one centimeter.

TABLE OF UNITS.

10 Milligrams,	*mg.*	=	1 Centigram	=	.15432 + oz. Troy.
10 Centigrams,	*cg.*	=	1 Decigram	=	1.54324 + " "
10 Decigrams,	*dg.*	=	1 ***Gram***	=	15.43248 + " "
10 GRAMS,	*G.*	=	1 Dekagram	=	.3527 + oz. Avoir.
10 Dekagrams,	*Dg.*	=	1 Hectogram	=	3.52739 + " "
10 Hectograms,	*Hg.*	=	1 { Kilogram or *Kilo.* }	=	2.20462 + lb.
10 Kilograms,	*Kg.*	=	1 Myriagram	=	22.04621 + "
10 Myriagrams,	*Mg.*	=	1 Quintal	=	220.46212 + "
10 Quintals,		=	1 { Tonneau or *Ton.* }	=	2204.6212 + "

The ***Kilogram*** or Kilo., which is little more than $2\frac{1}{5}$ lb. Avoir., is the *common* weight in trade. Heavy articles are weighed by the ***Tonneau***, which is 204 lb. more than a *common ton.*

Comparative Table of Units.

1 Inch	= .0254 meter.	1 Cu. foot	= .2832 Hectoliter.
1 Foot	= .3048 "	1 Cu. yard	= .7646 Steres.
1 Yard	= .9144 "	1 Cord	= 3.625 Steres.
1 Mile	= 1.6093 Kilometers.	1 Fl. ounce	= .02958 Liter.
1 Sq. inch	= .0006452 sq. meter.	1 Gallon	= 3.786 Liters.
1 Sq. foot	= .0929 "	1 Bushel	= .3524 Hectoliter.
1 Sq. yard	= .8361 "	1 Troy grain	= .0648 Gram.
1 Acre	= 40.47 Ares.	1 Troy lb.	= .373 Kilogram.
1 Sq. mile	= .259 Hectares.	1 Avoir. lb.	= .4536 Kilogram.
1 Cu. inch	= .01639 Liter.	1 Ton	= .9071 Tonneau.

EXAMPLES FOR PRACTICE.

451. Reduce

1. 84 lb. Avoir. to kilograms.
2. 37 T. to tonneau.
3. 96 bu. to hectoliters.
4. 75 fl. oz. to liters.
5. 89 cu. yd. to steres.
6. 328 acres to ares.
7. 4.0975 liters to cu. in.
8. 31.7718 sq. meters to sq. yd.
9. 272.592 liters to bushels.
10. 35.808 kilograms to Troy gr.
11. 133.75 steres to cords.
12. 33.307 steres to cu. ft.

13. If the price per gram is $.38, what is it per grain?

14. If the price per liter is $1.50, what is it per quart?

15. At 26.33 cents per hectoliter, what will be the cost of 157 bushels of peas?

16. When sugar is selling at 2.168 cents per kilogram, what will be the cost of 138 lb. at the same rate?

17. Reduce 834 grams to decigrams; to dekagrams.

18. In 84 hectoliters how many liters? how many centiliters?

19. A man travels at the rate of 28.279 kilometers a day. How many miles at the same rate will he travel in 45 days?

20. If hay is sold at $18.142 per ton, what is the cost of 48 tonneau at the same rate?

21. When a kilogram of coffee costs $1.1023, what is the cost of 148 lb. at the same rate?

DUODECIMALS.

452. *Duodecimals* are equal parts of a *linear, square* or *cubic* foot, formed by successively dividing by 12. Hence the following:

TABLE OF UNITS.

12 Thirds (‴)	= 1 Second	. 1″
12 Seconds	= 1 Prime	. 1′
12 Primes	= 1 Foot	. . *ft.*

1. ***Observe*** that each denomination in *duodecimals* may denote *length, surface,* or *volume.* Hence the highest denomination used must be marked so as to indicate whether the number represents *linear, surface* or *cubic* measure.

Thus, if the feet are marked ft., the lower denominations denote *length;* if marked sq. ft., *surface;* if marked cu. ft., *volume.*

2. Each of the following definitions should be *carefully studied* by drawing a diagram representing the unit defined. The diagram can be made on the blackboard on an enlarged scale.

453. A ***Linear Prime*** is *one-twelfth* of a foot; a ***Linear Second,*** *one-twelfth* of a linear prime; and a ***Linear Third,*** *one-twelfth* of a linear second.

454. A ***Surface Prime*** is *one-twelfth* of a *square* foot, and is 12 inches long and 1 inch wide, and is equal to 12 square inches.

455. A ***Surface Second*** is *one-twelfth* of a surface prime, and is 1 foot long and 1 *linear second* wide, which is equal to 1 *square inch.* Hence square inches are regarded as *surface seconds.*

456. A ***Surface Third*** is *one-twelfth* of a surface second, and is 1 foot long and 1 *linear third* wide, which is equal to 12 square seconds. Hence square seconds are regarded as *surface fourths.*

457. A ***Cubic Prime*** is *one-twelfth* of a cubic foot, and is 1 foot square by 1 inch thick, and is equal to a *board* foot.

458. A ***Cubic Second*** is *one-twelfth* of a cubic prime, and is 1 foot long by 1 inch square, and is equal to 12 *cubic* inches or a *board* inch.

459. A ***Cubic Third*** is *one-twelfth* of a cubic second, and is 1 foot long, 1 inch wide, and 1 *linear second* thick, and is equal to a cubic inch.

EXERCISE FOR PRACTICE.

460. Illustrate the following by diagrams on the blackboard:

1. 5 feet multiplied by 7 in. equals 35 *surface primes.*
2. 8 ft. multiplied by 4″ equals 32 *surface seconds.*
3. 7 feet multiplied by 6‴ equals 42 *surface thirds.*
4. 3 in. multiplied by 5 in. equals 15 *surface seconds.*
5. 4′ multiplied by 3″ equals 12 *surface thirds.*
6. From these examples deduce a rule for multiplying feet, inches, seconds, etc., by feet, inches, seconds, etc.

Multiply and explain the following:

7. 17 ft. 5′ 8″ by 8 ft. 9′ 7″.
8. 32 ft. 9′ 4″ by 6 ft. 5′ 11″.
9. 15 ft. 6′ 10″ by 9 ft. 4′ 8″.
10. 25 ft. 9′ 3″ by 14 ft. 7′ 2″.
11. 18 ft. 7′ 9″ by 12 ft. 8′ 5″.
12. 34 ft. 8′ by 26 ft. 4′ 9″.
13. 19 ft. 8′ 7″ by 2 ft. 5′ 9″ by 3 ft. 2′ 4″.
14. 48 ft. 9′ by 1 ft. 7′ 9″ by 2 ft. 8′ 5″.

Duodecimals are added and subtracted in the same manner as other compound numbers. Division being of little practical utility, is omitted. The pupil may, if desired, deduce a rule for division as was done for multiplication.

LONGITUDE AND TIME.

461. Since the earth turns on its axis *once* in *24 hours*, $\frac{1}{24}$ of 360°, or 15° of longitude must pass under the sun in 1 hour, and $\frac{1}{60}$ of 15°, or 15′ must pass under it in 1 minute of time, and $\frac{1}{60}$ of 15′, or 15″, must pass under it in 1 second of time. Hence the following

TABLE OF EQUIVALENTS.

A difference of	15°	in Long.	produces	a diff.	of	1 hr.	in time.
"	"	15′	"	"	"	1 min.	"
"	"	15″	"	"	"	1 sec.	"

Hence the following rule to find the difference of time between two places, when their difference of longitude is given:

462. RULE.—*Divide the difference of longitude of the two places by 15, and mark the quotient hours, minutes and seconds, instead of degrees, minutes and seconds.*

To find the difference of longitude when the difference of time is given.

463. RULE.—*Multiply the difference of time between the two places by 15, and mark the product degrees, minutes, and seconds, instead of hours, minutes, and seconds.*

Since the earth revolves from west to east, time is earlier to places west and later to places east of any given meridian.

EXAMPLES FOR PRACTICE.

464. Find the difference in time between the following:

1. Albany West Long. 73° 44′ 50″ and Boston W. Long. 71° 3′ 30″.

When the given places are on the same side of the first meridian, the difference of longitude is found by subtracting the lesser from the greater longitude.

2. Bombay East Long. 72° 54′ and Berlin East Long. 13° 23′ 45″.

3. New York W. Long. 74° 3′ and Chicago W. Long. 87° 37′ 4″.

4. San Francisco W. Long. 122° and St. Louis W. Long. 90° 15′ 15″.

5. Calcutta E. Long. 88° 19′ 2″ and Philadelphia W. Long. 75° 9′ 54″.

Observe, that when the given places are on *opposite sides* of the *first meridian,* the difference in longitude is found by adding the longitudes.

6. Constantinople E. Long. 28° 59′ and Boston W. Long. 71° 3′ 30″.

7. The difference in the time of St. Petersburg and Washington is 7 hr. 9 min. 19¼ sec. What is the difference in the longitude of the two places?

8. When it is 12 o'clock M. at New York, what time is it at a place 50° 24′ west?

9. In sailing from New Orleans to Albany, the chronometer lost 1 hr. 5 min. $10\frac{2}{3}$ sec. The longitude of Albany is 73° 44′ 50″. What is the longitude of New Orleans?

10. An eclipse is observed by two persons at different points, the one seeing it at 8 hr. 30 min. P. M., the other at 11 hr. 45 min. P. M. What is the difference in their longitude?

REVIEW AND TEST QUESTIONS.

465. 1. Define Related Unit, Denominate Number, Denominate Fraction, Denomination, and Compound Number.

2. Repeat Troy Weight and Avoirdupois Weight.

3. Reduce 9 bu. 3 pk. 5 qt. to quarts, and give a reason for each step in the process.

4. In 9 rd. 5 yd. 2 ft. how many inches, and why?

5. Repeat Square Measure and Surveyors' Linear Measure.

6. Reduce 23456 sq. in. to a compound number, and give a reason for each step in the process.

7. Define a cube, a rectangular volume, and a cord foot.

8. Show by a diagram that the contents of a rectangle is found by multiplying together its two dimensions.

9. Define a Board Foot, a Board Inch; and show by diagrams that there are 12 *board feet* in 1 cubic foot and 12 *board inches* in 1 board foot.

10. Reduce $\frac{3}{8}$ of an inch to a decimal of a foot, and give a reason for each step in the process.

11. How can a pound Troy and a pound Avoirdupois be compared?

12. Reduce .84 of an oz. Troy to a decimal of an ounce Avoirdupois, and give reason for each step in the process.

13. Explain how a compound number is reduced to a fraction or decimal of a higher denomination. Illustrate the abbreviated method, and give a reason for each step in the process.

ANSWERS.

The answers to oral exercises and the more simple examples have been omitted.

The answers for the exercises taken from the Arithmetical Tables, commence on page 392.

Art. 43.

1. $1525.
2. 1213.
3. 1599.
4. 5725.
5. 1542.
6. 322533.
7. $4030.
8. $2617.
9. $795.
10. $1330.

Art. 49.

1. $6217.65.
2. $3553.66.
3. $1004.94.
4. $75.38.
5. $906.75.
6. $312.09.
8. $607.65.
9. $179.31.
10. $5746.62.
11. $613.90.
12. $228.20.
13. $1751.22.
14. $8985.60.
15. $70.11.

Art. 69.

1. $1485.
2. 100 years.
3. 6269.
4. 6892 feet.
5. 5940.
6. 3502.
8. $550.
9. $1074.
10. $41874.
11. $4820.
12. $245.
13. $1634.
14. $1135.
15. 1005 bushels.
16. $262.64.
17. $166.38.
18. $11.06.
19. $95.12.
20. $272.59.
21. $901.92.

Art. 90.

6. $1995.
7. $3882.
8. $639.
9. 63360 feet.
10. $2556.
11. 1516 far.
12. $241.
13. $76.
14. $311.

Art. 99.

1. $24630.
2. $8300.
3. $72.
4. $1493.
5. 522270 gal.
6. 14136 bu.
7. 142692 days.
8. 8946 trees.
9. $9048.
10. $2373.
11. 35857536.
12. $1036800.
13. $11.25.
14. $635.27.
15. $72.25.
16. $5253.04 ; $669.
17. $21.99 gain.
18. $69.42.
19. $32.33 gain.
20. $2112 ; $345.60 gain.

Art. 124.

1. $985.
2. 231 ; 302.
3. 210 hours.
4. 147 barrels.
5. 121 months.
6. 25 weeks.
7. 134 baskets.
8. 86400 sec.
9. 480.
10. 47.
11. 59 dozen.
12. 69 sheep.
13. $178.
14. 76 barrels.

Art. 131.

1. 205.
2. 205.
3. 315.
4. 440.
5. 3158.
6. 832.
7. 1002.
8. 137.
9. 962.
10. 445.
11. 4455.
12. 4144.
13. 375.
14. 560.
15. 2247.
16. 4561.
17. 108.
18. 276.
19. 176.
20. 5362.
21. 90807.
22. 7967.
23. 1234.
24. 3147.
25. 46834.
26. 8643.
27. 7967.
28. 40367.
29. 14783.
30. 50406.
31. 39407.
32. 105070.

Art. 133.

2. 43 loads.
3. 15712.
4. 300 miles.
5. 36 acres.
6. 267 acres.
7. 758.
8. $1200.
9. 3600.

10. \$7080.
11. \$$491\frac{37}{52}$.
12. 275 acres.

Art. 157.

2. \$3384.
3. \$767.36.
4. \$735.56.
5. \$658.56.
6. \$20502.50.
7. \$632.20.
8. \$756.
9. \$331.89.
10. \$598.58.

Art. 158.

2. \$13.
3. \$3.
4. \$52.
5. \$7.
6. \$35.
7. \$18000.
8. \$72.

Art. 159.

2. \$1148.
3. \$1260.
4. \$3172.
5. 31104 miles.
6. \$162.
7. \$2496.

Art. 160.

2. 85 barrels.
3. 73 horses.
4. $161\frac{16}{38}$ acres.
5. 17 weeks.
6. 36 thousand.

Art. 161.

2. 2436 pounds.
3. 184 boxes.
4. 776 baskets.
5. 4900 pounds.
6. 168 days.
7. 168 cords.

Art. 162.

1. 36 pounds.
2. 712 cords.
3. \$192.
4. \$426.
5. \$3324.
6. \$470 gain.
7. \$1241.
8. \$504.
9. 60 ; 13.
10. 4714 ; 4262.
11. \$2096 ; \$1861.
12. \$4928 ; \$2466.
13. 50 days.
14. \$576.
15. \$56.
16. 6 bushels.
17. $149\frac{43}{87}$ acres ; $148\frac{63}{64}$ acres.

Art. 184.

1. 3, 7, 2, 5.
2. 3, 7, 2, 5, 3.
3. 2, 3, 7, 11.
4. 2, 3, 3, 7, 11.
5. 2, 5, 7, 3, 7.
6. 2, 2, 2, 3, 3, 113.
7. 2, 3, 11, 7, 7.
8. 7, 7, 89.
9. 3, 11, 2, 5, 7, 13.
10. 3, 3, 5, 5, 7, 7.
11. 5, 2, 5, 2, 2, 73.
12. 2, 2, 4007.
13. 5, 5, 13, 59.
14. 3, 3, 31, 37.
15. 2, 5, 5, 5, 5.
16. 2, 5, 2, 5, 2, 2, 2, 2, 2, 2.
17. 2, 5, 2, 2, 2, 2, 2, 7.
18. 2, 5, 2, 5, 2, 5.
19. 5, 3, 7, 43.
20. 2, 3, 7, 11, 17.
21. 2, 5, 3, 7, 11.
22. 2, 5, 5, 109.
23. 2, 2, 2, 2, 2, 2, 3, 3, 3, 3.
24. 5, 2, 2, 2, 229.
25. 2, 5, 11, 73.
26. 5, 7, 7, 17.
27. 2, 5, 2, 5, 5, 5, 5, 5.
28. 2, 5, 2, 5, 2, 5, 3, 3, 3, 3.
29. 2, 5, 2, 5, 2, 5, 2, 2, 2, 2, 2, 2.
30. 2, 5, 2, 5, 5, 7, 13.
31. 5, 5, 5, 5, 3, 3, 3.
32. 2, 5, 11, 13, 13.
33. 2, 5, 2, 3, 3, 7, 13.

Art. 188.

1. $1\frac{1}{2}$.
2. $16\frac{1}{7}$.
3. $15\frac{11}{27}$.
4. 20.
5. $5\frac{5}{48}$.
6. $166\frac{2}{3}$.
7. 200.
8. 27.
9. 2.
10. 20.
11. 42.
12. 4 barrels.
13. 30 weeks.
14. 6 shillings.
15. 80 loads.
16. 5 boxes.
17. 915 pounds.

Art. 196.

1. 15.
2. 5.
3. 21.
4. 39.
5. 4.
6. 18.
7. 17.
8. 81.
9. 75.
10. 34.
11. 33.
12. 46.
13. 35.
14. 5.
15. 3.
16. 12.
17. 45.
18. 24.

Art. 203.

1. 15.
2. 21.
3. 22.
4. 35.
5. 2.
6. 5.
7. 4.

8. 3.
9. 5.
10. 91.

Art. 210.

1. 8.
2. 3 feet.
3. 12 inches.
4. 36 feet.
5. 14 feet.
6. 12 barrels.
7. 14 yards.
8. \$38 ;
   214 acres ;
   165 acres.

Art. 217.

1. 2520.
2. 60 cents.
3. 36 quarts.
4. 168 rows.
5. \$798.
6. 36 feet.
7. 1650 feet.

Art. 221.

1. 182.
2. 330.
3. 390.
4. 2145.
5. 462.
6. 5005.
7. 2548.
8. 6006.

Art. 248.

8. $\frac{228}{5}$.
9. $\frac{585}{7}$.
10. $\frac{167}{12}$.
11. $\frac{613}{8}$.
12. $\frac{1363}{4}$.
13. $\frac{1314}{7}$.
14. $\frac{3699}{8}$.
15. $\frac{5561}{12}$.
16. $\frac{158}{45}$.
17. $\frac{1611}{213}$.
18. $\frac{407}{100}$.
19. $\frac{2087}{1000}$.

Art. 251.

4. $33\frac{7}{25}$.
5. $21\frac{33}{43}$.
6. $7\frac{4}{84}$.
7. $12\frac{49}{73}$,
8. $4\frac{37}{100}$.
9. $36\frac{82}{100}$.
10. $86\frac{20}{54}$.
11. $64\frac{275}{325}$.
12. $32\frac{41}{101}$.
13. $14\frac{356}{500}$.
14. $819\frac{1}{107}$.
15. $164\frac{197}{308}$.
16. $89\frac{256}{1009}$.
17. $64\frac{17}{32}$.
18. $6\frac{850}{909}$.

Art. 257.

7. $\frac{7}{15}$.
8. $\frac{6}{11}$.
9. $\frac{41}{57}$.
10. $\frac{13}{17}$.
11. $\frac{16}{25}$.
12. $\frac{5}{11}$.
13. $\frac{275}{903}$.
14. $\frac{35}{52}$.
15. $\frac{9}{17}$.
16. $\frac{117}{296}$.
17. $\frac{3}{4}$.
18. $\frac{123}{127}$.

Art 260.

5. $\frac{25}{30}$; $\frac{27}{30}$; $\frac{28}{30}$.
6. $\frac{16}{24}$; $\frac{18}{24}$; $\frac{20}{24}$; $\frac{21}{24}$.
7. $\frac{36}{90}$; $\frac{60}{90}$; $\frac{50}{90}$; $\frac{63}{90}$.
8. $\frac{45}{126}$; $\frac{78}{126}$; $\frac{98}{126}$.
9. $\frac{434}{756}$; $\frac{297}{756}$; $\frac{636}{756}$; $\frac{189}{756}$.
10. $\frac{224}{672}$; $\frac{588}{672}$; $\frac{560}{672}$; $\frac{432}{672}$; $\frac{72}{672}$; $\frac{357}{672}$.
11. $\frac{308}{396}$; $\frac{180}{396}$; $\frac{286}{396}$; $\frac{54}{396}$; $\frac{11}{396}$.
12. $\frac{486}{729}$; $\frac{324}{729}$; $\frac{189}{729}$; $\frac{48}{729}$; $\frac{31}{729}$; $\frac{72}{729}$.

Art. 264.

1. $2\frac{23}{60}$.
2. $3\frac{1}{14}$.
3. $\frac{271}{280}$.
4. $2\frac{7}{48}$.
5. $3\frac{5}{8}$.
6. $1\frac{27}{40}$.
7. $1\frac{73}{180}$.
8. $2\frac{3}{4}$.
9. $2\frac{3}{16}$.
10. $1\frac{23}{32}$.
11. $16\frac{5}{12}$.
12. $8\frac{11}{12}$.
13. $16\frac{17}{60}$.
14. $19\frac{14}{45}$.
15. $7\frac{95}{132}$.
16. $98\frac{23}{48}$.
17. $143\frac{47}{63}$.
18. $121\frac{45}{154}$.
19. $32\frac{1}{16}$.
20. $179\frac{1}{12}$ pounds.
21. $28\frac{17}{18}$ feet.
22. $5\frac{53}{72}$ yards.
23. \$$14\frac{7}{8}$.
24. $146\frac{97}{168}$ miles.
25. $185\frac{127}{168}$ yards.

Art. 268.

1. $1\frac{4}{9}$.
2. $\frac{23}{119}$.
3. $2\frac{13}{18}$.
4. $2\frac{27}{28}$.
5. $1\frac{7}{8}$.
6. $4\frac{7}{120}$.
7. $3\frac{137}{156}$.
8. $3\frac{1}{48}$.
9. $46\frac{20}{21}$.
10. $43\frac{35}{36}$.
11. $17\frac{5}{12}$.
12. $5\frac{7}{12}$.
13. $15\frac{1}{8}$ gallons.
14. \$$3\frac{2}{15}$,
15. $5\frac{1}{3}$ barrels.
16. $28\frac{11}{12}$.
17. $19\frac{5}{8}$ miles.
18. \$$4\frac{2}{15}$,
19. $96\frac{1}{8}$ pounds.

Art. 273.

10. $3\frac{51}{94}$.
11. $3\frac{11}{25}$.
12. $8\frac{2}{5}$.
13. $12\frac{7}{10}$.
14. $21\frac{115}{281}$.
15. 50.
16. 72.
17. $65\frac{655}{978}$.
18. $10\frac{90}{1001}$.

Art. 275.

1. \$266 ;
   \$$777\frac{7}{9}$;
   \$3780 ;
   \$$7778\frac{5}{9}$.
2. 7857 ;
   $51\frac{3}{10}$;
   $751\frac{1}{2}$;
   9 ;
   90.
3. \$360.
4. $142\frac{23}{36}$ acres.
5. \$580.
6. $335\frac{2}{15}$ yards.

Art. 278.

4. $\frac{5}{69}$; $\frac{6}{61}$; $\frac{7}{45}$; $\frac{29}{95}$.
5. $\frac{172}{511}$; $11\frac{11}{15}$; $\frac{25}{252}$; $\frac{3}{13}$.
6. \$$\frac{7}{20}$.
7. $\frac{3}{7}$.

Art. 281.

10. $959\frac{2}{5}$.
11. $952\frac{1}{2}$.
12. $44571\frac{7}{8}$.
13. 1534.
14. 73600.
15. 74800.

Art. 286.

1. $\frac{1}{6}$.
2. $\frac{5}{17}$.
3. $\frac{14}{45}$.
4. $\frac{4}{9}$.
5. $\frac{13}{20}$.
6. $\frac{2}{15}$.
7. $\frac{52}{561}$.
8. $\frac{450}{1819}$.
9. $\frac{2}{35}$.
10. $\frac{7}{144}$.
11. $\frac{3}{425}$.
12. $242\frac{17}{24}$.
13. $\frac{95}{252}$.
14. $\frac{1491}{21944}$.
15. $122\frac{2}{5}$.

16. \$171.50.
17. $\$68\frac{17}{24}$.
18. 1330104.
19. $\$1268\frac{7}{40}$.
20. $\$53\frac{39}{40}$; $\$110\frac{31}{45}$.
21. $\$262\frac{493}{1120}$.
22. $129\frac{313}{1155}$.
23. $5002\frac{1}{2}$.
24. 29 ct.
25. $69\frac{1}{15}$ tons.
26. $2\frac{23}{42}$.
27. $\$10\frac{19}{28}$.
28. $26\frac{173}{210}$.

Art. 289.

7. $\frac{1}{59}$.
8. $\frac{5}{432}$.
9. $\frac{3}{236}$.
10. $\frac{43}{2100}$.
11. $\frac{4}{365}$.
12. $\frac{1}{108}$.
13. $\$\frac{5}{56}$.
14. $\$\frac{1}{6}$.
16. $6\frac{58}{115}$.
17. $\$2\frac{13}{24}$; $\$22\frac{7}{8}$; $\$12\frac{17}{24}$; $\$68\frac{5}{8}$.
18. $\frac{1}{288}$.
19. $\frac{3}{64}$.
20. $\frac{1}{18}$.
21. $\frac{169}{540}$.

Art. 293.

24. $\frac{69}{79}$.
25. $1\frac{4}{5}$.
26. $1\frac{1}{4}$.
27. $\frac{26}{123}$.
28. $364\frac{1}{2}$.
29. $982\frac{2}{7}$.
30. $\frac{51}{367}$.
31. $1445\frac{2}{17}$.
32. $\frac{1250}{1679}$.
33. $825\frac{3}{25}$.
34. $\frac{31}{50}$.
35. $1\frac{5}{14}$.
36. 3540.
37. 11295.
38. $2\frac{6721}{13087}$.

Art. 296.

1. $\frac{4}{7}$.
2. $2\frac{43}{45}$.
3. $1\frac{47}{68}$.
4. $11\frac{16}{39}$.
5. $4807\frac{7}{9}$.
6. $20\frac{50}{213}$.
7. $18\frac{47}{53}$.
8. $4\frac{302}{747}$.
9. $50\frac{5}{6}$.
10. $3\frac{215}{299}$.
11. $5\frac{62}{63}$.
12. $3\frac{1121}{2003}$.
13. $14287\frac{1}{7}$.
14. $186\frac{42}{503}$.
15. $1813\frac{2}{13}$.
16. $2\frac{1192}{2019}$.
17. $978\frac{14}{37}$.
18. $363\frac{3}{5}$.
19. $4\frac{4}{21}$.
20. $16\frac{16}{21}$.
21. $54\frac{4}{9}$.
22. $7\frac{61}{72}$ bushels; $17\frac{11}{12}$ bushels; $171\frac{11}{64}$ bu.
23. $\$16\frac{2}{3}$; \$5; $\$94\frac{4}{9}$; $\$496\frac{8}{27}$.
24. $\$124\frac{17}{32}$.

Art. 303.

1. $\frac{65}{72}$.
2. $\frac{380}{469}$.
3. $\frac{510}{781}$.
4. $3\frac{1}{9}$.
5. $\frac{789}{1318}$.
6. $1\frac{433}{570}$.
7. $\frac{378}{865}$.
8. $\frac{735}{1352}$.
9. $\frac{1932}{2945}$.
10. $\frac{1}{63}$.
11. $2\frac{22}{75}$.
12. $\frac{19}{4000}$.

Art. 306.

1. $\frac{2}{3}$; $\frac{2}{3}$; $\frac{2\frac{3}{5}}{3}$; $\frac{2\frac{3}{8}}{3}$, $\frac{2\frac{2}{3}}{3}$, $\frac{2\frac{1}{2}}{3}$.
2. $\frac{5\frac{2}{3}}{7}$; $\frac{6\frac{1}{2}}{7}$; $\frac{6\frac{7}{9}}{7}$.
3. $\frac{4\frac{2}{3}}{7}$; $\frac{3\frac{8}{9}}{7}$; $\frac{4\frac{1}{5}}{7}$; $\frac{4\frac{1}{2}}{7}$; $\frac{2\frac{5}{8}}{7}$; $\frac{5\frac{5}{6}}{7}$.
4. $\frac{15\frac{25}{27}}{20}$; $\frac{7\frac{1}{6}}{9}$.
5. $\frac{50}{100}$; $\frac{75}{100}$; $\frac{70}{100}$; $\frac{80}{100}$; $\frac{45}{100}$.
6. $\frac{6\frac{2}{3}}{10}$; $\frac{8\frac{3}{4}}{10}$; $\frac{4\frac{1}{6}}{10}$; $\frac{9\frac{19}{39}}{10}$.
7. $\frac{71\frac{3}{7}}{100}$; $\frac{88\frac{8}{9}}{100}$; $\frac{58\frac{1}{3}}{100}$; $\frac{60}{100}$; $\frac{70\frac{60}{77}}{100}$.
8. $\frac{625}{1000}$; $\frac{760}{1000}$; $\frac{472}{1000}$; $\frac{935\frac{15}{31}}{1000}$.
9. $\frac{59\frac{33}{38}}{100}$; $\frac{16\frac{19}{66}}{100}$; $\frac{76\frac{12}{13}}{100}$; $\frac{57\frac{529}{703}}{100}$.
10. $\frac{166\frac{2}{3}}{100}$; $\frac{450}{100}$; $\frac{775}{100}$; $\frac{980}{100}$; $\frac{1700}{100}$; $\frac{80}{100}$; $\frac{70}{100}$; $\frac{230}{100}$.
11. $\frac{37\frac{1}{7}}{100}$; $\frac{70\frac{10}{17}}{100}$; $\frac{119\frac{9}{14}}{100}$; $\frac{217\frac{7}{29}}{100}$.

Art. 307.

1. $\frac{25}{30}$; $\frac{12}{30}$.
2. $\frac{12}{28}$; $\frac{10}{28}$; $\frac{18}{28}$; $\frac{4\frac{1}{2}}{28}$; $\frac{9}{28}$.
3. $\frac{24}{32}$; $\frac{14}{32}$; $\frac{7}{32}$.
5. $\frac{12}{15}$; $\frac{20}{25}$; $\frac{2}{2\frac{1}{2}}$; $\frac{3}{3\frac{3}{4}}$; $\frac{7}{8\frac{3}{4}}$.
6. $\frac{15}{25}$; $\frac{15}{21}$.
7. $3\frac{13}{24}$.
8. $\frac{10}{111}$.
9. $\$5272\frac{8}{21}$.
10. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{8}$; $\frac{1}{12}$.
11. $\frac{1}{30}$; $\frac{1}{6}$; $\frac{1}{40}$; $\frac{1}{12}$.
12. $\frac{3}{7}$.
13. 49.
14. $\$1614\frac{3}{5}$.
15. $40\frac{5}{7}$ tons.
16. $121\frac{2}{3}$ yards.
17. $\$63\frac{5}{9}$.
18. $25\frac{1}{5}$.
19. $\$289\frac{1}{6}$.
20. $47\frac{13}{16}$ miles; $38\frac{15}{16}$ miles.
21. $\$8113\frac{1}{14}$.
22. $556\frac{5}{36}$.
23. \$10500.
24. 504.
25. \$1800.
26. $123\frac{3}{7}$ cords.
27. $4\frac{124}{125}$ pounds; $8\frac{56}{225}$ pounds; $\frac{28}{75}$ pounds.
28. Increased by $\frac{5}{21}$.
29. $\$1\frac{1541}{4144}$.
30. $2\frac{11}{12}$ days.
31. $\$65\frac{11605}{15264}$.
32. 15 feet.
33. $1196\frac{229}{250}$ yds. $\$2\frac{8}{21}$.
34. $\$289\frac{53}{63}$.
35. $13\frac{11}{25}$ acres.
36. $\$5006\frac{1}{4}$.

Art. 340.

1. .4375.
2. .6.
3. .84375.
4. .36.
5. .712.
6. .4625.
7. .5008.
8. .053125.
9. $.3571\frac{3}{7}$.
10. $.4230\frac{10}{13}$.
11. $.1857\frac{1}{7}$.
12. $.3538\frac{6}{18}$.

13. $.0352\frac{16}{17}$.
14. $.0879\frac{11}{91}$.
15. $.1917\frac{19}{193}$.
16. $.1713\frac{37}{751}$.
17. $.\dot{4}2857\dot{1}$.
18. $.\dot{6}1904\dot{7}$.
19. $.5\dot{3}57142\dot{8}$.
20. $.\dot{8}4615\dot{3}$.
21. $.\dot{5}6410\dot{2}$.
22. $.32\dot{6}92307\dot{7}$.
23. $.8\dot{6}\dot{3}$.
24. $.\dot{3}921568627$-
450980̇.
25. $.4\dot{3}\dot{6}$.
26. $8.\dot{7}1428\dot{5}$.
27. $24.32\dot{1}4285\dot{7}$.
28. $32.4\dot{0}7692\dot{3}$.

Art. 343.

1. $\frac{43}{200}$.
2. $\frac{21}{25}$.
3. $\frac{3}{4}$.
4. $\frac{27}{5000}$.
5. $\frac{1}{80}$.
6. $\frac{4}{625}$.
7. $\frac{3}{3125}$.
8. $\frac{321}{40000}$.
9. $\frac{3}{4000}$.
10. $\frac{1}{16}$.
11. $\frac{16}{3125}$.
12. $\frac{26}{3125}$.

Art. 346.

1. $\frac{4}{11}$.
2. $\frac{26}{33}$.
3. $\frac{6}{11}$.
4. $\frac{124}{333}$.
5. $\frac{856}{999}$.
6. $\frac{5}{37}$.
7. $\frac{7}{37}$.
8. $\frac{12}{37}$.
9. $\frac{836}{999}$.
10. $\frac{488}{909}$.
11. $\frac{302}{1111}$.
12. $\frac{907}{1111}$.
13. $\frac{416}{11}$.
14. $\frac{337}{37}$.
15. $\frac{389}{111}$.
16. $\frac{985}{11}$.
17. $\frac{1973}{37}$.
18. $\frac{23735}{999}$.
19. $\frac{70298}{1111}$.
20. $\frac{2948}{101}$.
21. $\frac{670}{111}$.

Art. 349.

1. $\frac{47}{66}$.
2. $\frac{95}{99}$.
3. $\frac{241}{495}$.
4. $\frac{1081}{24975}$.
5. $\frac{127}{19800}$.
6. $\frac{217}{6600}$.
7. $\frac{4463}{124875000}$.
8. $\frac{1660371}{1999800000}$.
9. $\frac{3455459}{1666500000}$.
10. $\frac{1055}{198}$.
11. $\frac{1931}{198}$.
12. $\frac{73}{30}$.
13. $\frac{118}{15}$.
14. $\frac{269}{22}$.
15. $\frac{27}{5}$.

Art. 352.

1. 572 377.
2. 158.9656.
3. $538.26.
4. 105.9817.
5. 492.4198.
6. 16.62229.
7. $121.11.
8. 115.25493.

Art. 354.

1. 782.43.
2. 32.0445.
3. $37.26.
4. $16.65.
5. 37.124.
6. 374.960401.
7. 17.49603.
8. 6.93995.
9. .0894097.
10. .600091.
11. $72.091.
12. 94.1881.
13. 17.42.
14. 5.552.
15. $170.75.
16. $738.0225.

Art. 357.

1. 35.77.
2. 4.958.
3. 217.496.
4. $4075.26.
5. $58.555.
6. 286.625901.
7. 66.285618.
8. .029418.
9. .00748.
10. .36288.
11. .000072.
12. .0000252.
13. .003645.
14. .00000101
15. .001535.
16. $344.13\frac{1}{3}$.
17. $24.17\frac{7}{9}$.
18. $1.06375.
19. .0062.
20. .46075.
21. .00033575.
22. $7.47891.
23. $533.2114.
24. $685.6289.
25. $222.906\frac{2}{3}$ gr.
26. 52.16631 in.
27. 2.35644 yd.
28. $271.3017.
29. $5266.833+.
30. 1908.75.
31. $46070.
32. $33.88\frac{1}{2}$.
33. $4.96.
34. $362.90.

Art. 358.

1. .75.
2. .35.
3. .625.
4. .325.
5. .46875.
6. .1125.
7. $.\dot{7}1428\dot{5}$.
8. $.\dot{7}\dot{2}$.
9. $.8\dot{3}$.
10. .15.
11. $.079\dot{5}\dot{4}$.
12. $.\dot{3}0769\dot{2}$.

Art. 359.

1. 46.2857+.
2. 20.5652+.
3. 8.0721+.
4. .8640+.
5. 3.8235+.
6. .8604+.
7. 29.3661+.
8. 5.4844+.
9. 9.9009+.

Art. 363.

1. 17.0769.
2. 6.8548 ; 38.3680.
3. 1960.5945.
4. 1.0979.
5. .7583.
6. 6.0833.
7. 160.6284.
8. 142857.1428.
9. 4.1710.
10. 1.4743.
11. 73.3743.
12. 161.2496.
13. $5.8362.
14. 89.5901.
15. 161.1379 yd.
16. 5.1369.
17. 179.7866 lb.
18. 214.2327 yd.
19. 32.9685 tons.
20. .378.
21. $2320.4678.
22. $13450.

Art. 364.

1. $.7446.
2. $332.325.
3. $43.875.
4. $309.58.
5. $35.1125.
6. $281.567.
7. $53.8135.
8. .9166+.
9. .8888+.

10. .5.
11. .4888+.
12. .7714+.
13. .5050+.
14. .39.
15. .0199+.
16. 1.7666+.
17. .3478260+.
18. .03177+.
19. .3926+.
20. 11232.8110+.
21. $821.0125.
22. $94.22.
23. $3188.065.
24. $5.5566+.
25. 59.75 yards.
26. 439 bushels.
27. 61.44 yards; 82.5806+ yards; 48.9739+ yards.
28. $1\frac{16}{39}$.
29. 190.116 pounds.
30. $361.785.
31. $145.52.
32. $1.85.

Art. 377.

5. 27700 pounds.
6. 1885 drams.
7. 21900 grains.
8. 103240 grains.
9. 158328 grains.
10. 346547 pounds.
11. 5749 pennyweight.
12. 175393 pounds.
13. 217339 grains.
14. 5888000 ounces.
15. 7000 grains.
16. 39377 pennyweight.

Art. 380.

5. 6 lb. 9 oz. 3 pwt. 12 gr.
6. 438 T. 4 cwt. 45 lb.
7. 1017 lb. 2 oz.
8. 15 T. 14 cwt. 77 lb. 8 oz.
9. 48 lb. 6 oz. 4 dr. 2 sc. 7 gr.
10. 173 T. 8 cwt.
11. 35 lb. 9 oz. 17 pwt.
12. 1564 dr. 1 sc. 15 gr.
13. 27 T. 6 cwt. 4 oz.
14. 92 lb. 9 oz. 1 pwt. 14 gr.
15. 16 lb. 3 oz. 1 pwt. 21 gr.; $13\frac{529}{1400}$ lb. 16 lb. 3 oz. 2 sc. 5 gr.;
16. $7\frac{71}{175}$ lb.
17. 70 lb. 3 oz. 1 pwt. $23\frac{1}{2}$ gr.
18. 17 lb. 6 oz. 4 dr. 1 sc. $2\frac{1}{2}$ gr.
19. 5 lb. ℥10 ʒ6 ℈2 8 gr.

Art. 383.

1. 11 cwt. 11 lb. $1\frac{7}{9}$ oz.
2. ℈2 2 gr.
3. 8 oz. 14 pwt. $13\frac{1}{11}$ gr.
4. ℥8 ʒ4 ℈1 $14\frac{2}{7}$ gr.
5. 9.6 oz.
6. 17 cwt.
7. ℈2 16.4 gr.
8. 14 pwt. 14.4 gr.
9. 58 lb. $5\frac{1}{3}$ oz.
10. 15 lb. 10 oz.
11. 7 lb. 5 oz. 6 pwt. 16 gr.
12. 3 cwt. 70 lb.
13. lb. 5 ℥11 ʒ2 14.4 gr.
14. 13 T. 14 cwt. 28 lb. $9\frac{1}{7}$ oz.
15. 8 oz. 16 pwt. 17.664 gr.
16. 18 cwt. 71 lb. 3.2 oz.
17. 102 lb. 1 oz.
18. 1 lb. 15 pwt. 5 gr.
19. 86 lb. 9 oz. 13 pwt. 8 gr.; lb. 86 ℥9 ʒ5 ℈1.

Art. 386.

9. $\frac{3}{320000}$ T.; $\frac{21}{50000}$ T.; $\frac{1}{35}$ T.
10. $\frac{5}{864}$ lb.; $\frac{1}{360}$ lb.
11. $\frac{7}{96000}$ lb.; $\frac{1}{250}$ lb.
12. $\frac{3}{64}$ lb.; $\frac{3}{6400}$ cwt.
13. $\frac{1}{32}$ T.; $\frac{9}{200}$ T.; $\frac{1}{2500}$ T.; $\frac{17}{1000}$ T.; $\frac{43}{100000}$ T.; $\frac{1}{200000}$ T.; $\frac{3}{128000}$ T.

Art. 389.

1. $\frac{63}{250}$ T.
2. $\frac{457}{720}$ lb.
3. $\frac{259}{400}$ cwt.
4. $\frac{313}{360}$ lb.
5. $\frac{233}{288}$ lb.
6. $\frac{199}{2880}$ lb.
7. $\frac{15}{32}$ lb.
8. $\frac{593}{1440}$ lb.
9. $\frac{5759}{5760}$ lb.
10. $\frac{54}{125}$ T.
11. $\frac{41}{90}$.
12. $\frac{5}{8}$.
13. $.122\frac{2}{9}$.
14. .05375.
15. .4455 T.
16. $.7361\frac{1}{7}$ lb.
17. $.1138\frac{8}{9}$.
18. .81875 lb.
19. .634 T.
20. $.8121527\frac{7}{9}$ lb.
21. .9868+ lb.
22. .71498+.
23. $.5944\frac{4}{9}$.

Art. 391.

1. 74 T. 3 qr. 4 lb. 10 oz.
2. 33 lb. 2 oz. 1 sc. 12 gr.
3. 1 lb. 3 oz. 18 pwt. $20\frac{2}{7}$ gr.
4. 10 oz. 4 pwt. 9.6 gr.
5. 5 cwt. 20 lb. $12\frac{3}{4}$ oz.
6. 4 T. 3 cwt. 9 lb. 10 oz.
7. lb. 14 ℥2 ʒ4 ℈2 14 gr.
8. 49 cwt. 2 qr. 6 lb.
9. 10 T. 11 cwt. 42 lb. 15 oz.
10. 37 lb. 2 oz. 12 pwt. 18 gr.
11. lb. 40 ℥10 ℈1.
12. 8 lb. 11 oz. 19 pwt. 15.2 gr.
13. 80 lb. .5 oz.

14. 1 T. 14 cwt. 92 lb. 14 oz.

Art. 393.

1. 11 T. 18 cwt. 1 qr. 21 lb. 3 oz.
2. lb. 5 ℥9 ʒ6 ℈2 3 gr.
3. 11 lb. 4 oz. 13 pwt. 12 gr.
4. 4 lb. 6 oz. 11 pwt. 12 gr.
5. 1 T. 13 cwt. 62 lb. 8 oz.
6. ℥6 ʒ7 10 gr.
7. 5 lb. 10 oz. 7 pwt. 9 gr.
8. 3 cwt. 54 lb. 12 oz.
9. 9 oz. 8 pwt. 16.8 gr.
10. 7 T. 7 cwt. 45 lb.
11. lb. 7 ℥5 ʒ6 ℈1 5 gr.
12. lb. 3 ℥11 ʒ1 ℈2 10 gr.
13. 6 T. 12 cwt. 2 qr. 14 lb.

Art. 395.

1. 15 lb. 2 oz. 10 pwt. 20 gr.; 26 lb. 7 oz. 8 pwt. 23 gr.
2. 3 T. 9 cwt. 14 lb. 13 oz.; 4 T. 12 cwt. 19 lb. 12 oz.; 24 T. 4 cwt. 3 lb. 11 oz.
3. 18 T. 17 cwt. 52 lb.
4. 7 cwt. 66 lb. 5 oz.
5. 62 lb. 4 oz. 7 dr. 15 gr.
6. 1 cwt. 19 lb. $12\frac{4}{9}$ oz.
7. 1 cwt. 7 lb. $14\frac{2}{5}$ oz.
8. 3 T. 6 cwt. 92 lb.
9. 54 lb. 12 oz.
10. 25 lb. 14 oz.
11. 6 T. 3 cwt. 12 lb.
12. ℈2 11.12 gr.
13. 2 cwt. 59 lb. 6.72 oz.
14. 9 cwt. 72 lb. $3\frac{5}{9}$ oz.
15. $8\frac{24}{25}$ oz.
16. 50 lb.
17. 19.44 gr.
18. 78 T. 3 cwt. 16 lb.
19. 40 T. 5 cwt. 92 lb.

Art. 396.

1. 4 T. 17 cwt. 3 qr. 21 lb. 8 oz.; 1 T. 19 cwt. 18 lb. $9\frac{3}{5}$ oz.; 1 T. 4 cwt. 1 qr. 24 lb. 2 oz. 16 cwt. 1 qr. 7 lb. 12 oz.
2. 4 lb. 2 oz. 16 pwt. $13\frac{5}{7}$ gr.
3. ʒ1 5 gr.
4. ℥10 ʒ7 10 gr.; ℥7 ʒ2 $6\frac{2}{3}$ gr.; ℥3 ʒ5 $3\frac{1}{3}$ gr.; ℥1 ʒ2 ℈2 $13\frac{3}{4}$ gr.
5. 1 cwt. 10 lb. $11\frac{39}{43}$ oz.
6. 34.
7. 25.
8. 35 boxes.
9. $83\frac{33}{47}$.
10. $511\frac{7}{23}$; 1920.
11. ℥2 ʒ5 ℈1.
12. $12413\frac{23}{29}$; $10\frac{2}{3}$.

Art. 398.

1. 1386 in.
2. 5280 yd.
3. 21.85511 + mi.
4. 2100 l.
5. $84\frac{6}{7}$ in.
6. $55\frac{5}{9}$ l.
7. 1037.5 mi.
8. 4.3866 + statute mi.
9. 2544 in,
10. 245 statute mi.
11. 16575 l.
12. 233 rd. 3 yd. 10.8 in.
13. .000482 + mi.
14. $.\dot{4}64\dot{6}$.
15. 66 ch. 2 rd. 7 l. 1.76 in.
16. 2 mi. 48 ch. 3 rd. 24 l. 1.98 in.
17. 10 mi. 14 ch. 5 l.
18. 7 mi. 258 rd. 2 yd. $2\frac{10}{17}$ ft.
19. $1085.91\dot{6}$ statute mi.
20. $2\frac{1}{4}$ degrees; 155.625 statute mi.
21. $\frac{5}{72}$.
22. $.0\dot{3}\dot{6}$ mi.
23. $145.8\dot{3}$ statute mi.
24. $.\dot{0}60\dot{6}$.
25. .66 ft.

Art. 407.

1. 130680 sq. ft.
2. 15488000 sq. yd.
3. 4017.2 sq. yd.
4. 19200 sq. ch.
5. 448000 sq. l.
6. 8000 sq. l.
7. .00516528 + A.
8. .0001367 + sq. mi.
9. 284 A. 71 P. 3 sq. yd. 3 sq. ft. 36 sq. in.
10. 21.78 sq. ft.
11. 25 sq. mi. 457 A. 1 sq. ch. 6 P. $535\frac{5}{7}$ sq. l.
12. 81 P. 27 sq. yd. 7 sq. ft. 67.68 sq. in.
13. 4 A. 68 P. 6 sq. ft. $32\frac{11}{14}$ in.
14. 1 Tp. 12 sq. mi. 188 A. 9 sq. ch. 1 P.
15. 4 sq. mi. 319 A. 9 sq. ch. 8 P. $458\frac{1}{3}$ sq. l.
16. 9 sq. ch. 15 P. 624 sq. l. 35 sq. mi. 639 A. 9 sq. ch. 15 P. 624 sq. l.
17. 140 A. 6 sq. ch. 13 P. $89\frac{2}{7}$ sq. l.
18. 5 P. 28 sq. yd. 6 sq. ft. 108 sq. in.
19. 9 P. 5 sq. yd. 4 sq. ft. 108 sq. in.
20. 5 P. 29 sq. yd. 1 sq. ft. 18 sq. in.

21. 2 A. 3 sq. ch. 1 P. 375 sq. l.
22. 18 sq. yd. 1 sq. ft. 57$\frac{3}{5}$ sq. in.
23. 1 P. 2 sq. yd. 8 sq. ft. 14.4 sq. in.
24. 2230 sq. ft.
25. 384 boards.
26. 309$\frac{1}{3}$ yds.
27. $620.
28. $485.275.
29. 1096.98 tiles.
30. 1200 stones; 2129.72\frac{2}{9}$.
31. $500.76.
32. 11352 shingles.
33. $50.53.

Art. 415.

1. 167616 cu. in.
2. 2.43 cu. ft.
3. 867456 cu. in.
4. 32.6592 cu. in.
5. $.0\dot{2}\dot{2}$ cu. yd.
6. $.0\dot{2}9\dot{6}$ cu. yd.
7. 18 cu. ft. 1080 cu. in.
8. 9 cu. ft. 648 cu in.
9. 3 cu. yd. 18 cu. ft.
10. 58 cu. yd. 21 cu. ft. 1664 cu. in.
11. 4 cu. yd. 8 cu. ft. 432 cu. in.
12. 29 cu. yd. 22 cu. ft. 576 cu. in.
13. 5 cu yd. 14 cu. ft. 1080 cu. in.
14. 13 ft. 5$\frac{7}{9}$ in.
15. 48 ft.
16. 4 ft. 6 in.
17. 92808$\frac{17}{48}$ cu. ft.
18. 8273$\frac{31}{81}$ cu. yd.
19. 113$\frac{17}{36}$ cu ft.
20. 593$\frac{377}{486}$ cu. yd.
21. $233.6244+.
22. 87$\frac{82}{99}$ pch.

Art. 416.

1. 1584 cu. ft.
2. 336 cd. ft.
3. 4608 cu. ft.
4. 20.79 cu. ft.
5. 91$\frac{3}{7}$ cu. ft.
6. 1.3432098+.
7. 2.6337448+ cu. yd.
8. $.03\dot{2}3\dot{2}$ pch.
9. .005703125 cd.
10. .444$\frac{4}{9}$.
11. 2 cd. 3 cd. ft. 8 cu. ft. 972 cu. in.
12. 27$\frac{3}{7}$ bricks.
13. 16 cd. 7 cd. ft. 15 cu. ft. 345$\frac{3}{5}$ cu. in.
14. 91$\frac{7}{8}$ cd.
15. 53.0578\frac{1}{8}$.
16. 8 ft. 6$\frac{2}{5}$ in.
17. 108$\frac{4}{11}$ pch.
18. 160$\frac{45}{64}$ cd.
19. 17.21+ bricks.
20. 83387.3266+.
21. 405$\frac{1}{3}$ pch.
22. 2234\frac{2}{27}$.
23. $803.03631+.
24. $322.50.
25. $296.40.

Art. 420.

1. 26 B. ft.
2. 35 B. ft.
3. 24 B. ft.
4. 24 B. ft.
5. 28 B. ft. 2 B. in.
6. 42 B. ft. 6 B. in.
7. 7 B. ft. 6 B. in.
8. 42 B. ft. 6 B. in.
9. 29 B. ft. 2 B. in.
10. 18 B. ft. 9 B. in.
11. 40 B. ft. 6 B. in.
12. 44 B. ft. 4 B. in.
13. 32 B. ft. 7 B. in.
14. 13 B. ft. 6 B. in.
15. 504 B. ft.
16. 36 ft.
17. 31 ft. 2$\frac{2}{5}$ in.
18. $70.98.
19. $24.80625.
20. $85.008.

Art. 422.

1. 32 gi.; 64 gi.; 56 gi.; 28 gi.; 120 gi.
2. 128 pt.; 58 pt.; 110 pt.
3. O. 6 f℥ 5 fʒ 2 ♏15.
4. 2 bu. 3 pk. 1 pt.; 4 bu. 2 pk. 3 qt. 1 pt.; 9 bu. 7 qt.; 6 bu. 1 pk. 6 qt. 1 pt.; 11 bu. 2 qt.
5. Cong. .1377766+.
6. 369$\frac{33}{64}$ bush.
7. 371$\frac{23}{63}$ bbl.
8. Cong. 7.46972+.
9. 56$\frac{229}{252}$ bbl.
10. 1463$\frac{11}{32}$ bu.
11. .875 gal.
12. .064453125.
13. .00236+.
14. 523.21\frac{3}{7}$.
15. $424.52.

Art. 423.

1. 125.958+ bu.
2. 1244$\frac{197}{432}$ cu. ft.
3. 1282$\frac{14}{27}$ cu. ft.
4. 505$\frac{5}{16}$ cu. ft.
5. 2462.43$\frac{5}{9}$ cu. ft.
6. 601$\frac{3}{5}$ cu. ft.
7. 1001.475+ cu. ft.
8. 7.085+ ft.
9. 144$\frac{1}{96}$ oz.

Art. 427.

1. 99 yr. 4 mo. 19 da.
2. 22 yr. 3 mo. 14 da. 14 hr.
3. 66 yr. 6 mo. 13 da. 10 hr.
4. 118 yr. 5 mo. 17 da. 7 hr.
5. 7 yr. 9 mo. 1 da.
6. Feb. 22, 1732.

Art. 432.

1. 19663″.
2. 30600′.
3. 385740″.
4. $\frac{53}{225}$ s.
5. .025 cir.

6. 9 s. 28° 48′.
7. $\frac{1}{6}$ Cir.; $\frac{1}{4}$ Cir.; $\frac{1}{2}$ Cir.
8. 2 s. 12°.
9. 2 sextants; $2\frac{1}{2}$; $2\frac{3}{4}$; $4\frac{2}{15}$; $4\frac{11}{12}$.
10. 33 sex. 47°.
11. 64° 17′ $8\frac{4}{7}$″.
12. 383 yr. 8 mo. 20 da.
13. 37 yr. 5 mo. 23 da.
14. 94 doz.; 165 doz.
15. 1264 doz.; 2280 doz.
16. $29\frac{2}{3}$ doz.; $327\frac{5}{6}$ doz.; $141\frac{1}{4}$ doz.
17. 13320 sheets.
18. $.33\dot{3}$.

Art. 438.

1. 2736 far.
2. $83.745.
3. £.0375.
4. 34000 mills.
5. £.$002916\frac{2}{3}$.
6. .015.
7. 12s. 6d.
8. 16s. 9d. 2.4 far.
9. £1713.7573+.
10. $457.024.
11. £89 11s. 10d. .42+ far.
12. $10461.564.
13. 1172.0965\frac{1}{4}$; 4914 marks 45.08+ pfennig; 6073 fr. 3 ct. 8.9+m.
14. .417375.
15. .4632.
16. 647 fr. 6 dc. 6 ct. 8.3+ m.

Art. 451.

1. 38.1024 Kg.
2. 33.5627 Ton.
3. 33.8304 Hl.
4. 2.2185 L.
5. 68.0494 St.
6. 13274.16 A.
7. 250 cu. in.
8. 38 sq. yd.
9. 7.7353+ bu.
10. 552960 gr.
11. 36.8965+ cd.
12. 1176.15+ cu. ft.
13. $.024624.
14. $1.41975.
15. $14.5675+.
16. $1.357+.
17. 8340 dg.
83.4 Dg.
18. 8400 L.
840000 cl.
19. 790.75+ mi.
20. $960.
21. $74.00048+.

Art. 460.

7. 153 sq. ft. 8′ 9″ 3‴ 8⁗.
8. 212 sq. ft. 9′ 11″ 2‴ 8⁗.
9. 146 sq. ft. 2′ 1″ 10‴ 8⁗.
10. 376 sq. ft. 2′ 2″ 3‴ 6⁗.
11. 236 sq. ft. 9′ 11″ 2‴ 9⁗.
12. 915 sq. ft. 8″.
13. 156 cu. ft. 1′ 7″ 7‴ 6⁗ 11′′′′′.
14. 216 cu. ft. 8′ 11″ 2‴ 9⁗.

Art. 464.

1. 10 min. $45\frac{1}{3}$ sec.
2. 3 hr. 58 min. 1 sec.
3. 54 min, $16\frac{4}{15}$ sec.
4. 2 hr. 6 min. 59 sec.
5. 10 hr. 53 min. $55\frac{1}{15}$ sec.
6. 6 hr. 40 min. 10 sec.
7. 107° 19′ $48\frac{3}{4}$″.
8. 8 hr. 38 min. 24 sec.
9. 90° 2′ 30″.
10. 48° 45′.

ANSWERS FOR ARITHMETICAL TABLES.

Art. 39.

Observe, the answers to examples taken from the Arithmetical Tables are in every case arranged in the order the pupil is directed to take the examples from the Tables. The letters over the sets of answers indicate the columns of the Table used, and the black figures in the margin the number of the answer.

	A, B, C.	B, C, D.	C, D, E.	D, E, F.	E, F, G.	F, G, H.	G, H, I.	H, I, J.
1.	767.	1688.	1887.	1886.	877.	1787.	1885.	1863.
2.	1090.	1917.	2182.	1841.	1426.	2281.	1826.	2280.
3.	1439.	2407.	2088.	1902.	2042.	2439.	2407.	2092.
4.	1730.	2320.	2215.	2173.	1751.	2533.	2345.	2475.
5.	2021.	2228.	2292.	1942.	1442.	2443.	2445.	2469.
6.	2331.	2331.	2319.	2215.	1171.	2733.	2349.	2507.
7.	2604.	2059.	2604.	2064.	1665.	2672.	2737.	2387.
8.	2741.	2432.	2339.	2416.	2183.	2850.	2520.	2217.
9.	2703.	2048.	2501.	2035.	2370.	2720.	2216.	2181.
10.	2730.	2319.	2214.	2161.	2628.	2302.	2039.	2410.

Art. 40.

	A, B, C, D.	B, C, D, E.	C, D, E, F.	D, E, F, G.	E, F, G, H.	F, G, H, I.	G, H, I, J.
1.	13264.	22653.	26554.	25564.	15666.	26680.	26822.
2.	15792.	27940.	29427.	24295.	22977.	29790.	27927.
3.	21276.	32779.	27821.	28237.	22397.	33993.	29961.
4.	25804.	28058.	30610.	26129.	21322.	33240.	32428.
5.	28206.	32077.	30801.	28040.	20429.	34313.	33154.
6.	32928.	29295.	32983.	29860.	18630.	36323.	33256.
7.	35916.	29182.	31853.	28561.	25639.	36415.	34170.
8.	35026.	30286.	32894.	28968.	29706.	37084.	30866.
9.	36916.	29190.	31929.	29317.	33199.	32013.	30159.

Art. 41. *Example with five numbers in each.*

	A, B, C, D, E.	B, C, D, E, F.	C, D, E, F, G.	D, E, F, G, H.	E, F, G, H, I.	F, G, H, I, J.
1.	181411.	314140.	341433.	314362.	243644.	336469.
2.	226631.	336346.	363490.	334935.	249376.	393796.
3.	297622.	376258.	362615.	326186.	261888.	418914.
4.	337843.	378469.	384727.	347308.	273108.	431113.
5.	378053.	380569.	405729.	357326.	273287.	432903.
6.	427873.	378772.	387756.	377597.	276001.	460039.
7.	435129.	351331.	413346.	333495.	334979.	449819.
8.	448975.	389788.	397915.	379185.	391881.	418844.

Examples with six numbers in each.

	A, B, C, D, E.	B, C, D, E, F.	C, D, E, F, G.	D, E, F, G, H.	E, F, G, H, I.	F, G, H, I, J.
1.	250102.	401059.	400628.	406320.	263230.	432338.
2.	311474.	414783.	447868.	378724.	287271.	472739.
3.	377407.	474117.	441213.	412172.	321756.	517599.
4.	433819.	438237.	482416.	424105.	342082.	520862.
5.	476631.	466358.	463625.	436293.	362965.	529686.
6.	503820.	438250.	482541.	425453.	354565.	545688.
7.	533818.	438225.	482293.	422974.	429776.	497797.

Examples with seven numbers in each.

	A, B, C, D, E.	B, C, D, E, F.	C, D, E, F, G.	D, E, F, G, H.	E, F, G, H, I.	F, G, H, I, J.
1.	334945.	449496.	493006.	450109.	301125.	511291.
2.	391259.	312642.	526466.	464710.	347139.	571434.
3.	473383.	533885.	538902.	489069.	390630.	607348.
4.	532397.	524026.	540312.	502072.	431760.	617645.
5.	552578.	525836.	558410.	484149.	441529.	615335.
6.	602509.	525144.	551488.	514932.	449362.	593666.

Art. 66. *Examples with three figures.*

	A, B, C.	B, C, D.	C, D, E.	D, E, F.	E, F, G.	F, G, H.	G, H, I.	H, I, J.
1.	96.	38.	381.	188.	115.	153.	471.	293.
2.	257.	571.	288.	123.	233.	326.	265.	349.
3.	162.	380.	202.	20.	201.	15.	147.	473.
4.	70.	299.	8.	82.	182.	183.	169.	312.
5.	199.	6.	67.	333.	674.	262.	378.	222.
6.	162.	385.	152.	482.	183.	169.	309.	84.
7.	51.	494.	58.	422.	220.	197.	27.	268.
8.	162.	381.	191.	91.	91.	89.	106.	64.
9.	26.	260.	398.	21.	207.	70.	296.	34.
10.	226.	263.	369.	311.	111.	111.	114.	134.
11.	227.	274.	258.	416.	162.	377.	233.	329.

Art. 67. *Examples with four figures.*

	A, B, C, D.	B, C, D, E.	C, D, E, F.	D, E, F, G.	E, F, G, H.	F, G, H, I.	G, H, I, J.
1.	962.	381.	3812.	1885.	1153.	1529.	4707.
2.	2571.	5712.	2877.	1233.	2326.	3265.	2651.
3.	1620.	3798.	2020.	201.	2015.	147.	1473.
4.	701.	2992.	82.	818.	1817.	1831.	1688.
5.	1994.	67.	667.	3326.	6738.	2622.	3778.
6.	1615.	3848.	1518.	4817.	1831.	1691.	3084.
7.	506.	4942.	578.	4220.	2197.	1973.	268.
8.	1619.	3809.	1909.	909.	910.	894.	1064.
9.	260.	2602.	3979.	207.	2070.	704.	2966.
10.	2263.	3631.	3689.	3111.	1111.	1114.	1134.
11.	2274.	2742.	2584.	4162.	1623.	3767.	2329.

Art. 68. *Examples with six figures.*

	A, B, C, D, E, F.	B, C, D, E, F, G.	C, D, E, F, G, H.	D, E, F, G, H, I.	E, F, G, H, I, J.
1.	96188.	38115.	381153.	188471.	115293.
2.	257123.	571233.	287674.	123265.	232651.
3.	162020.	379799.	202015.	20147.	201473.
4.	70082.	299182.	8183.	81831.	181688.
5.	199333.	6374.	66738.	332622.	673778.
6.	161518.	384817.	151831.	481691.	183084.
7.	50578.	494220.	57803.	421973.	219732.
8.	161909.	380909.	190911.	90894.	91064.
9.	26021.	260207.	397930.	20704.	207034.
10.	226311.	363111.	368889.	311114.	111134.
11.	227416.	274162.	258377.	416233.	162329.

Art. 88. *Multiplicand three figures, multiplier one.*

	A, B, C.	B, C, D.	C, D, E.	D, E, F.	E, F, G.	F, G, H.	G, H, I.	H, I, J.
1.	1872.	1735.	942.	3565.	414.	3080.	2562.	2710.
2.	690.	2310.	3408.	4200.	1518.	2152.	3064.	5010.
3.	2765.	5736.	3384.	5184.	3402.	7776.	3240.	4374.
4.	3899.	2880.	6128.	4008.	6183.	5274.	3180.	6713.
5.	2922.	7875.	758.	5274.	4445.	5568.	5784.	5823.
6.	5488.	3476.	2073.	6433.	1560.	8622.	2930.	2607.
7.	5936.	3872.	4215.	3933.	3024.	4734.	7160.	4765.
8.	7173.	6846.	4710.	6872.	5382.	6702.	3472.	6165.
9.	4795.	4179.	7808.	6912.	4134.	6279.	7792.	2347.
10.	8865.	3428.	4046.	6312.	4480.	5802.	2712.	7047.
11.	4554.	4752.	8523.	1912.	5495.	7704.	3948.	5192.

Multiplicand four figures, multiplier one.

	A, B, C, D.	B, C, D, E.	C, D, E, F.	D, E, F, G.	E, F, G, H.	F, G, H, I.	G, H, I, J.
1.	11735.	6942.	23565.	21414.	11080.	11562.	42710.
2.	8310.	15408.	68200.	31518.	10152.	43064.	23010.
3.	23736.	57384.	45184.	45402.	43776.	43240.	58374.
4.	27880.	46128.	46008.	60183.	41274.	35180.	55713.
5.	43875.	8758.	68274.	29345.	69568.	41784.	86823.
6.	27476.	26073.	48433.	73560.	17622.	47930.	17607.
7.	67872.	24215.	75933.	35024.	22734.	63160.	44765.
8.	55846.	58710.	62872.	77382.	41902.	39472.	78165.
9.	67179.	47808.	87912.	46134.	48279.	71792.	29247.
10.	39428.	60046.	46312.	39480.	53802.	38712.	61047.
11.	60752.	53523.	37912.	33495.	70704.	59948.	45192.

Multiplicand six figures, multiplier one.

	A, B, C, D, E, F.	B, C, D, E, F, G.	C, D, E, F, G, H.	D, E, F, G, H, I.	E, F, G, H, I, J.
1.	1173565.	1041414.	3771080.	2141562.	692710.
2.	1108200.	2911518.	3410152.	4103064.	1523010.
3.	3165184.	6695402.	5083776.	3243240.	4378374.
4.	3346008.	5190183.	4601274.	2675180.	4815713.
5.	4388274.	4379345.	6069568.	3521784.	7826823.
6.	4808433.	6953560.	6227622.	4597930.	587607.
7.	7635933.	3875024.	5062734.	3503160.	1894765.
8.	6382872.	8807382.	5501902.	3439472.	5388165.
9.	8637912.	3586134.	6838279.	6151792.	2069247.
10.	7886312.	4289480.	3473802.	3158712.	8071047.
11.	3037912.	4163495.	8530704.	3349948.	6285192.

Art. 98. *Multiplicand five figures, multiplier three.*

	A, B, C, D, E.	B, C, D, E, F.	C, D, E, F, G.	D, E, F, G, H.	E, F, G, H, I.
1.	19997292.	18224326.	11925914.	48405130.	5306082.
2.	7812528.	24964200.	41432958.	45392832.	16448184.
3.	30306024.	63892864.	38805882.	57015456.	38675160.
4.	42270628.	33793448.	66641003.	46547784.	66318380.
5.	33691778.	80491534.	14794455.	56230768.	50960904.
6.	57906513.	37983603.	26155710.	72554862.	17529470.
7.	66601755.	41607383.	50458044.	43175954.	32892860.
8.	77870160.	75155712.	54154022.	77129442.	58311432.
9.	55474128.	47156952.	87529344.	74359399.	46764372.
10.	93353366.	41077142.	45448360.	67595752.	50578392.
11.	52327483.	53172332.	89761395.	22923024.	62615508.

Multiplicand six figures, multiplier five.

	A, B, C, D, E, F.	B, C, D, E, F, G.	C, D, E, F, G, H.	D, E, F, G, H, I.
1.	9042318325.	29574555914.	24764210975.	18119756082.
2.	13249639200.	21761400958.	55299024832.	25558832184.
3.	22816228864.	73350041882.	37777539456.	44623739160.
4.	48343909448.	43752666003.	45012729784.	58161088380.
5.	42380487534.	60605755455.	69768171768.	11496276904.
6.	33272295603.	73340935710.	20400148862.	34847711470.
7.	83027196383.	38071142044.	72554040954.	26215897860.
8.	47700798174.	95598260022.	60439965442.	59308535432.
9.	82339081154.	34613802344.	77142625399.	68960040372.
10.	58632758142.	81315762360.	27707044752.	62040262392.
11.	65994081332.	41008641395.	84813207024.	45366431508.

Art. 122. *Dividend three figures, divisor one.*

	A, B, C.	B, C, D.	C, D, E.	D, E, F.	E, F, G.	F, G, H.	G, H, I.	H, I, J.
1.	56.	98.	$173\frac{4}{5}$.	$86\frac{6}{8}$.	$158\frac{1}{6}$.	$55\frac{2}{9}$.	$243\frac{1}{4}$.	92.
2.	53.	41.	$51\frac{4}{5}$.	$159\frac{1}{4}$.	$41\frac{6}{9}$.	$107\frac{5}{7}$.	183.	$82\frac{1}{6}$.
3.	98.	154.	82.	$66\frac{1}{7}$.	$127\frac{3}{5}$.	$96\frac{2}{4}$.	$96\frac{3}{9}$.	$226\frac{1}{3}$.
4.	288.	192.	114.	283.	62.	$160\frac{2}{6}$.	$89\frac{2}{7}$.	28.
5.	128.	$85\frac{3}{8}$.	$209\frac{1}{4}$.	$41\frac{6}{9}$.	$125\frac{3}{6}$.	269.	$76\frac{4}{5}$.	424.
6.	53.	83.	$70\frac{2}{7}$.	$185\frac{1}{5}$.	89.	$84\frac{1}{8}$.	$184\frac{2}{4}$.	$48\frac{1}{8}$.
7.	159.	$40\frac{5}{9}$.	329.	$97\frac{2}{6}$.	$121\frac{2}{7}$.	165.	$119\frac{4}{8}$.	$113\frac{4}{5}$.
8.	142.	$105\frac{2}{5}$.	$34\frac{1}{8}$.	$184\frac{2}{4}$.	$42\frac{6}{9}$.	$169\frac{4}{5}$.	82.	103.
9.	197.	$134\frac{4}{7}$.	143.	$36\frac{5}{8}$.	$234\frac{2}{4}$.	43.	$439\frac{1}{2}$.	$113\frac{2}{7}$.
10.	237.	242.	$94\frac{1}{9}$.	$158\frac{1}{3}$.	$94\frac{1}{8}$.	$76\frac{2}{7}$.	$38\frac{4}{9}$.	$155\frac{1}{3}$.
11.	74.	232.	$40\frac{5}{7}$.	$171\frac{4}{5}$.	$198\frac{2}{3}$.	242.	114.	$141\frac{8}{6}$.

Dividend five figures, divisor one.

	A, B, C, D, E.	B, C, D, E, F.	C, D, E, F, G.	D, E, F, G, H.	E, F, G, H, I.	F, G, H, I, J.
1.	$3373\frac{4}{5}$.	$8586\frac{6}{8}$.	$14491\frac{3}{6}$.	$7721\frac{8}{9}$.	$23743\frac{1}{4}$.	6217.
2.	$4718\frac{1}{9}$.	$6159\frac{1}{4}$.	$5152\frac{7}{9}$.	$9017\frac{5}{7}$.	$12516\frac{1}{8}$.	$12582\frac{1}{6}$.
3.	13082.	13209.	$4927\frac{3}{5}$.	$11596\frac{2}{4}$.	$7096\frac{3}{9}$.	12893.
4.	9614.	$25616\frac{1}{3}$.	8562.	$14160\frac{2}{6}$.	$7089\frac{2}{7}$.	$10694\frac{6}{9}$.
5.	$19209\frac{1}{4}$.	$7597\frac{2}{9}$.	$13958\frac{5}{6}$.	18769.	$15076\frac{4}{5}$.	26924.
6.	$6070\frac{2}{7}$.	$4985\frac{1}{5}$.	$16422\frac{1}{3}$.	$11584\frac{1}{8}$.	$6684\frac{2}{4}$.	$8423\frac{1}{8}$.
7.	31829.	$6097\frac{2}{6}$.	9407.	$19498\frac{1}{3}$.	$10619\frac{4}{8}$.	$9913\frac{4}{5}$.
8.	$10659\frac{1}{8}$.	$13184\frac{2}{4}$.	$3042\frac{6}{9}$.	$12769\frac{4}{9}$.	$6415\frac{2}{6}$.	$9436\frac{3}{9}$.
9.	13143.	$11786\frac{5}{8}$.	$10734\frac{2}{4}$.	$3265\frac{2}{9}$.	$46939\frac{1}{2}$.	$5541\frac{6}{7}$.
10.	$10538\frac{5}{9}$.	$16158\frac{1}{3}$.	$10594\frac{1}{8}$.	$6790\frac{4}{7}$.	$8371\frac{7}{9}$.	7822.
11.	$8467\frac{2}{7}$.	$18571\frac{4}{5}$.	9532.	21492.	$9947\frac{2}{6}$.	$16141\frac{3}{6}$.

Art. 132. *Dividend four figures, divisor two.*

	A, B, C, D.	B, C, D, E.	C, D, E, F.	D, E, F, G.	E, F, G, H.	F, G, H, I.	G, H, I, J.
1.	$45\frac{21}{37}$.	$91\frac{44}{75}$.	$149\frac{52}{58}$.	$80\frac{69}{86}$.	$137\frac{44}{69}$.	$52\frac{85}{94}$.	$202\frac{40}{48}$.
2.	$49\frac{32}{86}$.	$35\frac{49}{69}$.	$41\frac{31}{94}$.	$130\frac{5}{49}$.	$38\frac{68}{97}$.	$103\frac{30}{73}$.	$152\frac{21}{36}$.
3.	$85\frac{14}{46}$.	$146\frac{48}{63}$.	$66\frac{21}{37}$.	$61\frac{63}{75}$.	$118\frac{14}{54}$.	$78\frac{45}{49}$.	$93\frac{30}{93}$.
4.	$240\frac{8}{24}$.	$167\frac{2}{46}$.	$108\frac{45}{63}$.	$223\frac{22}{38}$.	$57\frac{60}{86}$.	$143\frac{44}{67}$.	$79\frac{11}{79}$.
5.	$112\frac{67}{68}$.	$81\frac{33}{84}$.	$170\frac{45}{49}$.	$39\frac{9}{96}$.	$121\frac{36}{62}$.	$215\frac{7}{25}$.	74.
6.	$51\frac{16}{83}$.	$67\frac{13}{37}$.	$65\frac{51}{75}$.	$174\frac{45}{53}$.	$70\frac{13}{38}$.	$80\frac{18}{84}$.	$153\frac{41}{48}$.
7.	$129\frac{44}{49}$.	$39\frac{70}{92}$.	$253\frac{6}{26}$.	$87\frac{20}{67}$.	$116\frac{27}{73}$.	$130\frac{16}{38}$.	$112\frac{49}{85}$.
8.	$131\frac{12}{65}$.	$90\frac{53}{58}$.	$32\frac{50}{84}$.	$171\frac{5}{49}$.	$40\frac{49}{95}$.	$151\frac{36}{56}$.	$71\frac{28}{69}$.
9.	146.	$129\frac{32}{73}$.	$112\frac{37}{38}$.	$34\frac{82}{84}$.	$191\frac{28}{49}$.	$42\frac{15}{92}$.	$325\frac{18}{27}$.
10.	$225\frac{34}{42}$.	$167\frac{4}{29}$.	$91\frac{12}{93}$.	$125\frac{3}{38}$.	$86\frac{52}{87}$.	$67\frac{53}{79}$.	$37\frac{25}{93}$.
11.	$70\frac{48}{84}$.	$197\frac{26}{47}$.	$38\frac{9}{75}$.	$162\frac{10}{53}$.	$175\frac{18}{34}$.	$210\frac{24}{46}$.	$103\frac{51}{66}$.

Dividend six figures, divisor three.

	A, B, C, D, E, F.	B, C, D, E, F, G.	C, D, E, F, G, H.	D, E, F, G, H, I.	E, F, G, H, I, J.
1.	$222\frac{418}{758}$.	$1172\frac{157}{586}$.	$1000\frac{497}{869}$.	$1001\frac{279}{694}$.	$1001\frac{788}{948}$.
2.	$611\frac{603}{694}$.	$259\frac{584}{949}$.	$933\frac{51}{497}$.	$655\frac{234}{973}$.	$510\frac{133}{736}$.
3.	$616\frac{71}{637}$.	$2412\frac{138}{375}$.	$326\frac{582}{754}$.	$844\frac{511}{549}$.	$1295\frac{234}{493}$.
4.	$1245\frac{414}{463}$.	$1204\frac{344}{638}$.	$1774\frac{198}{386}$.	$979\frac{832}{867}$.	$730\frac{582}{679}$.
5	$905\frac{30}{849}$.	$1378\frac{445}{496}$.	$870\frac{598}{962}$.	$600\frac{384}{625}$.	$2991\frac{116}{252}$.
6.	$1133\frac{51}{375}$.	$331\frac{24}{753}$.	$915\frac{503}{538}$.	$2413\frac{146}{384}$.	$315\frac{265}{848}$.
7.	$687\frac{422}{926}$.	$1370\frac{59}{267}$.	$978\frac{301}{673}$.	$792\frac{460}{738}$.	$2206\frac{339}{385}$.
8.	$1460\frac{98}{584}$.	$621\frac{155}{849}$.	$553\frac{114}{495}$.	$772\frac{460}{956}$.	$676\frac{283}{569}$.
9.	$534\frac{201}{738}$.	$2455\frac{218}{384}$.	$504\frac{491}{849}$.	$597\frac{155}{492}$.	$1012\frac{669}{927}$.
10.	$3237\frac{34}{293}$.	$516\frac{745}{938}$.	$2190\frac{4}{387}$.	$540\frac{686}{879}$.	$950\frac{116}{793}$.
11.	$1248\frac{59}{475}$.	$1233\frac{37}{753}$.	$535\frac{278}{534}$.	$2484\frac{20}{346}$.	$1280\frac{369}{466}$.

Art. 205. *Examples with two numbers.*

1.	6.	2.	22.	88.	22.	7.	8.	56.	16.	48.	168.
2.	6.	6.	18.	6.	42.	8.	9.	9.	99.	63.	27.
3.	36.	132.	24.	20.	44.	9.	70.	10.	60.	60.	90.
4.	60.	40.	20.	35.	5.	10.	44.	44,	44.	44.	132.
5.	132.	132.	66.	66.	66.	11.	24.	24.	84.	84.	36.
6.	35.	105.	12.	3.	45.	12.	156.	26.	26.	91.	39.

Art. 206. *Examples with two numbers.*

1.	6.	6.	2.	2.	4.	18.	7.	6.	35.	1.	3.	21.	6.
2.	12.	6.	24.	6.	4.	18.	8.	6.	35.	3.	3.	9.	18.
3.	12.	12.	8.	60.	5.	3.	9.	6.	10.	24.	4.	60.	6.
4.	12.	24.	4.	42.	1.	3.	10.	12.	20.	24.	28.	12.	12.
5.	4.	3.	12.	6.	9.	3.	11.	12.	24.	2.	28.	21.	27.
6.	4.	35.	28.	12.	21.	15.							

Art. 207. *Examples with two numbers.*

1.	6.	36.	22.	16.	44.	198.	6.	2.	105.	3.	3.	63.	9.
2.	12.	30.	8.	6.	7.	3.	7.	6.	70.	16.	12.	48.	210.
3.	12.	132.	132.	6.	22.	264.	8.	66.	5.	33.	77.	3.	66.
4.	20.	15.	20.	12.	35.	195.	9.	42.	10.	24.	60.	36.	90.
5.	12.	8.	4.	6.	6.	24.	10.	12.	4.	2.	28.	3.	3.

Art. 208. *Examples with three numbers.*

1.	2.	2.	22.	22.	**7.**	8.	8.	16.	24.
2.	6.	6.	6.	6.	**8.**	9.	9.	9.	9.
3.	12.	12.	4.	4.	**9.**	10.	10.	60.	30.
4.	20.	20.	5.	5.	**10.**	44.	44.	44.	44.
5.	132.	66.	66.	66.	**11.**	24.	12.	84.	12.
6.	35.	3.	3.	3.	**12.**	26.	26.	13.	13.

Art. 209. *Examples with three numbers.*

1.	6.	6.	2.	2.	4.	18.	**6.**	2.	35.	1.	3.	21.	3.
2.	12.	6.	8.	6.	1.	3.	**7.**	6.	35.	1.	3.	3.	6.
3.	12.	12.	4.	6.	1.	3.	**8.**	6.	5.	3.	1.	3.	6.
4.	4.	3.	4.	6.	1.	3.	**9.**	6.	10.	24.	4.	12.	6.
5.	4.	1.	4.	6.	3.	3.	**10.**	12.	4.	2.	28.	3.	3.

Art. 222. *Examples with two numbers, according to* **205.**

1.	180.	2772.	1232.	176.	792.
2.	1680.	2520.	216.	756.	252.
3.	2772.	792.	5280.	5280.	3960.
4.	120.	120.	840.	2100.	6825.
5.	264.	792.	2772.	4158.	4752.
6.	420.	420.	5460.	16380.	4095.
7.	3360.	560.	1344.	1344.	1680.
8.	6930.	10395.	2079.	6237.	12474.
9.	1050.	840.	1200.	3600.	5040.
10.	660.	1320.	1408.	4620.	2640.
11.	3360.	10080.	2520.	1260.	3780.
12.	312.	1560.	1820.	1092.	2457.

Art. 222. *Examples with two numbers, according to* **206.**

1.	240.	1260.	5544.	4752.	1848.	1386.
2.	1008.	13860.	792.	4320.	4620.	5544.
3.	1260.	3960.	1320.	3360.	7700.	51480.
4.	660.	1320.	3960.	4620.	103950.	34320.
5.	4620.	9240.	12860.	11012.	20790.	102960.
6.	3360.	740.	1680.	2496.	4368.	32760.
7.	3168.	2520.	33264.	44352.	9072.	83160.
8.	6930.	3150.	23760.	69300.	45360.	20790.
9.	4620.	770.	2640.	23100.	7920.	55440.
10.	1848.	5280.	5544.	4620.	13860.	23760.
11.	2184.	6240.	32760.	5460.	3276.	7020.

Art. 222. *Examples with two numbers, according to* **207.**

1.	1260.	396.	1848.	5280.	440.	792.
2.	240.	840.	360.	3780.	2100.	8190.
3.	2772.	792.	792.	36960.	5940.	1584.
4.	420.	840.	840.	5460.	1575.	585.
5.	1056.	8140.	11088.	14784.	33264.	18480.
6.	13860.	315.	41580.	36036.	2835.	38610.
7.	3360.	1400.	1680.	4800.	5040.	2520.
8.	396.	13860.	2376.	2772.	124740.	4752.
9.	840.	16800.	5040.	2100.	5040.	3780.
10.	1716.	17160.	17160.	4004.	60060.	61776.

Art. 222. *Examples with three numbers, according to* **208.**

1.	140.	210.	84.	48.	48.
2.	90.	90.	90.	72.	72.
3.	1512.	432.	48.	48.	168.
4.	1440.	1440.	900.	150.	150.
5.	280.	2610.	360.	360.	72.
6.	378.	1890.	2160.	1680.	1680.
7.	126.	882.	1764.	392.	168.
8.	160.	224.	2016.	1008.	576.
9.	360.	1080.	1512.	567.	2268.
10.	1890.	1260.	720.	2640.	2640.
11.	495.	3465.	1386.	616.	264.
12.	1008.	1680.	3360.	1440.	864.

Art. 222. *Examples with three numbers, according to* **209.**

1.	252.	270.	840.	48.	72.	48.	168.
2.	1260.	864.	1080.	240.	1800.	120.	420.
3.	420.	30240.	360.	720.	600.	120.	1260.
4.	180.	70560.	1080.	90.	2400.	840.	6300.
5.	2520.	5040.	2016.	10584.	504.	3024.	576.
6.	1080.	5040.	1440.	7560.	5040.	14256.	6336.
7.	594.	6930.	3960.	41580.	55440.	7128.	792.
8.	2376.	18480.	41580.	4620.	5280.	792.	2376.