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ANNOUNCEMENT OF VOLUME IV — 1903

The May issue of the TEACHERS COLLEGE RECORD will deal with "Methods of Teaching French and German," according to the new or reform method, by Dr. Leopold Bahlsen, of Berlin, Lecturer on Methods of Teaching Modern Languages, Teachers College.

Other issues will discuss the principles which should govern the curriculum of primary and secondary schools, illustrated from outlines of courses as conducted in the Teachers College Schools, and followed by special issues dealing with specific work in the first, second, third, and fourth grades, and in the high school.

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MATHEMATICS IN THE ELEMENTARY SCHOOL

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The following outline of theory and of subject-matter is proposed rather as a basis for discussion with students in professional courses than as a fixed body of thought for use in the elementary school. Yet the course of work here outlined is followed to a very large extent in Teachers College in its Horace Mann School of observation and its Speyer School of practice, although the arrangement of topics is necessarily different in the two schools. The particular order here suggested is expected to apply more fully to the Horace Mann School than to the Speyer School.

I. CONTROLLING IDEAS THROUGHOUT THE CURRICULUM

The numerous changes in the curriculum of mathematics in the grades during recent years have been principally due to two causes, namely, a growing sympathy with children, and a more enlightened interpretation of the needs of society. The former has led, in many schools, to the omission or neglect of number work as a regular study during the first year or two of the course, on the plea that it is somewhat foreign to child nature; it has demanded less abstract problems, in order that truer interest might be aroused; and it has opposed especially long and difficult solutions, as in equation of payments and compound interest, on the ground that they were an unnecessary physical tax, besides giving a wrong idea of actual business life. The second cause of change, namely, the better interpretation of the needs of society, has likewise demanded less theoretical problems, and has urged the entire omission of topics that are not called for in the quantitative relations of daily life; as, for example, the table of troy weight, partnership, and cube root.

The question arises: Should these two ideas constitute the principal standard in the selection of a curriculum in mathematics? And, if so, dare they have fuller control than they have thus far obtained? Or, if neither of these should control, what standard should take their place?

A full discussion of these questions is out of place here, because it is a general educational problem that is involved, applying alike to all branches of knowledge, and in the main it should be assumed as settled, when an individual study is under consideration. Nevertheless, a brief mention of the former standard of selection of materials in mathematics for children, and a comparison of the same with the generally accepted standard in certain other studies, may suggest our answer to the proposed questions, and an outline of our argument.

Twenty years ago, acquaintance with arithmetical processes, and mental discipline, constituted the chief purposes of instruction in arithmetic. As to subject-matter, whatever problems promised best to acquaint pupils with these useful processes, and to furnish this mental training, were deemed acceptable. This is probably still the prevalent view.

But these aims, or standards of worth, are far from acceptable in certain other studies. Turn, for example, to literature, history, nature study, and art, as carried on in the grades. Who would rank business utility and mental discipline foremost among their purposes, and be guided primarily by these objects in the selection of topics in those subjects? Indeed, the whole point of view has radically changed as to these branches. The teacher's first aim is the excitement of a deep interest, possibly love, for these fields of human experience. That is the important, immediate, goal to be reached. When it comes to the subject-matter, those topics must be chosen which are capable of arousing interest;

this is one of the controlling ideas, and the teacher must become a close student of children in order to avoid serious error at this point.

But not everything that is thoroughly interesting can be accepted, and there is, therefore, a second controlling idea for selection. The subject-matter in each of those studies must reveal some side of life, and do it in such a way that the pupil feels forcibly the relation between it and practical living. The truths of literature must offer a guide for daily action; those of history must throw light upon and awaken sympathy for present social problems; nature study must lead into agriculture; and art must end in the real enjoyment of such pictures as should grace the home, as well as in the desire and ability to make home more attractive. Whatever matter contributes only slightly to this purpose of richer, more effective present living, must go; hence, non-classics in literature, numerous names and dates in history, mere sense-training object-lessons "of any sort" in nature study, and the bare mechanical drawing of geometric forms in art, are discarded. In other words, an actual weeding-out process is going on in these parts of our curriculum, with the object of searching out and retaining only those materials that (a) correspond with child nature, and (b) identify the child with actual life.

If these large purposes are accomplished, minor objects are also attained, including such as were above mentioned in connection with arithmetic. For instance, when genuine interest is assured, the prime condition for concentration of attention is fulfilled; hence, good mental discipline is bound to follow, provided, of course, the method of presenting the subject is good. In brief, good mental discipline, as an aim, has little to do with the choice of subject-matter; it is thought of only *after* this matter has been chosen, and in that sense is a subsidiary object. Who, for example, would allow the development of logical power to be a controlling factor in selecting literature and history topics? And even if it were so allowed, how could it be a clear guide?

Again, the business utility of a subject is of value; but if history leads into the heart of present social problems, if nature study opens to view many broad principles of agriculture, and if art makes the home more attractive, a good degree of business utility is attained, and much more. Utility taken in a narrow sense kills the spirit of any study, as we all know; but when each of these branches aims in this broad way to meet the needs of life, not only useful knowledge is acquired, but spirit and energy are aroused.

Enough has been said to indicate our answer to the query raised at the beginning of this article. Business utility and mental discipline should not rank as the primary aims in teaching mathematics to children; particularly in the selection of subject-matter. This study should stand on the same plane, should be controlled by the same broad ideas, as other studies. Accordingly, the child's interest in the quantitative side of life should be the highest immediate aim of the teacher of mathematics in the grades, just as his interest in the spiritual side is the highest immediate aim of the teacher of literature; and the nature of the child, together with the needs of society, should constitute the main standard in selecting subject-matter. This signifies that these two ideas should be given a fuller control in mathematics than has thus far been allowed.

What is there, however, in mathematics in the grades that is capable of appealing to interest? Has it a body of thought comparable in attractiveness to children to that in literature or history or nature study? That is the most fundamental question in this field at the present time. If there is no such body of thought, then of course we must drop back upon useful knowledge and mental discipline as our chief aims. To be sure, these are purposes that have been "conceived by the adult and forced upon the child," but what else would be left? On the other hand, if there is a rich body of thought here for the pupil, then it is time that we were finding it out.

Manual training has faced this same question and is in advance of arithmetic in its solution. It was not long ago that this study, like arithmetic, aimed mainly at utility, by leading children to make useful objects, and at mental discipline, by exercising the mind through the hand. But now there is a marked tendency to regard handwork as a means of opening up the industrial side of life. Through it, one is not only to develop some skill in carpentry, blacksmithing, masonry, basketry, and bent-iron work; but he is to be introduced into the constructive experiences of mankind, making possibly many excursions to factories, in order the more fully to comprehend and sympathize with this phase of human activity. And why is this not reasonable? Just as literature, relying upon words, tries to present man's highest aspirations, why should not some other study, relying largely upon observation and participation in manufacturing, try to present man's constructive experience? Little seems to be lost, and much gain is promised, by this change; for the thought-content of manual training is thereby increased far beyond the principles involved in the use of tools. The fact that we are distinguished as an industrial nation emphasizes the need of this advance, on the social side, while it promises an especially rich addition to the subject-matter of manual training. When this aim is more fully realized in practice, as it can be, manual training will stand on the same educational plane as other studies, and the instructor in that branch can then justly rely upon implanting a permanent interest in it, as do instructors of literature and nature study in their respective fields.

Again the question is before us: What is there in arithmetic that is capable of arousing the direct, immediate, interest of children? Ordinarily the part of any study that is depended upon to arouse interest, at least initial interest, is the part that is concrete, as the individual facts in history and nature study. The mere operations in arithmetic are abstract, as we know, and the principal other part of the study is the applied problems. Hence, these applied problems must be the concrete part of the subject, and if its content is to be greatly enriched, the change must be expected first of all in this particular.

But some persons, no doubt, will here raise the questions: Are the problems of arithmetic really concrete? Has arithmetic, in fact, any concrete part at all, as other studies have?

It becomes necessary to define what we mean by concrete. Some years ago it was found that a very large percentage of examples in our arithmetics were plainly abstract, consisting of unnamed units, such as $5\frac{1}{2} \times 3\frac{2}{3}$. To make these concrete they were made denominate; as, for example, If one barrel of apples cost $\$3\frac{2}{3}$, what will $5\frac{1}{2}$ barrels cost? But still there is the common complaint that these are practically as abstract as they were before. When, therefore, is an example concrete?

If one goes to a dictionary to learn the meaning of a list of new words, he is engaged in an abstract kind of work, because the words are then dissociated from their original, concrete, connections. If, on the other hand, one learns their meaning through the context, as most children do, the work is concrete, because the words are then approached and viewed in their individual relations. So, in general, a statement is truly concrete only when it is seen in its original associations; the moment it is "isolated from its origin and its goal, it becomes abstract."

Applied to mathematics, this means that problems are abstract when they are chosen at random, without the purpose of clarifying the individual situations in which they arose, but merely with the intent of giving exercises in processes. They are concrete only when they deal with real things and with actual, significant, situations. In the former case, the answers reached have no worth in themselves, and the work is unreal because the processes, which were originally a mere means, become the end. Motive for hard work on the part of pupils is no longer provided in the subject matter, and for that reason competition of child with child, an artificial incentive, must be largely relied upon. In the latter case the answers are the end, because they are valuable thoughts, like those in other studies.

For example, let us take for our topic a comparatively insignificant industry, namely, the manufacture and consumption of printers' ink in the United States:

1. At the present time about 30 million pounds of this ink are manufactured in the United States per year. How many tons is that? (*Answer*, 15,000.)

A single factory in New York makes 1½ million pounds. What part of the whole does it make? ($^1/_{20}$.) How many tons? (750.)

2. The worth of this one plant is about $125,000. At that rate, how much money is invested in this industry in the United States? ($2,500,000.)

3. This one factory employs 20 workmen, and 35 men in all, including salesmen and others. What, then, are the corresponding numbers employed in the entire industry? (700.)

4. The workmen in this factory, on the average, receive about $2 per day. At that rate what amount is paid per day to such workmen in the United States? Per year, of 300 days? The 35 men average about $5 per day. What would be the approximate pay roll per week, and per year, for all such factories together?

5. This one factory, in making the ink, consumes per month 250 bbls. of rosin brought mainly from the pine trees of Georgia and South Carolina. It costs $6 per bbl. What facts can you figure out from these statements? (3,000 bbls. are used in the one factory, and they cost $18,000. The cost for all the factories is $360,000.)

6. This one factory consumes per month 10,000 gal. of linseed oil, made from flax seed, and worth 75 cents per gal. What facts can you learn from this statement? (It costs the one factory $7,500 per month, or $90,000 per year, for such oil. It costs all the factories $1,800,000 per year, the total amount being 2,400,000 gal. The acreage of flax might also be considered.)

7. Four hundred tons of anthracite coal are burnt per year in this factory. What is its price, at the present rate?

8. This factory consumes 600 bbls. of water per day, 300 days in the year, mainly to keep cylinders cool. The city charges $1 per thousand cu. ft. of water. The barrels contain 50 gal. each, and 1 gal. of water weighs 8 lbs. A cubic foot of water weighs 62½ lbs. The factory has recently dug a well of its own. Estimate how much it saves by that means, supposing its pumping to cost nothing.

9. One and one-fourth lbs. of black ink will print 1000 eight-page newspapers of ordinary size. How much would an eight-page weekly paper, having a circulation of 12,000, consume per year? Find out the circulation of your principal county paper, and compute the amount of ink it uses.

10. How much would be consumed by one issue of the New York Morning Journal, a sixteen-page paper, having a circulation in 1898 of 309,472? One issue of the New York Sunday World, a fifty-six-page paper with a circulation in 1898 of 500,000? (2 tons and 375 lbs. for the World.)

11. The retail price of black ink varies from 4 to 10 cts. per lb. At 6 cents per lb., how much would one of the above named papers pay for ink, per year? (World, $262.50 per Sunday.)

12. The black part of black ink is like lampblack, and is made from gas. One pound of such black is worth 8 cents; and it takes 2000 cu. ft. of gas to make it. Gas in New York City costs $1 per thousand cu. ft. Is this lampblack probably manufactured in the city? If not, can you suggest where it might be made?

These examples, instead of being hypothetical, are actual cases, and in that sense are concrete. Some of the answers might well be worth remembering, like facts in history. Of course, if pupils have not become interested in New York City and its industries, this topic might be relatively unattractive to them. But that is a danger encountered in all studies, and suggests the desirable relationship in the grades between mathematics and other branches. Teachers in composition and in manual training try to select those topics to work upon, that have already become of interest to a class. These are recognized as branches of knowledge that are quite dependent upon others for suggestions as to subject matter; and the recognition of this dependence by no means detracts from their dignity. Indeed, dignity is not concerned here; this arrangement merely insures a greater degree of interest. Since elementary arithmetic is only the quantitative side of human experience, it is peculiarly dependent upon other fields for the content of its problems, although by no means dependent solely upon other school studies. The more the problems deal with interesting topics, the better; hence, correlation of the right kind is a matter of supreme importance to mathematics in the grades.

Further than that, the correlation of mathematics is of very great importance to other branches, and even necessary to the effectiveness of the curriculum as a whole. For instance, geography gives the causes for the location of factories in a certain place; nature study shows how the earthworm grinds up soil; and the United States history describes the campaigns of the armies in Virginia during the Civil War, when it was necessary to change the base of supplies. Now each of these topics has a quantitative side, and the picture is very incomplete in each case until that side has been presented. In fact the main part of the definition of factory must be gotten through arithmetic, as shown by the examples on ink manufacture, just given. The agricultural value of earthworms begins to be appreciated only when we measure the quantity of their castings on a given area; and the difficulties attending an army's change of its base of supplies are not clear until one knows how much bread, meat, flour, sugar and water 100,000 men consume in a day; and how many horses, wagons and men it takes to haul that amount of supplies a given distance in a given time. The single fact that it requires not less than 40 bbls. of salt per day for such an army throws much light on the matter. Figures give descriptions, just as words do, and our appreciation of the quantitative side of life is lamentably poor because we have made so little use of figures in school for this purpose. Indeed, many of our most pressing problems are largely quantitative, as for example, the maintenance of the Erie Canal in the State of New York. Why should we not know the relative cost and speed of transportation by rail and canal, together with cost of maintenance, as well as the parts of speech and the causes of our wars? And *must* we not possess such quantitative knowledge before we can be intelligent as to public policy in such matters?

This quantitative knowledge, too, calls for more than arithmetic alone, even in childhood. It requires mensuration, some geometry, and even a little algebra. Why should we try to maintain the barriers between these fields when they are so intimately related in daily life, and so simple in their elements? Why should the mediæval "Rule of False Position" still find place in our arithmetics under the form: "Let 100 per cent represent the cost"? It is in every way simpler to adopt the symbolism that for nearly three centuries has been conventional in mathematics, and say: "Let x represent the cost." In either case we employ a symbolism, but the second one is the simpler, the more easily handled in the solution, and hence the more widely accepted. Indeed, if we had adopted the symbolism of the simple equation long ago, in our own elementary arithmetic, topics like proportion and the applications of percentage would have been much more clearly understood, and the subjects of cube root and the progressions might have developed enough of value to have made it worth while to retain them. The fact is, there is no well defined line between our common arithmetic and algebra, and the examiner in the former who appends to a question the statement, "Do not solve by algebra," merely reveals his own ignorance. He might insist that $2:5=7:(?)$ is a problem in arithmetic, and that $\frac{x}{7}=\frac{5}{2}$ is one in algebra, but a little thought shows that they are identical. The fact is that, strictly speaking, algebra is a science treating of a certain class of functions, and that arithmetic is a science treating of numerical values. But arithmetic and algebra, as we commonly speak of them, are sciences each of which considerably overlaps the other, and rightly so.

The same overlapping is apparent for arithmetic and geometry. Geometry is not merely a modern form of Euclid, although, in England and America especially, we are wont to speak of it as such. The necessary mensuration of common life is a part of it, and as such is deserving of a much more scientific treatment than it has already had. This topic, however, is reserved for a subsequent article.

From these statements it follows that the subjects to be studied in mathematics should be selected with the same care as to age and present interests of the children concerned, as are classic poems. Answers would then be worth something, and children would not need to be blamed for "working for them" when they have nothing else for which to work. Then mathematics in the grades would be recognized as revealing one very interesting and large side of life, as does any other study worthy of pursuit.

And is this not natural? Children have a constant call for number in their play. In fact, one reason advanced for having no regular study called number in the first two years of school is that they learn nearly as much of this outside of school as in it. But it is not for the processes that they care; they wish quantitative facts, and the answer, if the problem is real, is after all the true goal. It is so with adults also; they are wanting the answer when they figure at all; it is this that furnishes the motive for accuracy. Why, then, should children in school be confined to processes, when the purpose of children outside of school, and of adults, is quite apart from processes?

The whole history of printed arithmetics shows how periodically has arisen the question as to the nature of the work to be demanded of pupils. While there has been a constant evolution towards a better treatment of the subject, a better understanding of the processes, a clearer presentation of the operations, and a recognition of the more valuable symbols of mathematics, there has not been a corresponding evolution in the nature of the problems. This has resulted from the ever present struggle between culture and utility; between mental discipline and the problem of daily bread; between aristocracy and trade. This struggle was seen in the earliest printed books, when the first arithmetics printed in Italy (Treviso, 1478) and in Germany (Bamberg, 1482) took the utility side of the case; while the great Italian treatise of Paciuoli (1494), and the first English arithmetic (Bishop Tonstall's, 1522), took the culture side. But this is a general historic truth: No arithmetic has flourished among a democratic people, and but two or three have had any great hold in schools that stood for pure culture, that did not minimize processes and magnify the study of those problems that revealed the actual quantitative side of the life of their day. Thus it was that Köbel and Riese in the 16th century in Germany had an enormous influence; that Trenchant and Savonne at the close of the same century in France dominated the teaching in that country; that Recorde did the same for England then, as Cocker did later; and that, with the great awakening of commerce in Holland in the 17th century, a flood of books of this class appeared, revealing in a most interesting way the actual life of the people of that period.

The problems of present arithmetic might well be of two kinds: first, those dealing with the quantitative side of matters of *local* interest; as the cost of blasting rock for a cellar in New York City, the quantity and cost of a mile of asphalt pavement in Buffalo, the quantity of water necessary for irrigating for a season an acre of land in Colorado, and the cost and quantity of materials necessary for fattening a herd of 25 cattle on an Ohio farm; second, those dealing with the quantitative side of matters of *general* interest; as the Pennsylvania Railroad system, an ocean steamship, the comparative cost of transportation of ore from Lake Superior to Pittsburg by rail and by water, the amount of freight carried on the Mississippi compared with that carried by the Illinois Central Railroad which parallels it, a great department store; the labor and money saved by the cotton gin and by other inventions concerned in cotton production; the sugar-cane and beet-sugar industries compared; and dairying and ranching. Since nearly all of the arithmetical processes are mastered by the end of the fifth year at school, the last three years of mathematics in the grades might be spent almost entirely in a study of important industries and other matters on the quantitative side: as mining, banking, investments, manufacture of clothing, government revenues, commission business, gardening and farming, the cost of paving, of water and of gas in different cities, the comparative cost of gas and electric light, a comparison of the steel industry in this country and in England, or a comparison of the growth of certain cities at home and abroad.

Such study would allow mathematics to culminate in two classes of generalizations: (1) the rules of arithmetic to which we are already accustomed, and (2) another large class of truths dealing with the material side of human affairs.

Furthermore; consider how such arithmetic would meet the "needs of society," since the subject-matter leads so directly into practical life. The relation, too, may be one of sympathy as well as of knowledge. A New York street car conductor very often rides 80 miles per day. When we recall this fact, together with the number of fares collected, the number of stops made, the number of persons helped on and off, and the number of hours consumed by all this, his life touches ours as it never did before; and this closer union means deeper sympathy. When, through actual data, we learn that the owner of city houses to rent must count upon their being idle from one-tenth to one-quarter of the time, we understand better the reason for his high prices. And when again, from actual facts, we know that even a careful grocer in a city expects to have 15 per cent of his fresh fruit spoil in hot weather, and to lose one-tenth of all the money that he credits, we are at least able to understand, if not always to condone, some of his actions.

The fact that the answers to many of these actual, living, problems can be only approximately correct, shows the necessity for a class of examples that do not "come out even," a class that really meet the "needs of society." A large percentage of business is of necessity merely a matter of estimates; while even in the most careful scientific experiments, the data can never be exact, and the results of computations cannot be closer to the truth than are these data.

It is by such a plan as the preceding that mathematics in the grades might accept the "interest of the child" and the "requirements of society" as its controlling ideas in the selection of subject-matter. And, under such control, the utility of the knowledge and the value of the mental discipline, using these terms in the common sense, would be enhanced rather than diminished; while much higher purposes in the study would also be accomplished.

But the above plan is far from practicable when present conditions are considered. In mathematics there must be more dependence upon text-books than in most other studies, and our text-books have been prepared along traditional lines. We can omit obsolete matter, but satisfactorily to add such topics as some of those mentioned, and to rearrange the whole on newer lines, is impossible without much time and effort.

The difficulty is particularly noticeable in the sixth, seventh, and eighth grades. The children there know the fundamental operations with both integers and fractions, and should be ready to devote all their time to applications needed in modern life, and the facts that arise from such application. But the arithmetics, instead, omit this latter side and often offer applications of past generations. For instance, they elaborate proportion, which is a mediæval method of solving simple equations, and neglect the geometric application to measuring figures of the same shape (see p. 62), which is really about the only application of importance outside of physics.

Under the present conditions, however, it is impossible to do more than keep clearly in mind the plan which we should like to follow, and to struggle toward it as best we can, here and there, in practice. It should be remembered, also, that it is the purpose of this publication to suggest suitable controlling ideas and a corresponding outline of subject matter, rather than to be a text-book in itself.

In conclusion, it is the mission of mathematics in the grades to make a large contribution to the knowledge of the children, and to appeal to their interest by the richness of its content. Therefore processes can be only a subordinate part of the subject-matter. Deep interest, surprise, and excitement should be produced by the valuable thoughts in this subject as in others. There is less danger in the criticism that an arithmetic is too much like an encyclopædia, than in the criticism that it is too much like a dictionary.

II. OUTLINE FOR THE FIRST FIVE GRADES

Grade I

1. *General Suggestions*

(*a*) In the spirit of the foregoing article, the purpose of mathematics for six-year-old children is to meet from their point of view, their daily need of number as it arises in their school studies and in their relations outside of school. And lest it may be felt that we are emphasizing the abstract, in speaking of number rather than measurement, of which latter we have recently been hearing so much, some explanation should be given. To a child, as to us, to measure anything is to count the number of times some arbitrary unit is contained in that thing. To find a ratio of one thing to another is to find how many times the one contains the other, or what part it is of the other; or, what is the same thing, to find how many times each contains some common unit. Hence the somewhat persistent argument that we should teach a child measurement rather than number, or ratio as the basis of number, is simply to say that we should teach number with due regard to its applications. To this we have already agreed; but to confine the early study of number to the measurement of lengths alone, and ratio to the consideration of an uninteresting series of blocks, is entirely foreign to our belief as expressed in the preceding article as to what arithmetic, at any stage, should be. Having now explained our understanding of the relation of number to measurement, we return to the work of the first grade.

Systematic instruction in this subject at this age is extremely difficult, owing to the danger of its being too formal. On that account, and also because of their peculiar need of other kinds of work, the children of the Speyer School should probably have no regular study of mathematics during the first school year. On the contrary, it should be quite incidental. But, as they are using a large number of materials that require some kind of measurement, they will incidentally acquire much knowledge in this field, provided the teacher is fairly attentive to the quantitative side of their experience.

The children of the Horace Mann School, on the other hand, begin the course with more mature minds, owing to their home training. They have heard more of business in a large way, they have come more closely in contact with nature, they have traveled and have listened to lively table conversation, and they can more safely run the danger of formal work. For them some regular study in this line is not only safe but valuable. But while the recitation period should be regular, the body of thought offered should make no attempt to constitute a system, such, for example, as that outlined by the "Grube Method." Such a plan is the logical arrangement of the adult mind, and ignores the need of motive on the part of the child. The two schools, therefore, cannot cover the same ground either during the first school year, or later. The following outline suggests the quality and arrangement of work; each of the schools should do as much or as little as is fitting. And what is true of these schools is true of others. It is always dangerous to lay down a strict programme for several schools; some latitude is always necessary. Even more dangerous is a series of books definitely limited as to grade work.

(*b*) Desirable Materials. The materials needed are: foot rules, yard sticks, 1-lb. weights with balance scales for weighing, toy or real money, pint and quart measures, building blocks of definite dimensions, including many inch cubes; Speer blocks, splints, materials for number games, and cardboard for making furniture to a scale. The use of fingers for counting should be discouraged, for the reason that they cannot later be removed entirely from reach, when not wanted. The same objection applies to the habit of making marks. But splints and other objects can be so removed and hence are unobjectionable. While materials should vary, in order to hold the interest, and should not lack esthetic value, they should not prove so attractive as to draw attention away from the number work. Much of the gaudily colored material often sold is objectionable on this ground.

(*c*) Variety is itself worthy of being regarded as a principle of education, and since very young children quickly tire of a single kind of work, it is important, where mathematics is a regular study, frequently to vary the materials and the nature of the problems.

(*d*) Normally in mathematics, written work should supplement oral work, by taking up those problems that are too difficult for oral treatment. Written numbers should receive very little attention in this grade, and should be undertaken mainly for the sake of variety and for exercise in penmanship.

(*e*) Accuracy, within a fraction of the large units used, is especially important at this age; it not only impresses the basal units of measurement on the mind, but is necessary for a proper appreciation of the spirit of this branch of knowledge. First impressions are peculiarly important in the attainment of this latter aim. Accuracy is much more important than speed, although it should be remembered that accuracy within small fractions of the units used finally becomes impossible, and the attempt to reach it in measuring is injurious physically.

(*f*) A text-book is unnecessary at this age, although it might secure additional variety in the Horace Mann School and prove helpful in assigning very simple kinds of seat work. On the whole, however, it is likely to increase the danger that the work will become too formal.

(*g*) A reasonable effort should be made to develop the number sense of each pupil; but such effort should stop short of persecution both in this and in higher grades, even though some children are thought to show "no ability to learn mathematics." Excellence in every study is not a *sine qua non* in the Elementary School.

(*h*) One of the principles of education especially applicable at this stage, though especially neglected until recently, is that of motor activity. Its application is called for both by the physical activity of the child, and by the nature of the subject, which requires actual measurement.

2. *The Mathematical Work.*

(*a*) The number-space is limited, extending from 1 to 100. The chief interest which children have in arithmetic, on entering school, is in counting, and this within the number-space above indicated.

(*b*) Counting: — Since much interest lies in counting, as seen in children's "counting out," "keeping score" in games, counting by 5's and by 10's, in being "It" in other games, and in referring to pages of their books, special attention is given to the number series in the space 1 to 100. The children are encouraged to count rapidly by units, and by the groups 2, 5, 10. The material objects of counting should vary so as to show a wide range of application. The emphasis laid upon counting appeals not only to the interest of the child, but it was historically the first stage in the development of the world's mathematics; and it is in harmony with the latest stage, which looks upon mathematics as the science of order, rather than the science of quantity.

In connection with writing, the common notation to 20 should be taught, with incidental use of written numbers to 100. On account of the clock-face, the Roman numerals to XII are taught in the second half-year.

The number idea precedes the symbol in the space 1 to 10. When the value of symbols is somewhat appreciated, the symbol properly becomes the more important of the two.

(*c*) Measuring: — All number is, of course, the result of measure; and as already suggested, there is no dividing line between counting and measuring. When the child counts the number of inches in a foot, he has measured the foot. Hence measuring appeals to his interests and needs at this time, and should form a considerable part of the work. The measures chiefly used in this grade, and with which the child should become familiar by frequent actual use, are the following:

Length, — the inch, foot, yard.

Capacity, — the pint, quart, gallon; the quart, peck, bushel.

Weight, — the ounce, pound.

Time, — the day, week.

The rod and mile are omitted, in this grade, because they are not within the child's range of interest. Similarly the gill, ton, month, year, second, minute (as $^1/_{60}$ of an hour). But of course no teacher should feel limited to the terms given; expressions like "5 minutes," "2 miles," and "6 square inches," may well be used whenever the need arises. Geographical considerations determine many questions of this kind; a child in the country being much more apt to know the width of a street in rods than a city child, and similarly such measures as the peck and bushel.

The metric system is not introduced in the primary grades, because it is not the one that children of this generation will chiefly use. It is needed for general information later in the course, but it would be a mistake to have it interfere with the common system here.

(*d*) Fractions: — As a result of attempts to measure, the fraction appears. It is seen in paper-folding, in the separating of groups of objects — as half the class, a quarter of the blocks, and in such other comparisons as the length of one stick compared with that of another. All this involves the idea of ratio, a fundamental notion in dealing with number, but one which it is neither necessary nor advisable to make very prominent, *per se,* with children.

The fractions with which children should become familiar in this grade are ½, ⅓, ¼, ¾. This does not, however, exclude the incidental use of such other fractions as may naturally enter into the work of the class; for if the child knows thirds, he knows ⅔ and $^3/_3$ as well as ⅓.

(*e*) Operations: — The only operation to which much attention need be given in this grade is *addition,* and this only as it is necessary in such problems as "5 inches and 4 inches are how many inches?" or "2 ft., 3 ft., and 5 ft. are how many feet?" In general, such problems should be arranged in columns, as is done in ordinary business life; the equation form, "5 in. + 4 in. = 9 in.," is relatively of less importance and should come in the second semester. It is a mistake, for various reasons, to attempt to treat the four fundamental processes simultaneously. They are not of equal difficulty; the child does not need them to an equal degree; and the world of business does not use them with equal frequency. Hence addition, the easiest, the most important, and the most interesting to the child, occupies the chief attention in this year. Incidentally, as needed in the simple problems proposed, the ideas of subtraction, multiplication, and division, are introduced; but the work, even in addition, is limited to the number-space 1 to 20, and no tables are learned.

So far as subtraction is treated, the "making change" method should be used. For example, "If you have 10 cents, and you buy a pencil costing 3 cents, how many cents have you left?" The child should see that he has 7 cents left, because 3 cents + 7 cents = 10 cents. This is the oral subtraction of business life, and it is the basis for the "Austrian subtraction" recommended in Grade III.

No attempt should be made in the first year to cover systematically all number relations within any set number-space.

(*f*) Symbols: — In Grade I there should not be any systematic attempt to have the symbols +, —, ×, ÷ used by the children. They may be used by the teacher, and explained, and made part of the lessons in writing, but it is not wise to give any considerable number of written exercises of the form $2 + 3 - 4 + 1 = ?$ Still more objectionable are forms like $2 \times 3 + 2$, and, particularly, $2 + 2 \times 3$.

The reasons for opposing such examples are as follows:

(1) These chains of operations enter very little into real mathematics. In practical life we never meet a problem like $2 \times 3 + 4 \times 6 \div 3 - 2$, nor do we find these symbols much used in algebra or the higher mathematics. Hence they should play but a very small part, if any, in the education of children.

(2) The teacher's personal judgment as to how a chain of operations ought to be treated has no validity unless supported by the conventions of mathematicians. For example, a teacher might say that he thinks that children should be taught that $2 + 2 \times 3 = 12$, taking the operations in the order stated; but the *mathematical convention* is that $2 + 2 \times 3 = 8$, the multiplication being performed first. If the child gets the wrong idea now, it will trouble him throughout his subsequent mathematical work. If such chains are to be given, they should be in forms that admit of no misunderstanding, as $2 \times 3 + 2$. But even these are open to such serious objection that they should not be recommended.

The symbol × is preferably read "times" when the multiplier comes first and "multiplied by" when it comes second, as

(*a*) 2 × $3, "2 times $3."

(*b*) $3 × 2, "$3 multiplied by 2."

This enables the sentence to be read from the left to the right. The reading

(*c*) $3 × 2, "2 times $3,"

has good authority, particularly in England and France, and among older American writers, but it is coming into disfavor because it is not in accord with the genius of our language. The reading first suggested (*a*) has the sanction of a rapidly growing number of our best writers, and is thus becoming conventional. There is the added reason for it that, in algebra, the numerical multiplier is generally put first, as in $3ab$. Practically, however, the method of reading the × is not so serious as teachers often suppose, because in business the × is used more commonly to mean "by," as in the expression 2 ft. × 5 ft., 3 × 9, where it does not indicate multiplication. Furthermore, it is well to know that in mathematics the symbol has lost all standing, except in the logical statement of an arithmetical solution. In algebra, and even in the theory of numbers, it has long since been nearly forgotten, the dot or the absence of sign having taken its place, as in $2 \cdot 4$, and $3ab$. Indeed, symbols +, —, ×, ÷, and =, were invented for algebra rather than for arithmetic, and their chief value is still there, although the × and ÷ have been generally discarded by algebraists. It is only in the written analysis of problems in the grammar school grades that arithmetic has much use for them.

3. *Problems Suggestive of the Type Desired*

As previously stated, arithmetic is merely the quantitative side of our experience, and those problems will be of most interest that are drawn from fields that have already become attractive. Hence they should be taken from other school studies, and from experiences of daily life outside of school. Since the curricula of the Horace Mann and the Speyer Schools are radically different in some respects, some of the following problems that are suitable for the one, will not prove fitting for the other school.

1. With toy dishes, or blocks to represent them, set a table for four persons. How many plates? knives? forks? napkins? How many spoons, with two for each person? How many of each are necessary for a family of four persons? For your family?

2. In your reader find the following pages as they are called: 5, 7, 21.

3. Count by 5-cent pieces; by dimes.

4. Tell the length and breadth of each of these blocks (building blocks). Name each by its measurement. For example, 2 × 4-inch block, 8-inch block. Use these names always in building, and, in general, call denominate numbers by their full names.

5. Make a plan of a room, the length being three 8-inch blocks, and the width two 8-inch blocks. Make other plans. Measure the dimensions of the school room, of the yard. Compare the school room in size with the home of the Pilgrims, and with an Eskimo hut.

6. Show a foot-measure. Draw lines, and point out objects, 1 ft. long. Do the same with the yard measure. Estimate the length, breadth, and height of objects, and then test by measurement. For example, — "Mary, estimate John's height; now measure to see how nearly right you were, making your answer correct within one inch." "Give what you think to be the height of this table, and measure as before." Give similar problems for chairs and doors. Draw the plan of a washcloth of suitable size; of an iron holder. Give their dimensions.

7. Find objects that you think weigh 1 lb. See if they do. Weigh various objects on scales. Likewise use other measures until the units of measurement are quite familiar.

8. A good milch cow averages about 10 quarts of milk per day. Show with the measures how much that would be. How many families could she supply with 1 quart for each? With 2 quarts?

9. Name things that the grocer sells by the quart, the pound, the bushel. Give such orders as your mother gives to grocers. Learn the prices, and pay for some of these things in toy or real money.

10. Do the same in connection with a bakery shop.

11. Using sand to represent sugar and other commodities, weigh out what different children call for.

12. Make measurements on paper for seed envelopes, boxes, and toy furniture of a certain size.

13. On the sand table lay out a garden according to some scale; include the walks and a garden plot for each child. Measure the growth of plants from time to time.

Problems in making a dirt digger

1. The stick is 10 inches long. If we use 2 inches for the point, how much is left for the handle and blade?

2. Half of the remaining part is for the handle; half for the blade. What is the length of each?

3. The stick is 1½ inches wide. The handle is to be half of that width. How wide is it?

4. How much must be cut off from each side, in order to leave the handle in the middle?

Seed marker

Cut strips 6 × 1 × ⅛ inch. Draw a line across the stick 1½ inches from the end. Find the center of the end. Draw a line from this point to each end of the cross line. Cut the stick.

Wool carders

1. Our board is 4 inches wide. How many rows of nails will it hold ½ inch from the edge and ½ inch apart?

2. How many nails are needed for each row, when the board is 4½ inches long and the nails are to be ½ inch from the edge and ½ inch apart?

Games

1. Draw on the floor or table three concentric circles of radii 3, 5, 8 inches, giving each circle a certain value. Now roll a marble so as to stop it in one of these circles, and count the value given in favor of pupil. Let each child keep his own score.

2. Bean-bag game. Have a board with a hole in it, and three bags of different sizes, each having a certain value. Throw these bags through the hole. Let each child keep his own score.

3. Ring toss. Same in principle as the preceding.

4. An exercise that children enjoy very much is counting by 1, 2, 5, or 10, while certain children are doing a given task. For example, see if we can count to 20 by 2's while the pencils are being collected. Teachers find this to be a valuable drill.

5. Shuffle board. Make bags out of heavy ticking. Fill them with sand, weighing respectively ½, 1, 1½, 2 lb. Vary from time to time the values placed upon bags. Have the captain keep the score.

Some of this work can best be done in other recitation periods than those for arithmetic, even though a separate period for arithmetic is planned. At such times it is important to keep in mind, as a pedagogical principle, that any one recitation should aim primarily to do work of one kind only, be it literature, learning to read, nature study, or something else; and that only as much apparently foreign subject matter should be allowed as is directly necessary for the accomplishment of this one aim. If this rule is not followed, a recitation period may be occupied with such a medley of matter that neither children nor teacher can tell what has been accomplished. The arithmetical games suggested should soon become part of the out-door play.

Grade II

1. *General Suggestions*

(*a*) Review the work of Grade I in the same spirit in which it is there taken. In general, the first three or four weeks of each year should be devoted to a review of the work of the preceding year. When this is done, the common complaint of the ignorance of children of the preceding work may cease.

It should also be remembered that the review should be a prominent factor in every lesson, occupying perhaps half the time in the primary grades. Drills, of course, are included under such reviews. Although much advance has been made in education, drill in the fundamental operations, even to the point of automatic work, has not yet been found unnecessary. In the higher grades, it is less necessary to plan so directly for reviews, because they are incidental to the solution of the problems.

Whatever part of the work outlined for Grade I has not been covered by the beginning of the second grade in the Speyer School will, of course, be taken up at the beginning of that year. In general, there is likely to be from six months to one full year's difference between the pupils of the two schools in mathematical ability and knowledge. This outline is planned primarily with reference to the Horace Mann School, when it comes to division of work into certain grades.

(*b*) In this grade the work in arithmetic comes to be treated more as a system, the child's interest having now been aroused to such a point as to permit it. This means that numerical tables are learned as such, and a more or less definite amount of work is laid out in the domain of pure number. As in other studies, however, this abstract part should follow a large amount of concrete matter.

(*c*) There is no radical departure, however, from the kind of work done in Grade I. There should be the same close relation to manual training, to other studies, and to the problems of daily life. The children should be encouraged to invent problems; the quantities of food, fuel, and clothing used, with cost, forming a basis for much of the work.

(*d*) Materials as before. Also cards with dots and figures. Dominoes and other games, including the number games, of the Cincinnati Game Company. Objective work in establishing number relations is still necessary. There is, for example, an advantage in counting by 10's, and in seeing bundles of splints. But there is a danger in carrying the objective work beyond the needs of the children, as there is in abolishing it too early. A support is necessary at first to a child learning to walk; it ceases to be necessary later; to tell when it ceases to be necessary and begins to be harmful, demands careful thought for each individual, — and so for objective teaching. But the children themselves may be relied upon for some aid in this matter, if a pride has been developed in them to abandon the use of objects as soon as possible.

(*e*) If the oral work is rapid and accurate, the written work will be so. But in quantity the oral work should greatly predominate. There is a temptation to have too much written arithmetic at this stage, simply because it is easily assigned for seat work. The great danger is that such young children, thus left alone at their seats, will drop into careless habits involving division of attention. No text-book is advisable, at least in the first part of this year, for reasons previously given.

(*f*) In the written work the same attention should be given to neatness and accuracy that is required in other written exercises. Indeed, these qualities are particularly important objects of written arithmetical work in this grade.

The equation form may now be introduced; as $2 + 5 = ?$, $2 + ? = 7$, $? + 8 = 8$, $2 \times ? = 6$, $6 \div ? = 2$; or with n (for number) in place of the " ?." But it should be remembered that these are not the forms of practical life, and more time should be directed to the forms more common; namely, the work in columns. Such symbols of operation are necessary later, when the child comes to the explanation of problems of some length; in this grade they enter only incidentally. If the teacher gives chains of operations like $2 + 3 - 1 + 2$, or $2 \times 3 - 4$, mathematical conventionalities must be observed; e.g., $2 + 3 \times 4 = 14$, not 20, and hence it would be better to write this $3 \times 4 + 2$, so that the operations could be taken in their order. (See p. 19.) Written work of this kind is not, however, very valuable; the oral treatment is better.

(*g*) Bibliography: — Smith: Teaching of Elementary Mathematics, Chaps. i-v; McClellan and Dewey: Psychology of Number, Chaps. i-ix; Phillips, in the Pedagogical Seminary, Oct., 1897.

2. *The Mathematical Work*

(*a*) The number-space, 1 to 10,000 for reading (as in larger street numbers), 1 to 1,000 for counting and writing, 1 to 100 for operations. The Roman numerals may be limited, as in Grade I, to the space I to XII, this being sufficient for the present uses of the children; although many city courses carry this system to C.

(*b*) Counting: — The counting of Grade I is carried forward. Counting by twos, from 2 to 20; by threes, from 3 to 30; by fours, from 4 to 40; the object being to lay the foundation for the multiplication table to 4 × 10. This counting will naturally and incidentally extend farther as an interesting exercise.

Also counting by twos from 1 to 9, by threes from 1 and 2 to 10, and by fours from 1, 2, 3, to 11; the object being to form the tables of addition. This work can easily be made an interesting exercise in games, and rapid oral work will have an interest *per se*. But for interest, much dependence should be placed on the proper kind of problems. The meaning of "dozen."

(*c*) Tables: — When the counting suggested in (*b*) has been carried out, the children know the table of addition, and also the table of multiplication through 4 × 10. The latter should, however, be learned and drilled upon as such. Also the table of 10's and 5's, the former to 100, the latter to 50. The subtraction table need not be developed and learned, since subtraction is made to depend upon addition as stated below.

(*d*) Measuring: — The measures chiefly used in this grade are those used in Grade I, with a few additions. They are:

Length, — the inch, foot, yard.
Capacity, — the gill, pint, quart, gallon;
the quart, peck, bushel;
the cubic inch.
Weight, — the ounce, pound.
Time, — the day, week, month;
minutes in hour, hours in day, days in week.
Surface, — the square inch, square foot.

Here, as in all grades, the children should become thoroughly familiar with the measures by actual use.

(*e*) Operations: — Addition of several one-figure num- 2
bers; or of two-figure numbers, neither column at first ex- 3
ceeding 9. Accustom the children to say, "5, 14, 21, 24, 26," 7
from below; and also, "2, 5, 12, 21, 26," from above. In 9
general the child should read a column as he reads a sentence, 5
never counting by units any more than he would spell by —
letters. 26

Subtraction has from the earliest times been taught in various ways. Some methods of treatment are more rapidly performed, others are more easily explained. Since, however, the child is not expected to learn or to repeat any but very simple explanations, and since the purpose of an explanation is to justify a process that soon should be merely mechanical, it is reasonable to expect that the shortest operation is the one that, in the long run, will be accepted by the world. This is seen in many of the other operations. For example, in the division of fractions, the world now inverts the divisor and multiplies, although for generations it divided by reducing the fractions to a common denominator.

Multiplication is in this year limited to one-figure factors, and to the domain 1×1 to 4×10.

Division is limited chiefly to "exact division," not involving mental "carrying." It is introduced only as needed, in simple and natural problems, as in taking ½, ⅓, etc.

The terms addend, sum, subtrahend, minuend, difference, multiplier, are used as necessary, but no formal definitions are given.

Fractions: Halves, thirds, fourths, fifths, sixths, eighths, tenths. The seventh and ninth are omitted, or are met only incidentally, there being almost no use for them in the measurements and the problems of this grade. In general, no inexact fractional parts, like ⅓ of 7; such cases may, however, enter into certain impromptu problems.

3. *Suggestive Problems*

1. Practice stepping off 2 feet at each step. Then, by stepping, find the width of the street in front of your school-building. What is the width of the widest streets near you? What seems to be the average width of streets in this city? In the same way find the width and depth of near-by lots. Show the size of an ordinary yard in a village.

2. How many children can be accommodated at the blackboard in your school-room at one time, counting 2½ feet for each child?

3. Measure your school-room. Make a floor plan of it, with blocks, according to some scale. Likewise measure and make a floor plan of some grocery store you have visited; also of a bakery.

4. Name the months in each of the four seasons. Each season is what part of one year?

5. Measure the growth of plants, twigs, etc., correct to within ½ inch.

6. Make a floor plan of the apartment, or of one of the floors of the house, in which you live. Use blocks or make a drawing to scale.

7. How many chairs are needed for a six-room apartment containing 4 persons? Explain your answer.

8. Give the size of a fair-sized napkin. Of a towel. Show by drawings. How many towels are needed to supply a family of four? Explain your answer.

9. Read the thermometer from time to time and explain what it signifies.

10. Count the number of grains on an ear of corn. One grain of corn often produces a stalk with three such ears.

11. A family in this city, burning coal in the range, often spends 25 cents per week for kindling, receiving 3 bundles of sticks for 5 cents. What is the cost per month? How many bundles are burned per week?

12. It takes 2½ gallons of milk to make 1 lb. of butter. One gallon weighs 9 lbs. How many pounds of milk make 1 lb. of butter? Milk costs us about 7 cents per quart. What is the worth of the milk necessary for 1 lb. of butter? What is the cost of one lb. of butter? How does this compare with the cost of the milk? Why this difference?

13. Give the dimensions of a pencil-box of suitable size for use.

14. Games, including dominoes, and the number games of the Cincinnati Game Company.

Grade III

1. *General Suggestions*

(*a*) Review the work of Grade II, devoting three or four weeks to it. See General Suggestions (*a*) under Grades I and II.

(*b*) The text-book is necessary in this grade. Some reasons for the delay have already been given. (See page 16.) The use of the text-book is that it may serve: — (1) as a reference book for facts, like a dictionary, as in the case of tables; (2) as a collection of problems to save the time of dictation. The problems often fail to appeal to interest and must be supplemented or omitted, but the choice of evils leads to using a book from now on. There is a serious danger, however, in introducing a book, of departing too radically from the child's daily interests.

(*c*) The work in this grade becomes still more systematic. The object of the year, so far as operations are concerned, is to become familiar with "the four processes" in the number-space 1 to 1,000, division being limited as set forth below. The object, so far as rich thought-content is concerned, is to acquaint the pupils with numerous separate facts of interest, and thus to reveal to them the quantitative side of a few large topics.

(*d*) Objective work becomes less and less needed in number relations of integers. But it is just as necessary as ever in presenting radically new subjects, as in reducing fractions, in studying new forms, and in taking up new measures.

(*e*) Written work is still subordinate, being confined largely to computations too extensive for purely mental treatment. A large amount of rapid oral analysis should be given in very abbreviated form. Elaborate analysis should be avoided both by the teacher and the pupil.

(*f*) There is a great advantage in the occasional use of oral analysis, giving the main steps without performing the operations. For example: If a book costs $2, how much will 36 such books cost? Answer: "36 books will cost 36 times $2."

Since children often fail to get the exact condition of a problem, it is well to have them restate it, in their own words, if it admits of such a change. It is well also to have them state the number of steps involved in the solution, showing very briefly what they are, and giving the approximate answer. The actual solution may then follow, or may be omitted entirely for the time. Such work forces children into the "thought-side" of arithmetic, and away from the impression that it is all "mere figuring"; in other words, it overcomes the tendency to become too formal. The oral solution of problems with only approximate answers may well increase in prominence as the work advances. One argument in its support is the fact that many of the problems of adults are actually solved only approximately; also, a very large percentage of adults use the approximate answer as a guide to correctness in the actual calculation.

(*g*) Geographical considerations should influence the subject here as in the other grades. (See Grade I, 2, (*c*).) For example, such words as penny, bit, shilling, nickel, are common as names of coins, in some parts of the country, but not in others.

(*h*) It is important to consider how far induction is legitimate in elementary arithmetic. The world induced before it deduced; and the child does the same. Even in the highest mathematics induction plays a great part, in discovery; then deduction comes to play its part in proving that the induction is correct. So in the primary grades induction is not only allowable, but it is the natural method. The child comes, however, in the later grades, to the period of more or less rigid deduction, because he is largely applying principles already familiar. Then, however, another kind of induction should be going on, since

(see page 50) he should be reaching new generalizations about industries and other phases of our quantitative experience. Truly inductive work in arithmetic requires that the rule be put in the background at first; concrete problems should first occupy the attention, and only after several of these have been solved, and the methods compared, should the rule itself be broached and worded. Even then there is a great danger in making altogether too much of memorizing rules, particularly those of operation. Teachers should, however, distinguish between a rule that is originated by the pupil, and one which is dogmatically given to him. The former has high value; the latter is dangerous.

For example, take this problem: Your garden is 24 yards long and 3 yards wide. How many feet of wire will it take to enclose it? A rectangle represents the plot of ground, and we have this column for addition:

$$\begin{array}{r} 24 \\ 24 \\ 3 \\ 3 \\ \hline \end{array}$$

Different devices may be employed to explain the process, as shown on page 31.

Another example might be: A soldier on a long march must carry a knapsack weighing 2 lbs; food in it weighing 4 lbs; a gun weighing 9 lbs; and a blanket weighing 14 lbs. How much must he carry in all?

Similar problems should be solved before an attempt is made, if ever, to phrase any rule of addition.

The following might be suitable problems in subtraction: — (1) A good carriage horse often weighs 1,025 lbs. A good dray horse often weighs 1,850 lbs. What is the number of pounds of difference? Why need there be such a difference in weight? (2) In May, 1902, anthracite coal sold at $5.25 per ton; in August, owing to the strike, at $10.00 per ton. How much advance was made in price? (3) For children who have been studying history: How many years is it since Henry Hudson sailed up the Hudson River?

In truly inductive work there is no feeling of hurry to reach the rule; the latter is gradually brought to light through numerous positively concrete examples.

(*i*) Types: — The question arises, How should the rule be recalled by the child? Shall he learn the teacher's rule verbatim? Or shall he memorize a wording of his own? Or shall he recall some practical problem, and reproduce the process there followed? Many teachers prefer the last plan. For example, the garden mentioned on page 29 is 24 yards long and 3 yards wide.

20 + 4		24		24
20 + 4		24		24
3	or	3	or	3
3		3		3
40 + 14 = 54		14		54
		40		
		54		

In subtraction, taking the example about the horses, we have:

	1,850	1,850 = 1,800 + 40 + 10
less	1,025	1,025 = 1,000 + 20 + 5
		800 + 20 + 5 = 825

For a discussion of the various methods of solution in subtraction see page 31. Whatever the method followed, the particular example is held in memory for some time as an aid to the solution of new problems. Being concrete, it can easily be remembered; and since it must be recalled many times for help, the rule itself gradually comes to light and may finally be carefully worded, — preferably in the pupil's own way. This plan avoids the premature learning of the generalization.

(*j*) Bibliography: — Smith: Teaching of Elementary Mathematics; and McClellan and Dewey: Psychology of Number, as before; McMurry: Method of Recitation, Chap. 2, p. 14.

2. *The Mathematical Work*

(*a*) The number-space: — 1 to 1,000,000 for reading and writing, 1 to 1,000 for operations. The Roman numerals I to D, but not to work out every numeral, the reading of chapter numbers being the important feature.

(*b*) Counting: — The counting of Grades I and II is continued sufficiently to complete the development of the multiplication table to 10 × 10, which is as far as it need be carried.

(*c*) Tables of abstract number: — The multiplication table as above, thoroughly learned and made the object of continued and rapid oral drill. The addition table reviewed daily, and also made the object of continued oral drill.

(*d*) Measuring: — In addition to the measures introduced in Grades I and II, the following are studied:

Length, — the rod and mile. Number of New York blocks to the mile, north and south, also east and west.

Weight, — the ton.

Time, — months in year;
 days in months, "Thirty days hath September," etc.;
 days in year;
 weeks (approximately) in the month and year.

Meaning of "quire" and "score."

Particular attention to the table of United States money, but always omitting the "eagle," and omitting the "mill" in the primary grades.

(*e*) Operations: — the four fundamental processes in the number-space 1 to 1,000, division being limited to divisors of two figures.

In addition, teachers are not limited to any one device. In explaining a process involving some mental retention of number, as in "carrying," it may be necessary to consider sticks in bundles of 10, as is often done, and to adopt also such devices as

$$\begin{array}{rcl} 26 = 20 + \;\;6 & & 26 \\ 39 = \underline{30} + \underline{\;\;9} & & \underline{39} \\ 50 + 15 = 65 & & 15 \\ & & \underline{50} \\ & & 65 \end{array}$$

in order to bring out the idea. Since no explanation should be required from the child at this stage, further than the answer to simple questions, all that is necessary is that he should (1) know how to perform the operation, and (2) feel that he has taken nothing on mere authority, beyond the conventionalities of the subject.

In subtraction, of the various methods suggested, the one that is now having the greatest favor is the so-called "Austrian method," which has the advantages of brevity and of requiring no subtraction table to be memorized. The general steps necessary to its development are:

(1) Meaning of difference, — that number which added to the smaller number produces the larger, — an idea sufficiently complete for children. For example; $9 - 5 = 4$ because $5 + 4 = 9$. By recalling the addition table, such differences are at once brought to mind. Apply to cases like $49 - 25$:

$$\begin{array}{r} 49 \\ 25 \\ \hline 24 \end{array}$$

Any subtraction of two-place numbers where no figure in the subtrahend signifies a larger amount than the corresponding figure in the minuend, follows this plan.

(2) Adding 10, 100, or 1,000 to each number does not change the difference. Inductively shown from

$$\begin{array}{rrr} 9 & 19 & 29 \\ 5 & 15 & 25 \\ \hline 4 & 4 & 4 \end{array}$$

(3) Consider the case of

$$\begin{array}{r} 52. \\ 27 \\ \hline 25 \end{array}$$

Excluding the advanced idea of negative numbers from this discussion, we may say, there is no number which added to 7 makes 2, but 5 added to 7 makes 12. We have now increased the 52 by 10, and we must add 10 to 27 so as not to change the difference. 3 (tens) and 2 (tens) are 5 (tens). Hence the difference is 25.

Like all innovations in our own long established mental processes, this may seem difficult; but looked at from the standpoint of the child it saves memorizing many facts, and it reduces subtraction to the much simpler operation of addition. The objection that it does not give the notion of "taking away," is hardly worth notice; it gives the result with a minimum of mental friction, and this is the practical end in view. The details of presentation are, of course, capable of great variation, involving the use of materials, and such devices as:

$52 = 50 + 2$ the difference being the same
$27 = 20 + 7$ as between

$$\begin{array}{rcr} 50 & + & 12 \\ 30 & + & 7 \\ \hline 20 & + & 5 \end{array}$$

In *short division,* it may frequently be necessary to use the "long division" form, showing that the former is only an abbreviation of the latter. And, in general, the abridgment naturally follows the more complete form in all operations. As in addition, an introduction to "carrying" often requires such a device as this:

```
27
36
——
13
50
——
63
```

and as in subtraction we have to resort to complete forms like this:

The difference between 52 and 27 equals that of

```
    40 + 12  or  50 + 12
and 20 +  7 and  30 +  7
                 ——    ——
                 20 +  5
```

so in multiplication we lead to

```
12            12
 7  through    7
——            ——
84            14
              70
              ——
              84
```

and in division we lead to

```
5)134
  ———
  26, and 4 remainder,
```

```
through      26
            ———
          5)134
            100
            ———
             34
             30
             ——
              4 remainder,
or through  5|100 + 30 + 4
             ————————————
             20 +  6, and 4 remainder.
```

No definite rule can be laid down for these cases. The teacher should use the more complete form whenever necessary for the class in hand, and come to the abridged form as soon as the pupils are ready for it.

We shall find that it is generally better in "long division" to put the quotient above the dividend. (See page 39.) In "short division" this could also be done were it worth while to change the well established custom of writing it underneath.

(*f*) Geometric forms. The square and rectangle, as met in manual training, basketry, paper-work, and elsewhere, are studied with reference to perimeter and area. The cubic inch and cubic foot. Study of the circle, including the diameter and the radius, but without mensuration. The details of the work in geometry are reserved for a subsequent article.

(*g*) Fractions and ratios are incidentally studied, carrying on the work of Grade II. There is no formal treatment of the subject. Such simple reductions as $\frac{2}{4}=\frac{1}{2}$, $\frac{2}{8}=\frac{1}{4}$, and such operations as $\frac{1}{2}+\frac{1}{4}$, $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{1}{2}-\frac{1}{4}$ are made part of the oral work.

The decimal form is used in writing United States money, but without any explanation of the decimal system. For the sake of practice, papers may be marked in per cent, and the term should be explained and used in other work.

3. *Suggestive Problems*

1. One gas jet burns about 5 cubic feet of gas per hour. How much gas is consumed per week in a house that burns 3 jets each evening from 7 to 10.30 o'clock? Estimate the amount that a street lamp burns in one month.

2. Gas costs $1 per thousand cubic feet. Estimate the cost of the different amounts of gas above mentioned. Estimate the cost of gas for street lights for a distance of one-quarter of a mile on one street.

3. A faucet actually leaked in a New York house at the rate of 1 pint of water every thirty seconds. How much would that amount to in one day of twenty-four hours? In one month? The water was allowed to run six months; how much was the waste?

4. Make out an actual bill for ice for your family for one month.

5. Live cattle sell at 7½ cents per lb. Good beef-steak now costs 28 cents per lb. What is the difference? Why is this difference?

6. Make yourself acquainted with some of the more common fish, and the average weight of each. Tell what one fish of each kind will cost at the customary price per pound.

7. Estimate the number of persons living in some apartment house near you, and explain how you do it.

8. Tell time by the clock, to minutes and fractions of a minute.

9. What distance do you walk each day in going to and from school once? Show how you find it.

10. In order to run a single ordinary rock drill, used for blasting, one engineer and two men are necessary. The leading man with the drill often receives $2.75 per day; his assistant, $1.50; the engineer, $3. One drill will keep a blacksmith busy about one-fourth of the time, sharpening the drills; he receives $3 per day. How much does the labor cost to run a single drill one day?

11. A business man often sends 18 letters per day, 6 days in the week, each requiring a 2-cent stamp. At that rate, what is his postage bill per month of four weeks?

12. Talk with a gardener to find out how large a lettuce bed it would take to supply all the lettuce your family would need for a season. Show how you estimate it.

13. Young chickens are now worth 20 cents per lb. What is the cost of one weighing 2½ lbs.? Cost per dozen?

14. The chicken industry:—

(*a*) Of the eggs that a farmer "sets" for raising chickens, seldom more than 4 out of 5 hatch out. If he sets 600, how many chickens does he get?

(*b*) Of those that are hatched out, about 1 out of 3 is drowned, killed by wild animals, or sickens and dies, before the age of 3 months is reached. How many grow up from the 600 eggs?

It would be well to continue a story like this, referring to the number eaten by a family, the number sold, the weight and price per pound, and the amount and cost of feed. Similar series of problems in raising a herd of sheep. Estimate the cost of feeding a horse in this city.

15. A motorman on a New York trolley car very frequently runs his car 80 miles per day. How many times does he stop if he averages one stop every third block, counting 20 blocks to one mile?

16. The conductor often collects 75 fares on one trip "up town," covering about 11 miles. How many dollars would this amount to, counting fares at 5 cents each?

Grade IV

1. *General Suggestions*

(*a*) Review the work of Grade III, devoting three or four weeks to it. See General Suggestion (*a*) under Grades I and II.

(*b*) On the use of the text-book, see General Suggestion (*b*) under Grade III.

(*c*) Touching processes, the work of the year is chiefly given to the ability to handle the four operations in general problems and to the addition and subtraction of common fractions, the essentially new work being the completion of "long division." As to quantitative facts of value, it is now possible to present the principal sides of many leading industries, and thus to build up as full and vivid a picture of many matters as is done in other studies, though always through figures rather than through words.

(*d*) This is the grade in which children often begin to lose interest in the subject, and well they may if processes only are depended upon to arouse life. It is the teacher's duty to plan much variety in method, and to make fair use of competition; but more than this should be done. Really valuable subjects should be selected; interest from any other source is likely to be largely artificial. The text-book, of course, contains but few such topics, and the teacher will simply have to do the best she can, under present circumstances, to find them herself.

(*e*) On the proper use of diagrams. These can be extensively used to advantage, and should be drawn and studied before the computation begins; otherwise, the child wanders about in the dark and gets no good from them. For example: — In finding the area of a floor, the diagram, if needed at all, is needed before the computation. In general, the children should become accustomed to thinking out the main steps to be taken and the approximate answer, before beginning the computation.

(*f*) For further suggestions, see Grades I, II, III.

(*g*) Bibliography: — Smith, McClelland and Dewey, and McMurry, as noted in the preceding grades.

2. *The Mathematical Work*

(*a*) The number space. No limits need be observed hereafter, except that number names beyond billions are of little value. Roman numerals up to MM being sufficient for dates, form the limit for this system in the grades.

(*b*) Tables. The addition and multiplication tables have already been learned, and are to be made the subject of constant review.

The following table of aliquot parts should become known this year, in anticipation of the work in decimal fractions for next year:

$$\tfrac{1}{2} = 0.50 = 50\%$$
$$\tfrac{1}{4} = 0.25 = 25\%$$
$$\tfrac{1}{8} = 0.12\tfrac{1}{2} = 12\tfrac{1}{2}\%$$
$$\tfrac{1}{5} = 0.20 = 20\%$$
$$\tfrac{1}{10} = 0.10 = 10\%$$
$$\tfrac{1}{3} = 0.33\tfrac{1}{3} = 33\tfrac{1}{3}\%$$

Such facts, like the rules of arithmetic, should be reached through study of concrete examples. The other plan, deductive as it is called, though not truly deductive, is to study each fact and the rules first; and, after having learned them, to apply them to examples.

(*c*) Measures. This year should see the tables of denominate numbers systematized, and the great basal units of arithmetic emphasized.

The following tables are given as the ones to be memorized, the chief units being italicized:

Length. 12 *in.* (12″) = 1 *ft.* (1′)
3 ft. = 1 *yd.*
5½ yds. or 16½ ft. = 1 *rd.*
320 rds. or 5280 ft. = 1 *mi.*

The surveyor's table is omitted, as being part of the technical education of a surveyor, and unnecessary in common life.

Surface. 144 *sq. in.* = 1 *sq. ft.*
9 sq. ft. = 1 *sq. yd.*
30¼ sq. yds. = 1 sq. rd.
160 sq. rds. = 1 *acre*
640 acres = 1 *sq. mi.*

In certain communities the study of township sections would have place.

Capacity.

Liquid.	Dry.	Cubic.
4 gi. = 1 *pt.*	2 pts. = 1 *qt.*	1728 *cu. in.* = 1 *cu. ft.*
2 pts. = 1 *qt.*	8 qts. = 1 *pk.*	27 cu. ft. = 1 *cu. yd.*
4 qts. = 1 *gal.*	4 pks. = 1 *bu.*	A "cord of wood" explained.

Weight. 16 *oz.* = 1 *lb.*
2000 lbs. = 1 *ton*

This table of avoirdupois weight is the one which the children will practically need. The long ton should be mentioned, but troy weight and apothecary's weight are technicalities of trade and are not needed in common life.

The child's original motive for studying these facts should be found largely in the needs arising in manual training and other studies, and in home experience; but the teacher should now collect the various facts into "tables," and drill upon them until they are properly fixed.

(*d*) Compound numbers are involved in the measures mentioned. In this grade the operations with them are confined chiefly to such simple reductions as are necessary in the applied problems mentioned; namely, feet to inches, pounds to ounces, days to weeks, and similar cases.

(*e*) Operations with integers. The essentially new feature of the year is the completion of "long division." The early printed arithmetics gave several methods of division, and the one now commonly used was finally adopted nearly two centuries after the beginning of printing, showing that it was by no means universally accepted as the best. Now a slight improvement on the common method has been suggested, and is rapidly meeting with favor. It is one of the forms often called by the name of "Austrian Method." The successive steps for explaining "long division" vary with the class. In a general way, they cover the following ground:

```
     Write                        Think

       78
     -----
    3)234
      210
      ---
       24                     3| 210 + 24
       24                      ---------
      ---                         70 +  8
```

The quotient of 200 ÷ 3 is not any number of 100's. The quotient of 23 tens ÷ 3 is 7 tens. And since the product of 3 and 7 tens is 21 tens, there remains 24 to be divided. 24 ÷ 3 = 8. And since the product of 3 and 8 is 24, there remains nothing to be divided.

Similarly for $625 \div 25$:

Write	*Think*

<pre>   25
25)625
   500
   --- 	
   125
   125
   ---</pre> 	
	<pre>25 |500 + 125
	    --------- 	
	     20 + 5</pre>

After this is understood, the abridgment as shown in the common "Austrian Method" should be used. For example: —

```
   25
  ----
25)625
   50
   ---
   125
   125
   ---
```

While the decimal fraction is not taken up in this grade, an example involving such fractions is given to show more fully the reason for adopting this particular algorism.

Required to divide 6.275 by 2.5:

OLD METHOD

```
2.5)6.275(2.51
    50
    ----
    1 27
    1 25
    ----
      25
      25
    ----
```

"Point off as many places in the quotient as the number of decimal places in the dividend exceeds that in the divisor."

COMMON AUSTRIAN METHOD

```
    2.51
25)62.75
   50
   ---
   127
   125
   ---
    25
    25
   ---
```

Dividend and divisor having been multiplied by such a power of 10 as makes the divisor a whole number, the decimal point in the quotient simply goes above that in the dividend.

The following method is recommended for the early work:

```
      2.51
  25)62.75
     50
     ----
     12.75
     12.5
     ----
      0.25
      0.25
     ----
```

The entire remainder is brought down each time, and the decimal point is preserved throughout.

In this grade the child is ready to consider the twofold notion of division, not very philosophically, but sufficiently to insure accuracy of statement. Division is the inverse of a simpler process, multiplication, just as subtraction is the inverse of a simpler process, addition. A complication arises, however, in division, which does not arise in subtraction, as is seen in the following: —

Addition and subtraction: —

$3 + $2 = $5; therefore, (a) $5 — $3 = $2
or, (b) $5 — $2 = $3.

These two inverses are essentially the same, because the numbers all represent dollars.

Multiplication and division: —

2 × $3 = $6; therefore, (c) $6 ÷ $3 = 2
or, (d) $6 ÷ 2 = $3.

These two inverses are different, because the numbers do not all represent dollars, as in the case of addition and subtraction.

These two cases have been dignified by different names, the first (c) being called "measuring" and the second (d) "partition." But it is unwise to use these names with children. The two cases are easily understood by children sufficiently to lead them to the use of correct forms, if only the teacher will use them.

(*f*) Fractions. The year's work includes the addition and subtraction of common fractions, generally with one figure in each term. Simple cases of multiplying fractions by integers are introduced. In many schools children in Grade IV go beyond this limit, but the eventual gain by so doing is not sufficient to overcome the loss in interest by carrying the child beyond his ordinary needs in this year. Improper fractions and mixed numbers are also introduced in this grade.

Sufficient work in factoring is introduced to allow the reduction of relatively simple fractions to lowest terms. This includes the tests of divisibility by 2, 3, 5, inductively presented. The idea of cancellation is also introduced.

The question as to the precedence of decimal and common fractions demands consideration at this point, although the former is met only incidentally in this grade. It is often argued that the decimal fraction, being merely an elaboration of the decimal notation with integers, should be taken up along with the latter; that since United States money is taken up in Grades I and II, decimal fractions also belong in those grades. But this is on the old theory that when a topic is first met it must then be mastered. It is one thing to know how to indicate values in United States money, and quite another thing to know how to divide one decimal fraction by another.

The needs of the child in Grade III require that he be shown how to write $1.25, and what the symbol means. They do not require that he know how to operate with decimal fractions, although he should know that ½ of ½ is ¼; ½ of ⅔ is ⅓. The idea of the decimal fraction is relatively so abstract that is was not until well along in the 18th century, fully one hundred fifty years after the first general use of the decimal point, that this form of the fraction found much favor in the business world or in the schools. Hence, it cannot be expected that children will readily grasp the idea of decimal fractions before the idea of common fractions is fairly well fixed in mind. Both present needs and propaedeutic considerations demand, however, that there be continued practice in the writing of numbers in dollars and cents, and in the incidental use of such forms as 0.50 and 50%, as set forth under (*b*) "Tables."

(*g*) Business forms. The simple form of bill, as used in retail trade, is introduced in this grade, for the reason that children at about this period begin practically to meet it in their reading, and in purchasing articles at stores.

(*h*) Oral Drill. There should be in this, as in every other grade, frequent drill in the addition of columns of figures, with a view to accuracy and fair rapidity; and in subtraction by the "making change" method. The three parts of arithmetic that people most often use are (1) addition of columns of figures, (2) making change, (3) the multiplication table; and a school day should seldom pass without some brisk exercise in these subjects. Above all, children should be urged to *read* columns of figures as they read words and sentences; they do not spell the words, neither should they slowly *count up* a column.

(*i*) Particular attention is given to the simple problems involving the use of United States money, manual training, the measurement of rooms for papering and carpeting (but without complications involving fractional widths and patterns, which practically are left to dealers), the common daily purchases of a family, and the common occupations. Maps are drawn to a scale. Much attention is given to accuracy, to a fair degree of speed, and to neatness of work.

Grade V

1. *General Suggestions*

(*a*) Review the work of Grade IV, as suggested in the notes for that grade.

(*b*) On the use of the text-book, see General Suggestion (*b*) under Grade III.

(*c*) The work of the year is chiefly directed to the fundamental operations with compound numbers and with fractions, and to the natural extension of the applications of arithmetic to various occupations, and to other quantitative phases of life.

(*d*) For further suggestions see Grades I-IV.

(*e*) Bibliography: — Smith's Teaching of Elementary Mathematics, as noted in preceding grades. On the geometry: Sundara Row's Geometric Paper Folding, for the spirit of the work; also, Beman and Smith: Higher Arithmetic, p. 66. On the formal statement of the work: Beman and Smith: Higher Arithmetic, p. 41.

2. *The Mathematical Work*

(*a*) Compound numbers. All of the necessary tables have now been taken. The reasons for omitting certain of those that formerly had place, have been stated. The kilowat and horsepower, as units of measure, are spoken of by more people in New York City to-day than are the scruple and the link; so that if we were to add to the number of the tables, it would not be in the direction of the apothecary's weight or the surveyor's measure, but rather in the direction of the newer units and the metric system. These, however, if taken at all, should enter into the work of the later grades, where the pupil's interest demands them.

The reduction of compound numbers to units of higher and lower denominations, "ascending" and "descending," is confined to numbers of not more than three denominations. The reasons for this are that in practical life we rarely use more than two, as feet and inches, yards and inches, pounds and ounces; and that one who can perform reductions with three denominations can easily perform those with more, if occasion ever demands. The recent change in custom, with respect to compound numbers, is quite marked. But a relatively short time ago it was not uncommon to see the area of a field stated in acres and rods, while now it is in acres and decimal parts of an acre; lengths were stated in rods and feet, but now in feet and decimals; and, in general, compound numbers were far more extensively used a generation ago than now.

The operations with compound numbers, generally involving only two denominations, and limited to three, are taken in this grade as suggested. The reasons for this limitation have just been given. When one considers the rarity of occasions for the use of such numbers, by himself or in ordinary business, he will be convinced that the time formerly devoted to the subject might better be spent on other portions of arithmetic, or on other subjects.

(*b*) Reduction of common fractions. The growth in the use of the decimal fraction has been so great during the past century, that much of the work formerly necessary in common fractions has now become almost obsolete. Text-books have been rather slow in recognizing these changes, usually being followers rather than leaders in any movement of this nature. It therefore becomes necessary for the teacher to orientate himself somewhat, before undertaking the work in fractions *per se*.

When arithmetics began to be printed, the sexagesimal fraction was used for all scientific purposes, as we use it now in degrees, minutes, and seconds. The common needs of trade demanded another fraction, the form of which was brought from the Arabs, and this was known as the common fraction. In cases where we, for example, now would use fractions like 0.432 and 0.9375, they were compelled to use $\frac{54}{125}$ and $\frac{15}{16}$. If these fractions should appear as $\frac{378}{875}$ and $\frac{195}{208}$ it was desirable to reduce them to lowest terms, and, if they were to be added, it was necessary to reduce them to common denominators, and preferably to the least common denominators. Hence arose the necessity for a study of fractions, and for finding the greatest common divisor and the least common multiple of two or more numbers. Now that the common fraction is used in only a few denominations, and those very small, the decimal fraction becoming more common in general, and nearly universal in scientific and monetary computations, there is no longer the practical necessity for any extensive treatment of factoring, and of divisors and multiples, on the part of children.

Hence the subject of factoring is limited to the ability to detect the factors 2, 3, 5, these being sufficient for any ordinary reduction of fractions to lowest terms. The subject of greatest common divisor is omitted, as a topic, for the reason that fractions formerly requiring the use of such divisors, are now reduced to the decimal form before operating, and the "practical (applied) problems" are always so palpably artificial as to be unworthy of attention. The subject of greatest common divisor should receive some attention, however, for the reason that the general information of pupils may still demand it for a time, but it is limited to the treatment by factoring as stated. There is a little more demand for the least common multiple, since this is necessary for finding the least common denominator of several fractions, and the addition and subtraction of common fractions is still important, within certain limits. For this, however, the factoring method is again ample.

(*c*) Operations with common fractions. While the conventional order of treatment is possibly justified on the ground of difficulty, and should probably be followed; it should not be felt to be the order of importance, and it is a fair question as to whether it is the order of difficulty. Other things being equal, addition and subtraction of common fractions are simpler than multiplication, when the denominators are alike, but otherwise they are usually not so. The operation $^2/_3$ of $^5/_7$ is easier to perform than that of $^5/_7 - ^2/_3$, and it is more important; while the explanation of one is about as difficult as that of the other. The natural order of presentation, however, would seem to be: Addition and subtraction of fractions having the same denominator; addition and subtraction of fractions not having the same denominator, and hence the necessity for a common denominator, and in particular for the least common denominator; multiplication; division.

It should be remembered, however, that the most important of these operations is that of multiplication, the others being used comparatively seldom. That is, we use "4 times 12½ cents," and expressions similar to this, more often than we add common fractions. It should also be remembered that however much we may be tempted to use other devices for division, like that of reducing to a common denominator and then dividing numerators, the practical plan is that of multiplying by the inverted divisor. This, therefore, should be the one with which children should become familiar; sufficient explanation being given to justify the process to their minds, but no elaborate explanation being required.

(*d*) Decimal fractions. As already stated, these are carried along with common fractions, so far as notation is concerned. But it now becomes necessary to study them *per se*, with respect chiefly to the operations.

Having become more or less familiar with the notation, it is not difficult for children to see that $^1/_{10}$, $^2/_{20}$, $^3/_{30}$, $^{10}/_{100}$, 0.10, 10%, ten hundredths, ten per cent, all have identically the same value, and all mean from this standpoint, exactly the same thing. Sometimes it is more convenient to write $^1/_{10}$, sometimes 10%, sometimes 0.10, as in the case of $2.10.

The addition and subtraction of decimal fractions usually offer no difficulties unless the teacher suggests them by some attempt at over-explanation.

The difficulty in multiplication has often been made the greater by the attempt to refer too much to the common-fraction notation. The most approved forms are the following:

(1) The decimal points are, as is natural, arranged in a column. Since 5 times hundredths is hundredths, the right hand number of the product is placed under hundredths. The rest of the work is identical with that of integers, the decimal point going under the others.

```
 6.25
 5.
─────
31.25
```

(2) In this case, since hundredths multiplied by tenths is thousandths, the right-hand figure of the product goes in the thousandths place.

```
6.25
0.5
─────
3.125
```

(3) In this case, since hundredths multiplied by hundredths is ten-thousandths, the right-hand figure of the product goes in the ten-thousandths place.

```
6.25
0.25
─────
0.3125
1.250
──────
1.5625
```

Not only is this arrangement the simplest and most natural, but there is an ulterior value that will be appreciated by computers. Since a large amount of computation in scientific matters need be carried to only two or three decimal places, this arrangement is the best for the approximate multiplications demanded. Thus in the approximate multiplication of 10.48 by 3.1416 we have, multiplying first by the units:

```
10.48
 3.1416
──────
31.44
 1.048
 0.419
 0.010
 0.006
──────
32.92
```

The result is correct to two places, and the unnecessary work is eliminated. The saving is more pronounced in more elaborate problems. Of course this latter work is not intended for this grade.

The division of decimal fractions has already been mentioned in Grade IV, 2, (*e*).

The operations with decimals are generally limited, for reasons already stated, to fractions having not more than three places.

Such simple aliquot parts (the name is not used) as .33⅓, .66⅔, .20, .25, are taken up in this grade.

(*e*) Business forms. Carrying forward the work of Grade IV, attention is given to the bill, the receipt, and the meaning of debit and credit.

(*f*) Oral work. See Grade IV, 2, (*h*).

(*g*) Geometry. The subject of geometry in the grades has of late attracted much attention. The nature of the work depends upon the answer to the question, Why should it be there? Its presence might be justified, (1) because the child's interests demand it, (2) because the child needs the logical training which it offers, (3) because general information demands it, or for similar reasons, all more or less connected, and all summed up in the statement that the child is ready for it. But for what is the child ready? For a knowledge of geometric forms certainly, and this he has with his manual training all through the grades, learning the names of simple geometric figures as a part of that work. It may be doubted if he needs to know of any other forms of plane geometry than those met in that way.

Furthermore he has interest in knowing how to measure certain of these forms, such as the parallelepiped and prism, which may be met even in sewing. In other words, mensuration should be undertaken rationally, such concrete problems being solved as are actually met in working with material things, this being the "laboratory method" in its truest form.

But he does not need at present, and he is not yet prepared for, the deductive proof of geometric propositions. Proof by things. As before stated, however, the subject of geometry is actual trial, and proof by logical reasoning, are two very different reserved for a later article.

So much being premised, the work in this grade is limited to the proper naming of the common geometric forms, particularly as met in the manual training, and to the mensuration of certain of these forms, as follows:

Length: circumference of circle compared with the diameter.
Area: square, rectangle, parallelogram, triangle, circle.
Volume: rectangular parallelepiped and prism.

Much interest may, however, be awakened by extending this work to the construction of regular polygons; tiles or hardwood floors offering an example of the hexagon, and the bee's cell showing its economic value.

It is very desirable to hold to the unity of mathematics, not letting the child feel that his study of geometry is separate from the study of arithmetic. Both are mathematics and are interrelated in life; the school too often separates such close interrelations.

(*h*) Formal work. Children now approach the stage of more formal reasoning about the work. As a consequence, the solution of applied problems should now appear in steps, neatly arranged and numbered. Accuracy of form is necessary to accuracy of reasoning. The following will illustrate what is meant:

Common Inaccuracies	The Accurate Form
1. $2 \times 25 = \$50$.	1. $2 \times \$25 = \$50$.
2. $2 \times \$25 = \$50. + \$2. = \$52$.	2. $2 \times \$25 = \$50$ $\$50 + \$2 = \$52$.
3. 2 ft. $\times$ 4 ft. = 8 ft., or 8 sq. ft.	3. 2×4 sq. ft. = 8 sq. ft., or $2 \times 4 \times 1$ sq. ft. = 8 sq. ft.
4. $100 \div 4 = \$25$.	4. $\$100 \div 4 = \$25$.
5. $\$100 \div 4 = 25$.	5. $\$100 \div \$4 = 25$, or $\$100 \div 4 = \$25$.
6. $100 \div \$4 = \$25$.	6. Impossible. See 5.
7. $\sqrt{144 \text{ sq. ft.}} = 12$ ft.	7. $\sqrt{144}$ ft. = 12 ft., or $\sqrt{144} \times 1$ ft. = 12 ft.
8. "As many times as 4 is contained in \$100."	8. "As many times as \$4 is contained in \$100" or "as 4 is contained in \$100."
9. "Tens o' thousandths."	9. "Ten thousandths."

III. GENERAL DISCUSSION OF THE WORK OF THE LAST THREE GRADES

(*a*) Modern Subject-Matter

The work of the last three grades is chiefly devoted to the application of arithmetic to the affairs of life. It is, therefore, desirable to know the nature of the problems demanded by the business of to-day, by the science, manual training, and other subjects in related courses in the school. This matter needs therefore to be considered at some length from the point of view of the teacher.

Knowing thus, in general, the kind of subjects that should constitute the principal topics for study in the sixth, seventh and eighth grades of school, we have a standard for judging the worth of the traditional topics and processes customarily dealt with in these years. In other words, we can decide as to what is really important in social life, and what is really used; and we can also decide what is not so useful, and what may even be cast aside as having slight value.

Thus far in the grades, the mathematics has had two prominent purposes, so far as knowledge is concerned; (a) acquaintance with certain processes; and (b) acquaintance with valuable facts of a quantitative nature. To the teacher the former may rightly have been a prominent object; just as in teaching literature the inculcation of the underlying truth may be the great purpose. But to the child, the answers to problems, if the problems have been well chosen, may have been the attractive aim; just as in literature it is the incidents with their "outcome," in the narrative that he reads. In other words, the teacher's object may be quite different from that of the child; and very often it is so.

From the beginning of the sixth grade, the mathematical processes as objects of thought should grow less prominent in the minds of both teacher and pupil, for the reason that most of these processes have already been mastered; the other phase of the work, meanwhile, should increase in prominence. In other words, the primary aim of teacher and pupil should now be increasingly a study of social and business life. Just as literature and history teach ethical and political principles of conduct; so arithmetic should now teach quantitative principles of business, and problems are the means for arriving at this knowledge.

In outlining the work for the next three grades, therefore, it is our duty to name not primarily the arithmetical processes to be treated, but the large topics that must be investigated on the quantitative side. Some of these, for example, are transportation by rail, by canal and ocean, with comparisons; corporations; government revenues and expenditures; manufacturing; farming. Modern arithmetics already show a tendency in this direction; for, while they discuss topics like ratio and proportion, discount, interest, profit and loss, which are mainly concerned with processes, they also take up topics like banking, stocks and bonds, and commissions, which are definite kinds of business. In fact, a modern arithmetic is a confused mixture of processes, and of the occupations of men, although it is evident all the time that there are very few new processes that need be taught. Our proposal amounts only to the suggestion that this confusion be remedied by a fuller acceptance of great business enterprises as topics of study.

And if this were done, the leading defect of our present study of arithmetic would be largely overcome. For every one now has the feeling that children begin to figure on a given problem too quickly, although it is certain that the work is none too accurate; they are at work with the pencil with altogether too little thought of the general conditions under which the actual business is conducted. This is because our very definition of arithmetic has been "figuring." If we adopted the other conception, namely, that arithmetic is a study aiming to bring one into an understanding and appreciation of leading business undertakings, the emphasis would naturally fall on the nature of each undertaking. This would insure more intelligence from the start, and give motive for the mechanical work in the solution of problems. The effect, too, would be that the processes themselves would be better understood; just as the conventional forms in written language are better learned when children have really interesting subjects to write about.

Aside from this argument, while many of these topics are among the most important themes of daily conversation, they are not and cannot be provided for in other studies, because an understanding of them is reached mainly by a study of quantity. It seems strange, under such circumstances, that arithmetic has been so long willing to confine itself exclusively to processes. With the examples of reform set by other studies along the same line, notably language work in its endeavor to enliven form through a richer content, it is fitting that a forward movement be made in arithmetic. Interest, therefore, may be as great an aim in arithmetic as in literature or in nature study; for the field of thought may be as inspiring.

Hence it is well to begin by enumerating some of the principal business undertakings that should constitute the course of study in arithmetic in the sixth, seventh, and eighth grades. A discussion of the various obsolete and of the acceptable methods or processes will next be in order. Then the special work of each year will follow.

Topics for study from which selections might well be made for the sixth, seventh, and eighth grades are as follows: —

Farm life: Dairy farm in New England; dimensions of a typical farm; cattle; milk; prices; butter; feed. Size of typical farm in Illinois; typical space given to each product; quantity of this product. Irrigated farms in Colorado; size; quantity and price of water; variety and quantity of products (compare with the two former); effect of irrigation on the price of land. Wheat farm in Washington; area; quantity; cost of production; prices; manner of renting land; variation from year to year. Fruit farms in New York; in Southern California; in New Jersey; in Florida. Cotton, rice, and sugar plantations.

Rainfall: Variations in different places and from year to year.

Mining: Nature of a mine; coal mine; dimensions; number of men; amount of coal; bituminous and anthracite mines compared. Iron ore in our own states; comparison with Great Britain. Copper output of Calumet and Hecla Mines; general comparisons. Gold and silver mining.

The Oil Industry: variety and quantity of products; comparison with Russia.

Lumbering.

Fishing.

Ranching: Cattle ranch; sheep ranch; dimensions; losses; prices.

Transportation: Wagon roads; extent and cost of various kinds of roads and pavement in various states and cities. Canal; extent; history of the Erie Canal on the quantitative side; amount of freight carried. Pennsylvania Railroad system; freight on the N. Y. Central Railroad compared with that on the Erie Canal; that on the Illinois Central Railroad compared with that on the Mississippi River. Ocean traffic; size of steamships; freight capacity; coal consumption. Elevators for grain.

Manufacture: Cotton; comparison of factories in the South and in Massachusetts; wages; prices. Wool; herd of sheep in Ohio; in Nebraska; factories; wages; prices. Leather. Clothing in New York; the sweat-shops. Sugar; cane and beet sugar compared. The Linen industry. The Silk industry. Gas compared with electricity as to cost; price of each in different cities.

Sale of Commodities: Grocery Store. Bakery. Department Store. Wholesale Store; dimensions; stock; traveling men.

Banking business: deposits; loans; exchange.

Corporations: stocks; bonds.

Rents: How farms are rented; amounts. Tenement houses in New York, with reference to recent reforms. Apartments in New York. Houses. Office buildings.

Insurance: fire; accident; life.

Revenues and expenses of New York City: taxes of various kinds; departments; employees; supplies; cost of elections; comparisons with other cities at home and abroad.

Revenues and expenses of the United States Government: Patent Office Department. Treasury Department; internal revenue; tariff; imports and exports; custom houses. War Department; its size; the cost of maintaining. Pension Department. Navy Department; warships; cost of navy. The capitol, and the various other expenses, such as those of the White House and the Department of State.

Immigration: Nationalities; restrictions.

Power for manufacturing; horse-power required in various factories; electricity and steam; Niagara Falls as a generator of power.

Labor: The cost at home and abroad; in various occupations.

Education: Cost of our schools; a great library; a university.

The cost of printing: The extent of the industry; consumption of ink and paper.

Public health: The effect of improved sewerage, of good water, of clean streets, of better ventilation.

The investigation of almost any one of these topics on the quantitative side calls into use many, and perhaps all, of the arithmetical processes that have thus far become familiar to the pupil. In arithmetic, as in geography, children are in the habit of forgetting one subject while learning another; but the above arrangement renders such forgetting impossible in arithmetic. Aside from that, however, text-books in arithmetic have too few "promiscuous examples." The examples and problems ordinarily bear so exclusively on one process that the pupils drop into a mechanical application of a single new rule before the series of examples is half completed, and the remainder of the problems are solved with the minimum amount of thought. A greater variety of work would prevent the application of the new rule, or of any particular new thought, to every example, and would thus compel the pupil to think at every step of his progress.

(*b*) Traditional Subject-Matter

The preceding discussion suggests an examination into the traditional topics of the more advanced arithmetics, with a view to determining their value. This examination must, in this case, be very brief and limited to the more prominent topics.

(1) *Percentage.* This subject, formerly a topic by itself, is merely one phase of decimal fractions, and should be so treated. A large part of business arithmetic involves the finding of per cents, so that the method is continually applied after it is once presented. The treatment of the subject by "cases," and the learning of definitions of terms like "amount," "difference," or even "percentage," may be considered obsolete. There is need to know what "per cent" means, namely, "hundredths" ("hundredth," or "of a hundredth," as in 6%, 1%, ½%), and there is occasionally some value in using the term "base." But the two leading problems of the subject are illustrated by two examples not requiring any elaborate vocabulary, namely:

1. 6% of \$250 is how much?
2. If 104% of $x = \$7.28$, what does x equal?

Practical problems in percentage rarely require any other forms.

(2) *Discounts.* (a) Commercial: Of high value, since it is used in all business, from extensive wholesale transactions down to so-called bargain sales.

(b) Bank: Generally used by those having occasion to borrow money at banks. Of high value. See (8) below.

(c) So-called "true" discount: The very name condemns it. It gives no true idea of business and should be omitted. It should, of course, be recognized that this form of discount is more advantageous to the borrower than bank discount, and on this account it has had many champions on theoretic grounds. For us, however, it is merely a question, in the grades, as to which is practically used.

(3) *Interest.* (a) Simple: Necessary in all business life. But in teaching this, as other similar subjects, the teacher needs to look at the matter in perspective. That is, one class of problems is very important; several others are relatively unimportant, and the teacher should present the subject with this in mind. The important point is that the pupils should know what simple interest is and how it is computed, with tables and without. It is unimportant, from the business standpoint, that the pupil should be able to find the time, given the principal, rate, and interest. Hence, while this latter case is valuable as fixing the former in mind, it deserves little emphasis.

(b) Compound: It is valuable for a child to know what compound interest is, and that it is allowed by savings banks in most states, and that the rate of interest is relatively low. It is well that there should be enough exercise in computing compound interest, with and without tables, to fix the idea in mind. This being accomplished, the subject has little practical value. To those who, either here or in other subjects of arithmetic, plead for mental gymnastics, it should again be said that arithmetic offers ample facilities for this valuable work without giving a false idea of business customs and values. Hence the value of this subject is relatively low.

(c) Annual: This is rarely used and hence is of low value. When met, however, in coupon notes, it involves no new principle.

(d) Partial Payments: This was formerly of high value, but the increase of banking facilities, and the simplification of banking methods, have diminished this value. In rural communities it still has considerable importance, the "United States Rule" being the only one generally recognized in most states. In order to secure the value of the subject, problems should involve common amounts, as a principal of $100 with payments like $10 and $25, rather than amounts like $251.42 and $19.79. If complicated multiplications are desired, let them enter where they belong, in problems involving scientific measurements. To introduce them here is to make the problem both unreal and uninteresting. Taught as suggested, the subject has a moderate value.

(4) *Insurance.* This is important in three general lines: fire, accident, and life insurance. As to the first, it is important that one class of problems should be emphasized, the usual one-year and three-year policies, at the prevailing rates in this part of the country, the premium being required. As to the second, it is important that the system be explained and a few simple problems solved. As to the third, life insurance, the classes of policies are now so numerous that only two or three standard cases need be considered. The "straight life," and the endowment policies are the most frequently found, and these should be considered in the light of the premiums and the returns. (See Beman & Smith's Higher Arithmetic, p. 177.) The value of this subject is high.

(5) *Stocks and bonds.* Since, in these days, most business requiring large capital is conducted by corporations, and the question of the rights and duties of capital are vital in economics, it is very necessary to relate arithmetic to this phase of modern life. The following matter should be, with due regard to sequence, explained to the class: Corporations, stock companies being special forms; directors and officers of stock companies; stock, preferred and common; dividends; certificate of stock; bonds, coupon and registered; interest; par, above and below par; usual par value of a share of stock and of a bond; how stocks and bonds are purchased; stock exchange; broker's commissions; newspaper quotations used in class; distinction between legitimate and illegitimate dealings in stock; the percentage against the gambler.

The problems should be confined to the usual cases, and may best be assigned with reference to the newspaper quotations.

The following errors are not uncommon in text-books, and should be avoided: Buying a fractional part of a share; paying a commission different from ⅛ per cent; sending a certain sum to a broker to be entirely invested, instead of ordering the broker to buy a certain number of shares and then paying him; quotations differing from the prevailing prices of standard stocks.

Taken up in the modern spirit, with blank forms of stock certificates and of bonds to be seen by the class, the subject has high economic value, although mathematically it offers nothing new. (For suggestions, see Beman & Smith's Higher Arithmetic, p. 171.)

(6) *Commission and brokerage.* In this is involved the element of the question of food supply for a city like New York, and from this practical standpoint it should be approached. The practical mathematical side of the case is simple, and the non-practical side, involving unusual business problems, should not be considered. (See Beman & Smith's Higher Arithmetic, p. 168.) The economic value is therefore high, although no new mathematical processes are involved.

(7) *Government revenues.* Instead of isolating the chapter on taxes, it is much better to consider this detail in connection with government revenues in general. There should then be explained to the class the following: The expenses of the United States Government, and where the money goes; the same for the state and the city; the sources of income to meet the expenses of the United States Government; internal revenues, including postage, and customs duties; the revenues for the state and city; taxes on real and personal property; inheritance taxes; prevailing tax rates; construction of a tax-table, and explanation of its use.

The problems should then relate to the common cases of direct taxation, of internal revenue, and of customs. To manufacture unreal problems of a complicated nature is to put out of relief the practical problems. Mathematical difficulties are better presented in other ways. From the standpoint suggested the subject has high economic and rather high mathematical value. (Beman & Smith's Higher Arithmetic, p. 163.)

(8) *Banking business.* Arithmetics quite generally separate bank discount from exchange, and each of these topics from numerous other features of banking, with which pupils should become acquainted. The ordinary citizen needs to know some-

thing of the following subjects: Savings banks versus general commercial banks; how to open a bank account; deposit slips and checks; certificates of deposit; interest on deposits; the banking business of express companies and the post office, as well as of banks, as shown in money orders and drafts.

If one is in business he will need to know: The process of borrowing money from a bank; bank discount; security; discounting notes that he may hold; collecting by means of drafts; sending money by means of drafts.

His general stock of information should also include a knowledge of the clearing houses in various cities, and particularly in New York, together with the volume of business transacted.

From this standpoint the economic and mathematical value of the subject is high. (Beman & Smith's Higher Arithmetic, p. 148.)

(9) *Profit and loss.* A very old name for a special chapter. At present the business expression "Profit and loss" has an entirely different meaning from that given in arithmetic. Since the chapter involves no new principles, pertains to no particular line of business, and involves no problems not naturally to be placed in miscellaneous business exercises, it may well be omitted as a special topic.

(10) *Equation of payments.* An old chapter, once of much practical value, but nearly obsolete in America. Improved banking facilities have made these long-standing accounts unnecessary.

(11) *Partnership.* This was formerly a very valuable subject, particularly before the invention of stock companies. Partnerships still exist; they are, indeed, more numerous than ever, but the style of problem set forth in the arithmetics, under this topic, has long been obsolete.

(12) *Longitude and Time.* This subject, formerly of much practical importance, ceased to have some of this value with the general adoption, at the close of the 19th century, of standard time in a large part of the civilized world. Practically, it is at present rather a part of geography than of arithmetic. Problems based on the 15° scheme should predominate. Other problems should generally refer to ships at sea and to observatories, becoming thus too technical for the grades.

(*c*) Summary Concerning Subject-Matter

These conventional subjects have been mentioned by somewhat the same names found in the ordinary arithmetics, so that the suggestions may be of service, no matter what book may be in use. The change in the standard chapters has recently been very marked. When the old "Rule of False," or "Rule of False Position," disappeared, many conservative teachers felt that there had been a great loss. When alligation disappeared, a few years ago, there was a similar protest. Now that equation of payments, and compound proportion, and unitary analysis, and profit and loss, and other chapters, are going, many very good teachers feel that nothing will be left of arithmetic. But in place of every business custom or mechanical process that becomes antiquated, new customs and new processes appear, and for these we need to be prepared. As the opportunity offers the following newer topics are introduced, the problems taken from current literature:

1. The question of the agricultural interests of the country, connecting with commissions, banking, taxes, and transportation.

2. The questions of fishing, lumbering, mining, manufacturing, and trade.

3. The question of transportation, connecting with the study of corporations, taxes, agriculture, mining, and banking.

4. The questions of labor and of labor organizations, and the relation of each to the questions above mentioned.

5. Modern treatment of the question of government revenues and expenditures.

6. Modern treatment of the problems of banking.

7. Modern treatment of problems involving corporations, as suggested above under stocks and bonds.

These topics are alive and are valuable for every citizen. It will be a better day for the schools when they replace the obsolete subjects to which reference has been made.

It is important to note that the topics named suggest our broader commercial, industrial, and social life as the field for arithmetic in the higher grades. Thereby, elementary mathematics is made to stand on the same plane as literature and other studies, for all these now culminate in rich generalizations. We should expect in each grade, therefore, a crude formulation of rules of business rather than rules for arithmetical processes.

For example, a study of farming would result in a knowledge of many principles, touching quantity, that guide the farmer; such as the proportionate division of his land for certain crops, the customary amount of stock, the variation in prices that may be expected. A study of rents would acquaint the pupil with methods of renting farms and tenements, the per cent of profit than can be expected, the dangers, and the customary losses. A study of insurance would likewise lead to some knowledge of the way in which risks are calculated, what rates are paid, and the provisions a man should make for the support of his family after his death. A text-book should word these principles with the same care with which rules for processes have heretofore been worded. Thus, mathematics in the grades would cease to be purely theoretical; but, on the contrary, would lead to an understanding of practical affairs.

But arithmetic in the grades can easily go farther than this. Why is it not a fair task to require a seventh or eighth grade pupil to discover by himself how much it costs to keep a horse or cow in a given community, or to estimate how much butter a cow might make, averaging 2½ gallons of milk per day, or to find the cost of fuel for the school, or to investigate other similar matters? Such a lesson might be assigned a week or more in advance, with the understanding that each pupil should collect the most reliable data he could get, and make a full report, including the answer to the problem, to the class. After some experience, the children might be left to their own resources as to where to go, how to make an appointment with a man, what questions to ask, how to know when all necessary questions have been put, and how to use the data collected. If such work were definitely required, the children would soon learn to do it properly.

Of course, the principal question here is whether or not such employment comes properly within the range of school duties. Waiving the fact that nature study and geography rightly call for excursions; is it not the purpose of the school to bring the pupil into an understanding and appreciation of community life, and, also, to identify him with that life as much as possible, including even the motor action? In other words, is not action, "doing" of one kind or another, the end-point of school instruction? Certainly this is the desired end-point in literature, in history, and in manual training; and if it is likewise true of arithmetic, might not this be one of the most effective ways for bringing the pupil into direct touch with the occupations and the people about him? Often such a task as that suggested above could be assigned to committees of the class, and some co-operative, administrative ability would need to be exercised before proper data could be collected. Why not? Even the kindergarten accepts the development of just such ability as one of its prominent objects; why not the grades? It takes time, but it is worth time. If the school is to cease being too theoretical, it must find in some manner an outlet in action. If data for live problems must be collected with care, it is only natural that pupils do some of this work. It is an artificial situation when nothing is left the pupil but to think and to figure. The fact that the teacher would need to be in close touch with the environment and reasonably well posted as to the facts, in order to carry on such work successfully, makes it all the more desirable. Whatever tends to make our teachers less "bookish," and to identify them with their surroundings, particularly in a quantitative way, is welcome.

(*d*) General Methods of Solving Problems

(1) General Analysis. By this we mean merely common sense applied to any ordinary problems. Of course this method is of the highest value. It excludes the memorizing of long and ultra-scientific forms of analysis, but includes a brief statement of the main steps, with reasons.

(2) Unitary Analysis. A special form of analysis, in which a series of terms is reduced to unity. For example: If two men do a piece of work in 4 days of 10 hours each, how many men will it require to do it in 5 days of 8 hours each? If the 4 days were each 1 hour long, it would take 20 men; if there were only 1 day, it would take 80 men; but since there are 5 days it will take 16 men, and since the days are 8 hours long, it will take 2 men.

This is a good plan for solving problems which, as given in arithmetics, are usually of a bad type. For example the antiquated problems of compound proportion easily yield to this method. These problems, however, are bad in that they give a false idea of business; for in business such extensive series are very rarely met. A single problem here corresponds to a series of problems in actual life.

(3) The equation method. This is the latest development in the science of arithmetic, as applied to daily problems.

The equation, as needed in arithmetic, rarely involves cases more difficult than the following, where *a, b,* and *c* stand for known numbers:—

$$ax + b = c,$$
$$\frac{x}{a} + b = c;$$

consequently children should become familiar with problems like the following:—

Twice a certain number is 58; what is the number?

Five times a certain number, together with 6, equals 41; what is the number?

Ten plus a sixth of a certain number equals 12; what is the number?

One-sixth of a certain number equals ⅖; what is the number?

Think of a number; multiply it by 5; add 4; tell me the result and I will tell you the number.

Problems like the above, while applying to no related subject, have in themselves a good deal of interest for children. They like to make up problems like the last one, giving them to one another as puzzle games.

This subject has not been sufficiently appreciated by teachers, but it is of highest value; for example, If 105½% of a certain sum is $2,110, what is the sum? Here, $1.055x = \$2,110$, $\therefore x = \$2,000$, a solution much simpler than the conventional one. It is evident that this problem, with little value itself, is suggestive of a long line of very real cases.

(4) Simple proportion. A favorite arithmetic method, under the name "Rule of Three," before the equation was generally known. It is merely a fractional equation, this fact being concealed from the pupil by a distinctive symbolism. As an instrument of business arithmetic it is antiquated and cumbersome. Its value in applied arithmetic lies in problems in physics, as in the pressure of gases, and in problems involving similar figures. Hence, in ordinary arithmetic it has slight value, although in geometry its

value is very great. It is generally better to translate it into the simple equation. It is easier to solve

$$\frac{x}{2} = \frac{5}{20},$$

$$\text{or } 20x = 10,$$

than to solve

$$20 : 5 = 2 : x,$$

although all these expressions are equivalent.

In other words, a proportion like

$$3 : 4 = 15 : x$$

is merely equivalent to

$$x : 15 = 4 : 3,$$

or to

$$\frac{x}{15} = \frac{4}{3}$$

which is solved by multiplying by 15, thus avoiding the old and ill-understood proportion rule.

As a preparation for this work it is not necessary to give any elaborate treatment of ratio. The concept, ratio, is valuable and necessary; the topic, ratio, is not, at least as usually treated.

The valuable application of proportion in these grades is very limited. The subject might be applied to problems like this: If $2 draw $0.06 interest in 1 year, how many dollars will draw $3.50 interest in the same time? But in the first place, the problem is not genuine; and furthermore, if it were, it could better be solved by simple analysis. And so with most of the conventional problems of proportion. There is one subject, however, to which proportion necessarily applies, the subject of similar figures. In as far as these have an interest for children, the subject of proportion is valuable in the grades. In particular, problems like the following are recommended, the figure always being drawn in advance of the solution:

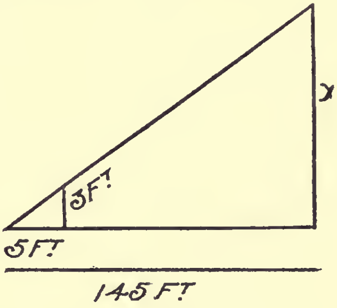
If a yard-stick, standing on the level pavement, casts a shadaw 5 ft. long, and the shadow of the gable of the school building at the same time is 145 ft. long, how high is the gable above the pavement?

The following form of solution is recommended:

1. Let x = the number of feet in height.

2. Then because the ratio of heights equals the ratio of shadow lengths,

$$\frac{x}{3} = \frac{145}{5}$$



3. Therefore,

$$x = \frac{3 \times 145}{5}$$
$$= 3 \times 29$$
$$= 87,$$ the number of feet required.

(5) Compound proportion. A method for solving certain obsolete problems, which were solved in the 16th and 17th centuries under a rule known by various names; as the double rule, the rule of five, the rule of seven. Practically of no value.

The unreal problems commonly found in text-books, and the entire subject of compound proportion, should be omitted.

(*e*) The Question of Geometry in the Grades

With the exclusion from arithmetic of a large amount of obsolete matter, which had gradually been accumulating up to within a few years, educators looked about for mathematical subjects to take its place. The most convenient material at hand was algebra and geometry, and since much of this was simpler than what had been excluded, it naturally found many advocates. As is usual in such cases, the movement to include algebra and geometry in the grades went to an extreme, with the result that children received a smattering of each and entered the high school with an exaggerated idea of their knowledge of these subjects or with a preconceived dislike for them. The question now begins to assume a more rational form: What is the value of these subjects in the grades, and, as a consequence, how should they be presented? The question as it relates to algebra has already been briefly answered. It remains to consider it as regards geometry.

Considering only the ability of children to demonstrate theorems or to solve problems, there is no question but that a considerable amount of demonstrative geometry can be introduced into the last three of the eight grades. Considering only the question of valuable mental gymnastics, the same answer would be valid. But if the subject is to be presented in the same way as in the high school, and if the child is there to go over the same ground, the effect upon him is not altogether happy. It is a common remark of teachers of geometry that they would prefer a pupil who had never studied geometry to one who had taken it in the grammar grades. The edge of interest is worn off, because nothing new is being mastered.

If, then, the subject is to be taken in the grades, it should be in such way as not only to appeal to the pupil's interests then, but to leave that impression of only partial mastery which acts as a stimulus to further consideration of the subject. What, then, are these interests, and how shall they be met?

In connection with the pupil's mathematical work it is convenient for him to solve an equation. Therefore the equation is introduced into his mathematics; not algebra as a separate topic, but such features of algebra as bear upon the work in hand. Likewise it is convenient for him to measure things. Therefore portions of geometry should come into his mathematics; not geometry as a separate science, but such features of it as bear upon the work in hand, and with the increase of a rational manual training this work is daily becoming more important.

This work in mensuration requires that the pupil should know the names of the common surfaces and solids. This part of geometry, therefore, is introduced in connection with mensuration. It is also necessary that he should know how to measure these common forms; hence, this work is also taken up, and in a manner quite scientific enough to merit the name geometric. As to the actual geometric facts absorbed by the pupil, this has been done for years. The new feature, developed of late years, is the discovery of the facts. Formerly it was merely to memorize a set of rules dogmatically given; now it is to work these rules out.

Bibliography: Smith's Teaching of Elementary Mathematics, as already noted; Beman & Smith's Higher Arithmetic, much of which is suggestive for grade work, both in business problems and in mensuration; on the geometry, consult Sundara Row, Geometric Paper Folding, much of which is suggestive for grade teachers, although the book is not entirely elementary.

IV. OUTLINE FOR THE LAST THREE GRADES

Grade VI

In this grade the reduction of common fractions to decimals, and vice versa, is the only topic in pure arithmetic demanding attention; except as the others enter into reviews. The year is given to applications, chiefly in percentage and denominate numbers.

At the beginning of the year the work of Grade V should be reviewed as suggested in the notes for that grade. On the subject of reviews in general, and the use of the text-book, consult the general suggestions of the preceding grades. It is particularly desirable to carry out the general policies as laid down in the notes on the work of all preceding grades, and hence teachers and those carrying on professional observation work should become familiar with those notes.

The following are some of the topics especially suitable for the sixth school grade: 1. Gardening; 2. Farming; 3. Rainfall, variations in different places and from year to year; 4. Mining; 5. Lumbering; 6. Fishing; 7. Ranching; 8. Transportation; 9. Manufacture and trade, introducing the metric system as connected with our rapidly growing foreign market.

Aside from the English-speaking world, the metric system is generally used in highly civilized countries. Our exports of guns, machinery, and all manufactured articles are dependent to quite a degree upon the use of metric measures in all descriptions of our goods. The foreign trade cannot be expected to surrender its extremely simple system to our cumbersome one; and whether we adopt the metric measures for our own use or not, we must adopt them for our export trade, if we wish to see it developed.

The traditional topics especially considered this year are: (1) Simple commercial transactions; bills, receipts, commercial discounts; (2) Simple investments; notes, mortgages, simple interest.

The drill with abstract numbers should always hereafter include the important problem of the addition of columns of figures, checking by adding in reverse order. It should also include subtraction, generally of numbers not exceeding thousands; multiplication of numbers like $13 \times 12\frac{1}{2}$, all these being cases frequently met in ordinary business; and division with one-figure or two-figure divisors. In general all this work should relate to the problems of daily life, rather than to tedious and artificially difficult problems intended merely as tasks.

In geometry, the work is confined to the measurement of surfaces and of solids, as indicated below; only commensurable magnitudes (for example, lines having a common measure) being considered. Paper-folding (Sundara Row, Geometric Exercises in Paper Folding, pages 1-7, 14-15), paper-cutting, and drawing, should be employed to illustrate mensuration propositions as in the earlier grades. The proofs of the following propositions, for commensurable lines, are entirely within the powers of pupils of this grade:

The area of a square a units on a side, is a^2 square units.

The area of a rectangle, a units high, b units long, is ab square units.

The area of a parallelogram a units high, b units long, is ab square units.

The area of a triangle, a units high, b units long, is $\frac{1}{2}ab$ square units.

This work is a proper part of mathematics for children of this age. The mathematical unity is preserved; the small amount of geometry not being looked upon as a separate study. For the sixth grade, besides the work above specified, the following should be included: Mensuration of the circle, cylinder, and the forms above mentioned; board measure (foot and thousand) and cord measure; application to manual training and to building.

The following problems are suggested as a type of a series devoted to a single subject:

1. A township in the western farming states is usually 6 miles square. How many square miles does it contain? Show by a drawing.

2. One square mile contains 640 acres of land and is often divided into four equal parts, called quarter-sections, for four farms. What is the size of each farm? What is the distance around each farm? Show by a drawing.

3. Divide 160 acres into 4 equal squares. How many acres in each?

How far is it around each? Show by a drawing. How long would it take you to walk around each?

4. Divide 40 acres into 4 equal squares. How many acres in each? What is the length, in feet, of one of its sides? The distance around it is what part of 1 mile?

5. Compare the area of some city block near you, with a 10 acre plot of ground. Make drawings to a scale.

6. One acre contains 160 square rods, or is equal to a square with each side 12.65 rods long. Mark out such an area somewhere near the school. How many feet around it?

7. How many acres in the block in which your school building stands? Pace it off, taking 2 ft. to a pace, in order to measure it.

8. The average farm in the United States contains about 140 acres. That is what part of one square mile? What part of one quarter section?

9. An ordinary city lot, 25 by 100 feet, is what part of an acre?

10. The land on which your school-building stands, including the yard, is what part of an acre? Estimate the area of other portions of land, and test by measurement to see how nearly right you are.

11. Good farm land in New York State can often be bought for about $60 per acre. According to that, what would be the price of the block of land in which your school building stands? Find out the approximate worth of this plot of land? Give some reasons for the difference in price, if there is a difference.

12. Plot out a 40 acre piece of land in your vicinity. Also one of 160 acres. Show that you are approximately right.

13. A certain farmer in Central Ohio has a farm of 160 acres. Show a plot of this ground, drawn on the scale of $^1/_{16}$ of one inch to one rod.

14. Mark off a suitable area for this man's orchard, yard and garden, barn and barn-yard, field for corn, field for clover and hay, field for wheat, one for pasture, and a timber lot.

15. It takes him about ¾ of a day to prepare an acre of land for planting corn. How long would it take to prepare the field that you have marked out for corn?

16. I takes him about ⅛ of a day to plant an acre of corn, and the same to sow an acre of wheat. How long would it take to plant and sow these two fields?

17. It takes a peck of corn to plant an acre, and 1¼ bushel of wheat to sow an acre. How much corn and wheat are necessary for your fields?

18. The average yield of corn per acre on this man's farm is about 40 bushels. How much corn would the above field yield?

19. The average yield of wheat per acre is about 18 bushels. How much wheat would the above field yield?

20. The highest prices that this farmer has received are 50 cents a bushel for corn and 80 cents a bushel for wheat. What facts can you discover about the possible income of the farmer?

Such a series suggests how arithmetic in the higher grades might give an extensive knowledge concerning an occupation, and at the same time furnish abundant drill in pure arithmetical work.

Grade VII

Review the work of Grade VI as suggested by a similar note under that grade.

The following topics in business arithmetic constitute a large part of this year's work: Commercial discount; per cent of gain or loss; commission and brokerage; taxes with customs, treated as suggested on p. 56; simple interest; compound interest as used in savings banks.

The spirit in which this work is treated is suggested in the notes on Grade VI. See Beman and Smith's Higher Arithmetic, the portion relating to business arithmetic being entirely suited to seventh and eighth-grade work.

The nature of the oral drill is set forth on p. 41.

Mensuration and geometry: Review mensuration of the circle and cylinder; add the pyramid and cone; in connection with this, review the geometric knowledge thus far secured, covering the ground laid down in Hanus, Geometry in the Grammar School, pages 37-46.

Ratio and proportion: See p. 61.

Grade VIII

1. *General Suggestions*

See p. 65. Children have now become mature enough to appreciate difficult business transactions, and these, together with certain geometry work, constitute this year's work.

2. *The Mathematical Work*

(*a*) Corporations: Organization, stocks and dividends, bonds and interest.

For this work, the best text-book is the financial page of a daily newspaper. It is not necessary to enter into all of the customs of the stock exchange; indeed only a few technical expressions are of any use to the average citizen. But a dozen problems involving the cost of ten shares of various specified stocks, and another dozen involving the amounts to be received from the sale of certain stocks, and a few relating to the purchase of bonds, all of the problems being based on the newspaper quotations and all involving the usual brokerage, will be interesting and valuable. Different pupils will have different answers, depending on the quotations used, which brings up the questions of the meaning of the expressions "highest," "lowest," "opening," and "closing," and of the causes of fluctuation from day to day. (See also p. 55.)

(*b*) Lending money: Notes, bonds, and mortgages; partial payments of a practical nature; writing notes and receipts.

(*c*) Banking: Starting accounts; bank books, deposit-slips, checks, the blanks being in actual use; bank discount; savings banks and compound interest.

It is now possible to procure school sets of blanks, including bank book, deposit slips, checks, notes, and drafts. The use of such sets adds greatly to the interest and the value of the work.

(*d*) Exchange treated from the modern standpoint; drafts, checks, money-orders.

The exchange rates of banks may be found on the financial page of any of the leading daily papers.

(*e*) Our foreign trade; its growth, customs, duties; the metric system reviewed, with the story of its origin, the extent of its use, its advantages; foreign exchange and the money systems of England, France and Germany; longitude and time reviewed in connection with study of geography of the countries involved.

(*f*) Insurance.

(*g*) Mensuration and geometry: Review mensuration of important figures, and cover the work laid down in Hanus, Geometry in the Grammar Grades, part 2, page 47-52. In connection with this work, consider square root as stated below.

(*h*) Square root approached through practical problems: Relation between the geometric and the algebraic forms; the latter used in explaining the process.

Since this article treats mainly of arithmetic in the grades, merely suggesting the use of algebra and geometry; and since in the eighth grade it is intended to review the arithmetic with special reference to its business applications, and to devote some time to elementary algebra and geometry, further consideration of the latter work is reserved for a subsequent article.

In general it may be said, however, that there is a manifest danger of overdoing the work in algebra and geometry in the eighth grade. Except so far as these subjects have practical application to problems of the life or the science that the pupils of this grade meet or have interest in, the work may better be left for the high school. Therefore the work in geometry may well be limited to mensuration and constructions, and that in algebra may well be directed to the applications of simple equations. Any elaborate treatment of algebraic functions, such as complicated fractions, or any considerable amount of work in literal equations, is of doubtful value at this time. Teachers who contribute to the stock of genuine applications of the simple equation, especially in the problems of business or of simple science, will perform a real service to education, for it is here that the greatest value of algebra for the eighth grade is to be found.

Teachers College
Columbia University

Teachers College is the professional school of Columbia University for the study of education and the training of teachers. The purpose of the College is to afford opportunity, both theoretical and practical, for the training of teachers of both sexes for elementary, secondary, and normal schools, of specialists in various branches of school work, and of principals, supervisors, and superintendents of schools.

Courses of Instruction

The College offers 56 courses in Education, including 6 courses on the History and Principles of Education, 4 courses on Educational Administration, 7 courses on Educational Psychology, and 25 courses on the theory and practice of teaching Biology, Domestic Art, Domestic Science, English, Fine Arts, French, Geography, German, Greek, History, Kindergarten, Latin, Manual Training, Mathematics, Music, Physical Science and Physical Education. Other courses of instruction supplementary to those above are as follows: Biology, 6 courses; Domestic Art, 5 courses; Domestic Science, 10 courses; English, 6 courses; Fine Arts, 14 courses; French, 3 courses; German, 2 courses; Geography, 4 courses; History, 4 courses; Kindergarten, 4 courses; Manual Training, 9 courses; Mathematics, 3 courses; Music, 5 courses; Physical Science, 4 courses, and Physical Education, 6 courses. Qualified students of Teachers College may also pursue University courses in History, Language and Literature, Natural Science, Mathematics, Philosophy, Psychology, Ethics, Anthropology, Music, Economics and Social Science.

Teachers College Schools

Teachers College maintains two schools of observation and practice: one, the Horace Mann School, the other known as the Speyer School. The Horace Mann School comprises three departments — a kindergarten for children of three to six years of age, an elementary school of eight grades, and a high school of four grades. The Speyer School consists of a kindergarten, elementary school, and special classes in sewing, cooking and manual training.

Courses of Study

Courses of Study are as follows: (1) A two-year Collegiate Course which if followed by a two-year professional course leads to the degree of B.S.; (2) Two-year professional courses leading to the Bachelor's diploma in (a) Secondary Teaching, (b) Elementary Teaching, (c) Kindergarten, (d) Domestic Art, (e) Domestic Science, (f) Fine Arts, (g) Music, and (h) Manual Training; (3) Graduate courses of one and two years, respectively, leading to the Master's and Doctor's diplomas in the several departments of the College. Students holding the degree of B.S. or A.B. may become candidates for A.M. and Ph.D.

Admission Requirements

The requirements for admission are as follows: (1) To the Collegiate Course — completion of a high-school course; (2) to the two-year courses — (*a, b,* and *c* above) completion of the Collegiate Course or its equivalent in an approved college or graduation from an approved normal school; (*d, e, f, g, h*) same as for (*a*) and (*b*) or two years of technical training or experience in teaching; (3) to the graduate courses — college graduation or its equivalent.

Fellowships and Scholarships

Tuition in graduate courses and courses leading to a degree, $150; in other courses, $100. The faculty annually awards 5 Fellowships of $650 each, 1 Scholarship of $400, 12 Scholarships of $150 each, and 4 Scholarships of $75 each.

For circulars and further information, address the Secretary.

JAMES E. RUSSELL, Ph.D., Dean.

Columbia University in the City of New York

Columbia University includes both a college and a university in the strict sense of the words. The college is Columbia College, founded in 1754 as King's College. The University consists of the Faculties of Law, Medicine, Philosophy, Political Science, Pure Science, and Applied Science.

The point of contact between the college and the university is the senior year of the college, during which year students in the college pursue their studies, with the consent of the college faculty, under one or more of the faculties of the university.

Barnard College, a college for women, is financially a separate corporation; but, educationally, is a part of the system of Columbia University.

Teachers College, a professional school for teachers, is also, financially, a separate corporation; and also, educationally, a part of the system of Columbia University.

Each college and school is under the charge of its own faculty, except that the Schools of Mines, Chemistry, Engineering, and Architecture are all under the charge of the Faculty of Applied Science.

For the care and advancement of the general interests of the university educational system, as a whole, a Council has been established, which is representative of all the corporations concerned.

I. THE COLLEGES.

Columbia College offers for men a course of four years, leading to the degree of Bachelor of Arts. Candidates for admission to the college must be at least fifteen years of age, and pass an examination on prescribed subjects, the particulars concerning which may be found in the annual Circular of Information.

Barnard College, founded in 1889, offers for women a course of four years, leading to the degree of Bachelor of Arts. Candidates for admission to the college must be at least fifteen years of age, and pass an examination on prescribed subjects, the particulars concerning which may be found in the annual Circular of Information.

II. THE UNIVERSITY.

In a technical sense, the Faculties of Law, Medicine, Philosophy, Political Science, Pure Science, and Applied Science, taken together constitute the university. These faculties offer advanced courses of study and investigation, respectively, in (*a*) private or municipal law, (*b*) medicine, (*c*) philosophy, philology, and letters, (*d*) history, economics, and public law, (*e*) mathematics and natural science, and (*f*) applied science. Courses of study under all of these faculties are open to members of the senior class in Columbia College. Certain courses under the non-professional faculties are open to women who have taken the first degree. These courses lead, through the Bachelor's degree, to the university degrees of Master of Arts and Doctor of Philosophy. The degree of Master of Laws is also conferred for advanced work in law done under the Faculties of Law and Political Science together.

III. THE PROFESSIONAL SCHOOLS.

The Faculties of Law, Medicine, and Applied Science conduct respectively the professional schools of Law, Medicine, and Mines, Chemistry, Engineering, and Architecture, to which students are admitted as candidates for professional degrees on terms prescribed by the faculties concerned. The faculty of Teachers College conducts professional courses for teachers, that lead to a diploma of the university.

1. THE SCHOOL OF LAW, established in 1858, offers a course of three years, in the principles and practice of private and public law, leading to the degree of Bachelor of Laws.

2. THE COLLEGE OF PHYSICIANS AND SURGEONS, founded in 1807, offers a course of four years, in the principles and practice of medicine and surgery, leading to the degree of Doctor of Medicine.

3. THE SCHOOL OF MINES, established in 1864, offers courses of study, each of four years, leading to a professional degree in mining engineering and in metallurgy.

4. THE SCHOOLS OF CHEMISTRY, ENGINEERING, AND ARCHITECTURE, set off from the School of Mines in 1896, offer respectively, courses of study, each of four years, leading to an appropriate professional degree, in analytical and applied chemistry; in civil, sanitary, electrical, and mechanical engineering; and in architecture.

5. TEACHERS COLLEGE, founded in 1888 and chartered in 1889, was included in the university in 1898. It offers the following courses of study: (*a*) graduate courses leading to the Master's and Doctor's diplomas in the several departments of the College; (*b*) professional courses, each of two years, leading to the Bachelor's diploma for Secondary Teaching, Elementary Teaching, Kindergarten, Domestic Art, Domestic Science, Fine Arts, Music, and Manual Training; (*c*) a collegiate course of two years, which, if followed by a two-year professional course, leads to the degree of Bachelor of Science. Certain of its courses may be taken, without extra charge, by students of the university in partial fulfilment of the requirements for the degrees of Bachelor of Arts, Master of Arts, and Doctor of Philosophy.

NICHOLAS MURRAY BUTLER, LL.D.,
President.

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