

VI *Curvarum Hyperbolicarum, æquationibus trium nominum utcunque definitarum, Quadratura generalis duplici Theoremate exhibita à D^o. Samuele Klingenstierna, Profess. Digniss. Math. in Acad. Upsal, & R. S. S. Communicante D^o. Jacobo Stirling, ejusdem etiam Soc. Doctiss. S.*

N.B. **C**URVÆ Hyperbolicæ, de quarum quadraturâ hic agitur ab Erud. Auctore, ad unum quasi genus reducuntur, ex communi quâ gaudent proprietate, quantumvis obscura sit nec satis per se determinata. Ad hoc enim genus refertur, omnis curva, cujus ordinata datum efficit rectangulum cum recta, quæ ex tribus partibus necessario diversis & ordine genitis constituitur. Diversæ partes esse intelliguntur, quæ ex diversis abscissæ potestatibus quomodocunque oriuntur; Ordine autem genitæ sunt, si modo ab ima ad summam potestatem æquis gradibus ascendant.

Species igitur determinantur ac definiuntur ex gradibus Potestatum determinatis & definitis.

Primas & simplicissimas hujus generis (ad quas etiam ceteræ omnes ultimo reducuntur) Neutonus ipse primus ex datis Circuli & Hyperbolæ areis dimensus est.

Cotesius deinde plures esse hujus generis Species, etiam in infinitum (secundum ordinem determinatum) progredientes detexit, quæ ad eandem quadraturæ

tura formam ac priores istæ & simpliciores reduci possint ; idque fecit ope Theorematis cujusdam novi de Inventione radicum æquationum binomialium, ex determinata quadam divisione circumferentiæ Circuli in partes æquales ; cujus Theorematis mentio facta est in Erudito suo Opere de Harmonia mensurarum.

Iisdem vestigiis insistendo D. Moivræus Theorema Cotesianum ulterius promovit ad inventionem radicum æquationum Trinomialium, idque adhibendo arcum circuli determinatæ magnitudinis vice circumferentiæ totius. Quo invento omnes hujus generis Species inter se commensurabiles esse secundum rationem quadraturæ suæ statim perspexit, Methodumque tradidit in exquisitis suis scriptis Miscellaneis nuper editis, quâ perveniatur ad quadraturam unius cujus libet formæ ex datis Circuli & Hyperbolæ quadraturis.

Ds. Kl. in Propositione sua, quæ sequitur, in unum collegit quicquid de quadraturis curvarum hujus generis antebac a prioribus inventum fuit. Verum tamen ita collegit non quasi sint variæ formæ sub uno genere, sed quasi una sit eademque forma generis ipsius. Theorema duplex est, quatenus quadratura referat ad aream vel citra, vel ultra ordinatam. Exhibetur in ipsis æquationis terminis sine reductione aut restrictione. Instituitur secundum Cotesii doctrinam, usurpando mensuras Angulorum & Rationum pro areis Circuli & Hyperbolæ. Traditur sine demonstratione, utpote cujus veritas facile innotescat ex Propositionibus Moivræanis.

Hæc de hujus doctrina fontibus indigitasse, non abs re fore judicatum est, ne, lectoribus inexercitatis, auctoris nimia brevitatis impedito esset.

P R O P O S I T I O.

Quadrare curvam, cujus abscissa est z & ordinata $\frac{c z^n \pm \frac{r}{n} z^{n-1}}{a^{2n} \pm a^{n-1} b z^n + z^{2n}}$, ubi n designat numerum quemlibet, r & n numeros quolibet integros & primos inter se, & denominator $a^{2n} \pm a^{n-1} b z^n + z^{2n}$ non potest resolvi in duos factores binomios.

In circumferentia circuli (*Tab. 2. Fig. 1.*) centro quovis O intervallo $OR = a$ descripta applicetur chorda $RT = b$, cui parallelus ducatur radius OP , ita quidem ut arcus PR sit quadrante major si habeatur $+b$, minor vero si habeatur $-b$. Incipiendo in puncto R , sumantur ordine tot arcus $R \overset{1}{R}, \overset{1}{R} \overset{2}{R}, \overset{2}{R} \overset{3}{R}, \overset{3}{R} \overset{4}{R}, \overset{4}{R} \overset{5}{R}, \dots$ &c. arci PR æquales, quot unitates continet fractio $\frac{r}{n}$, & a punctis $\overset{1}{R}, \overset{2}{R}, \overset{3}{R}, \overset{4}{R}, \overset{5}{R}, \dots$ &c. ducantur totidem rectæ $\overset{1}{R} \overset{1}{r}, \overset{2}{R} \overset{2}{r}, \overset{3}{R} \overset{3}{r}, \overset{4}{R} \overset{4}{r}, \overset{5}{R} \overset{5}{r}, \dots$ &c. radio OP parallela & rectæ OR occurrentes in punctis, $\overset{1}{r}, \overset{2}{r}, \overset{3}{r}, \overset{4}{r}, \overset{5}{r}, \dots$ &c. Deinde dividatur arcus PR in tot partes æquales quot sunt unitates in numero n , quarum illa quæ puncto P ad-

adjacet fit P A. Facto initio in puncto A dividatur integra circumferentia in tot partes æquales AB, BC, CD, DE, &c. quot sunt unitates in n ; sumtaque in ra-

dio OP, producto si opus ultra P, abscissa OS = $a \cdot \sqrt[n]{\frac{z}{a}}$,

jungantur SA, SB, SC, SD, SE, &c. ut & OA, OB, OC, OD, OE, &c. Denique sumantur arcus PAa, PBb, PCc, PDd, PEe, &c. qui sint ad arcus PA, PB, PC, PD, PE, &c. ut $n + r$ ad unitatem, & a punctis a, b, c, d, e , &c. ducantur tum rectæ $a\alpha, b\beta, c\gamma, d\delta, e\epsilon$, &c. parallelæ radio OP & occurrentes rectæ OR in punctis $\alpha, \beta, \gamma, \delta, \epsilon$, &c. tum etiam rectæ $a1, b2, c3, d4, e5$, &c. prioribus normales, & rectæ QO, quæ ad RO ducatur perpendicularis, occurrentes in punctis 1, 2, 3, 4, 5, &c.

His factis area curvæ cujus abscissa est z & or-

dinata $\frac{c z^{n+r} + \frac{r}{n} z^{n+r-1}}{a^{2n} \pm a^{n-1} b z^n + z^{2n}}$, erit $\frac{nc}{na^{n+1}} z^{\frac{r}{n}}$ in

$$\frac{R^I_r}{r-n} \times \left[\frac{a}{z} \right]^n - \frac{R^{II}_r}{r-2n} \times \left[\frac{a}{z} \right]^{2n} + \frac{R^{III}_r}{r-3n} \times \left[\frac{a}{z} \right]^{3n}$$

$$- \frac{R^{IV}_r}{r-4n} \times \left[\frac{a}{z} \right]^{4n} - \frac{R^V_r}{r-5n} \times \left[\frac{a}{z} \right]^{5n}, \text{ \&c.}$$

$$+ \frac{c}{n} a^{\frac{r}{n} - n - 1} \cdot \text{in} \left\{ \begin{array}{l} -a\alpha \text{ (SA:AO)} - a1 \text{ (+SAO)} \\ +b\beta \text{ (SB:BO)} + b2 \text{ (+SBO)} \\ -c\gamma \text{ (SC:CO)} + c3 \text{ (+SCO)} \\ +d\delta \text{ (SD:DO)} - d4 \text{ (-SDO)} \\ +e\epsilon \text{ (SE:EO)} + e5 \text{ (-SEO)} \end{array} \right\}$$

&c. &c.

Et

Et area curvæ cujus absciffa est x & ordinata

$$\frac{c x^{n-\frac{r}{n}n-1}}{a^{2n} \pm a^{n-1} b x^n + x^{2n}}, \text{ erit } \frac{nc}{na^{n+1}} x^{-\frac{r}{n}n} \text{ in}$$

$$\frac{\overset{i}{R}_r^i \times \left[\frac{x}{a} \right]^n}{r-n} - \frac{\overset{ii}{R}_r^{ii} \times \left[\frac{x}{a} \right]^{2n}}{r-2n} + \frac{\overset{iii}{R}_r^{iii} \times \left[\frac{x}{a} \right]^{3n}}{r-3n}$$

$$- \frac{\overset{iv}{R}_r^{iv} \times \left[\frac{x}{a} \right]^{4n}}{r-4n} - \frac{\overset{v}{R}_r^v \times \left[\frac{x}{a} \right]^{5n}}{r-5n}, \text{ \&c.}$$

$$+ \frac{c}{n} a^{-\frac{r}{n}n-n-1} \text{ in } \left. \begin{array}{l} -a\alpha (\text{SA:AO}) - a_1 (+\text{ASO}) \\ +b\beta (\text{SB:BO}) + b_2 (+\text{BSO}) \\ -c\gamma (\text{SC:CO}) + c_3 (+\text{CSO}) \\ +d\delta (\text{SD:DO}) - d_4 (-\text{DSO}) \\ +e\epsilon (\text{SE:EO}) + e_5 (-\text{ESO}) \end{array} \right\}$$

$\text{\&c.} \qquad \qquad \qquad \text{\&c.}$

Harum arearum prior adjacet absciffæ ad ordinatam terminatæ, posterior vero absciffæ ultra ordinatam productæ. Signa autem quantitatum has expreffiones ingredientium ita determinantur: 1. Rectæ $\overset{i}{R}_r^i, \overset{ii}{R}_r^{ii}, \overset{iii}{R}_r^{iii}, \text{\&c.}$ afficiuntur signis affirmativis, fi a punctis circumferentiæ $\overset{i}{R}, \overset{ii}{R}, \overset{iii}{R},$ tendunt secundum directionem $OP,$ negativis vero fi ab iisdem punctis secundum directionem contrariam PO procedunt. 2. Moduli rationum $a\alpha, b\beta, c\gamma, \text{\&c.}$ signa habent affirmativa, fi a punctis $a, b, c, \text{\&c.}$ tendunt secundum directionem $OP,$ negativa fi secundum contrariam. 3. E centro circuli O

G

cadat

cadat in chordam RT normalis OH . Et moduli angularum a 1, b 2, c 3, &c. signis gaudebunt affirmativis si a punctis $a, b, c, \&c.$ tendunt secundum directionem HO , negativis si secundum contrariam. 4. Producat radius PO donec circumferentiæ denuo occurrat in p , & anguli $SAO, SBO, SCO, \&c.$ ut & $ASO, BSO, CSO, \&c.$ sumi debent affirmative si existunt in semicirculo superiore PRp , negative si in inferiore. Et secundum has regulas signa quantitatum quibus areæ exprimentur nostræ figuræ accommodavimus.

VII. *Casus rarissimus Plicæ Polonicæ enormis à D. Abrahamo Vaterno, M. D. Prof. Anatom. Wittemberg. & R. S. S. per D. Conradum Sprengell, Equitem, M. D. R. S. S. & Coll. Med. Lond. Licent. communicatus. Vid. TAB. II. Fig. 2.*

FŒMINA rustica in Polonia, in terris Principis Radzivil, anno ætatis decimo quinto, viro nupta, incidit decimo octavo, in morbum Poloniæ Endemium, qui Plica Polonica a capillo inenodabili vocatur. Hanc Plicam per quinquaginta annos femina gestavit, ac per totum fere illud tempus dolore arthritico et contracturis tandemque marasmo universalis corporis afflicta tecto affixa fuit, tandemque senio confecta anno ætatis septuagesimo octavo diem suum obiit.

Fig. 2.



Fig. 1.

