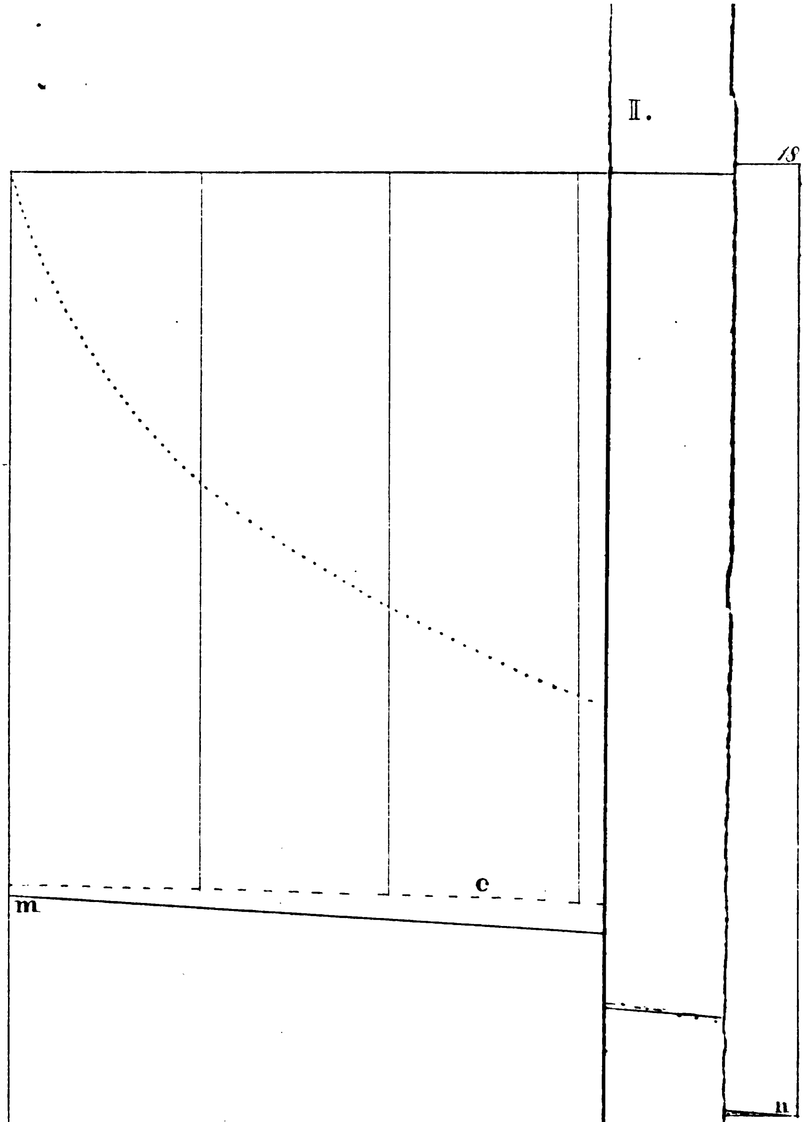

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AN

EXAMINATION

OF

PROFESSOR BARLOW'S REPORTS

ON

IRON RAILS,

ETC.

BY LIEUT. PETER LECOUNT, R.N., F.R.A.S.

AUTHOR OF "AN ESSAY ON THE LOCAL ATTRACTION OF IRON IN SHIPS," &c.

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TO

CAPTAIN C. R. MOORSOM, R. N.

THIS WORK

IS DEDICATED, AS A SMALL TRIBUTE

OF GRATITUDE

FOR A MUNIFICENT AND UNEXAMPLED GIFT,

WHICH

THE AUTHOR RECEIVED

FROM HIM,

ON THE 6TH OF AUGUST, 1827.

THROUGH THE LATE

ADMIRAL SIR ROBERT MOORSOM, K. C. B.

A gift which, in another quarter, was the Author's right; but which he in vain demanded from a scoundrel, whose name shall not disgrace a page, which carries that of two honest men,—but who, in due time, shall have a volume to himself.

NOTE FOR REFERENCES.—When the number of the page only is given, it implies that page in Mr. Barlow's First Report;—when a Roman L is attached, it is the page in this work.

PREFACE.

The mass of interested reports, to the prejudice of the Liverpool and Manchester Railway, having been now entirely removed from the minds of all those who are really conversant with the practical working of that concern, it follows, of course, that Railways not only have increased, but will continue to increase in all directions. At this moment, then, it becomes of paramount importance to construct them on principles which will ensure, to the enterprising individuals who embark their property in them, a fair return for the confidence which they may have placed, in what has so often been called, a delusive and ruinous speculation.

It becomes every one to add what he can, to the limited stock of knowledge we yet possess on this interesting subject; for it may be fairly said, that when they are scientifically constructed, and based upon the principle of making a proper return to those who advance their capital, they will introduce facilities of transit, which must have a most material effect upon the commercial prosperity of any country.

The talents and abilities of civil engineers are well known; yet, that differences of opinion should exist on some new points, is by no means surprising. This happens to be the case with respect to the best form of rail, chair, &c. The question may be thus stated,—the Liverpool and Manchester railway, although returning 10 per cent., has been worked hitherto at an expense which is, on all sides, admitted to be very great; and those who are interested in similar constructions, have naturally turned their thoughts to discover the best means of decreasing this outlay. Among others who have entered the arena, the talented Professor Barlow stands prominently forward. The two Reports which he has published are splendid specimens of mathematical talent of the first order; and, I unhesitatingly affirm, that no man, be he whom he may, can sit down and thoroughly make himself master of these investigations, without receiving benefit; and while he admires the powerful manner in which Mr. Barlow wields his analysis, he must be grateful for what has been there accomplished, and must only lament that the learned Professor had not time to perfect his work.

I am now to speak of my own intentions in the following pages. I am going to differ with several of the calculations and conclusions, delivered by the talented author of the Reports in question. I shall do so boldly. My object is to arrive at the truth, and I am certain that no man can be more anxious for that than the learned Professor himself. It will be for others to judge who is right; and, to enable as many as possible to enter on the enquiry, I shall not only make all my own calculations as plain as the subject will admit of, but shall explain several of Mr. Barlow's, and supply the steps which he has left out in his mathematical investigations as much as possible; for I am sensible no one can be qualified to enter on this intricate enquiry, with a proper chance of coming to a right conclusion, unless they master the whole of the subject from first to last.

I am only sorry that my time is so limited as to make me curtail, wherever the subject will admit of it, without injury. It is a wide field, and my hope is to see as many combatants as possible enter into it; provided they do so with the same intentions which actuate me: a desire for truth, and a feeling of perfect independence from all interested motives. It may be said on this, that I hold a situation in the Engineering department, under Captain Moorsom, in the London and Birmingham Railway. I reply, that so far from this influencing me, my opinions are in direct opposition to the whole of the steps lately decided on, with respect to rails, chairs, and blocks, at the Birmingham end of that Railway; I had, in fact, nothing whatever to do with the discussion upon them.

It must not be inferred here, that Professor Barlow is answerable for all those steps,—this is not the case. The principal point however is, that with respect to other companies, the question is, happily for them, still open to discussion. And, if I am fortunate enough to throw even a glowworm's gleam upon it, I shall think myself sufficiently rewarded, looking at its vast importance to the interests of so numerous a body as the proprietors of railway shares now are.

It requires a man of some nerve to face such a leviathan as Professor Barlow, on mathematical points, but it was necessary that some person should do it; and, it appears the lot has fallen upon Jonah, with what advantage others must judge. It remains only to add that I alone am responsible for the following pages, and that they have not the most distant connexion with the London and Birmingham Railway Company.

Constantine Cottage,
Wellington Road.
Birmingham, February, 1836.

ERRATA.

Page 35, line 11,—for *Cos* (2., read *Cos* (r.,

„ 38, „ bottom,—for $6\,\frac{6^3\,x^2}{a^5}$ read $6\,\frac{b^3\,x^2}{a^5}$

„ 39, „ 6,—for *time* read *term.*

„ 113, „ 6,—for $\frac{dx^3}{3}$ read $\frac{d''^3}{3}$

„ 113, „ bottom,—for $d'' + x^2$ read $d'' + \frac{x}{2}$

CHAP. I.

COMPARISON OF FISH-BELLIED AND PARALLEL RAILS.

Mr. Barlow's first report begins with a most essential investigation, namely, into the properties and the comparative advantages of the fish-bellied and the parallel rails, each of which shapes have their respective supporters, all of whom admit that the two rails are equally strong when the maximum depth of the fish-belly is the same as the depth of the parallel, but it is now supposed by some that they are not equally stiff.

Mr. Barlow therefore having stated, page 11, &c. that he intends to examine the transverse strength of iron, says—

"Before, however, proceeding to these experimental researches, there is one subject, rather of investigation than of experiment, on which I have thought it necessary to bestow some attention, it being one on which the opinions of practical men are much divided;—this is, the comparative advantages and disadvantages of what is called the fish-bellied rail, and that with parallel edges."

"EXAMINATION OF THE PROPERTIES, CURVATURE, AND RESISTANCE OF THE FISH-BELLIED RAIL.

"It is well known, both as a theoretical and mechanical fact, that if a beam be fixed with one end in a wall, or other immovable mass, to bear a weight suspended at the other end, the longitudinal section of such a bar (its breadth being uniform) should be a parabola; because, with that figure, every part of it will be strong in proportion to its strain, and thus one-third of the material may be saved. This form of construction is frequently adopted in the case of cast-iron beams in buildings, and with great advantage, as thereby one-third of the material is saved, while the strength is preserved, and the walls of the building relieved from a great unnecessary weight."

"This seems to have led to a somewhat similar principle of construction in what is called the fish-bellied rail; and the question here is, with what advantage? In the first place, it is to be remarked that the figure, which theory requires in this case, is not, as in the preceding, a parabola; for, as in the transit of the locomotive every part of the bar has, in succession, to bear the weight; and as the strain on any part of a beam supported at each end, and loaded in any part of its length, is as the rectangle of the two parts,—the strength being as the square of the depth,—it follows that the square of the depth ought to be every where proportional to the rectangle of the two parts, which is the known property of a semi-ellipse. The bar, therefore, in theory, ought to be a semi-ellipse, having its length equal to the transverse diameter, and the depth of the beam for its semi-conjugate, and there can be no doubt, that such a figure would be, to all intents and purposes, as strong in its ultimate resistance as a rectangular beam."

"But it is difficult to obtain this figure correctly in malleable iron, and many of what are called fish-bellied rails are but bad approximations to it, although others differ from it but slightly. The following is the general mode of manufacture." (*See plate* 2, *fig*. 1.)

"EF is the section of an iron roll; GH the section of another. This latter being hung on a false centre C, is turned down, leaving a groove of varying depth as shown in the figure. The cylinder GH being now again placed on its proper centre B, the bars are introduced between the two rolls at KL; and as the iron passes through, it acquires the variable depth shown in the lower roll. The inner circle, or bottom of the groove, is generally one foot in diameter, and the upper EF three feet in circumference; consequently, the figure is completed in a length of three feet, and there are commonly five such lengths in a bar. The computation of the ordinates to the curve thus formed is by no means difficult; for, calling the radius of the cylinder $CD=r$, and the distance of the centres $BC = d$ and x any angle LCD, we find the ordinate.

$$ID = BI - \sqrt{(r^2 + d^2 - 2rd \cos x)}.$$

And by this formula the ordinates of the curves have been computed for two different fish-bellied rails; the extreme depth in both being five inches, but the lesser depth in one three inches, and in the other three and three-quarter inches, the latter being that proposed by Mr. Stephenson for the London and Birmingham Railway. The ordinates are taken for each 10^0, or for every inch of the half-length, and in the last column are given the ordinates of the true ellipse."

"TABLE OF ORDINATES.

ABSCISSES.			ORDINATES in Fish-bellied Rail. Greatest Depth 5In. Least .. ditto 3 „	ORDINATES in Mr. Stephenson's Rail.	ORDINATES in the Ellipse.
Deg.		Inch.			
0	=	0	3·00	3·75	0
10	or	1	3·01	3·76	1·64
20	..	2	3·05	3·78	2·29
30	..	3	3·12	3·82	2·76
40	..	4	3·21	3·88	3·14
50	..	5	3·31	3·95	3·46
60	..	6	3·44	4·04	3·72
70	..	7	3·59	4·14	3·96
80	..	8	3·75	4·23	4·16
90	..	9	3·92	4·34	4·33
100	..	10	4·09	4·45	4·48
110	..	11	4·27	4·55	4·61
120	..	12	4·43	4·66	4·71
130	..	13	4·59	4·75	4·80
140	..	14	4·72	4·84	4.87
150	..	15	4·84	4·91	4.93
160	..	16	4·93	4·95	4·97
170	..	17	4·98	4·99	4·99
180	..	18	5·00	5·00	5·00

"We see by this table, (although it is impossible, with any proportions or degrees of eccentricity, to work out a true ellipse by this method,) that we may approximate towards it sufficiently near for practical purpose, as Mr. Stephenson has done; while on the other hand, without due precaution, we may so far deviate from it as to render the bar dangerously weak in the middle of its half-length."

"As far as relates to ultimate strength, there can be no doubt Mr. Stephenson's rail is equal to that of an elliptic rail, and consequently to that of a rectangular rail of the same depth; but there is still an important defect in all elliptical bars, viz. that although this form gives a uniform strength throughout, it is by no means so stiff as a rectangular bar of a uniform depth, equal to that of the middle of the curved bar, and it is the stiffness rather than the strength that is of importance; for the dimensions of the rail must so far exceed those which are barely *strong enough*, as to put the consideration of ultimate strength quite out of the question. The object, therefore, with a given quantity of metal, is to obtain the form least affected by deflection; and unfortunately the elliptical bar, although equally as strong as the rectangular bar of the same depth, as far as regards its ultimate resistance, is much less stiff. This will appear from the following investigation:"—(*See plate 2, fig. 2*).

“The deflections which beams sustain when supported at the ends and loaded in the middle, is the same as the ends would be deflected, if the beams were sustained in the middle, and equally loaded at the ends, each with half the weight; and the *law* of deflection is the same in the latter case, as when the beam is fixed in a wall and loaded at its end, although the *amount* is greater. At present, however, our inquiry is not the actual, but the relative deflection in two beams, one elliptical, and the other rectangular, of the same length, and of the same extreme depth—the breadth and load being also equal in each. It is quite sufficient, therefore, to consider the corresponding effects on two half beams, each fixed in an immovable mass, as represented in the preceding figures.”

“Now, in the first place, the elementary deflection at C is the same in both beams, because the lengths and loads are the same, and the depths at C A equal; but the whole deflection at any other point P, will be directly as MB^2, and inversely as MP^3. If, therefore, we call MB $= x$, and MP $= y$, the sum of all the deflections in the two beams will be $\int \frac{x^2}{y^3} \, d\,x \;\; \triangle$, $\triangle$ being the sine of deflection at C. But in fig. 1, y is constant and equal to d (the depth,) while in the latter,

$$y = \frac{d}{l} \sqrt{(2\,l\,x - x^2)}$$

l being the semi-transverse or length, and x any variable distance."

"The whole deflections, therefore, in the two cases, are,

Fig. 1. Deflection $= \int \frac{x^2 \, d\,x}{d^3} \triangle =$ (when $x = l$)

$$\frac{1}{3} \frac{l^3}{d^3} \triangle$$

And in Fig. 2:—

$$\text{Deflection} = \int \frac{x^2 \, dx \;\triangle}{\frac{d^3}{l^3} (2\,lx - x^2)^{\frac{3}{2}}} = \text{(when } x = l\text{)}$$

$$,41 \frac{l^3}{d^3} \triangle$$

"The deflections, therefore, in the two cases are with the same weights, as 33 to 41,* or nearly as 3 to 4, a result fully borne out by subsequent experiment. It is to be observed also, that this investigation applies only to the deflection when the weight is in the middle of the bar, and that it would be much greater in comparison with the parallel rail towards the middle of its half-length."

"This want of stiffness is, I should imagine, but badly compensated by the trifling saving of metal thus effected; for I find that an addition of little more than four pounds per yard, would convert this rail into a rectangular one of the same depth, which would have one-third more stiffness at its middle point, and probably one-half more, a little beyond the middle of the half-lengths. I am aware, objections are made to rectangular bars having so much depth of bearing

Note by Mr. Barlow,—"Experiments have been made from which it has appeared, that the fish-bellied rail was stiffer than the parallel rail, which is certainly possible, if the parallel rail be of inferior metal or of injudicious figure; but it is mechanically impossible if the parallel bar be made of the figure here assumed."

in their chairs, and this may be a practical defect, on which I shall offer no opinion; at all events, it is well to estimate properly both evils, and then to choose the least."

Mr. Barlow having thus come to the conclusion, that with the same load the deflection of the fish-bellied rail is to that of the parallel, as 41 to 33, I join my first issue on this point, and I say that weight for weight the fish-bellied rail will be deflected less by the same load than the parallel one will.

In the first place, Mr. Barlow's determination has nothing whatever to do with the real question. What has a mathematical ellipse to do with the form of the fish-bellied rail? the ellipse vanishes, or has no depth at one of its ends, whereas the fish-bellied rail has a depth (at 50lb. to the yard) of three inches and three quarters, and this alone might set the question at rest, so far as Mr. Barlow's numbers 41 and 33 are concerned.

The real question is, which is the stiffest, a fish-bellied rail of 50lb. to the yard, or a parallel rail of 50lb. to the yard? The mere saving of iron is of no consideration compared with the gain in the ultimate stability of the road, by lowering the rail in the chair an inch and a quarter; and it is the ultimate working of a railway, that all men of common forethought must necessarily look to.

The plate fronting the title page will shew this more clearly. *a a* is the parallel rail, *b b* is the mathematical ellipse, which is, by Mr. Barlow, compared with *a a*; and *c c* is the fish-bellied rail, which is the one which ought to have been compared; consequently all the iron contained between the lines *a a* and *c c* is left out in Mr. Barlow's solution of the problem. He correctly states the object to be "with a given quantity of metal to obtain the form least affected by deflection," but has not abided by this condition in his investigation.

In order to make this investigation plainer to those who are not particularly skilled in mathematical enquires, I will supply some of the steps of the process.

In the first place, there is an error of the press in the formula

$$\mathrm{ID} = \mathrm{BI} - \sqrt{(r^2 + d^2 - 2rd \cos x)}.$$

it should be

$$ID = BI - \sqrt{(r^2 + d^2 \pm 2rd \cos x)}.$$

where $2rd \cos x$ is $+$ from $0°$ to $90°$ of the semi circumference of the roller, (or in a 3 feet rail from 0 to 9 inches) and — in the next quadrant, $90°$ to $180°$ of the circumference, or from 9 to 18 inches of a 3 feet rail.

This formula is thus derived. *See plate 2, fig.* 3

By Euclid B 2; $c^2 = a^2 + b^2 \pm 2bd$ where $2bd$ is $+$ where the angle opposite c is obtuse, and — when that angle is acute, but $d = a \cos x$ when a is radius, hence $c^2 = a^2 + b^2 \pm 2ba \cos x$.

Mr. Barlow finds the "Sum of all the deflections in the two beams $= \int \frac{x^2}{y^3} dx \triangle$; the words "each *of*" are wanted here between "*in*" and "*the*"; it should stand the sum of all the deflections in each of the two beams is $\int \frac{x^2}{y^3} dx \triangle$ otherwise we should have $\frac{l^3}{3d^3} \triangle + ,41 \frac{l^3}{d^3} \triangle = \frac{x^3}{3y^3} \triangle = \frac{l^3}{3y^3} \triangle$ which is impossible, Mr. Barlow then substitutes for y^3 its value in the equation for the ellipse, which is

$$y = \frac{c\sqrt{tx - x^2}}{t}$$

Where t and c are the axes, y the ordinate, and x the absciss. Mr. Barlow's d = the semi-conjugate and his l = the semi-transverse, and substituting these letters in the above equation we get

$$y = \frac{2\,d\sqrt{2\,lx - x^2}}{2\,l}$$

which is the same as the one given by Mr. B.

We then have the deflection of a parallel rail when $x = l$, or when it is supported at every 3 feet, and l = 18 inches $= \frac{l^3}{3d^3}\,\triangle$

for the deflection in a mathematical ellipse we must integrate $\int \frac{x^2\,dx}{\frac{d^3}{l^3}\,(2\,lx - x^2)^{\frac{3}{2}}}$

this integral Mr. Barlow gives as $,41\,\frac{l^3}{d^3}\,\triangle$

In order to arrive at it, we have

$$\int \frac{x^2\,dx}{\frac{d^3}{l^3}(2\,lx - x^2)^{\frac{3}{2}}}$$

by clearing it of the denominator l^3 & making $2l = p$

$$= \frac{l^3}{d^3} \int \frac{x^2}{(px - x^2)^{\frac{3}{2}}} \cdot dx$$

which by changing the sign of the exponent in the denominator of the differential

$$= \frac{l^3}{d^3} \cdot \int x^2 \,(px - x^2)^{-\frac{3}{2}} \; dx$$

then as $\frac{x^2}{(px - x^2)^{\frac{3}{2}}}$ or its equal $x^2 . (px - x^2)^{-\frac{3}{2}}$

is equivalent to

$$\frac{x^{\frac{1}{2}}}{(p - x)^{\frac{3}{2}}}$$ or to $x^{\frac{1}{2}}\,(px - x)^{-\frac{3}{2}}$ we get

by squaring numerators and denominators

$$\frac{x^4}{(px - x^2)^3} = \frac{x}{(p - x)^3}$$ or $x^4 . (px - x^2)^{-3} =$

$x\,(p - x)^{-3}$ and extracting the square roots of the numerators and denominators of each of these expressions again (that is to say the fractional parts, the same which were squared before) we get $x^2 . (px - x^2)^{-\frac{3}{2}} dx$ or its equal $x^{\frac{1}{2}} . (p - x)^{-\frac{3}{2}} dx$.

We have now to expand $x^{\frac{1}{2}} . (p - x)^{-\frac{3}{2}}$ in a series, then to multiply each term of that series by dx and then integrating each term separately, we must multiply the result or sum of all the terms by $\frac{l^3}{d^3}$; we shall then have the sum of the deflections in a mathematical ellipse.

To expand $x^{\frac{1}{2}} (p - x)^{-\frac{3}{2}}$ we have the formula

$$\overline{P + QP}^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n} AQ + \frac{m - n}{2n}$$

$$BQ + \frac{m - 2n}{3n} CQ \times \frac{m - 3n}{4n} DQ \text{ \&c.}$$

Where Q = the second term of the binomial $p - x$ (or whatever it may be) divided by the first

$A = P^{\frac{m}{n}}$ = the first term of the right hand member.

$B = \frac{m}{n} AQ$ = the second term.

C = the third term and so on.

Now comparing $x^{\frac{1}{2}} . (p-x)^{-\frac{3}{2}}$ with the formula, we get when $(p-x)^{-\frac{3}{2}} = \overline{P + PQ}^{\frac{m}{n}}$

$P = p$

$Q = -\frac{x}{p}$

$\frac{m}{n} = -\frac{3}{2}$

and for the right hand member of the equation we have as follows.—

$A = p^{-\frac{3}{2}}$

$B = -\frac{3}{2} . p^{-\frac{3}{2}} . -\frac{x}{p}$

$C = -\frac{3}{2} . -\frac{5}{4} . p^{-\frac{3}{2}} . \frac{x^2}{p^2}$

and so on.

Also, $\frac{m-n}{2n} = -\frac{5}{4}$

$\frac{m-2}{3n} = -\frac{7}{6}$

and so on. Hence the terms of the series run

$x^{\frac{1}{2}} . p^{-\frac{3}{2}} = \frac{x^{\frac{1}{2}}}{p^{\frac{3}{2}}}$

$$x^{\frac{1}{2}} \cdot - \tfrac{3}{2} \cdot p - \tfrac{3}{2} \cdot - \frac{x}{p} = + \tfrac{3}{2} \cdot \frac{x^{\frac{3}{2}}}{p^{\frac{5}{2}}}$$

$$x^{\frac{1}{2}} \cdot - \tfrac{5}{4} \cdot - \tfrac{3}{2} \cdot p^{-\frac{3}{2}} - \frac{x}{p} \cdot - \frac{x}{p} = \tfrac{5}{4} \cdot \tfrac{3}{2} \cdot \frac{x^{\frac{5}{2}}}{p^{\frac{7}{2}}}$$

$$x^{\frac{1}{2}} \cdot - \tfrac{7}{6} \cdot - \tfrac{5}{4} \cdot - \tfrac{3}{2} \cdot p^{-\cdot\frac{3}{2}} \frac{x^2}{p^2} \cdot - \frac{x}{p} = \tfrac{7}{6} \cdot \tfrac{5}{4} \cdot \tfrac{3}{2} \cdot \frac{x^{\frac{7}{2}}}{p^{\frac{9}{2}}}$$

and so on for as many terms as may be considered necessary, which produces the series given by Mr. Barlow, p. 74, and we have then to integrate each term separately, by the usual method, that is to say, by adding 1 to the exponent of the unknown quantity x, dividing by that exponent so increased, and also dividing by or expunging dx.

We now convert the series thus obtained, into another, wherein l is substituted for x and $2\,l$ for p; the former, because the sum of the deflections are wanted when the rail is supported at every 3 feet, or when $l = 18$ inches; and the latter, because we before substituted p for $2\,l$; hence in the first term.

$$\tfrac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{p^{\frac{3}{2}}} = \tfrac{2}{3} \frac{l^{\frac{3}{2}}}{p^{\frac{3}{2}}} = \tfrac{2}{3} \frac{l^{\frac{3}{2}}}{(2l)^{\frac{3}{2}}}$$

squaring the latter part of the last fraction, we get $\frac{l^3}{(2l)^3} = \frac{l^3}{8\,l^3} = \frac{1}{8}$ and taking the square root and affixing the former part of the fraction, we get

$$\tfrac{2}{3} \cdot \frac{1}{\sqrt{8}} = \tfrac{2}{3} \cdot \frac{1}{2\sqrt{2}} = \frac{2}{6\sqrt{2}} = \frac{1}{3\sqrt{2}} = \tfrac{1}{3} \cdot \frac{1}{\sqrt{2}}$$

and as $\frac{1}{\sqrt{2}}$ occurs in each term, we may leave it out in summing the series, and multiply the whole by it afterwards.

In the same way

$$\tfrac{2}{5} \cdot \tfrac{3}{2} \cdot \frac{x^{\frac{5}{2}}}{p^{\frac{5}{2}}} = \tfrac{2}{5} \cdot \tfrac{3}{2}\, \frac{l^{\frac{5}{2}}}{(2\,l)^{\frac{5}{2}}} = \tfrac{2}{5} \cdot \tfrac{3}{2} \cdot \frac{l^{\frac{5}{2}}}{\sqrt{32}\,.\,l^{\frac{5}{2}}} = \tfrac{5}{2}$$

$$\cdot \tfrac{3}{2} \cdot \frac{1}{\sqrt{32}} = \tfrac{2}{5} \cdot \tfrac{3}{2} \cdot \frac{1}{4\sqrt{2}} = \tfrac{1}{5} \cdot \tfrac{3}{2} \cdot \frac{1}{2\sqrt{2}} =$$

$$\frac{1}{\sqrt{2}} \cdot \tfrac{1}{5} \cdot \tfrac{1}{2} \cdot \tfrac{3}{2}$$

Again, $\tfrac{2}{7} \cdot \tfrac{5}{4} \cdot \tfrac{3}{2} \cdot \frac{x^{\frac{7}{2}}}{p^{\frac{7}{2}}} = \tfrac{2}{7} \cdot \tfrac{5}{4} \cdot \tfrac{3}{2} \cdot \frac{l^{\frac{7}{2}}}{(2\,l)^{\frac{7}{2}}} = \tfrac{2}{7} \cdot \tfrac{5}{4} \cdot$

$$\frac{3}{2} \cdot \frac{\overline{l\,2}^{\,7}}{\sqrt{128}} \cdot \overline{l\,2}^{\,7} = \frac{2}{7} \cdot \frac{5}{4} \cdot \frac{3}{2} \cdot \frac{1}{\sqrt{128}} = \frac{2}{7} \cdot \frac{5}{4} \cdot$$

$$\frac{3}{2} \cdot \frac{1}{8\sqrt{2}} = \frac{1}{7} \cdot \frac{5}{4} \cdot \frac{3}{2} \cdot \frac{1}{4\sqrt{2}} = \frac{1}{7} \cdot \frac{5}{4} \cdot \frac{3}{2} \cdot \frac{1}{4} \cdot \frac{1}{\sqrt{2}}$$

Whence the law of the series becomes manifest, and as many terms as we like can be formed by merely following that law.

We have now to sum up this series, (and a very pleasant job it is.)

$\frac{1}{3} \cdot 1$ .... ,333333333

$\frac{1}{10} \cdot 1,5$ .... ,150000000

$\frac{1}{8} \cdot \frac{15}{8} = \frac{105}{224} =$ .... ,066964255

$\frac{1}{72} \cdot \frac{105}{48} = \frac{105}{4456} =$ .... ,030381944

$\frac{1}{176} \cdot \frac{945}{384} = \frac{945}{67584} =$ .... ,0139826

$\frac{1}{416} \cdot \frac{10395}{3840} = \frac{10395}{1597440} =$ .... ,00650729

$\frac{1}{960} \cdot \frac{135135}{46080} = \frac{135135}{44236800} =$ .... ,00305515

$\frac{1}{2176} \cdot \frac{2027025}{645120} = \frac{2027025}{1403781120} =$ .... ,00144397

$\frac{1}{19{,}2^{8}} = \frac{1}{4864} \cdot \frac{34459425}{10321920} = \frac{34459425}{50205818880} =$ .... ,00068636

SUM OF 9 TERMS .... ,606354902

and continuing a little farther we get ,607 for the sum of the whole.

Now $\sqrt{2} = 1{,}4142135624$, and the reciprocal of this, or $\frac{1}{1{,}41421356} = {,}707106$, &c. and $,707106 \times ,607 = ,42913342$.

From this it appears that the sum of the deflections have a greater difference than that which Mr. Barlow has given, viz. instead of being as 41 : 33 they are as 43 to 33, that is to say, when the deflection in the parallel rail is 33, the deflection in a mathematical ellipse is 43, the loads in each case being equal.

Mr. Barlow gives ,6095 instead of ,607 for the multiplier, this multiplied by $\frac{1}{\sqrt{2}}$ or $,707106 =$

,4309811, &c.

Mr. Barlow gets ,6095 by considering the series after a few terms as equal to a geometrical one, whose ratio is $\frac{1}{2}$, this means, I presume, that a few terms are got in the way I have done them, and the rest taken as a geometrical series, having the ratio $\frac{1}{2}$, for if the whole was summed as a series having that ratio, it would be ,666 $\times$,707106 = ,4714, giving the ratio 33 to 47.

Having thus seen what a parallel bar will deflect when compared with a mathematical ellipse, we must next ascertain what will be the deflection of the fish-bellied rail.

Taking the formula given by Mr. Barlow, in which the natural cosine of x is to be used, neglecting any radius which the formula supplies, and noting that $2\,rd$ must be added or subtracted, although Cos x may $= o$ we have

$$y = \mathrm{B\,I} - \sqrt{r^2 + d^2 \pm 2rd \operatorname{Cos} x}$$

where d is the excentricity of the roller, which is always equal to the minimum depth of the rail subtracted from the maximum and the remainder divided by 2, hence

$$r = 6$$
$$r^2 = 36$$
$$d = ,625 = \frac{5 - 3,75}{2}$$
$$d^2 = ,390625$$
$$2rd = 7,5$$

and $\mathrm{B\,I} = \dfrac{6+6+3,75+5}{2} = 10,375$, as may be seen on looking at the figure, l being taken as the radius of the roller as Mr. Barlow gives it, and 5 and 3,75 are the respective greatest and least depth of the rail.

This gives for the ordinate y, or the depth for the rail at the respective lengths or abcisses $= x$

x	y
0	3,750
1	3,759
2	3,785
3	3,826
4	3,884
5	3,955
6	4,039
7	4,133
8	4,236
9	4,343
10	4,451
11	4,559
12	4,662
13	4,756
14	4,840
15	4,907

x	y
16	4,958
17	4,989
18	5,000

for practical purposes empirical equations may be constructed which are sufficiently accurate, and rather easier to compute from, than the foregoing, thus for the 50lb. rail of 3 feet.

Putting $d =$ the minimum depth.

l = the ½ length = 18 inches.

x = the abcissa.

y = the ordinate.

$$y = d + \frac{x}{2}(1+x)\frac{d}{535,7} + \frac{x}{3}.(1+x)\cdot\frac{1+l-x}{1000} = 3,75 + \frac{x}{2}.(1+x)\cdot ,007 + \frac{x}{3}\cdot(1+x)\,.\frac{1+l-x}{1000}$$

this gives for y

					Difference in the two formulæ.
$x =$ 0	....	$y =$ 3,750	....	+	,000
1	....	3,769	....	+	,010
2	....	3,805	....	+	,020
3	....	3,856	....	+	,030
4	....	3,920	....	+	,036
5	....	3,955	....	+	,040
6	....	4,079	....	+	,040
7	....	4,170	....	+	,037
8	....	4,266	....	+	,030
9	....	4,365	....	+	,022
10	....	4,465	....	+	,014
11	....	4,564	....	+	,005
12	....	4,660	....	—	,002
13	....	4,751	....	—	,005
14	....	4,835	....	—	,005
15	....	4.910	....	+	,003
16	....	4,974	....	+	,016
17	....	5,025	....	+	,036
18	....	5,061	....	+	,061

These differences only amount to 4 hundredths of an inch in 3 cases, and the rails made by different manufacturers will differ sometimes nearly a tenth of an inch.

For those who wish more accuracy a rigid formula may be constructed from Mr. Barlow's, in which the data are derived from the rails, and not from the rollers as he has done; this may be useful as it will also enable the manufacturer to ascertain what dimensions his rollers must have, in order to make a rail of any given depth and length.

The rolls E F and *m n* should be both equal, and likewise equal to the required length of the rail, viz. in this case 3 feet, this on trial with the diameter of a 3 feet circle instead of $2r = 12$ which has been used to compute Mr. Barlow's ordinates by me, will be found to give the same results.

The roll G H should equal in diameter B I, let therefore

r = C D = radius of rolls E F and $m\,n$

d = Maximum depth of rail.

d' = Minimum depth.

e = CB = distance of centre $= \dfrac{d - d'}{2}$

z = angle L C D

$y = 2\,re$

$k = r^2 + e^2$

$x =$ Abscissa in inches.

$y =$ Ordinate in inches

$h =$ B I

then $h = \dfrac{2\,r + d + d'}{2}$

and $y = h - \sqrt{k \pm g \text{ Cos } z}$

but $z = \dfrac{2r\,.\,3{,}14159}{360}\,.\,10\,x = 1^0 \times 10\,x$ hence

we get $y = h - \sqrt{k \pm g \text{ Cos} \left(\dfrac{2r\,.\,3{,}14159}{360}\,.\,10x\right)}$

for every inch of x

$$= h - \sqrt{k \pm g \text{ Cos } \left(\frac{r\,.\,6{,}2831852}{360}\,.\,10\,x\right)}$$

$$= h - \sqrt{k \pm g \text{ Cos } (2\,.\,,0174533\,.\,10\,x)}$$

from which having the lengths and breadths, and least depths of any required rail, the size of the rolls may be determined, for their circumference will equal the length of the rail, viz. 3 feet, 5 feet. &c. and the rest is got from the equation above, which is general for all sized rails, only noting that when the length is any other than 3 feet, substitute $\frac{180}{a}$ x for 10 x where a = the inches in the $\frac{1}{2}$ length . thus for a 5 feet rail it will be r . ,0174533 . 6 x.

If we now apply either of these values of y in the equation $\frac{x^2}{y^3} dx \triangle$ and integrate it, we shall get the sum of the deflections in the fish-bellied rail.

With respect to the first one, or the empirical formula, the integration of it would most likely be just a few minutes amusement for Mr. Barlow; but I must confess the look of it alone is quite sufficient for me.

The second has also its difficulties, and when done, would, I think, be not so easily traced and computed by men of moderate mathematical abilities, as the method I am about to adopt. I am very anxious to make every thing as plain as the nature of the subject will allow, that as many persons as possible may not only be able to compute after me, but that they may be able to trace all the steps of the process of reduction, that I may be corrected if necessary.

If we look at the drawing of the 3 feet fish-bellied rail taken at 50lb. to the yard, and laid out from the correct ordinates given here, we shall see that it is very easy to draw a line from *m* to *n* so that an average slcping breadth may be assigned to the rail for each half length, and the fish-belly converted into an equal mass having the shape of the frustrum of a pyramid, we then have, calling the

Minimum depth $= a$

Maximum depth $= c$

and $c - a = d$

$$l : d = x : \frac{dx}{l}$$

where l is the half length or 18 inches, and we get for the depth or ordinate y at any length or abciss x

$$y = a + bx$$

where $b = \frac{c - a}{l} = \frac{c - a}{18}$

Substituting this value of y in the equation

$$\frac{x^2}{y^3} dx \triangle$$

we have $\dfrac{x^2}{(a+bx)^3} dx \triangle$

for the sum of the deflections in the fish-bellied rail.

In order to integrate this we have

$$\frac{x^2\,dx}{(a+bx)^3} = \frac{x^2\,dx}{1} \cdot \frac{1}{(a+bx)^3} = x^2\,dx\,.\,(a+bx)^{-3}$$

and expanding $(a+bx)^{-3}$ by the theorem

$$\overline{P+QP}^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n} \cdot AQ + \frac{m-n}{2n} \cdot BQ \text{ \&c.}$$

we have $P = a$

$$Q = \frac{bx}{a} = bx\,.a^{-1}$$

$$\frac{m}{n} = -3 \text{ or } m = -3 \;\&\; n = 1$$

$$A = a^{-3} = \frac{1}{a^3}$$

$$B = -3\,a^{-3}\,.\,bx\,.\,a^{-1} = -\,a^{-4}\,.\,bx = -\frac{3\,bx}{a^4}$$

$$C = -\frac{4}{2}\,.\,3\,a^{-3}\,bx\,.\,.\,a^{-1}\,.\,bx\,.\,a^{-1} = 6\,a^{-5}\,.\,b^2x^2$$

$= 6\,\dfrac{b^2x^2}{a^5}$ and so on, which when each term is multiplied by $x^2\,dx$ we have

$$x^2\,dx \cdot \left(\frac{1}{a^3} - \frac{3\,bx}{a^4} + \frac{6\,b^2\,x^2}{a^5} - \frac{10\,b^3\,x^3}{a^6} + \frac{15\,b^4\,x^4}{a^7} - \frac{21\,b^5\,x^5}{a^8} \ \&c. \right)$$

or $$\frac{x^2\,dx}{a^3} - \frac{3\,bx^3\,dx}{a^4} + \frac{6\,b^2\,x^4\,dx}{a^5} - \frac{10\,b^3\,x^5\,dx}{a^6} + \frac{15\,b^4\,x^6\,dx}{a^7} - \frac{21\,b^5\,x^7\,dx}{a^8} \ \&c.$$

and integrating each time we have

$$\frac{x^3}{3\,a^3} - \frac{3\,bx^4}{4\,a^4} + \frac{6\,b^2\,x^5}{5\,a^5} - \frac{10\,b^3\,x^6}{6\,a^6} + \frac{15\,b^4\,x^7}{7\,a^7} - \frac{21\,b^5\,x^8}{8\,a^8} + \frac{28\,b^6\,x^9}{9\,a^9} \ \&c.$$

If we now assume $a = 3{,}7$ which, by an inspection of the figure, is evidently too small, we get when $x = l = 18$ inches, and consequently $b = {,}07222$.

$x^3 = 5832$

$x^4 = 104976$

$x^5 = 1889568$

$x^6 = 3401224$

$x^7 = 612220032$

$x^8 = 11019960576$

$x^9 = 198359290168$

$x^{10} = 3570467223024$

$x^{11} = 64268410014432$

$x^{12} = 1156831380259776$

$x^{13} = 20822964844675968$

$a^3 = 50,653$

$a^4 = 187,4161$

$a^5 = 693,43957$

$a^6 = 2565,6641$

$a^7 = 9492,9657$

$a^8 = 35123,973$

$a^9 = 129958,7$

$a^{10} = 480847,2$

$a^{11} = 1779134,6$

$b^2 = ,0052157$

$b^3 = ,000376778$

$b^4 = ,000027205$

$b^5 = ,000001965$

$b^6 = ,00000014184$

$b^7 = ,0000000102437$

$b^8 = ,00000000074$

And from these values we get

$$\frac{5832}{151,959} - \frac{22744,1}{749,6644} + \frac{59132,5189}{3467,219785} -$$

$$\frac{128125,06376}{15393,98} + \frac{249831,689}{66450,7599} - \frac{454738,6732}{280991,784} \&c$$

or 38,3787 — 30,3452 + 17,0547 — 8,3231

+ 3,7596 — 1,6183 &c.

Whence we have, collecting the + and — terms, and subtracting the one from the other.

+	—
38,3787	30,3452
17,0547	8,3231
3,7596	1,6183
59,1930	40,2866
40,2866	
18,9064 say 18,92	

Again, taking $a = 3,8$ which I think to be the value of it, we have with $x = 18$ and $b = ,0666$.

$x^3 = 5832$

$x^4 = 104976$

$x^5 = 1889568$

$x^6 = 3401224$

$x^7 = 612220032$

$x^8 = 11019960576$

$x^9 = 198359290168$

$x^{10} = 3570467223024$

$x^{11} = 64268410014432$

$x^{12} = 1156831380259776$

$x^{13} = 20822964844675968$

$a^3 = 54{,}872$

$a^4 = 208{,}5136$

$a^5 = 792{,}35168$

$a^6 = 3010{,}936384$

$a^7 = 11441{,}5582592$

$a^8 = 43477{,}92138496$

$a^9 = 165216$

$a^{10} = 627821$

$a^{11} = 2385720$

$b^2 = {,}00443556$

$b^3 = {,}0002954083$

$b^4 = {,}00001967419$

$b^5 = {,}0000013103$

$b^6 = {,}00000008725$

$b^7 = {,}00000000581$

$b^8 = {,}000000000387$

And from these values we get

$$\frac{5832}{164,616} - \frac{20974,2048}{834,0544} + \frac{50287,7534}{3961,7584} -$$

$$\frac{100475}{18065,718} + \frac{180674,09}{80090,91} - \frac{303228,54}{347823,37} \text{ \&c.}$$

or $35,4279 - 25,1473 + 12,6933 - 5,5616$

$+ 2,2558 - ,8718$ &c.

Whence we have by collecting the terms

+	—
35,4279	25,1473
12,6933	5,5616
2,2558	,8718
50,3770	31,5807
31,5807	
18,7963 say 18,81	

Again if we roughly compute with $a = 4$; $x = 18$ and $b = ,055$ we have

$x^3 = 5832$

$x^4 = 104976$

$x^5 = 1889568$

$x^6 = 3401224$

$x^7 = 612220032$

$x^8 = 11019960576$

$a^3 = 64$

$a^4 = 256$

$a^5 = 1024$

$a^6 = 4096$

$a^7 = 16384$

$a^8 = 65536$

$b^2 = ,003025$

$b^3 = ,000166375$

$b^4 = ,00000915$

$b^5 = ,00000050325$

And from these values we get

$$\frac{5832}{192} - \frac{17321}{1024} + \frac{34295,6}{5120} - \frac{56460}{24576} + \frac{83813}{114688} - \frac{114607,6}{524288} \text{ \&c.}$$

$$\text{or } 30,37 - 16,915 + 6,698 - 2,297 + ,7308 - ,2186 \text{ \&c.}$$

Whence we have by collecting the terms.

+	—
30,37	16,915
6,698	2,297
,7308	,2186
37,7988	19,4306
19,4306	
18,3682	

When a is small the series converges very slowly; but by taking $a = 3$ and computing the first 8 terms, I was led to estimate that at about 3,3 the result of the calculation would be nearly equivalent to what the deflection is in the mathematical ellipse, and as it is desirable to get an approximation to this, let us try $a = 3,3$ $x = 18$ and $b = ,0944$. Working it roughly out, we have

$x^3 = 5832$

$x^4 = 104976$

$x^5 = 1889568$

$x^6 = 3401224$

$x^7 = 612220032$

$x^8 = 11019960576$

$x^9 = 198359290168$

$x^{10} = 3570467223024$

$x^{11} = 64268410014432$

$a^3 = 35,937$

$a^4 = 118,5921$

$a^5 = 391,35393$

$a^6 = 1291,467967$

$a^7 = 4261,8443$

$a^8 = 14064,086$

$a^9 = 46411,4838$

$a^{10} = 153157,89654$

$a^{11} = 504210,568$

$b^2 = ,00890816$

$b^3 = ,0008409303$

$b^4 = ,00007938379$

$b^5 = ,00000749383$

$b^6 = ,00000070541755$

$b^7 = ,00000006659$

$b^8 = ,000000006286$

from these values we get

$$\frac{5832}{107,811} - \frac{29729,2}{474,37} + \frac{100993,6}{1956,769} - \frac{286043}{7748,81} + \frac{728800}{29832,9} - \frac{1734212}{112512,7} + \frac{3917596}{417703} - \frac{8083538}{1531579} \text{ \&c.}$$

or $54,09 - 62,67 + 51,61 - 36,915 + 24,43 - 15,41 + 9,379 - 5,278$ &c.

Whence we have, by collecting the terms,

+	−
54,09	62,67
51,61	36,915
24,43	15,41
9,379	5,278
139,509	120,273
120,273	
19,236	

Now the next term + will be about 4, which added will make 24, hence taking the mean we have $\frac{19 + 24}{2} = 21,5$; and the mathematical ellipse is 20, so that 3,3 is too small, perhaps 3,4 would be nearer.

We see, therefore, that by drawing a line on the plate where the two rails and the ellipse are shewn, that we must not have it so that as much iron is taken in at the belly of the rail as is left out at the small end, for with $a = 3,3$ there is considerably more left out than is taken in, yet this approximates to the right value (and is too small rather than otherwise) for the frustrum of the pyramid to have a deflection equal to the mathematical ellipse; our estimated value above being 21,5, while that for the ellipse is 20.

In these examples I have repeated the numeral values of x, although it was not necessary, in order that those who choose to follow me may take them out conveniently near the same page where they find the values of a and b, and they may also integrate $\frac{x^2\,dx}{(a+bx)^3}$ a different way or continue the series farther if they choose.

Before gathering up our results, it will be as well to take another example, although one might almost as well be at Algiers again under old Exmouth, as have one's head knocked about in this way with all these figures.

Let us then take $a = 3{,}8$ that being my estimate of the frustrum of a pyramid equal to the 50lb. fish-bellied rail; and taking $x = 16{,}5$. Mr. Barlow having computed the 3 feet rails as 2 feet 9 inch ones, there being 3 inches deducted for the part buried in the chair, in order to get the clear bearing length, we then get $b = {,}0727$, whence we have nearly

$$x^3 = 4492{,}125$$
$$x^4 = 74120{,}0625$$
$$x^5 = 1222981{,}03125$$
$$x^6 = 20179187$$
$$x^7 = 332956586$$
$$x^8 = 5493783669$$
$$a^3 = 54{,}872$$
$$a^4 = 208{,}5136$$
$$a^5 = 792{,}35$$
$$a^6 = 3010{,}936$$
$$a^7 = 11441{,}5568$$
$$a^8 = 43447{,}89$$
$$b^2 = {,}00528529$$
$$b^3 = {,}00038424$$
$$b^4 = {,}000027934$$
$$b^5 = {,}0000020308$$

From these values we get

$$\frac{4492{,}125}{164{,}667} - \frac{16165{,}585}{834} + \frac{38781}{3961{,}76} -$$
$$\frac{77529}{18065{,}6} + \frac{136179}{80090{,}89} - \frac{234300}{347823} \text{ \&c.}$$

or $27{,}2885 - 19{,}383 + 9{,}789 - 4{,}2915 + 1{,}703 - {,}6736$ &c.

Whence we have, by collecting the terms,

+	—
27,2885	19,383
9,789	4,2915
1,703	,6736
38,7805	24,3481
24,3481	
14,4324	

We have next to see what depth a parallel rail must have to be equal in weight to a fish-bellied rail of 50lb. to the yard, for it is plain that the depth alone can be altered to bring them to a comparison, they must both have an equal head for the wheels to run on; in either case neither more nor less iron is required than is simply sufficient to fulfil that condition in a shape practically adapted for a rail; and the thickness of the centre rib must also be equal in both, as is perfectly evident, for this thickness is omitted in the differentials of the resistances; and we must consequently lessen the depth, in order to bring them to a state in which we can compare the deflections we shall have with an equal quantity of metal, in the one case shaped as a fish-bellied rail, in the other as a parallel one.

We have seen that, in reducing the fish-belly to the frustrum of a pyramid, the depths 5 inches in the middle and 3,8 inches at the end, approximate as close as possible, consequently a parallel rail must be just half the difference of these less in depth, or must be equal to $\frac{5 - 3,8}{2}$ $= \frac{1,2}{2} = 0,6$ less in depth than 5 inches.

We have then the fish-bellied 50lb. rail equal to 5 inches in the middle, and 3,8 inches at the end when 18 inches in its half length, and considered as the frustrum of a pyramid, or as a trapezoid; and the parallel rail of equal weight as 4,4 inches in depth, the heads and ribs of each being otherwise equal; therefore, substituting 18 for l and 4,4 for d in the equation $\frac{l^3}{3\,d^3}$ omitting $\triangle$ as common to both forms we have $\frac{5832}{255,552}$ $= 22,82$ for the deflection, while with the fish-bellied rail at 3,8 inches depth, as before explained, it is only 18,81, and at 3,7 inches depth only 18,92.

So that it appears that a parallel rail of 5 inches depth throughout will be deflected 15,55.

A mathematical ellipse 5 inches depth at a maximum, and having no depth at the end	=	20,062
A fish-bellied rail 5 inches and 3,8 inches in depth as explained before	=	18,81
The same rail 5 inches and 3,7 depth	=	18,92

If we compare the fish-belly of 50lb. weight and the parallel of 50lb. weight, the former having 5 and 3,8 depths, and the latter 4,4 inches we get

Deflection of fish-belly	=	18,81
Deflection of equivalent parallel	=	22,82

So that with 18 inches for the half length, in order to make the parallel deflect as little as the fish-belly, we must add to it a quantity of iron more than there is in the fish-belly, equal to a thickness of $\frac{3}{10}$ of an inch, for we have

$$\frac{l^3}{3\,d^3} = 18{,}81 \text{ ; whence putting}$$

$$56{,}43\ d^3 = l^3 = 5832$$

$$\text{or } d = \sqrt[3]{\frac{5832}{56{,}43}} = \sqrt[3]{103{,}35} = 4{,}69$$

And lastly, if we compare them with the deduction of 3 inches to get the clear bearing length as has been done by Mr. Barlow, and this is the best comparison, that is to say, with 16,5 for the half length, which is clearly the practical view of the case, we have taking 3,85 and 5 for the depth of the fish-belly l equalling 16,5

$$l^3 = 4492$$

$$3\,\frac{l^3}{d^3} = 17{,}57$$

whereas the fish-belly is only 14,4, and the quantity of iron to be added to the parallel, to make it as stiff as the fish-belly is

nearly $\frac{3}{10}$ as before, for $\frac{l^3}{3\,d^3} = 14{,}4$ or $43{,}2 d^3 =$

$l^3 = 4492$

$$\text{or } d^3 = \frac{4492}{43{,}2} = 104$$

$$\text{whence } d = \sqrt[3]{104} = 4{,}703$$

I have now, I believe, placed the question in such a position that all parties will be enabled to judge for themselves, and I have endeavoured to make the deductions and calculations on both sides of it equally plain to the comprehension of those who are only moderately versed in mathematics. Those who choose to continue either of the series farther, will find several elements carried on to a greater length than I have made use of them.

There are persons, I know, who are yet inclined to refer to statements originally made by the first inventors of the fish-bellied rail, in which they then put forth among its other advantages that it saved iron. In the present stage of railways this is a very insignificant matter; the problem is to construct them upon solid and lasting principles, so that they shall be worked at a reasonable annual cost, and by that means return to their spirited shareholders a fair sum upon their outlay; and when I say a fair sum, I mean it to be taken into consideration that these proprietors have in many instances advanced their capital in defiance of all the outcries which were made against railways almost up the present period, and when people who are perhaps themselves not scientific judges of the question risk thousands upon the credit and talents of the engineer, &c. these persons deserve a little more than three and a half per cent.

In the infancy of the fish-bellied rail, I dare say, that like all other new inventions, it was found necessary to put forth speculative advantages such as the saving of iron and other things, in order by all possible means to bring it into notice, and let its more substantive merits be seen; but we are not now searching the archives of first proposers; in fact while the world lasts, all who hope to benefit by the good of inventors, must submit to the evil of projectors; they must winnow what is put before the public, and separate the wheat from the chaff.

I recollect a few years ago, it was proposed by somebody to weld all the rails together in one length; only fancy such a line as the London and Birmingham railway, with the rails all in one length! 112 miles long!—By Hassler's experiments iron expands the ,0000069844 part of its length for each degree of Fahrenheit, and accordingly supposing the 4 rails each 112 miles long to expand towards one end, this would only amount to the trifling quantity of 170 feet between 20 and 60 Fahrenheit, or summer and winter, a tolerable quantity for the rails to be running in and out, at what would then be rather uniquely called the "Station," and as a change of 10^0 will often take place in a few minutes, in this best of all possible climates of ours (for Greenland bears, wild ducks, or mire snipes) we should have for that change, the end of the 4 rails at the "Station," trotting in and out, coaches, engines, and all, just as they stood upon them, a length of about 43 feet, or in other words, the road itself would occasionally take it into its head to travel faster than Sir Charles

Dance's best double barrelled steam carriage, and my countrymen, the cockneys, if they had made up their minds for a passage to Birmingham on a foggy morning, (that is to say about three hundred mornings in the year) would never be able to find the end of the railway for a minute together. Only think of a comfortable old London couple coming down, well cloaked over their eyes and ears, after waiting three months in vain for a fine day, to venture on an excursion a few miles, with all the little ones nicely bandaged, crossbarred, and barricaded up to their noses, to keep life and soul together, on what a Newfoundland dog might call a lovely spring morning; and, having duly reached the "Station," hat and bonnet boxes included, at last get a glimpse of the far-famed railroad, where they are to see such wonders,—when, lo! a wonder appears, for which they were not charged in their fare. The old lady (of either sex) begins to carefully put forth the best of her two bad feet to mount on the step of the carriage, when out comes a beam from Squire Phœbus, and presto, away flies the railroad, 50 feet or so in an instant, with all that is thereon, capsizing hats, wigs, bonnets, boxes, and all the "loveliest little children that ever was seen," and tumbling one half of them into the middle of next week.

In fact, the commencement of the road at the "Station," would never behave itself with common gravity, and lie still for a minute together; it would be like the index of a multiplying wheel barometer, eternally flying backwards and forwards before the eyes of the astonished natives; it would be a perfect ignis fatuus. George Stephenson, the giant progenitor of railways, would be utterly lost in astonishment; and it would puzzle even Sir David Brewster to give a sketch of the configurations of the carriages under this way of dealing with the optic nerve. The phenakistiscope would be a trifle to it, not to mention that upon some extraordinary warm day, when the thermometer got up to 71° and a half or 72°, which I am informed it does sometimes, the "Station" would not be large enough to hold the end of the road at all, and away the rails would start like Congreve rockets through a parish church or two, and a dozen of the next houses.—

Pretty visitors the four of them would be, those which are 75lb. to the yard especially, bundling through a side wall, when a lady, for instance, was giving a select tea-party, and walking in among her crockery; a handsome specimen of a new fashioned toasting fork, unless indeed one of the four happened to poke the said lady into her own fire and toast her.

But I think I hear the worthy professor calling "Question," "Question," and I ought to recollect that my attention now is on the matter-of-fact affairs of this life, and that no man has any business in the clouds; unless, as our friends say across the water, he can make sure of a flash of greased lightening to slide down upon again. Now then, I will be serious.

I consider it proved then, that where the deflection of a parallel rail is 3, the deflection of a mathematical ellipse cut out of that rail, and of course leaving behind a great portion of the iron, would be 4, or as Mr. Barlow has stated it, they are as 3 to 4; but turning from this impossible case, and looking to what can occur in practice on a railroad, these numbers require to be just reversed;

that is to say, when the deflection of a fish-bellied rail is 3, that of a parallel rail of equal weight is 4, the thickness of the shank and the shape of the head being identical in each.

CHAP. II.

THE LONGITUDINAL EXTENSION OF IRON.

Mr. Barlow next proceeds to determine, by experiment, what a bar will be extended in length under any given pressure, so as to preserve its elastic power; that is to say, on removing the stretching force, that the bar shall contract again to the same length which it was before the stretching force was applied to it.

In the description of the instrument which measured the expansion of the bar, under different weights, it will be seen on referring to Mr. Barlow's work, that its measures are related to correctness in the ratio of the arc to the tangent, it being a circular lever, acted upon by a straight line, and it might be supposed this would lead to errors. I have, however, investigated this point, and find that for the small arcs through which the index moved, this slight inaccuracy will not have any very sensible effect on the results. Those who wish to repeat the experiments, and to measure more minutely, may consult the description of an instrument I have used many years for that purpose, with great satisfaction, (Transactions of the Royal Astronomical Society, vol. 4, page 397, or Mechanic's Magazine, vol. 17, page 330). In a bar of cast iron so hard that a file will not touch it, length 1 foot, breadth $1\frac{1}{3}$ inches, depth $1\frac{1}{2}$ inches, this instrument will shew the deflection by a movement of the index of $\frac{1}{8}$ of an inch where the pressure is simply made with the finger and thumb.

Mr. Barlow then gives the following tables of his results, the stretching force being one of Bramah's hydraulic presses used in Woolwich dock-yard for proving chain cables, &c. "an excellent machine of its kind, capable of bearing a strain of one hundred tons, and is very sensible to a difference of strain of 1—8th of a ton."

"EXPERIMENTS ON THE LONGITUDINAL EXTENSION OF MALLEABLE IRON BARS, UNDER DIFFERENT DEGREES OF DIRECT TENSION.

TABLE I.

Bar No. 1, 1 inch square. February 21st.			Bar No. 2, 1 inch square. February 21st.		
Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.	Weight in Tons.	Index Readings	Parts of the whole Bar extended by each Ton.
2	zero		2	zero	
3	·0625	·0000625	3	·11	·0000733
4	·156	·0000935	4	·15	·0000800
5	·265	·0001090	5	·24	·0000900
6	·375	·0001100	6	·35	·0001100
7	not observed	mean.	7	·44	·0000900
8	·562	·0000935	8	·52	·0000800
9	not observed	mean.	9	·62	·0001000
10	·750	·0000940	10	·70	·0000800
11	·875	·0001250	11	·81	·0001100
			12	1·13	Elasticity injured

Bar No. 3, 1 inch diameter. February 23d.			Bar No. 4, 1 inch diameter. February 23d.		
Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.	Weight in Tons.	Index Readings.	Parts of the whole Bar extended by each Ton.
1	zero		1	zero	
2	·16	·0001600	2	·15	·0001500
3	·31	·0001500	3	·28	·0001300
4	·44	·0001300	4	·42	·0001400
5	·56	·0001200	5	·56	·0001400
6	·67	·0001100	6	·69	·0001300
7	·79	·0001200	7	·79	·0001000
8	·91	·0001200	8	·97	·0000800
9	·103	·0001200	9	·116	Elasticity destroyed

Mean extension per ton, per square inch.

Bar No. 1.	·0000982
No. 2.	·0000903
No. 3.	·0001010
No. 4.	·0000976
Mean of the four - - -	·0000967

TABLE II.

Bar No. 5, 2 inches square. February 28th.			Bar No. 6, 2 inches square. February 28th.			Bar No. 7, 2 inches square. February 29th.		
Weight in Tons.	Index Readings	Parts of the whole bar extended by each 4 Tons	Weight in Tons.	Index Readings.	Parts of the whole bar extended by each 4 Tons.	Weight in Tons.	Index Readings.	Parts of the whole bar extended by each 4 Tons.
4	zero		4	zero		4	zero	
6	·100		6	·090		6	·065	
8	·180	·000180	8	·150	·000150	8	·125	·000125
10	·240	·000140	10	·210	·000120	10	·175	·000110
12	·290	·000110	12	·250	·000100	12	.230	·000050
14	·350	·000110	14	·290	·000080	14	·280	·000050
16	·400	·000110	16	·335	·000085	16	·335	·000050
18	·450	·000110	18	·375	·000080	18	·385	·000105
20	·500	·000100	20	·410	·000075	20	·435	·000100
22	·550	·000100	22	·445	·000070	22	·480	·000095
24	·600	·000100	24	·485	·000075	24	·530	·000095
26	·650	·000100	26	·525	·000080	26	·575	·000095
28	·695	·000095	28	·565	·000080	28	·625	·000095
30	·740	·000090	30	·620	·000095	30	·670	·000095
32	·790	·000095	32	·660	·000095	32	·715	·000090
34	·825	·000085	34	·730	·000110	34	·755	·000085
36	·860	·000075	36		{Full elasticity.}	36	·805	·000090
38	·920	·000095	38			38	·850	·000095
40	1·05	·000145	40			40	·900	·000095
		{Elasticity exceeded}						{Elasticity perfect}

Mean extension per ton, per square inch, No. 5. ·0001082
No. 6. ·0000957
No. 7. ·0000841

Mean - - - - - - - - - - - - ·0000946
Mean of preceding Table - - - ·0000967"

"Collecting the results of these seven experiments and reducing them all to square inches, we find that the strain which was just sufficient to balance the elasticity of the iron, was in—

Bar, No. 1.	(re-manufactured iron)	10 tons.
2.	ditto	11 tons.
3.	New Bolt	11 tons.
4.	ditto	10 tons.
5.	(re-manufactured)	9·5 tons.
6.	ditto, from old furnace bars	8·25 tons.
7.	New bar, by Messrs. Gordon	10 tons.

We may consider, therefore, that the elastic power of good iron is equal to about ten tons per inch, and that this force varies from ten to eight tons in indifferent and bad iron. It appears, also, (considering ·000096 as representing in round numbers $\frac{1}{10000}$th) that a bar of iron is extended one ten-thousandth part of its length by every ton of direct strain per square inch of its section; and consequently, that its elasticity will be fully excited when stretched to the amount of one-thousandth part of its length."

The very nature of these experiments, depending so entirely on the quality of the iron experimented on in every case, they must necessarily be very numerous, in order to determine the range of tension; for instance (Mechanic's Magazine, vol. 4, page 444) the weight was found to be 16 tons, where the mode of trying it was also by the hydraulic press.

In the Tables before us, making the proper deductions for the weight where the index was set to zero, and also reducing these bars to one size in breadth and thickness, the first number in column 3, is the first in No. 2, divided by 1000, namely by 10, because the measuring index increased the quantity of expansion ten times, and by 100, because the bar was 100 inches long. The numbers, after the first, are the successive first differences of the numbers in column 2, in each case divided as before by 1000.

The mean of the last columns is taken thus; the whole is added up, and, for instance in bar No. 2, the sum is ,0008133, and this sum is divided by 9 for the weight in tons, the register or index, ranging from zero at 2 tons to ,00011 at 11 tons, and so for the rest.

I differ a little in this process from Mr. Barlow. In my opinion the mean would be best taken by dividing the maximum result obtained, by the weight, (from where the index was set at zero,) producing that result; thus, in bar No. 1, subtracting the weight when the index was set at zero from the tabular weight, we have 11 — 2 = 9 tons, and as this weight brings the index to ,875, we divide by 9 and afterwards by 1000 as before explained, and when this is done the result is as follows.

Bar No. 1. .... 9000),875(	=	,0000972
„ No. 2. .... 9000) ,81(	=	,0000810
„ No. 3. .... 8000)1,03(	=	,0001287
„ No. 4. .... 7000) ,97(	=	,0001386
		4),0004455
		,0001114

Again for Table 2, here the numbers must be divided by 4, in addition to the weight and 1000, in order to get the rate per square inch, to compare with Table 1.

No. 5	$\dfrac{,920}{34 \times 4 \times 1000}$	=	,00010824
„ 6	$\dfrac{,730}{30 \times 4 \times 1000}$	=	,00009732
„ 7	$\dfrac{,9}{36 \times 4 \times 1000}$	=	,00010000
			3),00030556
			,00010185

Hence the above means give

Table	1	=	,00011140
„	2	=	,00010185
			2),00021325
			,00010662

As far as determining the practical elasticities of the iron bars, these corrections are of no moment at all, but it will be seen hereafter what an effect minute quantities have on other determinations, and that in fixing laws derived partly through mathematical investigations, we cannot consider even these minute differences as matters not to be be taken into account.

I must confess, I was rather surprised when I saw in page 27, of Mr. Barlow's first report, that a bar furnished by Messrs. Gordons bore one ton less than some which were got elsewhere. I have seen Messrs. Gordons' iron subjected to much more severe shocks and weights than will be likely to take place on a railroad; namely, to the kicking of a long two and thirty pounder, heated by being fired twenty or thirty times with a full charge, which it stood admirably well; now any person at all acquainted with the mad bull diversions of these sort of implements, when hot with long and rapid firing, would be quite resigned to the efficacy of such a test; I should think it was as strong as any thing in the world, always excepting the ladies and gunpowder.

I do not, therefore, know any place where I should sooner go for a piece of good iron than to Messrs. Gordons; and accordingly we find, when the foregoing alterations are made, that the elasticities of the several bars are as follows, in which that of Messrs. Gordons is equal to any of the others, viz.

		TONS.
Bar No. 1	=	9
" " 2	=	9
" " 3	=	8
" " 4	=	7
" " 5	=	8½
" " 6	=	7½
" " 7	=	9

so that from these experiments I should prefer to say that the longitudinal elasticity of good and tolerable iron is from 6 to 9 tons, and that of indifferent and bad, a carte blanche.

In round numbers we may still say that a bar of iron is extended one ten thousandth part of its length for every ton of direct strain per square inch of its section.

CHAP. III.

ON THE POSITION OF THE NEUTRAL AXIS, AND THE DEFLECTION OF IRON BARS SUPPORTED AT THE ENDS AND LOADED IN THE MIDDLE.

When a bar of any kind is supported at the ends and loaded in the middle with a weight so as to bend it into the form of a curve, the upper fibres of the material are compressed together, and the lower fibres extended, and at some longitudinal line between the upper and under surfaces, the fibres will undergo neither compression nor extension, but may be considered in a state of rest; this line is called the neutral axis, and if we know the exact position of this for any material, it forms an essential element in theoretically determining the conditions under which that material will be deflected when any given weight is laid upon it, which it is able to bear with reference to the purpose for which it is intended.

Mr. Barlow has, therefore, made several experiments, in order to determine its position for iron bars, by ascertaining what weight gave them a certain measured deflection, to an extent which left them perfectly capable of restoring themselves to their original position; and the results of these, and the preceding experiments, are applied as elements in the following theoretical investigation, page 32. In copying which, I have, as before, corrected two or three trifling typographical errors, and added a few steps which were left out in the mathematical manipulation; (or cranipulation if the reader likes it better) for the benefit of those who may not have such a well furnished place to put their hats on as Mr. Barlow has. I hope my explanations will not be like Coke upon Lyttleton, where we often find the same puzzling gentleman with just the hind part of his wig turned before.

"EXPERIMENTS TO DETERMINE THE COMPARATIVE RESISTANCE OF MALLEABLE IRON TO EXTENSION AND COMPRESSION, AND THE POSITION OF THE NEUTRAL AXIS IN BARS SUBMITTED TO A TRANSVERSE STRAIN."

"Let A B, (see Fig. 4, Plate 2), represent an iron or any other bar supported at A and B, and loaded in the middle by a weight W, which deflects it; extending the fibres between *n* and *c d*, and compressing those between *n* and *c′ d′*. Now, supposing the system in equilibrio, ½ W acting at the extremity of the ½ length, or ¼ *l* W, is equivalent to the sum of all the resistances to extension in *n c d*, and to all those of compression in *n c′ d′*, each fibre acting on a lever equal to its distance from the neutral axis *n*. Consequently, as the quantity of extension of any fibre is as its distance from the neutral axis, and the lever by which it acts, being also as that distance, the actual resistance of a fibre at the distance, *x*, is as $\frac{x^2 t}{d'}$,

t being the tension of the lower fibre, and d' its depth below the neutral axis; and the sum of all these resistances will be $\int \frac{t\,x^2 dx}{d'} = \frac{1}{3} d'^2 t$, (when $x = d'$) or for the whole depth. In the same way, c being taken to denote the compression of the upper fibre, corresponding to the tension t, the sum of all the compressions will be,

$$\frac{1}{3} d''^2 c,$$

d'' denoting the depth of compression; hence the whole sum is,

$$\frac{1}{3} d''^2 c + \frac{1}{3} d'^2 t = \frac{1}{4} \mathrm{W}\, l;$$

but $d'' c = d' t$,* the quantity of resistance being

* "To prevent misapprehension, it may be proper to observe that c here, is not intended to represent the force requisite to compress a fibre the same quantity that the force t extends it; but simply, the force of the compression at c, corresponding to the tension t on the lower fibre. The equation, therefore $d'' c = d' t$ is equivalent to saying that the sum of all the forces in $n\,c'\,d'$ is equal to all the forces in $n\,c\,d$; or that $a\,g = n\,a'\,g'$; a, a', denoting the areas, and g, g' the distances of the centres of gravity from n, and taking $n\,t$ to denote the force which will compress a fibre to the same extent as the force t will extend it."

equal to that of extension; this, therefore, becomes

$$\frac{1}{3} d'' d' t + \frac{1}{3} d'^2 t = \frac{1}{4} l \text{ W}, \text{ or}$$

$$\frac{1}{3} (d'' + d') d' t = \frac{1}{4} l \text{ W}, \text{ or}$$

$$\frac{1}{3} d. d' t = \frac{1}{4} l \text{ W}.;$$

d being the whole depth, and d' the depth of tension; whence

$$d' = \frac{3 l \text{ W}}{4 d a b} = \text{depth of tension, and}$$

$d - d'$ the depth of compression,

consequently, $\frac{d'}{d - d'}$ the ratio, in which the neutral axis divides the sectional area in rectangular bars."

The reader will take notice that n in Mr. Barlow's note is a symbol, and has no reference to the n in the figure above.

Where t is taken as the tension of the lower fibre, what is meant is the t is that longitudinal tension which the material will bear, so as to retain the power of restoring itself to its original position, which tension has been determined in the preceding experiments.

The resistance to tension at any point x as for instance $\frac{x^2 t}{d'}$ multiplied by the rate of increase, or differential dx, equals the sum of all the resistances at that point.

In the same way the sum of all the compressions will be $\frac{1}{3} d''^2 c$ when $x = d''$

$\frac{1}{3} d''^2 c$ is reduced to $\frac{1}{3} d'' d' t$ when $d'' c = d' t$

in the same way, that, for instance, whenever any quantity $ab = cd$ then $a^2 b = a c d$

The reduction to $d' = \frac{3 l W}{4 d a t}$ is not very clear at first; a having been used in Mr. Barlow's note, to denote the areas of the parts compressed or extended in fig. 4, plate 2. I take it that the equation is $\frac{3 l W}{4 d t}$ for an unit of breadth, and that a is introduced in $\frac{3 l W}{4 d a t}$ to represent the breadth, and consequently generalize the formula in that respect.

It will be seen that $d = d'' + d'$ & $d'' = d - d'$ &c.

The following are Mr. Barlow's experiments, the first six of which were made on different parts of the bars, Nos. 5, 6, and 7, used in the experiments on longitudinal extension, the first three were measured by a scale, the rest by a micrometer screw, which, with the rest of the apparatus, Mr. Barlow fully describes.

"EXPERIMENTS MADE TO ASCERTAIN THE DEFLECTIONS DUE TO DIFFERENT TRANSVERSE STRAINS, AND THE WEIGHT WHICH FIRST PRODUCES A STRAIN EQUAL TO THE ELASTIC POWER, AND THENCE THE POSITION OF THE NEUTRAL AXIS."

"TABLE III."

PART 1. BAR NO. 5. Bearing 33 Inches. 2 Inches Square.

Weight in Tons.	Readings by Scale.	Deflections for each Half Ton.
No Weight.	1·96	
·875	1·92	·023
1·00	1·90	
1·50	1·90	·016
2·00	1·88	·020
2·50	1·86	·020
Weight removed.	returned to 1·96	
3·00	1 80	Elasticity injured.
Weight removed.	1·88	

PART 2. BAR NO. 5. Bearing 33 Inches. 2 Inches Square.

Weight in Tons.	Readings by Scale.	Deflections for each Half Ton.
No Weight.	1·95	
·750	1·92	·020
1·00	1·91	·020
1·50	1·89	·020
2·00	1·86	·030
2·50	1·84	·020
Weight removed.	returned to 1·95	
3·00	1·67	Elasticity injured.
Weight removed.	1·81	

PART 1. BAR NO. 6.

Weight in Tons.	Readings by Scale.	Deflections for each Half Ton.
No Weight.		
·50	1·56 ?	
1·0	1·50	
1·5	1·48	·020
2·0	1·45	·030
2·5	1·24	·210 } Elas. injd.
3·0		

PART 2. BAR NO. 6.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight.	·025	
·50	·043	·018
1·0	·068	·025
1·5	·091	·023
2·0	·128	·037 injd.
2·25	·178	·100
2·50	·313	·185

"TABLE III."

Part 1. Bar No. 7.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight.	·031	
·50	·053	·022
1·0	·077	·024
1·5	·096	·019
2·0	·126	·030
2·5	·147	·021
3·0	·211	·064injd.

Part 2. Bar No. 7.

Weight in Tons.	Readings in Micro. Screw.	Deflections for each Half Ton.
No Weight.	·025	
·50	·056	·031
1·0	·077	·021
1·5	·098	·021
2·0	·109	·011
2·5	·137	·028injd.
3·0	·180	

Part 3. Bar No. 7.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight.	·075 ?	
·50	·130	
1·0	·153	·023
1·5		·023
2·0	·199	·023
2·5	·220	·021
3·0	2·90	·070injd.

Part 2. Bar No. 7.
Reversed.

Weight in Tons.	Readings by Micro. Screw.	Deflections for each Half Ton.
No Weight.	·025	
·50	·054	·029
1·0	·092	·038
1·5	·153	·061
2·0	·235	·082

Elasticity clearly injured by the former experiment.

"It appears from these experiments, that both parts of the Bar No. 5, (whose direct elasticity was 9·5 tons,) had their restoring power just preserved with a transverse strain of two and a half tons, on a bearing length of thirty-three inches. Hence in the formula:—

$$d' = \frac{3\,l\,w}{4\,d\,a\,t}$$

we have l=33, w=2½, d=2, a=2, t=9·5, and d'=1·62 inches, depth of tension.

Consequently d''=·38 inches, depth of compression, and the ratio of the area of compression to tension . . . . 1 : 4·3

In the first part of Bar No. 6, w is not quite 2 tons, and t=8·5 tons; and hence the ratio . . . . 1 : 2·7

In the second part of the same bar, ditto 1 : 2·7

In the first, second, and third parts of Bar No. 7 w=2½ tons, and t=10 tons . 1 : 3·4

"As far as these experiments are authority, therefore, the neutral axis divides the sectional area of a rectangular bar in about the ratio of one to three and a half."

"In the following experiments, the iron was all supplied by Messrs. Gordon, and was of the same quality as the Bar No. 7,—its elasticity may therefore be taken as ten tons, but it was not determined by testing, as in the preceding experiments."

"TABLE IV."

BAR NO. 8.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections.	Deflections, each ½ Ton.	REMARKS.
inches.	inches.	inches.	tons.			
33	1.9	2	·123	·034		
			·250	·046		
			·500	·060		
			1·00	missed.	·019	
			1·50	·098	·019	
			2·00	·120	·022	Mean ·024
			2·25	·134	·028	w = 2.25. Neutral axis 1 : 3·4
			2·50	·151	·034	
			2·75	·176	·044	Elasticity injured with 2·50 T.

BAR NO. 9.

33	1.9	2	·250	·047		
			·500	·055	·016	
			1·00	·077	·022	
			1·50	·097	·020	Mean ·021
			2·00	·123	·026	
			2·25	·132	·018	w = 2·25. Neutral axis 1 : 3·4
			2·50	·145	·026	
			2·75	·164	·038	Elasticity injured with 2·50
			3·00	·210	·092	Ditto destroyed with 3·00

BAR NO. 10.

33	1·9	2	·500	·056		
			1·00	·076	·020	
			1·50	·095	·019	Mean ·024
			2·00	·124	·029	
			2·50	·151	·027	w = 2·5. Neutral axis 1 : 4.2
			3·00			

"DEDUCTIONS FROM THE THREE LAST EXPERIMENTS, CONFIRMED BY DIRECT OBSERVATION OF THE PLACE OF THE NEUTRAL AXIS."

"These experiments, like the former, imply, according to the formula, that the neutral axis lies at about one-fourth or one-fifth of the depth of the bar from its upper surface; but a method was adopted in these to discover, if possible, its position mechanically. With this view, a key-way, or groove, was cut in the side of the bar, one inch broad, and one-tenth of an inch deep,—thus reducing the breadth to 1·9 inches. To this key-way, or groove, was fitted a steel key, which might be moved easily; and when the strain was on, the key was introduced, which it was expected would be stopped at the point where the compression commenced, and this was accordingly found to be the case in two out of the three bars, but not in the third, the fitting not being sufficiently accurate. The other two, however, showed obviously a contraction of the groove, at about half an inch from the top, agreeing with the preceding computations. To make the results more certain, three other bars, exactly like the former, had deeper grooves cut, and the key more exactly fitted, and with these the results were as definite as could be desired. The key, as above stated, moved smoothly and easily before the experiment; but when two tons strain were on, and the key applied, it was stopped, and stuck at a definite point. The strain being then relieved, the key fell out by its own weight; the strain was again put on, the key sticking as before; the strain being relieved, the key again fell; and so on, as often as repeated. Precisely the same happened with all the three bars. One of them was then reversed, so that the part which had been compressed was now extended, and exactly the same result followed; showing, most satisfactorily, that our former computed situation of the neutral axis was very approximate. The measurements obtained in these experiments being tension 1·6, compression ·4, giving exactly the ratio of 1 to 4 in rectangular bars. These results seem the most positive of any hitherto obtained; still, there can be little doubt this ratio varies in iron of different qualities; but looking to the preceding experiments, it is probably always between 1 to 3, and 1 to 5."

In the ratio 1 : 4,3 the value of d'' is given = ,38. In $\frac{d'}{d-d'} = \frac{d'}{d''} = 2,7$ we have $d' = 1,456$ and $d'' = ,544$ and when the same equation gives 3.4 $d' = 1,547$ and $d'' = ,453$.

It will now be seen what a very considerable alteration these ratios will undergo when for t as Mr. Barlow has given it, is substituted t as I have stated it to be; and whether this is the correct value of it or not, every one can judge for himself, by examining my remarks on the first set of experiments to ascertain the longitudinal extension of iron bars.

	W tons.	d'	d''	$\frac{d'}{d''}$
Bar No. 5 1st part	8½	1,82	,18	1 : 10,1
" " " 2nd "	8½	1,82	,18	1 : 10,1
" " 6 1st "	7½	1,65	,35	1 : 4,6
" " " 2nd "	7½	1,65	,35	1 : 4,6
" " 7 1st "	9	1,7187	,2813	1 : 6,1
" " " 2nd "	9	1,7187	,2813	1 : 6,1
" " " 3rd "	9	1,7187	,2813	1 : 6,1
				7) 47,7
				1 : 6,81

This is an amazing difference from 1 : 4; and what is more, the experiments vary from 1 : 4½ to 1 : 10. In those which give 1 : 10, being the bar No. 5 in the previous experiments on longitudinal extension, we have clearly no right to assume, without some other proof than Mr. Barlow gives, that the bar will sustain a greater extension than that which is produced by a strain of 8½ tons per square inch; for at 40 tons, equal to 10 tons per square inch, the table states the elasticity to have been exceeded, and we have 4 tons to subtract from this, that being the strain on the bar by the table, when the index was set at zero. Thus, therefore, 36 tons divided by 4, would only give 9 tons per square inch, and we see, by the numbers in the column headed "parts of the whole bar extended by each 4 tons," that the numbers were increasing very fast, namely from ,000095 to ,000145 when the last two tons were applied; so that the permanent elasticity was evidently going before the point where it is set down as exceeded. Hence we can in fairness only take the bar to have preserved its full elasticity, at the point in the table opposite 38 tons, and deducting 4 for the zero we have the remainder 34 divided by 4=8½ tons per square inch, and by looking at bars Nos. 6 and 7, in the same table, we shall also find that on the face of what is there recorded, we cannot do otherwise than assign *t* as I have done.

In fact, the "feature upon which the question hinges," as C. A. C. Castlereagh used to say, is just this: what weight did the bar exactly bear when its elasticity was fully preserved? and to determine this, we ought to have a column given in the table, in which it should have been shewn what the index went back to, when the weight was removed; and this should have been done after each of the successive 2 ton strains were applied, or at any rate after those towards the end where the question might become at all doubtful; but as this has not been done, we can only take the experiments as they appear in the register, and then I say that for bar No. 5, we can in fairness assume nothing else than that the utmost tension which the bar bore, retaining the full power of contracting itself to its original length, when the tensile force was removed, was 8½ tons per square inch; and putting that weight in Mr. Barlow's formula, it gives the ratio of the neutral axis, or the ratio of the upper and lower areas into which it divides the bar, as 1 to 10.

If we now pursue the same method with the bars Nos. 8, 9, and 10, we shall find nearly as discordant results; for instance, in bar No. 8, taking t at Mr. Barlow's value, we have

$$d' = \frac{3\,l\,W}{4\,d\,a\,t} = \frac{3.\,33.\,2\frac{1}{4}}{4.\,2.\,1{,}9.\,10} = \frac{222{,}75}{152} = 1{,}465$$

$$\text{hence } d'' = {,}535 \text{ and } \frac{d'}{d''} = 1 : 2{,}7$$

Bar No. 9, is the same $= 1 : 2{,}7$

Bar No. 10, is $d' = \dfrac{3.\,33.\,2{,}5}{4.\,2.\,1{,}9.\,10} = \dfrac{247{,}5}{152} = 1{,}628$

$$\text{hence } d'' = {,}372 \text{ and } \frac{d'}{d''} = 1 : 4{,}38$$

But using my value for t, which for bar No. 7 is only 9 tons, and this is the elasticity Mr. Barlow directs us to take for bars Nos. 8, 9, and 10, we get

$$\text{No. 8} \quad d = \frac{3.\,33.\,2\frac{1}{4}}{4.\,2.\,1{,}9.\,9} = \frac{222{,}75}{136{,}8} = 1{,}628$$

$$\text{hence } d'' = {,}372 \text{ and } \frac{d'}{d''} = 1 : 4{,}38$$

No. 9 is the same $= 1 : 4{,}38$

$$\text{No. 10} \quad d' = \frac{3.\ 33.\ 2,5}{4.\ 2.\ 1,9.\ 9} = \frac{247,5}{136,8} = 1,8107$$

hence $d'' = ,1893$ & $\dfrac{d'}{d''} = 1 : 9,56$

and if as a further trial we make a the unknown quantity in the formula $\dfrac{3\,l\,W}{4\,d\,a\,t}$ we have, putting

$d' = \dfrac{33.\ 3.\ 2,25}{2.\ 4.\ \ 10a}$ and taking the assigned value of

$\dfrac{d'}{d''-d'}$ from the similar bar

$$\frac{d'}{2-d'} = 3,4$$

$$\text{or } d' = 6,8 - 3,4\,d' = \frac{6,8}{4,4} = 1,5454$$

hence $\dfrac{33\,.\,3\,.\,2,25}{2\,.\,4\,.\,10a} = 1.5454$ or $\dfrac{222,75}{80a} = 1,5454$

and a=1,8 instead of 1,9 which it did in reality by the table.

Mr. Barlow says page 43, that as the iron in these experiments was all of the same depth, "it was thought more satisfactory to make a few other experiments on bars of different breadths and depths. These are given in the following page. They were performed precisely like the last, and therefore require no particular description.

"EXPERIMENTS ON THE DEFLECTION OF MALLEABLE IRON BARS, UNDER DIFFERENT STRAINS.

Bar No. 11.

Distance of bearings.	Breadth.	Depth.	Weights.	Deflections.	Deflections, each ½ Ton.	REMARKS.
inches.	inches.	inches.	tons.			
33	1.5	3	·125	·043		
			·500	·059		
			1·00	·074	·015	Mean ·0103
			1·50	·083	·009	
			2·00	·095	·012	
			2·50	·101	·006	
			3·00	·109	·008	
			3·50	·120	·011	
			4·00	·131	·011	$w = 4\frac{1}{2}$. Neutral axis 1 : 4·9
			4·50	.148	·017	Elasticity preserved at 4½ Tons.

Bar No. 12.

33	1.5	3	0	0		
			·50	·017		
			1·00	·037		
			1·50	·052	·015	Mean ·0108
			2·00	·061	·009	
			2·50	·064	·003	
			3·00	·078	·014	
			3·50	·089	·011	
			4·00	·102	·013	$w = 4\frac{1}{2}$. Neutral axis 1 : 4.9
			4·50	·124	·022	Elasticity injured.

Bar No. 13.

33	1·5	2·5	0	·006		
			·50	·003	·024	Mean ·0173
			1·00	·050	·020	
			1·50	·060	·010	
			2·00	·074	·014	
			2·50	·093	·019	$w = 3$. Neutral axis 1 : 4·9.
			3·00	·110	·017	Elasticity preserved, 3 Tons.
			3·50	·149		
			7·5	Bent 8	inches,	

Examining these in the same way which we did the last, we find, using Mr. Barlow's value of t, that in Bar No. 11, $d' = \frac{3 \,.\, 33 \,.\, 4{,}25}{4 \,.\, 3 \,.\, 1{,}5 \,.\, 10} = \frac{420{,}75}{180} = 2{,}3375$ and consequently $d'' = {,}6625$

and $\frac{d'}{d''} = \frac{2{,}3375}{{,}6625} = 1 : 3{,}53$

Bar No. 12 = the same = 1 : 3,535

Bar 13 $d' = \frac{3 \,.\, 33 \,.\, 3}{4 \,.\, 1{,}5 \,.\, 2{,}5 \,.\, 10} = \frac{297}{150} = 1{,}98$

and consequently $d'' = {,}52$ and $\frac{d'}{d''} = \frac{1{,}98}{{,}52} = 1 : 3{,}8$

But as we have seen that t is only 9 tons, we have

Bar No. 11 $d' = \frac{3 \,.\, 33 \,.\, 4{,}25}{4 \,.\, 3 \,.\, 1{,}5 \,.\, 9} = \frac{420{,}75}{162} = 2{,}598$

and consequently $d'' = {,}402$ & $\frac{d'}{d''} = \frac{2{,}598}{{,}402} = 1 : 6{,}46$

Bar No. 12, the same = 1 : 6,46

Bar No. 13 $d' = \frac{3.\, 33.\, 3}{4.\, 1{,}5.\, 2{,}5.\, 9} = \frac{297}{135} = 2{,}2$

and consequently $d'' = {,}3$ and $\frac{d'}{d''} = \frac{2{,}2}{{,}3} = 1 : 7{,}33$

So that all the results are very discordant, ranging, as we before observed, from 1 : 4½ to 1 : 10 : a difference much too great to allow us to place any dependance on them. In fact, when we see what great variations in the results arise from small differences in the elements of the computation, we are entitled to say that this method of determining the position of the neutral axis, as far as these experiments go, leaves us exactly where we were before. For instance, when a difference of one ton in the longitudinal strain which a bar will bear, and yet be fully capable of restoring itself to its original length; will give the position of the neutral axis from 1 : 4,3 to 1 : 10,1, we must, I think, abandon the method altogether; for it must be recollected that it is a very difficult point to decide, even by experiment, where a bar of iron will exactly be able to restore itself to its original length, after having been stretched. And in this enquiry is also embodied another, viz. are we to demand this condition rigidly; or, to speak in round numbers, for practice? and, even then, the round numbers which will do for practice, when the iron is to be adapted for one use, will not be admissible when it is to be put to another. But if we are to rigidly demand the condition, then, from all the experiments which I have made for some years, we shall never find it occur that stretched iron will fully and completely return to the same point at which it was, before the stretching force was applied to it.

Mr. Barlow speaks doubtingly of these experiments himself, it will be observed; his expression is, page 39, "as far as these experiments are authority," to which I think we are now fully entitled to subjoin that they are of none whatever.

The mechanical mode of determining the question, which Mr. Barlow afterwards tried, is an exceedingly neat and ingenious method; and although he states one of the trials not to have been satisfactory, the rest appear to have been more so; from what I saw of them myself, I am inclined to think this, or some similar mechanical method, much the best for determining the question, and it ought to be set at rest as soon as possible, from its great utility.

We may also view the question this way; let us suppose that a railway bar, 33 inches long, is calculated to bear 9 tons without being permanently deflected, and let it be assumed that the neutral axis is 1 : 4, as Mr. Barlow thinks it is; then, giving the usual depth, 5 inches, and taking $t = 9$ tons, whatever the value of a is, that is to say, whatever mean breadth the bar may have assigned to it, if we call that breadth m, we have the formula $\frac{3\,l\,W}{4\,d\,a\,t}$ reduced to $\frac{3\,.\,33\,.\,9}{4\,.\,5\,.\,9\,.m}$ for the value of d', and as $\frac{d'}{d''} = 4$ or $d' = 4\ d'' = 4$ $(5 - d') = 20 - 4d'$ we have $d' + 4\ d' = 20$ or $d' = \frac{20}{5} = 4$ and $d'' = 1$; whence we get $\frac{3\,.\,33\,.\,9\,.}{4\,.\,5\,.\,9\,.m} = d' = 4$ or $\frac{891}{180m} = 4$ or $720m =$ 891 or $m = \frac{891}{720} = 1{,}24$ for the mean breadth.

Now let us suppose that the length of the bar is altered, as has lately been done on 75 miles of the London and Birmingham railroad, from 33 to 60 inches, (not 57 inches, as some people have said, it is not very easy to swear 3 inches off an iron rail, weighing 75 lbs. to the yard), we then have, taking in the alteration of the weight from 50 to

75 lbs. per yard, 1,74 for the corresponding average breadth, and we get, (putting *n* for the load the bar will bear)

$$\frac{3 \cdot 60 \cdot n}{4 \cdot 5 \cdot 9 \cdot 1{,}74} \text{ or } \frac{180\, n}{313} = 4 \text{ whence}$$

$$180n = 1252 \text{ or } n = \frac{1252}{180} = 7$$

or the load which the bar will sustain = 7 tons, while that which it ought to sustain is 9 tons. Not a very pleasant prospect for the shareholders. At this rate it will be better to lay down, at least, two lines of rails each way, instead of one, so that those persons who venture on the first journey may have a chance of getting back again, after they have put the road they first travelled by on the doctor's list.

Mr. Barlow in the following note, page 61, gives a third method of determining the neutral axis.

" We have the means of computing the position of the neutral line by the data obtained from the experiments, p. 42, which shew, that in rectangular bars the area is divided in the ratio of 1 to 4, or the area into the distance of the centre of gravity of the two parts as 1 to 4^2. But in enquiries of this kind, the less we have to depend on theory the better. I have, therefore, deduced the above position from the experiments on actual railway bars, p. 70, by considering the distance *nh* as unknown, and equating the formula in this shape with the mean elastic strength, which is found to be $8\frac{1}{4}$ tons. The equation is, therefore,

$$\tfrac{1}{3}\left\{5(5-x)9+\frac{11.(1-x)^2}{5-x}\right\}=\frac{8\frac{1}{4}\times 33}{4}$$

Whence we find $x = \cdot 47$, which may be considered as $\cdot 5$, without sensible error."

"EXPERIMENTS ON THE RESISTANCE AND DEFLECTION OF RAILWAY BARS.

Mr. Stephenson's Fish-bellied Rail, 50lbs. per Yard.

Bar No. 1.			Bar No. 2.		
Weights.	Deflections by Index	Deflections for each Ton.	Weights.	Deflections by Index.	Deflections for each Ton.
1	·035		1	·014	
2	·045	.010	2	·022	·008
3	·055	·010	3	·030	·008
4	·065	·010	4	·042	·012
5	·071	·006	5	·050	·008
6	·076	·005	6	·062	·012
7	·087	·011	7	·075	·013
7½	·095	·016	8	·085	·010
			9*	·101	·016
			10	*Elasticity injured.	
			11	.300	

Bar No 3.			Bar No. 4.		
Weights.	Deflections by Index	Deflections for each Ton.	Weights.	Deflections by Index.	Deflections for each Ton.
1	·018		1	·045	
2	·025	·007	2	·056	·011
3	·038	·013	3	·065	·009
4	·054	·016	4	·075	·010
5	·062	·008	5	·084	·009
6	·069	·007	6	·095	·011
7	·080	·011	7	·105	·010
8	·094	·014	8	·110	·005
8½	·100	·012	9	·116	·006
9*	·112	·018	10	·125	·009
9½	·118	·018	11	·165	
10	·126	·014			
11	·160	·034			
17	destroyed.				

Mean Deflections per Ton, Bar No. 1.	...	·0097
No. 2.	...	·0101
No. 3.	...	·0110
No. 4.	...	·0090
Mean	...	·0100

"TABLE CONTINUED."

Bar No. 5, Fish-bellied.
Great depth, 5 inch.
Less ditto, $3\frac{3}{4}$
Thickness of Rib, $\frac{9}{10}$.
Head estimated, 2 by 1.

Weight in Tons.	Deflection by Index	Deflection for each Ton.
1	·030	
2	·260	
3	·270	·010
4	·290	·020
5	·300	·010
6	·320	·020
7	·335	·015
8	·410	·060
Mean deflection per Ton to 7 Tons		·015
Ditto with $7\frac{1}{2}$ Tons.		107

Bar No. 6, Fish-bellied.
Great depth, $3\frac{1}{4}$ inch.
Less ditto, $2\frac{1}{4}$
Thickness of Rib, $\frac{7}{10}$.
Head estimated, 2 by $\frac{3}{4}$

Weight in Tons.	Deflection by Index	Deflection for each half Ton
0·5	·120	
1·0	·140	·020
1·5	·170	·030
2·0	·180	·010
2·5	·200	·020
3·0	·230	·030
3·5	·280	·050
4·0	·420	·140
Mean deflection per half Ton to 3 Tons		·022
Ditto with 3 Tons		·026

Bar No. 7, Fish-bellied.
Great depth, 3 inch.
Less ditto, 2
Thickness of Rib, $\frac{6}{10}$.
Head estimated, 2 by $\frac{1}{2}$

Weight in Tons.	Deflection by Index.	Deflection for each half Ton
0·5	·033	
1·0	·060	·027
	Re-adjusted	
1·5	·062	
2·0	·090	·028
2·5	·120	·030
3·0	·155	·035
3·5	·240	
4·0		
Mean deflection per half Ton to 2 Tons....		·030
Ditto, with 2 Tons		·060

"COMPARATIVE STRENGTH OF DIFFERENTLY-FORMED PARALLEL RAILS."

"Let A B C D (pl. 2 fig. 5) represent any rectangular rail with a bottom table; $n\,n$ its neutral axis; c the centre of compression, $c\,n$ being $\frac{2}{3}$ of $h\,n$.—Now, the tension of each fibre being as its distance from the neutral axis, and that of the lower fibre being given equal to t, the tension at any variable distance x will be $\frac{t\,x}{d'}$ (d' being taken to denote the whole depth $n\,s$), and therefore the sum of all the tensions will be,

$$\frac{t}{d'}\int x.\,dx \qquad (1)$$

which, therefore, become known, x being taken within its proper limits, according to the figure of the section.

"But as the effective resistance of each fibre is also as its depth below the line $n\,n$, the sum of all the resistances will be,

$$\frac{t}{d'}\int x^2.\,dx \qquad (2)$$

x being taken here also within its proper limits.

"And then to find the centre of tension, or that point into which, if all the tensions were collected, the whole resistance would be the same as in the actual case, this would be given by the formula;

$$\frac{\int x^2 .\, d x}{\int x .\, d x} \qquad (3)$$

which is precisely the expression for the centre of oscillation of a disc of the same figure.

"We have hence the following general rule for finding the resistance or the weight which any given bar or rail will support at its middle point, within the limits of its elastic power, that is,

Calling the integral of	formula (1) = A	
Ditto ditto	formula (2) = B	
Ditto ditto	formula (3) = D	
And the distance	$c\,n$ = C	

then, referring the sum of all the resistances B to the common centre of compression, we have

$$D \;::\; D+C \;::\; B \;:\; \frac{B\,(D+C)}{D}$$

which is the whole effect.

"For those who understand the integral calculus, this solution is sufficient; but as the article will probably be consulted principally by practical men, it will be more convenient to give a specific solution for a rail, embracing under one general figure all the usual forms, the only variations being in the depth, breadth, and thickness of the parts.

"Let A B C D (Plate 2, fig. 5), represent such a section, of which all the dimensions are given, as also the position of *n n* the neutral axis, the point *c* which is the centre of compression, *c n* being ⅔ds of *n h*, and the point *m*, which is in the centre of *r s*. The breadths *n n* and *m m* are also known. Then the resistance of the whole section referred to the common centre of compression *c*, may be considered to be made up of the three resistances.

"1st. Of the middle rib, continued through the head and foot tables, *v t z w*.

"2nd. Of the head A E F B, minus the breadth of the centre rib.

"3rd. Of the lower web, G C D H, also minus the continuation of the centre rib.

"Now, *t* being taken to represent the tension of iron per square inch, just within its limits of elasticity, we shall have

1. Resistance of ....$v\ t\ z\ w = \frac{1}{3}\ h\ s \ .\ n\ s \ .\ p\ q \ .\ t$

2. Resistance of AEFB$= \frac{1}{3} hx \,.\, nx \,.\, (nn - pq)\frac{nx}{ns}\, t$

"Now, let $nm + \frac{rs^2}{12\, nm} = \delta'$, and $\delta' + cn = \delta''$, then

3. Resistance of GCDH$= nm \,.\, rs \,.\, (mm - pq)\, \frac{\delta''}{d'}\, t$

"These three resistances being computed, let their sum be called s, and the clear bearing l; then $\frac{4\,s}{l} = w$, the load the bar ought to sustain at its middle point, for an indefinite time, without injury to its elasticity."

These formulæ are thus derived. "It has been shown generally, that if d' denote the depth of the lower fibre below $n\,n$, and its tension be made $= t$, and any variable distance $= x$, That $\frac{t}{d'} \int x\,dx$ = sum of all the tensions to a unit of breadth.

That $\frac{t}{d'} \int x^2 dx$ = sum of all the resistance referred to the axis n.

And $\dfrac{\frac{t}{d'} \int x^2 dx}{\frac{t}{d'} \int x\,dx} = \delta'$ distance of centre of tension.

"From which it follows,

that $\frac{t\delta'}{d'}\int x \,.\, dx =$ sum of all the resistances for a unit of breadth, x being taken in its ultimate state.

"Now, in the rib, when $x = d'$, $\delta' = \frac{2}{3}\, d'$ and $\int x \; dx = \frac{1}{2}\, d'^2$, whence the above becomes

$$\frac{1}{3}\, d'^2\, t:$$

but to refer this to the centre of compression c, we have (calling the whole depth d)—

$$\frac{2}{3}\, d' \;:\; \frac{2}{3}\, d \;::\; \frac{1}{3} d'^2\, t \;:\; \frac{1}{3}\, dd' t;$$

and introducing the breadth $p\,q$, it becomes—

$$\frac{1}{3}\;\; h s \,.\, n s \,.\, p q \,.\, t.$$

"In the same way, calling the tension at $x = t'$, and the breadth $(n n - p q)$, we have for the resistance of the head—

$$\frac{1}{3}\, h x \,.\, n x \,.\, (n n - p q)\; t';$$

but the tension at $x = \frac{n\,x}{n\,s}\, t$;

therefore, substituting this for t', we have

$$\frac{1}{3}\, h x \,.\, \overline{n\, x^2}\, \frac{(n\,n - p\,q)}{n\,s}$$

"For the lower web—

$$\frac{\frac{t}{d'}\int x^2\,dx}{\frac{t}{d'}\int x\;dx} = \delta'$$

Calling $n\,r = d''$, and x any variable distance below r, it becomes—

$$\frac{\int (d'' + x)^2\,dx}{\int (d'' + x)\;dx}$$

which, when $x = r\,s$, gives

$$\delta' = n\,m + \frac{r\,s^2}{12\,m\,n}$$

and $\frac{t}{d'}\int (d'' + x)\,dx = \frac{t\delta'}{d'}\,n\,m \,.\, r\,s$

whence the resistance referred to $n\,n$ is, for the breadth $(m\,m - p\,q)$

$$n\,m \,.\, r\,s\,(m\,m - p\,q)\,\frac{t\delta'}{d'}\;;$$

and calling $\delta' + n\,c = \delta''$, it is, when referred to c,

$$n\,m \,.\, r\,s\,(m\,m - p\,q)\,\frac{\delta''\,t}{d'}$$

which is the formula in question.

"Taking the result of the experiments at $w = 8\frac{1}{4}$, and the dimensions of that bar as known quantities, every thing in the above is given except the position of the line $n\,n$. Calling, therefore, $h\,n = x$, and substituting the proper numerical values for the other parts, we have,

$$\frac{1}{3}.5.(5-x).9+\frac{1}{3}(1-x)^2.\frac{1,1}{5-x}.10=\frac{33\;.\;8\frac{1}{4}}{4}$$

This reduces first to—

$$45\,(5-x)^2 + 11\,(1-x)^2 = 204{\cdot}18\,(5-x)$$

then to $x^2 - \dfrac{267{\cdot}82}{56}x = -\dfrac{115{\cdot}1}{56}$

whence the value of $x = ,484$.

"Here t is taken at ten tons, according to our first mean results; but if instead of this we consider it like x as an unknown quantity, the equation is

$$4{\cdot}5\,t\,(5-x)^2 + 1{\cdot}1\,t\,(1-x^2) = 204{\cdot}18\,(5-x.)$$

that is, t and x are dependent quantities, and every change in the value of t introduces a corresponding change in the value of x.

If $t = 10{\cdot}5$, then the equation is,

$$45\,(5-x)^2 + 11\,(1-x)^2 = 194{\cdot}54\,(5-x).$$

Whence $x = {\cdot}736$.

"Again, we may find x quite independently of these considerations, by taking the ratio of the surfaces of tension and compression found in p. 42, viz. 1 : 4; and these into the distances of their respective centres of gravity; or, which is the same, the whole quantity of compression to that of extension as 1^2 to 4^2.

"Considering this as a general law, and dividing our area accordingly, we have,

$$16\,x^2 = (1 - x)^2 + 3{\cdot}6\,(3 - x), \text{ or,}$$

$$16\,x^2 + 5{\cdot}6\,x = 11{\cdot}8$$

from which we find $x = {\cdot}723$.

"Hence it appears that whatever method is pursued, the resulting numbers are exceedingly approximative. It has, however, been thought best for the object in view, to derive our final data from that case most resembling the actual subject of inquiry,—which is that of Railway Bars having necessarily an upper table; and in these, t being taken as equal to ten tons in good iron, the neutral line may be considered to divide the area of the upper table into two equal parts; and on these are founded the rules given in p. 62.

In other cases it will be better to determine x, as in the last case, and proceed by the general rule.

"I know that it has been advanced, on theoretical principles, that at the commencement of strain the neutral axis is in the centre of gravity of the area of section, but this consideration does not enter into my investigation. I have not examined the question on theoretical, but on mechanical principles, with a view to one specific object, and have purposely avoided resting any point on mere hypothesis. Every thing is made to depend on experimental results; and from the uniformity and agreement of these, I have every confidence the rules founded on them will enable practical men to compute such cases as may occur, with all the precision that can be desired."

A few illustrations, &c. will, perhaps, render the above a little easier. In the first place, the use which is made of this investigation, I think clearly establishes the fact, that it is, even in the learned Professor's own hands, quite insufficient for the purpose; for, in the first place he lays it down, that in rectangular bars the neutral axis is as 1 : 4 in good iron, and p. 43, that under iron of different qualities, it is probably always between 1 : 3 and 1 : 5.

He next determines from the above experiments, that in railway bars without a lower web and 5 inches deep, it is ½ an inch below the upper surface, or 1 : 9; and thirdly he gives a calculated example for a 5 inch bar with a lower web, p. 64, in which he still takes it at half an inch below the upper surface, or 1 : 9.

So that if you take a bar of iron the same size and shape as the middle rib, this being a rectangular bar 5 inches deep, the neutral axis is as 1 : 4, next, if you put an upper head to it,- by fixing two properly shaped pieces of iron, which in conjunction with the upper part of the rib will form the required head, the neutral axis is directly as 1 : 9, and lastly if you in the same way fix pieces to the bottom, to give it a lower web, it will still be only 1 : 9. I am afraid the rising generation of sucking engineers will be not a little puzzled when they come to this.

With respect to the experiments given above, their results, taking the same method which I have before described, are as follows.

	Weights tons.	Deflected by index.	Deflection per ton.		
Bar No. 1.	$7\frac{1}{2}$	,095	$\frac{,095}{7\frac{1}{2}}$	=	,01266
„ „ 2.	8	,085	$\frac{,085}{8}$	=	,01065
„ „ 3.	$8\frac{1}{2}$	,100	$\frac{,100}{8\frac{1}{2}}$	=	,01176
„ „ 4.	10	,125	$\frac{,125}{10}$	=	,01250
	4)34				4),04757
Mean	$8\frac{1}{2}$ Tons		Mean		,01189 of an Inch.

And I must again repeat that this is the way to estimate the deflections; we are looking to practical effects, and what we want to know is the ultimate deflection which a railroad bar will undergo with any given pressure; for instance, ,065 with 4 tons ,095, with $7\frac{1}{2}$ tons, &c. and the mean deflection per ton can only be estimated by dividing the ultimate deflection by the weight producing it; the errors which the other mode adopted would lead us into, are visible on the face of things, for example in bar No. 5, Mr.

Barlow gives the mean deflection per ton as ,015 and the deflection for 7½ tons ,107; whereas, in the very same table, and only three lines above, this deduction of ,107 deflection for 7½ tons, it is shewn in the experiment that at 7 tons it was actually ,335 or 3 times greater than that which is deduced by this mode of proceeding for 7½ tons, there is some mistake here evidently.

In the mathematical investigation page 98 L, the expression $\frac{tx}{d'}$ is obtained from $d' : t = x : \frac{tx}{d'}$ that is, the whole depth below the neutral axis, is to the tension there, as any other depth (xx) to the tension at x.

The expressions $\frac{t}{d'}\int x\, dx$ and $\frac{t}{d'}\int x^2\, dx$ will be understood by comparing them with page 33; the resistance, &c. at any point multiplied by the rate of increase or differential, equals the sum of all the resistances; or, as it is in the latter expression $d' : t = x^2 : \frac{tx^2}{d'}$ which multiplied by the differential of x becomes $\frac{tx^2\,dx}{d'}$ or $\frac{t}{d'}\int x^2\,dx$

The equation $\frac{\int x^2\,dx}{\int x\,dx}$ is got from the two

preceding ones, or $\frac{\frac{t}{d'}\int x^2\,dx}{\frac{t}{d'}\int x\,dx}$; for the resistance being as the depth below the neutral axis, and the tension also as the same depth, it follows that the resistance divided by the tension equals the depth of the centre of tension at the distance x.

The integrals of the formulæ A B and D are as follows:—

$$A = \frac{tx^2}{2d'}$$

$$B = \frac{tx^3}{3d'}$$

$$D = \frac{2x^3}{3x^2}$$

In $D : D + C = B : \frac{(D + C)\,B}{D}$ that is supposing a tension t acting at a distance x on a fulcrum or centre of resistance at the neutral axis, then having the centre of this tension below the neutral axis (D), and the centre of compression above the same axis (C), to refer the tension to C we have D + C as the distance of the fulcrum; and considering the tension only, we get the effect by the proportion given, which may give a very good approximation perhaps, but the whole process is so strictly theoretical, that in the present state of our knowledge of such things, it is to say the least of it uncertain.

The proportion is, the distance of the centre of tension below the neutral axis, is to that distance plus the distance of the centre of compression above the neutral axis, so is the sum of the resistances opposed to the tension, to the weight or cause producing that tension.

$\frac{4\,s}{l}=w$ page 101 L, is got from $s=\frac{l\,w}{4\,a\,d^2}$ whence we have $4\,ad^2\,s=l\,w$ and $w=\frac{4\,a\,d^2\,s}{l}$ but $a\,d^2$ being the depth and breadth, are introduced before in the expression for the resistance, hence

$w=\frac{4\,s}{l}$

$\frac{2}{3}\,d' : \frac{2}{3}\,d = \frac{1}{3}\,d'^2\,t : \frac{1}{3}\,d\,d'\,t$ is the same as

$$D : D + C = B : \frac{B\,(D + C)}{D}$$

The tension of $x = \frac{nx}{ns}\,.\,t$ is from $ns : nx = t$

$$t' = \frac{nx}{ns}\,.\,t$$

I have corrected a typographical error in Mr. Barlow's $\frac{1}{3}\,hx\,.\,\overline{nx}^2\,.\,\frac{nn - pq}{ns}$ it should be $\frac{1}{3}$

$$hx\,.\,\overline{nx}^2\,.\,\frac{(nn - pq)}{ns}\,.\,t$$

The reduction of $\frac{\int (d'' + x)^2\,dx}{\int (d'' + x)\,dx} = \delta'$ is thus

For the Numerator we have

$$\int (d'' + x)^2\,dx = \frac{1}{3}\,(d'' + x)^3 + c$$

but when $x = o$ the integral must also $= o$ hence

$$\frac{1}{3}\,(d'' + o)^3 + c = o \text{ or } c = -\,\frac{d''^3}{3}$$

which combined with the integral gives

$$\int (d'' + x)^2\,dx = \frac{1}{3}\,d'' + x)^3 - \frac{d''^3}{3}$$

In the same manner for the denominator.

$$\int (d'' + x)\, dx = \frac{1}{2} (d'' + x)^2 + c$$

and we get $c = -\frac{d''^2}{2}$ hence the correct result is

$$\int (d'' + x)\, dx = \frac{1}{2} (d'' + x)^2 - \frac{d''^2}{2}$$

hence we get for the whole

$$\delta' = \frac{\frac{1}{3} (d'' + x)^3 - \frac{dx^3}{3}}{\frac{1}{2} (d'' + x)^2 - \frac{d''^2}{2}}$$

and actually involving we have after dividing both numerator and denominator by x

$$\delta' = \frac{2}{3} \left(\frac{3\, d''^2 + 3d''x + x^2}{2d'' + x}\right)$$

and multiplying respectively by 2 and 3 and dividing by 6

$$\delta' = \frac{d''^2 + d''x + \frac{1}{3} x^2}{d'' + \frac{1}{2}\, x}$$

but $d''^2 + d''\, x + \frac{1}{3}\, x^2 = d''^2 + d''\, x + \frac{1}{4}\, x^2 + \frac{1}{12}\, x^2$

and dividing this by $d'' + \frac{1}{2}\, x$ we get

$$\delta' = d'' + \frac{x}{2} + \frac{\frac{1}{12}\, x^2}{d'' + x^2} \quad \text{or}$$

$$\delta' = d'' + \frac{x}{2} + \frac{x^2}{12\,(d'' + \frac{x}{2})}$$

but $d'' = n\,r$, and when $x = r\,s$, $d'' + \frac{x}{2} = n\,m$ (see the figure,) consequently we then have

$$\delta' = n\,m + \frac{r\,s^2}{12\,m\,n}$$

I take it that the next line should stand

$$\frac{t\,\delta'}{d'} \int (d'' + x)dx = \frac{t\,\delta'}{d'}\;n\,m\,r\,s$$

for integrating we have as before

$$\frac{t\,\delta'}{d} \cdot \frac{2\,d''\,x + x^2}{2}$$

$$\text{or}\quad \frac{t\,\delta'}{d'} \cdot \overline{n\,r\,.\,r\,s + \frac{r\,s^2}{2}}$$

and $n\,r\,.\,r\,s + \frac{r\,s^2}{2} = n\,m\,.\,r\,s$ or dividing both sides by $r\,s$, $n\,r + \frac{r\,s}{2} = n\,m$ as may be seen by the figure.

Having now cleared up the mathematical investigation, let us see how it will look when brought to a numerical result.

The equation which is derived from

$$\left.\begin{array}{l}\frac{1}{3}\,hs\,.\,ns\,.\,pq\,.\,t \\ +\,\frac{1}{3}\,hx\,.\,\overline{n\,x}^2\,.\;\dfrac{n\,n-pq}{n\,s}\,.\,t\end{array}\right\}=s$$

$$\text{and } \frac{4\,s}{l}=w \quad \text{or } s=\frac{w\,.\,l}{4}$$

$$\text{is } \frac{1}{3}\left(5\,(5-x)\,9+\frac{11\,(1-x^2)}{5-x}\right)=\frac{8\frac{1}{4}+33}{4}$$

from which to determine x Mr. Barlow says p. 78

"Taking the result of the experiments, p. 70, at $w=8\frac{1}{4}$, and the dimensions of that bar as known quantities, every thing in the above is given except the position of the line $n\,n$. Calling, therefore, $h\,n=x$, and substituting the proper numerical values for the other parts, we have,

$$\frac{1}{3}\,.\,5\,.\,(5-x)\,.\,9+\frac{1}{3}\,(1-x)^2\,.\,\frac{1\cdot 1}{5-x}\,.\,10=\frac{33\,.\,8\frac{1}{4}}{4}$$

This reduces first to—

$$45\,(-x\,)^2+11\,(1-x)^2=204{\cdot}18\,(5-x)$$

then to $x^2-\dfrac{267{\cdot}82}{56}\,x=-\dfrac{115{\cdot}1}{56}$

whence the value of $x=\cdot 484$."

Here first $x^2-\dfrac{267{,}82}{56}\,x=-\dfrac{115{,}1}{56}$. Completing the square we have

$$x^2 - 4{,}782\,x + \overline{2{,}391}^2 = -2{,}055 + \overline{2{,}391}^2$$

$$\text{or}\quad x \pm 2{,}391 = \sqrt{-2{,}055 + \overline{2{,}391}^2}$$

$$= \sqrt{-2{,}055 + 5{,}717}$$

$$= \sqrt{3{,}662}$$

$$\text{or}\ x = \sqrt{3{,}662} \pm 2{,}391$$

$$= 1{,}9147 \pm 2{,}391 = -{,}4763 \text{ or } 4{,}3057$$

and if ,4763, is the right root we have the neutral axis as ,4763 : 4,5237, or as 1 to 9,55

But we have seen that t ought to be taken at 9 tons at the outside for the best iron, and with this value, and 8½ for w, as we have also found before, the other values are

$$hs = 5$$
$$ns = 5 - x$$
$$pq = {,}9$$
$$t = 9$$
$$hx = 1$$
$$nx = 1 - x$$
$$nn = 2$$
$$nn - pq = 1{,}1$$
$$w = 8{,}5$$

And with these we get in the formula for x

$$40{,}5\,(5-x)^2 + 9{,}9\,(1-x)^2 = 210{,}375\,(5-x)$$

$$\text{or } 40{,}5\,(25 - 10x + x^2) + 9{,}9\,(1 - 2x + x^2)$$
$$= 1051{,}875 - 210{,}375\,x$$

$$\text{or } 1012{,}5 - 405x + 40{,}5x^2 + 9{,}9 - 19{,}8x + 9{,}9x^2$$
$$= 1051{,}875 - 210{,}375\,x$$

$$\text{or } -29{,}475 - 214{,}425\,x + 50{,}4\,x^2 = o$$

$$\text{or } x^2 - 4{,}254\,x = {,}585$$

$$x^2 - 4{,}254\,x + \overline{2{,}127}^2 = {,}585 + \overline{2{,}127}^2$$

$$x - 2{,}127 = \sqrt{{,}585 + {,}2127^2}$$

$$= \sqrt{5{,}109} \qquad = 2{,}26$$

And $x = 2{,}26 + 2{,}127 = 4{,}387$

Now in the equation as Mr. Barlow has it, there being a choice of roots he has naturally taken ,4763, or the neutral axis is as 1 : 9½, but when my values of t and n are used, as in the second process, it is the root 4,387 which satisfies the equation; or the neutral axis is close to the bottom of the bar instead of close to the top.

I have calculated the above as parallel rails, although they are fish-bellies, in order to compare the results with Mr. Barlow, he having done the same.

As one more example, let us take the rail with lower web, (Plate 2, fig. 2), weight 50 lbs. depth 5 inches, thickness of rib ,6 and breadth of section of lower web 1,32.

Then $hs = 5$ and $\frac{1}{3}\, hs \,.\, ns \,.\, pq \,.\, t =$

$ns = 5 - x$

$pq = ,6$ $\quad \frac{1}{3} \,.\, 5 \,.\, (5-x) \,.\, ,6 \,.\, 9 = 45 - 9x$

$t = 9$

$hx = 1$ and $\frac{1}{3}\, hx \,.\, \overline{nx}^2 \,.\, \frac{nn - pq}{ns} \,.\, t =$

$nx = 1 - x$

$nn = 2$ $\quad \frac{1}{3} \,.\, (1-x)^2 \,.\, \frac{1,4}{5-x} \,.\, 9$

$nn - pq = 1,4$

$ns = 5 - x$

$mm - pq = ,72$ and $\gamma' = mn + \frac{rs^2}{12\, nm} =$

$nm = 4$

$rs^2 = 1$ $\quad 4 + \frac{1}{48} = 4,021$

$cn = \frac{2}{3}x$

$$\gamma'' = \gamma' + cn = \frac{2\,x}{3} + 4,021 =$$

$$\frac{2x + 12,063}{3}$$

hence $\frac{\gamma''}{\gamma'} = \frac{2\,x + 12,063}{12,063}$

and $nm \,.\, rs \,.\, (mm - pq) \,.\, \frac{\gamma''}{\gamma'} \,.\, t =$

$$4 \,.\, ,72 \,.\, 9 \,.\, \frac{2\,x + 12,063}{12,063} = 25,92 \,.\, \frac{2x}{12,063} +$$

$$25,92 = \frac{51.84\,x}{12.063} + 25,92 = \frac{51,84\,x + 312,7}{12,063}$$

and as $\frac{4s}{l} = w$, we have s, or the sum of the 3 equations above $= \frac{lw}{4} = \frac{8,75\ .\ 33}{4} = 72,19$ hence the whole stands $45 - 9\,x + \frac{4,2 - 8,4\,x + 4,2\,x^2}{5 - x} +$

$$\frac{51,84\,x + 312,7}{12,063} = 72,19 \quad \text{or}$$

$$2714 - 1085,67\,x + 108,567x^2 + 50,6646 - 101,3292\,x + 50,6646x^2 - 53,5x + 1563,5 + 51,84x^2 - 4354 + 870,83x = o$$

$$\text{or } 107,39\,x^2 - 369,669x = 25,835$$

$$\text{or} \quad x^2 - 3,4423x = ,2406$$

$$x^2 - 3,4423\,x + \overline{1,7211}^2 = ,2406 + \overline{1,7211}^2$$

$$x - 1,7211 = \sqrt{,2406 + \overline{1,7211}^2}$$

$$= \sqrt{,2406 + 2,9622}$$

$$= \sqrt{3,2028}$$

Whence $x = 1,7897 + 1,7211 = 3,5108$ or the neutral axis is as 3,5108 to 1,4892 or as 1 : 0,4 in the lower part of the bar again.

These examples will therefore sufficiently shew, that Mr. Barlow's third method fails completely; in fact, the process is far too theoretical in our present state of knowledge; for these not only discordant but impossible results be it observed, are all drawn from Mr. Barlow's own formulæ and his own experiments, substituting for the numbers derived from the latter, 9 tons instead of 10 tons, as the longitudinal elasticity of iron per square inch; which, as I have before shewn, is all we can get from the said experiments; and that this is the case, every one can see for himself; besides which, if so small a difference in the elasticity, makes the ratios so very discordant, we must necessarily reject the method, for no one can assign a mean resistance to even good iron, which will produce any thing like practical results.

I confess I was rather surprised, to find, in one or two places, Mr. Barlow stating, that he had avoided theory as much as possible, and grounded his results on the basis of experiments only. In this I must decidedly differ from him; the position of the neutral axis, enters as an element into almost every computation, and as is shewn above, we positively know nothing about it whatever; for even granting that the mechanical method of determining it is correct at 1 : 4, as Mr. Barlow thinks it is, this is only for rectangular bars, and we are still as far abroad as ever for railway bars, where Mr. Barlow himself takes it as 1 : 9, which ratio is got from the third method; and what that method leads to, we have seen, namely that it gives it close to the top of the bar, below the middle of the bar, and close to the bottom of the bar, all by the same formulæ; in fact "Chaos is come again."

Mr. Barlow p. 79 adverts to a fourth method for determining x, but as it rests on the assumption that the surface of extension, is to that of compression, as 1 : 4, of which we have no proof, it is of no use to enter on it at all.

It is so very easy to determine the neutral axis, by a visible and tangible process, that before long some one will do it I have no doubt; unfortunately, it is too often the case that people do not make experiments, they follow the mode of the honest Emeralder, try the thing at once, not in the closet but in the field, on the natural scale; like cutting a canal, and then looking where the water is to come from; or like Pat's tailor who made the coat first, and measured the man afterwards.

CHAP. IV.

ON THE STIFFNESS OF RECTANGULAR IRON BARS, AND THEIR DEFLECTION UNDER DIFFERENT WEIGHTS.

To arrive at this, Mr. Barlow adds to the experiments in the last chapter the following.

“EXPERIMENTS ON THE DEFLECTION OF MALLEABLE IRON BARS, UNDER DIFFERENT STRAINS.

Bar No. 11.

Distance of bearings.	Breadth.	Depth.	Weights.	Deflections.	Deflections, each ½ Ton.	REMARKS.
inches.	inches.	inches.	tons.			
33	1.5	3	·125	·043		
			·500	·059		
			1·00	·074	·015	
			1·50	·083	·009	
			2·00	·095	·012	
			2·50	·101	·006	Mean ·0103
			3·00	·109	·008	
			3·50	·120	·011	
			4·00	·131	·011	$w = 4\frac{1}{2}$. Neutral axis 1 : 4·9
			4·50	.148	·017	Elasticity preserved at 4½ Tons.

Bar No. 12.

Distance of bearings.	Breadth.	Depth.	Weights.	Deflections.	Deflections, each ½ Ton.	REMARKS.
33	1.5	3	0	0		
			·50	·017		
			1·00	·037		
			1·50	·052	·015	
			2·00	·061	·009	
			2·50	·064	·003	
			3·00	·078	·014	Mean ·0183
			3·50	·089	·011	
			4·00	·102	·013	$w = 4\frac{1}{2}$. Neutral axis 1 : 4.9
			4·50	·124	·022	Elasticity injured.

Bar No. 13.

Distance of bearings.	Breadth.	Depth.	Weights.	Deflections.	Deflections, each ½ Ton.	REMARKS.
33	1·5	2·5	0	·006		
			·50	·003	·024	
			1·00	·050	·020	
			1·50	·060	·010	
			2·00	·074	·014	Mean ·0173
			2·50	·093	·019	$w = 3$. Neutral axis 1 : 4·9.
			3·00	·110	·017	Elasticity preserved, 3 Tons.
			3·50	·149		
			7·5	Bent 8	inches.	

"To reduce the law of deflection from these results, we may have recourse to two well-known and well-established formulæ :—viz.

$$\frac{l\,w}{4\,a\,d^2} = S \text{ and } \frac{l^3\,w}{a\,d^3\,\delta} = E$$

$$\text{where } \left.\begin{aligned} S &= 9000 \\ E &= 91440000 \end{aligned}\right\} \text{for wrought iron.}$$

$$\begin{aligned} w &= \text{weight} \\ l &= \text{length} \\ a &= \text{breadth} \\ d &= \text{depth} \\ \delta &= \text{deflection.} \end{aligned}$$

S and E are both constant quantities for the same material, w being the greatest weight the bar will bear without injuring the elasticity; consequently, when l is also the same in both, $d\,\delta$ will be also constant, a being the breadth, d the depth, and δ the deflection. That is, all rectangular bars having the same bearing length, and loaded in their centre to the full extent of their elastic power, will be so deflected, that their deflection (δ) being multiplied by their depth (d) the product will be a constant quantity, whatever may be their breadths or other dimensions, provided their lengths are the same.

"Let us see how nearly our several results agree with this condition.

"In the several bars, Nos. 8, 9, 10, 11, 12, 13, multiplying the mean deflection for each half ton, by the number of half tons which excited its whole elasticity, and this again by the depth of the bar, we find

		depth.
No. 8, ultimate deflection	·108 × 2	= ·2160
No. 9, ...	·094 × 2	= ·1380
No. 10, ...	·120 × 2	= ·2400
No. 11, ...	·0876 × 3	= ·2628
No. 12, ...	·0918 × 3	= ·2754
No. 13, ...	·1038 × 2½	= ·2595
	6)	1·4417
	Mean	·2403

"There is rather a large discrepance in bar No. 9; the others are as approximative to the mean as can be expected in such cases.

"If we make the same trial on the three parts of bar No. 7, we have

1st part	·116	×	2	=	·2320
2d part	·105	×	2	=	·2100
3d part	·115	×	2	=	·2300
				3)	·6720
Mean				....	·2240
Former Mean				....	·2403
				2)	·4647
General Mean				....	·2323

"We may therefore say, that any malleable iron bar of 33 inches bearing, being strained to its full elasticity, will be so deflected, that its depth, multiplied by the deflection, due to 30 inches, will produce the decimal ·23; consequently $\frac{\cdot 23}{d}$ = the deflection, d being the whole depth in inches.

"In this form, however, it applies only to rectangular bars. To make it general, we must estimate it from the neutral axis, which in rectangular bars being $\frac{1}{5}$th of the depth below the upper surface, the above constant, when thus referred, becomes $\cdot 2323 \times \frac{4}{5} = \cdot 1858$. But, on the other hand, our instrument for measuring the deflection was but 30 inches long: it has therefore to be increased again in the ratio $30^2 : 33^2$, or as $10^2 : 11^2$ on this account; so that, ultimately, the formula is $d' \delta = \cdot 22$; d' denoting now the depth of the bar below the neutral axis, and in this form it is general for parallel rails of any section whatever."

In the experiment P. 81 L, I should say

No. 8 for mean $=$,024 read $\frac{,151}{5} = ,0302$

9 " ,021 " $\frac{,145}{5} = ,029$

10 " ,024 " $\frac{,151}{5} = ,0302$

for the mean deflection per ½ ton, multiplied by the number of ½ tons, should of course produce the deflection due to that weight; which it does not except when done as above; thus Bar No. 8, if ,024 is multiplied by 5, it gives ,12 instead of ,151 which was the actual deflection with 2½ tons.

In the same way taking the experiments P. 89 L within the range given we find

No. 11 for ,0103 read $\frac{,131-,074}{(4-)\ 2} = \frac{,057}{6} = 0095$

or more properly $\frac{,148}{9} = ,0164$, and if the last is left out $\frac{,131}{8} = ,0164$

12, for ,0108 read $\frac{,05}{5} = ,01$ or $\frac{,124}{9} = ,0138$ or $\frac{,102}{8} = ,01275$

13, for ,0173 read $\frac{,107}{5} = ,0214$ and leaving out the middle process above as the bar bent, the third process above gives $\frac{,110}{6} = ,0183$

The longitudinal resistance of these bars Mr. Barlow states to be the same as the last, or 10 tons by his estimation, and 9 tons by mine.

Hence the corrected mean results are

Bar No. 8	=	,0302
9	=	,029
10	=	,0302
11	=	,0164
12	=	,01275
13	=	,0183
		6),1368
	Mean	,0228

In like manner we find, subtracting the zero from the greatest deflection.

Bar No. 7, Part 1		=	,116
	,, 2	=	,084
	,, 3	=	,145
			3),345
		Mean	,115

If we do the same with

Bar No. 5, Part 1		=	,1
	,, 2	=	,11
No. 6,	,, 2	=	,066

No. 6 Part 1 is bad, and No. 7 Part 3 is doubtful, but I have included it as Mr. Barlow has done so.

Now substituting these values for Mr. Barlow's

	δ	d	w		
Bar 8	,0302	. 2	. 5	=	,302
9	,029	. 2	. 5	=	,29
10	,0302	. 2	. 5	=	,302
11	,0164	. 2	. 8½	=	,4182
12	,0127	. 3	. 8	=	,3048
13	,0183	. 2½	. 6	=	,2745
					6)1,8915
					,31525

Bar 7, Part 1	,116	. 2	=	,232
,, 2	,084	. 2	=	,168
,, 3	,145	. 2	=	,29
				3),690
				,23

Bar 5, Part	1	,1	.	2	=	,2
"	2	,11	.	2	=	,22
"	3	,066	.	2	=	,132
						3),552
						,184

And for a general mean

$$\begin{array}{r} 1{,}8915 \\ {,}69 \\ {,}552 \\ \hline 12)3{,}1335 \\ \hline \end{array}$$

Mean of the 12 = ,2611

now although a range of ,168 : ,418 is very great, yet till more experiments are made we must consider this the best approximation we can get.

We may therefore say, that a rectangular malleable iron bar, when loaded to the full extent of its elastic power, will have a deflection $\delta = \frac{,2611}{d}$, d being the depth of the bar in inches, or in round numbers

$$\delta = \frac{,26}{d}$$

Unfortunately, we cannot use this for bars of any other form; we are in ignorance, as has been shewn, of the position of the neutral axis: if it was at $\frac{1}{5}$ as Mr. Barlow has taken it, we should have $,2611 \, . \, \frac{4}{5} = ,20888$ or in round numbers $,21$ for the constant, whence

$$10^2 : 11^2 = ,20888 : = ,252$$

$$\text{or } \delta = \frac{,252}{d'} = \frac{,252}{\frac{4}{5}d} = \frac{,315}{d}$$

where d' is the depth below the neutral axis, Mr. Barlow's formula being $\frac{,22}{d'} = \frac{,22}{\frac{4}{5}d} = \frac{,275}{d}$; but unless we can be certain that the bar when deflected, took the shape of a parabola, and we take the deflection as an absciss, and the length for an ordinate, the proportion is perhaps as 10^3 to 11^3 and not as $10^2 : 11^2$ or $1000 : 1331 = ,20888 : ,3475$ or in Mr. Barlow's numbers

$$1000 : 1331 = ,1858 : ,2473$$

hence in this case his formula for bars not rectangular, is $\delta = \frac{,2473}{d'} = \frac{,3091}{d}$ and mine is

$$\delta = \frac{,3475}{d}$$

There is also this further anomaly; the deflection by Mr. Barlow's uncorrected numbers, is for a rectangular bar $\frac{,22}{d}$ and for a bar not rectangular, $= \frac{,275}{d}$; or the bar deflects more when its shape is not rectangular, as ,22 : ,275; yet Mr. Barlow in another place recommends a part of the iron of a rail, to be distributed in a bottom web, which, according to the above, would be doing harm, except that it would lower the rail in the chair a small quantity, and this is a very great desideratum; but we are at any rate going a round-about-way to get a small benefit, while we can have a great one, by a straight forward process, viz. by using the fish-bellied rail, and save expence too, a parallel rail taking a heavier chair than a fish-belly.

This lowering of the chair, is the very point to be most desired, for the wringing of the chair from the block, is found in practice to be directly as the height of the chair, or in a 50lb. rail increased one-third, and of course the loosening of the block in the ground is greater also with a high chair.

One half of the heavy expense of "maintenance of way," has, on the Liverpool and Manchester line, as far as wages are concerned, been laid out on raising sunk blocks, either by ramming them up or wedging up the chair with wood; and it is a curious fact, that the old 35 lb. fish-bellied rails on the Warrington line, although carrying weights, which place them on the verge of permanent deflection, have been found in better condition, than either the Wigan, or the Saint Helen's and Runcorn 42lb. rails; although the latter have carried lighter loads, and at less speed than the former.

We may say then, that in rectangular bars 30 inches long, the ultimate deflection is about $\frac{,26}{d}$ but in other shapes the formula will not answer: indeed, in rectangular bars, it can only be considered an approximation, for Mr. Barlow says p. 36, that the deflections experimentally given, were obtained by weighting the bars "till the successive deflections showed a tendency to increase in amount, which was taken as a sign of the elasticity being injured;" now, in order to obtain a correct result, this point should have been proved each time, by taking off the strain, and seeing whether the elasticity brought back the iron to the zero, from which the corresponding deflections were measured.

CHAP. V.

COMPARATIVE STRENGTH OF DIFFERENTLY-FORMED RAILS.

UNDER this head Mr. Barlow also calculates the comparative deflections as well as the strengths from his rules, which we have before examined; and as we have proved that the position of the neutral axis is unknown, and that we cannot tell whether it is at the top, in the middle, or at the bottom of a railway bar of any practical form, and as the neutral axis is a prominent element in all the calculations given in this chapter, it is plain that we can place no reliance on any of the results.

It may be well, however, to look a little at them. They stand as follows.

"(1) In Mr. Stephenson's rail, the greatest depth is 5 inches, with a plain rib, whose thickness is ·9 of an inch. Here,

$$\text{Resistance of Head}\left\{\begin{matrix}(2-\cdot 9)\times 10=11\\ (5-\tfrac{1}{2})\times 12=54\end{matrix}\right\}\frac{11}{54}=0\cdot 20$$

$$\text{Ditto of Rib} \;\ldots\ldots\ldots\ldots\; \frac{4\tfrac{1}{2}\times 5\times \cdot 9\times 10}{3}=67\cdot 50$$

$$\overline{67\cdot 7}$$

And $\dfrac{4\times 67\cdot 7}{33}=8\cdot 21$ tons, the greatest weight.

"Deflection with this weight $\dfrac{\cdot 22}{4\cdot 5}\times\dfrac{4}{3}=\cdot 066$

"(2) Parallel rail of the same depth and weight, viz. 50 lbs. per yard. Here the thickness of centre rib = ·78. Hence,

$$\begin{matrix}\text{Resistance of}\\ \text{Head.}\end{matrix}\left\{\begin{matrix}(2-\cdot 78)\times 10=12\cdot 2\\ (5-\tfrac{1}{2})\times 12=54\end{matrix}\right\}\frac{12\cdot 2}{54}=\overset{\text{TONS.}}{0\cdot 225}$$

$$\text{Ditto Rib} \;\ldots\ldots\; \frac{4\tfrac{1}{2}\times 5\times \cdot 78\times 10}{3} \;\ldots\ldots\; =58\cdot 5$$

$$\overline{58\cdot 725}$$

And $\dfrac{4\times 58\cdot 725}{33}=7\cdot 11$ tons, the greatest weight.

"Deflection with this weight $\frac{\cdot 22}{4\cdot 5} = \cdot 048$

"(3) Parallel rail with bottom web, the depth being still 5 inches, the thickness of rib ·6 of an inch, thickness or breadth of section of lower web 1·32, the weight being 50 lbs.

$$\text{Resistance of Head} \left\{ \begin{matrix} (2-\cdot 6) \times 10 = 14 \\ (5-\tfrac{1}{2}) \times 12 = 54 \end{matrix} \right\} \frac{14}{54} = \overset{\text{TONS.}}{0\cdot 26}$$

$$\text{Ditto of Rib} \ldots\ldots\ldots\ldots \quad \frac{4\tfrac{1}{2} \times 5 \times \cdot 6 \times 10}{3} = 45\cdot 00$$

$$\text{Lower Web} \left\{ \begin{matrix} (5-1) \times \cdot 72 \times 10 = 28\cdot 8 \\ 12(5-1)^2 + 24 = 216 = \text{1st No.} \\ 216 - 7 = 209 = \text{2d No.} \end{matrix} \right.$$

As 216 : 209 : : 28·8 : 27·94 .... 27·94

73·20

And $\frac{73\cdot 20 \times 4}{33} = 8\tfrac{3}{4}$ tons the greatest weight.

Deflection with this weight $\frac{\cdot 22}{4\cdot 5} = \cdot 048$.

"(4) As another example, let us take a parallel rail of 50lbs. per yard, depth $4\frac{1}{2}$ inches, thickness of rib $\frac{7}{10}$ th of an inch, and the bottom web 1·39.

$$\text{Resistance of Head} \left\{ \begin{matrix} (2-\cdot 7) \times 10 = 13 \\ (4\tfrac{1}{2}-\tfrac{1}{2}) \times 12 = 48 \end{matrix} \right\} \frac{13}{48} = \overset{\text{TONS.}}{0\cdot 27}$$

$$\text{Ditto of Rib} \ldots \quad \frac{4 \times 4\tfrac{1}{2} \times \cdot 7 \times 10}{3} = 42\cdot 00$$

Do. of lower Web . . $\left\{\begin{array}{l} 3\frac{1}{2}\times(1\cdot39-\cdot7)\times10=24\cdot15 \\ 12(3\frac{1}{2})^2+21=168=\text{1st No.} \\ \qquad 168-6=162=\text{2d No.} \end{array}\right.$

As 168 : 162 : : 24·15 : 23·28 = 23·28

65·55

$\frac{4\,.\,65\cdot55}{33}$ = 8 tons, nearly the greatest weight.

Deflections with this weight $\frac{.22}{4}$ = ·055.

The first curious result is, that in Example 1, a fish-bellied rail bears, when strained to the utmost, without permanent deflection, 8,21 tons. Taking the 2 sides of the head away from the middle rib, and leaving it, (the whole depth) we have $\frac{4\times67{,}5}{33}$ = 8,182 for the weight, differing only ,028 of a ton from what it was when the rail had its head complete. Now, as ,028 of a ton is about 62lb., and the whole weight it bears complete is 8,21 × 2240 = 18390, it follows that the rail complete will bear only $\frac{1}{296}$th more than the middle rib for the whole depth will bear; while if we look to the 4th Example, we find, that although the addition of the head to the middle rib, only makes the rail bear 62lbs. more, the addition of a less quantity of iron as a bottom web, makes it bear no less than 23 tons more. Yet in the 2nd Report p. 50, we find an experiment stated, in which ½ an inch of the lower web of a double T rail was cut away on each side, and that it "lost but little of its strength," although the iron thus abstracted, was "nearly ⅛ of the whole section."

In the next place Mr. Barlow compares a fish-bellied rail of 50 lbs. and 5 inches maximum depth, with a parallel one of 50lbs. and 5 inches depth, (Example 1 and 2), and to be able to do so, he abstracts ,12 of an inch from the entire thickness of the middle rib in the parallel rail. Now, I must say, I consider this unfair comparison to have been made inadvertently, for it could have only been done intentionally by one who entered into the question as a partisan, which Mr. Barlow expressly states he does not; how can he alter the breadth here when he solves the deflection for both rails in the 1st chapter, by working with the formula $\frac{x^2}{d^3}\,dx\,\triangle$ where the breadth is not included, but taken as equal in each rail.

The middle rib having a known and definite duty to perform, as all practical men are aware, cannot be altered in order to make the comparison; and even if it could, it is still wrong to take the deflection of the fish-belly as 4, when that of the parallel is 3. We have seen, chapter 1, that the ratio 3 to 4, is that which obtains between a parallel bar and a mathematical ellipse, and that with a fish-bellied bar, the ratio is nearly 5 to 6 when taken shape for shape, but when taken practically, that is to say, weight for weight, the numbers 3 to 4 have to be reversed, and it is the parallel bar which deflects 4, while the fish-belly only deflects 3; we cannot do as above, for by decreasing the rib in thickness, we also decrease the head, which is only sufficient for the wheel as it is, and Mr. Barlow's rules calculate the rib for the whole depth, not for the depth under the head.

In order to compare the two in practice, we have then two ways; either to make the rib of the fish-belly equal to that which Mr. Barlow has given to the parallel, or to keep the parallel rib the same thickness as the fish-belly, and in either case the result is nearly the same; in the first case we deepen the fish-belly, in the second case we make the parallel less deep.

By the first method, taking Mr. Barlow's numbers, and subtracting from the fish-bellied rib ,12 for a mean depth of 4,4 inches, we have ,587 of an inch to distribute along the bottom, when the breadth is ,9, which gives a depth at a maximum of 5,587, and leaving out the calculation of the heads, as they are so nearly equal, we have for a fish-bellied 50lb. rail by Mr. Barlow's method,

$$\frac{5{,}587 \cdot 5{,}087 \cdot 9}{3} = 85{,}26 \text{ and } \frac{4 \cdot 85{,}26}{33} = \overset{\text{TONS.}}{10{,}3}$$

greatest weight, and $\frac{,22}{5{,}087} \cdot \frac{6}{5} =,0518$ deflection.

While for the parallel of the same weight, the same length and thickness, and the same shaped head, we have $\frac{4{,}5 \cdot 5 \cdot 7{,}8}{3} = 58{,}5$ and $\frac{4 \cdot 58{,}5}{33}$ $=7{,}1$ tons greatest weight and $\frac{,22}{4{,}5} =,049$ deflection

Now if we try it the other way, we have the fish-belly with a thickness of ,9 =

$\frac{4,5 \, . \, 5 \, . \, 9}{3} = 67,5$ and $\frac{4 \, . \, 67,5}{33} = 8,18$ tons the greatest weight, and $\frac{,22}{4,5} \, . \, \frac{6}{5} = ,0587$ deflection; and with the parallel, subtracting ,587 from the depth, the same quantity which we added to the fish-belly, we have

$\frac{4,413 \, . \, 3,913 \, . \, 9}{3} = 51,8$ and $\frac{4 \, , \, 51,8}{33} = 6,3$ tons greatest weight, and $\frac{,22}{3,913} = ,0562$ deflection.

So that with Mr. Barlow's number for the thickness of the parallel rib, the rails are about equal in deflection, while the fish-belly is from 2 to 3 tons stronger. Let us now see whether this thickness is right. In chapter 1, we have found that ,6 is to be taken from the parallel, leaving its depth 4,4 in order to make it equal in weight to the fish-belly, and ,6 subtracted over 4 inches in depth and 33 in length = ,15, giving a thickness of ,75 only for the equivalent rib in the parallel; for looking at it practically, we must subtract our ,6 in this instance, not from head and rib, or 5 inches, but from rib only, say 4 inches. Calculating now with ,75 by Mr. Barlow's rules, and the first method, we have ,15 for a mean depth of 4,4 equal to ,73 to distribute along the bottom, giving a maximum depth of 5,73 and

$$\frac{5{,}73 \,.\, 5{,}23 \,.\, 9}{3} = 89{,}9 \text{ and } \frac{4 \,.\, 89{,}9}{33} = \quad 10{,}9 \text{ tons}$$

greatest weight, and

$$\frac{,22}{5{,}23} \cdot \frac{6}{5} \quad ,05 \text{ deflection ;}$$

while the parallel is weight 7,1 tons, deflection ,049, and by the second method subtracting ,73 from the depth, we have for the parallel

$$\frac{4{,}27 \,.\, 3{,}377 \,.\, 9}{3} = 48{,}27 \text{ and } \frac{4 \,.\, 48{,}27}{33} = 5{,}85 \text{ tons}$$

the greatest weight.

and $\frac{,22}{3{,}77} =$,0584 deflection.

While the fish-belly is weight 8,18, and deflection ,0587. So that although, through the position of the neutral axis being unknown, the formulæ do not, under any treatment, bring out the result they should by chapter 1, yet they will by no means give us the values Mr. Barlow has assigned to the fish-belly and parallel rails.

And that ,75 is not too small a number to represent the thickness of the rib in Mr. Barlow's 2d Example, may be shewn another way.

100 inches of a wrought iron bar 1 inch square weighed 28lb. or ,28lb. per square inch, and taking the head of the rails at 2 inches square, as Mr. Barlow has done, that is to say, 2 inches broad and 1 inch deep, we have for the head $2 . 36 = 72$ and $72 . ,28 = 20,16$lb. and $50 - 20,16 = 29,84$lbs. for the weight of the rib below the head, whatever its form may be.

Then $36 . 4 . ,28 . a = 29,84$ or $a = \frac{29,84}{36 . 4 . ,28}$ $= \frac{29,84}{40,32} = ,74$

These calculations are, of course, to be considered as only comparative ones, for we have seen that for any positive results, we have no neutral axis, and also the tension of iron is taken at 10 tons in Mr. Barlow's Examples, whereas the experiments only authorise us to take it at 9 tons.

We may get an approximation to the position of the neutral axis, in a fish-bellied rail, by a different mode of proceeding to that which has been adopted by Mr. Barlow, namely, by experiment alone. Those at page 71 appear to be very anomalous, for deducting in the 3rd for the zero, we have

$$\text{Bar No. } 5 = \frac{,335}{7} = ,0478$$

$$\text{,, } 6 = \frac{,230}{3} = ,0766$$

$$\text{,, } 7 = \frac{,155}{2} = ,0775$$

Now, for No. 5, Mr. Barlow gives the deflection at $7\frac{1}{2}$ tons $=$,107 while the above method gives it at ,0478 $\times$ $7\frac{1}{2}$ $=$,3585. and one glance at the table page 96 L bears this out to be the correct way.

Rejecting, therefore, page 71, and taking page 70 at a mean of ,01189 per ton, and $8\frac{1}{2}$ tons for the weight, and as we have seen that the deflection is about $= \frac{,26}{d'}$ where d' is the depth of the rail below the neutral axis, we have

$$\frac{,26}{d'} = ,01189.8,5$$

$$\text{or } \frac{,26}{d'} = ,101065$$

$$\text{and } ,26 = ,101065\,d'$$

$$\text{whence } d' = \frac{,26}{,101065} = 2,557$$

or the neutral axis, the depth of the rail being 5 inches, is as 2,443 to 2,557 or 1 : 1,047 a little above the middle of the bar.

From which, we should nearly have for parallel bars of the same depth (not weight) and without lower web

$$\frac{,26}{d'} \cdot \frac{6}{5} = ,101065$$

$$\text{or } 1,56 = ,505325\,d'$$

$$\text{and } d' = \frac{1,56}{,505325} = 3,087$$

or the neutral axis as 1,913 to 3,087 or 1 : 1,6; this agrees with what we have shewn in Chapter 1; and in the case of the 50lb. fishbelly, is directly from experiment; the parallel of the same depth is not so satisfactorily obtained, but those interested in such researches, can so easily derive the position by mechanical means, that it is not worth while to pursue the enquiry any farther; but we are justified in saying, that after the little which we know of the value of this element in the way

Mr. Barlow has determined it, and seeing how largely it affects the results of every computation both for strength and deflection, in the formulæ which he has given, we have no grounds for placing the least dependance upon these calculations, unless the learned Professor can reconcile the extraordinary anomalies which we have pointed out.

We have another example of the same sort, from the experiments p. 103 second report, where the measured deflection of a 62lb. parallel rail with bottom web, and depth 4½ inches, is with 11 tons as follows,

$$\begin{array}{rl} & ,074 \\ & ,086 \\ & ,055 \\ & ,077 \\ & ,071 \\ & ,067 \\ \hline 6\,) & ,430 \\ \hline \text{Rough Mean} & ,0717 \\ \hline\hline \end{array}$$

hence $\frac{,26}{d'} = ,0717$ or $,0717, d' = ,26$ whence d'

$= \frac{,26}{,0717} = 3,6$ and $4,5 - 3,6 = 1,9$ or the neutral axis as 1,9 : 3.6 or as 1 : 1,9; whereas, Mr. Barlow's rules give it 1 : 8 or a ratio more than 4 times too great, and if we had used $\frac{,22}{d'}$ Mr. Barlow's number, the results would have been still more anomalous.

If we are to make any use of the neutral axis it is clear that we must have it determined for each figure of section, whereas, Mr. Barlow by merely deducting ½ an inch from the total depth, would get the same deflection for all sections, whether with or without a lower web.

There is another curious thing in these experiments p. 103 second report.

'EXPERIMENTS MADE AT WOOLWICH TO ASCERTAIN THE STRENGTH AND STIFFNESS OF THE PARALLEL RAIL, WITH DOUBLE FLANCH, FOR THE NORTH UNION RAILWAY.

Weight per yard, 62lbs:; area of section, 6 inches; depth, $4\frac{1}{2}$ inches.

Results obtained from three single Experiments.

Weight	Deflection by Index.	Deflection for each ton.	Weight	Deflection by Index.	Deflection for each ton	Weight.	Deflection by Index.	Deflection for each ton.
1	·028		1	·035		1	·009	
2	·031	·003	2	·039	·004	2	·016	·007
3	·036	·005	3	·044	·005	3	·020	·004
4	·038	·002	4	·048	·004	4	·029	·009
5	·043	·005	5	·054	·006	5	·033	·004
6	·046	·003	6	·059	·005	6	·034	·001
7	·050	·004	7	·064	·005	7	·038	·004
8	·055	·005	8	·069	·005	8	·042	·004
9	·060	·005	9	·076	·007	9	·046	·004
10	·066	·006	10	·082	·006	10	·050	·004
11	·074	·008	11	·086	·004	11	·055	·005
12	·084	·010	12	·096	·010	12	·066	·011
Mean deflection, with 11 tons.		·051			·055			·051

Results obtained from the mean of three experiments.

Weight.	Deflection by Index.	Deflection for each Ton	Weight.	Deflection by Index.	Deflection for each Ton.	Weight.	Deflection by Index	Deflection for each ton.
1	·027		1	·021		1	·018	
2	·031	·004	2	·026	·005	2	·024	·006
3	·036	·005	3	·031	·005	3	·028	·004
4	·039	·003	4	·036	·005	4	·033	·005
5	·044	·005	5	·041	·005	5	·037	·004
6	·048	·004	6	·044	·003	6	·040	·003
7	·052	·004	7	·048	·004	7	·044	·004
8	·057	·005	8	·053	·005	8	·048	·004
9	·063	·006	9	·059	·006	9	·053	·005
10	·070	·007	10	·064	·005	10	·059	·006
11	·077	·007	11	·071	·007	11	·067	·008
12	·087	·010	12	·081	·010	12	·077	·010
Mean deflection, with 11 tons		·055			·055			·054

Computed mean deflection, ·055"

Now we have seen that the deflection of these rails is ,0717 for 11 tons, and it is for 10 tons

$$\begin{array}{r} ,066 \\ ,082 \\ ,050 \\ ,070 \\ ,064 \\ ,059 \\ \hline 6\,)\,,391 \\ \hline ,065 \\ \hline\hline \end{array}$$

In pages 50 and 51 second report, Mr. Barlow details as we have before said, the experiment of cutting nearly ⅛ of the whole section away from the lower web of a double T rail, and then after it is thus mutilated, it is found to be stronger than the mean strength assigned to it by Mr. Locke in previous experiments; which anomaly is attributed to the imperfection of the press employed to deflect the bar; this seems to imply that in Mr. Locke's hands, the press gave too small a deflection, although in another place it is mentioned as too large. Now, Mr. Barlow allows p. 15 second report, that it "may give pretty well the large and ultimate results," although it is "not sufficiently accurate for the purpose of the smaller strains;" the result here in question is tolerably large one would think, being 10 tons, yet we have as follows,

	Deflection with 10 tons.
Page 114 second report,	
"62lb. rail with ⅛ of the whole section cut away from the bottom web	,054"
Page 103 second report,	
"62lb. rail without mutilation	,065
62lb. rail by Mr. Locke without mutilation	,081"

Or too much rather than too little; and as a climax, Mr. Barlow himself p. 103 second report, states the deflection by computation &c., to be from ,051 to ,055 with 11 tons, although in the same page and only three lines above, the experimental deflection is registered from actual observation at ,0717. What have we here to do with calculation or hypothesis? We see the thing before our eyes; the rail does deflect ,0717, and why are we told that it only deflects ,055? We want to know what actually takes place in practice, not what *ought* to take place according to a mathematical formula and a computation:

A certain weight will come on the rail, and we require it to be strong enough to bear it; if it break down and upset a whole line of carriages, leaving the next train that comes up, to bring home the bones of the passengers in a competent number of coke sacks, what good will it do their relatives to be told that all this *ought* not to have happened, and that this can be demonstrated by a most approved mathematical formula; or that by (x, plus the rail, minus the square root of the stoker's old hat).dx all their bones *ought* to be safe and sound. Neither will the public be very fond of adventuring in these experiments on osteology. Passengers are not so plentiful as marines; when we get one of the latter killed on board ship, we have only to take his feather to the barracks, and there they will give us another man to wear it; but after expending a hundred or two of passengers, matters will assume rather a different aspect. The marines are used to it and don't mind it, but the passengers may very possibly have another way of thinking.

What then are we to say to pages 28 to 39, second report, or to the table of relative strengths and deflections of rails at 3, 4, 5, and 6 feet bearings, given in the second report, pages 70 and 71, for the purpose of assisting the directors of the London and Birmingham Railway in determining which length of bearing they should adopt.

At the first blush of the thing there are moral impossibilities in it, for in pages 70 and 71, leaving out the 3 feet 9 lengths we have

Bearings. Feet.	Weight per yard, of rails of equal strength.	1st diff.	Weight per yard, of rails of equal stiffness.	1st diff.
3	51,4 lbs.		51,4	
		12,5		20,6
4	63,9		72,0	
		3,5		20,0
5	67,4		92,0	
		11,6		30,5
6	79,0		122,5	

For instance, in rails all of the same general configuration, an addition of 20,6 of iron will make one at 4 feet bearings as stiff as one at 3 feet; by adding rather less, namely, 20lb. to the 4 feet rails, you get one as stiff for 5 feet bearings; but you must add 30½ lbs. to the 5 feet one, to get as stiff a one at 6 feet.

Again, it takes an average of 12lbs. to make a 3 or a 5 feet rail as strong as a 4 or a 6 feet; but by only adding $3\frac{1}{2}$lb. to a 4 feet you get as strong a one for 5 feet! In page 96 of the first report, Mr. Barlow surprises us enough by saying "We should be hard to believe that an increase of $\frac{1}{25}$ in the weight, could be made to add about $\frac{1}{9}$ to the strength, yet such is unquestionably the case," but in the above example this is beat hollow, and an increase of $\frac{1}{18}$ adds $\frac{1}{4}$ to the strength; namely, a 64lb. rail at 4 feet bearings, will, by adding $3\frac{1}{2}$lbs. to it, have exactly the same strength for a 5 feet bearing; these are not very felicitous instances of the advantage of "working by rule rather than by opinion," particularly when we recollect how these rules are affected by the position of the neutral axis, about which we positively know nothing more than Pat did, when he lost his kettle overboard, namely, that it was gone to the bottom; and so we know that there is such a thing as a neutral axis somewhere, as the Mevagissy Pilot said, when he was asked to point out where a certain rock was situated, "It's somewhere here abouts" said he, putting down the whole flat of his hand on the chart, and covering with it 2 or 300 square leagues of sea and land.

If we look at the neutral axis, as we have obtained it, from actual experiment, p. 145, L, we may guess it at about 2 : 3, for in one case it is in the middle of the bar, and in the other as 1 : 1,9. Now trying the 5 feet rail, given in page 31, second report, with this ratio and 9 tons as the longitudinal extension, we have by Mr. Barlow's rules, page 61.

$$\text{Head} \quad \frac{2 - {,}85 \,.\, 9}{3 \,.\, 12} = \frac{10{,}35}{36} = 0{,}29$$

$$\text{Rib} \quad \frac{5 - (5 - 2) \,.\, {,}85 \,.\, 9}{3} = \frac{114{,}75}{3} = 38{,}25$$

$$\text{Web} \quad 4 \,.\, (1{,}66 - {,}85) \,.\, 9 = 29{,}16$$

$$4 \,.\, 4 \,.\, 12 + 4 \,.\, 6 = \text{No. 1,} = 216$$

$$216 + 8 - 1 = \text{No. 2,} = 209$$

$$216 : 209 = 29{,}16 : \quad 28{,}2$$

$$\text{Total} \quad 66{,}77$$

$$\text{and} \quad \frac{66{,}74 \cdot 4}{60} = 4{,}45 \text{ tons greatest weight.}$$

rather an awkward conclusion for those concerned; and here be it observed, I have not altered the neutral axis at all in the 28 tons assigned to the lower web, but only in the head and rib. I have allowed 60 inches as the bearing length, not 57, because of the incomprehensible chairs with round bottoms which Mr. Barlow recommends, that being the practical effect of them; the alteration from a 3 feet rail on the present chairs, to a 5 feet one on the chairs above, being a jump from 2 feet 9 inches to 5 feet, or in the ratio of 36 to 216, a fearful fact against which we have only to balance an increase in weight of 17½lbs.

We have also to couple with this the following experiment; Mr. Barlow states, page 24, second report, that a wagon in a train, taking a lurch to itself, was found to throw up the index which measured the deflection, twice as much as the engine did, which bears 3 tons: now what becomes of the rail, when, instead of the light wagon taking the lurch, the heavy engine takes it? Yet, in the face of this experiment, Mr. Barlow only allows 7 tons strength to the rail, whereas the light wagon by his own statement requires 6 tons.

The presumed saving too, which looks remarkably well on paper, page 70, second report, turns out in practice to be a loss in the case of the London and Birmingham railway, of about £30,000, on the 75 miles, which are to be ornamented with this species of rail, block, chair, et hoc genus omnes.

As the efficiency of a structure is to be estimated at its weakest point, it is clear that the proper mode of proceeding in laying down a rail-road, is to test every rail as Mr. Barlow recommends; it must, however, be done in a different manner to what he says, page 95, viz. trying 50 or 60 a day, for at this rate it would take upwards of seven years to test the whole of the rails on the London and Birmingham line.

It must have been a mere slip of the pen which led Mr. Barlow to say, that when one wheel falls down from a higher part of the rail-road to a lower one, the other wheel bears for the time a double weight: a very rude experiment will shew this not to be the case, if it was, we should be badly off indeed, for the weight on one wheel being 3 tons, and this being doubled when the other wheel is falling, would be 6 tons, and this being, to say the least of it, increased to another

3 tons for lurching, would, at any rate, be 9 tons; whereas Mr. Barlow only recommends a strength of 7, and that these circumstances may take place at the same time is self-evident, unless we can enter into an amicable agreement with the engine, not to be taking unto itself a lurch just at the time one wheel happens to be off the ground; in fact the effect would be much more than the above, for it appears by the experiments that a light wagon in a lurch deflects the rail double the quantity which the heavy engine does, and taking the wagon at 1½ ton, and its load at 5, this would only be 6½ tons, or 1,6 ton on each wheel.

Before closing this chapter, I will add here a table of the elements often required by those who are obliged to recreate themselves with calculations of strength, deflection, &c.

Length inches.	Square.	Cube.	Depth inches.	Square.	Cube.
30	900	27000	$3\frac{1}{2}$	12,25	42,875
33	1089	35937	$3\frac{3}{4}$	14,0625	52,734375
36	1296	46656	4	16,	64,
42	1764	74088	$4\frac{1}{4}$	18,0625	76,765625
45	2025	91125	$4\frac{1}{2}$	20,25	91,125
48	2304	110592	$4\frac{3}{4}$	22,5625	107,171875
57	3249	185193	5	25,	125,
60	3600	216000	$5\frac{1}{4}$	27,5625	144,703125
69	4761	328509	$5\frac{1}{2}$	30,25	166,375
72	5184	373248	$5\frac{3}{4}$	33,0625	190,109375

The following will also be useful, extracted from Mr. Barlow's paper in the 3rd volume of "Transactions of the British Association."

"Let l, b, d, denote the length, breadth, and depth in inches in any beam, w the experimental breaking weight in pounds, then will $\frac{l\,w}{b\,d^2} = S$ be a constant quantity for the same material, and for the same manner of applying the straining force; but this constant is different in different modes of application. Or, making S constant in all cases for the same material, the above expression must be prefixed by a coefficient, according to the mode of fixing and straining.

"1. When the beam is fixed at one end, and loaded at the other,

$$\frac{l\,w}{b\,d^2} = S.$$

"2. When fixed the same but uniformly loaded,

$$\frac{1}{2} \times \frac{l\,w}{b\,d^2} = S.$$

"3. When supported at both ends, and loaded in the middle,

$$\frac{1}{4} \times \frac{l\,w}{b\,d^2} = S.$$

"4. Supported the same and uniformly loaded,

$$\frac{1}{8} \times \frac{l\,w}{b\,d^2} = S.$$

"5. Fixed at both ends, and loaded in the middle,

$$\frac{1}{6} \times \frac{l\,w}{b\,d^2} = S.$$

"6. Fixed the same, but uniformly loaded,

$$\frac{1}{12} \times \frac{l\,w}{b\,d^2} = S.$$

"7. Supported at the ends, and loaded at a point not in the middle. Then, $n\,m$ being the division of the beam at the point of application,

$$\frac{n\,m}{l^2} \times \frac{l\,w}{b\,d^2} = S.$$

"Some authors state the coefficients for cases

5 and 6 as $\frac{1}{8}$ and $\frac{1}{16}$, but both theory and practice have shewn these numbers to be erroneous.

"By means of these formulæ, and the value of S, given in the following Table, the strength of any given beam, or the beam requisite to bear a given load, may be computed. This column, however, it must be remembered, gives the ultimate strength, and not more than one third of this ought to be depended upon for any permanent construction.

"Retaining the same notation, but representing the constant by E, and the deflection in inches by δ, we shall have,

Case 1. $\frac{32}{1} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

2. $\frac{12}{1} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

3. $\frac{1}{1} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

4. $\frac{5}{8} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

5. $\frac{2}{3} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

6. $\frac{5}{12} \times \frac{l^3 w}{b\, d^3 \delta} = E.$

"Hence, again, from the column marked E in the following Table, the deflection a given load will produce in any case may be computed; or, the deflection being fixed, the dimensions of the beam may be found. Some authors, instead of this measure of elasticity, deduce it immediately from the formula $\frac{l^3 w}{3 b d^2 \delta} = \mathrm{E}$,

substituting for w the height in inches of a column of the material, having the section of the beam for its base, which is equal to the weight w, and this is then denominated the modulus of elasticity. It is useful in showing the relation between the weight and elasticity of different materials, and is accordingly introduced into the following Table.

"TABLE OF THE MEAN STRENGTH AND ELASTICITY OF VARIOUS MATERIALS, AS DEDUCED FROM THE MOST ACCURATE EXPERIMENTS.

Names of Materials.	Specific Gravity.	C Mean strength of cohesion on an inch section.	$S = \frac{l\,w}{4b\,d^2}$ Constants for transverse strains.	$E = \frac{l^3\,w}{b\,d^3}$ Constants for deflections.	Modulus of Elasticity.	Remarks.
Woods.		lbs.				
Acacia	710	......	1800	4609000	3739000	of English growth.
Ash	760	17000	2026	6580000	4988000	ditto.
Beech	700	11500	1560	5417000	4457000	ditto.
Birch, Common	700	......	1900	6570000	5406000	ditto.
——, American Black	750	......	1500	5700000	3388000	American.
Box	1000	20000				
Bullet Tree	1030	......	2650	10512000	5878000	Berbice.
Cabacully	900	......	2500	7427000	4759000	ditto
Deal, Christiana	680	11000	1550	6350000	5378000	
——, Memel...........	590	11000	1730	6420000	6268000	
Elm	540	5780	1030	2803000	3007000	English.
Fir, New England ...	550	12000	1100	5967000	6249000	
——, Riga.............	750	12600	1130	5314000	4080000	
——, Mar Forest	700	12000	1140	3400000	2797000	Scotland.
Green heart	1000	......	2700	10620000	6118000	Berbice.
Larch, Scotch	546	7000	1120	4200000	4480000	
Locust Tree	950	20580	3400	767000	4649000	America, South.
Mahogany..............	637	8000				
Norway Spars..........	580	12000	1470	5830000	5789000	
Oak, English from	700	9000	1200	3490000	2872000	Results very variable.
Oak, English to...	900	15000	2260	7000000	47020000	
——, African	980	14400	2000	9500000	55830000	
——, Adriatic	990	14000	1380	3880000	2257000	
——, Canadian	872	12000	1760	8590000	5674000	
——, Dantzic...........	760	14500	1450	4760000	3607000	
Pear Tree	646	9800				
Poon	600	14000	2200	6760000	6488000	East Indies.
Pine, Pitch	660	10500	1630	5000000	4364000	
——, Red	660	10000	1340	7360000	6423000	
Teak	750	15000	2460	9660000	7417000	East Indies.
Tonquin Bean..........	1050	......	2700	10620000	5826000	Berbice.
Iron.						
Iron, Cast from...	7200	16300	8100	69120000	5530000	
Iron, Cast to......	......	36000				
——, Malleable.........	7760	60000	9000	91440000	6770000	Mean of English and Foreign.
——, Wire..............	......	80000	......	....	....	

We see by these formulæ, what a difference there is between a bar fixed at the ends, and supported. Now, in a railroad, the bar, as far as these expressions are concerned, ought to be considered fixed, although we are obliged to take it as supported only, in order to be safe in practice.

CHAP. VI.

DEFLECTION OF RAILS UNDER DIFFERENT VELOCITIES OF THE ENGINE, AND CONCLUDING REMARKS.

From experiments made at the Liverpool and Manchester Railway, Mr. Barlow has deduced that the deflections of railways bars are the same at all velocities of the Engine, in fact the experiments shew that they are rather larger at great velocities than when the engine stands still; as this is an impossibility, leaving out, of course, all idea of impact and lurches, it will be well to look at this question a little, for which purpose the following table will be useful.

Velocity per hour miles.	Velocity per minute, yards.	Velocity per second, inches.	Pts. of a secd in which 18 inches are passed over	Pts. of a secd in which 24 inches are passed over.	Pts. of a secd in which 30 inches are passed over
10	293,33	176	$\frac{1}{9,8}$	$\frac{1}{7,33}$	$\frac{1}{5,86}$
20	586,66	352	$\frac{1}{19,6}$	$\frac{1}{14,7}$	$\frac{1}{11,73}$
30	879,99	528	$\frac{1}{29,3}$	$\frac{1}{22}$	$\frac{1}{17,6}$
40	1173,32	704	$\frac{1}{39,1}$	$\frac{1}{29,3}$	$\frac{1}{23,47}$
50	1466,65	880	$\frac{1}{48,9}$	$\frac{1}{36,7}$	$\frac{1}{29,3}$
60	1759,98	1056	$\frac{1}{58,7}$	$\frac{1}{44}$	$\frac{1}{35,2}$
70	2053,31	1232	$\frac{1}{68,4}$	$\frac{1}{51,3}$	$\frac{1}{41,1}$
80	2346,64	1408	$\frac{1}{78,2}$	$\frac{1}{58,7}$	$\frac{1}{46,9}$
90	2639,97	1584	$\frac{1}{88}$	$\frac{1}{66}$	$\frac{1}{52,8}$
100	2933,30	1760	$\frac{1}{97,8}$	$\frac{1}{73,3}$	$\frac{1}{58,7}$

or putting v for the velocity in yards per minute, v' for the velocity in yards per second, and a for the velocity in miles per hour, we have

$$\frac{1760 \, . \, a}{60} = 29{,}333 \, . \, a = v \; \& \; \frac{1760 \, . \, a}{3600} = {,}4888a = v'$$

Taking either of the right hand columns, according to the length of rail, for instance, the 18 inch column for a 3 feet rail &c. we have the number of inches through which the engine or any other body would fall by the action of gravity, in free space, in the time which it takes to pass over 18 inches by the formula,

$$s = t^2 \, . \, 193$$

Where t is the time in seconds, and s the space in inches; thus, at 20 miles an hour, and with a 3 feet rail, or in the time $\frac{1}{19,6}$ of a second, a body would fall $\left(\frac{1}{19,6}\right)^2 . 193 = \frac{1}{384,16}$.

$193 = \frac{193}{384,16} = ,5$; and at 30 miles an hour with a 3 feet rail, or in the time $\frac{1}{29,3}$ of a second a body would fall $\left(\frac{1}{29,3}\right)^2 . \ 193 = \frac{1}{858,49}$.

$193 = \frac{193}{858,49} = ,225$

And conversely knowing the distance an engine has to fall, from one rail at a joint being lower than the other, for instance, we have the space the engine will pass over without touching the lower rail, by the following formula.

$$t = \sqrt{\frac{s}{193}}$$

where t and s denote as before.

Thus when s = ,225, we have $\sqrt{\frac{225}{193}}$

$$= \sqrt{,001166} = ,0341 = \frac{1}{29,3} \text{ of a second}$$

in which at 30 miles an hour, we find by our table the engine will pass over 18 inches of tbe rail, describing in its fall a parabola modified by the action of the springs of the engine.

Mr. Locke, the Engineer of the Grand Junction railway, has also put this to the test of experiment, by bending a rail ,48 of an inch and then painting it, after which an engine and a train of carriages passed over this bend, and did not touch the paint for 22 inches.

There are two cases, in which this would affect a railway,—first, when the engine passes over a permanent curve, such as it would on an undulating railway, and second, when it passes over a variable curve, as it does when, by its own weight, it deflects the rail it is travelling on. These are perfectly distinct things, and should not have been confounded together. I see Dr. Lardner has been having a "flare-up" about that. The two cases are quite different, although, under certain restrictions, the same laws apply to both; in the one instance the engine passes over a ready made curve of a determinate form; in the other it has to make its own curve, and the form is unknown; it may be an arc of a circle, or a portion of a conic section, or a curve of a double curvature, or there may be a receding wave before the wheel, and a following one behind it, &c. &c. &c. About all which we know just nothing at all, simply because there has been no experiment made to ascertain anything about it, and we should do no practical good in entering into the subject till there has.

But that which we have to do with is this, the quantity which the engine would fall in free space, being in the case of a deflected rail, modified by the slope which that rail receives; if we assume, till we know something about it, that the slope is a plane, descending from one chair for 18 inches, or to the middle of the rail, and then ascending 18 inches to the next chair: we may investigate what would happen under these circumstances; our formula for the space then becomes,

$$S = \frac{H}{L} . 193 . t^2$$

Where H is the height of the plane and L its length.

Taking the deflection at ,1 of an inch, for an example, we have $\frac{H}{L}=\frac{1}{180}$ with a 3 foot rail,

hence $s=\frac{1}{180}\,.\,193\,.\,\frac{1}{384{,}16}=,00278$, when the velocity is 20 miles an hour, and

$$s=\frac{1}{180}\,.\,193\,.\,\frac{1}{858{,}49}=,00125$$ when the velocity is 30 miles an hour; or $\frac{1}{360}$ of an inch at 20 miles, and $\frac{1}{800}$ of an inch, at 30 miles an hour; and this is the space which gravity alone would cause the engine to descend in the same time that gravity and steam together carry it down 18 inches.

Let us next suppose that we have steam power enough to carry the engine along at a velocity so great, that gravity cannot bring it down the ,1 perpendicular, while steam is carrying it along the 18 inches horizontal, we shall find this velocity to be 44 miles an hour, for it takes $\frac{1}{43}$ of a second for a body to fall one-tenth of an inch by gravity, and $\frac{1}{43}$ second, : 18 inches, = 3600 seconds, : 2786400 inches, = 44 miles.

Thence it appears that at 44 miles an hour, and with a 3 foot rail, having a permanent curve of 1 tenth of an inch, the engine, after passing the chair, would not touch the rail till it came to the middle of it; and, of course, with a greater velocity, it would go proportionably beyond the middle before it touched it. We know that a velocity of 60 miles an hour has been attained; and with this, and a permanent curve, the engine would go along with a hop, step, and a jump, much like one of Marshall Vauban's ricochet shot, and in time perhaps would be able to leap a five-barred gate.

Now, in practice, the only difference between what we have supposed above (about the curve of the rail, not the five-barred gate), is, that instead of the engine going over a permanent curve of one-tenth of an inch, it has to make its own curve of the same depth; that is to say, its weight deflects the rail to that depth by our hypothesis. But one-tenth of an inch is, by experiment, found to be the stationary deflection, and if at 44 miles an hour the engine would not touch the first half of the rail, how can it be, confining the question as we before said, to weight only, that Mr. Barlow's experiments are correct when they shew a greater deflection at a rate of 20 or 30 miles an hour, then when the engine stood still? this is impossible; be the amount of deflection what it may, it must be a function of the velocity, and this agrees with some of the experiments, viz. page 13 and 14, second report, the mean of three trials of the Speedwell in motion, exclusive of joints, giving deflection ,0353, one of which trials being only ,027, while the same engine at rest gave ,04 nearly; the whole of these experiments, however, are very anomalous, and they evidently require to be repeated in much greater numbers, to enable us to form any practical conclusion whatever.

As, however, we see by the foregoing investigation that beyond a certain point, the greater the velocity the less the deflection, this shews that Mr. Barlow was right in abandoning his first recommendation, where looking to future expenses, he says, page 39, second report, " I must certainly prefer the large bars and longer bearings," for in addition to the reasons which he gives for this alteration, page 76, &c. second report, we see that with every increase in the length of the rail, we lose some of this decrease in the deflection from velocity; for instance, in a 5 feet rail it would take a velocity of about 75 miles an hour, to produce the effect described above on a 3 feet rail at 44 miles, so that in the late *improvements* we have been progressing like a crab at any rate, if not like a cow's tail. The French say us Englishmen are "Stupified by the climate and fattened with beer," this is rather severe certainly of Jean Crapaud, but it must be confessed, we do some most unaccountable things occasionally, for instance, we have a 50lb. fish-bellied rail in wear some time; it is thought advantageous to make this heavier and stronger; so far so good, and the fitting way to safely do so was proposed in February, 1835; but this prudent measure has descended to the tomb of all the Capulets, and now the first step is to substitute for the fish-belly, a parallel rail, which, weight for weight possesses less strength; we next add a few pounds of metal to the parallel, and then, for fear we should inadvertently have done a good thing by this, we put the blocks farther apart; thus giving with one hand and taking away with the other: and as a final godsend for luck, we put a chair to the rail, which makes the bearings 3 inches longer again; so that the "total of the whole" as the M. P. for Middlesex observes, is this; we first go from a bearing of 2 feet 9 inches to one of 4 feet, (including the effect of this unhappy chair) adding, for this increase in length, only 15lb. to the weight; and not contented with this, we jump to 5 feet with another addition of only 10 lbs. It requires no prophet to foretell the result of this mode of proceeding; not to mention that 75 miles is an awful length for a mere experiment, which in the present state of our knowledge, as it is the fashion to say, but as I should call it in the present state of our ignorance, this certainly is; for in the absence of proper experiments to determine the elements we have to work with, no trials for short lengths will settle the question.

I may as well remark here, that whenever I speak of a fishbellied rail, I mean that particular fish-belly which is based on elliptical ordinates.

The first rails of the fish-bellied form were much too weak in one particular point, and this was obviated by Mr. R. Stephenson, the talented Engineer of the London and Birmingham Railway, by collating, as far as possible, the ordinates with those of the ellipse; since which, of these, which Mr. Barlow properly calls "Mr. Stephenson's rails," there is hardly an instance of any having been broken in use, out of the thousands laid down now for years.

Mr. Barlow is therefore mistaken in speaking, p. 66, of the "many failures of fish-bellied rails, within a short distance of their point of support;" unless he means those of an improper form. He attributes this breaking to the deflection being greater than it is in a parallel at every other point except the middle; yet, p. 42, second report, the pressure is stated to be less as the slope is greater; and the former is explained to mean "that the change of direction of the tangent, is more rapid in that part." This does not clear up the contradiction; for it still stands that the force which urges the body being tangential to the rail, the change of direction of the tangent is more rapid, as the slope is greater, and this breaks the fish-bellied rail near the chair; yet, although this is done, the pressure is less.

Most of these various minutiæ, upon the right ordering of which, the ultimate chance of profitably working a railway so materially depends, may be totally neglected by those who select butt joints for their rails. There are some things which do not admit of argument, and I consider this to be one, that butt joints can ever compete with half lap ones, while the certain fact exists that we may, from alteration in the temperature alone, have a butt joint open nearly ¼ of an inch; while a half lap one, if properly made, will not have a tenth of that opening. At the velocities which are now attained, let alone those which we may reasonably hope to arrive at, a joint open nearly a quarter of an inch is almost as bad as a stage coach attempting to drive over the chasm of an earthquake, before encountering which it would be advisable for all those who could manage it, to give three cheers and jump overboard.

Temperature has more effect than this on iron, although that of England would not be very material perhaps. I was beginning to try some experiments on this at St. Helena, in the year 1818, but as the worthies there could not exactly understand what I was about, they denounced me as a spy of the Emperor's; and I suppose I may think myself very fortunate in not being made to swallow half a dozen two-and-thirty pound shot, or perhaps tried by a court martial, and, sinner that I was, found guilty, like Surgeon Stokoe, of calling Napoleon "the patient," or some other equally high crime and misdemeanor; "interfering with the preservation of the balance of power in Europe, and the safety and stability of his Majesty's government at home and abroad." (Vide any of the hero of Capri's St. Helena proclamations.)

With respect to a rail deflecting, we have another very interesting opinion brought forward by Mr. Barlow, p. 85, second report. "The greatest resistance a heavy load experiences, in consequence of the deflection of the bar over which it passes, is to the constant resistance it would experience in ascending an inclined plane, whose height is equal to the central deflection, as

,384 : 5:" and the sum of all the resistances, in the two cases is, that on the deflected bar they amount to only "half" what they are on an inclined plane.

Mr. Barlow restricts this comparison to bars of short lengths, and there are one or two other little things, which may probably affect the quantity of the result, which is given as one half, for instance, the springing up of the rail behind the wheel; but in the main question, as far as I can judge, Mr. Barlow is perfectly correct. Noting, as we have before done, that this has nothing to do with the slopes of cuttings and embankments.

Now, this important consideration, namely, that an engine is opposed by a constant plane, equal in height to the central deflection of the rail, and in length to the length of the rail, and that this resistance all takes place in the last half of the rail passed over, will at once shew what a serious thing deflection is. We may also see how it affects our preceding argument respecting the deflection being a function of the velocity, for the effect of gravity down the first half of the rail will be lessened one half; and yet, after the splendid triumph of the Liverpool and Manchester railway, and although such vast sums are now at stake in these constructions, some of which amount to nearly four millions, we have no means of computing this element for want of a few experiments! All is utter confusion and guess work; we are worse off than Barney's Brig, when she had both foretacks on board. For instance, Mr Barlow calculates the effect on the power of the engine thus, for the bars he patronizes.

Feet bearings.	Deflection at 3 Tons	Equivalent planes	Increase power per ton
3 „ 0	,024	1 in. 3000	,75 lbs.
3 „ 9	,037	„ 2432	,92
4 „ 0	,041	„ 2341	,96
5 „ 0	,064	„ 1875	1,2
6 „ 0	,082	„ 1756	1,3

The following table will enable us to see how this may alter through what we have before shewn, including the effect of the miraculous round-bottomed chair.

Length of bearing in inches:	1	2	3	4	5	6	7	8
33	,024	,026	,03	,02357	,0272	,0286	,072	,0858
42	,037	,050	,06	,047	,0538	,0569	,111	,1707
45	,041	,063	,072	,056	,064	,068	,123	,204
57	,064	,122	,139	,108	,123	,131	,192	,393
60	,074	,150	,171	,1335	,153	,162	,222	,486
69	,082	,210	,241	,188	,216	,228	,246	,684

Column 1, is Mr. Barlow's deflections, p. 30 second report &c., except for the 5 feet length, which is computed as the cube of the length, from the 4 feet 9 instance.

Column 2, the deflections by Mr. Barlow's formula $\frac{,275}{d} = \frac{,22}{d'}$ p. 47

Column 3, the deflections by the same formula, corrected to $\frac{,315}{d}$ p. 132 L.

Column 4, the deflections by Mr. Barlow's formula $\frac{,22}{d'}$ and the neutral axis taken ½ an inch from the head as he directs p. 60 &c.

Column 5, the deflections by column 4, corrected by simple proportion, to agree with the corrections in column 3, viz. $\frac{,275}{d} : \frac{,315}{d}$ = column 4 : column 5.

Column 6, mean of columns 3 and 5, which appear to me as the best approximations we can get.
Column 7, deflection with 9 tons from column 1.
Column 8, deflection with 9 tons from column 6.

The whole are reduced to 3 tons, and for their respective lengths of bearing.

The 5 feet example applies to 75 miles of the London and Birmingham railway; with a depth of 4,75 inches, and with the deflections above, we get the following constant planes to be overcome by Mr. Barlow's theory.

Column 1,	δ = ,074	$\frac{2l}{\delta}$	=	1 in. 1621
„ 2,	„ = ,150	„	=	„ 800
„ 3,	„ = ,171	„	=	„ 702
„ 4,	„ = ,1335	„	=	„ 899
„ 5,	„ = ,153	„	=	„ 791
„ 6,	„ = ,162	„	=	„ 741

This is probably too high an amount, as I have before remarked, but the subject ought to be taken into the most serious consideration of those persons, who are now undertaking new railways.

We also see that with Mr. Barlow's deflections at the full 60 inches, the amount with 9 tons is

,222; that 9 tons is a probable stress we have before shewn from Mr. Barlow's Liverpool experiments; yet, p. 70, a deflection of ,112 injures the elasticity of a 50lb. rail at 33 inches bearing, and by p. 103 second report, we find that a 62lb. rail at, it is to be presumed, 30 inches, deflects ,06 with 9 tons, being ,432 with 60 inches bearing.

Besides which, when the blocks are at 5 feet apart, if we allow an additional foot of surface to the joint blocks, we have 14080.5 + 28160.4 = 70400 + 112640 = 183040 square feet for the surface supporting the train for one hour, at as velocity of 20 miles; whereas, it is 70400.4 = 281600 square feet, when they are 3 feet apart, being a difference of 98560 square feet per hour, in favour of the short distance.

At p. 98 second report, Mr. Barlow investigates the form of rib and lower web, most advantageous for strength, and he states that if

a = breadth of head

b = breadth of rib

e = depth of lower web

x = depth of rib, and lower web, under the head.

$$\text{then } x^3 - \tfrac{3}{4}\left(\frac{a+eb}{b}\right)x^2 = -\frac{e^2 a}{4\,b}$$

gives a maximum strength; and as an example, with ,78 $= b$; the whole depth $= 5$; depth below neutral axis $= 4{,}5$; area $=$,78.4½ $= 3{,}51 = a$ and $e = 1$ the equation becomes

$$x^3 - 4{,}11\ x^2 = -1{,}12$$

or $x = 4{,}04$ and 4,04.,78 $= 3{,}15 =$ area of middle rib, and $a - b\,x = 3{,}51 - 3{,}15 = {,}36 =$ area of lower web,and also its breadth, the depth being 1. The equation is $x^3 - 4{,}125\ x^2 = -1{,}125$ as I have put it, for as it stands by a typographical error, x instead of x^2 it gives

$$65{,}94 - 16{,}6 = 49{,}34$$

If we try the same equation with a rail 4,54 deep

$$\begin{aligned} \text{then } a &= {,}71\ .\ 4{,}04 = 3{,}15 \\ b &= {,}78 \\ e &= 1 \end{aligned}$$

$$\frac{3}{4}\cdot\frac{{,}78\ .\ 3{,}15}{{,}78} = \frac{11{,}79}{3{,}12} = 3{,}779 \text{ and } -\frac{a}{4\,b}$$

$$= -\frac{3{,}15}{3{,}12} = -1{,}00961$$

and the equation is

$$x^3 - 3{,}779\ x^2 = - 1{,}01$$

Whence $x = 3{,}707$ and the whole depth of rail 4,207

With a rail 4,207 deep, we have

$$a = ,78 \,.\, 3{,}707 = 2{,}89$$
$$b = ,78$$
$$e = 1$$

$$\frac{3}{4} \cdot \frac{,78 + 2{,}89}{,78} = \frac{11{,}01}{3{,}12} = 3{,}53 \text{ and } - \frac{a}{4b}$$

$$= - \frac{2{,}89}{3{,}12} = - ,923$$

and $x^3 - 3{,}53\ x^2 = - ,923$

whence $x = 3{,}452$ and the whole depth 3,952

And with a rail = 3,952 we have

$$a = 78 \,.\, 3{,}452 = 2{,}6926$$
$$b = 78$$
$$e = 1$$

$$\frac{3}{4} , \frac{,78 + 2{,}6926}{,78} = \frac{10{,}4178}{3{,}12} = 3{,}34$$

and $x^3 - 3{,}34\ x^2 = - ,863$ and the whole depth 3.76

by the second example x has decreased ,333; by the third ,255; by the fourth ,192, &c. all these are only approximations, because ½ an inch ought not to have been subtracted, and afterwards added in each case, but the $\frac{1}{10}$ of the given depth according to Mr. Barlow; but, by what has gone before, we have just as much right to take the neutral axis $\frac{1}{2}$ an inch from the bottom, as $\frac{1}{2}$ an inch from the top.

Let us next see what we gain by this maxima and minima problem. The strength of the first example is, by Mr. Barlow, 7,3 tons, this would be different when the iron is taken at 9 tons instead of 10, but let us allow 10 tons for the sake of comparison; for the rail in its first shape without a lower web, and 5 inches deep; we then get

$$\text{Head}\quad \left.\begin{array}{l} (2 - ,78)\,.\,10 = 12,2 \\ (5 - \tfrac{1}{2})\,.\,12 = 54, \end{array}\right\} = ,226$$

$$\text{Rib}\quad \frac{5\,.\,4,5\,.\,,78\,.\,10}{3} = \qquad 58,5$$

$$\begin{array}{r} 58,726 \\ 4 \\ \hline 33)234,204 \\ \hline 7,12 \ \text{tons.} \end{array}$$

Hence we gain, it appears, 18 hundredths of a ton only for all our trouble, except that we see this curious thing, that the head in the two rails being precisely similar in breadth, depth, and shape, is ,024 of a ton weaker in a 5 inch rail than in a $4\frac{1}{2}$ one.

Let us compare this gain with what we lose by having a rail 60 inches long, instead of 57, that being one effect of the awful chairs, which Mr. Barlow has recommended.

If with 57 inches, it is, say 7 tons strength, then

$$\frac{7 \cdot 57}{60} = 6{,}65$$

or a loss of ,35th hundredths of a ton, being twice as much as we gained by taking the trouble to make the rail with a bottom web.

Mr. Barlow's problem should shew us in addition to what it does, the thickness of a bottom web which gives a maximum strength; but the fact is, we are working at the wrong end; we want experiments, not problems; if the heavens were to rain problems, none of them would help us after their failing in such able hands as Mr. Barlow's.

If we try the same formula with a 5 inch rail, and the neutral axis 1 inch from the top instead of ½ an inch, we have.

$$3{,}12\ a = \frac{3}{4} \cdot \frac{3{,}9}{,78} = \frac{11{,}7}{31{,}2} = 3{,}75\ \&\ -\frac{a}{4b} = -1,$$

$,78 = b$

$e = 1$

Whence $x^3 - 3{,}75\ x^2 = -1$

and $x = 3{,}67$ nearly, and the whole depth 4,67. By this it appears, that as the neutral axis is lowered, the rail must be deeper.

If we take the neutral axis in the middle, we have

$a = 2{,}5\ .\ ,78 = 1{,}95$ $\qquad \frac{3}{4} \cdot \frac{,78 + 1{,}95}{,78} = \frac{3}{4} \cdot \frac{2{,}73}{,78}$

$b = ,78$

$$= \frac{8{,}19}{3{,}12} = 2{,}62$$

$c = 1$

$$-\frac{a}{46} = -\frac{1{,}95}{3{,}12} = -,62$$

and $x^3 - 2{,}62\ x^2 = -,62$

Where $x = 2{,}522$, or the whole depth 5,022 nearly, so that here we should weaken our rail by making a bottom web, and for aught we know this is the correct view of the subject; for the neutral axis is, if any thing, more likely to be near the middle than any where else, by Mr. Barlow's experiments, see p. 146 L; yet upon these calculations, rails of 60 inches between the bearings are to be ventured on, without a corresponding increase in their weight; the longest ever used before, except for experiments, being 33 inches. In other words we are running a "pretty particular considerable" chance of cutting our own throats for the sake of an hypothesis. It will not be a bad thing for those passengers who first try their luck, to carry a fork along with them, then if they do break down they may be able to prick for the softest block to deposit their sterns on till the next train comes by; not a very comfortable prospect, unless perhaps for a moon-struck poet, who might, ad interim, compose a sonnet on a cauliflower, or any other interesting object in the back-ground.

We may see what increase of power will be required to ascend the planes, consequent on a 60 inch rail, for as my numbers average $\frac{1}{787}$ and as the increased power per ton from gravity is

$\frac{2240}{a}$ in pounds; where a is the denominator of the fraction expressing the slope, we have $\frac{2240}{787}$ = 2,85lbs. or about ⅓ of the effective power of the present engines on a level, which is 8¼ lbs. per ton; and although this is most probably too large, the enquiry is one of great moment to engine makers, as well as railway makers.

In every stage we must recollect how our errors are multiplied at high velocities, and provide accordingly. There is an instance of this in some rails lately laid down near Birmingham, by Mr. Forster of Stourbridge, who by sacrificing a portion of his profits, has produced an article ten to one superior in practical efficiency to other rails not a hundred miles off.

If Mr. Barlow's theory turns out to be correct, which I certainly think it will, there will be a fine field for projectors; together with "recoverers," instead of "discoverers;" or, as Troughton used to call them, "resurrection men." If the maximum power of boreing, possessed by other classes of persons, is represented by any number of horses, a projector will in general have a five horse power more; and if for "horses" we enunciated it "asses" we should often be not far out. Only fancy a cool, comfortable, well disposed dozen directors of a canal or a railroad, or anything else, who are willing to sign the requisite number of checks every day they meet, and sail quietly on in the world, broke in upon all of a sudden by a projector, possessing a hundred ass power of boreing, and bearing in his hand an invention which would rival the lucubrations of the learned pig, or ten to one might be worshipped without breaking the commandment, for it is a great chance if it is like anything in the heavens above, or the earth below, or the waters under the earth; and yet the inventor himself hugs it in his arms, as if it would pay the national debt, and looks as pleased and as proud as a dog with a tin tail.

It is time, however, that I should draw to a conclusion, and all my hope is that the discussion may lead to some practical good; to the public on the one hand, and to the pockets of the shareholders on the other. I have put together this work in the last few weeks, under the most unfavourable circumstances, and without being able to repeat some of the calculations; it has been written, in fact, from hand to mouth for the press, under the terror of a roar from the printer's devil for "copy," and in addition to my usual avocations; which consist in working twenty three hours in the week more than the negroes did when I was in the West Indies. I only regret I cannot, at present, extend the subject to blocks and chairs, as well as rails, in each of which there is nearly as much to say, but this must be reserved, at any rate for the present.

It only remains for me to add, that when I did make up my mind to the task, I sat down to it with right good humour; in fact, I have seen so many *improvements* lately that it was impossible to be very serious; but if there is anywhere an expression, which by any possible construction can appear in an offensive shape to Mr. Barlow, I most explicitly state, that such was farthest from my intention. I appear here as his antagonist, but I assure him, that among all his numerous acquaintance, there is no one who has a higher respect for his talents as a mathematician, or a greater esteem for his character as a man, than the author of these pages.

FINIS.

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PLATE II.

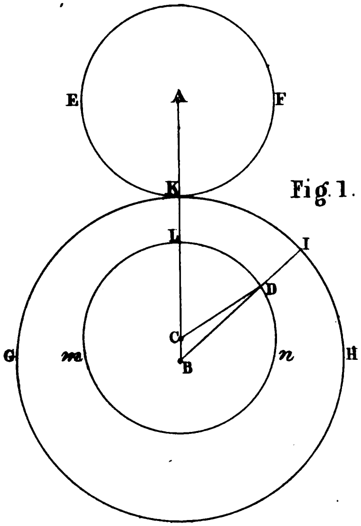


Fig. 1.

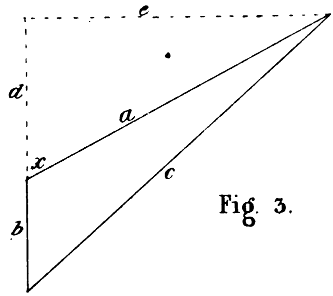


Fig. 3.

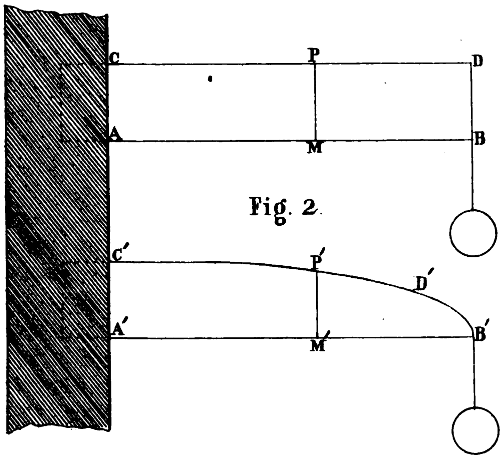


Fig. 2.

Fig. 5.

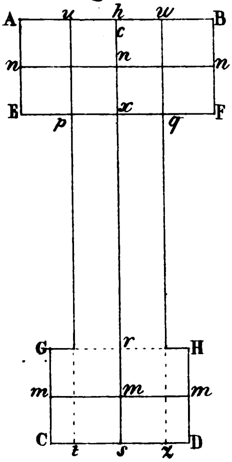
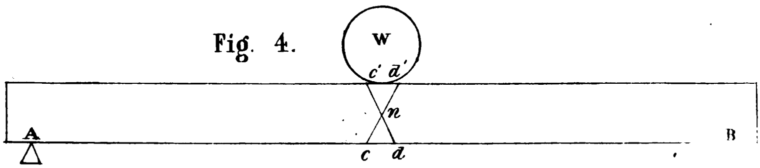


Fig. 4.



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