

# A Lunar Theory Reasserted<sup>1</sup>

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Until recently [Siegel, 1961; Senior and Siegel, 1961] we have not attempted to answer criticisms of our lunar theory in the belief that little is gained by a continual contest of words about what are, after all, only theories based on a limited amount of experimental data. In addition, it is probable that in the near future new experimental results will be obtained which will indicate with more certainty the structure and composition of the lunar surface, and which will then permit a more rigorous analysis of the scattering mechanism at radar wavelengths, and this would be the logical time to assess the merits of the rival theories. There would be little point in restating our own theory were it not for the fact that some of the more recent criticisms of it are based on an incorrect appreciation both of its origins and of its main points. This is particularly apparent in the recent paper by Winter [1962], and a brief restatement of our thesis is therefore necessary.

In two earlier papers [Senior and Siegel, 1959 and 1960], which we shall refer to as A and B respectively, a theory of lunar scattering was presented which was in accordance with all the experimental data available at that time (and, incidentally, since that time also), and which was both self-consistent and simple. The data did not appear to warrant the complications which are attendant on the introduction of statistics into the theory. A study of the paper by Winter [1962] only confirms us in that view, and it is perhaps one of the fallacies of the statistical approach that one can manufacture an understanding of a physical process by introducing enough arbitrary constants and functions into the theory by which one seeks to explain the process. Clearly any finite body of data can be "explained" if a sufficient number of undetermined constants are retained within the theory, but an approach of this type does not necessarily contribute to an understanding of the physical processes involved. If, on the other hand, the theory has a logical basis and only requires of the experimental data that it specify numerical values for a small number of parameters inherent in the scattering process, one criterion for assessing the theory is the degree to which all the experimental data can be satisfied. When viewed in this light, many of the recent lunar theories would not appear successful. For example, the agreement between theory and experiment in figures 3 and 4 of Winter's paper is hardly convincing and in addition there are aspects of the basic theory which must be criticised.

Before doing so, however, we would like to correct certain statements about our own theory which are contained in Winter's paper, and for this purpose it may be desirable to set out in summary form the salient features of the theory proposed in B:

- (i) the theory is in accordance with the results of all the lunar experiments made to date;
- (ii) it explains in a simple manner the nature of the specular return which is received;
- (iii) it leads to a radar cross section which has the observed pulse length dependence, and relates this dependence to the structure of the lunar surface;
- (iv) it enables the electromagnetic parameters of portions of the lunar surface to be derived; and
- (v) it is the only theory for which these parameters are consistent with the measurements [Salomonovich, 1960; Troitskii, 1960] of the thermal radiation from the lunar surface, and on the basis of this agreement estimates of the thermal conductivity and volumetric specific heats of the lunar surface were derived [Senior, Siegel, and Giraud, 1962].

One of the main criticisms leveled at this theory is the assumption of a small number of specular signals contributing to the initial peak return from the moon, and the occurrence of these signals was attributed to the presence of a corresponding number of smooth surfaces whose orientations are such as to provide specular contributions. These surfaces were termed "key scattering areas," and in A their number was estimated to be "of order ten or less." This conclusion was reached from a study of individual pulses obtained by Yaplee [1957, 1958], but because of the later work of Aarons et al. [1959] which became known to us in October 1959, a further analysis of Yaplee's data was carried out, and as a result of this the number of areas was increased to "somewhere between<sup>2</sup> 20 and 30." This revised number was published in B and was in no sense prompted by criticisms of Hughes [1960] and

<sup>1</sup> Because of the controversial nature of this subject matter Dr. Siegel was allowed, with the permission of Dr. Winter, to see the Winter manuscript "A theory of radar reflections from a rough moon." For this reason the two viewpoints appear here in the same issue.—*Editor.*

<sup>2</sup> The actual analysis produced the number 28, though the data cannot be relied upon to this degree of accuracy. In addition, the actual number contributing at any one instant may well vary with time, and would be expected to do so on our theory.

others, as stated by Winter. This can be verified by a study of the dates involved. Nor is Winter correct in implying that our theory requires the area around the sub-terrestrial point to differ in its scattering properties from the remainder of the moon. The only relevant factor is the orientation of such smooth areas as exist, and we believe that the distribution of slopes relative to the mean lunar sphere is such that no matter where in space the moon is observed from, a comparable collection of contributing areas would be found.

Winter further states that our theory "does not provide a functional formalism which permits quantitative comparison with experimental results obtained using a variety of pulse lengths," but rather would we assert that ours is one of the few (if not the only one) that does succeed in this regard. As pointed out in A and emphasized in B, the radar cross section of the moon observed under good propagation conditions and measured in terms of the peak return received from the lunar surface is of the form

$$\sigma = W |R|^2 \pi a^2 \quad (1)$$

where  $a$  is the radius of the mean lunar sphere,  $|R|^2$  is a typical power reflection coefficient and  $W$  is a factor which depends on the number of contributing areas and the degree of coherence between their individual returns. Because of the distribution in "depth" of these areas on the lunar surface,  $W$  is indeed a function of pulse length whose value can be deduced from Trexler's modulation loss law. Relative to its value for a 2  $\mu$ sec pulse,  $W$  is as follows:

TABLE 1

Pulse length	$W$
300 $\mu$ sec to cw	158
200 $\mu$ sec	126
30 $\mu$ sec	22
10 $\mu$ sec	3.2
2 $\mu$ sec	1

All of the available data on the moon's scattering properties are listed in table 2 and are in agreement with eq (1) to within  $\pm 3$  db if  $|R|^2$  is given the value  $5 \cdot 10^{-4}$ .

It is not clear to the present authors why the above does not provide the "functional relationship" which Winter denies us.

Turning now to the theory which Winter himself proposes, we are intrigued by his handling of the initial peak return. As the outcome of his statistical analysis, Winter obtains an equation (4.17) which gives merely the smooth sphere optics cross section, but does not associate any pulse length dependence to go with this. To "determine" the reflection coefficient he uses the parameters appropriate to a typical rock, but even when the permittivity is decreased by a factor 2 (to take account of the presumed granular character of a rock in vacuo), the resulting reflection coefficient would still yield specular results for small pulse lengths which are orders of magnitude greater than are observed. If he patches his equation with (7.4) to account for this discrepancy, we find that he has in fact merely duplicated the cross section values given in the very paper which he criticises. Moreover, it would appear that for the type of surface distribution which Winter advocates, the individual peaks in a short pulse return would be broader than the transmitted pulse, which is at variance with the experimental data. These criticisms are entirely apart from the fact that to use a smooth surface reflection coefficient with a roughness which the experimental data demands of a rough surface theory is without any theoretical foundation.

The above remarks are in no sense a general indictment of statistical theories, and indeed we are convinced that a statistical picture is the only basis on which to interpret the rough component measured by Pettengill [1960]. On the other hand, the use of statistics must not be regarded as a substitute for an underlying physical theory.

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TABLE 2. Power return from the moon

Frequency	Wavelength	$\frac{\sigma}{\pi a^2}$	Probable error (where known)	Pulse length	Source	"Modified" data	"Modified" data	"Modified" data	"Modified" data	"Modified" data
<i>Mc/s</i>	<i>m</i>									
10,000	0.03	$9 \times 10^{-2}$	2.5 db	cw	Kobrin (see Evans)	2-5 $\mu$ sec $5.7 \times 10^{-4}$	10 $\mu$ sec $1.8 \times 10^{-3}$	30 $\mu$ sec $1.3 \times 10^{-2}$	203 $\mu$ sec $7.2 \times 10^{-2}$	cw $9.4 \times 10^{-2}$
3,000	.10	$1.05 \times 10^{-1}$	3.5 db	cw	Kobrin (see Evans)	$6.6 \times 10^{-4}$	$2.1 \times 10^{-3}$	$1.5 \times 10^{-2}$	$8.3 \times 10^{-2}$	$1.0 \times 10^{-1}$
3,000	.10	$4 \times 10^{-4}$		5 $\mu$ sec	Hey and Hughes	$4 \times 10^{-4}$	$1.3 \times 10^{-3}$	$9.0 \times 10^{-3}$	$5.0 \times 10^{-2}$	$6.3 \times 10^{-2}$
2,860	.10	$3 \times 10^{-4}$	4 or 5 db	2 $\mu$ sec	Yaplee et al.	$3 \times 10^{-4}$	$9.5 \times 10^{-4}$	$6.7 \times 10^{-3}$	$3.8 \times 10^{-2}$	$4.7 \times 10^{-2}$
915	.33	$9 \times 10^{-2}$	3 db	msec	Aarons et al.	$5.7 \times 10^{-4}$	$1.8 \times 10^{-3}$	$1.3 \times 10^{-2}$	$7.2 \times 10^{-2}$	$9.0 \times 10^{-2}$
488	.61	$5 \times 10^{-2}$	3 db	cw	Blevis and Chapman	$3.2 \times 10^{-4}$	$1.0 \times 10^{-3}$	$7.2 \times 10^{-3}$	$4.0 \times 10^{-2}$	$5.1 \times 10^{-2}$
440	.68	$6.7 \times 10^{-2}$		100 $\mu$ sec	Pettengill and Henry	$6.7 \times 10^{-4}$	$2.1 \times 10^{-3}$	$1.5 \times 10^{-2}$	$8.4 \times 10^{-2}$	$1.1 \times 10^{-1}$
412.85	.73	$7.4 \times 10^{-2}$	1 db	cw	Fricker et al.	$4.7 \times 10^{-4}$	$1.5 \times 10^{-3}$	$1.0 \times 10^{-2}$	$5.9 \times 10^{-2}$	$7.4 \times 10^{-2}$
400	.75	$1 \times 10^{-1}$	3 db	msec	Leadabrand	$6.3 \times 10^{-4}$	$2.0 \times 10^{-3}$	$1.4 \times 10^{-2}$	$7.9 \times 10^{-2}$	$1.0 \times 10^{-1}$
300	1.00	$5 \sim 9 \times 10^{-2}$	4 db	cw	Trexler	$4.4 \times 10^{-4}$	$1.4 \times 10^{-3}$	$9.9 \times 10^{-3}$	$5.5 \times 10^{-2}$	$6.9 \times 10^{-2}$
201	1.49	$7 \times 10^{-2}$	3 db	msec	Aarons et al.	$4.4 \times 10^{-4}$	$1.4 \times 10^{-3}$	$9.9 \times 10^{-3}$	$5.5 \times 10^{-2}$	$6.9 \times 10^{-2}$
200	1.50	$6 \sim 10 \times 10^{-2}$	4 db	cw	Trexler	$5.0 \times 10^{-4}$	$1.6 \times 10^{-3}$	$1.1 \times 10^{-2}$	$6.3 \times 10^{-2}$	$7.9 \times 10^{-2}$
151	1.99	$5 \times 10^{-2}$		cw	Webb	$3.2 \times 10^{-4}$	$1.0 \times 10^{-3}$	$7.2 \times 10^{-3}$	$4.0 \times 10^{-2}$	$5.0 \times 10^{-2}$
120	2.50	$1 \times 10^{-1}$	3 db	30 msec	Evans	$6.3 \times 10^{-4}$	$2.0 \times 10^{-3}$	$1.4 \times 10^{-2}$	$7.9 \times 10^{-2}$	$1.0 \times 10^{-1}$
100	3.00	$1 \times 10^{-1}$	3 db	$\mu$ sec	Leadabrand	$6.3 \times 10^{-4}$	$2.0 \times 10^{-3}$	$1.4 \times 10^{-2}$	$7.9 \times 10^{-2}$	$1.0 \times 10^{-1}$

\*An estimated modulation loss correction of 100 (22-24db) has been applied.

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