N°. XVIII.

Obſervations on the Theory of Water Mills, &c. by W. WARING.

Read June 15, 1792.

BEING lately requeſted to make ſome calculations relative to mills; particularly Doct. Barker's conſtruction, as improved by James Rumſey, I found more difficulty in the attempt than I at firſt expected. It appeared neceſſary to inveſtigate new theorems for the purpoſe, as there are circumſtances peculiar to this conſtruction, which are not noticed, I believe, by any author; and the theory of mills, as hitherto publiſhed, is very imperfect, which I take to be the reaſon it has been of ſo little uſe to practical mechanics.

The firſt ſtep, then, toward calculating the power of any water-mill (or wind-mill) or proportioning their parts and velocities to the greateſt advantage, ſeems to be,

The correction of an eſſential miſtake adopted by writers on the Theory of Mills.

This is attempted with all the deference due to eminent authors, whoſe ingenious labours have juſtly raiſed their reputation and advanced the ſciences; but when any wrong principles are ſucceſſively publiſhed by a ſeries of ſuch pens, they are the more implicitly received, and more particularly claim a public rectification; which muſt be pleaſing, even to theſe candid writers themſelves.

George Atwood, M. A. F. R. S. in his maſterly treatiſe on the rectilinear motion and rotation of bodies, publiſhed ſo lately as 1784, continues this overſight, with its pernicious conſequences, through his propoſitions and corollaries (page 275 to 284.) although he knew the theory was ſuſpected: for he obſerves (page 382) "Mr. Smea-

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"ton in his paper on mechanic power (publiſhed in the "Philoſophical Tranſactions for the year 1776) allows, "that the theory uſually given will not correſpond with "matter of fact, when compared with the motion of ma-"chines; and ſeems to attribute this diſagreement, rather "to deficiency in the theory, than to the obſtacles which "have prevented the application of it to the complicated "motion of engines, &c. In order to ſatisfy himſelf con-"cerning the reaſon of this diſagreement he conſtructed a "ſet of experiments, which, from the known abilities "and Ingenuity of the author, certainly deſerve great con-"ſideration and attention from every one who is inter-"eſted in theſe inquiries." And notwithſtanding the ſame "learned author ſays, "The evidence upon which the "theory reſts is ſcarcely leſs than mathematical." I am ſorry to find, in the preſent ſtate of the ſciences, one of his abilities concluding (page 380) "It is not probable that the theory of motion, however inconteſtible its principles may be, can afford much aſſiſtance to the practical mechanic," although indeed his theory, compared with the above cited experiments, might ſuggeſt ſuch an inference. But to come to the point, I would juſt premiſe theſe

Definitions.

If a ſtream of water imping againſt a wheel in motion, there are three different velocities to be conſidered, appertaining thereto, viz.

Firſt, the abſolute velocity of the water:

Second, the abſolute velocity of the wheel:

Third, the relative velocity of the water to that of the wheel, *i. e.* the difference of the abſolute velocities; or the velocity with which the water overtates or ſtrikes the wheel.

Now the miſtake conſiſts in ſuppoſing the momentum, or force of the water againſt the wheel, to be in the *duplicate ratio of the relative velocity*: Whereas.

Prop. I.

The force of an invariable ſtream, impinging againſt a Mill-Wheel in motion is in the *ſimple direct proportion of the relative velocity.*

For, if the relative velocity of a fluid againſt a ſingle plane be varied, either by the motion of the plane, or of the fluid from a given aperture, or both, then, the number of particles acting on the Plane in a given time, and likewiſe the momentum of each particle, being reſpectively as the relative velocity, the force on both theſe accounts, muſt be in the *duplicate* ratio of the relative velocity, agreeably to the common theory, with reſpect to this *ſingle plane*; but, the number of theſe planes, or parts of the wheel acted on in a given time, will be as the velocity of the wheel, or *inverſely as the relative velocity*; therefore, the moving force of the wheel muſt be in the ſimple direct ratio of the relative volocity. Q. E. D.

Or, the propoſition is manifeſt from this conſideration; that, while the ſtream is invariable, whatever be the velocity of the wheel, the ſame number of particles or quantity of the fluid, muſt ſtrike it ſome where or other in a given time; conſequently, the variation of force is *only* on account of the varied impingent velocity of the ſame body, occaſioned by a change of motion in the wheel; that is, the momentum is as the relative velocity.

Now, this true principal ſubſtituted for the erroneous one in uſe, will bring the theory to agree remarkably with the notable experiments of the ingenious Smeaton, before mentioned, publiſhed in the Philoſophical Tranſactions of the Royal ſociety of London for the year 1751, Vol. 51, for which the honorary annual medal was adjudged

judged by the ſociety, and preſented to the author by their preſident. An inſtance or two of the Importance of this correction may be adduced as follow.

Prop. II.

The velocity of a wheel, moved by the impact of a ſtream, muſt be half the velocity of the fluid, to produce the greateſt poſſible effect.

For, let $\left\{ \begin{array}{l} V = \text{the velocity, } M = \text{the momentum of the fluid} \\ v = \text{the velocity, } P = \text{the power of the wheel.} \end{array} \right.$

Then, $V - v =$ their relative velocity, by definition 3d. and, as $V : V - v :: M : \frac{M}{V} \times \overline{V - v} = P$ (*Prop.* 1.) which $\times\, v = P\, v = \frac{M}{V} \times V\, v - v^2 =$ a maximum; hence $V\, v - v^2 =$ a maximum, and its fluxion, (v being the variable quantity) $= V\, \dot{v} - 2 v \dot{v} = 0$; therefore $v = \frac{1}{2} V$, that is, the velocity of the wheel = half that of the fluid, at the place of impact, when the effect is a maximum. Q. E. D.

The uſual theory gives $v = \frac{1}{3} V$; where the error is not leſs than one third of the true velocity of the wheel!

This propoſition is applicable to underſhot wheels, and correſponds with the accurate experiments before cited, as appears from the Author's concluſion, (Philoſophical Tranſactions for 1776 page 457) viz. " The velocity of " the wheel, which, according to M. Parents determina- " tion, adopted by Deſaguliers and Maclaurin, ought to " be no more than one third of that of the water, varies " at the maximum in the experiments of Table I. be- " tween one third and one half; but in all the caſes there " related, in which the moſt work is performed in propor- " tion to the water expended and which approach the near- " eſt to the circumſtances of great works when properly " executed, the maximum lies much nearer one half than " one third, *one half ſeeming to be the true maximum*, if " nothing were loſt by the reſiſtance of the air, the ſcatte- " ring of the water carried up by the wheel, &c." Thus

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he fully ſhews the common theory to have been very defective; but, I believe, none have ſince pointed out wherein the deficiency lay, nor how to correct it; and now we ſee the agreement of the true theory with the reſult of his experiments.

I might proceed with this correction through ſeveral propoſitions, &c. and ſhew their coincidence with thoſe experiments; but muſt leave that, at preſent, for ſuch as have more leiſure; my view being only to ſhew where this perplexing difficulty crept in, in order that thoſe who may have occaſion to uſe the theory in future, or inſtruct young men in the principles of mechanics, may make any uſe of theſe hints they pleaſe: I will, however, juſt add one problem, as I have it by me; though it may not be the moſt ſuitable I could have choſen.

Prop. III. Fig. 1. Plate 4.

Given, the momentum (M) and volocity (V) of the fluid at I, the place of impact; the radius (R=IS) of the wheel ABC; the radius (r=DS) of the ſmall wheel DEF on the ſame axle or ſhaft; the weight (W) or reſiſtance to be overcome at D, and the Friction (F) or force neceſſary to move the wheel without the weight; *required* the velocity (x) of the wheel, &c.

Here we have $V : V - x :: M : M\times\frac{V-x}{V}$ = the acting force at I in the direction KI, as before. (prop. 2.) now, $R : r :: W : \frac{rW}{R}$ = the power at I neceſſary to counterpoiſe the weight W; hence, $\frac{rW}{R}+F$ = the whole reſiſtance oppoſed to the action of the fluid at I; which deducted from the moving force, leaves $M\times\frac{V-x}{V}-\frac{rW}{R}-F$,= the accelerating force of the machine; which, when the motion becomes uniform, will be evaneſcent or=O; therefore, $M\times\frac{V-x}{V}=\frac{rW}{R}+F$, which gives $x=V\times\overline{1\ \frac{rW}{MR}\ \frac{F}{M}}$ = the true velocity required; or, if we reject the friction, then $x=V\times\overline{1-\frac{rW}{MR}}$ is the theorem

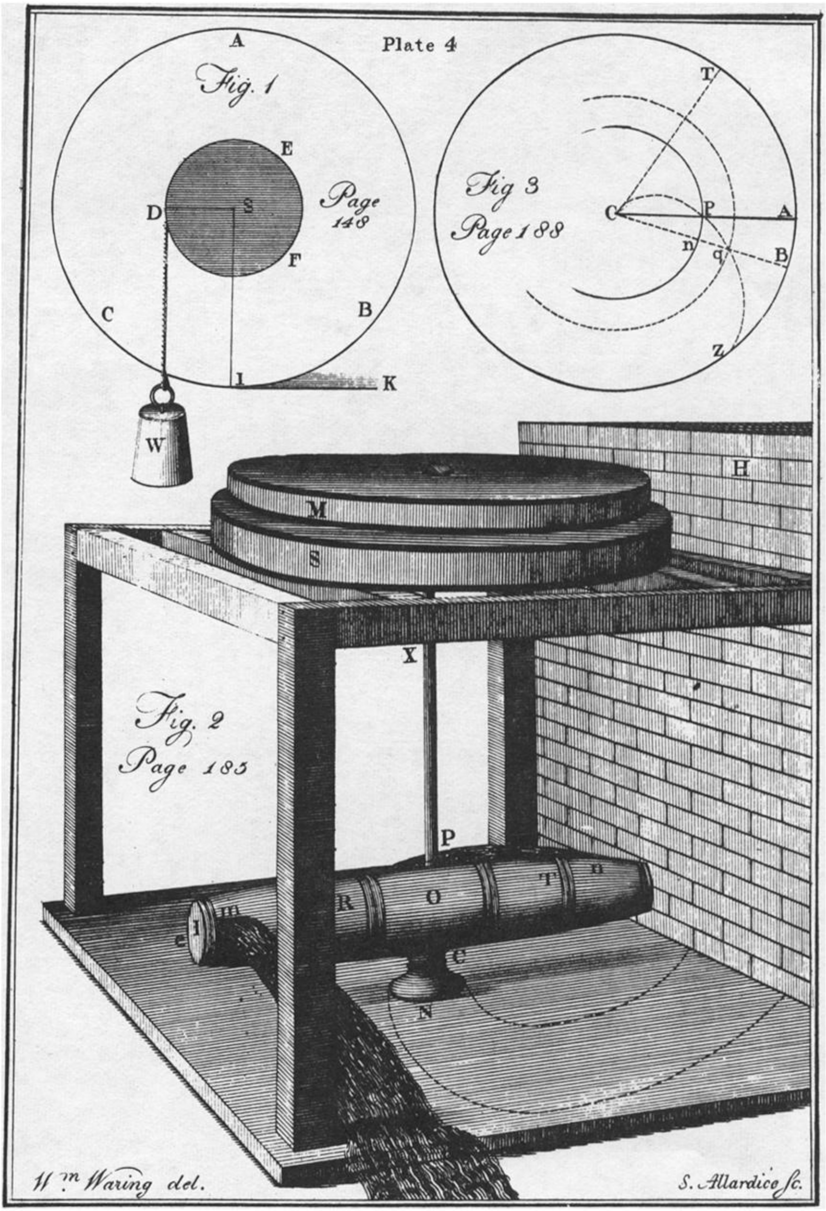
Plate 4
Fig. 1
Page 148
A
E
D
S
F
C
B
I
K
W
Fig 3
Page 188
T
C
P
A
n
q
B
Z
Fig. 2
Page 185
H
M
S
X
P
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O
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o
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orem for the velocity of the wheel. This, by the common theory would be $x = V \times \overline{1 - \sqrt{\frac{rW}{MR}}}$, which is too little by $V\sqrt{\frac{rW}{MR}} - V\frac{rW}{MR}$: No wonder why we have hitherto derived so little advantage from the theory.

Corol. 1. If the weight (W) or resistance be required, such as just to admit of that velocity which would produce the greatest effect; then, by substituting $\frac{1}{2}V$ for its equivalent x (by prop. II.) we have $\frac{1}{2}V = V \times \overline{1 - \frac{rW}{MR} - \frac{F}{M}}$; hence $W = \frac{\frac{1}{2}M - F}{r} \times R$; or, if $F = 0$, $W = \frac{MR}{2r}$; but theorists make this $\frac{4MR}{9r}$, where the error is $\frac{MR}{18r}$.

Corol. 2. We have also $r = \frac{\frac{1}{2}M - F}{W} \times R$; or, rejecting friction, $r = \frac{MR}{2W}$, when the greatest effect is produced, instead of $r = \frac{4MR}{9W}$, as has been supposed: this is an important theorem in the construction of Mills.

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Philadelphia, 7th, 9th mo. 1790.

Astronomical