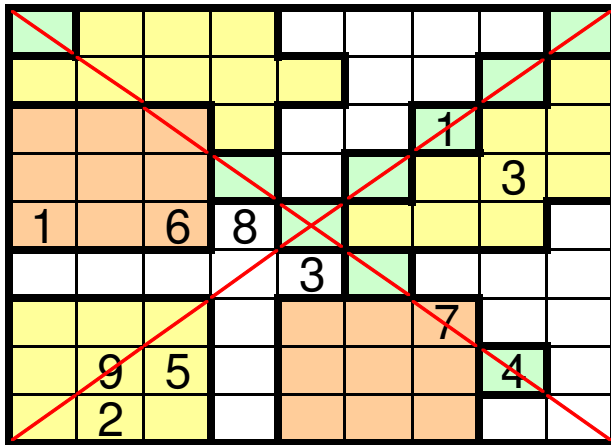# Proof of the Unique Solution of the 11-clue X Pandemonion submitted to Wikipedia
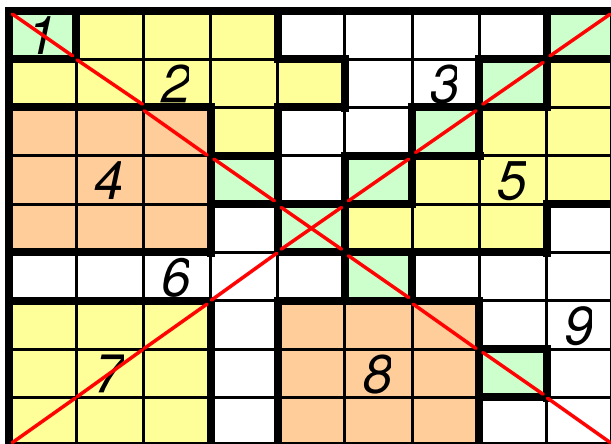
The Puzzle

The Rules

1. Every row, column and cluster must contain the numbers 1 to 9.
2. The green cluster is in nine parts. It must also contain the numbers 1 to 9.
3. The numbers 1 to 9 must appear on both red diagonal lines.

Identifying the clusters

Giving each cluster a number will make the task of explanation much easier.

The Proof

Entry Number   1

The cells at D5 and I5 are "intrusion" cells. They are the only two in the clusters at the bottom half of the puzzle (i.e. clusters 6, 7, 8 and 9) to push into the upper half of the puzzle. In order to be compensated, i.e. in order to ensure that the numbers in those cells are fully represented in rows 6, 7, 8 and 9 the numbers in those cells must be replicated in green (cluster-1) cells at F6 and H8. As H8 is already occupied 8 must occupy F6.

Entry Number   2

The corollary of the fact stated at entry number 1 is that 4 (from H8) must be replicated at I5.

Entry Number 3

G4 is the only cell available to 4 in cluster 5.

Entry Number 4

Cells E2 and E6 act similarly to those at D5 and I5 (see entry number 1) which means that the numbers therein must be replicated at green (cluster-1) cells at A1 and D4. 3 at E6 requires replication at A1 or D4 but 3 at H4 blocks D4 so 3 must occupy A1.

Entry Number 5

All other cells on this diagonal are blocked including by the fact that 3 at A1 is located in a cluster-1 cell. So, 3 must occupy C7.

Entry Number 6

Now 8 is blocked to all other cells on this diagonal. So, it must occupy A9 ....

Entry Number 7

.... as is 4, which must occupy D6.

Entry Number 8

D4 is the only green (cluster-1) cell available to 9. All the other green cells are blocked to 9 by virtue of the fact that they share the same diagonal as 9 at B8.

Entry Number   9

	A	B	C	D	E	F	G	H	I
1	3								
2					**9**				
3							1		
4				9			4	3	
5	1		6	8					4
6				4	3	8			
7			3				7		
8		9	5					4	
9	8	2							

9 must occupy E2 for reasons set out at entry number 4.

Entry Number   10

	A	B	C	D	E	F	G	H	I
1	3								
2					9				
3			**2**				1		
4				9			4	3	
5	1		6	8	**5**				4
6				4	3	8			
7			3				7		
8		9	5					4	
9	8	2							

The numbers still available on the diagonal from A1 to I9 are: 1, 2, 5 and 6. Note that 1 at G3 and 6 at C5 create "intersections" on this diagonal at C3 and E5. This means that 1 and 6 must occupy B2 and I9 in some order, leaving 2 and 5 to C3 and E5. 5 at C8 blocks C3, leaving 5 to E5 and, consequently, 2 to C2.

Entry Number   11

	A	B	C	D	E	F	G	H	I
1	3								
2					9				
3			2				1		
4				9			4	3	
5	1	**3**	6	8	5				4
6				4	3	8			
7			3				7		
8		9	5					4	
9	8	2							

B5 is the only cell in row 5 available to 3.

Entry Number   12

A3 is the only cell available to 9 in cluster 4 which, in turn, makes B3 the only cell available to 4 in the same cluster.

Entry Number   13

1s at G3 and A5 block all cells in cluster 5 other than I2 and I4. We established at entry number 10 that 1 and 6 occupy B2 and I9 in some order. That order is not important at the moment but, in combination, they form an "intersection" at I2, thus excluding 1 and 6 from this cell. So, 1 must occupy I4.

Entry Number   14

The recent placement of 1 at I4 now blocks I9, which forces 1 to occupy B2 on that diagonal which, in turn, permits 6 to occupy the last cell on the diagonal - at I9.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1		
4				9			4	3	1
5	1	3	6	8	5				4
6				4	3	8			
7			3				7		
8		9	5					4	
9	8	2	**1**						6

C9 is the only cell available to 1 in cluster 7.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	**6**	
4				9			4	3	1
5	1	3	6	8	5				4
6			**9**	4	3	8			
7			3				7		
8		9	5					4	
9	8	2	1						6

H3 is the only cell available to 3 in cluster 5 while C6 is the only cell available to 9 in cluster 6.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4				9		**6**	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7			3				7		
8		9	5					4	
9	8	2	1						6

F4 is the only green (cluster-1) cell available to 6.

Entry Number   18

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4				9	**2**	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7			3				7		
8		9	5					4	
9	8	2	1						6

E4 is the only cell available to 2 in row 4.

Entry Number   19

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4				9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	**4**	**6**	3				7		
8	**7**	9	5					4	
9	8	2	1						6

A7 is the only cell available to 4 in cluster 7 and A8 is the only cell in the same cluster available to 7. So, when 4 and 7 are placed 6 must occupy the only remaining cell in cluster 7 - B7.

Entry Number   20

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4				9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3				7		
8	7	9	5					4	**3**
9	8	2	1						6

I8 is the only cell available to 3 in cluster 9 and in column I.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	**5**			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3				7		
8	7	9	5					4	3
9	8	2	1						6

5, 7 and 8 are the numbers still available in cluster 4. 7 at A8 and 8 at A9 block A4, leaving only 5 to occupy this cell.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3				7		**9**
8	7	9	5					4	3
9	8	2	1						6

I7 is the only cell available to 9 in column I.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3				7	**8**	9
8	7	9	5					4	3
9	8	2	1						6

H7 is the only cell available to 8 in cluster 9.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3		**1**		7	8	9
8	7	9	5					4	3
9	8	2	1		**4**				6

Look around!, all numbers other than 1 are blocked at E7 and other than 4 at E9.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3		1	**5**	7	8	9
8	7	9	5			**2**		4	3
9	8	2	1		4				6

Look around!, all numbers other than 2 are blocked at F8. Then, with 2 located in F8, 5 is revealed at F7.

	A	B	C	D	E	F	G	H	I
1	3								
2		1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6			9	4	3	8			
7	4	6	3	**2**	1	5	7	8	9
8	7	9	5			2		4	3
9	8	2	1		4				6

2 is the last remaining number in row 7.

	A	B	C	D	E	F	G	H	I
1	3								
2	**2**	1			9				
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6	**6**		9	4	3	8			
7	4	6	3	2	1	5	7	8	9
8	7	9	5			2		4	3
9	8	2	1		4				6

A2 is the only cell available to 2 in cluster 2, thus forcing 6 to complete column A at A6.

	A	B	C	D	E	F	G	H	I
1	3								**2**
2	2	1			9			**7**	
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6	6		9	4	3	8			
7	4	6	3	2	1	5	7	8	9
8	7	9	5			2		4	3
9	8	2	1		4				6

I1 is the only green (cluster-1) cell available to 2, leaving 7 to take the last green cell at H2.

	A	B	C	D	E	F	G	H	I
1	3								2
2	2	1			9			7	
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6	6		9	4	3	8			**7**
7	4	6	3	2	1	5	7	8	9
8	7	9	5			2		4	3
9	8	2	1		4				6

I6 is the only cell available to 7 in cluster 9.

	A	B	C	D	E	F	G	H	I
1	3								2
2	2	1			9			7	
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6	6	**5**	9	4	3	8			7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	**1**		2		4	3
9	8	2	1	**7**	4				6

Look around!, B6 is blocked to all numbers other than 5 and D8 is blocked to all numbers other than 1. With these numbers revealed, 7 completes cluster at 6 D9.

	A	B	C	D	E	F	G	H	I
1	3								2
2	2	1			9			7	
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5				4
6	6	5	9	4	3	8	**2**	**1**	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1		2		4	3
9	8	2	1	7	4			**5**	6

H9 is the only cell available to 5 in cluster 9, cell G6 is blocked to all numbers other than 2, leaving 1 to complete cluster 9 at H6.

	A	B	C	D	E	F	G	H	I
1	3								2
2	2	1			9			7	
3	9	4	2				1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5	**7**	**9**	**2**	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1		2		4	3
9	8	2	1	7	4			5	6

F5 is the only cell available to 7 in cluster and H7 is the only cell available to 2, leaving 9 to complete row 5 at G5.

Entry Number   33

F9 is the only cell available to 9 in row 9, leaving 3 to complete the row at G9.

Entry Number   34

9 completes column H at H1.

Entry Number   35

F3 is blocked to all numbers other than 3.

Entry Number   36

	A	B	C	D	E	F	G	H	I
1	3							9	2
2	2	1			9			7	
3	9	4	2	**5**		3	1	6	
4	5			9	2	6	4	3	1
5	1	3	6	8	5	7	9	2	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1		2		4	3
9	8	2	1	7	4	9	3	5	6

D3 is blocked to all numbers other than 5.

Entry Number   37

	A	B	C	D	E	F	G	H	I
1	3							9	2
2	2	1			9			7	**5**
3	9	4	2	5	**7**	3	1	6	**8**
4	5			9	2	6	4	3	1
5	1	3	6	8	5	7	9	2	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1		2		4	3
9	8	2	1	7	4	9	3	5	6

I2 is the only cell available to 5 in cluster 5 and column I, leaving 8 to completer the cluster and the column which, in turn, forces 7 to complete row 3 at E3.

Entry Number   38

	A	B	C	D	E	F	G	H	I
1	3					**1**		9	2
2	2	1			9	**4**		7	5
3	9	4	2	5	7	3	1	6	8
4	5			9	2	6	4	3	1
5	1	3	6	8	5	7	9	2	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1		2		4	3
9	8	2	1	7	4	9	3	5	6

F1 is the last cell available to 1 in cluster 3 and column F, leaving 4 to complete column F at F2.

Entry Number   39

	A	B	C	D	E	F	G	H	I
1	3	7	4	6	8	1	5	9	2
2	2	1	8	3	9	4	6	7	5
3	9	4	2	5	7	3	1	6	8
4	5	8	7	9	2	6	4	3	1
5	1	3	6	8	5	7	9	2	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1	6	2	8	4	3
9	8	2	1	7	4	9	3	5	6

We have sufficient entries to complete the puzzle without controversy.

The completed X Pandemonion

	A	B	C	D	E	F	G	H	I
1	3	7	4	6	8	1	5	9	2
2	2	1	8	3	9	4	6	7	5
3	9	4	2	5	7	3	1	6	8
4	5	8	7	9	2	6	4	3	1
5	1	3	6	8	5	7	9	2	4
6	6	5	9	4	3	8	2	1	7
7	4	6	3	2	1	5	7	8	9
8	7	9	5	1	6	2	8	4	3
9	8	2	1	7	4	9	3	5	6