PC 2

Report No. 74

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF MINES
HELIUM ACTIVITY
HELIUM RESEARCH CENTER

INTERNAL REPORT

DESIGN CALCULATIONS FOR A BERYLLIUM-COPPER

ISOCHORIC COOLING CELL

BY

John E. Miller

BRANCH Branch of Fundamental Research

PROJECT NO. 4330

DATE May 1965

AMARILLO, TEXAS

Report No. 74

HELIUM RESEARCH CENTER

INTERNAL REPORT

DESIGN CALCULATIONS FOR A BERYLLIUM-COPPER ISOCHORIC COOLING CELL

By

John E. Miller

Branch of Fundamental Research

Project 4330

May 1965

CONTENTS

ILLUSTRATIONS

TABLES

DESIGN CALCULATIONS FOR A BERYLLIUM-COPPER ISOCHORIC COOLING CELL

by

John E. Miller[1/]

ABSTRACT

This report contains design considerations, criteria, and calculations for a cell to be used in an isochoric cooling apparatus. The apparatus is to be used to determine PVT data for helium and helium-nitrogen mixtures at temperatures from 77° K to 300° K and pressures up to 15,000 psi. Minor changes would make the cell suitable for other purposes, it could be used in an isothermal Burnett apparatus.

INTRODUCTION

One of the advantages of the isochoric cooling method is that a large number of data points can be taken in a short period of time (3).[2/] To facilitate thermal equilibrium, however, it is desirable to have the cell made out of a high thermal-conductivity material. The time required to change the temperature of the cell can be minimized by keeping the weight of the cell low, but the weight of the cell is primarily determined by its internal volume and safety factor. The internal volume of the cell needs to be large compared to the [illegible]

1/ Research Chemist, Helium Research Center, Bureau of Mines, Amarillo, Texas.

2/ Underlined numbers in parentheses refer to items in the bibliography at the end of this report.

Work on manuscript completed May 1965

volume; the safety factor (ratio of burst pressure to working pressure) is arbitrary to some extent, although a value of 4 is often used.

The cell will be continuously cylced from room temperature to liquid nitrogen temperature, so it seems logical that welding the cell together would be preferable to having a mechanical seal; there would be less trouble with leaks and the initial volume calibration of the cell should remain valid.

DESIGN CRITERIA AND SPECIAL PROPERTIES OF Be-Cu ALLOYS

The above considerations, along with a desired working pressure of about 15,000 psi, seem to limit the choice of materials for the cell to either a Be-Cu alloy or an annealed austinetic stainless steel.

Be-Cu has several advantages over stainless steel, especially with respect to thermal conductivity and yield strength. Also, the high-temperature strength of Be-Cu is much better than that for stainless steel. The only way to increase the yield strength of stainless steel is by cold-work; cold-rolled roundstock is available, but the increased strength is lost if the metal is heated above the annealing temperature (therefore, cold-rolled roundstock should not be welded). Be-Cu can be welded and then heat-treated to give a high yield strength.

The only objectionable property of certain Be-Cu alloys is low impact strength, but this deficiency may not be too serious for a small, laboratory-scale cell.

Be-Cu roundstock is available in four tempers: A, H, AT, and HT. The best choice is the AT condition; it is easier to machine than condition A. The properties of H and HT are obtained by cold work, so these tempers should not be welded.

Condition AT is available in five Be-Cu alloys, but only two need be considered: high-strength alloy 25 and high-conductivity alloy 50. The Brush Beryllium Company, Cleveland, Ohio, has recommended that we construct the cell from an upset forging of alloy 25, condition AT (private communication from L. Dean Alspach, Manager of the Market Division, Brush Beryllium Company, Cleveland, Ohio). According to (7), the impact strength of alloy 25 AT from a casting is about 3 ft-lbs on the Charpy K scale (-300°F to 70°F). The impact strength for forged alloy 25 AT is probably about the same. The ASME code requires a minimum impact strength of 15 ft-lbs on the Charpy K scale.

Alloy 50 AT is not as strong as alloy 25 AT (its average 0.2% yield strength is 90,000 psi compared to 160,000 psi for alloy 25 AT); but its room-temperature thermal conductivity is twice as high as that for alloy 25, and its 0.2% yield strength is about twice as high as that for austenitic stainless steel. The impact strength of alloy 50 is not available at this time, but it is probably better than that for alloy 25.

A comprehensive bibliography on cylinder design and behavior is given in an addendum to Helium Research Center Internal Report

No. 67 (5). Additional design calculations for an isochoric cooling apparatus are given in (6).

PARTIAL STRESS-STRAIN CURVES FOR Be-Cu ALLOYS 25 AND 50, CONDITION AT

The average tensile properties for alloys 25 and 50 are given in table 1. The values quoted in table 1 are taken from data sheets furnished by the Brush Beryllium Company, Cleveland, Ohio, and the Beryllium Corporation, Reading, Pennsylvania.

The data in table 1 can be used to construct the initial part of the stress-strain curve for each alloy. A detailed description of the method can be found in Helium Research Center Internal Report No. 73 (4); it contains an example calculation for 303 stainless steel. The conversion of engineering stress-engineering strain to true stress - true strain is summarized in table 2.

The true stress - true strain curve above the elastic limit can be represented by an equation of the form (8):

$$\sigma = c\varepsilon^{n} \qquad (1)$$

and if yield strength at two offsets is known, the constants "c" and "n" can be computed. Substituting the appropriate data from table 2 into equation (1) gives:

$$\text{alloy 25:} \quad \sigma = 3{,}050{,}000\ \varepsilon^{0.6459} \qquad (2)$$

$$\text{alloy 50:} \quad \sigma = 1{,}112{,}000\ \varepsilon^{0.5071} \qquad (3)$$

TABLE 1. - Average tensile properties of Be-Cu alloys 25 AT and 50 AT

Description	Alloy 25 AT	Alloy 50 AT
Ultimate tensile strength, psi	178,000	110,000
0.002% yield strength, psi	112,000	65,000
0.2% yield strength, psi	160,000	90,000
Young's Modulus, E, psi	18.5×10^{6}	17.5×10^{6}
Poisson's ratio, u	0.3	0.3
density, ρ, gm/cc	8.221	8.747
0.01% yield strength 1/	116,760	72,500

1/ Computed from equations (5) and (6), but not converted back to engineering stress. The error is less than 0.7%, and is negligible as far as design purposes are concerned.

TABLE 2. - True stress-true strain values at two offsets for Be-Cu alloys 25 AT and 50 AT

Description	Alloy 25 AT		Alloy 50 AT	
Offset	0.002%	0.2%	0.002%	0.2%
Engineering stress, σ', 1/ psi	112,000	160,000	65,000	90,000
Engineering strain, ϵ' 2/	6.074×10^{-3}	3.648×10^{-3}	3.734×10^{-3}	7.143×10^{-3}
True stress, σ, psi 3/	112,680	161,700	65,243	90,643
True strain, ε 4/	6.055×10^{-3}	10.592×10^{-3}	3.727×10^{-3}	7.118×10^{-3}

1/ $\sigma' = \text{load/original area}$

2/ $\epsilon' = \Delta L/L_o$

3/ $\sigma'(1 + \epsilon')$

4/ $\epsilon = \ln(1 + \epsilon')$

The elastic line of the stress-strain curve is:

$$\sigma = E\varepsilon \tag{4}$$

The true stress - true strain function for alloy 25 AT is:

$$\begin{aligned} \sigma(\text{psi}) = f(\varepsilon) &= 18.5\text{x}10^{6}\varepsilon && 0 \le \varepsilon \le 0.006000 \\ &= 3{,}050{,}000\varepsilon^{0.6459} && .006000 < \varepsilon < \text{about } 0.014 \end{aligned} \tag{5}$$

The true stress - true strain function for alloy 50 AT is:

$$\begin{aligned} \sigma(\text{psi}) = f(\varepsilon) &= 17.5\text{x}10^{6}\varepsilon && 0 \le \varepsilon \le 0.003600 \\ &= 1{,}112{,}000\varepsilon^{0.5071} && 0.003600 < \varepsilon < \text{about } 0.010 \end{aligned} \tag{6}$$

The stress - strain curves for alloys 25 and 50, as given by equations (5) and (6), are given in figures 1 and 2.

WALL THICKNESS CALCULATIONS

The initial elastic breakdown pressure, Py, for a cylinder is (1), (8):

$$Py = \frac{\sigma_{y.01\%}}{\sqrt{3}} \; \frac{K^2 - 1}{K^2} \tag{7}$$

where $\sigma_{y.01\%}$ is the engineering yield stress at 0.01% offset and K = b/a = outside radius/inside radius.

The burst pressure for a number of materials is given by (1):

$$P_{burst} = \frac{2\sigma_{y.0.2\%}}{\sqrt{3}} \left(2 - \frac{\sigma_{y.0.2\%}}{\sigma_{ult.}}\right) \ln K, \tag{8}$$

FIGURE 1. - True stress - log strain curve for Be-Cu alloy 25 AT

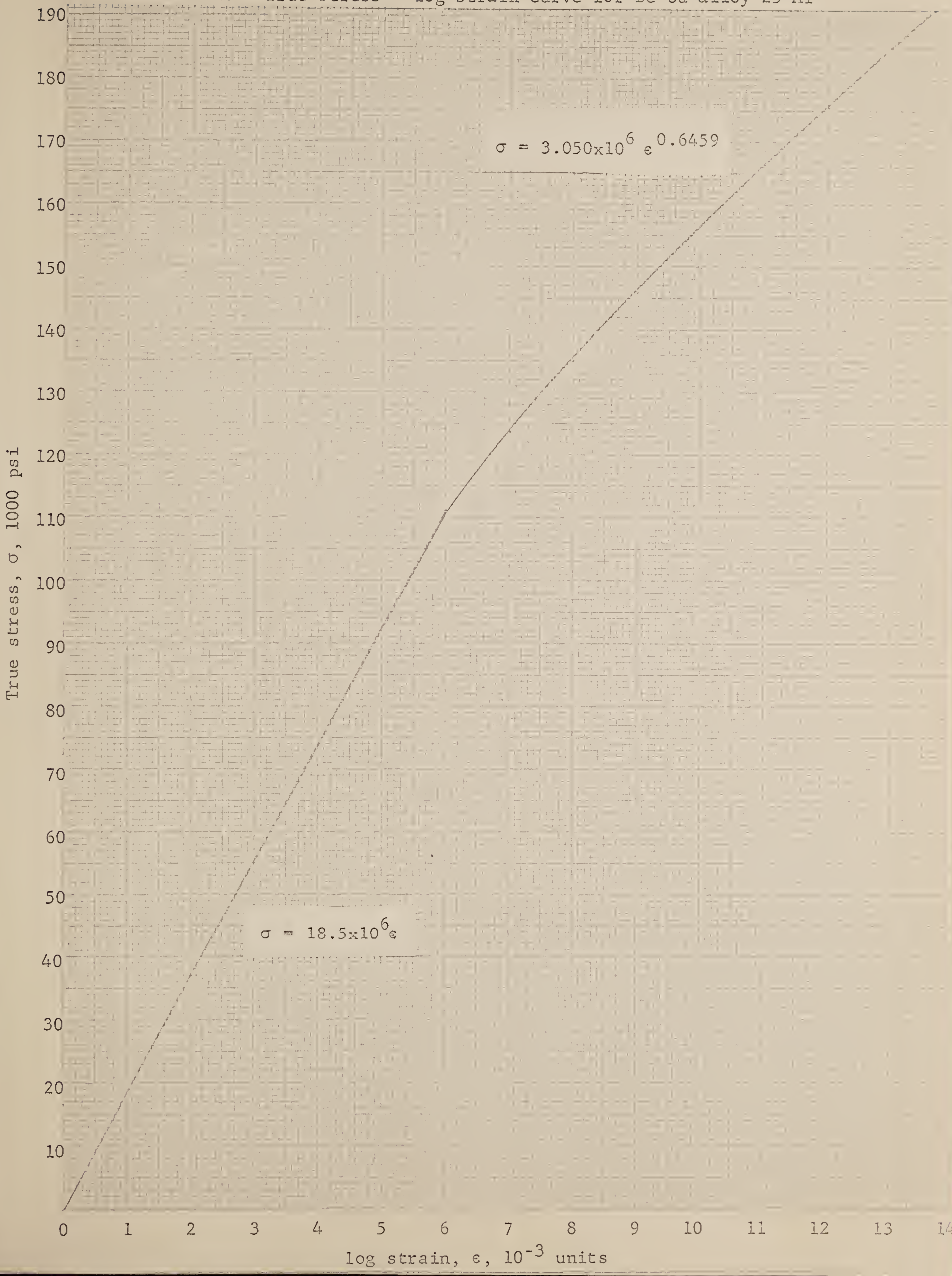
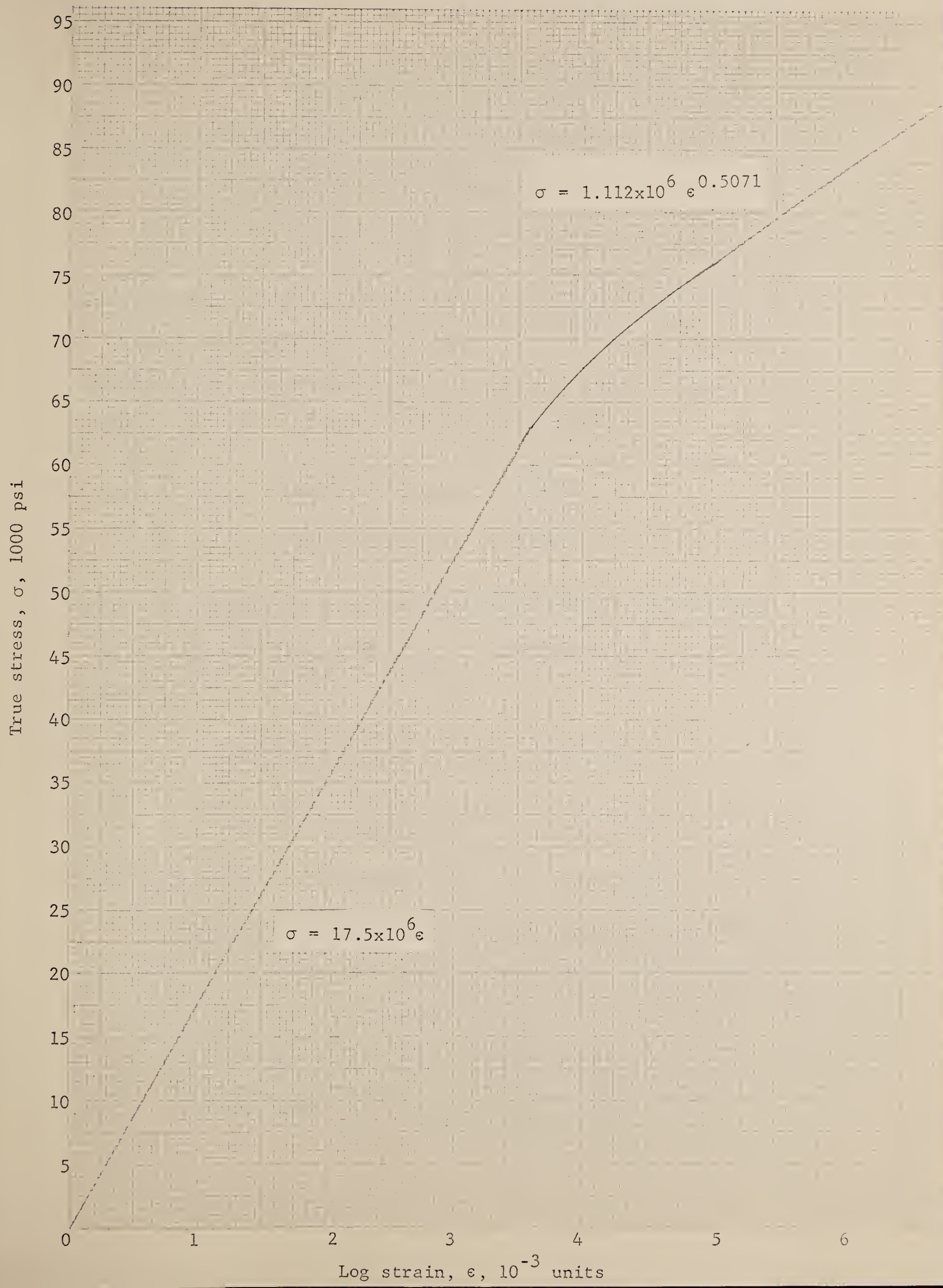


FIGURE 2. - True stress - log strain for Be-Cu Alloy 50 AT



where $\sigma_{y0.2\%}$ = engineering stress at 0.2% offset

$\sigma_{ult.}$ = ultimate tensile strength

The ratio of b/a is fixed by choosing the working pressure, P_w, to be 50% of P_y. The desired working pressure is 15,000 psi, so P_y = 30,000 psi. A hydrostatic test at 1.5 P_w = 22,500 psi; this does not exceed Py. Estimated values of $\sigma_{y.01\%}$ are given in table 1. Substituting these values into equation (7) gives:

$$\text{alloy 25 AT: } K = 1.3423 \quad (9)$$

$$\text{alloy 50 AT: } K = 1.8787 \quad (10)$$

An estimate of the burst pressure can now be made by substituting (9) and (10) into (8), along with the appropriate tensile strengths from table 1:

$$\text{alloy 25 AT: burst pressure} = 59{,}890 \text{ psi} \quad (11)$$

$$\text{alloy 50 AT: burst pressure} = 77{,}450 \text{ psi} \quad (12)$$

The safety factors would be 3.99 and 5.16, respectively.

The dimensions "a" and "b" are fixed by choosing the weight of the cell to be 3000 grams and the internal length of the cylinder as 6 inches. The total end thickness of the cylinder is taken as 2(b-a) + 0.75 inches; each end is to be at least as thick as the wall, and 0.75 inches is added to allow for drilling an inlet fitting hole, resistance

thermometer hole, etc.

The total volume of the cylinder is:

$$V_T = \pi b^2[2(b-a) + 6.75] \text{ in}^3 \quad (13)$$

$$V_T = 51.482b^2[2(b-a) + 6.75] \text{ cc, (a \& b are still in inches)}, \quad (14)$$

The internal volume is:

$$V_i = \pi a^2(6) \text{ in}^3 \quad (15)$$

$$V_i = 51.482a^2(6) \text{ cc, (a is still in inches)} \quad (16)$$

The volume of metal, in cc, is (14) minus (16):

$$V_M = 51.482 \left\{ b^2[2(b-a) + 6.75] - a^2(6) \right\} \text{ cc} \quad (17)$$

The weight of metal, in grams, is (17) multiplied by the metal density, ρ_m g/cc (given in table 1):

$$\text{Weight of metal} = 51.482\rho_m \left\{ b^2[2(b-a) + 6.75] - a^2(6) \right\} \quad (18)$$

In equation (18), values have been specified for everything except "a" and "b", but the ratios of b/a are given by equations (9) and (10) for the two alloys being considered. Therefore, "b" can be eliminated by substituting: "Ka". Equation (18) can now be solved for "a". The results of this calculation are:

alloy 25 AT: $a = 0.9806$ inch, $b = 1.3163$ inch (19)

alloy 50 AT: $a = 0.5593$ inch, $b = 1.0508$ inch (20)

The capacity of the cylinders is found by substitution into equation (16):

alloy 25 AT: $V_i = 297cc$

alloy 50 AT: $V_i = 97cc$

The results of the design calculations are summarized in table 3.

BORE PRESSURE-BORE DEFORMATION CURVES AT PRESSURES ABOVE P_y

A recent paper (8) gives a method for computing bore deformation in the plastic region. This method is adapted in (4) to computing bore deformation in the elastic-plastic region. The method is described in detail, and an example calculation is given in (4), so only results will be given in this report. The only change in nomenclature is: internal radius is called "R" in (4), while it is called "a" in this report.

The bore pressure for the alloy 25 AT cylinder described in table 3 is:

$$P = \frac{18.5\times10^6}{\sqrt{3}}\left[\epsilon\left(1 - \frac{\sqrt{3}\,\epsilon}{4} + \frac{(\sqrt{3}\,\epsilon)^2}{36}\right)\right]_{\epsilon_b}^{0.006} - \frac{3.050\times10^6}{\sqrt{3}}\int_{0.006}^{\epsilon_a}\frac{\epsilon^{0.6459}\,d\epsilon}{1-e^{\sqrt{3}\,\epsilon}} \quad (21)$$

TABLE 3. - Summary of design calculations for two Be-Cu cylinders

Description	Alloy 25 AT	Alloy 50 AT
Working pressure, psi	15,000	15,000
Initial elastic breakdown, psi	30,000	30,000
Burst pressure, psi	59,890	77,450
Internal radius, inches	0.9806	0.5593
External radius, inches	1.3163	1.0508
Bottom end thickness, inches	0.3357	0.4915
Top end thickness, inches	1.0857	1.2415
Weight of cylinder, grams	3,000	3,000
Capacity, cc	297	97
P_y/P_w	2.00	2.00
P_B/P_w	3.99	5.16
Hoop stress, σ_T, psi	3.4944P(psi)	1.7907P(psi)
σ_T at 15,000 psi, psi	52,420	26,860
σ_T at P_w/ultimate tensile strength	0.294[1/]	0.244[1/]

1/ This ratio would be 0.250 for a cylinder designed according to ASME rules.

$$P = 10.681\times10^6 \left\{0.00598443 - \epsilon_b\left[1 - \frac{\sqrt{3}\ \epsilon_b}{4} + \frac{\sqrt{3}\ \epsilon_b{}^2}{36}\right]\right\}$$

$$+ 1.7609\times10^6 \left\{\epsilon_a^{0.6459}\left[1.548294 - \frac{\sqrt{3}\ \epsilon_a}{3.2917} + \frac{(\sqrt{3}\ \epsilon_a)^2}{31.7505} - \frac{(\sqrt{3}\ \epsilon_a)^4}{3345.0}\right] - 0.056746\right\} \quad (22)$$

The bore pressure for the alloy 50 AT cylinder described in table 3 is:

$$P = \frac{17.5\times10^6}{\sqrt{3}}\left[\epsilon\left(1 - \frac{\sqrt{3}\ \epsilon}{4} + \frac{(\sqrt{3}\ \epsilon)^2}{36}\right)\right]_{\epsilon_b}^{0.0036} - \frac{1.112\times10^6}{\sqrt{3}}\int_{0.0036}^{\epsilon_a}\frac{\epsilon^{0.5071}d\epsilon}{1 - e^{\sqrt{3}\ \epsilon}} \quad (23)$$

Equation (23) reduces to:

$$P = 10.104\times10^6\left[\epsilon\left(1 - \frac{\sqrt{3}\ \epsilon}{4} + \frac{(\sqrt{3}\ \epsilon)^2}{36}\right)\right]_{\epsilon_b}^{0.0036} \quad (24)$$

$$+ 642{,}136\left[\epsilon^{0.5071}\left\{1.971869 - \frac{\sqrt{3}\ \epsilon}{3.01427} + \frac{(\sqrt{3}\ \epsilon)^2}{30.0856} - \frac{(\sqrt{3}\ \epsilon)^4}{3245.13}\right\}\right]_{0.0036}^{\epsilon_a}$$

The results of the bore pressure-bore deformation calculations from equations (22) and (24) are given in table 4; the curves are plotted in figures 3 and 4. The elastic lines are slightly different from those that would be computed from exact elastic theory because the Poisson ratio correction was neglected to simplify the calculations. Also, notice that the bore pressure entries in the first line of table 4 are computed from proportional limit strains; this gives another estimate for Py that is independent of equation (7). The agreement is not exact (28,263 and 26,005 psi versus 30,000 psi), but

it is good, considering that simplifying assumptions have been made in equations (22) and (24). A hydrostatic test at 22,500 psi does not exceed any of the estimates for Py.

If it should ever be necessary to estimate a permanent bore deformation, Py must be taken as the first bore pressure entry in table 4. The permanent set caused by some pressure, $P_i > Py$, is found by subtracting the elastic unloading displacement from the μ_i corresponding to P_i; this is discussed in (4). For example, if the alloy 25 AT cylinder were pressured up to 37,551 psi, then depressured; the permanent bore deformation would be estimated as:

$$0.007000 - 37.551\times10^{3}/5.5325\times10^{6}$$

$$= 0.007000 - 0.006787$$

$$= 0.00021 \text{ inch}$$

The slope of the elastic line is $28.263\times10^{3}/5.1085\times10^{-3}$ or 5.5325×10^{6}. The new internal radius would then be 0.9806 + 0.00021 = 0.98081 in.

SUGGESTIONS FOR CONSTRUCTION OF THE CELL

The best place for the weld is probably at about the mid-section. This would make it easier to machine the inside ends of the cylinder; this is difficult when the depth is large because of vibration in the cutting tool.

Be-Cu will oxidize to form beryllium oxide, so the weld should be made in an inert atmosphere; it would be desirable to heat-treat

TABLE 4. - Tabulated values of bore pressure-bore deformation

Alloy 25 AT			Alloy 50 AT		
μ_a, inches 1/	ϵ_a 2/	P, psi 3/	μ_a, inches	ϵ_a	P, psi
0.0051085	0.0060000	28,263	0.0017466	0.0036000	26,005
.0052	.0061071	28,832	.00185	.0038123	27,528
.0054	.0063413	29,921	.00195	.0040183	28,966
.0056	.0065756	30,975	.00205	.0042233	30,337
.0058	.0068096	31,998	.00220	.0045327	32,326
.0060	.0070436	32,990	.00240	.0049440	34,823
.0062	.0072778	33,955	.00260	.0053546	37,164
.0065	.0076288	35,353	.00280	.0057661	39,377
.0070	.0082135	37,551			
.0075	.0087979	39,606			

1/ μ_a = displacement of bore from original zero pressure radius; permanent bore deformation = μ_a - elastic unloading displacement

2/ ϵ_a = log strain at the bore

3/ P = bore pressure required to cause ϵ_a and μ_a

FIGURE 3. - Bore deformation - bore pressure for Be-Cu alloy 25 AT
K = 1.3423
IR = 0.9806 inch

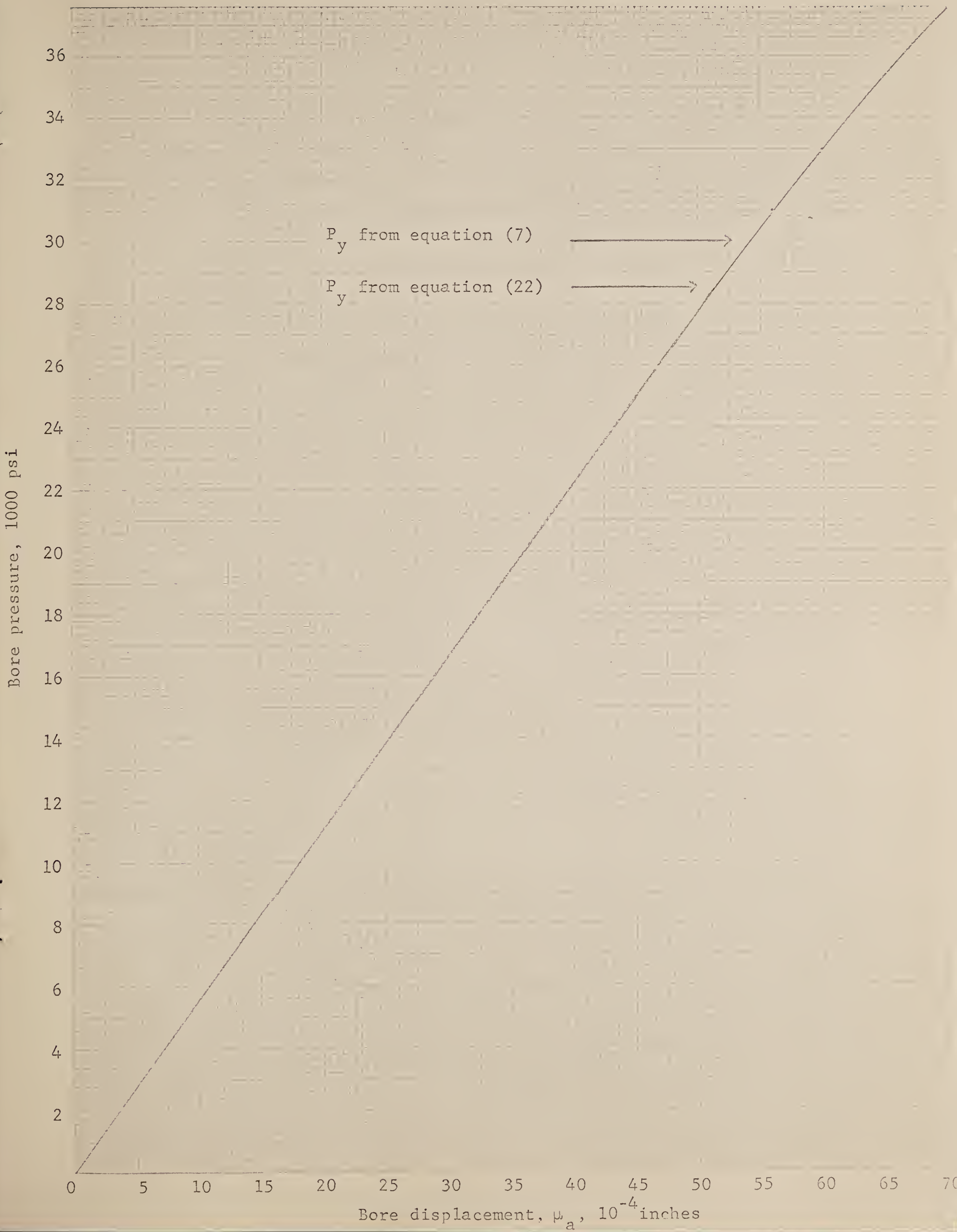
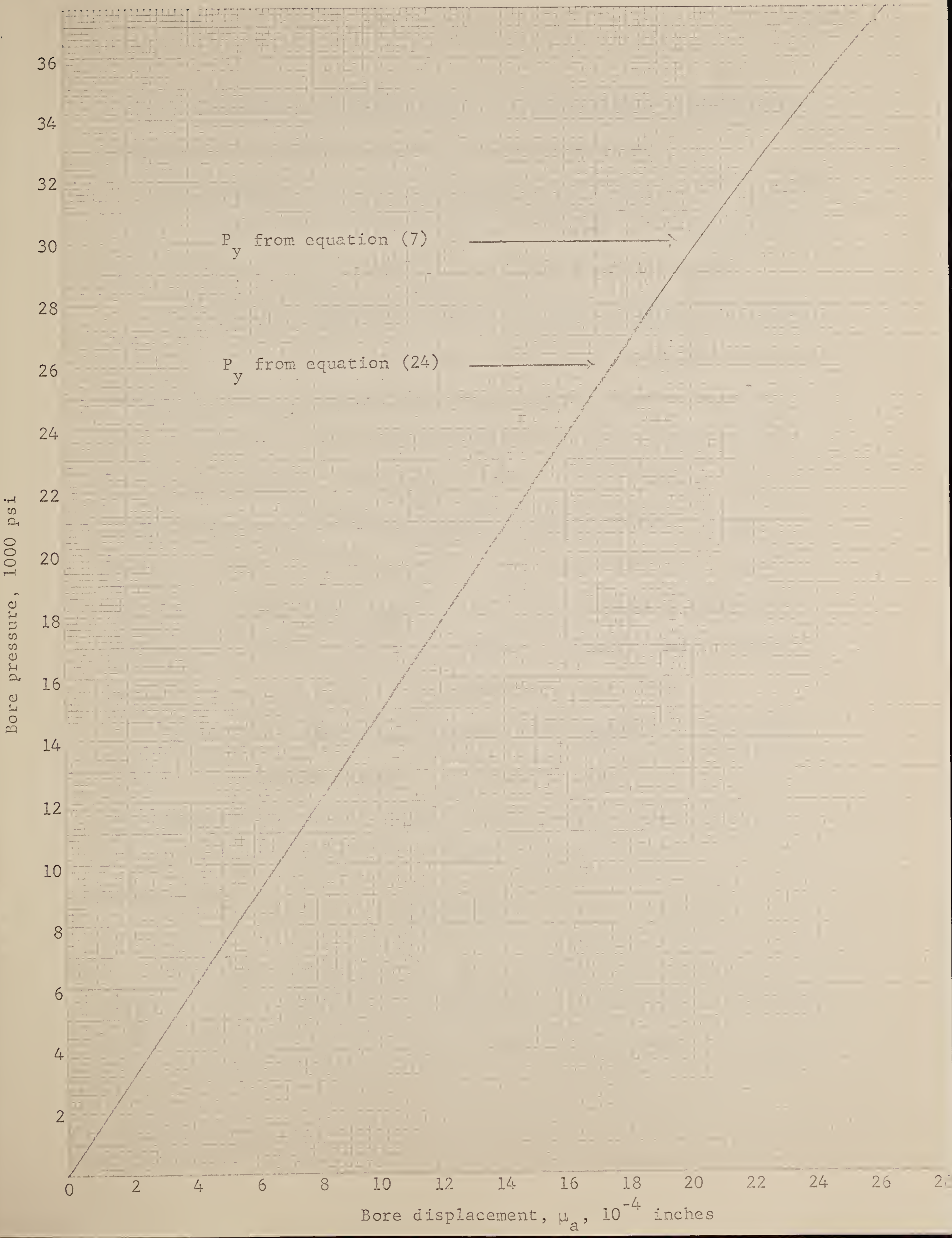


FIGURE 4. - Bore deformation - bore pressure for Be-Cu alloy 50 AT
K = 1.8787
IR = 0.5593 inch



the cell in an inert atmosphere also. This procedure eliminates the necessity of pickling the cylinder after construction.

Sharp corners should be eliminated on the inside of the cylinder to help prevent stress intensification. It might be desirable to polish the inside surface before the cylinder is welded together.

The capillary connection at the top of the cylinder will probably be an NBS type threaded, cone-sleeve connection. This type fitting is available in 1/8" capillary tube from the High Pressure Equipment Co., Erie, Pennsylvania. The actual connection to the cylinder could be made with an adapter, 1/8" female x 1/4" male; this adapter could be screwed into the cylinder or it could be silver brazed. Silver brazing should also be done with an inert atmosphere on the inside of the cell; it can be done after the cell has been welded together, but before the final heat treatment. Optimum clearance between parts to be silver brazed is 0.001 to 0.003 inch for Be-Cu alloys.

Figure 5 is a preliminary sketch of the suggested construction details. The drawing is not to scale and no dimensions are shown, but they would be easy to figure from the data in table 3 after a choice is made as to what kind of cell will be built.

CONCLUSIONS

Aside from the question of impact strength, the choice between an alloy 25 AT cylinder or an alloy 50 AT cylinder will be a hard decision. Both cylinders described in table 3 were sized for economic

operation at liquid helium temperature; both were conservatively designed to insure stability of the volume calibration; both will retain their strength up to about 500°F; both are better than a stainless steel cylinder; there is little difference in difficulty of construction.

The large capacity of the 25 AT cylinder is its greatest asset; the 50 AT cylinder would probably give better performance with respect to temperature control.

The problem of impact strength may not be a serious disadvantage for a small cylinder to be used in a laboratory apparatus. A Be-Cu bomb for use at 4.2°K and 14-Kbar (about 200,000 psi) is mentioned in (2); this bomb is probably made from an upset forging of alloy 25 AT. As long as conditions that will cause brittle failure are absent, the impact strength is not "used" anyway. For example, impact strength is determined by cutting a deep notch in a strip of metal - the notch is then struck smartly with a pendulum; as far as I know, we don't plan to cut a deep notch in our isochoric cooling cell - and then strike it smartly with a pendulum. The cell will be completely protected by one or two dewars, so the chances of it being struck by a pendulum are extremely remote (notch or no notch). Otherwise, the cell is used under conditions of essentially static loading - which means that there are not very many ways that a high stress can be applied suddenly over a small area.

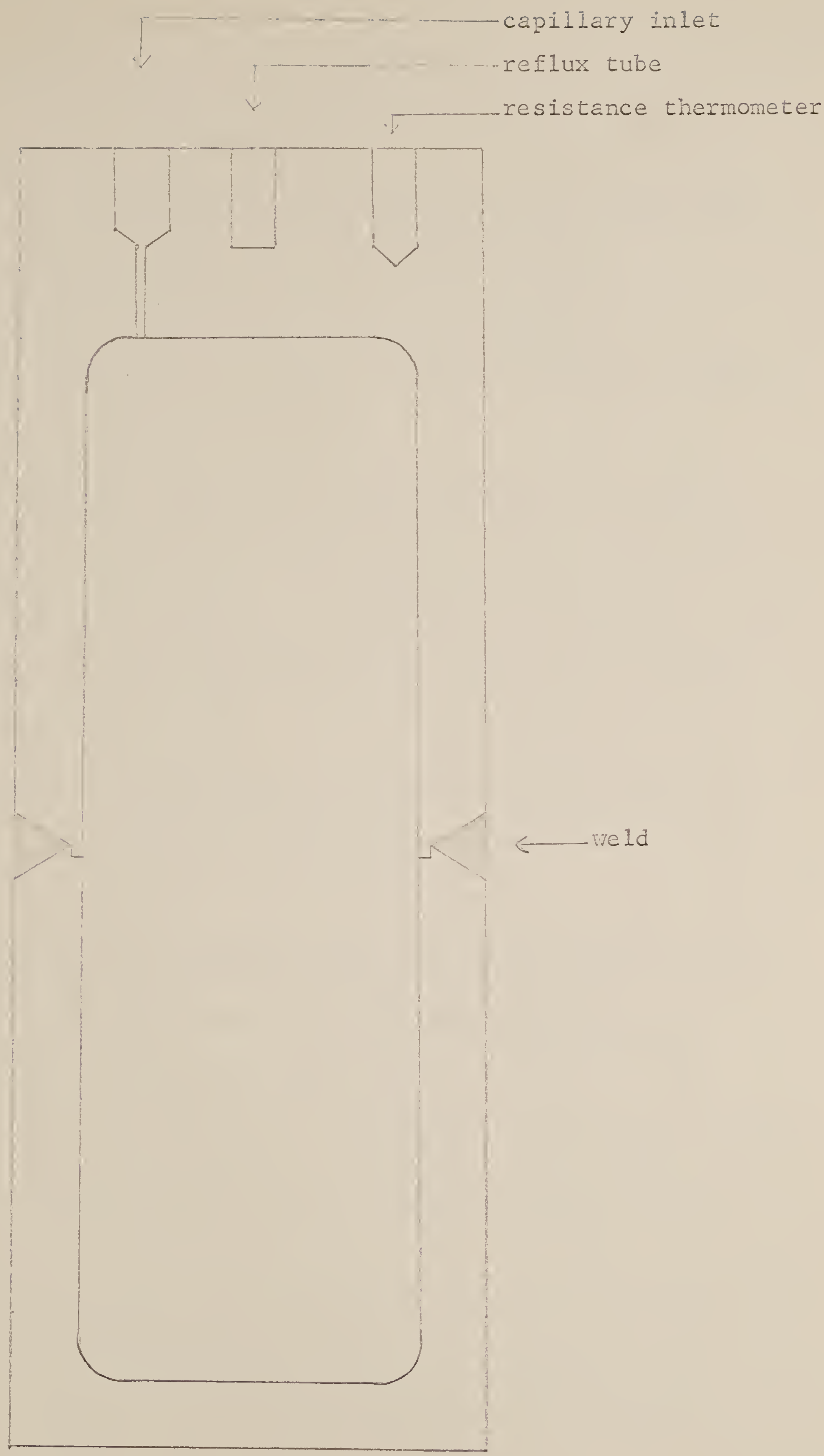


FIGURE 5. - Preliminary sketch showing cylinder construction details

BIBLIOGRAPHY

1. Faupel, J. H. Yield and Bursting Characteristics of Heavy-Wall Cylinders. Trans. ASME, v. 78, 1956, pp. 1031-1064.

2. Goree, W. S., B. McDowell, and T. A. Scott. Seals for Low Temperature High Pressure Systems. Review of Scientific Instruments, February 1965, pp. 99-101.

3. Goodwin, Robert D. Apparatus for Determination of Pressure-Density-Temperature Relations and Specific Heats of Hydrogen to 350 atmospheres at Temperatures Above 14°K. J. Research of the NBS - Engineering and Instrumentation, v. 65C, No. 4, October-December 1961, pp. 231-243.

4. Miller, John E. Elastic Distortion Correction for the Burnett Apparatus. Helium Research Center Internal Report No. 73, May 1965.

5. _____. Design of the Jacketed Bombs for the Isothermal Burnett Apparatus. Helium Research Center Internal Report No. 67, January 1965, with addendum, May 1965.

6. _____. Preliminary Design Calculations for an Isochoric Cooling Apparatus. Helium Research Center Memorandum Report No. 69, May 1965.

7. National Bureau of Standards. Cryogenic Materials Data Handbook.

8. Svennson, N. L. The Bursting Pressure of Cylindrical and Spherical Vessels. J. Appl. Mech., March 1958, pp. 89-96.