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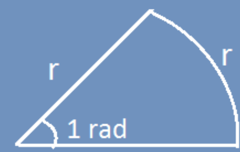
WHAT IS RADIAN ?

A central angle (simply angle made by a line with some axis (say x-axis) such that length of line is equal to length of arc on which particle is moving . this angle is equal to 1 radian.
and

$$1 \text{ rad} = 57.3^\circ \quad \text{approx}$$



so $180^\circ = 3.1415......$ rad

or $180^\circ = \pi$ rad

Value of Pi

To find value of pi we will use trigonometric functions and we will recall the calculus (differentiation, integration)

we start with derivative of inverse sine function

$$\frac{d}{dx} \sin x = \cos x$$

recall

$$\frac{d}{dx} \cos x = -\sin x$$

let $\sin y = x$ ....(1)

then $y = \sin^{-1}x$ ....(2)

differntiate (1) w.r.t 'x'

$$\cos y \frac{dy}{dx} = 1$$

or

$$\frac{dy}{dx} = \frac{1}{\cos y} \quad ....(3)$$

now from (1)

$\sin y = x$

as

$$\cos y = \sqrt{1 - \sin^2 y}$$

so

$$\cos y = \sqrt{1 - x^2}$$

using value of cosy in (3)

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

from (2)

$y = \sin^{-1}x$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx} \sin^{-1}x$$

also

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

so

$$\frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1 - x^2}}$$

integrating bothsides
w.r.t 'x'

$$\int \frac{d}{dx} \sin^{-1}x \, dx = \int \frac{1}{\sqrt{1 - x^2}} \, dx$$

$$\sin^{-1}x = \int \left(1 - x^2\right)^{-1/2} dx$$

using binomial series on right side

$$\left(1 + x\right)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \ldots\ldots\ldots\ldots$$

$$\sin^{-1}x = \int \left(1 - x^2\right)^{-1/2} dx \qquad -1 \leq x \leq 1$$

$$= \int \left(1 + \frac{x^2}{2} + \frac{(1)(3)}{2^2(2!)} x^4 + \frac{(1)(3)(5)}{2^3(3!)} x^6 + \ldots\ldots\right) dx$$

so

$$\sin^{-1}x = x + \frac{x^3}{2(3)} + \frac{(1)(3)}{2^2(2!)(5)} x^5 + \frac{(1)(2)(3)}{2^3(3!)(7)} x^7 + \ldots\ldots\ldots$$

so

$$\sin^{-1}x = x + \frac{x^3}{2(3)} + \frac{(1)(3)}{2^2(2!)(5)}x^5 + \frac{(1)(2)(3)}{2^3(3!)(7)}x^7 + \ldots\ldots$$

put x=1

$$\sin^{-1}1 = 1 + \frac{1}{2(3)} + \frac{(1)(3)}{2^2(2!)(5)}(1) + \frac{(1)(2)(3)}{2^3(3!)(7)}(1) + \ldots\ldots$$

or

$$\frac{\pi}{2} = 1 + \frac{1}{2(3)} + \frac{(1)(3)}{2^2(2!)(5)}(1) + \frac{(1)(2)(3)}{2^3(3!)(7)}(1) + \ldots\ldots$$

using any calculator or computer we can find sum of this series and hence value of pi .

Pi is of Fundamental importance in Physics it is used to calculate electric field intensity due to a charge(Columb's law), calculate drag force on a spherical body (Stoke's law) and at many other places.