XIth HAND WRITTEN MATERIALS

CHAPTER – 12

INTRODUCTION TO PROBABILITY THEORY

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CHAPTER - 12.

INTRODUCTION TO PROBABILITY THEORY

Exercise: 12.1

1) An experiment has the four possible mutually exclusive and exhaustive outcomes A, B, C and D. Check whether the following assignments of probability are permissible.

(i) $P(A) = 0.15$, $P(B) = 0.30$, $P(C) = 0.43$, $P(D) = 0.12$.

Sol

$P(A)$, $P(B)$, $P(C)$ and $P(D) \geq 0$

Also, $P(S) = P(A) + P(B) + P(C) + P(D)$

$= 0.15 + 0.30 + 0.43 + 0.12$

$= 1$

$\therefore$ The assigment of probability is permissible.

(ii) $P(A) = 0.22$, $P(B) = 0.38$, $P(C) = 0.16$, $P(D) = 0.34$

Sol

$P(A) = 0.22 \geq 0$, $P(B) = 0.38 \geq 0$

$P(C) = 0.16 \geq 0$, $P(D) = 0.34 \geq 0$

$P(S) = 0.22 + 0.38 + 0.16 + 0.34 = 1.1 \neq 1$

$\therefore$ The assignment of probability is not permissible.

(iii) $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{5}$, $P(C) = -\frac{1}{5}$, $P(D) = \frac{1}{5}$

Sol

Since $P(C) = \frac{-1}{5}$ is negative, the assigment of probability is not permissible.

2) If two coins are tossed simultaneously, then find the probability of getting

(i) one head and on tail

(ii) at most two tails.

Sol

(i) $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

Let A be the event of getting one head and one tail

$\therefore A = \{HT, TH\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(ii) Let B be the event of getting atmost two tails.

$\therefore B = \{HH, HT, TH, TT\}$

$n(B) = 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{4} = 1$

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8) Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is a mango and the other is an apple (ii) both are of the same variety.

Sol

$S = \{5 \text{ mangoes and } 4 \text{ apples}\}$

$n(S) = 9C_2$

[$\because$ 2 fruits are taken from 9 fruits]

(i) Let A be the event of getting one mango and one apple.

$\therefore n(A) = {}^5C_1 \times {}^4C_1 = 5 \times 4 = 20$

$P(A) = \frac{n(A)}{n(S)} = \frac{20}{9C_2} = \frac{20}{\frac{9\times8}{2\times1}} = \frac{20\times2}{9\times8} = \frac{5}{9}$

(ii) Let B be the event of getting 2 fruits from the same variety.

$\therefore n(B) = {}^5C_2 + {}^4C_2$

$= \frac{5\times4}{2\times1} + \frac{4\times3}{2\times1} = \frac{20}{2} + \frac{12}{2}$

$= 10 + 6 = 16$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{16}{9C_2} = \frac{16}{\frac{9\times 8}{2\times 1}}$$

$$= \frac{16\times 2}{9\times 8} = \frac{4}{9}$$

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4) What is the chance that (i) non-leap year (ii) leap year should have fifty three sundays?

Sol (i) An ordinary year consists of 365 days.

365 days = 52 weeks + 1 day.

52 weeks will have 52 sundays.

That remaining one sunday can be selected from the following ways.

1	2	3	4
sunday	Monday	tuesday	wednesday
5	6	7	
Thursday	Friday	Saturday.	

$\therefore$ probability that an ordinary year may contain 53 sundays $= \frac{1}{7}$

(ii) A leap year contains 366 days

366 days = 52 weeks + 2 days.

These two days can be selected from the following combinations:

1) Monday & Tuesday
2) Tuesday & Wednesday
3) Wednesday & Thursday
4) Thursday & Friday
5) Friday & Saturday
6) Saturday & Sunday
7) Sunday & Monday.

$\therefore$ Required probability $= \dfrac{\text{No. of favourable events}}{\text{Total No. of events}}$

$= \dfrac{2}{7}$ (Ans)

5) Eight coins are tossed once, find the probability of getting.

(i) exactly two tails (ii) atleast two tails

(iii) atmost two tails.

Sol Since 8 coins are tossed,

(i) $n(S) = 2^8 = 256$

Let A be the event of getting exactly two tails,

$\therefore n(A) = 8C_2 = \dfrac{8\times7}{2\times1} = 4\times7$

$\therefore P(S) = \dfrac{n(A)}{n(S)} = \dfrac{4\times7}{2^8} = \dfrac{2^2\times7}{2^8} = \dfrac{7}{2^8\times2^{-2}} = \dfrac{7}{2^6} = \dfrac{7}{64}$

(ii) Let B be the event of getting atleast two tails

$n(B) = {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= n(S) - [{}^8C_0 + {}^8C_1]$

$= 2^8 - [1+8] = 2^8 - 9 = 256 - 9 = 247$

$\therefore P(B) = \dfrac{n(B)}{n(S)} = \dfrac{247}{256}$

(iii) Let C be the event of getting atmost two tails.

$\therefore n(C) = 8C_0 + 8C_1 + {}^8C_2 = 1 + 8 + \dfrac{8\times7}{2\times1}$

$= 9 + 28 = 37$

$\therefore P(C) = \dfrac{n(C)}{n(S)} = \dfrac{37}{256}$

6) An integer is chosen at random from the first 100 positive integer. What is the probability that the integer chosen is a prime or multiple of 8? (396)

Sol Let $S = \{1, 2, 3, \ldots, 100\}$

$n(S) = 100$

Let A be the event of getting a prime Number and B be the event of getting multiple of 8.

$\therefore A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

$n(A) = 25$

$B = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$

$n(B) = 12$

$n(A \text{ or } B) = n(A) + n(B) = 25 + 12 = 37$

$\therefore P(A \text{ or } B) = \dfrac{n(A) + n(B)}{n(S)} = \dfrac{37}{100}$

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7) A bag contains 7 red and 4 black balls, 3 balls are drawn at random. Find the probability that (i) all are red (ii) one red and 2 black.

Sol $S = \{7 \text{ red}, 4 \text{ black balls}\}$

$n(S) = {}^{11}C_3$

[$\because$ 3 balls are drawn out of 11 balls]

(i) Let A be the event of getting 3 red balls.

$\therefore n(A) = {}^{7}C_3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{7}C_3}{{}^{11}C_3} = \frac{\frac{7 \times 6 \times 5}{3 \times 2 \times 1}}{\frac{11 \times 10 \times 9}{3 \times 2 \times 1}}$$

$$= \frac{7\times6\times5}{11\times10\times9} = \frac{7}{33}$$

(ii) Let B be the event of getting one red and 2 Black balls.

$$\therefore n(B) = 7C_1 \times 4C_2 = 7 \times \frac{4\times3}{2\times1} = 42$$

$$P(A) = \frac{n(B)}{n(S)} = \frac{42}{11C_3} = \frac{42}{\frac{11\times10\times9}{3\times2\times1}} = \frac{42\times3\times2}{11\times10\times9} = \frac{14}{55}$$

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8) A single card is drawn from a pack of 52 cards. what is the probability that

(i) The card is an ace or a king

(ii) The card will be 6 or a smaller

(iii) The card is either a queen or a 9?

Sol

$$S = \{\text{pack of 52 cards}\}$$

$$n(S) = 52.$$

(i) P(ace card or a king card) = P(ace card) + P(king card)

[Since They are mutuall exclusive]

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

(ii) P(card will be 6 or smaller)

$$= \frac{5+5+5+5}{52} = \frac{20}{52} = \frac{5}{13}$$

[$\because$ 5 cards which are 6 or smaller from each variety]

(iii) P(queen card or 9)

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

9) A cricket club has 16 members of whom only 5 can bowl. What is the probability that in a team of 11 members atleast 3 bowlers are selected?

Sol)

$S = \{\text{member of cricket club}\}$

$n(S) = {}^{16}C_{11}$

[∵ 11 members must be selected for a team]

Let A be the event of selecting atleast 3 bowlers.

Atleast 3 bowlers can be selected as follows:

	11 non bowlers	5 bowlers	
(i)	8	3	${}^{11}C_8 \times {}^5C_3 = {}^{11}C_3 \times {}^5C_2 = 1650$
(ii)	7	4	${}^{11}C_7 \times {}^5C_4 = {}^{11}C_4 \times {}^5C_1 = 1650$
(iii)	6	5	${}^{11}C_6 \times {}^5C_5 = {}^{11}C_5 \times 1 = 462$
			3762

No. of favourable cases = 3762

$n(A) = 3762$

$n(S) = {}^{16}C_{11} = {}^{16}C_5$

$$= \frac{16 \times 15 \times 13 \times 14 \times 12}{5 \times 4 \times 3 \times 2} = 4368$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3762}{4368} = \frac{627}{728}$$

10) (i) The odd that the event A occurs is 5 to 7, find P(A).

(ii). Suppose $P(B) = \frac{2}{5}$. Express the odds that the event B occurs. (399)

Sol The odd of an event A are a : b in favour of an event and

$$P(A) = \frac{a}{a+b}$$

(i) Given that odd of an event A occurs is 5 : 7

$$\therefore P(A) = \frac{5}{5+7} = \frac{5}{12}.$$

(ii) Given $P(B) = \frac{2}{5} = \frac{2}{2+3}$ $[\because a = 2 \text{ and } b = 3]$

$\therefore$ The odds that the event B occurs is 2 to 3.

Exercise : 12.2

1) If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$, then find (i) $P(\bar{A})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap B)$ (iv) $P(\bar{A} \cup \bar{B})$

Sol Given $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{8}$

(i) $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$

(ii) $P(A \cup B) = P(A) + P(B)$

$[\because$ A and B are mutually exclusive, $P(A \cap B) = 0]$

$$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}.$$

(iii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$= P(B) - 0 = \frac{1}{8}$$

$[\because$ A and B are mutually exclusive events, $P(A \cap B) = 0]$

(iv) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$ [By de morgan's law]

$= 1 - P(A \cap B)$

$= 1 - 0 = 1$

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2) If A and B are two events associated with a random experiment for which $P(A) = 0.35$, $P(A \text{ or } B) = 0.85$ and $P(A \text{ and } B) = 0.15$. Find

(i) $P(\text{only } B)$ (ii) $P(\bar{B})$ (iii) $P(\text{only } A)$

Sol Given $P(A) = 0.35$

$P(A \text{ or } B) = P(A \cup B) = 0.85$

$P(A \text{ and } B) = P(A \cap B) = 0.15$

(i) $P(\text{only } B)$: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.85 = 0.35 + P(B) - 0.15$

$P(B) = 0.85 - 20 = 0.65$

$P(\text{only } B) = P(A \cup B) - P(A)$

$= 0.85 - 0.35 = 0.50$

(ii) $P(\bar{B}) = 1 - P(B) = 1 - 0.65 = 0.35$

(iii) $P(\text{only } A) = P(A \cup B) - P(B)$

$= 0.85 - 0.65 = 0.20$

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3) A die is thrown twice. Let A be the event, 'First die shows 5' and B be the event, 'second die shows 5'. Find $P(A \cup B)$.

Sol $S = \{(1,1), (1,2), (1,3), \ldots (6,6)\}$

$n(S) = 36$

A: first die shows 5

$A = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

$n(A) = 6$

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$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

B: second die shows 5

$B = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5)\}$

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$$

$(A \cap B) = \{(5,5)\}$

$n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{\cancel{P(A \cap B)}\ n(S)} = \frac{1}{36}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{6+6-1}{36} = \frac{12-1}{36} = \frac{11}{36}$$

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4) The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then finds the probability of

(i) $P(A \cup B)$ (ii) $P(A \cap \bar{B})$ (iii) $P(\bar{A} \cap B)$

Sol)

Given $P(A) = 0.5$

$P(B) = 0.3$

Since A and B are mutually exclusive events

$P(A \cap B) = 0$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (2m)

$= 0.5 + 0.3 - 0 = 0.8$

(ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= 0.5 - 0 = 0.5$

(iii) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= 0.3 - 0 = 0.3$

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5) A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96.

(i) what is the probability that a fire engine is available when needed?

(ii) What is the probability that neither is available when needed?

Sol

(i) Given $P(A) = 0.96$, $P(B) = 0.96$

P(Fire engine is available when needed)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.96 + 0.96 - P(A) \cdot P(B)$$

$$= 0.96 + 0.96 - (0.96)(0.96)$$

$$= 1.92 - 0.9216$$

$$P(A \cup B) = 0.9984$$

(ii) P(Neither fire engine is available when needed)

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.9984 = 0.0016$$

$$= 0.0016$$

b) The probability that a new railway bridge will get an award for its design is 0.48, the (403) probability that it will get an award for the efficient use of material is 0.36 and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards.

Sol) Let the events be as follows:

A: Getting an award for its design

B: getting an award for efficient use of materials.

Given, $P(A) = 0.48$, $P(B) = 0.36$ and $P(A \cap B) = 0.2$.

(i) P(getting atleast one of the two awards)

$$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.48 + 0.36 - 0.2$$

$$= 0.84 - 0.2$$

$$= 0.64.$$

(ii) P(getting only one of the awards)

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B)$$

$$= 0.48 + 0.36 - 2(0.2)$$

$$= 0.84 - 0.4 = 0.44.$$

Exercise 12.3

1) Can two events be mutually exclusive and independent simultaneously?

Sol) Two events cannot be mutually exclusive and independent simultaneously. since for mutually exclusive events $P(A \cap B) = 0$ and for independent events $P(A \cap B) = P(A) \cdot P(B)$.

2) If A and B are two events such that $P(A \cup B) = 0.7$, $P(A \cap B) = 0.2$ and $P(B) = 0.5$, then show that A and B are independent.

Sol) Given $P(A \cup B) = 0$

$P(A \cap B) = 0.2$, $P(B) = 0.5$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = P(A) + 0.5 - 0.2$$

$$0.7 = P(A) + 0.3$$

$$P(A) = 0.7 - 0.3 = 0.4$$

Now $P(A) \cdot P(B) = (0.4)(0.5)$

$= 0.20$ and $P(A \cap B) = 0.2$

$$P(A \cap B) = P(A) \cdot P(B)$$

Thus, A and B are independent events.

3) If A and B are two independent events such that $P(A \cup B) = 0.6$, $P(A) = 0.2$, find $P(B)$

Sol) Given A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B) \longrightarrow ①$$

$P(A \cap B) = 0.6$, $P(A) = 0.2$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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$$0.6 = 0.2 + P(B) - P(A)\,P(B)$$

$$0.6 - 0.2 = P(B)[1 - 0.2]$$

$$0.4 = P(B)[0.8]$$

$$P(B) = \frac{0.4}{0.8} \times \frac{10}{10} = \frac{4}{8} = \frac{1}{2} = 0.5$$

Thus, $P(B) = 0.5$

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4) If $P(A) = 0.5$, $P(B) = 0.8$ and $P(B/A) = 0.8$, find $P(A|B)$ and $P(A \cup B)$

Sol

Given $P(A) = 0.5$, $P(B) = 0.8$, $P(B/A) = 0.8$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$0.8 = \frac{P(A \cap B)}{0.5}$$

$$P(A \cap B) = (0.8)(0.5) = 0.4$$

(i) Now $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.4}{0.8} = \dfrac{1}{2} = 0.5$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.5 + 0.8 - 0.4$$

$$= 1.3 - 0.4 = 0.9$$

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5) If for two events A and B, $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$ and $A \cup B = S$ (sample space), find the conditional probability $P(A/B)$.

Sol Given $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{5}$, $(A \cup B) = S$

$$P(A \cup B) = P(S) = 1$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$1 = \frac{3}{4} + \frac{2}{5} - P(A \cap B)$$

(Ans)

$$P(A \cap B) = \frac{3}{4} + \frac{2}{5} - 1$$

$$P(A \cap B) = \frac{15 + 8 - 20}{20} = \frac{23 - 20}{20} = \frac{3}{20}$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{20}}{\frac{2}{5}} = \frac{3}{20} \times \frac{5}{2}$$

$$= \frac{3}{8}$$

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6) A problem in mathematics is given to three students whose chance to solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$. (i) what is the probability that the problem is solved ?

(ii) what is the probability that exactly one of them will solve it ?

Sol Let the three students be A, B, C respectively.

$$P(\text{A solved the problem}) = P(A) = \frac{1}{3}$$

$$P(\text{A will not solve the problem}) = P(\bar{A}) = 1 - (P(A))$$

$$= -\frac{1}{3} + 1 = \frac{2}{3}$$

$$P(\text{B solves the problem}) = P(B) = \frac{1}{4}$$

$$P(\text{B will not solve the problem}) = P(\bar{B}) = 1 - P(B)$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(\text{C solves the problem}) = P(C) = \frac{1}{5}$$

$$P(\text{C will not solve the problem}) = P(\bar{C}) = 1 - P(C)$$

$$= 1 - \frac{1}{5} = 4/5$$

P(none of them will solve the problem)

$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$

$\therefore$ P(Problem is solved) $= 1 - \frac{2}{5} = \frac{3}{5}$

(ii) P(Exactly one of them will solve it)

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= P(A) \cdot P(\bar{B}) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(B) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(\bar{B}) \cdot P(C)$

$= \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5}$

$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} = \frac{12+8+6}{60} = \frac{26}{60} = \frac{13}{30}$

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7) The probability that a car being filled with petrol will also need an oil change is 0.30, the probability that it needs a new oil filter is 0.40 and the probability that both the oil and filter need changing is 0.15

(i) If the oil had to be changed, what is the probability that a new oil filter is needed?

(ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

Sol Let the event can be defined follows:

B: car being filled with petrol will also need an oil change.

$P(B) = 0.30$

E_1: car needs a new oil filter.

$P(E_1) = 0.40$

(AOB)

$\therefore P(B \cap E) = 0.15$

(i) If the oil had to be changed, the probability that a new oil filter is needed

$$= P(E/B) = \frac{P(E_1 \cap B)}{P(B)} = \frac{0.15}{0.30} = \frac{1}{2} = 0.5$$

(ii) If a new filter is needed, the probability that the oil has to be changed.

$$= P(B/E) = \frac{P(B \cap E)}{P(E_1)} = \frac{0.15}{0.40} = 0.375$$

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8) One bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. If one ball is drawn from each bag, find the probability that (i) Both are white (ii) Both are black (iii) one white and one black.

Sol (i) Bag A has 5 white and 3 black balls (Totally 8 balls)

Bag B contains 4 white and 6 Black balls (Totally 10 balls)

$\therefore$ P(white ball from I bag) $= \frac{5}{8}$

P(white ball from II bag) $= \frac{4}{10}$

$\therefore$ Required probability $= \frac{5}{8} \times \frac{4}{10} = \frac{20}{80} = \frac{1}{4}$

(ii) P(Black ball from I bag) $= \frac{3}{8}$

P(Black ball from II bag) $= \frac{6}{10}$

$\therefore$ Required probability $= \frac{3}{8} \times \frac{6}{10} = \frac{18}{80} = \frac{9}{40}$ (409)

(iii) P(white ball from I bag and black ball from II bag) + P(Black ball from I bag and white ball from II bag)

$$= \frac{5}{8} \times \frac{6}{10} + \frac{3}{8} \times \frac{4}{10} = \frac{30}{80} + \frac{12}{80} = \frac{42}{80} = \frac{21}{40}$$

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9) Two thirds of students in a class are boys and rest girls. It is known that the probability of a girl getting a first grade is 0.85 and that of boys is 0.70. Find the probability that a student chosen at random will get first grade marks.

Sol

P(selecting a boy) $= \frac{2}{3}$

P(Selecting a girl) $= \frac{1}{3}$

P(Boy getting I class) $= 0.70$

P(girl getting II class) $= 0.85$

P(student getting I class mark) = P(selection a boy) × P(Boy getting I class) + P (selecting a girl) × P(girl getting I class)

$$= \frac{2}{3} \times 0.70 + \frac{1}{3} \times 0.85 = \frac{1.4}{3} + \frac{0.85}{3}$$

$$= \frac{2.25}{3} = 0.75$$

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10) Given $P(A) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(B)$ if

(i) A and B are mutually exclusive

(ii) A and B are independent events

(iii) $P(A/B) = 0.4$

(iv) $P(B/A) = 0.5$

Sol. Given $P(A) = 0.4$, $P(A \cup B) = 0.7$. (4M)

(i) Given A and B are mutual exclusive events.

$$\therefore P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.4 + P(B) - 0$$

$$P(B) = 0.7 - 0.4$$

$$P(B) = 0.3$$

(ii) Given A and B are independent events

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) - P(B) - P(A \cap B)$$

$$0.7 = 0.4 - P(B) - P(A) \cdot P(B)$$

$$0.7 - 0.4 = P(B) [1 - P(A)]$$

$$0.3 = P(B) [1 - 0.4]$$

$$0.3 = P(B) (0.6)$$

$$P(B) = \frac{0.3}{0.6} = 0.5$$

(iii) Give $P(A/B) = 0.4$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$0.4 = \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

$$[\because P(A \cup B) = P(A) + P(B) - P(A \cap B)]$$

$$= \frac{0.4 + P(B) - 0.7}{P(B)}$$

$$0.4 P(B) = 0.4 + P(B) - 0.7$$

$$0.3 = P(B) - (0.4)(P(B))$$

$$0.3 = P(B) [1 - 0.4]$$

$$0.3 = P(B) (0.6)$$

$$P(B) = \frac{0.3}{0.6} = 0.5$$

(iv) Given $P(B/A) = 0.5$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$0.5 = \frac{P(A) + P(B) - P(A \cup B)}{0.4}$$

$$(0.5)(0.4) = 0.4 + P(B) - 0.7$$

$$0.2 = P(B) - 0.3$$

$$P(B) = 0.2 + 0.3$$

$$P(B) = 0.5$$

---

11) A year is selected at random. what is the probability that

(i) it contains 53 sundays

(ii) it is a leap year which contains 53 sundays.

Sol (i) A year has 365 days = 52 weeks + 1 day

That 1 day may be sunday or monday or Tuesday or wednesday or Thursday or Friday or saturday

$$\therefore n(S) = 7, \; n(A) = 1$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

A year is selected at random.

It may be an ordinary year for which the probability is $\frac{3}{4}$ or a leap year for which the probability is $\frac{1}{4}$

$$\therefore \text{Required probability} = \frac{3}{4} \times \frac{1}{7} + \frac{1}{4} \times \frac{2}{7} = \frac{3}{28} + \frac{2}{28} = \frac{5}{28}$$

(ii) leap year which contains 53 sundays.

A leap year has 366 days = 52 weeks + 2days.

Those two days may be sunday and monday (or) monday and Tuesday (or) Tuesday and wednesday or wednesday and Thursday (or) Thursday and Friday or Friday and saturday or saturday and sunday.

$$\therefore n(S) = 7, \quad n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{7}$$

$\therefore$ Probability that the year selected in a leap year and it contains 53 sundays.

$$= \frac{1}{4} \times \frac{2}{7} = \frac{1}{14}$$

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12) Suppose the chances of hitting a target by a person x is 3 times in 4 shots, by y is 4 times in 5 shots and by z is 2times in 3 shots. They fire simultaneously exactly one time. what is the probability that the target is damaged by exactly 2 hits?

sol

$$P(\text{target is hit by } x) = P(x) = \frac{3}{4}$$

$$\therefore P(\bar{x}) = 1 - P(x) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(\text{target is hit by } y) = P(y) = \frac{4}{5}$$

$$P(\bar{y}) = 1 - P(y) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$P(\text{target is hit by } z) = P(z) = \frac{2}{3}$$

$$P(\bar{Z}) = 1 - P(Z) = 1 - \frac{2}{3} = \frac{1}{3}$$

P(target is hit by exactly 2 hits)

$$= P(X \cap Y \cap \bar{Z}) + P(X \cap \bar{Y} \cap Z) + P(\bar{X} \cap Y \cap Z)$$

$$= P(X) \cdot P(Y) \cdot P(\bar{Z}) + P(X) \cdot P(\bar{Y}) \cdot P(Z) + P(\bar{X}) \cdot P(Y) \cdot P(Z)$$

$$= \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{5} \times \frac{2}{3} + \frac{1}{4} \times \frac{4}{5} \times \frac{2}{3}$$

$$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{12+6+8}{60} = \frac{26}{60} = \frac{13}{30}$$

Exercise : 12.4

1) A factory has two machines - I and II. Machine - I produces 60% of items and machines - II produces 40% of the items of the total output. Further 2% of the items produced by machine - I are defective whereas 4% produced by machine - II are defective. If an item is drawn at random what is the probability that it is defective?

Sol: Let A_1 be the event that the items are produced by Machine - I.

Let A_2 be the event that the items are produced by Machine - II.

Let B be the event of drawing a defective item.

Given $P(A_1) = \frac{60}{100}$

$$P(B/A_1) = \frac{2}{100}$$

$$P(A_2) = \frac{40}{100}, \quad P(B/A_2) = \frac{4}{100}$$

we have to find the total probability of event B. Since A_1, A_2 are mutually exclusive and exhaustive. we, have (414)

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2)(P(B/A_2))$$

$$P(B) = \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{4}{100}$$

$$P(B) = \frac{120}{10000} + \frac{160}{10000} = \frac{280}{10000}$$

$$P(B) = 0.028.$$

---

2) There are two identical urns containing respectively 6 blacks and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if it the ball is black, what is the probability that it is from the first urn?

Sol

Let A_1 be the event of selecting urn - I and A_2 be the event of selecting urn - II

Let B be the event of selecting a black ball.

$$P(A_1) = \frac{1}{2}, \quad P(B/A_1) = \frac{6}{10}$$

$$P(A_2) = \frac{1}{2} \quad P(B/A_2) = \frac{2}{4} = \frac{1}{2}$$

(i) $P(B) = P(A_1) \cdot (P(B/A_1)) + P(A_2) \cdot P(B/A_2)$

$$= \frac{1}{2} \times \frac{6}{10} + \frac{1}{2} \times \frac{1}{2}$$

$$P(B) = \frac{3}{10} + \frac{1}{4} = \frac{12+10}{40} = \frac{22}{40} = \frac{11}{20}$$

(ii) By Bayes theorem, black ball from urn-II

$$= P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)\cdot P(B/A_2)}$$ (4.15)

$$= \frac{\frac{1}{2}\times\frac{6}{10}}{P(B)} = \frac{\frac{3}{10}}{\frac{11}{20}} = \frac{3}{10}\times\frac{20}{11} = \frac{6}{11}$$

$$\therefore P(A_1/B) = \frac{6}{11}$$

---

3) A firm manufactures PVC pipes in three plants viz, X, Y, and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production.

(i) find the probability that the selected pipe is a defective one.

(ii) If the selected pipe is a defective, then what is the probability that it was produced by plant Y?

Sol Let the events be defined as follows:

A_1 : production of PVC pipe from plant X

A_2 : production of PVC pipe from plant Y

A_3 : Production of PVC pipe from plant Z

B : Selecting a defective pipe

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Given $P(A_1) = \dfrac{2000}{2000 + 3000 + 5000}$

$$= \frac{2000}{10000} = \frac{2}{10}$$

$$P(A_2) = \frac{3000}{10000} = \frac{3}{10}$$

$$P(A_3) = \frac{5000}{10000} = \frac{5}{10}$$

$$P(B/A_1) = \frac{3}{100}$$

$$P(B/A_2) = \frac{4}{100}$$

$$P(B/A_3) = \frac{2}{100}$$

(i) $P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)$

$$= \frac{2}{10} \times \frac{3}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{5}{10} \times \frac{2}{100}$$

$$= \frac{6}{1000} + \frac{12}{1000} + \frac{10}{1000} = \frac{28}{1000} = \frac{7}{250}$$

(ii) By Bayes theorem, probability of selecting defective pipe from plant Y.

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{3}{10} \times \frac{4}{100}}{P(B)} = \frac{\frac{12}{1000}}{\frac{7}{250}} = \frac{12}{1000} \times \frac{250}{7}$$

$$\therefore P(A_2/B) = \frac{3}{7}$$

4) The chance of A, B, C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been imporoved, what is the probability that B was appointed as the manager? (417)

Sol Let the event be the defined as follows:

A1 : Event of A becoming a Manager

A2 : Event of B becoming a Manager

A3 : Event of C becoming a Manager

B : Event of office canteen will be improved

To find $P(A2/B)$

Given $P(A1) = \dfrac{5}{5+3+2} = \dfrac{5}{10}$

$P(B/A1) = 0.4$

$P(A2) = \dfrac{3}{10}$

$P(B/A2) = 0.5$

$P(A3) = 2/10$

$P(B/A3) = 0.3$

By Bayes's Theorem,

$$P(A2/B) = \frac{P(A2)\,P(B/A2)}{P(A1)(P(B/A1)) + P(A2)\,P(B/A2) + P(A3).\,P(B/A3)}$$

$$= \frac{\frac{3}{10} \times 0.5}{\frac{5}{10} \times 0.4 + \frac{3}{10} \times 0.5 + \frac{2}{10} \times 0.3}$$

$$= \frac{\frac{1.5}{10}}{\frac{2}{10} + \frac{1.5}{10} + \frac{6}{10}} = \frac{\frac{1.5}{10}}{\frac{4.1}{10}}$$

$$P(A_2/B) = \frac{1.5}{10} \times \frac{10}{4.1} = \frac{1.5}{4.1} \times \frac{10}{10} = \frac{15}{41}$$

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5) An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) If the husband is watching the television, the wife is also watching the television.

Sol) Let the event be defined as follows:

A_1 : Event of wife and watching the television

A_2 : Wife not watching the television

B : Husband is watching the television

Given $P(A_1) = 0.60$

$P(B/A_1) = 0.4$

$P(A_2) = 1 - 0.60 = 0.40$

$P(B/A_2) = 0.30$

(i) P(Husband watching the television)

$$P(B) = P(A_1) . P(B/A_1) + P(A_2) . P(B/A_2)$$

$$P(B) = (0.40)(0.60) + (0.30).(0.40)$$

$$P(B) = 0.24 + 0.12$$

$$P(B) = 0.36$$

$$P(B) = \frac{36}{100} = \frac{9}{25}$$

(ii) P(if the husband is watching, the wife is also watching the television)

$$P(A_1/B) = \frac{P(A_1) P(B/A_1)}{P(A_1)(P(B/A_1)) + P(\bar{A}_2) . P(B/\bar{A}_2)}$$

$$P(A_1/B) = \frac{(0.40)(0.60)}{P(B)} = \frac{0.24}{\frac{9}{25}}$$

$$P(A_1/B) = \frac{24}{100} \times \frac{25}{9} = \frac{2}{3}$$