

III. *An Extract of a Letter from the Reverend Dr. John Wallis to Richard Waller, Esq; Secretary to the Royal Society, concerning the Spaces in the Cycloid, which are perfectly Quadrable.*

Oxford, August 22. 1695.

S I R,

I Find it is thought by most, that there is no other part of the *Semicycloid* Figure (adjacent to the Curve) that is capable of being Geometrically Squared, but these two, *viz.*

1. The Segment AbV , Figur. 1. taking $AV = \frac{1}{2} Aa$, (which was first observed by Sir *Christopher Wren*, and after him by *Hugenius* and others) and it is $= \frac{1}{2} s R = \frac{1}{2} R^2 \sqrt{3}$.

2. The Trilinear $A d D$ (taking dD , in the Parallel dDC , passing through the Center C ,) which is $= R^2$.

But it is otherwise (as I have shewed in my Treatise *De Cycloide*, and that *De Motu*; the Figures of which latter I retain here, so far as they concern this occasion;) there being other portions of it, equally capable of Quadrature.

In order to which, I there shew (*De Motu, Cap. 5. Prop. 20. A. pag. 802, 803, 804.*) that not only the *Cycloid* is Triple to the Circle Generant, (which was known before) but that the *respective parts* of that are Triple to those of this: which is the Foundation on which I build my whole Process concerning the *Cycloid* in both Treatises, (and which is not pretended, that I know of, to have been observed, or known, by any body before me:.) That is $b\beta a A$ Figur. 1, triple to the Sector BaA (taking $b\beta$ parallel to Ba) where ever, in the Curve $A\tau$, we take the point b .

I then

I then shew, that the *Cycloid* is a Figure compounded of these two; the Semicircle $AD\alpha$, and the Trilinear $AD\alpha\tau bA$, lying between the two Curves $AD\alpha$ and $Ad\tau$, (and therefore, to Square any part of these, is the same as to Square the respective part of the *Cycloid*.)

I shew further (*Ibidem*, pag. 804.) that this Trilinear is but a distorted Figure, (by reason of the Semicircle thrust in between it and its Axis) which being restored to its due Position (by taking out the Semicircle into a different Figure, as *Figur. 2.* and thrusting the Lines bB home to the Axis, so as that BV be the same point) is the same with $A\tau\alpha$, *Figur. 3.* (the Parallelograms $b\beta\alpha B$ being set upright, which in the *Cycloid* stand sloping; and the Circular Archs $b\beta$, *Figur. 1.*, becoming streight-lines in *Figur. 3.* and the Lines bB being, in both, equal to the respective Archs $B\beta$, every where;) which therefore I call *Trilineum Restitutum* (the Trilinear restored to its due Position, which Figure I do not find that any before me has considered :) So that to Square any part of this, is the same as to Square the respective part of the *Cycloid*, (or of the Trilinear in the *Cycloid* :) That which in the *Cycloid* lies between two Archs of the Circle Generant in different Positions, answering to that which, in the restored Figure, lies between the respective streight-lines.

And therefore $AdDA = \tau d\delta\tau$, *Figur. 1.*, $= AdDA = \tau d\delta\tau$, *Figur. 3.*, $= R^2$. And $AbkdA$, $\tau b k \delta \tau$, *Figur. 1.*, $= AbkdA$, $\tau b k \delta \tau$, *Figur. 3.*, $= sR$. And bkd , *Figur. 1.*, $= bkd$, *Figur. 3.*, $= R^2 - sR$, *Ibidem*, Cap. 17. B. pag. 756. Where, if b be taken above $dkDC$ (passing through the Center C ,) these Figures are within the *Cycloid*, and within the restored Figure; but without them, if b be taken below that Line, and adjacent to the Curve $Ab\tau$, in both cases.

By

By R , I understand the Radius of the Circle Generant ; and by s , the Right Sine of the Arch BA , whose Versed Sine is VA .

And, where ever in my whole Discourse of the *Cycloid*, or the Restored Trilinear (which is a Figure of Archs, and a Figure of Versed Sines) the Arch a is no Ingredient in the designation ; such part or portion of them is capable of being Geometrically squared. But when I exclude a , I do therein exclude P (for that is an Arch also) and $f = a + s$, and $e = a - s$, because a is therein included.

Mr. *Caswell*, (not being aware that I had squared these Figures) had done the same by a Method of his own, (which he shewed me lately) which I would have inserted here, but that he thought it not necessary ; and instead thereof, hath given me the Quadrature of a Portion of the *Epicycloid* (which you will receive with this) and, I think, it is purely new.

IV. The Quadrature of a Portion of the *Epicycloid*. By Mr. *Caswell*.

Suppose DPV to be half of an exterior *Epicycloid*, VB its Axis, V the Vertex, VLB half of the generant Circle, E its Center ; DB the Base, C its Center : Bise& the Arc of the Semicircle VB in L , and on the Center C through L draw a Circle cutting the *Epicycloid* in P : Then I say the Curvilinear Triangle VLP will be $= BEq$ in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle will be to the Curvilinear Triangle VLP , as CB the Semidiameter of the Base, to CE : which CE in the exterior *Epicycloid* is the

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Fig. 2

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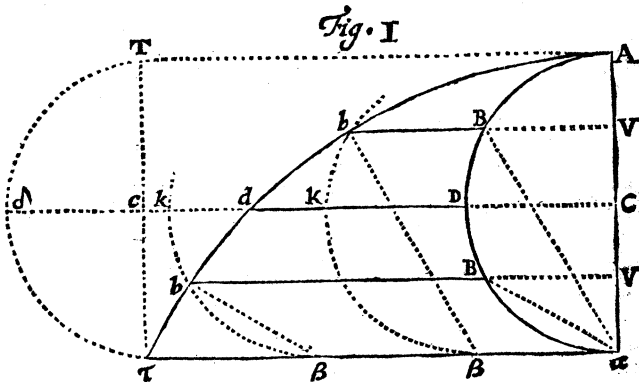
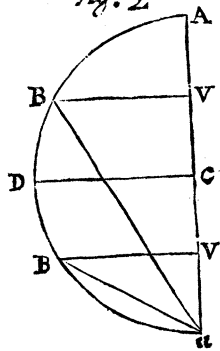


Fig. 1

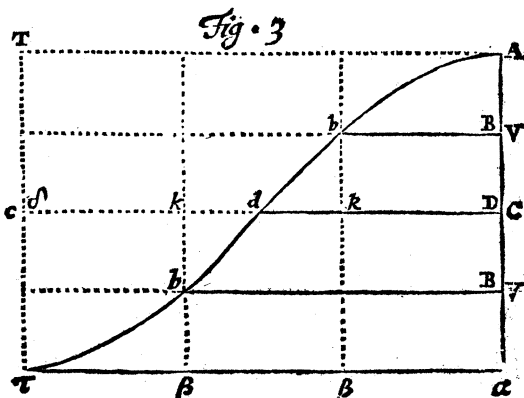
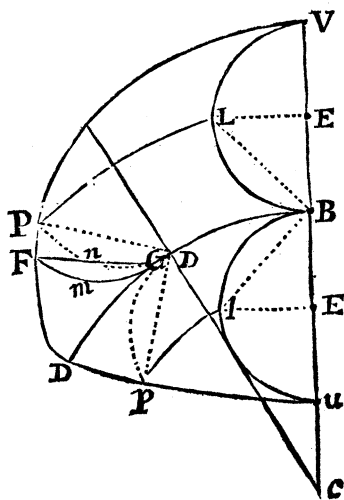


Fig. 3