THE OBSERVATION OF GRAVITY AT SEA

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THE OBSERVATION OF GRAVITY

AT SEA

A Thesis

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By

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DEDICATION

To my wife

Margaret

Preface

Knowledge of the gravity field of the earth is of prime importance to the many branches of the physical earth sciences. Chiefly, it is the geodesist or the geodesist-geophysicist who is responsible for the analytic study of the gravity field. Merely the collection of observational data on an evenly distributed world-wide basis has been a long labored after task. After the collection of data comes the reduction and analysis of the material. An increase in our understanding of the earth's gravity field is, therefore, arrived at only after much effort from the geodesists of many lands.

The knowledge gained is worthy of the labor it costs. Geologically speaking, gravity anomalies help support or refute the interpretation of the geological structure of an area.

Geodetically, the gravity anomalies amassed for the earth allow us to determine the shape of the geoid. We may also determine the flattening of the reference ellipsoid, establish a world geodetic datum, answer the question of triaxiality, determine the deflections of the verticals, and establish intercontinental ties. In the present "age of space" precise geodetic knowledge cannot be over emphasized.

Since the world ocean areas represent some 70-75% of the earth's surface, the problem of observing gravity at sea is a very pertinent one for the geodesist. Unfortunately, it is not possible to extrapolate values of Δg (gravity anomalies) into the vast unobserved areas of, for instance, the South Pacific Ocean with a truly acceptable degree of accuracy. Rather, we need observational gravity data from the entire surface of the earth. It shall be the purpose of this paper to describe and discuss the techniques available to this end for observations at sea.

The Vening Meinesz Pendulum Apparatus, for years our only acceptable at sea measuring device, will not be emphasized as strongly in the following pages as the new sea surface gravity meters. My reasoning is simply that our professional literature extensively discusses the Vening Meinesz Apparatus, while information about the revolutionary surface meters is only slowly appearing in the texts and journals of geodesy and geophysics.

The bibliography to this paper is divided into three sections. The first section lists by reference number and author all references available and used in the preparation of this thesis. The second section is a reference chart listing numbered sections of this paper and the reference numbers from the bibliography pertinent to each section of the paper. The remaining section of the bibliography is a listing of references which were unavailable to me for one reason or another, but which may be helpful to the reader who has access to them.

It is a pleasure for me to acknowledge the advice and encouragement given me in preparing this thesis by Dr. Ivan Mueller of the Ohio State University and more particularly his stimulating lectures on physical geodesy which first led me to this subject.

I also wish to acknowledge my indebtedness to the United States Navy for allowing me to temporarily "abandon" my professional seafaring duties to pursue the study of geodesy and for the financial assistance rendered, without which this course of study would have been impossible.

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1. Introduction

A basic purpose of gravimetry is to study the gravity field of the earth. Since this sets a requirement for world-wide gravity observations, various techniques had to be developed to make observations possible over a wide range of conditions.

Before going further, however, we should first define the quantity we are attempting to observe. Gravity is the resultant of centrifugal force due to the earth's rotation and the attraction of mass. It is this acceleration g (gravity) which we try to measure. Gravity may be observed in the absolute sense, but this ~~must~~ may be done only on land. Absolute gravity has been measured with an accuracy of one milligal, but such observations are tedious, time-consuming, and expensive. Relative gravity, on the other hand, can be observed quickly and accurately with gravimeters between absolute stations. The relative gravity is the measured difference at a field station from the absolute gravity at a base station.

It is relative gravity that we are measuring at sea. On land such measurements can be made with an accuracy of 0.02 milligal; at sea under the best conditions 3 to 4 milligals is the best obtainable accuracy.

The problems inherent in ocean observations are formidable. Interestingly , there would be little difference between land observations and sea observations were it not for the movements of the observation platform. The motion of the ship caused mainly by wave motion are transformed into vertical, horizontal, and rotational accelerations. Figure 1 illustrates the resultant ship motions which cause the various acceleration disturbances to a shipboard observation platform.

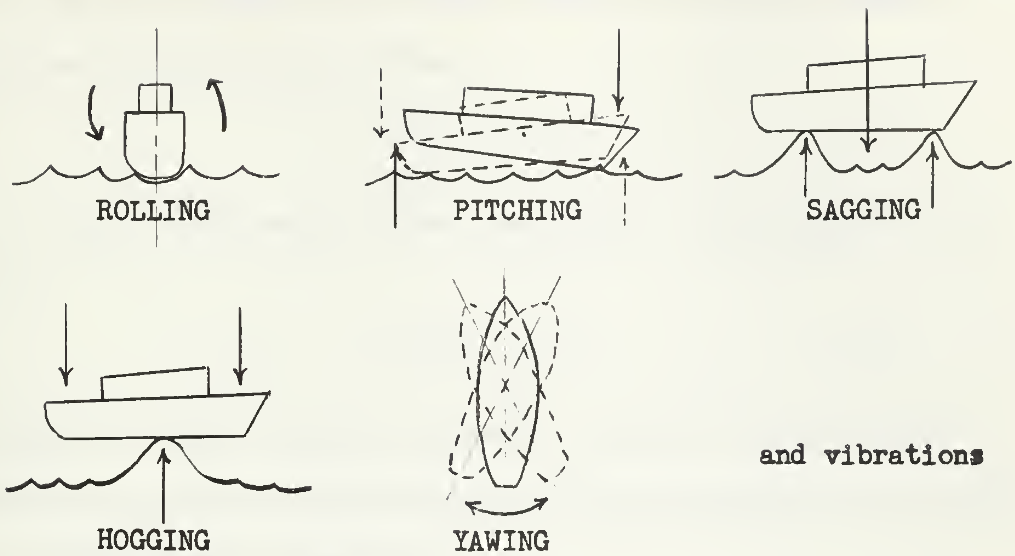
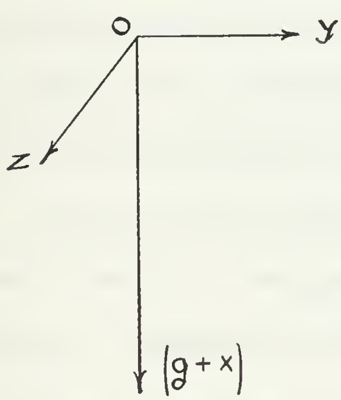


figure 1

The accelerations acting on a sea gravity observation instrument will be initially defined by the coordinate system in Figure 2.

$\ddot{z}$ and $\ddot{y}$ then represents the horizontal disturbance accelerations and $\ddot{x}$ represents the vertical disturbance accelerations that is



$$\left.\begin{aligned} \dot{x} &= \frac{\partial x}{\partial t} \quad ; & \ddot{x} &= \frac{\partial^2 x}{\partial t^2} \\ \dot{y} &= \frac{\partial y}{\partial t} \quad ; & \ddot{y} &= \frac{\partial^2 y}{\partial t^2} \\ \dot{z} &= \frac{\partial z}{\partial t} \quad ; & \ddot{z} &= \frac{\partial^2 z}{\partial t^2} \end{aligned}\right\} \quad (1)$$

figure 2

We are trying with our measuring apparatus to measure g, but the acceleration disturbances $\ddot{x}$, $\ddot{z}$ and $\ddot{y}$ add vectorially to g. We will see later on that $\ddot{z}$ and $\ddot{y}$ may be separately observed and deducted

from the value we obtain for g, but $\ddot{x}$ cannot. The relationship of gravity and vertical acceleration is

$$\frac{1}{T}\int_0^T (g + \ddot{x})\, dt \qquad (2)$$

where T is the time length of the observation. The integral of (2) then is

$$g + \left[\dot{x}\right]_0^T \Big/ T \qquad (3)$$

Navigation errors resulting from the travel of the observation platform on or over the vast ocean areas also contribute to the problem of accurate sea observations.

A part of gravity is centrifugal force, and the effect of centrifugal force on the gravity measured is increased or decreased by the east-west component of the moving platform's velocity. This effect is called the Eötvös effect. Computation of the Eötvös effect is dependent upon accurate course and speed knowledge; in practice this correction proves to be a large source of error.

There are available three approaches to gravity observations at sea. One may observe with a pendulum apparatus in a submarine, or with a gravity meter on the ocean floor, or with a special gravity meter either on a ~~surface~~ ship or in an aircraft. Each approach has its limitations as well as its particular strong points. We shall see that at the present time only the last approach holds promise of being able to give us extensive gravity observations over the ocean areas in a relatively short period of time.

The remaining portion of chapter 1 will outline briefly some of the historical development of gravity at sea techniques. The subsequent

chapters will present a more detailed presentation of the Vening Meinesz Pendulum Apparatus, the Askania-Werke Seagravimeter Gss2 after Graf and the La Coste-Romberg Air-Sea gravity meter.

1.1 Early History and Development - The Pendulum Apparatus

In the early 1920's a Dutch scientist began wrestling with the problem of determining precisely the deflections of the vertical in his country's geodetic network. The single pendulum apparatus normally used to measure gravity incident to the computation of the deflections of the verticals proved unsatisfactory. The pendulum was too much disturbed by the microseisms of the waterlogged Netherlands soil to give the desired first order accuracy. To eliminate terrain disturbances on the pendulum he conceived the idea of swinging two pendulums in anti-phase from the same stand. Fortunately for us, this scientist, F. A. Vening Meinesz, saw rather startling possibilities in this new apparatus for marine observations.

His first experiments were conducted on several cruises in the North Sea on board a small steamer. Vening Meinesz soon found that despite many improvements on his apparatus, the mechanical vibrations of the steamer could not be damped out. A submarine was the only answer.

By enlisting the aid and the interest of the Royal Dutch Navy the first submarine gravity survey expedition was set up in 1923. Vening Meinesz selected the East Indies as the ideal proving ground for his apparatus. His reasoning was simple: the East Indies area offered the greatest elevation difference between the tops of island mountains and the continuous for deeps (troughs) oceanward from the archipelagos. A Stückrath Pendulum Apparatus was used, being four

pendulums swinging in two planes. Technologically, the cruise allowed Vening Meinesz to perfect the photographic recording of each separate pendulum, improve the gimbal suspension system, improve the pendulum bearings, and set up a technique of timing the pendulums from multiple chronometers.

Scientifically, the cruise was more rewarding than could possibly have been hoped in the planning stage. Data from the gravity observations revealed a thin line of gravity anomalies along the ocean-side of the great sinuous island archipelago. In magnitude the anomalies were greater than any other gravity anomalies known. Most amazing, however, was their definitely negative sign all along the belt line. (The over-all field was slightly plus: [19].) Initially this discovery would appear to upset the theory of isostasy. Careful analysis of the new gravity field information, however, tends to strengthen the isostatic theory through revised theories of mountain building and structural geology.

The initial cruise and several subsequent cruises (1923-1927) were thus very significant. The Vening Meinesz Pendulum Apparatus for gravity observations at sea evolved and was acknowledged as our only device workable at sea for such observations.

In 1936 the USS Barracuda voyaged through the West Indies area with the Vening Meinesz Pendulum Apparatus, and as a result of their observations it was shown that the West Indian arc is tectonophysically similar to the East Indian arc [19]. A significant advancement in the observation technique was made by the use of the Bell Laboratory's crystal chronometer [19]. This new time piece made the chronometer rate a negligible error.

B. C. Browne of Cambridge, England published in 1937 the results of his theoretical studies of free and forced oscillations of the gimbal suspension system [4]. He derived a correction now referred to as the Browne or second order acceleration correction, which must always be applied to the observed gravity obtained from a free pendulum apparatus. In 1937-1938 Vening Meinesz [68] (and later, Browne and Cooper [5]) attempted to prove Browne's theoretical studies of 1937, but was only partially successful. Vening Meinesz did, however, design a long period pendulum which, when added to his three pendulum apparatus, measured the second order horizontal accelerations. J. C. Harrison has investigated the vertical accelerations for periods between 5.9 and 12.8 seconds and the horizontal accelerations for periods of 3.30 to 6.85 seconds [26]. His results give direct support to the correctness of Browne's second order correction.

1.2 Underwater Gravimeters

The late 1930's saw the beginning development of gravimeters for under water observations. These underwater gravimeters or "bottom sitters" are good to depths of approximately 100 fathoms. The coastal area out to the 100 fathom curve, that is, the shelf area, represents some 7.6% of the ocean area [14p 20]; frequently, submarine operations are impossible over the shallower portions of the shelf (i.e., in-shore areas, gulfs, bays, reefs, lagoons, some territorial waters, etc.) Hence, underwater meters are a necessity.

The basic construction of an underwater meter classifies it as one of four categories [67p 113]:

1. Housing containing both instrument and observer.

2. Housing containing only the instrument.

3. Pressurized housing with direct gravity leveling.

4. Pressurized housing with remote control leveling.

One of the earliest underwater gravimeters was manufactured by the Gulf Research and Development Company [46]. It was in use by the late 1930's. It is the remote control type, enclosed in a pressurized housing, and quite complicated. Specifications for this instrument are listed in Table I.

The Finnish Geodetic Institute recently completed a gravity survey in the Baltic Sea area with a Gulf underwater gravimeter. Honkasalo [36] in reporting the survey noted that the meter functioned so well that a drift check was needed only every third day. The m/s Aranda, a specially outfitted Finnish oceanographic ship, has two screws - a forescrew and an after screw - thereby allowing the ship to be well positioned over the gravity station. Claimed positional accuracy is 20 meters; 178 stations were measured, proving the workability of the underwater meter in large shallow areas.

Many different underwater gravimeters are available. Generally, the meter is a conventional land meter, as in the case of the Gulf underwater gravimeter, and only the housing and control units are original. Some of the most interesting housing schemes are those described by Frowe [21]. Figure 3 shows a cylindrical diving bell of the most elaborate type. The bell consists of two water tight cylindrical steel chambers with a top hatch leading into the inner chamber. The outer chamber is flooded by the observer to produce

Specifications of the Gulf Underwater Gravimeter

TYPE METER	Pressure housing; Remote control leveling; Gulf Land Meter
RANGE (depth)	Tested to 700'
LEVEL CONTROL	Remote; accuracy 10" assured; motor drive against gimbals in 2 independent directions. Max. limit 15° off center to any 1 direction. 15° to 15° in 3 minutes; back lash < 1 second; clamping-permanet magnet remote operated. Level circuit controled by photo-voltaic cells.
RECORDING	Continuous 35mm; adopted Leica camera, with sp. access hole
WATER ALARM	Red light on control panel actuated by bell float in housing
HOUSING ASSEMBLY	Aluminum; approx. 300 lbs (25-30 submerged). Up to 350 lbs ballast on legs as needed.
SUPPORT	Tripod legs with flat disk feet.
OPERATOR CONSOLE UNIT	Seven meters, 4 pilot lights, 9 switches, 2 relays, 2 motor driven controls for fine level adjustment.
POWER SUPPLY	2 - 12 volt storage batteries for power and lights 1 - 12 volt storage battery for temperature control
TEMPERATURE CONTROL	Thermal lagging and thermostat control. Accurate to 0.001°C for short period and .0X°C for long periods.
CABLE CONDUCTOR	1 cable — 19 conductors, 4 for common - ground or as spare conductors, 15 for control-control 4 motors (3 reversable) 3 light circuits, thermostat control, 15° level limit, water alarm, clamp signal, two photocell circuits.
ACCURACY	Read, accuracy to 0.02 mgal SE of obs. measured gravity 0.3 mgal.
OBSERVATIONS	Record is 20 STATIONS/DAY Average 6 STATIONS/DAY.

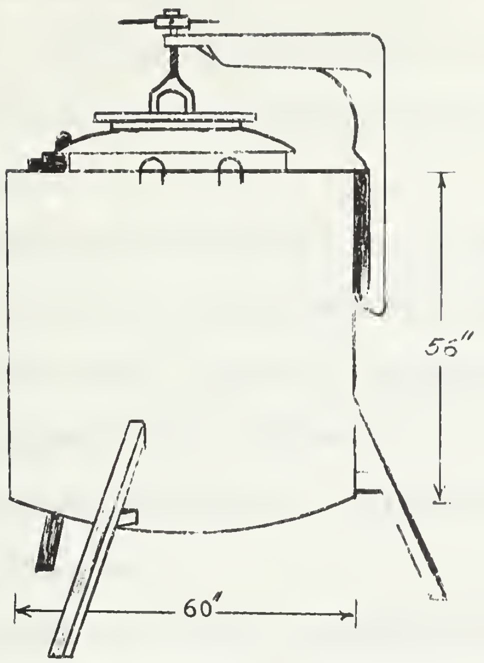
TABLE 1

negative buoyancy. The gross weight (observer + meter + bell and weights) is 5,000 pounds, of which 2,000 pounds is lead ballast. The bell's displacement is 5,400 pounds resulting in a net buoyancy of 400 pounds. Safety provisions include two-way phones, high pressure air exhausts, and two separate air breathing sources. Design depth is 250 feet; test depth is 500 feet. Tripod legs firmly support the bell on the bottom after lowering by a work boat crane. The cylindrical diving bell has been used successfully, notably along the southern coast of Cuba.

The conical shallow water bell shown in figure 4 is good for depths to 60 feet. It has excellent stability characteristics, making it particularly useful on soft mud bottoms. The design of the ballast system is variable, allowing for a flat extended skirt around the base of the bell, or the tripod legs as shown in the figure.

Figure 5 shows an open bottom diving bell designed by V. W. Humphrey [21] p 4. This bell is limited to depths of less than 33 feet, because the air pressure within must be greater than the water pressure. The open bottom design was successful in a gravity survey of the north end of Lake Maracaibo, where an elevated tripod support was undesirable due to strong winds and rough water. Operation in this bell is rather fatiguing, and only a limited number of dives per day is possible.

In addition to the underwater gravity meters rather expensive work boat equipment is needed. The boat must be able to be located and anchored over the observation point. This requires a minimum of two anchors (bower and kedge). Some work boats are fitted with

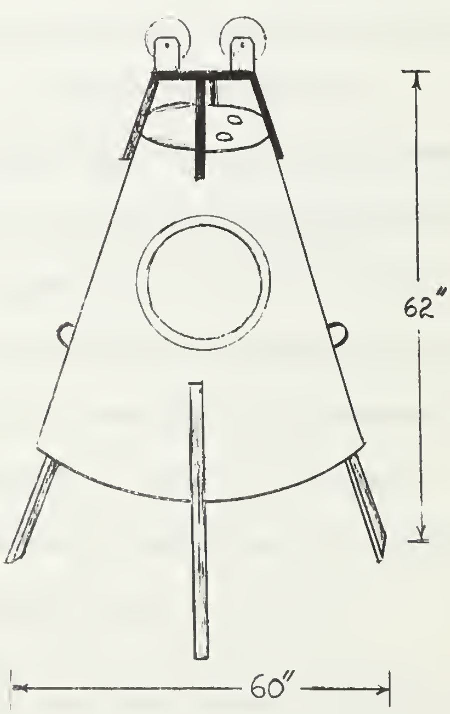


Cylindrical Diving Bell
Bell with self-contained water ballast chamber. Good between 18 - 250' on a coral or hard sand bottom.

figure 3

Conical Diving Bell
Limited to shallow waters. Bell has excellent stability characteristics on soft mud bottoms.

figure 4



scudders (extension stilts at the four corners of the barge-like hull). A minimum operating depth of 2½ feet is possible on a specially designed work boat of the Gulf Research and Development Company.

Gravity meters used for the accurate measurement of earth tides are a form of underwater gravity meters. They will not be discussed in this paper, since the information obtained from them is not direct gravity field information.

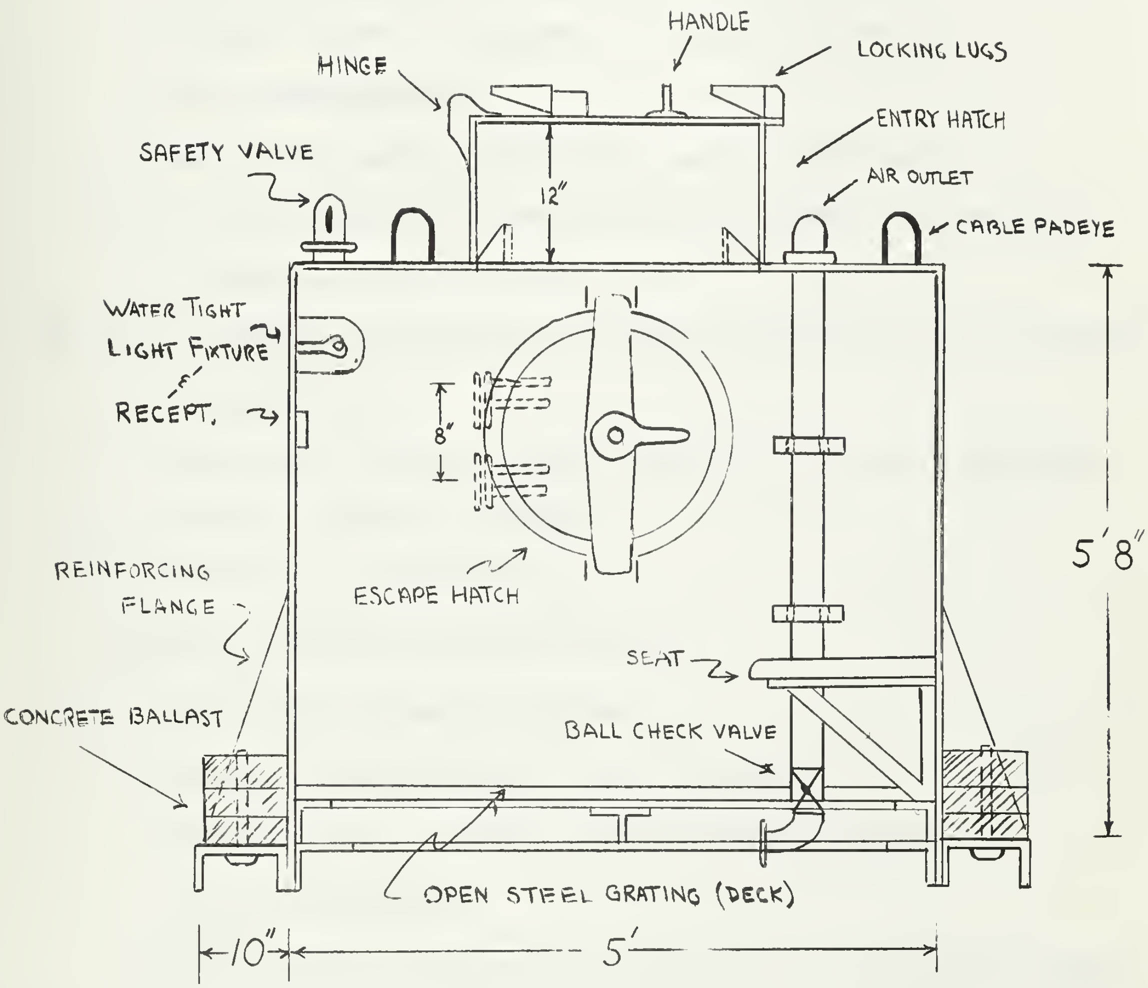
1.3 Historical Development of the Graf and La Coste-Romberg Surface Gravity Meters.

I shall only briefly mention the sea surface meter developments here. The evolution of these meters is very interesting but goes hand in hand with understanding the operation and resulting accuracies which I will describe in some detail in subsequent chapters.

Prior to 1957 there had been no significant advance in the measurement of gravity at sea since the Vening Meinesz Pendulum Apparatus of 1927. Pendulum measurements, however accurate, are slow and costly. Availability of submarines for observations are understandably limited. The number of at sea pendulum stations is lamentably few compared to the coverage geodetic investigations require. Mr. B. C. Browne, secretary of Section IV of the International Commission of Gravimetry, prompted by this state of affairs, said in 1955:

"Reports on work carried out during the last three years were received. Although the large amount of work was generally appreciated, it was clear that the number of stations was still too few for geodetic purposes."

The answer to the problem lies in the invention of measuring apparatus capable of observations for a surface ship. Design difficulties are considerable. Such a meter would have to correct for acceler-



An open bottom type diving bell designed by V. W. HUMPHREY. Air pressure must always exceed water pressure, and dives are limited to around 33'.

figure 5

ation forces up to 100,000 milligals. Specification for a surface meter may be very generally listed as follows: [29]

General Specifications

1. Goal of 1 milligal observation accuracy

2. Heavily damped to suppress horizontal and vertical acceleration disturbances

3. Very stable meter drift checks may be weeks apart

4 a. Not affected by horizontal accelerations and stabilized along the true vertical

or 4 b. Mounted in gimbals and allowed to follow the total apparent acceleration

5. Dissimilar natural periods of meter and waves to eliminate possible resonance effects

6. Portable or semi-portable

Additional desirable specifications

1. Quick and continuous readings

2. Automatic Browne second order correction

3. Operable with or without gyro stabilized platform

4. Aircraft adaptable

The years 1957 and 1958 may well prove to be historically remarkable years for physical geodesy. It was then that gravity measurements were obtained from two different meters on surface ships. The initial results were not without appreciable error, but indications from the observations point to an imminent technological breakthrough in oceanic gravity field measurement.

Dr. Anton Graf of West Germany, through the efforts of Dr. J. Lamar Worzel at the Lamont Geological Observatory and of the U.S.

Navy, was able to test his new meter on the submarine Becuna [25]. Previous tests by Graf on the Starnberger Sea and in the Adriatic Sea from Venice to Trieste had convinced him of the worthy potential of his gravimeter.

The Becuna tests conducted from Palma, Majorca to Portsmouth, England observed 19 Mediterranean stations with good weather and short waves and 40 Atlantic stations with typically rough weather. The Atlantic was so severe that observations across the Atlantic Ridge were impossible. Observations were made as nearly simultaneously as possible with the Graf meter and a Vening Meinesz Apparatus. The drift factor of the Graf meter was unknown. Fifty-nine comparisons were made.

No. obs.	Differences from V.M. Pendulum obs.	Possible Cause
3	Large (magnitude not reported)	1. Poor depth control 2. Excessive weather
14	5-9 milligals	1. Unknown 2. Depth control 3. Short obs. time
39	0-3 milligals	--------

Table 2

From table 2 it can be seen that the Graf meter showed great promise. His next development steps were to add a second order correction, correct drift uncertainty, and mount the instrument on a stabilized platform.

In 1957 Worzel was able to conduct some initial tests on the USS Compass Island (EAG 153). The ship ran a course south east out from

New York following the earlier Vening Meinesz stations along the Hudson Canyon as nearly as possible. The results were encouraging and warranted more extensive testing.

The spring of 1958 saw the Compass Island out from New York bound for the Mediterranean. Two Graf gravimeters were on board operating from a stabilized platform. The ship is fin stabilized resulting in a normal roll of less than one degree. Pitch is also less than one degree. A fin stabilized ship has a small metal fin protruding below the water line on each side of the hull. The fins are gyrostabilized perpendicular to the true vertical. As the ship rolls, counter torque is created by the fins moving in opposition to the roll to maintain their relationship to the true vertical; hence, the true roll of the ship is damped. A 90% reduction in roll has been achieved by the utilization of the gyro-fin stabilization system on board the Compass Island [2 p.378]. Worzel reports that the instrument platform is capable of staying within one minute of the true vertical. A base station closure of eleven milligals at New York 35 days after sailing was considered to be an acceptable magnitude.

The Compass Island report [82] concludes that "continuous gravity measurements at sea with a precision of ± 5 milligals can be made with the Graf Sea Gravimeter mounted on a stable platform on a surface vessel." I am inclined to believe that the claimed accuracy of ± 5 milligals is somewhat optimistic. The Eötvös effect and navigational error introduce at least 3 to 5 milligals average errors in submarine observations. Unfortunately, we know little of the observational

conditions (i.e., weather, operator technique, sea state, etc.), so an estimate of accuracy is difficult to arrive at.

The cruise furnished Graf with valuable insight into the problem areas of his meter, and in 1961 he published a paper on the improvements he had built into his latest Askania Gss2 Sea Gravimeter [25]. Significant improvements were a 500% increase in damping and the ability to handle vertical accelerations up to 100,000 milligals in magnitude.

During the period that the Graf meter was being developed, Dr. Lucien La Coste of La Coste-Romberg in Texas was similarly developing a sea gravimeter of his own. The testing and evaluating of La Coste's meter closely parallels the type of development previously described for the Graf meter.

In 1955 the first testing took place, being conducted off the southwest coast of California aboard the U.S. submarine Tilefish. Former pendulum stations were occupied as closely as navigation techniques permitted. A second cruise in that year on board the USS Baya from Hawaii to San Diego included observations across the Murray Fracture zone. Initial conclusions were that scatter between pendulum and meter observations is comparable to the normal scatter between two pendulum observations. There was no accuracy advantage to the meter over the pendulum, but operations and the reduction of results with the meter are very much simpler, and a continuous relative gravity profile is obtainable. Both cruises pointed up the need for a meter capable of handling vertical and horizontal acceleration disturbances up to 100,000 milligals.

The Texas Agricultural and Mechanical College ship Hidalgo put out to sea in the spring of 1958 for a Caribbean IGY cruise. On board was the now greatly improved La Coste meter. Drift had been brought to a very acceptable level of approximately one milligal per month. The Browne second order correction was to be measured by a long period pendulum and computed and deducted from the observation by an analog computer. The six week cruise was followed by cruises in the Horizon, a craft of the University of California. The first Horizon cruise gave the observers a root mean square difference between pendulum stations and surface meter stations along the southern coast of California of 4.4 milligals. Observation speeds started at 3.5 knots and increased to 8.5 knots and finally to 11.5 knots. Harrison reports that navigation during the three day cruise was exceptionally good, and estimated navigation accuracies were as good as 0.25 knots and 0.5 miles for part of the observations [28 p 1876]. The meter proved to give quite valid free air gravity profile when compared to a detailed bathymetric chart and when compared against itself in two track recrossings (only 3 and 2 milligal difference).

Navigation makes comparisons between pendulum and surface observations of some questionable value. On the preceeding cruise navigation for surface observations was quite good, but it is exceedingly difficult to equate how accurate the pendulum stations were located in the first place. At any rate, those who participated in the Horizon cruises felt that under reasonable conditions and good navigational control measurements accurate to ± 5 milligals could be obtained. [28 p 1880].

Subsequent cruises were and continue to be made, testing, evaluating, and further developing the La Coste-Romberg meters. 1961 saw a comparison test between the La Coste-Romberg and Askania Graf meters aboard the USS Aragonese in the Mediterranean sponsored by the Office of Naval Research. Final results are yet to be published, and the preliminary report is quite inconclusive [15]. Dehlinger of Texas A & M participated in the Aragonese cruises and prior cruises with the meter in the Gulf of Mexico. He has reported [15 p 4] that yawing or fish tailing of the ship (see figure 1) seems to seriously affect the meter's true reading of g. The accelerometer recording the horizontal acceleration may not record the yawing effect, yet this effect may be seen by the meter; hence, the proper magnitude horizontal accelerations would not be averaged out.

Similarly, the U.S. Coast and Geodetic Survey is conducting surface ship surveys with a La Coste-Romberg meter No. 11 on board the C. and G.S. Ship Pioneer in the Pacific. A preliminary report by H. Orlin [45] indicates the significant error sources requiring greater attention are: navigational errors, Eötvös correction, yawing effect of ship's motion, sensing ability of horizontal accelerometers, and sea state limitations. Orlin shortly will publish a detailed study of theoretical and practical considerations in the La Coste meter. It should add significantly to the literature in this field.

Thus, the development of the Graf and La Coste meters continues. Perhaps the key to success for these meters is the complete study of the accelerations affecting the ultimate meter accuracies. Chapters 3

and 4 will describe the construction of these two meters.

1.4 Some Other Measuring Devices Under Development.

Although the meters previously discussed remain the most promising, mention should be made of some of the other meter types being developed.

Professor C. Tsuboi in collaboration with T. Tomada is developing a meter with a mass suspended from a bifilar suspension and maintained in rotational oscillation [87]. The period of the oscillations is the result of gravity. Actually, a difference in period from a crystal quartz oscillator signal is continuously made, compared and recorded. The suspended mass is highly damped producing a period of about 50 seconds; therefore, vertical accelerations are greatly reduced. The physical damper is a special silicone oil in a dual suspension designed to eliminate surface tension. The meter is gimbal suspended. No horizontal correction is considered. The temperature change is linear thus reducing temperature control to 0.5°C. One observation takes 50 minutes. Initial tests in a bay area gave an approximate 3 milligals to the known values.

Dr. Yu D. Bulanzhe is developing an apparatus with two sets of three pendulums giving two observations per station. The horizontal and vertical accelerations are measured with a forty-second pendulum [87].

There is a Nørgaard type of heavily damped meter being tested, but little about this has been published in the English journals. The Nørgaard land gravimeters are very simple in design, being an E shaped quartz frame as shown in figure 6. Along the top and bottom edges of the E is a quartz beam suspended on a thread. At the end of the

beam is a mirror. A fixed mirror is attached to the middle arm of the E. As gravity changes the pull on the quartz frame, the beam is moved. The two mirrors, fixed and moved, display a change in angle, which is scaled to a change in gravity. I suspect the sea type Nørgaard meter will be a variation of the land meter.

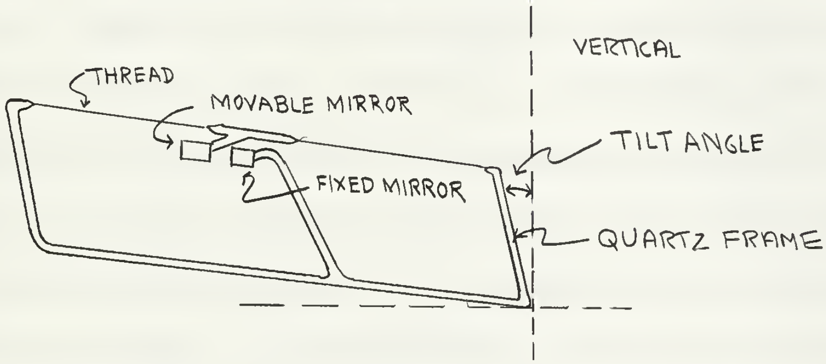
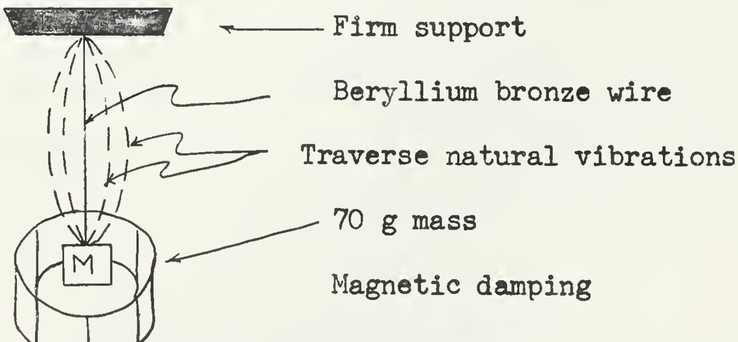


figure 6 Nørgaard Land Gravimeter

Mention should also be made of the vibrating string type gravimeters tested by R.L.G. Gilbert in 1948 [23] and A. M. Lozinskaya in 1958 [41]. The string gravimeter measures the change in frequency of the natural vibrations of a wire stretched by a weight. The Lozinskaya meter is shown schematically in figure 7. The Gilbert meter is very similar.



Lozinskaya String Gravimeter

figure 7

The traverse natural vibrations shown in figure 7 depend upon the

tension created by the 70 gram weight which in turn is dependent upon the gravity acting on it. Therefore the natural vibration variations can be related to changes in gravity. Both Gilbert's and Lozinskaya's gravimeters have shown promise in initial testing, but to my knowledge only the latter meter is being further developed.

E. I. Popov has conducted initial sea surface tests with a quartz type "GAL" gravity meter in 1958 [47]. Considerable difficulty was encountered with the vertical and horizontal acceleration disturbances. The mean quadratic errors of the observations when compared to the observational results of a 4 pendulum type apparatus was ± 3.5 to ± 14.0 milligals. Specific details of the "GAL" gravimeter are unknown to me.

Dr. B. J. Collette of the Netherlands Geodetic Commission hopes to test a surface towed meter in 1962 [50]. The results will undoubtedly be published.

The foregoing is by no means a complete survey of the development of sea gravimeters. Rather it should serve to demonstrate the variety of instrumentation approaches possible to this problem of ocean gravity observations.

2. The Vening Meinesz Pendulum Apparatus

2.1 Some Theoretical Considerations [67,68]

The Vening Meinesz Pendulum Apparatus in its most simple form is three pendulums hung from the same horizontal support, swinging in the same vertical plane (figure 8). The coordinate system in figure 1 and the equations from (1) define the notation used to describe the vertical acceleration disturbances ($\ddot{x}$) and the horizontal acceleration disturbances ($\ddot{y}$ and $\ddot{z}$). The measurement of gravity from the observed periods of the pendulum apparatus is affected by the $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ accelerations as well as the gravity. If we can determine the effect of these acceleration disturbances on the observed gravity result we may be able to eliminate them or measure the magnitude of their effect. In the latter case, we can then deduct from the observed value of gravity that portion which was not gravity but acceleration disturbance.

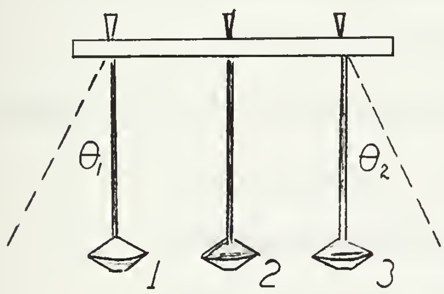


figure 8

Vening Meinesz developed his apparatus so that in swinging the two outer pendulums (Numbers 1 and 3 in Figure 8) in anti-phase the first order horizontal accelerations will be eliminated. The two outer pendulums are observed with the common middle pendulum (No. 2) at rest, creating two pendulum pairs. The angles of elongation (amplitude) of the two pairs are recorded as the angles of elongations of two "fictitious" pendulums. The actual pendulums must be isochronous (or very

nearly so). From the equation of motion for the two outer pendulums, considering only a horizontal disturbance:

$$l_1\ddot{\theta}_1 + g\theta_1 + \ddot{y} = 0 \qquad (4)$$

and

$$l_2\ddot{\theta}_2 + g\theta_2 + \ddot{y} = 0 \qquad (5)$$

Since the pendulums are swung in anti-phase and are isochronous ($l_1 = l_2$), we need the difference (4) - (5)

$$l(\ddot{\theta}_1 - \ddot{\theta}_2) + g(\theta_1 - \theta_2) = 0 \qquad (6)$$

where: θ_1 and θ_2 are the angles of elongation, $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are the second derivatives of θ_1 and θ_2 with respect to time. l is the mathematical pendulum length. The length of the mathe- mathematical pendulum is equal to the reduced length of the physical pendulum, and their periods are equal. The reduced length of the physical pendulum is from the pivot point to the pendulum center of gravity.

It can be seen from (6) that $\ddot{y}$ has been eliminated.

From (2) it was shown that gravity and the vertical acceleration disturbance ($\ddot{x}$) are observed as one quantity by the measuring apparatus. The integral of this relationship (3) was:

$$g + \left[\dot{x}\right]_0^T \Big/ T$$

We can express $[\dot{x}]$ as $\dot{x}_0$ at t = 0 (beginning) and $\dot{x}_1$ at t = T (end) where T is the total observation time. Or

$$\frac{\dot{x}_1 - \dot{x}_0}{T} = \frac{\underline{\text{difference in vertical velocities}}}{\text{length of observation}} \qquad (7)$$

By making T large compared to $\dot{x}_1 - \dot{x}_0$ we can make the first order vertical acceleration disturbance, $\ddot{x}$, negligible. The practical time for T is a

minimum of 30 minutes. It should be noted that this places a minimum half-hour observation time on a Vening Meinesz Pendulum observation.

In 1937 B. C. Browne [4] pointed out that the second order term for $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ were not negligible as originally assumed by Vening Meinesz. If the period of the disturbance is longer than the pendulum period, the apparatus will measure the resultant acceleration of the components of $g+\ddot{x}$, $\ddot{y}$ and $\ddot{z}$. Using the notation of Browne, the resultant acceleration, G (t), is shown in Figure 9. The magnitude of G (t) is expressed by

$$/G(t)/ = \left[(g+\ddot{x})^2 + \ddot{y}^2 + \ddot{z}^2\right]^{1/2} \qquad (8)$$

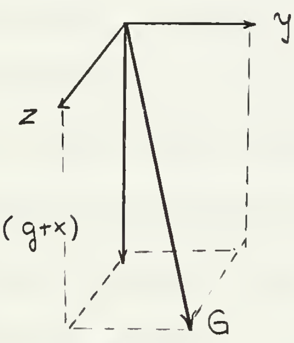


figure 9

Expanding, then multiplying the right side of (8) by g^2/g^2 we get

$$g\left[1 + \frac{2\ddot{x}}{g} + \frac{\ddot{x}^2}{g^2} + \frac{\ddot{y}^2}{g^2} + \frac{\ddot{z}^2}{g^2}\right]^{1/2} \qquad (9)$$

It is assumed that the accelerations $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ are small compared to g. We may therefore consider $\ddot{x}^2/g^2$ to be negligible and rewrite the horizontal accelerations as $(\ddot{y}^2 + \ddot{z}^2)/g^2$. By a binomial series expansion $(a+x)^n = a^n + na^{n-1}x + \ldots$ of (9).

$$/G(t)/ = g\left(1 + \frac{\ddot{x}}{g} + \frac{\ddot{y}^2 + \ddot{z}^2}{2g^2}\right) \qquad (10)$$

The mean G(t), $\overline{G}$, is the resultant acceleration over the observation period of $T \geqq 30$ minutes as can be expressed by

$$\overline{G} = g\left(1 + \frac{[\dot{x}]}{Tg} + \frac{\overline{\ddot{y}^2} + \overline{\ddot{z}^2}}{2g^2}\right) \qquad (11)$$

From the previous discussion of the first order vertical acceleration $\ddot{x}$ and from (7) we see that $[\dot{x}]/T\,g$ is negigible. Therefore

$$\overline{G} = g\left(1 + \frac{\overline{\ddot{y}^2} + \overline{\ddot{z}^2}}{2g^2}\right) \qquad (12)$$

Thus it remains for us to determine the second order horizontal acceleration disturbances $(\ddot{y}^2 + \ddot{z}^2)$. The problem of determining $\ddot{y}$ and $\ddot{z}$ is that we have no stationary reference on board ship. Browne aptly stated the problem as follows:

"The real difficulty lies in the fact that the measurements are made relative to a set of axes moving with the ship; and as the result is required relative to a set of axes rotating with the Earth, we must made some observation of the relative motion of the two sets of axes". k

Fortunately, Vening Meinesz was able to construct a set of special long period pendulums for his apparatus that would record the position of the vertical necessary to the determination of $\ddot{y}$ and $\ddot{z}$. The long period pendulum unit will be described in paragraph 2.22.

The second order vertical accelerations also must be considered. The vertical accelerations are estimated from the fluctuations of the chronometer time marks shown on the photographic record and/or from accelerometer records. The well known formula:

$$T = \pi\sqrt{\frac{l}{g}} \qquad (13)$$

shows the period of the pendulum (T) varies inversely with the square root of the acceleration. A linear approach to the variation of T with g is therefore out of the question. Jefferies solution for the second order vertical acceleration disturbances [4] is developed from (10). The phase velocity at any instant is given by $(G(t)/l)^{1/2}$ and can be approximated by

$$(g/l)^{1/2}\left(1 + \frac{\ddot{x}}{2g} + \frac{\ddot{y}^2 + \ddot{z}^2}{4g^2} - \frac{\ddot{x}^2}{8g^2}\right) \qquad (14)$$

and the second order vertical acceleration correction is

$$+\frac{\ddot{x}^2}{4g^2} \qquad (15)$$

The complete second order correction as developed by Browne may be shown as:

$$dg = \left(\frac{[\ddot{x}^2]}{2} - \frac{[\ddot{y}^2] + [\ddot{z}^2]}{g} \right) \Big/ 2g \qquad (16)$$

The final group of accelerations to be discussed are the rotational accelerations. The resulting effects of rotational movements are two fold:

1. Relative accelerations are given to different parts of the apparatus. The unequal forces can thereby invalidate the reading. Theoretically, this effect could be minimized by using an infinitely small apparatus, thereby making the rotational effect equal to all parts of the apparatus. The Vening Meinesz outer pendulums are 26 cm apart, so the rotational effect must be considered.
2. The apparatus tilts away from the true vertical. Tilting changes the magnitude of the component of the resultant acceleration in the direction of the axes. If we can measure the tilt, we can correct this portion of the observation.

The functions of the rotational movements are recorded by utilizing two independent and heavily damped auxiliary pendulums at right angles to each other as the standard against which the swinging plane of the main pendulum is compared and recorded photographically.

2.2 The Apparatus

The apparatus is made up of three distinct units bonded together, one on top of the other as shown in Figure 10. The photographic recording unit is the upper-most section, then the slow pendulum apparatus, and finally the pendulum box. The entire apparatus is

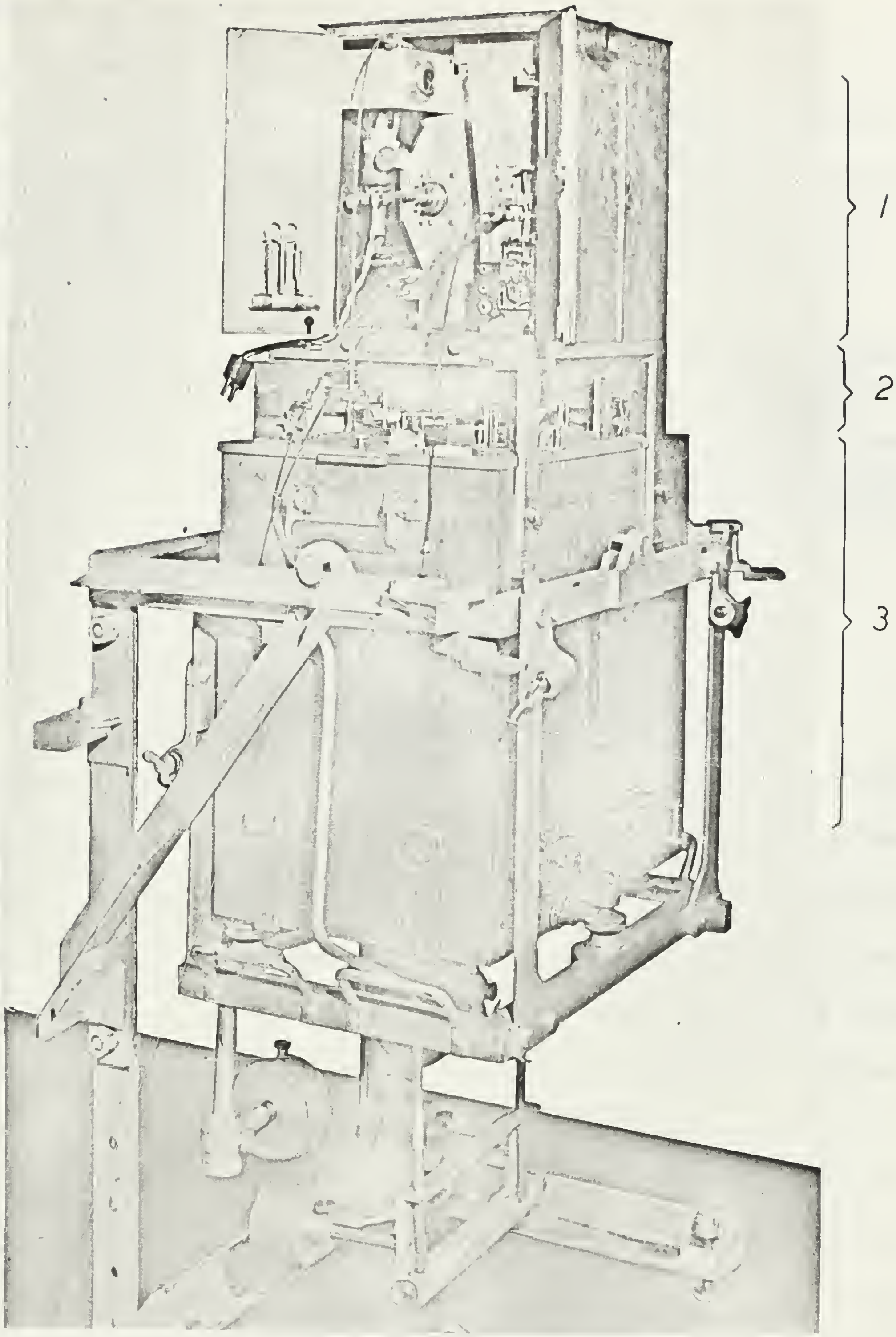
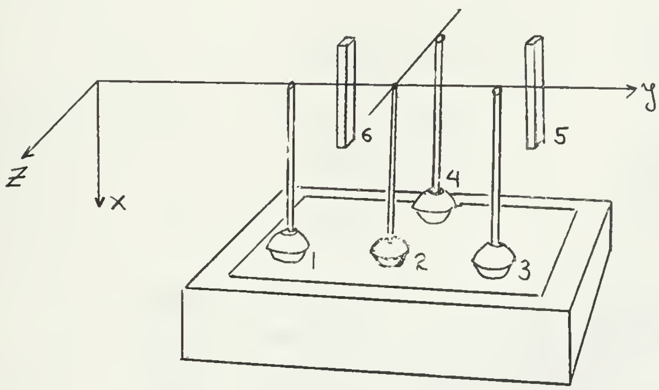


figure 10   Vening Meinesz Pendulum Apparatus.
1. Recording Section, 2 Slow Pendulum Section 3, Pendulum Box.

suspended from gimbals in an angular metal frame.

2.21 The Pendulum System

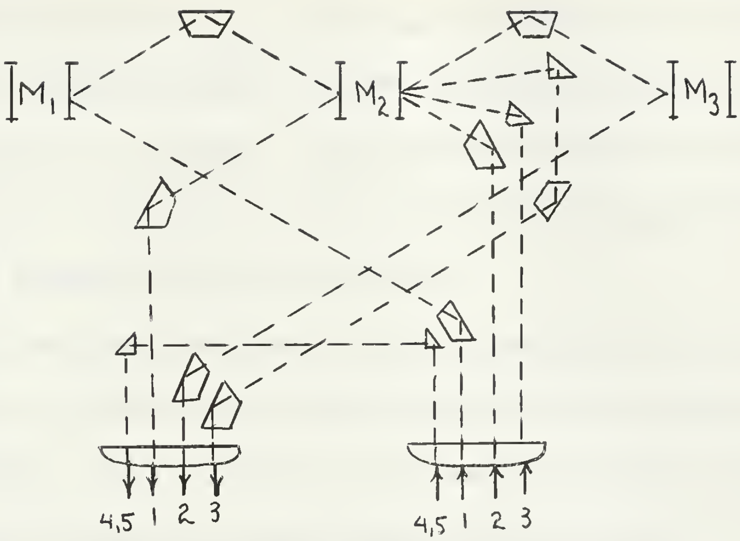
There are seven pendulums (plus one dummy pendulum) within the pendulum box: three main pendulums, two auxiliary pendulums, and two damped pendulums. The pendulum arrangement is shown in Figure 11.



1,2,3	main pendulums
4	dummy pendulum with thermometer
5	auxiliary-damped pendulums (z plane)
6	auxiliary-damped pendulums (y plane)

figure 11 Pendulum Arrangement

The three main pendulums, numbers 1, 2, and 3, are of the half second, brass Sterneck type. They are 13 cm. apart and aligned in the same vertical swinging plane. Numbers 1 and 3, swinging in anti-phase and compared separately to pendulum No. 2, create the two recorded angles of elongation of the fictitious pendulums. An optical system reflects light rays from mirrors on top of the main and auxiliary pendulums, so that the fictitious angles of elongation, $\theta_1 - \theta_2$, $\theta_2 - \theta_3$, θ_2 the inside air temperature, and the tilt β of the swinging plane are recorded on a photographic record. Figure 12 shows the paths of the light ray through a schematic of the prism arrangement. Not shown in the figure is the optical arrangement for recording the two horizontal pendulums in the slow pendulum section used in the computation of the Browne



No. 1 records $\theta_1 - \theta_2$ elongation
No. 2 records $\theta_2 - \theta_3$ elongation
No. 3 records θ_2 elongation
No. 4 records inside air temperature
No. 5 records tilt angle (β) of the swinging plane from the vertical
M_1, M_2, M_3 are mirrors atop each main pendulum.

figure 12 Light ray path inside the Vening Meinesz pendulum box.

By permission from The Earth and Its Gravity Field, by Heiskanen and Vening Meinesz, 1958, McGraw-Hill Book Company, Inc.

correction.

One auxiliary pendulum (No. 5 in figure 11) swings in a plane parallel to the main swinging plane, and the angle of elongation between it and pendulum no. 2 is recorded. The auxiliary pendulum is heavily damped to make it independent of the local disturbances affecting the main pendulums through the apparatus frame.

The second auxiliary pendulum swings perpendicular to the main swinging plane, and thus we can record the tilt β of the main plane from the vertical. This pendulum is also heavily damped, and the damping system is similarly independent of the apparatus.

An examination of the damping system discloses that both auxiliary pendulums have a small damping pendulum inside. The periods of the inner damping pendulum and outer auxiliary pendulum are decidedly different. The damping pendulums are constructed with a little fin at the lower end which drags in a small oil pot attached to the outer auxiliary pendulum, thereby damping the auxiliary pendulum.

When observations are not taking place, the seven pendulums should be lifted off their knife edge suspensions. This is done by turning a small hand wheel at the base of the pendulum box. The lifting is done simultaneously for all the pendulums, that is, for the three main pendulums and the two damping pendulums. After the damping pendulums are lifted a short distance, pins in the inner damping pendulums engage slots in the outer auxiliary pendulums lifting the outer pendulums off their knife edges. Clever fixing levers for clutching the pendulums and their bulbs have been incorporated into the apparatus design by the inventor. The lifting and clutching operations are

interlocked so that they cannot be performed in improper order.

Amplitude may be given the three main pendulums simultaneously or just to the outer pair by levers moved against the pendulum bulbs. The mechanical amplitude levers are designed to enable the observer to set up, either by hand or mechanically, any pendulum swinging arrangement he may desire.

Temperature control is provided by a brass box within a brass box arrangement. The area between the two boxes is packed with insulation. Submarine observations are conducted just after a dive, and the temperature gradient in the boat is apt to rise steeply.

A heating coil is provided on the bottom of the pendulum box allowing the interior box temperature to be brought to the anticipated post dive temperature of the boat well before observations start.

The interior temperature is monitored by one thermometer in a dummy Sterneck Pendulum and a recording thermometer. The temperature from the thermometer in the dummy pendulum is used in the computations.

2.22 The Slow Pendulum Section

There are two brass horizontal slow pendulum rods in this section. They are 25 cm long and positioned at right angles to each other. The knife edges are of steel pivoted on supports of vidia, a hard metal alloy. As in the case of the pendulums described in the preceding section, the two horizontal pendulums must also be lifted off their knife edges, when observations are not in progress. Lifting levers are operated by a small hand wheel at the edge of the slow pendulum section. A second pair of manually operated levers

firmly clamps the pendulum after the lifting operation.

When the pendulums are unclasped, balanced levers gently press against the upper and lower edges of the pendulum eliminating any initial amplitude in the pendulums before observations start.

The pendulums are air damped which is sufficient to reduce the amplitude by approximately two-thirds. Both pendulums have small vanes which move in a close fitting case; hence, as the pendulums swing, air drag on the vanes reduces their amplitude.

The period of the pendulum must be known, and if their construction is such that large changes of periods can occur, then frequent determination of the period is necessary. Ideally, the periods should first be determined accurately by land observations. This will allow us to know the magnitude of the change at sea and therefore judge its importance. The Vening Meinesz long period pendulums have small grooves 1.08 mm on either side of the knife edge. Small ball bearing balls may be added to the grooves and will cause a discernible deviation on the record. The period may then be determined from

$$T_{\text{seconds}} = 25.2\sqrt{S_{cm}} \qquad (17)$$

Pendulum equilibrium may be roughly read from a scale on each pendulum housing; adjustments, if necessary, may be made by turning fine screws at either end of the pendulum.

2.23 The Recording Section

The photographic recording section is the uppermost unit of the Vening Meinesz Apparatus. As seen from Figure 12 the recording section must record from the pendulum box the following information:

1. $\theta_1 - \theta_2$; $\theta_2 - \theta_3$

2. θ_2

3. Temperature of inside air

4. Tilt of swinging plane from vertical

It must also record from the long period pendulum section:

1. The amplitude of long period pendulum 1

2. The amplitude of long period pendulum 2

An example of a photographic pendulum record is shown in Figure 13.

The light source for the apparatus is variable and is dependent upon the D.C. or A.C. current supply. To the light ray circuit is added a crystal chronometer, such as the type developed by the Bell Telephone Laboratories in New York [71]. As the light rays leave the recording section, they are interrupted by a phonic motor driven shutter triggered and synchronized with the chronometer. Usually, the ray path is broken about four times a second as shown in Figure 13. The ray path then enters the pendulum section, and the long period pendulum unit then is directed back into the recording section to the photographic paper. The recording paper is fixed from one roll to another roll and is operated by a wound clock mechanism. Two recording speeds are available, but the records indicate the slower speed produces a better record.

2.3 Summary-Corrections, Accuracies, Conclusions

The following corrections must be known and applied. Formulas for their computation are very well treated in the literature and will not be repeated here [67].

1. Temperature correction

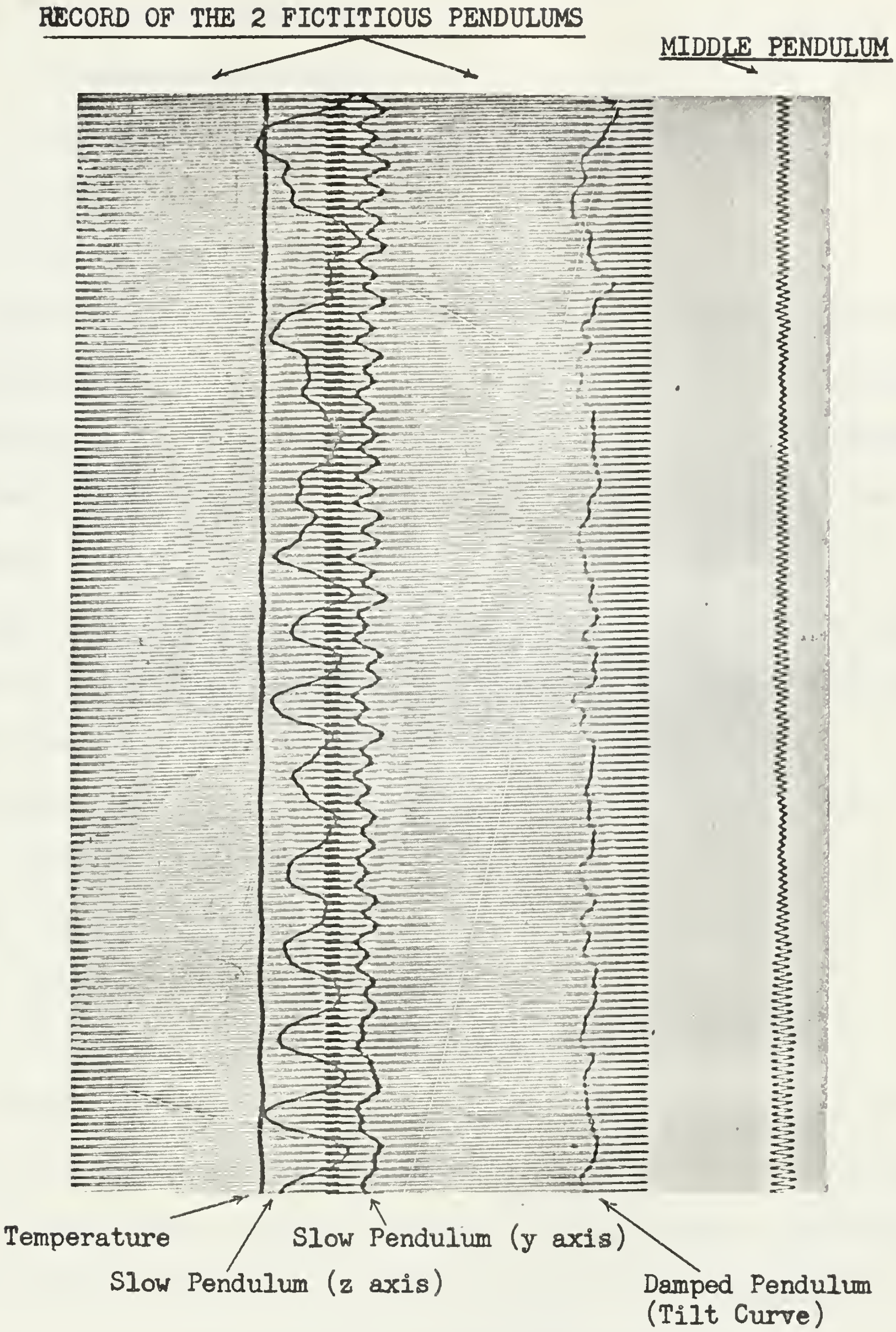


figure 13. A portion of a Vening Meinesz Pendulum

Apparatus record, actual size. Read from top to bottom. The time marks show as clear spaces on the fictitious pendulum tracings.

2. Air density correction

3. Isochronism correction

4. Chronometer rate

5. Tilt of the swinging plane

6. Amplitude correction

The above corrections, when applied to photographed period of the fictitious pendulum, render the observed gravity at the observation depth. The values are then corrected up to sea surface(geoid). The Browne second order correction and the Eötvös correction are then applied.

Each resulting value of gravity is averaged from over one mile, since one observation requires a minimum of thirty minutes, and speed during the observation is at least two knots.

Serious error sources in the Vening Meinesz Apparatus are:

1. The Eötvös correction

2. Poor positional data

3. Changes in pendulum length

4. Estimation of vertical and horizontal accelerations

The presence of error sources logically makes us question the accuracy of the pendulum observations as a whole. Just what accuracy can we expect under average conditions? It was earlier stated that the apparatus was capable of measuring g to 1 or 2 milligals, and that the Eötvös correction added at least another 2 milligals. The following chart by Ewing and his Lamont Geological Laboratory group [18] gives their estimation of individual errors.

Accuracy of Observation	ΔT (seconds)	Δg (milligals)
Temperature	$\pm 5.0 \times 10^{-7}$	$\pm$ 1.0
Air density	3.0×10^{-7}	0.6
Isochronous corr.	0.1×10^{-7}	0.02
Chronometer rate	1.0×10^{-7}	0.2
Tilt (β)	0.1×10^{-7}	0.02
Amplitude	2.0×10^{-7}	0.4
Period measure	2.0×10^{-7}	0.4
Browne corr.		1.0
Eötvös corr.		2.0
Corr. to sea surf.		0.2
Geographic position		2.0
Base station closure		1.5

Table 3

They conclude by stating that they feel "the uncertainty in a free air anomaly for a typical observation is estimated at $\pm$ 3.6 mgals." Their opinion was published following their reduction of 594 sea gravity stations.

Such accuracy is to the present time the best available for sea observations. Probably the readings are high rather than low; that is, the effect of horizontal accelerations are not entirely seen and/or eliminated from the observation. In the study of the new sea-surface gravimeter we will see that the horizontal accelerations require great analysis, before we can produce meters that will equal or surpass the Vening Meinesz Pendulum Apparatus. The acceleration disturbances may now be measured more accurately than off the photographic record by accelerometers, but this introduces a new problem of assuming that the accelerometers see and record all the acceleration seen and felt by the apparatus.

The advantages of the apparatus are simply summarized. The apparatus gives us our most accurate values of g in the open sea areas. At present, it is the "standard" to which other devices are aspiring, with some modifications (i.e., surface rather than sub surface operation). The disadvantages may also be simply stated. Notwithstanding the fact that the apparatus is our most accurate tool, the error sources are nevertheless larger than desired. Continuous gravity profiles are not possible, and a single observation requires a minimum of thirty minutes on a submerged platform. The computation of the results is also fairly lengthy, and immediate results are not readily available.

3. The Askania Seagravimeter Gss2 after Graf

3.1 The Gravimeter Design [1]

The Graf meter measures the acceleration due to gravity by noting the deflections of a long thin aluminum beam (hereafter referred to as "weighbeam"). The weighbeam is supported and held horizontal by horizontally stressed helical springs on either side and by a fine measuring spring attached to the top front corner. Four tough fiber wires radiate out from a support to each side of the weighbeam preventing horizontal movements. Vertical accelerations of the beam are strongly damped by a magnetic field from a strong permanent magnet.

Also shown along with the weighbeam in Figure 14 is a photocell lamp and a simple optics system. The light ray bundle produced from this lamp is directed through the optics system and through a diaphragm fixed to the weighbeam, to two photoelectric cells. If the weighbeam is at the null point (horizontal), the light received and the voltages produced by the photoelectric cells will be equal. The cells are wired in opposition, so that at the null their voltages cancel each other out, and a zero reading is recorded by a special device. Should the weighbeam tilt, then the light received by the photoelectric cells will be unequal, and a voltage difference is produced. The voltage difference is recorded after passing through a D.C. amplifier stage. The record therefore shows the displacement of the weighbeam from zero (null point) caused by small changes in gravity.

[1] Sea gravimeter specifications and design as described herein are primarily from reference [85], which is essentially a factory manual on the meter furnished me through the U.S. Navy Hydrographic Office by Askania Werkes. All illustrations of the Askania Seagravimeter Gss2 after Graf are by courtesy of Askania Werke, Berlin-Friedenau.

[illegible]

[illegible]

[illegible]

[illegible]

Larger changes in gravity are adjusted for by adjusting the fine measuring spring as measured by a precision glass scale (Figure 14).

The Graf seagravimeter comprises the following equipment:

1. The seagravimeter unit and power cable
2. A stand for free gimbal suspension or a gyro stabilized platform [25 p. 1814].
3. Instrument cabinets
    a) The recording apparatus
    b) The power supply and D.C. amplifier
4. Transportation cases
5. Optional equipment
    a) Sine lift
    b) Power pack

The seagravimeter unit is shown from the inside out in Figures 15 to 18. The measuring unit (fig. 16) consists of the measuring system just described inside a pressure tight cylindrical housing (fig. 17). Figure 18 shows the measuring unit encased by a thermally insulated housing and suspended from a stand complete with balancing weights for free gimbal suspension. The thermal housing is provided with two separate thermostats to insure temperature control.

The front end view of the meter in Figure 19 shows the cable connection at the top center of the picture with a small free gimbal suspension joint just behind it. The rails on either side of the meter are hand rails. The knob at the very base of the meter is for thermostatic control with a selection of 25°C, 30°C, 35°C, or 40°C possible.

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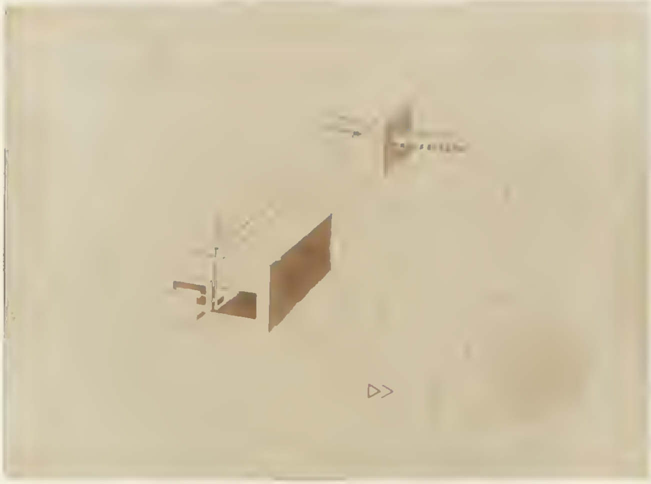

figure 14 Schematic view of measuring system, 1 photocell lamp and optics, 2 diaphragm, 3 photoelectric cells, 4 damping magnet, 5 measuring spring and scale, 6 torsion spring, 7 fibers for constraining weighbeam motion, 8 weighbeam, 9 amplifier, 10 recorder.

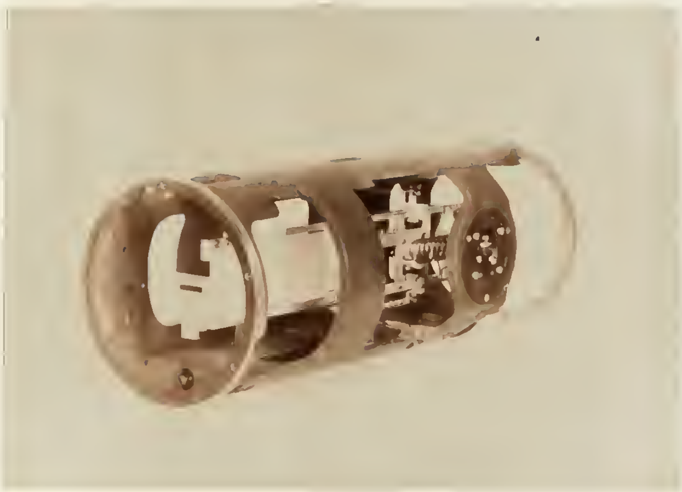

figure 15 The weighbeam in the system carrier

figure 16 The measuring unit without the pressure vessel.

figure 17 The measuring unit in the pressure vessel.

figure 18 Gimbal-mounted Seagravimeter with frame balancing weights.

figure 19 Front view

figure 20 View in micrometer (scale reading 37.735)

Just above the thermostat control is the measuring range shift control. The control is locked on setting by a control cap that may be screw locked on. Upon removal of the protective cap an outer knob on the control (see white dot in Figure 19) is set on "free". The inner knob is now set to the amount of range adjustment needed. The adjustment amount is viewed through the small peepsight north west of the measuring range shift control. After adjustment the outer knob is set to "fixed" and the protective cap replaced.

The scale shown schematically in Figure 14 is read from the projecting eyepiece shown top left in Figure 19, and the scale view seen is shown in Figure 20. The scale division (37 in Figure 20) is straddled by the two hairlines by rotating the knob just below and to the left of the eye piece (Figure 19). Scale adjustments are made by turning the knob to the left of the eyepiece and can be locked in position with the lever shown in the illustration.

The window on the east edge of the face is for viewing the level vials and thermometer.

The instrument cabinet is shown in Figure 21. It is compact, rigidly constructed and easily maintained, since both sections are on easily removable pull out chassis. Specifications for this instrument will not be listed here. Suffice to say, it is built for shipboard operations and as such is a rugged, stable unit.

Askania Werke has recently shown the Gss2 Gravimeter mounted on a new gyrostabilized platform [25 p.1814]. Such a platform adds greatly to the cost of a survey expedition but gives much greater flexibility to the meter (i.e. it can operate in heavier sea states

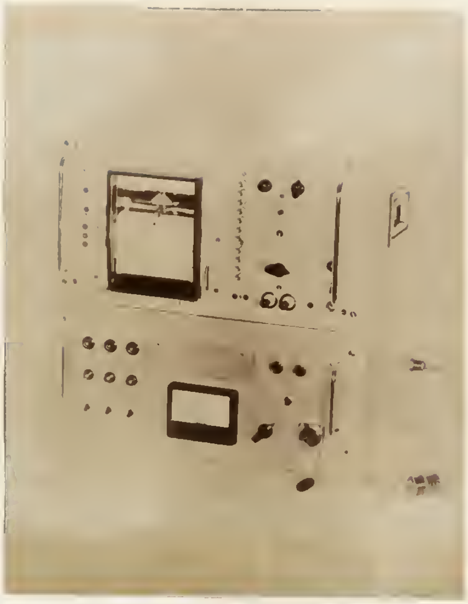

figure 21. Instrument cabinet. Top: Enograph recording apparatus, Bottom: power supply unit with built in D.C. amplifier

figure 22. A typical record of a linearisation test. Acceleration $\pm$30,000 mgals. Approximate period, T = 7.5 sec. Scale value $\approx$1 mgal/mm. Chart speed 5 mm/min. Chart width 120 mm. Read chart from bottom to top.

and from smaller vessels than if hung from gimbals).

3.2 Linearisation and Calibration

The difficult requirement to fulfill is the observation of gravity despite the periodic vertical accelerations of up to 100,000 milligals. The submarine platform escaped these excesive variations by submerging to a less disturbed depth. How does the Graf Gss2 meter hope to compensate for these variations? Dr. Graf has approached the problem by trying to make the acceleration disturbance gravity relationship a linear one. To accomplish this the basic instrument design of the weighbeam was shown by field test to be correct, providing the system damping is correct. In this regard it should be mentioned that the damping of the vertical accelerations by a heavy magnet is not sufficient alone. Electrical current damping is provided, the voltage difference signal sent by the photoelectric cells to the D.C. amplifier. This electrical damping occurs just before and after the D.C. amplifier in the system circuit. Recently, the damping magnet itself was redesigned to incorporate the best known properties. As a result, all meters manufactured since July, 1960 have a damping increase of 500% percent [25 p. 1814]. The following table by Graf and Schulze will emphasize this point [25 p. 1815].

Period Sec.	Attenuation Ratio *	
	Without Electrical Damping	With Electrical Damping
6	1/ 340	1/ 186,000
8	1/ 190	1/ 74,000
10	1/ 130	1/ 38,000
	* $\frac{\text{Apparent gravity change}}{\text{True gravity change}}$	

Table 4

The linearity of each Graf meter is determined at the factory and may be periodically redetermined by the purchaser with an optional sine lift mechanism available from Askania Werke. Laboratory tests with the Graf meter, subjected to accelerations up to 100,000 milligals at periods of 6 and 10 seconds showed a remarkable difference of only $\pm$ 2 to 3 milligals between the static (at rest) and dynamic sine lift (up and down motion) response [25 p. 1818]. Sea motion translated ship motion is not always smoothly sinusoidal as mentioned above but is frequently an irregularly alternating motion. After imparting jerky irregular motions to the factory laboratory test, Graf reports that "the mean value of the response curve is almost exactly equal to the static response." It appears that for all practical purposes the linearity relationship Dr. Graf sought has been achieved.

It follows that the linearisation of the seagravimeter is dependent upon the adjustment of the photocell lamp. If the lamp is moved or replaced, a linearisation calibration must be performed and the new standard recorded for future use. The photocell lamp should last several years, so with care in building the instrument linearisation checks can be held to a minimum. Figure 22 shows a typical linearisation test record. At the bottom we can see the meter actions as the sine lift is set in motion. After approximately six minutes the meter settles down to a dynamic value. The lift is stopped and the static value measured. If perfect linearity were attainable, static response would equal the dynamic response. The recording stylus is then set over (horizontal movement of pen on chart) and the test repeated with a different period and/or acceleration.

Calibration of any measuring device is an understood necessity. The Graf seagravimeter is factory calibrated along a calibration line, and the results are given on the test certificate. To eliminate the need for recalibration checks a simple internal test device is included within the meter.

Below the helical springs is a "ball-container box", rectangular in shape, brass, and containing an accurately weighed ball. The ball may be positioned in either of two precisely located three point supports within the box. The meter is arrested and tilted, and the ball will roll from one support to the other. The weighbeam responds to the resulting torque, and the angular amount thus defined has a definite milligal value previously determined during calibration. Comparison between the test calibration data and ball test data gives a check on consistency of the original calibration of the instrument.

3.3 Operational Considerations

Because of the heavy damping system acting on the weighbeam the meter does not read out the true gravity change Δg_o but an apparent change Δg_o^*. In addition the damping causes a time lag (Δt) between the time a gravity anomaly is passed over and the time the corresponding amplitude is recorded on the enograph record. The operational capability of the Graf meter is not impaired, however, if the damping effects can be correctly analysed, observed, and applied.

Graf and Schulze [25] have recently shown that the observer can expect gravity observations in any one of three forms:

1. Sinusoidal change
2. Step change
3. Linear change

and that the torques affecting the weighbeam may be expressed as in Figure 23.

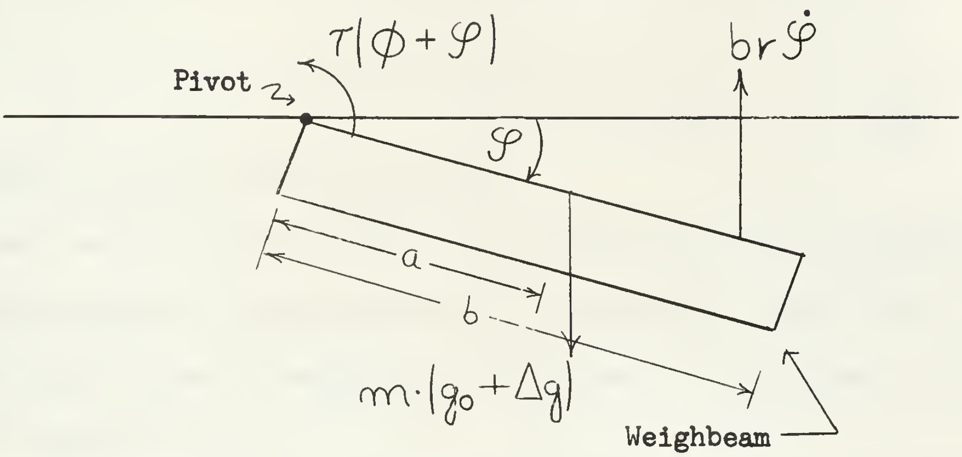

figure 23

$\tau(\phi + \varphi)$ = torque of the torsion spring

τ = torsion constant

ϕ = initial twist angle

φ = deflection angle

$ma(g_0 + \Delta g)$ = torque produced by gravity at CG

m = mass

a = distance from axis to CG

g = acceleration of gravity

$br\dot{\varphi}$ = torque of damping center

r = damping constant

b = lever arm

With the measuring system at rest:

$$m g_0 a = \tau \phi \tag{18}$$

and as gravity changes, a deflection φ is introduced:

$$\Delta g^{\star} = (\tau/ma)\,\varphi \tag{19}$$

Since $\Delta g^{*} \neq \Delta g_o$, the equation of motion of the weigh-beam is needed to examine the three forms of gravity change mentioned above.

$$I\ddot{\varphi} + b^2 r\dot{\varphi} + \tau\varphi = ma\Delta g \tag{20}$$

Where I is the moment of inertia.

They then developed a differential expression for the equation of motion for each gravity change form (sinusoidal, step, linear) and solved for φ to get an expression for Δg and for convenience. B = time constant = $b^2 r/\tau$.

For sinusoidal changes within the period I sec < T<30 seconds it was shown that D, the damping number $(\Delta g_o^{*}/\Delta g_o)$ is approximately linearly dependent upon T.

$$D \approx \frac{1}{\omega B} = \frac{T}{2\pi B} \tag{21}$$

For greater periods $\Delta g_o^{*}/\Delta g_o$ will render a percentage factor of the amount of the amplitude that has been recorded.

The damping of a sinusoidal gravity change also causes a time lag (Δt). The phase difference Ψ may be expressed:

$$\cos\Psi \approx \frac{1}{\sqrt{1+\omega^2 B^2}} \tag{22}$$

and the time lag from

$$\Delta t = \Psi T/2\pi \tag{23}$$

Considering now the step type change in gravity and accepting B, the time constant as previously defined, we can see that B is proportional to the damping force,

$$B = b^2 r/\tau \tag{24}$$

Should a sudden step change occur, the meter will record a marked

horizontal change on the enograph record. The response curve then traced is nearly exponential, and the time constant B can be computed by matching its curve to the best fitting exponential function.

The linear gravity change question must be solved before we really know the time lag between the true gravity and apparent gravity

Apparent $g = \Delta g^{*}$ at t_2

True $g = \Delta g$ at t_1

True linear g change $= \Delta g = ct_1$; c = constant

Apparent g change (indicated)

$$\Delta g^{*} = CB\left(e^{-t_2/B} + t_2/B - 1\right)$$

If we let $\Delta g^{*} = \Delta g$ then $t_2 - t_1 = \Delta t = B\left(1 - e^{-t_2/B}\right)$

$$t_2 >> B$$

$\Delta t = B$ and Δt is independent of the constant linearity (25) rate C.

There is inherent in the Graf type meter a new disturbance factor - the cross coupling affect. It is produced from a combining of the horizontal and vertical accelerations into a rotational couple of the beam about the axis. Disregarding the Graf meter, it is ideally shown in Figure 24 [40 p. 93] .

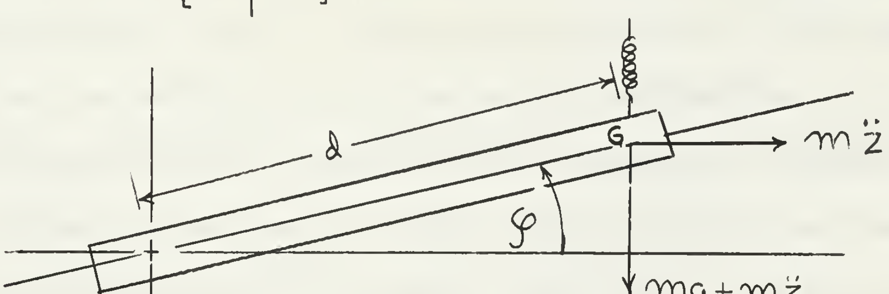

figure 24

$$C.C. = md\left[\left(g+\ddot{x}\right)\cos\varphi + \ddot{z}\sin\varphi \right] \tag{26}$$

The cross coupling effect for the Graf meter is given by

$$\Delta g = \frac{1}{2} E\, v_0\, h_0\, D \cos\left(\Psi - \lambda\right) \tag{27}$$

where:

E = static sensitivity of gravimeter (5×10^{-6} mgal)

$D = \Delta g_0^{*} / v_0$ = amplitude reduction factor from magnetic damping

Ψ = phase difference, vertical

λ = phase difference, horizontal

v_0, h_0 = vertical, horizontal accelerations

The quantities v_0, h_0, and λ are directly observable from accelerometers, and Ψ is obtained from (22). C.C. effects are present when the periods of the vertical and horizontal accelerations are equal or very nearly so. The periodic deflection of the weighbeam

$$\varphi = \varphi_0 \sin\left(\omega t + \Psi\right) \tag{28}$$

and the effect of h combine to produce a counterclockwise torque. If $v_0 = h_0$ = 100,000 mgal, D = 0.005, Cos $(\Psi - \lambda)$ = 0.5 then $\Delta g \cong 62.5$ mgal.

If accelerometers measure h, then the direction and magnitude are readily known. The C.C. effect may be made zero, if the weighbeam is kept perpendicular to the horizontal accelerations. Since the accelerometer gives us the information necessary to eliminate cross coupling, the problem is largely solved. Also, if we observe on opposite headings (180°), the cross couple effect will mean out, providing the period accelerations remain constant over both tracks.

There remain only a few additional considerations. The question of gimbal or stabilized platform suspension has been discussed from the general view point. The Graf meter's Browne second order correction is

$$\Delta g = \frac{h_o^2}{4g} = \left(\frac{g}{4}\right) \theta^2 \tag{29}$$

If the meter is mounted on a gyro platform, a leveling correction must be considered. It is caused by horizontal accelerations deviating the gyro platform from a true horizontal. The Harrison-developed formula [25] is:

$$\Delta g = \frac{1}{2} \epsilon_o \, h_o \cos(\lambda - \beta) \tag{30}$$

Where $\epsilon = \epsilon_o \cdot \sin(\omega t + \beta)$ = platform deviation

$\lambda - \beta$ = phase angle between ϵ and h.

In practice a platform deviation $\leq 1.6'$ is tolerable [25 p.1820].

Usually, the choice of meter suspension systems is limited strictly by what is available. If a choice is available, the enforced leveling system is superior and should be selected. Observations in a greater sea state are possible, since the CC effect and the leveling errors are linear to h, whereas in the Browne correction the second order correction increases to its limit by the square of h. Also, the C.C. correction and leveling correction are quite small; however, the Browne correction can build in magnitude to several hundred milligals.

3.4 Operational Capabilities of the Seagravimeter

The first consideration under the actual operation of the meter is the running in or warm up time required. Askania-Werke lists the following MINIMUM times required by each component of the seagravimeter:

Thermostat	24 hours
Gravimeter lamp (photocell lamp)	48 hours
Enograph, Amplifier, Gravimeter upon release	10 minutes
Gravimeter after a slight adjustment of measuring spindle	6 minutes

For safe performance the meter should be firmly secured in its observation location.

Reasonably good temperature control should be provided within the observation area. The meter should be located as near the shipboard center of gravity as possible but not in close proximity to heavy vibration-producing equipment.

The Askania Gss2 meter is capable of observation from submerged as well as surface vessels. It is not suited for airborne observations. Throughout observation periods an attendent should be present to mark course, speed, and time at 15 minute intervals on the record.

The U.S. Navy Hydrographic Office, one of the principal users of the Graf meter, reduces their observations in the following way [42] :

1. Obtain gravity reading in terms of the meter factor.

   The number of counter divisions which represents the tension of the spring in calibrated dial divisions. Each reading is the average (minimum) of 6 minutes and is scaled off the enograph record.

2. Convert meter readings (dial divisions) to milligals.

   Based on calibration tables furnished by the factory, original for each meter.

3. Reduce meter readings in milligals to uncorrected observed gravity.

[illegible]

Compute relative gravity difference from sea observation and original port base station. Add difference algebraically to base station value. Correct for elevation if necessary.

4. Compute Eötvös correction.

   Form of correction depends on speed; see section 5.1

5. Apply Eötvös correction to obtain corrected observed gravity.

After obtaining the observed gravity (corrected to sea level if applicable), it is compared to the normal gravity (γ_0) obtained from the International Gravity Formula.

$$\gamma = 978.0490\ (1 + 0.0052884\ \sin^2\phi - 0.0000059\ \mathrm{Sin}^2\ 2\phi)\ \mathrm{cm/sec}^2 \tag{31}$$

The normal gravity is computed to the spheroid. The observed gravity is reduced by an appropriate reduction (free air, Bouguer, isostatic, etc.) to the geoid. The difference then between the observed g_0 and theoretical, γ is the gravity anomaly Δg.

$$\Delta g = g_0 - \gamma \tag{32}$$

Unfortunately, a truly definitive set of data from which dependable accuracy figures may be obtained are not available to this author. Data published seldom, if ever, list course, sea state, wind, current, precise averaging time, and so on. Without such information true accuracy figures cannot be arrived at. That the meter is capable of ± 10 milligal accuracy is quite certain. Most of Worzel's later reports assign an overall accuracy of ± 5 milligals, but usually navigational control was unusually good. After surveying much literature on the subject, I believe a figure of ± 8 milligals is reasonable for surface observations in the open sea. Undoubtedly the accuracy will

[illegible]

[illegible]

[illegible]

[illegible]

increase in the next few years. If the Seagravimeter is used on board a submarine, accuracy of $\pm$ 3-4 milligals is obtainable.

4. The La Coste-Romberg Air-Sea Gravity Meter

4.1 The Gravity Meter Design [2]

The La Coste-Romberg meter is of the spring type proven so successful for land gravity meters. A beam or arm, pivoted at one end and supported at the other end by a spring, responds by a deflection to changes in the acceleration of gravity and/or vertical and horizontal accelerations. A critical requirement for this type of meter is to insure a linear response in the spring over a wide milligal range. The top end of the spring is attached to a "semi-fixed" plate attached to the meter frame. The plate has a finely calibrated measuring screw within it, such that movement of the screw causes a vertical movement of the measuring beam. Figure 25 shows in a very idealized way this arrangement.

The main control box regulates the calibrated measuring screw; it is so adjusted as to make the beam move up and down past the zero mark (null point). Seen on the control box in Figure 25 are the beam indicator and timing indicator. The former shows the beam deflection directly on the microammeter; the latter shows in integrated form the deflection on the timing microammeter.

The monitoring recorder is used to indicate those portions of the continuous record that must be eliminated when the beam strikes its stops, as in a turn or sudden acceleration. It produces a continuous time marked record of the beam position.

The horizontal accelerometers and horizontal accelerometer control box are just what the names imply. The control box has the added

[2] The basic references from which this material was collected are [39], [28], [30] and [50].

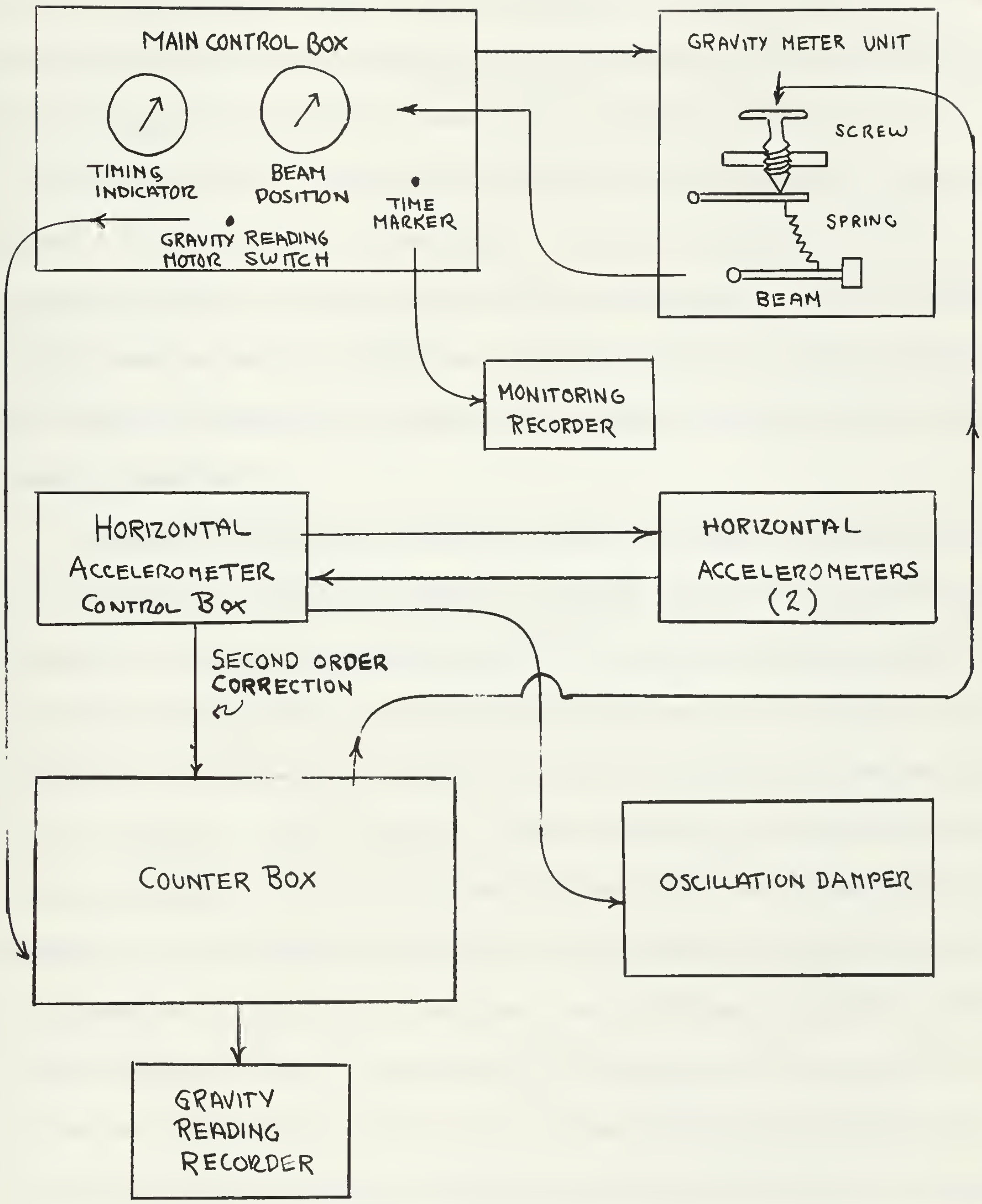

figure 25. Block diagram of the La Coste-Romberg Air-Sea Gravity Meter.

function of converting horizontal acceleration information into orders to the oscillation damper. These "orders" or inputs to the oscillation damper cause small horizontal translations in its servo-mechanism to the suspension gimbals, reducing its natural period [39 p. 312].

The counter box receives the horizontal accelerometer control box input (the second order correction), and this is applied to the counter readings (spring tension) from the main control box. The corrected spring tension and the reference time marks for each zero crossing of the timing microammeter are then continuously recorded by the gravity reading recorder.

To make the foregoing instrument design workable, La Coste devised an automatic averaging system to achieve the proper beam correction from the horizontal accelerometers [39]. Since the meter measures the resultant of gravity and acceleration disturbances, we should know the component of acceleration parallel to this path and also perpendicular to it (see 5.4). The solution of these equations is performed by an analog computer. It was mentioned earlier that the spring tension is adjusted so as to make the beam move repeatedly past the null. The acceleration of gravity is obtained from recording the time variation of the spring tension. Obviously, the variation of the spring is affected not only by the gravity but by the bothersome vertical and horizontal accelerations. The vertical accelerations may be averaged out from the record by considering the entire observation record. The horizontal accelerations are electronically determined by two horizontal accelerometers measuring the deflection angle between the instantaneous vertical and two mutually perpendicular long period

beams (similar to Vening Meinesz long period apparatus).

The analog computer receives the horizontal accelerometer outputs and solves the equation to determine the correct interval of time over which the horizontal average is correct. In effect Browne's second order horizontal correction for the La Coste Meter

$$-\left|g/2\right| \theta^2 \tag{33}$$

is automatically and continuously computed, applied to the spring tension, and recorded independently. The record will display the corrected spring tension in binary-decimal coded form and the previously mentioned time marks.

4.2 Operational Considerations

Drift in any gravity meter is important and particularly so in the case of seagravity meters, where base checks may be weeks, even months apart. The La Coste meter has proven very stable, and in its presently developed state it displays no appreciable drift at all [15 p.4], [32 p.4].

Either the gimbal suspension system or a gyrostabilized platform system is possible with the La Coste Meter. However, it has been developed primarily along gimbal suspension lines, in which case the horizontal accelerations are handled as described above.

If the meter is positioned on a stabilized platform, the horizontal accelerations are eliminated (or nearly so) by forcing the platform to remain very level. The degree of leveling precision is quite critical. For example, La Coste has shown that if a tilt (ϕ) of $1'$ occurs from a horizontal acceleration (A_h) of some 100,000 milligals,

the resulting error is $A_h \phi$ or about thirty milligals [39 p. 311].

The possibility of cross coupling effects must also be considered. In the case of long period wave motions or matching of vertical and horizontal acceleration periods such serious cross coupling effects could be produced. La Coste has been able to greatly minimize the cross coupling effect by placing the meter-sensing element below the gimbals at a distance exactly equal to the length of a simple model pendulum with a period matching the period of the meter in the gimbal suspension [29 p. 272].

If the meter is mounted on an enforced leveling type platform, the cross coupling effect can be made unimportant by a slow continual rotation in the meter about the vertical axis.

Additional error sources in the La Coste-Romberg Meter are listed below. I am indebted to Dr. J. C. Harrison for making these known to me.

1. If the pivot axis of the horizontal pendulum used for vertical reference is not horizontal, errors will result.

If this should occur, the ever present yawing of the surface ship will be transmitted to the meter as an acceleration. Care in manufacture and aligning of the pivot axes eliminates this error.

2. If the distance of the sensing element below the gimbal suspension is not equal to the length of a model pendulum of the same period as the meter in suspension, an error will result.

In addition to the cross coupling effect the meter will not respond well for wave periods less than three seconds. Since the distance of the meter below its suspension is fixed, the free period is altered

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

by changing the moment of inertia.

3. If long period accelerations are present, error will be introduced by the inability of the horizontal pendulum to function efficiently as a vertical reference.

Harrison explains that while periods longer than 20 seconds are not present in the ocean wave spectrum, they may be produced by the hunting of the auto-pilot or a snaking course of the ship from short period buffeting waves. If after damping and circuit improvements are made in the meter, motions of long periods (greater than one minute) are found with correspondingly large amplitudes, the meter will have to be operated from a stabilized platform or with vastly more responsive accelerometers.

4.3 Operational Capabilities-Conclusions

It should be apparent to the reader by now that there is a dearth of information about the La Coste Meter compared to the Graf Meter. The La Coste Meter is being developed under military contracts, and as a result, little specific information is available; however, some operational specifications may be listed here and some general conclusions drawn.

The La Coste-Romberg Air Sea Gravity Meter is capable of handling vertical and horizontal accelerations to 100,000 milligals. It has a reading range of some 6,000 milligals. Averaging time varies with observational speed and the steepness of the horizontal gradient of gravity, but ten minute intervals at twelve knots has proven quite successful. This renders a gravity value every two miles and is a procedure used by the gravity survey of the U.S. Coast and

Geodetic Survey ship Pioneer [45] .

Harrison and Spiess [33] have compared the La Coste-Romberg Meter simultaneously with underwater gravimeters at 27 stations in the Gulf of California. The mean differences between the underwater and surface meters was -2.7 ± 1.5 milligals. A similar comparison of nine land readings with a geodetic meter gave -1.1 ± 2.6 milligals.

Orlin [45p12] and Harrison [28p5] working independently on separate evaluations of the La Coste-Romberg Gravity Meter have reported the attainable accuracy to be $\pm$ 7 to 8 milligals for ocean observations. As with the Graf Seagravimeter increased ability to sense out and correct all disturbing accelerations and increased navigational control is needed to reduce the accuracy error to the ultimate goal of 1 to 2 milligals.

4.4 Airborne Observations

Airborne gravity observations are at the present time less accurate than those attainable on the surface. Obviously, however, a high flying, fast moving aircraft could survey an area faster than any surface method and could also reach areas otherwise inaccessible through either natural or political reasons. Since geodesy requires only mean values of gravity for many of the barren $1^\circ \times 1^\circ$ squares of the earth, an airborne meter with an accuracy of around 10 milligals should suffice.

Thompson and La Coste [62] have listed the problems of airborne observations to be solved:

1. Type of observation required
2. Navigation problem - speed, position, elevation

3. Stability of aircraft as observation platforms

4. Eötvös effect - correction

5. Gravimeter

Problem 1 was discussed above. Problem 2 is solvable using moderately sophisticated electronic equipment (i.e., radar doppler system, Decca, Shoran, ground tracking, etc.). Present aircraft systems are capable of positional accuracy for latitude, longitude to ± 0.25 mile, course to ± 0.5 degrees, ground speed to ± 1 knot and elevation by radio altimeter to ± 25 feet [62 p. 306]. These accuracies exceed the positional accuracy needed, since a change in gravity with latitude is only 1 to 2 milligals per mile. The mean gravity would not be affected by a positional error of a few miles within a $1^{\circ} \times 1^{\circ}$ square.

Table 5 gives an indication of the magnitude of Eötvös correction we can expect at aircraft speeds. (Values given in the table are somewhat conservative.) Thompson and La Coste [62 p 307] feel that to determine ground speed to one knot is sufficient for an Eötvös correction to ± 5 milligals. To support their belief they developed the following formulas:

$$\Delta g = \frac{R_{\phi} + h}{R_{\phi}^{2}} \left[\left(V_{\phi} + v_{e} \right)^{2} + v_{n}^{2} - V_{\phi}^{2} \right] \qquad (34)$$

where: Δg = Eötvös correction in milligals

R_{ϕ} = Radius of earth to ϕ point

V_{ϕ} = Tangential velocity at the surface at the ϕ point

v_{e} = Easterly component of the ground speed of the aircraft

v_n = Northerly (or southerly) component of the ground speed of the aircraft

h = elevation above mean sea level;let v = $\sqrt{v_e^2 + v_n^2}$ = ground speed of aircraft then $\Delta g = \frac{R_\phi + h}{R_\phi^2}\left[2V_\phi v_e + v^2\right]$ (35)

and $\frac{R_\phi + h}{R_\phi}$ corrects velocity from the surface to the aircraft altitude. (36)

V_ϕ and R_ϕ are precise quantities; v_e , v_n and v may be in error. By differentiating Δg with respect to the easterly component they showed the error to be:

$$\frac{\partial(\Delta g)}{\partial v_e} = 2\,\frac{R_\phi + h}{R_\phi^2}\left(V_\phi + V_e\right) \tag{37}$$

and considering h negligible compared to R_ϕ

$$\left.\begin{aligned} \frac{\partial(\Delta g)}{\partial v_e} &= \frac{2}{R_\phi}\left(V_\phi + v_e\right) \quad \text{for easterly course} \\ \frac{\partial(\Delta g)}{\partial v_e} &= \frac{2}{R_\phi}\left(V_\phi - v_e\right) \quad \text{for westerly course} \end{aligned}\right\} \tag{38}$$

From the above formulas they determined some theoretical errors, for example:

v_e = 0 at the equator = 7.5 mgal/knot error

v_e = 450 kts easterly at equator = 11 mgal/knot error

v_e = 450 kts westerly at equator = 3.7 mgal/knot error

The feasibility of their approach is based really on the radial component:

$$V_\phi^2 / R_\phi$$

which is the tangential velocity at a point on the earth's surface and the earth radius to that point. The correction of this value at

the earth's surface to the aircraft height is done by:

$$R_\phi + h / R_\phi$$

and it is at this point that we should dissent. The correction term will not adjust the centrifugal force at the surface to the aircraft elevation (see section 5.1). We need to know the diminishing effect of centrifugal force with altitude. If a rigid pole were extended normal from a point on the earth to a great height and if an object were then placed unfixed on the top, we would find that the object would not rotate with the earth but would fall behind.

What the magnitude of error is in neglecting the change in centrifugal force I do not know. Thompson and La Coste are using a fixed coordinate system in assuming the aircraft is rigidly fixed to the rotating earth, therefore the centrifugal force at the surface and at h are equal. Since the centrifugal force does diminish with altitude,a moving coordinate system should be assumed.

The meter used in the test flights at Edwards Air Force Base, California in 1959 was the La Coste-Romberg Air Sea Gravity Meter No. 5 (This meter is under continual development; meter No. 11 is already in use by the U.S. Coast and Geodetic Survey on the Pioneer.) The aircraft was an Air Force KC-135 jet tanker. The project has been aptly described in professional literature [62], and only the highlights will be mentioned here.

Under normal atmospheric conditions accelerations were handled by the associated accelerometers. Horizontal accelerations were less than 30 milligals, but the corresponding periods approached 42 seconds and caused some problems, since they should be well under one minute

to be properly averaged. The vertical accelerations were much larger, as expected, but could be averaged out after operations smoothed down. When the accelerations caused the beam to hit the stops, the meter needed about three minutes to recover. Interestingly, the Askania ground camera tracking range disclosed considerable variation in the aircraft's ground speed; ± 5 knots over periods of 1/2 to 2 minutes. This varying speed greatly affects the Eötvös correction and may be an undisclosed error in areas where ground tracking is unavailable.

The averaging system on the La Coste surface meter when adapted for air use had to be speeded up somewhat. The analog computer now produces an integral of the beam correction, and this is continuously recorded from which the counter readings are obtained. For reduction an averaging time of five minutes was used. A computer was utilized to smooth out the integral record of the beam correction. Then the difference between the integral of the adjusted spring tension and the integral of the average beam correction renders the corrected change in gravity.

Thompson and Szabo [63] have reported that some 350 hours were flown in tests with the La Coste-Romberg meter in 1960. Much of this work was done in C-130 turbo prop aircraft of the 1370th Photo Mapping Wing. One of the more interesting problems confronting them was in locating the center of gravity of the aircraft. It was not near the wing section in the fuselage, as might be expected, but forward in the pilot's cockpit. Reportedly, flight lines of continuous gravity profiles of over 400 miles have been flown [63 p.128]. Also, the gradient of gravity was determined for the first time.

From the foregoing it is apparent that regardless of the present errors in airborne observation, the system as a whole opens an entirely new approach to gathering the gravity data geodesy has been seeking. Now that the door leading to this new field has been opened, research development work should proceed rapidly.

5. Corrections

Some error sources are pertinent not only to a particular meter or apparatus but to the field of sea observations as a whole. They are: the Eötvös correction, the navigation problem, the latitude corrections, the second order corrections, and the reduction to sea level. Their importance cannot be over-emphasized, because at the present time it is these very corrections, singly and collectively, that largely restrict the accuracy of sea-surface meters. This chapter will discuss these corrections, but in truth each one represents an extensive field of study. It will be seen that there is some degree of overlap between these error sources; for example, the navigation problem with its inherent errors puts errors into the Eötvös corrections and latitude corrections.

5.1 The Eötvös Effect

The effects of the east-west component of the moving platform's velocity and the centrifugal force of the earth's rotation on the gravity is the Eötvös effect. The Eötvös correction is additive for eastward and subtractive for westward components of the moving platform. The Eötvös correction is always present and may be the largest single error source in sea observations. Theoretically, a very slow moving platform such as a submarine traveling in a true meridional direction would produce no Eötvös effect, but in the practical sense such conditions are unattainable.

Increased observation speeds of the platform increase the magnitude of the Eötvös correction. The new surface-air gravity meters are capable of continuous recordings over long distances. This re-

quires precise speed and course information for long periods. With good navigational control (see section 5.2) the Eötvös correction should be accurate to about 2 to 3 milligals.

The Eötvös correction is usually derived in the following manner:

$$\omega = \frac{2\pi}{T}$$

where ω = angular velocity of the earth
T = siderial day
a = centrifugal accelerations
R = radius of latitude circle through observation point
ϕ = latitude of observation point

then $a = \omega^2 R$

differentiating,
$da = 2\omega R d\omega$

The change in gravity $\Delta g = da \cos\phi = 2\omega R \cos\phi \, d\omega$

The east-west linear velocity component of the observing platform is V and $V = R d\omega$

$$\therefore \quad \Delta g = 2\omega V \cos\phi \qquad (39)$$

This expression for the Eötvös correction is legitimate for submarine observations, because the speed is quite slow. Obviously, this formula is not rigorous enough to expose the proper Eötvös correction of fast moving platforms on varied courses. Actual true course must be introduced into the formula and a more rigorous expression for speed developed.

In his work with the Graf sea gravimeter Worzel has recently developed such a formula for use with surface observations. It has certain limitations with regard to airborne observations which will be discussed later.

The Eötvös correction by Worzel is derived from two conditions. Condition I is to express gravity (in milligals) at a stationary point on the rotating earth. Condition II is to then consider gravity from a platform moving along course C with speed S on the

rotating earth.

Condition I

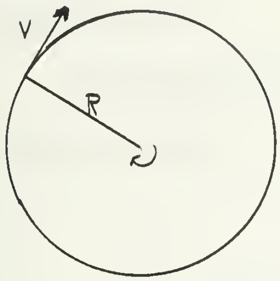

figure 26

$$CF = \frac{V^2}{r}$$

and $V = \omega r$

$$\therefore \; CF = \frac{\omega^2 r^2}{r} = \omega^2 r$$

where : (considering unit mass)
V = linear velocity (cm sec^{-1})
ω = rotational speed (cm sec^{-1})
r = distance at ϕ from rotation axis

$$\text{then} \quad \left.\begin{aligned} CF_{equ.} &= \omega^2 R \\ CF_{pole} &= 0 \end{aligned}\right\} \qquad (40)$$

$\therefore$ The centrifugal force at point A (figure 27) or any point from $\phi = 0°$ to $\phi = \pm 90°$ must be a function of latitude or

$$0 < CF_A < \omega^2 R \qquad (41)$$

From the figure $X = R \cos \phi$, and the component of CF_A directly opposite to gravitation gives us

$$\omega^2 R \cos^2 \phi \qquad (42)$$

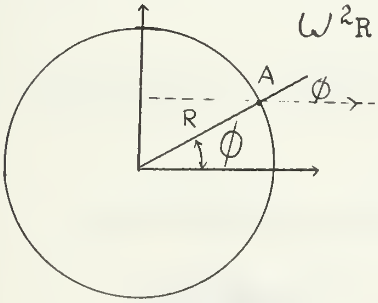

figure 27

Then at a stationary point on the rotating earth the gravity, g, is

$$g = \frac{K^2 M}{R^2} - \omega^2 R \cos^2 \phi \qquad (43)$$

where the first term is the expression for the component of gra-

vitation, and the second term is the component of centrifugal force.

Condition II

Observing platform moving along course C with speed S on a rotating earth.

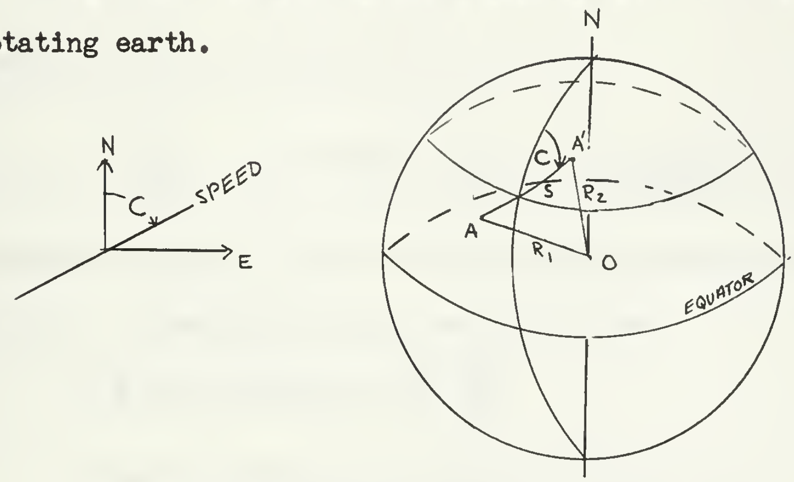

figure 28

Assumptions:

1. $R_1 = R_2$ because of the relatively short distance A-A$'$ $\therefore$ $R_1 \cos \phi = R_2 \cos \phi = R \cos \phi$ (44)

the tangential component of S to the parallel is

$$S \sin C \tag{45}$$

and the resultant angular velocity of the moving platform is

$$\omega + \frac{S \sin C}{R \cos \phi} \tag{46}$$

the tangential component of S to the meridian is

$$S \cos C \tag{47}$$

$\therefore$ Centrifugal acceleration due to movement along the meridian is

$$\left(\frac{S \cos C}{R}\right)^2 R = \frac{S^2 \cos^2 C}{R} \tag{48}$$

We may now express gravity for the moving platform from the gravitation and (44), (46) and (48).

$$g_M = \frac{K^2 M}{R^2} - \left(\omega + \frac{S\sin C}{R\cos\phi}\right)^2 R\cos^2\phi - \frac{S^2\cos^2 C}{R} \qquad (49)$$

The Eötvös correction then is

$$\Delta g = g - g_M \qquad (50)$$

Writing (50) from (43) and (49)

$$\Delta g = \left[\frac{K^2 M}{R^2} - \omega^2 R\cos^2\phi\right] - \left[\frac{K^2 M}{R^2} - R\left(\omega + \frac{S\sin C}{R\cos\phi}\right)^2 \cos^2\phi - \frac{S^2\cos^2 C}{R}\right] \qquad (51)$$

Consider the second term and expand the second term within it and multiply through with R and $\cos^2\phi$. Then

$$\Delta g = 2\omega S\sin C\cos\phi + \frac{S^2}{R} \qquad (52)$$

In order to get Δg in milligals and be able to use S in knots we must modify (52) to include a conversion factor. The nautical mile (6080.20 feet) is the accepted marine distance measure, and since marine navigation uses knots exclusively, it is convenient to express S in knots. From the first term of (52) we can show that

$$2\omega = \frac{4\pi \text{ radians}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec.}} = \frac{4\pi \text{ radians}}{86{,}400 \text{ solar seconds}} \qquad (53)$$

The S in the first term of (52) is in knots, and we want Δg in milligals, so knots must be expressed cm/sec. 1 nautical mile = 185,325 cm. $\therefore$

$$\frac{185{,}325 \text{ cm}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{185{,}325 \text{ cm}}{3600 \text{ sec}} \qquad (54)$$

Combining (53) and (54) we obtain a constant for 2ω with the conversion for knots to give Δg in milligals.

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$\frac{4\,\pi \text{ radians}}{86{,}400 \text{ seconds}} \cdot \frac{185{,}325 \text{ cm}}{3600 \text{ sec}} = 7.487 \text{ cm sec}^{-2} \qquad (55)$$

Substituting the constant obtained from (55) into (52) we have the Eötvös correction in practical form.

$$\Delta g = 7.487 \text{ S} \sin \text{C} \cos \phi + \text{S}^2/\text{R} \qquad (56)$$

where: Δg = correction in milligals

S = speed in knots

C = true course made good

ϕ = latitude

R = radius of the earth

The second term of (56)

$$\frac{\text{S nautical miles}^2/\text{hr}^2}{\text{R nautical miles}}$$

must similarly be converted from NM/hr^2 to cm/sec^2

$$\frac{1 \text{ cm}}{\text{sec}^2} = \frac{1}{185{,}325 \text{ cm}} \cdot \frac{\text{NM}}{\text{sec}^2} = \frac{3600^2}{185{,}325} \cdot \frac{\text{NM}}{\text{hr}^2} = \frac{12.96 \times 10^6}{18.5325 \times 10^4} =$$

$$0.06994 \text{ NM/hr}^2$$

Example:

S = 18 knots, R = 3,430 nautical miles

$$\frac{\text{S}^2}{\text{R}} = \frac{18^2}{3{,}430 \times 0.07} \cong 1.3 \text{ milligals}$$

Table 5 shows the Eötvös correction computed from (56). As may be seen from the table Δg decreases with an increase in ϕ and increases with speed. The second term S^2/R may be neglected for slow speeds. The second term rapidly becomes significant, however, with an increase in speed.

Most interesting is the effect of the second term at high speeds,

ϕ	3 Kts Submarine Spd. Second Term equals .0		12 Kts Surf. Ship (slow), Second Term equals .6		18 Kts Surf. Ship (fast), Second Term equals 1.3		275 Kts An Aircraft Spd., Second Term equals 314.1	
	1st Term	Δg	1st Term	Δg	1st Term	Δg	1st Term	Δg
0°	22.5	22.5	89.8	90.4	134.8	136.1	2,058.9	2373.0
10°	22.1	22.1	88.5	89.1	132.7	134.0	2,027.6	2341.7
20°	21.1	21.1	84.4	85.0	126.6	127.9	1,934.8	2248.9
30°	19.5	19.5	77.8	78.4	116.7	118.0	1,783.1	2097.2
40°	17.2	17.2	68.8	69.4	103.2	104.5	1,577.2	1891.3
30°	14.4	14.4	57.8	58.4	86.6	87.9	1,323.5	1637.6
60°	11.2	11.2	44.9	45.5	67.4	68.7	1,029.5	1343.6
70°	7.7	7.7	30.7	31.3	46.1	47.4	704.2	1018.3
80°	3.9	3.9	15.6	16.2	23.4	24.7	357.5	671.6
90°	0	0	0	.6	0	1.3	0	314.1

The Eötvös correction as a function of ϕ and S along a maximum effect course of 090° True. Δg MGALS $= 7.487\ S \sin C \cos\phi + S^2/R$.

Table 5

even when the platform is moving in a meridional direction.

Given: $S = 20$ knots
$C = 000^{\circ}$ True
$\phi = 40^{\circ}N$

$$\text{Then} \quad \Delta g = 7.487 \text{ (20 kts) } (0)(.6428) \quad \frac{400}{0.7 \times 344}$$

$$\Delta g = \quad 0 \quad + \quad 1.7 \text{ mgals}$$

The Eötvös corrections for airborne observations given in Table 5 are not rigorous, but they serve to give an indication of the magnitude of correction we can expect for air observations. It may be remembered that Worzel developed a surface Eötvös correction from two points of view: 1. a stationary platform on a rotating spherical earth and 2. a platform moving along a course with a speed on a rotating spherical earth. Referring to section 4.4 we see that we no longer have a stationary and a moving condition for the airborne correction. At altitude h the platform will never have a stationary condition but will fall behind the rotating earth. A more rigorous formula for the Eötvös correction is needed.

5.2 The Navigation Problem

We must know where the observation took place (ϕ, λ, depth) for it to be of complete value, and positioning on the high seas is a significant problem. The mariner is quite happy with a two-three mile navigational accuracy; greater accuracy is of little value to him compared to the effort necessary to achieve it. Geodetically, we may need only a mean anomaly value for an area (ex. $1^{\circ} \times 1^{\circ}$), so some positioning inaccuracies may be tolerated. A detailed geologic study of an area, however, requires explicit positioning. We must position as accurately as possible; the value may not be apparent today, but

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

tomorrow it may be urgent.

Latitude and longitude may be determined by a number of ways: celestial fixes, electronically (radar, Loran, Decca, etc.), bathymetric charts and soundings, terrestrial navigation, dead reckoning. It is apparent that inshore, we should use electronics, soundings, and terrestrial fixes as available and in combined form, an accuracy of one mile in position and a half knot in speed should be expected. Such accuracy is the standard we must attempt to achieve on the open sea, although two miles in position and one knot in speed is frequently accepted [39].

Given good sky conditions and weather conditions, navigational accuracy of 1-1/2 miles is quite attainable by celestial observations.

Unfortunately such navigation conditions seldom prevail for long, and if we are beyond electronic coverage from Loran or similar systems, our accuracy rapidly drops off. For example: at $\phi = 30^\circ$:

a one mile navigational error $\approx$1 milligal error

a one knot error in speed $\approx$ 7 milligal error

It has generally been accepted that positional accuracy will be 1-2 miles, speed 0.5-1 knot.

For the quadratic errors of the maximum and minimum positional accuracy we obtain

$$\left.\begin{aligned} m_{max.} &= \sqrt{2^2 + 7^2} = 7.3 \text{ mgal} \\ m_{min.} &= \sqrt{1^2 + 3.5^2} = 3.6 \text{ mgal} \end{aligned}\right\} \qquad (57)$$

Therefore the positional data accepted yields a mean error of approximately 3.6 to 7.3 milligals.

Undoubtedly, one of the best gravity expeditions from the standpoint of navigational control was in May 1948 on board the HMS Talent when R.I.B. Cooper participated in a gravity survey in the English Channel [6]. Of course, in such a waterway high accuracy should be expected. Position was determined by Decca, and Cooper reports an accuracy of ± 100 yards which corresponds to 0.05 milligals north or south. The submarine was equipped with taut wire gear for ground speed control. The taut wire gear is a long reel of piano gauge wire housed internally in the submarine and fed out through an opening in the bottom of the boat by a weight. Before diving the submarine obtains accurate Decca and visual (if possible) fixes. The craft then dives and from her established position reels out the wire while cruising close to the bottom. By measuring the wire strung out over the observation period they were able to obtain ground speed control of ± 0.05 knots which is equivalent to 0.2 milligal error east or west.

Open sea navigation, we may then conclude, is the limiting accuracy factor of sea observations at the present time. Sophisticated navigational systems are in existence, but they are not available for general expeditions, and the accuracies are classified. The USS Compass Island, for example, has the SINS (Ships Inertial Navigational System) developed at MIT [2 p. 378]. It is an all weather, all latitude, day-night system which continuously determines ϕ and λ, true north, and ship's ground speed. Aside from the cost and installations requirements, it undoubtedly answers or closely answers the navigational problem.

We may further illustrate accuracy requirements by considering the

maximum allowable error in speed to produce a desired accuracy of one milligal (other error sources neglected).

differentiating the Eötvös correction (56)

$$d\Delta g = \left[7.487 \text{ Sin C Cos} \phi + \frac{2S}{R} \right] \cdot dS \qquad (58)$$

solve for dS

$$dS = \frac{d\Delta g}{7.487 \text{ Sin C Cos } \phi + \frac{2S}{R}} \qquad (59)$$

Table 6 gives some indication of the critical accuracy with which we need to know the speed. To get one milligal accuracy at 50° latitude we need to know our programed 18 knot speed to 0.2 of a knot. The adjoining table shows that if we decrease accuracy requirements from one milligal down to five milligals, the allowable speed error increases accordingly.

The selection of observation speeds of surface vessels is limited by two factors: one, the speed capability of the ship itself, and two, the speed limitation caused by the PDR (position depth recorder) or other sounding equipment.

The U.S. Coast and Geodetic Survey ships generally use 12 knots as their average observation speed. This speed works well with the capability of the ship and it is a good compromise between very slow speeds with excessive rolling and high speeds with excessive vibration. The U.S. Navy's gyro fin stabilized USS Compass Island (EAG 153) is a converted Mariner class hull. She is a big vessel, 17,600 tons, and is capable of speeds up to 20 knots 2 p. 378. Because the sea gravity meter used was on a stable platform, and because the ship is also stabilized (gyro fin) sustained observation speeds of 18 knots were possible.

[illegible]

Maximum allowable error in speed to give less than 1 mgal error. Computation is based on formula (59) using a maximum effect course of 090° True.

ϕ	KNOTS			
	3	12	18	275
	ds	ds	ds	ds
0°	0.133	0.132	0.131	0.102
10°	.135	.133	.133	.104
20°	.142	.140	.139	.107
30°	.154	.152	.151	.114
40°	.174	.171	.167	.125
50°	.207	.204	.202	.141
60°	.265	.260	.255	.166
70°	.387	.376	.369	.207
80°	.755	.715	.690	.280
Cross section at ϕ = 40° as allowable error increases				
2 mgal	0.347	0.343	0.334	0.249
3 mgal	.521	.514	.501	.374
4 mgal	.694	.686	.668	.499
5 mgal	.868	.857	.835	.624

Table 6

[illegible]

5.3 The Latitude Effect

The latitude effect (normal gravity correction) is due to the fact that the semi-minor axis of the earth (b) through the poles is approximately 21 km shorter than the semi-major axis (a). There is a corresponding increase in the acceleration of gravity along the earth's surface as the latitude (ϕ) increases from 0° to 90°. The increase from the equator to the poles is approximately 5,000 milligals. It follows, then , that as the observation platform moves in a meridional direction, there is a rate increase (or decrease) in g per unit traveled, see chart 1. As the platform moves along a course out from a meridian course, the latitude correction will decrease with the cosine of the course bearing. The latitude correction will be zero when the course is due east or west.

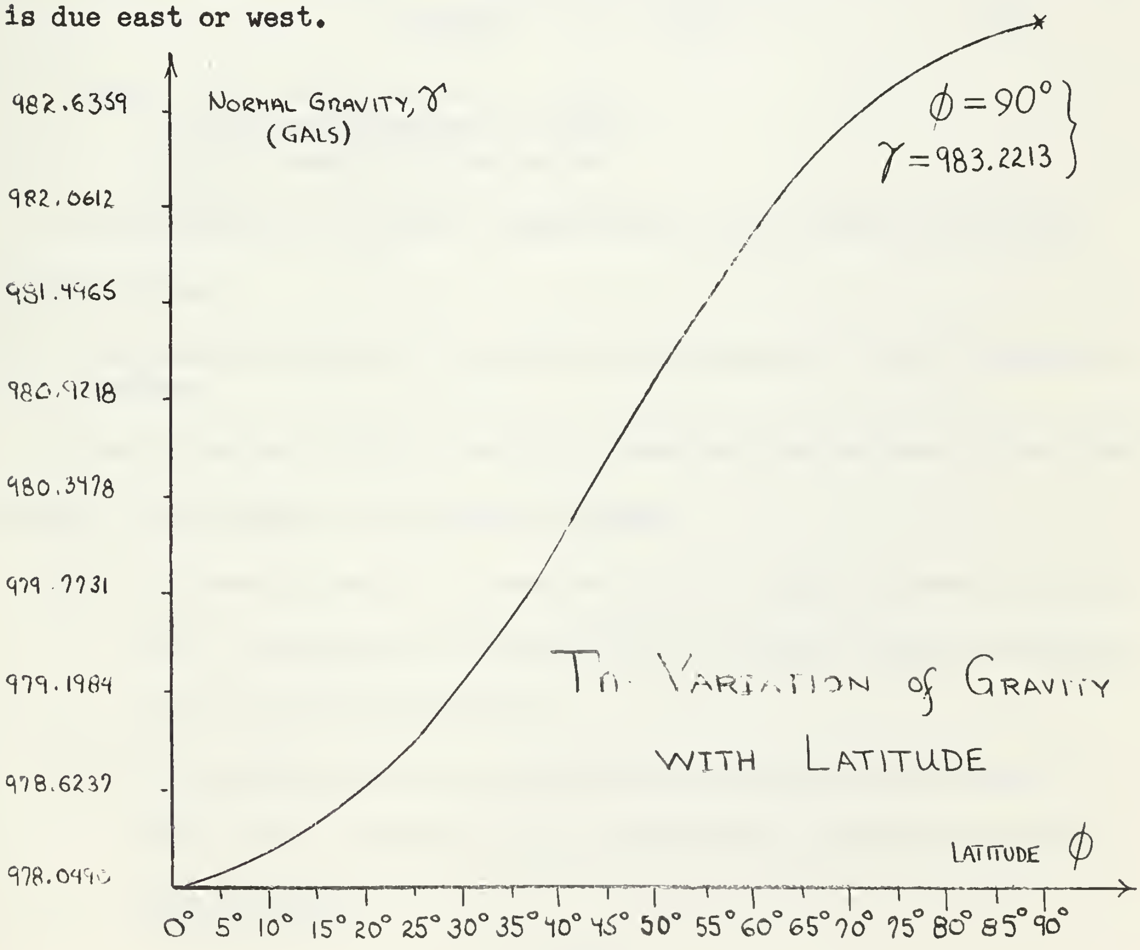

Chart 1

The International Gravity Formula (31) clearly shows the variation of normal gravity (γ) with latitude (ϕ). And we may conclude that the value of gravity that we measure is a function of latitude at the observation point. With a gravimeter such as the Graf Seagravimeter the latitude effect is automatically corrected for by a mechanical device. The Enograph recorder will then indicate the gravity anomalies [85 p.21].

5.4 Reduction to Sea Level

The reduction to sea level is necessary for submarine observations. Surface observations are observed practically at sea level (geoid) hence eliminate the need for such a reduction. The free air and Bouguer reduction may be combined and expressed as [34 p. 158]

$$-0.09406\left(1 - \frac{3}{2} \cdot \frac{1.027}{\rho_m}\right)d \qquad (60)$$

where: d = depth in feet
ρ_m = mean density of earth
1.027 = density of sea water

The reduction of airborne observations is not within the scope of this paper.

5.5 Corrections Necessary to Gravimeters because of their Suspension Systems

Gravity meters of the type to be described can be used at sea from either of two types of suspension systems.

(1.) The gravity meter is suspended from a gimbal system, in which case it measures the acceleration of gravity along the instantaneous apparent vertical.

(2.) The gravity meter is supported on a stabilized platform, in which case it measures the acceleration of gravity along the true vertical.

5.51 Gimbal Suspension

In the first case where the meter is hung freely, the instantaneous apparent vertical is the resultant of gravity and the vertical and horizontal accelerations. As was shown in the description of the Pendulum Apparatus, the measured value of g is too large, and a second order Browne correction must be computed and the value deducted from the observed quantity. In the following development of this correction it should be borne in mind that the gravity meter in gimbal suspension is actually a pendulum, is treated as such, and possesses a particular natural period. What follows is the correction term, developed by La Coste [30p.222] for a gravity meter in gimbal suspension.

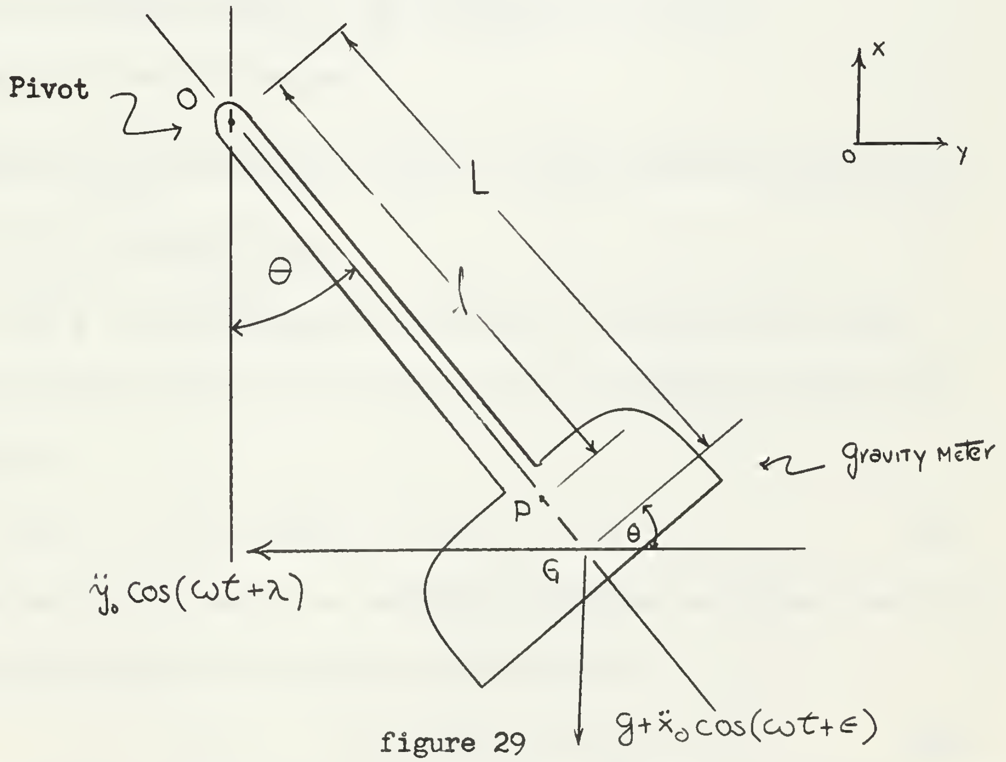

figure 29

Where: O is the suspension pivot

[illegible]

G is the center of gravity

P is the sensing element of the meter

M is the mass of the system

k is the radius of gyration about O

and OG = L; OP = ℓ and P lies along line OG

The first step is to develop the equation of motion for the suspended system. For this we need the expressed accelerations which are:

$$\left.\begin{aligned} \text{horiz. accel.} &= \ddot{y} = \ddot{y}_0 \cos(\omega t + \lambda) \\ \text{vertical accel.} &= \ddot{x} = \ddot{x}_0 \cos(\omega t + \epsilon) \end{aligned}\right\} \qquad (61)$$

Where $\omega = \frac{2\pi}{T}$ = angular frequency, λ & ϵ = phase angles

Also $$\frac{L^2 + k^2}{L} = L_0 = \frac{g}{\omega_0^2} \qquad (62)$$

and the natural period of the pendulum is $2\pi/\omega_0$

The equation of motion becomes

$$\ddot{\theta} + \beta\dot{\theta} + \left(\frac{1}{L_0}\right)\left\{g + \ddot{x}_0\cos(\omega t + \epsilon)\right\}\sin\theta = \left(\frac{\ddot{y}_0}{L_0}\right)\cos(\omega t + \lambda)\cos\theta \qquad (63)$$

Where β is the friction term

Since θ is a very small angle, let Sin $\theta = \theta$ and cos $\theta = 1$ and neglect $\ddot{x}$ with respect to g, since it can be averaged out with time, we now get

$$\ddot{\theta} + \beta\dot{\theta} + \omega_0^2\theta = -\left(\frac{\ddot{y}_0}{L_0}\right)\cos(\omega t + \lambda) \qquad (64)$$

La Coste now sets A and γ as arbitrary notations and ϕ as the phase angle of the forced oscillation developed from

$$\tan(\phi - \lambda) = \beta\omega/(\omega_0^2 - \omega^2) \qquad (65)$$

And referring to the equation of motion, the solution is separated into

[illegible]

two approaches:

damped free oscillations

$$\theta_n = A \exp.\left(-\frac{1}{2}\beta t\right) \cos\left(\left(\omega_0^2 - \frac{1}{4}\beta^2\right)^{\frac{1}{2}} t - \lambda\right) \tag{66}$$

and forced oscillations

$$\theta_d = -\left(\frac{\ddot{y}_0}{L_0}\left[\left(\omega_0^2 - \omega^2\right)^2 + \beta\,\omega^2\right]^{-\frac{1}{2}} \cos\left(\omega t - \emptyset\right)\right) \tag{67}$$

Assume

1. The natural periods are maintained (usually eliminated by damping after a settling period of the meter after starting) at a constant level.
2. The gimbal friction β is negligible.

Then

$$\theta_n = B \cos\left(\omega_0 t - \gamma\right)$$

$$\theta_d = - \frac{\ddot{y}_0}{\left[L_0\left(\omega_0^2 - \omega^2\right)\right]\left[\cos\left(\omega t + \lambda\right)\right]}$$

$$\theta = \theta_n - \theta_d \tag{68}$$

The component of acceleration parallel to the axis OG along which the gravity meter measures is

$$\left\{g + \ddot{x}_0 \cos(\omega t + \epsilon)\right\} \cos\theta - \ddot{y}_0 \cos(\omega t + \lambda) \sin\theta + l\,\dot{\theta}^2 \tag{69}$$

and by substituting in the approximate values we get

$$\left\{g + \ddot{x}_0 \cos(\omega t + \epsilon)\right\}\left(1 - \frac{1}{2}\theta^2\right) - \ddot{y}_0 \cos(\omega t + \lambda) \sin\theta + l\,\dot{\theta}^2 \tag{70}$$

Averaged over a long period with respect to the periods

$$g - \frac{1}{2} g \bar{\theta}^2 + \frac{\ddot{y}_0^2}{2 L_0 (\omega_0^2 - \omega^2)} + \frac{1}{2}\, l\, \frac{(B^2 \omega_0^2 + \ddot{y}_0^2 \omega^2)}{L_0^2 (\omega_0^2 - \omega^2)^2} \tag{71}$$

To simplify the above expression set $l = L_0 = g/\omega_0^2$

then

$$g - \frac{1}{2} g \bar{\theta}^2 + \frac{1}{2} g \left\{B^2 + \frac{\ddot{y}_0^2 \omega_0^4}{g^2 (\omega_0^2 - \omega^2)}\right\} \tag{72}$$

However $$\bar{\Theta}^2 = \left[B\cos(\omega_0 t - \gamma) - \frac{\ddot{y}_0}{L_0(\omega_0^2 - \omega^2)} \cos(\omega t + \lambda) \right]^2$$

$$= \frac{1}{2}\left[B^2 + \frac{\ddot{y}_0^2\, \omega_0^4}{g^2(\omega_0^2 - \omega^2)} \right] \qquad (73)$$

So the measured acceleration is

$$g + \frac{1}{2} g \bar{\Theta}^2 \qquad (74)$$

The acceleration perpendicular to the axis may be similarly derived. La Coste has determined its effect to be negligible. He also determined (74) is accurate to 1 mgal for $\ddot{x}_0$ and $\ddot{y}_0$ for accelerations to 60,000 mgals (assuming dissimilar period between disturbances and natural meter period [30] .

5.52 Stabilized Platform

If our meter is on a stabilized platform, that is, the vertical of the meter axis is stabilized along the true vertical, it will measure $g + \ddot{x}$.

If stabilization were perfect we would get g directly, but since such perfection is unattainable, a small tilt angle Θ must be considered. Θ is formed between the meter axis and the true vertical axis. Again the accelerations are expressed from (61). Harrison has developed the enforced leveling correction as follows: [30 p. 217]

$$\Theta = \theta_0 \; \theta_1 \cos(pt + \phi) \qquad (75)$$

Where

p = period

t = time

ϕ = phase angle of forced oscillations

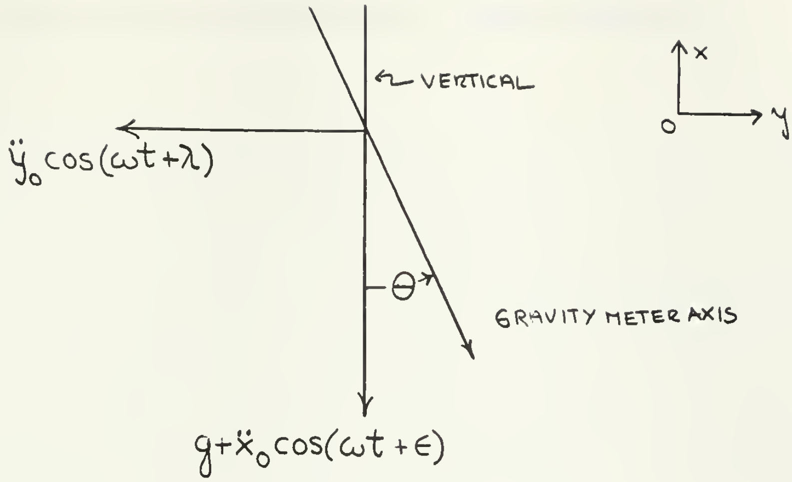

figure 30

From figure 30 the total measure acceleration is

$$(g+\ddot{x})\cos\theta - \ddot{y}\sin\theta \tag{76}$$

or since θ is small

$$g\left\{1-\frac{1}{2}\left[\theta_0+\theta_1\cos(pt+\phi)\right]^2\right\}+\ddot{x}_0\cos(\omega t+\epsilon)$$
$$-\ddot{y}\cos(\omega t+\lambda)\left[\theta_0+\theta_1\cos(pt+\phi)\right] \tag{77}$$

Average over a long time compared to periods we get

$$g\left\{1-\frac{1}{2}\theta_0^2-\frac{1}{4}\theta_1^2\right\}-\frac{1}{2}\theta_1\ddot{y}_0\left\{\cos\left[(\omega+p)t+\phi+\lambda\right]+\cos\left[(\omega-p)t+\lambda-\phi\right]\right\} \tag{78}$$

if $\omega = p$

$$g\left[1-\frac{1}{2}\theta_0^2-\frac{1}{4}\theta_1^2\right]-\frac{1}{2}\theta_1\ddot{y}_0\cos(\lambda-\phi) \tag{79}$$

Harrison points out that θ_0 must be less than 5' for 1 mgal accuracy, and θ_1 is much more critical [30 p.218].

Given $\ddot{y}_0$ = 100,000 mgal

$\lambda - \phi = 0$

then θ_1 must be less than 4" for 1 mgal accuracy.

6. Summary, Conclusions, and Recommendations

6.1 Summary

Although many approaches resulting in a myriad of instrument types have attempted to solve the problem of efficient, accurate gravity observations at sea, only three such available instruments have proven themselves practical. The Vening Meinesz Pendulum Apparatus, the Askania Sea Gravimeter GSS2 after Graf, and the LaCoste-Romberg Air-Sea Gravity Meter, being the workable instruments, are summarized by general characteristics in Table 7.

6.2 Conclusions and Recommendations

The design of all three measuring devices is more than adequate for stationary observations, but only the Pendulum Apparatus provides a consistently acceptable accuracy value of around $\pm$3-5 milligals from a moving platform at sea. The two gravimeters are only now approaching this accuracy, and the ultimate goal of one milligal is indeed a long way off. However, the speed of readout, simplicity of operation, production of a continuous gravity profile record, and most important, the portability and surface ship usefulness make the gravimeter the potential answer to our need for fast, accurate gravity data over the vast ocean expanses.

A detailed analytic study of all known and suspected acceleration-disturbance forces is a fundamental prerequisite to the improvement of sea gravimeter accuracy. That this is an acknowledged fact is demonstrated by the extensive development cruises previously described as well as those planned for the future. Specifically, the horizontal, vertical, rotational and/or cross coupling effects must be more

INSTRUMENT	MEASURING SYSTEM	OBSERVATION PLATFORM	SUSPENSION or MOUNTING	ACCELERATION DISTURBANCES VERTICAL	HORIZONTAL	TYPE OBSERV.	RECORD	EÖTVÖS CORRECTION	DRIFT	RANGE	TEMP CONTROL	ESTIMATED ACCURACY (IDEAL CONDITIONS)
Vening Meinesz Pendulum Apparatus (1923)	3 brass, $\frac{1}{2}$ sec. STERNECK type pendulums.	Submarine	Gimbals	handles up to 7,000 mgals Determined from record	1. Measured by mutually long period pendulums ($\approx 26''$) or 2. Accelerometers	Single point obs. 30 min./obs.	1. Photographic 1 $\Theta_2 - \Theta_1$ 2 $\Theta_2 - \Theta_3$ 3 Θ_2 4 Temperature 5 Tilt (β) 6 } Amplitudes of 7 } Long P. Pendulums	formula (39)	—	—	$\pm$ 0.0X°C	Harbor $\pm$1 mgal open sea $\pm$ 3-5 mgals
ASKANIA Seagravimeter Gss 2 after Graf (1957)	Long thin aluminum "Weigh-beam" held by horizontally stressed helical springs & 8 wire fibers	1. Submarine 2. Surface ship (up to sea state 4) 3. (has airborne potential)	1. Gimbals or 2. Gyro stabilized platform	handles up to 100,000 mgals Determined from record	1. Use stabilized platform or 2. Accelerometers	Continuous profile averaging depends upon steepness of horiz. gradient of gravity $\approx$10-15 min.	1. ENOGRAPH - cont. viewable record Deflection of weighbeam with attendent disturbances	formula (56) No. of terms dependent on speed	Unreported estimate 1-5 mgal per mo.	4,000 mgals additional 3,000 mgal adjustment	$\pm$ 0.01°C	Harbor $\pm$ 1-5 mgals open sea $\pm$7-8 mgals
La Coste - Romberg Air-Sea gravity Meter (1958)	Pivoted mass supported by a precision spring (La Coste Suspension)	1. Submarine 2. Surface ship (up to sea state 4) 3. Aircraft (with normal atmosphere conditions)	1. Servo operated gimbals or 2. Gyro stabilized platform	handles up to 100,000 mgals Determined from record	Analog computer continually observes, computes and deducts correction	1. Surface same as Gss 2 2. Air averaging time $\approx$ 5 min.	1. ESTERLINE-ANGUS Recorder using 16 digit biniary code 1. Beam correction 2. Time marks 3. Corrected spring tension.	formula (56) No. of terms dependent on speed (34)	Negligible	6,000 mgals	$\pm$0.0X°C	Harbor $\pm$ 0.5-1 mgal open sea $\pm$7-8 mgals Airborne $\pm$ 10 mgals

General specifications for the Vening Meinesz Pendulum Apparatus, the Askania Seagravimeter GSS2 after Graf and the La Coste-Romberg Air-Sea Gravity Meter.

Table 7

thoroughly known. More information is needed on ship motion and shipboard vibration. And, of course, highly sensitive accelerometers are needed to feel out and record all hidden disturbances to the meter sensing element. It should be borne in mind that the study of disturbance periods from a few seconds outward to the energy limit of the disturbance is essential. The effect of the periods and phrasing of the periods, from one acceleration compared to another acceleration, must be known.

We never really know how accurate a measuring instrument is until it can be compared against a standard set of values while observing under actual field conditions. At the present time no "true" set of values exist. Seagravimeter observations are compared to corresponding pendulum observations, but this is not a rigorous test of accuracy. Not only may the navigation of the testing ship be erroneous with respect to the pendulum station, but the pendulum station itself may be incorrectly charted. In addition our standard values should be more accurate than the ± 3-4 milligal pendulum accuracy.

To correct the above situation H. Orlin [45 p. 12] reports that the U.S.C. and G.S. may establish a well defined gravity field test area on the Pacific Coast. Gravity values would be determined by underwater gravimeters for a coastal area ($\leq$ 100 fathoms) covering a large milligal range. The geographic location of the test area would be such that the various electronic navigation systems could be used to precisely pilot the testing ships through the area. It is hoped that if such a test area is successfully established on the west coast, that a similar area will be established on the east coast.

The drift of a seagravimeter either should be negligibly small or, if it exists, should be kept linear. The foregoing conditions must be maintained for a minimum period of 4 to 5 weeks. Once the port base station has been left behind, there is no opportunity for drift checks until the ship reaches another port base station. It may prove advantageous, therefore, to consider the establishment of anchored drift check station buoys, as first suggested to me by Dr. U. Uotila of the Ohio State University. A small submerged sealed buoy anchored in shallower mid ocean areas (mid ocean ridges, sunken level topped volcanoes - guyots) could be triggered by a ship transmission to emit identifying signals. The vessel with a meter on board could then make precise runs over the buoy and compare its observed gravity to the previously established value for the station. Obviously, such a device could fulfill many scientific missions. For example, it could continuously record current information, temperature, and pressure, be useful in long range sound transmission experiments, provide precise navigational checks, and assist in establishing distance ties. The cost of such an undertaking would be fairly large, but the scientific benefit could be very great.

Aside from allowing a surface ship gravity meter to obtain mid ocean drift checks for a particular cruise, the sea time of the seagravimeter could be greatly increased. For example, a gravimeter as portable as the Graf Seagravimeter could be transferred at sea from one ship to another. Having participated in many at sea transfers of ammunition, electronic equipment, and personnel, the author is convinced that transfer of the gravimeter and its supporting equipment is feasible. Naturally, an observing team would also have to accompany

the seagravimeter as it is transferred about.

The true accuracy of such sea drift checks is undoubtedly not equal to the accuracy of checks at port base stations, but it should extend the continuous at sea periods for a gravity meter.

Heiskanen in a recent paper published a map of the world which was divided into $5^{\circ} \times 5^{\circ}$ blocks of mean free air anomalies [35]. As might be expected, most continental areas and well traveled ocean routes were well filled in with mean values. But the vast ocean areas of the South Pacific, much of the South Atlantic, the Indian Ocean and similar remote areas were largely untouched by gravity surveys. Before the role of gravimetric geodesy in determining the shape of the earth can be fulfilled, these areas must be surveyed. It is to this end, of course, that the perfection of sea gravity meters (and airborne meters) is so eagerly anticipated.

Many different countries and agencies are obtaining gravity field information. Fortunately, cooperation exists between most of these groups, so that the material can be collected and analyzed by such organizations as the Institute of Geodesy, Photogrammetry, and Cartography at Ohio State University.

Toward this end Vajk and Van der Sleen have proposed "standardization of gravity survey procedures" [66]. Among the procedures suggested for standardization is the reporting of gravity bench marks. This includes:

1. Designation of the station
2. Date bench mark set
3. Geographic location

4. Location with respect to landmark

5. Station elevation

6. Observed gravity value

7. Density correction factor used

8. Bouguer anomaly

9. Other information

The authors of this proposal were, in the main, concerned with land gravity surveys, but by expanding item 9 we could include those items peculiar to sea observations. They are:

1. Observation course and speed

2. Wind velocity and direction

3. Sea state and direction

4. Depth

5. Navigation control and estimate of positional accuracy

The foregoing information would permit the analysist of the data to estimate the true accuracy of the observational data. These accuracy estimates for the data coming partly from external conditions, such as sea state and wind velocity, will improve as the meters are tested over a wide range of field conditions.

Although the accuracies for the La Coste-Romberg and Graf meters are presently only about $\pm$ 7-8 milligals, improvements may reasonably be expected in the near future. Gravity observations at sea, then, must be pushed ahead expeditiously. The tools long awaited have been developed to the point where the large scale collection of gravity data is practical.

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2. Cross Reference Chart

SECTION	REFERENCE NUMBER							
1	44	49	53	64	65	69	73	76
	77	84						
1.1	16	17	18	19	20	24	26	27
	54	57	60	70	71	72	74	75
	78	80	86					
1.2	4	5	8	9	14	21	34	36
	38	46						
1.3	1	2	7	11	13	15	28	39
	45	58	78	81	82	87		
1.4	1	11	23	41	47	48	50	83
	87							
2	67	68						
2.1	3	4	22	26	30	51	52	70
2.2	3	10	16	17	18	30	34	54
	56	57	71	72	74	80		
2.3	18	19	24	34	54	55	56	57
	71	72	74	75	80			
3.1	25	30	37	79	81	82	85	
3.2	25	85						
3.3	25	50	79	81	82	85		
3.4	15	25	42	79	81	82	83	85
4.1	30	39	43	58				
4.2	28	33	39	58				
4.3	15	28	31	37	39	43	45	50
	58	83						
4.4	50	61	62	63				
5.1	3	30	34					
5.2	2	6	24					

SECTION	REFERENCE NUMBER						
5.3	3	34					
5.4	34						
5.5	12	30	40				
6	35	45	53	59	64	65	66

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