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COMPRESSED AIR

THEORY AND COMPUTATIONS

UNIV. OF
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BY

ELMO G. HARRIS, C.E.

PROFESSOR OF CIVIL ENGINEERING, MISSOURI SCHOOL OF MINES,
IN CHARGE OF COMPRESSED AIR AND HYDRAULICS;
MEMBER OF AMERICAN SOCIETY OF
CIVIL ENGINEERS

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PREFACE

THIS volume is designed to present the mathematical treatment of the problems in the production and application of compressed air.

It is the author's opinion that prerequisite to a successful study of compressed air is a thorough training in mathematics, including calculus, and the mathematical sciences, such as physics, mechanics, hydraulics and thermodynamics. Therefore no attempt has been made to adapt this volume to the use of the self-made mechanic except in the insertion of more complete tables than usually accompany such work. Many phases of the subject are elusive and difficult to see clearly even by the thoroughly trained; and serious blunders are liable to occur when an installation is designed by one not well versed in the technicalities of the subject.

As one advocating the increased application of compressed air and the more efficient use where at present applied, the author has prepared this volume for college-bred men, believing that such only, and only the best of such, should be entrusted with the designing of compressed-air installations.

The author claims originality in the matter in, and the use of, Tables I, II, III, V, VI, VII and IX, in the chapter on friction in air pipes and in the chapter on the air-lift pump.

Special effort has been made to give examples of a practical nature illustrating some important points in the use of air or bringing out some principles or facts not usually appreciated.

Acknowledgment is herewith made to Mr. E. P. Seaver for tables of Common Logarithms of Numbers taken from his Handbook.

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SYMBOLS

For ready reference most of the symbols used in the text are assembled and defined here.

p = intensity of pressure (absolute), usually in pounds per square foot. Compressed-air formulas are much simplified by using pressures measured from the absolute zero. Hence where ordinary gage pressures are given, p = gage pressure + atmospheric pressure. In the majority of formulas p must be in pounds per square foot, while gage pressures are given in pounds per square inch. Then p = (gage pressure + atmospheric pressure in pounds per square inch) × 144.

v = volume — usually in cubic feet.

Where sub-a is used, thus p_a, v_a, the symbol refers to free air conditions.

r = ratio of compression or expansion $= \dfrac{\text{higher pressure}}{\text{lower pressure}}$.

The lower pressure is not necessarily that of the atmosphere.

t = absolute temperature = Temp. F. + 460.6.

n = an empirical exponent varying from 1 to 1.41.

$\log_e$ = hyperbolic logarithm = (common log.) × 2.306.

W = work — usually in foot-pounds per second.

Q = weight of air passed in unit time.

w = weight of a cubic unit of air.

Other symbols are explained where used.

FORMULAS

For convenience of reference the principal formulas appearing in the text are collected here with the article and page where demonstration and complete explanation can be found.

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COMPRESSED AIR

CHAPTER I

Formulas for Work

Art. 1. Temperature Constant or Isothermal Conditions. From the laws of physics (Boyle's Law) we know that while the temperature remains unchanged the product pv remains constant for a fixed amount (weight) of air. Hence to determine the work done on or by air confined in a cylinder, or like conditions, when conditions are changed from p_1v_1 to p_2v_2 we can write

$$p_1v_1 = p_xv_x = p_2v_2,$$

sub x indicating variable intermediate conditions.

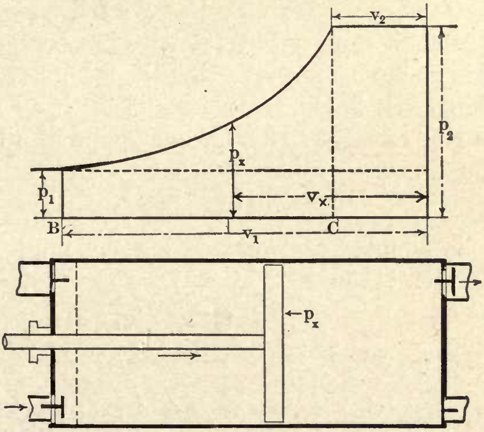

Fig. 1.

Whence $p_x = \frac{p_1v_1}{v_x}$ and $dW = p_xAdl = p_xdv_x$ since $Adl = dv$; A being the area of cylinder, therefore $dW = p_1v_1\frac{dv_x}{v_x}$, and work of compression or expansion between points *B* and *C* (Fig. 1) is the integral of this, or

$$W = p_1v_1 \int_{v_2}^{v_1} \frac{dv_x}{v_x} = p_1v_1 (\log_e v_1 - \log_e v_2)$$

$$= p_1v_1 \log_e \frac{v_1}{v_2} = p_1v_1 \log_e \frac{p_2}{p_1} = p_1v_1 \log_e r = p_2v_2 \log_e r.$$

Note that this analysis is only for the work against the *front* of the piston while passing from *B* to *C*. To get the work done during the entire stroke of piston from *B* to *D* we must note that throughout the stroke (in case of ordinary compression) air is entering behind the piston and following it up with pressure p_1. Note also that after the piston reaches *C* (at which time valve *f* opens) the pressure in front is constant and $= p_2$ for the remainder of the stroke. Hence the complete expression for work done by, or against, the piston is

$$p_1v_1 \log_e r - p_1v_1 + p_2v_2;$$

but since $p_1v_1 = p_2v_2$, the whole work done is

$$W = p_1v_1 \log_e r \quad \text{or} \quad p_2v_2 \log_e r. \tag{1}$$

Note that the operation may be reversed and the work be done by the air against the piston, as in a compressed-air engine, without in any way affecting the formula.

Forestalling Art. 2, Eq. (4), we may substitute for pv in Eq. (1) its equivalent, 53.17 t, for one pound of air and get for one pound

$$W = 53.17\, t \times \log_e r. \tag{1a}$$

This may be adopted for common logs by multiplying by 2.3026. It then becomes

$$W = (122.61 \log_{10} r)\, t, \tag{1b}$$

$$(\log 122.61 = 2.0878852.)$$

Note that in solving by logs the log of log r must be taken. Values of the parenthesis in Eq. (1b) are given in Table I

For the special temperature of 60° F. (1b) becomes for one pound of air

$$W = 63737 \log_{10} r, \tag{2}$$

$$\log 63737 = 4.8043894.$$

Example 1a. What will be the work in foot-pounds per stroke done by an air compressor displacing 2 cubic feet per stroke, compressing from $p_a = 14$ lbs. per sq. inch to a gage pressure = 70 lbs.; compression isothermal, $T = 60°$ F.?

Solution (*a*):

Inserting the specified numerals in Eq. (1) it becomes

$$W = 144 \times 14 \times 2 \times \log_e \frac{70 + 14}{14} = 4032 \times 1.79 = 7217.$$

Solution (*b*): By Tables I and II.

By Table II the weight of a cubic foot of air at 14 lbs. and 60° is .07277, and .07277 × 2 = .14554. The absolute t is $460 + 60 = 520$, and $r = 6.0$.

Then in Table I, column 11, opposite $r = 6$ we find 95.271, whence

$$W = 95.271 \times 520 \times .14554 = 7208.$$

The difference in the two results is due to dropping off the fraction in temperature.

Art. 2. Temperature Varying.

The conditions are said to be *adiabatic* when, during compression or expansion, no heat is allowed to enter in, or escape from, the air although the temperature in the body of confined air changes radically during the process.

Physicists have proved that under adiabatic conditions the following relations hold:

$$\frac{p_1 v_1}{p_2 v_2} = \frac{t_1}{t_2}, \tag{3}$$

and since for one pound of air at 32° F. $pv = 26{,}214$ and $t = 492.6$, we get for one pound at any pressure, volume and temperature,

$$pv = 53.17\,t. \tag{4}$$

While formulas (3) and (4) are very important, they do not apply to the actual conditions under which compressed air is worked, for in practice we get neither isothermal nor adiabatic conditions but something intermediate.

For such conditions physicists have discovered that the following holds nearly true:

$$p_1 v_1^n = p_x v_x^n = p_2 v_2^n, \tag{5}$$

sub x indicating any intermediate stage and the exponent n varying between 1 and 1.41 according to the effectiveness of the cooling in case of compression or the heating in case of expansion. From this basic formula (5) the formulas for work must be derived.

As in Art. (1) $dW = p_x dv_x = p_1v_1^n \frac{dv_x}{v_x^n} = p_1v_1^n (v_x^{-n})\, dv_x$.

Therefore

$$W' = p_1v_1^n \int_{v_2}^{v_1} v_x^{-n} dv_x = p_1v_1^n\left(\frac{v_1^{1-n} - v_2^{1-n}}{1-n}\right) = p_1v_1^n\left(\frac{v_2^{1-n} - v_1^{1-n}}{n-1}\right).$$

Now since $p_1v_1^n \times v_2^{1-n} = p_2v_2^n \times v_2^{1-n} = p_2v_2$ and $p_1v_1^n v_1^{1-n} = p_1v_1$ the expression becomes

$$W' = \frac{p_2v_2 - p_1v_1}{n-1},$$

which represents the work done in compression or expansion between B and C, Fig. 1. To this must be added the work of expulsion, p_2v_2, and from it must be subtracted the work done by the air entering behind the piston, p_1v_1. Hence the whole net work done in one stroke is

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_1v_1 \qquad (6)$$

$$= \frac{n}{n-1}(p_2v_2 - p_1v_1). \qquad (7)$$

Equation (7) is in convenient working form and may be used when the data are in pressures and volumes, but it is common to express the compression or expansion in terms of r. For such cases a convenient working formula is gotten as follows:

From Eq. (5) $\quad p_2v_2 = \frac{p_1v_1 \times v_1^{n-1}}{v_2^{n-1}}.$

Also $\quad r = \frac{p_2}{p_1} = \frac{v_1^n}{v_2^n}$, therefore $\frac{v_1}{v_2} = r^{\frac{1}{n}}$,

and $\quad \frac{v_1^{n-1}}{v_2^{n-1}} = r^{\frac{n-1}{n}}$, therefore $p_2v_2 = p_1v_1r^{\frac{n-1}{n}}$,

and Eq. (7) becomes $W = \frac{n}{n-1} p_1v_1\left[r^{\frac{n-1}{n}} - 1\right]. \qquad (8)$

The most common uses of equations (7) and (8) are when air is compressed from free air conditions, then p_1 and v_1 become p_a and v_a. This case must be carefully distinguished from the case of incomplete expansion as presented in Art. 3.

In perfectly adiabatic conditions $n = 1.41$, but in practice the compressor cylinders are water-jacketed and thereby part of the heat of compression is conducted away, so that n is less than 1.41. For such cases Church assumes $n = 1.33$ and Unwin assumes $n = 1.25$. Undoubtedly the value varies with size and proportions of cylinders, details of water-jacketing, temperature of cooling water and speed of compressors. Hence precision in the value of n is not practicable. Fortunately the work does not vary as much as n does.

For one pound of air at initial temperature of 60° F. Eq. (8) gives in foot-pounds,

$$\text{When } n = 1.41,\ W = 95{,}193\ (r^{0.29} - 1). \qquad (8b)$$

$$\text{When } n = 1.25,\ W = 138{,}405\ (r^{0.2} - 1). \qquad (8c)$$

Common log of 95,193 = 4.978606.

Common log of 138,405 = 5.141141.

The above special values will be found convenient for approximate computations. For compound compression see Art. 12.

If in Eq. (8) we substitute for pv its value, $53.17\,t$, for one pound, we get

$$W = \left[\left(\frac{n}{n-1}\right) 53.17 \left(r^{\frac{n-1}{n}} - 1\right)\right] \times t = Kt, \qquad (8d)$$

where
$$k = \frac{n}{n-1} \times 53.17 \left(r^{\frac{n-1}{n}} - 1\right).$$

Table I gives values of K for $n = 1.25$ and $n = 1.41$ and for values of r up to 10, varying by one-tenth. The theoretic work in any case is $K \times Q \times t$, where Q is the number of pounds passed and t is the absolute lower temperature. Further explanation accompanies the table.

The difference between isothermal and adiabatic compression (and expansion) can be very clearly shown graphically as in Fig. 2. In this illustration the terminal points are correctly placed for a ratio of 5 for both the compression and expansion curve.

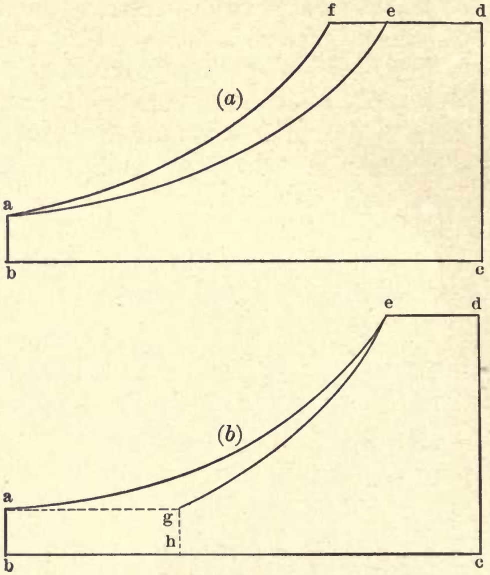

Fig. 2.

Note that in the compression diagram (*a*), the area between the two curves *aef* represents the work lost in compression due to heating, and the area between the two curves *aeghb* in (*b*) represents the work lost by cooling during expansion. The isothermal curve, *a e*, will be the same in the two cases.

Such illustrations can be readily adapted to show the effect of reheating before expansion, cooling before compression, heating during expansion, etc.

Example 2a. What horse power will be required to compress 1000 cubic feet of free air per minute from $p_a = 14.5$ to a gage pressure $= 80$, when $n = 1.25$ and initial temperature $= 50°$ F.?

Solution. From Table II, interpolating between 40° and 60° the weight of one cubic foot is .07686 and the weight of 1000 is 76.86 –. The r from above data is 6.5. Then in

Table I opposite $r = 6.5$ in column 9 we find .3658. Then

$$\text{Horse power} = .3658 \times \frac{76.86}{100} \times 510 = 143.$$

The student should check this result by Eq. (8) or (8d) without the aid of the table.

Art. 3. Incomplete Expansion.

When compressed air is applied in an engine as a motive power its economical use requires that it be used expansively in a manner similar to the use of steam. But it is never practicable to expand the air down to the free air pressure, for two reasons: — first, the increase of volume in the cylinders would increase both cost and friction more than could be balanced by the increase in power; and second, unless some means of reheating be provided, a high ratio of expansion of compressed air will cause a freezing of the moisture in and about the ports.

The ideal indicator diagram for incomplete expansion is shown in Fig. 3. In such diagrams it is convenient and simplifies the demonstrations to let the horizontal length represent volumes. In any cylinder the volumes are proportional to the length.

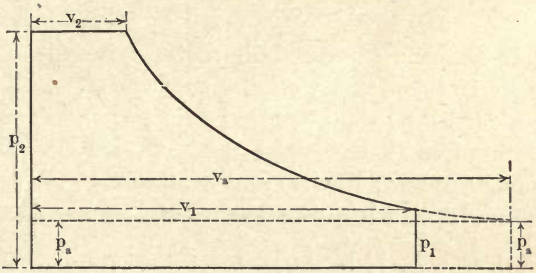

Fig. 3.

Air at pressure p_2 is admitted through that part of the stroke represented by v_2 — thence the air expands through the remainder of the stroke represented by v_1, the pressure dropping to p_1. At this point the exhaust port opens and the pressure drops to that of the free air. The dotted portion would be added to the diagram if the expansion should be carried down to free air pressure.

To write a formula for the work done by the air in such a case we will refer to Eq. (6) and its derivation. In the case of simple compression or complete expansion it is correctly written

$$W = \frac{p_2v_2 - p_av_a}{n-1} + p_2v_2 - p_av_a,$$

which would give work in the case represented by Fig. 1 when there is a change of temperature, but in such a case as is represented by Fig. 3 the equation must be modified thus:

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_av_1, \tag{9}$$

the reason being apparent on inspection.

In numerical problems under Eq. (9) there will be known p_2v_2, n, and either p_1 or v_1. The unknown must be computed from the relations from Eq. (5):

$$p_1 = p_2\left(\frac{v_2}{v_1}\right)^n \quad \text{or} \quad v_1 = v_2\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}.$$

Example 3a. A compressed-air motor takes air at a gage pressure = 100 lbs. and works with a cut-off at ¼ stroke. What work (ft.-lbs.) will be gotten per cu. ft. of compressed air, assuming free air pressure = 14.5 lbs. and $n = 1.41$?

Solution. Applying Eq. (9) and noting that all pressures are to be multiplied by 144 and that the pressure at end of stroke $= p_1 = 114.5\left(\frac{\frac{1}{4}}{1}\right)^{1.41} = 16.3$ and that $v_1 = 4\,v_2$, we get

$$W = 144\left(\frac{114.5\times1 - 16.3\times4}{.41} + 114.5\times1 - 14.5\times4\right) = 25{,}444.$$

Art. 4. Effect of Clearance: In Compression.

It is not practicable to discharge all of the air that is trapped in the cylinder; there are some pockets about the valves that the piston cannot enter, and the piston must not be allowed to strike the head of the cylinder. This clearance can usually be determined by measuring the water that can be let into the cylinder in front of the piston when at the end of its stroke; but the construction of each compressor must be studied before this can be undertaken intelligently, and it is not done with equal ease in all machines.

To formulate the effect of this clearance in the operation of the machine,

Let v = volume of piston displacement (= area of piston × length of stroke),

Let cv = clearance, c being a percentage.

Then $v + cv$ is the volume compressed each stroke. But the clearance volume cv will expand to a volume rcv as the piston recedes, so that the fresh air taken in at each stroke will be $v + cv - rcv$, and the volumetric efficiency will be

$$E_v = \frac{v + cv - rcv}{v} = 1 + c(1 - r). \qquad (10)$$

When $E_v = 0$ $c = \frac{1}{r - 1}$ and no air will be discharged.

Theoretically (as the word is usually used) clearance does not cause a loss of work, but practically it does, insomuch as it requires a larger machine, with its greater friction, to do a given amount of effective work.

Example 4a. A compressor cylinder is 12″ diam. × 16″ stroke. The clearance is found to hold $1\frac{1}{4}$ pints of water $= \frac{1.25}{8} \times 231 = 36$ cubic inches; therefore $c = \frac{36}{113 \times 16}$ $= 0.02.$

Piston area

Then by Eq. (10) when $r = 7$

$$E = 1 + 0.02(1 - 7) = 88\%.$$

Such a condition is not abnormal in small compressors, and the volumetric efficiency is further reduced by the heating of air during admission as considered in Art. 6.

Art. 5. Effect of Clearance and Compression in Expansion Engines.

Fig. 4 is an ideal indicator diagram illustrating the effect of clearance and compression in an expansion engine.

In this diagram the area E shows the effective work, D the effect of clearance, B the effect of back pressure of the atmosphere and C the effect of compression on the return stroke.

The study of effect of clearance in an expansion engine differs from the study of that in compression, due to the fact that the volume in the clearance space is exhausted into the atmosphere at the end of each stroke.

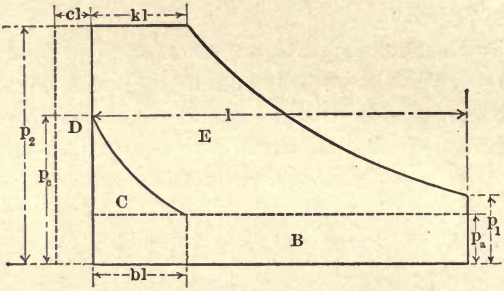

Fig. 4.

If the engine takes full pressure throughout the stroke the air (or steam) in the clearance is entirely wasted; but when the air is allowed to expand as illustrated in the diagram some useful work is gotten out of the air in the clearance during the expansion.

The loss due to clearance in such engine is modified by the amount of compression allowed in the back stroke. If the compression $p_c = p_2$, the loss of work due to clearance will be nothing, but the effective work of the engine will be considerably reduced, as will be apparent by a study of a diagram modified to conform to the assumption.

While the formula for work that includes the effect of clearance and compression will not be often used in practice its derivation is instructive and gives a clear insight into these effects.

The symbols are placed on the diagram and will not need further definition.

The effective work E will be gotten by subtracting from the whole area the separate areas B, C and D. From Art. 2, after making the proper substitutions for the volumes, there results

$$\text{Total area} = l\left[\frac{p_2(c+k) - p_1(1+c)}{n-1} + p_2(c+k)\right].$$

$$\text{Area } B = lp_a,$$
$$\text{Area } D = lp_2c,$$
$$\text{Area } C = l\left[\frac{p_cc - p_a(b+c)}{n-1} - p_ab\right].$$

Subtracting the last three from the first and reducing their results:

$$\frac{\text{Work}}{Al} = \frac{1}{n-1}[c(p_2+p_a-p_c-p_1)+n(p_2k+p_ab-p_a)-(p_1-p_a)]$$
$$= \text{Mean effective pressure.}$$

The actual volume ratio before and after expansion is

$$\frac{v_2}{v_1} = \frac{cv_1+kv_1}{cv_1+v_1} = \frac{c+k}{c+1}.$$

This is the ratio with which to enter Table I to get r and t and from r the unknown pressure p_1. Similarly for the compression curve the ratio of volumes is $\frac{c}{b}$, and p_c can be found as indicated above.

Art. 6. Effect of Heating Air as it Enters Cylinders. When a compressor is in operation all the metal exposed to the compressed air becomes hot even though the water jacketing is of the best. The entering air comes in contact with the admission valves, cylinder head and walls and the piston head and piston rod, and is thereby heated to a very considerable degree. In being so heated the volume is increased in direct proportion to the absolute temperature (see Eq. (5)), since the pressure may be assumed constant and equal that of the atmosphere. Hence a volume of cool free air less than the cylinder volume will fill it when heated. This condition is expressed by the ratio

$$\frac{v_a}{v_c} = \frac{t_a}{t_c} \quad \text{or} \quad v_a = v_c\frac{t_a}{t_c},$$

where v_c and t_c represent the cylinder volume and temperature. The volumetric efficiency as effected by the heating is

$$E_v = \frac{v_a}{v_c} = \frac{t_a}{t_c}.$$

Example 6a. Suppose in Example 4*a* the outside free air temperature is 60° F. and in entering the temperature rises to 160° F., then

$$\frac{t_a}{t_c} = \frac{460 + 60}{460 + 160} = 84 \text{ per cent.}$$

Then the final volumetric efficiency would be 88 × 84 = 74% nearly.

The volumetric efficiency of a compressor may be further reduced by leaky valves and piston.

In Arts. 4 and 6 it is made evident that the volumetric efficiency of an air compressor is a matter that cannot be neglected in any case where an installation is to be intelligently proportioned. It should be noted that the volumetric efficiency varies with the various makes and sizes of compressors and that the catalog volume rating is always based on the *piston displacement.*

These facts lead to the conclusion that much of the uncertainty of computations in compressed-air problems and the conflicting data recorded is due to the failure to determine the actual amount of air involved either in terms of net volume and temperature or in pounds.

Methods of determining volumetric efficiency of air compressors are given in Chapter III.

The loss of work due to the air heating as it enters the compressor cylinder is in direct proportion to the loss of volumetric efficiency due to this cause. In Example 6*a* only 84% of the work done on the air is effective.

By the same law any cooling of the air before entering the compressor effects a saving of power. See Art. 9.

Art. 7. Change of Temperature in Compression or Expansion.

Eq. (4) may be written

$$p_1v_1 = ct_1;\ p_2v_2 = ct_2$$

and Eq. (5) may be factored thus,

$$p_1v_1v_1^{n-1} = p_2v_2v_2^{n-1}.$$

Substituting we get

$$ct_1v_1^{n-1} = ct_2v_2^{n-1}.$$

Whence
$$t_2 = t_1\left(\frac{v_1}{v_2}\right)^{n-1} \qquad (11)$$

and
$$t_2 = t_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = t_1 r^{\frac{n-1}{n}}, \qquad (11a)$$

since from Eq. (5) $\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$.

It is possible to compute n from Eq. (11) by controlling the v_1 and v_2 and measured t_1 and t_2.

Table I, columns 5 and 6, is made up from Eq. (11a) and columns 3 and 4 from Eq. (5) as just written.

Example 7. What would be the temperature of air at the end of stroke when $r = 7$ and initial temperature $= 70° F.$?

Solution. Referring to Table I in line with $r = 7$ note that

$$\frac{t_2}{t_1} = \begin{cases} 1.4758 \text{ when } n = 1.25 \\ \qquad \therefore\ t_2 = (460 + 70) \times 1.4758 - 460 = 322° \text{ F.} \\ 1.7585 \text{ when } n = 1.41 \\ \qquad \therefore\ t_2 = (460 + 70) \times 1.7585 - 460 = 472° \text{ F.} \end{cases}$$

From the same table the volume of one cubic foot of free air when compressed and *still hot* would be respectively 0.21 and 0.25, while when the compressed air is cooled back to 70° its volume would be 0.143.

Art. 8. Density at Given Temperature and Pressure.

By Eq. (4) $pv = 53.17\,t$ for one pound, and the weight of one cubic foot

$$= w = \frac{1}{v} = \frac{p}{53.17\,t}. \qquad (12)$$

Note that p must be the absolute pressure in pounds per square foot, and t the absolute temperature. When gage pressures are used and ordinary Fahrenheit temperature the formula becomes

$$w = \frac{144}{53.17}\left(\frac{p_g + p_a}{460 + \text{F}}\right)$$
$$= 2.708\left(\frac{p_g + p_a}{460.6 + F}\right). \qquad (12a)$$

Table III is made up from Eq. (12).

Art. 9. Cooling Water Required.

In isothermal changes, since pv is constant, evidently there is no change in the *mechanical energy* in the body of air as measured by the *absolute* pressure and using the term "mechanical energy" to distinguish from heat energy. Hence evidently all the work delivered to the air from outside must be abstracted from the air in some other form, and we find it in the heat absorbed by the cooling water. Therefore,

$$\frac{pv \log_e r}{780} = \text{(B.T.U's)}$$

of work done on compressed air $= 35.5 \log r$ (B.T.U's) per pound of air compressed from temperature of 60° F. If the water is to have a rise of temperature $T°$ (T being small, else the assumption of isothermal changes will not hold), then

$$\frac{pv \log_e r}{780\ T} = \text{Pounds of water required in same time.}$$

Example 8a. How many cubic feet of water per minute will be required to cool 1000 cubic feet of free air per minute, air compressed from $p_a = 14.2$ to $p_g = 90°$ gage, initial temperature of air $= 50°$ F. and rise in temperature of cooling water $= 25°$?

Solution:

$$\frac{144 \times 14.2 \times 1000 \times \log_e\left(\dfrac{90+14.2}{14.2}\right)}{780 \times 25 \times 62.5} = 24 \text{ cu. ft. per min.}$$

It is practically possible to attain nearly isothermal conditions by spraying cool water into the cylinder during compression. In such a case this article would enable the designer to compute the quantity of water necessary and therefrom the sizes of pipes, pumps, valves, etc.

Art. 10. Reheating and Cooling.

In any two cases of change of state of a given weight of air, assuming the ratio of change in pressure to be the same, the work done (in compression or expansion) will be directly proportional to the volume, as will be evident by examination of the formulas for work. Also at any given pressure the volumes will be directly proportional to the absolute temperatures. Hence the work done either in compression or expansion (ratio of change in pressures being the same in each case) will be directly proportional to the absolute initial temperatures.

Thus if the temperature of the air in the intake end of one compressor is 160° F. and, in another 50° F., the work done on equal weights of air in the two cases will be in the proportion of 460 + 150 to 460 + 50, or 1.2 to 1; that is, the work in the first case is 20% more than that in the second case. This is equally true, of course, for expansion.

The facts above stated reveal a possible and quite practicable means of great economy of power in compressing air and in using compressed air.

The opportunities for economy by cooling for compression are not as good as in heating before the application in a motor, but even in compression marked economy can be gotten at almost no cost by admitting air to the compressor from the coolest convenient source, and by the most thorough water-jacketing with the coolest water that can be conveniently obtained.

In all properly designed compressor installations the air is supplied to the machine through a pipe from outside the building to avoid the warm air of the engine room. In winter the difference in temperature may exceed 100°, and this simple device would reduce the work of compression by about 20%. For the effect of intercoolers and interheaters see Art. 12 on compounding.

By reheating before admitting air to a compressed-air engine of any kind the possibilities of effecting economy of power are greater than in cooling for compression, the reason being that heating devices are simpler and less costly than any means of cooling other than those cited above.

The compressed air passing to an engine can be heated to any desired temperature; the only limit is that temperature that will destroy the lubrication. Suppose the normal temperature of the air in the pipe system is 60° F. and that this is heated to 300° F. before entering the air engine, then the power is increased 46%. Reheating has the further advantage that it makes possible a greater ratio of expansion without the temperature reaching freezing point.

The devices for reheating are usually a coil or cluster of pipes through which the air passes while the pipe is exposed to the heat of combustion from outside. Ordinary steam boilers may be used, the air taking the place of the steam and water.

Unwin suggests reheating the air by burning the fuel *in the compressed air* as suggested in the cut.

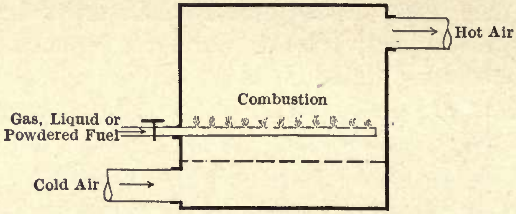

Fig. 4 A.

Even when the details are worked out such a device would be simple and inexpensive. The theoretic advantages of such a device are that all the heat would go into the air, the gases of combustion (if solid or liquid fuel be used) would increase the volume, and the combustion occurring in compressed air would be very complete.

The author has no knowledge of any such devices having been used in practice.

The power efficiency of the fuel used in reheaters is very much greater than that of the fuel used in steam boilers. Unwin states that it is five or six times as much. The chief reason is that none of the heat is absorbed in evaporation as in a steam boiler.

In many of the applications of compressed air reheating is impracticable, and efficiency is secondary to convenience — but in large fixed installations, such as mine pumps, reheating should be applied.

Art. 11. Compounding.

In steam-engine designs compounding is resorted to to economize power by saving steam, while in air compressors and compressed-air engines compounding is resorted to for the twofold purpose of economizing power and controlling temperature, both objects being accomplished by reducing the extreme change of temperature. The economic principles involved in compound steam engines and in compound air engines are quite different, the reasons underlying the latter being much more definite.

The air is first compressed to a moderate ratio in the low-pressure cylinder, whence it is discharged into the "intercooler," where most of the heat developed in the first stage is absorbed and thereby the volume materially reduced, so that in the second stage there will be less volume to compress and a less injurious temperature.

The changes occurring and the manner in which economy is effected in compression may be most easily understood by reference to Fig. 5, which represents ideal indicator diagrams from the two cylinders, superimposed one over the other, the scale being the same in each, the dividing line being *kb*.

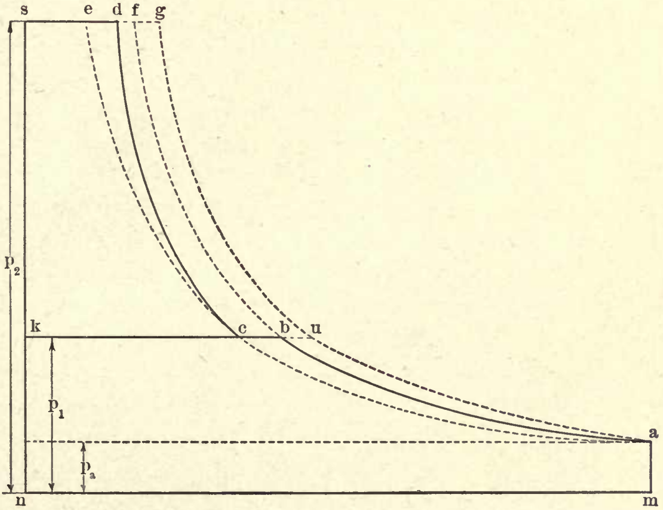

Fig. 5.

In this diagram, Fig. 5,

abk is the compression line in the first-stage or low-pressure cylinder,

cds is the compression line in the second-stage or high-pressure cylinder,

bc is the reduction of volume in the intercooler,

abf would be the pressure line if no intercooling occurred,
The area *cdfb* is the work saved by the intercooler,
ace would be the compression line for isothermal compression,
aug would be the compression line for adiabatic compression.

The diagram Fig. 5 is correctly proportioned for $r = 6$.

Fig. 6 is a diagram drawn in a manner similar to that used in Fig. 5 and is to illustrate the changes and economy effected by compounding with heating when compressed air is applied in an engine. It is assumed that the air is "preheated," that is, heated once before entering the high-pressure cylinder and again heated between the two cylinders.

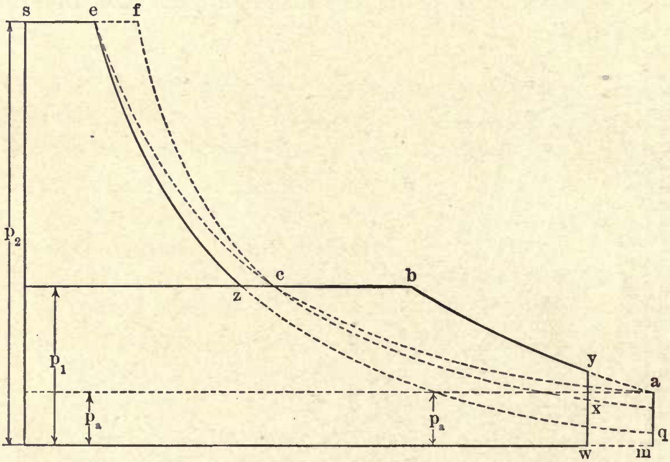

Fig. 6.

In this diagram, Fig. 6,

se is the volume of compressed air at normal temperature,
sf is the volume of compressed air after preheating,
fc is the expansion line in the high-pressure cylinder,
cb is the increase of volume in the interheater,
by is the expansion line in low-pressure cylinder,
ezq would be the adiabatic expansion line without any heating,
efcz is work gained by preheating,
cbyx is work gained by interheating.

In no case is it economical to expand down to atmospheric pressure. Hence the diagram is shown cut off with pressure still above that of free air.

The diagram Fig. 6 is proportioned for preheating and reheating 300° F.

Art. 12. Proportions for Compounding.

It is desirable that equal work be done in each stage of compounding. If this condition be imposed, Eq. (8) indicates that the r must be the same in each stage, for on the assumption of complete intercooling the product pv will be the same at the beginning of each stage.

If then r_1 be the ratio of compression in the first stage, the pressure at end of first stage will be $r_1 p_a = p_1$, and the pressure at end of second stage $= r_1 p_1 = r_1^2 p_a = p_2$, and similarly at end of third stage the pressure will be $p_3 = r_1^3 \, p_a$, or

In two-stage work $r_1 = \left(\frac{p_2}{p_a}\right)^{\frac{1}{2}} = r_2^{\frac{1}{2}}$.

In three-stage work $r_1 = \left(\frac{p_3}{p_a}\right)^{\frac{1}{3}} = r_3^{\frac{1}{3}}$.

Let v_1 = free air intake per stroke in low-pressure cylinder or first stage,

v_2 = piston displacement in second stage,

v_3 = piston displacement in third stage,

r_1 = ratio of compression in each cylinder.

Then, assuming complete intercooling,

$$v_2 = \frac{v_1}{r_1} \quad \text{and} \quad v_3 = \frac{v_2}{r_1} = \frac{v_1}{r_1^2},$$

or

$$\frac{v_2}{v_1} = \frac{1}{r_1} \quad \text{and} \quad \frac{v_3}{v_1} = \frac{1}{r_1^2}.$$

The length of stroke will be the same in each cylinder; therefore the volumes are in the ratio of the squares of diameters, or

$$\frac{d_2^2}{d_1^2} = \frac{1}{r_1} \quad \text{and} \quad \frac{d_3^2}{d_1^2} = \frac{1}{r_1^2}.$$

Hence

$$d_2 = \frac{d_1}{r_1^{\frac{1}{2}}} \quad \text{and} \quad d_3 = \frac{d_1}{r_1}. \qquad (12b)$$

If the intention to make the work equal in the different cylinders be strictly carried out it will be necessary to make the first-stage cylinder enough larger to counteract the effect of volumetric efficiency. Thus if volumetric efficiency be 75%, the volume (or area) of the intake cylinder should be one-third larger. Note that the volumetric efficiency is chargeable entirely to the intake or low-pressure cylinder. Once the air is caught in that cylinder it must go on.

Example 12. Proportion the cylinders of a compound two-stage compressor to deliver 300 cu. ft. of free air per minute at a gage pressure = 150. Free air pressure = 14.0, R.P.M. = 100, stroke 18″, piston rod $1\frac{3}{4}''$ diameter, volumetric efficiency = 75%.

Solution. From the above data the net intake must be 3 cu. ft. per revolution. Add to this the volume of one piston rod stroke (= .025 cu. ft.) and divide by 2. This gives the volume of one piston stroke 1.512. The volume of one foot of the cylinder will be $\frac{12}{18} \times 1.512 = 1.008$ cu. ft. From Table X the nearest cylinder is 14″ diam., the total ratio of compression $= \frac{150 + 14}{14} = 11.71$, and the ratio in each stage is $(11.71)^{\frac{1}{2}} = 3.7 = r_1$, and by (12b)

$$d_2 = \frac{d_1}{(r_1)^{\frac{1}{2}}} = \frac{14}{1.92} = 7.3'', \text{say } 7\tfrac{3}{8}'', \text{for the high-pressure cylinder.}$$

Now we must increase the low-pressure cylinder by one-third to allow for volumetric efficiency. The volume per foot will then be 1.344, which will require a cylinder about $15\frac{5}{8}''$ diameter. Note that the diameter of the high-pressure cylinder will not be affected by the volumetric efficiency.

Art. 13. Work in Compound Compression.

Assuming that the work is the same in each stage, Eq. (8) can be adapted to the case of multistage compression thus: —

In two-stage work

$$W = \frac{n}{n-1} p_a v_a \left(r_1^{\frac{n-1}{n}} - 1 \right) \times 2 \qquad (13)$$

$$= \frac{n}{n-1} p_a v_a \left(r_2^{\frac{n-1}{2n}} - 1 \right) \times 2. \qquad (13a)$$

In three-stage work

$$W = \frac{n}{n-1} p_a v_a \left(r_1^{\frac{n-1}{n}} - 1 \right) \times 3 \tag{13b}$$

$$= \frac{n}{n-1} p_a v_a \left(r_3^{\frac{n-1}{3n}} - 1 \right) \times 3. \tag{13c}$$

Note that $r_2 = \frac{p_2}{p_a}$ and $r_3 = \frac{p_3}{p_a}$ and also that $p_a v_a = p_1 v_1 = p_2 v_2$, etc., assuming complete intercooling.

Laborious precision in computing the work done on or by compressed air is useless, for there are many uncertain and changing factors: n is always uncertain and changes with the amount and temperature of the jacket water, the volumetric efficiency, or actual amount of air compressed, is usually unknown, the value of p_a varies with the altitude, and r is dependent on p_a.

Art. 14. Work under Variable Intake Pressure.

There are some cases where air compressors operate on air in which the intake pressure varies and the delivery pressure is constant. This is true in case of exhaust pumps taking air out of some closed vessels and delivering it into the atmosphere. It is also the condition in the "return-air" pumping system in which one charge of air is alternately forced into a tank to drive the water out and then exhausted from the tank to admit water. For full mathematical discussion of this pump see Trans. Am. So. C. E., Vol. 54, p. 19. The following formulas and others more complex were first worked out to apply to that pumping system.

In such cases it is necessary to determine the maximum rate of work in order to design the motive power.

First assume the operation as being isothermal. Then in Eq. (1), viz.

$$W = p_x v \log_e \frac{p_1}{p_x}$$

p_x is variable, while v and p_1 are constant. In this formula W becomes zero when p_x is zero and again when $p_x = p_1$, since log 1 is zero. To find when the work is maximum, differentiate and equate to zero; thus differential of

$$v\,(p_x \log p_1 - p_x \log p_x) = v\left[\log p_1 dp_x - \left(p_x \frac{dp_x}{p_x} + \log p_x dp_x.\right)\right]$$

Equate this to zero and get $\log p_1 = 1 + \log p_x$,

or $\log \dfrac{p_1}{p_x} = 1$, therefore $\dfrac{p_1}{p_x} = e = 2.72$.

That is, when $r = 2.72$ the work is a maximum.

When the temperature exponent n is to be considered the study must be made in Eq. (8), viz.

$$W = \frac{n}{n-1}\, p_x v \left[\left(\frac{p_1}{p_x}\right)^{\frac{n-1}{n}} - 1\right]. \tag{8}$$

Differentiating this with respect to p_x and equating to zero, the condition for maximum work becomes $\left(\dfrac{p_1}{p_x}\right)^{\frac{n-1}{n}} = n$. Insert this in (8) and the reduced formula becomes

$$W = n\, p_x v. = \frac{p_1 v}{n^{\frac{1}{n-1}}}$$

From the above expressions for maximum the following results:

When $n = 1.41$ the maximum occurs when $r = 3.26$.
When $n = 1.25$ the maximum occurs when $r = 3.05$.
When $n = 1.$ the maximum occurs when $r = 2.72$.

In practice $r = 3$ will be a safe and convenient rule.

Exercise 14*a*. Air is being exhausted out of a tank by an exhaust pump with capacity = 1 cu. ft. per stroke. At the beginning the pressure in the tank is that of the atmosphere = 14.7 lbs. per sq. in. Assume the pressure to drop by intervals of one pound and plot the curve of work with p_x as the horizontal ordinate and W as the vertical, using the formula $W = p_x v \log \dfrac{p_a}{p_x}$.

Exercise 14*b*. As in 14*a* plot the curve by Eq. (8) with $n = 1.25$.

Art. 15. Exhaust Pumps.

In designing exhaust pumps the following problems may arise.

Given a closed tank and pipe system of volume V under pressure p_0 and an exhaust pump of stroke volume v, how many strokes will be necessary to bring the pressure down to p_m?

The analytic solution is as follows, assuming isothermal conditions in the volume V.

The initial product of pressure by volume is p_0V. After the first stroke of the exhaust pump this air has expanded into the cylinder of the pump and pressure has dropped to p_1 under the law that pressure by volume is constant.

Hence $(V+v)\,p_1 = p_0V$, or $p_1 = \dfrac{p_0V}{V+v}$ at end of first stroke,

$$(V+v)\,p_2 = p_1V, \text{ or } p_2 = \frac{p_1V}{V+v} = p_0\left(\frac{V}{V+v}\right)^2$$

at end of second stroke,

$$(V+v)\,p_3 = p_2V, \qquad \text{or} \qquad p_3 = p_2\frac{V}{V+v} = p_0\left(\frac{V}{V+v}\right)^3$$

at end of third stroke, etc.

$$\text{Finally} \qquad p_m = p_0\left(\frac{V}{V+v}\right)^m \quad \text{and} \quad m = \frac{\log \dfrac{p_m}{p_0}}{\log\left(\dfrac{V}{V+v}\right)}. \qquad (14)$$

m is the required number of strokes.

Example 15a. A closed tank containing 100 cu. ft. of air at atmospheric pressure (= 14.5 lbs. per sq. in.) is to be exhausted down to 5 lbs. by a pump making 1 cu. ft. per stroke. How many strokes required?

Solution. $\dfrac{p_m}{p_0} = \dfrac{5}{14.5}$ and $\dfrac{V}{V+v} = \dfrac{100}{101}$.

$$\begin{array}{rl} \log 5 = & 0.69897 \\ \log 14.5 = & \underline{1.16136} \\ & \bar{1}.53761 \end{array} \qquad \begin{array}{rl} \log 100 = & 2.00000 \\ \log 101 = & \underline{2.00432} \\ & \bar{1}.99568 \end{array}$$

These two logarithms may be written thus:

$$\begin{array}{l} -1 + 0.53761 = -\ .46239 \\ -1 + 0.99568 = -\ .00432 \end{array} \text{ and } \frac{.46239}{.00432} = 107 = m.$$

If the volumetric efficiency of the machine be E, then the number of strokes would be $107 \div E$.

The results found under Arts. 14 and 15 serve well to illustrate the curious mathematical gymnastics that compressed air is subject to, and should encourage the investigator who likes such work, and should put the designer on guard.

Art. 16. Efficiency when Air is Used without Expansion.

In many applications of compressed air convenience and safety are the prime requisites, so that power efficiency receives little attention at the place of application. This is so with such apparatus as rock drills, pneumatic hammers, air hoists and the like. The economy of such devices is so great in replacing human labor that the cost in power is little thought of. Further, the necessity of simplicity and portability in such apparatus would bar the complications needed to use the air expansively. There are other cases, however, notably in pumping engines and devices of various kinds, where the plant is fixed, the consumption of air considerable and the work continuous, where neglect to work the air expansively may not be justified.

In any case the designer or purchaser of a compressed-air plant should know what is the sacrifice for simplicity or low first cost when the proposition is to use the air at full pressure throughout the stroke and then exhaust the cylinder full of compressed air.

Let p be the absolute pressure on the driving side of the piston and p_a be that of the atmosphere on the side next the exhaust. Then the effective pressure is $p - p_a$ and the effective work is $(p - p_a)\,v$, while the least possible work required to produce this air is $pv \log_e r$.

Hence the efficiency is $E = \dfrac{(p - p_a)\,v}{pv \log_e r}$.

Dividing numerator and denominator by $p_a v$ this reduces to

$$E = \frac{r - 1}{r \log_e r}. \qquad (15)$$

This is the absolute limit to the efficiency when air is used without expansion and without reheating. It cannot be reached in practice.

Table VI represents this formula. Note that the efficiency decreases as r increases. Hence it may be justifiable to use low-pressure air without expansion when it would not be if the air must be used at high pressure.

Clearance in a machine of this kind is just that much compressed air wasted. If clearance be considered, Eq. (15) becomes

$$E = \frac{r-1}{(1+c)\, r \log_e r} \qquad (15a),$$

where c is the percentage of clearance. In some machines, if this loss were a visible leak, it would not be tolerated.

Art. 17. Variation of Atmospheric Pressure with Altitude.

In most of the formulas relating to compressed-air operations the pressure p_a, or weight w_a, of free air is a factor. This factor varies slightly at any fixed place, as indicated by barometer readings, and it varies materially with varying elevations.

To be precise in computations of work or of weights or volumes of air moved, the factors p_a and w_a should be determined for each experiment or test, but such precision is seldom warranted further than to get the value of p_a for the particular locality for ordinary atmospheric conditions. This however should always be done. It is a simple matter and does not increase the labor of computation. In many plants in the elevated region p_a may be less than 14.0 lbs. per sq. in., and to assume it 14.7 would involve an error of more than 5%.

Direct reading of a barometer is the easiest and usual way of getting atmospheric pressure, but barometers of the aneroid class should be used with caution. Some are found quite reliable, but others are not. In any case they should be checked by comparison with a mercurial barometer as frequently as possible.

If m be the barometer reading in inches of mercury and F be the temperature (Fahrenheit), the pressure in pounds per sq. in. is

$$p_a = \frac{14.794}{30}\, m\, [1 - .0001\,(F - 32)]$$

$$= .4931\, m\, [1 - .0001\,(F - 32)]. \qquad (16)$$

The information in Table II will usually obviate the need of using Eq. (16).

In case the elevation is known and no barometer available the problem can be solved as follows:

Let p_s = pressure of air at sea level,
w_s = weight of air at sea level,
p_x, w_x be like quantities for any other elevation.

Then in any vertical prism of unit area and height dh we have

$$p_x + dp_x = p_x + w_x dh,$$

or

$$dp_x = w_x dh.$$

But

$$\frac{w_x}{w_s} = \frac{p_x}{p_s}; \text{ therefore } dp_x = \frac{w_s}{p_s} p_x dh,$$

or

$$dh = \frac{p_s}{w_s}\frac{dp_x}{p_x}, \text{ and therefrom } h = \frac{p_s}{w_s} \times \log \frac{p_s}{p_a},$$

where p_a is the pressure at elevation h above sea level. Substitute for w_s its equivalent

$$w_s = \frac{p_s}{53.17\,t} \quad \text{and we get} \quad \frac{h}{53.17\,t} = \log \frac{p_s}{p_a}.$$

Whence

$$\log_e p_a = \log_e p_s - \frac{h}{53.17\,t}.$$

Making $p_s = 14.745$ and adopting to common logarithm and Fahrenheit temperatures,

$$\log_{10} p_a = 1.16866 - \frac{h}{122.4\,(T + 460)}. \qquad (17)$$

Table V is made up by formula 17.

CHAPTER II

MEASUREMENT OF AIR

Art. 18. General Discussion.

Progress in the science of compressed-air production and application has evidently been hindered by a lack of accurate data as to the amount of compressed air produced and used.

The custom is almost universal of basing computations on, and of recording results as based on, catalog rating of compressor volumes — that is, on piston displacement.

The evil would not be so great if all compressors had about the same volumetric efficiency, but it is a fact that the volumetric efficiency varies from 60 per cent to 90 per cent, depending on the make, size, condition and speed of the machine; no wonder, then, that calculations often go wrong and results seem to be inconsistent.

There are problems in compressed-air transmission and use for the solution of which accurate knowledge of the volume or weight of air passing is absolutely necessary. Chief among these are the determination of friction factors in air pipes and the efficiency of pumps, including air lifts.

Purchasers may be imposed upon, and no doubt often are, in the purchase of compressors with abnormally low volumetric efficiencies. Contracts for important air-compressor installation should set a minimum limit for the volumetric efficiency, and the ordinary mechanical engineer should have knowledge and means sufficient to test the plant when installed.

There is little difficulty in the measurement of air. The calculations are a little more technical, but the apparatus is as simple and the work much less disagreeable than in measurements of water.

In many text-books theoretic formulas are presented for the flow of air at high pressures through orifices into the atmosphere. Such formulas are complicated by the necessity of considering change of volume and temperature, and even where the proper empirical coefficients are found the formulas are unwieldy.

Art. 19. Apparatus for Measuring Air by Orifice.

Present indications are that the standard method of determining flow of air will require the pressure to be reduced to less than one foot head of water in order that change of volume and temperature may be neglected and the formula simplified thereby.

Experiments under such circumstances show coefficients even more constant than those for standard orifices for measuring water. The coefficients given in Art. 20 were determined at McGill University by methods and apparatus described first in Trans. Am. So. Mech. E., Vol. 27, Dec., 1905, and later in *Compressed Air*, Sept., 1906, p. 4187.

Having the coefficients once determined, the necessary apparatus is simple and inexpensive. The essentials are shown in Fig. 7.

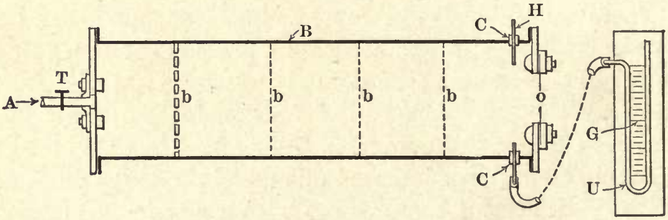

Fig. 7.

A = Compressed-air pipe,
B = Closed box or cylinder.
T = Throttle,
b = Baffle boards or screen,
H = Thermometer,
C = Cork,
O = Orifice in thin metal plate (Standard),
U = Bent glass tube containing colored water,
G = Scale of inches.

The box B may be made of any convenient light material. The pressure is only a few ounces and the tendency to leak slight. The purpose of the throttle T is to control the pressure against which the compressor works. The appropriate orifice O can be determined by a preliminary computation, assuming i at say 5″. See Eq. (18).

In testing a compressor it should be run until every part is at its normal running temperature. By means of the throttle T the compressor can be worked under various pressures and speed and thereby its individual curves of volumetric efficiency obtained.

Art. 20. Formula for Standard Orifice under Low Pressure.

Let p_a = air pressure in lbs. per sq. in. inside the box,
Q = weight of air passing per second,
w_a = weight of a cubic foot of air in pounds,
d = diameter of orifice in inches,
i = pressure as read on water gage in inches,
t = absolute temperature Fahrenheit's scale, inside box,
c = the experimental coefficient.

Where changes of density and temperature can be neglected the theoretic velocity through the orifice is $v = \sqrt{2\,gh}$ where h is the head of air of uniform density that would produce the pressure head i.

Hence $h = \frac{i}{12} \times \frac{62.5}{w_a}$; therefore $v = \sqrt{2\,g\frac{i}{12} \times \frac{62.5}{w_a}}$.

But $Q = w_a \times av$ where a = area of orifice in sq. ft. $= \frac{\pi d^2}{4 \times 144}$.

Inserting these values and putting w_a under the radical there results

$$Q = \frac{\pi d^2}{4 \times 144}\sqrt{2\,g\frac{i}{12}\,\frac{62.5\,w_a{}^2}{w_a}}.$$

But $$w_a = \frac{p_a'}{53.17\,t}.$$ $p_a' = 144 \times p_a$

Therefore $$Q = .0136 d^2 \sqrt{\frac{i}{t}\,p_a'} = .1632\,d^2 \sqrt{\frac{i}{t}\,p_a} \qquad (18)$$

where p_a is in lbs. per sq. in.

The pressure due to i (= .036 i) should be included in p_a. If the work is at sea level and pressure i be neglected, $p_a = 14.7 \times 144$ and the formula becomes

$$Q = .6299\, d^2 \sqrt{\frac{i}{t}}, \tag{18a}$$

which is the formula published by McGill University.

This is the theoretic formula. To it must be applied the experimental coefficient c as given in Table VIII. Note that c varies but little from 0.60, and the same c can be used in Eq. (18) and (18a).

Example 20a. In a run with the apparatus shown in Fig. 7 the following was recorded: $d = 2.32''$; $i = 4.6''$; $T = 186°$ F. inside drum, $T = 86°$ F. in free air. Elevation 1200′. Find the weight and volume of free air passing.

Solution. From Table II, interpolating for 86° in the line with 1200 elevation we get $w_a = .0700$ and p_a for free air $= 14.1$. Add the pressure due to i ($= .036 \times 4.6$) and we get the corrected $p_a = 14.26$. In Table VIII the coefficient for $d = 2.32$ and $i = 4.6$ is 0.599. These numbers inserted in (18) give

$$Q = .599 \times .1632 \times (2.32)^2 \sqrt{\frac{4.6}{646} \times 14.26}$$
$$= .1684 \text{ pound per second}$$

and the free air volume

$$= \frac{.1684}{.0700} \times 60 = 144.3 \text{ cu. ft. per minute.}$$

By Eq. (18a) $Q = .1747$.

Art. 21. Air Measurement in Tanks.

The amount of air put into or taken out of a closed tank or system of tanks and pipes, of known volume, can be accurately determined by Eq. (3), viz.,

$$\frac{p_a v_a}{p_x v_x} = \frac{t_a}{t_x} \quad \text{or} \quad v_a = \frac{p_x t_a}{p_a} \frac{v_x}{t_x}.$$

By this means the volume of air delivered into a closed system by a compressor can be very accurately determined.

The process would be as follows: Determine the volumes of all tanks, pipes, etc., to be included in the closed system, open all to free air and observe the free-air temperature; then switch the delivery from the compressor into the closed system; count the strokes of the compressor until the pressure is as high as desired; then shut off the closed tank and note pressure and temperatures of each separate part of the volume. Then the formula above will give the volume of free air which compressed and heated would occupy the tanks. From this subtract the volume of free air originally in the tanks; the remainder will be what the compressor has delivered into the system. Note that the compressor should be running hot and at normal speed and pressure when the test is made for its volumetric efficiency.

Usually the temperature changes will be considerable, but if the system is tight, time can be given for the temperature to come back to that of the atmosphere, thus saving the necessity of any temperature observations.

Where a convenient closed-tank system is available this method is recommended.

This method — that is, Eq. (3) as stated above — was used to determine the quantity of air passing the orifices in the experiments by which the coefficients were determined as given in Art. 20, Table VII.

Example 21a. A tank system consists of one receiver 3′ diam. × 12′, one air pipe 6″ × 40′, one 4″ × 4000′ and a second receiver at end of pipe 2′ diam. × 8′. A compressor 12″ × 18″ with $1\frac{1}{2}''$ piston rod puts the air from 1250 revolutions into the system, after which the pressure is 80 gage and temperature in first receiver 200°, while in other parts of the tank system it is 60°. Temperature of outside air being 50°, $p_a = 14.5$ per sq. in. Find volumetric efficiency of the compressor.

Solution. Volumes (from Table X):

1st receiver	84.84 cu. ft.	
6″ pipe	7.84	382.16
4″ pipe	349.20	
2nd receiver	25.12	
Total	467.00 in tank system.	

Piston displacement in one revolution = 2.338 cu. ft. (piston rod deducted).

By formula $v_a = \left(\frac{p_x t_a}{p_a}\right) \times \frac{v}{t_x}$ note that the quantity in parenthesis is constant and therefore a slide rule can be conveniently used, otherwise work by logarithms.

v_a in first receiver $= \frac{(80+14.5)(460+50)}{14.5} \times \frac{84.84}{460+200} = 417.2$

v_a in 6″ pipe, 4″ pipe and second receiver with total volume 382.16 and $t = 60°$ = ... 2447.1

Total.................................. 2864.3

Original volume of free air...................... 467

Volume of free air added................. 2397.3

$$2397.3 \div 2.338 = 1028.$$

Therefore the volumetric efficiency is

$$E = 1028 \div 1250 = 82\%.$$

CHAPTER III

FRICTION IN AIR PIPES

Art. 22. In the literature on compressed air many formulas can be found that are intended to give the friction in air pipes in some form. Some of these formulas are, by evidence on their face, unreliable, as for instance when no density factor appears; the origin of others cannot be traced and others are in inconvenient form. Tables claiming to give friction loss in air pipes are conflicting, and reliable experimental data relating to the subject are quite limited.

In this article and the next are presented the derivation of rational formulas for friction in air pipes with full exposition of the assumptions on which they are based. The coefficients were gotten from the data collected in Appendix B.

Art. 23. The Formula for Practice.

The first investigation will be based on the assumption that volume, density and temperature remain constant throughout the pipe.

Evidently these assumptions are never correct; for any decrease in pressure is accompanied by a corresponding increase in volume even if temperature is constant. (The assumption of constant temperature is always permissible.) However, it is believed that the error involved in these assumptions will be less than other unavoidable inaccuracies involved in such computations.

Let f = lost pressure in pounds per sq. in.,
l = length of pipe in feet,
d = diameter of pipe in inches,
s = velocity of air in pipe in feet per second,
r = ratio of compression in atmospheres,
c = an empirical coefficient including all constants.

Experiments have proved that fluid friction varies very nearly with the square of the velocity and directly with the density. Hence if k be the force in pounds necessary to force atmospheric air ($r = 1$) over one square foot of surface at a velocity of one foot per second, then at any other velocity and ratio of compression the force will be

$$F_1 = ks^2r,$$

and the force necessary to force the air over the whole interior of a pipe will be

$$F = \frac{\pi d}{12} l \times krs^2,$$

and the work done per second, being force multiplied by distance, is

$$\text{Work} = \frac{\pi dl}{12} \times krs^3.$$

Now if the pressure at entrance to the pipe is f pounds per sq. in. greater than at the other end, the work per second due to this difference (neglecting work of expansion in air) is

$$\text{Work} = f\,\frac{\pi d^2}{4}\,s.$$

Equating these two expressions for work there results

$$f\,\frac{\pi d^2}{4}\,s = \frac{\pi d}{12}\,lkrs^3,$$

or
$$f = \frac{4}{12}\,k\,\frac{l}{d}\,rs^2. \tag{19}$$

Now the volume of compressed air, v, passing through the pipe is, in cubic feet,

$$v = \frac{\pi d^2}{4 \times 144}\,s$$

and the volume of free air, v_a, is rv.

Therefore
$$v_a = \frac{\pi d^2}{4 \times 144} \times rs$$

and
$$s^2 = \frac{(4 \times 144)^2 v_a^2}{\pi^2 d^4 r^2}.$$

Insert this value of s^2 in Eq. (19) and reduce and there results

$$f = \frac{4}{12} k \left(\frac{4 \times 144}{\pi}\right)^2 \frac{l}{d^5} \frac{v_a^2}{r},$$

or

$$f = c \frac{l}{d^5} \frac{v_a^2}{r}, \tag{20}$$

where c is the experimental coefficient and includes all constants.

From Eq. (20),

$$d = \left(\frac{clv_a^2}{fr}\right)^{\frac{1}{5}}. \tag{21}$$

From the data collected in Appendix B the following results were computed. In this r and s are mean results and c is the average of all the runs made on each pipe.

d	c	r	s	t
½	.092	2.4 to 8.0	29 to 70	60° F.
¾	.076	1.5 to 10.2	35 to 100	100
1	.084	1.3 to 10.8	10 to 50	80
2	.080	2.0 to 10.6	5 to 28	80
3	.072	4	12 to 100	60
4	.066	7	28	35
5	.057	5	30	86
6	.066	4.5	33	70
8	.061	4.5	20	70
12	.047	7.5	20	....

An examination of the data in Appendix B shows that the coefficient c is independent of r and of s. If it is affected by the temperature it cannot be detected in these data. In relation to the diameters c evidently increases as the diameter decreases. A plot of diameters and c on coördinate paper gives a straight line and reveals the relation $c = .0866 - .0033\,d$ as most nearly averaging the results. This gives the following values for c*:

Diameters	½	1	1½	2	2½	3	4	5	6	8	10	12
Coefficients	.085	.083	.081	.080	.079	.078	.073	.070	.067	.060	.053	.047

Formulas (20) and (21) would be theoretically a little more accurate if v_a were expressed in terms of the actual weight of air passing. This would involve the observed free air pressure and temperature at the time considered. Such a

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* See Appendix C, page 122.

modification renders the formula much more laborious and would probably add nothing to its value for practical purposes.

Table IX and Plates 0, I, II, III, and IV are based on formula (20).

Art. 24. Theoretically Correct Friction Formula.

The theoretically correct formula for friction in air pipes must involve the work done in expansion while the pressure is dropping.

Let p_1 and p_2 be the absolute pressures at entrance and discharge of the pipe respectively and let Q be the total weight of air passing per second.

Then the total energy in the air at entrance is

$$p_a v_a \log \frac{p_1}{p_a} + \frac{Q s_1^2}{2g}$$

and at discharge the energy is

$$p_a v_a \log \frac{p_2}{p_a} + \frac{Q s_2^2}{2g}.$$

The difference in these two values must have been absorbed in friction in the pipe. Hence, using the expression for work done in friction that was derived in Art. 23, we get

$$\frac{\pi d}{12} lkrs^3 = p_a v_a \left(\log \frac{p_1}{p_a} - \log \frac{p_2}{p_a}\right) + \frac{Q}{2g}(s_2^2 - s_1^2).$$

Numerical computations will show the last term, viz. $\frac{Q}{2g}(s_2^2 - s_1^2)$ is relatively so small that it can be neglected in any case in practice without appreciable error. Hence by a simple reduction we get

$$\log_e \frac{p_1}{p_2} = \frac{\pi k}{12 p_a} \times \frac{dlrs^3}{v_a} \quad \text{but} \quad v_a = \frac{\pi d^2}{4 \times 144} rs,$$

which when substituted gives

$$\log_e \frac{p_1}{p_2} = \frac{4 \times 144\, k}{12\, p_a} \times \frac{l}{d} s^2,$$

or considering p_a as constant,

$$\log_{10} \frac{p_1}{p_2} = c_1 \frac{l}{d} s^2$$

or

$$\log_{10} p_2 = \log_{10} p_1 - c_1 \frac{l}{d} s^2. \qquad (22)$$

In Eq. (22) c_1 is the experimental coefficient and includes all constants. s is the velocity in the air pipe and varies slightly increasing as the pressure drops. All efforts so far have failed to get a formula in satisfactory shape that makes allowance for the variation in s.

In working out c_1 from experimental data s should be the mean between the s_1 and s_2, and when using the formula the s may be taken as about 5 per cent greater than s_1.

Note that in the solution of Eq. (22) common logarithms should be used for convenience, allowing the modulus, 2.3+, to go into the constant c_1.

The working formula may be put in a different and possibly a more convenient form, thus. In the expression

$$\log_e \frac{p_1}{p_2} = \frac{\pi k}{12} \times \frac{dl}{p_a v_a} rs^3$$

substitute for s its value

$$s = \frac{4 \times 144\, v_a}{\pi d^2 r}$$

and reduce and we get

$$\log p_2 = \log p_1 - c_2 \frac{l v_a{}^2}{p_a d^5 r^2}. \tag{23}$$

Still another form is gotten thus. The whole weight of air passing is $v_a \times w_a = Q$, and by Eq. (12)

$$Q = v_a \frac{p_a}{53.17\, t} \text{ and therefore } v_a = \frac{53.17\, tQ}{p_a}.$$

Also

$$r_x = \frac{p_x}{p_a} \text{ and } w_a = \frac{p_a}{53.17\, t}.$$

Substitute these in (23) and it reduces to

$$\log p_2 = \log p_1 - c_2 \frac{t_a l}{w_a d^5} \left(\frac{Q}{p_x}\right)^2 \tag{24}$$

In ordinary practice $\frac{t_a}{w_a}$ may be taken as constant. If this be done Eq. (24) becomes

$$\log p_2 = \log p_1 - c_3 \frac{l}{d^5} \left(\frac{Q}{p_x}\right)^2. \tag{24a}$$

If $t_a = 525$ and $w_a = .075$, then $c_3 = 7000\, c_2$.

In (24) and (24a) p_x varies between p_1 and p_2. Careful computations by sections of a long pipe show p_x to vary as ordinates to a straight line. Modifying the formulas to allow for this variation renders them unmanageable. In working out the coefficient p_x may be taken as a mean between p_1 and p_2, and in using the formula p may be taken as p_1 less half of the assumed loss of pressure.

As before suggested, common logarithms should be used in all the equations of this article.

Finally it should be said that extreme refinement in computing friction in air pipes is a waste of labor, for there are too many variables in practical conditions to warrant much effort at precision.

A study of the data collected in Appendix B gives values for c_2, Eq. (24), that, for pipes three to twelve inches diameter, conform closely to the expression

$$c_2 = .0124 - .0004\,d,$$

which gives the following:

$d'' =$	3	4	5	6	8	10	12
$C_2 =$	.0112	.0108	.0104	.0100	.0092	.0084	.0080
$C_3 =$	78.4	75.6	72.8	70.0	64.4	58.8	56.0

With these coefficients p_x in equations (24) and (24a) is to be taken in pounds per square inch.

Equations (24) and (24a) are theoretically more correct than Eq. (20) and the coefficients of the former will not vary so much as those for the latter, but when the coefficients are correctly determined for Eq. (20) it is much easier to compute and can be adapted to tabulation, while Eq. (24) cannot be tabulated in any simple way.

Example 24a. Apply formulas (20) and (24) to find the pressure lost in 1000′ of 4″ pipe when transmitting 1200 cu. ft. free air per minute compressed to 150 gage when atmospheric conditions are $p_a = 14.0$, $w_a = .073$ and $t_a = 540$.

Solution by Eq. (20): $r = \frac{150 + 14}{14} = 11.71$. By Table IX divide 23.44 by 11.71 and the result, 2 pounds, is the pressure lost per 1000′.

Solution of Eq. (24): The coefficient for a 4″ pipe is .0108, and $\log p_1 = \log (150 + 14) = 2.214844$.

Then $\log p_2 = 2.214844 - .0108 \dfrac{540}{.073} \times \dfrac{1000}{(4)^5} \left(\dfrac{1200}{60} \times \dfrac{.073}{164}\right)^2$.

The log of the last term is $\bar{3}.791193$ and its corresponding number is .006183.

$$2.214844 - .006183 = 2.208661 = \log p_2.$$

Whence $\quad p_2 = 161.7+ \quad$ and $\quad p_1 - p_2 = 2.3$.

Art. 25. Efficiency of Power Transmission by Compressed Air.

In the study of propositions to transmit power by piping compressed air, persons unfamiliar with the technicalities of compressed air are apt to make the error of assuming that the loss of power is proportional to the loss of pressure, as is the case in transmitting power by piping water. Following is the mathematical presentment of the subject:

p_1 = absolute air pressure at entrance to transmission pipe,
p_2 = absolute air pressure at end of transmission pipe,
v_1 = volume of compressed air entering pipe at pressure p_1,
v_2 = volume of compressed air discharged from pipe at pressure p_2.

Then crediting the air with all the energy it can develop in isothermal expansion, the energy at entrance is $p_1v_1 \log \dfrac{p_1}{p_a} = p_1v_1 \log r_1$, and at discharge the energy is $p_2v_2 \log \dfrac{p_2}{p_a} = p_2v_2 \log r_2$.

Hence $\quad$ efficiency $E = \dfrac{p_2v_2 \log_e r_2}{p_1v_1 \log_e r_1} = \dfrac{\log_e r_2}{\log_e r_1}$. $\quad$ (25)

Common logs may be used since the modulus cancels. The varying efficiencies are illustrated by the following tables.

$p_a = 14.5. \quad p_1 = 87. \quad r_1 = 6. \quad \log r_1 = .7781.$

p_2	85	80	75	70	65	60
r_2	5.86	5.52	5.17	4.83	4.48	4.14
$\log r_2$..	.7679	.7419	.7135	.6839	.6513	.6170
E	.987	.953	.917	.879	.837	.793

$p_a = 14.5.$ $p_1 = 145.$ $r_1 = 10.$ $\log r_1 = 1.000.$

p_2	140	135	130	125	120
r_2	9.66	9.31	8.97	8.62	8.28
$\log r_2$	.9850	.9689	.9528	.9355	.9185
E	.98	.97	.95	.93	.92

The above examples illustrate the advantage in transmitting at high pressure. Of course the work cannot be fully recovered in either case without expanding down to atmospheric pressure, and to do this in practice heating would be necessary. It should be understood also that by reheating this efficiency can be exceeded.

It should be noted also that the above does not apply in cases where the air is applied without expansion. In such cases the efficiency of transmission alone would be

$$E = \frac{(p_2 - p_a)\,v_2}{(p_1 - p_a)\,v_1} = \frac{r_1\,(r_2 - 1)}{r_2\,(r_1 - 1)}.$$

Example 25a. What diameter of pipe will transmit 5000 cu. ft. of free air per minute compressed to 100 lbs. gage, with a loss of 10 per cent of its energy in 2500 feet of pipe, assuming $p_a = 14.0$?

Solution.

$$r_1 = \frac{114}{14} = 8.15; \text{ then by Eq. (25) } \frac{\log r_2}{\log 8.15} = \frac{90}{100}.$$

Whence $\log r_2 = 0.8200$; $r_2 = 6.6$, and $6.6 \times 14 = 92.4$.

$$f = 114 - 92.4 = 21.6 = \text{loss of pressure.}$$

By Eq. (21),

$$\log d = \frac{1}{5}\left[\log (.06 \times 2500) \times \left(\frac{5000}{60}\right)^2 - \log\left(21.6 \times \frac{8.15 + 6.6}{2}\right)\right]$$

$$= .7602, \text{ whence } d = 5.75''.$$

Otherwise go into Table IX with loss for 1000 ft. $= \frac{21.6}{2.5}$ $= 8.64$, and $8.64 \times r = 8.64 \times 7.37 = 63$, (7.37 being the mean r). Then opposite 5000 in the first column find nearest value to 63, which is 55 in the 6″ column; showing the required pipe to be a little less than 6″.

CHAPTER IV

OTHER AIR COMPRESSORS

Art. 26. Hydraulic Air Compressors.—*Displacement Type.* Compressors of this type are of limited capacity and low efficiency, as will be shown. They are therefore of little practical importance. However, since they are frequently the subject of patents and special forms are on the market, their limitations are here shown for the benefit of those who may be interested.

Omitting all reference to the special mechanisms by which the valves are operated, the scheme for such compressors is to admit water under pressure into a tank in which air has been trapped by the valve mechanisms. The entering water brings the air to a pressure equal to that of the water; after which the air is discharged to the receiver, or point of use. When the air is all out the tank is full of water, at which time the water discharge valves open, allowing the water to escape and free air to enter the tank again, after which the operation is repeated. Usually these operations are automatic. The efficiency of such compression is limited by the following conditions.

Let P = pressure of water above atmosphere, or ordinary gage pressure,

V = volume of the tank.

Then $P + p_a$ = absolute pressure of air when compressed. The energy represented by one tank full of water is PV and by one tank full of free air when compressed to $P + p_a$ is

$$p_a V \log_e \frac{P + p_a}{p_a} = p_a V \log_e r.$$

Therefore the limit of the efficiency is

$$E = \frac{p_a V \log_e r}{PV} = \frac{p_a \log_e r}{P}.$$

But $P = p_1 - p_a$, where p_1 is the absolute pressure of the compressed air. Inserting this and dividing by p_a the expression becomes

$$E = \frac{\log_e r}{r - 1} = \frac{\log_{10} r \times 2.3}{r - 1}. \qquad (26)$$

Table VII is made up from formula (26).

The practical necessity of low velocities for the water entering and leaving the tanks renders the capacity of such compressors low in addition to their low efficiency.

Should the problem arise of designing a large compressor of this class an interesting problem would involve the time of filling and emptying the tank under the varying pressure and head. Since it is not likely to arise space is not given it.

It is possible to increase the efficiency of this style of compressor by carrying air into the tank with the water by induced current or Sprengle pump action — a well-known principle in hydraulics. At the beginning of the action water is entering the tank under full head with no resistance, and certainly additional air could be taken in with the water.

Art. 27. Hydraulic Air Compressors.—*Entanglement Type.*

A few compressors of this type have been built comparatively recently and have proven remarkably successful as regards efficiency and economy of operation, but they are limited to localities where a waterfall is available, and the first cost of installation is high.

The principle involved is simply the reverse of the air-lift pump, and what theory can be applied will be found in Art. 33 on air-lift pumps.

Fig. 8 illustrates the elements of a hydraulic air compressor.

h is the effective water fall.

H is the water head producing the pressure in the compressed air.

t is a steel tube down which the water flows.

S is a shaft in the rock to contain the tube t and allow the water to return.

R is an air-tight hood or dome, either of metal or of natural rock, in which the air has time to separate from the water.

A is the air pipe conducting the compressed air to point of use.

b is a number of small tubes open at top and terminating in a throat or contraction, in the tube *t*.

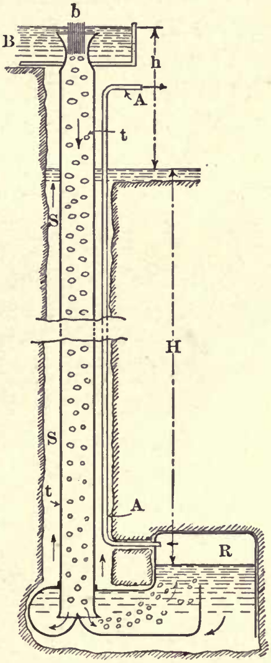

Fig 8.

By a well-known hydraulic principle, when water flows freely down the tube *t* there will occur suction in the contraction. This draws air in through the tubes *b*, which air becomes entangled in the passing water in a myriad of small bubbles; these are swept down with the current and finally liberated under the dome *R*, whence the air pipe *A* conducts it away as compressed air.

The variables involved practically defy algebraic manipulation, so that clear comprehension of the principles involved must be the guide to the proportions.

Attention to the following facts is essential to an intelligent design of such a compressor.

1. Air must be admitted freely — all that the water can entangle.
2. The bubbles must be as small as possible.
3. The velocity of the descending water in the tube *t* should be eight or ten times as great as the *relative* ascending velocity of the bubble.

The ascending velocity of the bubble *relative* to the water increases with the volume of the bubble, and therefore varies throughout the length of the tube, the volume of any one bubble being smaller at the bottom of the tube than at the top. For this reason it would be consistent to make the lower end of the tube *t* smaller than the top.

Efficiencies as high as 80 per cent are claimed for some of these compressors, which is a result hardly to have been expected.

The great advantage of this method of air compression lies in its low cost of operation and in its continuity. Mechanical compressors operated by the water power could be built for less first cost and probably with as high efficiency, but cost of operation would be much higher.

Art. 28. Centrifugal Air Compressors.

With the perfection of the steam turbine it has become practicable to deliver air at several atmospheres pressure through centrifugal machines. Such machines are not yet common, but doubtless in a few years they will become the standard machine where large volumes of air are needed at low and constant pressure. The simplicity, compactness and low first cost of such machines assure them a popularity.

The theory of centrifugal fans or air compressors would involve matter not appropriate to the purpose of this volume and is therefore omitted.

In testing centrifugal compressors or blowers the orifice measurement, Art. 20, is the only practicable scheme. If the coefficients have not been determined for orifices sufficiently large to pass the volume of air, then more than one orifice can be placed in the orifice box. It is not necessary of course that these orifices all be of one size.

The volume of air delivered and the efficiency of centrifugal fans and blowers is a matter little understood, seldom known, and often far from what is assumed or claimed. The remedy for this is to be found in intelligent use of the orifice, large and small; and for such purposes the determination of orifice coefficients such as shown in Table V should be extended to orifices all the way up to two feet in diameter in order to test very large ventilating fans.

Some theoretic discussion of centrifugal fans can be found in Trans. Am. So. C. E., Vol. 51, p. 12. See also "Turbo Compressors," *Compressed Air*, June, 1909, p. 5364, and *Engineering Magazine*, Vol. 39, p. 237.

CHAPTER V

SPECIAL APPLICATIONS OF COMPRESSED AIR

IN this chapter attention is given only to those applications of compressed air that involve technicalities — with which the designer or user may not be familiar, or by the discussion of which misuse of compressed air may be discouraged and a proper use encouraged.

Art. 29. The Return-Air System.

In the effort to economize in the use of compressed air by working it expansively in a cylinder the designer meets two difficulties: first, the machine is much enlarged when proportioned for expansion; second, it is considerably more complicated; and third, unless reheating is applied the expansion is limited by danger of freezing.

To avoid these difficulties it has been proposed to use the air at a high initial pressure, apply it in the engine without expansion, and exhaust it into a pipe, returning it to the intake of the compressor with say half of its initial pressure remaining. The diagram Fig. 9 will assist in comprehending the system.

To illustrate the operation and theoretic advantages of the system assume the compressor to discharge air at 200 pounds pressure and receive it back through R at 100 pounds. Then the ratio of compression is only 2 and yet the effective pressure in the engine is 100 pounds.

Evidently then with a ratio of compression and expansion of only 2 the trouble and loss due to heating are practically removed; and the efficiency in the engine even without a cut-off would be, by Eq. (15) 72 per cent. By the above discussion the advantages of the system are apparent, and where a compressor is to run a single machine, as for instance a pump, the advantage of this return-air system will surely outweigh the disadvantage of two pipes and the high pressure, but where one compressor installation is to serve various purposes such as rock drills, pumps, machine shops, etc., the system cannot be applied. There should be a receiver on each air pipe near the compressor.

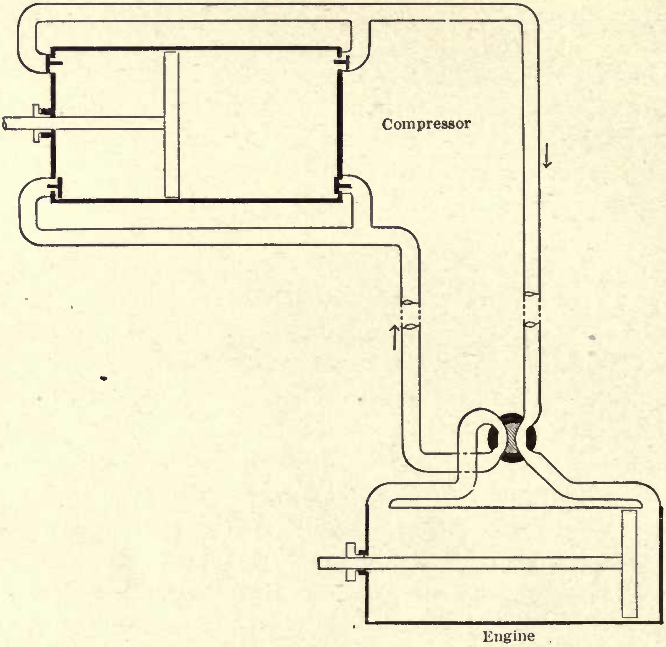

Fig. 9.

Art. 30. The Return-Air Pumping System.

Following the preceding article it is appropriate to mention the return-air pumping system. The economic principle involved is different from that of the return-air system in general.

The scheme is illustrated in Fig. 10. It consists of two tanks near the source of water supply. Each tank is connected with the compressor by a single air pipe, but the air pipes pass through a switch whereby the connection with the discharge and intake of the compressor can be reversed, as is apparent on the diagram. In operation, the compressor discharges air into one tank, thereby forcing the water out while it is exhausting the air from the other tanks, thereby drawing the water in. The charge of air will adjust itself so that when one tank is emptied the other will be filled, at which time the switch will automatically reverse the operation.

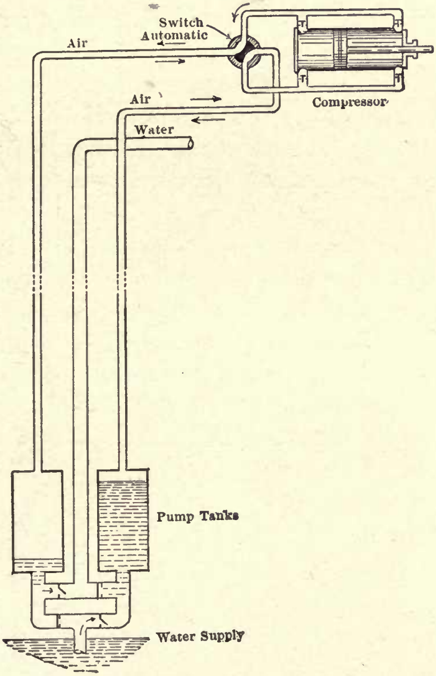

Fig. 10.

The economic advantage of the system lies in the fact that the expansive energy in the air is not lost as in the ordinary displacement pump (Art. 31). The compressor takes in air at varying degrees of compression while it is exhausting the tank.

The mathematical theory and derivation of formulas for proportioning this style of pump are quite complicated but interesting. Since the system is patented, further discussion would seem out of place. It will be found in Trans. Am. So. C. E., Vol. 54, p. 19.

Art. 31. Simple Displacement Pump. *First known as the Shone ejector pump.*

In this style of pump the tank is submerged so that when the air escapes it will fill by gravity. The operation is simply to force in air and drive the water out. When the tank is emptied of water, a float mechanism closes the compressed-air inlet and opens to the atmosphere an outlet through which the air escapes, allowing the tank to refill. Various mechanisms are in use to control the air valve automatically. The chief troubles are the unreliable nature of float mechanisms and the liability to freezing caused by the expansion of the escaping air. Some of the late designs seem reliable.

The limit of efficiency of this pump is given by formula 15 and Table VI. The pump is well adapted to many cases where pumping is necessary under low lifts. In case of drainage of shallow mines and quarries, lifting sewerage, and the like, one compressor can operate a number of pumps placed where convenient; and each pump will automatically stop when the tank is uncovered and start again when the tank is again submerged.

CHAPTER VI

THE AIR-LIFT PUMP

Art. 32. The air-lift pump was introduced in a practical way about 1891, though it had been known previously, as revealed by records of the Patent Office. The first effort at mathematical analysis appeared in the *Journal of the Franklin Institute* in July, 1895, with some notes on patent claims. In 1891 the United States Patent Office twice rejected an application for a patent to cover the pump on the ground that it was *contrary to the law of physics and therefore would not work.* Altogether the discovery of the air-lift pump served to show that at that late date all the tricks of air and water had not been found out.

The air lift is an important addition to the resources of the hydraulic engineer. By it a greater quantity of water can be gotten out of a small deep well than by any other known means, and it is free from the vexatious and expensive depreciation and breaks incident to other deep well pumps. While the efficiency of the air lift is low it is, when properly proportioned, probably better than would be gotten by any other pump doing the same service.

The industrial importance of this pump; the difficulty surrounding its theoretic analysis; the diversity in practice and results; the scarcity of literature on the subject; and the fact that no patent covers the air lift in its best form, seem to justify the author in giving it relatively more discussion than is given on some better understood applications of compressed air.

Art. 33. Theory of the Air-lift Pump.

An attempt at rational analysis of this pump reveals so many variables, and some of them uncontrollable, that there seems little hope that a satisfactory rational formula will ever be worked out. However, a study of the theory will reveal *tendencies* and better enable the experimenter to interpret results.

In Fig. 11, P is the water discharge or reduction pipe with area a, open at both ends and dipped into the water. A is the air pipe through which air is forced into the pipe, P, under pressure necessary to overcome the head D. b is a bubble liberated in the water and having a volume O which increases as the bubble approaches the top of the pipe.

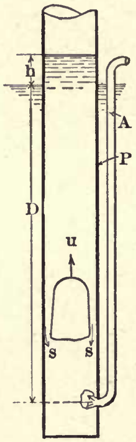

Fig. 11.

The motive force operating the pump is the buoyancy of the bubble of air, but its buoyancy causes it to slip through the water with a *relative velocity u.*

In one second of time a volume of water $= au$ will have passed from above the bubble to below it and in so doing must have taken some absolute velocity s in passing the contracted section around the bubble.

Equating the work done by the buoyancy of the bubble in ascending, to the kinetic energy given the water descending we have

$$wOu = wau\,\frac{s^2}{2\,g} \quad \text{where } w = \text{weight of water,}$$

or

$$\frac{O}{a} = \frac{s^2}{2\,g}. \tag{a}$$

$\frac{s^2}{2\,g}$ is the equivalent of the head h at top of the pipe which is necessary to produce s, therefore $h = \frac{O}{a}$.

Suppose the volume of air, O, to be divided into an infinite number of small particles of air, then the volume of a particle divided by a would be zero and therefore s would be zero; but the sum of the volumes $(= O)$ would reduce the specific gravity of the water, and to have a balance of pressure between the columns inside and outside the pipe the equation

$$wO = wah \text{ must hold.}$$

Hence again $h = \frac{O}{a}$, showing that the head h depends only on the volume of air in the pipe and not on the manner of its subdivision.

The slip, u, of the air relative to the water constitutes the chief loss of energy in the air lift. To find this apply the law of physics, that forces are proportional to the velocities they can produce in a given mass in a given time. The force of buoyancy wO' of the bubble causes in one second a downward velocity s in a weight of water wau. Therefore

$$\frac{wO}{wau} = \frac{s}{g}.$$

Whence $u = \frac{O}{a}\frac{g}{s}$. But $\frac{O}{a} = \frac{s^2}{2\,g}$ as proved above.

Therefore

$$u = \frac{s}{2} = \sqrt{\frac{O}{a}\frac{g}{2}}. \qquad \text{(b)}$$

This shows that the slip varies with the square root of the volume of the bubble. It is therefore desirable to reduce the size of the bubbles by any means found possible.

If $u = \frac{s}{2}$, then the bubble will occupy half the cross section of the pipe. This conclusion is modified by the effect of surface tension, which tends to contract the bubble into a sphere. The law and effect of this surface tension cannot be formulated nor can the volume of the bubbles be entirely controlled. Unfortunately, since the larger bubbles slip through the water faster than the small ones, they tend to coalesce; and while the conclusions reached above may approximately exist about the lower end of an air lift, in the upper portion, where the air has about regained its free volume, no such decorous proceeding exists, but instead there is a succession of more or less violent rushes of air and foamy water.

The losses of energy in the air lift are due chiefly to two causes: first, the slip of the bubbles, through the water, and second, the friction of the water on the sides of the pipe. As one of these decreases the other increases, for by reducing the velocity of the water the bubble remains longer in the pipe and has more time to slip.

The best proportion for an air lift is that which makes the sum of these two losses a minimum. Only experiment can determine what this best proportion is. It will be affected somewhat by the average volume of the bubbles. As before said, any means of reducing this volume will improve the results.

Art. 34. Design of Air-lift Pumps.

The variables involved in proportioning an air-lift pump are: —

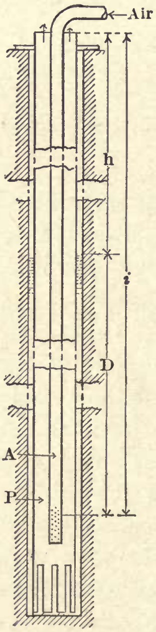

Fig. 12.

Q = volume of water to be lifted,
h = effective lift from free water surface outside the discharge pipe,
$l = D + h$ = total length of water pipe above air inlet,
D = Depth of submergence = depth at which air is liberated in water pipe measured from free water surface outside the discharge pipe.
v_a = volume per second of free air forced into well,
a = area of water pipe,
A = area of air pipe,
O = volume of the individual bubbles.

The designer can control A, a, $D + h$ and v_a but he has little control over O, and cannot foretell what D and Q will be until the pump is in and tested.

When the pump is put in operation the free water surface in the well will always drop. What this drop will be depends first on the geology and second on the amount, Q, of water taken out. In very favorable conditions, as in cavernous limestone, very porous sandstone or gravel, the drop may be only a few feet, but in other cases it may be so much as to prove the well worthless. In any case it can be determined by noting the drop in the air pressure when the water begins flowing. If this drop is p pounds, the drop of water surface in the well is $2.3 \times p$ feet.

Unless other and similar wells in the locality have been tested, the designer should not expect to get the best proportion with the first set of piping, and an inefficient set of piping should not be left in the well.

The following suggestions for proportioning air lifts have proved safe in practice, but, of course, are subject to revision as further experimental data are obtained. (See Figs. 13 and 14.)

Air Pipe. Since in the usually very limited space high velocities must be permitted we may allow a velocity of about 30 ft. per second in the air pipe.

Submergence. The ratio $\frac{D}{D+h}$ is defined as the Submergence ratio. Experience seems to indicate that this should be not less than one-half; and about 60 per cent is a common rule of practice. Probably the efficiency will increase with the submergence. The cost of the extra depth of well necessary to get this submergence is the most serious handicap to the air-lift pump.

Ratio $\frac{v_a}{Q}$.

Let D = depth of submergence and h = effective lift = nD. Then the energy in the compressed air is

$p_a v_a \log_e\left(\frac{D+33.3}{33.3}\right)$, $\frac{D+33.3}{33.3}$ being the ratio of compression, $= r$, and the effective work in water lifted is

$$wQh = 62.5\, QnD.$$

If E be the efficiency of the system, then

$$62.5 \times Q \times nD = E \times 2100\, v_a \times 2.3 \log_{10}(r),$$

cubic foot units being used and common logs.

Whence
$$\frac{v_a}{Q} = \frac{1}{77.3}\,\frac{n}{E}\,\frac{D}{\log_{10} r}. \qquad (27)$$

Several apparently well proportioned wells are on record, see Art. 37, in which D is from 350 to 500 feet, n about $\frac{2}{3}$

and E 40 to 50 per cent. Taking $n = \frac{2}{3}$ and $E = 45$ per cent, Eq. (27) reduces to

$$\frac{v_a}{Q} = \frac{D}{50 \log_{10} r}. \qquad (27a)$$

From which the following table is computed.

h	D	l	$\frac{v_a}{Q}$	h	D	l	$\frac{v_a}{Q}$
				167	250	417	5.4
6.6	10	16.6	1.8	200	300	500	6.1
33.	50	83.3	2.5	233	350	583	6.6
66.	100	166.0	3.4	267	400	667	7.2
100.	150	250.0	4.1	300	450	750	7.8
133.	200	333.	4.8	333	500	833	8.4
....	....	....	....	366	550	916	8.9

This table is reproduced in the curve plate V. It should be used only with full recognition of the assumptions on which it is based, and with due regard to what follows about velocities in the water pipe. The table has been verified for h between 200 and 400 feet. For lower lifts it would be expected that a better efficiency could be obtained—the best data that can be found seem to indicate that such is the case. In consideration of this the dotted line on plate V may be a better guide than the full line.

Velocity in the Water Pipe.

This is the factor that most affects the efficiency, but unfortunately, owing to the usual small area in the well, the velocity cannot always be kept within the limits desired. The complicated action and varying conditions in a well make the designer entirely dependent on the results of experience in fixing the allowable velocities in the discharge pipes.

The velocity of the ascending column of mixed air and water should certainly be not less than twice the velocity at which the bubble would ascend in still water. This would probably put the most advantageous *least* velocity in any air lift at between five and ten feet, and this would occur where the air enters the discharge pipe.

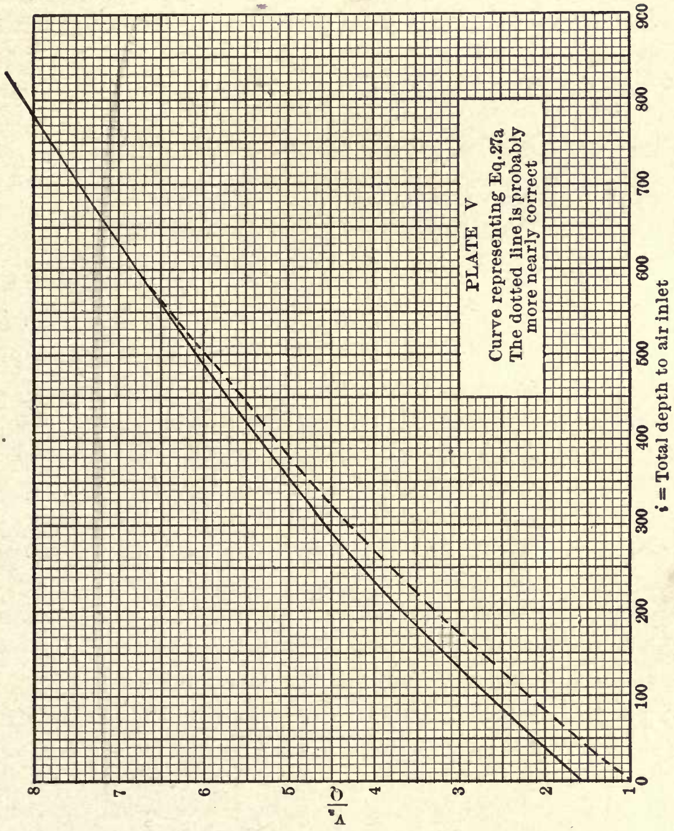
PLATE V
Curve representing Eq.27a
The dotted line is probably more nearly correct
i = Total depth to air inlet
0
100
200
300
400
500
600
700
800
900
$\frac{Q}{V_a}$
1
2
3
4
5
6
7
8

The velocity at any section of the pipe will be

$$s = \frac{Q + v}{a},$$

where Q and v are the volumes of water and air respectively and a the effective area of the water pipe. s increases from bottom to top probably very nearly according to the formula

$$s = v_a \left(1 - \frac{x}{l}\left[1 - \frac{1}{r}\right]\right) \qquad (28)$$

where

r = ratio of compression under running conditions,
l = total length of discharge pipe above air inlet,
x = distance down from top of discharge pipe to section where velocity is s.

The formula (28) is based on the assumption that the volume of air varies as the ordinate to a straight line while ascending the pipe through length l. As the volume of each bubble increases in ascending the pipe, the velocity of the mixture of water and air should also increase in order to keep the sum of losses due to slip of bubble and friction of water a minimum; but for deep wells with the resultant great expansion of air the velocity in the upper part of the pipe will be greater than desired, especially if the discharge pipe be of uniform diameter. Hence it will be advantageous to increase the diameter of the discharge pipe as it ascends. The highest velocity (at top) probably should never exceed twenty feet per second if good efficiency is the controlling object.

Good results have been gotten in deep wells with velocities about six feet at air inlet and about twenty feet at top. (See Art. 37.)

Fig. 13 shows the proportions and conditions in an air lift at Missouri School of Mines.

The flaring inlet on the bottom should never be omitted. Well-informed students of hydraulics will see the reason for this, and the arguments will not be given here.

The numerous small perforations in the lower joint of the air pipe liberate the bubbles in small subdivisions and some advantage is certainly gotten thereby.

No simpler or cheaper layout can be designed, and it has proved as effective as any. It is the author's opinion that nothing better has been found where submergence greater than 50 per cent can be had.

Art. 35. The Air Lift as a Dredge Pump.

The possibilities in the application of the air lift as a dredge pump do not seem to have been fully appreciated. This may be due to its being free from patents and therefore no one being financially interested in advocating its use.

With compressed air available a very effective dredge can be rigged up at relatively very little cost and one that can be adapted to a greater variety of conditions than those in common use, as the following will show.

Suggestions:

Clamp the descending air pipe to the outside of the discharge pipe. Suspend the discharge pipe from a derrick and connect to the air supply with a flexible pipe (or hose). With such a rigging the lower end of the discharge pipe can be kept in contact with the material to be dredged by lowering from the derrick; the point of operation can be quickly changed within the reach of the derrick, and the dredge can operate in very limited space. In dredging operations the lift of the material above the water surface is usually small, hence a good submergence would be available. The depth from which dredging could be done is limited only by the weight of pipe that can be handled.

Art. 36. Testing Wells with the Air Lift.

The air lift affords the most satisfactory means yet found for testing wells, even if it is not to be permanently installed. Such a test will reveal, in addition to the yield of water, the position of the free water surface in the well at every stage of the pumping, this being shown by the gage pressures. However, some precautions are necessary in order properly to correct the gage readings for friction loss in the air pipe.

The length of air pipe in the well and any necessary corrections to gage readings must be known.

The following order of proceeding is recommended.

At the start run the compressor very slowly and note the pressure p_1 at which the gage comes to a stand. This will indicate the submergence before pumping commences, since there will be practically no air friction and no water flowing at the point where air is discharged. Now suddenly speed up the compressor to its prescribed rate and again note the gage pressure p_2 before any discharge of water occurs. Then $p_2 - p_1 = p_f$ is the pressure lost in friction in the air pipe. When the well is in full flow the gage pressure p_3 indicates the submergence plus friction, or submergence pressure $p_s = p_3 - p_f$. The water head in feet may be taken as $2.3 \times p$. Then, knowing the length of air pipe, the distance down to water can be computed for conditions when not pumping and also while pumping.

Art. 37. Data on Operating Air Lifts.

In Figs. 13 and 14 are shown the controlling numerical data of two air lifts at Rolla, Mo. These pumps are perhaps unusual in the combination of high lift and good efficiency. The data may assist in designing other pumps under somewhat similar circumstances.

The figures down the left side show the depth from surface. The lower standing-water surface is maintained while the pump is in operation; the upper where it is not working.

The broken line on the right shows, by its ordinate, the varying velocities of mixed air and water as it ascends the pipe.

The pump Fig. 13 delivers 120 gallons per minute with a ratio $\frac{\text{Free air}}{\text{Water}} = 6.0$. The submergence is 58 per cent and

$$\text{efficiency} = \frac{\text{Net energy in water lift}}{pv \log_e r} = 50 \text{ per cent.}$$

The pump Fig. 14 delivers 290 gallons per minute with a ratio $\frac{\text{Free air}}{\text{Water}} = 5.2$. Submergence = 64 per cent and

$$\text{efficiency} = \frac{\text{Net energy in water lift}}{pv \log_e r} = 45 \text{ per cent.}$$

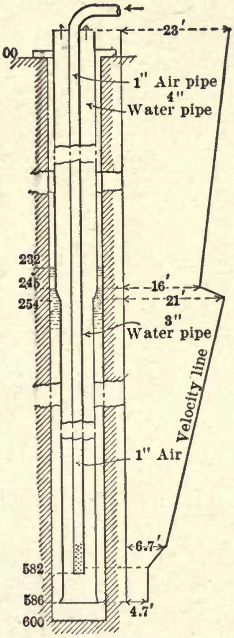

Fig. 13.

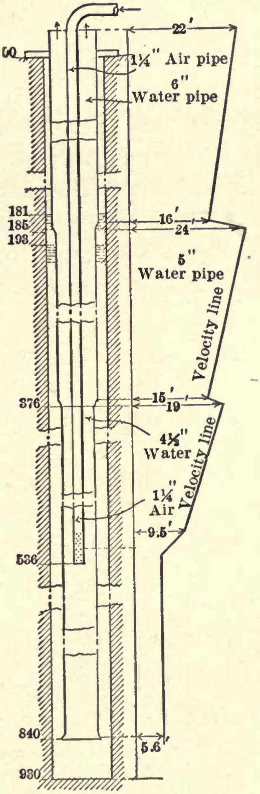

Fig. 14.

The volumes of air used in the above data are the actual volumes delivered by the compressor. The volumetric efficiencies of the compressors by careful tests proved to be about 72 per cent.

CHAPTER VII

EXAMPLES AND EXERCISES

Art. 38. The following combined example includes a solution of many of the types of problems that arise in designing compressed-air plants. The student will find it well worth while to become familiar with every step and detail of the solutions, which are given more fully than would be necessary except for a first exercise.

Example 26. An air-compressor plant is to be installed to operate a mine pump under the following specifications:

1. Volume of water = 1500 gallons per minute.
2. Net water lift = 430 feet.
3. Length of water pipe = 1280 feet.
4. Diameter of water pipe = 10 inches.
5. Length of air pipe = 1160 feet.
6. Atmospheric pressure = 14.0 pounds per sq. in.
7. Atmospheric temperature 50° F.
8. Loss in transmission through air line = 8 per cent of the $pv \log_e r$ at compressor.
9. Mechanical efficiency of the pump = 90 per cent as revealed by the indicators on the air end and the known work delivered to the water.
10. Average piston speed of pump = 200 feet per minute.
11. Mechanical efficiency of the air compressor = 85 per cent as revealed by the indicator cards.
12. R.P.M. of air compressor = 90 and volumetric efficiency = 82 per cent.
13. In compression and expansion $n = 1.25$.

Preliminary to the study of the problems involving the air we must determine:

(*a*) *Total pressure head against which the pump must work.*

By the methods taught in hydraulics the friction head in a pipe 10 inches in diameter, 1280 feet long, delivering 1500 gallons per minute, is about 20 feet. Therefore the total head = 450 feet.

(b) *Total work* (W_1) *delivered to the water in one minute.* $W_1 = 1500 \times 8\frac{1}{3} \times 450 = 5{,}625{,}000$ foot-pounds.

(c) *Total work* (W) *required in air end of pump.*

By specification 9, $W = \frac{W_1}{.90} =$ **6,250,000** ft.-lbs. = 190 horse power.

For the purpose of comparison, two air plants will be designed; the first, designated *d*, as follows:

(*d*) Compression single-stage to 80 pounds gage. No reheating. No expansion in air end of pump. Pump direct acting without fly wheels.

Determine the following:

(*d*1) *Air pressure at pump and pressure lost in air pipe.*

By specification 8 and Eq. (25),

$$\frac{92}{100} = \frac{\log \frac{p_2}{14}}{\log \frac{80+14}{14}}, \text{or} \log \frac{p_2}{14} = .92 \log 6.72.$$

Whence, using common logs, $\log \frac{p_2}{14} = 0.76118$ and

$$p_2 = 80.78.$$

Then lost pressure $= p_1 - p_2 = 94 - 80.78 =$ **13.22** $=$ **f,** and gage pressure at pump $= 80 - 13.22 =$ **66.78.**

(*d*2) *Ratio between areas of air and water cylinders in pump.*

The pressure due to 450 feet head $= 450 \times .434 = 194.3$, say 195 pounds, per sq. in.; and since pressure by area must be equal on the two ends, $\frac{\text{area air end}}{\text{area water end}} = \frac{195}{66.78} =$ **3** nearly.

(*d*3) *Volume of compressed air used in the pump. Cubic feet per minute:*

Evidently from solution (*d*2) the volume of compressed air used in the pump will be three times that of the water pumped, or

$$v = \frac{1500}{7.48} \times 3 = \textbf{601.6} \text{ cu. ft. per min.}$$

(*d*4) *Diameters of air cylinder and of water cylinder.*

Since the piston speed is limited to 200 feet per min. (spec. 10) and the volume is 1500 gallons, we have, when all is reduced to inch units and letting a = area of water cylinder, $a \times 200 \times 12 = 1500 \times 231$. Whence $a = 144$ sq. in. which requires a diameter of about **13⅝** inches.

The area of air cylinder is by d_2 three times that of the water cylinder, which gives a diameter **23½** inches for the air cylinder.

(*d*5) *Volume of free air.*

From $d1$, r at the pump = 5.76. Therefore

$$v_a = 601.6 \times 5.76 = \mathbf{3465} \text{ cu. ft. per min.}$$

(*d*6) *Diameter of air pipe.*

The mean r in the air pipe is $\frac{5.76 + 6.72}{2} = 6.24$. Using this in Eq. (21) with $c = .06$, we get $d = \mathbf{5}$ inches.

Or using plate III with $r \times 13.22 \div 1.160$ or $r \times \frac{13.22}{1.160}$ as vertical ordinate and 3465 as horizontal ordinate, the intersection falls near the 5-inch line.

(*d*7) *Horse power required in steam end of compressor.*

By table II the weight per foot of free air is .07422 pound per cu. ft. Total weight of air compressed = Q

$$Q = .07422 \times 3465 = \mathbf{257} \text{ pounds per min.}$$

In table I opposite $r = 6.72$ in column 9 find by interpolation .3736. Then

$$\text{Horse power} = 2.57 \times .3736 \times (460 + 50) = 489.6 \text{ in air end} = \frac{489.6}{.85} = \mathbf{576} \text{ in steam end.}$$

The second plant will be designated by the letter e and will be two-stage compression to **200** pounds gage at air compressor, will be reheated to **300°** at the pump and used expansively in the pump; the expansion to be such that the temperature will be **32°** at end of stroke.

(*e*1) *Air pressure at pump.*

Apply Eq. (25) as in $d1$. In this case r_1 (at the compressor) = 15.3 and r_2 (at the pump) = 12.3. Therefore pressure at the pump = 12.3 × 14 = **172.3** and the lost pressure = 214 − 172.3 = **41.7** = f.

(*e*2) *Point of cut-off in air end of pump = fraction of stroke during which air is admitted.*

By Eq. (11) viz. $\frac{t_2}{t_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$, putting in numbers we get $\frac{492}{760} = \left(\frac{v_1}{v_2}\right)^{.25}$ whence $\frac{v_1}{v_2} = \mathbf{.176}$, which is the point of cut-off, and $v_2 = \mathbf{5.68}\ v_1$.

Or go into table I in column 5, find the ratio $\frac{760}{492} = 1.545$, and in same horizontal line in column 3 find .176.

(e3) *Volume of compressed hot air admitted to air end of pump.*

Apply Eq. (9) viz. Work $= \frac{p_1v_1 - p_2v_2}{n-1} + p_1v_1 - p_av_2$.

In this we have Work = 6,250,000, $v_2 = 5.68\ v_1$, $p_1 = 214$, $n - 1 = .25$, $p_a = 14$, and p_2 must be found by Eq. (11a), or it may be gotten from table I by noting that for a temperature ratio of 1.545 the pressure ratio is 8.8 and $\frac{1}{r} = .1136$, therefore $p_2 = .1136 \times 172.3 = \mathbf{19.57}$. This would give gage pressure = 5.57.

Inserting these numerals in Eq. (9) we get

$$6{,}250{,}000 = 144\ v_1 \left(\frac{172.3 - 5.68 \times 19.57}{.25} + 172.3 - 14 \times 5.68\right).$$

Whence $v_1 = \mathbf{128.6}$ cu. ft. per min.

(*e*4) *Diameter of air cylinder of pump when air and water pistons are direct connected.*

Since expansion ratio is 5.68 (see *e*2) and the volume before cut-off is 128.6, the total piston displacement is 128.6 × 5.68 = **730.8** cu. ft. per min. When the air and water pistons are direct connected they must travel through equal distances, therefore the air piston travels through 200 ft. per min. (spec. 10). Then if a = area of piston in sq. ft. we have

$$200\ a = 730.8 \quad \text{and} \quad a = 3.654 \text{ sq. ft.}$$

By table X the diameter is 26 inches nearly.

(*e*5) *Volume of cool compressed air used by pump, cu. ft. per min.*

By *e*3 the volume of hot compressed air is 128.6, and since under constant pressure volumes are proportional to absolute temperatures, we have

$$\frac{v}{128.6} = \frac{510}{760}\cdot \quad \text{Whence } v = \mathbf{86.3} \text{ cu. ft. per min.}$$

(*e*6) *Volume of free air used.*

From *e*1 the ratio of compression at the pump is 12.3 and from *e*5 the volume of cool compressed air is 86.3, therefore the volume of free air is 86.3 × 12.3 = **1061.6.**

(*e*7) *Diameter of air pipe.*

$$\text{The } r \text{ for Eq. (21) is } \frac{12.3 + 15.3}{2} = \mathbf{13.8.}$$

Applying Eq. (21) with coefficient $c = .07$ we have

$$d = \left(\frac{.07 \times 1160 \times \left(\frac{1061.6}{60}\right)^2}{41.7 \times 13.8}\right)^{\frac{1}{5}} = \mathbf{2.13} \text{ inches.}$$

(*e*8) *Horse power required in steam end of compressor.*

By *d*7 the weight per cu. ft. of free air is .07422 and by *e*6 the volume of free air compressed is 1061.6. Therefore the total weight compressed is .07422 × 1061.6 = **78.8** pounds per min., and the initial absolute temperature is 510.

In the two-stage compression $r_2 = 15.3$, and assuming equal work in the two stages the $r_1 = \sqrt{15.3} = 3.91$ nearly. (See Art. 12.) Then going into Table I with $r = 3.91$ in column 9 find .2525. Hence horse power = .2525 × 78.8 × 510 = 101.5 for one stage, and for the two stages 101.5 × 2 = 203, and (spec. 11) $\frac{203}{.85}$ = **238.8** horse power in steam end.

(*e*9) *Diameter of air compressor cylinders, assuming* **3-***foot strokes and* 2½-*inch piston rods, equal work in the two cylinders and allowing for volumetric efficiency.*

By *e*6 the free air volume is 1061.6 and (spec. 12) the volumetric efficiency = 82 per cent. Therefore the piston displacement = $\frac{1061.6}{.82}$ = 1294.6 cu. ft. per min.

By spec. 12 the R.P.M. = 90. Therefore the displacement per revolution = 14.7, nearly, for the low-pressure cylinder. Add to this the volume of one piston rod length of 3 feet, which is 3 × .0341 = 0.1023. Whence the volume per revolution must be 14.8 or for one stroke 7.4. Whence the area $= \frac{7.4}{3} = 2.466$ sq. ft. By Table X the diameter is **21¼** inches nearly for low-pressure cylinders.

The high-pressure cylinder must take in the net volume of air compressed to $r \doteq 3.91$ (see *e*8). Therefore the net volume per revolution $= \frac{1061.6}{90 \times 3.91} = 3.02$. Add one piston rod volume and get 3.12 per revolution or 1.56 per stroke and an area of 0.53 sq. ft. By Table X this requires a diameter of 10 inches nearly.

(*e*10) *Temperature of air at end of each compression stroke.* In Table I the ratio of temperatures for $r = 3.91$ is 1.313. Hence the higher temperature = 510 × 1.313 = 669 absolute = 209 F.

EXERCISES

1. (*a*) Assuming isothermal conditions, how many revolutions of a compressor 16″ stroke, 14″ diameter, double acting, would bring the pressure up to 100 lbs. gage in a tank 4 feet diam. × 12 feet length, atmospheric pressure = 14.5 per sq. in.?

(*b*) What would be the horse power of such a compressor running at 100 R.P.M.?

(*c*) What would be the horse power if the compression were adiabatic?

(*d*) What weight of air would be passed per minute when R.P.M. = 100 and $T = 60°$ F.?

2. The air end of a pump (operated by compressed air) is 20″ diam. by 30″ stroke, R.P.M. = 50, cut-off at ¼ stroke, free air pressure = 14.0, $T_a = 60°$, compressed air delivered at 75 lbs. gage, $T = 60°$ and $n = 1.41$.

(*a*) Find work done in horse power.

(*b*) Find weight handled per minute.

(*c*) Find temperature of exhaust (degrees F).

3. With atmospheric pressure, $p_a = 14.7$, and $T_a = 50°$, under perfect adiabatic compression, what would be the pressure (gage) and temperature (F.) when air is compressed to
(*a*) $\frac{1}{2}$ its original volume ?
(*b*) $\frac{1}{4}$ its original volume ?
(*c*) $\frac{1}{6}$ its original volume ?
(*d*) $\frac{1}{8}$ its original volume ?
(*e*) $\frac{1}{10}$ its original volume ?

4. With $p_a = 14.1$ and $T_a = 60°$ what will be the pressure of a pound of air when its volume = 3 cu. ft. ?

5. What would be the theoretic horse power to compress 10 pounds of air per minute from $p_a = 14.3$ and $T_a = 60°$ to 90 pounds gage ?
(*a*) Compression isothermal.
(*b*) Compression adiabatic.

6. Find the point of cut-off when air is admitted to a motor at 250° F. and expanded adiabatically until the temperature falls to 32° F.

7. What is the weight of 1 cu. ft. of air when $p_a = 14.0$ and $T_a = -10°$?

8. A compressor cylinder is 20″ diam. by 26″ stroke double acting. Clearance = 0.8%, piston rod = 2″, R.P.M. = 100, atmospheric pressure, $p_a = 14.3$, atmospheric temperature $= T_a = 60°$ F., and gage pressure = 98 lbs.

Determine the following:
(*a*) *Compression isothermal.*
    1*a*. Volume of free air compressed, cu. ft. per min.
    2*a*. Volume of compressed air, cu. ft. per min.
    3*a*. Work of compression, ft.-lbs. per min.
    4*a*. Lbs. of cooling water, $T_1 = 50°$, $T_2 = 75°$.
(*b*) $n = 1.25$ and air heated to 100° while entering.
    1*b*. Volume of free air compressed per min.
    2*b*. Volume of cool compressed air per min.
    3*b*. Work done in compression.
    4*b*. Temperature of air at discharge.

9. The cylinder of a compressed-air motor is 18″ × 24″, the R.P.M. = 90, air pressure 100 pounds gage. In the

motor the air is expanded to four times its original volume (cut-off at $\frac{1}{4}$), with $n = 1.25$.

(*a*). Determine the horse power and final temperature when initial $T = 60°$ F.

(*b*). Determine the horse power and final temperature when initial $T = 212°$ F.

10. Observations on an air compressor show the intake temperature to be 60° F., the $r = 7$ and the discharge temperature $= 300$ F. What is the n during compression?

Hint. Use Eq. (11a) with n unknown.

11. In a compressed-air motor what percentage of power will be gained by heating the air before admission from 60° to 300° F. ?

12. If air is delivered into a motor at 60° F. and the exhaust temperature is not to fall below 32° F., what ratio of expansion can be allowed? What could be allowed if initial temperature were 300° ? What would be the ratio of work gotten in the two cases assuming $n = 1.25$?

13. A compressed-air locomotive system is estimated to require 4000 cu. ft. per min. of free air compressed to 500 pounds gage in three stages with complete cooling between stages.

Assume $n = 1.25$, $p_a = 14.5$, $T_a = 60°$, Vol. Eff. = 80 per cent, Mechanical Eff. = 85 per cent and R.P.M. = 60.

Compute the volume of piston stroke in each of the three cylinders and the total horse power required of the steam end.

14. A compressor is guaranteed to deliver 4 cu. ft. of free air per revolution at a pressure of 116 (absolute). To test this the compressor is caused to deliver into a closed system consisting of a receiver, a pipe line and a tank. Observed conditions are as follows:

	Receiver.	Pipe.	Tank.
Pressures at start (ab.)	14.5	14.5	14.5
Temperatures at start (F.)	60.0	60.0	60.0
Pressures at end (ab.)	116.0	116.0	116.0
Temperatures at end (F.)	150.0	90.0	60.0
Volumes (cu. ft.)	50.0	10.0	100.0

How many revolutions of the compressor should produce this effect ?

15. Find the discharge in pounds per minute through a standard orifice when $d = 2''$, $i = 5''$, $t = 600°$ and $p_a = 14.0$.

16. What diameter of orifice should be supplied to test the delivery of a compressor that is guaranteed to deliver 1000 cu. ft. per min. of free air ?

17. What is the efficiency of transmission when air pressure drops from 100 to 90 pounds (gage) in passing through a pipe system ?

18. A compressor must deliver 100 cu. ft. per min. of compressed air at a pressure = 90 pounds, gage, at the terminus of a pipe 3000 ft. long and 3″ diameter. $p_a = 14.4$, $T_a = 60°$ F.

(*a*) Assuming a Vol. Eff. = 75 per cent, what must be the piston displacement of the compressor ?

(*b*) What pressure is lost in transmission ?

(*c*) What horse power is necessary in steam end of compressor if $n = 1.25$ and the mechanical efficiency = 85 per cent?

(*d*) What would be the efficiency of the whole system if air is applied in the motor without expansion, the efficiency to be reckoned from steam engine to work done in motor ?

19. It is proposed to convey compressed air into a mine a distance of 5000′. The question arises: Which is better, a 3″ or a 4″ pipe?

Compare the propositions financially, using the following data: Nominal capacity of the plant = 1000 cu. ft. free air per min., Vol. Eff. of compressor = 80 per cent, $n = 1.25$ gage pressure at compressor = 100, weight of free air $w_a = .074$, $p_a = 14.36$, weight of 3″ pipe = 7.5 and of 4″ pipe = 10.7 pounds per foot. Cost of pipe in place = 4 cents per pound. Cost of one horse power in form of $pv \log r$ for 10 hours per day for one year = \$150. Plant runs 24 hours per day. Rate of interest = 6 per cent.

20. Air enters a 4″ pipe with 60 feet velocity and 80 pounds gage pressure; the air pipe is 1500 feet long; $p_a = 14.7$.

(*a*) Find the efficiency of transmission.

(*b*) Find horse power delivered at end of pipe in form $pv \log r$ when $T = 60°$ F.

(*c*) Find horse power delivered at end of pipe in form $P_g \times v$.

21. An air pipe is to be 2000 feet long and must deliver 50 horse power at the end with a loss of 5 per cent of the $pv \log r$ as measured at compressor. The pressure at compressor is 75 pounds gage. $p_a = 14.7$. Find diameter of pipe.

22. Modify 21 to read: 50 horse power . . . with loss of 5 per cent of the energy in form $P_g \times v$, where P_g is gage pressure, and find diameter of air pipe.

23. In case 21 let pressure at compressor be 250 pounds gage and find diameter of air pipe.

24. The air cylinder of a compressed-air pump is 20″ diam. by 30″ stroke. The machine is double acting and makes 50 R.P.M. The cut-off is to be so adjusted that the temperature of exhaust shall be 30°. $p_a = 14.5$ and the r at pump $= 8$.

(*a*) Find cut-off when initial temperature is 60° F.

(*b*) Find cut-off when initial temperature is 250° F.

(*c*) Find horse power in case (*a*).

(*d*) Find horse power in case (*b*).

(*e*) In case (*a*) find efficiency in applying the $pv \log r$ of cool air.

(*f*) In case (*b*) find efficiency in applying the $pv \log r$ of cool air.

(*g*) Find the volumes of free air used in cases (*a*) and (*b*).

√ **25.** A compound mine pump is to receive air at 150 lbs. gage; this is to be reheated from 60° to 250° F., let into the H.P. cylinder of the pump and expanded until the temperature is 32°, then exhausted into an interheater where the temperature is again brought to 250°. It then goes into the L.P. cylinder and is expanded down to atmospheric pressure $= 14.5$, (ab.).

(*a*) Find point of cut-off in each cylinder, $n = 1.25$.

(*b*) If the air is compressed in two stages with $n = 1.25$, what will be the efficiency of the system, neglecting friction losses and clearance?

(*c*) How much free air will be required to operate the pump if it is to deliver 250 horse power, assuming the efficiency of the pump to be 80 per cent reckoned from the work in the air end?

(*d*) If the pump strokes be 60 per min. and 60″ long, fix diameters of air cylinders in case (*c*).

26. Compute the horse power of a motor passing one pound of air per minute admitted at 200° F. and 116.0 pounds (ab.) $r = 8$, the air to be expanded until pressure drops to 29 pounds (ab.), $r = 2$.

27. A pump to be operated by compressed air must deliver 1000 gallons of water per minute against a net head of 200′ through 800′ of 10″ pipe. The pump is double acting, 30″ stroke, 50 strokes per min. The air is reheated to 275° F. before entering the pump. The cut-off is so adjusted that with $n = 1.25$ the temperature at exhaust = 36° F. Mec. Effi. of pump = 80%. Air pressure at compressor = 90 pounds gage, $p_a = 14.4$. Length of air pipe = 2000′. Permissible loss in transmission = 7 per cent of the $pv \log r$ at compressor. Mec. Effi. of compressor = 85 per cent. Vol. Effi. = 80 per cent.

(*a*) Proportion the cylinders of the pump.

(*b*) Determine the volume of free air used.

(*c*) Determine the diameter of air pipe.

28. Compare the volume displacement of two air compressors, one at sea level and the other at 12,000 feet elevation; the compressors to handle the same weight of air.

29. (*a*) An exhaust pump has an effective displacement of 3 cu. ft. per revolution. How many revolutions will reduce the pressure in a gas tank from 30 to 5 pounds absolute? Volume of tank = 400 cu. ft. when $p_a = 14.7$?

(*b*) If the pump is delivering the gas under a constant pressure of 30 pounds, what is the maximum rate of work done by the pump — foot pounds per revolution?

PLATES AND TABLES

NOTES ON TABLE I.

The table is the solution of formulas 11, 11a, 8a and 1a.

When the weight of air passed and its initial temperature are known, the table covers all conditions including elevation above sea level, reheating, and compounding.

In compounding, either compression or expansion, the same weight goes through each cylinder. Then knowing the initial t and the r for each cylinder, find from the table the work done in each cylinder and add. Usually the r and t are assumed the same for each cylinder — then take out the work for one stage and multiply by the number of stages.

The table does not include friction in the machine nor the effect of clearance in expansion motors.

The table is equally applicable to compression or expansion provided the correct r be taken in cases of expansion.

Example. Air is received at such a pressure that $r = 8$. What should be the cut-off in order that the temperature drop from 60° to 32° F.? Expansion adiabatic.

The ratio of temperatures is 1.057, which by linear interpolation corresponds to a volume ratio of .871 or cut-off at about $\frac{7}{8}$.

What would be the pressure at exhaust?

The two ratios above correspond to a $\frac{1}{r} = .825$. Therefore the final pressure is .825 × initial pressure.

To find the foot-pounds of work per pound of air compressed multiply the number opposite the r in column 7, 8 or 11 as the case may be by the absolute initial temperature, t.

To find the weight compressed, go into Table II with known atmospheric conditions and the cubic feet capacity of the machine.

To find the horse power per hundred pounds of air passed per minute, multiply the number opposite r in column 9, 10 or 12 as the case may be by the absolute lower temperature, t.

TABLE I. GENERAL TABLE RELATING TO AIR COMPRESSION AND EXPANSION

Ratio of Compression or Expansion.	Ratio of Less to Greater Volume — Air Cool.	Ratio of Less to Greater Volume — Temp. Changing. $\frac{v_2}{v_1} = \left(\frac{1}{r}\right)^{\frac{1}{n}}$		Ratio of Greater to Less Temperature. — Temperatures Absolute. $\frac{t_2}{t_1} = (r)^{\frac{n-1}{n}}$		Work Factor. Air Heated by Compression.				Work Factor for Isothermal Compression.	
						$K = 53.17 \frac{n}{n-1} \times \left(r^{\frac{n-1}{n}} - 1\right)$ Factor K for one pound.		H.P. Factor per 100 Pounds per Minute $= \frac{K}{330}$.		$K = 53.17 \log_e r$ Factor K for one pound.	H.P. Factor per 100 Lbs. per Min. $\frac{K}{330}$
		$n =$ 1.25	$n =$ 1.41	$n =$ 1.25	$n =$ 1.41	$n =$ 1.25	$n =$ 1.41	$n =$ 1.25	$n =$ 1.41		
r	$\frac{1}{r}$	$\frac{v_2}{v_1}$	$\frac{v_2}{v_1}$	$\frac{t_2}{t_1}$	$\frac{t_2}{t_1}$	Ft.-Lbs.	Ft.-Lbs.	H.P.	H.P.	Ft.-Lbs.	H.P.
1	1.0000	1.000	1.000	1.000	1.000	0.0	0.0	.0	.0	0.0	.0
1.1	.9091	.927	.935	1.019	1.028	5.131	5.140	.0155	.0155	5.068	.0153
1.2	.8333	.862	.877	1.037	1.054	9.863	9.932	.0298	.0301	9.694	.0293
1.3	.7692	.812	.830	1.054	1.079	14.329	14.450	.0434	.0437	13.950	.0422
1.4	.7143	.764	.787	1.070	1.103	18.503	18.766	.0560	.0568	17.890	.0542
1.5	.6667	.723	.750	1.085	1.125	22.465	22.827	.0680	.0691	21.559	.0653
1.6	.6250	.687	.717	1.100	1.146	26.186	26.704	.0793	.0809	24.991	.0757
1.7	.5882	.654	.686	1.112	1.166	29.775	30.417	.0902	.0921	28.214	.0855
1.8	.5555	.625	.659	1.125	1.186	33.178	33.985	.1005	.1029	31.252	.0947
1.9	.5263	.598	.634	1.137	1.205	36.421	37.422	.1104	.1134	34.127	.1034
2.0	.5000	.574	.612	1.149	1.223	39.530	40.733	.1198	.1235	36.855	.1117
2.1	.4762	.552	.590	1.160	1.240	42.536	43.897	.1289	.1330	39.450	.1196
2.2	.4545	.532	.571	1.171	1.259	45.407	46.988	.1376	.1424	41.912	.1270
2.3	.4348	.514	.553	1.181	1.273	48.199	49.970	.1461	.1514	44.287	.1342
2.4	.4166	.496	.537	1.191	1.289	50.884	52.878	.1542	.1602	46.548	.1411
2.5	.4000	.480	.522	1.202	1.304	53.462	55.676	.1620	.1687	48.720	.1476
2.6	.3846	.466	.508	1.211	1.319	55.988	58.402	.1697	.1769	50.805	.1539
2.7	.3704	.452	.493	1.220	1.334	58.434	61.054	.1771	.1850	52.811	.1600
2.8	.3571	.439	.481	1.229	1.348	60.800	63.651	.1843	.1929	54.745	.1659
2.9	.3448	.427	.469	1.237	1.362	63.086	66.175	.1912	.2006	56.612	.1715
3.0	.3333	.415	.458	1.246	1.375	65.319	68.626	.1979	.2080	58.414	.1770
3.1	.3226	.405	.448	1.254	1.388	67.499	71.158	.2045	.2156	60.157	.1823
3.2	.3125	.394	.438	1.262	1.401	69.626	73.400	.2110	.2224	61.845	.1874
3.3	.3030	.385	.428	1.270	1.414	71.700	75.686	.2173	.2294	63.481	.1924
3.4	.2941	.376	.419	1.277	1.426	73.720	77.936	.2234	.2362	65.087	.1972
3.5	.2857	.367	.411	1.285	1.438	75.688	80.131	.2294	.2428	66.610	.2019
3.6	.2778	.359	.403	1.292	1.450	77.628	82.307	.2352	.2494	68.108	.2064
3.7	.2703	.351	.395	1.299	1.461	79.516	84.411	.2410	.2557	69.564	.2108
3.8	.2632	.343	.388	1.306	1.473	81.350	86.496	.2465	.2621	70.982	.2151
3.9	.2564	.337	.381	1.313	1.484	83.158	88.544	.2520	.2683	72.364	.2193
4.0	.2500	.330	.374	1.319	1.495	84.939	90.510	.2574	.2743	73.710	.2234
4.1	.2439	.323	.367	1.326	1.506	86.694	92.472	.2627	.2802	75.023	.2274
4.2	.2381	.317	.361	1.332	1.516	88.395	94.434	.2678	.2862	76.304	.2312
4.3	.2326	.311	.355	1.339	1.526	90.043	96.346	.2729	.2919	77.555	.2350
4.4	.2273	.306	.349	1.345	1.537	91.691	98.202	.2779	.2976	78.776	.2387
4.5	.2222	.300	.344	1.351	1.547	93.312	100.012	.2828	.3031	79.972	.2424
4.6	.2174	.295	.338	1.357	1.557	94.882	101.823	.2875	.3085	81.141	.2459
4.7	.2128	.290	.333	1.363	1.566	96.424	103.616	.2922	.3140	82.284	.2494
4.8	.2083	.285	.328	1.368	1.576	97.966	105.371	.2969	.3193	83.404	.2528

TABLE I (*Continued*).

1	2	3	4	5	6	7	8	9	10	11	12
4.9	.2041	.280	.324	1.374	1.586	99.481	107.109	.3015	.3246	84.500	.2561
5.0	.2000	.276	.319	1.380	1.595	100.943	108.811	.3059	.3297	85.574	.2593
5.1	.1961	.272	.315	1.385	1.604	102.405	110.493	.3103	.3348	86.627	.2625
5.2	.1923	.267	.310	1.391	1.613	103.841	112.157	.3147	.3398	87.660	.2657
5.3	.1887	.263	.306	1.396	1.622	105.260	113.830	.3180	.3449	88.673	.2687
5.4	.1852	.259	.302	1.401	1.631	106.673	115.440	.3232	.3498	89.666	.2717
5.5	.1818	.256	.298	1.406	1.640	108.013	117.010	.3273	.3546	90.642	.2747
5.6	.1786	.252	.294	1.411	1.648	109.353	118.570	.3314	.3593	91.600	.2776
5.7	.1754	.248	.291	1.416	1.657	110.683	120.114	.3354	.3640	92.541	.2805
5.8	.1722	.245	.287	1.421	1.665	112.003	121.632	.3394	.3686	93.466	.2833
5.9	.1695	.242	.284	1.426	1.673	113.305	123.150	.3433	.3732	94.375	.2860
6.0	.1667	.238	.280	1.431	1.681	114.581	124.640	.3472	.3777	95.271	.2887
6.1	.1639	.235	.277	1.436	1.689	115.831	126.113	.3510	.3822	96.147	.2914
6.2	.1613	.232	.274	1.440	1.697	117.080	127.576	.3548	.3866	97.012	.2940
6.3	.1587	.229	.271	1.445	1.705	118.303	129.030	.3585	.3910	97.863	.2966
6.4	.1562	.226	.268	1.449	1.713	119.573	130.466	.3622	.3953	98.700	.2991
6.5	.1538	.223	.265	1.454	1.721	120.723	131.880	.3658	.3997	99.524	.3016
6.6	.1515	.221	.262	1.458	1.728	121.920	133.300	.3694	.4039	100.336	.3040
6.7	.1492	.219	.259	1.464	1.736	123.063	134.710	.3729	.4082	101.134	.3065
6.8	.1471	.216	.256	1.467	1.744	124.205	136.090	.3764	.4124	101.920	.3088
6.9	.1449	.213	.254	1.471	1.751	125.348	137.450	.3799	.4165	102.700	.3112
7.0	.1428	.211	.251	1.476	1.758	126.492	138.800	.3833	.4206	103.465	.3135
7.1	.1408	.208	.249	1.480	1.766	127.608	140.120	.3867	.4246	104.219	.3158
7.2	.1389	.206	.246	1.484	1.773	128.708	141.430	.3900	.4286	104.963	.3181
7.3	.1370	.204	.244	1.488	1.780	129.789	142.710	.3933	.4327	105.696	.3203
7.4	.1351	.202	.241	1.492	1.787	130.878	143.979	.3966	.4363	106.420	.3225
7.5	.1333	.199	.239	1.496	1.794	131.941	145.239	.3998	.4401	107.133	.3246
7.6	.1316	.197	.237	1.500	1.801	132.995	146.489	.4030	.4439	107.837	.3268
7.7	.1299	.195	.235	1.504	1.807	134.043	147.732	.4062	.4477	108.539	.3289
7.8	.1282	.193	.233	1.508	1.814	135.063	148.976	.4093	.4514	109.219	.3310
7.9	.1266	.191	.231	1.512	1.821	136.091	150.217	.4124	.4552	109.896	.3330
8.0	.1250	.189	.228	1.516	1.828	137.110	151.427	.4155	.4589	110.565	.3350
8.1	.1236	.188	.226	1.519	1.834	138.111	152.633	.4185	.4625	111.225	.3370
8.2	.1220	.186	.224	1.523	1.841	139.093	153.823	.4215	.4661	111.875	.3390
8.3	.1205	.184	.223	1.527	1.847	140.076	155.010	.4245	.4698	112.522	.3410
8.4	.1190	.182	.221	1.531	1.854	141.060	156.178	.4275	.4733	113.158	.3429
8.5	.1176	.180	.219	1.534	1.861	142.017	157.348	.4304	.4768	113.788	.3448
8.6	.1163	.179	.217	1.538	1.867	142.974	158.508	.4333	.4804	114.410	.3465
8.7	.1149	.177	.215	1.541	1.873	143.931	159.658	.4362	.4838	115.023	.3487
8.8	.1136	.176	.214	1.545	1.879	144.862	160.800	.4390	.4873	115.633	.3504
8.9	.1124	.174	.212	1.548	1.885	145.780	161.927	.4418	.4906	116.233	.3522
9.0	.1111	.172	.210	1.552	1.891	146.700	163.041	.4446	.4941	116.827	.3540
9.1	.1099	.171	.208	1.555	1.897	147.627	164.147	.4474	.4974	117.415	.3558
9.2	.1087	.170	.207	1.559	1.903	148.557	165.236	.4502	.5007	117.996	.3576
9.3	.1072	.168	.205	1.562	1.909	149.554	166.334	.4532	.5041	118.571	.3593
9.4	.1064	.167	.204	1.565	1.915	150.312	167.431	.4555	.5074	119.138	.3610
9.5	.1058	.165	.202	1.569	1.921	151.188	168.520	.4582	.5107	119.702	.3627
9.6	.1042	.164	.201	1.572	1.927	152.066	169.589	.4609	.5139	120.259	.3644
9.7	.1031	.162	.299	1.575	1.933	152.944	170.650	.4635	.5171	120.810	.3661
9.8	.1020	.161	.298	1.578	1.939	153.794	171.700	.4661	.5213	121.355	.3677
9.9	.1010	.160	.296	1.582	1.944	154.645	172.754	.4686	.5235	121.895	.3693
10.0	.1000	.159	.295	1.585	1.950	155.495	173.789	.4712	.5266	122.429	.3710

NOTES ON TABLE II.

The purpose of this table is to determine the weight of air compressed by a machine of known cubic feet capacity. It is to be used in connection with Table I for determining power or work.

The barometric readings and elevations are made out for a uniform temperature of 60° F. and are subject to slight errors but not enough to materially affect results. Table V gives more accurately the relation between elevation temperature and pressure.

TABLE II. — WEIGHTS OF FREE AIR UNDER VARIOUS CONDITIONS

Approximate Barometric Reading. $T = 60$.	Atmospheric Pressure.	Weight of One Cubic Foot at Given Temperature (Fahr.)							Approximate Elevation. $T = 60°$.
		− 20°	00°	20°	40°	60°	80°	100°	
30.52	15.0	.09211	.08811	.08444	.08108	.07796	.07508	.07240	−600
30.32	14.9	.09150	.08753	.08388	.08054	.07744	.07458	.07192	−400
30.12	14.8	.09089	.08694	.08331	.08000	.07693	.07408	.07144	−200
29.91	14.7	.09027	.08635	.08275	.07945	.07640	.07358	.07095	00
29.71	14.6	.08965	.08576	.08219	.07895	.07589	.07308	.07047	200
29.50	14.5	.08903	.08517	.08163	.07837	.07536	.07258	.06999	400
29.30	14.4	.08842	.08458	.08107	.07783	.07484	.07208	.06950	600
29.10	14.3	.08781	.08400	.08050	.07729	.07432	.07158	.06902	800
28.90	14.2	.08719	.08341	.07994	.07675	.07380	.07108	.06854	1000
28.69	14.1	.08659	.08282	.07938	.07621	.07329	.07058	.06806	1200
28.49	14.0	.08597	.08224	.07882	.07567	.07277	.07008	.06758	1400
28.28	13.9	.08535	.08165	.07825	.07513	.07225	.06957	.06709	1600
28.08	13.8	.08474	.08106	.07769	.07459	.07173	.06907	.06661	1800
27.88	13.7	.08412	.08048	.07713	.07405	.07120	.06857	.06612	2000
27.67	13.6	.08351	.07989	.07656	.07350	.07068	.06807	.06564	2100
27.47	13.5	.08289	.07930	.07600	.07296	.07016	.06757	.06516	2300
27.27	13.4	.08228	.07871	.07544	.07242	.06965	.06707	.06468	2500
27.06	13.3	.08167	.07813	.07487	.07189	.06913	.06657	.06420	2700
26.86	13.2	.08106	.07754	.07431	.07135	.06861	.06607	.06371	2900
26.66	13.1	.08044	.07695	.07375	.07080	.06809	.06557	.06323	3100
26.45	13.0	.07983	.07637	.07319	.07026	.06757	.06507	.06274	3300
26.25	12.9	.07921	.07578	.07262	.06972	.06705	.06457	.06226	3500
26.05	12.8	.07860	.07518	.07206	.06918	.06652	.06407	.06178	3700
25.84	12.7	.07798	.07460	.07150	.06862	.06600	.06357	.06130	4000
25.64	12.6	.07737	.07401	.07094	.06810	.06549	.06307	.06082	4200
25.44	12.5	.07676	.07343	.07038	.06756	.06497	.06257	.06033	4400
25.23	12.4	.07615	.07284	.06981	.06702	.06445	.06207	.05985	4600

1/20 + 5

.0

TABLE II. — *Continued.*

25.03	12.3	.07553	.07225	.06925	.06648	.06393	.06157	.05937	4800
24.83	12.2	.07492	.07166	.06868	.06594	.06341	.06107	.05889	5000
24.62	12.1	.07430	.07108	.06812	.06540	.06289	.06057	.05840	5200
24.42	12.0	.07369	.07049	.06756	.06486	.06237	.06007	.05792	5400
24.22	11.9	.07307	.06990	.06699	.06432	.06185	.05957	.05744	5600
24.01	11.8	.07246	.06932	.06643	.06378	.06133	.05907	.05696	5800
23.81	11.7	.07184	.06873	.06587	.06324	.06081	.05857	.05647	6100
23.60	11.6	.07123	.06812	.06530	.06270	.06029	.05807	.05599	6300
23.40	11.5	.07061	.06755	.06474	.06216	.05977	.05757	.05551	6500
23.20	11.4	.07000	.06693	.06418	.06161	.05925	.05707	.05502	6800
22.99	11.3	.06938	.06638	.06362	.06108	.05873	.05656	.05454	7100
22.79	11.2	.06877	.06579	.06305	.06054	.05821	.05606	.05406	7300
22.59	11.1	.06816	.06520	.06249	.06000	.05769	.05556	.05358	7600
22.38	11.0	.06754	.06462	.06193	.05945	.05717	.05506	.05310	7900
22.18	10.9	.06692	.06403	.06136	.05891	.05665	.05456	.05261	8100
21.98	10.8	.06632	.06344	.06080	.05837	.05613	.05406	.05213	8400
21.77	10.7	.06571	.06285	.06024	.05783	.05561	.05356	.05164	8600
21.57	10.6	.06510	.06226	.05968	.05729	.05509	.05306	.05116	8900
21.37	10.5	.06448	.06168	.05911	.05675	.05457	.05256	.05068	9100
21.16	10.4	.06386	.06109	.05855	.05621	.05405	.05206	.05020	9400
20.96	10.3	.06325	.06050	.05799	.05567	.05353	.05156	.04972	9600
20.76	10.2	.06263	.05991	.05743	.05513	.05301	.05106	.04923	9900
20.55	10.1	.06202	.05933	.05686	.05459	.05249	.05056	.04875	10100
20.35	10.0	.06141	.05874	.05630	.05405	.05198	.05006	.04827	10400
20.15	9.9	.06079	.05816	.05572	.05351	.05146	.04956	.04779	10700
19.94	9.8	.06017	.05757	.05517	.05297	.05094	.04906	.04730	11000
19.74	9.7	.05956	.05698	.05461	.05243	.05041	.04856	.04682	11200
19.53	9.6	.05894	.05639	.05404	.05188	.04990	.04806	.04633	11500
19.33	9.5	.05833	.05580	.05348	.05134	.04937	.04756	.04585	11800
19.13	9.4	.05772	.05522	.05292	.05081	.04886	.04706	.04538	12100
18.93	9.3	.05711	.05463	.06236	.05027	.04834	.04655	.04489	12400
18.72	9.2	.05649	.05404	.05179	.04972	.04782	.04605	.04440	12700
18.52	9.1	.05587	.05345	.05123	.04918	.04730	.04555	.04392	13000
18.31	9.0	.05526	.05286	.05067	.04864	.04678	.04505	.04344	13400

NOTE ON TABLE III.

The table is designed to compute readily weights of compressed air by formula 12, Art. 8, viz., $w = \frac{p}{53.17\,t}$. If p is given in pounds per square inch the formula becomes $w = \frac{144 \times p}{53.17 \times t}$.

The value $\frac{p}{t}$ can most readily be obtained with the slide rule.

TABLE III. — WEIGHTS OF COMPRESSED AIR

Pounds per Cubic Foot.

The Ratio $\frac{p}{t}$ is for absolute pressure in pounds per square inch and absolute temperature Fahrenheit. (See Note at foot of previous page.)

$\frac{p}{t}$	w	$\frac{p}{t}$	w	$\frac{p}{t}$	w	$\frac{p}{t}$	w
.000	0.0000	.255	.6906	.510	1.3813	.765	2.0718
.005	.0135	.260	.7041	.515	1.3947	.770	2.0853
.010	.0271	.265	.7177	.520	1.4083	.775	2.0988
.015	.0406	.270	.7312	.525	1.4219	.780	2.1125
.020	.0542	.275	.7447	.530	1.4355	.785	2.1260
.025	.0677	.280	.7583	.535	1.4490	.790	2.1395
.030	.0813	.285	.7719	.540	1.4625	.795	2.1530
.035	.0948	.290	.7852	.545	1.4760	.800	2.1665
.040	.1083	.295	.7989	.550	1.4895	.805	2.1798
.045	.1218	.300	.8125	.555	1.5030	.810	2.1950
.050	.1354	.305	.8260	.560	1.5166	.815	2.2071
.055	.1489	.310	.8395	.565	1.5312	.820	2.2207
.060	.1625	.315	.8531	.570	1.5437	.825	2.2343
.065	.1760	.320	.8666	.575	1.5572	.830	2.2480
.070	.1896	.325	.8801	.580	1.5707	.835	2.2615
.075	.2031	.330	.8937	.585	1.5843	.840	2.2750
.080	.2166	.335	.9072	.590	1.5980	.845	2.2885
.085	.2302	.340	.9208	.595	1.6115	.850	2.3020
.090	.2437	.345	.9343	.600	1.6250	.855	2.3155
.095	.2573	.350	.9478	.605	1.6385	.860	2.3290
.100	.2708	.355	.9613	.610	1.6520	.865	2.3425
.105	.2843	.360	.9749	.615	1.6654	.870	2.3561
.110	.2979	.365	.9884	.620	1.6792	.875	2.3698
.115	.3114	.370	1.0020	.625	1.6927	.880	2.3833
.120	.3250	.375	1.0155	.630	1.7062	.885	2.3970
.125	.3385	.380	1.0290	.635	1.7198	.890	2.4105
.130	.3520	.385	1.0425	.640	1.7333	.895	2.4240
.135	.3656	.390	1.0561	.645	1.7468	.900	2.4375
.140	.3792	.395	1.0697	.650	1.7603	.905	2.4510
.145	.3927	.400	1.0833	.655	1.7739	.910	2.4645
.150	.4062	.405	1.0968	.660	1.7875	.915	2.4780
.155	.4197	.410	1.1103	.665	1.8010	.920	2.4917
.160	.4333	.415	1.1240	.670	1.8145	.925	2.5052
.165	.4468	.420	1.1375	.675	1.8280	.930	2.5187
.170	.4603	.425	1.1510	.680	1.8415	.935	2.5323
.175	.4739	.430	1.1645	.685	1.8550	.940	2.5459
.180	.4875	.435	1.1780	.690	1.8680	.945	2.5594
.185	.5010	.440	1.1917	.695	1.8822	.950	2.5730
.190	.5145	.445	1.2052	.700	1.8959	.955	2.5865
.195	.5281	.450	1.2177	.705	1.9094	.960	2.6000
.200	.5416	.455	1.2323	.710	1.9229	.965	2.6135
.205	.5551	.460	1.2457	.715	1.9365	.970	2.6270
.210	.5687	.465	1.2594	.720	1.9500	.975	2.6405
.215	.5822	.470	1.2730	.725	1.9635	.980	2.6541
.220	.5958	.475	1.2865	.730	1.9770	.985	2.6670
.225	.6094	.480	1.3000	.735	1.9905	990	2.6813
.230	.6229	.485	1.3135	.740	2.0042	.995	2.6949
.235	.6364	.490	1.3270	.745	2.0177	1.000	2.7084
.240	.6499	.495	1.3416	.750	2.0312		
.245	.6635	.500	1.3542	.755	2.0448		
.250	.6771	.505	1.3677	.760	2.0582		

TABLE IV.* — SPECIAL TABLE RELATING TO STAGE COMPRESSION FROM FREE AIR AT 14.7 POUNDS PRESSURE AND 62° TEMPERATURE.

Compression adiabatic but cooled between stages.

			Single Stage.			Two Stage.		
Gage Pressure.	Ratio of Compression.	Weight of One Cubic Foot at Temperature 62° F.	Mean Effective Pressure.	Final Temperature, Fahrenheit.	Horse Power to Compress One Cu. Ft. of Free Air per Minute.	Ratio of Compression in Each Stage.	Final Temperature in Each Stage, Fahrenheit.	Horse Power to Compress One Cu. Ft. of Free Air per Minute.
P_g	r	w	M.E.P.	T_1	H.P.	$\sqrt{r}$	T_2	H.P.
5	1.34	.1020	4.50	108	.0197			
10	1.68	.1279	8.30	144	.0362			
15	2.02	.1537	11.51	177	.0045			
20	2.36	.1796	14.40	207	.0628			
25	2.70	.2055	17.00	235	.0742			
30	3.04	.2313	19.40	259	.0845			
35	3.38	.2572	21.65	280	.0944			
40	3.72	.2831	23.60	303	.1030			
45	4.06	.3090	25.50	321	.1112			
50	4.40	.3348	27.50	341	.1195	2.10	180	.1063
55	4.74	.3607	29.10	358	.1268	2.17	189	.1123
60	5.08	.3866	30.75	373	.1339	2.25	196	.1184
65	5.42	.4124	32.30	392	.1408	2.33	200	.1235
70	5.76	.4383	33.80	405	.1472	2.40	207	.1286
75	6.10	.4642	35.18	420	.1532	2.47	214	.1329
80	6.44	.4901	36.55	434	.1590	2.54	222	.1372
85	6.78	.5159	37.90	447	.1650	2.60	227	.1410
90	7.12	.5418	39.10	461	.1705	2.67	233	.1462
95	7.46	.5676	40.35	473	.1758	2.73	238	.1500
100	7.80	.5935	41.65	485	.1812	2.79	242	.1542
105	8.14	.6194	42.30	497	.1841	2.85	246	.1578
110	8.48	.6453	43.75	508	.1908	2.90	251	.1615
115	8.82	.6712	45.16	519	.1965	2.99	256	.1648
120	9.16	.6971	46.00	530	.2008	3.02	259	.1681
125	9.50	.7230	47.05	540	.2045	3.08	262	.1710
130	9.84	.7488	47.80	550	.2085	3.14	266	.1740
135	10.18	.7747	48.85	560	.2135	3.19	269	.1775
140	10.52	.8005	49.90	569	.2176	3.24	272	.1810
145	10.86	.8264	51.00	578	.2220	3.29	276	.1837
150	11.20	.8522	51.70	587	.2255	3.35	280	.1865

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* The table is limited to the special initial condition of air specified in the caption. The assumption of 14.7 as atmospheric pressure makes the weights and work a little in excess of average conditions. However, it is a valuable and very instructive table.

TABLE IV (*Continued*).

Gage Pressure.	Ratio of Compression.	Weight of One Cubic Foot of Air at 62° F.	Two Stage.			Three Stage.		
			Ratio of Compression in Each Stage.	Final Temperature in Each Stage, Fahrenheit.	Horse Power to Compress One Cu. Ft. of Free Air per Minute.	Ratio of Compression in Each Stage.	Final Temperature in Each Stage, Fahrenheit.	Horse Power to Compress One Cu. Ft. of Free Air per Minute.
P_g	r	w	$(r)^{\frac{1}{2}}$	T_2	H.P.	$(r)^{\frac{1}{3}}$	T_3	H.P.
100	7.8	.5936	2.79	242	.1542	1.98	176	.1450
150	11.2	.8522	3.35	280	.1865	2.24	200	.1752
200	14.6	1.1110	3.82	308	.2110	2.44	215	.1965
250	18.0	1.3697	4.24	332	.2315	2.62	226	.2140
300	21.4	1.6285	4.63	353	.2490	2.78	241	.2295
350	24.8	1.8872	4.98	370	.2640	2.92	251	.2418
400	28.2	2.1459	5.31	386	.2770	3.04	259	.2535
450	31.6	2.4048	5.61	399	.2895	3.16	267	.2630
500	35.0	2.6634	5.91	412	.2915	3.27	275	.2730
550	38.4	2.9221				3.37	281	.2830
600	41.8	3.1810				3.47	287	.2910
650	45.2	3.4395				3.56	292	.2960
700	48.6	3.6982				3.64	297	.3025
750	52.0	3.9570				3.73	302	.3090
800	55.4	4.2155				3.80	307	.3150
850	58.8	4.4745				3.83	312	.3210
900	62.2	4.7330				3.96	316	.3260
950	65.6	4.9920				4.03	320	.3315
1000	69.0	5.2510				4.10	324	.3360
1050	72.4	5.5095				4.17	328	.3400
1100	75.8	5.7684				4.23	331	.3445
1150	79.2	6.0270				4.29	334	.3490
1200	82.6	6.2855				4.36	337	.3525
1250	86.0	6.5445				4.41	341	.3570
1300	89.4	6.8030				4.47	344	.3615
1350	92.8	7.0620				4.52	347	.3660
1400	96.2	7.3210				4.58	350	.3685
1450	99.6	7.5795				4.64	353	.3710
1500	103.0	7.8382				4.70	356	.3740
1550	106.4	8.0965				4.75	359	.3780
1600	109.8	8.3550				4.79	361	.3820
1650	113.2	8.6140				4.83	363	.3850
1700	116.6	8.8730				4.87	365	.3880
1750	120.0	9.1320				4.93	367	.3915
1800	123.4	9.3900				4.97	369	.3940
1850	126.8	9.6485				5.02	371	.3965

TABLE V. — VARYING PRESSURES WITH ELEVATIONS.

Solution of formula 17, Art. 17, viz. $\log_{10} p_a = 1.16866 - \dfrac{h}{122.4\, t}$.

Elevation in Feet.	Pressure in Pounds per Square Inch.		
	Temp. 50° F.	Temp 35° F.	Temp. 20° F.
0	14.70	14.70	14.70
1000	14.17	14.15	14.14
2000	13.66	13.63	13.99
3000	13.16	13.12	13.07
4000	12.69	12.63	12.57
5000	12.23	12.16	12.09
5280	12.10	12.03	11.96
6000	11.78	11.71	11.63
7000	11.36	11.27	11.18
8000	10.95	10.85	10.75
9000	10.55	10.45	10.33
10000	10.17	10.06	9.94
12500	9.28	9.15	9.02
15000	8.46	8.32	8.18

TABLE VI.* — HIGHEST LIMIT TO EFFICIENCY WHEN COMPRESSED AIR IS USED WITHOUT EXPANSION, ASSUMING ATMOSPHERIC PRESSURE = 14.5 POUNDS PER SQUARE INCH.

r	*h*	*E*	*r*	*h*	*E*	*r*	*h*	*E*
1.2	6.66	91.4	5.2	140.0	49.0	9.2	273.3	40.2
1.4	13.3	84.9	5.4	146.6	48.3	9.4	280.0	39.9
1.6	20.0	79.8	5.6	153.3	47.7	9.6	286.6	39.6
1.8	26.6	75.6	5.8	160.0	47.0	9.8	293.3	39.3
2.0	33.3	72.0	6.0	166.6	46.5	10.0	300.0	39.0
2.2	40.0	69.2	6.2	173.3	46.0	10.25	308.3	38.6
2.4	46.6	66.7	6.4	180.0	45.5	10.50	316.6	38.5
2.6	53.3	61.9	6.6	186.6	45.0	10.75	325.0	38.0
2.8	60.0	62.4	6.8	193.3	44.5	11.00	333.3	37.9
3.0	66.6	60.7	7.0	200.0	44.0	11.25	341.6	37.7
3.2	73.3	59.1	7.2	206.6	43.6	11.50	350.0	37.4
3.4	80.0	57.8	7.4	213.3	43.1	11.75	353.3	37.1
3.6	86.6	56.4	7.6	220.0	42.8	12.00	366.6	36.9
3.8	93.3	55.2	7.8	226.6	42.4	12.25	375.0	36.7
4.0	100.0	54.1	8.0	233.3	42.0	12.50	383.3	36.4
4.2	106.6	53.1	8.2	240.0	41.7	12.75	391.6	36.2
4.4	113.3	52.1	8.4	246.6	41.4	13.0	400.0	36.0
4.6	120.0	51.3	8.6	253.3	41.1	14.0	433.3	35.2
4.8	126.6	50.5	8.8	260.0	40.8	15.0	466.6	34.5
5.0	133.3	49.7	9.0	266.6	40.5	16.0	500.0	33.8

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* This table reveals the limit of efficiency when air is applied without utilizing any of its expansive energy.

The column headed *r* gives the ratio of compression, while that headed *h* gives the water head equivalent to a pressure given by the ratio *r* on the assumption that one atmosphere is a pressure of 14.5 pounds per square inch or a water head of 33.3 feet, this being more nearly the average condition than 14.7, which is so commonly taken.

It should be understood that this efficiency cannot be reached in practice, — it being reduced by friction of air and machinery and by clearance in any form of engine.

TABLE VII. — EFFICIENCY OF DIRECT HYDRAULIC AIR COMPRESSORS.

Formula 26, Art. 25, viz. $E = \frac{2.3 \log_{10} r}{r - 1}$.

Water Head.	Gage Pressure.	Absolute Pressure.	Atmospheres $= r$	Efficiency, E
0.0	0.0	14.5	1	1.00
33.3	14.5	29.0	2	.69
66.6	29.0	43.5	3	.55
100.0	43.5	58.0	4	.46
133.3	58.0	72.5	5	.40
166.6	72.5	87.0	6	.36
200.0	87.0	101.5	7	.33
233.3	101.5	116.0	8	.30
266.0	116.0	130.5	9	.28
300.0	130.5	145.0	10	.26

TABLE VIII. — COEFFICIENT "c" FOR VARIOUS HEADS AND DIAMETERS.

d''	$i = 1''$	$i = 2''$	$i = 3''$	$i = 4''$	$i = 5''$
$\frac{5}{16}$	0.603	0.606	0.610	0.613	0.616
$\frac{1}{2}$	0.602	0.605	0.608	0.610	0.613
1	0.601	0.603	0.605	0.606	0.607
$1\frac{1}{2}$	0.601	0.601	0.602	0.603	0.603
2	0.600	0.600	0.600	0.600	0.600
$2\frac{1}{2}$	0.599	0.599	0.599	0.598	0.598
3	0.599	0.598	0.597	0.596	0.596
$3\frac{1}{2}$	0.599	0.597	0.596	0.595	0.594
4	0.598	0.597	0.595	0.594	0.593
$4\frac{1}{2}$	0.598	0.596	0.596	0.593	0.592

Table VIII gives the experimental coefficients for orifices for determining the weight of air passing by formula:

$$\text{Weight } (Q) = 0.6299\, c d^2 \sqrt{\frac{i}{t}}$$

Q = Weight of air passing in pounds per second.
c = Experimental coefficient.
d = Diameter of orifice in inches.
i = Difference of pressure inside and outside of orifice in inches of water.
t = Absolute temperature of air back of orifice.

TABLE IX. — FRICTION IN AIR PIPES.

Divide the number corresponding to the diameter and volume by the ratio of compression. The result is the loss in pounds per square inch in 1000 feet of pipe.

Cubic Feet of Free Air per Minute.	Diameter of Pipe in Inches.								
	½	¾	1	1¼	1½	1¾	2	2½	3
6	27.3	35.4	.83	.26					
12	108.3	14.26	3.32	1.05					
24		56.64	13.28	4.20	1.71	.78			
36		126.4	29.86	9.45	3.84	1.75			
48		226.6	53.15	16.80	6.83	3.12	1.60		
60			84.94	26.26	10.70	4.87	2.50		
72			119.8	37.90	15.40	7.03	3.62	1.17	
84			163.7	51.46	20.90	9.55	4.91	1.59	
96				67.21	27.30	12.48	6.41	2.07	
108				85.06	34.55	15.80	8.12	2.62	
120				105.0	42.67	19.50	10.00	3.25	
150				164.1	66.53	30.47	15.66	5.06	1.85
180					96.00	43.87	22.54	7.28	2.67
210					130.7	59.71	30.70	9.91	3.63
	1¾	2	2½	3	3½	4	4½	5	6
240	78.00	40.09	12.94	4.74	2.13				
270	98.70	50.72	16.48	6.00	2.70				
300	121.8	62.62	20.23	7.41	3.33				
330		75.78	24.57	8.97	4.03				
360		90.29	29.12	10.67	4.80				
390		105.5	34.20	12.53	5.63				
420		122.8	39.64	14.52	6.53	2.87			
450			45.50	16.67	7.49	3.30			
480			51.88	18.97	8.53	3.75			
510			58.44	21.42	9.62	4.23			
540			65.39	24.01	10.79	4.75			
570			73.00	26.75	12.02	5.29	2.94		
600			80.90	29.64	13.32	5.86	3.25		
660			97.90	35.87	16.12	7.09	3.93		
720			116.50	42.68	19.19	8.43	4.68		
780				50.10	22.50	9.00	5.50	3.25	
840				58.10	26.11	11.48	6.37	3.76	
900				66.70	29.98	13.18	7.32	4.32	
960				75.88	34.10	15.00	8.32	4.92	
1020				85.65	38.50	16.93	9.40	5.55	2.23
1080				96.04	43.17	18.98	10.53	6.22	2.50
1140				107.00	48.10	21.15	11.74	6.93	2.79
1200					53.29	23.44	13.01	7.68	3.09
1320					64.49	28.36	15.74	9.29	3.73
1440					76.74	33.75	18.73	11.06	4.44
1560					90.05	39.61	21.89	12.98	5.22
1680					104.45	45.95	25.50	16.78	6.05

TABLE IX (*Continued*).

Cubic Feet of Free Air per Minute.	Diameter of Pipe in Inches						
	4	4½	5	6	8	10	12
1800	52.73	28.23	17.82	6.95	1.65		
1920	60.00	33.30	19.66	7.90	1.87		
2040	67.74	37.59	22.20	8.92	2.12		
2160	75.94	42.15	24.89	10.00	2.37		
2280	84.60	46.95	27.65	11.14	2.64		
2400	93.74	52.02	30.72	12.35	2.93	0.96	
2520		53.38	33.87	13.61	3.23	1.06	
2640		62.96	37.17	14.94	3.55	1.16	
2780		68.81	40.66	16.33	3.88	1.27	
2880		74.92	44.78	17.78	4.22	1.38	
3000		81.30	48.00	19.29	4.58	1.50	
3300		98.37	58.08	23.34	5.54	1.81	
3600			69.13	27.78	6.59	2.16	0.87
3900			81.13	32.60	7.74	2.53	1.02
4200			94.09	37.81	8.97	2.94	1.18
4500				43.41	10.30	3.37	1.36
4800				49.39	11.72	3.84	1.54
5100				55.76	13.23	4.34	1.74
5400				62.51	14.83	4.86	1.95
5700				69.62	16.53	5.41	2.18
6000				77.18	18.31	6.00	2.41
6600					22.16	7.26	2.92
7200					26.37	8.64	3.47
7800					30.95	10.10	4.07
8400					35.90	11.76	4.73
9000					41.20	13.50	5.40
9600					46.88	15.36	6.17
10200					52.92	17.34	6.97
10800					59.36	19.44	7.81
11400					66.11	21.66	8.70
12000					73.25	24.00	9.64
13200						29.04	11.67
14400						34.56	13.89
15600						40.56	16.30
16800						47.40	18.90
18000						54.00	21.70
19200						61.43	24.70
20400						69.36	27.87
21000						77.75	31.25
22800						86.64	34.82
24000						96.00	38.58

This table represents the author's formula 20, Chap. IV.,

$$f = c \frac{l}{d^5} \frac{v_a^2}{r}.$$

f = Loss of pressure in pounds per square inch.
c = An experimental coefficient.
l = Length of pipe in feet.
d = Diameter of pipe in inches.
v_a = Cubic feet of free air passing per second.
r = Ratio of compression from free air.

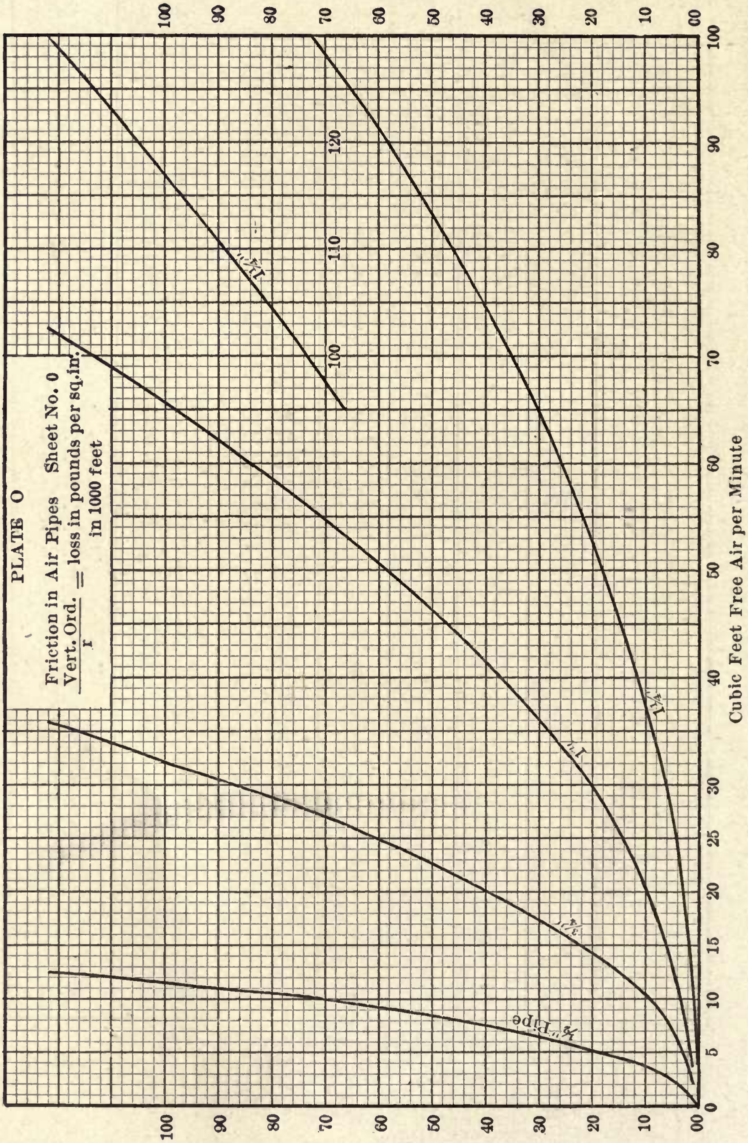

PLATE O

Friction in Air Pipes Sheet No. 0

$\frac{\text{Vert. Ord.}}{r}$ = loss in pounds per sq.in. in 1000 feet

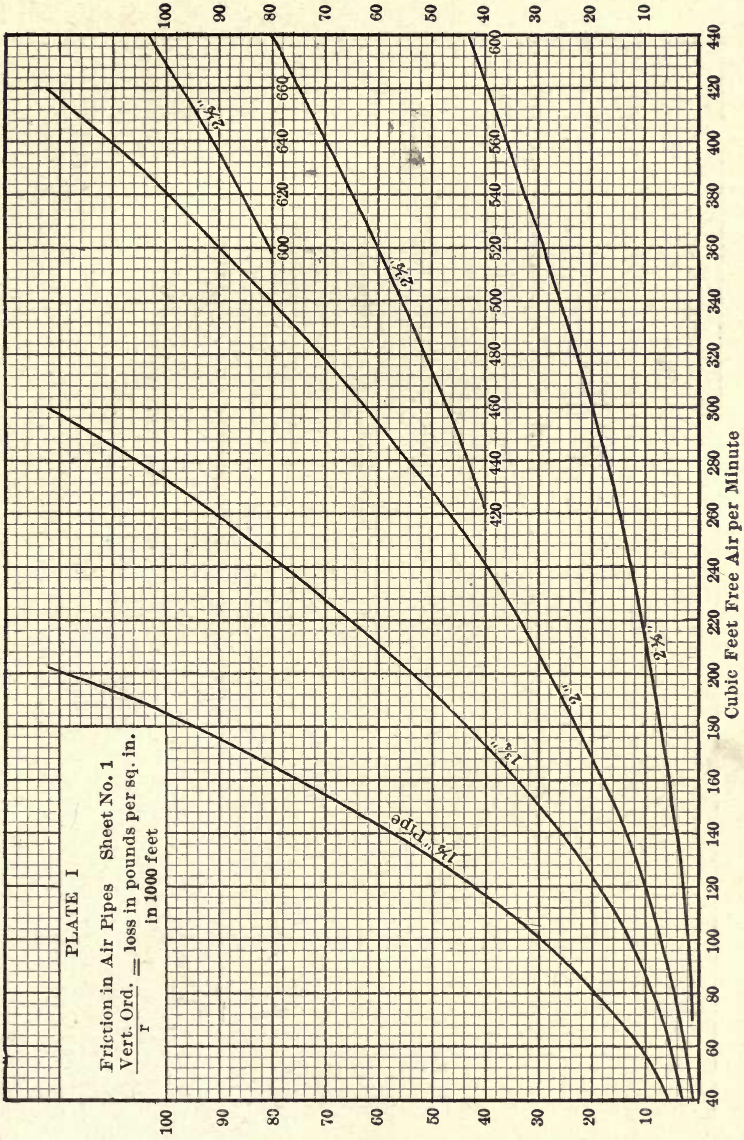

PLATE I

Friction in Air Pipes Sheet No. 1

$\frac{\text{Vert. Ord.}}{r}$ = loss in pounds per sq. in. in 1000 feet

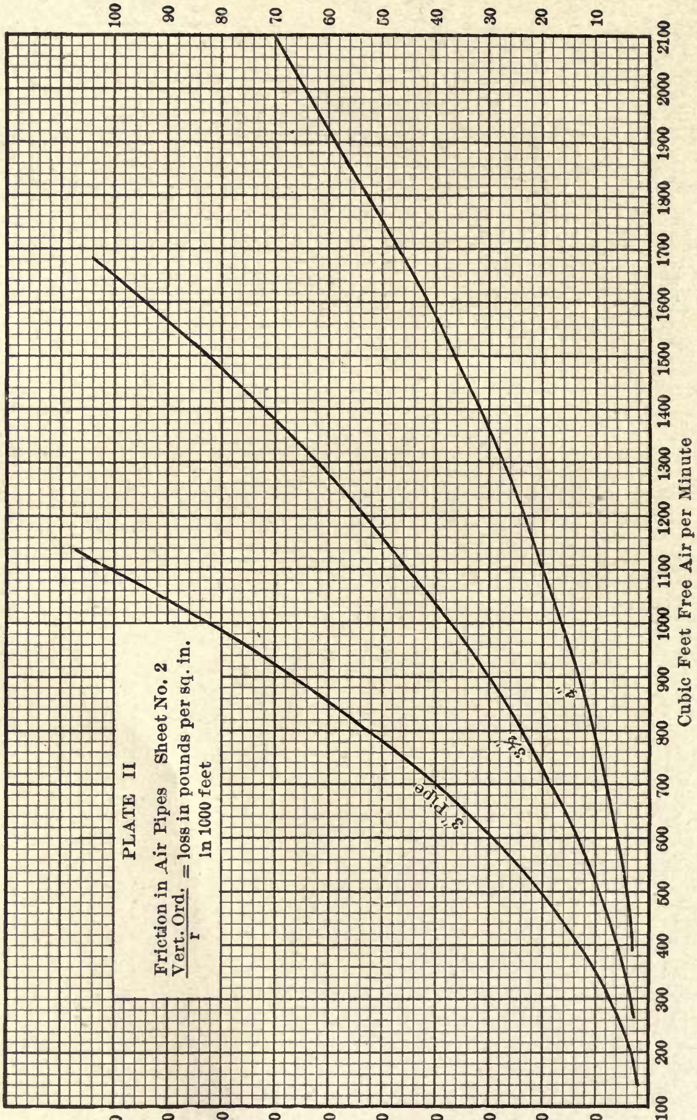

PLATE II

Friction in Air Pipes Sheet No. 2

$\frac{\text{Vert. Ord.}}{r}$ = loss in pounds per sq. in. in 1000 feet

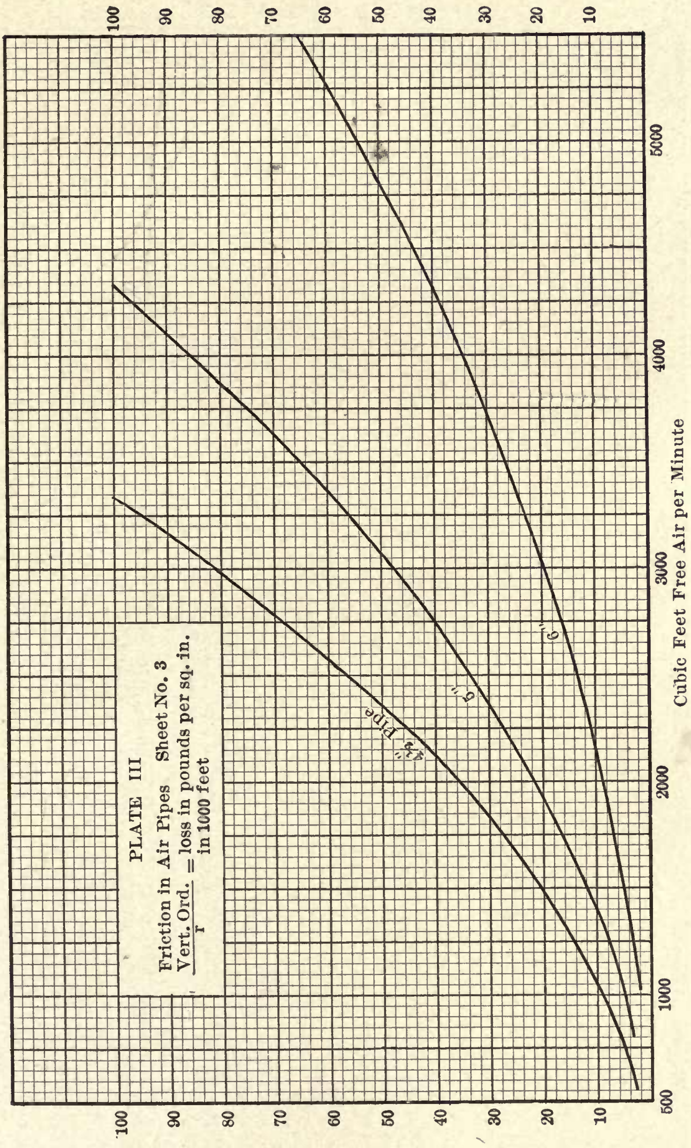

PLATE III

Friction in Air Pipes Sheet No. 3

$\frac{\text{Vert. Ord.}}{r}$ = loss in pounds per sq. in. in 1000 feet

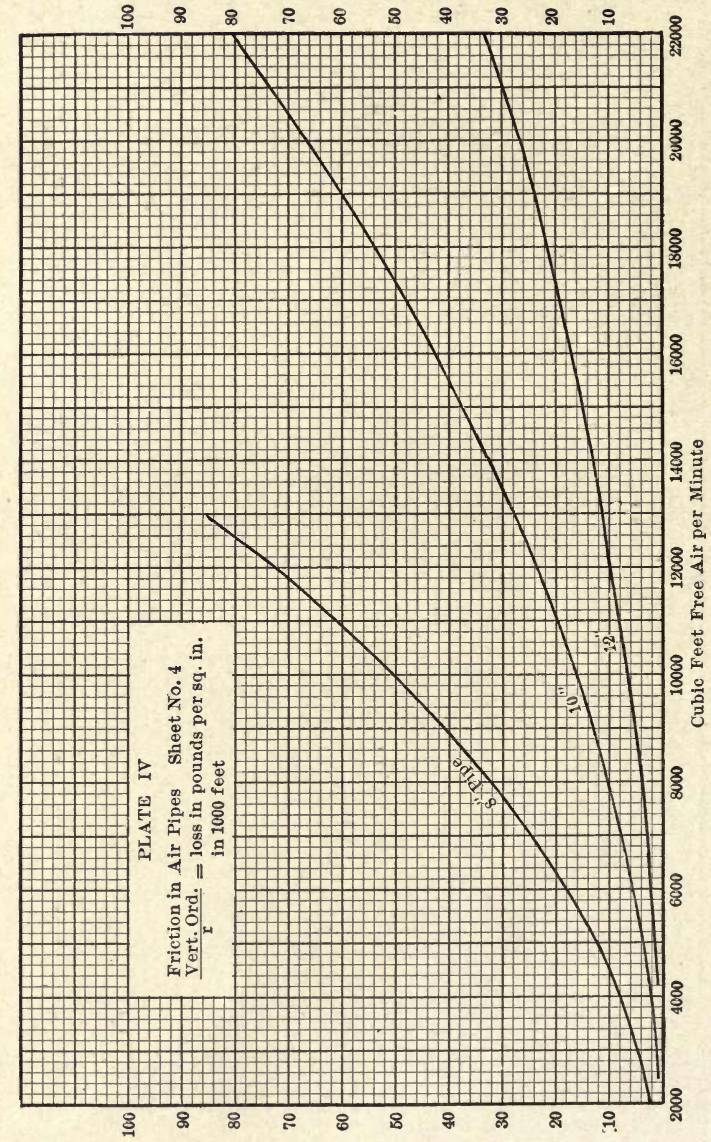
PLATE IV
Friction in Air Pipes Sheet No. 4
Vert. Ord./r = loss in pounds per sq. in. in 1000 feet
8" Pipe
10"
12"
100
90
80
70
60
50
40
30
20
10
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
22000
Cubic Feet Free Air per Minute

TABLE X.—TABLE OF CONTENTS OF PIPES IN CUBIC FEET AND IN U. S. GALLON.

Diam. in Inches.	Diam. in Decimals of a Foot.	For 1 Foot in Length.		Diam. in Inches.	Diam. in Decimals of a Foot.	For 1 Foot in Length.	
		Cubic Feet. Also Area in Square Feet.	Gallons of 231 Cubic Inches.			Cubic Feet. Also Area in Square Feet.	Gallons of 231 Cubic Inches.
¼	.0208	.0003	.0026	11.	.9167	.6600	4.937
5/16	.0260	.0005	.0040	¼	.9375	.6903	5.163
⅜	.0313	.0008	.0057	½	.9583	.7213	5.395
7/16	.0365	.0010	.0078	¾	.9792	.7530	5.633
½	.0417	.0014	.0102	12.	1 Foot	.7854	5.876
9/16	.0469	.0017	.0129	½	1.042	.8523	6.375
⅝	.0521	.0021	.0159	13.	1.083	.9218	6.895
11/16	.0573	.0026	.0193	½	1.125	.9940	7.435
¾	.0625	.0031	.0230	14.	1.167	1.069	7.997
13/16	.0677	.0036	.0270	½	1.208	1.147	8.578
⅞	.0729	.0042	.0312	15.	1.250	1.227	9.180
15/16	.0781	.0048	.0359	½	1.292	1.310	9.801
1.	.0833	.0055	.0408	16.	1.333	1.396	10.44
¼	.1042	.0085	.0638	½	1.375	1.485	11.11
½	.1250	.0123	.0918	17.	1.417	1.576	11.79
¾	.1458	.0168	.1250	½	1.458	1.670	12.50
2.	.1667	.0218	.1632	18.	1.500	1.767	13.22
¼	.1875	.0276	.2066	½	1.542	1.867	13.97
½	.2083	.0341	.2550	19.	1.583	1.969	14.73
¾	.2292	.0413	.3085	½	1.625	2.074	15.52
3.	.2500	.0491	.3673	20.	1.666	2.182	16.32
¼	.2708	.0576	.4310	½	1.708	2.292	17.15
½	.2917	.0668	.4998	21.	1.750	2.405	17.99
⅔	.3125	.0767	.5738	½	1.792	2.521	18.86
4.	.3333	.0873	.6528	22.	1.833	2.640	19.75
¼	.3542	.0985	.7370	½	1.875	2.761	20.65
½	.3750	.1105	.8263	23.	1.917	2.885	22.58
¾	.3958	.1231	.9205	½	1.958	3.012	21.53
5.	.4167	.1364	1.020	24.	2.000	3.142	23.50
¼	.4375	.1503	1.124	25.	2.083	3.409	25.50
½	.4583	.1650	1.234	26.	2.166	3.687	27.58
¾	.4792	.1803	1.349	27.	2.250	3.976	29.74
6.	.5000	.1963	1.469	28.	2.333	4.276	31.99
¼	.5208	.2130	1.594	29.	2.416	4.587	34.31
½	.5417	.2305	1.724	30.	2.500	4.909	36.72
¾	.5625	.2485	1.859	31.	2.583	5.241	39.21
7	.5833	.2673	1.999	32.	2.666	5.585	41.78
¼	.6042	.2868	2.144	33.	2.750	5.940	44.43
½	.6250	.3068	2.295	34	2.833	6.305	47.17
¾	.6458	.3275	2.450	35.	2.916	6.681	49.98
8.	.6667	.3490	2.611	36.	3.000	7.069	52.88
¼	.6875	.3713	2.777	37.	3.083	7.468	55.86
½	.7083	.3940	2.948	38.	3.166	7.876	58.92
¾	.7292	.4175	3.125	39.	3.250	8.296	62.06
9.	.7500	.4418	3.305	40.	3.333	8.728	65.29
¼	.7708	.4668	3.492	41.	3.416	9.168	68.58
½	.7917	.4923	3.682	42.	3.500	9.620	71.96
¾	.8125	.5185	3.879	43.	3.583	10.084	75.43
10.	.8333	.5455	4.081	44.	3.666	10.560	79.00
¼	.8542	.5730	4.286	45.	3.750	11.044	82.62
½	.8750	.6013	4.498	46.	3.833	11.540	86.32
¾	.8958	.6303	4.714	47.	3.916	12.048	90.12
				48.	4.000	12.566	94.02

TABLE XI—CYLINDRICAL VESSELS, TANKS, CISTERNS, ETC.

Diameter in Feet and Inches, Area in Square Feet, and U. S. Gallons Capacity for One Foot in Depth.

$$1 \text{ gallon} = 231 \text{ cubic inches} = \frac{1 \text{ cubic foot}}{7.4805} = 0.13368 \text{ cubic feet.}$$

Diam.		Area.	Gals.	Diam.		Area.	Gals.	Diam.		Area.	Gals.
Ft.	In.	Sq. Ft.	1 Ft. Depth.	Ft.	In.	Sq. Ft.	1 Ft. Depth.	Ft.	In.	Sq. Ft.	1 Ft. Depth.
1		.785	5.89	5	5	23.04	172.38	17	6	240.53	1799.3
1	1	.922	6.89	5	6	23.76	177.72	17	9	247.45	1851.1
1	2	1.069	8.00	5	7	24.48	183.15	18		254.47	1903.6
1	3	1.227	9.18	5	8	25.22	188.66	18	3	261.59	1956.8
1	4	1.396	10.44	5	9	25.97	194.25	18	6	268.80	2010.8
1	5	1.576	11.79	5	10	26.73	199.92	18	9	276.12	2065.5
1	6	1.767	13.22	5	11	27.49	205.67	19		283.53	2120.9
1	7	1.969	14.73	6		28.27	211.51	19	3	291.04	2177.1
1	8	2.182	16.32	6	3	30.68	229.50	19	6	298.65	2234.0
1	9	2.405	17.99	6	6	33.18	248.23	19	9	306.35	2291.7
1	10	2.640	19.75	6	9	35.78	267.69	20		314.16	2350.1
1	11	2.885	21.58	7		38.48	287.88	20	3	322.06	2409.2
2		3.142	23.50	7	3	41.28	308.81	20	6	330.06	2469.1
2	1	3.409	25.50	7	6	44.18	330.48	20	9	338.16	2529.6
2	2	3.687	27.58	7	9	47.17	352.88	21		346.36	2591.0
2	3	3.976	29.74	8		50.27	376.01	21	3	354.66	2653.0
2	4	4.276	31.99	8	3	53.46	399.88	21	6	363.05	2715.8
2	5	4.587	34.31	8	6	56.75	424.48	21	9	371.54	2779.3
2	6	4.909	36.72	8	9	60.13	449.82	22		380.13	2843.6
2	7	5.241	39.21	9		63.62	475.89	22	3	388.82	2908.6
2	8	5.585	41.78	9	3	67.20	502.70	22	6	397.61	2974.3
2	9	5.940	44.43	9	6	70.88	530.24	22	9	406.49	3040.8
2	10	6.305	47.16	9	9	74.66	558.51	23		415.48	3108.0
2	11	6.681	49.98	10		78.54	587.52	23	3	424.56	3175.9
3		7.069	52.88	10	3	82.52	617.26	23	6	433.74	3244.6
3	1	7.467	55.86	10	6	86.59	647.74	23	9	443.01	3314.0
3	2	7.876	58.92	10	9	90.76	678.95	24		452.39	3384.1
3	3	8.296	62.06	11		95.03	710.90	24	3	461.86	3455.0
3	4	8.727	65.28	11	3	99.40	743.58	24	6	471.44	3526.6
3	5	9.168	68.58	11	6	103.87	776.99	24	9	481.11	3598.9
3	6	9.621	71.97	11	9	108.43	811.14	25		490.87	3672.0
3	7	10.085	75.44	12		113.10	846.03	25	3	500.74	3745.8
3	8	10.559	78.99	12	3	117.86	881.65	25	6	510.71	3820.3
3	9	11.045	82.62	12	6	122.72	918.00	25	9	520.77	3895.6
3	10	11.541	86.33	12	9	127.68	955.09	26		530.93	3971.6
3	11	12.048	90.13	13		132.73	992.91	26	3	541.19	4048.4
4		12.566	94.00	13	3	137.89	1031.5	26	6	551.55	4125.9
4	1	13.095	97.96	13	6	143.14	1070.8	26	9	562.00	4204.1
4	2	13.635	102.00	13	9	148.49	1110.8	27		572.56	4283.0
4	3	14.186	106.12	14		153.94	1151.5	27	3	583.21	4362.7
4	4	14.748	110.32	14	3	159.48	1193.0	27	6	593.96	4443.1
4	5	15.321	114.61	14	6	165.13	1235.3	27	9	604.81	4524.3
4	6	15.90	118.97	14	9	170.87	1278.2	28		615.75	4606.2
4	7	16.50	123.42	15		176.71	1321.9	28	3	626.80	4688.8
4	8	17.10	127.95	15	3	182.65	1366.4	28	6	637.94	4772.1
4	9	17.72	132.56	15	6	188.69	1411.5	28	9	649.18	4856.2
4	10	18.35	137.25	15	9	194.83	1457.4	29		660.52	4941.0
4	11	18.99	142.02	16		201.06	1504.1	29	3	671.96	5026.6
5		19.63	146.88	16	3	207.39	1551.4	29	6	683.49	5112.9
5	1	20.29	151.82	16	6	213.82	1599.5	29	9	695.13	5199.9
5	2	20.97	156.83	16	9	220.35	1648.4	30		706.86	5287.7
5	3	21.65	161.93	17		226.98	1697.9				
5	4	22.34	167.12	17	3	233.71	1748.2				

TABLE XII. — STANDARD DIMENSIONS OF WROUGHT-IRON WELDED PIPE.

(National Tube Works.)

Nominal Inside Diameter	Actual Outside Diameter.	Actual Inside Diameter.	Internal Area.		External Area.	
Ins.	Ins.	Ins.	Sq. In.	Sq. Ft.	Sq. In.	Sq. Ft.
⅛	.405	.270	.057	.0004	.1288	.0009
¼	.540	.364	.104	.0007	.2290	.0016
⅜	.675	.493	.191	.0013	.3578	.0025
½	.840	.622	.304	.0021	.554	.0038
¾	1.050	.824	.533	.0037	.866	.0060
1	1.315	1.048	.861	.0060	1.358	.0094
1¼	1.660	1.380	1.496	.0104	2.164	.0150
1½	1.900	1.610	2.036	.0141	2.835	.0197
2	2.375	2.067	3.356	.0233	4.430	.0308
2½	2.875	2.468	4.780	.0332	6.492	.0451
3	3.500	3.067	7.383	.0513	9.621	.0668
3½	4.000	3.548	9.887	.0689	12.566	.0875
4	4.500	4.026	12.730	.0884	15.904	.1104
4½	5.000	4.508	15.961	.1108	19.635	.1364
5	5.563	5.045	19.986	.1388	24.301	.1688
6	6.625	6.065	28.890	.2006	34.472	.2394
7	7.625	7.023	38.738	.2690	45.664	.3171
8	8.625	7.981	50.027	.3474	58.426	.4057
9	9.625	8.937	62.730	.4356	72.760	.5053
10	10.75	10.018	78.823	.5474	90.763	.6303
11	11.75	11.000	95.033	.6600	108.434	.7530
12	12.75	12.000	113.098	.7854	127.677	.8867
13	14	13.25	137.887	.9577	153.938	1.0690
14	15	14.25	159.485	1.1075	176.715	1.2272
15	16	15.25	182.665	1.2685	201.062	1.3963
17	18	17.25	239.706	1.6229	254.470	1.7671
19	20	19.25	291.040	2.0211	314.159	2.1817
21	22	21.25	354.657	2.4629	380.134	2.6398
23	24	23.25	424.558	2.9483	452.390	3.1416

TABLE XIII.—HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1.01	.00995	1.57	.45108	2.13	.75612	2.69	.98954
1.02	.01980	1.58	.45742	2.14	.76081	2.70	.99325
1.03	.02956	1.59	.46373	2.15	.76547	2.71	.99695
1.04	.03922	1.60	.47000	2.16	.77011	2.72	1.00063
1.05	.04879	1.61	.47623	2.17	.77473	2.73	1.00430
1.06	.05827	1.62	.48243	2.18	.77932	2.74	1.00796
1.07	.06766	1.63	.48858	2.19	.78390	2.75	1.01160
1.08	.07696	1.64	.49470	2.20	.78846	2.76	1.01523
1.09	.08618	1.65	.50078	2.21	.79299	2.77	1.01885
1.10	.09531	1.66	.50681	2.22	.79751	2.78	1.02245
1.11	.10436	1.67	.51282	2.23	.80200	2.79	1.02604
1.12	.11333	1.68	.51879	2.24	.80648	2.80	1.02962
1.13	.12222	1.69	.52473	2.25	.81093	2.81	1.03318
1.14	.13103	1.70	.53063	2.26	.81536	2.82	1.03674
1.15	.13977	1.71	.53649	2.27	.81978	2.83	1.04028
1.16	.14842	1.72	.54232	2.28	.82418	2.84	1.04380
1.17	.15700	1.73	.54812	2.29	.82855	2.85	1.04732
1.18	.16551	1.74	.55389	2.30	.83291	2.86	1.05082
1.19	.17395	1.75	.55962	2.31	.83725	2.87	1.05431
1.20	.18232	1.76	.56531	2.32	.84157	2.88	1.05779
1.21	.19062	1.77	.57098	2.33	.84587	2.89	1.06126
1.22	.19885	1.78	.57661	2.34	.85015	2.90	1.06471
1.23	.20701	1.79	.58222	2.35	.85442	2.91	1.06815
1.24	.21511	1.80	.58779	2.36	.85866	2.92	1.07158
1.25	.22314	1.81	.59333	2.37	.86289	2.93	1.07500
1.26	.23111	1.82	.59884	2.38	.86710	2.94	1.07841
1.27	.23902	1.83	.60432	2.39	.87129	2.95	1.08181
1.28	.24686	1.84	.60977	2.40	.87547	2.96	1.08519
1.29	.25464	1.85	.61519	2.41	.87963	2.97	1.08856
1.30	.26236	1.86	.62058	2.42	.88377	2.98	1.09192
1.31	.27003	1.87	.62594	2.43	.88789	2.99	1.09527
1.32	.27763	1.88	.63127	2.44	.89200	3.00	1.09861
1.33	.28518	1.89	.63658	2.45	.89609	3.01	1.10194
1.34	.29267	1.90	.64185	2.46	.90016	3.02	1.10526
1.35	.30010	1.91	.64710	2.47	.90422	3.03	1.10856
1.36	.30748	1.92	.65233	2.48	.90826	3.04	1.11186
1.37	.31481	1.93	.65752	2.49	.91228	3.05	1.11514
1.38	.32208	1.94	.66269	2.50	.91629	3.06	1.11841
1.39	.32930	1.95	.66783	2.51	.92028	3.07	1.12168
1.40	.33647	1.96	.67294	2.52	.92426	3.08	1.12493
1.41	.34359	1.97	.67803	2.53	.92822	3.09	1.12817
1.42	.35066	1.98	.68310	2.54	.93216	3.10	1.13140
1.43	.35767	1.99	.68813	2.55	.93609	3.11	1.13462
1.44	.36464	2.00	.69315	2.56	.94001	3.12	1.13783
1.45	.37156	2.01	.69813	2.57	.94391	3.13	1.14103
1.46	.37844	2.02	.70310	2.58	.94779	3.14	1.14422
1.47	.38526	2.03	.70804	2.59	.95166	3.15	1.14740
1.48	.39204	2.04	.71295	2.60	.95551	3.16	1.15057
1.49	.39878	2.05	.71784	2.61	.95935	3.17	1.15373
1.50	.40547	2.06	.72271	2.62	.96317	3.18	1.15688
1.51	.41211	2.07	.72755	2.63	.96698	3.19	1.16002
1.52	.41871	2.08	.73237	2.64	.97078	3.20	1.16315
1.53	.42527	2.09	.73716	2.65	.97454	3.21	1.16627
1.54	.43178	2.10	.74194	2.66	.97833	3.22	1.16938
1.55	.43825	2.11	.74669	2.67	.98208	3.23	1.17248
1.56	.44469	2.12	.75142	2.68	.98582	3.24	1.17557

TABLE XIII. *Continued.* — HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3.25	1.17865	3.81	1.33763	4.37	1.47476	4.93	1.59534
3.26	1.18173	3.82	1.34025	4.38	1.47705	4.94	1.59737
3.27	1.18479	3.83	1.34286	4.39	1.47933	4.95	1.59939
3.28	1.18784	3.84	1.34547	4.40	1.48160	4.96	1.60141
3.29	1.19089	3.85	1.34807	4.41	1.48387	4.97	1.60342
3.30	1.19392	3.86	1.35067	4.42	1.48614	4.98	1.60543
3.31	1.19695	3.87	1.35325	4.43	1.48840	4.99	1.60744
3.32	1.19996	3.88	1.35584	4.44	1.49065	5.00	1.60944
3.33	1.20297	3.89	1.35841	4.45	1.49290	5.01	1.61144
3.34	1.20597	3.90	1.36098	4.46	1.49515	5.02	1.61343
3.35	1.20896	3.91	1.36354	4.47	1.49739	5.03	1.61542
3.36	1.21194	3.92	1.36609	4.48	1.49962	5.04	1.61741
3.37	1.21491	3.93	1.36864	4.49	1.50185	5.05	1.61939
3.38	1.21788	3.94	1.37118	4.50	1.50408	5.06	1.62137
3.39	1.22083	3.95	1.37371	4.51	1.50630	5.07	1.62334
3.40	1.22378	3.96	1.37624	4.52	1.50851	5.08	1.62531
3.41	1.22671	3.97	1.37877	4.53	1.51072	5.09	1.62728
3.42	1.22964	3.98	1.38128	4.54	1.51293	5.10	1.62924
3.43	1.23256	3.99	1.38379	4.55	1.51513	5.11	1.63120
3.44	1.23547	4.00	1.38629	4.56	1.51732	5.12	1.63315
3.45	1.23837	4.01	1.38879	4.57	1.51951	5.13	1.63511
3.46	1.24127	4.02	1.39128	4.58	1.52170	5.14	1.63705
3.47	1.24415	4.03	1.39377	4.59	1.52388	5.15	1.63900
3.48	1.24703	4.04	1.39624	4.60	1.52606	5.16	1.64094
3.49	1.24990	4.05	1.39872	4.61	1.52823	5.17	1.64287
3.50	1.25276	4.06	1.40118	4.62	1.53039	5.18	1.64481
3.51	1.25562	4.07	1.40364	4.63	1.53256	5.19	1.64673
3.52	1.25846	4.08	1.40610	4.64	1.53471	5.20	1.64866
3.53	1.26130	4.09	1.40854	4.65	1.53687	5.21	1.65058
3.54	1.26412	4.10	1.41099	4.66	1.53902	5.22	1.65250
3.55	1.26695	4.11	1.41342	4.67	1.54116	5.23	1.65441
3.56	1.26976	4.12	1.41585	4.68	1.54330	5.24	1.65632
3.57	1.27257	4.13	1.41828	4.69	1.54543	5.25	1.65823
3.58	1.27536	4.14	1.42070	4.70	1.54756	5.26	1.66013
3.59	1.27815	4.15	1.42311	4.71	1.54969	5.27	1.66203
3.60	1.28093	4.16	1.42552	4.72	1.55181	5.28	1.66393
3.61	1.28371	4.17	1.42792	4.73	1.55393	5.29	1.66582
3.62	1.28647	4.18	1.43031	4.74	1.55604	5.30	1.66771
3.63	1.28923	4.19	1.43270	4.75	1.55814	5.31	1.66959
3.64	1.29198	4.20	1.43508	4.76	1.56025	5.32	1.67147
3.65	1.29473	4.21	1.43746	4.77	1.56235	5.33	1.67335
3.66	1.29746	4.22	1.43984	4.78	1.56444	5.34	1.67523
3.67	1.30019	4.23	1.44220	4.79	1.56653	5.35	1.67710
3.68	1.30291	4.24	1.44456	4.80	1.56862	5.36	1.67896
3.69	1.30563	4.25	1.44692	4.81	1.57070	5.37	1.68083
3.70	1.30833	4.26	1.44927	4.82	1.57277	5.38	1.68269
3.71	1.31103	4.27	1.45161	4.83	1.57485	5.39	1.68455
3.72	1.31372	4.28	1.45395	4.84	1.57691	5.40	1.68640
3.73	1.31641	4.29	1.45629	4.85	1.57898	5.41	1.68825
3.74	1.31909	4.30	1.45861	4.86	1.58104	5.42	1.69010
3.75	1.32176	4.31	1.46094	4.87	1.58309	5.43	1.69194
3.76	1.32442	4.32	1.46326	4.88	1.58515	5.44	1.69378
3.77	1.32707	4.33	1.46557	4.89	1.58719	5.45	1.69562
3.78	1.32972	4.34	1.46787	4.90	1.58924	5.46	1.69745
3.79	1.33237	4.35	1.47018	4.91	1.59127	5.47	1.69928
3.80	1.33500	4.36	1.47247	4.92	1.59331	5.48	1.70111

TABLE XIII *Continued.*—HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
5.49	1.70293	6.05	1.80006	6.61	1.88858	7.17	1.96991
5.50	1.70475	6.06	1.80171	6.62	1.89010	7.18	1.97130
5.51	1.70656	6.07	1.80336	6.63	1.89160	7.19	1.97269
5.52	1.70838	6.08	1.80500	6.64	1.89311	7.20	1.97408
5.53	1.71019	6.09	1.80665	6.65	1.89462	7.21	1.97547
5.54	1.71199	6.10	1.80829	6.66	1.89612	7.22	1.97685
5.55	1.71380	6.11	1.80993	6.67	1.89762	7.23	1.97824
5.56	1.71560	6.12	1.81156	6.68	1.89912	7.24	1.97962
5.57	1.71740	6.13	1.81319	6.69	1.90061	7.25	1.98100
5.58	1.71919	6.14	1.81482	6.70	1.90211	7.26	1.98238
5.59	1.72098	6.15	1.81645	6.71	1.90360	7.27	1.98376
5.60	1.72277	6.16	1.81808	6.72	1.90509	7.28	1.98513
5.61	1.72455	6.17	1.81970	6.73	1.90658	7.29	1.98650
5.62	1.72633	6.18	1.82132	6.74	1.90806	7.30	1.98787
5.63	1.72811	6.19	1.82294	6.75	1.90954	7.31	1.98924
5.64	1.72988	6.20	1.82455	6.76	1.91102	7.32	1.99061
5.65	1.73166	6.21	1.82616	6.77	1.91250	7.33	1.99198
5.66	1.73342	6.22	1.82777	6.78	1.91398	7.34	1.99334
5.67	1.73519	6.23	1.82937	6.79	1.91545	7.35	1.99470
5.68	1.73695	6.24	1.83098	6.80	1.91692	7.36	1.99606
5.69	1.73871	6.25	1.83258	6.81	1.91839	7.37	1.99742
5.70	1.74047	6.26	1.83418	6.82	1.91986	7.38	1.99877
5.71	1.74222	6.27	1.83578	6.83	1.92132	7.39	2.00013
5.72	1.74397	6.28	1.83737	6.84	1.92279	7.40	2.00148
5.73	1.74572	6.29	1.83896	6.85	1.92425	7.41	2.00283
5.74	1.74746	6.30	1.84055	6.86	1.92571	7.42	2.00418
5.75	1.74920	6.31	1.84214	6.87	1.92716	7.43	2.00553
5.76	1.75094	6.32	1.84372	6.88	1.92862	7.44	2.00687
5.77	1.75267	6.33	1.84530	6.89	1.93007	7.45	2.00821
5.78	1.75440	6.34	1.84688	6.90	1.93152	7.46	2.00956
5.79	1.75613	6.35	1.84845	6.91	1.93297	7.47	2.01089
5.80	1.75786	6.36	1.85003	6.92	1.93442	7.48	2.01223
5.81	1.75958	6.37	1.85160	6.93	1.93586	7.49	2.01357
5.82	1.76130	6.38	1.85317	6.94	1.93730	7.50	2.01490
5.83	1.76302	6.39	1.85473	6.95	1.93874	7.51	2.01624
5.84	1.76473	6.40	1.85630	6.96	1.94018	7.52	2.01757
5.85	1.76644	6.41	1.85786	6.97	1.94162	7.53	2.01890
5.86	1.76815	6.42	1.85942	6.98	1.94305	7.54	2.02022
5.87	1.76985	6.43	1.86097	6.99	1.94448	7.55	2.02155
5.88	1.77156	6.44	1.86253	7.00	1.94591	7.56	2.02287
5.89	1.77326	6.45	1.86408	7.01	1.94734	7.57	2.02419
5.90	1.77495	6.46	1.86563	7.02	1.94876	7.58	2.02551
5.91	1.77665	6.47	1.86718	7.03	1.95019	7.59	2.02683
5.92	1.77834	6.48	1.86872	7.04	1.95161	7.60	2.02815
5.93	1.78002	6.49	1.87026	7.05	1.95303	7.61	2.02946
5.94	1.78171	6.50	1.87180	7.06	1.95444	7.62	2.03078
5.95	1.78339	6.51	1.87334	7.07	1.95586	7.63	2.03209
5.96	1.78507	6.52	1.87487	7.08	1.95727	7.64	2.03340
5.97	1.78675	6.53	1.87641	7.09	1.95869	7.65	2.03471
5.98	1.78842	6.54	1.87794	7.10	1.96009	7.66	2.03601
5.99	1.79009	6.55	1.87947	7.11	1.96150	7.67	2.03732
6.00	1.79176	6.56	1.88099	7.12	1.96291	7.68	2.03862
6.01	1.79342	6.57	1.88251	7.13	1.96431	7.69	2.03992
6.02	1.79509	6.58	1.88403	7.14	1.96571	7.70	2.04122
6.03	1.79675	6.59	1.88555	7.15	1.96711	7.71	2.04252
6.04	1.79840	6.60	1.88707	7.16	1.96851	7.72	2.04381

TABLE XIII. *Continued.* — HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
7.73	2.04511	8.30	2.11626	8.87	2.18267	9.44	2.24496
7.74	2.04640	8.31	2.11746	8.88	2.18380	9.45	2.24601
7.75	2.04769	8.32	2.11866	8.89	2.18493	9.46	2.24707
7.76	2.04898	8.33	2.11986	8.90	2.18605	9.47	2.24813
7.77	2.05027	8.34	2.12106	8.91	2.18717	9.48	2.24918
7.78	2.05156	8.35	2.12226	8.92	2.18830	9.49	2.25024
7.79	2.05284	8.36	2.12346	8.93	2.18942	9.50	2.25129
7.80	2.05412	8.37	2.12465	8.94	2.19054	9.51	2.25234
7.81	2.05540	8.38	2.12585	8.95	2.19165	9.52	2.25339
7.82	2.05668	8.39	2.12704	8.96	2.19277	9.53	2.25444
7.83	2.05796	8.40	2.12823	8.97	2.19389	9.54	2.25549
7.84	2.05924	8.41	2.12942	8.98	2.19500	9.55	2.25654
7.85	2.06051	8.42	2.13061	8.99	2.19611	9.56	2.25759
7.86	2.06179	8.43	2.13180	9.00	2.19722	9.57	2.25863
7.87	2.06306	8.44	2.13298	9.01	2.19834	9.58	2.25968
7.88	2.06433	8.45	2.13417	9.02	2.19944	9.59	2.26072
7.89	2.06560	8.46	2.13535	9.03	2.20055	9.60	2.26176
7.90	2.06686	8.47	2.13653	9.04	2.20166	9.61	2.26280
7.91	2.06813	8.48	2.13771	9.05	2.20276	9.62	2.26384
7.92	2.06939	8.49	2.13889	9.06	2.20387	9.63	2.26488
7.93	2.07065	8.50	2.14007	9.07	2.20497	9.64	2.26592
7.94	2.07191	8.51	2.14124	9.08	2.20607	9.65	2.26696
7.95	2.07317	8.52	2.14242	9.09	2.20717	9.66	2.26799
7.96	2.07443	8.53	2.14359	9.10	2.20827	9.67	2.26903
7.97	2.07568	8.54	2.14476	9.11	2.20937	9.68	2.27006
7.98	2.07694	8.55	2.14593	9.12	2.21047	9.69	2.27109
7.99	2.07819	8.56	2.14710	9.13	2.21157	9.70	2 27213
8.00	2.07944	8.57	2.14827	9.14	2.21266	9.71	2.27316
8.01	2.08069	8.58	2.14943	9.15	2.21375	9.72	2.27419
8.02	2.08194	8.59	2.15060	9.16	2.21485	9.73	2.27521
8.03	2.08318	8.60	2.15176	9.17	2.21594	9.74	2.27624
8.04	2.08443	8.61	2.15292	9.18	2.21703	9.75	2.27727
8.05	2.08567	8.62	2.15409	9.19	2.21812	9.76	2.27829
8.06	2.08691	8.63	2.15524	9.20	2.21920	9.77	2.27932
8.07	2.08815	8.64	2.15640	9.21	2.22029	9.78	2.28034
8.08	2.08939	8.65	2.15756	9.22	2.22138	9.79	2.28136
8.09	2.09063	8.66	2.15871	9.23	2.22246	9.80	2.28238
8.10	2.09186	8.67	2.15987	9.24	2.22351	9.81	2.28340
8.11	2.09310	8.68	2.16102	9.25	2.22462	9.82	2.28442
8.12	2.09433	8.69	2.16217	9.26	2.22570	9.83	2.28544
8.13	2.09556	8.70	2.16332	9.27	2.22678	9.84	2.28646
8.14	2.09679	8.71	2.16447	9.28	2.22786	9.85	2.28747
8.15	2.09802	8.72	2.16562	9.29	2.22894	9.86	2.28849
8.16	2.09924	8.73	2.16677	9.30	2.23001	9.87	2.28950
8.17	2.10047	8.74	2.16791	9.31	2.23109	9.88	2.29051
8.18	2.10169	8.75	2.16905	9.32	2.23216	9.89	2.29152
8.19	2.10291	8.76	2.17020	9.33	2.23323	9.90	2.29253
8.20	2.10413	8.77	2.17134	9.34	2.23431	9.91	2.29354
8.21	2.10535	8.78	2.17248	9.35	2.23538	9.92	2.29455
8.22	2.10657	8.79	2.17361	9.36	2.23645	9.93	2.29556
8.23	2.10779	8.80	2.17475	9.37	2.23751	9.94	2.29657
8.24	2.10900	8.81	2.17589	9.38	2.23858	9.95	2.29757
8.25	2.11021	8.82	2.17702	9.39	2.23965	9.96	2.29858
8.26	2.11142	8.83	2.17816	9.40	2.24071	9.97	2.29958
8.27	2.11263	8.84	2.17929	9.41	2.24177	9.98	2.30058
8.28	2.11384	8.85	2.18042	9.42	2.24284	9.99	2.30158
8.29	2.11505	8.86	2.18155	9.43	2.24390		

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580

Pp. Pts.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

No.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	963	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
	30									

Pp. Pts.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777

Pp. Pts.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40 140	157	175	192	209	226	243	261	278	295	18
253	312	329	346	364	381	398	415	432	449	466	1 1.8
254	483	500	518	535	552	569	586	603	620	637	2 3.6 3 5.4
255	654	671	688	705	722	739	756	773	790	807	4 7.2
256	824	841	858	875	892	909	926	943	960	976	5 9.0 6 10.8
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	7 12.6
258	41 162	179	196	212	229	246	263	280	296	313	8 14.4 9 16.2
259	330	347	363	380	397	414	430	447	464	481	
260	497	514	531	547	564	581	597	614	631	647	
261	664	681	697	714	731	747	764	780	797	814	
262	830	847	863	880	896	913	929	946	963	979	17
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	1 1.7 2 3.4
264	42 160	177	193	210	226	243	259	275	292	308	3 5.1 4 6.8
265	325	341	357	374	390	406	423	439	455	472	5 8.5
266	488	504	521	537	553	570	586	602	619	635	6 10.2
267	651	667	684	700	716	732	749	765	781	797	7 11.9 8 13.6
268	813	830	846	862	878	894	911	927	943	959	9 15.3
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	
270	43 136	152	169	185	201	217	233	249	265	281	
271	297	313	329	345	361	377	393	409	425	441	16
272	457	473	489	505	521	537	553	569	584	600	1 1.6
273	616	632	648	664	680	696	712	727	743	759	2 3.2
274	775	791	807	823	838	854	870	886	902	917	3 4.8 4 6.4
275	933	949	965	981	996	*012	*028	*044	*059	*075	5 8.0
276	44 091	107	122	138	154	170	185	201	217	232	6 9.6
277	248	264	279	295	311	326	342	358	373	389	7 11.2 8 12.8
278	404	420	436	451	467	483	498	514	529	545	9 14.4
279	560	576	592	607	623	638	654	669	685	700	
280	716	731	747	762	778	793	809	824	840	855	
281	871	886	902	917	932	948	963	979	994	*010	15
282	45 025	040	056	071	086	102	117	133	148	163	1 1.5
283	179	194	209	225	240	255	271	286	301	317	2 3.0 3 4.5
284	332	347	362	378	393	408	423	439	454	469	4 6.0
285	484	500	515	530	545	561	576	591	606	621	5 7.5
286	637	652	667	682	697	712	728	743	758	773	6 9.0 7 10.5
287	788	803	818	834	849	864	879	894	909	924	8 12.0
288	939	954	969	984	*000	*015	*030	*045	*060	*075	9 13.5
289	46 090	105	120	135	150	165	180	195	210	225	
290	240	255	270	285	300	315	330	345	359	374	
291	389	404	419	434	449	464	479	494	509	523	14
292	538	553	568	583	598	613	627	642	657	672	1 1.4 2 2.8
293	687	702	716	731	746	761	776	790	805	820	3 4.2
294	835	850	864	879	894	909	923	938	953	967	4 5.6
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	5 7.0 6 8.4
296	47 129	144	159	173	188	202	217	232	246	261	7 9.8
297	276	290	305	319	334	349	363	378	392	407	8 11.2 9 12.6
298	422	436	451	465	480	494	509	524	538	553	
299	567	582	596	611	625	640	654	669	683	698	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	15
306	572	586	601	615	629	643	657	671	686	700	1 1.5 2 3.0
307	714	728	742	756	770	785	799	813	827	841	3 4.5
308	855	869	883	897	911	926	940	954	968	982	4 6.0 5 7.5
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9.0
310	49 136	150	164	178	192	206	220	234	248	262	7 10.5 8 12.0
311	276	290	304	318	332	346	360	374	388	402	9 13.5
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	14
317	50 106	120	133	147	161	174	188	202	215	229	1 1.4 2 2.8
318	243	256	270	284	297	311	325	338	352	365	3 4.2
319	379	393	406	420	433	447	461	474	488	501	4 5.6
320	515	529	542	556	569	583	596	610	623	637	5 7.0 6 8.4
321	651	664	678	691	705	718	732	745	759	772	7 9.8 8 11.2
322	786	799	813	826	840	853	866	880	893	907	9 12.6
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	13
328	587	601	614	627	640	654	667	680	693	706	1 1.3 2 2.6
329	720	733	746	759	772	786	799	812	825	838	3 3.9
330	851	865	878	891	904	917	930	943	957	970	4 5.2 5 6.5
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	6 7.8
332	52 114	127	140	153	166	179	192	205	218	231	7 9.1 8 10.4
333	244	257	270	284	297	310	323	336	349	362	9 11.7
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	12
339	53 020	033	046	058	071	084	097	110	122	135	1 1.2 2 2.4
340	148	161	173	186	199	212	224	237	250	263	3 3.6
341	275	288	301	314	326	339	352	364	377	390	4 4.8 5 6.0
342	403	415	428	441	453	466	479	491	504	517	6 7.2
343	529	542	555	567	580	593	605	618	631	643	7 8.4 8 9.6
344	656	668	681	694	706	719	732	744	757	769	9 10.8
345	782	794	807	820	832	845	857	870	882	895	
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	
355	55 023	035	047	060	072	084	096	108	121	133	13 1 1.3 2 2.6 3 3.9 4 5.2 5 6.5 6 7.8 7 9.1 8 10.4 9 11.7
356	145	157	169	182	194	206	218	230	242	255	
357	267	279	291	303	315	328	340	352	364	376	
358	388	400	413	425	437	449	461	473	485	497	
359	509	522	534	546	558	570	582	594	606	618	
360	630	642	654	666	678	691	703	715	727	739	
361	751	763	775	787	799	811	823	835	847	859	
362	871	883	895	907	919	931	943	955	967	979	
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	12 1 1.2 2 2.4 3 3.6 4 4.8 5 6.0 6 7.2 7 8.4 8 9.6 9 10.8
367	467	478	490	502	514	526	538	549	561	573	
368	585	597	608	620	632	644	656	667	679	691	
369	703	714	726	738	750	761	773	785	797	808	
370	820	832	844	855	867	879	891	902	914	926	
371	937	949	961	972	984	996	*008	*019	*031	*043	
372	57 054	066	078	089	101	113	124	136	148	159	
373	171	183	194	206	217	229	241	252	264	276	
374	287	299	310	322	334	345	357	368	380	392	
375	403	415	426	438	449	461	473	484	496	507	
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	11 1 1.1 2 2.2 3 3.3 4 4.4 5 5.5 6 6.6 7 7.7 8 8.8 9 9.9
378	749	761	772	784	795	807	818	830	841	852	
379	864	875	887	898	910	921	933	944	955	967	
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	
381	58 092	104	115	127	138	149	161	172	184	195	
382	206	218	229	240	252	263	274	286	297	309	
383	320	331	343	354	365	377	388	399	410	422	
384	433	444	456	467	478	490	501	512	524	535	
385	546	557	569	580	591	602	614	625	636	647	
386	659	670	681	692	704	715	726	737	749	760	
387	771	782	794	805	816	827	838	850	861	872	
388	883	894	906	917	928	939	950	961	973	984	10 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	
391	218	229	240	251	262	273	284	295	306	318	
392	329	340	351	362	373	384	395	406	417	428	
393	439	450	461	472	483	494	506	517	528	539	
394	550	561	572	583	594	605	616	627	638	649	
395	660	671	682	693	704	715	726	737	748	759	
396	770	780	791	802	813	824	835	846	857	868	
397	879	890	901	912	923	934	945	956	966	977	
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	
399	60 097	108	119	130	141	152	163	173	184	195	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	11 1 1.1 2 2.2 3 3.3 4 4.4 5 5.5 6 6.6 7 7.7 8 8.8 9 9.9
409	172	183	194	204	215	225	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	10 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0
422	531	542	552	562	572	583	593	603	613	624	
423	634	644	655	665	675	685	696	706	716	726	
424	737	747	757	767	778	788	798	808	818	829	
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	236	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	9 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	

LOGARITHMS OF NUMBERS

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66 087	096	106	115	124	134	143	153	162	172	10 1 1.0
459	181	191	200	210	219	229	238	247	257	266	2 2.0
460	276	285	295	304	314	323	332	342	351	361	3 3.0
461	370	380	389	398	408	417	427	436	445	455	4 4.0 5 5.0
462	464	474	483	492	502	511	521	530	539	549	6 6.0
463	558	567	577	586	596	605	614	624	633	642	7 7.0 8 8.0
464	652	661	671	680	689	699	708	717	727	736	9 9.0
465	745	755	764	773	783	792	801	811	820	829	
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	9
472	394	403	413	422	431	440	449	459	468	477	1 0.9
473	486	495	504	514	523	532	541	550	560	569	2 1.8
474	578	587	596	605	614	624	633	642	651	660	3 2.7 4 3.6
475	669	679	688	697	706	715	724	733	742	752	5 4.5
476	761	770	779	788	797	806	815	825	834	843	6 5.4 7 6.3
477	852	861	870	879	888	897	906	916	925	934	8 7.2
478	943	952	961	970	979	988	997	*006	*015	*024	9 8.1
479	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	8
486	664	673	681	690	699	708	717	726	735	744	1 0.8
487	753	762	771	780	789	797	806	815	824	833	2 1.6
488	842	851	860	869	878	886	895	904	913	922	3 2.4 4 3.2
489	931	940	949	958	966	975	984	993	*002	*011	5 4.0
490	69 020	028	037	046	055	064	073	082	090	099	6 4.8 7 5.6
491	108	117	126	135	144	152	161	170	179	188	8 6.4
492	197	205	214	223	232	241	249	258	267	276	9 7.2
493	285	294	302	311	320	329	338	346	355	364	
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	9 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
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LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9
550	74 036	044	052	060	068	076	084	092	099	107
551	115	123	131	139	147	155	162	170	178	186
552	194	202	210	218	225	233	241	249	257	265
553	273	280	288	296	304	312	320	327	335	343
554	351	359	367	374	382	390	398	406	414	421
555	429	437	445	453	461	468	476	484	492	500
556	507	515	523	531	539	547	554	562	570	578
557	586	593	601	609	617	624	632	640	648	656
558	663	671	679	687	695	702	710	718	726	733
559	741	749	757	764	772	780	788	796	803	811
560	819	827	834	842	850	858	865	873	881	889
561	896	904	912	920	927	935	943	950	958	966
562	974	981	989	997	*005	*012	*020	*028	*035	*043
563	75 051	059	066	074	082	089	097	105	113	120
564	128	136	143	151	159	166	174	182	189	197
565	205	213	220	228	236	243	251	259	266	274
566	282	289	297	305	312	320	328	335	343	351
567	358	366	374	381	389	397	404	412	420	427
568	435	442	450	458	465	473	481	488	496	504
569	511	519	526	534	542	549	557	565	572	580
570	587	595	603	610	618	626	633	641	648	656
571	664	671	679	686	694	702	709	717	724	732
572	740	747	755	762	770	778	785	793	800	808
573	815	823	831	838	846	853	861	868	876	884
574	891	899	906	914	921	929	937	944	952	959
575	967	974	982	989	997	*005	*012	*020	*027	*035
576	76 042	050	057	065	072	080	087	095	103	110
577	118	125	133	140	148	155	163	170	178	185
578	193	200	208	215	223	230	238	245	253	260
579	268	275	283	290	298	305	313	320	328	335
580	343	350	358	365	373	380	388	395	403	410
581	418	425	433	440	448	455	462	470	477	485
582	492	500	507	515	522	530	537	545	552	559
583	567	574	582	589	597	604	612	619	626	634
584	641	649	656	664	671	678	686	693	701	708
585	716	723	730	738	745	753	760	768	775	782
586	790	797	805	812	819	827	834	842	849	856
587	864	871	879	886	893	901	908	916	923	930
588	938	945	953	960	967	975	982	989	997	*004
589	77 012	019	026	034	041	048	056	063	070	078
590	085	093	100	107	115	122	129	137	144	151
591	159	166	173	181	188	195	203	210	217	225
592	232	240	247	254	262	269	276	283	291	298
593	305	313	320	327	335	342	349	357	364	371
594	379	386	393	401	408	415	422	430	437	444
595	452	459	466	474	481	488	495	503	510	517
596	525	532	539	546	554	561	568	576	583	590
597	597	605	612	619	627	634	641	648	656	663
598	670	677	685	692	699	706	714	721	728	735
599	743	750	757	764	772	779	786	793	801	808

Pp. Pts.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
609	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
611	604	611	618	625	633	640	647	654	661	668	
612	675	682	689	696	704	711	718	725	732	739	
613	746	753	760	767	774	781	789	796	803	810	
614	817	824	831	838	845	852	859	866	873	880	
615	888	895	902	909	916	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
623	449	456	463	470	477	484	491	498	505	511	
624	518	525	532	539	546	553	560	567	574	581	
625	588	595	602	609	616	623	630	637	644	650	
626	657	664	671	678	685	692	699	706	713	720	
627	727	734	741	748	754	761	768	775	782	789	
628	796	803	810	817	824	831	837	844	851	858	
629	865	872	879	886	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	
639	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
641	686	693	699	706	713	720	726	733	740	747	
642	754	760	767	774	781	787	794	801	808	814	
643	821	828	835	841	848	855	862	868	875	882	
644	889	895	902	909	916	922	929	936	943	949	
645	956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	
662	086	092	099	105	112	119	125	132	138	145	
663	151	158	164	171	178	184	191	197	204	210	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
664	217	223	230	236	243	249	256	263	269	276	
665	282	289	295	302	308	315	321	328	334	341	
666	347	354	360	367	373	380	387	393	400	406	
667	413	419	426	432	439	445	452	458	465	471	
668	478	484	491	497	504	510	517	523	530	536	
669	543	549	556	562	569	575	582	588	595	601	
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	
685	569	575	582	588	594	601	607	613	620	626	
686	632	639	645	651	658	664	670	677	683	689	
687	696	702	708	715	721	727	734	740	746	753	
688	759	765	771	778	784	790	797	803	809	816	
689	822	828	835	841	847	853	860	866	872	879	
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	
708	85 003	009	016	022	028	034	040	046	052	058	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
709	065	071	077	083	089	095	101	107	114	120	
710	126	132	138	144	150	156	163	169	175	181	
711	187	193	199	205	211	217	224	230	236	242	
712	248	254	260	266	272	278	285	291	297	303	
713	309	315	321	327	333	339	345	352	358	364	
714	370	376	382	388	394	400	406	412	418	425	
715	431	437	443	449	455	461	467	473	479	485	
716	491	497	503	509	516	522	528	534	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
723	914	920	926	932	938	944	950	956	962	968	
724	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	064	070	076	082	088	
726	094	100	106	112	118	124	130	136	141	147	
727	153	159	165	171	177	183	189	195	201	207	
728	213	219	225	231	237	243	249	255	261	267	
729	273	279	285	291	297	303	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
731	392	398	404	410	415	421	427	433	439	445	
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
737	747	753	759	764	770	776	782	788	794	800	
738	806	812	817	823	829	835	841	847	853	859	
739	864	870	876	882	888	894	900	906	911	917	
740	923	929	935	941	947	953	958	964	970	976	
741	982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87 040	046	052	058	064	070	075	081	087	093	
743	099	105	111	116	122	128	134	140	146	151	
744	157	163	169	175	181	186	192	198	204	210	
745	216	221	227	233	239	245	251	256	262	268	
746	274	280	286	291	297	303	309	315	320	326	
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
770	649	655	660	666	672	677	683	689	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
788	653	658	664	669	675	680	686	691	697	702	
789	708	713	719	724	730	735	741	746	752	757	
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	
812	956	961	966	972	977	982	988	993	998	*004	
813	91 009	014	020	025	030	036	041	046	052	057	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
814	062	068	073	078	084	089	094	100	105	110	
815	116	121	126	132	137	142	148	153	158	164	
816	169	174	180	185	190	196	201	206	212	217	
817	222	228	233	238	243	249	254	259	265	270	
818	275	281	286	291	297	302	307	312	318	323	
819	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
832	92 012	018	023	028	033	038	044	049	054	059	
833	065	070	075	080	085	091	096	101	106	111	
834	117	122	127	132	137	143	148	153	158	163	
835	169	174	179	184	189	195	200	205	210	215	
836	221	226	231	236	241	247	252	257	262	267	
837	273	278	283	288	293	298	304	309	314	319	
838	324	330	335	340	345	350	355	361	366	371	
839	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	
858	349	354	359	364	369	374	379	384	389	394	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
859	399	404	409	414	420	425	430	435	440	445	
860	450	455	460	465	470	475	480	485	490	495	
861	500	505	510	515	520	526	531	536	541	546	
862	551	556	561	566	571	576	581	586	591	596	
863	601	606	611	616	621	626	631	636	641	646	
864	651	656	661	666	671	676	682	687	692	697	
865	702	707	712	717	722	727	732	737	742	747	
866	752	757	762	767	772	777	782	787	792	797	
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	
872	052	057	062	067	072	077	082	086	091	096	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
873	101	106	111	116	121	126	131	136	141	146	
874	151	156	161	166	171	176	181	186	191	196	
875	201	206	211	216	221	226	231	236	240	245	
876	250	255	260	265	270	275	280	285	290	295	
877	300	305	310	315	320	325	330	335	340	345	
878	349	354	359	364	369	374	379	384	389	394	
879	399	404	409	414	419	424	429	433	438	443	
880	448	453	458	463	468	473	478	483	488	493	
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
887	792	797	802	807	812	817	822	827	832	836	
888	841	846	851	856	861	866	871	876	880	885	
889	890	895	900	905	910	915	919	924	929	934	
890	939	944	949	954	959	963	968	973	978	983	
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95 036	041	046	051	056	061	066	071	075	080	
893	085	090	095	100	105	109	114	119	124	129	
894	134	139	143	148	153	158	163	168	173	177	
895	182	187	192	197	202	207	211	216	221	226	
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96 047	052	057	061	066	071	076	080	085	090	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
914	095	099	104	109	114	118	123	128	133	137	
915	142	147	152	156	161	166	171	175	180	185	
916	190	194	199	204	209	213	218	223	227	232	
917	237	242	246	251	256	261	265	270	275	280	
918	284	289	294	298	303	308	313	317	322	327	
919	332	336	341	346	350	355	360	365	369	374	
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	
935	081	086	090	095	100	104	109	114	118	123	
936	128	132	137	142	146	151	155	160	165	169	
937	174	179	183	188	192	197	202	206	211	216	
938	220	225	230	234	239	243	248	253	257	262	
939	267	271	276	280	285	290	294	299	304	308	
940	313	317	322	327	331	336	340	345	350	354	
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Pp. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
963	363	367	372	376	381	385	390	394	399	403	
964	408	412	417	421	426	430	435	439	444	448	
965	453	457	462	466	471	475	480	484	489	493	
966	498	502	507	511	516	520	525	529	534	538	
967	543	547	552	556	561	565	570	574	579	583	
968	588	592	597	601	605	610	614	619	623	628	
969	632	637	641	646	650	655	659	664	668	673	
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
981	167	171	176	180	185	189	193	198	202	207	
982	211	216	220	224	229	233	238	242	247	251	
983	255	260	264	269	273	277	282	286	291	295	
984	300	304	308	313	317	322	326	330	335	339	
985	344	348	352	357	361	366	370	374	379	383	
986	388	392	396	401	405	410	414	419	423	427	
987	432	436	441	445	449	454	458	463	467	471	
988	476	480	484	489	493	498	502	506	511	515	
989	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	

APPENDIX A

The following notes and tables relating to drill capacities and losses due to valves, elbows and tees are taken from the Ingersoll-Rand catalog.

DRILL CAPACITY TABLES

The following tables are to determine the amount of free air required to operate rock drills at various altitudes with air at given pressures.

The tables have been compiled from a review of a wide experience and from tests run on drills of various sizes. They are intended for fair conditions in ordinary hard rock, but owing to varying conditions it is impossible to make any guarantee without a full knowledge of existing conditions.

In soft material where the actual time of drilling is short, more drills can be run with a given sized compressor than when working in hard material, when the drills would be working continuously for a longer period, thereby increasing the chance of all the drills operating at the same time.

In tunnel work, where the rock is hard, it has been the experience that more rapid progress has been made when the drills were operated under a high air pressure, and that it has been found profitable to provide compressor capacity in excess of the requirements by about 25 per cent. There is also a distinct advantage in having a compressor of large capacity, in that it saves the trouble and expense of moving the compressor as the work progresses, and will not interfere with the progress of the work by crowding the tunnel.

No allowance has been made in the tables for loss due to leaky pipes, or for transmission loss due to friction, but the capacities given are merely the displacement required, so that when selecting a compressor for the work required these matters must be taken into account.

Table I gives cubic feet of free air required to operate one drill of a given size and under a given pressure.

Table II gives multiplication factors for altitudes and number of drills by which the air consumption of one drill must be multiplied in order to give the total amount of air.

TABLE I. — CUBIC FEET OF FREE AIR REQUIRED TO RUN ONE DRILL OF THE SIZE AND AT THE PRESSURE STATED BELOW

Gage Pressure, Pounds	SIZE AND CYLINDER DIAMETER OF DRILL												
	A35	A32	B	C	D	D	D	E	F	F	G	H	H9
	2″	$2\frac{1}{4}$″	$2\frac{1}{2}$″	$2\frac{3}{4}$″	3″	$3\frac{1}{8}$″	$3\frac{3}{16}$″	$3\frac{1}{4}$″	$3\frac{1}{2}$″	$3\frac{5}{8}$″	$4\frac{3}{4}$″	5″	$5\frac{1}{2}$″
60	50	60	68	82	90	95	97	100	108	113	130	150	164
70	56	68	77	93	102	108	110	113	124	129	147	170	181
80	63	76	86	104	114	120	123	127	131	143	164	190	207
90	70	84	95	115	126	133	136	141	152	159	182	210	230
100	77	92	104	126	138	146	149	154	166	174	199	240	252

TABLE II.—MULTIPLIERS TO DETERMINE CAPACITY OF COMPRESSOR REQUIRED TO OPERATE FROM 1 TO 70 ROCK DRILLS AT ALTITUDES COMPARED WITH SEA LEVEL

Altitude above Sea Level.	NUMBER OF DRILLS																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	50	60	70
Ft.	MULTIPLIERS																		
0	1.	1.8	2.7	3.4	4.1	4.8	5.4	6.0	6.5	7.1	8.1	9.5	11.7	13.7	15.8	21.4	25.5	29.4	33.2
1000	1.03	1.85	2.78	3.5	4.22	4.94	5.56	6.18	6.69	7.3	8.34	9.78	12.05	14.1	16.3	22.0	26.26	30.3	34.2
2000	1.07	1.92	2.89	3.64	4.39	5.14	5.78	6.42	6.95	7.60	8.67	10.17	12.52	14.66	16.9	22.9	27.28	31.46	35.52
3000	1.10	1.98	2.97	3.74	4.51	5.28	5.94	6.6	7.15	7.81	8.91	10.45	12.87	15.07	17.38	23.54	28.05	32.34	36.52
4000	1.14	2.05	3.08	3.88	4.67	5.47	6.15	6.84	7.41	8.09	9.23	10.83	13.34	15.62	18.01	24.4	29.07	33.52	37.8
5000	1.17	2.10	3.16	3.98	4.8	5.62	6.32	7.02	7.61	8.31	9.48	11.12	13.69	16.03	18.49	25.04	29.84	34.4	38.84
6000	1.20	2.16	3.24	4.08	4.9	5.76	6.48	7.2	7.8	8.52	9.72	11.4	14.04	16.44	18.96	25.68	30.6	35.4	39.84
7000	1.23	2.21	3.32	4.18	5.04	5.9	6.64	7.38	7.99	8.73	9.96	11.68	14.39	16.85	19.43	26.32	31.36	36.16	40.84
8000	1.26	2.27	3.40	4.28	5.17	6.05	6.8	7.56	8.19	8.95	10.21	11.97	14.74	17.26	19.9	26.96	32.13	37.04	41.83
9000	1.29	2.32	3.48	4.39	5.29	6.19	6.96	7.74	8.38	9.16	10.45	12.26	15.09	17.67	20.38	27.6	32.9	37.92	42.83
10000	1.32	2.38	3.56	4.49	5.41	6.34	7.13	7.92	8.58	9.37	10.69	12.54	15.44	18.08	28.86	28.25	33.66	38.8	43.82
12000	1.37	2.47	3.7	4.66	5.62	6.57	7.4	8.22	8.9	9.73	11.1	13.02	16.03	18.77	21.64	29.32	34.94	40.28	45.48
15000	1.43	2.57	3.86	4.86	5.86	6.86	7.72	8.58	9.3	10.15	11.58	13.58	16.73	19.59	22.59	30.6	36.46	42.04	47.47

Example.—Required the amount of free air necessary to operate thirty 5-inch "H" drills at 9000 feet altitude, using to operate these drills air at a gage pressure of 80 pounds per square inch.

From Table I we find, when operating the drills at 80 pounds gage pressure at sea level, that one 5-inch "H" drill requires 190 cubic feet of free air per minute.

From Table II we also find that the factor for 30 drills at 9000 ft. altitude is 20.38; multiplying 190 cubic feet by 20.38 gives 3872 cubic feet free air per minute, which is the displacement of a compressor for the above outfit under average conditions, to which must be added pipe line losses, such as friction and leakage.

GLOBE VALVES, TEES AND ELBOWS

The reduction of pressure produced by globe valves is the same as that caused by the following additional lengths of straight pipe, as calculated by the formula:

$$\text{Additional length of pipe} = \frac{114 \times \text{diameter of pipe}}{1 + (36 \div \text{diameter})}$$

Diameter of pipe	1	1½	2	2½	3	3½	4	5	6	inches
Additional length	2	4	7	10	13	16	20	28	36	feet
	7	8	10	12	15	18	20	22	24	inches
	44	53	70	88	115	143	162	181	200	feet

The reduction of pressure produced by elbows and tees is equal to two-thirds of that caused by globe valves. The following are the additional lengths of straight pipe to be taken into account for elbows and tees. For globe valves multiply by $\frac{3}{2}$.

Diameter of pipe	1	1½	2	2½	3	3½	4	5	6	inches
Additional length	2	3	5	7	9	11	13	19	24	feet
	7	8	10	12	15	18	20	22	24	inches
	30	35	47	59	77	96	108	120	134	feet

These additional lengths of pipe for globe valves, elbows and tees must be added in each case to the actual lengths of straight pipe. Thus a 6-inch pipe, 500 feet long, with 1 globe valve, 2 elbows and 3 tees, would be equivalent to a straight pipe $500 + 36 + (2 \times 24) + (3 \times 24) = 656$ feet long.

APPENDIX B

In the following tables are collected all the reliable data that the author has been able to find relative to friction in air pipes.

In these tables the significance of the symbols is as follows:

No = Reference number of the experiment.

p_1 = Absolute pressure at first station on the pipe = pounds per square inch.

p_2 = Absolute pressure at second station on the pipe = pounds per square inch.

$p_m = \frac{p_1 + p_2}{2}$ = mean pressure in pipe between stations.

$f = p_1 - p_2$ = pressure lost between stations = pounds per square inch.

r = Mean ratio of compression between stations.

v_a = Cubic feet of free air passing per second.

v_m = Cubic feet of compressed air passing per second.

s = Velocity of air in pipe = feet per second.

Q = Weight in pounds of air passing per second.

d = Diameter of pipe in inches.

l = Length of pipe in feet.

c = Coefficient in formula (20), Art. 23, *viz.*, $f = c \frac{l}{d^5} \frac{v_a^2}{r}$.

DATA ON FRICTION IN AIR PIPES

No.	p_1	p_2	T_m	f	v_a	r	v_m	S	Q	C	
	EXPERIMENTS BY GUTTERMUTH & REIDLER, PARIS, 1890. $d = 11.811''$										
+ 1	106.57	77.17	91.87	20.45	135.26	5.49	24.64	28.44	10.07	.0375	$d = 11.811''$ $l = 54141'$
2	114.37	78.79	96.58	27.12	135.70	6.59	20.60	27.03	10.21	.0540	
3	119.44	100.55	109.98	14.13	109.30	7.49	14.57	19.13	8.22	.0506	
4	116.13	107.02	111.57	6.67	79.03	7.60	10.40	13.45	5.94	.0667	
+ 5	114.37	108.78	111.57	3.97	74.51	8.78	8.49	11.15	4.85	.0376	
+ 6	116.13	111.72	113.92	3.46	98.64	7.75	12.72	16.70	7.42	.0346	$d = 11.811''$ $l = 14446'$
+ 7	119.44	116.86	118.16	2.35	66.38	8.04	8.26	10.85	4.99	.0747	
8	114.37	105.40	109.88	7.14	135.70	7.44	18.24	23.95	10.21	.0567	
9	119.44	113.19	116.31	5.11	109.30	8.27	13.21	18.14	8.22	.0689	
10	113.93	110.25	112.09	2.96	84.25	7.62	11.05	14.50	6.34	.0627	
+11	108.34	105.84	107.08	1.66	89.26	7.29	12.25	16.08	6.71	.0364	
+12	112.45	108.65	110.54	2.55	79.05	7.53	10.50	13.78	5.97	.0360	$d = 11.811''$ $l = 28737'$
+13	102.90	100.55	101.72	2.35	109.30	6.92	15.79	20.73	8.22	.0280	$d = 11.811$ $l = 10958'$
+14	107.75	107.16	107.57	0.59	79.05	7.31	10.81	14.17	5.97	.0141	
	EXPERIMENTS BY STOCKALPER AT ST. GOTHARD TUNNEL.										
1	83.32	77.03	79.67	6.29	33.07	5.35	6.53	19.32	2.67	.0634	$d = 7.874''$ $l = 15092'$ $T = 70°$ F.
2	63.94	60.71	62.99	3.23	22.00	4.00	5.51	16.30	1.78	.0586	
3	56.45	53.65	55.05	2.79	18.36	3.49	5.26	15.57	1.48	.0580	
4	77.13	73.50	75.31	3.63	33.07	4.68	7.06	37.14	2.69	.0664	$d = 5.96''$ $l = 1713'$ $T = 80°$ F.
+ 5	60.71	59.69	60.97	1.03	22.00	3.75	5.86	30.82	1.78	.0351	
6	53.65	52.11	59.88	1.54	18.36	3.29	5.58	29.34	1.48	.0660	

The experiments marked + seem abnormal as compared to all other experiments.

DATA ON FRICTION IN AIR PIPES (Continued)

No.	p_1	p_2	p_m	f	v_a	r	v_m	S	Q	C	
	EXPERIMENTS BY DEVILLEZ AT LEVANT DU FLENU, 1879										
1	78.60	78.34	78.48	0.24	4.72	5.36	0.88	6.70	0.343	.0516	$d = 4.921''$
2	76.60	75.21	75.91	1.39	11.54	5.48	2.23	16.99	0.84	.0478	$l = 3219'$
3	73.66	70.13	71.85	3.63	18.11	4.88	3.71	28.01	1.32	.0486	$T = 86°$ F.
4	69.75	64.80	67.28	4.95	18.34	4.57	4.01	30.35	1.34	.0604	
5	64.93	60.98	62.96	3.95	16.79	4.28	3.92	29.69	1.22	.0537	
6	62.69	59.81	61.25	2.88	13.83	4.32	3.32	25.14	1.01	.0563	
7	59.76	54.86	57.31	4.90	17.53	3.90	4.50	34.03	1.27	.0557	
8	56.45	51.83	54.13	4.60	15.94	3.68	4.33	32.52	1.16	.0599	
+9	54.27	52.52	53.40	1.75	18.90	7.71	2.45	18.55	0.65	.0738	
10	52.73	51.93	52.33	0.80	6.88	3.57	1.93	13.59	0.50	.0543	
11	50.74	57.02	58.88	3.73	13.36	3.25	4.11	30.42	0.97	.0623	
12	79.95	79.09	79.52	0.86	8.57	5.42	1.58	11.98	0.64	.0563	
13	77.67	77.08	76.38	2.59	14.09	5.20	2.71	20.53	1.05	.0607	
14	75.20	73.60	74.40	1.60	10.82	5.06	2.14	16.19	0.81	.0621	
+15	66.19	58.78	62.49	7.41	23.81	4.25	5.60	40.32	1.77	.0408	
16	49.94	43.86	46.90	6.07	16.73	3.19	5.24	39.71	1.25	.0623	

DATA ON FRICTION IN AIR PIPES (Continued)

No.	p_1	p_2	p_m	f	v_a	r	v_m	S	Q	C	
	EXPERIMENTS BY LORENS AT OFFENBACH, 1892										
1	99.50	96.78	98.14	2.72	17.34	5.86	2.96	30.71	1.39	.0498	
2	99.78	96.65	98.20	3.12	15.98	6.62	2.39	28.39	1.28	.0538	$d = 3.937''$
3	100.01	97.39	98.78	2.79	14.81	6.73	2.20	26.05	1.19	.0824	$l = 981'$
4	100.90	98.21	99.56	2.69	17.27	6.77	2.55	30.16	1.38	.0588	$T = 36°$ F.
5	101.25	98.49	99.87	2.76	16.01	6.78	2.36	27.86	1.28	.0704	
6	100.93	98.40	99.66	2.52	17.41	6.77	2.57	30.37	1.39	.0545	
7	100.61	98.12	99.66	2.48	14.75	6.77	2.18	25.81	1.18	.0769	
8	100.93	98.40	99.66	2.53	16.52	6.80	2.43	28.82	1.23	.0605	
9	101.24	98.49	99.86	2.48	15.30	6.80	2.25	26.63	1.23	.0769	
10	100.69	97.93	99.31	2.76	14.81	6.76	2.19	25.94	1.19	.0820	
11	100.20	97.48	98.81	2.72	15.29	6.71	2.28	26.94	1.23	.0751	
	EXPERIMENTS BY DEVILLEZ AT LEVANT DU FLENU, 1879										
1	74.56	66.12	70.34	8.44	14.65	4.79	3.06	67.95	1.12	.0654	
2	63.52	62.33	62.92	1.19	4.95	3.15	1.57	25.67	0.38	.0723	$d = 2.874''$
3	61.96	55.30	58.62	6.64	10.89	3.99	2.73	60.59	0.83	.0776	$l = 564'$
4	59.12	43.27	51.23	15.92	16.83	3.48	4.83	107.2	1.28	.0681	$T = 62°$ F
5	56.44	56.18	56.31	0.264	2.17	3.81	0.57	12.61	0.17	.0742	
6	46.56	42.14	54.40	4.51	7.92	3.02	2.62	57.86	0.60	.0755	

APPENDIX C

During 1910 and 1911, an extensive series of experiments were made at Missouri School of Mines to determine the laws of friction of air in pipes under three inches in diameter; the chief object being to determine the coefficient "c" in the formula $f = c\frac{l}{d^5}\frac{v_a^2}{r}$. (See Art. 23.)

The general scheme is illustrated in Fig. 15, in which the parts are lettered as follows:

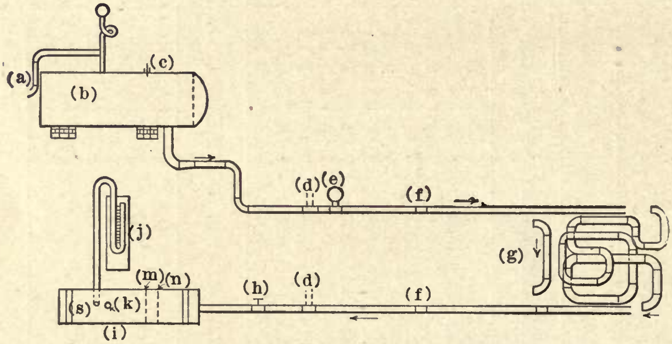


Fig. 15. Diagram Illustrating Assembled Apparatus.

a, is the compressed-air supply pipe.

b, a receiver of about 25 cubic feet capacity.

c, a thermometer set in receiver.

d and *d*, points of attachment of differential gauge.

f and *f*, lengths of straight pipe going to and from the group of fittings.

e, the pressure gauge.

g, the group of fittings — varied in different experiments.

h, the throttle valve to control pressure.

I, the orifice drum for measuring air, with the attachments as in Fig. 7.

On each set of fittings there were made ten or twelve runs with varying pressures and quantities of air in order to show the relation of f to $\frac{v_a{}^2}{r}$ over as wide a field as possible.

The data of each run was worked up and recorded in tabular form. Three of these tables, relating to 1-inch pipe and fittings, are shown herewith as example. It should be recorded that in the series of runs and checks some puzzling inconsistencies developed, but not more noticeable than appears in the data from European experiments on larger pipe. (See Appendix B.)

In these tables the symbols are as follows:

z = Head, in inches of mercury, in differential gauge.
f = Lost pressure in pounds per square inch.
p_2 = Gauge pressure at entrance to pipe.
r_m = Mean ratio of compression in pipe.
i = Water head, in inches, in U tube on orifice drum.
T_c = Temperature (centigrade) in drum.
d_o = Diameter, in inches, of orifice in drum.
v_a = Volume of free air passing (cubic feet per second).
S = Velocity of compressed air in pipe (feet per second).
f' = Value of f when corrected for temperature.

EXPERIMENTS AT MISSOURI SCHOOL OF MINES — 1911

TABLE III. — ACTUAL DIAMETER OF PIPE = 1.07″. LENGTH PIPE = 80′.

Fittings: 2 elbows, 13 nipples (reamed ends).

No.	z'' (Hg)	f	p_2	r_2	r_m	i	T_c	d_o''	$\frac{v_a^2}{r_m}$	S	f'
1	50.5 (H_2O)	1.82	22	2.58	2.56	1.9	13.0	1.50	.186	35	1.83
2	7.0	3.44	24	2.74	2.62	4.2	14.0	"	.403	65	3.44
3	9.7	4.77	25	2.79	2.62	5.8	13.0	"	.560	76	4.77
4	1.3	0.64	52	4.69	4.67	1.4	13.0	"	.072	17	0.62
5	3.5	1.22	50	4.58	4.54	3.4	13.0	"	.124	28	1.18
6	5.7	2.86	51	4.64	4.54	4.8	13.0	"	.264	39	2.75
7	2.3	1.13	75	6.33	6.30	3.4	13.5	"	.134	24	1.05
8	1.5	0.74	75	6.33	6.31	2.0	13.5	"	.079	19	0.69
9	4.3	2.11	75	6.33	6.26	5.9	14.0	"	.240	32	1.96
10	0.8	0.39	100	8.09	8.08	1.3	15.0	"	.040	12	0.35
11	1.9	0.93	100	8.09	8.07	3.2	15.0	"	.099	18	0.84
12	3.3	1.62	100	8.09	8.04	5.5	15.0	"	.135	24	1.45
13	0.7	0.34	125	9.86	9.84	1.3	16.0	"	.021	8	0.30
14	1.8	0.85	125	9.86	9.83	3.5	16.0	"	.083	15	0.73
15	2.3	1.13	124	9.80	9.76	6.5	17.0	"	.165	21	0.98
16	0.8	0.39	150	11.64	11.63	1.3	18.0	"	.028	8	0.33
17	1.4	0.69	150	11.64	11.61	3.0	18.0	"	.032	9	0.58
18	2.9	1.43	150	11.64	11.61	6.6	18.5	"	.140	18	1.30

EXPERIMENTS AT MISSOURI SCHOOL OF MINES — 1911

TABLE IV. — ACTUAL DIAMETER OF PIPE = 1.07″. LENGTH PIPE = 80′.

Fittings: 10 elbows, 9 nipples (unreamed ends).

No.	z'' (Hg)	f	p_2	r_2	r_m	i	T_e	d_e''	$\frac{v_a^2}{r_m}$	S	f'
1	4.6	2.26	22	2.56	2.48	1.9	12.0	1.50	0.197	47	2.25
2	7.8	3.85	24	2.70	2.56	3.34	12.0	"	0.321	59	3.81
3	14.3	7.05	24	2.70	2.35	5.9	12.0	"	0.618	108	7.00
4	5.2	2.57	50	4.54	4.45	4.3	12.0	"	0.236	38	2.45
5	2.0	0.99	50	4.54	4.50	1.5	12.0	"	0.079	22	0.94
6	10.0	4.92	51	4.60	4.42	7.5	12.5	"	0.418	51	4.70
7	1.7	0.84	75	6.31	6.28	1.9	13.0	"	0.071	17	0.77
8	4.2	2.07	75	6.31	6.24	4.4	13.0	"	0.175	28	1.90
9	7.2	3.55	75	6.31	6.19	7.2	14.0	"	0.287	36	3.25
10	2.1	1.04	100	8.08	8.05	2.5	14.0	"	0.075	16	0.95
11	3.2	1.58	100	8.08	8.03	4.3	15.0	"	0.131	21	1.38
12	5.5	2.71	100	8.08	7.98	7.0	15.0	"	0.209	27	2.31
13	1.3	0.64	125	9.85	9.83	2.2	15.0	"	0.053	12	0.57
14	2.0	0.98	125	9.85	9.82	2.9	16.0	"	0.089	14	0.84
15	5.2	2.56	125	9.85	9.76	8.0	16.0	"	0.201	24	2.20
16	1.2	0.59	150	11.65	10.63	2.0	17.0	"	0.029	9	0.49
17	2.0	0.99	150	11.65	10.62	3.9	17.5	"	0.090	15	0.82
18	4.7	2.32	150	11.65	10.57	8.6	17.5	"	0.198	22	1.92

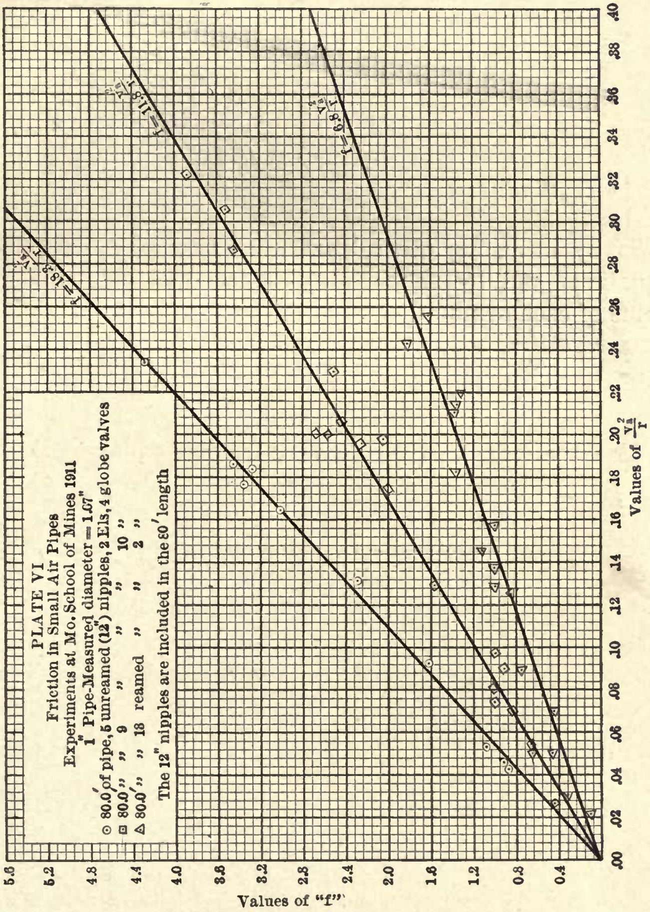
EXPERIMENTS AT MISSOURI SCHOOL OF MINES — 1911

TABLE V. — ACTUAL DIAMETER OF PIPE = 1.07″. LENGTH PIPE = 80′.

Fittings: 4 globe valves, 2 elbows, 5 nipples (unreamed ends).

No.	z'' (Hg)	f	p_2	r_2	r_m	i	T_e	d_o''	$\frac{v_a^2}{r_m}$	S	f'
1	7.2	3.55	21	2.49	2.36	1.8	10.0	1.50	0.189	47	3.52
2	20.2	10.00	20	2.42	2.07	4.3	10.0	"	0.442	71	9.90
3	31.2	15.40	24	2.70	2.15	7.3	9.0	"	0.852	107	15.24
4	2.1	1.20	45	4.20	4.16	0.9	9.0	"	0.053	19	1.07
5	11.0	5.41	45	4.20	4.01	4.4	9.5	"	0.218	43	5.20
6	20.2	10.00	46	4.26	3.91	8.3	9.5	"	0.529	61	9.60
7	3.2	1.58	72	6.10	6.05	2.1	10.5	"	0.035	12	1.48
8	7.2	3.55	70	5.96	5.84	4.3	11.0	"	0.183	29	3.32
9	13.9	6.88	70	5.96	5.13	7.9	12.0	"	0.342	40	6.40
10	1.8	0.89	100	8.09	8.07	1.6	13.0	"	0.049	13	0.81
11	5.2	2.55	100	8.09	8.00	4.3	14.0	"	0.131	21	2.32
12	9.5	4.67	100	8.09	7.93	7.5	14.0	"	0.233	23	4.25
13	2.0	0.98	124	9.80	9.77	1.7	16.0	"	0.043	14	0.87
14	5.4	2.66	125	9.86	9.77	4.9	16.0	"	0.077	18	2.36
15	7.8	3.84	124	9.80	9.67	7.0	16.0	"	0.177	22	3.40
16	2.1	1.04	150	11.64	11.61	2.2	17.0	"	0.045	10	0.90
17	3.8	1.88	150	11.64	11.58	4.4	18.0	"	0.092	15	1.62
18	7.0	3.45	150	11.64	11.53	7.8	18.0	"	0.164	16	2.99

On platting the values of f and $\frac{v_a^2}{r}$ as corresponding co-ordinates, it becomes apparent that they are related to each other in all cases as ordinates to a straight line; which could have been anticipated from the established laws of fluid frictions. This is shown on Plate VI.



From this plate we get the following three equations:

$$80.0\,K + 2\,e + 5\,u + 4\,g = 18.3,$$

$$80.0\,K + 10\,e + 9\,u = 11.8,$$

$$80.0\,K + 2\,e + 13\,m = 6.8,$$

in which $K\frac{v_a^2}{r}$ = resistance due to one foot of pipe;

$e\frac{v_a^2}{r}$ = resistance due to one elbow;

$m\frac{v_a^2}{r}$ = resistance due to one extra ferrule or joint with ends reamed;

$u\frac{v_a^2}{r}$ = resistance due to one extra ferrule or joint with ends unreamed;

$g\frac{v_a^2}{r}$ = resistance due to one globe valve.

So by attaching other lengths or fittings we get other equations and by simple algebra can find the numerical value of each symbol.

Then $$Kl\frac{v_a^2}{r} = c\frac{l}{d^5}\frac{V_a^2}{r} \quad \text{or} \quad c = d^5K.$$

Also the length of pipe giving friction equal to that of one elbow is $\frac{e}{k}$, and so with other fittings.

These experiments covered standard galvanized pipes of 2, 1½, 1, ¾, and ½ inch diameter. With each size pipe, runs were made to find friction loss in ordinary elbows, 45° elbows, globe valves, return bends, unreamed joints, and reamed joints. For each combination, data was taken for platting twelve to eighteen points, altogether about eight hundred. The results as a whole are satisfactory for the 2-, 1½-, and 1-inch pipes.

For the ¾- and ½-inch pipes, especially the ½-inch pipe, the results were so irregular, erratic, and conflicting that the results finally recorded cannot be accepted as final. In the light of these results, it is not probable that a satisfactory coefficient will ever be gotten for pipes under 1 inch; the reason being that in pipes of such small diameter, irregularities have relatively much greater effect than in larger pipes, and the probability of obstructions lodging in such pipes is relatively greater. In the ½-inch pipe and fitting, unreamed joints were found at which four-tenths of the area was obstructed, and this with a knife edge. No doubt consistent results could have been gotten by using only pipes that had been "plugged and reamed," and selected filling, but these results would not have been a safe guide for practice unless such preparation of the pipe be specified.

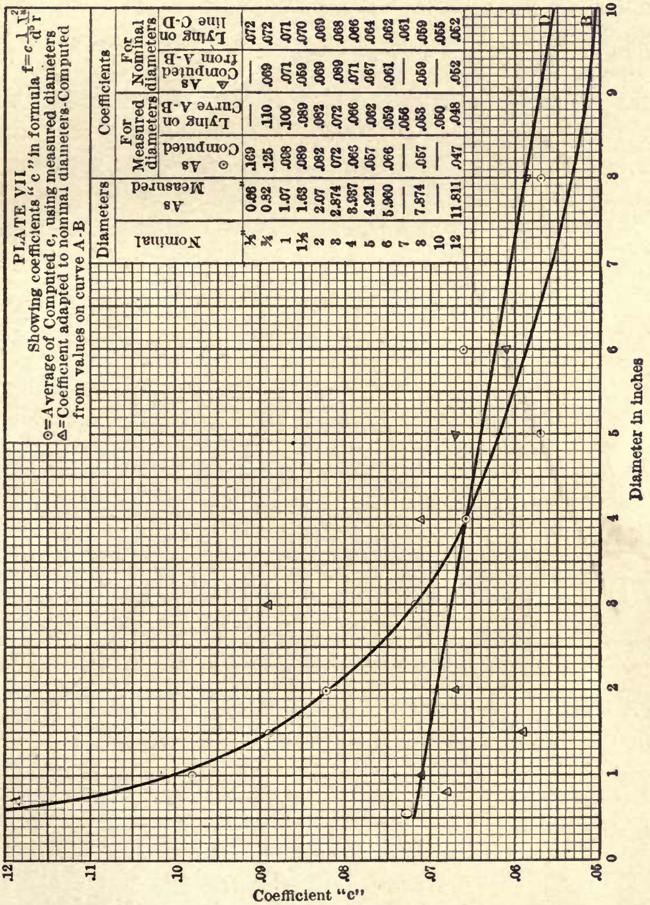


PLATE VII
Showing coefficients "c" in formula $f=c\frac{1}{d^3}\frac{L^2}{r}$
⊙=Average of Computed c, using measured diameters
△=Coefficient adapted to nominal diameters-Computed from values on curve A-B

Diameters		Coefficients			
		For Measured diameters		For Nominal diameters	
Nominal	As Measured	⊙ As Computed	Lying on Curve A-B	△ As Computed from A-B	Lying on line C-D
½"	0.66"	.169	—	—	.072
¾	0.82	.125	.110	.069	.072
1	1.07	.098	.100	.071	.071
1½	1.63	.089	.089	.059	.070
2	2.07	.082	.082	.069	.069
3	2.874	072	.072	.089	.068
4	3.937	.066	.066	.071	.066
5	4.921	.057	.062	.067	.064
6	5.960	.066	.059	.061	.062
7	—	—	.056	—	.061
8	7.874	.057	.053	.059	.059
10	—	—	.050	—	.055
12	11.811	.047	.048	.052	.052

The results of these researches are embodied in Plate VII. They show the averages of such data as seem worthy of consideration. The data for pipes exceeding 2 inches diameter are taken from the matter recorded in Appendix B. Verification of these by the use of the sensitive differential gauge is desirable.

Table IX and Plates 0 to IV of this volume were worked out with coefficients differing slightly from those here recommended, but the errors are probably well within those ordinarily effecting results in practice. Until the results of further research are available, the author recommends the use, in practice, of the coefficients taken from the curve *AB*, Plate VII.

In the series of experiments referred to, the results worked out for the resistance of fittings were more erratic than those for straight pipes. Hence no clain is made for precision or finality in the results here presented. However, two important conclusions are reached. One is that the resistance of globe valves has heretofore been underestimated, and the importance of reaming small pipe has not been appreciated.

TABLE OF LENGTHS OF PIPE IN FEET THAT GIVE RESISTANCE EQUAL THAT OF VARIOUS FITTINGS

Diameter of Pipe.	90° Elbows.	Unreamed Joints, Two Ends.	Reamed Joints, Two Ends.	Return Bends.	Globe Valves.
$\frac{1}{2}$	10.0	2 to 4	1.0	10.0	20.0
$\frac{3}{4}$	7.0	"	1.0	7.0	25.0
1	5.0	"	1.0	5.0	40.0
$1\frac{1}{2}$	4.0	"	1.0	4.0	45.0
2	3.5	"	1.0	3.5	47.0

A series of runs were made on 50-foot lengths of rubber-lined armored hose such as is used to connect with compressed-air tools. The scheme was the same as that described for pipes and fittings; and the range of $\frac{v_a^2}{r}$ was the same. The average results are here given. This includes the resistance in a 50-foot length with the metallic end couplings. In these end connections a considerable contraction occurs. For the half-inch hose the end couplings are quarter-inch. The excessive resistance in the half-inch hose may have been due to these end contractions or to some other obstruction. It is a further illustration of the fact that reliable coefficients cannot be gotten for pipes of half-inch diameter and less.

Diameter of hose in inches	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$
Resistance in 50-foot joints	$950.0\frac{v_a^2}{r}$	$20.0\frac{v_a^2}{r}$	$4.5\frac{v_a^2}{r}$	$2.6\frac{v_a^2}{r}$

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