Lesson 6: 
Modeling the Web as a graph

Unit 1: 
Reviewing basic terms from graph theory

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Introduction to Web Science Part 2
Emerging Web Properties
Completing this unit you should

• Be familiar with a set theoretic way of denoting a graph

• Know at least 4 different types of graphs

• Have practiced your abilities in reading and writing mathematical formulas
Let’s approach graph based models like text

1. Descriptive modeling
   - Understand the topology
   - Look at distributions of occurrence
   - Find measures to quantify what we have seen

2. Linear algebra
   - Maybe useful for predictions

3. Generative modeling
   - Find a process to generate the descriptive model
Definition of a graph

• A Graph $G(V, E)$ consists of two sets
  – $V$ the set of vertices or nodes
  – $E \subseteq V \times V$ the set of edges
  – $V$ is usually finite, thus is $E$

• A Graph is called undirected if:
  – $\forall e = (v, u) \in E \exists e' \in E : e' = (u, v)$
  – How it this expressed as a sentence?
Definition of a labeled graph

- A graph is called **labeled** if it is a **vertex-labeled** graph or an **edge-labeled** graph.

- A Graph $G(V, E)$ is called
  - **vertex-labeled** if there is a labeling function $l : V \rightarrow L$ with $L$ a set of labels.
  - **edge-labeled** if the labeling function has the edges as a domain i.e.: $l : E \rightarrow L$

- Can you think of some examples?
Definition of weighted graphs

- A **weighted** Graph is an edge-labeled graph with a **labeling function** $\omega : E \rightarrow \mathbb{R}$ such that every label is a real number.

- The following choices for $L$ are pretty common
  - $L = \mathbb{N}$
  - $L = \{0, 1\}$
  - $L = \{-1, 1\}$
Definition of a graph

- A Graph $G(V, E)$ is called if and only if

  - Property 1
    - $\exists U_1, U_2 \subset V$
    - $V = U_1 \cup U_2$
    - $U_1 \cap U_2 = \emptyset$

  - Property 2

    $\forall e = (u, v) \in E : u \in U_1 \land v \in U_2 \lor v \in U_1 \land u \in U_2$
Definition of a bipartite graph

- A Graph \( G(V, E) \) is called bipartite if and only if
  
  - Vertices have a disjoint split
    - \( \exists U_1, U_2 \subseteq V \)
    - \( V = U_1 \cup U_2 \)
    - \( U_1 \cap U_2 = \emptyset \)
  
  - Such that all edges cross the disjoint sets

\[
\forall e = (u, v) \in E : u \in U_1 \land v \in U_2 \lor v \in U_1 \land u \in U_2
\]
Thank you for your attention!

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