

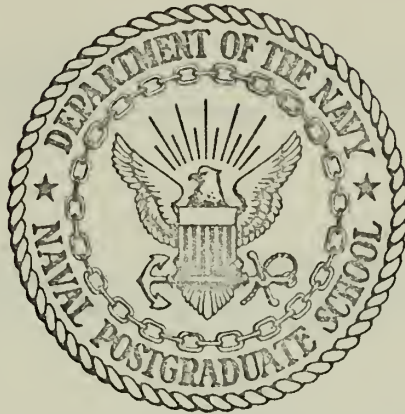
A PARAMETER OPTIMIZATION APPROACH TO AIR-  
CRAFT GUST ALLEVIATION

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

A PARAMETER OPTIMIZATION APPROACH TO  
AIRCRAFT GUST ALLEVIATION

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March 1972

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A Parameter Optimization Approach to  
Aircraft Gust Alleviation

by

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## ABSTRACT

This research indicates that a knowledge of angle of attack ( $\alpha$ ) and/or gust perturbation angle of attack ( $\alpha_g$ ) is of prime importance for a longitudinal autopilot designed to alleviate the effect of vertical gusts. However, accurate measurement of  $\alpha$  and  $\alpha_g$  is extremely difficult. This research indicates that the readily measurable normal acceleration factor ( $n$ ) and pitch rate ( $\hat{q}$ ) can be used to provide  $\alpha$  and  $\alpha_g$  information indirectly.





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## SYMBOLS

$a_{C_z}$	= Linear acceleration of aircraft center of mass in z direction, ft/sec.
$\bar{c}$	= Length of mean aerodynamic chord, ft.
$c_{m_\eta}$	= $\frac{\partial c_m}{\partial \eta} \circ$
$c_{L_\alpha}$	= $\frac{\partial c_L}{\partial \alpha} \circ$
$g$	= Local acceleration of gravity, ft/sec.
$h$	= Altitude, (ft).
$\bar{e}_O^2$	= Mean-square value of output of system.
$K1 \rightarrow K4$	= Control system gains.
$L_w$	= Gust characteristic length, ft.
$m$	= Aircraft mass, Slugs.
$\frac{n}{n^2}$	= $a_{C_z}/g$ , normal acceleration factor, ft/sec <sup>2</sup> /g.
$\Delta \bar{n}^2$	= $\bar{n}^2_{\text{unalleve.}} - \bar{n}^2$ , (ft/sec <sup>2</sup> /g) <sup>2</sup> .
$\bar{n}^2_{\text{unalleve.}}$	= Value of $\bar{n}^2$ with no autopilot utilized, (ft/sec <sup>2</sup> /g) <sup>2</sup> .
$\hat{q}$	= Pitch rate, rad/airsec.
$\hat{q}_g$	= Pitch rate due to vertical gust, rad/airsec.
$U_O$	= Aircraft forward velocity, ft/sec.
$u$	= Aircraft forward velocity perturbation, ft/sec.
$xyz$	= Stability axis system.
$w$	= Velocity perturbation along z axis, ft/sec.
$w_g$	= Gust vertical velocity, ft/sec.
$\alpha_g$	= $w_g/U_O$ , angle of attack perturbation due to gust, measured at aircraft center of mass, rad.





- $\eta$  = Elevator deflection, measured from trim, rad.
- $\overline{\eta^2}$  = Mean-square value of elevator deflection (Rad)<sup>2</sup>.
- $\delta_f$  = DLC (direct lift control) deflection.
- $\delta_t$  = Throttle deflection.
- $\Omega$  = Spatial frequency, rad/ft.
- $\hat{\omega}$  =  $\Omega U_o t^* = \Omega U_o \frac{\bar{C}}{2U_o}$ , Frequency, Rad/AirSec.
- $\sigma_w$  = Root mean-square value of vertical gust velocity, ft/sec.
- $\theta$  = Pitch angle perturbation, Rad.
- $\rho$  = Atmospheric density, Slugs/Ft<sup>3</sup>.



## ACKNOWLEDGEMENTS

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## I. INTRODUCTION

### A. LONGITUDINAL AUTOPILOTS

#### 1. Basic Ideas in Longitudinal Control

Since the beginning of powered flight, methods have been sought to reduce pilot effort in controlling aircraft and to automate various aspects of aircraft control. The autopilots and stability augmentation systems of today are realizations of such research.

Autopilots and stability augmentation systems are usually classified as longitudinal or lateral. As the names imply, longitudinal (lateral) autopilots are concerned with the longitudinal (lateral) flight regulation. This dual classification is justified in flight conditions where the existence of pure longitudinal/lateral motion is ensured.

Blakelock [Ref. 1] offers a very readable introduction to autopilot design. He discusses three basic types of longitudinal controllers, each designed for different longitudinal operations: a displacement autopilot where aircraft pitch angle is fed back and compared with a reference pitch angle; a pitch orientational autopilot where aircraft pitch rate is fed back and compared with some desired pitch rate; and an acceleration control system where the acceleration of the center of gravity is fed back and compared with a reference acceleration. A good treatment of the feedbacks that may be used in longitudinal control systems can be found in Ref. 2, Sec. II. Table I (obtained from Ref. 2) indicates the different feedbacks which can be utilized to control longitudinal motion.



The research summarized here is concerned with autopilots designed to alleviate the effect of vertical gusts on a large jet transport in steady, wings-level flight. Two flight conditions are considered, high altitude cruise and landing approach. By reducing the aircraft's gust response it is possible to provide for a smoother ride, aircraft structural fatigue problems can be alleviated, and the ability of the aircraft to function as a gun platform can be improved.

## 2. Longitudinal Control to Alleviate Gusts

Appendix B offers an introduction to the statistical design concepts utilized in this study. As Table I indicates many feedback parameters such as pitch, pitch rate, center of gravity acceleration, etc., can be utilized as feedbacks in a controller designed to reduce gust response. Hess [Ref. 3] discussed a two-parameter gust alleviation system shown in Fig. 1. He analyzed this system using a performance index given by:

$$I = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\alpha g}(\hat{\omega}) \left| \frac{n}{\alpha_g}(s) \right|_{s=j\hat{\omega}}^2 d\hat{\omega} + \int_{-\infty}^{\infty} \phi_{\alpha g}(\hat{\omega}) \left| \frac{\eta}{\alpha_g}(s) \right|_{s=j\hat{\omega}}^2 d\hat{\omega}$$

where  $\hat{\omega}$  is the non-dimensionalized frequency and the transfer functions  $\frac{n}{\alpha_g}(s)$  and  $\frac{\eta}{\alpha_g}(s)$  are obtained from the aircraft's equation of motion as discussed in Appendix C.  $\phi_{\alpha g}(\hat{\omega})$  is the non-dimensionalized power spectral density of the gust field and was obtained from Equation 1 found in Appendix B. Hess's study utilized a large, rigid jet transport. The characteristics for the two flight conditions considered may be found in Appendix A. His results are shown in Table II.





Hess also proposed an optimal control system, Fig. 2, which showed considerable improvement over the two-parameter feedback system above. For analysis of this system Hess used a performance index given by:

$$I = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [qn^2(t) + ru^2(t)] dt$$

The aircraft was the same as used in analysis of the two-parameter system. Table III shows results of his study for a  $q/r$  equal to 10, a value was selected by a root-square locus technique. Note the improvement in  $\frac{\Delta n^2}{\overline{n^2_{unalleviated}}}$  which Hess's optimal system maintains over the two-parameter system of Table II.

#### B. BASIS OF THIS RESEARCH

A major obstacle to the implementation of Hess's optimal gust alleviation system is the difficulty in measuring  $\alpha_g$ . Attempts to design instruments that will give satisfactory on-line measurements of  $\alpha_g$  have been unsuccessful. This report investigated the effect of eliminating  $\alpha_g$  information on the performance of the alleviation system. Figure 3 shows Hess's optimal gust alleviation system with gust information removed. This is the system utilized in this study.



## II. SOLUTION PROCEDURES

### A. METHODS OF SOLUTION

Optimal feedback design can be classified as free configuration or fixed configuration. The latter is often referred to as parameter optimization. In the free configuration procedure the designer is not restricted in any way. The final design depends only upon the input and the form of the performance criteria. With the parameter optimization techniques the configuration is chosen using feedbacks which are readily obtainable and then the feedback parameters are optimized to minimize the performance criteria.

As noted in Ref. 2, for autopilot design the parameter optimization technique is probably the superior method considering current state of the art. This technique yields an autopilot which fits the "real world" situation. This research utilized the parameter optimization procedure since the form of the longitudinal controller was prespecified (Fig. 3).

### B. PERFORMANCE INDEX

The performance index chosen for the optimization procedure was given by:

$$I = \frac{1}{2\pi} \left[ \int_{-\infty}^{\infty} \phi_n(\hat{\omega}) d\hat{\omega} + \int_{-\infty}^{\infty} \phi_\eta(\hat{\omega}) d\hat{\omega} \right] = \bar{n}^2 + \bar{\eta}^2 \quad (1)$$

where  $\hat{\omega}$  denotes the non-dimensionalized frequency of the random gusts. This performance index is identical to that used by Hess in his evaluation of the optimal controller with  $q/r = 10$ . It is a sum of the mean square



normal acceleration factor and the mean square elevator deflection. The latter was included to prevent large elevator excursions. Equation (1) can be manipulated as indicated in Appendix B to a more usable form:

$$I = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\alpha g}(\hat{\omega}) \left| \frac{n}{\alpha_g}(j\hat{\omega}) \right|^2 d\hat{\omega} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\alpha g}(\hat{\omega}) \left| \frac{\eta}{\alpha_g}(j\hat{\omega}) \right|^2 d\hat{\omega} \quad (2)$$

$\phi_{\alpha g}$  is obtained from Equation (1) in Appendix B by substituting  $\hat{\omega} = \Omega_0 t^*$ . This non-dimensionalized form is:

$$\phi_{\alpha g}(\hat{\omega}) = \frac{\sigma_w^2 c}{L_w U_0^2} \frac{1}{\hat{\omega}^2 + (c^2/4L_w^2)} \quad (3)$$

The transfer functions  $\frac{n}{\alpha_g}(j\hat{\omega})$  and  $\frac{\eta}{\alpha_g}(j\hat{\omega})$  are obtained from the aircraft's longitudinal equations of motion, the equation for normal acceleration and the mathematical equation of the autopilot. Appendix C shows how this was done.

For the purposes of determining the aerodynamic gust effects, the aircraft was idealized as a segment of the x-axis. This idealization has been found to give good results as long as the shortest vertical-gust wave length is greater or equal to eight times the aircraft length [Ref. 4, pg. 323].

### C. INTEGRATION OF THE PERFORMANCE INDEX

Substitution of Equation (3) and the transfer functions into Equation (2) produced an equation whose form is frequently found in system analysis. Tables for solution of these equations are readily available, e.g. Ref. 5.



Appendix D shows how a computer program was written to solve for the performance index, which in this case took on the functional form of  $I_4(K_1, K_2, K_3, L_w)$ .

#### D. OPTIMIZATION

Integration of the performance index for a given characteristic gust length ( $L_w$ ) produced a function which contained the variables  $K_1$ ,  $K_2$ , and  $K_3$ , i.e. the feedback gains. The function obtained could be thought of as a performance index surface with dimensions  $K_1$ ,  $K_2$  and  $K_3$ . The optimum solution would be found where the surface produced a minimum value. A computer program was written to solve for this minimum by a method of steepest descent. Appendix D indicates how this was accomplished.





### III. RESULTS AND DISCUSSION

#### A. PERFORMANCE COMPARISONS

The results of the computer computation are presented in Table IV. The performance surface at most points was very shallow making it difficult to rapidly find a minimum value on the surface. It was found that the performance index was a weak function of K2, but a strong function of K1, K3 and  $L_w$ . K1, K2 and K3 were found to be very dependent on the flight condition, i.e. cruise or landing approach.

Comparing Table IV with Tables II and III shows that the three-parameter system has the largest performance index. Figures 5 and 6 indicate that the  $\frac{\Delta \bar{n}^2}{\bar{n}^2}_{\text{unalleve}}$  performance initially compares favorably with the optimal and the two-parameter system, but the performance rapidly deteriorates as  $L_w$  increases. This indicates that a knowledge of  $\alpha_g$  becomes increasingly important as  $L_w$  gets larger.

Note that in the two-parameter system of Fig. 1 the feedbacks  $n$  and  $\hat{q}$  give a knowledge of  $\alpha$  and  $\alpha_g$ . This can be seen from the equation for the normal acceleration factor which can be written as:

$$D(\alpha + \alpha_g) = \hat{q} - \frac{g\bar{c}}{2U_o^2} n$$

#### B. TRANSIENT PERFORMANCE

Figures 6 and 7 present the locus of roots of the characteristic equation obtained when the feedback gains K1 and K3 are set at values obtained for a gust characteristic length of 1000 feet while K2 was allowed to vary. This could be done because the performance index was



a weak function of  $K_2$ . A value of the variable  $K_2$  could be selected which would give the three-parameter system better transient performance than the two-parameter system, but this transient performance was still inferior to the optimal control system. A new root locus diagram would have to be drawn for each gust length and flight condition. In an effort to circumvent the drawing of multiple root locus diagrams, the feedback gains  $K_1$  and  $K_3$  were set at values obtained for  $L_w$  equal to 3000 feet. It is found, however, that by doing this performance was unacceptably degraded, particularly in the lower ranges of  $L_w$ .

#### C. STABILITY

It was found that the three-parameter system was stable throughout the range of feedback gains considered here.

#### D. POSSIBLE ALTERNATE SOLUTION

Comparing the three-parameter longitudinal controller of this research with the optimal controller (Fig. 2) and the two-parameter system (Fig. 1) it appears that  $\alpha_g$  is an important parameter for a gust alleviation system. Since, using present technology, it is not possible to measure  $\alpha_g$  aboard the aircraft some alternative method of obtaining this information must be employed.

It was noted previously (Sec. III, Part A) that a knowledge of  $n$  and  $\hat{q}$  would provide information on  $\alpha$  and  $\alpha_g$ . This would explain why the RMS performance of the two parameter feed back system tends to approach the RMS performance of the optimal controller as the characteristic gust length increases.



It is felt that by combining the optimal controller and the two-parameter system a gust alleviation system could be built that would have performance comparable to the optimal controller. Indeed, Hess [Ref. 6] has shown this to be the case. The modified gust alleviation system appears in Fig. 4. This system would have the advantage of utilizing feedback information that is readily obtainable on-board the aircraft.



#### IV. CONCLUSIONS

1. The longitudinal gust alleviation system evaluated by this report is a stable, usable system and could be designed to provide better transient performance than the two-parameter system of Fig. 1. However, the two-parameter system and the optimal controller of Fig. 2 both exhibit better RMS performance.
2.  $\alpha_g$  provides important information for a gust alleviation system. Knowledge of  $\alpha_g$  becomes increasingly important as the gust characteristic length increases, i.e. as the random gust field departs from white noise.
3. To provide for a gust alleviation system which uses readily obtainable information, and still produce satisfactory performance, the system presented in Fig. 4 is proposed.





Table I  
LONGITUDINAL COMPETING SYSTEMS

FEEDBACK OR CROSSFEED	PRIMARY FUNCTIONS PERFORMED	PRACTICAL DESIGN PROBLEMS
$\hat{q} \rightarrow \eta$	<ol style="list-style-type: none"> <li>1. Increase short-period damping.</li> <li>2. Reduce pitch response to gusts.</li> </ol>	Gain adjustment with flight condition.
$\theta \rightarrow \eta$	<ol style="list-style-type: none"> <li>1. Increase short-period damping and frequency.</li> <li>2. Reduce pitch response to gusts.</li> <li>3. Increase phugoid damping.</li> <li>4. Stabilize tuck mode.</li> </ol>	<ol style="list-style-type: none"> <li>1. Gain adjustment with flight condition.</li> <li>2. Increased <math>h</math> and <math>a_z</math> response to vertical gusts.</li> </ol>
$a_{C_z} \rightarrow \eta$	<ol style="list-style-type: none"> <li>1. Increase short-period damping and frequency.</li> <li>2. Reduce <math>h</math> and <math>a_z</math> response to gusts.</li> </ol>	<ol style="list-style-type: none"> <li>1. Severe gain adjustment with flight condition.</li> <li>2. Sensor location adequate for all flight conditions.</li> <li>3. Structural mode feedback.</li> <li>4. Increased <math>\theta</math> response to vertical gusts.</li> </ol>
$\alpha \rightarrow \eta$	<ol style="list-style-type: none"> <li>1. Increase short-period damping and frequency.</li> <li>2. Reduce <math>h</math> and <math>a_z</math> response to vertical gusts.</li> </ol>	<ol style="list-style-type: none"> <li>1. Gain adjustment with flight conditions.</li> <li>2. Sensor instrumentation. <ol style="list-style-type: none"> <li>a. Determination of operating point.</li> <li>b. Errors due to aerodynamic interference.</li> <li>c. Elaborate sensor complex required to suppress gust inputs.</li> </ol> </li> </ol>
$u \rightarrow \eta$	Stabilize tuck mode.	Sensor instrumentation (see above).



Table I (Continued)  
LONGITUDINAL COMPETING SYSTEMS

FEEDBACK OR CROSSFEED	PRIMARY FUNCTIONS PERFORMED	PRACTICAL DESIGN PROBLEMS
$\alpha \rightarrow \delta_T$	Prevent altitude instability.	Sensor instrumentation (see above).
$u \rightarrow \delta_T$	Prevent altitude instability.	Sensor instrumentation (see above).
$a_{C_z} \rightarrow \delta_F$	Reduce $h$ and $a_z$ response to gusts.	<ol style="list-style-type: none"> <li>1. Severe gain adjustment with flight condition.</li> <li>2. Sensor location adequate for all flight conditions.</li> <li>3. Structural mode feedback.</li> <li>4. Probable drag penalty due to direct lift control surface.</li> </ol>
$\eta \rightarrow \delta_T$	Prevent altitude instability.	Will not work if stick force/knot gets too low.



Table II  
TWO-PARAMETER LONGITUDINAL AUTOPILOT

Cruising Flight						
$K_1$	$K_2$	$L_w$	$\overline{n^2}$	$\overline{\eta^2}$	$\frac{\overline{\Delta n^2}}{\overline{n^2}_{unalle.}}$	$\sigma_w$
Rad/Ft/Airs <sup>2</sup> /g	Rad/Rad/Airs	Ft	(Ft/Sec <sup>2</sup> /g) <sup>2</sup>	(Rad) <sup>2</sup>		(Ft/Sec)
.36	675	500	.0466	.00282	.27	10
.36	475	1000	.0277	.00153	.37	10
.36	400	2000	.0151	.000818	.41	10
.36	375	3000	.0139	.000501	.24	10
.36	350	4000	.00790	.000435	.44	10
.36	350	5000	.00637	.000348	.44	10
.36	350	6000	.00534	.000290	.44	10

Landing Approach

$K_1$	$K_2$	$L_w$	$\overline{n^2}$	$\overline{\eta^2}$	$\frac{\overline{\Delta n^2}}{\overline{n^2}_{unalle.}}$	$\sigma_w$
.74	275	500	.0384	.0138	.21	10
.74	200	1000	.0209	.00690	.30	10
.74	175	2000	.0109	.00351	.41	10
.74	175	3000	.00736	.00234	.37	10
.74	175	4000	.00556	.00176	.38	10
.74	175	5000	.00447	.00141	.38	10
.74	175	6000	.00373	.00117	.39	10



Table III  
OPTIMAL LONGITUDINAL CONTROLLER  
Cruising Flight

$q/r$	$I_w$ Ft	$K_1$ Rad/Rad	$K_2$ Rad/Rad/Airs	$K_3$ Rad/Rad	$K_4$ Rad/Rad	$\overline{n^2}$ (Ft/Sec <sup>2</sup> /g) <sup>2</sup>	$\overline{n^2}$ (Rad) <sup>2</sup>	$\frac{\overline{\Delta n^2}}{n^2}$ Unall.	$\sigma_w$ Ft/Sec
10.0	500	44.06	1231.	-3.454	37.4	4.008 10 <sup>-2</sup>	2.790 10 <sup>-3</sup>	.37	10
10.0	1000	44.06	1231.	-3.454	37.4	2.252 10 <sup>-2</sup>	1.441 10 <sup>-3</sup>	.48	10
10.0	2000	44.06	1231.	-3.454	37.4	1.226 10 <sup>-2</sup>	7.303 10 <sup>-4</sup>	.53	10
10.0	3000	44.06	1231.	-3.454	37.4	8.589 10 <sup>-3</sup>	4.891 10 <sup>-4</sup>	.53	10
10.0	4000	44.06	1231.	-3.454	37.4	6.702 10 <sup>-3</sup>	3.679 10 <sup>-4</sup>	.52	10
10.0	5000	44.06	1231.	-3.454	37.4	5.552 10 <sup>-3</sup>	2.950 10 <sup>-4</sup>	.51	10
10.0	6000	44.06	1231.	-3.454	37.4	4.779 10 <sup>-3</sup>	2.463 10 <sup>-4</sup>	.50	10





Table III (Continued)  
OPTIMAL LONGITUDINAL CONTROLLER  
Landing Approach

$g/r$	$L_w$ Ft	$K_1$ Rad/Rad	$K_2$ Rad/Rad/Airs	$K_3$ Rad/Rad	$K_4$ Rad/Rad	$\overline{n^2}$ (Ft/Sec <sup>2</sup> /g) <sup>2</sup>	$\overline{n^2}$ (Rad) <sup>2</sup>	$\frac{\overline{\Delta n^2}}{n^2}$ Unall.	$\sigma_w$ Ft/Sec
10.0	500	15.02	279.2	-1.253	13.1	3.143 10 <sup>-2</sup>	8.482 10 <sup>-3</sup>	.35	10
10.0	1000	15.02	279.2	-1.253	13.1	1.712 10 <sup>-2</sup>	4.369 10 <sup>-3</sup>	.43	10
10.0	2000	15.02	279.2	-1.253	13.1	9.097 10 <sup>-3</sup>	2.219 10 <sup>-3</sup>	.46	10
10.0	3000	15.02	279.2	-1.253	13.1	6.282 10 <sup>-3</sup>	1.490 10 <sup>-3</sup>	.46	10
10.0	4000	15.02	279.2	-1.253	13.1	4.847 10 <sup>-3</sup>	1.123 10 <sup>-3</sup>	.46	10
10.0	5000	15.02	279.2	-1.253	13.1	3.997 10 <sup>-3</sup>	9.017 10 <sup>-4</sup>	.45	10
10.0	6000	15.02	279.2	-1.253	13.1	3.393 10 <sup>-3</sup>	7.542 10 <sup>-4</sup>	.44	10



Table IV  
THREE PARAMETER LONGITUDINAL CONTROLLER  
Cruising Flight

$L_W$ Ft.	K1 Rad/Rad	K2 $\frac{\text{Rad/Rad}}{\text{Airs}}$	K3 Rad/Rad	$\overline{n^2}$ (Ft/Sec <sup>2</sup> /g) <sup>2</sup>	$\overline{\eta^2}$ (Rad) <sup>2</sup>	$\frac{\overline{\Delta n^2}}{\overline{n^2}}$ unall
500	1.59	688.0	-2.57	.0461	.0001	.276
1000	1.60	688.0	-2.57	.0324	.0001	.259
2000	0.795	688.0	-3.08	.0207	*	.201
3000	0.568	688.0	-3.07	.0150	*	.176
4000	0.452	688.0	-2.99	.0120	*	.143
5000	0.393	688.0	-2.96	.0100	*	.123
6000	0.447	688.0	-2.99	.0085	*	.115

$\sigma = 10$  Ft/Sec

\* - Value less than .0001

Landing Approach

$L_W$ Ft.	K1 Rad/Rad	K2 $\frac{\text{Rad/Rad}}{\text{Airs}}$	K3 Rad/Rad	$\overline{n^2}$ (Ft/Sec <sup>2</sup> /g) <sup>2</sup>	$\overline{\eta^2}$ (Rad) <sup>2</sup>	$\frac{\overline{\Delta n^2}}{\overline{n^2}}$ unall
500	0.651	400.0	-1.00	.0356	.0011	.262
1000	0.785	400.0	-1.13	.0219	.0008	.270
2000	0.255	400.0	-0.996	.0124	.0004	.250
3000	0.165	400.0	-0.939	.0086	.0003	.265
4000	0.165	400.0	-0.939	.0066	.0003	.236
5000	0.098	400.0	-0.974	.0054	.0002	.250
6000	0.085	400.0	-0.953	.0045	.0002	.250

$\sigma = 10$  Ft/Sec



FIGURE 1

Two Parameter Gust Alleviation System

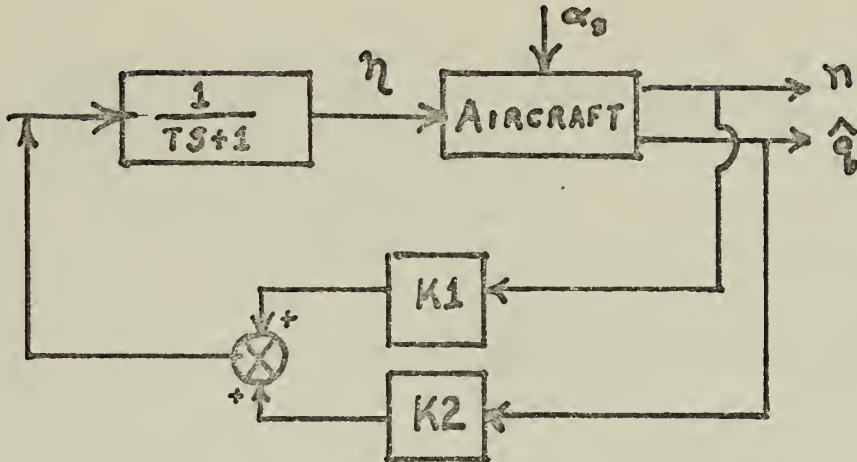


FIGURE 2

Optimum Gust Alleviation System

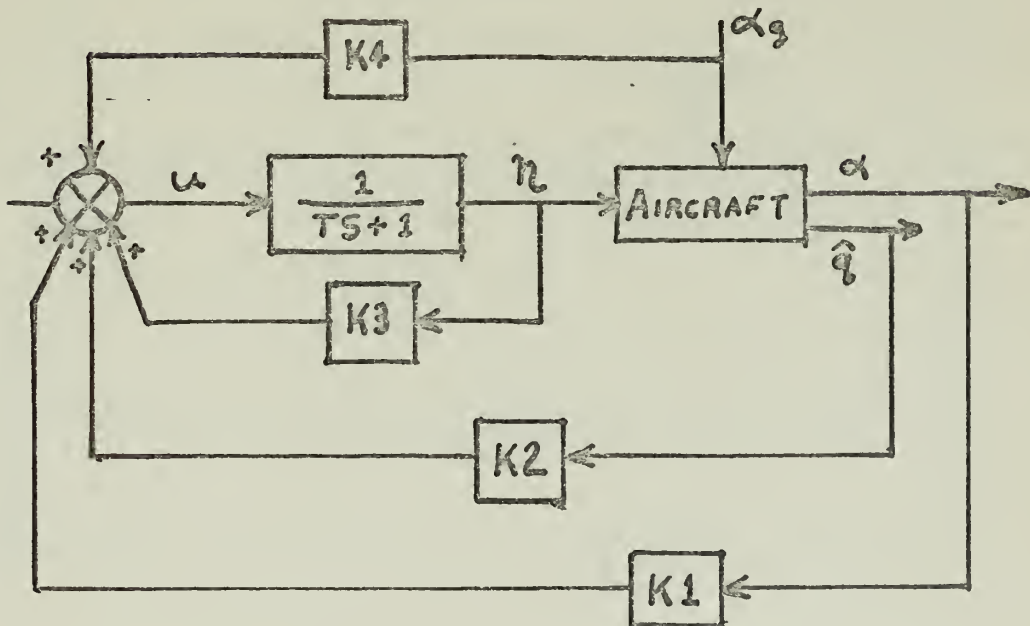




FIGURE 3

Three Parameter Gust Alleviation System

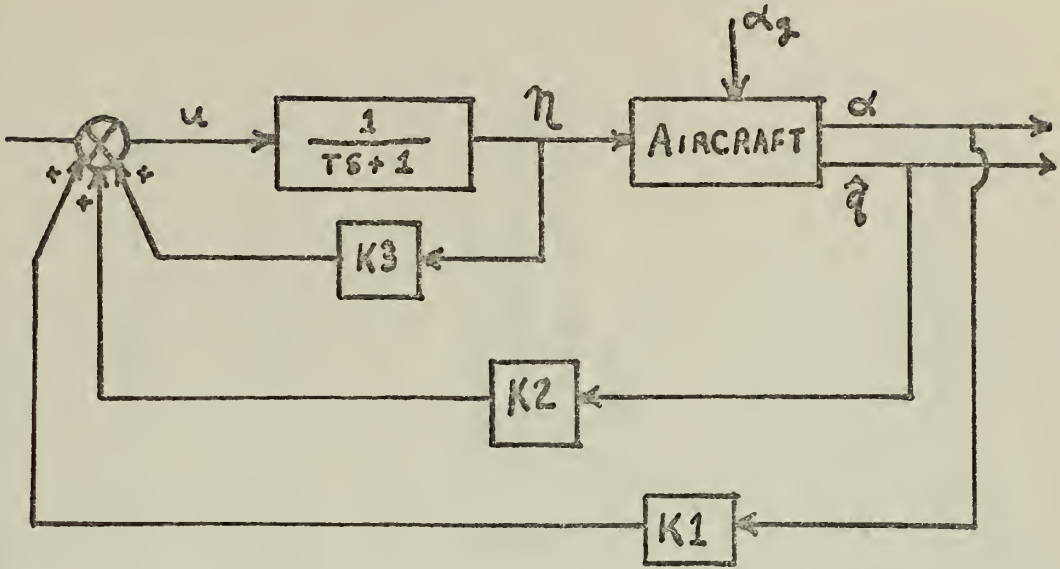


FIGURE 4

Proposed Gust Alleviation System

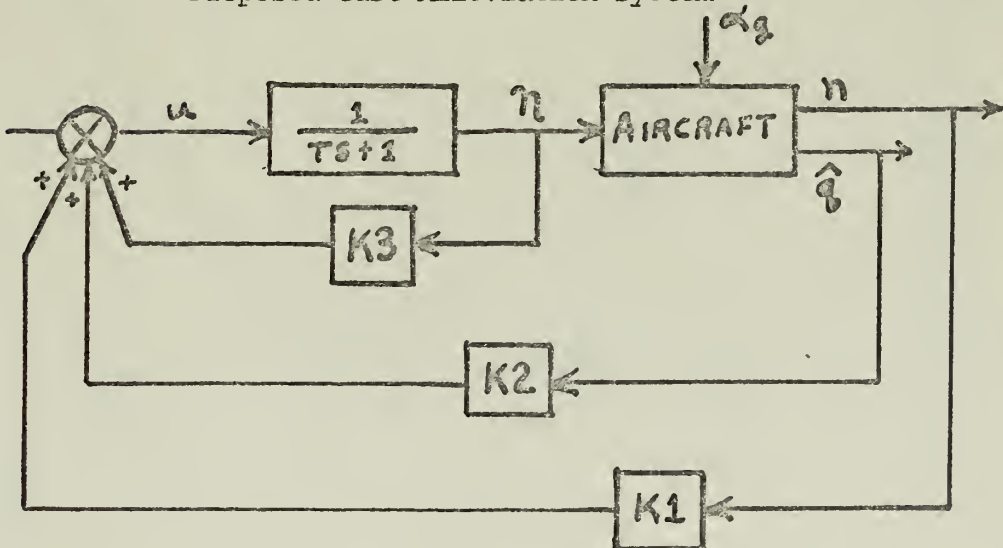






FIGURE 5. PERFORMANCE INDEX COMPARISONS

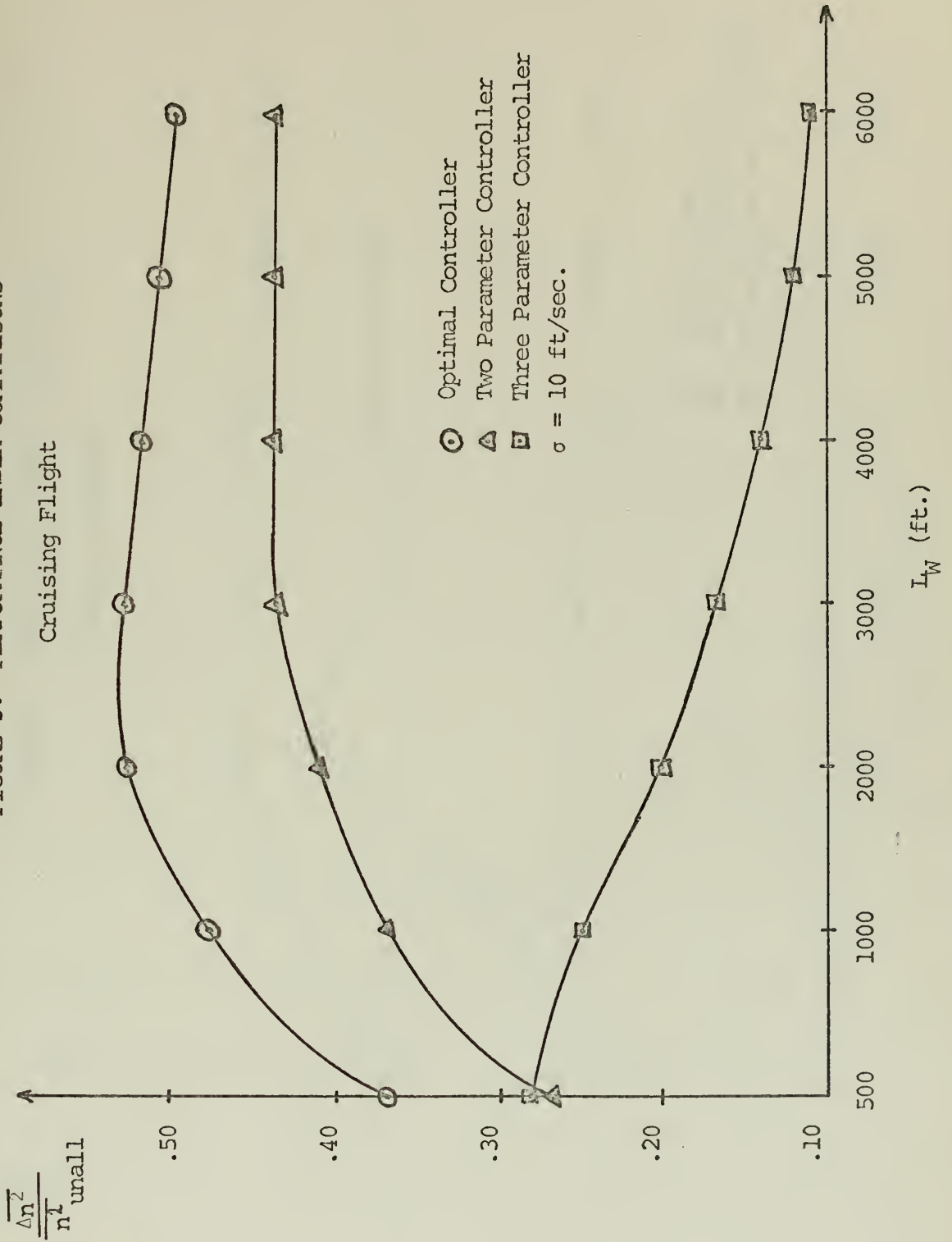




FIGURE 6. PERFORMANCE INDEX COMPARISONS

Landing Approach

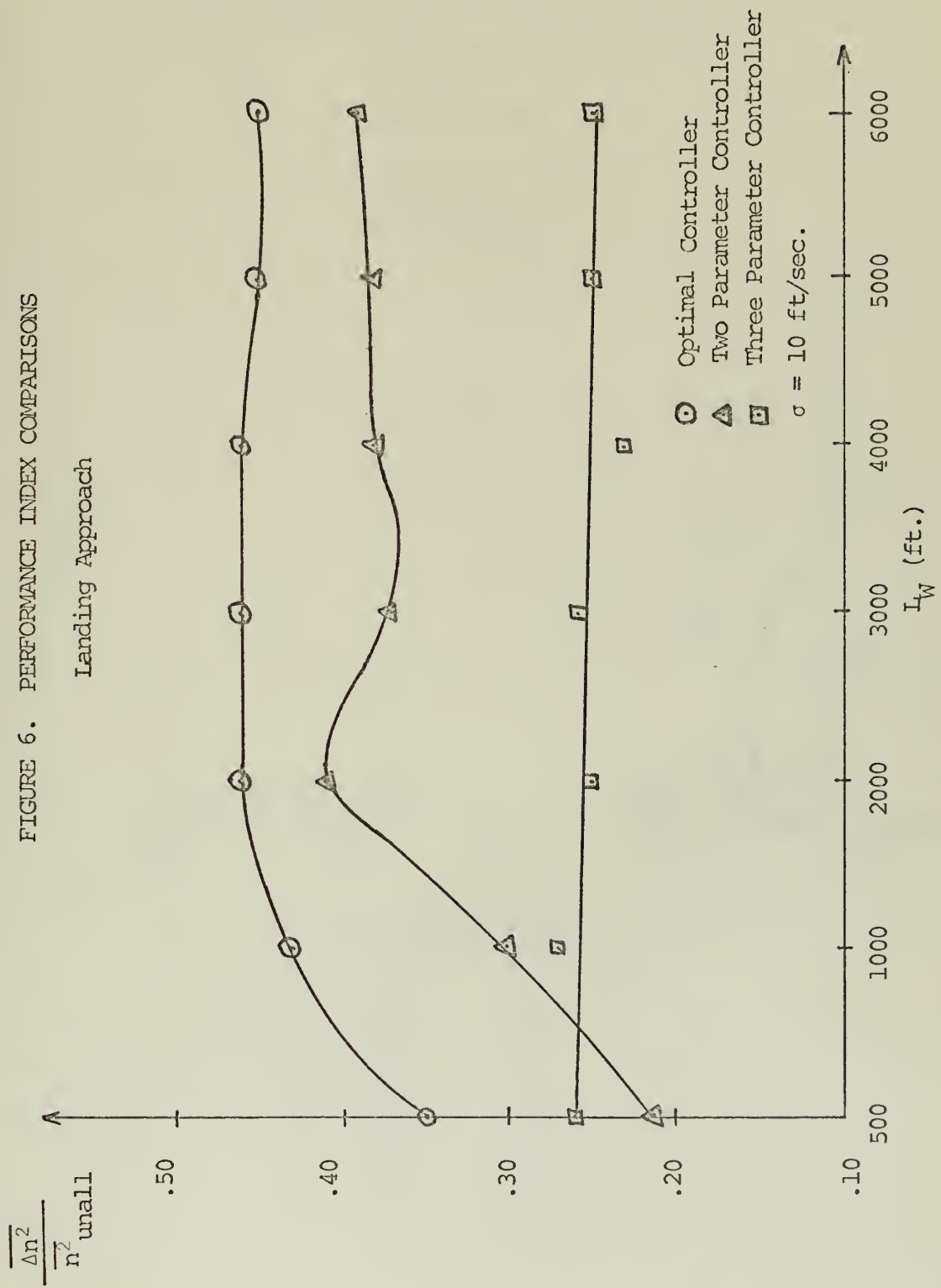




FIGURE 7

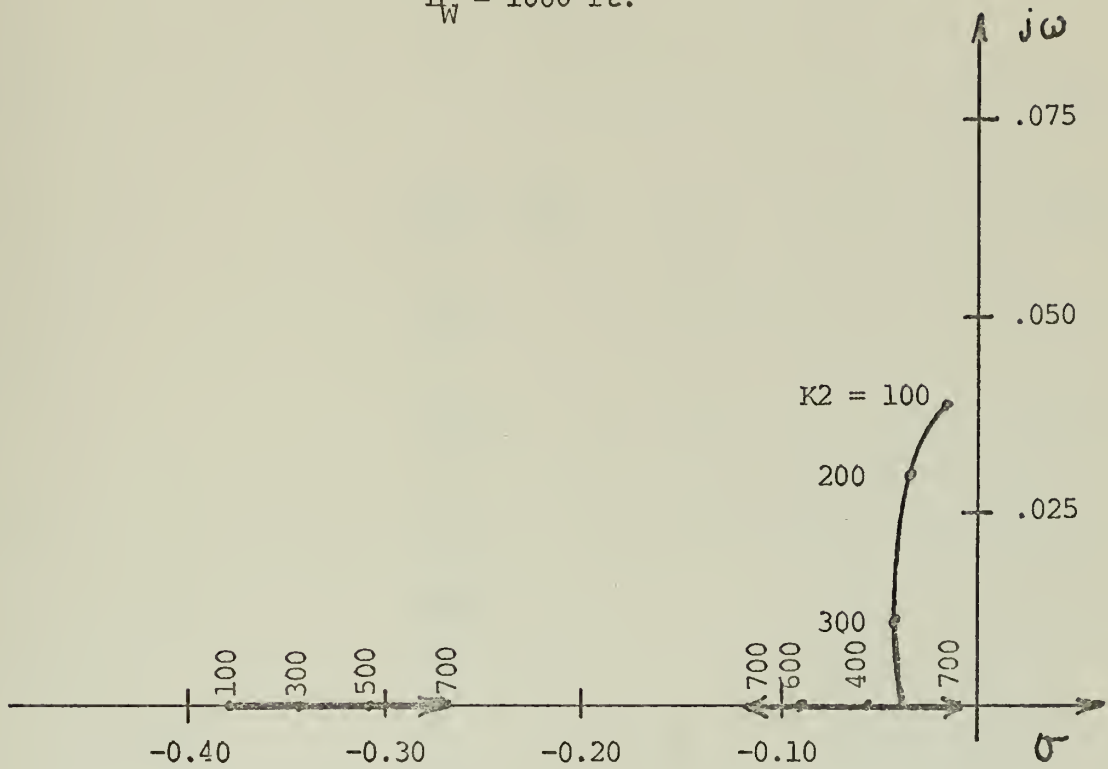
Three Parameter Gust Alleviation System

ROOT LOCUS

CRUISING FLIGHT

$K_1 = .785, K_3 = -1.13$

$I_W = 1000 \text{ ft.}$



PERFORMANCE INDEX	
K2	P.I.
100	.0336
200	.0330
300	.0327
400	.0326
500	.0325
600	.0325
700	.0325



FIGURE 8

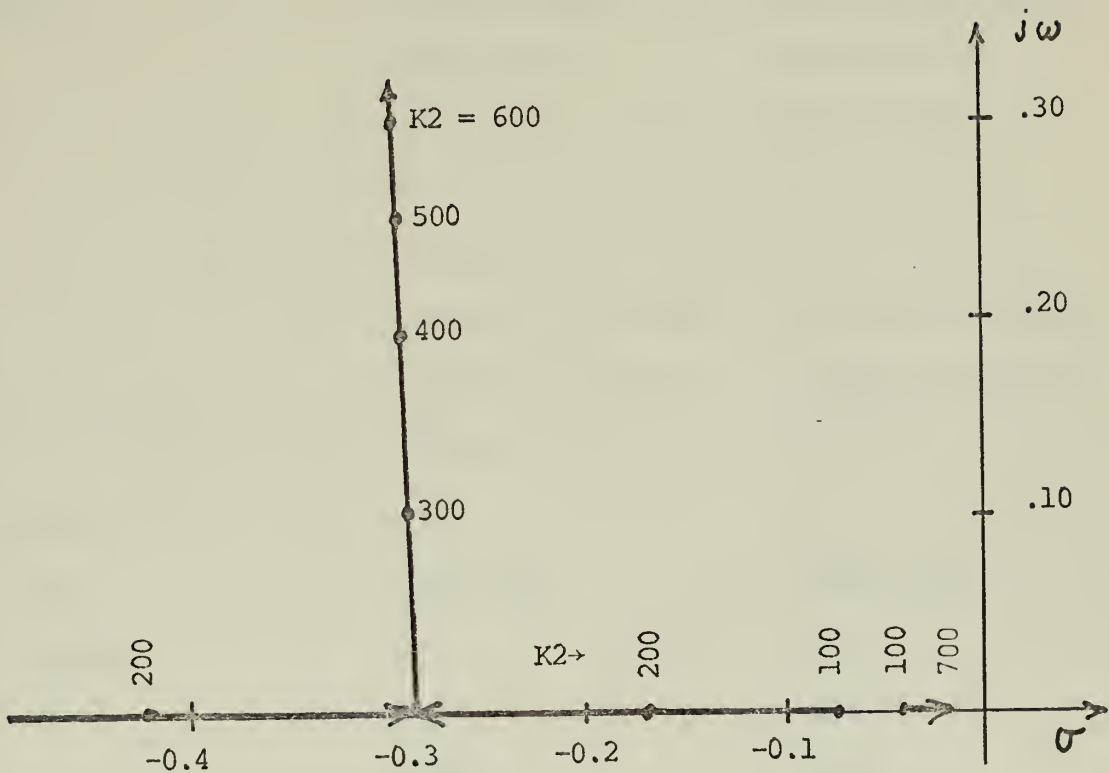
Three Parameter Gust Alleviation System

ROOT LOCUS

LANDING APPROACH

$K_1 = .785, K_3 = -1.13$

$L_W = 1000 \text{ ft.}$



PERFORMANCE INDEX	
K2	P.I.
100	.0237
200	.0230
300	.0228
400	.0227
500	.0228
600	.0229
700	.0231





APPENDIX A

Aircraft Characteristics

The aircraft utilized in this report was the large, rigid, jet transport used by Etkin to illustrate application of theory he presented in Chapter 6 of Ref. 4. The cruising flight data were obtained from Ref. 4 and the landing approach data were obtained from Ref. 7.

Parameter	Cruising Flight	Landing Approach
W	1000,000 lbs.	1000,000 lbs.
S	1667 sq. ft.	1667 sq. ft.
A	7	7
Alt.	30,000 ft.	0 ft.
$U_o$	500 mph. or 733 fps.	200 mph. or 294 fps.
$\rho$	0.000889 slugs/ft. <sup>3</sup>	0.002377 slugs/ft. <sup>3</sup>
$\bar{c}/2$	7.7 ft.	7.7 ft.
$\mu = m/\rho s \bar{c}/2$	272	102
$t^* = \bar{c}/2U_o$	0.0105 sed.	0.0262 sec.
$i_B = B/\rho s \bar{c}/2$	1900	710
$C_{L_o}$	0.25	0.59
$C_{D_o}$	0.0188	0.0295
$C_{z_\alpha}$	-4.9	-4.8
$C_{z_{\dot{\alpha}}}$	0	0
$C_{m_\alpha}$	-0.488	-0.478
$C_{m_{\dot{\alpha}}}$	-4.2	-4.2



Aircraft Characteristics (Continued)

Parameter	Cruising Flight	Landing Approach
$C_{m\dot{q}}$	-22.9	-22.9
$C_{z\dot{q}}$	0	0
$C_{z\eta}$	-0.24	-0.24
$C_{m\eta}$	-0.72	-0.72
$C_{m\dot{\eta}}$	0	0



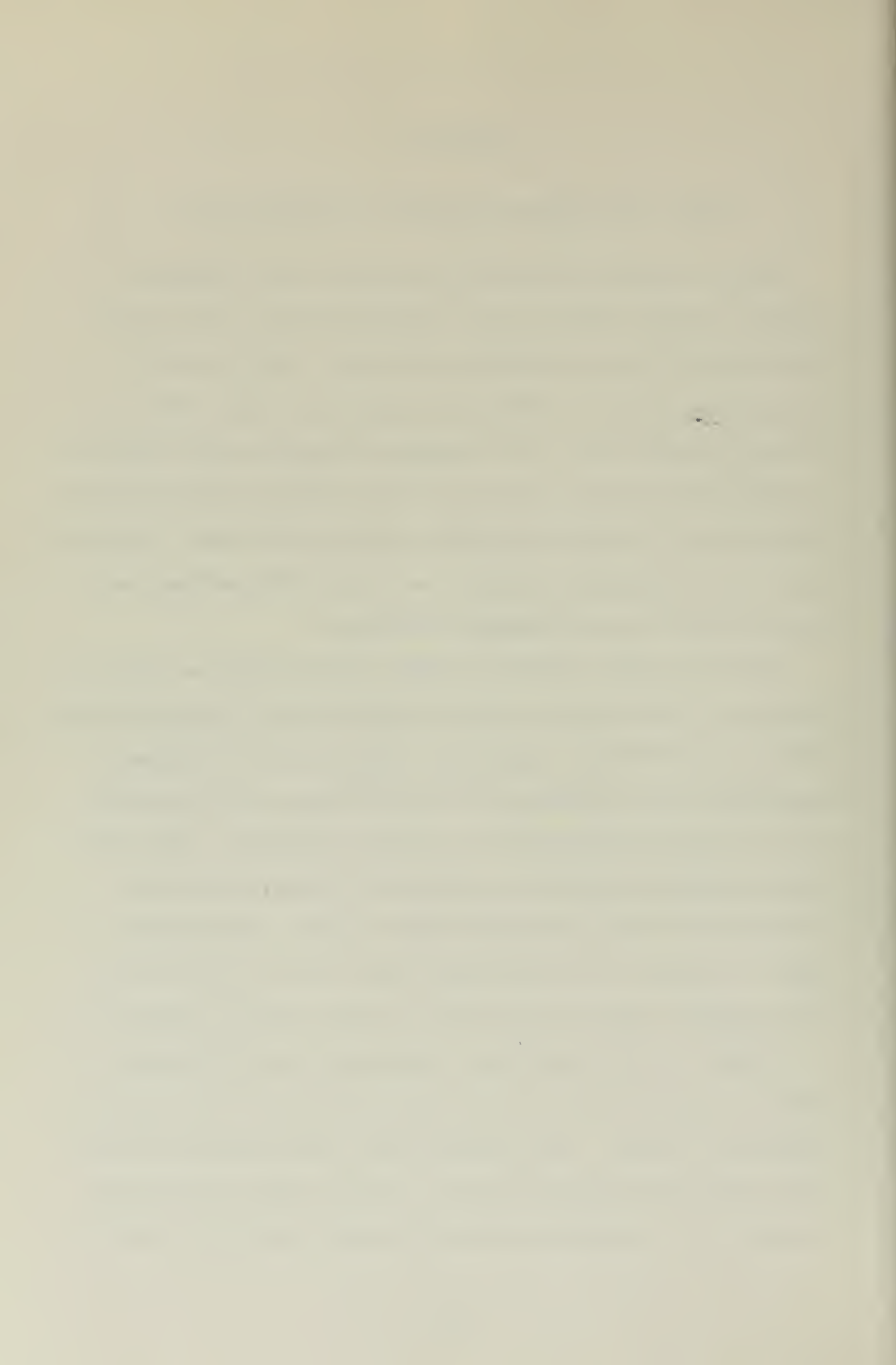
## APPENDIX B

### Random Gust Fields and Their Use in Autopilot Design

Aircraft controllers designed to behave well when disturbed by sinusoidal, step or impulsive gust disturbances often perform poorly when subjected to actual atmospheric conditions. This has led to seeking better methods of defining the turbulence or gust fields. It has been discovered that if the atmospheric disturbance was treated as a random process and the aircraft controller designed using statistical concepts many of the above difficulties could be eliminated. Controllers could then be analytically designed, and built, which performed well when subjected to actual atmospheric conditions.

Statistical design techniques brought with them many new terms and concepts not used in the era of the so-called classical autopilot design. Terms such as stationary, homogeneous, isotropic, frozen turbulence, mean-square error, correlation functions and power spectral densities are now included in the analytical designer's vocabulary. This writer will not attempt to provide any mathematical derivations of these concepts and will not even discuss them at length. The interested reader is referred to the bibliography found in Ref. 4, pp. 339-340. A very readable review of this subject is found in Ref. 1, Chapter 9.

In order to utilize statistical techniques in gust alleviation design an expression for the power spectral density of the atmospheric turbulence is needed. Such expressions have been developed which show good agreement with spectra measured in cumulus clouds, thunderstorms and clear air. The spectra used for the random vertical gust field of



this report is found in Ref. 3, given by:

$$\phi_{wg}(\Omega) = 2\sigma_w^2 L_w \frac{1}{1 + (L_w \Omega)^2} \quad (1)$$

where  $\sigma_w^2$  is the mean square value of the vertical gust velocity.  $L_w$  is the characteristic gust length, and  $\Omega$  is the spatial frequency.

Equation (1) can be written in non-dimensional form as:

$$\phi_{wg}(\hat{\omega}) = \frac{\sigma_w^2 c^2}{L_w} \frac{1}{(\hat{\omega}^2 + (c^2/4L_w^2))} \quad (2)$$

Although this expression is not as comprehensive as many expressions found for  $\phi_{wg}$ , it is used because it is easy to manipulate and it agrees fairly well with measured atmospheric power spectra. A form of Equation (1) was originally suggested in Ref. 8.  $\sigma_w$  has been found to vary from 0 to 40 feet per second and  $L_w$  from 500 to 6000 feet.

The power spectral density of any of the aircraft longitudinal motion variables such as angle of attack ( $\alpha$ ) can be obtained:

$$\phi_{\alpha\alpha}(\hat{\omega}) = \phi_{wg}(\hat{\omega}) \left| \frac{\alpha}{w_g}(\hat{\omega}) \right|^2 \quad (3)$$

Here  $\frac{\alpha}{w_g}(j\hat{\omega})$  represents an aerodynamic transfer function, with  $s = j\hat{\omega}$ , and relates angle of attack and gust vertical velocity.





In addition, mean square values of longitudinal output variables can be obtained as:

$$\overline{\alpha^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{\alpha\alpha}(\hat{\omega}) d\hat{\omega} \quad (4)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{w_g}(\hat{\omega}) \left| \frac{\alpha}{w_g}(j\hat{\omega}) \right|^2 d\hat{\omega} \quad (5)$$

Equations (3) and (5) provide the necessary tools for simple analysis and parameter optimization studies.



## APPENDIX C

### Determination of Transfer Functions

$$\frac{\hat{n}}{\alpha_g}(j\omega) \text{ and } \frac{\hat{\eta}}{\alpha_g}(j\omega)$$

#### I. Longitudinal Equations of Motion.

The aircraft longitudinal equations below were obtained from Ref. 4, page 133:

$$\text{X force: } (2\mu D - 2C_{L_O} \tan\theta_O - C_{X_u})\hat{u} - C_{X_\alpha} \alpha + C_{L_O} \theta = 0$$

$$\text{Z force: } (2C_{L_O} - C_{Z_u})\hat{u} + (2\mu D - C_{Z_\alpha} D - C_{Z_\alpha})\alpha - [(2\mu + C_{Z_q})D - C_{L_O} \tan\theta_O]\theta - C_{Z_\eta} \eta = 0$$

$$\text{Moment: } -C_{m_u} \hat{u} - (C_{m_\alpha} D + C_{m_\alpha})\alpha + (i_B D^2 - C_{m_q} D)\theta$$

$$- (C_{m_\eta} D + C_{m_\eta})\eta = 0$$

$$\hat{q} - D\theta = 0 \quad t^* = \ell/u_O \quad \ell = \bar{c}/2$$

Substituting  $\alpha = \alpha + \alpha_g$ ,  $\hat{q} = \hat{q} + \hat{q}_2$  and  $\hat{p} = \hat{p} = \hat{p}_g$  ;

Neglecting changes in forward velocity and considering the X-force equation to be identically satisfied yields Equations (1) and (2) below.

The equations are shown as they appear after the Laplace transform has been taken with zero initial condition assumed:

$$(2\mu s - C_{Z_\alpha} s - C_{Z_\alpha})\alpha - (2\mu + C_{Z_q})\hat{q} - C_{Z_\eta} \eta = [C_{Z_\alpha} s + C_{Z_\alpha} - C_{Z_q} s]\alpha_g \quad (1)$$

$$-(C_{m_\alpha} s + C_{m_\alpha})\alpha + (i_B s - C_{m_q})\hat{q} - [C_{m_\eta} s + C_{m_\eta}]\eta \quad (2)$$

$$= [C_{m_\alpha} s + C_{m_\alpha} - C_{m_q} s]\alpha_g$$



note: (a)  $\hat{q} + \hat{q}_g = D\theta$

$$\hat{q}_g = \frac{d(\omega_g)dt}{dt dx} = \frac{d \frac{\omega_g}{U_o}}{dt}$$

$$\hat{q}_g = -D\alpha_g$$

(b) The  $\alpha_g$  terms above are included on the right side of Equations (1) and (2) to indicate their use as forcing functions.

## II. Acceleration Equation.

The equation for the linear acceleration of the aircraft center of mass in the z direction is obtained from:

$$a_{c_z} = \dot{W} + PV - QU$$

by neglecting velocities in the y direction, assuming small disturbances, and using  $\omega = u_o \sin\alpha$  Equation (3) is obtained:

$$a_{c_z} = \frac{U_o}{t^*} (D\alpha - \hat{q}) \quad (3)$$

Now one can define the normal acceleration factor:

$$n(\hat{t}) = \frac{-a_{c_z}}{g} = \frac{2u_o^2}{2c} (\hat{q} - D\alpha)$$

or in Laplace form

$$n(s) = \frac{2U_o^2}{2c} (\hat{q} - s\alpha) \quad (4)$$



### III. Mathematical Model of Longitudinal Control System.

The equation for the longitudinal control system was obtained from the block diagram of the system and is given by:

$$K_1\alpha + K_2\hat{q} + (K_3 - T^s - 1)\eta = 0 \quad (5)$$

Note that the elevator-serro combination is approximated by a first order system with a time constant (T) of 1/10 second. T in the non-dimensionalized form is given by:

$$T^* = T/t^* = \frac{2TU_0}{c}$$

### IV. Simultaneous Solution.

From Equations (1), (2), (4), and (5) a system of equations is obtained of the form:

$$\begin{aligned} A_{11}\alpha + A_{12}\hat{q} + A_{13}\eta + A_{14}n &= b_1\alpha_g \\ A_{21}\alpha + A_{22}\hat{q} + A_{23}\eta + A_{24}n &= b_2\alpha_g \\ A_{31}\alpha + A_{32}\hat{q} + A_{33}\eta + A_{34}n &= b_3\alpha_g \\ A_{41}\alpha + A_{42}\hat{q} + A_{43}\eta + A_{44}n &= b_4\alpha_g \end{aligned} \quad (6)$$

where

$$\begin{aligned} A_{11} &= 2\mu^s - C_{z\dot{\alpha}}^s - C_{z\alpha} & A_{21} &= C_{m\dot{\alpha}}^s - C_{m\alpha} \\ A_{12} &= -2\mu - C_{z\dot{q}} & A_{22} &= i_B^s - C_{mq} \\ A_{13} &= -C_{z\eta} & A_{23} &= -C_{m\dot{\eta}} - C_{m\eta} \\ A_{14} &= 0 & A_{24} &= 0 \\ b_1 &= C_{z\dot{\alpha}}^s + C_{z\alpha} - C_{z\alpha}^s & b_2 &= C_{m\dot{\alpha}}^s + C_{m\alpha} - C_{mq}^s \end{aligned}$$





$$A_{31} = K1$$

$$A_{32} = K2$$

$$A_{33} = K3 - T^*s - 1$$

$$A_{34} = 0$$

$$b_3 = 0$$

$$A_{41} = -2U_0^2 s / g\bar{c}$$

$$A_{42} = 2U_0^2 / g\bar{c}$$

$$A_{43} = 0$$

$$A_{44} = -1$$

$$b_4 = 0$$

The transfer function  $\bar{n}(s)/\alpha_g(s) \Big|_{s=j\omega}$  is given by:

$$\frac{\bar{n}}{\alpha_g}(s) \Big|_{s=j\omega} = \frac{\begin{vmatrix} A_{11} & A_{12} & A_{13} & b_1 \\ A_{21} & A_{22} & A_{23} & b_2 \\ A_{31} & A_{32} & A_{33} & b_3 \\ A_{41} & A_{42} & A_{43} & b_4 \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix}} \quad (7)$$

$\bar{\eta}(s)/\alpha_g(s) \Big|_{s=j\omega}$  is obtained in a similar manner.



APPENDIX D

Computer Program

I. The computer program was written to provide a solution to a performance index given by:

$$I_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\alpha g} \left| \frac{n}{\alpha g} (j\hat{\omega}) \right|^2 d\hat{\omega} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{\alpha g}(\hat{\omega}) \left| \frac{n}{\alpha g} (j\hat{\omega}) \right|^2 d\hat{\omega} \quad (1)$$

where

$$I_{\alpha g}(\hat{\omega}) = \frac{\sigma^2 \bar{C}}{L_w U_0^2} \frac{1}{\omega^2 + \frac{c^2}{4L_w^2}} \quad (2)$$

$$\left| \frac{n}{\alpha g} (j\hat{\omega}) \right|^2 = \left| \frac{x1(j\hat{\omega})^2 + x2(j\hat{\omega}) + x3}{x4(j\hat{\omega})^3 + x5(j\hat{\omega})^2 + x6(j\hat{\omega}) + x7} \right| \quad (3)$$

$$\left| \frac{n}{\alpha g} (j\hat{\omega}) \right|^2 = \left| \frac{z1(j\hat{\omega})^3 + z2(j\hat{\omega})^2 + z3(j\hat{\omega}) + z4}{x4(j\hat{\omega})^3 + x5(j\hat{\omega})^2 + x6(j\hat{\omega}) + x7} \right| \quad (4)$$

In Equations (3) and (4), x1 through x7 and z1 through z4 contain the variables K1, K2, and K3 and coefficients of these variables. The coefficients are functions of the aircraft stability derivatives and the gust characteristic length.

It can be seen that the performance index can be written in the form:

$$I_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{C(j\hat{\omega})C(-j\hat{\omega})}{d(j\hat{\omega})d(-j\hat{\omega})} d\omega \quad (5)$$



This integral is evaluated for  $n = 1$  through  $n = 10$  in Ref. 5. For the performance index of this problem  $I_n = I_4$  where

$$I_4 = \frac{c_3^2(-d_0^2 d_3 + d_0 d_1 d_2) + (c_2^2 - 2c_1 c_3) d_0 d_1 d_4 + (c_1^2 - 2c_0 c_2) d_0 d_3 d_4 + c_0^2(-d_1 d_4^2 + d_2 d_3 d_4)}{2d_0 d_4(-d_0 d_3^2 - d_1^2 d_4 + d_1 d_2 d_3)} \quad (6)$$

$$c(j\hat{\omega}) = c_3(j\hat{\omega})^3 + c_2(j\hat{\omega})^2 + c_1(j\hat{\omega}) + c_0$$

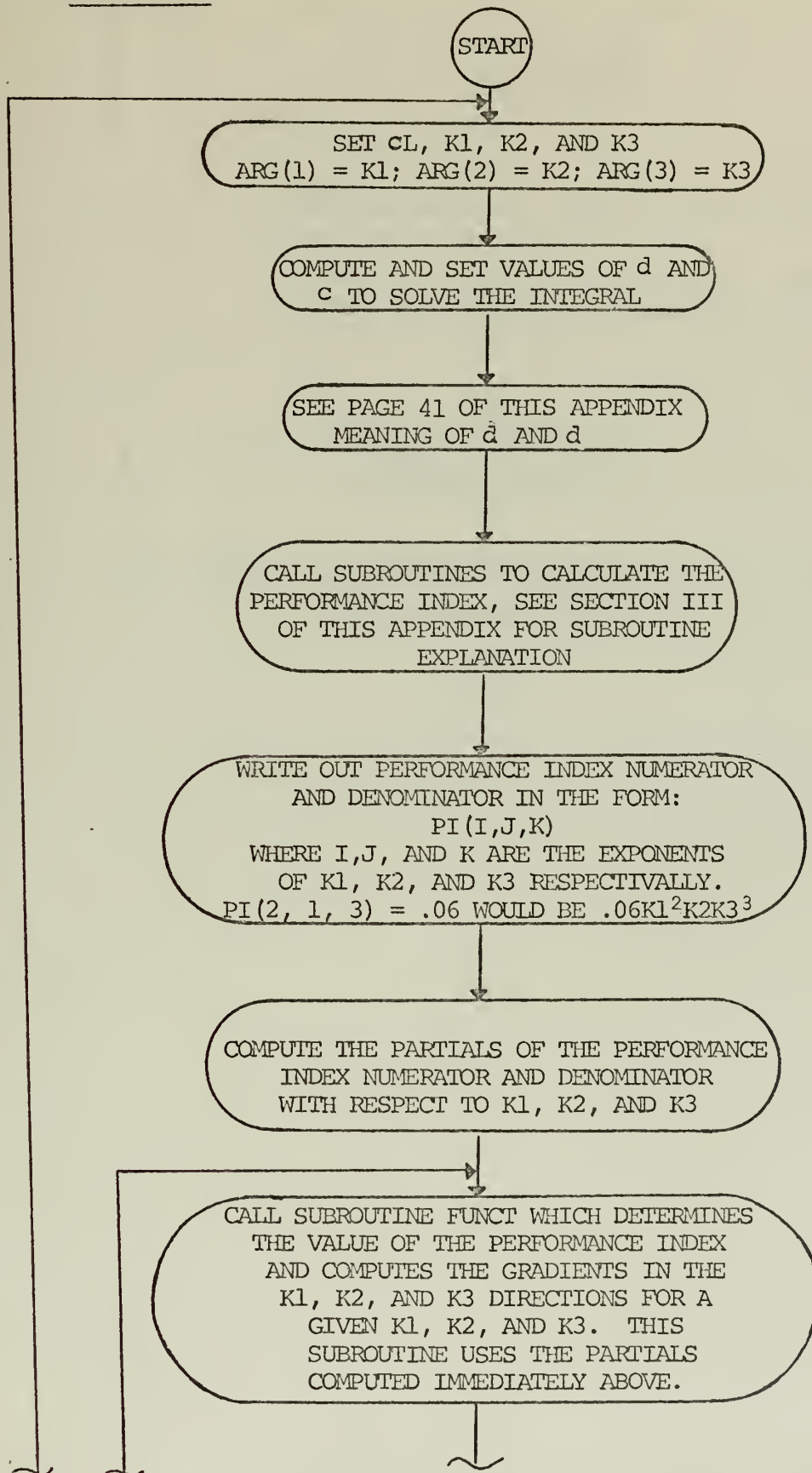
$$d(j\hat{\omega}) = d_4(j\hat{\omega})^4 + d_3(j\hat{\omega})^3 + d_2(j\hat{\omega})^2 + d_1(j\hat{\omega}) + d_0$$

The  $I_4$  equation was programmed and solved on the digital computer. The program was written so that the type aircraft could be changed, i.e. all stability derivatives were retained in the computation of  $x_1$  through  $x_7$  and  $z_1$  through  $z_4$ . The gust characteristic length (CL) was set at the beginning of the program. To facilitate handling of the variables  $K_1$ ,  $K_2$ , and  $K_3$  their coefficients were stored in 3 dimensional arrays according to the exponents of the variables. For example, the coefficient of  $K_1^2 K_2 K_3^3$  would be stored in an array at location (3,2,4). The value stored in this location was printed out as  $PI(2,1,3) = (\text{value stored})$ . The arrays were manipulated to solve for the performance index in terms of  $K_1$ ,  $K_2$ , and  $K_3$ .

A minimum value of  $PI(K_1, K_2, K_3)$  was determined by a steepest descent method. A starting point for the minimization procedure was obtained by computing values of the performance index as  $K_1$ ,  $K_2$ , and  $K_3$  varied over a large range. The point where the performance index was a minimum was then used as a starting point for the steepest descent method. Several other starting points were utilized to help ensure that a proper minimum was reached.

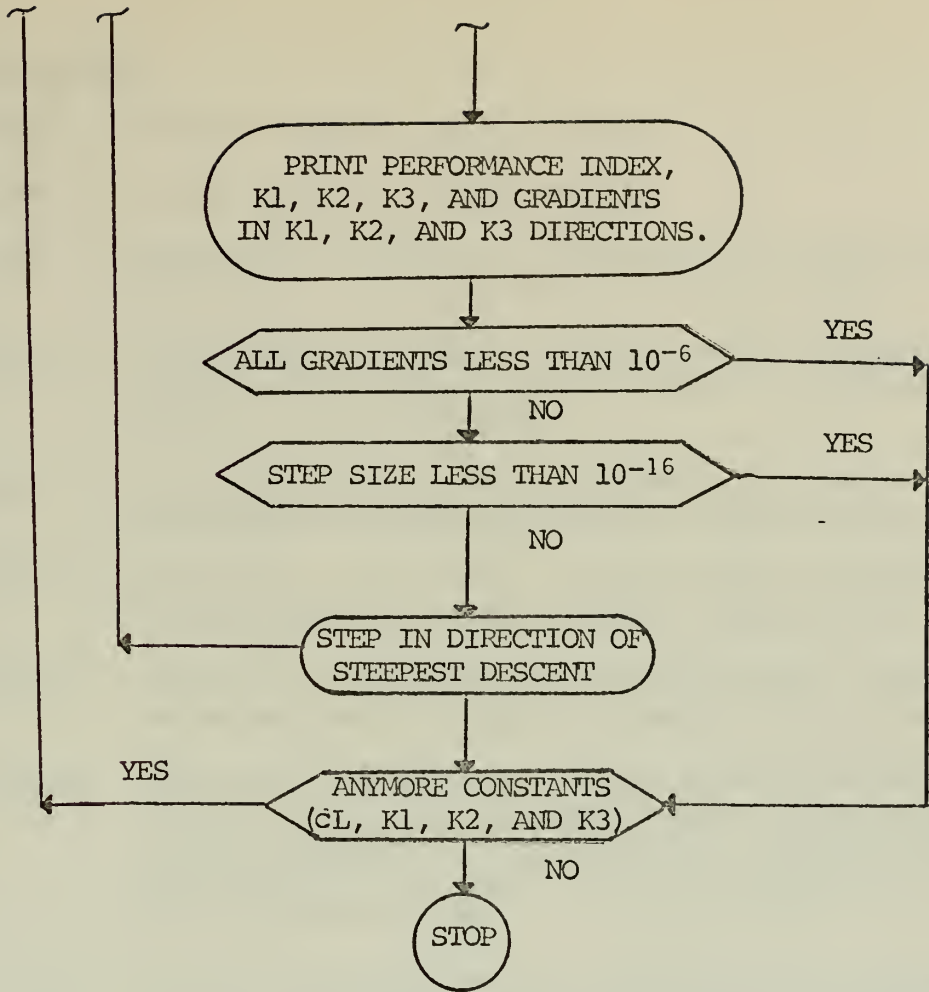


II. Flow Chart











### III. Subroutines

- ZERO3 - Places zeros in a 3x3x3 matrix.
- ZERO4 - Places zeros in a 4x4x4 matrix.
- MULT1 - Multiplies a first order polynomial by a first order polynomial and stores answer in a 3x3x3 matrix.
- MULT2 - Determines the product of a first order polynomial times a first order polynomial times 2.0 and stores answer in a 3x3x3 matrix.
- MULT3 - Multiplies a first order polynomial by a second order polynomial and stores answer in a 4x4x4 matrix.
- SUBT1 - Subtracts a third order polynomial from a third order polynomial and stores answer in a 4x4x4 matrix.
- SUBT2 - Subtracts a second order polynomial from a second order polynomial and stores answer in a 4x4x4 matrix.
- STUFFP - Determines the product of a second order polynomial times a third order polynomial times a constant and stores the answer in a 6x6x6 matrix. Separate 6x6x6 matrices hold the numerator and the denominator of the performance index.
- FUNCT - Takes the performance index numerator and denominator arrays, the specific values of K1, K2, and K3, and the arrays holding the partials of the performance index numerator and denominator and calculates the value of the performance index and the gradients in the K1, K2 and K3 directions.



```

// EXEC FORTCLG, REGION.GO=130K
// FORT
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION PIN(6,6,6), PID(6,6,6), PIL(6,6,6), D2(2,2,2), D3(2,2,2), D4(2,2,2)
DIMENSION DC(2,2,2), D1(2,2,2), C1(2,2,2), C2(2,2,2), C3(2,2,2)
DIMENSION CC(2,2,2), CI(2,2,2), HOLD32(3,3,3)
DIMENSION HOLD31(3,3,3), HOLD42(4,4,4)
DIMENSION HOLD41(4,4,4), PRTLK3(6,6,6), PRTLK1(6,6,6)
DIMENSION PRTLK2(6,6,6), PRLDK2(6,6,6), PRLDK1(6,6,6)
DIMENSION ARG(3), GRAD(3)
ILOOP=-1

```

THIS PROGRAM SOLVES:

$$I = \frac{+INFINITY \int_{-INFINITY}^C (JW)C(-JW) DW}{2 \int_{-INFINITY}^D (JW)D(-JW) DW}$$

FOR N=4. REFERENCE - NEWTON, G.C. JR, AND OTHERS, "ANALYTICAL DESIGN OF LINEAR FEEDBACK CONTROLS", P.372, JOHN WILEY & SONS, 1957. AND K3 IF DESIRED, THE PROGRAM WILL LOOK FOR THE VALUES OF K1, K2, AND K3 WHICH MAKES I A MINIMUM VALUE BY A METHOD OF STEEPEST DESCENT. IN THIS PROGRAM I REFERRED TO AS PI (PERFORMANCE INDEX).

PROGRAM PARAMETERS

```

ITPFLT - ITPFLT=1      INCLUDES ONLY CRUISING STABILITY DERIVATIVES
ITPELT=0              INCLUDES LANDING APPROACH DERIVATIVES ONLY
IMIN - IMIN=1         INCLUDES ROUTING TO MINIMIZE PI
IELRMS - IELRMS=1    SKIPS MINIMIZATION ROUTINE
CL - CL=#             INCLUDES THE ELEVATOR CONTRIBUTION TO THE PI
                                SKIPS THE ELEVATOR CONTRIBUTION
                                PUT # 500.0, 1000.0, 2000.0, 3000.0, 4000.0, THAT
                                5000.0, OR 6000.0 AND PROGRAM WILL USE ONLY THAT
                                CHARACTERISTIC LENGTH
                                PROGRAM WILL ITERATE THRU ALL 7 VALUES OF CL
                                CL=0.0

```

SET ITPFLT, IMIN, IELRMS, AND CL BELOW.

```

CL=6000.0
ITPELT=1
IMIN=1
IELRMS=1

```

IF(ITPFLT.EC.1)GO TO 1

CC



```

C  * * * * *
C  CONSTANTS FOR THE LANDING APPROACH
C  WRITE(6,3)
C  FORMAT(' ', 'LANDING APPROACH')
C  UZERO=294.0
C  T=0.1
C  GRAV=32.2
C  SIGMA=10.0
C  CBAR=15.4
C  CZQ=0.0
C  CIR=710.0
C  CZA=-4.8
C  CMQ=-22.9
C  CMU=102.0
C  CZN=-0.24
C  CMAD=-4.2
C  CMND=0.0
C  CMN=-0.72
C  CMA=-0.478
C  * * * * *
C  END OF CONSTANTS FOR LANDING APPROACH

```

```

C  GO TO 6
C  * * * * *
C  CONSTANTS FOR CRUISING FLIGHT
C  WRITE(6,4)
C  FORMAT(' ', 'CRUISING FLIGHT')
C  UZERO=733.0
C  T=0.1
C  GRAV=32.2
C  SIGMA=10.0
C  CBAR=15.4
C  CZQ=0.0
C  CIR=1900.0
C  CZA=-4.9
C  CMQ=-22.9
C  CMU=272.0
C  CZN=-0.24
C  CMAD=-4.2
C  CMND=0.0
C  CMN=-0.72
C  CMA=-0.488
C  * * * * *
C  END OF CONSTANTS FOR CRUISING FLIGHT

```

```

C  6 CONTINUE
C  CHECK CL AND SET UP PROGRAM FOR VARYING CL OR FOR MAKING ONLY

```





C ONE PASS THRU PROGRAM WITH A SPECIFIC CL.

```
CLH=CL
IF (CL .EQ. 500.0) GO TO 800
IF (CL .EQ. 0.0) GO TO 800
ICLPIC=FIX(SNGL(CL/1000.0))
ILOOP=ILOOP+1+ICLPIC
GO TO(801,802,803,804,805,806),ICLPIC
8 IF (CLH.NE.0.0) ILOOP=7
DO I=1,5
10 WRITE(6,I)
11 FORMAT(6,'0')
5 FORMAT('0', CHARACTERISTIC LENGTH =',F7.1)
```

C ZERO ARRAYS

```
DO I=1,2
DO J=1,2
DO K=1,2
D1(I,J,K)=0.0
D2(I,J,K)=0.0
D3(I,J,K)=0.0
D4(I,J,K)=0.0
C1(I,J,K)=0.0
C2(I,J,K)=0.0
C3(I,J,K)=0.0
```

120  
110  
100

```
CONTINUE
CONTINUE
DO I=1,6
DO J=1,6
DO K=1,6
PIN(I,J,K)=0.0
PID(I,J,K)=0.0
PRTLK3(I,J,K)=0.0
PRTLK2(I,J,K)=0.0
PRLDK3(I,J,K)=0.0
PRLDK2(I,J,K)=0.0
PRLDK1(I,J,K)=0.0
CONTINUE
CONTINUE
```

150  
140  
130



PROGRAM CONSTANTS

```

A=SIGMA*(CZAD-CZQ)*CIB
Y=CBAR/(2.0*CL)
TIMND=2.0*T*UZERO/CBAR

X1K100=(CZAD-CZQ)*CIB
X2K100=CZA*CIB-CZQ*CMQ+CZAD*CMQ
X3K100=CZA*CMQ
X4K000=CIB*TIMND*(2.0*CMU-CZAD)
X5K001=CIB*(CZAD-2.0*CMU)
X5K010=-2.0*CMU*CMND
X5K000=CIB*(2.0*CMU-CZAD)-TIMND*(CZA*CIB+2.0*CMU*CMAD+CZQ*CMQ)
X5K001=X5K000-TIMND*(CZA*D+2.0*CMU*CMAD+CZQ*CMQ)
X6K001=CZA*CIB+2.0*CMU*CMQ-CZAD*CMQ
X6K001=X6K001+CMAD*(2.0*CMU+CZQ)
X6K100=-CZN*CIB
X6K100=X6K100-CMND*(2.0*CMU+CZQ)
X6K010=CZAD*CMND+CZA*CMND-CZN*CMAD-2.0*CMU*CMN
X6K000=-CMQ*(2.0*CMU-CZA*TIMND-CZAD)-CZA*CIB
X6K000=X6K000-TIMND*(2.0*CMU*CMN+CZA+CZQ*CMN)-CMAD*(2.0*CMU+CZQ)
X7K001=-CZA*CMQ
X7K001=X7K001+CMN*(2.0*CMU+CZQ)
X7K010=CMN*(CZAD+CZA)-CZN*CMN
X7K100=X7K100-2.0*CMU*CMN-CZQ*CMND
X7K000=X7K000+CMQ
X7K000=X7K000-CMA*(2.0*CMU+CZQ)
ACCFAC=2.0*UZERO/(GRAV*CBAR)
Z1K001=ACCFAC*(CIB*(CZAD-CZQ))
Z1K001=ACCFAC*(CMND*(CZAD-CZQ))
Z1K000=ACCFAC*(TIMND*(2.0*CMU*(CMQ-CMAD)+CZAD*CMAD-CZAD*CMQ))
Z1K000=Z1K000+ACCFAC*(CMAD*TIMND*(-2.0*CMU-CZQ)+CZAD-CZQ))
Z1K000=Z1K000+ACCFAC*(CMQ*TIMND*(2.0*CMU+CZAD))
Z1K000=Z1K000+ACCFAC*(CIB*(CZAD-CZA*TIMND))
Z2K001=ACCFAC*(2.0*CMU*CMAD-CZAD*CMAD-2.0*CMU*CMQ+CZAD*CMQ)
Z2K001=Z2K001+ACCFAC*(CMAD*(2.0*CMU-CZQ)+2.0*CMU*CMQ+CZAD*CMQ)
Z2K010=ACCFAC*(CZN*(CMQ-CMAD)+CMN*(CZAD-CZQ)+CZA*CMND)
Z2K100=ACCFAC*(CMND*(CZAD-CZQ))
Z2K000=ACCFAC*(TIMND*(CZA*CMAD-2.0*CMU*CMN+CZAD*CMN)-CZA*CMQ)+(CMA
1D*(CZAD-2.0*CMU*CMAD+2.0*CMU*CMQ-CZAD*CMQ))
Z2K000=Z2K000+ACCFAC*(CMAD*(-2.0*CMU-CZQ)+CZAD+CZA*TIMND)
Z2K000=Z2K000+ACCFAC*(CMA*TIMND*(CZAD-2.0*CMU-CZQ-2.0*CMU))
Z2K000=Z2K000+ACCFAC*(CMQ*(2.0*CMU+CZQ+CZAD+CZA*TIMND-CZQ))
Z2K000=Z2K000-ACCFAC*(CMA*CMN-CZQ-CZA*CMQ)

```



```

Z3K010=ACCFAC*(CZA*CMN-CMA*CMND)
Z3K100=ACCFAC*(CZN*CMQ-CZM*CMND)
Z3K100=Z3K100+ACCFAC*(CZAD*CMN-CZQ*CMN+CZA*CMND)
Z3K000=2.0*ACCFAC*(CZA*CMQ-CMA*CMND)
Z4K100=-ACCFAC*CMND
Z4K100=Z4K100+ACCFAC*(CZA*CMN)

```

```

C
C
C
C
SET VALUES OF D AND C FOR COMPUTATION OF THE DENOMINATOR OF THE PI AND
THE NUMERATOR OF THE ACCELERATION CONTRIBUTION TO THE PI.

```

```

D4(1,1,1)=X4K000
D3(1,1,2)=X5K001
D3(1,2,1)=X5K012
D3(1,1,1)=X5K000+X4K000*Y
D2(1,1,2)=X6K001+X5K001*Y
D2(1,2,1)=X6K010+X5K010*Y
D2(2,1,1)=X6K100
D2(1,1,1)=X6K000+X5K000*Y
D1(1,1,2)=X7K001+X6K001*Y
D1(1,2,1)=X7K010+X6K010*Y
D1(2,1,1)=X7K100+X6K100*Y
D1(1,1,1)=X7K000+X6K000*Y
D0(1,1,2)=X7K001*Y
D0(2,1,1)=X7K100*Y
D0(1,1,1)=X7K000*Y
C1(1,1,1)=Z3K000
C1(1,2,1)=Z3K010
C1(1,1,1)=Z3K001
C2(1,1,1)=Z2K000
C2(1,2,1)=Z2K010
C2(1,1,2)=Z2K001
C3(1,1,1)=Z1K000
C3(1,1,2)=Z1K001
C3(1,2,1)=Z1K010
JELAG=0

```

```

C
C
ROUTINE TO CALCULATE PI STARTS HERE AND ENDS AT STATEMENT 370

```

```

CALL ZERR03(HOLD31)
CALL ZERR04(HOLD41)
CALL MULT1(HOLD31,D0,D0)
CALL MULT3(HOLD41,D3,HOLD31)
CALL ZERRC3(HOLD31)
CALL MULT1(HOLD31,D1,D2)
CALL ZERR04(HOLD42)

```



```

CALL MULT3(HOLD42, D0, HOLD31)
CALL SUBT1(HOLD42, HOLD41)
CALL ZER03(HOLD31)
CALL MULTI(HOLD31, C3, C3)
CALL STUFFP(PIN, HOLD31, HOLD42, A)
CALL ZER03(HOLD31), C2, C2)
CALL ZER03(HOLD32)
CALL MULT2(HOLD32, C1, C3)
CALL SUBT1(HOLD31, HOLD32)
CALL ZER03(HOLD32, D0, D1)
CALL ZER04(HOLD41)
CALL MULT3(HOLD41, D4, HOLD32)
CALL STUFFP(PIN, HOLD31, HOLD41, A)
IF(JFLAG.EQ.0.1)CALL STUFFP(PIL, HOLD31, HOLD41, A)
CALL ZER03(HOLD31), C1, C1)
CALL ZER03(HOLD32)
CALL MULT2(HOLD32, C0, C2)
CALL SUBT2(HOLD31, HOLD32)
CALL ZER03(HOLD32)
CALL ZER04(HOLD41)
CALL MULT3(HOLD41, D4, HOLD32)
CALL STUFFP(PIN, HOLD31, HOLD41, A)
IF(JFLAG.EQ.0.1)CALL STUFFP(PIL, HOLD31, HOLD41, A)
CALL ZER03(HOLD31), C0, C0)
CALL ZER03(HOLD32)
CALL MULTI(HOLD41), D2, D3)
CALL ZER04(HOLD41)
CALL MULT3(HOLD41, D4, HOLD32)
CALL ZER03(HOLD32), D1, D4)
CALL ZER04(HOLD42)
CALL MULT3(HOLD42, D4, HOLD32)
CALL SUBT1(HOLD41, HOLD42)
CALL STUFFP(PIN, HOLD31, HOLD41, A)
IF(JFLAG.EQ.0.1)CALL STUFFP(PIL, HOLD31, HOLD41, A)
IF(JFLAG.EQ.0.1)GO TO 300
DO I=1,6
DO J=1,6
LM=J-I
NK=K-I
IF(PIN(I,J,K).EQ.0.0)GO TO 190

```

160





```

195 WRITE(6,195) L, M, N, PIN(I, J, K)
190 FORMAT(1, ' ', ACCELERATION NUMERATOR(1, 311, ' ) = ', D16.6)
180 CONTINUE
170 CONTINUE

```

```

C
C SKIP ELEVATOR CONTRIBUTION TO PI IF DESIRED.
C

```

```

C IF(IELRMS.EQ.0)GO TO 336
C

```

```

C ZER0 ARRAYS.
C

```

```

196 DO 200 I=1,6
DO 210 J=1,6
DO 220 K=1,6
CO(I, J, K)=0.0
C1(I, J, K)=0.0
C2(I, J, K)=0.0
C3(I, J, K)=0.0

```

```

220 CONTINUE
210 CONTINUE
200 CONTINUE

```

```

C
C THE PORTION OF THE PROGRAM BETWEEN HERE AND THE NEXT COMMENT CARD
C COMPUTES THE CONTRIBUTION OF THE ELEVATOR DEFLECTION TO THE PI.
C

```

```

Z2K010=2.0*CMU*(CMAD-C*Q)+CZAD*CMG-CMAD*CZQ
Z2K100=CIB*(CZAD-CZQ)
Z3K010=2.0*CMU*(CMA+CZA*CMQ-CMA*CMQ)
Z3K100=2.0*CMU*(CMAD-C*Q)+CZA*CIB+CZQ*CMAD-CZAD*CMQ
Z4K010=2.0*CMU*(CMA+CZQ*CMQ-CMA-CZA*CMQ)
C2(1,2,1)=Z2K010
C2(2,1,1)=Z2K100
C1(1,2,1)=Z3K010
C1(2,1,1)=Z3K100
C0(2,1,1)=Z4K100
JFLAG=JFLAG+1
IF(JFLAG.EQ.1)GO TO 160

```

```

300 DO 310 I=1,6
DO 320 J=1,6
DO 330 K=1,6
L=I-1
M=J-1
N=K-1

```

```

IF(PIL(I, J, K).EQ.0.0)GO TO 330
WRITE(6,335) L, M, N, PIL(I, J, K)
335 FORMAT(1, ' ', ELEVATOR NUMERATOR(1, 311, ' ) = ', D16.6)
330 CONTINUE

```



```

320 CONTINUE
310 CONTINUE
C *** ELEVATOR COMPUTATION ENDS HEPE
C
336 A=1.0
      ZER03(HOLD31)
      CALL MULTI(HOLD31,D1,D2)
      CALL ZER04(HOLD41)
      CALL MULTI(HOLD41,D3,HOLD31)
      CALL ZER03(HOLD31)
      CALL MULTI(HOLD31,D1,D1)
      CALL ZER04(HOLD42)
      CALL MULTI(HOLD42,D4,HOLD31)
      CALL ZER03(HOLD31)
      CALL MULTI(HOLD31,D0,D3)
      CALL ZER04(HOLD42)
      CALL MULTI(HOLD42,D3,HOLD31)
      CALL ZER03(HOLD31)
      CALL MULTI(HOLD31,D0,D4)
      CALL STUFFP(PID,HOLD31,HOLD41,A)
DO 340 I=1,6
DO 350 J=1,6
DO 360 K=1,6
L=I-1
M=J-1
N=K-1
IF(PIN(I,J,K).EQ.0.0)GO TO 360
WRITE(6,365)L,M,N,PIN(I,J,K)
FORMAT(3I,1,1,'PERFORMANCE INDEX NUMERATOR(',3I1,')=',D16.6)
365 CONTINUE
360 CONTINUE
DO 370 I=1,6
DO 380 J=1,6
DO 390 K=1,6
IF(PID(I,J,K).EQ.0.0)GO TO 390
L=I-1
M=J-1
N=K-1
WRITE(6,395)L,M,N,PID(I,J,K)
FORMAT(3I,1,1,'PERFORMANCE INDEX DENOMINATOR(',3I1,')=',D16.6)
395 CONTINUE
390 CONTINUE
370 CONTINUE
C

```



```

C START ROUTINE TO COMPUTE THE PARTIALS OF THE PI NUMERATOR AND
C DENOMINATOR WITH RESPECT TO K1, K2, AND K3.
C ROUTINE ENDS AT STATEMENT VALUE 700.

```

```

DO 700 I=1,6
DO 710 J=1,6
DO 720 K=1,6
IF((PIN(I,J,K).EQ.0.0).AND.(PID(I,J,K).EQ.0.0))GO TO 720
L=I-1
M=J-1
N=K-1
FL=DFLOAT(L)
FM=DFLOAT(M)
FN=DFLOAT(N)
IF(N.EQ.0)GO TO 760
IF(PIN(I,J,K).NE.0.0)PRTLK3(I,J,N)=FN*PIN(I,J,K)
IF(PID(I,J,K).NE.0.0)PRLDK3(I,J,N)=FN*PID(I,J,K)
IF(M.EQ.0)GO TO 750
IF(PIN(I,J,K).NE.0.0)PRTLK2(I,M,K)=FM*PIN(I,J,K)
IF(PID(I,J,K).NE.0.0)PRLDK2(I,M,K)=FM*PID(I,J,K)
IF(L.EQ.0)GO TO 740
IF(PIN(I,J,K).NE.0.0)PRTLK1(L,J,K)=FL*PIN(I,J,K)
IF(PID(I,J,K).NE.0.0)PRLDK1(L,J,K)=FL*PID(I,J,K)
CONTINUE
700 CONTINUE
710 CONTINUE
720 CONTINUE
760
750
740

```

```

C START ROUTINE TO MINIMIZE AND PRINT PERFORMANCE INDEX. ROUTINE ENDS
C AT STATEMENT VALUE 800.

```

```

STPSIZ=5
WRITE(6,780)
WRITE(6,781)
780 FORMAT('0',T4,'PERFORMANCE')
781 FORMAT('0',T4,'INDEX',T19,'K1',T34,'K2',T49,'K3',T64,'GRAD1',T79,
C GRAD2',T94,'GRAD3',T109,'TEST1',T121,'TEST2')
C CALL FUNCT(ARG,VAL,GRAD,PIN,PID,PRTLK1,PRTLK2,PRTLK3,PRLDK1,PRLDK2
C ,PRLDK3)
795 CONTINUE
WRITE(6,782)VAL,ARG(1),ARG(2),ARG(3),GRAD(1),GRAD(2),GRAD(3)
782 FORMAT('0',T4,D11.5,T19,D11.5,T34,D11.5,T49,D11.5,T79,D1
C 1.5,T94,D11.5)

```

```

C SKIP MINIMIZATION PROCEDURE IF DESIRED.
C
IF(IMIN.EQ.0)GO TO 800
STEP=STPSIZ

```



```

SARG2=ARG(2)
SARG1=ARG(1)
SARG3=ARG(3)
SGRD1=GRAD(1)
SGRD2=GRAD(2)
SGRD3=GRAD(3)
SVAL=VAL
IF((DABS( GRAD(1) ) .LE. ID-6) .AND. (DABS( GRAD(2) ) .LE. ID-6) .AND. (DABS( G
CRAD(3) ) .LE. ID-6) ) GO TO 789
SUMSQ=GRAD(1)**2+GRAD(2)**2+GRAD(3)**2
UNITGR=DSQRT(SUMSQ)
DLTAK1=-STEP*SGRD1/UNITGR
DLTAK2=-STEP*SGRD2/UNITGR
DLTAK3=-STEP*SGRD3/UNITGR
IF(ARG(3) .GT. 1.2) ARG(3)=1.2
IF( ARG(1) .EQ. 0.0) K3=K3 GREATER THAN 1.2 - LIMITING K3 TO 1.2'
ARG(1)=SARG1+DLTAK1
CALL FUNCT(ARG,VAL,GRAD,PIN,PID,PRTLK1,PRTLK2,PRTLK3,PRLDK1,PRLDK2
,PRLDK3)
IF((VAL-SVAL) .LT. 0.0) GO TO 795
STEP=STEP/2.0
IF(STEP .LE. ID-66) GO TO 791
GO TO 786
786 WRITE(6,790)
789 WRITE(6,790)
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803 IF (ILOOP.EQ.2)GO TO 8
    CL=3000.0
    ARG(1)=.5
804 IF (ILOOP.EQ.3)GO TO 8
    CL=4000.0
    ARG(1)=.5
805 IF (ILOOP.EQ.4)GO TO 8
    CL=5000.0
    ARG(1)=.5
    IF (ILOOP.EQ.5)GO TO 8
806 IF (ILOOP.EQ.6)GO TO 8
    CL=6000.0
    ARG(2)=325.0
    ARG(1)=1.6
    ARG(3)=-2.60
    IF (ILOOP.EQ.6)GO TO 8
    STOP
    SEND

```

```

SUBROUTINE FUNCT (ARG,VAL,GRAD,PIN,PID,PRTLK1,PRTLK2,PRTLK3,PRLDK1,
1PRLDK2,PRLDK3)
    DIMENSION REAL*8 (A-H,Q-Z)
    DIMENSION ARG(3), GRAD(3)
    DIMENSION PIN(6,6,6), PID(6,6,6)
    DIMENSION PRTLK1(6,6,6), PRTLK2(6,6,6), PRTLK3(6,6,6)
    DIMENSION PRLDK1(6,6,6), PRLDK2(6,6,6), PRLDK3(6,6,6)
    VALN=PIN(1,1,1)
    VALD=PID(1,1,1)
    GRADK1=PRTLK1(1,1,1)
    GRADK2=PRTLK2(1,1,1)
    GRADK3=PRTLK3(1,1,1)
    DGRDK1=PRLDK1(1,1,1)
    DGRDK2=PRLDK2(1,1,1)
    DGRDK3=PRLDK3(1,1,1)
    IF (ARG(1).EQ.0.0).AND. (ARG(2).EQ.0.0).AND. (ARG(3).EQ.0.0))GO TO
1240
    DO 100 I=1,6
    DO 110 J=1,6
    DO 120 K=1,6
    L=I-I
    M=J-I
    N=K-I
    IF ((L.EQ.0.0).AND. (M.EQ.0.0).AND. (N.EQ.0.0))GO TO 120
    IF ((L.EQ.0.0).AND. (M.EQ.0.0).AND. (N.EQ.0.0))GO TO 10
    IF ((L.EQ.0.0).AND. (M.NE.0.0).AND. (N.EQ.0.0))GO TO 20
    IF ((L.NE.0.0).AND. (M.EQ.0.0).AND. (N.EQ.0.0))GO TO 30
    IF ((L.NE.0.0).AND. (M.NE.0.0).AND. (N.EQ.0.0))GO TO 40

```



```

IF((L.NE.0).AND.(M.EQ.0).AND.(N.NE.0))GO TO 50
IF((L.NE.0).AND.(M.NE.0).AND.(N.EQ.0))GO TO 60
MULTK=(ARG(1)**L)*(ARG(2)**M)*(ARG(3)**N)
GO TO 150
10 MULTK=ARG(3)**N
GO TO 150
20 MULTK=ARG(2)**M
GO TO 150
30 MULTK=(ARG(2)**M)*(ARG(3)**N)
GO TO 150
40 MULTK=ARG(1)**L
GO TO 150
50 MULTK=(ARG(1)**L)*(ARG(3)**N)
GO TO 150
60 MULTK=(ARG(1)**L)*(ARG(2)**M)
150 IF(PIN(I,J,K).EQ.0.0)GO TO 160
VALN=VALN+PIN(I,J,K)*MULTK
160 IF(PID(I,J,K).EQ.0.0)GO TO 180
VALD=VALD+PID(I,J,K)*MULTK
180 IF(PRTLK1(I,J,K).EQ.0.0)GO TO 190
GRADK1=GRADK1+PRTLK1(I,J,K)*MULTK
190 IF(PRTLK2(I,J,K).EQ.0.0)GO TO 200
GRADK2=GRADK2+PRTLK2(I,J,K)*MULTK
200 IF(PRTLK3(I,J,K).EQ.0.0)GO TO 210
GRADK3=GRADK3+PRTLK3(I,J,K)*MULTK
210 IF(PRLDK1(I,J,K).EQ.0.0)GO TO 220
DGRDK1=DGRDK1+PRLDK1(I,J,K)*MULTK
220 IF(PPLDK2(I,J,K).EQ.0.0)GO TO 230
DGRDK2=DGRDK2+PRLDK2(I,J,K)*MULTK
230 IF(PRLDK3(I,J,K).EQ.0.0)GO TO 120
DGRDK3=DGRDK3+PRLDK3(I,J,K)*MULTK
120 CONTINUE
110 CONTINUE
100 CONTINUE
240 IF(VALD.EQ.0.0)WRITE(6,1002)VALN,VALD,GRADK1,GRADK2,GRADK3,DGRDK1,
1 DGRDK2,DGRDK3
1002 16.4)
1 IF(VALD.EQ.0.0)GO TO 500
VAL=VALN/VALD
VALD2=VALD*VALD
GRAD(1)=-((VALD*GRADK1)-(VALN*DGRDK1))/VALD2
GRAD(2)=-((VALD*GRADK2)-(VALN*DGRDK2))/VALD2
GRAD(3)=-((VALD*GRADK3)-(VALN*DGRDK3))/VALD2
CONTINUE
RETURN
END
500

```



```

SUBROUTINE ZERO4(DUM1)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DUM1(4,4,4)
DO 100 I=1,4
DO 110 J=1,4
DO 120 K=1,4
DUM1(I,J,K)=0.0
CONTINUE
CONTINUE
RETURN
END
120
110
100

```

```

SUBROUTINE ZERO3(DUM1)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DUM1(3,3,3)
DO 100 I=1,3
DO 110 J=1,3
DO 120 K=1,3
DUM1(I,J,K)=0.0
CONTINUE
CONTINUE
RETURN
END
120
110
100

```

```

SUBROUTINE MULTI(DUM1,DUM2,DUM3)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DUM1(3,3,3), DUM2(2,2,2), DUM3(2,2,2)
J=1
K=1
L=1
DO 100 I=1,4
DUM1(J,K,L)=DUM2(J,K,L)*DUM3(1,1,1)+DUM1(J,K,L)
DUM1(J+1,K,L)=DUM2(J,K,L)*DUM3(2,1,1)+DUM1(J+1,K,L)
DUM1(J,K+1,L)=DUM2(J,K,L)*DUM3(1,2,1)+DUM1(J,K+1,L)
DUM1(J,K,L+1)=DUM2(J,K,L)*DUM3(1,1,2)+DUM1(J,K,L+1)
J=1
K=1
L=1
IF(I.EQ.1)J=2
IF(I.EQ.2)K=2
IF(I.EQ.3)L=2
CONTINUE
100

```



```
RETURN  
END
```

```
SUBROUTINE MULT2(DUM1, DUM2, DUM3)  
IMPLICIT REAL*8(A-H,O-Z)  
DIMENSION DUM1(3,3,3), DUM2(2,2,2), DUM3(2,2,2)  
J=1  
K=1  
L=1  
DO I=1,4  
DUM1(J,K,L)=2.0*DUM2(J,K,L)*DUM3(1,1,1)+DUM1(J,K,L)  
DUM1(J+1,K,L)=2.0*DUM2(J,K,L)*DUM3(2,1,1)+DUM1(J+1,K,L)  
DUM1(J,K+1,L)=2.0*DUM2(J,K,L)*DUM3(1,2,1)+DUM1(J,K+1,L)  
DUM1(J,K,L+1)=2.0*DUM2(J,K,L)*DUM3(1,1,2)+DUM1(J,K,L+1)  
J=1  
K=1  
L=1  
IF(I.EQ.1)J=2  
IF(I.EQ.2)K=2  
IF(I.EQ.3)L=2  
CONTINUE  
RETURN  
END
```

100

```
SUBROUTINE MULT3(DUM1, DUM2, DUM3)  
IMPLICIT REAL*8(A-H,O-Z)  
DIMENSION DUM1(4,4,4), DUM2(2,2,2), DUM3(3,3,3)  
J=1  
K=1  
L=1  
DO I=1,4  
DUM1(J,K,L)=DUM2(J,K,L)*DUM3(1,1,1)+DUM1(J,K,L)  
DUM1(J,K,L+1)=DUM2(J,K,L)*DUM3(1,1,2)+DUM1(J,K,L+1)  
DUM1(J,K+1,L)=DUM2(J,K,L)*DUM3(1,2,1)+DUM1(J,K+1,L)  
DUM1(J+1,K,L)=DUM2(J,K,L)*DUM3(2,1,1)+DUM1(J+1,K,L)  
DUM1(J,K+1,L+1)=DUM2(J,K,L)*DUM3(1,2,2)+DUM1(J,K+1,L+1)  
DUM1(J+1,K,L)=DUM2(J,K,L)*DUM3(2,1,2)+DUM1(J+1,K,L)  
DUM1(J+1,K+1,L)=DUM2(J,K,L)*DUM3(2,2,1)+DUM1(J+1,K+1,L)  
DUM1(J+2,K,L)=DUM2(J,K,L)*DUM3(3,1,1)+DUM1(J+2,K,L)  
DUM1(J,K+2,L)=DUM2(J,K,L)*DUM3(1,3,1)+DUM1(J,K+2,L)  
DUM1(J,K,L+2)=DUM2(J,K,L)*DUM3(1,1,3)+DUM1(J,K,L+2)  
J=1  
K=1  
L=1
```





```

IF(I.EQ.1)J=2
IF(I.EQ.2)K=2
IF(I.EQ.3)L=2
CONTINUE
RETURN
END
100

```

```

SUBROUTINE SUBT1(DUM1,DUM2)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DUM1(4,4,4), DUM2(4,4,4)
DO 100 I=1,4
DO 110 J=1,4
DO 120 K=1,4
DUM1(I,J,K)=DUM1(I,J,K)-DUM2(I,J,K)
CONTINUE
CONTINUE
CONTINUE
RETURN
END
120
110
100

```

```

SUBROUTINE SUBT2(DUM1,DUM2)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION DUM1(3,3,3), DUM2(3,3,3)
DO 100 I=1,3
DO 110 J=1,3
DO 120 K=1,3
DUM1(I,J,K)=DUM1(I,J,K)-DUM2(I,J,K)
CONTINUE
CONTINUE
CONTINUE
RETURN
END
120
110
100

```

```

SUBROUTINE STUFFP(PIX,DHOLD1,DHOLD2,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION PIX(6,6,6), DHOLD1(3,3,3), DHOLD2(4,4,4)
DC 600 L=1,10
I=1
J=1
K=1
IF(L.EQ.1)K=3
IF(L.EQ.2)J=3

```



```

I=3
I=2
J=2
J=2
K=2
J=2
I=2
J=2
K=2
X=DHOLD(I,J,K)*A
PIX(I,J,K)+PIX(I,J,K+1)
PIX(I,J,K+1)+PIX(I,J,K+2)
PIX(I,J,K+2)+PIX(I,J,K+3)
PIX(I,J,K+3)+PIX(I,J,K+4)
PIX(I,J,K+4)+PIX(I,J,K+5)
PIX(I,J,K+5)+PIX(I,J,K+6)
PIX(I,J,K+6)+PIX(I,J,K+7)
PIX(I,J,K+7)+PIX(I,J,K+8)
PIX(I,J,K+8)+PIX(I,J,K+9)
PIX(I,J,K+9)+PIX(I,J,K+10)
PIX(I,J,K+10)+PIX(I,J,K+11)
PIX(I,J,K+11)+PIX(I,J,K+12)
PIX(I,J,K+12)+PIX(I,J,K+13)
PIX(I,J,K+13)+PIX(I,J,K+14)
PIX(I,J,K+14)+PIX(I,J,K+15)
PIX(I,J,K+15)+PIX(I,J,K+16)
PIX(I,J,K+16)+PIX(I,J,K+17)
PIX(I,J,K+17)+PIX(I,J,K+18)
PIX(I,J,K+18)+PIX(I,J,K+19)
PIX(I,J,K+19)+PIX(I,J,K+20)
PIX(I,J,K+20)+PIX(I,J,K+21)
PIX(I,J,K+21)+PIX(I,J,K+22)
PIX(I,J,K+22)+PIX(I,J,K+23)
PIX(I,J,K+23)+PIX(I,J,K+24)
PIX(I,J,K+24)+PIX(I,J,K+25)
PIX(I,J,K+25)+PIX(I,J,K+26)
PIX(I,J,K+26)+PIX(I,J,K+27)
PIX(I,J,K+27)+PIX(I,J,K+28)
PIX(I,J,K+28)+PIX(I,J,K+29)
PIX(I,J,K+29)+PIX(I,J,K+30)
PIX(I,J,K+30)+PIX(I,J,K+31)
PIX(I,J,K+31)+PIX(I,J,K+32)
PIX(I,J,K+32)+PIX(I,J,K+33)
PIX(I,J,K+33)+PIX(I,J,K+34)
PIX(I,J,K+34)+PIX(I,J,K+35)
PIX(I,J,K+35)+PIX(I,J,K+36)
PIX(I,J,K+36)+PIX(I,J,K+37)
PIX(I,J,K+37)+PIX(I,J,K+38)
PIX(I,J,K+38)+PIX(I,J,K+39)
PIX(I,J,K+39)+PIX(I,J,K+40)
PIX(I,J,K+40)+PIX(I,J,K+41)
PIX(I,J,K+41)+PIX(I,J,K+42)
PIX(I,J,K+42)+PIX(I,J,K+43)
PIX(I,J,K+43)+PIX(I,J,K+44)
PIX(I,J,K+44)+PIX(I,J,K+45)
PIX(I,J,K+45)+PIX(I,J,K+46)
PIX(I,J,K+46)+PIX(I,J,K+47)
PIX(I,J,K+47)+PIX(I,J,K+48)
PIX(I,J,K+48)+PIX(I,J,K+49)
PIX(I,J,K+49)+PIX(I,J,K+50)
PIX(I,J,K+50)+PIX(I,J,K+51)
PIX(I,J,K+51)+PIX(I,J,K+52)
PIX(I,J,K+52)+PIX(I,J,K+53)
PIX(I,J,K+53)+PIX(I,J,K+54)
PIX(I,J,K+54)+PIX(I,J,K+55)
PIX(I,J,K+55)+PIX(I,J,K+56)
PIX(I,J,K+56)+PIX(I,J,K+57)
PIX(I,J,K+57)+PIX(I,J,K+58)
PIX(I,J,K+58)+PIX(I,J,K+59)
PIX(I,J,K+59)+PIX(I,J,K+60)
PIX(I,J,K+60)+PIX(I,J,K+61)
PIX(I,J,K+61)+PIX(I,J,K+62)
PIX(I,J,K+62)+PIX(I,J,K+63)
PIX(I,J,K+63)+PIX(I,J,K+64)
PIX(I,J,K+64)+PIX(I,J,K+65)
PIX(I,J,K+65)+PIX(I,J,K+66)
PIX(I,J,K+66)+PIX(I,J,K+67)
PIX(I,J,K+67)+PIX(I,J,K+68)
PIX(I,J,K+68)+PIX(I,J,K+69)
PIX(I,J,K+69)+PIX(I,J,K+70)
PIX(I,J,K+70)+PIX(I,J,K+71)
PIX(I,J,K+71)+PIX(I,J,K+72)
PIX(I,J,K+72)+PIX(I,J,K+73)
PIX(I,J,K+73)+PIX(I,J,K+74)
PIX(I,J,K+74)+PIX(I,J,K+75)
PIX(I,J,K+75)+PIX(I,J,K+76)
PIX(I,J,K+76)+PIX(I,J,K+77)
PIX(I,J,K+77)+PIX(I,J,K+78)
PIX(I,J,K+78)+PIX(I,J,K+79)
PIX(I,J,K+79)+PIX(I,J,K+80)
PIX(I,J,K+80)+PIX(I,J,K+81)
PIX(I,J,K+81)+PIX(I,J,K+82)
PIX(I,J,K+82)+PIX(I,J,K+83)
PIX(I,J,K+83)+PIX(I,J,K+84)
PIX(I,J,K+84)+PIX(I,J,K+85)
PIX(I,J,K+85)+PIX(I,J,K+86)
PIX(I,J,K+86)+PIX(I,J,K+87)
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PIX(I,J,K+88)+PIX(I,J,K+89)
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PIX(I,J,K+93)+PIX(I,J,K+94)
PIX(I,J,K+94)+PIX(I,J,K+95)
PIX(I,J,K+95)+PIX(I,J,K+96)
PIX(I,J,K+96)+PIX(I,J,K+97)
PIX(I,J,K+97)+PIX(I,J,K+98)
PIX(I,J,K+98)+PIX(I,J,K+99)
PIX(I,J,K+99)+PIX(I,J,K+100)
CONTINUE
RETURN
END
600

```



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KEY WORDS

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Use of Random Gust Fields  
Autopilot Design

LINK A

LINK B

LINK C

ROLE

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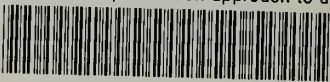
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