# THE MATHEMATICAL SOLUTION FOR THE BEAL CONJECTURE 

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#### Abstract

- The Beal conjecture is an unsolved number problem. It was formulated in 1993 by Andrew Beal ,A Banker and an amature mathematician during investigating generalization of Fermats Last Theorem [1] [2] since 1997. He offered a monentary prize for an impressive proof of this conjecture . At present no suitable proof has been produced. In this article I also provided the clear and a systematic proof of this conjecture.


Keywords: Proof ;Beal Conjecture: Number theory

## 1.Introduction

IN 1993 , Andrew Beal, a Texas Banker and a number theory Mathematician formulated a number theory which he named "Beal conjecture". After 4 years in In 1997 he publicly announced to offer a $\$ 5,000$ prize money for the peer review proof. By now he increased the prize money to a sum of $\$ 1,000,000$. Many number of experts, mathematicians and scholars have published their research articles on the proof of this number problem but they could not present a suitable proof. I am also presenting a proof in this article .

## 2.The Beal conjecture

Let $A, B, C, X, Y, Z$ be positive integers, with $X, Y, Z$ greater than 2. The Beal conjecture states that

If :
$\mathbf{A}^{\mathbf{X}}+\mathbf{B}^{\mathbf{Y}}=\mathbf{C}^{\mathbf{Z}}$
Then A,B,C must have a common prime factor and all variables should have distinct value.

## 3. Conventions

Author has used symbols and conventions to proof the conjecture where $\mathbf{O}_{\mathbf{a}}, \mathbf{O}_{\mathbf{b}}$ and $\mathbf{O}_{\mathbf{c}}$ are odd symbols. And $\mathbf{E}_{\mathbf{a}}, \mathbf{E}_{\mathbf{b}}$ and $\mathbf{E}_{\mathbf{c}}$ are even symbols . where as 1,3,5,7, $\ldots$. represent odd no. and $2,4,6,8 \ldots \ldots$ represent even numbers .

## 4. Conditions of addition operations

I observed three different conditions for solving conjecture
Condition 1:
$\mathrm{O}_{\mathrm{a}}+\mathrm{O}_{\mathrm{b}}=\mathrm{E}_{\mathrm{c}}$
Condition 2:
$\mathrm{O}_{\mathrm{a}}+\mathrm{E}_{\mathrm{b}}=\mathrm{O}_{\mathrm{c}}$
Condition 3 :
$\mathrm{E}_{\mathrm{a}}+\mathrm{E}_{\mathrm{b}}=\mathrm{E}_{\mathrm{c}}$

## 5.Verification

Here author has solved the equation very clearly, and found out a common prime factor in the equations as well as represented distinct exponents according to original definition of Beal conjecture which can be seen in the numerical examples. Hence Beal conjecture is quite true.

## 6.The proof

Below is given the proof for the Beal conjecture using the three conditions .
6.1 Condition 1
$\mathrm{O}_{\mathrm{a}}+\mathrm{O}_{\mathrm{b}}=\mathrm{E}_{\mathrm{c}}$
Let ' $\mathrm{O}_{\mathrm{a}}{ }^{\text {' Represent }} \mathrm{A}$
Let ' $\mathrm{O}_{\mathrm{b}}{ }^{\text {' Represent }} \mathrm{B}$
Let ' $\mathrm{E}_{\mathrm{c}}$ 'Represent C
$\mathrm{A}+\mathrm{B}=\mathrm{C}$

Let $\mathrm{A}=1$, Let $\mathrm{B} \neq 1$ and Let $\mathrm{C} \neq 1$ where $\mathrm{B}=\mathrm{B}_{1}{ }^{\mathrm{y}}$ and $\mathrm{y}=1$
Let $\mathrm{C}=\mathrm{C}_{1}{ }^{\mathrm{z}}$ and $\mathrm{z} \geq 3$
Equation 2 is an ordinary addition operation and variables are represented in exponential forms.
$1+\mathrm{B}_{1}{ }^{\mathrm{y}}=\mathrm{C}_{1}^{\mathrm{z}}$
According to author's observation equation is multiplied 3 times by $\mathrm{B}_{1}{ }^{z}$


By applying the laws of indices I can have
$\left(\mathrm{B}_{1}\right)^{\mathrm{z}+\mathrm{z}+\mathrm{z}}+\left(\mathrm{B}_{1}\right)^{\mathrm{y}+\mathrm{z}+\mathrm{z}+\mathrm{z}}=\left(\mathrm{C}_{1} \times \mathrm{B}_{1}\right)^{\mathrm{z}+\mathrm{z}+\mathrm{z}+\mathrm{z}}$
$\left(\mathrm{B}_{1}\right)^{3 \mathrm{z}}+\left(\mathrm{B}_{1}\right)^{\mathrm{y}+3 \mathrm{z}}=\left(\mathrm{C}_{1} \times \mathrm{B}_{1}\right)^{4 \mathrm{z}}$
Now after adjusting the exponents of equation 5 it can be formed Beal equation .
According to author's observation $B_{1}$ is a common prime factor in above equation.
Comment - condition 1 clearly proof the Beal equations .

### 6.1.1 Numerical examples

1. Derivation of Beal conjecture from the equation
$1+7=8$
Solution -

$$
1+7=8
$$

$1+7^{1}=2^{3}$
Multiplying through by $7^{\mathbf{3}} 3$ times
$1 \times 7^{3} \times 7^{3} \times 7^{3}+7^{1} \times 7^{3} \times 7^{3} \times 7^{3}=2^{3} \times 7^{3} \times 7^{3} \times 7^{3}$
Applying the rules of indices

$$
7^{9}+7^{10}=(7 \times 7 \times 7 \times 2)^{3}
$$

Adjusting the exponents of equation it can be proved Beal conjecture
$7^{9}+(49)^{5}=(686)^{3}$
Now 7 is common prime factor in the above equation.
Beal conjecture is true for condition 1
2. Derivation of Beal equation from example
$1+127=128$

## Solution

$1+127=128$
$1+127^{1}=2^{7}$
Multiplying the equation three times by $127^{7}$
$1 \times 127^{7} \times 127^{7} \times 127^{7}+127^{1} \times 127^{7} \times 127^{7} \times 127^{7}=2^{7} \times 127^{7} \times 127^{7} \times 127^{7}$
Applying the rules of indices
$(127)^{21}+(127)^{22}=(2 \times 127 \times 127 \times 127)^{7}$
$127^{21}+(16129)^{11}=(4096766)^{7}$
127 is a common prime factor of above equation.

### 6.2 Condition 2

$\mathrm{O}_{\mathrm{a}}+\mathrm{E}_{\mathrm{b}}=\mathrm{O}_{\mathrm{c}}$
Let ' $\mathrm{O}_{\mathrm{a}}$ ' represent A
Let ' $\mathrm{E}_{\mathrm{b}}$ ' represent B
Let ' $\mathrm{O}_{\mathrm{c}}$ ' represent C
$\mathrm{A}+\mathrm{B}=\mathrm{C}$
Let $\mathrm{A}=1$, Let $\mathrm{B} \neq 1$, and Let $\mathrm{C} \neq 1$

Equation 6 is an ordinary addition operation and $\mathrm{A}, \mathrm{B}$ and C are an exponential format.
Let $B=B_{1}{ }^{\mathrm{y}}$, where $\mathrm{y} \geq 3$
Let $\mathrm{C}=\mathrm{C}_{1}{ }^{\mathrm{z}}$ where $\mathrm{z}=1$
$1+\mathrm{B}_{1}{ }^{\mathrm{y}}=\mathrm{C}_{1}{ }^{\mathrm{z}}$
$c_{1}{ }^{\mathrm{y}}$ multiplies the equation two times .
So Beal conjecture can be made .
$1 \times \mathrm{C}_{1}{ }^{\mathrm{y}} \times \mathrm{C}_{1}{ }^{\mathrm{y}}+\mathrm{B}_{1}{ }^{\mathrm{y}} \times \mathrm{C}_{1}{ }^{\mathrm{y}} \times \mathrm{C}_{1}{ }^{\mathrm{y}}=\mathrm{C}_{1}^{\mathrm{z}} \times \mathrm{C}_{1}{ }^{\mathrm{y}} \times \mathrm{C}_{1}{ }^{\mathrm{y}}$
By applying laws of indices I have
$\left(\mathrm{C}_{1}\right)^{\mathrm{y}+\mathrm{y}}+\left(\mathrm{B}_{1} \times \mathrm{C}_{1} \times \mathrm{C}_{1}\right)^{\mathrm{y}}=\left(\mathrm{C}_{1}\right)^{z+\mathrm{y}+\mathrm{y}}$
$\mathrm{C}_{1}{ }^{2 \mathrm{y}}+\left(\mathrm{B}_{1} \times \mathrm{C}_{1} \times \mathrm{C}_{1}\right)^{\mathrm{y}}=\left(\mathrm{C}_{1}\right)^{2 \mathrm{y}+\mathrm{z}}$
$\mathrm{C}_{1}$ is a common prime factor of above equation .
After adjusting the exponents of equation 9 Beal conjecture can be formed.
Comment - Beal conjecture is true for condition 2

### 6.2.1 numerical example

Derivation of Beal conjecture from equation $1+16=17$

## Solution -

$1+16=17$
$1+2^{4}=17^{1}$
Multiplying the equation 2 times by $17^{4}$
$1 \times 17^{4} \times 17^{4}+2^{4} \times 17^{4} \times 17^{4}=17^{1} \times 17^{4} \times 17^{4}$
By applying the law of indices I can get
$17^{4+4}+(2 \times 17 \times 17)^{4}=17^{1+4+4}$
$17^{8}+(578)^{4}=17^{9}$

Adjusting the powers of equation we can get Beal conjecture
$17^{8}+(578)^{4}=(4913)^{3}$
Comment - 17 is the common prime factor in the above equation.

## Solution 2 -

$1+256=257$
$1+4^{4}=257$
Multiplying through by $257^{4}$
$1 \times 257^{4} \times 257^{4}+4^{4} \times 257^{4} \times 257^{4}=257^{1} \times 257^{4} \times 257^{4}$
By applying the rules of indices
$257^{4+4}+(4 \times 257 \times 257)^{4}=(257)^{1+4+4}$
$257^{8}+(264196)^{4}=(257)^{9}$
Or $(257)^{8}+(264196)^{4}=(16974593)^{3}$
Comment - 257 is the common prime factor in the above equation .

## Other examples-

$81+256=337$
$3^{4}+4^{4}=337$
Multiply the equation two times by $337^{4}$
$3^{4} \times 337^{4} \times 337^{4}+4^{4} \times 337^{4} \times 337^{4}=337^{1} \times 337^{4} \times 337^{4}$
By applying the law of indices I can get
$(3 \times 337 \times 337)^{4}+2^{8} \times 337^{8}=337^{1+4+4}$
$(340707)^{4}+(674)^{8}=(337)^{9}$
Comment - 337 is common prime factor in above equation.
Example 2-
$729+616=1409$
$9^{3}+6^{3}=1409$
Multiplying through $1409^{3}$ two times
$9^{3} \times 1409^{3} \times 1409^{3}+6^{3} \times 1409^{3} \times 1409^{3}=1409^{1} \times 1409^{3} \times 1409^{3}$
Applying the rules of indics we can get
$3^{6} \times 1409^{6}+(6 \times 1409 \times 1409)^{3}=(1409)^{1+3+3}$
$(4227)^{6}+(11911686)^{3}=(1409)^{7}$
Comment - 1409 is a common prime factor in the above equation.

### 6.3 Condition 3

$\mathrm{E}_{\mathrm{a}}+\mathrm{E}_{\mathrm{b}}=\mathrm{E}_{\mathrm{c}}$
Let ' $\mathrm{E}_{\mathrm{a}}$ ' Represent A
Let ' $\mathrm{E}_{\mathrm{b}}$ ' Represent B
Let ' $\mathrm{E}_{\mathrm{c}}$ ' Represent C
$A+B=C$
Let $\mathrm{A} \neq 1$, Let $\mathrm{B} \neq 1$ and Let $\mathrm{C} \neq 1$
$\mathrm{A}=\mathrm{B}$ and $\mathrm{C}=2 \mathrm{~A}=2 \mathrm{~B}$
Where $x \geq 3, y \geq 3$ and $z \geq 4$
Where equation 10 is an ordinary addition operation and variables are in an exponential form
$\mathrm{A}_{1}{ }^{\mathrm{x}}+\mathrm{B}_{1}{ }^{\mathrm{y}}=\mathrm{C}_{1}{ }^{\mathrm{z}}$
According to author observation $\mathrm{C}_{1}{ }^{\mathrm{x}}$ or $\mathrm{C}_{1}{ }^{\mathrm{y}}$ or $\mathrm{A}_{1}{ }^{\mathrm{x}}$ or $\mathrm{B}_{1}{ }^{\mathrm{y}}$ multiplies through the equation two times.
$\mathrm{A}_{1}{ }^{\mathrm{x}} \times \mathrm{A}_{1}{ }^{\mathrm{x}} \times \mathrm{A}_{1}{ }^{\mathrm{x}}+\mathrm{B}_{1}{ }^{\mathrm{y}} \times \mathrm{B}_{1}{ }^{\mathrm{y}} \times \mathrm{B}_{1}{ }^{\mathrm{y}}=\mathrm{C}_{1}{ }^{\mathrm{z}} \times \mathrm{B}_{1}{ }^{\mathrm{y}} \times \mathrm{B}_{1}{ }^{\mathrm{y}}$

Applying the rule of indices
$\left(A_{1}\right)^{x+x+x}+\left(B_{1}\right)^{y+y+y}=\left(C_{1}\right)^{z+y+y}$
$\left(\mathrm{A}_{1}\right)^{3 \mathrm{x}}+\left(\mathrm{B}_{1}\right)^{3 \mathrm{y}}=\left(\mathrm{C}_{1}\right)^{\mathrm{z}+2 \mathrm{y}}$
$\mathrm{A} 1=\mathrm{B}_{1}=\mathrm{C}_{1}$ is the common prime factor of above equation.
Comment- $\mathrm{A}_{1}=\mathrm{B}_{1}=\mathrm{C}_{1}$ are common prime factor in above equation and Beal conjecture is true for condition 3 .

### 6.3.1 Numerical example

Derivation of Beal conjecture from equation
$8+8=16$

## Solution -

$8+8=16$
$2^{3}+2^{3}=2^{4}$
Multiplying through $2^{3}$ two times.
$2^{3} \times 2^{3} \times 2^{3}+2^{3} \times 2^{3} \times 2^{3}=2^{4} \times 2^{3} \times 2^{3}$
Applying the rule of indices I can get
$2^{3+3+3}+2^{3+3+3}=2^{4+3+3}$
$2^{9}+2^{9}=2^{10}$
Adjusting the exponents of the equation I can get a suitable Beal equation with different powers.
$8^{3}+2^{9}=4^{5}$
Comment- 2 is common prime factor of above equation.

## Example 2-

$32+32=64$

$$
2^{5}+2^{5}=2^{6}
$$

Multiplying two times by $2^{5}$
$2^{5} \times 2^{5} \times 2^{5}+2^{5} \times 2^{5} \times 2^{5}=2^{5} \times 2^{5} \times 2^{6}$
Now applying the rules of indices
$2^{5+5+5}+2^{5+5+5}=2^{5+5+6}$
$2^{15}+2^{15}=2^{16}$
Or
$8^{5}+2^{15}=4^{8}$
Comment- 2 is common prime factor in the above equation and it also satisfies truly the Beal conjecture.

## 7. conclusion-

Author very clearly proved the Beal conjecture. According to real definition of theorem represented a common prime factor with distinct exponents. So Beal conjecture is quite true for all three conditions.

## References

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