Plate A

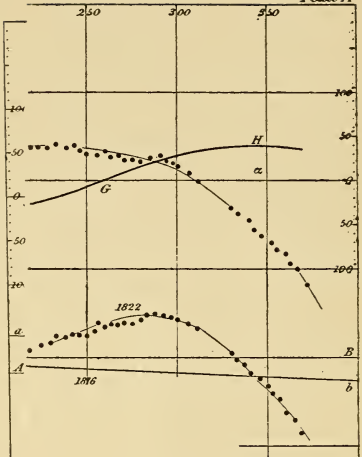


Fig 2

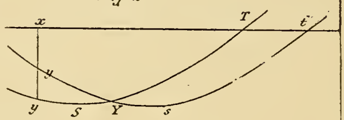
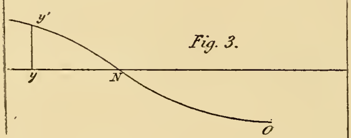


Fig. 3.



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OUTLINES OF ASTRONOMY

BY

rederick William

SIR JOHN F. W. HERSCHEL, Bart.

PART TWO

NEW YORK
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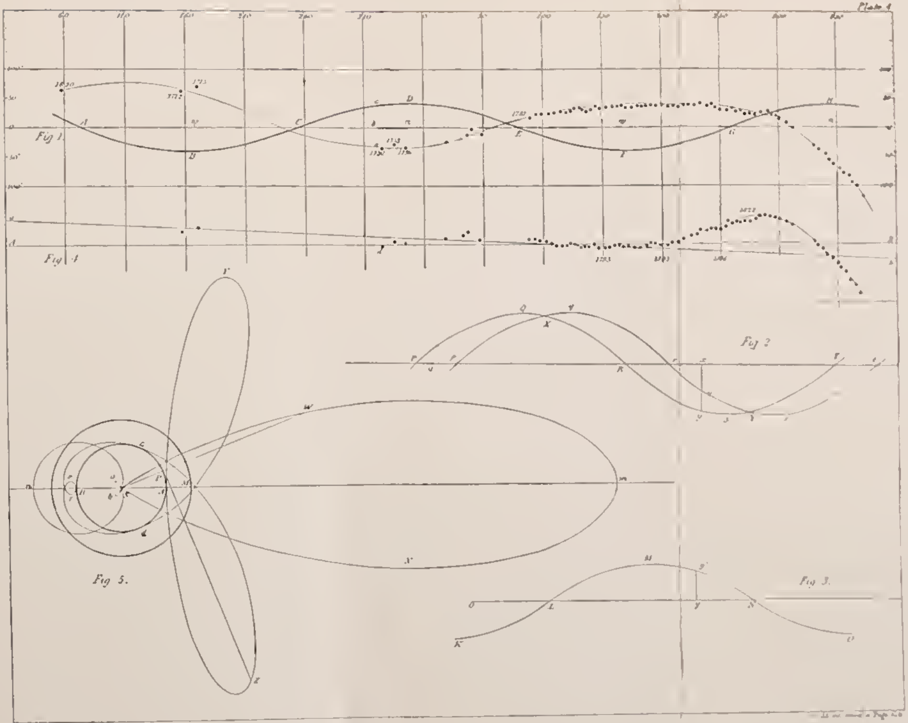
Plate 1



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CHAPTER XI

OF COMETS

Great Number of Recorded Comets—The Number of those Unrecorded Probably much Greater—General Description of a Comet—Comets without Tails, or with more than One—Their Extreme Tenuity—Their Probable Structure—Motions Conformable to the Law of Gravity—Actual Dimensions of Comets—Periodical Return of Several—Halley's Comet—Other Ancient Comets Probably Periodic—Encke's Comet—Biela's—Faye's—Lexell's—De Vico's—Brorsen's—Peters's—Great Comet of 1843—Its Probable Identity with Several Older Comets—Great Interest at Present Attached to Cometary Astronomy, and its Reasons—Remarks on Cometary Orbits in General—Great Comets, of 1858, 1861 and 1862

(554.) The extraordinary aspect of comets, their rapid and seemingly irregular motions, the unexpected manner in which they often burst upon us, and the imposing magnitudes which they occasionally assume, have in all ages rendered them objects of astonishment, not unmixed with superstitious dread to the uninstructed, and an enigma to those most conversant with the wonders of creation and the operations of natural causes. Even now, that we have ceased to regard their movements as irregular, or as governed by other laws than those which retain the planets in their orbits, their intimate nature, and the offices they perform in the economy of our system, are as much unknown as ever. No distinct and satisfactory account has yet been rendered of those immensely voluminous appendages which they bear about with them, and which are known by the name of their tails (though improperly, since they often precede them in their motions), any more than of several other singularities which they present.

(555.) The number of comets which have been astronomically observed, or of which notices have been recorded in history, is very great, amounting to several hundreds,[1] and when we consider that in the earlier ages of astronomy, and indeed in more recent times, before the invention of the telescope, only large and conspicuous ones were noticed; and that, since due attention has been paid to the subject, scarcely a year has passed without the observation of one or two of these bodies, and that sometimes two and even three have appeared at once; it will be easily supposed that their actual number must be at least many thousands. Multitudes, indeed, must escape all observation, by reason of their paths traversing only that part of the heavens which is above the horizon in the daytime. Comets so circumstanced can only become visible by the rare coincidence of a total eclipse of the sun—a coincidence which happened, as related by Seneca, sixty-two years before Christ, when a large comet was actually observed very near the sun. Several, however, stand on record as having been bright enough to be seen with the naked eye in

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[1] See catalogues in the Almagest of Riccioli; Pingré's Cometographie; Delambre's Astron. vol. iii.; Astronomische Abhandlungen, No. 1 (which contains the elements of all the orbits of comets which have been computed to the time of its publication, 1823); also a catalogue, by the Rev. T. J. Hussey. Lond. and Ed. Phil. Mag. vol. ii. No. 9, *et seq.* In a list cited by Lalande from the 1st vol. of the Tables de Berlin, 700 comets are enumerated. See also notices of the Astronomical Society and Astron. Nachr. *passim.* A great many of the more ancient comets are recorded in the Chinese Annals, and in some cases with sufficient precision to allow of the calculation of rudely approximate orbits from their motions so described.

the daytime, even at noon and in bright sunshine. Such were the comets of 1402, 1532 and 1843, and that of 43 B.C. which appeared during the games celebrated by Augustus in honor of Venus shortly after the death of Cæsar, and which the flattery of poets declared to be the soul of that hero taking its place among the divinities.

(556.) That feelings of awe and astonishment should be excited by the sudden and unexpected appearance of a great comet, is no way surprising; being, in fact, according to the accounts we have of such events, one of the most imposing of all natural phenomena. Comets consist for the most part of a large and more or less splendid, but ill-defined nebulous mass of light, called the head, which is usually much brighter toward its centre, and offers the appearance of a vivid *nucleus*, like a star or planet. From the head, and in a direction *opposite to that in which the sun is situated* from the comet appear to diverge two streams of light, which grow broader and more diffused at a distance from the head, and which most commonly close in and unite at a little distance behind it, but sometimes continue distinct for a great part of their course; producing an effect like that of the trains left by some bright meteors, or like the diverging fire of a sky-rocket (only without sparks or perceptible motion). This is the tail. This magnificent appendage attains occasionally an immense apparent length. Aristotle relates of the tail of the comet of 371 B.C., that it occupied a third of the hemisphere, or 60°; that of A.D. 1618 is stated to have been attended by a train no less than 104° in length. The comet of 1680, the most celebrated of modern times, and on many accounts the most remarkable of all, with a head not exceeding in brightness a star of the second magnitude, covered with its tail an extent of more than 70° of the heavens, or, as some accounts state, 90°; that of the comet of 1769 extended 97°, and that of the last *great* comet (1843) was estimated at about 65° when longest. The figure (*fig.* 2, Plate II.) is a representation of the comet of 1819—by no means one of the most considerable, but which was, however, very conspicuous to the naked eye.

(557.) The tail is, however, by no means an invariable appendage of comets. Many of the brightest have been observed to have short and feeble tails, and a few great comets have been entirely without them. Those of 1585 and 1763 offered no vestige of a tail; and Cassini describes the comets of 1665 and 1682 as being as round[2] and as well defined as Jupiter. On the other hand, instances are not wanting of comets furnished with many tails or streams of diverging light. That of 1744 had no less than six, spread out like an immense fan, extending to a distance of nearly 30° in length. The small comet of 1823 had two, making an angle of about 160°, the brighter turned as usual from the sun, the fainter toward it, or nearly so. The tails of comets, too, are often somewhat curved, bending, in general, toward the region which the comet has left, as if moving somewhat more slowly, or as if resisted in their course.

(558.) The smaller comets, such as are visible only in telescopes, or with difficulty by the naked eye, and which are by far the most numerous, offer very frequently no appearance of a tail, and appear only as round or some-

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[2] This description, however, applies to the "disk" of the head of these comets as seen in a telescope. Cassini's expressions are, "aussi rond, aussi net, et aussi clair que Jupiter" (where it is to be observed that the latter epithet must by no means be translated *bright*). To understand this passage fully, the reader must refer to the description given further on, of the "disk" of Halley's comet, after its perihelion passage in 1835–6.

what oval vaporous masses, more dense toward the centre, where, however, they appear to have no distinct nucleus, or anything which seems entitled to be considered as a solid body. This was shown in a very remarkable manner in the case of the comet discovered by Miss Mitchell in 1847, which on the 5th of October in that year passed *centrally* over a star of the fifth magnitude: *so* centrally that with a magnifying power of 100 it was impossible to determine in which direction the extent of the nebulosity was greatest. The star's light seemed in no degree enfeebled; yet such a star would be completely obliterated by a moderate fog, extending only a few yards from the surface of the earth. And since it is an observed fact, that even those larger comets which have presented the appearance of a nucleus have yet exhibited *no phases*, though we cannot doubt that they shine by the reflected solar light, it follows that even these can only be regarded as great masses of thin vapor, susceptible of being penetrated through their whole substance by the sunbeams, and reflecting them alike from their interior parts and from their surfaces. Nor will any one regard this explanation as forced, or feel disposed to resort to a phosphorescent quality in the comet itself, to account for the phenomena in question, when we consider (what will be hereafter shown) the enormous magnitude of the space thus illuminated, and the extremely small *mass* which there is ground to attribute to these bodies. It will then be evident that the most unsubstantial clouds which float in the highest regions of our atmosphere, and seem at sunset to be drenched in light, and to glow throughout their whole depth as if in actual ignition, without any shadow or dark side, must be looked upon as dense and massive bodies compared with the filmy and all but spiritual texture of a comet. Accordingly, whenever powerful telescopes have been turned on these bodies, they have not failed to dispel the illusion which attributes *solidity* to that more condensed part of the head, which appears to the naked eye as a nucleus; though it is true that in some a very minute stellar point *has* been seen, indicating the existence of something more substantial.

(559.) It is in all probability to the feeble coercion of the elastic power of their gaseous parts, by the gravitation of so small a central mass, that we must attribute this extraordinary development of the atmospheres of comets. If the earth, retaining its present size, were reduced, by any internal change (as by hollowing out its central parts) to one thousandth part of its actual mass, its coercive power over the atmosphere would be diminished in the same proportion, and in consequence the latter would expand to a thousand times its actual bulk; and indeed much more, owing to the still further diminution of gravity, by the recess of the upper parts from the centre.[3] An atmosphere, however, free to expand equally in all directions, would envelope the nucleus spherically, so that it becomes necessary to admit the action of other causes to account for its enormous extension in the direction of the tail—a subject to which we shall presently take occasion to recur.

(560.) That the luminous part of a comet is something in the nature of a smoke, fog or cloud, suspended in a transparent atmosphere, is evident from a fact which has

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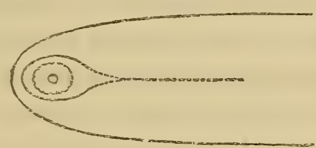
[3] Newton has calculated (Princ. III. p. 512) that a globe of air of ordinary density at the earth's surface, of one inch in diameter, if reduced to the density due to the altitude above the surface of one radius of the earth, would occupy a sphere exceeding in radius the orbit of Saturn. The tail of a great comet, then, for aught we can tell, may consist of only a very few pounds or even ounces of matter.

been often noticed, viz.—that the portion of the tail where it comes closest to, and surrounds the head, is yet separated from it by an interval less luminous, as if sustained and kept off from contact by a transparent stratum, as we often see one layer of clouds over another with a considerable clear space between. These, and most of the other facts observed in the history of comets, appear to indicate that the structure of a comet, as seen in section in the direction of its length, must be that of a hollow envelope, of a parabolic form, inclosing near its vertex the nucleus and head, something as represented in the annexed figure. This would



account for the apparent division of the tail into two principal lateral branches, the envelope being oblique to the line of sight at its borders, and therefore a greater depth of illuminated matter being there exposed to the eye. In all probability, however, they admit great varieties of structure, and among them may very possibly be bodies of widely different physical constitution, and there is no doubt that one and the same comet at different epochs undergoes great changes, both in the disposition of its materials and in their physical state.

(561.) We come now to speak of the motions of comets. These are apparently most irregular and capricious. Sometimes they remain in sight only for a few days, at others for many months; some move with extreme slowness, others with extraordinary velocity; while not infrequently, the two extremes of apparent speed are exhibited by the same comet in different parts of its course. The comet of 1472 described an arc of the heavens of 40° of a great circle[4] in a single day. Some pursue a direct, some a retrograde, and others a tortuous and very irregular course; nor do they confine themselves, like the planets, within any certain region of the heavens, but traverse indifferently every part. Their variations in apparent size, during the time they continue visible, are no less remarkable than those of their velocity; sometimes they make their first appearance as faint and slow-moving objects, with little or no tail; but by degrees accelerate, enlarge, and throw out from them this appendage, which increases in length and brightness till (as always happens in such cases) they approach the sun, and are lost in his beams. After a time they again emerge, on the other side, receding from the sun with a velocity at first rapid, but gradually decaying. It is for the most part after thus passing the sun, that they shine forth in all their splendor, and that their tails acquire their greatest length and development; thus indicating plainly the action of the sun's rays as the exciting cause of that extraordinary emanation. As they continue to recede from the sun, their motion diminishes and the tail dies away, or is absorbed into the head, which itself grows continually feebler, and is at length altogether lost sight of, in by far the greater number of cases never to be seen more.

(562.) Without the clew furnished by the theory of gravitation, the enigma of these seemingly irregular and capricious movements might have remained for ever unresolved. But Newton, having demonstrated the possibility of any conic section whatever being described about the sun, by a

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[4] 120° in extent in the former editions. But this was the arc described *in longitude*, and the comet at the time referred to had great north latitude.

body revolving under the dominion of that law, immediately perceived the applicability of the general proposition to the case of cometary orbits; and the great comet of 1680, one of the most remarkable on record, both for the immense length of its tail and for the excessive closeness of its approach to the sun (within one-sixth of the diameter of that luminary), afforded him an excellent opportunity for the trial of his theory. The success of the attempt was complete. He ascertained that this comet described about the sun as its focus an elliptic orbit of so great an excentricity as to be indistinguishable from a parabola (which is the extreme, or limiting form of the ellipse when the axis becomes infinite), and that in this orbit the areas described about the sun were, as in the planetary ellipses, proportional to the times. The representation of the apparent motions of this comet by such an orbit, throughout its whole observed course, was found to be as satisfactory as those of the motions of the planets in their nearly circular paths. From that time it became a received truth, that the motions of comets are regulated by the same general laws as those of the planets—the difference of the cases consisting only in the extravagant elongation of their ellipses, and in the absence of any limit to the inclinations of their planes to that of the ecliptic, or of any general coincidence in the direction of their motions from west to east, rather than from east to west, like what is observed among the planets.

(563.) It is a problem of pure geometry, from the general laws of elliptic or parabolic motion, to find the situation and dimensions of the ellipse or parabola which shall represent the motion of any given comet. In general, three complete observations of its right ascension and declination, with the times at which they were made, suffice for the solution of this problem (which is, however, by no means an easy one) and for the determination of the elements of the orbit. These consist, *mutatis mutandis*, of the same data as are required for the computation of the motion of a planet (that is to say, the longitude of the perihelion, that of the ascending node, the inclination to the ecliptic, the semi-axis, excentricity, and time of perihelion passage, as also whether the motion is direct or retrograde); and, once determined, it becomes very easy to compare them with the whole observed course of the comet, by a process exactly similar to that of art. 502, and thus at once to ascertain their correctness, and to put to the severest trial the truth of those general laws on which all such calculations are founded.

(564.) For the most part, it is found that the motions of comets may be sufficiently well represented by parabolic orbits—that is to say, ellipses whose axes are of infinite length, or, at least, so very long that no appreciable error in the calculation of their motions, during all the time they continue visible, would be incurred by supposing them actually infinite. The parabola is that conic section which is the limit between the ellipse on the one hand, which returns into itself, and the hyperbola on the other, which runs out to infinity. A comet, therefore, which should describe an elliptic path, however long its axis, must *have* visited the sun before, and must again return (unless disturbed) in some determinate period—but should its orbit be of the hyperbolic character, when once it had passed its perihelion, it could never more return within the sphere of our observation, but must run off to visit other systems, or be lost in the immensity of space. There is no instance of a comet whose orbit has been very carefully calculated by more than one computist being *proved* to have described a hyperbola, though several have been suspected of doing so.[5] Many have been well ascertained to move in ellipses. These latter, in so far as their orbits can remain unaltered by the attractions of the planets, must be regarded as permanent members of our system.

(565.) We must now say a few words on the actual dimensions of comets. The calculation of the diameters of their heads, and the lengths and breadths of their tails, offers not the slightest difficulty when once the elements of their orbits are known, for by these we know their real distances from the earth at any time, and the true direction of the tail, which we see only foreshortened. Now calculations instituted on these principles lead to the surprising fact, that the comets are by far the most voluminous bodies in our system. The following are the dimensions of some of those which have been made the subjects of such inquiry.

(566.) The tail of the great comet of 1680, immediately after its perihelion passage, was found by Newton to have been no less than 20000000 of leagues in length, and to have occupied only two days in its emission from the comet's body! a decisive proof this of its being darted forth by some active force, the origin of which, to judge from the direction of the tail, must be sought in the sun itself. Its greatest length amounted to 41000000 leagues, a length much exceeding the whole interval between the sun and earth. The tail of the comet of 1769 extended 16000000 leagues, and that of the great comet of 1811, 36000000. The portion of the head of this last, comprised within the transparent atmospheric envelope which separated it from the tail, was 180000 leagues

[5] For example that of 1723, calculated by Burckhardt; that of 1771, by both Burckhardt and Encke; and the second comet of 1818, by Rosenberg and Schwabe.

in diameter. It is hardly conceivable, that matter once projected to such enormous distances should ever be collected again by the feeble attraction of such a body as a comet—a consideration which accounts for the surmised progressive diminution of the tails of such as have been frequently observed.

(567.) The most remarkable of those comets which have been ascertained to move in elliptic orbits is that of Halley, so called from the celebrated Edmund Halley, who, on calculating its elements from its perihelion passage in 1682, when it appeared in great splendor, with a tail 30° in length, was led to conclude its identity with the great comets of 1531 and 1607, whose elements he had also ascertained. The intervals of these successive apparitions being 75 and 76 years, Halley was encouraged to *predict* its reappearance about the year 1759. So remarkable a prediction could not fail to attract the attention of all astronomers, and, as the time approached, it became extremely interesting to know whether the attractions of the larger planets might not materially interfere with its orbitual motion. The computation of their influence from the Newtonian law of gravity, a most difficult and intricate piece of calculation, was undertaken and accomplished by Clairaut, who found that the action of Saturn would retard its return by 100 days, and that of Jupiter by no less than 518, making in all 618 days, by which the expected return would happen later than on the supposition of its retaining an unaltered period—and that, in short, the time of the expected perihelion passage would take place within a month, one way or other, of the middle of April, 1759.—It actually happened on the 12th of March in that year. Its next return was calculated by several eminent geometers,[6] and fixed successively for the 4th, the 7th,

[6] Damoiseau, Pontecoulant, Rosenberger and Lehmann.

the 11th, and the 26th of November, 1835; the two latter determinations appearing entitled to the higher degree of confidence, owing partly to the more complete discussion bestowed on the observations of 1682 and 1759, and partly to the continually improving state of our knowledge of the methods of estimating the disturbing effect of the several planets. The last of these predictions, that of M. Lehmann, was published on the 25th of July. On the 5th of August the comet first became visible in the clear atmosphere of Rome as an exceedingly faint telescopic nebula, within a degree of its place as predicted by M. Rosenberger for that day. On or about the 20th of August it became generally visible, and, pursuing very nearly its calculated path among the stars, passed its perihelion on the 16th of November; after which, its course carrying it south, it ceased to be visible in Europe, though it continued to be conspicuously so in the southern hemisphere throughout February, March, and April, 1836, disappearing finally on the 5th of May.

(568.) Although the appearance of this celebrated comet at its last apparition was not such as might be reasonably considered likely to excite lively sensations of terror, even in superstitious ages, yet, having been an object of the most diligent attention in all parts of the world to astronomers, furnished with telescopes very far surpassing in power those which had been applied to it at its former appearance in 1759, and indeed to any of the greater comets on record, the opportunity thus afforded of studying its physical structure, and the extraordinary phenomena which it presented when so examined have rendered this a memorable epoch in cometic history. Its first appearance, while yet very remote from the sun, was that of a small round or somewhat oval nebula, quite destitute of tail, and having a minute point of more concentrated light excentrically situated within it. It was not before the 2d of October that the tail began to be developed, and thenceforward increased pretty rapidly, being already 4° or 5° long on the 5th. It attained its greatest apparent length (about 20°) on the 15th of October. From that time, though not yet arrived at its perihelion, it decreased with such rapidity, that already on the 29th it was only 3°, and on November the 5th 2½° in length. There is every reason to believe that before the perihelion, the tail had altogether disappeared, as, though it continued to be observed at Pulkowa up to the very day of its perihelion passage, no mention whatever is made of any tail being then seen.

(569.) By far the most striking phenomena, however, observed in this part of its career, were those which, commencing simultaneously with the growth of the tail, connected themselves evidently with the production of that appendage and its projection from the head. On the 2d of October (the very day of the first observed commencement of the tail) the nucleus, which had been faint and small, was observed suddenly to have become much brighter, and to be in the act of throwing out a jet or stream of light from its anterior part, or that turned *toward* the sun. This ejection after ceasing a while was resumed, and with much greater apparent violence, on the 8th, and continued, with occasional intermittences, so long as the tail itself continued visible. Both the form of this luminous ejection, and the direction in which it issued from the nucleus, meanwhile underwent singular and capricious alterations, the different phases succeeding each other with such rapidity that on no two successive nights were the appearances alike. At one time the emitted jet was single, and confined within narrow limits of divergence from the nucleus. At others it presented a fan-shaped or swallow-tailed form, analogous to that of a gas-flame issuing from a flattened orifice: while at others again two, three or even more jets were darted forth in different directions.[7] (See figures *a*, *b*, *c*, *d*, Plate I. *fig.* 4, which represent, highly magnified, the appearances of the nucleus with its jets of light, on the 8th, 9th, 10th, and 12th of October, and in which the direction of the anterior portion of the head, or that fronting the sun, is supposed alike in all, viz. toward the upper part of the engraving. In these representations the head itself is omitted, the scale of the figures not permitting its introduction: *e* represents the nucleus and head as seen October 9th on a less scale.) The direction of the principal jet was observed meanwhile to oscillate to and fro on either side of a line directed to the sun in the manner of a compass-needle when thrown into vibration and oscillating about a mean position, the change of direction being conspicuous even from hour to hour. These jets, though very bright at their point of emanation from the nucleus, faded rapidly away, and became diffused as they expanded into the coma, at the same time curving backward as streams of steam or smoke would do, if thrown out from narrow orifices, more or less obliquely in opposition to a powerful wind, against which they were unable to make way, and, ultimately yielding to its force, so as to be drifted back and confounded in a vaporous train, following the general direction of the current.[8]

(570.) Reflecting on these phenomena, and carefully con-

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[7] See the exquisite lithographic representations of these phenomena by Bessel, Astron. Nachr. No. 302, and the fine series by Schwabe in No. 297 of that collection, as also the magnificent drawings of Struve, from which our figures *a*, *b*, *c*, *d*, are copied.

[8] On this point Schwabe's and Bessel's drawings are very express and unequivocal. Struve's attention seems to have been more especially directed to the scrutiny of the nucleus.

sidering the evidence afforded by the numerous and elaborately executed drawings which have been placed on record by observers, it seems impossible to avoid the following conclusions. 1st. That the matter of the nucleus of a comet is powerfully excited and dilated into a vaporous state by the action of the sun's rays, escaping in streams and jets at those points of its surface which oppose the least resistance, and in all probability throwing that surface or the nucleus itself into irregular motions by its reaction in the act of so escaping, and thus altering its direction.

2dly. That this process chiefly takes place in that portion of the nucleus which is turned toward the sun; the vapor escaping chiefly in that direction.

3dly. That when so emitted, it is prevented from proceeding in the direction originally impressed upon it, by some force directed *from* the sun, drifting it back and carrying it out to vast distances behind the nucleus, forming the tail or so much of the tail as can be considered as consisting of material substance.

4thly. That this force, whatever its nature, acts unequally on the materials of the comet, the greater portion remaining unvaporized, and a considerable part of the vapor actually produced, remaining in its neighborhood, forming the head and coma.

5thly. That the force thus acting on the materials of the tail cannot possibly be identical with the ordinary gravitation of matter, being centrifugal or repulsive, as respects the sun, and of an energy very far exceeding the gravitating force toward that luminary. This will be evident if we consider the enormous velocity with which the matter of the tail is carried backward, in opposition both to the motion which it had as part of the nucleus, and to that which it acquired in the act of its emission, both which motions have to be destroyed in the first instance, before any movement in the contrary direction can be impressed.

6thly. That unless the matter of the tail thus repelled from the sun be retained by a peculiar and highly energetic attraction to the nucleus, differing from and exceptional to the ordinary power of gravitation, it must leave the nucleus altogether; being in effect carried far beyond the coercive power of so feeble a gravitating force as would correspond to the minute mass of the nucleus; and it is therefore very conceivable that a comet may lose, at every approach to the sun, a portion of that peculiar matter, whatever it be, on which the production of its tail depends, the remainder being of course less excitable by the solar action, and more impassive to his rays, and therefore, *pro tanto*, more nearly approximating to the nature of the planetary bodies.

7thly. That considering the immense distances to which at least some portion of the matter of the tail is carried from the comet, and the way in which it is dispersed through the system, it is quite inconceivable that the whole of that matter should be reabsorbed—that therefore it must lose during its perihelion passage some portion of its matter, and if, as would seem far from improbable, that matter should be of a nature to be repelled from, not attracted by, the sun, the remainder will, by consequence, be, *pro quantitate inertiæ*, more energetically attracted to the sun than the mean of both. If then the orbit be elliptic, it will perform each successive revolution in a shorter time than the preceding, until, at length, the whole of the repulsive matter is got rid of.—But to return to the comet of Halley.

(571.) After the perihelion passage, the comet was lost sight of for upward of two months, and at its reappearance (on the 24th of January, 1836) presented itself under quite a different aspect, having in the interval evidently undergone some great physical change which had operated an entire transformation in its appearance. It no longer presented any vestige of tail, but appeared to the naked eye as a hazy star of about the fourth or fifth magnitude, and in powerful telescopes as a small, round, well defined disk, rather more than 2′ in diameter, surrounded with a nebulous *chevelure* or coma of much greater extent. Within the disk, and somewhat excentrically situated, a minute but bright nucleus appeared, from which extended toward the posterior edge of the disk (or that remote from the sun) a short vivid luminous ray. (See *fig.* 1, Plate VI.) As the comet receded from the sun, the coma speedily disappeared, as if absorbed into the disk, which, on the other hand, increased continually in dimensions, and that with such rapidity, that in the week elapsed from January 25th to February 1st (calculating from micrometrical measures, and from the known distance of the comet from the earth on those days), the actual volume or *real solid content* of the illuminated space had dilated in the ratio of upward of 40 to 1.[9] And so it continued to swell out with undiminished rapidity, until, from this cause alone, it ceased to be visible, the illumination becoming fainter as the magnitude increased; till at length the outline became indistinguishable from

[9] On the night of the 22d of January the comet was observed by M. Boguslawski of Breslau, *as a star* of the sixth magnitude, a bright concentrated point, which showed no disk with a magnifying power of 140, and that it actually *was* the comet he assured himself by turning his telescope the next night on the place where he saw it (which he had carefully noted and registered). It was gone. From his observation it appears then that at 17h 50m M. T. at Breslau, Jan. 22, the diameter of the nucleus with its envelope was rigorously *nil*, at which moment, within an hour one way or the other, the process of formation of the envelope must have commenced.

simple want of light to trace it. While this increase of dimension proceeded, the form of the disk passed, by gradual and successive additions to its length in the direction opposite to the sun, to that of a paraboloid, as represented in *g*, *fig.* 1, Plate VI., the anterior curved portion preserving its planetary sharpness, but the base being faint and ill-defined. It is evident that had this process continued with sufficient light to render the result visible, a tail would have been ultimately reproduced; but the increase of dimension being accompanied with diminution of brightness, a short, imperfect, and as it were rudimentary tail only was formed, visible as such for a few nights to the naked eye, or in a low magnifying telescope, and that only when the comet itself had begun to fade away by reason of its increasing distance.

(572.) While the parabolic envelope was thus continually dilating and growing fainter, the nucleus underwent little change, but the ray proceeding from it increased in length and comparative brightness, preserving all the time its direction along the axis of the paraboloid, and offering none of those irregular and capricious phenomena which characterized the jets of light emitted anteriorly, previous to the perihelion. If the office of those jets was to feed the tail, the converse office of conducting back its successively condensing matter to the nucleus would seem to be that of the ray now in question. By degrees this also faded, and the last appearance presented by the comet was that which it offered at its first appearance in August, viz. that of a small round nebula with a bright point in or near the centre.

(573.) Besides the comet of Halley, several other of the great comets recorded in history have been surmised with more or less probability to return periodically, and therefore to move in elongated ellipses around the sun. Such is the great comet of 1680, whose period is estimated at 575 years, and which has been considered, with at least a high *primâ facie* probability, to be identical with a magnificent comet observed at Constantinople and in Palestine, and referred by contemporary historians, both European and Chinese, to the year A.D. 1106; with that of A.D. 531, which was seen at noonday close to the sun; with the comet of 43 B.C., already spoken of as having appeared after the death of Cæsar, and which was also observed in the daytime; and finally with two other comets, mention of which occurs in the Sibylline Oracles, and in a passage of Homer, and which are referred, as well as the obscurity of chronology and the indications themselves will allow, to the years 618 and 1194 B.C. It is to the assumed near approach of this comet to the earth about the time of the Deluge, that Whiston ascribed that overwhelming tide-wave to whose agency his wild fancy ascribed that great catastrophe—a speculation, it is needless to remark, purely visionary. These coincidences of time are certainly remarkable, especially when it is considered how very rare are the appearances of comets of this class. Professor Encke, however, has discussed, with all possible care, the observations recorded of the comet of 1680, taking into consideration the perturbations of the planets (which are of trifling importance, by reason of the great inclination of its orbit to the ecliptic), and his calculations show that no elliptic orbit, with such a period as 575 years, is competent to represent them within any probable or even possible limits of error, the most probable period assigned by them being 8814 Julian years. Independent of this consideration, there are circumstances recorded of the comet of A.D. 1106 incompatible with its motion in any orbit identical with that of the comet of 1680, so that the idea of referring all these phenomena to one and the same comet, however seducing, must be relinquished.

(574.) Another great comet, whose return about the year 1848 had been considered by more than one eminent authority in this department of astronomy[10] highly probable, is that of 1556, to the terror of whose aspect some historians have attributed the abdication of the Emperor Charles V. This comet is supposed to be identical with that of 1264, mentioned by many historians as a great comet, and observed also in China—the conclusion in this case resting upon the coincidence of elements calculated on the observations, such as they are, which have been recorded. On the subject of this coincidence Mr. Hind has entered into many elaborate calculations, the result of which is strongly in favor of the supposed identity. This probability is further increased by the fact of a comet, with a tail of 40° and a head bright enough to be visible after sunrise, having appeared in A.D. 975: and of two others having been recorded by the Chinese annalists in A.D. 395 and 104. It is true that if these be the same, the mean period would be somewhat short of 292 years. But the effect of planetary perturbation might reconcile even greater differences, and though up to the time of our writing (1858) no such comet has yet been observed, two or three years must yet elapse, in the opinion of those best competent to judge, before its return must be considered hopeless.

(575.) In 1661, 1532, 1402, 1145, 891 and 243 great comets appeared—that of 1402 being bright enough to be seen at noonday. A period of 129 years would conciliate all these appearances, and should have brought back the comet

[10] Pingré, Comètographie, i. 411; Lalande, Astr. 3185.

in 1789 or 1790 (other circumstances agreeing). That no such comet was observed about that time is no proof that it did not return, since, owing to the situation of its orbit, had the perihelion passage taken place in July it might have escaped observation. Mechain, indeed, from an elaborate discussion of the observations of 1532 and 1661, came to the conclusion that these comets were not the same; but the elements assigned by Olbers to the earlier of them differ so widely from those of Mechain for the same comet on the one hand, and agree so well with those of the last named astronomer for the other,[11] that we are perhaps justified in regarding the question as not yet set at rest.

(576.) We come now, however, to a class of comets of short period, respecting whose return there is no doubt, inasmuch as two at least of them have been identified as having performed successive revolutions round the sun; have had their return predicted already several times; and have on each occasion scrupulously kept to their appointments. The first of these is the comet of Encke, so called from Professor Encke of Berlin, who first ascertained its periodical return. It revolves in an ellipse of great excentricity (though not comparable to that of Halley's), the plane of which is inclined at an angle of about 13° 22′ to the plane of the ecliptic, and in the short period of 1211 days, or about $3\frac{1}{3}$ years. This remarkable discovery was made on the occasion of its fourth recorded appearance, in 1819. From the ellipse then calculated by Encke, its return in 1822 was predicted by him, and observed at Paramatta, in New South Wales, by M. Rümker, being invisible in Europe: since which it has been repredicted and reobserved in all the principal observatories, both in the

[11] See Schumacher's Catal. Astron. Abhandl. i.

northern and southern hemispheres, as a phenomenon of regular occurrence.

(577.) On comparing the intervals between the successive perihelion passages of this comet, after allowing in the most careful and exact manner for all the disturbances due to the actions of the planets, a very singular fact has come to light, viz. that the periods are continually diminishing, or, in other words, the mean distance from the sun, or the major axis of the ellipse, dwindling by slow and regular degrees at the rate of about $0^{d}{\cdot}11176$ per revolution. This is evidently the effect which would be produced by a resistance experienced by the comet from a very rare ethereal medium pervading the regions in which it moves; for such resistance, by diminishing its actual velocity, would diminish also its centrifugal force, and thus give the sun more power over it to draw it nearer. Accordingly this is the solution proposed by Encke, and at present generally received. Should this be really the case, it will, therefore, probably fall ultimately into the sun, should it not first be dissipated altogether—a thing no way improbable, when the lightness of its materials is considered. The considerations adduced at the end of art. 570 would seem, however, to open out another possible explanation of the phenomenon in question, not necessarily leading to such a catastrophe.

(578.) By measuring the apparent magnitude of this comet at different distances from the sun, and thence, from a knowledge of its actual distance from the earth at the time, concluding its real volume, it has been ascertained to contract in bulk as it approaches to, and to expand as it recedes from, that luminary. M. Valz, who was the first to notice this fact, accounts for it by supposing it to undergo a real compression or condensation of volume arising from the pressure of an ethereal medium which he conceives to grow more dense in the sun's neighborhood. But such a hypothesis is evidently inadmissible, since it would require us to assume the exterior of the comet to be in the nature of a skin or bag impervious to the compressing medium. The phenomenon is analogous to the increase of dimension above described, as observed in the comet of Halley, when in the act of receding from the sun, and is doubtless referable to a similar cause, viz. the alternate conversion of evaporable matter into the states of visible cloud and invisible gas, by the alternating action of cold and heat. This comet has no tail, but offers to the view only a small ill-defined nucleus, excentrically situated within a more or less elongated oval mass of vapors, being nearest to that vertex which is toward the sun.

(579.) Another comet of short period is that of *Biela*, so called from M. Biela, of Josephstadt, who first arrived at this interesting conclusion on the occasion of its appearance in 1826. It is considered to be identical with comets which appeared in 1772, 1805, etc., and describes its very excentric ellipse about the sun in 2410 days, or about $6\frac{3}{4}$ years; and in a plane inclined 12° 34′ to the ecliptic. It appeared again, according to the prediction, in 1832 and in 1846. Its orbit, by a remarkable coincidence, very nearly intersects that of the earth; and had the latter, at the time of its passage in 1832, been a month in advance of its actual place, it would have passed through the comet—a singular rencontre, perhaps not unattended with danger.[12]

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[12] Should calculation establish the fact of a resistance experienced also by this comet, the subject of periodical comets will assume an extraordinary degree of interest. It cannot be doubted that many more will be discovered, and by their resistance questions will come to be decided, such as the following:—What

(580.) This comet is small and hardly visible to the naked eye, even when brightest. Nevertheless, as if to make up for its seeming insignificance by the interest attaching to it in a physical point of view, it exhibited, at its appearance in 1846, a phenomenon which struck every astronomer with amazement, as a thing without previous example in the history of our system.[13] It was actually seen to separate itself into two distinct comets, which, after thus parting company, continued to journey along amicably through an arc of upward of 70° of their apparent orbit, keeping all the while within the same field of view of the telescope pointed toward them. The first indication of something unusual being about to take place might be, perhaps, referred to the 19th of December, 1845, when the comet appeared to Mr. Hind pear-shaped, the nebulosity being unduly elongated in a direction inclining northward. But on the 13th of January, at Washington in America, and on the 15th and subsequently in every part of Europe, it was distinctly seen to have become double; a very small and faint cometic body, having a nucleus of its own, being observed appended to it, at a distance of about

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is the law of density of the resisting medium which surrounds the sun? Is it at rest or in motion? If the latter, in what direction does it move? Circularly round the sun, or traversing space? If circularly, in what plane? It is obvious that a circular or vorticose motion of the ether would *accelerate some comets and retard others*, according as their revolution was, relative to such motion, direct or retrograde. Supposing the neighborhood of the sun to be filled with a material fluid, it is not conceivable that the circulation of the planets in it for ages should not have impressed upon it some degree of rotation in their own direction. And this may preserve them from the extreme effects of accumulated resistance.—*Note to edition of* 1833.

[13] Perhaps not quite so. To say nothing of a singular surmise of Kepler, that two great comets *seen at once* in 1618, might be a single comet separated into two, the following passage of Hevelius cited by M. Littrow (Nachr. 564) does really seem to refer to some phenomenon bearing at least a certain analogy to it. "In ipso disco," he says (Cometographia, p. 326), "*quatuor vel quinque* corpus cula quædam sive nucleos reliquo corpore aliquanto densiores ostendebat."

2′ (in arc) from its centre, and in a direction forming an angle of about 328° with the meridian, running northward from the principal or original comet (see art. 204). From this time the separation of the two comets went on progressively, though slowly. On the 30th of January, the apparent distance of the nucleus had increased to 3′, on the 7th of February to 4′, and on the 13th to 5′, and so on, until on the 5th of March the two comets were separated by an interval of 9′ 19″, the apparent direction of the line of junction all the while varying but little with respect to the parallel.[14]

(581.) During this separation, very remarkable changes were observed to be going on, both in the original comet and its companion. Both had nuclei, both had short tails, parallel in direction, and nearly perpendicular to the line of junction, but whereas at its first observation on January 13th, the new comet was extremely small and faint in comparison with the old, the difference both in point of light and apparent magnitude diminished. On the 10th of February, they were nearly equal, although the day before the moonlight had effaced the new one, leaving the other bright enough to be well observed. On the 14th and 16th, however, the new comet had gained a decided superiority of light over the old, presenting at the same time a sharp and starlike nucleus, compared by Lieut. Maury to a diamond spark. But this state of things was not to continue. Already, on the 18th, the old comet had regained its superiority, being nearly twice as bright as its companion, and offering an unusually bright and starlike nucleus. From

[14] By far the greater portion of this increase of apparent distance was due to the comet's increased proximity to the earth. The real increase reduced to a distance = 1 of the comet was at the rate of about 3″ per diem.

this period the new companion began to fade away, but continued visible up to the 15th of March. On the 24th the comet appeared again single, and on the 22d of April both had disappeared.

(582.) While this singular interchange of light was going forward, indications of some sort of communication between the comets were exhibited. The new or companion comet, besides its tail, extending in a direction parallel to that of the other, threw out a faint arc of light which extended as a kind of bridge from the one to the other; and after the restoration of the original comet to its former pre-eminence, it, on its part, threw forth additional rays, so as to present (on the 22d and 23d February) the appearance of a comet with three faint tails forming angles of about 120° with each other, one of which extended toward its companion.

(583.) Professor Plantamour, director of the observatory of Geneva, having investigated the orbits of both these comets as separate and independent bodies, from the extensive and careful series of observations made upon them, arrived at the conclusion that the increase of distance between the two nuclei, *at least during the interval from February* 10*th to March* 22*d*, was simply apparent, being due to the variation of distance from the earth, and to the angle under which their line of junction presented itself to the visual ray; the real distance during all that interval (neglecting small fractions) having been on an average about thirty-nine times the semidiameter of the earth, or less than two-thirds the distance of the moon from its centre. From this it would appear that already, at this distance, the two bodies had ceased to exercise any perceptible amount of perturbative gravitation on each other; as, indeed, from the probable minuteness of cometary masses we might reasonably expect. It may well be supposed that astronomers would not allow so remarkable a duplication to pass unwatched at the next return of the comet in 1852. In August and September of that year both nuclei were observed by Professor Challis at Cambridge, Secchi at Rome, and M. Struve, presenting, as regards direction, the same relative situation with regard to each other, so that we have here the historical proof of a permanent addition to the members of our system taking place at a definite instant under our very eye.[15] (Plate VI. *fig.* 2.)

(584.) A third comet, of short period, has been added to our list by M. Faye, of the observatory of Paris, who detected it on the 22d of November, 1843. A very few observations sufficed to show that no parabola would satisfy the conditions of its motion, and that to represent them completely, it was necessary to assign to it an elliptic orbit of very moderate excentricity. The calculations of M. Nicolai, subsequently revised and slightly corrected by M. Leverrier, have shown that an almost perfect representation of its motions during the whole period of its visibility would be afforded by assuming it to revolve in a period of $2717^d{\cdot}68$ (or somewhat less than $7\frac{1}{2}$ years) in an ellipse whose excentricity is 0·55596, and inclination to the ecliptic 11° 22′ 31″; and taking this for a basis of further calculation, and by means of these data and the other elements of the orbit estimating the effect of planetary perturbation during the revolution now in progress, he fixed its next return to the perihelion for the 3d of April, 1851, with a probable error one way or

[15] According to the elements of this comet deduced by M. Santini, taking into account all planetary perturbation, its two heads ought to have passed their perihelion on January 27 and January 29, respectively, 1866. Its appearance was anxiously and perseveringly looked for, but in vain; nor has any probable cause been assigned for its disappearance!

other not exceeding one or two days. This prediction has been strikingly verified. It actually passed its perihelion on the 1st of April, 1851, having been rediscovered by Professor Challis at Cambridge in November, 1850, and followed beyond the perihelion by M. Otto Struve up to March 4, 1851.

(585.) The effect of planetary perturbation on the motion of comets has been more than once alluded to in what has been above said. Without going minutely into this part of the subject, which will be better understood after the perusal of a subsequent chapter, it must be obvious, that as the orbits of comets are very excentric, and inclined in all sorts of angles to the ecliptic, they must, in many instances, if not actually intersect, at least pass very near to the orbits of some of the planets. We have already seen, for instance, that the orbit of Biela's comet so nearly intersects that of the earth, that an actual collision is not impossible, and indeed (supposing neither orbit variable) must in all likelihood happen in the lapse of some millions of years. Neither are instances wanting of comets having actually approached the earth within comparatively short distances, as that of 1770, which on the 1st of July of that year was within little more than seven times the moon's distance. The same comet, in 1767, passed Jupiter at a distance only one 58th of the radius of that planet's orbit, and it has been rendered extremely probable that it is to the disturbance its former orbit underwent during that appulse that we owe its appearance within our own range of vision. This exceedingly remarkable comet was found by Lexell to describe an elliptic orbit with an excentricity of 0·7858, with a periodic time of about five years and a half, and in a plane only 1° 34′ inclined to the ecliptic, having passed its perihelion on the 13th of August,

1770. Its return of course was eagerly expected, but in vain, for the comet has never been certainly identified with any comet since seen. Its observation on its first return in 1776 was rendered impossible by the relative situations of the perihelion and of the earth at the time, and before another revolution could be accomplished (as has since been ascertained), viz. about the 23d of August, 1779, by a singular coincidence it again approached Jupiter within one 491st part of its distance from the sun, being nearer to that planet by one-fifth than its fourth satellite. No wonder, therefore, that the planet's attraction (which at that distance would exceed that of the sun in the proportion of at least 200 to 1) should completely alter the orbit and deflect it into a curve, not one of whose elements would have the least resemblance to those of the ellipse of Lexell. It is worthy of notice that by this rencontre with the system of Jupiter's satellites, none of *their* motions suffered any perceptible derangement—a sufficient proof of the smallness of its mass. Jupiter, indeed, seems, by some strange fatality, to be constantly in the way of comets, and to serve as a perpetual stumbling-block to them.

(586.) On the 22d of August, 1844, Signor de Vico, director of the observatory of the Collegio Romano, discovered a comet, the motions of which, a very few observations sufficed to show, deviated remarkably from a parabolic orbit. It passed its perihelion on the 2d of September, and continued to be observed until the 7th of December. Elliptic elements of this comet, agreeing remarkably well with each other, were accordingly calculated by several astronomers, from which it appears that the period of revolution is about 1990 days, or 5½ (5·4357) years, which (supposing its orbit undisturbed in the interim) would bring it back to the perihelion on or about the 13th of January, 1850, on which occasion, however, by reason of its unfavorable situation with respect to the sun and earth, it could not be observed. As the assemblage and comparison of these elements, thus computed independently, will serve better, perhaps, than any other example, to afford the student an idea of the degree of arithmetical certainty capable of being attained in this branch of astronomy, difficult and complex as the calculations themselves are, and liable to error as *individual* observations of a body so ill defined as the smaller comets are for the most part, we shall present them in a tabular form, as on the next page: the elements being as usual; the time of perihelion passage, longitude of the perihelion, that of the ascending node, the inclination to the ecliptic, semiaxis and excentricity of the orbit, and the periodic time.

This comet, when brightest, was visible to the naked eye, and had a small tail. It is especially interesting to astronomers from the circumstance of its having been rendered exceedingly probable by the researches of M. Leverrier, that it is identical with one which appeared in 1678 with some of its elements considerably changed by perturbation. This comet is further remarkable, from having been concluded by Messrs. Laugier and Mauvais, to be identical with the comet of 1585 observed by Tycho Brahe, and possibly also with those of 1743, 1766 and 1819.

(587.) Elliptic elements have in like manner been assigned to the comet discovered by M. Brorsen, on the 26th of February, 1846, which, like that last mentioned, speedily after its discovery began to show evident symptoms of deviation from a parabola. These elements, with the names of their respective calculators, are as follows. The dates are for February, 1846, Greenwich time.

ELEMENTS OF THE PERIODICAL COMET OF DE VICO

Computed by	Nicolai.	Hind.	Goldschmidt.	Faye.	Schubert.	Brunnow.
	1844, Sep.	1844, Sep.	1844, Sep.	1844, Sep.	1844, Sep.	1844, Sep.
Perihelion passage . .	$2^d.47594$	$2^d.50412$	$2^d.45430$	$2^d.48745$	$2^d.56348$	$2^d.47402$
Longitude of perihelion .	342° 31′ 5″·5	342° 32′ 40″·1	342° 29′ 44″·9	342° 31′ 15″·5	342° 34′ 31″·5	342° 30′ 49″·6
Longitude of ☊ . .	63 48 48·9	63 52 24·1	63 48 55·2	63 49 30 6	63 54 40·8	63 49 0·1
Inclination . . .	2 54 45·8	2 54 27·1	2 55 1·9	2 54 45·0	2 52 51·8	2 54 50·3
Semiaxis	3·09853	3·08582	3·11111	3·09946	3·02612	3·10295
Excentricity . . .	0·61716	0·61566	0·61861	0·61726	0·60866	0·61788
Period (days) . . .	1992	1980	2400	1993	1923	1996

Computed by	Brunnow.	Hind.	Van Willingen and De Haan.
Perihelion passage . .	25d ·37794	25d ·33109	25d ·02227
Long. of Perihelion . .	116° 28′ 34″·0	116° 28′ 17″·8	116° 23′ 52″·9
Long. of ☊	102 39 36·5	102 45 20·9	103 31 25·7
Inclination . . .	30 55 6·5	30 49 3·6	30 30 30·2
Semiaxis	3·15021	3·12292	2·87052
Excentricity . . .	0·79363	0·79771	0·77313
Period (days) . . .	2042	2016	1776

This comet is faint, and presents nothing remarkable in its appearance. Its chief interest arises from the great similarity of its *parabolic* elements to those of the comet of 1532, the place of the perihelion and node, and the inclination of the orbit, being almost identical.

(588.) Elliptic elements have also been calculated by M. D'Arrest, for a comet discovered by M. Peters, on the 26th of June, 1846, which go to assign it a place among the comets of short period, viz. 5804d·3, or very nearly 16 years. The excentricity of the orbit is 0·75672, its semiaxis 6·32066, and the inclination of its plane to that of the ecliptic 31° 2′ 14″. This comet passed its perihelion on the 1st of June, 1846.

(589.) By far the most remarkable comet, however, which has been seen during the present century, is that which appeared in the spring of 1843, and whose tail became visible in the twilight of the 17th of March in England as a great beam of nebulous light, extending from a point above the western horizon, through the stars of Eridanus and Lepus, under the belt of Orion. This situation was low and unfavorable; and it was not till the 19th that the head was seen, and then only as a faint and ill-defined nebula, very rapidly fading on subsequent nights. In more southern latitudes, however, not only the tail was seen, as a magnificent train of light extending 50° or 60° in length; but the head and nucleus appeared with extraordinary splendor, exciting in every country where it was seen the greatest astonishment and admiration. Indeed, all descriptions agree in representing it as a stupendous spectacle, such as in superstitious ages would not fail to have carried terror into every bosom. In tropical latitudes in the northern hemisphere, the tail appeared on the 3d of March, and in Van Diemen's Land, so early as the 1st, the comet having passed its perihelion on the 27th of February. Already on the 3d the head was so far disengaged from the immediate vicinity of the sun, as to appear for a short time above the horizon after sunset. On this day when viewed through a 46-inch achromatic telescope it presented a planetary disk, from which rays emerged in the direction of the tail. The tail was double, consisting of two principal lateral streamers, making a very small angle with each other, and divided by a comparatively dark line, of the estimated length of 25°, prolonged however on the north side by a divergent streamer, making an angle of 5° or 6° with the general direction of the axis, and traceable as far as 65° from the head. A similar though fainter lateral prolongation appeared on the south side. A fine drawing of it of this date by C. P. Smyth, Esq., of the Royal Observatory, C.G.H., represents it as highly symmetrical, and gives the idea of a vivid cone of light, with a dark axis, and nearly rectilinear sides, inclosed in a fainter cone, the sides of which curve slightly outward. The light of the nucleus at this period is compared to that of a star of the first or second magnitude; and on the 11th, of the third; from which time it degraded in light so rapidly, that on the 19th it was invisible to the naked eye, the tail all the while continuing brilliantly visible, though much more so at a distance from the nucleus, with which, indeed, its connection was not then obvious to the unassisted sight—a singular feature in the history of this body. The tail, subsequent to the 3d, was generally speaking a single straight or slightly curved broad band of light, but on the 11th it is recorded by Mr. Clerihew, who observed it at Calcutta, to have shot forth a lateral tail nearly twice as long as the regular one but fainter, and making an angle of about 18° with its direction on the southern side. The projection of this ray (which was not seen either before or after the day in question) to so enormous a length (nearly 100°) in a single day conveys an impression of the intensity of the forces acting to produce such a velocity of material transfer through space, such as no other natural phenomenon is capable of exciting. It is clear that *if we have to deal here with matter, such as we conceive it*, viz. *possessing inertia—at all*, it must be under the dominion of forces incomparably more energetic than gravitation, and of quite a different nature.

(590.) There is abundant evidence of the comet in question having been seen in full daylight, and in the sun's immediate vicinity. It was so seen on the 28th of February, the day after its perihelion passage, by every person on board the H.E.I.C.S. "Owen Glendower," then off the Cape, as a short dagger-like object close to the sun a little before sunset. On the same day at $3^h\ 6^m$ P.M., and consequently in full sunshine, the distance of the nucleus from the sun was actually measured with a sextant by Mr. Clarke of Portland, United States, the distance centre from centre being then only 3° 50′ 43″. He describes it in the following terms: "The nucleus and also every part of the tail were as well defined as the moon on a clear day. The nucleus and tail bore the same appearance, and resembled a perfectly pure white cloud without any variation, except a slight change near the head, just sufficient to distinguish the nucleus from the tail at that point." The denseness of the nucleus was so considerable, that Mr. Clarke had no doubt it might have been visible upon the sun's disk, had it passed between that and the observer. The length of the visible tail resulting from these measures was 59′ or not far from double the apparent diameter of the sun; and as we shall presently see that on the day in question the distance from the earth of the sun and comet must have been very nearly equal, this gives us about 1700000 miles for the linear dimensions of this the densest portion of that appendage, making no allowance for the foreshortening, which at that time was very considerable.

(591.) The elements of this comet are among the most remarkable of any recorded. They have been calculated by several eminent astronomers, among whose results we shall specify only those which agree best; the earlier attempts to compute its path having been rendered uncertain by the difficulty attending exact observations of it in the first part of its visible career. The following are those which seem entitled to most confidence:

	Encke.	Plantamour	Knorre.	Nicolai.	Peters.
Perihel. pass., 1843 Feb., mean time at Greenwich . . .	27d 45096	27d·42935	27d·39638	27d·43023	27d·41319
Long. of perihel. .	279° 2′ 30″	278° 18′ 3″	278° 28′ 25″	278° 36′ 33″	279° 59′ 7″
Long. of ☊ . . .	4 15 5	0 51 4	1 48 3	1 37 55	3 55 17
Inclination . . .	35 12 38	35 8 56	35 35 29	35 36 29	35 15 42
Perihel. dist. . .	0·00522	0·00581	0·00579	0·00558	0·00428
Motion	Retrograde.	Retrograde.	Retrograde.	Retrograde.	Retrograde.

(592.) What renders these elements so remarkable is the smallness of the perihelion distance. Of all comets which have been recorded this has made the nearest approach to the sun. The sun's radius being the sine of his apparent semidiameter (16′ 1″) to a radius equal to the earth's mean distance = 1, is represented on that scale by 0·00466, which falls short of 0·00534, the perihelion distance found by taking a mean of all the foregoing results, by only 0·00068, or about one-seventh of its whole magnitude. The comet, therefore, approached the *luminous* surface of the sun within about a seventh part of the sun's radius! It is worth while to consider what is implied in such a fact. In the first place, the intensity both of the light and radiant heat of the sun at different distances from that luminary increase proportionally to the spherical area of the portion of the visible hemisphere covered by the sun's disk. This disk, in the case of the earth, at its mean distance has an angular diameter of 32′ 1″·1. At our comet in perihelio the apparent angular diameter of the sun was no less than 121° 32′. The ratio of the spherical surfaces thus occupied (as appears from spherical geometry) is that of the squares of the sines of the fourth parts of these angles to each other, or that of 1 : 47042. And in this proportion are to each other the amounts of light and heat thrown by the sun on an equal area of exposed surface on our earth and at the comet in equal instants of time. Let any one imagine the effect of so fierce a glare as that of 47000 suns such as we experience the warmth of, on the materials of which the earth's surface is composed. To form some practical idea of it we may compare it with what is recorded of Parker's great lens, whose diameter was 32½ inches, and focal length six feet eight inches. The effect of this, supposing all the light and heat transmitted, and the focal concentration perfect (both conditions very imperfectly satisfied), would be to enlarge the sun's effective angular diameter to 23° 26′, which, compared on the same principle with a sun of 32′ in diameter, would give a multiplier of only 1915, and when increased sevenfold (as was usually the case), by interposing a concentrating lens, 13405 instead of 47000. The heat to which the comet was subjected therefore surpassed that in the focus of the lens in question, on the lowest calculation, in the proportion of $24\frac{1}{2}$ to 1 without, or $3\frac{1}{2}$ to 1 with the concentrating lens. Yet that lens, so used, melted carnelian, agate, and rock crystal!

(593.) To this extremity of heat however the comet was exposed but for a short time. Its actual velocity in perihelio was no less than 366 miles per second, and the whole of that segment of its orbit above (*i.e.* north of) the plane of the ecliptic, and in which, as will appear from a consideration of the elements, the perihelion was situated, was described in little more than two hours; such being the whole duration of the time from the ascending to the descending node, or in which the comet had north latitude. Arrived at the descending node, its distance from the sun would be already doubled, and the radiation reduced to one-fourth of its maximum amount. The comet of 1680, whose perihelion distance was 0·0062, and which therefore approached the sun's surface within one-third part of his radius (more than double the distance of the comet now in question) was computed by Newton to have been subjected to an intensity of heat 2000 times that of red-hot iron—a term of comparison indeed of a very vague description, and which modern thermotics do not recognize as affording a legitimate measure of radiant heat.[16]

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[16] A transit of the comet of 1843 over the sun's disk must probably have taken place shortly after its passage through its descending node. It is greatly

(594.) Although some of the observations of this comet were vague and inaccurate, yet there seem good grounds for believing that its whole course cannot be reconciled with a parabolic orbit, and that it really describes an ellipse. Previous to any calculation, it was remarked that in the year 1668 the tail of an immense comet was seen in Lisbon, at Bologna, in Brazil, and elsewhere, occupying nearly the same situation among the stars, and at the same season of the year, viz. on the 5th of March and the following days. Its brightness was such that its reflected trace was easily distinguished on the sea. The head, when it at length came in sight, was comparatively faint and scarce discernible. No precise observations were made of this comet, but the singular coincidence of situation, season of the year, and physical resemblance, excited a strong suspicion of the identity of the two bodies, implying a period of 175 years *within a day or two more or less.* This suspicion has been converted almost into a certainty by a careful examination of what is recorded of the older comet. Locating on a celestial chart the situation of the head, concluded from the direction and appearance of the tail, when only that was seen, and its visible place, when mentioned, according to the descriptions given, it has been found practicable to derive a rough orbit from the course thus laid down: and this agrees in all its features so well with that of the modern comet as nearly to remove all doubt on the subject. Comets, moreover, are recorded to have been seen in A.D. 268, 442–

to be regretted that so interesting a phenomenon should have passed unobserved. Whether it be possible that some offset of its tail, darted off so late as the 7th of March, when the comet was already far south of the ecliptic, should have crossed that plane and been seen near the Pleiades, may be doubted. Certain it is, that on the evening of that day, a decidedly cometic ray *was* seen in the immediate neighborhood of those stars by Mr. Nasmyth. (Ast. Soc. Notices, vol. v. p. 270.)

43, 791, 968, 1143, 1317, 1494, which may have been returns of this, since the period above mentioned would bring round its appearance to the years 268, 443, 618, 793, 968, 1143, 1318, and 1493, and a certain latitude must always be allowed for unknown perturbations.

(595.) But this is not the only comet on record whose identity with the comet of '43 has been maintained. In 1689 a comet bearing a considerable resemblance to it was observed from the 8th to the 23d of December, and from the few and rudely observed places recorded, its elements had been calculated by Pingré, one of the most diligent inquirers into this part of astronomy.[17] From these it appears that the perihelion distance of that comet was very remarkably small, and a sufficient though indeed rough coincidence in the places of the perihelion and node tended to corroborate the suspicion. But the inclination (69°) assigned to it by Pingré appeared conclusive against it. On recomputing the elements, however, from his data, Professor Pierce has assigned to that comet an inclination widely differing from Pingré's, viz. 30° 4′,[18] and quite within reasonable limits of resemblance. But how does this agree with the longer period of 175 years before assigned? To reconcile this we must suppose that these 175 years comprise at least eight returns of the comet, and that in effect a mean period of $21^{y}{\cdot}875$ must be allowed for its return. Now it is

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17 Author of the "Cométographie," a work indispensable to all who would study this interesting department of the science.

18 United States Gazette, May 29, 1843. Considering that all the observations lie *near* the descending node of the orbit, the proximity of the comet at that time to the sun, and the loose nature of the recorded observations, no doubt almost any given inclination *might* be deduced from them. The true test in such cases is not to ascend from the old incorrect data to elements, but to descend from known and certain elements to the older data, and ascertain whether the recorded phenomena can be represented by them (perturbations included) within fair limits of interpretation. Such is the course pursued by Clausen.

worth remarking that this period calculated backward from 1843·156 will bring us upon a series of years remarkable for the appearance of great comets, many of which, as well as the imperfect descriptions we have of their appearance and situation in the heavens, offer at least no obvious contradiction to the supposition of their identity with this. Besides those already mentioned as indicated by the period of 175 years, we may specify as probable or possible intermediate returns, those of the comets of 1733?[19], 1689 above mentioned, 1559?, 1537,[20] 1515,[21] 1471, 1426, 1405–6, 1383, 1361, 1340,[22] 1296, 1274, 1230,[23] 1208, 1098, 1056, 1034, 1012,[24] 990?,[25] 925?, 858??, 684,[26] 552, 530,[27] 421, 245 or 247,[28] 180[29], 158. Should this view of the subject be the true one, we may expect its return about the end of 1864 or beginning of 1865, in which event it will be observable in the Southern Hemisphere both before and after its perihelion passage.[30]

(596.) M. Clausen, from the assemblage of all the obser-

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[19] P. Passage 1733·781. The great southern comet of May 17th seems too early in the year.

[20] P. P. 1536·906. In January 1537, a comet was seen in Pisces.

[21] P. P. 1515·031. A comet *predicted* the death of Ferdinand the Catholic. He died Jan. 23, 1515.

[22] P. P. 1340·031. Evidently a southern comet, and a very probable appearance.

[23] P. P. 1230·656, was perhaps a return of Halley's.

[24] P. P. 1011·906. In 1012, a very great comet in the southern part of the heavens. "Son éclat blessait les yeux." (Pingré Cométographie, from whom all these recorded appearances are taken.)

[25] P. P. 990·031. "Comète fort épouvantable," *some year* between 989 and 998.

[26] P. P. 683·781. In 684, appeared two or three comets. Dates begin to be obscure.

[27] Two distinct comets appeared in 530 and 531, the former observed in China, the latter in Europe.

[28] P. P. 246·281; both southern comets of the Chinese annals. The year of one or other may be wrong.

[29] P. P. 180·656. Nov. 6, A.D. 180. A southern comet of the Chinese annals.

[30] Clausen, Astron. Nachr. No. 415. Mr. Cooper's remarks on this comet in his Catalogue of Comets (notes, p. xviii.) go to assign by far the greatest weight of probability to a period of 35·1y for this comet.

vations of this comet known to him, has calculated elliptic elements which give the extraordinarily short period of 6·38 years. And in effect it has been suggested that a still further subdivision of the period of 21·875 into three of 7·292 years would reconcile this with other remarkable comets. This seems going too far, but at all events the possibility of representing its motions by so short an ellipse will easily reconcile us to the admission of a period of 21 years. That it should only be visible in certain apparitions, and not in others, is sufficiently explained by the situation of its orbit.

(597.) We have been somewhat diffuse on the subject of this comet, for the sake of showing the degree and kind of interest which attaches to cometic astronomy in the present state of the science. In fact, there is no branch of astronomy more replete with interest, and we may add more eagerly pursued at present, inasmuch as the hold which exact calculation gives us on it may be regarded as completely established; so that whatever may be concluded as to the motions of any comet which shall henceforward come to be observed, will be concluded on sure grounds and with numerical precision; while the improvements which have been introduced into the calculation of cometary perturbation, and the daily increasing familiarity of numerous astronomers with computations of this nature, enable us to trace their past and future history with a certainty, which at the commencement of the present century could hardly have been looked upon as attainable. Every comet newly discovered is at once subjected to the ordeal of a most rigorous inquiry. Its elements, roughly calculated within a few days of its appearance, are gradually approximated to as observations accumulate, by a multitude of ardent and expert computists. On the least indication of a deviation from a parabolic orbit, its elliptic elements become a subject of universal and lively interest and discussion. Old records are ransacked, and old observations reduced, with all the advantage of improved data and methods, so as to rescue from oblivion the orbits of ancient comets which present any similarity to that of the new visitor. The disturbances undergone in the interval by the action of the planets are investigated, and the past, thus brought into unbroken connection with the present, is made to afford substantial ground for prediction of the future. A great impulse meanwhile has been given of late years to the discovery of comets by the establishment in 1840,[31] by his late Majesty the King of Denmark, of a prize medal to be awarded for every such discovery, to the first observer (the influence of which may be most unequivocally traced in the great number of these bodies which every successive year sees added to our list), and by the circulation of notices, by special letter,[32] of every such discovery (accompanied, when possible, by an ephemeris), to all observers who have shown that they take an interest in the inquiry, so as to insure the full and complete observation of the new comet so long as it remains within the reach of our telescopes. Among the observers who have been most successful in the discovery of comets, we find no less than 29 discovered by Pons, 14 by Messier, and 10 by Méchain, 8 by De Vico, 8 by Miss C. Herschel—who, however, is not the only female observer of these bodies, the comet of 1847 having been independently detected by *two* ladies, Miss Maria Mitchell, of Nantucket,

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[31] See the announcement of this institution in Astron. Nachr. No. 400.

[32] By the late Prof. Schumacher, Director of the Royal Observatory of Altona.

U. S., and Madame Rümker, of Hamburg, the priority lying with the American astronomess.

(598.) It is by no means merely as a subject of antiquarian interest, or on account of the brilliant spectacle which comets occasionally afford, that astronomers attach a high degree of importance to all that regards them. Apart even from the singularity and mystery which appertains to their physical constitution, they have become, through the medium of exact calculation, unexpected instruments of inquiry into points connected with the planetary system itself, of no small importance. We have seen that the movements of the comet Encke, thus minutely and perseveringly traced by the eminent astronomer whose name is used to distinguish it, has afforded ground for believing in the presence of a resisting medium filling the whole of our system. Similar inquiries, prosecuted in the cases of other periodical comets, will extend, confirm, or modify our conclusions on this head. The perturbations, too, which comets experience in passing near any of the planets, may afford, and have afforded, information as to the magnitude of the disturbing masses, which could not well be otherwise obtained. Thus the approach of this comet to the planet Mercury in 1838 afforded an estimation of the mass of that planet the more precious, by reason of the great uncertainty under which all previous determinations of that element labored. Its approach to the same planet in the year 1848 was still nearer. On the 22d of November their mutual distance was only fifteen times the moon's distance from the earth.

(599.) It is, however, in a physical point of view that these bodies offer the greatest stimulus to our curiosity. There is, beyond question, some profound secret and mystery of nature concerned in the phenomenon of their tails.

Perhaps it is not too much to hope that future observation, borrowing every aid from rational speculation, grounded on the progress of physical science generally (especially those branches of it which relate to the ethereal or imponderable elements), may erelong enable us to penetrate this mystery, and to declare whether it is really *matter* in the ordinary acceptation of the term which is projected from their heads with such extravagant velocity, and if not impelled, at least *directed* in its course by a reference to the sun, as its point of avoidance. In no respect is the question as to the materiality of the tail more forcibly pressed on us for consideration, than in that of the enormous sweep which it makes round the sun in perihelio, in the manner of a straight and rigid rod, in defiance of the law of gravitation, nay, even of the received laws of motion, extending (as we have seen in the comets of 1680 and 1843) from near the sun's surface to the earth's orbit, yet whirled round unbroken: in the latter case through an angle of 180° in little more than two hours. It seems utterly incredible that in such a case it is one and the same material object which is thus brandished. If there could be conceived such a thing as a *negative shadow*, a momentary impression made upon the luminiferous ether behind the comet, this would represent in some degree the conception such a phenomenon irresistibly calls up. But this is not all. Even such an extraordinary excitement of the ether, conceive it as we will, will afford no account of the projection of lateral streamers; of the effusion of light from the nucleus of a comet toward the sun; and its subsequent *re*jection; of the irregular and capricious mode in which that effusion has been seen to take place; none of the clear indications of alternate evaporation and condensation going on in the immense regions of space occupied by the tail and coma—none, in short, of innumerable other facts which link themselves with almost equally irresistible cogency to our ordinary notions of matter and force.

(600.) The great number of comets which appear to move in parabolic orbits, or orbits at least indistinguishable from parabolas during their description of that comparatively small part within the range of their visibility to us, has given rise to an impression that they are bodies extraneous to our system, wandering through space, and merely yielding a local and temporary obedience to its laws during their sojourn. What truth there may be in this view, we may never have satisfactory grounds for deciding. On such a hypothesis, our elliptic comets owe their permanent denizenship within the sphere of the sun's predominant attraction to the action of one or other of the planets near which they may have passed, in such a manner as to diminish their velocity, and render it compatible with elliptic motion.[33] A similar cause acting the other way, might with equal probability give rise to a hyperbolic motion. But whereas in the former case the comet would remain in the system, and might make an indefinite number of revolutions, in the latter it would return no more. This may possibly be the cause of the exceedingly rare occurrence of a hyperbolic comet as compared with elliptic ones.

(601.) All the planets without exception, and almost all the satellites, circulate in one direction. Retrograde comets, however, are of very common occurrence, which certainly would go to assign them an exterior or at least an independent origin. Laplace, from a consideration of all the cometary orbits known in the earlier part of the

[33] The velocity in an ellipse is always less than in a parabola, at equal distances from the sun; in a hyperbola always greater.

present century, concluded, that the mean or average situation of the planes of all the cometary orbits, with respect to the ecliptic, was so nearly that of perpendicularity, as to afford no presumption of any cause biasing their directions in this respect. Yet we think it worth noticing that among the comets which are as yet known to describe elliptic orbits, not one whose inclination is under 17° is retrograde; and that out of thirty-six comets which have had elliptic elements assigned to them, whether of great or small excentricities, and without any limit of inclination, only five are retrograde, and of these, only two, viz. Halley's and the great comet of 1843, can be regarded as satisfactorily made out. Finally, of the 125 comets whose elements are given in the collection of Schumacher and Olbers, up to 1823, the number of retrograde comets under 10° of inclination is only 2 out of 9, and under 20°, 7 out of 23.[34] A plane of motion, therefore, nearly coincident with the ecliptic, and a periodical return, are circumstances eminently favorable to direct revolution in the cometary as they are decisive among the planetary orbits. [Here also we may notice a very curious remark of Mr. Hind (Ast. Nachr. No. 724), respecting periodic comets, viz. that so far as at present known, they divide themselves for the most part into two families—the one having periods of about 75 years, corresponding to a mean distance about that of Uranus; the other corresponding more nearly with those of the asteroids, and with a mean distance between these small planets and Jupiter. The former group consists of four members, Halley's comet revolving in 76 years, one discovered by Olbers in 74, De Vico's 4th comet in

[34] So in edition of 1850. See, however, Appendix, Table IV., for a more recent view of these statistical particulars.

73, and Brorsen's 3d in 75 respectively. Examples of the latter group are to be seen in App. Table IV. at the end of this volume.]

We may add, too, a marked tendency in the major axes of periodical comets to group themselves about a certain determinate direction in space, that is to say, a line pointing to the sphere of the fixed stars northward to 70° long, and 30° N. lat. or nearly toward the star λ Persei (in the Milky Way), and in the southern to a point (also in the Milky Way) diametrically opposite. (Ast. Nachr. No. 853.)

(601 *a.*) The third great comet of the present century (those of 1811 and 1843 being the other two) appeared from June 2, 1858, to January, 1859, being known as Donati's comet, from its first discoverer. Its head was remarkably brilliant; and its tail, like a vast *aigrette* or gracefully-curved plume, extended, when longest, over a space of upward of 30°. Its curvature was very marked, deflecting toward the region quitted by the comet, as if left behind (no *proof*, as generally supposed, of any *resistance* experienced in its motion, but a necessary consequence of the combination of its impulse outward from the sun with the proper velocity of the comet at the moment of its emission). The American observers speak of two long, narrow, perfectly straight rays of faint light, tangents to the limiting curves of the aigrette at its quitting the head. The phenomena of the nucleus under high magnifying powers were very complex and remarkable. In each of the years 1861, 1862, appeared great comets: that of 1861, through, or very near whose tail the earth passed on the 30th of June, was remarkable for the great length and straightness of one side of its tail; that of 1862 for the high condensation of the nucleus and of the single jet issuing from it.

PART II

OF THE LUNAR AND PLANETARY PERTURBATIONS

"Magnus ab integro sæclorum nascitur ordo."—VIRG. *Pollio.*

CHAPTER XII

Subject Propounded—Problem of Three Bodies—Superposition of Small Motions—Estimation of the Disturbing Force—Its Geometrical Representation—Numerical Estimation in Particular Cases—Resolution into Rectangular Components—Radial, Transversal and Orthogonal Disturbing Forces—Normal and Tangential—Their Characteristic Effects—Effects of the Orthogonal Force—Motion of the Nodes—Conditions of their Advance and Recess—Cases of an Exterior Planet Disturbed by an Interior—The Reverse Case—In Every Case the Node of the Disturbed Orbit Recedes on the Plane of the Disturbing on an Average—Combined Effect of Many Such Disturbances—Motion of the Moon's Nodes—Change of Inclination—Conditions of its Increase and Diminution—Average Effect in a Whole Revolution—Compensation in a Complete Revolution of the Nodes—Lagrange's Theorem of the Stability of the Inclinations of the Planetary Orbits—Change of Obliquity of the Ecliptic—Precession of the Equinoxes Explained—Nutation—Principle of Forced Vibrations

(602.) IN the progress of this work, we have more than once called the reader's attention to the existence of inequalities in the lunar and planetary motions not included in the expression of Kepler's laws, but in some sort supplementary to them, and of an order so far subordinate to those leading features of the celestial movements, as to require, for their detection, nicer observations, and longer-continued comparison between facts and theories, than suffice for the establishment and verification of the elliptic theory. These inequalities are known, in physical astronomy, by the name of *perturbations.* They arise, in the case of the primary planets, from the mutual gravitations of these planets toward each other, which derange their elliptic motions round the sun; and in that of the secondaries, partly from the mutual gravitation of the secondaries of the same system similarly deranging their elliptic motions round their common primary, and partly from the unequal attraction of the sun and planets on them and on their primary. These perturbations, although small, and, in most instances, insensible in short intervals of time, yet when accumulated, as some of them may become, in the lapse of ages, alter very greatly the original elliptic relations, so as to render the same elements of the planetary orbits, which at one epoch represented perfectly well their movements, inadequate and unsatisfactory after long intervals of time.

(603.) When Newton first reasoned his way from the broad features of the celestial motions, up to the law of universal gravitation, as affecting all matter, and rendering every particle in the universe subject to the influence of every other, he was not unaware of the modifications which this generalization would induce upon the results of a more partial and limited application of the same law to the revolutions of the planets about the sun, and the satellites about their primaries, as their *only* centres of attraction. So far from it, his extraordinary sagacity enabled him to perceive very distinctly how several of the most important of the lunar inequalities take their origin, in this more general way of conceiving the agency of the attractive power, especially the retrograde motion of the nodes, and the direct revolution of the apsides of her orbit. And if he did not extend his investigations to the mutual perturbations of the planets, it was not for want of perceiving that such perturbations *must* exist, and *might* go the length of producing great derangements from the actual state of the system, but was owing to the then undeveloped state of the practical part of astronomy, which had not yet attained the precision requisite to make such an attempt inviting, or indeed feasible. What Newton left undone, however, his successors have accomplished; and, at this day, it is hardly too much to assert that there is not a single perturbation, great or small, which observation has become precise enough clearly to detect and place in evidence which has not been traced up to its origin in the mutual gravitation of the parts of our system, and minutely accounted for, in its numerical amount and value, by strict calculation on Newton's principles.

(604.) Calculations of this nature require a very high analysis for their successful performance, such as is far beyond the scope and object of this work to attempt exhibiting. The reader who would master them must prepare himself for the undertaking by an extensive course of preparatory study, and must ascend by steps which we must not here even digress to point out. It will be our object, in this chapter, however, to give some general insight into the nature and manner of operation of the acting forces, and to point out what are the circumstances which, in some cases, give them a high degree of efficiency—a sort of *purchase* on the balance of the system; while, in others, with no less amount of intensity, their effective agency in producing extensive and lasting changes is compensated or rendered abortive; as well as to explain the nature of those admirable results respecting the stability of our system, to which the researches of geometers have conducted them; and which, under the form of mathematical theorems of great simplicity and elegance, involve the history of the past and future state of the planetary orbits during ages of which, contemplating the subject in this point of view, we neither perceive the beginning nor the end.

(605.) Were there no other bodies in the universe but the sun and one planet, the latter would describe an exact ellipse about the former (or both round their common centre of gravity), and continue to perform its revolutions in one and the same orbit forever; but the moment we add to our combination a third body, the attraction of this will draw both the former bodies out of their mutual orbits, and, by acting on them unequally, will disturb their relation to each other, and put an end to the rigorous and mathematical exactness of their elliptic motions, not only about a fixed point in space, but about one another. From this way of propounding the subject, we see that it is not the whole attraction of the newly introduced body which produces perturbation, but *the difference* of its attractions on the two originally present.

(606.) Compared to the sun, all the planets are of extreme minuteness; the mass of Jupiter, the greatest of them all, being not more than about one 1100th part that of the sun. Their attractions on each other, therefore, are all very feeble, compared with the presiding central power, and the effects of their disturbing forces are proportionally minute. In the case of the secondaries, the chief agent by which their motions are deranged is the sun itself, whose mass is indeed great, but whose disturbing influence is immensely diminished by their near proximity to their primaries, compared to their distances from the sun, which renders the *difference* of attractions on both extremely small, compared to the whole amount. In this case the greatest part of the sun's attraction, viz. that which is common to both, is exerted to retain both primary and secondary in their common orbit about itself, and prevent their parting company. Only the small overplus of force on one as compared with the other acts as a disturbing power. The mean value of this overplus, in the case of the moon disturbed by the sun, is calculated by Newton to amount to no higher a fraction than $\frac{1}{638000}$ of gravity at the earth's surface, or $\frac{1}{179}$ of the principal force which retains the moon in its orbit.

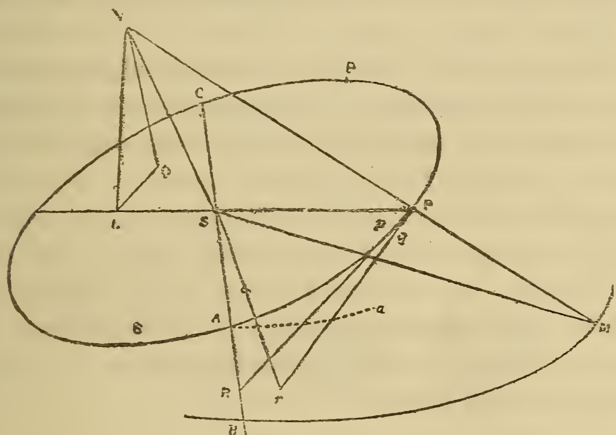
(607.) From this extreme minuteness of the intensities of the disturbing, compared to the principal forces, and the consequent smallness of their *momentary* effects, it happens that we can estimate each of these effects separately, as if the others did not take place, without fear of inducing error in our conclusions beyond the limits necessarily incident to a first approximation. It is a principle in mechanics, immediately flowing from the primary relations between forces and the motions they produce, that when a number of very minute forces act at once on a system, their joint effect is the sum or aggregate of their separate effects, at least within such limits, that the original relation of the parts of the system shall not have been materially changed by their action. Such effects supervening on the greater movements due to the action of the primary forces may be compared to the small ripplings caused by a thousand varying breezes on the broad and regular swell of a deep and rolling ocean, which run on as if the surface were a plane, and cross in all directions without interfering, each as if the other had no existence. It is only when their effects become accumulated in lapse of time, so as to alter the primary relations or data of the system, that it becomes necessary to have especial regard to the changes correspondingly introduced into the estimation of their momentary efficiency, by which the *rate* of the subsequent changes is affected, and periods or cycles of immense length take their origin. From this consideration arise some of the most curious theories ot physical astronomy.

(608.) Hence it is evident, that in estimating the disturbing influence of several bodies forming a system, in which one has a remarkable preponderance over all the rest, we need not embarrass ourselves with combinations of the disturbing powers one among another, unless where immensely long periods are concerned; such as consist of many hundreds of revolutions of the bodies in question about their common centre. So that, in effect, so far as we propose to go into its consideration, the problem of the investigation of the perturbations of a system, however numerous, constituted as ours is, reduces itself to that of a system of three bodies: a predominant central body, a disturbing, and a disturbed; the two latter of which may exchange denominations, according as the motions of the one or the other are the subject of inquiry.

(609.) Both the intensity and direction of the disturbing force are continually varying, according to the relative situation of the disturbing and disturbed body with respect to the sun. If the attraction of the disturbing body M, on the central body S, and the disturbed body P (by which designations, for brevity, we shall hereafter indicate them), were equal, and acted in parallel lines, whatever might otherwise be its law of variation, there would be no deviation caused in the elliptic motion of P about S, or of each about the other. The case would be strictly that of art. 454; the attraction of M, so circumstanced, being at every moment exactly analogous in its effects to terrestrial gravity, which acts in parallel lines, and is equally *intense* on all bodies, great and small. But this is not the case of nature. Whatever is stated in the subsequent article to that last cited, of the disturbing effect of the sun and moon, is, *mutatis mutandis*, applicable to every case of perturbation; and it must be now our business to enter, somewhat more in detail, into the general heads of the subject there merely hinted at.

(610.) To obtain clear ideas of the manner in which the disturbing force produces its various effects, we must ascertain at any given moment, and in any relative situations of the three bodies, its direction and intensity as compared with the gravitation of P toward S, in virtue of which latter force alone P would describe an ellipse about S regarded as fixed, or rather P and S about their common centre of gravity in virtue of their mutual gravitation to each other. In the treatment of the problem of three bodies, it is convenient, and tends to clearness of apprehension, to regard one of them as fixed, and refer the motions of the others to it as to a relative centre. In the case of two planets disturbing each other's motions, the sun is naturally chosen as this fixed centre; but in that of satellites disturbing each other, or disturbed by the sun, the centre of their primary is taken as their point of reference, and the sun itself is regarded in the light of a very distant and massive satellite revolving about the primary in a *relative* orbit, equal and similar to that which the primary describes *absolutely* round the sun. Thus the generality of our language is preserved, and when, referring to any particular central body, we speak of an exterior and an interior planet, we include the cases in which the former is the sun and the latter a satellite; as, for example, in the Lunar theory. It is a principle in dynamics, that the relative motions of a system of bodies *inter se* are no way altered by impressing on all of them a common motion or motions, or a common force or forces accelerating or retarding them all equally in common directions, *i.e.* in parallel lines. Suppose, therefore, we apply to all the three bodies, S, P, and M, alike, forces equal to those with which M and P attract S, but in opposite directions. Then will the relative motions both of M and P about S be unaltered; but S, being now urged by equal and opposite forces to and from both M and P, will remain at rest. Let us now consider how either of the other bodies, as P, stands affected by these newly introduced forces, in addition to those which before acted on it. It is clear that now P will be simultaneously acted on by *four* forces; first, the attraction *of* S in the direction P S; secondly, an additional force, in the same direction, equal to its attraction *on* S; thirdly, the attraction *of* M in the direction P M; and fourthly, a force parallel to M S, and equal to M's attraction *on* S. Of these, the first two following the same law of the inverse square of the distance S P, may be regarded as one force, precisely as if the sum of the masses of S and P were collected in S; and in virtue of their joint action, P will describe an ellipse about S, except in so far as that elliptic motion is disturbed by the other two forces. Thus we see that in this view of the subject the *relative* disturbing force acting on P is no longer the mere single attraction of M, but a force resulting from the composition of that attraction with M's attraction on S transferred to P in a contrary direction.

(611.) Let C P A be part of the relative orbit of the disturbed, and M B of the disturbing body, their planes intersecting in the line of nodes S A B, and having to each other the inclination expressed by the spherical angle P A a. In M P, produced if required, take M N : M S :: M S^2 : M P^2. Then, if S M[1] be taken to represent, in quantity and direction, the accelerative attraction of M on S, M S will represent in quantity and direction the new force applied to P, parallel to that line, and N M will represent on the same scale the accelerative attraction of M on P. Consequently, the disturbing force acting on P will be the



resultant of two forces applied at P, represented respectively by N M and M S, which by the laws of dynamics are equivalent to a single force represented in *quantity* and *direction* by N S, *but having* P *for its point of application.*

(612.) The *line* N S is easily calculated by trigonometry, when the relative situations and real distances of the bodies are known; and the *force* expressed by that line is directly comparable with the attractive forces of S on P by

[1] The reader will be careful to observe the order of the letters, where forces are represented by lines. M S represents a force acting from M toward S, S M from S toward M.

the following proportions, in which M, S, represent the masses of those bodies which are supposed to be known, and to which, at equal distances, their attractions are proportional:

Disturbing force : M's attraction on S :: N S : S M;

M's attraction on S : S's attraction on M :: M : S;

S's attraction on M : S's attraction on P :: S P² : S M²;

by compounding which proportions we collect as follows:

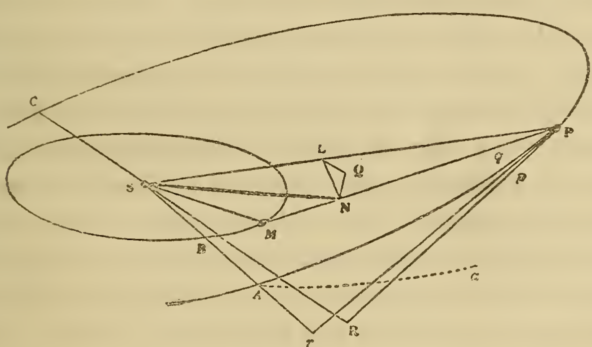
Disturbing force : S's attraction on P :: M . N S . S P² : S . S M³.

A few numerical examples are subjoined, exhibiting the results of this calculation in particular cases, chosen so as to exemplify its application under very various circumstances, throughout the planetary system. In each case the numbers set down express the proportion in which the central force retaining the disturbed body in its elliptic orbit exceeds the disturbing force, to the nearest whole number. The calculation is made for three positions of the disturbing body, viz. at its greatest, its least, and its mean distance from the disturbed.

Disturbing Body.	Disturbed Body.	Ratio at the greatest Distance : 1.	Ratio at the mean Distance : 1.	Ratio at the least Distance : 1.
The Sun	The Moon	90	179	89
Jupiter	Saturn	354	312	128
Jupiter	The Earth	95683	147575	53268
Venus	The Earth	255208	210245	26833
Neptune	Uranus	57420	56592	5519
Mercury	Neptune	845	845	845
Jupiter	Ceres	6433	6937	1033
Saturn	Jupiter	20248	21579	3065

(613.) If the orbit of the disturbing body be circular, S M is invariable. In this case, N S will continue to represent the disturbing force *on the same invariable scale*, whatever may be the configuration of the three bodies with respect to each other. If the orbit of M be but little elliptic, the same will be nearly the case. In what follows throughout this chapter, except where the contrary is expressly mentioned, we shall neglect the excentricity of the disturbing orbit.

(614.) If P be nearer to M than S is, M N is greater than M P, and N lies in M P prolonged, and therefore on the opposite side of the plane of P's orbit from that on which M is situated. The force N S therefore urges P toward M's plane, and toward a point X, situated between S and M, in the line S M. If the distance M P be equal to M S as when P is situated, suppose at D or E, M N is also equal to M P



or M S, so that N coincides with P, and therefore X with S, the disturbing forces being in these cases directed *toward* the central body. But if M P be greater than M S, M N is less than M P, and N lies between M and P, or on the same side of the plane of P's orbit that M is situated on. The force N S, therefore, applied at P, urges P toward the contrary side of that plane toward a point in the line M S produced, so that X now shifts to the further side of S. In all cases, the disturbing force is wholly effective in the plane M P S, in which the three bodies lie.

It is very important for the student to fix distinctly and bear constantly in his mind these relations of the disturbing agency considered as a *single unresolved force*, since their recollection will preserve him from any mistakes in conceiving the mutual actions of the planets, etc., on each other. For example, in the figures here referred to, that of art. 611, corresponds to the case of a nearer disturbed by a more distant body, as the earth by Jupiter, or the moon by the Sun; and that of the present article to the converse case: as, for instance, of Mars disturbed by the earth. Now, in this latter class of cases, whenever M P is greater than M S, or S P greater than 2 S M, N lies on the same side of the plane of P's orbit with M, so that N S, the disturbing force, contrary to what might at first be supposed, always urges the disturbed planet out of the plane of its orbit toward the opposite side to that on which the disturbing planet lies. It will tend greatly to give clearness and definiteness to his ideas on the subject, if he will trace out on various suppositions as to the relative magnitude of the disturbing and disturbed orbits (supposed to lie in one plane) the form of the oval about M considered as a fixed point, in which the point N lies when P makes a complete revolution round S.

(615.) Although it is necessary for obtaining in the first instance a clear conception of the action of the disturbing force, to consider it in this way as a single force having a definite direction in space and a determinate intensity, yet as that direction is continually varying with the position of N S, both with respect to the radii S P, S M, the distance P M, and the direction of P's motion, it would be impossible, by so considering it, to attain clear views of its dynamical effect after any considerable lapse of time, and it therefore becomes necessary to resolve it into other equivalent forces acting in such directions as shall admit of distinct and separate consideration. Now this may be done in several different modes. First, we may resolve it into three forces acting in fixed directions in space rectangular to one another, and by estimating its effect in each of these three directions separately, conclude the total or joint effect. This is the mode of procedure which affords the readiest and most advantageous handle to the problem of perturbations when taken up in all its generality, and is accordingly that resorted to by geometers of the modern school in all their profound researches on the subject. Another mode consists in resolving it also into three rectangular components, not, however, in fixed directions, but in variable ones, viz. in the directions of the lines N Q, Q L, and L S, of which L S is in the direction of the radius vector S P, Q L in a direction perpendicular to it, and in the plane in which S P and a tangent to P's orbit at P both lie; and lastly, N Q in a direction perpendicular to the plane in which P is at the instant moving about S. The first of these resolved portions we may term the *radial* component of the disturbing force, or simply the radial disturbing force; the second the *transversal*; and the third the *orthogonal*.[2] When the disturbed orbit is one of small excentricity, the transversal component acts nearly in the direction of the tangent to P's orbit at P, and is therefore confounded with that resolved component which we shall presently describe (art. 618) under the name of the *tangential* force. This is the mode

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[2] This is a term coined for the occasion. The want of some appellation for this component of the disturbing force is often felt.

of resolving the disturbing force followed by Newton and his immediate successors.

(616.) The immediate actions of these components of the disturbing force are evidently independent of each other, being rectangular in their directions; and they affect the movement of the disturbed body in modes perfectly distinct and characteristic. Thus, the radial component, being directed to or from the central body, has no tendency to disturb either the plane of P's orbit, or the equable description of areas by P about S, since the law of areas proportional to the times is not a character of the force of gravity only, but holds good equally, whatever be the force which retains a body in an orbit, *provided only* its direction is always toward a fixed centre.[3] Inasmuch, however, as its law of variation is not conformable to the simple law of gravity, it alters the elliptic form of P's orbit, by directly affecting both its curvature and velocity at every point. In virtue, therefore, of the action of this disturbing force, the orbit deviates from the elliptic form by the approach or recess of P to or from S, so that the effect of the perturbations produced by this part of the disturbing force falls wholly on the radius vector of the disturbed orbit.

(617.) The transversal disturbing force represented by Q L, on the other hand, has no direct action to draw P to or from S. Its whole efficiency is directed to accelerate or retard P's motion in a direction at right angles to S P. Now the area momentarily described by P about S, is, *cœteris paribus*, directly as the velocity of P in a direction perpendicular to S P. Whatever force, therefore, increases this transverse velocity of P, accelerates the description of

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[3] Newton, i. 1.

areas, and *vice versâ*. With the area A S P is directly connected, by the nature of the ellipse, the angle A S P described or to be described by P from a fixed line in the plane of the orbit, so that any change in the rate of description of areas ultimately resolves itself into a change in the amount of angular motion about S, and gives rise to a departure from the elliptic laws. Hence arise what are called in the perturbational theory equations (*i.e.* changes or fluctuations to and fro about an average quantity) of the mean motion of the disturbed body.

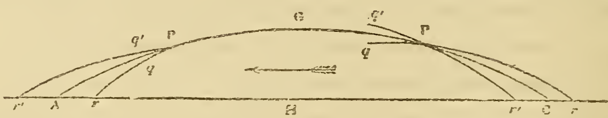
(618.) There is yet another mode of resolving the disturbing force into rectangular components, which, though not so well adapted to the computation of results, in reducing to numerical calculation the motions of the disturbed body, is fitted to afford a clearer insight into the nature of the modifications which the form, magnitude, and situation of its orbit undergo in virtue of its action, and which we shall therefore employ in preference. It consists in estimating the components of the disturbing force, which lie in the plane of the orbit, not in the direction we have termed radial and transversal, *i.e.* in that of the radius vector P S and perpendicular to it, but in the direction of a tangent to the orbit at P, and in that of a normal to the curve, and at right angles to the tangent, for which reason these components may be called the *tangential* and *normal* disturbing forces. When the orbit of the disturbed body is circular, or nearly so, this mode of resolution coincides with or differs but little from the former, but when the ellipticity is considerable, these directions may deviate from the radial and transversal directions to any extent. As, in the Newtonian mode of resolution, the effect of the one component falls wholly upon the approach and recess of the body P to the central body S, and of the other wholly on the rate of description of areas by P round S, so in this which we are now considering, the direct effect of the one component (the normal) falls wholly on the curvature of the orbit at the point of its action, increasing that curvature when the normal force acts inward, or toward the concavity of the orbit, and diminishing it when in the opposite direction; while, on the other hand, the tangential component is directly effective on the velocity of the disturbed body, increasing or diminishing it according as its direction conspires with or opposes its motion. It is evident enough that where the object is to trace simply the changes produced by the disturbing force, in *angle* and *distance* from the central body, the former mode of resolution must have the advantage in perspicuity of view and applicability to calculation. It is less obvious, but will abundantly appear in the sequel that the latter offers peculiar advantages in exhibiting to the eye and the reason the momentary influence of the disturbing force on the *elements* of the orbit itself.

(619.) Neither of the last-mentioned pairs of resolved portions of the disturbing force tends to draw P out of the plane of its orbit P S A. But the remaining or orthogonal portion N Q acts directly and solely to produce that effect. In consequence, under the influence of this force, P must quit that plane, and (the same cause continuing in action) must describe a *curve of double curvature* as it is called, no two consecutive portions of which lie in the same plane passing through S. The effect of this is to produce a continual variation in those elements of the orbit of P on which *the situation of its plane* in space depends, *i.e.* on its inclination to a fixed plane, and the position in such a plane of the node or line of its intersection therewith. As this, among all the various effects of perturbation, is that which is at once the most simple in its conception, and the easiest to follow into its remoter consequences, we shall begin with its explanation.

(620.) Suppose that up to P (arts. 611, 614) the body were describing an undisturbed orbit C P. Then at P it would be moving in the direction of a tangent P R to the ellipse P A, which prolonged will intersect the plane of M's orbit somewhere in the line of nodes, as at R. Now, at P, let the disturbing force parallel to N Q act momentarily on P; then P will be deflected in the direction of that force, and instead of the arc P p, which it would have described in the next instant if undisturbed, will describe the arc P q lying in the state of things represented in art. 611 below, and in art. 614 above, P p with reference to the plane P S A. Thus, by this action of the disturbing force, the plane of P's orbit will have shifted its position in space from P S p (an elementary portion of the old orbit) to P S q, one of the new. Now the line of nodes S A B in the former is determined by prolonging P p into the tangent P R, intersecting the plane M S B in R, and joining S R. And in like manner, if we prolong P q into the tangent P r, meeting the same plane in r, and join S r, this will be the new line of nodes. Thus we see that, under the circumstances expressed in the former figure, the momentary action of the orthogonal disturbing force will have caused the line of nodes to *retrograde* upon the plane of the orbit of the disturbing body, and under those represented in the latter to advance. And it is evident that the action of the other resolved portions of the disturbing force will not in the least interfere with this result, for neither of them tends either to carry P out of its former plane of motion, or to prevent its quitting it. Their influence would merely go to transfer the points of intersection of the tangents P p or P q from R or r to R′ or r', points nearer to or further from S than R r, but in the same lines.

(621.) Supposing, now, M to lie to the left instead of the right side of the line of nodes in *fig.* 1, P retaining its situation, and M P being less than M S, so that X shall still lie between M and S. In this situation of things (or *configuration*, as it is termed of the three bodies with respect to each other), N will lie *below* the plane A S P, and the disturbing force will tend to raise the body P above the plane, the resolved orthogonal portion N Q in this case acting upward. The disturbed arc P q will therefore lie above P p, and when prolonged to meet the plane M S B, will intersect it in a point *in advance* of R; so that in this configuration the node will advance upon the plane of the orbit of M, provided always that the latter orbit remains fixed, or, at least, does not itself shift its position in such a direction as to defeat this result.

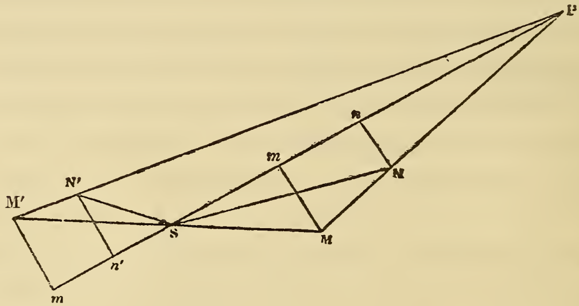
(622.) Generally speaking, the node of the disturbed orbit will recede *upon any plane which we may consider as fixed*, whenever the action of the orthogonal disturbing force tends to bring the disturbed body nearer to that plane; and *vice*



versâ. This will be evident on a mere inspection of the annexed figure, in which C A represents a semicircle of the projection of the fixed plane as seen from S on the sphere of the heavens, and C P A that of the plane of P's undisturbed orbit, the motion of P being in the direction of the arrow, from C the ascending, to A the descending node. It is at once seen, by prolonging P q, P q' into arcs of great circles, P r, P r' (forward or backward, as the case may be) to meet C A, that the node will have retrograded through the arc A r, or C r, whenever P q lies between C P A and C A, or when the perturbing force carries P toward the fixed plane, but will have advanced through A r' or C r' when P q' lies above C P A, or when the disturbing impulse has lifted P above its old orbit or away from the fixed plane, *and this without any reference to whether the undisturbed orbitual motion of* P *at the moment is carrying it toward the plane* C A *or from it*, as in the two cases represented in the figure.

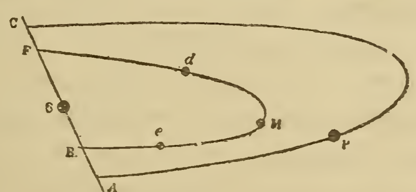
(623.) Let us now consider the mutual disturbance of two bodies M and P, in the various configurations in which they may be presented to each other and to their common central body. And first, let us take the case, as the simplest, where the disturbed orbit is exterior to that of the disturbing body (as in *fig.* art. 614), and the distance between the orbits greater than the semiaxis of the smaller. First, let both planets lie on the same side of the line of nodes. Then (as in art. 620) the direction of the whole disturbing force, and therefore also that of its orthogonal component, will be toward the opposite side of the plane of P's orbit from that on which M lies. Its effect therefore will be to draw P out of its plane in a direction *from* the plane of M's orbit, so that in this state of things the node will advance on the latter plane, however P and M may be situated in these semicircumferences of their respective orbits. Suppose, next, M transferred to the opposite side of the line of nodes, then will the direction of its action on P, with respect to the plane of P's orbit, be reversed, and P in quitting that plane will now approach to instead of receding from the plane of M's orbit, so that its node will now recede on that plane.

(624.) Thus, while M and P revolve about S, and in the course of many revolutions of each are presented to each other and to S in all possible configurations, the node of P's orbit will always advance on M's when both bodies are on the same side of the line of nodes, and recede when on the opposite. They will therefore, on an average, advance and recede during equal times (supposing the orbits nearly circular). And, therefore, if their advance were at each instant of its duration equally rapid with their recess at each corresponding instant during that phase of the movement, they would merely oscillate to and fro about a mean posi-



tion, without any permanent motion in either direction. But this is not the case. The rapidity of their recess in every position favorable to recess is greater than that of their advance in the corresponding opposite position. To show this, let us consider any two configurations in which M's phases are diametrically opposite, so that the triangles P S M, P S M′, shall lie in one plane, having any inclination to P's orbit, according to the situation of P. Produce P S, and draw M m M′ m' perpendicular to it, which will therefore be equal. Take M N : M S : : M S² : M P², and M′ N′ : M′ S : : M′ S² : M′ P² : then, if the orbits be nearly circles, and therefore M S = M′ S, N′ M′ will be less than M N; and therefore (since P M′ is greater than P M) P N′ : P M′ in a greater ratio than P N : P M; and consequently, by similar triangles, drawing N n, N′ n' perpendicular to P S, N′ n' : M′ m' in a greater ratio than N n : M m, and therefore N′ n' is greater than N n. Now the plane P M M′ intersects P's orbit in P S, and being inclined to that orbit at the same angle through its whole extent, if from N and N′ perpendiculars be conceived let fall on that orbit, these will be to each other in the proportion of N n, N′ n'; and therefore the perpendicular from N′ will be greater than that from N. Now since by art. 611 N′ S and N S represent in quantity and direction the total disturbing forces of M′ and M on P respectively, therefore these perpendiculars express (art. 615) the orthogonal disturbing forces, the former of which tends (as above shown) to make the nodes recede, and the latter to advance; and therefore the preponderance in every such pair of situations of M is in favor of a retrograde motion.

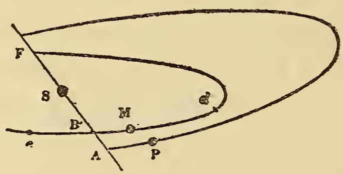
(625.) Let us next consider the case where the distance between the orbits is less than the semiaxis of the interior, or in which the least distance of M from P is less than M S. Take any situation of P with respect to the line of nodes



A C. Then two points d and e, distant by less than 120°, can be taken on the orbit of M equidistant from P with S. Suppose M to occupy successively every possible situation in its orbit, P retaining its place;—then, if it were not for the existence of the arc d e, in which the relations of art. 624

are reversed, it would appear by the reasoning of that article that the motion of the node is direct when M occupies any part of the semiorbit F M B, and retrograde when it is in the opposite, but that the retrograde motion on the whole would predominate. Much more then will it predominate when there exists an arc *d* M *e* within which if M be placed, its action will produce a retrograde instead of a direct motion.

(626.) This supposes that the arc *d e* lies wholly in the semicircle F *d* B. But suppose it to lie, as in the annexed figure, partly within and partly without that circle. The *greater* part *d* B necessarily lies within it, and not only so, but within that portion, the point of M's orbit nearest to P, in which, therefore, the retrograding force has its maximum,



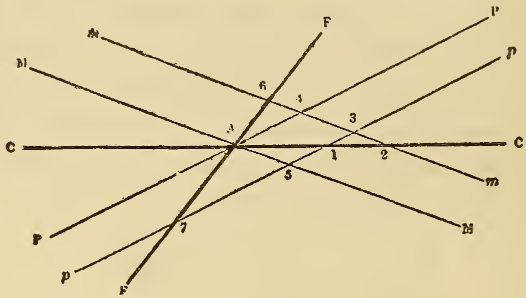
is situated. Although, therefore, in the portion B *e*, it is true, the retrograde tendency otherwise general over the whole of *that* semicircle (art. 624) will be reversed, yet the effect of this will be much more than counterbalanced by the more energetic and more prolonged retrograde action over *d* B; and, therefore, in this case also, on the average of every possible situation of M, the motion of the node will be retrograde.

(627.) Let us lastly consider an interior planet disturbed by an exterior. Take M D and M E (*fig.* of art. 611), each equal to M S. Then first, when P is between D and the node A, being nearer than S to M, the disturbing force acts toward M's orbit on the side on which M lies, and the node recedes. It also recedes when (M retaining the same situation) P is in any part of the arc E C from E to the other node, because in that situation the direction of the disturbing force, it is true, is reversed, but that portion of P's orbit being also reversely situated with respect to the plane of M's, P is still urged toward the latter plane, but on the side opposite to M. Thus (M holding its place), whenever P is anywhere in D A or E C, the node recedes. On the other hand, it advances whenever P is between A and E or between C and D, because, in these arcs, only one of the two determining elements (viz. the direction of the disturbing force with respect to the plane of P's orbit; and the situation of the one plane with respect to the other as to above and below) has undergone reversal. Now first, whenever M is anywhere but in the line of nodes, the sum of the arcs D A and E C exceeds a semicircle, and *that* the more, the nearer M is to a position at right angles to the line of nodes. Secondly, the arcs favorable to the recess of the node comprehend those situations in which the orthogonal disturbing force is most powerful, and *vice versâ*. This is evident, because as P approaches D or E, this component decreases, and vanishes at those points (612). The movement of the node itself also vanishes when P comes to the node, for although in this position the disturbing orthogonal force neither vanishes nor changes its direction, yet, since at the instant of P's passing the node (A) the recess of the node is changed into an advance, it must necessarily at that point be stationary.[4] Owing to both these causes, therefore (that

[4] It would seem, at first sight, as if a change *per saltum* took place here, but the continuity of the node's motion will be apparent from an inspection of the annexed figure, where $b\ a\ d$ is a portion of P's disturbed path near the

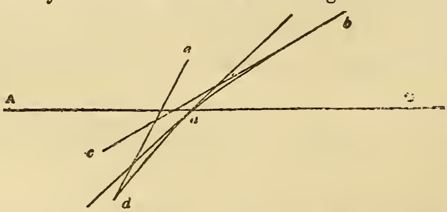
the node recedes during a longer time than it advances, and that a more energetic force acting in its recess causes it to recede more rapidly), the retrograde motion will preponderate on the whole in each complete synodic revolution of P. And it is evident that the reasoning of this and the foregoing articles is no way vitiated by a moderate amount of excentricity in either orbit.

(628.) It is therefore a general proposition, that on the average of each complete synodic revolution, the node of



every disturbed planet recedes upon the orbit of the disturbing one, or, in other words, that in every pair of orbits, the node of each recedes upon the other, and of course upon any intermediate plane which we may regard as fixed. On a plane not intermediate between them, however, the node of one orbit will advance, and that of the other will recede.

node A, concave toward the plane G A. The momentary place of the moving node is determined by the intersection of the tangent *b e* with A G, which as



b passes through *a* to *d*, recedes from A to *a*, rests there for an instant, and then advances again.

Suppose, for instance, C A C to be a plane intermediate between P P and M M the two orbits. If *p p* and *m m* be the new positions of the orbits, the node of P on M will have receded from A to 5, that of M on P from A to 4, that of P and M on C C respectively from A to 1 and from A to 2. But if F A F be a plane not intermediate, the node of M on that plane has receded from A to 6, but that of P will have *advanced* from A to 7. If the fixed plane have not a common intersection with those of both orbits, it is equally easy to see that the node of the disturbed orbit may either recede on both that plane and the disturbing orbit or advance on the one and recede on the other, according to the relative situation of the planes.

(629.) This is the case with the planetary orbits. They do not all intersect each other in a common node. Although perfectly true, therefore, that the node of any one planet would recede on the orbit of any and each other by the individual action of that other, yet, when all act together, recess on one plane may be equivalent to advance on another, so that the motion of the node of any one orbit on a given plane, arising from their joint action, taking into account the different situations of all the planes, becomes a curiously complicated phenomenon whose law cannot be very easily expressed in words, though reducible to strict numerical statement, being, in fact, a mere geometrical result of what is above shown.

(630.) The nodes of all the planetary orbits on the *true* ecliptic, as a matter of fact, are retrograde, though they are not all so on a fixed plane, such as we may conceive to exist in the planetary system, and to be a plane of reference unaffected by their mutual disturbances. It is, however, to the ecliptic, that we are under the necessity of referring their movements from our station in the system; and if we would transfer our ideas to a fixed plane, it becomes necessary to take account of the variation of the ecliptic itself, produced by the joint action of all the planets.

(631.) Owing to the smallness of the masses of the planets and their great distances from each other, the revolutions of their nodes are excessively slow, being in every case less than a single degree per century, and in most cases not amounting to half that quantity. It is otherwise with the moon, and that owing to two distinct reasons. First, that the disturbing force itself arising from the sun's action (as appears from the table given in art. 612) bears a much larger proportion to the earth's central attraction on the moon than in the case of any planet disturbed by any other. And secondly, because the synodic revolution of the moon, within which the average is struck (and always on the side of recess), is only 29¹ days, a period much shorter than that of any of the planets, and vastly so than that of several among them. All this is agreeable to what has already been stated (arts. 407, 408) respecting the motion of the moon's nodes, and it is hardly necessary to mention that, when calculated, as it has been, *à priori*, from an exact estimation of all the acting forces, the result is found to coincide with perfect precision with that immediately derived from observation, so that not a doubt can subsist as to this being the real process by which so remarkable an effect is produced.

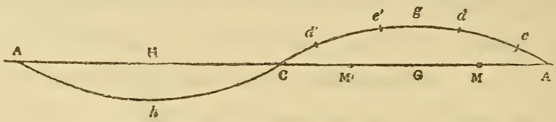
(632.) So far as the physical condition of each planet is concerned, it is evident that the position of their nodes can be of little importance. It is otherwise with the mutual inclinations of their orbits with respect to each other, and to the equator of each. A variation in the position of the ecliptic, for instance, by which its pole should shift its distance from the pole of the equator, would disturb our seasons. Should the plane of the earth's orbit, for instance, ever be so changed as to bring the ecliptic to coincide with the equator, we should have perpetual spring over all the world; and, on the other hand, should it coincide with a meridian, the extremes of summer and winter would become intolerable. The inquiry, then, of the variations of inclination of the planetary orbits *inter se*, is one of much higher practical interest than those of their nodes.

(633.) Referring to the figures of art. 610 *et seq.*, it is evident that the plane S P q, in which the disturbed body moves during an instant of time from its quitting P, is differently inclined to the orbit of M, or to a fixed plane, from the original or undisturbed plane P S p. The difference of absolute position of these two planes in space is the angle made between the planes P S R and P S r, and is therefore calculable by spherical trigonometry, when the angle R S r or the momentary recess of the node is known, and also the inclination of the planes of the orbits to each other. We perceive, then, that between the momentary change of inclination and the momentary recess of the node there exists an intimate relation, and that the research of the one is in fact bound up in that of the other. This may be, perhaps, made clearer, by considering the orbit of P to be not merely an imaginary line, but an actual circle or elliptic hoop of some rigid material, without inertia, on which, as on a wire, the body P may slide as a bead. It is evident that the position of this hoop will be determined at any instant, by its inclination to the ground plane to which it is referred, and by the place of its intersection therewith, or node. It will also be determined by the momentary direction of P's motion, which (having no inertia) it must obey; and any change by which P should, in the next instant, alter its orbit, would be equivalent to a shifting, bodily, of the whole hoop, changing at once its inclination and nodes.

(634.) One immediate conclusion from what has been pointed out above, is that where the orbits, as in the case of the planetary system and the moon, are slightly inclined to one another, the momentary variations of the inclination are of an order much inferior in magnitude to those in the place of the node. This is evident on a mere inspection of our figure, the angle R P r being, by reason of the small inclination of the planes S P R and R S r, necessarily much smaller than the angle R S r. In proportion as the planes of the orbits are brought to coincidence, a very trifling angular movement of P p about P S as an axis will make a great variation in the situation of the point r, where its prolongation intersects the ground plane.

(635.) Referring to the figure of art. 622, we perceive that although the motion of the node is retrograde whenever the momentary disturbed arc P Q lies between the planes C A and C G A of the two orbits, and *vice versâ*, indifferently whether P be in the act of receding from the plane C A, as in the quadrant C G, or of approaching to it, as in G A, yet the same identity as to the character of the change does not subsist in respect of the inclination. The inclination of the disturbed orbit (*i.e.* of its momentary element) P q or P q', is measured by the spherical angle P r H or P r' H. Now in the quadrant C G, P r H is less, and P r' H greater than P C H; but in G A, the converse. Hence this rule: 1st, If the disturbing force urge P toward the plane of M's orbit, and the undisturbed motion of P carry it also toward that plane; and 2dly, if the disturbing force urge P from that plane, while P's undisturbed motion also carries it from it, in either case the inclination momentarily increases; but if, 3dly, the disturbing force act to, and P's motion carry it from—or if the force act from, and the motion carry it to, that plane, the inclination momentarily diminishes. Or (including all the cases under one alternative) if the action of the disturbing force and the undisturbed motion of P with reference to the plane of M's orbit be of the same character, the inclination increases; if of contrary characters, it diminishes.

(636.) To pass from the momentary changes which take place in the relations of nature to the accumulated effects produced in considerable lapses of time by the continued action of the same causes, under circumstances varied by these very effects, is the business of the integral calculus. Without going into any calculations, however, it will be easy for us to demonstrate, from the principles above laid down, the leading features of this part of the planetary theory, viz. the periodic nature of the change of the inclinations of two orbits to each other, the re-establishment of their original values, and the consequent oscillation of each plane about a certain mean position. As in explaining the motion of the nodes, we will commence, as the simplest case, with that of an exterior planet disturbed by an interior one at less than half its distance from the central body. Let A C A′ be the great circle of the heavens into which M's orbit seen from S is projected, extended into a straight line, and A g C h A′ the corresponding projection of the orbit of P so seen. Let M occupy some fixed situation, suppose in the semicircle A C, and let P describe a complete revolution from A through g C h to A′. Then while it is between A and g or in its first quadrant, its motion is *from* the plane of M's orbit, and at the same time the orthogonal force acts *from* that plane: the inclination, therefore (art. 635) increases. In the second quadrant the motion of P is *to*, but the force continues to act from, the plane, and the inclination again decreases. A similar alternation takes place in its course through the quadrants C *h* and *h* A. Thus the plane of P's orbit oscillates to and fro about its mean position twice in each revolution of P. During this process if M held a fixed position at G, the forces being symmetrically alike on either side, the extent of these oscillations would be exactly equal, and the inclination at the end of one revolution of P would revert precisely to its original value. But if M be elsewhere, this will not be the case, and in a single revolution of P, only a partial



compensation will be operated, and an overplus on the side, suppose of diminution, will remain outstanding. But when M comes to M′, a point equidistant from G on the other side, this effect will be precisely reversed (supposing the orbits circular). On the average of both situations, therefore, the effect will be the same as if M were divided into two equal portions, one placed at M and the other at M′, which will annihilate the preponderance in question and effect a perfect restoration. And on an average of all possible situations of M, the effect will in like manner be the same as if its mass were distributed over the whole circumference of its orbit, forming a ring, each portion of which will exactly destroy the effect of that similarly situated on the opposite side of the line of nodes.

(637.) The reasoning is precisely similar for the more complicated cases of arts. 625 and 627. Suppose that owing either to the proximity of the two orbits (in the case of an exterior disturbed planet) or to the disturbed orbit being interior to the disturbing one, there were a larger or less portion, *d e*, of P's orbit in which these relations were reversed. Let M be the position of M′ corresponding to *d e*, then taking G M′ = G M, there will be a similar portion *d′ e′* bearing precisely the same reversed relation to M′, and therefore, the actions of M′ M will equally neutralize each other in this as in the former state of things.

(638.) To operate a complete and rigorous compensation, however, it is necessary that M should be presented to P in every possible configuration, not only with respect to P itself, but to the line of nodes, to the position of which line the whole reasoning bears reference. In the case of the moon, for example, the disturbed body (the moon) revolves in $27^d \cdot 322$, the disturbing (the sun) in $365^d \cdot 256$, and the line of nodes in $6793^d \cdot 391$, numbers in proportion to each other about as 1 to 13 and 249 respectively. Now in 13 revolutions of P, and one of M, if the node remained fixed, P would have been presented to M so nearly in every configuration as to operate an almost exact compensation. But in 1 revolution of M, or 13 of P, the node itself has shifted $\frac{13}{249}$ or about $\frac{1}{19}$ of a revolution, in a direction opposite to the revolutions of M and P, so that although P has been brought back to the same configuration with respect to M, both are $\frac{1}{19}$ of a revolution in advance of the same configuration as respects the node. The compensation, therefore, will not be exact, and to make it so, this process must be gone through 19 times, at the end of which both the bodies will be restored to the same relative position, not only with respect to each other, but to the node. The fractional parts of entire revolutions, which in this explanation have been neglected, are evidently no further influential than as rendering the compensation thus operated in a revolution of the node slightly inexact, and thus giving rise to a compound period of greater duration, at the end of which a compensation almost mathematically rigorous will have been effected.

(639.) It is clear then, that if the orbits be circles, the lapse of a very moderate number of revolutions of the bodies will very nearly, and that of a revolution of the node almost exactly, bring about a perfect restoration of the inclinations. If, however, we suppose the orbits excentric, it is no less evident, owing to the want of symmetry in the distribution of the forces, that a perfect compensation will not be effected either in one or in any number of revolutions of P and M, independent of the motion of the node itself, as there will always be some configuration more favorable to either an increase of inclination than its opposite is unfavorable. Thus will arise a change of inclination which, were the nodes and apsides of the orbits fixed, would be always progressive in one direction until the planes were brought to coincidence. But, 1st, half a revolution of the nodes would of itself reverse the direction of this progression by making the position in question favor the opposite movement of inclination; and, 2dly, the planetary apsides are themselves in motion with unequal velocities, and thus the configuration whose influence destroys the balance, is, itself, always shifting its place on the orbits. The variations of inclination dependent on the excentricities are therefore, like those independent of them, periodical, and being, moreover, of an order more minute (by reason of the smallness of the excentricities) than the latter, it is evident that the total variation of the planetary inclinations must fluctuate within very narrow limits. Geometers have accordingly demonstrated by an accurate analysis of all the circumstances, and an exact estimation of the acting forces, that such is the case; and this is what is meant by asserting the stability of the planetary system as to the mutual inclinations of its orbits. By the researches of Lagrange (of whose analytical conduct it is impossible here to give any idea), the following elegant theorem has been demonstrated:—

"*If the mass of every planet be multiplied by the square root of the major axis of its orbit, and the product by the square of the tangent of its inclination to a fixed plane, the sum of all these products will be constantly the same under the influence of their mutual attraction.*" If the present situation of the plane of the ecliptic be taken for that fixed plane (the ecliptic itself being variable like the other orbits), it is found that this sum is actually very small: it must, therefore, always remain so. This remarkable theorem alone, then, would guarantee the stability of the orbits of the greater planets; but from what has above been shown of the tendency of each planet to work out a compensation on every other, it is evident that the minor ones are not excluded from this beneficial arrangement.

(640.) Meanwhile, there is no doubt that the plane of the ecliptic does actually vary by the actions of the planets. The amount of this variation is about 48″ per century, and has long been recognized by astronomers, by an increase of the latitudes of all the stars in certain situations, and their diminution in the opposite regions. Its effect is to bring the ecliptic by so much per annum nearer to coincidence with the equator; but from what we have above seen, this diminution of the obliquity of the ecliptic will not go on beyond certain very moderate limits, after which (although in an immense period of ages, being a compound cycle resulting from the joint action of all the planets) it will again increase, and thus oscillate backward and forward about a mean position, the extent of its deviation to one side and the other being less than 1° 21′.

(641.) One effect of this variation of the plane of the ecliptic—that which causes its nodes on a fixed plane to change—is mixed up with the precession of the equinoxes, and indistinguishable from it, except in theory. This last-mentioned phenomenon is, however, due to another cause, analogous, it is true, in a general point of view, to those above considered, but singularly modified by the circumstances under which it is produced. We shall endeavor to render these modifications intelligible, as far as they can be made so without the intervention of analytical formulæ.

(642.) The precession of the equinoxes, as we have shown in art. 312, consists in a continual retrogradation of the node of the earth's equator on the ecliptic; and is, therefore, obviously an effect so far analogous to the general phenomenon of the retrogradation of the nodes of the orbits on each other. The immense distance of the planets, however, compared with the size of the earth, and the smallness of their masses compared to that of the sun, puts *their* action out of the question in the inquiry of its cause, and we must, therefore, look to the massive though distant sun, and to our near though minute neighbor, the moon, for its explanation. This will, accordingly, be found in their disturbing action on the redundant matter accumulated on the equator of the earth, by which its figure is rendered spheroidal, combined with the earth's rotation on its axis. It is to the sagacity of

Newton that we owe the discovery of this singular mode of action.

(643.) Suppose in our figure (art. 611) that instead of one body, P, revolving round S, there were a succession of particles not coherent, but forming a kind of fluid ring, free to change its form by any force applied. Then, while this ring revolved round S in its own plane, under the disturbing influence of the distant body M (which now represents the moon or the sun, as P does one of the particles of the earth's equator), two things would happen: 1st, its figure would be bent out of a plane into an undulated form, those parts of it within the arcs D A and E C being rendered more inclined to the plane of M's orbit, and those within the arcs A E, C D, less so than they would otherwise be; 2dly, the nodes of this ring, regarded as a whole, without respect to its change of figure, would retreat upon that plane.

(644.) But suppose this ring, instead of consisting of discrete molecules free to move independently, to be rigid and incapable of such flexure, like the *hoop* we have supposed in art. 633, but having inertia, then it is evident that the effort of those parts of it which tend to become more inclined will act through the medium of the ring itself (as a mechanical engine or lever) to counteract the effort of those which have *at the same instant* a contrary tendency. In so far only, then, as there exists an excess on the one or the other side will the inclination change, an average being struck at every moment of the ring's motion; just as was shown to happen in the view we have taken of the inclinations, in every complete revolution of a single disturbed body, under the influence of a fixed disturbing one.

(645.) Meanwhile, however, the nodes of the rigid ring will retrograde, the general or average tendency of the nodes of every molecule being to do so. Here, as in the other case, a struggle will take place by the counteracting efforts of the molecules contrarily disposed, propagated through the solid substance of the ring; and thus at every instant of time, an average will be struck, which being identical in its nature with that effected in the complete revolution of a single disturbed body, will, in every case, be in favor of a recess of the node, save only when the disturbing body, be it sun or moon, is situated in the plane of the earth's equator.

(646.) This reasoning is evidently independent of any consideration of the cause which maintains the rotation of the ring; whether the particles be small satellites retained in circular orbits under the equilibrated action of attractive and centrifugal forces, or whether they be small masses conceived as attached to a set of imaginary spokes, as of a wheel, centring in S, and free only to shift their planes by a motion of those spokes perpendicular to the plane of the wheel. This makes no difference in the *general* effect; though the different velocities of rotation, which may be impressed on such a system, may and will have a very great influence both on the absolute and relative magnitudes of the two effects in question—the motion of the nodes and change of inclination. This will be easily understood, if we suppose the ring *without* a rotatory motion, in which extreme case it is obvious that so long as M remained fixed there would take place no recess of nodes at all, but only a tendency of the ring to tilt its plane round a diameter perpendicular to the position of M, bringing it toward the line S M.

(647.) The motion of such a ring, then, as we have been considering, would imitate, so far as the recess of the nodes goes, the precession of the equinoxes, only that its nodes would retrograde far more rapidly than the observed precession, which is excessively slow. But now conceive this ring to be loaded with a spherical mass enormously heavier than itself, placed concentrically within it, and cohering firmly to it, but indifferent, or very nearly so, to any such cause of motion; and suppose, moreover, that instead of one such ring there are a vast multitude heaped together around the equator of such a globe, so as to form an elliptical protuberance, enveloping it like a shell on all sides, but whose mass, taken together, should form but a very minute fraction of the whole spheroid. We have now before us a tolerable representation of the case of nature;[5] and it is evident that the rings, having to drag round with them in their nodal revolution this great inert mass, will have their velocity of retrogradation proportionally diminished. Thus, then, it is easy to conceive how a motion similar to the precession of the equinoxes, and, like it, characterized by extreme slowness, will arise from the causes in action. It may seem at first sight paradoxical that the *whole* effect of the external attraction should terminate in the production of such a movement, without producing any change in the inclination of the equator to the ecliptic. But a due consideration of the reasoning in arts. 636, 637 will make it evident that for every particle in the revolving ring (in every situation of the disturbing body) whose change of motion would tend to create

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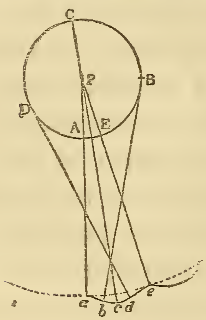
[5] That a perfect sphere would be so inert and indifferent as to a revolution of the nodes of its equator under the influence of a distant attracting body appears from this—that the direction of the resultant attraction of such a body, or of that single force which, opposed, would neutralize and destroy its whole action, is necessarily in a line passing through the centre of the sphere, and, therefore, can have no tendency to turn the sphere one way or other. It may be objected by the reader, that the whole sphere may be conceived as consisting of rings parallel to its equator, of every possible diameter, and that, therefore, its nodes should retrograde even without a protuberant equator. The inference is incorrect, but our limits will not allow us to go into an exposition of the fallacy. We should, however, caution him, generally, that no dynamical subject is open to more mistakes of this kind, which nothing but the closest attention, in every varied point of view, will detect.

a change of inclination in one direction, there exists another, exercising an equal tendency of an opposite kind.

(648.) Now a recess of the node of the earth's equator, upon a given plane, corresponds to a conical motion of its axis round a perpendicular to that plane. But in the case before us, that plane is not the ecliptic, but the moon's orbit for the time being; and it may be asked how we are to reconcile this with what is stated in art. 317 respecting the nature of the motion in question. To this we reply, that the nodes of the lunar orbit, being in a state of continual and rapid retrogradation, while its inclination is preserved nearly invariable, the point in the sphere of the heavens round which the pole of the earth's equator revolves (with that extreme slowness characteristic of the precession) is itself in a state of continual circulation round the pole of the ecliptic, with that much more rapid motion which belongs to the lunar node. A glance at the annexed figure will explain this better than words. P is the pole of the ecliptic, A the pole of the moon's orbit, moving round the small circle A B C D in 19 years; *a* the pole of the earth's equator, which at each moment of its progress has a *direction* perpendicular to the varying position of the line A *a*, and a *velocity* depending on the varying intensity of the acting causes during the period of the nodes. This velocity, however, being extremely small, when A comes to B, C, D, E, the line A *a* will have taken up the positions B *b*, C *c*, D *d*, E *e*, and the earth's pole *a* will thus, in one tropical revolution of the node, have arrived at *e*, having described not an exactly circular arc *a e*, but a single undulation of a wave-shape or



epicycloidal curve, *a b c d e*, with a velocity alternately greater and less than its mean motion, and this will be repeated in every succeeding revolution of the node.

(649.) Now this is precisely the kind of motion which, as we have seen in art. 325, the pole of the earth's equator really has round the pole of the ecliptic, in consequence of the joint effects of precession and nutation, which are thus uranographically represented. If we superadd to the effect of lunar precession that of the solar, which alone would cause the pole to describe a circle uniformly about P, this will only affect the undulations of our waved curve, by extending them in length, but will produce no effect on the depth of the waves, or the excursions of the earth's axis to and from the pole of the ecliptic. Thus we see that the two phenomena of nutation and precession are intimately connected, or rather both of them essential constituent parts of one and the same phenomenon. It is hardly necessary to state that a rigorous analysis of this great problem, by an exact estimation of all the acting forces and summation of their dynamical effects, leads to the precise value of the coefficients of precession and nutation, which observation assigns to them. The solar and lunar portions of the precession of the equinoxes, that is to say, those portions which are uniform, are to each other in the proportion of about 2 to 5.

(650.) In the nutation of the earth's axis we have an example (the first of its kind which has occurred to us) of a periodical movement in one part of the system, giving rise to a motion having the same precise period in another. The motion of the moon's nodes is here, we see, represented, though under a very different form, yet in the same exact periodic time, by a movement of a peculiar oscillatory kind impressed on the solid mass of the earth. We must not let the opportunity pass of generalizing the principle involved in this result, as it is one which we shall find again and again exemplified in every part of physical astronomy, nay, in every department of natural science. It may be stated as "the principle of forced oscillations, or of forced vibrations," and thus generally announced:—

If one part of any system connected either by material ties, or by the mutual attractions of its members, be continually maintained by any cause, whether inherent in the constitution of the system or external to it, in a state of regular periodic motion, that motion will be propagated throughout the whole system and will give rise, in every member of it, and in every part of each member, to periodic movements executed in equal period, with that to which they owe their origin, though not necessarily synchronous with them in their maxima and minima.[6]

The system may be favorably or unfavorably constituted for such a transfer of periodic movements, or favorably in some of its parts and unfavorably in others; and accordingly as it is the one or the other, the *derivative* oscillation (as it may be termed) will be imperceptible in one case, of appreciable magnitude in another, and even more perceptible in its visible effects than the original cause in a third; of this last kind we have an instance in the moon's acceleration, to be hereafter noticed.

(651.) It so happens that our situation on the earth, and the delicacy which our observations have attained, enable us to make it as it were an instrument to *feel* these forced vibrations—these derivative motions, communicated from various

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[6] See a demonstration of this theorem for the forced vibrations of system, connected by material ties of imperfect elasticity, in my treatise on Sound, Encyc. Metrop. art. 323. The demonstration is easily extended and generalized to take in other systems.

quarters, especially from our near neighbor, the moon, much in the same way as we detect, by the trembling of a board beneath us, the secret transfer of motion by which the sound of an organ pipe is dispersed through the air, and carried down into the earth. Accordingly, the monthly revolution of the moon, and the annual motion of the sun, produce, each of them, small *nutations* in the earth's axis, whose periods are respectively half a month and half a year, each of which, in this view of the subject, is to be regarded as one portion of a period consisting of two equal and similar parts. But the most remarkable instance, by far, of this propagation of periods, and one of high importance to mankind, is that of the tides, which are forced oscillations, excited by the rotation of the earth in an ocean disturbed from its figure by the varying attractions of the sun and moon, each revolving in its own orbit, and propagating its own period into the joint phenomenon. The explanation of the tides, however, belongs more properly to that part of the general subject of perturbations which treats of the action of the radial component of the disturbing force, and is therefore postponed to a subsequent chapter.

CHAPTER XIII

THEORY OF THE AXES, PERIHELIA, AND EXCENTRICITIES

"Incipiunt magni procedere menses."—VIRG. *Pollio.*

Variation of Elements in General—Distinction Between Periodic and Secular Variations—Geometrical Expression of Tangential and Normal Forces—Variation of the Major Axis Produced only by the Tangential Force—Lagrange's Theorem of the Conservation of the Mean Distances and Periods—Theory of the Perihelia and Excentricities—Geometrical Representation of their Momentary Variations—Estimation of the Disturbing Forces in Nearly Circular Orbits—Application to the Case of the Moon—Theory of the Lunar Apsides and Excentricity—Experimental Illustration—Application of the Foregoing Principles to the Planetary Theory—Compensation in Orbits very Nearly Circular—Effects of Ellipticity—General Results—Lagrange's Theorem of the Stability of the Excentricities

(652.) IN the foregoing chapter we have sufficiently explained the action of the orthogonal component of the disturbing force, and traced it to its results in a continual displacement of the plane of the disturbed orbit, in virtue of which the nodes of that plane alternately advance and recede upon the plane of the disturbing body's orbit, with a general preponderance on the side of advance, so as after the lapse of a long period to cause the nodes to make a complete revolution and come around to their former situation. At the same time the inclination of the plane of the disturbed motion continually changes, alternately increasing and diminishing; the increase and diminution however compensating each other, nearly in single revolutions of the disturbed and disturbing bodies, more exactly in many, and with perfect accuracy in long periods, such as those of a complete revolution of the nodes and apsides. In the present and following chapters we shall endeavor to trace the effects of the other components of the disturbing force—those which act in the plane (for the time being) of the disturbed orbit, and which tend to derange the elliptic form of the orbit, and the laws of elliptic motion in that plane. The small inclination, generally speaking, of the orbits of the planets and satellites to each other, permits us to separate these effects in theory one from the other, and thereby greatly to simplify their consideration. Accordingly, in what follows, we shall throughout neglect the mutual inclination of the orbits of the disturbed and disturbing bodies, and regard all the forces as acting and all the motions as performed in one plane.

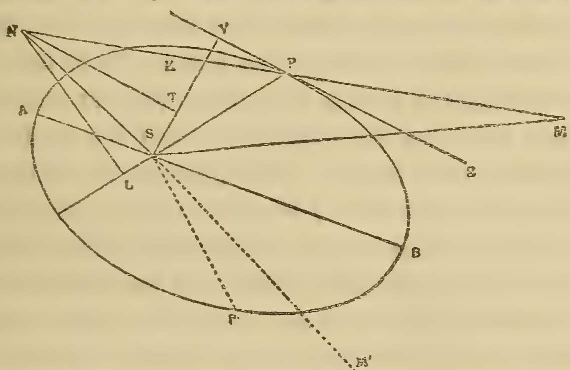
(653.) In considering the changes induced by the mutual action of two bodies in different aspects with respect to each other on the magnitudes and forms of their orbits and in their positions therein, it will be proper in the first instance to explain the conventions under which geometers and astronomers have alike agreed to use the language and laws of the elliptic system, and to continue to apply them to disturbed orbits, although those orbits so disturbed are no longer, in mathematical strictness, ellipses, or any known curves. This they do, partly on account of the convenience of conception and calculation which attaches to this system, but much more for this reason—that it is found, and may be demonstrated from the dynamical relations of the case, that the departure of each planet from its ellipse, as determined at any epoch, is capable of being truly represented, by supposing the ellipse itself to be slowly variable, to change its magnitude and excentricity, and to shift its position and the plane in which it lies according to certain laws, while the planet all the time continues to move in this ellipse, just as it would do if the ellipse remained invariable and the disturbing forces had no existence. By this way of considering the subject, the whole effect of the disturbing forces is regarded as thrown upon the orbit, while the relations of the planet to that orbit remain unchanged. This course of procedure, indeed, is the most natural, and is in some sort forced upon us by the extreme slowness with which the variations of the elements, at least where the planets only are concerned, develop themselves. For instance, the fraction expressing the excentricity of the earth's orbit changes no more than 0·00004 in its amount in a *century*; and the place of its perihelion, as referred to the sphere of the heavens, by only 19′ 39″ in the same time. For several years, therefore, it would be next to impossible to distinguish between an ellipse so varied and one that had not varied at all; and in a single revolution, the difference between the original ellipse and the curve really represented by the varying one, is so excessively minute, that, if accurately drawn on a table, six feet in diameter, the nicest examination with microscopes, continued along the whole outlines of the two curves, would hardly detect any perceptible interval between them. Not to call a motion so minutely conforming itself to an elliptic curve, *elliptic*, would be affectation, even granting the existence of trivial departures alternately on one side or on the other; though, on the other hand, to neglect a variation, which continues to accumulate from age to age, till it forces itself on our notice, would be wilful blindness.

(654.) Geometers, then, have agreed in each single revolution, or for any moderate interval of time, to regard the motion of each planet as elliptic, and performed according to Kepler's laws, with a reserve in favor of those very small and transient fluctuations which take place within that time, but at the same time to regard all the *elements* of each ellipse as in a continual, though extremely slow, state of change; and, in tracing the effects of perturbation on the system, they take account principally, or entirely, of this change of the elements, as that upon which any material change in the great features of the system will ultimately depend.

(655.) And here we encounter the distinction between what are termed secular variations, and such as are rapidly periodic, and are compensated in short intervals. In our exposition of the variation of the inclination of a disturbed orbit (art. 636), for instance, we showed that, in each single revolution of the disturbed body, the plane of its motion underwent fluctuations to and fro in its inclination to that of the disturbing body, which nearly compensated each other; leaving, however, a portion outstanding, which again is nearly compensated by the revolution of the disturbing body, yet still leaving outstanding and uncompensated a minute portion of the change which requires a whole revolution of the node to compensate and bring it back to an average or mean value. Now, the first two compensations which are operated by the planets going through the succession of configurations with each other, and therefore in comparatively short periods, are called periodic variations; and the deviations thus compensated are called *inequalities depending on configurations;* while the last, which is operated by a period of the node (one of the *elements*), has nothing to do with the configurations of the individual planets, requires a very long period of time for its consummation, and is, therefore, distinguished from the former by the term *secular* variation.

(656.) It is true, that, to afford an exact representation of the motions of a disturbed body, whether planet or satellite, both periodical and secular variations, with their corresponding inequalities, require to be expressed; and, indeed, the former even more than the latter; seeing that the secular inequalities are, in fact, nothing but what remains after the mutual destruction of a much larger amount (as it very often is) of periodical. But these are in their nature transient and temporary: they disappear in short periods, and leave no trace. The planet is temporarily drawn from its orbit (its slowly varying orbit), but forthwith returns to it, to deviate presently as much the other way, while the varied orbit accommodates and adjusts itself to the average of these excursions on either side of it; and thus continues to present, for a succession of indefinite ages, a kind of medium picture of all that the planet has been doing in their lapse, in which the expression and character is preserved; but the individual features are merged and lost. These periodic inequalities, however, are, as we have observed, by no means neglected, but it is more convenient to take account of them by a separate process, independent of the secular variations of the elements.

(657.) In order to avoid complication, while endeavoring to give the reader an insight into both kinds of variations, we shall for the present conceive all the orbits to lie in one plane, and confine our attention to the case of two only, that of the disturbed and disturbing body, a view of the subject which (as we have seen) comprehends the case of the moon disturbed by the sun, since any one of the bodies may be regarded as fixed at pleasure, provided we conceive all its motions transferred in a contrary direction to each of the others. Let therefore A P B be the undisturbed elliptic orbit of a planet P; M a disturbing body, join M P, and supposing M K = M S take M N : M K :: M K² : M P². Then if S N be joined, N S will represent the disturbing force of M on P, on the same scale that S M represents M's attraction on S. Suppose Z P Y a tangent at P, S Y perpendicular to it, and N T, N L perpendicular respectively to S Y and P S produced. Then will N T represent the tangential, T S the normal, N L the transversal, and L S the radial components of the disturbing force. In circular orbits or orbits only slightly elliptic, the directions P S L



and S Y are nearly coincident, and the former pair of forces will differ but slightly from the latter. We shall here, however, take the general case, and proceed to investigate in an elliptic orbit of any degree of excentricity the momentary changes produced by the action of the disturbing force in those elements on which the magnitude, situation, and form of the orbit depend (*i.e.* the length and position of the major axis and the excentricity), in the same way as in the last chapter we determined the momentary changes of the inclination and node similarly produced by the orthogonal force.

(658.) We shall begin with the momentary variation in the *length* of the axis, an element of the first importance, as on it depend (art. 487) the periodic time and mean angular motion of the planet, as well as the average supply of light and heat it receives in a given time from the sun, any permanent or constantly progressive change in which would alter most materially the conditions of existence of living beings on its surface. Now it is a property of elliptic motion performed under the influence of gravity, and in conformity with Kepler's laws, that if the velocity with which a planet moves at any point of its orbit be given, and also the distance of that point from the sun, the major axis of the orbit is thereby also given. It is no matter in what *direction* the planet may be moving at that moment. This will influence the excentricity and the position of its ellipse, but not its length. This property of elliptic motion has been demonstrated by Newton, and is one of the most obvious and elementary conclusions from his theory. Let us now consider a planet describing an indefinitely small arc of its orbit about the sun, under the joint influence of its attraction, and the disturbing power of another planet. This arc will have some certain curvature and direction, and, therefore, may be considered as an arc of a certain ellipse described about the sun as a focus, for this plain reason—that whatever be the curvature and direction of the arc in question, an ellipse may always be assigned, whose focus shall be in the sun, and which shall coincide with it throughout the whole interval (supposed indefinitely small) between its extreme points. This is a matter of pure geometry. It does not follow, however, that the ellipse thus instantaneously determined will have the same elements as that similarly determined from the arc described in either the previous or the subsequent instant. If the disturbing force did not exist, this would be the case; but, by its action, a variation of the element from instant to instant is produced, and the ellipse so determined is in a continual state of change. Now when the planet has reached the end of the small arc under consideration, the question whether it will in the next instant describe an arc of an ellipse having the same or a varied axis will depend, not on the new *direction* impressed upon it by the acting forces—for the axis, as we have seen, is independent of that direction—not on its change of distance from the sun, while describing the former arc—for the elements of that arc are accommodated to it, so that one and the same axis must belong to its beginning and its end. The question, in short, whether in the next arc it shall take up a new major axis or go on with the old one will depend solely on this—whether its *velocity* has or has not undergone a change by the action of the *disturbing* force. For the central force residing in the focus can impress on it no such change of velocity as to be incompatible with the permanence of its ellipse, seeing that it is by the action of that force that the velocity is maintained in that due proportion to the distance which elliptic motion, as such, requires.

(659.) Thus we see that the momentary variation of the major axis depends on nothing but the momentary deviation from the law of elliptic velocity produced by the disturbing force, without the least regard to the direction in which that extraneous velocity is impressed, or the distance from the sun at which the planet may be situated, at the moment of its impression. Nay, we may even go further, for, as this holds good at every instant of its motion, it will follow, that after the lapse of any time, however great, the total amount of change which the axis may have undergone will be determined only by the total deviation produced by the action of the disturbing force in the velocity of the disturbed body from that which it would have had in its undisturbed ellipse, at the same distance from the centre, and that therefore the total amount of change produced in the axis in any lapse of time may be estimated, if we know at every instant the efficacy of the disturbing force to alter the velocity of the body's motion, and *that* without any regard to the alterations which the action of that force may have produced in the other elements of the motion in the same time.

(660.) Now it is not the whole disturbing force which is effective in changing P's velocity, but only its tangential component. The normal component tends merely to alter the curvature of the orbit or to deflect it into conformity with a circle of curvature of greater or lesser radius, as the case may be, and in no way to alter the velocity. Hence it appears that the *variation of the length of the axis is due entirely to the tangential force*, *and is quite independent on the normal.* Now it is easily shown that as the velocity increases, the axis increases (the distance remaining unaltered[1]) though not in the same exact proportion. Hence it follows that if the tangential disturbing force conspires with the motion of P, its momentary action increases the axis of the disturbed orbit, whatever be the situation of P in its orbit, and *vice versâ.*

(661.) Let A S B (*fig.* art. 657) be the major axis of the ellipse A P B, and on the opposite side of A B take two

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[1] If a be the semiaxis, r the radius vector, and v the velocity of P in any point of an ellipse, a is given by the relation $v^2 = \frac{2}{r} - \frac{1}{a}$, the units of velocity and force being properly assumed.

points P′ and M′, similarly situated with respect to the axis with P and M on their side. Then if at P′ and M′ bodies equal to P and M be placed, the forces exerted by M′ on P′ and S will be equal to those exerted by M on P and S, and therefore the tangential disturbing force of M′ on P′ exerted in the direction P′ Z′ (suppose) will equal that exerted by M on P in the direction P Z. P′ therefore (supposing it to revolve in the same direction round S as P) will be retarded (or accelerated, as the case may be) by *precisely* the same force by which P is accelerated (or retarded), so that the variation in the axis of the respective orbits of P and P′ will be equal in amount, but contrary in character. Suppose now M's orbit to be circular. Then (*if the periodic times of* M *and* P *be not commensurate, so that a moderate number of revolutions may bring them back to the same precise relative positions*) it will necessarily happen, that in the course of a very great number of revolutions of both bodies, P will have been presented to M on one side of the axis, at some one moment, in the same manner as at some other moment on the other. Whatever variation may have been effected in its axis in the one situation will have been reversed in that symmetrically opposite, and the ultimate result, on a general average of an infinite number of revolutions, will be a complete and exact compensation of the variations in one direction by those in the direction opposite.

(662.) Suppose, next, P's orbit to be circular. If now M's orbit were so also, it is evident that in one complete synodic revolution, an exact restoration of the axis to its original length would take place, because the tangential forces would be symmetrically equal and opposite during each alternate quarter revolution. But let M, during a synodic revolution, have *receded* somewhat from S, then will its disturbing power have become gradually weaker, so that, in a synodic revolution the tangential force in each quadrant, though reversed in direction being inferior in power, an exact compensation will not have been effected, but there will be left an outstanding uncompensated portion, the excess of the stronger over the feebler effects. But now suppose M to approach by the same gradations as it before receded. It is clear that this result will be reversed; since the uncompensated stronger actions will all lie in the opposite direction. Now suppose M's orbit to be elliptic. Then during its recess from S or in the half revolution from its perihelion to its aphelion, a continual uncompensated variation will go on accumulating in one direction. But from what has been said, it is clear that this will be destroyed, during M's approach to S in the other half of its orbit, so that here again, on the average of a multitude of revolutions during which P *has been* presented to M *in every situation for every distance of* M *from* S, the restoration will be effected.

(663.) If neither P's nor M's orbit be circular, and if moreover the directions of their axes be different, this reasoning, drawn from the symmetry of their relations to each other, does not apply, and it becomes necessary to take a more general view of the matter. Among the fundamental relations of dynamics, relations which presuppose no particular law of force like that of gravitation, but which express in general terms the results of the action of *force* on *matter* during *time*, to produce or change *velocity*, is one usually cited as the "Principle of the conservation of the *vis viva*," which applies directly to the case before us. This principle (or rather this theorem) declares that if a body subjected at every instant of its motion to the action of forces directed to fixed centres (no matter how numerous), and having their intensity dependent only on the distances from their respective centres of action, travel from one point of space to another, the velocity which it has on its arrival at the latter point will differ from that which it had on setting out from the former, by a quantity depending only on the different relative situations of these two points in space, without the least reference to the form of the curve in which it may have moved in passing from one point to the other, whether that curve have been described freely under the simple influence of the central forces, or the body have been compelled to glide upon it, as a bead upon a smooth wire. Among the forces thus acting may be included any constant forces, acting in parallel directions, which may be regarded as directed to fixed centres infinitely distant. It follows from this theorem, that, if the body return to the point P from which it set out, its velocity of arrival will be the same with that of its departure; a conclusion which (for the purpose we have in view) sets us free from the necessity of entering into any consideration of the laws of the disturbing force, the change which its action may have induced in the form of the orbit of P, or the successive steps by which velocity generated at one point of its intermediate path is destroyed at another, by the reversed action of the tangential force. Now to apply this theorem to the case in question, let M be supposed to retain a fixed position during one whole revolution of P. P then is acted on, during that revolution, by three forces: 1st, by the central attraction of S directed always to S; 2d, by that to M, always directed to M; 3d, by a force equal to M's attraction on S; but in the direction M S, which therefore is a constant force, acting always in parallel directions. On completing its revolution, then, P 's velocity, and therefore the major axis of its orbit, will be found unaltered, at least neglecting that excessively minute difference which will result from the non-arrival after a revolution at the *exact* point of its departure by reason of the perturbations in the orbit produced in the interim by the disturbing force, which for the present we may neglect.

(664.) Now suppose M to revolve, and it will appear, by a reasoning precisely similar to that of art. 662, that whatever uncompensated variation of the velocity arises in successive revolutions of P during M's recess from S will be destroyed by contrary uncompensated variations arising during its approach. Or, more simply and generally thus: whatever M's situation may be, for every place which P can have, there must exist some other place of P (as P′), in which the action of M shall be precisely reversed. Now *if the periods be incommensurable*, in an indefinite number of revolutions of both bodies, for every possible combination of situations (M, P) there will occur, *at some time or other*, the combination (M, P) which neutralizes the effect of the other, when carried to the general account; so that ultimately, and when very long periods of time are embraced, a complete compensation will be found to be worked out.

(665.) This supposes, however, that in such long periods the orbit of M is not so altered as to render the occurrence of the compensating situation (M, P′) impossible. This would be the case if M's orbit were to dilate or contract indefinitely by a variation in *its* axis. But the same reasoning which applies to P, applies also to M. P retaining a *fixed* situation, M's velocity, and therefore the axis of its orbit, would be exactly restored at the end of a revolution of M; so that for every position P M there exists a compensating position P M′. Thus M's orbit is maintained of the same magnitude, and the possibility of the occurrence of the compensating situation (M, P′) is secured.

(666.) To demonstrate as a rigorous mathematical truth the complete and absolute ultimate compensation of the variations in question, it would be requisite to show that the minute outstanding changes due to the non-arrivals of P and M at the same *exact* points at the end of each revolution, cannot accumulate in the course of infinite ages in one direction. Now it will appear in the subsequent part of this chapter, that the effect of perturbation on the excentricities and apsides of the orbits is to cause the former to undergo only periodical variations, and the latter to revolve and take up in succession every possible situation. Hence in the course of infinite ages, the points of arrival of P and M at fixed lines of direction, S P, S M, in successive revolutions, though at one time they will approach S, at another will recede from it, fluctuating to and fro about mean points from which they never greatly depart. And if the arrival of either of them as P, at a point *nearer* S, at the end of a complete revolution, cause an *excess* of velocity, its arrival at a more distant point will cause a deficiency, and thus, as the fluctuations of distance to and fro ultimately balance each other, so will also the excesses and defects of velocity, though in periods of enormous length, being no less than that of a complete revolution of P's apsides for the one cause of inequality, and of a complete restoration of its excentricity for the other.

(667.) The dynamical proposition on which this reasoning is based is general, and applies equally well to cases wherein the forces act in one plane, or are directed to centres anywhere situated in space. Hence, if we take into consideration the inclination of P's orbit to that of M, the same reasoning will apply. Only that in this case, upon a complete revolution of P, the variation of inclination and the motion of the nodes of P's orbit will prevent its returning to a point in the exact *plane* of its original orbit, as that of the excentricity and perihelion prevent its arrival at the same exact *distance* from S. But since it has been shown that the inclination fluctuates round a mean state from which it never departs much, and since the node revolves and makes a complete circuit, it is obvious that in a complete period of the latter the points of arrival of P at the same longitude will deviate as often and by the same quantities above as below its original point of departure from exact coincidence; and, therefore, that on the average of an infinite number of revolutions, the effect of this cause of non-compensation will also be destroyed.

(668.) It is evident, also, that the dynamical proposition in question being general, and applying equally to any *number* of fixed centres, as well as to any distribution of them in space, the conclusion would be precisely the same whatever be the number of disturbing bodies, only that the periods of compensation would become more intricately involved. We are, therefore, conducted to this most remarkable and important conclusion, viz. that the major axes of the planetary (and lunar) orbits, and, consequently, also their mean motions and periodic times, are subject to none but periodical changes; that the length of the year, for example, in the lapse of infinite ages, has no preponderating tendency either to increase or diminution—that the planets will neither recede indefinitely from the sun, nor fall into it, but continue, so far as their mutual perturbations at least are concerned, to revolve forever in orbits of very nearly the same dimensions as at present.

(669.) This theorem (the *Magna Charta* of our system), the discovery of which is due to Lagrange, is justly regarded as the most important, as a single result, of any which have hitherto rewarded the researches of mathematicians in this application of their science; and it is especially worthy of remark, and follows evidently from the view here taken of it, that it would not be true but for the influence of the perturbing forces on other elements of the orbit, viz. the perihelion and excentricity, and the inclination and nodes; since we have seen that the revolution of the apsides and nodes, and the periodical increase and diminution of the excentricities and inclinations, are both essential toward operating that final and complete compensation which gives it a character of mathematical exactness. We have here an instance of a perturbation of one kind operating on a perturbation of another to annihilate an effect which would otherwise accumulate to the destruction of the system. It must, however, be borne in mind, that it is the smallness of the excentricities of the more influential planets, which gives this theorem its *practical* importance, and distinguishes it from a mere barren speculative result. Within the limits of ultimate restoration, it is this alone which keeps the periodical fluctuations of the axis to and fro about a mean value within moderate and reasonable limits. Although the earth might not fall into the sun, or recede from it beyond the present limits of our system, any considerable increase or diminution of its mean distance, to the extent, for instance, of a tenth of its actual amount, would not fail to subvert the conditions on which the existence of the present race of animated beings depends. Constituted as our system is, however, changes to anything like this extent are utterly precluded. The greatest departure from the mean value of the axis of any planetary orbit yet recognized by theory or observation (that of the orbit of Saturn disturbed by Jupiter), does not amount to a thousandth part of its length.[2] The effects of these fluctuations, however, are very sensible, and manifest themselves in alternate accelerations and retardations in the angular motions of the disturbed about the central body, which cause it alternately to outrun and to lag behind its *elliptic* place in its orbit, giving rise to what are called equations in its motion, some of the chief instances of which will be hereafter specified when we come to trace more particularly in detail the effects of the tangential force in various configurations of the disturbed and disturbing bodies, and to explain the consequences of a near approach to commensurability in their periodic times. An exact commensurability in this respect, such, for instance, as would bring both planets round to the same configuration in two or three revolutions of one of them, would appear at first sight to destroy one of the essential elements of our demonstration. But even supposing such an exact adjustment to subsist at any epoch, it could not remain permanent, since by a remarkable property of perturbations of this class, which geometers have demonstrated, but the reasons of which we cannot stop to explain, any change produced on the axis of the disturbed planet's orbit is necessarily accompanied by a change *in the contrary direction* in that of the disturbing, so

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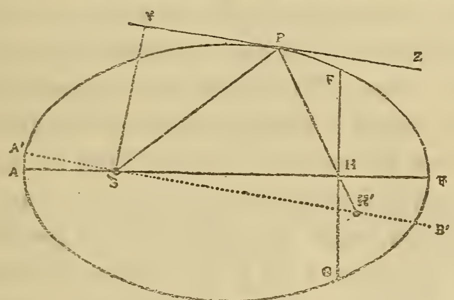
[2] Greater deviations will probably be found to exist in the orbits of the small extra-tropical planets. But these are too insignificant members of our system to need special notice in a work of this nature.

that the periods would recede from commensurability by the mere effect of their mutual action. Cases are not wanting in the planetary system of a certain approach to commensurability, and in one very remarkable case (that of Uranus and Neptune) of a considerably near one, not near enough, however, in the smallest degree to affect the validity of the argument, but only to give rise to inequalities of very long periods, of which more presently.[3]

(670.) The variation of the length of the axis of the disturbed orbit is due solely to the action of the tangential disturbing force. It is otherwise with that of its excentricity and of the position of its axis, or, which is the same thing, the longitude of its perihelion. Both the normal and tangential components of the disturbing force affect these ele-



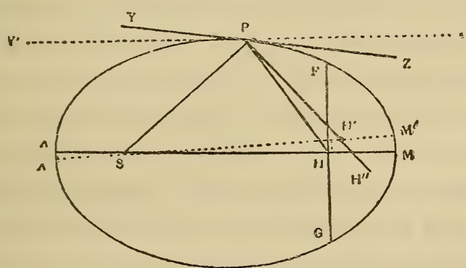
ments. We shall, however, consider separately the influence of each, and, commencing, as the simplest case, with that of the tangential force;—let P be the place of the disturbed planet in its elliptic orbit A P B, whose axis at the moment is A S B and focus S. Suppose Y P Z to be a tangent to this orbit at P. Then, if we suppose A B = 2 a, the other focus of the ellipse, H, will be found by making the

[3] 41 revolutions of Neptune are nearly equal to 81 of Uranus, giving rise to an inequality having 6805 years for its period.

angle Z P H = Y P S or Y P H = 180°—Y P Z, or S P H = 180°—2 Y P S, and taking P H = 2 a – S P. This is evident from the nature of the ellipse, in which lines drawn from any point to the two foci make equal angles with the tangent, and have their sum equal to the major axis. Suppose, now, the tangential force to act on P and to increase its velocity. It will therefore increase the axis, so that the new value assumed by a (viz. a') will be greater than a. But the tangential force does not alter the angle of tangency, so that to find the new position (H′) of the upper focus, we must measure off along *the same line* P H, a distance P H′ (= 2 a' – S P) greater than P H. Do this then, and join S H′ and produce it. Then will A′ B′ be the new position of the axis, and ½ S H′ the new excentricity. Hence we conclude, 1st, that the new position of the perihelion A′ will deviate from the old one A toward the same side of the axis A B on which P is when the tangential force acts to increase the velocity, whether P be moving from perihelion to aphelion, or the contrary. 2dly, That on the same supposition as to the action of the tangential force, the excentricity increases when P is between the perihelion and the perpendicular to the axis F H G drawn through the upper focus, and diminishes when between the aphelion and the same perpendicular. 3dly, That for a given change of velocity, *i.e.* for a given value of the tangential force, the momentary variation in the place of the perihelion is a maximum when P is at F or G, from which situation of P to the perihelion or aphelion, it decreases to nothing, the perihelion being stationary when P is at A or B. 4thly, That the variation of the excentricity due to this cause is complementary in its law of increase and decrease to that of the perihelion, being a maximum for a given tangential force when

P is at A or B, and vanishing when at G or F. And lastly, that where the tangential force acts to diminish the velocity, all these results are reversed. If the orbit be very nearly circular[4] the points F, G, will be so situated that, although not at opposite extremities of a *diameter*, the *times* of describing A F, F B, B G, and G A will be all equal, and each of course one quarter of the whole periodic time of P.

(671.) Let us now consider the effects of the normal component of the disturbing force upon the same elements. The direct effect of this force is to increase or diminish the curvature of the orbit at the point P of its action, without



producing any change on the velocity, so that the length of the axis remains unaltered by its action. Now, an increase of curvature at P is synonymous with a decrease in the angle of tangency S P Y when P is approaching toward S, and with an increase in that angle when receding from S. Suppose the former case, and while P approaches S (or is moving from aphelion to perihelion), let the normal force act *inward* or toward the concavity of the ellipse. Then will the tangent P Y by the action of that force have taken up the position P Y′. To find the corresponding position H′ taken up by the focus of the orbit so disturbed,

[4] So nearly that the cube of the excentricity may be neglected.

we must make the angle S P H′ = 180° − 2 S P Y′, or, which comes to the same, draw P H″ on the side of P H opposite to S, making the angle H P H″ = twice the angle of deflection Y P Y′ and in P H″ take P H′ = P H. Joining, then, S H′ and producing it, A′ S H′ M′ will be the new position of the axis, A′ the new perihelion, and ½ S H the new excentricity. Hence we conclude, 1st, that the normal force acting *inward*, and P moving *toward* the perihelion, the new direction S A′ of the perihelion is in advance (with reference to the direction of P's revolution) of the old—or the apsides advance—when P is anywhere situated between F and A (since when at F the point H′ falls upon H M between H and M). When P is at F the apsides are stationary, but when P is anywhere between M and F the apsides retrograde, H′ in this case lying on the opposite side of the axis. 2dly, That the same directions of the normal force and of P's motion being supposed, the excentricity increases while P moves through the whole semi-ellipse from aphelion to perihelion—the rate of its increase being a maximum when P is at F, and nothing at the aphelion and perihelion. 3dly, That these effects are reversed in the opposite half of the orbit, A G M, in which P passes from perihelion to aphelion or recedes from S. 4thly, That they are also reversed by a reversal of the direction of the normal force, outward, in place of inward. 5thly, That here also the variations of the excentricity and perihelion are complementary to each other; the one variation being most rapid when the other vanishes, and *vice versa*. 6thly, And lastly, that the changes in the situation of the focus H produced by the actions of the tangential and normal components of the disturbing force are at right angles to each other in every situation of P, and therefore where the tangential force is most efficacious (in proportion to its intensity) in varying either the one or the other of the elements in question, the normal is least so, and *vice versâ.*

(672.) To determine the momentary effect of the whole disturbing force then, we have only to resolve it into its tangential and normal components, and estimating by these principles separately the effects of either constituent on both elements, add or subtract the results according as they conspire or oppose each other. Or we may at once make the angle H P H″ equal to twice the angle of deflection produced by the normal force, and lay off P H″ = P H + twice the variation of a produced in the same moment of time by the tangential force, and H″ will be the new focus. The momentary velocity generated by the tangential force is calculable from a knowledge of that force by the ordinary principles of dynamics; and from this, the variation of the axis is easily derived.[5] The momentary velocity generated by the normal force in its own direction is in like manner calculable from a knowledge of that force, and dividing this by the linear velocity of P at that instant, we deduce the angular velocity of the tangent about P, or the momentary variation of the angle of tangency S P Y, corresponding.

(673.) The following *résumé* of these several results in a tabular form includes every variety of case according as P is approaching to or receding from S; as it is situated

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[5] $\frac{1}{a} = \frac{2}{r} - v^2$, and $\frac{1}{a'} = \frac{2}{r} - v'^2 \therefore \frac{1}{a'} - \frac{1}{a} = v^2 - v'^2 = (v + v')(v - v')$ or when infinitesimal variations only are considered $\frac{a' - a}{a^2} = 2v(v' - v)$ or $a' - a = 2a^2 v(v' - v)$ from which it appears that the variation of the axis arising from a given variation of velocity is independent of r, or is the same at whatever distance from S the change takes place, and that *cœteris paribus* it is *greater* for a given change of velocity (or for a given tangential force) in *the direct ratio of the velocity itself.*

in the arc F A G of its orbit *about the perihelion* or in the remoter arc G M F *about the aphelion*, as the tangential force *accelerates* or *retards* the disturbed body, or as the normal acts *inward* or outward with reference to the concavity of the orbit.

EFFECTS OF THE TANGENTIAL DISTURBING FORCE.

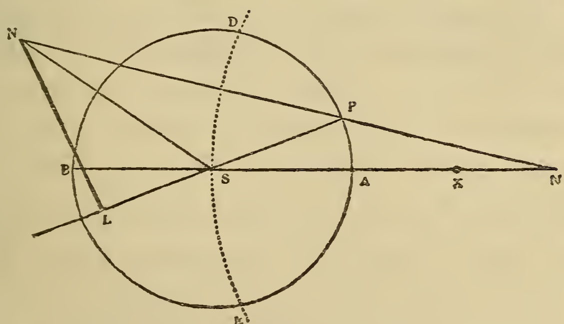
Direction of P's motion.	Situation of P in orbit.	Action of Tangential Force.	Effect on Elements.
Approaching S.	Anywhere	Accelerating P.	Apsides recede
Ditto.	Ditto	Retarding P.	advance
Receding from S.	Ditto	Accelerating P.	advance
Ditto	Ditto	Retarding P.	recede
Indifferent.	About Aphelion	Accelerating P.	Excentr. decreases
Ditto	Ditto	Retarding P.	increases
Ditto	About Perihelion	Accelerating P.	increases
Ditto	Ditto	Retarding P.	decreases

EFFECTS OF THE NORMAL DISTURBING FORCE.

Direction of P's motion.	Situation of P in orbit.	Action of Normal Force.	Effect on Elements.
Indifferent	About Aphelion	Inward	Apsides recede
Ditto	Ditto	Outward	advance
Ditto	About Perihelion	Inward	advance
Ditto	Ditto	Outward	recede
Approaching S.	Anywhere	Inward	Excentr. increases
Ditto	Ditto	Outward	decreases
Receding from S.	Ditto	Inward	decreases
Ditto	Ditto	Outward	increases

(674.) From the momentary changes in the elements of the disturbed orbit corresponding to successive situations of P and M, to conclude the total amount of change produced in any given time is the business of the integral calculus, and lies far beyond the scope of the present work. Without its aid, however, and by general considerations of the periodical recurrence of configurations of the same character, we have been able to demonstrate many of the most interesting conclusions to which geometers have been conducted, examples of which have already been given in the reasoning by which the permanence of the axes, the periodicity of the inclinations, and the revolutions of the nodes of the planetary orbits have been demonstrated. We shall now proceed to apply similar considerations to the motion of the apsides, and the variations of the excentricities. To this end we must first trace the changes induced on the disturbing forces themselves, with the varying positions of the bodies, and here as in treating of the inclinations we shall suppose, unless the contrary is expressly indicated, both orbits to be very nearly circular, without which limitation the complication of the subject would become too embarrassing for the reader to follow, and defeat the end of explanation.

(675.) On this supposition the directions of S P and S Y, the perpendicular on the tangent at P, may be regarded as coincident, and the normal and radial disturbing forces

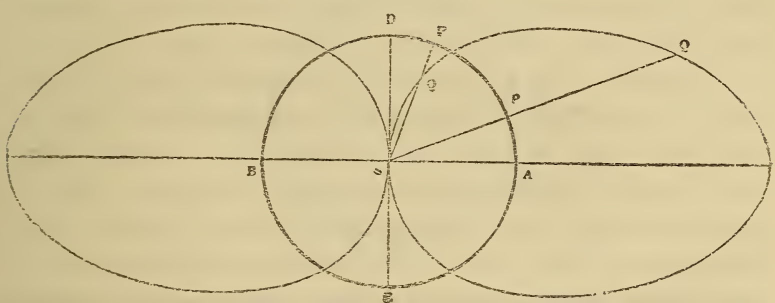


become nearly identical in quantity, also the tangential and transversal, by the near coincidence of the points T and L (*fig.* art. 657). So far then as the *intensity* of the forces is concerned, it will make very little difference in which way the forces are resolved, nor will it at all materially affect our conclusions as to the effects of the normal and tangential forces, if in estimating their *quantitative* values, we take advantage of the simplification introduced into their numerical expression by the neglect of the angle P S Y, *i.e.* by the substitution for them of the radial and transversal components. The *character* of these effects depends (arts. 670, 671) on the *direction* in which the forces act, which we shall suppose normal and tangential as before, and it is only on the estimation of their quantitative effects that the error induced by the neglect of this angle can fall. In the lunar orbit this angle never exceeds 3° 10′, and its influence on the quantitative estimation of the acting forces may therefore be safely neglected in a first approximation. Now M N being found by the proportion M P^2 : M S^2 :: M S : M N, N P (=M N−M P) is also known, and therefore N L=N P. sin N P S=N P . sin (A S P+S M P) and L S=P L−P S=N P. cos N P S−P S=N P . cos (A S P+S M P) −S P become known, which express respectively the tangential and normal forces on the same scale that S M represents M's attraction on S.[6] Suppose P to revolve in the direction E A D B. Then, by drawing the figure in various situations of P throughout the whole circle, the reader will easily satisfy himself—1st. That the tangential force accelerates P, as it moves from E toward A, and from D

[6] $MS = R$; $SP = r$; $MP = f$; $ASP = \theta$; $AMP = M$; $MN = \frac{R^3}{f^2}$; $NP = \frac{R^3 - f^3}{f^2} = (R - f)\left(1 + \frac{R}{f} + \frac{R^2}{f^2}\right)$; whence we have $NL = (R - f) . \sin(\theta + M) \left(1 + \frac{R}{f} + \frac{R^2}{f^2}\right)$; $LS = (R - f) . \cos(\theta + M) . \left(1 + \frac{R}{f} + \frac{R^2}{f^2}\right) - r$. When R and f, owing to the great distance of M, are nearly equal, we have $R - f = PV$, $\frac{R}{f} = 1$ nearly, and the angle M may be neglected; so that we have $NP = 3\,PV$.

toward B, but retards it as it passes from A to D, and from B to E. 2dly. That the tangential force vanishes at the four points A, D, E, B, and attains a maximum at some intermediate points. 3dly. That the normal force is directed outward at the syzygies A, B, and inward at the points D, E, at which points respectively its outward and inward intensities attain their maxima. Lastly, that this force vanishes at points intermediate between A D, D B, B E, and E A, which points, when M is considerably remote, are situated nearer to the quadrature than the syzygies.

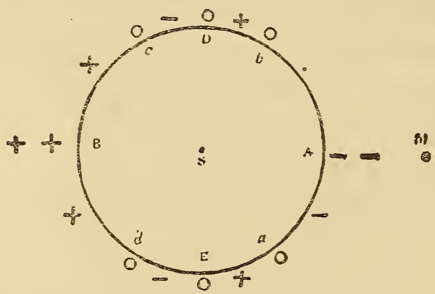
(676.) In the lunar theory, to which we shall now proceed to apply these principles, both the geometrical representation and the algebraic expression of the disturbing forces admit of great simplification. Owing to the great distance of the sun M, at whose centre the radius of the



moon's orbit never subtends an angle of more than about 8′, N P may be regarded as parallel to A B. And D S E becomes a straight line coincident with the line of quadratures, so that V P becomes the cosine of A S P, to radius S P, and N L=N P . sin A S P; L P=N P . cos A S P. Moreover, in this case (see the note on the last article) N P=3 P V=3 S P . cos A S P; and consequently N L =3 S P . cos A S P . sin A S P=$\frac{3}{2}$ S P . sin 2 A S P, and

$L S = S P (3 . \cos A S P^2 - 1) = \frac{1}{2} S P (1 + 3 . \cos 2 A S P)$ which vanishes when $\cos A S P^2 = \frac{1}{3}$, or at 64° 14′ from the syzygy. Suppose through every point of P's orbit there be drawn $S Q = 3 S P . \cos A S P^2$, then will Q trace out a certain looped oval, as in the figure, cutting the orbit in four points 64° 14′ from A and B respectively, and P Q will always represent in quantity and direction the normal force acting at P.

(677.) It is important to remark here, because upon this the whole lunar theory and especially that of the motion of the apsides hinges, that all the acting disturbing forces, at equal angles of elongation A S P of the moon from the sun, are *cæteris paribus* proportional to S P, the moon's distance from the earth, and are therefore greater when



the moon is near its apogee than when near its perigee; the extreme proportion being that of about 28 : 25. This premised, let us first consider the effect of the normal force in displacing the lunar apsides. This we shall best be enabled to do by examining separately those cases in which the effects are most strongly contrasted, viz. when the major axis of the moon's orbit is directed toward the sun, and when at right angles to that direction. First, then, let the line of apsides be directed to the sun as in the annexed figure, where A is the perigee, and take the arcs A a, A b, B c, B d each=64° 14′. Then while P is between a and b the normal force acting outward, and the moon being near its perigee, by art. 671, the apsides will recede, but when between c and d, the force there acting outward, but the moon being near its apogee, they will advance. The rapidity of these movements will be respectively at its maxima at A and B, not only because the disturbing forces are then most intense, but also because (see art. 671) they act most advantageously at those points to displace the axis. Proceeding from A and B toward the neutral points a b c d the rapidity of their recess and advance diminishes, and is nothing (or the apsides are stationary) when P is at either of these points. From b to D, or rather to a point some little beyond D (art. 671) acts inward, and the moon is still near perigee, so that in this arc of the orbit the apsides advance. But the rate of advance is feeble, because in the early part of that arc the normal force is small, and as P approaches D and the force gains power, it acts disadvantageously to move the axis, its effect vanishing altogether when it arrives beyond D at the extremity of the perpendicular to the upper focus of the lunar ellipse. Thence up to c this feeble advance is reversed and converted into a recess, the force still acting inward, but the moon now being near its apogee. And so also for the arcs d E, E a. In the figure these changes are indicated by ++ for rapid advance, — — for rapid recess, + and — for feeble advance and recess, and 0 for the stationary points. Now if the forces were equal on the sides of + and — it is evident that there would be an exact counterbalance of advance and recess on the average of a whole revolution. But this is not the case. The force in apogee is greater than that in perigee in the proportion of 28 : 25, while in the quadratures about D and E they are equal. Therefore, while the feeble movements + and — in the neighborhood of these points destroy each other almost exactly, there will necessarily remain a considerable balance in favor of advance, in this situation of the line of apsides.

(678.) Next, suppose the apogee to lie at A, and the perigee at B. In this case it is evident that, so far as the direction of the motions of the apsides is concerned, all the conclusions of the foregoing reasoning will be reversed by the substitution of the word perigee for apogee, and *vice versâ;* and all the signs in the figure referred to will be changed. But now the most powerful forces act on the side of A, that is to say, still on the side of advance, this condition also being reversed. In either situation of the orbit, then, the apsides advance.

(679.) (Case 3.) Suppose, now, the major axis to have the situation D E, and the perigee to be on the side of D. Here, in the arc *b c* of P's motion the normal force acts inward, and the moon is near perigee, consequently the apsides advance, but with a moderate rapidity, the maximum of the inward normal force being only half that of the outward. In the arcs A *b* and *c* B the moon is still near perigee, and the force acts outward, but though powerfully toward A and B, yet at a constantly increasing disadvantage (art. 671). Therefore in these arcs the apsides recede, but moderately. In *a* A and B *d* (being toward apogee) they again advance, still with a moderate velocity. Lastly, throughout the arc *d a*, being about apogee with an inward force, they recede. Here as before, if the perigee and apogee forces were equal, the advance and recess would counterbalance; but as in fact the apogee forces preponderate, there will be a balance on the entire revolution in favor of recess. The same reasoning of course holds good if the perigee be toward E. But now, between these cases and those in the foregoing articles, there is this difference, viz. that in this the dominant effect results from the inward action of the normal force in quadratures, while in the others it results from its outward, and doubly powerful action in syzygies. The recess of the apsides in their quadratures arising from the action of the normal force will therefore be less than their advance in their syzygies; and not only on this account, but also because of the much less extent of the arcs *b c* and *d a* on which the balance is mainly struck in this case, than of *a b* and *c d*, the corresponding most influential arcs in the other.

(680.) In intermediate situations of the line of apsides, the effect will be intermediate, and there will of course be a situation of them in which on an average of a whole revolution, they are stationary. This situation it is easy to see will be nearer to the line of quadratures than of syzygies, and the preponderance of advance will be maintained over a much more considerable arc than that of recess, among the possible situations which they can hold. On every account, therefore, the action of the normal force causes the lunar apsides to progress *in a complete revolution of* M or in a synodical year, during which the motion of the sun round the earth (as we consider the earth at rest) brings the line of syzygies into all situations with respect to that of apsides.

(681.) Let us next consider the action of the tangential force. And as before (Case 1), supposing the perigee of the moon at A, and the direction of her revolution to be

A D B E, the tangential force *retards* her motion through the quadrant A D, in which she *recedes* from S, therefore by art. 670, the apsides *recede.* Through D B the force *accelerates*, while the moon still *recedes*, therefore they *advance.* Through B E the force retards, and the moon approaches, therefore they continue to advance, and finally throughout the quadrant E A the force accelerates and the moon approaches, therefore they recede. In virtue therefore of this force, the apsides recede, during the description of the arc E A D, and advance during D B E, but the force being in this case as in that of the normal force more powerful at apogee, the latter will preponderate, and the apsides will advance on an average of a whole revolution.

(682.) (Case 2.) The perigee being toward B, we have to substitute in the foregoing reasoning approach to S, for recess from it, and *vice versâ*, the accelerations and retardations remaining as before. Therefore the results, as far as direction is concerned, will be reversed in each quadrant, the apsides advance during E A D and recede during D B E. But the situation of the apogee being also reversed, the predominance remains on the side of E A D, that is, of advance.

(683.) (Case 3.) Apsides in quadratures, perigee near D.—Over quadrant A D, approach and retardation, therefore advance of apsides. Over D B recess and acceleration, therefore again advance; over B E recess and retardation with recess of apsides, and lastly over E A approach and acceleration, producing their continued recess. Total result: advance during the half revolution A D B, and recess during B E A, the acting forces being more powerful in the latter, whence of course a preponderant recess. The same result when the perigee is at E.

(684.) So far the analogy of reasoning between the action of the tangential and normal forces is perfect. But from this point they diverge. It is not here as before. The recess of the apsides in quadratures does not now arise from the predominance of feeble over feebler forces, while that in syzygies results from that of powerful over powerful ones. The maximum accelerating action of the tangential force is equal to its maximum retarding, while the inward action of the normal at its maximum is only half the maximum of its outward. Neither is there that difference in the extent of the arcs over which the balance is struck in this, as in the other case, the action of the tangential force being inward and outward alternately over equal arcs, each a complete quadrant. Whereas, therefore, in tracing the action of the normal force, we found reason to conclude it much more effective to produce progress of the apsides in their syzygy, than in their quadrature situations, we can draw no such conclusion in that of the tangential forces: there being, as regards *that* force, a *complete symmetry* in the four *quadrants*, while in regard of the normal force the symmetry is only a *half-symmetry* having relation to two *semicircles*.

(685.) Taking the average of many revolutions of the sun about the earth, in which it shall present itself in every possible variety of situations to the line of apsides, we see that the effect of the normal force is to produce a rapid advance in the syzygy of the apsides, and a less rapid recess in their quadrature, and on the whole, therefore, a moderately rapid general advance, while that of the tangential is to produce an equally rapid advance in syzygy, and recess in quadrature. Directly, therefore, the tangential force would appear to have no ultimate influence in causing either increase or diminution in the *mean motion* of the apsides resulting from the action of the normal force. It does so, however, indirectly, conspiring in that respect with, and greatly increasing, an indirect action of the normal force in a manner which we shall now proceed to explain.

(686.) The sun moving uniformly, or nearly so, in the same direction as P, the line of apsides when in or near the syzygy, in advancing follows the sun, and therefore remains materially longer in the neighborhood of syzygy than if it rested. On the other hand, when the apsides are in quadrature they recede, and moving therefore contrary to the sun's motion, remain a shorter time in that neighborhood, than if they rested. Thus the advance, already preponderant, is made to preponderate more by its longer continuance, and the recess, already deficient, is rendered still more so by the shortening of its duration.[7] Whatever cause, then, increases directly the rapidity of both advance and recess, *though it may do both equally*, aids in this indirect process, and it is thus that the tangential force becomes effective through the medium of the progress already produced, in doing and aiding the normal force to do that which alone it would be unable to effect. Thus we have perturbation exaggerating perturbation, and thus we see what is meant by geometers, when they declare that a considerable part of the motion of the lunar apsides is due to the square of the disturbing force, or, in other words, arises out of a second approximation in which the influence of the first in altering the data of the problem is taken into account.

(687.) The curious and complicated effect of perturbation, described in the last article, has given more trouble to geometers than any other part of the lunar theory. Newton

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[7] Newton, Princ. i. 66. Cor. 8.

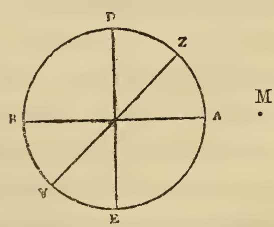
himself had succeeded in tracing that part of the motion of the apogee which is due to the direct action of the radial force; but finding the amount only half what observation assigns, he appears to have abandoned the subject in despair. Nor, when resumed by his successors, did the inquiry, for a very long period, assume a more promising aspect. On the contrary, Newton's result appeared to be even minutely verified, and the elaborate investigations which were lavished upon the subject without success began to excite strong doubts whether this feature of the lunar motions could be explained at all by the Newtonian law of gravitation. The doubt was removed, however, almost in the instant of its origin, by the same geometer, Clairaut, who first gave it currency, and who gloriously repaired the error of his momentary hesitation, by demonstrating the exact coincidence between theory and observation, when the effect of the tangential force is properly taken into the account. The lunar apogee circulates in $3232^{d}{\cdot}575343$, or about $9\frac{1}{2}$ years.

(688.) Let us now proceed to investigate the influence of the disturbing forces so resolved on the excentricity of the lunar orbit, and the foregoing articles having sufficiently familiarized the reader with our mode of following out the changes in different situations of the orbit, we shall take at once a more general situation, and suppose the line of apsides in any position with respect to the sun, such as Z Y, the perigee being at Z, a point between the lower syzygy and the quadrature next following it, the direction of P's motion as all along supposed being A D B E. Then (commencing with the normal force) the momentary change of excentricity will vanish

at *a*, *b*, *c*, *d*, by the vanishing of that force, and at Z and Y by the effect of situation in the orbit annulling its action (art. 671). In the arcs Z *b* and Y *d* therefore the change of excentricity will be small, the acting force nowhere attaining either a great magnitude or an advantageous situation within their limits. And the force within these two arcs having the same character as to inward and outward, but being oppositely influential by reason of the approach of P to S in one of them and its recess in the other, it is evident that, so far as these arcs are concerned, a very near compensation of effects will take place, and though the apogeal arc Y *d* will be somewhat more influential, this will tell for little upon the average of a revolution.

(689.) The arcs *b* D *c* and *d* E *a* are each much less than a quadrant in extent, and the force acting inward throughout them (which at its maximum in D and E is only half the outward force at A, B) degrades very rapidly in intensity toward either syzygy (see art. 676). Hence whether Z be between *b c* or *b* A, the effects of the force in these arcs will not produce very extensive changes on the excentricity, and the changes which it does produce will (for the reason already given) be opposed to each other. Although, then, the arc *a d* be further from perigee than *b c*, and therefore the force in it is greater, yet the predominance of effect here will not be very marked, and will moreover be partially neutralized by the small predominance of an opposite character in Y *d* over Z *b*. On the other hand, the arcs *a* Z, *c* Y are both larger in extent than either of the others, and the seats of action of forces doubly powerful. Their influence, therefore, will be of most importance, and their preponderance one over the other (being opposite in their tendencies) will decide the question whether on an average of the revolution, the excentricity shall increase or diminish. It is clear that the decision must be in favor of *c* Y, the apogeal arc, and, since in this the force is *outward* and the moon *receding* from the earth, an *increase* of the excentricity will arise from its influence. A similar reasoning will, evidently, lead to the same conclusion were the apogee and perigee to change places, for the directions of P's motion as to approach and recess to S will be indeed reversed, but at the same time the dominant forces will have changed sides, and the arc *a* A Z will now give the character to the result. But when Z lies between A and E, as the reader may easily satisfy himself, the case will be altogether different, and the reverse conclusion will obtain. Hence the changes of excentricity emergent on the average of single revolutions from the action of the normal force will be as represented by the signs + and — in the figure above annexed.

(690.) Let us next consider the effect of the tangential force. This retards P in the quadrants A D, B E, and accelerates it in the alternate ones. In the whole quadrant A D, therefore, the effect is of one character, the perigee being less than 90° from every point in it, and in the whole quadrant B E it is of the opposite, the apogee being so situated (art. 670). Moreover, in the middle of each quadrant, the tangential force is at its maximum. Now, in the other quadrants, E A and D B, the change from perigeal to apogeal vicinity takes place, and the tangential force, however powerful, has its effect annulled by situation (art. 670), and this happens more or less nearly about the points where the force is a



maximum. These quadrants, then, are far less influential on the total result, so that the character of that result will be decided by the predominance of one or other of the former quadrants, and will lean to that which has the apogee in it. Now in the quadrant B E the force *retards* the moon and the moon is in apogee. Therefore the excentricity increases. In this situation therefore of the apogee, *such* is the average result of a complete revolution of the moon. Here again also if the perigee and apogee change places, so will also the character of all the partial influences, arc for arc. But the quadrant A D will now preponderate instead of D E, so that under this double reversal of conditions the result will be identical. Lastly, if the line of apsides be in A E, B D, it may be shown in like manner that the excentricity will diminish on the average of a revolution.

(691.) Thus it appears, that in varying the excentricity, precisely as in moving the line of apsides, the direct effect of the tangential force conspires with that of the normal, and tends to increase the extent of the deviations to and fro on either side of a mean value which the varying situation of the sun with respect to the line of apsides gives rise to, having for their period of restoration a synodical revolution of the sun and apse. Supposing the sun and apsis to start together, the sun of course will outrun the apsis (whose period is nine years), and in the lapse of about $(\frac{1}{4} + \frac{1}{32})$ part of a year will have gained on it 90°, during all which interval the apse will have been in the quadrant A E of our figure, and the excentricity continually decreasing. The decrease will then cease, but the excentricity itself will be a minimum, the sun being now at right angles to the line of apsides. Thence it will increase to a maximum when the sun has gained another 90°, and again attained the line of apsides, and so on alternately. The actual effect on the numerical value of the lunar excentricity is very considerable, the greatest and least excentricities being in the ratio of 3 to 2.[8]

(692.) The motion of the apsides of the lunar orbit may be illustrated by a very pretty mechanical experiment, which is otherwise instructive in giving an idea of the mode in which orbitual motion is carried on under the action of central forces variable according to the situation of the revolving body. Let a leaden weight be suspended by a brass or iron wire to a hook in the under side of a firm beam, so as to allow of its free motion on all sides of the vertical, and so that when in a state of rest it shall just clear the floor of the room, or a table placed ten or twelve feet beneath the hook. The point of support should be well secured from wagging to and fro by the oscillation of the weight, which should be sufficient to keep the wire as tightly stretched as it will bear, with the certainty of not breaking. Now, let a very small motion be communicated to the weight, not by merely withdrawing it from the vertical and letting it fall, but by giving it a slight impulse sidewise. It will be seen to describe a regular ellipse about the point of rest as its centre. If the weight be heavy, and carry attached to it a pencil, whose point lies exactly in the direction of the string, the ellipse may be transferred to paper lightly stretched and gently pressed against it. In these circumstances, the situation of the major and minor axes of the ellipse will remain for a long time very nearly the same, though the resistance of the air and the stiffness of the wire will gradually diminish its dimensions and excentricity. But if the impulse communicated to the weight be

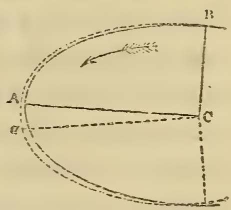
[8] Airy, Gravitation, p. 106.

considerable, so as to carry it out to a great angle (15° or 20° from the vertical), this permanence of situation of the ellipse will no longer subsist. Its axis will be seen to shift its position at every revolution of the weight, advancing in the same direction with the weight's motion, by a uniform and regular progression, which at length will entirely reverse its situation, bringing the direction of the longest excursions to coincide with that in which the shortest were previously made; and so on, round the whole circle; and, in a word, imitating to the eye, very completely, the motion of the apsides of the moon's orbit.

(693.) Now, if we inquire into the cause of this progression of the apsides, it will not be difficult of detection. When a weight is suspended by a wire, and drawn aside from the vertical, it is urged to the lowest point (or rather in a direction at every instant perpendicular to the wire) by a force which varies as the sine of the deviation of the wire from the perpendicular. Now, the sines of very small arcs are nearly in the proportion of the arcs themselves; and the more nearly, as the arcs are smaller. If, therefore, the deviations from the vertical be so small that we may neglect the curvature of the spherical surface in which the weight moves, and regard the curve described as coincident with its projection on a horizontal plane, it will be then moving under the same circumstances as if it were a revolving body attracted to a centre by a force varying directly as the distance; and, in this case, the curve described would be an ellipse, having its centre of attraction not in the focus, but in the centre,[9] and the apsides of this ellipse would remain fixed. But if the excursions of the weight from the vertical be considerable, the force urging it toward the

[9] Newton, Princip. i. 47.

centre will deviate in its law from the simple ratio of the distances; being as the *sine*, while the distances are as the *arc*. Now the sine, though it continues to increase as the arc increases, yet does not increase so fast. So soon as the arc has any sensible extent, the sine begins to fall somewhat short of the magnitude which an exact numerical proportionality would require; and therefore the force urging the weight toward its centre or point of rest at great distances falls, in like proportion, somewhat short of that which would keep the body in its precise elliptic orbit. It will no longer, therefore, have, at those greater distances, the same command over the weight, *in proportion to its speed*, which would enable it to deflect it from its rectilinear tangential course into an ellipse. The true path which it describes will be *less curved in the remoter parts* than is consistent with the elliptic figure, as in the annexed cut; and, therefore, it will not so soon have its motion brought to be again at right angles to the radius. It will require a longer continued action of the central force to do this; and before it is accomplished, more than a quadrant of its revolution must be passed over in angular motion round the centre. But this is only stating at length, and in a more circuitous manner, that fact which is more briefly and summarily expressed by saying that *the apsides of its orbit are progressive.* Nothing beyond a familiar illustration is of course intended in what is above said. The case is not an exact parallel with that of the lunar orbit, the disturbing force being simply radial, whereas in the lunar orbit a transversal force is also concerned, and even were it otherwise, only a confused and indistinct view of apsidal motion can



be obtained from this kind of consideration of the curvature of the disturbed path. If we would obtain a clear one, the two foci of the instantaneous ellipse must be found from the laws of elliptic motion performed under the influence of a force directly as the distance, and the radial disturbing force being decomposed into its tangential and normal components, the momentary influence of either in altering their positions and consequently the directions and lengths of the axis of the ellipse must be ascertained. The student will find it neither a difficult nor an uninstructive exercise to work out the case from these principles, which we cannot afford the space to do.

(694.) The theory of the motion of the planetary apsides and the variation of their excentricities is in one point of view much more simple, but in another much more complicated than that of the lunar. It is simpler, because owing to the exceeding minuteness of the changes operated in the course of a single revolution, the angular position of the bodies with respect to the line of apsides is very little altered by the motion of the apsides themselves. The line of apsides neither follows up the motion of the disturbing body in its state of advance, nor *vice versa*, in any degree capable of prolonging materially their advancing or shortening materially their receding phase. Hence no second approximation of the kind explained in art. 686, by which the motion of the lunar apsides is so powerfully modified as to be actually doubled in amount, is at all required in the planetary theory. On the other hand, the latter theory is rendered more complicated than the former, at least in the cases of planets whose periodic times are to each other in a ratio much less than 13 to 1, by the consideration that the disturbing body shifts its position with respect to the line of apsides by a much greater angular quantity in a revolution of the disturbed body than in the case of the moon. In that case we were at liberty to suppose (for the sake of explanation), without any very egregious error, that the sun held nearly a fixed position during a single lunation. But in the case of planets whose times of revolution are in a much lower ratio this cannot be permitted. In the case of Jupiter disturbed by Saturn for example, in one sidereal revolution of Jupiter, Saturn has advanced in its orbit with respect to the line of apsides of Jupiter by more than 140°, a change of direction which entirely alters the conditions under which the disturbing forces act. And in the case of an exterior disturbed by an interior planet, the situation of the latter with respect to the line of the apsides varies even more rapidly than the situation of the exterior or disturbed planet with respect to the central body. To such cases then the reasoning which we have applied to the lunar perturbations becomes totally inapplicable; and when we take into consideration also the excentricity of the orbit of the disturbing body, which in the most important cases is exceedingly influential, the subject becomes far too complicated for verbal explanation, and can only be successfully followed out with the help of algebraic expression and the application of the integral calculus. To Mercury, Venus, and the earth indeed, as disturbed by Jupiter, and planets superior to Jupiter, this objection to the reasoning in question does not apply; and in each of these cases therefore we are entitled to conclude that the apsides are kept in a state of progression by the action of all the larger planets of our system. Under certain conditions of distance, excentricity, and relative situation of the axes of the orbits of the disturbed and disturbing planets, it is perfectly possible that the reverse may happen, an instance of which is afforded by Venus, whose apsides recede under the combined action of the earth and Mercury more rapidly than they advance under the joint actions of all the other planets. Nay, it is even possible under certain conditions that the line of apsides of the disturbed planet, instead of revolving always in one direction, may librate to and fro within assignable limits, and in a definite and regularly recurring period of time.

(695.) Under any conditions, however, as to these particulars, the view we have above taken of the subject, enables us to assign at every instant, and in every configuration of the two planets, the momentary effect of each upon the perihelion and excentricity of the other. In the simplest case, that in which the two orbits are so nearly circular, that the relative situation of their perihelia shall produce no appreciable difference in the intensities of the disturbing forces, it is very easy to show that whatever temporary oscillations to and fro in the positions of the line of apsides, and whatever temporary increase and diminution in the excentricity of either planet may take place, the final effect on the average of a great multitude of revolutions, presenting them to each other in all possible configurations, must be *nil*, for both elements.

(696.) To show this, all that is necessary is to cast our eyes on the synoptic table in art. 673. If M, the disturbing body, be supposed to be successively placed in two diametrically opposite situations in its orbit, the aphelion of P will stand related to M in one of these situations precisely as its perihelion in the other. Now the orbits being so nearly circles as supposed, the distribution of the disturbing forces, whether normal or tangential, is symmetrical relative to their common diameter passing through M, or to the line of syzygies. Hence it follows that the half of P's orbit "about perihelion" (art. 673) will stand related to all the acting forces in the one situation of M, precisely as the half "about aphelion" does in the other: and also, that the half of the orbit in which P "approaches S," stands related to them in the one situation precisely as the half in which it "recedes from S" in the other. Whether as regards, therefore, the normal or tangential force, the conditions of advance or recess of apsides, and of increase or diminution of excentricities, are reversed in the two supposed cases. Hence it appears that whatever situation be assigned to M, and whatever influence it may exert on P in that situation, that influence will be annihilated in situations of M and P, diametrically opposite to those supposed, and thus, on a general average, the effect on both apsides and excentricities is reduced to nothing.

(697.) If the orbits, however, be excentric, the symmetry above insisted on in the distribution of the forces does not exist. But, in the first place, it is evident that if the excentricities be moderate (as in the planetary orbits), by far the larger part of the effects of the disturbing forces destroys itself in the manner described in the last article, and that it is only a residual portion, viz. that which arises from the greater proximity of the orbits at one place than at another, which can tend to produce permanent or secular effects. The precise estimation of these effects is too complicated an affair for us to enter upon; but we may at least give some idea of the process by which they are produced, and the order in which they arise. In so doing, it is necessary to distinguish between the effects of the normal and tangential forces. The effects of the former are greatest at the point of conjunction of the planets, because the normal force itself is there always at its maximum; and although, where the conjunction takes place at 90° from the line of apsides, its effect to move the apsides is nullified by situation, and when *in* that line its effect on the excentricities is similarly nullified, yet, in the situations rectangular to these, it acts to its greatest advantage. On the other hand, the tangential force vanishes at conjunction, whatever be the place of conjunction with respect to the line of apsides, and where it is at its maximum its effect is still liable to be annulled by situation. Thus it appears that the normal force is most influential, and mainly determines the character of the general effect. It is, therefore, at conjunction that the most influential effect is produced, and therefore, on the long average, those conjunctions which happen about the place where the orbits are nearest will determine the general character of the effect. Now, the nearest points of approach of two ellipses which have a common focus may be very variously situated with respect to the perihelion of either. It may be at the perihelion or the aphelion of the disturbed orbit, or in any intermediate position. Suppose it to be at the perihelion. Then, if the disturbed orbit be *interior* to the disturbing, the force acts outward, and therefore the apsides recede: if exterior, the force acts inward, and they advance. In neither case does the excentricity change. If the conjunction take place at the aphelion of the disturbed orbit, the effects will be reversed: if intermediate, the apsides will be less, and the excentricity more affected.

(698.) Supposing only two planets, this process would go on till the apsides and excentricities had so far changed as to alter the point of nearest approach of the orbits so as either to accelerate or retard and perhaps reverse the motion of the apsides, and give to the variation of the excentricity a corresponding periodical character. But there are many planets all disturbing one another. And this gives rise to variations in the points of nearest approach of all the orbits taken two and two together, of a very complex nature.

(699.) It cannot fail to have been remarked, by any one who has followed attentively the above reasonings, that a close analogy subsists between two sets of relations; viz. that between the inclinations and nodes on the one hand, and between the excentricity and apsides on the other. In fact, the strict geometrical theories of the two cases present a close analogy, and lead to final results of the very same nature. What the variation of excentricity is to the motion of the perihelion, the change of inclination is to the motion of the node. In either case, the period of the one is also the period of the other; and while the perihelia describe considerable angles by an oscillatory motion to and fro, or circulate in immense periods of time round the entire circle, the excentricities increase and decrease by comparatively small changes, and are at length restored to their original magnitudes. In the lunar orbit, as the rapid rotation of the nodes prevents the change of inclination from accumulating to any material amount, so the still more rapid revolution of its apogee effects a speedy compensation in the fluctuations of its excentricity, and never suffers them to go to any material extent; while the same causes, by presenting *in quick succession* the lunar orbit in every possible situation to all the disturbing forces, whether of the sun, the planets, or the protuberant matter at the earth's equator, prevent any secular accumulation of small changes, by which, in the lapse of ages, its ellipticity might be materially increased or diminished. Accordingly, observation shows the *mean* excentricity of the moon's orbit to be the same now as in the earliest ages of astronomy.

(700.) The movements of the perihelia, and variations of excentricity of the planetary orbits, are interlaced and complicated together in the same manner and nearly by the same laws as the variations of their nodes and inclinations. Each acts upon every other, and every such mutual action generates its own peculiar period of circulation or compensation, and every such period, in pursuance of the principle of art. 650, is thence propagated throughout the system. Thus arise cycles upon cycles, of whose compound duration some notion may be formed, when we consider what is the length of one such period in the case of the two principal planets—Jupiter and Saturn. Neglecting the action of the rest, the effect of their mutual attraction would be to produce a secular variation in the excentricity of Saturn's orbit, from 0·08409, its *maximum*, to 0·01345, its *minimum* value: while that of Jupiter would vary between the narrow limits, 0·06036 and 0·02606: the greatest excentricity of Jupiter corresponding to the least of Saturn, and *vice versâ*. The period in which these changes are gone through would be 70414 years. After this example, it will be easily conceived that many millions of years will require to elapse before a complete fulfilment of the joint cycle which shall restore the whole system to its original state as far as the excentricities of its orbits are concerned.

(701.) The place of the perihelion of a planet's orbit is of little consequence to its well-being; but its excentricity is most important, as upon this (the axes of the orbits being permanent) depends the mean temperature of its surface, and the extreme variations to which its seasons may be liable. For it may be easily shown that the *mean annual* *amount* of light and heat received by a planet from the sun is, *cæteris paribus*, as the minor axis of the ellipse described by it. Any variation, therefore, in the excentricity, by changing the minor axis will alter the *mean* temperature of the surface. How such a change will also influence the extremes of temperature appears from art. 368 *et seq.* Now it may naturally be inquired whether (in the vast cycle above spoken of, in which, at some period or other, conspiring changes may accumulate on the orbit of one planet from several quarters) it may not happen that the excentricity of any one planet—as the earth—may become exorbitantly great, so as to subvert those relations which render it habitable to man, or to give rise to great changes, at least, in the physical comfort of his state. To this the researches of geometers have enabled us to answer in the negative. A relation has been demonstrated by Lagrange between the masses, axes of the orbits, and excentricities of each planet, similar to what we have already stated with respect to their inclinations, viz. *that if the mass of each planet be multiplied by the square root of the axis of its orbit, and the product by the square of its excentricity, the sum of all such products throughout the system is invariable;* and as, in point of fact, this sum is extremely small, so it will always remain. Now, since the axes of the orbits are liable to no secular changes, this is equivalent to saying that no one orbit shall increase its excentricity, unless at the expense of a common fund, the whole amount of which is, and must forever remain, extremely minute.[10]

[10] There is nothing in this relation, however, taken *per se*, to secure the smaller planets—Mercury, Mars, Juno, Ceres, etc.—from a catastrophe, could they accumulate on themselves, or any one of them, the whole amount of this *excentricity fund*. But that can never be: Jupiter and Saturn will always retain the lion's share of it. A similar remark applies to the *inclination fund* of art. 639. These *funds*, be it observed, can never get into debt. Every term of them is essentially positive.

(701 *a.*) (1865.) The actual numerical computation of the limiting excentricities of the planets, taking into account all their mutual reactions, was attempted by Lagrange in 1782; but owing to an erroneous assumption of the mass of Venus, his results were rendered uncertain. M. Leverrier, in a remarkable memoir published in 1843, has resumed the subject with the advantage of perfectly reliable *data*, and has obtained the following, as the *superior* limits of excentricities of the seven principal *then known* planets—viz. for that of Mercury, 0·225646; Venus, 0·086716; the Earth, 0·077747; Mars, 0·142243; Jupiter, 0·061548; Saturn, 0·084919; and Uranus, 0·064646. And it is remarkable that although the erroneous assumption in question has vitiated Lagrange's conclusions as to the secular progression in the magnitudes of the excentricities, the superior limits arrived at by him agree very nearly indeed with these. For the *inferior* limit of that of the Earth's orbit, M. Leverrier assigns 0·003314, being the nearest approach it will make to the circular form. This will be attained in 23980 years from the epoch A.D. 1800, for which the calculations are instituted; *i.e.* in A.D. 25780. The triple period of the excentricities of Jupiter, Saturn, and Uranus, taken as a group, is 900,000 years (uncertain to 4000±). The maxima and the minima of that of Saturn are separated by an interval of 34647 years (uncertain to 117±), and its next minimum will happen in A.D. 17914, at which epoch its value will be 0·0136. In the Appendix the reader will find the elements of the earth's orbit, calculated for intervals of 10,000 years from 100,000 years before A.D. 1800 to 100,000 after that date by M. Leverrier, and the excentricities by Mr. Croll for 1,000,000 years before and after the same epoch.

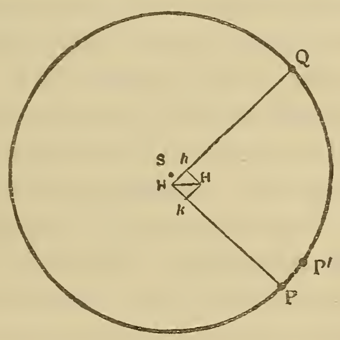
CHAPTER XIV

Of the Inequalities Independent of the Excentricities—The Moon's Variation and Parallactic Inequality—Analogous Planetary Inequalities—Three Cases of Planetary Perturbation Distinguished—Of Inequalities Dependent on the Excentricities—Long Inequality of Jupiter and Saturn—Law of Reciprocity Between the Periodical Variations of the Elements of both Planets—Long Inequality of the Earth and Venus—Variation of the Epoch—Inequalities Incident on the Epoch Affecting the Mean Motion—Interpretation of the Constant Part of these Inequalities—Annual Equation of the Moon—Her Secular Acceleration—Lunar Inequalities Due to the Action of Venus—Effect of the Spheroidal Figure of the Earth and Other Planets on the Motions of their Satellites—Of the Tides—Masses of Disturbing Bodies Deducible from the Perturbations they Produce—Mass of the Moon, and of Jupiter's Satellites, how Ascertained—Perturbations of Uranus Resulting in the Discovery of Neptune—Determination of the Absolute Mass and Density of the Earth

(702.) To calculate the actual place of a planet or the moon, in longitude and latitude at any assigned time, it is not enough to know the changes produced by perturbation in the elements of its orbit, still less to know the *secular* changes so produced, which are only the outstanding or uncompensated portions of much greater changes induced in short periods of configuration. We must be enabled to estimate the actual effect on its longitude of those periodical accelerations and retardations in the rate of its mean angular motion, and on its latitude of those deviations above and below the mean plane of its orbit, which result from the continued action of the perturbative forces, not as compensated in long periods, but as in the act of their generation and destruction in short ones. In this chapter we purpose to give an account of some of the most prominent of the *equations* or inequalities thence arising, several of which are of high historical interest, as having become known by observation previous to the discovery of their theoretical causes, and as having, by their successive explanations from the theory of gravitation, removed what were in some instances regarded as formidable objections against that theory, and afforded in all most satisfactory and triumphant verifications of it.

(703.) We shall begin with those which compensate themselves in a synodic revolution of the disturbed and disturbing body, and which are independent of any permanent excentricity of either orbit, going through their changes and effecting their compensations in orbits slightly elliptic, almost precisely as if they were circular. These inequalities result, in fact, from a circulation of the true upper focus of the disturbed ellipse about its mean place in a curve whose form and magnitude the principles laid down in the last chapter enable us to assign in any proposed case. If the disturbed orbit be circular, this mean place coincides with its centre: if elliptic, with the situation of its upper focus, as determined from the principles laid down in the last chapter.

(704.) To understand the nature of this circulation, we must consider the joint action of the two elements of the disturbing force. Suppose H to be the place of the upper focus, corresponding to any situation P of the disturbed body, and let P P′ be an infinitesimal element of its orbit, described in an instant of time. Then supposing no disturbing force to act, P P′ will be a portion of an ellipse, having H for its focus, equally whether the point P or P′ be regarded. But now let the disturbing forces act during the instant of describing P P′. Then the focus H will shift its position to H′, to find which point we must recollect, 1st. What is demonstrated in art. 671, viz. that the effect of the normal force is to vary the position of the line P′ H so as to make the angle H P H′ equal to double the variation of the angle of tangency due to the action of that force, without altering the distance P H: so that in virtue of the normal force alone, H would move to a point h, along the line H Q, drawn from H to a point Q, 90° in advance of P (because S H being exceedingly small, the angle P H Q may be taken as a right angle when P S Q is so), H *approaching* Q if the normal force act *outward*, but receding



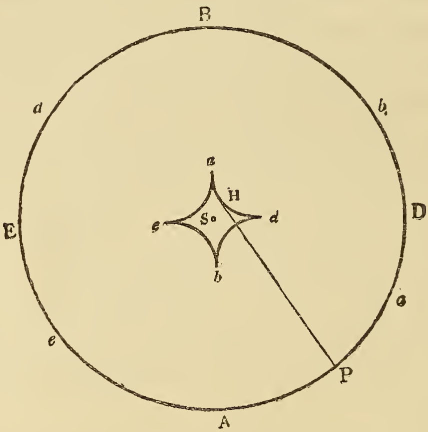
from Q if inward. And similarly the effect of the tangential force (art. 670) is to vary the position of H in the direction H P or P H, according as the force retards or accelerates P's motion. To find H′ then from H draw H P, H Q, to P and to a point of P's orbit 90° in advance of P. On H Q take H h, the motion of the focus due to the normal force, and on H P take H k, the motion due to the tangential force; complete the parallelogram H H′, and its diagonal H H′ will be the element of the true path of H in virtue of the joint action of both forces.

(705.) The most conspicuous case in the planetary system to which the above reasoning is applicable, is that of the moon disturbed by the sun. The inequality thus arising is known by the name of the moon's variation, and was discovered so early as about the year 975 by the Arabian astronomer Aboul Wefa.[1] Its magnitude (or the extent of fluctuation to and fro in the moon's longitude which it produces) is considerable, being no less than 1° 4′, and it is otherwise interesting as being the first inequality produced by perturbation, which Newton succeeded in explaining by the theory of gravity. A good general idea of its nature may be formed by considering the direct action of the disturbing forces on the moon, supposed to move in a circular orbit. In such an orbit undisturbed, the velocity would be uniform; but the tangential force acting to accelerate her motion through the quadrants *preceding* her conjunction and opposition, and to retard it through the alternate quadrants, it is evident that the velocity will have two maxima and two minima, the former at the syzygies, the latter at the quadratures. Hence at the syzygies the velocity will exceed that which corresponds to a circular orbit, and at quadratures will fall short of it. The true orbit will therefore be less curved or more flattened than a circle in syzygies, and more curved (*i.e.* protuberant beyond a circle) in quadratures. This would be the case even were the normal force not to act. But the action of that force increases the effect in question, for at the syzygies, and as far as 64° 14′ on either side of them, it acts outward, or in counteraction of the earth's attraction, and thereby prevents the orbit from being so much curved as it otherwise would be; while at quadratures, and for 25° 46′ on either side of them, it acts inward, aiding the earth's at-

[1] Sedillot, Nouvelles Recherches pour servir à l'Histoire de l'Astronomie chez les Arabes.

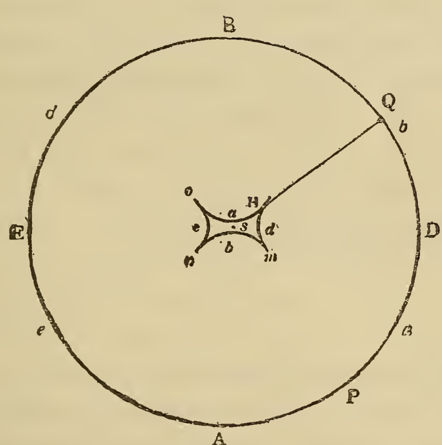
traction, and rendering that portion of the orbit more curved than it otherwise would be. Thus the joint action of both forces distorts the orbit from a circle into a flattened or elliptic form, having the longer axis in quadratures, and the shorter in syzygies; and in this orbit the moon moves faster than with her mean velocity at syzygy (*i.e.* where she is nearest the earth) and slower at quadratures where furthest. Her *angular* motion about the earth is therefore for both reasons greater in the former than in the latter situation. Hence at syzygy her true longitude seen from the earth will be in the act of gaining on her mean—in quadratures of losing, and at some intermediate points (not very remote from the octants) will neither be gaining nor losing. But at these points, *having been gaining or losing* through the whole previous 90°, the *amount of gain or loss* will have attained its maximum. Consequently at the octants the true longitude will deviate most from the mean in excess and defect, and the inequality in question is therefore *nil* at syzygies and quadratures, and attains its maxima in advance or retardation at the octants, which is agreeable to observation.

(706.) Let us, however, now see what account can be rendered of this inequality by the simultaneous variations of the axis and excentricity as above explained. The tangential force, as will be recollected, is *nil* at syzygies and quadratures, and a maximum at the octants, accelerative in the quadrants E A and D B, and retarding in A D and B E. In the two former then the axis is in process of lengthening; in the two latter, shortening. On the other hand the normal force vanishes at (a, b, d, e) 64° 14′ from the syzygies. It acts outward over e A a, b B d, and inward over a D b and d E e. In virtue of the tangential force, then, the point H moves toward P when P is in A D, B E, and from it when in D B, E A, the motion being *nil* when at A, B, D, E, and most rapid when at the octant D, at which points, therefore (so far as this force is concerned), the focus H would have its mean situation. And in virtue of the normal focus, the motion of H in the direction H Q will be at its maximum of rapidity toward Q at A, or B, from Q at D or E, and *nil* at *a*, *b*, *d*, *e*. It will assist us in following out these indications to obtain a notion of the



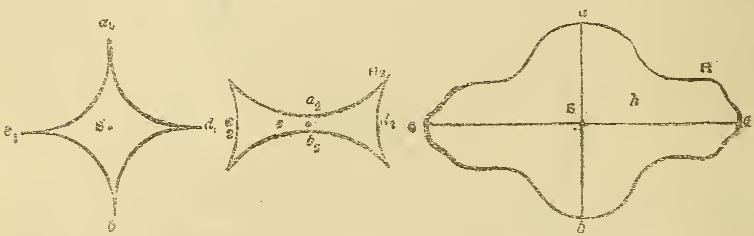
form of the curve really described by H, if we trace separately the paths which H would pursue in virtue of either motion separately, since its true motion will necessarily result from the superposition of these partial motions, because at every instant they are at right angles to each other, and therefore cannot interfere. First, then, it is evident, from what we have said of the tangential force, that when P is at A, H is for an instant at rest, but that as P removes from A toward D, H continually approaches P along their line of junction H P, which is, therefore, at each instant a tangent to the path of H. When P is in the octant, H is at its mean distance from P (equal to P S), and is then in the act of approaching P most rapidly. From thence to the quadrature D the movement of H toward P decreases in rapidity till the quadrature is attained, when H rests for an instant, and then begins to reverse its motion, and travel *from* P at the same rate of progress as before *toward* it. Thus it is clear that, in virtue of the tangential force alone, H would describe a four-cusped curve, *a*, *d*, *b*, *e*, its direction of motion round S in this curve being opposite to that of P, so that A and *a*, D and *d*, B and *b*, E and *e*, shall be corresponding points.

(707.) Next as regards the normal force. When the moon is at A the motion of H is toward D, and is at its



maximum of rapidity, but slackens as P proceeds toward D and as Q proceeds toward B. To the curve described, H Q will be always a tangent, and since at the neutral point of the normal force (or when P is 64° 14′ from A, and Q 64° 14′ from D), the motion of H is for an instant *nil* and is then reversed, the curve will have a cusp at l corresponding, and H will then begin to travel along the arc $l\ m$, while P describes the corresponding arc from neutral point to neutral point through D. Arrived at the neutral point between D and B, the motion of H along Q H will be again arrested and reversed, giving rise to another cusp at m, and so on. Thus, in virtue of the normal force acting alone, the path of H would be the four-cusped, elongated curve $l\ m\ n\ o$, described with a motion round S the reverse of P's, and having a, d, b, e, for points corresponding to A, B, D, E, places of P.

(708.) Nothing is now easier than to superpose these motions. Supposing H_1, H_2 to be the points in either curve corresponding to P, we have nothing to do but to set from off S, S h equal and parallel to S H_1 in the one curve and from h, h H equal and parallel to S H_2 in the other. Let this be done for every corresponding point in the two



curves, and there results an oval curve $a\ d\ b\ e$, having for its semiaxis $Sa = Sa_1 + Sa_2$; and $Sd = Sd_1 + Sd_2$. And this will be the true path of the upper focus, the points a, d, b, e, corresponding to A, D, B, E, places of P. And from this it follows, 1st, that at A, B, the syzygies, the moon is in perigee in her momentary ellipse, the lower focus being nearer than the upper. 2dly, That in quadratures D, E, the moon is in apogee in her *then* momentary ellipse, the upper focus being then nearer than the lower. 3dly, That H revolves in the oval $a\ d\ b\ e$ the contrary way to P in its orbit, making a complete revolution from syzygy to syzygy in one synodic revolution of the moon.

(709.) Taking 1 for the moon's mean distance from the earth, suppose we represent Sa_1 or Sd_1 (for they are equal) by $2a$, Sa_2 by $2b$, and Sd_2 by $2c$, then will the semiaxes of the oval $a\ d\ b\ e$, Sa and Sd be respectively $2a + 2b$ and $2a + 2c$, so that the excentricities of the momentary ellipses at A and D will be respectively $a + b$ and $a + c$. The total amount of the effect of the tangential force on the *axis*, in passing from syzygy to quadrature, will evidently be equal to the length of the curvilinear arc $a_1\ d_1$ (*fig.* art. 708), which is necessarily less than $Sa_1 + Sd_1$ or $4a$. Therefore the total effect on the *semiaxis* or distance of the moon is less than $2a$, and the excess and defect of the greatest and least values of this distance thus varied above and below the mean value $S\ A = 1$ (which call α) will be less than a. The moon then is moving at A *in* the *perigee* of an ellipse whose semiaxis is $1 + \alpha$ and excentricity $a + b$, so that its actual distance from the earth there is $1 + \alpha - a - b$, which (because α is less than a) is less than $1 - b$. Again, at D it is moving *in apogee* of an ellipse whose semiaxis is $1 - \alpha$ and excentricity $a + c$, so that its distance then from the earth is $1 - \alpha + a + c$, which (a being greater than α) is greater than $1 + c$, the latter distance exceeding the former by $2a - 2\alpha + b + c$.

(710.) Let us next consider the corresponding changes induced upon the angular velocity. Now it is a law of elliptic motion that at different points of different ellipses, each differing very little from a circle, the angular velocities are to each other as the square roots of the semiaxes directly, and as the squares of the distances inversely. In this case the semiaxes at A and D are to each other as $1 + a$ to $1 - a$, or as $1 : 1 - 2a$, so that their square roots are to each other as $1 : 1 - a$. Again, the distances being to each other as $1 + a - a - b : 1 - a + a + c$, the inverse ratio of their squares (since a, a, b, c, are all very small quantities) is that of $1 - 2a + 2a + 2c : 1 + 2a - 2a - 2b$, or as $1 : 1 - 4a - 4a - 2b - 2c$. The angular velocities then are to each other in a ratio compounded of these two proportions, that is, in the ratio of

$$1 : 1 + 3a - 4a - 2b - 2c,$$

which is evidently that of a greater to a less quantity. It is obvious also, from the constitution of the second term of this ratio, that the normal force is far more influential in producing this result than the tangential.

(711.) In the foregoing reasoning the sun has been regarded as fixed. Let us now suppose it in motion (in a circular orbit), then it is evident that at equal angles of *elongation* (of P from M seen from S), equal disturbing forces, both tangential and normal, will act: only the syzygies and quadratures, as well as the neutral points of the normal force, instead of being points fixed in longitude on the orbit of the moon, will advance on that orbit with a uniform angular motion equal to the angular motion of the sun. The cuspidated curves $a_1\, d_1\, b_1\, e_1$ and $a_2\, d_2\, b_2\, e_2$, *fig.* art. 708, will, therefore, no longer be re-entering curves; but each will have its cusps *screwed round* as it were in the direction of the sun's motion, so as to increase the angles between them in the ratio of the synodical to the sidereal revolution of the moon (art. 418). And if, in like manner, the motions in these two curves, thus separately described by H, be compounded, the resulting curve, though still (loosely speaking) a species of oval, will not return into itself, but will make successive spiroidal convolutions about S, its furthest and nearest points being in the same ratio more than 90° asunder. And to this movement that of the moon herself will conform, describing a species of elliptic spiroid, having its least distances always in the line of syzygies and its greatest in that of quadratures. It is evident also, that, owing to the longer continued action of both forces, *i.e.* owing to the greater arc over which their intensities increase and decrease by equal steps, the branches of each curve between the cusps will be longer, and the cusps themselves will be more remote from S, and in the same degree will the dimensions of the resulting oval be enlarged, and with them the amount of the inequality in the moon's motion which they represent.

(712.) In the above reasoning the sun's distance is supposed so great, that the disturbing forces in the semi-orbit nearer to it shall not sensibly differ from those in the more remote. The sun, however, is actually nearer to the moon in conjunction than in opposition by about one two-hundredth part of its whole distance, and this suffices to give rise to a very sensible inequality (called the *parallactic inequality*) in the lunar motions, amounting to about 2′ in its effect on the moon's longitude, and having for its period one synodical revolution or one lunation. As this inequality, though subordinate in the case of the moon to the great inequality of the variation with which it stands in connection, becomes a prominent feature in the system of inequalities corresponding to it in the planetary perturbations (by reason of the very great variations of their distances from conjunction to opposition), it will be necessary to indicate what modifications this consideration will introduce into the forms of our focus curves, and of their superposed oval. Recurring then to our figures in arts. 706, 707, and supposing the moon to set out from E, and the upper focus, in each curve from *e*, it is evident that the intercuspidal arcs *e a*, *a d*, in the one, and *e o*, *o a l*, *l d*, in the other, being described under the influence of more powerful forces, will be greater than the arcs *d b*, *b e*, and *d m*, *m b n*, *n e* corresponding in the other half revolution. The two extremities of these curves then, the initial and terminal places of *e* in each, will not meet, and the same conclusion will hold respecting those of the compound oval in which the focus really revolves, which will, therefore, be as in the annexed figure. Thus, at the end of a complete lunation, the focus will have shifted its place from *e* to *f* in a line parallel to the line of quadratures. The next revolution, and the next, the same thing

would happen. Meanwhile, however, the sun has advanced in its orbit, and the line of quadratures has changed its situation by an equal angular motion. In consequence, the next terminal situation (*g*) of the forces will not lie in the line *e f* prolonged, but in a line parallel to the new situation of the line of quadratures, and this process continuing, will evidently give rise to a movement of circulation of the point *e*, round a mean situation in an annual period; and this, it is evident, is equivalent to an annual circulation of the central point of the compound oval itself, in a small orbit about its mean position S. Thus we see that no permanent and indefinite increase of excentricity can arise from this cause; which would be the case, however, but for the annual motion of the sun.

(713.) Inequalities precisely similar in principle to the variation and parallactic inequality of the moon, though greatly modified by the different relations of the dimensions of the orbits, prevail in all cases where planet disturbs planet. To what extent this modification is carried will be evident, if we cast our eyes on the examples given in art. 612, where it will be seen that the disturbing force in conjunction often exceeds that in opposition in a very high ratio (being in the case of Neptune disturbing Uranus more than ten times as great). The effect will be, that the orbit described by the centre of the compound oval about S, will be much greater relatively to the dimensions of that oval itself, than in the case of the moon. Bearing in mind the nature and import of this modification, we may proceed to inquire, apart from it, into the number and distribution of the undulations in the contour of the oval itself arising from the alternations of direction *plus* and *minus* of the disturbing forces in the course of a synodic revolution. But first it should be mentioned that, in the case of an exterior disturbed by an interior planet, the disturbing body's angular motion exceeds that of the disturbed. Hence P, though advancing in its orbit, recedes relatively to the line of syzygies, or, which comes to the same thing, the neutral points of either force overtake it in succession, and each, as it comes up to it, gives rise to a cusp in the corresponding *focus curve*. The angles between the successive cusps will therefore be to the angles between the corresponding neutral points for a fixed position of M, in the same constant ratio of the synodic to the sidereal period of P, which, however, is now a ratio of less inequality. These angles then will be contracted in amplitude, and, for the same reason as before, the excursions of the focus will be diminished, and the more so the shorter the synodic revolution.

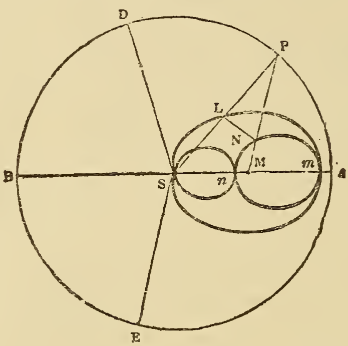
(714.) Since the cusps of either curve recur, in successive synodic revolutions in the same order, and at the same angular distances from each other, and from the line of conjunction, the same will be true of all the corresponding points in the curve resulting from their superposition. In that curve, every cusp, of either constituent, will give rise to a convexity, and every intercuspidal arc to a relative concavity. It is evident then that the compound curve or true path of the focus so resulting, but for the cause above mentioned, would return into itself, whenever the periodic times of the disturbing and disturbed bodies are commensurate,

because in that case the synodic period will also be commensurate with either, and the arc of longitude intercepted between the sidereal place of any one conjunction, and that next following it, will be an aliquot part of 360°. In all other cases it would be a non-re-entering, more or less undulating and more or less regular, spiroid, according to the number of cusps in each of the constituent curves (that is to say, according to the number of neutral points or changes of direction from inward to outward, or from accelerating to retarding, and *vice versâ*, of the normal and tangential forces), in a complete synodic revolution, and their distribution over the circumference.

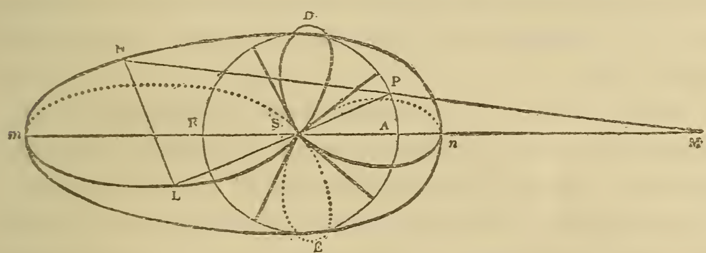
(715.) With regard to these changes, it is necessary to distinguish three cases, in which the perturbations of planet by planet are very distinct in character. 1st. When the disturbing planet is exterior. In this case there are four neutral points of either force. Those of the tangential force occur at the syzygies, and at the points of the disturbed orbit (which we shall call points of equidistance), equidistant from the sun and the disturbing planet (at which points, as we have shown, art. 614, the total disturbing force is always directed inward toward the sun). Those of the normal force occur at points intermediate between these last-mentioned points, and the syzygies, which, if the disturbing planet be *very* distant, hold nearly the situation they do in the lunar theory, *i.e.* considerably nearer the quadratures than the syzygies. In proportion as the distance of the disturbing planet diminishes, two of these points, viz. those nearest the syzygy, approach to each other, and to the syzygy, and in the extreme case, when the dimensions of the orbits are equal, coincide with it.

(716.) The second case is that in which the disturbing planet is interior to the disturbed, but at a distance from the sun greater than half that of the latter. In this case there are four neutral points of the tangential force, and only two of the normal. Those of the tangential force occur at the syzygies, and at the points of equidistance. The force retards the disturbed body from conjunction to the first such points after conjunction, accelerates it thence to the opposition, thence again retards it to the next point of equidistance, and finally again accelerates it up to the conjunction. As the disturbing orbit contracts in dimension the points of equidistance approach; their distance from syzygy from 60° (the extreme case) diminishing to nothing, when they coincide with each other, and with the conjunction. In the case of Saturn disturbed by Jupiter, that distance is only 23° 33′. The neutral points of the normal force lie somewhat beyond the quadratures, on the side of the opposition, and do not undergo any very material change of situation with the contraction of the disturbing orbit.

(717.) The third case is that in which the diameter of the disturbing interior orbit is less than half that of the disturbed. In this case there are only two points of evanescence for either force. Those of the tangential force are the syzygies. The disturbed planet is accelerated throughout the whole semi-revolution from conjunction to opposi-



tion, and retarded from opposition to conjunction, the maxima of acceleration and retardation occurring not far from quadrature. The neutral points of the normal force are situated nearly as in the last case; that is to say, beyond the quadratures toward the opposition. All these varieties the student will easily trace out by simply drawing the figures, and resolving the forces in a series of cases, beginning with a very large and ending with a very small diameter of the disturbing orbit. It will greatly aid him in impressing on his imagination the general relations of the subject, if he construct, as he proceeds, for each case, the elegant and symmetrical ovals in which the points N and L (*fig.* art. 675) always lie, for a fixed position of M, and of which the annexed figure expresses the forms they respectively assume in the third case now under consideration. The second only differs from this, in having the common vertex *m* of both ovals outside of the disturbed orbit A P, while in the case of an exterior disturbing planet the oval *m* L assumes a four-lobed form; its lobes respectively touching the oval *m* N in its vertices, and cutting the orbit A P in the points of equidistance and of tangency (*i.e.* where M P S is a right angle) as in this figure.



(718.) It would be easy now, bearing these features in mind, to trace in any proposed case the form of the spiroid curve, described, as above explained, by the upper focus. It will suffice, however, for our present purpose to remark, 1st, That between every two successive conjunctions of P and M the same general form, the same subordinate undulations, and the same terminal displacement of the upper focus are continually repeated. 2dly, That the motion of the focus in this curve is retrograde whenever the disturbing planet is exterior, and that in consequence the apsides of the momentary ellipse also recede, with a mean velocity such as, but for that displacement, would bring them round at each conjunction to the same relative situation with respect to the line of syzygies. 3dly, That in consequence of this retrograde movement of the apse, the disturbed planet, apart from that consideration, would be twice in perihelio and twice in aphelio in its momentary ellipse in each synodic revolution, just as in the case of the moon disturbed by the sun—and that in consequence of this and of the undulating movement of the focus H itself, an inequality will arise, analogous, *mutatis mutandis* in each case, to the moon's variation, under which term we comprehend (not exactly in conformity to its strict technical meaning in the lunar theory) not only the principal inequality thus arising, but all its subordinate fluctuations. And on this the parallactic inequality thus violently exaggerated is superposed.

(719.) We come now to the class of inequalities which depend for their existence on an appreciable amount of *permanent* excentricity in the orbit of one or of both the disturbing and disturbed planets, in consequence of which all their conjunctions do not take place at equal distances either from the central body or from each other, and therefore that symmetry in every synodic revolution on which depends the exact restoration of both the axis and excentricity to their original values at the completion of each such revolution no longer subsists. In passing from conjunction to conjunction, then, there will no longer be effected either a complete restoration of the upper focus to the same relative situation, or of the axis to the same length which they respectively had at the outset. At the same time it is not less evident that the differences in both respects are only what remain outstanding after the compensation of by far the greater part of the deviations to and fro from a mean state which occur in the course of the revolution; and that they amount to but small fractions of the total excursions of the focus from its first position, or of the increase and decrease in the length of the axis effected by the direct action of the tangential force—so small, indeed, that, unless owing to peculiar adjustments they be enabled to accumulate again and again at successive conjunctions in the same direction, they would be altogether undeserving of any especial notice in a work of this nature. Such adjustments, however, would evidently exist if the periodic times of the planets were exactly commensurable; since in that case all the possible conjunctions which could ever happen (the elements not being materially changed) would take place at fixed points in longitude, the intermediate points being never visited by a conjunction. Now, of the conjunctions thus distributed, their relations to the lines of symmetry in the orbits being all dissimilar, some one must be more influential than the rest on each of the elements (not necessarily the *same* upon *all*). Consequently, in a complete cycle of conjunctions, wherein each has been visited in its turn, the influence of that one on the element to which it stands so especially related will preponderate over the counteracting and compensating influence of the rest, and thus, although in such a cycle as above specified, a further and much more exact compensation will have been effected in its value than in a single revolution; still that compensation will not be complete, but a portion of the effect (be it to increase or to diminish the excentricity or the axis, or to cause the apse to advance or to recede) will remain outstanding. In the next cycle of the same kind this will be repeated, and the result will be of the same character, and so on, till at length a sensible and ultimately a large amount of change shall have taken place, and in fact until the axis (and with it the mean motion) shall have so altered as to destroy the commensurability of periods, and the apsides have so shifted as to alter the place of the most influential conjunction.

(720.) Now, although it is true that the mean motions of no two planets are exactly commensurate, yet cases are not wanting in which there exists an approach to this adjustment. For instance, in the case of Jupiter and Saturn, a cycle composed of five periods of Jupiter and two of Saturn, although it does not *exactly* bring about the same configuration, does so pretty nearly. Five periods of Jupiter are 21663 days, and two periods of Saturn, 21519 days. The difference is only 146 days, in which Jupiter describes, on an average, 12°, and Saturn about 5°; so that after the lapse of the former interval they will only be 7° from a conjunction in the same parts of their orbits as before. If we calculate the time which will exactly bring about, on the average, three conjunctions of the two planets, we shall find it to be 21760 days, their synodical period being 7253·4 days. In this interval Saturn will have described 8° 6′ in excess of two sidereal revolutions, and Jupiter the same angle in excess of five. Every third conjunction, then, will take place 8° 6′ in advance of the preceding, which is near enough to establish, not, it is true, an identity with, but still a great approach to the case in question. The excess of action, for several such triple conjunctions (7 or 8) in succession, will lie the same way, and at each of them the elements of P's orbit and its angular motion will be similarly influenced, so as to accumulate the effect upon its longitude; thus giving rise to an irregularity of considerable magnitude and very long period, which is well known to astronomers by the name of the great inequality of Jupiter and Saturn.

(721.) The arc 8° 6′ is contained $44\frac{4}{9}$ times in the whole circumference of 360°; and accordingly, if we trace round this particular conjunction, we shall find it will return to the same point of the orbit in so many times 21760 days, or in 2648 years. But the conjunction we are now considering is only one out of three. The other two will happen at points of the orbit about 123° and 246° distant, and *these points also will advance* by the same arc of 8° 6′ in 21760 days. Consequently the period of 2648 years will bring them *all* round, and in that interval each of them will pass through that point of the two orbits from which we commenced: hence *a conjunction* (one or other of the three) will happen at that point once in one-third of this period, or in 883 years; and this is, therefore, the cycle in which the "great inequality" would undergo its full compensation, did the elements of the orbits continue all that time invariable. Their variation, however, is considerable in so long an interval; and, owing to this cause, the period itself is prolonged to about 918 years.

(722.) We have selected this inequality as the most remarkable instance of this kind of action on account of its magnitude, the length of its period, and its high historical interest. It had long been remarked by astronomers, that on comparing together modern with ancient observations of Jupiter and Saturn, the mean motions of these planets did not appear to be uniform. The period of Saturn, for instance, appeared to have been lengthening throughout the whole of the seventeenth century, and that of Jupiter shortening—that is to say, the one planet was constantly lagging behind, and the other getting in advance of its calculated place. On the other hand, in the eighteenth century, a process precisely the reverse seemed to be going on. It is true the whole retardations and accelerations observed were not very great; but, as their influence went on accumulating, they produced, at length, material differences between the observed and calculated places of both these planets, which as they could not then be accounted for by any theory, excited a high degree of attention, and were even, at one time, too hastily regarded as almost subversive of the Newtonian doctrine of gravity. For a long while this difference baffled every endeavor to account for it; till at length Laplace pointed out its cause in the near commensurability of the mean motions, as above shown, and succeeded in calculating its period and amount.

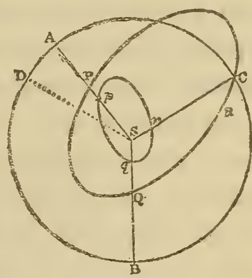
(723.) The inequality in question amounts at its maximum, to an alternate gain and loss of about 0° 49′ in the longitude of Saturn, and a corresponding loss and gain of about 0° 21′ in that of Jupiter. That an acceleration in the one planet must necessarily be accompanied by a retardation in the other, might appear at first sight self-evident, if we consider, that action and reaction being equal, and in contrary directions, whatever momentum Jupiter communicates to Saturn in the direction P M, the same momentum must Saturn communicate to Jupiter in the direction M P. The one, therefore, it might seem to be plausibly argued, will be dragged forward, whenever the other is pulled back in its orbit. The inference is correct, *so far as the general and final result goes;* but the reasoning by which it would, on the first glance, appear to be thus summarily established is fallacious, or at least incomplete. It is perfectly true that whatever momentum Jupiter communicates directly to Saturn, Saturn communicates an equal momentum to Jupiter in an opposite linear direction. But it is not with the absolute motions of the two planets in space that we are now concerned, but with the relative motion of each separately, with respect to the sun regarded as at rest. The *perturbative* forces (the forces which disturb these relative motions) do not act along the line of junction of the planets (art. 614). In the reasoning thus objected to, the attraction of each on the sun has been left out of the account,[2] and it remains to be shown that these attractions neutralize and destroy each other's effects in considerable periods of time, as bearing upon the result in question. Suppose then that we for a moment abandon the point of view, in which we have hitherto all along considered the subject, and regard the sun as free to move, and liable to be displaced by the attractions of the two planets. Then will the movements of all be performed about the common centre of gravity, just as they would have been about the sun's centre regarded as immovable, the sun all the while circulating in a small orbit (with a motion compounded of the two elliptic motions it would have in virtue of their separate attractions) about the same centre. Now in this case M still disturbs P, and P, M, but the whole *disturbing* force now acts along their line of junction, and since it remains true that whatever momentum M generates in P, P will generate the same in M in a contrary direction; it will also be strictly true that, so far as a disturbance of their elliptic motions *about the common centre of gravity of the system is alone regarded*, whatever disturbance of velocity is generated in the one, a contrary dis-

[2] We are here reading a sort of recantation. In the edition of 1833 the remarkable result in question *is* sought to be established by this vicious reasoning. The mistake is a very natural one, and is so apt to haunt the ideas of beginners in this department of physics, that it is worth while expressly to warn them against it.

turbance of velocity (only in the inverse ratio of the masses and modified, though never contradicted, by the directions in which they are respectively moving), will be generated in the other. Now when we are considering only inequalities of long period comprehending many complete revolutions of both planets, and which arise from changes in the axes of the orbits, affecting their *mean motions*, it matters not whether we suppose these motions performed about the common centre of gravity, or about the sun, which never departs from that centre to any material extent (the mass of the sun being such in comparison with that of the planets, that that centre always lies within his surface). The *mean* motion therefore, regarded as the average angular velocity during a revolution, is the same whether estimated by reference to the sun's centre, or to the centre of gravity, or, in other words, the relative mean motion referred to the sun is identical with the absolute mean motion referred to the centre of gravity.

(724.) This reasoning applies equally to every case of mutual disturbance resulting in a long inequality such as may arise from a slow and long-continued periodical increase and diminution of the axes, and geometers have accordingly demonstrated as a consequence from it, that the proportion in which such inequalities affect the *longitudes* of the two planets concerned, or the maxima of the excesses and defects of their longitudes above and below their elliptic values, thence arising, in each, are to each other in the inverse ratio of their masses multiplied by the square roots of the major axes of their orbits, and this result is confirmed by observation, and will be found verified in the instance immediately in question as nearly as the uncertainty still subsisting as to the masses of the two planets will permit.

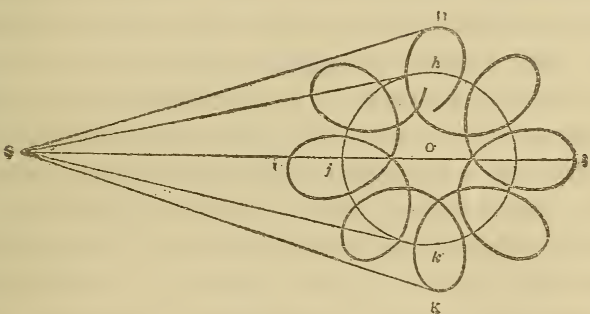
(725.) The inequality in question, as has been observed in general (art. 718), would be much greater, were it not for the partial compensation which is operated in it in every triple conjunction of the planets. Suppose P Q R to be Saturn's orbit, and *p q r* Jupiter's; and suppose a conjunction to take place at P *p*, on the line S A; a second at 123° distance, on the line S B; a third at 246° distance, on S C; and the next at 368°, on S D. This last-mentioned conjunction, taking place nearly in the situation of the first, will produce nearly a repetition of the first effect in retarding or accelerating the planets; but the other two, being in the most remote situations possible from the first, will happen under entirely different circumstances as to the position of the perihelia of the orbits. Now, we have seen that a presentation of the one planet to the other in conjunction, in a variety of situations, tends to produce compensation; and, in fact, the greatest possible amount of compensation which can be produced by only three conjunctions is when they are thus equally distributed round the centre. Hence we see that it is not the whole amount of perturbation which is thus accumulated in each triple conjunction, but only that small part which is left uncompensated by the intermediate ones. The reader, who possesses already some acquaintance with the subject, will not be at a loss to perceive how this consideration is, in fact, equivalent to that part of the geometrical investigation of this inequality which leads us to seek its expression in terms of the third order, or involving the cubes and products of three dimensions of the excentricities and inclinations: and how the continual accumula-



tion of small quantities, during long periods, corresponds to what geometers intend when they speak of small terms receiving great accessions of magnitude by the introduction of large coefficients in the process of integration.

(726.) Similar considerations apply to every case of approximate commensurability which can take place among the mean motions of any two planets. Such, for instance, is that which obtains between the mean motion of the earth and Venus—13 times the period of Venus being very nearly equal to 8 times that of the earth. This gives rise to an extremely near coincidence of every fifth conjunction, in the same parts of each orbit (within $\frac{1}{240}$th part of a circumference), and therefore to a correspondingly extensive accumulation of the resulting uncompensated perturbation. But, on the other hand, the part of the perturbation thus accumulated is only that which remains outstanding after passing the equalizing ordeal of five conjunctions equally distributed round the circle; or, in the language of geometers, is dependent on powers and products of the excentricities and inclinations of the fifth order. It is, therefore, extremely minute, and the whole resulting inequality, according to the elaborate calculations of Mr. Airy, to whom it owes its detection, amounts to no more than a few seconds at its maximum, while its period is no less than 240 years. This example will serve to show to what minuteness these inquiries have been carried in the planetary theory.

(727.) That variations of long period arising in the way above described are necessarily accompanied by similarly periodical displacements of the upper focus, equivalent in their effect to periodical fluctuations in the magnitude of the excentricity, and in the position of the line of apsides, is evident from what has been already said respecting the motion of the upper focus under the influence of the disturbing forces. In the case of circular orbits the *mean* place of H coincides with S the centre of the sun, but if the orbits have any independent ellipticity, this coincidence will no longer exist—and the *mean* place of the upper focus will come to be inferred from the average of all the situations which it actually holds during an entire revolution. Now the fixity of this point depends on the equality of each of the branches of the cuspidated curves, and consequent equality of excursion of the focus in each particular direction, in every successive situation of the line of conjunction. But if there be



some one line of conjunction in which these excursions are greater in any one particular direction than in another, the mean place of the focus will be displaced, and if this process be repeated, that mean place will continue to deviate more and more from its original position, and thus will arise a circulation of the *mean* place *of the focus for a revolution* about another mean situation, the average of all the former mean places during a complete cycle of conjunctions. Supposing S to be the sun, O the situation the upper focus would have, had these inequalities no existence, and H K the path of the upper focus, which it pursues about O by reason of them, then it is evident that in the course of a complete cycle of the inequality in question, the excentricity will have fluctuated between the extreme limits S J and S I and the direction of the longer axis between the extreme position S H and S K, and that if we suppose *i j h k* to be the corresponding mean places of the focus, *i j* will be the extent of the fluctuation of the mean excentricity, and the angle *h s k*, that of the longitude of the perigee.

(728.) The periods then in which these fluctuations go through their phases are necessarily equal in duration with that of the inequality in longitude, with which they stand in connection. But it by no means follows that their maxima all coincide. The variation of the axis to which that of the mean motion corresponds, depends on the tangential force only whose maximum is not at conjunction or opposition, but at points remote from either, while the excentricity depends both on the normal and tangential forces, the maximum of the former of which is at the conjunction. That particular conjunction, therefore, which is most influential on the axis, is not so on the excentricity, so that it can by no means be concluded that either the maximum value of the axis coincides with the maximum, or the minimum of the excentricity, or with the greatest excursion to or fro of the line of apsides from its mean situation, all that can be safely asserted is, that as either the axis or the excentricity of the one orbit varies, that of the other will vary in the opposite direction.

(729.) The primary elements of the lunar and planetary orbits, which may be regarded as variable, are the longitude of the node, the inclination, the axis, excentricity, longitude of the perihelion, and epoch (art. 496). In the foregoing articles we have shown in what manner each of the first five of these elements is made to vary, by the direct action of the perturbing forces. It remains to explain in what manner the last comes to be affected by them. And here it is necessary, in the first instance, to remove some degree of obscurity which may be thought to hang about the sense in which the term itself is to be understood in speaking of an orbit, every other element of which is regarded as in a continual state of variation. Supposing, then, that we were to reverse the process of calculation described in arts. 499 and 500 by which a planet's heliocentric longitude in an elliptic orbit is computed for a given time; and setting out with a heliocentric longitude ascertained by observation, all the *other* elements being known, we were to calculate either what mean longitude the planet had at a given epochal time, or, which would come to the same thing, at what moment of time (thenceforward to be assumed as an epoch) it had a given mean longitude. It is evident that by this means the epoch, if not otherwise known, would become known, whether we consider it as the moment of time corresponding to a convenient mean longitude, or as the mean longitude corresponding to a convenient time. The latter way of considering it has some advantages in respect of general convenience, and astronomers are in agreement in employing, as an element under the title "Epoch of the mean longitude," the mean longitude of the planet so computed for a fixed date; as, for instance, the commencement of the year 1800, mean time at a given place. Supposing now all the elements of the orbit invariable, if we were to go through this reverse process, and thus ascertain the epoch (so defined) from any number of different perfectly correct heliocentric longitudes, it is clear we should always come to the same result. One and the same "epoch" would come out from all the calculations.

(730.) Considering then the "epoch" in this light, as merely a result of this reversed process of calculation, and not as the direct result of an observation instituted for the purpose at the precise epochal moment of time (which would be, generally speaking, impracticable), it might be conceived subject to variation in two distinct ways, viz. dependently and independently. Dependently it must vary, as a necessary consequence of the variation of the other elements; because, if setting out from one and the same observed heliocentric longitude of the planet, we calculate back to the epoch with two different sets of intermediate elements, the one set consisting of those which it had immediately before its arrival at that longitude, the other that which it takes up immediately after (*i.e.* with an unvaried and a varied system), we cannot (unless by singular accident of mutual counteraction) arrive at the same result; and the difference of the results is evidently the variation of the epoch. On the other hand, however, it cannot vary independently; for since this is the only mode in which the unvaried and varied epochs can become known, and as both result from direct processes of calculation involving only given data, the results can only differ by reason of the difference of those data. Or we may argue thus. The change in the path of the planet, and its place in that path so changed, at any future time (supposing it to undergo no further variation), are entirely owing to the change in its velocity and direction, produced by the disturbing forces at the point of disturbance; now these latter changes (as we have above seen) are *completely* represented by the momentary change in the situation of the upper focus, taken in combination with the momentary variation in the plane of the orbit; and these therefore express the total effect of the disturbing forces. There is, therefore, no direct and specific action on the epoch as an independent variable. It is simply left to accommodate itself to the altered state of things in the mode already indicated.

(731.) Nevertheless, should the effects of perturbation by inducing changes on these other elements affect the mean longitude of the planet in any other way than can be considered as properly taken account of, by the varied periodic time due to a change of axis, such effects must be regarded as incident on the epoch. This is the case with a very curious class of perturbations which we are now to consider, and which have their origin in an alteration of the average distance at which the disturbed body is found at every instant of a complete revolution, distinct from, and not brought about by the variation of the major semi-axis, or *momentary* "*mean distance*" which is an imaginary magnitude, to be carefully distinguished from the average of the actual distances now contemplated. Perturbations of this class (like the moon's variation, with which they are intimately connected) are independent on the excentricity of the disturbed orbit; for which reason we shall simplify our treatment of this part of the subject, by supposing that orbit to have no permanent excentricity, the upper focus in its successive displacements merely revolving about a mean position coincident with the lower. We shall also suppose M very distant, as in the lunar theory.

(732.) Referring to what is said in arts. 706 and 707, and to the figures accompanying those articles, and considering first the effect of the tangential force, we see that besides the effect of that force in changing the length of the axis, and consequently the periodic time, it causes the upper focus H to describe, in each revolution of P, a four-cusped curve, *a*, *d*, *b*, *e*, about S, all whose intercuspidal arcs are similar and equal. This supposes M fixed, and at an invariable distance—suppositions which simplify the relations of the subject, and (as we shall afterward show) do not affect the general nature of the conclusions to be drawn. In virtue, then, of the excentricity thus given rise to, P will be at the perigee of its momentary ellipse at syzygies and in its apogee at quadratures. *Apart, therefore, from the change arising from the variation of axis*, the distance of P from S will be less at syzygies, and greater at quadratures, than in the original circle. But the average of all the distances during a whole revolution will be unaltered; because the distances of *a*, *d*, *b*, *e* from S being equal, and the arcs symmetrical, the approach in and about perigee will be equal to the recess in and about apogee. And, in like manner, the effect of the changes going on in the length of the axis itself, on the average in question, is *nil*, because the alternate increases and decreases of that length balance each other in a complete revolution. Thus we see that the tangential force is excluded from all influence in producing the class of perturbations now under consideration.

(733.) It is otherwise with respect to the normal force. In virtue of the action of that force the upper focus describes, in each revolution of P, the four-cusped curve (*fig.* art. 707), whose intercuspidal arcs are alternately of very unequal extent, arising, as we have seen, from the longer duration and greater energy of the outward than of the inward action of the disturbing force. Although, therefore, in perigee at syzygies and in apogee at quadratures, the apogeal recess is much greater than the perigeal approach, inasmuch as S d greatly exceeds S a. On the average of a whole revolution, then, the recesses will preponderate, and the average distance will therefore be greater in the disturbed than in the undisturbed orbit. And it is manifest that this conclusion is quite independent of any change in the length of the axis, which the normal force has no power to produce.

(734.) But neither does the normal force operate any change of linear velocity in the disturbed body. When carried out, therefore, by the effect of that force to a greater distance from S, the angular velocity of its motion round S will be diminished: and contrariwise when brought nearer. The average of all the momentary angular motions, therefore, will decrease with the increase in that of the momentary distances; and in a higher ratio, since the angular velocity, under an equable description of areas, is inversely as the square of the distance, and the disturbing force, being (in the case supposed) directed to or from the centre, does not disturb that equable description (art. 616). Consequently, on the average of a whole revolution, the angular motion is slower, and therefore the time of completing a revolution, and returning to the same longitude, longer than in the undisturbed orbit, and *that* independent of and without any reference to the length of the momentary axis, and the "periodic time" or "mean motion" dependent thereon. We leave to the reader to follow out (as is easy to do) the same train of reasoning in the cases of planetary perturbation, when M is not very remote, and when it is interior to the disturbed orbit. In the latter case the preponderant effect changes from a retardation of angular velocity to an acceleration, and the dilatation of the average dimensions of P's orbit to a contraction.

(735.) The above is an accurate analysis, according to strict dynamical principles, of an effect which, speaking roughly, may be assimilated to an alteration of M's gravitation toward S by the mean preponderant amount of the outward and inward action of the normal forces constantly exerted—nearly as would be the case if the mass of the disturbing body were formed into a ring of uniform thickness, concentric with S and of such diameter as to exert an action on P everywhere equal to such mean preponderant force, and in the same direction as to inward or outward. For it is clear that the action of such a ring on P, will be the difference of its attractions on the two points P and S, of which the latter occupies its centre, the former is excentric. Now the attraction of a ring on its centre is manifestly equal in all directions, and therefore, estimated in any one direction, is zero. On the other hand, on a point P out of its centre, if *within* the ring, the resulting attraction will always be *outward*, toward the nearest point of the ring, or directly from the centre.[3] But if P lie without the ring, the

[3] As this is a proposition which the equilibrium of Saturn's ring renders not merely speculative or illustrative, it will be well to demonstrate it; which may be done very simply, and without the aid of any calculus. Conceive a spherical shell, and a point within it; every line passing through the point, and terminating both ways in the shell, will, of course, be equally inclined to its surface at either end, being a chord of a spherical surface, and, therefore symmetrically related to all its parts. Now, conceive a small double cone, or pyramid, having its apex at the point, and formed by the conical motion of such a line round the point. Then will the two portions of the spherical shell, which form the bases of both the cones, or pyramids, be similar and equally inclined to their axes. Therefore their areas will be to each other as the squares of their distance from the common apex. Therefore their attractions on it will be equal, because the attraction is as the attracting matter directly, and the square of its distance inversely. Now, these attractions act in opposite directions, and therefore counteract each other. Therefore the point is in equilibrium between them; and as the same is true of every such pair of areas into which the spherical shell can be broken up, therefore the point will be in equilibrium *however situated within* such a spherical shell. Now take a ring, and treat it similarly, breaking its

resulting force will act always *inward*, urging P toward its centre. Hence it appears that the mean effect of the radial force of the ring will be different in its direction, according as the orbit of the disturbing body is exterior or interior to that of the disturbed. In the former case it will act in diminution, in the latter in augmentation of the central gravity.

(736.) Regarding, still, only the mean effect, as produced in a great number of revolutions of both bodies, it is evident that such an increase of central force will be accompanied with a diminution of periodic time and distance of a body revolving with a stated velocity, and *vice versâ*. This, as we have shown, is the first and most obvious effect of the radial part of the disturbing force, when exactly analyzed. It alters permanently, and by a certain mean amount, the distances and times of revolution of all the bodies composing the planetary system, from what they would be, did each planet circulate about the sun uninfluenced by the attraction of the rest; the angular motion of the interior bodies of the system being thus rendered less, and those of the exterior greater, than on that supposition. The latter effect, indeed, might be at once concluded from this obvious consideration—that all the planets revolving interiorly to any orbit may be considered as adding to the general aggregate of the attracting matter within, which is not the less efficient for being distributed over space, and maintained in a state of circulation.

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circumference up into pairs of elements, the bases of *triangles* formed by lines passing through the attracted point. Here the attracting elements being *lines*, not *surfaces*, are in the *simple* ratio of the distances, not the *duplicate*, as they should be to maintain the equilibrium. Therefore it will not be maintained, but the *nearest* elements will have the superiority, and the point will, on the whole, be urged toward the nearest part of the ring. The same is true of every *linear* ring, and is therefore true of any assemblage of concentric ones forming a flat annulus, like the ring of Saturn.

(737.) This effect, however, is one which we have no means of measuring, or even of detecting, otherwise than by calculation. For our knowledge of the periods of the planets is drawn from observations made on them in their actual state, and therefore under the influence of this, which may be regarded as a sort of *constant part* of the perturbative action. Their observed mean motions are therefore affected by the whole amount of its influence; and we have no means of distinguishing this by observation from the direct effect of the sun's attraction, with which it is blended. Our knowledge, however, of the masses of the planets assures us that it is extremely small; and this, in fact, is all which it is at all important to us to know, in the theory of their motions.

(738.) The action of the sun upon the moon, in like manner, tends, by its mean influence during many successive revolutions of both bodies, to increase permanently the moon's distance and periodic time. But this general average is not established, either in the case of the moon or planets, without a series of subordinate fluctuations, which we have purposely neglected to take account of in the above reasoning, and which obviously tend, in the average of a great multitude of revolutions, to neutralize each other. In the lunar theory, however, some of these subordinate fluctuations are very sensible to observation. The most conspicuous of these is the moon's annual equation; so called because it consists in an alternate increase and decrease in her longitude, corresponding with the earth's situation in its annual orbit; *i.e.* to its angular distance from the perihelion, and therefore having a year instead of a month, or aliquot part of a month, for its period. To understand the mode of its production, let us suppose the sun, still holding a fixed position in longitude, to approach gradually nearer to the earth. Then will all its disturbing forces be gradually increased in a very high ratio compared with the diminution of the distance (being inversely as its cube; so that its effects of every kind are three times greater in respect of any change of distance, than they would be by the simple law of proportionality). Hence, it is obvious that the focus H (art. 707) in the act of describing each intercuspidal arc of the curve *a*, *d*, *b*, *e*, will be continually carried out further and further from S; and the curve, instead of returning into itself at the end of each revolution, will open out into a sort of cuspidated spiral, as in the figure annexed. Retracing now the reasoning of art. 733 as adapted to this state of things, it will be seen that so long as this dilatation goes on, so long will the differences between M's recess from S in aphelio and its approach in perihelio (which is equal to the difference of

consecutive long and short semidiameters of this curve) also continue to increase, and with it the average of the distances of M from S in a whole revolution, and consequently also the time of performing such a revolution. The reverse process will go on as the sun again recedes. Thus it appears that, as the sun approaches the earth, the mean angular motion of the moon on the average of a whole revolution will diminish, and the duration of each lunation will therefore exceed that of the foregoing, and *vice versâ.*

(739.) The moon's orbit being supposed circular, the sun's orbitual motion will have no other effect than to keep the moon longer under the influence of every gradation of the disturbing force, than would have been the case had his situation in longitude remained unaltered (art. 711). The same effects, therefore, will take place, only on an increased scale in the proportion of the increased time; *i.e.* in the proportion of the synodic to the sidereal revolution of the moon. Observation confirms these results, and assigns to the inequality in question a maximum value of between 10′ and 11′, by which the moon is at one time in advance of, and at another behind, its mean place, in consequence of this perturbation.

(740.) To this class of inequalities we must refer one of great importance, and extending over an immense period of time, known by the name of the *secular acceleration of the moon's mean motion.* It had been observed by Dr. Halley, on comparing together the records of the most ancient lunar eclipses of the Chaldean astronomers with those of modern times, that the period of the moon's revolution at present is sensibly shorter than at that remote epoch; and this result was confirmed by a further comparison of both sets of observations with those of the Arabian astronomers of the eighth and ninth centuries. It appeared, from these comparisons, that the rate at which the moon's mean motion increases is about 11 seconds per century—a quantity small in itself, but becoming considerable by its accumulation during a succession of ages. This remarkable fact, like the great equation of Jupiter and Saturn, had been long the subject of toilsome investigation to geometers. Indeed, so difficult did it appear to render any exact account of, that while some were on the point of again declaring the theory of gravity inadequate to its explanation, others were for rejecting altogether the evidence on which it rested, although quite as satisfactory as that on which most historical events are credited. It was in this dilemma that Laplace once more stepped in to rescue physical astronomy from its reproach, by pointing out the real cause of the phenomenon in question, which, when so explained, is one of the most curious and instructive in the whole range of our subject—one which leads our speculations further into the past and future, and points to longer vistas in the dim perspective of changes which our system has undergone and is yet to undergo, than any other which observation assisted by theory has developed.

(741.) The year is not an exact number of lunations. It consists of twelve and a fraction. Supposing then the sun and moon to set out from conjunction together; at the twelfth conjunction subsequent the sun will not have returned precisely to the same point of its annual orbit, but will fall somewhat short of it, and at the thirteenth will have overpassed it. Hence in twelve lunations the gain of longitude during the first half year will be somewhat under and in thirteen somewhat over compensated. In twenty-six it will be nearly twice as much overcompensated, in thirty-nine not quite so nearly three times as much, and so on, until, after a certain number of such multiples of a lunation have elapsed, the sun will be found half a revolution in advance, and in place of receding further at the expiration of the next, it will have begun to approach. From this time every succeeding cycle will destroy some portion of that overcompensation, until a complete revolution of the sun in excess shall be accomplished. Thus arises a subordinate or rather supplementary inequality, having for its period as many years as is necessary to multiply the deficient arc into a whole revolution, at the end of which time a much more exact compensation will have been operated, and so on. Thus after a moderate number of years an almost perfect compensation will be effected, and if we extend our views to centuries we may consider it as quite so. Such at least would be the case if the solar ellipse were invariable. But that ellipse is kept in a continual but excessively slow state of change by the action of the planets on the earth. Its axis, it is true, remains unaltered; but its excentricity is, and has been since the earliest ages, diminishing; and this diminution will continue, as is explained in art. 701 *a*, till the excentricity has attained its minimum value, 0·003314; after which it will again open out into an ellipse, increasing in excentricity up to 0·077747, and then again decrease. The time required for these evolutions, though calculable, has not been calculated, further than to satisfy us that it is not to be reckoned by hundreds or by thousands of years. It is a period, in short, in which the whole history of astronomy and of the human race occupies but as it were a point, during which all its changes are to be regarded as uniform. Now, it is by this variation in the excentricity of the earth's orbit that the secular acceleration of the moon is caused. The compensation above spoken of (even after the lapse of centuries) will now, we see, be only imperfectly effected, owing to this slow shifting of one of the essential data. The steps of restoration are no longer identical with, nor equal to, those of change. The struggle up hill is not maintained on equal terms with the downward tendency. The ground is all the while slowly sliding beneath the feet of the antagonists. During the whole time that the earth's excentricity is diminishing, a preponderance is given to the reaction over the action; and it is not till that diminution shall cease, that the tables will be turned, and the process of ultimate restoration will commence. Meanwhile, a minute, outstanding, and uncompensated effect in favor of acceleration is left at each recurrence, or near recurrence, of the same configurations of the sun, the moon, and the solar perigee. These accumulate, and at length affect the moon's longitude to an extent not to be overlooked.

(742.) The phenomenon, of which we have now given an account, is another and very striking example of the propagation of a periodic change from one part of a system to another. The planets, with one exception, have no direct appreciable action on the lunar motions as referred to the earth. Their masses are too small, and their distances too great, for their difference of action on the moon and earth ever to become sensible. Yet their effect on the earth's orbit is thus, we see, propagated through the sun to that of the moon; and, what is very remarkable, the transmitted effect thus indirectly produced on the angle described by the moon round the earth is more sensible to observation than that directly produced by them on the angle described by the earth round the sun.

(743.) Referring to the reasoning of art. 738, we shall perceive that if, owing to any other cause than its elliptic motion, the sun's distance from the earth be subject to a periodical increase and decrease, that variation will give rise to a lunar inequality of equal period analogous to the annual equation. It thus happens that very minute changes impressed on the orbit of the earth, by the direct action of the planets (provided their periods, though not properly speaking secular, be of considerable length), may make themselves sensible in the lunar motions. The longitude of that satellite, as observed from the earth, is, in fact, singularly sensible to this kind of reflected action, which illustrates in a striking manner the principle of forced vibrations laid down in art. 650. The reason of this will be readily apprehended, if we consider that however trifling the increase of her longitude which would arise in a single revolution, from a minute and almost infinitesimal increase of her mean angular velocity, that increase is not only repeated in each subsequent revolution, but is reinforced during each by a similar fresh accession of angular motion generated in its lapse. This process goes on so long as the angular motion continues to increase, and only begins to be reversed when lapse of time, bringing round a contrary action on the angular motion, shall have destroyed the excess of velocity previously gained, and begun to operate a retardation. In this respect, the advance gained by the moon on her undisturbed place may be assimilated, during its increase, to the space described from rest under the action of a continually accelerating force. The velocity gained in each instant is not only effective in carrying the body forward during each subsequent instant, but new velocities are every instant generated, and go on adding their cumulative effects to those before produced.

(744.) The distance of the earth from the sun, like that of the moon from the earth, may be affected in its average value estimated over long periods embracing many revolutions, in two modes, conformably to the theory above delivered. 1st, it may vary by a variation in the length of the axis major of its orbit, arising from the direct action of some tangential disturbing force on its velocity, and thereby producing a change of mean motion and periodic time in virtue of the Keplerian law of periods, which declares that the periodic times are in the sesquiplicate ratio of the mean distances. Or, 2dly, it may vary by reason of that peculiar action on the average of actual distances during a revolution, which arises from variations of excentricity and perihelion only, and which produces that sort of change in the mean motion which we have characterized as incident on the epoch. The change of mean motion thus arising, has nothing whatever to do with any variation of the major axis. It does not depend on the change of distance by the Keplerian law of *periods*, but by that of *areas*. The altered mean motion is not sub-sesquiplicate to the altered axis of the ellipse, which in fact does not alter at all, but is *subduplicate* to the altered *average of distances* in a revolution; a distinction which must be carefully borne in mind by every one who will clearly understand either the subject itself, or the force of Newton's explanation of it in the 6th Corollary of his celebrated 66th Proposition. In whichever mode, however, an alteration in the mean motion is effected, if we accommodate the general sense of our language to the specialties of the case, it remains true that every change in the mean motion is accompanied with a corresponding change in the mean distance.

(745.) Now we have seen (art. 726) that Venus produces in the earth a perturbation in longitude, of so long a period (240 years) that it cannot well be regarded without violence to ordinary language, otherwise than as an equation of the mean motion. Of course, therefore, it follows that during that half of this long period of time, in which the earth's motion is retarded, the distance between the sun and earth is on the increase, and *vice versâ*. Minute as is the equation in question, and consequent alteration of solar distance, and almost inconceivably minute as is the effect produced on the moon's mean angular velocity in a single lunation, yet the great number of lunations (1484), during which the effect goes on accumulating in one direction, causes the moon at the moment when that accumulation has attained its maximum to be very sensibly in advance of its undisturbed place (viz. by 23″ of longitude), and after 1484 more lunations, as much in arrear. The calculations by which this curious result has been established, formidable from their length and intricacy, are due to the industry, as the discovery of its origin is to the sagacity, of Professor Hansen.

(746.) The action of Venus, just explained, is indirect, being as it were a sort of reflection of its influence on the earth's orbit. But a very remarkable instance of its influence, in actually perturbing the moon's motions by its direct attraction, has been pointed out, and the inequality due to it computed by the same eminent geometer.[4] As the details of his processes have not yet appeared, we can here only explain, in general terms, the principle on which the result in question depends, and the nature of the peculiar adjustment of the mean angular velocities of the earth and Venus which render it effective. The disturbing forces of Venus on the moon are capable of being represented or expressed (as is indeed generally the case with all the forces concerned in producing planetary disturbance) by the substitution for them of a series of other forces, each having a period or cycle within which it attains a maximum in one direction, decreases to nothing, reverses its action, attains a maximum in the opposite direction, again decreases to nothing, again reverses its action and reattains its former magnitude, and so on. These cycles differ for each particular constituent or *term*, as it is called, of the total forces considered as so broken up into partial ones, and generally speaking, every combination which can be formed by subtracting a multiple of the mean motion of one of the bodies concerned from a multiple of that of the other, and, when there are three

[4] Astronomische Nachrichten, No. 597.

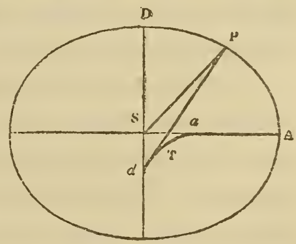
bodies disturbing one another, every such triple combination becomes, under the technical name of an argument, the cyclical representative of a force acting in the manner and according to the law described. Each of these periodically acting forces produces its perturbative effect, according to the law of the superposition of small motions, as if the others had no existence. And if it happen, as in an immense majority of cases it does, that the cycle of any particular one of these partial forces has no relation to the periodic time of the disturbed body, so as to bring it to the same, or very nearly the same point of its orbit, or to any situation favorable to any particular form of disturbance, over and over again when the force is at its maximum; that force will, in a few revolutions, neutralize its own effect, and nothing but fluctuations of brief duration can result from its action. The contrary will evidently be the case, if the cycle of the force coincide so nearly with the cycle of the moon's anomalistic revolution, as to bring round the maximum of the force acting in one and the same direction (whether tangential or normal) either accurately, or very nearly indeed to some definite point, as, for example, the apogee of her orbit. Whatever the effect produced by such a force on the angular motion of the moon, if it be not exactly compensated in one cycle of its action, it will go on accumulating, being repeated over and over again under circumstances very nearly the same, for many successive revolutions, until at length, owing to the want of precise accuracy in the adjustment of that cycle to the anomalistic period, the maximum of the force (in the same phase of its action) is brought to coincide with a point in the orbit (as the perigee), determinative of an opposite effect, and thus, at length, a compensation will be worked out; in a time, however, so much the longer as the difference between the cycle of the force and the moon's anomalistic period is less.

(747.) Now, in fact, in the case of Venus disturbing the moon, there exists a cyclical combination of this kind. Of course the disturbing force of Venus on the moon varies with her distance from the earth, and this distance again depends on her configuration with respect to the earth and the sun, *taking into account the ellipticity of both their orbits.* Among the combinations which take their rise from this latter consideration, and which, as may easily be supposed, are of great complexity, there is a term (an exceedingly minute one), whose argument or cycle is determined by subtracting 16 times the mean motion of the earth from 18 times that of Venus. The difference is so very nearly the mean motion of the moon in her anomalistic revolution, that whereas the latter revolution is completed in $27^d\ 13^h\ 18^m\ 32{\cdot}3^s$, the cycle of the force is completed in $27^d\ 13^h\ 7^m\ 35{\cdot}6^s$, differing from the other by no more than $10^m\ 56{\cdot}7^s$, or about one 3625th part of a complete period of the moon from apogee to apogee. During half of this very long interval (that is to say, during about 136½ years), the perturbations produced by a force of this character, go on increasing and accumulating, and are destroyed in another equal interval. Although therefore excessively minute in their actual effect on the angular motion, this minuteness is compensated by the number of repeated acts of accumulation, and by the length of time during which they continue to act on the longitude. Accordingly M. Hansen has found the total amount of fluctuation to and fro, or the value of the equation of the moon's longitude, so arising to be $27{\cdot}4''$. It is exceedingly interesting to observe that the two equations considered in these latter paragraphs, account satisfactorily for the only remaining material differences between theory and observation in the modern history of this hitherto rebellious satellite. We have not thought it necessary (indeed it would have required a treatise on the subject) to go into a special account of the almost innumerable other lunar inequalities which have been computed and tabulated, and which are necessary to be taken into account in every computation of her place from the tables. Many of them are of very much larger amount than these. We ought not, however, to pass unnoticed, that the parallactic inequality, already explained (art. 712), is interesting, as affording a measure of the sun's distance. For this equation originates, as there shown, in the fact that the disturbing forces are not precisely alike in the two halves of the moon's orbit nearest to and most remote from the sun, all their values being greater in the former half. As a knowledge of the relative dimensions of the solar and lunar orbits enables us to calculate *à priori* the amount of this inequality, so a knowledge of that amount deduced by the comparison of a great number of observed places of the moon with tables in which every inequality *but this* should be included, would enable us conversely to ascertain the ratio of the distances in question. Owing to the smallness of the inequality, this is not a very accurate mode of obtaining an element of so much importance in astronomy as the sun's distance, but were it larger (*i.e.* were the moon's orbit considerably larger than it actually is), this would be, perhaps, the most exact method of any by which it could be concluded.

(748.) The greatest of all the lunar inequalities, produced by perturbation, is that called the *evection*. It arises directly from the variation of the excentricity of her orbit, and from the fluctuation to and fro in the general progress of the line of apsides, caused by the different situation of the sun, with respect to that line (arts. 685, 691). Owing to these causes the moon is alternately in advance, and in arrear of her elliptic place by about 1° 20′ 30″. This equation was known to the ancients, having been discovered by Ptolemy, by the comparison of a long series of observations handed down to him from the earliest ages of astronomy. The mode in which the effects of these several sources of inequality become grouped together under one principal argument common to them all, belongs, for its explanation, rather to works specially treating of the lunar theory than to a treatise of this kind.

(749.) Some small perturbations are produced in the lunar orbit by the protuberant matter of the earth's equator. The attraction of a sphere is the same as if all its matter were condensed into a point in its centre; but that is not the case with a spheroid. The attraction of such a mass is neither exactly directed to its centre, nor does it exactly follow the law of the inverse squares of the distances. Hence will arise a series of perturbations, extremely small in amount, but still perceptible in the lunar motions; by which the node and the apogee will be affected. A more remarkable consequence of this cause, however, is a small nutation of the lunar orbit, exactly analogous to that which the moon causes in the plane of the earth's equator, by its action on the same elliptic protuberance. And, in general, it may be observed, that in the systems of planets which have satellites, the elliptic figure of the primary has a tendency to bring the orbits of the satellites to coincide with its equator —a tendency which, though small in the case of the earth, yet in that of Jupiter, whose ellipticity is very considerable, and of Saturn especially, where the ellipticity of the body is reinforced by the attraction of the rings, becomes predominant over every external and internal cause of disturbance, and produces and maintains an almost exact coincidence of the planes in question. Such, at least, is the case with the nearer satellites. The more distant are comparatively less affected by this cause, the difference of attractions between a sphere and spheroid diminishing with great rapidity as the distance increases. Thus, while the orbits of all the interior satellites of Saturn lie almost exactly in the plane of the ring and equator of the planet, that of the external satellite, whose distance from Saturn is between sixty and seventy diameters of the planet, is inclined to that plane considerably. On the other hand, this considerable distance, while it permits the satellite to retain its actual inclination, prevents (by parity of reasoning) the ring and equator of the planet from being perceptibly disturbed by its attraction, or being subjected to any appreciable movements analogous to our nutation and precession. If such exist, they must be much slower than those of the earth; the mass of this satellite being, as far as can be judged by its apparent size, a much smaller fraction of that of Saturn than the moon is of the earth; while the solar precession, by reason of the immense distance of the sun, must be quite imperceptible.

(750.) The subject of the tides, though rather belonging to terrestrial physics than properly to astronomy, is yet so directly connected with the theory of the lunar perturbations, that we cannot omit some explanatory notice of it, especially since many persons find a strange difficulty in conceiving the manner in which they are produced. That the sun, or moon, should by its attraction heap up the waters of the ocean under it, seems to them very natural. That it should at the same time heap them up on the opposite side seems, on the contrary, palpably absurd. The error of this class of objectors is of the same kind with that noticed in art. 723, and consists in disregarding the attraction of the disturbing body on the mass of the earth, and looking on it as wholly effective on the superficial water. Were the earth indeed absolutely fixed, held in its place by an external force, and the water left free to move, no doubt the effect of the disturbing power would be to produce a single accumulation vertically under the disturbing body. But it is not by its whole attraction, but by the difference of its attractions on the superficial water at both sides, and on the central mass, that the waters are raised: just as in the theory of the moon, the difference of the sun's attractions on the moon and on the earth (regarded as movable and as obeying that amount of attraction which is due to its situation) gives rise to a *relative* tendency in the moon to recede from the earth in conjunction and opposition, and to approach it in quadratures. Referring to the figure of art. 675, instead of supposing A D B E to represent the moon's orbit, let it be supposed to represent a section of the (comparatively) thin film of water reposing on the globe of the earth, in a great circle, the plane of which passes through the disturbing body M, which we shall suppose to be the moon. The disturbing force on a particle at P will then (exactly as in the lunar theory) be represented in amount and direction by N S, on the same scale on which S M represents the moon's whole attraction on a particle situated at S. This force, applied at P, will urge it in the direction P X parallel to N S; and therefore, when compounded with the direct force of gravity which (neglecting as of no account in this theory the spheroidal form of the earth) urges P toward S, will be equivalent to a single force deviating from the direction P S toward X. Suppose P T to be the direction of this force, which, it is easy to see, will be directed toward a point in D S *produced*, at an extremely small distance below S, because of the excessive minuteness of the disturbing force compared to gravity.[6] Then if this be done at every point of the quadrant A D, it will be evident that the direction P T of the resultant force will be always that of a tangent to the small cuspidated curve *a d* at T, to which tangent the surface of the ocean at P must everywhere be perpendicular, by reason of that law of hydrostatics



which requires the direction of gravity to be everywhere perpendicular to the surface of a fluid *in equilibrio*. The form of the curve D P A, to which the surface of the ocean will tend to conform itself, so as to place itself everywhere in equilibrio under two acting forces, will be that which always has P T for its radius of curvature. It will therefore be slightly less curved at D, and more so at A, being in fact no other than an ellipse, having S for its centre, *d a* for its *evolute*, and S A, S D for its longer and shorter semiaxes respectively; so that the whole surface (supposing it covered with water) will tend to assume, as its form of equilibrium, that of an oblongated ellipsoid, having its longer axis

[6] According to Newton's calculation, the maximum disturbing force of the sun on the water does not exceed one 25736400th part of its gravity. That of the moon will therefore be to this fraction as the cube of the sun's distance to that of the moon's directly, and as the mass of the sun to that of the moon inversely, *i.e.* as $(400)^3 \times 0{\cdot}011364 : 354936$, which, reduced to numbers, gives, for the moon's maximum of power to disturb the waters, about one 12560000th of gravity, or somewhat less than 2½ times the sun's.

directed toward the disturbing body, and its shorter of course at right angles to that direction. The difference of the longer and shorter semiaxes of this ellipsoid due to the moon's attraction would be about 58 inches: that of the ellipsoid, similarly formed in virtue of the sun, about 2½ times less, or about 23 inches.

(751.) Let us suppose the moon only to act, and to have no orbitual motion; then if the earth also had no diurnal motion, the ellipsoid of equilibrium would be quietly formed, and all would be thenceforward tranquil. There is never time, however, for this spheroid to be fully formed. Before the waters can take their level, the moon has advanced in her orbit, both diurnal and monthly (for in this theory it will answer the purpose of clearness better, if we suppose the earth's diurnal motion transferred to the sun and moon in the contrary direction), the vertex of the spheroid has shifted on the earth's surface, and the ocean has to seek a new bearing. The effect is to produce an immensely broad and excessively flat wave (not a circulating *current*), which follows, or endeavors to follow, the apparent motions of the moon, and must, in fact, by the principle of forced vibrations, imitate, by equal though not by *synchronous* periods, all the periodical inequalities of that motion. When the higher or lower parts of this wave strike our coasts, they experience what we call high and low water.

(752.) The sun also produces precisely such a wave, whose vertex tends to follow the apparent motion of the sun in the heavens, and also to imitate its periodic inequalities. This solar wave co-exists with the lunar—is sometimes superposed on it, sometimes transverse to it, so as to partly neutralize it, according to the monthly synodical configuration of the two luminaries. This alternate mutual reinforcement and destruction of the solar and lunar tides cause what are called the spring and neap tides—the former being their sum, the latter their difference. Although the real amount of either tide is, at present, hardly within the reach of exact calculation, yet their proportion at any one place is probably not very remote from that of the ellipticities which would belong to their respective spheroids, could an equilibrium be attained. Now these ellipticities, for the solar and lunar spheroids, are respectively about two and five feet; so that the average spring tide will be to the neap as 7 to 3, or thereabout.

(753.) Another effect of the combination of the solar and lunar tides is what is called the *priming* and *lagging* of the tides. If the moon alone existed, and moved in the plane of the equator, the tide-day (*i.e.* the interval between two successive arrivals at the same place of the same vertex of the tide-wave) would be the lunar day (art. 143), formed by the combination of the moon's sidereal period and that of the earth's diurnal motion. Similarly, did the sun alone exist, and move always on the equator, the tide-day would be the mean solar day. The actual tide-day, then, or the interval of the occurrence of two successive *maxima* of their superposed waves, will vary as the separate waves approach to or recede from coincidence; because, when the vertices of two waves do not coincide, their joint height has its maximum at a point intermediate between them. This variation from uniformity in the lengths of successive tide-days is particularly to be remarked about the time of the new and full moon.

(754.) Quite different in its origin is that deviation of the time of high and low water at any port or harbor, from the culmination of the luminaries, or of the theoretical maximum of their superposed spheroids, which is called the "establishment" of that port. If the water were without inertia, and free from obstruction, either owing to the friction of the bed of the sea, the narrowness of channels along which the wave has to travel before reaching the port, their length, etc., the times above distinguished would be identical. But all these causes tend to create a difference, and to make that difference not alike at all ports. The observation of the establishments of harbors is a point of great maritime importance; nor is it of less consequence, theoretically speaking, to a knowledge of the true distribution of the tide-waves over the globe. In making such observations, care must be taken not to confound the time of "slack water," when the current caused by the tide ceases to flow visibly one way or the other, and that of *high* or *low water*, when the level of the surface ceases to rise or fall. These are totally distinct phenomena, and depend on entirely different causes, though in land-locked places they may sometimes coincide in point of time. They are, it is feared, too often mistaken one for the other by practical men; a circumstance which, whenever it occurs, must produce the greatest confusion in any attempt to reduce the system of the tides to distinct and intelligible laws.

(755.) The declination of the sun and moon materially affects the tides at any particular spot. As the vertex of the tide-wave tends to place itself vertically under the luminary which produces it, when this vertical changes its point of incidence on the surface, the tide-wave must tend to shift accordingly, and thus, by monthly and annual periods, must tend to increase and diminish alternately the principal tides. The period of the moon's nodes is thus introduced into this subject; her excursions in declination in one part of that period being 29°, and in another only 17°, on either side the equator.

(756.) Geometry demonstrates that the efficacy of a luminary in raising tides is inversely proportional to the cube of its distance. The sun and moon, however, by reason of the ellipticity of their orbits, are alternately nearer to and further from the earth than their mean distances. In consequence of this, the efficacy of the sun will fluctuate between the extremes 19 and 21, taking 20 for its mean value, and that of the moon between 43 and 59. Taking into account this cause of difference, the highest spring tide will be to the lowest neap as 59+21 to 43−19, or as 80 to 24, or 10 to 3. Of all the causes of differences in the height of tides, however, local situation is the most influential. In some places the tide-wave, rushing up a narrow channel, is suddenly raised to an extraordinary height. At Annapolis, for instance, in the Bay of Fundy, it is said to rise 120 feet. Even at Bristol the difference of high and low water occasionally amounts to 50 feet.

(757.) It is by means of the perturbations of the planets, as ascertained by observation and compared with theory, that we arrive at a knowledge of the masses of those planets which having no satellites, offer no other hold upon them for this purpose. Every planet produces an amount of perturbation in the motions of every other, proportioned to its mass, and to the degree of advantage or *purchase* which its situation in the system gives it over their movements. The latter is a subject of exact calculation; the former is unknown, otherwise than by observation of its effects. In the determination, however, of the masses of the planets by this means, theory lends the greatest assistance to observation, by pointing out the combinations most favorable for eliciting this knowledge from the confused mass of superposed inequalities which affect every observed place of a planet; by pointing out the laws of each inequality in its periodical rise and decay; and by showing how every particular inequality depends for its magnitude on the mass producing it. It is thus that the mass of Jupiter itself (employed by Laplace in his investigations, and interwoven with all the planetary tables) has been ascertained, by observations of the derangements produced by it in the motions of the ultra-zodiacal planets, to have been insufficiently determined, or rather considerably mistaken, by relying too much on observations of its satellites, made long ago by Pound and others, with inadequate instrumental means. The same conclusion has been arrived at, and nearly the same mass obtained, by means of the perturbations produced by Jupiter on Encke's comet. The error was one of great importance; the mass of Jupiter being by far the most influential element in the planetary system, after that of the sun. It is satisfactory, then, to have ascertained, as Mr. Airy has done, the cause of the error; to have traced it up to its source, in insufficient micrometric measurements of the greatest elongations of the satellites; and to have found it disappear when measures, taken with more care and with infinitely superior instruments, are substituted for those before employed.

(758.) In the same way that the perturbations of the planets lead us to a knowledge of their masses, as compared with that of the sun, so the perturbations of the satellites of Jupiter have led, and those of Saturn's attendants will no doubt hereafter lead, to a knowledge of the proportion *their* masses bear to their respective primaries. The system of Jupiter's satellites has been elaborately treated by Laplace; and it is from his theory, compared with innumerable observations of their eclipses, that the masses assigned to them, in art. 540, have been fixed. Few results of theory are more surprising than to see these minute atoms weighed in the same balance, which we have applied to the ponderous mass of the sun, which exceeds the least of them in the enormous proportion of 65000000 to 1.

(759.) The mass of the moon is concluded, 1st, from the proportion of the lunar to the solar tide, as observed at various stations, the effects being separated from each other by a long series of observations of the relative heights of spring and neap tides which, we have seen (art. 752), depend on the proportional influence of the two luminaries. 2dly, from the phenomenon of nutation, which, being the result of the moon's attraction alone, affords a means of calculating her mass, independent of any knowledge of the sun's. Both methods agree in assigning to our satellite a mass about one seventy-fifth that of the earth.[6]

(760.) Not only, however, has a knowledge of the perturbations produced on other bodies of our system enabled us to estimate the mass of a disturbing body already known to exist, and to produce disturbance. It has done much more, and enabled geometers to satisfy themselves of the existence, and even to indicate the situation of a planet previously unknown, with such precision, as to lead to its immediate discovery on the very first occasion of pointing a telescope to the place indicated. We have already (art. 506) had occasion to mention in general terms this great discovery; but its importance, and its connection with the subject before us, call for a more specific notice of the circumstances

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[6] Laplace, Expos. du Syst. du Monde, pp. 285, 300. Later researches have shown that this is somewhat too large, about one 88th being the value at present received.

attending it. When the regular observation of Uranus, consequent on its discovery in 1781, had afforded some certain knowledge of the elements of its orbit, it became possible to calculate backward into time past, with a view to ascertain whether certain stars of about the same apparent magnitude, observed by Flamsteed, and since reported as *missing*, might not possibly be this planet. No less than six ancient observations of it as a supposed star were thus found to have been recorded by that astronomer—one in 1690, one in 1712, and four in 1715. On further inquiry, it was also ascertained to have been observed by Bradley in 1753, by Mayer in 1756, and no less than twelve times by Le Monnier, in 1750, 1764, 1768, 1769, and 1771, all the time without the least suspicion of its planetary nature. The observations, however, so made, being all circumstantially registered, and made with instruments the best that their respective dates admitted, were quite available for correcting the elements of the orbit, which, as will be easily understood, is done with so much the greater precision the larger the arc of the ellipse embraced by the extreme observations employed. It was, therefore, reasonably hoped and expected, that, by making use of the data thus afforded, and duly allowing for the perturbations produced since 1690, by Saturn, Jupiter, and the inferior planets, elliptic elements would be obtained, which, taken in conjunction with those perturbations, would represent not only all the observations up to the time of executing the calculations, but also all future observations, in as satisfactory a manner as those of any of the other planets are actually represented. This expectation, however, proved delusive. M. Bouvard, one of the most expert and laborious calculators of whom astronomy has had to boast, and to whose zeal and indefatigable industry we owe the tables of Jupiter and Saturn in actual use, having undertaken the task of constructing similar tables for Uranus, found it impossible to reconcile the ancient observations above mentioned with those made from 1781 to 1820, so as to represent both series by means of the same ellipse and the same system of perturbations. He therefore rejected altogether the ancient series, and grounded his computations solely on the modern, although evidently not without serious misgivings as to the grounds of such a proceeding, and "leaving it to future time to determine whether the difficulty of reconciling the two series arose from inaccuracy in the older observations, or whether it depend on some extraneous and unperceived influence which may have acted on the planet."

(761.) But neither did the tables so calculated continue to represent, with due precision, observations subsequently made. The "error of the tables" after attaining a certain amount, by which the true longitude of Uranus was in advance of the computed, and which advance was steadily maintained from about the year 1795 to 1822, began, about the latter epoch, rapidly to diminish, till, in 1830–31, the tabular and observed longitudes agreed. But, far from remaining in accordance, the planet, still losing ground, fell, and continued to fall behind its calculated place, and that with such rapidity as to make it evident that the existing tables could no longer be received as representing, with any tolerable precision, the true laws of its motion.

(762.) The reader will easily understand the nature and progression of these discordances by casting his eye on *fig.* 1, Plate A, in which the horizontal line or *abscissa* is divided into equal parts, each representing 50° of heliocentric longitude in the motion of Uranus round the sun, and in which the distances between the horizontal lines represent each 100″ of error in longitude. The result of each year's observation of Uranus (or of the mean of all the observations obtained during that year) in longitude, is represented by a black dot placed above or below the point of the *abscissa*, corresponding to the mean of the observed longitudes for the year: above, if the observed longitude be in excess of the calculated, below if it fall short of it, and on the line if they agree; and at a distance from the line corresponding to their difference on the scale above mentioned.[7] Thus in Flamsteed's earliest observations in 1690, the dot so marked is placed above the line at 65″·9 above the line, the observed longitude being so much greater than the calculated.

(763.) If, neglecting the individual points, we draw a curve (indicated in the figure by a fine unbroken line) through their general course, we shall at once perceive a certain regularity in its undulations. It presents two great elevations above, and one nearly as great intermediate depression below the medial line or abscissa. And it is evident that these undulations would be very much reduced, and the errors in consequence greatly palliated, if each dot were removed in the vertical direction through a distance and in the direction indicated by the corresponding point of the curve A, B, C, D, E, F, G, H, intersecting the abscissa at points 180° distant, and making equal excursions on either side. Thus the point *a* for 1750 being removed

[7] The points are laid down from M. Leverrier's comparison of the whole series of observations of Uranus, with an ephemeris of his own calculation, founded on a complete and searching revision of the tables of Bouvard, and a rigorous computation of the perturbations caused by all the known planets capable of exercising any influence on it. The differences of longitude are *geocentric*, but for our present purpose it matters not in the least whether we consider the errors in heliocentric or in geocentric longitude.

upward or in the direction toward b through a distance equal to $c\ b$ would be brought almost to precise coincidence with the point d in the abscissa. Now, this is a clear indication that a very large part of the differences in question is due, not to perturbation, but simply to error in the elements of Uranus which have been assumed as the basis of calculation. For such excesses and defects of longitude alternating over arcs of 180° are precisely what would arise from error in the excentricity, or in the place of the perihelion, or in both. In ellipses slightly excentric, the true longitude alternately exceeds and falls short of the mean during 180° for each deviation, and the greater the excentricity, the greater these alternate fluctuations to and fro. If then the excentricity of a planet's orbit be assumed erroneously (suppose too great) the observed longitudes will exhibit a less amount of such fluctuation above and below the mean than the computed, and the difference of the two, instead of being, as it ought to be, always *nil*, will be alternately + and − over arcs of 180°. If then a difference be observed following such a law, it may arrive from erroneously assumed excentricity, provided always the longitudes at which they agree (supposed to differ by 180°) be coincident with those of the perihelion and aphelion; for in elliptic motion nearly circular, these are the points where the mean and true longitudes agree, so that any fluctuation of the nature observed, if this condition be not satisfied, cannot arise from error of excentricity. Now the longitude of the perihelion of Uranus in the elements employed by Bouvard is (neglecting fractions of a degree) 168°, and of the aphelion 348°. These points then, in our figure, fall at ω and a respectively, that is to say, nearly half way between A C, C E, E G, etc. It is evident therefore that it is not to error of excentricity that the fluctuation in question is mainly due.

(764.) Let us now consider the effect of an erroneous assumption of the *place* of the perihelion. Suppose in *fig*. 2, Plate A, $o\ x$ to represent the longitude of a planet, and $x\ y$ the excess of its true above its mean longitude, due to ellipticity. Then if R be the place of the perihelion, and P, or T, the aphelion in longitude, y will always lie in a certain undulating curve P Q R S T, above[8] P T between R and T, and below it between P and R. Now suppose the place of the perihelion shifted forward to r, or the whole curve shifted bodily forward into the situation $p\ q\ r\ s\ t$, then at the same longitude $o\ x$, the excess of the true above the mean longitude will be $x\ y'$ only; in other words, this excess will have diminished by the quantity $y\ y'$ below its former amount. Take therefore in o N (*fig*. 3, Plate A) $o\ y = o\ x$ and $y\ y'$ always$= y\ y'$ in *fig*. 2, and having thus constructed the curve K L M N O, the ordinate $y\ y'$ will always represent the effect of the supposed change of perihelion. It is evident (the excentricity being always supposed small) that this curve will consist also of alternate superior and inferior waves of 180° each in amplitude, and the points L, N of its intersection with the axis will occur at longitudes corresponding to X, Y intermediate between the *maxima* Q, q and S, s of the original curves, that is to say (if these intervals Q q, S s, or R r to which both are equal, be very small) very nearly at 90° from the perihelion and aphelion. Now this agrees with the conditions of the case in hand, and we are therefore authorized to conclude that the major portion of the errors in question *has* arisen from error in the place of the perihelion of Uranus itself, and not from

[8] The curves, figs. 2, 3, are inverted in the engraving.

perturbation, and that to correct this portion, the perihelion must be shifted somewhat forward. As to the amount of this shifting, our only object being explanation, it will not be necessary here to inquire into it. It will suffice that it must be such as shall make the curve A B C D E F G as nearly as possible similar, equal, and opposite to the curve traced out by the dots on the other side. And this being done, we may next proceed to lay down a curve of the residual differences between observation and theory in the mode indicated in art. 763.

(765.) This being done, by laying off at each point of the line of longitudes an ordinate equal to the difference of the ordinates of the two curves in *fig.* 1, when on opposite, and their sum when on the same side of the abscissa, the result will be as indicated by the dots in *fig.* 4. And here it is at once seen that a still further reduction of the differences under consideration would result, if, instead of taking the line A B for the line of longitudes, a line *a b* slightly inclined to it were substituted, in which case the whole of the differences between observation and theory from 1712 to 1800 would be annihilated, or at least so far reduced as hardly to exceed the ordinary errors of observation; and as respects the observation of 1690, the still outstanding difference of about 35″ would not be more than might be attributed to a not very careful observation at so early an epoch. Now the assumption of such a new line of longitudes as the correct one is in effect equivalent to the admission of a slight amount of error in the periodic time and epoch of Uranus; for it is evident that by reckoning from the inclined instead of the horizontal line, we in effect alter all the apparent outstanding errors by an amount proportional to the time before or after the date at which the two lines intersect (viz. about 1789). As to the direction in which this correction should be made, it is obvious by inspection of the course of the dots, that if we reckon from A B, or any line parallel to it, the observed planet on the long run keeps falling more and more behind the calculated one; *i.e.* its assigned mean angular velocity by the tables is too great and must be diminished, or its periodic time requires to be increased.

(766.) Let this increase of period be made, and in correspondence with that change let the longitudes be reckoned on a b, and the residual differences from that line instead of A B, and we shall have then done all that can be done in the way of reducing and palliating these differences, and that, with such success, that up to the year 1804 it might have been safely asserted that positively no ground whatever existed for suspecting any disturbing influence. But with this epoch an action appears to have commenced, and gone on increasing, producing an acceleration of the motion in longitude, in consequence of which Uranus continually gains on its elliptic place, and continued to do so till 1822, when it ceased to gain, and the excess of longitude was at its maximum, after which it began rapidly to lose ground, and has continued to do so up to the present time. It is perfectly clear, then, that in this interval some extraneous cause must have come into action which was not so before, or not in sufficient power to manifest itself by any marked effect, and that *that* cause must have ceased to act, or rather begun to reverse its action, in or about the year 1822, the reverse action being even more energetic than the direct.

(767.) Such is the phenomenon in the simplest form we are *now* able to present it. Of the various hypotheses formed to account for it, during the progress of its development, none seemed to have any degree of rational probability but that of the existence of an exterior, and hitherto undiscovered, planet, disturbing, according to the received laws of planetary disturbance, the motion of Uranus by its attraction, or rather superposing its disturbance on those produced by Jupiter and Saturn, the only two of the old planets which exercise any sensible disturbing action on that planet. Accordingly, this was the explanation which naturally, and almost of necessity, suggested itself to those conversant with the planetary perturbations who considered the subject with any degree of attention. The idea, however, of setting out from the observed anomalous deviations, and employing them as data to ascertain the distance and situation of the unknown body, or, in other words, to resolve the inverse problem of perturbations, "*given the disturbances to find the orbit, and place in that orbit* of the *disturbing planet*," appears to have occurred only to two mathematicians, Mr. Adams in England and M. Leverrier in France, with sufficient distinctness and hopefulness of success to induce them to attempt its solution. Both succeeded, and their solutions, arrived at with perfect independence, and by each in entire ignorance of the other's attempt, were found to agree in a surprising manner when the nature and difficulty of the problem is considered; the calculations of M. Leverrier assigning for the heliocentric longitude of the disturbing planet for the 23d September, 1846, 326° 0′, and those of Mr. Adams (brought to the same date) 329° 19′, differing only 3° 19′; the plane of its orbit deviating very slightly, if at all, from that of the ecliptic.

(768.) On the day above mentioned—a day forever memorable in the annals of astronomy—Dr. Galle, one of the astronomers of the Royal Observatory at Berlin, received a letter from M. Leverrier, announcing to him the result he had arrived at, and requesting him to look for the disturbing planet in or near the place assigned by his calculation. He did so, and *on that very night actually found it.* A star of the eighth magnitude was seen by him and by M. Encke in a situation where no star was marked as existing in Dr. Bremiker's chart, then recently published by the Berlin Academy. The next night it was found to have moved from its place, and was therefore assuredly a planet. Subsequent observations and calculations have fully demonstrated this planet, to which the name of Neptune has been assigned, to be really that body to whose disturbing attraction, according to the Newtonian law of gravity, the observed anomalies in the motion of Uranus were owing. The geocentric longitude determined by Dr. Galle from this observation was 325° 53′, which, converted into heliocentric, gives 326° 52′, differing 0° 52′ from M. Leverrier's place, 2° 27′ from that of Mr. Adams, and only 47′ from a mean of the two calculations.

(769.) It would be quite beyond the scope of this work, and far in advance of the amount of mathematical knowledge we have assumed our readers to possess, to attempt giving more than a superficial idea of the course followed by these geometers in their arduous investigations. Suffice it to say, that it consisted in regarding, as unknown quantities, to be determined, the mass, and all the elements of the unknown planet (supposed to revolve in the same plane and the same direction with Uranus), except its major semiaxis. This was assumed in the first instance (in conformity with "Bode's law," art. 505, and certainly at the time with a high *primâ facie* probability) to be double that of Uranus, or 38·364 radii of the earth's orbit.

Without *some* assumption as to the value of this element, owing to the peculiar form of the analytical expression of the perturbations, the analytical investigation would have presented difficulties apparently insuperable. But besides these, it was also necessary to regard as unknown, or at least as liable to corrections of unknown magnitude of the same order as the perturbations, all the elements of Uranus itself, a circumstance whose necessity will easily be understood, when we consider that the received elements could only be regarded as provisional, and must *certainly* be erroneous, the places from which they were obtained being affected by at least some portions of the very perturbations in question. This consideration, though indispensable, added vastly both to the complication and the labor of the inquiry. The axis (and therefore the mean motion) of the one orbit, then, being known very nearly, and that of the other thus hypothetically assumed, it became practicable to express in terms, partly algebraic, partly numerical, the amount of perturbation at any instant, by the aid of general expressions delivered by Laplace in his "*Mécanique Céleste*" and elsewhere. These, then, together with the corrections due to the altered elements of Uranus itself, being applied to the tabular longitudes, furnished, when compared with those observed, a series of *equations*, in which the elements and mass of Neptune, and the corrections of those of Uranus entered as the *unknown quantities*, and by whose resolution (no slight effort of analytical skill) all their values were at length obtained. The calculations were then repeated, reducing at the same time the value of the assumed distance of the new planet, the discordances between the given and calculated results indicating it to have been assumed too large; when the results were found to agree better, and the solutions to be, in fact, more satisfactory. Thus, at length, elements were arrived at for the orbit of the unknown planet, as below.

	Leverrier.	Adams.
Epoch of Elements	Jan. 1, 1847.	Oct. 6, 1846.
Mean longitude in Epoch . . .	318° 47′ 4	323° 2′
Semiaxis Major	36·1539	37·2474
Excentricity	0.107610	0·120615
Longitude of Perihelion . . .	284° 45′ 8	299° 11′
Mass (the Sun being 1)	0·00010727	0·00015003

The elements of M. Leverrier were obtained from a consideration of the observations up to the year 1845, those of Mr. Adams, only as far as 1840. On subsequently taking into account, however, those of the five years up to 1845, the latter was led to conclude that the semiaxis ought to be still much further diminished, and that a mean distance of 33·33 (being to that of Uranus as 1 : 0·574) would probably satisfy all the observations very nearly.[9]

(770.) On the actual discovery of the planet, it was, of course, assiduously observed, and it was soon ascertained that a mean distance, even less than Mr. Adams's last presumed value, agreed better with its motion; and no sooner were elements obtained from direct observation, sufficiently approximate to trace back its path in the heavens for a considerable interval of time, than it was ascertained to have been observed as a star by Lalande on the 8th and 10th of May, 1795, the latter of the two observations, however, having been rejected by him as faulty, by reason of its non-agreement with the former (a consequence of the motion of the planet in the interval). From these observations,

[9] In a letter to the Astronomer Royal, dated Sept. 2, 1846—*i.e.* three weeks previous to the optical discovery of the planet.

combined with those since accumulated, the elements calculated by Prof. Walker, U. S., result as follows:

Epoch of Elements . . .	Jan. 1, 1847, M. noon, Greenwich
Mean longitude at Epoch . .	328° 32′ 44″·2
Semiaxis Major	30·0367
Excentricity	0·00871946
Longitude of the Perihelion .	47° 12′ 6″·50
Ascending Node . . .	130° 4′ 20″·81
Inclination	1° 46′ 58″·97
Periodic time	164·6181 tropical year
Mean annual Motion . . .	2°·18688

(771.) The great disagreement between these elements and those assigned either by M. Leverrier or Mr. Adams will not fail to be remarked; and it will naturally be asked how it has come to pass, that elements so widely different from the truth should afford anything like a satisfactory representation of the perturbation in question, and that the true situation of the planet in the heavens should have been so well, and indeed accurately, pointed out by them. As to the latter point, any one may satisfy himself by half an hour's calculation that both sets of elements do really place the planet, on the day of its discovery, not only in the longitudes assigned in art. 763, *i.e.* extremely near its apparent place, but also at a distance from the Sun very much more approximately correct than the *mean* distances or semiaxes of the respective orbits. Thus the radius vector of Neptune, calculated from M. Leverrier's elements for the day in question, instead of 36·1539 (the mean distance) comes out almost exactly 33; and indeed, if we consider that the excentricity assigned by those elements gives for the perihelion distance 32·2634, the longitude assigned to the perihelion brings the whole arc of the orbit (more than 83°), described in the interval from 1806 to 1847 to lie within 42° one way or the other of the perihelion, and therefore, during the whole of that interval, the hypothetical planet would be moving within limits of distance from the sun, 32·6 and 33·0. The following comparative tables of the relative situations of Uranus, the real and hypothetical planet, will exhibit more clearly than any lengthened statement, the near imitation of the motion of the former by the latter within that interval. The longitudes are heliocentric.[10]

A. D.	Uranus.	Neptune.		Leverrier.		Adams.	
	Long.	Long.	Rad. Vec.	Long.	Rad. Vec.	Long.	Rad. Vec.
1805·0	197°·8	235°·9	30·3	241°·2	33·1	246°·5	34·2
1810·0	220·9	247·0	30·3	251·1	32·8	255·9	33·7
1815·0	243·2	258·0	30·3	261·2	32·5	265·5	33·3
1820·0	264·7	268·8	30·2	271·4	32·4	275·4	33·1
1821·0	269·0	271·0	30·2	273·5	32·3	277·4	33·0
1822·0	273·3	273·2	30·2	275·6	32·3	279·5	33·0
1823·0	277·6	275·3	30·2	277·6	32·3	281·5	32·9
1824·0	281·8	277·4	30·2	279·7	32·3	283·6	32·9
1825·0	285·8	279·6	30·2	281·8	32·3	285·6	32·8
1830·0	306·1	290·5	30·1	292·1	32·3	296·0	32·8
1835·0	326·0	301·4	30·1	302·5	32·4	306·3	32·8
1840·0	345·7	312·2	30·1	312·6	32·6	316·3	32·9
1845·0	365·3	323·1	30·0	322·6	32·9	326·0	33·1
1847·0	373·3	327·6	30·0	326·5	33·1	329·3	33·2

(772.) From this comparison it will be seen that Uranus arrived at its conjunction with Neptune at or immediately before the commencement of 1822, with the calculated planet of Leverrier at the beginning of the following year 1823, and with that of Adams about the end of 1824. Both the theoretical planets, and especially that of M. Leverrier, therefore, during the whole of the above interval of time, so far as the *directions* of their attractive forces on Uranus are concerned, would act nearly on it as the true planet must have done. As regards the intensity of the relative disturbing forces, if we estimate these by the principles of art.

[10] The calculations are carried only to tenths of degrees, as quite sufficient for the object in view.

612 at the epochs of conjunction, and for the commencement of 1805 and 1845, we find for the respective denominators of the fractions of the sun's attraction on Uranus regarded as unity, which express the *total* disturbing force, N S, in each case, as below:

			1805.	Conjunction.	1845.
Neptune with	Peirce's mass	$\frac{1}{19840}$	27540	7508	32390
	Struve's mass	$\frac{1}{14496}$	20244	5519	23810
Leverrier's theoretical Planet, mass		$\frac{1}{9322}$	20837	5193	19935

The masses here assigned to Neptune are those respectively deduced by Prof. Peirce and M. Struve from observations of the satellite discovered by Mr. Lassell, made with the large telescopes of Fraunhofer in the observatories of Cambridge, U. S., and Pulkova respectively. These it will be perceived differ very considerably, as might reasonably be expected in the results of micrometrical measurements of such difficulty, and it is not possible at present to say to which the preference ought to be given. As compared with the mass assigned by M. Struve, an agreement on the whole more satisfactory could not have been looked for within the interval immediately in question.

(773.) Subject then to this uncertainty as to the real mass of Neptune, the theoretical planet of Leverrier must be considered as representing with quite as much fidelity as could possibly be expected in a research of such exceeding delicacy, the particulars of its motion and perturbative action during the forty years elapsed from 1805 to 1845, an interval which (as is obvious from the rapid diminution of the forces on either side of the conjunction indicated by the numbers here set down) comprises all the most influential range of its action. This will, however, be placed in full evidence by the construction of curves representing the normal and tangential forces on the principles laid down (as far as the normal constituent is concerned) in art. 717, one slight change only being made, which, for the purpose in view, conduces greatly to clearness of conception. The force L S (in the figure of that article) being supposed applied at P in the direction L S, we here construct the curve of the normal force by erecting at P (*fig.* 5, Plate A) P W always perpendicular to the disturbed orbit, A P, at P, measured from P in the same direction that S lies from L, and equal in length to L S. P W then will always represent both the direction and magnitude of the normal force *acting at* P. And in like manner, if we take always P Z on the tangent to the disturbed orbit at P, equal to N L of the former figure, and measured in the same direction from P that L is from N, P Z will represent both in magnitude and direction the tangential force acting at P. Thus will be traced out the two curious ovals represented in our figure of their proper forms and proportions for the case in question. That expressing the normal force is formed of four lobes, having a common point in S, viz. S W *m* X S *a* S *n* S *b* S W, and that expressing the tangential, A Z *c f* B *e d* Y A Z, consisting of four mutually intersecting loops, surrounding and touching the disturbed orbit in four points, A B *c d*. The normal force acts outward over all that part of the orbit, both in conjunction and opposition, corresponding to the portions of the lobes *m*, *n*, exterior to the disturbed orbit, and inward in every other part. The figure sets in a clear light the great disproportion between the energy of this force near the conjunction, and in any other configuration of the planets; its exceedingly rapid degradation as P approaches the point of neutrality (whose situation is 35° 5′ on either side of the conjunction, an arc described synodically by Uranus in $16^{y}\cdot72$); and the comparatively short duration and consequent inefficacy to produce any great amount of perturbation, of the more intense part of its inward action in the small portions of the orbit corresponding to the lobes *a*, *b*, in which the line representing the inward force exceeds the radius of the circle. It exhibits, too, with no less distinctness, the gradual development, and rapid degradation and extinction of the tangential force from *its* neutral points, *c*, *d*, on either side up to the conjunction, where its action is reversed, being accelerative over the arc *d* A, and retardative over A *c*, each of which arcs has an amplitude of 71° 20′, and is described by Uranus synodically in $34^{y}\cdot00$. The insignificance of the tangential force in the configurations remote from conjunction throughout the arc *c* B *d* is also obviously expressed by the small comparative development of the loops *e*, *f*.

(774.) Let us now consider how the action of these forces results in the production of that peculiar character of perturbation which is exhibited in our curve, *fig*. 4, Plate A. It is at once evident that the increase of the longitude from 1800 to 1822, the cessation of that increase in 1822, and its conversion into a decrease during the subsequent interval is in complete accordance with the growth, rapid decay, extinction at conjunction, and subsequent reproduction in a reversed sense of the tangential force: so that we cannot hesitate in attributing the greater part of the perturbation expressed by the swell and subsidence of the curve between the years 1800 and 1845—all that part, indeed, which is symmetrical on either side of 1822—to the action of the tangential force.

(775.) But it will be asked—has then the normal force (which, on the plain showing of *fig.* 5, is nearly twice as powerful as the tangential, and which does not reverse its action, like the latter force, at the point of conjunction, but, on the contrary, is there most energetic) no influence in producing the observed effects? We answer, very little, within the period to which observation had extended up to 1845. The effect of the tangential force on the longitude is direct and immediate (art. 660), that of the normal indirect, consequential, and cumulative with the progress of time (art. 734). The effect of the tangential force on *the mean motion* takes place through the medium of the change it produces on the axis, and is transient: the reversed action after conjunction (supposing the orbits circular) exactly destroying all the previous effect, and leaving the mean motion on the whole unaffected. In the passage through the conjunction, then, the tangential force produces a sudden and powerful acceleration, succeeded by an equally powerful and equally sudden retardation, which done, its action is completed, and no trace remains in the subsequent motion of the planet that it ever existed, for its action on the perihelion and excentricity is in like manner also nullified by its reversal of direction. But with the normal force the case is far otherwise. Its *immediate* effect on the angular motion is *nil.* It is not till it has acted long enough to produce a perceptible change in the distance of the disturbed planet from the sun that the angular velocity begins to be sensibly affected, and it is not till its whole outward action has been exerted (*i.e.* over the whole interval from neutral point to neutral point) that its maximum effect in lifting the disturbed planet away from the sun has been produced, and the full amount of diminution in angular velocity it is capable of causing has been developed. *This* continues to act in producing a retardation in longitude long after the normal force itself has reversed its action, and from a powerful outward force has become a feeble inward one. A certain portion of this perturbation is incident on the epoch in the mode described in art. 731 *et seq.*, and permanently disturbs the mean motion from what it would have been, had Neptune no existence. The rest of its effect is compensated in a single synodic revolution, not by the reversal of the action of the force (for that reversed action is far too feeble for this purpose), but by the effect of the permanent alteration produced in the excentricity, which (the axis being unchanged) compensates by increased proximity in one part of the revolution, for increased distance in the other. Sufficient time has not yet elapsed since the conjunction to bring out into full evidence the influence of this force. Still its commencement is quite unequivocally marked in the more rapid descent of our curve *fig.* 4, subsequent to the conjunction than ascent previous to that epoch, which indicates the commencement of a series of undulations in its future course *of an elliptic character*, consequent on the altered excentricity and perihelion (the total and ultimate effect of this constituent of the disturbing force) which will be maintained till within about 20 years from the next conjunction, with the exception, perhaps, of some trifling inequalities about the time of the opposition, similar in character, but far inferior in magnitude to those now under discussion.

(776.) Posterity will hardly credit that, with a full knowledge of all the circumstances attending this great discovery —of the calculations of Leverrier and Adams—of the communication of its predicted place to Dr. Galle—and of the new planet being actually found by him in that place, in the remarkable manner above commemorated; not only have doubts been expressed as to the validity of the calculations of those geometers, and the legitimacy of their conclusions, but these doubts have been carried so far as to lead the objectors to attribute the acknowledged fact of a planet previously unknown occupying that precise place in the heavens at that precise time, to sheer accident! [11] What share accident may have had in the successful issue of the calculations, we presume the reader, after what has been said, will have little difficulty in satisfying himself. As regards the time when the discovery was made, much has also been attributed to fortunate coincidence. The following considerations will, we apprehend, completely dissipate this idea, if still lingering in the mind of any one at all conversant with the subject. The period of Uranus being 84·0140 years, and that of Neptune 164·6181, their synodic revolution (art. 418), or the interval between two successive conjunctions, is

[11] These doubts seem to have originated partly in the great disagreement between the predicted and real elements of Neptune, partly in the near (*possibly* precise) commensurability of the mean motions of Neptune and Uranus. We conceive them however to be founded in a total misconception of the nature of the problem, which was not, from such obviously uncertain indications as the observed discordances could give, to determine as astronomical quantities the axis, excentricity and mass of the disturbing planet; but practically to discover where to look for it; when, if once found, these elements would be far better ascertained. To do this, *any axis, excentricity, perihelion and mass, however wide of the truth*, which would represent, even roughly, the amount, but *with tolerable correctness the direction* of the disturbing force during the very moderate interval when the departures from theory were really considerable, would equally serve their purposes; and with an excentricity, mass and perihelion disposable, it is obvious that any assumption of the axis between the limits 30 and 38, nay, even with a much wider inferior limit, would serve the purpose. In his attempt to assign an inferior limit to the axis, and in the value so assigned, M. Leverrier, it must be admitted, was not successful. Mr. Adams, on the other hand, influenced by no considerations of the kind which appear to have weighed with his brother geometer, fixed ultimately (as we have seen) *before the actual discovery of the planet*, on an axis not very egregiously wrong. Still it were to be wished, for the satisfaction of all parties, that some one would undertake the problem *de novo*, employing formulæ not liable to the passage through infinity, which, technically speaking, hampers, or may be supposed to hamper, the *continuous* application of the usual perturbational formulæ when cases of commensurability occur.

171·58 years. The late conjunction having taken place about the beginning of 1822; that next preceding must have happened in 1649, or more than 40 years before the first recorded observation of Uranus in 1690, to say nothing of its discovery as a planet. In 1690, then, it must have been effectually out of reach of any perturbative influence worth considering, and so it remained during the whole interval from thence to 1800. From that time the effect of perturbation began to become sensible, about 1805 prominent, and in 1820 had nearly reached its maximum. At this epoch an alarm was sounded. The maximum was not attained—the event, so important to astronomy, was still in progress of development—when the fact (anything rather than a striking one) was noticed, and made matter of complaint. But the time for discussing its cause with any prospect of success was not yet come. Everything turns upon the precise determination of the epoch of the maximum, when the perturbing and perturbed planet were in conjunction, and upon the law of increase and diminution of the perturbation itself on either side of that point. Now it is always difficult to assign the time of the occurrence of a maximum by observations liable to errors bearing a ratio far from inconsiderable to the whole quantity observed. Until the lapse of some years from 1822 it would have been impossible to have fixed that epoch with any certainty, and as respects the law of degradation and total arc of longitude over which the sensible perturbations extend, we are hardly yet arrived at a period when this can be said to be completely determinable from observation alone. In all this we see nothing of accident, unless it be accidental that an event which must have happened between 1781 and 1953, actually happened in 1822; and that we live in an age when astronomy has reached that perfection, and its cultivators exercise that vigilance which neither permit such an event, nor its scientific importance, to pass unnoticed. The blossom had been watched with interest in its development, and the fruit was gathered in the very moment of maturity.[12]

(776 *a*.) In the foregoing chapters we have enumerated and described the several bodies so far as known of which our system consists, and have shown how their mutual distances from and their motions with respect to each other may be determined, and their masses compared with that of the central body, and ultimately with that of our own planet as a unit of reference; but nothing has been said respecting the means by which that unit itself can be brought into comparison with the mass, weight or inertia of those portions of its substance which we see and handle on its surface. This datum—the total weight of the earth itself—the number of times that its entire mass exceeds that of a pound of lead or other matter—or in other words (its bulk being accurately known) its mean density—remains up to this point of the present work undetermined, and is the one thing wanting to complete our knowledge of the data of our system and fully to connect astronomy with ordinary mechanics. We shall now therefore proceed to explain the methods by which this has been accomplished.

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[12] The student who may wish to see the perturbations of Uranus produced by Neptune, as computed from a knowledge of the elements and mass of that planet, such as we now know to be pretty near the truth, will find them stated at length from the calculations of Mr. Walker (of Washington, U. S.), in the "Proceedings of the American Academy of Arts and Sciences," vol. i. p. 334 *et seq*. On examining the comparisons of the results of Mr. Walker's formulæ with those of Mr. Adams's theory in p. 342, he will perhaps be surprised at the enormous difference between the actions of Neptune and Mr. Adams's "hypothetical planet" on the longitude of Uranus. This is easily explained. Mr. Adams's perturbations are deviations from Bouvard's orbit of Uranus, as it stood immediately previous to the late conjunction. Mr. Walker's are the deviations from a mean or undisturbed orbit freed from the influence of the long inequality resulting from the near commensurability of the motions.

(776 *b*.) The principle which at once suggests itself to every mind is to measure the direct attraction, if it be possible, of some known mass, at some known distance, on some other. We say, if it be possible, because whatever notion we may form *à priori* of the weight of the earth as estimated in pounds or tons, it is clearly something enormous; and moreover, since it follows from the law of gravitative attraction[13] that the attractions of spheres of equal density on points at their surface are to each other as their radii, the attraction of a globe a foot in diameter, of the same average density of the earth, on a material point at its own surface would only amount to the 41,849,280th part of the weight of such material point; and therefore its attraction on a spherical body, suppose also a foot in diameter, placed in contact with it, would only amount to one 167,-397,120th part of the weight of such body. Now when we have to deal with fractions of such an order of minuteness, all ordinary modes of directly measuring forces and weights break down, and the utmost resources of invention and art must be taxed even to render them perceptible, to say nothing of their precise determination.

(776 *c*.) The first and most obvious mode of producing a magnified result is to augment, in as high a ratio as possible, the attracting mass; and therefore to substitute some great natural mass of the most suitable form which can be found, for an artificial sphere. And as the resources afforded by the integral calculus furnish the means of calculating the attraction of a body of any size and figure of known materials on a point anywhere situated without it, the idea naturally enough suggested itself to take some large mountain,

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[13] Princip. bk. 1, prop. 72.

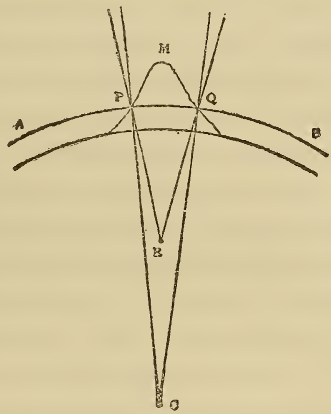
of as regular a shape as might be found, for the attracting body, and to measure its attraction, on a principle pointed out by Newton,[14] by the deviation from verticality of a plumb-line suspended near it, which will necessarily be drawn aside toward the mountain. As the deflection to be expected however, even in the case of a very large mountain, is still exceedingly minute, the working out of this idea into practice calls for very exact and refined astronomical observations.

(776 *d.*) In the first place the question arises *in limine*, how are we to ascertain, at any place, what *is* a vertical direction? The *deviated* plumb-line, it is obvious, cannot give us this information, nor can levels, for the surface of still water is always at right angles to the single force, whatever that may be, which results from a combination of all the forces acting on it—in other words, to the direction of the *deviated* plumb-line. Here it is that our knowledge of the figure and dimensions of the earth stand us in stead. We cannot, it is true, remove the mountain so as to find where the plumb-line would point, or the level rest in its absence; but we can shift our station to the opposite side, and by sidereal observation ascertain whether the direction of the plumb-line has varied by more or less than the amount of change due to such a change of station on the globe. Thus then we proceed:—

Suppose M the mountain, A B a circle of latitude passing through two stations P, Q, at its foot (or rather at such heights on its slope as shall correspond to the maxima of its lateral attraction), at each of which let observations be made with a portable zenith sector alternately established at one

[14] Treatise on the System of the World in a popular way (1728).

and the other of the zenith distances of some star passing very near the zenith of the mountain (so as to free the observations from uncertainty of refraction). Were there no lateral attraction, the plumb-lines at both stations would point directly to the centre of curvature C of the terrestrial spheroid (art. 219), and the angle between them, P C Q, would be the difference of latitudes of the stations. Now the dimensions and ellipticity of the earth as a whole being known, this latter difference can be independently determined by a trigonometrical survey instituted for the pur-



pose, a base being measured, and the meridional distance P Q ascertained by triangulation (art. 274 *et seq.*), which, converted into seconds of latitude, gives the difference in question; to which, were there no *local attraction*, the observed difference of zenith distances ought to correspond. But this will not be the case. The mountain will attract the plummet both ways inward, into situations P R, Q R, including a greater angle than P C Q, and this being the observed angle or apparent difference of zenith distances—subtracting from it the difference of latitude so independently obtained, the excess will represent the sum of the two deviations north and south due to the attraction required. The mountain has then to be surveyed, and modelled, and mineralogical specimens taken from every accessible part of it, and their specific gravities determined; and thus, no matter with what amount of calculation (for it is no light task), the total lateral attraction is computed in units of some definite scale; such, for instance, that each unit shall represent the total attraction of a sphere of 1lb. weight, on a point 1 foot distant from its centre. The sum of all these units, each reduced to a horizontal direction, is the total lateral attraction of the mountain, and is therefore to the total vertical attraction of the earth as the tangent of the deviation (taken so as to divide the total observed difference in the ratio of the computed attractions at either station) is to radius.

(776 *e.*) The process is laborious and costly—requires excellent instruments and the co-operation of more than one practiced observer. It has, however, been put in execution on several occasions; viz., 1st, by the French Academicians, Bouguer and La Condamine, who, in the course of their operations in Peru for the measurement of an arc of the meridian (art. 216), instituted observations of the kind above described on Chimborazo in 1738. Their means of observation, however, were not such as to afford any distinct result, though a deviation of the plumb-line to the amount of about 11″ appears to have been obtained. 2d, by Maskelyne, in 1774, on the mountain Schehallien in Scotland, a mountain, not indeed of any great magnitude, being only about 3000 feet in altitude, but well situated, and otherwise well adapted for the experiment. It was successful. A joint amount of the lateral deviations on either side, of 11″·6, was well ascertained to be produced by the local attraction, and the calcu-

lations being executed (by Dr. Hutton and subsequently by Professor Playfair), a result entitled to some reliance was obtained, according to which the mean density of the earth comes out 4·713 times that of water at the surface. More recently, we find a series of observations instituted by Sir H. James, Superintendent of the Ordnance Survey, on Arthur's Seat near Edinburgh,[15] by which, from a deflexion of 2″·21 observed on the north and of 2″·00 on the south side of that mountain, a mean density results of 5·316.

(776 *f*.) Observations of the time of oscillation of a pendulum afford (see art. 235) a direct measure of the force by which the oscillating mass is urged vertically downward. Hence it follows that if this time be very precisely determined, both at the summit and at the foot of a mountain or elevated tableland, the attraction of the mass of such mountain or tableland vertically downward will become known. For gravity decreasing inversely as the square of the distance would be enfeebled by the increase of that distance in a proportion which can be precisely calculated from the known height of the upper station; and therefore, could the pendulum be supported in the air at that height, the increase of its time of oscillation, under those circumstances, would be exactly known by calculation. But being supported on a mountain mass, protruding above the level surface of the terrestrial spheroid, the attraction of that mass acts on it in addition to the so diminished force of general gravitation, and prevents it from losing on the sea-level rate so much as it would do were the mountain devoid of attractive power. Experiments of this nature have been made by the Italian astronomers Plana and Carlini on Mont Cenis in

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[15] Phil. Tr. 1856, p. 591.

Savoy, and the result, all computations executed, have given 4·950 for the mean density in question.

(776 *g*.) But it is also possible to descend below, as well as to rise above, the general sea-level, and to observe the pendulum at great depths below that level, as in deep mines. It was shown by Newton[16] that the attraction of a hollow, spherical, homogeneous shell on a point however situate *within it*, is simply *nil*, *i.e.* that the material point so placed is equally attracted by it in all directions. Hence by descending below the surface, we set ourselves free of the attraction of *the whole spherical shell* exterior to the point of observation, and the remaining attraction is the same as that of the whole interior mass collected in its centre. This may, or may not, be less than the attraction of *the whole earth* on a point at *its* surface. It will be less if the earth be homogeneous or of the same density throughout; for in that case Newton has shown[17] that the attractive forces of the whole sphere, and of the interior sphere, each on a point on its own surface, are to each other as their radii. But if the internal portions of the earth be more dense than the external (as they must be if the foregoing determinations be any approach to truth), it may be greater. The experiment has been made, on three several occasions, by the present Astronomer Royal (Mr. Airy). On the first two in the Dolcoath mine in Cornwall at a depth of 1200 feet—a clock and pendulum were transported alternately to the bottom and the mouth of the shaft. On both these occasions the arrangements were defeated; on the first, by the accidental combustion of the packages of instruments in mid-air while in the act of raising them from below,

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16 Princip. Lib. i. Prop. 70.

17 *Ib.*, 73.

attended with their precipitation down the shaft of the mine; on the other, by the subsidence of a mass of rock, "many times the size of Westminster Abbey," during the experiments, deluging the mine with water and forcing a premature conclusion. The third attempt (in the Harton Coal Pit, near South Shields, 1200 feet in depth) proved perfectly successful, and the oscillations of the pendulum below being compared with those of the clock above, by the immediate transmission of the beats of the latter down the mine by an electric wire, the great difficulty (that of the exact transmission of time) was annihilated. The result of this experiment was that a pendulum vibrating seconds at the mouth of the pit, would *gain* $2\frac{1}{4}$ sec. per day at its bottom; and the final result (of which the calculations have very recently been published[18]) gives 6·565 for the mean density of the earth.

(776 *h.*) The difference between these several results is considerable, and even the interval between the last mentioned and the highest of the others pretty large: it is bridged over, however, so to speak, and the interval partly filled up, by the results of a totally different class of experiments of a much more curious and artificial nature, which we have now to describe. We have already seen (art. 234) that the force of gravity may be brought directly into comparison with other material forces by using as an intermedium the elastic power of a spring. What is true of gravitation to the whole mass of the earth is equally so of gravitation toward any material mass, as a leaden ball. It may be measured by equilibrating it with the tension of a spring; provided, 1st, that we can frame a spring so deli-

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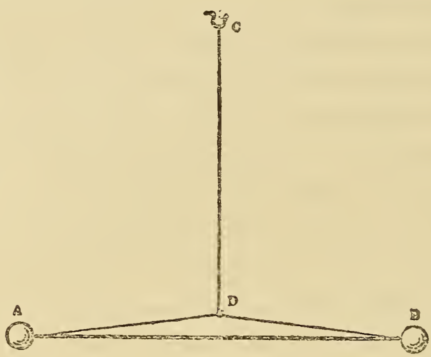
[18] Phil. Trans. 1856, p. 297.

cate as to be visibly and measurably affected by so minute a force; 2dly, that the force can be so applied as to be the *only* force tending to bend the spring, a condition which implies that it shall act on it, not vertically, but horizontally, so as to eliminate the weight of the spring, or at least prevent its being mixed up with the result; and, 3dly, that we shall possess some independent means of measuring the elastic power of the spring itself. All these conditions are satisfied by the balance of torsion, devised by Michell with a view to this inquiry, and applied, after his death, to the intended purpose by Cavendish, in the celebrated experiment usually cited as "the Cavendish Experiment." [19]

(776 *i.*) The apparatus consists essentially of a long wooden rod made so as to unite great strength with little weight; carrying at its extremities two equal balls A, B,



and suspended in a horizontal situation by a wire no thicker than necessary securely to sustain the weight, from a point over its centre of gravity, the wire being arranged as in the figure, so as to relieve the rod of the weight of the balls, its

[19] Phil. Trans. 1798, 469. Cavendish expressly states that Michell's invention of this beautiful instrument, and his communication of it to him, was antecedent to the publication of Coulomb's researches.

office being solely to keep them apart at a given horizontal distance. It is evident that when suspended from C, and allowed to take its position of equilibrium undisturbed by any external force, the rod will assume such a situation that the wire C D shall be quite devoid of torsion; but that if the rod A B be disturbed from this neutral position, C D remaining vertical, the elastic force of the wire called into action by the torsion so induced will tend to bring it back to the point of departure by a force proportional to the angle of torsion. When so disturbed then, and abandoned to itself, it will oscillate backward and forward in horizontal arcs, the oscillations being all performed in equal times; and from the time observed to be occupied in each oscillation, the weights of the balls and that of the rod being known, we are able, from dynamical principles, to determine the *motive force* by which the wire acts on the balls, or the force of torsion. Suppose, now, two heavy leaden spheres to be brought, laterally, up nearly into contact, the one with A, the other with B, but on opposite sides of them, they will attract A, B, and their attractions will conspire in twisting the wire the same way; so that the point of rest will be changed from the original neutral point to one in which the torsion shall just counterbalance the attractions. By shifting the attracting balls alternately to the one and the other sides of A B, these will assume positions of rest, alternately on opposite sides of the original neutral point, and equidistant from it, so that the deviation, if any, shall thus become doubled in its effect on the readings off of a scale marked by a pointer at the end of the rod, which may be observed through a telescope placed at a distance, so that the approach of the observer's person may create no disturbance.

(776 *j*.) Practically, the observation is not so simple as in the above statement. The balls can hardly ever be brought completely to rest; and the neutral point has to be concluded by noting the extremes of the arc of oscillation, perpetually diminishing by the resistance of the air. And when the attracting balls are brought into action, their attraction (acting laterally, according to the inverse squares of the distances) mixes itself with the force of torsion, to produce a compound law of force, under whose influence the times, velocities, and arcs have a different relation from those due to the torsion alone, and which, when investigated rigorously, lead to calculations of great complexity. Fortunately, the extreme minuteness of the attractive forces dispenses with a rigorous solution of this problem, and allows of a very simple and ready approximation, quite exact enough for the purpose. But besides these, a host of disturbing influences, arising from currents of air induced by difference of temperature, has to be contended with or guarded against, so as to render the experiment one of great difficulty and full of niceties, the mere enumeration of which here, however, would lead us far beyond our limits.[20]

(776 *k*.) The experiment, as conducted by Cavendish, afforded as its final result 5·480. Repeated since, with greater precautions, by Professor Reich, 5·438 was obtained; and still more recently, by the late F. Baily, in a series of experiments exhibiting an astonishing amount of skill and patience in overcoming the almost innumerable

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[20] The reader is warned to be on his guard against accepting as correct an account of the principle of the Cavendish experiment, professing to emanate from one very high astronomical authority, and passed without note or comment (and therefore so far sanctioned) by another, but which involves a total misconception of its true nature (Arago, *Lezione di Astronomia tradutte ed annotate di E. Capocci*, Napoli, 1851, p. 238).

obstacles to complete success, 5·660; a result undoubtedly preferable to the two former. Thus the final result of the whole inquiry will stand as below, the densities concluded being arranged in order of magnitude:

Schehallien experiment, by Maskelyne, calculated by Playfair,	D = 4·713
Carlini from pendulum on Mount Cenis (corrected by Giulio) .	4·950
Col. James from attraction of Arthur's Seat	5·316
Reich, repetition of Cavendish experiment	5·438
Cavendish, result 5·480, corrected by Mr. Baily's recomputation .	5·448
Baily's repetition of Cavendish experiment	5·660
Airy from pendulum in Harton coal-pit	6·565
General mean	5·441[21]
Mean of greatest and least .	5·639

(776 *l.*) Calculating on $5\frac{1}{2}$ as a result sufficiently approximative and convenient for memory; taking the mean diameter of the earth, considered as a sphere, at 7912·41 miles, and the weight of a cubic foot of water at 62·3211 lbs.; we find for its solid content in cubic miles, 259,373 millions, and for its weight in tons of 2240 lbs. avoird. each, 5842 trillions ($=5842\times10^{18}$).

[21] Newton, by one of his astonishing divinations, had already expressed his opinion that the mean density of the earth would be found to be between five and six times that of water. (Princ. iii. 10.)

PART III

OF SIDEREAL ASTRONOMY

CHAPTER XV

Of the Fixed Stars—Their Classification by Magnitudes—Photometric Scale of Magnitudes—Conventional or Vulgar Scale—Photometric Comparison of Stars—Distribution of Stars over the Heavens—Of the Milky Way or Galaxy—Its Supposed Form that of a Flat Stratum Partially Subdivided—Its Visible Course among the Constellations—Its Internal Structure—Its Apparently Indefinite Extent in Certain Directions—Of the Distance of the Fixed Stars—Their Annual Parallax—Parallactic Unit of Sidereal Distance—Effect of Parallax Analogous to that of Aberration—How Distinguished from it—Detection of Parallax by Meridional Observations—Henderson's Application to α Centauri—By Differential Observations—Discoveries of Bessel and Struve—List of Stars in which Parallax has been Detected—Of the Real Magnitudes of the Stars—Comparison of their Lights with that of the Sun

(777.) Besides the bodies we have described in the foregoing chapters, the heavens present us with an innumerable multitude of other objects, which are called generally by the name of stars. Though comprehending individuals differing from each other, not merely in brightness, but in many other essential points, they all agree in one attribute—a high degree of permanence as to apparent relative situation. This has procured them the title of "fixed stars"; an expression which is to be understood in a comparative and not an absolute sense, it being certain that many, and probable that all, are in a state of motion, although too slow to be perceptible unless by means of very delicate observations, continued during a long series of years.

(778.) Astronomers are in the habit of distinguishing the stars into classes, according to their apparent brightness. These are termed magnitudes. The brightest stars are said to be of the first magnitude; those which fall so far short of the first degree of brightness as to make a strongly marked distinction are classed in the second; and so on down to the sixth or seventh, which comprise the smallest stars visible to the naked eye, in the clearest and darkest night. Beyond these, however, telescopes continue the range of visibility, and magnitudes from the 8th down to the 16th are familiar to those who are in the practice of using powerful instruments; nor does there seem the least reason to assign a limit to this progression; every increase in the dimensions and power of instruments, which successive improvements in optical science have attained, having brought into view multitudes innumerable of objects invisible before; so that, for anything experience has hitherto taught us, the number of the stars may be really infinite, in the only sense in which we can assign a meaning to the word.

(779.) This classification into magnitudes, however, it must be observed, is entirely arbitrary. Of a multitude of bright objects, differing probably, intrinsically, both in size and in splendor, and arranged at unequal distances from us, one must of necessity appear the brightest, one next below it, and so on. An order of succession (relative, of course, to our local situation among them) *must* exist, and it is a matter of absolute indifference, where, in that infinite progression downward, from the one brightest to the invisible, we choose to draw our lines of demarcation. All this is a matter of pure convention. Usage, however, has established such a convention; and though it is impossible to determine exactly, or *à priori*, where one magnitude ends and the next begins, and although different observers have differed in their magnitudes, yet, on the whole, astronomers have restricted their first magnitude to about 23 or 24 principal stars; their second to 50 or 60 next inferior; their third to about 200 yet smaller, and so on; the numbers increasing very rapidly as we descend in the scale of brightness, the whole number of stars already registered down to the seventh magnitude, inclusive, amounting to from 12000 to 15000.

(780.) As we do not see the actual disk of a star, but judge only of its brightness by the total impression made upon the eye, the apparent "magnitude" of any star will, it is evident, depend, 1st, on the star's distance from us; 2d, on the absolute magnitude of its illuminated surface; 3d, on the intrinsic brightness of that surface. Now, as we know nothing, or next to nothing, of any of these data, and have every reason for believing that each of them may differ in different individuals, in the proportion of many millions to one, it is clear that we are not to expect much satisfaction in any conclusions we may draw from numerical statements of the number of individuals which have been arranged in our artificial classes antecedent to any general or definite *principle* of arrangement. In fact, astronomers have not yet agreed upon any principle by which the magnitudes may be photometrically classed *à priori*, whether for example a scale of brightnesses decreasing in geometrical progression should be adopted, each term being one-half of the preceding, or one-third, or any other ratio, or whether it would not be preferable to adopt a scale decreasing as the squares of the terms of a harmonic progression, *i.e.* according to the series $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$, etc. The former would be a purely photometric scale, and would have the apparent advantage that the light of a star of any magnitude would bear a fixed proportion to that of the magnitude next above it, an advantage, however, merely apparent, as it is certain, from many optical facts, that the unaided eye forms very different judgments of the proportions existing between bright lights, and those between feeble ones. The latter scale involves a physical idea, that of supposing the scale of magnitudes to correspond to the appearance of a first magnitude standard star, removed successively to twice, three times, etc., its original or standard distance. Such a scale, which would make the nominal magnitude a sort of index to the *presumable* or average distance, on the supposition of an equality among the real lights of the stars, would facilitate the expression of speculative ideas on the constitution of the sidereal heavens. On the other hand, it would at first sight appear to make too small a difference between the lights in the lower magnitudes. For example, on this principle of nomenclature, the light of a star of the seventh magnitude would be thirty-six 49ths of that of one of the sixth, and of the tenth 81 hundredths of the ninth, while between the first and the second the proportion would be that of four to one. So far, however, from this being really objectionable, it falls in well with the general tenor of the optical facts already alluded to, inasmuch as the eye (in the absence of disturbing causes) does actually discriminate with greater precision between the relative intensities of feeble lights than of bright ones, so that the fraction $\frac{36}{49}$, for instance, expresses quite as great a step downward (physiologically speaking) from the sixth magnitude, as $\frac{1}{4}$ does from the first. As the choice, therefore, so far as we can see, lies between these two scales, in drawing the lines of demarcation between what may be termed the *photometrical magnitudes* of the stars, we have no hesitation in adopting, and recommending others to adopt, the latter system in preference to the former.

(781.) The conventional magnitudes actually in use among astronomers, so far as their usage is consistent with itself, conforms moreover very much more nearly to this than to the geometrical progression. It has been shown[1] by direct photometric measurement of the light of a considerable number of stars from the first to the fourth magnitude, that if M be the number expressing the magnitude of a star on the above system, and *m* the number expressing the magnitude of the same star in the loose and irregular language at present conventionally or rather provisionally adopted, so far as it can be collected from the conflicting authorities of different observers, the difference between these numbers, or M—*m*, is the same in all the higher parts of the scale, and is less than half a magnitude ($0^{m}\cdot 414$). The standard star assumed as the unit of magnitude in the measurements referred to, is the bright southern star *α* Centauri, a star somewhat superior to Arcturus in lustre. If we take the distance of this star for unity, it follows that when removed to the distances 1·414, 2·414, 3·414, etc., its apparent lustre would equal those of average stars of the 1st, 2d, 3d, etc., magnitudes, *as ordinarily reckoned*, respectively.

(782.) The difference of lustre between stars of two consecutive magnitudes is so considerable as to allow of many intermediate gradations being perfectly well distinguished. Hardly any two stars of the first or of the second magnitude would be judged by an eye practiced in such comparisons

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[1] See "Results of Observations made at the Cape of Good Hope," etc., p. 371. By the Author.

to be exactly equal in brightness. Hence, the necessity, if anything like accuracy be aimed at, of subdividing the magnitudes and admitting fractions into our nomenclature of brightness. When this necessity first began to be felt, a simple bisection of the interval was recognized, and the intermediate degree of brightness was thus designated, viz. 1.2 m, 2.3 m, and so on. At present it is not infrequent to find the interval trisected thus: 1 m, 1.2 m, 2.1 m, 2 m, etc., where the expression 1.2 m denotes a magnitude intermediate between the first and second, but nearer 1 than 2; while 2.1 m designates a magnitude also intermediate, but nearer 2 than 1. This may suffice for common parlance, but as this department of astronomy progresses toward exactness, a decimal subdivision will of necessity supersede these rude forms of expression, and the magnitude will be expressed by an integer number followed by a decimal fraction; as, for instance, 2.51, which expresses the magnitude of γ Geminorum on the vulgar or conventional scale of magnitudes, by which we at once perceive that its place is almost exactly half way between the 2d and 3d average magnitudes, and that its light is to that of an average first magnitude star in that scale (of which α Orionis in its usual or normal state[2] may be taken as a typical specimen) as $1^2 : (2{\cdot}51)^2$, and to that of α Centauri as $1^2 : (2{\cdot}924)^2$, making its place in the photometric scale (so defined) 2·924. Lists of stars northern and southern, comprehending those of the vulgar first, second, and third magnitudes, with their magnitudes decimally expressed in both systems, will be found at the end of this work. The light of a star of the sixth magnitude may be roughly stated as about the hundredth part of one of the

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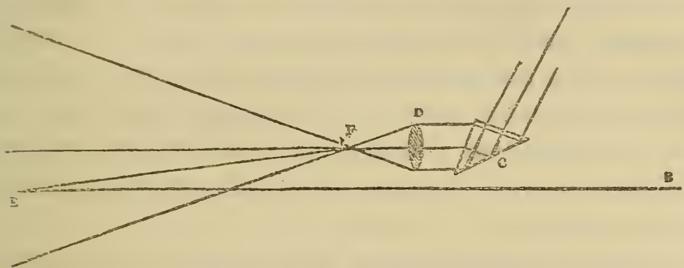
[2] In the interval from 1836 to 1839 this star underwent considerable and remarkable fluctuations of brightness.

first. Sirius would make between three and four hundred stars of that magnitude.

(783.) The exact photometrical determination of the comparative intensities of light of the stars is attended with many and great difficulties, arising partly from their differences of color; partly from the circumstance that no invariable standard of *artificial* light has yet been discovered; partly from the physiological cause above alluded to, by which the eye is incapacitated from judging correctly of the proportion of two lights, and can only decide (and that with not very great precision) as to their equality or inequality; and partly from other physiological causes. The least objectionable method hitherto proposed would appear to be the following. A natural standard of comparison is in the first instance selected, brighter than any of the stars, so as to allow of being equalized with any of them by a *reduction* of its light optically effected, and at the same time either invariable, or at least only *so* variable that its changes can be exactly calculated and reduced to numerical estimation. Such a standard is offered by the planet Jupiter, which, being much brighter than any star, subject to no phases, and variable in light only by the variation of its distance from the sun, and which moreover comes in succession above the horizon at a convenient altitude simultaneously with all the fixed stars, and in the absence of the moon, twilight, and other disturbing causes (which fatally affect all observations of this nature), combines all the requisite conditions. Let us suppose, now, that Jupiter being at A and the star to be compared with it at B, a glass prism C is so placed that the light of the planet deflected by *total internal reflection* at its base, shall emerge parallel to B E, the direction of the star's visual ray. After reflection, let it be received on a lens D, in whose focus F it will form a small bright star-like image capable of being viewed by an eye placed at E, so far out of the axis of the cone of diverging rays as to admit of seeing at the same time, and with the same eye, and so comparing, this image with the star seen directly.



By bringing the eye nearer to or further from the focus F, the apparent brightness of the focal point will be varied in the inverse ratio of the square of the distance E F, and therefore may be *equalized*, as well as the eye can judge of such equalities, with the star. If this be done for two stars several times alternately, and a mean of the results taken, by measuring E F, their relative brightness will be obtained: that of Jupiter, the temporary standard of comparison, being altogether eliminated from the result.

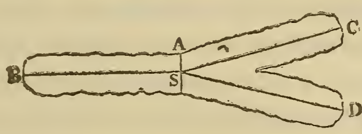
(784.) A moderate number of well-selected stars being thus photometrically determined by repeated and careful measurements, so as to afford an ascertained and graduated scale of brightness among the stars themselves, the intermediate steps or grades of magnitude may be filled up, by inserting between them, according to the judgment of the eye, other stars, forming an ascending or descending *sequence*, each member of such a sequence being brighter than that below, and less bright than that above it; and thus at length, a scale of numerical magnitudes will become established, complete in all its members, from Sirius, the brightest of the stars, down to the least visible magnitude.[3] It were much to be wished that this branch of astronomy, which at present can hardly be said to be advanced beyond its infancy, were perseveringly and systematically cultivated. It is by no means a subject of mere barren curiosity, as will abundantly appear when we come to speak of the phenomena of variable stars; and being moreover one in which amateurs of the science may easily chalk out for themselves a useful and available path, may naturally be expected to receive large and interesting accessions at their hands.

(785.) If the comparison of the apparent magnitudes of the stars with their numbers leads to no immediately obvious conclusion, it is otherwise when we view them in connection with their local distribution over the heavens. If indeed we confine ourselves to the three or four brightest classes, we shall find them distributed with a considerable approach to impartiality over the sphere: a marked preference however being observable, especially in the southern hemisphere, to a zone or belt, following the direction of a great circle passing through ε Orionis and α Crucis. But if we take in the whole amount visible to the naked eye, we shall perceive a great increase of number as we approach the borders of the Milky Way. And when we come to telescopic magnitudes, we find them crowded beyond imagina-

[3] For the method of combining and treating such sequences, where accumulated in considerable numbers, so as to eliminate from their results the influence of erroneous judgment, atmospheric circumstances, etc., which often give rise to contradictory arrangements in the order of stars differing but little in magnitude, as well as for an account of a series of photometric comparisons (in which, however, not Jupiter, but the moon, was used as an intermediate standard), see the work above cited, note on p. 353. (Results of Observations, etc.) Prof. Heis of Munster is, so far as we are aware, the only observer who has adopted and extended the method of sequences there employed.

tion, along the extent of that circle, and of the branches which it sends off from it; so that in fact its whole light is composed of nothing but stars of every magnitude, from such as are visible to the naked eye down to the smallest point of light perceptible with the best telescopes.

(786.) These phenomena agree with the supposition that the stars of our firmament, instead of being scattered in all directions indifferently through space, form a stratum of which the thickness is small, in comparison with its length and breadth; and in which the earth occupies a place somewhere about the middle of its thickness, and near the point where it subdivides into two principal laminæ, inclined at a small angle to each other (art. 302). For it is certain that, to an eye so situated, the apparent



density of the stars, supposing them pretty equally scattered through the space they occupy, would be least in a direction of the visual ray (as S A), perpendicular to the lamina, and greatest in that of its breadth, as S B, S C, S D; increasing rapidly in passing from one to the other direction, just as we see a slight haze in the atmosphere thickening into a decided fog bank near the horizon, by the rapid increase of the mere length of the visual ray. Such is the view of the construction of the starry firmament taken by Sir William Herschel, whose powerful telescopes first effected a complete analysis of this wonderful zone, and demonstrated the fact of its entirely consisting of stars.[4]

[4] Thomas Wright of Durham (Theory of the Universe, London, 1750) appears so early as 1734 to have entertained the same general view as to the con-

So crowded are they in some parts of it, that by counting the stars in a single field of his telescope, he was led to conclude that 50000 had passed under his review in a zone two degrees in breadth, during a single hour's observation. In that part of the Milky Way which is situated in 10h. 30m. R.A. and between the 147th and 150th degree of N. P. D., upward of 5000 stars have been reckoned to exist in a square degree. The immense distances at which the remoter regions must be situated will sufficiently account for the vast predominance of small magnitudes which are observed in it.

(787.) The course of the Milky Way as traced through the heavens by the unaided eye, neglecting occasional deviations and following the line of its greatest brightness as well as its varying breadth and intensity will permit, conforms as nearly as the indefiniteness of its boundary will allow it to be fixed, to that of a great circle inclined at an angle of about 63° to the equinoctial, and cutting that circle in R. A. 6h. 47m. and 18h. 47m., so that its northern and southern poles respectively are situated in R. A. 12h. 47m. N. P. D. 63° and R. A. 0h. 47m. N. P. D. 117°. Throughout the region where it is so remarkably subdivided (art. 186), this great circle holds an intermediate situation between the two great streams; with a nearer approximation however to the brighter and continuous stream, than to the fainter and interrupted one. If we trace its course in order of right ascension, we find it traversing the constellation Cassiopeia, its brightest part passing about two degrees to the north of the star δ of that constellation, *i.e.* in about 62° of north

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stitution of the Milky Way and starry firmament, founded, quite in the spirit of just astronomical speculation, on a partial resolution of a portion of it with a "one-foot reflector" (a reflector one foot in focal length). See an account of this rare work by Mr. de Morgan in Phil. Mag. Ser. 3, xxxii. p. 241, *et seq.*

declination, or 28° N. P. D. Passing thence between γ and ϵ Cassiopeiæ it sends off a branch to the south-preceding side, toward α Persei, very conspicuous as far as that star, prolonged faintly toward ϵ of the same constellation, and possibly traceable toward the Hyades and Pleiades as remote outliers. The main stream, however (which is here very faint), passes on through Auriga, over the three remarkable stars, ϵ, ζ, η, of that constellation called the Hœdi, preceding Capella, between the feet of Gemini and the horns of the Bull (where it intersects the ecliptic nearly in the Solstitial Colure) and thence over the club of Orion to the neck of Monoceros, intersecting the equinoctial in R. A. 6h. 54m. Up to this point, from the offset in Perseus, its light is feeble and indefinite, but thenceforward it receives a gradual accession of brightness, and where it passes through the shoulder of Monoceros and over the head of Canis Major it presents a broad, moderately bright, very uniform, and to the naked eye, starless stream up to the point where it enters the prow of the ship Argo, nearly on the southern tropic.[5] Here it again subdivides (about the star m Puppis), sending off a narrow and winding branch on the preceding side as far as γ Argûs, where it terminates abruptly. The main stream pursues its southward course to the 123d parallel of N.P.D., where it diffuses itself broadly and again subdivides, opening out into a wide fan-like expanse, nearly 20° in breadth formed of interlacing branches, all which terminate abruptly, in a line drawn nearly through λ and γ Argûs.

[5] In reading this description a celestial globe will be a necessary companion. It may be thought needless to detail the course of the Milky Way verbally, since it is mapped down on all celestial charts and globes. But in the generality of them, indeed in all which have come to our knowledge, this is done so very loosely and incorrectly, as by no means to dispense with a verbal description.

(788.) At this place the continuity of the Milky Way is interrupted by a wide gap, and where it recommences on the opposite side it is by a somewhat similar fan-shaped assemblage of branches which converge upon the bright star η Argûs. Thence it crosses the hindfeet of the Centaur, forming a curious and sharply defined semicircular concavity of small radius, and enters the Cross by a very bright neck or isthmus of not more than 3 or 4 degrees in breadth, being the narrowest portion of the Milky Way. After this it immediately expands into a broad and bright mass, inclosing the stars α and β Crucis, and β Centauri, and extending almost up to α of the latter constellation. In the midst of this bright mass, surrounded by it on all sides, and occupying about half its breadth, occurs a singular dark pear-shaped vacancy, so conspicuous and remarkable as to attract the notice of the most superficial gazer, and to have acquired among the early southern navigators the uncouth but expressive appellation of the *coal-sack*. In this vacancy which is about 8° in length, and 5° broad, only one very small star visible to the naked eye occurs, though it is far from devoid of telescopic stars, so that its striking blackness is simply due to the effect of contrast with the brilliant ground with which it is on all sides surrounded. This is the place of nearest approach of the Milky Way to the South Pole. Throughout all this region its brightness is very striking, and when compared with that of its more northern course already traced, conveys strongly the impression of greater proximity, and would almost lead to a belief that our situation as spectators is separated on all sides by a considerable interval from the dense body of stars composing the Galaxy, which in this view of the subject would come to be considered as a flat ring or some other re-entering form of immense and irregular breadth and thickness, within which we are excentrically situated, nearer to the southern than to the northern part of its circuit.

(789.) At α Centauri, the Milky Way again subdivides,[6] sending off a great branch of nearly half its breadth, but which thins off rapidly, at an angle of about 20° with its general direction, toward the preceding side, to η and d Lupi, beyond which it loses itself in a narrow and faint streamlet. The main stream passes on increasing in breadth to γ Normæ, where it makes an abrupt elbow and again subdivides into one principal and continuous stream of very irregular breadth and brightness on the following side, and a complicated system of interlaced streaks and masses on the preceding, which covers the tail of Scorpio, and terminates in a vast and faint effusion over the whole extensive region occupied by the preceding leg of Ophiuchus, extending northward to the parallel of 103° N. P. D., beyond which it cannot be traced; a wide interval of 14°, free from all appearance of nebulous light, separating it from the great branch on the north side of the equinoctial of which it is usually represented as a continuation.

(790.) Returning to the point of separation of this great branch from the main stream, let us now pursue the course of the latter. Making an abrupt bend to the following side, it passes over the stars ι Aræ, θ and ι Scorpii, and γ Tubi to γ Sagittarii, where it suddenly collects into a vivid oval mass about 6° in length and 4° in breadth, so excessively rich in stars that a very moderate calculation makes their number exceed 100,000. Northward of this mass, this

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[6] All the maps and globes place this subdivision at β Centauri, but erroneously.

stream crosses the ecliptic in longitude about 276°, and proceeding along the bow of Sagittarius into Antinous has its course rippled by three deep concavities, separated from each other by remarkable protuberances, of which the larger and brighter (situated between Flamsteed's stars 3 and 6 Aquilæ) forms the most conspicuous patch in the southern portion of the Milky Way visible in our latitudes.

(791.) Crossing the equinoctial at the 19th hour of right ascension, it next runs in an irregular, patchy, and winding stream through Aquila, Sagitta and Vulpecula up to Cygnus; at ϵ of which constellation its continuity is interrupted, and a very confused and irregular region commences, marked by a broad dark vacuity, not unlike the southern "coal-sack," occupying the space between ϵ, α, and γ Cygni, which serves as a kind of centre for the divergence of three great streams; one, which we have already traced; a second, the continuation of the first (across the interval) from α northward, between Lacerta and the head of Cepheus to the point in Cassiopeia whence we set out, and a third branching off from γ Cygni, very vivid and conspicuous, running off in a southern direction through β Cygni, and s Aquilæ almost to the equinoctial, where it loses itself in a region thinly sprinkled with stars, where in some maps the modern constellation Taurus Poniatovski is placed. This is the branch which, if continued across the equinoctial, might be supposed to unite with the great southern effusion in Ophiuchus already noticed (art. 789). A considerable offset, or protuberant appendage, is also thrown off by the northern stream from the head of Cepheus directly toward the pole, occupying the greater part of the quartile formed by α, β, ι, and δ of that constellation.

(792.) We have been somewhat circumstantial in describing the course and principal features of the Via Lactea, not only because there does not occur anywhere (so far as we know) any correct account of it, but chiefly by reason of its high interest in sidereal astronomy, and that the reader may perceive how very difficult it must necessarily be to form any just conception of the real, solid form, as it exists in space, of an object so complicated, and which we see from a point of view so unfavorable. The difficulty is of the same kind which we experience when we set ourselves to conceive the real shape of an auroral arch or of the clouds, but far greater in degree, because we know the laws which regulate the formation of the latter, and limit them to certain conditions of altitude—because their motion presents them to us in various aspects, but chiefly because we contemplate them from a station considerably below their general plane, so as to allow of our mapping out some kind of ground-plan of their shape. All these aids are wanting when we attempt to map and model out the Galaxy, and beyond the obvious conclusion that its form must be, generally speaking, *flat*, and of a thickness small in comparison with its area in length and breadth, the laws of perspective afford us little further assistance in the inquiry. Probability may, it is true, here and there enlighten us as to certain features. Thus when we see, as in the coal-sack, a sharply defined oval space free from stars, insulated in the midst of a uniform band of not much more than twice its breadth, it would seem much less probable that a conical or tubular hollow traverses the whole of a starry stratum, continuously extended from the eye outward, than that a *distant* mass of comparatively moderate thickness should be simply perforated from side to side, or that an oval vacuity should be seen foreshortened in a *distant* foreshortened area, not really exceeding two or three times its own breadth. Neither can we without obvious improbability refuse to admit that the long lateral offsets which at so many places quit the main stream and run out to great distances, are either planes seen edgewise, or the convexities of curved surfaces viewed tangentially, rather than cylindrical or columnar excrescences bristling up obliquely from the general level. And in the same spirit of probable surmise we may account for the intricate reticulations above described as existing in the region of Scorpio, rather by the accidental crossing of streaks thus originating, at very different distances, or by a cellular structure of the mass, than by real intersections. Those cirrous clouds which are often seen in windy weather, convey no inapt impression either of the kind of appearance in question, or of the structure it suggests. It is to other indications, however, and chiefly to the telescopic examination of its intimate constitution and to the law of the distribution of stars, not only within its bosom, but generally over the heavens, that we must look for more definite knowledge respecting its true form and extent.

(793.) It is on observations of this latter class, and not on merely speculative or conjectural views, that the generalization in art. 786, which refers the phenomena of the starry firmament to the system of the Galaxy as their embodying fact, is brought to depend. The process of "gauging" the heavens was devised by Sir W. Herschel for this purpose. It consisted in simply counting the stars of all magnitudes which occur in single fields of view, of 15′ in diameter, visible through a reflecting telescope of 18 inches aperture, and 20 feet focal length, with a magnifying power of 180°: the points of observation being very numerous and taken indiscriminately in every part of the surface of the sphere visible in our latitudes. On a comparison of many hundred such "gauges" or local enumerations it appears that the density of starlight (or the number of stars existing on an average of several such enumerations in any one immediate neighborhood) is least in the pole of the *Galactic circle*,[7] and increases on all sides, with the *Galactic polar distance* (and that nearly equally in all directions) down to the Milky Way itself, where it attains its maximum. The progressive rate of increase in proceeding from the pole is at first slow, but becomes more and more rapid as we approach the plane of that circle according to a law of which the following numbers, deduced by M. Struve from a careful analysis of all the gauges in question, will afford a correct idea:

Galactic[8] North Polar Distance	Average Number of Stars in a Field 15′ in Diameter
0°	4·15
15°	4·68
30°	6·52
45°	10 36
60°	17·68
75°	30·30
90°	122·00

From which it appears that the *mean* density of the stars in the galactic circle exceeds in a ratio of very nearly 30 to 1 that in its pole, and in a proportion of more than 4 to 1 that in a direction 15° inclined to its plane.

(794.) These numbers fully bear out the statement in art. 786 and even draw closer the resemblance by which that statement is there illustrated. For the rapidly increasing density of a fog-bank as the visual ray is depressed toward the plane of the horizon is a consequence not only

[7] From γαλα, γαλακτος, milk; meaning the great circle spoken of in art. 787, to which the course of the Via Lactea most nearly conforms. This circle is to sidereal what the invariable ecliptic is to planetary astronomy—a plane of ultimate reference, the ground-plane of the sidereal system.

[8] Etudes d'Astronomie Stellaire, p. 71. M. Struve maintains the Galactic circle to be a small, not a great, circle of the sphere. The appeal is to the eyesight. I retain my own conviction.

of the mere increase in length of the foggy space traversed, but also of an actual increase of density in the fog itself in its lower strata. Now this very conclusion follows from a comparison *inter se* of the numbers above set down, as M. Struve has clearly shown from a mathematical analysis of the empirical formula, which faithfully represents their law of progression, and of which he states the result in the following table, expressing the densities of the stars at the respective distances, 1, 2, 3, etc., from the galactic plane, taking the mean density of the stars in that plane itself for unity.

Distances from the Galactic Plane.	Density of Stars.	Distances from the Galactic Plane.	Density of Stars.
0·00	1·00000	0·50	0·08646
0·05	0·48568	0·60	0·05510
0·10	0·33288	0·70	0·03079
0·20	0·23895	0·80	0·01414
0·30	0·17980	0·866	0.00532
0·40	0·13021		

The unit of distance, of which the first column of this table expresses fractional parts, is the distance at which such a telescope is capable of rendering just *visible* a star of average magnitude, or, as it is termed, its *space-penetrating* power. As we ascend therefore from the galactic plane into this kind of stellar atmosphere, we perceive that the density of its parallel strata decreases with great rapidity. At an altitude above that plane equal to only one-twentieth of the telescopic limit, it has already diminished to one-half, and at an altitude of 0·866, to hardly more than one-two-hundredth of its amount in that plane. So far as we can perceive there is no flaw in this reasoning, if only it be granted, 1st, that the level planes are continuous, and of equal density throughout; and, 2dly, *that an absolute and definite limit* *is set to telescopic vision, beyond which, if stars exist, they elude our sight, and are to us as if they existed not:* a postulate whose probability the reader will be in a better condition to estimate, when in possession of some other particulars respecting the constitution of the Galaxy to be described presently.

(795.) A similar course of observation followed out in the southern hemisphere, leads independently to the same conclusion as to the law of the visible distribution of stars over the southern galactic hemisphere, or that half of the celestial surface which has the south galactic pole for its centre. A system of gauges, extending over the whole surface of that hemisphere taken with the same telescope, field of view and magnifying power employed in Sir William Herschel's gauges, has afforded the average numbers of stars *per* field of 15′ in diameter, within the areas of zones encircling that pole at intervals of 15°, set down in the following table:

Zones of Galactic South Polar Distance	Average Number of Stars per Field of 15′
0° to 15°	6·05
15 to 30	6·62
30 to 45	9·08
45 to 60	13·49
60 to 75	26·29
75 to 90	59·06

(796.) These numbers are not directly comparable with those of M. Struve, given in art. 793, because the latter correspond to the limiting polar distances, while these are the averages for the included zones. That eminent astronomer, however, has given a table of the average gauges appropriate to *each degree* of north galactic polar distance,[9] from which it is easy to calculate averages for the whole extent

[9] Etudes d'Astronomie Stellaire, p. 34.

of each zone. How near a parallel the results of this calculation for the northern hemisphere exhibit with those above stated for the southern will be seen by the following table:

Zones of Galactic North Polar Distance	Average Number of Stars per Field of 15′ from M. Struve's Table
0° to 15°	4·32
15 to 30	5·42
30 to 45	8·21
45 to 60	13·61
60 to 75	24·09
75 to 90	53·43

It would appear from this that, with an almost exactly similar law of apparent density in the two hemispheres, the southern were somewhat richer in stars than the northern, which may, and not improbably does arise, from our situation not being precisely in the middle of its thickness, but somewhat nearer to its northern surface.

(797.) When examined with powerful telescopes, the constitution of this wonderful zone is found to be no less various than its aspect to the naked eye is irregular. In some regions the stars of which it is wholly composed are scattered with remarkable uniformity over immense tracts, while in others the irregularity of their distribution is quite as striking, exhibiting a rapid succession of closely clustering rich patches separated by comparatively poor intervals, and indeed in some instances by spaces absolutely dark *and completely void of any star*, even of the smallest telescopic magnitude. In some places not more than 40 or 50 stars on an average occur in a "gauge" field of 15′, while in others a similar average gives a result of 400 or 500. Nor is less variety observable in the character of its different regions in respect of the magnitudes of the stars they exhibit, and the proportional numbers of the larger and smaller magnitudes associated together, than in respect of their aggregate numbers. In some, for instance, extremely minute stars, though never altogether wanting, occur in numbers so moderate as to lead us irresistibly to the conclusion that in these regions we see *fairly through* the starry stratum, since it is impossible otherwise (supposing their light not intercepted) that the numbers of the smaller magnitudes should not go on continually increasing ad infinitum. In such cases moreover the ground of the heavens, as seen between the stars, is for the most part perfectly dark, which again would not be the case, if innumerable multitudes of stars, too minute to be individually discernible, existed beyond. In other regions we are presented with the phenomenon of an almost uniform degree of brightness of the individual stars, accompanied with a very even distribution of them over the ground of the heavens, both the larger and smaller magnitudes being strikingly deficient. In such cases it is equally impossible not to perceive that we are looking *through* a sheet of stars nearly of a size, and of no great thickness compared with the distance which separates them from us. Were it otherwise we should be driven to suppose the more distant stars uniformly the larger, so as to compensate by their greater intrinsic brightness for their greater distance, a supposition contrary to all probability. In others again, and that not infrequently, we are presented with a double phenomenon of the same kind, viz. a tissue as it were of large stars spread over another of very small ones, the intermediate magnitudes being wanting. The conclusion here seems equally evident that in such cases we look through two sidereal sheets separated by a starless interval.

(798.) Throughout by far the larger portion of the extent of the Milky Way in both hemispheres, the general blackness of the ground of the heavens on which its stars are projected, and the absence of that innumerable multitude and excessive crowding of the smallest visible magnitudes, and of glare produced by the aggregate light of multitudes too small to affect the eye singly, which the contrary supposition would appear to necessitate, must, we think, be considered unequivocal indications that its dimensions *in directions where these conditions obtain*, are not only not infinite, but that the space-penetrating power of our telescopes suffices fairly to pierce through and beyond it. It is but right however to warn our readers that this conclusion has been controverted, and that by an authority not lightly to be put aside, on the ground of certain views taken by Olbers as to a defect of perfect transparency in the celestial spaces, in virtue of which the light of the more distant stars is enfeebled more than in proportion to their distance. The extinction of light thus originating, proceeding in geometrical progression while the distance increases in arithmetical, a limit, it is argued, is placed to the space-penetrating powers of telescopes, far within that which distance alone apart from such obscuration would assign. It would lead us too far aside of the objects of a treatise of this nature to enter upon any discussion of the grounds (partly metaphysical) on which these views rely. It must suffice here to observe that the objection alluded to, if applicable to any, is equally so to every part of the galaxy. We are not at liberty to argue that at one part of its circumference our view is limited by this sort of cosmical veil which extinguishes the smaller magnitudes, cuts off the nebulous light of distant masses, and closes our view in impenetrable darkness; while at another we are compelled by the clearest evidence telescopes can afford to believe that star-strewn vistas *lie open*, exhausting their powers and stretching out beyond their utmost reach, as is proved by that very phenomenon which the existence of such a veil would render impossible, viz. infinite increase of number and diminution of magnitude, terminating in complete irresolvable nebulosity. Such is, in effect, the spectacle afforded by a very large portion of the Milky Way in that interesting region near its point of bifurcation in Scorpio (arts. 789, 792) where, through the hollows and deep recesses of its complicated structure, we behold what has all the appearance of a wide and indefinitely prolonged area strewed over with discontinuous masses and clouds of stars which the telescope at length refuses to analyze.[10] Whatever other conclusions we may draw, this must anyhow be regarded as the direction of the greatest linear extension of the ground plan of the galaxy. And it would appear to follow, also, as a not less obvious consequence, that in those regions where that zone is clearly resolved into stars well separated and *seen projected on a black ground*, and where by consequence it is *certain*, if the foregoing views be correct, that we look out beyond them into space, the smallest visible stars appear as such, not by reason of excessive distance, but of a real inferiority of size or brightness.[11]

(799.) When we speak of the comparative remoteness of certain regions of the starry heavens beyond others, and of our own situation in them, the question immediately

[10] It would be doing great injustice to the illustrious astronomer of Pulkova (whose opinion, if we here seem to controvert, it is with the utmost possible deference and respect) not to mention that at the time of his writing the remarkable essay already more than once cited, in which the views in question are delivered, he could not have been aware of the important facts alluded to in the text, the work in which they are described being then unpublished.

[11] Professor Loomis (Progress of Astronomy, 1850, p. 141), with the facts adduced before him, arrives at a contrary conclusion. Astronomers will judge of the validity of his objections. Professor Argelander (Astron. Nachr. 996) has cited me in support of Olbers' theory, in direct opposition to my own opinion, here (as I should have thought distinctly enough) recorded.

arises, what is the distance of the nearest fixed star? What is the scale on which our visible firmament is constructed? And what proportion do its dimensions bear to those of our own immediate system? To these questions astronomy has at length been enabled to afford an answer.

(800.) The diameter of the earth has served us for the base of a triangle, in the *trigonometrical survey* of our system (art. 274), by which to calculate the distance of the sun; but the extreme minuteness of the sun's parallax (art. 357) renders the calculation from this "ill-conditioned" triangle (art. 275) so delicate, that nothing but the fortunate combination of favorable circumstances, afforded by the transits of Venus (art. 479), could render its results even tolerably worthy of reliance. But the earth's diameter is too small a base for direct triangulation to the verge even of our own system (art. 526), and we are, therefore, obliged to substitute the *annual parallax* for the diurnal, or, which comes to the same thing, to ground our calculation on the relative velocities of the earth and planets in their orbits (art. 486), when we would push our triangulation to that extent. It might be naturally enough expected, that by this enlargement of our base to the vast diameter of the earth's orbit, the next step in our survey (art. 275) would be made at a great advantage;—that our change of station, from side to side of it, would produce a considerable and easily measurable amount of annual parallax in the stars, and that by its means we should come to a knowledge of their distance. But, after exhausting every refinement of observation, astronomers were, up to a very late period, unable to come to any positive and coincident conclusion upon this head; and the amount of such parallax, even for the nearest fixed star examined with the requisite attention, remained mixed up with, and concealed among, the errors incidental to all astronomical determinations. The nature of these errors has been explained in the earlier part of this work, and we need not remind the reader of the difficulties which must necessarily attend the attempt to disentangle an element not exceeding a few tenths of a second or at most a whole second from the host of uncertainties entailed on the results of observations by them: none of them individually perhaps of greater magnitude, but embarrassing by their number and fluctuating amount. Nevertheless, by successive refinements in instrument making, and by constantly progressive approximation to the exact knowledge of the uranographical corrections, that assurance has been obtained, even in the earlier years of the present century, viz. that no star visible in northern latitudes, to which attention had been directed, manifested an amount of parallax exceeding a single second of arc. It is worth while to pause for a moment to consider what conclusions would follow from the admission of a parallax to this amount.

(801.) Radius is to the sine of 1″ as 206265 to 1. In this proportion then *at least* must the distance of the fixed stars from the sun exceed that of the sun from the earth. Again, the latter distance, as we have already seen (art. 357), exceeds the earth's radius in the proportion of 23984 to 1. Taking therefore the earth's radius for unity, a parallax of 1″ supposes a distance of 4947059760 or nearly five thousand millions of such units: and lastly, to descend to ordinary standards, since the earth's radius may be taken at 4000 of our miles, we find 19788239040000 or about twenty billions of miles for our resulting distance.

(802.) In such numbers the imagination is lost. The only mode we have of conceiving such intervals at all is by the time which it would require for light to traverse them. Light, as we know (art. 545), travels at the rate of a semidiameter of the earth's orbit in $8^m\ 13^s\cdot3$. It would, therefore, occupy 206205 times this interval or 3 years and 83 days to traverse the distance in question. Now as this is an inferior limit which it is already ascertained that even the brightest and therefore probably the nearest stars exceed, what are we to allow for the distance of those innumerable stars of the smaller magnitudes which the telescope discloses to us! What for the dimensions of the galaxy in whose remoter regions, as we have seen, the united lustre of myriads of stars is perceptible only in powerful telescopes as a feeble nebulous gleam!

(803.) The space-penetrating power of a telescope or the comparative distance to which a given star would require to be removed in order that it may appear of the same brightness in the telescope as before to the naked eye, may be calculated from the aperture of the telescope compared with that of the pupil of the eye, and from its reflecting or transmitting power, *i.e.* the proportion of the incident light it conveys to the observer's eye. Thus it has been computed that the space-penetrating power of such a reflector as that used in the star-gauges above referred to is expressed by the number 75. A star then of the sixth magnitude removed to 75 times its distance would still be perceptible *as a star* with that instrument, and admitting such a star to have 100th part of the light of a standard star of the first magnitude, it will follow that such a standard star, if removed to 750 times its distance, would excite in the eye, when viewed through the gauging telescope, the same impression as a star of the sixth magnitude does to the naked eye. Among the infinite multitude of such stars in the remoter regions of the galaxy, it is but fair to conclude that innumerable individuals equal in intrinsic brightness to those which immediately surround us must exist. The light of such stars then must have occupied upward of 2000 years in travelling over the distance which separates them from our own system. It follows then that when we observe the places and note the appearances of such stars, we are only reading their history of two thousand years' anterior date thus wonderfully recorded. We cannot escape this conclusion but by adopting as an alternative an intrinsic inferiority of light in *all* the smaller stars of the galaxy. We shall be better able to estimate the probability of this alternative when we shall have made acquaintance with other sidereal systems whose existence the telescope discloses to us, and whose analogy will satisfy us that the view of the subject here taken is in perfect harmony with the general tenor of astronomical facts.

(804.) Hitherto we have spoken of a parallax of $1''$ as a mere limit below which that of any star yet examined assuredly, or at least very probably falls, and it is not without a certain convenience to regard this amount of parallax as a sort of unit of reference, which, connected in the reader's recollection with a parallactic unit of distance from our system of 20 billions of miles, and with a $3\frac{1}{4}$ year's journey of light, may save him the trouble of such calculations, and ourselves the necessity of covering our pages with such enormous numbers, when speaking of stars whose parallax has actually been ascertained with some approach to certainty, either by direct meridian observation or by more refined and delicate methods. These we shall proceed to explain, after first pointing out the theoretical peculiarities which enable us to separate and disentangle its effects from those of the uranographical corrections, and from other causes of error which being periodical in their nature add greatly to the difficulty of the subject. The effects of precession and proper motion (see art. 852) which are uniformly progressive from year to year, and that of nutation which runs through its period in nineteen years, it is obvious enough, separate themselves at once by these characters from that of parallax; and, being known with very great precision, and being certainly independent, as regards their causes, of any individual peculiarity in the stars affected by them, whatever small uncertainty may remain respecting the numerical elements which enter into their computation (or in mathematical language their *coefficients*), can give rise to no embarrassment. With regard to aberration the case is materially different. This correction affects the place of a star by a fluctuation annual in its period, and therefore, so far, agreeing with parallax. It is also very similar in the *law* of its variation at different seasons of the year, parallax having for its apex (see arts. 343, 344) the apparent place of the sun in the ecliptic, and aberration a point in the same great circle 90° behind that place, so that in fact the formulæ of calculation (the coefficients excepted) are the same for both, substituting only for the sun's longitude in the expression for the one, that longitude diminished by 90° for the other. Moreover, in the absence of *absolute certainty* respecting the nature of the propagation of light, astronomers have hitherto considered it necessary to assume at least as a *possibility* that the velocity of light may be to some slight amount dependent on individual peculiarities in the body emitting it.[12]

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[12] In the actual state of astronomy and photology this necessity can hardly be considered as still existing, and it is desirable, therefore, that the practice of astronomers of introducing an unknown correction for the constant of aberration

(805.) If we suppose a line drawn from the star to the earth at all seasons of the year, it is evident that this line will sweep over the surface of an exceedingly acute, oblique cone, having for its axis the line joining the sun and star, and for its base the earth's annual orbit, which, for the present purpose, we may suppose circular. The star will therefore appear to describe each year about its mean place regarded as fixed, and in virtue of parallax alone, a minute ellipse, the section of this cone by the surface of the celestial sphere, perpendicular to the visual ray. But there is also another way in which the same fact may be represented. The apparent orbit of the star about its mean place as a centre, will be precisely that which it would appear to describe, if seen from the sun, supposing it really revolved about that place in a circle exactly equal to the earth's annual orbit, in a plane parallel to the ecliptic. This is evident from the equality and parallelism of the lines and directions concerned. Now the effect of aberration (disregarding the slight variation of the earth's velocity in different parts of its orbit) is precisely similar in law, and differs only in amount, and in its bearing reference to a direction 90° different in longitude. Suppose, in order to fix our ideas, the maximum of parallax to be 1″ and that of aberration 20·5″, and let A B, *a b*, be two circles imagined to be described separately, as above, by the star about its mean place S, in virtue of these two causes respectively, γ being a line parallel to that of the line of equinoxes. Then if in virtue of parallax alone, the star would be found at *a* in the smaller orbit, it would in virtue of aberration alone be found at A, in the larger, the angle *a* S A being a right angle. Draw-

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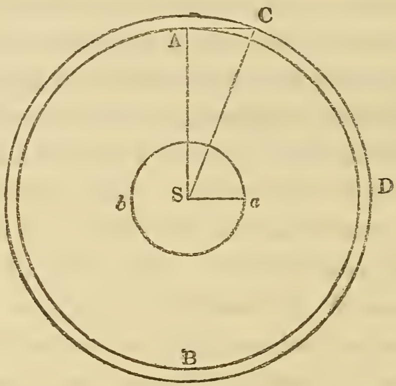
into their "equations of condition" for the determination of parallax, should be disused, since it actually tends to introduce error into the final result.

ing then A C equal and parallel to S a, and joining S C, it will in virtue of both simultaneously be found in C, *i.e.* in the circumference of a circle whose radius is S C, and at a point in that circle, in advance of A, the aberrational place, by the angle A S C. Now since S A : A C : : 20·5 : 1, we find for the angle A S C 2° 47′ 35″, and for the length of the radius S C of the circle representing the compound motion 20″·524. The difference (0″·024) between this and S C, the radius of the aberration circle, is quite imperceptible, and even supposing a quantity so minute to be capable of detec-



tion by a prolonged series of observations, it would remain a question whether it were produced by parallax or by a specific difference of aberration from the general average 20″·5 in the star itself. It is therefore to the difference of 2° 48′ between the angular situation of the displaced star in this hypothetical orbit, *i.e.* in the *arguments* (as they are called) of the joint correction (γ S C) and that of aberration alone (γ S A), that we have to look for the resolution of the problem of parallax. The reader may easily figure to himself the delicacy of an inquiry which turns wholly (even when stripped of all its other difficulties) on the *precise* determination of a quantity of this nature, and of such very moderate magnitude.

(806.) But these other difficulties themselves are of no trifling order. All astronomical instruments are affected by differences of temperature. Not only do the materials of which they are composed expand and contract, but the masonry and solid piers on which they are erected, nay even the very soil on which these are founded, participate in the general change from summer warmth to winter cold. Hence arise slow oscillatory movements of exceedingly minute amount, which levels and plumb-lines afford but very inadequate means of detecting, and which *being also annual in their period* (after rejecting whatever is merely casual and momentary) mix themselves intimately with the matter of our inquiry. Refraction too, besides its casual variations from night to night, which a long series of observations would eliminate, depends for its theoretical expression on the constitution of the strata of our atmosphere, and the law of the distribution of heat and moisture at different elevations, which cannot be unaffected by difference of season. No wonder then that mere meridional observations should, almost up to the present time, have proved insufficient, except in one very remarkable instance, to afford unquestionable evidence, and satisfactory quantitative measurement of the parallax of any fixed star.

(807.) The instance referred to is that of α Centauri, one of the brightest and for many other reasons, one of the most remarkable of the southern stars. From a series of observations of this star, made at the Royal Observatory of the Cape of Good Hope in the years 1832 and 1833, by Professor Henderson, with the mural circle of that establishment, a parallax to the amount of an entire second was concluded on his reduction of the observations in question after his return to England. Subsequent observations by Mr. Maclear, partly with the same, and partly with a new and far more efficiently constructed instrument of the same description made in the years 1839 and 1840, have fully confirmed the reality of the parallax indicated by Professor Henderson's observations, though with a slight diminution in its concluded amount, which comes out equal to $0''\cdot9128$ or about $\frac{10}{11}$ths of a second; *bright stars in its immediate neighborhood being unaffected by a similar periodical displacement, and thus affording satisfactory proof that the displacement indicated in the case of the star in question is not merely a result of annual variations of temperature.* As it is impossible at present to answer for so minute a quantity as that by which this result differs from an exact second, we may consider the distance of this star as approximately expressed by the parallactic unit of distance referred to in art. 804.

(808.) A short time previous to the publication[13] of this important result, the detection of a sensible and measurable amount of parallax in the star N° 61 Cygni of Flamsteed's catalogue of stars was announced by the celebrated astronomer of Königsberg, the late M. Bessel.[14] This is a small and inconspicuous star, hardly exceeding the sixth magnitude, but which had been pointed out for especial observation by the remarkable circumstance of its being affected by a *proper motion* (see art. 852), *i.e.* a regular and continually progressive annual displacement among the surrounding stars to the extent of more than $5''$ per annum, a quantity so very much exceeding the average of similar minute an-

[13] Prof. Henderson's paper was read before the Astronomical Society of London, Jan. 3, 1839. It bears date Dec. 24, 1838.

[14] Astronomische Nachrichten, Nos. 365, 366, Dec. 13, 1838.

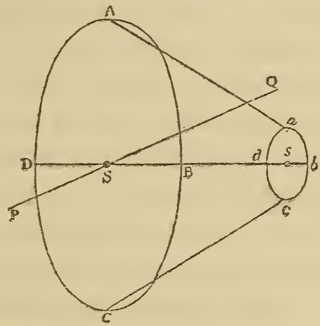
nual displacements which many other stars exhibit, as to lead to a suspicion of its being actually nearer to our system. It is not a little remarkable that a similar presumption of proximity exists also in the case of α Centauri, whose unusually large proper motion of nearly 4″ per annum is stated by Professor Henderson to have been the motive which induced him to subject his observations of that star to that severe discussion which led to the detection of its parallax. M. Bessel's observations of 61 Cygni were commenced in August, 1837, immediately on the establishment at the Königsberg observatory of a magnificent heliometer, the workmanship of the celebrated optician Fraunhofer of Munich, an instrument especially fitted for the system of observation adopted; which being totally different from that of direct meridional observation, more refined in its conception, and susceptible of far greater accuracy in its practical application, we must now explain.

(809.) Parallax, proper motion, and specific aberration (denoting by the latter phrase that part of the aberration of a star's light which may be supposed to arise from its individual peculiarities, and which we have every reason to believe at all events an exceedingly minute fraction of the whole) are the only uranographical corrections which do not necessarily affect alike the apparent places of two stars situated in, or *very nearly in*, the same visual line. Supposing then two stars at an immense distance, the one behind the other, but otherwise so situated as to appear very nearly along the same visual line, they will constitute what is called a star *optically double*, to distinguish it from a star *physically double*, of which more hereafter. Aberration (that which is common to all stars), precession, nutation, *nay, even refraction, and instrumental causes of apparent dis-*

placement, *will affect them alike*, or so very nearly alike (if the minute difference of their apparent places be taken into account) as to admit of the difference being neglected, or very accurately allowed for, by an easy calculation. If then, instead of attempting to determine by observation the place of the nearer of two *very unequal* stars (which will probably be the larger) by direct observation of its right ascension and polar distance, we content ourselves with referring its place to that of its remoter and smaller companion by *differential observation*, *i.e.* by measuring only its *difference* of situation from the *latter*, we are at once relieved of the necessity of making these corrections, and from all uncertainty as to their influence on the result. And for the very same reason, errors of adjustment (art. 136), of graduation, and a host of instrumental errors, which would for this delicate purpose fatally affect the absolute determination of either star's place, are harmless when only the difference of their places, each equally affected by such causes, is required to be known.

(810.) Throwing aside therefore the consideration of all these errors and corrections, and disregarding for the present the minute effect of specific aberration and the uniformly progressive effect of proper motion, let us trace the effect of the differences of the parallaxes of two stars thus juxtaposed, or their apparent relative distance and position at various seasons of the year. Now the parallax being inversely as the distance, the dimensions of the small ellipses apparently described (art. 805) by each star on the concave surface of the heavens by parallactic displacement will differ—the nearer star describing the larger ellipse. But both stars lying very nearly in the same direction from the sun, these ellipses will be similar and similarly situated.

Suppose S and *s* to be the positions of the two stars as seen from the sun, and let A B C D, *a b c d*, be their parallactic ellipses; then, since they will be at all times similarly situated in these ellipses, when the one star is seen at A, the other will be seen at *a*. When the earth has made a quarter of a revolution in its orbit, their apparent places will be B *b*; when another quarter, C *c*; and when another, D *d*. If, then, we measure carefully, with micrometers adapted



for the purpose, their apparent situation with respect to each other, at different times of the year, we should perceive a periodical change, both in the *direction* of the line joining them, and in the *distance* between their centres. For the lines A *a* and C *c* cannot be parallel, nor the lines B *b* and D *d* equal, unless the ellipses be of equal dimensions, *i.e.* unless the two stars have the same parallax, or are equidistant from the earth.

(811.) Now, micrometers, properly mounted, enable us to measure very exactly both the distance between two objects which can be seen together in the same field of a telescope, and the position of the line joining them with respect to the horizon, or the meridian, or any other determinate direction in the heavens. The double image micrometer, and especially the heliometer (arts. 200, 201), is peculiarly adapted for this purpose. The images of the two stars formed side by side, or in the same line prolonged, however momentarily displaced by temporary refraction or instrumental tremor, *move together*, preserving their relative situation, the judgment of which is no way disturbed by such irregular movements. The heliometer also, taking in a greater range than ordinary micrometers, enables us to compare one large star with more than one adjacent small one, and to select such of the latter among many near it, as shall be most favorably situated for the detection of any motion in the large one, not participated in by its neighbors.

(812.) The star examined by Bessel has two such neighbors, both very minute, and therefore probably very distant, most favorably situated, the one (s) at a distance of 7′ 42″, the other (s') at 11′ 46″ from the large star, and so situated, that their directions from that star make nearly a right angle with each other. The effect of parallax therefore would necessarily cause the two distances S s and S s' to vary so as to attain their maximum and minimum values alternately at three-monthly intervals, and this is what was actually observed to take place, the one distance being always most rapidly on the increase or decrease when the other was stationary (the uniform effect of proper motion being understood of course to be always duly accounted for). This alternation, though so small in amount as to indicate, as a final result, a parallax, or rather a difference of parallaxes between the large and small stars of hardly more than one-third of a second, was maintained with such regularity as to leave no room for reasonable doubt as to its cause, and having been confirmed by the further continuance of these observations, and quite recently by the exact coincidence between the result thus obtained, and that deduced by M.

Peters from observations of the same star at the observatory of Pulkova,[15] is considered on all hands as fully established. The parallax of this star finally resulting from Bessel's observation is 0″·348, so that its distance from our system is very nearly three parallactic units. (Art. 804.)

(813.) The bright star α Lyræ has also near it, at only 43″ distance (and therefore within the reach of the parallel wire or ordinary double image micrometer), a very minute star, which has been subjected since 1835 to a severe and assiduous scrutiny by M. Struve, on the same principle of differential observation. He has thus established the existence of a measurable amount of parallax in the large star, less indeed than that of 61 Cygni (being only about ¼ of a second), but yet sufficient (such was the delicacy of his measurements) to justify this excellent observer in announcing the result as at least highly probable, on the strength of only five nights' observation, in 1835 and 1836. This probability, the continuation of the measures to the end of 1838 and the corroborative, though not in this case precisely coincident, result of M. Peters's investigations have converted into a certainty. M. Struve has the merit of being the first to bring into practical application this method of observation, which, though proposed for the purpose, and its great advantages pointed out by Sir William Herschel so early as 1781,[16] remained long unproductive of any result, owing partly to the imperfection of micrometers for the measurement of distance, and partly to a reason which we shall presently have occasion to refer to.

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[15] With the great vertical circle by Ertel.

[16] It has been referred even to Galileo. But the general explanation of Parallax in the Systema Cosmicum, Dial. iii. p. 271 (Leyden edit. 1699), to which the reference applies, does not touch any of the peculiar features of the case, or meet any of its difficulties.

(814.) If the component individuals S, *s* (*fig.* art. 810) be (as is often the case) very close to each other, the parallactic variation of their *angle of position*, or the extreme angle included between the lines A a, C c, may be very considerable, even for a small amount of difference of parallaxes between the large and small stars. For instance, in the case of two adjacent stars 15″ asunder, and otherwise favorably situated for observation, an annual fluctuation to and fro in the apparent direction of their line of junction to the extent of half a degree (a quantity which could not escape notice in the means of numerous and careful measurements) would correspond to a difference of parallax of only ⅛ of a second. A difference of 1″ between two stars apparently situated at 5″ distance might cause an oscillation in that line to the extent of no less than 11°, and if nearer one proportionally still greater. This mode of observation has been applied to a considerable number of stars by Lord Wrottesley, and with such an amount of success, as to make its further application desirable. (Phil. Trans. 1851.[17])

(815.) The following are some of the principal fixed stars to which parallax has been up to the present time more or less probably assigned:

			″	
α	Centauri		0·976	(Henderson, corrected by Peters.)
61	Cygni		0·348	(Bessel.)
	Lalande	21258	0·260	(Krüger.)
	Oeltzen	17415–6	0·247	(Krüger.)
α	Lyræ		0·155	(W. Struve, corrected by O. Struve.)
	Sirius		0·150	(Henderson, corrected by Peters.)
70 *p*	Ophiuchi		0·16	(Krüger.)
ι	Ursæ Majoris		0·139	(Peters.)
	Arcturus		0·127	ditto
	Polaris		0·067	ditto
	Capella		0·046	ditto

[17] See Phil. Trans. 1826, p. 266 *et seq.*, and 1827, for a list of stars well adapted for such observation, with the times of the year most favorable.—The list in Phil. Trans. 1826 is incorrect.

Although the extreme minuteness of the last four of these results deprives them of much numerical reliance, it is at least certain that the parallaxes by no means follow the order of magnitudes, and this is further shown by the fact that α Cygni, one of M. Peters's stars, shows absolutely no indications of any measurable parallax whatever.

(816.) From the distance of the stars we are naturally led to the consideration of their real magnitudes. But here a difficulty arises, which, so far as we can judge of what optical instruments are capable of effecting, must always remain insuperable. Telescopes afford us only negative information as to the apparent angular diameter of any star. The round, well-defined, planetary disks which good telescopes show when turned upon any of the brighter stars are phenomena of diffraction, dependent, though at present somewhat enigmatically, on the mutual interference of the rays of light. They are consequently, so far as this inquiry is concerned, mere optical illusions, and have therefore been termed *spurious* disks. The proof of this is that telescopes of different apertures and magnifying powers, when applied for the purpose of measuring their angular diameters, give different results, the greater aperture (even with the same magnifying power) giving the smaller disk. That the true disk of even a large and bright star can have but a very minute angular measure, appears from the fact that in the occultation of such a star by the moon, its extinction is *absolutely instantaneous*, not the smallest trace of gradual diminution of light being perceptible. The apparent or spurious disk also remains *perfectly round* and *of its full size* up to the instant of disappearance, which could not be the case were it a real object. If our sun were removed to the distance expressed by our parallactic unit (art. 804), its apparent diameter of 32′ 1″·5 would be reduced to only 0″·0093, or less than the hundredth of a second, a quantity which we have not the smallest reason to hope any practical improvement in telescopes will ever show as an object having distinguishable *form*.

(817.) There remains therefore only the indication which the quantity of light they send to us may afford. But here again another difficulty besets us. The light of the sun is so immensely superior in intensity to that of any star, that it is impracticable to obtain any direct comparison between them. But by using the moon as an intermediate term of comparison it may be done, not indeed with much precision, but sufficiently well to satisfy in some degree our curiosity on the subject. Now *α* Centauri has been directly compared with the moon by the method explained in art. 783. By a mean of eleven such comparisons made in various states of the moon, duly reduced and making the proper allowance on photometric principles for the moon's light lost by transmission through the lens and prism, it appears that the mean quantity of light sent to the earth by a full moon exceeds that sent by *α* Centauri in the proportion of 27408 to 1. Now Wollaston, by a method apparently unobjectionable, found[18] the proportion of the sun's light to that of the full moon to be that of 801072 to 1. Combining these results, we find the light sent us by the sun to be to that sent by *α* Centauri as 21,955,000,000, or about twenty-two thousand millions to 1. Hence from the parallax assigned above to that star, it is easy to conclude that its intrinsic splendor, as compared with that of our sun at equal distances, is 2·3247, that of the sun being unity.[19]

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[18] Wollaston, Phil. Trans. 1829, p. 27.

[19] *Results of Astronomical Observations at the Cape of Good Hope, etc.*, art.

(818.) The light of Sirius is four times that of α Centauri, and its parallax only $0''\cdot15$. (Art. 230.) This in effect ascribes to it an intrinsic splendor equal to 169·35 times that of α Centauri, and therefore 393·7 times that of our sun.[20]

CHAPTER XVI

Variable and Periodical Stars—List of those Already Known—Irregularities in their Periods and Lustre when Brightest—Irregular and Temporary Stars—Ancient Chinese Records of Several—Missing Stars—Double Stars—Their Classification—Specimens of each Class—Binary Systems—Revolution Round each other—Describe Elliptic Orbits under the Newtonian Law of Gravity—Elements of Orbits of Several—Actual Dimensions of their Orbits—Colored Double Stars—Phenomenon of Complementary Colors—Sanguine Stars—Proper Motion of the Stars—Partly Accounted for by a Real Motion of the Sun—Situation of the Solar Apex—Agreement of Southern and Northern Stars in Giving the Same Result—Principles on which the Investigation of the Solar Motion Depends—Absolute Velocity of the Sun's Motion—Supposed Revolution of the Whole Sidereal System Round a Common Centre—Systematic Parallax and Aberration—Effect of the Motion of Light in Altering the Apparent Period of a Binary Star

(819.) Now, for what purpose are we to suppose such magnificent bodies scattered through the abyss of space? Surely not to illuminate *our* nights, which an additional moon of the thousandth part of the size of our own would

278, p. 363. If only the results obtained near the quadratures of the moon (which is the situation most favorable to exactness) be used, the resulting value of the intrinsic light of the star (the sun being unity) is 4·1586. On the other hand, if only those procured near the full moon (the worst time for observation) be employed, the result is 1·4017. Discordances of this kind will startle no one conversant with photometry. That α Centauri really emits more light than our sun must, we conceive, be regarded as an established fact. To those who may refer to the work cited it is necessary to mention that the quantity there designated by M, expresses, on the scale there adopted, 500 times the actual illuminating power of the moon at the time of observation, that of the mean full moon being unity.

[20] See the work above cited, p. 367.—Wollaston makes the light of Sirius one 20,000-millionth of the sun's. Steinheil by a very uncertain method found $\odot = (3286500)^3 \times$ Arcturus.

do much better, nor to sparkle as a pageant void of meaning and reality, and bewilder us among vain conjectures. Useful, it is true, they are to man as points of exact and permanent reference; but he must have studied astronomy to little purpose, who can suppose man to be the only object of his Creator's care, or who does not see in the vast and wonderful apparatus around us provision for other races of animated beings. The planets, as we have seen, derive their light from the sun; but that cannot be the case with the stars. These doubtless, then, are themselves suns, and may, perhaps, each in its sphere, be the presiding centre round which other planets, or bodies of which we can form no conception from any analogy offered by our own system, may be circulating.

(820.) Analogies, however, more than conjectural, are not wanting to indicate a correspondence between the dynamical laws which prevail in the remote regions of the stars and those which govern the motions of our own system. Wherever we can trace the law of periodicity—the regular recurrence of the same phenomena in the same times—we are strongly impressed with the idea of rotatory or orbitual motion. Among the stars are several which, though no way distinguishable from others by any apparent change of place, nor by any difference of appearance in telescopes, yet undergo a more or less regular periodical increase and diminution of lustre, involving in one or two cases a complete extinction and revival. These are called periodical stars. The longest known and one of the most remarkable is the star *Omicron*, in the constellation Cetus (sometimes called Mira Ceti), which was first noticed as variable by Fabricius in 1596. It appears about twelve times in eleven years, or more exactly in a period of

$331^d\ 8^h\ 4^m\ 16^s$; remains at its greatest brightness about a fortnight, being then on some occasions equal to a large star of the second magnitude; decreases during about three months, till it becomes completely invisible to the naked eye, in which state it remains about five months: and continues increasing during the remainder of its period. Such is the general course of its phases. It does not always however return to the same degree of brightness, nor increase and diminish by the same gradations, neither are the successive intervals of its maxima equal. From the recent observations and inquiries into its history by M. Argelander, the mean period above assigned would appear to be subject to a cyclical fluctuation embracing eighty-eight such periods, and having the effect of gradually lengthening and shortening alternately those intervals to the extent of twenty-five days one way and the other.[1] The irregularities in the degree of brightness attained at the maximum are probably also periodical. Hevelius relates[2] that during the four years between October, 1672, and December, 1676, it did not appear at all. It was unusually bright on October 5, 1839 (the epoch of its maximum for that year according to M. Argelander's observations), when it exceeded α Ceti and equalled β Aurigæ in lustre. When near its minimum its color changes from white to a full red.

(821.) Another very remarkable periodical star is that called Algol, or β Persei. It is usually visible as a star of the second magnitude, and such it continues for the space of $2^d\ 13\frac{1}{2}^h$, when it suddenly begins to diminish in splendor, and in about $3\frac{1}{2}$ hours is reduced to the fourth magnitude, at which it continues about 15^m. It then begins

[1] Astronom. Nachr. No. 624.

[2] Lalande's Astronomy, Art. 794.

again to increase, and in $3\frac{1}{2}$ hours more is restored to its usual brightness, going through all its changes in $2^d\ 20^h\ 48^m\ 54^s{\cdot}7$. This remarkable law of variation certainly appears strongly to suggest the revolution round it of some opaque body, which when interposed between us and Algol, cuts off a large portion of its light; and this is accordingly the view taken of the matter by Goodricke, to whom we owe the discovery of this remarkable fact,[3] in the year 1782; since which time the same phenomena have continued to be observed, but with this remarkable additional point of interest; viz. that the more recent observations as compared with the earlier ones indicate a diminution in the periodic time. The latest observations of Argelander, Heis and Schmidt even go to prove that this diminution is not uniformly progressive, but is actually proceeding with accelerated rapidity, which however will probably not continue, but, like other cyclical combinations in astronomy, will by degrees relax, and then be changed into an increase, according to laws of periodicity which, as well as their causes, remain to be discovered. The first minimum of this star in the year 1844 occurred on January 3, at $4^h\ 14^m$ Greenwich mean time.[4]

(822.) The star δ in the constellation Cepheus is also subject to periodical variations, which, from the epoch of its first observation by Goodricke in 1784 to the present

[3] The same discovery appears to have been made nearly about the same time by Palitzch, a farmer of Prolitz, near Dresden—a peasant by station, an astronomer by nature—who, from his familiar acquaintance with the aspect of the heavens, had been led to notice among so many thousand stars this one as distinguished from the rest by its variation, and had ascertained its period. The same Palitzch was also the first to rediscover the predicted comet of Halley in 1759, which he saw nearly a month before any of the astronomers, who, armed with their telescopes, were anxiously watching its return. These anecdotes carry us back to the era of the Chaldean shepherds. Montanari in 1669, and Maraldi in 1694, had already noticed a fluctuation of brightness in Algol.

[4] Ast. Nach. No. 472.

time, have been continued with perfect regularity. Its period from minimum to minimum is $5^d\ 8^h\ 47^m\ 39^s{\cdot}5$, the first or epochal minimum for 1849 falling, on January 2, $3^h\ 13^m\ 37^s$ M. T. at Greenwich. The extent of its variation is from the fifth to between the third and fourth magnitudes. Its increase is more rapid than its diminution, the interval between the minimum and maximum of its light being only $1^d\ 14^h$, while that from the maximum to the minimum is $3^d\ 19^h$.

(823.) The periodical star β Lyræ, discovered by Goodricke also in 1784, has a period which has been usually stated at from $6^d\ 9^h$ to $6^d\ 11^h$, and there is no doubt that in about this interval of time its light undergoes a remarkable diminution and recovery. The more accurate observations of M. Argelander however have led him to conclude[5] the true period to be $12^d\ 21^h\ 53^m\ 10^s$, and that in this period a double maximum and minimum takes place, the two maxima being nearly equal and both about the 3·4 magnitude, but the minima considerably unequal, viz. 4·3 and 4·5m. In addition to this curious subdivision of the whole interval of change into two semiperiods we are presented in the case of this star with another instance of slow alteration of period, which has all the appearance of being itself periodical. From the epoch of its discovery in 1784 to the year 1840 the period was continually lengthening, but more and more slowly, till at the last-mentioned epoch it ceased to increase, and has since been slowly on the decrease. As an epoch for the least or absolute minimum of this star, M. Argelander's calculations enable us to assign 1846 January $3^d\ 0^h\ 9^m\ 53^s$ G. M. T.

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[5] Astron. Nachr. No. 624. See also the valuable papers by this excellent astronomer in A. N. Nos. 417, 455, etc.

(824.) The following list comprises most of the variable stars at present known:

Star.	R. A. 1850.		N. P. D. 1850.		Magn.		Period, Days.	Discovered by
					Max.	Min.		
	h	m						
T ? Pisc.	0	24	76°	23′	9·5	11	242±	Luther, 1855.
α Cass.	0	32	34	17	2	2·5	79·1	Birt, 1831.
S ? Pisc.	1	10	81	51	9	13	. .	Hind, 1851.
R ? Pisc.	1	23	87	54	7·5	9·5	343	Hind, 1850.
o Ceti	2	12	93	40	2	12	331·336	Fabricius, 1596.
β Pers.	2	58	49	38	2·3	4·5	2·8673	Goodricke, 1782.
λ Taur.	3	52	77	56	4	5·4	4±	Baxendell, 1848.
R ? Taur.	4	20	80	10	8	13·5	. .	Hind, 1849.
S ? Taur.	4	22	80	22	8·0	12·5	257	
R ? Orio.	4	51	82	6	9	12·5	237 ?	Hind, 1848.
ε Auri.	4	51	46	24	3	4	250±	Heis, 1846.
	4	53	105	2	7	. .	. .	Schmidt, 1855.
α Orio.	5	47	82	38	1	1·5	. .	J. Herschel, 1836.
ζ Gemi.	6	55	69	13	3·7	4·5	10·15	Schmidt, 1847.
R ? Gemi.	6	58	67	4	7	11	370	Hind, 1848.
R ? Can. *m.*	7	0	79	44	8	. .	. .	Argelander, 1854.
S ? Can. *m.*	7	25	81	22	8·1	. .	. .	Hind, 1856.
S ? Gemi.	7	34	66	12	9	13·5	295	Hind, 1848.
T ? Gemi.	7	40	65	54	9	13·5	287	Hind, 1848.
U ? Gemi.	7	46	67	37	9	13·5	100 ?	Hind, 1855.
R ? Canc.	8	8	77	51	6	10	380	Schwerd, 1829.
S ? Canc.	8	35	70	25	8	10·5	9·484	Hind, 1848.
S ? Hyd.	8	46	86	22	8·5	13·5	260	Hind, 1848.
	8	48	69	35	8·5	12	. .	Hind, 1850.
T ? Hyd.	8	49	98	39	8·5	10·5	240±	Hind, 1851.
α Hyd.	9	20	98	0	2·5	3	55	J. Herschel, 1837.
3 Leon.	9	21	81	10	6	0	78	Smyth,—?
ψ Leon.	9	36	75	18	6	0	Long.	Montanari, 1667.
R ? Leon.	9	39	77	53	5	10	313 ?	Koch, 1782.
R ? Urs. M.	10	34	20	26	7·5	13	301·35	Pogson, 1853.
η Argus.	10	39	148	54	1	4	46 years ?	Burchell, 1823.
α Urs. M.	10	54	27	26	1·5	2	Long.	Lalande, 1786.
R ? Comæ.	11	57	70	23	8·0	. .	. .	
δ Urs. M.	12	8	32	8	2	2·5	Long.	
21 Virg.	12	26	98	28	. .	. .	. .	
R ? Virg.	12	31	82	11	6·5	11	145·724	Harding, 1809.
S ? Urs. M.	12	37	28	5	7	12	221·750	Pogson, 1853.
U ? Virg.	12	43	83	38	7·8	. .	. .	
υ Hydr.	13	22	112	20	4	10	495	Maraldi, 1704.
S ? Virg.	13	25	96	25	5·5	11	377 ?	Hind, 1852.
η Urs. M.	13	42	39	56	1·5	2	Long.	Lalande, 1786.
Libr.	14	45	101	45	8	9·5	. .	Schumacher,—?
β Urs. *m.*	14	51	15	14	2	2·5	Long.	Struve, 1838.
δ Libræ.	14	53	97	55	. .	. .	. .	

Star.	R. A. 1850.		N. P. D. 1850.		Magn.		Period, Days.	Discovered by
					Max.	Min.		
	h	m						
S ? Serp. .	15	15	75°	9′	8	10	367	Harding, 1828.
R ? Cor. B.	15	42	61	23	6	. .	323	Pigott, 1795.
R ? Serp. .	15	44	74	24	6·5	10	359	Harding, 1826.
R ? Scorp. .	16	9	112	20	9·0	. .	. .	Chacornac, —?
S ? Scorp. .	16	9	112	20	9	12	. .	Chacornac, 1855.
80 Mess. .	16	9	112	26	. .	. .	. .	Pogson, 1860.
S ? Ophi. .	16	26	106	52	9·3	13·5	220 ?	Pogson, 1854.
* Nova .	16	51	102	39	4·5	13·5	. .	Hind, 1848.
R ? Ophi. .	16	59	105	53	8	13	396 ?	Pogson, 1853.
α Herc. .	17	8	75	26	3·1	3·7	66·33 ?	W. Herschel, 1795.
κ Cor. A.	18	23	128	50	3	6	Long.	Halley, 1676.
R ? Scut. .	18	39	95	51	5	9	61	Pigott, 1795.
β Lyræ .	18	45	56	49	3·5	4·5	12·914	Goodricke, 1784.
13 Lyræ .	18	51	46	15	4·3	4·6	48	Baxendell, 1856.
R ? Aqui. .	18	59	82	0	6·5	. .	. .	
R ? Cygn. .	19	33	40	8	8	14	415·50	Pogson, 1812.
χ Cygn. .	19	45	57	28	5	11	406·06	Kirch, 1687.
η Aqui. .	19	45	89	22	3·3	4·7	7·1763	Pigott, 1784.
η Cygn. .	19	51	55	19	4·5	5·5	Long.	J. Herschel, 1842.
R ? Capr. .	20	3	104	42	9·5	13·5	. .	Hind, 1848.
34 Cygn. .	20	12	52	26	3	6	18 years ?	Janson, 1600.
Delph.	20	33	77	49	8	8—	274 ?	Struve, 1823.
Urs. *m.*	20	38	1	20	5	11	. .	Pogson, 1853.
Aquar .	20	42	95	42	. .	. .	. .	Goldschmidt.
T ? Capr. .	21	15	105	47	9·0	. .	274	
BAC 7582 .	21	39	31	54	3	6	Long.	W. Herschel, 1782.
S ? Pega. .	22	15	82	44	8·5	13·5	. .	Hind, 1848.
δ Ceph. .	22	24	32	21	3·7	4·7	5·3664	Goodricke, 1784.
β Pega. .	22	57	62	44	2	2·5	41	Schmidt, 1848.
R ? Pega. .	22	59	80	16	8·5	13·5	350	Hind, 1848.
R ? Aqua.	23	37	106	6	6·5	10	388·50	Harding, 1810.
R ? Cass. .	23	51	39	26	6	14	434 ?	Pogson, 1853.

(826.) Irregularities similar to those which have been noticed in the case of *o* Ceti, in respect of the maxima and minima of brightness attained in successive periods, have been also observed in several others of the stars in the foregoing list. *χ* Cygni, for example, is stated by Cassini to have been scarcely visible throughout the years 1699, 1700, 1701, at those times when it was expected to be most conspicuous. No. 59 Scuti is sometimes visible to the naked eye at its minimum, and sometimes not so, and its maximum is also very irregular. Pigott's variable star in Corona is stated by M. Argelander to vary for the most part so little that the unaided eye can hardly decide on its maxima and minima, while yet after the lapse of whole years of these slight fluctuations, they suddenly become so great that the star completely vanishes. The variations of α Orionis, which were most striking and unequivocal in the years 1836–1840, within the years since elapsed became much less conspicuous. In January, 1849, they had recommenced; and on December 5, 1852, Mr. Fletcher observed α Orionis brighter than Capella, and actually the largest star in the Northern hemisphere. The star called U Geminorum, in the list above given, is stated by Mr. Pogson to be subject to alternations or twinklings of light from the ninth to the thirteenth magnitude, in intervals from nine to fifteen seconds, neighboring stars of equal brightness remaining steady!

(827.) These irregularities prepare us for other phenomena of stellar variation, which have hitherto been reduced to no law of periodicity, and must be looked upon, in relation to our ignorance and inexperience, as altogether casual; or, if periodic, of periods too long to have occurred more than once within the limits of recorded observation. The phenomena we allude to are those of *Temporary Stars*, which have appeared, from time to time, in different parts of the heavens, blazing forth with extraordinary lustre; and after remaining awhile apparently immovable, have died away, and left no trace. Such is the star which, suddenly appearing some time about the year 125 B.C., and which was visible in the daytime, is said to have attracted the attention of Hipparchus, and led him to draw up a catalogue of stars, the earliest on record. Such, too, was the star which appeared, A.D. 389, near α Aquilæ, remaining for three weeks as bright as Venus, and disappearing entirely. In the years 945, 1264, and 1572, brilliant stars appeared in the region of the heavens between Cepheus and Cassiopeia; and, from the imperfect account we have of the places of the two earlier, as compared with that of the last, which was well determined, as well as from the tolerably near coincidence of the intervals of their appearance, we may suspect them, with Goodricke, to be one and the same star, with a period of 312 or perhaps of 156 years. The appearance of the star of 1572 was so sudden, that Tycho Brahe, a celebrated Danish astronomer, returning one evening (the 11th of November) from his laboratory to his dwelling-house, was surprised to find a group of country people gazing at a star, which he was sure did not exist half an hour before. This was the star in question. It was then as bright as Sirius, and continued to increase till it surpassed Jupiter when brightest, and was visible at midday. It began to diminish in December of the same year, and in March, 1574, had entirely disappeared. So, also, on the 10th of October, 1604, a star of this kind, and not less brilliant, burst forth in the constellation of Serpentarius, which continued visible till October, 1605.

(828.) Similar phenomena, though of a less splendid character, have taken place more recently, as in the case of the star of the third magnitude discovered in 1670, by Anthelm, in the head of the Swan; which, after becoming completely invisible, reappeared, and, after undergoing one or two singular fluctuations of light, during two years, at last died away entirely, and has not since been seen.

(829.) On the night of the 28th of April, 1848, Mr. Hind observed a star of the fifth magnitude or 5·4 (very conspicuous to the naked eye) in a part of the constellation Ophiuchus (R.A. $16^h\ 51^m\ 1^s{\cdot}5$, N. P. D. 102° 39′ 14″), where, from perfect familiarity with that region, he was certain that up to the 5th of that month no star so bright as 9·10m. previously existed. Neither has any record been discovered of a star being there observed at any previous time. From the time of its discovery it continued to diminish, without any alteration of place, and before the advance of the season rendered further observation impracticable, was nearly extinct. Its color was ruddy, and was thought by many observers to undergo remarkable changes, an effect probably of its low situation.

(830.) The alterations of brightness in the southern star η Argûs, which have been recorded, are very singular and surprising. In the time of Halley (1677) it appeared as a star of the fourth magnitude. Lacaille, in 1751, observed it of the second. In the interval from 1811 to 1815, it was again of the fourth; and again from 1822 to 1826 of the second. On the 1st of February, 1827, it was noticed by Mr. Burchell to have increased to the first magnitude, and to equal α Crucis. Thence again it receded to the second; and so continued until the end of 1837. All at once in the beginning of 1838 it suddenly increased in lustre so as to surpass all the stars of the first magnitude except Sirius, Canopus, and α Centauri, which last star it nearly equalled. Thence it again diminished, but this time not below the 1st magnitude until April, 1843, when it had again increased so as to surpass Canopus, and nearly equal Sirius in splendor. In May, 1863 [as well as in the years 1866–68], according to Mr. Abbott [and Mr. John Tebbutt, Jr.], it was only of the 6th magnitude.[6] Professor Loomis consid-

[6] Notices of R. Ast. Soc. xxiv. p. 6; xxv. p. 192; xxviii. p. 200, and p. 266.

ers it as periodical, the interval of the minima being about seventy years.[7] "A strange field of speculation," it has been remarked, "is opened by this phenomenon. The temporary stars heretofore recorded have all become totally extinct. Variable stars, so far as they have been carefully attended to, have exhibited periodical alternations, in some degree at least regular, of splendor and comparative obscurity. But here we have a star fitfully variable to an astonishing extent, and whose fluctuations are spread over centuries, apparently in no settled period, and with no regularity of progression. What origin can we ascribe to these sudden flashes and relapses? What conclusions are we to draw as to the habitability of a system depending for its supply of light and heat on so uncertain a source?" Speculations of this kind can hardly be termed visionary, when we consider that, from what has before been said, we are compelled to admit a community of nature between the fixed stars and our own sun; and reflect that geology testifies to the fact of extensive changes having taken place at epochs of the most remote antiquity in the climate and temperature of our globe difficult to reconcile with the operation of secondary causes, such as a different distribution of sea and land, but which would find an easy and natural explanation in a slow variation of the supply of light and heat afforded by the sun itself.

(831.) The Chinese annals of Ma-touan-lin,[8] in which stand *officially* recorded, though rudely, remarkable astronomical phenomena, supply a long list of "strange stars," among which, though the greater part are evidently comets, some may be recognized as belonging in all probability to

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[7] Notices of Royal Astr. Soc. xxix. p. 298.
[8] Translated by M. Edward Biot, Connoissance des Temps, 1846.

the class of *Temporary Stars* as above characterized. Such is that which is recorded to have appeared in A.D. 173, between α and β *Centauri*, which (no doubt, scintillating from its low situation) exhibited "the five colors," and remained visible from December in that year till July in the next. And another which these annals assign to A.D. 1011, and which would seem to be identical with a star elsewhere referred to A.D. 1012, "which was of extraordinary brilliancy, and remained visible in the southern part of the heavens during three months,"[9] a situation agreeing with the Chinese record, which places it low in Sagittarius. Among several less unequivocal is one referred to B.C. 134, in Scorpio, which may possibly have been Hipparchus's star. [Lastly, on May 12, 1866, a star of the second magnitude was unexpectedly noticed by Mr. Birmingham (at Tuam) near ϵ Coronæ. It diminished rapidly, having been seen by Mr. Huggins on May 15, 16, 17, 18, 19, 20 respectively as 3·6, 4·2, 4·9, 5·3, 5·7, and 6·2m. After dwindling to 10m. it again recovered so far as to have been seen on October 5 by M. Schmidt as 7m. Its place for 1866 was R. A. $16^h\ 54^m$; N. P. D. 63° 42′. Its spectrum was twofold, exhibiting both positive and negative lines, indicating at once the presence of flame and absorptive vapors.]

(832.) On a careful re-examination of the heavens, and a comparison of catalogues, many stars are now found to be missing; and although there is no doubt that these losses have arisen in the great majority of instances from mistaken entries, and in some from planets having been mistaken for stars, yet in some it is equally certain that there is no mis-

[9] Hind, Notices of the Astronomical Society, viii. 156, citing Hepidannus. He places the Chinese star of 173 A.D. between α and β *Canis Minoris*, but M. Biot distinctly says, α, β *pied oriental du Centaure.*

take in the observation or entry, and that the star has really been observed, and as really has disappeared from the heavens. The whole subject of variable stars is a branch of practical astronomy which has been too little followed up, and it is precisely that in which amateurs of the science, and especially voyagers at sea, provided with only good eyes, or moderate instruments, might employ their time to excellent advantage. Catalogues of the comparative brightness of the stars in each constellation have been constructed by Sir Wm. Herschel, with the express object of facilitating these researches, and the reader will find them, and a full account of his method of comparison, in the Phil. Trans. 1796, and subsequent years.

(833.) We come now to a class of phenomena of quite a different character, and which give us a real and positive insight into the nature of at least some among the stars, and enable us unhesitatingly to declare them subject to the same dynamical laws, and obedient to the same power of gravitation, which governs our own system. Many of the stars, when examined with telescopes, are found to be double, *i.e.* to consist of two (in some cases three or more) individuals placed near together. This might be attributed to accidental proximity, did it occur only in a few instances; but the frequency of this companionship, the extreme closeness, and, in many cases, the near equality of the stars so conjoined, would alone lead to a strong suspicion of a more near and intimate relation than mere casual juxtaposition. The bright star Castor, for example, when much magnified, is found to consist of two stars of nearly the third magnitude, within 5″ of each other. Stars of this magnitude, however, are not so common in the heavens as to render it otherwise than excessively improbable that, if scattered at random, they would fall so near. But this improbability becomes immensely increased by a consideration of the fact, that this is only one out of a great many similar instances. Michell, in 1767, applying the rules for the calculation of probabilities to the case of the six brightest stars in the group called the Pleiades, found the odds to be 500000 to 1 against their proximity being the mere result of a random scattering of 1500 stars (which he supposed to be the total number of stars of that magnitude in the celestial sphere[10]) over the heavens. Speculating further on this, as an indication of physical connection rather than fortuitous assemblage, he was led to surmise the possibility (since converted into a certainty, but at that time, antecedent to any observation) of the existence of compound stars revolving about one another, or rather about their common centre of gravity. M. Struve, pursuing the same train of thought as applied specially to the cases of double and triple combinations of stars, and grounding his computations on a more perfect enumeration of the stars visible down to the 7th magnitude, in the part of the heavens visible at Dorpat, calculates that the odds are 125 to 1 against any two stars, from the 1st to the 7th magnitude inclusive, out of the whole possible number of binary combinations then visible, falling (if fortuitously scattered) within 4″ of each other. Now the number of instances of such binary combinations actually observed at the date of this calculation was already 91, and many more have since been added to the list. Again, he calculates that the odds against any such stars fortuitously scattered, falling within 32″ of a third, so as to constitute a

[10] This number is considerably too small, and in consequence, Michell's odds in this case materially overrated. But enough will remain, if this be rectified, fully to bear out his argument. Phil. Trans. vol. 57.

triple star, is not less than 4340 to 1. Now, four such combinations occur in the heavens; viz. θ Orionis, σ Orionis, 11 Monocerotis, and ζ Cancri. The conclusion of a physical connection of some kind or other is therefore unavoidable.

(834.) Presumptive evidence of another kind is furnished by the following consideration. Both α Centauri and 61 Cygni are "Double Stars." Both consist of two individuals, nearly equal, and separated from each other by an interval of about a quarter of a minute. In the case of 61 Cygni, the stars exceeding the 7th magnitude, there is already a *prima facie* probability of 9 to 1 against their apparent proximity. The two stars of α Centauri are both at least of the 2d magnitude, of which altogether not more than about 50 or 60 exist in the whole heavens. But, waiving this consideration, both these stars, as we have already seen, have a proper motion so considerable that, supposing the constituent individuals unconnected, one would speedily leave the other behind. Yet at the earliest dates at which they were respectively observed these stars were not perceived to be double, and it is only to the employment of telescopes magnifying at least 8 or 10 times, that we owe the knowledge we now possess of their being so. With such a telescope Lacaille, in 1751, was barely able to perceive the separation of the two constituents of α Centauri, whereas, had one of them only been affected with the observed proper motion, they should then have been 6′ asunder. In these cases then some physical connection may be regarded as proved by this fact alone.

(835.) Sir William Herschel has enumerated upward of 500 double stars, of which the individuals are less than 32″ asunder. M. Struve, prosecuting the inquiry with instruments more conveniently mounted for the purpose, and wrought to an astonishing pitch of optical perfection, has added more than five times that number. And other observers have extended still further the catalogue of "Double Stars," without exhausting the fertility of the heavens. Among these are a great many in which the distance between the component individuals does not exceed a single second. They are divided into classes by M. Struve (the first living authority in this department of astronomy), according to the proximity of their component individuals. The first class comprises those only in which the distance does not exceed 1″; the 2d those in which it exceeds 1″ but falls short of 2″; the 3d class extends from 2″ to 4″ distance; the 4th from 4″ to 8″; the 5th from 8″ to 12″; the 6th from 12″ to 16″; the 7th from 16″ to 24″, and the 8th from 24″ to 32″. Each class he again subdivides into two sub-classes of which the one under the appellation of *conspicuous* double stars (*Duplices lucidæ*) comprehends those in which both individuals exceed the $8\frac{1}{4}$ magnitude, that is to say, are *separately* bright enough to be easily seen in any moderately good telescope. All others, in which one or both the constituents are below this limit of easy visibility, are collected into another sub-class, which he terms *residuary* (*Duplices reliquæ*). This arrangement is so far convenient, that after a little practice in the use of telescopes as applied to such objects, it is easy to judge what optical power will probably suffice to resolve a star of any proposed class and either sub-class, or would at least be so if the second or residuary sub-class were further subdivided by placing in a third sub-class "delicate" double stars, or those in which the companion star is so very minute as to require a high degree of optical power to perceive it, of which instances will presently be given.

(836.) The following may be taken as specimens of each class. They are all taken from among the lucid, or conspicuous stars, and to such of our readers as may be in possession of telescopes, and may be disposed to try them on such objects, will afford a ready test of their degree of efficiency.

Class I., 0″ to 1″.

γ Coronæ Bor.	η Coronæ.	τ Ophiuchi.	Atlas Pleiadum.
γ Centauri.	η Herculis.	ϕ Draconis.	4 Aquarii.
γ Lupi.	λ Cassiopeiæ.	ϕ Ursæ Majoris.	5 Aquarii.
ϵ Arietis.	λ Ophiuchi.	χ Aquilæ.	42 Comæ.
ζ Herculis.	π Lupi.	ω Leonis.	52 Arietis.
γ 2 Andromedæ.	λ Cygni.	ϕ Andromedæ.	66 Piscium

Class II., 1″ to 2″.

γ Circini.	ζ Bootis.	ξ Ursæ Majoris.	2 Camelopardi.
δ Cygni.	ι Cassiopeiæ.	π Aquilæ.	32 Orionis.
ϵ Chamæleontis.	ι 2 Cancri.	σ Coronæ Bor.	52 Orionis.

Class III., 2″ to 4″.

α Piscium.	γ Virginis.	ζ Aquarii.	μ Draconis.
β Hydræ.	δ Serpentis.	ζ Orionis.	μ Canis.
γ Ceti.	ϵ Bootis.	ι Leonis.	ρ Herculis.
γ Leonis.	ϵ Draconis.	ι Trianguli.	σ Cassiopeiæ.
γ Coronæ Aus.	ϵ Hydræ.	κ Leporis.	44 Bootis.

Class IV., 4″ to 8″.

α Crucis.	θ Phœnicis.	ξ Cephei.	μ Eridani.
α Herculis.	κ Cephei.	π Bootis.	70 Ophiuchi.
α Geminorum.	λ Orionis.	ρ Capricorni.	12 Eridani.
δ Geminorum.	μ Cygni.	υ Argus.	32 Eridani.
ζ Coronæ Bor.	ξ Bootis.	ω Aurigæ.	95 Herculis.

Class V., 8″ to 12″.

β Orionis.	ζ Antliæ.	ι Orionis.
γ Arietis.	η Cassiopeiæ.	f Eridani.
γ Delphini.	θ Eridani.	2 Canum Ven.

Class VI., 12″ to 16″.

α Centauri.	γ Volantis.	κ Bootis.
β Cephei.	η Lupi.	8 Monocerotis.
β Scorpii.	ζ Ursæ Major.	61 Cygni.

Class VII., 16″ to 24″.

α Canum Ven.	θ Serpentis.	24 Comæ.
ϵ Normæ.	κ Coronæ Aus.	41 Draconis.
ζ Piscium.	χ Tauri.	61 Ophiuchi.

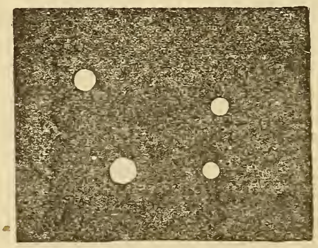
Class VIII., 24″ to 32″.

δ Herculis.	κ Herculis.	χ Cygni.
η Lyræ.	ψ Draconis.	23 Orionis.

(837.) Among the most remarkable triple, quadruple, or multiple stars (for such also occur), may be enumerated,

γ Andromedæ.	θ Orionis.	ξ Scorpii.
ϵ Lyræ.	μ Lupi.	11 Monocerotis.
η Cancri.	μ Bootis.	12 Lyncis.

Of these, γ Andromedæ, μ Bootis, and μ Lupi appear in telescopes, even of considerable optical power, only as ordinary double stars; and it is only when excellent instruments are used that their smaller companions are subdivided and found to be, in fact, extremely close double stars. ϵ Lyræ offers the remarkable combination of a double-double star. Viewed with a telescope of low power it appears as a coarse and easily divided double star, but on increasing the magnifying power, each individual is perceived to be beautifully and closely double, the one pair being about $2\frac{1}{2}''$, the other



about 3″ asunder. Each of the stars ζ Cancri, ξ Scorpii, 11 Monocerotis, and 12 Lyncis consists of a principal star, closely double, and a smaller and more distant attendant, while θ Orionis presents the phenomenon of four brilliant principal stars, of the respective 4th, 6th, 7th, and 8th magnitudes, forming a trapezium, the longest diagonal of which is 21″·4, and accompanied by two excessively minute and very close companions (as in the above figure), to perceive *both* which is one of the severest tests which can be applied to a telescope.

(838.) Of the "delicate" sub-class of double stars, or those consisting of very large and conspicuous principal stars, accompanied by very minute companions, the following specimens may suffice:

α 2 Cancri.	α Polaris.	ι Ursæ Majoris.	γ Bootis.
α 2 Capricorni.	α Scorpii.	κ Circini.	φ Virginis.
α Indi.	β Aquarii.	κ Geminorum.	χ Eridani.
α Lyræ.	γ Hydra.	μ Persei.	16 Aurigæ.

(839.) To the amateur of astronomy the double stars offer a subject of very pleasing interest, as tests of the performance of his telescopes, and by reason of the finely contrasted colors which many of them exhibit, of which more hereafter. But it is the high degree of physical interest which attaches to them, which assigns them a conspicuous place in modern astronomy, and justifies the minute attention and unwearied diligence bestowed on the measurement of their angles of position and distances, and the continual enlargement of our catalogues of them by the discovery of new ones. It was, as we have seen, under an impression that such combinations, if diligently observed, might afford a measure of parallax through the periodical variations it might be expected to produce in the relative situation of the small attendant star, that Sir W. Herschel was induced (between the years 1779 and 1784) to form the first extensive catalogues of them, under the scrutiny of higher magnifying powers than had ever previously been applied to such purposes. In the pursuit of this object, the end to which it was instituted as a means was necessarily laid aside for a time, until the accumulation of more abundant materials should have afforded a choice of stars favorably circumstanced for systematic observation. Epochal measures, however, of each star, were secured, and, on resuming the subject, his attention was altogether diverted from the original object of the inquiry by phenomena of a very unexpected character, which at once engrossed his whole attention. Instead of finding, as he expected, that annual fluctuation to and fro of one star of a double star with respect to the other—that alternate annual increase and decrease of their distance and angle of position, which the parallax of the earth's annual motion would produce—he observed, in many instances, a regular progressive change; in some cases bearing chiefly on their distance—in others on their position, and advancing steadily in one direction, so as clearly to indicate either a real motion of the stars themselves, or a general rectilinear motion of the sun and whole solar system, producing a parallax of a higher order than would arise from the earth's orbitual motion, and which might be called systematic parallax.

(840.) Supposing the two stars, and also the sun, in motion independently of each other, it is clear that for the interval of several years, these motions must be regarded as rectilinear and uniform. Hence, a very slight acquaintance with geometry will suffice to show that the *apparent motion* of one star of a double star, referred to the other as a centre, and mapped down, as it were, on a plane in which that other shall be taken for a fixed or zero point, can be no other than a right line. This, at least, must be the case if the stars be independent of each other; but it will be otherwise if they have a physical connection, such as, for instance, real proximity and mutual gravitation would establish. In that case, they would describe orbits round each other, and round their common centre of gravity; and therefore the apparent path of either, referred to the other as fixed, instead of being a portion of a straight line, would be bent into a curve concave toward that other. The ob·

served motions, however, were so slow, that many years' observation was required to ascertain this point; and it was not, therefore, until the year 1803, twenty-five years from the commencement of the inquiry, that anything like a positive conclusion could be come to respecting the rectilinear or orbitual character of the observed changes of position.

(841.) In that, and the subsequent year, it was distinctly announced by him, in two papers, which will be found in the Transactions of the Royal Society for those years,[11] that there exist sidereal systems, composed of two stars revolving about each other in regular orbits, and constituting what may be termed *binary stars*, to distinguish them from double stars generally so called, in which these physically connected stars are confounded, perhaps, with others only *optically* double, or casually juxtaposed in the heavens at different distances from the eye; whereas the individuals of a binary star are, of course, equidistant from the eye, or, at least, cannot differ more in distance than the semi-diameter of the orbit they describe about each other, which is quite insignificant compared with the immense distance between them and the earth. Between fifty and sixty instances of changes, to a greater or less amount, in the angles of position of double stars, are adduced in the memoirs above mentioned; many of which are too decided, and too regularly progressive, to allow of their nature being misconceived. In particular, among the more conspicuous stars —Castor, γ Virginis, ξ Ursæ, 70 Ophiuchi, σ and η Coronæ, ξ Bootis, η Cassiopeiæ, γ Leonis, ζ Herculis, δ Cygni, μ Bootis, ε 4 and ε 5 Lyræ, λ Ophiuchi, μ Draconis, and ζ Aquarii, are enumerated as among the most remarkable

[11] The announcement was in fact made in 1802, but unaccompanied by the observations establishing the fact.

instances of the observed motion; and to some of them even periodic times of revolution are assigned; approximative only, of course, and rather to be regarded as rough guesses than as results of any exact calculation, for which the data were at the time quite inadequate. For instance, the revolution of Castor is set down at 334 years, that of γ Virginis at 708, and that of γ Leonis at 1200 years.

(842.) Subsequent observation has fully confirmed these results. Of all the stars above named, there is not one which is not found to be fully entitled to be regarded as binary; and, in fact, this list comprises nearly all the most considerable visible in our latitudes which have yet been detected, though (as attention has been closely drawn to the subject, and observations have multiplied) it has, of late, received large accessions. Upward of a hundred double stars, certainly known to possess this character, were enumerated by M. Mädler in 1841,[12] and more are emerging into notice with every fresh mass of observations which come before the public. They require excellent telescopes for their effective observation, being for the most part so close as to necessitate the use of very high magnifiers (such as would be considered extremely powerful microscopes if employed to examine objects within our reach), to perceive an interval between the individuals which compose them.

(843.) It may easily be supposed, that phenomena of this kind would not pass without attempts to connect them with dynamical theories. From their first discovery, they were naturally referred to the agency of some power, like that of gravitation, connecting the stars thus demonstrated to be in a state of circulation about each other; and the extension of the Newtonian law of gravitation to these remote

[12] Dorpat Observations, vol. ix. 1840 and 1841.

systems was a step so obvious, and so well warranted by our experience of its all-sufficient agency in our own, as to have been expressly or tacitly made by every one who has given the subject any share of his attention. We owe, however, the first distinct system of calculation, by which the elliptic elements of the orbit of a binary star could be deduced from observations of its angle of position and distance at different epochs, to M. Savary, who showed,[13] that the motions of one of the most remarkable among them (ξ Ursæ) were explicable, within the limits allowable for error of observation, on the supposition of an elliptic orbit described in the short period of 58¼ years. A different process of computation conducted Professor Encke[14] to an elliptic orbit for 70 Ophiuchi, described in a period of seventy-four years. M. Mädler has especially signalized himself in this line of inquiry (see Table). Several orbits have also been calculated by Mr. Hind, Admiral Smyth, Mr. Jacob, Mr. Powell, M. Villarceau, Professors Winnecke and Klinkerfues; and the author of these pages has himself attempted to contribute his mite to these interesting investigations.[15] The following may be stated as the chief results which have been obtained in this branch of astronomy:[16]

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[13] Connoiss. des Temps, 1830. [14] Berlin Ephem. 1832.

[15] Mem. R. Ast. Soc. vols. v. and xviii.

[16] The "position of the node" in col. 4 expresses the angle of position (see art. 204) of the line of intersection of the plane of the orbit, with the plane of the heavens on which it is seen projected. The "inclination" in col. 6 is the inclination of these two planes to one another. Col. 5 shows the angle actually included *in the plane of the orbit*, between the line of nodes (defined as above) and the line of apsides. The elements assigned in this table to ω Leonis, ξ Bootis and Castor must be considered as very doubtful. Some cause of perturbation has been suspected to exist in the movements of p Ophiuchi. Mr. Jacob, comparing some old (and no doubt very rude) observations by Richaud and Feuillé, in 1690 and 1709, draws a similar conclusion in the case of the system of α Centauri. Comparing the more modern (and only reliable observations), however, this opinion seems hardly entitled to the confidence with which he insists on it. A very few years' additional observation will decide the question. This magnificent double star well merits the most careful and diligent attention of astronomers.

Star's Name.		Apparent Semiaxis	Excentricity.	Position of Node.	Perihelion from Node on Orbit.	Inclination	Period in Years	Perihelion Passage	By whom computed
1. ζ Herculis	a	1″·208	0·43200	19° 25′	276° 39′	44° 5′	30·216	1830·417	Mädler.
1. Ditto	b	1·254	0·44821	34 21	104 55	43 43	36·357	1830·48	Villarceau.
2. μ Cor. B.	a	0·902	0·28900	20 6	215 11	59 28	42·500	1807·21	Mädler.
2. Ditto	b	0·9567	0·28652	22 18	215 29	60 40	43·677	1850·336	Winnecke.
3. ζ Cancri	a	1·013	0·47430	10 31	227 9	65 39	42.501	1805·67	Villarceau.
3. Ditto	b	1·030	. .	18 23	. .	48 36	58.94	1815·53	Winnecke.
4. ξ Ursæ	a	3·857	0·41640	95 22	131 38	50 40	58.262	1817·25	Savary.
4. Ditto	b	3·278	0·37770	97 47	134 22	56 6	60.720	1816·73	Herschel, junior.
4. Ditto	c	2·417	0·41350	98 52	130 48	54 56	61·464	1816·44	Mädler.
4. Ditto	d	2.439	0·43148	95 50	128 57	52 49	61·576	1816·86	Villarceau.
5. ω Leonis	a	0·857	0·64338	135 11	185 27	46 33	82·533	1849·76	Ditto.
5. Ditto	b	. .	0·72248	169 12	. .	60 13	227·77	1841·405	Klinkerfues.
6. p Ophiuchi	a	4·328	0·43007	147 12	125 22	46 25	73·862	1806·88	Encke.
6. Ditto	b	4·392	0·46670	137 2	145 46	48 5	80.340	1807·06	Herschel, junior.
6. Ditto	c	4·704	0·49149	125 22	155 44	57 55	94·370	1808·791	Schur.
6. Ditto	d	. .	0·546	296 30	7 29	49 56	98·146	1806·92	E. B. Powell.
7. ξ Bootis	.	12·560	0·59374	359 59	100 59	80 5	117·140	1779·88	Herschel, junior.
8. δ Cygni	.	1·811	0·60667	24 54	243 24	46 23	178·700	1862·87	Hind.
9. γ Virginis	.	3·580	0·87952	5 33	313 45	23 36	182·120	1836·43	Herschel, junior.
10. Castor	a	8·086	0·75820	58 6	97 29	70 3	252·660	1855·83	Ditto.
10. Ditto	b	7·008	0·79725	23 5	87 37	70 58	232.124	1913·90	Mädler.
10. Ditto	c	6·300	0·24050	11 24	356 22	43 14	632·270	1699·26	Hind.
11. σ Cor. B.	a	3·918	0·69978	25 7	64 38	29 29	608·450	1826.60	Mädler.
11. Ditto	b	5·194	0·72560	21 3	69 24	25 39	736·880	1826·48	Hind.
11. Ditto	c	2·94	0·38870	3 8	96 53	45 6	240·00	1829·70	E. B. Powell.
11. Ditto	d	2·717	0·30880	1 57	101 57	46 47	195·12	1831·17	Jacob.
12. α Centauri	a	15·500	0·95000	86 7	291 22	47 56	77·000	1851·50	Ditto.
12. Ditto	b	13·570	0·77520	17 22	26 2	62 53	81·40	1859·63	Hind.
12. Ditto	c	30·000	0·96600	177 50	. .	77 50	75·30	1858·00	E. B. Powell.

(844.) Of the stars in the above list, that which has been most assiduously watched, and has offered phenomena of the greatest interest, is γ Virginis. It is a star of the vulgar 3d magnitude (3·08=Photom. 3·494), and its component individuals are very nearly equal, and as it would seem in some slight degree variable, since, according to the observations of M. Struve, the one is alternately a little greater and a little less than the other, and occasionally exactly equal to it. It has been known to consist of two stars since the beginning of the eighteenth century; the distance being then between six and seven seconds, so that any tolerably good telescope would resolve it. When observed by Herschel in 1780, it was 5″·66, and continued to decrease gradually and regularly till at length, in 1836, the two stars had approached so closely as to appear perfectly round and single under the highest magnifying power which could be applied to most excellent instruments—the great refractor at Pulkova alone, with a magnifying power of 1000, continuing to indicate by the wedge-shaped form of the disk of the star its composite nature. By estimating the ratio of its length to its breadth and measuring the former, M. Struve concludes that, at this epoch (1836·41), the distance of the two stars, centre from centre, might be stated at 0″·22. From that time the star again opened, and is now again a perfectly easily separable star. This very remarkable diminution and subsequent increase of distance has been accompanied by a corresponding and equally remarkable increase and subsequent diminution of relative angular motion. Thus, in the year 1783 the apparent angular motion hardly amounted to half a degree per annum, while in 1830 it had increased to 5°, in 1834 to 20°, in 1835 to 40°, and about the middle of 1836 to upward of 70° per annum, or at the rate of a degree in five days. This is in entire conformity with the principles of dynamics, which establish a necessary connection between the angular velocity and the distance, as well in the apparent as in the real orbit of one body revolving about another under the influence of mutual attraction; the former varying inversely as the square of the latter, in both orbits, whatever be the curve described and whatever the law of the attractive force. It fortunately happens that Bradley, in 1718, had noticed and recorded in the margin of one of his observation books, the apparent direction of the line of junction of the two stars, as seen on the meridian in his transit telescope, viz. parallel to the line joining two conspicuous stars α and δ of the same constellation, as seen by the naked eye. This note, rescued from oblivion by the late Professor Rigaud, has proved of singular service in the verification of the elements above assigned to the orbit, which represent the whole series of recorded observations that date up to the end of 1846 (comprising an angular movement of nearly nine-tenths of a complete circuit), both in angle and distance, with a degree of exactness *fully equal to that of observation itself.* No doubt can, therefore, remain as to the prevalence in this remote system of the Newtonian law of gravitation.

(845.) The observations of ξ Ursæ Majoris are equally well represented by M. Mädler's elements (4 c of our table), thus fully justifying the assumption of the Newtonian law as that which regulates the motions of their binary systems. And even should it be the case, as M. Mädler appears to consider, that in one instance at least (that of p Ophiuchi), deviations from elliptic motion, too considerable to arise from mere error of observation, exist (a position we are by no means prepared to grant[17]), we should rather be disposed to look for the cause of such deviations in perturbations arising (as Bessel has suggested) from the large or central star itself being actually a close and hitherto unrecognized double star than in any defect of generality in the Newtonian law.

(846.) If the great length of the periods of some of these bodies be remarkable, the shortness of those of others is hardly less so. ζ Herculis has already completed two revolutions since the epoch of its first discovery, exhibiting in its course the extraordinary spectacle of a sidereal occultation, the small star having twice been hidden behind or before the large one. η Coronæ, ζ Cancri, ξ Ursæ and α Centauri have each performed more than one entire circuit, and 70 Ophiuchi and γ Virginis have accomplished by far the larger portion of one in angular motion. If any doubt, therefore, could remain as to the reality of their orbitual motions, or any idea of explaining them by mere parallactic changes, or by any other hypothesis than the agency of centripetal force, these facts must suffice for their complete dissipation. We have the same evidence, indeed, of their rotations about each other, that we have of those of Uranus and Neptune about the sun; and the correspondence between their calculated and observed places in such very elongated ellipses, must be admitted to carry with it proof

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[17] p Ophiuchi belongs to the class of very unequal double stars, the magnitudes of the individuals being 4 and 7. Such stars present difficulties in the exact measurement of their angles of position which even yet continue to embarrass the observer, though, owing to later improvements in the art of executing such measurements, their influence is confined within much narrower limits than in the earlier history of the subject. In simply placing a fine single wire parallel to the line of junction of two such stars it is easily possible to commit an error of 3° or 4°. By placing them between two parallel thick wires such errors are in great measure obviated. [The elements by Schur, in our table, art. 843, represent with the exactness of observation itself the whole series of positions and distances observed from 1779 to 1866.]

of the prevalence of the Newtonian law of gravity in their systems, of the very same nature and cogency as that of the calculated and observed places of comets round the central body of our own.

(847.) But it is not with the revolutions of bodies of a planetary or cometary nature round a solar centre that we are now concerned; it is with that of sun round sun—each, perhaps, at least in some binary systems where the individuals are very remote and their period of revolution very long, accompanied with its train of planets and *their* satellites, closely shrouded from our view by the splendor of their respective suns, and crowded into a space bearing hardly a greater proportion to the enormous interval which separates *them*, than the distances of the satellites of our planets from their primaries bear to their distances from the sun itself. A less distinctly characterized subordination would be incompatible with the stability of their systems, and with the planetary nature of their orbits. Unless closely nestled under the protecting wing of their immediate superior, the sweep of their other sun in its perihelion passage round their own might carry them off, or whirl them into orbits utterly incompatible with the conditions necessary for the existence of their inhabitants. It must be confessed, that we have here a strangely wide and novel field for speculative excursions, and one which it is not easy to avoid luxuriating in.

(848.) The discovery of the parallaxes of α Centauri and 61 Cygni, both which are above enumerated among the "conspicuous" double stars of the 6th class (a distinction fully merited in the case of the former by the brilliancy of both its constituents), enables us to speak with an approach to certainty as to the absolute dimensions of both their orbits, and thence to form a probable opinion as to the general scale on which these astonishing systems are constructed. The distance of the two stars of 61 Cygni subtends at the earth an angle which, since the earliest micrometrical measures in 1781, has varied hardly 1″ either way from a mean value 16″·5. On the other hand, the angle of position has altered since the same epoch by nearly 50°, so that it would appear probable that the true form of the orbit is not far from circular, its situation at right angles to the visual line, and its periodic time probably not short of 500 years. Now, as the ascertained parallax of this star is 0″·348, which is, therefore, the angle the radius of the earth's orbit would subtend if equally remote, it follows that the mean distance between the stars is to that radius, as 16″·5 : 0″·348, or as 47·41 : 1. The orbit described by these two stars about each other undoubtedly, therefore, greatly exceeds in dimensions that described by Neptune about the sun. Moreover, supposing the period to be five centuries (and the distance being actually on the increase, it can hardly be less) the general propositions laid down by Newton,[18] taken in conjunction with Kepler's third law, enable us to calculate the sum of the masses of the two stars, which, on these data, we find to be 0·353, the mass of our sun being 1. The sun, therefore, is neither vastly greater nor vastly less than the stars composing 61 Cygni.

(849.) The data in the case of α Centauri are more uncertain. Since the year 1822, the distance has been steadily and pretty rapidly decreasing at an average rate of about half a second per annum, and that with little change till lately in the angle of position.[19] Hence, it follows evidently

[18] Principia, l. i. Props. 57, 58, 59.

[19] In the 70 years between Lacaille's observations and 1822, there exists no record of any observed angle of position.

that the plane of its orbit passes nearly through the earth, and (the distance about the middle of 1834 having been $17\frac{1}{2}''$) it is very probable that either an occultation, like that observed in ζ Herculis, or a close appulse of the two stars, will take place about the year 1859. As the observations we possess afford no sufficient grounds for a satisfactory calculation of elliptic elements we must be content to assume what, at all events, they fully justify, viz. that the major semiaxis must exceed $12''$, and is very probably considerably greater. Now this with a parallax of $0''\cdot913$ would give for the real value of the semiaxis 13·15 radii of the earth's orbit, as a minimum. The real dimensions of their ellipse, therefore, cannot be so small as the orbit of Saturn; in all probability exceed that of Uranus; and may possibly be much greater than either.

(850.) The parallel between these two double stars is a remarkable one. Owing no doubt to their *comparative* proximity to our system, their apparent proper motions are both unusually great, and for the same reason probably rather than owing to unusually large dimensions, their orbits appear to us under what, for binary double stars, we must call unusually large angles. Each consists, moreover, of stars, not very unequal in brightness, and in each both the stars are of a high yellow approaching to orange color, the smaller individual, in each case, being also of a deeper tint. Whatever the diversity, therefore, which may obtain among other sidereal objects, these would appear to belong to the same family or genus.[20]

[20] Similar combinations are very numerous. Many remarkable instances occur among the double stars catalogued by the author in the 2d, 3d, 4th, 6th and 9th volumes of Trans. Roy. Ast. Soc. and in the volume of Southern observations already cited. See Nos. 121, 375, 1066, 1907, 2030, 2146, 2244, 2772, 3853, 3935, 3998, 4000, 4055, 4196, 4210, 4615, 4649, 4765, 5003, 5012, of

(851.) Many of the double stars exhibit the curious and beautiful phenomenon of contrasted or complementary colors.[21] In such instances, the larger star is usually of a ruddy or orange hue, while the smaller one appears blue or green, probably in virtue of that general law of optics, which provides, that when the retina is under the influence of excitement by any bright, colored light, feebler lights, which seen alone would produce no sensation but of whiteness, shall for the time appear colored with the tint complementary to that of the brighter. Thus a yellow color predominating in the light of the brighter star, that of the less bright one in the same field of view will appear blue; while, if the tint of the brighter star verge to crimson, that of the other will exhibit a tendency to green—or even appear as a vivid green, under favorable circumstances. The former contrast is beautifully exhibited by ι Cancri—the latter by γ Andromedæ,[22] both fine double stars. If, however, the colored star be much the less bright of the two, it will not materially affect the other. Thus, for instance, η Cassiopeiæ exhibits the beautiful combination of a large white star, and a small one of a rich ruddy purple. It is by no means, however, intended to say, that in all such cases one of the colors is a mere effect of contrast, and it may be easier suggested in words, than conceived in imagination, what variety of illumination *two suns*—a red and a green, or a yellow and a

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these catalogues. The fine binary star, B. A. C. No. 4923, has its constituents 15″ apart, the one 6*m.* yellow, the other 7*m.* orange.

[21] "——— other suns, perhaps,
With their attendant moons thou wilt descry,
Communicating male and female light
(Which two great sexes animate the world),
Stored in each orb, perhaps, with some that live."
—*Paradise Lost*, viii. 148.

[22] The small star of γ Andromedæ is close double. Both its individuals are green: a similar combination, with even more decided colors, is presented by the double star, h. 881.

blue one—must afford a planet circulating about either; and what charming contrasts and "grateful vicissitudes"—a red and a green day, for instance, alternating with a white one and with darkness—might arise from the presence or absence of one or other, or both, above the horizon. Insulated stars of a red color, almost as deep as that of blood,[23] occur in many parts of the heavens, but no green or blue *star* (of any decided hue) has, we believe, ever been noticed unassociated with a companion brighter than itself. Many of the red stars are variable.

(852.) Another very interesting subject of inquiry, in the physical history of the stars, is their proper motion. It was first noticed by Halley, that three principal stars, Sirius, Arcturus, and Aldebaran, are placed by Ptolemy, on the strength of observations made by Hipparchus, 130 years B.C., in latitudes respectively 20′, 22′, and 33′ more *northerly* than he actually found them in 1717.[24] Making due allowance for the diminution of obliquity of the ecliptic in the interval (1847 years) they ought to have stood, if really fixed, respectively 10′, 14′, and 0′ more *southerly*. As the circumstances of the statement exclude the supposition of error of transcription in the MSS., we are necessitated to

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[23] The following are the R. ascensions and N. P. distances for 1830, of some of the most remarkable of these sanguine or ruby stars:

R. A.	N. P. D.	R. A.	N P. D.	R. A.	N. P. D.
h. m. s.	° ′ ″	h. m. s.	° ′ ″	h. m. s.	° ′ ″
4 40 53	61 46 21	10 52 10	107 24 40	21 37 18	31 59 47
4 51 51	105 2 4	12 37 31	148 45 47	21 37 20	52 54 47
5 38 29	136 32 15	16 29 44	122 2 0	21 15 37	48 8 12
9 27 56	152 2 48	20 7 8	111 50 11		
9 48 31	130 47 12	21 15 36	48 22 40		

Of these No. 6 (in order of right ascension) is in the same field of view with α Hydræ et Crateris, and No. 7, with β Crucis. No. 2 (in the same order) is variable.

[24] Phil. Trans. 1717, vol. xxx. fol. 736.

admit a southward motion in latitude in these stars to the very considerable extent, respectively, of 37′, 42′, and 33′, and this is corroborated by an observation of Aldebaran at Athens, in the year A.D. 509, which star, on the 11th of March in that year, was seen immediately after its emergence from occultation by the moon, in such a position as it could not have had if the occultation were not nearly central. Now, from the knowledge we have of the lunar motions, this could not have been the case had Aldebaran at that time so much southern latitude as at present. *À priori*, it might be expected that apparent motions of some kind or other should be detected among so great a multitude of individuals scattered through space, and with nothing to keep them fixed. Their mutual attractions even, however inconceivably enfeebled by distance, and counteracted by opposing attractions from opposite quarters, must in the lapse of countless ages produce *some* movements—some change of internal arrangement—resulting from the difference of the opposing actions. And it is a fact, that such apparent motions are really proved to exist by the exact observations of modern astronomy. Thus, as we have seen, the two stars of 61 Cygni have remained constantly at the same, or very nearly the same, distance, of 15″, for at least fifty years past, although they have shifted their local situation in the heavens, in this interval of time, through no less than 4′ 23″, the annual proper motion of each star being 5″·3; by which quantity (exceeding a third of their interval) this system is every year carried bodily along in some unknown path, by a motion which, for many centuries, must be regarded as uniform and rectilinear. Among stars not double, and no way differing from the rest in any other obvious particular, ϵ Indi,[25]

[25] D'Arrest. Astr. Nachr., No. 618; Argelander Do. No. 475.

Groomb. 1830, and μ Cassiopeiæ are to be remarked as having the greatest proper motions of any yet ascertained, amounting respectively to $7''\cdot74$, $7''\cdot75$ and $3''\cdot74$ of annual displacement. And a great many others have been observed to be thus constantly carried away from their places by smaller, but not less unequivocal motions.[26]

(853.) Motions which require whole centuries to accumulate before they produce changes of arrangement, such as the naked eye can detect, though quite sufficient to destroy that idea of mathematical fixity which precludes speculation, are yet too trifling, as far as practical applications go, to induce a change of language, and lead us to speak of the stars in common parlance as otherwise than fixed. Small as they are, however, astronomers, once assured of their reality, have not been wanting in attempts to explain and reduce them to general laws. No one, who reflects with due attention on the subject, will be inclined to deny the high probability, nay certainty, that the *sun* as well as the stars must have a proper motion in *some* direction; and the inevitable consequence of such a motion, if unparticipated by the rest, must be a slow *average* apparent tendency of all the stars to the vanishing point of lines parallel to that direction, and to the region which he is leaving, however greatly individual stars might differ from such average by reason of their own peculiar proper motion. This is the necessary effect of perspective; and it is certain that it must be detected by observation, if we knew accurately the apparent proper motions of all the stars, and if we were sure that they were independent, *i.e.* that the whole firmament, or at least

[26] The reader may consult "a list of 314 stars having, or supposed to have, a proper motion of not less than about $0''\cdot5$ of a great circle" (*per annum*) by the late F. Baily, Esq. *Trans. Ast. Soc.* v. p. 158.

all that part which we see in our own neighborhood, were not drifting along together, by a general *set* as it were, in one direction, the result of unknown processes and slow internal changes going on in the sidereal stratum to which our system belongs, as we see motes sailing in a current of air, and keeping nearly the same relative situation with respect to one another.

(854.) It was on this assumption, tacitly made indeed, but necessarily implied in every step of his reasoning, that Sir William Herschel, in 1783, on a consideration of the apparent proper motions of such stars as could at that period be considered as tolerably (though still imperfectly) ascertained, arrived at the conclusion that a relative motion of the sun, among the fixed stars in the direction of a point or parallactic apex, situated near λ Herculis, that is to say, in R. A. $17^h\ 22^m = 260°\ 34'$, N. P. D. $63°\ 43'$ (1790), would account for the chief observed apparent motions, leaving, however, some still outstanding and not explicable by this cause; and in the same year Prevost, taking nearly the same view of the subject, arrived at a conclusion as to the solar apex (or point of the sphere toward which the sun relatively advances), agreeing nearly in polar distance with the foregoing, but differing from it about 27° in right ascension. Since that time methods of calculation have been improved and concinnated, our knowledge of the proper motions of the stars has been rendered more precise, and a greater number of cases of such motions have been recorded. The subject has been resumed by several eminent astronomers and mathematicians: viz. 1st, by M. Argelander, who, from the consideration of the proper motions of 21 stars exceeding 1″ per annum in arc, has placed the solar apex in R. A. 256° 25′, N. P. D. 51° 23′; from those of 50 stars between 0″·5 and 1″·0, in 255° 10′, 51° 26′; and from those of 319 stars having motions between 0″·1 and 0″·5 per annum, in 261° 11′, 59° 2′; 2dly, by M. Luhndahl, whose calculations, founded on the proper motions of 147 stars, give 252° 53′, 75° 34′; and, 3dly, by M. Otto Struve, whose result, 261° 22′, 62° 24′, emerges from a very elaborate discussion of the proper motions of 392 stars. All these places are for A.D. 1790.

(855.) The most probable mean of the results obtained by these three astronomers is (for the same epoch) R. A. = 259° 9′, N. P. D. 55° 23′. Their researches, however, extending only to stars visible in European observatories, it became a point of high interest to ascertain how far the stars of the southern hemisphere not so visible, treated independently on the same system of procedure, would corroborate or controvert their conclusion. The observations of Lacaille, at the Cape of Good Hope, in 1751 and 1752, compared with those of Mr. Johnson at St. Helena, in 1829–33, and of Henderson at the Cape in 1830 and 1831, have afforded the means of deciding this question. The task has been executed in a masterly manner by Mr. Galloway, in a paper published in the Philosophical Transactions for 1841 (to which we may also refer the reader for a more particular account of the history of the subject than our limits allow us to give). On comparing the records, Mr. Galloway finds eighty-one southern stars not employed in the previous investigations above referred to, whose proper motions in the intervals elapsed appear considerable enough to assure us that they have not originated in error of the earlier observations. Subjecting these to the same process of computation he concludes for the place of the solar apex, for 1790, as follows: viz. R. A. 260° 1′, N. P. D. 55°, 37′, a result so nearly identical with that afforded by the northern hemisphere, as to afford a full conviction of its near approach to truth, and what may fairly be considered a demonstration of the physical cause assigned.

(856.) Of the mathematical conduct of this inquiry the nature of this work precludes our giving any account; but as the philosophical principle on which it is based has been misconceived, it is necessary to say a few words in explanation of it. Almost all the greatest discoveries in astronomy have resulted from the consideration of what we have elsewhere termed RESIDUAL PHENOMENA,[27] of a quantitative or numerical kind, that is to say, of such portions of the numerical or quantitative results of observation as remain outstanding and unaccounted for after subducting and allowing for all that would result from the strict application of known principles. It was thus that the grand discovery of the precession of the equinoxes resulted as a residual phenomenon, from the imperfect explanation of the return of the seasons by the return of the sun to the same apparent place among the fixed stars. Thus, also, aberration and nutation resulted as residual phenomena from that portion of the changes of the apparent places of the fixed stars which was left unaccounted for by precession. And thus again the *apparent* proper motions of the stars are the observed *residues* of their apparent movements outstanding and unaccounted for by strict calculation of the effects of precession, nutation, and aberration. The nearest approach which human theories can make to perfection is to diminish this residue, this *caput mortuum* of observation, as it may be considered, as much as practicable,

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[27] Discourse on the Study of Natural Philosophy (1833). *Cab. Cyclopædia*, No. 14.

and, if possible, to reduce it to nothing, either by showing that something has been neglected in our estimation of known causes, or by reasoning upon it as a new fact, and on the principle of the inductive philosophy ascending from the effect to its cause or causes. On the suggestion of any new cause hitherto unresorted to for its explanation, our first object must of course be to decide whether such a cause would produce *such* a result *in kind*: the next, to assign to it such an intensity as shall account for the greatest possible amount of the residual matter in hand. The proper motion of the sun being suggested as such a cause, we have two things disposable—its direction and velocity, both which it is evident, if they ever become known to us at all, can only be so by the consideration of the very phenomenon in question. Our object, of course, is to account, if possible, for the *whole* of the observed proper motions by the proper assumption of these elements. If this be impracticable, what remains unaccounted for is a residue of a more recondite kind, but which, so long as it *is* unaccounted for, we must regard as purely casual, seeing that, for anything we can perceive to the contrary, it might with equal probability be one way as the other. The theory of chances, therefore, necessitates (as it does in all such cases) the application of a general mathematical process, known as "the method of least squares," which leads, as a matter of strict geometrical conclusion, to the values of the elements sought, *which, under all the circumstances, are the most probable.*

(857.) This is the process resorted to by all the geometers we have enumerated in the foregoing articles (arts. 854, 855). It gives not only the direction in space, but also the *velocity* of the solar motion, estimated on a scale conformable to that in which the velocity of the sidereal motions to be explained are given; *i.e.* in seconds of arc as subtended at the average distance of the stars concerned, by its annual motion in space. But here a consideration occurs which tends materially to complicate the problem, and to introduce into its solution an element depending on suppositions more or less arbitrary. The distance of the stars being, except in two or three instances, unknown, we are compelled either to restrict our inquiry to *these*, which are too few to ground any result on, or to make some supposition as to the relative distances of the several stars employed. In this we have nothing but general probability to guide us, and two courses only present themselves, either, 1st, To class the distances of the stars according to their magnitudes, or apparent brightnesses, and to institute separate and independent calculations for each class, including stars assumed to be equidistant, or nearly so: or, 2dly, To class them according to the observed amount of their apparent proper motions, on the presumption that those which appear to move fastest are really nearest to us. The former is the course pursued by M. Otto Struve, the latter by M. Argelander. With regard to this latter principle of classification, however, two considerations interfere with its applicability, viz. 1st, that we see the *real* motion of the stars foreshortened by the effect of perspective; and 2dly, that that portion of the total *apparent* proper motion which arises from the real motion of the sun depends, not simply on the absolute distance of the star from the sun, but also on its angular apparent distance from the solar apex, being, *cœteris paribus*, as the sine of that angle. To execute such a classification correctly, therefore, we ought to know both these particulars for each star. The first is evidently out of our reach. We are, therefore, for that very reason, compelled to regard it as casual, and to assume that on the average of a great number of stars it would be uninfluential on the result. But the second cannot be so summarily disposed of. By the aid of an approximate knowledge of the solar apex, it is true, approximate values may be found of the simply apparent portions of the proper motions, supposing all the stars equidistant, and these being subducted from the total observed motions, the residues might afford ground for the classification in question.[28] This, however, would be a long, and to a certain extent precarious system of procedure. On the other hand, the classification by apparent brightness is open to no such difficulties, since we are fully justified in assuming that, on a general average, the brighter stars are the nearer, and that the exceptions to this rule are casual in that sense of the word which it always bears in such inquiries, expressing solely our ignorance of any ground for assuming a bias one way or other on either side of a determinate numerical rule. In Mr. Galloway's discussion of the southern stars the consideration of distance is waived altogether, which is equivalent to an admission of complete ignorance on this point, as well as respecting the real directions and velocities of the individual motions.

(858.) The velocity of the solar motion which results from M. Otto Struve's calculations is such as would carry it over an angular subtense of 0″·3392 if seen at right angles from the average distance of a star of the first magnitude.

[28] M. Argelander's classes, however, are constructed without reference to this consideration, on the sole basis of the total apparent amount of proper motion, and are, therefore, *pro tanto*, questionable. It is the more satisfactory then to find so considerable an agreement among his partial results as actually obtains.

If we take, with M. Struve, senior, the parallax of such a star as probably equal to 0″·209,[29] we shall at once be enabled to compare this annual motion with the radius of the earth's orbit, the result being 1·623 of such units. The sun then advances through space (relatively, at least, among the stars), carrying with it the whole planetary and cometary system with a velocity of 1·623 radii of the earth's orbit,[30] or 154,185,000 miles *per annum*, or 422,000 miles (that is to say, nearly its own semidiameter) *per diem:* in other words, with a velocity a very little greater than one-fourth of the earth's annual motion in its orbit.

(859.) Another generation of astronomers, perhaps many, must pass away before we are in a condition to decide from a more precise and extensive knowledge of the proper motions of the stars than we at present possess, how far the direction and velocity above assigned to the solar motion deviates from exactness, whether it continue uniform, and whether it show any sign of deflection from rectilinearity; so as to hold out a prospect of one day being enabled to trace out an arc of the solar orbit, and to indicate the direction in which the preponderant gravitation of the sidereal firmament is urging the central body of our system. An analogy for such deviation from uniformity would seem to present itself in the alleged existence of a similar deviation in the proper motions of Sirius and Procyon, both which stars are considered to have varied sensibly in this respect within the limits of authentic and dependable observation. Such, indeed, would appear to be the amount of evidence for this as a matter of *fact* as to have given rise

[29] Etudes d'Astronomie Stellaire, p. 107.

[30] Mr. Airy (Mem. Ast. Soc. xxviii.) makes this velocity materially greater. See. however, Note L.

to a speculation on the probable circulation of these stars round opaque (and therefore invisible) bodies at no great distances from them respectively, in the manner of binary stars: [and it has been recently shown by M. Peters (Ast. Nachr. 748) that, in the case of Sirius, such a circulation, performed in a period of 50·093 years in an ellipse whose excentricity is 0·7994, the perihelion passage taking place at the epoch A.D. 1791·431, would reconcile the observed anomalies, and reduce the residual motion to uniformity. See Note J.]

(860.) The whole of the reasoning upon which the determination of the solar motion in space rests, is based upon the entire exclusion of any *law* either derived from observation or assumed in theory, affecting the amount and direction of the real motions both of the sun and stars. It supposes an absolute non-recognition, in those motions, of any general directive cause, such as, for example, a common circulation of all about a common centre. Any such limitation introduced into the conditions of the problem of the solar motion would alter *in toto* both its nature and the form of its solution. Suppose for instance that, conformably to the speculations of several astronomers, the whole system of the Milky Way, including our sun, and the stars, our more immediate neighbors, which constitute our sidereal firmament, should have a general movement of rotation in the plane of the galactic circle (any other would be exceedingly improbable, indeed hardly reconcilable with dynamical principles), being held together in opposition to the centrifugal force thus generated by the mutual gravitation of its constituent stars. Except we at the same time admitted that the scale on which this movement proceeds is so enormous that all the stars whose proper motions we include in our calculations go together in a body, so far as that movement is concerned (as forming too small an integrant portion of the whole to differ sensibly in their relation to its central point); we stand precluded from drawing any conclusion whatever, not only respecting the absolute motion of the sun, but respecting even its relative movement among those stars, until we have established some law, or at all events framed some hypothesis having the provisional force of a law, connecting the whole, or a part of the motion of each individual with its situation in space.

(861.) Speculations of this kind have not been wanting in astronomy, and recently an attempt has been made by M. Mädler to assign the local centre in space, round which the sun and stars revolve, which he places in the group of the Pleiades, a situation in itself utterly improbable, lying as it does no less than 26° out of the plane of the galactic circle, out of which plane it is almost inconceivable that any *general* circulation can take place. In the present defective state of our knowledge respecting the proper motion of the smaller stars, especially in right ascension (an element for the most part far less exactly ascertainable than the polar distance, or at least which has been hitherto far less accurately ascertained), we cannot but regard all attempts of the kind as to a certain extent premature, though by no means to be discouraged as forerunners of something more decisive. The question, as a matter of fact, whether a rotation of the galaxy in its own plane exist or not might be at once resolved by the assiduous observation both in R. A. and polar distance of a considerable number of stars of the Milky Way, judiciously selected for the purpose, and *including all magnitudes*, down to the smallest distinctly identifiable, and capable of being observed with normal accuracy: and we would recommend the inquiry to the special attention of directors of permanent observatories, provided with adequate instrumental means, in both hemispheres. Thirty or forty years of observation perseveringly directed to the object in view, could not fail to settle the question.[31]

(862.) The solar motion through space, if real and not simply relative, must give rise to uranographical corrections analogous to parallax and aberration. The solar or systematic parallax is no other than that part of the proper motion of each star which is simply apparent, arising from the sun's motion, and until the distances of the stars be known, must remain inextricably mixed up with the other or real portion. The systematic aberration, amounting at its maximum (for stars 90° from the solar apex) to about 5″, displaces all the stars in great circles diverging from that apex through angles proportional to the sines of their respective distances from it. This displacement, however, is permanent, and therefore uncognizable by any phenomenon, so long as the solar motion remains invariable; but should it, in the course of ages, alter its direction and velocity, both the direction and amount of the displacement in question would alter with it. The change, however, would become mixed up with other changes in the apparent proper motions of the stars, and it would seem hopeless to attempt disentangling them.

(863.) A singular, and at first sight paradoxical effect of the progressive movement of light, combined with the

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[31] An examination of the proper motions of the stars of the B. Assoc. Catal. in the portion of the Milky Way nearest either pole (where the motion should be almost wholly in R. A.) indicates no distinct symptom of such a rotation. If the question be taken up fundamentally, it will involve a redetermination from the recorded proper motions, both of the precession of the equinoxes and the change of obliquity of the ecliptic.

proper motion of the stars, is, that it alters the apparent periodic time in which the individuals of a binary star circulate about each other.[32] To make this apparent, suppose them to circulate round each other in a plane perpendicular to the visual ray in a period of 10,000 days. Then if both the sun and the centre of gravity of the binary system remained fixed in space, the relative apparent situation of the stars would be exactly restored to its former state after the lapse of this interval, and if the angle of position were 0° at first, after 10,000 days it would again be so. But now suppose that the centre of gravity of the star were in the act of receding in a direct line from the sun with a velocity of one-tenth part of the radius of the earth's orbit *per diem*. Then at the expiration of 10,000 days it would be more remote from us by 1000 such radii, a space which light would require 5·7 days to traverse. Although really, therefore, the stars would have arrived at the position 0° at the exact expiration of 10,000 days, it would require 5·7 days more for the notice of that fact to reach our system. In other words, the period would appear to us to be 10,005·7 days, since we could only conclude the period to be completed when to us as observers the original angle of position was again restored. A contrary motion would produce a contrary effect.

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[32] Astronomische Nachrichten, No. 520, by the Author.

CHAPTER XVII

OF CLUSTERS OF STARS AND NEBULÆ

Of Clustering Groups of Stars—Globular Clusters—Their Stability Dynamically Possible—List of the most Remarkable—Classification of Nebulæ and Clusters—Their Distribution over the Heavens—Irregular Clusters—Resolvability of Nebulæ—Theory of the Formation of Clusters by Nebulous Subsidence—Of Elliptic Nebulæ—That of Andromeda—Annular and Planetary Nebulæ—Double Nebulæ—Nebulous Stars—Connection of Nebulæ with Double Stars—Insulated Nebulæ of Forms not Wholly Irregular—Of Amorphous Nebulæ—Their Law of Distribution Marks them as Outliers of the Galaxy—Nebulæ and Nebulous Group of Orion—Of Argo—Of Sagittarius—Of Cygnus—The Magellanic Clouds—Singular Nebula in the Greater of Them—Variable Nebulæ—The Zodiacal Light—Shooting Stars—Speculations on the Dynamical Origin of the Sun's Heat

(864.) WHEN we cast our eyes over the concave of the heavens in a clear night, we do not fail to observe that here and there are groups of stars which seem to be compressed together in a more condensed manner than in the neighboring parts, forming bright patches and clusters, which attract attention, as if they were there brought together by some general cause other than casual distribution. There is a group, called the Pleiades, in which six or seven stars may be noticed, if the eye be directed full upon it; and many more if the *eye be turned carelessly aside* while *the attention* is kept directed[1] upon the group. Telescopes show fifty or sixty

[1] It is a very remarkable fact that the centre of the visual area is far less sensible to feeble impressions of light, than the exterior portions of the retina. Few persons are aware of the extent to which this comparative insensibility extends, previous to trial. To estimate it, let the reader look alternately full at a star of the fifth magnitude, and beside it; or choose two, equally bright, and about 3° or 4° apart, and look full at one of them, the probability is, he will see *only the other*. The fact accounts for the multitude of stars with which we are impressed by a general view of the heavens; their paucity when we come to count them.

large stars thus crowded together in a very moderate space, comparatively insulated from the rest of the heavens. The constellation called Coma Berenices is another such group, more diffused, and consisting on the whole of larger stars.

(865.) In the constellation Cancer, there is a somewhat similar, but less definite, luminous spot, called Præsepe, or the bee-hive, which a very moderate telescope—an ordinary night-glass, for instance—resolves entirely into stars. In the sword-handle of Perseus, also, is another such spot, crowded with stars, which requires rather a better telescope to resolve into individuals separated from each other. These are called clusters of stars; and, whatever be their nature, it is certain that other laws of aggregation subsist in these spots, than those which have determined the scattering of stars over the general surface of the sky. This conclusion is still more strongly pressed upon us, when we come to bring very powerful telescopes to bear on these and similar spots. There are a great number of objects which have been mistaken for comets, and, in fact, have very much the appearance of comets without tails: small round, or oval nebulous specks, which telescopes of moderate power only show as such. Messier has given, in the *Connois. des Temps* for 1784, a list of the places of 103 objects of this sort; which all those who search for comets ought to be familiar with, to avoid being misled by their similarity of appearance. That they are not, however, comets, their fixity sufficiently proves; and when we come to examine them with instruments of great power—such as reflectors of eighteen inches, two feet, or more in aperture—any such idea is completely destroyed. They are then, for the most part, perceived to consist entirely of stars crowded together so as to occupy almost a definite outline, and to run up to a blaze of light in the centre, where their condensation is usually the greatest. (See *fig.* 1, Plate II., which represents (somewhat rudely) the thirteenth nebula of Messier's list (described by him as *nébuleuse sans étoiles*), as seen in a reflector of 18 inches aperture and 20 feet focal length.) Many of them, indeed, are of an exactly round figure, and convey the complete idea of a globular space filled full of stars, insulated in the heavens, and constituting in itself a family or society apart from the rest, and subject only to its own internal laws. It would be a vain task to attempt to count the stars in one of these *globular clusters.* They are not to be reckoned by hundreds; and on a rough calculation, grounded on the apparent intervals between them at the borders, and the angular diameter of the whole group, it would appear that many clusters of this description must contain, at least, five thousand stars, compacted and wedged together in a round space, whose angular diameter does not exceed eight or ten minutes; that is to say, in an area not more than a tenth part of that covered by the moon.

(866.) Perhaps it may be thought to savor of the gigantesque to look upon the individuals of such a group as suns like our own, and their mutual distances as equal to those which separate our sun from the nearest fixed star: yet, when we consider that their united lustre affects the eye with a less impression of light than a star of the fourth magnitude (for the largest of these clusters is barely visible to the naked eye), the idea we are thus compelled to form of their distance from us may prepare us for almost any estimate of their dimensions. At all events, we can hardly look upon a group thus insulated, thus *in seipso totus, teres, atque rotundus*, as not forming a system of a peculiar and definite character. Their round figure clearly indicates the existence of some general bond of union in the nature of an attractive force; and, in many of them, there is an evident acceleration in the rate of condensation as we approach the centre, which is not referable to a merely uniform distribution of equidistant stars through a globular space, but marks an intrinsic *density* in their state of aggregation, greater in the centre than at the surface of the mass. It is difficult to form any conception of the dynamical state of such a system. On the one hand, without a rotatory motion and a centrifugal force, it is hardly possible not to regard them as in a state of progressive collapse. On the other, granting such a motion and such a force, we find it no less difficult to reconcile the apparent sphericity of their form with a rotation of the whole system round any single axis, without which internal collisions might at first sight appear to be inevitable. If we suppose a globular space filled with equal stars, uniformly dispersed through it, and very numerous, each of them attracting every other with a force inversely as the square of the distance, the resultant force by which any one of them (those at the surface alone excepted) will be urged, in virtue of their joint attractions, will be directed toward the common centre of the sphere, and will be directly as the distance therefrom. This follows from what Newton has proved of the *internal* attraction of a homogeneous sphere. (See also note on art. 735.) Now, under such a law of force, each particular star would describe a perfect ellipse about the common centre of gravity as its centre, and *that*, in whatever plane and whatever direction it might revolve. The condition, therefore, of a rotation of the cluster, as a mass, about a single axis would be unnecessary. Each ellipse, whatever might be the proportion of its axis, or the inclination of its plane to the others, would be invariable *in every particular*, and all would be described in one common period, so that at the end of every such period, or *annus magnus* of the system, every star of the cluster (except the superficial ones) would be exactly re-established in its original position, thence to set out afresh, and run the same unvarying round for an indefinite succession of ages. Supposing their motions, therefore, to be so adjusted at any one moment as that the orbits should not intersect each other, and so that the magnitude of each star, and the sphere of its more intense attraction, should bear but a small proportion to the distance separating the individuals, such a system, it is obvious, might subsist, and realize, in great measure, that abstract and ideal harmony, which Newton, in the 89th Proposition of the First Book of the *Principia*, has shown to characterize a law of force directly as the distance.[2]

(867.) The following are the places, for 1830, of the principal of these remarkable objects, as specimens of their class:

R. A.			N. P. D.		R. A.			N. P. D.		R. A.			N. P. D.	
h.	m.	s.	°	′	h.	m.	s.	°	′	h.	m.	s.	°	′
0	16	25	163	2	15	9	56	87	16	17	26	51	143	34
9	8	33	154	10	15	34	56	127	13	17	28	42	93	8
12	47	41	159	57	16	6	55	112	33	11	26	4	114	2
13	4	30	70	55	16	23	2	102	40	18	55	49	150	14
13	16	38	136	35	16	35	37	53	13	21	21	43	78	34
13	34	10	60	46	16	50	24	119	51	21	24	40	91	34

Of these, by far the most conspicuous and remarkable is ω *Centauri*, the fifth of the list in order of right ascension. It is visible to the naked eye as a dim round cometic object about equal to a star 4·5m., though probably if con-

[2] See also *Quarterly Review*, No. 94, p. 540.

centred in a single point, the impression on the eye would be much greater. Viewed in a powerful telescope it appears as a globe of fully 20′ in diameter, very gradually increasing in brightness to the centre, and composed of innumerable stars of the 13th and 15th magnitudes (the former probably being two or more of the latter closely juxtaposed). The 11th in order of the list (R. A. $16^h\ 35^m$) is also visible to the naked eye in *very* fine nights, between η and ζ Herculis, and is a superb object in a large telescope. Both were discovered by Halley, the former in 1677 and the latter in 1714.

(868.) It is to Sir William Herschel that we owe the most complete analysis of the great variety of those objects which are generally classed under the common head of Nebulæ, but which have been separated by him into—1st. Clusters of stars, in which the stars are clearly distinguishable; and these, again, into globular and irregular clusters; 2d. Resolvable nebulæ, or such as excite a suspicion that they consist of stars, and which any increase of the optical power of the telescope may be expected to resolve into distinct stars; 3d. Nebulæ, properly so called, in which there is no appearance whatever of stars; which, again, have been subdivided into subordinate classes, according to their brightness and size; 4th. Planetary nebulæ; 5th. Stellar nebulæ; and, 6th. Nebulous stars. The great power of his telescopes disclosed the existence of an immense number of these objects before unknown, and showed them to be distributed over the heavens, not by any means uniformly, but with a marked preference for a certain district, extending over the northern pole of the galactic circle, and occupying the constellations Leo, Leo Minor, the body, tail, and hind legs of Ursa Major, Canes Venatici, Coma Berenices, the preceding leg of Bootes, and the head, wings, and shoulder of Virgo. In this region, occupying about one-eighth of the whole surface of the sphere, one-third of the entire nebulous contents of the heavens are congregated. On the other hand, they are very sparingly scattered over the constellations Aries, Taurus, the head and shoulders of Orion, Auriga, Perseus, Camelopardalus, Draco, Hercules, the northern part of Serpentarius, the tail of Serpens, that of Aquila, and the whole of Lyra. The hours 3, 4, 5, and 16, 17, 18, of right ascension in the northern hemisphere are singularly poor, and, on the other hand, the hours 10, 11, and 12 (but especially 12), extraordinarily rich in these objects. In the southern hemisphere a much greater uniformity of distribution prevails, and with exception of two very remarkable centres of accumulation, called the Magellanic clouds (of which more presently), there is no very decided tendency to their assemblage in any particular region.

(869.) Clusters of stars are either globular, such as we have already described, or of irregular figure. These latter are, generally speaking, less rich in stars, and especially less condensed toward the centre. They are also less definite in outline; so that it is often not easy to say where they terminate, or whether they are to be regarded otherwise than as merely richer parts of the heavens than those around them. Many, indeed the greater portion of them, are situated in or close on the borders of the Milky Way. In some of them the stars are nearly all of a size, in others extremely different; and it is no uncommon thing to find a very red star much brighter than the rest, occupying a conspicuous situation in them. Sir William Herschel regards these as globular clusters in a less advanced state of condensation, conceiving all such groups as approaching, by their mutual attraction, to the globular figure, and assembling themselves together from all the surrounding region, under laws of which we have, it is true, no other proof than the observance of a gradation by which their characters shade into one another, so that it is impossible to say where one species ends and the other begins. Among the most beautiful objects of this class is that which surrounds the star $\varkappa$ Crucis, set down as a nebula by Lacaille. It occupies an area of about one 48th part of a square degree, and consists of about 110 stars from the 7th magnitude downward, eight of the more conspicuous of which are colored with various shades of red, green, and blue, so as to give to the whole the appearance of a rich piece of jewelry.

(870.) Resolvable nebulæ can, of course, only be considered as clusters either too remote, or consisting of stars intrinsically too faint to affect us by their individual light, unless where two or three happen to be close enough to make a joint impression, and give the idea of a point brighter than the rest. They are almost universally round or oval—their loose appendages, and irregularities of form, being as it were extinguished by the distance, and the only general figure of the more condensed parts being discernible. It is under the appearance of objects of this character that all the greater globular clusters exhibit themselves in telescopes of insufficient optical power to show them well; and the conclusion is obvious, that those which the most powerful can barely render resolvable, and even those which, with such powers as are usually applied, show no sign of being composed of stars, would be completely resolved by a further increase of optical power. In fact, this probability has almost been converted into a certainty by the magnificent reflecting telescope constructed by Lord Rosse, of six feet in aperture, which has resolved or rendered resolvable multitudes of nebulæ which had resisted all inferior powers. The sublimity of the spectacle afforded by that instrument of some of the larger globular and other clusters enumerated in the list given in art. 867, is declared by all who have witnessed it to be such as no words can express.

(871.) Although, therefore, nebulæ do exist, which even in this powerful telescope appear *as* nebulæ, without any sign of resolution, it may very reasonably be doubted whether there be really any essential physical distinction between nebulæ and clusters of stars, at least in the nature of the matter of which they consist,[3] and whether the distinction between such nebulæ as are easily resolved, barely resolvable with excellent telescopes, and altogether irresolvable with the best, be anything else than one of degree, arising merely from the excessive minuteness and multitude of the stars, of which the latter, as compared with the former, consist. The first impression which Halley, and other early discoverers of nebulous objects received from their peculiar aspect, so different from the keen, concentrated light of mere stars, was that of a phosphorescent vapor (like the matter of a comet's tail) or a gaseous and (so to speak) elementary form of luminous sidereal matter.[4] Admitting the existence of such a medium, dispersed in some cases irregularly through vast regions in space, in others confined to narrower and more definite limits, Sir W. Herschel was led to speculate on its gradual subsidence and condensation by the effect of its own gravity, into more or less regular spherical or spheroidal forms, denser

[3] See Note K. [4] Halley, Phil. Trans. xxix. p. 390.

(as they must in that case be) toward the centre. Assuming that in the progress of this subsidence, local centres of condensation, subordinate to the general tendency, would not be wanting, he conceived that in this way solid nuclei might arise, whose local gravitation still further condensing, and so absorbing the nebulous matter, each in its immediate neighborhood, might ultimately become stars, and the whole nebula finally take on the state of a cluster of stars. Among the multitude of nebulæ revealed by his telescopes, every stage of this process might be considered as displayed to our eyes, and in every modification of form to which the general principle might be conceived to apply. The more or less advanced state of a nebula toward its segregation into discrete stars, and of these stars themselves toward a denser state of aggregation round a central nucleus, would thus be in some sort an indication of age. Neither is there any variety of aspect which nebulæ offer, which stands at all in contradiction to this view. Even though we should feel ourselves compelled to reject the idea of a gaseous or vaporous "nebulous matter," it loses little or none of its force. Subsidence, and the central aggregation consequent on subsidence, may go on quite as well among a multitude of discrete bodies under the influence of mutual attraction, and feeble or partially opposing projectile motions, as among the particles of a gaseous fluid.

(872.) The "*nebular hypothesis*," as it has been termed, and the *theory of sidereal aggregation* stand, in fact, quite independent of each other, the one as a physical conception of processes which may yet, for aught we know, have formed part of that mysterious chain of causes and effects antecedent to the existence of separate self-luminous solid bodies; the other, as an application of dynamical principles to cases of a very complicated nature no doubt, but in which the possibility or impossibility, at least, of certain general results may be determined on perfectly legitimate principles. Among a crowd of solid bodies of whatever size, animated by independent and partially opposing impulses, motions opposite to each other *must* produce collision, destruction of velocity, and subsidence or near approach toward the centre of preponderant attraction; while those which conspire, or which remain outstanding after such conflicts, *must* ultimately give rise to circulation of a permanent character. Whatever we may think of such collisions as events, there is nothing in this conception contrary to sound mechanical principles. It will be recollected that the appearance of central condensation among a multitude of separate bodies in motion, by no means implies permanent proximity to the centre in each any more than the habitually crowded state of a market-place, to which a large proportion of the inhabitants of a town must frequently or occasionally resort, implies the permanent residence of each individual within its area. It is a fact that clusters thus centrally crowded do exist, and therefore the conditions of their existence must be dynamically possible, and in what has been said we may at least perceive some glimpses of the manner in which they are so. The actual intervals between the stars, even in the most crowded parts of a resolved nebula, to be seen at all by us, must be enormous. Ages, which to us may well appear indefinite, may easily be conceived to pass without a single instance of collision, in the nature of a catastrophe. Such may have gradually become rarer as the system has emerged from what must be considered its chaotic state, till at length, in the fulness of time, and under the prearranging guidance of that DESIGN which pervades universal nature, each individual may have taken up such a course as to annul the possibility of further destructive interference.

(873.) But to return from the regions of speculation to the description of facts. Next in regularity of form to the globular clusters, whose consideration has led us into this digression, are elliptic nebulæ, more or less elongated. And of these it may be generally remarked, as a fact undoubtedly connected in some very intimate manner with the dynamical conditions of their subsidence, that such nebulæ are, for the most part, beyond comparison more difficult of resolution than those of globular form. They are of all degrees of excentricity, from moderately oval forms to ellipses so elongated as to be almost linear, which are, no doubt, edge-views of very flat ellipsoids. In all of them the density increases toward the centre, and as a general law it may be remarked that, so far as we can judge from their telescopic appearance, their internal strata approach more nearly to the spherical form than their external. Their resolvability, too, is greater in the central parts, whether owing to a real superiority of size in the central stars or to the greater frequency of cases of close juxtaposition of individuals, so that two or three united appear as one. In some the condensation is slight and gradual, in others great and sudden; so sudden, indeed, as to offer the appearance of a dull and blotted star, standing in the midst of a faint, nearly equable elliptic nebulosity, of which two remarkable specimens occur in R. A. $12^h\ 10^m\ 33^s$, N. P. D. $41° 46'$, and in $13^h\ 27^m\ 28^s$, $119° 0'$ (1830).

(874.) The largest and finest specimens of elliptic nebulæ which the heavens afford are that in the girdle of Andromeda (near the star ν of that constellation) and that discovered in 1783, by Miss Carolina Herschel, in R. A. 0^h 39^m 12^s, N. P. D. 116° 13′. The nebula in Andromeda (Plate II. *fig.* 3) is visible to the naked eye, and is continually mistaken for a comet by those unacquainted with the heavens. Simon Marius, who noticed it in 1612 (though it appears also to have been seen and described as oval, in 995), describes its appearance as that of a candle shining through horn, and the resemblance is not inapt. Its form, as seen through ordinary telescopes, is a pretty long oval, increasing by insensible gradations of brightness, at first very gradually, but at last more rapidly, up to a central point, which, though very much brighter than the rest, is decidedly not a star, but nebula of the same general character with the rest in a state of extreme condensation. Casual stars are scattered over it, but with a reflector of 18 inches in diameter, there is nothing to excite any suspicion of its consisting of stars. Examined with instruments of superior defining power, however, the evidence of its resolvability into stars may be regarded as decisive. Mr. G. P. Bond, assistant at the observatory of Cambridge, U. S., describes and figures it as extending nearly $2\frac{1}{2}°$ in length, and upward of a degree in breadth (so as to include two other smaller adjacent nebulæ), of a form, generally speaking, oval, but with a considerably protuberant irregularity at its north following extremity, very suddenly condensed at the nucleus almost to the semblance of a star, and though not itself clearly resolved, yet thickly sown over with visible minute stars, so numerous as to allow of 200 being counted within a field of 20′ diameter in the richest parts. But the most remarkable feature in his description is that of two perfectly straight, narrow, and comparatively or totally obscure streaks which run nearly the whole length of one side of the nebula, and (though slightly divergent from each other) nearly parallel to its longer axis. These streaks (which obviously indicate a stratified structure in the nebula, if, indeed, they do not originate in the interposition of imperfectly transparent matter between us and it) are not seen on a general and cursory view of the nebula; they require attention to distinguish them,[5] and this circumstance must be borne in mind when inspecting the very extraordinary engraving which illustrates Mr. Bond's account. The figure given in our Plate II. *fig.* 3, is from a rather hasty sketch, and makes no pretensions to exactness. A similar, but much more strongly marked case of parallel arrangement than that noticed by Mr. Bond in this, is one in which the two semiovals of an elliptically formed nebula appear cut asunder and separated by a broad obscure band parallel to the longer axis of the nebula, in the midst of which a faint streak of light parallel to the sides of the cut appears: it is seen in the southern hemisphere in R. A. $13^h\ 15^m\ 31^s$, N. P. D. 132° 8′ (1830). The nebulæ in $12^h\ 27^m\ 3^s$, 63° 5′, and $12^h\ 31^m\ 11^s$, 100° 40′ present analogous features.

(875.) Annular nebulæ also exist, but are among the rarest objects in the heavens. The most conspicuous of this class is to be found almost exactly half way between β and γ Lyræ, and may be seen with a telescope of moderate power. It is small and particularly well defined, so as to have more the appearance of a flat oval solid ring than of a nebula. The axes of the ellipse are to each other in the proportion of about 4 to 5, and the opening occupies about half or rather more than half the diameter. The central vacuity is not quite dark, but is filled in with faint nebula, like a gauze stretched over a hoop. The powerful

[5] Trans. American Acad., vol. iii. p. 80.

telescopes of Lord Rosse resolve this object into excessively minute stars, and show filaments of stars adhering to its edges.[6]

(876.) PLANETARY NEBULÆ are very extraordinary objects. They have, as their name imports, a near, in some instances, a perfect resemblance to planets, presenting disks round, or slightly oval, in some quite sharply terminated, in others a little hazy or softened at the borders. Their light is in some perfectly equable, in others mottled and of a very peculiar *texture*, as if curdled. They are comparatively rare objects, not above four or five and twenty having been hitherto observed, and of these nearly three-fourths are situated in the southern hemisphere. Being very interesting objects we subjoin a list of the most remarkable.[7] Among these may be more particularly specified the sixth in order, situated in the Cross. Its light is about equal to that of a star of the 6·7 magnitude, its diameter about 12″, its disk circular or very slightly elliptic, and with a clear, sharp, well-defined outline, having exactly the appearance of a planet with the exception only of its

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[6] The places of some remarkable annular nebulæ (for 1830) are,

	R. A.			N. P. D.			R. A.			N. P. D.	
1.	10^h	16^m	36^s	107°	48′	4.	17^h	19^m	2^s	113°	37′
2.	12	42	52	47	57	5.	18	47	13	57	11
4.	17	10	39	128	18	6.	20	9	33	59	57

[7] Places for 1830 of twelve of the most remarkable planetary nebulæ.

	R. A.	N. P. D.		R. A.	N. P. D.		R. A.	N. P. D.
	h. m. s.	° ′		h. m. s.	° ′		h. m. s.	° ′
1.	7 34 2	104 20	5.	11 4 49	34 4	9.	19 34 21	104 33
2.	9 16 39	147 35	6.	11 41 56	146 14	10.	19 40 19	39 54
3.	9 59 52	129 36	7.	15 5 18	135 1	11.	20 54 53	102 2
4.	10 16 36	107 47	8.	19 10 9	83 46	12.	23 17 44	48 24

color, which is a fine and full blue verging somewhat upon green. And it is not a little remarkable that this phenomenon of a blue color, which is so rare among *stars* (except when in the immediate proximity of yellow stars) occurs, though less strikingly, in three other objects of this class, viz. in No. 4, whose color is sky-blue, and in Nos. 11 and 12, where the tint, though paler, is still evident. Nos. 2, 7, 9, and 12, are also exceedingly characteristic objects of this class. Nos. 3, 4, and 11 (the latter in the parallel of ν Aquarii, and about 5^m preceding that star), are considerably elliptic, and (respectively) about 38″, 30″ and 15″ in diameter. On the disk of No. 3, and very nearly in the centre of the ellipse, is a star 9m, and the texture of its light, being velvety, or as if formed of fine dust, clearly indicates its resolvability into stars. The largest of these objects is No. 5, situated somewhat south of the parallel of β Ursæ Majoris and about 12^m following that star. Its apparent diameter is 2′ 40″, which, supposing it placed at a distance from us not more than that of 61 Cygni, would imply a linear one seven times greater than that of the orbit of Neptune. The light of this stupendous globe is perfectly equable (except just at the edge, where it is slightly softened), and of considerable brightness. Such an appearance would not be presented by a globular space uniformly filled with stars or luminous matter, which structure would necessarily give rise to an apparent increase of brightness toward the centre in proportion to the thickness traversed by the visual ray. We might, therefore, be induced to conclude its real constitution to be either that of a hollow spherical shell or of a flat disk, presented to us (by a highly improbable coincidence) in a plane precisely perpendicular to the visual ray.

(877.) Whatever idea we may form of the real nature of such a body, or of the planetary nebulæ in general, which all agree in the absence of central condensation, it is evident that the intrinsic splendor of their surfaces, *if continuous*, must be almost infinitely less than that of the sun. A circular portion of the sun's disk, subtending an angle of 1′, would give a light equal to that of 780 full moons; while among all the objects in question there is not one which can be seen with the naked eye. M. Arago has surmised that they may possibly be envelopes shining by *reflected* light, from a solar body placed in their centre, invisible to us by the effect of its excessive distance; removing, or attempting to remove the apparent paradox of such an explanation, by the optical principle that an illuminated surface is equally *bright* at all distances, and, therefore, if large enough to subtend a measurable angle, can be equally well seen, whereas the central body, subtending no such angle, has its effect on our sight diminished in the inverse ratio of the square of its distance.[8] The immense optical powers applied by Lord Rosse and Mr. Lassell to the examination of these enigmatical objects have hitherto only added to the mystery which hangs about them, by disclosing caprices of structure in several of them of the most extraordinary nature.[9]

(878.) Double nebulæ occasionally occur—and when such

[8] With due deference to so high an authority we must demur to the conclusion. Even supposing the envelope to reflect and scatter (equally in all directions) half the light of the central sun, the portion of the light so scattered which would fall to our share could not exceed the remaining half which that sun itself would still send to us by direct radiation. But this, *ex hypothesi*, is too small to affect the eye with any luminous perception, when concentrated in a point, much less then could it do so if spread over a surface many million times exceeding in angular area the apparent disk of the central sun itself. (See Annuaire du Bureau des Longitudes, 1842, p. 409, 410, 411.)

[9] See the figures in their papers, Phil. Trans. 1850 and 1861, and Mem. Ast. Soc. vol. xxxvi.

is the case, the constituents most commonly belong to the class of spherical nebulæ, and are in some instances undoubtedly globular clusters. All the varieties of double stars, in fact, as to distance, position, and relative brightness, have their counterparts in double nebulæ; besides which the varieties of form and gradation of light in the latter afford room for combinations peculiar to this class of objects. Though the conclusive evidence of observed relative motion be yet wanting, and though from the vast scale on which such systems are constructed, and the probable extreme slowness of the angular motion, it may continue for ages to be so, yet it is impossible, when we cast our eyes upon such objects, or on the figures which have been given of them,[10] to doubt their physical connection. The argument drawn from the comparative rarity of the objects in proportion to the whole extent of the heavens, so cogent in the case of the double stars, is infinitely more so in that of the double nebulæ. Nothing more magnificent can be presented to our consideration, than such combinations. Their stupendous scale, the multitude of individuals they involve, the perfect symmetry and regularity which many of them present, the utter disregard of complication in thus heaping together system upon system, and construction upon construction, leave us lost in wonder and admiration at the evidence they afford of infinite power and unfathomable design.

(879.) Nebulæ of regular forms often stand in marked and symmetrical relation to stars, both single and double. Thus we are occasionally presented with the beautiful and striking phenomenon of a sharp and brilliant star concentrically surrounded by a perfectly circular disk or atmos-

[10] Phil. Trans. 1833, Plate vii.

phere of faint light, in some cases dying away insensibly on all sides, in others almost suddenly terminated. These are *Nebulous Stars*. Fine examples of this kind are the 45th and 69th nebulæ of Sir Wm. Herschel's fourth class[11] (R. A. 7^h 19^m 8^s. N. P. D. 68° 45′, and 3^h 58^m 36^s, 59° 40′) in which stars of the 8th magnitude are surrounded by photospheres of the kind above described respectively of 12″ and 25″ in diameter. Among stars of larger magnitudes, 55 Andromedæ and 8 Canum Venaticorum may be named as exhibiting the same phenomenon with more brilliancy, but perhaps with less perfect regularity.

(880.) The connection of nebulæ with double stars is in many instances extremely remarkable. Thus in R. A. 18^h 7^m 1^s, N. P. D. 109° 56′, occurs an elliptic nebula having its longer axis about 50″, in which, symmetrically placed, and rather nearer the vertices than the foci of the ellipse, are the equal individuals of a double star, each of the 10th magnitude. In a similar combination noticed by M. Struve (in R. A. 18^h 25^m, N. P. D. 25° 7′), the stars are unequal and situated precisely at the two extremities of the major axis. In R. A. 13^h 47^m 33^s, N. P. D. 129° 9′, an oval nebula of 2′ in diameter has very near its centre a close double star, the individuals of which, slightly unequal, and about the 9·10 magnitude, are not more than 2″ asunder. The nucleus of Messier's 64th nebula is "strongly suspected" to be a close double star—and several other instances might be cited.

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[11] The classes here referred to are not the species described in art. 868, but lists of nebulæ, eight in number, arranged according to brightness, size, density of clustering, etc., in one or other of which all nebulæ were originally classed by him. Class I. contains "Bright nebulæ"; II. "Faint do."; III. "Very faint do."; IV. "Planetary nebulæ, stars with bars, milky chevelures, short rays, remarkable shapes, etc."; V. "Very large nebulæ"; VI. "Very compressed rich clusters"; VII. "Pretty much compressed do."; VIII. "Coarsely scattered clusters."

(881.) Among the nebulæ which, though deviating more from symmetry of form, are yet not wanting in a certain regularity of figure, and which seem clearly entitled to be regarded as systems of a definite nature, however mysterious their structure and destination, by far the most remarkable are the 27th and 51st of Messier's Catalogue.[12] The former consists of two round or somewhat oval nebulous masses united by a short neck of nearly the same density. Both this and the masses graduate off, however, into a fainter nebulous envelope which completes the figure into an elliptic form, of which the interior masses with their connection occupy the lesser axis. Seen in a reflector of 18 inches in aperture, the form has considerable regularity; and though a few stars are here and there scattered over it, it is unresolved. Lord Rosse, viewing it with a reflector of double that aperture, describes and figures it as resolved into numerous stars with much intermixed nebula; while the symmetry of form, by rendering visible features too faint to be seen with inferior power, is rendered considerably less striking, though by no means obliterated.

(882.) The 51st nebula of Messier, viewed through an 18-inch reflector, presents the appearance of a large and bright globular nebula, surrounded by a ring at a considerable distance from the globe, very unequal in brightness in its different parts, and subdivided through about two-fifths of its circumference as if into two laminæ, one of which appears as if turned up toward the eye out of the plane of the rest. Near it (at about a radius of the ring distant) is a small bright round nebula. Viewed through

[12] Places for 1830: R. A. 19h. 52m. 12s., N. P. D. 67° 44′, and R. A. 13h. 22m. 59s., N. P. D. 41° 56′.

the 6-feet reflector of Lord Rosse the aspect is much altered. The interior, or what appeared the upturned portion of the ring, assumes the aspect of a nebulous coil or convolution tending in a spiral form toward the centre, and a general tendency to a spiroid arrangement of the streaks of nebula connecting the ring and central mass which this power brings into view, becomes apparent, and forms a very striking feature. The outlying nebula is connected by a narrow nebulous arc with the ring, and the whole has a resolvable character. (See Plate VI. *fig.* 3.) Both Lord Rosse and Mr. Lassell have found this spiral character, even still more marked, to belong to many other nebulæ: sufficiently numerous, in fact, to form a class apart, of which Messier's 99th nebula is a fine specimen.

(883.) We come now to a class of nebulæ of totally different character. They are of very great extent, utterly devoid of all symmetry of form—on the contrary, irregular and capricious in their shapes and convolutions to a most extraordinary degree, and no less so in the distribution of their light. No two of them can be said to present any similarity of figure or aspect, but they have one important character in common. They are all situated in, or very near, the borders of the Milky Way. The most remote from it is that in the sword-handle of Orion, which being 20° from the galactic circle, and 15° from the visible border of the Via Lactea, might seem to form an exception, though not a striking one. But this very situation may be adduced as a corroboration of the general view which this principle of localization suggests. For the place in question is situated in the prolongation of that faint offset of the Milky Way which we traced (art. 787) from α and ε Persei toward Aldebaran and the Hyades, and in the zone of Great Stars noticed in art. 785 as an appendage of, and probably bearing relation to that stratum.

(884.) From this it would appear to follow, almost as a matter of course, that they must be regarded as outlying, very distant, and as it were detached fragments of the great stratum of the Galaxy, and this view of the subject is strengthened when we find on mapping down their places that they may all be grouped in four great masses or nebulous regions—that of Orion, of Argo, of Sagittarius, and of Cygnus. And thus, inductively, we may gather some information respecting the structure and form of the Galaxy itself, which, could we view it as a whole, from a distance such as that which separates us from these objects, would very probably present itself under an aspect quite as complicated and irregular.

(885.) The great nebula surrounding the stars marked θ 1 in the sword-handle of Orion was discovered by Huyghens in 1656, and has been repeatedly figured and described by astronomers since that time. Its appearance varies greatly (as that of all nebulous objects does) with the instrumental power applied, so that it is difficult to recognize in representations made with inferior telescopes, even principal features, to say nothing of subordinate details. Until this became well understood, it was supposed to have changed very materially, both in form and extent, during the interval elapsed since its first discovery. No doubt, however, now remains that these supposed changes have originated partly from the cause above mentioned, partly from the difficulty of correctly drawing, and, still more, engraving such objects, and partly from a want of sufficient care in the earlier delineators themselves in faithfully copying that which they really did see. Our figure (Plate IV.

fig. 1) is reduced from a larger one made under very favorable circumstances, from drawings taken with an 18-inch reflector at the Cape of Good Hope, where its meridian altitude greatly exceeds what it has at European stations. The area occupied by this figure is about one 25th part of a square degree, extending in R. A. (or horizontally) 2^m of time, equivalent almost exactly to 30′ in arc, the object being very near the equator, and 24′ vertically, or in polar distance. The figure shows it reversed in declination, the northern side being lowermost, and the preceding toward the right hand. In form, the brightest portion offers a resemblance to the head and yawning jaws of some monstrous animal, with a sort of proboscis running out from the snout. Many stars are scattered over it, which for the most part appear to have no connection with it, and the remarkable sextuple star θ 1 Orionis, of which mention has already been made (art. 837), occupies a most conspicuous situation close to the brightest portion, at almost the edge of the opening of the jaws. It is remarkable, however, that within the area of the trapezium no *nebula* exists. The general aspect of the less luminous and cirrous portion is simply nebulous and irresolvable, but the brighter portion immediately adjacent to the trapezium, forming the square front of the head, is shown with the 18-inch reflector broken up into masses (very imperfectly represented in the figure), whose mottled and curdling light evidently indicates by a sort of granular texture its consisting of stars, and when examined under the great light of Lord Rosse's reflector, or the exquisite defining power of the great achromatic at Cambridge, U. S., is evidently perceived to consist of clustering stars. There can therefore be little doubt as to the whole consisting of stars, too minute to be discerned individually even with these powerful aids, but which become visible as points of light when closely adjacent in the more crowded parts in the mode already more than once suggested.

(886.) The nebula is not confined to the limits of our figure. Northward of θ about 33′, and nearly on the same meridian are two stars marked C 1 and C 2 Orionis, involved in a bright and branching nebula of very singular form, and south of it is the star ι Orionis, which is also involved in strong nebula. Careful examination with powerful telescopes has traced out a continuity of nebulous light between the great nebula and both these objects, and there can be little doubt that the nebulous region extends northward, as far as ϵ in the belt of Orion, which is involved in strong nebulosity, as well as several smaller stars in the immediate neighborhood. Professor Bond has given a beautiful figure of the great nebula in Trans. American Acad. of Arts and Sciences, new series, vol. iii., and Lord Oxmantown a superb one in Phil. Tr. 1868.

(887.) The remarkable variation in lustre of the bright star η in Argo, has been already mentioned. This star is situated in the most condensed region of a very extensive nebula or congeries of nebular masses, streaks and branches, a portion of which is represented in *fig.* 2, Plate IV. The whole nebula is spread over an area of fully a square degree in extent, of which that included in the figure occupies about one-fourth, that is to say, 28′ in polar distance, and 32′ of arc in R. A., the portion not included being, though fainter, even more capriciously contorted than that here depicted, in which it should be observed that the preceding side is toward the right hand, and the southern uppermost. Viewed with an 18-inch reflector, no part of this strange object shows any sign of resolution into stars, nor in the brightest and most condensed portion adjacent to the singular oval vacancy in the middle of the figure is there any of that curdled appearance or that tendency to break up into bright knots with intervening darker portions which characterize the nebula of Orion, and indicate its resolvability. The whole is situated in a very rich and brilliant part of the Milky Way, so thickly strewed with stars (omitted in the figure), that in the area occupied by the nebula, not less than 1200 have been actually counted, and their places in R. A. and P. D. determined. Yet it is obvious that these have no connection whatever with the nebula, being, in fact, only a simple continuation over it of the general ground of the galaxy, which on an average of two hours in right ascension in this period of its course contains no less than 3138 stars to the square degree, all, however, distinct, and (except where the object in question is situated) seen projected on a perfectly dark heaven, without any appearance of intermixed nebulosity. The conclusion can hardly be avoided, that in looking at it we see through, and beyond the Milky Way, far out into space, through a starless region, disconnecting it altogether from our system. "It is not easy for language to convey a full impression of the beauty and sublimity of the spectacle which this nebula offers, as it enters the field of view of a telescope fixed in right ascension, by the diurnal motion, ushered in as it is by so glorious and innumerable a procession of stars, to which it forms a sort of climax," and in a part of the heavens otherwise full of interest. One other bright and very remarkably formed nebula of considerable magnitude precedes it nearly on the same parallel, but without any traceable connection between them.

(888.) The nebulous group of Sagittarius consists of several conspicuous nebulæ[13] of very extraordinary forms by no means easy to give an idea of by mere description. One of them (*h*, 1991[14]) is singularly trifid, consisting of three bright and irregularly formed nebulous masses, graduating away insensibly externally, but coming up to a great intensity of light at their interior edges, where they inclose and surround a sort of three-forked rift, or vacant area, abruptly and uncouthly crooked, and quite void of nebulous light. A beautiful triple star is situated precisely on the edge of one of these nebulous masses just where the interior vacancy forks out into two channels. A fourth nebulous mass spreads like a fan or downy plume from a star at a little distance from the triple nebula.

(889.) Nearly adjacent to the last described nebula, and no doubt connected with it, though the connection has not yet been traced, is situated the 8th nebula of Messier's Catalogue. It is a collection of nebulous folds and masses, surrounding and including a number of oval dark vacancies, and in one place coming up to so great a degree of brightness, as to offer the appearance of an elongated nucleus. Superposed upon this nebula, and extending in one direction beyond its area, is a fine and rich cluster of scattered stars, which seem to have no connection with it, as the nebula does not, as in the region of Orion, show any tendency to congregate about the stars.

(890.) The 19th nebula of Messier's Catalogue, though

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[13] About R. A. 17h. 52m., N. P. D. 113° 1′, four nebulæ, No. 41 of Sir Wm. Herschel's 4th class, and Nos. 1, 2, 3, of his 5th all connected into one great complex nebula.—In R. A. 17h. 53m. 27s., N. P. D. 114° 21′, the 8th, and in 18h. 11m., 106° 15′, the 17th of Messier's Catalogue.

[14] This number refers to the catalogue of nebulæ in Phil. Trans. 1833. The reader will find figures of the several nebulæ of this group in that volume, plate iv. *fig.* 35, in the Author's "Results of Observations, etc., at the Cape of Good Hope." Plates i. *fig.* 1, and ii. *figs.* 1 and 2, and in Mason's Memoir in the collection of the American Phil. Soc., vol. vii. art. xiii.

some degrees remote from the others, evidently belongs to this group. Its form is very remarkable, consisting of two loops like capital Greek Omegas, the one bright, the other exceedingly faint, connected at their bases by a broad and very bright band of nebula, insulated within which by a narrow comparatively obscure border, stands a bright, resolvable knot, or what is probably a cluster of exceedingly minute stars. A very faint round nebula stands in connection with the upper or convex portion of the brighter loop.

(891.) The nebulous group of Cygnus consists of several large and irregular nebulæ, one of which passes through the double star *k* Cygni, as a long, crooked, narrow streak, forking out in two or three places. The others,[15] observed in the first instance by Sir W. Herschel and by the author of this work as separate nebulæ, have been traced into connection by Mr. Mason, and shown to form part of a curious and intricate nebulous system, consisting, 1st, of a long, narrow, curved, and forked streak, and, 2dly, of a cellular effusion of great extent, in which the nebula occurs intermixed with, and adhering to stars around the borders of the cells, while their interior is free from nebula, and almost so from stars.

(892.) The Magellanic clouds, or the nubeculæ (major and minor), as they are called in the celestial maps and charts, are, as their name imports, two nebulous or cloudy masses of light, conspicuously visible to the naked eye, in the southern hemisphere, in the appearance and brightness of their light not unlike portions of the Milky Way of the same apparent size. They are, generally speaking, round, or somewhat oval, and the larger, which deviates most

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[15] R. A. 20h. 49m. 20s., N. P. D. 58° 27′.

from the circular form, exhibits the appearance of an axis of light, very ill defined, and by no means strongly distinguished from the general mass, which seems to open out at its extremities into somewhat oval sweeps, constituting the preceding and following portions of its circumference. A small patch, visibly brighter than the general light around, in its following part, indicates to the naked eye the situation of a very remarkable nebula (No. 30 Doradûs of Bode's Catalogue), of which more hereafter. The greater nubecula is situated between the meridians of $4^h\ 40^m$ and $6^h\ 0^m$ and the parallels of 156° and 162° of N. P. D., and occupies an area of about 42 square degrees. The lesser, between the meridians[16] 0 28 and $1^h\ 15^m$ and the parallels of 162° and 165° N. P. D., covers about ten square degrees. Their degree of brightness may be judged of from the effect of strong moonlight, which totally obliterates the lesser, but not quite the greater.

(893.) When examined through powerful telescopes, the constitution of the nubeculæ, and especially of the nubecula major, is found to be of astonishing complexity. The general ground of both consists of large tracts and patches of nebulosity in every stage of resolution, from light, irresolvable with 18 inches of reflecting aperture, up to perfectly separated stars like the Milky Way, and clustering groups sufficiently insulated and condensed to come under the designation of irregular, and in some cases pretty rich clusters. But besides those, there are also nebulæ in abundance, both regular and irregular; globular clusters in every state of condensation; and objects of a nebulous character quite peculiar, and which have no analogue in any other region of the heavens. Such is the concentration of these

[16] It is laid down nearly an hour wrong in all the celestial charts and globes.

objects, that in the area occupied by the nubecula major, not fewer than 278 nebulæ and clusters have been enumerated, besides 50 or 60 outliers, which (considering the general barrenness in such objects of the immediate neighborhood) ought certainly to be reckoned as its appendages, being about $6\frac{1}{2}$ per square degree, which very far exceeds the average of any other, even the most crowded parts of the nebulous heavens. In the nubecula minor, the concentration of such objects is less, though still very striking, 37 having been observed within its area, and 6 adjacent, but outlying. The nubeculæ, then, combine, each within its own area, characters which in the rest of the heavens are no less strikingly separated—viz. those of the galactic and the nebular system. Globular clusters (except in one region of small extent) and nebulæ of regular elliptic forms are comparatively rare in the Milky Way, and are found congregated in the greatest abundance in a part of the heavens, the most remote possible from that circle; whereas, in the nubeculæ, they are indiscriminately mixed with the general starry ground, and with irregular though small nebulæ.

(894.) This combination of characters, rightly considered, is in a high degree instructive, affording an insight into the probable comparative distance of *stars* and *nebulæ*, and the real brightness of individual stars as compared one with another. Taking the apparent semidiameter of the nubecula major at 3°, and regarding its solid form as, roughly speaking, spherical, its nearest and most remote parts differ in their distance from us by a little more than a tenth part of our distance from its centre. The brightness of objects situated in its nearer portions, therefore, cannot be *much* exaggerated, nor that of its remoter *much* enfeebled, by their difference of distance; yet within this globular space we have collected upward of 600 stars of the 7th, 8th, 9th, and 10th magnitudes, nearly 300 nebulæ, and globular and other clusters, *of all degrees of resolubility*, and smaller scattered stars innumerable of every inferior magnitude, from the 10th to such as by their multitude and minuteness constitute irresolvable nebulosity, extending over tracts of many square degrees. Were there but one such object, it might be maintained without utter improbability that its apparent sphericity is only an effect of foreshortening, and that in reality a much greater proportional difference of distance between its nearer and more remote parts exists. But such an adjustment, improbable enough in one case, must be rejected as too much so for fair argument in two. It must, therefore, be taken as a demonstrated fact, that stars of the 7th or 8th magnitude and irresolvable nebula may co-exist within limits of distance not differing in proportion more than as 9 to 10, a conclusion which must inspire some degree of caution in admitting, *as certain*, many of the consequences which have been rather strongly dwelt upon in the foregoing pages.

(895.) Immediately preceding the centre of the nubecula minor, and undoubtedly belonging to the same group, occurs the superb globular cluster, No. 47 Toucani of Bode, very visible to the naked eye, and one of the finest objects of this kind in the heavens. It consists of a very condensed spherical mass of stars, *of a pale rose color*, concentrically inclosed in a much less condensed globe of white ones, 15′ or 20′ in diameter. This is the first in order of the list of such clusters in art. 867.

(896.) Within the nubecula major, as already mentioned, and faintly visible to the naked eye, is the singular nebula (marked as the star 30 Doradûs in Bode's Catalogue) noticed by Lacaille as resembling the nucleus of a small comet. It occupies about one-500th part of the whole area of the nubecula, and is so satisfactorily represented in Plate V. *fig.* 1, as to render further description superfluous. See art. 896 *a*, Note K.

(897.) We shall conclude this chapter by the mention of two phenomena, which seem to indicate the existence of some slight degree of nebulosity about the sun itself, and even to place it in the list of nebulous stars. The first is that called the zodiacal light, which may be seen any very clear evening soon after sunset, about the months of March, April, and May, or at the opposite seasons before sunrise, as a cone or lenticularly-shaped light, extending from the horizon obliquely upward, and following generally the course of the ecliptic, or rather that of the sun's equator. The apparent angular distance of its vertex from the sun varies, according to circumstances, from 40° to 90°, and the breadth of its base perpendicular to its axis from 8° to 30°. It is extremely faint and ill defined, at least in this climate, though better seen in tropical regions, but cannot be mistaken for any atmospheric meteor or aurora borealis. It is manifestly in the nature of a lenticularly-formed envelope,[17] surrounding the sun, and extending beyond the orbits of Mercury and Venus, and nearly, perhaps quite, attaining that of the earth, since its vertex has been seen fully 90° from the sun's place in a great circle. It may be conjectured to be no other than the denser part of that medium,

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[17] I cannot imagine upon what grounds Humboldt persists in ascribing to it the form of a ring encircling the sun. For a most elaborate series of observations of the zodiacal light, by the Rev. G. Jones, see "United States Japan Expedition," vol. iii. 4°. Washington, 1856. It contains 357 plates of its appearance.

which, we have some reason to believe, resists the motion of comets; loaded, perhaps, with the actual materials of the tails of millions of those bodies, of which they have been stripped in their successive perihelion passages (art. 566). An *atmosphere* of the sun, in any proper sense of the word, it cannot be, since the existence of a gaseous envelope propagating pressure from part to part; subject to mutual friction in its strata, and therefore rotating in the same or nearly the same time with the central body; and of such dimensions and ellipticity, is utterly incompatible with dynamical laws. If its particles have inertia, they must necessarily stand with respect to the sun in the relation of separate and independent minute planets, each having its own orbit, plane of motion, and periodic time. The total mass being almost nothing compared to that of the sun, mutual *perturbation* is out of the question, though *collisions* among such as may cross each other's paths may operate in the course of indefinite ages to effect a subsidence of at least some portion of it into the body of the sun or those of the planets.

(898.) Nothing prevents that these particles, or some among them, may have some tangible size, and be at very great distances from each other. Compared with planets visible in our most powerful telescopes, rocks and stony masses of great size and weight would be but as the impalpable dust which a sunbeam renders visible as a sheet of light when streaming through a narrow chink into a dark chamber. It is a fact, established by the most indisputable evidence, that stony masses and lumps of iron do occasionally, and indeed by no means infrequently, fall upon the earth from the higher regions of our atmosphere (where it is obviously impossible they can have been generated), and that they have done so from the earliest times of history. Four instances are recorded of persons being killed by their fall. A block of stone fell at Ægos Potamos, B.C. 465, as large as two millstones; another at Narni, in 921, projected, like a rock, four feet above the surface of the river, into which it was seen to fall. The emperor Jehangire had a sword forged from a mass of meteoric iron which fell, in 1620, at Jalandar, in the Punjab.[18] Sixteen instances of the fall of stones in the British Isles are well authenticated to have occurred since 1620, one of them in London. In 1803, on the 26th of April, thousands of stones were scattered by the explosion into fragments of a large fiery globe over a region of twenty or thirty square miles around the town of L'Aigle, in Normandy. The fact occurred at mid-day, and the circumstances were officially verified by a commission of the French government.[19] These, and innumerable other instances,[20] fully establish the general fact; and after vain attempts to account for it by volcanic projection, either from the earth or the moon, the planetary nature of these bodies seems at length to be almost generally admitted. The heat which they possess when fallen, the igneous phenomena which accompany them, their explosion on arriving within the denser regions of our atmosphere, etc., are all sufficiently accounted for on physical principles, by the condensation of the air before them in consequence of their enormous velocity, and by the relations of air in a highly attenuated state to heat.[21]

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[18] See the emperor's own very remarkable account of the occurrence, translated in Phil. Trans. 1793, p. 202.

[19] See M. Biot's report in Mém. de l'Institut. 1806.

[20] See a list of upward of 200, published by Chladni, Annales du Bureau des Longitudes de France, 1825.

[21] Edinburgh Review, Jan. 1848, p. 195. It is very remarkable that no new chemical element has been detected in any of the numerous meteorolites which have been subjected to analysis.

(899.) Besides stony and metallic masses, however, it is probable that bodies of very different natures, or at least states of aggregation, are thus circulating round the sun. Shooting stars, often followed by long trains of light, and those great fiery globes, of more rare, but not *very* uncommon occurrence, which are seen traversing the upper regions of our atmosphere, sometimes leaving trains behind them remaining for many minutes, sometimes bursting with a loud explosion, sometimes becoming quietly extinct, may not unreasonably be presumed to be bodies extraneous to our planet, which only become visible on becoming ignited in the act of grazing the confines of our atmosphere. Among the last-mentioned meteors, however, are some which can hardly be supposed solid masses. The remarkable meteor of August 18, 1783, traversed the whole of Europe, from Shetland to Rome, with a velocity of about 30 miles per second, at a height of 50 miles from the surface of the earth, with a light greatly surpassing that of the full moon, and a real diameter of fully half a mile. Yet with these vast dimensions, it made a sudden bend in its course; it changed its form visibly, and at length quietly separated into several distinct bodies, accompanying each other in parallel courses, and each followed by a tail or train.

(900.) There are circumstances in the history of shooting stars, which very strongly corroborate the idea of their extraneous or *cosmical* origin, and their circulation round the sun in definite orbits. On several occasions they have been observed to appear in unusual, and, indeed, astonishing numbers, so as to convey the idea of a shower of rockets, and brilliantly illuminating the whole heavens for hours together, and that not in one locality, but over whole continents and oceans, and even (in one instance) in both hemispheres. Now it is extremely remarkable that, whenever this great display has been exhibited (at least in modern times), it has uniformly happened on the night between the 12th and 13th, or on that between the 13th and 14th of November. Such cases occurred in 1799, 1832, 1833, and 1834. On tracing back the records of similar phenomena, it has been ascertained, moreover, that more often those identical nights, but sometimes those immediately adjacent, have been, time out of mind, habitually signalized by such exhibitions. Another annually recurring epoch, in which, though far less brilliant, the display of meteors is more *certain* (for that of November is often interrupted for a great many years), is that of the 10th of August, on which night, and on the 9th and 11th, numerous, large, and bright shooting stars, with trains, are almost sure to be seen. Other epochs of periodic recurrence, less marked than the above, have also been to a certain extent established.

(901.) It is impossible to attribute such a recurrence of identical dates of very remarkable phenomena to accident. Annual periodicity, irrespective of geographical position, refers us at once to the place occupied by the earth in its annual orbit, and leads direct to the conclusion that at that place the earth incurs a liability to *frequent* encounters or concurrences with a stream of meteors in their progress of circulation round the sun. Let us test this idea by pursuing it into some of its consequences. In the first place then, supposing the earth to plunge, in its yearly circuit, into a uniform *ring* of innumerable small meteor-planets, of such breadth as would be traversed by it in one or two days; since during this small time the motions, whether of the earth or of each individual meteor, may be taken as uniform and rectilinear, and those of all the latter (at the place and time) parallel, or very nearly so, it will follow that the relative motion of the meteors referred to the earth as at rest, will be also uniform, rectilinear, and *parallel.* Viewed, therefore, from the centre of the earth (or from any point in its circumference, if we neglect the diurnal velocity as very small compared with the annual) they will all appear to diverge from a common point, *fixed in relation to the celestial sphere*, as if emanating from a sidereal apex (art. 115).

(902.) Now this is precisely what actually happens. The meteors of the 12th–14th of November, or at least the vast majority of them, describe apparently arcs of great circles, passing through or near γ Leonis. No matter what the situation of that star with respect to the horizon or to its east and west points may be at the time of observation, the paths of the meteors all appear to diverge from that star. On the 9th–11th of August the geometrical fact is the same, the apex only differing; B Camelopardali being for that epoch the point of divergence. As we need not suppose the meteoric ring coincident in its plane with the ecliptic, and as for a *ring* of meteors we may substitute an elliptic annulus of any reasonable excentricity, so that both the velocity and direction of each meteor may differ to any extent from the earth's, there is nothing in the great and obvious difference *in latitude* of these apices at all militating against the conclusion.

(903.) If the meteors be uniformly distributed in such a ring or elliptic annulus, the earth's encounter with them in every revolution will be certain, if it occur once. But if the ring be broken, if it be a succession of groups revolving in an ellipse in a period *not* identical with that of the earth, years may pass without a rencounter; and when such happen, they may differ to any extent in their intensity of character, according as richer or poorer groups have been encountered.

(904.) No other plausible explanation of these highly characteristic features (the annual periodicity and divergence from a common apex, *always alike for each respective epoch*) has been even attempted, and accordingly the opinion is generally gaining ground among astronomers that shooting stars belong to their department of science, and great interest is excited in their observation and the further development of their laws. The first connected and systematic series of observations of them, having for their object to trace out their *relative* paths with respect to the earth, are those of Benzenberg and Brandes, who, by noting the instants and apparent places of appearance and extinction, as well as the precise apparent paths among the stars, of individual meteors, from the extremities of a measured base line nearly 50,000 feet in length, were led to conclude that their heights at the instant of their appearance and disappearance vary from 16 miles to 140, and their relative velocities from 18 to 36 miles per second, velocities so great as clearly to indicate an independent planetary circulation round the sun. [A very remarkable meteor or bolide, which appeared on the 19th August, 1847, was observed at Dieppe and at Paris, with sufficient precision to admit of calculation of the elements of its orbit in absolute space. This calculation has been performed by M. Petit, director of the observatory of Toulouse, and the following *hyperbolic* elements of its orbit *round the sun* are stated by him (Astr. Nachr. 701) as its result; viz. Semimajor axis $= -0{\cdot}3240083$; excentricity $= 3{\cdot}95130$; perihelion distance $= 0{\cdot}95626$; inclination to plane of the earth's equator, $18^\circ\ 20'\ 18''$; ascending node on the same plane, $10^\circ\ 34'\ 48''$; motion direct. According to this calculation, the body would have occupied no less than 37340 years in travelling from the distance of the nearest fixed star supposed to have a parallax of 1″.]

(905.) It is by no means inconceivable that the earth approaching to such as differ but little from it in direction and velocity, may have attached many of them to it as permanent satellites, and of these there *may* be some so large, and of such texture and solidity, as to shine by reflected light, and become visible (such, at least, as are very near the earth) for a brief moment, suffering extinction by plunging into the earth's shadow; in other words, undergoing total eclipse. Sir John Lubbock is of opinion that such is the case, and has given geometrical formulæ for calculating their distances from observations of this nature.[22] The observations of M. Petit would lead us to believe in the existence of at least one such body, revolving round the earth, as a satellite, in about 3 hours 20 minutes, and therefore at a distance equal to 2·513 radii of the earth from its centre, or 5,000 miles above its surface.[23] (See Note N.)

(905 *a*.) In art. 400 the generation of heat by friction is suggested as affording a possible explanation of the supply of solar heat, without actual combustion. A very old doctrine, advocated on grounds anything rather than reasonable or even plausible by Bacon, but afterward worked into a circumstantial and elaborate theory by the elder Seguin, which makes heat to consist in a continual, rapid, vibratory or gyratory motion of the particles of bodies, has of late been put forward into great prominence by Messrs. Mayer and Joule and Sir W. Thomson; according to this theory motion once generated or however originating, is never

[22] Phil. Mag., Lond. Ed. Dub. 1848, p. 80.
[23] Comptes Rendus, Oct. 12, 1846, and Aug. 9, 1847.

destroyed, but continues to subsist in the form of "vis viva" among the molecules of bodies, even when by their impact or mutual obstruction they *appear* to have been brought to rest.[24] The "vis viva" only takes another form, and is disseminated, as increased vibratory or gyratory movement, among their molecules; as such it is heat, or light, or both, and is communicated to the molecules of the luminiferous ether, and so distributed throughout that ether, constituting the phenomena of radiant light and heat. Granting a few postulates (not very easy of conception, and still less so of admission when conceived), this theory is not without its plausibility, and certainly does (on its own conventions) give a consistent account of the production of heat by friction and impact. It has been applied by Mr. Waterson and Sir W. Thomson to explain the evolution of solar light and heat, as follows. According to the former, the meteorolites which, revolving in very excentric or cometic orbits, arrive within the limits of the solar atmosphere are precipitated on the sun's surface in such abundance, and with such velocity, as to generate in the way above described the totality of the emitted radiants. Sir W. Thomson, undismayed as would appear by the perpetual battery thus kept up on the sun's surface (on every square foot of which, on Mr. Waterson's view of the subject, a weight of matter equal to 5 lbs. would require to be delivered per hour with a velocity of 390 miles per second, covering the whole surface with stony or other solid material, to the depth of 12 feet *per annum*, if of the density of granite), prefers to consider the nebula of the zodiacal light in a vaporous state as continually subsiding into the

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[24] On this point see a paper by the Author on the absorption of light, Lond. and Ed. Phil. Mag. and Journ., 3d series, vol. iii. No. 18, Dec. 1833.

sun, by gradual spiral approach, until suddenly meeting with greatly increased resistance in its atmosphere (as arriving in a state of more rapid revolution) by friction *on the external envelope or photosphere of its surface* (art. 389), produces there the heat and light actually observed; whereas the theory of Mr. Waterson would place its origin on the solid surface itself, contrary to the observed fact.[25] Our readers will judge for themselves what degree of support the telescopic aspect of the sun's surface as described in arts. 386–395, and especially 387 *a*, Note G, affords.

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[25] The quantity of matter annually required to be deposited on the sun, whether in a pulverulent, liquid, or vaporous form, by Sir W. Thomson's theory, is nearly double of that called for by Mr. Waterson's, viz. 24 feet of granite per annum, *i.e.* a mile in 260 years; so that the sun's apparent diameter would be increasing at the rate of about 1″ per 100,000 years on this hypothesis.

In the "Manuel de la Science, ou Annuaire du Cosmos" for 1859, by the Abbé Moigno (a work of high interest, and, generally speaking, of great impartiality in the discussion of claims to scientific priority), pp. 85, 6, 7, 2me *partie*, this article is so translated (probably for want of a perfect appreciation of the force of the expressions used in it) as to convey an unqualified adhesion to the theory in question and to M. Seguin's doctrine. This, however (especially the latter, as stated at length in Pt. I. pp. 224 *et seq.*), I am very far from prepared to give:—and the English reader will, I presume, consider the terms employed quite sufficiently guarded, even as respects the general principle; to say nothing of the specialties of M. Seguin's theory.—[*Note added*, 1859.]

PART IV

OF THE ACCOUNT OF TIME

CHAPTER XVIII

Natural Units of Time—Relation of the Sidereal to the Solar Day Affected by Precession—Incommensurability of the Day and Year—Its Inconvenience—How Obviated—The Julian Calendar—Irregularities at its first Introduction—Reformed by Augustus—Gregorian Reformation—Solar and Lunar Cycles—Indiction—Julian Period—Table of Chronological Eras—Rules for Calculating the Days Elapsed Between Given Dates—Equinoctial Time—Fixation of Ancient Dates by Eclipses

(906.) TIME, like distance, may be measured by comparison with standards of any length, and all that is requisite for ascertaining correctly the length of any interval, is to be able to apply the standard to the interval throughout its whole extent, without overlapping on the one hand, or leaving unmeasured vacancies on the other; to determine, without the possible error of a unit, the number of integer standards which the interval admits of being interposed between its beginning and end; and to estimate precisely the fraction, over and above an integer, which remains when all the possible integers are subtracted.

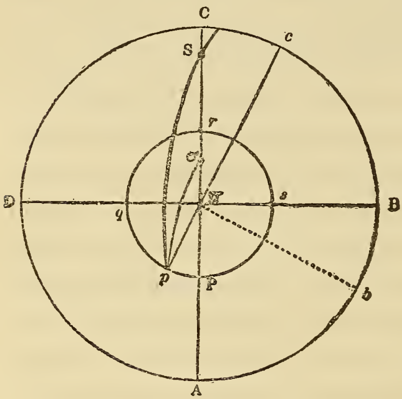
(907.) But though all standard units of time are equally possible, theoretically speaking, yet all are not, practically, equally convenient. The solar day is a natural interval which the wants and occupations of man in every state of society force upon him, and compel him to adopt as his fundamental unit of time. Its length as estimated from the departure of the sun from a given meridian, and its next return to the same, is subject, it is true, to an annual fluctuation in excess and defect of its mean value, amounting at its maximum to full half a minute. But except for astronomical purposes, this is too small a change to interfere in the slightest degree with its use, or to attract any attention, and the tacit substitution of its mean for its true (or variable) value may be considered as having been made from the earliest ages, by the ignorance of mankind that any such fluctuation existed.

(908.) The time occupied by one complete rotation of the earth on its axis, or the mean[1] sidereal day, may be shown, on dynamical principles, to be subject to no variation from any external cause, and although its duration would be shortened by contraction in the dimensions of the globe itself, such as might arise from the gradual escape of its internal heat, and consequent refrigeration and shrinking of the whole mass, yet theory, on the one hand, has rendered it almost certain that this cause cannot have effected any perceptible amount of change during the history of the human race; and, on the other, the comparison of ancient and modern observations affords every corroboration to this conclusion. From such comparisons, Laplace has concluded that the sidereal day has not changed by so much as one hundredth of a second since the time of Hipparchus. The mean sidereal day therefore possesses in perfection the essential quality of a standard unit, that of complete *invariability*. The same is true of the mean sidereal year, if estimated upon an average sufficiently large to compensate the minute fluctuations arising from the periodical variations of the major axis of the earth's orbit due to planetary perturbation (art. 668).

(909.) The mean solar day is an immediate derivative of

[1] The true sidereal day is variable by the effect of nutation; but the variation (an excessively minute fraction of the whole) compensates itself in a revolution of the moon's nodes.

the sidereal day and year, being connected with them by the same relation which determines the synodic from the sidereal revolutions of any two planets or other revolving bodies (art. 418). The *exact* determination of the ratio of the sidereal to the solar day, which is a point of the utmost importance in astronomy, is, however, in some degree, complicated by the effect of precession, which renders it necessary to distinguish between the absolute time of the earth's rotation on its axis (the real natural and invariable standard of comparison), and the *mean* interval between two successive returns of a given star to the same meridian, or rather of a given meridian to the same star, which not only differs by a minute quantity from the sidereal day, but is actually not the same for all stars. As this is a point to which a little difficulty of conception is apt to attach, it will be necessary to explain it in some detail. Suppose then π the pole of the ecliptic, and P that of the equinoctial, A C the solstitial colure at any given moment of time, and P p q r the small circle described by P about π in one revolution of the equinoxes, *i.e.* in 25870 years, or 9448300 solar days, all projected on the plane of the ecliptic A B C D. Let S be a star anywhere situated on the ecliptic, or *between* it and the small circle P q r. Then if the pole P were at rest, a meridian of the earth setting out from P S C, and revolving in the direction C D, will come again to the star after the exact lapse of one sidereal day, or one rotation of



the earth on its axis. But P is not at rest. After the lapse of one such day it will have come into the situation (suppose) p, the vernal equinox B having retreated to b, and the colure P C having taken up the new position $p\ c$. Now a conical movement impressed on the axis of rotation of a globe already rotating is equivalent to a rotation impressed on the whole globe round the axis of the cone, in addition to that which the globe has and retains round its own independent axis of revolution. Such a new rotation, in transferring P to p, being performed round an axis passing through π, will not alter the situation of that point of the globe which has π in its zenith. Hence it follows that $p\ \pi\ c$ passing through π will be the position taken up by the meridian P π C after the lapse of an exact sidereal day. But this does not pass through S, but falls short of it by the hour-angle $\pi\ p$ S, which is yet to be described before the meridian comes up to the star. The meridian, then, has lost so much on, or lagged so much behind, the star in the lapse of that interval. The same is true whatever be the arc P p. After the lapse of any number of days, the pole being transferred to p, the spherical angle $\pi\ p$ S will measure the total hour angle which the meridian has lost on the star. Now when S lies anywhere between C and r, this angle continually increases (though not uniformly), attaining 180° when p comes to r, and still (as will appear by following out the movement beyond r) increasing thence till it attains 360° when p has completed its circuit. Thus in a whole revolution of the equinoxes, the meridian will have lost one exact revolution upon the star, or in 9448300 sidereal days will have reattained the star only 9448299 times: in other words, the length of the day measured by the mean of the successive arrivals of any star outside of the circle P $p\ q\ r$

on one and the same meridian is to the absolute time of rotation of the earth on its axis as 9448300 : 9448299, or as 1·00000011 to 1.

(910.) It is otherwise of a star situated *within* this circle, as at σ. For such a star the angle $\pi\ p\ \sigma$, expressing the lagging of the meridian, increases to a maximum for some situation of p between q and r, and decreases again to o at r; after which it takes an opposite direction, and the meridian begins to get in advance of the star, and continues to get more and more so, till p has attained some point between s and P, where the advance is a maximum, and thence decreases again to o when p has completed its circuit. For any star so situated, then, the mean of all the days so estimated through a whole period of the equinoxes is an absolute sidereal day, as if precession had no existence.

(911.) If we compare the sun with a star situated in the ecliptic, the sidereal year is the mean of all the intervals of its arrival at that star throughout indefinite ages, or (without fear of sensible error) throughout recorded history. Now, if we would calculate the synodic sidereal revolution of the sun and of a meridian of the earth *by reference to a star so situated*, according to the principles of art. 418, we must proceed as follows: Let D be the length of the mean solar day (or synodic day in question), d the mean sidereal revolution of the meridian *with reference to the same star*, and y the sidereal year. Then the arcs described by the sun and the meridian in the interval D will be respectively $360° \frac{D}{y}$ and $360° \frac{D}{d}$. And since the latter of these exceeds the former by precisely 360°, we have

$$360° \frac{D}{d} = 360° \frac{D}{y} + 360°;$$

whence it follows that

$$\frac{D}{d} = 1 + \frac{D}{y} = 1{\cdot}00273780,$$

taking the value of the sidereal year y as given in art. 383, viz. $365^d\ 6^h\ 9^m\ 9{\cdot}6^s$. But, as we have seen, d is not the absolute *sidereal day*, but exceeds it in the ratio 1·00000011 : 1. Hence to get the value of the mean solar day, as expressed in absolute sidereal days, the number above set down must be increased in the same ratio, which brings it to 1·00273791, which is the ratio of the solar to the sidereal day actually in use among astronomers.

(912.) It would be well for chronology if mankind would, or could have contented themselves with this one invariable, natural, and convenient standard in their reckoning of time. The ancient Egyptians did so, and by their adoption of a historical and official year of 365 days have afforded the only example of a practical chronology, free from all obscurity or complication. But the return of the seasons, on which depend all the more important arrangements and business of cultivated life, is not conformable to such a multiple of the diurnal unit. Their return is regulated by the *tropical year*, or the interval between two successive arrivals of the sun at the vernal equinox, which, as we have seen (art. 383), differs from the sidereal year by reason of the motion of the equinoctial points. Now this motion is not absolutely unifom, because the ecliptic, upon which it is estimated, is gradually, though very slowly, changing its situation in space under the disturbing influence of the planets (art. 640). And thus arises a variation in the *tropical* year, which is dependent on the place of the equinox (art 383). The *tropical* year is actually about $4{\cdot}21^s$

shorter than it was in the time of Hipparchus. This absence of the most essential requisite for a standard, viz. invariability, renders it necessary, since we cannot help employing the tropical year in our reckoning of time, to adopt an arbitrary or artificial value for it, so near the truth as not to admit of the accumulation of its error for several centuries producing any practical mischief, and thus satisfying the ordinary wants of civil life; while, for scientific purposes, the tropical year, so adopted, is considered only as the representative of a certain number of integer days and a fraction—the day being, in effect, the only standard employed. The case is nearly analogous to the reckoning of value by guineas and shillings, an artificial relation of the two coins being fixed by law, near to, but scarcely ever exactly coincident with, the natural one, determined by the relative market price of gold and silver, of which either the one or the other—whichever is really the most invariable, or the most in use with other nations—may be assumed as the true theoretical standard of value.

(913.) The other inconvenience of the tropical year as a greater unit is its incommensurability with the lesser. In our measure of space all our subdivisions are into aliquot parts: a yard is three feet, a mile eight furlongs, etc. But a year is no *exact* number of days, nor an integer number with any exact fraction, as one-third or one-fourth, over and above; but the surplus is an *incommensurable* fraction, composed of hours, minutes, seconds, etc., which produces the same kind of inconvenience in the reckoning of time that it would do in that of money, if we had gold coins of the value of twenty-one shillings, with odd pence and farthings, and a fraction of a farthing over. For this, however, there is no remedy but to keep a strict register of the surplus fractions; and, when they amount to a whole day, cast them over into the integer account.

(914.) To do this in the simplest and most convenient manner is the object of a well-adjusted calendar. In the Gregorian calendar, which we follow, it is accomplished with considerable simplicity and neatness, by carrying a little further than is done above, the principle of an assumed or artificial year, and adopting *two* such years, both consisting of an exact integer number of days, viz. one of 365 and the other of 366, and laying down a simple and easily remembered rule for the order in which these years shall succeed each other in the civil reckoning of time, so that during the lapse of at least some thousands of years the sum of the integer artificial, or Gregorian, years elapsed shall not differ from the same number of real tropical years by a whole day. By this contrivance, the equinoxes and solstices will always fall on days similarly situated, and bearing the same name in each Gregorian year; and the seasons will forever correspond to the same months, instead of running the round of the whole year, as they must do upon any other system of reckoning, and used, in fact, to do before this was adopted, as a matter of ignorant haphazard in the Greek and Roman chronology, and of strictly defined and superstitiously rigorous observance in the Egyptian.

(915.) The Gregorian rule is as follows:—The years are denominated *as years current* (*not as years elapsed*) from the midnight between the 31st of December and the 1st of January immediately subsequent to the birth of Christ, according to the chronological determination of that event by Dionysius Exiguus. Every year whose number is not divisible by 4 without remainder, consists of 365 days; every year which *is* so divisible, but is not divisible by 100, of

366; every year divisible by 100, but not by 400, again of 365; and every year divisible by 400, again of 366. For example, the year 1833, not being divisible by 4, consists of 365 days; 1836 of 366; 1800 and 1900 of 365 each; but 2000 of 366. In order to see how near this rule will bring us to the truth, let us see what number of days 10000 Gregorian years will contain, beginning with the year A.D. 1. Now, in 10000, the numbers not divisible by 4 will be $\frac{3}{4}$ of 10000 or 7500; those divisible by 100, but not by 400, will in like manner be $\frac{3}{4}$ of 100, or 75; so that, in the 10000 years in question, 7575 consist of 365, and the remaining 2425 of 366, producing in all 3652425 days, which would give for an average of each year, one with another, $365^{d}\cdot2425$. The actual value of the tropical year (art. 383), reduced into a decimal fraction, is 365·24224, so the error in the Gregorian rule on 10000 of the present tropical years, is 2·6, or $2^{d}\ 14^{h}\ 24^{m}$; that is to say, less than a day in 3000 years; which is more than sufficient for all human purposes, those of the astronomer excepted, who is in no danger of being led into error from this cause. Even this error is avoided by extending the wording of the Gregorian rule one step further than its contrivers probably thought it worth while to go, and declaring that years divisible by 4000 should consist of 365 days. This would take off two integer days from the above calculated number, and 2·5 from a larger average; making the sum of days in 100000 Gregorian years, 36524225, which differs only by a single day from 100000 real tropical years, such as they exist at present.[2]

(916.) In the historical dating of events there is no year A.D. 0. The year immediately previous to A.D. 1 is always called B.C. 1. This must always be borne in mind in

[2] See note A at the end of the chapter.

reckoning chronological and astronomical intervals. The sum of the nominal years B.C. and A.D. must be diminished by 1. Thus, from January 1, B.C. 4713, to January 1, A.D. 1582, the years elapsed are not 6295, but 6294.

(917.) As any distance along a highroad might, though in a rather inconvenient and roundabout way, be expressed without introducing error by setting up a series of milestones, at intervals of unequal lengths, so that every fourth mile, for instance, should be a yard longer than the rest, or according to any other fixed rule; taking care only to mark the stones so as to leave room for no mistake, and to advertise all travellers of the difference of lengths and their order of succession; so may any interval of time be expressed correctly by stating in what Gregorian years it begins and ends, *and whereabout in each.* For this statement coupled with the declaratory rule, enables us to say how many integer years are to be reckoned at 365, and how many at 366 days. The latter years are called bissextiles, or leap-years, and the surplus days thus thrown into the reckoning are called *intercalary or leap-days.*

(918.) If the Gregorian rule, as above stated, had always and in all countries been known and followed, nothing would be easier than to reckon the number of days elapsed between the present time, and any historical recorded event. But this is not the case; and the history of the calendar, with reference to chronology, or to the calculation of ancient observations, may be compared to that of a clock, going regularly when left to itself, but sometimes forgotten to be wound up; and when wound, sometimes set forward, sometimes backward, either to serve particular purposes and private interests, or to rectify blunders in setting. Such, at least, appears to have been the case with the Roman calendar, in which our own originates, from the time of Numa to that of Julius Cæsar, when the lunar year of 13 months, or 355 days, was augmented at pleasure to correspond to the solar, by which the seasons are determined, by the arbitrary intercalations of the priests, and the usurpations of the decemvirs and other magistrates, till the confusion became inextricable. To Julius Cæsar, assisted by Sosigenes, an eminent Alexandrian astronomer and mathematician, we owe the neat contrivance of the two years of 365 and 366 days, and the insertion of one bissextile after three common years. This important change took place in the 45th year before Christ, which he ordered to commence on the 1st of January, *being the day of the new moon immediately following the winter solstice of the year before.* We may judge of the state into which the reckoning of time had fallen, by the fact, that to introduce the new system it was necessary to enact that the previous year, 46 B.C., should consist of 445 days, a circumstance which obtained for it the epithet of "the year of confusion."

(919.) Had Cæsar lived to carry out into practical effect, as Chief Pontiff, his own reformation, an inconvenience would have been avoided, which at the very outset threw the whole matter into confusion. The words of his edict establishing the Julian system have not been handed down to us, but it is probable that they contained some expression equivalent to "every fourth year," which the priests misinterpreting after his death to mean (according to the sacerdotal system of numeration) as *counting the leap year newly elapsed as* No. 1, *of the four*, intercalated *every third* instead of every 4th year. This erroneous practice continued during 36 years, in which therefore 12 instead of 9 days were intercalated, and an error of three days produced; to rectify which, Augustus ordered the suspension of all intercalation during three complete *quadriennia*—thus restoring, as may be presumed his intention to have been, the Julian dates for the future, and re-establishing the Julian system, which was never afterward vitiated by any error, till the epoch when its own inherent defects gave occasion to the Gregorian reformation. According to the Augustan reform the years A.U.C. 761, 765, 769, etc., which we now call A.D. 8, 12, 16, etc., are leap years. And starting from *this* as a certain fact (for the statements of the transaction by classical authors are not so precise as to leave *absolutely no doubt* as to the previous intermediate years), astronomers and chronologists have agreed to reckon backward in unbroken succession on this principle, and thus to carry the Julian chronology into past time, *as if* it had never suffered such interruption, and *as if* it were certain[3] that Cæsar, by way of securing the intercalation as a matter of precedent, made his initial year 45 B.C. a leap year. Whenever, therefore, in the relation of any event, either in ancient history, or in modern, previous to the change of style, the time is specified in our modern nomenclature, it is always to be understood as having been identified with the assigned date by threading the mazes (often very tangled and obscure ones, of special and national chronology, and referring the day of its occurrence to its place in the Julian system *so interpreted.*

(920.) Different nations in different ages of the world have of course reckoned their time in different ways, and

[3] Scaliger, Ideler, and the best authorities consider it probable. A strong, if not decisive, argument in its favor, is that Augustus *evidently intending to reinstate the Julian idea,* and with a clear view of the recent inconveniences present to his mind, did actually direct the future intercalations to take place in *odd* years U.C. Such then, no doubt, must have been Cæsar's intention. For the correction of Roman dates during the fifty-two years between the Julian and Augustan reformations, see Ideler, "Handbuch der Mathematischen und Technischen Chronologie," which we take for our guide throughout this chapter.

from different epochs, and it is therefore a matter of great convenience that astronomers and chronologists (as they have agreed on the uniform adoption of the Julian system of years and months) should also agree on an epoch antecedent to them all, to which, as to a fixed point in time, the whole list of chronological eras can be differentially referred. Such an epoch is the noon of the 1st of January, B.C. 4713, which is called the epoch of the Julian period, a cycle of 7980 Julian years, to understand the origin of which, we must explain that of three subordinate cycles, from whose combination it takes its rise, by the multiplication together of the numbers of years severally contained in them; viz. the Solar and Lunar cycles, and that of the indictions.

(921.) The Solar cycle consists of 28 Julian years, after the lapse of which the same days of the week on the Julian system would always return to the same days of each month throughout the year. For four such years consisting of 1461 days, which is not a multiple of 7, it is evident that the least number of years which will fulfil this condition must be seven times that interval, or 28 years. The place in this cycle for any year A.D., as 1849, is found by adding 9 to the year, and dividing by 28. The remainder is the number sought, 0 being counted as 28.

(922.) The Lunar cycle consists of 19 years or 235 lunations, which differ from 19 Julian years of 365¼ days only by about an hour and a half, so that, supposing the new moon to happen on the first of January, in the first year of the cycle, it will happen on that day (or within a very short time of its beginning or ending) again after a lapse of 19 years; and all the new moons in the interval will run on the same days of the month as in the preceding cycle. This period of 19 years is sometimes called the Metonic cycle, from its discoverer Meton, an Athenian mathematician, a discovery duly appreciated by his countrymen, as insuring the correspondence between the lunar and solar years, the former of which was followed by the Greeks. Public honors were decreed to him for this discovery, a circumstance very expressive of the annoyance which a lunar year of necessity inflicts on a civilized people, to whom a regular and simple calendar is one of the first necessities of life. A cycle of $4 \times 19 = 76$ years was proposed by Callippus as a supposed improvement on the Metonic, but in this interval the errors accumulated to 6 hours and in $4 \times 76 = 304$ years to an entire day. To find the place of a given year in the lunar cycle (or as it is called the Golden Number), add 1 to the number of the year A.D., and divide by 19, the remainder (or 19 if exactly divisible), is the Golden Number.

(923.) The cycle of the indictions is a period of 15 years used in the courts of law, and in the fiscal organization of the Roman empire, under Constantine and his successors, and thence introduced into legal dates, as the Golden Number, serving to determine Easter, was into ecclesiastical ones. To find the place of a year in the indiction cycle, add 3 and divide by 15. The remainder (or 15 if 0 remain) is the number of the indictional year.

(924.) If we multiply together the numbers 28, 19, and 15, we get 7980, and, therefore, a period or cycle of 7980 years will bring round the years of the three cycles again in the same order, so that each year shall hold the same place in all the three cycles as the corresponding year in the foregoing period. As none of the three numbers in question have any common factor, it is evident that no two years in the same compound period can agree in all the three particulars: so that to specify the numbers of a year in each of these cycles is, in fact, to specify the year, if within that long period; which embraces the entire of authentic chronology. The period thus arising of 7980 Julian years, is called the Julian period, and it has been found so useful, that the most competent authorities have not hesitated to declare that, through its employment, light and order were first introduced into chronology.[4] We owe its invention or revival to Joseph Scaliger, who is said to have received it from the Greeks of Constantinople. The first year of the current Julian period, or that of which the number in each of the three subordinate cycles is 1, was the year 4713 B.C., and the noon of the 1st of January of that year, for the meridian of Alexandria, is the chronological epoch, to which all historical eras are most readily and intelligibly referred, by computing the number of integer days intervening between that epoch and the noon (for Alexandria) of the day, which is reckoned to be the first of the particular era in question. The meridian of Alexandria is chosen as that to which Ptolemy refers the commencement of the era of Nabonassar, the basis of all his calculations.

(925.) Given the year of the Julian period, those of the subordinate cycles are easily determined as above. Conversely, given the years of the solar and lunar cycles and of the indiction, to determine the year of the Julian period proceed as follows:—Multiply the number of the year in the solar cycle by 4845, in the lunar by 4200, and in the Cycle of the Indictions by 6916, divide the sum of the products by 7980, and the remainder is the year of the Julian period sought.

(926.) The following table contains these intervals for some of the more important historical eras:—

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[4] Ideler, Handbuch, etc., vol. i. p. 37.

Intervals in Days between the Commencement of the Julian Period, and that of some other remarkable chronological and astronomical Eras[5]

Names by which the Era is usually cited	First day current of the Era	Chronological Designation of the Year	Current Year of the Julian Period	Interval, days elapsed. For days current add 1
Julian Epochs	*Julian Dates*	B.C.		
Julian period	Jan. 1.	4713	1	0
Creation of the world (Usher) . .	(Jan. 1.)	4004	710	258,963
Era of the Deluge (Aboulhassan Kuschiar)	Feb. 18.	3102	1612	588,466
Ditto Vulgar Computation . . .	(Jan. 1.)	2348	2366	863,817
Era of Abraham (Sir H. Nicolas) .	Oct. 1.	2015	2699	985,718
Destruction of Troy, (ditto) . .	July 12.	1184	3530	1,289,160
Dedication of Solomon's Temple .	(May 1.)	1015	3699	1,350,815
Olympiads (mean epoch in general use)	July 1.	776	3938	1,438,171
Building of Rome (Varronian epoch, U.C.)	April 22.	753	3961	1,446,502
Era of Nabonassar	Feb. 26.	747	3967	1,448,638
Eclipse of Thales	May 28.	585	4129	1,507,900
Eclipse of Larissa	May 19.	557	4157	1,518,118
Metonic cycle (Astronomical epoch) .	July 15.	432	4282	1,563,831
Callippic cycle Do. (Biot) . .	June 28.	330	4384	1,599,608
Philippic era, or era of Philip Aridæus	Nov. 12.	324	4390	1,603,398
Era of the Seleucidæ	Oct. 1.	312	4402	1,607,739
Eclipse of Agathocles	Aug. 15.	310	4404	1,608,422
Cæsarean era of Antioch . . .	Sept. 1.	49	4665	1,703,770
Julian reformation of the Calendar .	Jan. 1.	45	4669	1,704,987
Spanish era	Jan. 1.	38	4676	1,707,544
Actian era in Rome	Jan. 1.	30	4684	1,710,466
Actian era of Alexandria	Aug. 29.	30	4684	1,710,706
Vulgar or Dionysian era	Jan. 1.	A.D. 1	4714	1,721,424
Era of Diocletian	Aug. 29.	284	4997	1,825,030
Hejira (astronomical epoch, new moon)	July 15.	622	5335	1,948,439
Era of Yezdegird	June 16.	632	5345	1,952,063
Eclipse of Sticklastad	Aug. 31.	1030	5743	2,097,508
Gelalæan era (Sir H. Nicolas) . .	March 14.	1079	5792	2,115,235
Last day of Old Style (Catholic nations)	Oct. 4.	1582	6295	2,299,160
Last day of Old Style in England .	Sept. 2.	1752	6465	2,361,221
Gregorian Epochs	*Gregorian Dates*			
New Style in Catholic nations . .	Oct. 15.	1582	6295	2,299,161
Ditto in England	Sept. 14.	1752	6465	2,361,222
Commencement of the 19th century. Epoch of Bode's catalogue of stars.	Jan. 1.	1801	6514	2,378,862
Epoch of the catalogue of stars of the R. Astronomical Society . .	Jan. 1.	1830	6543	2,389,454
Epoch of the catalogue of the British Association	Jan. 1.	1850	6563	2,396,759

[5] See note B at the end of this chapter.

(927.) The determination of the exact interval between any two given dates, is a matter of such importance, and, unless methodically performed, is so very liable to error, that the following rules will not be found out of place. In the first place it must be remarked, generally, that a date, whether of a day or year, always expresses the day or year *current* and not *elapsed*, and that the designation of a year by A.D. or B.C. is to be regarded as the *name* of that year, and *not as a mere number uninterruptedly designating the place of the year in the scale of time.* Thus in the date, Jan. 5, B.C. 1, Jan. 5 does not mean that 5 days of January in the year in question have elapsed, but that 4 have elapsed, and the 5th is current. And the B.C. 1, indicates that *the first day* of the year so named (the first year current before Christ), preceded the first day of the vulgar era by one year. The scale of A.D. and B.C., as already observed, is not continuous, the year 0 in both being wanting; so that (supposing the vulgar reckoning correct) our Saviour was born in the year B.C. 1.

(928.) *To find the year current of the Julian period* (J.P.) *corresponding to any given year current* B.C. *or* A.D. If B.C., subtract the number of the year from 4714; if A.D., add its number to 4713. For examples, see the foregoing table.

(929.) *To find the day current of the Julian period corresponding to any given date, Old Style.* Convert the year B.C. or A.D. into the corresponding year J.P. as above. Subtract 1 and divide the number so diminished by 4, and call Q the integer quotient, and R the remainder. Then will Q be the number of entire *quadriennia* of 1461 days each, and R the residual years, *the first of which is always a leap year.* Convert Q into days by the help of the first of the annexed tables, and R by the second, and the sum will be the interval between the Julian epoch, and the commencement, January 1, of the year. Then find the days intervening between the beginning of January 1, and that of the date-day by the third table, using the column for a leap year, where R=0, and that for a common year when R is 1, 2 or 3. Add the days so found to those in Q+R, and the sum will be, the days elapsed of the Julian period, the number of which increased by 1 gives the day current.

TABLE 1.—Multiples of 1461, the days in a Julian *Quadriennium*

1	1461	4	5844	7	10227
2	2922	5	7305	8	11688
3	4383	6	8766	9	13149

TABLE 2.—Days in Residual years

0	0
1	366
2	731
3	1096

TABLE 3.—Days elapsed from Jan. 1, to the 1st of each Month

	In a common Year	In a leap Year		In a common Year	In a leap Year
Jan. 1 . .	0	0	July 1 . .	181	182
Feb. 1 . .	31	31	Aug. 1 . .	212	213
March 1 .	59	60	Sept. 1 . .	243	244
April 1 . .	90	91	Oct. 1 . .	273	274
May 1 . .	120	121	Nov. 1 . .	304	305
June 1 . .	151	152	Dec. 1 . .	334	335

EXAMPLE.—What is the current day of the Julian period corresponding to the last day of Old Style in England, on September 2, A.D. 1752.

1752		1000	1,461,000
4713		600	876,600
6465	year current.	10	14,610
1		6	8,766
4)6464	years elapsed.	R=0	0
Q=1616		Jan. 1. to Sept. 1.	244
R= 0		Sept. 1. to Sept. 2.	1
			2,361,221 days elapsed.
		Current day the	2,361,222[d].

(930.) To find the same for any given date, New Style. Proceed as above, considering the date as a Julian date, and disregarding the change of style. Then from the resulting days, subtract as follows:—

For any date of New Style, antecedent to March 1, A.D. 1700			.	10 days.
After Feb. 28, 1700 and before March 1, A.D. 1800			. . . .	11 days.
" 1800	"	" 1900	. . . .	12 days.
" 1900	"	" 2100	. . . .	13 days, etc.

(931.) *To find the interval between any two dates, whether of Old or New Style, or one of one, and one of the other.* Find the day current of the Julian period corresponding to each date, and their difference is the interval required. If the dates contain hours, minutes and seconds, they must be annexed to their respective days current, and the subtraction performed as usual.

(932.) The Julian rule made every fourth year, without exception, a bissextile, *beginning with* the year J. P. 1, *which is to be accounted as such.* This is, in fact, an over-correction; it supposes the length of the tropical year to be $365\frac{1}{4}^{d}$, which is too great, and thereby induces an error of 7 days in 900 years, as will easily appear on trial. Accordingly, so early as the year 1414, it began to be perceived that the equinoxes were gradually creeping away from the 21st of March and September, where they ought to have always fallen had the Julian year been exact, and happening (as it appeared) too early. The necessity of a fresh and effectual reform in the calendar was from that time continually urged, and at length admitted. The change (which took place under the popedom of Gregory XIII.) consisted in the omission of ten[6] nominal days after the 4th of October, 1582 (so that the next day was called the

[6] See note C at the end of this chapter.

15th, and not the 5th), and the promulgation of the rule already explained for future regulation. The change was adopted immediately in all Catholic countries; but more slowly in Protestant. In England, "the change of style," as it was called, took place after the 2d of September, 1752, eleven nominal days being then struck out; so that, the last day of Old Style being the 2d, the first of New Style (the next day) was called the 14th, instead of the 3d. The same legislative enactment which established the Gregorian year in England in 1752, shortened the preceding year, 1751, by a full quarter. *Previous to that time, the year was held to begin with the 25th March*, and the year A.D. 1751 did so accordingly; but that year was not suffered to run out, but was supplanted on the 1st January by the year 1752, which (as well as every subsequent year) it was enacted should commence on that day, so that our English year 1751 was in effect an "annus confusionis," and consisted of only 282 days. Russia is now the only country in Europe in which the Old Style is still adhered to, and (another secular year having elapsed) the difference between the European and Russian dates amounts, at present, to 12 days.

(933.) It is fortunate for astronomy that the confusion of dates, and the irreconcilable contradictions which historical statements too often exhibit, when confronted with the best knowledge we possess of the ancient reckonings of time, affect recorded observations but little. An astronomical observation, of any striking and well-marked phenomenon, carries with it, in most cases, abundant means of recovering its exact date, when any tolerable approximation is afforded to it by chronological records; and, so far from being abjectly dependent on the obscure and often contradictory dates, which the comparison of ancient authorities indicates, is often itself the surest and most convincing evidence on which a chronological epoch can be brought to rest. Remarkable eclipses, for instance, now that the lunar theory is thoroughly understood, can be calculated back for several thousands of years, without the possibility of mistaking the day of their occurrence. And, whenever any such eclipse is so interwoven with the account given by an ancient author of some historical event, as to indicate precisely the interval of time between the eclipse and the event, and at the same time completely to identify the eclipse, that date is recovered and fixed forever. This has been done in the cases of four very remarkable total eclipses of the sun (the dates of which are accordingly entered as epochs in the table of Chronological Eras, art. 926), which have given rise (at least one of them) to much discussion and diversity of opinion among astronomers, but which have at length been definitively settled by Mr. Airy on the occasion of the recent publication of Prof. Hansen's Lunar Tables, the accuracy of which is such as to justify the most entire reliance on the results of such calculations grounded upon them.

(933 *a.*) The solar eclipse designated as that "of Thales" is the celebrated one which is stated by Herodotus to have been predicted by that philosopher, and to have caused the suspension of a battle between the Medes and Lydians, followed by a treaty of peace. Only a *total* eclipse, as Mr. Baily had clearly shown, could have so attracted the attention of the combatants; and in a very remarkable memoir on the subject (Phil. Trans. 1811) that eminent astronomer was led, by the use of the best tables then in existence, to identify this eclipse with the total one of September 30th, B.C. 610, which, according to those tables, must have

passed over the mouth of the River Halys (where it had all along been assumed, though without any positive grounds for the assumption, the battle was fought). The same conclusion having been arrived at by M. Oltmanns, the point was supposed to be settled. Prof. Hansen's tables, however, throw the path of the shadow in this eclipse altogether out of Asia Minor, and even north of the Sea of Azof. On the other hand the eclipse of B.C. 585, which was also total, passed, according to those tables, over Issus, a locality satisfying all the circumstantial and general military conditions of the narrative even better than the Halys, and at this spot there can now be little or no doubt the battle was really fought.

(933 *b.*) The total eclipse of the sun which was witnessed by the fleet of Agathocles in his escape from Syracuse, blockaded by the Carthaginians, on the second day of his voyage to Cape Bon, had been considered by Mr. Baily in the memoir above cited, and found to be incompatible (according to the then existing tables) with the year B.C. 310, *supposing the former eclipse to have been rightly identified.* This having now been shown not to be the case, it is all the more satisfactory to find that, under very reasonable and natural suppositions respecting Agathocles' voyage, the total eclipse which did undoubtedly pass on the date assigned very near the southern corner of Sicily might have enveloped his fleet, and that no other eclipse by possibility could have done so.

(933 *c.*) The "eclipse of Larissa" is related by Xenophon to have caused the capture of the Asiatic city of that name, by producing a panic terror in its Median defenders, of which the Persian besiegers took advantage. The site of Larissa has been satisfactorily identified by Mr. Layard

with NIMROUD, and a pyramid of stone close to it (the city being of burned brick on a substructure of stone) is expressly mentioned by Xenophon, the remains of which still exist; as well as those of a neighboring castle near Mespila (Mosul) built of *shelly stone* (also expressly mentioned *as such* by the Greek historian). According to Hansen's tables the total eclipse of May 19th, B.C. 557, *passed centrally over Nimroud*, and—the total shadow being in this instance a very small one, not exceeding some 25 miles in diameter —we are thus presented with a datum, in those remote times, having all the precision of a most careful modern observation, not only for establishing a chronological epoch, but for affording a point of reference in the history of the moon's motion. The eclipse of Sticklastad is of no historical importance.[7]

(934.) The days thus parcelled out into years, the next step to a perfect knowledge of time is to secure the identification of each day, by imposing on it a name universally known and employed. Since, however, the days of a whole year are too numerous to admit of loading the memory with distinct names for each, all nations have felt the necessity of breaking them down into parcels of a more moderate extent; giving names to each of these parcels, and particularizing the days in each by numbers, or by some especial indication. The lunar month has been resorted to in many instances; and some nations have, in fact, preferred a lunar to a solar chronology altogether, as the Turks and Jews continue to do to this day, making the year consist of 12 lunar months, or 354 days. Our own division into twelve

[7] The solar eclipse in the first year of the Peloponnesian War, which was total at Athens, "some stars becoming visible," according to Thucydides, deserves to be recomputed. See Heis. He supposes the eclipse not total, and the "stars" to have been planets.

unequal months is entirely arbitrary, and often productive of confusion, owing to the equivoque between the lunar and calendar month.[8] The intercalary day naturally attaches itself to February as the shortest month.

(935.) Astronomical time reckons from the noon of the current day; civil from the preceding midnight, so that the two dates coincide only during the earlier half of the astronomical and the latter of the civil day. This is an inconvenience which might be remedied by shifting the astronomical epoch to coincidence with the civil. There is, however, another inconvenience, and a very serious one, to which both are liable, inherent in the nature of the day itself, which is a local phenomenon, and commences at different instants of absolute time, under different meridians, whether we reckon from noon, midnight, sunrise, or sunset. In consequence, all astronomical observations require in addition to their date, to render them comparable with each other, the longitude of the place of observation from some meridian, commonly respected by all astronomers. For geographical longitudes, the Isle of Ferroe has been chosen by some as a common meridian, indifferent (and on that very account offensive) to all nations. Were astronomers to follow such an example, they would probably fix upon Alexandria, as that to which Ptolemy's observations and computations were reduced, and as claiming on that account the respect of all while offending the national egotism of none. But even this will not meet the whole difficulty. It will still remain doubtful, on a meridian of 180° remote from that of Alexandria, what

[8] "A month in law is a lunar month or twenty-eight days" (!! see Art. 418), "unless otherwise expressed."—*Blackstone,* ii. chap. 9: "a lease for twelve months is only for forty-eight weeks."—*Ibid.*; yet the same eminent authority (Introd. § 3) informs us that "the law is the perfection of reason," and that "what is not reason is not law."

day is intended by any given date. Do what we will, when it is Monday the 1st of January, 1849, in one part of the world, it will be Sunday the 31st of December, 1848, in another, so long as time is reckoned by local hours. This equivoque, and the necessity of specifying the geographical locality as an element of the date, can only be got over by a reckoning of time which refers itself to some event, real or imaginary, common to all the globe. Such an event is the passage of the sun through the vernal equinox, or rather the passage of an imaginary sun, supposed to move with perfect equality, through a vernal equinox supposed free from the inequalities of nutation, and receding upon the ecliptic with *perfect* uniformity. The actual equinox is variable, not only by the effect of nutation, but by that of the inequality of precession resulting from the change in the plane of the ecliptic due to planetary perturbation. Both variations are, however, periodical, the one, in the short period of 19 years, the other, in a period of enormous length, hitherto uncalculated, and whose maximum of fluctuation is also unknown. This would appear, at first sight, to render impracticable the attempt to obtain from the sun's motion any rigorously uniform measure of time. A little consideration, however, will satisfy us that such is not the case. The solar tables, by which the apparent place of the sun in the heavens is represented with almost absolute precision from the earliest ages to the present time, are constructed upon the supposition that a certain angle, which is called "the sun's mean longitude" (and which is in effect the sum of the mean sidereal motion of the sun, *plus* the mean sidereal motion of the equinox in the opposite direction, *as near as it can be obtained* from the accumulated observations of twenty-five centuries), increases with rigorous uniformity as time advances. The conversion of this mean longitude into time at the rate of 360° to the mean tropical year (such as the tables assume it), will therefore give us both the unit of time, and the uniform measure of its lapse, which we seek. It will also furnish us with an epoch, not indeed marked by any real event, but not on that account the less positively fixed, being connected, through the medium of the tables, with every single observation of the sun on which they have been constructed and with which compared.

(936.) Such is the simplest abstract conception of equinoctial time. It is the mean longitude of the sum of *some one approved set of solar* tables, converted into time at the rate of 360° to the tropical year. Its unit is the mean tropical year which those tables assume, *and no other*, and its epoch is the mean vernal equinox of these tables for the current year, or the instant when the mean longitude of the tables is rigorously 0, according to the assumed mean motion of the sun and equinox, the assumed epoch of mean longitude, and the assumed equinoctial point on which the tables have been computed, *and no other.* To give complete effect to this idea, it only remains to specify the particular tables fixed upon for the purpose, which ought to be of great and admitted excellence, since, once decided on, the very essence of the conception is that *no subsequent alteration in any respect should be made, even when the continual progress of astronomical science shall have shown any one or all of the elements concerned to be in some minute degree erroneous* (as necessarily they must), *and shall have even ascertained the corrections they require* (to be themselves again corrected, when another step in refinement shall have been made).

(937.) Delambre's solar tables (in 1828), when this mode of reckoning time was first introduced,[9] appeared entitled to this distinction. According to these tables, the sun's mean longitude was 0°, or the mean vernal equinox occurred, in the year 1828, on the 22d of March at $1^h\ 2^m\ 59^s{\cdot}05$ mean time at Greenwich, and therefore at $1^h\ 12^m\ 20^s{\cdot}65$ mean time at Paris, or $1^h\ 56^m\ 34^s{\cdot}55$ mean time at Berlin, at which instant, therefore, the equinoctial time was $0^d\ 0^h\ 0^m\ 0^s{\cdot}00$, being the commencement of the 1828th year current of equinoctial time, if we choose to date from the mean tabular equinox, nearest to the vulgar era, or of the 6541st year of the Julian period, if we prefer that of the first year of that period.

(938.) Equinoctial time then dates from the mean vernal equinox of Delambre's solar tables, and its unit is the mean tropical year of these tables ($365^d{\cdot}242264$). Hence, having the fractional part of a day expressing the difference between the mean local time at any place (suppose Greenwich) on any one day between two consecutive mean vernal equinoxes, that difference will be the same for every other day in the same interval. Thus, between the mean equinoxes of 1828 and 1829, the difference between equinoctial and Greenwich time is $0^d{\cdot}956261$ or $0^d\ 22^h\ 57^m\ 0^s{\cdot}95$, which expresses the equinoctial day, hour, minute, and second, corresponding to mean noon at Greenwich on March 23, 1828, and for the noons of the 24th, 25th, etc., we have only to substitute 1d, 2d, etc., for 0^d, retaining the same decimals of a day, or the same hours, minutes, etc., up to and including March 22, 1829. Between Greenwich noon of the 22d and 23d of March, 1829, the 1828th equinoctial year terminates, and the 1829th commences. This happens

[9] On the instance of the author of these pages.

at $0^d \cdot 286003$, or at $6^h\ 51^m\ 50^s \cdot 66$ Greenwich mean time, after which hour, and until the next noon, the Greenwich hour added to equinoctial time $364^d \cdot 956261$ will amount to more than 365·242264, a complete year, which has therefore to be subtracted to get the equinoctial date in the next year, corresponding to the Greenwich time. For example, at $12^h\ 0^m\ 0^s$ Greenwich mean time, or $0^d \cdot 500000$, the equinoctial time will be $364 \cdot 9562610 + \cdot 500000 = 365 \cdot 456261$, which being greater than 365·242264, shows that the equinoctial year current has changed, and the latter number being subtracted, we get $0^d \cdot 213977$ for the equinoctial time of the 1829th year current corresponding to March 22, 12^h Greenwich mean time.

(939.) Having, therefore, the fractional part of a day for any one year expressing the equinoctial hour, etc., at the mean noon of any given place, that for succeeding years will be had by subtracting $0^d \cdot 242264$, and its multiples, from such fractional part (increased if necessary by unity), and for preceding years by adding them. Thus, having found 0·198525 for the fractional part for 1827, we find for the fractional parts for succeeding years up to 1853 as follows[10]:—

1828	·956261	1835	·260413	1842	·564565	1848	·110981
1829	·713997	1836	·018149	1843	·322301	1849	·868717
1830	·471733	1837	·775885	1844	·080037	1850	·626453
1831	·229469	1838	·533621	1845	·837773	1851	·384189
1832	·987205	1839	·291357	1846	·595509	1852	·141925
1833	·744941	1840	·049093	1847	·353245	1853	·899661
1834	·502677	1841	·806829				

[10] These numbers differ from those in the Nautical Almanac, and would require to be substituted for them, to carry out the idea of equinoctial time as above laid down. In the years 1828–1833, the late eminent editor of that work, Dr. Young, used an equinox slightly differing from that of Delambre, which accounts for the difference in those years. In 1834, it would appear that a deviation both from the principle of the text and from the previous practice of that ephemeris took place, in deriving the fraction for 1834 from that for 1833, which has been ever since perpetuated. It consisted in rejecting the mean longitude of Delambre's tables, and adopting Bessel's correction of that

element. The effect of this alteration was to insert 3m 3s·68 *of purely imaginary time* between the end of the equinoctial year 1833 and the beginning of 1834, or, in other words, to make the intervals between the noons of March 22 and 23, 1834, 24h 3m 3s·68, when reckoned by equinoctial time. In 1835, and in all subsequent years, a further departure from the principle of the text took place by substituting Bessel's tropical year of 365·2422175 for Delambre's. Thus the whole subject fell into confusion. Under the present eminent superintendent of the Nautical Almanac a compromise has been effected—a fixed equinoctial year of 365·242216 mean solar days has been adopted (and it is to be hoped will henceforward be adhered to), and corrections stated by which the data in the Almanacs for 1828–1834 may be brought into consistency with those in after years. According to this arrangement the fractional parts in question for 1828–1856 will run as follows:

1828	·958321	1836	·020593	1843	·325081	1850	·629569
1829	·716105	1837	.778377	1844	·082865	1851	·387353
1830	·473889	1838	·536161	1845	·840649	1852	·145137
1831	·231673	1839	·293945	1846	·598433	1853	·902921
1832	·989457	1840	·051729	1847	·356217	1854	·660705
1833	·747241	1841	.809513	1848	·114001	1855	·418489
1834	·505025	1842	·567297	1849	·871785	1856	·176273
1835	·262809						

[*Note* A *on Art.* 915.

A rule proposed by Omar, a Persian astronomer of the court of Gelaleddin Melek Schah, in A.D. 1079 (or more than five centuries before the reformation of Gregory), deserves notice. It consists in interpolating a day, as in the Julian system, every fourth year, only postponing to the 33d year the intercalation which on that system would be made in the 32d. This is equivalent to omitting the Julian intercalation altogether in each 128th year (retaining all the others). To produce an accumulated error of a day on this system would require a lapse of 5000 years, so that the Persian astronomer's rule is not only far more simple, but materially more exact than the Gregorian.]

[*Note* B *on Table, Art.* 926.

The civil epochs of the Metonic cycle and the Hejira are each one day later than the astronomical, the latter being the epochs of the absolute *new moons,* the former those of the earliest possible visibility of the lunar crescent in a tropical sky. M. Biot has shown that the solstice and new moon not only coincided on the day here set down as the commencement of the Callippic cycle, but that, by a happy coincidence, a *bare* possibility existed of seeing the crescent moon at Athens *within that day, reckoned from midnight to midnight.*]

[*Note* C *on Art.* 932.

The reformation of Gregory was, after all, incomplete. Instead of 10 days he ought to have omitted 12. The interval from Jan. 1, A.D. 1, to Jan. 1, A.D. 1582, reckoned as Julian years, is 577460 days, and as tropical, 577448, with an error not exceeding 0d·01, the difference being 12 days, whose omission would have completely restored the Julian epoch. But Gregory assumed for his fixed point of departure, not that epoch, but one later by 324 years, viz. Jan. 1, A.D. 325, the year of the Council of Nice; assuming which, the difference of the two reckonings is 9d·505, or, to the nearest whole number, 10 days. To such as may have occasion, in an isolated case, to compute the interval between the beginnings of two proposed years, or the number of days elapsed from Jan. 1st 0h. of the one to Jan. 1st 0h. of the other, the following formulæ will be found useful, in which it is to be observed that the notation $\underline{\underline{\frac{m}{n}}}$ is used to express the *integer portion* only of the quotient when one number m is divided by another n, and where D is the number of days required.

Case 1.—Years before Christ—from B.C. x to A.D. 1.

$$D = x \,.\, 365 + \underline{\underline{\frac{x+4}{4}}}$$

Case 2.—Years after Christ—from A.D. 1 to A.D. x.

$$\text{(A)} \;\ldots\ldots\; D = (x-1) \,.\, 365 + \underline{\underline{\frac{x-1}{4}}}$$

$$\text{(B)} \;\ldots\ldots\; D = (x-1) \,.\, 365 + \underline{\underline{\frac{x-1}{4}}} - \underline{\underline{\frac{x-200}{100}}} + \underline{\underline{\frac{x}{400}}}$$

where the formula (A) is to be used if the later date (x) is *before* the change of style in that particular country for which the calculation is made, and (B) if *after*.]

[*Note* D, *Art.* 223 *a*.

(223 *a*.) Since the table of measured arcs in art. 216 was compiled, vast additions have been made to our knowledge of the true figure and dimensions of our globe; especially through the extension of the two great arcs of Russia and India, the former of which has been enlarged to an amplitude of 25° 20′, the latter to 21° 21′. The whole series of geodesical measurements of any authority, executed in all parts of the world (these and the French arc, the next in magnitude, of course included), have been lately combined under one general and comprehensive system of calculation

by Capt. A. R. Clarke, R.E., in an elaborate memoir (Mem. R. Ast. Soc. vol. xxix. 1860), of which the final result (including some slight subsequent corrections) may be stated as follows:

The earth is not exactly an ellipsoid of revolution. The equator itself is slightly elliptic, the longer and shorter diameters being respectively 41,852,864 and 41,843,096 feet. The ellipticity of the equatorial circumference is therefore $\frac{1}{4283}$, and the excess of its longer over its shorter diameter about two miles. The vertices of the longer diameter are situated in longitudes 14° 23′ E. and 194° 23′ E. of Greenwich, and of its shorter in 104° 23′ and 284° 23′ E. The polar axis of the earth is 41,707,796 feet in length, and consequently the most elliptic meridian (that of long. 14° 23′ and 194° 23′) has for its ellipticity $\frac{1}{287 \cdot 5}$, and the least so (that of long. 104° 23′ and 284° 23′) an ellipticity of $\frac{1}{308 \cdot 3}$.

General Schubert also (Mem. Imp. Acad. Petersb. 1859) has arrived at a somewhat similar conclusion, by a totally different (and, as appears to us, less general and definitive) system of calculation. He makes the ellipticity of the equator $\frac{1}{8885}$, and places the vertices of its longer axis 26° 41′ to the eastward of Capt. Clarke's. His polar axis, as deduced from each of the three great meridian arcs, the Russian, Indian and French respectively, is 41,711,019, 41,712,534 and 41,697,496 feet, the mean of which, giving to each a weight proportional to the length of the arc from which it is deduced,[11] is 41,708,710.

A very remarkable consequence follows from these results. If we reduce the polar axis last found to Brit-

[11] This is not Gen. Schubert's mode of procedure. He arbitrarily excludes the result of the French arc, and gives the Russian double the weight of the Indian—a procedure manifestly unfair.

ish imperial inches, the result is 500,504,520, exceeding 500,500,000 by 4520 in. On the other hand Capt. Clarke's result similarly reduced is 500,493,552, falling short of 500,500,000 by 6448 in., so that we may take 500,500,000 inches for the length of the earth's polar axis, with every probability of being within 150 yards of the truth. Were our imperial standard of length, then, increased by *exactly one-thousandth part*, the inch, foot, yard, etc., retaining their present relative proportions, our inch would then be, with *all but mathematical precision*, one five-hundred-millionth part of the earth's polar axis, a unit common to all the world. And, what is still more remarkable, if at the same time our standard of weight were increased by one 2,600th part (or one-seventh of a grain on the ounce), an ounce of distilled water, at our present standard temperature of 62° Fahr., would occupy *precisely* one-thousandth part of our (*then*) cubic foot, and our half-pint *precisely* one-hundredth. Thus, by a change such as would be absolutely unfelt in any commercial transaction, we should be put in possession of a *modular* or *geometrical* system (which we might decimalize if thought proper) far superior both in principle and in accuracy to anything which has yet been devised on the important subject of a national system of weights and measures, the French metrical system not excepted.]

[*Note* E *on Art.* 234.

Mr. Broun of Edinburgh, and M. Babinet of Paris, have each separately and independently devised extremely ingenious applications of the principle of the torsion balance, to bring into equilibrium the force of gravity and the elastic force of a metallic spring, with a view to utilize the method here suggested of ascertaining the variation of gravity. It were much to be desired that both their methods should be brought to the test of practical trial.]

(357 *a.*) A series of concerted observations of the differences of apparent declinations between the planet Mars and neighboring fixed stars, set on foot at the instance of M. Winnecke, during its last opposition, which took place under circumstances of proximity to the earth particularly favorable to the determination of its parallax, has resulted in assigning to that planet a parallax greater by about one-twenty-seventh part than that which would correspond to its distance as computed for the time by the hitherto assumed dimensions of the planetary orbits. The conclusion of course is, that these dimensions (including the earth's distance from the sun) have been overestimated by that fractional part of their value; that the sun's parallax, in place of $8''\cdot6$, or more exactly $8''\cdot5766$, should be set down at $8''\cdot8953$, and its distance at 91,600,000 instead of 95,000,000 miles. Now it is strongly corroborative of this conclusion, and, at the same time, affords a striking instance of the way in which the several departments of science depend on and illustrate one another, that a correction in the same direction, and to nearly the same amount, is indicated by a recent redetermination, by direct experiment, of the velocity of light by M. Foucault, who finds that velocity not only less than that concluded by M. Fizeau's experiments (see art. 545), but even less than the commonly received estimate of 192,000 miles per second, by about the same fractional part. Light, as shown in art. 545, travels over the diameter of the earth's orbit in $16^{m}\ 26^{s}\cdot6$; and the time remaining the same, a diminished speed corresponds to a diminished diameter, and therefore the sun's distance computed from the velocity so determined, and the time as given by observation, will come to be diminished in the

same proportion. These and several other conspiring indications lead to an extremely strong presumption that all the dimensions of our system have been overrated, and should be diminished by about one-twenty-eighth part.

(357 *b b*.) In strong corroboration, or rather in full confirmation, of this presumption, it must be especially noticed, that quite recently the whole subject of the reduction of the transit observations of 1769 has been resumed by Mr. Stone in a memoir for which the gold medal of the Astronomical Society for 1869 has been awarded to him, in which he has clearly shown that the received result of those observations has been vitiated by a misinterpretation of the expressions used by the observers in describing the phenomena of the external and internal contacts of the limb of Venus with that of the sun, which are complicated with certain optical appearances materially influencing the estimation of their times of happening; and that when those expressions are taken according to their real and legitimate meaning, and duly calculated on, they afford a value for the solar parallax of 8″·91 with a probable error of 0″·03 (in place of the hitherto received value, 8″·5776), agreeing very precisely with the value (8″·943) deduced by him from the assemblage of comparative observations of Mars in his opposition of 1862, instituted at Greenwich, the Cape of Good Hope and Williamstown, Victoria, N. S. W.

(357 *c*.) The distances being diminished in any ratio, the estimated masses will require to be diminished also, in the ratio of the cubes of the distances; for *all* the distances will have to be reckoned on a new scale, and among the rest the diameters of the orbits of all satellites (the moon excepted); and as the squares of the periodic times are as the sums of the masses directly and the cubes of the

distances inversely, the times remaining unchanged the masses must be diminished in the same proportion as those cubes.

(357 *d.*) The time is not yet come for a complete and final determination of the exact "coefficient of diminution" to be applied to our planetary elements. The whole matter is too recent, and we must wait for the next transits of Venus in 1874 and 1882 for a full and precise settlement of this important question. Meanwhile, therefore, and provisionally, the reader will bear in mind that in all our numerical statements of distances (those of the fixed stars and the velocity of light inclusive, but that of the moon excepted), as well in the text as in the tables of elements, the numbers set down have to be diminished by one twenty-eighth of their value, and in the case of the masses (the moon's excepted) in the ratio of $27^3 : 28^3$ or of 0·89664 to 1.

(357 *e.*) The superficial reader (one of a class too numerous) may think it strange and discreditable to science to have erred by nearly four millions of miles in estimating the sun's distance. But such may be reminded that the error of 0″·32 in the sun's parallax, on which the correction turns, corresponds to the apparent breadth of a human hair at 125 feet, or of a sovereign at 8 miles off, and that, moreover, *this error has been detected* and the correction applied; and that the detection and correction have originated with the *friends and not with the enemies of science.*]

[*Note* G, *Art.* 387 *a.*

This curious appearance of the "pores" of the sun's surface has lately received a most singular and unexpected interpretation from the remarkable discovery of Mr. J. Nasmyth, who, from a series of observations published in the Memoirs of the Lit. and Phil. Society of Manchester for 1862, made with a reflecting telescope of his own construction under very high magnifying powers and under exceptional circumstances of tranquillity and definition, has come to the conclusion that these pores are the polygonal interstices between certain luminous objects of an exceedingly definite shape and general uniformity of size, whose form (at least as seen in projection, in the central portions of the disk) is that of the oblong leaves of a willow tree. These cover the whole disk of the sun (except in the space occupied by spots) in countless millions, and lie crossing each other in every imaginable direction. A representation copied from the figure in Mr. Nasmyth's memoir (the engraver being aided by photographs from his original drawings obligingly supplied by him for the purpose) of the structure of the general luminous surface, is given in our Fig. 1, Plate B, while Fig. 2 of the same plate exhibits their arrangement at the borders and in the penumbræ of a spot. This most astonishing revelation has been confirmed to a certain considerable extent, and with some modifications as to the form of the objects, their exact uniformity of size, and resemblance of figure, by Messrs. De la Rue, Pritchard, and Stone, in England, and M. Secchi in Rome. Mr. Stone compares them to rice grains, others to bits of straw. They strongly suggest the idea of solid bodies sustained in *in equilibrio* at a definite level (determined by their density) in a transparent atmosphere *passing by every gradation of density from that of a liquid to that of the rarest gas* by reason of its heat and the enormous, superincumbent pressure (as in the experiments of M. Cagniard de la Tour on the vaporization of liquids under high pressure); their luminosity being a *consequence* of their solidity;

transparent and colorless fluids radiating no light from their interior however hot.[12]

(387 *b.*) In speaking of the intimate nature of the sun's luminous envelope, the remarkable phenomenon witnessed on the 1st of September, 1859, by two independent observers, Mr. Carrington and Mr. Hodgson, ought not to be passed in silence. These two gentlemen viewing the sun, each at his own residence and without previous concert, at the same instant of time on that day, were both surprised by the sudden appearance, in the immediate confines of a large irregular spot, of what seemed to be two luminous clouds, *much more brilliant than the general surface of the sun.* They lasted about five minutes, and disappeared almost instantaneously, sweeping in that interval among the details of the spot, with a visible progressive motion, over a space which could not be estimated at less than 35,000 miles. The magnetic needle, as was afterward ascertained, underwent a considerable and sudden disturbance at that very time; and the phenomenon, indeed, occurred during the continuance of one of the most remarkable and universal "magnetic storms" on record.

(387 *c c.*) To Mr. Carrington, also, we owe a long-continued and most elaborate series of observations of the solar spots (contained in a memoir recently presented to the Royal Society), continued through a whole period of their maximum and minimum frequency: arriving at the conclusion that the period of rotation of a spot is depen-

[12] Such is the view of their nature and of that of the solar photosphere suggested by the author in a paper "On the Solar Spots," published in the Quarterly Journal of Science for April, 1864. M. Faye, in a Memoir read to the French Academy in January, 1865, has arrived at a conclusion nearly analogous. It ought to be noticed that Mr. Dawes still professes himself dissatisfied as to the existence of these objects.

dent on its heliographical latitude; those on and near the sun's equator being carried round more swiftly than those in northern and southern latitudes. The empirical law at which he arrives as a mean expression of all his observations assigns for the movement of rotation *per diem* of a spot in heliographical latitude l

$$856' - 165' (\sin l)^{1.75},$$

so that a spot on the equator will make a complete sidereal revolution in $24^{d}{\cdot}202$—one in N. or S. latitude 15° in $25^{d}{\cdot}44$, and in 30° N. or S. in $26^{d}{\cdot}24$.

(387 *d d.*) The confinement of the spots to a region limited both ways in latitude and rarely beyond 30° on either side of the solar equator, as well as the frequency of their arrangement along parallels of latitude, has already (art. 393) given us ground to conclude the existence of a circulation in the solar atmosphere relatively to the solid body of the sun, and to surmise an analogy between the cause of that circulation and that of the trade winds in our own atmosphere. It has been suggested[13] that, owing to oblateness in the solar atmosphere, its greater depth in the equatorial regions might be conceived to oppose the free escape of heat more than in the polar, and so might give rise to a permanent inequality of temperature in the two regions from which movements analogous to those winds must of necessity originate. Were this the case, however, one of the results would (of equal necessity) be the production of an equatorial region of calm. Supposing our earth so covered with cloud that a spectator without could never see the solid body, and could judge only of its rotation from the observed circulation of dark masses of its clouds; he might,

[13] Results of Astronomical Observations at the Cape of Good Hope, p. 434.

it is true, conclude correctly the time of rotation of the solid body from observation of cloudy masses in the higher strata either on the equator or in high latitudes; but in the two intermediate zones north and south of the equator, he would conclude a greater rapidity of rotation, owing to the general westerly tendency of the upper aërial currents in those zones. In proceeding then from the poles toward the equator, he would find, first, an apparent acceleration, then in certain latitudes N. and S. a maximum of rotary velocity, and thence up to the equator a comparative retardation. Mr. Carrington's law is therefore incompatible with this supposed analogy, and we must look elsewhere for its explanation. The only one which seems at all satisfactory is that of external force impressing such a movement; and thus we fall back upon the frictional impulse of circulating planetary matter in process of subsidence into and absorption by the central body. The rotation of the sun, it will be remembered, is very much slower than that of a planet revolving just clear of its surface—a fact perfectly in unison with that theory of the formation of our system to which the term "the nebular hypothesis" (arts. 871, 872) has been applied, according to which the central body has resulted from the aggregation of all the matter assembling from every quarter, whose movements conflicting would destroy each other, leaving only as a surplus that small portion of rotation in one direction which remained outstanding. The same considerations render an equally plausible account of the intense heat of the central body.]

[*Note* H, *Art.* 430 *a.*

The enormous size and vast depth of many of the lunar craters, far surpassing, in both respects, anything observed on our globe, are certainly very striking features, but are easily reconcilable with what we know of the special conditions which obtain on the moon's surface. For while on the one hand the force of volcanic explosion and ejection is nowise dependent on the total mass of the planetary body on which the volcano may subsist, the repressing power to prevent an outbreak, which is the weight of the incumbent matter, is only about one-sixth of what an equal mass of overlying matter would exert on the earth, the force of gravity on the moon being less than that on the earth in that proportion. Again, when disrupted and scattered in fragments, the force generating their velocity of projection being the same, and therefore also that velocity, the broken fragments, stones, scoriæ, etc., ejected would be hurled to far greater distances, being less powerfully coerced by gravity, in the same ratio. Most of the ejected matter would therefore fly out beyond the rims of the craters, instead of falling back to refill them. And lastly, we have to add, the absence of the resistance of an atmosphere—that powerful coercer of projectile range here on earth.

(430 *b b.*) Changes in the aspect and configuration of particular portions of the lunar disk have been often suspected, but no satisfactory evidence had been obtained of anything which could not be accounted for, either by the difference of optical power in the telescopes used, or by difference of presentation to the solar rays, or to those reflected from the earth. Quite recently, however, on the 16th of October, 1866, the crater marked as A in Lohrmann's Chart (sec. iv.), and designated by Mädler under the name "Linnæus" (a crater five miles and a half in diameter, and very deep, and which had served those selenographers as a zero-point of the first class for their

micrometrical measurements), has been declared by Professor J. Schmidt, Director of the Observatory at Athens, to have altogether disappeared, and to be replaced by a smooth surface unmarked by any shadow! Subsequent observations made by him in November and December, under the most favorable circumstances as to solar illumination, failed to show any signs of the missing crater, though other much smaller ones in the neighborhood were readily perceived. The most plausible conjecture as to the cause of this disappearance seems to be the filling up of the crater from beneath by an effusion of viscous lava, which, overflowing the rim on all sides, may have so flowed down the outer slope as to efface its ruggedness and convert it into a gradual declivity casting no stray shadow.]

[*Note* I *on Art.* 437.

Our Plate C exhibits the appearance of a very rough and volcanic portion of the moon's surface as modelled from telescopic observation by Mr. Nasmyth, the engraving being taken from the photograph of the original model kindly furnished by him for the purpose. A very ingenious idea of Mr. Wheatstone has enabled the photographer to produce *stereoscopic* views of the moon, presenting it, not as a flat disk, but as a sphere, with all the mountains in full relief, and with all the appareance of a real object. Owing to the libration of the moon (art. 435) the same point of her surface is seen sometimes on one side of the centre of her disk, and sometimes on the other, the effect being the same as if, the moon remaining fixed, the eye were shifted from right to left through an angle equal to the total libration. Now this is the condition on which stereoscopic vision depends: so that by choosing two epochs *in different lunations* in which the moon shall be presented in the two aspects best adapted for the purpose, *and in the same phase of illumination* (which the annual motion of the earth renders possible, by bringing the moon to the same elongation from the sun, in different parts of her own elliptic orbit), and taking separate and independent photographs of it in each aspect, the two stereoscopically combined, so completely satisfy all the requisite conditions as to show *the spherical* form just as a giant might see it whose stature were such that the interval between his eyes should equal the distance between the place where the earth stood when one view was taken, and that to which it would have to be removed (our moon being fixed) to get the other. Nothing can surpass the impression of *real cor-*

poreal form thus conveyed by some of these pictures as taken by Mr. De la Rue with his powerful reflector, the production of which (as a step in some sort taken by man outside of the planet he inhabits) is one of the most remarkable and unexpected triumphs of scientific art.

Mr. Birt has recently bestowed much pains on the frequent and minute scrutiny of particular and limited regions of the lunar surface.]

[*Note* J.

(859 *a*.) It is not *necessary* that the companion body producing these disturbances of proper motion should be non-luminous. Nothing prevents that it should be a small companion star which, though invisible to ordinary telescopes, or lost in the brightness of the disturbed luminary, may become visible through telescopes of increased power. Antares has such a minute companion at only 12″ distance, α Lyræ at 43″, and Procyon at 46″. A disturbance in the regular progression of proper motion of the conspicuous star, periodical in its nature, both in R. A. and in N. P. D., would arise from the displacement of the larger star around the common centre of gravity, however comparatively massive, and (mass for mass) greater the greater the distance of the individuals. It was not, therefore, without much interest that astronomers received the announcement of the recent detection of a small companion of Sirius, by Mr. Alvan Clarke, a most eminent and successful constructor of large achromatic telescopes, by means of an instrument of this description constructed by him of 18 inches aperture. According to Messrs. Rutherford, Bond and Chacornac, this companion is at present situated at about 10″ distance, nearly following Sirius. It remains to be examined, however, by a series of observations of distance and position continued for many years, 1st, Whether this star really *is* a satellite or binary companion of the principal star—a con-

clusion to which some have *jumped* (not unnaturally); and, 2d, Whether, *if so*, its attraction will explain the observed inequalities of proper motion, which by no means follows of course: far more so, as M. Goldschmidt assures us that, with a telescope of very inferior power to that of Mr. Clarke, he has detected no less than *six* small companions of Sirius at distances from 10″ to 60″. Should this be verified, we have our choice of disturbing influences; and it would be hard if the minute displacements, respecting which Messrs. Auwers and Peters agree (or any other) could not be plausibly explained *antecedently* to the indispensable verification of a real physical connection. Meanwhile the conclusions of these geometers rest upon observed inequalities in R. A. only. Prof. Safford, of Ann Arbor, U. S., has, however, investigated those in polar distance, and finds that they are alike reconcilable with an elliptic, orbital motion, with one not incompatible with that previously assigned, and with the hypothesis that the newly discovered star, so far as it has yet been observed, is really the disturbing body.

(859 *b b.*) Spectrum analysis (see Note M) has of late been applied by various observers with great diligence and success to the light of the fixed stars, and even to that of the nebulæ. Those of the stars are found to exhibit (variously for different stars) many of those fixed lines which are considered characteristic of chemical elements such as are found in our planet. In applying this method of examination to Sirius, however, Mr. Huggins has found that the brightest of three lines characteristic of hydrogen corresponds to a position in the spectrum of that star very slightly differing from the position of the same line in the solar spectrum, and that, in point of fact, the index of re-

fraction of the prism employed for that line in the stellar spectrum is less, by a minute but measurable quantity, than in the solar. To interpret this observation, it must be remembered that the undulatory theory of light postulates, as a condition indispensable to its interpretation of the different refrangibility of the rays, that the velocity of propagation of a luminous undulation *within* the refracting medium shall depend (according to some law depending on the physical nature of the medium) on its wave-length on arriving at the refracting surface; or, in other words, on the number of its undulations per second which are incident on that surface; the longer waves or less number per second corresponding to the lower degree of refrangibility. Suppose now a certain ray to originate in Sirius from some vibratory movement of a particle of its matter producing isochronous impulses of a certain determinate frequency on the luminiferous ether. Sirius being at any definite distance from the earth at the moment of the first impulse, that impulse will reach the prism at a certain moment; and were the star and earth relatively at rest, its successors would follow it up and reach the refracting surface at intervals of time precisely equal to those of their origination. But if the star and earth be receding from each other with any uniform velocity, the next succeeding impulses, having each in succession a greater and greater distance to travel, will require a longer and longer time to arrive; or, in other words, the intervals of time between the arrivals of successive impulses will exceed the intervals between their original production, each by as much time as it would take a ray of light to travel the distance run over by Sirius from the earth between two consecutive impulses. Arrived at the prism then, the ray consisting of these vibrations will have a lower refrangibility than if the distance of the star from the earth had remained unaltered; and the amount of this difference being ascertained (by a most nice and delicate process of observation), the ratio of the relative velocity of recess of the star from the earth to that of light can be ascertained. Referring for details to Mr. Huggins' memoir,[14] we may state, as his final conclusion, 41·4 miles per second for this relative velocity, of which, at the time of observation, 12 miles per second were due to the motion of the earth in its orbit; leaving $29^{m}{\cdot}4$ per second, or $2{,}540{,}000^{m}$ daily, for the increase of distance between Sirius and our system. The validity of this conclusion rests, of course, on the assumption (for in the absence of observation of other lines in the spectrum it can be only such) that the fixed line observed is due to hydrogen, and not to some other (unknown) chemical element.]

[*Note* K, *Art.* 896 *a*.

Several objects observed as nebulæ are now missing from the heavens. They are such as have been (for the most part) only once observed, and may reasonably be supposed to have been telescopic comets. This, indeed, has in one instance proved to have been really the case; as, by tracing back the path of the 2d comet of 1792 to the date of the observation of a nebula discovered by Maskelyne on Feb. 14, 1793, but which is now missing, it appears to have really occupied that place (R. A. $2^{h}\ 39^{m}$, N. P. D. 46° 15′) at that time. But, besides these, there are cases in which a nebula, undoubtedly such, has either disappeared and reappeared in the same place, or has undergone some

[14] Phil. Trans. 1868.

remarkable change of brightness; or, lastly, has been observed as a conspicuous object in a part of the heavens so well known as to make it exceedingly improbable that it should have escaped all previous observation.

(896 *b.*) On the 11th Oct., 1852, Mr. Hind discovered a nebula in Taurus, previously unnoticed—in R. A. 4^h 14^m, N. P. D. 70° 49′ (1860). He saw it repeatedly, and in 1855 and 1856 it was reobserved by M. D'Arrest. On the 3d Oct., 1861, M. D'Arrest missed it. "Hujus nebulæ . . . ne umbram quidem," he says, "detegere valeo. Attamen semel ac sæpius a me annis 1855 et 1856 observata est, ejusque locus quater determinatus." On Dec. 29, 1861, it was again seen, though with the utmost difficulty, in the great Pulkova refractor by M. Otto Struve, after which it had so far increased in brightness on March 22, 1862, as to bear a faint illumination of the wires.

(896 *c.*) On Sept. 1, 1859, Mr. Tuttle discovered a nebula not previously observed, in R. A. 18^h 23^m 55^s, N. P. D. 15° 29′ 48″ (1860). This nebula is described by M. Auwers as pretty bright and elongated in form. On the night of Sept. 24, 1862, it appeared to M. D'Arrest so brilliant and remarkable that he considers it impossible it should have been overlooked (if then so conspicuous) in the sweeps made by my father and myself over that part of the heavens.

(896 *d.*) Mons. Chacornac has recently announced in the "Bulletin Météor. de Paris," under date of April 28, 1863, the discovery of a nebula in Taurus, in R. A. 5^h 29^m 4^s, N. P. D. 68° 52′ 20″ (1860), so conspicuous as to render its non-previous discovery most improbable, in a portion of the heavens so frequently under inspection, if always of its present brightness.

(896 *e.*) Certainly the last place in the heavens in which the discovery of a new nebula would have been expected, is within the cluster of the Pleiades. Yet here, close over Merope, one of the more conspicuous stars of that cluster, on Oct. 19, 1859, Mr. Tempel observed a large bright nebula which he took for a comet, and was only undeceived when, on observing it next night, he found it unchanged in place. On Dec. 31, 1860, it was seen, though with some difficulty, by himself and Dr. Pope, with the six-feet refractor at Marseilles. Its place for 1860 is R. A. $3^h\ 37^m\ 52^s$, N. P. D. 66° 40′ 13″. M. Auwers describes it as 15′ in extent and triangular in form—but conceives that it *might* have escaped previous notice by reason of its proximity to so bright a star as Merope. Mr. Hind states also that he has often suspected nebulosity about some of the smaller outlying stars of the Pleiades.

(896 *f.*) Not less singular and startling is the observation by Mr. Pogson of the bright and very conspicuous and well-known nebula, the 80th of Messier's Catalogue, often observed, and described as a compressed and beautiful globular cluster of very minute stars, in R. A. $16^h\ 8^m\ 41^s$, N. P. D. 112° 37′ 34″ (1860). While examining the neighborhood of this object on May 28, 1860, his attention was arrested "by the startling appearance of a star, $7{\cdot}8^m$, *in the place which the nebula* had previously occupied." He had seen the nebula so recently as May 9, with the same telescope and power, and it presented nothing unusual. On June 10th the stellar appearance had vanished, but the cluster yet shone with unusual brilliancy and condensation. Prof. Luther and M. Auwers had also perceived the change so early as May 21st, when it was rated as a star of the 6·7 magnitude. On June 10th, the nebula had dis-

appeared to Mr. Pogson, though M. Auwers never quite lost sight of it, and could perceive that the star was excentric. The occurrence of a temporary or a variable star in so peculiar a situation is assuredly very remarkable.

(896 *g*.) Lastly, by a letter from Mr. E. B. Powell, of Madras, an observer of too much experience and note to be easily deceived or to speak on light grounds, I am informed that the southern end of the very remarkable lemniscate-shaped vacuity close to the bright central star in the nebula about η Argus (see Plate IV. *fig*. 2), which, when the drawings were made from which that figure was taken, was *closed* and terminated by a strong and sharply cut outline (marked by a small star in the *upper edge* of that vacuity), is now decidedly open! More recent observations, however (by Lieut. J. Herschel), with an achromatic telescope of five inches aperture, accompanied with careful drawings of the appearance of the nebula (taken on Nov. 22–3, 1868), make it evident—1. That the lemniscate still exists, as such, though (as might be expected) not so strongly defined as when seen with an instrument of superior power; 2. That the relative situations of 48 out of 49 stars in the immediate vicinity of η, laid down in the drawings, in respect of the principal star, have undergone no material change, the 49th being a very minute star of doubtful identity; and, lastly, that the principal star η itself, though greatly diminished in lustre, occupies most decidedly its old situation, pretty deeply immersed in the brightest portion of the nebula on the following or eastern side of the lemniscate, and not (as stated in the last edition of this work, on what we considered sufficient authority) within the lemniscate or its remains, and *out of the nebulosity*.

This is perhaps the right place to mention that a general catalogue of nebulæ and clusters of stars (5,078 in number) in order of R. A., and brought up to 1860, with precession for 1880, and descriptions, prepared by the author of this work, has been published by the Royal Society as Part I. of the Philosophical Transactions for 1864.

(896 *h h.*) Spectral observation, as already mentioned, has been recently applied to the brighter nebulæ. The light, even of the brightest of these objects, is so excessively feeble (for it will be borne in mind that *telescopes afford no means of increasing the intrinsic brightness of a surface*) that any perception of delicate, hair-like, dark lines in their spectra like the fixed lines in that of the sun, is not to be expected. The phenomena which they exhibit, however, are very peculiar, and more in analogy with those of flame, or incandescent gases, than with solar or stellar sources of light. The brighter globular cluster, indeed, and those nebulæ of irregular forms, which are either clearly resolved, or evidently of a resolvable character, give stellar spectra, *i.e.* trains of light of all gradations of refrangibility. It is otherwise with many of those nebulæ of Sir W. Herschel's 4th class, to which the designation "Planetary Nebulæ" has been applied, and with some others of an irresolvable character, among which are the great nebulæ in Orion and Argo. The light of these is either monochromatic, that is, consisting of rays of one definite refrangibility (corresponding in all of them hitherto observed to the nitrogen line in the solar spectrum, or to the light emitted by nitrogen gas rendered incandescent by the electric discharge), or composed of this and of two, and in some few cases of three, other such monochromatic rays, one of which corresponds to one of the hydrogen

lines of the solar spectrum. Such, in brief summary, are the remarkable and important results obtained by Mr. Huggins,[15] and fully corroborated by the observations of Lieut. J. Herschel, R.E., made at Bangalore, with the great advantage of an Indian sky, by the aid of a spectroscopic apparatus furnished (with the telescope already mentioned) by the Royal Society for observation of the solar eclipse of August 18, 1868.[16] If any hesitation should remain as to the certainty of conclusions from the scrutiny of objects so excessively faint, it will be removed by a fact recorded by the last-named observer, viz.—that on removing the slit or limiting aperture of the spectroscope, and viewing through the prism the whole field of the telescope directed to Messier's 46th cluster, a rich and brilliant assemblage of stars, including among them the planetary nebula H. IV. 39, the latter was seen as a faint patch of light in the midst of an infinity of *streaks*, the continuous spectra of the individual stars. "Nothing," he remarks, "could have been more conclusive as a test."[17] Had the light of the nebula not been monochromatic or nearly so, its dilatation by the prism would have precluded its being seen at all as a definite object.]

[*Note* L, *Art.* 858 *a*.

A fundamentally different method of treating the problem of the sun's proper motion from any of those described in art. 857, has been adopted by the present Astronomer Royal, by which the assumption of an approximate knowl-

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[15] Phil. Trans. 1864, 1868.
[16] Proceedings of the Royal Society, xvi. p. 451, and xvii. pp. 58, 103.
[17] Ibid. xvii. 306.

edge of the situation of the "solar apex," is altogether dispensed with. It consists in referring the absolute proper annual motions both of the sun and the stars used in the inquiry, to linear co-ordinates fixed in space, and treating the question as a purely geometrical one, according to the rules of the calculus of probabilities, basing his procedure on two distinct assumptions, between which the truth must lie, viz.—1. That *all* the "irregularities of proper motion" (meaning thereby all the residual amounts of annual movement in each case, which are not accountable for by solar motion) are mere results of error of observation and are *not* caused by any real motions in the stars. This is evidently an extreme supposition.—2. That *none* of such residual movements are due to error of observation, but all originate in real stellar movement. This is as clearly an extreme supposition the other way. Assuming then M. Struve's classification of the stars according to a scale of distances which has at least no *primâ facie* improbability, and using for the purpose those 113 stars of a catalogue prepared with great care by Mr. Main, and published in the "Memoirs of the Astronomical Society," which indicate great proper motion, he arrives at the following conclusions as to the situation of the apex, and the annual parallactic motion of the sun as seen from a star of the first magnitude, on each of these two suppositions.

$$\text{1st Supposition}\left\{\begin{array}{l}\text{Solar apex in R.A. } 256^\circ\, 54';\ \text{N.P.D. } 50^\circ\, 31'.\\ \text{Parallactic motion } 1''{\cdot}269.\end{array}\right.$$

$$\text{2d Supposition}\left\{\begin{array}{l}\text{Solar apex R.A. } 261^\circ\, 29';\ \text{N.P.D. } 65^\circ\, 16'.\\ \text{Parallactic motion } 1''{\cdot}912.\end{array}\right.$$

So far as the situation of the apex is concerned, both these results stand in what (considering the nature of the subject) may be called good accordance with those of our art. 854. The parallactic motion (or which comes to the same, the actual velocity of the solar motion) is, indeed, much greater than that of art. 858. But this evidently results from the restriction of the inquiry to *stars of great proper motion.*

(858 *b.*) These results were deduced by Mr. Airy in a memoir communicated to the Roy. Ast. Soc. in 1859. The subject has since been resumed (avowedly in extension of the same principle to a larger list of stars), by Mr. Dunkin, who, using the same geometrical formulæ, but basing his results on the observed proper motions of 1167 stars, 819 in the northern and 348 in the southern hemisphere, *of all magnitudes, and all (sensible) amounts of proper motion,* arrives at the following results on either of Mr. Airy's two extreme suppositions:

1st Supposition { Solar apex in R.A. 261° 14′; N.P.D. 57° 5′. Parallactic motion 0″·3346.

2d Supposition { Solar apex 263° 44′; N.P.D. 65° 0′. Parallactic motion 0″·4103.

Agreeing remarkably (as to the second supposition, which is by far the more reasonable of the two) with the other, and, as regards both, exhibiting an almost perfect accordance in respect of parallactic motion with M. Struve's results as given in art. 858.

(858 *c.*) A very extraordinary circumstance remains to be noticed. From the general agreement of all the results of the investigations of so many astronomers and mathematicians, entering on the inquiry in such various ways, and employing such a multitude of stars so variously combined, there cannot remain a shadow of doubt either as to the *reality* of the solar motion, or as to its direction in space toward a point very near to R. A. 259°, N. P. D. 56°. As

to its velocity there is also every reason to believe that it is not extravagantly over- or under-estimated in the statement above given. But when we come to ascertain by calculation how large a portion of the whole proper motions of the stars; how much of that general *residuum* or *caput mortuum* alluded to in art. 856, as left outstanding after precession, aberration, and nutation, have exercised their solvent influences on it, remains yet unaccounted for; we shall find it includes by far the larger part of the total phenomenon of stellar proper motion. The sum of the squares of the total residua (in seconds of arc), *uncorrected* for the proper motion of the sun, for example, in Mr. Dunkin's 1167 stars are, in R. A., 78·7583, and in N. P. D. 63·2668. And when *corrected* for the effect of that motion (so concluded), they are represented in R. A. by 75·5831, and in N. P. D. by 60·9084. No one need be surprised at *this*. If the sun move in space, why not also the stars? and if so it would be manifestly absurd to expect that any movement could be assigned to the sun by any system of calculation which should account for more than a very small portion of the totality of the observed displacements. But what is indeed astonishing in the whole affair, is, that among all this chaotic heap of miscellaneous movement, among all this drift of cosmical atoms, of the laws of whose motions we know absolutely nothing, it should be possible to place the finger on one small portion of the sum total, to all appearance indistinguishably mixed up with the rest, and to declare with full assurance that this particular portion of the whole is due to the proper motion of our own system.]

[*Note* M *on Art.* 400, *note*[33].

The reference of the dark lines in the solar spectrum to absorptive action in the sun's atmosphere has of late received a most unexpected confirmation, and it may now be considered as almost certain that they owe their origin to the presence in that atmosphere of the vapors of metals and metalloids *identical* with those which exist here on earth. These vapors, or many of them, have been shown by Kirchoff, Bunsen and Fizeau to possess the singular property, when present in an *unburned* (or metallic) state in a flame, of destroying in the spectrum of that flame rays of *precisely* the refrangibilities of those which they themselves when burning emit in peculiar abundance. Though there is something so enigmatical as almost to appear self-contradictory in the facts adduced —the conclusion, especially as applied to the most conspicuous of all the lines (one double one in the yellow, marked D by Fraunhofer, and which owes its origin to *sodium*) seems inevitable. The spectra of some of the stars seem to indicate the presence of chemical elements *not* identifiable with any terrestrial ones.]

[*Note* N, *Art.* 905 *a a.*

(905 *a a.*) Since the publication of the later editions of this work, meteoric phenomena have engaged the assiduous attention of many zealous and devoted observers, and have acquired an especial interest from the reappearance, in 1866, of the great November display mentioned in art. 900, under circumstances which render it an epoch in what may henceforward be very properly termed METEORIC ASTRONOMY. Before giving any account of these, however, it will be proper to mention that by the exertions of a Committee of the British Association, consisting of Mr. Brayley, Mr. Glaisher, Mr. Greg, and Mr. A. S. Herschel, acting in conjunction with numerous coadjutors scattered over the area of our island (and in correspondence with M. Heis, Prof. Haidinger, and other continental observers and meteorologists), observing on the plan originally followed by Benzenburg and Brandes (much improved, however, and singularly facilitated by the use of a series of charts specially constructed on the gnomonic projection for the purpose

by the last-named member of the committee), a vast collection of observations for the determination of the heights of appearance and disappearance, the velocities, and paths, of individual meteors, has been accumulated, and a considerable number of additional radiant points corresponding to dates of periodic recurrence other than those of August 10 and November 13, determined. As regards the heights of appearance and disappearance, and the velocities (understanding, of course, the *relative* velocities resulting from the simultaneous motions of the earth and the meteor), the general result of these observations seems to be: 1st, to assign a height intermediate between 20 and 130 British statute miles above the earth's surface, for that to which the luminous or visible portion of the trajectory of *non-detonating* meteors is confined, with *average* heights of first appearance and final disappearance of 70 and 54 miles respectively, so far corroborating the evidence afforded by auroral phenomena of the extension of our atmosphere much beyond the usually assigned limits of 45 miles; 2dly, a relative velocity intermediate between 17 and 80 miles per second, with a general average of about 34 miles, fully bearing out the earlier conclusions of Benzenburg and Brandes; and 3dly, a series of radiant points and annual epochs of which the following (exclusive of those of Aug. 10 and Nov. 13) are the most remarkable:

Jan. 2d . . .	R.A. 234° . . .	N. dec. 51°
April 20th . . .	" 277° . . .	" 35°
Oct. 18th . . .	" 90° . . .	" 16°
Dec. 12th . . .	" 105° . . .	" 30°

(905 *b b.*) As regards the November display:—On tracing back the records of meteoric phenomena so far as they have been preserved by history or tradition, it has been as-

certained (chiefly by the laborious researches of Prof. Newton of New Haven, U. S.) that no less than twelve[18] such displays, well characterized, have been *noticed* and recorded as occurring from the year A.D. 902 onward down to 1833, both inclusive, viz. in the years A.D. 902, 934, 1002, 1101, 1202, 1366, 1533, 1602, 1698, 1799, 1832, 1833; all which are comprised within a chain of epochs breaking the interval between 902 and 1833 into periods of 32, 33, or 34 years each, corresponding to an average of 33·24 ($33\frac{1}{4}$) years, or of four such occurrences in 133 years. As to the calendar *dates* of the displays, the earliest, in A.D. 902, bears the date Oct. 13, O.S., and the others advance (with some considerable irregularities) in the calendar up to 1833, Nov. 13, N.S. Converting these dates into Julian days current (arts. 929, 930), we find them to be respectively 2,050,799 and 2,390,879, the difference of which, 340,068 days, exceeds 931 tropical years ($=340,040^d$) by 28 days; so that the dates advance in the calendar at an average rate of 28 days in 931 years, or almost exactly 3 days in a century. The general impression resulting from the intervals of 33 and 34 years between the great displays of 1799 and 1832, 1833, that a similar one might be expected in 1866 or 1867, was by this converted almost into a certainty; and on the strength of this induction a grand meteoric exhibition on the night between Nov. 13 and 14, 1866, was announced as almost sure to take place, and all observers were forewarned to be on the watch. The verification of this prediction will be fresh in the recollection of our readers, and the

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[18] That of A.D. 931, Oct. 16, O.S., is here omitted. It seems to have been but a feeble exhibition, an irregular precursor of the more normal one of Oct. 14, 934.

spectacle presented by the heavens on that night, though falling short of what the glowing and no doubt exaggerated descriptions of the phenomenon of 1799 might have led some to expect, was such as can never be forgotten by those who witnessed it. Those who were not so fortunate will do well to be on the watch on the same anniversary in the current year 1867.[19]

(905 *c.c.*) Attention being especially directed to the situation of the radiant on this occasion, it was fixed (in reference to the ecliptic) in long. 142° 35′; lat. 10° 27′ N., at a point between the stars ζ and ϵ Leonis, and somewhat above the star marked x in that constellation in Bode's Catalogue. Now the longitude of the earth at that time, as seen from the sun, was 51° 28′, so that the radiant (in confirmation of a remark made by Prof. Encke on the occasion of the display of 1833), if projected on the plane of the ecliptic, would be almost exactly in the direction of a tangent to the earth's orbit at the moment, or in "the apex of the earth's way." Hence it follows, that, regarding each meteor as a small planet, it must have been revolving (*in a retrograde direction, so as to meet the earth*) either in a circle concentric with the earth's orbit (a thing in itself most improbable, and which would bring about a rencounter every year, contrary to observation) or in an ellipse having either its perihelion or its aphelion coincident, or very nearly so, with the point of rencounter at the descending node, in longitude 51° 28′;

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[19] On that occasion (Nov. 13 and 14, 1867) the principal display took place in longitudes much westward of our island. At Bloomington, Indiana, U. S., 525 were seen by Prof. Kirkwood between midnight and 5h. 15m. A.M. Off Martinique, they appeared as a brilliant shower, and at Trinidad, according to Commander Chimmo, 1600 were counted between 2h. A.M. and daylight; while at Nassau in the Bahamas, Captain Stuart and his co-observers registered 1040 between 1h. 0m. and 5h. 34m. A.M.

and as a necessary consequence with its major axis lying in or very *nearly* in the plane of the ecliptic.

(905 *d d.*) Admitting the meteors to be revolving planetules, the recurrence of these rencounters at average intervals of 4 in 133 years is explicable on two distinct hypotheses as to the kind of ellipse described by the meteoric group. It may be either one very nearly approaching to a circle described in a period not very different from a sidereal year, or a very elongated one described in the exact period of $33\frac{1}{4}$ of such years. We will consider the cases separately. The first supposition admits of the adoption of two distinct ellipses:—1st, that suggested by Prof. Newton, in which the rencounter takes place at the *aphelion* of an ellipse described in $354^d \cdot 57$, or $10^d \cdot 67$ short of a sidereal year, corresponding to a semiaxis 0·981, and an excentricity 0·0204; or, 2dly, that proposed by the writer of an article on Meteoric Showers in the "Edinburgh Review" for January, 1867, where it is supposed to happen at the *perihelion* of an ellipse described in $376^d \cdot 56$, or $11^d \cdot 33$ *more* than a year—corresponding to a semiaxis 1·021, and an excentricity 0·0192. In the first of these ellipses, a meteor revolving would in each sidereal year gain 10° 50′ in its orbit on a complete revolution, and in the other would lose as much; so that at the end of 33 years, in the former case it would be found to have overshot the original point of rencounter by 2° 30′, and in the latter to fall short of it by just so much: and tracing it round from revolution to revolution, it will be found in either case that after a series of intervals succeeding each other in the cycle 33, 33, 33, 34 years, the meteor will always be found so near the original point of rencounter that an extension of the whole group so as to occupy 11° in their common orbit will render extremely probable, and one of 22° will insure its

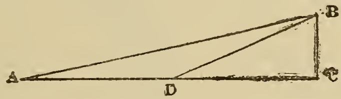
penetration by the earth at some point or other, with a probability of such penetration taking place twice in two successive years.

(905 *e e.*) The other hypothesis, suggested by Sig. Schiaparelli (Director of the Observatory at Milan), is that of a rencounter at or very near to the *perihelion* of an ellipse of $33\frac{1}{4}$ years, corresponding to a semiaxis 10·340, and an excentricity 0·9033; the rencounter in this, as in the other, taking place also at the descending node. Such a period, combined with an extent of the group on the orbit such as would occupy somewhat more than a year in passing through the node (*i.e.* $\frac{4}{133}$ of its whole circumference), would bring round rencounters in precisely the same cycle of years, with a fair probability, and if of twice that extent with a certainty of their happening. An increased extent of the group to somewhat beyond this would give rise to a frequent occurrence of two or even three rencounters in annual succession, and would therefore cover the whole series of recorded instances.

(905 *f f.*) The regular advance of three days per century in the calendar date of the phenomenon is partly accounted for by the greater length of the sidereal year (which brings the earth round to the same point in its orbit) as compared with the tropical year (which brings it to the same *longitude*, reckoned from the receding equinox, by which the calendar is regulated). This accounts for $1^d{\cdot}4$ per century; the remaining $1^d{\cdot}6$ must arise from a slow and regular advance in the place of the node to the amount of 1° 36′ per century, or 57″·6 per annum, due probably to planetary perturbation, and chiefly, no doubt, to the disturbing action of the earth itself in its successive passages through the group.

(905 *g g.*) On either of the two former orbits the velocity

of the meteors will be very nearly equal to that of the earth in a retrograde direction, whence it will readily appear that the true inclination of the orbit will be almost exactly double the apparent—that is to say, 20° 54′. In the case of the long ellipse, the velocity of the meteor in perihelio will be found to be to that of the earth as 1·371 : 1, so that supposing AC to be the earth's orbit, and BD that of the meteor, the



apparent inclination BAD being 10° 27′, and the sides BD, DA, respectively, 1·371 and 1, we shall find the angle DBA=7° 13′, and therefore the true inclination BDC=18° 31′.

(905 *h h.*) The supposition of minute planetary bodies revolving in a nearly circular orbit of almost exactly the dimensions of the earth's in a retrograde direction, and at an inclination not greater than that of some of the asteroids, stands in such strong opposition to all the analogies of our system, as to render it in itself highly improbable; add to which, that (as no perturbative action could possibly have flung them from without into such an orbit) they must be supposed to have so circulated for countless ages, during which time their innumerable rencounters with the earth must have torn the group to pieces, and scattered all its members which escaped extinction into orbits of every different inclination and excentricity. On the other hand, the ellipse of $33^{y} \cdot \frac{1}{4}$ has a decidedly cometary character; and in such, retrograde motion is not uncommon. By a most singular coincidence, remarked almost simultaneously by Messrs. Peters and Schiaparelli (a coincidence too close

and striking to admit of hesitation as to their community of origin), the elements of the first comet of 1866, discovered by M. Tempel, coincide almost precisely, in every particular except in the date of the perihelion passage, with those we have just derived from the very simple considerations adduced. The parallel is as below:

	Meteoric Orbit	Tempel's Comet
Perihelion passage . . .	Nov. 13, 1866 . . .	Jan. 11, 1866
Perihelion distance . . .	0·9893[20] . . .	0·9765
Excentricity . . .	0·9033 . . .	0·9054
Semiaxis major . . .	10·340 . . .	10·324
Inclination . . .	18° 31′ . . .	17° 18′·1
Long. descending node . .	51° 28′ . . .	51° 26′·1
Periodic time . . .	33y·25 . . .	33·176
Motion . . .	Retrograde . . .	Retrograde[21]

(905 *i i.*) It will not fail to have been observed that a major semiaxis 10·34 with a perihelion distance 1 will throw the aphelion of the meteoric orbit to a distance from the sun=19·68, that is to say, but a short distance beyond the orbit of Uranus; while the fact of the axis major itself lying exactly or at least very nearly in the plane of the ecliptic, a plane itself very little inclined to the orbit of Uranus, will insure a very near appulse of the meteors to that planet whenever their two mean motions may have brought or may hereafter bring them to the corresponding parts of their orbits, allowing for the change (if any) in the position of the axis. We say, if any, for it is not of necessity the same

[20] This is the earth's radius vector on Nov. 13.

[21] The computations of Sig. Schiaparelli, founded on a somewhat different (and, we are inclined to think, less accurate) situation of the radiant of last November, lead him to assign the date Nov. 10 for the perihelion passage, and to the perihelion itself the longitude 56° 25′·9, again agreeing well with that of the comet in question, which is 60° 28′. But we have preferred (avoiding all niceties, which, in the actual uncertainty as to the *exact* place of the radiant, are, after all, premature), for the sake of perspicuity, to present the chain of reasoning in a form requiring almost no calculation.

as that of the node, and, though calculable, has not as yet been calculated. This, however, does not affect the conclusion that such near appulse must at *some* former time have taken place, and will do so again. M. Leverrier, to whom these considerations seem to have occurred independently, has concluded that it *did* take place about the year A.D. 126; and the motion of both bodies being very slow at that time (the velocity of the meteors being in aphelio only 0·07 of that of the earth, or only 1·32 mile per second), they would remain for a long time within the influence of the planet's disturbing power, while at the same time that power would be acting at the greatest advantage to produce deflection from their line of motion. Hence that illustrious astronomer was led to conclude, that, just as Jupiter on a similar occasion seized on and threw into an orbit of short period Lexell's comet (see art. 585), so at that epoch a wandering group of planetules, whose existence would, but for that meeting, have never become known to us, was deflected into the ellipse they actually describe. Sig. Schiaparelli, on the other hand, considering that the semi-minor axis of the meteoric ellipse is but small (0·441), so that by reason of the moderate inclination the meteor in its course can never rise much more than 1½ radius of the earth's orbit above its plane; appears disposed to attribute their present form of orbit to the attraction of Jupiter or Saturn, within whose disturbing influence he considers that they must at some period or other have passed, an opinion which appears to us less probable, inasmuch as the disturbing force would in that case have tended chiefly in a direction at right angles to the plane of their motion, and (by reason of the much greater velocities of both bodies) have acted for a much shorter time.

(905 *jj.*) For the meteors of the 10th of August, adopting as the place of the radiant the star *k* Persei, assigned by the observations of Mr. A. S. Herschel in 1863, and *assuming the orbit to be a parabola* (an assumption which determines the velocity at the moment of rencounter, being to that of the earth regarded as describing a circle, in the constant ratio of $\sqrt{2} : 1$), a velocity which he found to agree tolerably well with that directly determined by Mr. Herschel and his co-observers on the same occasion, M. Schiaparelli has also computed the elements of *their* orbit. And again, by a coincidence hardly less striking, these are found to agree with the elements of the great comet of 1862, as the following comparison will show:

	August Meteors, Schiaparelli's Elements.	Comet III. 1862. Elements of Oppolzer.
Passage through descending Node	1866, Aug. 10·75	—
Perihelion Passage	— July 23·62	1862, Aug. 22·9
Longitude of Perihelion . . .	343° 38′	344° 41′
Longitude of Ascending Node .	138° 16′	137° 27′
Inclination	64° 3′	66° 25′
Perihelion Distance	0·9643	0·9626
Period	—	123·74
Motion	Retrograde	Retrograde

Without supposing the orbit absolutely parabolic, an ellipse of long period (say 124 years) would equally well satisfy the conditions; but to make the rencounter annual, a complete annular or elliptic stream of meteors would be required. The radiant point of the August meteors, however, seems hardly so definite as that of the November group, the determination in different years by different observers differing considerably. Both these considerations would seem to authorize the ascription of a far higher antiquity to the introduction of this assemblage into our system, giving time not only for the individual meteors to gain or lose upon each other in consequence of minute differences in their periodic time, so as to draw out the original group into a stream, but to disperse themselves over considerable differences of inclination and excentricity by the effect of the earth's perturbative action, while all the phenomena of the November group point to a much more recent origin.

[*Note* O, *Art.* 395.

(395 *b b.*) The total solar eclipse of August 18, 1868, which, commencing not far from Aden, at the entrance of the Red Sea, traversed the whole peninsula of India from Malwa to Masulipatam, and pursued its course eastward and southward across the Malayan peninsula, to the extreme northern point of Australia, afforded an excellent opportunity for the critical examination of the marginal protuberances as well as the phenomena of the corona; which, if seen at all from a station on the central line, could not be held to originate in the earth's atmosphere by reason of the great breadth of the total shadow (at least 115 miles). Accordingly, it was eagerly seized, and competent observers, well furnished with every requisite instrument and means of observation and record, took up their stations at points on or very nearly adjacent to the central line. The unusual duration of the total obscuration, being nearly six minutes, allowed ample time for making all the necessary observations, as well as for securing photographs, which last desideratum was successfully accomplished at Guntoor in India, by skilled photographers under the direction of Major Tennant, as also at Aden. The final results may be thus briefly stated.—1. The darkness was by no means so great as was expected: doubtless, owing to the

great amount of light emitted by the corona and the marginal prominences. The light of the former gave a continuous spectrum, and moreover was found to be distinctly and strongly polarized; everywhere in a plane passing through the point examined and the sun's centre. This establishes beyond all possible doubt its origin in reflection from a solar atmosphere exterior to the photosphere (or at least of some sort of envelope, whether atmospheric or nebulous in its nature and connection with the sun) of vast extent, though probably of small density, and is altogether opposed to its origination in that of our earth; our sky light so near the sun exhibiting no trace of polarization.—2. The red prominences, which were numerous and most remarkable, showed no sign of polarization, and were, therefore, self-luminous. One of them, the most conspicuous, projected like a horn or tall excrescence to the distance of 3′ 10″ from the true limb of the sun, which corresponds to a vertical elevation of 90,200 miles above the level of the photosphere. Its outline, as photographed at Guntoor, was as in the annexed diagram, indicating by its markings a spiral form, like that which might be conceived to result from a combined rotatory and ascensional movement, as of a vast column of ignited vapor rushing upward with a swirl from the photosphere into the higher regions of a non-luminous atmosphere.—3. The light of the prominences subjected to spectroscopic examination was in accordance with this idea. It gave no continuous spectrum, but appeared to consist of distinct *monochromatic* rays, or definite

bright lines (characteristic of incandescent gases). Of these, M. Janssen, stationed at Guntoor, saw six, in the red, yellow, green, blue, and violet regions of the spectrum; two of them corresponding to Fraunhofer's lines C, F, indicative of hydrogen. Major Tennant, at the same station, perceived only four, viz.: C in the red, and D in the orange (corresponding respectively to hydrogen and sodium)—one in the green near F (hydrogen), and a fourth seen with difficulty in the blue near to G. Lieutenant Herschel at Jamkandi (where the total phase of the eclipse was much interfered with by passing clouds) perceived distinctly three vivid lines, red, orange, and blue, and no others, nor any trace of a continuous spectrum. The orange line proved by measurement to coincide precisely with D, the others approximated to C and F, and probably, the difficult circumstances of the measurements considered, were coincident with those lines. M. Rayet at Wah Tonne, in the Malayan peninsula, noted no less than nine brilliant lines corresponding to the solar dark lines B, D, E, b, F, two adjacent to G, and one between b and F (probably Barium). These observations are quite decisive as to the gaseous nature and vehement incandescence of the prominences, and indicate astonishingly powerful ascensional movements of what might be called flame (were combustion possible) in an atmosphere reposing on the photosphere.—4. Besides these hard and sharply defined prominences, were also seen ranges irregular in form, of what might perhaps be considered cloudy or vaporous matter, of less intensity and softer outline.

(395 *c c*.) Reasoning on the monochromatic character of the light emitted by incandescent gases, and speculating on the extreme probability of the solar prominences being

in the nature of tumultuous ejections of such gases, it had early occurred both to Mr. Huggins and Mr. Lockyer that they might possibly become the subject of spectroscopic study, as appendages to the sun's limb, or in the umbræ of spots unilluminated by photospheric light, without the necessity of waiting for the rare occurrence of a total eclipse. Accordingly, during the two years immediately preceding that of August 18, 1868, the former made several attempts with various spectroscopic and other contrivances (such as viewing the projected image of the sun's border through combinations of colored glasses, etc.) to obtain a view of them, though without success; and the latter had applied for and obtained from the Royal Society a grant for the construction of an apparatus for the purpose, which, however, was not completed till after the occurrence of the eclipse. Meanwhile the actual observation of the monochromatic character of their light, and the exact coincidence of their lines with situations which in the spectrum of the photosphere are marked by a deficiency of light, at once suggested to M. Janssen, as it did also to Lieutenant Herschel, the possibility of discerning, or at all events of, as it were, feeling them out, the former by means of the spectroscope, the latter by combinations of colored glasses. M. Janssen, on the day immediately following the eclipse, put his conception into practice, and at once succeeded. Placing the slit of his spectroscope so as partly to be illuminated by the edge of the photosphere, and partly by the light (from whatever origin) exterior to it, he found the spectrum of the former portion in the immediate neighborhood of the ray C to be crossed (as might be expected) by that dark line. At the point of the limb to which his examination was first directed, nothing was seen beyond. But, on shifting the point of examination gradually along the limb, a small dot of ruddy light was perceived, in exact continuation outward of the dark line, which, on continuing the movement of the spectroscope along the limb in the same direction, gradually lengthened, and then again shortened: thus revealing the existence of a prominence giving out that particular monochromatic red, whose form and outline he was thus enabled to trace out. Directing his attention, in like manner, to the dark line F, the same phenomenon was repeated, in the tint proper to that region of the spectrum. At some points it was also observed that the bright line of the prominence encroached upon and extended into the corresponding dark line of the photosphere.

(395 *d d.*) Mr. Lockyer's apparatus having meanwhile been completed, he was at length enabled to announce (on October 20) that after a number of failures which made the attempt seem hopeless, he had at length succeeded in observing, as part of the spectrum of a solar prominence, three bright lines, one absolutely coincident with C, one near D, and one nearly coincident with F. On February 16, 1869, another practical step in the same direction was made by Mr. Huggins, who, limiting, by an ingenious contrivance, the light admitted to his spectroscope to rays of about the refrangibility C, widening the slit sufficiently to admit of the whole prominence being included in its field, and absorbing the light of other refrangibilities so admitted by a ruby glass, was enabled distinctly to perceive *at one view*, the form of the prominence. Almost immediately after, Mr. Lockyer succeeded, by merely widening the slit of his spectroscope, *without the use of any absorptive media*, in obtaining a clear view of the forms in ques-

tion. "The solar and atmospheric spectra being hidden, and the image of the wide slit alone being visible, the telescope or slit is moved slowly, and the strange shadow-forms flit past. Here one is reminded, by the fleecy infinitely delicate cloud-films, of an English hedgerow with luxuriant elms; here of a densely intertwined tropical forest, the intimately interwoven branches threading in all directions; the prominences generally expanding as they mount upward, and changing slowly, indeed almost imperceptibly." Lastly, on the 4th, 5th and 6th of May, Lieutenant Herschel found the spectrum of the solar envelope to be visible without difficulty, and without other aid than the spectroscope adapted to his telescope, and was enabled to form a general picture of the distribution of the luminous region surrounding the sun. Two prominences were in particular examined, one of which formed a luminous cloud floating 1′ or 2′ above the surface. He perceived also (now for the first time) a fourth line near G (since seen repeatedly), and subsequently another between F and G. On this last occasion, having at first swept round the sun and found nothing particularly worthy of remark, on returning to the point of departure, he perceived the lines much more brilliant and intense than usual, and further scrutiny satisfied him that he had been witness to a "violent and spasmodic eruption of vapor" lasting only a few minutes! The mode in which he was enabled thus to discern the forms of the solar clouds consisted in giving to the telescope a vibratory motion up and down, on the principle of the persistence of luminous impressions on the retina, by which the perception of the total form of an object results from the mental combination of a series of linear sections of its area. And he describes the appearance of these solar clouds as "very similar to terrestrial—fleecy, irregular-shaped, and illuminated; just such as eclipses have told us they are." We have thus a new chapter of solar physics opened out, the commencement, doubtless, of a series of grand discoveries as to the nature and constitution of the great central body of our system. Mr. Huggins has also applied spectrum analysis to the comæ and tails of comets in which he considers satisfactory proof to exist of the presence of carbon.

LIST OF PLATES

APPENDIX

APPENDIX

I. LISTS OF NORTHERN AND SOUTHERN STARS, WITH THEIR APPROXIMATE MAGNITUDES, ON THE VULGAR AND PHOTOMETRIC SCALE

1. NORTHERN STARS

Star.	Mag.		Star.	Mag.		Star.	Mag.		Star.	Mag.	
	Vulg.	Phot.		Vulg.	Phot.		Vulg.	Phot.		Vulg.	Phot.
Arcturus . .	0·77	1·18	γ Cassiopeiæ .	2·52	2·93	η Draconis . .	3·02	3·43	ε Aurigæ (Var)	3·37	3·78
Capella . .	1·0:	1·4:	α Andromedæ.	2·54	2·95	β Draconis . .	3·06	3·47	γ Lyncis . .	3·39	3·80
Lyra . . .	1·0:	1·4:	α Cassiopeiæ .	2·57	2·98	β Arietis . .	3·09	3·50	ζ Draconis .	3·40	3·81
Procyon . .	1·0:	1·4:	γ Geminorum .	2·59	3·00	γ Pegasi . . .	3·11	3·52	π Herculis . .	3·41	3·82
α Orionis . .	1·0:	1·43	Algol (Var.) .	2·62	3·03	ε Virginis? . .	3·14	3·55	β Canis Min.?	3·41	3·82
Aldebaran .	1·1:	1·5:	ε Pegasi . . .	2·62	3·03	θ Aurigæ . .	3·17	3·58	ζ Tauri . . .	3·42	3·83
α Aquilæ . .	1·28	1·69	γ Draconis . .	2·62	3·03	β Herculis . .	3·18	3·59	δ Draconis .	3·42	3·83
Pollux . . .	1·6:	2·0:	β Leonis . . .	2·63	3·04	α Canum Ven .	3·22	3·63	μ Geminorum.	3·42	3·83
Regulus . .	1·6:	2·0:	α Ophiuchi . .	2·63	3·04	β Ophiuchi . .	3·23	3·64	γ Bootis . .	3·43	3·84
α Cygni . . .	1·90	2·31	β Cassiopeiæ .	2·63	3·04	δ Cygni . . .	3·24	3·65	ε Geminorum.	3·43	3·84
Castor . . .	1·94	2·35	γ Cygni . . .	2·63	3·04	ε Persei . . .	3·26	3·67	δ Herculis . .	3·44	3·85
ε Ursæ (Var.) .	1·95	2·36	α Pegasi . . .	2·65	3·06	η Tauri? . . .	3·26	3·67	δ Geminorum.	3·44	3·85
α Ursæ (Var.) .	1·96	2·37	β Pegasi . . .	2·65	3·06	ζ Persei . . .	3·27	3·68	q Orionis . .	3·45	3·86
α Persei . . .	2·07	2·48	α Coronæ . .	2·69	3·10	ζ Herculis . .	3·28	3·69	β Cephei . .	3·45	3·86
η Ursæ (Var.) .	2·18	2·59	γ Ursæ . . .	2·71	3·12	ι Aurigæ . .	3·29	3·70	θ Ursæ . . .	3·45	3·86
γ Orionis . .	2·18	2·59	β Ursæ . . .	2·77	3·18	γ Ursæ Minor .	3·30	3·71	ι Ursæ . . .	3·46	3·87
β Tauri . . .	2·28	2·69	ε Bootis . . .	2·80	3·21	η Pegasi . . .	3·31	3·72	η Aurigæ . .	3·46	3·87
Polaris . .	2·28	2·69	ε Cygni . . .	2·88	3·29	ζ Aquilæ . . .	3·32	3·73	γ Lyræ . . .	3·47	3·88
γ Leonis . . .	2·34	2·75	α Cephei . . .	2·90	3·31	β Cygni . . .	3·33	3·74	η Geminorum.	3·48	3·89
α Arietis . .	2·40	2·81	α Serpentis . .	2·92	3·33	γ Persei . . .	3·34	3·75	γ Cephei . .	3·48	3·89
ζ Ursæ . . .	2·43	2·84	δ Leonis . . .	2·94	3·35	μ Ursæ . . .	3·35	3·76	κ Ursæ . . .	3·49	3·90
β Andromedæ.	2·45	2.86	γ Aquilæ . .	2·98	3·39	β Triang. Bor. .	3·35	3·76	ε Cassiopeiæ .	3·49	3·90
β Aurigæ . .	2·48	2·89	δ Cassiopeiæ .	2·99	3·40	δ Persei . . .	3·36	3·77	θ Aquilæ . .	3·50	3·91
γ Andromedæ.	2·50	2·91	η Bootis . . .	3·01	3·42	ψ Ursæ . . .	3·36	3·77			

2. SOUTHERN STARS

Star.	Mag. Vulg.	Mag. Phot.	Star.	Mag. Vulg.	Mag. Phot.	Star.	Mag. Vulg.	Mag. Phot.	Star.	Mag. Vulg.	Mag. Phot.
Sirius . . .	0·08	0·49	α Hydræ . .	2·30	2·71	κ Scorpii . .	2·91	3·32	δ Corvi . . .	3·22	3·63
η Argus (Var.).	—	—	δ Canis . . .	2·32	2·73	η Centauri . .	2·91	3·32	β Eridani . .	3·26	3·67
Canopus . .	0·29	0·70	α Pavonis . .	2·33	2·74	κ Argus . . .	2·94	3·35	θ Argus . .	3·26	3·67
α Centauri . .	0·59	1·00	β Gruis . . .	2·36	2·77	β Corvi . . .	2·95	3·36	β Hydri . . .	3·27	3·68
Rigel . . .	0·82	1·23	σ Sagittarii. .	2·41	2·82	β Scorpii . . .	2·96	3·37	ε Corvi . . .	3·28	3·69
α Eridani . .	1·09	1·50	δ Argus . . .	2·42	2·83	ζ Centauri . .	2·96	3·37	β Aræ . . .	3·31	3·72
β Centauri. .	1·17	1·58	β Ceti	2·46	2·87	ζ Ophiuchi . .	2·97	3·38	α Toucani. . .	3·32	3·73
α Crucis . . .	1·2	1·6	λ Argus . . .	2·46	2·87	α Aquarii . .	2·97	3·38	β Capricorni .	3·32	3·73
Antares . .	1·2	1·6	θ Centauri . .	2·54	2·95	π Argus . . .	2·98	3·39	ρ Argus . . .	3·32	3·73
Spica . . .	1·38	1·79	β Canis . . .	2·58	2·99	δ Centauri . .	2·99	3·40	π Scorpii . .	3·35	3·76
Fomalhaut .	1·54	1·95	κ Orionis . .	2·59	3·00	α Leporis . .	3·00	3·41	β Leporis . .	3·35	3·76
β Crucis . . .	1·57	1·98	δ Orionis . .	2·61	3·02	δ Ophiuchi . .	3·00	3·41	γ Lupi . . .	3·36	3·77
α Gruis . . .	1·66	2·07	γ Centauri . .	2·68	3·09	ζ Sagittarii .	3·01	3·42	υ Scorpii . .	3·37	3·78
γ Crucis . . .	1·73	2·14	ε Scorpii . .	2·71	3·12	π Ophiuchi . .	3·05	3·46	ι Orionis . .	3·37	3·78
ε Orionis . . .	1·84	2·25	ζ Argus . . .	2·72	3·13	β Libræ . . .	3·07	3·48	α Aræ . . .	3·40	3·81
ε Canis . . .	1·86	2·27	α Phœnicis . .	2·78	3·19	γ Virginis . .	3·08	3·49	π Sagittarii .	3·40	3·81
λ Scorpii . .	1·87	2·28	ι Argus . . .	2·80	3·21	μ Argus . . .	3·08	3·49	α Museæ . .	3·43	3·84
ζ Orionis . .	2·01	2·42	α Lupi . . .	2·82	3·23	δ Sagittarii . .	3·11	3·52	α Hydri? . .	3·44	3·85
β Argus . . .	2·03	2·44	ε Centauri . .	2·82	3·23	α Libræ . . .	3·12	3·53	τ Scorpii . .	3·44	3·85
γ Argus . . .	2·08	2·49	η Canis . . .	2·85	3·26	λ Sagittarii . .	3·13	3·54	ζ Hydræ . .	3·45	3·86
ε Argus . . .	2·18	2·59	β Aquarii . .	2·85	3·26	β Lupi	3·14	3·55	γ Hydræ . .	3·46	3·87
α Trianguli A.	2·23	2·64	δ Scorpii . .	2·86	3·27	α Columbæ . .	3·15	3·56	β Trianguli .	3·46	3·87
ε Sagittarii .	2·26	2·67	η Ophiuchi . .	2·89	3·30	ι Centauri . .	3·20	3·61	σ Scorpii . .	3·50	3·91
θ Scorpii . .	2·29	2·70	γ Corvi . . .	2·90	3·31	δ Capricorni .	3·20	3·61	τ Argus . .	3·50	3·91

II. SYNOPTIC TABLE OF THE ELEMENTS OF THE PLANETARY SYSTEM

N.B.—a denotes the mean distance from the sun, that of the earth being taken for unity; P the mean sidereal period in mean solar days; e the excentricity in decimal parts of the semiaxis; i the inclination of the orbit to the ecliptic; ☊ the longitude of the ascending node; ω that of the perihelion from node on orbit; L the mean longitude of the planet at the moment of the epoch E, for which the elements are stated; M the denominator of the fraction expressing the mass of the planet, that of the sun being 1; D the diameter in miles; △ the density, that of the earth being 1; T the time of rotation on its axis; d the mean angular equatorial diameter of the body of the planet, at its mean distance from the earth, in seconds; ϵ the ellipticity of the spheroid, as a fraction of the equatorial diameter; γ the inclination of the axis of rotation to the plane of the ecliptic; H the mean intensity of light and heat received from the sun, that received by the earth being 1. The asteroids are numbered in their order of discovery.

		a	P	e	i		
					°	′	″
☉	The Sun						
☿	Mercury . .	0·3870981	87·9692580	0·2054925	7	0	9·1
♀	Venus . . .	0·7233316	224·7007869	0·0068722	3	23	28·5
⊕	Earth . . .	1·0000000	365·2563612	0·0167918			
♂	Mars . . .	1·5236923	686·9796458	0·0931125	1	51	6·2
1	Ceres . . .	2·7664313	1680·650	0·0805096	10	36	27·8
2	Pallas . . .	2·7686906	1683·103	0·2400116	34	43	8·7
3	Juno . . .	2·6707543	1594·221	0·2565949	13	1	9·0
4	Vesta . . .	2·3616682	1325·640	0·0890879	7	7	55·7
5	Astræa. . .	2·5778960	1511·800	0·1872123	5	19	5·7
6	Hebe . . .	2·4246290	1379·007	0·2028856	14	46	38·0
7	Iris	2·3860360	1346·212	0·2312145	5	27	53·3
8	Flora . . .	2·2013860	1193·004	0·1567040	5	53	8·0
9	Metis . . .	2·3860333	1346·211	0·1238156	5	36	7·3
10	Hygeia. . .	3·1511972	2043·200	0·0993911	3	48	47·1
11	Parthenope .	2·4524062	1402·770	0·0990197	4	37	0·9
12	Victoria . .	2·3344916	1302·756	0·2178214	8	23	1·5
13	Egeria . . .	2·5762970	1510·400	0·0867520	16	31	54·9
14	Irene . . .	2·5863965	1519·288	0·1686725	9	7	27·3
15	Eunomia . .	2·6441373	1570·452	0·1868308	11	43	35·2
16	Psyche. . .	2·9263840	1823·493	0·1341254	3	3	58·6
17	Thetis . . .	2·4732811	1420·721	0·1276441	5	36	5·6
18	Melpomene .	2·2956321	1270·430	0·2174420	10	8	58·6
19	Fortuna . .	2·4415662	1393·481	0·1572326	1	32	30·2
20	Massilia . .	2·4089122	1365·621	0·1442977	0	41	11·1
21	Lutetia . .	2·4347444	1387·643	0·1617091	3	5	21·0
22	Calliope . .	2.9094987	1812·699	0·1019602	13	45	28·4
23	Thalia . . .	2·6282154	1556·291	0·2321817	10	13	13·1
24	Themis. . .	3·1439415	2036·147	0·1170627	0	48	51·6
25	Phocea . .	2·3999822	1358·033	0·2547900	21	34	54·9
26	Proserpina .	2·6552100	1580·319	0·0871572	3	35	38·0
27	Euterpe . .	2·3463460	1312·760	0·1736210	1	35	30·3
28	Bellona . .	2·7784700	1691·986	0·1500986	9	21	26·3
29	Amphitrite .	2·5536647	1490·542	0·0737850	6	7	49·8

		a	P	e	i
					° ′ ″
30	Urania . .	2·3654133	1328·795	0·1276142	2 6 1·8
31	Euphrosyne .	3·1571421	2048·988	0·2181494	26 26 0·0
32	Pomona . .	2·5887137	1521·332	0·0802144	5 28 51·6
33	Polyhymnia .	2·8650620	1771·328	0·3381960	1 56 18·9
34	Circe . . .	2·6870950	1608·877	0·1056142	5 26 37·5
35	Leucothea . .	3·0060548	1903·680	0.2136315	3 10 32·1
36	Atalanta . .	2·7479006	1663·792	0·2971309	18 41 51·6
37	Fides . . .	2·6421873	1568·674	0·1748927	3 7 10·5
38	Leda . . .	2·7401483	1656·758	0·1554322	6 58 25·3
39	Lætita . . .	2·7699820	1683·862	0·1111018	10 20 58·3
40	Harmonia . .	2·2677226	1247·335	0·0463143	4 15 52·3
41	Daphne . .	2·7665526	1680·725	0·2690810	16 3 13·7
42	Isis	2·439999	1392·14	0·2256143	8 34 29·8
43	Ariadne . .	2·203358	1194·61	0·1675933	3 27 39·1
44	Nysa . . .	2·424741	1379·10	0·1503545	3 41 40·9
45	Eugenia . .	2·720510	1638·98	0·0892361	6 34 57·0
46	Hestia . . .	2·526151	1466·52	0·1646660	2 17 36·7
47	Aglaia . . .	2·880197	1785·38	0·1324005	5 0 9·6
48	Doris . . .	3·109420	2002·71	0·0769467	6 29 40·2
49	Pales . . .	3·080259	1974·60	0·2385024	3 8 45·1
50	Virginia . .	2·649741	1575·45	0·2873290	2 47 36·9
51	Nemausa . .	2·365579	1328·94	0·0662400	3 47 53·0
52	Europa . .	3·099773	1993·39	0·1014745	7 24 40·3
53	Calypso . .	2·620957	1549·84	0·2038207	5 6 42·7
54	Alexandra .	2·714864	1633·88	0·1966174	11 47 11·3
55	Pandora . .	2·759621	1674·45	0·1420334	7 13 28·1
56	Melete . . .	2·597110	1528·74	0·2370434	8 1 50·8
57	Mnemosyne .	3·157288	2049·13	0·1041157	15 8 1·6
58	Concordia . .	2·694441	1615·48	0·0401686	5 1 6·7
59	Olympia . .	2·713870	1632·98	0·1173539	8 37 41·3
60	Echo . . .	2·393088	1352·19	0·1847386	3 34 18·7
61	Denae . . .	3·003978	1901·71	0·1684354	18 16 32·9
62	Erato . . .	3·130695	2023·29	0·1709820	2 12 25·9
63	Ausonia . .	2·393785	1352·78	0·1247228	5 46 54·5
64	Angelina . .	2·680557	1603·01	0·1290952	1 19 51·5
65	Maximiliana .	3·419842	2309·98	0·1201719	3 28 9·5
66	Maia . . .	2·663544	1587·77	0·1339165	3 2 24·6
67	Asia . . .	2·420277	1375·29	0·1848482	5 59 27·3
68	Leto . . .	2·722658	1640·92	0·1697847	8 10 16·5
69	Hesperia . .	2·994932	1893·12	0·1745242	8 28 25·0
70	Panopea . .	2·629115	1557·09	0·1950190	11 31 56·5
71	Niobe . . .	2·756159	1671·30	0·1737289	23 18 29·5
72	Feronia . .	2·274958	1253·31	0·1164641	5 25 55·2
73	Clytie . . .	2·665541	1589·56	0·0454381	2 24 57·6
74	Galatea . .	2·575212	1509·45	0·2554310	3 15 43·8
75	Eurydice . .	2·665856	1589·84	0·3054377	4 59 8·8
76	Freia . . .	3·188960	2080·04	0·0302110	2 13 3·0
77	Frigga . . .	2·673752	1596·91	0·1358136	2 27 55·2
78	Diana . . .	2·626316	1554·60	0·2066804	8 39 47·0
79	Eurynome . .	2·444173	1395·71	0·1953329	4 36 50·5
80	Sappho . .	2·296375	1271·05	0·2002178	8 36 47·9

		a	P	e	i		
					°	′	″
81	Terpsichore .	2·850428	1757·77	0·2100461	7	55	22·0
82	Alcmene . .	2·764144	1678·57	0·2299055	2	50	34
♃	Jupiter . . .	5·2027760	4332·5848212	0·0481626	1	18	51·3
♄	Saturn . . .	9·5387861	10759·2198174	0·0561502	2	29	35·7
♅	Uranus . .	19·1823900	30686·8208296	0·0466683	0	46	28·4
♆	Neptune . .	30·05660	60186·6385	0·0084962	1	47	0·6

SYNOPTIC TABLE OF ELEMENTS (*Continued*)

	☊			ω			L			E	
☉	°	′	″	°	′	″	°	′	″		d
☿	45	57	30·9	74	21	46·9	166	0	48·6	1801. Jan.	1·0, G.
♀	74	54	12·9	128	43	53·1	11	33	3·0	Do.	
⊕	.		.	99	30	5·0	100	39	10·2	Do.	
♂	48	0	3·5	332	23	56·6	64	22	55·5	Do.	
1	80	48	47·9	148	27	14·1	25	44	50·6	1864. Oct.	12·0, G.
2	172	42	15·1	122	4	48·1	354	44	38·9	1864. Sept.	6·0, G.
3	170	52	4·9	54	44	9·6	268	18	46·8	1864. June,	8·0, G.
4	103	23	11·7	250	38	10·3	52	34	2·6	1863. Nov.	23·0, G.
5	141	25	43·5	135	20	26·3	262	24	22·5	1864. June,	26·0, G.
6	138	37	21·4	15	22	12·6	269	8	16·9	1862. May,	31·0, B.
7	259	48	13·3	41	19	33·5	8	9	6·5	1862. Sept.	12·0, W.
8	110	17	48·6	32	54	28·3	68	48	31·9	1848. Jan.	1·0, B.
9	68	31	58·4	71	32	19·0	248	1	48·7	1863. May,	30·0, B.
10	286	43	55·0	234	49	57·7	316	54	16·7	1862. Apr.	26·0, G.
11	125	5	56·1	317	21	4·7	304	57	4·5	1862. July,	21·0, B.
12	235	33	34·9	301	56	36·0	312	33	52·4	1857. Aug.	2·0, G.
13	43	19	45·4	119	0	16·2	111	51	59·4	1864. Jan.	10·0, P.
14	86	42	8·8	180	6	33·9	98	57	33·4	1862. June,	1·0, G.
15	293	58	20·7	27	37	24·7	107	0	24·5	1862. Jan.	30·0, W.
16	150	34	1·6	14	44	57·6	226	7	32·6	1863. Apr.	29·5, W.
17	125	21	37·2	260	40	8·1	245	5	1·1	1860. July,	12·0, B.
18	150	4	30·7	15	9	58·0	167	39	20·9	1858. Mar.	6·0, G.
19	211	27	18·2	30	29	32·3	41	51	13·5	1860. Nov.	8·0, B.
20	206	42	30·8	98	26	22·4	351	2	39·8	1863. Aug.	29·5, B.
21	80	30	56·2	326	27	0·6	349	21	33·0	1860. Jan.	19·0, G.
22	66	36	21·8	56	34	13·1	224	46	26·6	1860. Jan.	0·0, B.
23	67	39	12·0	124	9	6·7	141	3	59·3	1862. Feb.	19·0, B.
24	36	11	48·1	139	49	24·2	62	24	39·7	1862. Oct.	23·0, G.
25	214	3	0·1	302	53	58·3	108	4	27·7	1863. Jan.	13·0, B.
26	45	55	3·4	234	50	23·9	61	14	30·0	1860. Feb.	6·0, G.
27	93	46	22·1	87	40	8·9	311	56	4·5	1863. July,	23·0, B.
28	144	41	9·9	122	55	29·6	66	3	57·4	1862. Mar.	24·0, B.
29	356	28	37·9	57	23	12·1	283	40	58·5	1863. June,	30·0, B.
30	308	16	8·4	30	52	49·6	73	59	9·5	1862. Dec.	17·0, B.
31	31	28	40·0	94	15	6·4	155	10	34·1	1862. Mar.	9·0, B.
32	220	47	37·3	194	2	32·3	6	29	1·5	1862. Oct.	1·0, W.
33	9	6	44·1	342	27	53·9	285	39	12·6	1863. May,	28·5, W.
34	184	42	23·0	150	14	45·2	105	34	22·0	1863. Jan.	3·0, B.

SYNOPTIC TABLE OF ELEMENTS (*Continued*).

	☊			ω			L			E
	°	′	″	°	′	″	°	′	″	d
35	355	51	20·9	201	26	27·1	69	52	4·3	1863. Nov. 16·5, B.
36	359	10	46·4	42	38	20·0	71	20	46·7	1861. Jan. 0·0, B.
37	8	9	37·4	66	4	28·2	42	34	35·2	1856. Jan. 0·0, B.
38	296	27	34·9	100	51	44·3	112	58	27·6	1856. Jan. 0·0, B.
39	157	20	6·6	2	11	36·7	146	41	34·5	1856. Jan. 1·0, B.
40	93	34	23·7	1	2	41·7	225	47	29·8	1863. May, 12·0, B.
41	179	2	30·1	220	5	20·1	335	3	21·7	1862. Sept. 23·435, B.
42	84	31	6·9	317	59	39·2	247	46	19·5	1860. Jan. 1·0, B.
43	264	35	52·7	277	50	50·4	132	1	30·2	1863. Jan. 0·0, B.
44	131	3	5·2	111	28	29·3	116	18	19·1	1860. Jan. 28·0, B.
45	148	5	51·8	229	51	52·7	294	35	2·8	1858. Jan. 0·0, B.
46	181	33	41·1	354	43	56·3	86	7	34·3	1863. Jan. 0·0, B.
47	4	11	52·2	313	53	0·8	116	34	11·4	1859. June, 17·0, B.
48	185	14	13·2	76	52	38·5	16	6	47·0	1858. Feb. 3·0, B.
49	290	29	4·2	32	31	40·6	159	34	55·3	1860. Jan. 28·0, B.
50	173	36	12·2	9	51	30·3	89	16	52·2	1863. Jan. 0·0, B.
51	175	39	11·2	175	10	53·1	177	11	25·3	1858. Mar. 25·5, B.
52	129	57	16·5	101	54	57·2	136	20	51·4	1858. Jan. 0·0, B.
53	144	1	52·5	92	47	35·8	239	19	16·8	1863. May, 31·5, B.
54	313	49	0·7	294	53	46·1	149	40	21·6	1861. Jan. 8·0, B.
55	10	57	29·1	11	28	38·6	28	26	57·6	1858. Dec. 30·0, B.
56	194	25	59·4	293	37	37·9	62	16	45·7	1862. Dec. 18·0, B.
57	200	5	25·1	52	53	13·0	28	35	25·6	1860. Jan. 1·0, B.
58	161	11	39·8	180	17	24·0	162	28	26·1	1860. Jan. 0·0, B.
59	170	22	31·4	16	54	44·4	190	6	18·6	1863. Jan. 0·0, B.
60	191	59	47·3	98	30	17·3	232	27	11·8	1863. Jan. 0·0, B.
61	334	16	57·9	342	44	12·7	73	6	25·4	1862. Jan. 0·0, B.
62	126	12	55·1	34	0	8·2	176	1	23·2	1863. Mar. 25·5, B.
63	338	3	48·9	269	41	0·7	177	7	9·1	1861. Mar. 5·0, B.
64	311	4	46·9	123	43	50·3	182	57	5·4	1861. May, 28·0, B.
65	158	53	48·7	258	22	17·1	192	17	21·4	1861. Mar. 18·0, B.
66	8	13	12·5	38	13	4·7	188	42	32·5	1861. May, 27·0, W.
67	202	40	10·1	306	18	47·9	313	38	23·3	1862. Jan. 0·0, B.
68	44	37	54·7	358	57	32·0	282	33	47·2	1862. Jan. 0·0, B.
69	186	59	4·0	111	8	42·0	164	2	21·0	1861. June, 3·0, B.
70	48	16	27·8	299	47	31·6	250	34	16·1	1861. June, 0·0, B.
71	316	18	48·4	221	58	46·8	322	15	20·8	1861. Sept. 25·5, B.
72	207	37	13·1	309	48	37·9	22	52	55·9	1863. Jan. 0·0, B.
73	7	32	18·9	61	33	51·1	189	47	11·9	1862. May, 26·0, W.
74	200	29	20·2	6	42	36·5	0	31	29·4	1862. Sept. 16·0, B.
75	359	52	19·1	334	40	12·1	4	40	23·1	1862 Nov. 0·0, W.
76	212	29	32·5	67	10	17·9	28	48	2·8	1862. Oct. 24·5, G.
77	2	7	1·7	58	9	1·3	39	25	26·4	1863. Jan. 0·0, B.
78	334	2	34·2	121	13	59·2	173	41	43·8	1863. May, 8·5, B.
79	206	42	40·0	44	20	33·1	45	50	13·3	1864. Jan. 1·0, G.
80	218	31	18·8	355	7	20·7	61	25	32·3	1865. Dec. 3·0. B.
81	2	31	45·1	48	17	29·6	29	35	16·4	1864. Nov. 13·0, B.

SYNOPTIC TABLE OF ELEMENTS (*Continued*).

	☊			ω			L			E		
	°	′	″	°	′	″	°	′	″			d
82	26	50	33	131	12	48	94	4	41	1865.	Jan.	0·0, B.
♃	98	26	18·9	11	8	34·6	112	15	23·0	1801.	Jan.	1·0, G.
♄	111	56	37·4	89	9	29·8	135	20	6·5		Do.	
♅	72	59	35·3	167	31	16·1	177	48	23·0		Do.	
♆	130	7	31·9	43	17	30·3	335	6	0·4	1850.	Jan.	1·0, G.

SYNOPTIC TABLE OF ELEMENTS (*Continued*).

See 357 *a*, Note F.

	M	D	*d*	Δ	T			γ			H	ε
			″		h	m	s	°	′	″		
☉	1	882000	1923·64	0·25	609	7	—	82	45	—	45940	
☿	4865751	3183	6·89	1·14	21	5	—	.	.	.	6·674	
♀	401839	8108	17·55	0·84	23	21	—	.	.	?	1·911	
⊕	359551	7925·648	. .	1·00	23	56	4·1	66	32	6	1·000	298
♂	2680337	4546	6·46	0·72	24	37	22·7	59	41	49	0·431	62
♃	1047·871	90734	37·91	0·24	9	55	21·3	86	54	30	0·036	16·84
♄	3501·600	76791	17·50	0·11	10	16	0·4	61	49	—	0·011	
♅	20470	35307	3·91	0·20	.	.	.	.	.	.	0·003	
♆	18780?	39793?	2·88	0·15	.	.	.	.	.	.	0·00	

NAMES OF DISCOVERERS AND DATES OF DISCOVERY OF THE ASTEROIDS

Ceres	Piazzi	Jan. 1, 1801.
Pallas	Olbers	Mar. 28, 1802.
Juno	Harding	Sept. 1, 1804.
Vesta	Olbers	Mar. 29, 1807.
Astræa	Hencke	Dec. 8, 1845.
Hebe	Hencke	July 1, 1847.
Iris	Hind	Aug. 13, 1847.
Flora	Hind	Oct. 18, 1847.
Metis?	Graham	Apr. 25, 1848.
Hygeia	Gasparis	Apr. 12, 1849.
Parthenope	Gasparis	May 11, 1850.
Victoria	Hind	Sept. 13, 1850.
Egeria?	Gasparis	Nov. 12, 1850.
Irene	Hind	May 19, 1851.
Eunomia	Gasparis	July 29, 1851.
Psyche	Gasparis	Mar. 17, 1852.
Thetis	Luther	Apr. 17, 1852.
Melpomene	Hind	June 24, 1852.
Fortuna	Hind	Aug. 22, 1852.
Massilia	Gasparis	Sept. 19, 1852.
Lutetia	Goldschmidt	Nov. 15, 1852.
Calliope	Hind	Nov. 16, 1852.
Thalia	Hind	Dec. 15, 1852.
Themis?	Gasparis	Apr. 5, 1853.
Phocea	Chacornac	Apr. 6, 1853.
Proserpina	Luther	May 5, 1853.
Euterpe	Hind	Nov. 8, 1853.
Bellona	Luther	Mar. 1, 1854.
Amphitrite	Marth	Mar. 1, 1854.
	Pogson	Mar. 1, 1854.
Urania	Hind	July 22, 1854.
Euphrosyne	Ferguson	Sept. 1, 1854.
Pomona	Goldschmidt	Oct. 26, 1854.
Polyhymnia	Chacornac	Oct. 28, 1854.
Circe	Chacornac	Apr. 6, 1854.
Leucothea	Luther	Apr. 19, 1855.
Atalanta	Goldschmidt	Oct. 5, 1855.
Fides	Luther	Oct. 5, 1855.
Leda	Chacornac	Jan. 12, 1856.
Lætitia?	Chacornac	Feb. 8, 1856.
Harmonia	Goldschmidt	Mar. 1, 1856.
Daphne	Goldschmidt	May 22, 1856.
Isis?	Pogson	May 23, 1856.
Ariadne	Pogson	Apr. 15, 1857.
Nysa	Goldschmidt	May 27, 1857.
Eugenia?	Goldschmidt	June 28, 1857.
Hestia?	Pogson	Aug. 16, 1857.
Aglaia	Luther	Sept. 15, 1857.
Doris	Goldschmidt	Sept. 19, 1857.

Pales?	Goldschmidt	Sept. 19, 1857.
Virginia	Ferguson	Oct 4, 1857.
Nemuasa?	Laurent	Jan. 22, 1858.
Europa	Goldschmidt	Feb. 4, 1858.
Calypso	Luther	Apr. 4, 1858.
Alexandra	Goldschmidt	Apr. 11, 1858.
Pandora	Searle	Sept. 10, 1858.
Melete	Goldschmidt	Sept. 9, 1857.
Mnemosyne	Luther	Sept. 22, 1859.
Concordia	Luther	Mar. 24, 1860.
Olympia	Chacornac	Sept. 13, 1860.
Echo	Ferguson	Sept. 15, 1860.
Danäe	Goldschmidt	Sept. 9, 1860.
Erato	Lesser	Sept. 14, 1860.
Ausonia	Gasparis	Feb. 11, 1861.
Angelina	Tempel	Mar. 6, 1861.
Maximiliana	Tempel	Mar. 10, 1861.
Maia	Tuttle	Apr. 10, 1861.
Asia	Pogson	Apr. 18. 1861.
Leto	Luther	Apr. 29, 1861.
Hesperia	Schiaparelli	Apr. 29, 1861.
Panopea	Goldschmidt	May 5, 1861.
Niobe	Luther	Aug. 13, 1861.
Feronia	Peters	Jan. 29, 1862.
Clytie	Tuttle	Apr. 7, 1862.
Galatea	Tempel	Aug. 29, 1862.
Eurydice	Peters	Sept. 22, 1862.
Freia	D'Arrest	Nov. 14, 1862.
Frigga	Peters	Nov. 12, 1862.
Diana	Luther	Mar. 15, 1863.
Eurynome	Watson	Sept. 14, 1863.
Sappho	Pogson	May 3, 1864.
Terpischore	Tempel	Sept. 30, 1864.
Alcmene	Luther	Nov. 27, 1864.

Note.—Many of the names of the Asteroids appear to us very unhappily chosen. Thus, confusion is very likely to arise in printing or speaking, between Iris and Isis, Lutetia and Lætitia, Thetis and Metis, Thetis and Themis, Vesta and Hestia, Hygeia and Egeria, Egeria and Eugenia, Pallas and Pales. Is it too much to hope that the discoverers of the interfering members of these pairs will reconsider their names? It is not yet too late: the Nymphs, Dryads, Oceanidæ, etc., afford an infinite choice of classic names, graceful and euphonious. Metis is known to few as a mythological name, Pales to fewer as that of a female divinity, Nemausa to none as the name of anybody (the ancient name of Nismes was Nemaus*us*).

III

SYNOPTIC TABLE OF THE ELEMENTS OF THE ORBITS OF THE SATELLITES, SO FAR AS THEY ARE KNOWN[1]

1. THE MOON

Mean distance from the earth	60r·27343300
Mean sidereal revolution	27d·321661418
Mean synodical ditto	29d·530588715
Excentricity of orbit	0·054908070
Mean revolution of nodes	6793d·391080
Mean revolution of apogee	3232d·575343
Mean longitude of node at epoch	13° 53′ 17″·7
Mean longitude of perigee at ditto	266 10 7 ·5
Mean inclination of orbit	5 8 39 ·96
Mean longitude of moon at epoch	118 17 8 ·3
Mass, that of the earth being 1,	0·011364
Diameter in miles	2164·6
Density, that of the earth being 1,	0·55654

[1] The distances are expressed in equatorial radii of the primaries. The epoch is Jan. 1, 1801, unless otherwise expressed. The periods, etc., are expressed in mean solar days.

2. SATELLITES OF JUPITER.

Sat.	Sidereal Revolution.				Mean Distance.	Inclination of Orbit to a fixed Plane proper to each.			Inclination of the fixed Plane to Jupiter's Equator.			Retrograde Revolution of Nodes on the fixed Plane	Mass, that of Jupiter being 1,000,000,000.
	d	h	m	s		°	′	″	°	′	″	Years.	
1	1	18	27	33·506	6·04853	0	0	0	0	0	6	. . .	17328
2	3	13	14	36·393	9·62347	0	27	50	0	1	5	29·9142	23235
3	7	3	42	33·262	15·35024	0	12	20	0	5	2	141·7390	88497
4	16	16	31	49·702	26·99835	0	14	58	0	24	4	531·0000	42659

The excentricities of the 1st and 2d Satellites are insensible, those of the 3d and 4th small, but variable, in eonsequenee of their mutual perturbation.

3. SATELLITES OF SATURN.

Name and order of Satellite.	Sidereal Revolution.				Mean Distance.	Epoch of Elements.	Mean Longitude at the Epoch.			Excentricity.	Perisaturnium.		
	d	h	m	s d			°	′	″		°	′	″
1. Mimas .	0	22	37	22·9	3·3607	1790·0	256	58	48				
2. Enceladus .	1	8	53	6·7	4·3125	1836·0	67	41	36				
3. Tethys .	1	21	18	25·7	5·3396	Ditto	313	43	48	0·04 ?	54 ?		
4. Dione .	2	17	41	8·9	6·8398	Ditto	327	40	48	0·02 ?	42 ?		
5. Rhea . .	4	12	25	10·8	9·5528	Ditto	353	44	0	0·02 ?	95 ?		
6. Titan . .	15	22	41	25·2	22·1450	1830·0	137	21	24	0·029314	256	38	11
7. Hyperion .	21	7	7	40·8	26·7834	. . .	. .	. .	. .	Rather large.			
8. Iapetus .	79	7	53	40·4	64·3590	1790·0	269	37	48				

The longitudes are reckoned in the plane of the ring from its *descending* node with the ecliptic. The first seven satellites move in, or very nearly in, its plane; that of the 8th is inclined to it at an angle about half-way intermediate between the planes of the ring and of the planet's orbit. The apsides of Titan have a direet motion of 30′ 28″ per annum in longitude (on the ecliptie).

The diseovery of Hyperion is quite reeent, having been made on the same night (Sept. 19, 1848), by Mr. Lassell, of Liverpool, and Prof. Bond, of Cambridge, U. S. Its distance and period are as yet hardly more than eonjecture. Messrs. Kater, Encke and Lassell agree in representing the ring of Saturn as subdivided by several narrow dark lines, besides the broad black divisions which ordinary telescopes show.

4. SATELLITES OF URANUS

(Lassell, Astr. Soc. Notices, xiii. 151)

No.	Name.	Sidereal Revolution.				Mean Distance.	Nodes and Inclinations.
		d	h	m	s		
1	Ariel	2	12	29	20·66	7·40	The orbits are inclined at an angle of about 78° 58′ to the ecliptic in a plane whose ascending node is in long. 165° 30′ (Equinox of 1798). Their motion is *retrograde*. The orbits are nearly circular.
2	Umbriel	4	3	28	8·00	10·31	
3	Titania	8	16	56	31·30	16·92	
4	Oberon	13	11	7	12·6	22·56	

5. SATELLITES OF NEPTUNE

1. Sidereal revolution = 5d. 21h. 2m. 43s. Longitude of ☊ = 175° 40′? Longitude of perihelion = 177° 30′? Inclination to ecliptic = 151° 0′? Excentricity = 0·10597? Mean distance about 12 radii of the planet.—(*Hind*, *Astr. Soc. Notices*, xv. 47.)

IV

ELEMENTS OF PERIODICAL COMETS

	Halley's.	Encke's.	Biela's.	Faye's.	De Vico's.	Brorsen's.
	1835. Nov. 15.	1845. Aug. 9.	1846. Feb. 11.	1843. Oct. 17.	1844. Sept. 2.	1846. Feb. 25.
τ	$22^h\ 41^m\ 22^s$	$15^h\ 11^m\ 11^s$	$0^h\ 2^m\ 50^s$	$3^h\ 42^m\ 16^s$	$11^h\ 36^m\ 53^s$	$9^h\ 13^m\ 35^s$
ω	304° 31′ 32″	157° 44′ 21″	109° 5′ 47″	49° 34′ 19″	342° 31′ 15″	116° 28′ 34″
☊	55 9 59	334 19 33	245 56 58	209 29 19	63 49 31	102 39 36
ι	17 45 5	13 7 34	12 34 14	11 22 31	2 54 45	30 55 7
a	17·98796	2·21640	3·50182	3·81179	3·09946	3·15021
ϵ	0·967391	0·847436	0·755471	0·555962	0·617256	0·793629
P	27865d·74	1205d·23	2393d·52	2718d·26	1993d·09	2042·24
	Retrograde.	Direct.	Direct.	Direct.	Direct.	Direct.

τ is the time of perihelion passage; ω the longitude of the perihelion; and ☊ that of the ascending node for the epoch of the perihelion; ι, the inclination to the ecliptic; a, the semiaxis; ϵ, the excentricity; P, the period in days.

N.B.—The reader will find a complete list of elements of all known comets up to June, 1847, by *all* their several computors, in Prof. Encke's edition of Olbers's "Abhandlung über die leichtsete und bequemste Methode die Bahn eines Cometen zu berechnen." The list is compiled by Dr. Galle. It contains orbits of 178 distinct comets. From an examination of these orbits we collect the following, as a more correct statement of cometary statistics than that in art. 601, viz.:—Retrograde comets under 10° inclination, 3 out of 15; under 20°, 9 out of 29. Retrograde comets, moving in orbits sensibly elliptic, under 17° inclination, 0 out of 9. In such orbits, of all inclinations from 0 to 90°, 11 out of 37. Thus we see that the induction of that article is materially strengthened by the enlarged field of comparison. Mr. Cooper has also given a catalogue of 198 comets with *all* their computed elements, and copies and interesting notes (Dublin, 1852). While these sheets are passing through the press, we learn that the second comet of 1851, discovered by M. D'Arrest, has been seen again on its predicted return on the 4th Dec. 1857, at the Cape, by Mr. Maclear.

Elements of the Orbit of the Earth as computed by M. Leverrier for Intervals of 10,000 Years, from 100,000 Years before A.D. 1800 to 100,000 Years after that date. (Connaissance des Temps pour l'an 1843.)

Epoch.	Excentricity.	Longitude of Perihelion.		Inclination.			Longitude of ☊	
		°	′	°	′	″	°	′
— 100,000	0·0473	316	18	3	45	31	96	34
— 90,000	0·0452	340	2	2	42	19	76	17
— 80,000	0·0398	4	13	1	18	58	73	47
— 70,000	0 0316	27	22	1	13	58	136	8
— 60,000	0·0218	46	8	2	36	42	136	29
— 50,000	0·0131	50	14	3	40	11	116	9
— 40,000	0·0109	28	36	4	3	1	91	59
— 30,000	0·0151	5	50	3	41	51	66	49
— 20,000	0·0188	44	0	2	44	12	41	34
— 10,000	0·0187	78	28	1	24	35	16	39
0	0·0168	99	30	0	0	0	0	0
+ 10,000	0·0155	134	14	1	14	26	148	15
+ 20,000	0·0047	192	22	2	7	46	124	29
+ 30,000	0·0059	318	47	2	33	19	100	29
+ 40,000	0·0124	6	25	2	27	53	75	31
+ 50,000	0 0173	38	3	1	51	54	48	13
+ 60,000	0·0199	64	31	0	51	52	10	47
+ 70,000	0·0211	71	7	0	34	35	220	38
+ 80,000	0·0188	101	38	1	45	40	170	15
+ 90,000	0·0176	109	19	2	40	56	139	3
+ 100,000	0·0189	114	5	3	2	57	109	57

Excentricity and Longitude of the Perihelion of the Earth's Orbit for a Million of Years Past and to Come, as computed by Mr. Croll (*Phil. Mag.* April, 1866)

Number of Years before Epoch 1800.	Excentricity.	Longitude of Perihelion.		Number of Years after Epoch 1800.	Excentricity.	Longitude of Perihelion.	
		°	′			°	′
1,000,000	0·0151	248	22	50,000	0·0173	38	12
950,000	0·0517	97	51	100,000	0·0191	114	50
900,000	0·0102	135	2	150,000	0·0353	201	57
850,000	0·0747	239	28	200,000	0·0076	303	30
800,000	0·0132	343	49	250,000	0·0286	350	54
750,000	0·0575	27	18	300,000	0·0158	179	29
700,000	0·0220	208	13	350,000	0·0098	201	40
650,000	0 0226	141	29	400,000	0·0429	6	9
600,000	0·0417	32	34	450,000	0·0231	98	37
550,000	0·0166	251	50	500,000	0·0534	157	26
500.000	0·0388	193	56	550,000	0·0259	287	31
450,000	0·0308	356	52	600,000	0·0395	285	43
400,000	0·0170	290	7	650,000	0·0169	144	3
350,000	0·0195	182	50	700,000	0·0375	17	12
300,000	0·0424	23	29	750,000	0·0195	0	53
250,000	0·0258	59	39	800,000	0·0639	140	38
200,000	0·0569	168	18	850,000	0·0144	176	41
150,000	0·0332	242	56	900,000	0·0659	291	16
100,000	0·0473	316	2	950,000	0·0086	115	13
50,000	0·0131	50	3	1,000,000	0·0528	57	21
0,000	0·0168	99	30				

ELEMENTS OF ASTEROIDS DISCOVERED SINCE 1864

		a	P	*e*	*i*		
					°	′	″
83	Beatrice . .	2·437244	1389·78	0·0859251	5	0	3·0
84	Clio	2·361823	1325·77	0·2361542	9	22	25·5
85	Io	2·683884	1579·14	0·1908907	11	53	9·7
86	Semele . . .	3·090786	1984·73	0·2049113	4	47	44·6
87	Sylvia . . .	3·494110	2385·66	0·0823511	10	50	56·9
88	Thisbe . . .	2·768885	1682·89	0·1651097	5	14	30·1
89	Julia . . .	2·549835	1487·19	0·1803041	16	11	25·5
90	Antiope . .	3·138789	2031·15	0·1747166	2	16	47·8
91	Ægina . . .	2·491717	1436·63	0·0662013	2	9	31·6
92	Undina . . .	3·191618	2082·65	0·1031710	9	57	3·0
93	Minerva . .	2·755910	1671·07	0·1402670	8	36	31·8
94	Aurora . . .	3·160410	2052·12	0·0889312	8	5	18·5
95	Arethusa . .	3·068786	1963·58	0·1465338	12	51	1·5
96	Ægle . . .	3·054500	1949·71	0·1402763	16	6	31·1
97	Clotho . . .	2·668621	1592·31	0·2568941	11	44	58·4
98	Ianthe . . .	2·684535	1606·58	0·1891891	15	42	35·1
99							
100	Hecate . . .	2·993973	1892·21	0·1690456	6	9	50·4
101	Helena . . .	2·573119	1507·61	0·1394040	10	4	19·5
102	Miriam . . .	2·662367	1586·72	0·2547668	5	6	3·3
103	Hera . . .	2·702265	1622·52	0·0806707	5	21	35·2
104	Clymene . .	3·179809	2071·09	0·1973404	2	53	26·7
105	Artemis . .	2·379975	1341·06	0·1761972	21	38	59·0
106	Dione . . .	3·201010	2091·84	0·1950237	4	41	33·2

	☊			ω			L			E
	°	′	″	°	′	″	°	′	″	
83	27	33	31·2	193	49	32·0	339	49	35·1	1866. Oct. 1·5, B.
84	337	22	1·1	339	11	58·1	353	48	43·6	1865. Nov. 13·0, B.
85	203	53	27·0	322	35	52·7	102	36	43·6	1867. Jan. 0·0, B.
86	87	55	52·1	28	39	6·7	39	12	57·1	1866. Jan. 20·0, B.
87	76	24	4·6	336	59	9·2	251	32	58·3	1866. May 16·42, B.
88	277	42	52·5	308	50	52·4	304	59	21·7	1866. Aug. 4·5, B.
89	311	30	11·7	353	17	8·1	330	44	57·1	1866. Aug. 31·0, B.
90	71	14	59·6	301	1	28·5	351	44	29·6	1866. Oct. 28·0, B.
91	11	41	34·8	68	54	3·3	281	27	18·8	1869. June 30·0, B.
92	102	52	32·7	333	21	56·0	278	25	58·8	1867. Jan. 0·0, B.
93	5	4	11·4	275	38	16·3	342	40	15·4	1867. Oct. 2·0, B.
94	4	34	36·4	44	37	19·5	159	47	3·2	1870. Jan. 0·0, B.
95	244	22	31·4	30	22	34·4	127	59	11·2	1869. Jan. 0·0, B.
96	322	51	4·3	164	16	51·5	275	11	5·1	1870. Jan. 0·0, B.
97	160	36	34·8	65	33	36·0	126	8	50·0	1868. Jan. 0·0, B.
98	354	16	43·2	147	43	7·5	150	25	16·9	1868. Jan. 0·0, B.
99										
100	128	16	59·3	343	0	46·1	316	7	52·7	1868. July. 11·0, B.
101	343	35	0·1	328	40	51·0	346	33	18·1	1868. Sept. 14·0, B.
102	211	32	43·2	355	9	10·2	303	28	54·0	1868. Jan. 0·0, B.
103	135	56	56·4	326	15	4·8	0	2	16·1	1868. Oct. 13·0, B.
104	43	46	42·1	62	11	55·4	18	24	14·2	1868. Sept. 14·0, B.
105	187	54	1·8	242	36	17·8	348	54	8·0	1868. Oct. 13·0, B.
106	62	42	38.9	35	37	53·6	22	25	0·2	1868. Oct. 11·0, B.

NAMES OF DISCOVERERS AND DATES OF DISCOVERY

83	Beatrice	Gasparis	April 26, 1865.
84	Clio	Luther	Aug. 25, 1865.
85	Io	Peters	Sept. 19, 1865.
86	Semele	Tietjen	Jan. 4, 1866.
87	Sylvia	Pogson	May 16, 1866.
88	Thisbe	Peters	June 15, 1866.
89	Julia	Stephan	Aug. 6, 1866.
90	Antiope	Luther	Oct. 1, 1866.
91	Ægina	Stephan	Nov. 4, 1866.
92	Undina	Peters	July 26, 1867.
93	Minerva	Watson	Aug. 24, 1867.
94	Aurora	Watson	Sept. 6, 1867.
95	Arethusa	Luther	Nov. 23, 1867.
96	Ægle	Coggia	Feb. 17, 1868.
97	Clotho	Tempel	Feb. 17, 1868.
98	Ianthe	Peters	April 18, 1868.
99		Borelly	May 28, 1868.
100	Hecate	Watson	July 11, 1868.
101	Helena	Watson	Aug. 15, 1868.
102	Miriam	Peters	Aug. 23, 1868.
103	Hera	Watson	Sept. 7, 1868.
104	Clymene	Watson	Sept. 13, 1868.
105	Artemis	Watson	Sept. 16, 1868.
106	Dione	Watson	Oct. 10, 1868.

TABLE OF NUMBERS IN FREQUENT USE AMONG ASTRONOMERS (See Note F)*

	Number or Multiplier	Logarithm	Log. Reciprocal
Circumference of a circle to diameter 1 π	3·1415927	0·4971499	9·5028501
No. of degrees in circular arc=radius .	57·2957795	1·7581225	8·2418775
No. of seconds in circular arc	206264·8	5·3144251	4·6855749
No. of seconds in the whole circumference, 360°	1296000	6·1126050	3·8873950
No. of sine of 1′ to radius =1 . . .	0·0002909	6·4637261	3·5362739
No. whose natural logarithm is 1 . ε	2·7182818	0·4342945	9·5657055
Multiplier to reduce common to natural logarithms	2·3025851	0·3622149	9·6377851
Amplitude of the probability curve for probability $=\frac{1}{2}$	0·47694	9·6963040	0·3036960
Multiplier to reduce French metres to British feet	3·28090	0·5159929	9·4840071
Multiplier to reduce French metres to British inches	39·37079	1·5951741	8·4048259
Multiplier to reduce French toises to British feet	6·394593	0·8058129	9·1941871
Multiplier to reduce French grammes to British grains	15·423460	1·1884351	8·8115649
Multiplier to reduce French litres to British cubic inches	61·027043	1·7855223	8·2144777
Length of seconds pendulum, London, vacuum, sea-level, in inches . . .	39·13929	1·5926129	8·4073871
Velocity (in feet per 1″) generated by gravity in 1″ (lat. 35° 16′) . . .	32·18169	1·5076088	8·4923912
Earth's mean diameter in British standard miles	7912·410	3·8983088	6·1016912
Mean barometric pressure at sea-level on 1 square inch in lbs.	14·7304	1·1682145	8·8317855
Weight of 1 cubic inch of distilled water, 62° Fahr., bar. 30 inches, in grains	252·458	2·4021892	7·5978108
Specific gravity of mercury at 32° Fahr., (water at 40°)	13·596	1·1334112	8·8665888
Velocity of sound in dry air at 32° Fahr., in feet per second	1089·42	3·0371964	6·9628036
Velocity of light in vacuo in British standard miles per second	191515*	5·2822029	4·7177971
Multiplier to reduce sidereal days to mean solar days	0·9972696	9·9988126	0·0011874
Multiplier to reduce sidereal year to mean solar days	365·2563612	2·5625977	7·4374023
Multiplier to reduce tropical year to mean solar days	365·2422414	2·5625809	7·4374191
Multiplier to reduce mean synodic lunar months to mean solar days . . .	29·5305887	1·4702726	8·5297274
The sun's mean equatorial horizontal parallax	8″·5776*	0·9333658	9·0666342

TABLE OF NUMBERS IN FREQUENT USE AMONG ASTRONOMERS (*Continued*)

	Number or Multiplier	Logarithm	Log. Reciprocal
The moon's mean equatorial horizontal parallax	3422″·325	3·5343212	6·4656788
The sun's mean apparent semidiameter .	961″·820	2·9830938	7·0169062
The moon's mean apparent semidiameter	934″·685	2·9706652	7·0293348
Constant of aberration	20″·4452	1·3105914	8·6894086
Maximnm of nutation of obliquity of ecliptic	9″·2236	0·9649005	9·0350995
Maximum of nutation in longitude . .	17″·2524	1·2368495	8·7631505
Mean annual precession of the equinoxes (for 1790)	50″·23492	1·7010057	8·2989943
Constant of refraction (=ref. at 45° alt, bar. 29·6, therm. 50° Fahr.) . . .	57″·524	1·7598491	8·2401509
Mean horizontal refraction	1980″	3·2966652	6·7033348
Mean obliquity of the ecliptic, Jan. 1, 1860	23° 27′ 27″·38	———	———

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N.B. The references are to the articles, not to the pages.
... attached to a reference number indicates that the reference extends to the article cited, and several subsequent in succession

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Plate I

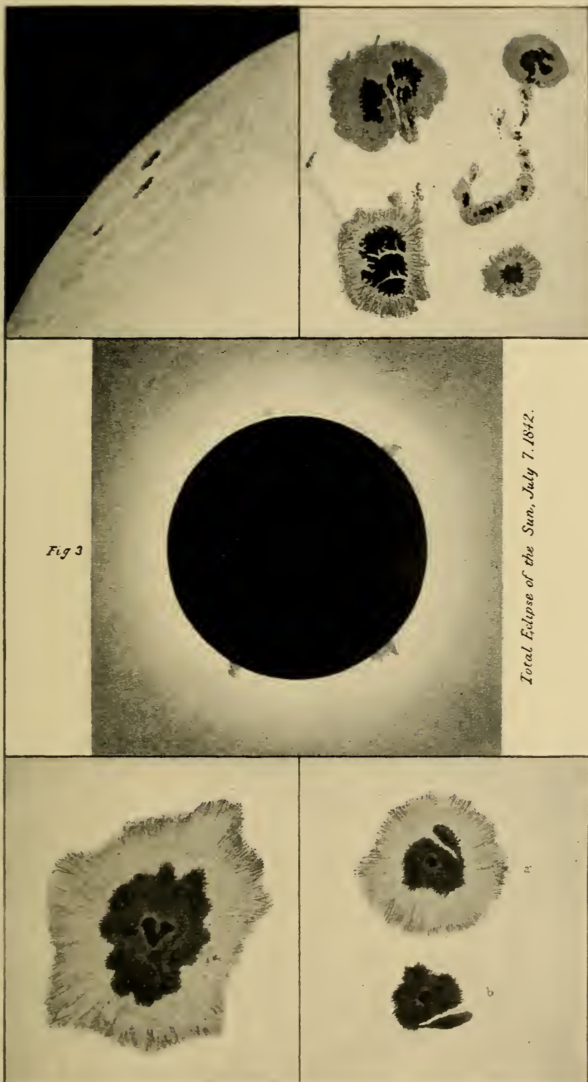


Fig 1

Fig. 2.

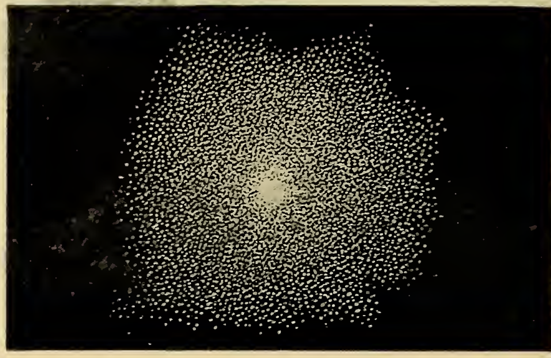
Fig 3

Fig 4

Fig 5.

H.Adlard.sc.

Plate. II.



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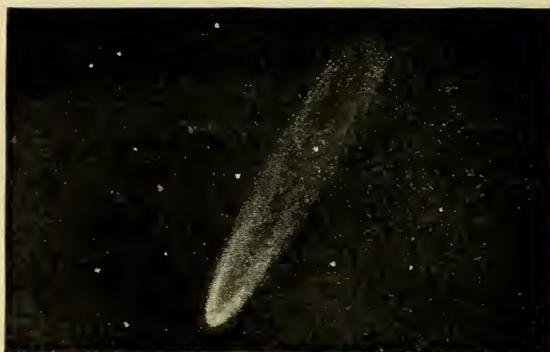

Fig. 2.

Fig. 3.

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Plate III

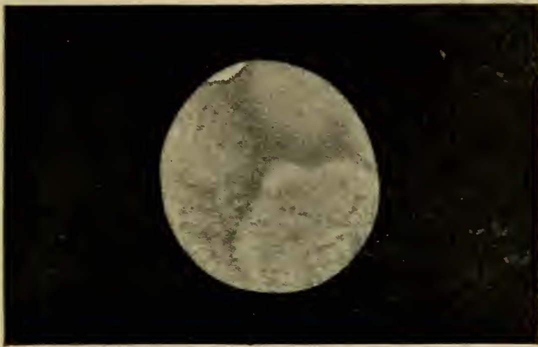

Fig. 1

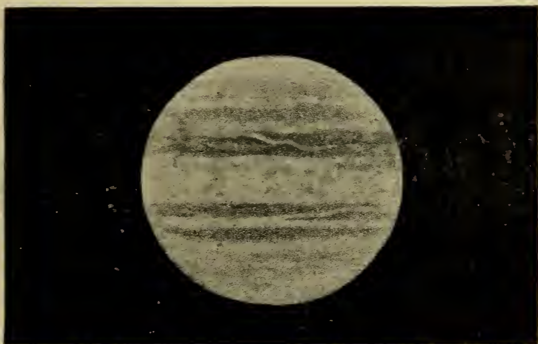

Fig. 2.

Fig. 3

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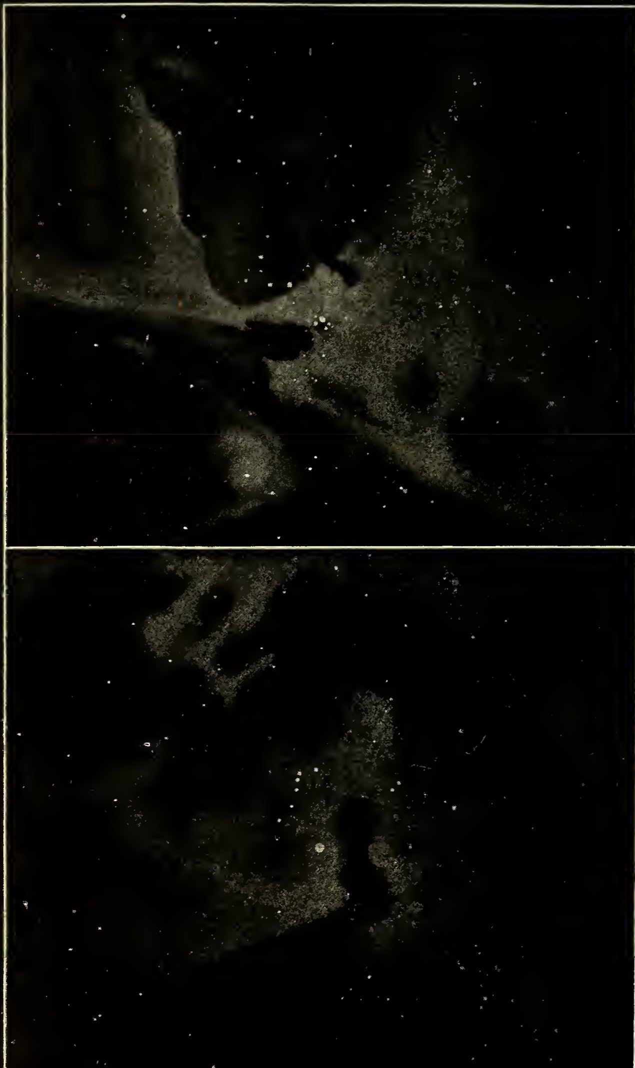
Plate. IV

SOUTH

FOLLOWING



PRECEDING

Fig 1.

Fig 2

NORTH

Plate V

Fig. 2

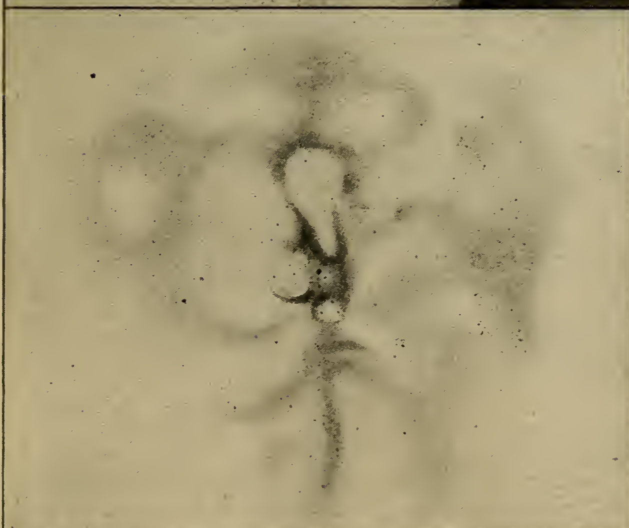


Fig. 1

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Plate VI

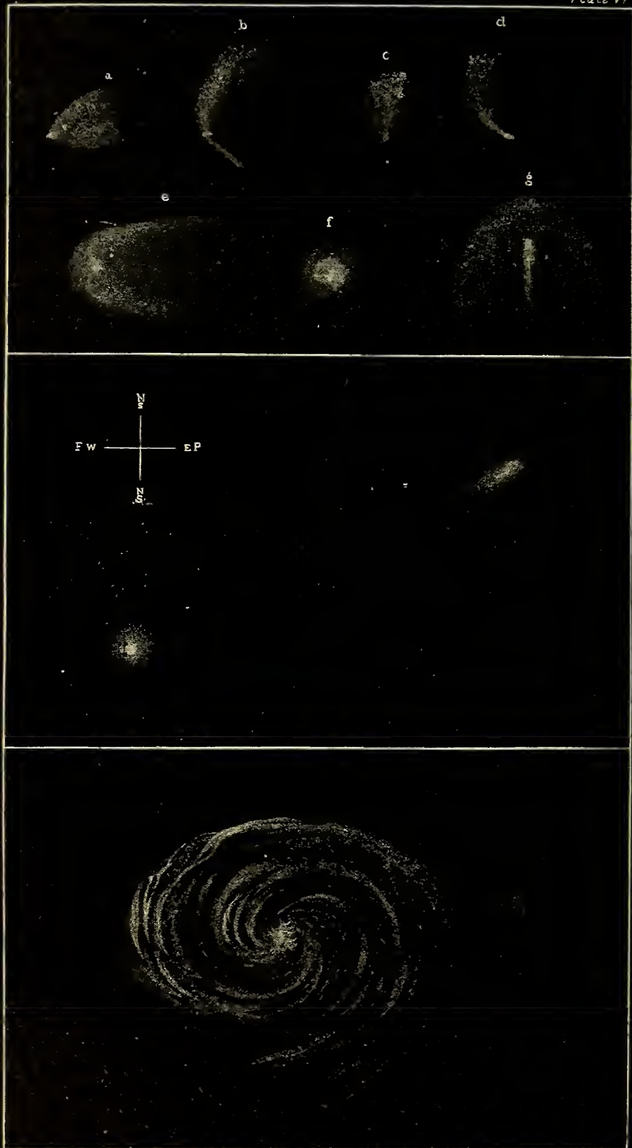

Fig. 1

Fig 2

Fig 3

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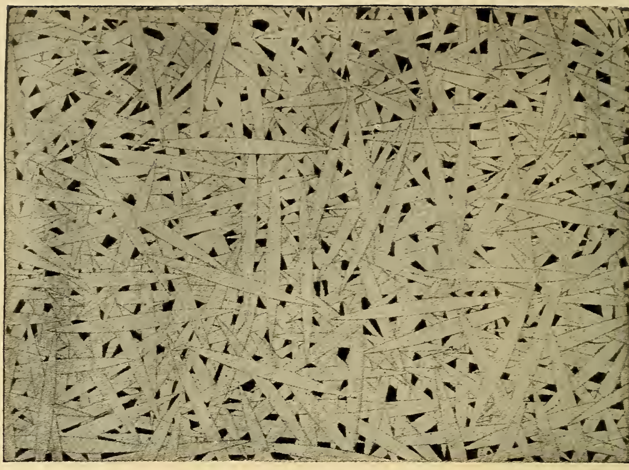
A portion of the Moon's surface, from a model by Mr. Nasmyth.

Plate B

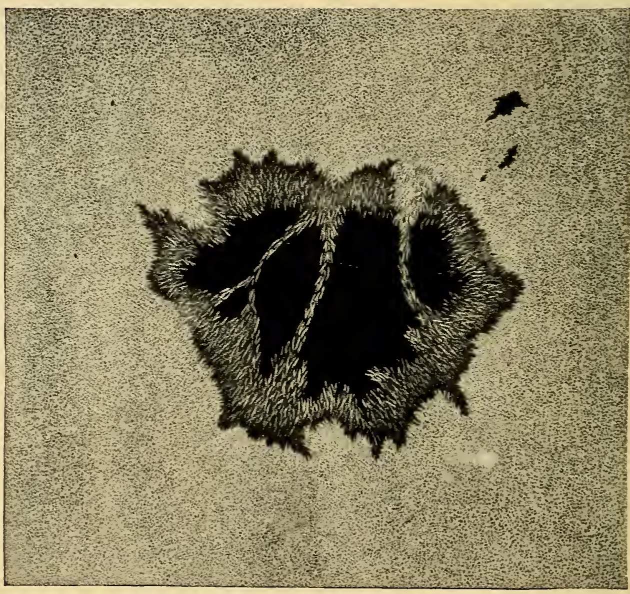
Fig. 1



SCALE OF 60.000 MILES

10 0 10 20 30 40 50

Fig. 2



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