CYCLOMATHESIS:

OR AN

EASY INTRODUCTION

To the ſeveral Branches of the

MATHEMATICS.

Being principally deſigned for the Inſtruction of young Students, before they enter upon the more abſtruſe and difficult parts thereof.

W. Emerson .... vide p 5.

Scribere laus magna eſt: ſed ſcriptis addere lucem,
Hoc verò egregiæ dexteritatis opus. Ruſ. Med.

LONDON,
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to his MAJESTY.
MDCCLXIII.

General Introduction

Concerning the

NATURE, USEFULNESS, and CERTAINTY

OF THE

MATHEMATICS.

AS man is endued with the noble faculty of reaſon, and likewiſe with a ſtrong innate deſire of knowledge; it is natural for him to exert this his diſtinguiſhing talent in the purſuit of knowledge. Truth alone is the object of knowledge; for it is impoſſible to know a falſe thing to be true: and evidence is the certain mark or criterion of truth; and this conſiſts in the perception of the agreement or diſagreement of our ideas in the mind, according as the things in nature agree or diſagree. As there is no ſtronger paſſion in the human ſoul than the love of truth, and no greater deſire for any thing than to find it out; ſo, when it is found, there is no greater pleaſure to the underſtanding, than the contemplation thereof in the ſeveral branches of ſcience; even when the ſearch of it is attended with the greateſt labour and pains. Truth is of ſuch a nature, as always to be conſiſtent with itſelf, and needs nothing to enforce or recommend it, but its own native evidence. It is but one ſimple, uniform, invariable thing; whilſt its oppoſite, falſhood, is infinitely various, inconſiſtent, and contradictory. As truth is what all men admire, and every one aims at; and error what every man hates, that is not blinded by ſelf-intereſt; it is neceſſary that we take care never to receive any thing for truth, which does not bring its proper evidence along with it. For it is evidence alone that can gain our aſſent, and remove all our doubts; and when that appears, the mind can neither expect nor deſire any thing further. By the help of this we are enabled to diſtinguiſh truth from falſhood, right from wrong; and we likewiſe have a power of ſuſpending our aſſent till that evidence appears; and when it does appear, it compels our aſſent, and carries abſolute conviction. Truth, when expreſſed in words, is the ſame thing as a true propoſition; and, as evidence is a neceſſary voucher for truth, we ought never to give aſſent to a doubtful or obſcure propoſition; but ſhould deny it as long as we can, and not give our judgment as long as we can withhold it, in ſuch things as we can have an evident knowledge of.

Now ſince truth is of ſo amiable a nature, and ſo deſirable to the underſtanding, it will be aſked where it is to be found, and how ſhall we come to the knowledge of it? I anſwer, it is to be found in the writings of the mathematicians, where the method of finding it is clearly explained. In the mathematical ſciences truth appears moſt conſpicuous, and ſhines in its greateſt luſtre. In other ſciences it is either ſelf-evident, and then it affords little pleaſure to the mind; or elſe it appears with ſo much obſcurity, that falſhood is often miſtaken inſtead of it. The evidence for it is ſo dim, that it is only ſeen as in a miſt; and truth, ſeen through ſuch a dull medium, will hardly be known to be truth; the mind will be loſt in doubt and obſcurity, and will be be unable to make any certain concluſion. But in the mathematics, all their demonſtrations are free from any obſcurity, every ſtep has a clear and intuitive evidence; and where that falls ſhort, the matter is thrown out as not deſerving a place among mathematical truths.

The manner whereby truth is found out, is by reaſoning, which is performed by firſt laying down, as a foundation, certain evident principles, or ſuch as cannot be denied; and then proceeding from theſe by ſeveral ſteps till they come at the concluſion; which ſteps are ſo to be linked with each other, and laid in ſuch order, that the underſtanding may perceive their connection and agreement; which being every where true and right, the concluſion muſt infallibly be true: for all the parts being locked together by truth; the laſt reſult, though never ſo long, muſt be equally true.

Thus mathematicians, from a few plain and ſimple principles, and a continued chain of reaſoning, proceed to the diſcovery and demonſtration of truths that appear at firſt ſight beyond human capacity. The art of finding proofs, and the admirable methods they have invented for finding out and laying in order, thoſe intermediate ideas that ſhew the connection of the ſeveral ſteps of the proof, or the ſeveral links of this chain of reaſoning; is that which has carried them ſo far, and produced ſuch wonderful and unexpected diſcoveries. In this ſcience there appears to be an inexhauſtible fund in the ſeveral branches thereof; any one of which a man may purſue as far as he pleaſes, and ſtill improve his knowledge further and further: and thus, by the help of truths already known, more and more may ſtill be found out *ad infinitum*.

When the mind works on mathematical ideas, it works ſecurely, which cannot be done in other things ſo truly; becauſe one cannot keep ſo ſtrictly to the definitions, or the meaning of words, in other ſubjects;

where the ideas are often confounded. But mathematicians take care not to confound theirs; for none ever miſtook the idea of a ſquare for that of a circle. Therefore mathematical demonſtrations are the moſt proper means to cleanſe the mind from errors, and to give it a reliſh of truth; which is the natural food and nouriſhment of the underſtanding.

Reaſoning, which is the exerciſe of reaſon, is beſt learned from the examples and practices of the mathematicians. It is certain, that no ſort of human knowledge can lay ſo juſt a claim to an unſhaken evidence and certainty, or boaſt ſo great a ſtrength of its demonſtrations, or produce ſuch a multitude of undeniable truths, as the mathematics. All that beautiful analogy, and that harmonious connection and conſiſtency, is quite loſt in other ſciences. Wherefore it is no wonder that greater improvements have been made in the mathematical ſciences, than in all the reſt put together. By following their methods, a habit of right reaſoning is obtained by frequent practice, like other things; and the cauſe why many people reaſon ſo badly is, for want of practice, due attention, and conſideration. They proceed in that tract which chance has put them into, being ignorant of true ſcience, and of thoſe univerſal invariable principles, upon which true reaſoning depends: as is evident from the many inſtances of falſe reaſoning and ignorance, wherewith the diſcourſes and writings of mankind abound.

In purſuance of our reaſoning in the mathematical way, we are often forced to draw diagrams, in order to repreſent the thing in queſtion; likewiſe to form ideas of the ſeveral parts, compound them, divide them, abſtract from them; to conſult the memory, to ſee what has been done and what is to do; to inſpect tables, books, inſtruments, *&c.* to call up all ſuch axioms, theorems, experiments, and obſervations, as are already known, and which can be uſeful to us. And then the mind examines, compares, methodizes, and alters them; till the ſeries be laid in a proper order, from the firſt principles to the laſt concluſion. For the principal thing required in ſtrict reaſoning is, to lay the ſeveral ſteps in due order, to ſee that they be firmly connected, and properly expreſſed, without any rhetorical flouriſhes, and to aim at truth by the ſhorteſt method. This indeed requires cool, ſedate, and ſober thinking; as alſo frequent application and practice, without which nothing can be done to the purpoſe. To which we may add, a fixt, conſtant, and firm reſolution to embrace truth wherever we find it; and to ſhun error and falſehood, when we find ourſelves in danger of falling into them.

There is but one method of true reaſoning, ſuch as has been deſcribed; but the grounds of falſe reaſoning are many, ſuch as theſe, want of faculties, want of learning, defects of memory, want of due reflection, not connecting the ſteps of the proof, truſting too much to the ſenſes, paſſions, appetites, prejudices, cuſtom, ſelf-intereſt, errors of education, wrong ſtating the queſtion, not underſtanding the terms, want of proofs, vulgar received opinions, weak authorities, precipitancy of judgment, *&c.* theſe will frequently diſturb us in our ſearch after truth, and are apt to biaſs the mind in reaſoning upon all other ſubjects; but few or none of them intrude in the mathematical ſciences. Mathematicians never attempt to reſolve any problems without proper *data*.

It muſt be owned that the progreſs of this ſort of knowledge is but ſlow, owing to the difficulty of the ſeveral branches that come under conſideration; but then it is ſure and certain; the acquiſition here gained is real knowledge. For this reaſon it is the work of ages to bring even a ſingle branch to perfection: and every ſucceeding age improves upon the foregoing.

And therefore it is no wonder if the ancients have, in many caſes, made uſe of round-about methods to encompaſs their ends, and given us long and tedious demonſtrations, and laid down many propoſitions, either of no uſe, or too ſimple and trifling to be taken notice of. Whence moſt of their inventions may be demonſtrated ſhorter, propounded eaſier, diſpoſed in a better method, and taught in a more compendious way.

But beſides the pleaſure a man finds in the ſearch and attaining of knowledge, and the agreeable ſurprize the mind is affected with, at the diſcovery of new and difficult truths; the advantages ariſing to mankind from theſe ſciences, in all the parts of human life, are endleſs. By help thereof we are able to keep our accounts regular and juſt, and manage all our tranſactions with one another; to caſt up and calculate immenſe ſums, for nothing lies without the power of numbers; to meaſure and divide lands and eſtates; and alſo all manner of ſurfaces or ſolids; to meaſure inacceſſible diſtances and altitudes, and find the hight of the clouds; to build houſes, caſtles, &c. by which we enjoy the principal delights of life, and ſecurity of health; to make fortifications to defend us from the enemy; to make guns and other inſtruments of war, and to ſhew how to uſe them in our defence; to reſolve all manner of pleaſant and ſubtle queſtions; to build ſhips, and by the help of wind and ſails, and the rules of art, to ſail upon the ſea, and find our way through it to diſtant countries, and traffick with foreign nations, whereby our wealth is increaſed; to contrive inſtruments to weigh and meaſure all ſorts of commodities, and give every man his juſt weight and meaſure; to make engines for raiſing and removing huge bodies; to invent innumerable machines, uſeful in private life, and neceſſary for our living commodiouſly, ſuch as clocks, watches, jacks, pumps, &c. to make dials and other inſtruments for keeping a regular account of time; to make ephemerides and chronological tables, to ſhew and account for the return of the various ſeaſons of the year, and to keep account of remarkable tranſactions and events; to deſcribe the ſeveral countries of the earth, and make maps and repreſentations thereof, and even to meaſure the whole earth and ſea; to account for the riſing and falling of the tides; to number the ſtars, and range them in their proper order; to meaſure the magnitude and diſtances of the planets, and explain the laws of their motion, and ſet bounds to their wandering courſes; to aſcertain the ſituation of all the great bodies of the univerſe, and ſhew the fabrick and conſtruction of the whole world; and to admire that wonderful power that contrived and framed it; to lead us through the dark mazes of nature, and through the intricate labyrinths and hidden ſecrets of philoſophy; to make proper inſtruments to improve the ſight, and even reſtore it in old age; and to magnify ſmall bodies, imperceptible to the naked eye, and make them become viſible; and to cauſe remote inviſible things to appear to us large and diſtinct; to give the true repreſentation or draught of any object, ſuch as towers, caſtles, trees, towns, *&c.* and to fix in the mind a method and habit of right reaſoning, a thing of the utmoſt conſequence, without which a man can hardly be called a rational creature.

The time would fail me in attempting to enumerate all the uſes and advantages of mathematical learning; and no words can fully expreſs the praiſes of that ſcience, which wanders through the heavens, the earth, and the ſeas: nor is it poſſible to ſet any bounds to ſo extenſive a ſcience. In this age, the number of its admirers and profeſſors are many, and daily increaſe more and more. Moſt people ſeem to be inſpired with the love of mathematical learning, and to be inamoured with its charms, and to court its favours and acquaintance. For this reaſon the ingenious and polite, of all ages, have applied themſelves to it, and made it a great part of their ſtudies. And in many countries princes and noblemen have practiſed and exerciſed themſelves herein. Nay, the greateſt kings and philoſophers, charmed with the beauties of this ſcience, have not only beſtowed a great part of their time, in the contemplation and ſtudy thereof, but have travelled abroad to diſtant places, in order to make improvements in this admirable knowledge. And have, at great expence, ſent perſons of learning and ſagacity into remote countries, on purpoſe to make new diſcoveries in ſome branch or other of this ſcience: a ſcience truly divine, being converſant in nothing but truth; which ſo ſatisfactorily inſtructs and informs the underſtanding, and feeds the mind with ſuch variety of pleaſures and recreations, and productive of ſuch infinite advantages to mankind.

And upon account of the great importance of this art, and its utility to all ranks of men; *foreign nations* have honoured men of eminent learning therein, with peculiar marks of their eſteem, by allowing public ſalaries to perſons of diſtinguiſhed merit, who have by their writings inſtructed mankind, and ſpent their time and money in improving and explaining the ſeveral branches of this ſcience; as motives and encouragements for promoting theſe arts, and proſecuting their ſtudies therein, and as rewards for the labour and difficulties they undergo in travelling through theſe craggy paths, in order to increaſe the common ſtock of knowledge, for the public good.

Having ſaid thus much concerning the certainty, extent, and uſe, of the mathematics; I ſhall now proceed to explain the nature thereof more particularly, and to deſcribe the ſeveral methods that mathematicians proceed in, for the diſcovery of truth; by by which will appear the excellency of mathematical reaſoning, above all others in the world; and conſequently that it is juſtly to be eſteemed the beſt pattern, that men can propoſe to follow.

Mathematics is a ſcience that conſiders and treats of all kinds of quantities whatever, that can be numbered or meaſured. That part which treats of numbering is called *arithmetic:* and that which concerns meaſuring is called *geometry*. Theſe two, which are converſant about multitude and magnitude, are called pure or abſtract mathematics, becauſe they inveſtigate and demonſtrate the properties of abſtract numbers and magnitudes of all ſorts: theſe two are the foundation of all the other parts. And when theſe two parts are applied to particular ſubjects, they are called *mixt mathematics*. Mathematics is alſo called *ſpeculative*, ſo far as it is concerned in finding out true propoſitions; and *practical*, as theſe relate to uſe, and are applied to practice.

Quantity is whatever will admit of increaſe or decreaſe; or is capable of any ſort of calculation or menſuration.

A propoſition is ſomething propoſed to be proved; or ſomething required to be done.

A theorem is a demonſtrative propoſition, wherein the nature and property of a thing is propoſed to be proved. And a ſet of ſuch theorems is called a *theory*.

A problem is a queſtion requiring ſomething to be done. *A limited problem* is that which has but one anſwer. *An unlimited problem* is that which has an infinite number of anſwers. *A determinate problem* is that which has a certain number of anſwers.

Solution of a problem, is the anſwer given to it. *A numerical ſolution* is the anſwer in numbers. *A geometrical ſolution* is an anſwer by the principles of geometry. *A mechanical* ſolution is one which is gained by trials.

A lemma is a ſhort preparatory propoſition, laid down in order to ſhorten the demonſtration of the main propoſition which follows it.

A corollary, or *conſectary*, is a conſequence drawn from a propoſition already demonſtrated.

A ſcholium is a remark made on any propoſition, corollary, or other diſcourſe.

Principles are the firſt grounds, rules, or foundations, of any ſcience; as definitions, axioms, poſtulates, and hypotheſes.

A definition is the explication of any word or term, in any ſcience; every definition ought to be clear, and contain no word or term but what is perfectly underſtood.

An axiom, or *maxim*, is a ſelf-evident propoſition. Theſe appear to be true at firſt hearing, and no body can deny them, without contradicting common ſenſe and reaſon. Here nothing ought to be allowed for an axiom, but what is clear and ſelf-evident: as this, *the whole is greater than a part*. Out of an infinite number of ſelf-evident truths that occur to the mind, men ſelect ſuch as are general, and of moſt uſe in demonſtrating any ſcience, and lay them up in ſtore, to have recourſe to, as need requires. And though men in their reaſoning do not always mention ſuch and ſuch axioms; yet the mind perceives the force of them, and what they mean, without ſtopping to repeat the words, or name them.

A poſtulate, or *petition*, is ſomething required to be done, which is ſo eaſy, that no body will diſpute it.

An hypotheſis is a ſuppoſition aſſumed to be true, by which a man is to argue, and build his reaſoning upon.

Demonſtration is the collecting the ſeveral proofs and arguments, and laying them in ſuch order, as to ſhew the truth of the propoſition under conſideration. Theſe proofs are to be drawn only from firſt principles,

principles, and from propoſitions already demonſtrated. Here we muſt keep ſtrictly to one and the ſame ſenſe of each definition; and when nothing is admitted but definitions, and axioms, and ſuch poſtulates and hypotheſes as are agreeable to the nature of the thing; and the conſtruction of figures in geometrical ſubjects; and demonſtrated propoſitions; and when the ſeveral arguments, or ſteps, are rightly connected together, ſo as one is plainly ſeen to be directly inferred from another, through the whole ſeries or chain of reaſoning: the concluſion at laſt obtained muſt be certain and true. Thus one truth is drawn from another, and from theſe a third, and thus continuing to deduce truths from truths, through the whole train of truths, we come at laſt to the concluſion or truth ſought after.

A direct, *poſitive*, or *affirmative demonſtration*, is that which concludes with the certain and direct proof of the propoſition in hand. This kind of demonſtration is moſt ſatisfactory to the mind; and therefore is called an *oſtenſive demonſtration.*

A negative, or *indirect demonſtration*, is that which ſhews a propoſition to be true, by ſome abſurdity which would neceſſarily follow if the propoſition advanced ſhould be falſe: this is called *reductio* and *abſurdum*; and ſhews the abſurdity and falſhood of all ſuppoſitions, but that contained in the propoſition. This is frequently made uſe of for eaſe and brevity's ſake, and to avoid a long perplext *oſtenſive* demonſtration. But although this ſort equally convinces the mind, and forces aſſent, yet it does not equally enlighten it. For it does not ſo much demonſtrate the truth itſelf directly, as the conſequent abſurdity or impoſſibility of the oppoſite ſuppoſition; whence it follows certainly (though indirectly) that the propoſition is true. When, at the ſame time, the original reaſon of its truth, or by what intrinſic cauſe it comes to be ſo, remains quite obſcure and in the dark.

A geometrical demonſtration, is that which depends on the principles of geometry.

It has been ſhewn, that when the firſt principles are all true, upon which the reaſoning relies; and all the ſteps truly and evidently connected together; that the concluſion we come to at laſt, muſt neceſſarily be true.

But if we lay down a falſe hypotheſis, and argue upon it as true, although we carry on our reaſoning ever ſo rightly, yet the concluſion will moſt certainly be falſe. For from falſe premiſes nothing but falſhood can follow. And therefore, on the contrary, when we argue from a precarious hypotheſis, and conduct our reaſoning with the greateſt rigour of truth, and at laſt come to a falſe concluſion; we may be aſſured, the hypotheſis we argued from is falſe. For there is no other poſſible cauſe for falling into a falſe concluſion. And this is the foundation of that way of reaſoning before mentioned, called *reductio ad abſurdum vel impoſſibile.* And this teaches us how to detect falſe hypotheſes.

Again, if our hypotheſis and other principles be all true; and we happen to reaſon wrong, either by giving a falſe meaning to any term, or making uſe of falſe propoſitions, in the courſe of our reaſoning; or not connecting the ſeveral ſteps rightly together; then falſhood and not truth muſt again be the concluſion; except it be by mere chance, that one error may correct another. And if our firſt principles and reaſoning be both falſe; it is a thouſand to one but the concluſion will be falſe, and truth here muſt have a poor chance for appearing.

Method is the art of diſpoſing a train of arguments, in a right order, either to find out the truth, or falſhood of a propoſition; or to demonſtrate it to others, when we have found it out. This is either analytical or ſynthetical.

Analyſis, or *the analytic method*, is the art of finding out the truth of a propoſition, by ſuppoſing the thing to be done; and going back ſtep by ſtep, till we arrive at ſome known truth. This is called the *method of invention*, or *reſolution*, and is generally uſed in algebra.

Syntheſis, or *the ſynthetic method*, is the ſearching out truth, by firſt laying down ſome ſimple and eaſy principles, and purſuing the conſequences till we come at the concluſion. This method begins at the moſt ſimple and eaſy things, and proceeds to the more compounded and general. It is alſo called the *method of compoſition*, and is contrary to the analytic method; as this proceeds from known principles to an unknown concluſion; whilſt the other goes in a retrograde order from the thing ſought, as if it was known, to ſome known principle. And therefore when any truth has been found out by the analytic method; it may be demonſtrated in a backward order, by ſyntheſis.

Thus you have an account of the rules and methods, whereby the mathematicians manage this their ſcience, and handle their ſeveral ſubjects. Methods ſo clear and inſtructive, that they may juſtly challenge the world to produce any others, of equal perſpicuity, evidence, and certainty. And the ſtructures they erect thereby are equally ſtrong and impregnable, as well as admirable and ſurprizing. For in the firſt place, they premiſe ſome general principles to begin with, as definitions, axioms, &c. from theſe they derive ſome ſimple and eaſy propoſitions; and from theſe others are drawn ſtill harder; and then by degrees they arrive at the more difficult ones; what goes before being always helpful for finding out the following. Thus a chain of arguments is carried on in an uninterrupted ſeries, and their truth confirmed by infallible reaſoning. Then the moſt general and uſeful propoſitions are collected together, and drawn up in order, and put into a body or magazine, and reſerved for uſe, to be called forth, as occaſion requires, for the inveſtigation and demonſtration of others. Thus they form ſo many ſyſtems of mathematical truths, according to the various ſubjects they examine; which muſt ſtand as principles for finding out new ones, or as teſts for trying the truth of others. For any propoſition being once proved true, muſt eternally remain true, and can never vary: it being the nature and eſſence of truth to continue invariable.

Now theſe ſeveral ſyſtems, or branches of the mathematics, that is, the diviſion of the mathematical ſciences, have been differently made and reckoned up, by different men. But the principal branches or parts thereof, at leaſt thoſe of moſt uſe, may be reckoned to be theſe: arithmetic, geometry, proportion, trigonometry, projection of the ſphere, menſuration, ſurveying, guaging, dialling, gunnery, geography, conic ſections and curve lines, navigation, mechanics, optics, perſpective, chronology, algebra, centripetal forces, aſtronomy, fluxions, increments. I have already publiſhed ſeveral of theſe in ſeparate tracts; and from the regard I always had for theſe arts, and the great deſire I have of ſeeing them flouriſh; I intend from time to time, in the courſe of this work, to publiſh the reſt, as ſoon as they can be got ready for the preſs. Which done, I doubt not but the young ſtudent will be furniſhed with a compleat courſe of the mathematics, ſufficient to inſtruct him in his progreſs, through theſe difficult paths, and to make him fit and able to read larger, and more elaborate treatiſes.

* * *

A

TREATISE

OF

ARITHMETIC,

CONTAINING

All the PRACTICAL PARTS thereof;

BOTH IN

WHOLE NUMBERS,
VULGAR FRACTIONS,
AND DECIMALS.

LIKEWISE

The THEORY of NUMBERS,

And their Principal Properties, demonſtrated in a plain and eaſy manner.

Doctores, elementa velint ut diſcere prima. HOR.

THE

PREFACE.

HE that would make any conſiderable progreſs in the mathematics, muſt begin at the firſt principles, and proceed gradually forward from one branch of that ſcience to another; according as they are naturally connected together, and have a dependance upon one another. This will make the progreſs as eaſy, ſhort, and intelligible, as the nature of the thing will admit of. Whilſt he that takes a contrary courſe, will always be involved in difficulty, doubt, and obſcurity; the knowledge he gains will be imperfect; and for want of evidence, the mind will want that conviction which is neceſſary for eſtabliſhing truth.

Arithmetic may be juſtly ſaid to be the baſis of all the other parts of mathematics. All things of whatever kind they are, may be reduced to numbers, and their quantities and proportions, calculated by numbers. All other branches have need of arithmetic, ſome way or other; and would often be at a ſtand without it. Yet arithmetic has no need of them, but ſtands ſolely upon its own principles. In all parts of the mathematics, no problem of any ſort is deemed to be compleatly ſolved, till it be calculated arithmetically, and its value brought out in numbers. And ſince it is of ſuch conſequence, it is abſolutely neceſſary for the young ſtudent, who would lay a good foundation for attaining a competent knowledge in the mathematics, firſt of all to make himſelf ac-

quainted with all the parts of arithmetic, and the nature and properties of numbers: without which it would be in vain for him to attempt any thing.

And as it is of ſuch great uſe in the ſciences, ſo it is equally ſerviceable in human actions and employments. He muſt be very little verſed in the common affairs of life, that does not know the great uſefulneſs of arithmetic in every inſtance thereof. No buſineſs can be carried on without the help of numbers; no trade or commerce exerciſed without regular accounts: ſo that in all ſituations of life, arithmetic is a neceſſary accompliſhment.

As to the enſuing treatiſe, I have in the firſt book, fully, and yet very conciſely handled all the parts of common arithmetic; and have made all the rules thereof, as ſhort as poſſible, ſo as to be intelligible; and the reader cannot fail of underſtanding them, by means of the examples there given, which I ſuppoſe are ſufficient for that end, and no more. I have alſo endeavoured to give the reaſons for the ſeveral operations in the fundamental parts of this art, which cannot miſs pleaſing the reader, as he will have his judgment and underſtanding informed, at the ſame time he is learning the practice.

In the ſecond book, I have delivered the ſubſtance of what Euclid *and others have written about the properties of numbers, adding whatever I thought of any conſequence in the theory of numbers. And here I have for the moſt part demonſtrated the propoſitions of* Euclid *after a different manner from him, and often more generally. And though the theory ought to precede the practice, in any ſcience: yet here it was hardly poſſible to obſerve that rule. For there is not only frequent uſe made of multiplication, diviſion,* &c. *but there is a good deal of abſtract reaſoning about the properties of num-*

numbers, which could not well be underſtood, till the reader was well acquainted with the operations of arithmetic; which is the reaſon I have put it laſt. I know of nothing that is wanting in this treatiſe, except it be a greater variety of examples; and this would require more room; and the intelligent reader can eaſily ſupply theſe of himſelf; to whom I wiſh ſucceſs, anſwerable to his endeavours.

W. Emerſon.

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THE

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BOOK II.

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ARITHMETIC.

DEFINITIONS.

1. *ARITHMETIC* is the art of computing by numbers; it is called *vulgar* or *common Arithmetic*, when it treats of whole numbers.

2. *Unity* is that by which every thing is called one; and a unit is the beginning of number.

3. *Number* is a multitude of units: by this every thing is reckoned.

4. An *integer* is any whole thing.

5. A *whole number* is a preciſe number without any parts annext.

6. A *mixt number* is a whole number with ſome part annext.

7. A *fraction* is a part or parts of an unit.

8. A *proper fraction* is leſs than a unit.

9. An *improper fraction* is greater than a unit.

10. An *aliquot part* is that which is contained a preciſe number of times in another.

Cor. Hence 1 is an aliquot part of any number: but a number cannot be called an aliquot part of itſelf.

11. An *aliquant part* is ſuch as is contained in another, ſome number of times, with ſome part or parts over.

12. One number is ſaid to be *multiple* of another, when it contains it a preciſe number of times.

13. One number is ſaid *to meaſure* another, when it is contained in the other a preciſe number of times, without a remainder. The ſaid meaſure is alſo a *diviſor.*

Cor. Any number is a meaſure to itſelf. And 1 is a meaſure to any number.

14. An *even number* is that whoſe half is a whole number.

15. An *odd number* is that which cannot be divided into two equal whole numbers.

Cor. The numbers one, two, three, four, *&c.* are alternately odd and even for ever.

16. A *prime number* is that which can only be meaſured by a unit.

17. *Numbers* are ſaid to be *prime to one another*, when only a unit meaſures both. Theſe are alſo called *coprimes.*

Cor. Therefore 1 is prime to every number.

18. A *compoſite number* is that produced by multiplying ſeveral other numbers together, called *factors* or *multipliers.* Alſo what is produced by ſuch multiplication, is called a *product.*

19. Numbers are ſaid to be *compoſed to one another*, when ſome number (greater than a unit) meaſures both.

20. A *plane number* is the number produced by multiplying two other numbers.

21. A *ſolid number* is the product of three numbers.

22. A *ſquare number* is the product of a number by itſelf.

23. A *cube number* is the product of a number, and its ſquare.

24. *Like or ſimilar plane or ſolid numbers*, are thoſe whoſe ſides or multipliers are proportional.

25. A *perfect number* is that which is equal to the ſum of all its aliquot parts.

26. The *power* of any number, ſignifies, that the number (called the *root*) ſhall be ſo often multiplied, as is denoted by the number (or index) expreſſing the power. Thus the 2d power of 5, is 5 multiplied by 5, or 25; the 3d power of 5, is 25 multiplied by 5, *&c.*

27. Four numbers are ſaid to be *proportional*, or in the *ſame proportion*, when comparing two and two; the firſt is the ſame multiple, or the ſame part or parts of the ſecond, as the third is of the fourth, thus: 6, 2, 9 and 3, are proportional; for 6 contains 2 thrice, and 9 contains 3 thrice. Alſo 4, 6, 10, 15, are proportional; for 6 is once and half 4, and 15 is once and half 10. And the ſeveral numbers are called the *terms* of the proportion; and the quotient ariſing, by dividing the former by the latter number, is called the *Ratio.*

28. Numbers are ſaid to be in *continual proportion*, or in *geometrical progreſſion*, when the firſt has the ſame proportion to the ſecond, as the ſecond to the third, and as the third to the fourth, and ſo on, thus: 2, 6, 18, 54, *&c.* are continual proportionals.

29. *Mean proportionals* are all the intermediate terms, between the *extremes*, in a geometrical progreſſion.

30. *Surds* are ſuch numbers as have no exact roots.

NOTATION.

1. The characters by which numbers are expreſſed, are theſe ten: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; 0 is called a *cypher*; and the reſt, or rather all of them, are called *figures*, or *digits*. The names and ſignification of theſe characters, and the origin or generation of the numbers they ſtand for, are here ſet down:

0 nothing.
1 one, or a ſingle thing called a unit.
then $1 + 1 = 2$ two.
$2 + 1 = 3$ three.
$3 + 1 = 4$ four.
$4 + 1 = 5$ five.
$5 + 1 = 6$ ſix.
$6 + 1 = 7$ ſeven.
$7 + 1 = 8$ eight.
$8 + 1 = 9$ nine.

then $9 + 1 =$ ten, which has no ſingle character; and thus by the continual addition of 1, all numbers are generated.

2. The value of any number depends not on the figure or figures alone, but upon the figures and places where they ſtand, jointly. And the order of places is backward from the right hand towards the left. The firſt place is called the place of units; the ſecond, tens; the third, hundreds; the fourth, thouſands; the fifth, ten thouſands; the ſixth, hundred thouſands; the ſeventh, millions; and ſo on. Thus in the number 765487654; 4 in the firſt place ſignifies only 4; 5 in the ſecond place ſignifies five tens or fifty; 6 in the 3d place ſignifies ſix hundred; 7 in the 4th place is ſeven thouſand; 8 in the 5th place is eighty thouſand; 4 in the 6th place is four hundred thouſand; 5 in the 7th place is five millions; and ſo on.

3. A cypher, though of no value by itſelf, yet it occupies a place, and advances the figures on the left hand into higher places, from whence they have a greater value. Thus 3 ſignifies only 3, but 30 ſignifies 3 tens or thirty, and 300 ſignifies 3 hundred.

4. The values of all figures increaſe in a tenfold proportion from the right hand towards the left, each following place being ten times greater than the foregoing. Thus in the number 33333333; 3 in the firſt place is three; in the ſecond, 30 thirty; in the

third,

third, 300 three hundred; in the fourth, 3000 three thouſand; in the fifth, 30000 thirty thouſand, *&c.* And thus 1 ſignifies one, 10 ſignifies ten, 100 ſignifies a hundred, 1000 ſignifies a thouſand, and ſo on; and in general, ten units make 1 ten, ten tens make 1 hundred, ten hundred make 1 thouſand, *&c.*

5. Hence, placing 1, 2, 3, *&c.* cyphers on the right hand of any number, makes it ten, a hundred, a thouſand times, *&c.* greater than before. But placing cyphers on the left hand does not alter the value, becauſe every figure remains in the ſame place as before.

This method of expreſſing numbers, by the different values of the figures in different places, is an admirable invention; without which it had been neceſſary to have as many different characters, as there are numbers to be expreſſed; which would have been impoſſible.

AXIOMS.

1. If two numbers are equal to a third, they are equal to one another.

2. If equal numbers be added to equal numbers, the wholes will be equal.

3. If from equal numbers the ſame or equal numbers be taken away, the remainders will be equal.

4. Thoſe numbers are equal, which are the ſame multiple of equal numbers.

5. Thoſe numbers are equal, which are the ſame part of equal numbers.

6. The ſame powers, or the ſame roots of equal numbers, are equal.

7. Unity or 1 neither multiplies nor divides; that is, the product or quotient is ſtill the ſame number.

8. If a number be compoſed of two numbers, multiplied together; either of them meaſures it by the other.

9. If a number meaſures ſeveral other numbers; it

it likewiſe meaſures the ſum (or difference) of theſe numbers.

10. If a number meaſures another; it alſo meaſures every number which that other meaſures.

11. If a number meaſures the whole, and a part taken away; it alſo meaſures the reſidue.

The Signification of other Characters here uſed.

Characters.	Signification.
$+$	*more, and, to be added,* being an affirmative ſign. Thus $7+3$ ſignifies 3 added to 7; and $A+B$ denotes the ſum of A and B.
$-$	*leſs, leſſened by, abating,* being a negative ſign. Thus $7-3$ means 3 taken out of 7, and $A-B$ denotes the remainder, when B is ſubtracted from A.
$\times$	*multiplied by,* as 7×3 ſignifies 7 times 3; alſo $A\times B$ or AB, is the product of A and B multiplied together. Where note, if letters ſtand to denote numbers, they are commonly ſet together, like letters in a word.
$\div$	*divided by,* thus $6\div 3$ ſignifies 6 divided by 3; alſo 3) 6 (ſignifies 6 divided by 3; alſo $\frac{6}{3}$ ſignifies 6 divided by 3; and in general $A\div B$, or B) A (, or $\frac{A}{B}$, is the quotient of A divided by B.
A^2	*the ſquare of* A, that is, AA.
A^3	*the cube of* A, that is, AAA.
A^n	*the* n^{th} *power* of A, the index n being any number.
$\surd$	*the ſquare root,* thus $\sqrt{16}$ is the ſquare root of 16, and $\sqrt{A}$ is the ſquare root of A.

Characters.	Signification.
$\sqrt[3]{}$	*the cube root*, as $\sqrt[3]{8}$ is the cube root of 8, and $\sqrt[3]{A}$ is the cube root of A.
$=$	*equal to*, as $7+3=10$, 7 and 3 equal to 10.
$::$	*A note of proportion*, thus $2:3::4:6$, ſignifies 2 is to 3, as 4 to 6; and $A:B::a:b$, A is to B, as a to b, ſometimes written thus, A—B—a—b.
∺	*continual proportionals*, $A:B:C:D$ ∺, A, B, C, D are in continual proportion.
$\overline{A+B+C}$	the ſum of A, B, and C; a line drawn over ſeveral numbers, denotes the ſum of them.

BOOK I.

The Practice of Arithmetic.

CHAP. I.

The fundamental Rules of common or vulgar Arithmetic.

PROBLEM I.

To read or expreſs any Number written.

THIS is called *Numeration*, and is eaſily performed by help of the following table, which ſhews the names of the ſeveral places, and conſequently of the figures ſtanding there, as explained before in the Notation.

NUMERATION TABLE.

&c.	Tens of billions	Billions, or millions of millions	Hundred thouſands of millions	Ten thouſands of millions	Thouſands of millions	Hundreds of millions	Tens of millions	Millions	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units
	4	3	8	7	6	5	4	3	8	7	6	5	4	3

RULE.

RULE.

1. Begin at the units place, and divide, or rather diſtinguiſh your number into periods of 6 figures apiece, called *grand periods*, or *double periods*. The firſt period to the right is units, the ſecond millions, the third bi-millions, the fourth tri-millions, the 5th, 6th, *&c.* quadri-millions, quinti-millions, ſexti-millions, ſepti millions, octi-millions, noni-millions, deci-millions, *&c.*

2. Likewiſe diſtinguiſh theſe grand periods into two parts, called *ſingle periods* of three figures apiece; in theſe write (or ſuppoſe to be written) units over the firſt place, tens over the ſecond place, and hundreds over the third place.

3. Begin to read at the left hand, expreſſing hundreds, tens, units, as you come to the reſpective places where theſe figures are; and at the end of each ſingle period (on the left hand) always pronounce thouſands; and at the end of the grand period, expreſs its title or ſurname belonging to it; proceeding thus to the right hand where the number ends.

Ex. 1.

Read the number 50765.

t u h t u
50 765

Having diſtinguiſhed the number into periods, and written u over units, t over tens, h over hundreds, it will be read thus: fifty thouſand, ſeven hundred and ſixty-five.

Ex. 2.

To read 43876543876543.

t u h t u h t u
43 876 543 876 543

Forty three bi-millions, eight hundred and ſeventy ſix thouſand, five hundred and forty three millions, eight hundred and ſeventy ſix thouſand, five hundred and forty three.

Ex. 3.

Read this number 2418579643219004613254768096.

```
     h t u    h t u    h t u    h t u    h t u
2418 579643 219004 613254 768096
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Two thouſand, four hundred and eighteen quadri-millions;
Five hundred ſeventy nine thouſand, ſix hundred forty three tri-millions;
Two hundred nineteen thouſand, and four bi-millions;
Six hundred thirteen thouſand, two hundred fifty four millions;
Seven hundred ſixty eight thouſand, and ninety ſix.

PROBLEM II.

To add whole numbers together.

Addition is the rule by which ſeveral numbers are put together, in order to find the ſum of them all.

RULE.

1. Place all the numbers ſo, that units may ſtand under units, tens under tens, hundreds under hundreds, *&c.* and draw a line underneath.

2. Begin at the units place, and reckon up all the figures in that place from the bottom to the top, and what overplus there is above even tens, ſet down, and carry ſo many to the next row as there were tens.

3. Reckon up all the figures in the place of tens, together with what you carried, and ſet down the overplus, carrying the tens to the next row; and ſo proceed to the laſt.

4. If you don't chooſe to reckon forward, you may make a prick when you have reckoned to ten or more, carrying on the overplus; and then add ſo many to the next row as you have pricks.

Ex. 1.

Let theſe numbers be added together:

$$\begin{array}{r} 9482 \\ 590 \\ 307 \\ 85 \\ \hline 10464 \\ \hline \end{array}$$

Beginning at 5, ſay the ſum of 5 and 7 is 12 and 2 is 14, ſet down 4 and carry 1. The ſum of 1 and 8 is 9 and 9 is 18 and 8 is 26, ſet down 6 and carry 2. Then 2 and 3 is 5 and 5 is 10 and 4 is 14, ſet down 4 and carry 1. Laſtly, 1 and 9 is 10, which being the laſt, ſet it down.

The reaſon of carrying the tens to the next place is plain; for the ſum of 5, 7 and 2 being 14, the 4 belongs to the units, and the 1 to the tens. Again, the ſum of 1, 8, 9 and 8 being 26, which are tens, the 6 belongs to the tens, and the 2 to the next ſuperior place, which is hundreds. Then the ſum of 2, 3, 5 and 4 being 14, *viz.* 14 hundreds, the 4 belongs to that place, and the 1 to the place above, which is thouſands. Laſtly, the ſum of 1 and 9 is 10, that is 10 thouſand, that is 0 in the place of thouſands, and 1 in the place of ten thouſands. In ſhort, thus:

The ſum of the row of units	14
The ſum of the row of tens	250
The ſum of the row of hundreds	1200
The ſum of the row of thouſands	9000
total	10464

Ex. 2.

Add theſe numbers together.

350709
31806500
339087
46011
2935

ſum 32545242

The *proof* of Addition is this: begin at the top, and add all the numbers downwards, by the ſame rule as you added them upwards before; then if the total ſums agree, the work is right.

PROB-

PROBLEM III.

To add numbers of ſeveral denominations together.

RULE.

1. Place the numbers ſo, that thoſe of the ſame denomination may ſtand directly under one another, then draw a line under them.

2. Begin at the loweſt denomination firſt, and reckon upwards till you get as many as makes one of the next denomination above; then make a prick, and carry the overplus, or exceſs, to the next figures; and ſo reckon forward, always pricking when you have as many as makes one of the next denomination. Proceed thus till that denomination is finiſhed, and ſet down the overplus at bottom.

3. Reckon your pricks in the denomination you have finiſhed, and carry ſo many, to be added to the next denomination, which muſt be added up by the ſame rule; and ſo of the reſt. In the laſt denomination, add them up as whole numbers.

Ex. 1. *Money.*

Add theſe ſums of money together.

	£.	*s.*	*d.*
	57	6	8·
	127	14·	0
	0	9	$6\frac{1}{2}$
	17	0	$3\frac{3}{4}$
ſum	202	10	$6\frac{1}{4}$

Note, 4 farthings make 1 penny, 12 pence 1 ſhilling, 20 ſhillings 1 pound.

Ex. 2. *Troy Weight.*

	oz.	*pwts.*	*grs.*
	207	13	19
	81	0	11
	157	15	6
	31	9	20
total	477	19	8

Note. In Troy weight, 24 grains make a penny-weight, 20 penny-weights an ounce, 12 ounces a pound.

Ex. 3. *Apothecary's Weight.*

	oz.	*drs.*	*ſcr.*	*grs.*
	15	7	2	15
	3	4	0	12
	0	0	1	18
	1	5	1	3
total	21	2	0	8

Note, In Apothecary's weight, 20 grains make a ſcruple (℈), 3 ſcruples a dram (ʒ), 8 drams an ounce (℥), 12 ounces a pound (℔).

Ex. 4. *Averdupoize leſſer weight.*

	lb.	*oz.*	*dr.*
	15	11	12
	4	10	0
	12	0	13
	0	15	9
total	33	6	2

Note, 16 drams make an ounce, 16 ounces a pound.

Ex. 5. *Averdupoize greater weight.*

	tuns	*hunds.*	*ſto.*	*lb.*
	570	18·	6	11·
	38	7·	2·	0
	92	0	6	3
	12	15	0	10
total	714	1	7	10

Note, 14 pounds make a ſtone, 8 ſtone 1 hundred weight, 20 hundred weight 1 tun.

Ex. 6. *Long Meaſure.*

	yds.	*feet*	*inch.*
	37	2·	11·
	7	0	3
	8	1	10·
	4	2·	5
total	58	1	5

Note, 3 barley-corns make an inch, 12 inches a foot, 3 feet a yard; alſo 5½ yards make a pole, 22 yards a chain, 10 chains a furlong, 8 furlongs a mile.

Liquid Meaſure.

2 pints make a quart, 2 quarts a pottle, 2 pottles a gallon, 8½ gallons a firkin or anker, 6 firkins a hogſhead of ale, 63 gallons a hogſhead of wine.

Dry Meaſure.

2 pints make a quart, 2 quarts a pottle, 2 pottles a gallon, 2 gallons a peck, 4 pecks a buſhel, 8 buſhels a quarter, 4 quarters a chaldron, 10 quarters a laſt.

SCHOLIUM.

If a long liſt of numbers is to be added up, divide

vide it into ſeveral parcels, and add them ſeparately; and then add all theſe parcels together.

The *proof* of this rule is the ſame as the laſt; only in reckoning downward, make croſſes inſtead of pricks, to avoid confuſion.

PROBLEM IV.

To ſubtract one whole number from another.

Subtraction is the taking one number from another, to find their difference.

RULE.

1. Place the greater number uppermoſt, and the other under it, ſo as units may be under units, tens under tens, *&c.* and draw a line under them.

2. Begin at the right hand or place of units, and ſubtract the lower figure from the upper, and ſet down the difference underneath them; do the ſame with the reſt of the figures.

3. When the lower figure is greater, borrow 10, and add it to the upper number, from which ſubtract the lower, and ſet down the remainder; carry 1 to be added to the next lower figure, and ſubtract the ſum from the upper, and ſet down the remainder; and ſo on from one row to another.

Ex. 1.

	£.
from	270481467
take	31065363
rem.	239416104

The reaſon of this operation is plain, only when the lower number is leſs, 10 is added to the upper number, as here, 5 is leſs than 1, therefore 1 is borrowed from 8 to make 11, then 5 from 11 remains 6; then the next figure 6 ought in reality to be taken from 7, inſtead of 8; but the difference will be the ſame, whether you take 6 from 7, or add the 1 borrowed to 6, and take the ſum 7 out of 8, in either caſe 1 remains.

Ex. 2.

from 30076058972
take 17078032863

rem. 12998026109

Ex. 3.

One born in 1682, *how old is he in* 1763?

1763
1682

81 anſwer.

The *proof* of Subtraction is to add the remainder to the leſſer number, which ought to make up the greater, if the work be right.

PROBLEM V.

To ſubtract numbers of different denominations.

RULE.

1. Place the numbers, ſo that the greater may be uppermoſt, and that thoſe of the ſame denomination may ſtand directly under one another, and draw a line under them.

2. Begin at the loweſt denomination, and take the lower number from the upper one, and ſet down the difference, or remainder, underneath. Do the ſame with the next denomination, and ſo on till the laſt, which muſt be ſubtracted as whole numbers.

3. When the lower number in any denomination happens to be the greater, borrow 1, that is, add as many

many to the upper number as makes one of the next higher denomination, and then ſubtract the lower number, and ſet down the remainder. Then carry 1, and add it to the lower number of the next denomination, and then ſubtract as before.

Ex. 1. *Money.*

	£.	*s.*	*d.*
from	241	9	$6\frac{1}{4}$
take	82	6	3
rem.	159	3	$3\frac{1}{4}$

Ex. 2. *Money.*

from	3794	0	$3\frac{1}{2}$
take	129	5	$10\frac{3}{4}$
rem.	3664	14	$4\frac{3}{4}$

Ex. 3. *Troy Weight.*

	lb.	*oz.*	*pwts.*	*grs.*
from	19	12	15	18
take	13	11	17	7
rem.	6	0	18	11

PROBLEM VI.

To multiply one whole number by another.

Multiplication is taking the *multiplicand*, or number to be multiplied, ſo many times as there are units in the *multiplier*; and the reſult is called the *product.* Multiplication is a compendious method of addition, and is performed by help of the following table, which muſt be got by heart.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The uſe of the table is this: find one figure on the ſide of the table, and the other at top; then in the angle of meeting is their product. Thus the product of 5 and 7 is 35; and the product of 9 times 8 is 72.

1. A GENERAL RULE.

1. Place the multiplier under the multiplicand, the units under units, &c. and draw a line under them.

2. You muſt multiply from the right hand to the left, thus: begin with the units or loweſt figure of the multiplier, by which multiply the loweſt figure of the multiplicand, and ſet down the overplus above the tens, and carry the tens. Then multiply the 2d figure of the multiplicand by the ſame, adding ſo many units, as you had tens to carry; and ſet down the overplus, and carry the tens as before. Do thus

till

till you come to the laſt figure, whoſe product muſt be ſet down entire.

3. Then take the ſecond figure of the multiplier, and multiply by this as you did before; ſetting the firſt figure of the product under the figure you multiply with; do ſo with the reſt of the figures in the multiplier; ſetting the firſt figure of each product under, or in the ſame place as the figure you multiply by. Or, which is the ſame thing, ſetting each product ſo many places back towards the left hand, as the multiplying figure is diſtant from the firſt figure.

4. Laſtly, add all theſe products together, for the product of the two numbers given.

Note, you may eaſily multiply by 12 in one line, as if it was a ſingle figure, if you get by heart all the products of all the natural numbers by 12, as far as 9.

Ex. 1.

multiply	60735
by	7
product	425145

Explanation.

7 times 5 is 35; ſet down 5 and carry 3. 7 times 3 is 21 and 3 I carry is 24; ſet down 4 and carry 2. 7 times 7 is 49 and 2 carried is 51; ſet down 1 and carry 5. 7 times 0 is 0 but 5 is 5; ſet down 5 and carry 0. 7 times 6 is 42, which ſet down.

Ex. 2.

multiply	2760325
by	37072
	5520650
	19322275
	193222750
	8280975
product	102330768400

Demonſtration of the rule.

In Ex. 1. 7 multiplying 5 produces 35, the 5 will fall in the place of units, and the 3 belongs to the tens. Then 7 multiplying 3 in the 2d place, or place of tens, produces 21, of which 1 belongs to the tens, to which the 3 carried being alſo tens, muſt be added, which makes 4 tens; and the 2 belongs to the 3d place, or hundreds. Then 7 multiplying 7 in the third place, makes 49, the 9 belongs to the 3d place, to which add the 2, which alſo belongs to the 3d place, the ſum is 51; 1 belongs to the third place and 5 to the 4th place. Then 7 times 0 is 0, (in the 4th place) but 5 is 5. Laſtly, 7 times 6 is 42, the 2 belongs to the 5th place, and 4 to the 6th. Theſe particular products will ſtand thus:

$$\begin{array}{r}
60735 \\
7 \\
\hline
35 \\
21\,. \\
49\,.\,. \\
0\,.\,.\,. \\
42\,.\,.\,.\,. \\
\hline
425145 \\
\hline
\end{array}$$

And in Ex. 2. 2 multiplying 5 produces 10, the 0 is in the place of units, and ſo on. Again, 7 multiplying 5 makes 35, the 5 is in the 2d place, becauſe the multiplier is really 70. Again, 7 in the 4th place multiplying 5 makes 35, and the 5 will be in the 4th place, becauſe you really multiply by 7000, and ſo for all the reſt.

Ex. 3.

If 1 *hogſhead coſt* 13 *pound, what will* 18 *coſt?*

```
          18
         ----
         104
         13
         ----
answ.    234   pounds.
         ----
```

2 RULE.

When one or both the numbers end with cyphers, neglect the cyphers and multiply the remaining figures as before; and to the product, annex the cyphers that are in both numbers.

Ex. 4.

```
multiply     507300
by             4020
            -------
              10146
             20292
           ----------
product    2039346000
           ----------
```

3 RULE.

When any number is to be multiplied by 10, 100, 1000, *&c.* annex ſo many cyphers at the end of the number, as there are in the multiplier.

Ex. 5.

Multiply 23079 by 100, the product is 2307900.

4 RULE.

In large multiplications, make a table of the multiplicand multiplied by all the 9 digits. Then you have no more to do, but to take out the reſpective product for each figure of the multiplier, and add them all together.

TABLE.

1	70500768
2	141001536
3	211502304
4	282003072
5	352503840
6	423004608
7	493505376
8	564006144
9	634506912

Ex. 6.

multiply 70500768
by 50431

70500768
211502304
282003072
352503840

product 3555424231008

The *proof* of Multiplication, is by making the multiplicand to be the multiplier; then if the product comes out the ſame as before, your work is right.

That two numbers will give the ſame product, whichever is the multiplier, will appear thus: ſuppoſe the numbers 4 and 36. Then 36 times 1 is the ſame with once 36; and therefore 36 times 1+1+1+1, or 36 times 4 is the ſame with 4 times 36; and ſo of others.

SCHOLIUM.

There is a way of proving multiplication by caſting away the nines, which though not infallible, ſerves to confirm the other, and is very expeditious. It is thus, ſee Ex. 4. make a croſs, and add all the figures or digits of the multiplicand together, as units, thus 5+7+3=15, throw away the nines, and ſet the remainder 6 on one ſide of the croſs. Do the ſame with the multiplier 4+2=6, ſet the remainder on the other ſide of the croſs. Do the like with the product, and ſet the remainder at top. Laſtly, multi-

multiply the figures on the ſides, and throw away the nines, and ſet the remainder at bottom, which muſt be the ſame with the top, if the work is right.

Ex. 6.

PROBLEM VII.

To multiply numbers of different denominations, by a given number.

1 RULE.

If the multiplier be a ſingle figure; begin at the loweſt denomination, and multiply it by the given number, and ſee how many of the next denomination is contained in the product; ſet down the odds, and carry ſo many to the next. Then multiply the next denomination, adding what you carried; and ſet down the odds. Proceed thus till all be multiplied.

This method is rather reckoning than multiplying.

Ex. 1. *Money.*

	£.	*s.*	*d.*
multiply	49	13	10
by			7
product	347	16	10

Ex. 2. *Weight.*

	c.	*ſt.*	*lb.*
multiply	11	2	13
by			6
product	68	1	8

2 RULE.

If the multiplier be a great number made up of ſeveral others multiplied together. Multiply ſucceſſively by the parts, inſtead of the whole.

Ex. 3.

	£.	s.	d.	
multiply	127	13	9	by 45.
			5	
	638	8	9	
			9	
product	5745	18	9	

3 RULE.

If the multiplier is not compoſed of others; find two or more numbers, whoſe product comes neareſt: then multiply as before, and add what is wanting, or ſubtract what is over.

Ex. 4.

	£.	s.	d.	
multiply	7	12	10	by 47.
			6	
	45	17	0	
			8	
	366	16	0	
ſubtract	7	12	10	
product	359	3	2	

PROBLEM VIII.

To divide one whole number by another.

Diviſion teaches to find how often one number, called the *diviſor*, is contained in another, called the *dividend*. Or it ſhews how to find ſuch a part of the dividend as the diviſor expreſſes. The number here ſought is called the *quotient*.

1. A GENERAL RULE.

1. Set down the dividend, and the diviſor on the left hand of it, within a crooked line; alſo make another crooked line on the right hand, for the quotient.

2. Enquire how oft the firſt figure of the diviſor is contained in the firſt figure of the dividend, or in the two firſt figures, when that of the diviſor is greater; and place the anſwer in the quotient.

3. Multiply the whole diviſor by the quotient figure, and ſet the product orderly under the dividend towards the left hand, and ſubtract it therefrom. But *note*, if this product be greater than that part of the dividend; a leſs figure muſt be placed in the quotient.

4. Make a prick under the next figure of the dividend to mark it, and bring it down, annexing it to the remainder; then this number is called the *dividual.*

5. Seek how oft the diviſor is contained in the dividual, and ſet the anſwer in the quotient; then multiply and ſubtract as before; and proceed thus till all the figures in the dividend are brought down one by one. And *note*, for every figure brought down, a figure (or a cypher) muſt be placed in the quotient.

Note, ſince there is a neceſſity of trial, to find out the true quotient figure; therefore, before it be ſet down, multiply 2 or 3 figures of the diviſor on the left hand, by that figure in mind, to ſee if it exceed the dividual.

Ex. 1.

Divide 14122 by 46.

```
46) 14122 (307 the quotient.
    138··
    ———
      322
      322
      ———
        ·
```

Expla-

Explanation.

Firſt I aſk how oft 4 in 1, which is no times at all: then how oft 4 in 14, which is 3 times; then I place 3 in the quotient, and then multiply 46 by 3, and ſet the product 138 under 141, and ſubtracting there remains 3. Then I prick the 2 and bring it down to 3, which then is 32 for a dividual; then enquiring how oft 4 in 3, the anſwer is 0, which I place in the quotient. Then I prick, and bring down the next figure 2, and the dividual is now 322, then I aſk how oft 4 in 32, the anſwer would be 8; but then 46 multiplied by 8 would exceed 322, therefore I place 7 in the quotient, by which I multiply 46, and the product is 322; and that ſubtracted from 132, leaves nothing. Then 307 is the quotient.

Ex. 2.

Divide 18972584 by 6023.

```
6023) 18972584 (3150 the quotient.
      18069 ·:·
      -----
        9035
        6023
        ----
        30128
        30115
        -----
          134 the remainder.
          ---
```

Demonſtration of the rule.

In Ex. 1. ſince 46 is contained 3 times in 141, therefore it is contained 300 times in 14122; that is, 3 muſt be in the third place.

Alſo ſince 46 is contained 7 times in the remainder 322; therefore 46 is contained in the whole dividend 307 times.

And

And in Ex. 2. ſince 6023 is contained 3 times in 18972; it is contained 3000 times in 18972584; and 100 times in the remainder 903584, and 50 times in the next remainder 301284; and 0 times in the laſt remainder 134. Therefore the diviſor is contained in the whole dividend, 3150 times.

2 RULE.

When the diviſor ends with cyphers, cut them off, and likewiſe cut off as many places of the dividend on the right hand; and perform the diviſion by the remaining figures. And when the diviſion is finiſhed, annex the figures cut off to the remainder.

Ex. 3.

Divide 745678 by 30400.

```
304|00) 7456|78 (24 quotient.
        608
        ----
        1376
        1216
        ----
         16078 remainder.
```

3 RULE.

To divide by 10, 100, 1000, &c. cut off from the dividend ſo many places as the diviſor has cyphers; and that will be the quotient; and the figures cut off the remainder.

Ex. 4.

Divide 78607 by 100.

The quotient is 786, and 07 remaining.

4 RULE.

When you have a large dividend, and your diviſor is often repeated; make a table of all the products

ducts of the diviſor and the nine digits; which is done by continually adding the diviſor. By this table diviſion may be wrought by inſpection, only by the help of addition and ſubtraction. For you have no more to do, but only to take out of the table the number always the next leſs than each dividual, and the quotient figure along with it; which numbers are to be continually ſubtracted from theſe dividuals, as in the general rule.

Ex. 5.

Divide 40377982057 by 35016.

TABLE.

1	35016
2	70032
3	105048
4	140064
5	175080
6	210096
7	245112
8	280128
9	315144
10	350160

35016) 40377982057 (1153129.
35016
53619
35016
186038
175080
109582
105048
45340
35016
103245
70032
332137
315144
16993 remains.

5 RULE.

When you are to divide by a ſingle figure, you need not ſet down the operation at large, but perform it in mind; the ſame may be done with 12.

Ex. 6.

```
7) 30721
     4388 quotient.
          5 rem.
```

Thus 30721 divided by 7, the quotient is 4388, and 5 remaining.

Diviſion is proved by multiplying the diviſor and quotient together, and adding the remainder, when there is any; which muſt be equal to the dividend, when the work is right.

Or it may be proved by caſting away nines, as in multiplication. Caſt away the nines in the diviſor and quotient, and ſet the remainders on the ſides of the croſs. Do the ſame with the dividend, and ſet the remainder at top. Multiply the figures on the ſides, throw away the nines, and ſet the remainder at bottom, which muſt be equal to the top. See Ex. 1. *Note*, if there be a remainder, it muſt be added to the product, on the ſides of the croſs, and the nines thrown out as before.

PROBLEM IX.

To divide a number of different denominations by a given number.

1 RULE.

If the diviſor be a ſingle figure, begin at the higheſt denomination, which divide by the given diviſor, and ſet the anſwer in the quotient, and to be of the ſame denomination; what remains muſt be be multiplied by the number of parts in the next inferior denomination, and added to the given number of that denomination, and then divide as before. Proceed thus through all the denominations.

Ex. 1.

£. s. d.
Divide 58 10 3 into 7 parts, what is 1 part?

£. s. d. £. s. d.
7) 58 10 3 (8 7 2.
56
2=40
7) 50 (7
49
1=12
7) 15 (2
14
1

Explanation.

Say how oft 7 in 58, 8 times; which ſet in the quotient, then 8 times 7 is 56, which ſubtracted from 58, leaves 2. But 2 pounds are 40 ſhillings, to which add 10, the ſum is 50. Then ſay how oft is 7 in 50, anſwer 7 times, which ſet in the quotient for ſhillings; then 7 times 7 is 49, which taken from 50 leaves 1 ſhilling, or 12 pence, to which add 3, the ſum is 15. Then ſay how oft 7 in 15, the anſwer is 2, which ſet in the quotient for pence, then 2 times 7 is 14, which taken from 15, 1 remains. So the anſwer is 8 *l.* 7 *s.* 2 *d.*; and 1 penny remaining.

Ex.

Ex. 2.

c. *ſt.* *lb.*
What is the 6th part of 72 6 11?
c. *ſt.* *lb.* *c.* *ſt.* *lb.*
6) 72 6 11 (12 0 15 the quotient.
72
7) 6 (0
0
6=84
6) 95
90
5 remains.

2 RULE.

If the diviſor be a great number made up of ſeveral others by multiplication. Divide ſucceſſively by the parts, inſtead of the whole.

Ex. 3.

£. *s.* *d.*
Divide 320 12 8 by 35.
£. *s.* *d.* 7) £. *s.* *d.*
5) 320 12 8 (64 2 $6\frac{1}{4}$ (9 3 $2\frac{1}{2}$ quotient.
320   63
0 = 0   1=20
5) 12   7) 22 (3
10   21
2=24   1=12
5) 32   7) 18 (2
30   14
2=8   4=16
5) 8 (1   7) 17 (2
5   14
3 rem.   3 rem.

PROBLEM X.

To extract the ſquare root.

1. A GENERAL RULE.

1. Begin at the units place, and point every other figure on the top, dividing it into ſeveral periods.

2. Find the greateſt ſquare that is contained in the firſt period, towards the left hand. Set the root in the quotient, and ſubtract the ſquare from the figures of that period.

3. To the remainder bring down the two figures under the next point, for a *reſolvend*. This is always to be repeated.

4. Double the quotient for a diviſor, and ſee how oft it is contained in the reſolvend (excepting the laſt figure); and ſet the anſwer in the quotient, and alſo after the diviſor. This muſt always be repeated; for a new diviſor muſt be found for every figure.

5. Then multiply this whole diviſor by that quotient figure, and ſubtract the product from the whole reſolvend; but if that product be greater, a leſs figure muſt be placed in the quotient. Proceed thus till all the figures or periods be brought down.

6. *Note*, inſtead of doubling the quotient every time for a diviſor, you may always add the laſt quotient figure to the laſt diviſor, for a new diviſor; and proceed as before.

Ex. 1.

Extract the ſquare root of 393129.

```
        . . .
       393129 (627 the root.
       36 · ·
       ------
 122)  331
 +2    244
       ----
1247)  8729
       8729
       ----
```

Expla-

Explanation.

The neareſt ſquare to 39 the firſt pointing, is 36, whoſe root 6 I place in the quotient; and ſubtract the ſquare 36 from 39, the remainder is 3.

Then I bring down 31, the next point, and annex it to 3, and the reſolvend is 331. Then I double the quotient for a diviſor, which is 12; and I ſeek how oft 12 in 33, the anſwer is 2, which I place in the quotient, and alſo after 12; then the diviſor becomes 122; and 122 multiplied by 2 produces 244, which I ſubtract from 331, the remainder is 87.

Laſtly, I bring down 29, the next point, and the reſolvend is 8729. Then I either double the quotient 62, which is 124; or I add the quotient figure 2 to 122, the laſt diviſor, which is 124; and this is a new diviſor. Then I aſk how oft 124 in 872, the anſwer is 7 times. Then I multiply 1247 by 7, and ſubtract the product 8729 from 8729, and there remains 0. So the root is exactly 627.

Ex. 2.

Extract the root of 733120000.

```
          • • • • •
          733120000 (27076 the root.
          4 • • •
          ---
   47)    333
   +7     329
  ----    ---
   5407)   41200
     +7    37849
   ----    -----
   54146)   335100
            324876
            ------
             10224 rem.
            ------
```

Ex. 3.

What is the root of 3272869681?

```
             .  .  .  .  .
            3272869681 (57209 root.
            25  .  :  :  :
            --
   107)  772
      7  749
   ----  ---
   1142)  2386
       2  2284
  -----  ----
  114409) 1029681
          1029681
          -------
            ....
          -------
```

2 RULE.

When more than half the figures of the root are found; all the reſt will be found as truly by plain diviſion; as is ſhewn more at large in the extraction of the roots of decimal fractions. But if common diviſion be uſed, you muſt bring down as many figures, as there were periods to come down, when you began with diviſion.

Ex. 4.

Let 14876008357020684 be given.

```
 . . . . . . . . . .
 14876008357020684 (121967243
 1 . . . .  . . . . .
 ---
22) 48
+2  44
---  ---
241) 476
 +1  241
---  ---
2429)23500
  +9 21861
----  -----
24386) 163983
    +6 146316
-----  ------
divifor 24392)  176675
                170744
                ------
                 59317
                 48784
                 -----
                105330
                 97568
                 -----
                  77622
                  73176
                  -----
                   4446
                   ----
```

The proof is, to multiply the root by itſelf, and add the remainder; which muſt be equal to the number given to be extracted, if the work be right.

PROBLEM XI.

To extract the cube root.

RULE.

1. Begin at the units place, and point every third figure; that is, the 1ſt, 4th, 7th, *&c.* miſſing two places.

2. Find the neareſt leſs root of the figures of the firſt punctation on the left hand, ſubtract its cube from the number given; to the remainder annex the next figure, for the reſolvend.

3. Take $\frac{1}{3}$ of the reſolvend for a dividend.

4. And for a diviſor, take the ſquare of the root, added to half the root, (or rather added to the product of the root, and the next quotient figure, leaving out the laſt figure of the product).

5. Divide the ſaid dividend by that diviſor, the quotient is the ſecond figure of the root.

6. Begin the operation anew, *viz.* cube the two figures of the root, and ſubtract the cube from the given number, annexing another figure, for the reſolvend.

7. Take the third part of the reſolvend for a dividend, and the ſquare of the root added to half the root (or rather added to the product of the root, and next quotient figure, ſtriking off the laſt figure of the product) for a diviſor.

8. This diviſion gives another figure of the root, but the diviſion is to be continued on to two figures, by the contraction in diviſion of decimals, or otherwiſe.

9. Repeating the operation with 4 figures in the root, you will get 4 more by a new diviſion, which gives 8 figures in the root; and from 8 to 16, *&c.* always double.

10. *Note*, when the cube exceeds the number given, a leſs figure muſt be writ in the quotient. And obſerve every diviſion gives one figure, and the reſt are found by continuing the diviſion, and dropping a figure of the diviſor every time.

Ex. 1.

Extract the cube root of 7892485271.

```
                  .  .  .  .
                 7892485271 (19 = 1 root
                 1                                   19
                ---                                  19
            3) 68  reſolvend                        ---
   diviſor  1) 22 (9                                171
           +1  18                                   19
           --  ---                                  ---
true diviſor 2  4                                   361
           --------                                  19
                 .  .                               ---
                78924 (                            3249
                6859                                361
                ----                               ----
           3) 10334 reſolvend                      6859
   diviſor 361) 3445 (91                           ----
          +17   3402
          ---   ----
true diviſor 378)  43        hence the root is 1991.
                ----
```

Then 1991 cubed is 7892485271, and therefore 1991 is exactly the root required.

Explanation.

1 being the greateſt cube contained in 7, the firſt point; ſubtract 1 there remains 6, to which annex 8, and the reſolvend is 68, the third part is 22 for a dividend. Then 1 the ſquare of the root being a diviſor, ſay how oft 1 in 22, the quotient would give more than 10, but ſince we can have no figure above 9, we will take 9 by gueſs for the quotient; then 9 times the root 1 is 9, which is very near 10, throw away the 0 and add 1 to the root 1, which makes 2

for the true diviſor; then to have the true quotient figure, ſay how oft 2 in 22, *anſ.* 9 times, for we can take no more; therefore 9 is rightly taken.

Then the root 19 being ſquared gives 361, and cubed is 6859. This cube ſubtracted from 78924 leaves 10334 the reſolvend, which divided by 3 gives 3445 for a dividend; and 361 is the diviſor, and the quotient is 9; then the root 19 multiplied by 9 gives 171, therefore add 17 to 361 gives 378 for the exact diviſor. Then by dividing you will get 91: and the root 1991.

Ex. 2.

To extract the cube root of 28373625.

$\dot{2}837\dot{3}62\dot{5}$ (30 = 1 root

27

3) 13 900 = ſquare

9) 4 (0 27000 = cube

$\dot{2}83\dot{7}36$ 30 root

27000 5 quotient

3) 13736 150

900) 4579 (5

15 4575 therefore 305 is the root, which cubed gives 28373625, exact.

diviſor 915 4

All the root might have been had at once by bringing down another figure, and that is becauſe the ſecond figure happens to be 0.

Thus $283\dot{7}$

27

3) 137

9) 4·5 (05

Ex.

Ex. 3.

To extract the cube root of 8302348000000.

· · · · ·
8302348000000 (202 = 1 root
8
3) 3
4) 1 (02
0
10

then 202 ſquared is 40804, and cubed is 8242408.

· ·
83023480
8242408
3) 599400
40804) 199800 (48
½ root 101 163620
40905 36180
32724
3456

therefore the root is 20248, or very near 20249.

Ex. 4.

Extr. the cube root of 118248245000000000000000.

· · · · · · · ·
118248 245000 000000 000000 (49
64
3) 542 reſolvend
16) 180 (9 4
3 171 9
diviſor 19 9 3|6

Then

Then 49 ſquared is 2401, and cubed is 117649.

```
                 ·  ·
                1182480
                117649
                -------
             3) 5990 reſolvend
                ----
diviſor 2401) 1996 (08; and the root is 4908.
              1920
              ----
                76
        ----------
```

Then the ſquare of 4908 is 24088464, and its cube 118226181312, therefore proceed

```
                     ·  ·  ·  ·
                  1182482450000
                  118226181312
                  -------------
                3) 220636880 reſolvend
        24088464)  73545626 (3052         4908
            1472   72269808                  3
        --------   --------             ------
diviſor 24089936    1275818             1472|4
                    1204496
                    -------
                      71322
                      48180
                      -----
                      23142, &c.
```

Therefore the root is 49083052, or very near 49083053.

The proof of your work is, to multiply the root by itſelf and the product by the root; which muſt equal, or nearly equal, the number given to be extracted.

CHAP. II.

VULGAR FRACTIONS.

DEFINITIONS.

1. A *FRACTION* is ſome part or parts of an integer or whole thing, repreſented by 1; as $\frac{3}{4}$ is a fraction denoting three fourth parts of an integer or 1. Every fraction conſiſts of two numbers, placed one above the other, with a line between them, as in this fraction $\frac{3}{4}$. The lower number 4 is called the *denominator*, and ſhows how many parts the integer is divided into; the upper number 3 is called the *numerator*, and expreſſes how many of theſe parts the fraction conſiſts of. And both numerator and denominator are called *terms* of the fraction.

2. A *proper fraction* is that where the numerator is leſs than the denominator, as $\frac{3}{4}$.

3. An *improper fraction* is that wherein the denominator is leſs than, or equal to, the numerator, as $\frac{4}{3}$ or $\frac{3}{3}$, *&c.*

4. A *ſingle fraction* is that which conſiſts of but one numerator and one denominator.

5. A *compound fraction*, or fraction of a fraction, is that whoſe parts are vulgar fractions, connected with the word *of*, as $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$.

6. A *mixt number* is a whole number with a fraction annexed, as $15\frac{2}{3}$.

7. *Denomination* is the name of any integer or thing. Thus pounds, ſhillings and pence are ſeveral denominations; where ſhillings are of a lower denomination than pounds, and higher than pence.

SCHOLIUM.

Any fraction, as $\frac{3}{4}$, may be conſidered either as $\frac{1}{4}$ of the number 3, or as $\frac{3}{4}$ of 1. For $\frac{1}{4}$ of 3 being thrice as much as $\frac{1}{4}$ of 1, and $\frac{3}{4}$ of 1 being alſo thrice as much as $\frac{1}{4}$ of 1; it follows, that $\frac{1}{4}$ of 3, and $\frac{3}{4}$ of 1 ſignify the ſame quantity.

Likewiſe in any fraction as $\frac{3}{4}$, the numerator 3 may be conſidered as a dividend, and the denominator 4 as a diviſor. For as $\frac{3}{4}$ ſignifies the fourth part of 3, it intimates a diviſion by 4; therefore 3 becomes a dividend and 4 a diviſor, by the nature of diviſion, and $\frac{3}{4}$ repreſents the quotient.

When an integer is divided into any number of parts (denoted by the denominator); the fewer or more parts taken, the leſs or greater is the fraction, that is, the leſs or greater the numerator, the leſs or greater is the fraction. And if the number of parts taken be the ſame as the integer is divided into, that is, if the numerator be equal to the denominator, then that fraction will be equal to the whole or integer. Thus 2 halfs, 3 thirds, *&c.* that is, $\frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$ *&c.* is equal the whole thing, or equal to 1 the integer. And therefore when the numerator is leſs or greater than the denominator, the fraction is leſs or greater than 1.

From what has been ſaid, if one fraction or mixt number as $18\frac{11}{14}$, be to be divided by another as $4\frac{3}{5}$, it may be written thus, $\frac{18\frac{11}{14}}{4\frac{3}{5}}$, and if any ſuch fractional quantity as this $\frac{18\frac{11}{14}}{14\frac{3}{5}}$ occur, it denotes a diviſion of the number $18\frac{11}{14}$ by $4\frac{3}{5}$.

PROBLEM I.

To reduce a fraction into another of equal value.

RULE.

Multiply (or divide) both terms of the fraction by one and the ſame number, and you will have a new fraction equivalent to the fraction given.

Example.

Let the fraction be $\frac{3}{5}$, multiply both terms

by

by 6 produces $\frac{18}{30}$ for the new fraction; that is, $\frac{3}{5}=\frac{3\times 6}{5\times 6}=\frac{18}{30}$. On the contrary, in the fraction $\frac{18}{30}$, divide both terms by 6, gives $\frac{3}{5}$, with is equivalent to $\frac{18}{30}$.

For in the fraction $\frac{3}{5}$, it is plain the 5th part of 3 is all one as the 10th part of 6, or the 15th part of 9, and ſo on; that is, the 5th part of 3, is the ſame as the $\overline{6\times 5\text{th}}$ part (30th part) of 6 × 3 or 18.

Or thus, in the improper fraction $\frac{4}{2}$, 4 contains 2 as oft as 3 times 4 (12), contains 3 times 2 (6); that is, $\frac{4}{2}=2$ for the quotient, and $\frac{12}{6}=2$ for the quotient, therefore $\frac{4}{2}=\frac{12}{6}$, &c.

In like manner it is evident that 3 pennies contain 1 penny, as oft as 3 groats contain 1 groat; or as oft as 3 ſhillings contain 1 ſhilling. That is, $\frac{3}{1}=\frac{3\times 4}{1\times 4}=\frac{3\times 12}{1\times 12}$, &c.

And the ſame holds equally true for diviſion, that is, $\frac{3\times 12}{1\times 12}=\frac{3}{1}$, &c.

PROBLEM II.

To reduce a whole number to the form of a fraction.

RULE.

Place 1 under it for a denominator.

Example.

Suppoſe 7 is the whole number, then it becomes $\frac{7}{1}$ for the fractional quantity required.

PRO-

PROBLEM III.

To reduce a whole number to a fraction of a given denominator.

RULE.

Multiply the whole number by the given denominator, and under the product write the ſame denominator.

Example.

Suppoſe 7 to have the denominator 11.

$$\begin{array}{r} 7 \\ 11 \\ \hline 77, \end{array} \qquad \text{then } \frac{7\times 11}{11} \text{ or } \frac{77}{11} \text{ is the fraction required.}$$

For $\frac{7\times 11}{11} = \frac{7}{1} = 7$.

PROBLEM IV.

To reduce a compound fraction into a ſingle one.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, of the ſingle fraction.

Ex. 1.

Let the fraction be $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{2}{7}$.

$$\begin{array}{rr} 2 & 7 \\ 3 & 5 \\ \hline 6 & 35 \\ 1 & 2 \\ \hline 6 & 70 \end{array} \qquad \text{then } \frac{1\times 3\times 2}{2\times 5\times 7} = \frac{6}{70} \text{ the ſingle fraction.}$$

For $\frac{1}{5}$ of $\frac{2}{7}$ is the ſame as $\frac{2}{7}$ divided by 5, or $\frac{2}{5\times 7}$, therefore $\frac{3}{5}$ thereof will be 3 times as much or $\frac{3\times 2}{5\times 7}$. Laſtly, the whole fraction being now $\frac{3\times 2}{5\times 7}$, the

the $\frac{1}{2}$ of it is $\frac{3\times2}{5\times7}$ divided by 2, or $\frac{1\times3\times2}{2\times5\times7}=\frac{6}{70}$.

Ex. 2.

What fraction of a pound is $3\frac{1}{2}d.$?

$3\frac{1}{2}d.=\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound,

that is, $3\frac{1}{2}d.=\frac{7\times1\times1}{2\times12\times20}=\frac{7}{480}$ of a pound.

And thus $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a pound is $\frac{24}{60}$ or $\frac{8}{20}$ of a pound or 20 ſhillings, that is, 8 ſhillings. For $\frac{4}{5}$ of a pound is 16 ſhillings, and $\frac{3}{4}$ of 16 ſhillings is 12 ſhillings, and $\frac{2}{3}$ of 12 ſhillings is 8 ſhillings.

PROBLEM V.

To reduce a mixt number into an improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; and the ſum is a new numerator, and the denominator the ſame as before.

Example.

The mixt number is $32\frac{5}{7}$.

$$\begin{array}{r} 32 \\ \underline{7} \\ 224 \\ \underline{+5} \\ 229 \end{array}$$

then $\frac{32\times7+5}{7}=\frac{229}{7}$ is the fraction required.

For 32 wholes or $\frac{32}{1}=\frac{23\times7}{7}=\frac{224}{7}$ or 224 ſevenths, to which if the other 5 ſevenths be added, the whole is 229 ſevenths or $\frac{229}{7}$.

PRO-

PROBLEM VI.

To reduce an improper fraction into a whole or mixt number.

RULE.

Divide the numerator by the denominator, and the quotient is the whole number. Then what remainder there is, place it over the denominator, and annex this fraction to the quotient before found.

Example.

Let $\frac{631}{16}$ be propoſed; 631 divided by 16 gives 39 for the quotient, and 7 remaining, therefore $39\tfrac{7}{16}=\frac{631}{16}$ as required.

16) 631 ($39\tfrac{7}{16}$
48
151
144
7

For the fraction $\frac{631}{16}$ ſignifying 631 ſixteenths, therefore every 16 makes 1, and therefore the quotient 39 ſhows how many ones are contained in the number, and the 7 ſixteenths which remains, muſt therefore be placed as a fraction.

PROBLEM VII.

To find the greateſt common diviſor for the numerator and denominator of a fraction, or for any two numbers.

1. RULE.

1 RULE.

Divide the greater by the leſſer, and the laſt diviſor by the remainder, and ſo on continually till nothing remain; then the laſt diviſor is that required.

Or in dividing take the neareſt quotient, and the difference between the dividend and that multiple, for the next diviſor, *&c.*

Ex. 1.

Let $\frac{252}{364}$ be propoſed; dividing according to rule, the laſt diviſor is 28, which is the greateſt number that will divide both numerator and denominator, without a remainder.

Note, if the laſt diviſor be 1, the 2 numbers are prime to one another.

```
252) 364 (1
     252
     ———
     112) 252 (2
          224
          ———
           28) 112 (4
               112
               ———
                ..
               ———
```

For ſince 28 meaſures 112, it likewiſe meaſures twice 112, or 224; and therefore 28 meaſures 224 + 28, or 252.

Again, ſince 28 meaſures 112 and 252, therefore it meaſures 252 + 112, or 364; and ſo on. Therefore 28 meaſures both 252 and 364.

Now 28 is the greateſt common meaſure; for if there be a greater G, then ſince G meaſures 252 and 364, it alſo meaſures the remainder 112, and ſince G meaſures 112 and 252, it alſo meaſures the remainder 28, that is, the greater meaſures the leſs, which is abſurd.

2 RULE.

If the numbers given be mixt numbers, or fractions; reduce them to a common denominator; and take the two new numerators, and proceed as in the firſt rule to find their greateſt common meaſure; make it a numerator, under which put the common denominator; and that fraction will be the greateſt common meaſure ſought.

Ex. 2.

Let $9\frac{3}{4}$ and 13 be propoſed.

Theſe reduced to a common denominator are $\frac{39}{4}$ and $\frac{52}{4}$, then 39) 52 (1
39
13) 39 (3 ſo $\frac{13}{4}$ is the greateſt
39 common meaſure of
0 $9\frac{3}{4}$ and 13.

PROBLEM VIII.

To reduce a fraction to its leaſt terms.

1. A GENERAL RULE.

Find the greateſt common meaſure, by which divide both terms of the fraction; the quotients will be the terms of the fraction required.

Ex. 1.

Let the fraction be $\frac{252}{364}$, whoſe greateſt common meaſure is 28, diviſion being performed, we have $\frac{9}{13}$, that is, $\frac{252}{364} = \frac{9}{13}$.

$$\begin{array}{l} 28)\ 252\ (9 \\ 252 \\ \hline 0 \end{array} \qquad \begin{array}{l} 28)\ 364\ (13 \\ 28 \\ \hline \ 84 \\ \ 84 \\ \hline 0 \end{array} \qquad \frac{9}{13}\ \text{the fraction.}$$

PARTICULAR RULES.

2 RULE.

When the terms of the fraction are even numbers, divide them by 2 continually.

Ex. 2.

$\frac{48}{272}$, being continually halfed is $\frac{48}{272}\Big|\frac{24}{136}\Big|\frac{12}{68}\Big|\frac{6}{34}\Big|\frac{3}{17}$, therefore $\frac{48}{232}=\frac{3}{17}$.

3 When both terms end with 5; or one with 5, and the other with a cypher; divide both by 5.

Ex. 3.

As $\frac{225}{475}$; $5)\ \frac{225}{475}\ (\frac{45}{95}\ (\frac{9}{19}$.

4. When both terms end with cyphers, cut off equal cyphers in both.

Ex. 4.

As $\frac{10000}{25700}$, which becomes $\frac{100}{257}$.

5. If you can eſpy any number which will divide both terms, divide by that number.

Ex. 5.

As $\frac{21}{39}$, divide by $3\ )\ \frac{21}{39}\ (\frac{7}{13}$.

6. For expedition, try all numbers 2, 3, 4, 5, *&c.* till you find ſome that will divide both, if any there be.

Ex. 6.

As $\frac{119}{168}$; trying 2, 3, 4, 5, 6, none of them will do, but trying 7 it ſucceeds, $7\big)\,\frac{119}{168}\,\big(\frac{17}{24}$.

PROBLEM IX.

To reduce fractions of different denominators, to thoſe of equal value, having a common denominator.

1. A GENERAL RULE.

Multiply each numerator by all the denominators except its own, for a new numerator; then multiply all the denominators together for a new denominator.

Ex. 1.

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, become $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

$$\begin{array}{cccc} 2 & 3 & 4 & 3 \\ 4 & 3 & 4 & 4 \\ \hline 8 & 9 & 16 & 12 \\ 5 & 5 & 3 & 5 \\ \hline 40 & 45 & 48 & 60 \end{array}$$

For in each fraction, both terms are multiplied by the ſame number; and therefore its value is not altered.

PARTICULAR RULES.

2 RULE.

Divide the denominators by their greateſt common diviſor; and multiply both terms of each fraction, by all the other quotients, which will produce as many new fractions. This is the beſt rule for 2 fractions, as

Ex. 2.

$\frac{5}{12}, \frac{7}{18}$. Divide by 6) $\underset{2}{\frac{5}{12}}, \underset{3}{\frac{7}{18}}$, the quotients are 2, 3. Then $\frac{5\times3}{12\times3}=\frac{15}{36}$, and $\frac{7\times2}{18\times2}=\frac{14}{36}$.

3 RULE.

In ſeveral fractions, divide all the denominators by their greateſt common diviſor, ſetting the quotients underneath; then find the leaſt number which all theſe quotients can meaſure; and divide this number ſeverally by all theſe quotients, and ſet theſe new quotients underneath. Then multiply the terms of each fraction by its new quotient, gives the correſpondent fraction required, and all theſe will be in their leaſt terms.

Ex. 3.

3) $\frac{13}{36}\ \frac{1}{24}\ \frac{11}{18}\ \frac{7}{12}\ \frac{4}{9}$, the greateſt com. diviſor is 3.
$12\ \ 8\ \ 6\ \ 4\ \ 3$, the leaſt number they meaſure is 24.
$2\ \ 3\ \ 4\ \ 6\ \ 8$
$\frac{26}{72}\ \frac{3}{72}\ \frac{44}{72}\ \frac{42}{72}\ \frac{32}{72}$ the fractions required.

It is evident each of theſe is of the ſame value as that given, having both its terms multiplied alike. And they will be in the leaſt terms, becauſe 24 is the leaſt number that the firſt quotients meaſure.

SCHOLIUM.

By this problem the greateſt of two or more fractions may be diſcovered.

PROBLEM X.

Several fractions being given; to find as many whole numbers, in the ſame proportion.

RULE.

Reduce the fractions to a common denominator, then the ſeveral numerators will be to one another as the fractions given.

Example.

Suppoſe $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. Theſe are reduced to $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, therefore the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, are as the numbers 6, 4, and 3.

PROBLEM XI.

To find the value of a vulgar fraction in known parts of the integer.

RULE.

Multiply the numerator by the number of parts contained in the integer, and divide the product by the denominator, the quotient ſhews the known parts. If there be any remainder, multiply it by the next inferior denomination, and divide by the denominator as before: and continue this work till you come at the loweſt denomination.

Example.

What is $\frac{3}{17}$ of a pound ſterl.? Anſ. 3*s.* 6*d.* $1\frac{7}{17}$*f.*

3
20
17) 60 (3 ſhillings
51
9
12
18
9
17) 108 (6 pence
102
6
4
17) 24 ($1\frac{7}{17}$ farthings.
17
7

PROBLEM XII.

To reduce a fraction of one denomination to the fraction of another denomination.

RULE.

1. From a leſs to a greater denomination; multiply the *denominator* by all the denominations, from that given, to that ſought.

2. From a greater to a leſs denomination; multiply the *numerator* by all the denominations, from that given, to that ſought.

Ex. 1.

Given $\frac{3}{5}$ of a penny; what fraction of a pound is it?

Anſw. $\frac{3}{5 \times 12 \times 20} = \frac{3}{1200}$ of a pound.

Ex. 2.

$\frac{3}{5}$ of a pound, what is that of a penny?

Anſ. $\frac{3 \times 20 \times 12}{5} = \frac{720}{5}$ of a penny.

For $\frac{3}{5}$ of a penny is $\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{3}{5 \times 12 \times 20}$.

And $\frac{3}{5}$ of a pound reduced to pence is $\frac{3}{5} \times 20 \times 12$.

PROBLEM XIII.

To add fractions together.

1. A GENERAL RULE.

Reduce compound fractions to ſingle ones; mixt numbers to improper fractions; and fractions of different denominators to a common denominator. Then add the numerators, and ſubſcribe the common denominator.

Ex. 1.

What is the ſum of $\frac{2}{9}$ and $\frac{3}{9}$?

$$\begin{array}{ll} \text{to} & 2 \\ \text{add} & \underline{3} \\ & 5 \end{array} \quad \text{anſ. } \frac{5}{9}.$$

Ex. 2.

What is the ſum of $\frac{3}{4}$ and $\frac{3}{5}$?

When reduced to a common denominator they are $\frac{15}{20}$ and $\frac{12}{20}$,

$$\begin{array}{ll} \text{to} & 15 \\ \text{add} & \underline{12} \\ & 27 \end{array} \quad \text{the ſum } \frac{27}{20} \text{ or } 1\tfrac{7}{20}.$$

Ex. 3.

What is the ſum of $\frac{1}{3}$ of $\frac{1}{4}$, and $\frac{3}{8}$, and $1\frac{1}{4}$?

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, alſo $1\frac{1}{4} = \frac{5}{4}$. Then $\frac{1}{12}$, $\frac{3}{8}$ and $\frac{5}{4}$, reduced to a common denominator, are $\frac{2}{24}$, $\frac{9}{24}$ and $\frac{30}{24}$.

$$\begin{array}{r} 2 \\ 9 \\ \underline{30} \\ 41 \end{array}$$

the ſum $\frac{41}{24}$ or $1\frac{17}{24}$.

PARTICULAR RULES.

2 RULE.

When many fractions are given, firſt add two of them, and to the ſum add a third, and to that ſum a fourth, and ſo on.

Ex. 4.

Add together $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

$\frac{2}{3}$ and $\frac{3}{4}$ are reduced to $\frac{8}{12}$ and $\frac{9}{12}$, whoſe ſum is $\frac{17}{12}$. Then

$\frac{17}{12}$ and $\frac{4}{5}$ are reduced to $\frac{85}{60}$ and $\frac{48}{60}$, whoſe ſum is $\frac{133}{60}$.

Then $\frac{133}{60}$ and $\frac{5}{6}$ are reduced to $\frac{133}{60}$ and $\frac{50}{60}$, whoſe ſum is $\frac{183}{60}$ or $3\frac{3}{60}$, the ſum of all the four fractions.

3 RULE.

When mixt numbers are to be added, firſt add the fractions to the fractions; and then the whole numbers by themſelves.

Ex. 5.

Let $3\frac{1}{2}$, $4\frac{1}{3}$, and $10\frac{3}{8}$ be added.

$\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{8}$ are reduced to $\frac{12}{24}$, $\frac{8}{24}$ and $\frac{9}{24}$,

$$\begin{array}{r} 12 \\ 8 \\ \underline{9} \\ 29 \end{array} \qquad \frac{29}{24}$$

or $1\frac{5}{24}$ is the ſum of the fractions,

to which add the whole numbers $\left\{\begin{array}{l} 1\frac{5}{24} \\ 3 \\ 4 \\ 10 \end{array}\right.$

the ſum $18\frac{5}{24}$

4 RULE.

In fractions of different denominations, reduce them to thoſe of a common denomination, and then to a common denominator. Then add the numerators, and ſubſcribe the common denominator.

Ex. 6.

Add together

$\frac{3}{5}$ of a pound, $\frac{5}{10}$ of a ſhilling, and $\frac{7}{8}$ of a penny.

$\frac{5}{10}$ of a ſhilling is $\frac{5}{200}$ of a pound, and $\frac{7}{8}$ of a penny is $\frac{7}{1920}$ of a pound.

Then,

$\frac{3}{5}$, $\frac{5}{200}$ and $\frac{7}{1920}$ are reduced to $\frac{5760}{9600}$, $\frac{240}{9600}$, $\frac{35}{9600}$.

$$\begin{array}{r} 5760 \\ 240 \\ 35 \\ \hline 6035 \end{array}$$

The ſum of the fractions is $\frac{6035}{9600}$ of a pound, or $\frac{1207}{1920}$ in leſs terms.

Or the fractions may be reduced to ſhillings, or pence.

PROBLEM XIV.

To ſubtract one fraction from another.

1. A GENERAL RULE.

Reduce compound fractions to ſingle ones; mixt numbers to improper fractions; and fractions of different denominations to thoſe of the ſame denomination; and laſtly, fractions of different denominators to a common denominator.

Then ſubtract the numerators, and ſubſcribe the common denominator.

Ex. 1.

From $\frac{4}{5}$ take $\frac{2}{5}$.

from 4
take 2

$\frac{}{2}$, the remainder is $\frac{2}{5}$.

Ex. 2.

From $\frac{6}{13}$ take $\frac{3}{8}$.

Reduced to $\frac{48}{104}$, $\frac{39}{104}$.

$$\begin{array}{ll} \text{from} & 48 \\ \text{take} & \underline{39} \\ & 9 \end{array}, \text{ the rem.} = \frac{9}{104}.$$

Ex. 3.

Take $\frac{2}{3}$ of $\frac{4}{5}$ from $\frac{2}{3}$.

$\frac{2}{3}$ of $\frac{4}{5}$ is reduced to $\frac{8}{15}$.

Then $\frac{8}{15}$ and $\frac{2}{3}$ are reduced to $\frac{8}{15}$ and $\frac{10}{15}$.

$$\begin{array}{c} 10 \\ \underline{8} \\ 2 \end{array}. \text{ The remainder is } \frac{2}{15}.$$

Ex. 4.

From $25\frac{3}{8}$, take $21\frac{1}{4}$.

Reduced to $\frac{203}{8}$ and $\frac{85}{4}$.

$$\begin{array}{r} 203 \\ \underline{85} \\ 118 \end{array}, \text{ the rem.} = \frac{118}{4} = 29\tfrac{2}{4}, \text{ or } 29\tfrac{1}{2}.$$

Ex. 5.

From $\frac{1}{3}$ of a pound take $\frac{7}{9}$ of a ſhilling.

$\frac{1}{3}$ of a pound $= \frac{20}{3}$ of a ſhilling.

$$\begin{array}{r} 20 \\ \underline{7} \\ 13 \end{array}, \text{ the rem.} = \frac{13}{3} \text{ of a ſhilling} = 4\tfrac{1}{3} \text{ ſhilling.}$$

Or $\frac{7}{9}$ of a ſhilling may be reduced to pounds, &c.

PARTI-

PARTICULAR RULES.

2 RULE.

In mixt numbers, take the fraction from the fraction, and the whole number from the whole number, remembring to reduce the fractions to a common denominator: and if the fraction to be ſubtracted is leſs, borrow 1.

Ex. 6.

Take $21\frac{1}{4}$ from $25\frac{3}{8}$.

$\frac{1}{4}$ is reduced to $\frac{2}{8}$. Then

from $25\frac{3}{8}$
take $21\frac{2}{8}$

remains $4\frac{1}{8}$

Ex. 7.

From $108\frac{3}{4}$ take $92\frac{5}{6}$.

$\frac{3}{4}$ and $\frac{5}{6}$ reduced to a com. denom. are $\frac{9}{12}$ and $\frac{10}{12}$.

from $108\frac{9}{12}$ or $107\frac{21}{12}$
take $92\frac{10}{12}$ $92\frac{10}{12}$

remains $15\frac{11}{12}$ $15\frac{11}{12}$

here as 10 is greater than 9; add 1, that is, $\frac{12}{12}$ to 9 makes $\frac{21}{12}$, then 10 from 21, remains 11 twelfths, then carry 1 to 2 makes 3; and 3 from 8, remains 5, 9 from 10 remains 1.

Ex. 8.

From $272\frac{7}{12}$ take 14.

$272\frac{7}{12}$
14

remains $258\frac{7}{12}$

Ex.

Ex. 9.

Take $59\frac{5}{9}$ from 120.

$$\begin{array}{lrcr} & 120 & \text{or} & 119\frac{9}{9} \\ & 59\frac{5}{9} & & 59\frac{5}{9} \\ \hline \text{remains} & 60\frac{4}{9} & & 60\frac{4}{9} \\ \hline \end{array}$$

3 RULE.

A fraction from 1 or an integer; ſubtract the numerator from the denominator, the remainder is the numerator to be placed over the given denominator.

Ex. 10.

Take $\frac{17}{23}$ from 1.

$$\begin{array}{r} 23 \\ 17 \\ \hline 6 \end{array}$$

Then the remainder is $\frac{6}{23}$.

4 RULE.

A proper fraction from any whole number; ſubtract the numerator from the denominator, for the numerator of the fraction, which is to be annext to the whole number leſſened by 1.

Ex. 11.

Take $\frac{17}{23}$ from 57, the remainder is $56\frac{6}{23}$.

$$\begin{array}{lr} \text{from} & 57 \\ \text{take} & 0\frac{17}{23} \\ \hline \text{rem.} & 56\frac{6}{23} \\ \hline \end{array}$$

The reaſon of the rules in addition and ſubtraction, is evident; for when fractions are reduced to the ſame denominator, they have the ſame name; therefore as 2 ſhillings and 3 ſhillings make 5 ſhillings, ſo ſo 2 twentieths and 3 twentieths, make 5 twentieths. And 2 twentieths from 3 twentieths leaves 1 twentieth. That is, $\frac{2}{20} + \frac{3}{20} = \frac{5}{20}$, and $\frac{3}{20} - \frac{2}{20} = \frac{1}{20}$. And for the ſame reaſon $\frac{2}{9}$ and $\frac{3}{9}$ make $\frac{5}{9}$. And $\frac{2}{5}$ from $\frac{4}{5}$, remains $\frac{2}{5}$, *&c.*

PROBLEM XV.

To multiply fractions together.

1. A GENERAL RULE.

Reduce mixt numbers to fractions; then multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Ex. 1.

Multiply $\frac{2}{3}$ by $\frac{5}{7}$. The product is $\frac{2\times5}{3\times7} = \frac{10}{21}$.

Ex. 2.

Multiply $7\frac{1}{2}$ by $\frac{3}{4}$.

$7\frac{1}{2}$ is reduced to $\frac{15}{2}$; then the product is $\frac{15\times3}{2\times4} = \frac{45}{8}$, or $5\frac{5}{8}$.

Ex. 3.

Multiply $3\frac{4}{7}$ by 13.

Theſe are reduced to $\frac{25}{7}$ and $\frac{13}{1}$.

$$\begin{array}{r@{\qquad}r} 25 & 7 \\ \underline{13} & \underline{1} \\ 75 & 7 \\ \underline{25} & \\ \underline{325} & \end{array}$$

the product is $\frac{325}{7}$, or $46\frac{3}{7}$.

PARTICULAR RULES.

2 RULE.

When the numerator of one and denominator of the other, can be divided by any number; take the quotients inſtead thereof.

Ex. 4.

Multiply $\frac{3}{8}$ by $\frac{4}{7}$.

Divide by 4. $4)\frac{3}{\underset{2}{8}}\times\frac{\overset{1}{4}}{7}$, then $\frac{3}{2}\times\frac{1}{7}=\frac{3}{14}$ the product.

Ex. 5.

Multiply $\frac{3}{8}$ by $\frac{4}{9}$.

$\begin{matrix}3)\\4)\end{matrix}\frac{3}{8}\times\frac{4}{9}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ the product.

3 RULE.

A mixt number or fraction, to multiply by a whole number; multiply the whole number by the whole number; and then multiply the numerator by the ſaid whole number, and divide by the denominator, and add this quotient to the former product.

Ex. 6.

Multiply $\frac{3}{4}$ by 9. Then $\frac{3\times9}{4}=\frac{27}{4}$ the product.

$$\begin{array}{rl} & 3 \\ & \underline{9} \\ 4) & 27\ (6\tfrac{3}{4}\text{ the product.} \\ & \underline{24} \\ & \underline{3} \end{array}$$

Ex. 7.

Multiply $3\frac{4}{7}$ by 13.

$$\begin{array}{r} 3 \\ 13 \\ \hline 39 \end{array} \qquad \begin{array}{r} 13 \\ 4 \\ \hline 7)\ 52\ (7\tfrac{3}{7} \\ 49 \\ \hline 3 \end{array} \qquad \begin{array}{r} 39 \\ 7\tfrac{3}{7} \\ \hline 46\tfrac{3}{7} \end{array}$$ the product.

4 RULE.

When a fraction is to be multiplied by a number which happens to be the ſame with the denominator; take the numerator for the product.

Ex. 8.

Multiply $\frac{3}{5}$ by 5, the product is 3.

5 RULE.

When ſeveral fractions are to be multiplied; ſtrike out ſuch multipliers as are found both in the numerators and denominators.

Ex. 9.

Multiply theſe $\frac{2}{7}$, $\frac{14}{15}$, $\frac{5}{8}$.

That is, $\frac{2\times14\times5}{7\times15\times8}$.

This becomes $\frac{1\times2\times1}{1\times3\times4}$, or $\frac{1\times1\times1}{1\times3\times2} = \frac{1}{6}$.

For 2 and 8 become 1 and 4, 14 and 7 become 2 and 1, and 5 and 15 become 1 and 3; by dividing reſpectively by 2, 7, and 5.

A fraction is multiplied by any number, by multiplying the numerator by that number, or dividing the denominator by it, when it can be done;

as to multiply $\frac{3}{4}$ by 9, the product is $\frac{27}{4}$. For ſince 3 of any denomination multiplied by 9 produces 27 of that denomination, therefore 3 fourths multiplied by 9 produces 27 fourths, or $\frac{27}{4}$. And ſince $\frac{3}{4} = \frac{3\times9}{4\times9} = \frac{27}{36}$, therefore if $\frac{3}{4}$ or $\frac{27}{36}$ be multiplied by 9, the product is $\frac{27\times9}{36}$, or $\frac{27\times9}{4\times9} = \frac{27}{4}$, the ſame as dividing 36 (the denominator of $\frac{27}{36}$) by 9.

The reaſon of the general rule is this; $\frac{2}{3}$ multiplied by $\frac{5}{7}$, makes $\frac{2\times5}{3\times7}$ or $\frac{10}{21}$. For to take $\frac{2}{3}$ once we ſhall have juſt $\frac{2}{3}$, but to take $\frac{2}{3}$ only $\frac{1}{7}$ of a time, we ſhall only have $\frac{2}{3\times7}$, or $\frac{2}{21}$, becauſe dividing any fraction by any number as 7, is but multiplying the denominator by that number 7. Again, taking $\frac{5}{7}$ of $\frac{2}{3}$ is taking 5 times as much as $\frac{1}{7}$, that is, 5 times $\frac{2}{21}$, and this will be $\frac{2\times5}{21}$, becauſe multiplying any fraction by any number 5, is the ſame as multiplying the numerator by that number 5; and therefore the product is $\frac{10}{21}$.

And in the particular contracted rules, ſince both numerator and denominator are divided by the ſame numbers, the fraction will be of the ſame value.

Multiplication of fractions is only reducing a compound fraction to a ſingle one, for to multiply $\frac{2}{3}$ by $\frac{5}{7}$, is no more than to take $\frac{5}{7}$ of $\frac{2}{3}$.

In multiplication of proper fractions, the product is leſs than either the multiplier or multiplicand. As if $\frac{2}{3}$ be multiplied by $\frac{5}{7}$; if $\frac{2}{3}$ be multiplied by 1, the product will be juſt $\frac{2}{3}$; but if $\frac{2}{3}$ be taken not ſo much as once, as only $\frac{5}{7}$ of a time, the product will be leſs than $\frac{2}{3}$. And for the ſame reaſon it will be leſs than $\frac{5}{7}$, if $\frac{2}{3}$ be the multiplier.

PROBLEM XVI.

To divide one fraction by another.

1 A GENERAL RULE.

Reduce compound fractions to ſingle ones, mixt numbers to improper fractions, and fractions of different denominations to thoſe of the ſame denomination. Then multiply the denominator of the diviſor by the numerator of the dividend, for a new numerator; alſo multiply the numerator of the diviſor by the denominator of the dividend, for a new denominator; the new fraction is the quotient.

Ex. 1.

Divide $\frac{5}{8}$ by $\frac{3}{7}$.

$$\left.\frac{3}{7}\right)\ \frac{5}{8}\ \left(\frac{7\times5}{3\times8}=\frac{25}{24}=1\tfrac{11}{24}.\right.$$

Ex. 2.

Divide $\frac{3}{5}$ of a pound by $\frac{8}{9}$ of a ſhilling.

$\frac{8}{9}$ of a ſhilling is reduced to $\frac{8}{180}$ of a pound $=\frac{2}{45}$ of a pound. $\left.\frac{2}{45}\right)\ \frac{8}{9}\ \left(\frac{360}{18}=20.\right.$

Ex.

Ex. 3.

Divide $11\frac{2}{3}$ by $2\frac{3}{4}$.

Theſe are reduced to $\frac{35}{3}$ and $\frac{11}{4}$.

$$\frac{11}{4}\Big)\ \frac{35}{3}\ \Big(\frac{140}{33} = 4\tfrac{8}{33}.$$

Ex. 4.

Divide 7 by $\frac{3}{5}$.

$$\frac{3}{5}\Big)\ \frac{7}{1}\ \Big(\frac{35}{3} = 11\tfrac{2}{3}.$$

PARTICULAR RULES.

2 RULE.

When it can be done, divide the numerator of the dividend by the numerator of the diviſor, and the denominator by the denominator, for the quotient.

Ex. 5.

Divide $\frac{8}{15}$ by $\frac{2}{3}$.

$$\frac{2}{3}\Big)\ \frac{8}{15}\ \Big(\frac{4}{5} \text{ the quotient.}$$

3 RULE.

When the two numerators, or the two denominators, can be divided by any number; take the quotients inſtead thereof.

Ex. 6.

Divide $\frac{12}{27}$ by $\frac{8}{5}$.

$$\overset{2}{\frac{8}{5}}\Big)\ \overset{3}{\frac{12}{27}}\ \Big(\frac{15}{54}.$$

Ex. 7.

Divide $\frac{8}{9}$ by $\frac{2}{45}$.

$$\frac{\overset{1}{2}}{\underset{5}{45}}\Big)\;\frac{\overset{4}{8}}{\underset{1}{9}}\;\Big(\frac{20}{1} = 20.$$

4 RULE.

A fraction by a whole number; multiply the denominator by the whole number.

Ex. 8.

Divide $\frac{13}{15}$ by 7, the quotient $\frac{13}{15\times 7} = \frac{13}{105}$.

5 RULE.

If the denominators are equal, place the numerator of the dividend over the numerator of the diviſor, for the quotient.

Ex. 9.

Divide $\frac{8}{19}$ by $\frac{3}{19}$, the quotieni is $\frac{8}{3}$, or $2\frac{2}{3}$.

To demonſtrate that $\frac{5}{8}$ divided by $\frac{3}{7}$, gives $\frac{35}{24}$ in the quotient, let them be reduced to a common denominator, then $\frac{3}{7} = \frac{24}{56}$, and $\frac{5}{8} = \frac{35}{56}$; then it is plain $\frac{5}{8}$ divided by $\frac{3}{7}$ is the ſame as $\frac{35}{56}$ divided by $\frac{24}{56}$. But 35 fifty ſixths contain 24 fifty ſixths, as oft as 35 contains 24, therefore the quotient is $\frac{35}{24}$ or $\frac{7\times 5}{3\times 8}$, as by the rule.

Alſo a fraction is divided by a whole number by multiplying the denominator by that number. As if $\frac{13}{15}$ be divided by 7, the quotient is $\frac{13}{15\times 7} = \frac{13}{105}$.

For

For $\frac{13}{15}=\frac{13\times7}{15\times7}=\frac{91}{105}$: now if we take the 7th part of $\frac{13}{15}$, or its equal $\frac{91}{105}$, this is the ſame as dividing 91 hundred and fifths by 7, and the quotient is 13 hundred and fifths, or $\frac{13}{105}=\frac{13}{15\times7}$. And hence a fraction is divided by a whole number, by dividing the numerator by that number, when it can be done; for $\frac{91}{105}$ divided by 7, gives $\frac{13}{105}$ for the quotient.

In diviſion of fractions, if the diviſor be a proper fraction, the quotient will always be greater than the dividend. For it is evident, when any quantity or dividend is to be divided by 1, the quotient will be equal to the dividend: therefore if it is divided by a proper fraction, which is leſs than 1, the quotient will then be greater than the dividend: for a leſs diviſor will be oftener contained in the dividend, than a greater diviſor.

PROBLEM XVII.

To extract the ſquare root of a fraction, &c.

RULE.

1. Reduce them to the leaſt terms; then extract the root of the numerator for a new numerator; and the root of the denominator for a new denominator.

2. When they have not exact roots, add an equal number of cyphers to both terms, and then extract: or

3. When neither numerator nor denominator has an exact root, multiply the numerator by the denominator, and extract the root of the product, for a numerator, and under it place the ſaid denominator.

4. To find the fractional part of the root of a whole number nearly, take the remainder for a numerator, and twice the root (+ 1 if you will) for a denominator, of the fractional part.

Or more exactly, make twice the remainder a numerator; and add 1 to 4 times the root, for a denominator.

Ex. 1.

Extract the ſquare root of $\frac{50}{18}$.

Here $\frac{50}{18}=\frac{25}{9}$, and the root of 25 is 5, and the root of 9 is 3; therefore the root of $\frac{25}{9}$ is $\frac{5}{3}$, or $1\frac{2}{3}$.

Ex. 2.

Extract the root of $5\frac{3}{16}$.

$5\frac{3}{16}=\frac{83}{16}$, then the root is $\frac{\sqrt{83}}{4}=\frac{9}{4}$ nearly.

Or thus.

$\frac{83}{16}=\frac{83000}{16000}$, and the root of $\frac{83000}{16000}$ is $\frac{\sqrt{1328000000}}{16000}$ $=\frac{36441}{16000}=\frac{9110}{4000}=\frac{911}{400}$ near.

Ex. 3.

To extract the root of $\frac{2}{3}$.

Here $\frac{2}{3}=\frac{20000}{30000}$. But the root of 20000 is 141; and the root of 30000 is 173;

Therefore the root of $\frac{2}{3}$ is $\frac{141}{173}$.

Or thus.

$\frac{2}{3}=\frac{200}{300}$, and $200 \times 300 = 60000$, whoſe root is 245, then the root is $\frac{245}{300}=\frac{49}{60}$.

Ex. 4.

Extract the root of $27\frac{3}{5}$.

$27\frac{3}{5}=\frac{138}{5}$, and $138 \times 5 = 690$, and the root of 690 is 26, then the root is $\frac{26}{5} = 5\frac{1}{5}$, nearly, but too ſmall.

Ex.

Ex. 5.

Extract the root of 22, or $\frac{22}{1}$.

$2\dot{2}$ ($4\frac{6}{8}$, or $4\frac{6}{9}$ the root.
16

rem. 6

Or thus.

$2\dot{2}$ ($4\frac{12}{17}$ the root.
16

$$\begin{array}{cc} 6 & 4 \\ 2 & 4 \\ \hline 12 & 16+1=17. \end{array}$$

Ex. 6.

To extract the root of 253.

$\dot{2}5\dot{3}$ ($15\frac{28}{30}$, or $15\frac{28}{31}$ the root.
1

25) 153
125 or more exactly $15\frac{56}{61}$ is the root.

28

Ex. 7.

Extract the root of $\frac{7}{8}$.

Here $8 \times 7 = 56$. And the root of 56 is $7\frac{7}{14}$, or $7\frac{7}{15}$.

$5\dot{6}$ ($7\frac{7}{14} = 7\frac{1}{2}$. And the root is $\frac{7\frac{1}{2}}{8} = \frac{15}{16}$.
49

7 or more exactly $\frac{7\frac{14}{29}}{8}$.

PROBLEM XVIII.

To extract the cube root of a fraction.

RULE.

1. Reduce the fraction to the leaſt terms; then extract the roots of the numerator and denominator, if they have any, for the numerator and denominator of the fraction.

2. If they have not exact roots, add an equal number of cyphers to both terms, and then extract: or
3. If neither of them have exact roots, multiply the numerator by the ſquare of the denominator, and extract the root of the product for a numerator, and under it place the ſaid denominator. And here you may add cyphers to both, before you begin, as before.
4. To find the fractional part of the cube root of a whole number; make the remainder a numerator, and thrice the ſquare of the root a denominator.

Or more exactly, make twice the remainder a numerator, and add 3 times the root to 6 times its ſquare, for a denominator.

But the moſt general method is to reduce the fraction to a decimal, and then extract the root, as hereafter.

Ex. 1.

Extract the cube root of $\frac{1}{27}$.

The root of 1 is 1, and the root of 27 is 3, then $\frac{1}{3}$ is the root.

Ex. 2.

To extract the root of $\frac{24}{375}$.

$\frac{24}{375}$ is reduced to $\frac{8}{125}$, whoſe root is $\frac{2}{5}$.

Ex. 3.

Extract the root of $\frac{2}{3}$.

$\frac{2}{3} = \frac{20000}{30000}$, the root of 20000 is 27, and the root of 30000 is 31, therefore the root of $\frac{2}{3}$ is $\frac{27}{31}$.

Or thus.

$2 \times 3 \times 3 = 18$. And the root is $\frac{\sqrt[3]{18}}{3}$. But $\sqrt[3]{18}$ $= 2\frac{5}{6}$.

$1\dot{8}$ $(2\frac{10}{12} = 2\frac{5}{6}$ the numerator.

$$\begin{array}{r} 8 \\ \hline 10 \\ \hline 2\times3=\ 6 \\ 4\times6=24 \\ \hline 30 \end{array}$$

or rather $2\frac{20}{30} = 2\frac{2}{3}$ for the numerator, and the root is $\frac{2\frac{2}{3}}{3} = \frac{8}{9}$.

Ex. 4.

Extract the cube root of $13\frac{4}{7}$.

$13\frac{4}{7}$ is reduced to $\frac{95}{7}$, then $\frac{95}{7} = \frac{95000}{7000}$.

The root of $95\dot{0}0\dot{0}$ is 45 the numerator.

And the root of $\dot{7}00\dot{0}$ is 19 the denominator,

And the root $\frac{45}{19} = 2\frac{7}{19}$.

Otherwiſe.

$95 \times 7 \times 7 = 4655$, whoſe root is 16 or 17; therefore the root is between $\frac{16}{7}$ and $\frac{17}{7}$.

Or thus.

$$\begin{array}{l} \dot{4}65\dot{5}\ (16 \\ 4096 \\ \hline \end{array}$$

rem. 559, and thrice the ſquare of $16 = 768$, and the root is $16\frac{559}{768} = 16\frac{8}{11}$ nearly, the numerator. Therefore the root of $13\frac{4}{7}$ is $\frac{16\frac{8}{11}}{7} = 2\frac{30}{77}$.

CHAP. III.

DECIMAL FRACTIONS.

Notation.

A *DECIMAL FRACTION* is a fraction whoſe denominator is 1 with one or more cyphers; thus, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{27}{100}$, $\frac{9}{1000}$, are decimal fractions.

Here 1, or the integer, is always ſuppoſed to be divided into 10, 100, 1000, *&c.* equal parts; or, which is the ſame thing, 1 is ſuppoſed to be divided into 10 equal parts, and each of theſe parts into 10 equal parts, and each of theſe into 10 parts more, and ſo on, by a continual ſubdiviſion.

A decimal fraction is expreſſed without the denominator, by writing only the numerator and prefixing a point on the left hand of it. And the number of places in the numerator is always equal to the number of cyphers in the denominator; thus .3 ſignifies $\frac{3}{10}$, .03 ſignifies $\frac{3}{100}$, .37 ſignifies $\frac{37}{100}$, and .004 ſignifies $\frac{4}{1000}$; therefore when the numerator hath not ſo many places as the denominator has cyphers, the void places muſt be filled up with cyphers towards the left hand. And from hence is diſcovered how many cyphers the denominator conſiſts of.

Cyphers on the right hand of a decimal do neither increaſe nor diminiſh the value; thus .3 and .30 and .300, *&c.* are all equal, becauſe $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000}$, *&c.* as is plain from vulgar fractions: and therefore decimals

decimals are ſoon reduced to a common denominator, by annexing cyphers.

The *notation* of decimal fractions, will be plain from the following table.

tenth parts	hundred parts	thouſand parts	10 thouſand parts	100 thouſand parts	million parts	10 million parts	
.3	2	8	5	0	7	6	*&c.*

As in whole numbers, the 1ſt place contains units, the ſecond place to the left, tens; the third, hundreds; *&c.* So in decimals the order of places is contrary, for the firſt place in decimals is tenths; the 2d place to the right is hundred parts; the 3d, is thouſand parts; *&c.* And as whole numbers increaſe from the right hand to the left in decuple proportion, or decreaſe from the left to the right in a ſubdecuple proportion; ſo decimals alſo increaſe from the right to the left in a decuple proportion, and decreaſe from the left to the right in the ſame ſubdecuple proportion. Thus in the table above, 3 ſignifies $\frac{3}{10}$, 2 ſignifies $\frac{2}{100}$, 8 ſignifies $\frac{8}{1000}$.

But in reading any decimal, as .328, we do not ſay 3 tenths, 2 hundredths, 8 thouſands; but firſt reduce them all to the denominator of the greateſt; and call them all by that name. Thus $\frac{3}{10} = \frac{300}{1000}$, $\frac{2}{100} = \frac{20}{1000}$, and $\frac{8}{1000}$ remains the ſame; and collecting them together, we have $\frac{328}{1000}$, that is, three hundred and twenty eight thouſand parts: for .300 +.020+.008 =.328.

A *mixt number*, is made up of a whole number and a decimal, which are ſeparated from one another by a point. Thus 32.17 ſignifies $32\frac{17}{100}$. And 5.03 ſignifies $5\frac{3}{100}$.

Hence any mixt number, as 5.03, may be expreſſed thus, $\frac{503}{100}$, or $\frac{5030}{1000}$, or $\frac{50300}{10000}$, *&c.* and 32.17 $=\frac{32.17}{1}=\frac{321.7}{10}=\frac{3217}{100}=\frac{32170}{1000}$, *&c.*

Numeration, or the reading of decimals, is the very ſame as that of whole numbers, only adding the name of the parts ſignified by the decimal. Thus 328.328 ſignifies 328 thouſands, and 328 thouſand parts.

Since decimals as well as whole numbers decreaſe to the right hand in a ſubdecuple proportion, therefore decimals have the ſame properties as whole numbers, and are ſubject to the ſame rules of operation. For in any whole number, the ſeveral parts of it are, in effect, but decimal parts of one another.

PROBLEM I.

To add decimal fractions.

RULE.

Place all the points directly under each other, then tenths will be under tenths, and hundred parts under hundredths, *&c.* then add them together as if they were whole numbers; and laſtly, put a point under the other points, which will prick off the number of decimal places in the ſum.

Ex.

Ex. 1.

.3527
62.013
.002
.5
ſum 62.8677

Ex. 2.

.0035
.02761
.81017
.22
.017
ſum 1.07828

Ex. 3.

32.
5.07
.81
.20571
.0035
ſum 38.08921

PROBLEM II.

To ſubtract one decimal from another.

RULE.

Place the greater number uppermoſt, the points under the points, tenths under tenths, *&c.* then ſubtract

ſubtract as in whole numbers; placing the point of ſeparation under the other points.

Ex. 1.

from .4302
take .257
rem. .1732

Ex. 2.

from 17.203
take .07542
rem. 17.12758

Ex. 3.

from 29.
take .0545
rem. 28.9455

PROBLEM III.

To multiply decimals together.

1. A GENERAL RULE.

Multiply the decimals as if they were whole numbers; and from the product cut off as many decimal places, as there are in both numbers. If there be not ſo many places, make them out with cyphers on the left.

Ex.

Ex. 1.

.9087
.852

18174
45435
72696

product .7742124

Ex. 2.

23.17
2.016

13902
2317
4634

product 46.71072

Ex. 3.

.09047
.00125

45235
18094
9047

product .0001130875

Ex. 4.

.003479
5081.

3479
27832
17395

product 17.676799

To prove the truth of the rule, let 9087 be multiplied by 852; theſe are equivalent to $\frac{9087}{10000}$ and $\frac{852}{1000}$, whence if the numerators be multiplied together, and the denominators alſo, the product will be $\frac{7742124}{10000000}$, that is, .7742124 conſiſting of as many decimal places as there are cyphers, that is, of as many places as are in both the numbers.

For the ſame reaſon $\frac{2717}{100}$ multiplied by $\frac{2016}{1000}$, produces $\frac{4671072}{100000}$, or 46.71072.

PARTICULAR RULES *for contracting the work.*

2 RULE.

In large decimals, you muſt multiply in a contrary order, thus: Begin with the left hand figure of the multiplier, by which multiply the whole multiplicand.

Then prick off the laſt figure of the multiplicand on the right, and multiply the reſt by the next figure of the multiplier on the left.

Then prick off another figure of the multiplicand, and multiply the reſt by the next figure of the multiplier. Go on thus with all the figures of the multiplier; always pricking off a figure in the multiplicand, at each multiplying. And obſerve what is to be carried from the preceding figure, when you begin each multiplication.

Set the firſt figure of each product directly in a line under one another, to be added together.

Laſtly, when you multiply by the units place, obſerve what place of the multiplicand it begins with; and cut off ſo many decimals, in the product.

Or, obſerve the places of any two decimals that begin the multiplication, and the ſum of them gives the number of decimal places in the product.

Note,

Note, inſtead of pricking off the figures gradually in the multiplicand; you may know where to begin to multiply every time thus: If the firſt figure on the left of the multiplier, begins with the firſt figure on the right of the multiplicand; then the 2d figure begins with the 2d; and the 3d with the 3d; and ſo on.

Ex. 1.

	• • • • •
multiply	76.84375
by	8.21054
	61475000
	1536875
	76843
	3842
	307
product	630.92867

Ex. 2.

	• • • • •
multiply	.3570643
by	.0210576
	7141286
	357064
	17853
	2499
	214
product	.007518916

Explanation.

In Ex. 1. 8 multiplying the whole multiplicand, gives 61475000 for the product. Then prick off 5, and multiply by 2, ſaying 2 times 5 is 10, carry 1, and 2 times 7 is 14 and 1 is 15, 2 times 3 is 6 and 1 is 7, &c. and the product is 1536875. Again, prick off 7, and ſay once 3 is 3, once 4 is 4, &c. and that product is 76843. Then prick off 3, and ſay 0 times 4 is 0; again, prick off 4, and ſay 5 times 4 is 20, carry 2, then 5 times 8 is 40, and 2 is 42, &c. and the product is 3842. Laſtly, prick off 8, and ſay 4 times 8 is 32, carry 3; then 4 times 6 is 24 and 3 is 27, 4 times 7 is 28, and 2 is 30, and that product is 307. And the ſum of all 630.92867. And ſince 8 the units begins with 5 in the 5th place, there muſt be 5 places of decimals.

And ſince 2 begins to multiply at 7, 1 at 3, 0 at 4, 5 at 8, and 4 at 6; it is plain the firſt figure of each product will be in the 5th place of decimals; becauſe the ſum of the places of the two multipliers always makes 5.

In the 2d Ex. 2 begins to multiply at 3, 1 at 4, 0 at 6, 5 at 0, 7 at 7, 6 at 5. Where the ſum of both places makes 9; therefore there are 9 places of decimals.

Ex. 3.

. . . .
multiply 17.002576|830
by .35608204

51007730
8501288
1020154
13602
340
7

product 6.0543121

3 RULE.

When any decimal is to be multiplied by 10, 100, 1000; &c. remove the ſeparating point ſo many

many places to the right hand, as there are cyphers.

Ex. 8.

multiply	32.075
by	10
product	320.75

Ex. 9.

multiply	25.7
by	1000
product	25700.

4 RULE.

In large multiplications, make a table of all the products of the multiplicand by the 9 digits; and then the ſeveral products, are eaſily taken out of the table and writ down, as directed in multiplication of whole numbers.

PROBLEM IV.

To divide one decimal by another.

1. A GENERAL RULE.

Divide as if they were whole numbers. Then cut off as many decimal places in the quotient, as the number of decimal places in the dividend exceeds the number in the diviſor; if there are not ſo many in the diviſor, prefix ſo many cyphers.

Or thus, the firſt figure of the quotient (or indeed any quotient figure) is of the ſame degree as that *figure* of the dividend, under which the units place of the product ſtands.

Annex cyphers to the dividend, when there are not places ſufficient. Likewiſe by continually annexing cyphers, the diviſion may be continued as far as you pleaſe.

Ex. 1.

Divide 13.4 by 3207.3

```
3207.3) 13.400000 (.00417
        128292··
        --------
          57080 dividual.
          32073
          -----
          250070
          224511
          ------
           25559
           -----
```

Explanation.

As the dividend wants places, I add cyphers at pleaſure; and there being ſix places of decimals in the dividend, and 1 in the diviſor; there will be 5 in the quotient; therefore 2 cyphers muſt be prefixt before 417, and the quotient is .00417 as required.

Or thus, ſince 9 the units place (of the product of the diviſor by 4) ſtands under the third place of decimals, therefore 4 is in the third place of decimals.

Ex. 2.

Divide 271.5 by 5.746

5.746) 271.50000 (47.25

22984 · · ·

41660

40222

14380

11492

28880

28730

150 &c.

Ex. 3.

Divide .4368 by .0078

.0078) .4368 (56.

390 ·

468

468

· ·

Ex. 4.

Divide .052701 by 36.

36) .052701 (.001463

36 · · ·

167

144

230

216

141

108

33

To

To prove the rule; ſince the number of decimals in the dividend is equal to the number in both diviſor and quotient; it follows that the quotient contains as many as the dividend exceeds the diviſor.

Again, the quotient contains as many decimals, as 12829 (the product of 3207. by 4) contains, (for there are none in 3207 the diviſor); and that is, as many as are in the dividend 13.400, under which it ſtands to be ſubtracted; therefore it follows, that the quotient figure 4 is of the ſame degree as 9, the product of the units place of the diviſor, or as (0) the figure above it in the dividend. Therefore 4 the quotient figure is in the 3d place of decimals.

2. RULE.

To contract the work in large diviſions, inſtead of pricking one down from the dividend, prick one figure off the diviſor each operation; and in multiplying leave out theſe figures prickt off, only you muſt have regard to what is to be carried from the figure laſt prickt off.

Note, if the firſt figure in the quotient begins to multiply at the firſt figure in the diviſor, then the 2d begins at the 2d, the 3d at the 3d, *&c.*

Ex. 5.

```
 • ••••••
76.84375) 630.92878 (8.210541
           61475000
           --------
            1617878
            1536875
            -------
              81003
              76843
              -----
               4159
               3842
               ----
                317
                307
                ---
                 10
                  7
                 --
                  3
```

Explanation.

Here 8 is multiplied into 76.84375; then 2 is multiplied into 76.8437 (carrying 1); then 1 is multiplied into 76.843; the multiplication of 7684 by 0, is omitted; then 768 by 5; then 76 by 4, laſtly 7 by 1.

3 RULE.

To divide by 10, 100, 1000, *&c.* remove the ſeparating point, ſo many places to the left hand as there are cyphers.

Ex. 6.

Divide 32.075 by 10.
quotient 3.2075

Ex. 7.

Divide 25.7 by 1000.
quotient .0257

4 RULE.

In large diviſions, make a table of the products of the diviſor and all the 9 figures. And then diviſion will be wrought by inſpection; for the ſeveral products are eaſily taken out of the table, as you want them, according to the directions in diviſion of whole numbers.

PROBLEM V.

To reduce or change a vulgar fraction to a decimal fraction.

RULE.

Add cyphers at pleaſure to the numerator, repreſenting ſo many places of decimals; and then divide by the denominator, as far as you pleaſe.

Ex. 1.

Reduce $\frac{3}{4}$ to a decimal.

```
4) 3.0000 (.7500, or .75
   28 . . .
   --
    20
    20
    --
     .00
     --
```

Ex. 2.

Reduce $13\frac{4}{7}$ to a decimal or mixt number.

```
7) 4.000000 (.571428
   35 . . . . .
   --
    50
    49   then 13 4/7 = 13.571428
    --
     10
      7
     --
      30
      28
      --
       20
       14
       --
        60
        56
        --
         4 &c.
         --
```

Ex. 3.

To reduce $\frac{16}{3}$ to decimals.

3) 16.00000 (5,333 &c. = $\frac{16}{3}$.
15 . . .
10
9
10
9
10
9
1 &c.

Ex. 4.

To change $\frac{1}{243}$ to a decimal.

243) 1.00000000 (.004115 = $\frac{1}{243}$.
972 . . . . .
280
243
370
243
1270
1215
55 &c.

SCHOLIUM.

To reduce a decimal to a vulgar fraction, is no more than dividing by the greateſt common meaſure; the denominator of the decimal being 10, 100, 1000, &c.

PROBLEM VI.

To reduce the known part or parts of any integer to a decimal.

RULE.

Begin at the laſt part, and reduce it to a vulgar fraction, of the next ſuperior denomination, and ſo to a decimal. Then take that, and the next part, if there is any, which alſo reduce to a decimal of the next ſuperior denomination; and ſo on to the laſt.

Ex. 1.

What decimal of a ſhilling is three half-pence?

3 half-pence is $= 1\frac{1}{2}d. = 1.5d.$, then $\frac{1.5}{12}d. =$ the fraction of a ſhilling, by dividing, $\frac{1.5}{12} = .125$ the decimal of a ſhilling.

$$\begin{array}{l} 12)\ 1.500\ (.125 \\ \qquad 12 \\ \hline \qquad\ 30 \\ \qquad\ 24 \\ \hline \qquad\quad 60 \\ \qquad\quad 60 \\ \hline \qquad\qquad . \\ \hline \end{array}$$

Ex. 2.

Reduce $6s.\ 3\frac{1}{4}d.$ to the decimal of a pound.

Here $\frac{1}{4}$ of a penny $= .25$, and $3\frac{1}{4}$ or 3.25 divided by 12, that is, $\frac{3.25}{12} = .270833$ the fraction of a ſhilling; and $6s.\ 3\frac{1}{4}d.$ or 6.270833 divided by

20

$20\left(\frac{6.270833}{20}\right)$ is $= .31354166$ the decimal of a pound.

Ex. 3.

What decimal of a hundred weight is 3 *ſt.* 7 *lb.* 9 *oz.*; at 14 *lb.* to the ſtone.

9 *oz.* $= \frac{9}{16}$ *lb.* $= .5625$ *lb.*, and $\frac{7.5625}{14} = .540178$ *ſt.*

and $\frac{3.540178}{8} = .442522$ hundreds.

Hence the following decimal table is made.

Money. 1 *l.* the integer. 1 *s.* = .05 1 *d.* = .00416667 1 *f.* = .00104167	*Averdupoiſe weight.* 1 *lb.* the integer. 1 *oz.* = .0625 1 *dr.* = .00390625
Troy weight. 1 *lb.* the integer. 1 *oz.* = .0833333 1 *pwt.* = .0041666 1 *gr.* = .0001736	*Averdupoiſe weight.* 1 hundred the integer. 1 *qr.* = .25 1 *lb.* = .00892857 1 *oz.* = .00055803
Apothecary's weight. 1 *oz.* the integer. 1 *dr.* = .125 1 *ſcr.* = .0416666 1 *gr.* = .0020833	*Long meaſure.* A yard the integer. 1 *f.* = .3333333 1 *in.* = .0277777
Time. 1 day the integer. 1 *ho.* = .0416666 1 *min.* = .0006944 1 *ſec.* = .0000115	*Square and ſolid meaſure.* 1 *in.* = .006945, the decimal of a ſquare foot. 1 *in.* = .0005787, the decimal of a cubic foot.

PRO-

PROBLEM VII.

To find the value of a decimal in known parts of the integer.

RULE.

Multiply the decimal by the number of parts contained in the next inferior denomination, gives the parts required : and if the decimal cut off be multiplied by the next lower denomination, you'll have the parts of that denomination ; and ſo on.

Ex. 1.

How much money is .732 of a pound?

.732 *l.*
20

14.640 *s.* Anſ. 14 *s.* 7 *d.* $2\frac{7}{10}$ *f.*
12

7.680 *d.*
4

2.72 *f.*

Ex. 2.

What weight is 5.7305 *lb.* averdupoiſe?

5.7305 *lb.*
16 Anſ. 5 *lb.* 11 *oz.* 11 *dr.*

43830
7305

11.6880 *oz.*
16

4128
688

11.008 *dr.*

PROBLEM VIII.

To change a common diviſor into a common multiplier.

RULE.

Divide 1 by that diviſor, the quotient is a multiplier. If the diviſor be a vulgar fraction, invert it, making the numerator the denominator, *&c.*

Ex. 1.

If 2150.4 be a diviſor, what is the multiplier to effect the ſame thing?

$$\begin{array}{lll} 2150.4) & 1.000000000 & (.00046503 \text{ the multiplier.} \\ & 86016\cdots & \\ & \overline{139840} & \\ & 129024 & \\ & \overline{\ \ 108160} & \\ & \ \ 107520 & \\ & \overline{\ \ \ \ \ 64000} & \\ & \ \ \ \ \ 64512 & \\ & \overline{} & \end{array}$$

Ex. 2.

If $\frac{5}{8}$ be a diviſor, what is the multiplier?

$\frac{5}{8}) \ \frac{1}{1} \ (\frac{8}{5}$ the multiplier = 1.6

PROBLEM IX.

To extract the ſquare root of a decimal, or mixt number.

RULE.

Annex cyphers on the right hand as many as you pleaſe, and begin at the units place and point every other

other figure both to the left and right. Then proceed to extract in all reſpects as if it was a whole number; and cut off as many whole numbers in the root, as there are points in the whole number, and as many decimals, as points in the decimals. And the operation may be continued as far as you will, by adding pairs of cyphers.

Ex. 1.

Extract the root of 2211.8209

```
         .  .   .  .
        2211.8209 (47.03 the exact root.
        16 .   . .
       ----
   87)  611
        609
       ----
 9403)  28209
        28209
        -----
        . . . .
```

Ex. 2.

What is the ſquare root of 10 ?

```
          .  .  .
        10.0000 (3.16227 &c. the root.
         9   . .
        ---
   61)   100
   +1     61
  ----   ---
  626)   3900
  +6]    3756
  ----   ----
  6322)   14400
    +2    12644
  -----   -----
  63242)   175600
     +2    126484
  ------   ------
  632447)  4911600
           4427129
           -------
            484471
```

Ex. 3.

Extract the ſquare root of ,001234

```
 .  .  .  .
0.001234 (.0351283362 the root near.
      9 .
 ----  ----
  65)  334
  +5   325
 ----  ----
  701)   900
   +1    701
  ----   ----
  7022)  19900
    +2   14044
         -----
  70248)  585600
   ....   561984
          ------
           23616
           21074
           -----
            2542
            2107
            ----
             435
             421
             ---
              14
              14
             ---
               .
             ---
```

Expla-

Ex. 4.

To extract the ſquare root of $\frac{7}{9}$.

$\frac{7}{9}$ reduced to a decimal is .777777, *&c.*

$0.\dot{7}7\dot{7}7\dot{7}7\dot{7}7$ (.8819171, *&c.* the root.
64 $\cdot$ $\cdot$

168) 1377
1344

1761) 3377
1761

17629) 161677
$\cdot\cdot\cdot$ 158661

3016
1763

1253
1233

20
17

3

PROBLEM IX.

To extract the cube root of a decimal, or mixt number.

RULE.

Add cyphers at pleaſure on the right hand, that the decimals may conſiſt of 3, 6, 9, 12, *&c.* places; and begin at the units place and point every third figure

figure both to the left and right hand. Then extract the root as if it was a whole number; and the extraction may be continued as far as you will, by ſtill adding ternaries of cyphers. At laſt cut off as many places of whole numbers, as there are points in the whole numbers, and the like for decimals.

Note, if you deſire the laſt quotient to go true to more places of figures, do thus; add half the laſt quotient to the laſt root, and ſquare the ſum for a diviſor, and divide over again.

Ex. 1.

Extract the cube root of 146708.483

$14\dot{6}70\dot{8}.48\dot{3}00\dot{0}$ (52.74 the root.
125

3) 217
25) 72 (2
2) 54
27) 18

52 = 1ſt root.
2704 = ſquare.
140608 = cube.

$14\dot{6}70\dot{8}.4$
140608

3) 61004
2704) 20334 (74
½ the root 26 19110
2730 1224
1092
132

Ex. 2.

What is the cube root of 2?

$\dot{2}.00\dot{0}00\dot{0}$ (1.259921 *&c.* the root.
1
3) 10
1) 3 (3 too much.
3
0

$\dot{2}.00\dot{0}$
1.728
3)2720
144) 906 (60, too
7 906 much.
151 ...

1 root = 12
ſquare = 144
cube = 1728

2 root =1259
ſquare =1585081
cube =1995616979

$\dot{2},00\dot{0}00\dot{0}00\dot{0}0$
1995616979
3) 43830210
1585081) 14610070 (92106
1133 14275926
1586214) 334144
:... 317243
16901
15862
1039
951
88

Ex. 3.

What is the cube root of .0001357?

0.000135700000 (.05138 *&c.* the root.
125
3) 107
25) 35 (1
2 27
27 8
1357000
132651
3) 30490
2601) 10163 (38
15 7848
2616 2315
2093
222

1 root 51
ſquare 2601
cube 132651

Ex. 4.

Extract the cube root of $13\frac{2}{3}$.

Reduce $\frac{2}{3}$ to a decimal, and the number is 13.666666

13.666666 *&c.* (2.390 *&c.*
8
3) 56
4) 18
1 15
5 3
136666
12167
3) 14996
529) 4998 (90
21 4950
550 48

1 root 23
ſquare 529
cube 12167

Or thus.

$$23+\frac{\cdot 90}{2}=23\cdot\dot{4}\dot{5}\dot{0}$$

23.45

46900
7035
938
117

549.90
diviſor.

549.9) 499.80 (9089
· · · 494 91 and the root 2.3908

489
439

50

Ex. 5.

What is the cube root of 171.46776406?

· · · ·
171.467764060 (5.5
125

3) 464
25.) 155 (5
2 135

27 20

1 root	5
ſquare	25
cube	125

· ·
171.4677
166 375

3) 50927
3025) 16975 (55
27 15260

3052) 1715
· 1526

189

2 root	55
ſquare	3025
cube	166375

and the root = 5.555

Or

Or thus.

```
3055·3|2) 169750 (55558
 · · ·    152767
          ------
           16983
           15276
           -----
            1707
            1528
            ----
             179
             153
             ---
              26
```

and the root = 5·55555 &c. = $5\frac{5}{9}$.

```
    55·
  + ·275
 --------
  55·2750
  55·275
 --------
  2763750
   276375
    11055
     3869
      276
 --------
 3055·325
  diviſor.
```

CHAP. IV.

Several Practical Rules in Arithmetic.

PROBLEM I.

To reſolve a queſtion in reduction.

R*Eduction deſcending* is when ſome integers of a greater denomination are to be reduced to thoſe of a leſs.

Reduction aſcending is when the leſſer denomination is to be reduced to the greater.

RULE.

In reduction deſcending, multiply continually by all the denominations from the given one to that ſought; adding to each product by the way, thoſe of the ſame denomination with itſelf, if ſuch there be.

In reduction aſcending, where the quantity is to be reduced to a higher denomination; divide continually by all the denominations from the given one to that ſought. Sometimes both rules muſt be uſed promiſcuouſly as occaſion requires.

Ex. 1.

In 415 pounds, how many pence?

$$\begin{array}{r} 415 \\ 20 \\ \hline 8300 \\ 12 \\ \hline 16600 \\ 8300 \\ \hline \end{array}$$

Anſwer 99600 pence.

Ex.

Ex. 2.

In 3076*l.* 13*s.* $11\frac{1}{4}$*d.* how many ſhillings, pence, and farthings?

3076 —— 13 —— $11\frac{1}{4}$
20

61533 ſhillings adding 13
12

123077
61533

738407 pence adding 11
4

2953629 farthings adding 1

Ex. 3.

In 354*lb.* 0*oz.* 16*dw.* 15*gr.* how many grains?
12

708
354

4248 ounces
20

84976 pennyweights
24

339919
169952

2039439 grains

Ex. 4.

In 48067 ounces averdupoiſe, how many hundred weight?

```
               14)      8)
16) 48067 (3004lb. (214ſt. (26C. 6ſt. 8lb. 3oz.
    48···   28··    16·
    -----   ----    ---
      067    20      54
       64    14      48
      ---   ----    ---
        3     64      6
        -     56      -
             ---
               8
               -
```

Ex. 5.

In 11923 pence, how many pounds?

```
                 20)
12) 11923 (993 ſhillings (49 pounds.
    108··   80·
    -----   ---
      112   193
      108   180      Anſ. 49l. 13s. 7d.
      ---   ---
       43    13
       36     -
       --
        7
        -
```

Ex. 6.

In 207*l.* 15*s.* 6*d.* how many pieces (at 7*s.* 3½*d.* per piece) gowlands (at 7 pieces per gowland) and ringlets (at 11 gowlands a ringlet)?

```
7s. 3½d.      207l. 15s. 6d.
 12              20
----           -----
 87             4155
  2               12
----           -----
175             8316
halfpence      4155
               -----
               49866
                   2  7)           11)
               -----
     175) 99732 (569 pieces (81 gowl. (7 ringl.
          875     56            77
          ----    --            --
          1223     9             4
          1050     7            --
          ----    --
           1732    2
           1575   --
           ----
            157
            ---
```

Ex. 7.

If 27 pounds be divided among 31 perſons, what is the ſhare of each?

```
           27l.
           20
          ----
     31)  540 (17s. 5d. of. anſwer.
          31
          ---
          230
          217
          ---
           13
           12
          ---
     31)  156 (5
          155
          ---
            1
            4
            -
     31)    4 (0
```

Ex. 8.

In 8769 dollars, at 4*s.* 7*d.* per dollar, how many groats, ſhillings, crowns, and pounds?

4*s.* 7*d.* 8769
12 55
55 pence 43845
43845
4) 482295 (120573 groats.
rem. 3 pence
5) 4)
3) 120573 (40191 ſhil. (8038 crowns (2009 pounds.
0 1 rem. 2 rem.

The proof of reduction is to work the queſtion backwards.

PROBLEM II.

To reſolve a queſtion in the rule of three.

Here are three numbers given to find a fourth in proportion. If a greater number requires a greater, or a leſs requires a leſs, it is called the *rule of three direct.*

But if a greater requires a leſs, or a leſs requires a greater number; it is called the *rule of three inverſe.*

1. A GENERAL RULE.

1. To ſtate the queſtion, place the three given terms ſo, that the firſt and third may be of one name, the third being that which aſks the queſtion. And the ſecond muſt be of the ſame name with the fourth term ſought. And let them be reduced to their loweſt denomination, where the firſt and third muſt be of the ſame.

2. Then ſay, if the firſt term give or require the ſecond, what does the third give or require. If *more* be required, mark the *leſſer* extreme; if *leſs* be required, mark the *greater* extreme, for a *diviſor.* Multiply the other two numbers together, and divide

by this diviſor. The quotient is the anſwer, of the ſame denomination with the ſecond term.

3. What remains will either make a fractional part; or it muſt be reduced to a lower denomination, and divided as before.

Ex. 1.

If 18 lb. of Sugar coſt 12 ſhillings, what will 150 coſt?

lb. *ſh.* *lb.*
18 : 12 : : 150.

Here, if 18 lb. coſt 12 ſhillings, 150 lb. muſt coſt more, therefore divide by 18 the leſſer extreme.

* 18 12 150
12
300
150
18) 1800 (100 ſhillings
18
00
20) 100 (5*l.* the anſwer.
100
..

Ex. 2.

If 35 yards of cloth coſt 39*l.* 7*s.* 6*d.* how many yards may be bought for 19*l.* 2*s.* 6*d.*?

* 39*l.* 7*s.* 6*d.* : 35 *yds.* : : 19*l.* 2*s.* 6*d.*
20 20
787 382
12 12
9450 : 35 : : 4590
35
22950
13770
9450) 160650 (17 *yds.* anſw.
9450
66150
66150
..

Ex.

Ex. 3.

If $40\frac{1}{2}$ *lb.* of tobacco coſt 3 *l.* how much can I buy for 7 *l.* 15 *s.*?

* 3 *l.* : 40 *lb.* 8 *oz.* : : 7 *l.* 15 *s.*
20   16   20

60 : 648 : : 155
155

3240
3240
648

16)
60) 100440 (1674 ounces (104 *lb.* 10 *oz.*
16
74
64
10

Or thus by vulgar fractions.

* 3 : $40\frac{1}{2}$: : $7\frac{3}{4}$:
that is 3 : $\frac{81}{2}$: : $\frac{31}{4}$:

81   4
31   2
81   8
243

$\frac{3}{1}$) $\frac{2511}{8}$ ($\frac{837}{8}$ = $104\frac{5}{8}$ *lb.*

Or

Or thus by decimals.

3*l.* : 40.5*lb.* : : 7.75*l.*

40.5

3875

3100

3) 313875 (104.625 = 104*l.* 10*oz.*

16

3750

625

10.000

Ex. 4.

If 6 men be 10 days in finiſhing a piece of work, how long will 8 men be?

6*m.* : 10*d.* : : 8*m.* *

Here 8 men will be leſs time than 6, therefore more requires leſs; and 8, the greater extreme, is the diviſor.

6 : 10 : 8 *

6

8) 60 ($7\frac{4}{8}$ = $7\frac{1}{2}$ days.

56

4

Ex. 5.

If I lend a perſon 300*l.* for a year, how long ought he to lend me 500*l.* to requite me?

300*l.* : 365*d.* : 500*l.* *

300

5|00) 1095|00 (219 days.

Here leſs time is required, and 500 the diviſor, by the inverſe rule.

Ex.

Ex. 6.

How many yards of cloth, a yard and a quarter broad, will line a piece of tapeſtry 10 yards long, and $3\frac{1}{2}$ broad?

$3\frac{1}{2}b. : 10l. :: 1\frac{1}{4}b.$ *

7 that is, $\frac{7}{2} : 10 :: \frac{5}{4}$ *.

10

70

$\frac{5}{4}) \frac{70}{2} \left(\frac{280}{10} = 28\right.$ *yds.*

2 RULE *for contracting the work.*

When the diviſor and either of the other terms, can be exactly divided by ſome common diviſor; then divide them, and take the quotients inſtead of theſe terms. And proceed thus as oft as you can.

Ex. 7.

If 63 gallons of brandy coſt 42*l.* what will 72 gallons coſt? Here 63 is the diviſor.

Divide by 9) * 63 : 42 :: 72
7) * 7 : 42 :: 8
* 1 : 6 :: 8 : 48*l.* anſ.

Ex. 8.

There is a paſture which will feed 18 horſes for 7 weeks; how long will it feed 42 horſes? Here 42 is the diviſor, and the rule inverſe.

7) 18 : 7 :: 42 *
6) 18 : 1 :: 6 *
3 : 1 :: 1 * : 3 weeks; anſwer.
3
1) 3 (3

Ex. 9.

If $\frac{3}{8}$ of a yard coſt 27 ſhillings; what will $\frac{7}{8}$ of a yard coſt?

$\frac{1}{8})*\frac{3}{8} : 27 :: \frac{7}{8}$
3)*3 : 27 :: 7
*1 : 9 :: 7 : 63*s.* anſwer.

The

The proof is made by multiplying the quotient by the diviſor, adding the remainder; which muſt be equal to the product of the other two numbers.

PROBLEM III.

To reſolve a queſtion in the double rule or compound rule of three.

RULE.

1. Here, as in the ſingle rule of three, put that term into the ſecond place, which is of the ſame denomination with that ſought; and the terms of ſuppoſition one above another in the firſt place; alſo the terms of demand in the ſame order, one above another, in the third place. Then the firſt and third of every row will be of one name, and muſt be reduced to the ſame denomination, *viz.* the loweſt concerned.

2. Then proceed with each row as with ſo many ſeparate queſtions in the ſingle rule of three, in order to find out the ſeveral diviſors; uſing the ſecond term in common for each of them. That is, in any row, ſay, if the firſt term give the ſecond, does the third require more or leſs? if *more*, mark the *leſſer* extreme; if *leſs*, the *greater*, for a diviſor.

3. Multiply all theſe diviſors together for a diviſor; and all the reſt of the numbers together, for a dividend. The quotient is the anſwer, and of the ſame name with the ſecond term.

4. To contract the work, when the ſame numbers are concerned in both diviſor and dividend, throw them out of both. Or divide any numbers by their greateſt common diviſor, and take the quotients inſtead of them.

Ex. 1.

If 16 horſes in 6 days eat up 9 buſhels of oats; how many horſes muſt there be to eat up 24 buſhels in 7 days?

* 9 *b.*——16 *b.*——24 *b.*
6 *d.*——————7 *d.* *

9
7
63 diviſor

24
16
144
24
384
6
63) 2304 ($36\frac{12}{21}$ horſes; anſwer.
189
414
378
36

Explanation.

Say, if 9 buſhels ſerve 16 horſes, 24 buſhels will ſerve more horſes, therefore mark the leſſer extreme 9 for a diviſor.

Again, ſay if 6 days require 16 horſes to eat up any quantity, 7 days will require fewer horſes to eat them; ſo mark the greater extreme 7 for diviſor.

Then $9 \times 7 = 63$ for diviſor, and $16 \times 24 \times 6 = 2304$ for a dividend; and the quotient is $36\frac{12}{21}$ horſes $= 36\frac{4}{7}$.

Ex.

Ex. 2.

If 9 ſtudents ſpend 12 pounds in 8 months, how much will ſerve 24 ſtudents 16 months?

*9 *ſt.*——12 *l.*——24 *ſt.*
*8 *m.*——————16 *m.*
72 144
24
384
12
768
384
72) 4608 (64 pounds; anſwer.
432
288
288

Or thus by contraction.

3)*9 *ſt.*——12 *l.*——24 *ſt.*
8)*8 *m.*——————16 *m.*

And further.

3)*3——12——8
*1——————2
and then *1—— 4——8
*1——————2
diviſor 1 16
4
64 anſwer.

Ex. 3.

If 8 men be 6 days in digging 24 yards of earth; how many men muſt there be to dig 18 yards in 3 days?

d. *m.* *d.*
3) 6———8———3 *
6)*24*y.*——————18*y.*

Contracted.

2*d.*———8*m.*———1*d.*
4)*4*y.*——————3*y.*

Further contracted.

2*d.*———2*m.*———1*d.**
* 1*y.* ——————— 3*y.*
diviſor 1 2

6
2

12 men; anſwer.

Ex. 4.

If a garriſon of 6000 men may have each 15 ounces of bread to laſt 16 weeks, how much muſt 5000 men have a-piece to laſt 24 weeks?

1000) 6000*m.*———15*oz.*———5000*m.* *
8) 16*w.*——————————24*w.* *

Contracted.

3) 6———15———5 *
2——————3 *

Further contracted.

1 2———3———1 * 3
1 2 ————— 1 * 2

1 diviſor 6
2

Anſwer 12 ounces.

Ex.

Ex. 5.

What principal will gain 20 pounds in 8 months, at 5 per cent. per annum?

12 *m.*——100 *l.*——8 *m.* *
* 5*g.*————————20*g.*

Here the principal is 100*l.* and the time 12 months.

Dividend = / Diviſor = $\frac{12\times100\times20}{8\times5}$ = (by contraction) $\frac{3\times100\times4}{2\times1}$

$= \frac{3\times100\times2}{1\times1}$ = 600*l.* principal, the anſwer.

Ex. 6.

If the carriage of 5 hundred weight coſt 3*l.* 7*s.* 6*d.* for 150 miles, what will the carriage of 7¾ hundred weight come to for 64 miles?

* 5*h.*——3*l.* 7*s.* 6*d.*——7*h.* 3*q.*
* 150*m.* ——————————— 64*m.*
reduced * 20————810*d.*————31*q.*
* 150*m.* ——————————— 64

Dividend / Diviſor $\frac{810\times31\times64}{20\times150} = \frac{27\times31\times32}{10\times5}$, by contraction.

```
10      27                      20
 5      31               12) 535 (44 ſhill. (2 pnds.
--      --                   48    40
50      27                   --    --
        81                   55     4
        --                   48    --
       837                   --
        32                    7
       ---                   --
      1674                        Anſ. 2l. 4s. 7½d.
     2511
     ----
5|0) 2678|4 (535 pence
     ----
       34
        4
       --
 50)  136 (2
```

Ex. 7.

If the carriage of 150 feet of wood, that weighs 3 ſtone a foot, comes to 3 l. for 40 miles, how much will the carriage of 54 feet of free ſtone, that weighs 8 ſtone a foot, coſt for 25 miles?

* 150 *f.*——3 *l.*——54 *f.*

* 3 *ſt* ———————— 8 *ſt.*

* 40 *m.* ———————— 25 *m.*

$$\begin{matrix}\text{Dividend} \\ \text{Diviſor}\end{matrix} \; \frac{54 \times 8 \times 25 \times 3}{150 \times 3 \times 40} = \frac{54 \times 1 \times 25 \times 1}{150 \times 1 \times 5} = \frac{54 \times 5}{150}$$

$$= \frac{54}{30} = \frac{18}{10}.$$

10) 18 (1 *l.* 16 *s.* anſwer.

10

8

20

10) 160 (16

160

Ex. 8.

If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardneſs and $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep; in how many days of 9 hours, will 24 men dig a trench, of 4 degrees of hardneſs, and $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep?

248 *m.*——$5\frac{1}{2}$ *d.*——24 *m.* *

11 *h.* ———————— 9 *h.* *

7 *deg.* ———————— 4 *deg.* *

* $232\frac{1}{2}$ *l.* ———————— $337\frac{1}{2}$ *l.*

* $3\frac{2}{3}$ *w.* ———————— $5\frac{3}{5}$ *w.*

* $2\frac{1}{3}$ *deep* ———————— $3\frac{1}{2}$ *deep.*

$$\begin{matrix}\text{Dividend} \\ \text{Diviſor}\end{matrix} \; \frac{248.11.7.5\frac{1}{2}.337\frac{1}{2}.5\frac{3}{5}.3\frac{1}{2}}{232\frac{1}{2}.3\frac{2}{3}.2\frac{1}{3}.24.9.4} = \text{(by reducing)}$$

$$\frac{248.11.7.\frac{11}{2}.\frac{675}{2}.\frac{7}{2}.\frac{28}{5}}{\frac{465}{2}.\frac{11}{3}.\frac{7}{3}.24.9.4} = \frac{248.11.7.11.675.7.28}{465.11.7.24.9.4} \times \frac{2.9}{8.5}$$

$$=$$

$$= \frac{248.11.675.7.28.2}{465.24.4.8.5} = \frac{31.11.675.7.28}{465.24.2.5}$$

$$= \frac{31.11.135.7.7}{93.6.2.5} = \frac{11.27.7.7}{3.6.2} = \frac{11.9.7.7}{6.2} = \frac{11.3.7.7}{2.2}$$

$= \frac{1617}{4} = 404\frac{1}{4}$ days, the anſwer. All this by throwing equal quantities out of both numerator and denominator.

The proof of this rule is, by multiplying the quotient and all the diviſors together; whoſe product muſt be equal to the product of all the other numbers, when the work is right.

SCHOLIUM.

Any queſtion in the compound rule of three may alſo be reſolved at ſeveral operations, by the ſingle rule of three, but with more labour, thus:

The queſtion being rightly ſtated, take the three terms in the firſt row, and find a fourth term, by the ſingle rule. Make this the ſecond term in the ſecond row; from theſe three terms in the ſecond row find a fourth term. Proceed thus to the laſt.

As if the queſtion in Ex. 1. was propoſed, ſay, if 9 buſhels ſerve 16 horſes any time, how many horſes will 24 buſhels ſerve for the ſame time; they will ſerve more horſes, and therefore 9 is the diviſor, and the anſwer is $42\frac{2}{3}$ horſes.

Again ſay, if 6 days require $42\frac{2}{3}$ horſes to eat up any quantity, how many do 7 days require. Here fewer horſes are required, therefore 7 is diviſor, and the anſwer is $36\frac{4}{7}$ horſes.

PROBLEM IV.

To reſolve a queſtion by the rule of practice.

When a queſtion in the rule of three has 1 for the firſt term, it is more expeditiouſly reſolved, by taking ſome aliquot part or parts of the thing propoſed: and this is called the *rule of practice.*

1. A GENERAL RULE.

Firſt value the integers, obſerving to multiply integers by integers; and for the inferior denominations take what aliquot part you can get, and for what is wanting take parts of that part, and ſo on. Then ſum up the whole.

Ex. 1.

What will 37*c.* 3*q.* 12*lb.* come to, at 5*l.* 15*s.* $7\frac{1}{2}$*d.* the hundred weight?

	5*l.*	15*s.*	$7\frac{1}{2}$*d.*	
	37*c.*	3*q.*	12 *lb.*	
	185	0	0 .	37*c.* at 5*l.*
	18	10	0 .	37 at 10*s.*
	9	5	0 .	37 at 5*s.*
37*c.* at 1*s.* - - 1*l.* 17*s.*		18	6 .	37 at 6*d.*
		4	$7\frac{1}{2}$.	37 at $1\frac{1}{2}$*d.*
	2	17	$9\frac{3}{4}$.	price of $\frac{1}{2}$*c.*
	1	8	11 .	pr. of 1 *q.*
price of $\frac{1}{7}$*q.* - - 4 $1\frac{4}{7}$		12	$4\frac{1}{2}$.	pr. of 12 *lb.*
tot.	218	17	$2\frac{3}{4}$.	anſ.

Explanation.

Firſt I multiply 37 by 5 gives 185*l.* Then ſince 15*s.* is $\frac{3}{4}$ of a pound, or $\frac{1}{2}$ and $\frac{1}{2}$ of that. Therefore I take half 37 which 18*l.* 10*s.* and half of that which is 9*l.* 5*s.* and the fifth part of 9*l.* 5*s.* is 1*l.* 17*s.* the price at 1*s.* the hundred weight. Then becauſe $7\frac{1}{2}$*d.* is the half of a ſhilling, and a fourth of that half. Therefore half of 1*l.* 17*s.* is 18*s.* 6*d.* and $\frac{1}{4}$ of that is 4*s.* $7\frac{1}{2}$*d.*: ſo now the integers are valued.

Then

Then $\frac{3}{4}$ of a hundred being a half and half of that half, I take half of 5*l.* 15*s.* 7$\frac{1}{2}$*d.* which is 2*l.* 17*s.* 9$\frac{3}{4}$*d.* and half that 1*l.* 8*s.* 11*d.* Laſtly, ſince 12*lb.* is $\frac{12}{28}$ or $\frac{3}{7}$ of a quarter, I take $\frac{1}{7}$ of 1*l.* 8*s.* 11*d.* which is 4*s.* 1$\frac{4}{7}$*d.* and triple that is 12*s.* 4$\frac{1}{2}$*d.* the price of 12 pounds. And the ſum of all theſe, is 218*l.* 17*s.* 2$\frac{3}{4}$*d.*

PARTICULAR RULES.

2 RULE.

Sometimes the value may be eaſily found by reckoning the price ſome even number above what is given, which done, take ſome aliquot part for what it is above, and ſubtract it from the former.

Ex. 2.

If a pound of tobacco coſts 11*d.* what is a hundred weight?

	£.	*s.*	*d.*	
112*l.* (at 1*s.*) —— ——	5	12	0	
112*l.* (at 1*d.* is $\frac{1}{12}$) ſubt.	0	9	4	
	5	2	8	anſ.

3 RULE.

When the price is ſhillings, or pounds and ſhillings. Firſt multiply the quantity by the pounds, if there be any; then multiply by half the (even) number of ſhillings, obſerving to write double the product of the firſt figure for ſhillings, and the reſt of the product for pounds, And for an odd ſhilling take $\frac{1}{20}$ of the quantity.

Ex. 3.

What comes 413 yards to, at 2 ſhilling a yard?

413
1

Anſ. 41*l.* : 6*s.*

Ex. 4.

If an ounce coſts 12 ſhillings, what will 76 coſt?

76
6

Anſ. 45*l.* : 12*s.*

Ex. 5.

What is the price of 796 groſs, at 13*s.* the groſs?

796
6

477 : 12 at 12 ſhillings.
39 : 16 at 1 ſhilling.

Anſ. 517*l.* : 08*s.*

Ex. 6.

If a hundred weight coſt 2*l.* 17*s.* what will 238 coſt?

238
2 17

476 00 at 2*l.*
190 08 at 16*s.*
11 18 at 1*s.*

Anſ. 678 6

4 RULE.

When the price is pence, or ſhillings and pence. Multiply the quantity by the ſhillings, if there be any. Then for the pence take ſome aliquot part or parts of the quantity propoſed.

Ex. 7.

What comes 472 ounces to, at 8 *d.* an ounce?

3) 472 (157*s.* 4*d.* at 4*d.*

157 4 . at 4*d.*

20) 314*s.* 8*d.*

Anſ. 15*l.* 14*s.* 8*d.*

Ex. 8.

What will 74 yards of cloth coſt, at 13 *s.* 9*d.* the yard?

74

13 9

222

74

962 0 at 13*s.*

2) 74 —— 37 0 at 6*d.*

4) 74 —— 18 6 at 3*d.*

20) 1017*l.* 6*d.*

Anſ. 50*l.* 17*s.* 6*d.*

Ex. 9.

What comes 150 hundred weight to, at 2 *l.* 11 *s.* 8½ *d.* or 51 *s.* 8 *d.* the hundred?

150
51 8

150
750

7650 *s.*
2) 150 —— 75
3) 75 —— 25

20) 7750 *s.*
Anſ. 387 *l.* 10 *s.*

5 RULE.

When the price is an aliquot part or parts of a pound; then take ſuch aliquot parts of the quantity propoſed.

Ex. 10.

What does 63 gallons come to, at 5 ſhillings a gallon?

l. *s.*
4) 63 (15 : 15 anſ.

Ex. 11.

If I gain 13 *s.* 4 *d.* for a dozen, what do I gain for 100 dozen?

3) 100——33 6 8 at 6 *s.* 8 *d.*
33 6 8

Anſ. 66 13 4

6 RULE.

If farthings be concerned in the price, take ſuch aliquot parts as you can find; or parts of aliquot parts.

Ex. 12.

What comes 371 gallons to, at 13$\frac{1}{2}$*d.* per gallon?

	s.		*d.*	
	371		0	at 1 ſhilling.
8) 371 ——	46		4$\frac{1}{2}$	at 1$\frac{1}{2}$*d.*
20)	417		4$\frac{1}{2}$	
Anſ.	20	17	4$\frac{1}{2}$	

Ex. 13.

How much money can I get for 347 French crowns, at 4*s.* 5$\frac{1}{4}$*d.* a piece?

	347			
	4		5$\frac{1}{4}$	
	1388		0	at 4*s.*
3) 347 ·· (4)	115		8	at 4*d.*
4)	28		9	at 1*d.*
	7		2$\frac{1}{4}$	at $\frac{1}{4}$*d*
20)	1539		7$\frac{1}{4}$	
Anſ.	76	19	7$\frac{1}{4}$	

The proof of this rule is to work the queſtion by different methods.

SCHOLIUM.

Other queſtions that may occur, are eaſily reſolved by the rules of compound multiplication.

When it happens that the firſt term is more than 1; work by the foregoing rules as if the firſt term was

was 1; and at laſt divide by that term, according to the rules of compound diviſion. But ſuch queſtions as theſe are beſt reſolved by the rule of three.

PROBLEM V.

To reſolve a queſtion in the ſingle rule of fellowſhip.

The ſingle rule of fellowſhip, is that which determines how much gain or loſs, is due to every partner concerned; by having the whole gain or loſs, and their particular ſtocks, given.

1. A GENERAL RULE.

Say by the rule of three, as the whole ſtock : is to the whole gain or loſs : : ſo is every man's particular ſtock : to his particular part of the gain or loſs.

Ex. 1.

Two partners A, and B, make a ſtock of 56 pounds; A puts in 24*l.*; and B 32*l.* They gain 7*l.* by trade. What is the gain of each?

```
       24
       32
       ——
(1)    56  :  7  : :  24
                       7
                     ———
                56)  168  (3l. = A's gain.
                     168
                     ———
                      .

(2)    56  :  7  : :  32
                       7
                     ———
                56)  224  (4l. = B's gain.
                     224
                     ———
                     ...
                     ———
```

Ex. 2.

Three men A, B, C, freight a ſhip with wine; A had 284 tuns; B, 140, and C, 64. By a ſtorm at ſea, they were obliged to caſt 100 tuns overboard. What loſs does each ſuſtain?

A 284
B 140
C 64
———
(1) 488 : 100 : : 284
100
———
488) 28400 ($58\frac{96}{488}$ tuns = A's loſs.
2440·
———
4000
3904
———
96

(2) 488 : 100 : : 140
100
———
488) 14000 ($28\frac{336}{488}$ tuns = B's loſs.
976·
———
4240
3904
———
336

(3) 488 : 100 : : 64
100
———
488) 6400 ($13\frac{56}{488}$.
488·
———
1520
1464
———
56

2 RULE.

Where many partners are concerned; find the ſhare of 1 integer, by dividing the whole gain or loſs by the whole ſtock, and the quotient will be a common multiplier; by that multiply every man's part of the ſtock, and it will give his ſhare of the loſs or gain.

Ex. 3.

Four men trade together, A puts in 200*l.* B 150, C 85, D 70; and they gain 60*l.* What is the ſhare of each?

```
A  200     505) 60.0 (.11881 a common multiplier.
B  150           505
C   85           ----
D   70            950
   ----           505
   505            ----
                  4450
                  4040
                  ----
                   4100
                   4040
                   ----
                    600

.11881        .11881        .11881        .11881
   200           150            85            70
---------     ---------     ---------     ---------
23.762|00      59405         59405        8.31670
    20        11881         95048            20
---------     ---------     ---------     ---------
15.24         17.82150      10.09885      6.334|00
   12            20            20            12
---------     ---------     ---------     ---------
 2.88         16.43|00      1.977|00       4.008
---------        12            12         ---------
              ---------     ---------
                516         11.724
              ---------     ---------
```

A gains	23 *l.*	15 *s.*	2.9 *d.*
B	17	16	5.2
C	10	1	11.8
D	8	6	4.1
	60	0	0

Ex. 4.

Five captains plundered the enemy of 1200*l.* The firſt had 20 men, the ſecond 40, the third 55, the fourth 55, the fifth 70. What muſt each captain have in proportion to his number of ſoldiers?

1	20
2	40
3	55
4	55
5	70
	240

240) 1200 (5
1200
..

20	40	55	70
5	5	5	5
100*l.*	200*l.*	275*l.*	350

the firſt gets	100*l.*
the ſecond	200
the third	275
the fourth	275
the fifth	350
	1200

3 RULE.

When there are a great number of partners; the beſt way is to make a table, after this manner. Divide the gain or loſs by the whole ſtock, to find what is the gain or loſs of 1. Then by continual addition of this, make your table as far as 10; then by the continual addition of the gain or loſs of 10, continue the table through all the tens to 100: add in like manner, for all the hundreds to 1000, if there be occaſion. Then you have no more to do, but take every man's ſhare out of the table (at once or oftener) and write it down.

Ex. 5.

There is a certain townſhip, which is to raiſe a tax of 56*l.* 8*s.* 3*d.* To find what each much pay

towards

towards it, the inhabitants being rated as in the following table.

Perſons.	Rent.	Perſ.	Rent.	Perſ.	Rent.
	l.		*l.* *s.*		*l.* *s.*
A	150	I	30	R	4
B	125	K	24	S	3–10
C	100	L	15–10	T	3
D	100	M	12	V	3
E	87	N	12	U	2
F	80	O	12	W	1–10
G	63	P	7	X	1–10
H	40	Q	5–10	Y	1
				Z	1
	745		118 0		20 10
					118 0
					745 0
					883 10

Here 883*l.* 10*s.* = 883.5*l.*
and 56*l.* 8*s.* 3*d.* = 56.4125*l.*

```
883.5) 56.4125 (.0638513
       53010           20
       -----    -----------
        34025   1.277026|0
        26505           12
        -----   ----------
         7520   3.324312
         7068          4
         ----   ----------
          452   1.297248
          441
          ---
          115
           88
          ---
           27
```

So

So 1*l.* is 1*s.* 3*d.* 1.297*f.* whence the following table is made.

£.	£.	s.	d.	f.
0½	0	0	7	2.648
1	0	1	3	1.297
2	0	2	6	2.594
3	0	3	9	3.891
4	0	5	1	1.188
5	0	6	4	2.485
6	0	7	7	3.782
7	0	8	11	1.079
8	0	10	2	2.376
9	0	11	5	3.673
10	0	12	9	0.97
20	1	5	6	1.94
30	1	18	3	2.91
40	2	11	0	3.88
50	3	3	10	0.85
60	3	16	7	1.82
70	4	9	4	2.79
80	5	2	1	3.76
90	5	14	11	0.73
100	6	7	8	1.7
200	12	15	4	3.4
300	19	3	1	1.1

&c.

	£.	s.	d.	f.
Hence the ſhare of A for 100 is	6	7	8	1.7
for 50 is	3	3	10	0.85
total ſhare of A	9	11	6	2.55

The ſhare of B

	£.	s.	d.	f.
for 100	6	7	8	1.7
20	1	5	6	1.94
5	0	6	4	2.48
whole ſhare of B	1	19	7	2.42

and

and ſo on with the reſt; whence we get the following bill.

	£.	s.	d.	f.		£.	s.	d.	f.
A	9	11	6	2.55	O	0	15	3	3.56
B	7	19	7	2.12	P	0	8	11	1.08
C	6	7	8	1.70	Q	0	7	0	1.13
D	6	7	8	1.70	R	0	5	1	1.19
E	5	11	1	0.84	S	0	4	5	2.54
F	5	2	1	3.76	T	0	3	9	3.89
G	4	0	5	1.71	V	0	3	9	3.89
H	2	11	0	3.88	U	0	2	6	2.59
I	1	18	3	2.91	W	0	1	10	3.95
K	1	10	7	3.13	X	0	1	10	3.95
L	0	19	9	2.11	Y	0	1	3	1.30
M	0	15	3	3.56	Z	0	1	3	1.30
N	0	15	3	3.56					
						2	17	5	2.37
	53	10	9	1.53		53	10	9	1.53
						56	8	2	3.9

true to the 10th part of a farth.

The proof is made, by adding together all the ſhares, which muſt be equal to the whole gain or loſs.

PROBLEM VI.

To reſolve a queſtion by the double rule of fellowſhip.

The double rule of fellowſhip, is that which determines how much gain or loſs is due to every partner concerned; by having the whole gain or loſs, and the particular ſtocks, and their times of continuance, given.

1 RULE.

Multiply every man's ſtock, by the time it is employed; then by the rule of three, ſay, as the ſum of theſe products : to the whole gain or loſs : : ſo each of theſe products : to each man's gain or loſs.

Ex. 1.

Three merchants, A, B, C, enter into partnerſhip. A puts in 65*l.* for 8 months; B 78*l.* for 12; and C 84 for 4 months, and 6*l. viz.* 90*l.* for 2 months. They gain 166*l.* 12*s.* What is each man's ſhare of the gain?

65	78	84	90	520
8	12	4	2	936
A = 520	B = 936	336	180	516
			336	1972
			C = 516	

1972 —— 166.6 —— 520

520

33320
8330

1972) 86632.0 (43*l.* 18*s.* $7\frac{1}{2}$*d.* for A.
7888

7752
5916

1836
20

1972) 36720 (18
1972

17000
15776

1224
12

1972) 14688 ($7\frac{8}{12}$
13804

884

Again, 1972——166.6——936
936

9996
4998
14994

1972) 155937.6 ($79\frac{15}{197}$ = 79*l.* 1*s.* $6\frac{1}{4}$*d.*
13804 for B.

17897
17748

149

Laſtly, 1972 — 166.6*l.* — 516 — 43*l.*:11*s.*:$10\frac{1}{4}$*d.*
for C.

Ex. 2.

Four men, A, B, C, D, hold a paſture in common, for which they pay 60*l.* A had 24 oxen 32 days; B 12 oxen 48 days; C 16 oxen for 24 days; and B had 10 oxen for 30 days. What muſt each pay?

$24 \times 32 = 768$
$12 \times 48 = 576$
$16 \times 24 = 384$
$10 \times 30 = 300$

2028

Then 2028 : 60*l.* : : ſo each product : to its ſhare.
That is 169 : 5*l.* : : 768 : $22\frac{122}{169}$
and 169 : 5 : : 576 : $17\frac{7}{169}$
169 : 5 : : 384 : $11\frac{61}{169}$
169 : 5 : : 300 : $8\frac{148}{169}$

		£.	*s.*	*d.*
Hence there is paid by	A,	22	14	$5\frac{1}{4}$
	B,	17	0	10
	C,	11	7	$2\frac{1}{2}$
	D,	8	17	$6\frac{1}{4}$

2 RULE.

When many people are concerned; divide the whole gain or loſs, by the firſt term or ſum of the products; the quotient is a common multiplier, by which multiplying the ſeveral products, you'll have the ſeveral ſhares.

Ex. 3.

Four merchants trade after this manner.

A puts in 100*l.* for 8 months.

B puts in 80*l.* for 5 months, and then puts in 40*l.* more for 3 months longer.

C puts in 176*l.* for 4 months, and then takes out 50*l.* for four months more.

D puts in 230*l.* for 6 months, and then takes out the whole.

They gained 212*l.* 10*s.*; then what is the gain of each merchant.

The ſeveral products of the ſtock and time will be as follows.

100 × 8 —— 800 for A.

80 × 5 —— 400
120 × 3 —— 360

760 for B.

176 × 4 —— 704
126 × 4 ſubt. 504

1208 for C.

230 × 6 —— 1380 for D.

ſum 4148

4148) 212.50 (.05123 the ſhare of 1 pound being a common multiplier.

.05123	.05123	.05123	.05123
800	760	1208	1380
40.984	30738	40984	40984
for A.	35861	614760	15369
			5123
	38.9348	61.88584	70.6974
	for B.	for C.	for D.

	£.	s.	d.
Hence A's ſhare is	40	19	8
B's	38	18	8¼
C's	61	17	8½
D's	70	13	11¼

The proof is had, by adding all the parts of the gain or loſs together, which muſt be equal to the whole.

PROBLEM VII.

To reſolve a queſtion in the rule of alligation medial.

Alligation medial teaches how to find the mean rate of a mixture, when the particular quantities mixt, and their ſeveral rates are given.

RULE.

Multiply the quantities of the mixture by their reſpective prices, and divide the ſum of the products by the ſum of the quantities, gives the mean rate.

Ex. 1.

A man would mix 10 buſhels of wheat, at 4 ſhillings a buſhel, with 8 buſhels of rye at 2 *s.* 8 *d.* a buſhel.

a buſhel. At what price muſt the mixture be ſold?

£. d.
10 × 48 = 480 the wheat.
8 × 32 = 256 the rye.

18 736

18) 736 ($40\frac{8}{9}$, or 3*s.* 5*d.* a buſhel very near,
72 the price of the miſlegin.

16
0

16

Ex. 2.

A vintner would mix 30 gallons of Malaga, at 7*s.* 6*d.* the gallon; with 18 gallons of Canary, at 6*s.* 9*d.*; and 27 gallons of white wine, at 4*s.* 3*d.* how muſt the mixture be ſold?

90 × 30 = 2700
81 × 18 = 1458
51 × 27 = 1377

75) 5535 ($73\frac{1}{5}$*d.* or 6*s.* $1\frac{1}{5}$*d.* per gallon.
525

285
225

60

The proof is made, by finding the value of the whole mixture at the mean price; which muſt be equal to the total value of the ſeveral ingredients.

PRO-

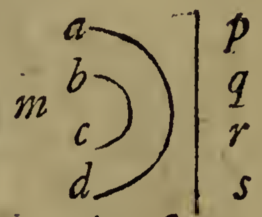
PROBLEM VIII.

To reſolve a queſtion in the rule of alligation alternate.

Alligation alternate ſhows how to find the particular quantities concerned in any mixture; when the particular rates of each ſort, and alſo the mean rate, are given.

Preparation.

Set down the ſeveral rates in order from the greateſt to the leaſt, as the letters *a*, *b*, *c*, *d*; and the mean price (*m*) behind in its due order.



Couple every two rates together by an arch, ſo as one rate may be greater and another leſs than the mean, till they be all coupled. Where *note*, that one rate may be coupled with ſeveral others one by one, as oft as you will.

Take the difference between each rate and the mean rate, and place it *alternately*, that is, againſt all its yoke-fellows. Do thus with all the rates; then the differences will ſtand as *p*, *q*, *r*, *s*. When ſeveral differences happen to ſtand againſt one rate, add them all together. Then,

1 RULE.

When no quantity is given of any of theſe ſorts; the numbers (or differences) ſtanding againſt the ſeveral rates, are the quantities required.

Ex. 1.

A man would mix wheat at 4*s.* a buſhel, with rye at 2 *s.* 8 *d.* a buſhel; to ſell it at 3 *s.* 6 *d.* per buſhel. How much of each muſt he take?

d.
42 48 | 10 buſhels of wheat } the anſwer.
32 | 6 buſhels of rye, }

Ex. 2.

A vintner would mix Malaga at 7*s.* 6*d.* per gallon, with Canary at 6*s.* 9*d.* and white wine at 4*s.* 3*d.*; to ſell it at 5*s.* 2*d.* per gallon. What quantity of each muſt he take?

	d.				
62	90	11	11	qrts. Malaga	anſwer.
	81	11	11	Canary	
	51	19.28	47	w. wine	

Explanation of Ex. 2.

The difference between 62 and 51 is 11, which I ſet againſt 81, and alſo againſt 90. The difference between 62 and 81 is 19, which I place againſt 51. The difference between 62 and 90 is 28, which I alſo ſet againſt 51. Then 19 added to 28 is 47. So the differences, to work by, will be 11, 11, 47.

2 RULE.

In alligation partial, where one of the quantities (to be mixed) is given. Say, by the rule of three;

As the difference ſtanding againſt the price of the given quantity:
To the given quantity : :
So are the ſeveral other differences :
To the reſpective quantities required.

Ex. 3.

I would mix 10 buſhels of wheat at 5*s.* with rye at 3*s.* 6*d.* and barley at 2*s.* 4*d.*; to be ſold at 4*s.* per buſhel. How much rye and barley muſt I take?

48	wheat	60	6.20	26
	rye	42	12	12
	barley	28	12	12

Then 26 : 10 : : 12 : 4$\frac{8}{13}$ buſhels of rye and of barley.

Ex. 4.

How much Malaga at 7*s.* 6*d.* the gallon, ſherry at 5*s.* white wine at 4*s.* 3*d.* muſt be mixt with 24 gallons of Canary at 6*s.* 9*d.*; that the whole may be ſold for 6*s.* per gallon?

72	Malaga 90	12
	Canary 81	21
	ſherry 60	18
	w. wine 51	9

Or thus.

72	Malaga 90	21
	Canary 81	12
	ſherry 60	9
	w. wine 51	18

&c.

Then the quantity of Canary being given, ſay by the firſt method, 21 : 24 :: ſo is each difference : to its reſpective quantity; that is,

As 7 : 8 :: { 12 : $13\frac{5}{7}$ gal. Malaga
18 : $20\frac{4}{7}$ ſherry
9 : $10\frac{2}{7}$ w. wine } anſwer.

Or thus, by the latter method.

As 12 : 24 :: { 21 : 42 gal. Malaga.
that is, 1 : 2 :: 9 : 18 ſherry.
18 : 36 w. wine.

3 RULE.

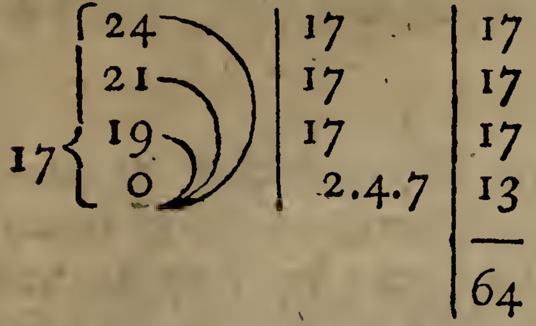
In alligation total, where the total ſum of the quantities (to be mixt) is given; add up all the differences together, then ſay by the rule of three,

As the ſum of the differences :
To the quantity given ::
So every particular difference :
To its reſpective quantity.

Ex. 5.

A goldſmith would mix gold of 24 carracts, with ſome of 21 carracts, and with ſome other of 19 carracts

racts fine, and with a due quantity of allay; ſo that 190 ounces might bear 17 carracts fine. How much of each ſort muſt he take?



here allay is to be reckoned 0 carracts.

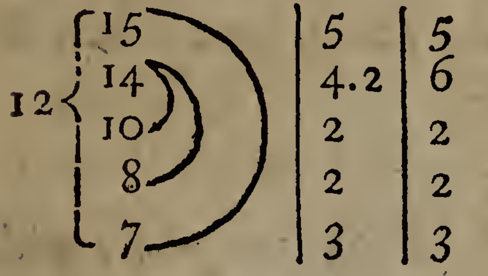
$$\text{Then } 64 : 190 :: \left\{ \begin{array}{l} 17 : \overset{oz.}{50\tfrac{15}{32}} \text{ of the 3 ſorts of gold.} \\ 13 : 38\tfrac{19}{32} \text{ of allay.} \end{array} \right.$$

Ex. 6.

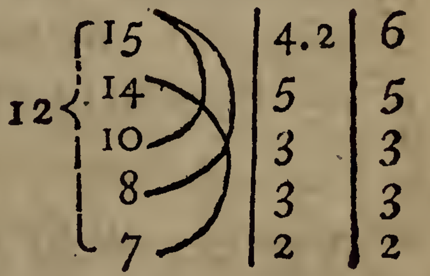
A mixture of wine is to be made up conſiſting of 130 quarts, from theſe five ſorts, whoſe prices are 7*d.*, 8*d.*, 10*d.*, 14*d.*, and 15*d.* a quart: and the whole is to be ſold at 12*d.* a quart. Quere, how much of each?

Here being 5 quantities concerned, they will admit of ſeveral alternations.

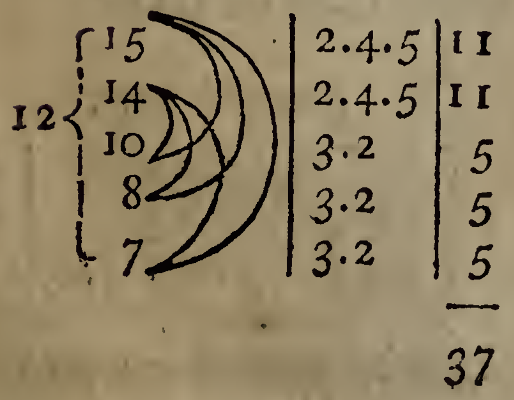
firſt way.



ſecond way.



third way.



&c.

The

The operation, by the laſt way, is thus.

$$37 : 130 :: \left\{ \begin{array}{l} 11 : 38\tfrac{24}{37} \text{ qrts. of wine at } 15d. \text{ and } 14d. \\ 5 : 17\tfrac{21}{37} \text{ quarts, at } 10d., 8d., \text{ and } 7d. \end{array} \right.$$

SCHOLIUM.

Although the ſeveral ways of combining or coupling the rates, as before directed, afford ſo many different ſolutions to the queſtion; yet they do not give all the anſwers the queſtion is capable of. To remedy which, and to make the method more general; you may repeat any two alternate (or correſponding) differences as often as you will; and the like for any other two, &c. This will give a great variety of ſolutions, from which the eaſieſt, and moſt ſuitable may be ſelected. Or rather proceed by the following rule.

4 RULE, UNIVERSALLY.

Having coupled the rates as before directed; then inſtead of any couple of the differences, take any equimultiples thereof; that is, multiply them both by any number you will; do the like for any other couple, &c. By this means, you'll have a new ſet of differences, to work with.

Ex. 7.

A grocer would mix 12 *lb.* of ſugar at 10 *d.*, with two other ſorts of 8 *d.*, and 5 *d.*; ſo that the mixture may be ſold at 7 *d.* How much muſt he take?

common way.

$$7 \left\{ \begin{array}{l} 10 \\ 8 \\ 5 \end{array} \right. \left| \begin{array}{l} 2 \\ 2 \\ 1.3 \end{array} \right| \begin{array}{l} 2 \\ 2 \\ 4 \end{array}$$

general way.

$$7 \left\{ \begin{array}{l} 10 \\ 88 \\ 55 \end{array} \right. \left| \begin{array}{l} 2 \times 2 \\ 2 \times 3 \\ 1 \times 2.3 \times 3 \end{array} \right| \begin{array}{l} 4 \\ 6 \\ 11 \end{array}$$

Here the couple of differences againſt 10 and 5 being 2 and 1, I multiply them both by 2, and they become 4 and 2. Again, the couple againſt 8 and 5, being 2 and 3, I multiply them both by 3, and they become 6 and 9. Then you will have 4, 6, 11 for a new ſet of differences. Therefore

$$4 : 12 :: \left\{ \begin{array}{lll} 6 : 18\,lb. & \text{at} & 8\,d. \\ 11 : 33\,lb. & \text{at} & 5\,d. \end{array} \right.$$

Ex. 8.

A farmer would mix wheat at 4*s.* with rye at 3*s.* and barley at 2*s.* and oats at 1*s.* per buſhel; to have a quantity of 120 buſhels, to be ſold at 2*s.* 4*d.* the buſhel. How much of each muſt he take?

$$28 \left\{ \begin{array}{lr} & d. \\ \text{wheat} & 48 \\ \text{rye} & 36 \\ \text{barley} & 24 \\ \text{oats} & 12 \end{array} \right. \quad \begin{array}{|r|r} \\ 16 \times 3 & 48 \\ 4 \times 5 & 20 \\ 8 \times 5 & 40 \\ 20 \times 3 & 60 \\ \hline & 168 \end{array}$$

$$\text{Then } 168 : 120, \text{ or } 7 : 5 :: \left\{ \begin{array}{lll} 48 : 34\tfrac{2}{7} & \text{buſh.} & \text{wheat.} \\ 20 : 14\tfrac{2}{7} & & \text{rye.} \\ 40 : 28\tfrac{4}{7} & & \text{barley.} \\ 60 : 42\tfrac{6}{7} & & \text{oats.} \end{array} \right.$$

The proof is had by finding the value of the whole mixture at the mean rate; which muſt be equal to the total value of the ſeveral ſimples. And moreover, in *alligation total*, the ſum of the particulars, muſt agree with the ſum given.

PROBLEM IX.

To reſolve a queſtion in the ſingle rule of falſe.

This rule makes a ſingle ſuppoſition of ſome falſe number to reſolve the queſtion, by means whereof the true number or numbers are found out.

RULE.

RULE.

Suppoſe ſome fit number, and proceed with this according to the tenor of the queſtion. Then ſay by the rule of three,

As the falſe number reſulting :
To the true number given : :
So the whole or any part of the falſe number :
To the whole or reſpective part of the number ſought.

Ex. 1.

A man would divide 30 crowns among 3 perſons; ſo that the firſt ſhould have half; the ſecond, a third; and the third, a fourth part. To find each one's ſhare.

Take a number which is diviſible by 2, 3, 4; ſuppoſe 12, then 2) 12 (6 . 3) 12 (4 . 4) 12 (3.

$$\begin{array}{c|c|l} 1 & 6 & \\ 2 & 4 & \text{Then } 13 : 30 :: \left\{\begin{array}{l} 6 : 13\frac{11}{13} \text{ firſt ſhare.} \\ 4 : 9\frac{3}{13} \text{ ſecond ſhare.} \\ 3 : 6\frac{12}{13} \text{ third ſhare.} \end{array}\right. \\ 3 & 3 & \\ \hline & 13. & \end{array}$$

Ex. 2.

A, B, and C buy a parcel of timber, which coſts 48 *l.* and it is agreed that B ſhall pay a third part more than A, and C a fourth more than B. What ſum muſt each pay?

Suppoſe A pays 3, then B pays 4, and C pays 5. But 3 + 4 + 5 = 12, which ſhould be 48. Therefore ſay,

$$\text{As } 12 : 48, \text{ or as } 1 : 4 :: \left\{\begin{array}{l} 3 : 12, \text{ A's ſhare.} \\ 4 : 16, \text{ B's ſhare.} \\ 5 : 20, \text{ C's ſhare.} \end{array}\right.$$

Ex. 3.

There are 3 cocks, A, B, C, belonging to a ciſtern; A can fill it in 1 hour, B in 2, and C in 3. In what time will they all fill it?

Suppoſe

Suppoſe they fill it in half an hour; then ſay,

hour. ciſtern. hour.

As 1 —— 1 —— $\frac{1}{2}$ —— $\frac{1}{2}$ ciſtern for A.

2 —— 1 —— $\frac{1}{2}$ —— $\frac{1}{4}$ ciſtern for B.

3 —— 1 —— $\frac{1}{2}$ —— $\frac{1}{6}$ ciſtern for C.

But $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$ ciſtern, which ſhould be 1 ciſt.

Therefore $\frac{11}{12}$ ciſt. : 1 ciſt. : : $\frac{1}{2}$ hour : $\frac{6}{11}$ hour the time ſought.

The proof of this rule is made, by ſumming up the ſeveral parts, which muſt be equal to the whole.

PROBLEM X.

To reſolve a queſtion in the double rule of falſe.

This rule reſolves queſtions, by making two ſuppoſitions of falſe numbers; by means of which, the true number, which anſwers the queſtion, is found out.

RULE.

1. Take ſome number by gueſs, for a firſt ſuppoſition, and try if it will anſwer the queſtion. If not, ſet the error under it, and mark it with + if it exceeds the truth, or with — if it fall ſhort. Then make a ſecond ſuppoſition with another number, and proceed the ſame way with it. (It is uſual to ſet a croſs between them).

2. Multiply alternately the firſt number by the 2d error, and the 2d number by the 1ſt error. And divide the ſum of the products by the ſum of the errors, when the errors are of different kinds, (that is, when one is greater and the other leſs than the truth;)

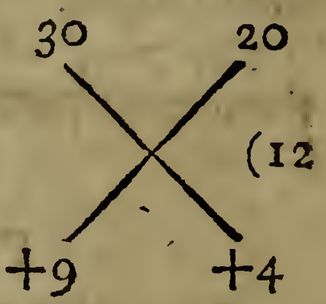
truth;) or the difference of the products by the difference of the errors, when both errors are of one kind; and the quotient is the true number ſought, for which the ſuppoſitions were made.

In ſhort thus, *addito diſſimiles, ſubtrahitoque pares.*

Ex. 1.

A workman agreed to thraſh 60 buſhels of corn, part of it wheat, and part oats; at the rate of 2 *d.* per buſhel for the wheat, and $1\frac{1}{2}$ *d.* for the oats. At laſt he received 8 ſhillings for his labour. How much of each did he thraſh?

1. Firſt, I ſuppoſe there are 30 buſhels of wheat; then there are alſo 30 buſhels of oats.



Price of the wheat	60 pence.
Price of the oats	45 pence.
too much	105
which ſhould be 8 *s.* or	96
1 error	+9

2. Again, I ſuppoſe 20 buſh. of wheat, the pr. 40 *d.*
then there is 40 buſhels of oats, pr. 60

whole price, too much	100
	96
2 error	+4

Then

```
Then  30     20
       4      9
     ----   ----
     120    180
     ----   120
       9    ----
       4  5).60(12 bushels the wheat.
     ----    60| 48 the oats.
       5    ---  --
                 60
                 --
```

Ex. 2.

A man hired a labourer for 40 days, on condition that he ſhould have 20 pence for every day he wrought, and forfeit 10 pence for every day he idled. At laſt he received 41 *s.* 8 *d.* for his labour. How many days did he work, and how many was he idle?

```
1. Suppoſe he wrought 24 days          480 pence.
   then he idled      16               160
                                       ---
                     received but      320
   inſtead of 41 s. 8 d. or            500
                                       ---
                     1 error ſhort    —180
                                       ---

2. Suppoſe he wrt. 32 days  640 pence.
         idled      8        80    24      32
                            ---
         ſhould receive     560              (30
         inſtead of         500
                            ---
    2 error above           +60| —180      +60
                            ---

                                          180
```

180 24
32 60

36 1440
54

180 5760
+60 +1440

240) 7200 (30 days he wrought, conſequently
720 he idled 10 days.

..

Ex. 3.

Two merchants, A, B, lay out an equal ſum of money in trade. A gains 126*l.* and B loſes 87*l.* And A's money is now double to B's. What did each lay out?

1. Suppoſe each lays out 200*l.*
then 200 200
126 87

A's money=326 113=B's money.
226 2

1 error +100 226

200 250
(300
+100 +50

100 25000 10000
50 10000

50) 15000 (300 £. the anſ.
150..

2. Suppoſe each lays out 250*l.*
then 250 250
126 87

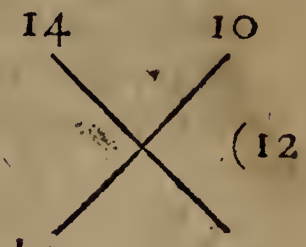
A's money = 376 163 = B's money.
326 2

2 error + 50 326

Ex. 4.

A perſon finding ſeveral beggars at his door, gave each of them 3 pence a-piece, and had 5 pence remaining. He would have given them 4 pence a-piece, but he wanted 7 pence to do it. How many beggars were there?

1. Suppoſe 14 beggars. 14
3 4
42 56
+5 −7
his money=47 49 his mon.
49 —— alſo.
1 error + 2



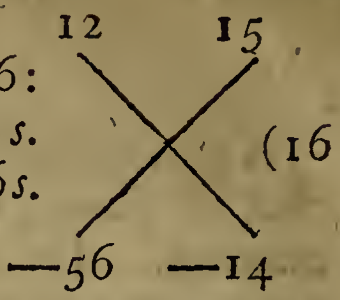
+2 −2
2|28 20
2|20
4)48(12 beggars the anſwer.

2. Suppoſe 10 beggars. 10
3 4
30 40
+5 −7
35 33
33
2 error −2

Ex. 5.

A and B play at cards; A ſtakes B 8*s.* to 6*s.* every game. After 28 games they leave off play, and find that neither of them are winners. How many games did each win?

1. Suppoſe A won 12, then B won 16: and A wins 72*s.* and (B wins 128*s.* or A) loſes 128*s.* that is, he loſes 56*s.* therefore 1ſt error = — 56.



2. Suppoſe A wins 15 games, and B 13, then A wins 90*s.* and loſes 104: ſo the ſecond error is —14.

56|840 168
14|168
—-——
42)672(16 games for A.
42
——
252
252 and 12 for B.
——

2 RULE.

You muſt proceed as directed in the 1ſt rule, till you have found the errors, and their ſigns, then

1. Multiply the difference of the ſuppoſed numbers, by the leaſt error, and divide the product by the difference of the errors, if they are like; or by the ſum if unlike: The quotient is the correction of the number belonging to the leaſt error.

2. Obſerve whether this be the leſſer or greater number, as alſo whether the errors have like or unlike ſigns.

If it is the leſſer number, and like ſigns, ſubtract the correction; if unlike ſigns, add it.

If the greater number, and like ſigns, add the correction; if unlike ſigns, ſubtract it: ſo you'll have the true number required.

Or in other words,

If like ſigns, ſubtract from the leſſer, or add to the greater number.

Unlike ſigns, add to the leſſer, or ſubtract from the greater number; to get the true number.

Ex.

Ex. 6.

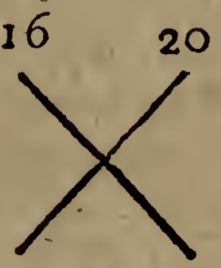
A certain man being aſked what was the age of his four ſons; anſwered, that his eldeſt was 4 years older than the ſecond, and the ſecond 5 years older than the third, and the third 6 years elder than the fourth, which was half the age of the eldeſt. How old was each?

1. Suppoſe 16 for the eldeſt, then
the youngeſt is 1
half the eldeſt 8

1 error —7

2. Suppoſe 20 for the eldeſt,
the youngeſt 5
half the eldeſt 10

2 error —5



20
—7 16 —5

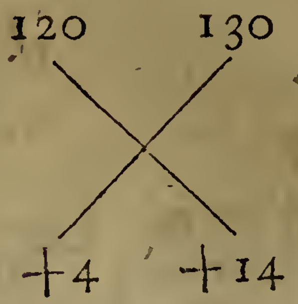
7 4
5 5
(cor.
2) 20 (10 the
20

anſ. 30
the eldeſt.

Ex. 7.

Two perſons diſcourſing of their money; ſays A, if you will give me 25*l.* I ſhall have as much as you; ſays B, if you will give me 22*l.* I ſhall have twice as much as you. How much had each?

	1 *Sup.*	2 *Sup.*
A has	120	130
add	25	25
B has left	145	155
add	25	25
B had at firſt	170	180
add	22	22
B has now	192	202
A has left	98	108
double	196	216
1 error	+4	2 er. +14



130
—120

14 10
—4 4

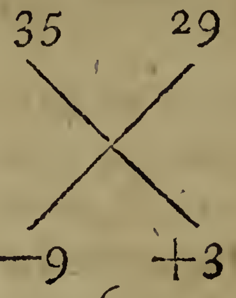
10) 40 (4 cor.

120
—4

116 A's mon.

Ex. 8.

There is a crown weighing 60*lb.* which is made of gold, braſs, tin, and iron. The weight of the gold and the braſs together is 40*lb.* of the gold and tin, 45; of the gold and iron 36. Quere, how much gold was in it?



	1 *Sup.*	2 *Sup.*
Gold	35*lb.*	29*lb.*
Braſs	5	11
Tin	10	16
Iron	1	7
	51	63
	60	60
	1 er. —9.	2 er. +3

9 6
3 3

12) 18 (1½ = cor.
12

6

35
29

6

29
+1½

anſ. 30½ gold.

Ex. 9.

A factor delivers 6 French crowns, and 2 dollars for 45 ſhillings. And at another time 9 French crowns, and 5 dollars for 76 ſhill. What is the value of each?

1. Suppoſe 5*s.* = 1 crown. 2. Suppoſe 7*s.* = 1 crown.

45
(1) $6\times5=30$

2 doll. = 15
1 doll. = $7\frac{1}{2}$

$9\times5=45$
$5\times7\frac{1}{2}=37\frac{1}{2}$

$82\frac{1}{2}$
76

1 error + $6\frac{1}{2}$

45
(2) $6\times7=42$

2 doll. 3
1 doll. = $1\frac{1}{2}$

$9\times7=63$
$5\times1\frac{1}{2}=7\frac{1}{2}$

$70\frac{1}{2}$
76

2 er. —$5\frac{1}{2}$

5 7

$+6\frac{1}{2}$ $-5\frac{1}{2}$

7
5

2
$5\frac{1}{2}$

12)11($\frac{11}{12}$ = cor.
7

a crown = $6\frac{1}{12}$
a dollar = $4\frac{1}{4}$.

Ex. 10.

To find the logarithm of 740326.

5.8694077 5.8694664

—26 +74

1. I ſuppoſe 5.8694077 to be its log.; but by a table of logarithms, it proves only to be the logarithm of 740300.

740326
740300

1 error — 26

2. I ſuppoſe 5.8694664 for the log. but this by the table is the log. of 740400.

740400
740326

2 error +74

5.8694664
5.8694077

.0000587
26

3522
1174

74
26

100).0015262
(152

the

5.8694077
.0000152 cor.

the logarithm ſought 5.8694229

The proof of this rule is, by trying the number found, according to the conditions of the queſtion, in the ſame manner as you find out the errors. And if it agree, the work is right.

SCHOLIUM.

It will ſometimes ſhorten the work, by ſuppoſing one of the numbers 0, and you may ſuppoſe the other 1, if you pleaſe. A great many queſtions may be reſolved by this rule, which cannot be reſolved by any other rules in arithmetic. But there are many queſtions, where it cannot be certainly known, whether they can be reſolved by it or not, till they be tried.

The rule is founded upon this ſuppoſition, that the firſt error is to the ſecond; as the difference between the true and firſt ſuppoſed number, to the difference between the true and ſecond ſuppoſed number. When this does not happen, the rule of falſe does not give the exact anſwer, except the two ſuppoſed numbers be taken very near the true one: as in the laſt example.

In the rule of falſe, whatever operations the queſtion requires to be performed with the number ſought, and any given number or numbers; the ſame operations in every reſpect are to be made with the two ſuppoſed numbers, and the ſame given numbers. From the reſult of theſe three operations, are collected the errors, which are nothing elſe, but the differences between the true reſult, and each of the falſe reſults. Hence if the errors are unlike, the true number lies between the ſuppoſed numbers:

and if the errors are like, the true number lies without them both.

The rule of falſe, eſpecially the latter, will reſolve any the moſt difficult queſtion, by many trials; provided the queſtion can any way be proved, if the true reſolution was given. But then the ſuppoſed numbers muſt be taken near the truth. And after each operation is over, you muſt take the laſt reſult for one of the next ſuppoſed numbers; and the neareſt of the two former (or that with the leaſt error), for the other. And by repeating this proceſs, the anſwer will continually approximate to the true number, within any degree of exactneſs you pleaſe. For this reaſon it is of prodigious ſervice in the abſtruſer parts of the mathematics. For in many difficult problems, there is hardly any other way to come at a ſolution, but by this *method of trial and error*.

PROBLEM XI.

To reſolve a queſtion in the rule of exchange.

When ſeveral different ſorts of things are compared together, as to their value; this rule teaches to find, how many of one ſort is equal to a given number of another ſort.

RULE.

Place the terms in two perpendicular columns, ſo that there may not be found in either column, two terms of one kind. Then the numbers in the leſſer column muſt be multiplied for a diviſor; and the numbers in the greater column, where the odd term is, for a dividend. The quotient is the anſwer.

Note, to abridge the work, throw out any numbers that you can find in both columns.

Ex. 1.

If 6 *lb.* of ſugar be equal in value to 7 *lb.* of raiſons, 5 pound of raiſons to 4 yards of ribbon, 10 yards of ribbon to 40 nutmegs, and 7 nutmegs to 18 pence; what is 3 pound of ſugar worth?

6 ſug.	7 raiſ.	2100) 33600 (16 pence.
5 raiſ.	4 rib.	
10 r.b.	40 nut.	
7 nut.	10 pnce.	
	3 ſug.	
2100		
	33600	

Or thus.

$$\frac{7\times4\times40\times10\times3}{6\times5\times10\times7}=\frac{4\times40\times3}{6\times5}=\frac{4\times8}{2}=16.$$

Ex. 2.

If 3 pair of gloves be worth 2 yards of lace, 3 yards of lace equal to 7 dozen of buttons, 6 dozen of buttons to 2 penknives, and 21 penknives to 18 pair of buckles; how many pair of gloves is equal to 28 pair of buckles?

3 gloves	2 lace	
3 lace	7 buttons	504) 31752 (63.
6 buttons	2 pence	
21 pence	18 buckles	
28 buckles		
	504	
31752		

Or thus.

$$\frac{3\times3\times6\times21\times28}{2\times7\times2\times18}=\frac{3\times21\times28}{2\times7\times2}=3\times21=63.$$

Ex. 3.

If 9 ſhillings Engliſh be equal in value to 2 French crowns, and 1 French crown to 3 livrés, and 4 livrés to 3 guilders, and 9 guilders to 4 rix dollars, and 4 rix dollars to 3 Barcelona ducats; what is 5 Barcelona ducats worth in Engliſh money?

9 ſhil. Eng. = 2 Fr. cr.
1 Fr. cr. 3 liv.
4 liv. 3 guil.
9 guil. 4 rix doll.
4 rix doll. 3 Barc. duc.
5 Barc. duc.

$$\frac{9\times1\times4\times9\times4\times5}{2\times3\times3\times4\times3}$$

$$\frac{9\times4\times9\times5}{2\times3\times3\times3}$$

$$\frac{4\times3\times5}{2}=2\times3\times5$$

= 30 ſhillings.

PROBLEM XII.

To reſolve a queſtion by help of a table of logarithms.

Logarithms are a certain ſet of artificial numbers, fitted to the ſeries of natural numbers, and formed into a table; whoſe property is ſuch, that they perform the ſame thing by addition and ſubtraction, which the natural numbers do by multiplication and diviſion.

A logarithm conſiſts of two parts, a decimal fraction and an integer. The decimal part is always affirmative, the integer may be either affirmative or negative, and is called the *characteriſtic.* It always ſhews how far the firſt figure of the abſolute number is diſtant from the units place. Thus when the characteriſtic is 0, 1, 2, 3, *&c.* the firſt figure of the correſponding number will be units, tens, hundreds, thouſands, *&c.* reſpectively. And if it be —1, —2, —3, *&c.* then the firſt figure of the number belonging, is in the firſt, ſecond, third, *&c.* place of decimals.

In

In many tables, the characteriſtic is not ſet down, becauſe it is eaſily ſupplied, for any given number, from the rule before mentioned; by only conſidering how many places of integers, &c.; the given number conſiſts of.

Though the decimal part of the log. is always affirmative, yet in ſome particular caſes, where the characteriſtic is negative, it is neceſſary to reduce it to another form, where the whole is negative. Thus the log. —2.3406424 which ſignifies the ſame as —2.+.3406424, is reduced to —1.—.6593576, or —1.6593576, where the whole is negative; which is done by ſubtracting the decimal from 1. But when the operation is over, it muſt be reduced to its original form. Or it may be otherways reduced ſo as to be expreſſed in two parts, without making the decimal negative, by adding equal numbers to both the negative and affirmative part. Thus —2.3406424 is equivalent to —3.+1.3406424, or = —4.+2.3406424 = —5.+3.3406424, &c. where the latter part is entirely affirmative: and this way is more commodious for ſome ſort of operations.

Having a number given to find its log. and the contrary. Look through the column of numbers, till you find the given number, againſt this is its logarithm. Or when the log. is given, look through the column of logarithms till you find it, or the neareſt thereto, and againſt it is the number. Thus if the number is 2191, the log. is 3.3406424. And if the log. be 2.8241900, the number is 667.1; and ſo of others. But if the number exceed the table, that is, if it conſiſts of more than 4 places, proceed as in Ex. 10. Prob. 10, to find the log. or the contrary.

The table of logarithms is too large for this book, its principal uſe being in trigonometrical operations. See my Trigonometry, Edit. 2.

1 RULE.

After the queſtion is reſolved in form, and the numbers are ready for operation. To find the product of any numbers multiplied together. Set down all the numbers and their logarithms againſt them; then add all the logarithms together. When you come at the characteriſtics, add what you carried, to the affirmatives, and take the difference between the ſum of the affirmatives, and the ſum of the negatives, and ſet it down with the ſign of the greater. This is the characteriſtic of the product; whoſe number muſt be found in the table.

Ex. 1.

What is the product of 37 × 250?

	37	1.5682017
	250	2.3979400
prod.	9250	3.9661417

Ex. 2.

What is the product of 7 × 486 × .0042?

	7	0.8450980
	486	2.6866363
	.0042	—3.6232493
prod. near	14.29	+1.1549836

2 RULE.

When a quantity appears in form of a fraction, to find the quotient ariſing by dividing the numerator by the denominator. Subtract the log. of the denominator from the log. of the numerator. If you carry 1, add it to the lower charact. if +, or ſubtract it, if —; which done, if the charact. have unlike ſigns, add them with the ſign of the upper; if like ſigns,

ſigns, ſubtract with the ſame ſign; except the lower be the greater, and then with a contrary ſign.

If either numerator or denominator is any product of certain numbers, its log. muſt be found by Rule 1.

Ex. 3.

What is the value of $\frac{438}{73}$?

	438	- -	2.6414741
	73	- -	1.8633229
quotient	6	- -	60.7781512

Ex. 4.

Divide 125 by 3125.

	125	- -	2.0969100
	3125	- -	3.4948500
quotient	.04	- -	—2.6020600

Ex. 5.

Divide 342 by .035.

	342	- -	2.5340261
	.035	- -	—2.5440680
quot.	9771	- -	3.9899581

Ex. 6.

What is the value of $\frac{.54 \times .0157}{48}$?

	.54	- -	—1.7323938
	.0157	- -	—2.1958996
product		- -	—3.9282934
	48	- -	1.6812412
quot.	0001766	- -	—4.2470522

3 RULE.

When a number is to be ſquared, cubed, *&c.* multiply its log. by the index of the power. Obſerving, when the characteriſtic is negative, to ſubtract what you carry thither. Then find the number anſwering.

Ex. 7.

What is the ſquare root of 2 ?

426 - - 2.6294096
2

ſquare 181500 - - 5.2588192

Ex. 8.

What is the cube of .405 ?

.405 - - —1.66074550
3

cube .06643 - - —2.8223650

Ex. 9.

To find the 4th power of .09054.

.09054 - - —2.956840445

4th power .0000672 - - —5.8273620

4 RULE.

When any root is to be extracted; divide the log. of the number by the index of the root. Remembring to reduce the log. if the characteriſtic be negative, when there is occaſion.

Ex. 10.

What is the ſquare root of 2?

2 - - - 2) 0.3010300
root 1.414 - - 0.1505150

Ex. 11.

Find the ſquare root of 4823.

4823 - - 2)3.6833173
root 69.45 - - 1.8416586

Ex. 12.

What is the cube root of .005832?

.005832 - - 3)—3.7658175
root .18 - - - —1.2552725

Ex. 13.

To find the cube root of .02456.

.02456 - - —2.3902284
reduced - - 3)—1.6097716 all neg.
reduce this back —0.5365905
root .2907 - - —1.4634095

Or thus.

The log. —2.3902284 is equal to —3.+1.3902284
3)—3.+1.3902284
root .2907 - - —1. 4634095

Ex. 14.

What is the 5th root of .004705?

.004705 - - —3.6725596
reduced to 5)—5+2.6725596
root .3424 - - —1.5345119

---

5 RULE.

When in the ſolution of a queſtion, you come at ſome compound quantity, conſiſting of products, powers, roots, &c. connected by the ſigns + and —; they muſt be wrought ſeparately by the foregoing rules, and the numbers found and collected, according to the ſigns.

Ex. 15.

To find the number expreſſed by this quantity.

$$\frac{350\times20\times11-108\times13^2}{11\times13\times15}.$$

This is the ſame as the two quantities $\frac{350\times20\times11}{11\times13\times15} - \frac{108\times13\times13}{11\times13\times15}$. That is $\frac{350\times20}{13\times15} - \frac{108\times13}{11\times15}$.

350	2.5440680	13	1.1139433
20	1.3010300	15	1.1760913
	3.8450980		2.2900346
ſubt.	2.2900346		
35.90	1.5550634		

the firſt part.

108	2.0334238	11	1.0413927
13	1.1139433	15	1.1760913
	3.1473671		2.2174840
ſubt.	2.2174840		
8.509	0.9298831		

the ſecond part. Then from 35.900
take 8.509

the number ſought, 27.391

Ex. 16.

Suppoſe in a certain queſtion, I come to this concluſion for the number ſought, $\frac{12 \times 37 \times 20 + \overline{25}^{3}}{37 \times 25 - 12 \sqrt{\overline{37 \times 20}}}$, what is the number?

n.	*log.*
12	1.0791812
37	1.5682017
20	1.3010300
	3.9484129
numb.	8880.

37	1.5682015
25	1.3979400
	2.9661417
numb.	925.0

n.	*log.*
25	1.3979400
	3
	4.1938200
numb.	15620

37	1.5682017
20	1.3010300
	2.8692317
half	1.4346158
12	1.0791812
	2.5137970
numb.	326.4

The ſolution becomes $\frac{8880 + 15620}{925.0 - 326.4} = \frac{24500}{598.6}$

log. 4.3891661

log. 2.7771367

1.6120294

the numb. 40.93 anſwer.

PROBLEM XIII.

To reſolve the uſual queſtions about the intereſt of money, and annuities.

Intereſt is the money paid for the uſe or loan of any ſum or principal; and is generally eſtimated at ſo much per hundred for a year, as 4 per cent. 5 per cent.; *&c.* which is called the *rate of intereſt.*

Simple intereſt is that which is charged only upon the principal, for any length of time after it is due.

Compound intereſt, or intereſt upon intereſt, is that which ariſeth from both principal and intereſt; this ſuppoſes that the intereſt itſelf, ſhall alſo gain intereſt, after the time it becomes due.

Rebate is the abatement made by paying a ſum of money before it is due.

Amount is the quantity of money in arrear, conſiſting of the principal or annuity, together with its intereſt, forborn for ſome time after it is due.

Several queſtions in the buſineſs of intereſt being very difficult to reſolve ſolely by arithmetic; I have therefore inſerted the four following tables; by help of which all the common queſtions relating to intereſt and annuities may very ſpeedily be reſolved, for any numbers that come within the reach of theſe tables.

Their uſe is eaſy and evident at ſight: for the rate of intereſt being found at the top, and the time of continuance on the ſide; at the angle of meeting, you have the amount of 1 pound, (Tab. 1 and 3); or of 1 pound annuity (Tab. 2 and 4), at either ſimple or compound intereſt. But their uſefulneſs will more clearly appear from the following rules and examples.

1 RULE.

When the ſimple intereſt for days, is required; divide the rate by 100, to have the rate for 1 *l.* then multiply the principal, the rate for 1 pound, and the number of days, continually; and divide the product by 365; the quotient is the intereſt.

Ex. 1.

What is the intereſt of 160*l.* for 85 days, at 3 per cent.?

$\frac{3}{100} = .03$ the rate of 1*l.*

	£. s. d.
160	365) 408 (1 : 2 : 4¼ anſwer.
.03	365
4.80	43
8 5	20
24 0	860 (2
384	730
408.0	130
	12
	1560 (4.2
	1460
	100
	73

2 RULE.

To find the preſent worth of 1*l.* in money, due any number of years hence; or of 1*l.* annuity to continue any number of years, at a given rate either of ſimple or compound intereſt.

For 1*l. in money.* Look into Tab. I. for ſimple intereſt, or Tab. III. for compound intereſt, and under the given rate, and againſt the number of years, you'll find a number for a diviſor, by this divide 1, the quotient is the preſent worth.

For 1*l. annuity.* Conſult the Tables I. and II. for ſimple intereſt; or III. and IV. for compound intereſt. And under the given rate, and againſt the number of years, in both tables, you'll find two numbers, which take out, and divide the latter by the former, for the preſent worth.

Ex.

Ex. 2.

What is the preſent worth of 1 *l.* due 14 years hence, at 4 per cent. at ſimple or compound intereſt?

Num. Tab. I. - - 1.56) 1.000 (.64102 the preſ. worth at ſimp. inter.
936
640
624
160
156
400

Num. Tab. III. - - 1.73167) 1.000000 (.577476 the preſ. worth at comp. intereſt.
865835
134165
121217
12948
12122
826
693
133
121
12

Ex. 3.

What is the preſent worth of 1 *l.* annuity to continue 14 years, at 5 per cent. ſimple and compound intereſt?

Tab. II. - - / Tab. I. - - - $\frac{18.55}{1.7}$ = preſent worth at ſimp. intereſt.

That is, 1.7) 18.55 (10.91176 the preſent worth at ſimple intereſt.
17 . .
155
153
20
17
30
17
130
119
11

Tab. IV. - - / Tab. III. - - $\frac{19.59863}{1.97993}$ = preſent worth at comp. inter.

That is, 1.97993) 19.59863 (9.89865 the preſ. worth at compound intereſt.
17 81937
1 77926
1 58394
19532
17819
1713
1584
129
118
11

3 RULE.

3 RULE.

Queſtions, where principal, annuity, amount, &c. are concerned, are likewiſe to be ſolved by the tables. For there are ſimilar numbers in the tables analogous to thoſe given; and therefore having three terms given, a proportion or analogy muſt be made by the rule of three, between the numbers given in the queſtion, and thoſe in the proper table, for the ſame rate and time, in order to find the 4th term, which is either the thing itſelf which is ſought, or it will ſhew it by the table. And as 1 is commonly a term in the proportion, the queſtion will generally be ſolved by multiplication or diviſion.

If any thing is wanting to make the proportion, or to carry on the proceſs, it muſt be found from what is given in the queſtion.

Ex. 4.

If 250*l.* be put out to intereſt, what will it amount to in 21 years, at 4*l.* per cent. ſimple or compound intereſt?

By Tab. I. the amount of 1*l.* for 21 years, at 4 per cent. is 1.84; therefore ſay, as 1 *(principal)* : 1.84 *(amount)* : : 250 *(principal)* : 1.84 x 250 = 460, the amount required, at ſimple intereſt.

Again, by Tab. III. the amount of 1*l.* is 2.27877; Therefore ſay, as 1 *(pr.)* : 2.27877 *(am.)* : : 250 *(pr.)* : 2.27877 x 250 = 569.6925*l.* the amount required, at compound intereſt.

Ex. 5.

What principal put out for 21 years will amount to 460*l.* at 4 per cent. ſimple intereſt?

By Tab. I. the amount of 1*l.* is 1.84 for the given time and rate; then ſay, 1.84 *am.*—1 *pr.*—460 *am.*— $\frac{460}{1.84} = 250$*l.* the principal ſought.

Ex. 6.

In what time will 250*l.* amount to 569.6925*l.* being put out at 4 per cent. compound intereſt?

Say, as 250 *pr.* : 569.6925 *am.* : : 1 *pr.* : $\frac{569.6925}{250}$ = 2.27877 the amount of 1*l.* Seek this number in Tab. III. col. 4 per C. and you'll find it againſt 21 years, the time ſought.

Ex. 7.

At what rate of ſimple intereſt will 250*l.* amount to 460*l.* in 21 years?

By Tab. I. ſay, 250 *pr.* — 460 *am.* — 1 *pr.* — $\frac{460}{250}$ = 1.84, the amount of 1*l.*; which being ſought for againſt 21 years, will fall in col. 4 per C. the rate of intereſt required.

Ex. 8.

If 320*l.* yearly rent be forborn for 12 years, what will be in arrear at that time, at 4½ per cent. ſimple and compound intereſt?

By Tab. II. the amount of 1*l.* annuity for 12 years is 14.97; then ſay, 1 *an.* — 14.97 *am.* — 320 *an.* — 14.97 × 320 = 4790.4*l.* the arrear ſought, at ſimple intereſt.

Again, by Tab. IV. the amount of 1*l.* annuity is 15.46403; therefore ſay, as 1 *rent* — 15.46403 *am.* — 320*r.* — 15.46403 × 320 = 4948.49*l.* the amount, at compound intereſt.

Ex. 9.

What yearly rent being forborn 12 years, will amount to 4948.49, at 4½ per cent. comp. intereſt?

By Tab. IV. the amount of 1*l.* annuity is 15.46403; then ſay, as 15.46403 *am.* — 1 *r.* — 4948.49 *am.* — $\frac{4948.49}{15.46403}$ = 320*l.* the rent ſought

Ex. 10.

In what time will 320*l.* yearly rent, amount to 4790.4*l.* at $4\frac{1}{2}$ per cent. ſimple intereſt?

Say, 320 *rent* — 4790.4 *am.* — 1 *rent* — $\frac{4790.4}{320}$ = 14.97, the amount of 1 *l.* annuity; which being found in col. $4\frac{1}{2}$ per C. Tab. II. ſtands over-againſt 12 years, the time ſought.

Ex. 11.

At what rate of compound intereſt, does 320*l.* rent, amount to 4948.49*l.* in 12 years?

Say, as 320 *rent* — 4948.49 *am.* — 1 *rent* — $\frac{4948.49}{320}$ = 15.46403 the amount of 1 *l.* annual rent. Seek this number over-againſt 12 years in Tab. IV. and it is found under $4\frac{1}{2}$ per C. the rate ſought.

Ex. 12.

What is the preſent worth of 65*l.* a year, to continue 40 years, at 5 per cent. ſimple and compound intereſt?

By Rule 2, find the preſent worth of 1 *l.* annuity at ſimple intereſt, for the time and rate given, which is $\frac{79}{3}$; then ſay,

As 1 *an.* — $\frac{79}{3}$ *pr.* — 65 *an.* — $\frac{65 \times 79}{3}$ = 1711.66 the preſent worth ſought, at ſimple intereſt.

Again, by Rule 2, find the preſent worth of 1 *l.* annuity at compound intereſt, which is $\frac{120.79977}{7.03999}$; then ſay,

1 *an.* — $\frac{120.7}{7.0}$&c. *pr.* — 65 *an.* — $\frac{120.79977 \times 65}{7.03999}$ = 1115.34, the preſent worth ſought, at comp. intereſt.

Ex. 13.

What annuity to continue 40 years, will 1711.66 *l.* ready money purchaſe, at 5 per cent. ſimple intereſt?

By Rule 2, find the preſent worth of 1 *l.* annuity, which is $\frac{79}{3}$; then ſay, $\frac{79}{3}$ *pr.*—1 *an.*—1711.66 *pr.*—$\frac{3\times1711.66}{79}$ = 65 *l.* the annuity required.

Ex. 14.

How long may one have a leaſe of 65 *l.* a year, for 1711.66 *l.* ready money, at 5 per cent. ſimple intereſt?

Say, as 65 *rent* — 1711.66 *pr.*— 1 *rent* — $\frac{1711.66}{65}$ = 26.33, the preſent worth of 1 *l.* annuity, for an unknown time. Then,

Take ſome year by gueſs, and find the amount by Tab. II. and the preſent worth of that amount, by Tab. I. If this agrees not with 26.33, try again, and by a few eaſy trials you'll come to the truth.

In ſhort thus, ſet down the correſpondent numbers in Tab. II. and I. fractionwiſe, to approach continually to 26.33, which at laſt you'll obtain.

Suppoſe 30 years - - $\frac{51.75}{2.5}$ = 20. &*c.* too little.

38 years - - $\frac{73.15}{2.9}$ = 25.2 &*c.* too little.

40 years - - $\frac{79}{3}$ = 26.33 juſt. So 40 years is the time required.

Ex. 15.

If one give 1115.34 *l.* ready money, for the purchaſe of an annuity of 65 *l.* a year, to continue 40 years; what is the rate at compound intereſt?

Say, as 65 *an.* — 1115.34 *pr.* — 1 *an.* — $\frac{1115.34}{65}$ = 17.159, the preſent worth of 1 *l.* annuity, at an unknown rate.

Take ſome rate of intereſt by gueſs, and find the amount for 40 years by Tab. IV; and the preſent worth of that amount by Tab. III. repeat this work with other rates, till the reſult be 17.159.

Or in ſhort thus, ſet down the correſpondent numbers in Tab. IV. and III. fractionwiſe, and you will approach to the rate ſought by a few trials. Thus,

Suppoſe 3 per cent. - - $\frac{75.4}{3.2}$ = 23, too great.

4 per cent. - - $\frac{95.0}{4.8}$ = 19.8, too great.

5 per cent. - - $\frac{120.799}{7.0399}$ = 17.159, juſt.

Therefore 5 per cent. is the rate required.

4 RULE.

When freehold eſtates are to be valued; divide 1 by the rate of 1 *l.* the quotient ſhows how many years purchaſe it is worth, at compound intereſt.

Or if the annuity or rent be required; multiply the purchaſe money by the rate of 1 *l.* for the annuity.

Ex. 16.

What is an eſtate at 30*l.* a year worth, at $3\frac{1}{2}$ per cent.?

Here $\frac{1}{.035}$ = 28.571 years purchaſe.

Or 28.571 × 30 = 857.13*l.* the purchaſe money.

Ex. 17.

What annuity can I buy for 857.13*l.* at $3\frac{1}{2}$ per cent.?

Here 857.13 × .035 = 29.999*l.* or 30*l.* the annuity.

5 RULE.

5 RULE.

When ſeveral ſums of money are out at ſimple intereſt, and are to be paid in, at different times; to find the time, when the whole may be paid in at once, without loſs to the debtor or creditor.

Multiply every ſum of money by the time it is to continue; and divide the ſum of the products, by the total ſum of all the money, the quotient will be the mean time of payment.

And the ſame rule holds true, very near; when ſeveral ſums of money are due at different times, only it makes the mean time a ſmall matter too big.

Ex. 18.

I have three ſums of money let out to intereſt, for different times; *viz*, 50*l.* continues for 2 years, 40*l.* for $3\frac{1}{2}$ years, and 20*l*, for $4\frac{1}{2}$ years. But it is now agreed, that they ſhall be all paid at once. The queſtion is, when muſt I receive the whole together?

50	40	20	50	100	
2	$3\frac{1}{2}$	$4\frac{1}{2}$	40	140	
100	120	80	20	90	
	20	10	110)	330	(3 years; anſwer.
	140	90		330	

Ex. 19:

A man has three ſeveral ſums of money due at different times, 50*l.* at the end of 5 months, 84*l.* at the end of 10 months, and 36*l*, a year and half hence. But he would receive them all at once; in what time ſhall he receive the whole ſum?

```
 50    84    38          50    250
  5    10    18          84    840
---   ---   ---          36    684
250   840   304         ---    ---
---   ---    38         170) 1774(10.43 months, nearly;
            ---                170       the answer.
            684                ---
                                74
                                68
                               ---
                                 60
                                ---
```

The proof, in all queſtions of intereſt, is to change the *data*, and work the queſtion backwards.

SCHOLIUM.

It is contrary to law to let out money at compound intereſt. Yet in the valuation of annuities, it is always the cuſtom to allow compound intereſt; for by ſimple intereſt, they would be overvalued.

TAB. I.

A table of the amount of 1 pound for years, at ſimple intereſt.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1.03	1.035	1.04	1.045	1.05
2	1.06	1.070	1.08	1.090	1.10
3	1.09	1.105	1.12	1.135	1.15
4	1.12	1.140	1.16	1.180	1.20
5	1.15	1.175	1.20	1.225	1.25
6	1.18	1.210	1.24	1.270	1.30
7	1.21	1.245	1.28	1.315	1.35
8	1.24	1.280	1.32	1.360	1.40
9	1.27	1.315	1.36	1.405	1.45
10	1.30	1.350	1.40	1.450	1.50
11	1.33	1.385	1.44	1.495	1.55
12	1.36	1.420	1.48	1.540	1.60
13	1.39	1.455	1.52	1.585	1.65
14	1.42	1.490	1.56	1.630	1.70
15	1.45	1.525	1.60	1.675	1.75
16	1.48	1.560	1.64	1.720	1.80
17	1.51	1.595	1.68	1.765	1.85
18	1.54	1.630	1.72	1.810	1.90
19	1.57	1.665	1.76	1.855	1.95
20	1.60	1.700	1.80	1.900	2.00
21	1.63	1.735	1.84	1.945	2.05
22	1.66	1.770	1.88	1.990	2.10
23	1.69	1.805	1.92	2.035	2.15
24	1.72	1.840	1.96	2.080	2.20
25	1.75	1.875	2.00	2.125	2.25
26	1.78	1.910	2.04	2.170	2.30
27	1.81	1.945	2.08	2.215	2.35
28	1.84	1.980	2.12	2.260	2.40
29	1.87	2.015	2.16	2.305	2.45
30	1.90	2.050	2.20	2.350	2.50

Tab. I.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
31	1.93	2.085	2.24	2.395	2.55
32	1.96	2.120	2.28	2.440	2.60
33	1.99	2.155	2.32	2.485	2.65
34	2.02	2.190	2.36	2.530	2.70
35	2.05	2.225	2.40	2.575	2.75
36	2.08	2.260	2.44	2.620	2.80
37	2.11	2.295	2.48	2.665	2.85
38	2.14	2.330	2.52	2.710	2.90
39	2.17	2.365	2.56	2.755	2.95
40	2.20	2.400	2.60	2.800	3.00
41	2.23	2.435	2.64	2.845	3.05
42	2.26	2.470	2.68	2.890	3.10
43	2.29	2.505	2.72	2.935	3.15
44	2.32	2.540	2.76	2.980	3.20
45	2.35	2.575	2.80	3.025	3.25
46	2.38	2.610	2.84	3.070	3.30
47	2.41	2.645	2.88	3.115	3.35
48	2.44	2.680	2.92	3.160	3.40
49	2.47	2.715	2.96	3.205	3.45
50	2.50	2.750	3.00	3.250	3.50
51	2.53	2.785	3.04	3.295	3.55
52	2.56	2.820	3.08	3.340	3.60
53	2.59	2.855	3.12	3.385	3.65
54	2.62	2.890	3.16	3.430	3.70
55	2.65	2.925	3.20	3.475	3.75
56	2.68	2.960	3.24	3.520	3.80
57	2.71	2.995	3.28	3.565	3.85
58	2.74	3.030	3.32	3.610	3.90
59	2.77	3.065	3.36	3.655	3.95
60	2.80	3.100	3.40	3.700	4.00

TAB. II.

A table of the amount of 1 pound annuity for years, at ſimple intereſt.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1.00	1.000	1.00	1.000	1.00
2	2.03	2.035	2.04	2.045	2.05
3	3.09	3.105	3.12	3.135	3.15
4	4.18	4.210	4.24	4.270	4.30
5	5.30	5.350	5.40	5.450	5.50
6	6.45	6.525	6.60	6.675	6.75
7	7.63	7.735	7.84	7.945	8.05
8	8.84	8.980	9.12	9.260	9.40
9	10.08	10.260	10.44	10.620	10.80
10	11.35	11.575	11.80	12.025	12.25
11	12.65	12.925	13.20	13.475	13.75
12	13.98	14.310	14.64	14.970	15.30
13	15.34	15.730	16.12	16.510	16.90
14	16.73	17.185	17.64	18.095	18.55
15	18.15	18.675	19.20	19.725	20.25
16	19.60	20.200	20.80	21.400	22.00
17	21.08	21.760	22.44	23.120	23.80
18	22.59	23.355	24.12	24.885	25.65
19	24.13	24.985	25.84	26.695	27.55
20	25.70	26.650	27.60	28.550	29.50
21	27.30	28.350	29.40	30.450	31.50
22	28.93	30.085	31.24	32.395	33.55
23	30.59	31.855	33.12	34.385	35.65
24	32.28	33.660	35.04	36.420	37.80
25	34.00	35.500	37.00	38.500	40.00
26	35.75	37.375	39.00	40.625	42.25
27	37.53	39.285	41.04	42.795	44.55
28	39.34	41.230	43.12	45.010	46.90
29	41.18	43.210	45.24	47.270	49.30
30	43.05	45.225	47.40	49.575	51.75

Tab. II.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
31	44.95	47.275	49.60	51.925	54.25
32	46.88	49.360	51.84	54.320	56.80
33	48.84	51.480	54.12	56.760	59.40
34	50.83	53.635	56.44	59.245	62.05
35	52.85	55.825	58.80	61.775	64.75
36	54.90	58.050	61.20	64.350	67.50
37	56.98	60.310	63.64	66.970	70.30
38	59.09	62.605	66.12	69.635	73.15
39	61.23	64.935	68.64	72.345	76.05
40	63.40	67.300	71.20	75.100	79.00
41	65.60	69.700	73.80	77.900	82.00
42	67.83	72.135	76.44	80.745	85.05
43	70.09	74.605	79.12	83.635	88.15
44	72.38	77.110	81.84	86.570	91.30
45	74.70	79.650	84.60	89.550	94.50
46	77.05	82.225	87.40	92.575	97.75
47	79.43	84.835	90.24	95.645	101.05
48	81.84	87.480	93.12	98.760	104.40
49	84.28	90.160	96.04	101.920	107.80
50	86.75	92.875	99.00	105.125	111.25
51	89.25	95.625	102.00	108.375	114.75
52	91.78	98.410	105.04	111.670	118.30
53	94.34	101.230	108.12	115.010	121.90
54	96.93	104.085	111.24	118.395	125.55
55	99.55	106.975	114.40	121.825	129.25
56	102.20	109.900	117.60	125.300	133.00
57	104.88	112.860	120.84	128.820	136.80
58	107.59	115.855	124.12	132.385	140.65
59	110.33	118.885	127.44	135.995	144.55
60	113.10	121.950	130.80	139.650	148.50

TAB. III.

A table of the amount of 1 pound for years, at compound intereſt.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1.03000	1.03500	1.04000	1.04500	1.05000
2	1.06090	1.07122	1.08160	1.09202	1.10250
3	1.09273	1.10872	1.12486	1.14116	1.15762
4	1.12551	1.14752	1.16986	1.19252	1.21550
5	1.15927	1.18769	1.21665	1.24618	1.27628
6	1.19405	1.22925	1.26532	1.30226	1.34009
7	1.22987	1.27228	1.31593	1.36086	1.40710
8	1.26677	1.31681	1.36857	1.42210	1.47745
9	1.30477	1.36290	1.42331	1.48609	1.55132
10	1.34391	1.41060	1.48024	1.55297	1.62889
11	1.38423	1.45997	1.53945	1.62285	1.71034
12	1.42576	1.51107	1.60103	1.69588	1.79585
13	1.46853	1.56395	1.66507	1.77219	1.88565
14	1.51259	1.61869	1.73167	1.85194	1.97993
15	1.55797	1.67535	1.80094	1.93528	2.07893
16	1.60470	1.73398	1.87298	2.02237	2.18287
17	1.65285	1.79467	1.94790	2.11338	2.29202
18	1.70243	1.85749	2.02582	2.20848	2.40662
19	1.75350	1.92250	2.10685	2.30786	2.52695
20	1.80611	1.98979	2.19112	2.41171	2.65330
21	1.86029	2.05943	2.27877	2.52024	2.78596
22	1.91610	2.13151	2.36992	2.63365	2.92526
23	1.97359	2.20611	2.46471	2.75216	3.07152
24	2.03279	2.28333	2.56330	2.87601	3.22510
25	2.09378	2.36324	2.66583	3.00543	3.38635
26	2.15659	2.44596	2.77247	3.14068	3.55567
27	2.22129	2.53157	2.88337	3.28201	3.73345
28	2.28793	2.62017	2.99870	3.42970	3.92013
29	2.35656	2.71188	3.11865	3.58403	4.11613
30	2.42726	2.80679	3.24340	3.74532	4.32194

TAB. III.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
31	2.50008	2.90503	3.37313	3.91386	4.53804
32	2.57508	3.00671	3.50806	4.08998	4.76494
33	2.65233	3.11194	3.64838	4.27403	5.00319
34	2.73190	3.22086	3.79431	4.46636	5.25335
35	2.81386	3.33359	3.94609	4.66735	5.51601
36	2.89828	3.45026	4.10393	4.87738	5.79181
37	2.98523	3.57102	4.26809	5.09686	6.08141
38	3.07478	3.69601	4.43881	5.32622	6.38548
39	3.16703	3.82537	4.61636	5.56590	6.70475
40	3.26204	3.95926	4.80102	5.81636	7.03999
41	3.35990	4.09783	4.99306	6.07810	7.39199
42	3.46069	4.24126	5.19278	6.35161	7.76159
43	3.56452	4.38970	5.40049	6.63744	8.14967
44	3.67145	4.54334	5.61651	6.93612	8.55715
45	3.78159	4.70236	5.84117	7.24825	8.98501
46	3.89504	4.86694	6.07482	7.57442	9.43426
47	4.01189	5.03728	6.31781	7.91527	9.90597
48	4.13225	5.21359	6.57053	8.27145	10.40127
49	4.25622	5.39606	6.83335	8.64367	10.92133
50	4.38390	5.58492	7.10668	9.03263	11.46740
51	4.51542	5.78040	7.39095	9.43910	12.04077
52	4.65088	5.98271	7.68659	9.86386	12.64281
53	4.79041	6.19211	7.99405	10.30774	13.27495
54	4.93412	6.40883	8.31381	10.77158	13.93869
55	5.08215	6.63314	8.64637	11.25631	14.63563
56	5.23461	6.86530	8.99222	11.76284	15.36741
57	5.39165	7.10558	9.35191	12.29217	16.13578
58	5.55340	7.35428	9.72599	12.84532	16.94257
59	5.72000	7.61168	10.11502	13.42335	17.78970
60	5.89160	7.87809	10.51963	14.02741	18.67918

TAB. IV.

A table of the amount of 1 pound annuity for years, at compound intereſt.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.03000	2.03500	2.04000	2.04500	2.05000
3	3.09090	3.10622	3.12160	3.13702	3.15250
4	4.18363	4.21494	4.24646	4.27819	4.31012
5	5.30913	5.36246	5.41632	5.47071	5.52563
6	6.46841	6.55015	6.63297	6.71689	6.80191
7	7.66242	7.77941	7.89829	8.01915	8.14201
8	8.89233	9.05169	9.21422	9.38001	9.54911
9	10.15910	10.36849	10.58279	10.80211	11.02656
10	11.46388	11.73139	12.00611	12.28821	12.57789
11	12.80779	13.14199	13.48635	13.84118	14.20679
12	14.19203	14.60196	15.02580	15.46403	15.91713
13	15.61779	16.11303	16.62684	17.15991	17.71298
14	17.08632	17.67698	18.29191	18.93211	19.59863
15	18.59891	19.29568	20.02359	20.78405	21.57856
16	20.15688	20.97103	21.82453	22.71934	23.65749
17	21.76159	22.70501	23.69751	24.74171	25.84036
18	23.41443	24.49969	25.64541	26.85508	28.13238
19	25.11687	26.35718	27.67123	29.06356	30.53900
20	26.87037	28.27968	29.77808	31.37142	33.06595
21	28.67648	30.26947	31.96920	33.78314	35.71925
22	30.53678	32.32890	34.24797	36.30338	38.50521
23	32.45288	34.46041	36.61789	38.93703	41.43047
24	34.42647	36.66653	39.08260	41.68919	44.50200
25	36.45926	38.94986	41.64591	44.56521	47.72710
26	38.55304	41.31310	44.31174	47.57064	51.11345
27	40.70963	43.75906	47.08421	50.71132	54.66912
28	42.93092	46.29063	49.96758	53.99333	58.40258
29	45.21885	48.91080	52.96628	57.42303	62.32271
30	47.57541	51.62268	56.08494	61.00707	66.43885

Tab. IV.

Yea.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
31	50.00268	54.42947	59.32833	64.75239	70.76079
32	52.50276	57.33450	62.70147	68.66624	75.29883
33	55.07784	60.34121	66.20953	72.75622	80.06377
34	57.73018	63.45315	69.85791	77.03025	85.06696
35	60.46208	66.67401	73.65222	81.49662	90.32031
36	63.27594	70.00760	77.59831	86.16396	95.83632
37	66.17422	73.45787	81.70224	91.04134	101.62814
38	69.15945	77.02889	85.97033	96.13820	107.70954
39	72.23423	80.72490	90.40915	101.46442	114.09502
40	75.40126	84.55028	95.02551	107.03032	120.79977
41	78.66330	88.50954	99.82653	112.84669	127.83976
42	82.02320	92.60737	104.81960	118.92479	135.23175
43	85.48389	96.84863	110.01238	125.27640	142.99334
44	89.04841	101.23833	115.41288	131.91384	151.14300
45	92.71986	105.78167	121.02939	138.84996	159.70015
46	96.50146	110.48403	126.87057	146.09821	168.68516
47	100.39650	115.35097	132.94539	153.67263	178.11942
48	104.40839	120.38826	139.26320	161.58790	188.02539
49	108.54065	125.60184	145.83373	169.85936	198.42066
50	112.79687	130.99790	152.66708	178.50303	209.34799
51	117.18077	136.58284	159.77377	187.53566	220.81539
52	121.69620	142.36323	167.16472	196.97477	232.85616
53	126.34708	148.34595	174.85130	206.83863	245.49897
54	131.13749	154.53806	182.84536	217.14637	258.77392
55	136.07162	160.94689	191.15917	227.91796	272.71262
56	141.15377	167.58003	199.80554	239.17427	287.34825
57	146.38838	174.44533	208.79776	250.93710	302.71566
58	151.78003	181.55092	218.14967	263.22928	318.85144
59	157.33343	188.90519	227.87566	276.07460	335.79402
60	163.05344	196.51688	237.99068	289.49795	353.58372

CHAP. V.

A collection of queſtions to exerciſe the ſeveral rules of arithmetic.

Queſt. 1.

A Merchant buys 890*C.* 3*q.* groſs weight of goods, but tare is to be ſubtracted at the rate of 14*lb.* to the hundred of groſs weight, how much neat weight will remain?

Groſs weight is the weight of the goods, together with the cheſt, bag, *&c.*

Tare is the cheſt, bag, but, caſk, *&c.* which contains the goods.

Neat weight is the weight of the goods alone.

$890\frac{3}{4} \times 8 = 7126$ ſtone, and 14*lb.* $=$ 1 ſtone, and 112*lb.* $=$ 8*ſt.*

then 8*ſt.* : 1 *ta.* : : 7126*ſt.* : $\frac{7126}{8} = 89\frac{3}{4}$ ſtone, the tare.

from 7126
take 89¾
remains 7036¼ the neat weight.

Queſt. 2.

A merchant buys 235*lb.* weight of goods, but is to have an additional allowance of 4*lb.* tret for every 100*lb.* weight of goods. Then how much weight does he receive of all?

Tret is the allowance made to the buyer, of ſo much per hundred, *&c.* over and above. And *Cloſ* another allowance of the ſame kind.

to 100
add 4

Say as, 100 : 104 : : 235 : 244.4*lb.* Anſwer.

Queſt. 3.

If 200*lb.* weight of goods coſt 3*l.* at what price muſt a pound be ſold, to gain 10*l.* in the hundred laid out?

100
10

100 : 110 : : 3 : 3.3 advanced price.
200 : 3.3 : : 1 : .0165*l.* the price of 1*lb.*
but .0165*l.* = 3.96 pence, near 4*d.* a pound.

Queſt. 4.

How much ſugar, at 8*d.* a pound, may be bought for 10*C.* weight of tobacco, at 3*l.* the *C.*?

1*C.* : 3*l.* : : 10*C.* : 30*l.* the value of the tobacco.

then, ſince 8*d.* is $\frac{1}{3}$ of a pound,

$\frac{1}{30}$*l.* : 1*lb.* : : 30*l.* : 30 × 30 = 900*lb.* of ſugar.

Queſt. 5.

Two merchants, A and B, *barter* with one another thus, A has 43 yards of broad cloth, worth 9*s.* 2*d.* per yard, but in *barter* he will have 11*s.* a yard. B has ſhaloon, worth 2*s.* per yard, which he charges at 2*s.* 6*d.* How much ſhaloon muſt A receive for his cloth; and what does he gain or loſe by the bargain?

In this queſtion, firſt find what the cloth comes to at the advanced price; then how much ſhaloon, at its advanced price, may be bought for that money; and laſtly the true value of both.

1 *y.* : 11 *s.* : : 43 *y.* : 473 *s.* the price of the cloth.
$2\frac{1}{2}$ *s.* : 1 *y.* : : 473 *s.* $189\frac{1}{5}$ yards of the ſhaloon received.
then 1 *y.* : $9\frac{1}{6}$ *s.* : : 43 *y.* : $394\frac{1}{6}$ = 394 *s.*—2 *d.* the value of the cloth.
and 1 *y.* : 2 *s.* : : $189\frac{1}{5}$ *y.* : $378\frac{2}{5}$ = 378 *s.*—$4\frac{3}{4}$ *d.* the value of the ſhaloon.
diff. 15 *s.*—$9\frac{1}{4}$ *d.*

So A loſes 15 *s.*—$9\frac{1}{4}$ *d.* by the bargain.

Queſt. 6.

A hath 100 pieces of ſilk worth 3 *l.* a-piece; but he charges them at 4 *l.* a-piece, and barters them with B for wool worth 7 *l.*—10 *s.* the C weight. How much wool muſt A receive from B for the ſilk, that both may be equal gainers?

In this queſtion the price of B's wool muſt be advanced in the ſame proportion as A's ſilk.

3 *l.* : 4 *l.* : : $7\frac{1}{2}$ *l.* : 10 *l.* the advanced price of the wool.
then 100 *l.* × 4 = 400 *l.* the value of the ſilk.
10 *l.* : 1 *C.* : : 400 *l.* : 40 *C.* the quantity of wool.

Queſt. 7.

How many ducats, at 5 *s.*—6 *d.* may be had for 250 dollars, at 4 *s.*—3 *d.* a-piece?

66 *d.* = a ducat, 51 *d.* = 1 dollar.
250 × 51 = 12750 *d.* the value of 250 dollars.

$$\frac{12750}{66} = \underline{193\tfrac{2}{11}} \text{ ducats.}$$

Queſt. 8.

A man would exchange 200*l.* for dollars, at 54*d.* ducats at 68*d.* and crowns at 73*d.* and would have 2 ducats and 3 crowns for 1 dollar. How many of each muſt he have?

$$\begin{array}{rrl}
 & 54 = & 1 \text{ dollar} \\
2 \times 68 = & 136 = & 2 \text{ ducats} \\
3 \times 73 = & \underline{219} = & 3 \text{ crowns} \\
 & 409 = & \text{ſum.}
\end{array}
\qquad
\begin{array}{r}
200 \\
\underline{20} \\
4000 \\
\underline{12} \\
48000d.
\end{array}$$

Now it is plain, as oft as 409 is contained in 48000, ſo often 1 dollar, 2 ducats, and 3 crowns muſt be taken.

$$\frac{48000}{409} = 117\tfrac{147}{409} \text{ the dollars.}$$

$$234\tfrac{294}{409} \text{ the ducats.}$$

$$352\tfrac{32}{409} \text{ the crowns.}$$

Queſt. 9.

A man buys 120 ſtaves at 3 a penny, and afterwards 120 more for 2 a penny; how muſt he ſell them out to loſe nothing?

$$\begin{array}{rrl}
3)\ 120 = & 40d. & \text{for the firſt bargain.} \\
2)\ 120 = & 60d. & \text{for the ſecond bargain.} \\
\overline{240} & \overline{100} &
\end{array}$$

100*d.* : 240*ſt.* : : 1*d.* : $2\frac{2}{5}$*ſt.* per penny;
that is, 12 ſtaves for 5 pence.

Queſt. 10.

A tradeſman begins the world with 1000*l.* and finds that he can gain 1000*l.* in 5 years by land trade alone, and that he can gain 1000*l.* in 8 years by ſea trade alone; and likewiſe that he ſpends 1000*l.* in $2\frac{1}{2}$ years by gaming. How long will his eſtate laſt, if he follows all three?

$\frac{1000}{5} = 200$ his gain by land trade in 1 year.

$\frac{1000}{8} = 125$ his gain by ſea trade in 1 year.

325 his whole gain.

$\frac{1000}{2\frac{1}{2}} = 400$ his loſs by gaming in 1 year.

the difference 75 his loſs by all three in 1 year.

then 75*l.* : 1 *y.* : : 1000*l.* : $13\frac{1}{3}$ years his eſtate will laſt.

Queſt. 11.

There were 25 coblers, 20 taylors, 18 weavers, and 12 combers, ſpent 133 ſhillings at a meeting; to which reckoning 5 coblers paid as much as 4 taylors, 12 taylors as much as 9 weavers, and 6 weavers as much as 8 combers; how much did each company pay?

Find 4 numbers by the rule of three to expreſs theſe proportions, as theſe,

coblers,	taylors,	weavers,	combers,
5	4	3	4

that is, 5 coblers paid as much as 4 taylors, or 3 weavers, or 4 combers. Suppoſe each company paid

paid 1 ſhilling, then, by the ſingle rule of falſe,

1 man in each company will pay	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$
which multiply by the number of men	25	20	18	12
produces	5	5	6	3

whoſe ſum is 19; then it will be

$$19 : 133 :: \left\{\begin{array}{l} 5 : 35s.\text{ for the coblers.} \\ 5 : 35\text{————taylors.} \\ 6 : 42\text{————weavers.} \\ 3 : 21\text{————combers.} \end{array}\right.$$

Queſt. 12.

There is an iſland 72 miles about, and two footmen ſet out together to travel round it the ſame way. A travels 9 miles a day, and B 7. To find the time they will be together again.

It is plain A will overtake B when he leads him the circumference of the iſland.

A—— 9
B—— 7

2 miles gained by A in 1 day.

then 2*m.* : 1*d.* :: 72*m.* : 36 days, the Anſwer.

Queſt. 13.

There is an iſland 73 miles round, and 3 footmen all ſtart together, to travel the ſame way about it. A travels 5 miles a day, B 8, and C 10. When will they all come together again?

B —— 8
A —— 5

B gains 3 miles a day of A.

C—10

C—10
A— 5

C gains 5 miles a day of A.

then 3 *m.* : 1 *d.* : : 73 *m.* : $24\frac{1}{3}$ days when A and B [meet,
and 5 : 1 : : 73 : $14\frac{3}{5}$ days when A and C [meet,

Now $24\frac{1}{3}$ days being the period of B's meeting with A, and $14\frac{3}{5}$ days, the period of C's meeting with A; and they can never meet but at the end of theſe periods. Therefore B and C can never both meet with A, but when ſome number of B's periods is equal to ſome number of C's periods. Therefore find two whole numbers which are in the ſame proportion, as $24\frac{1}{3}$ to $14\frac{3}{5}$, which will be 365 and 219. Therefore after 365 of B's periods, or 219 of A's; all three men will meet again, and not before, as 365 and 219 are in their leaſt terms. Therefore the time of meeting is $219 \times 24\frac{1}{3} = 5329$ days.

Queſt. 14.

A clock hath two hands or pointers, the firſt, A, goes round once in 12 hours, the ſecond, B, once in an hour. Now, if they both ſet forward together, in what time will they meet again?

Here A goes only $\frac{1}{12}$ of the circumference in an hour.

And B goes the whole circumference in an hour.
So B gains $\frac{11}{12}$ of A in that time.

Therefore $\frac{11}{12}$C : 1 *h.* : : 1 C : $\frac{12}{11}$*h.* $= 1\frac{1}{11}$*h.* $=$ 1 *h.* : $5\frac{5}{11}$*m.* the Anſwer.

Queſt. 15.

A greyhound is courſing a hare, which is 100 of her leaps before him; and the hare takes 4 leaps for every 3 leaps of the greyhound; but 2 of the greyhound's leaps are equal to 3 of the hare's. How many leaps muſt he take before he catch her?

2 *gr.* : 3 *ha.* : : 3 *gr.* : $4\frac{1}{2}$ hare's leaps = 3 of the greyhound's.

Therefore, for every 3 leaps of the greyhound, the hare loſes $\frac{1}{2}$ of one of hers. Therefore

$\frac{1}{2}$ *h.* : 3 *gr.* : : 100 *l.* : 600 of the greyhound's leaps; the Anſwer.

Queſt. 16.

Four merchants, A, B, C, D, gain 2000 *l.* by trade, whereof $\frac{1}{2}$ of A's ſhare is equal to $\frac{3}{4}$ of B's, $\frac{4}{5}$ of C's, and $\frac{5}{6}$ of D's. What ſhare had each?

Take a number at pleaſure, and divide in proportion to their ſhares, then proceed by the ſingle rule of falſe.

$$\begin{array}{ll} A & 120 \\ B & 80 \\ C & 75 \\ D & 72 \\ \hline & 347 : 2000 : : \left\{ \begin{array}{lll} 120 : 691\frac{223}{347} & \text{for} & A. \\ 80 : 461\frac{33}{347} & & B. \\ 75 : 432\frac{96}{347} & & C. \\ 72 : 414\frac{342}{347} & & D. \end{array} \right. \end{array}$$

Queſt. 17.

Two merchants together make up a ſtock of 600 *l.* A's ſtock continued in company 9 months, and B's 11, they gain 200 *l.* which they divide equally. How much did each put in?

Since

Since the gains are equal, A's ſtock multiplied by his time 9, is equal to B's ſtock multiplied by his time 11; therefore A's ſtock is to B's ſtock as 11 to 9.

$$\begin{array}{l} 11 \\ \underline{9} \\ 20 : 600 :: \left\{ \begin{array}{l} 11 : 330 \text{ A's ſtock.} \\ 9 : 270 \text{ B's ſtock.} \end{array} \right. \end{array}$$

Queſt. 18.

An apothecary has ſeveral ſimples, A hot in 3 degrees, B hot in 1, C temperate, D cold in 2; and he intends to make up 17 drams, to be in 1 degree of cold. How much of each muſt be taken?

Put 1, 2, 3, &c. for the 4th, 3d, 2d, &c. degree of cold, and proceed by the rule of alligation.

$$4 \left\{ \begin{array}{l|l|l} 8 & 1 & 1 \\ 6 & 1 & 1 \\ 5 & 1 & 1 \\ 3 & 1.2.4 & \underline{7} \end{array} \right.$$

$$10 : 17 :: \left\{ \begin{array}{l} 1 : 1\tfrac{7}{10} \text{ of A, B, C.} \\ 7 : 11\tfrac{9}{10} \text{ of D.} \end{array} \right.$$

Queſt. 19.

A factor delivers 6 French crowns and 4 dollars for 53*s.*—6*d.* and at another time 4 French crowns and 6 dollars for 49*s.*—10*d.* What was the value of each?

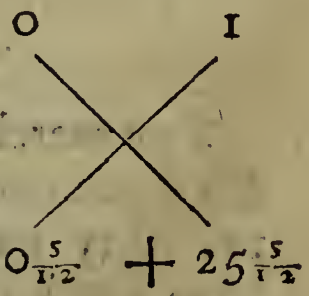
Suppoſe, by the double rule of falſe, there are 0 French crowns;

then 4 doll. $= 53\frac{1}{2}$, 1 doll. $= 13\frac{3}{8}$.

and 4 cr. $+$ 6 doll. $= 80\frac{1}{4}$

$$\underline{49\tfrac{10}{12}}$$

1 cr. $+\ 30\frac{5}{12}$



Again,

Again, ſuppoſe 1 crown, then 4 dollars = $47\frac{1}{2}$,
and 1 dollar = $11\frac{7}{8}$,
and 4 crowns + 6 dollars = $75\frac{1}{4}$
$49\frac{10}{12}$

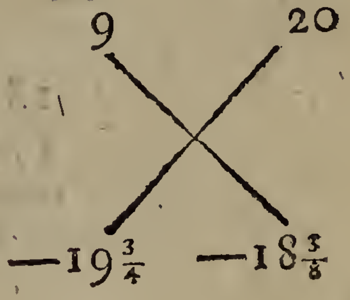
2 er. + $25\frac{5}{12}$

diff. er. 5) $30\frac{5}{12}$ ($6\frac{1}{12}$ = 6*s.*—1*d.* the value of a crown.
and $4\frac{1}{4}$ or 4*s.*—3*d.* = a dollar.

Queſt. 20.

Three companies of ſoldiers paſſing by a ſhepherd, the firſt takes half his flock and half a ſheep, the ſecond takes half the remainder and half a ſheep, the third takes half the laſt remainder and half a ſheep; after which the ſhepherd had 20 ſheep remaining. How many had he at firſt?

By the double rule of falſe, ſuppoſe two numbers, as follows.



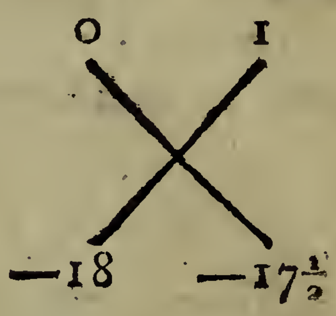
1 ſup. 9	2 ſup. 20
5	$10\frac{1}{2}$
1 rem. 4	1 rem. $9\frac{1}{2}$
$2\frac{1}{2}$	$4\frac{3}{4}$
2 rem. $1\frac{1}{2}$	2 rem. $4\frac{1}{4}$
$1\frac{1}{4}$	$2\frac{5}{8}$
3 rem. $\frac{1}{4}$	3 rem. $1\frac{5}{8}$
20	20
1 er.—$19\frac{3}{4}$	2 er.—$18\frac{3}{8}$

20
9
11
$18\frac{3}{8}$
88
$19\frac{3}{4}$ 11
$18\frac{3}{8}$ $4\frac{1}{8}$
$1\frac{3}{8}$) $202\frac{1}{8}$ (147
20
167 ſheep, the Anſwer.

Queſt. 21.

There is a fiſh whoſe head is 9 inches in length, and his tail is as long as his head and half his body, and his body as long as his head and tail. How long was the fiſh?

1 ſup. body	0	2 ſup. body	1
head	9	head	9
½ body	0	½ body	½
tail	9	tail	9½
body	18	body	18½
	0		1
1 er.	—18	2 er.	—17½



18
17½

½) 18 (36 the body.
27
9

72 the whole fiſh.

9
18

tail 27

Queſt. 22.

There is an annuity of 75*l.* in *reverſion*, which is not to commence for ſeven years, and then it is to continue for 14 years; what is the preſent value of it at 4 per cent. compound intereſt?

head 9
tail 9 + ½ b
body 9 + 9 + ½ b
½ b = 18 b = 36

27
36
72

Find

Find the preſent worth of the annuity of 1 *l.* for 14 years, and then the preſent worth of that ſum of money for 7 years, which multiply by the annuity.

By Tab. III. and IV. the preſent worth of 1 *l.* annuity is $\frac{18.29191}{1.73167} = 10.56313$. Then by Tab. III. the preſent worth of 1 *l.* 7 years hence, is $\frac{1}{1.31593}$, this multiplied by 10.56313 gives $\frac{10.56313}{1.31593} =$ 8.02713, the preſent worth of 1 *l.* annuity in reverſion; laſtly, $8.02713 \times 75 = 602.035$ *l.* the preſent value required.

Queſt. 23.

There is a houſe rented at 25 *l.* a year for 21 years; but the tenant is deſirous to pay 100 *l.* fine (or preſent money). How much rent then muſt he pay, allowing 5 per cent. compound intereſt?

By Tab. III. and IV. the preſent worth of 1 *l.* annuity for 21 years, is $\frac{35.71925}{2.78596}$; then ſay,

$\frac{35.71925}{2.78596}$ (*pr.*) : 1 *l.* (*an.*) : : 100 *l.* (*pr.*) : $\frac{278.596}{35.7192}$ $= 7.7997$ *l.* the rent anſwering the fine of 100 *l.*

then from 25.0000
take 7.7997

remains 17.2003 the rent ſought.

BOOK

BOOK II.

The Theory of Numbers.

CHAP. I.

Numbers produced by addition, ſubtraction, multiplication, and diviſion. Of odd and even numbers. Prime and compoſite numbers. Numbers that are prime to one another; and ſuch as meaſure others. Powers and products of ſquares, cubes, &c.

PROP. I.

If A *and* B *be two numbers; then* A *added to* B *is the ſame ſum as* B *added to* A.

FOR if both of them be reſolved into its units, and placed in a right line, they will count to the ſame number, begin at which end you will.

B, 5.	A, 3.
A, 3.	B, 5.
8	8

Cor. *Hence if ſeveral numbers are to be added together, they will amount to the ſame ſum, whatever order they are placed in. Or if ſeveral numbers are to be ſubtracted, it is the ſame thing, whether they be ſubtracted one after another, or all together.*

PROP. II.

If two numbers A, B, *are to be multiplied together; the product of* A *multiplied by* B, *is equal to the product of* B *multiplied by* A.

A,	3.	B,	5.
B,	5.	A,	3.
	15		15

For A times 1 = to the units in A = 1ce A.
And A times B = B times that product, that is = B times A.

Cor. 1. *If ſeveral numbers are to be multiplied together; they will make the ſame product, in whatever order they are multiplied.*

Cor. 2. *If ſeveral numbers,* A, B, C, *are to be multiplied together; it is the ſame thing, whether* A *be multiplied by the product of the reſt* BC; *or* A *be multiplied firſt by* B, *and the product by* C; *and ſo on.*
For by either method the product will be ABC.

Cor. 3. *And on the contrary, if a number* ABC *is to be divided by another* BC; *it is the ſame thing whether* ABC *is divided by* BC *at once; or it be divided firſt by one factor* B, *and then the quotient by another factor* C, *and ſo on.*

For $\frac{ABC}{BC} = A$ (Ax. 8); and $\frac{ABC}{B} = AC$ (Ax. 8), and then $\frac{AC}{C} = A$ (Ax. 8), that is, $= \frac{ABC}{BC}$.

PROP. III.

If the number S, *be made up of the parts* A, B, C; *the product of* S, *by any number* M, *is equal to the ſum of the ſeveral products, made by multiplying ſeparately, each particular part* A, B, C, *by* M.

For $M \times S = M \times \overline{A+B+C}$ (Ax. 4) $= \overline{A+B+C} \times M$ (Pr. 2). But $\overline{A+B+C}$ times M is nothing elſe but taking M as oft as there are units in $A+B+C$; that is, as oft as there are units in A, and alſo as oft as there are units in B, and alſo in C; and that is, $AM + BM + CM$. Therefore $MS = AM + BM + CM =$ (Pr. 2) $MA + MB + MC$.

$$\begin{array}{lcl} & & A,\ \ B,\ \ C, \\ S,\ 13 & = & 3+\ 4+\ 6 \\ M,\ 5 & & \qquad\quad 5 \\ \hline 65 & = & 15+20+30 \\ \hline \end{array}$$

Cor 1. *If* D *be the difference of two numbers* A *and* B; *then* D *multiplied by any number* M, *is equal to the difference of the products, of* A *by* M, *and* B *by* M.

$$\begin{array}{lcl} & & A,\ \ \ B, \\ D,\ 2 & = & 9 - 7 \\ M,\ 5 & & \qquad 5 \\ \hline 10 & = & 45-35 \\ \hline \end{array}$$

Cor. 2. *If* $S = A + B + C$, *and* $M = F + G$; *then the product of the wholes,* $S \times M =$ *ſum of the products of all the parts of one, by all the parts of the other,* $FA + FB + FC + GA + GB + GC$.

For $SM = MA + MB + MC = \overline{F+G} \times A + \overline{F+G} \times B + \overline{F+G} \times C = FA + GA + FB + GB + FC + GC$.

PROP. IV.

The quotient ariſing by dividing the ſum of two or more numbers $(A+B)$, *by any diviſor* D; *is equal to the ſum of the quotients ariſing by dividing the parts* A, B, *ſeparately by the ſame diviſor. That is,*

$$\frac{A+B}{D} = \frac{A}{D} + \frac{B}{D}.$$

$$\frac{A+B}{D}.\ \frac{A}{D} + \frac{B}{D}$$

$$\frac{9}{3} = \frac{3}{3} + \frac{6}{3}.$$

For

For let the whole be called S, then ſince A + B = S, any part of A, together with the ſame part of B = the like part of S (Ax. 5); that is,

$$\frac{A}{D}+\frac{B}{D}=\frac{S}{D}=\frac{A+B}{D}.$$

PROP. V.

If any multitude of even numbers be added together, the ſum will be even.

For ſince an even number may be divided into two equal whole numbers, let theſe numbers be 2A, 2B, 2C, *&c.* then the ſum will be 2A + 2B + 2C, *&c.*; and the half is A + B + C, *&c.* a whole number (Def. 14).

Cor. *If an even number be taken from an even number, the remainder is even.*

PROP. VI.

If an even multitude of odd numbers be added together, their ſum is even.

For theſe odd numbers may be repreſented by 2A + 1, 2B + 1, *&c.* And the ſum of 2A and 2B, *&c.* is an even number (Pr. 5). And an even number of units, is an even number. Therefore their ſum is an even number.

$$\begin{array}{r} 9\\ 7\\ 5\\ 3\\ \hline 24\\ \hline \end{array}$$

Cor. *An odd multitude of odd numbers added together makes an odd number.*

$$\begin{array}{r} 3\\ 5\\ 7\\ \hline 15 \end{array}$$

PROP. VII.

If there be taken an even number from an odd number, or an odd number from an even number; the remainder is odd.

For let 2A be an even number, then ſince 2A taken from an even number, leaves an even number (Cor. Pr. 5); therefore 2A taken from that even number and 1 more, will leave 1 more; that is, an odd number will remain: and alſo 2A+1 (an odd number) taken from that even number, 1 leſs will remain; that is, an odd number remains.

7	10
4	7
—	—
3	3
—	—

Cor. *If an odd number be taken from an odd number, the remainder is even.*

PROP. VIII.

If an odd number be multiplied by an odd number, the product will be odd.

For the product conſiſts of an odd number taken an odd number of times, and therefore is odd (Cor. Pr. 6).

Cor. 1. *If an odd number be divided by an odd number, the quotient will be odd.*

Cor. 2. *Every number is odd, which meaſures an odd number. Or an even number cannot meaſure an odd number.*

PROP. IX.

If an even number be multiplied by any number, even or odd, the product will be even.

For the product conſiſts of the even number taken ſo many times as there are units in the multiplier, and therefore will be even (Pr. 5).

6	6
2	3
—	—
12	18

Cor. 1. *If an even number be divided by an odd number, the quotient will be even.*

Cor.

Cor. 2. *If an odd number meaſures an even number, it ſhall alſo meaſure half of it.*

Cor. 3. *If an odd number* A, *be prime to any number* B, *it ſhall be prime to its double* 2B.

For no even number can meaſure A (Cor. 2. Pr. 8); and an odd number which meaſures 2B, will alſo meaſure B (Cor. 2); and then A and B would not be prime.

Cor. 4. *A number which is prime to any in a double progreſſion, is prime to them all.*

PROP. X.

If there be two numbers, A *the greater, and* B *the leſſer, and the leſſer* B *be continually taken from the greater* A; *and the remainder* C *from* B; *and the next remainder* D *from* C; *and the next remainder* E *from* D, *and ſo on, till nothing remains. I ſay, the laſt number* E *that remained, will be the greateſt common meaſure of the numbers* A *and* B.

For E meaſures D, ſince 0 remains; and it alſo meaſures C which is ſome multiple (once or oftener) of D with E over (Ax. 10, 11). For the ſame reaſon it meaſures B, which is a multiple of C with D over; and laſtly, it meaſures A, which is a multiple of B with C over. Therefore E is a common meaſure.

```
27)75(2
   54
   ——
   21)27(1
      21
      ——
       6)21(3
         18
         ——
          3)6(2
            6
            —
            0
```

And it is the greateſt; for if there was one F greater than E, then ſince F is ſuppoſed to meaſure A and B, it alſo meaſures C (Ax. 11); and for the ſame reaſon ſince F meaſures both B and C, it alſo meaſures D; and ſince it meaſures both C and D, it alſo meaſures E, the greater the leſs; which is abſurd.

Cor. 1. *If there be two numbers given, and the greater be divided by the leſs; and then the leſſer divided by the remainder; and this remainder by the next remainder, and ſo on, ſtill making the laſt remainder a diviſor. By proceeding thus, if* 1 *remains at laſt; then the two given numbers are prime to one another.*

Ex. 28 *and* 19.

```
19)28(7
   19
   --
    9)19(2
      18
      --
       1)9(9
         9
         -
         0
```

Cor. 2. *If a number* F *meaſures ſeveral numbers, it will alſo meaſure their greateſt common meaſure* E.

This is plain from the demonſtration of this prop. For if F meaſures A and B, it alſo meaſures E, the greateſt common meaſure of theſe two quantities. And if F meaſures E and a third number: it meaſures their greateſt common meaſure; that is, it meaſures the greateſt common meaſure of all the three numbers; and ſo on.

PROP. XI.

If the number N *be the leaſt, which ſeveral other numbers meaſure; theſe numbers ſhall only meaſure all the multiples of* N, *but no other number beſides.*

For ſince they meaſure N, they ſhall alſo meaſure 2N, 3N, *&c.* or in general *r*N (Ax. 10), *r* being any number.

But they can meaſure no other number as P; for take *r*N the neareſt multiple to P; then ſince they meaſure both *r*N and P, they will alſo meaſure their difference (Ax. 9). But that difference is leſs than N;

therefore N is not the leaſt number which they meaſure; contrary to the hypotheſis.

Cor. *If ſeveral numbers meaſure any number; the leaſt which they meaſure ſhall alſo meaſure the ſame number; that is, their leaſt common dividend, ſhall alſo meaſure it.*

PROP. XII.

If N *be the leaſt number (or the leaſt common dividend) that ſeveral prime numbers,* A, B, C, *meaſure: no other prime* D *ſhall meaſure the ſame.*

For if the prime D meaſures it, then D muſt be a factor in N, as well as A, B, C, are; and then N would not be the leaſt number, which A, B, C, meaſure.

PROP. XIII.

If two numbers, A, B, *be prime to one another; the number* C, *which meaſures one of them* A, *will be prime to the other* B.

For if C and B be not prime to one another, let D meaſure both. But becauſe D meaſures C, it alſo meaſures A (Ax. 10); conſequently A and B are not prime to one another: contrary to the hypotheſis.

A, 9. B, 4.
C, 3. D . .

PROP. XIV.

If two numbers, A, B, *be prime to any number* C, *their product* AB *will be prime to it.*

For no numbers can meaſure AB and C, but ſuch (prime) factors as A, B, and C, are made up of. But in A and C there are none that are common to both; becauſe A and C are prime to one another; nor in B and C for the

A, 5. C, 8.
B, 3.
———
AB, 15.

ſame reaſon. Therefore let A be denoted by the factors P and Q; that is, let A = PQ, and B = RS; and alſo C = EF; then AB = PQRS. Now it is evident that PQRS and EF are prime to one another, becauſe there is no factor common to both, therefore their equals AB and C are prime to one another.

Cor. 1. *If ſeveral numbers, how many ſo ever,* A, B, C, D, *&c. be each of them prime to any number* F; *their product,* ABCD *&c. will alſo be prime to the ſame* F.

For (by this prop.) AB and C are both prime to F; therefore ABC is prime to F. Again, ABC and D are both prime to F; therefore ABCD is prime to F.

Cor. 2. *If one number* A *be prime to another* F; *its ſquare, cube, or any power* A^n, *ſhall alſo be prime to the ſame number* F.

This is evident from Cor. 1. by ſuppoſing A, B, C, D, *&c.* all equal.

PROP. XV.

If two numbers, A, B, *be prime to one number* C, *and alſo to another* D; *their products* AB *and* CD *ſhall alſo be prime to one another.*

For AB is prime to C, and alſo to D (Pr. 14); therefore AB is prime to CD.

Cor. 1. *If ſeveral numbers,* A, B, C, D, *&c. be prime to each of the numbers* F, G, H, I, *&c. then their products,* ABCD, *and* FGHI, *&c. will be prime to one another.*

For (by this prop.) AB is prime to FG, and ſince AB and C are prime to FG and H; therefore ABC is prime to FGH. Again, ſince ABC and D, are prime

prime to FGH and I, therefore ABCD is prime to FGHI, *&c.*

Cor. 2. *If two numbers,* A, F, *be prime to one another; then any power of one* A^m, *will be prime to any power of the other* F^n.

This follows from Cor. 1. by ſuppoſing B, C, D, *&c.* = A, and G, H, I, *&c.* = F.

PROP. XVI.

If two numbers, A, B, *be prime to one another, and each of them meaſures ſome number* D; *then their product* AB *ſhall meaſure the ſame number* D.

For ſince A and B are prime to one another, there is no factor common to both; and ſince they both of them meaſure D, therefore they both are factors in D. Therefore let D = ABF, then A and B meaſure ABF, and it appears that AB meaſures ABF or D.

Cor. *If ſeveral numbers* A, B, C, *&c. be prime to one another; and each of them meaſures another* D; *then their product* ABC, *&c. ſhall meaſure the ſame number* D.

PROP. XVII.

If two numbers, A, B, *be prime to one another; their ſum* A + B *will be prime to either of them.*

If you deny it, let D be the common meaſure of A and A + B, then it will meaſure the reſidue B (Ax. 11). Therefore A, B, are not prime: againſt the hypotheſis.

Cor. *If a number be prime to one of its parts; it is alſo prime to the remaining part.*

PROP. XVIII.

If the number A *be prime to* B; *then* A *ſhall meaſure no multiple of* B, *leſs than* A × B; *or whoſe multiplier is leſs than* A.

Let r be any number, and ſuppoſe r times B, or rB to be ſome multiple of B. Now the numbers A, B, being prime to one another, there is no factor common to both A and B: therefore if A meaſures rB, it muſt meaſure r alone. But it can never meaſure r leſs than itſelf: therefore r muſt be equal to A, or to ſome multiple of A.

Cor. *If* A, B, *be prime to one another*; *then* A *ſhall meaſure all the multiples of* AB, *and no other multiples of* B *beſides.*

PROP. XIX.

More prime numbers may be found, than any propoſed multitude, A, B, C.

Let N be the leaſt number which A, B, C, meaſure; then if N + 1 be a prime number, another prime is found. But if it is a compoſite number, then ſome other prime, as D, meaſures it, and ſo the prime D is found.

PROP. XX.

Let M *be any number,* 1, 2, 3, 4, *&c. then* M × 6 — 1, *and* M × 6 + 1, *conſtitute a ſeries, which contains all prime numbers above* 3.

For thoſe left out of the ſeries are no primes. For 6M + 2, and 6M — 2, are not primes, being diviſible by 2. Alſo 6M + 3, and 6M — 3, being diviſible by 3, are no primes. But theſe are all the numbers left out.

PROP.

PROP. XXI.

No number is a ſquare number, that conſiſts not of two equal factors; nor a cube, that conſiſts not of three equal factors: and ſo for higher powers.

This appears from the definition of ſquare and cube numbers; and other higher powers. For a ſquare requires to have two equal multipliers, or elſe a ſquare could not be produced; and a cube muſt have three. And ſo on.

Cor. 1. *There is no ſuch thing as the exact ſquare root of* 2, 3, 5, 6, 7, 8, 10, *&c.* *Nor the exact cube root of* 2, 3, 4, 5, 6, 7, 9, *&c.*

For there are no ſuch factors to be found in theſe numbers, and infinite others. For example, the two factors in 2, are 1 and 2; in 3, 1 and 3; in 6, 2 and 3, *&c.* and therefore they are no ſquares. Again, the three factors in 2, are 1, 1, and 2; in 3, are 1, 1, and 3; in 12, they are 2, 2, and 3, *&c.* which are no cubes.

Cor. 2. *All numbers are ſurds, whoſe roots are not ſome of the natural ſeries,* 1, 2, 3, 4, 5, 6, *&c.* ad infinitum.

PROP. XXII.

The ſum of two numbers differing by a unit, is equal to the difference of their ſquares.

Let N and $N+1$ be the numbers;

multiply - - $N+1$

by - - $N+1$

the ſquare of $N+1$ - - $NN+N+N+1$

the ſquare of N - - - NN ſubtract

remains - - - - $N+\overline{N+1}$,

the ſum of the two numbers.

Cor.

Cor. *The differences of the ſquares of* 0, 1, 2, 3, 4, *&c. proceed by the odd numbers,* 1, 3, 5, 7, *&c.*

PROP. XXIII.

The ſum of any number of terms (n), of the ſeries of odd numbers 1, 3, 5, 7, *&c. is equal to the ſquare (nn) of that number.*

Set down the ſeries of ſquares, and their differences, according to Cor. Pr. 21. and by adding them we ſhall have

0	1^2	2^2	3^2	4^2	5^2	6^2	7^2.
	1	3	5	7	9	11	13

$0+1$ or the ſum of 1 term $= 1^2$ or 1,
$1+3$ or the ſum of 2 terms $= 2^2$ or 4,
$4+5$ or the ſum of 3 terms $= 3^2$ or 9,
$9+7$ or the ſum of 4 terms $= 4^2$ or 16,
$16+9$ or the ſum of 5 terms $= 5^2$ or 25, and ſo on.
Whence it is plain, let n be what number you will, the ſum of n terms will be $= nn$.

PROP. XXIV.

The ſum of two numbers multiplied by their difference, is equal to the difference of their ſquares.

Let the numbers be A, E; which multiplied together will produce AA—EE (Prop. 3, and Cor. 1).

$$\begin{array}{rr} A+E & \\ A-E & \\ \hline AA+AE & \\ -AE & -EE \\ \hline AA & -EE \\ \hline \end{array}$$

Cor. *The difference of the ſquares of two numbers, is diviſible, by either the ſum or difference of theſe numbers.*

PROP.

PROP. XXV.

The ſum of two cube numbers is diviſible by the ſum of their roots. Or the ſum of any two numbers will meaſure the ſum of their cubes.

Let the numbers be A, E; multiply $AA-AE+EE$
by $A+E$

$$\begin{array}{r} \hline A^3-A^2E+AEE \qquad\quad \\ +A^2E-AEE+E^3 \\ \hline \end{array}$$

(by Pr. 3. and Cor.) product, $\begin{array}{l} A^3 - - - - +E^3 \\ \hline \end{array}$

Therefore A^3+E^3 is diviſible by $A+E$ (Ax. 8).

PROP. XXVI.

The difference of any two numbers will meaſure the difference of their cubes.

If A, E, be the numbers; mult. $AA+AE+EE$
by $A-E$

$$\begin{array}{r} \hline A^3+A^2E+AEE \qquad\quad \\ -A^2E-AEE-E^3 \\ \hline \end{array}$$

the product (Pr. 3) $\begin{array}{l} A^3 - - - -E^3 \\ \hline \end{array}$

Therefore the product A^3-E^3 is diviſible by $A-E$ (Ax. 8).

PROP. XXVII.

The product of two ſquare numbers, is a ſquare number; and of two cube numbers, a cube number: and ſo on.

For $AA \times BB = AABB = AB \times AB$, the ſquare of AB.

Alſo $A^3 \times B^3 = AAABBB = ABABAB$, the cube of AB, and ſo of others.

Cor.

Cor. *If a ſquare number divide or meaſure a ſquare number; or a cube number a cube number*; &c. *the quotient will be a ſquare, or cube number*, &c. *reſpectively.*

For $\frac{AABB}{BB} = AA$ (Ax. 8), the ſquare of A.

and $\frac{A^3B^3}{B^3} = A^3$, the cube of A; *&c.*

PROP. XXVIII.

Every power of a ſquare number is a ſquare number; and every power of a cube number is a cube number: and ſo on.

For AA or A^2 is the ſquare of A; and $\overline{AA}^2$ or A^4 is the ſquare of AA. $\overline{AA}^3$ or A^6 is the ſquare of A^3. $\overline{AA}^5$ or A^{10} is the ſquare of A^5, *&c.*

Again, $\overline{AAA}^2$ or A^6 is the cube of AA: and $\overline{AAA}^3$ or A^9 is the cube of A^3: alſo $\overline{AAA}^4$ or A^{12} is the cube of A^4, *&c.* and ſo of others.

CHAP. II.

Of proportional numbers, and thoſe in geometrical progreſſion. Mean proportionals. Like plane and ſolid numbers.

PROP. XXIX.

If four quantities, A, B, C, D, *are proportional; the product of the means is equal to the product of the extremes,* AD = BC.

FOR ſince A : B : : C : D; then $\frac{A}{B} = \frac{C}{D} = r$ (Def. 27); and $A = Br$, $C = Dr$ (Ax. 4, 5). Whence $AD = BrD$, and $BC = BDr$ (Ax. 4); therefore $AD = BC$ (Ax. 1).

Cor. 1. *The firſt is to the third, as the ſecond to the fourth;* A : C : : B : D.

For ſince AD = BC, then $\frac{AD}{CD} = \frac{BC}{CD}$ (Ax. 5), that is, $\frac{A}{C} = \frac{B}{D}$, or A : C : : B : D.

Cor. 2. *The ſecond is to the firſt, as the fourth to the third, or* B : A : : D : C.

For ſince BC = AD, $\frac{BC}{AC} = \frac{AD}{AC}$ (Ax. 5), that is, $\frac{B}{A} = \frac{D}{C}$.

Cor. 3. $A : B : : A + C : B + D : : A - C : B - D$.

For ſince $\frac{A}{B} = r$, and $A = Br$, $C = Dr$; then $A + C = Br + Dr = \overline{B + D} \times r$ (Ax. 2); therefore $\frac{A + C}{B + D} = r = \frac{A}{B}$ (Ax. 1).

In

In like mannner $A - C = Br - Dr = \overline{B - D} \times r$, and $\frac{A - C}{B - D} = r = \frac{A}{B}$, whence $A : B :: A + C : B + D :: A - C : B - D$ (Def. 27).

Cor. 4. *If any like parts or multiples of* A *and* B *be denoted by* r, *then* $A : B :: rA : rB$.

For $\frac{rA}{A} = r = \frac{rB}{B}$; therefore $rA : A :: rB : B$ (Def. 27); and $rA : rB :: A : B$ (Cor. 1).

Cor 5. *If* $A : B :: C : D$; *then* D *can only be a whole number, when* A *meaſures the product* BC.

For $AD = BC$, and $D = \frac{BC}{A}$ (Ax. 5).

Cor. 6. *If three numbers*, A, B, C, *are in continual proportion; then the ſquare of the mean is equal to the product of the extremes*, $BB = AC$.

This is plain, by ſuppoſing the two middle terms to be equal; and then the fourth becomes the third.

PROP. XXX.

If two numbers, A, B, *are prime to one another, no other numbers can be found in that proportion, but what are ſome multiple of* A *and* B.

Let C, D be others in the ſame proportion, then ſince $A : B :: C : D$, then $AD = BC$ (Pr. 29); and $D = \frac{BC}{A}$ (Ax. 5). Now D can only be a whole number, when A meaſures BC (Cor. 5. Pr. 29). But A being prime to B, there is no factor common to both; therefore if A meaſures BC, it muſt meaſure C alone; that is, C is ſome multiple of A, and conſequently D is ſome multiple of B.

A, 5.	B, 3.
C, 10.	D, 6.

Cor. 1. *Numbers*, A, B, *that are prime to one another, are the leaſt of all numbers in the ſame proportion.*

Cor. 2. *Numbers*, A, B, *that are the leaſt in a given proportion, are prime to one another.*

For

For if they are not prime, they may be reduced to leſs numbers in the ſame proportion.

PROP. XXXI.

If there be a ſeries of numbers, A, B, C, D, *(greater than* 1) *in continual proportion; and the extremes* A,D *prime to one another; there cannot be found another number in the ſame proportion.*

Let E be another term, if poſſible; then A : B : : D : E; and A : D : : B : E (Cor. 1. Pr. 29); but A, D, are prime to one another by ſuppoſition; therefore B, E are multiples of A and D (Pr. 30.); therefore A meaſures B. And ſince A meaſures B, therefore B meaſures C, and C meaſures D (Def. 27); therefore A meaſures D (Ax. 10). Therefore A and D are not prime to one another: contrary to the hypotheſis.

A	B	C	D	E
8	12	18	27	

Cor. 1. *If two numbers (greater than* 1) *be prime to one another, there cannot be found a third number in the ſame proportion.*

PROP. XXXII.

If there be ſeveral numbers, A, B, C, D, *in continual proportion, and the extremes* A, D *prime to one another; then theſe numbers are the leaſt of all numbers in the ſame proportion. And the contrary.*

For let E, F, G, H, be other numbers in the ſame proportion. Then ſince A : B : : E : F, therefore A : E : : B : F : : C : G : : D : H (Cor. 1. Pr. 29). And A : D : : E : H (ib.). But A and D are prime to one another, by ſuppoſition, and therefore the leaſt in that proportion (Cor. 1. Pr. 30.) therefore E, H are greater than A, D; and all of them, A, B, C, D, are leſs than E, F, G, H.

A	B	C	D
8	12	18	27
E	F	G	H

On the contrary, if A, B, C, D are the leaſt in that proportion, then A and D are prime to one another. For if you ſuppoſe E, H to be prime to one another, then E, F, G, H will be the leaſt in that proportion: contrary to the hypotheſis.

Cor. *If* A, B, C, D *be in continual proportion, and the extremes* A, D *prime to one another; then all other numbers*, E, F, G, H, *in the ſame proportion, muſt be ſome multiple of* A, B, C, D.

For it being A : D : : E : H, and A, D being prime to one another (this Prop.), E, H muſt be ſome multiple of A, D (Pr. 30). Therefore E, F, G, H are multiple of A, B, C, D.

PROP. XXXIII.

In a ſeries of numbers the leaſt in continual proportion; if there be three numbers, the extremes are ſquares; if four, cubes; and in general if there be n numbers, the extremes are the $\overline{n - 1}^{th}$ *powers of two numbers, which are the leaſt in that proportion.*

A, 4 : B, 6 : C, 9.
A, 8 : B, 12 : C, 18 : D, 27.

For let A, B be the leaſt in that proportion, then AA, AB, BB are continual proportionals, in the ſame proportion of A to B (Cor. 4. Pr. 29). And ſince A, B are prime to one another (Cor. 2. Pr. 30), AA and BB will be prime to one another (Cor. 2. Pr. 15); therefore AA, AB, and BB are the leaſt in the proportion of A to B (Pr. 28); where the extremes are ſquares.

For the ſame reaſon A^3, A^2B, AB^2, B^3 are the leaſt in continual proportion of A to B; where the extremes are the cubes of A and B. And ſo of others.

Cor. 1. *Between two ſquare numbers there is one mean proportional; between two cubes, two means. And in general, between two* n^{th} *powers, there are* $\overline{n - 1}$ *means.*

For

For between AA and BB there is the mean AB, and between the cubes A^3 and B^3 are the means A^2B, AB^2. And ſo forward.

Cor. 2. *In a ſeries of numbers, the leaſt in continual proportion; two numbers, which are the leaſt in that proportion, meaſure all the means.*

For both A and B meaſure AB, the mean of three proportionals. Alſo both A and B meaſure A^2B and AB^2, the two means of four proportionals. And ſo on.

Cor. 3. *If there be three numbers the leaſt in continual proportion, the ſum of any two is prime to the other.*

For in the numbers AA, AB, BB no number can meaſure any one of them, and alſo the ſum of the other two.

PROP. XXXIV.

In a ſeries of numbers in continual proportion, if the firſt meaſure not the ſecond; neither ſhall any one meaſure any other.

A : B : C : D : E
16 24 36 54 81
F G H I

I ſay, for example, B does not meaſure E. For, as E is the fourth from B, take the four numbers, F, G, H, I, the leaſt in that proportion; then B : C : : F : G; therefore B : F : : C : G : : D : H : : E : I (Cor. 1. Pr. 29); and B : E : : F : I (ib.). But F, I are prime to one another (Pr. 32). Therefore F does not meaſure I (except F be 1), and conſequently B does not meaſure E.

Here F is not 1, for A : B : : F : G. If F was 1, F would meaſure G, and A meaſure B; contrary to the hypotheſis.

Cor. *If the firſt meaſure the laſt, it ſhall alſo meaſure the ſecond.*

For if you ſay it meaſures not the ſecond, then it ſhall not meaſure the laſt: againſt the hypotheſis.

PROP. XXXV.

If between two numbers there fall ſeveral mean proportionals; ſo many ſhall alſo fall between two other numbers, having the ſame proportion.

27 : 36 : 48 : 64
54 : 72 : 96 : 128

For ſuppoſe the four quantities, A^3, A^2B, AB^2, B^3, to be the leaſt in that proportion. Then, ſince A^3 and B^3 are prime to one another (Pr. 32), all other numbers, in that proportion, muſt be ſome multiple thereof (Cor. Prop. 32). Take any number, r, and let rA^3, rB^3 be the extremes; then rA^2B and rAB^2will be the means (Cor. 4. Pr. 29). And the like for any other number of mean proportionals.

PROP. XXXVI.

If between two numbers, prime to one another, there fall ſeveral mean proportionals; ſo many ſhall alſo fall between either of them and a unit. And the contrary.

For in the four proportional numbers, A^3, A^2B, AB^2, B^3, there are two means, A^2B, AB^2, between A^3 and B^3, which ſuppoſe to be prime. Now put $A = 1$, then the four proportionals become 1, B, B^2, B^3; where B and BB are the two means. Again, put $B = 1$, then the four proportionals become A^3, A^2, A, 1; where A and AA are the two means.

And on the contrary, between A^3 and B^3 two mean proportionals fall (Cor. 1. Prop. 33). And ſo of others.

PROP. XXXVII.

If there be a ſeries of numbers continually proportional; and the firſt be a ſquare, the third ſhall be a ſquare. If the firſt be a cube, the fourth ſhall be a cube. If the firſt be a fourth power, the fifth ſhall be a fourth power.

Let $AA : B : C$; then $AAC = BB$ (Cor. 6. Pr. 29), and $C = \frac{BB}{AA}$; therefore C is a ſquare (Cor. Pr. 27).

Again, let $A^3 : B : C : D$; then $BB = A^3C$ (Cor. 6. Pr. 29), and $B^3 = A^3BC$ (Ax. 4), and $BC = \frac{B^3}{A^3}$ (Ax. 5). Alſo $A^3D = BC$ (Pr. 29), and conſequently $A^3D = \frac{B^3}{A^3}$, and $D = \frac{B^3}{A^6}$; therefore D is a cube (Cor. Pr. 27).

Likewiſe if $A^4 : B : C : D : E$. Then $C = \frac{BB}{A^4}$, and $A^4E = BD = CC = \frac{B^4}{A^8}$, and $E = \frac{B^4}{A^{12}}$, a fourth power, whoſe root is $\frac{B}{A^3}$. And ſo on.

PROP. XXXVIII.

In a ſeries of numbers continually proportional, beginning at 1; any prime number, that meaſures the laſt, ſhall meaſure all the reſt after the unit.

Let the ſeries be $1 : A : AA : A^3 : A^4 : A^5$; and let the prime P meaſure A^5; then if you deny that P meaſures A, then P is prime to A, and therefore it is prime to A^5 (Cor. 2. Pr. 14); contrary to the hypotheſis.

Cor. 1. *If any number meaſures the laſt and not the firſt (after the unit), it is a compoſite number.*

Cor. 2. *If the firſt term (after the unit) be a prime, no other prime ſhall meaſure the laſt.*

Cor. 3. *In a ſeries of continual proportionals from 1, if the term next 1 be a prime; no number ſhall meaſure the laſt, but thoſe in that ſeries.*

For A, A^2, A^3, &c, all meaſure A^5; and no others do, becauſe A is a prime number (Cor. 2. Pr. 14).

PROP. XXXIX.

If four numbers are proportional, and three of them ſquares, the fourth is a ſquare; and if three of them be cubes, the fourth is a cube; and ſo on.

Suppoſe $AA : BB :: CC : D$, then $AAD = BBCC$ (Pr. 29), and $D = \frac{BBCC}{AA}$ (Ax. 5); therefore D is a ſquare (Cor. Pr. 27).

Again, $A^3 : B^3 :: C^3 : D$; then $A^3D = B^3C^3$, and $D = \frac{B^3C^3}{A^3}$, and D is a cube (Cor. Pr. 27).

Cor. *Hence the proportion of a ſquare number to one not ſquare, cannot be expreſſed by two ſquare numbers; neither can the proportion of a cube number to one not cube, be expreſſed by two cube numbers.*

PROP. XL.

The product of two like plane numbers is a ſquare number; and of three like ſolid numbers, a cube; &c.

Let ab, AB be two like plane numbers; then ſince $a : A :: b : B$, we ſhall have $aB = Ab$ (Pr. 29). But $ab \times AB = aBbA = Ab \times bA$, or $aB \times aB$, a ſquare, whoſe root is aB or Ab.

Again, let abc, ABC, EFG, be three like cube numbers; then ſince $a : b :: A : B$, and $a : c :: E : G$; alſo $B : C :: F : G$; therefore $aB = bA$, $aG = cE$, and $CF = BG$; then $abc \times ABC \times EFG = a \times bA \times cE \times BG \times CF = a \times aB \times aG \times BG \times BG = a^3B^3G^3$, a cube, whoſe root is aBG or aCF, or bAG, or bCE, or cAF, or cBE.

Cor. 1. *If the product of two numbers be a ſquare; or of three numbers a cube; they are ſimilar plane or ſolid numbers.*

For if it is not $a : A :: b : B$, then it is not $aB = Ab$, but rather $aB = Db$, and then we ſhould not have $aB \times bA$, or $aB \times aB$, a ſquare number (but rather $aB \times bD$); contrary to the hypotheſis.

Cor. 2. *Two diſſimilar plane numbers cannot produce a ſquare.*

For a ſquare is only produced from ſimilar numbers (Cor. 1).

Cor. 3. *If the ſquare of a number,* A, *be a cube, the number itſelf,* A, *is a cube.*

For A^3 is a cube by nature, and A^2 is a cube by ſuppoſition; therefore $\frac{A3}{A^2}$ or A is a cube (Cor. Pr. 27).

Cor. 4. *If any number meaſure or divide a ſquare number; the quotient will be a plane number, ſimilar to the diviſor.*

PROP. XLI.

Between two like plane numbers there is one mean proportional; between two like ſolid numbers there are two means; and ſo on.

Let ab, AB be two like plane numbers; then

theſe numbers are proportional $\begin{cases} a : A \\ \quad b : B \end{cases}$

whence theſe are proportional $\Big\}$ $ab : Ab : AB$ (Cor. 4. Pr. 29).

Again, let abc, ABC be two ſimilar ſolid numbers; then

theſe numbers are proportional $\begin{cases} a : A \\ \quad b : B \\ \qquad c : C \end{cases}$

whence theſe are proportional $\Big\{$ $abc : Abc : ABc : ABC$ (Cor. 4. Pr. 29).

And ſo on for others.

Cor. 1. *Theſe are like plane numbers, that have one mean proportional between them; and like ſolid numbers, that have two means: And ſo on.*

For ſince $ab : Ab : AB$; therefore $abAB = AbAb$ (Pr. 29), and $aB = Ab$ (Ax. 5); alſo $\frac{aB}{AB} = \frac{Ab}{AB}$ (ib.) or $\frac{a}{A} = \frac{b}{B}$, therefore $a : A :: b : B$ (Def. 27).

Likewiſe $abc \times ABc = Abc \times Abc$, or $aB = Ab$, whence $a : A :: b : B$; alſo $abc \times ABC = Abc \times ABc$, or $aC = Ac$, whence $a : A :: c : C$. And ſo of others.

Cor. 2. *Between two nonſimilar numbers, one or more means cannot be found.*

For if there were any means, the numbers would be ſimilar (Cor. 1).

PROP. XLII.

Like plane numbers are to one another, as the ſquares of their ſimilar ſides or factors; and like ſolid numbers are as their cubes; and ſo on.

For if ab, AB be ſimilar planes, then $a : A :: b : B$, and $aB = Ab$; but $ab : AB :: aab : aAB$ or $AAb :: aa : AA$ (Cor. 4. Pr. 29).

Again, if abc, ABC are ſimilar cubes, then ſince $aB = Ab$, and $aC = Ac$, therefore $abc : ABC :: aa \times abc : aa \times ABC$ (Cor. 4. Pr. 29) $:: a^3 \times bc : A \times Ab \times Ac :: a^3 : A^3$ (Cor. 4. Pr. 29).

Cor. *No numbers prime to one another, except ſquares, are ſimilar plane numbers.*

For if they be ſimilar plane numbers, they are not prime; for if a be prime to A, yet b and B are ſome equal multiple of a, A; and therefore are not prime to one another (Pr. 30).

PROP. XLIII.

If a number of any power meaſures another number of the ſame power; then the root of the firſt will meaſure the root of the laſt. And the contrary.

For in the continual proportionals, A^3, A^2B, AB^2, B^3; ſince A^3 meaſures B^3, it alſo meaſures A^2B the ſecond term (Cor. Pr. 34). But ſince $A^3 : A^2B :: A : B$ (Cor. 4. Pr. 25); therefore if A^3 meaſures A^2B, A will meaſure B (Def. 27). On the contrary, if if A meaſures B, A^3 will meaſure A^2B; and A^2B, AB^2; and $AB,^2$ B^3: therefore A^3 meaſures B^3, (Ax. 10).

Cor. *If the power does not meaſure the power, neither ſhall the root meaſure the root; and the contrary.*

For if you ſay A meaſures B, then ſhall A^3 meaſure B^3; contrary to the hypotheſis.

And if you ſay that A^3 meaſures B^3, then A will meaſure B; likewiſe againſt the hypotheſis.

CHAP. III.

The properties of particular numbers. Diviſors and aliquot parts. Circulating numbers, and ſuch as terminate, or run on ad infinitum *by diviſion.*

PROP. XLIV.

ALL the powers of any number, ending in 5, will alſo end in 5: and if a number ends in 6, all its powers end in 6.

For 5 times 5 is 25. And 6 times 6 is 36.

PROP. XLV.

No number is a ſquare, that ends in 2, 3, 7, or 8.

This is plain by ſquaring all the natural numbers to 10.

PROP. XLVI.

Any even ſquare number is diviſible by 4.

The root is even (Pr. 9), therefore let $2n$ be the root, then $4nn$ is the ſquare of it; and 4 meaſures or divides $4nn$.

Cor. *A number conſiſting of two, three, &c. even ſquares, is diviſible by 4.*

PROP. XLVII.

An odd ſquare number, divided by 4, leaves a remainder of 1.

The root of an odd ſquare is odd (Pr. 8), therefore let $2n+1$, be the root, which multiplied by itſelf,

itſelf, gives the ſquare $4nn + 4n + 1$, but 4 will meaſure $4nn + 4n$, and 1 will remain.

Cor. *If a number conſiſting of two odd ſquares, be divided by* 4, *it leaves a remainder of* 2; *of three odd ſquares, it leaves a remainder of* 3.

PROP. XLVIII.

In every ſquare number, the number of diviſors is odd; in nonquadrate numbers, even.

Let 36 (*aabb*) be a ſquare number; now ſince any diviſor and its quotient, are two diviſors; therefore if they be ſet down together, you will find them to proceed by couples, till you come to the ſquare root, where the diviſor and quotient are the ſame, and therefore that makes an odd one. But in a number not ſquare, there is no ſuch odd diviſor, for they proceed by couples to the laſt, and make an even number of diviſors.

1	36	1	*aabb*
2	18	*a*	*abb*
3	12	*b*	*aab*
4	9	*aa*	*bb*
6		*ab*	

Cor. *If the number of diviſors be odd, it is a ſquare number; if even, it is no ſquare.*

PROP. XLIX.

Any power of a prime number hath as many aliquot parts, as is the dimenſion of its power.

As if a be a prime, then any power as a^3 contains the 3 aliquot parts 1, a, aa. Alſo a^4 contains theſe, 1, a, aa, a^3, which are 4; and ſo on.

Cor. *The number of diviſors in any power of a prime number, is equal to the index of the next ſuperior power thereof.*

For it is 1 more than the number of aliquot parts.

PROP.

PROP. L.

In any number made up of different primes or their powers; the number of diviſors thereof, is equal to the continual product of the indices of the next ſuperior powers of theſe primes.

For the diviſors of a^3, are $1, a, aa, a^3$ (Cor. Pr. 48); that is 4. And the diviſors of a^3b^2, are ſuch as are produced by multiplying $1, a, aa, a^3$, by each of the diviſors in b^2, that is, by $1, b, bb$, which will make 4×3 or 12 diviſors.

$$\left.\begin{matrix} 1, & a, & aa, & a^3 \\ b, & ba, & baa, & ba^3 \\ bb, & bba, & bbaa, & bba^3 \end{matrix}\right\} 12.$$

Likewiſe the diviſors in a^3b^2c, are had by multiplying theſe twelve into $1, c$, the two diviſors of c, which will be $4\times3\times2=24$; and ſo on.

Cor. *If the powers of ſeveral different prime numbers be multiplied together; the number of diviſors in the product, is equal to the product made by the number of diviſors in each power, multiplied together.*

For the number of diviſors in a^3 is 4, in b^2 is 3, in c is 2; and in a^3b^2c is $4\times3\times2=24$.

PROP. LI.

Any number divided by 9, will leave the ſame remainder, as the ſum of its figures or digits divided by 9.

Let there be any number, as 7604; this ſeparated into ſeveral parcels becomes $\overline{7000}+600+4$; but $7000=7\times1000=7\times999+1=7\times999+7$. In like manner $600=6\times99+6$. Therefore $7604=7\times999+7+6\times99+6+4=7\times999+6\times99+7+6+4$. Therefore $\frac{7604}{9}=\frac{7\times999+6\times99}{9}+\frac{7+6+4}{9}$ (Ax. 5); but $7\times999+6\times99$ is evidently diviſible by 9, therefore 7604 divided by 9 leaves the remainder $7+6+4$ to be divided by 9, which is nothing elſe but the ſum of the digits $7+6+0+4$. And the ſame holds for any other number.

Cor. 1. *If any number is diviſible by* 9, *the ſum of its figures or digits is diviſible by* 9. *And the contrary.* For then the remainder will be nothing, in both of them.

Cor. 2. *Any number divided by* 9, *leaves the ſame remainder, as when all the figures of it are any way tranſpoſed, and then divided by* 9.
For the ſum of the digits ſtill remains the ſame.

PROP. LII.

Any number divided by 3, *will leave the ſame remainder, as the ſum of its figures or digits divided by* 3.

For ſuppoſe any number, as 7604, and proceeding as in the laſt Prop. we have $7604=7\times999+6\times99+7+6+4=7\times3\times333+6\times3\times33+7+6+4$, and $\frac{7604}{3}=\frac{21\times333+18\times33}{3}+\frac{7+6+4}{3}$. But it is evident $21\times333+18\times33$ is diviſibly by 3, conſequently there remains only $7+6+4$ to be divided by 3, which is the ſum of the digits, as was propoſed.

Cor. 1. *If any number is diviſible by* 3, *the ſum of its digits is alſo diviſible by* 3: *and the contrary.*
For in both caſes nothing will remain.

Cor. 2. *Any number divided by* 3, *leaves the ſame remainder as it would do, when its digits are tranſpoſed and placed in any other order.*
For the ſum of the digits remains the ſame in any poſition.

PROP.

PROP. LIII.

If any two numbers are ſeparately divided by 9, *and the two remainders multiplied together, and that product divided by* 9, *this laſt remainder will be the ſame, as if you divide the product of the two firſt numbers by* 9.

For let $9A + a$, and $9B + b$, be two numbers; a, b, being the two remainders. Then the product of the two numbers is $9 \times 9AB + 9Ab + 9Ba + ab$. But $9 \times 9\ AB + 9Ab + 9Ba$ is diviſible by 9; therefore there is no remainder but what is had by dividing ab by 9.

Cor. *This Prop. holds equally true for the number* 3; *and is demonſtrated the ſame way.*

PROP. LIV.

If one number be divided by another prime to it, and the diviſion continued on indefinitely; the number of figures which circulate (or return again) in the quotient, will be always leſs than the number of units in the diviſor.

Suppoſe 6 divided by 7; here the diviſor being 7, the remainder muſt be always leſs than it, and muſt be either 1, 2, 3, 4, 5, or 6. So that in the 7th place, if not before, one of theſe remainders muſt needs return a ſecond time; and the ſame remainder returning, as before, a repetition of the ſame figures muſt return again in the quotient: and ſo forward. And it is evident the ſame will hold for any diviſor; the number of remainders, being always leſs than the number of units in it.

$$
\begin{array}{l}
7)\,6.0\,(857142,857142,8 \quad \&c.\\
\underline{56}\\
40\\
\underline{35}\\
50\\
\underline{49}\\
10\\
\underline{7}\\
30\\
\underline{28}\\
20\\
\underline{14}\\
6\ \&c.
\end{array}
$$

PROP. LV.

If one number divide another prime to it, the quotient will end after a certain number of figures, when the diviſor is compounded of 2 or 5, or both: In all other caſes, the quotient will never end.

For ſince dividing by any power of 2 is equivalent to dividing, firſt by 2, and then the quotient by 2, and ſo on; alſo dividing by any power of 5 is the ſame as dividing firſt by 5, and then the quotient by 5, and ſo forward; and laſtly, ſince any number may be divided by 2 or 5, at moſt by adding a cypher: therefore it is plain, when the diviſor is a compoſite number made up of the powers of 2 and 5, if the diviſion be performed continually by the ſingle numbers 2, and 5, as often as they are involved; that ſo many ſeveral operations will end the diviſion, and the quotient be at an end.

On the contrary; any number P that is prime to 2 and 5, will be prime to 2 × 5 or 10 (Prop. 14). And the ſame being prime to 10, will be prime to 100, 1000, 10000, &c. *ad infinitum* (Cor. 2. Pr. 14); and therefore P can meaſure none in that ſeries. Likewiſe if Q be prime to P, then P will be prime to 10Q, 100Q, &c. (Pr. 14). So that P can ſtill meaſure none in this laſt ſeries. Whence if P divide any of theſe, the quotient will continue without end. Yet the numbers will at laſt circulate, according to Prop. 54.

PROP. LVI.

In any circulating number, the whole circulating or repeating part, running on for ever; is equal to a vulgar fraction whoſe numerator is the number repeating (or the repetend), and denominator as many 9's as there are figures in the repetend.

As in the number 24.35076 5076 5076 5076 *&c.* ad infinitum; 5076 5076 5076 *&c.* $= \frac{5076}{9999} = \frac{564}{1111}$, *in the leaſt terms.*

For let C = whole circulating part, R = repetend or repeating figures 5076; then from the whole circulating part, that is,

from .5076 5076 5076 5076 5076 *&c.* = C,

take .5076 5076 5076 5076 *&c.* $= \frac{1}{10000}$C;

rem. .5076 = R.

But this taking away from C the 10000th part of itſelf, is equivalent to multiplying C by $1 - \frac{1}{10000}$ or by $\frac{10000-1}{10000}$, that, is by $\frac{9999}{10000}$, where there are as many cyphers and 9's, as there are places of figures in the repetend. Therefore $\frac{9999}{10000}C = R = .5076$, and $C = \frac{10000 \times .5076}{9999} = \frac{5076.}{9999.} = .5076\ 5076\ 5076$ *&c. ad infinitum.* And it is evident from the proceſs, that it holds equally for any circulating number.

Cor. 1. *The circulation may be ſuppoſed to begin at any figure of the repetend, and therefore* 24.35076 5076 5076 *&c. for ever, is* $= 24.3\frac{5076}{9999} = 24.35\frac{0765}{9999} = 24.350\frac{7650}{9999} = 24.3507\frac{6507}{9999} = 24.35076\frac{5076}{9999}$ &c.

Cor. 2. *Hence if the repetend be divided by as many 9's as it conſiſts of places; the quotient will be the whole circulating part, or the figures of the repetend, repeated over and over for ever.*

For $\frac{5076}{9999} = C$.

Cor. 3. *And if the whole circulating part be multiplied by a number conſiſting of as many 9's, as there be places in the repetend (conſidered as a decimal); the product will be the repetend.*

For

For 9999 C = 5076, and .9999 C = .5076, the firſt repetend.

Cor. 4. *If any circulating number be multiplied by any given number, the product will be a circulating number; and the repetend will conſiſt of the ſame number of figures as before.*

For in the circulating number 5076 5076 *&c.* every repetend 5076 being equally multiplied, muſt produce the ſame product. And if theſe products conſiſt of more places, the overplus in each being alike, is carried to the next, ſo that each product is equally increaſed, and therefore every four places continue alike. And the ſame holds for any other number. For example, 5076 × 13 = 65988, but the 6 belongs to the firſt place of the next repetend; which being every where added, the repetend now appears to be 5994.

$$\begin{array}{r} 65988 \\ 65988 \\ 659886 \\ \hline 6599459945994 \end{array}$$

But the ſame thing does not hold in diviſion.

Cor. 5. *If you take any prime number (except* 2 *and* 5) *for a diviſor; and by it divide* 1.0000 &c. *till* 1 *remains, or divide* .99999 &c. *till* 0 *remains; the number of cyphers or nines made uſe of, will be equal to the number of figures in the repetend; when the dividend is any number which is prime to the diviſor.*

For in dividing 1.00 *&c.* by any number, when 1 remains, the figures in the quotient begin then to repeat over again, as you had 1 at firſt to begin with. And ſince 999 *&c.* is leſs by 1 than 1000 *&c.* therefore 0 muſt remain here when the repeating figures are at their period. Whatever number of repeating figures we have when this dividend is 1; we ſhall have the ſame number of figures in the repetend, whatever the dividend be, by Cor 4. Therefore altering the dividend at pleaſure, does not alter the number of places in the repetend, the diviſor continuing the ſame; provided the diviſor and dividend be prime to one another. For when the contrary happens, the quotient will circulate in fewer figures.

Cor. 6. *If a circulating decimal has a repetend of any number of figures, it may be conſidered as having a repetend of twice or thrice that number of figures, or any multiple thereof.*

Thus in the number 4.137,37,37, having the repetend 37 of 2 places; it may be conſidered as having the repetend 3737, or 373737; of 4 or 6 places, *&c.*

Cor. 7. *If two or more numbers be added together, that have repetends of equal places; the ſum will have a repetend of the ſame number of places.*

This appears from Cor. 1, and by the reaſoning in Cor. 4. For every column of periods or repetends amounts to the ſame ſum.

PROP. LVII.

If A, B, *be two numbers, prime to one another; and each of them divides a number prime to it, and gives in the quotients two repetends of* C *and* D *places: I ſay, the ſame number divided by the product* AB, *will give a repetend of ſo many places, as is denoted by the leaſt dividend of* C *and* D.

For let N be the leaſt number that C, D, divide; and let $a \times C = N = b \times D$. Now it is plain that a periods of C will end with b periods of D; and therefore they both terminate together after N places, if they begin together, as they may be ſuppoſed to do (Cor. 1. Pr. 56). And they do not end ſooner, becauſe N is the leaſt dividend. Therefore the repetend conſiſts of N places, and no more.

To make it plainer, ſuppoſe $\frac{1}{11\times37}$ or $\frac{1}{407}$ to be the fraction propoſed. Then ſince $\frac{1}{11} = 09$ *&c.*

repeats in 2 places, and $\frac{1}{37} = .027$, *&c.* repeats in three places. And the leaſt common dividend of 2 and 3 is 6, therefore we may ſuppoſe them both to repeat in 6 places (Cor. 6. Pr. 56). And ſince 99 is diviſible by 11; therefore 99,99,99 is alſo diviſible by 11; and ſince 999 is diviſible by 37, therefore 999,999, is alſo diviſible by 37. Therefore 999999 is diviſible both by 11 and 37; and therefore it is diviſible by 11×37 or 407 (Prop. 16). And therefore the repetend of $\frac{1}{407}$ will conſiſt of 6 places (Cor. 5. Pr. 56).

Cor. *If the ſeveral diviſors* A, B, C, &c. *be prime to one another, and repeat in* E, F, G, &c. *places, reſpectively. And if* N *be the leaſt dividend of* E, F, G, &c. *then if the product* ABC, &c. *be made a diviſor, the quotient will repeat in* N *places.*

This follows from Cor. Prop. 16, and the reaſoning in this Prop.

CHAP. IV.

Numerical Problems.

PROBLEM I.

To find the greateſt common meaſure of two or more numbers.

RULE.

TAKE two of the numbers, and divide the greater by the leſſer, and the leſſer by the remainder, and the laſt diviſor by the laſt remainder, and ſo on, till nothing remain: then the laſt diviſor is the greateſt common meaſure of theſe two numbers.

If there be more numbers, take the number laſt found and another of the given numbers, and find their greateſt common meaſure as before: then this is the greateſt common meaſure of the three given numbers. And ſo on. This proceſs is plain from Prop. 10.

Ex. 1.

Find the greateſt common meaſure of 72 *and* 60.

60)72(1
   60

12)60(5
   60

So 12 is the greateſt common meaſure of 72 and 60.

Ex. 2.

To find the greateſt common meaſure of 72,60 *and* 28.

Find

Find 12 the greateſt common meaſure of 72 and 60; then find the greateſt common meaſure of 12 and 28.

12)28(2
24

4)12(3
12

.

So 4 is the greateſt common meaſure of 72,60, and 28.

PROBLEM II.

Two or more numbers being given, to find the leaſt numbers, that have the ſame proportion with them.

RULE.

Divide the ſeveral numbers by their greateſt common meaſure; and the quotients will be the numbers required. By Cor. 1. Pr. 30.

Ex. 1.

Let 12 and 18 be propoſed, then 6 is the greateſt common meaſure, found by Prob. 1.

6) 12 (2 6) 18 (3.

Then 2 and 3 are the numbers ſought.

Ex. 2.

Let 6, 4, and 8 be the numbers given; their greateſt common diviſor is 2.

2) 6 (3 2) 4 (2 2) 8 (4.

Then 3, 2, 4, are proportional to 6, 4, and 8, and the leaſt in that proportion.

PROBLEM III.

Two or more numbers being given, to find out their leaſt common dividend.

RULE.

Take two of the numbers, and divide their product by the greateſt common meaſure of theſe numbers; the quotient is the anſwer for theſe two numbers.

Then take a third number and the laſt quotient, and divide their product by their greateſt common meaſure; and the quotient is the leaſt number which theſe three numbers meaſure. And ſo on.

For let the two numbers be A, B; let P, Q, be the leaſt in that proportion, M their greateſt common meaſure; then $PM = A$, $QM = B$. Then AQ or $\frac{AB}{M}$ is the leaſt number A and B can divide or meaſure.

If you ſuppoſe F to be leſs; let $\frac{F}{A} = G$, $\frac{F}{B} = H$, or $F = AG$ or BH, then by proportion P : Q :: A : B :: AG or BH : BG :: H : G (Cor 4. Pr. 29). But P meaſures H; and Q meaſures G (Prop. 30). And Q : G :: AQ : AG. And ſince Q meaſures G, therefore AQ or $\frac{AB}{M}$ meaſures AG or F; that is, the greater meaſures the leſs; which is abſurd.

And if there be three numbers A, B, C; let $D = \frac{AB}{M}$ be the leaſt dividend of A and B, and let E be the leaſt that C and D meaſure. Then E will be the leaſt that A, B, C, meaſure.

For if you ſay there is a leſs, as F; then ſince D is the leaſt that A, B, meaſure; therefore D meaſures F (Cor. Pr. 11); and ſince E is the leaſt that C, D meaſure; therefore E meaſures F, the greater the leſs: which is abſurd.

Ex.

Ex. 1.

To find the leaſt number which 12 and 15 meaſure, or their leaſt dividend.

The greateſt common meaſure is 3.

$$\begin{array}{r} 12 \\ 15 \\ \hline 60 \\ 12 \\ \hline 3)180(60,\ \text{anſ.} \\ 18\cdot \\ \hline 0 \end{array}$$

Ex. 2.

To find the leaſt number that 12, 15, and 24 meaſure.

60 is the leaſt dividend of 12 and 15. Then the greateſt common meaſure of 60 and 24 is 12.

$$\begin{array}{l} \quad 24 \\ \quad 60 \\ \hline 12)1440(120,\ \text{the leaſt common dividend.} \end{array}$$

PROBLEM IV.

To find out the leaſt numbers continually proportional, as many as ſhall be required, in a given proportion.

RULE.

Find A, B, the leaſt numbers in the given proportion (Prob. 2); then A^2, AB, B^2, will be the three leaſt; and A^3, A^2B, AB^2, B^3, will be the four leaſt numbers. And in general if $n+1$ denote the number of terms required, then A^n, $A^{n-1}B$, $A^{n-2}B^2$, $A^{n-3}B^3$, *&c.* to B^n will be the numbers ſought.

This is plain from Prop. 33. and Cor. 1.

Ex. 1.

To find three the leaſt numbers in proportion as 8 to 12. Two the leaſt are 2 and 3, therefore the 3 numbers are 4 : 6 : 9.

Ex. 2.

To find the four leaſt numbers, as 4 to 6.

Anſ. 8 : 12 : 18 : 27.

Ex. 3.

To find five the leaſt numbers, as 2 to 3.

Anſ. 16 : 24 : 36 : 54 : 81.

PROBLEM V.

Several proportions being given in the leaſt terms; to find out the leaſt numbers that continue theſe proportions.

RULE.

Let A : B, C : D, E : F be the ſeveral proportions;

$$\begin{array}{ccccccc} A & : & B & & & & \\ & & C & : & D & & \\ & & & & E & : & F \\ \hline ACE & : & BCE & : & BDE & : & BDF. \end{array}$$

The ſeveral proportions being placed as in the margin; multiply the two firſt terms A, B, by the leading terms of all the other proportions, C, E; this gives the two firſt terms.

Multiply the latter term D in the ſecond proportion, by ſuch factors as the firſt term C is multiplied by: this is the third term.

Multiply the latter term F in the third proportion, by ſuch factors as the former E is multiplied by, for the fourth term. And proceed thus through all the proportions.

Laſtly, divide all by their greateſt common meaſure, when there is any ſuch. By Cor. 4. Pr. 29.

Ex. 1.

Let the proportions be 6 : 5, and 10 : 9.

$$\begin{array}{rrcrcr} & 6 & : & 5 & & \\ & & & 10 & : & 9 \\ \hline \text{common diviſor } 5) & 60 & : & 50 & : & 45 \\ \text{anſwer} & 12 & : & 10 & : & 9 \end{array}$$

Ex. 2.

Suppoſe 6 : 5, and 4 : 3, and 2 : 7.

$$\begin{array}{lrcrcrcr} & 6 & : & 5 & & & & \\ & & & 4 & : & 3 & & \\ & & & & & 2 & : & 7 \\ \hline & 2\times4\times6 & : & 2\times4\times5 & : & 2\times5\times3 & : & 5\times3\times7 \\ \text{anſ. - -} & 48 & : & 40 & : & 30 & : & 105 \\ \hline \end{array}$$

PROBLEM VI.

To reſolve a number into all its component parts or factors.

RULE.

Divide the number by 2 as oft as you can, then by 3, then by 5, by 7, and all the ſmalleſt prime numbers, till you get a prime number in the quotient. Then you have all the compounding prime numbers, which being continually multiplied, produce the number given. Def. 18.

Ex. 1.

Let 60 be propoſed.

$2)\,60\,\overset{2}{(30}\,\overset{3}{(15}\,(5.$ then $2\times2\times3\times5=60.$

Ex. 2.

What are the component parts of 360?

$$\begin{array}{l} \quad\;\; 2 \quad\;\; 2 \quad\; 3 \quad\; 3 \quad 5 \\ 2)\,360\,(180\,(90\,(45\,(15\,(5\,(1. \end{array}$$

Therefore $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

PROBLEM VII.

To find all the juſt diviſors of a given number.

RULE.

Divide it and all the ſucceeding quotients by the ſmalleſt prime numbers in order, till the laſt quotient be 1. Then you have all the prime diviſors. Then multiply every two together, and every three, and every four, and ſo on. And thus you will have all the compound diviſors thereof.

This follows from Prop. 50.

Ex. 1.

What are all the diviſors of 48?

$$\begin{array}{l} \quad\;\; 2 \quad\; 2 \quad\; 2 \quad 3 \\ 2)\,48\,(24\,(12\,(6\,(3\,(1. \end{array}$$

Then 1, 2, 2, 2, 2, 3, are all the prime diviſors, and 1×2, 1×3, 2×2, 2×3, and $2\times2\times2$, $2\times2\times3$, and $2\times2\times2\times2$, $2\times2\times2\times3$, and $2\times2\times2\times2\times3$; that is, 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48, are all the diviſors.

Ex. 2.

What are all the diviſors of $abbc^3$?

The ſimple diviſors are 1, *a*, *b*, *b*, *c*, *c*, *c*. And all the diviſors will be 1, *a*, *b*, *c*, *ab*, *ac*, *bc*, *abb*, *abc*, *acc*, *bb*, *cc*, *bbc*, *bcc*, c^3, *abbc*, *abcc*, *bbcc*, ac^3, bc^3, $abbc^2$, abc^3, bbc^3, $abbc^3$.

PROBLEM VIII.

To find a number that ſhall have a given multitude of diviſors.

RULE.

Take the powers of as many prime numbers as is convenient, ſo that their indices being each leſſened by 1, and then multiplied together, may be equal to the number of diviſors, I ſay, theſe powers all multiplied together is the number ſought. And the leſſer the primes, the leſſer the number will be.

This is plain by Prop. 50.

Example.

To find a number having 20 diviſors.

Here $20 = 10\times2 = 5\times4 = 5\times2\times2$. Then take a, b, c, d, &c. and any of theſe a^{19}, a^9b, a^4b^3, a^4bc, will do. Let $a = 2$, $b = 3$, $c = 5$. Then 2^{19}, $2^9\times3$, 2^43^3, $2^4\times3\times5$; that is, 524288, 1536, 432, 240, will any of them anſwer the queſtion.

SCHOLIUM.

The number of aliquot parts, being 1 leſs, is found the ſame way. And by this operation it appears how to find all the different ways it can be denoted: which in this example are but four. But any prime numbers may be uſed in each of theſe ways.

PROBLEM IX.

To reduce a given fraction, or a given ratio, to the leaſt terms; and as near as may be, of the ſame value.

1 RULE.

Let A, B, be the two numbers. Divide the latter B by the former A, and you will have 1 for A; and ſome number and a fraction annext, for B, call this C. Place theſe in the firſt ſtep.

Then ſubtract the fractional parts, from the denominator, and what remains put after C + 1, with a negative ſign. Then throw away the denominator, and place 1 and that laſt number in the ſecond ſtep. This is the foundation of all the reſt.

If the fractional parts in both be nearly equal, add theſe two ſteps together; if not, multiply the leſſer by ſuch a number as will make the fractional parts, in both, nearly equal, and then add. And this multiplier is found by dividing the greater fraction by the leſſer, ſo far as to get an integer quotient. When you add the ſteps together, you muſt ſubtract the fractional parts from one another, becauſe they have contrary ſigns.

The proceſs is to be continued on, the ſame way, adding the laſt ſtep, or its multiple, to a foregoing ſtep, *viz.* to that which has the leaſt fraction.

Note. The ratios thus found will be alternately greater and leſſer than the true one, but continually approaching nearer and nearer. And that is the neareſt in ſmall numbers, which precedes far larger numbers: and the exceſs or defect of any one is viſible in the operation.

Ex. 1.

To find the ratio of 10000 to 7854, in ſmall numbers.

	A	B	
1	1	0+.7854	
2	1	1—.2146,	firſt ratio.
	.2146)	.7854 (3	
3	3	3—.6438	
4	4	3+.1416,	2d ratio.
5	5	4—.0730,	3d ratio.
6	9	7+.0686,	4th ratio.
7	14	11—.0044,	5th ratio.
	.0044)	.0686 (15	
8	210	165—.0660	
9	219	172+.0026,	6th ratio.
10	233	183—.0018,	7th ratio.
11	452	355+.0008,	8th ratio.
	.0008)	0018 (2	
12	904	710+.0016	
13	1137	893—.0002,	9th ratio.
	.0002)	.0008 (4	
14	4548	3572—.0008	
15	5000	3927±.0000,	10th ratio.

Explanation.

The ratio of 10000 to 7854 is the ſame as 1 to 0+.7854 or 1 to 1—.2146; here 1 and 1 is the firſt ratio. But 2146 being leſs than 7854, divide the latter by the former, and you get 3 in the quotient, then multiply 1 and 1—.2146 by 3, produces 3 and 3—.6438 as in the 3d ſtep. This third ſtep added to the firſt ſtep produces 4 and 3 for the integers, and ſubtracting the fractional parts, leaves .1416. So

So the 4th ſtep is 4 and 3 $+ .1416$; and the integers 4 and 3 is the 2d ratio. In this manner it is continued to the end; and the ſeveral ratios approximating nearer and nearer, are $\frac{1}{1}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{9}{7}$, $\frac{14}{11}$, $\frac{219}{172}$, $\frac{233}{183}$, $\frac{452}{355}$, $\frac{1137}{893}$, and $\frac{5000}{3927}$. Here $\frac{14}{11}$ is the neareſt in ſmall numbers, the defect being only $\frac{44}{10000}$.

Ex. 2.

To find the ratio of 268.8 to 282 in the leaſt numbers.

$$\begin{array}{l} 2688)\,2820\,(1\frac{132}{2688} = 2 - \frac{2556}{2688}. \\ \underline{2688} \\ \underline{132} \end{array}$$

1	1	1 + 0132,	firſt ratio.
2	1	2 − 2556	
	132) 2556 (19		
3	19	19 + 2508	
4	20	21 − 48,	2d ratio.
	48) 132 (2		
5	40	42 − 96	
6	41	43 + 36,	3d ratio.
7	61	64 − 12,	4th ratio.
	12) 36 (3		
8	183	192 − 36	
9	224	235	, 5th ratio.

So the ſeveral ratios are $\frac{1}{1}$, $\frac{20}{21}$, $\frac{41}{43}$, $\frac{61}{64}$, $\frac{224}{235}$. And the defect or exceſs is plain by inſpection, *e. g.* $\frac{41}{43}$ differs from the truth only $\frac{36}{2688}$ parts; and $\frac{20}{21}$, but 48 ſuch parts.

The reaſon of this proceſs is evident from Cor. 3. Pr. 29. For if the terms of equal ratios be added together, the ſums will be in the ſame ratio.

2 RULE.

Divide the greater number by the leſſer, and the diviſor by the remainder, and the laſt diviſor by the laſt remainder, and ſo on till 0 remain. Then

1 divided by the firſt quotient, gives the firſt ratio.

And the terms of the firſt ratio multiplied by the ſecond quotient, and 1 added to the denominator, gives the ſecond ratio.

And in general, the terms of any ratio, multiplied by the next quotient, and the terms of the foregoing ratio added, gives the next ſucceeding ratio.

Ex. 3.

Let the numbers be 10000 and 31416, or the ratio $\frac{10000}{31416}$.

```
10000)31416(3
      30000
      -----
       1416)10000(7
             9912
             ----
               88)1416(16
                  88
                  --
                  536
                  528
                  ---
                    8)88(11
                      88
                      --
                       0
                      --
```

Then $\frac{1}{3}$ = firſt or leaſt ratio.

$\frac{1\times7}{3\times7+1}$ or $\frac{7}{22}$ = ſecond ratio.

$\frac{7\times16+1}{22\times16+3}$ or $\frac{113}{355}$ = third ratio.

$\frac{113\times11+7}{355\times11+22}$ or $\frac{1250}{3927}$ = fourth ratio.

Ex. 4.

The ratio of 268.8 to 282 is required.

2688)2820(1
2688
132)2688(20
264
48)132(2
96
36)48(1
36
12)36(3
36
0

Then $\frac{1}{1}$ = firſt ratio.

$\frac{1\times20}{1\times20+1}$ or $\frac{20}{21}$ = 2d ratio.

$\frac{20\times2+1}{21\times2+1}$ or $\frac{41}{43}$ = 3d ratio.

$\frac{41\times1+20}{43\times1+21}$ or $\frac{61}{64}$ = 4th ratio.

$\frac{61\times3+41}{64\times3+43}$ or $\frac{224}{235}$ = 5th ratio.

To prove the truth of this rule, let $\frac{10000}{31416}$ be the ratio propoſed; this is reduced to $\frac{1}{3.1416}$. It is plain that $\frac{1}{3}$ is the firſt ratio, or that expreſſed in the leaſt terms. Now inſtead of 3 take $3\frac{1416}{10000}$ or $3\frac{1}{7}$, which is more exact than 3. Then inſtead of $\frac{1}{3}$ we ſhall have $\frac{1}{3\frac{1}{7}}$ or $\frac{1\times7}{3\times7+1}=\frac{7}{22}$ for the 2d ratio. Now inſtead of 7 take $7\frac{88}{1416}$ or nearly $7\frac{1}{16}$, which is nearer than 7. Then $\frac{1\times7}{3\times7+1}$ becomes $\frac{1\times7\frac{1}{16}}{3\times7\frac{1}{16}+1}$ or $\frac{1\times7\times16+1}{3\times7\times16+16+3}=\frac{7\times16+1}{22\times16+3}$ for the third ratio, which is equal to the 2d ratio multiplied by 16, + the 1ſt ratio. Again, for 16 take $16\frac{8}{88}$ or $16\frac{1}{11}$, which will be more exact ſtill; then $\frac{7\times16+1}{22\times16+3}$ becomes $\frac{7\times16\frac{1}{11}+1}{22\times16\frac{1}{11}+3}$ or $\frac{7\times16\times11+11+7}{22\times16\times11+3\times11+22}$ $=\frac{\overline{7\times16+1}\times11+7}{\overline{22\times16+3}\times11+22}$ for the 4th ratio, which is equal to the 3d ratio multiplied by 11, + the 2d ratio. And ſo forward, if there were more.

PROBLEM X.

To reduce a decimal to a vulgar fraction.

RULE.

Place the decimal as a numerator over 1 and as many cyphers as there are figures, for a denominator. Then reduce it to the loweſt terms.

If the decimal circulate, place the figures of the repetend for a numerator, and as many 9's for a denominator: and reduce as before. This appears from Prop. 56.

Ex. 1.

Let .3065 be propoſed.

$.3065 = \frac{3065}{10000}$, divide by 5, then $\frac{613}{2000}$ is the fraction required.

Ex. 2.

To reduce 6.32309309309 *&c.* to the form of a vulgar fraction.

Here 6.32309309 *&c.* $= 6.32\frac{309}{999} = 6.32\frac{103}{333} = 6\frac{32\frac{103}{333}}{100} = 6\frac{10759}{33300}$.

PROBLEM XI.

Having a vulgar fraction given in the loweſt terms, and the denominator a prime (neither 2 *nor* 5); *to find the number of figures that circulate, by dividing the numerator by the denominator.*

RULE.

Divide 9999 *&c.* by the denominator till 0 remains, then the number of 9's made uſe of, will be equal to the number of places in the repetend.

By Cor. 5. Prop. 56.

Ex. 1.

Suppoſe $\frac{287}{37}$, to be given.

```
37)99999(027.
   74
   ———
   259
   259
   ———
```

Here are three nines uſed, therefore the repetend conſiſts of 3 places.

Ex. 2.

Let $\frac{1}{11}$ be propoſed.

11)999(9. Here are 2 nines made uſe of, therefore
   99 the repetend has 2 places.
   ——
   0
   ——

Ex. 3.

Let $\frac{2}{7}$ be given.

7)9999999(142857. Here are 6 nines, and the repetend conſiſts of 6 places.

PROBLEM XII.

Having a vulgar fraction in the loweſt terms, and the denominator made up of two or more different primes (neither 2 nor 5); to find the number of figures circulating, by dividing thereby.

RULE.

Find the number for each ſingle prime in the denominator, by Prob. 11. Then find the leaſt dividend of all theſe numbers, by Prob. 3. And that is the number of figures circulating.

This appears by Prop. 57. and Cor.

Ex. 1.

Let $\frac{13}{11 \times 37}$ be propoſed.

The repetend by 11 conſiſts of 2 places; and that by 37 of 3 places; and 6 is the leaſt number that 2 and 3 divide; therefore if 13 be divided by 407, the repetend in the quotient will conſiſt of 6 places.

Ex. 2.

Let the fraction be $\frac{1}{3\times7\times11\times37}$ or $\frac{1}{8547}$.

The repetend by 3, 7, 11, and 37, is 1, 6, 2, 3, reſpectively; and the leaſt number which 1, 6, 2, and 3 meaſure, is 6, for the number of places in the repetend.

SCHOLIUM.

It is not my deſign here to ſhew the ſeveral ways of working with circulating numbers, or repeating decimals. It is ſufficient for me to explain the general principles thereof; that the reader may have an idea of the nature of them. For almoſt all operations may be as ſpeedily performed by the ſhort rules delivered in multiplication and diviſion of decimals. They that would ſee more of it may conſult Mr. *Cun*'s treatiſe of circulating numbers.

FINIS.

ERRATA.

Page	*Line*	*Read*
79	8	*Ex.* 5.
	17	*Ex.* 6.
	27	In Ex. 5th.
80	18	In Ex. 6th.
	22	*Ex.* 7.