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Mathematics

1. Unit on the circle
2. Teaching Aids for Geometry
3. First Day Display for Geometry
4. Poems on Mathematics
5. Reading in Mathematics
6. Bibliography for Secondary Mathematics Teachers
7. Expansion Levels in Mathematics
8. A unit on the Circle.

W. A. Scott

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File: Mathematics

Office of the Superintendent
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Portland 8, Oregon

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A UNIT
on
THE CIRCLE

Overview

This unit presents the circle, with the lines, angles and arcs related to it, on the tenth or eleventh grade level. It is designed to cover approximately five weeks'work and is usable with most of the newer plane geometry texts. The unit aims to give more emphasis to the position of the circle in nature, mechanics and art without slighting the geometry of a circle as a plane figure.

Submitted by
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OBJECTIVES

I. General

A. Understandings

1. Truth is often approached by subjecting an hypothesis to the battering of time.
   a. Individuals with superior insight and sensitivity become intuitively aware that certain relationships or situations must be true. If by assuming them to be true, accepted facts can be explained; and if no known data conflicts with the assumption, scientists will accept the possible truth as a workable hypothesis.
      (1) The pupil can give examples of hypotheses resulting from intuitive insight such as:
         (a) Matter is composed of continually moving molecules held in relative position by the interaction of forces.
         (b) An electric current consists of a flow of electrons.
         (c) Human happiness is most dependent upon inner attitude, etc.
   b. If no evidence of the future is contrary to the hypothesis, the hypothesis may be accepted as truth.

2. Conclusions are arrived at through a process of logic
   a. The pupil can pick out and state assumptions, the truth of which is too apparent to require proof.
   b. The pupil can state the conditions existing as true in any given situation
   c. The pupil can build upon the given and prove the truth of other existing relationships in harmony with the basic assumptions.

3. Conclusions approach truth in proportion to the truth of the basic assumptions and the correctness of the logic.
   a. The pupil can start with simple, rather insignificant assumptions he knows to be false and by correct logic arrive at important conclusions which are strikingly not true.

4. New data may contradict assumptions accepted as axiomatic and cause all thought structures built upon them to totter.
   a. Pupils can recall the shift of thinking and methods of solving world and social situations when such ideas as follow were demonstrated to be false.
      (1) The earth is flat.
      (2) Evil spirits cause disease.
      (3) The white race was made to rule.
      (4) The sun revolves around the earth.
      (5) Kings receive their right to rule from God.
   b. Pupils speculate upon the affect upon present scientific framework of giving up assumptions that:
      (1) Matter and energy are distinct.
      (2) Light travels in straight lines.
      (3) Distance between two fixed points is constant.
      (4) Time can go only forward, etc.

5. Understanding requires that all words used have exact meanings.
   a. The pupil can discover contradictions in single statements in advertisement and can reword them to avoid ambiguity.

6. Intuition often leads to false conclusions.
   (The pupil may be shown this in a manner such as used in the introduction to "The Education of T. C. MITS." by Lieber

7. Statements are not always true in converse.
   a. The pupil can distinguish between a statement and its converse.
   b. The pupil can state the converse to given truths.
   c. The pupil can give sets of statements and converses which are not true.

8. Form occurs in nature because of properties inherent in the form.
   a. The pupil can give examples of geometric form in nature and give the functions in the fulfillment of which the form evolved.
   b. The pupil recognizes that form in non living matter results from an interplay of forces.

9. As man discovers the inherent properties of form, he can put figures to his own use related to those properties.
   a. The pupil can name geometric form which man has put to use and tell wherein man takes advantage of the properties of the form.

10. Mathematics has developed with civilization
    a. The pupil realizes that man developed mathematics gradually in response to the pressure of his environment

B. Attitudes and Appreciations

1. Pupils distinguish between conclusions based upon emotion and reason and is aware that the former must be avoided as unreliable.

2. The pupil looks at form in the world of living and non living matter seeing it not as just existing but as resulting from function, in living matter, and action of forces, in non living matter. His whole view of matter is thus colored.

3. The pupil enjoys expressing geometric relations in algebraic symbols.

4. The pupil gets satisfaction in noting the use of geometric figure in architecture and design and can discover geometric figure as subtle directors of accent and line in painting.

5. The pupil shows satisfaction in good form in work of others and his own.

6. The pupil gets satisfaction in precise use of words.

C. Skills and Abilities

1. The pupil expresses his thought in exact and concise language.

2. The pupil spells correctly the words associated with the area covered by the unit.

3. The pupil constructs geometric figures related to the unit.

4. The pupil expresses geometric relationship in algebraic symbols.

5. The pupil sees and states relations correctly and draws conclusions logically.

6. The pupil distinguishes between good and bad argument.

II. Specific

A. Understandings

1. The pupil recognizes that a circle is produced when --
   a. A plane cuts a cone at right angles to the axis
   b. A plane cuts a cylinder at right angles to the axis
   c. A plane cuts a sphere at any angle
   d. A point moves in a plane so as always to be a given distance from a fixed point.

2. The pupil sees that in nature a sphere is produced when matter is suspended in fluid state and cohesive forces are greater than adhesive forces.
   a. He recognizes the earth, raindrop, oil droplet, etc., as examples.

3. He recognizes a sphere as presenting the least surface for a given volume.
   a. He again sees earth, raindrop, drops of mercury, etc., as examples.
   b. He realizes that this property of least exposed surface is an advantage to living protoplasm
      (1) Protoplasm forms spherical spores
      (2) Seeds often spherical
      (3) Fruit most often spherical

4. The pupil accepts the reasonableness of a circle, as a projection of a sphere upon a plane, showing properties similar to those of a sphere.
   a. He accepts the circle as a symbol of inner unity.
   b. He realizes a circle shows the least circumference for a given area or the greatest area for a given circumference.

5. The pupil recognizes that a circle represents a type of symmetry in which any line through the center is an axis of symmetry and in which there can be rotation without change of position.
   a. He realizes that this symmetry is evidenced in simple animals of Phyla Porifera, Coelenterata and Eshinodermata.
   b. He realizes that this property makes a wheel so useful as a basis for a vehicle.

6. The pupil sees a circle as a limiting case of a regular polygon as the number of sides approaches infinity.

7. The pupil is aware that rolling friction is much less than sliding friction and that the wheel, as a circle, takes advantage of this. (Wheel vs. runner, ordinary bearing vs. ball bearings)

8. The pupil understands certain relationships which exist between variables related to a circle.
   a. The length of a chord decreases as it moves further from the center.
   b. Arcs are measured in terms of central angles and are not proportional to the length of the curve.
   c. The circumference varies directly as the radius. (May be left until later)
   d. The area varies directly as the square of the radius. (May be left until later)

9. The pupil understands that direction of motion along a circle at any given point is along the tangent to the circle at that point.

10. The pupil realizes that a circle is definitely determined if:
   a. The center and radius are given
   b. Three points through which it must pass are given.

B. Attitudes and Appreciations
   1. He sees the appropriations of a circle as a symbol of continuity
      a. As used in the wedding ring
      b. As used in primitive language to represent God
      c. As used to represent biological cycles (CO_2- O_2) or life cycles of plants.
      d. As representing repeating cycles in motors, etc.
      e. As used to designate time.

   2. He enjoys design based upon circles as found in windows, floors, arches, bridges, towers and tunnels.

   3. He recognizes the influence of the properties of the circle as used in the wheel upon the advancement of civilization.

   4. He values neatness and form.

C. Skills and Abilities
   1. He is able to name and define all lines, angles and curves related to the circle.

   2. He is able to recognize and state relations within the circle
      a. Between angles and arcs
      b. Between length of chord and distance from the center
      c. Between chords and arcs
      d. Between tangents and radii

   3. He can illustrate the relations which might exist between two or more circles, i. e., externally tangent, internally tangent, concentric.

   4. He is able to make accurate constructions related to one or more circles.
      a. Bisect arcs, chords, angles
      b. Inscribe and circumscribe regular polygons of 3 sides, 4 sides, 6 sides, 8 sides
      c. Circumscribe a circle about or inscribe a circle in a triangle.
      d. Construct tangents to circles from outside points or at a point on the circle.

5. He can apply all understandings of relationships to specific, numerical problems.
6. He can copy various designs based upon circles.
7. He originates his own designs based upon circles.
8. He can prove in formal demonstration various theorems related to the circle.
9. He speaks clearly and explains while he draws or writes in board demonstration.
10. He can spell all words related to geometric terms.

APPROACH TO THE LEARNING SITUATION

The suggested approach material includes much more than it would be either necessary or desirable to use with any one class. The choice would be directed by the teacher's knowledge of the background and interest of the group, by the equipment at hand and by the arrangement of the classroom. Suggestion "III" could be affectively handled only if the room contained tables or work space other than small desks. Such material as not worked into the approach can be anticipated as a probable part of the learning activities.

At all times in work with students, whether in approach or learning activities, it is understood that desirable responses or suggestions which do not come spontaneously from the group should be drawn from them in appropriate manner. The more subtly this can be achieved, the more the students will retain their feeling of self direction. This feeling is, of course, both a means and an end in unit teaching.

The need to include certain activities in a fairly definite order in the learning experiences, even if they must be suggested by the teacher, is greater in mathematics and science than in English or in the social studies. In the former fields a definite body of understanding and skills often comprehended only in a specific sequence must be held by every member in the group. This might mean more teacher direction than is ideally desirable.

Evaluation, through observation of what the student does and how he says what he says, is a continuous process. The results should determine the direction of teacher influence upon subsequent activity.

I. Through design - Windows, floor, metal, leather, etc,

Display pictures selected to include combinations of circles and other geometric forms such that if a student desired to duplicate them, he would need to be able to:

1. Find the center of curvature of an arc.
2. Draw a tangent to a circle at a point on the circle
3. Draw circles tangent to each other
4. Draw concentric circles

5. Inscribe or circumscribe regular polygons

6. Divide a circle into equal arcs

7. Make equal arcs on equal circles

8. Bisect arcs

The better students will remember enough from work with the circle in Math 1 so that discussion can lead the students to list the understandings and skills which they would need to have in order to use the circle in design.

The first felt need should be that of gaining vocabulary so that further discussion could proceed more intelligently.

II. Through examining or listing articles usually constructed in circular shape or the cross-section of which is a circle. This includes cans, dishes, clocks, dials, wheels, roller bearings, etc. Discussion should lead to the advantages obtained in each case because of the properties of the circle. Properties such as:

1. Circle encloses greatest area with given perimeter--economy in construction when given volume can be obtained with least surface.

2. Circle is divided into equal parts by equal periods of revolution.

3. Circle withstands stress with certain type of reinforcement for rigidity.

4. Circle presents possibility for reducing friction both in a single wheel vs. sliding friction or in roller bearings vs. plain bearings.

The above might be handled somewhat as follows:

A. Hold up a can from which the top has been removed with the question, "Why are the cans on the grocer's shelf shaped like this?" Speculation should end in

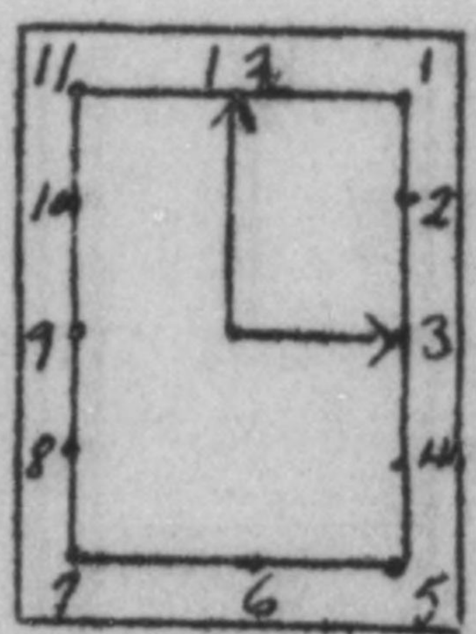
1. Filling can with liquid and by transferring liquid into several other rectangular containers; noting height at which it stands and then determining the surface area of all the containers of equal volumes; finally discovering the cylinder gives least surface. (If a spherical container is available the discovery could be extended to show that the sphere gives the least surface for a given volume. Discussion could bring out why a sphere is not used for containers.)

2. Noting fewer points of vulnerability from bump ing, etc., less apt to loosen solder, etc.
   a. Noting that this property of the solid is also a property of the base. The circle gives the advantage of fewer points of vulnerability. This quality is taken advantage of by man in making dishes and bottles circular, by nature in seeds and fruits.

B. Try to get emotional reaction to drawings of polygons and circles. Which tends to most keep eye and attention directed toward a single point? See if some students get the feeling of the circle as most representing a whole, with every point drawn toward the center, where as in a polygon lines of sight branch outward in direction of sides. Circle is thus a symbol of inner completeness, self sufficiency (symbol for God). Forces within circle direct toward the center as the cohesive forces in cooling masses like the earth; or in liquids suspended in fluids like rain drops in air; oil in water, mercury, etc.

Note that as a self centered unit a circle is not so good as used in flower beds in the midst of rectangular lawns. To carry feeling from a circle outward into surrounding area or space and cause it to be part of a larger unit requires radiating lines. (Circle with rays may be a better symbol for God--a force of great central unity extending its sphere of influence in every direction.

C. Have sketch such as below on board with question, "Why don't we make a clock face like this?"



Students, of course, state that all points wouldn't be same distance from the center and that equal distances between numbers wouldn't make equal divisions in time. Students are thus reminded that in using form man utilizes the properties of that form.

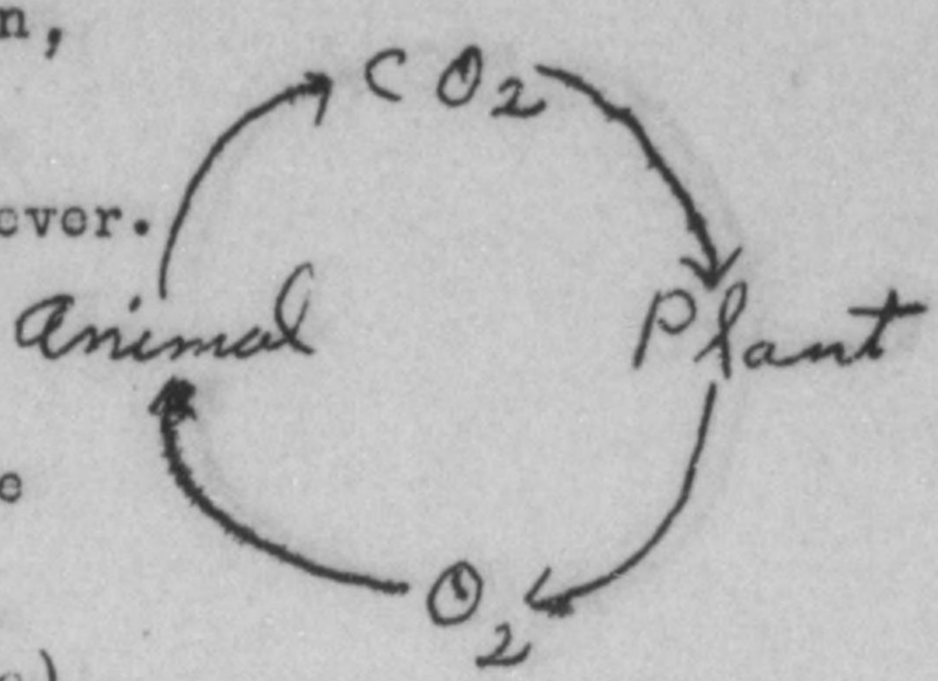
Discussion can direct students to list other places in which man uses the circle along with the properties most useful in each. Someone will present wheel as man's greatest use of circle and the students may find difficulty in stating the properties used. Have students demonstrate the properties as follows:

1. Have three equal masses, one spherical, one cylindrical, one rectangular arranged with attachment for pulling with spring balance. Have students discover that the force required to pull a given mass varies as the contact surface.

2. Have two elliptical or square substitutes for wheels joined by an axle through center and two wheels similar ly joined. Have students note that the axle of the wheels remains always the same distance from the surface over which it moves. Bring the students to state that the path of the center of a rolling wheel is a line parallel to the surface over which it moves and a distance equal to the radius above that surface.

Discussion should also bring out the fact that in using form for desirable properties, man must often offset undesirable qualities. Students should bring up and try to answer questions like, "Why spokes on a wheel?" "Why attach structures like mud guards and fenders on wheels?" Students will bring out fact that direction of movement at any point is tangent to the circle at that point.

D. A diagram of a simple cycle such as this can be placed on the board with the question, "why are cycles so often represented by using a circle?" Students should bring out fact that the exchange of gases goes on forever. (Circle is a symbol of continuity--without beginning, without end). What is the only thing which man's philosophy considers as without beginning or end? (God) (Therefore primitive symbol represented God with a circle). What human social relationship is supposed to continue without end? (Marriage) Therefore what is the symbol of marriage? (Plain ring!)



This, and more, can be pulledtogether with a suggestion from someone that, since we use circles in so many ways, it should be worth while getting in mind the properties of a circle which makes it so useful in art and mechanics and nature.

III. Arouse interest in the circle by use of displays with proper questions. Some of these could be placed at tables in the room to be used only by the group at that table. Others could be posted so as to be accessible to all. Suggestions follow:

A. Pictures -- Rose window, bridges, etc. with questions, "What properties of a circle are used in these?" "What would you need to be able to do with a circle in order to copy this?"

B. Models from molding clay--a sphere, a cone, a cylinder--and a knife. Also a sign requesting that each student cut two plane cross sections, one which would and one which would not produce a circle.

C. A sphere, a cylinder, and a rectangular block of equal weight with attached device for pulling with a spring balance. The accompanying request could be, "Discover one reason why wheels are so valuable."

D. Pictures of animals such as:

1. Sponges

2. Coelenterates -- hydra, anemone, jelly fish, etc.

3. Echinoderms -- sea urchin, sand dollar, star fish

Question-- "What characteristic due to form is common to all these animals?"

E. Horizontal cross sections of fruit such as apple, orange, lemon, poppy pod and/or other large seed pods showing divisions of circle with lines radiating from center.

Questions, "What are the advantages to the plant of the arrangements common to these fruits?" "What characteristics of a circle produce these advantages?"

F. A clock with circular face, a set of roller bearings, a system of cogged wheels.

Questions, "Why is a circle used in the dial?" "Why are spheres used in the bearings?" "Why are circles used in the cogged wheels?"

G. A poster with a large circle with several lines and angles, such as radius, diameter, chord, arc, tangent, and secant; all numbered.

Question, "How many of these parts can you name?"

H. A card with such questions as:

1. Why are plates round?

2. Why is a can cylindrical?

3. Why is a ring an appropriate part of a marriage ceremony?

4. If you wanted to enclose the largest garden with a given fence, what shape would you make it?

I. A wheel that can be turned (such as a small grind stone) by water, or a small weight fastened to a string. A card suggests that with the above equipment the students discover the direction of motion at any instant on a revolving circle. Also the question, "How does the principle you discovered apply to a set of cog wheels?

Interest aroused by these approaches should lead, some more readily than others, into the circle unit. Such of this as is not used as approach material and is related to the objectives should be included at appropriate times during the progress of the unit.

PROBABLE LEARNING EXPERIENCES

I. The students will list the things they would have to learn about the circle in order to understand why and how it is used in the way it is. One member of class (chosen by them) would first put them down in whatever order they were given. Later the group would make an attempt to list in the order of attack. They would probably arrange the first few somewhat as follows:

A. Review vocabulary associated with circle.

B. Discover some relationships between the lines, arcs, and angles associated with the circle.

C. Learn how to (make) construct accurately

1. Tangents

2. Chords in certain positions

3. Equal arcs on equal circles (or same circle)

4. Inscribed and circumscribed polygons of 6, 3, 12, 4, 8 sides.

D. Find out if Portland has any striking examples of circles in church window design or bridge or architecture construction.

E. See if any striking examples of isolated circles in lawn design are in Portland.

F. See if they can construct original designs based upon the circle.

G. Etc.

II. The students will organize their activity. They will probably suggest that:

A. Every one learn the vocabulary

B. Every one discover experimentally certain relationships.

C. Everyone prove the relationships true in a series of theorems.

D. Certain ones--groups or individuals--locate places in Portland where circle is used.

1. In design in windows, floor coverings, bridges, arches, etc.

2. In landscaping at Lambert's gardens, sunken gardens, Washington Park, private residences--noting emotional response to same.

3. In quilt and other needle art designs

4. In pictures at art museum or store displays

5. In design books in library

They will organize for excursions in and out of school time, usually in groups of two. They will set time limits for reports on findings. (After reports, if any of special note are discovered, they will arrange for larger groups to visit location)

E. They may divide into groups and prepare final roundup demonstrations showing the properties of a circle as illustrated in nature and mechanical uses and as proved through geometric reasoning. Six possible ones are listed below.

1. Circle as a <u>wheel</u> - properties used.

a. Contact in one point--little friction. (Demonstrate with weight solids of varying contact surface as indicated in approach)

b. Center (axle) always same distance from point of contact. (Demonstrate as in approach)
c. Direction of motion at any point is in direction of tangent to the circle at that point.
   (1) Used in cogged wheels
   (2) Counteracted in banking of curves
   (3) Made unobjectionable with use of mudguards and fenders on cars
   (4) Demonstrated in variety of ways
      (a) Small revolving wheel upon which liquid can be placed--like grind stone
      (b) Small weight on string, revolved and then string released
      (c) Horizontal wheel arranged so it can be turned with considerable speed is placed above a larger paper upon which marks can be made.
      Small objects such as pith balls are dropped upon wheel, forced to edge and then off when wheel is revolved. The observed position of leaving wheel and crossing edge of paper below are connected by a line. Barring error of observation, this should be tangent to the wheel at point where object left wheel.
d. Wheel has neutral equilibrium--it therefore comes to rest in any position, since its center of gravity is neither raised nor lowered by motion and requires same force to move it from any position.
   (Student could demonstrate the types of equilibrium with advantages of each as is done in "Modern Physics" by Charles E. Dull, pgs. 129 and 130, or as can be found in most physics textbooks.)

2. Circle in design
   a. Pupils display pictures of circles in design which they have found. Each states what the design must have been able to do with circles to have created the design.
   b. Pupils display original designs and state the geometric properties of the circle which were used in the construction.

3. Circle in nature
   a. Sphere produced in non living matter as a result of cohesive forces
      (1) Pupils display oil suspended as droplets in water
      (2) Molten lead dropped into cold water through sieve
      (3) Mercury separated into small amounts
      (4) A soap bubble--a toy balloon
      (surface tension produces solid containing the given volume in the least surface)
   b. Pupils display cross sections of fruits showing divisions radiating from center--lemons, oranges, apples, lily pods, poppy pods, etc. (Nourishment, to maturing seed, and protection best given by circular form with radiating carpels.)

4. Circle and geometric proof
Students will decide upon one of truths related to circle which can be demonstrated as true in particular cases by actual measurement and then proved true under all situations by formal reasoning.

5. Circle as a cross section of certain solids
Students demonstrate with molding clay models of spheres, cones and cylinders how circles may be produced as plane sections of each.

6. Circle shows least perimeter for given area.
Students could take several equal masses of molding clay and shape various figures all of equal thickness (could be circular, rectangular, square, five or any number of sides.) Perimeter of each could be measured and circle verified as least for all considered.

Some of this demonstration work may be a repetition of material in the approach but it will be more meaningful since students have acquired supposedly a background for understanding.

The students will organize for demonstrations and set up standards. This planning, or at least the detail of it, might be best carried on after the unit has progressed two or three weeks. The suggestion that this be done should come from the group. At least one class period several days before demonstrations come off must be used to check with each group to see that each has a definite plan and knows where to get material needed. Another period should give each group a chance to go through its demonstration for itself before the final demonstration for the entire class.

III. The pupils will attack, with the help of the teacher, the learning activities they suggested to be carried on by all under II "1", "2" and "3".

A. The student will construct circles and will note:
1. It is determined by a given center and radius
2. It is the path of a point which moves so that it will remain a distance "r" from a fixed point, the center.

B. Students will review names applied to lines and angles associated with the circle--radius, diameter, chord, tangent, secant, central angles, inscribed angles, angles between two chords, two tangents, two secants, a chord and a tangent, a secant and a tangent. They will draw (not construct) each.

C. Students will discover experimentally (that is (1) by constructing the first situation true and (2) observing that as far as crude measurement is concerned, the second is true) That in a circle:
1. Equal central angles have equal arcs
2. Equal arcs have equal central angles
3. Equal chords have equal arcs
4. Equal arcs have equal chords
5. Equal chords are equidistant from the center
6. Chords equidistant from the center are equal
7. A tangent to a circle is perpendicular to the radius at point of contact.

8. A line perpendicular to a radius at its extremity is tangent to the circle.
9. Tangents to a circle from an external point are equal and make equal angles with the line joining that point to the center.
10. If two circles intersect the line joining their centers is the perpendicular bisector of the common chord.
11. Parallel lines intercept equal arcs on a circle.

(It may be advisable to take "1" through "6" in one group and return to the experimental work with the others after geometric proof and application of "1" through "6")

As each of these is verified experimentally, it is added to the list of apparent truths worthy of being proved.

D. The pupils will arrange what seems to them a reasonable order of attempted proof. Perhaps this order will be altered as proof progress and it is discovered that certain ones need to be based upon the acceptance of others not yet proved.

E. Proofs are developed for major truths (theorems) and closely associated corallaries.

F. Applications are solved in connection with each theorem or group of theorems. (Most of our modern texts have many from which to choose.)

IV. Reports by special committees are called for by chairmen at appropriate intervals.

V. The culminating activity of group displays and demonstrations is carried on.

EVALUATION

I. Specific Understandings

A. Have students select from several clay models those from which a circle can be produced by cutting along a plane. Have another student describe how such cutting should be done. Have still a different student make the cuts.

The group which shows how circles may be produced as part of its final demonstration can be judged by observing the ease and accuracy with which the members produce, and explain the production of the circular sections.

B. and C. For the group whose final activity is directly related to these objectives, the evaluation would be based upon judging how adequately the points given under these objectives are presented to the group.

A question such as follows could be included in a test.

"Name five places or situations in nature where a circle or a sphere is produced and state the characteristics of the circle or sphere which causes it to be the existant form in each case.

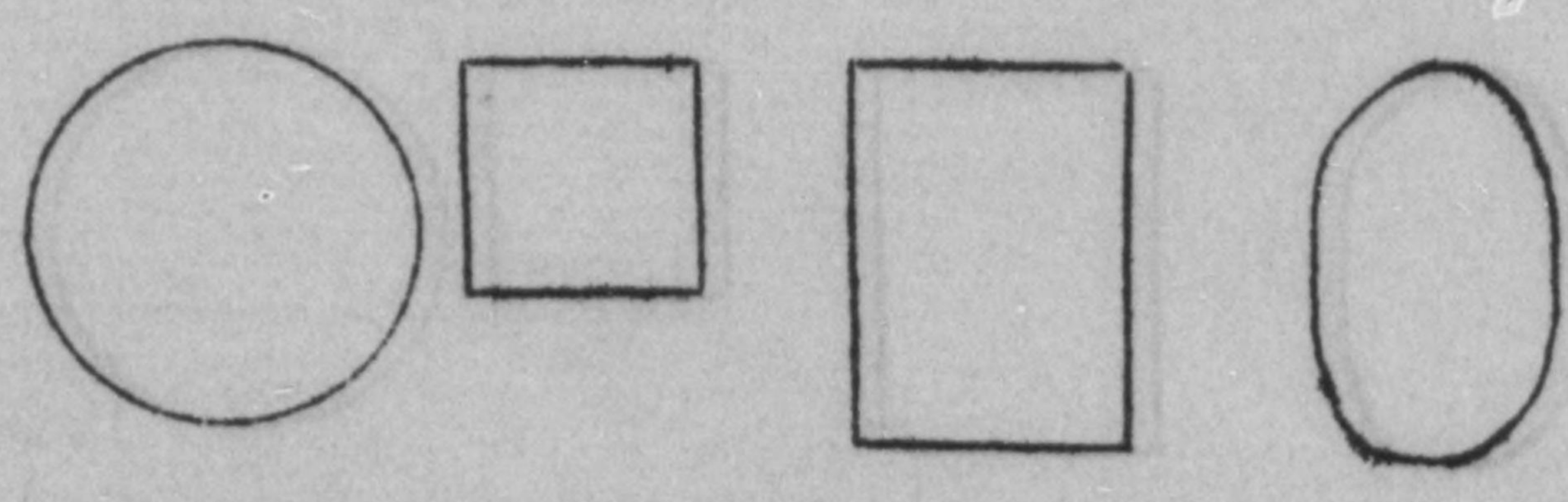
D. "D1" is difficult to evaluate. When reports are given upon circles in design or in landscaping or when cross sections of spherical fruits are examined, one can watch for such comments as, "That circle doesn't tie in with the rest of it." "That circle is too complete in itself." "Each ring seems so finished."

In the learning activity associated with "D2", each student could be handed four equal lengths of string (8 in. to 16 in.) and a couple dozen pins with the suggestion that each shape four plane figures having equal circumferences and having areas which the student can determine. Each is to discover which gives the greatest area for the given circumference. (The students who assist in directing this activity should demonstrate, to the teacher, before time that they can carry it to completion.) The evaluating would be done at the time of the activity by observing the reaction of the youngsters to the results obtained.

A question such as "If you wish to enclose the largest possible grazing area with a given amount of fence, what shape would you make it?"

E. Include questions such as follows in tests.

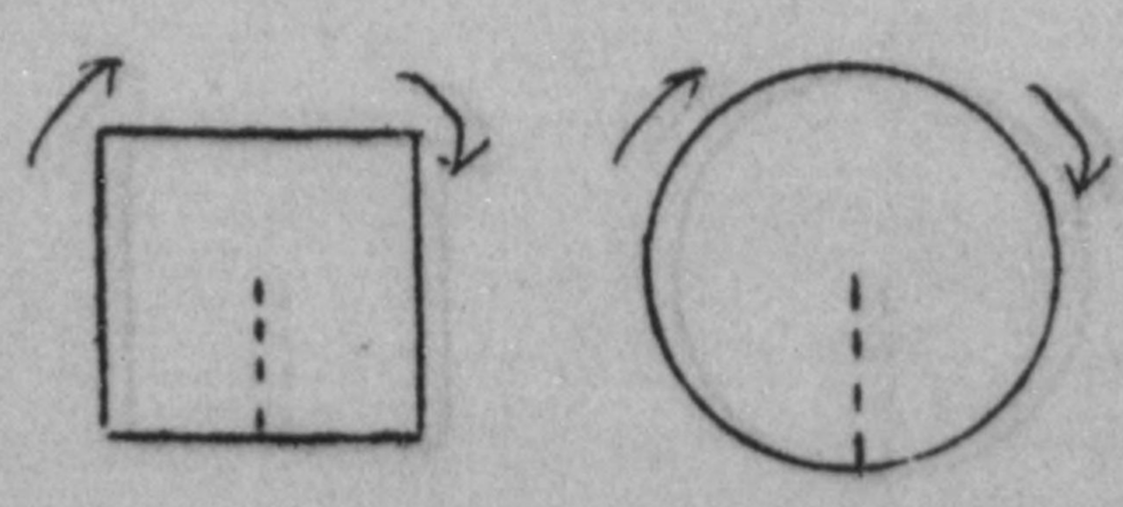
1. Draw the axes of symmetry in each of these figures. Indicate with words if the figure is not adequate.



2. Indicate the center of gravity of each of the above figures.

3. Which of the above shapes has its center of gravity neither raised nor lowered if it is rotated?

4. Place the square in a position from which it could be more easily rotated than a circle.



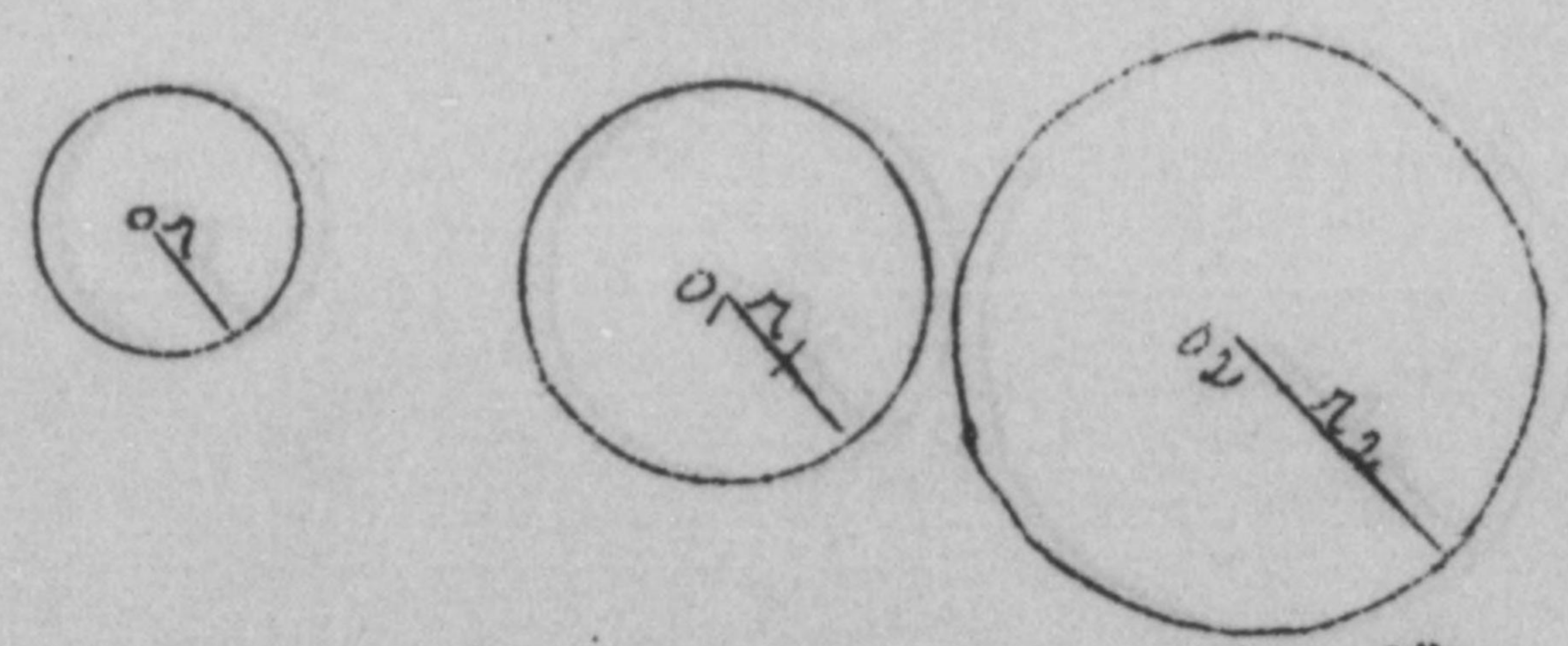
5. Name two animals which show the same type of symmetry as a circle.

F. Include such questions as the following in a test?
1. Show by means of a drawing (it need not be constructed accurately) how a regular polygon becomes more and more nearly a circle as the number of sides increases.

G. Observe the interest shown in the spring balance readings during demonstration. Note which students associate the results with the use of ball bearings or with the advantage of runners over wheels on ice.

When asked to list the properties of a circle made use of in the wheel, note how many remember that a circle has only one point of contact with a plane surface over which it moves and as a result a wheel has little friction.

H. Include test questions such as:
1.



Given: 3 circles "O", "O_1", "O_2" with $\begin{cases} r = 1" \\ r_1 = 2" \\ r_2 = 3" \end{cases}$
with circumferences C, C_1, C_2
and areas A, A_1, A_2 respectively
C_1 = how many times C ?
C_2 = how many times C ?
A_2 = how many times A ?
A_2 = how many times A_1 ?

2. Given circle "O" with radius equal 5 inches. How many chords can be drawn 4 inches from O?
Arrange in order of size, beginning with the smallest, chords with distance from the center 3", 4", 2" 1", 4½" respectively.

3. Show by a sketch that arcs of circles may have the same number of degrees but not the same length.

4. If you know an arc of a circle is 5 ft. long, what else must you also know before you can draw the arc?

I. Students who share in the demonstration related to this objective will display understanding (or lack of it). Watch others for their eagerness to give illustrations such as--a car which goes off a curve goes in a direction tangent to the curve, youngsters thrown off the centrifugal wheel in the fun house go in a direction tangent to the wheel.

J. At any time one student is directing another to draw a circle, be sure he gives adequate data to definitely determine the circle.

II. Attitudes and Appreciations

A. Observe expressions and attitudes when these points come up in the discussion. Look for evidence of special interest and eagerness--a lighting up of the eyes, an appreciative smile.

B. Note whether or not pupil makes more than the required number of designs or gets satisfaction from the balance or intricacies of pattern of those which he finds.

Those who belong to the group which investigates the circle in design can be judged by their degree of enthusiasm for and delight in the picture, structure or what ever they have found about the city.

C. Evaluate by observation as in "1".

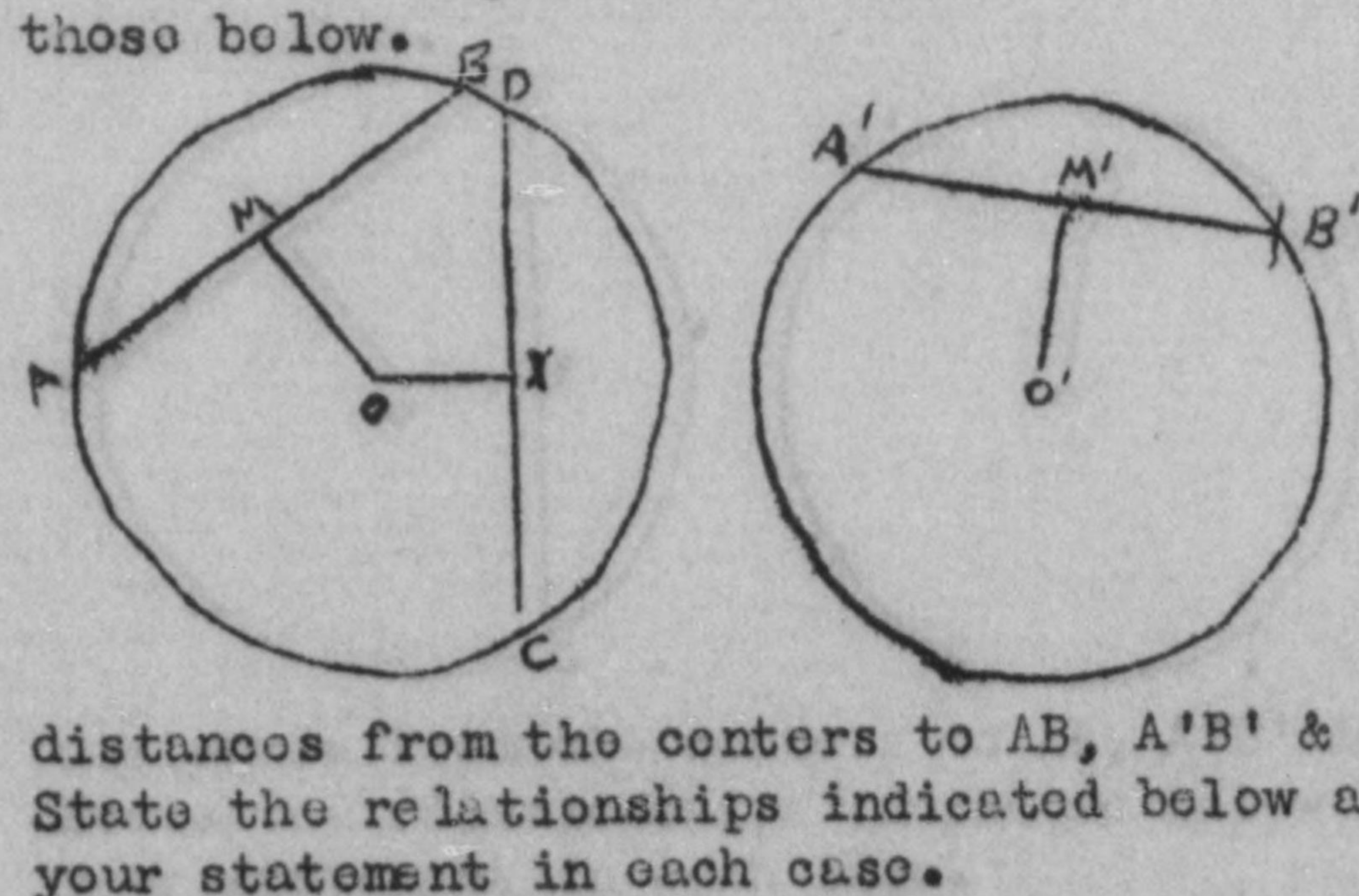
D. Note the degree to which the pupil gets satisfaction from turning in neat work and comments favorably upon work of others which shows special neatness or individual and appealing form.

III. Skills and Abilities

A. Pupils could be asked to make a labeled drawing which would include all the parts listed on the board or could be asked to name all the parts numbered in a drawing on the board.

B. & C. Note the responses to a group of questions and problems such as those below.

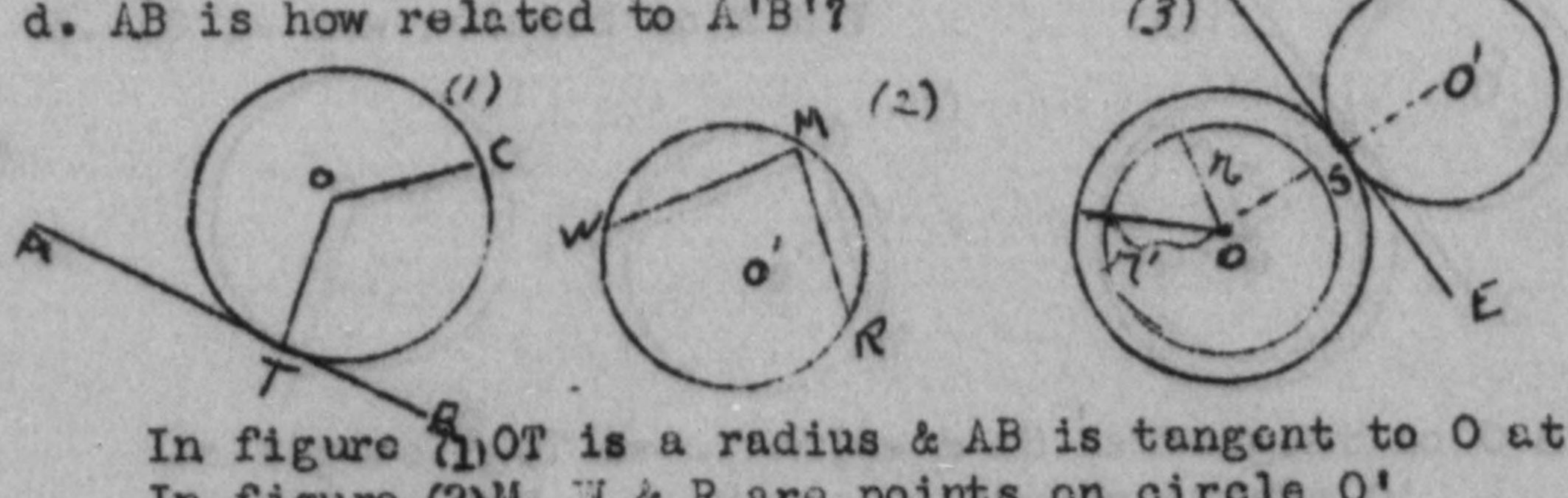
1.



In these figures ⊙ "O" ⊙ "O'" AB = A'B' OX < OM OM, O'M' & OX represent the distances from the centers to AB, A'B' & CD respectively. State the relationships indicated below and <u>give the reason</u> for your statement in each case.

a. OM is how related to AB?
b. OM is how related to O'M'?
c. A'B' is how related to DC?
d. $\overset{\frown}{AB}$ is how related to $\overset{\frown}{A'B'}$?

2.



In figure (1) OT is a radius & AB is tangent to O at T
In figure (2) M, W & R are points on circle O'
In figure (3) O is the center of the circles with radii

"r" & "r'"; radii OS & O'S are both ⊥ to DE at S.
Fill in the blanks in the following statements and give the reason in each case.
a. OT is ________ to AB
b. ∟TOC ________ TC
c. Angle WMR is inscribed in ________.
d. Angle WMR intercepts ________.
e. Arc MR is ________ by chord MR.
f. In figure (3) O & O' are ______ ______ at ______.
g. The circles with radii r & r' are ______ circles.

D. The students make the constructions listed and the teacher notes the degree of accuracy.

E. Include in a test numerical problems of the types included in class discussion.

F. Note the sureness and accuracy with which the pupil recognizes what needs to be constructed, lays out the order in which constructions should be made and then carries on the required constructions in copying a design.

G. Note to what extent the designs produced by the student are his own creations. Watch as many as possible if they are making designs during class time. Listen for such comments as "This came out of my very own head." on out of class constructions.

H. Observe the ease with which each:
1. States assumptions and situations to be proved
2. Outlines plan of attack
3. Goes through a logical sequence of statements
4. Justifies each statement with appropriate reasons

Make these observations when student is:
1. Assisting the group in developing the proof for a theorem or problem
2. Demonstrates a proof at the board
3. Writes a complete proof without help in class.

I. Make sure that each pupil can be heard clearly in all parts of the room. Observe whether or not he uses the device of explaining what he is doing and why as he draws lines or writes statements in board demonstrations.

J. Check the spelling of all geometric terms in all written work.

BIBLIOGRAPHY

Student

* Bakst, Aaron, Mathematics, Its Magic and Mastery, Van Nostrand Co., 1941

Bartoo and Osborn, Plane Geometry, Webster Publishing Co., 1939

Berkhoff and Beatley, Basic Geometry, Scott Foresman and Co., 1941

* Fawcett, Nature of Proof, 13th Year Book, Bureau of Publications, Teachers College, Columbia University

Herberg and Orleans, A New Geometry, D. C. Heath, 1940

Hogben, L. T., Mathematics for the Million, Norton, 1937

* Hudson, Hilda, Ruler and Compasses

* Lieber, H. G. and Lieber, L. R., The Education of T. C. Mits

McCormack, Joseph P., Plane Geometry, Appleton Century, 1940

Morgan, Foberg, Beckenridge, Plane Geometry, Houghton Mifflin, 1943.

* Nyberg, Fundamentals of Plane Geometry, American Book Co., 1944

Schorling, Clark, Smith, Modern School Geometry, World Book Co., 1938

Seymour & Smith, Plane Geometry, MacMillan, 1942

* Strader, Rhoads, Modern Trend Geometry, Winston, 1940.

* Sykes, Mabel, Problems in Geometry

* Welchons and Krickenberger, Plane Geometry, Ginn & Co., 1940

Bower and Robinson, Dynamic Physics, Rand McNally, 1942

* Dull, C. E., Modern Physics, Holt, 1939

* Curtis, Caldwell & Sherman, Everyday Biology, Ginn & Co.

Ritchie, Biology and Human Affairs, World Book Co.

Smith, Ella Thea, Exploring Biology, Harcourt, Brace & Co.

Almost any physics text for the secondary level and any biology text which includes invertebrate classification could be substituted for those listed.

* Indicates those regarded as best of ones listed.

MATHEMATICAL SKILLS

ELEMENTARY TRIGONOMETRY -
Scale drawings. - Similar geometric figures
Ratio and trigonometric functions

ELEMENTARY ALGEBRA:
Four fundamentals using signed numbers.
Theory of equations.
Equation building and solution from problem situations.

ELEMENTARY GEOMETRY:
Formula
Geometric construction of more usual figures.
Evolution of the theorem.
Some primary geometric laws and their use in proof.

TAKE OFF

EXPANSION LEVELS
Group "B"

MATHEMATICAL SKILLS - ADVANCED COM... PROB...

INDIVIDUAL & IMMEDIATE COMM.

INDIVIDUAL
FAMILY
HOME CASES

Level of minimum requirement-

CENTRAL THEME

ADAPTATION
to central proj.

AREAS & VOLUMES: (Proj. Framework of a building)
Scale drawing- construction plans of structure.
Drill in Decimals- Statements, costs, discounts.
Areas- floor plan, estimated bill of lumber.
Volumes- Board feet, basement, foundation etc.

PREPARATION
for central
project.

EVOLUTION: Linear unit, surface unit, space unit.
Development of the formula and formal plan for work.
Scale drawings, Use of standard units including metric

ORIENTATION

ORIENTATION to new environment.
Develop ability to interpret and execute instruction.
Simple geometric construction in group under direct instruction.
Check, compare and encourage constructive criticism of each others work. Practically no use of text.

Build Up

EVALUATION

TEST:
Diagnostic tests
Complete room record for each pupil. Suggest card form

Expansion Levels in Mathematics

EXPANSION LEVELS

Group "B"

ATHEMATICAL SKILLS - ADVANCED COMMUNITY PROB.

INDIVIDUAL & IMMEDIATE COMM.

INDIVIDUAL FAMILY HOME CASES

TAKE OFF

ADAPTATIONS IN SPECIALIZED FIELDS

Federal income- Duties, excises, imposts, TVA
Federal loans- Farm loans, HOLC, RFC, FHA

NATIONAL FINANCE

Wage levels- budgets, expense accts., union vs non-un
Savings- banking, savings acct., P.S. stamps & bonds
U.S. Savings Bonds. (comp & simple int.)
Investments- Business investments, costs margin s.p.
Insurance- life, health, accident.
INCOME Bonds, stocks, old age security, etc.
Property cost and upkeep. Taxes (use local set up)
Community services,-cost - sources of income-Taxes

COMMUNITY INTEREST IN HOME AND INDIVIDUAL.

Buying or building a house- loans, mortgages, FHA loans
Furnishing- installment vs cash, Protection-insurance
Upkeep- depreciation, services, etc.

HOME FINANCE AND MAINTENANCE.

CENTRAL THEME

...ework of a building)
... plans of structure.
...ts, costs, discounts.
...d bill of lumber.
...nt, foundation etc.
...nit, space unit.
...mula and formal plan for work.
... standard units including metric

...xecute instruction.
...roup under direct instruction.
...ructive criticism of each
...f text.

...1. Suggest card form

Group "A"	FOUR FUND problem situation	1. Elementary vocational 2. Standard time belts and travel 3. Simple problems in areas & volumes
	FOUR FUND. Community prob.	1. Wages, savings, percent, 2. Banking, personal loans, etc. 3. Buying with discounts. 4. Making change, marking goods, etc.
	FOUR FUND. Home problems	1. Cost of food clothes 2. Wise buying 3. Cost of utilities, meter reading, etc. 4. Cost of operating a car.
PICK UP*	FOUR FUND. introduction	Drill over the four fundamentals using elementary problems. Drill on reading & construction of nos.

...ls in Mathematics

File: Mathematics

SELECTED BIBLIOGRAPHY FOR MATHEMATICS TEACHERS IN SECONDARY SCHOOLS

History of Mathematics

Bell, E.T. Men of Mathematics. Simon and Schuster, 1937
Biographies of men who created modern mathematics(not a true history). With photographs.

(6) Hogben, L.T. Mathematics for the Millions. Norton, 1937
Popular history with emphasis on social and applied mathematics

Other histories by Rouse Ball, Cantor, Heath, Sullivan, Dantzig, Cajori, Smith and Neugebauer are more comprehensive and more technical.
Geo. Darton

General Reading

Miller, Denning. Popular Mathematics. Coward-McCann, Inc. N.Y. 1942
The understanding and enjoyment of mathematics

(1) Bakst, Aaron Mathematics, Its Magic and Mastery. Van Nostrand Co. 1941
Makes mathematics interesting and shows how mathematics applies to every-day living.

(8) Lieber, H.G. and Lieber, L.R. The Education of T.C. MITS.
(The Celebrated Man In the Street) Interesting portrayal of mathematic concepts to the layman.
Non Euclidean Geo + The 4th Demention

Professional Books

Prof. Include
Butler, Chas.H. and Wren, F.L. The teaching of Secondary Mathematics.
McGraw Hill, 1941
Discusses the place and function, improvement and evaluation, and the teaching of mathematics.

Minnick, J.H. Teaching Mathematics in Secondary Schools. Prentice Hall,
Principles and methods.

Schorling, R. The Teaching of Mathematics. Ann Arbor Press, 1936.
General but comprehensive.

P.E.A. Mathematics in General Education Appleton-Century Co. 1940
Discusses place of Mathematics in general education and how to evaluate student growth in Mathematics.

N.C.ofT.M. Fifteenth Yearbook. The place of Mathematics in Secondary Education. Bureau of Pub. T.C.C.U. 1940

Professional Magazines

National Council of Teachers of Mathematics. The Mathematics Teacher.
8 issues, $2.00. General, research and council activiteis.

Central Associations of Science and Mathematics Teachers. School Science and Mathematics. 9 issues, $3.00.
Comprehensive, problems and new activities

654
456
198

Bibliography for Secondary Mathematics Teachers

Selected Bibliography for Mathematics Teachers in Secondary Schools (cont.)

Mathematical Recreations.

Collins Fredrick A. Fun with Figures.

Ball, Rouse W.W. Mathematical Recreations and Essays. Macmillan, 1911
Excellent puzzling problems and essays, but old.

Jones, Samuel I. Mathematical Wrinkles. Author Publisher, Nashville, Tenn. $3.00 1929. Best handbook of puzzles, tricks, and math curiosities with answers and illustrations.

Licks, H.E. Recreations in Mathematics. Van Nostrand Co, Inc. 1917
Introduction to mathematical enquiries and recreations.

Macmahon, P.A. New Mathematical Pastimes. Cambridge U. Press. 1930
Elementary geometric permutations and combinations.

Steinhaus, H. Mathematical Snapshots. G.E. Stechert and Co. Interesting geometrical illustrations with simple brief explanations.

Some "Brain Teasers" are found in Schorling's The Teaching of Mathematics, in Lasley and Mudds, The New Applied Mathematics, and others.

Meneker, F.E. How Smart are You

Sources of Applied Mathematics

1. Fifty Typical problems in Mathematics and Physics. Florida State Department of Education. Attention Paul Eddy, Editor, Tallahassee, Florida.
2. Flight Preparation Training Series. McGraw-Hill, 1943. (Supervised by the Bureau of Aeronautics. U.S. Navy).
   - Mathematics for Pilots ($.75)
   - Physics Manual for Pilots. ($.90)
   - Principles of Flying($1.50)
   - Aerology for Pilots($1.25)
   - Air Navigation (Seven Parts, $1.00 each)
   - Psychology of Flying.
3. Naidich, J. Mathematics for the Aviation Trades. McGraw Hill, 1942.
4. Mathematics Essential for the War Effort. Bulletin No. 40, State Dept. of Education. Tallahassee, Fla., 1942.
5. Seventeenth Yearbook of the National Council of Teachers of Mathematics. A Source Book of Mathematical Applications. Bureau of Pub., Teachers College, Columbia University, New York, N.Y., 1943. ($2.00)
6. Seimens, Cornelius H. Aeronautics Workbook. Boston: Ginn and Company, 1942
7. Some Military Applications of Elementary Mathematics. Prepared vy the Dept. of Mathematics. U.S. Military Academy, West Point, N.Y.
8. The Air-Age Education Series. New York: The Macmillan Series. 1942.
   1. Aviation Education Research Group, Teachers College, Columbia U. Science of Pre-Flight Aeronautics for High Schools. 868 p.(general)
   2. Aviation Educational Research Group, Teachers College, U. of Nebraska Elements of Pre-Flight Aeronautics for High Schools. 556 p.(general)
   3. Osteyee, George. Mathematics in Aviation. 186 p.
   4. Cartwright, C. et al. Bibliography of Aviation Education Materials.
   5. War Department, Technical Manual. Mathematics for Pilot Trainees. TM 1-900, Washington, April, 1942. (10 cents)

A Thousand Problems in Aviation by (?)

READING IN MATHEMATICS

Types of reading:

Reading for details
Reading for main ideas
Reading for organization of ideas
Reading to understand principles
Reading to follow directions
Reading to solve problems
Reading to understand definitions
Reading to understand and interpret graphic material

General Rules for Reading Mathematics Problems.

Read slowly and carefully
Read more than once.
Read first to know what you are asked to find.
Read next to find the facts you will need to use.
Decide what you will do to get the answer;
add, subtract, multiply, or divide.

Suggestions on Getting Students to Read.
Reporting time and credit
Suggested techniques
Example

How to read in

Reading in Mathematics

5

The Adventures of Little An Alysis

An Alysis was a little girl who didn't like to work;
She didn't like to go to school, and her studies she would shirk,
But of all the studies she liked least, she despised geometry.
She couldn't tell a circle from a rhombus or a square;
She thought hypotenuse was a beast owned by Pythagoras,
In fact, she didn't know a thing. She was the dumbest lass.

Now An Alysis had a cousin whose mane was Polygon,
And Poly was the smartest girl you'd find in Oregon.
She knew a tangent was a line that touched a circle once;
And, since she knew her axioms, she had never been the dunce.

Now Poly knew her cousin An was very dumb indeed.
And thought that she would help her out, since she was so much in need.
She told her first of angles, the obtuse and acute;
She taught her how to make an arc, and how to find square root.
She taught her just a lot of things, and in the end, you mind,
Our little An Alysis was the brightest you could find.

(written by Carol Culver in a plane geometry class at Iowa State Teacher's College
Taken from the Mathematics Teacher.

Pythagoras

When several hundred years ago
The learned Pythagoras
Sat in his outdoor studio
Before his eager class,

He talked of angles, right and straight,
Diagonals and sides;
Gave council how to calculate
With formulas as guides,

An area or hypotenuse,
A volume or a base;
Geometry's eternal use
For every populace.

by Ola Eskerlson, Berger, Texas.

Poem

To learn arithmetic
Takes work; it's not a trick.
To make an A
Our problems we must state.
We learn percent and rate
By working very late
Each single day.

When we reach Algebra
We feel we've traveled far
The road of Math.
No student ere resigns,
But journeys on by signs,
Sometimes with tears and whines
The upward path.

Euclid has made us love
His theorems to prove
In geometry.
Hypothesis we know
Conclusion we must show
So we can quote below,
Q.E.D.

By Rosalie Tutwiler,
from the Mathematics Teacher.

For I dipt into the future, far as human eye could see
Saw the vision of the world an the wonder that would be:

Saw the heavens fill with commerce, argosies of magic sails,
Pilots of the purple twilight, dropping down with costly bales;

Heard the heavens fill with shouting, and there rain'd a ghastly dew.
From the nation's airy navies grappling in the central blue:

Far along the world-wide whisper of the south wind rushing warm,
With the standards of the peoples plunging thro' the thunderstorm:

Till the war-drum throbb'd no longer, and the battle flags were furl'd
In the Parliament of man, the Federation of the world.

There the common sense of most shall hold a fretful realm in awe,
And the kindly earth shall slumber, lapt in universal law.

from Locksley Hall, by Tennyson, 1842

Poems on Mathematics

4

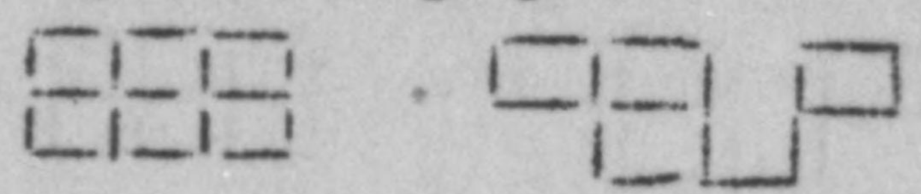
3

Real OELY Sporty LPPE CTTCT

CTTCQ CTLPL ELRY

SUGGESTED FIRST DAY DISPLAY FOR GEOMETRY.

1. Small printed sign bearing this jingle:
   "If you plant cube roots in the ground, what grows?
   And where has a polygon, who knows?
   And where does an angle get its degree?
   And is it B.A. or Ph.D. ? "
2. Sign and 17 matches with a sketch of their arrangement and the puzzle: Can you Match this? Make five whole squares out of six by re-arranging 3 of the match sticks.



(answer to be put elsewhere in room)

3. Can you guess..when this was written?
   The above is a caption for a large printed copy of "For I dipt into the future"..by Tennyson from Locksley Hall. Several dates were listed, and the correct one, 1842, is put with the answer to #2 above.

4. Can You Figure..this one out?
   Write the value 100, using only the figures from 1-9, and the signs of arithmetic. (this is a sign, with small figures on the bottom of the poster made from paper cut-outs and bearing the four arithmetic signs.
   (ans.--- 1-2-3-4-5-6-7-8-9= 100

5. Geometry goes Hollywood.
   Two mounted line drawings from a paint book of pictures from Walt Disney's "Fantasia". Appropriate questions are printed on each.

6. Lettered sign bearing "A Song for Geometry" to tune "Maryland"
   "Oh Geometry, Geometry,
   A solid subject, you'll agree
   And what care I how plane it be,
   If it be not plain to me?

   And angles: if your brain's acute,
   Your work is right and sure to suit,
   But if you chance to be obtuse,
   To toil and weep is not much use."

7. Mounted photograph: A New England Church, Labelled "An American Church". Questions from the back typed and clipped to picture:
   a. In what section of our country did this type of construction originate?
   b. Find the six geometric figures used in this building.
   c. Where do you see parallel lines, symmetry, a sphere?

8. An engineering drawing: "Applied Geometry".

First Day Display for Geometry

TEACHING AIDS FOR GEOMETRY

FLAT PICTURES (filed by number in folder..Mounted)

Subject	File	Number	Applications
Armadillo	Animals	1	Arcs, hexagons, and parallel lines
Rug design	Geometry	1	Navajo..quadrilaterals, polygons
Cartoons	Geometry	2	symmetry in walking in step
Garden, formal	Geometry	3	symmetry, arcs, cones, proportion
New Eng. church	Geometry	4	symmetry, triangle, cone, sphere, rectangle
New Delhi	Geometry	5	symmetry, proportional lines, arcs
Italian cities	Geometry	6,7, 8	figures seen in buildings, proportional lines in mosaic, and city plans.
Mod. church	Geometry	9	Proportional lines in skyscraper.
Am. Gothic	Geometry	10	Circular window, spires, buttresses
Hindu bldg.	Geometry	11	Comparison Occidental gate & fence to Oriental arches, spires, etc.
French Gothic	Geometry	12	Circular window..proportional lines.
Design in Ind.	Geometry	13	cylinders, other figures in designs
Mod. House	Geometry	14	Parallel lines in modern architecture
Am. R.R.	Geometry	15	parallel lines, triangles, rectangles.
China caves	Geometry	16	Arcs, triangles, rectangles in sculpture
African design	Geometry	17	symmetry, polygon, circle, prop. lines.
Mosaic, Italy	Geometry	18	Polygon, symmetry, proportion
Dam, Russia	Geometry	19	semicircles and papallel and perpendiculars
Dist., Time	Geometry	20	How far is an hour?
Artillery	Geometry	21	Precision in firing, angles, etc.
Drafting	Geometry	22	parallel lines with "Grid machine"
Paint book	un-numbered		Fantasia, by Walt Disney

POSTER FOR BULLETIN BOARD

Skills essential for Navy
1. Common fractions.(problems)
2. Conversion metric into British Units.
3. Averages, ratio, proportion, percentages.
4. Pythagorean theorem and square root.(no proofs)
5. Transposition in equations.
6. Problems using simple equations.
7. Non-Algebraic graphs.
8. Triangles of velocities.

MOTION PICTURES

EYES OF SCIENCE..deals with optics and optical instruments.
3 reels, silent, U. of California

HOW OUR EYES DECEIVE US..optical illusions
½ reel, silent, U. of Colorado, Boulder, Colorado.

GEOMETRY IN ACTION...geometric forms.
1 reel, sound, Bald Eagle Film Products, 104 Howe St.Annex New Haven, Conn.

Precisely So..accuracy in weights and measures
1 reel, sound. Chevrolet Motor division, General Motors
Apply to local dealer for film.

2

Teaching Aids for Geometry

Teaching aids for Geometry, continued

Interest getting ~~First~~ Day Techniques

Puzzles
Games
Tricks
Field Trips
Visual Aids — Snow white & the 7 devils (In Mathematics books)
Plays and Drama
Bulletin Board
Meeting Individual needs.

Devices and Supplies

1. Angle measurement device..hinged sticks with meter stick.
2. Colored craft paper.
3. Small boards to make pantograph and "teeter" board.
4. Brace and bit and clamps for #3 above.
5. Mechanical drawing sets.
6. T-squares and drawing boards.
7. Masking tape to make models from toothpicks for "geometric figures". and to hold paper to drawing boards. Secure from auto supply stores.
8. Colored chalk.
9. Soap to be carved into crystal models.
10. Maps showing the four projections: Lambert, polar, gnomic and mercator.
11. Meter sticks and blackboard compasses.
12. Heavy cardboard for making scrap book covers.
13. Magnet and glass and iron filings to use for illustration of inductive reasoning.
14. Textbooks for Commercial Law for use in explanation of deductive thinking.
15. Carpenter's square and level and weight for plumb line.
16. Toothpicks and paper rectangles (atoms) to be put together as molecules to illustrate synthesis in chemistry. Relate this to synthesis in problem solving.
17. Indian baskets to illustrate polygons.
18. Architect's rule to show an application of proportion.
19. Thumbtacks and cord, or stakes and rope to lay out the figures for study of areas of polygons.
20. Mirrors, candles and other equipment from laboratory.
21. Chemicals, scales, graduated cylinders, etc., to use in making problems on proportion as vivid as possible.
22. Paper, scissors and masking tape to make cylinders and cones.

Additional Books for Teacher Reference

Burton, William H., The Guidance of Learning Activities, D. Appleton Century Co., 1944

Butler, Chas. H. and Wren, F. L., The Teaching of Secondary Mathematics McGraw Hill, 1941

Bureau of Publications, Teachers College, Columbia University, N. Y. City, Mathematics in Modern Education, 11th Year Book, 1936