ENGINEERING DESCRIPTIVE GEOMETRY

Engineering Descriptive Geometry

A Treatise on Descriptive Geometry as the Basis of Mechanical Drawing, Explaining Geometrically the Operations Customary in the Draughting Room

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PREFACE.

The aim of this work is to make Descriptive Geometry an integral part of a course in Mechanical or Engineering Drawing.

The older books on Descriptive Geometry are *geometrical* rather than *descriptive.* Their authors were interested in the subject as a branch of mathematics, not as a branch of drawing.

Technical schools should aim to produce engineers rather than mathematicians, and the subject is here presented with the idea that it may fit naturally in a general course in Mechanical Drawing. It should follow that portion of Mechanical Drawing called *Line Drawing,* whose aim is to teach the handling of the drawing instruments, and should precede courses specializing in the various branches of Drawing, such as Mechanical, Structural, Architectural, and Topographical Drawing, or the "Laying Off" of ship lines.

The various branches of drawing used in the different industries may be regarded as dialects of a common language. A drawing is but a written page conveying by the use of lines a mass of information about the geometrical shapes of objects impossible to describe in words without tedium and ambiguity. In a broad sense all these branches come under the general term Descriptive Geometry. It is more usual, however, to speak of them as branches of Engineering Drawing, and that term may well be used as the broad label.

The term Descriptive Geometry will be restricted, therefore, to the common geometrical basis or ground work on which the various industrial branches rest. This ground work of mathematical laws is unchanging and permanent.

The branches of Engineering Drawing have each their own abbreviations, and special methods adapting them to their own particular fields, and these conventional methods change from time to time, keeping pace with changing industrial methods.

Descriptive Geometry, though unchanged in its principles, has recently undergone a complete change in point of view. In changing its purpose from a *mathematical* one to a *descriptive* one, from being a training for the geometrical powers of a mathematician to being a foundation on which to build up a knowledge of some branch of Engineering Drawing, the number and position of the planes of projection commonly used are altered. The object is now placed behind the planes of projection instead of in front of them, a change often spoken of as a change from the "1st quadrant" to the "3d quadrant," or from the French to the American method. We make this change, regarding the 3d quadrant method as the only natural method for American engineers. All the principles of Descriptive Geometry are as true for one method as for the other, and the industrial branches, as Mechanical Drawing, Structural Drawing, etc., as practiced in this country, all demand this method.

In addition, the older geometries made practically no use of a third plane of projection, and we take in this book the further step of regarding the use of three planes of projection as the rule, not the exception. To meet the common practice in industrial branches, we use as our most prominent method of treatment, or tool, the use of an auxiliary plane of projection, a device which is almost the draftsman's pet method, and which in books is very little noticed.

As the work is intended for students who are but just taking up geometry of three dimensions, in order to inculcate by degrees a power of visualizing in space, we begin the subject, not with the mathematical point in space but with a solid tangible object shown by a perspective drawing. No exact construction is based on the perspective drawings which are freely used to make a realistic appearance. As soon as the student has grasped the idea of what orthographic projection is, knowledge of how to make the projection is taught by the constructive process, beginning with the point and passing through the line to the plane. To make the subject as tangible as possible, the finite straight line and the finite portion of a plane take precedence over the infinite line and plane. These latter require higher powers of space imagination, and are therefore postponed until the student has had time to acquire such powers from the more naturally understood branches of the subject.

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T. W. J.

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CHAPTER I.

NATURE OF ORTHOGRAPHIC PROJECTION.

1. **Orthographic Projection.**—The object of Mechanical Drawing is to represent solids with such mathematical accuracy and precision that from the drawing alone the object can be built or constructed without deviating in the slightest from the intended shape. As a consequence the "working drawing" is the ideal sought for, and any attempt at artistic or striking effects as in "show drawings" must be regarded purely as a side issue of minor importance. Indeed mechanical drawing does not even aim to give a picture of the object as it appears in nature, but the views are drawn for the mind, not the eye.

The shapes used in machinery are bounded by surfaces of mathematical regularity, such as planes, cylinders, cones, and surfaces of revolution. They are not random surfaces like the surface of a lump of putty or other surfaces called "shapeless." These definite shapes must be represented on the flat surface of the paper in an unmistakable manner.

The method chosen is that known as "*orthographic projection.*" If a plane is imagined to be situated in front of an object, and from any salient point, an edge or corner, a perpendicular line, called a "projector," is drawn to the plane, this line is said to project the given point upon the plane, and the foot of this perpendicular line is called the projection of the given point. If all salient points are projected by this method, the *orthographic drawing* of the object is formed.

2. **Perspective Drawing.**—The views we are accustomed to in artistic and photographic representations are "Perspective Views." They seek to represent objects exactly as they appear in nature. In their case a plane is supposed to be erected between the human eye and the object, and the image is formed on the plane by supposing straight lines drawn from the eye to all salient points of

the object. Where these lines from the eye, or "Visual Rays," as they are called, pierce the plane, the image is formed.

Fig. 1 represents the two contrasted methods applied to a simple object, and the customary nomenclature.

An orthographic view is sometimes called an "Infinite Perspective View," as it is the view which could only be seen by an eye at an infinite distance from the object. "The Projectors" may then be considered as parallel visual rays which meet at infinity, where the eye of the observer is imagined to be.

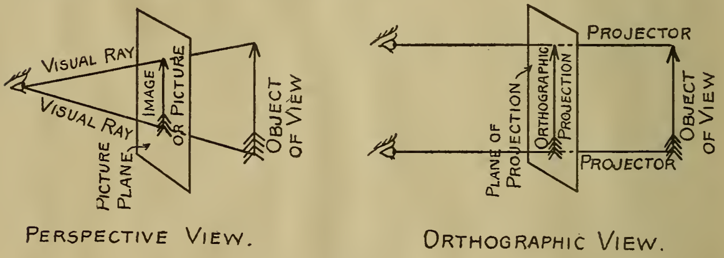

FIG. 1.

3. The Regular Orthographic Views.—Since solids have three "dimensions," length, breadth and thickness, and the plane of the paper on which the drawing is made has but two, a single orthographic view can express two only of the three dimensions of the object, but must always leave one indefinite. Points and lines at different distances from the eye are drawn as if lying in the same plane. From one view only the mind can imagine them at different distances by a kind of guess-work. If two views are made from different positions, each view may supplement the other in the features in which it is lacking, and so render the representation entirely exact. Theoretically two views are always required to represent a solid accurately.

To make a drawing all the more clear, other views are generally advisable, and three views may be taken as the average requirement for single pieces of machinery. Six regular views are possible, however, and an endless number of auxiliary views and "sections" in addition. For the present, we shall consider only the "regular views," which are six in number.

4. Planes of Projection.—A *solid object* to be represented is supposed to be surrounded by planes at short distances from it, the planes being perpendicular to each other. From each point of every salient edge of the object, lines are supposed to be drawn perpendicular to each of the surrounding planes, and the succession of points where these imaginary projecting lines cut the planes are supposed to form the lines of the drawings on these planes. One of the planes is chosen for the plane of the paper of the actual drawing. To bring the others into coincidence with it, so as to have all of them on one flat sheet, they are imagined to be unfolded from about the object by revolving them about their lines of intersection with each other. These lines of intersection, called "axes of projection," separate the flat drawing into different views or elevations.

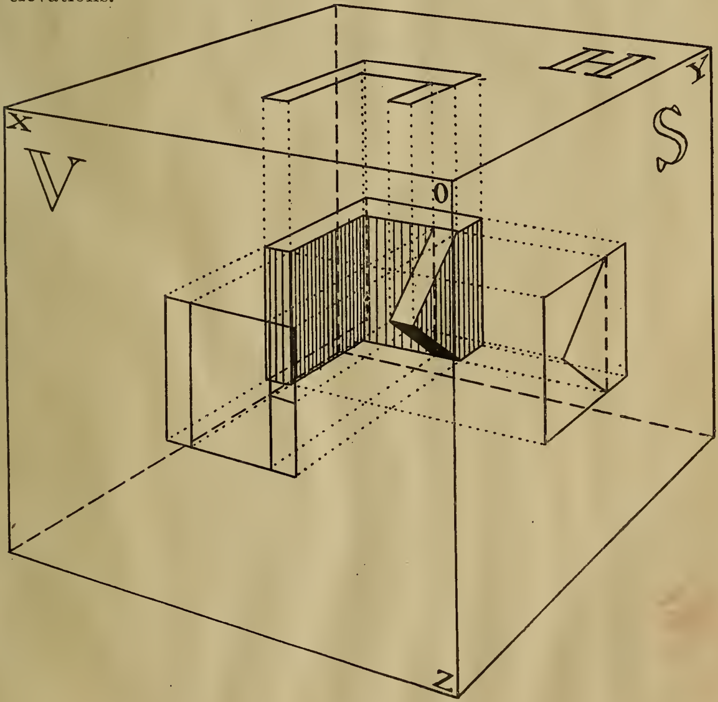

FIG. 2.

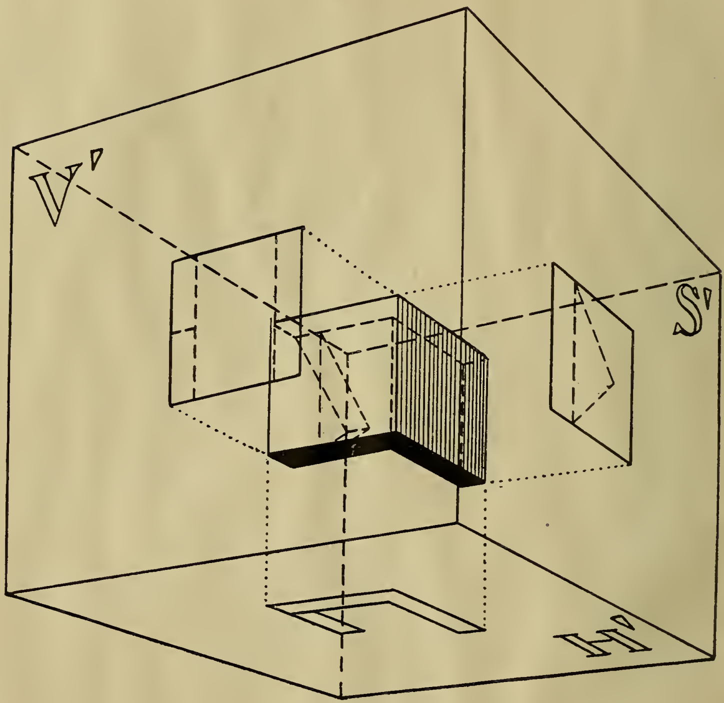

FIG. 2a.

Fig. 2 is a true perspective drawing of a solid object and the planes as they are supposed to surround it. This figure is not a mechanical drawing, but represents the mental process by which the mechanical drawing is supposed to be formed by the projection of the views on the planes. In this case the planes are supposed to be in the form of a perfect cube. The top face of the cube shows the drawing on that face projected from the solid by fine dotted lines. Remember that these fine dotted lines are supposed to be perpendicular to the top plane. This drawing on the top plane is called the "plan." On the front of the cube the "front view" or "front elevation" is drawn, and on the right side of the cube is the "right side elevation." Three other views are supposed to be drawn on the other faces of the cube, but they are shown on Fig. 2a, which is the perspective view of the cube from the opposite point of view, that is, from the back and from below instead of from in front and from above.

This method of putting the object to be drawn in the center of a cube of transparent planes of projection is a device for the imagination only. It explains the nature of the "projections," or "views," which are used in engineering drawing.

5. Development or Flattening Out of the Planes of Projection.— Now imagine the six sides of the cube to be flattened out into one plane forming a grouping of six squares as in Fig. 3. What we

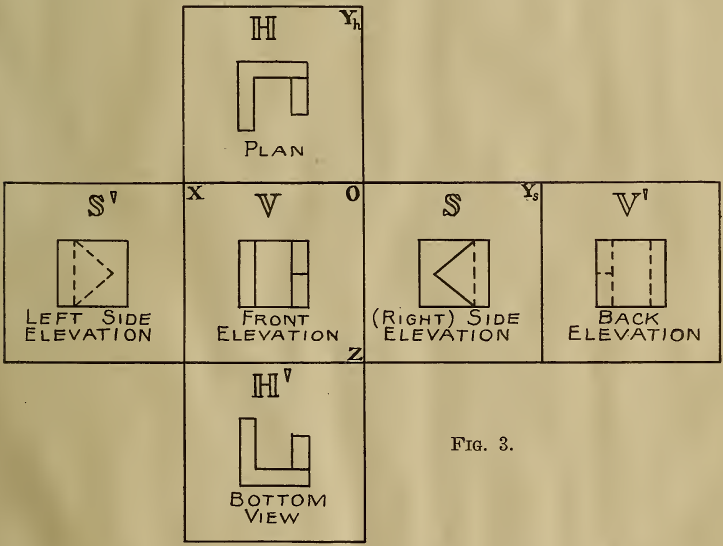

FIG. 3.

have now is a description or mechanical drawing of the object showing six "views." The object itself is now dispensed with and its projections are used to represent it. These six views are what we call the "regular views." With one slight change they correspond to the regular set of drawings of a house which architects make.

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The set of six "regular" projections would not be altered by passing the transparent planes at unequal distances from each other, so long as they surround the object and are mutually perpendicular. They may form a rectangular parallelopiped instead of a cube without altering the nature of the views.

It will be noticed also that views on opposite faces of the cube differ but little. Corresponding lines in the interior may in one case be full lines and in the other "broken lines." Broken lines (formed by dashes about $\frac{1}{8}''$ long, with spaces of $\frac{1}{16}''$) represent parts concealed by nearer portions of the object itself. All edges project upon the plane faces of the cube, forming lines on the draw-

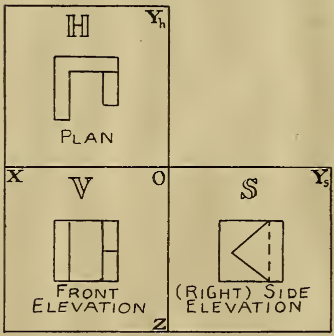


FIG. 4.

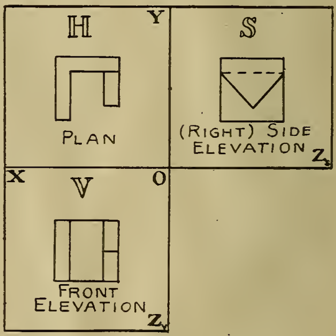


FIG. 5.

ings, the edges concealed by nearer portions of the object forming broken lines.

6. The Reference Planes and Principal Views.—In drawings of parts of machinery six regular views are usually unnecessary. The three views shown in Fig. 2 are the "Principal Views," and others are needed only occasionally. The planes of those views are the "Reference Planes."

These views, when flattened from their supposed position about the object into one plane, give the grouping in Fig. 4.

Another arrangement of the same views, obtained by unfolding the planes of the cube in a different order, is shown in Fig. 5. These two arrangements are standard in mechanical drawing, and are those most used.

7. The Nomenclature.—The nomenclature adopted is as follows: The "Reference Planes," or three principal planes of projection, are called from their position, the Horizontal Plane, or H, the Vertical Plane, or V, and the (right) Side Plane, or S. The plane S is by some called the "Profile Plane." The point *O* (Fig. 2), in which they meet, is the "Origin" of coordinates. The line *OX*, in which H and V intersect, is called the "Axis of *X*," or "Ground Line." The line *OY*, in which H and S meet, is called the "Axis of *Y*," and the line *OZ*, in which V and S meet, is called the "Axis of *Z*." The three axes together are called the "Axes of Projection."

Since drawings are considered as held vertically before the face, it is considered that the plane V coincides at all times with the "Plane of the Paper." In unfolding the planes from their positions in Fig. 2 to that in Fig. 4, it is considered that the plane H has been revolved about the axis of *X* (line *OX*), through an angle of 90°, until it stands vertically above V. In the same way S is considered to be revolved about the line *OZ*, or axis of *Z*, until it takes its place to the right of V.

The arrangement in Fig. 5 corresponds to a different manner of revolving the plane S. It is revolved about the axis of *Y* (*OY*) until it coincides with the plane H, and is then revolved with H, about the axis of *X*, until both together come into the plane of the paper, or V.

The three other faces of the original cube of planes of projection are appropriately called H′, V′, and S′. On account of the similarity of the views on them, to those on H, V and S, they are but little used. S′ alone is fairly common since a grouping of planes H, V and S′ is at times more convenient than the standard group H, V and S.

8. Meaning of "Descriptive Geometry."—The aim of Engineering or Mechanical Drawing is to represent the shapes of solid objects which form parts of structures or machines. It shows rather the shapes of the *surfaces* of the objects, surfaces which are usually composed of plane, cylindrical, conical, and other surfaces. In the drawing room, by the application of mathematical laws and principles, views are constructed. These are usually Plan, Front

Elevation, and Side Elevation, and are exactly such views as would be obtained if the object itself were put within a cage of transparent planes, and the true projections formed.

It is these mathematical laws or rules which form the subject known as Descriptive Geometry. A drawing made in such a way as to bring out clearly these fundamental laws of projection, by the use of axes of projection, etc., may be conveniently called a "Descriptive Drawing."

In the practical application of drawing to industrial needs, short-cuts, abbreviations, and special devices are much used (their nature depending on the special branch of industry for which the drawing is made). In addition, the axes of projection are usually omitted or left to the imagination, no particular effort being made to show the exact mathematical basis provided the drawing itself is correct. Such a drawing is a typical "Mechanical Drawing." By the addition of axes of projection and similar devices, it may be converted into a strict "Descriptive Drawing."

9. The Descriptive Drawing of a Point in Space.—The imaginary process of making a descriptive drawing consists in putting the object within a cube of transparent planes, and projecting points and lines to these planes. In practice the projections are formed all on a single sheet of paper, which is kept in a perfectly flat shape, by the application of rules of a geometrical kind derived from the imaginary process. The key to the practical process is in these rules. The first subject of exact investigation should be the manner of representing a point in space by its projections and the fixing of its position as regards the "reference planes" by the use of coordinate distances.

Figs. 6 and 7 show the imaginary and the practical processes of representing P by its projections.

Fig. 6 is a perspective drawing showing the cube of planes, or rather the three sides of the cube regularly used for reference planes. The cube must be of such size that the point P falls well within it. The perpendicular projectors of P are PP_h, PP_v and PP_s. The origin and the axes of projection are all marked as on Fig. 2.

In Fig. 7 the "field" of the drawing, that part of the paper devoted to it, is prepared by drawing two straight lines at right angles to represent the axes of projection, lettering the horizontal line XOY_s and the vertical one ZOY_h. This field corresponds to that of Fig. 4, the outer edges of the squares being eliminated since there is no need to confine each plane to the size of any particular cube. If more field is needed, the lines are simply extended. It must be remembered that these axes are quite different from the coordinate axes used in plane analytical geometry, or graphic algebra. These divide the field of the drawing into four

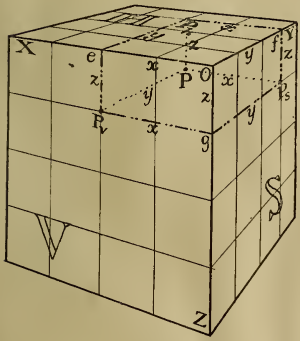


FIG. 6.

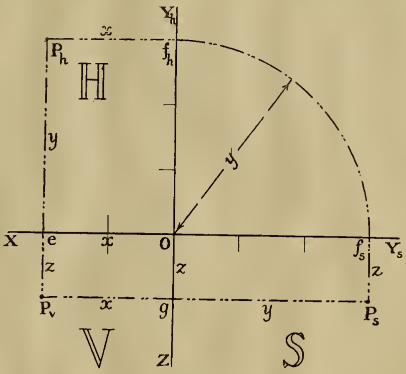


FIG. 7.

quadrants, of which three represent *three different planes,* mutually perpendicular, the fourth being useful only for the purposes of construction.

Usually the upper left quadrant, the "North-West," represents $\mathbb{H}$; the lower left quadrant, or "South-West," represents $\mathbb{V}$, and the lower right quadrant, or "South-East," represents $\mathbb{S}$.

On occasion the axes may be lettered XOZ_s horizontally and Z_vOY vertically, to correspond to Fig. 5, the upper right quadrant now representing $\mathbb{S}$.

10. Coordinates of a Point in Space.—A *point in space* is located by its perpendicular distances from the three planes of projection, that is to say, by the length of its projectors. These distances are called the coordinates of the point, and are designated by *x, y* and *z*. In the example given, these values are 2, 3 and 1. In Fig. 6 PP_s, the 𝕊 projector of *P*, is two units long, or $x=2$. The perpendicular distance to the plane 𝕍, the 𝕍 projector, PP_v, is three units long. $y=3$. In the same way PP_h, the ℍ projector, is one unit long. $z=1$.

In describing the point *P*, it is sufficient to state that it is the point for which $x=2$, $y=3$, and $z=1$. This is abbreviated conveniently by calling it the point *P* (2, 3, 1), the coordinates, given in the bracket, being taken always in the order *x, y, z*.

The projectors, the true coordinate distances, are shown in Fig. 6 by lines of *dots*, not dashes.

If in each plane ℍ, 𝕍 and 𝕊, perpendicular lines are drawn (dashes, not dots) from the projections of *P* to the axes, we shall have the lines P_he and P_hf, P_ve and P_vg, P_sg and P_sf. These lines meet in pairs at *e, g,* and *f,* forming a complete rectangular parallelopiped of which *P* and *O* are the extremities of a diagonal. The other corners of the parallelopiped are P_h, P_v, P_s, *e, f* and *g*.

Each coordinate, *x, y* and *z,* appears in *four* places along four edges of the parallelopiped, as is marked in Fig. 6.

The distances *x, y* and *z* are all considered positive in the case shown.

In Fig. 7, the descriptive drawing of the point *P, P* itself does not appear, being represented by its projections, P_h, P_v and P_s. The true projectors (shown in Fig. 6 by lines of *dots*) do not appear, but in place of each coordinate *three* distances equal to it do appear, so that in Fig. 7 *x, y* and *z* each appear in three places as is there marked. Thus *x* appears as P_hf_h, *eO*, and P_vg. As all these are measured to the left from the vertical axis, ZOY_h, it follows that P_hep_v is a straight line, or P_h is vertically above P_v. It is often said that P_v "projects vertically" to P_h. In the same way P_v "projects horizontally" to P_s. The distance *y* appears as eP_h, Of_h, Of_s, and gP_s. The point *f* appears double due to the axis of *Y* itself doubling. To represent the original coincidence of f_h and f_s, a quadrant of a circle with center at *O* is often used to connect them.

11. Three Laws of Projection for $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$.—The three relations shown by Fig. 7 amount to three laws governing the projections of a point in the three views, and must always be rigidly observed. They may seem easy and obvious when applied to one point, but when dealing with a multitude of points it is not easy to observe them unfailingly.

They may be thus tabulated:

(1) P_h must be vertically above P_v.

(2) P_s must be on the same horizontal line as P_v.

(3) P_s must be as far to the right of OZ as P_h is above OX.

From these laws it follows that if two projections of a point are given, the third is easily found. In Fig. 7, if two of the corners of the figure $P_h f_h f_s P_s P_v$ are given, the figure can be graphically completed. Much of the work of actual mechanical drawing consists in correctly locating two of the projections of a point by plotting or measuring, and of finding the other projection by the application of these laws or of this construction. Constant checking of the points between the various views of a drawing is a vital principle in drawing.

On the drawing board the horizontal projection of P_v to P_s is naturally done by the T-square alone, and the vertical projection of P_h to P_v by T-square and triangle. There are two methods of carrying out the third law in addition to the graphical construction of Fig. 7. Fig. 8 shows a graphical method which makes use of a 45° line, OL, in the construction space, instead of the quadrant of a circle. It is easier to execute, but the meaning is not so clearly shown. The third method is by the use of the dividers directly to transfer the x coordinate from whichever place it is first plotted, to the other view in which it appears.

12. Paper Box Diagrams.—When studying a descriptive drawing, such as Fig. 8, imagine as you look at P_v that the real point P lies *back of the paper,* at a distance equal to eP_h.

Whenever figures in the text following seem hard to grasp, carry out the following scheme. Trace the figure on thin paper, or on tracing cloth. Using Fig. 8 as an example, and supposing it to have been traced on semitransparent paper, hold the paper before you and fold the top half back 90° on the line XOY_s. Then, viewing P_h from above, imagine the true point P to lie below the paper at a distance equal to eP_v, in the same way as you imagine P to lie back of P_v at a distance equal to eP_h.

After flattening the paper, fold the right half back 90° on the line ZOY_h, and, viewing P_s from the right, imagine P to lie back of P_s a distance P_vg. Finally, crease the paper on the line OL, OL itself forming a groove, not a ridge, and bend the paper on all

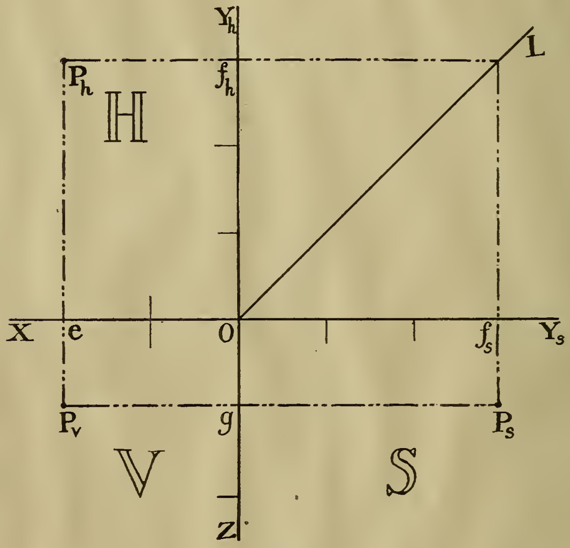


FIG. 8.

the creases at once, so that H and S fold back into positions at right angles to V and to each other at the same time.

The "construction space" Y_hOY_s is thus folded away inside and OY_h and OY_s come in contact with each other. Fig. 9 shows the final folding partly completed.

No diagram, however complicated, can remain obscure if studied from all sides in this manner.

To have a convenient name, these space diagrams may be called "Paper Box Diagrams."

Figs. 4 and 5 make good paper box diagrams, while Fig. 3 may be traced and folded into a perfect cube which, if held in proper position, will give the exact views shown in Figs. 2 and 2a, omitting the solid object supposed to be seen in the center of those figures.

13. Zero Coordinates.—Points having zero coordinates are sometimes perplexing. If one coordinate is zero, the point in question is on one of the reference planes, and indeed coincides with one of its own projections. Since x is the length of the orthographic projector of the point P upon the plane $\mathfrak{S}$, if $x=0$, this projector disappears and the point P and its $\mathfrak{S}$ projection P_s coincide. If

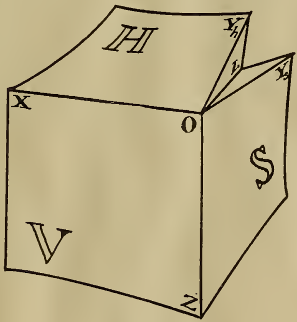


FIG. 9.

the point Q $(0, 3, 1)$ is to be plotted it will be found to coincide with P_s in Fig. 6. The descriptive drawing will correspond with Fig. 7 with all lines to the left of ZOY_h omitted, and with the lettering changed as follows: For P_s put Q_s (and Q), for f_h put Q_h, for g put Q_v. The student should make this diagram on cross-section paper and should study out for himself the similar cases for the points Q' $(2, 0, 1)$ [P_v in Fig. 6] and Q'' $(2, 3, 0)$ [P_h in Fig. 6] and should proceed from them to more general cases, assuming ordinates at will, using cross-section paper for rapid sketch work of this kind.

If two coordinates are zero, the point lies on one of the axes, on that axis, in fact, which corresponds to the ordinate which is not zero. Thus the point R $(2, 0, 0)$ is the point e of Fig. 6, R_h and R_v are at e, and R_s is at 0.

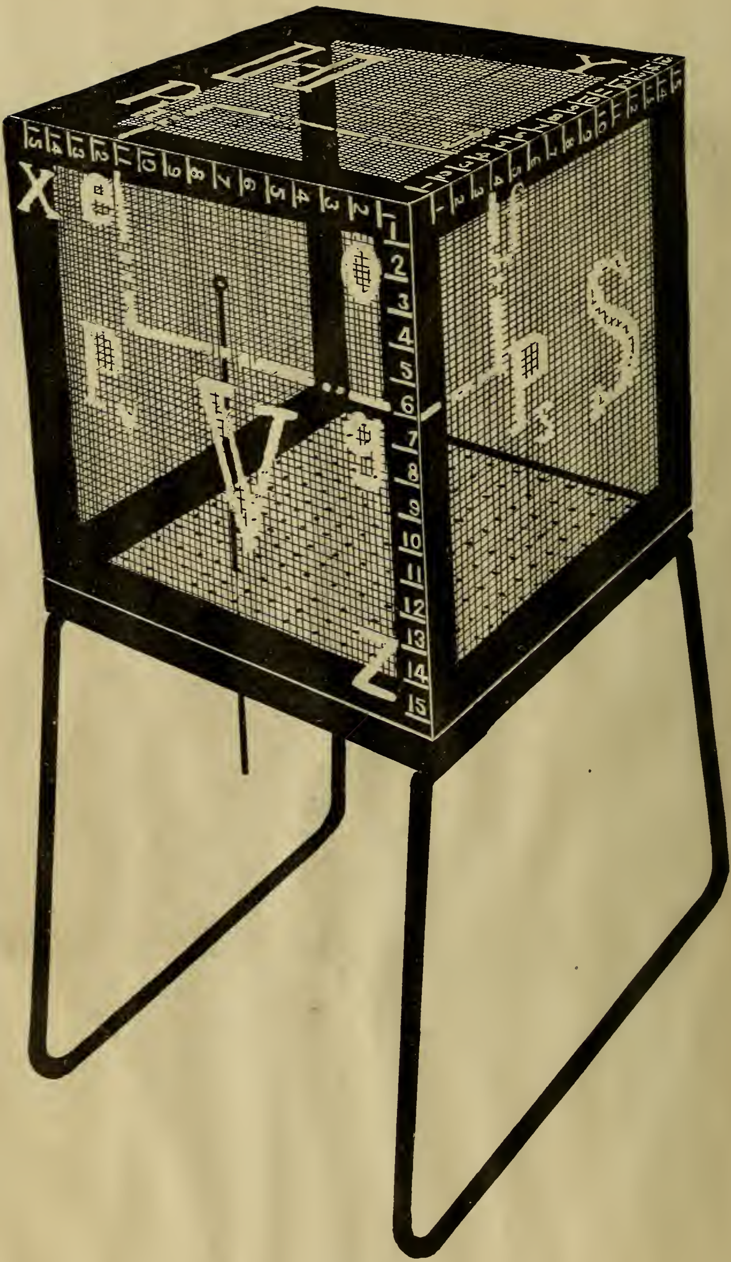


FIG. 10.

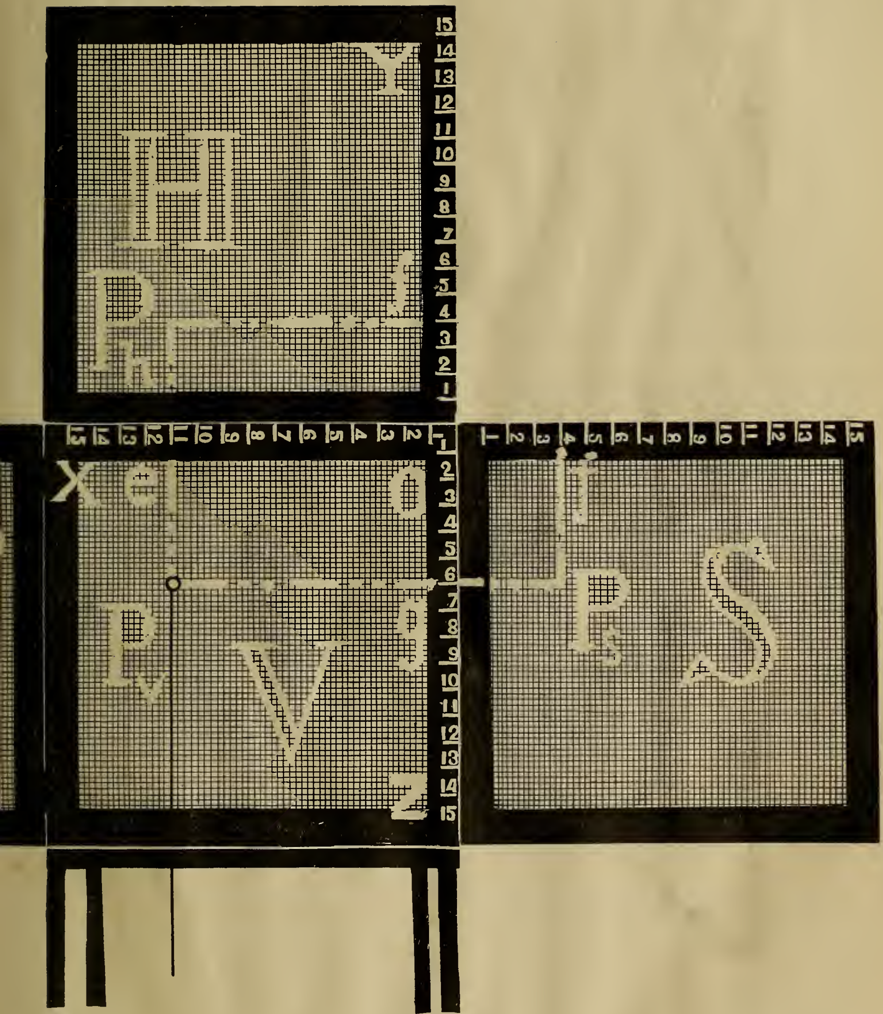


Fig. 10a.

Wire-mesh Cage.

If possible, it is very desirable to have cages similar to Fig. 10, formed of wire-mesh screens, representing the planes $\mathbb{H}$, $\mathbb{V}$, $\mathbb{S}$ and $\mathbb{S}'$. On these screens chalk marks may be made and the planes, being hinged together, may afterward be brought into coincidence with $\mathbb{V}$, as represented in Fig. 10a.

In order to plot points in space within the cage, pieces of wire about 20 inches long, with heads formed in the shape of small loops or eyes, are used as *point markers*. They may be set in holes drilled in the base of the cage at even spaces of 1″ in each direction, so that a marker may be set to represent any point whose x or y coordinates are even inches. To adjust the marker to a required z coordinate, it may be pulled down so that the wire projects through the base, lowering the head the required amount. z may vary fractionally.

In Fig. 10 a point marker is set to the point P (11, 4, 6), and the lines on the screens have been put on with chalk, to represent all the lines analogous to those of Fig. 6.

Fig. 10a represents the descriptive drawing produced by the development of the screens in Fig. 10. It is analogous to Fig. 7.

Several points may be thus marked in space and soft lead wire threaded through the loops, so that any plane figure may be shown in space, and its corresponding orthographic projections may be drawn on the planes in chalk.

Problems I.

1. Plot by the use of the wire markers the three points, A, B and C, whose coordinates are (5, 12, 11), (3, 3, 3), and (12, 4, 8), and draw the projections on the screens in chalk. By joining point to point we have a triangle and its projections. Use lead wire for joining the points, and chalk lines for joining the projections.
2. Form the triangle as above with the following coordinates: (11, 3, 2), (12, 6, 12) and (14, 12, 7).
3. Form the triangle as above with the following coordinates: (7, 0, 11), (9, 9, 0) and (2, 2, 3).
4. Form the triangle as above with the following coordinates: (0, 11, 13), (14, 3, 3) and (14, 13, 0).

(The following examples may be solved on coordinate paper, or plotted in inches on the blackboard.)

5. Make the descriptive drawing of a triangle in three views by plotting the vertices and joining them by straight lines. The vertices are the points A (1, 10, 8), B (5, 6, 8), C (9, 2, 4).

6. Make the descriptive drawing as above using the points

A (12, 2, 5), B (0, 8, 6), C (4, 6, 0).

7. Make the descriptive drawing as above using the points

A (3, 4, 2), B (13, 8, 10), C (5, 10, 14).

8. The four points A (3, 3, 3), B (3, 3, 15), C (15, 3, 15), and D (15, 3, 3) form a square. Make the descriptive drawing. Why are two projections straight lines only? What are the coordinates of the center of the square?

9. The four points A (12, 2, 12), B (2, 2, 12), C (7, 14, 12), and D (7, 6, 2) are the corners of a solid tetrahedron. Make the descriptive drawing, being careful to mark concealed edges by broken lines.

10. Make the descriptive drawing of the tetrahedron A (2, 3, 2), B (9, 8, 3), C (4, 8, 9), D (1, 3, 6), marking concealed edges by broken lines.

11. Make the descriptive drawing of the tetrahedron A (3, 2, 4), B (6, 8, 2), C (8, 1, 8), D (2, 7, 8).

12. Plot the points A (12, 7, 7), B (8, 13, 5), C (2, 9, 2), and D (6, 3, 4). Why is the $\mathbf{V}$ projection a straight line?

13. Make the descriptive drawing of the tetrahedron A (13, 5, 3), B (1, 5, 3), C (7, 2, 6), D (7, 8, 6). To which axis is the line AB parallel? To which axis is CD parallel?

14. Plot and join the points A (11, 3, 3), B (3, 3, 3), C (7, 9, 7), and D (15, 9, 7). Do AC and BD meet at a point or do they pass without meeting?

CHAPTER II.

ORTHOGRAPHIC PROJECTION OF THE FINITE STRAIGHT LINE.

14. The Finite Straight Line in Space: One not Parallel to any Reference Plane, or an " Oblique Line."—A line of any kind consists merely of a succession of points. Its orthographic projection is the line formed by the projections of these points.

In the case of a straight line, the orthographic projection is itself a straight line, though in some cases this straight line may degenerate to a single point, as mathematicians express it.

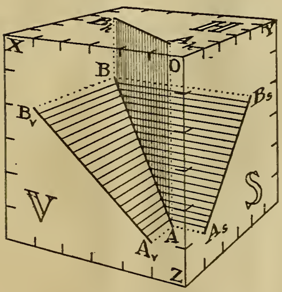


FIG. 11.

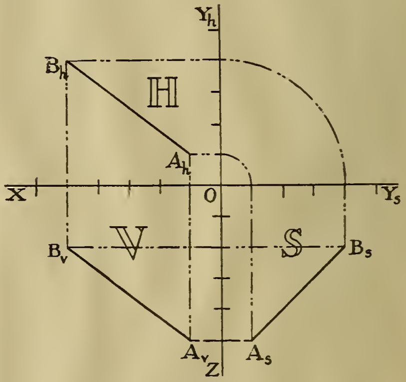


FIG. 12.

To find the H, V and S projections of a finite straight line in space, the natural course is to project the extremities of the line on each reference plane and to connect the projections of the extremities by straight lines. We shall not consider this as requiring proof here. It is common knowledge that a straight line cannot be held in any position that will make it appear curved, and orthographic projection is, as shown by Fig. 1, only a special case of perspective projection. The strict mathematical proof is not exactly a part of this subject.

The projectors from the different points of a straight line form a plane perpendicular to the plane of projection. This "projector-plane," of course, contains the given line. If the straight line is a limited or finite line the projector-plane is in the form of a quadrilateral having two right angles. Thus in Fig. 11 the ℍ projectors of the straight line AB form the figure AA_hB_hB, having right angles at A_h and B_h. These projector-planes AA_hB_hB, AA_sB_sB, and AA_vB_vB are shown clearly in this perspective drawing, in which they are shaded.

Fig. 12 is the descriptive drawing of the same line AB which has been selected as a "line in space," that is, as one which does not obey any special condition. In such general cases the projections are all shorter than the line itself. As drawn, the extremities are A (1, 1, 5) and B (5, 4, 2).

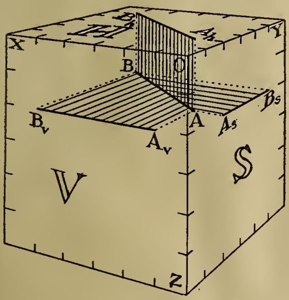


FIG. 13.

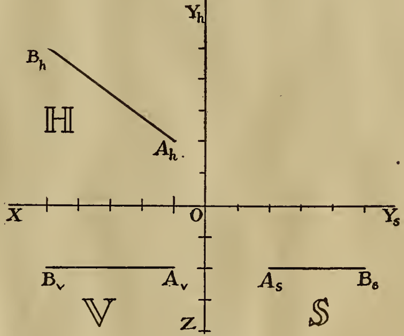


FIG. 14.

15. Line Parallel to One Reference Plane, or Inclined Line.— A line which is parallel to one reference plane, but is not parallel to an axis, appears projected at its true length on that reference plane only.

Figs. 13 and 14 show a line five units long, connecting the points A (1, 2, 2) and B (5, 5, 2). A_hB_h is also five units in length but A_vB_v is but four and A_sB_s is three. The projector-plane AA_hB_hB is a rectangle.

The student should construct on coordinate paper the two similar cases. For example: the line C (4, 2, 1), D (1, 2, 5) is parallel to 𝕍; E (2, 1, 2), F (2, 5, 5) is parallel to 𝕊.

16. Line Parallel to One of the Axes and thus Parallel to Two Reference Planes.—If a finite straight line is parallel to one of the axes of projection, its projection on the two reference planes which intersect at that axis, will be equal in length to the line itself. Its projection on the other reference plane will be a simple point.

Fig. 15 is the perspective drawing and Fig. 16 the descriptive drawing, of a line parallel to the axis of X, four units in length. Its extremities are the points A (1, 2, 2) and B (5, 2, 2). In $\mathbb{H}$

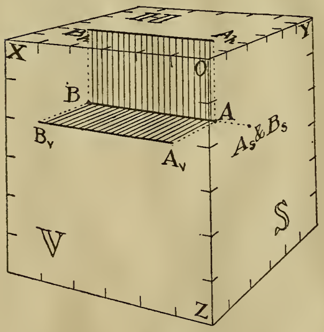


FIG. 15.

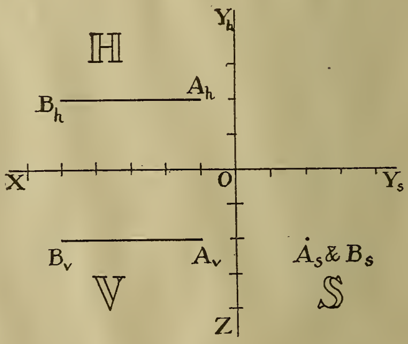


FIG. 16.

and $\mathbb{V}$ its projections are four units long. The projector-planes AA_hB_hB and AA_vB_vB are rectangles. The $\mathbb{S}$ projector-plane degenerates to a single line BAA_s. It will be seen that the coordinates of the extreme points of the line differ only in the value of the x coordinate. In fact, any point on the line will have the y and z coordinates unchanged. It is the line (x variable, 2, 2).

The student should construct for himself descriptive drawings of lines parallel to the axis of Y and the axis of Z, using preferably "coordinate paper" for ease of execution. Good examples are the lines C (1, 1, 1), D (1, 5, 1) and E (3, 1, 1), F (3, 1, 4). Points on the line CD differ only as regards the y coordinate. It is a line parallel to the axis of Y. EF is parallel to the axis of Z and z alone varies for different points along the line.

17. Foreshortening.—The projection of a line oblique to the plane of projection is shorter than the original line. This is called foreshortening. The $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$ projections of Fig. 12, and the $\mathbb{V}$ and $\mathbb{S}$ projections of Fig. 14, are foreshortened. It is a loose method of expression, but a common one, to say that a line is foreshortened when it is meant that a certain projection of a line is shorter than the line itself. When subscripts are omitted and A_hB_h is called AB, it is natural to speak of the line AB as appearing "foreshortened" in the plan view or projection on $\mathbb{H}$. This inexact method of expression is so customary that it can hardly be avoided, but with this explanation no misconception should be possible.

18. Inclined and Oblique Lines.—The words Inclined and Oblique are taken generally to mean the same thing, but in this subject it becomes necessary to draw a distinction, in order to be able to specify without chance of misunderstanding the exact nature of a given line or plane.

A line will be described as:

Parallel to an axis, when parallel to any axis. As a special case a line parallel to the axis of Z may be called simply *vertical.*

Inclined, when parallel to a reference plane, but not parallel to an axis. The line AB, Fig. 13, is an illustration.

Oblique, when not parallel to any reference plane or axis. The typical "line in space" is oblique. AB, of Fig. 11, illustrates this case.

19. Inclined and Oblique Planes.—A plane will be called:

Horizontal, when parallel to $\mathbb{H}$. The $\mathbb{V}$ projector-plane in Fig. 15 is of this kind.

Vertical, when parallel to $\mathbb{V}$ or $\mathbb{S}$. The $\mathbb{H}$ projector-plane in Fig. 15 is of this kind.

Inclined, when perpendicular to one reference plane only. The $\mathbb{H}$ projector-plane of Fig. 13 is of this kind.

Oblique, when not perpendicular to any reference plane. Planes of this kind will appear later on.

The surface of the solid of Fig. 2 is composed of vertical, horizontal, and inclined planes (but no oblique plane). Its edges are lines, parallel to the axes of X, Y and Z; and inclined lines (because parallel to 𝕊); but no oblique lines.

20. The Point on a Given Line.—It is self-evident that if a given point is on a given line, *all* the projections of the point must lie on the projections of the line.

If the middle point of a line AB is projected, as C in Fig. 17, its projections C_h, C_v, and C_s bisect the projections of the line. The reason for this appears when we consider the true shape of the

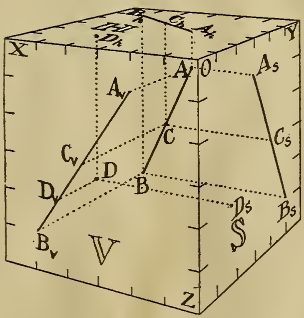


FIG. 17.

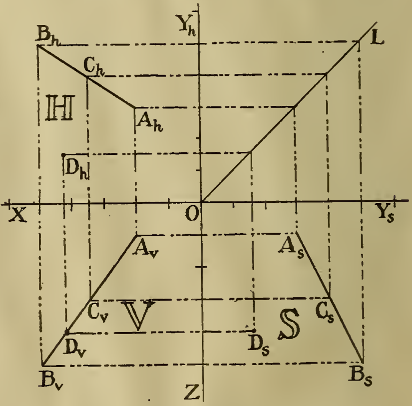


FIG. 18.

projector-planes, all three of which appear distorted in the perspective drawing, Fig. 17, and which do not appear at all on the descriptive drawing, Fig. 18. In Fig. 17 AA_hB_hB is a quadrilateral, having right angles at A_h and B_h, it is therefore a trapezoid. CC_h is parallel to AA_h and BB_h, and since it bisects AB at C, it must also bisect A_hB_h at C_h. The result of this is that in Fig. 18, where the projections which do appear are of their true size, C_h bisects A_hB_h, C_v bisects A_vB_v, and C_s bisects A_sB_s.

This principle applies to other points than the bisector. Since all ℍ projectors are parallel to each other, if any point divides AB into unequal parts, the projections of the point will divide the projectors of AB in parts having the same ratio. A point one-tenth of the distance from A to B will, by its projections, mark off one-tenth of the distance on A_hB_h, A_vB_v, etc.

The points illustrated in Figs. 17 and 18 are A $(2, 3, 1)$, B $(5, 5, 5)$ and C $(3\frac{1}{2}, 4, 3)$. It will be noticed that the x coordinate of C is the mean of those of A and B, and the y and z coordinates of C also are the mean of the y and z coordinates of A and B.

Unless all three of the projections of a point fall on the projections of a line, the point is not in the given line. If one of the projections of the point be on the corresponding projection of the line, one other projection of both point and line should be examined. If in this second projection it is found that the point does not lie on the line, it shows that the point in space lies in one of the projector-planes.

Thus the point D in Fig. 18 has its $\mathbb{V}$ projection on A_vB_v, but its $\mathbb{H}$ and $\mathbb{S}$ projections are not on A_hB_h and A_sB_s. D is not a point in the line AB but is on the $\mathbb{V}$ projector-plane of AB, as is clearly shown on Fig. 17.

In the case illustrated, D_v bisects B_vC_v. The plotting of the $\mathbb{V}$ projection of a point is governed only by its x and z coordinates. D_v bisects B_vC_v because its x and z coordinates are the means of the x and z coordinates of B and C. The y coordinate of D, however, has no connection with the y coordinates of B and C.

21. The Isometric Diagram.—A device to obtain some of the realistic appearance of a true perspective drawing without the excessive labor of its construction is known as "isometric" drawing.

A full explanation of this kind of drawing will follow later, but for present purposes we may regard it as a simplified perspective of a cube in about the position of that in Figs. 2, 6, 11, etc., but turned a little more to the left. Vertical lines are unchanged. Lines which are parallel to the axis of X, and which in the perspective drawing incline up to the left at various angles, are all made parallel and incline at 30° to the horizontal. In the same way lines parallel to the axis of Y are drawn at 30° to the horizontal, inclining up to the right.

Fig. 19 shows the shape of a large cube divided into small unit cubes. In plotting points the same scale is used in all three directions, that is, for distances parallel to all three axes. Fig. 19a shows the point P (2, 3, 1) plotted in this manner, so that the figure is equivalent to the true perspective drawing, Fig. 6.

It is not intended that the student should make any true perspective drawing while studying or reciting from this book. If any of the space diagrams here shown by true perspective drawings

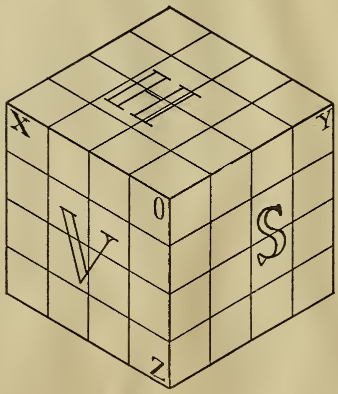


FIG. 19.

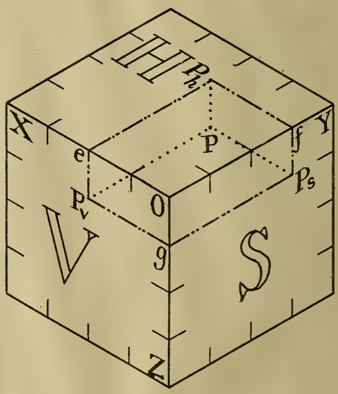


FIG. 19a.

must be reproduced, the corresponding isometric drawing should be substituted.

For rapid sketch work, especially ruled paper, called "isometric paper," is very convenient. It has lines parallel to each of the three axes. With such paper it is easy to pick out lines corresponding to those of Fig. 19.

An excellent exercise of this kind is to sketch on isometric paper the shaded solid shown in Fig. 2, taking the unit square of the paper for 1″ and considering the solid to be cut from a 10″ cube, the thickness of the walls left being 2″, and the height of the triangular portion being 6″. The solid may be sketched in several positions.

Problems II.

(For solution with wire-mesh cage, or cross-section paper, or on blackboard.)

15. A line connects the points A (5, 2, 6) and B (5, 12, 6). What are the coordinates of the point C, the center of the line?

What are the coordinates of a point D on the line, one-tenth of the way from A to B?

16. Same with points A (6, 6, 2) and B (6, 6, 12).

17. Draw the line AB whose extremities are A (2, 7, 4) and B (14, 2, 4). On what view does its true length appear? What is this length? What are the coordinates of a point C on the line one-third of its length from A?

18. With the same line A (2, 7, 4), B (14, 2, 4), state what is the true shape of the ℍ projector-plane. Give length of each edge and state what angles are right angles. Same for 𝕍 projector-plane.

19. Same as Problem 18, with line A (4, 2, 2), B (4, 11, 8).

20. With the line of Problem 19, state what is the true shape of the ℍ and 𝕊 projector-planes, giving length of each edge, and state which angles are right angles.

21. Same as Problem 17, with line A (0, 4, 8), B (9, 4, 1).

22. The ℍ projection of C (8, 2, 6) lies on the ℍ projection of the line A (10, 1, 9), B (2, 5, 2). Is the point on the line?

23. Same as Problem 22, with line A (2, 1, 8), B (8, 10, 5), and point C (4, 4, 7).

24. A triangle is formed by joining the points A (6, 3, 1), B (10, 3, 10) and C (2, 3, 10). In what view or views does the true length of AB appear? In what view or views does the true length of BC appear? Mark the center of the triangle (one-third the distance from the center of the base BC to the vertex A) and give its coordinates.

25. Same with points A (5, 9, 6), B (5, 3, 1) and C (5, 3, 12).

26. Same with points A (10, 1, 4), B (7, 10, 4) and C (1, 4, 4).

27. The 𝕍 projections of the points A (8, 1, 2), B (10, 3, 8), C (4, 3, 10) and D (2, 1, 4) form a square. Draw the projections and connect them point to point. What are the coordinates of the center where AC and BD intersect?

28. Plot the parallelogram A (11, 3, 3), B (3, 3, 3), C (7, 9, 7), D (15, 9, 7). The diagonals intersect at E. Give the coordinates of E. Describe the ℍ projector-planes of AB, AC, CD, giving lengths of sides of each quadrilateral. Is the plane of the figure inclined or oblique? Is AC an inclined or oblique plane?

29. Plot the quadrilateral

A (11, 10, 3), B (3, 10, 11), C (7, 2, 7), D (11, 4, 3).

Is the plane of the figure horizontal, vertical, inclined or oblique?
Is the line AB horizontal, vertical, inclined or oblique?
Is the line BC horizontal, vertical, inclined or oblique?
Is the line CD horizontal, vertical, inclined or oblique?
Is the line DA horizontal, vertical, inclined or oblique?

CHAPTER III.

THE TRUE LENGTH OF A LINE IN SPACE.

22. The Use of an Auxiliary Plane of Projection.—To find the true length of a "line in space," or oblique straight line, an auxiliary plane of projection is of great value, and is constantly used in all branches of Engineering Drawing.

A typical solution is shown by Figs. 20 and 21. The essential feature is the selection of a new plane of projection, called an

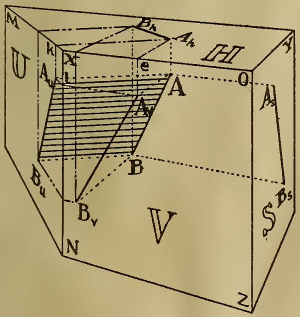


FIG. 20.

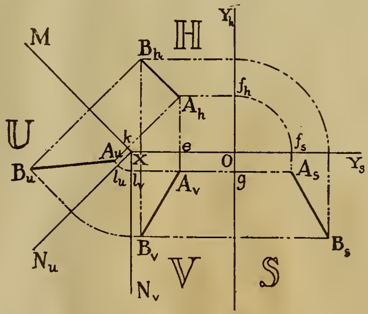


FIG. 21.

auxiliary plane, and denoted by U, which must be parallel to the given line and easily revolved into coincidence with one of the regular planes of reference.

This auxiliary plane is passed parallel to one of the projector-planes. In Fig. 20 the plane S′ of the cube of planes has been replaced by a plane U, parallel to the H projector-plane, AA_hB_hB. Like that plane, U is also perpendicular to H, and XM, the line of intersection of U and H, is parallel to A_hB_h. The distance of the plane U from the projector-plane may be taken at will and in the practical work of drawing it is a matter of convenience, choice being governed by the desire to make the resulting figures clear and separated from each other. In Fig. 20 the auxiliary plane 𝕌 has been established by selecting a point X in ℍ for it to pass through. 𝕌 is an "inclined plane," not an "oblique plane."

23. Traces of the Auxiliary Plane 𝕌.—The auxiliary plane 𝕌 cuts the plane 𝕍 in a line XN, parallel to the axis of Z. The lines of intersection of 𝕌 with the reference planes, are called the "traces" of 𝕌. Since there are three reference planes, there may be as many as three traces of 𝕌. In the case illustrated in Fig. 20, there are, however, but two traces. Only one of these three possible traces of 𝕌 can be an inclined line. In Fig. 20 the trace XM alone is an "inclined" line.

We shall see later that the auxiliary plane may be taken perpendicular to 𝕍 or to 𝕊 as alternative methods. In every case there is but one inclined trace, that on the plane to which 𝕌 is perpendicular. It is this trace which has the greatest importance in the process. For the sake of uniformity, M and N will be assigned as the symbols for marking the traces of an auxiliary plane of projection.

24. The 𝕌 Projectors.—A new system of projectors, AA_u, BB_u, etc., project the line AB upon the plane 𝕌. These projectors, being perpendicular distances between a line and a plane parallel to it, are all equal, and the projector-plane AA_uB_uB of Fig. 20 is in reality a rectangle. A_uB_u is therefore equal in length to AB, or AB is projected upon 𝕌 without foreshortening.

25. Development of the Auxiliary Plane 𝕌.—The descriptive drawing, Fig. 21, is the drawing of practical importance, which is based on the perspective diagram, Fig. 20, which shows the mental conception of the process employed. In practical work, of course, Fig. 21 alone is drawn, and it is constructed by geometrical reasoning deduced from the mental process exhibited by Fig. 20.

In the process of flattening out or "developing" the planes of projection, 𝕌 is generally considered as attached or hinged to the "inclined trace," XM in this example. In Fig. 21 𝕌 has been revolved about XM, bringing it into the plane of ℍ, the trace XN having opened out to two lines. N separates into two points and is marked N_u as a point in 𝕌 and N_v as a point in 𝕍, analogous to Y_h and Y_s in the development of the reference planes. The space N_uXN_v, like Y_hOY_s, may be considered as construction space.

26. Fourth Law of Projection—that for Auxiliary Plane, $\mathbb{U}$.— It will be seen from Fig. 20 that AA_heA_v is a rectangle and that eA_v is equal to A_hA. On the descriptive drawing, Fig. 21, these two lines, eA_v and eA_h, form one line perpendicular to OX. This is in accordance with the first law of projection of Art. 11.

As the plane $\mathbb{U}$ is perpendicular to $\mathbb{H}$ we have the same relation there, and AA_hkA_u, Fig. 20, is a rectangle. kA_u is therefore equal to A_hA, and in the development, Fig. 21, A_uk and kA_h form one line A_ukA_h perpendicular to XM.

If from A_u and A_v, Fig. 20, perpendiculars are let fall upon the intersection of $\mathbb{U}$ and $\mathbb{V}$ (the trace XN) they will meet at the common point l, both kA_ulX and XlA_ve being rectangles. In the descriptive drawing, Fig. 21, A_ul is perpendicular to XN_u, l_ul_v is the arc of a circle, center at X, and l_vA_v is perpendicular to XN_v.

The following law of projection governs the position of A_u in the plane $\mathbb{U}$:

(4) From the regular projections of A draw perpendiculars to the traces of $\mathbb{U}$. These lines continued into the field of $\mathbb{U}$ intersect at A_u. One of these lines is carried across the construction space by the arc of a circle whose center is the meeting point of the traces of $\mathbb{U}$.

27. The Graphical Application of this Law to a Point.—The procedure for locating the projection A_u on the descriptive drawing, Fig. 21, after the location of the plane $\mathbb{U}$ has been determined, is as follows: From the adjacent projections of the point draw lines perpendicular to the traces of the plane $\mathbb{U}$. Continue one of these lines across the trace. Swing the foot of the other perpendicular to the duplicated trace, and continue it by a line perpendicular to this trace to meet the line first mentioned. Their intersection is the projection of the point on $\mathbb{U}$. In Fig. 21, this requires A_hkA_u to be drawn perpendicular to XM, and the line $A_vl_vl_uA_u$ to be traced as shown.

28. The True Length of a Line.—The procedure for finding *the true length of a line* consists in first drawing, Fig. 21, a line parallel to one of the projections of the line to act as the trace of the auxiliary plane. Where this trace intersects an axis of projection perpendicular lines are erected, one perpendicular to the axis, one perpendicular to the trace. These lines are the two developed positions of the other trace of the plane 𝕌. Then locate the extremities of the given line on the auxiliary plane 𝕌. The line joining the extremities is the required projection of the line on 𝕌, and is equal in length to the given line.

29. Alternative Method of Developing the Auxiliary Plane, 𝕌.— A modification of this construction is shown in the descriptive drawing, Fig. 22, in which the plane 𝕌 has been revolved about the vertical trace XN until it coincides with the plane 𝕍. XM separates into two lines, XM_h and XM_u. k, of Fig. 20, becomes k_h and k_u, and the space M_hXM_u is construction space. A is on

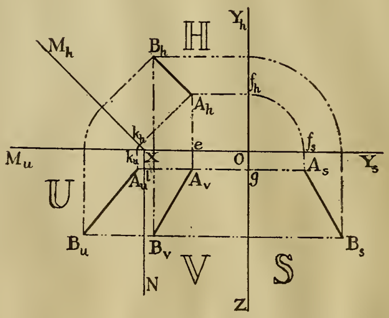


FIG. 22.

a horizontal line drawn through A_v. A_hk_h is perpendicular to XM_h. k_hk_u is the arc of a circle having X as a center, and k_uA_u is perpendicular to XM_u. A_u is thus located.

This method of development of the planes is much less common in practical drawing than the other, because, as a rule, it is less convenient than the first method. In such cases as occur it offers no particular difficulty. Both Figs. 21 and 22 are solutions of the problem of finding the true length of the oblique line AB by projection on an auxiliary inclined plane, 𝕌.

30. Alternative Positions of the Plane 𝕌.—We saw that the exact position of the plane 𝕌, so long as it remained perpendicular to ℍ and parallel to A_hB_h, was left to choice governed by practical considerations. $\mathbb{U}$ itself, however, may be taken perpendicular to $\mathbb{V}$ and parallel to A_vB_v, or it may be taken perpendicular to $\mathbb{S}$ and parallel to A_sB_s. To get an entire grasp of the subject the student is advised to trace Fig. 21 on thin paper, or plot it on coordinate paper, points A, B, and X being $(6, 6, 2)$, $(10, 10, 8)$ and $(11, 0, 0)$, and fold the figure into a paper box diagram, the construction spaces N_uXN_v and Y_hOY_s being creased in the middle and folded out of the way. Fig. 22 will serve equally well. The final result will be a paper box exactly similar to Fig. 20.

The variation in which $\mathbb{U}$ is perpendicular to $\mathbb{V}$ may be plotted, passing the new inclined trace of $\mathbb{U}$ (lettered YM) through the point $(0, 0, 3)$ parallel to A_vB_v. Fold this figure into a paper box, the paper being cut along a line YN perpendicular to YM.

The other variation may be plotted with the inclined trace of $\mathbb{U}$ on the plane $\mathbb{S}$, parallel to A_sB_s and passing through the point $(0, 0, 6)$ (f_s of Fig. 21). Letter this trace Y_sM and draw Y_sN perpendicular to it, inclining up to the right. The paper must be cut on this line to enable it to be properly folded.

31. The Method Applied to a Plane Figure.—The special value of this use of the auxiliary plane is seen when one operation serves to give the true length of a number of lines at once, and thus shows a whole plane figure in its true shape.

In Fig. 23 the polygon $ABCDE$ is shown by its projection, the point A alone being lettered. It is noticeable that in $\mathbb{V}$ the edges all form one straight line. The $\mathbb{V}$ projector-planes of the various edges are therefore all parts of the same plane, and the polygon itself is a plane figure placed perpendicular to $\mathbb{V}$. It may be said the polygon is "seen on edge" in $\mathbb{V}$.

An auxiliary plane $\mathbb{U}$ has been taken parallel to the plane of the polygon, and therefore perpendicular to $\mathbb{V}$. The trace XM being parallel to the $\mathbb{V}$ projections of the edges, this auxiliary plane serves to show the true length of all the edges at once. The projection on $\mathbb{U}$ is the true shape of the polygon $ABCDE$. In the case illustrated, the $\mathbb{U}$ projection discloses the fact that the polygon is a regular pentagon, a fact not realized from the regular projections, owing to the foreshortening to which they are subject.

This figure is well adapted to tracing and folding into a paper box diagram.

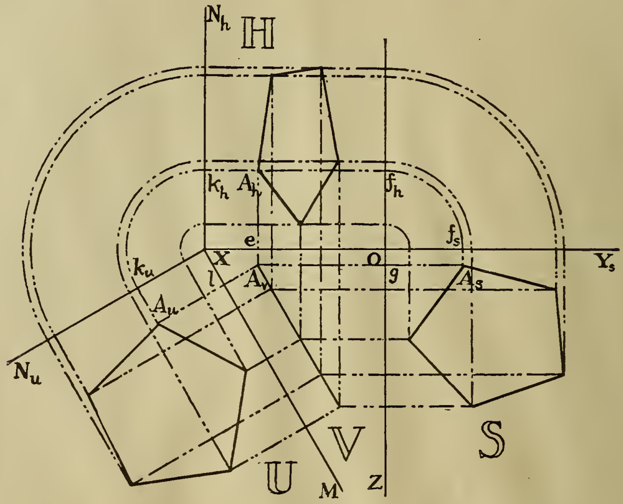


FIG. 23.

32. The True Length of a Line by Revolving About a Projector.—A second method of finding the true length of a line seems in a way simpler, but proves to be of much less value in practical work. The method consists in supposing an oblique line AB to be revolved about a projector of some point in the line until it becomes parallel to one of the planes of reference. In this new position it is projected to the reference plane as of its true length.

In Fig. 24 the V projector-plane of the line AB has been shaded for emphasis (A is the point $(1, 1, 5)$, and B is the point $(5, 4, 2)$). The projector AA_v has been selected at will, and the V projector-plane (of which the line AB is one edge) has been rotated about AA_v as an axis until it has come into the position $A_vB'_vB'A$. In its new position, AB' projects to H as $A_hB'_h$. This is the true length of the line. During its rotation the point B has moved to B', but in so doing it has not revolved about A as its center, but about the point b on A_vA extended. bA_v is equal in length to BB_v.

B_v moves to B'_v, revolving about A_v as a center. In Fig. 25, the corresponding descriptive drawing, the original projections are shown as full lines and the projections of the line after the rotation has occurred are shown by long dashes.

In $\mathbb{V}$, A_vB_v swings about A_v as a pivot until in its new position $A_vB'_v$ it is parallel to OX. In $\mathbb{H}$, B_h moves in a line parallel to OX (since in Fig. 24 the motion of B takes place entirely in the plane of bBB', which is parallel to $\mathbb{V}$), and as B'_h must be vertically above B'_v the motion terminates where a line drawn vertically

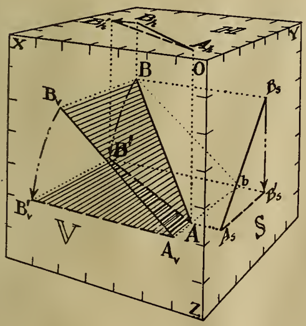


FIG. 24.

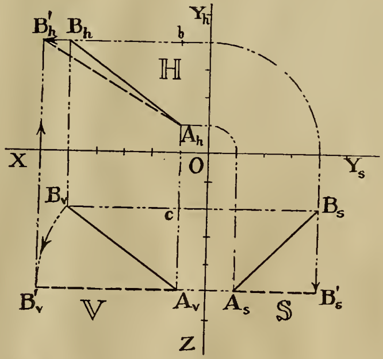


FIG. 25.

up from B'_v meets the horizontal line $B_hB'_h$. Joining A_h and B'_h, the new $\mathbb{H}$ projection is the true length of the given line. The $\mathbb{S}$ projection is of no interest in this case. The $\mathbb{H}$ and $\mathbb{V}$ projections of Fig. 25 show the graphical process corresponding to the theory of this rotation. In $\mathbb{V}$, B_v moves to B'_v, whence a vertical projector meeting a horizontal line of motion from B_h determines B'_h, the new position of B_h. $A_hB'_h$ is the true length of the line. The arrow-heads on the broken lines make these steps clear.

33. Variations in the Method.—The method is subject to wide variations. The same projector-plane AA_vB_vB, Fig. 24, revolving about the same projector AA_v, might start in the opposite direction and swing to a position parallel to $\mathbb{S}$. The graphical process of Fig. 25 would then confine itself to $\mathbb{V}$ and $\mathbb{S}$ instead of $\mathbb{V}$ and $\mathbb{H}$.

In addition, the rotation might have been about BB_v as an axis or about the 𝕍 projector of any point in AB or AB extended. Finally, the ℍ projector-plane or the 𝕊 projector-plane might have been selected and made to revolve into position. There are six distinct varieties of the process, each one subject to great modifications.

This method can be applied to a plane figure which appears "on edge" in one of the regular views. In Fig. 26 a polygon lies in a

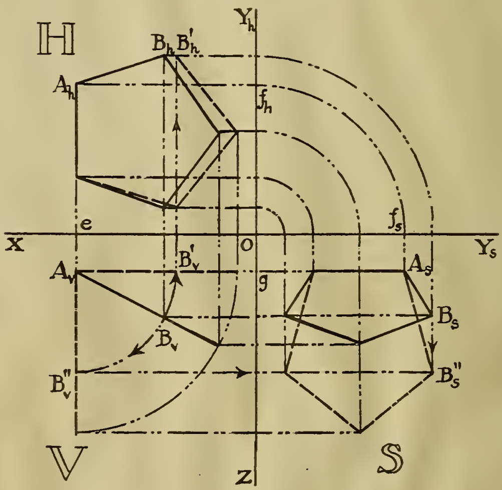


FIG. 26.

plane perpendicular to 𝕍. There are two varieties of the process applicable in this case. Choosing the 𝕍 projector of the point A for the axis of rotation, the whole polygon may be rotated up parallel to ℍ, thence its true shape projected upon ℍ, or it may be revolved down until parallel to 𝕊, thence its true shape projected upon 𝕊. Both methods are shown, though of course in practice one at a time should be enough.

34. A Projector-Plane Used as an Auxiliary Plane.—The two processes for finding the true length of a line differ in this respect.

In one the line is projected on a plane which is revolved into *coincidence* with one of the reference planes, by revolving about a line in that reference plane. In the second process, a projection plane is itself revolved about a projector, that is, about a line *perpendicular* to one reference plane, to a position *parallel* to a second reference plane. The line in its new position is projected on the latter plane.

A method which is a modification of the first process is in many cases very simple. A projector-plane is itself used as an auxiliary plane, and is revolved into coincidence with the plane to which it is perpendicular by rotation about its trace in that plane. In Fig. 23, for example, instead of passing XM parallel to A_vC_v, A_vC_v would be extended to the axis of X, and used itself for the inclined trace of the auxiliary plane. XN would be moved to the right and other slight modifications made.

As in the second method, a projector-plane is here rotated; but it is not rotated about a *projector,* but about a *projection* (its trace), and the real similarity of the process is with the first method, that of the auxiliary plane of projection. It is but a special case of this kind.

In practical drawing, it rarely happens that one of the projector-planes can be thus used itself with advantage as an auxiliary plane of projection. It leads usually to an overlapping of views and it will not be found so useful as the more general method.

For the continuation of this study, all these methods should be at the students' finger ends.

35. The True Length of a Line by Constructing a Right Triangle.—These methods of finding the true length of a line are generally used for the true lengths of many lines in one operation, or for the true shape of a plane figure. When a *single* line is wanted, the construction of a right triangle from lines whose true lengths appear on the drawing is sometimes resorted to. In Fig. 24 the triangle ABb is a right triangle, AB being the hypotenuse and AbB the right angle. In the descriptive drawing, Fig. 25, A_vB_v is equal in length to Ab of Fig. 24, and A_hb is easily found, equal to Ab of Fig. 24. These lines may be laid off at any convenient place as the sides of a right triangle, and the hypotenuse measured to give the true length of AB. Mathematically the hypotenuse is the square root of the sum of the squares of the sides. In the case illustrated A_vB_v is 5 (itself the square root of $\overline{A_vc}^2+\overline{B_vc}^2$, or $\sqrt{3^2+4^2}$) and A_hb is 3. The length AB is therefore $\sqrt{5^2+3^2} = \sqrt{34} = 5.83$.

Problems III.

(For use with wire-mesh cage, cross-section paper, or blackboard.)

30. A square in a position similar to the pentagon of Fig. 26 has the corners A (10, 12, 2), B (2, 12, 8), C (2, 2, 8) and D (10, 2, 2). Find its true shape by the use of an auxiliary plane.

31. A square is in a position similar to the pentagon of Fig. 23. The corners are A (9, 3, 3), B (9, 13, 3), C (3, 13, 11), and D (3, 3, 11). Find its true shape by revolving into a plane paralel to $\mathbb{H}$.

32. Plot the triangle A (11, 3, 2), B (12, 6, 12), C (14, 12, 7). Find its true shape by the use of an auxiliary plane perpendicular to $\mathbb{H}$.

33. Plot the triangle A (13, 14, 8), B (10, 10, 0), C (7, 6, 8). Show the true shape of the triangle by revolving it about AA_h until in a plane parallel to $\mathbb{S}$. Find the true shape by projection on a plane $\mathbb{U}$ perpendicular to $\mathbb{H}$, whose inclined trace passes through the point (0, 16, 0). (With the wire-mesh cage turn plane $\mathbb{S}$ to serve for this auxiliary plane $\mathbb{U}$.)

34. Same with triangle A (9, 7, 8), B (12, 11, 13), C (15, 15, 2).

35. Plot the right triangle A (14, 4, 3), B (14, 10, 3), C (6, 4, 9). Revolve it about BB_v into a plane parallel to $\mathbb{H}$ and project its true shape on $\mathbb{H}$. (With the wire-mesh cage put markers at points A, B, C and C', the new position of C.)

36. Plot the right triangle A (9, 3, 6), B (9, 3, 0), C (15, 11, 6). Revolve it about AB until in a plane parallel to $\mathbb{V}$ and plot C', the new position of the vertex. Revolve it about the same axis into a plane parallel to $\mathbb{S}$, and plot C'', the new position of the vertex. (With the wire-mesh cage put point markers at A, C, C' and C''.)

37. Plot the square A $(14, 8, 2)$, B $(10, 2, 7\frac{1}{5})$, C $(10, 14, 7\frac{1}{5})$, D $(8, 8, 12\frac{2}{5})$. The diagonal is 12 units long. Revolve it about AA_v into a plane parallel to $\mathbb{H}$, and project its true shape on $\mathbb{H}$. (With wire-mesh cage put point markers at A, B, C, D, B', C', and D'.)

38. Plot the triangle A $(12, 2, 14)$, B $(2, 2, 14)$, C $(7, 7, 2)$. Revolve it about ABA_s into a plane parallel to $\mathbb{H}$, and project the true shape on $\mathbb{V}$. (With wire-mesh cage put markers at points A, B, C and A'. On coordinate paper or blackboard show true shape by projection on an auxiliary plane $\mathbb{U}$ perpendicular to $\mathbb{S}$, through the point $(0, 8, 0)$.)

(For use on coordinate paper or blackboard, not wire-mesh cage.)

39. The triangle A $(3, 7, 11)$, B $(13, 2, 13)$, C $(5, 2, 1)$ is a triangle in an oblique plane. Find its true shape as follows: BC appears at its true length in $\mathbb{V}$. Draw A_vD_v perpendicular to B_vC_v. AD is an oblique line, but it is perpendicular to BC since its $\mathbb{V}$ projector-plane AA_vD_vD is perpendicular to BC. Find the true length of AD by any method. On $\mathbb{V}$ extend A_vD_v to E_v, making D_vE_v equal to the true length of AD. $E_vB_vC_v$ is the true shape of the triangle ABC.

CHAPTER IV.

PLANE SURFACES AND THEIR INTERSECTIONS AND DEVELOPMENTS.

36. The Omission of the Subscripts, h, v, **and** s—In a descriptive drawing a point does not itself appear but is represented by its projections on the reference planes. This fact has been emphasized in the previous chapters. In the more complicated drawings which now follow it will save time and will prevent overloading the figures with lettering, to omit the subscripts h, v, and s, and to refer to a point and its projections by the same letter. Thus "A_v" or "the point A in 𝕍" are expressions which call attention to the projection of A on 𝕍, but a diagram will show only the letter A at that place. If at any time it is necessary to be more precise the subscripts may be restored. They should be used if any confusion is caused by their omission.

If the projections of two points coincide, it is sometimes advisable to indicate which point is behind the other in that view by forming the letter of fine dots. Referring back to Fig. 14, the projections of A and B on 𝕊 coincide. On this system subscripts are omitted and the letter B (on 𝕊 only) is formed of dots, as in Fig. 27.

37. Intersecting Plane Faces.—Many pieces of machines and structures which form the subjects of mechanical drawings, are pieces all of whose surfaces are portions of planes, each portion or face having a polygonal outline.

In making such drawings there arise problems as to the exact points and lines of intersection, which can be solved by applying the laws of projection treated of in the preceding chapters. How these intersections are determined from the usual data will now be shown.

38. A Pyramid Cut by a Plane.—As a simple example let us suppose that it is required to find where a plane perpendicular to 𝕍, and inclined at an angle of 30° with ℍ, intersects an hexagonal pyramid with axis perpendicular to ℍ. Fig. 27 is the drawing of the pyramid, having the base *ABCDEF* and vertex *P*. The cutting plane is an inclined plane such as we have used for an auxiliary plane, and its traces are therefore similar to those of an auxiliary plane. *KL* is the inclined trace on 𝕍 and *KK′* and *LL′* are the traces parallel to the axis of *Y*. The problem is to find the shape of the polygonal intersection *abcdef* in ℍ and 𝕊, and its true shape.

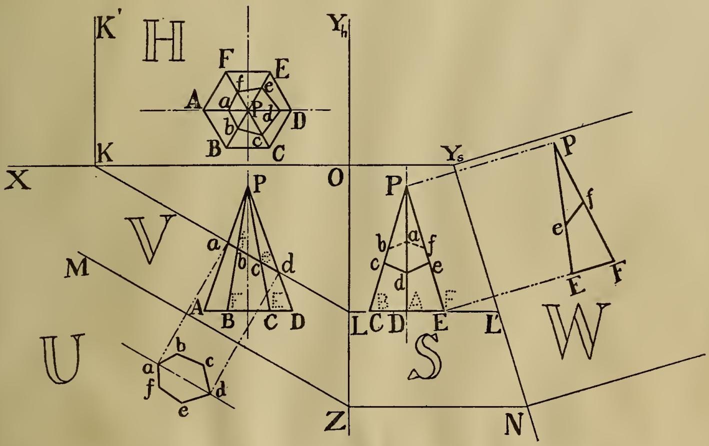


FIG. 27.

The method of solution of all such problems is to take into consideration each edge of the pyramid in turn, and to trace the points where they pierce the plane. Thus, the edge *PA* pierces the given plane at *a*, whose projection on 𝕍 is first located; for the given plane is *seen on edge* in 𝕍, and *PA* cannot pierce the plane at any other point consistent with that condition. *a*, once located in 𝕍, can be projected horizontally to the line *PA* in 𝕊 and vertically to *PA* in ℍ.

The true shape of the polygon *abcdef* may be shown on an auxiliary plane, 𝕌, whose traces are *ZM* and *ZN*. In Fig. 27 the projection of the pyramid on 𝕌 is incomplete. As it is only to show the polygon *abcdef* the rest of the figure is omitted.

39. Intersecting Prisms.—As an example of somewhat greater difficulty let it be required to find the intersection of two prisms, one, the larger, having a pentagonal base, parallel to H; and the other a triangular base, parallel to S. The axes intersect at right angles, and the smaller prism pierces the larger.

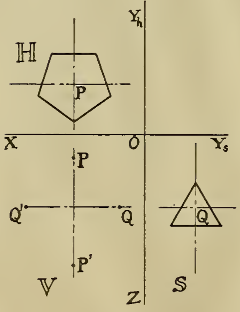


FIG. 28.

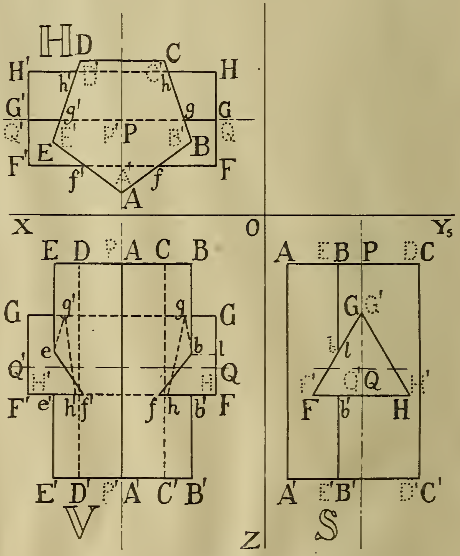


FIG. 29.

The known elements or data of the problem are shown recorded as a descriptive drawing in Fig. 28. It shows the projection of the pentagon on H, of the triangle on S, and of the axes on V. The problem is to complete the drawing to the condition of Fig. 29, shown on a larger scale. The corners of the pentagonal prism are *ABCDEF* and *A'B'C'D'E'F'* and its axis is *PP'*. The corners of the triangular prism are *FGH* and *F'G'H'* and its axis is *QQ'*.

40. Points of Intersection.—The general course in solving the problem of the intersection of the prisms is to consider each edge of each prism in turn, and to trace out where each edge pierces the various plane faces of the other prism. When all such points of intersection have been determined, they are joined by lines to give the complete line of intersection of the prisms.

To determine where a given edge of one prism cuts a given plane face of the other prism, that view in which the given plane face is seen as a line only, or is "seen on edge," as is said, must be referred to. Taking the hexagonal prism first, the edges *AA′*, *CC′*, and *DD′* entirely clear the triangular prism, as is disclosed by the plan view on ℍ where they appear "on end" or as single points only. They, therefore, have no points of intersection with the triangular prism and in 𝕍 and 𝕊 these lines may be drawn as uninterrupted lines, being made full or broken according to the rule at the end of Art. 5. *BB′*, as may be seen in ℍ, meets the small prism. This line when drawn in 𝕊, where the plane faces *FF′G′G* and *FF′H′H* are seen on edge, meets those faces at *b* and *b′*. From 𝕊 these points are projected to 𝕍. The edge *BB′* consists really of two parts, *Bb* and *b′B′*. *EE′* meets the same two faces at points *e* and *e′* determined in the same way.

FF′, when drawn in ℍ, is seen to pierce the plane face *AA′B′B* at *f* and *AA′E′E* at *f′*. These points, located in ℍ, are projected vertically down to 𝕍. *GG′* in ℍ pierces *BB′C′C* at *g*, and *EE′D′D* at *g′*. *h* and *h′* on the line *HH′* are similarly determined first in ℍ and are projected down to 𝕍.

41. Lines of Intersection.—Having found the points of intersection of the edges, we determine the lines of intersection of the plane surfaces by considering the intersections of *plane* with *plane*, instead of *line* with *plane*. *BB′* is one line of the plane *AA′B′B*, and pierces the plane *FF′G′G* (seen on edge in 𝕊) at *b*. *b* is therefore a point of both planes. *FF′* is a line of the plane *FF′G′G*, and it pierces the plane *AA′B′B* (seen on edge in ℍ) at *f*. *f* is also a point common to both planes. Since these two points are in both planes, they are points on the line of intersection of the two planes. We therefore connect *b* and *f* by a straight line in 𝕍, but do not extend it beyond either point because the planes are themselves limited.

By the same kind of reasoning *b* and *g* are found to be points common to *BB′C′C* and *FF′G′G*, and are therefore joined by a straight line, *bg* in 𝕍. *gh* also is the line of intersection of two planes, and the student should follow for himself the full process of reasoning which proves it. *e*, *f′*, and *g′* are points similar to *b*, *f*, and *g*. Since the original statement required the triangular prism to *pierce* the pentagonal one, *gg′*, *ff′*, and *hh′* are joined by broken lines representing the concealed portions of the edges *GG′*, *FF′*, and *HH′* of the small prism. Had it been stated that the object was one solid piece instead of two pieces, these lines would not exist on the descriptive drawing.

42. Use of an Auxiliary Plane of Projection.—To find the intersection of solids composed of plane faces, it is essential to have

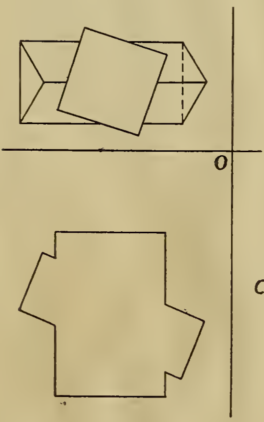


FIG. 30.

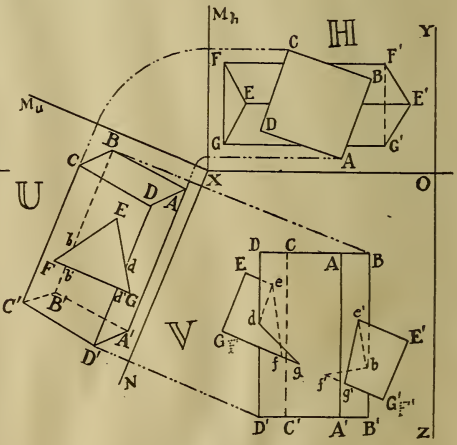


FIG. 31.

views in which the various plane faces are seen on edge. To obtain such views, an auxiliary plane of projection is often needed.

Fig. 30 shows the data of a problem which requires the auxiliary view on 𝕌 in order to show the side planes of the triangular prism "on edge." (These planes are oblique, not inclined, and therefore do not appear "on edge" on any reference plane.) Fig. 31 shows the complete solution, the object drawn being one solid piece and not one prism piercing another prism. *b* and *d* are located by the use of the view on 𝕌. In this case and in many similar cases in practical drawing, the complete view on 𝕌 need not be constructed.

The use of 𝕌 is only to give the position of *b* and *d*, which are then projected to 𝕍. The construction on 𝕌 of the square ends of the square prism are quite superfluous, and would be omitted in practice. In fact, the view 𝕌 would be only partially constructed in pencil, and would not appear on the finished drawing in ink.

After the method is well understood, there will be no uncertainty as to how much to omit.

43. A Cross-Section.—In practical drawing it often occurs that useful information about a piece can be given by imagining it *cut by a plane surface,* and the shape of this plane intersection drawn. In machine drawing, such a section showing only the material actually cut by the plane and nothing beyond, is called a "cross-section."

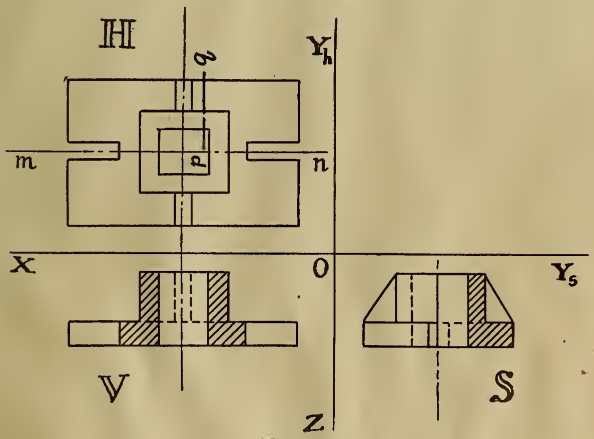


FIG. 32.

In other branches of drawing other names for the same kind of a section are used, The "contour lines" of a map are of this nature, as well as the "water lines" of a hull drawing in Naval Architecture.

44. Sectional Views.—The cross-section is used freely in machinery drawing, but a "sectional view," which is a view of a cross-section, with all those parts of the piece which lie beyond the plane of the section as well, is much more common.

These sectional views are sometimes made additional to the regular views, but often replace them to some extent. Fig. 32 is a good example. It represents a cast-iron structural piece shown by plan, and two sectional views. The laws of projection are not altered, but the views bear no relation to each other in one respect. One view is of the whole piece, one is of half the piece, and one is of three-quarters of it. The amount of the object imagined to be cut away and discarded in each view is a matter of independent choice.

In the example the projection on 𝕍 is a view of half of the piece, imagining it to have been cut on a plane shown in ℍ by the line *mn*. The half between *mn* and *OX* has been discarded, and the drawing shows the far half. The actual section, the cross-section on the line *mn*, is an imaginary surface, not a true surface of the object, and it is made distinctive by "hatching." This hatching is a conventional grouping of lines which show also the material of which the piece is formed. For this subject, consult tables of standards as given in works on Mechanical Drawing. This projection on 𝕍 is not called a "Front Elevation," but a "Front Elevation in Section," or a "Section on the Front Elevation."

The view projected on 𝕊 is called a "Side Elevation, Half in Section," or a "Half-Section on the Side Elevation." Since a section generally means a sectional view of the object *with half removed,* a half-section means a view of the object with *one-quarter removed.* If, in ℍ, the object is cut by a plane whose trace is *np* and another whose trace is *pq,* and the N. E. corner of the object is removed, it will correspond to the condition of the object as seen in 𝕊.

Sections are usually made on the center lines, or rather on central planes of the object. When strengthening ribs or "webs" are seen in machine parts, it is usual to take the plane of the section just in front of the rib rather than to cut a rib or web which lies on the central plane itself. This position of the imaginary saw-cut is selected rather than the adjacent center line.

When the plane of a section is not on a center line, or adjacent to one, its exact location should be marked in one of the views in which it appears "on edge," and reference letters put at the extremities. The section is then called the "section on the line *mn*."

The passing of these section planes causes problems in intersection to arise, which are similar to those treated in Articles 37-42.

45. Development of a Prism.—It is often desired to show the true shape of all the plane faces of a solid object in one view, keeping the adjacent faces in contact as much as possible. This is called *developing the surface on a plane,* and is particularly useful for all objects made of sheet-metal, as the development forms a pattern for cutting the metal, which then requires only to be bent into shape and the edges to be joined or soldered.

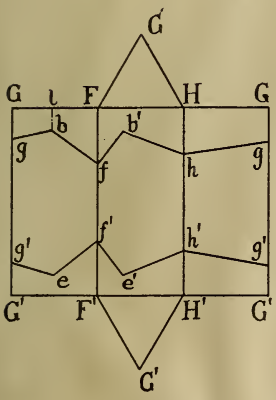


Fig. 33.

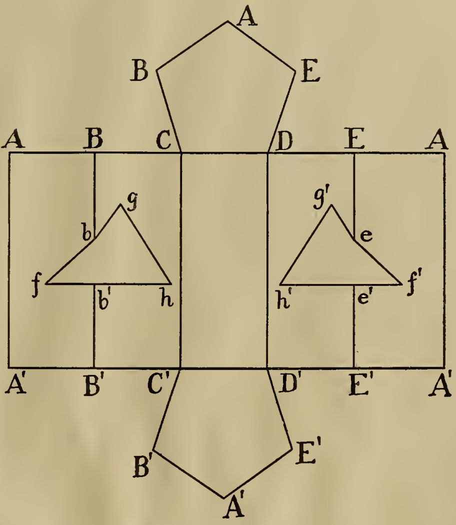


Fig. 34.

Development is a process already applied to the planes of projection themselves when these planes were revolved about axes until all coincided in one plane. The same operation applied to the surfaces of the solid itself produces the development.

The two prisms of Fig. 29 afford good subjects for development. Fig. 33 shows the developed surface of the triangular prism, the lines *g-g* and *g′-g′* showing the lines of intersection with the other prism. In this figure it is considered that the surface of the triangular prism is cut along the lines *GG′, GF, G′F′, GH,* and

G'H'; and the four outer planes unfolded, using the lines bounding *FF'H'H* as axes, until the entire surface is flattened out on the plane of *FF'H'H*.

Fig. 34 shows the development of the large prism of Fig. 29, with the holes where the triangular prism pierces it when the two are assembled. The surface of the prism is cut on the line *AA'*, and on other lines as needed, and the surfaces are flattened out by unfolding on the edges not cut.

The construction of these developments is simple, since the surfaces are all triangles or pentagons whose true shapes are given; or are rectangles, the true length of whose edges are already known.

In Fig. 33 the distances *Gg, G'g', Ff, F'f'* are taken directly from 𝕍 in Fig. 29. The points *b* and *e* are plotted as follows: The perpendicular distance *bl* to the line *GF* is taken from 𝕍, Fig. 29, and *Gl* is taken from *Gl* in 𝕊, Fig. 29. The other points are plotted in the same manner.

46. Development of a Pyramid.—Fig. 35 shows the development of the point of the pyramid, Fig. 27, cut off by the intersecting

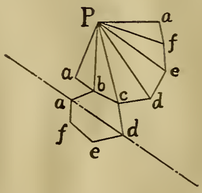


FIG. 35.

plane whose trace is *KL*. The base is taken from the projection on 𝕌, where its true shape is given. Each slant side must have its true shape determined, either as a whole plane figure (Art. 31), or by having all three edges separately determined (Art. 28 or Art. 32) In this case *Pa* and *Pd* are shown in true length in 𝕍, Fig. 27, and it is only necessary to determine the true lengths of *Pb* and *Pc* (or their equivalents *Pf* and *Pe*) to have at hand all the data for laying out the development. The face *Pef* may be conveniently shown in its true shape on an auxiliary plane 𝕎, Fig. 27, perpendicular to 𝕊 and cutting 𝕊 in a trace Y_sX as shown.

Problems IV.

(For use with wire-mesh cage, or on cross-section paper or blackboard.)

40. Plot the projections of the points A (9, 3, 16), B (6, 3, 16), C (6, 8, 16), D (9, 18, 16), and E (0, 3, 4), F (0, 3, 8), G (0, 8, 8), H (0, 8, 4). Join the projections A to E, B to F, C to G, etc. (With wire-mesh cage use stiff wire to represent the lines AE, BF, etc.) Show how to find the true shape of every plane surface of the figure contained between the 4 lines, and the planes 𝕊 and ℍ′. On cross-section paper or on blackboard show how to draw the development of the surface of the solid.

41. Same as Problem 40, with points A (10, 8, 0), B (8, 10, 0), C (12, 14, 0), D (14, 12, 0) on ℍ and E (10, 8, 16), F (6, 12, 15), G (6, 14, 15), H (12, 10, 14) on ℍ′.

42. Draw the tetrahedron whose four corners are A (16, 2, 13), B (6, 2, 13), C (11, 14, 13) and D (11, 7, 1). It is intersected by a plane perpendicular to 𝕍 cutting 𝕍 in a trace passing through the origin, making an angle of 30° with OX. Draw the trace of the plane on 𝕍. Where are its traces on ℍ and 𝕊? Show the ℍ and 𝕊 projections of the line of intersection of the plane and tetrahedron.

43. A solid is in the form of a pyramid whose base is a square of 10″, and whose height is 8″. The corners are A (16, 2, 10), B (10, 2, 2), C (2, 2, 8) and D (8, 2, 16) and the vertex E (9, 10, 9). It is intersected by a plane perpendicular to ℍ, whose trace on ℍ passes through the origin making an angle of 30° with OX. Draw the 𝕍 and 𝕊 projections of the intersections of the pyramid and plane. Where is the trace of the cutting plane on 𝕍?

44. A plane ℍ′ is parallel to ℍ at a distance of 16 inches. A square prism has its base in ℍ, points A (8, 2, 0), B (3, 7, 0), C (8, 12, 0), D (13, 7, 0). Its other base is in ℍ′, points $A'B'C'D'$ having same x and y coordinates and z coordinates 16.

A plane 𝕊′ is parallel to 𝕊 at a distance of 16″. A triangular prism has its base in 𝕊, points E (0, 5, 8), F (0, 13, 2), *G* (0, 13, 14); and its other base in $\mathcal{S}'$, points *E'*, *F'*, *G'* having *x* coordinates 16, and *y* and *z* coordinates unchanged.

Make the drawing of the intersecting prisms considering the triangular prism to be solid and parts of the square prism cut away to permit the triangular one to pass through.

(For use on cross-section paper or blackboard, not wire-mesh cage.)

45. A sheet-iron coal chute connects a square port, *A* (2, 4, 2), *B* (2, 12, 2), *C* (2, 4, 10), *D* (2, 12, 10), with a square hatch, *E* (14, 6, 16), *F* (14, 10, 16), *G* (10, 10, 16), *H* (10, 6, 16). The corners form lines *AE, BF, CG, DH* and the side plates are bent on the lines *AH* and *BG*. Draw the development of the surface.

46. Draw the development of the tetrahedron in Problem 42 with the line of intersection marked on it.

47. Draw the development of the pyramid in Problem 43 with the line of intersection marked on it.

48. Draw the development of the square prism of Problem 44 with the line of intersection marked on it.

49. Draw the development of the triangular prism of Problem 44 with the line of intersection marked on it.

CHAPTER V.

CURVED LINES.

47. The Simplest Plane Curve, the Circle.—The geometrical natures of the common curves are supposed to be understood. Descriptive Geometry treats of the nature of their orthographic projections. The curves now considered are plane curves, that is, every point of the curve lies in the same plane. It is natural, therefore, that the relation of the plane of the curve to the plane of projection governs the nature of the projection.

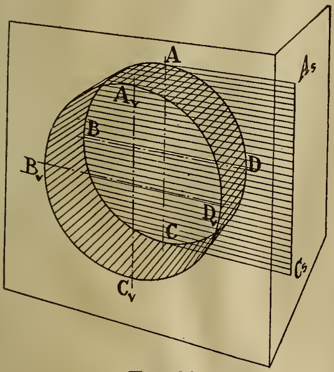


FIG. 36.

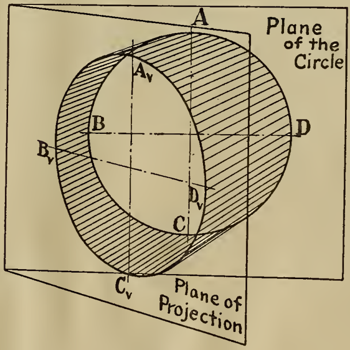


FIG. 37.

The simplest plane curve is a circle. Figs. 36 and 37 show the three forms in which it projects upon a plane. In Fig. 36, a perspective drawing, we have a circle projected upon a parallel plane of projection (that in the position customary for $\mathbb{V}$). The projectors are of equal length and the projection is itself a circle exactly equal to the given circle.

On a second plane of projection (that in the position of $\mathbb{S}$) perpendicular to the plane of the circle the projection is a straight line equal in length to the diameter of the circle, *AC*. The projectors for this second plane of projection form a projector-plane.

In Fig. 37 the circle is in a plane inclined at an angle to the plane of projection. The projectors are of varying lengths. There must be one diameter of the circle, however, that marked *AC*, which is parallel to the plane of projection. The projectors from these points are of equal length, and the diameter *AC* appears of its true length on the projection as A_vC_v.

The diameter *BD* at right angles to *AC*. has at its extremity *B* the shortest projector, and at the extremity *D* the longest projector. On the projection, *BD* appears greatly foreshortened as B_vD_v, though still at right angles to the projection of *AC* and bisected by it.

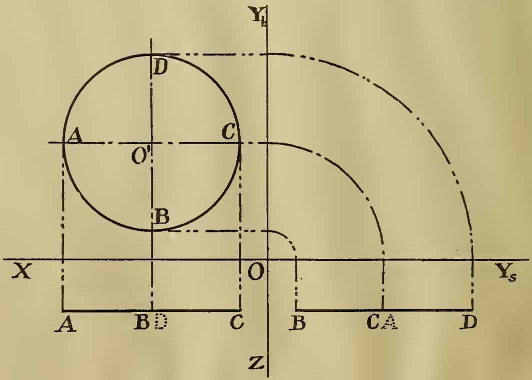


FIG. 38.

The true shape of the projection is an ellipse, of which A_vC_v is the major axis and B_vD_v is the minor axis. No matter at what angle the plane of projection lies, the projection of a circle is an ellipse whose major axis is equal to the diameter of the circle.

For convenience the two planes of projection in Fig. 36 have been considered as $\mathbb{V}$ and $\mathbb{S}$, and the projections lettered accordingly. The plane of projection in Fig. 37 has been treated as if it were $\mathbb{V}$, and the ellipse so lettered. It must be remembered that the three forms in which the circle projects upon a plane, as a circle, as a line, and as an ellipse, cover all possible cases, and the relations between the plane of the circle and the plane of projection shown in the two figures are intended to be perfectly general and not confined to $\mathbb{V}$ and $\mathbb{S}$ alone.

48. The Circle in a Horizontal or Vertical Plane.—Passing now to the descriptive drawing of a circle, the simplest case is that of a circle which lies in a plane parallel to ℍ, 𝕍 or 𝕊. The projections are then of the kind shown in Fig. 36, two projections being lines and one the true shape of the circle. Fig. 38 shows the case for a circle lying in a horizontal plane. The true shape appears in ℍ. The 𝕍 projection shows the diameter *AC*, the 𝕊 projection shows the diameter *BD*.

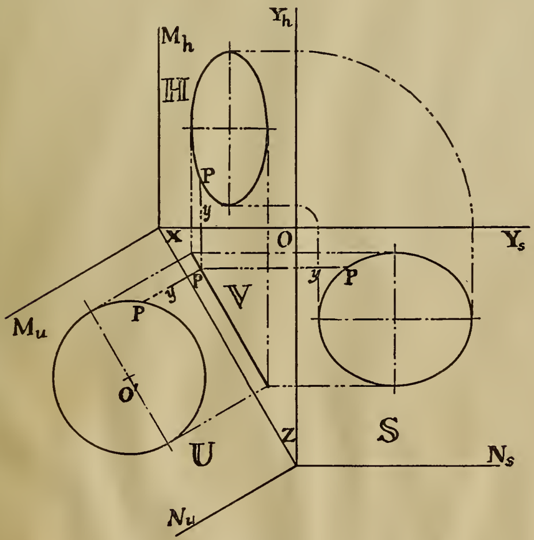


Fig. 39.

49. The Circle in an Inclined Plane.—Fig. 39 shows the circle lying in an inclined plane, perpendicular to 𝕍, and making an angle of 60° with ℍ. The 𝕍 projectors, lying in the plane of the circle itself, form a projector-plane and the 𝕍 projection is a straight line equal to a diameter of the circle. As the plane of the circle is oblique to ℍ and 𝕊, these projections on ℍ and 𝕊 are ellipses whose major axes are equal to the diameter of the circle. Of course, for any point of the curve, as *P*, the laws of projection hold, as is indicated. The true shape of the curve can be shown by projection on any plane parallel to the plane of the circle. It is here shown on the auxiliary plane U, taken as required. If the drawing were presented with projections H, V and S, as shown, one might at first suspect that it represented an ellipse and not a circle; but, if a number of points were plotted on U, the existence of a center O' could be proved by actual test with the dividers.

50. The Circle in an Oblique Plane.—When a circle is in an oblique plane, all three projections are ellipses, as in Fig. 40. The noticeable feature is that the three major axes are all equal in length.

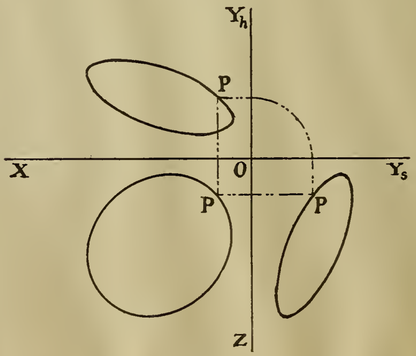


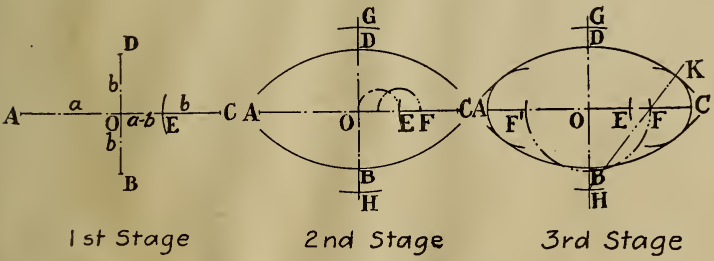
FIG. 40.

When an *ellipse* is in an oblique plane, its three projections are also ellipses, but the major axes will be of unequal lengths. The proof of this fact must be left until later. The fact that the three projections have their major axes equal must be taken at present as sufficient evidence that the curve itself is a circle.

51. The Ellipse: Approximate Representation.—The ellipse is little used as a shape for machine parts. It appears in drawings chiefly as the projection of a circle. Some properties of ellipses are very useful and should be studied for the sake of reducing the labor of executing drawings in which ellipses appear.

An approximation to a true ellipse by circular arcs, known as the "draftsman's ellipse," may be constructed when the major axis $2a$ and the minor axis $2b$, Fig. 41, of an ellipse are known.

The steps in the process are shown in Fig. 41. The center of the ellipse is at O. The major axis is AC, equal to $2a$. The minor axis is DB, equal to $2b$. From C, one end of the major axis, lay off CE, equal to b. The point E is at a distance equal to $a-b$ from O and at a distance equal to $2a-b$ from A. This last distance is the *radius* of a circular arc which is used to approximate to the flat sides of the ellipse. It may be called the "side arc." Setting the compass to the distance AE and using D and B as centers, points H and G are marked on the minor axis, extended, for use as centers for the "side arcs." These arcs are now drawn (passing through the points D and B), as shown in the 2nd stage of the process.



THE "DRAFTSMAN'S ELLIPSE."

FIG. 41.

By use of the bow spacer, the distance OE is bisected and the half added to itself, giving the point F (distant $\frac{3}{2}$ $(a-b)$ from O). F is the center of a circular arc which approximates to the end of the true ellipse. With F as center, and FC as radius, describe this arc. If this work is accurate, this "end arc" will prove to be tangent to the side arcs already drawn, as shown in the 3rd stage of the process. If desired, the exact point of tangency of the two arcs, K, may be found by joining the centers H and F and extending the line to K. F is swung about O as center by compass or dividers to F', for the center of the other "end arc." In inking such an ellipse, the arcs must be terminated exactly at the points of tangency, K and the three similar points.

This method is remarkably accurate for ellipses whose minor

5

axes are at least two-thirds the length of their major axes. It should always be used for such wide ellipses, and if the character of the drawing does not require great accuracy, it may be used even when the minor axis is but half the length of the major axis. For all narrow ellipses, exact methods of plotting should be used.

52. The Ellipse: Exact Representation.—The true and accurate methods of plotting an ellipse are shown in Figs. 42, 43, and 44. Fig. 42 is a convenient method when the major axis AC and minor axis BD are given, bisecting each other at O. Describe circles with centers ot O, and with diameters equal to AC and BD. From O draw *any* radial line. From the point where this radial line meets the larger circle draw a vertical line, and from the point where it cuts the smaller circle draw a horizontal line. Where these lines

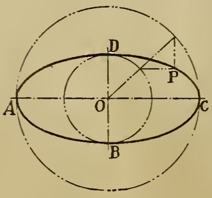


FIG. 42.

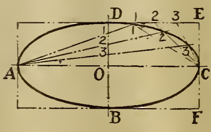


FIG. 43.

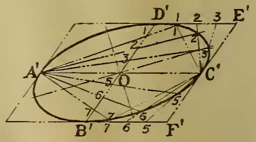


FIG. 44.

meet at P is located a point on the ellipse. By passing a large number of such radial lines sufficient points may be found between D and C to fully determine the quadrant of the ellipse. Having determined one quadrant, it is generally possible to transfer the curve by the pearwood curves with less labor than to plot each quadrant.

With the same data a second method, Fig. 43, is more convenient for work on a large scale when the T-square, beam compass, etc., are not available.

Construct a rectangle using the given major and minor axes as center lines. Divide DE into any number of equal parts (as here shown, 4 parts), and join these points of division with C. Divide DO into the same number of equal parts (here, 4). From A draw lines through these last points of division, extending them to the first system of lines intersecting the first of the one system with the first of the other, the second with the second, etc. These intersections, 1, 2, 3, are points on the ellipse.

The third method, an extension or generalization of the second, is very useful when an ellipse is to be inscribed in a parallelogram, the major and minor axes being unknown in direction and magnitude. Lettering the parallelogram $A'B'C'D'$ in a manner similar to the lettering in Fig. 43, the method is exactly the same as before, $D'E'$ and $D'O$ being divided into an equal number of parts and the lines drawn from C' and A'. The actual major and minor axes, indicated in the figure, are not determined in any manner by this process.

53. The Helix.—The curve *in space* (not a plane curve) which is most commonly used in machinery, is the helix. This curve is described by a point revolving uniformly about an axis and at the same time moving uniformly in the direction of that axis. It is popularly called a "cork-screw" curve, or "screw thread," or even, quite incorrectly, a "spiral."

The helix lies entirely on the surface of a cylinder, the radius of the cylinder being the distance of the point from the axis of rotation, and the axis of the cylinder the given direction.

Fig. 45 represents a cylinder on the surface of which a moving point has described a helix. Starting at the top of the cylinder, at the point marked 0, the point has moved uniformly completely around the cylinder at the same time that it has moved the length of the cylinder at a uniform rate. The circumference of the top circle of the cylinder has been divided into twelve equal parts by radii at angles of 30°, the apparent inequality of the angles being due to the perspective of the drawing. The points of division are marked from 0 to 11, point 12 not being numbered, as it coincides with point 0. The length of the cylinder is divided into twelve equal parts on the vertical line showing the numbers from 0 to 12, and at each point of division a circle, parallel to the top base, is described about the cylinder. The helix is the curve shown by a heavy line. From point 0, which is the zero point of both movements, the first twelfth part of the motion carries the point from 0 to 1 around the circumference, and from 0 to 1 axially downward, at the same time. The true movement is diagonally across the curved rectangle to the point marked 1 on the helix. This movement is continued step by step to the points 2, 3, etc. In the position chosen in Fig. 45, points 0, 1, 2, 3, 4, 12 are in full view, points 5 and 11 are on the extreme edges, and the intermediate

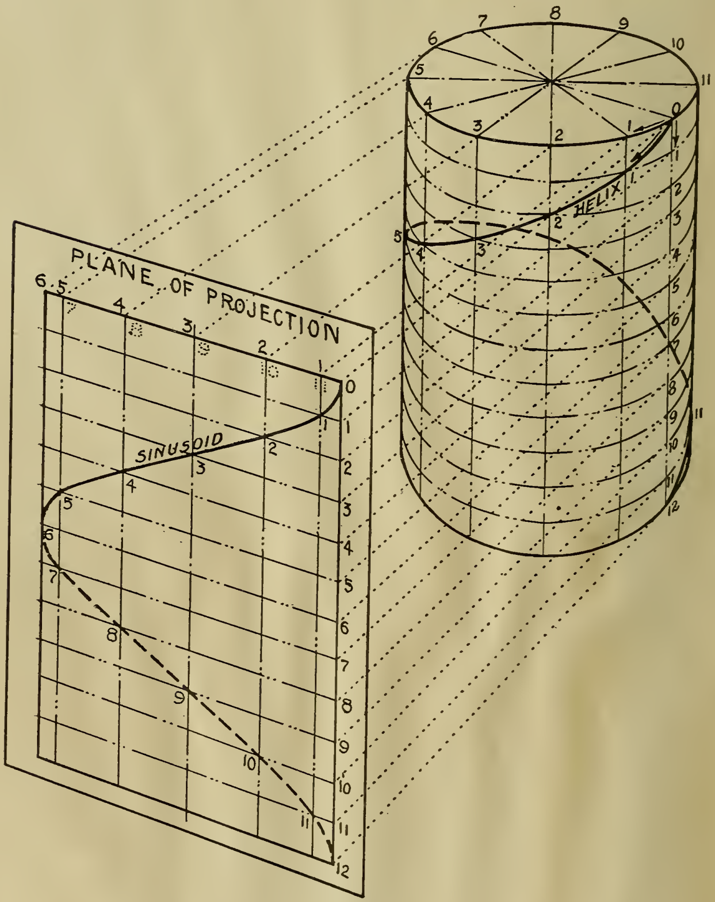


FIG. 45.

points (from 6 to 10) are on the far side of the cylinder. The construction lines for these latter points have been omitted, in order to keep the figure clear.

54. Projections of the Helix.—The projection of this curve on a plane parallel to the axis of the cylinder is shown to the left. The circles described about the cylinder become equidistant parallel straight lines. The axial lines remain straight but are no longer equally spaced, and the curve is a kind of continuous diagonal to the small rectangles formed by these lines on the plane of projection.

The projection of the helix on any plane *perpendicular* to the axis of the cylinder is a circle coinciding with the projection of the cylinder itself. The top base is such a plane and on it the projection of the helix coincides with the circumference of the base.

55. Descriptive Drawing of the Helix.—The typical descriptive drawing of a helix is shown in Fig. 46. The axis of the cylinder is perpendicular to $\mathbb{H}$, and the top base is parallel to $\mathbb{H}$. The helix in $\mathbb{H}$ appears as a circle. In $\mathbb{V}$ it appears as on the plane of projection in Fig. 45, but this view is no longer seen obliquely as is there represented.

This $\mathbb{V}$ projection of the helix is a plane curve of such importance as to receive a separate name. It is called the "sinusoid." Since the motion of the describing point is not limited to one complete revolution, it may continue indefinitely. The part drawn is one complete portion and any addition is but the repetition of the same moved along the axial length of the curve. The proportions of the curve may vary between wide limits depending on the relative size of the radius of the cylinder to the axial movement for one revolution. This axial distance is known as the "pitch" of the helix.

In Fig. 46 the pitch is about three times the radius of the helix. In Fig. 47, a short-pitch helix is represented, the pitch being about $\frac{3}{4}$ the radius, and a number of complete rotations being shown.

The proportions of the helix depends therefore on the radius and on the pitch. To execute a drawing, such as Fig. 46, describe first the view of the helix which is a circle. Divide the circumference into any number of equal parts (12 or 24 usually). From these points of division project lines to the other view or views. Divide the pitch into the same number of equal parts, and draw lines perpendicular to those already drawn. Pass a smooth curve through the points of intersection of these lines, forming the continuous diagonal. In Figs. 45 and 46 the helix is a "right-hand helix." The upper part of Fig. 47 shows a left-hand helix, the motion of rotation being reversed, or from 12 to 11 to 10, etc. The ordinary

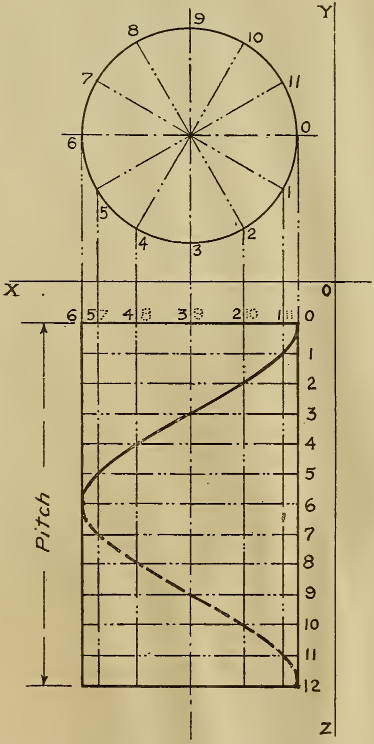


FIG. 46.

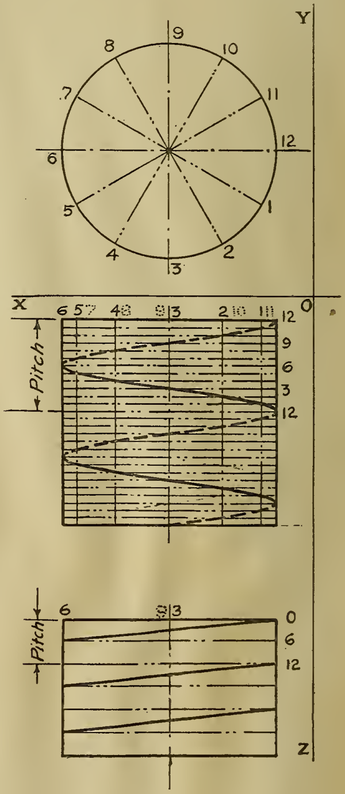


FIG. 47.

screw thread used in machinery is a very short-pitched right-hand helix. It is so short indeed that it is customary to represent the curve by a straight line passing through those points which would be given if the construction were reduced to dividing the circumference and the pitch into 2 equal parts. This is shown in the lower part of Fig. 47, where only the points 0, 6 and 12 have been used.

The concealed portion of the helix is then omitted entirely, no broken line for the hidden part being allowed by good practice.

56. The Curved Line in Space.—A curve in space may sometimes be required, one which follows no known mathematical law, but which passes through certain points given by their coordinates. For example, in Fig. 48, four points, A $(12, 1, 9)$, B $(5, 4, 6)$,

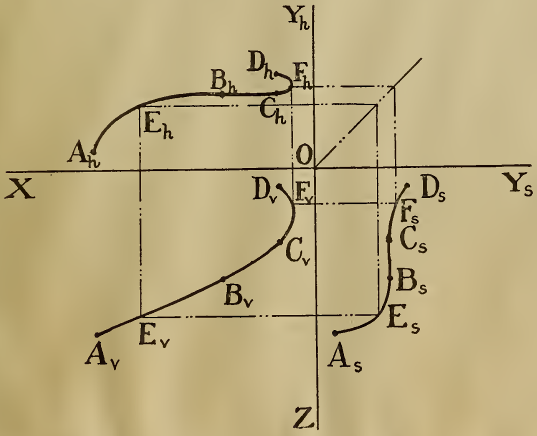


FIG. 48.

C $(2, 4, 4)$ and D $(2, 5, 1)$, were taken as given and a "smooth curve," the most natural and easy curve possible, has been passed through them. It is fairly easy to pass smooth curves through the projections of the 4 points on each reference plane, but it is essential that not only should the original points obey the laws of projection of Art. 11, but every intermediate point as well. The views must check therefore point by point and the process of tracing the curve must be carried out about as follows: The projections of the 4 points on $\mathbb{V}$ and $\mathbb{S}$ are seen to be more evenly extended than those on $\mathbb{H}$, and smooth curves are made to pass through them by careful fitting with the draftsman's curves. The view on $\mathbb{S}$ cannot now be put in at random, but must be constructed to correspond to the other views. To fill in the wide gap between A_h and B_h an intermediate point is taken, as E_v on A_vB_v. By a horizontal line E_s is defined. From E_v and E_s the $\mathbb{H}$ projection (E_h) is plotted by the regular method of checking the projections of a point. As many such intermediate points may be taken as may seem necessary in each case.

To define the sharp turn on the curve between C_h and D_h, one or more extra points, as F_h, should be plotted from the $\mathbb{V}$ and $\mathbb{S}$ projections. Thus every poorly defined part is made definite and the views of the line mutually check. The work of "laying out" the lines of a ship on the "mold-loft floor" of a shipbuilding plant is of this kind, with the exception that the curves are chiefly plane curves, not curves in space.

Problems V.

(For blackboard or cross-section paper.)

50. Make the descriptive drawing of a circle lying in a plane parallel to $\mathbb{S}$, center at C (3, 6, 7) and radius 5.

51. Make the descriptive drawing of a circle lying in a plane perpendicular to $\mathbb{V}$, making an angle of 45° with $\mathbb{H}$ (the trace in $\mathbb{H}$ passing through the points (18, 0, 0), and (0, 0, 18)). The center of the circle is at C (9, 6, 9), and the radius is 5. (Make the $\mathbb{V}$ projection first, then a projection on an auxiliary plane $\mathbb{U}$. From these views construct the $\mathbb{H}$ and $\mathbb{S}$ projections, using 8 or 9 points.

52. Make the descriptive drawing of a circle in a plane perpendicular to $\mathbb{H}$, the trace in $\mathbb{H}$ passing through the points (12, 0, 0) and (0, 16, 0). The center is at (6, 8, 10) and the radius 8. (Draw plan and auxiliary view showing true shape first, and from those views construct projections on $\mathbb{V}$ and $\mathbb{S}$.)

53. An ellipse lies in a plane passing through the axis of Y, making angles of 45° with $\mathbb{H}$ and $\mathbb{S}$. The $\mathbb{H}$ projection is a circle, center at (10, 10, 0) and radius 8. Prove that the $\mathbb{S}$ projection is also a circle and find the true shape of the ellipse by revolving the plane of the ellipse into the plane $\mathbb{H}$.

54. An ellipse lies in a plane passing through the axis of Y, making an angle of 60° with ℍ and 30° with 𝕊. The ℍ projection is a circle, center at (8, 8, 0), radius 6. Find the true shape of the ellipse. Construct the view on 𝕊 by projecting points for center and for the extremities of the axes of the ellipse. Pass a draftsman's ellipse through those points. Show that no appreciable error can be observed.

55. Construct a draftsman's ellipse, on accurate cross-section or coordinate paper, with major axis 24 units, and minor axis 12 units. Perform the accurate plotting of the true ellipse on the same axes by the method of Fig. 43, using 6 divisions for DE and EC. Note the degree of accuracy of the approximate process.

56. On coordinate paper, plot an ellipse by the method of Fig. 43, the major axis being 16 units long and the minor axis 8 units. Plot another ellipse whose major axis is 18 and whose minor axis is 12. (To divide the semi-minor axis of 6 units into 4 equal parts, use points of division on the vertical line CE instead of OD. CE being twice as far from A as OD, 12 units must be used for the whole length, and these divided into 4 parts.)

57. On isometric paper pick out a rhombus like the top of Fig. 19, but having 8 units on each side. Inscribe an ellipse by plotting by the method of Fig. 44.

58. Make the descriptive drawing of a helix whose axis is perpendicular to 𝕊 through the point (0, 7, 7). The pitch of the helix is 12, and the initial point is (2, 5, 2). Draw the ℍ and 𝕍 projections of a right-hand helix, numbering the points in logical order.

59. Connect the 4 points A (10, 8, 10), B (8, 10, 6), C (6, 9, 4) and D (2, 2, 4) by a smooth curve, filling out poorly defined portions in 𝕊 from the ℍ and 𝕍 projections.

CHAPTER VI.

CURVED SURFACES AND THEIR ELEMENTS.

57. Lines Representing Curved Surfaces.—To represent solids having curved surfaces, it is not enough to represent the actual corners or edges only. Hitherto only edges have appeared on descriptive drawings, and it has been a feature of the drawings that every point represented on one projection must be represented on the other projections, the relation between projections being strictly according to rule. We now come to a class of lines which do not appear on all three views, lines due to the curvature of the surfaces.

The general principle, called the "Principle of Tangent Projectors," governing this new class of lines is as follows: In projecting a curved surface to a given plane of projection (by perpendicular projectors, of course) all points, and only those points, whose projectors are tangent to the curved surface should be projected. A good illustration of this principle is shown in Fig. 45, where the cylinder is projected upon the plane of projection. The top and bottom bases are edges, and project under the ordinary rules, but along the straight line $0, 1, 2, \ldots, 12$ the curved surface of the cylinder is itself perpendicular to the plane of projection. If from any point on this line a projector is drawn to the plane of projection (as is shown in the figure for the points 1, 2, 3, etc.), this projector is *tangent to the cylinder*. The whole line therefore projects to the plane of projection. The projection of the cylinder on a plane parallel to its axis is therefore a rectangle, two of its sides representing the circular bases and the two other edges representing the curved sides of the cylinder.

58. The Right Circular Cylinder.—The complete descriptive drawing of a cylinder is therefore as shown in Fig. 49. This cylinder is a right circular cylinder. Mathematicians consider that the cylinder is "generated" by revolving the line AA' about PP', the axis of the cylinder. The generating line in any particular position is called an "element" of the surface. Thus *AA′*, *BB′*, *CC′*, etc., are elements.

When the cylinder is projected upon V, *AA′* and *CC′* are the elements which appear in V because the V projectors of all points along those lines are tangent to the cylinder, as can be seen from the view on H. The elements which are represented by lines on S are *BB′* and *DD′*.

The right circular cylinder may also be considered as generated by moving a circle along an axis perpendicular to its own plane through its center.

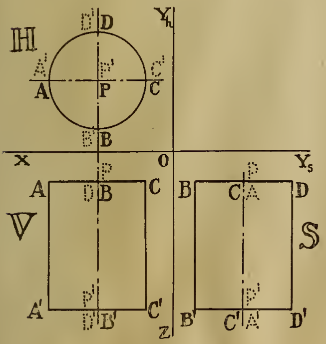


FIG. 49.

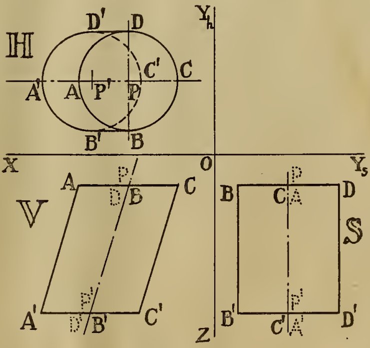


FIG. 50.

In Fig. 45 consider the top base of the cylinder to be moved down the cylinder. Each successive position of the circle is a "circular element" of the cylinder. The circles through the points 1, 2, 3, etc., are simply circular elements of the cylinder taken at equal distances apart.

59. The Inclined Circular Cylinder.—Fig. 50 shows an *inclined circular cylinder*. It has circular and straight line elements as before, though it cannot be generated by revolving a line about another at a fixed distance, but can be generated by moving the circle *ABCD* obliquely to *A′B′C′D′*, the center moving on the axis *PP′*. The straight elements are all parallel to the axis. The cross-section of a cylinder is a section taken perpendicular to the axis.

In this case the cross-section is an ellipse, and for this reason the Inclined Circular Cylinder is sometimes called the Elliptical Cylinder.

60. Straight and Inclined Circular Cones.—If a generating line *AP*, Fig. 51, meets an axis *PP′* at a point *P*, and is revolved about it, it will generate a Straight Circular Cone. The cone has both straight and circular elements, the circular elements increasing in size as they recede from the vertex *P*. The base *ABCD* is one of the elements.

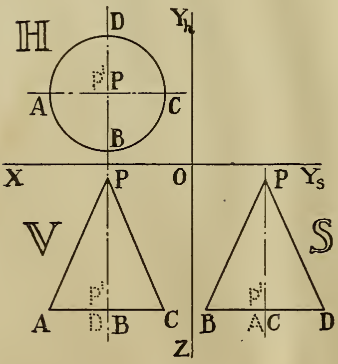


FIG. 51.

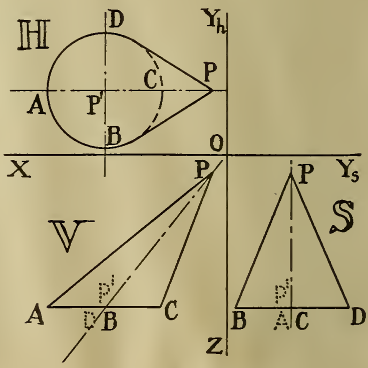


FIG. 52.

The Inclined Circular Cone (Fig. 52) has straight and circular elements, but it is not generated by revolving a line about the axis. The circular elements move obliquely along the axis *PP′* and increase uniformly as they recede from the vertex *P*.

61. The Sphere.—The Sphere can be generated by revolving a semicircle about a diameter. Each point generates a circle, the radii of the circles for successive points having values varying between 0 and the radius of the sphere. Since the sphere can be generated by using any diameter as an axis, the number of ways in which the surface can be divided into circular elements is infinite.

62. Surfaces of Revolution.—In general, any line, straight or curved, may be revolved about an axis, thus creating a *surface of revolution*. Every point on the "generating line" creates a "circular element" of the surface, and the plane of each circular element is perpendicular to the axis of the surface.

The straight circular cylinder is a simple case of the general class of surfaces of revolution. To generate it a straight line is revolved about a parallel straight line. The different points of the generating line create the circular elements of the cylinder, and

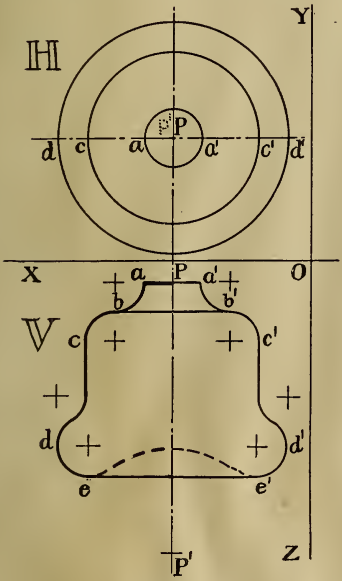


FIG. 53.

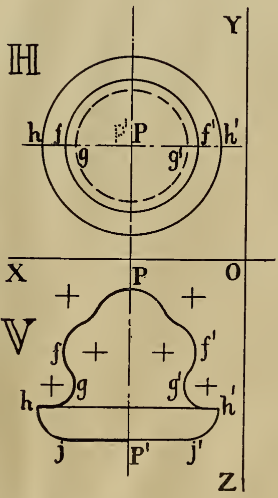


FIG. 54.

the different positions of the generating line mark the straight elements. The cone and the sphere are also surfaces of revolution, as they are generated by revolving a line about an axis.

If a circle be revolved about an axis in its own plane, but entirely exterior to the circle, a solid, called an "anchor ring," is generated. A small portion of this surface, part of its inner surface, is often spoken of as a "bell-shaped surface," from its similarity to the flaring edge of a bell.

Any curved line may create a surface of revolution, but in designs of machinery lines made up of parts of circles and straight lines are most frequently used. Figs. 53 and 54 show two examples which illustrate well the application of the Principle of Tangent Projectors. The generating line is emphasized and the centers of the various arcs are marked.

Any angular point on the generating line, as *a* (Fig. 53), creates a circular edge on the surface. This edge appears as a circle on the plan (as *aa′* on ℍ), and as a straight line, equal to the diameter, on the elevation (as *aa′* on 𝕍). See also the point *h* (Fig. 54). In addition, any portion of the generating line which is perpendicular to the axis, as *b* (Fig. 53), even if for an infinitely short distance only, creates a line on the side view, as *bb′* on 𝕍, but no corresponding circle on ℍ. A 𝕍 projector from any point on the circular element created by the point *b* is tangent to the surface, and therefore creates a point on the drawing, but an ℍ projector is not tangent to the surface. *e* is a similar point, and so also is *j* of Fig. 54.

Any point, as *c*, Fig. 53, where the generating line is parallel to the axis for a finite, or for an infinitely small distance, generates a circular element, from every point of which the ℍ projectors are tangent to the surface, but the 𝕍 projectors are not. A circle *cc′* appears, therefore, on the plan for this element of the surface of revolution, but no straight line on the side view. *d* is a similar point, as are also *f* and *g*, on Fig. 54.

63. The Helicoidal Surface.—If a line, straight or curved, is made to revolve uniformly about an axis and move uniformly along the axis at the same time, every point in the line will generate a helix of the same pitch. The surface swept up is called a Helicoidal Surface.

The generating line chosen is usually a straight line intersecting the axis. The surfaces used for screw threads are nearly all of this kind. Fig. 55 gives an example of a sharp V-threaded screw, the two surfaces of the thread having been generated by lines inclined at an angle of 60° to the axis. Fig. 56 shows a square thread, the generating lines of the two helicoidal surfaces being perpendicular to the axis. Any particular position of the straight line is a "straight element" of the helicoidal surface.

64. Elementary Intersections.—In executing drawings of machinery it is often necessary to determine the *line of intersection* of two surfaces, plane or curved. The simplest lines of intersection are such as coincide with elements of a curved surface. They may

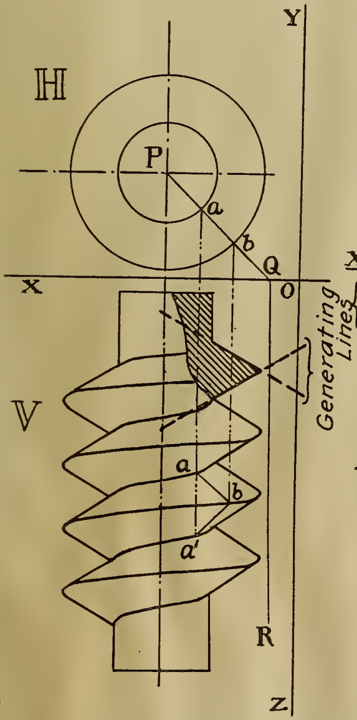


FIG. 55.

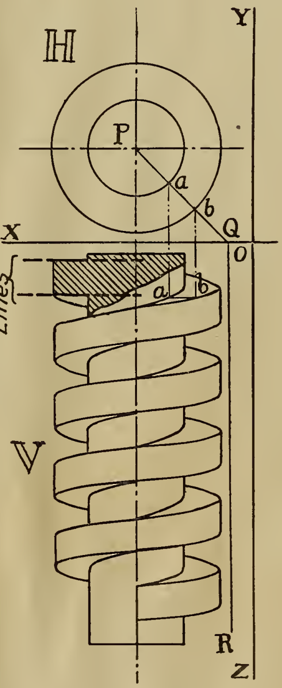


FIG. 56.

be called "Elementary Intersections." An elementary intersection may arise when a curved surface is intersected by a plane, so placed as to bear some simple relation to the surface itself.

In Fig. 49, any plane perpendicular to the axis of the cylinder intersects it in a circular element of the cylinder, and any plane parallel to the axis of the cylinder (or containing it) intersects it (if it intersects it at all) in two straight line elements of the cylinder.

In Fig. 50 any plane parallel to the base of the cylinder intersects it in a circular element, and any plane parallel to the axis, or containing it, intersects it in straight elements of the cylinder.

In Fig. 51 or 52 any plane parallel to the base of the cone intersects it in a circular element, and any plane containing the vertex of the cone (if it intersects at all) intersects the cone in straight elements.

In Fig. 53 or 54 any plane perpendicular to the axis of the surface of revolution intersects it in a circular element.

In Fig. 55 or 56 any plane containing the axis of the screw intersects the helicoidal surfaces in straight elements. The plane perpendicular to $\mathbb{H}$, cutting $\mathbb{H}$ in a trace PQ, and cutting $\mathbb{V}$ in a trace QR, cuts the helicoidal surfaces at each convolution in straight elements. Only ab and $a'b$ are marked on the figure.

Problems VI.

(For blackboard or cross-section paper or wire-mesh cage.)

60. Draw the projections of a cylinder whose axis is P (6, 2, 6), P' (6, 16, 6), and radius 5. Draw the intersection of this cylinder with a plane parallel to $\mathbb{H}$, at 4 units from $\mathbb{H}$, and with a plane parallel to $\mathbb{V}$, 10 units from $\mathbb{V}$.

61. An inclined circular cylinder has its bases parallel to $\mathbb{S}$. Its axis is P (2, 7, 7), P' (14, 7, 13). Its radius is 5. Draw the $\mathbb{V}$ and $\mathbb{S}$ projections and the intersections with a plane parallel to $\mathbb{S}$, 6 units from $\mathbb{S}$, and with a plane parallel to $\mathbb{V}$, 3 units from $\mathbb{V}$.

62. Draw a cone with vertex at P (4, 8, 8), center of base at P' (16, 8, 8), and radius 6, base-line in a plane parallel to $\mathbb{S}$. Draw the intersection with a plane parallel to $\mathbb{S}$, 12 units from $\mathbb{S}$, and with a plane perpendicular to $\mathbb{S}$, whose trace in $\mathbb{S}$ passes through the points (0, 8, 8) and (0, 14, 0).

63. An oblique cone has its vertex at P (16, 8, 4), its base in a plane parallel to $\mathbb{H}$, center at P' (8, 8, 16), and radius 5. Draw the intersection with a plane parallel to $\mathbb{H}$, 12 units from $\mathbb{H}$, and with a plane containing the axis and the point (16, 0, 16).

64. A cone has an axis P (8, 2, 2), P' (8, 14, 10). Its base is in a plane parallel to 𝕍, 10 units from 𝕍 and its radius is 6 units. Draw the intersection with a plane containing the vertex and the points (0, 14, 12) and (16, 14, 12).

65. A surface of revolution is formed by revolving a circle whose center is at (12, 8, 8) and radius 3 units, lying in a plane parallel to 𝕍, about an axis perpendicular to ℍ at the point (8, 8, 0). It is cut by a plane parallel to ℍ at a distance of 6 units from ℍ. Draw the intersections.

66. A sphere has its center at (8, 8, 9) and radius 5 units. Draw the intersection with a cylinder whose axis is P (8, 8, 0), P' (8, 8, 16), and whose radius is 4 units, its bases being planes perpendicular to its axis.

67. A sphere has its center at (8, 8, 8) and radius 5 units. Find its intersection with a cone whose vertex is P (0, 8, 9), center of base is (16, 8, 8), and radius of base 6 units, the base being in a plane 𝕊′ parallel to 𝕊.

68. In Fig. 53 let the generating line *Pabcde* be revolved about *ee′* as an axis. Assume any dimension for the line and draw the 𝕍 and 𝕊 projections of the surface of revolution thus formed. Draw the intersection with a plane parallel to 𝕊 just to the right of *d*.

69. In Fig. 54 let the generating line *Pfgh* be revolved about *hh′* as an axis. Assume any dimensions for the line and draw the 𝕍 and 𝕊 projections of the surface of revolution formed.

CHAPTER VII.

INTERSECTIONS OF CURVED SURFACES.

65. The Method of the Intersection of the Intersections.—The determination of the line of intersection of two curved surfaces (or of a curved surface and a plane), when not an "Elementary Intersection," is of much greater difficulty and requires a clear understanding of the nature of the curved surfaces themselves, and some little ingenuity in applying general principles.

The method generally relied upon for the solution is the use of *auxiliary intersecting planes* so chosen as to cut elementary intersections with each of the given surfaces. These elementary intersections are drawn and the *points of intersection of the intersections* are identified and recorded as points on the required line of intersection. This method is spoken of as "finding the intersection of the intersections." When a number of auxiliary planes have been used in this way, a smooth curve is passed through the points on the required intersection of the surfaces, as described in Art. 55. It should not be necessary, however, to interpolate points to fill out gaps as was done in Fig. 48 for *E* and *F*. This can be done better by the use of more auxiliary intersecting planes. Examples of this method will make it clear.

66. An Inclined Circular Cylinder Cut by an Inclined Plane.—In Fig. 57 an inclined cylinder, axis *PP′*, is cut by a plane perpendicular to $\mathbb{V}$, and inclined to $\mathbb{H}$. The traces of this plane are *IJ* in $\mathbb{H}$, *JK* in $\mathbb{V}$, and *KL* in $\mathbb{S}$.

It is an Inclined Plane (see Art. 19), not an Oblique Plane. Having the descriptive drawing of the cylinder and the traces of the plane given, the problem is to draw the line of intersection of the surfaces. It is well-known that in this case the line of intersection is an ellipse, but the method of determining it permits the ellipse to be plotted whether it is recognized as such or not. No use is to be made of previous knowledge of the nature of the curve of intersection of any of the cases treated in this and the next chapter.

Two variations of the method are applicable in this case. In the first method, auxiliary intersecting planes may be taken *parallel to the axis of the cylinder.* The simplest method of doing this is to take auxiliary planes parallel to $\mathbb{V}$, since the axis itself is parallel

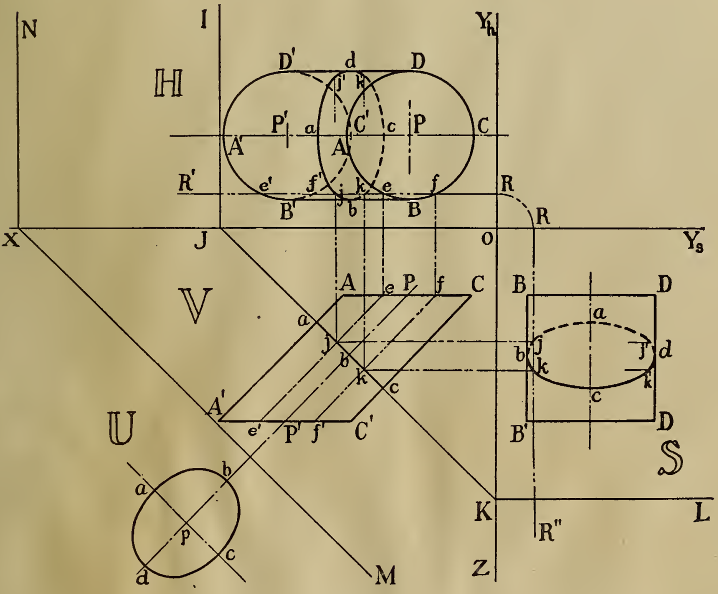


FIG. 57.

to $\mathbb{V}$. Let $R'R$ be the trace on $\mathbb{H}$, and RR'' the trace on $\mathbb{S}$ of a plane parallel to $\mathbb{V}$. We may call this plane simply "R."

Let e and f be the points where $R'R$ cuts the top base of the cylinder. Project these points from $\mathbb{H}$ to $\mathbb{V}$ and in $\mathbb{V}$ draw ee' and ff' parallel to PP'. These straight elements of the cylinder are the lines of intersection of the auxiliary plane with the cylinder. As a check on the work, e' and f', where $R'R$ in $\mathbb{H}$ cuts the bottom base of the cylinder, should project vertically to e' and f' in $\mathbb{V}$.

The auxiliary plane cuts the given plane JK in a line of intersection whose projection on 𝕍 coincides with JK itself.

The points j and k, where ee' and ff' intersect JK, are the "*intersections of the intersections*," and are therefore points on the line of intersection of the cylinder and the plane K. Project j and k to $R'R$ on ℍ and to RR'' on 𝕊. These are points on the required curves in ℍ and 𝕊. By extending in ℍ the projecting lines of j and k as far above the axis PP' as j and k are below it, j' and k', points on the upper half of the cylinder, symmetrical with j and k on the lower half, are found. The construction is equivalent to passing a second auxiliary plane parallel to PP' at the same distance from PP' as R, but on the other side.

By passing a number of planes similar to R, a sufficient number of points are located to define accurately the ellipse $abcd$ in ℍ and 𝕊.

The true shape of this ellipse is shown in 𝕌, a plane parallel to JK, at any convenient distance. In the example chosen, the plane JK has been taken perpendicular to PP', so that the ellipse $abcd$ is the true *cross-section* of the cylinder. Nothing in the method depends on this fact and it is perfectly general and applicable to any inclined plane.

A variation may be made by passing the auxiliary planes perpendicular to 𝕍 and parallel to PP'. ee' in 𝕍 may be taken as the trace of such a plane. The intersections of this auxiliary with both surfaces should be traced and the intersection of the intersections identified and recorded as a point of the curve required. j and j' are the points thus found. This method indeed requires the same construction lines as before, but gives a different explanation to them.

67. A Second Method Using Circular Elements of the Cylinder.— A plane parallel to the base of the cylinder and therefore, in this case, parallel to ℍ, will cut the cylinder in a line of intersection which is one of the circular elements of the cylinder. Let $T'T$ and TT'', in Fig. 58, be the traces of a plane "T" parallel to ℍ. The axis of the cylinder PP' pierces the plane T at p. p is therefore the center of the circle of intersection of the auxiliary plane T with the cylinder. Project p to ℍ, and using p as a center and with a radius equal to pt, describe the circle as shown.

The planes T and JK are both perpendicular to $\mathbb{V}$ or "seen on edge" in $\mathbb{V}$. Their line of intersection is therefore perpendicular to $\mathbb{V}$, or is "seen on end" in $\mathbb{V}$, as the point j. Project j to $\mathbb{H}$, where it appears as the line jj'. This line is the intersection of the two planes.

The points j and j', where this line of intersection jj' meets the circular intersection whose center is at p, are the "intersections of the intersections," and are points on the required curve.

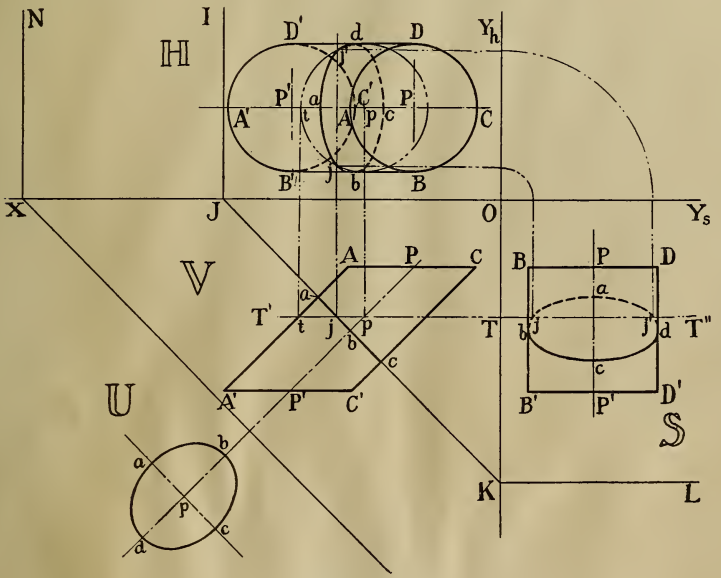


FIG. 58.

Planes like T, at various heights on the cylinder, determine pairs of points on the curve of intersection on $\mathbb{H}$. From $\mathbb{H}$ and $\mathbb{V}$ the points may be plotted on $\mathbb{S}$ by the usual rules of projection, thus completing the solution.

68. Singular or Critical Points.—It is nearly always found that one or two points on the line of intersection may be projected directly from some one view to the others without new construction lines. In this case a and c in $\mathbb{V}$, Fig. 57, may be projected at once to ℍ and 𝕊. They correspond theoretically to points determined by a central plane, cutting ℍ in a trace *PP′*. *b* and *d* may also be projected directly, as they correspond to planes whose traces in ℍ are *BB′* and *DD′*. These critical points should always be the first points identified and recorded, though usually no explanation will be given, as they should be obvious to any one who has grasped the general method.

69. A Cone Intersected by an Inclined Plane.—Fig. 59 shows

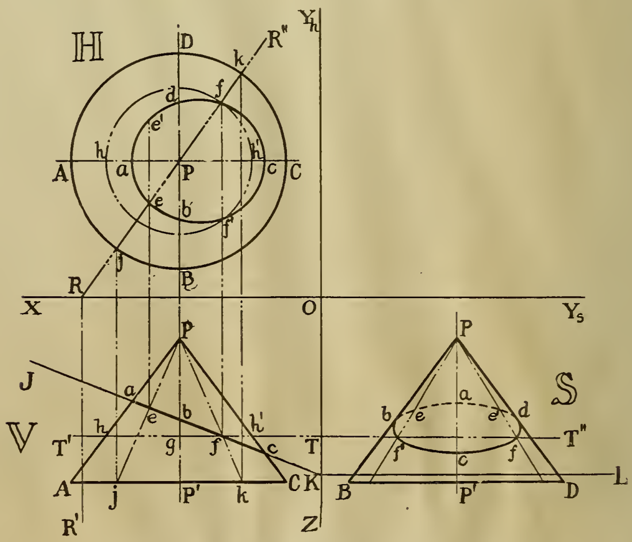


FIG. 59.

the descriptive drawing of a right circular cone intersected by an inclined plane whose traces are *JK* and *KL*. Two methods of solution are shown.

A plane *R, containing the axis PP′*, and therefore perpendicular to ℍ, is shown by its traces *R′R* and *RR″*. It intersects the cone in the elements *Pj* and *Pk*. From ℍ project these points *j* and *k* to 𝕍, and draw the elements in 𝕍. The 𝕍 projection of the intersection of *R* with the plane *JK* is the line *JK*, and the points *e* and *f* are the intersections of the intersections. *e* and *f* are now projected to the plan, where they necessarily lie on the line RR''. Symmetrical points e' and f' are also plotted and all four points transferred to the side elevation.

A plane T *perpendicular to the axis* PP' whose traces are $T'T$ and TT''' may be used instead of R. Its intersection with the cone is a circle, seen on edge in the front elevation as the line hh'. Its center is g, and radius is gh. Draw this circle in the plan. The

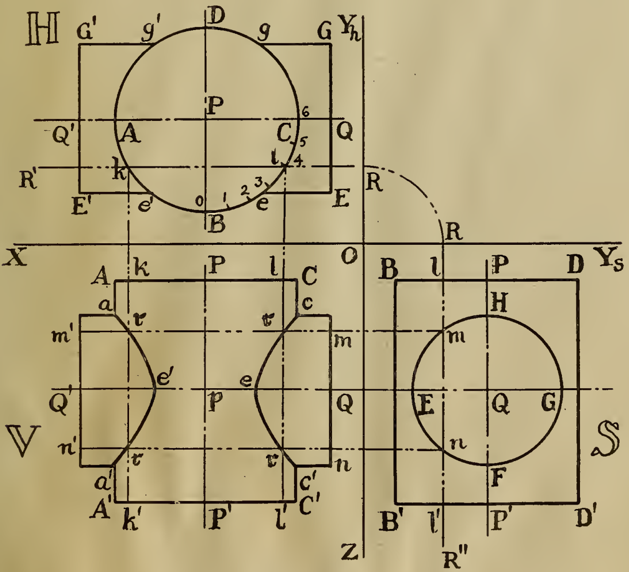


FIG. 60.

intersection of T with the plane JK is a line, seen on end, as the point f of the front elevation. Draw $f'f$ in H as this line. The points f and f' are the intersections of the intersections.

70. Intersection of Two Cylinders.—Fig. 60 shows the intersection of two cylinders. Since they are right cylinders, and their axes are at right angles, planes parallel to any one of the three reference planes will cut only straight or circular elements of the cylinders. By the solution, Fig. 60, auxiliary planes parallel to V have been chosen, the traces of one being $R'R$ and RR''. This plane intersects the vertical cylinder in the lines kk' and ll', and it intersects the horizontal cylinder in the lines mm' and nn'. The intersections of these intersections are the points marked r.

If the axes of the cylinders do not meet but pass at right angles, no new complication is introduced. If the axes of the cylinders meet at an angle, and one or both cylinders are inclined, the choice

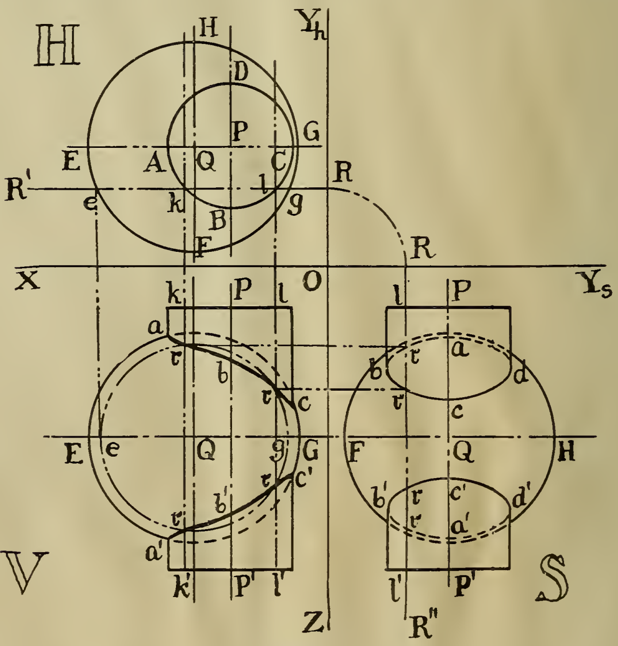


Fig. 61.

of methods may be greatly reduced, but one method is always possible. To discover it, try planes parallel to the axes of both cylinders, or parallel to one axis and to one plane of reference; or in some manner bearing a definite relation to the nature of the surfaces.

71. Intersection of a Cylinder and a Sphere.—In Fig. 61 a sphere is intersected by a cylinder, whose axis PP' does not pass through the center of the sphere at Q. In the solution, Fig. 61, auxiliary planes parallel to 𝕍 have been chosen, the traces of one of them being *R'R* and *RR"*. The plane *R* cuts the sphere in a circle whose diameter is *eg*, as given by the plan. This circle is described in 𝕍. The intersections of this circle with the elements of the cylinder *kk'* and *ll'* are the points marked *r*, points on the required curve of intersection.

In this case the points are first determined on the front elevation and then projected to the side elevation. Solutions by planes parallel to ℍ or to 𝕊 may be made, requiring however different construction lines.

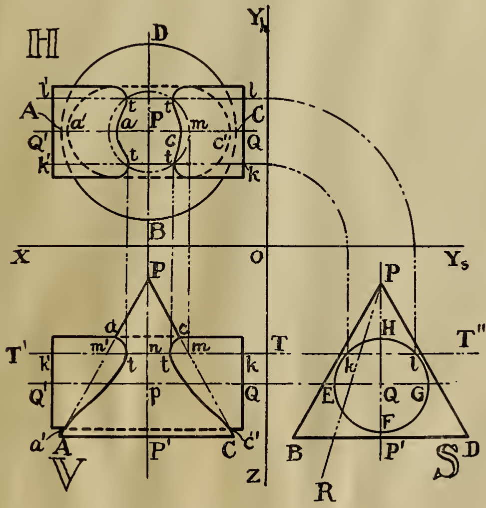


FIG. 62.

72. Intersection of a Cone and a Cylinder: Axes Intersecting.— In Fig. 62 a cone and a cylinder intersect at right angles. The solution chosen is by horizontal planes, as *T*.

An alternate solution is by planes perpendicular to 𝕊, and containing the point *P*. The planes must cut both surfaces, and their traces, where seen on edge, as *PR*, Fig. 62, must cut the projections of both surfaces. These two solutions hold good even if the axes do not meet but pass each other at right angles.

If the axes are not at right angles, modifications must be made, and the search for a system of planes making elementary intersections with *both* surfaces requires some ingenuity and thought.

73. Intersection of a Cone and Cylinder: Axes Parallel.—A simple case is shown in Fig. 63. Two methods of solution are available. In one, horizontal planes are used. Each plane, such as *T*,

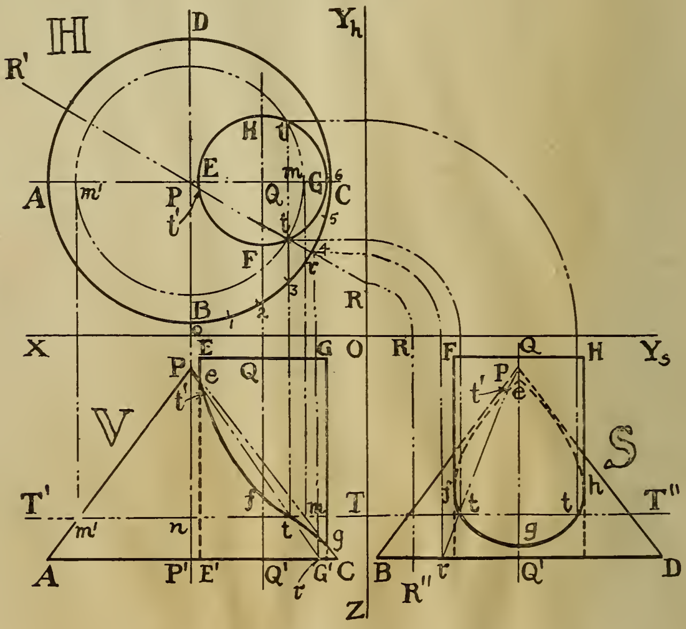


FIG. 63.

makes circular intersections, with both cone and cylinder, the intersections intersecting at points *t* and *t*. A second method is by planes perpendicular to ℍ, containing the axis *PP'*. One plane "*R*" is shown by its traces *R'R* in ℍ and *RR''* in 𝕊, this plane being taken so as to give the same point *t* on the curve and another point *t'*. In the execution of drawings of this class it is natural to take the auxiliary planes at regular intervals if the planes are parallel to each other, or at equal angles if the planes radiate from a central axis.

Problems VII.

70. An inclined cylinder has one base in ℍ and one in a plane parallel to ℍ. Its axis is P (11, 8, 0), P' (5, 8, 16). Its radius is 4 units. It is intersected by a plane perpendicular to 𝕍, its trace passing through the points (5, 0, 0) and (11, 0, 16). Draw the three projections and show one intersecting auxiliary plane by construction lines.

71. A cone has its vertex in ℍ at (6, 6, 0) and its base in a plane parallel to ℍ, center at (6, 6, 12), and radius 5. It is intersected by a plane containing the axis of Y and making angles of 45° with ℍ and 𝕊. Draw the projections.

72. A cone has its vertex at (2, 14, 16) and its base is a circle in ℍ, center at (8, 8, 0), and radius 6. Find its intersection with a vertical plane 4 units from 𝕊.

73. A right circular cylinder has its base in 𝕊, center at (0, 8, 8), and radius 4. Its axis is 16 units long. Another right cylinder has its base in ℍ, center at (8, 8, 0), radius 5, and axis 16 units long. Draw their lines of intersection, the smaller cylinder being supposed to pierce the larger.

74. A right circular cylinder has its base in 𝕊, center at (0, 7, 8), and radius 4. Its axis is 16 units long. Another right cylinder has its base in ℍ, center at (8, 9, 0), radius 5, and axis 16 units long. Draw their line of intersection, the smaller cylinder being supposed to pierce the larger.

75. Two inclined circular cylinders of 3 units' radius have their bases in ℍ and in ℍ′ (16 units from ℍ). The axis of one is P (4, 8, 0), P' (12, 8, 16), and of the other is Q (12, 8, 0), Q' (4, 8, 16). Prove that their line of intersection consists of a circle in a plane parallel to ℍ and an ellipse in a plane parallel to 𝕊.

76. A sphere has its center at (8, 9, 8), and radius 6 units. A vertical right circular cylinder has its top base in ℍ, center at (8, 6, 0), radius 4, and length 16 units. Find the intersections of the surfaces.

77. A right circular cylinder, axis P (0, 8, 9), P' (16, 8, 9), radius 5, is pierced by a right circular cone. The base of the cone

is in a plane 16 units from $\mathbb{H}$, center at Q (8, 18, 16), and radius 6. The vertex of the cone is at Q (8, 8, 0). Find the lines of intersection.

78. An inclined cylinder has an oblique line P (0, 11, 5), P' (16, 5, 11) for its axis. The radius of the circular base is 4 units and the planes of the bases are $\mathbb{S}$, and $\mathbb{S}'$ parallel to $\mathbb{S}$ at 16 units' distance. The cylinder is cut by a plane parallel to $\mathbb{V}$ at 7 units' distance from $\mathbb{V}$. Draw the three projections of the cylinder and the line of intersection.

79. An inclined cylinder has an oblique line P (0, 11, 5), P' (16, 5, 11) for its axis. The radius of the circular base is 4 units, and the planes of the bases are $\mathbb{S}$ and $\mathbb{S}'$ parallel to $\mathbb{S}$ at 16 units' distance. The cylinder is cut by a plane perpendicular to $\mathbb{V}$, its trace passing through the points (2, 0, 0) and (14, 0, 16). Draw the three projections.

CHAPTER VIII.

INTERSECTIONS OF CURVED SURFACES; CONTINUED.

74. Intersection of a Surface of Revolution and an Inclined Plane.—In Figs. 64 and 65 a surface of revolution is shown. It is

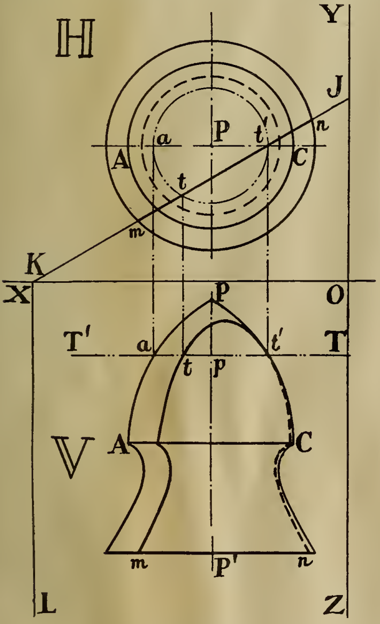


Fig. 64.

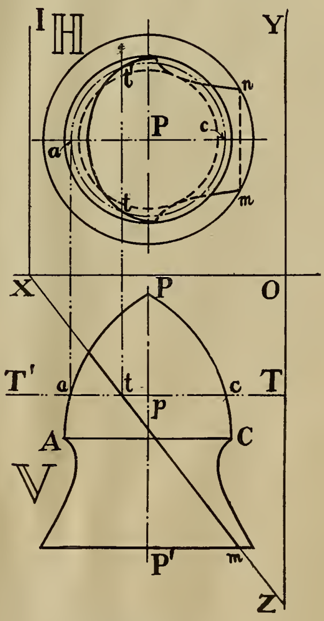


Fig. 65.

cut by an inclined plane perpendicular to ℍ in the first case, and by one perpendicular to 𝕍 in the second case. The planes are given by their traces, and the problem is to find the curves of intersection. Both solutions make use of cutting planes perpendicular to *PP′*, the axis of revolution of the curved surface.

In Fig. 64 a plane *T*, taken at will perpendicular to *PP'*, cuts the surface of revolution in a circular element seen as the straight line *at'* in 𝕍. *a* is projected to ℍ and the circle *att'* drawn. The inclined plane whose traces are *JK* and *KL* is intersected by the plane *T* in a line whose horizontal projection is the line *KL* itself. *t* and *t'* (on ℍ) are therefore the intersections of the intersections and are projected to the front elevation, giving points on the required line of intersection. A system of planes such as *T* defines points enough to fully determine the curve, *mtt'n*.

In Fig. 65 the given plane has the traces *IX* and *XZ*. The plane *T* intersects the surface of revolution on the circle *atct'*, and it

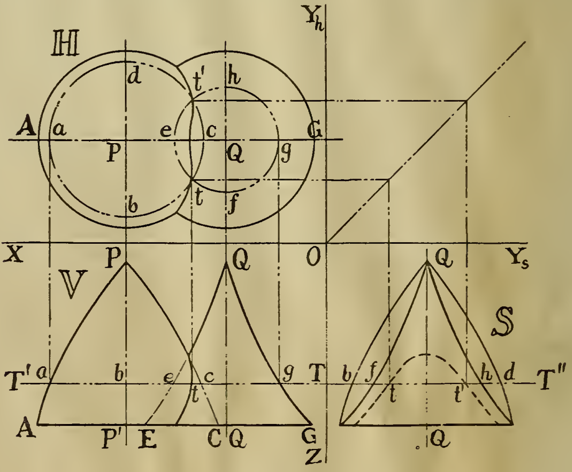


FIG. 66.

intersects the plane in the line *tt'*, seen on end in 𝕍 as the point *t*. *t* and *t'* in ℍ are points on the required curve of intersection, *mtt'n*.

The point of this surface of revolution *APC* has been given a special name. It is an "ogival point." The generating line *AP* is an arc of 60°, center at *C*, and conversely the generating line *PC* has its center at *A*. The shell used in ordnance is usually a long cylinder with an ogival point. A double ogival surface is produced by revolving an arc of 120° about its chord.

75. Intersection of Two Surfaces of Revolution: Axes Parallel.—This problem is illustrated in Fig. 66, where two surfaces of revolution are shown. A horizontal plane T cuts both surfaces in circular elements. These elements are drawn in ℍ as circles *abcd* and *efgh*. t and t' are the intersections of the intersections. From ℍ t and t' are projected to 𝕍 and 𝕊. The problem in Art. 73 is but a special case of this general problem. In addition to the solution by horizontal planes another solution is there possible, due to special properties of the cone and cylinder.

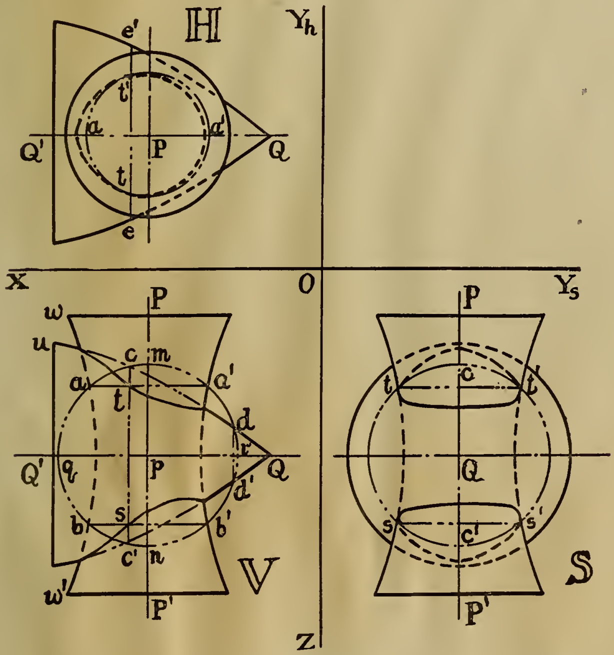


FIG. 67.

76. Intersection of Two Surfaces of Revolution: Axes Intersecting.—An example of two surfaces of revolution whose axes intersect is given by Fig. 67. A surface is formed by the revolution of the curve ww' about the vertical axis PP', and another surface by revolving the curve uQ about the horizontal axis QQ'. The intersection of the axes PP' and QQ' is the point p. The peculiarity of this case is that no plane can cut both surfaces in circular elements. However, a sphere described with the point of intersection of the axes as a center, if of proper size, will intersect both surfaces in circular elements. V is parallel to both axes and on this projection a circle is described with p as center representing a sphere. The radius is chosen at will. To keep the drawing clear, this sphere *has not been described on plan or front elevation*, as it would be quite superfluous in those views.

The sphere has the peculiarity that it is a surface of revolution, using any diameter as an axis. The curve ww' and the semicircle $mabn$ are in the same plane with the axis PP'. When both axes are revolved about PP', a and b, their points of intersection, generate circular elements, which are common to the sphere and to the vertical surface of revolution. Therefore, these circles are the intersections of the sphere and the vertical surface. The H and S projections of these circles are next drawn.

The curve uQ and the semicircle $qcdr$ are in the same plane with the axis QQ'. When both axes are revolved about QQ', their intersections, c and d, generate circles which are common to both surfaces, or are their lines of intersection. The circle generated by c is drawn in H and S, but that generated by d is not needed.

The three circles aa', bb', and cc' appear as straight lines on V, but from them the points t and s, the intersections of the intersections, are determined. These are points on the required curve in V.

The circle aa' appears as a circle $ata't'$ in H, and as a line tt' in S. The circle cc' appears as a circle $ctc't'$ in S, and as a line ee' in H. These circles intersect in H at t and t', and in S at t and t' and s and s'. These are points on the required curves in H and S.

For the complete solution, a number of auxiliary spheres, differing slightly in radius, must be used.

77. Intersection of a Cone and a Non-Circular Cylinder.—A non-circular cylinder is a surface created by a line which moves always parallel to itself, being guided by a curve lying in a plane perpendicular to the generating line. This curve, called the directrix, is usually a closed curve. The cross-section of such a cylinder is everywhere similar to the directrix.

This fact may be utilized to advantage in some cases. In Fig. 68, an oblique cone and a non-circular cylinder intersect. The directrix of the cylinder is a pointed oval curve, *abcd* in H. Horizontal planes, as *T″T*, intersect the cylinder in a curve identical in shape with its directrix, so that its projection on H coincides with the projection of the directrix on H. The intersection with the cone is a circle, *mt′tn*, and the intersections of the intersections are the points *t*.

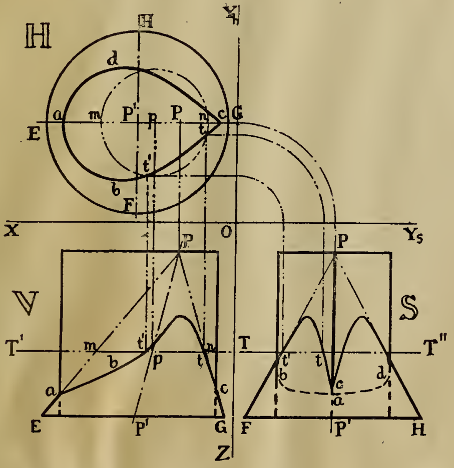


FIG. 68.

78. Alteration of a Curve of Intersection by a Fillet.—In Fig. 69 a hollow cone and a non-circular cylinder, *abcd* in H, intersect. On the left half the unmodified curve of intersection is traced by the method of the preceding article, no construction lines being shown however, as the case is very simple. On the right half the curve is modified by a fillet or small arc of a circle which fills in the angular groove. The fillet whose center is at *g* modifies that point of the line of intersection marked *c*. The top of the circular arc marks the point where an H or S projector is tangent to the surface.

The corresponding crest to the fillet at other positions on the curve of intersection is traced as follows: If a line drawn through k and parallel to PG, the generating line of the cone, is used as a new generator it will by its rotation about PP' create a new cone, on the surface of which the required line of the crests of the

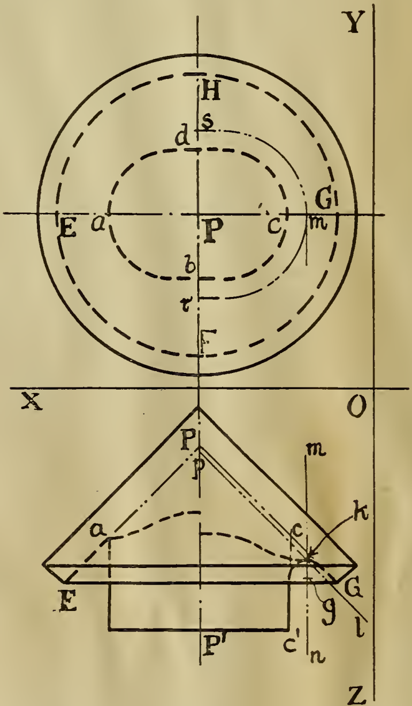


FIG. 69.

fillets must lie. If a line mn, parallel to cc', the generating line of the cylinder, is moved parallel to cc', and at a constant distance from the surface of the non-circular cylinder, it will generate a new non-circular cylinder on the surface of which the required path of the point k must lie. The directrix of this new cylinder is drawn in ℍ, the line rms, as shown. The intersection of these two

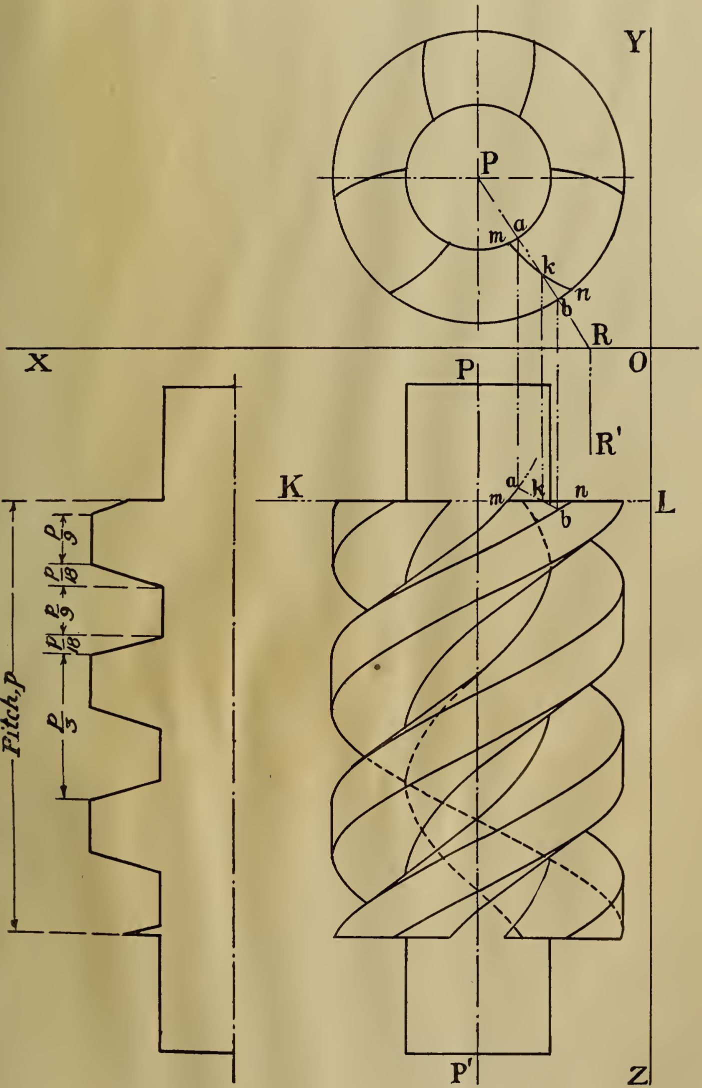


FIG. 70.

new surfaces, found by the method used above (or by planes perpendicular to $\mathbb{H}$ through the axis PP'), is the required path of k or the line which appears on $\mathbb{V}$ and $\mathbb{S}$. The line *rms*, representing the same path on $\mathbb{H}$, is not properly a line of the drawing and is not inked except as a construction line.

79. Intersection of a Helicoidal Surface and a Plane.—In Fig. 70 there is shown a long-pitched screw having a triple thread, such as is often employed for a "worm." To the left is shown a partial longitudinal section giving the generating lines. In $\mathbb{V}$ the concealed parts of the helical edges are omitted, except in the cases of one of the smaller and one of the larger edges. The plane whose trace on $\mathbb{V}$ is KL is perpendicular to the axis, and terminates the screw threads. The intersection of this plane with the screw threads is the curve of intersection to be drawn on $\mathbb{H}$. It is determined by passing planes containing the axis of the worm. One of these is shown by its traces PR and RR'.

From points a and b in the plan corresponding points are plotted on the front elevation, a falling on the helix of small diameter (extended in this case), and b on the helix of large diameter. This element ab of the helix is seen to pierce the plane KL at k. This point k is projected to the plan and is one of the points on the required curve *mkn*.

Problems VIII.

(For units, use inches on blackboard or wire-mesh cage, or small squares on cross-section paper.)

80. An anchor-ring is formed by revolving a circle of 6 units' diameter about a vertical axis, so that its center moves in a circle of 10 units' diameter, center at Q (8, 8, 8). The anchor-ring is intersected by a plane parallel to $\mathbb{V}$ through the point A (8, 6, 8) and by another plane parallel to $\mathbb{V}$ through the point B (8, 4, 8). Draw the projections of the ring, the traces of the planes and the lines of intersection.

81. The same anchor-ring is intersected by a plane perpendicular to $\mathbb{V}$, having a trace passing through the points C (0, 0, 2) and D (8, 0, 8). Make the descriptive drawing and show the true shape of the lines of intersection.

82. The same anchor-ring is intersected by a right circular cylinder, axis P (12, 8, 0), P' (12, 8, 16), and diameter of 4 units. Make the descriptive drawing of the anchor-ring, imagining it to be pierced by the cylinder.

83. An anchor-ring has an axis P (0, 8,8), P' (16, 8, 8). Its center moves in a plane 10 units from 𝕊 describing a circle of 8 units' diameter. The radius of the describing circle is 3 units. It is intersected by an ogival point whose axis is a vertical line Q (7, 8, $3\frac{7}{8}$), Q' (7, 8, 16). The generating line of the ogival point is an arc of 60°, with center at (0, 8, 16), and radius 14 units, so that the point Q is the vertex and point Q' is the center of the circular base of 7 units' radius. The axes intersect at P (7, 8, 8). Draw the projections and the line of intersection, front and side elevations only.

84. The line P (4, 13, 8), P' (16, 8, 8) is the chord of an arc of 45°, whose radius is 13 units. The arc is the generating line of a surface of revolution of which PP' is the axis. Draw the projection on ℍ. Draw the end view on an auxiliary plane 𝕌 perpendicular to PP', the trace of 𝕌 on ℍ intersecting OX at (16, 0, 0). The surface is intersected by a plane perpendicular to ℍ and containing the line PP'. Draw the line of intersection in 𝕍.

85. The same surface is intersected by a plane perpendicular to ℍ whose trace in ℍ passes through the points (4, 10, 0) and (16, 5, 0). Draw the line of intersection on 𝕍.

86. The line P (3, 8, 8), P' (13, 8, 8) is the chord of an arc of 60°, radius 10 units. It is the axis of revolution of a surface of which the arc is the generating line. It is intersected by a right circular cone having its vertex at Q (8, 8, 2), and center of base at Q' (8, 8, 12), radius of base 6 units. Draw the line of intersection.

87. A non-circular cylinder has its straight elements, length 16 units, perpendicular to ℍ, passing through the points of a smooth curve through the points A (14, 6, 0), B (12, 4, 0), C (10, 4, 0), D (8, 5, 0), E (5, 8, 0), F (2, 13, 0). It is pierced by a cylinder whose base is in 𝕍, whose axis is perpendicular to 𝕍 at the point (8, 0, 8), and whose radius is 5 units and length 14 units. Find the line of intersection in 𝕊.

88. The line P (8, 8, 2), P (8, 8, 14) is the axis of a right circular cylinder of 6″ diameter. Projecting from the cylinder is an helicoidal surface, of 12 units' pitch, of which G (5, 8, 2), G' (1, 8, 2) is the generating line. The helicoid is intersected by a plane perpendicular to ℍ whose trace in ℍ passes through the points (5, 0, 0) and (16, 11, 0). Draw the plan and front elevation of the cylinder and helicoid and plot the line of intersection with the plane.

89. The helicoidal surface of Problem 87 is intersected by a right circular cylinder whose axis Q (12, 8, 2), Q' (12, 8, 14) is parallel to PP'. The radius of the cylinder is 3 units. Draw the line of intersection.

CHAPTER IX.

DEVELOPMENT OF CURVED SURFACES.

80. Meaning of Development as Applied to Curved Surfaces.— Many curved surfaces may be developed on a plane in a manner similar to the development of prisms and pyramids explained in Articles 45 and 46. By development, is meant flattening out, without *stretching or otherwise distorting the surface.* If a curved surface is developed on a plane and this portion of the plane, called "the development of the surface," is cut out, this development may

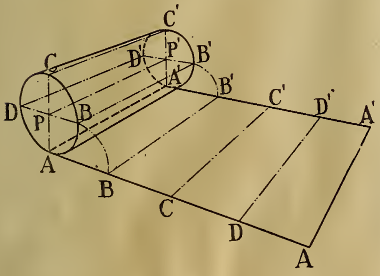


FIG. 71.

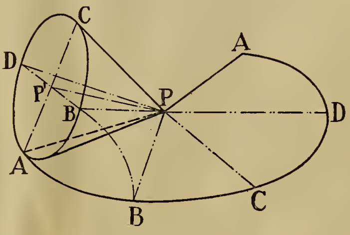


FIG. 72.

be bent into the shape of the surface itself. The importance of the process comes from the fact that many articles of sheet metal are so made. If a sheet of paper is bent in the hands to any fantastic shape, it will always be found that through every point of the paper a straight line may be drawn on the surface in some one direction, the greatest curvature of the surface at this point being in a direction at right angles to this straight line element through the point. The surfaces which can be formed by twisting a plane surface without distortion are called surfaces of single curvature. The curved surfaces, therefore, which are capable of development are only those which are surfaces of single curvature and have straight line elements, but not by any means all of these. All forms of cylinders and cones, right circular, oblique circular, or non-circular, may be developed. The helicoidal surfaces, illustrated by Figs. 55 and 56, though having straight elements, cannot be developed, nor can the hyperboloid of revolution, a surface generated by revolving a straight line about a line not parallel nor intersecting. Figs. 71 and 72 are perspective drawings showing the process of rolling out or developing a right circular cylinder and a right circular cone.

81. Rectification of the Arc of a Circle.—In developing curved surfaces it frequently happens that the whole or part of the circumference of a circle is rolled out into a straight line. Since the surface must not be stretched or compressed, the straight line must be equal in length to the arc of the circle. This process of finding a straight line equal to a given arc is called *rectifying the arc.* No

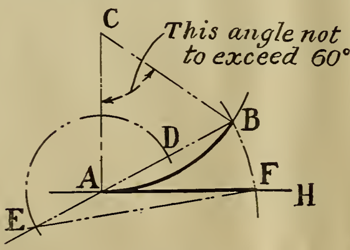


FIG. 73.

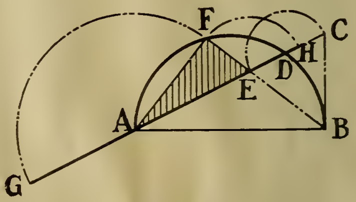


FIG. 74.

absolutely exact method is possible, but methods are known which are so nearly exact as to lead to no appreciable error. These have the same *practical* value as if geometrically perfect.

In Fig. 73, AB is the arc of a circle, center at C. For accurate work the arc should not exceed 60°. It is required to find a straight line equal to the given arc. Draw AH, the tangent at one extremity, and draw AB, the chord. Bisect AB at D. Produce the chord and set off AE equal to AD. With E as a center, and with EB as a radius, describe the arc BF, meeting AH at F. Then $AF =$ arc AB, within one-tenth of one per cent.

In this figure, and in the two following ones, the arc and the straight line equal to it are made extra heavy for emphasis.

82. Rectifying a Semicircle.—A second method, applicable particularly to a semicircle, was recently devised by Mr. George Pierce. In Fig. 74 the semicircle *AFB* is to be rectified. A tangent *BC*, equal in length to the radius, is drawn at one extremity. Join *AC*, cutting the circumference at *D*. Lay off *DE*=*DC*, and join *BE*, producing *BE* to the circumference at *F*. Join *AF*. Then the triangle *AEF*, shown lightly shaded, has its periphery equal to the semicircle *AFB*, within one twenty-thousandth part. The periphery may be conveniently spread into one line by using *A* and *E* as centers, and with *AF* and *EF* as radii, swinging *F* to the left to *G* and to the right to *H* on the line *AF* extended. *GH* is the rectified length of the semicircle.

83. To Lay Off an Arc Equal to a Given Straight Line.—This inverse problem, namely to lay off on a given circle an arc equal to

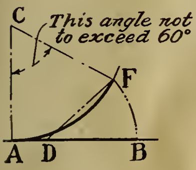


FIG. 75.

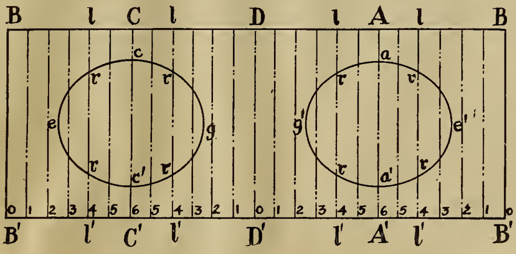


FIG. 76.

a given straight line, frequently arises. In Fig. 75 a line *AB* is given. It is required to find an arc of a given radius *AC* equal to the given line *AB*. At *A* erect a perpendicular, making *AC* equal to the given radius, and with *C* as a center describe the arc *AF*. On *AB*, take the point *D* at one-fourth of the total distance from *A*. With *D* as center and *DB* as a radius, draw the arc *BF*, meeting *AF* at *F*. *AF* is the required arc, equal to *AB*.

This process is also accurate to one-tenth of one per cent if the arc *AF* is not greater than 60°. If in the application of this process to a particular case the arc *AF* is found to be greater than 60°, the line *AB* should be divided into halves, thirds or quarters, and the operation applied to the part instead of to the whole line.

84. Development of a Straight Circular Cylinder.—In Fig. 60 let the intersecting cylinders represent a large sheet-iron ventilating pipe, with two smaller pipes entering it from either side. Such a piece is called by pipe fitters a "cross." The problem is to find the shape of a flat sheet of metal which, when rolled up into a cylinder, will form the surface of the vertical pipe, with the openings already cut for the entrance of the smaller pipes. Before developing the large cylinder, it must be considered as cut on the straight element BB'. After the pipe is formed from the development used as a pattern, the element BB' will be the location of a longitudinal seam.

A rectangle, Fig. 76, is first drawn, the height BB' being equal to the height of the cylinder and the horizontal length being equal to the circumference of the base $BCDA$. (This length may be best found by Mr. Pierce's method, which gives the half-length, BD.) On the drawing, Fig. 60, the base $BCDA$ must be divided into equal parts, 24 parts being usually taken, as they correspond to arcs of 15°, which are easily and accurately constructed with the draftsman's triangles. Only 6 of these 24 parts are required to be actually marked on Fig. 60, as the figure is doubly symmetrical and each quadrant is similar to the others. On Fig. 76 the line $BCDAB$ is divided into 24 parts also, the numbering of the lines of division running from 0 to 6 and back to 0 for each half-length of the development. In V of Fig. 60, draw the elements corresponding to the points of division. The elemnt II' already drawn corresponds to No. 4, and BB' and CC' correspond to Nos. 0 and 6. The others are not drawn in Fig. 60, to avoid complicating the figure, but would have to be drawn in practice before constructing the development. On the four elements which are numbered 4 on the development, Fig. 76, lay off the distances Ir equal to Ir in Fig. 60. On the two elements, Fig. 76, numbered 6, lay off Cc or Aa equal to Cc of Fig. 60, and imagine the proper distances to be laid off on elements numbered 3 and 5. Smooth curves through the points thus plotted are the ovals which must be cut out of the sheet of metal to give the proper-shaped openings for the small pipes.

When it is known in advance that the surface of such a cylinder as that in Fig. 60 must be developed, it is often possible to so choose the system of auxiliary intersecting planes used to define the curve of intersection as to give the required equally spaced straight elements for the development.

The smaller cylinder may be developed in the same way. A new system of equally spaced straight elements would probably have to be chosen for this cylinder.

85. Development of a Right Circular Cone.—The cone of Fig. 63 has been selected for this illustration. Imagine it to be cut on the element *PB* and flattened into a plane. The surface takes the

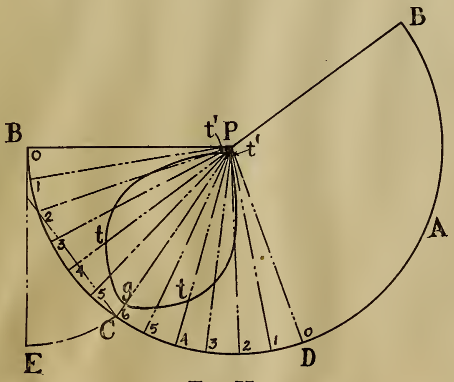


FIG. 77.

form of a sector of a circle, the radius of the sector being the slant height of the cone (or length of the straight element), and the arc of the sector being equal in length to the circumference of the base of the cone. Several means of finding the length of the arc of the sector are available.

The most natural method is to rectify the circumference of the base and then, with the slant height as radius, to draw an arc and to lay out on the arc a length equal to this rectified circumference. In Fig. 63 suppose that the semi-circumference *ABC* (in ℍ) has been rectified by Pierce's method. In Fig. 77 let an arc be drawn with radius *PB* equal to *PB* in 𝕊, Fig. 63, and from *B* draw a tangent *BE* equal to one-half the rectified length of the semi-circumference. Find the arc *BC* equal to *BE* by the method of Art.

83, Fig. 75. *BC* is one-fourth of the required arc, and corresponds to the quadrant *BC* in ℍ, Fig. 63. Divide the arc *BC* and the quadrant *BC* into the same number of equal parts, numbering them from 0 to 6, if 6 parts are chosen. Repeat the divisions in the arc *CD* (equal to *BC*), numbering the points of division from 6 down to 0, this duplication of numbers being due to the symmetry of the ℍ projection of Fig. 63, about the line *APC*. In Fig. 63, as in Fig. 77, the points 0 to 6 are all supposed to be joined to *P*, the only straight elements actually shown there being *P*0, *P*4, and *P*6.

On the elements *P*4 of the development lay off the *true length* of the line *Pt* (and the true length of the line *Pt'* also). *Pt* is an oblique line, but if its ℍ projector-plane (*Pt* in ℍ, Fig. 63) be revolved up to the position *Pm*, the point *t* in 𝕍 moves to *m*, and *Pm* is the true length of *Pt*. The distance *Pg* (𝕍, in Fig. 63) is laid off on *P*6 of the development.

When the proper distances have been laid off on the elements *P*2, *P*3 and *P*5, a smooth curve may be drawn through the points. The sector, with this opening cut in it, is the pattern for forming the cone out of sheet iron or any thin material.

If the ratio of *PA* to *P'A* in 𝕍, Fig. 63, can be exactly determined, the most accurate method of getting the angle of the sector is by calculation, for the degrees of arc in the development are to the degrees in the base of the cone (360°) as the radius of the base of the cone is to the slant height. In this case *P'A* is $\frac{3}{5}$ *PA*. The sector in Fig. 75 subtends $\frac{3}{5} \times 360°$, or 216°. In the use of this method a good protractor is required to lay out the arc.

Problems IX.

90. Draw an arc of 60° with 10 units' radius. At one end draw a tangent and on the tangent lay off a length equal to the given line. On the tangent lay off a length of 8 units, and find the length of arc equal to this distance.

91. An arc of 12 units' radius, one of 9 units' radius, and a straight line are all tangent at the same point. Find on the tangent the straight line equal in length to 45° of the large arc. Find the length on the other arc equal to this length on the tangent and show that it is an arc of 60°.

92. Rectify a semicircle of 10 units' radius and compare the length with the calculated length, 31.4 units.

93. A rectangle 31.4 units by 12 units is the developed area of a cylinder of 10 units' diameter. A diagonal line is drawn on the development, which is then rolled into cylindrical form. Plot the form taken by the diagonal and show that it is a helix of 12 units' pitch.

94. A right circular cone has a base of 10 units' diameter, and a vertical height of 12 units. Its slant height is 13 units. Calculate the angle of the sector which is the developed surface of the cone. Find this angle by rectifying the circumference of the cone, and by finding the arc equal to the rectified length. (This last operation must be performed on one-third or one-quarter of the rectified length, to keep the accuracy within one-tenth of one per cent.)

95. A semicircle, radius 10 units, is rolled up into a cone. What is the radius of the base? What is the slant height? What is the relation between the area of the curved surface of the cone and the area of the base?

96. A right circular cylinder, such as Fig. 49, is of 6.367 units' diameter, and 12 units' height. It is intersected by a plane perpendicular to V through the points C and A'. Draw plan, front elevation and the development of the surface.

97. A right circular cone, like that of Fig. 51, has its front elevation an equilateral triangle, each side being 10 units in length. From A_v a perpendicular is drawn to P_vC_v cutting it at E. If this line represents a plane perpendicular to V, draw the development of the cone with the line of intersection of the cone and plane traced on the development.

98. A right circular cylinder, standing in a vertical position, as in Fig. 49, diameter 7 units, and length 10 units, is pierced from side to side by a square hole $3\frac{1}{2}$ units on each edge, the axis of the hole and the axis of the cylinder bisecting each other at right angles. Draw the development of the surface.

99. A sheet of metal 22 units square with a hole 11 units square cut out of its middle, the sides of the hole being parallel to the edges of the sheet, is rolled up into a cylinder. Draw the plan, front and side elevations of the cylinder.

CHAPTER X.

STRAIGHT LINES OF UNLIMITED LENGTH AND THEIR TRACES.

86. Negative Coordinates.—We have dealt only with points having positive or zero coordinates, and the lines and planes have been

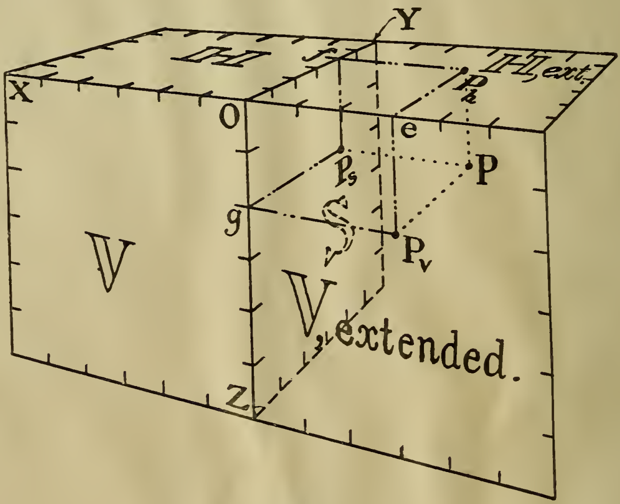


FIG. 78.

limited in their extent, or, if infinite, have extended indefinitely only in the positive directions. As it becomes necessary at times to trace lines and planes in their course, no matter if they cross the reference planes into new regions of space, the use and meaning of negative coordinates must be explained. The value of the x coordinate of a point is the length of the S projector or perpendicular distance from the point to the side reference plane S. (See Figs. 6 and 7, Art. 9.) If this value decreases gradually to zero, the point moves towards $\mathbb{S}$ until it lies in $\mathbb{S}$ itself. If this value becomes negative, it is clear that the point crosses the side reference plane into a space to the right of it.

For example, a point P, having a variable x coordinate, but having its y coordinate always equal to 4 and its z coordinate equal to 2, is a point moving on a line parallel to the axis of X. If x decreases to zero, it is on $\mathbb{S}$ at the point marked P_s in Fig. 78. If the x coordinate decreases further, reaching a value of -3, it moves to the point P in that figure. Fig. 78 is the perspective drawing of a point P $(-3, 4, 2)$. The y and z projectors cannot project the point P to $\mathbb{H}$ and $\mathbb{V}$ in their customary positions, but

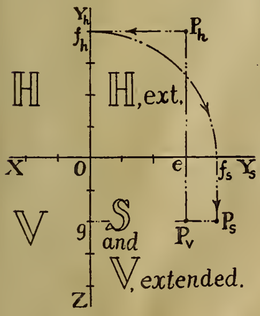


FIG. 79.

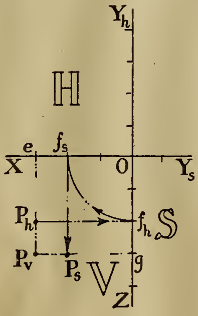


FIG. 80.

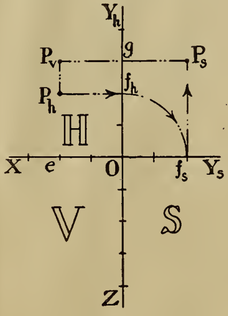


FIG. 81.

project it upon parts of those planes extended beyond the axes of Y and Z, as shown. In Fig. 79, the corresponding descriptive drawing, it must be understood that the plane $\mathbb{H}$, extended, has been revolved with $\mathbb{H}$, about the axis of X, into the plane of the paper, $\mathbb{V}$, and $\mathbb{S}$ has been revolved as usual about the axis of Z, coming into coincidence with $\mathbb{V}$, extended. This "development" of the planes of reference is exactly as described in Art. 7. It is noticeable that the x coordinate of P is laid off to the right of the origin instead of to the left. P_h lies, therefore, in the quadrant which usually represents no plane of projection, and P_v lies in the quadrant which usually represents $\mathbb{S}$. P_s lies in its customary place, since both y and z, the coordinates which alone appear in $\mathbb{S}$, are positive.

It is evident that the laws of projection for $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$, Art. 11, have not been altered, but simply extended. P_h and P_v are in the same vertical line; P_v and P_s are in the same horizontal line; and the construction which connects P_h and P_s still holds good.

In Fig. 79 the space marked $\mathbb{S}$ represents not only $\mathbb{S}$ but $\mathbb{V}$ extended as well.

In Fig. 80 is represented a point P (3, −2, 3), having a negative y coordinate. The point is *in front* of $\mathbb{V}$, at 2 units' distance, not *behind* $\mathbb{V}$. The projection on $\mathbb{H}$, instead of being *above* the axis of X a distancee of 2 units, is *below* it by the same amount. So also the projection on $\mathbb{S}$ is to the *left* of the axis of Z, a distance of 2 units, instead of the the *right* of it. After developing the reference planes in the manner of Art. 7, plane $\mathbb{H}$, extended, has come into coincidence with $\mathbb{V}$, and plane $\mathbb{S}$, extended, has also come into coincidence with $\mathbb{V}$. Thus the field representing $\mathbb{V}$ represents also the other two reference planes, extended.

In Fig. 81 a point P (2, 2, −3) having a negative z coordinate is represented. The point is *above* $\mathbb{H}$ 3 units, instead of *below* $\mathbb{H}$, at the same perpendicular distance. P projects upon $\mathbb{V}$ on $\mathbb{V}$ extended above the axis of X. After developing the reference planes, plane $\mathbb{H}$ comes into coincidence with $\mathbb{V}$ extended. P_s is on $\mathbb{S}$ extended above the axis of Y, and therefore after development it occupies the so-called "construction space."

Points having two or three negative coordinates may be dealt with in the same manner, but are little likely to arise in practice.

It is evident that subscripts must be used invariably, to prevent confusion whenever negative values are encountered.

87. Graphical Connection Between P and P_s.—In Figs. 79, 80 and 81, P_h and P_s are connected by a construction line $P_hf_hf_sP_s$ in a manner which is an extension of that shown by Fig. 7, Art. 9. Note that the quadrant of a circle connecting P_h and P_s must be described always on the construction space or on the field devoted to $\mathbb{V}$, never on the fields devoted to $\mathbb{H}$ or $\mathbb{S}$.

88. Traces of a Line of Unlimited Length, Parallel to an Axis.—A straight line which has no limit to its length, but extends indefinitely in either direction, must necessarily have some points whose coordinates are negative. In passing from positive to negative regions the line must pass through some plane of reference (having one of its coordinates zero at that point), and the point where it pierces a plane of reference is called the *trace* of the line on that plane of reference, the word trace being used to indicate a "track" or print showing the passage of the line.

Lines parallel to the axes have been used freely already. An ℍ projector is simply a vertical line or line parallel to the axis of Z. Any perspective figure showing a point P and its horizontal projection P_h will serve as an illustration of this line, as PP_h in Fig. 6, Art. 9.

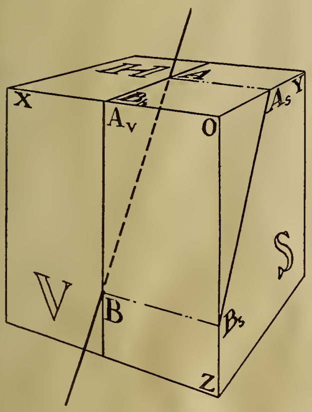


FIG. 82.

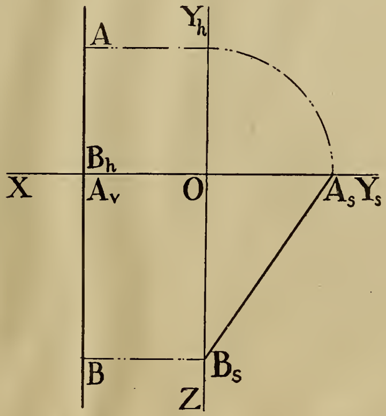


FIG. 83.

Imagine PP_h to be extended in both directions as an unlimited straight line. Then P_h is the trace of the line on ℍ. In Fig. 7, the point P_h itself is the ℍ projection of the line. P_ve, extended in both directions, is the vertical projection and P_sf_s is the side projection. Thus it is seen that a vertical line has but one trace, that on the plane to which it is perpendicular. PP_v may be taken as an illustration of a line parallel to the axis of Y, and PP_s of one parallel to the axis of X. A better example of this latter case is shown in Figs. 15 and 16, Art. 16. The line BAA_s, perpendicular to 𝕊, has its trace on 𝕊 at A_s.

89. Traces of an Inclined Straight Line.—An inclined line such as *AB* in Figs. 82 and 83 pierces two reference planes as at *A* and *B*, but as it is parallel to the third reference plane, 𝕊, it has no trace on 𝕊. The peculiarity of the descriptive drawing of this line, Fig. 83, is the apparent coincidence of the ℍ and 𝕍 projections as one vertical line. The 𝕊 projection is required to determine the traces *A* and *B*.

90. Traces of an Oblique Straight Line: The ℍ and 𝕍 Traces.—An oblique line, if unlimited in length, must pierce each of the reference planes, since it is oblique to all three. Any line is com-

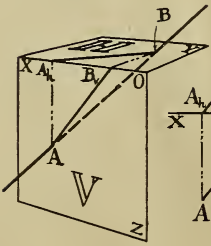


FIG. 84.

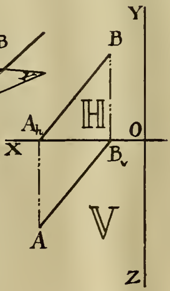


FIG. 85.

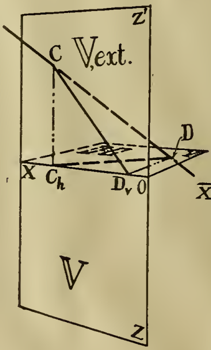


FIG. 86.

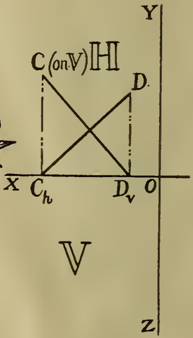


FIG. 87.

pletely defined when two points on the line are given. If two traces of a straight line are given, the third trace cannot be assumed, but must be constructed from the given conditions by geometrical process. It will always be found that of the three traces of an oblique line one trace at least has some negative coordinate.

As the complete relation between the three traces is somewhat complicated, the relation between two traces, as, for instance, ℍ and 𝕍 traces, must be considered first. Two cases are shown, the first by Figs. 84 and 85, and the second by Figs. 86 and 87. The line *AB* is the line whose traces are *A* (5, 0, 4) and *B* (2, 4, 0). The line *CD* is the line whose traces are *C* (7, 0, 5) and *D* (2, 4, 0).

From the descriptive drawing of AB, Fig. 85, it is seen that the ℍ projection of the line cuts the axis of X vertically above the trace on 𝕍, and that the 𝕍 projection cuts the axis of X vertically under the trace on ℍ. It may be noted that the two right triangles A_hBB_v and B_vAA_h have the line A_hB_v on the axis of X as their common base. From the descriptive drawing of the line CD, Fig. 87, it is seen that the effect of the vertical trace C having a negative z coordinate simply puts C (on 𝕍) above C_h, instead of below it. The two right triangles C_hDD_v and D_vCC_h have the line C_hD_v on the axis of X, as their common base, but the latter triangle is above the axis instead of in its normal position.

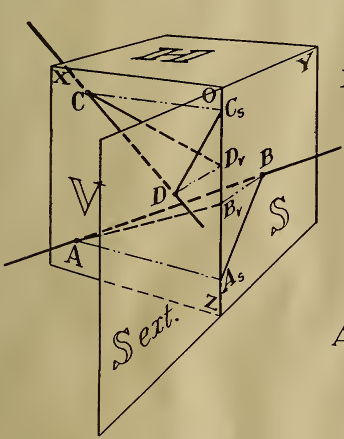


FIG. 88.

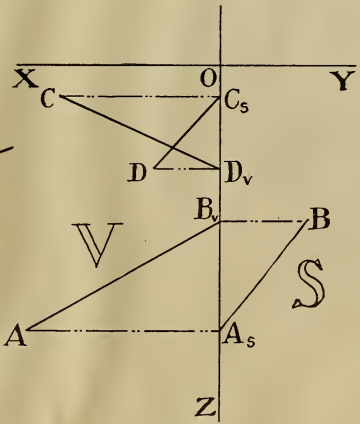


FIG. 89.

91. Traces of an Oblique Straight Line: The 𝕍 and 𝕊 Traces.— Figs. 88 and 89 show two lines piercing 𝕍 and 𝕊.

The line AB pierces 𝕍 at A and 𝕊 at B. The two right triangles A_sAB_v and B_vBA_s have their common base A_sB_v on the axis of Z.

The line CD pierces 𝕍 at C and 𝕊 extended at D, the point D having a negative y coordinate. The right triangles C_sCD_v and D_vDC_s have their base D_vC_s in common on the axis of Z, but in the descriptive drawing D_vDC_s lies to the left of the axis of Z instead of to the right, owing to the point D having a negative y coordinate.

92. Traces of an Oblique Straight Line: The ℍ and 𝕊 Traces.—Figs. 90 and 91 show two lines piercing ℍ and 𝕊.

The line AB pierces ℍ at A and 𝕊 at B. The triangles A_sAB_h and B_hBA_s have their common base A_sB_h on the axis of Y, Fig. 90, but in the descriptive drawing the duplication of the axis of Y causes this base A_sB_h to separate into two separate bases, one on OY_h and one on OY_s. Otherwise, there has been no change.

The line CD pierces ℍ at C and 𝕊 extended at D, the point D having a negative z coordinate. In Fig. 90 C_sCD_h and D_hDC_s have

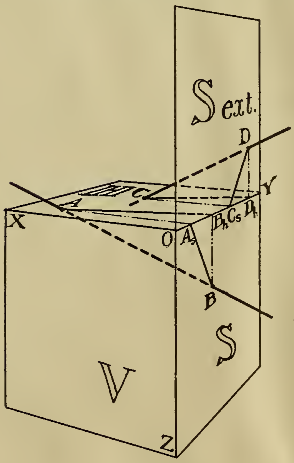


FIG. 90.

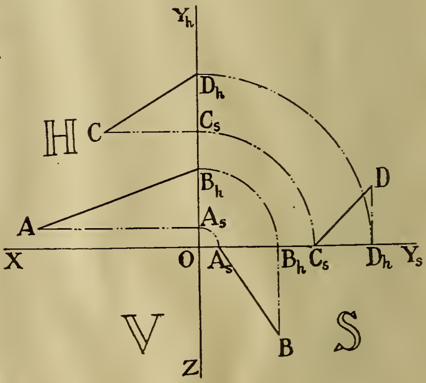


FIG. 91.

their common base C_sD_h on the axis of Y, but in the descriptive drawing C_sD_h appears in two places. The triangle D_hDC_s lies above 𝕊 in the "construction space," or on 𝕊 extended, since D has a negative z coordinate.

93. Three Traces of an Oblique Straight Line.—Figs. 92 and 93 show an oblique straight line ABC piercing 𝕍 at A, ℍ at B, and 𝕊 extended at C. Since the line is straight, the three projections of the line AB_vC_v, A_sB_sC and A_hBC_h are all straight lines. In the perspective drawing, Fig. 92, part of the 𝕍 projection is on 𝕍 extended and part of the 𝕊 projection on 𝕊 extended.

In the descriptive drawing, Fig. 93, the relation between A and B is the same as that in Fig. 85, as shown by the two triangles A_hAB_v and B_vBA_h, or the quadrilateral A_hAB_vB. The relation between A and C, as shown by the quadrilateral A_sAC_vC, is the same as that between A and B, Fig. 89, as shown by the quadrilateral A_sAB_vB. The relation between B and C, Fig. 93, as shown by the two triangles B_sBC_h and C_hCB_s, is the same as that between C and D of Fig. 91, as shown by the triangles C_sCD_h and D_hDC_s. No new feature has been introduced.

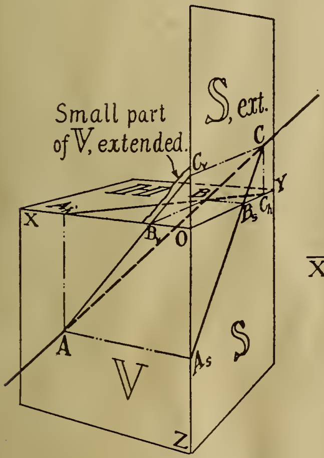


FIG. 92.

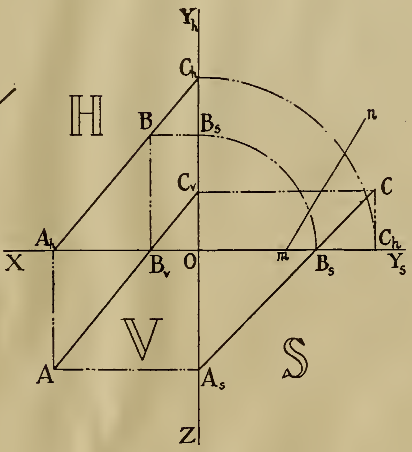


FIG. 93.

94. Paper Box Diagram.—To assist in understanding Figs. 92 and 93, a model in space should be made and studied from all sides. The complete relation of the traces is then quickly grasped. Construct the descriptive drawing, Fig. 93, on coordinate paper, using, as coordinates for A, B and C, $(15, 0, 12)$, $(5, 12, 0)$, and $(0, 18, -6)$. Fold into a paper box after the manner of Fig. 9, Art. 12, having first cut the paper on some such line as mn, so that the part of the paper on which C is plotted may remain upright, serving as an extension to S. It will be found that a straight wire or long needle or a thread may be run through the points A, B and C, thus producing a model of the line and all its projections.

95. Intersecting Lines.—If two lines intersect, their point of intersection, when projected upon any plane of reference, must necessarily be the point of intersection of the projections on that plane. For example, a line AB intersects a line CD at E. Project E upon a plane of reference, as ℍ. Then E_h must be the point of intersection of A_hB_h and C_hD_h. In the same way E_v must be the point of intersection of A_vB_v and C_vD_v, and E_s of A_sB_s and C_sD_s.

To determine whether two lines given by their projections meet in space or pass without meeting, the projections on at least two reference planes must be extended (if necessary) till they meet. Then for the lines themselves to intersect, the points of intersection of the two pairs of projections must obey the rules of projection of a point in space (Art. 11). Thus if A_hB_h and C_hD_h are given and meet at a point vertically above the point of intersection of A_vB_v and C_vD_v, the two lines really meet at a point whose projections are the intersections of the given projections. If this condition is not filled the lines pass without meeting, the intersecting of the projections being deceptive.

96. Parallel Lines.—If two lines are parallel, the projections of the lines on a reference plane are also parallel (or coincident). For, the two lines make the same angle with the plane of projection; their projector-planes are parallel; and the projections themselves are parallel.

Thus if a line AB is parallel to another line CD, then A_hB_h must be parallel to C_hD_h, A_vB_v to C_vD_v, and A_sB_s to C_sD_s. If the two lines lie in a plane perpendicular to a plane of projection—for example, perpendicular to ℍ—then the ℍ projector-planes coincide and the ℍ projections also coincide. The 𝕍 and 𝕊 projections are parallel but not coincident.

If two lines do not fill the conditions of intersecting or of parallel lines, they must necessarily be lines which pass at an angle without meeting.

Problems X.

100. Plot the points A $(8, 6, -4)$, B $(7, -3, 5)$, C $(-7, 0, 12)$.

101. Plot the points A $(6, -10, 3)$, B $(0, 0, -5)$, C $(-6, 5, 4)$.

102. Make a descriptive drawing of a line 26 units long through the point P $(-8, 4, 9)$, perpendicular to $\mathbb{S}$. What traces does it have? What are the coordinates of its middle point?

103. A line is drawn from P $(12, 5, 16)$ perpendicular to $\mathbb{H}$. Make the descriptive drawing of the line, and of a line perpendicular to it, drawn from Q $(0, 0, 8)$. What is the length of this perpendicular line, and where are its traces?

104. A straight line extends from A $(8, 12, 0)$ through D $(8, 6, 8)$ for a distance of 20 units. Make the descriptive drawing of the line. Where are its traces and its middle point?

105. A straight line pierces $\mathbb{H}$ at A $(8, 6, 0)$ and $\mathbb{V}$ at B $(8, 0, 12)$. Draw its projections. Where is its trace on $\mathbb{S}$? What are the coordinates of D, its middle point?

106. A straight line extends from E $(15, 6, 16)$ through A $(3, 6, 0)$ to meet $\mathbb{S}$. Make the descriptive drawing and mark the traces on $\mathbb{H}$ and $\mathbb{S}$.

107. Draw the lines A $(16, 11, 8)$, B $(4, 8, 2)$; C $(12, 5, 10)$, D $(0, 2, 4)$; and E $(11, 3, 0)$, F $(5, 15, 16)$. Which pair meet, which are parallel, and which pass at an angle? What are the coordinates of the point of intersection of the pair which meet?

108. The points A $(8, 0, 12)$, B $(0, 8, 6)$ and C $(-8, 16, 0)$ are the traces of a straight line. Make the descriptive drawing of the line.

109. The points A $(8, -4, 0)$, D $(4, 4, 6)$ and E $(2, 8, 9)$ are on a straight line. Find the trace B where it pierces $\mathbb{V}$ and the trace C where it pierces $\mathbb{S}$.

CHAPTER XI.

PLANES OF UNLIMITED EXTENT: THEIR TRACES.

97. Traces of Horizontal and Vertical Planes.—The lines of intersection of a plane with the reference planes are called its traces. Planes of unlimited extent may be of three kinds, parallel to a reference plane, inclined, or oblique. Unlimited planes of the first two classes have been dealt with already, but for the sake of precision may be treated here again to advantage.

A horizontal plane is one parallel to ℍ, and the trace of such a plane on 𝕍 is a line parallel to the axis of *X*, and the trace on 𝕊 is a line parallel to the axis of *Y*. These traces meet the axis of *Z* at the same point and appear on the descriptive drawing as one continuous line. There is of course no trace on ℍ. In Fig. 58, Art. 67, the plane *T*, represented by its traces *T′T* on 𝕍 and *TT″* on 𝕊, is a horizontal plane. These traces are not only the intersections of *T* with ℍ and 𝕊, but *T* is "seen on edge" in those views. Every point of the plane *T*, when projected upon 𝕍, lies somewhere on the line *T′T*, extended indefinitely in either direction.

A vertical plane parallel to 𝕍 has for its traces a line on ℍ parallel to the axis of *X*, and on 𝕊 a line parallel to the axis of *Z*, with no trace on 𝕍. These traces meet the axis of *Y* at the same point, and appear on the descriptive drawing as two lines at right angles to this axis, the point on *Y* separating into two points as usual. In Fig. 57, Art. 66, a vertical plane *R*, parallel to 𝕍, is represented by its traces *R′R* on ℍ and *RR″* on 𝕊.

A vertical plane parallel to 𝕊 has for its trace on ℍ a line parallel to the axis of *Y*, and for its trace on 𝕍 a line parallel to the axis of *Z*, with no trace on 𝕊. These traces meet the axis of *X* at the same point and appear on the descriptive drawing as one continuous line.

98. Traces of Inclined Planes.—Inclined planes are those perpendicular to one reference plane, but not to two reference planes. The auxiliary planes of projection have been of this kind. In

Fig. 20, Art. 22, the plane 𝕌, perpendicular to ℍ, has the line *MX* for its trace on ℍ, and *XN* for its trace on 𝕍. In the descriptive drawing, Fig. 21, *MX* and XN_v are these traces.

If in Fig. 20 both 𝕌 and 𝕊 are imagined to be extended towards the eye, they will intersect in a line parallel to *OZ*. This 𝕊 trace will be on 𝕊 extended, and every point of it will have the same negative *y* coordinate. Of the three traces of 𝕌, two are vertical lines, and one only, *MX*, is an inclined line. The plane in Fig. 64, Art. 74, may be taken as a second example of an inclined plane perpendicular to ℍ. The trace on 𝕊 is not a negative line in this case, but is a vertical line on 𝕊 to the right of the axis of *Z* at a distance equal to *OJ*.

In Fig. 57, Art. 66, *IJ*, *JK* and *KL* are the three traces of an inclined plane perpendicular to 𝕍. In every case of an inclined plane the inclined trace is on that reference plane to which it is perpendicular, and shows the angles of the inclined plane with one or both of the other reference planes.

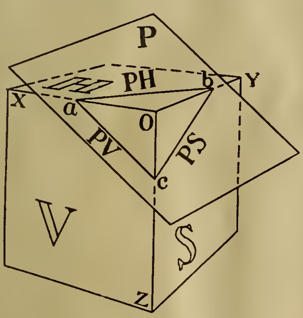


FIG. 94.

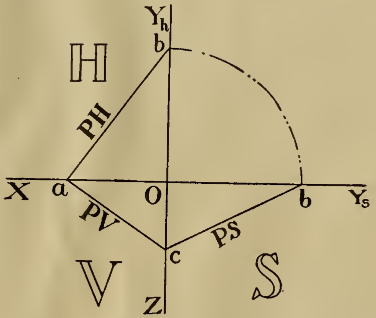


FIG. 95.

99. Traces of an Oblique Plane: All Traces "Positive."—The general case of an oblique plane is shown in Fig. 94. The plane *P* is represented as cutting the cube of reference planes in the lines marked *PH*, *PV* and *PS*. These lines are the traces of the plane *P*, and may be understood to extend indefinitely, the plane itself extending in all directions without limit. They are shown limited in Fig. 94 in order to make a more realistic appearance. *PH*, *PV* and *PS* are used to define the three traces.

Where *PH* and *PV* meet we have a point common to three planes, *P*, ℍ and 𝕍. Since it is common to ℍ and 𝕍 it is on the line of intersection of ℍ and 𝕍, or in other words it is on the axis of *X*. This point is marked *a*. In the same way *PH* and *PS* meet at *b* on the axis of *Y*, and *PV* and *PS* meet at *c* on the axis of *Z*.

The descriptive drawing, Fig. 95, is obvious from the explanation of the perspective drawing. From Fig. 95 it is evident that if two traces of a plane are given the third trace can be determined

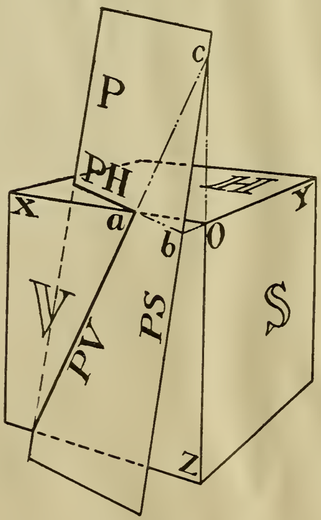


FIG. 96.

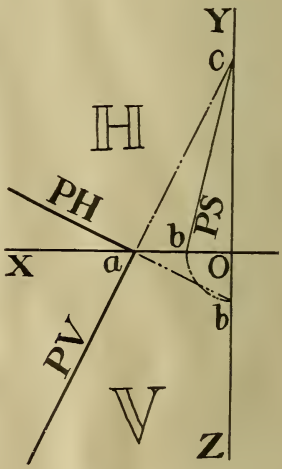


FIG. 97.

by geometrical construction. Thus, if *PH* and *PV* are given, *PS* may be defined by extending *PH* to *b* on the axis of *Y* and extending *PV* to *c* on the axis of *Z*. The line joining *bc* is the required trace of the plane on 𝕊. If any two points on one trace are given, and any one point on a second trace, the whole figure may be completed. Thus any two points on *PH* define that line and enable *a* and *b* to be found. A third point on *PV*, taken in conjunction with *a*, defines *PV*, and enables *c* to be located. *bc*, as before, defines the trace *PS*. This is an application of the general principle that three points determine a plane.

100. Traces of an Oblique Plane: One Trace "Negative."—In Figs. 94 and 95 the plane *P* has been so selected that all traces have positive positions. These are the portions usually drawn. Of course each trace may be extended in either direction, points on the trace then having one or more negative coordinates. Any trace having points all of whose coordinates are positive, or zero, may be called a positive trace.

In Fig. 96 a plane *P* is shown, intersecting 𝕳 and 𝖁 in the "positive" traces *PH* and *PV*. The third trace, *PS*, in this case, has no point all of whose coordinates are positive. In the descriptive drawing, Fig. 97, the two positive traces, meeting at *a* on the

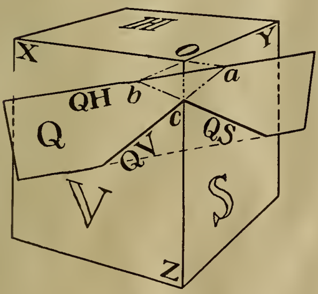


FIG. 98.

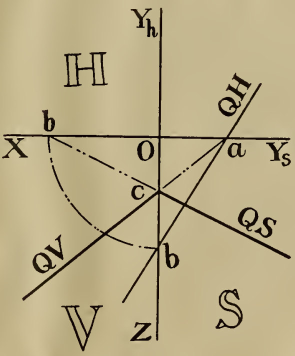


FIG. 99.

axis of *X*, are usually considered as fully representing the plane *P*. From these lines *PH* and *PV*, alone, the imagination is relied upon to "see the plane *P* in space," as shown by Fig. 96.

In Fig. 98, the plane *Q* is represented. Ordinarily the positive traces *QV* and *QS*, meeting at *c* on the axis of *Z*, are the only traces shown in the descriptive drawing, Fig. 99, and are considered to indicate perfectly the path of the plane *Q*.

101. Position of the Negative Trace.—The negative trace *PS*, in Fig. 96, is shown as one of the edges of the rectangular plate representing the unlimited plane *P*. This line *PS* has been determined by extending *PH* to meet the axis of *Y* (extended) at *b*, and by extending *PV* to meet the axis of *Z* (extended) at *c*. The line joining *b* and *c* is the trace *PS*. It will be noted that in finding the location of *PS* in Fig. 97, *PV* has been extended to cut the axis of *Z* (extended up from *ZO*) at *c* and *PH* has been extended to cut the axis of *Y* (extended down from *YO*) at *b*. *b* has been rotated 90° about the origin, and the points *b* and *c* thus plotted (on 𝕊 extended) have been found to give the line *PS*. Every step of the process and the lettering of the figure have been similar to those used in finding *PS* from *PH* and *PV* in Art. 98.

In Fig. 98, the negative trace is *QH*, the top line of the rectangular plate representing the unlimited plane *Q*. *QH* has been determined as follows: *QV* extended meets the axis of *X* extended at *a*, and *QS* extended meets the axis of 𝕍 extended at *b*. The line *ab* is therefore the trace on ℍ, or *QH*. In the descriptive drawing the same process of extending *QV* to *a* and *QS* to *b* determines the line *QH*, a line every point of which has some negative coordinate. Of course *QH* must be considered as drawn on parts of the plane ℍ extended over 𝕍, 𝕊, and the so-called construction space. In finding the negative traces, it is imperative to letter the diagrams uniformly, keeping *a* for the intersection of the plane with the axis of *X*, *b* for that with the axis of *Y*, and *c* for that with the axis of *Z*. With this rule *b* will always be the point which is doubled by the separation of the axis of *Y* into two lines, and the arc *bb* will always be described in the construction space or in the quadrant devoted to 𝕍, never in those devoted to ℍ and 𝕊.

102. Parallel Planes.—If two planes are parallel to each other, their traces on ℍ, 𝕍 and 𝕊 are parallel each to each. This proposition may be proved as follows: If we consider two planes *P* and *Q* parallel to each other and each intersecting the plane ℍ, the lines of intersection with ℍ (*PH* and *QH*) cannot meet, for, if they did meet, the planes themselves would meet and could not then be parallel planes. *PH* and *QH* must therefore be parallel lines described on ℍ. Thus, if a plane *P* and a plane *Q* are parallel, then *PH* and *QH* are parallel, *PV* and *QV* are parallel, and *PS* and *QS* are parallel.

The method of finding the true length of a line by its projection upon a plane parallel to itself, treated in Chapter III, is really the process of passing a plane parallel to a projector-plane of the given line. Thus in Fig. 21, Art. 25, the auxiliary plane 𝕌 has its horizontal trace XM parallel to A_hB_h, and the vertical trace of the ℍ projector-plane, if drawn, would be parallel to XN_v.

103. The Plane Containing a Given Line.—If a line lies on a plane, the trace of the line on any plane of reference (the point where it pierces the plane of reference) must lie on the trace of the plane on that plane of reference. Thus, if the line EF, Fig. 100, lies on the plane P, then A, the trace of EF on ℍ, lies on PH, the trace of P on ℍ; and B, the trace of EF on 𝕍, lies on PV, the trace of P on 𝕍.

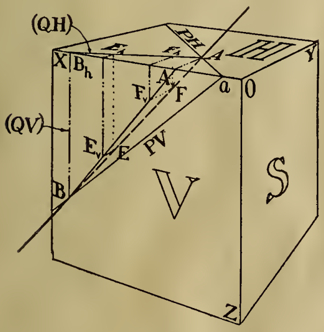


FIG. 100.

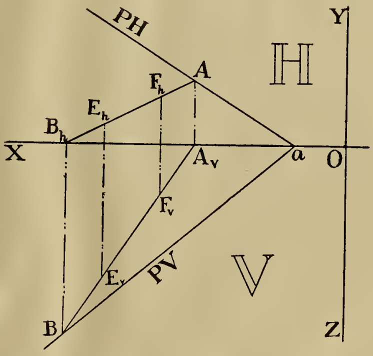


FIG. 101.

From this fact it follows that to pass a plane which will contain a given line it is necessary to find two traces of the line and to pass a trace of the plane through each trace of the line. As an infinite number of planes may be passed through a given line, it is necessary to have some second condition to define a single plane. For example, the plane may be made also to pass through a given point or to be perpendicular to a reference plane.

In Fig. 100, if only the line EF is given and it is required to pass a plane P, containing that line, and containing also some point, as a, on the axis of X, the process is as follows: Extend the line EF to A and B, its traces on ℍ and 𝕍. Join Ba and aA. These are the traces of the required plane P. In the descriptive drawing, Fig. 101, the corresponding operation is performed. A and B must be determined as in Art. 90, and joined to a. These lines represent the traces of a plane containing the line EF and the chosen point a.

To pass a plane Q containing the line EF and also perpendicular to $\mathbb{H}$ (Figs. 100 and 101), the trace of Q on $\mathbb{H}$ must coincide with the projection of EF on $\mathbb{H}$, for the required plane perpendicular to $\mathbb{H}$ is the $\mathbb{H}$ projector-plane of the line. Its traces are therefore AB_h and B_hB.

The traces of a plane containing EF and perpendicular to $\mathbb{V}$ are BA_v and A_vA.

104. The Line or Point on a Given Plane.—To determine whether a *line* lies on a given plane is a problem the reverse of that just treated. It amounts simply to determining whether the traces of the line lie on the traces on the plane. Thus, in Fig. 101, if PV and PH are given, and the line EF is given by its projections, the traces of EF must be found, and if they lie on PH and PV the line is then known to lie on the given plane P.

To determine whether a given *point* lies on a given plane is almost as simple. Join one projection of the point with any point on the corresponding trace of the plane. Find the other trace of the line so formed, and see whether it lies on the other trace of the given plane. Thus in Fig. 101, if the traces PH and PV and the projections of any one point, as E, are given, select some point on PH, as A, and join E_hA and E_vA_v. Find the trace B. If it lies on PV, the point E itself lies on P.

To draw on a given plane a line subject to some other condition, such as parallel to some plane of reference, is always a problem in constructing a line whose traces are on the traces of the given plane, and which yet obeys the second condition, whatever it may be.

105. The Plane Containing Two Given Lines.—From the last article, if a plane contains *two* given lines, the traces of the plane must contain the traces of the lines themselves. The given lines must be intersecting or parallel lines, or the solution is impossible.

In Fig. 102 two lines, AB and AC, are given by their projections. They intersect at A, since A_h, the intersection of the $\mathbb{H}$ projections, is vertically above A_v, the intersection of the 𝕍 projections. Extend the lines to E, F, G and H, their traces on ℍ and 𝕍. Join the ℍ traces, E and G, and produce the line also to a on the axis of X. Join the 𝕍 traces, H and F, and extend the line HF also to a. Ea and aH are the traces of a plane P containing both lines, AB and AC. The meeting of the two traces at a is a test of the accuracy of the drawing.

This process may be applied to a pair of parallel lines, but not of course to two lines which pass at an angle without meeting.

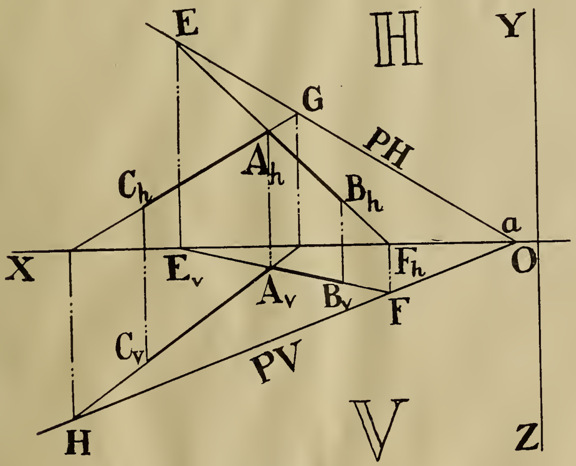


FIG. 102.

106. The Line of Intersection of Two Planes.—If two planes P and Q are given by their traces, their line of intersection must pass through the point where the ℍ traces meet and the point where the 𝕍 traces meet. Thus, in Fig. 103, PH and QH meet at A and PV and QV meet at B. A and B are points on the required line of intersection of P and Q, and since A is on ℍ and B is on 𝕍, they are the ℍ and 𝕍 traces of the line of intersection. AB_h and BA_v are therefore the projections, and should be marked PQ_h and PQ_v.

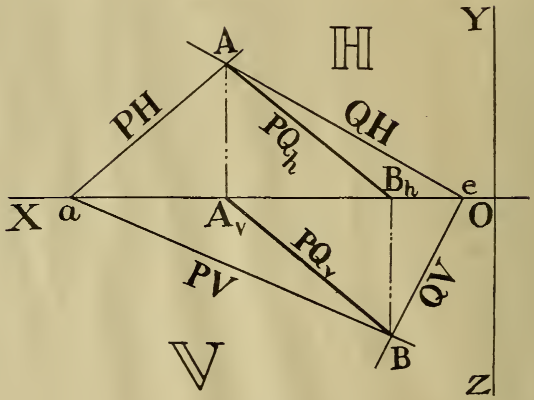


FIG. 103.

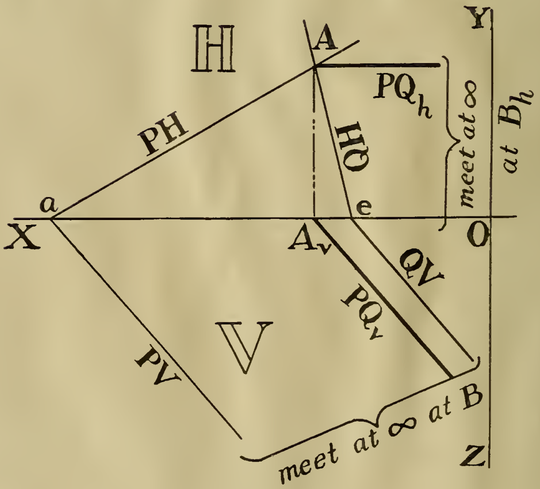


FIG. 104.

107. Special Case of the Intersection of Two Planes: Two Traces Parallel.—The construction must be varied a little in the special case when two of the traces of the planes are parallel. In Fig. 104 the traces *PV* and *QV* are parallel. In carrying out the construction as in Fig. 100, it is necessary to join A_v with *B*. But the point *B* is the intersection of *PV* and *QV*, which are parallel, and therefore is a point at an infinite distance in the direction of those lines, as indicated by the bracket on Fig. 104. To join A_v with *B* at infinity means to draw a line through A_v parallel to *PV* and *QV*.

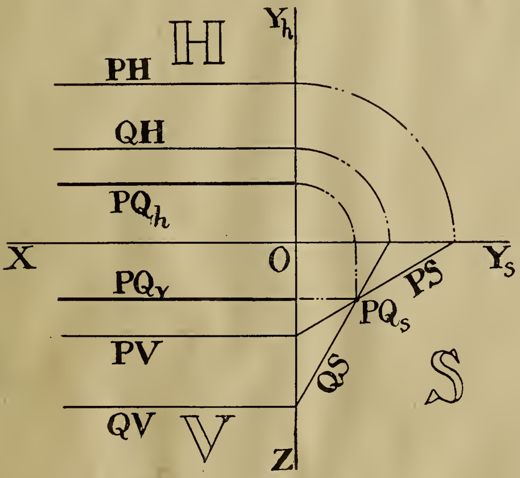


FIG. 105.

From *B*, at infinity, a perpendicular must be supposed to be drawn to the axis of *X*, intersecting it at B_h. B_h is therefore at an infinite distance to the right on the axis of *X* (extended). To join the point *A* with the point B_h means, therefore, to draw a line through *A* parallel to the axis of *X*. These lines are the required projections of *PQ*.

108. Special Case of the Intersection of Two Planes: Four Traces Parallel.—Another special case arises when the four traces (on two planes of projection) are parallel. It is then necessary to refer to a third plane of projection. In Fig. 105 the planes *P* and

Q have their four traces on ℍ and 𝕍 all parallel. The planes are inclined planes perpendicular to 𝕊, and if their traces are drawn on 𝕊, their intresection is the line PQ. In 𝕊 both P and Q are "seen on edge," so their line of intersection is "seen on end." From PQ_s, PQ_v and PQ_h are drawn by projection.

109. The Point of Intersection of a Line and a Plane.—The simple cases of this problem have been previously explained and used. If the plane is horizontal, vertical or inclined, there is

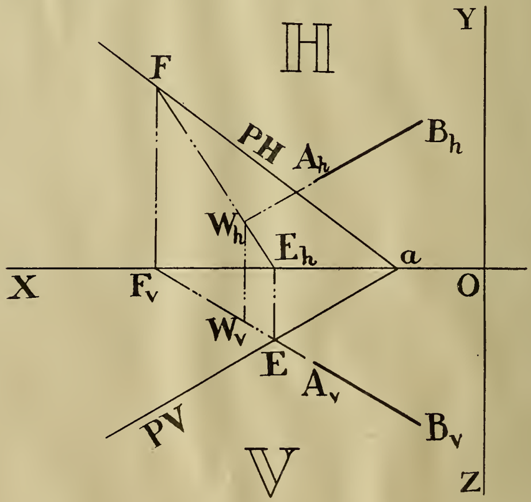


FIG. 106.

always one view at least in which it is seen on edge. In that view the given line is seen to pierce the given plane at a definite point from which, by the rules of projection, the other views of the point of intersection are easily determined. Thus in Fig. 27, Art. 38, the point a, where PA pierces the plane KL, is determined first in 𝕍 and then projected to ℍ and 𝕊.

The general case of this problem may be solved as in Fig. 106. A plane P is given by its traces PH and PV. A line AB is given by its projections. It is required to find where AB pierces P. The solution is as follows: Let a plane perpendicular to 𝕍 be passed through the projection A_vB_v. According to Art. 103 the traces of this plane are B_vF_v and F_vF. Draw the line of intersection of this plane with the plane P (Art. 106) as follows: B_vF_v and PV intersect at E. F and E are the traces of the line of intersection of the two planes. Complete the drawing of the line of intersection in ℍ, as FE_h.

Referring to the horizontal projection, A_hB_h is seen to intersect FE_h, the ℍ projection of the line of intersection, at W_h. Since both FE and AB are lines which lie in the vertical projector-plane through AB, this point of intersection, W_h, is the projection of the true point of intersection, W, of those two lines. From W_h project to W_v for the other projection of W. This point W which lies on P and is on the line AB is the required point.

Problems XI.

(For blackboard or cross-section paper or wire-mesh cage.)

110. Plot the point A (4, 7, 9). Pass a horizontal plane P through the point A, and draw the traces of P. Pass a vertical plane Q, parallel to 𝕍, and draw its traces. Pass an inclined plane R, perpendicular to ℍ, making an angle of 45° with OX.

111. Plot the line A (8, 2, 4), B (2, 6, 16). Pass an inclined plane P perpendicular to ℍ through this line and draw the traces of P. At C, the middle point of AB, pass a plane Q perpendicular to P and to ℍ, and draw QH and QV.

112. The plane P cuts the axes at the points a (10, 0, 0), b (0, 5, 0) and c (0, 0, 15). Pass a plane Q parallel to P, through the point a' (6, 0, 0).

113. A plane P has its trace on ℍ through the points A (12, 12, 0) and b (0, 6, 0). Its trace on 𝕍 passes through the point c (0, 0, 12). Draw the three traces. Draw three traces of a plane Q, parallel to P through the point c' (3, 0, 0).

114. An indefinite line contains the points A (11, 2, 6) and B (5, 6, 0). Pass a plane P perpendicular to ℍ containing this line and draw the traces PH, PV and PS. Pass a plane Q containing this line and the point a' (2, 0, 0). Draw the traces QH and QV. Draw the negative trace QS on 𝕊 extended over ℍ.

115. A plane P cuts the axis of X at a $(4, 0, 0)$, the axis of Y at b $(0, 6, 0)$, and the axis of Z at c $(0, 0, -12)$. Draw its traces. Draw the $\mathbb{V}$ and $\mathbb{S}$ traces of a plane Q parallel to P and containing the line A $(1, 4, 11)$, B $(4, 1, 14)$.

116. An inclined plane, perpendicular to $\mathbb{H}$, has for its $\mathbb{V}$ and $\mathbb{S}$ traces lines parallel to OZ at positive distances of 15 and 5 units. An inclined plane Q perpendicular to $\mathbb{H}$ has its $\mathbb{V}$ and $\mathbb{S}$ traces parallel to OZ at distances of 12 units and 8 units. Draw all three traces and the projection of PQ, their line of intersection.

117. Draw the traces of a plane P, containing the points A $(8, 1, 3)$, B $(4, 5, 1)$ and C $(2, 4, 3)$. Does the point D $(4, 1, 5)$ lie on this plane?

118. The traces of a plane P are lines through the points a $(10, 0, 0)$, b $(0, 15, 0)$ and E $(14, 0, 6)$. A plane Q has its traces through the points a' $(2, 0, 0)$, E, and F $(7, 5, 0)$. Draw the projections of their line of intersection, PQ.

119. The plane P cuts the axes at a $(12, 0, 0)$, b $(0, 12, 0)$ and c $(0, 0, 12)$. Where does the line E $(1, 5, 12)$, F $(5, 3, 6)$ pierce the plane?

CHAPTER XII.

VARIOUS APPLICATIONS.

110. Traces of an Inclined Plane Perpendicular to an Oblique Plane.—One of the most general devices used in the drafting room is the auxiliary plane of projection, and it is often advantageous to pass this plane perpendicular to some plane of the drawing in

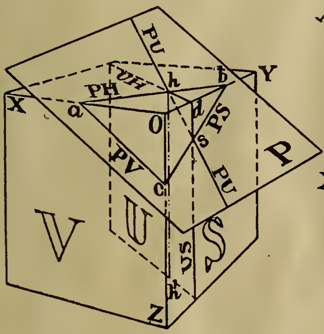


FIG. 107.

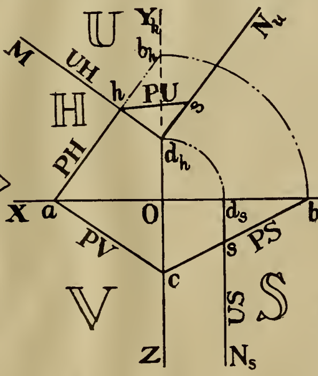


FIG. 108.

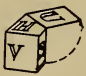


FIG. 109.

order to get the advantage of showing that plane "on edge." Thus in Fig. 31, Art. 42, the plane U has been taken perpendicular to the long rectangular faces of the triangular prism, in order to show clearly where *BB′* and *DD′* pierce those planes. The manner of passing the plane U was fairly clear in that case from the simplicity of the figure. However, as it is not always clear how to pass a plane perpendicular to an oblique plane, the general method may well be explained here. In Fig. 107 the plane *P*, previously shown in Fig. 94, is represented, and an auxiliary plane U, perpendicular to it and to H, is shown. The traces of *P* are *PH, PV* and *PS* as before, and the traces of U are *UH* and *US*. It must be understood that the $\mathbb{H}$ traces of these planes, *PH* and *UH*, are *perpendicular to each other*, as this condition is essential if *P* and $\mathbb{U}$ are to be planes perpendicular to each other.

Fig. 108 is the descriptive drawing corresponding to the perspective drawing, Fig. 107. At some point *h* on *PH* a line Md_h has been drawn perpendicular to *PH*. This line is the inclined trace of a plane $\mathbb{U}$ perpendicular to $\mathbb{H}$. The other traces of $\mathbb{U}$ are parallel to the axis of *Z* (Art. 98). One of these, the trace on $\mathbb{S}$, is shown by the line d_sN_s, parallel to *OZ*, d_h and d_s being two representations of the same point *d* in Fig. 107, just as b_h and b_s represent the point *b*, duplicated. Md_h may be called *UH* and d_sN_s may be called *US*. *UH* and *US* are the traces of an inclined plane $\mathbb{U}$, perpendicular to the oblique plane *P*.

The proof that *P* and $\mathbb{U}$ are perpendicular to each other is as follows: If, in Fig. 107, a line *hh′* is drawn perpendicular to $\mathbb{H}$ at the point *h*, it will lie in the plane $\mathbb{U}$. The angle *ahh′* will then be an angle of 90°, and by construction the angle *ahd* is also 90°. Thus the line *ah* is perpendicular to two intersecting lines described in the plane $\mathbb{U}$ and is therefore perpendicular to $\mathbb{U}$ itself. The plane *P* contains the line *PH* and is thus perpendicular to $\mathbb{U}$.

111. An Auxiliary Plane of Projection Perpendicular to an Oblique Plane.—To utilize the inclined plane $\mathbb{U}$ as an auxiliary plane of projection, its developed position must be shown by drawing d_hN_u perpendicular to *UH*. This line is the duplicate position of d_sN_s or *US*. In developing the planes, $\mathbb{U}$ is first revolved on *UH* as an axis into the plane of $\mathbb{H}$ as shown in Fig. 109, and then with $\mathbb{H}$ into the plane of the paper, $\mathbb{V}$. The trace of *P* on $\mathbb{U}$, or *PU*, is the line of intersection of the planes, and is shown clearly in Fig. 107. This line passes through *h* where *PH* and *UH* meet, and through *s* where *PS* and *US* meet. In Fig. 108, d_hs is laid off on d_hN_u, equal to d_ss, and the line *hs* is the required trace of *P* on $\mathbb{U}$, or *PU*. The actual line *PU*, in Fig. 108, is only that part of *PU*, in Fig. 107, which is between *h* and *s*, shown as a broken line.

The important part in this process is that $\mathbb{U}$ is taken perpendicular to *P*, so that *P* is "seen on edge" on $\mathbb{U}$. By this process the plane *P*, which is *oblique* when $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$ are considered, becomes an *inclined* plane when only 𝕳 and 𝖀 are considered. As it is easier to deal with inclined than with oblique planes, we

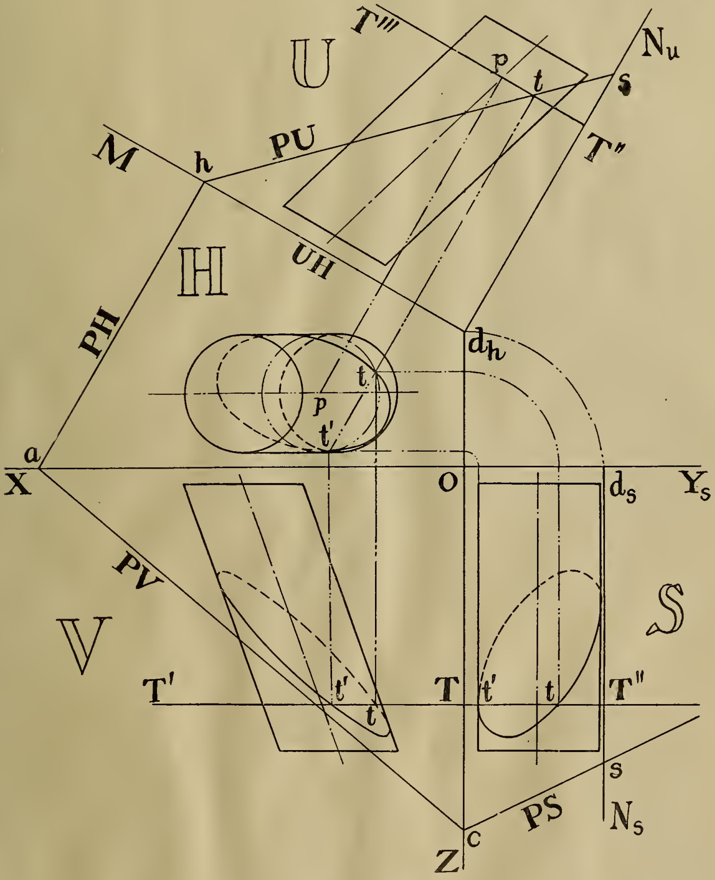


FIG. 110.

may now treat *P* as inclined to 𝕳 and perpendicular to 𝖀 in further operations.

Fig. 108 is well adapted to making a paper box diagram which, when folded, will give most of the lines of Fig. 107. To reconstruct Fig. 108, plot the points a (18, 0, 0), b (0, 18, 0), c (0, 12, 0), d (0, 6, 0), h (6, 12, 0) and s (0, 6, 8). The line d_hN_u is at an angle of 45° with ZOY_h and the construction space $Y_sOd_hN_u$ can be folded away inside by creasing or cutting it on several lines.

112. Intersection of an Oblique Plane and a Cylinder.—An example of the use of an auxiliary view on which an oblique plane is seen on edge is shown in Fig. 110. An inclined cylinder is intersected by an oblique plane P given by its traces PH, PV and PS. It is required to describe on the cylinder the curve of intersection of the plane and the cylinder. The solution is as follows: An auxiliary plane $\mathbb{U}$, perpendicular to P and to $\mathbb{H}$, is chosen, and PU is drawn upon $\mathbb{U}$ as in Fig. 108. PU is the view of P "seen on edge" in $\mathbb{U}$. Auxiliary cutting planes parallel to $\mathbb{H}$ are used for the determination of the required line of intersection. The traces of one of the planes are drawn, as $T'T$ in $\mathbb{V}$, TT'' in $\mathbb{S}$, and $T''T'''$ in $\mathbb{U}$. This latter trace is parallel to d_hM (or UH), because T is parallel to $\mathbb{H}$, and the distance d_hT'' is equal to d_sT'' in $\mathbb{S}$. $T''T'''$ cuts the axis of the cylinder at p. p is projected to $\mathbb{H}$, and the circular element described in $\mathbb{H}$, with p as a center, is the intersection of the auxiliary plane T and the cylinder. In $\mathbb{U}$ the planes P and T are both seen "on edge," intersecting in a line seen on end. This point projected to $\mathbb{H}$ gives this line of intersection of P and T as tt'.

The intersections of the intersections are therefore the points t and t', where the circle and the straight line meet.

113. The Angle between Two Oblique Lines.—This problem of finding the angle between two oblique lines is shown in Fig. 111. Let two lines AB and AC, meeting at A, be given by their $\mathbb{H}$ and $\mathbb{V}$ projections. It is required to find the true angle between them.

By the process of Art. 105, Fig. 102, the traces of the plane containing AB and AC are found and the lines are all lettered according to Fig. 102.

An auxiliary plane of projection, $\mathbb{U}$, is passed perpendicular to PV, and therefore perpendicular to both P and $\mathbb{V}$, and is revolved into the plane $\mathbb{V}$. The projections of AB and AC on this plane fall in the single line $A_uC_uB_u$, since P, the plane of the lines, is "seen on edge" on 𝕌. A portion of the plane P is now revolved about the 𝕌 projector of the point A into a position parallel to XM. In 𝕌, C_u moves to C'_u and B_u to B'_u, revolving about A as their center. In 𝕍, B_v moves to B'_v and C_v to C'_v, both parallel to XM. This is the process of finding the true length of a line by

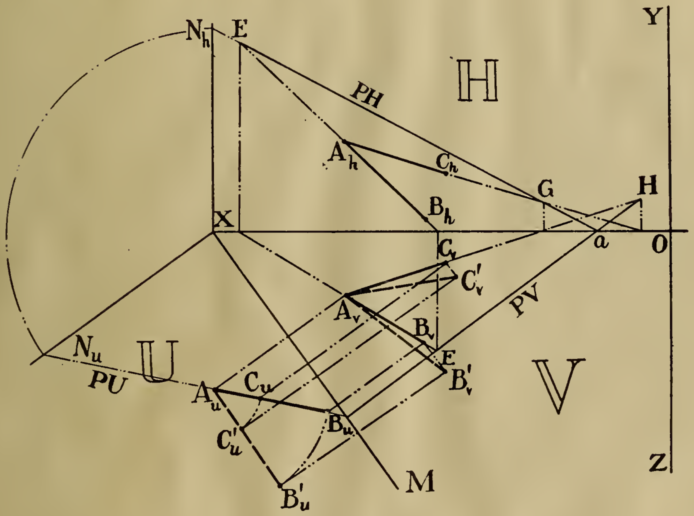


FIG. 111.

revolving about a projector, as in Art. 32. $A_vB'_v$ is the true length of AB; $A_vC'_v$ is the true length of AC; and $B'_vA_vC'_v$ is the true angle between the lines.

This process makes it possible to find the *true shape* of any figure described on an oblique plane.

114. A Plane Perpendicular to an Inclined Line.—It is often advantageous to pass a plane perpendicular to a line in order to use the plane as a plane of projection, on which the given line will be seen on end as a point. The method of passing a plane perpendicular to an inclined line is shown in Fig. 112. Let AB be an inclined line, lying in a plane parallel to $\mathbb{V}$, so that A_hB_h is parallel to the axis of X. It is required to find the traces of a plane P perpendicular to AB. The essential point is that the traces of the plane must be perpendicular to the corresponding projections of

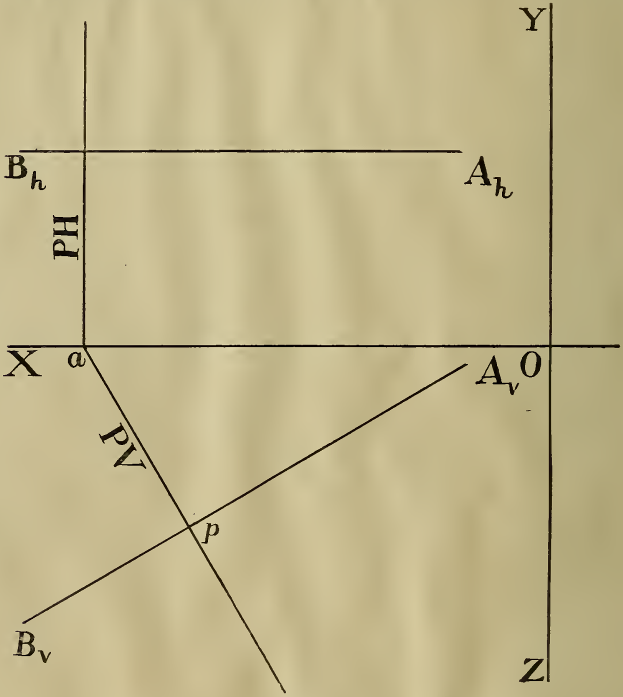


FIG. 112.

the line. Thus, choose some point p on the inclined projection of the line, in this case on A_vB_v, and through p draw a perpendicular to A_vB_v, to serve as the trace of P. At a, where this trace PV meets the axis of X, erect a perpendicular to PH. These lines PV and PH are the traces of an inclined plane perpendicular to AB and to $\mathbb{V}$. It is noticeable that the inclined trace of the plane is on that reference plane which shows the inclined projection of the line.*

115. Application of a Plane Perpendicular to a Line.—In Fig. 113 an application of an inclined plane perpendicular to an inclined line is made for the purpose of finding the line of intersection between an inclined cone and an inclined cylinder whose axes do not meet.

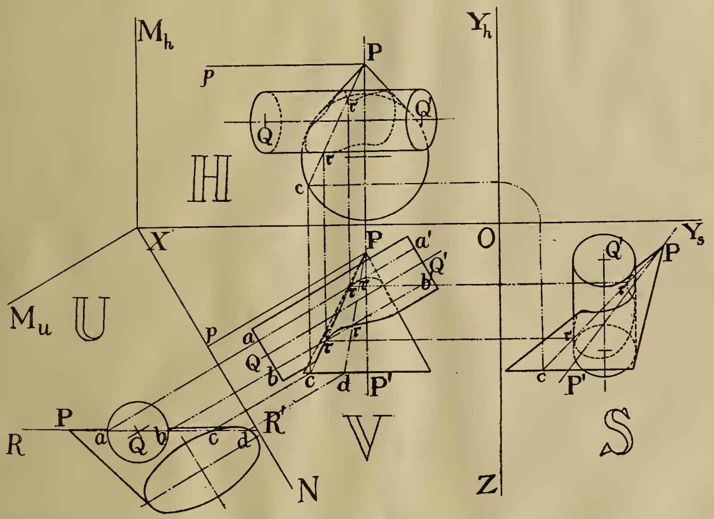


FIG. 113.

If from P, the vertex of the cone, a line Pp is drawn parallel to QQ', as shown, any plane which contains this line and cuts both

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* A proof that P is perpendicular to AB is as follows: AB is the line of intersection of its own ℍ projector-plane, and its own 𝕍 projector-plane. P is perpendicular to both these projector-planes. For, P is perpendicular to 𝕍 and therefore to the ℍ projector-plane, which is parallel to 𝕍; the 𝕍 projector-plane is perpendicular to 𝕍, so that it is seen on edge on 𝕍 just as is P itself; apA_v is therefore the true angle between these two planes, and by construction is a right angle. P is therefore perpendicular to both projector-planes and therefore to the line AB, which is their line of intersection.

surfaces will cut only simple elements of the surfaces. For such a plane contains the vertex of the cone, and therefore, if it cuts the cone, will cut it in *straight* elements; and such a plane is parallel to QQ', and therefore, if it cuts the cylinder, cuts only straight elements. No other planes can be found which cut simple elements and can be used to determine the line of intersection.

If a plane 𝕌 is passed perpendicular to Pp at any point p, and is used as an auxiliary plane of projection, Pp will be seen on end as the point P, and any plane R through P seen on edge in 𝕌, as RR', will cut only straight elements on the two curved surfaces. The complete projections of the cone and cylinder have been shown on 𝕌, and the plane R cuts the bases at a, b, c and d. These points projected to 𝕍 enable the elements to be drawn there, and the intersections of the intersections are the four points marked r. From 𝕍 these points are projected to ℍ and 𝕊. Two of these points r have been projected to the other views to show the necessary construction lines.

116. A Plane Perpendicular to an Oblique Line.—To pass a plane perpendicular to an oblique line, it is only necessary to draw the traces of the plane perpendicular to the corresponding projections of the line. In Fig. 114, let AB be an oblique line. At any point on A_hB_h draw a perpendicular line PH. From a, where PH meets the axis of X, draw PV perpendicular to AB.*

A paper box diagram traced from Fig. 114, or constructed on coordinate paper, using the coordinates A (10, 4, 4) and B (6, 8, 2), C (2, 12, 0) and D (14, 0, 6), and a (8, 0, 0), will assist materially in understanding the problem.

The oblique plane P is not serviceable as an auxiliary plane of projection.

117. The Application of Axes of Projection to Mechanical Drawings.—Descriptive Geometry is a geometrical science, the science dealing primarily with orthographic projection, while Mechanical Drawing is the art of applying these principles to the

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* The proof of this construction is more difficult than in the corresponding case of an inclined line, but it depends as before on the line AB being the intersection of its ℍ and 𝕍 projector-planes, and these planes themselves being perpendicular to P.

needs of engineers and mechanics in the pursuit of industries. Mechanical Drawing includes therefore many abbreviations and conventional representations, which seek to curtail unnecessary work and often to convey information as to methods of manufacture and other such commercial considerations foreign to the strict scientific study.

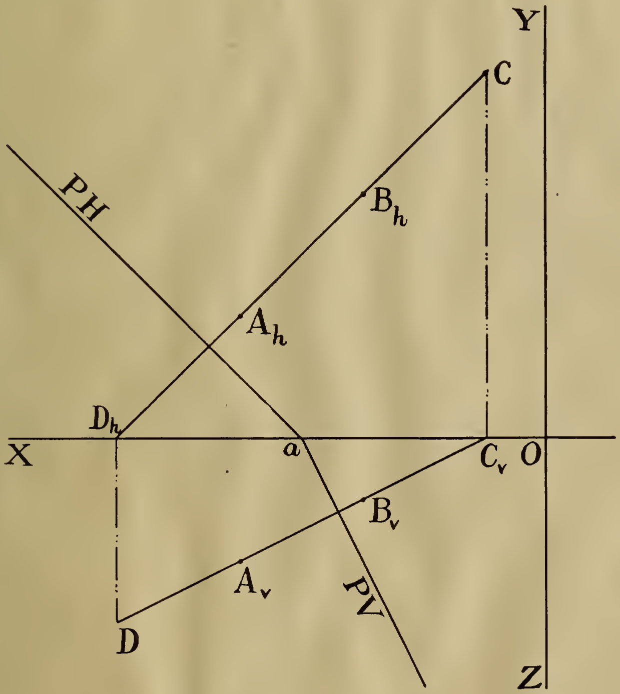


FIG. 114.

In Mechanical Drawing many lines necessary to the strict execution of a descriptive drawing are omitted as unnecessary to the application of the principles, when once the principles have been fully grasped. A noteworthy omission is the axes of projection, which, though absent, still govern the rules for making the drawing. Instead of measuring distances from the axes for every point on the drawing, the "center lines" of the different views (which really represent central planes) are laid off and distances from these center lines are thereafter used. This is the regular procedure in drawing-room practice. That this difference is purely one of omission is clear from the fact that axes of projection may always be inserted in a mechanical drawing. If two views only of a piece are presented, any line between them (perpendicular to the lines of projection from one view to another) may be selected as the axis of X, and any convenient point on that line as the origin of coordinates.

If three views are given, as, for example, Fig. 32, Art. 44, supposing the axes to be there omitted, a ground line XOY_s may be selected at will, dividing the fields of ℍ and 𝕍. The other line must be determined as follows: By the dividers take the vertical distance from OX to the center line mn, and lay off this distance horizontally to the left from the center line of the side elevation. The line ZOY_h may be drawn. All y coordinates of points will now check correctly, measured parallel to the two axes of Y, if the original drawing itself is accurate.

It is thus evident that in applying Descriptive Geometry to practical mechanical drawing we may fall back upon the use of axes of projection whenever the lack of them is felt.

118. Practical Application of Descriptive Geometry.—Many draftsmen have picked up a knowledge of Descriptive Geometry without direct study of the science. This is largely due to the fact that, till very recently, all books on Descriptive Geometry were based on a system of planes of projection which are analogous to the methods of practical drawing in use on the continent of Europe, but which are little used in England, and hardly at all in the United States of America. It will be found, however, that in American drafting rooms all the usual devices of draftsmen are applications, sometimes almost unconscious applications, of the principles covered in the preceding chapters. The favorite device is the application of an inclined auxiliary plane of projection, suitably chosen; next in importance is the rotation of the object to show some true shape; while other applications are used less frequently. The methods of determining lines of intersection of planes and curved surfaces are exactly those described in Chapters IV, VII and VIII.

Problems XII.

(For use on blackboard, with cross-section paper or wire-mesh cage.)

120. The plane P has its traces through the points a (14, 0, 0), b (0, 14, 0) and c (0, 0, 7). Pass a plane Q, perpendicular to P and to $\mathbb{H}$, through the point A (5, 7, 0). If Q is to be used as an auxiliary plane of projection, draw the trace of P on Q when Q has been revolved into coincidence with $\mathbb{H}$.

121. Draw the traces of a plane P cutting the axes at the points a (12, 0, 0), b (0, 8, 0) and c (0, 0, 12). Draw the traces of an auxiliary plane, $\mathbb{U}$, perpendicular to PH at the point A (3, 6, 0). Is the point B $(6, 1, 4\frac{1}{2})$ on the plane P?

122. The $\mathbb{H}$ trace of a plane P passes through the points A (12, 5, 0) and B (6, 2, 0). Its $\mathbb{V}$ trace passes through C (9, 0, 6). Pass an inclined plane perpendicular to $\mathbb{H}$ and perpendicular to P, through the point D (5, 9, 7).

123. Of a plane P, HP, the horizontal trace, passes through the points A (5, 3, 0) and B (13, 9, 0), and VP passes through C (12, 0, 11). Complete the traces of P and draw the traces of a plane perpendicular to VP at the point D (8, 0, 8). Prove that the line E (9, 6, 1), F (6, 3, 2) lies on the plane P.

124. A sphere of radius 7 units has its center at C (8, 8, 8). A plane P cuts the axes of projection at a (26, 0, 0), b (0, 13, 0) and c (0, 0, 13). Pass an auxiliary plane of projection $\mathbb{U}$, perpendicular to $\mathbb{H}$ and to P, cutting the axis of X at d (16, 0, 0). Draw the trace of P on $\mathbb{U}$. The circle of intersection of the sphere and the plane P is seen on edge on $\mathbb{U}$. Show the elliptical projection of this circle, on $\mathbb{H}$, by passing auxiliary cutting planes parallel to $\mathbb{U}$. (If this problem is solved by use of wire-mesh cage, the point a is inaccessible, but PH passes through E (16, 5, 0), and PV through F (16, 0, 5). The plane $\mathbb{S}'$ can be turned to serve as $\mathbb{U}$.)

125. Find the true shape of the triangle A (3, 2, 6), B (9, 6, 2), C (8, 0, 0). Find the traces of two of the sides of the triangle and pass the plane $\mathbb{U}$ perpendicular to the plane of the triangle and perpendicular to $\mathbb{H}$, and through the point D (0, 7, 0).

126. Find the true shape of the triangle A (7, 6, 1), B (4, 2, 9), C (10, 2, 3). Find the traces of two of the sides of the triangle and pass the plane $\mathbb{U}$ perpendicular to the plane of the triangle and perpendicular to $\mathbb{H}$, and through the point D (0, 1, 0).

127. Draw the traces of a plane P perpendicular to $\mathbb{V}$ and to the line A (2, 6, 9), B (8, 6, 5) at C (11, 6, 3). If this plane is used as an auxiliary plane of projection, what is the projection of AB on it?

128. Draw the traces of a plane P perpendicular to $\mathbb{H}$ and to the line A (3, 9, 6), B (13, 4, 6), at C (17, 2, 6), a point on AB. (If wire-mesh cage is used for the solution, turn $\mathbb{S}'$ to serve as $\mathbb{U}$ and draw on it the view of A_uB_u.)

129. Draw the three traces of a plane P perpendicular to the oblique line A (8, 12, 5), B (14, 3, 7). Show that all three traces are perpendicular to the corresponding projections of AB.

CHAPTER XIII.

THE ELEMENTS OF ISOMETRIC SKETCHING.

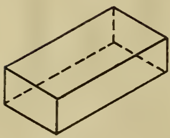
119. Isometric Projection.—There is one special branch of Orthographic Projection which is of peculiar value for representing forms which consist wholly or mainly of plane faces at right angles to each other. Ordinary orthographic views are projections upon planes parallel to the principal plane faces of the object, as shown in Fig. 2, Art. 4. If, however, instead of the regular planes of projection, the object is projected upon a new plane of projection, making the *same angle with each of the regular planes,* an entirely different result is obtained, called an *"isometric projection."* This view has the useful property that it has all the air of a perspective and may, with certain restrictions, be used alone without other views as a full representation of the object.

In Art. 21 the method of converting the perspective drawings of this treatise into isometric sketches was explained in a rough and unscientific way. In this chapter there is explained the method of making isometric sketches from models, as a step to making orthographic drawings or isometric drawings.

120. Isometric Sketches of Rectangular Objects.—Figs. 19 and 19a are the isometric drawings of a cube. Since the line of sight from the eye to the point O makes equal angles with $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$, the three planes must subtend the same angle at O. XOY, YOZ and XOZ are each 120°, though representing angles of 90° on the cube. Since opposite edges of $\mathbb{H}$ are parallel, it follows that each face of the cube is a rhombus and that the cube appears as a regular hexagon, all edges appearing of exactly the same length. This fact is the basis of the name "isometric," meaning "equal-measured."

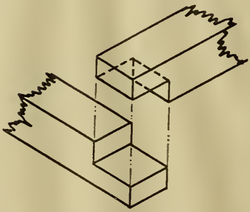
Figs. 115, 116 and 117 are sketches of other objects, all of whose corners are right angles. The angles at these corners appear therefore like those of the cube, either as 60° or 120° on the isometric sketch.

In making the isometric sketch from a model having rectangular faces, the first step is to put the object approximately in the iso-



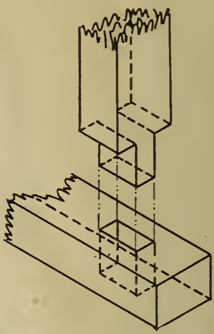
Brick

Fig. 115.



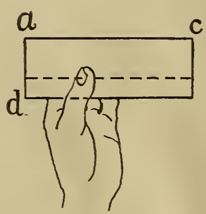
Half Joint

Fig. 116.



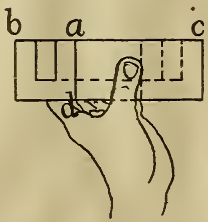
Mortise & Tenon Joint.

Fig. 117.



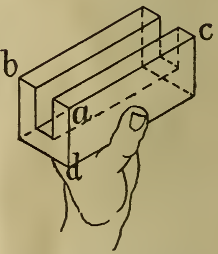
Position for Orthographic Projection.

Fig. 118.



Turned 45° about a vertical axis.

Fig. 119.



Tilted 35°-44' about an horizontal axis.

Fig. 120.

metric position. At any projecting corner imagine a line to project from the corner so as to make equal angles with the three edges which meet at the given corner. View the object by sighting along this imaginary line and begin the sketch from that view.

If there is any difficulty in finding this line of vision directly, the object may be turned horizontally through an angle of 45° and tilted down through an angle of 35° 44′. This operation is the basis of the method of finding the "isometric projection."

Figs. 118, 119 and 120 show the steps in passing from the orthographic position to the isometric position, the model used being a rectangular block with a lengthwise groove cut in one face.

121. Isometric Axes.—It will be noticed in the previous isometric figures that all lines are drawn in one of three general directions. One of these directions is usually taken as vertical and the other two directions make angles of 120° with the vertical. These three directions are known as the *isometric axes.* In this sense the word axis means a direction, not a line.

In plotting points from a selected origin, the x coordinates are plotted up and to the left, the y coordinates up and to the right, and the z coordinates vertically downward, as in Fig. 19a.

122. Isometric Paper.—Paper ruled in the direction of the isometric axes is called isometric paper, and is of great assistance in making isometric sketches. The lines divide the paper into small equilateral triangles.

In sketching, the sides of these equilateral triangles are taken to represent unit distances, exactly or at least approximately. Thus, if the model shown in Fig. 120 is a block $3'' \times 3'' \times 8''$, with a $2'' \times 1''$ groove lengthwise along one face, some point a on the paper is selected, and from it distances are taken along the isometric axes, so that each unit space represents one inch.

From a three units are counted vertically downward, eight up, and to the right, and one unit, followed by a gap in the line of one unit, and then a second unit, up to the left. Thus all lines of the sketch follow the ruled lines as long as the dimensions of the model are in even inches.

An isometric sketch made in this manner, particularly if spaces have been exactly counted off according to the dimensions of the piece, is practically an isometric drawing. If fully dimensioned, a sketch on plain paper proportioned by the eye is nearly as good as one in which spaces are counted exactly. Such sketches serve all purposes, though of course more difficult to make than those on isometric paper.

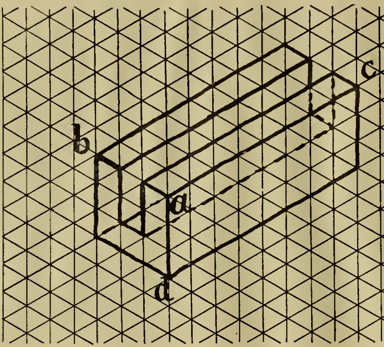


FIG. 121.

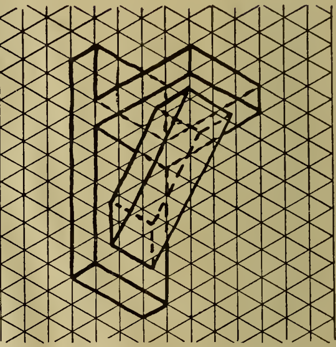


FIG. 122.

123. Non-Isometric Lines in Isometric Sketching. — Objects which have a *few* faces and edges oblique to the principal plane faces may still be shown by isometric sketching. In such cases it is always well to circumscribe a set of rectangular planes about the

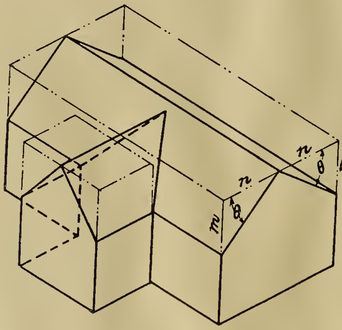


FIG. 123.

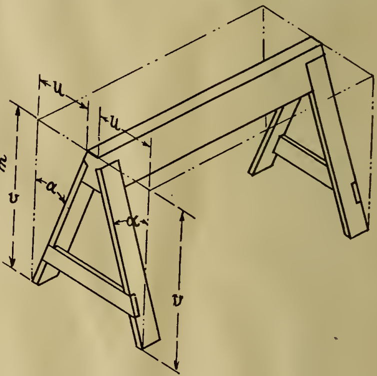


FIG. 124.

oblique parts of the object to aid the imagination. Dimension extension lines should be used for this purpose. In using isometric paper this squaring up is done by the lines of the paper.

Figs. 122, 123 and 124 are good examples of oblique lines and faces. Figs. 123 and 124 show also the circumscribed isometric lines which "square up" the oblique parts.

124. Angles in Isometric Sketching.—In isometric sketching angles do not, as a rule, appear of their true magnitude. Thus the 90° angles on the faces of the brick appear in Fig. 115 as 60° or 120°, but not as 90°. In general, the lengths of oblique or inclined lines depend on position, and are not subject to measurement by scale.

The lines which square up oblique parts are useful in giving the *tangent* of the angle of an oblique surface. Thus in Fig. 124, the angle a differs in reality from the angle as it appears in either place marked, but the *tangent* of a is $\frac{u}{v}$. In Fig. 123, $\theta = \tan^{-1}\frac{m}{n}$. In practice angles are often given by their tangents. Thus the slope of

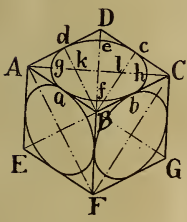


FIG. 125.

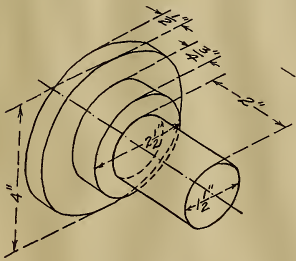


FIG. 126.

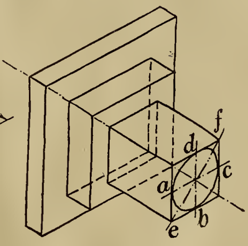


FIG. 127.

a roof is given as "one in two" or the gradient of a railroad as "three per cent."

125. Cylindrical Surfaces in Isometric Sketching.—In orthographic drawings circles appear commonly on planes parallel to the three planes of projection. To illustrate the position and appearance of circles in isometric drawing in the three typical cases, Fig. 125 represents the isometric sketch of a cube, having a circle inscribed in each square face.

Each of the faces of the cube is perpendicular to the isometric axis given by the intersection of the other two faces. Thus the square *ABCD* is perpendicular to the edge *BF*. The circle *abcd,* inscribed in the square *ABCD*, appears as an ellipse, whose minor axis, *ef*, lies on the diagonal *BD* of the square, *BD* appearing as a continuation of the edge *FB*. In all three cases, then, the minor axis of the ellipse lies in the same direction, on the sketch, as that isometric axis to which the plane of the circle is in reality perpendicular.

The major axis is necessarily perpendicular to the minor axis, and lies on the other diagonal of the square.

Since the cylinder is the surface most used in engineering, the rule may be applied to cylinders as follows: The ellipse which represents the circular base of any cylinder must be so sketched that its minor axis is in line with the axis or center line of the cylinder. Fig. 126 is an isometric sketch of a piece composed of cylinders. All the ellipses are seen to follow this rule.

In sketching cylindrical parts of objects, it is necessary to imagine them squared up by the use of isometric lines and planes. Thus the first steps in sketching the piece of Fig. 126 are shown in Fig. 127. The circumscribing of a square about a circle in the object corresponds to circumscribing a rhombus about the ellipse in the isometric sketch. It now remains to inscribe an ellipse in the rhombus. This ellipse must be tangent to the rhombus at the *middle* of each side. To sketch the ellipse, as for example the small end in Fig. 127, draw the diagonals of the rhombus to get the directions of the major and minor axes, and find the middle points of the sides (by center lines, through the intersection of the diagonals). It is now easy to sketch the ellipse, having *four* points given, the *direction* of passing through those points, and the directions of the major and minor axes.

126. Isometric Sketches from Orthographic Sketches.—A good exercise consists in making isometric sketches from orthographic sketches or drawings. The three coordinate directions, x, y and z, must be kept in mind at all times. Fig. 128, as an example, is most instructive. From the orthographic sketches, Fig. 128, the isometric sketch, Fig. 129, is to be made. A point a is selected to represent a point a on the orthographic views. The line *ab* is an x dimension and is plotted up to the left; *ac* is a y dimension, and is plotted up to the right; while *ad* is a z dimension, and is plotted vertically downward. The semicircle is inscribed in a half-rhombus, tangent at *b*, *e* and *f*.

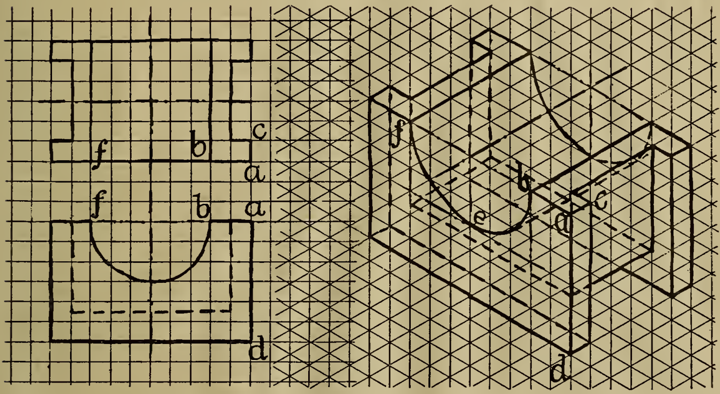


FIG. 128. FIG. 129.

The cross-section lines of Fig. 128 and the isometric lines of Fig. 129 are represented as overlapping between the figures. Some isometric paper is ruled in this manner, so that it may be used for both purposes.

Problems XIII.

(For blackboard or isometric paper.)

130. Make an isometric sketch of the angle piece, Fig. 130, using the spaces for 1″ distances.

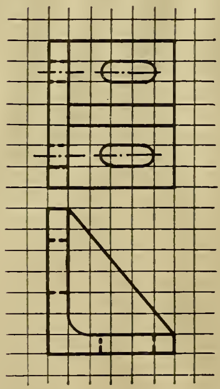


FIG. 130.

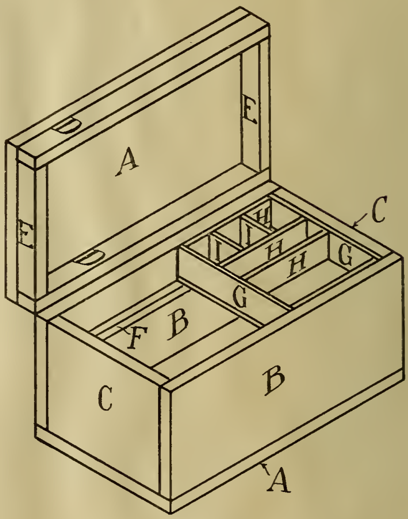


FIG. 131.

131. Measure the tool-chest, Fig. 131, and make a bill of material, tabulating the boards used, and recording their sizes, giving dimensions in the order: width, thickness, length, thus:

Mark.	Name.	Size.	Number.
A.	Top of Chest.	14″ × 1″ × 24″.	2.

132. A parallelopiped, $9'' \times 6'' \times 3''$, has a 3″ square hole from center to center of the largest faces, and a 2″ bore-hole centrally from end to end. Make an isometric sketch.

133. Let Fig. 3, Art. 5, represent a model cut from a 12″ cube by removing the center, leaving the thickness of the walls 3″. Let the angular point form a triangle whose base is 12″ and altitude 8″. Make an isometric sketch.

134. A cube of 10″ has a 6″ square hole piercing it centrally from one side to the other, and a 4″ bore-hole piercing it centrally from side to side at right angles to the larger hole. Make an isometric sketch.

135. A grating is made by nailing slats $\frac{3}{4}'' \times \frac{1}{2}'' \times 12''$, spaced $\frac{1}{2}''$ apart, on three square pieces, $1\frac{1}{4}''$ square, 22″ long, spaced $4\frac{1}{8}''$ apart. Make an isometric sketch.

136. Make orthographic sketches of the bracket, Fig. 122. Views required are plan and front elevation. (On cross-section paper use the unit distance for the unit of the isometric paper. On blackboard let each unit of the isometric paper be represented by a distance of 2″.)

137. Make isometric sketches of Fig. 11, Art. 14, and Fig. 24, Art. 32.

138. Make isometric sketches of Fig. 13, Art. 15, and of Fig. 82, Art. 89. In Fig. 82 let A be the point (9, 8, 0) and B the point (9, 0, 12).

139. Make an isometric sketch of Fig. 71, Art. 84, the diameter of the cylinder being 7 units and the length 14 units.

140. Make an isometric sketch of Fig. 92, Art. 93, using the coordinates given in Art. 94.

CHAPTER XIV.

ISOMETRIC DRAWING AS AN EXACT SYSTEM.

127. The Isometric Projection on an Oblique Auxiliary Plane.— The sketches previously considered have generally had no exact scale. Those drawn on isometric paper have a certain scale according to the distance which one unit space of the paper actually represents.

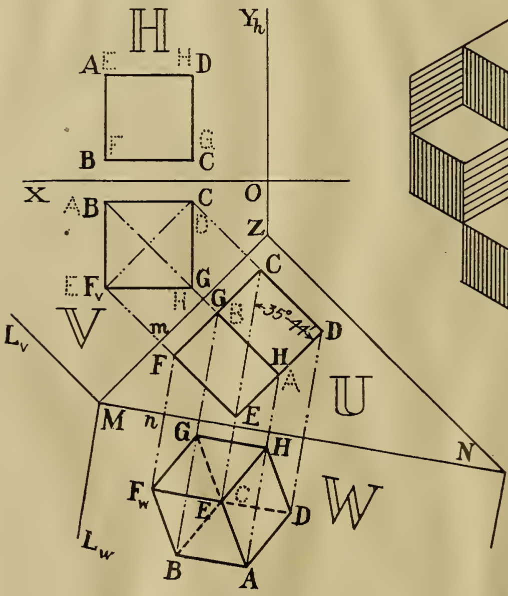


Fig. 132.

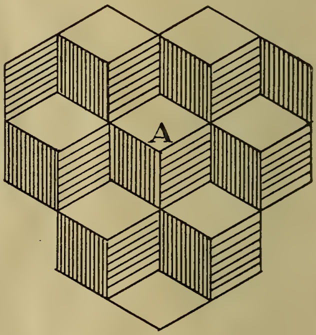


Fig. 133.

If the isometric projection is derived from an orthographic drawing of the usual kind by the laws of projection, the isometric projection so formed has of course the same scale as the original drawing.

In Fig. 132 an isometric projection of a cube is derived from the orthographic drawing by the use of an inclined plane of projection, 𝕌, and an oblique auxiliary plane of projection 𝕎. The aim is to produce the projection on a plane making the same angle with all three edges of the cube meeting at any one corner. This plane must be perpendicular to a diagonal of the cube. In Fig. 132 this diagonal is the line *EC*, a true diagonal, passing through the center of the cube, not a diagonal of one face of the cube.

The first, or inclined, auxiliary plane 𝕌 is taken parallel to the 𝕍 projection of *EC*, and therefore perpendicular to 𝕍 and making an angle of 45° with ℍ and 𝕊. The projection of *EC* on 𝕌 shows its true length.

The second, or oblique, auxiliary plane 𝕎 is taken perpendicular to *EC*. It is oblique as regards ℍ and 𝕍, but, as *EC* is a line parallel to 𝕌, and 𝕎 is perpendicular to *EC*, 𝕎 is perpendicular to 𝕌. As regards 𝕍 and 𝕌, 𝕎 is an inclined plane, having its inclined trace *MN* on 𝕌, the trace on 𝕍 being a line ML_v, perpendicular to *ZM*, the trace of 𝕌 on 𝕍. The construction of this second projection is therefore according to the usual methods. Any point, as *F*, is projected by a perpendicular line across the trace *MN* and the distance nF_w is laid off equal to mF_v.

The projection on 𝕎 is the isometric projection of the cube and is full-size if the plan and front elevation are full-size projections. The edges are all foreshortened, however, and measure only $\frac{83}{100}$ of their true length.

128. The Angles of the Auxiliary Planes.—The plane 𝕌 makes an angle of 45° with the plane ℍ. The plane 𝕎 makes an angle of 35° 44′ with 𝕊, or (90° − 35° 44′) with 𝕍. If the side of the cube is taken as 1, the length of the diagonal of the face of the cube is $\sqrt{2}$, and the length of the diagonal of the cube is $\sqrt{3}$. The first angle is that angle whose tangent is $\frac{1}{1}$, or whose sine is $\frac{1}{\sqrt{2}}$. The second angle is that angle whose tangent is $\frac{1}{\sqrt{2}}$ and whose sine is $\frac{\sqrt{2}}{\sqrt{3}}$.

129. The Isometric Projection by Rotating the Object.—In Fig. 134 is shown a method of deriving the isometric projection by turning the object. The plan, front, and side elevations are drawn with the object turned through an angle of 45° from the natural position (that in which the faces of the cube are all parallel to the reference planes). The side elevation shows the true length of one diagonal of the cube, AG. Some point on AG extended, as K, is taken as a pivot, and the whole object is tilted down through an angle of 35° 44′, bringing AG into a horizontal position, $A'G'$. The

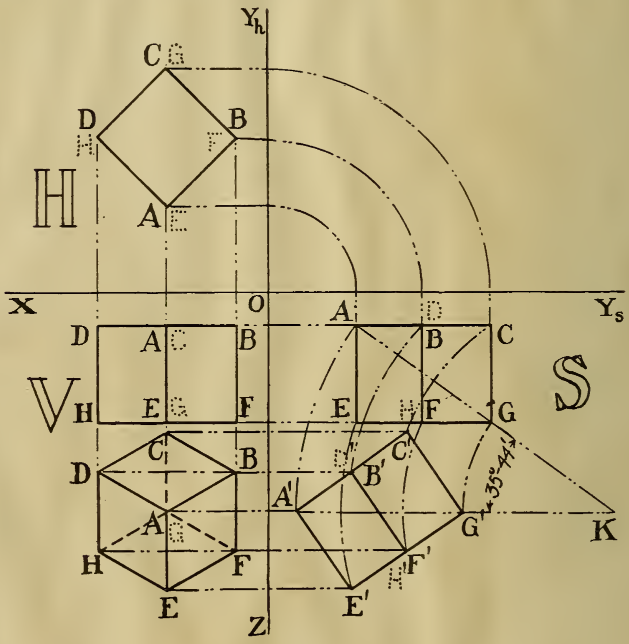


FIG. 134.

new projection of the object in 𝕍 is the isometric projection. This process of turning the object corresponds to the turning of the object in isometric sketching, as shown in Figs. 118, 119 and 120.

The isometric projection of the cube has all eight edges of the *same* length, but foreshortened from the *true* length in the ratio of $\sqrt{3}$ to $\sqrt{2}$.

Any object of a rectangular nature may be treated by either process to obtain the isometric projection.

130. The Isometric Drawing.—To make a practical system of drawing capable of representing rectangular objects in an unmistakable manner in one view, the fact that all edges are foreshortened alike is seized upon, but the disagreeable ratio of foreshortening is obviated by ignoring foreshortening altogether.

An *isometric drawing* is one constructed as follows: On three lines of direction, called *isometric axes,* making angles of 120° with each other, the true lengths of the edges of the object are laid off. These lengths, however, are only those which are mutually at right angles on the object. All other lines are altered in shape or length. An isometric drawing is distinct from an isometric projection, as it is larger in the proportion of 100 to 83 ($\sqrt{3}:\sqrt{2}$). The isometric drawing of a 1″ cube is a hexagon measuring 1″ on each edge.

131. Requirement of Perpendicular Faces.—An isometric drawing, being a single view, cannot really give "depth," or tell exactly the relative distances of different points of the object from the eye. It absolutely requires that the object drawn shall have its most prominent faces, at least, mutually perpendicular. The mind must be able to assume that the object represented is of this kind, or the drawing will not be "read" correctly. Even on this assumption, in some cases isometric drawing of rectangular objects may be misunderstood if some projecting angle is taken as a reentrant one. Thus in Fig. 133 we have a drawing which might be taken as the pattern of inlaid paving or other flat object. If it is taken as an isometric drawing and the various faces are *assumed* to be perpendicular to each other, it becomes the drawing of a set of cubes. Curiously enough, it can be taken to represent either 6 or 7 cubes, according as the point *A* is taken as a raised point or as a depressed one. In other words, it even requires one to know just *how* the faces are perpendicular to each other to be able to take the drawing in the way intended.

This requirement of perpendicular faces limits the system of drawing to one class of objects, but for that class it is a very easy, direct, and readily understood method. Untrained mechanics can follow isometric drawings more easily than orthographic drawings.

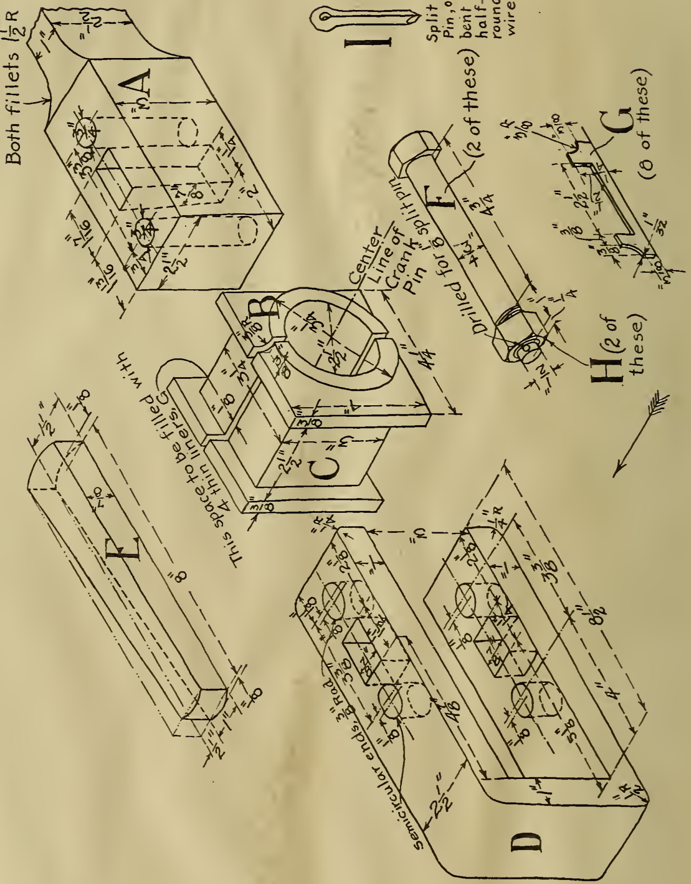

Both fillets $1\frac{1}{2}$" R
A
B
C
D
E
F
(2 of these)
Drilled for $\frac{1}{8}$" split pin
Center Line of Crank Pin
G
(8 of these)
H (2 of these)
I
Split Pin, of bent half-round wire.
This space to be filled with 4 thin liners, G
Semicircular ends, $\frac{3}{8}$" Rad.

Fig. 135.

132. The Representation of the Circle.—In executing isometric drawings, the circle, projected as an ellipse, is the one drawback to the system. To minimize the labor, an approximate ellipse must be substituted for an exact one, even at the expense of displeasing a critical eye. The system, if used, is used for *practical* purposes where beauty must be sacrificed to speed. In Fig. 125 the rhombus *ABCD* is the typical rhombus in which the ellipse must be inscribed. The exact method is shown in Fig. 43, but requires too much time for constant use. The following draftsman's ellipse, devised to be exactly tangent to the rhombus at the middle point of each side, is reasonably accurate. From *B*, one extremity of the short diagonal of the rhombus, drop perpendiculars *Bd* and *Bc* upon opposite sides, cutting the long diagonal at *k* and *l*. With *B* as a center and *Bd* as a radius, describe the arc *dc*. Similarly, with *D* as a center, describe the arc *ba*. With *k* and *l* as centers, and *kd* as a radius, describe the arcs *ad* and *cb*. The resulting oval has the correct major axis within one-eighth of 1 per cent, and has the correct minor axis within $3\frac{1}{2}$ per cent.

This draftsman's ellipse is exact where required, namely, on the two diameters *ac* and *db*, which are isometric axes, and it is practically exact at the extremity of the major axis.

133. Set of Isometric Sketches.—Fig. 135 is a set of isometric sketches of the details of the strap end of a small connecting-rod, from which to make orthographic drawings. The isometric sketch is much clearer than the corresponding orthographic sketch, and the set shows clearly how the pieces are assembled.

The orthographic drawing of the *assembled* rod end is much easier to make than the assembled isometric drawing. It is in fact clearer for the mechanic than the *assembled isometric* drawing would be, for the number of lines would in that case be quite confusing. It illustrates well the fact that isometric sketches and drawings should be limited to fairly simple objects.

Another noteworthy fact is that center lines, which should always mark symmetrical parts in orthographic drawings, should be used in isometric drawing only when *measurements are recorded from them.*

The sketch as given is taken directly from an examination paper used at the U S. Naval Academy for a two-hour examination. On account of the shortness of the period, only one orthographic view, the front elevation, is required, but if time were not limited, a plan also should be drawn.

The following explanation of the sheet is printed on the original:

"*Explanation of Mechanism.*—The isometric sketches represent the parts of the strap end of a connecting-rod for a small engine. In assembling, *A, B, C,* and *D* are pushed together, with the thin metal liners, *G,* filling the space between *B* and *C*. The tapered key, *E,* is driven in the $\frac{7}{8}''$ holes of *A* and *D,* which will be found to be in line, except for a displacement of $\frac{1}{8}''$ which prevents the key from being driven down flush with the top of the strap *D*. The two bolts, *F,* are inserted in their holes, nuts *H* screwed on, and split pins (which are not drawn) inserted in the $\frac{1}{8}''$ holes, locking the nuts in place. In time the bore of the brasses *B* and *C* wears to oval form. To restore to circular form, one or two liners would be removed and the strap replaced. The key driven in would then draw the parts closer by the thickness of the liners removed.

"*Drawing (to be Orthographic, not Isometric).*—On a sheet $14'' \times 11''$ make in ink a working drawing of the front elevation of the rod end assembled, viewed in the direction of the arrow. Put paper with long dimension horizontal. Put center of bore of brasses 4″ from left edge of paper and 5″ from top edge. No sketch, no legend, no dimensions."

Problems XIV.

140. An ordinary brick measures $8'' \times 4'' \times 2\frac{1}{2}''$. Make an orthographic drawing and an isometric *projection* after the manner of Fig. 132, Art. 125. Contrast it with the isometric *drawing,* Art. 128.

141. Make the isometric projection of the brick, $8'' \times 4'' \times 2\frac{1}{2}''$, turning it through the angles of 45° and 35° 44′, as in Fig. 133, Art. 127.

142. From Fig. 135 make a plan and front elevation of the strap *D*.

143. From Fig. 135 make a plan and front elevation of the stub end *A*.

144. From Fig. 135 make a plan and front elevation of the brass *C*.

SET OF DESCRIPTIVE DRAWINGS.

The following four drawing sheets are designed to be executed in the drawing room to illustrate those principles of Descriptive Geometry which have the most frequent application in Mechanical or Engineering Drawing.

The paper used should be about $28'' \times 22''$, the drawing-board of the same size, and the blade of the T-square $30''$.

To lay out the sheets find the center, approximately, draw center lines, and draw three concentric rectangles, measuring $24'' \times 18''$, $22'' \times 16''$, and $21'' \times 15''$. The outer rectangle is the cutting line to which the sheets are to be trimmed. The second one is to be inked for the border line. The inner one is described in pencil only as a "working line," or line outside of which no part of the actual figures should extend. The center lines and other fine lines, including dimensions, may extend beyond the working line. In the lower right corner reserve a rectangle $6'' \times 3''$, touching the working lines, for the legend of the drawing.

In making the drawings three widths of line are used.

The actual *lines of the figures* must be "standard lines" or lines not quite one-hundredth of an inch thick. The thin metal erasing shield may be used as a gauge for setting the right-line pen, by so adjusting the pen that the shield will slowly slip from between the nibs, when inserted and allowed to hang vertically. *Visible edges* are full lines. *Hidden edges* are broken lines; the dashes $\frac{1}{8}''$ long and spaces $\frac{1}{32}''$ long.

The *extra-fine lines* are described with the pen adjusted to as fine a line as it will carry continuously. The *axes of projection* are fine full lines. The *dimension lines* are long dashes, $\frac{1}{2}''$ to $1''$ long, with $\frac{1}{8}''$ spaces. The *center lines* are long dashes with fine dots between the dashes, or are dash-dot lines. The *construction lines* are long dashes with two dots between, or are dash-dot-dot lines. When auxiliary cutting planes are used, one only, together with its corresponding projection lines, should be inked in this manner.

The *extra-heavy lines* are about two-hundredths of an inch thick, and are for two purposes: for *shade lines,* if used; and for *paths of sections,* or lines showing where sections have been taken, as *np,* Fig. 32. These paths of sections should be formed of dashes about $\frac{1}{2}''$ long.

SHEET I: PRISMS AND PYRAMIDS.

Lay out the sheet and from the center of the sheet plot three origins: The first origin $5\frac{1}{4}''$ to the left and $4\frac{1}{2}''$ above the center of the sheet; the second $8''$ to the right and $1\frac{1}{2}''$ above the center; and the third $4''$ to the left and $4''$ below the center. Pass vertical and horizontal lines through these points to act as axes of projection.

First Origin: Pentagonal Prism and Inclined Plane.

Describe a pentagonal prism, the axis extending from P ($2''$, $1\frac{3}{4}''$, $1''$) to P' ($2''$, $1\frac{3}{4}''$, $2\frac{1}{2}''$). The top base is a regular pentagon inscribed in a circle of $1\frac{1}{2}''$ radius, one corner of the pentagon being at A ($2''$, $\frac{1}{4}''$, $\frac{1}{4}''$). Draw three views of the prism. Draw the traces of a plane P, perpendicular to $\mathbb{V}$, its trace on $\mathbb{V}$ passing through the point c ($0''$, $0''$, $2\frac{1}{2}''$) and making an angle of $60°$ with the axis of Z. Draw on the side elevation the line of intersection of the prism and the plane P. Show the true shape of the polygonal line of intersection on an auxiliary plane $\mathbb{U}$, perpendicular to $\mathbb{V}$, its traces on $\mathbb{V}$ passing through the point ($0''$, $0''$, $4\frac{1}{2}''$). On $\mathbb{U}$ show only the section cut by the plane. Draw the development of the surface of the prism, with the line of intersection described on it. Draw the left edge of the development [representing A ($2''$, $\frac{1}{4}''$, $\frac{1}{4}''$), A' ($2''$, $\frac{1}{4}''$, $2\frac{1}{2}''$)] as a vertical line $\frac{1}{2}''$ to the right of the axis of Y, and use the top working edge of the sheet as the top line of the development.

Second Origin: Octagonal Prism and Triangular Prism.

Describe an octagonal prism, the axis extending from P ($2\frac{1}{4}''$, $1\frac{3}{8}''$ $\frac{1}{4}''$) to P' ($2\frac{1}{4}''$, $1\frac{3}{8}''$, $4\frac{1}{4}''$). The octagonal base is circumscribed about a circle of $2\frac{1}{4}''$ diameter, one flat side being parallel to the axis of X. Describe a triangular prism, its axis extending from Q ($0''.52$, $1\frac{3}{8}''$, $1\frac{1}{4}''$) to Q' ($3''.98$, $1\frac{3}{8}''$, $3\frac{1}{4}''$), intersecting PP' at its middle point and making an angle of 60° with it. The triangular base is in a plane perpendicular to QQ' and is circumscribed about a circle of 1″ diameter. One corner is at J ($1''$, $1\frac{3}{8}''$, $0''.38$). Draw the $\mathbb{H}$, $\mathbb{V}$, and $\mathbb{S}$ projections of the prisms and a complete projection on a plane $\mathbb{U}$, taken perpendicular to QQ', and whose trace on $\mathbb{V}$ passes through the point ($6''$, $0''$, $0''$). Draw the triangular prism as if piercing the octagonal prism.

Third Origin: Hexagonal Pyramid and Square Prism.

Describe an hexagonal pyramid, vertex at P ($1\frac{3}{4}''$, $2''$, $\frac{1}{2}''$), center of base at P' ($1\frac{3}{4}''$, $2''$, $3''$). The hexagonal base is in a plane parallel to $\mathbb{H}$ and is circumscribed about a circle $2\frac{1}{2}''$ in diameter, one corner being at A ($1\frac{3}{4}''$, $0''.56$, $3''$). Projecting from the sides of the pyramid are two portions of a square prism, whose axis is Q ($\frac{1}{4}''$, $2''$, $2\frac{1}{4}''$), Q' ($3\frac{1}{4}''$, $2''$, $2\frac{1}{4}''$). The square base is in a plane parallel to $\mathbb{S}$ and measures 1″ on each edge, and its edges are parallel to the axes of Y and Z. Letter the edges GG', HH', etc., the point G being ($\frac{1}{4}''$, $1\frac{1}{2}''$, $1\frac{3}{4}''$), H ($\frac{1}{4}''$, $2\frac{1}{2}''$, $1\frac{3}{4}''$), etc. Draw the object as if cut from one solid piece of material, the prism not piercing the pyramid.

The views required are plan, front elevation, and side elevation, and also an auxiliary projection on a plane $\mathbb{U}$, perpendicular to $\mathbb{H}$. The $\mathbb{H}$ trace of $\mathbb{U}$ makes an angle of 120° with the axis of X at the point X ($2\frac{5}{8}''$, $0''$, $0''$).

Draw also the developments of the surfaces. Place the vertex of the developed pyramid at a point $\frac{1}{2}''$ to the right and $3\frac{1}{4}''$ above the origin, and the point A $\frac{1}{2}''$ to the right and $0''.56$ above the origin. Mark the line of intersection with the prism on this development.

Between the side elevation and the legend space, draw the development of the square prism, placing the long edges, GG', HH', etc., in a vertical position. Describe the line of intersection on the development. Let the edge which has been opened out be GG', and let the middle portion of the prism, which does not in reality exist, be drawn with construction lines.

General Directions for Completing the Sheet.

In inking the sheet show one line of projection for the determination of one point on each line of intersection. Shade the figure, except the developments.

In the legend space make the following legend:

	SHEET I.	(Block letters 15/32″ high.)
	DESCRIPTIVE GEOMETRY.	(All caps 3/16″ high.)
	PRISMS AND PYRAMIDS.	(All caps 9/32″ high.)
Name (signature).	Class.	(Caps 1/8″ high, lower case 1/12″ high.)
	Date.	(Caps 1/8″ high, lower case 1/12″ high.)

SHEET II: CYLINDERS, ETC.

Lay out cutting, border, and working lines, and legend space as before.

Plot four points of origin as follows: First origin, 6″ to the left and 4″ above the center of the sheet; second origin, $4\frac{3}{4}$″ to the right and $4\frac{1}{2}$″ above the center; third origin, $6\frac{1}{2}$″ to the left and $3\frac{1}{2}$″ below the center; fourth origin, $6\frac{1}{4}$″ to the right and $4\frac{1}{4}$″ below the center.

First Origin: Intersecting Right Cylinders.

Draw the three views of two intersecting right cylinders. The axis of one is P ($2\frac{1}{2}$″, 2″, $\frac{1}{2}$″), P' ($2\frac{1}{2}$″, 2″, $3\frac{1}{2}$″), and its diameter is 3″. The axis of the other is Q ($\frac{1}{2}$″, $1\frac{3}{4}$″, 2″), Q' ($4\frac{1}{2}$″, $1\frac{3}{4}$″, 2″), and its diameter $2\frac{3}{8}$″. Determine the line of intersection in $\mathbb{V}$ by planes parallel to $\mathbb{V}$ at distances of $\frac{3}{4}$″, 1″, $1\frac{1}{4}$″, etc.

Second Origin: Inclined Cylinder and Inclined Plane.

Draw three views of an inclined circular cylinder, cut by a plane. The axis of the cylinder is P (3.73″, $1\frac{3}{4}$″, $\frac{1}{2}$″), P' (2″, $1\frac{3}{4}$″, $3\frac{1}{2}$″). The base is a circle, diameter $2\frac{1}{2}$″, in a plane parallel to $\mathbb{H}$. The plane cutting the cylinder is perpendicular to $\mathbb{V}$, and its trace in $\mathbb{V}$ passes through the middle point of PP', and inclines up to the left at an angle of 30° with OX. Plot the intersection in $\mathbb{H}$, $\mathbb{V}$, and $\mathbb{S}$ and find the true shape of the ellipse by an auxiliary plane of projection perpendicular to $\mathbb{V}$ through the point (3″, 0″, 4″).

Third Origin: Right Circular Cone and Inclined Plane.

Draw a right circular cone, vertex at P (2″, $1\frac{3}{4}$″, $\frac{1}{2}$″), center of base at P' (2″, $1\frac{3}{4}$″, 4″), diameter of base 3″. The cone is intersected by a plane perpendicular to $\mathbb{S}$, having its trace in $\mathbb{S}$ parallel to the extreme right element of the cone and through the point (0″, $2\frac{1}{4}$″, 4″). Draw the line of intersection in plan and front elevation, and show the true shape of the curve by projection on an auxiliary plane $\mathbb{U}$ perpendicular to $\mathbb{S}$, its trace passing through the point (0″, $2\frac{1}{4}$″, 0″).

Fourth Origin: Ogival Point, Vertical Plane and Inclined Plane.

Let $\mathbb{S}$ lie to the right of $\mathbb{H}$ and make no use of $\mathbb{V}$. The problem is to draw two views of a $3\frac{1}{2}$″ ogival shell, intersected by two planes. The ogival point is generated by revolving 60° of arc of $3\frac{1}{2}$″ radius about an axis perpendicular to $\mathbb{H}$ at the point (2″, $1\frac{3}{4}$″, 0″). The initial position of the generating arc is as follows: The center is at D (0″, $3\frac{1}{2}$″, $3\frac{1}{2}$″), one extremity is at B (0″, 0″, $3\frac{1}{2}$″), and one is at P (2″, $1\frac{3}{4}$″, 0.46″). The cylindrical body of the shell extends from the ogival point to the right in the side elevation, a distance of $\frac{3}{4}$″. Two planes, T and R, intersect the shell. T is parallel to and $1\frac{1}{2}$″ from $\mathbb{S}$. R is perpendicular to $\mathbb{S}$, and its $\mathbb{S}$ trace passes through the origin, and makes angles of 45° with the axis of Y and the axis of Z. Draw: The traces of T and R; the side elevation; the line of intersection of T with the shell; and, on the plan, the line of intersection of R with the shell.

General Directions for Completing the Sheet.

In inking the sheet show one cutting plane for the determination of each line of intersection, and show clearly how one point is determined in each view of each figure. Shade the figure except the developments.

In the legend space make the following legend:

SHEET II.	(Block letters 15/32″ high.)
DESCRIPTIVE GEOMETRY.	(All caps 3/16″ high.)
INTERSECTIONS OF CYLINDERS, ETC.	(All caps 9/32″ high.)
Name (signature). Class.	(Caps 1/8″ high, lower case 1/12″ high.)
Date.	(Caps 1/8″ high, lower case 1/12″ high.)

SHEET III: SURFACES OF REVOLUTION.

Lay out center lines, cutting, border and working lines, and legend space as before.

Plot five points of origin as follows: First origin, 6¾″ to the left and 3¾″ above the center of the sheet; second origin, 1½″ to the right and 6″ above the center; third origin, 8¾″ to the right and 5¾″ above the center; fourth origin, 5½″ to the left and 4¾″ below the center; fifth origin, 7¾″ to the right of the center of the sheet on the horizontal center line.

First Origin: Sphere and Cylinder.

Draw a sphere pierced by a right circular cylinder. The center of the sphere is at (2″, 2″, 2″), its diameter 3½″. The axis of the cylinder is *P* (2″, 1⅛″, ⅛″), *P′* (2″, 1⅛″, 3⅞″). Draw the sphere and cylinder in $\mathbb{H}$, $\mathbb{V}$ and $\mathbb{S}$, and determine the line of intersection by passing planes parallel to $\mathbb{V}$ at distances of ½″, ¾″, 1⅛″ and 1⅝″.

Second Origin: Forked End of Connecting-Rod.

The forked end of a connecting rod has the shape of a surface of revolution, faced off at the sides to a width of 1⅛″, as shown in Fig. 136. The centers *a, b,* and *c* are points (2″, 1″, 0″), (2″, 0″, ¾″), and (2″, 0″, 1″). The arc which has *d* as a center is *tangent* at its ends to the adjacent arc and to the side of the 1″ cylinder.

Determine the continuation of the line of intersection of the plane and surface at *w*, by passing planes parallel to $\mathbb{H}$ at distances from $\mathbb{H}$ of 2¼″, 2⅜″, 2½″, 2⅝″ and 2¾″. Draw no side view.

Third Origin: Stub End of Connecting-Rod.

The stub end of a connecting-rod is a surface of revolution faced off at the sides to a width of $1\frac{1}{2}''$, and pierced by bore-holes parallel to its axis as shown in Fig. 137. Centers are at a ($1\frac{7}{8}''$, $1''$, $0''$), b ($3''$, $1''$, $0''$), c ($\frac{3}{4}''$, $1''$, $0''$), d ($3\frac{3}{8}''$, $0''$, $1\frac{7}{8}''$), and e ($\frac{3}{8}''$, $0''$, $1\frac{7}{8}''$). Determine the continuation of the line of intersection at w by passing planes parallel to H at distances from H of $1\frac{1}{16}''$, $1\frac{1}{8}''$, $1\frac{3}{16}''$,

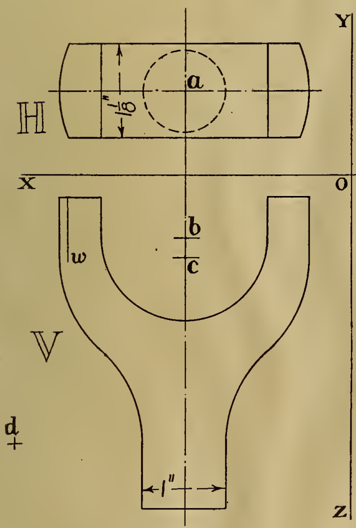


FIG. 136.

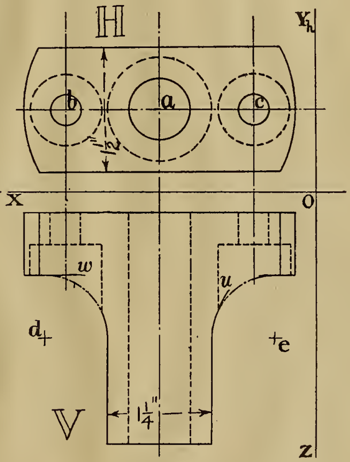


FIG. 137.

$1\frac{1}{4}''$, and $1\frac{5}{61}''$. Draw also the side view and determine the appearance of the edge marked u, where the large part of the bore-hole intersects the surface of revolution, by means of the same system of planes with two additional planes.

Fourth Origin: Right Circular Cylinder and Cone.

A right circular cone is pierced by a right circular cylinder, the axes intersecting at right angles, as in Fig. 62, Art. 72. The axis of the cone is P ($2\frac{1}{8}''$, $2\frac{1}{8}''$, $\frac{1}{4}''$), P' ($2\frac{1}{8}''$, $2\frac{1}{8}''$, $2\frac{3}{4}''$). The base, in a plane parallel to H, is a circle of $3\frac{3}{4}''$ diameter. The axis of the cylinder is Q ($\frac{1}{4}''$, $2\frac{1}{8}''$, $1\frac{5}{8}''$), Q' ($4''$, $2\frac{1}{8}''$, $1\frac{5}{8}''$), and its diameter is $1\frac{1}{2}''$.

Draw three views of the figures, determining the line of intersection by planes parallel to H. It is best not to pass these planes at equal intervals, but through points at equal angles on the base of the cylinder. Divide the base of the cylinder in S into arcs of 30°, and in numbering the points let that corresponding to F, in Fig. 62, be numbered 0 and let H be numbered 6. Insert intermediate points from 1 to 5 on both sides, so that the horizontal planes used for the determination of the curve of intersection are seven in number, the lowest passing through the point 0, the second through the two points 1, the third through the two points 2, etc. Determine the curve of intersection by these planes.

Draw the development of the surface of the cylinder, cutting the surface on the element OO' (or FF' in Fig. 62). Place this line of the development vertically on the sheet, the point O being $1''$ to the left and $7\frac{1}{2}''$ below the center of the sheet, and O' being $1''$ to the left and $3\frac{3}{4}''$ below the center of the sheet.

Draw the development of the surface of the cone Note that the radius of the base, the altitude, and the slant height are in the ratio of 3 : 4 : 5. To get equally spaced elements on the surface of the cone, divide the arc corresponding to BC in H, Fig. 62, into five equal spaces. Number the point B 0 and C 5, and the intermediate points in series. Since the cone is symmetrical about two axes at right angles, one quadrant may represent all four quadrants. Put the vertex of the developed surface $5''$ to the left of the center of the sheet and $1''$ below it, and consider it cut on the line $P0$ or PB. Locate the point 0 $5''$ to the left of the center of the sheet and $4\frac{1}{8}''$ below it. Divide the development into four quadrants and then divide each quadrant into five parts, numbering the 21 points 0, 1, 2, 3, 4, 5, 4, 3, 2, 1, 0, 1, 2, 3, 4, 5, 4, 3, 2, 1, 0.

Fifth Origin: Cone and Double Ogival Point.

In this figure a right circular cone pierces a double ogival point. The cone has a vertical axis, PP', the vertex P being at ($3''$, $1\frac{1}{2}''$,

$\frac{1}{4}''$), and P', the center of the base, at ($3''$, $1\frac{1}{2}''$, $3\frac{1}{2}''$). The base is a circle of $2\frac{1}{2}''$ diameter lying in a horizontal plane.

The ogival point has an axis of revolution, Q ($\frac{3}{8}''$, $1\frac{1}{2}''$, $2''$), Q' ($5\frac{5}{8}''$, $1\frac{1}{2}''$, $2''$), $5\frac{1}{4}''$ long. The generating line is an arc of $4''$ radius of which QQ' is the chord, and in its initial position the arc has its center at ($3''$, $1\frac{1}{2}''$, $5''.02$). Draw three views of the cone piercing the double ogival surface, and determine the line of intersection by means of three auxiliary cutting spheres, centered at p, the intersection of PP' and QQ'. Use diameters of $2\frac{1}{2}''$, $2\frac{1}{4}''$, and $2\frac{1}{16}''$. This curve appears on the U. S. Navy standard $3''$ valve.

General Directions for Completing the Sheet.

In inking the sheet show one cutting plane or sphere for the determination of each line of intersection, and show clearly how one point is determined in each view of each figure. Shade the figures, except the developments.

In the legend space record the following legend:

SHEET III.	(Block letters 15/32″ high.)
DESCRIPTIVE GEOMETRY.	(All caps 3/16″ high.)
INTERSECTIONS OF SURFACES OF REVOLUTION.	(All caps 9/32″ high.)
Name (signature). Class.	(Caps 1/8″ high, lower case 1/12″ high.)
Date.	(Caps 1/8″ high, lower case 1/12″ high.)

SHEET IV: CONES, ANCHOR RING AND HELICOIDAL SURFACES.

Lay out center lines, cutting, border, working lines, and legend space as before.

From the center of the sheet plot origins as follows: First origin, $3\frac{1}{8}''$ to the left of the center and $3\frac{15}{16}''$ above the center; second origin, $5\frac{1}{2}''$ to the right of the center and $3''$ above the center; third origin, $10\frac{1}{2}''$ to the right of the center and $4\frac{1}{4}''$ above the center; fifth origin, $3''$ to the right of the center and $6''$ below the center.

First Origin: Intersecting Inclined Cones.

Draw two intersecting inclined cones. The first cone has its vertex at P $(1'', 1\frac{7}{8}'', \frac{1}{4}'')$, and the center of its base at P' $(2\frac{11}{16}'', 1\frac{7}{8}'', 4\frac{3}{4}'')$. The base is a circle of $3\frac{3}{8}''$ diameter, lying in a plane parallel to $\mathbb{H}$. The second cone has its vertex at Q $(5'', 1\frac{7}{8}'', 2\frac{1}{2}'')$, and the center of its base at Q' $(\frac{1}{2}'', 1\frac{7}{8}'', 3\frac{1}{4}'')$. The base is a circle of $3''$ diameter lying in a plane parallel to $\mathbb{S}$. Draw plan, front elevation, side elevation, and an auxiliary projection on a plane $\mathbb{U}$, perpendicular to the line PQ, the trace of $\mathbb{U}$ on $\mathbb{V}$, passing through the point M $(7\frac{1}{4}'', 0'', 0'')$. Determine the line of intersection of the cones by auxiliary cutting planes containing the line PQ, and treat the problem on the supposition that the cone PP' pierces the cone QQ'.

Second Origin: Helicoidal Surface for Screw Propeller.

A right vertical cylinder, $1\frac{1}{2}''$ in diameter, has for its axis P $(2\frac{1}{2}'', 2\frac{1}{2}'', \frac{1}{2}'')$, P' $(2\frac{1}{2}'', 2\frac{1}{2}'', 3\frac{1}{2}'')$. Projecting from the cylinder is a line A $(3\frac{1}{4}'', 2\frac{1}{2}'', \frac{1}{2}'')$, B $(4\frac{1}{2}'', 2\frac{1}{2}'', \frac{1}{2}'')$. This line, moving uniformly along the cylinder, and about it clockwise, describes one complete turn of a helicoidal surface of $3''$ pitch. Draw plan and front elevation of the figure. This helicoid is intersected by an elliptical cylinder of which the generating line is perpendicular to $\mathbb{H}$ and the directrix is an ellipse lying in $\mathbb{H}$, having its major axis C $(2'', 2\frac{1}{2}'', 0'')$, D $(\frac{1}{2}'', 2\frac{1}{2}'', 0'')$, and minor axis E $(1\frac{1}{4}'', 3'', 0'')$, F $(1\frac{1}{4}'', 2'', 0'')$. Find the intersection of the two surfaces. Ink in full lines only the circular cylinder and the intersection. This portion of a helicoidal surface is similar to that which is used for the acting surface of the ordinary marine screw propeller, of 3 or 4 blades.

Third Origin: Worm Thread Surface.

A worm shaft is a right cylinder, $1\frac{1}{2}''$ in diameter, its axis being P $(1\frac{3}{4}'', 1\frac{3}{4}'', \frac{1}{4}'')$, P' $(1\frac{3}{4}'', 1\frac{3}{4}'', 8\frac{3}{4}'')$. A triple right-hand worm thread, of the same profile as in Fig. 70, projects from the cylinder along the middle $6''$ of its length. The pitch of the thread is $4\frac{1}{2}''$, so that each thread has more than a complete turn. The outside diameter of the worm is 3″. Make a complete drawing of the plan and front elevation, as in Fig. 70, letting the worm thread begin at any point on the circumference.

Fourth Origin: Anchor Ring and Planes.

An anchor ring, R, is formed by revolving a circle of $1\frac{1}{4}''$ diameter, lying in a plane parallel to $\mathbb{V}$ and with its center at A $(1\frac{1}{8}'', 2\frac{3}{8}'', \frac{7}{8}'')$, about an axis perpendicular to $\mathbb{H}$ and piercing $\mathbb{H}$ at the point B $(2\frac{3}{8}'', 2\frac{3}{8}'', 0'')$. Draw plan, front elevation, right side elevation (to the right of $\mathbb{H}$), and left side elevation (to the left of $\mathbb{H}$) on a plane $\mathbb{S}'$, $4\frac{3}{4}''$ from $\mathbb{S}$. A plane P, parallel to $\mathbb{S}$ at $\frac{5}{8}''$ from $\mathbb{S}$, cuts the ring. Draw the trace of P on $\mathbb{H}$, and the intersection of P and the ring on $\mathbb{S}$. A second plane P', parallel to $\mathbb{S}$ at $1\frac{1}{8}''$ distance, cuts the ring. Draw the trace $P'H$ and the intersection $P'R$ on $\mathbb{S}$. A third plane Q is parallel to $\mathbb{V}$ at $1\frac{3}{4}''$ distance from $\mathbb{V}$. Draw the trace QH and the intersection QR on $\mathbb{V}$. A fourth plane, Q', is parallel to $\mathbb{V}$ at 2″ distance. Draw the trace $Q'H$ and the intersection $Q'R$ on $\mathbb{V}$. An inclined plane T is perpendicular to $\mathbb{S}$ and $\mathbb{S}'$, its trace on $\mathbb{S}'$ passing through the point C $(4\frac{3}{4}'', 2\frac{3}{8}'', \frac{7}{8}'')$, and inclining down to the right at such an angle as to be tangent to the projection on $\mathbb{S}'$ of the generating circle when its center is at D $(2\frac{3}{8}'', 1\frac{1}{8}'', \frac{7}{8}'')$. Draw the trace of T on $\mathbb{S}'$, and the intersection TR on $\mathbb{H}$. Find the true shape of TR by means of an auxiliary plane of projection $\mathbb{U}$ perpendicular to $\mathbb{S}'$, cutting $\mathbb{S}'$ in a trace parallel to TS' through the point on $\mathbb{S}'$ whose coordinates are E $(4\frac{3}{4}'', 0'', 1\frac{1}{4}'')$.

General Directions for Completing the Sheet.

Ink the sheet uniform with the preceding sheets, and in the legend space record the following legend:

SHEET IV.	(Block letters 15/32″ high.)
DESCRIPTIVE GEOMETRY.	(All caps 3/16″ high.)
CONES, ANCHOR RING AND HELICOIDS.	(All caps 9/32″ high.)
Name (signature). Class.	(Caps 1/8″ high, lower case 1/12″ high.)
Date.	(Caps 1/8″ high, lower case 1/12″ high.)