intellus
LEARNING
OPEN COURSES

Static Equilibrium and Elasticity

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at:
https://openstax.org/details/books/university-physics-volume-1

Conditions for Static Equilibrium, Part 1

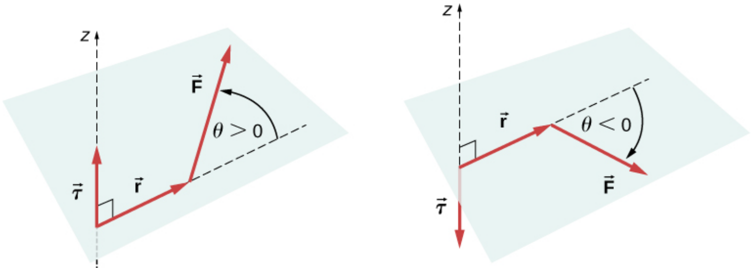
A rigid body is in **equilibrium** when it has zero linear and angular acceleration in some inertial reference frame. **Static equilibrium** is when a rigid body is at rest in an inertial reference frame. Because the laws of physics are identical in all inertial frames, there is no real distinction between the two.

There are two conditions for equilibrium. First, a body must be in translational equilibrium, meaning it experiences no net external force, $\sum_k \vec{\mathbf{F}}_k = \vec{\mathbf{0}}$. Second, it must be in rotational equilibrium, meaning it experiences no net external torque, $\sum_k \vec{\boldsymbol{\tau}}_k = \vec{\mathbf{0}}$. Though torque is dependent on the origin of the reference frame, it can be shown that when translating from a reference frame where equilibrium holds to another reference frame, equilibrium still holds.

Conditions for Static Equilibrium, Part 2

For planar equilibrium problems about a fixed axis, the six scalar equilibrium equations reduce to only three. By convention, rotation is about the z-axis, and only the forces in the $x - y$ plane apply non-zero torque, resulting in the three scalar equilibrium conditions:

- $F_{1x} + F_{2x} + \cdots + F_{Nx} = 0$
- $F_{1y} + F_{2y} + \cdots + F_{Ny} = 0$
- $\tau_{1z} + \tau_{2z} + \cdots + \tau_{Nz} = 0.$



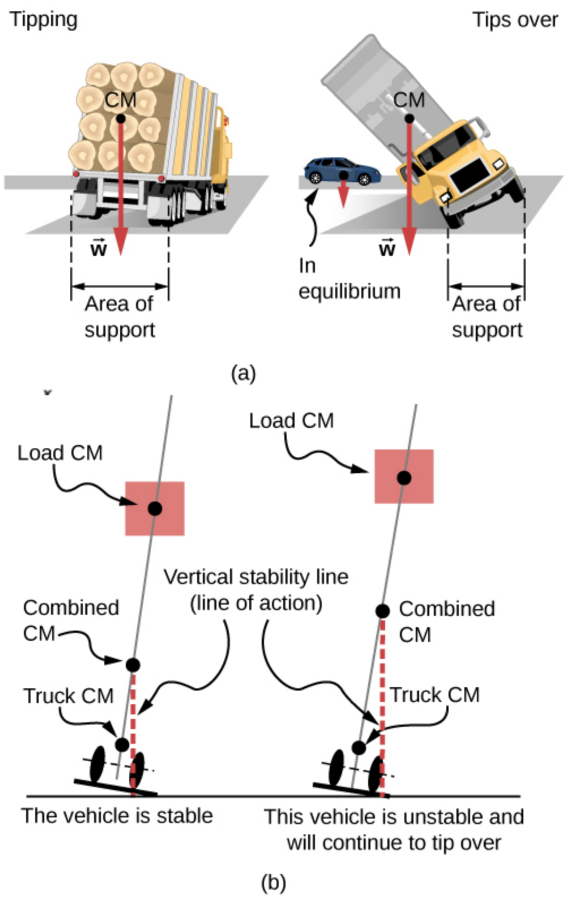
Counterclockwise rotation the sense of torque: $+\tau$
(a)

Clockwise rotation the sense of torque: $-\tau$
(b)

(University Physics Volume 1. OpenStax. Figs. 12.2.)

Gravitational Torque

Frequently, one of the forces acting on a body is its weight. The weight vector is attached to the **center of gravity**, which is identical to the center of mass for cases where the gravitational field is constant over the object. When the center of mass is located off the axis of rotation, the object's weight applies a **gravitational torque**. For tall, heavy objects like large trucks, this torque may be sufficient to tip the body.



(University Physics Volume 1. OpenStax. Figs. 12.3.)

Problem-Solving Strategy for Static Equilibrium

- Identify the object(s) to be analyzed. Identify the forces acting on the object, the known quantities both explicit and implicit, and the questions that need to be answered.
- Draw a free-body diagram. Choose the xy-reference frame and include all the forces acting on the body. Choose the rotation axis and pivot point to simplify calculations as much as possible.
- Set up the equations of equilibrium using the free-body diagram.
- Simplify and solve the system of equations for unknown quantities.
- Evaluate the expression for the unknown quantities that you obtained. Check for reasonable values and correct units.

Stress and Strain, Part 1

Stress and strain describe an object's deformation. **Stress** is defined as force per unit area. The SI unit of stress is the Pascal (Pa), equal to one newton per meter squared.

- **Tensile stress** is when a pulling force is applied to stretch an object.
- **Compressive stress** is when a pushing force is applied to squeeze an object.
- When a compressive force is applied from all sides, the stress is said to be **bulk stress** or **volume stress**.
- When a force is applied tangent to the surface of an object, the stress is called **shear stress**.

Stress and Strain, Part 2

Strain is the quantity that describes deformation. Strain is unitless because it is given as a fractional change in length, volume, or geometry.

- Tensile stress creates **tensile strain**.
- Bulk stress creates **bulk strain** or **volume strain**.
- Shear stress creates **shear strain**.

Elastic Modulus

Stress and strain are related by the **elastic modulus** at low levels of deformation by the equation, stress = (elastic modulus) $\times$ strain. The elastic modulus for tensile stress is called **Young's modulus**. For bulk stress, the elastic modulus is called the **bulk modulus**. For shear stress, the elastic modulus is called the **shear modulus**. The elastic moduli of a material must be measured in a laboratory.

material	Young's modulus $\times 10^{10}\text{Pa}$	bulk modulus $\times 10^{10}\text{Pa}$	shear modulus $\times 10^{10}\text{Pa}$
bone (tension)	1.6	0.8	8.0
iron	21.0	16.0	7.7
water		0.22	

Tensile and Compressive Stress and Strain

For a force acting with a magnitude of $F_{\perp}$ perpendicular to an object with cross-sectional area A, the tensile stress on the object is defined by the relationship tensile stress $= \frac{F_{\perp}}{A}$. The resulting strain is the object's fractional change in length, tensile strain $= \frac{\Delta L}{L_0}$, where ΔL is the change in length of the object and L_0 is the object's initial length. Compressive stress and strain are defined identically, except that for compressive stress and strain, we take absolute values of the right-hand sides of each equation. Young's modulus, Y, is the measured ratio of stress to strain for a given material, $Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_{\perp}/A}{\Delta L/L_0} = \frac{F_{\perp}L_0}{A\Delta L}$.

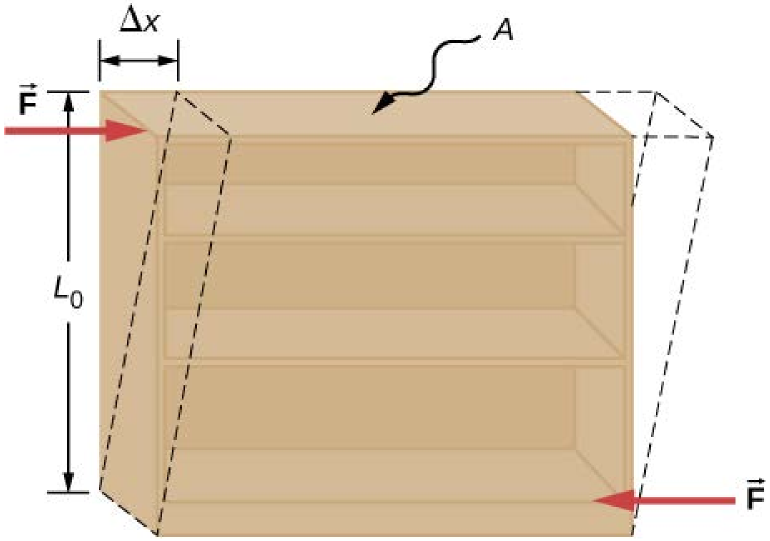
Bulk Stress and Strain

Bulk stress is the result of an increase in **pressure**, or perpendicular force per unit area, on the entire surface of an object. In air, we experience a **normal pressure**, p_0, of one atmosphere. When we dive underwater, the pressure increases by an amount, Δp. This change in pressure results in a stress, bulk stress $= \Delta p$.

The bulk strain on an object is its fractional change in volume, bulk strain $= \frac{\Delta V}{V_0}$, and the bulk modulus, B, is the ratio of bulk stress to bulk strain for a given material, $B = -\Delta p \frac{\Delta V}{V_0}$. The reciprocal of the bulk modulus, k, called the **compressibility**, is given by $k = \frac{1}{B} = -\frac{\Delta V / V_0}{\Delta p}$.

Shear Stress and Strain

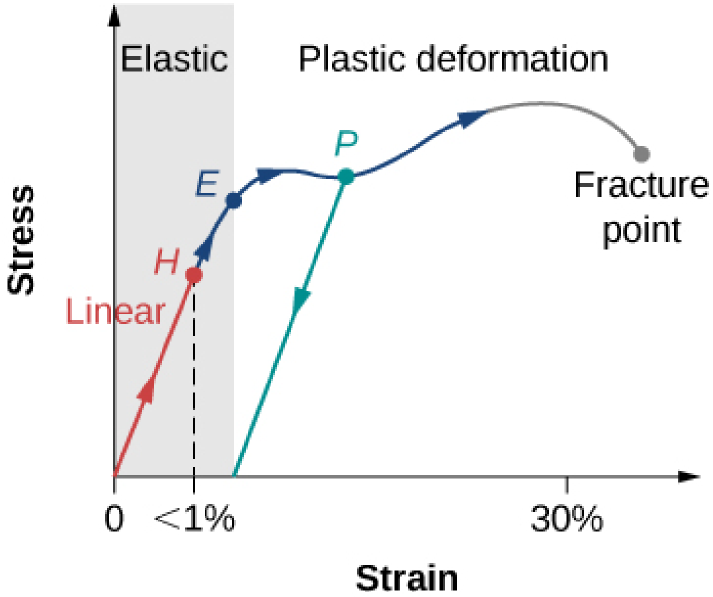
For a surface of area A, a force of magnitude $F_{\|}$ parallel to the surface causes a stress given by shear stress $= \frac{F_{\|}}{A}$. The shear strain is the size of the deformation parallel to the force divided by the transverse direction over which the deformation occurs, shear strain $= \frac{\Delta x}{L_0}$, and the shear modulus is the ratio, $S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F_{\|} L_0}{A \Delta x}$.



(University Physics Volume 1. OpenStax. Figs. 12.24.)

Elasticity and Plasticity

The linear stress-strain relation remains true up a **proportionality limit,** below which it is **elastic** and will return to its original shape when a load is removed. The **elastic limit** is the stress beyond which a material exhibits **plastic behavior** and permanently deforms, and after reaching its **ultimate stress**, the material breaks. These behaviors can be seen in a **stress-strain diagram**.



(University Physics Volume 1. OpenStax. Figs. 12.25.)

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

intellus LEARNING OPEN COURSES