



UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF MINES HELIUM ACTIVITY HELIUM RESEARCH CENTER

INTERNAL REPORT

CORRECTION FOR NON-UNIFORMITY OF THE BORE

OF A CAPILLARY TUBE VISCOSIMETER

BY

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CORRECTION FOR NON-UNIFORMITY OF THE BORE OF A CAPILLARY TUBE VISCOSIMETER

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J. E. Miller, $\frac{1}{R}$ R. A. Guereca, $\frac{2}{r}$ H. P. Richardson, $\frac{1}{2}$ and J. L. Gordon $\frac{1}{2}$

ABSTRACT

Analytic solutions for correction of non-uniformity in viscosimeter capillary bores are presented for the following cases: ellipse, cone, sine wave, sawtooth wave, square wave, and the general case. Rapid, accurate estimates of the correction factor $(1+\alpha)$ may be obtained from a graph of percent relative deviation in the bore versus α . Comparison of the analytic solutions to previously published data on glass capillary tubes illustrates the convenience of the analytic solutions. The general case is used to estimate $(1+\alpha)$ for a section of stainless steel capillary tubing. The radius of a glass capillary may be determined by filling the bore with mercury and measuring the electrical resistance. The correction factor for nonuniformity of the bore is shown to be $\alpha/3$ using the resistance method.

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Work on manuscript completed October 1966.

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INTRODUCTION

At a given temperature (T) and pressure (P), for a steady-state laminar volumetric flowrate, Q, through a capillary of length, L_T , and resulting pressure drop, $\triangle P$, the Poiseuille equation (1) for viscosity, η , is derived,

$$I = \left[\frac{\pi \triangle P r^{4}}{8 Q L_{T}} \right]_{(T,P)}, \qquad (1)$$

by assuming that the tube bore is a perfect right circular cylinder with radius r. Because real capillary bores are non-uniform, a correction is applied to equation (1). This correction is computed by assuming that if deviations in the bore are small, then the pressure drop, dP_i, over length, dx_i, will still be proportional to $1/r_i^4$ because the radial component of velocity may be considered negligible (<u>1</u>).^{3/}

<u>3/</u> Underlined numbers in parentheses refer to items in the list of references at the end of this report.

If this is true, $(L_T/r^4)^{-1}$ in (1) can be replaced by

$$\begin{bmatrix} x_i = L_T \\ \int \frac{dx_i}{x_i = 0} \frac{dx_i}{r_i^4} \end{bmatrix}$$

ENTRADUCT LON

At a gluen temperature (I) and pressure (P), for a stoady-state laminar volumetric flowrate, Q, through a capillary of length, L_P, and resulting pressure drop, (P, the Poiseuille equation (!) for visuosity, N, is derived.

by assuming that the tube bare is a perfect right circular cylinder with redius r. Because real capillary bares are non-uniform, a correction is applied to equation (1). This correction is computed by assuming that if deviations in the bare are small, then the pressure drop, dr_1 , over length, dx_1 , will still be proportional to $1/r_1^2$ because the radial component of velocity may be considered negligible (1). $\frac{3}{2}$

3/ Underlined numbers is parentheses refer to items in the list of references at the end of this report.

If this is true, (Ly/r") " in (1) can be replaced by

or r⁴ in (1) can be replaced by

$$\begin{bmatrix} x_i = L_T \\ \frac{1}{L_T} & \int \frac{dx_i}{x_i = 0} & \frac{dx_i}{r_i} \end{bmatrix}$$

When the radial component of velocity is not negligible, the actual velocity distribution must be determined and equation (1) rederived because dP_i/dx_i along the cylinder axis will be perturbed. In this report, all calculations of the correction for non-uniformity of the bore are based on the assumption that the expression in (2) is valid.

In most of the previous work with absolute capillary flow viscosimeters, a root-mean-square radius, r_{ms} , is determined by measuring the internal volume and length of the capillary; then r_{ms} of a right circular cylinder with the same length and volume is computed. The dimensionless correction for non-uniformity of the bore (δ) is defined so that

$$\frac{r_{ms}^{4}}{\delta} = \begin{bmatrix} x_{i} = L_{T} \\ \frac{1}{L_{T}} & \int_{1}^{0} \frac{dx_{i}}{\frac{1}{L_{T}}} \\ \frac{1}{L_{T}} & x_{i} = 0 \\ 1 \end{bmatrix}$$
(3)

6

(2)

ur r in (1) can be replaced by



When the radial component of velocity is not negligible, the setual valuatty distribution must be determined and equation (1) rederived because dP_1/da_1 along the cylinder axis will be perturbed. In this resport, all calculations of the correction for non-uniformity of the bore are based on the assumption that the expression in (2) is valid. In cost of the previous work with absolute radiilary flow viscostinuters, a root-mean-equate radius, r_{m_1} , is determined by measuring the internal volume and length of the capillary; then r_{m_1} of a slight eitenlar cylinder with the same length and volume is composed. The dimensionless correction for non-uniform is composed. The

$$= \begin{bmatrix} x_{i} = L_{T} \\ r_{ms}^{4} & \int_{1}^{4} \frac{dx_{i}}{I_{T}} \\ T & x_{i} = 0 & r_{i}^{4} \end{bmatrix} .$$
(4)

As a consequence of equation (3), the viscosity equation (1) assumes the form

δ

$$\eta = \frac{\pi \Delta P r_{ms}^4}{8 Q L_{T} \delta} . \qquad (1a)$$

For a right circular cylinder, $\delta = 1$; when $r_i \neq a$ constant, $\delta > 1$. As is well known, the pressure drop through an actual capillary bore is greater than would be observed in the corresponding perfect cylinder.

There are a few cases where equation (4) may be solved exactly. Barr (1) gives the solutions for an ellipse, cone, and cone with elliptical cross sections. In this report, solutions for a sine wave, sawtooth wave, square wave, and the general case are presented, together with those for the cone and ellipse. All solutions are reduced to the same form:

$$\delta -1 = \alpha = f(K) , \qquad (5)$$

or



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Ret a right sizedar cylinder, b = 1; skan r, * a constant, b = 1. As is well known, the pressure drup through an solumi combiliary have is groater than would be observed in the crriety and a perfect.

There are a factors where openation (0) may be solved exactly. Marr (1) gives the solutions for an elitare, cone, and cone with elitorical trade sections. In this report, solutions for a sink wave, and cold do the square wave, and the general case are presented, topschar with them is the case are aligned, All relations are reduced to the same form.

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where K is roughly the relative deviation from symmetry. Figure 1 is a

Figure 1.-Graphs of the bore profiles.

graphic representation of the bore profiles.

EXACT SOLUTIONS OF EQUATION (4)

Ellipse

In this case, the semi-axes r_{max} and r_{min} are at right angles and there is no rotation of axes down the length of the bore. The area of a cross section is $\left[\pi r_{max} r_{min} \right]$, so r_{m} is the radius of a circle having the same area:

$$\pi r_m^2 = \pi r_{max} r_{min} ; \qquad (6)$$

$$r_{\rm m}^4 = r_{\rm max}^2 r_{\rm min}^2$$
 (7)

According to Barr (1), evaluation of (2) gives

$$\frac{2 r_{\max}^{3} r_{\min}^{3}}{r_{\max}^{2} + r_{\min}^{2}},$$
 (8)

so equation (4) becomes:

$$\delta = 1 + \alpha_{E} = r_{\max}^{2} r_{\min}^{2} \left(\frac{r_{\max}^{2} + r_{\min}^{2}}{2 r_{\max}^{3} r_{\min}^{3}} \right).$$
(9)

where K is roughly the relative deviation from symmetry. Figure 1 is a

figure 1.-Graphe of the bore profides.

rephic representation of the same profiles.

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In this case, the same area that and the are at right suches and there is the recallen of true down the techth of the bace. The stees of a cross soution is [= 1 may form], so r is the reduce of s streight having the same story

scanding to may (i), analyzation of (2) given







Cone



Sine wave



Sawtooth wave

Square wave

FIGURE 1. - Graphs of the Bore Profiles.



Eliloso (cross section)



Sine wave

.



Sone



iquare wave Soutooth wave

FIGURE 1. - Grophs of the Bone Profiles.

$$K_{E} = \frac{r_{max} - r_{min}}{r_{min}}, \qquad (10)$$

$$\delta = 1 + \alpha_{E} = \frac{1 + K_{E} + \frac{K_{E}^{2}}{2}}{1 + K_{E}} = 1 + \frac{K_{E}^{2}}{2(1 + K_{E})} , \quad (11)$$

$$\alpha_{\rm E} = \frac{\kappa_{\rm E}^2}{2 \ (1+\kappa_{\rm E})} = \frac{1}{2} \left(\kappa_{\rm E}^2 - \kappa_{\rm E}^3 + \kappa_{\rm E}^4 - \kappa_{\rm E}^5 + \kappa_{\rm E}^6 \pm \cdots \right) \ . \ (12)$$

Cone

The cone is assumed to have circular cross sections with

$$r_{i} = r_{min} (1 + bx)$$
 (13)

At

 $x = 0, r_{i} = r_{min};$

at

$$K = L_T, r_i = r_{max}$$

Also,

$$r_{m}^{2} = \frac{1}{L_{T}} \int_{0}^{L_{T}} r_{i}^{2} dx = \frac{r_{min}^{2}}{L_{T}} \int_{0}^{L_{T}} (1 + bx)^{2} = \frac{r_{max}^{2} + r_{max}r_{min} + r_{min}^{2}}{3} . (14)$$

If

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Evaluating,

$$\int_{0}^{L_{T}} \frac{dx}{r_{i}^{4}} = \int_{0}^{L_{T}} \frac{dx}{(1+bx)^{4}} = \frac{L_{T}}{3} \left[\frac{r_{\max}^{2} + r_{\max}r_{\min} + r_{\min}^{2}}{3 r_{\max}^{3} r_{\min}^{3}} \right].$$
(15)

Then substitution into (4) gives

$$\delta = 1 + \alpha_{\rm C} = \left[\frac{r_{\rm max}^2 + r_{\rm max}r_{\rm min} + r_{\rm min}^2}{3} \right]^2 \left[\frac{r_{\rm max}^2 + r_{\rm max}r_{\rm min} + r_{\rm min}^2}{3 r_{\rm max}^3 r_{\rm min}^3} \right]$$
$$= \frac{\left[\frac{r_{\rm max}^2 + r_{\rm max}r_{\rm min} + r_{\rm min}^2}{3} \right]^3}{r_{\rm max}r_{\rm min}^3} .$$
(16)

If

$$K_{\rm C} = \frac{r_{\rm max} - r_{\rm min}}{r_{\rm min}}$$
(17)

is substituted into (16):

$$5 = 1 + \alpha_{\rm C} = \frac{\left[(1 + K_{\rm C})^3 - 1 \right]^3}{27 K_{\rm C}^3 (1 + K_{\rm C})^3}$$

$$= 1 + \frac{K_{C}^{2}}{1 + K_{C}} + \frac{K_{C}^{4}}{3 (1 + K_{C})^{2}} + \frac{K_{C}^{6}}{27 (1 + K_{C})^{3}}$$
(18)

Evaluating.

$$12 \cdot \left[\frac{12^{2} + n \ln^{2} + n \ln^{2} + n \ln^{2} + n \ln^{2} + n \ln^{2}}{1 + n \ln^{2} + n \ln^{2} + 1}\right] \cdot \frac{14}{1 + n \ln^{2} + n \ln^{2} + 1}$$

Then subscritting (4) gives

$$= 1 + 2 = \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{100} + \frac{1}{100} \end{bmatrix} \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{100} + \frac{1}{1000} \end{bmatrix}$$

$$= 1 + 2 = \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{100} + \frac{1}{1000} \end{bmatrix} \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} \end{bmatrix} \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{1000} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1000} + \frac{1}{1000} \frac{1}{100$$

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$$\frac{1}{2} \left[\frac{1}{2} - \frac{1}{2} \frac{1}{2} + \frac{1}{2} \right] = \frac{1}{2} \left[\frac{1}{2} + \frac{1}{2} \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{1}{2}$$

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5

and

$$\alpha_{\rm C} = \frac{\kappa_{\rm C}^2}{1+\kappa_{\rm C}} + \frac{\kappa_{\rm C}^4}{3(1+\kappa_{\rm C})^2} + \frac{\kappa_{\rm C}^6}{27(1+\kappa_{\rm C})^3}$$
$$= \kappa_{\rm C}^2 - \kappa_{\rm C}^3 + \frac{4}{3}\kappa_{\rm C}^4 - \frac{15}{9}\kappa_{\rm C}^5 + \frac{55}{27}\kappa_{\rm C}^6 - \frac{22}{9}\kappa_{\rm C}^7 \pm \cdots$$
(20)

4/ K. R. Van Doren, research mathematician, Helium Research Center, Bureau of Mines, Amarillo, Tex., contributed substantially in developing this solution.

The equation of a sine wave deviation superimposed on a right circular cylinder is

$$r_{i} = r_{0} + C_{1} \sin C_{2} x_{.}$$
 (21)

Then

$$r_{m}^{2} = \frac{1}{L_{T}} \int_{0}^{L_{T}} (r_{0} + C_{1} \sin C_{2}x)^{2} dx$$
 (22)

$$= r_0^2 + \frac{2 r_0^2 r_1}{C_2 L_T} (1 - \cos C_2 L_T) + \frac{C_1^2}{2} - \frac{C_1^2}{4 C_2 L_T} \sin 2 C_2 L_T, \quad (23)$$

 $\frac{x_0^2}{x_0} + \frac{x_0^2}{3(1+x_0)^2} + \frac{x_0^2}{27(1+x_0)^3}$

K. R. Van Doren, research methemacicien, Helium Research Genter, Burgau of Mines, Amerillo, Tax., contributed substantially in devaloping this solution.

The equation of a sine wave divintion superisposed on a right circular cylicher is

Recti

(22)

 $\frac{2}{50} = \frac{2}{5_{0}} \frac{1}{5_{0}} \left(1 - \cos \phi_{2} t_{p}\right) + \frac{\phi_{1}}{2} - \frac{\phi_{1}^{2}}{2} + \frac{\phi_{2}^{2}}{5_{0}} \frac{\phi_{1}}{1} + \frac{\phi_{2}}{2} + \frac{\phi_{2}}{5_{0}} \frac{\phi_{1}}{1} \right)$ (23)

bas

and for an integral number of cycles,

$$r_m^2 = r_0^2 + \frac{C_1^2}{2}$$
 (24)

The other integral,

$$\int_{0}^{L} \frac{dx}{(r_{0} + C_{1} \sin C_{2} x)^{4}},$$
 (25)

can be solved by making the following substitutions:

$$u = C_2 x ; (26)$$

$$Z = \tan \frac{u}{2} .$$
 (27)

Then

$$L_{T} \qquad Z = \tan \frac{C_{2}L_{T}}{2}$$

$$\int_{0}^{1} \frac{dx}{(r_{0} + C_{1} \sin C_{2}x)^{4}} = \frac{2}{C_{2}} \qquad \int_{0}^{1} \frac{(1 + z^{2})^{3} dz}{(r_{0} + 2 C_{1}z + r_{0}z^{2})^{4}} \qquad (28)$$

and the numerator can be expanded to give four integrals that can be solved by standard methods, although each of the four must be broken up into two parts to prevent integration across a discontinuity. The answer to (28) is a very long expression, but for an integral number of cycles it is, after making the substitution:

$$K_{S} = \frac{C_{1}}{r_{0}} = \frac{r_{max} - r_{0}}{r_{0}} = \frac{r_{0} - r_{min}}{r_{0}},$$
 (29)

and for an integral number of cycles,

The other integral,

ran be solved by making the following substitution;

$$\frac{\alpha_{1}}{\alpha_{1}} = \frac{\alpha_{1}}{\alpha_{2}} = \frac{\alpha_{1}}{\alpha_{1}} = \frac{\alpha_{1}}{\alpha$$

and the numerator can be appended to give four integrals that con be solved by standard methods, sithough each of the four must be broken up into two parts to prevent integration strong a discontinuity. The

in it. it. after which the substitution:

$$\frac{m^2 m^2 - q^2}{q^2} = \frac{q^2 - m^2}{q^2} = \frac{1^2}{q^2} = \frac{1^2}{q^2}$$

2.7

$$\int_{0}^{L_{T}} \frac{dx}{(r_{0} + C_{1} \sin C_{2}x)^{4}} = \frac{L_{T} (2 + 3K_{S}^{2})}{2 r_{0}^{4} (1 - K_{S}^{2})^{7/2}}.$$
 (30)

Substituting (29), (30), and (24) into (4) gives:

$$\delta = 1 + \alpha_{\rm S} = \frac{\left(1 + \frac{{\rm K}_{\rm S}^2}{2}\right)^2 \left(2 + 3{\rm K}_{\rm S}^2\right)}{\left(1 - {\rm K}_{\rm S}^2\right)^{7/2}} \,. \tag{31}$$

Equation (31) can be simplified to give

$$\delta = 1 + \alpha_{\rm S} = 1 + 6 \, {\rm K}_{\rm S}^2 + 18.375 \, {\rm K}_{\rm S}^4 + 29.375 \, {\rm K}_{\rm S}^6 + \cdots \qquad (32)$$

Sawtooth Wave

A sawtooth wave superimposed on a cylinder may be solved using the same techniques. If $K_{\rm T}$ is defined as

$$K_{\rm T} = \frac{r_{\rm max} - r_0}{r_0} = \frac{r_0 - r_{\rm min}}{r_0}, \qquad (33)$$

it can be shown that

$$r_{\rm m}^2 = r_0^2 \left(1 + \frac{K_{\rm T}^2}{3}\right)$$
 (34)

and

$$\int_{0}^{1} \frac{d\pi}{(x_{0} + t_{1} \sin \theta_{1} \pi)^{2}} = \frac{L_{p}(2 + 3k_{0}^{2})}{1 c_{0}^{2}(1 - k_{0}^{2})^{2}}$$

abbattanting (23), (24), -und (26) into (6) gives:

Equation (32) can be also lifted to give

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$$\delta = 1 + \alpha_{\rm T} = \frac{r_{\rm m}^4}{L_{\rm T}} \int_0^{\rm T} \frac{dx}{r_{\rm i}^4} = \frac{\left(1 + \frac{K_{\rm T}^2}{3}\right)^3}{\left(1 - K_{\rm T}^2\right)^3}$$
(35)

or

$$\delta = 1 + \alpha_{\rm T} = 1 + 4 \, {\rm K}_{\rm T}^2 + \frac{28}{3} \, {\rm K}_{\rm T}^4 + \frac{460}{27} \, {\rm K}_{\rm T}^6 + \frac{244}{9} \, {\rm K}_{\rm T}^8 + \cdots \qquad (36)$$

Square Wave

If the square wave deviation is symmetrical to r₀,

 $r_i = r_0 (1 \pm K_W)$ (37)

and

$$r_{m}^{2} = \frac{1}{L_{T}} \left[\int_{0}^{L_{T}} r_{0}^{2} (1 + K_{W})^{2} dx + \int_{L_{T}}^{L_{T}} r_{0}^{2} (1 - K_{W})^{2} \right]$$

$$= r_0^2 (1 + K_W^2)$$
(38)

for an even number of cycles. Also,

$$\frac{1}{L_{T}} \int_{0}^{L_{T}} \frac{dx}{r_{i}^{4}} = \frac{1}{L_{T}} \left[\frac{L_{T}}{2 r_{0}^{4} (1 + K_{W})^{4}} + \frac{L_{T}}{2 r_{0}^{4} (1 - K_{W})^{4}} \right]. \quad (39)$$



Then

$$\delta = 1 + \alpha_{W} = \frac{(1 + K_{W}^{2})^{2}}{2} \left[\frac{(1 - K_{W})^{4} + (1 + K_{W})^{4}}{(1 - K_{W})^{4}(1 + K_{W})^{4}} \right]$$
$$= \frac{(1 + K_{W}^{2})^{2}(1 + 6 K_{W}^{2} + K_{W}^{4})}{(1 - K_{W}^{2})^{4}}$$
$$= 1 + 12 K_{W}^{2} + 56 K_{W}^{4} + 164 K_{W}^{6} + 368 K_{W}^{8} + \cdots$$
(40)

Table 1 summarizes analytical solutions for all cases discussed. Table 2 shows values for α for various values of K. Figure 2 shows a

Figure 2.-Values for K versus α .

graphic representation of K versus Q.

General Case

In some cases it may not be possible to find an analytic expression of $r_i = f(x)$ for substitution into (4). Then (2) must be evaluated by graphical or numerical methods. Swindells, Coe, and Godfrey (5) replaced the integral by a summation. Using our notation, they let

$$\frac{1}{L_{T}} \int_{0}^{L_{T}} \frac{dx}{r_{i}^{4}} = \frac{1}{n} \sum_{1}^{n} \frac{1}{r_{i}^{4}}$$
(41)

Then

$$\frac{2}{2}\left(\frac{1}{2}+\frac{1}{2}\right)^{2}\left(\frac{1}{2}+\frac{1$$

.

(00) +
$$\frac{1}{2}$$
 est + $\frac{1}{2}$ sol + $\frac{1}{2}$ sol + $\frac{1}{2}$ + $\frac{1}{2}$ = . . . (30)

•

Table 2 shows values for o for verious values of K. Figure 2 shows a

Figure 2. -Values for K tereus Q.

graphic representation of X wright o

Genetal Comp

In some cases if why not be possible to find an analytic expression of $r_1 = f(x)$ for expecticution into (4). Then (2) must be evaluated by graphical or summarical methods. Swindells, Dos, and Godfrey (5) replaced the integral by a schwatton. Using our notation, they lat

Type of bore 1/	K	$\delta - 1 = \alpha$
Ellipse	r _{max} - r _{min} r _{min}	$\frac{1}{2} \left(K^2 - K^3 + K^4 - K^5 + K^6 \pm \cdots \right)$
Cone	r _{max} - r _{min} r _{min}	$K^2 - K^3 + \frac{4}{3} K^4 - \frac{15}{9} K^5 + \frac{55}{27} K^6 \pm \cdots$
Sine wave	r _{max} - r ₀ r ₀	$6 \text{ k}^2 + 18.375 \text{ k}^4 + 29.375 \text{ k}^6 + \cdot \cdot \cdot$
Sawtooth wave	<u>r_{max} - r₀</u> r ₀	$4 K^{2} + \frac{28}{3} K^{4} + \frac{460}{27} K^{6} + \cdots$
Square wave	r _{max} - r ₀ r ₀	$12 \text{ k}^2 + 56 \text{ k}^4 + 164 \text{ k}^6 + \cdot \cdot$
General	<u>r</u> - r _m r _m	- 24 K _{avg} $\pm \cdots$, or 24 $\left(\frac{r_{m} - r_{avg}}{r_{m}} \right)$

TABLE 1.-Summary of analytic solutions for $\delta - 1 = \alpha = f(K)$

1/ See figure 1.

1.

Sine wave						
	0 ² - xee ² 0 ²	+ An # + An				
		- 275 - 22				
Ellipse,Cone,Sawtooth wave,Sine wave,Square wave $\alpha_{\rm E}$ $\alpha_{\rm C}$ $\alpha_{\rm T}$ $\alpha_{\rm S}$ $\alpha_{\rm W}$ 0.01 $2/0.005$ 0.010.040.060.12.02.02.04.16.24.48.04.08.16.64.961.92.06.18.361.442.164.32.06.32.642.563.847.68.10.501.004.006.0012.00.12.721.445.768.6417.28.14.981.967.8411.7623.52.161.282.5610.2415.3630.72.181.623.2312.9619.4438.94.202.003.9916.0024.0048.00.222.424.8319.3629.0458.21.242.885.7523.0434.5669.31.263.376.7427.0440.5681.38.283.917.8231.3647.0494.42.304.498.9736.0054.00108.45.48166496192.61836144216432.83263256384768	к, % <u>1/</u>	<u></u>	8	$\delta - 1 = \alpha = f(K), 1$	0 ⁻⁶ units	
--	----------------	----------------	----------------	------------------------------------	-----------------------	----------------
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		Ellipse,	Cone,	Sawtooth wave,	Sine wave,	Square wave,
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		α _E	α _C	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	α _s	α _W
02 02 04 $.16$ $.24$ $.48$ 04 08 $.16$ $.64$ $.96$ 1.92 06 $.18$ $.36$ 1.44 2.16 4.32 06 $.32$ $.64$ 2.56 3.84 7.68 10 $.50$ 1.00 4.00 6.00 12.00 12 $.72$ 1.44 5.76 8.64 17.28 $.14$ $.98$ 1.96 7.84 11.76 23.52 $.16$ 1.28 2.56 10.24 15.36 30.72 $.18$ 1.62 3.23 12.96 19.44 38.94 $.20$ 2.00 3.99 16.00 24.00 48.00 $.22$ 2.42 4.83 19.36 29.04 58.21 $.24$ 2.88 5.75 23.04 34.56 69.31 $.26$ 3.37 6.74 27.04 40.56 81.38 $.28$ 3.91 7.82 31.36 47.04 94.42 $.30$ 4.49 8.97 36.00 54.00 108.45 $.4$ 8 16 64 96 192 $.6$ 18 36 144 216 432 $.8$ 32 63 256 384 768	0.01	2/0.005	0.01	0.04	0.06	0.12
.04 $.08$ $.16$ $.64$ $.96$ 1.92 $.06$ $.18$ $.36$ 1.44 2.16 4.32 $.06$ $.32$ $.64$ 2.56 3.84 7.68 $.10$ $.50$ 1.00 4.00 6.00 12.00 $.12$ $.72$ 1.44 5.76 8.64 17.28 $.14$ $.98$ 1.96 7.84 11.76 23.52 $.16$ 1.28 2.56 10.24 15.36 30.72 $.18$ 1.62 3.23 12.96 19.44 38.94 $.20$ 2.00 3.99 16.00 24.00 48.00 $.22$ 2.42 4.83 19.36 29.04 58.21 $.24$ 2.88 5.75 23.04 34.56 69.31 $.26$ 3.37 6.74 27.04 40.56 81.38 $.28$ 3.91 7.82 31.36 47.04 94.42 $.30$ 4.49 8.97 36.00 54.00 108.45 $.4$ 8 16 64 96 192 $.6$ 18 36 144 216 432 $.8$ 32 63 256 384 768	.02	. 02	.04	.16	.24	. 48
.06 $.18$ $.36$ 1.44 2.16 4.32 $.06$ $.32$ $.64$ 2.56 3.84 7.68 $.10$ $.50$ 1.00 4.00 6.00 12.00 $.12$ $.72$ 1.44 5.76 8.64 17.28 $.14$ $.98$ 1.96 7.84 11.76 23.52 $.16$ 1.28 2.56 10.24 15.36 30.72 $.18$ 1.62 3.23 12.96 19.44 38.94 $.20$ 2.00 3.99 16.00 24.00 48.00 $.22$ 2.42 4.83 19.36 29.04 58.21 $.24$ 2.88 5.75 23.04 34.56 69.31 $.26$ 3.37 6.74 27.04 40.56 81.38 $.28$ 3.91 7.82 31.36 47.04 94.42 $.30$ 4.49 8.97 36.00 54.00 108.45 $.4$ 8 16 64 96 192 $.6$ 18 36 144 216 432 $.8$ 32 63 256 384 768	.04	.08	.16	.64	.96	1.92
.06 $.32$ $.64$ 2.56 3.84 7.68 $.10$ $.50$ 1.00 4.00 6.00 12.00 $.12$ $.72$ 1.44 5.76 8.64 17.28 $.14$ $.98$ 1.96 7.84 11.76 23.52 $.16$ 1.28 2.56 10.24 15.36 30.72 $.18$ 1.62 3.23 12.96 19.44 38.94 $.20$ 2.00 3.99 16.00 24.00 48.00 $.22$ 2.42 4.83 19.36 29.04 58.21 $.24$ 2.88 5.75 23.04 34.56 69.31 $.26$ 3.37 6.74 27.04 40.56 81.38 $.28$ 3.91 7.82 31.36 47.04 94.42 $.30$ 4.49 8.97 36.00 54.00 108.45 $.4$ 8 16 64 96 192 $.6$ 18 36 144 216 432 $.8$ 32 63 256 384 768	.06	.18	.36	1.44	2.16	4.32
.10 $.50$ 1.00 4.00 6.00 12.00 $.12$ $.72$ 1.44 5.76 8.64 17.28 $.14$ $.98$ 1.96 7.84 11.76 23.52 $.16$ 1.28 2.56 10.24 15.36 30.72 $.18$ 1.62 3.23 12.96 19.44 38.94 $.20$ 2.00 3.99 16.00 24.00 48.00 $.22$ 2.42 4.83 19.36 29.04 58.21 $.24$ 2.88 5.75 23.04 34.56 69.31 $.26$ 3.37 6.74 27.04 40.56 81.38 $.28$ 3.91 7.82 31.36 47.04 94.42 $.30$ 4.49 8.97 36.00 54.00 108.45 .4 8 16 64 96 192 $.6$ 18 36 144 216 432 $.8$ 32 63 256 384 768	.06	.32	.64	2.56	3.84	7.68
.12.721.445.768.6417.28.14.981.967.8411.7623.52.161.282.5610.2415.3630.72.181.623.2312.9619.4438.94.202.003.9916.0024.0048.00.222.424.8319.3629.0458.21.242.885.7523.0434.5669.31.263.376.7427.0440.5681.38.283.917.8231.3647.0494.42.304.498.9736.0054.00108.45.48166496192.61836144216432.83263256384768	.10	.50	1.00	4.00	6.00	12.00
.14.98 1.96 7.84 11.76 23.52 .16 1.28 2.56 10.24 15.36 30.72 .18 1.62 3.23 12.96 19.44 38.94 .20 2.00 3.99 16.00 24.00 48.00 .22 2.42 4.83 19.36 29.04 58.21 .24 2.88 5.75 23.04 34.56 69.31 .26 3.37 6.74 27.04 40.56 81.38 .28 3.91 7.82 31.36 47.04 94.42 .30 4.49 8.97 36.00 54.00 108.45 .48 16 64 96 192 .6 18 36 144 216 432 .8 32 63 256 384 768	.12	.72	1.44	5.76	8.64	17.28
.16 1.28 2.56 10.24 15.36 30.72 .18 1.62 3.23 12.96 19.44 38.94 .20 2.00 3.99 16.00 24.00 48.00 .22 2.42 4.83 19.36 29.04 58.21 .24 2.88 5.75 23.04 34.56 69.31 .26 3.37 6.74 27.04 40.56 81.38 .28 3.91 7.82 31.36 47.04 94.42 .30 4.49 8.97 36.00 54.00 108.45 .48 16 64 96 192 .6 18 36 144 216 432 .8 32 63 256 384 768	.14	.98	1.96	7.84	11.76	23.52
.18 1.62 3.23 12.96 19.44 38.94 .20 2.00 3.99 16.00 24.00 48.00 .22 2.42 4.83 19.36 29.04 58.21 .24 2.88 5.75 23.04 34.56 69.31 .26 3.37 6.74 27.04 40.56 81.38 .28 3.91 7.82 31.36 47.04 94.42 .30 4.49 8.97 36.00 54.00 108.45 .48 16 64 96 192 .6 18 36 144 216 432 .8 32 63 256 384 768	.16	1.28	2.56	10.24	15.36	30.72
$\begin{array}{cccccccccccccccccccccccccccccccccccc$.18	1.62	3.23	12.96	19.44	38.94
$\begin{array}{cccccccccccccccccccccccccccccccccccc$.20	2.00	3.99	16.00	24.00	48.00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$.22	2.42	4.83	19.36	29.04	58.21
$\begin{array}{cccccccccccccccccccccccccccccccccccc$.24	2.88	5.75	23.04	34.56	69.31
$\begin{array}{cccccccccccccccccccccccccccccccccccc$.26	3.37	6.74	27.04	40.56	81.38
.304.498.9736.0054.00108.45.48166496192.61836144216432.83263256384768	.28	3.91	7.82	31.36	47.04	94.42
.48166496192.61836144216432.83263256384768	.30	4.49	8.97	36.00	54.00	108.45
.61836144216432.83263256384768	.4	8	16	64	96	192
.8 32 63 256 384 768	.6	18	36	144	216	432
	.8	32	63	256	384	768
1.0 50 99 400 600 1,200	1.0	50	99	400	600	1,200
1.2 71 142 576 864 1,728	1.2	71	142	576	864	1,728
1.4 96 193 784 1,177 2,352	1.4	96	193	784	1,177	2,352
1.6 126 252 1,024 1,538 3,072	1.6	126	252	1,024	1,538	3,072
1.8 159 318 1,297 1,946 3,894	1.8	159	318	1,297	1,946	3,894
2.0 196 392 1,601 2,403 4,800	2.0	196	392	1,601	2,403	4,800
2.2 237 474 1,938 2,908 5,821	2.2	237	474	1,938	2,908	5,821
2.4 281 562 2,307 3,462 6,931	2.4	281	562	2,307	3,462	6,931
2.6 330 659 2,708 4,064 8,138	2.6	330	659	2,708	4,064	8,138
2.8 382 763 3,142 4,715 9,442	2.8	382	763	3,142	4,715	9,442
3.0 437 874 3,608 5,415 10,845	3.0	437	874	3,608	5,415	10,845

TABLE 2. -Values for $\delta - 1 = \alpha = f(K)$

 $\frac{1}{2}$ See table 1 for definition of K. $\frac{2}{2} \alpha = 0.005 \times 10^{-6}, \delta = 1.000000005.$

MERE 3 - Values for 5-1 - a - 6(8)

											100.45	15:00							
									. 20.2										

A = 0.002 - 10-P. V = 1.000000000





RELATIVE DEVIATION, K, percent

1

$$r_{\rm m}^2 = \frac{1}{n} \sum_{\rm l}^{\rm n} r_{\rm l}^2$$
, (42)

5/

<u>5</u>/ In this case, r_m is a root-mean-square radius determined, for example, from a large number of small, equally spaced samples, n. It is used later in the section "Calculation of δ for a 19-foot Section of Stainless Steel Capillary Tubing."

then

$$\delta = 1 + \alpha_{g} = \frac{r_{m}^{4}}{n} \sum_{i=1}^{n} \frac{1}{r_{i}^{4}}.$$
 (43)

In this case, it is convenient to let

$$r_{i} = r_{m} (1 + K_{i}) .$$
 (44)

Substitution of (44) into (42) gives

$$r_{\rm m}^2 = \frac{r_{\rm m}^2}{n} \sum_{\rm 1}^{\rm n} (1 + K_{\rm i})^2$$
(45)

$$n = \sum_{i=1}^{n} (1 + 2 K_{i} + K_{i}^{2}) = n + 2 \sum_{i=1}^{n} K_{i} + \sum_{i=1}^{n} K_{i}^{2}$$
(46)

or

and

and

•

$$2 \sum_{i=1}^{n} \kappa_{i} + \sum_{i=1}^{n} \kappa_{i}^{2} = 0.$$
 (47)

Substitution of (44) into (43) gives

$$\delta = 1 + \alpha_{g} = \frac{r_{m}^{4}}{n} \sum_{1}^{n} \frac{1}{r_{m}^{4} (1 + K_{i})^{4}} = \frac{1}{n} \sum_{1}^{n} \frac{1}{(1 + K_{i})^{4}}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1+4 K_{i}+6 K_{i}^{2}+4 K_{i}^{3}+K_{i}^{4}}$$

.

$$= \frac{1}{n} \sum_{i=1}^{n} (1 - 4 K_{i} + 10 K_{i}^{2} - 20 K_{i}^{3} + 35 K_{i}^{4} - 56 K_{i}^{5} + 84 K_{i}^{6}$$

- 120
$$K_{i}^{7} \pm \cdots$$
)

$$= \frac{1}{n} \left[n - 4 \sum_{i=1}^{n} K_{i} + 10 \sum_{i=1}^{n} K_{i}^{2} - 20 \sum_{i=1}^{n} K_{i}^{3} + 35 \sum_{i=1}^{n} K_{i}^{4} \pm \cdots \right].$$
(48)

From (47),

$$4 \sum_{i=1}^{n} \kappa_{i} = 2 \sum_{i=1}^{n} \kappa_{i}^{2}$$

so

$$\delta = 1 + \alpha_{g} = \frac{1}{n} \left[n + 12 \sum_{i=1}^{n} K_{i}^{2} - 20 \sum_{i=1}^{n} K_{i}^{3} + 35 \sum_{i=1}^{n} K_{i}^{4} \pm \cdots \right]. \quad (49)$$

The average K_i is

$$K_{avg} = \frac{1}{n} \sum_{i=1}^{n} K_{i} = -\frac{1}{2n} \sum_{i=1}^{n} K_{i}^{2}$$
 (50)

so

$$\sum_{1}^{n} K_{i}^{2} = -2n K_{avg} .$$
 (51)

Substituting (51) into (49) gives

$$\delta = 1 + \alpha_{g} = 1 - 24 K_{avg} - \frac{20}{n} \sum_{i=1}^{n} K_{i}^{3} + \frac{35}{n} \sum_{i=1}^{n} K_{i}^{4} \pm \cdots$$

$$\delta \approx 1 - 24 K_{avg}, \qquad (52)$$

which is a very good approximation because K_1^3 is usually negligible and is both plus and minus so the sum tends to cancel. Then the sum of

$$-4\sum_{l=1}^{n} x_{l} = 2\sum_{l=1}^{n} x_{l}^{2}$$

The avocage K, is .

$$R_{avg} = \frac{1}{n} \sum_{l}^{n} R_{l} = -\frac{1}{2n} \sum_{l}^{n} R_{l}^{2}$$

ante (2) and (5) antentitude

$$= \frac{1+\alpha_{B}}{1+\alpha_{B}} = \frac{1-2\alpha_{B}}{1+\alpha_{B}} \sum_{n=1}^{\infty} \sum_{n=1}^{\infty} \frac{1+\alpha_{n}}{2} + \frac{1+\alpha_{$$

(52)

which is a very good approximation because K_1^2 is usually negligible and is both plus and minus so the sum tends to cancel. Then the sum of

20

1

 K_i^3 is negligible also. Equation (52) is essentially the same as equation (IV-8) in Flynn (2).

A more convenient equation to compute δ can be derived from (50):

 $K_{avg} = \frac{\sum_{i=1}^{n} r_{i}}{n r_{m}} - 1.$ (53)

But

$$r_{avg} = \frac{1}{n} \sum_{i=1}^{n} r_{i}, \qquad (54)$$

so

$$x_{avg} = \frac{n r_{avg}}{n r_{m}} - 1 = \frac{r_{avg} - r_{m}}{r_{m}}$$

and

$$\delta \simeq 1 - 24 K_{avg} = 1 + \frac{24 (r_m - r_{avg})}{r_m} = 1 + \frac{12\sigma^2}{r_m^2},$$
 (55)

where

$$\sigma = \left[\frac{1}{n} \sum_{i=1}^{n} (r_i - r_{avg})^2\right]^{\frac{1}{2}}$$

Equation (41) is essentially a trapezoidal rule integration

$$\int_{a}^{b} y \, dx \cong h\left(\frac{1}{2}y_{0} + y_{1} + y_{2} + \cdots + y_{n-1} + \frac{1}{2}y_{n}\right);$$

 \mathbf{x}_1^{λ} is negligible also. Equation (52) is essentially the same as equation (IV-8) in Flynn (2).

a more convenient equation to compute 6 can be derived from (50):

$$\tilde{K}_{avg} = \frac{\tilde{Z}}{\tilde{Z}_{1}} - 1.$$
 (53)

But

80

$$K_{avg} = \frac{n r_{avg}}{n r_{m}} = 1 = \frac{r_{avg} - r_{g}}{r_{m}}$$

Barnis

$$\delta = 1 - 2\tilde{h} \, \kappa_{avg} = 1 + \frac{2\delta \, (\tau_m - \kappa_{avg})}{\kappa_m} = 1 + \frac{12\sigma^2}{\kappa_m}, \quad (55)$$

Vitere

Equation (41) is essentially a trapezoidal rule integration

$$ds = h\left(\frac{1}{2}, y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}, y_n\right)$$

21

$$h = \frac{b-a}{n}$$

So

$$\frac{1}{L_{T}} \int_{0}^{L_{T}} \frac{dx}{r_{i}^{4}} = \frac{L_{T}}{n L_{T}} \left(\frac{1}{2r_{0}^{4}} + \frac{1}{r_{1}^{4}} + \frac{1}{r_{2}^{4}} + \cdots + \frac{1}{r_{n-1}^{4}} + \frac{1}{2r_{n}^{4}} \right)$$

and if $r_0 \cong r_n$, or choose $r_0 = r_n$,

$$\frac{1}{L_{T}} \int_{0}^{L_{T}} \frac{dx}{r_{i}^{4}} = \frac{1}{n} \left(\frac{1}{r_{1}^{4}} + \frac{1}{r_{2}^{4}} + \cdots + \frac{1}{r_{n}^{4}} \right) = \frac{1}{n} \sum_{1}^{n} \frac{1}{r_{i}^{4}}$$

Also, (44) is derived from

$$r_{m}^{2} = \frac{1}{L_{T}} \int_{0}^{L_{T}} r_{i}^{2} dx \approx \frac{L_{T}}{nL_{T}} \left(\frac{r_{0}^{2}}{2} + r_{1}^{2} + r_{2}^{2} + \cdots + r_{n-1}^{2} + \frac{r_{n}^{2}}{2} \right)$$
$$r_{m}^{2} \approx \frac{1}{n} \sum_{i=1}^{n} r_{i}^{2} .$$

These equations are for n equally spaced samples. One must know the exact profile of $r_i = f(x)$ to get an "exact" value for either r_m or δ . Simpson's rule gives nearly the same result as the trapezoidal rule in the evaluation of (42) and (43).

Due to the way K is defined, K will always be negative, so δ is always greater than one. It is necessary to use r computed from

$$T = \frac{1}{2} + \frac{1}{2} +$$

These squatters are for n equally spaced samples. One must know the exact profile of $r_1 = \ell(x)$ is get as "exact" value for either r_2 or b fingern's rule gives marriy the same result as the trapecoidal rule (b) and musture for evaluation of (42), and (43).

Due to the way K1 is defined. K will always be negative, so d

(42) to get a valid estimate of δ from (43). Otherwise, the estimate for δ will be in error by the ratio of the two r's raised to the fourth power and it is possible to compute values of δ that are less than one. The use of (42) and (43) with relative r_i measurements will give a valid estimate for δ ; this is essentially the procedure used by Swindells, Coe, and Godfrey (5).

It is interesting to note that one cannot let K_i be constant in this derivation. Because of the way K_i is defined, (47) would give:

$$2 n K + n K^2 = 0$$
 (56)

or

K (K + 2) = 0

and

$$K = 0, -2$$

The first case corresponds to $r_i = r_m$, which is a perfect cylinder. The second case gives $r_i = -r_m$, which is impossible. If $K_i = \pm$ constant, one should use (40) for the square wave.

SAMPLE CALCULATIONS

Four examples are given to illustrate the use of equations and figure 2 in this report. Some relative diameter measurements given in (5) are used for the first three examples. These measurements are shown (42) to get a valid setimate of 5 from (43). Otherwise, the estimate for 5 mill be in error by the ratio of the two r's raised to the fourth puset and it is possible to compute values of 5 that are less than one. The use of (42) and (43) with relative r₁ measurements will give a valid estimate for 5: this is essentially the procedure used by Swindelly fore, and Godfrey (5).

It is intereating to note that one cannot lot K, be constant in this derivation. Secause of the way K, is defined, (57) would give:

Sno

$$X_{i} = 0_{i} - 2_{i}$$

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SAMPLE CALCULATIONS

Four examples are given to libratrate the use of equations and figure 2 in this report. Some relative diameter measurements given in (5) are used for the first three examples. These measurements are shown in table 3 and are graphed in figure 3; the deviations are roughly

Figure 3.-Cross-sectional area of bore at regular positions along the capillaries.

sinusoidal with

$$K_{\rm S} = \frac{r_{\rm max} - r_0}{r_0} \simeq \frac{562.6 - 561.625}{561.625} \simeq 0.00174$$
, (57)

= 0.174 percent .

From the sine wave curve of figure 2, $\alpha = 18 \times 10^{-6}$ and $\delta = 1.000018$; (5) gives $\delta = 1.000021$.

The data in table 3 may be substituted into equations (42) and (43) giving $\delta = 1.00002086$; using the approximation of (52) and (53) yields

$$K_{avg} = \frac{6188.625}{6188.630382} - 1 = -0.86966 \times 10^{-6}$$
(58)

and

$$\delta = 1 - 24 \text{ K}_{\text{avg}} = 1.00002087, \qquad (59)$$

so the approximate equation gives very good results in this case.

The third example is for capillary tube number 2.5a of (5); cross-sectional areas are seen to be similar to a sawtooth deviation. The radius is proportional to the square root of area, so K_T may be taken as:

in table 3 and are graphed in figure 3; the deviations are roughly

Figure 3. -Gross-sectional area of bors at regular positions along the

tity Isblosmis

$$\frac{r_{max} - r_0}{r_0} = \frac{562.6 - 501.625}{561.625} = 0.00176$$
, (57)

. D.174 percent .

From the sine wave curve of figure 2. $\alpha = 18 \times 10^{\circ}$ and $\delta = 1.000018$; (5) gives $\delta = 1.000021$.

The data in table 3 may be substituted into equations (42) and (43) events in 6 = 1.00002086; using the approximation of (52) and (53) yields

$$K_{ava} = \frac{6138.623}{6188.630332} - 1 = -0.85966 \times 10^{-6}$$
 (38

bris

so the approximate equation gives very good results in this case. The third example is for capillary tube number 2.5a of (5); crosssectional areas are seen to be similar to a sawtooth deviation. The radius is proportional to the square root of area, so K_{c} may be taken as

Position along tube	Mean diameter	Area of cross section
cm		
1	1,123.75	126 281
5	1,123.25	126 169
10	1,124.75	126 506
15	1,127.25	127 069
20	1,125.75	126 731
25	1,125.00	126 563
30	1,125.00	126 563
35	1,124.25	126 394
40	1,127.75	127 182
45	1,127.00	127 013
49	1,123.50	126 225
Source: Swindells	I G I R Coo an	d T B Codfrou

TABLE 3.-Variations in cross-sectional area along capillary 2.5, arbitrary units

> ource: Swindells, J. G., J. R. Coe, and T. B. Godfrey. Absolute Viscosity of Water at 20° C. NBS J. Res., v. 48, No. 1, January 1952, p. 16.

Beacs1, 2	

TABLE 3. -Vertectors in prose-scultors! eren slong capillacy 2.5.

Abaclute Viscosity of Water at 20° C. Has J. Langer of 20° C. Has J.



Source: Swindells, J. G., J. R. Coe, and T. B. Godfrey. Absolute Viscosity of Water of 20° C. NBS J. Res., v. 48, No. 1, Jonuary 1952, p. 16.

•



$$x_{\rm T} = \frac{\sqrt{1271} - \sqrt{1266}}{\sqrt{1266}} = 0.00197$$
 (60)

Then from figure 2, $\delta = 1.000016$. Swindells, Coe, and Godfrey (5) give $\delta = 1.000023$.

The last example, from $(\underline{3})$, is the profile of a glass capillary bore, figure 4. This deviation curve is roughly a sine wave with an

Figure 4.-Profile of a glass capillary bore.

average amplitude of about

$$\frac{9.141 \times 10^{-3} - 8.968 \times 10^{-3}}{2} = 8.65 \times 10^{-5} \text{ cm} . \quad (61)$$

Then

$$K_{\rm S} \cong \frac{8.65 \times 10^{-5}}{9.028 \times 10^{-3}} \cong 9.58 \times 10^{-3} \text{ or } 0.96 \%.$$
 (62)

Then from figure 2, $\alpha = 0.00055$ and $\delta = 1.00055$. The value quoted in (3) is $\delta = 1.0006$ (determined by graphical integration with a planimeter).

CALCULATION OF δ FOR A 19-FOOT SECTION OF STAINLESS STEEL CAPILLARY TUBING

A 19-foot section of 347 stainless steel capillary tubing supplied by Superior Tube Company, $\frac{6}{}$ Norristown, Pa., was cut from one

6/ Trade names are used for identification only and endorsement by the



Then finds figure 2. 5 = 1.000016. Swindelle, dou, and Godfrey (1) give

The last example, from (1), is the profile of a gloss capfilary bore, figure 4. This deviation corve is roughly a size wave with an

Figure 4. -Fredile of a glass capillary bore.

rverage asplitude of about

9.141 x.10" - 8.968 x 10" - 8.55 x 10" - (61)

minit

8.65 x 10⁻³ = 9.58 x 10⁻³ or 0.96 % . (64)

Then from figure 3, x = 0.00035 and x = 1.00035. The value quoted in (3) is 6 = 1.0006 (decembed by graphics) integration with a planim-

> CARCOLATION OF & FOR A 19-FOOT SECTION OF STAINLESS STREE CAPILLARY INDIAC

A 19-cuet meetion of 307 stainings steel capillary tubics, applied by Superior Nube Company, $\frac{67}{2}$ Karristown, Pa., see cut from one

// Trade names are used for identification only and epdorsenent by the



FIGURE 4. - Profile of a Glass Capillary Bore.

Source: Giddings, John G. The Viscosity of Light Hydrocarbon Mixtures at High Pressures: The Methane-Propane System. Ph. D. thesis, William Marsh Rice University, Houston, Texas, May 1963, 94+ pp. Copyright 1963.



Bureau of Mines is not implied.

end of a 220-foot length of cold-drawn (onto a mandrel) tubing. This section was sent to the Atomic Energy Commission Pantex Plant, Amarillo, Tex., where the section was cut into 57 4-inch samples. Six internal diameter measurements, three at each end, were made on each sample, for a total of 114 cross sections or 342 diameter measurements. The individual measurements are listed in Helium Research Center Internal Report No. 92 (4).

The deviations of the average radius per foot from the overall average radius are shown in figure 5. The only trend noted is a net

Figure 5.-Profile of a stainless steel capillary bore.

change in average radius from the 2nd to the 19th foot. Equations (42) and (43) were used to estimate δ . The calculation was performed on an IBM 1620 computer giving δ = 1.00004360, using all 342 diameter measurements, and δ = 1.00003442, using the average radius for each of the 114 cross sections. Using the approximation in (55)^{7/} gives δ =

$$\frac{7}{r_{avg}} = 0.01519883040$$
, r_{ms} (all 342 diameters) = 0.01519885800,
 $r_{ms} = 0.01519885219$ inch for 114 cross sections.

1.00004358 for all 342 measurements and δ = 1.00003441 for the 114 cross sections. From figure 5,

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FIGURE 5. - Profile of a Stainless Steel Capillary Bore.



$$K_{\rm S} \cong \frac{40 \times 10^{-6}}{152 \times 10^{-4}} = 0.263 \%$$

and figure 2 gives $\delta = 1.000042$.

The net change in radius, about 0.26 percent, in the 19-foot section may have been due to the fact that the section came from one end of a batch. It is not known which way the mandrel wire was withdrawn, with respect to the 19-foot length, but the only two possibilities are that essentially either all or none of the mandrel passed through it. It seems likely that a section from the center of a typical 200-foot batch would be more uniform than either of the end sections. Also, the 19foot end section of stainless steel capillary tubing was more uniform than some specially selected glass capillaries reported in the literature. For instance, values of δ up to 1.0047 for glass precision bore capillary tubing are reported in (3).

In the near future, the Helium Research Center plans to measure r_{ms} , for use in equation (1a), by a gravimetric method using mercury. The r_{ms}^4 thus determined should be very close to the r^4 in equation (1). At the same time, measurement errors in r_i will be minimized by using equation (43) to compute δ . These measurements will be taken on a 208-foot long section of capillary being used to evaluate gas viscosities.

APPLICATION TO ELECTRIC RESISTANCE MEASUREMENTS

Sometimes the radius of a glass capillary bore is determined by filling the bore with mercury and measuring the electric resistance of $x = \frac{40 \times 10^{-6}}{152 \times 10^{-4}} = 0.1$

and figure 2 gives 5 = 1.000042.

The net change in reding, about 0.26 percent, in the 19-foot enction may have been due to the fact that the section came from one and a a batch. It is not known shift way the anothel wire was withdrewn, with reapect to the 19-foot inspil, but the only two possibilities are that essentially either all or once of the mendral pessed through it. It seems likely that a section from the conter of a hypigal 200-foot batch would be note sufform then wither of the and sections. Also, the 19foot and section of statoless steel capillary tubing was more uniform than some specially salected glass capillary tubing was more uniform cure. For instance, values of 2 as to 10000 for glass precision bore capillary tubing TR toported in (3).

In the ever future, the Melium Monearth Center plane to desembly the for use in equation (1a), by a gravimetric method using moreury. The reactions decommined should be very close to the r^A in equation (1). At the seme time, measurement errors in r, will be claimined by using equation (43) to compute 6. These measurements will be taken on a 20. foot long section of capiliary boing used to evaluate gap viscosities.

PELICATION TO ELECTRIC RESTSTANCE MEASUREMENTS

Sometimes the ratios of a gines capillary bore is determined by

the mercury (5). Then the resistance, R, is related to radius, r, by

$$R = \frac{\rho L_{T}}{\pi r^{2}}, \qquad (63)$$

where ρ is the resistivity of mercury and L_T is length of the bore. Because of non-uniformity of the bore the actual resistance will be greater than the resistance of the equivalent right circular cylinder by the factor $(1 + \Delta)$, where

$$1 + \Delta = \frac{r_{m}^{2}}{L_{T}} \int_{0}^{L_{T}} \frac{dx}{r_{i}^{2}} \cong \frac{r_{m}^{2}}{n} \int_{1}^{n} \frac{1}{r_{i}^{2}}.$$
 (64)

Using the same method as was used to obtain (55) gives:

$$1 + \triangle = 1 + 8 \left(\frac{r_{m} - r_{avg}}{r_{m}} \right) = 1 - 8 K_{avg},$$
 (65)

and

$$\Delta = \frac{1}{3} \alpha . \tag{66}$$

Therefore, the values of α given in tables 1 and 2 and shown in figure 2 may be divided by three to get the corresponding correction for electric resistance.

SUMMARY

In this report, all of the analytic solutions for δ are independent of length or number of cycles for a regular deviation. The correction the mercury (2). Then the resistance, R, is related to radius, r. by

$$R = \frac{p L_T}{\pi r^2}, \quad (63)$$

where p is the restativity of mercary and L_T is length of the bore. Because of non-uniformity of the bore the actual realstance will be greater than the realstance of the equivalent right circular cylinder by the factor $(1 + \Delta)$, where

$$+\Delta = \frac{1}{4\pi} \sum_{i=1}^{n} \frac{1}{2} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2}$$
(64)

Using the same method as was used to obtain (55) gives:

(23)
$$\cdot \frac{1}{6} = 1 + 8 \left(\frac{2\sqrt{2}}{6} - \frac{1}{6} + 2\sqrt{2} \right) = 1 - 8 \frac{1}{6} + 2\sqrt{6} = 1 + 8 \frac{1}{6} + 2\sqrt{6} +$$

ben

$$\Delta = \frac{1}{3} \sigma \, . \qquad (66)$$

Therefore, the values of & given in tables I and 2 and shown in Figure 2 may be divided by three to get the corresponding correction for electric resistance.

SUMWAY IS

In this report, all of the analytic solutions for 5 are independent of langth or number of cycles for a regular deviation. The correction factor, δ , usually will be negligible; deviations of 0.2 percent in the root-mean-square radius lead to corrections of less than 0.005 percent in computing the viscosity, η . For relatively large deviations, the equations in this report may lead to incorrect estimates of δ because the radial component of velocity may not be negligible.

The graph of percent relative deviation, K, versus α makes it possible to obtain rapid, accurate estimates of δ . The most conservative estimate is given by the square wave function; the most realistic estimate probably is given by the sine wave deviation.

If one can replace r^4 in equation (1) by the quantity in equation (2), it is not necessary to compute δ . On the other hand, if one has measured a value for r_{ms} , an estimate for δ may be computed from equation (43) or (55) but in both cases it is necessary to use r_m computed from equation (42) to get a valid estimate for δ . The estimated value for δ can then be applied to equation (1) with r_{ms}^4/δ substituted for r^4 . factor, b, usually will be amplighted in contentions of 0.2 payment in the most-mean-advance radius lead to corrections of less than 0.005 percent in computing the viscosity, b. For relatively large deviations, the equations in this report may hand to iterarreat sothers bacame the radial emponent of velocity may out be negligible. The graph of porteest relative deviation, X, versus 1 makes it the assisting to sotate relative deviation, X, versus 1 makes it estimates to sotate report he square wave functions the most conserve the assistance is glaved by the square wave function; the most conserve of the set motion is glaved by the square wave function; the most conserve the assistance is glaved by the sine wave destructor, it can have the structs probably is glaved by the sine wave destructor, if our conserve is negligible is accounted to the other hand, it can have destructed to the pay of the sine wave destructor (1) it is an observe to manying 5. On the other hand, it can have destructed from equation (1) by the quantity in equation destructed from equate them (43) or (35) hue to both reases it is meassance to use r_a-computed from equation (42)-sto get a valid certaeted for 6. The estimated value
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