Tractive Effort and Acceleration of Automobile Vehicles on Land, Air and Water.

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(MEMBER).

INTRODUCTORY.

THE subject of the present paper does not in itself possess any vast degree of novelty, neither is it one that offers great scope for original thought. It has been the author's endeavour to deal with some of the more generally neglected aspects of the subject, and to demonstrate the method employed by him for the measurement and recording of acceleration and tractive effort, also to show the uses and value to the automobile engineer of acceleration diagrams. A considerable number of actual curves taken on vehicles of various kinds are given.

There are three quantities relating to the traction of a vehicle whose algebraic sum is zero, namely, the resistance, the acceleration and the tractive effort. When the tractive effort is equal and opposite to the resistance there is zero acceleration. If the tractive effort is equal to the acceleration there must be zero resistance; and finally, if there is no tractive effort the resistance represents the acceleration, which under these conditions is of course negative.

In order to put the above facts into proper algebraic form we require to define the signs, and the units in which the various quantities are measured. Thus the tractive effort and resistance are commonly given in terms of *pounds per ton of gross load*, and quite frequently they are measured positive in opposite sense, thus the tractive effort is reckoned positive in the direction of motion and the resistance is positive in the reverse direction. Again, the acceleration is usually given in terms of gravity, that is in *poundals per lb.*, or in other words in *feet per second per second*, in brief, *ft./sec.*^{2}. Now this is all very well in its way, and there would be no need to object to the variety of signs and units, were it not that the three quantities form part of one account; and the position is comparable to the purchase of goods, where we have the balance of the account between the three quantities *the price of the article*, *the money paid*, and *the change given.* What should we think of the shopkeeper who marked his goods in £ *s. d.* and tendered change in francs and centimes? More than this, what should we think of the members of a trade or profession in which such a confused method was an established custom?

Let us assume that we deal with units of one kind in the measurement of tractive effort, resistance, and acceleration. For the moment, it does not matter what the particular unit may be that we ultimately select. The principles of account-keeping are the same in England, France, and Germany—that is, whether the unit is the sovereign, the franc, or the mark; similarly, it is of no importance whether we adopt *poundals per lb.*, *pounds per ton*, or *per cent. of weight*, so long as we express all quantities by one kind of unit.

Let us take the direction of motion as positive for all purposes, and let P be the co-efficient of traction, R the co-efficient of resistance, and f the acceleration; then $P+R$ is the acceleration—that is—

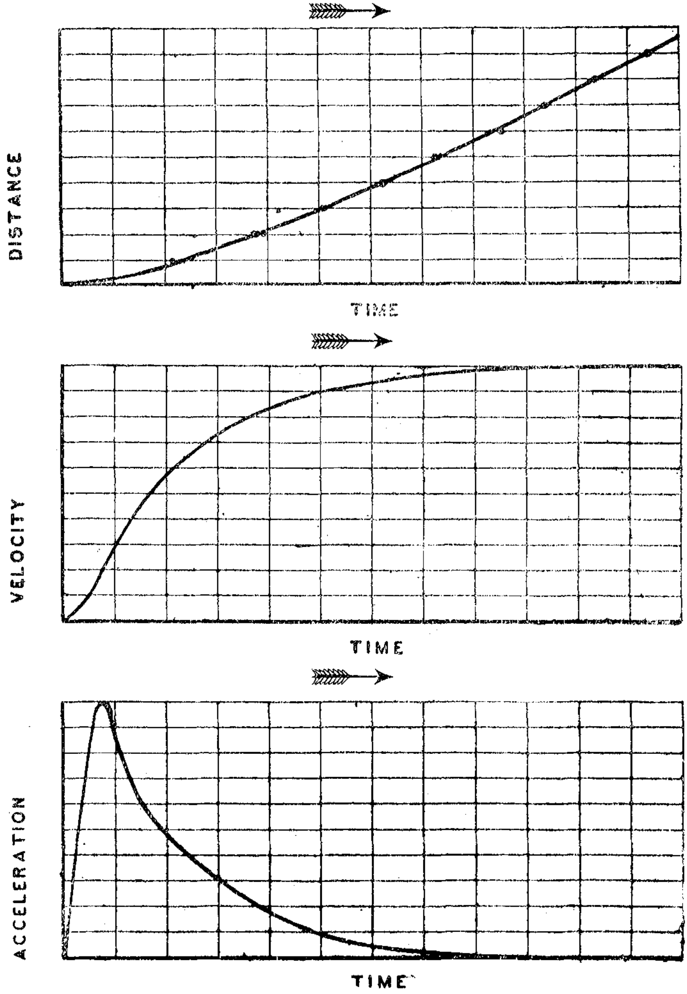
$$P+R=f.$$

So long as the applied force is one of propulsion P is positive. P only becomes negative when the brake (or some equivalent) is employed. R is always negative. Thus, if in any given case P is 3 (say poundals per lb.) and the resistance is 2 poundals per lb.—that is, $R=-2$

$$\text{then } f=3-2=1.$$

That is to say, f will be 1 poundal per lb., or one ft./sec. per sec.

The reason that *poundals per lb.* have been employed as the unit in the foregoing example is that this unit (as has already been pointed out), is in effect the unit commonly employed as a measure of acceleration in the form *ft./sec. per sec.*, or *ft./sec.*^{2}, and it is at the same time a form of expression more intelligible as a tractive effort or co-efficient; we are accustomed to *pounds per ton*, we are also familiar with *units per cent.;* one poundal per lb. is approximately equal to 70 pounds per ton, and roughly the equivalent of 3 per cent. In the present paper this unit is employed throughout in the abbreviated form pdls./lb., and a table is



Figs. 1, 2 and 3.

given in the appendix giving approximate equivalents in pounds per ton, p. 149.

MEASUREMENT OF ACCELERATION.

In years gone by the only known method of measurement of acceleration was by observations of the time required to attain a given velocity, the mean acceleration during the period comprised by the observation being thus obtained. The same method in another form involves observations of the displacement of the vehicle from its point of rest, the time being recorded for a number of measured points along the track; from data so obtained a plotting is made (Fig. 1), and the curve so given is transformed by successive differentiation first into a velocity curve (Fig. 2), and then into an acceleration curve (Fig. 3).

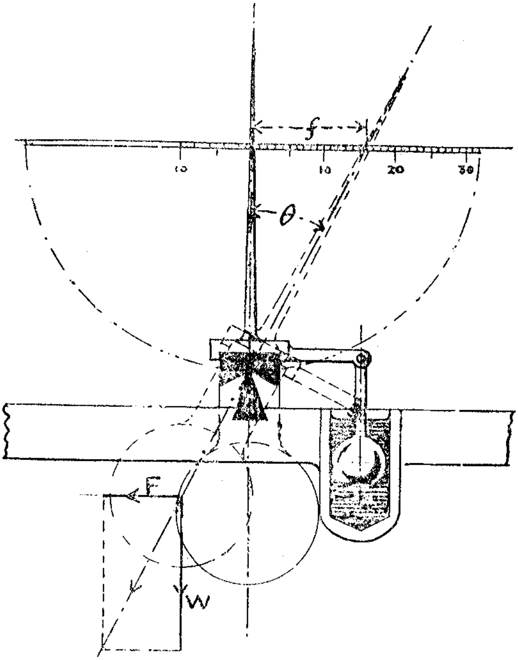


FIG. 4.

But there is now a method by which acceleration diagrams can be taken directly, and the foregoing process may be then inverted (if desired), and by integration the velocity and displacement curves can be plotted.

It is evident that when a body or vehicle undergoes acceleration positive or negative, the force of acceleration is shared by every portion of the body or vehicle in direct proportion to the mass, and if we provide means for measuring the force of acceleration of any small part of the vehicle whose mass is known, we at once know the rate of acceleration.

Thus, supposing that we arrange 1 lb. mass on a horizontal frictionless track at any convenient point on the vehicle, and measure the horizontal inertia forces in the line of motion by means of a spring balance (which we may suppose graduated to poundals), then the spring balance will read the acceleration that the vehicle is undergoing at every instant of time. If instead of the frictionless track and the spring balance we substitute a pendulum (Fig. 4), it is no longer necessary to know the mass, for whatever the size of the bob the acceleration is given in terms of gravity by the tangent of the angle of deflection; thus, in the figure the force W represents an acceleration of the bob of the pendulum $= g$, that is,

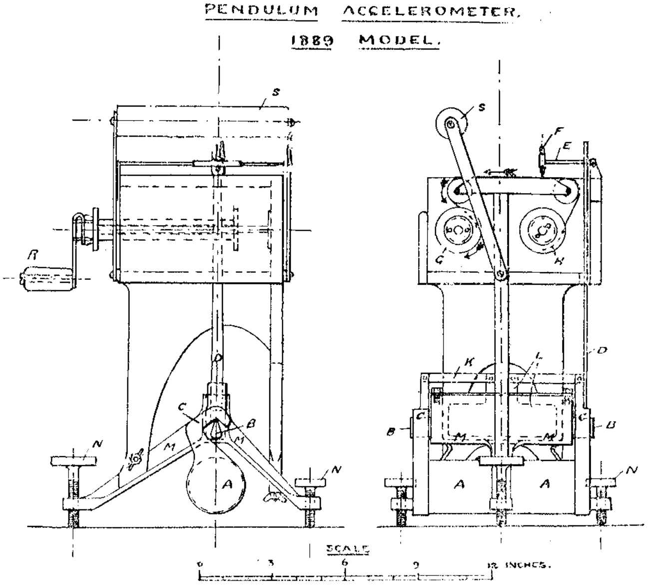


FIG. 5.

32·2, and under the influence of a force of acceleration F the position of the pendulum is determined by a simple parallelogram of forces, and F/W (see Fig. 4) is the tangent of the angle of deflection. Hence, if the pendulum be fitted with a pointer (Fig 4) recording on a horizontal scale graduated to read 32·2 when tan θ is unity, the instrument will show the acceleration f directly at every instant. An instrument embodying the above principles was originated by the author somewhat over twenty years ago, and is appropriately termed the "Pendulum (recording) Accelerometer."

THE PENDULUM ACCELEROMETER.

The author's original instrument was a home-made affair constructed in 1889 (Fig. 5). The elementary components of this old machine are functionally identical with a more modern and effective design dating 1904 (Fig. 6). Referring to Figs. 5 and 6, we have the pendulum A suspended on knife edges B, carried on a frame M provided with levelling screws N, and carrying a dash pot L. In the earlier model the dash pot consisted merely of a vane working in a trough; in the later model it takes the form of a spherical piston working in a short cylinder which it approximately fits; in both cases the charge consists of a highly viscous oil. The recording portion of the instrument consists of a "tan.-angle mechanism" E (given separately in Figs. 7 and 8), and of a continuous recording outfit G H. In the early model no clockwork was fitted, an omission of course rectified in the later machine shown in Fig. 9, Plate XIX.

The dash pot is necessary in order to prevent the setting up of oscillations by the vibrations of the vehicle; it is adjusted to the just dead-beat condition by varying the viscosity of the oil, a thick and a thin oil being carried. This is a detail that could probably be improved by the provision of an adjustable bye-pass.

The pendulum should be made as short as possible, firstly, in order that its oscillation period shall be rapid, and therefore easily damped, and, secondly, so that the motion of the pendulum bob shall differ as little as possible from that of the vehicle. It is evident that any movement of the pendulum bob relatively to the rest of the vehicle constitutes a source of error. It is, strictly speaking, the acceleration of the pendulum bob that is recorded, and the more nearly this approximates to that of the vehicle the better. The error from this cause is, in actuality, quite negligible

—the length of the pendulum in the modern machine is but $1\frac{1}{4}$ inches.

Attention may be called to the two forms of "tan.-angle mechanism," shown in Figs. 7 and 8. The 1889 type is merely a mechanical realisation of the requirements of the problem of the most obvious kind. It suffers from excessive friction; it required a ten pound bob and three inches pendulum length to operate the pencil without serious error. The improved type designed for the 1904 instrument is less obvious, and introduces an error (in practice quite negligible). It is, however, practically frictionless, except as concerns the pencil point.

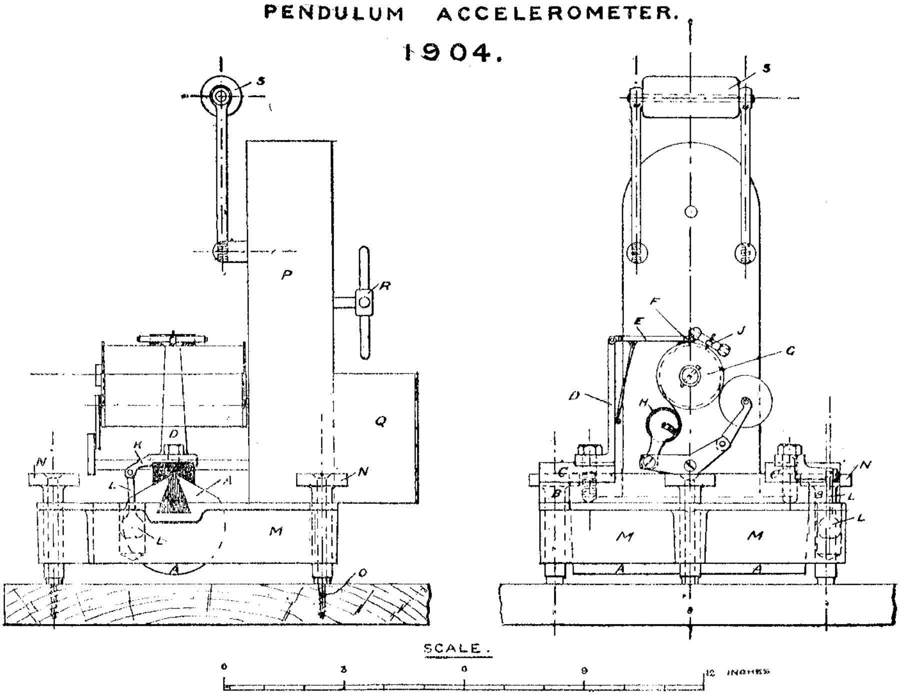
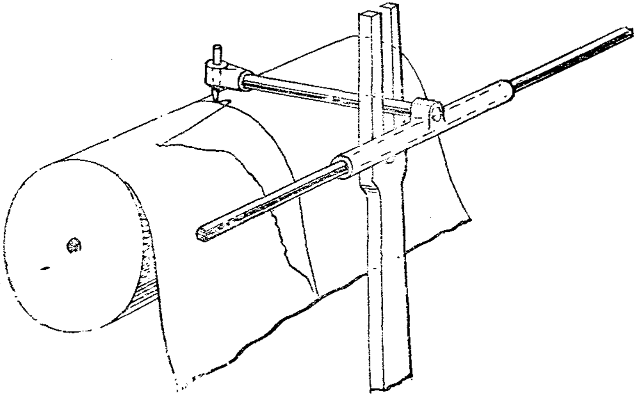


FIG. 6.

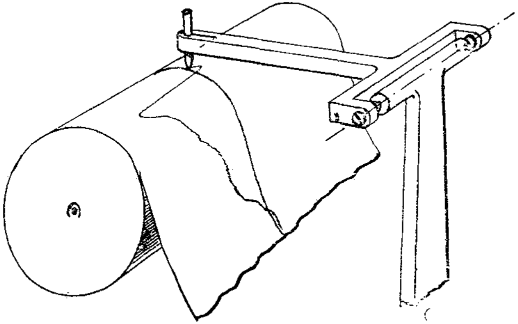
We will now turn our attention to a few of the diagrams taken by the aid of the Pendulum Accelerometer. The group of brake diagrams given in Fig. 10 were taken on the railway in 1889 by means of the original instrument; the time scale is therefore

TAN.-ANGLE MECHANISM.



1889

FIG. 7.



1904

FIG. 8.

irregular; the diagrams read from left to right. The acceleration is of course negative, and its magnitude is indicated in each case in the figure. The chief point arising from an inspection of these diagrams is the *sudden* change of acceleration at the instant of coming to rest. The same feature is shown also by the further group of railway brake diagrams given in Fig. 11, taken more recently on the Great Western Railway (local service) by the aid of the newer instrument. In all these diagrams we have the brake effort increased right up to the moment of stopping; this appears still to be the custom on trains serving the local and suburban traffic. If there were no elasticity in the structure of the train and track this change would be absolutely sudden; *until* the

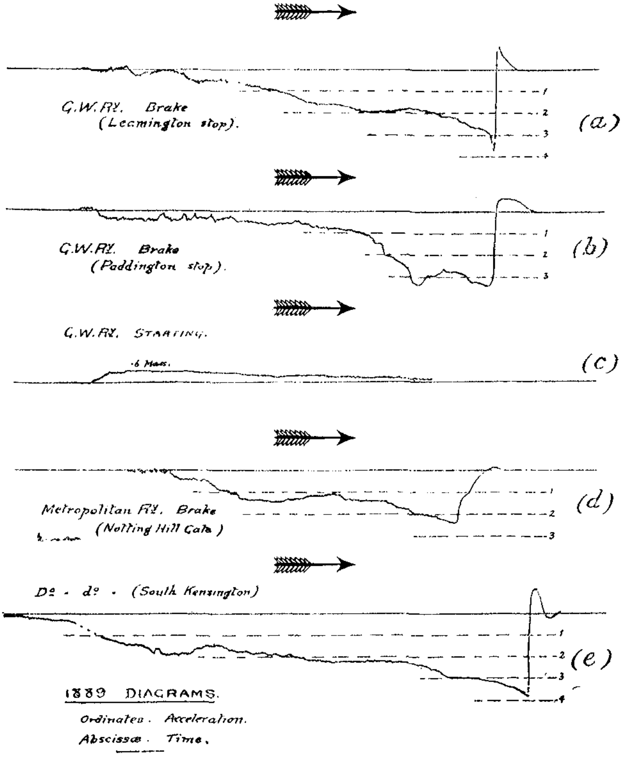


FIG. 10.

instant of stopping the negative acceleration would be that due to the brake application, *from* the instant of stopping the acceleration is zero. It is evident that a passenger standing in a vehicle always requires to adopt a position complementary to that of the pendulum; in other words, the pendulum indicates from instant to instant the apparent direction of gravity, and the passenger needs to stand at the same inclination as that of the pendulum. So long as the changes in the position of the pendulum are slow this fact presents no difficulty, but when an approximately instantaneous change takes place the passenger has no time to accommodate himself, and, standing in an inclined position, he finds himself quite suddenly unsupported, and the well known result follows. At the time the author made these early investigations the "jerk" experienced when a train stops was currently attributed to the recoil of the buffer springs, though doubtless there were many engineers in the country who knew better; in reality the "jerk" is, as has been shown, due to quite a different cause, and if the buffer springs have any influence at all, it is in the direction of ameliorating the conditions.

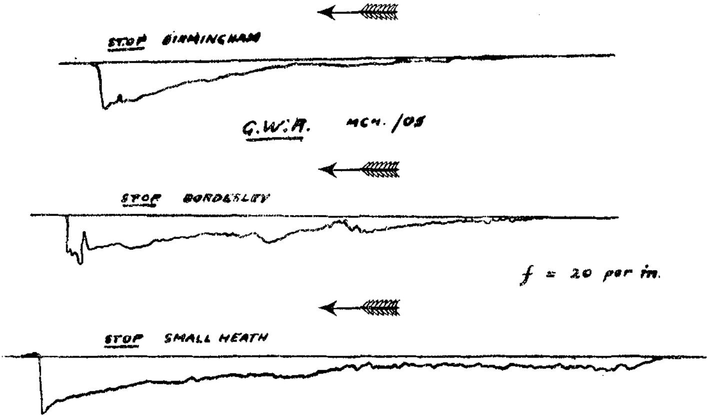


FIG. 11.

It is now well understood that the correct manner to brake a train or other kind of vehicle (if the comfort of passengers is considered) is to withdraw the brake somewhat as the vehicle comes to rest, the ideal condition being that at the instant of coming to rest the brake effort should be zero. In practice the most that can be done is to very materially diminish the extent of the brake effort, as shown by diagrams Figs. 12 and 13. It is, in fact, comparatively rarely that even this is done ; it is so easy for a driver to stop at a given place by reducing speed to about 5 or 10 m.p.h. and then virtually " clamp " the vehicle at the desired spot by the full application of the brake. It requires, on the other hand, very great judgment and skill and perfect mechanism to stop at an exact place with a tailing off brake diagram, and in the case of railways it is a question whether any great improvement will be made in this direction unless some automatic mechanism actuated from the track itself is adoped.

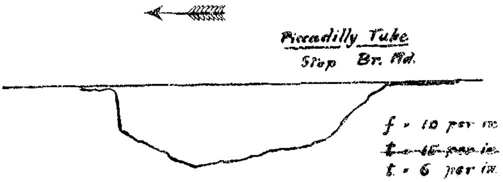


FIG. 12.

In the regulations relating to the management of urban railways it is not unknown to find that some stated acceleration must not be exceeded, the intention being the avoidance of discomfort and, more than this, of danger to the passengers. Now, while it is quite true that there must be a limit to the acceleration permissible dictated by considerations of safety, it is more than probable that this limit is far higher than is generally supposed, and that the *rate of change of acceleration*, *i.e.*, the *slope* of the curve, is the quantity that primarily causes trouble.

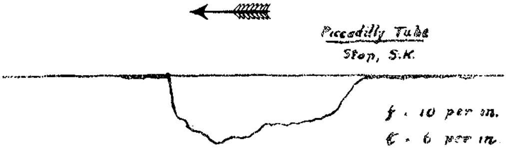


FIG. 13.

We may bear in mind that the consequence of any given acceleration applied continuously to the vehicle has precisely the same effect,* so far as the passengers are concerned, as the tilting of the

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* This statement is not, academically speaking, accurate ; there is in actual fact a slight increase in the *apparent weight*, which, however, does not become sensible unless the apparent angle is more than what is termed a *small angle*. The argument is unaffected.

vehicle through an angle whose tangent represents the uniform applied acceleration in terms of gravity. Now, if we suppose that the vehicle be tilted by slowly tilting the track, so that the passenger has ample time in which to accommodate himself to the change, it is obvious that the vehicle will begin to slide along the track before the passenger will have the least difficulty in maintaining his equilibrium, for the co-efficient of friction of shoe leather on wood is far greater than that of the steel wheels on the track. The interpretation of this is that so long as the rate of change of acceleration is gradual, there can be no danger, *per se*, in any acceleration that can possibly be applied to a vehicle on rails, even when driving or braking from all wheels up to the skidding point.

On the other hand we know by experience that a sudden unexpected drop of but one or two poundals per lb. is sufficient to throw a man down. We see, therefore, that it is the *rate of change* of acceleration that should be limited, and the acceleration curve should be of smooth contour, never exceeding some specified gradient; then there need be no artificial limit set on the maximum acceleration.

There is one further point of importance in connection with the present branch of the subject. Since a sudden change of acceleration results in the passenger's loss of equilibrium, much depends (if such sudden change takes place) on what the actual acceleration is immediately afterwards. Thus, if the sudden change occurs at the instant of coming to rest, the consequences are generally trivial, as the apparent position of the car is horizontal. If, however, the *same change* were to happen at the instant of starting, the passenger might be thrown from one end of a saloon to the other, for during the period subsequent to the initial start the apparent position of the car is at an inclination whose tangent represents the starting acceleration, and the position, so far as the senses of the passenger are concerned, is that he has been thrown down a steep hill.

In vehicles such as urban electric tramcars, a magnetic brake is frequently fitted, and when this is used the acceleration is not limited by the ordinary considerations; the slipping limit no longer exists, for the friction is not dependent upon the weight of the vehicle, but upon the magnetic suction, which may be made as great as we please. If too powerful a magnetic brake is employed to the full, it may be just as dangerous to the life of the passengers as an actual collision.*

SOME POINTS IN THE READING OF ACCELEROMETER DIAGRAMS.

So long as the road surface or track on which the vehicle is being run is level, no ambiguity arises in the reading of accelerometer diagrams; the instrument being once set true to the datum line by means of the levelling screws, the trace of the recording point accurately represents acceleration positive or negative, and this is, as before stated, the balance between the propulsion and resistance co-efficients. When, however, the instrument is operated on an inclined road, the conditions are somewhat different, the readings no longer give true acceleration, *but they continue to give the correct difference between the co-efficients of resistance and propulsion.* This is a very important and rather subtle point; it is so unexpected, that when published in the author's paper to the Physical Society in 1904, a well-known engineer in the midlands

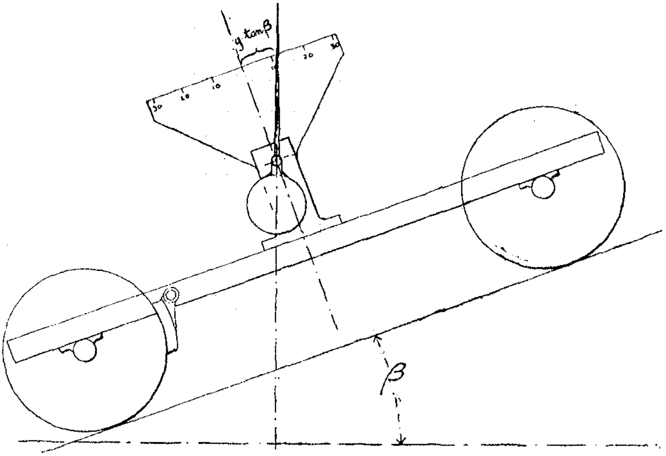

FIG. 14.

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* Passengers have been thrown from one end of a tramcar to the other, and others have been unseated, by the injudicious use of such a brake. In this case we probably have both the sudden change of acceleration and an excessive maximum immediately succeeding: the first throws the passenger down, and the second prevents the least chance of his recovery.

disputed it as a fact, and was not satisfied until he had actually rigged up a rough model and demonstrated the matter to his satisfaction.

Referring to Fig. 14, in which a car is represented as standing on the side of a hill, whose slope is β, with the brake applied, it is evident that, since the instrument has been set to zero with the car on a level, the pencil will record g tan. β, and this will be the correct measure of the total brake and frictional effort necessary to prevent the car running backward down the hill. If, now, the brake be removed, and we suppose frictional resistance and resistance of other kinds absent, the actual acceleration due to gravity will be g sin. β, as the car runs freely down the slope*; and this will for small angles neutralise the direct effect of the slope, so that the accelerometer will at once read zero; in other words, the direction of the apparent plumb is, under the conditions supposed, at right angles to the road surface. We thus see that when the road resistance is zero, or, similarly, if the road resistance and tractive effort are equal, the accelerometer reads zero on an incline just as it would on a level, and consequently any acceleration recorded on an incline represents the difference between the applied co-efficients of traction and resistance, just as is the case when the track is truly level. The only essential is that the instrument is set to zero at some known level place, or, alternatively, the levelling may be done on a slope, two readings being taken and the car turned end for end.

Evidently the diagrams obtained with the car travelling over sloping or undulating ground are useless for integration for the determination of the velocity curve, since the accelerations due to the slope are not recorded.

A source of error of some importance exists in the case of road vehicles, especially those fitted with light or "easy" suspension, owing to the changes of longitudinal trim with changes of acceleration. The accelerative forces may be taken as applied at the ground level, it is in fact at the road surface that the forces of traction are applied to the system *from without*; the centre of mass being necessarily above this level, a couple is set up which gives rise to a change in the weight distribution on the front and rear axles, and these changes of weight distribution result in the changes of trim aforesaid. A long wheel base and low centre of gravity, such as are now commonly employed, reduce the error under

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* The original text read g tan. β. Compare discussion, p. 150.

discussion, but it is desirable for accurate work to fit a car with specially stiff springs and to work on as smooth a road surface as possible.

THE ANALYSIS OF THE ACCELEROMETER DIAGRAM.

By means of an accelerometer diagram taken under the best conditions, *i.e.*, on a truly level road surface, it is possible to effect a complete analysis of the forces concerned in traction.

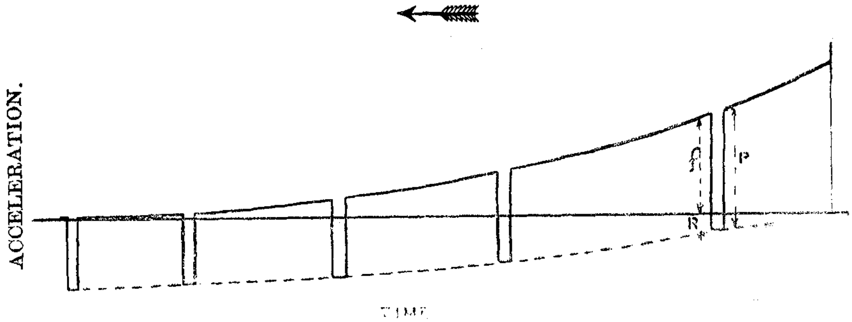


FIG. 15.

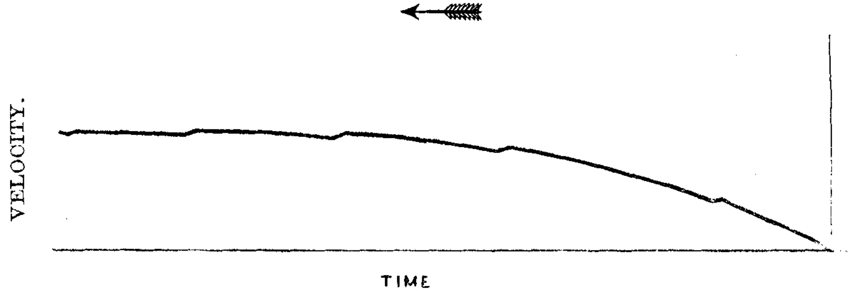
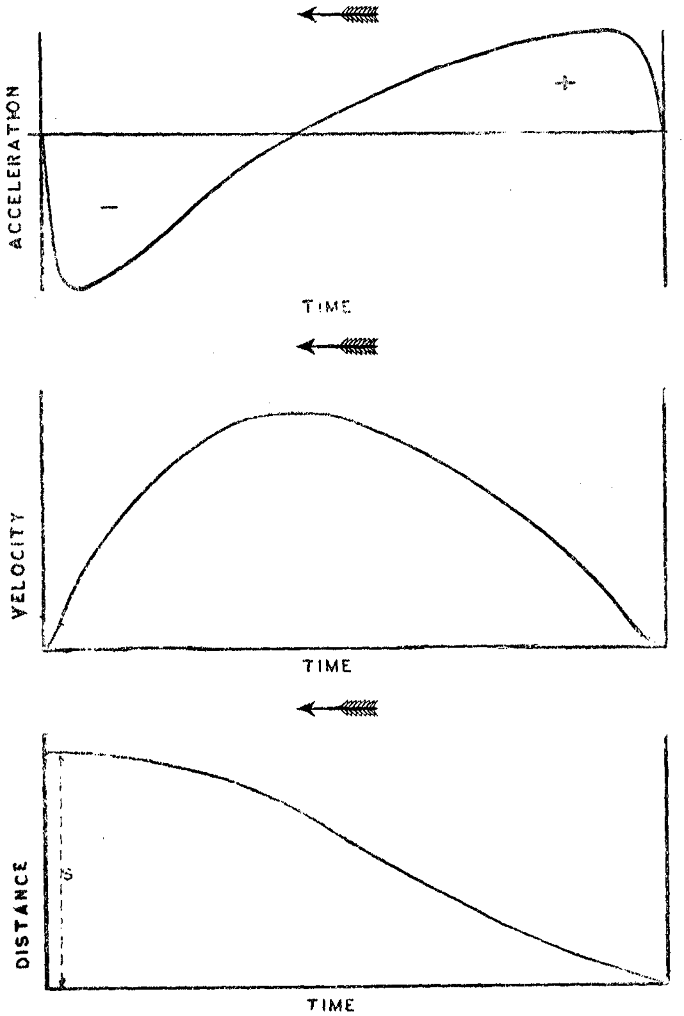


FIG. 16.

Referring to Fig. 15, let us assume a diagram taken on an ordinary motor vehicle with an intermittent withdrawal of the clutch, so that we have alternations that at intervals give respectively the net acceleration and the tractive resistance. Then we can readily calculate the velocity at every point (Fig. 16), the areas above the datum line in Fig. 15 representing plus velocity or velocity gained, and those below minus velocity or velocity lost. Then for each and all of the several velocities so calculated the original diagram gives the values of the three main quantities that constitute the traction account, *P* (the propulsion co-efficient), *R* (the co-efficient of resistance) and *f* (the acceleration), and the diagram also shows graphically the fact that these three quantities constitute an account that must always balance.



Figs. 17, 18 and 19.

It would manifestly be possible to plot and further analyse the resistance from the data obtained, to show its components variable and constant in respect of velocity.

As an ocular demonstration is always of value and interest, an accelerometer has been mounted on wheels to run on the lecture table, in order that diagrams may be taken in the presence of members of the Institution. An interesting demonstration both of the action of the instrument and of the relationship and integration of acceleration curves to those of velocity and displacement may be given. Thus at first sight it might seem difficult for a man completely enclosed in a box, and entirely cut off from the outer world, to have much knowledge of his movements, yet let him be armed with an accelerometer and a planimeter and the thing becomes easy enough.

Thus in Fig. 17, we have a simple starting and stopping diagram, the car starts from rest and comes to rest. Now, firstly, it may be observed that the areas cut off by the curve above and below the datum line must be equal, for these areas represent velocity gained and lost, and the initial state is the same as the final state. If we measure the area of successive portions of the curve, we can plot the curve of velocity (Fig. 18); we see this rises from zero, reaches a maximum at the point where the acceleration curve (Fig. 17) cuts datum, and then again falls to zero. Now the area cut off by the velocity curve in turn represents displacement; again using the planimeter we obtain the curve (Fig. 19). This represents the displacement of the vehicle at successive intervals of time (abscissae represent time throughout), and the total displacement S is the distance travelled. This seems a very indirect way of measuring distance, and certainly it is not very accurate—for long distances the error, which is cumulative, would be horrible—still, as a lecture table demonstration it is of interest. (See foot of p. 147, and Fig. 31, p. 148.)

Characteristic Features of Accelerometer Diagrams.

When we examine in detail the starting diagram taken by means of the accelerometer from a vehicle of any kind we are nearly always confronted with some characteristic features belonging to the particular mode of propulsion employed. Thus in diagrams taken from an ordinary petrol motor car we have the definite series of separate sections of the diagram representing the different gears employed; during the period of gear change the acceleration falls to zero or even shows the negative effort due to the road resistance. Further than this, there is commonly found in each section or portion of the diagram a break or drop in the curve; that is to say, the first portion of the curve is higher than the last portion, and the passing of the one into the other is not gradual; there is a definite point where the fall takes place. This is due to the flywheel effect; during the first part of the application of the gear the torque of the engine is augmented by the torque of the flywheel, the clutch in fact is slipping. From the time the clutch takes hold the torque is that due to the work done in the cylinder alone, and this is the second stage of the curve. These effects are very clearly evidenced in Figs. 20, 21, 22, and 23. In the Lanchester hand control it will

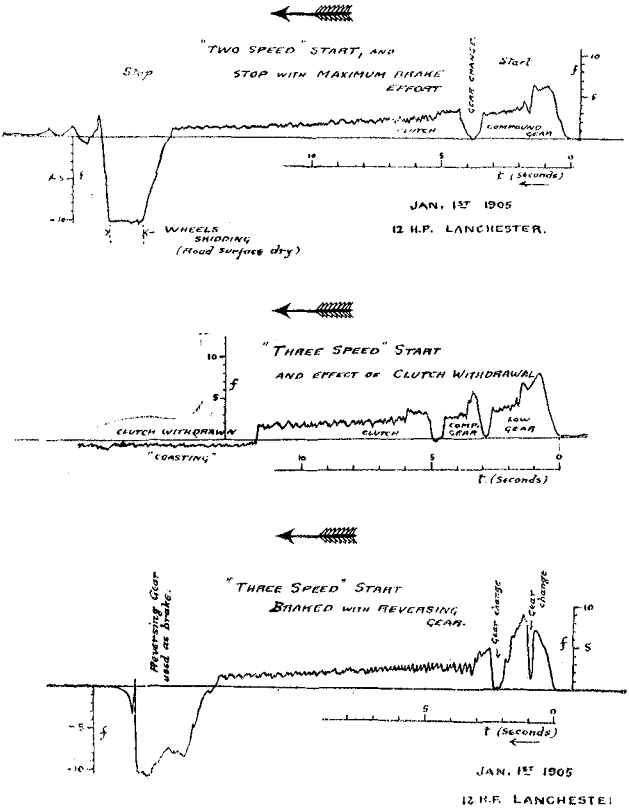


FIG. 20.

be remarked how little of the possible starting area is lost, the gear changes are represented by narrow gashes in the curve (Fig. 20), whereas the ordinary foot control commonly involves "valleys" of quite considerable duration (Figs. 21, 22, and 23).

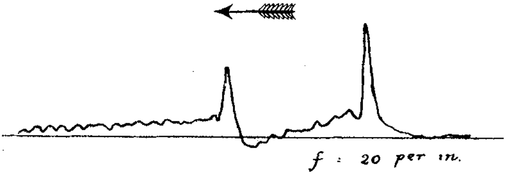


FIG. 21.

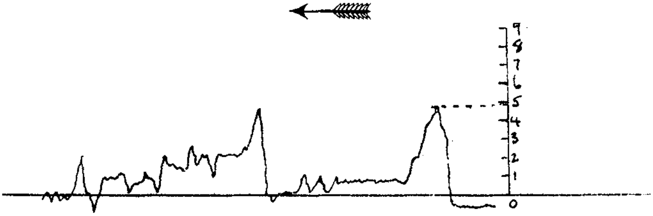


FIG. 22. Charron Taxi-Cab.

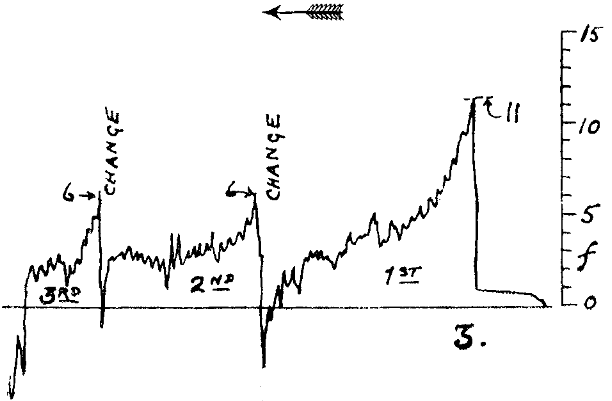


FIG. 23.

Next let us turn to the petrol-electric combination (known as the "Auto-mixte," or Electric Auxiliary system).

Fig. 24 is a group of diagrams taken from the new Daimler omnibus; (*a*) gives a start from engine standing, and the magnetic brake then applied to the full. In (*b*) the vehicle was started from engine running, and brought to rest by the application of the front wheel brake (foot applied). In (*c*) the start was from engine running and the stop by magnetic brake, the driver being instructed

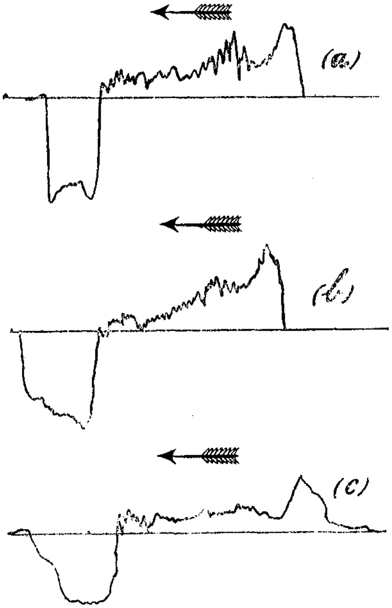


FIG. 24.

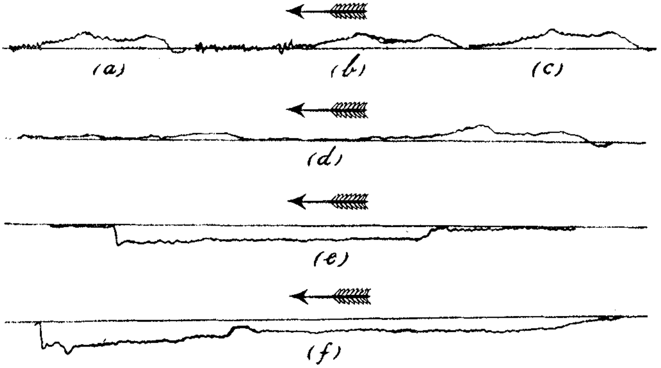


FIG. 25.

to act as if in service; this diagram shows well the capacity of the system for delicate handling. The withdrawal of the brake at the moment of stopping is a particularly good example of this desirable feature; the maximum brake application in this particular case is, however, not so good. The scale of these curves is, $f=10$ per in. and $t=15$ per in.

In Fig. 25 is a group of diagrams taken from a Daimler railway coach running on the London, Brighton and South Coast Railway. (*a*), (*b*), (*c*) and (*d*) are starting curves. Here we see an initial rise in which the acceleration runs up to about two pdls./lb., we then have a drop to about one pdl./lb. where the automatic clutch regulator comes into operation, then at a later point the acceleration again reaches about two pdls./lb., this is where the clutch is finally taking hold and just before slipping ceases. These features are always observable to some degree, though sometimes not to the extent shown in the examples given.

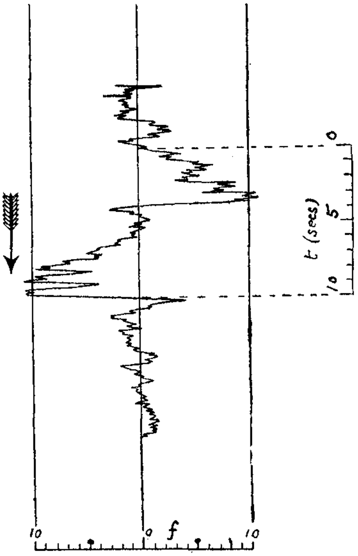
(*e*) (Fig. 25) is a brake diagram, and (*f*) is a stopping diagram in which the first portion of the curve is due to the use of the dynamo as motor, the energy absorbed passing into the accumulators; the later part of the curve (to the left of the "notch") being the brake diagram proper. The scale is $f=20$ per in., $t=15$ per in.

In the case of an ordinary railway train the distinctive features are less marked. The acceleration never appears to exceed one pdl./lb., and commonly remains round about half or three-quarters during the initial period of getting up speed; it falls to zero slowly as the resistance rises. The most interesting feature observed in the railway diagram is met with when the locomotive driving wheels slip. The acceleration immediately falls to about half its previous value, and only picks up again when the wheels resume their proper grip of the metals.

Almost every new type of vehicle to which the accelerometer is applied yields up the secrets of its mode of traction, and the weakness of each particular type, which in some cases of each individual design, is exposed in so graphic a manner as to render very great help to the designer in effecting desirable improvements. In this manner the superiority of one mode of control, or sometimes even of one carburettor, over another may be more readily demonstrated than by any other method.

FUTURE APPLICATIONS.

There are many uses to which an accelerometer may be put other than for the direct measurement of acceleration or starting effort. At present the author of this paper has not had occasion to carry his investigations beyond the more immediate applications of the instrument already given, but a few suggestions will not be out of place as to other possible uses. It may be remarked parenthetically that even the *imaginary* use of the accelerometer in certain cases is of material assistance to the mind in the understanding of the conditions of a problem.



DAIMLER 38 h.p.
Rounding "S" Bend ~
7/1/10

FIG. 26.

In the utilization of the accelerometer for some new purpose, the first step is to make a rough estimate of the magnitude of the maximum acceleration to be recorded, in order that the instrument may be set to a suitable scale. The adjustment of the instrument in this respect is effected by fitting a packing piece of appropriate thickness between the clockwork carrying the recording drum and the base casting on which the knife edges are mounted. When the instrument is set up without packing, the scale is 10 ft./sec.^{2} per inch, and the maximum plus or minus 17.

Let us suppose that we apply the instrument to a flying machine. We know in this case the tractive effort will exceed 5 pdls./lb., for this is roughly the resistance coefficient. Let us take it that it may reach double this value. We may decide to use the instrument either with or without packing. Then, firstly, we set the instrument to zero, with the flying machine in running order and trim on a level platform. We then take diagrams just as with a motor car, either starting effort and resistance diagrams if the ground is undulating, or true acceleration diagrams if an accurately level ground is available. It is doubtful whether acceleration diagrams can be effectively taken when a machine is in flight, and also whether they would serve any useful purpose, but as a means of investigating the starting conditions and the propeller thrust at low speeds, the method should certainly be found of service.

It is not possible to use the pendulum accelerometer directly on an ordinary hydraulic lift, owing to the fact that all accelerations are vertical. By making the lift tow a trolley carrying the accelerometer by means of a cord passing round a pulley the difficulty may be overcome. It would be quite possible to construct a special form of accelerometer comprising a vertically moving weight supported by a spring for elevator work, but the pendulum principle is not applicable.

The pendulum accelerometer may be used transversely on a vehicle to measure the lateral stresses to which the tyres and structure of the vehicle are subjected when turning corners or taking curves (Fig. 26). Here, again, it is not the centrifugal force that is measured unless the road happens to be level (laterally), it is the actual tangential force. Thus, if a car were travelling round Brooklands track at a speed exactly appropriate to the banking, the accelerometer (arranged transversely) would read zero. Conversely if the car were merely standing on the track, the accelerometer would correctly record the lateral effort exerted by the tyres in preventing the vehicle from sliding sideways.

The use of an accelerometer for recording the effort at right angles to the direction of motion, brings clearly into prominence the advantages of "transition" curves, such as are adopted by engineers in laying out a railway track. It is evident that if two straight sections of the line were connected by the arc of a circle, the accelerometer would record a sudden change at the points of juncture, which we know to be a most objectionable feature, and this sudden change could not be obviated or mitigated by the banking, for the banking cannot begin *suddenly*. By the adoption of a form of curve in which the radius of curvature changes gradually after the manner of a hyperbola of which the two straight limbs are the asymptotes, the difficulty is overcome, and the banking may be fitted everywhere to the curvature. The accelerometer might be readily employed in the manner now proposed to survey existing lines and determine where defects exist, and where improvements might with advantage be made. When used in this way, the most rigid test would be obtained by placing the instrument directly over a bogie trunnion, or, if the initial condition of the track is not perfect enough to permit of reasonable diagrams being obtained in this position, a first approximation showing only the more serious defects could be obtained if the instrument be set up about the centre of the coach, midway between the two bogies.

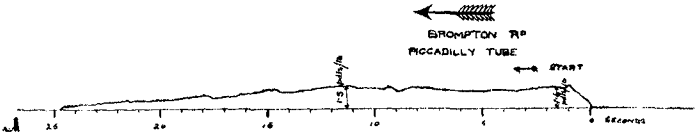


FIG. 27.

The accelerometer may be used for ships and launches; but where a considerable change of trim takes place in high speed craft by the sucking down of the stern, the readings will be vitiated to the extent of the change of trim, unless precautions are taken to level the instrument to the running condition. Under ordinary conditions the acceleration in ships is pretty well ascertained by the torque of the engine and pitch of the screw. In certain cases, however, as where a propeller of variable pitch is used, or where the torque at low velocities is not known with precision, the results would be valuable. The value of the coefficient of resistance in vessels of different classes varies from upwards of 3 pdl./lb. in high speed craft to as little as 0·03 pdl./lb. for slow

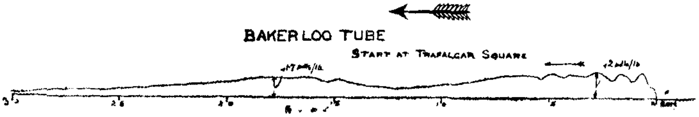


FIG. 28.

tramp steamers. These figures represent also approximately the initial accelerations at the moment of starting.

Figs. 27 and 28 are from two London tube railways where continuous current multiple unit electric traction is employed. The controller notches, and series and parallel running are easily noted.

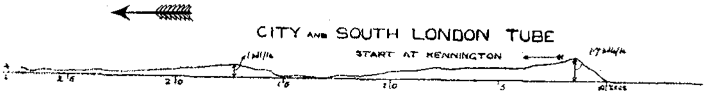


FIG. 29.

Fig. 29 is from a continuous current railway using locomotives.

Fig. 30 is from a single-phase multiple unit train. This is controlled by variation of the transformer windings, the steps being sharply marked. Two braking diagrams of this railway are shown superposed. The dotted one, though spread over a longer time than the other, was very disagreeable to the passengers owing to the abrupt finish. The one in full lines was a good example of rapid yet smooth stopping.

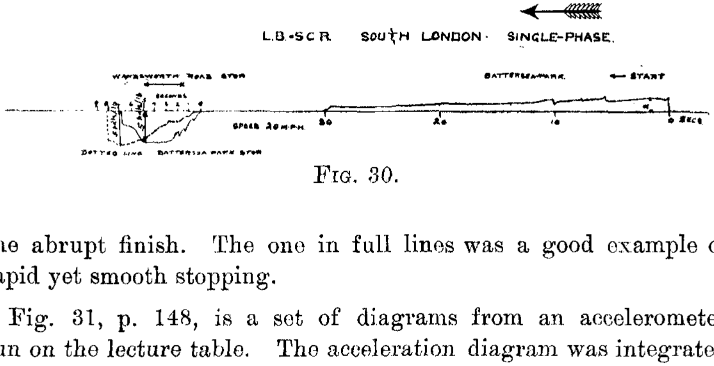


FIG. 30.

Fig. 31, p. 148, is a set of diagrams from an accelerometer run on the lecture table. The acceleration diagram was integrated by the planimeter and plotted as a velocity diagram, this being integrated similarly and plotted as a distance diagram. The calculated distance was 7 ft. 3 in., the actual measurement being 7 ft. 6 in.

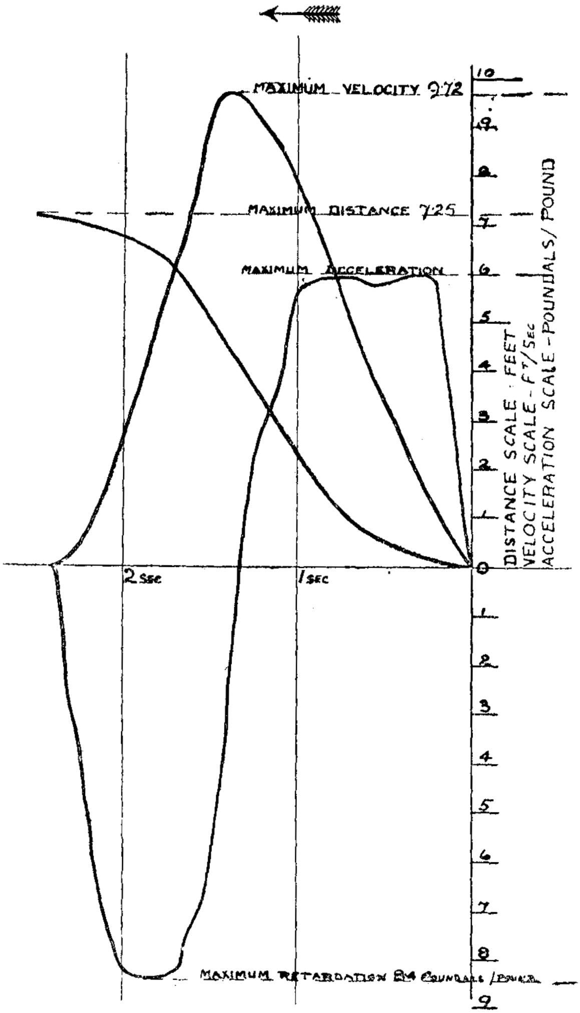

MAXIMUM VELOCITY 9·72
MAXIMUM DISTANCE 7·25
MAXIMUM DECELERATION
MAXIMUM RETARDATION 8·4 POUNDALS /POUND
2 SEC
1 SEC
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
DISTANCE SCALE : FEET
VELOCITY SCALE - FT/SEC
ACCELERATION SCALE - POUNDALS/ POUND

FIG. 31.

APPENDIX.

APPROXIMATE EQUIVALENTS.

Poundals per lb.	Pounds per ton.	Poundals per lb.	Pounds per ton.
·1	6·9	2·6	180
·2	13·9	2·7	187
·3	20·8	2·8	194
·4	27·8	2·9	201
·5	34·7	3·0	208
·6	41·7	3·1	215
·7	48·6	3·2	222
·8	55·6	3·3	229
·9	62·5	3·4	236
1·0	69·5	3·5	243
1·1	76·4	3·6	250
1·2	83·4	3·7	257
1·3	90·4	3·8	264
1·4	97·2	3·9	271
1·5	104·0	4·0	278
1·6	111·2	4·1	285
1·7	118·0	4·2	292
1·8	125·0	4·3	299
1·9	132·0	4·4	306
2·0	139·0	4·5	313
2·1	146·0	4·6	319
2·2	153·0	4·7	326
2·3	160·0	4·8	333
2·4	167·0	4·9	340
2·5	173·0	5·0	347

THE DISCUSSION.

The President: We have had to-night what we expected from Mr. Lanchester, a brilliant paper. He has dealt with a most difficult subject in his inimitable manner. I shall reserve any remarks I have to make until other speakers have had an opportunity of taking part in the discussion.

Mr. H. E. Wimperis, in opening the Discussion, said: I thank you for giving me the opportunity of making a few remarks. I am only a guest here, and I appreciate your courtesy, Mr. President, and that of the Institution, very much.

I have carefully read Mr. Lanchester's earlier paper on the subject in the *Philosophical Magazine*, and I am glad to see that his experiments have been extended. Before going on to describe some of the tests which I have made myself, I will refer to a remark on page 136 of the paper. In his description of the diagram, Fig. 14, Mr. Lanchester says that "the actual acceleration due to gravity will be g tan. β"; that cannot be right, for if β were 45 degrees, you would have the acceleration down this slope coming out at 32 ft. per second per second. A correction ought to run through that page, I think.

We have had much given us to-night on the scientific side of this subject, and I want to tell the Institution something of the purely practical side which appeals to automobile engineers.

The most important measurement you can make on motor cars is probably that of the coasting retardation, because if you take that measurement in ft. per second per second, and multiply it by 70, you at once get the resistance overcome in pounds per ton. There must be many persons who have had to watch cars on test, and they will know that purchasers often specify that the consumption must not exceed a certain figure per mile or per gross ton-mile, particularly in the case of tests with special fuels. When the road test is made there is no foretelling what the weather is going to be, whether the roads will be dry or whether they will be heavy and sticky, so offering great resistance. It is not fair, therefore, to compare one vehicle with another unless you make measurements of the road resistance during the tests, and there is a necessity for some instrument which can do that. This necessity came home very strongly to me when I had occasion to watch a number of official tests of this kind, and I was driven to invent an instrument for the purpose. In conjunction with Mr. Elphinstone, of Messrs. Elliott Bros., I experimented in this direction. Mr. Lanchester's instrument would have been too big for us; we wanted one not bigger than a man's two fists, which would give correct readings, be quite cheap to make, and be fool-proof.

As a result of our investigations, we were able to get out a little instrument, which was carefully tested first by ourselves and afterwards by the War Office officials. The tests were so successful that now both the War Office and the Crown Agents, who between them are very large purchasers of cars, have instruments for use on official tests. I have already made a great many tests myself. Only to-day I have been watching the official trials of a 2-ton motor waggon which weighed 4·35 tons. I may explain that to measure the coasting retardation, the instrument must be placed in some convenient part of the car, and levelled with a screw. Then the car is run along the road, up or down, it matters little which, the clutch is withdrawn, the reading of the pointer taken, and that is all. The scale on the dial of the instrument used gives the resistance in pounds per ton, so that no multiplication or other calculation of any kind is necessary.

In the course of my experiments I have found that the resistance of clean wood paving (with tramlines) is 70 lb. per ton. On passing on to thoroughly good dry hard macadam, the resistance is not affected; on a very muddy and sticky road, the resistance jumps up to 95 lb. per ton; on road metal partly rolled, with the rollers still at work, it goes up to 120 lb. per ton, while on loose road metal not rolled at all it increases at once up to 200 lb. per ton. These measurements are of real importance. The resistances measured include the road resistance, the road wheel bearing friction, and everything up to the clutch. Suppose that there is a specified fuel consumption for the test, and that a day is chosen on which the resistance is never more than 70 lb. per ton. Naturally a high fuel efficiency is obtained, and the car will run many miles to the gallon of petrol consumed. If, however, the day were to be such as would cause the road to be very muddy and sticky, corresponding say to 95 lb. per ton resistance, the fuel consumption, as measured by gross ton-miles per gallon, would drop; that is to say, the fuel consumed would be greater in proportion to the distance covered and the load carried. To work to a definite standard in the testing of vehicles, measurements must be made of the road resistance, and account taken of the circumstances of each trial, in order to be able to make fair comparisons.

There is another important point; if this resistance is measured, and the speed is also noted by means of a speedometer, the brake horse-power given off by the engine at that speed can be calculated. Furthermore, if fuel economy is also being measured, a comparison of the brake horse-power with the fuel used immediately gives the brake thermal efficiency of the engine, which cannot otherwise be measured on an ordinary road test. I do not believe that the brake thermal efficiency can be got at practically in any other way, unless days and days are given up to research, whereas these records are obtained with this instrument during an ordinary 50 mile test.

I now come to another point based on measurements which I have made, and I notice that Mr. Lanchester has got nearly the same results. The greatest braking retardation I have measured is 10 ft. per second per second. Such rapid braking is very uncomfortable, as Mr. Lanchester says. In a chain-driven vehicle, by putting the foot brake down hard, this retardation of 10 ft. per second per second can be obtained. The biggest driving pull ever obtained on the chain of such a vehicle as that which I have mentioned, is small in comparison with the pull on a chain which results from putting on the brake, and it is interesting to measure the relation of the two. The pull on the chain in this case is, I find, about five times as great when braking with the foot brake, as it is on the hardest drive the engine could give. Everyone has known that some such effect as this is produced, but it is interesting to get actual figures.

I should like to say, if Mr. Lanchester will not mind a little criticism, that I do not think the "damping" arrangement of the dash-pot in his instrument is a good one. It is necessary to have two kinds of oil in use, and any changes in the temperature of the atmosphere on the day of observation, will change the viscosity of the oil. I have introduced into my instrument what is admitted to be the most perfect form of damping arrangement there is, namely, the magnetic brake; that is a magnet operating on a copper disc.

Mr. LANCHESTER: Can you give us a brief description of your instrument, Mr. Wimperis?

The PRESIDENT: We are also curious to know the exact size.

Mr. WIMPERIS: The instrument is about 4 in. across, and the depth 3 in. It can be put in the pocket.

The PRESIDENT: Do you fix it on the dash board?

Mr. WIMPERIS: You can fix it there if you like, but I have so far found it more convenient to carry it loose.

The general appearance of the instrument is shown in Fig. 32, Plate XIX.; the internal mechanism is illustrated diagrammatically in Fig. 33, and the "compensating principle" is shown in Fig. 34.

The general nature of the internal mechanism may be described as a lop-sided copper disc, mounted on a vertical axis and controlled in its rotation by a coiled spring. The acceleration causes the heavier side of the disc to lag behind, and so partially wind up the spring. The degree to which the spring is wound up measures the acceleration. Any tendency of the disc to oscillate is checked by a magnetic field at right angles to the plane of the disc. Besides these parts there is the "compensating balance," which causes the instrument to record absolutely correctly even when travelling rapidly around curves, or when the road is cambered in one direction or the other.

With the aid of Fig. 33, the method by which the parts work together can be explained. The copper disc *D* has a hole *H* cut in it in order to throw the centre of gravity slightly out of the centre of the disc. On its axis is fastened a gear wheel G_1, which moves with the copper disc. This gear wheel is in mesh with a second gear wheel G_2, which carries the reading pointer *N*. This pointer travels over the scale seen in Fig. 32. *M* is the magnet for damping the motions of the disc, and *L*, *L*, *L* are the three legs upon which the instrument stands, that on the right hand side being adjustable by means of a screw.

The spring which is coiled up by the rotation of the disc is not shown, but it lies in a horizontal plane just above the disc.

It is now necessary to describe the mode of action of the compensating device. If, when the disc is deflected by an acceleration, a second acceleration occurs at right angles to the first and in the plane of the paper it will also produce a force on the disc which will have a moment about the axis, and so tend to wind or unwind the spring, which then must give false indications of the acceleration in the direction in which the instrument is set to record. This would be a very serious fault, as such transverse accelerations are very common in practice, and often of considerable amount—sometimes as much as 10 ft. per second per second. The writer, therefore, added the gear wheels G_1, and G_2, and so arranged the weight of the pointer N, which is fixed to the gear wheel G_2, that the product of weight by the distance of the centre of gravity from the axis of rotation is just the same for the element "copper disc + gear wheel G_1" as it is for the element "pointer N + gear wheel G_2." This equality of mass moments makes the system equivalent to two equal copper discs geared together at their circumferences as shown in Fig. 34.

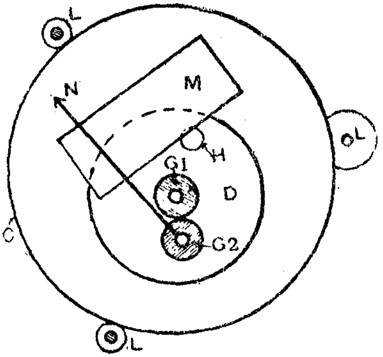


FIG. 33.

Inspection of this Fig. 34 will show that forces in the direction P will cause the two discs to roll together, but forces in the direction Q cause no rotation whatever. Forces perpendicular to the paper (considerable as they are known to be) can, of course, cause no rotation of the disc, so that the instrument records *only* the forces in one of the three directions of space, and is not concerned with what may be happening in the other two directions. Thus the dial of the instrument may even be held vertical, so allowing an acceleration of 32 ft. per second per second to act across the dial, without the readings being affected at all.

Continuing, Mr. WIMPERIS said: I think I may claim to have produced what is the first instrument for recording accelerations in one direction of space only. Every other instrument is affected by vertical or side oscillations or by the camber of the road. The damping magnet is powerful enough to damp out any tendency for the needle to swing, and even on the District Railway, which is very "jolty," the pointer is quite steady.

There is only one other thing I have to add. I could have wished that Mr. Lanchester had not introduced the expression "poundals." I do not think most engineers know what poundals are. Everyone can understand a train gaining a velocity of so many miles per hour every second, or an acceleration of so many feet per second per second, but I question if there are many engineers who understand what $7\frac{1}{2}$ poundals per pound means. Although the "sums" Mr. Lanchester gives may be nicer to work out in this way, I do not think any real practical advantage can be got from the use of this term.

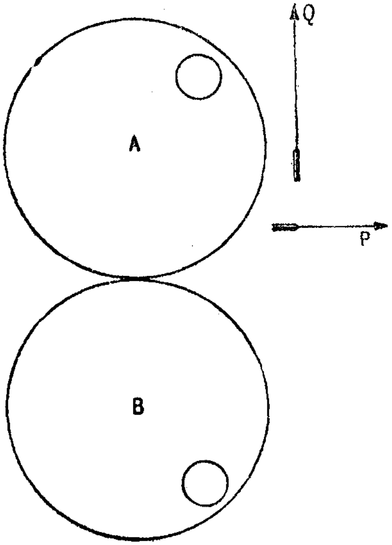


Fig. 34.

Mr. L. A. Legros: I think Mr. Lanchester would add greatly to the interest of his paper if he could give us, in addition to the diagrams he has shown, one or two taken from the Tube or the Metropolitan Railways, and also from the Electric Trams with the magnetic brake.* I have seen some extremely quick stops made, and every motorist knows how very inconvenient it is for a tram to stop quickly in front of his car. Trams with slipper-brakes can usually stop more quickly than trams with rim-brakes. On the Continent many trams are fitted with magnetic rim-brakes so arranged that when a wheel, which is just on the point of skidding, begins to slip, the brake is automatically released. The result is that the brake can give the maximum effect and this can be extremely unpleasant as Mr. Lanchester has pointed out.

With regard to the jerk, a number of years ago there were endless discussions about it, and I think the matter was pretty well laid to rest at that time.

Mr. LANCHESTER: What date?

Mr. LEGROS: About 1884.

Continuing, Mr. LEGROS said: In the ordinary railway train the brake blocks are arranged as shoes on both sides of the wheels. When the train is moving and the brakes are applied, the forces exerted on the brake blocks by the wheel form a couple p q which tends to rotate the frame of the carriage, Fig. 35, so that the load on the springs at the leading end is increased, and that at the trailing end is decreased to form the balancing couple P Q. As soon as the carriage comes to rest, this couple P Q at once causes the wheels to rotate backwards till equilibrium is restored. When the brakes are released the energy stored in the buffer-springs may cause a further backward movement of the carriages.

Mr. E. G. E. BEAUMONT: I would like to add my personal expression of objection to the use of "poundal" units. Mr. Wimperis has suggested that most engineers do not know the meaning of the poundal and, although I do not agree with this statement, it is evident that the gravitation units are more commonly employed as being the more convenient at present.

Mr. Lanchester has pointed out in his paper that the poundal/lb. is the equivalent of the force which effects acceleration of the pound mass at the rate of one foot per second per second, and from this it may readily be understood that conversion from the one unit to the other is simple. Mr. Lanchester

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* These have since been added by the Author in the text.

stated that there was an advantage in using the poundal unit because the same unit was then employed to represent the different quantities in acceleration calculations; does he, however, consider that quite true, inasmuch as it would usually be necessary first to convert the quantities representing, for instance, tractive force and resistance. It is, I think, easier to imagine or realise the value of velocities in feet per second and efforts in pounds than in the poundal units.

With Mr. Legros, I hope that other diagrams will be added to the paper, for they serve as records of the rates of acceleration *realised with vehicles as now constructed. I was much struck* by the very low rates of acceleration of the steam railway trains. The comparison with the rates of acceleration of automobiles is much in favour of the latter, but it must be remembered that the rates for the electric lines are even higher, at any rate approaching the rates obtained with the Daimler Automixte omnibus when starting with the dynamotor only.

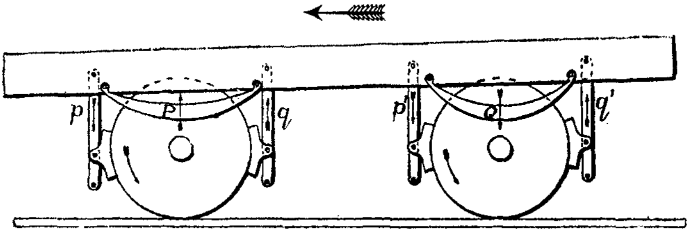

FIG. 35.

May I ask whether Mr. Lanchester has used his instrument on the new single-phase alternating current electric trains of the L. B. & S. C. railway? I have noticed there that the acceleration is not uniform, there being a peculiar jerk at some of the controller positions. As Mr. Lanchester has possessed himself of several secrets he might, perhaps, be induced to acquire that also.

The PRESIDENT (Dr. H. S. HELE-SHAW): This is the right time to propose a vote of thanks to Mr. Lanchester, and I need hardly say that I am sure you will pass such a vote with acclamation. Personally, I have a bone to pick with Mr. Lanchester for not being present on one or two occasions when we had hoped that he would have been, but on one occasion he was ill. To-night, however, he has redeemed everything by giving such a splendid and interesting paper, and Mr. Wimperis has added to its interest. It is more than twenty years since Mr. Lanchester first used his instrument, and nobody can deprive him of the credit of being the first to investigate this subject.

I would like to make a further suggestion, and to ask Mr. Lanchester whether he has considered the vertical pivot. I hope he will also say a word or two about the vertical versus the horizontal movement, since in Mr. Lanchester's instrument it is horizontal, and in that of Mr. Wimperis it is vertical.

I think, perhaps, Mr. Lanchester will allow me to say that I hope he will go over the paper, and put an arrow on every figure to indicate which way the curve is taken, as this will make his diagrams much clearer.* With these remarks, I ask you to give Mr. Lanchester a very hearty vote of thanks for one of the most interesting papers we have ever had before us.

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* This addition has now been made to the diagrams.

COMMUNICATED.

Mr CHAS. E. G. HOUSE wrote: There seems to be a physical objection to the method of the direct recording of acceleration, because of the impossibility of recording an acceleration instantaneously from the start. The recording point must always begin by darting out from the zero position to whatever the reading is. Now, if a car started off with simple harmonic motion, its acceleration would, of course, be a maximum at the instant of starting. It may be that it is impossible to commence with definite acceleration in this way, and Mr. Lanchester evidently is of this opinion if one may judge from the curves on pages 125 and 138. In coming to rest the same thing might occur, only the other way round. Now, suppose a car were to come to rest according to harmonic motion, the rate of change of acceleration would be infinite, and yet it seems difficult to believe that it would have the effect of unsettling a passenger in the car to the extent stated in the foot-note to page 135. Perhaps Mr. Lanchester has an explanation of the difficulty.

It would be interesting to learn what steps have been taken to ascertain the degree to which the results are affected by the lag of the pendulum. True, the lag is only a short distance behind the rest of the vehicle, but this occurs in a small period of time, so that the velocity might be relatively great. It must be remembered in this connection that there is no restoring force on a pendulum when in its central position.

It may not be generally known that it has been proposed to use as an accelerometer an instrument like a spirit-level, but having the curvature much greater, and this affords a roughly accurate apparatus for all ordinary purposes. It can be readily calibrated by tilting in the manner explained by Mr. Lanchester. The vertical shocks tend to spoil the readings, but the transverse shocks which Mr. Wimperis has taken such pains to counteract, are entirely absent. The results also cannot be automatically recorded, and in this respect it is not inferior to the Wimperis instrument.

In connection with the use of a spirit-level, I notice that

Mr. Lanchester suggests (page 146) that the accelerometer could be used on a launch, taking precautions to level the instrument to change of trim when the suck-down occurs. I would point out, however, that the spirit-level no longer reads correctly when the change occurs, but perhaps he would use a gyrostat for the purpose. The change of acceleration when the stern sucks down would be interesting to ship designers.

It would appear that valuable information could be obtained if the accelerometer were applied to the measurement of the vertical accelerations during the springing movements of a car going over a bumpy road. Mr. Lanchester's instrument in its present form could not be used, but that of Mr. Wimperis is quite well adapted for the purpose. The springs could be readjusted to get rid of the gravitation constant if there were any danger of running off the scale. Further applications to lifts, etc. could be made.

It would be interesting to be able to observe rotational acceleration also, and find out the efficacy of clutches, spring-drives, and such devices, but it must be admitted that it would not be so immediately useful to the designer as the present instrument, besides being very difficult to construct.

AUTHOR'S REPLY TO THE DISCUSSION AND WRITTEN CONTRIBUTION.

Mr. F. W. Lanchester: I was pleased to see Mr. Wimperis present, and to hear him take so prominent a part in the discussion. I have at the outset to thank him for having called attention to a slight error; $g \tan \beta$ should read $g \sin \beta$, but the error is one not vital to the argument. For small angles the difference between $\sin \beta$ and $\tan \beta$ is negligible. The main fact is undoubtedly correct, and the argument is not materially affected by the error.

Mr. Wimperis: The facts are undoubtedly correct.

Mr. Lanchester: The next point Mr. Wimperis mentioned was that if you multiply the ft./sec. per second by 70 it gives lb. per ton. That is perfectly correct; I have given the same figure in my paper on page 125. The more exact figure is 69·5 as given in the comparative table in the Appendix.

I have been much interested in hearing the description of the Elliott instrument. The reason why the question of vertical acceleration is of comparatively small importance on the pendulum instrument is that the road in general is not undergoing permanent undulations of considerable magnitude; one does not pick out for acceleration tests, the parts of a road where there are great departures from the straight. The lesser vibrations shown in most of the diagrams are due in the main to the undulations in the road; this is an effect of the vertical accelerations mentioned by Mr. Wimperis. If my instrument had been arranged with rotation about a vertical axis, it would have been absolutely necessary to apply a corrective mechanism such as that used by Elliott Bros. in the Wimperis instrument. The Wimperis method possesses incidentally the great beauty of preventing the rotational acceleration from affecting the instrument and giving rise to error.

Mr. Wimperis: That is an unintentional virtue.

Mr. Lanchester: It is very candid of you to say so. The point appealed to me as constituting in principle the foundation of the balancing of the original Lanchester engine—a method of killing lateral oscillation by two parts rotating in opposite senses. In this engine there are two parallel intergeared crankshafts one vertically over the other, rotating in opposite directions, the two crank-pins being arranged in phase in respect of their horizontal motion. Balance weights are provided whose motions neutralise in the vertical plane but combine in respect of their horizontal component, the motion of the common centre of gravity of the balance weights thus being a harmonic motion in one straight line. To the two crankshafts respectively are attached flywheels of equal moment of inertia just as in the balancing components of the Wimperis Accelerometer.

In my *Aerial Flight*, Vol. 1, page 344, I have proposed to use a similar device in connection with the Dines method of measuring wind resistance in order to avoid errors due to acceleration.

On the question of the dash-pot I certainly agree with Mr. Wimperis that the electrical "Faraday disc" dash-pot is an ideal device where it can be conveniently applied. I am a little doubtful whether the power necessary to render a 10 lb. pendulum dead beat would not be rather more than it would be possible to obtain from the Faraday disc without adding unduly to the bulkiness of the instrument. The objections to the viscous oil dash-pot are, so far as my experience goes, more apparent than real. The carrying of oil of two different viscosities, which certainly appears to be a clumsy expedient, gives so little trouble in practice that I have never even added a screw regulated leak, as I had at one time intended to do. It is perhaps worth while pointing out that a leaky piston and viscous oil is a far better form of dash-pot than a well-fitted piston and a regulated efflux aperture, as the law of resistance in the former approaches more closely to the viscous law, that is, the resistance varies directly as the velocity. In brief, the essential difference between Mr. Wimperis's instrument and my own is that the former is not a self-recording instrument, while the pendulum recording accelerometer has to make its mark on a paper roll, and a pendulum of considerable weight is necessary. The copper disc dash-pot would, I think, add still more weight to that of an already too heavy instrument.

As to the hostility to the use of the term "poundal" expressed by Mr. Wimperis and others, I have always looked at the matter as a question of exactitude of thought. To put the matter as a straightforward question:—Do you wish to know what you are doing, or do you not? If you do, use absolute units; whether British absolute units or c. g. s. are adopted is of practically no consequence. If you do not, then use any units you like. The engineer who shirks the absolute unit is not only shirking the employment of the gravitation constant to convert his results into the units of commerce—he has to do his division or multiplication by g inside the equation if he does not do it outside—if the required result is to be obtained in lb. force. What he is actually shirking is the *understanding of both the equation and the process which he uses.* There are cases in which it is impossible to form a clear conception of the meaning of an equation in which the use of absolute units is ignored. To illustrate this point let us take the formula as ordinarily given for centrifugal force—

$$F = \frac{m\,v^2}{gr}.$$

If we apply this formula under ordinary conditions of life it tells us that a body of mass m moving with a velocity v in a circle whose radius is r will exert a radial force F on the string by which its motion is restrained, the force F being given by the equation in terms of the attraction of the earth for one lb. mass. Now supposing that the question be one of a body at a great altitude, then g will be less than at the surface of the earth, and the same equation will give a different value for the force F although we know as an actual fact this force is exactly the same as before; the reason for this different value is that at the supposed high altitude the force of gravity is less, so that the attraction of one lb. mass (the unit in which the force is measured) has altered. We have therefore an equation expressing the value of a definite constant force by different units for varying position relatively to the earth's surface according to the local variations of g. Again, let us suppose that we are a few "light years" distance from the earth (a "light year" is the astronomer's measuring staff); here g becomes virtually zero, and by the equation F therefore becomes infinite, and the equation tells us the very interesting fact that the force F consists of an infinite number of zero units! Of course, we really know that the force F is exactly the same as before. In other words, by importing g into the equation we introduce endless confusion. The proper form of the expression in using absolute units is—

$$F = \frac{m\,v^2}{r}.$$

In this case we have the result in poundals, and the expression may be applied to bodies terrestrial or celestial alike.

If we wish to make concessions to the blindfold symbol-monger who prefers not to know what his formulæ mean there is no need to introduce g into the formula at all as is so frequently and erroneously done. For practical purposes the approximate value of g on the surface of the earth may be used, that is to say, 32·2, or perhaps better still, the rabidly "practical" man may be informed that the *poundal* means *half an ounce,* and with that information he will be quite happy.

In reply to Mr. Legros, two stopping diagrams taken from a tube railway are given in Figs. 12 and 13 of the paper. They appear to show that on the tube railways the brakes are partially eased off as the cars come to rest. Further Tube Railway diagrams will be added to the paper before publication in the proceedings (Figs. 27, 28 and 29).

In his explanation of the "jerk," I think that Mr. Legros is at fault. Let us imagine a car coming to rest with a uniform negative acceleration, then there will be no "jerk" so long as the car is in motion; and further, if the negative acceleration of the brake were continued to make the car go backwards as soon as it comes momentarily to rest no "jerk" would be felt at all. Now, the effect of the springs described by Mr. Legros will be to cause a slight continuation of the negative acceleration after the instant of stopping, and it appears to me that this must at least ameliorate the conditions, and to some extent actually diminish the "jerk" rather than be the cause of the "jerk" as suggested by Mr. Legros.

Like Mr. Wimperis, Mr. E. G. E. Beaumont worries me as to the name I have given the units of acceleration; it appears that he wants me to call them ft./sec. per sec. It really does not matter, *they are ft./sec. per sec.*, and they are *equally poundals/lb.* The whole of my reasons for expressing units in this particular form are given in the paper. My plea is that the tractive resistances and rate of acceleration should be expressed in like units, and my reason (as stated in the paper) is that I prefer not to pay for an article in £ s. d., and to receive the change in a mixture of francs and pesetas. There has of late been some agitation in favour of the adoption of the French system of units; to accept such inconsistencies as those to which attention is directed, and then to agitate, as some few engineers are doing, for the adoption of the metric system, is like straining at one gnat and being prepared to swallow many camels. Mr. Beaumont asks me whether I have used my instrument on the L. B. & S. C. electrified section. I have not "poached" there yet, but I intend doing so, and will, if possible, add the results obtained in time for publication in the proceedings (Fig. 30).

Dr. Hele-Shaw has suggested that I might with advantage have put arrows on the diagrams indicating the direction of time. I assent. The diagrams have been taken at different times, and collected in order to illustrate this paper; hence their lack of uniformity. The suggested arrows will be added before final publication.

In reply to Dr. Hele-Shaw's question as to the comparison between the vertical and the horizontal pivot, I may say that with the horizontal pivot as used by me, the balancing by double rotation is avoided, as errors due to road undulations are unimportant. With a vertical pivot the errors would be those due to road curvature, and would be far too serious to neglect; hence the use of a vertical pivot involves of necessity a more elaborate construction of instrument.

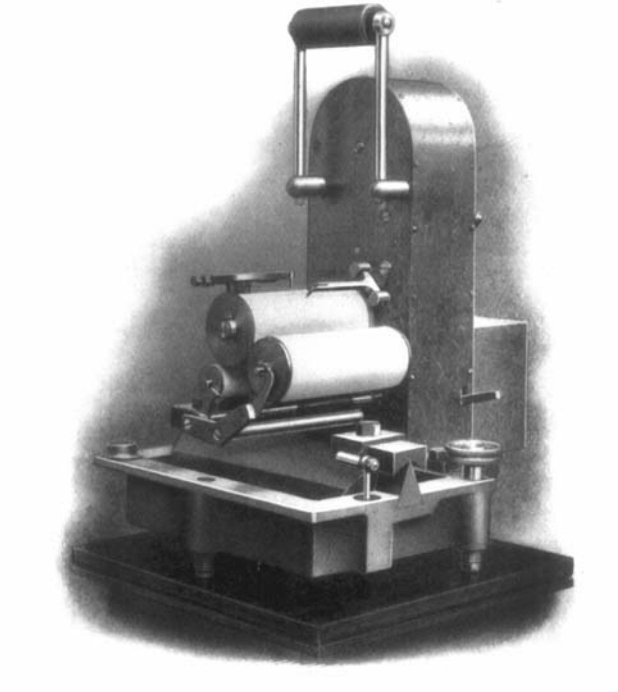
Mr. LANCHESTER wrote in reply to Mr. Charles E. G. House: Mr. House raises an objection. He says it is impossible to record an acceleration instantaneously from the start, as the recording point must always begin by darting out from the zero position. This assumes that at the start there is an absolutely sudden change of acceleration, which is scarcely ever the case. The suggestion as to a car starting with a simple harmonic motion is open to the reply that cars do not start in this manner. Mr. House's objection carries most weight in the case of the sudden stop at the termination of the brake diagram. Here the recording point, instead of falling instantly to zero, takes an appreciable time. In this particular case we know definitely that the acceleration does fall with almost complete suddenness, and therefore the diagrams show the total extent of Mr. House's objection in the departure of the falling brake curve from the plumb; it does not appear to be a serious error.

If the pendulum were a yard long the position would be far worse, and it is mainly in order to minimise the error from the cause stated that the pendulum is made as short as it is.

To be strictly accurate the acceleration diagram gives the *acceleration of the pendulum bob* rather than that of the vehicle, and it is quite obvious that for general engineering purposes where a stop or start occupies many feet or yards, the greatest difference between the motion of the pendulum bob and that of the vehicle—a fraction of an inch—is quite negligible.

With regard to Mr. House's objections to the use of an accelerometer on a launch, I pointed out the difficulty he mentioned in my paper. The accelerometer could be adjusted to work accurately so long as the trim of the vessel is constant. In some launches the error in using the accelerometer would not be great; in others the changes of trim under varying conditions of propeller thrust would render the readings of but little use. The gyroscope could be used as Mr. House suggests; for instance, by providing a "Tower steady platform" (an apparatus which is controlled by a gyroscope), the accelerometer could be used with quite satisfactory results.

I do not quite understand Mr. House's suggestions of applying the accelerometer to record vertical accelerations during the springing movements of a car going over a bumpy road. If information on this point were wanted it would be more effective to take a direct displacement curve by means of link work from the axle of the car recording direct on a chronograph cylinder on the body.



Lanchester Pendulum (recording) Accelerometer.
Fig. 9.

Wimperis (non-recording) Accelerometer.
Fig. 32.