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Vectors

Module Overview

Acknowledgments

This presentation is based on and includes content derived from the following OER resource:

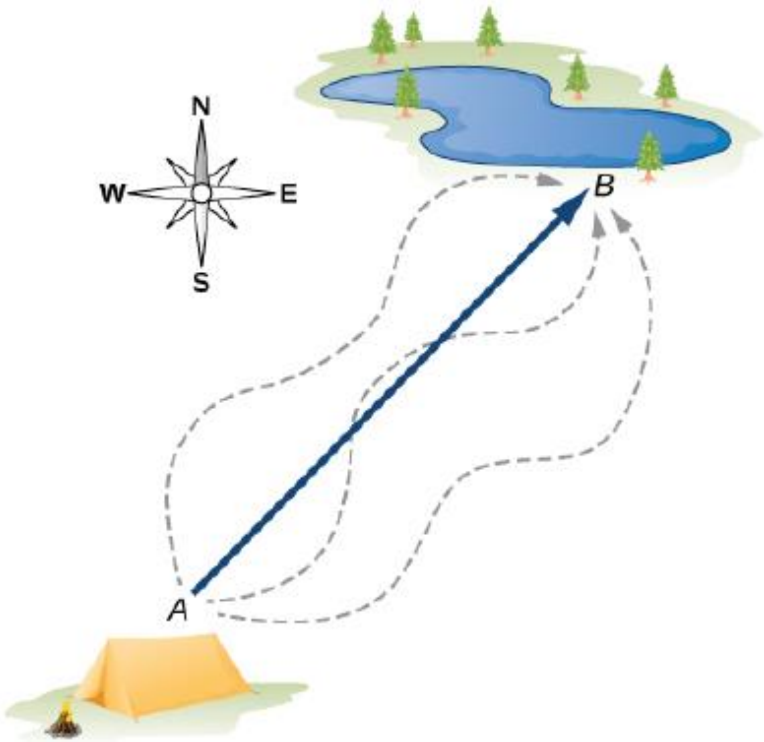
University Physics Volume 1

An OpenStax book used for this course may be downloaded for free at: https://openstax.org/details/books/university-physics-volume-1

Scalars and Vectors, Part 1

A **scalar** is a physical quantity that can be represented by only a single number and unit.

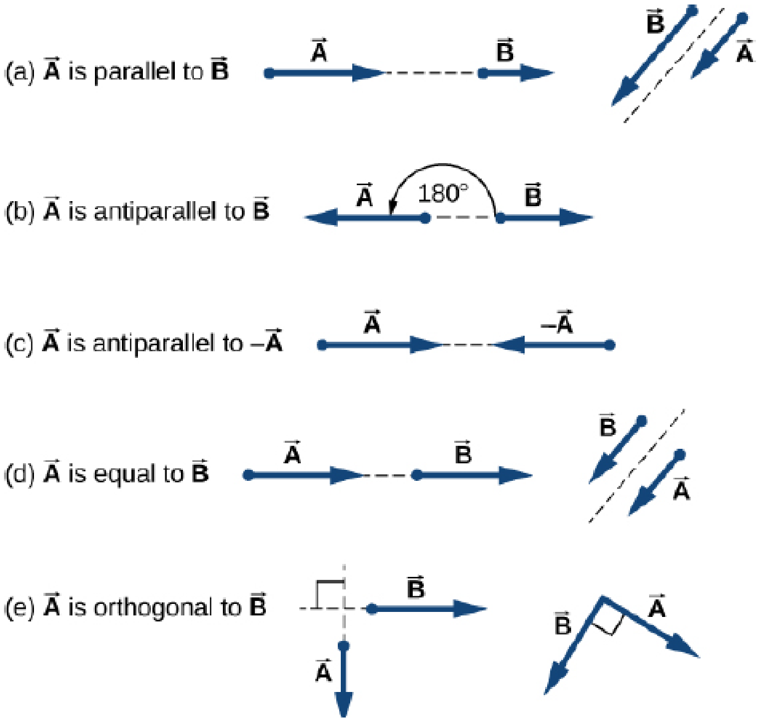
A **vector** is a physical quantity that is represented by a number of units and a direction. Vectors are represented by bold letters with an over arrow, like $\vec{\mathbf{A}}$, and their lengths are given by the **magnitude**, denoted $|\vec{\mathbf{A}}|$ or A.



(University Physics Volume 1. OpenStax. Fig. 2.3)

Scalars and Vectors, Part 2

Vectors are **parallel** if they have the same direction, **antiparallel** if they have opposite directions, and **orthogonal** if they are perpendicular. Vectors are equal if they are both parallel and have the same magnitude, $|\vec{\mathbf{A}}| = |\vec{\boldsymbol{B}}|$.



(University Physics Volume 1. OpenStax. Fig. 2.5)

Algebra of Vectors in One Dimension, Part 1

In one dimension, vectors can be multiplied by scalars, added to other vectors, or subtracted from other vectors. Equality between different vectors can be shown in a **vector equation**, where both sides of the equation are vectors, similarly to a **scalar equation**, where both sides of the equation are numbers.

operation	equation
scalar multiplication	$\vec{\mathbf{A}} = \alpha\vec{\mathbf{B}}$
vector sum	$\vec{\mathbf{D}}_{\mathrm{AC}} = \vec{\mathbf{D}}_{\mathrm{AB}} + \vec{\mathbf{D}}_{\mathrm{BC}}$
difference of two vectors	$\vec{\mathbf{D}}_{\mathrm{AB}} = \vec{\mathbf{D}}_{\mathrm{AC}} - \vec{\mathbf{D}}_{\mathrm{BC}} = \vec{\mathbf{D}}_{\mathrm{AC}} + (-\vec{\mathbf{D}}_{\mathrm{BC}})$

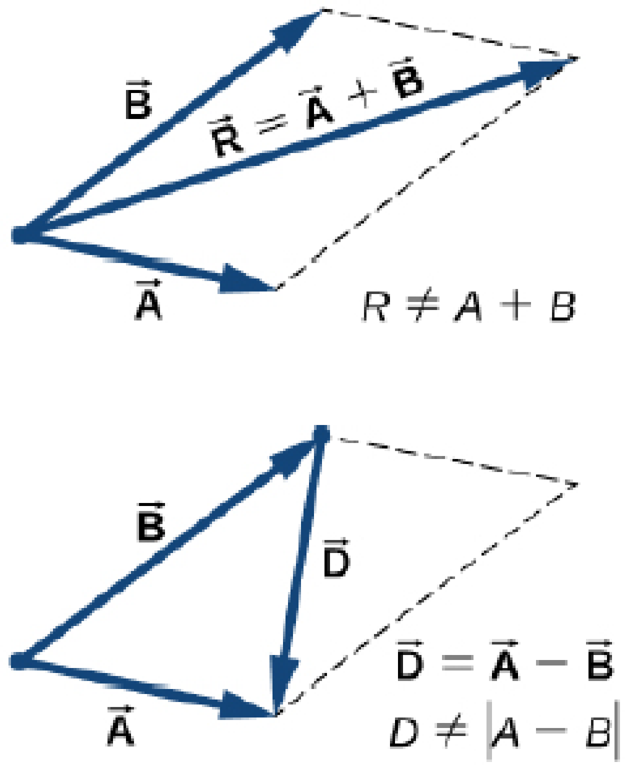
Algebra of Vectors in One Dimension, Part 2

The result of a vector operation like the vector sum is the **resultant**.
A vector with magnitude equal to 1 is called a **unit vector**.

property	description	equation
commutative	Vectors can add in any order.	$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$
associative	Vector addition can be regrouped.	$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$
distributive	Scalars can be distributed.	$\alpha_{\mathbf{1}}\vec{\mathbf{A}} + \alpha_{\mathbf{2}}\vec{\mathbf{A}} = (\alpha_{\mathbf{1}}+\alpha_{\mathbf{2}})\,\vec{\mathbf{A}}$

Algebra of Vectors in Two Dimensions, Part 1

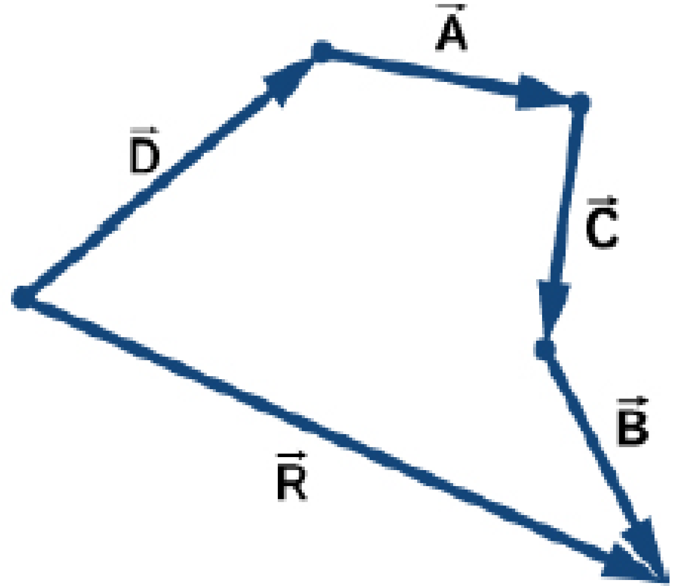
In two dimensions, all the same operations can be done to vectors as in one dimension, but the construction of vector sums and differences is more subtle. The **parallelogram rule** is used to construct vector sums and differences from two vectors.



(University Physics Volume 1. OpenStax. Fig. 2.10)

Algebra of Vectors in Two Dimensions, Part 2

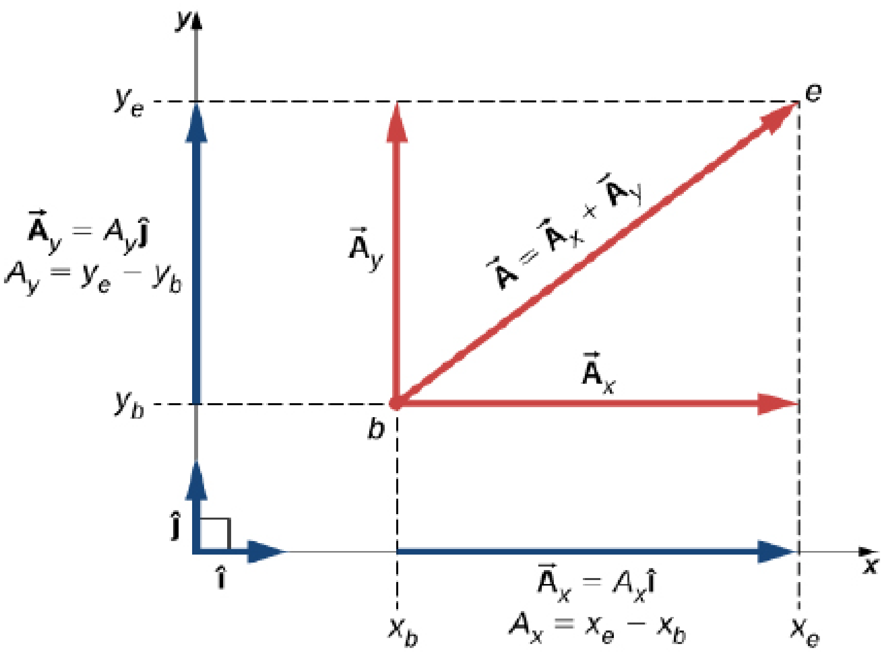
Tail-to-head geometric construction is a generalization of the parallelogram rule that gives us the ability to quickly add more than two vectors. To add multiple vectors, place the tail of each vector at the head of the preceding one. The vector from the first tail to the last head is the resultant vector.



(University Physics Volume 1. OpenStax. Fig. 2.12)

Coordinate Systems and Vector Components, Part 1

A vector $\vec{\mathbf{V}}$ can be expressed in the Cartesian coordinate system by its **vector components**, $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. The magnitudes of the vector components, V_x and V_y, are called the **scalar components** of the vector. The **unit vectors of the axes**, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, define orthogonal directions in the plane.



(University Physics Volume 1. OpenStax. Fig. 2.16)

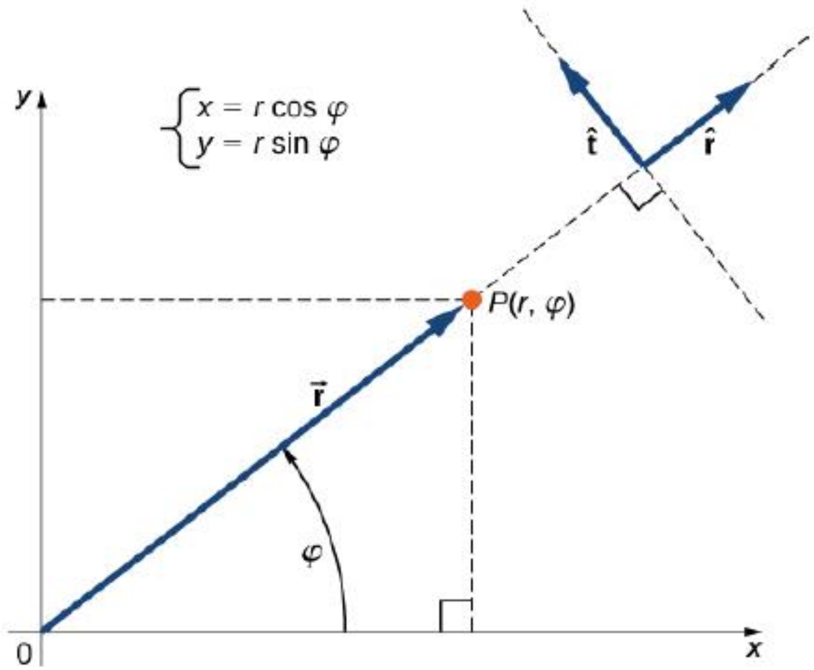
Coordinate Systems and Vector Components, Part 2

A vector $\vec{\mathbf{A}}$ can be written in **component form** as $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$. The scalar components of a vector are given by the differences in the initial and final coordinate values, $A_x = x_e - x_b$ and $A_y = y_e - y_b$, where *e* stands for end and *b* stands for beginning.

The magnitude of a vector $\vec{\mathbf{A}}$ can be written in terms of its scalar components as $A = \sqrt{A_x^2 + A_y^2}$. The **direction angle** θ_A is the angle a vector makes with the positive x-axis measured counterclockwise. In terms of scalar components, it has the value $\theta_A = \tan^{-1}({}^{A_y}/_{A_x})$.

Polar Coordinates

The **polar coordinate system** is often used to describe rotation. **Polar coordinates** are written in terms of a **radial coordinate**, r, which denotes distance from the origin, and an angle, φ, which denotes the angle between the vector and the positive x-axis. Polar coordinates can be rewritten in terms of Cartesian coordinates.



(University Physics Volume 1. OpenStax. Fig. 2.20)

Vectors in Three Dimensions

In three dimensions, we need three unit vectors and three components to specify a vector. The third unit vector, corresponding to the z-axis, is called $\hat{\mathbf{k}}$. The magnitude and components of vectors in three dimensions are defined analogously to vectors in two dimensions.

$$A_z = z_e - z_b$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Algebra of Vectors in Three Dimensions

- Scalar multiplication is distributive over a vector sum.
- The generalization of zero as a vector is called the **null vector**, $\vec{\mathbf{0}}$. It has all of its components equal to zero. Two vectors are **equal vectors** when their difference is the null vector.
- Many vectors can be summed analytically by adding their corresponding Cartesian components.
- The unit vector pointing in the direction of a given vector $\vec{\mathbf{V}}$ is given by dividing by the length of the vector, $\hat{\mathbf{V}} = \frac{\vec{\mathbf{V}}}{V}$.

The Scalar (Dot) Product

The **scalar product**, or dot product, of two vectors is defined mathematically as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = ABcos(\varphi)$, where φ is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. The scalar product of orthogonal vectors is equal to 0. The scalar product is commutative, meaning that $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$, and distributive, meaning that $\overrightarrow{\mathbf{A}} \cdot (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$.

In terms of scalar components of both vectors, the scalar product can be written simply as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$. From the definition of the scalar product, $\cos(\varphi)$ can be solved for yielding the expression, $\cos(\varphi) = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB}$.

The Vector (Cross) Product, Part 1

The **vector product**, or cross product, of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as the vector orthogonal to both vectors with magnitude $\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = ABsin(\varphi)$, where φ is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. The direction of the cross product is given by the **corkscrew right-hand rule**: Place a corkscrew in the plane defined by the two vectors, and turn the corkscrew from the first vector to the second vector. The vector product points in the direction that the corkscrew progresses.

The Vector (Cross) Product, Part 2

The vector product of parallel or antiparallel vectors is equal to the null vector. The vector product is anticommutative, meaning $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$, and distributive, meaning $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \vec{\mathbf{C}}$.

In terms of scalar components, the vector product can be expressed by the equation, $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_x B_z - A_z B_x)\hat{\mathbf{j}} + (A_x B_y - A_y B_z)\hat{\mathbf{k}}$.

How to Study this Module

- Read the syllabus or schedule of assignments regularly.
- Understand key terms; look up and define all unfamiliar words and terms.
- Take notes on your readings, assigned media, and lectures.
- As appropriate, work all questions and/or problems assigned and as many additional questions and/or problems as possible.
- Discuss topics with classmates.
- Frequently review your notes. Make flow charts and outlines from your notes to help you study for assessments.
- Complete all course assessments.

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