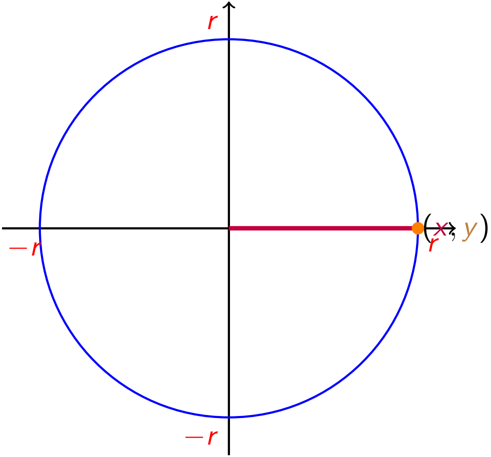
Circles

June 5, 2014

Building the Equation



Building the Equation

Conclusion: the equation of a circle centred at the origin with radius r is given by

$$x^2 + y^2 = r^2.$$

Translations

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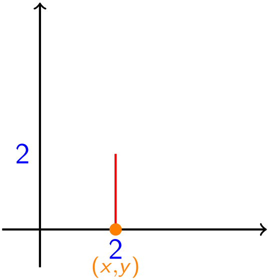
The set of points we're interested in is the set of (x, y) such that the distance between (x, y) and (a, b) is fixed at some radius r, that is:

$$(x - a)^2 + (y - b)^2 = r^2.$$

Example of a Translation: $(x-2)^2 + (y-2)^2 = 4$



Semicircles

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The equations we have used to describe a circle do not classify as functions, since for any fixed x such that $-r \leq x \leq r$, we have

$$x^2 + y^2 = r^2 \implies y^2 = r^2 - x^2 \implies y = \pm\sqrt{r^2 - x^2}.$$

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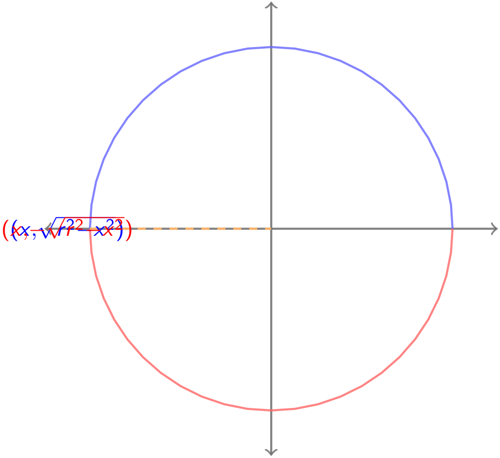
If we only choose one solution for y, we obtain a semi-circle, which is a function from $[-r, r] \to [0, r]$ or $[-r, r] \to [-r, 0]$.

Semicircles



Summary

- The circle centred at (a, b) with radius r has equation

$$(x - a)^2 + (y - b)^2 = r^2.$$

- Semicircles are given by solving for y and choosing either the positive square root (upper semicircle) or negative square root (lower semicircle). Centred at the origin, this appears as

$$y = \begin{cases} \sqrt{r^2 - x^2} & \text{for upper semicircle} \\ -\sqrt{r^2 - x^2} & \text{for lower semicircle.} \end{cases}$$