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INTERNAL REPORT

ELASTIC DISTORTION OF THE HIGH-PRESSURE COMPRESSIBILITY

BOMBS OVER THE TEMPERATURE RANGE 0° TO 80° C

BY

Ted C. Briggs

BRANCH Fundamental Research

PROJECT NO. 4330

DATE February 1966

AMARILLO, TEXAS

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ELASTIC DISTORTION OF THE HIGH-PRESSURE COMPRESSIBILITY BOMBS OVER THE TEMPERATURE RANGE 0° TO 80° C

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Ted C. Briggs[1/]

ABSTRACT

This report contains experimentally-observed jacket-pressure distortion coefficients and smoothed jacket-pressure distortion coefficients for the present Burnett type compressibility apparatus for the temperature range 0° to 80° C. Values for Young's modulus and the internal-pressure distortion coefficients were derived from the experimental observations and are recorded in this report as functions of temperature.

INTRODUCTION

A correction for elastic pressure distortion of the high-pressure bombs must be applied to compressibility data obtained by the Burnett method. Elastic pressure distortion of the bombs is dependent upon the dimensions of the bombs, Young's modulus of the bomb material, and Poisson's ratio for the bomb material. Young's

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modulus is temperature dependent; therefore, elastic distortion of the bombs is a function of the bomb temperature. Elastic distortion of the high-pressure compressibility bombs has been evaluated at 30° C (3)[2/] and at 0° C (4). The purpose of this report is to

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2/ Underlined numbers in parentheses refer to items in the list of references at the end of this report.

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present elastic distortion data between 0° and 30° C, to extend the data to 80° C, and to present smoothed data for Young's modulus, the jacket-pressure distortion coefficient, and the internal-pressure distortion coefficient for the temperature range 0° to 80° C.

MEASUREMENT OF THE JACKET-PRESSURE DISTORTION COEFFICIENT

The change of volume V_1 of the compressibility apparatus can be represented by equation (1). Volume V_1 consists of a jacketed bomb along with unjacketed fittings, tubing, and valve (3).

$$\frac{\Delta V_1}{V_1^o} = \alpha_1 P_i - \beta_1 P_j \qquad (1)$$

The terms of equation (1) are:

ΔV_1 = change of isothermal volume of V_1 due to pressure distortion

V_1^o = isothermal volume of V_1 at zero internal and jacket pressures

α_1 = V_1 internal-pressure distortion coefficient

β_1 = V_1 jacket-pressure distortion coefficient

P_i = internal pressure

P_j = jacket pressure

Experimental determination of the jacket-pressure distortion coefficient at 30° C was described in a previous report (2). The same experimental procedure was used for measurements made at other temperatures.

Volume V_1 of the compressibility apparatus was filled with helium to some initial pressure. The bomb bath was controlled at the desired temperature, and the internal gas pressure was measured with a piston gage. Jacket pressure was increased in increments of 1000 psi from 15 to 5000 psia. Time was allowed for gas in V_1 to reach thermal equilibrium after each increase in jacket pressure. Internal cylinder gas pressure was measured for each jacket pressure. Experimental observations for all of the runs are recorded in table 1. Observations at 30° and 0° C were reported previously in references (2) and (4) respectively, and are recorded again in table 1 of this report.

EVALUATION OF THE JACKET-PRESSURE DISTORTION COEFFICIENT

The jacket-pressure distortion coefficient β_1 of equation (1) can be evaluated from equation (2), reference (2), page 6.

$$\beta_1 = \frac{d\ln P_i}{dP_j}\left[1 - \frac{d\ln Z_i}{d\ln P_i}\right] \qquad (2)$$

The term $\frac{d\ln P_i}{dP_j}$ of equation (2) was evaluated for each of the

TABLE 1. - Experimental jacket-pressure distortion coefficient data

0° C

Run No. He-0-β_1-1

P_j,psia	P_i,psia
15	10253.48
1000	10255.52
2000	10257.56
3000	10259.52
4000	10261.57
5000	10263.61

Run No. He-0-β_1-2

P_j,psia	P_i,psia
15	10134.63
1000	10136.52
2000	10138.59
3000	10140.54
4000	10142.55
5000	10144.57

Run No. He-0-β_1-3

P_j,psia	P_i,psia
15	10104.89
1000	10106.86
2000	10108.82
3000	10110.71
4000	10112.77
5000	10114.82

Run No. He-0-β_1-4

P_j,psia	P_i,psia
15	5167.670
1000	5168.579
2000	5169.521
3000	5170.413
4000	5171.341
5000	5172.279

Run No. He-0-β_1-5

P_j,psia	P_i,psia
15	5109.585
1000	5110.471
2000	5111.378
3000	5112.290
4000	5113.178
5000	5114.060

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

10° C

Run No. He-10-β_1-1		Run No. He-10-β_1-2		Run No. He-10-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	8156.165	15	7310.857	15	6063.216
1000	8157.712	1000	7312.219	1000	6064.312
2000	8159.246	2000	7313.590	2000	6065.433
3000	8160.793	3000	7314.939	3000	6066.545
4000	8162.166	4000	7316.319	4000	6067.635
5000	8163.774	5000	7317.670	5000	6068.749

20° C

Run No. He-20-β_1-1		Run No. He-20-β_1-2		Run No. He-20-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	6860.523	15	5227.388	15	5403.651
1000	6861.720	1000	5228.285	1000	5404.606
2000	6862.944	2000	5229.247	2000	5405.546
3000	6864.145	3000	5230.152	3000	5406.510
4000	6865.425	4000	5231.059	4000	5407.482
5000	6866.714	5000	5232.027	5000	5408.453

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

30° C

Run No. He-30-β_1-1

P_j,psia	P_i,psia
15	7280.597
1000	7281.996
2000	7283.312
3000	7284.685
4000	7286.032
5000	7287.382

Run No. He-30-β_1-2

P_j,psia	P_i,psia
15	7103.142
1000	7104.404
2000	7105.714
3000	7106.994
4000	7108.248
5000	7109.571

Run No. He-30-β_1-3

P_j,psia	P_i,psia
15	7702.140
1000	7703.567
2000	7704.962
3000	7706.378
4000	7707.767
5000	7709.224

Run No. He-30-β_1-4

P_j,psia	P_i,psia
15	7516.373
1000	7517.732
3000	7520.510
4000	7521.887
5000	7523.335

Run No. He-30-β_1-5

P_j,psia	P_i,psia
15	3478.731
1000	3479.283
3000	3480.462
4000	3481.047
5000	3481.643

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

40° C

Run No. He-40-β_1-1		Run No. He-40-β_1-2		Run No. He-40-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	7471.045	15	5556.079	15	5102.579
1000	7472.321	1000	5557.059	1000	5103.489
2000	7473.671	2000	5558.065	2000	5104.394
3000	7474.989	3000	5559.050	3000	5105.297
4000	7476.308	4000	5560.020	4000	5106.204
5000	7477.716	5000	5560.997	5000	5107.089

50° C

Run No. He-50-β_1-1		Run No. He-50-β_1-2		Run No. He-50-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	6489.329	15	5021.613	15	5599.489
1000	6490.452	1000	5022.461	1000	5600.463
3000	6492.813	2000	5023.318	2000	5601.442
4000	6493.958	3000	5024.204	3000	5602.415
5000	6495.061	4000	5025.047	4000	5603.393
		5000	5025.953	5000	5604.413

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

40° C

Run No. He-40-β_1-1		Run No. He-40-β_1-2		Run No. He-40-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	7471.045	15	5556.079	15	5102.579
1000	7472.321	1000	5557.059	1000	5103.489
2000	7473.671	2000	5558.065	2000	5104.394
3000	7474.989	3000	5559.050	3000	5105.297
4000	7476.308	4000	5560.020	4000	5106.204
5000	7477.716	5000	5560.997	5000	5107.089

50° C

Run No. He-50-β_1-1		Run No. He-50-β_1-2		Run No. He-50-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	6489.329	15	5021.613	15	5599.489
1000	6490.452	1000	5022.461	1000	5600.463
3000	6492.813	2000	5023.318	2000	5601.442
4000	6493.958	3000	5024.204	3000	5602.415
5000	6495.061	4000	5025.047	4000	5603.393
		5000	5025.953	5000	5604.413

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

60° C

Run No. He-60-β_1-1		Run No. He-60-β_1-2		Run No. He-60-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	5336.011	15	6232.679	15	5583.208
1000	5336.970	1000	6233.777	1000	5584.190
2000	5337.890	2000	6234.916	2000	5585.160
3000	5338.874	3000	6235.994	3000	5586.125
4000	5339.762	4000	6237.069	4000	5587.116
5000	5340.701	5000	6238.204	5000	5588.087

70° C

Run No. He-70-β_1-1		Run No. He-70-β_1-2		Run No. He-70-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	4533.651	15	5309.645	15	7343.050
1000	4534.391	1000	5310.558	1000	7344.374
3000	4535.937	2000	5311.472	2000	7345.661
4000	4536.735	3000	5312.401	3000	7347.048
5000	4537.516	4000	5313.288	4000	7348.361
		5000	5314.172	5000	7349.702

TABLE 1. - Experimental jacket-pressure distortion coefficient data (Con.)

80° C

Run No. He-80-β_1-1		Run No. He-80-β_1-2		Run No. He-80-β_1-3	
P_j,psia	P_i,psia	P_j,psia	P_i,psia	P_j,psia	P_i,psia
15	5499.807	15	6038.315	15	5155.755
1000	5500.906	1000	6039.330	1000	5156.646
2000	5501.924	2000	6040.445	2000	5157.539
3000	5502.787	3000	6041.480	3000	5158.364
4000	5503.717	4000	6042.661	4000	5159.261
5000	5504.842	5000	6043.662	5000	5160.386

experimental runs by least squares calculation of the slope of a linear plot of ln P_i versus P_j. Least squares calculated values of $\frac{d\ln P_i}{dP_j}$ are listed in table 2 along with the standard deviations of $\frac{d\ln P_i}{dP_j}$. The term $\frac{d\ln Z_i}{d\ln P_i}$ of equation (2) can be evaluated, provided compressibility data for helium are available. PVT data obtained by Wiebe, Gaddy, and Heins as presented in Helium Research Center Internal Report No. 71 (6) were used to evaluate $\frac{d\ln Z_i}{d\ln P_i}$ for all of the runs except those at 30° C. Compressibility factors for helium at 30° C were computed from Johnson's data as recorded in Helium Research Center Memorandum Report No. 42 (5). Equation (3), reference (2), page 6 was used to calculate values of $\frac{d\ln Z_i}{d\ln P_i}$.

$$\frac{d\ln Z_i}{d\ln P_i} = \frac{B'P_i + 2C'P_i^2}{1 + B'P_i + C'P_i^2} \qquad (3)$$

Numerical values used in this report for the constants B' and C' of equation (3) are listed in table 3.

Average values of $\frac{d\ln Z_i}{d\ln P_i}$ for each of the runs were calculated from equation (3) and are recorded in table 2. Values for the jacket-pressure distortion coefficient β_1 were calculated using equation (2) and are tabulated in table 2. The average β_1, average standard deviation of β_1, standard deviation of a single β_1, and the standard deviation of the average β_1 for each temperature are recorded in table 2. The average β_1 for a particular temperature is the numerical average of the individual β_1 values at that

TABLE 2. - Experimental jacket-pressure distortion coefficients

0° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-0-β_1-1	$1.9750\text{x}10^{-7} \pm 0.0068\text{x}10^{-7}$	0.24180
He-0-β_1-2	$1.9694\text{x}10^{-7} \pm 0.0078\text{x}10^{-7}$	0.24002
He-0-β_1-3	$1.9618\text{x}10^{-7} \pm 0.0128\text{x}10^{-7}$	0.23957
He-0-β_1-4	$1.7848\text{x}10^{-7} \pm 0.0057\text{x}10^{-7}$	0.14774
He-0-β_1-5	$1.7592\text{x}10^{-7} \pm 0.0052\text{x}10^{-7}$	0.14642

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-0-β_1-1	$1.4974\text{x}10^{-7} \pm 0.0052\text{x}10^{-7}$	$-0.0043\text{x}10^{-7}$
He-0-β_1-2	$1.4967\text{x}10^{-7} \pm 0.0059\text{x}10^{-7}$	$-0.0050\text{x}10^{-7}$
He-0-β_1-3	$1.4918\text{x}10^{-7} \pm 0.0097\text{x}10^{-7}$	$-0.0099\text{x}10^{-7}$
He-0-β_1-4	$1.5211\text{x}10^{-7} \pm 0.0049\text{x}10^{-7}$	$+0.0194\text{x}10^{-7}$
He-0-β_1-5	$1.5016\text{x}10^{-7} \pm 0.0044\text{x}10^{-7}$	$-0.0001\text{x}10^{-7}$

Average $\beta_1 = 1.5017\text{x}10^{-7} \pm 0.0051\text{x}10^{-7}$ psia^{-1}

Average standard deviation of $\beta_1 = \pm 0.0060\text{x}10^{-7}$ psia^{-1}

Standard deviation of a single $\beta_1 = \pm 0.0114\text{x}10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

10° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-10-β_1-1	$1.8581 \times 10^{-7} \pm 0.0169 \times 10^{-7}$	0.20156
He-10-β_1-2	$1.8678 \times 10^{-7} \pm 0.0034 \times 10^{-7}$	0.18639
He-10-β_1-3	$1.8288 \times 10^{-7} \pm 0.0037 \times 10^{-7}$	0.16202

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-10-β_1-1	$1.4836 \times 10^{-7} \pm 0.0135 \times 10^{-7}$	-0.0283×10^{-7}
He-10-β_1-2	$1.5197 \times 10^{-7} \pm 0.0028 \times 10^{-7}$	$+0.0078 \times 10^{-7}$
He-10-β_1-3	$1.5325 \times 10^{-7} \pm 0.0031 \times 10^{-7}$	$+0.0206 \times 10^{-7}$

Average β_1 = $1.5119 \times 10^{-7} \pm 0.0147 \times 10^{-7}$ psia^{-1}

Average standard deviation of β_1 = $\pm\, 0.0065 \times 10^{-7}$ psia^{-1}

Standard deviation of a single β_1 = $\pm\, 0.0254 \times 10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

20° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-20-β_1-1	$1.8051\times10^{-7} \pm 0.0151\times10^{-7}$	0.17253
He-20-β_1-2	$1.7750\times10^{-7} \pm 0.0087\times10^{-7}$	0.13970
He-20-β_1-3	$1.7796\times10^{-7} \pm 0.0045\times10^{-7}$	0.14345

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-20-β_1-1	$1.4937\times10^{-7} \pm 0.0125\times10^{-7}$	-0.0213×10^{-7}
He-20-β_1-2	$1.5270\times10^{-7} \pm 0.0075\times10^{-7}$	$+0.0120\times10^{-7}$
He-20-β_1-3	$1.5243\times10^{-7} \pm 0.0039\times10^{-7}$	$+0.0093\times10^{-7}$

Average β_1 = $1.5150\times10^{-7} \pm 0.0107\times10^{-7}$ psia^{-1}

Average standard deviation of β_1 = $\pm 0.0080\times10^{-7}$ psia^{-1}

Standard deviation of a single β_1 = $\pm 0.0185\times10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

30° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-30-β_1-1	$1.8635\times10^{-7} \pm 0.0077\times10^{-7}$	0.17460
He-30-β_1-2	$1.8113\times10^{-7} \pm 0.0056\times10^{-7}$	0.17149
He-30-β_1-3	$1.8371\times10^{-7} \pm 0.0060\times10^{-7}$	0.18181
He-30-β_1-4	$1.8525\times10^{-7} \pm 0.0076\times10^{-7}$	0.17866
He-30-β_1-5	$1.6811\times10^{-7} \pm 0.0076\times10^{-7}$	0.09684

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-30-β_1-1	$1.5381\times10^{-7} \pm 0.0064\times10^{-7}$	$+0.0218\times10^{-7}$
He-30-β_1-2	$1.5007\times10^{-7} \pm 0.0046\times10^{-7}$	-0.0156×10^{-7}
He-30-β_1-3	$1.5031\times10^{-7} \pm 0.0049\times10^{-7}$	-0.0132×10^{-7}
He-30-β_1-4	$1.5215\times10^{-7} \pm 0.0062\times10^{-7}$	$+0.0052\times10^{-7}$
He-30-β_1-5	$1.5183\times10^{-7} \pm 0.0069\times10^{-7}$	$+0.0020\times10^{-7}$

Average $\beta_1 = 1.5163\times10^{-7} \pm 0.0068\times10^{-7}$ psia^{-1}

Average standard deviation of $\beta_1 = \pm 0.0058\times10^{-7}$ psia^{-1}

Standard deviation of a single $\beta_1 = \pm 0.0152\times10^{-7}$ psia^{-1}

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[illegible]	[illegible]	[illegible]

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TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

40° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-40-β_1-1	$1.7865\times10^{-7} \pm 0.0097\times10^{-7}$	0.17342
He-40-β_1-2	$1.7747\times10^{-7} \pm 0.0061\times10^{-7}$	0.13791
He-40-β_1-3	$1.7723\times10^{-7} \pm 0.0053\times10^{-7}$	0.12873

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-40-β_1-1	$1.4767\times10^{-7} \pm 0.0080\times10^{-7}$	-0.0403×10^{-7}
He-40-β_1-2	$1.5300\times10^{-7} \pm 0.0053\times10^{-7}$	$+0.0130\times10^{-7}$
He-40-β_1-3	$1.5442\times10^{-7} \pm 0.0046\times10^{-7}$	$+0.0272\times10^{-7}$

Average $\beta_1 = 1.5170\times10^{-7} \pm 0.0206\times10^{-7}$ psia^{-1}

Average standard deviation of $\beta_1 = \pm 0.0060\times10^{-7}$ psia^{-1}

Standard deviation of a single $\beta_1 = \pm 0.0356\times10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

50° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-50-β_1-1	$1.7802\times10^{-7} \pm 0.0114\times10^{-7}$	0.15141
He-50-β_1-2	$1.7298\times10^{-7} \pm 0.0076\times10^{-7}$	0.12327
He-50-β_1-3	$1.7576\times10^{-7} \pm 0.0060\times10^{-7}$	0.13471

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-50-β_1-1	$1.5107\times10^{-7} \pm 0.0097\times10^{-7}$	-0.0053×10^{-7}
He-50-β_1-2	$1.5166\times10^{-7} \pm 0.0067\times10^{-7}$	$+0.0006\times10^{-7}$
He-50-β_1-3	$1.5208\times10^{-7} \pm 0.0052\times10^{-7}$	$+0.0048\times10^{-7}$

Average β_1 = $1.5160\times10^{-7} \pm 0.0042\times10^{-7}$ psia^{-1}

Average standard deviation of β_1 = $\pm 0.0072\times10^{-7}$ psia^{-1}

Standard deviation of a single β_1 = $\pm 0.0051\times10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

60° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-60-β_1-1	$1.7597\times10^{-7} \pm 0.0112\times10^{-7}$	0.12581
He-60-β_1-2	$1.7716\times10^{-7} \pm 0.0082\times10^{-7}$	0.14258
He-60-β_1-3	$1.7498\times10^{-7} \pm 0.0037\times10^{-7}$	0.13054

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-60-β_1-1	$1.5383\times10^{-7} \pm 0.0098\times10^{-7}$	$+0.0121\times10^{-7}$
He-60-β_1-2	$1.5190\times10^{-7} \pm 0.0070\times10^{-7}$	-0.0072×10^{-7}
He-60-β_1-3	$1.5214\times10^{-7} \pm 0.0032\times10^{-7}$	-0.0048×10^{-7}

Average $\beta_1 = 1.5262\times10^{-7} \pm 0.0061\times10^{-7}$ psia^{-1}

Average standard deviation of $\beta_1 = \pm 0.0067\times10^{-7}$ psia^{-1}

Standard deviation of a single $\beta_1 = \pm 0.0105\times10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

70° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-70-β_1-1	$1.7137\times10^{-7} \pm 0.0068\times10^{-7}$	0.10623
He-70-β_1-2	$1.7115\times10^{-7} \pm 0.0087\times10^{-7}$	0.12124
He-70-β_1-3	$1.8166\times10^{-7} \pm 0.0066\times10^{-7}$	0.15696

Run No.	β_1, psia^{-1}	Deviation from average β_1, psia^{-1}
He-70-β_1-1	$1.5317\times10^{-7} \pm 0.0061\times10^{-7}$	$+0.0093\times10^{-7}$
He-70-β_1-2	$1.5040\times10^{-7} \pm 0.0076\times10^{-7}$	-0.0184×10^{-7}
He-70-β_1-3	$1.5315\times10^{-7} \pm 0.0056\times10^{-7}$	$+0.0091\times10^{-7}$

Average β_1 = $1.5224\times10^{-7} \pm 0.0092\times10^{-7}$ psia^{-1}

Average standard deviation of β_1 = $\pm 0.0064\times10^{-7}$ psia^{-1}

Standard deviation of a single β_1 = $\pm 0.0159\times10^{-7}$ psia^{-1}

TABLE 2. - Experimental jacket-pressure distortion coefficients (Con.)

80° C

Run No.	$\frac{d\ln P_i}{dP_j}$	Average $\frac{d\ln Z_i}{d\ln P_i}$
He-80-β_1-1	$1.7938\text{x}10^{-7} \pm 0.0363\text{x}10^{-7}$	0.12187
He-80-β_1-2	$1.7899\text{x}10^{-7} \pm 0.0165\text{x}10^{-7}$	0.13151
He-80-β_1-3	$1.7667\text{x}10^{-7} \pm 0.0409\text{x}10^{-7}$	0.11552

Run No.	β_1,psia^{-1}	Deviation from average β_1,psia^{-1}
He-80-β_1-1	$1.5752\text{x}10^{-7} \pm 0.0319\text{x}10^{-7}$	$+0.0111\text{x}10^{-7}$
He-80-β_1-2	$1.5545\text{x}10^{-7} \pm 0.0143\text{x}10^{-7}$	$-0.0096\text{x}10^{-7}$
He-80-β_1-3	$1.5626\text{x}10^{-7} \pm 0.0362\text{x}10^{-7}$	$-0.0015\text{x}10^{-7}$

Average β_1 = $1.5641\text{x}10^{-7} \pm 0.0159\text{x}10^{-7}$ psia^{-1}

Average standard deviation of β_1 = $\pm$ $0.0275\text{x}10^{-7}$ psia^{-1}

Standard deviation of a single β_1 = $\pm$ $0.0104\text{x}10^{-7}$ psia^{-1}

TABLE 3. - Constants for equation (3)

Temperature,°C	B',atm^{-1}		C',atm^{-2}	
0	$5.24903\text{x}10^{-4}$		$-4.2030\text{x}10^{-8}$	
10	$5.04530\text{x}10^{-4}$		$-3.9885\text{x}10^{-8}$	
20	$4.85437\text{x}10^{-4}$		$-3.7824\text{x}10^{-8}$	
30	$4.74342\text{x}10^{-4}$	(5)	$-4.3388\text{x}10^{-8}$	(5)
40	$4.50683\text{x}10^{-4}$		$-3.3971\text{x}10^{-8}$	
50	$4.34843\text{x}10^{-4}$		$-3.2182\text{x}10^{-8}$	
60	$4.19920\text{x}10^{-4}$		$-3.0482\text{x}10^{-8}$	
70	$4.03993\text{x}10^{-4}$		$-2.8868\text{x}10^{-8}$	
80	$3.92554\text{x}10^{-4}$		$-2.7338\text{x}10^{-8}$	

temperature. The average standard deviation of β_1 is the numerical average of the individual standard deviations. The standard deviation of a single β_1 is calculated from the individual deviations from the average β_1 ignoring the information available in the individual standard deviations. For a specific set of data, the average standard deviation of β_1 and the standard deviation of a single β_1 are usually not identical; one is usually larger than the other. The deviation of the average β_1 is obtained by dividing the larger of the two quantities by the square root of the number of β_1 values in the set.

The average experimental jacket-pressure distortion coefficients were fitted to equation (4) by the method of least squares.

$$\beta_1 = x_1 + x_2 t \tag{4}$$

The least squares calculated values of the constants are

$$x_1 = 1.50096\text{x}10^{-7} \pm 0.00700\text{x}10^{-7}$$

$$x_2 = 5.0533\text{x}10^{-11} \pm 1.4708\text{x}10^{-11}$$

The average experimental jacket-pressure distortion coefficients, the standard deviations of the average experimental jacket-pressure distortion coefficients, the least squares calculated values of the jacket-pressure distortion coefficients, the calculated standard deviations of the least squares calculated jacket-pressure distortion coefficients, and the difference between the calculated and experimental jacket-pressure distortion coefficients are recorded in table 4. The data contained in table 4 are

TABLE 4. - Experimental and calculated jacket-pressure distortion coefficients

t,°C	Experimental β_1,psia^{-1}	Calculated β_1,psia^{-1}
0	$1.5017 \times 10^{-7} \pm 0.0051 \times 10^{-7}$	$1.5010 \times 10^{-7} \pm 0.0070 \times 10^{-7}$
10	$1.5119 \times 10^{-7} \pm 0.0147 \times 10^{-7}$	$1.5060 \times 10^{-7} \pm 0.0058 \times 10^{-7}$
20	$1.5150 \times 10^{-7} \pm 0.0107 \times 10^{-7}$	$1.5111 \times 10^{-7} \pm 0.0048 \times 10^{-7}$
30	$1.5163 \times 10^{-7} \pm 0.0068 \times 10^{-7}$	$1.5161 \times 10^{-7} \pm 0.0041 \times 10^{-7}$
40	$1.5170 \times 10^{-7} \pm 0.0206 \times 10^{-7}$	$1.5212 \times 10^{-7} \pm 0.0038 \times 10^{-7}$
50	$1.5160 \times 10^{-7} \pm 0.0042 \times 10^{-7}$	$1.5262 \times 10^{-7} \pm 0.0041 \times 10^{-7}$
60	$1.5262 \times 10^{-7} \pm 0.0061 \times 10^{-7}$	$1.5313 \times 10^{-7} \pm 0.0048 \times 10^{-7}$
70	$1.5224 \times 10^{-7} \pm 0.0092 \times 10^{-7}$	$1.5363 \times 10^{-7} \pm 0.0058 \times 10^{-7}$
80	$1.5641 \times 10^{-7} \pm 0.0159 \times 10^{-7}$	$1.5414 \times 10^{-7} \pm 0.0070 \times 10^{-7}$

t,°C	$\Delta\beta_1$,Cal.-Exp.,psia^{-1}
0	-0.0007×10^{-7}
10	-0.0059×10^{-7}
20	-0.0039×10^{-7}
30	-0.0002×10^{-7}
40	$+0.0042 \times 10^{-7}$
50	$+0.0102 \times 10^{-7}$
60	$+0.0051 \times 10^{-7}$
70	$+0.0139 \times 10^{-7}$
80	-0.0227×10^{-7}

illustrated graphically in figure 1. The small circles of figure 1 represent the average experimental jacket-pressure distortion coefficients. The x's represent the standard deviations of the average experimental jacket-pressure distortion coefficients. The solid line illustrated in figure 1 represents the least squares calculated jacket-pressure distortion coefficients. The two dashed curved lines of figure 1 illustrate the least squares calculated standard errors in the calculated jacket-pressure distortion coefficients.

CALCULATION OF YOUNG'S MODULUS FROM THE MEASURED VALUES OF THE JACKET-PRESSURE DISTORTION COEFFICIENT

Young's modulus can be calculated from equation (5), reference (3), page 15.

$$E = \frac{(5 - 4\sigma) b_b^2}{\beta_1' (b_b^2 - a_b^2)} \qquad (5)$$

E = Young's modulus

σ = Poisson's ratio

β_1' = bomb jacket-pressure distortion coefficient

b_b = bomb isothermal external radius

a_b = bomb isothermal internal radius

$$\beta_1' = \beta_1 \cdot \frac{V_1^o}{V_{b_1}^o} \qquad (6)$$

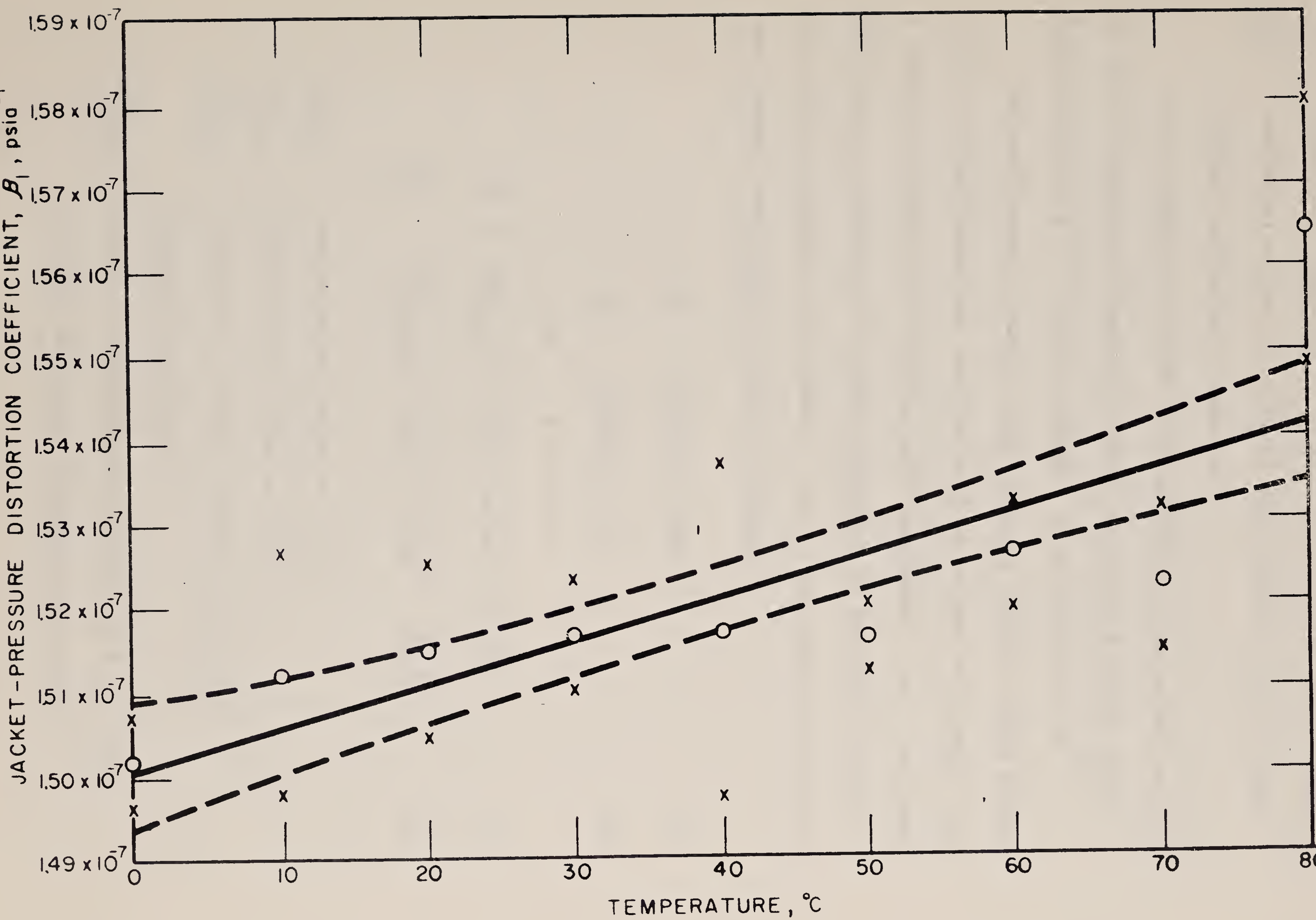

FIGURE 1. - Jacket-Pressure Distortion Coefficients as a Function of Temperature

V_1^o = isothermal volume of V_1 at zero internal and jacket pressures

$V_{b_1}^o$ = isothermal volume of jacketed bomb portion of V_1 at zero internal and jacket pressures

A base temperature of 30° C was arbitrarily selected, and all of the PVT apparatus dimensions listed in Helium Research Center Internal Report No. 77 (3) are assumed to be correct at 30° C.

The bomb external radius, bomb internal radius, tubing dimensions, fitting dimensions, and the various volumes all change with a change of temperature from the base temperature. Let γ represent the thermal coefficient of linear expansion of the bomb material. The various components of volumes V_1 and V_2 are assumed to have the same thermal coefficient of linear expansion.

$$b_b = (b_b)_{30}(1 + \gamma\Delta t) \tag{7}$$

$$a_b = (a_b)_{30}(1 + \gamma\Delta t) \tag{8}$$

$$\Delta t = t - 30,° \text{ C}$$

$$V_1^o = (V_1^o)_{30}(1 + 3\gamma\Delta t + 3\gamma^2\Delta t^2 + \gamma^3\Delta t^3) \tag{9}$$

$$V_{b_1}^o = (V_{b_1}^o)_{30}(1 + 3\gamma\Delta t + 3\gamma^2\Delta t^2 + \gamma^3\Delta t^3) \tag{10}$$

$(b_b)_{30}$ = bomb external radius at 30° C

$(a_b)_{30}$ = bomb internal radius at 30° C

$(V_1^o)_{30}$ = volume of V_1 at zero internal and jacket pressures and at 30° C

$(V_{b_1}^o)_{30}$ = volume of jacketed bomb portion of V_1 at zero internal and jacket pressures and at 30° C

Substitution of equations (9) and (10) into equation (6) yields equation (11).

$$\beta_1' = \beta_1 \cdot \frac{(V_1^o)_{30}}{(V_{b_1}^o)_{30}} \tag{11}$$

Substitution of equations (7) and (8) into equation (5) gives equation (12).

$$E = \frac{(5 - 4\sigma)(b_b)_{30}^2}{\beta_1'[(b_b)_{30}^2 - (a_b)_{30}^2]} \tag{12}$$

$(V_{b_1}^o)_{30} = 4.649 \text{ in}^3$

$(V_1^o)_{30} = 4.757 \text{ in}^3$

$\sigma = 0.305$ (1)

$(a_b)_{30} = 0.50$ in

$(b_b)_{30} = 1.25$ in

Values for the bomb jacket-pressure distortion coefficient β_1' were calculated from the experimental values of the V_1 jacket-pressure distortion coefficient β_1 using equation (11). Calculated values of β_1' are recorded in table 5. Values of Young's modulus E were calculated from the values of β_1' using equation (12). For purposes of identification, the values of Young's modulus calculated from equation (12) are designated as observed values. The observed values of Young's modulus are recorded in table 5. The observed values of Young's modulus were fitted to equation (13) by the method

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TABLE 5. - Bomb jacket-pressure distortion coefficients and Young's modulus at various temperatures

t,°C	β_1',psia^{-1}	E,Obs.,psi
0	$1.5366 \times 10^{-7} \pm 0.0052 \times 10^{-7}$	$29.285 \times 10^{6} \pm 0.099 \times 10^{6}$
10	$1.5470 \times 10^{-7} \pm 0.0150 \times 10^{-7}$	$29.089 \times 10^{6} \pm 0.282 \times 10^{6}$
20	$1.5502 \times 10^{-7} \pm 0.0109 \times 10^{-7}$	$29.029 \times 10^{6} \pm 0.204 \times 10^{6}$
30	$1.5515 \times 10^{-7} \pm 0.0070 \times 10^{-7}$	$29.004 \times 10^{6} \pm 0.131 \times 10^{6}$
40	$1.5522 \times 10^{-7} \pm 0.0211 \times 10^{-7}$	$28.991 \times 10^{6} \pm 0.395 \times 10^{6}$
50	$1.5512 \times 10^{-7} \pm 0.0043 \times 10^{-7}$	$29.010 \times 10^{6} \pm 0.080 \times 10^{6}$
60	$1.5617 \times 10^{-7} \pm 0.0062 \times 10^{-7}$	$28.815 \times 10^{6} \pm 0.115 \times 10^{6}$
70	$1.5578 \times 10^{-7} \pm 0.0094 \times 10^{-7}$	$28.887 \times 10^{6} \pm 0.174 \times 10^{6}$
80	$1.6004 \times 10^{-7} \pm 0.0163 \times 10^{-7}$	$28.118 \times 10^{6} \pm 0.287 \times 10^{6}$

t,°C	E,Cal.,psi	ΔE,Cal.-Obs.,psi
0	$29.294 \times 10^{6} \pm 0.129 \times 10^{6}$	$+0.009 \times 10^{6}$
10	$29.199 \times 10^{6} \pm 0.107 \times 10^{6}$	$+0.110 \times 10^{6}$
20	$29.104 \times 10^{6} \pm 0.088 \times 10^{6}$	$+0.075 \times 10^{6}$
30	$29.009 \times 10^{6} \pm 0.075 \times 10^{6}$	$+0.005 \times 10^{6}$
40	$28.914 \times 10^{6} \pm 0.070 \times 10^{6}$	-0.077×10^{6}
50	$28.819 \times 10^{6} \pm 0.075 \times 10^{6}$	-0.191×10^{6}
60	$28.724 \times 10^{6} \pm 0.088 \times 10^{6}$	-0.091×10^{6}
70	$28.629 \times 10^{6} \pm 0.107 \times 10^{6}$	-0.258×10^{6}
80	$28.534 \times 10^{6} \pm 0.129 \times 10^{6}$	$+0.416 \times 10^{6}$

of least squares.

$$E = x_2 + x_3 t \qquad (13)$$

The values of the least squares calculated constants of equation (13) are:

$$x_2 = 29.29396 \times 10^6 \pm 0.12896 \times 10^6$$

$$x_3 = -9.4933 \times 10^3 \pm 2.7086 \times 10^3$$

Values of Young's modulus calculated from equation (13) are recorded in table 5 along with the calculated standard deviations of Young's modulus E. Poisson's ratio is assumed to be independent of temperature for the calculations of this report. The data of table 5 are illustrated graphically in figure 2. The observed values of Young's modulus are represented by the small circles in figure 2. The x's represent the standard deviations of the observed values of Young's modulus. The straight line shown in figure 2 represents the least squares calculated values of Young's modulus. The two curved dashed lines in figure 2 represent the limits of the calculated standard deviations of the least squares calculated values of Young's modulus.

CHANGE OF VOLUMES V_1 AND $V_1 + V_2$ AS A FUNCTION OF THE INTERNAL AND JACKET PRESSURES

The change of volume V_1 at 30° C as a function of internal and jacket pressures is given by equation (14), reference (3), page 19.

$$\left(\frac{\Delta V_1}{V_1^o}\right)_{30} = 1.1455 \times 10^{-7} P_i - 1.5161 \times 10^{-7} P_j \qquad (14)^{3/}$$

P_i and P_j in equation (14) are in psia

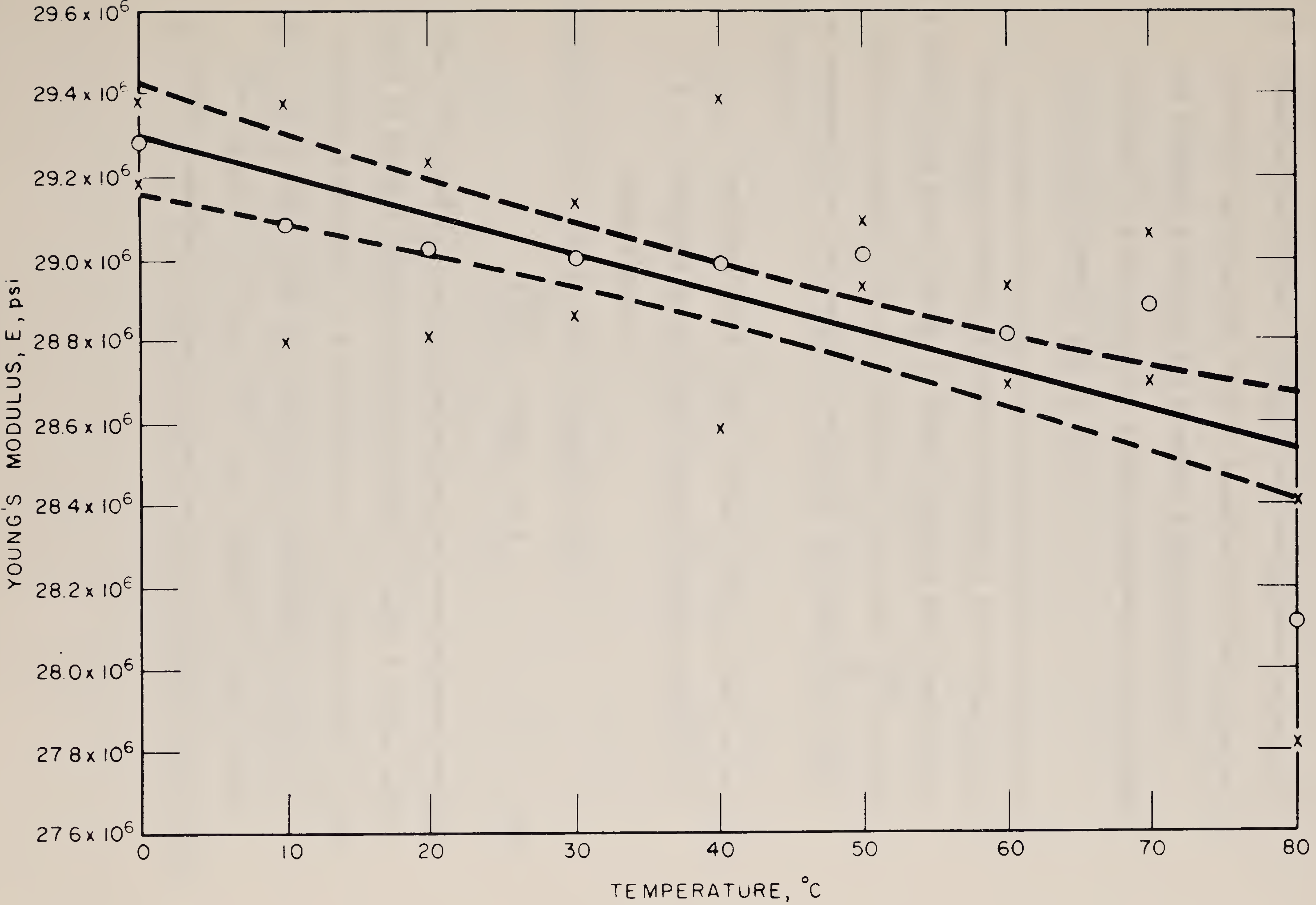

FIGURE 2 - Young's Modulus as a Function of Temperature

3/ Equation (14) differs slightly from the equation reported in reference (3), page 19, because equation (14) was adjusted for the difference between the observed Young's modulus of 29.004×10^6 psi and the least squares smoothed value of Young's modulus of 29.009×10^6 psi at 30° C.

Any change in the distortion coefficients with temperature is assumed to be due to change of Young's modulus with temperature; therefore, equation (14) is multiplied by the ratio $\frac{E_{30}}{E_t}$ to obtain an expression for the change of volume V_1 at temperatures other than 30° C.

E_{30} = smoothed least squares value of Young's modulus at 30° C

E_t = smoothed least squares value of Young's modulus at temperature t° C

Equation (14) is in the form of equation (15).

$$\left(\frac{\Delta V_1}{V_1^o}\right)_t = \alpha_1 P_i - \beta_1 P_j \qquad (15)$$

Values of α_1 and β_1 were calculated from the least squares smoothed values of Young's modulus and are recorded in table 6 in the units of psia and atmospheres. A consistent unit of pressure must be used for P_i and P_j when using the distortion coefficients of table 6.

The change of $V_1 + V_2$ at 30° C as a function of the internal and jacket pressures is given by equation (16), reference (3), page 20.

TABLE 6. - Smoothed internal-pressure and jacket-pressure distortion coefficients

t,°C	$\frac{E_{30}}{E_t}$	α_1,psia^{-1}	β_1,psia^{-1}	α_{1+2},psia^{-1}
0	0.9902710	1.1344×10^{-7}	1.5013×10^{-7}	1.1349×10^{-7}
10	0.9934929	1.1380×10^{-7}	1.5062×10^{-7}	1.1386×10^{-7}
20	0.9967358	1.1418×10^{-7}	1.5112×10^{-7}	1.1424×10^{-7}
30	1.0000000	1.1455×10^{-7}	1.5161×10^{-7}	1.1461×10^{-7}
40	1.0032856	1.1493×10^{-7}	1.5211×10^{-7}	1.1499×10^{-7}
50	1.0065928	1.1531×10^{-7}	1.5261×10^{-7}	1.1537×10^{-7}
60	1.0099220	1.1569×10^{-7}	1.5311×10^{-7}	1.1575×10^{-7}
70	1.0132732	1.1607×10^{-7}	1.5362×10^{-7}	1.1613×10^{-7}
80	1.0166468	1.1646×10^{-7}	1.5413×10^{-7}	1.1652×10^{-7}

t,°C	β_{1+2},psia^{-1}	α_1,atm^{-1}	β_1,atm^{-1}	α_{1+2},atm^{-1}	β_{1+2},atm^{-1}
0	1.5025×10^{-7}	1.6671×10^{-6}	2.2063×10^{-6}	1.6678×10^{-6}	2.2081×10^{-6}
10	1.5074×10^{-7}	1.6724×10^{-6}	2.2135×10^{-6}	1.6733×10^{-6}	2.2153×10^{-6}
20	1.5123×10^{-7}	1.6780×10^{-6}	2.2209×10^{-6}	1.6789×10^{-6}	2.2225×10^{-6}
30	1.5173×10^{-7}	1.6834×10^{-6}	2.2281×10^{-6}	1.6843×10^{-6}	2.2298×10^{-6}
40	1.5223×10^{-7}	1.6890×10^{-6}	2.2354×10^{-6}	1.6899×10^{-6}	2.2372×10^{-6}
50	1.5273×10^{-7}	1.6946×10^{-6}	2.2427×10^{-6}	1.6955×10^{-6}	2.2445×10^{-6}
60	1.5324×10^{-7}	1.7002×10^{-6}	2.2501×10^{-6}	1.7011×10^{-6}	2.2520×10^{-6}
70	1.5374×10^{-7}	1.7058×10^{-6}	2.2576×10^{-6}	1.7066×10^{-6}	2.2594×10^{-6}
80	1.5426×10^{-7}	1.7115×10^{-6}	2.2651×10^{-6}	1.7124×10^{-6}	2.2670×10^{-6}

$$\left(\frac{\Delta(V_1 + V_2)}{V_1^o + V_2^o}\right)_{30} = 1.1461\text{x}10^{-7}P_i - 1.5173\text{x}10^{-7}P_j \qquad (16)^{4/}$$

P_i and P_j in equation (16) are in psia

4/ Equation (16) is based on the smoothed value of $29.009\text{x}10^6$ psi for Young's modulus at 30° C and differs slightly from the equation based on the observed value of $29.004\text{x}10^6$ psi as reported in reference (3), page 20.

Equation (16) is multiplied by the ratio $\frac{E_{30}}{E_t}$ to obtain an expression for the change of volume $V_1 + V_2$ at temperatures other than 30° C. Equation (16) is in the form of equation (17).

$$\left[\frac{\Delta(V_1 + V_2)}{V_1^o + V_2^o}\right]_t = \alpha_{1+2}P_i - \beta_{1+2}P_j \qquad (17)$$

Values of α_{1+2} and β_{1+2} were calculated from the least squares smoothed values of Young's modulus and are recorded in table 6 in the units of psia and atmospheres.

DISCUSSION OF RESULTS

The scatter of the experimental jacket-pressure distortion coefficients is somewhat larger than expected. The maximum calculated standard deviation in a least squares calculated jacket-pressure distortion coefficient is about 0.5 percent. The maximum standard deviation of an average experimental jacket-pressure distortion coefficient is about 1.4 percent. The minimum deviation

of an average experimental jacket-pressure distortion coefficient is about 0.3 percent.

The values obtained for Young's modulus in this work are in general agreement with values reported in the literature. Garofalo, Malenock, and Smith (8) report values of $28.5x10^6$ and $28.2x10^6$ psi for 304 stainless steel at 75° F, $28.0x10^6$ and $28.7x10^6$ psi for 316 stainless steel at 75° F, and $28.5x10^6$ and $28.7x10^6$ psi for 347 stainless steel at 75° F. Garofalo, et al. report values for $\frac{dE}{dt}$ ranging between $-9.0x10^3$ and $-12.6x10^3$ psi per degree C for various steels. These authors state, "Poisson's ratio does not change significantly with temperature." Garofalo, Malenock, and Smith report values of Poisson's ratio ranging from 0.23 to 0.33 for various 18-8 stainless steels at 75° F.

Frederick (7) shows in one of his graphs a value of about $29x10^6$ psi for Young's modulus of 18-8 stainless steel at room temperature, and a $\frac{dE}{dt}$ of about $-10.6x10^3$ psi per degree C for the same steel. Frederick presents a graph showing an increase in Poisson's ratio with increasing temperature for 18-8 stainless steel. His graph shows a value of Poisson's ratio of about 0.297 at room temperature and a change of roughly 0.0001 per degree C.

The high-pressure compressibility bombs are constructed of 303 stainless steel; therefore, the smoothed value of Young's modulus of $29.009x10^6$ psia at 30° C seems reasonable. A change of Young's modulus $\frac{dE}{dt}$ of $-9.49x10^3$ psi per degree C was recorded in this report and falls within the range of values reported in the literature.

The calculations of this report are based, in part, upon PVT data for helium reported in the literature. This is necessary in order to obtain initial estimates of the pressure distortion coefficients. The pressure distortion coefficients of this report will be used in the evaluation of helium compressibility factors from data obtained in the Helium Research Center PVT laboratory. The compressibility factors obtained at the Helium Research Center can then be used to recalculate the pressure distortion coefficients.

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