That quality may become y^e subject of mathematical investigation, it is not enough y^t it may be greater or less. The affections of the mind have this property of quantity and a foundness for y^e appearance of precision for which mathematics is eminent has led some to make use of mathematical reasonings in comparing them. Thus M^r Hucheson says y^t gratitude is directly as y^e product of the favour & y^e dignity of y^e conferrer & inversely as y^e merit of y^e receiver. $G \div \frac{FD}{M}$; but this is rediculous and conveys no meaning. Mathematics is not y^e science of quality but of measure. "Now these quantities are reciprocal of measures."

Measurable quantity is of two kinds one may be measured by itself, such as extension, duration, number, & proportion. Other quantity cannot be thus measured of this kind are velocity, density, force. In order therefore y^t these may be made y^e subject of mathematical reasoning they must be invariably connected with some quantity of y^e first kind, in such a manner y^t y^e quantity is always increased & dimmished when they are, & y^t each determined degree of this is invariably accompanied by a certain deter mined magnitude of the other. In y manner motion may be mathematically considered, because it may be accurately measured by space & velocity. How shall we measure force, vis insita, vis inertiae? By velocity. no; for they are not invariably connected. A body moving w a determined velocity has a determined vis insita, but it will not produce such a change of velocity in a great as in a small body. We must measure it by y quantity of motion. The connection is invariable, for the effect is always produced.

But is there always a determined quantity of y one corresponding to a determined quantity of the other? This is so strictly true that we have no notion of any degree of y one but by means of the other. Nay we have no notion of any degree of y one but in y degree of the other. For y force of a man & y inherent force of mr, are not y objects of our preception. perhaps they are not even existance, & when we say y mr has an inherent force or an Inertia, it is only a short way of saying that, in particular situations, motion is produced, or changed, by it in other mr.

It is in y way only y we are intitled to say that the vis insita & vis inertiae of mr are equal. We only mean to say in a fewer words y y changes produced in y moving & resisting body are equal.

As it is only by y quantity of y effect y we judge of the quantity of the power, so it is only by y direction of the power. Thus if a body from rest moves in y direction AB we say that y power acts in y direction AB. In y same manner, if a body moving uniformly in y direction has its velocity increased without any change in its direction, we say y it acts in y direction AB. If the velocity is dimineshed we say that it acts in y direction BA. If y body, w by moving uniformly during a very small space of time would have got to B, by y action of a power is found at C at that instant of time in w it would otherwise have been at B, then drawing BC we say y y power acts in direction BC, & take BC for its measure.

When motion is observed to have any invariable relation to the situation & distance of any particular body, y cause of such motion is ascribed to that body as a quality or power. Such powers are classed under y general term of y Affections of matter. Thus magnetism, electricity, gravity, are considered as affections of mr, by w it is supposed to produce motion in other bodies at a distance,

altho' we have not yet been able to perceive any mate-
-rial connection between them. We therefore class those
affections of m^r & y^e powers of animals along w^t y^e powers
of m^r hitherto considered, & look on them all as properties
of one kind, capable of mutual comparision, by means of
their common measure, y^e motion w^h they produce.
However it may appear illogical to compute things so dif-
-ferent as impulse & pressure, the impropriety vanishes
when we reflect y^t we are only computing their effects.

The exertions of power are very differently termed,
according to y^e reference we make of y^e motions w^h are
conceived as their effects. If y^e motions are directed to
a point, y^e power is termed attraction - If from a
point repulsion. These metaphors are borrowed from
animal life, and there is no danger of abusing y^e use
of them if we always consider them merely as express-
-ive of the effect, & not of the manner in w^h it is produced.
Were there no changes of motion but such as have been
hitherto considering, & are produced by means of im-
-pulse, we could have no notion of any powers but
inertia & inherent force. But observation points out
many w^h have not been reduced to this head.

If a ball lies on a table & I without striking it only pressed
it on one side, it moves when I pull a string fastened to a
body, it moves.
If I press or pull w^t greater force, it moves faster. If I follow
it w^t y^e pressure or pull, it accellerates in its motion.
When a body is in motion if I press or pull it backwards,
it retards; & y^e more, or y^e longer I press or pull it retards
the more, & may even stop.
when a stone is no longer supported it falls, a spring
disangaged drives a body before it. Iron approaches or avoids
a magnet, and light bodies approach or shun an electri-
-fied body. All these changes of motion are also ascribed to
powers called gravity, elasticity, magnetism, electricity
& these, to distinguish them from impulse, are called
pressing powers. Farther such powers are suggested even
without any production of a change of motion. When I
wrestle w^t another, & feel y^t in order to prevent being thrown
I must exert power. This is also transferred by analogy
to m^r, & when known power produces no motion, it is
conceived as opposed by another power. The vis inertia
of A may not only be deduced from the change w^h it
produces on a moving body B, but also from the

change in A's motion not being such y B shall suffer no change. Without considering y B moving w the velocity 2 suffers a diminution 1 & may only observe y B does not acquire y change 2. In like manner pressing powers are deduced not only from the changes w they produce but also from y changes w they prevent. Thus y cohesion of m^r in a string is deduced not only from giving motion to a body w I pull, but also from preventing a suspended heavy body from falling.

The solidity of m^r is not only deduced from y rest produced in y body w strikes it but from supporting y weight w lies on it.

When powers are conceived as residing in m^r & y effect as produced by their exertion, then y body conceived as possessing y power is said to act on the body in w y effect is perceived.

No term in Nat. philos. is used in a more loose & indetermined sense than y term action. It is a term borrowed from animal life. A man exerting his powers in strict language is said to act. The mind is immediately employed on y body, & properly speaking, acts only on it. But as its action on y body is only w a view of producing some effect on external things, we overlook y instrument, and attentive only to y ultimate object, are said to act on other bodies.

In y strict sense of y word therefore activity means the production of motion where there was none before either in y agent or thing acted on: an active being is one y can do this.

From y by analogy we have transferred y Idea & y term to y production of motion in one body by means of motion in another, & y moving body is said to act on y in w y motion follows. Farther, as we find ourselves obliged to act in order to diminish or destroy motion so a body is analogically said to act when it diminishes or destroys y motion of another body.

A man is not said to act unless he produces some effect. Thought is the effect of y thinking principle acting and motion is y effect of y mind's acting on the body. If we attempt to fix our attention neither on y mind nor on y effect, but on the procedure in producing it, we have no object of contemplation.

The exertion is an event it is not ye object of perception. when therefore we speak of action as distinct from ye agent we usually mean nothing but ye effect. Sometimes indeed we mean mean ye fact of producing it, but ye is a very obscure conception. There is action therefore only in so far as we conceive ye production of an effect. It requires a violent action in me to begin motion in a slide; but I never conceive myself as acting while sliding along. In like manner action is conceived as necessary in order to begin motion in mr. but none to continue it. A body in motion therefore is not acting, for it is producing no effect. Its own motion is not ye continual production of an effect, but ye continuation of an effect already produced. We indeed conceive motion as ye effect of an action but there would be no effect if ye body were not moving. Motion therefore is not action, but ye effect of an action.

Action therefore must be restored to ye production of a change of motion, & is perceived by us in no other way than in ye change produced. It is therefore an inaccuracy of analogy to say ye in a moving body there is a power of preserving motion. There is indeed a power of changing ye motion of another body, but not yet exerted or brought into action.

Action is sometimes considered as a quantity & made ye subject of mathematical investigation.

This obliges us to agree on a measure of action. The change of motion is a proper measure both of ye power & ye action, and is indeed ye only thing usualy meant by either term.

The actions of animals have different names. If in boxing I strike my antagonist, I am properly said to act. But if I only parry his blow I am said to resist tho' ye exertion of my strength is ye same. This distinction is also analogically applied to mr. When a moving body changes the motion of another & ye action is mentioned in reference to ye change it is called action, impulse. But when considered in relation to its mentaining its own condition, it is called resistance. In ye sense both a moving and a resisting body are said both to act & to resist each other & any change of motion.

Sir I. N. considering ye there was in both cases ye same exertion of power, called resistance by ye more abstract term reaction. The distinction therefore between action, or impulse & reaction, or resistance, is a distinction not in ye thing, but in ye reference we make of ye event, as changing ye situation of other body, or a mentaining its own.

Since then ye measure of ye action or impulse & of ye reaction or resistance is one & ye same, namely by the effect or change produced, & since in

all communications of motion y change is equal & opposite
we infer that in the communication of motion
by impulse, action is equal & contrary to re-
action.

With respect to pressing powers, we also call
their exertion action. Gravity, Electricity &c are said
to act on bodies. This action is called by y distinctive
name of pressure, & y opposite action is termed Resistance.
But these are distinctions not in y thing but in
thought. When my strength is exerted in order to
throw y antagonist w whom I wrestle, I act; but when
in order to prevent his throwing me, I resist. They are
best distinguished by y, that when, upon one power ceasing
to act, motion follows, y action of y other power is called
action. When motion does not follow, it is called resistance.
Thus when a string breaks & a body falls, Gravity is
said to have acted. when I cease to press a body to y table, &
y body still lies there, y solidity of y table is conceived as
only to have resisted. We also conceive reaction to take
place in such opposition of pressing powers. we know y
pressing powers are greater & less, but when no chance of
motion ensues from their exertion, we have no measure
of their quantity. Were an obstacle immoveable, no

change of motion produced in y impelling body would
give us any notion of y quantity of its resistance, tho
it would of y action of y resistance. But if known
quantities of motion are just suffering to overcome them
or known powers just suffering to ballance them, we
have proper measures of their quantity. Thus I call
y cohesion of y string or y table double w will just sup-
port a double weight, or just extinguish a double quantity
of motion. Since then these effects are y only way in w we
conceive y existence of such powers or measure their quantity
we must affirm y in y mutual actions of
pressing powers action is equal and
contrary to reaction.

But it remains a question to be decided not from
metaphysical principles but from experiment, whether in
all y exertions of a pressing power there is a concomitant
exertion of an equal & contrary one. This must be consi-
dered as already demonstrated in all those w ballance
each other & in all those w tho they do not ballance
each other yet oppose each others exertions in pro-
ducing motion. For as it only from y ballanceing a
known power in whole or in part that we judge of y
existence of y other it can only be by y quantity

w. it ballances ÿ we can judge of its quantity. Accordingly ÿ is confirmed by experiment a peice of clay w. I press to ÿ table w. my finger is equally flattened on both sides. But it does not necessarily follow ÿ in cases where motion in ÿ one is not opposed by motion in ÿ other there should always be an equal & contrary action. Sir I. N. thinks ÿ it does, & he gives following demonstration. Suppose **A** attracts **B**, but ÿ **B** does not attract **A**. interpose some obstacle it will be uniformly pressed, & will move to ÿ side where it is not pressed. And as ÿ inequality of pressure still continues, it will go on in ÿ derection w. an accellerated velocity, w., says he, is absurd, & contrary to ÿ first law of motion.

But there is no absurdity in any matter of fact, if it does not involve a contradiction. There is therefore no absurdity in saying that a body when left to itself should begin to move, & go on accellerating forever. Some of the instances too w. are adduced in support of ÿ law are not proper. A man in one boat pushing or pulling another boat makes both recede or approach without equal quantities of motion. But in ÿ case ÿ man either pushes one boat with his hand & ÿ other w. his feet, or pulls on boat to his hand & pushes ÿ other w. his feet. All other instances where there is a material connection between ÿ bodies w. acquire equal & contrary quantities of motion are of ÿ same kind.

There are here other experiments & observations w. are not liable to ÿ objection. Magnets & iron mutually approach & recede. So do light bodies & electrified bodies The same is perceived between ÿ Earth & Moon, & causes ÿ tide. The same will be afterwards shown of ÿ Earth, Moon & sun, &c. in short, between all ÿ bodies of ÿ solar system. It will be shown ÿ there is a mutual tendency of all ÿ planets to ÿ sun and of ÿ sun to all ÿ planets, & ÿ all mutually tend to each other, & from ÿ tendency they are all disturbed in ÿ regularity of their motions. It also appears from ÿ observations of Messieurs Buger & Condamine in Peru, & of Maskelyne in Scotland, ÿ ÿ different parts of this globe tend to each other, & by ÿ mutuall tendency are kept together.

Since then ÿ fact may be affirmed to take place invariably in all ÿ exertions of pressing powers which we can observe, we may now assume as ÿ second general law of motion that

Every action of bodies is ac=
=companied by an equal and con=
trary reaction.

For ye resolution of forces see Book
from page 21 to page 38.

Such then are ye general laws of motion, ye principle from wch we are afterwards to reason synthetically, in deducing ye subordinate laws of motion, in investigating ye powers & in explaining ye phaenomena of nature.

Motions are either free or constrained a body is said to move freely, when it moves in ye path & w ye velocity wch its own inherent force & the acting powers would determine. Thus ye planets, terrestrial projectiles move freely. A motion is constrained when it is not allowd to follow ye path along wch its own inherent force or ye action of ye solicitating power would determine it to move. Thus a body descending along an inclined plane, or swinging by a string has its motion constrained.

It is evidently proper to consider ye free motions of bodies first, as it is from them yt we get our knowledge of the active powers of nature.

Secondly not only single bodies move, but also col: collection of bodies have, besides their particular motions, a motion wch belongs to ye whole assemblage. Thus besides the diurnal revolution of ye Earth round its axis and the revolution of ye moon round ye Earth, the assemblage has a general motion round the sun. In like manner that assemblage of mr called a ship has a particular motion of rolling pitching, of its several parts & the general progressive motion in its course. It is evi=dent yt these must be complex, and therefore to be post=poned to ye motions of single bodies. Such an assemblage of bodies, connected together by powers of mutual tendency or by ye cohesion of mr I shall call by ye general name of a system.

Lastly motions may be uniform or varied. As our original notions of bodies are taken from uniform motion, & as every variety is ascribed to some power as its cause it follows yt uniform motions are to be first considered as ye foundations of our future reasonings.

I shall therefore in ye course of Dynamics consider first ye free & secondly ye constrained motion of single bodies & then ye free & constrained motions of systems, in each of wch classes I shall consider first ye uniform motion, & then ye varied & take the varities in ye order of their simplicity – first

of the free motions of a single body — see book from page 47 to page 50 —

I of uniform Motion.

we have already seen y^t motion is of its own nature uniform, & therefore when a body is in motion & affected by no external cause, its motion will have y^e properties formerly demonstrated of uniform motions.

I. A body therefore in motion must, if left to itself, describe equal spaces in equal times.

II. A body in motion must describe spaces proportional to y^e times.

III. In different uniform motions

$$S \div TV$$

$$T \div \frac{S}{V}$$

$$V \div \frac{S}{T}$$

In all these cases our determinations proceed on the composition of y^e quantities w^{ch} have the proportions of y^e times with quantities w^{ch} have the proportions of the times with quantities w^{ch} have y^e proportions of y^e velocities. The proportions of y^e spaces are easily had, because y^e spaces are y^e objects of our perception. But how shall we determine y^e proportions of the times & how are the rendered capable of a composition — This is a question of the greatest importance in Nat. Philos. for without a knowledge of the proportions of times we can't pronounce any thing with regard to motion w^{ch} will be susceptible of any application.

As y^e proportions of time are not immediately perceivable of themselves. The only method w^{ch} we can employ is to find out a proper measure of its parts, some proper quantity so intimately connected to it as y^t we can demonstrate y^t they increase & decrease together, & y^t a certain determined quantity of y^e one always corresponds to a certain determined quantity of y^e other. This measure must be well known in order to be usefull, & y^e comparison must be simple. From y^s it follows y^t uniform motion is the simplest measure of time. on y^e one hand the properties of the parts of a straight line is what of all others we conceive to y^e greatest distinction & farther there is no proportion so clearly perceived as y^t of equality. now in uniform motion the proportion of the parts of time is equal to the proportion of the parts of space run over in them. Uniform motion therefore gives us at once ye means of composing the proportions of ye parts of time to a set of proportions wh we understand best of any, & also to do yt in the simplest manner.

Moreover, independent of its simplicity, it is ye most most natural measure we can employ; for there is no proportion wh we know so accurately as yt of the parts of space, & in general, any motion, whose law is given, would lead us to a discovery of ye proportion of ye parts of time by means of ye relation which subsists, in consequence of yt law between ye spaces & times. If therefore there is any kind of motion in wh ye analogy between ye time & space is given, independent of any hypothesis & by ye very nature of motion, & if yt kind of motion is ye only one in wh ye analogy is given independent of any hypotheses, it is necessarily the most natural measure of time.

Now it is only uniform motion wh has ye character. For motion is of its own nature uniform, & it is accelerated or retarded only by ye actions of external cause, in wh case it is susceptible of an infinity of laws of vaiation. No varied motion therefore becomes ye proper measure of time till we have ascertained ye law of its variation or its difference from uniform motion, & yt supposes us already in possession of the measure of time.

But how shall we know whether any motion is uniform or varied, yt we may employ or reject it. In answer to this I observe yt we are equally ignorant of ye law of all motions wh are not uniform; & ye result is that we cannot know exactly, & wh ye ultimate strictness ye proportion of ye parts of time. But still uniform motion is ye proper & simple measure of them. Not being therefore able to find an accurate measure of time, we must endeavour to find one yt will be nearly so amoung ye motions wh we judge most uniform.

we have three ways of judging of ye uniformity of motion.

1. If a body moves over equal spaces in times wh we have reason to think equal, yt is in wh repeated experiment shews ye equal effects are produced wh we have reason to think should employ ye same time The time of emptying a sand glass is reasonably supposed by us to be always the same. If therefore in these times ye Sun is observ'd to move thro equal arches, we have reason to think his motion uniform. Accordingly yt was ye first invention for measuring time & tho' dayly motion of ye sun being found always to

employ y^e same number of intervals of emptying y^e glass, was substituted as a proper measure. For besides inequalities w^c could be observed in different glasses, the observations made w^t them at different places could not be compared w^tout refering theme to some common measure. Thus y^e measures were changed, & whereas a day was formerly counted so many glasses, a glass was afterward counted such a part of a day—

2. when we have reason to believe y^t y^e action of any accelerating or retarding cause w^c we know however to exist is constant. It is in y^e manner, by uniting these two methods, y^t y^e motion of y^e Earth round her axis is reckoned y^e most accurate measure of time; & y^e supposition is not only not contradicted by y^e other ultimate motions, but found perfectly to agree w^t them. We have found reason for rejecting y^e Sun's diurnal & annual motion because we have discovered y^e existence of accelerating & retard

X

retarding causes w^ch are too considerable not to produce sensi[ble]
irregularities. Whereas in y^e motion of y^e Earth, all y^e mo-
-tions w^ch are performed on its surface, & w^ch must indeed
disturb its regularity, & y^e motion of y^e tides, cannot produce
a deviation w^ch will bear any sensible proportion to the
motion of y^e Earth.

3. when we compare y^e supposed uniform motion
w^th other motion & observe the same law in both – Thus
if we see two bodies move in such a manner y^t y^e
spaces w^ch they describe in y^e same time are always in a
given ratio and y^t y^s holds whether y^e bodies have both
begun to move at y^e same time or at different times, &
when we can see no cause of their motion but y^e motion
once begun then we reasonably judge these motions to be
uniform. All these circumstances united have led us to
prefer y^e motion of y^e Earth round its axis as y^e proper
measure of time. Subdivisions of a revolution are had by
means of y^e fixed stars, which, as they do not change their relative
places are proper measures of y^e parts of space. By this means
we are enabled to discover y^t y^e motion of y^e sun & moon
are not uniform & to determine y^e laws of their variation, &
thus arrive at a knowledge of y^e changing forces the principle
object of nat. philosophy. – For varied motions & se book from p. 53 to 6[?]

Theorem Fig. 13

Let a body A move from rest along AC by an accelerating force directed to C. Let there be a curve KDF such that the area ABDK is always proportional to the square of y^e velocity w^{ch} y^e body has acquired at B. Then I say y^t y^e accelerating force at any point B is always proportional to the ordinate BD

Demonstration

Take BE an infinetely small part of AC, & draw the ordinate EF. Call y^e space AB s, and y^e area ABDK v^2. Then BE $= \dot{s}$, & BEFD $= 2v\dot{v}$, and BD $= \frac{2v\dot{v}}{\dot{s}}$

$$\text{Now } F = \frac{\dot{v}}{t}$$

$$\text{and } \dot{s} = vt$$

$$\text{therefore } F \times \dot{s} = \frac{\dot{v}}{t} \times vt, = v\dot{v}$$

$$\text{and } F = \frac{v\dot{v}}{\dot{s}}$$

$$\text{But } BD = \frac{2v\dot{v}}{\dot{s}}$$

$$\text{therefore } BD = 2F$$

$$\text{and } BD \text{ is always} \div F$$

Coroll^y If $V^2 \div s$ then y^e power is constant for then $A \div x'$ & consequently $y \div x', \div 1$

Theorem

Let a body A move as before, & let BD be always proportional to y^e force at any point B. Then y^e area ABDK will always be proportional to y^e square of y^e velocity at B.

This is y^e converse of y^e former, & is demonstrated in y^e very same way, only reversing y^e steps.

Corollary 1. Then y^e square root of $\sqrt{ABDK}$ is equal to y^e velocity acquired by moving thro' AB

Corollary 2^d Draw another ordinate MN. Then MNDB is equal to y^e square of y^e velocity acquired at B. the square of y^e velocity acquired at M; and y^e square root of $\sqrt{MNDB}$ is equal to y^e increment of y^e velocity acquired in moving over the space MB

Corollary 3^d If F constant then y^e space is proportional to y^e square of y^e velocity. For then $y \div 1, \div x^0$; and $A \div x'$, but $A \div v^2$, or $S \div v^2$

Theorem

Let a body A move as before, & let there be a curve LHI such y^t y^e ordinate BH is always inversely proportional to y^e velocity acquired at B. Then the area ABHL will always be proportional to the time of moving from A to B.

Demonstration

In AC take an infinitely small part BE and draw the ordinate EI then $BH = \frac{1}{v}$

and $BE = s$

and $BEIH = \frac{1}{v} \times s, = \frac{s}{v}$

But $v = \frac{s}{t}$

therefore $vt = s$

and $t = \frac{s}{v}$

or $t = BEIH$

and $\int t$, or $t = \int BEIH$ or $ABHL$

Corollary

1, If F is constant then $s \div t^2$. For then $x \div v^2$, Then is $x \div \frac{1}{y^2}$, or $y^2 \div \frac{1}{x}$, or $y \div \frac{1}{x^{1/2}}$, or $y \div x^{-\frac{1}{2}}$; therefore $A \div x^{1/2}$, or $x \div A^2, = t^2$.

II. If F is constant then $v \div t$. For then $s \div v^2$ and it has just now been shown ye $s \div$ to t^2 therefore $v^2 \div t^2$, and $v \div t$. Such a force therefore produces an uniformly accelerated motion. For if $v \div t$, them $v \div t$.

III. If F is constant, then ye time of describing any given space a is double of the time in wh ye space would be uniformly described wth ye velocity v acquired at ye end of ye accelerated motion. Here v is also supposed known, & $\frac{a}{v}$ is the time T of the uniform motion. Now since v is given y is given, for $= \frac{1}{v}$. Then $A = 2a\frac{1}{v}$, or $\frac{2a}{v}$. But $A = t$ therefore $t = \frac{2a}{v}, = 2T$. Hence if any space a is described by the action of a constant force F in a time t, twice ye space will be described in ye same time uniformly wth ye velocity acquired at ye end of ye accelerated motion.

Cory Draw any other ordinate MO then $MBHO$ = time of moving from M to B

Theorem

The spaces thro' which any two constant forces F and φ must impell a body in motion to communicate ye same velocity v are reciprocally as ye forces.

Let AB, αB be ye spaces, & AF, $\alpha\varphi$ be ye forces. Then ye curves expressing ye law of ye forces are straight lines FD, φE. The velocities are supposed equal therefore ye areas $ABDF$, $\alpha BE\varphi$, wh are as ye squares of ye velocities must be equal. therefore $AF : \alpha\varphi = \alpha B : AB$.

Of Curvilineal motions

Since a body of itself tends to move in a straight line, it cannot move in a curve wthout ye continual action of some transverse force whose action, being combined wth ye inherent force of the body, makes it continually to deflect from the rectilineal path, wh it every mome tends to pursue along the tangent to ye point of the curve where it is —

Free curvilineal motions therefore are always indications of ye continual action of a pressing power. The degree or quantity of this pressing power is to be deduced from ye effect wch it produces, it is from ye deflexion which it occasions from a straight line this again must be measured by ye distances of ye point where ye body really is from that point of ye tangent to ye curve to wch it would have arrived in the same instant had ye pressing power not acted on it —

The deviation of ye pressing power will be deduced from ye position of ye line joining these two points —

This line is in all cases ye subtence of ye angle of contact at ye point from which the deflections is supposed to begin.

But as ye pressing power is unknown, & its exertions may vary according to ~~any~~ any law and as ye deflexion from the straight line at ye end of any given time, or at the end of any arch of the curve, is ye effect of ye accumulated, & perhaps varied action of ye power, it is evident yt it will not give us a precise notion of ye exertion of ye power at any instant of its action.

It follows then yt ye only way of determining ye wth accuracy is to consider ye subtense of ye angle of contact in its nascent or evanascent state, when ye body has described a nascent arc of ye curve, in ye case ye ratio of ye inherent force to ye transverse force is ye same wth ye ratio of ye nascent arch to ye nascent subtense of ye angle of contact, wch again is ye same wth ye ratio of ye nascent chord to ye nascent subtense of ye angle of contact —

From all yt it follows yt our investigation of those powers of nature wch produce curvilineal motion must proceed on ye fluxionary proportions of ye curve described —

On ye other hand suppose ye nature of ye transverse force to be known we can at all times tell what curvilineal motions will insue upon its acting on a body moving in any known direction wth a known velocity. For if we know ye degree of ye force, we know ye motion wch it will produce on a given body, & consequently we know ye proportion between ye proportion between ye motion & yt of ye body. That is we know ye ratio of ye nascent subtense of ye angle of contact to the nascent chord of ye curve along wch it tends at ye moment to direct ye body. Farther if we know ye direction of ye transverse force we shall know ye position of ye subtense — nothing more is necessary to determine any curve whatever.

1. In general we can affirm yt ye body will deflect continually & its path will be a curve —

2. Its path will be concave to y^e side to w^h y^e force is directed—
3. Its deflection from y^e tangent will be so much y^e greater as y^e ratio of y^e transverse force to y^e inherent force is greater—
4. Its deflection will be greater as y^e transverse force is more nearly perpendicular to y^e direction of y^e bodies motion—

Instances of free curvilineal motions are frequent, & of great dignity. all bodies projected on y^e surface of y^e Earth in directions oblique to y^e horizon— all y^e planetary motions— Therefore they deserve our accurate examination, as they serve fore discovering y^e greatest powers in nature—

However variable these curvilineal motions may be, there is one general principle in w^h they all agree & by it we are enabled w^t case to investigate y^e nature of y^e transverse power w^h produce them.—

If a body A revolves in any curve ABC (fig. 14) & if it be found from observation that there is a power S such y^t, drawing from any three points A, B, C, the straight lines AS, BS, CS, y^e areas SAB, SAC are proportional to y^e times of describing the curvilianeal bases AB, BC; then y^e transverse power w^h causes the deflection from y^e rectilineal motion is a power continually directed to y^e point S,

Since areas are described proportional to the times equal areas are described in equal times.

Let therefore AB, BC be two nascent arches described in two equal times. The arch AB may be conceived as ultimately coinciding w^h its tangent ABD— Take BD equal to AB ~~equal~~ draw BC, DS, & BE equal & parallel to DC.— & join CE.— DE indicates by its direction y^e direction of y^e force w^h acted at B. For y^e body being actuated by its perseverance only would have gone on to D in a time equal to that of describing AB, that is, in y^e time of describing BC. BE therefore is y^e direction of y^e transverse force acting at B. But since AB is equal to BD the triangle SAB is equal to y^e triangle SBD. But SAB is equal to SBC therefore SBC is equal to SBD, & DC is parallel to BS. Therefore y^e line BE lies in y^e line BS, & y^e transverse force at B is directed to S—

On y^e other hand if it is previously known y^t y^e transverse force is always directed to S, it may be demonstrated y^t y^e body will describe y^e curvilineal bases of areas proportional to y^e times in w^h y^e bases are described— The demonstration is just y^e inverse of this—

These two theorems were discovered by Sir I. N. & make y^e foundation of all his discoveries in y^e just movements of Nature. The first serves for y^e discoveries of y^e determining y^e motions occasioned by y^e forces already discovered.—

For, in y^e first place, you must observe y^t altho' y^e measure of the transverse force is a deflection from the rectilineal motion, yet y^t cannot be discovered till its direction is known. Secondly knowing y^e property of y^e areas & y^e times, we can at any time assign y^e power in w^h a revolving body is to be found, by means of its known motion during any given time. Suppose, for instance, that y^e planet Mars acted on by a force directed to the sun has in one day described y^e arch AC, & I want to know in what point of his orbit he will be after three days. Draw a line SG making y^e area CSG = 3ASB, and G is y^e point of his orbit to w^h he will arrive at y^e end of three days.

When y^e directions of transverse forces always pass thro' a given point, they are called y^e central forces, & y^e point is called y^e center of forces; if they are always directed to y^t point, & y^e curve is consequently concave to y^e point, they are called centripetal forces, & by analogy attractions. If they are directed from y^e point, & y^e curve is convex to it, they are centrifugal forces, & by analogy repulsions. Instances of both occur in Magnetism & Electricity.

Of Central forces

If a body revolves in a curve by means of a force directed to a point, its velocity is always inversely proportional to a perpendicular let fall from y^e centre of forces on y^e tangent drawn thro y^e place of y^e body. For, [fig. 15] let AB, CD be two nascent arches described in equal times; they may be considered as coinciding w^h y^e tangents BH, DG. Since y^e triangles SAB, SCD are equal, their bases AB, DC are reciprocally proportional to their heights SH, SG. But as their bases are described in y^e same time they are as y^e velocity w^h w^h they are described: therefore y^e velocity at A : velo^y at C = SG : SH.

If a body actuated by a centripetal force move in any curve, & another body actuated by y^e same force moves in y^e line passing thro' y^e centre of forces, & if their velocities at any equal distances from y^e centre be equal, they will also be equal in all or equal distances from it.

Let y^e straight line AS pass thro' y^e centre of forces, & let the bodies have equal velocities in I and D equally distant from S. Join IS, & taking DE a small part of DS describe y^e circle ENK. Draw KS, and draw NT perpendicular to the curve in T.

Since DS = IS, y^e centripetal forces at D & I are equal. Let them be represented by y^e equal lines DE, IN. Resolve IN, into NT, w^h produces no change of velocity along y^e curve, &

IT, w accelerates y velocity in it. The accellerations in equal moments of time are proportional to y accelerating forces. Therefore this acceleration in D is to that in I — DE : IT, but in different small times the accelerations are as the forces & times jointly (for in small times y forces may be considered as constant.) But since y velocities at I and D are equal y times of describing DE and IK will be as DE, IK therefore y acceleration in D is to y acceleration in I as $DE \times DE$ or DE^2, or $IN^2 : IT \times IK$. But since the angular velocities are inversely proportional to y square of y distance from y centre of forces for y angular velocities at A & C (fig. 15) are as y increments of y angles ISA, ISC w were formerly demonstrated to be as $\frac{AE}{AS} : \frac{CF}{CS}$. Now since $AE \times AS = CF \times CS$ on account of the equality of areas then $\frac{AE}{AS} : \frac{CF}{CS} = \frac{AE \times AE \times AS}{AS} : \frac{CF \times CF \times CS}{CS} = AE^2 : CF^2$. But since $AE \times AS = CF \times CS$, then $AE : CF = CS : AS$ and $AE^2 : CF^2 = CS^2 : AS^2$ therefore y angular velocities are reciprocally proportional to their distances.

An Orbit lies all in one plane

It has been shewn y y nature of y central force was to be deduced from that of the curve in w it incited y body to move now these are susceptible of an infinite variety, & there is no occasion for examining what kind of central force would make a body describe each. There is no way of connecting them w will enable us to deduce a general method of investigation applicable to all.

If a thread revolves round a fixed point in a plane, it will describe a circle. If it gradually untwists from a cylinder it will describe a spiral. If it untwists from a Cycloid it will describe an equal Cycloid, and so on of other curves. Every curve will produce another curve by a thread gradually untwisting itself from it. on the other hand there is no curve, concave to one side, (w all orbits will be) but what may be described by y gradually untwisting of a thread from some other curve. Every curve may in this manner be considered as y evolute of some other curve Fig. 16 no 2

Now while the thread is unfolding, it may, in every moment of its motion, be considered as y momentary radius of a circle, whose centre is that point of y evolving curve w is just going to quit. The extremity of y thread therefore is at y same time describing y nascent arch of y curve called y evolute, and y nascent arch of y circle. Consequently these two nascent arches coincide.

From of ye genesis of ye curve it appears yt any body, while it describes ye nascent arch of the curve, may be considered as describing ye nascent arch of ye coincident or equicurve circle. If therefore we can find a general expression for ye centeral force directed to any point of a circle, we at ye same time find ye expression for ye central force in a coincident arch of a curve, where ye line drawn to the centre of forces is similarly placed w respect to the tangent. & all yt now remains is to determine from ye properties of ye curve ye Chord of ye enquicurve circle.

Let two bodies A, F (fig. 17) revolve in the circles ABE, FGK by means of central forces directed to any points S and T, and let AB, FG be nascent arches discribed in equal times — draw ye chords ASE, FTK, and ye chords AB, FG, BE, GK and ye tangents AC, FH — and draw BC, GH parallel to AE, FK, and BD, GI parallel to AC, FH ——

It is evident yt CB or AD is ye proper measure of ye centripetal force at A. & yt FI is ye measure of ye centripetal force at F. By ye property of ye circle EA:AB = AB:AD, and KF:FG = FG:FI therefore

$$AD = \frac{AB^2}{AE}, \text{ and } FI = \frac{FG^2}{KF}$$

But since ye arches AB, FG are described in equal times, they are as ye velocities at A & F, and AD, FI are as ye forces at A & F and AE, KF are ye chords at A & F therefore

$$F,A : F,F = \frac{V^2A}{CA} : \frac{V^2F}{CF}$$

Hence we deduce ye following general theorem and formula —

Central forces are directly as ye squares of ye velocities. & inversely as the chords of ye equicurve circle passing thro ye centre of forces —

The formula is $F = \frac{v^2}{C}$

Cor. 1 — If S is ye centre of ye circle then C is equal to twice ye radius. But as halves have ye same proportion w their wholes C will always be proportional to R, or to the distance D from ye centre of forces —

2. The velocity will be every where equal & ye motion uniform. For ye perpendicular to ye tangents, w is always reciprocally proportional to ye velocity, is always ye same or radius —

3. In different circles ye central force is directly proportional to ye distance and inversely proportional to ye square of ye periodic time P.

For the velocity is directly as ye circumference described and inversely as ye time of describing it. or $v \div \frac{D}{P}$. because D or radius $\div$ to ye circumference. Therefore $v^2 \div \frac{D^2}{P^2}$, and since $F \div \frac{V^2}{C}$, and $C = D$, then $F \div \frac{\frac{D^2}{P^2}}{D}$, or $F \div \frac{D}{P^2}$.

Cor. 4. If a body fall along ye radius with ye uniform action of ye force wh retains it in ye circumference, it will acquire a velocity equal to that wh it revolves in ye circumference. Let AL be ye space thro' wh it must fall in order to acquire ye velocity of ye revolution, and let AF be the arch described uniformly during ye time T of falling along AL. Then $AF = 2AL$, and $AF^2 = 4AL^2$. Let AB be a nascent arch, and AD be its versed sine, or ye space thro' wh ye centripetal force would cause ye body to fall in ye time t of describing AB. Then we have ye proportion $AD : AL = t^2 : T^2$, or on account of ye uniform motion in ye circumference

$$AD : AL = AB^2 : AF^2 = \frac{AB^2}{AG} : \frac{AF^2}{AG}.$$

But it has been Demonstrated yt $\frac{AB^2}{AG} = AD$. Therefore $\frac{AF^2}{AG} = AL$, and therefore $\frac{4AL^2}{AG} = AL$, or $\frac{4AL}{AG} = 1$ & therefore $AG = 4AL$, & $AC = 2AL$.

Cor. 5 The periodic time is to ye time of falling thro' AL as ye circumference of a circle to ye radius. For $AF = R.VP:T$; $AF = AFA : AF$ = circumference : Radius. But $T; AF = T; AL$. Therefore $P:T; AL$ = Circumference : Rad.

These two formulas $F \div \frac{v^2}{C}$ & $F \div \frac{D}{P^2}$ are all yt is necessary for investigating ye central forces which produce any motions wh we can observe. The first serves for discovering ye law of ye central forces producing a revolution in any one orbit and ye second for comparing together the forces wh produce revolutions in different orbits.

Let it be always remembered yt in order to avoid error, it is proper to consider F not as ye immediate symbol for any metaphysical being existing in ye body or the agent, but as a symbol expressing merely ye quantity of approach wh ye revolving body makes to ye centre of forces in some small given time, suppose a second.

This way of considering it has also ye peculiar advantage yt we can compare every accelerating force whatever [& a central force is an accelerating force whose direction is transverse to ye motion of ye body] wh the fall of heavy bodies let us take for ye term of comparison a second.

In this time a heavy body falls thro' 16.1 feet. If therefore F is taken as a symbol of ye effect of gravity F = 16.1 feet. Let us express this however by ye letter G. G then is the measure of gravity considered as an accelerating force, & ye proportion of G to $\frac{V^2}{D}$ will always express the proportion of gravity to the accelerating force, or $\frac{V}{DG} = F$, that is whatever is ye nature of ye centripetal force, ye difference of ye forces with a body would revolve in ^a quiescent curve, & another body revolve in ye same curve, while it revolves uniformly round ye centre of forces, are in ye inverse triplicate ratio of ye distance from ye centre, that is to say $\frac{V^2}{DG}$, or $\frac{\frac{1}{16.1}V^2}{D}$ will be ye number of feet wh the central, or other accelerating force F will cause the body to move thro' in a second.