

證明 AB, BC, CD ヲ正多角形ノ相連續セル三邊トセヨ.  $\angle B$  及  $\angle C$  ノ二等分線ノ交點ヲ  $O$  トシ,  $O$  ヨリ  $BC$  ニ垂線ヲ下シ, 其足ヲ  $L$  トセヨ. 然ルトキハ  $O$  ハ内接圓及外接圓ノ中心,  $L$  ハ  $BC$  ノ中點ニシテ  $OL$  ハ内接圓ノ半徑,  $OC$  ハ外接圓ノ半徑ナルコト明カナリ.

$\angle BOC$  ハ, 中心  $O$  ニ於テ各邊ヲ見込ム角ノ和即チ 4 直角ノ  $\frac{1}{n}$  ニ等シキヲ以テ

$$\angle BOC = \frac{2\pi}{n} \text{「レデアン」}$$

$$\therefore \angle BOL = \frac{1}{2} \angle BOC = \frac{\pi}{n}$$

$$\therefore a = 2 \cdot BL = 2 \cdot OL \tan \angle BOL = 2r \tan \frac{\pi}{n}$$

$$\therefore r = \frac{a}{2 \tan \frac{\pi}{n}} = \frac{a}{2} \cot \frac{\pi}{n}$$

$$\text{又 } a = BC = 2 \cdot BL = 2R \sin \angle BOL = 2R \sin \frac{\pi}{n}$$

$$\therefore R = \frac{a}{2 \sin \frac{\pi}{n}} = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$$

### 8. 正 $n$ 邊形ノ面積

正  $n$  邊形ノ一邊ノ長サヲ  $a$  トシ, 面積ヲ  $S$  トスレバ次ノ公式アリ.

$$S = \frac{na^2}{4} \cot \frac{\pi}{n}$$

證明  $S = n \cdot \triangle BOC = n \times \frac{1}{2} OL \cdot BC$   
 $= n \cdot OL \cdot BL = n \cdot BL \cot \angle LOB \cdot BL$   
 $= n \cdot \frac{a^2}{4} \cot \frac{\pi}{n}$

### (第四) 級數, 不等式等

9.  $\sin a + \sin(a + \beta) + \sin(a + 2\beta) + \dots + \sin(a + (n-1)\beta)$

ヲ求ムルコト.

解  $2 \sin a \sin \frac{\beta}{2} = \cos\left(a - \frac{\beta}{2}\right) - \cos\left(a + \frac{\beta}{2}\right)$

$$2 \sin(a + \beta) \sin \frac{\beta}{2} = \cos\left(a + \frac{\beta}{2}\right) - \cos\left(a + \frac{3\beta}{2}\right)$$

$$2 \sin(a + 2\beta) \sin \frac{\beta}{2} = \cos\left(a + \frac{3\beta}{2}\right) - \cos\left(a + \frac{5\beta}{2}\right)$$

.....

$$2 \sin(a + (n-1)\beta) \sin \frac{\beta}{2} = \cos\left(a + \frac{2n-3}{2}\beta\right) - \cos\left(a + \frac{2n-1}{2}\beta\right)$$

此總テノ等式ヲ邊々相加ヘ, 且ツ與ヘラレタル級數ノ和ヲ  $S$  トスレバ

$$2S \sin \frac{\beta}{2} = \cos\left(a - \frac{\beta}{2}\right) - \cos\left(a + \frac{2n-1}{2}\beta\right)$$

$$= 2 \sin\left(a + \frac{n-1}{2}\beta\right) \sin \frac{n\beta}{2}$$

$$\therefore S = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin\left(a + \frac{n-1}{2}\beta\right) \dots \dots \dots (1)$$



10.  $\cos a + \cos(a + \beta) + \cos(a + 2\beta) + \dots + \cos(a + (n-1)\beta)$

ヲ求ムルコト.

解

$$2\cos a \sin \frac{\beta}{2} = \sin\left(a + \frac{\beta}{2}\right) - \sin\left(a - \frac{\beta}{2}\right)$$

$$2\cos(a + \beta) \sin \frac{\beta}{2} = \sin\left(a + \frac{3\beta}{2}\right) - \sin\left(a + \frac{\beta}{2}\right)$$

$$2\cos(a + 2\beta) \sin \frac{\beta}{2} = \sin\left(a + \frac{5\beta}{2}\right) - \sin\left(a + \frac{3\beta}{2}\right)$$

.....

$$2\cos(a + (n-1)\beta) \sin \frac{\beta}{2} = \sin\left(a + \frac{2n-1}{2}\beta\right) - \sin\left(a + \frac{2n-3}{2}\beta\right)$$

ソコテ與ヘラレタル級數ノ和ヲ S トスレバ

$$2S \sin \frac{\beta}{2} = \sin\left(a + \frac{2n-1}{2}\beta\right) - \sin\left(a - \frac{\beta}{2}\right)$$

$$= 2\cos\left(a + \frac{n-1}{2}\beta\right) \sin \frac{n\beta}{2}$$

$$\therefore S = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cos\left(a + \frac{n-1}{2}\beta\right) \dots \dots (2)$$

例 1.  $\sin a + \sin 3a + \sin 5a + \dots + \sin(2n-1)a$  ヲ求ムルコト.

解 (1)ニ於テ  $a=a, \beta=2a$  トオケバ,此和ハ

$$\frac{\sin \frac{n \cdot 2a}{2}}{\sin \frac{2a}{2}} \sin\left(a + \frac{n-1}{2} \cdot 2a\right) = \frac{\sin na}{\sin a} \sin na = \frac{\sin^2 na}{\sin a}$$

例 2.  $\cos a - \cos(a + \beta) + \cos(a + 2\beta) - \dots$  ノ n 項ノ和ヲ求ムルコト.

解 此級數ノ和ハ次ノ如ク書クコトヲ得.

$$\cos a + \cos(a + \beta + \pi) + \cos(a + 2\beta + 2\pi) + \cos(a + 3\beta + 3\pi) + \dots$$

ソコテ (2)ニ於テ  $\beta$ ノ代リニ  $\beta + \pi$ ヲオケバ求ムル和ハ

$$\frac{\sin \frac{n(\beta + \pi)}{2}}{\sin \frac{\beta + \pi}{2}} \cos\left\{a + \frac{n-1}{2}(\beta + \pi)\right\}$$

$$= \frac{\sin \frac{n(\beta + \pi)}{2}}{\cos \frac{\beta}{2}} \cos\left\{a + \frac{(n-1)(\beta + \pi)}{2}\right\}$$

例 3.  $S = \operatorname{cosec} a + \operatorname{cosec} 2a + \operatorname{cosec} 4a + \dots + \operatorname{cosec} 2^{n-1}a$

ヲ求ムルコト.

解  $\operatorname{cosec} a = \frac{1}{\sin a} = \frac{\sin \frac{a}{2}}{\sin \frac{a}{2} \sin a} = \frac{\sin\left(a - \frac{a}{2}\right)}{\sin \frac{a}{2} \sin a}$

$$= \frac{\sin a \cos \frac{a}{2} - \cos a \sin \frac{a}{2}}{\sin \frac{a}{2} \sin a} = \cot \frac{a}{2} - \cot a$$

$$\therefore \operatorname{cosec} a = \cot \frac{a}{2} - \cot a$$

$$\operatorname{cosec} 2a = \cot a - \cot 2a$$

$$\operatorname{cosec} 4a = \cot 2a - \cot 4a$$

.....

$$\operatorname{cosec} 2^{n-1}a = \cot 2^{n-2}a - \cot 2^{n-1}a$$

故ニ加法ニヨリテ  $S = \cot \frac{a}{2} - \cot 2^{n-1}a$



## 11. 不等式ノ證明ノ例

例 1.  $a \tan^2 \theta \neq b$  ナルトキハ  $a^2 \tan^2 \theta + b^2 \cot^2 \theta > 2ab$   
ナルコトヲ證明セヨ.

證明  $a^2 \tan^2 \theta + b^2 \cot^2 \theta = (a \tan \theta - b \cot \theta)^2 + 2ab$

然ルニ  $a \tan^2 \theta \neq b \quad \therefore a \tan \theta \neq b \cot \theta$

$\therefore a \tan \theta - b \cot \theta \neq 0 \quad \therefore a^2 \tan^2 \theta + b^2 \cot^2 \theta > 2ab$

是ニヨリテ  $a^2 \tan^2 \theta + b^2 \cot^2 \theta$  ハ  $a \tan^2 \theta = b$  ナルトキ極小ニシテ其値ハ  $2ab$  ナリ.

例 2.  $1 + \sin^2 \alpha + \sin^2 \beta > \sin \alpha + \sin \beta + \sin \alpha \sin \beta$  ナルコトヲ證明セヨ. 但シ  $\sin \alpha, \sin \beta$  ハ何レモ  $1$ ニ等シカラズトス.

證明  $\sin \alpha - 1 \neq 0 \quad \therefore (1 - \sin \alpha)^2 > 0$

$\therefore 1 + \sin^2 \alpha > 2 \sin \alpha$

同様ニ  $1 + \sin^2 \beta > 2 \sin \beta$

又  $\sin^2 \beta + \sin^2 \alpha \geq 2 \sin \beta \sin \alpha$

$\therefore 2(1 + \sin^2 \alpha + \sin^2 \beta) > 2(\sin \alpha + \sin \beta + \sin \alpha \sin \beta)$

$\therefore 1 + \sin^2 \alpha + \sin^2 \beta > \sin \alpha + \sin \beta + \sin \alpha \sin \beta$

例 3.  $a, b$  ヲ二ツノ正ノ數トシ,  $a > b$  トスレバ, 一般

ニ

$$a \operatorname{cosec} \theta > b \cot \theta + \sqrt{a^2 - b^2}$$

ナルコトヲ證明セヨ.

證明  $a \operatorname{cosec} \theta - b \cot \theta = \frac{a - b \cos \theta}{\sin \theta}$

$$= \frac{a \left( \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} \right) - b \left( \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right)}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$= \frac{(a+b) \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} + \frac{(a-b) \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$= \frac{1}{2} (a+b) \tan \frac{\theta}{2} + \frac{1}{2} (a-b) \cot \frac{\theta}{2}$$

然ルニ  $a+b > 0, a-b > 0$  ナルヲ以テ例 1ニヨリテ, 此式ハ一般ニ

$$2 \sqrt{\frac{1}{2}(a+b)} \sqrt{\frac{1}{2}(a-b)} = \sqrt{a^2 - b^2}$$

ヨリ大ナリ. 即チ一般ニ

$$a \operatorname{cosec} \theta - b \cot \theta > \sqrt{a^2 - b^2}$$

$\therefore a \operatorname{cosec} \theta > b \cot \theta + \sqrt{a^2 - b^2}$

## 12. 極大, 極小ノ問題ノ例

例 1.  $a \cos \theta + b \sin \theta$  ノ最大數値ヲ求ムルコト.

解  $a = r \cos \alpha, b = r \sin \alpha$  トオケバ

$$r^2 = a^2 + b^2, \tan \alpha = \frac{b}{a}$$

$\therefore a \cos \theta + b \sin \theta = r \cos \theta \cos \alpha + r \sin \theta \sin \alpha$

$$= r \cos(\theta - \alpha)$$

故ニ此數値ノ最大ナルハ

$$\cos(\theta - \alpha) = \pm 1 \quad \text{從テ } \theta - \alpha = n\pi$$

ナルトキ, 即チ



$$\theta = n + n\pi$$

ナルトキ ( $n$  は任意ノ整数又ハ 0) = シテ、其絶対値ハ

$$r = \sqrt{a^2 + b^2}$$

ナリ。

例 2.  $a \cos(\alpha + \theta) + b \cos(\beta + \theta)$  ノ最大數値ヲ求ムル

コト。

$$\begin{aligned} \text{解 } a \cos(\alpha + \theta) + b \cos(\beta + \theta) &= a(\cos\alpha \cos\theta - \sin\alpha \sin\theta) \\ &\quad + b(\cos\beta \cos\theta - \sin\beta \sin\theta) \end{aligned}$$

$$= (a \cos\alpha + b \cos\beta) \cos\theta - (a \sin\alpha + b \sin\beta) \sin\theta$$

ナルヲ以テ例 1 = ヨリ求ムル最大數値ハ

$$\begin{aligned} &\sqrt{(a \cos\alpha + b \cos\beta)^2 + (a \sin\alpha + b \sin\beta)^2} \\ &= \sqrt{a^2 + b^2 + 2ab \cos(\alpha - \beta)} \end{aligned}$$

ナリ。

例 3.  $n$  箇ノ正ノ角  $\theta_1, \theta_2, \theta_3, \dots, \theta_n$  ノ和ガ一定ノ角  $\alpha$  ( $\alpha < 2\pi$  トス) = 等シキトキ

$$\text{(第一)} \quad \sin\theta_1 \sin\theta_2 \sin\theta_3 \dots \sin\theta_n$$

$$\text{(第二)} \quad \sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \dots + \sin\theta_n$$

ノ最大値ハ  $\theta_1 = \theta_2 = \theta_3 = \dots = \theta_n = \frac{\pi}{n}$  ナルトキナルコトヲ證明セヨ。

證明 (第一) 先ヅ角ガ二ツノ場合ヲ證明セン。

$$\begin{aligned} \text{サテ } \sin\theta_1 \sin\theta_2 &= \frac{1}{2} \cos(\theta_1 - \theta_2) - \frac{1}{2} \cos(\theta_1 + \theta_2) \\ &= \frac{1}{2} \cos(\theta_1 - \theta_2) - \frac{1}{2} \cos\alpha \end{aligned}$$

是ハ  $\cos(\theta_1 - \theta_2) = 1$  ナルトキ最大ナルコト明カナリ。

$$\text{サテ } \cos(\theta_1 - \theta_2) = 1 \quad \text{ヨリ}$$

$$\theta_1 - \theta_2 = 2n\pi$$

$$\text{即チ } \theta_1 = \theta_2 + 2n\pi$$

ヲ得、然ルニ  $\theta_1 + \theta_2 < \alpha < 2\pi$  ナルヲ以テ

$$n = 0 \quad \therefore \theta_1 = \theta_2$$

即チ  $\sin\theta_1 \sin\theta_2$  ハ  $\theta_1 = \theta_2$  ナルトキニ最大値ヲ有ス。

次ニ角ガ三ツ以上アル場合ヲ證明セン。

$$\sin\theta_1 \sin\theta_2 \sin\theta_3 \dots \sin\theta_n \dots \dots \dots (1)$$

= 於テ  $\theta_1, \theta_2, \theta_3, \dots, \theta_n$  ノ中ノ二ツガ相等シカラザレバ上ノ證明ニヨリ其二ツノ角ヲ相等シカラシムレバ  $\theta_1 + \theta_2 + \theta_3 + \dots + \theta_n$  ハ變ゼズシテ、其積ヲ増加セシムルコトヲ得。故ニ(1)ハ

$$\theta_1 = \theta_2 = \theta_3 = \dots = \theta_n = \frac{\alpha}{n}$$

ナルトキ最大値ヲ有ス。

(第二) 矢張り、先ヅ角ノ二ツノ場合ヲ證明セン。

$$\text{サテ } \sin\theta_1 + \sin\theta_2 = 2 \sin \frac{\theta_1 + \theta_2}{2} \cos \frac{\theta_1 - \theta_2}{2}$$

$$= 2 \sin \frac{\alpha}{2} \cos \frac{\theta_1 - \theta_2}{2}$$

是ハ  $\cos \frac{\theta_1 - \theta_2}{2} = 1$  ナルトキ最大ナルコト明カナリ。

ソコデ(第一)ノ最初ノ部分ノ證明ニ倣ヒ  $\sin\theta_1 + \sin\theta_2$

ハ  $\theta_1 = \theta_2$  ナルトキニ最大ナルコトヲ知ル。

角ガ  $n$  箇ノ場合ハ(第一)ノ後ノ部分ノ證明ニ倣ヒテ證明スルコトヲ得。



例 4.  $A, B, C$  ヲ三角形ノ三ツノ角トシ

$$\sin A + \sin B + \sin C$$

ノ最大值ヲ求ムルコト.

解 前例ニ於テ  $\theta_1 = A, \theta_2 = B, \theta_3 = C, a = 180^\circ$  トオケル

$$\sin A + \sin B + \sin C \text{ ノ 最大値ハ } 3 \sin \frac{180^\circ}{3} = 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$\sin A \sin B \sin C \text{ ノ 最大値ハ } \left(\sin \frac{180^\circ}{3}\right)^3 = \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{8}$$

### 13. 附録(第一)乃至(第四)ニ關スル雜題

1. 三角形ノ三邊ヲ 3, 5 及 6 トスルトキ, 此三角形ノ内接圓及外接圓ノ半徑ヲ求メヨ (45年, 陸軍士官候補生).

$$\text{解 } 2s = 3 + 5 + 6 = 14 \quad \therefore s = 7$$

$$\therefore S = \sqrt{7(7-3)(7-5)(7-6)} = \sqrt{7 \times 4 \times 2 \times 1} = 2\sqrt{14}$$

$$\therefore \text{内接圓ノ半徑} = \frac{2\sqrt{14}}{7}, \quad \text{外接圓ノ半徑} = \frac{3 \times 5 \times 6}{4 \times 2\sqrt{14}} = \frac{45\sqrt{14}}{56}$$

2. 一邊ノ長サ  $a$  尺ナル正多角形アリ, 其邊數ガ  $n$  ナルトキハ此多角形ノ面積幾何ナルカ (45年, 海軍機關).

$$\text{答 } \frac{na^2 \cot \frac{\pi}{n}}{4} \quad (\text{第 8 節ヲ見ヨ})$$

$$3. \quad r = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2}$$

ナルコトヲ證明セヨ.

解 附録第 2 節ノ圖ニ於ケル, 三角形 AIE ニ於テ  $\angle IAE = \frac{A}{2}, IE = r, AE = s - a$  (AE + BD + CD) ヲ三角形ノ周ノ半ニ等シキ

$$\text{ニヨル, 而シテ } IE = AE \tan \frac{A}{2} \quad \therefore r = (s-a) \tan \frac{A}{2}$$

同様ニ  $r = (s-b) \tan \frac{B}{2}, r = (s-c) \tan \frac{C}{2}$  ヲ證明スルコトヲ得.

$$\therefore r = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2}$$

$$4. \quad r = a \sec \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = \frac{2a \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}{\sin A}$$

ナルコトヲ證セヨ.

解 附録第 2 節ノ圖ニ於テ  $\angle IBD = \frac{B}{2}, \angle ICD = \frac{C}{2}$

$$\therefore r \cot \frac{B}{2} + r \cot \frac{C}{2} = BD + CD = a \quad \therefore r = \frac{a}{\cot \frac{B}{2} + \cot \frac{C}{2}}$$

$$\therefore r = \frac{a}{\frac{\cos \frac{B}{2}}{\sin \frac{B}{2}} + \frac{\cos \frac{C}{2}}{\sin \frac{C}{2}}} = \frac{a}{\frac{\sin \frac{C}{2} \cos \frac{B}{2} + \cos \frac{C}{2} \sin \frac{B}{2}}{\sin \frac{B}{2} \sin \frac{C}{2}}} = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\sin \frac{B+C}{2}}$$

$$= \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}} = a \sec \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\text{次ニ } r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}} = \frac{2a \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}} = \frac{2a \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}{\sin A}$$

$$5. \quad r_1 = a \sec \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = \frac{2a \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}{\sin A}$$

ナルコトヲ證明セヨ.

解 附録第 3 節ノ圖ニ於テ  $\angle I_1 E D_1 = 90^\circ - \frac{B}{2}, \angle I_1 C D_1 = 90^\circ - \frac{C}{2}$

$$\therefore r_1 \cot \left(90^\circ - \frac{B}{2}\right) + r_1 \cot \left(90^\circ - \frac{C}{2}\right) = BD_1 + CD_1 = a \quad \therefore r_1 \tan \frac{B}{2} + r_1 \tan \frac{C}{2} = a$$

$$\therefore r_1 = \frac{a}{\tan \frac{B}{2} + \tan \frac{C}{2}} = \frac{a}{\frac{\sin \frac{B}{2}}{\cos \frac{B}{2}} + \frac{\sin \frac{C}{2}}{\cos \frac{C}{2}}} = \frac{a \cos \frac{B}{2} \cos \frac{C}{2}}{\sin \frac{B+C}{2}}$$



$$= \frac{a \cos \frac{B}{2} \cos \frac{C}{2}}{\cos \frac{A}{2}} = a \sec \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$\text{次} = r_1 = \frac{a \cos \frac{B}{2} \cos \frac{C}{2}}{\cos \frac{A}{2}} = \frac{2a \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}} = \frac{a \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}{\sin A}$$

6.  $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$  ナルコトヲ證明セヨ.

解  $r_1 = \frac{S}{s-a}, r_2 = \frac{S}{s-b}, r_3 = \frac{S}{s-c}$

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{s-a}{S} + \frac{s-b}{S} + \frac{s-c}{S} = \frac{3s-(a+b+c)}{S} = \frac{3s-2s}{S} = \frac{s}{S}$$

然ルニ  $r = \frac{S}{s}$  従テ  $\frac{1}{r} = \frac{s}{S} \therefore \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

7.  $r_1 + r_2 + r_3 - r = 4R$  ナルコトヲ證明セヨ.

解  $r_1 + r_2 = \frac{2a \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}{\sin A} + \frac{2b \sin \frac{B}{2} \cos \frac{C}{2} \cos \frac{A}{2}}{\sin B}$  (5ヲ見ヨ)

然ルニ  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$$\begin{aligned} \therefore r_1 + r_2 &= 4R \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} + 4R \sin \frac{B}{2} \cos \frac{C}{2} \cos \frac{A}{2} \\ &= 4R \cos \frac{C}{2} \left( \sin \frac{A}{2} \cos \frac{B}{2} + \cos \frac{A}{2} \sin \frac{B}{2} \right) = 4R \cos \frac{C}{2} \sin \frac{A+B}{2} \\ &= 4R \cos \frac{C}{2} \cos \frac{C}{2} = 4R \cos^2 \frac{C}{2} \end{aligned}$$

同様ニ  $r_3 - r = 4R \sin \frac{C}{2} \cos \frac{A}{2} \cos \frac{B}{2} - 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

$$\begin{aligned} &= 4R \sin \frac{C}{2} \left( \cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2} \right) \\ &= 4R \sin \frac{C}{2} \cos \frac{A+B}{2} = 4R \sin^2 \frac{C}{2} \end{aligned}$$

$$\therefore r_1 + r_2 + r_3 - r = 4R \cos^2 \frac{C}{2} + 4R \sin^2 \frac{C}{2} = 4R (\cos^2 \frac{C}{2} + \sin^2 \frac{C}{2})$$

$$\therefore r_1 + r_2 + r_3 - r = 4R$$

8. 三角形 ABC = 於テ A ヲ通ル中線ノ長サヲ m トスレバ

$$m = \frac{1}{2} \sqrt{b^2 + c^2 + 2bc \cos A}$$

ナルコトヲ證明セヨ.

解  $b^2 + c^2 = 2m^2 + 2\left(\frac{a}{2}\right)^2 \therefore 2(b^2 + c^2) = 4m^2 + a^2 \dots \dots (1)$

然ルニ  $a^2 = b^2 + c^2 - 2bc \cos A$  之ヲ(1)ニ代入スレバ

$$2(b^2 + c^2) = 4m^2 + b^2 + c^2 - 2bc \cos A \therefore b^2 + c^2 = 4m^2 - 2bc \cos A$$

$$\therefore 4m^2 = b^2 + c^2 + 2bc \cos A \therefore m = \frac{1}{2} \sqrt{b^2 + c^2 + 2bc \cos A}$$

9. 三角形 ABC ノ角 A 及其外角ノ二等分線ガ BC 及其延長ニ出會フ點ヲ夫々 D, D' トシ, AD=f, AD'=f' トスレバ

$$f = \frac{2bc}{b+c} \cos \frac{A}{2}, \quad f' = \frac{2bc}{b-c} \sin \frac{A}{2}$$

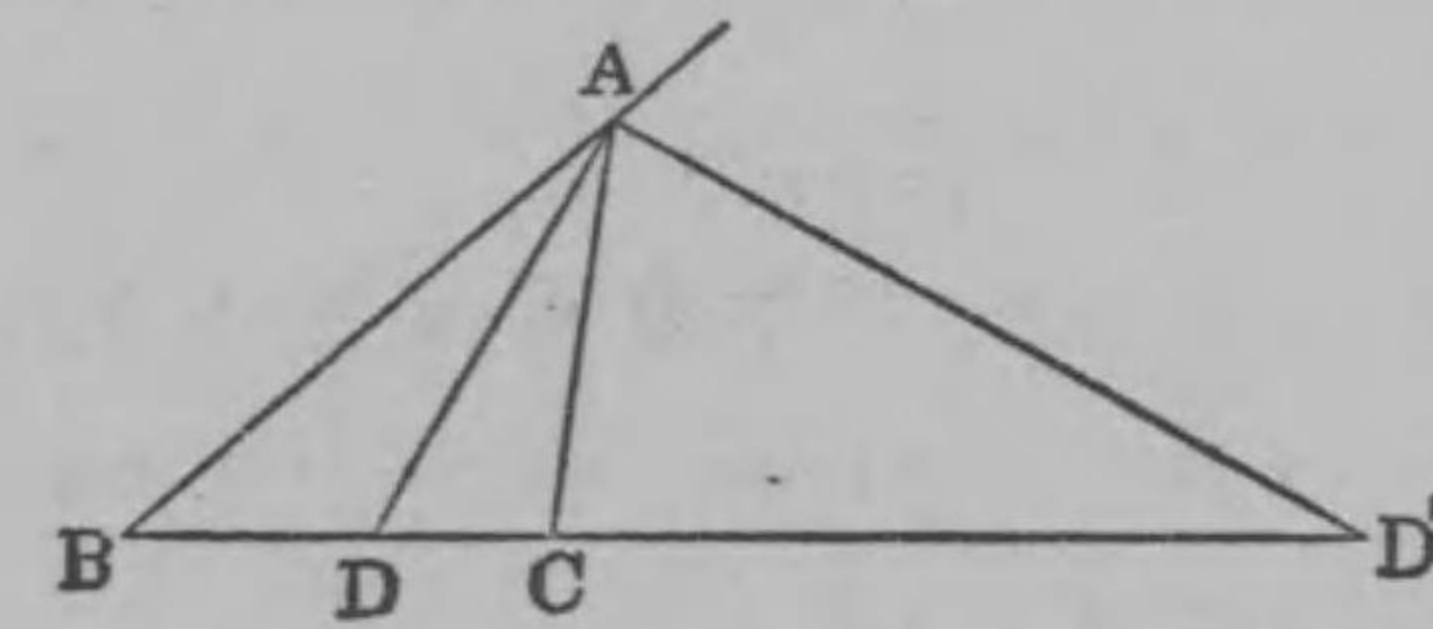
ナルコトヲ證明セヨ.

解 三角形 ABD = 於テ  $\triangle ABD = \frac{1}{2} AB \cdot AD \sin \frac{A}{2} = \frac{cf}{2} \sin \frac{A}{2}$

同様ニ  $\triangle ACD = \frac{bf}{2} \sin \frac{A}{2}$

$$\therefore \triangle ABD + \triangle ACD = \frac{cf}{2} \sin \frac{A}{2} + \frac{bf}{2} \sin \frac{A}{2}$$

$$\therefore S = \frac{f}{2} (b+c) \sin \frac{A}{2}$$



$$\therefore \frac{1}{2} bc \sin A = \frac{f}{2} (b+c) \sin \frac{A}{2} \therefore f = \frac{bc \sin A}{(b+c) \sin \frac{A}{2}} = \frac{2bc \cos \frac{A}{2}}{b+c}$$



又假  $\psi = AB > AC$  即チ  $c > b$  トスレバ  $D'$  ハ  $EC$  ノ延長上ニ在リ、而シテ

$$\Delta ABD' - \Delta ACD' = \frac{1}{2}ef'\sin\left(90^\circ + \frac{A}{2}\right) - \frac{1}{2}bf'\sin\left(90^\circ - \frac{A}{2}\right) \\ = \frac{1}{2}\left(cf'\cos\frac{A}{2} - bf'\cos\frac{A}{2}\right)$$

$$\therefore S = \frac{f'}{2}(c-b)\cos\frac{A}{2} \quad \therefore \frac{1}{2}bc\sin A = \frac{f'}{2}(c-b)\cos\frac{A}{2}$$

而シテ  $b > c$  ナルトキハ明カニ  $\frac{1}{2}bc\sin A = \frac{f'}{2}(b-c)\cos\frac{A}{2}$

$$\therefore f' = \frac{bc\sin A}{(b-c)\cos\frac{A}{2}} = \frac{2bc\sin\frac{A}{2}}{b-c}$$

10.  $r_1 = r_2 + r_3 + r$  ナル如キ三角形ハ直角三角形ナルコトヲ證明セヨ。

$$\text{解 } 7 \text{ ノ解ト同様ニシテ } r_2 + r_3 = 4R\cos^2\frac{A}{2}, \quad r_1 - r = 4R\sin^2\frac{A}{2}$$

$$\therefore r_2 + r_3 - (r_1 - r) = r_2 + r_3 + r - r_1 = 4R\left(\cos^2\frac{A}{2} - \sin^2\frac{A}{2}\right) = 4R\cos A$$

然ルニ題意ニヨリテ  $r_1 = r_2 + r_3 + r$  ナルヲ以テ  $r_2 + r_3 + r - r_1 = 0$

$$\therefore 4R\cos A = 0 \quad \text{然ルニ } R \neq 0 \quad \therefore \cos A = 0$$

然ルニ  $A$  ハ三角形ノ一ツノ角ナルヲ以テ  $A = 90^\circ$  ナリ。

11. 三角形ノ一ツノ邊  $a$ , 他ノ二邊ノ和  $b+c$  及内接圓ノ半徑  $r$  ガ與ヘラレタルトキ此三角形ノ二邊  $b, c$  及ビ三ツノ角  $A, B, C$  ヲ計算セヨ(45年東京高等工業)。

解  $a$  モ  $b+c$  モ與ヘラレタルヲ以テ  $a+b+c=2s$  チ知ルコトヲ得、從テ  $s$  チ知ルコトヲ得。サテ三角形  $ABC$  ニ於テ

$$r = (s-a)\tan\frac{A}{2} \quad (3 \text{ チ見ヨ}) \text{ ナルヲ以テ, } r \text{ ト } s-a \text{ チ知レバ之ニヨリテ } \frac{A}{2} \text{ チ計算スルコトヲ得。即チ } A \text{ チ求ムルコトヲ得。}$$

$$\text{次ニ正弦法則 } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \Rightarrow \frac{a}{\sin A} = \frac{b+c}{\sin B + \sin C}$$

$$\therefore \frac{b+c}{a} = \frac{\sin B + \sin C}{\sin A} = \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}}$$

$$\therefore \cos\frac{B-C}{2} = \frac{b+c}{a}\sin\frac{A}{2}, \quad \text{然ルニ } A \text{ ハ既ニ計算シ, } a, b+c \text{ ハ與ヘラ}$$

レタルモノナルヲ以テ、此式ニヨリテ  $\frac{B-C}{2}$  チ計算スルコトヲ得。然ルニ  $B+C=180^\circ-A$  ナルガユエニ  $A$  チ知レバ  $B+C$  ハ直チニ之ヲ求ムルコトヲ得。從テ  $B, C$  チ求ムルコトヲ得。  
 $a, A, B, C$  チ知レバ正弦法則ニヨリテ  $b, c$  チ計算スルコトヲ得。

12.  $B, a, b+c$  ヲ知リテ三角形ヲ解クコト。

$$\text{解 } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \therefore \frac{a}{\sin A} = \frac{b+c}{\sin B + \sin C}$$

$$\therefore \frac{a}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{b+c}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}} \quad \therefore \frac{a}{\sin\frac{A}{2}} = \frac{b+c}{\cos\frac{B-C}{2}} \dots\dots(1)$$

$$\text{然ルニ } C = 180^\circ - A - B \quad \therefore B - C = B - 180^\circ + A + B = A + 2B - 180^\circ$$

$$\therefore \frac{B-C}{2} = \frac{A}{2} + B - 90^\circ \quad \therefore \cos\frac{B-C}{2} = \cos\left(\frac{A}{2} + B - 90^\circ\right) = \sin\left(\frac{A}{2} + B\right)$$

$$\therefore (1) \Rightarrow \frac{a}{\sin\frac{A}{2}} = \frac{b+c}{\sin\left(\frac{A}{2} + B\right)} \quad \therefore \frac{b+c}{a} = \frac{\sin\left(\frac{A}{2} + B\right)}{\sin\frac{A}{2}} = \cos B + \sin B \cot\frac{A}{2}$$

$$\therefore \sin B \cot\frac{A}{2} = \frac{b+c}{a} - \cos B \quad \therefore \cot\frac{A}{2} = \frac{b+c}{a} \operatorname{cosec} B - \cot B$$

然ルニ  $a, b+c, B$  ハ與ヘラレタルモノナルヲ以テ之ニヨリテ  $\frac{A}{2}$ , 從テ  $A$  チ計算スルコトヲ得。サスレバ  $A, B, a$  チ知ルヲ以テ既ニ脱ケル仕方ニヨリテ此三角形ヲ解クコトヲ得。



13.  $\alpha, A, S$ ヲ知リテ三角形ヲ解クコト.

解  $S = \frac{1}{2}bc \sin A \therefore bc = \frac{2S}{\sin A}$  然ルニ  $S, A$ ハ與ヘラレタルモノナルヲ以テ之ニ依リテ  $bc$ ヲ計算スルコトヲ得. 然ルニ正弦法則ニヨリテ  $\frac{a^2}{\sin^2 A} = \frac{bc}{\sin B \sin C} \therefore \sin B \sin C = \frac{bc \sin^2 A}{a^2}$

$$\therefore \cos(B-C) - \cos(B+C) = \frac{2bc \sin^2 A}{a^2} \therefore \cos(B-C) = \cos(B+C) + \frac{2bc \sin^2 A}{a^2}$$

然ルニ  $a, bc, A$ ヲ知ルヲ以テ  $B+C$ モ知レ, 從テ此式ニヨリテ  $B-C$ ヲ計算スルコトヲ得, 從テ  $B, C$ ヲ求ムルコトヲ得. サスレバ  $B, C, a$ ヲ知ルヲ以テ既ニ説ケル仕方ニヨリテ此三角形ヲ解クコトヲ得.

14. 圓ニ内接スル四邊形  $ABCD$ ニ於テ  $\angle CAD = \alpha, \angle BAC = \beta, \angle ABD = \gamma$ トセバ  $CD = \frac{AB \sin \alpha}{\sin(\alpha + \beta + \gamma)}$ ナルコトヲ

證セヨ(41年, 山口高等商業).

解 三角形  $DCB$ ニ於テ

$$\frac{CD}{\sin \angle CBD} = \frac{BC}{\sin \angle BDC}$$

然ルニ  $\angle CBD = \angle CAD = \alpha$

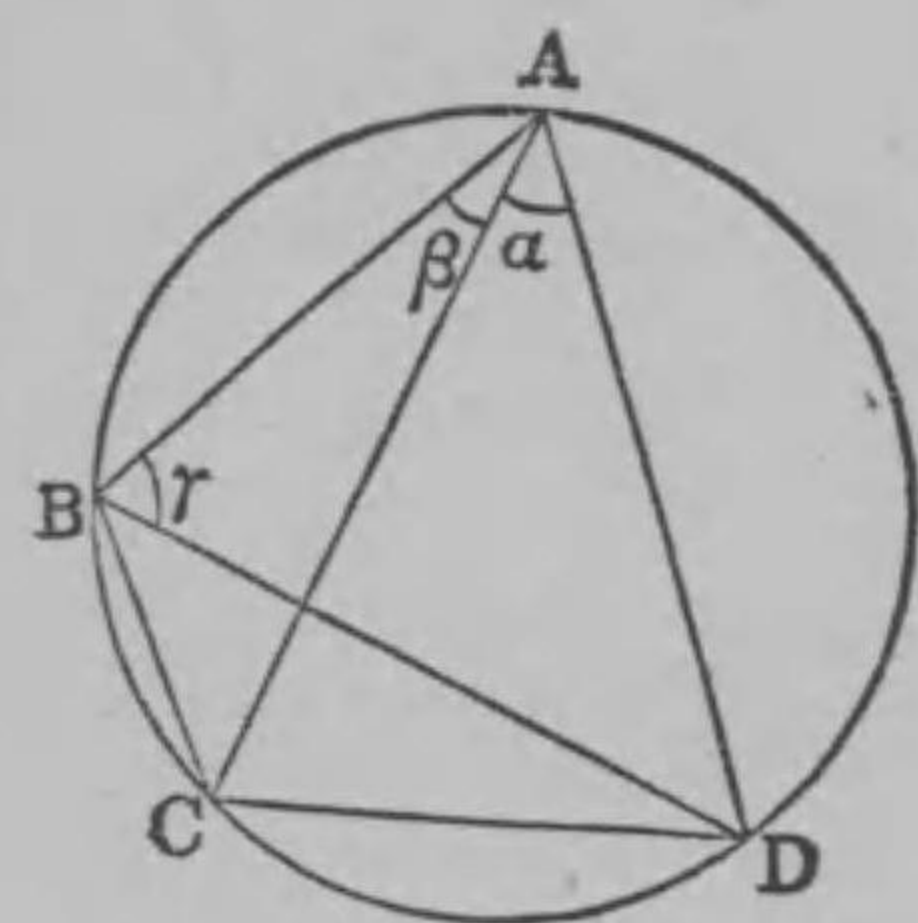
又  $\angle BDC = \angle BAC = \beta$

$$\therefore \frac{CD}{\sin \alpha} = \frac{BC}{\sin \beta} \therefore CD = \frac{BC \sin \alpha}{\sin \beta}$$

次ニ三角形  $ABD$ ニ於テ

$$\angle D = 180^\circ - (\alpha + \beta + \gamma)$$

又三角形  $ABC$ ニ於テ  $\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \beta}$



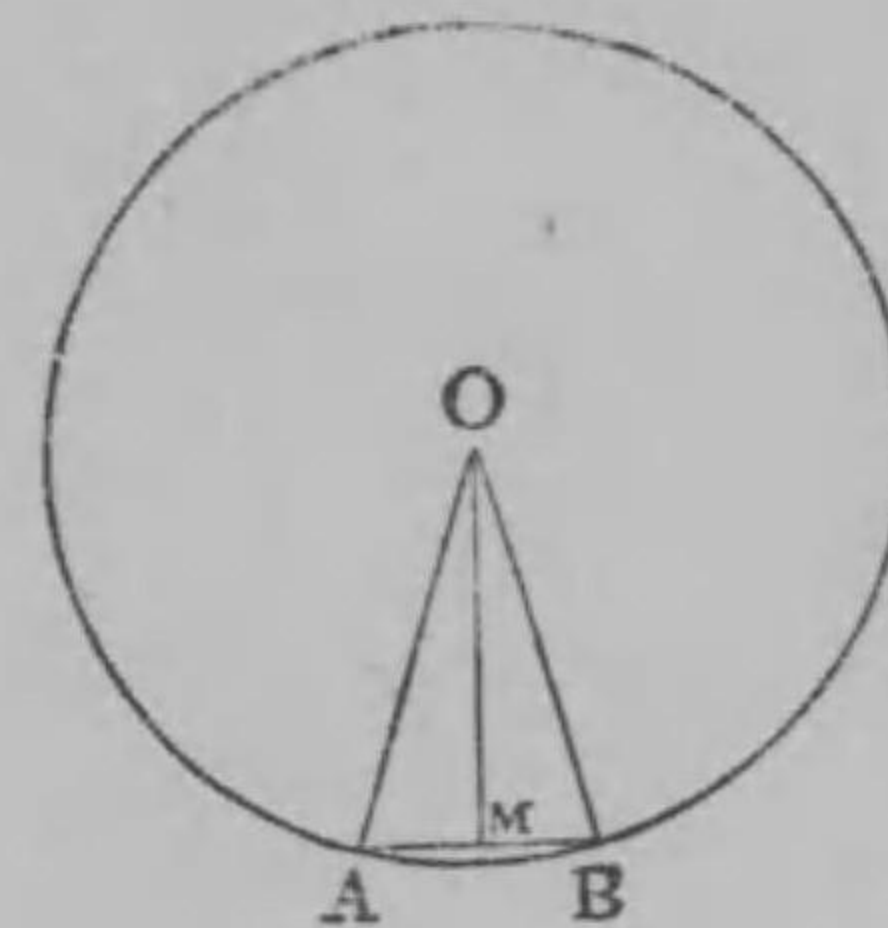
然ルニ  $\angle ACB = \angle ADB = 180^\circ - (\alpha + \beta + \gamma)$

$$\therefore \frac{AB}{\sin(\alpha + \beta + \gamma)} = \frac{BC}{\sin \beta} \therefore BC = \frac{AB \sin \beta}{\sin(\alpha + \beta + \gamma)}$$

$$CD = \frac{AB \sin \beta}{\sin(\alpha + \beta + \gamma)} \times \frac{\sin \alpha}{\sin \beta} = \frac{AB \sin \alpha}{\sin(\alpha + \beta + \gamma)}$$

15. 圓ノ面積ト之ニ内接スル正十六角形ノ面積トノ比ヲ小數點以下二位マテ算出セヨ(42年, 大阪高等工業).

解  $AB$ ヲ中心  $O$  半徑  $r$ ナル圓ニ内接スル正十六邊形ノ一邊トセヨ.



サスレバ  $\angle AOB = 360^\circ \div 16 = 22.5$

ソコテ  $O$ ヨリ  $AB$ ニ垂線ヲ引キ  $AB$ トノ交點ヲ  $M$ トスレバ

$$OM = r \cos \frac{22.5}{2}, \quad AM = r \sin \frac{22.5}{2}$$

$$\therefore \triangle OAB = \frac{1}{2} AB \times OM = AM \cdot OM$$

$$= r^2 \cos \frac{22.5}{2} \sin \frac{22.5}{2} = \frac{r^2}{2} \sin 22.5$$

故ニ圓  $O$ ノ内接正十六邊形ノ面積ハ  $8r^2 \sin 22.5$  ナリ.

故ニ圓ノ面積ト之ニ内接スル正十六邊形ノ面積トノ比ハ

$$\pi r^2 : 8r^2 \sin 22.5 = \pi : 8 \sin 22.5$$

$$\text{然ルニ } \sin 22.5 = \sqrt{\frac{1 - \cos 45^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} = \frac{\sqrt{2} - \sqrt{2}}{2}$$

$$\therefore \pi : 8 \sin 22.5 = \pi : 8 \times \frac{\sqrt{2} - \sqrt{2}}{2} = \pi : 4\sqrt{2} - \sqrt{2} = \pi : \sqrt{2} + \sqrt{2} : 4\sqrt{2} = \pi\sqrt{4+2\sqrt{2}} : 8$$

此値ヲ小數第二位マテ求ムルニハ先ヅ  $\pi\sqrt{4+2\sqrt{2}}$ ヲ同シ位マテ求メザルベカラズ, ソレニハ  $\pi$ 及  $\sqrt{4+2\sqrt{2}}$ ヲ各小數第四位マテ求ムルヲ要ス. サテ  $4+2\sqrt{2} = 4+2 \times 1.414214 \dots = 6.828428 \dots$

ソコテ  $\pi\sqrt{4+2\sqrt{2}}$ ヲ小數第二位マテ求ムル實地計算ハ次ノ如シ.



6.82	84	2	2.6131	3.1415
4			46	1316.2
282			6	628.40
276			521	188.49
684			1	3.14
521			522	94
1632				3
1566				8.20
66				
52				

故ニ求ムル比ハ  $8.20 \dots + 8 = 1.02 \dots$  答 1.02

16. 一邊ノ長サ  $a$  尺ナル  $n$  邊ノ正多角形ニ内接及ビ外接スルニツノ圓ノ周ノ間ニ夾マレタル部分ノ面積ヲ求メ且ツ之ヲ最モ簡單ナル形ニテ表セ(41年,海軍機關).

解 前問ノ解ノ圖ニ於ケル  $AB$  サ正  $n$  邊形ノ一邊トシ,其長サヲ  $a$  尺トセヨ. サスレバ  $OA$  ハ外接圓ノ半徑ニシテ,  $OM$  ハ内接圓ノ半徑ナリ. 然ルニ  $\angle AOM = \frac{2\pi}{n} \div 2 = \frac{\pi}{n}$

$\therefore OA = \frac{AB}{2} \operatorname{cosec} \frac{\pi}{n} = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$  尺, 又  $OM = \frac{AB}{2} \cot \frac{\pi}{n} = \frac{a}{2} \cot \frac{\pi}{n}$  尺

故ニ求ムル面積ノ平方尺ノ數ハ

$$\begin{aligned} \pi \cdot OA^2 - \pi \cdot OM^2 &= \left(\frac{a}{2} \operatorname{cosec} \frac{\pi}{n}\right)^2 \pi - \left(\frac{a}{2} \cot \frac{\pi}{n}\right)^2 \pi \\ &= \frac{a^2 \pi}{4} \left(\frac{1}{\sin^2 \frac{\pi}{n}} - \frac{\cos^2 \frac{\pi}{n}}{\sin^2 \frac{\pi}{n}}\right) = \frac{a^2 \pi}{4} \cdot \frac{1 - \cos^2 \frac{\pi}{n}}{\sin^2 \frac{\pi}{n}} \\ &= \frac{a^2 \pi}{4} \cdot \frac{\sin^2 \frac{\pi}{n}}{\sin^2 \frac{\pi}{n}} = \frac{a^2 \pi}{4} \end{aligned}$$

17. 次ノ無限級數ノ和ヲ最モ簡單ナル形ニテ表ハセ(43年,仙臺高等工業).

$$a \sin \theta, a \sin \theta \cos \theta, a \sin \theta \cos^2 \theta, a \sin \theta \cos^3 \theta, \dots$$

解 是ハ初項ガ  $a \sin \theta$ , 公比ガ  $\cos \theta$  ナル無限等比級數ナルヲ以テ

其和ハ  $\frac{a \sin \theta}{1 - \cos \theta} = \frac{a \times 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = a \cot \frac{\theta}{2}$  ナリ.

18. 次ノ式ヲ證明セヨ(43年,陸軍主計候補生).

$$\sin 2a + \sin 4a + \sin 6a = \frac{\cos a - \cos 7a}{2 \sin a}$$

解 此左邊ハ附錄(第四)ノ第9節ニ於ケル級數ノ  $\alpha = 2a, \beta = 2a, n = 3$  ナル場合ナルユエ,其積ハ  $\frac{\sin 3a}{\sin a} \sin 4a = \frac{\cos a - \cos 7a}{2 \sin a}$  即チ右邊ニ等シ但シ試験問題トシテ此問題ヲ解ク場合ニハ上ニイヘル第9節ノ解ニ倣ヒテ解クベキナリ.

19.  $\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7} = \frac{1}{2}$  ナルコトヲ證明セヨ.

解  $\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7}$  ハ附錄(第四)ノ第10節ニ於ケル級數ノ  $\alpha = \frac{\pi}{7}, \beta = \frac{2\pi}{7}, n = 3$  ナル場合ナルユエ,其和ハ  $\frac{\sin \frac{3\pi}{7}}{\sin \frac{\pi}{7}} \cos \frac{2\pi}{7} = \frac{\sin \frac{6\pi}{7}}{2 \sin \frac{\pi}{7}}$

然ルニ  $\sin \frac{6\pi}{7} = \sin \left(\pi - \frac{6\pi}{7}\right) = \sin \frac{\pi}{7}$

$\therefore$  上ニ得タル和ハ  $\frac{1}{2}$  ニ等シ.

20.  $\tan \theta + \cot \theta$  ガ最小ナルトキノ  $\theta$  ノ正ノ最小角ヲ求ム(45年,海軍兵).



解  $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$ , 此値ハ  $\sin 2\theta$  最大ナルトキ即チ  $\sin 2\theta = 1$  ナルトキハ最小ナリ。然ルニ  $\sin 2\theta = 1$  ニ適スル  $2\theta$  ノ正ノ最小角ハ  $\frac{\pi}{2}$  ナリ。  $\therefore \theta$  ノ正ノ最小角ハ  $\frac{\pi}{4}$  ナリ。

## 數及ビ三角函數

ノ

## 四桁ノ對數表



數	0	1	2	3	4	5	6	7	8	9	比例部分			
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374				
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	<b>43 42 41 39</b>			
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	<b>1</b> 4.3 4.2 4.1 3.9			
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	<b>2</b> 8.6 8.4 8.2 7.8			
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	<b>3</b> 12.9 12.6 12.3 11.7			
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	<b>4</b> 17.2 16.8 16.4 15.6			
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	<b>5</b> 21.5 21.0 20.5 19.5			
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	<b>6</b> 25.8 25.2 24.6 23.4			
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	<b>7</b> 30.1 29.4 28.7 27.3			
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	<b>8</b> 34.4 33.6 32.8 31.2			
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	<b>9</b> 38.7 37.8 36.9 35.1			
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	<b>38 37 36 35</b>			
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	<b>1</b> 3.8 3.7 3.6 3.5			
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	<b>2</b> 7.6 7.4 7.2 7.0			
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	<b>3</b> 11.4 11.1 10.8 10.5			
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	<b>4</b> 15.2 14.8 14.4 14.0			
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	<b>5</b> 19.0 18.5 18.0 17.5			
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	<b>6</b> 22.8 22.2 21.6 21.0			
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	<b>7</b> 26.6 25.9 25.2 24.5			
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	<b>8</b> 30.4 29.6 28.8 28.0			
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	<b>9</b> 34.2 33.3 32.4 31.5			
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	<b>34 33 32 31</b>			
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	<b>1</b> 3.4 3.3 3.2 3.1			
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	<b>2</b> 6.8 6.6 6.4 6.2			
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	<b>3</b> 10.2 9.9 9.6 9.3			
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	<b>4</b> 13.6 13.2 12.8 12.4			
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	<b>5</b> 17.0 16.5 16.0 15.5			
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	<b>6</b> 20.4 19.8 19.2 18.6			
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	<b>7</b> 23.8 23.1 22.4 21.7			
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	<b>8</b> 27.2 26.4 25.6 24.8			
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	<b>9</b> 30.6 29.7 28.8 27.9			
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	<b>29 \ 28 27</b>			
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	<b>1</b> 2.9 2.8 2.7			
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	<b>2</b> 5.8 5.6 5.4			
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	<b>3</b> 8.7 8.4 8.1			
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	<b>4</b> 11.6 11.2 10.8			
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	<b>5</b> 14.5 14.0 13.5			
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	<b>6</b> 17.4 16.8 16.2			
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	<b>7</b> 20.3 19.6 18.9			
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	<b>8</b> 23.2 22.4 21.6			
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	<b>9</b> 26.1 25.2 24.3			
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152				
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235				
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316				
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396				

數	0	1	2	3	4	5	6	7	8	9	比例部分			
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474				
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	<b>26 25 24 23</b>			
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	<b>1</b> 2.6 2.5 2.4 2.3			
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	<b>2</b> 5.2 5.0 4.8 4.6			
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	<b>3</b> 7.8 7.5 7.2 9.9			
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	<b>4</b> 10.4 10.0 9.6 9.2			
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	<b>5</b> 13.0 12.5 12.0 11.5			
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	<b>6</b> 15.6 15.0 14.4 13.8			
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	<b>7</b> 18.2 17.5 16.8 16.1			
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	<b>8</b> 20.8 20.0 19.2 18.4			
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	<b>9</b> 23.4 22.5 21.6 20.7			
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	<b>22 21 19 18</b>			
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	<b>1</b> 2.2 2.1 1.9 1.8			
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	<b>2</b> 4.4 4.2 3.8 3.6			
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	<b>3</b> 6.6 6.3 5.7 5.4			
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	<b>4</b> 8.8 8.4 7.6 7.2			
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	<b>5</b> 11.0 10.5 9.5 9.0			
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	<b>6</b> 13.2 12.6 11.4 10.8			
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	<b>7</b> 15.4 14.7 13.3 12.6			
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	<b>8</b> 17.6 16.8 15.2 14.4			
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	<b>9</b> 19.8 18.9 17.1 16.2			
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	<b>17 16 15 14</b>			
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	<b>1</b> 1.7 1.6 1.5 1.4			
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	<b>2</b> 3.4 3.2 3.0 2.8			
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	<b>3</b> 5.1 4.8 4.5 4.2			
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	<b>4</b> 6.8 6.4 6.0 5.6			
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	<b>5</b> 8.5 8.0 7.5 7.0			
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	<b>6</b> 10.2 9.6 9.0 8.4			
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	<b>7</b> 11.9 11.2 10.5 9.8			
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	<b>8</b> 13.6 12.8 12.0 11.2			
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	<b>9</b> 15.3 14.4 13.5 12.6			
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	<b>13 12 11</b>			
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	<b>1</b> 1.3 1.2 1.1			
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	<b>2</b> 2.6 2.4 2.2			
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	<b>3</b> 3.9 3.6 3.3			
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	<b>4</b> 5.2 4.8 4.4			
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	<b>5</b> 6.5 6.0 5.5			
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	<b>6</b> 7.8 7.2 6.6			
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	<b>7</b> 9.1 8.4 7.7			
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	<b>8</b> 10.4 9.6 8.8			
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	<b>9</b> 11.7 10.8 9.9			
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863				
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908				
98	9912	9917	9921	9926	9930	9933	9939	9943	9948	9952				
99	9959	9961	9965	9969	9974	9978	9983	9987	9991	9996				



表中ニナキ 6° 30' 未滿ノ角(之ヲ x' トス)ノ正弦  
及ビ正切ノ對數ト x' ノ對數間ニハ次ノ關係アリ  
 $\log \sin x' = \log x + S$   $\log x = \log \sin x' - S$   
 $\log \tan x' = \log x + T$   $\log x = \log \tan x' - T$   
 但シ S 及ビ T ハ其角ニ最モ近キ表中ニアル角ニ  
 應ズル S 及ビ T ナリ

角	正弦	S	正切	T	餘切	餘弦
0° 0'	-∞		-∞		+∞	0.0000
10'	4.4637	4.4637	3.4637	4.4637	2.5363	0.0000
20'	3.7648	4.4637	3.7648	4.4637	2.2352	0.0000
30'	3.9408	4.4637	3.9409	4.4637	2.0591	0.0000
40'	2.0658	4.4637	2.0658	4.4637	1.9342	0.0000
50'	2.1627	4.4637	2.1627	4.4638	1.8372	0.0000
1° 0'	2.2419	4.4637	2.2419	4.4638	1.7581	1.9999
10'	2.3088	4.4637	2.3089	4.4638	1.6911	1.9999
20'	2.3668	4.4637	2.3669	4.4638	1.6331	1.9999
30'	2.4179	4.4637	2.4181	4.4638	1.5819	1.9999
40'	2.4637	4.4637	2.4638	4.4638	1.5362	1.9998
50'	2.5050	4.4637	2.5053	4.4639	1.4947	1.9998
2° 0'	2.5428	4.4636	2.5431	4.4639	1.4569	1.9997
10'	2.5776	4.4636	2.5779	4.4639	1.4221	1.9997
20'	2.6097	4.4636	2.6101	4.4640	1.3899	1.9996
30'	2.6397	4.4636	2.6401	4.4640	1.3599	1.9996
40'	2.6677	4.4636	2.6682	4.4640	1.3318	1.9995
50'	2.6940	4.4635	2.6945	4.4641	1.3055	1.9995
3° 0'	2.7188	4.4635	2.7194	4.4641	1.2806	1.9994
10'	2.7423	4.4635	2.7429	4.4642	1.2571	1.9993
20'	2.7645	4.4635	2.7652	4.4642	1.2348	1.9993
30'	2.7857	4.4635	2.7865	4.4642	1.2137	1.9992
40'	2.8059	4.4634	2.8067	4.4643	1.1933	1.9991
50'	2.8251	4.4634	2.8261	4.4644	1.1739	1.9990
4° 0'	2.8436	4.4634	2.8446	4.4644	1.1554	1.9989
10'	2.8613	4.4633	2.8624	4.4645	1.1376	1.9989
20'	2.8783	4.4633	2.8795	4.4646	1.1205	1.9988
30'	2.8946	4.4633	2.8960	4.4646	1.1040	1.9987
40'	2.9104	4.4632	2.9118	4.4647	1.0882	1.9986
50'	2.9256	4.4632	2.9272	4.4647	1.0728	1.9985
5° 0'	2.9403	4.4632	2.9420	4.4648	1.0580	1.9983
10'	2.9545	4.4631	2.9563	4.4649	1.0437	1.9982
20'	2.9682	4.4631	2.9701	4.4650	1.0299	1.9981
30'	2.9816	4.4630	2.9836	4.4650	1.0164	1.9980
40'	2.9945	4.4630	2.9966	4.4651	1.0034	1.9979
50'	2.0070	4.4630	1.0093	4.4652	0.9907	1.9977
6° 0'	1.0192	4.4629	1.0216	4.4653	0.9784	1.9976
10'	1.0311	4.4629	1.0336	4.4654	0.9664	1.9975
20'	1.0426	4.4628	1.0453	4.4655	0.9547	1.9973
30'	1.0539	4.4628	1.0567	4.4656	0.9433	1.9972
餘弦	S	餘切	T	正切	正弦	角

比例部分

111 109 108 107

1	11.1	10.9	10.8	10.7
2	22.2	21.8	21.6	21.4
3	33.3	32.7	32.4	32.1
4	44.4	43.6	43.2	42.8
5	55.5	54.5	54.0	53.5
6	66.6	65.4	64.8	64.2
7	77.7	76.3	75.6	74.9
8	88.8	87.2	86.4	85.6
9	99.9	98.1	97.2	96.3

105 104 102 101

1	10.5	10.4	10.2	10.1
2	21.0	20.8	20.4	20.2
3	31.5	31.2	30.6	30.3
4	42.0	41.6	40.8	40.4
5	52.5	52.0	51.0	50.5
6	63.0	62.4	61.2	60.6
7	73.5	72.8	71.4	70.7
8	84.0	83.2	81.6	80.8
9	94.5	93.6	91.8	90.9

99 98 97 95

1	9.9	9.8	9.7	9.5
2	19.8	19.6	19.4	19.0
3	29.7	29.4	29.1	28.5
4	39.6	39.2	38.8	38.0
5	49.5	49.0	48.5	47.5
6	59.4	58.8	58.2	57.0
7	69.3	68.6	67.9	66.5
8	79.2	78.4	77.6	76.0
9	89.1	88.2	87.3	85.5

94 93 91 89

1	9.4	9.3	9.1	8.9
2	18.8	18.6	18.2	17.8
3	28.2	27.9	27.3	26.7
4	37.6	37.2	36.4	35.6
5	47.0	46.5	45.5	44.5
6	56.4	55.8	54.6	53.4
7	65.8	65.1	63.7	62.3
8	75.2	74.4	72.8	71.2
9	84.6	83.7	81.9	80.1

87	86	85	84	82	81	79	78	77	76	75	74	73	71	69	
1	8.7	8.6	8.5	8.4	8.2	8.1	7.9	7.8	7.7	7.6	7.5	7.4	7.3	7.1	6.9
2	17.4	17.2	17.0	16.8	16.4	16.2	15.8	15.6	15.4	15.2	15.0	14.8	14.6	14.2	13.8
3	26.1	25.8	25.5	25.2	24.6	24.3	23.7	23.4	23.1	22.8	22.5	22.2	21.9	21.3	20.7
4	34.8	34.4	34.0	33.6	32.8	32.4	31.6	31.2	30.8	30.4	30.0	29.6	29.2	28.4	27.6
5	43.5	43.0	42.5	42.0	41.0	40.5	39.5	39.0	38.5	38.0	37.5	37.0	36.5	35.5	34.5
6	52.2	51.6	51.0	50.4	49.2	48.6	47.4	46.8	46.2	45.6	45.0	44.4	43.8	42.6	41.4
7	60.9	60.2	59.5	58.8	57.4	56.7	55.3	54.6	53.9	53.2	52.5	51.8	51.1	49.7	48.3
8	69.6	68.8	68.0	67.2	65.6	64.8	63.2	62.4	61.6	60.8	60.0	59.2	58.4	56.8	55.2
9	78.3	77.4	76.5	75.6	73.8	72.9	71.1	70.2	69.3	68.4	67.5	66.6	65.7	63.9	62.1

比例部分

68 67 66 65

1	6.8	6.7	6.6	6.5
2	13.6	13.4	13.2	13.0
3	20.4	20.1	19.8	19.5
4	27.2	26.8	26.4	26.0
5	34.0	33.5	33.0	32.5
6	40.8	40.2	39.6	39.0
7	47.6	46.9	46.2	45.5
8	54.4	53.6	52.8	52.0
9	61.2	60.3	59.4	58.5

64 63 61 59

1	6.4	6.3	6.1	5.9
2	12.8	12.6	12.2	11.8
3	19.2	18.9	18.3	17.7
4	25.6	25.2	24.4	23.6
5	32.0	31.5	30.5	29.5
6	38.4	37.8	36.6	35.4
7	44.8	44.1	42.7	41.3
8	51.2	50.4	48.8	47.2
9	57.6	56.7	54.9	53.1

58 57 56 55

1	5.8	5.7	5.6	5.5
2	11.6	11.4	11.2	11.0
3	17.4	17.1	16.8	16.5
4	23.2	22.8	22.4	22.0
5	29.0	28.5	28.0	27.5
6	34.8	34.2	33.6	33.0
7	40.6	39.9	39.2	38.5
8	46.4	45.6	44.8	44.0
9	52.2	51.3	50.4	49.5

角	正弦	差	正切	通變	餘切	差	餘弦
6° 30'	1.0539	109	1.0567	111	0.9433	1	1.9972
40'	1.0548	107	1.0678	108	0.9322	2	1.9971
50'	1.0755	104	1.0786	105	0.9214	1	1.9969
7° 0'	1.0859	104	1.0891	105	0.9109	1	1.9968
10'	1.0961	102	1.0995	104	0.9005	2	1.9966
20'	1.1060	99	1.1096	101	0.8904	2	1.9964
30'	1.1157	97	1.1194	98	0.8806	1	1.9963
40'	1.1252	95	1.1291	97	0.8709	2	1.9961
50'	1.1345	93	1.1385	94	0.8615	2	1.9959
8° 0'	1.1436	91	1.1478	93	0.8522	1	1.9958
10'	1.1525	89	1.1569	91	0.8431	2	1.9956
20'	1.1612	87	1.1658	89	0.8342	2	1.9954
30'	1.1697	85	1.1745	87	0.8255	2	1.9952
40'	1.1781	84	1.1831	86	0.8169	2	1.9950
50'	1.1863	82	1.1915	84	0.8085	2	1.9948
9° 0'	1.1943	80	1.1997	82	0.8003	2	1.9946
10'	1.2022	79	1.2078	81	0.7922	2	1.9944
20'	1.2100	78	1.2158	80	0.7842	2	1.9942
30'	1.2176	76	1.2236	78	0.7764	2	1.9940
40'	1.2251	75	1.2313	77	0.7687	2	1.9938
50'	1.2324	73	1.2389	76	0.7611	2	1.9936
10° 0'	1.2397	73	1.2463	74	0.7537	2	1.9934
10'	1.2468	71	1.2536	73	0.7464	3	1.9931
20'	1.2538	70	1.2609	73	0.7391	2	1.9929
30'	1.2606	68	1.2680	71	0.7320	2	1.9927
40'	1.2674	68	1.2750	70	0.7250	3	1.9924
50'	1.2740	66	1.2819	69	0.7181	2	1.9922
11° 0'	1.2806	66	1.2887	68	0.7113	3	1.9919
10'	1.2870	64	1.2953	66	0.7047	2	1.9917
20'	1.2934	64	1.3020	67	0.6980	3	1.9914
30'	1.2997	63	1.3085	65	0.6915	2	1.9912
40'	1.3058	61	1.3149	64	0.6851	3	1.9909
50'	1.3119	61	1.3212	63	0.6788	2	1.9907
12° 0'	1.3179	60	1.3275	63	0.6725	3	1.9904
10'	1.3238	59	1.3336	61	0.6664	3	1.9901
20'	1.3296	58	1.3397	61	0.6603	2	1.9899
30'	1.3353	57	1.3458	59	0.6542	3	1.9896
40'	1.3410	56	1.3517	59	0.6483	3	1.9893
50'	1.3466	55	1.3576	58	0.6424	3	1.9890
13° 0'	1.3521		1.3634		0.6366		1.9887
餘弦	差	餘切	通變	正切	差	正弦	角



角	正弦	差	正切	通差	餘切	差	餘弦	比例部分
13° 0'	1.3521	54	1.3634	57	0.6366	3	1.9887	0' 77°
10'	1.3575	54	1.3691	57	0.9309	3	1.9884	50'
20'	1.3629	53	1.3748	56	0.6252	3	1.9881	40'
30'	1.3682	52	1.3804	55	0.6196	3	1.9878	30'
40'	1.3734	52	1.3859	55	0.6141	3	1.9875	20'
50'	1.3786	51	1.3914	54	0.6086	3	1.9872	10'
14° 0'	1.3837	50	1.3968	53	0.6032	3	1.9869	0' 76°
10'	1.3887	50	1.4021	53	0.5979	3	1.9866	50'
20'	1.3937	49	1.4074	53	0.5926	3	1.9863	40'
30'	1.3986	49	1.4127	53	0.5873	4	1.9859	30'
40'	1.4035	49	1.4178	51	0.5822	3	1.9856	20'
50'	1.4083	48	1.4230	52	0.5770	3	1.9853	10'
15° 0'	1.4130	47	1.4281	51	0.5719	4	1.9849	0' 75°
10'	1.4177	47	1.4331	50	0.5669	3	1.9846	50'
20'	1.4223	46	1.4381	50	0.5619	3	1.9843	40'
30'	1.4269	46	1.4430	49	0.5570	4	1.9839	30'
40'	1.4314	45	1.4479	49	0.5521	3	1.9836	20'
50'	1.4359	45	1.4527	48	0.5473	4	1.9832	10'
16° 0'	1.4403	44	1.4575	48	0.5425	4	1.9828	0' 74°
10'	1.4447	44	1.4622	47	0.5378	4	1.9825	50'
20'	1.4491	42	1.4669	47	0.5331	4	1.9821	40'
30'	1.4533	43	1.4716	46	0.5284	3	1.9817	30'
40'	1.4576	42	1.4762	46	0.5238	3	1.9814	20'
50'	1.4618	41	1.4808	45	0.5192	4	1.9810	10'
17° 0'	1.4659	41	1.4853	45	0.5147	4	1.9806	0' 73°
10'	1.4700	41	1.4898	45	0.5102	4	1.9802	50'
20'	1.4741	40	1.4943	44	0.5057	4	1.9798	40'
30'	1.4781	40	1.4987	44	0.5013	4	1.9794	30'
40'	1.4821	40	1.5031	44	0.4969	4	1.9790	20'
50'	1.4861	39	1.5075	43	0.4925	4	1.9786	10'
18° 0'	1.4900	39	1.5118	43	0.4882	4	1.9782	0' 72°
10'	1.4939	38	1.5161	42	0.4839	4	1.9778	50'
20'	1.4977	38	1.5203	42	0.4797	4	1.9774	40'
30'	1.5015	37	1.5245	42	0.4755	5	1.9770	30'
40'	1.5052	38	1.5287	42	0.4713	4	1.9765	20'
50'	1.5090	36	1.5329	41	0.4671	4	1.9761	10'
19° 0'	1.5126	37	1.5370	41	0.4630	5	1.9757	0' 71°
10'	1.5163	36	1.5411	40	0.4589	4	1.9752	50'
20'	1.5199	36	1.5451	40	0.4549	5	1.9748	40'
30'	1.5235	35	1.5491	40	0.4509	4	1.9743	30'
40'	1.5270	36	1.5531	40	0.4469	5	1.9739	20'
50'	1.5306	35	1.5571	40	0.4429	4	1.9734	10'
20° 0'	1.5341	34	1.5611	39	0.4389	5	1.9730	0' 70°
10'	1.5375	34	1.5650	39	0.4350	4	1.9725	50'
20'	1.5409	34	1.5689	38	0.4311	5	1.9721	40'
30'	1.5443	34	1.5727	39	0.4273	5	1.9716	30'
40'	1.5477	33	1.5766	38	0.4234	5	1.9711	20'
50'	1.5510	33	1.5804	38	0.4196	4	1.9706	10'
21° 0'	1.5543	33	1.5842	38	0.4158	4	1.9702	0' 69°
餘弦	差	餘切	通差	正切	差	正弦	角	比例部分

比例部分	角	正弦	差	正切	通差	餘切	差	餘弦	角
39 38 37 36	21° 0'	1.5543	33	1.5842	37	0.4158	5	1.9702	0' 69°
1 3.9 3.8 3.7 3.6	10'	1.5576	33	1.5879	38	0.4121	5	1.9697	50'
2 7.8 7.6 7.4 7.2	20'	1.5609	32	1.5917	37	0.4083	5	1.9692	40'
3 11.7 11.4 11.1 10.8	30'	1.5641	32	1.5954	37	0.4046	5	1.9687	30'
4 15.6 15.2 14.8 14.4	40'	1.5673	31	1.5991	37	0.4009	5	1.9682	20'
5 19.5 19.0 18.5 18.0	50'	1.5704	32	1.6028	36	0.3972	5	1.9677	10'
6 23.4 22.8 22.2 21.6	22° 0'	1.5736	31	1.6064	36	0.3936	5	1.9672	0' 68°
7 27.3 26.6 25.9 25.2	10'	1.5767	31	1.6100	36	0.3900	6	1.9667	50'
8 31.2 30.4 29.6 28.8	20'	1.5796	30	1.6136	36	0.3864	5	1.9661	40'
9 35.1 34.2 33.3 22.4	30'	1.5828	31	1.6172	36	0.3828	5	1.9656	30'
1 3.5 3.4 3.3 3.2	40'	1.5859	30	1.6208	35	0.3792	5	1.9651	20'
2 7.0 6.8 6.6 6.4	50'	1.5889	30	1.6243	36	0.3757	6	1.9646	10'
3 10.5 10.2 9.9 9.6	23° 0'	1.5919	29	1.6279	35	0.3721	5	1.9640	0' 67°
4 14.0 13.6 13.2 12.8	10'	1.5948	29	1.6314	34	0.3686	6	1.9633	50'
5 17.5 17.9 16.5 16.0	20'	1.5978	29	1.6348	35	0.3652	5	1.9629	40'
6 21.0 20.4 19.8 19.2	30'	1.6007	29	1.6383	34	0.3617	6	1.9624	30'
7 24.5 23.8 23.1 22.4	40'	1.6036	29	1.6417	35	0.3583	5	1.9618	20'
8 28.0 27.2 26.4 25.6	50'	1.6065	28	1.6452	34	0.3548	6	1.9613	10'
9 31.5 30.6 29.7 28.8	24° 0'	1.6093	28	1.6486	34	0.3514	5	1.9607	0' 66°
1 3.1 2.9 2.8 2.7	10'	1.6121	28	1.6520	33	0.3480	6	1.9602	50'
2 6.2 5.8 5.0 5.4	20'	1.6149	28	1.6553	34	0.3447	6	1.9596	40'
3 9.3 8.7 8.4 8.1	30'	1.6177	28	1.6587	33	0.3413	6	1.9590	30'
4 12.4 11.6 11.2 10.8	40'	1.6205	27	1.6620	34	0.3380	5	1.9584	20'
5 15.5 14.5 14.0 13.5	50'	1.6232	27	1.6654	33	0.3346	6	1.9579	10'
6 18.6 17.4 16.8 16.2	25° 0'	1.6259	27	1.6687	33	0.3313	6	1.9573	0' 65°
7 21.7 20.3 19.6 18.9	10'	1.6286	27	1.6720	32	0.3280	6	1.9567	50'
8 24.8 23.2 22.4 21.6	20'	1.6313	27	1.6752	33	0.3248	6	1.9561	40'
9 27.9 26.1 25.2 24.3	30'	1.6340	26	1.6785	32	0.3215	6	1.9555	30'
1 2.6 2.5 2.4 2.3	40'	1.6366	26	1.6817	33	0.3183	6	1.9549	20'
2 5.2 5.0 4.8 4.6	50'	1.6392	26	1.6850	32	0.3150	6	1.9543	10'
3 7.8 7.5 7.2 6.9	26° 0'	1.6418	26	1.6882	32	0.3118	7	1.9537	0' 64°
4 10.4 10.0 9.6 9.2	10'	1.6444	26	1.6914	32	0.3086	6	1.9530	50'
5 13.0 12.5 12.0 11.5	20'	1.6470	25	1.6946	31	0.3054	6	1.9524	40'
6 15.6 15.0 14.4 13.8	30'	1.6495	26	1.6977	32	0.3023	6	1.9518	30'
7 18.2 17.5 16.8 16.1	40'	1.6521	25	1.7009	31	0.2991	7	1.9512	20'
8 20.8 20.0 19.2 18.4	50'	1.6546	24	1.7040	32	0.2960	6	1.9505	10'
9 23.4 22.5 21.6 20.7	27° 0'	1.6570	25	1.7072	31	0.2928	7	1.9499	0' 63°
1 4.4 4.3 4.2 4.1	10'	1.6595	25	1.7103	31	0.2897	6	1.9492	50'
2 8.8 8.6 8.4 8.2	20'	1.6620	24	1.7134	31	0.2866	7	1.9486	40'
3 13.2 12.9 12.6 12.3	30'	1.6644	24	1.7165	31	0.2835	6	1.9479	30'
4 17.6 17.2 16.8 16.4	40'	1.6668	24	1.7196	30	0.2804	7	1.9473	20'
5 22.0 21.5 21.0 20.5	50'	1.6692	24	1.7226	31	0.2774	7	1.9466	10'
6 26.4 25.8 25.2 24.6	28° 0'	1.6716	24	1.7257	30	0.2743	6	1.9459	0' 62°
7 30.8 30.1 29.4 28.7	10'	1.6740	23	1.7287	30	0.2713	7	1.9453	50'
8 35.2 34.4 33.6 32.8	20'	1.6763	24	1.7317	31	0.2683	7	1.9446	40'
9 39.6 38.7 37.8 36.9	30'	1.6787	23	1.7348	30	0.2652	7	1.9439	30'
1 4.4 4.3 4.2 4.1	40'	1.6810	23	1.7378	30	0.2622	7	1.9432	20'
2 8.8 8.6 8.4 8.2	50'	1.6833	23	1.7408	30	0.2592	7	1.9425	10'
3 13.2 12.9 12.6 12.3	29° 0'	1.6856	23	1.7438	29	0.2562	7	1.9418	0' 61°
4 17.6 17.2 16.8 16.4	餘弦	差	餘切	通差	正切	差	正弦	角	比例部分



角	正弦	差	正切	通差	餘切	差	餘弦	比例部分
29° 0'	1.6856	22	1.7438	29	0.2562	7	1.9418	0' 61'
10'	1.6878	23	1.7467	30	0.2533	7	1.9411	50'
20'	1.6901	23	1.7497	29	0.2503	7	1.9404	40'
30'	1.6923	23	1.7526	30	0.2474	7	1.9397	30'
40'	1.6946	22	1.7556	29	0.2444	7	1.9390	20'
50'	1.6968	22	1.7585	29	0.2415	7	1.9383	10'
30° 0'	1.6990	22	1.7614	30	0.2386	8	1.9376	0' 60'
10'	1.7012	21	1.7644	29	0.2356	7	1.9368	50'
20'	1.7033	22	1.7673	28	0.2327	7	1.9361	40'
30'	1.7055	21	1.7701	29	0.2299	8	1.9353	30'
40'	1.7076	21	1.7730	29	0.2270	7	1.9346	20'
50'	1.7097	21	1.7759	29	0.2241	8	1.9338	10'
31° 0'	1.7118	21	1.7788	28	0.2212	7	1.9331	0' 59'
10'	1.7139	21	1.7816	29	0.2184	8	1.9323	50'
20'	1.7160	21	1.7845	28	0.2155	8	1.9315	40'
30'	1.7181	20	1.7873	29	0.2127	7	1.9308	30'
40'	1.7201	21	1.7902	28	0.2098	8	1.9300	20'
50'	1.7222	20	1.7930	28	0.2070	8	1.9292	10'
32° 0'	1.7242	20	1.7958	28	0.2042	8	1.9284	0' 58'
10'	1.7262	20	1.7986	28	0.2014	8	1.9276	50'
20'	1.7282	20	1.8014	28	0.1986	8	1.9268	40'
30'	1.7302	20	1.8042	28	0.1958	8	1.9260	30'
40'	1.7322	20	1.8070	27	0.1930	8	1.9252	20'
50'	1.7342	19	1.8097	28	0.1903	8	1.9244	10'
33° 0'	1.7361	19	1.8125	28	0.1875	8	1.9236	0' 57'
10'	1.7380	20	1.8153	27	0.1847	9	1.9228	50'
20'	1.7400	19	1.8180	28	0.1820	8	1.9219	40'
30'	1.7419	19	1.8208	27	0.1792	8	1.9211	30'
40'	1.7438	19	1.8235	28	0.1765	9	1.9203	20'
50'	1.7457	19	1.8263	27	0.1737	8	1.9194	10'
34° 0'	1.7476	18	1.8290	27	0.1710	9	1.9186	0' 56'
10'	1.7494	19	1.8317	27	0.1683	8	1.9177	50'
20'	1.7513	18	1.8344	27	0.1656	9	1.9169	40'
30'	1.7531	19	1.8371	27	0.1629	9	1.9160	30'
40'	1.7550	18	1.8398	27	0.1602	9	1.9151	20'
50'	1.7568	18	1.8425	27	0.1575	8	1.9142	10'
35° 0'	1.7586	18	1.8452	27	0.1548	9	1.9134	0' 55'
10'	1.7604	18	1.8479	27	0.1521	9	1.9125	50'
20'	1.7622	18	1.8506	27	0.1494	9	1.9116	40'
30'	1.7640	17	1.8533	26	0.1467	9	1.9107	30'
40'	1.7657	18	1.8559	27	0.1441	9	1.9098	20'
50'	1.7675	17	1.8586	27	0.1414	9	1.9089	10'
36° 0'	1.7692	18	1.8613	26	0.1387	01	1.9080	0' 54'
10'	1.7710	17	1.8639	27	0.1361	9	1.9070	50'
20'	1.7727	17	1.8666	26	0.1334	9	1.9061	40'
30'	1.7744	17	1.8692	26	0.1308	01	1.9052	30'
40'	1.7761	17	1.8718	27	0.1282	9	1.9042	20'
50'	1.7778	17	1.8745	26	0.1255	01	1.9033	10'
37° 0'	1.7795	17	1.8771	26	0.1229	01	1.9023	0' 53'
	餘弦		餘切	通差	正切	差	正弦	角

比例部分	角	正弦	差	正切	通差	餘切	差	餘弦	比例部分
	37° 0'	1.7795	16	1.8771	26	0.1229	9	1.9023	0' 53'
	10'	1.7811	17	1.8797	27	0.1203	10	1.9014	50'
	20'	1.7828	16	1.8824	26	0.1176	9	1.9004	40'
	30'	1.7844	17	1.8850	26	0.1150	10	1.8995	30'
	40'	1.7861	16	1.8876	26	0.1124	10	1.8985	20'
	50'	1.7877	16	1.8902	26	0.1098	10	1.8975	10'
	38° 0'	1.7893	17	1.8928	26	0.1072	10	1.8965	0' 52'
	10'	1.7910	16	1.8954	26	0.1046	10	1.8955	50'
	20'	1.7926	15	1.8980	26	0.1020	10	1.8945	40'
	30'	1.7941	16	1.9006	26	0.0994	10	1.8935	30'
	40'	1.7957	16	1.9032	26	0.0968	10	1.8925	20'
	50'	1.7973	16	1.9058	26	0.0942	10	1.8915	10'
	39° 0'	1.7989	15	1.9084	26	0.0916	10	1.8905	0' 51'
	10'	1.8004	16	1.9110	25	0.0890	11	1.8895	50'
	20'	1.8020	15	1.9135	26	0.0865	10	1.8884	40'
	30'	1.8035	15	1.9161	26	0.0839	10	1.8874	30'
	40'	1.8050	16	1.9187	25	0.0813	11	1.8864	20'
	50'	1.8066	15	1.9212	26	0.0788	10	1.8853	10'
	40° 0'	1.8081	15	1.9238	26	0.0762	11	1.8843	0' 50'
	10'	1.8096	15	1.9264	25	0.0736	11	1.8832	50'
	20'	1.8111	14	1.9289	26	0.0711	11	1.8821	40'
	30'	1.8125	15	1.9315	26	0.0685	10	1.8810	30'
	40'	1.8140	15	1.9341	25	0.0659	11	1.8800	20'
	50'	1.8155	14	1.9366	26	0.0634	11	1.8789	10'
	41° 0'	1.8169	15	1.9392	25	0.0608	11	1.8778	0' 49'
	10'	1.8184	14	1.9417	26	0.0583	11	1.8764	50'
	20'	1.8198	15	1.9443	25	0.0557	11	1.8756	40'
	30'	1.8213	14	1.9468	26	0.0532	12	1.8745	30'
	40'	1.8227	14	1.9494	25	0.0506	11	1.8733	20'
	50'	1.8241	14	1.9419	25	0.0481	11	1.8722	10'
	42° 0'	1.8255	14	1.9544	26	0.0456	12	1.8711	0' 48'
	10'	1.8269	14	1.9570	25	0.0430	11	1.8699	50'
	20'	1.8283	14	1.9595	26	0.0405	12	1.8688	40'
	30'	1.8297	14	1.9621	25	0.0379	11	1.8676	30'
	40'	1.8311	13	1.9646	25	0.0354	12	1.8665	20'
	50'	1.8324	14	1.9671	26	0.0321	12	1.8653	10'
	43° 0'	1.8338	13	1.9697	25	0.0303	12	1.8641	0' 47'
	10'	1.8351	14	1.9722	25	0.0278	11	1.8629	50'
	20'	1.8365	13	1.9747	25	0.0253	12	1.8618	40'
	30'	1.8378	13	1.9772	26	0.0228	12	1.8606	30'
	40'	1.8391	14	1.9798	25	0.0202	12	1.8594	20'
	50'	1.8405	13	1.9823	25	0.0177	13	1.8582	10'
	44° 0'	1.8418	13	1.9848	26	0.0152	12	1.8569	0' 46'
	10'	1.8431	13	1.9874	25	0.0126	12	1.8557	50'
	20'	1.8444	13	1.9899	25	0.0101	13	1.8545	40'
	30'	1.8457	12	1.9924	25	0.0076	12	1.8532	30'
	40'	1.8469	13	1.9949	26	0.0051	13	1.8520	20'
	50'	1.8482	13	1.9975	25	0.0025	12	1.8507	10'
	45° 0'	1.8495		0.0000		0.0000		1.8495	0' 45'
		餘弦	差	餘切	通差	正切	差	正弦	角



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