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A DIGITAL ANALYSIS OF INTERNAL WAVES AT OCEAN STATION "P"

by

Denny Jackson Denham

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THESIS

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A Digital Analysis of Internal Waves

at

Ocean Station "P"

by

Denny Jackson Denham
Lieutenant, United States Naval Reserve
B.S., Leland Stanford Junior University, 1962

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

Methods of investigating internal waves from bathythermograph information are discussed. A series of 651 hourly observations taken at Ocean Station "P" is analyzed by two methods to determine the characteristics of the internal waves present. One method involves a study of the fluctuations in depth of six selected isotherms; the other involves a study of the fluctuations in depth of the top, center, and bottom of the thermocline. An objective method is used to determine the depths of these features from the bathythermograph data.

Spectra of these features are calculated by Fast Fourier transform, hanned, and smoothed, and these spectra are compared with previously published results obtained from a different method of analysis of the same time series.

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ACKNOWLEDGEMENTS

The topic for this thesis was suggested by Assistant Professor Noel E. J. Boston, of the Naval Postgraduate School, Monterey, California, who had developed the concept of a Gaussian definition of the thermocline. His assistance in obtaining data records and in interpreting the results of data reduction contributed greatly to the successful completion of the research. Assistant Professor Ted Green and Assistant Professor Warren Denner, also of the Naval Postgraduate School provided invaluable guidance in the early stages of the research.

The data analysis was performed at the Naval Postgraduate School Computer Facility, the staff of which was most cooperative and helpful. Mrs. Patricia Johnson of the Computer Facility was particularly helpful in advising and guiding the method of the efficient creation of the program and in frequently lending assistance in troubleshooting the program in its intermediate stages of development.

To these people and organizations the author expresses his sincere thanks.

I. INTRODUCTION

Tidal period internal waves in the deep ocean were observed as long ago as 1907, by the Swedish oceanographer Pettersson [1909]. Little work was done in describing these waves or investigating their cause before 1950, although Stokes developed the theoretical treatment which could describe internal waves--neglecting geostrophic effects-- in 1847. This theory was used by Ekman [1904] in explaining the "dead water" phenomenon in Norwegian fjords. Although internal waves have been observed to exist for more than half a century, only recently have mathematical tools and equipment been developed which enable investigators to more closely analyze these waves in terms of periodic components and energy distribution. Although oceanographers are now in a position to say a great deal about the various frequencies which are present in a spectrum of internal waves, the presence of many of these frequencies and the causes of them have not yet been explained.

The most useful tools for analysis of internal wave data are Fourier analysis and power spectrum analysis. Fourier analysis, although a tool of mathematicians for decades, required great expenditure of time to be carried out before the advent of high-speed digital computers, and has only recently been brought into use in analyzing internal wave records. Power spectrum analysis, developed and refined by Blackman and Tukey [1959] has been used by electronic and communications engineers for several years, but has only recently been applied to ocean wave analysis. Programming either of these methods for use on a digital computer makes possible analysis of large amounts of data accurately in a reasonably short time.

In order to describe internal waves, it is at least necessary to obtain a time or space series of observations for analysis. The most common method of obtaining a time series is to observe temporal changes in thermal structure at a fixed location. This is sometimes done from weather observation ships located at deep ocean stations. One of these, Ocean Station "P", located in the northeastern Pacific Ocean at 145° West Longitude, 50° North Latitude and maintained by the Department of Transport of the Dominion of Canada, has been making bathythermograph observations at a maximum interval of 12 hours since July 1952. From time to time, bathythermograph observations are made from the ship occupying Station "P" at four hour intervals, or more frequently. The resulting bathythermograph data provides an excellent time series for analysis of internal waves.

The data for use in this thesis consisted of hourly bathythermograph observations, obtained with mechanical bathythermographs from 2000 GMT, 5 August 1961 to 2200 GMT, 1 September 1961, a total of 651 hourly observations. The data were furnished in digital format on punched cards by the National Oceanographic Data Center, Washington, D. C. Digital analysis was performed on an IBM 360 computer at the Naval Postgraduate School Computer Facility, Monterey, California.

Bathythermograms of the data used in this thesis may be found in Manuscript Report Series No. 106, published by the Fisheries Research Board of Canada, Programmed by the Canadian Committee on Oceanography.

II. OBTAINING DATA FOR POWER SPECTRUM ANALYSIS

One of the most commonly used methods of obtaining data for analysis in describing internal waves has been following the depth fluctuations of selected isotherms. This method was used by Tabata [1965] at Ocean Station "P" in the northeastern Pacific Ocean, and by Seiwell [1965] in the North Atlantic Ocean. Although this method has been quite successful in the past, its validity depends on rather specific conditions. Internal waves propagate along isopycnal surfaces, and have their greatest amplitude at a density discontinuity. An isothermal surface will closely approximate an isopycnal surface for propagation of an internal wave only if the water column is nearly isohaline and the wave is of small amplitude. In this case one can approximate the change in density due to salinity and the change in density due to pressure fluctuations by zero. In choosing the isotherms of interest, Tabata selected isotherms which were within the permanent thermocline, assuming that the thermocline would oscillate as a unit, although phase differences between thermoclines would not be indicated in the power spectrum. Seiwell, in the North Atlantic, chose the 20°C isotherm as being close to defining the top of the thermocline, or depth of the mixed layer.

Defant [1932] in examining the data collected by METEOR anchor stations, used a similar method, although he examined temperature fluctuations at fixed depths. In order to compare his results with that of other investigators, one must select a depth near the top of the thermocline, since other investigators used isotherms near the top of the thermocline for analysis. Decreases in temperature would indicate the approach of a crest, and increases in temperature the approach of a trough.

LaFond and Rao [1954] obtain their raw data from the mixed layer depth, disregarding any changes above or below this depth, which corresponds to the top of the permanent thermocline. This would appear to be a more realistic approach, since the top of the thermocline will approximate a density discontinuity, and internal waves should have their greatest amplitudes at this point, and thus be most easily detected.

In this thesis, the fluctuations in depth of the top, center, and bottom of the thermocline were used to investigate internal waves. The depths of these features were obtained using a method originated by Boston [1966] in which the thermocline is made to approximate a Gaussian distribution of depths and related temperatures. The depths thus obtained were subjected to a Fourier analysis, and the coefficients obtained were hanned and smoothed over discrete frequency bands.

For comparison of results of isotherm analysis, the depths of six selected isotherms were followed through the time series and subjected to analytical procedures identical to those used in the spectral analysis of the thermocline.

III. OBTAINING THERMOCLINE DEPTHS

A. THE GAUSSIAN THERMOCLINE

A gaussian, or normal, distribution of temperature (T) as a function of elevation (Z) is given by the equation

$$T(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} e^{-\frac{z^2}{2}} dz$$

where Z is positive upwards. The frequency function corresponding to this distribution function is

$$\frac{dT(Z)}{dZ} = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}}$$

Using a basic statistical approach, defining a normalized variable

$$Z' = \frac{Z-m}{\sigma}$$

where m is the mean of the Z values, and σ is the standard deviation of the array of Z values, we obtain a new frequency function which is

$$\frac{dT(Z')}{dZ} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(Z-m)^2}{2\sigma^2}}$$

This function has the following characteristics:

1. It is symmetric about the point Z = m
2. It has two symmetric points of inflection at $Z = m \pm \sigma$
3. It has a maximum rate of change at $Z = m \pm \sigma\sqrt{3}$

A change in the numerical value of m causes a displacement of the curve in the vertical direction, but does not alter its form.

A change in σ has the effect of altering the scale in the vertical direction. The smaller the value of σ, the more concentrated the curve is about the point Z = m.

If the water surface is defined to be Z = 0, and the depth of the water considered to be infinite, then the function defining the vertical temperature distribution is

$$T(Z') = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{(Z-m)^2}{2\sigma^2}} dZ$$

The center of the curve T(Z') versus Z is found at the point Z = -m.

If this curve is considered as a bathythermograph trace, the following definitions may be made. The center of the distribution of T(Z') versus Z is the center of the thermocline, called Z_c. The top and bottom of the thermocline will correspond to the points of maximum rate of change of slope of this distribution. If Z_t is used to designate the depth of the top of the thermocline and Z_{bt} to designate the depth of the bottom of the thermocline, these definitions become

$$Z_c = -m$$

$$Z_t = -m+\sigma$$

$$Z_{bt} = -m-\sigma$$

These features are illustrated in Figure 1A.

To obtain the necessary values to apply these definitions, consider the first and second moments of the (T,Z) curve about the origin, Z = 0. The nth moment of the curve about the origin is, by definition, the mean value of the array Z^n, called $\overline{Z^n}$, where n is a positive integer. With a normal curve, the first moment is identical with the mean, which corresponds to the center of the thermocline.

$$Z_c = \frac{1}{T(0) - T(-\infty)} \int_{T(-\infty)}^{T(0)} Z(T)\,dT$$

where T(0) is the temperature at the surface and $T(-\infty)$ is the temperature of the bottom of the lower isothermal layer which, so far, has been assumed to be the bottom of the ocean. The second moment

$$\overline{Z^2} = \frac{1}{T(0) - T(-\infty)} \int_{T(-\infty)}^{T(0)} Z(T)^2\,dT$$

can be used to compute σ , since the variance of the distribution, σ^2 , is defined as

$$\sigma^2 = \overline{Z^2} - m^2 = \overline{Z^2} - Z_c^2$$

In practice, several approximations to this exact solution must be made. The temperature $T(-\infty)$ is measured at the bottom of the bathythermograph trace. The differentials, dT, become finite differences, δT, and the integrals are replaced by summations.

In changing from an integral representation to a summation over finite intervals, a certain amount of error in the value of σ is introduced, due to the fact that summation in this fashion is not a continuous process, whereas integration is.

To compensate for this loss of accuracy, a weighting factor must be introduced in all computations involving σ. Grosfils [1968] calls this factor sk, and bases the size of the factor on the size of δT.

For $\delta T \geq 1.0^\circ$ sk = 1.30

For $\delta T \leq 0.5^\circ$ sk = 1.47

Intermediate values of δT will have intermediate values of sk, but these values were chosen as being those most commonly encountered. Using this factor, the equations defining the top and bottom of the thermocline become

$$Z_t = -m + \sigma\ sk$$

$$Z_{bt} = -m - \sigma\ sk$$

Using this approach to define the thermocline, one can fully describe a thermocline knowing the temperature at the surface, the temperature at the bottom of the bathythermograph cast, and depths corresponding to some constant temperature interval.

The center of the Gaussian thermocline can now be defined as

$$Z_c = \overline{Z(T_n)_i} = \frac{1}{T_1 - T_{bt}} \sum Z(T_n)_i$$

and since

$$\overline{Z(T_n)_i^2} = \frac{1}{T_1 - T_{bt}} \sum Z(T_n)_i^2$$

then

$$\sigma^2 = \overline{Z(T_n)_i^2} - Z_c^2$$

Using these factors, the top and the bottom of the Gaussian thermocline may readily be determined.

In the event the distribution of depth with temperature is not Gaussian, the inaccuracies built into this objective definition are intolerable. Whether or not this distribution is Gaussian can be determined by computing the third moment of the (T,Z) curve about the origin. This, however, is unnecessary when one applies a non-Gaussian approach to the thermal structure.

B. THE NON-GAUSSIAN THERMOCLINE

The approach that was used by Boston and by Grosfils was based on depth data given at constant temperature intervals. The data used for this thesis were temperature data given at constant depth intervals. This difference does not affect the treatment of the data except for the determination of the temporary center of the thermocline. The details of the method used in this thesis to find the center of the thermocline are given in Section IV.

When the depth of the center of the thermocline is found, by whatever method the data requires, the data is then processed as two Gaussian thermoclines--one based on the depth distribution of temperatures above and including the depth Z_c, the other based on the depth distribution of temperatures below and including the depth Z_c.

In order to locate Z_t, only the upper portion of the (T,Z) curve is considered. This portion of the curve is "reflected" about Z_c, to produce the curve shown in Figure 1B. From this curve σ^2 is computed by methods previously described and, using this value, Z_t may be calculated.

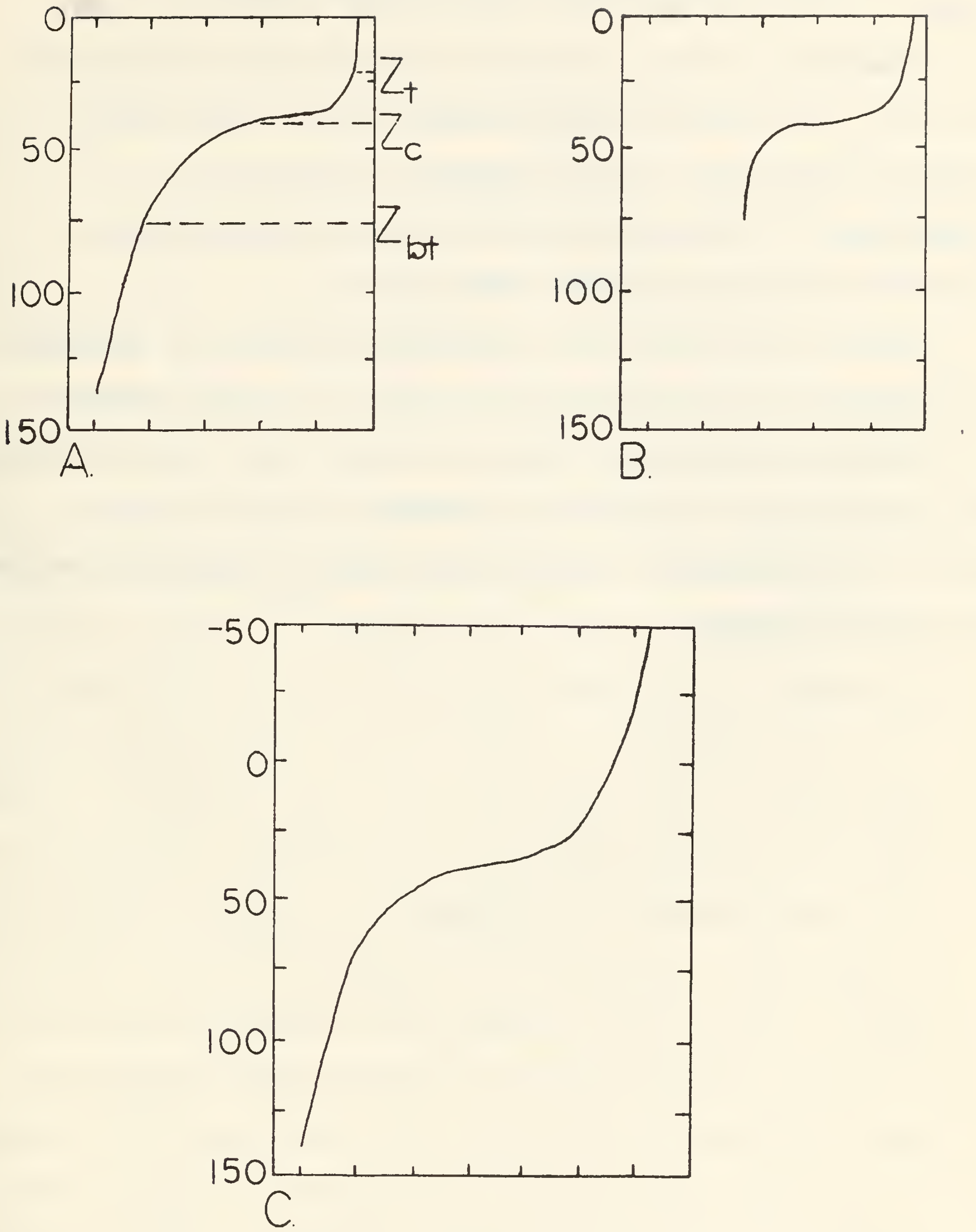

FIGURE 1

In order to determine Z_{bt}, a similar procedure is followed, except that in this case, the lower portion of the (T,Z) curve is "reflected" about Z_c, to produce the curve shown in Figure 1C. Note that it is possible for this resulting curve to include negative depths, but since the depths are squared in computing σ^2, this does not result in a narrowing of the distribution through an underestimate of σ.

It can be seen that in the event the thermocline is Gaussian, treating it in this fashion will not alter the results obtained by treating it as Gaussian. Thus, every (T,Z) input may be treated as a non-Gaussian distribution, regardless of its true form. This is the method which was used in processing the bathythermogram data for this thesis.

IV. DIGITAL METHODS USED TO OBTAIN DEPTH INPUTS

The bathythermograph data was obtained from the National Oceanographic Data Center in digital format. For each bathythermograph trace, the digital data was contained on three or four computer data cards, including the sequential number of the observation, the date and hour of the observation, temperature in degrees Centigrade at each five meters of depth, and such other bits of synoptic information as the location of the observation, the ship making the observation, and a number identifying the cruise during which the data was taken (Figure 2).

The first phase of data reduction was to transfer the sequential number of the observation, the time and date, and the temperatures at the first twenty depths--which correspond to temperatures from the surface to a depth of 95 meters--to a computer data card. This was not essential for data reduction, but was done for convenience as it reduced the size of the data deck by three quarters. By inspection of bathythermograms for the entire series of observations the permanent thermocline was in all cases determined to be shallower than 95 meters in depth.

The data deck thus obtained was then analyzed to obtain the depths of the top, center, and bottom of the thermocline using a method based on a digital computer program developed by Grosfils [1968]. Grosfils' program, however, was written for data consisting of depths corresponding to equally spaced temperatures, and since the NODC format differs from this, some modifications had to be made in the program.

```
NODC   Consec.
I.D.   No.             Date      Time  Lat.   Long.                               Card No.

07042  390   3 18 10P  05 22 08 61 0200  5000N 14500W   13 2                                                      A 0104 02
                48        STONETOWN                                                                        02
                135 135 135 135 134 133 131 127 110 086 072 068 065 061 058 056 054 052 050 049          5 03
                049 049 049 048 048 048 047 046                                                          5 04

07042  391   3 18 10P  05 22 08 61 0300  5000N 14500W   13 2                                                      A 0104 02
                48        STONETOWN                                                                        02
                136 136 136 136 136 135 133 125 088 079 073 070 067 062 059 057 055 052 051 051          5 03
                051 051 051 051 051 050 049 049                                                          5 04

07042  392   3 18 10P  05 22 08 61 0400  5000N 14500W   13 2                                                      A 0104 02
                48        STONETOWN                                                                        02
Z(m)             0   5  10  15  20  25  30  35  40
T(°C)x10        135 136 136 136 136 136 134 130 111 085 076 071 068 067 062 057 054 051 049 049          5 03
                049 049 049 049 049 046 045 045                                                          5 04

07042  393   3 18 10P  05 22 08 61 0500  5000N 14500W   13 2                                                      A 0104 02
                48        STONETOWN                                                                        02
                136 136 136 135 135 135 131 103 083 073 069 067 064 060 057 054 052 050 050 050          5 03
                050 050 050 049 048 047 046 046                                                          5 04
```

FIGURE 2
NODC DATA FORMAT

The computer first reads the data from the data deck, then calls subroutine INTERP, which fits straight lines through the given data points, then determines the depths of temperatures at intervals of 0.1°C from the surface to 95 meters. The temperature is read by the computer as an integer--in tenths of degrees--and left in this form for the interpolation process, to eliminate truncation error in manipulating the temperatures.

In the program written by Grosfils, the center of the thermocline was defined as the point where the second difference of depths was greater than zero. Due to the format of the data used in his program, however, a great deal more information was known about the thermocline than is given in the NODC format. For this reason, the following method was used to define the center of the thermocline, utilizing the array of temperatures before interpolation.

In subroutine INTERP, the first differences of temperatures were calculated, from the surface (corresponding to T_1) downward. The ith first difference is defined as

$$\Delta T_i = T_i - T_{i+1}$$

The first time the ith first difference is less than -0.3°C, T_i is stored as ITTOP and the computer then begins calculating first differences again at a depth of 45 meters. When the ith first difference then becomes greater than -0.2°C, the computer stores T_{i+1} as ITBOT. The temperature of the center of the thermocline is then defined as the arithmetic mean of ITTOP and ITBOT, and is stored as ITC. The computer then interpolates depths and temperatures as descrived previously.

Once the interpolation is completed, subroutine PURGE is called to insure that only one depth is recorded for each temperature value, which would not be the case if any temperature appeared more than once in the data which was read in. PURGE examines the interpolated temperatures, and in the event that any temperature is greater than or equal to a temperature at a shallower depth, discards the corresponding depth. After proceeding through the entire interpolated trace in this manner, the trace is reduced to one in which there is only one depth corresponding to any given temperature, and it will be the shallowest depth at which this temperature appears. Although the program at this point has destroyed all information about inversions, this was not felt to be an unreasonable step. The raw data were all examined for inversions prior to the interpolation step, and in no case were inversions present greater than $+0.1^{\circ}$C in five meters of depth.

At this point, subroutine FINDZC is called, and it examines the temperature and depth information thus computed and chooses the depth of the first (shallowest) temperature which is less than or equal to ITC. It stores this depth as ZC, the depth of the center of the thermocline. This depth, along with all the remaining temperature and depth information, is returned to the main program.

Using this information, the main program calls subroutine NONGA, which uses the procedure described in Section II to find the top and bottom of the thermocline through the use of subroutine GAUSS1 and GAUSS2, respectively.

For comparison with Tabata's results, the depth fluctuations of selected isotherms were also subjected to Fourier analysis. Depth information for this analysis was obtained in a manner similar to that described above, except that for depths of isotherms, subroutine FINDZS is called from INTERP after PURGE and the depths are determined in a manner similar to that used to find ZC.

Fluctuations of isotherms and thermoclines with time are shown in Figures 3 and 4, respectively.

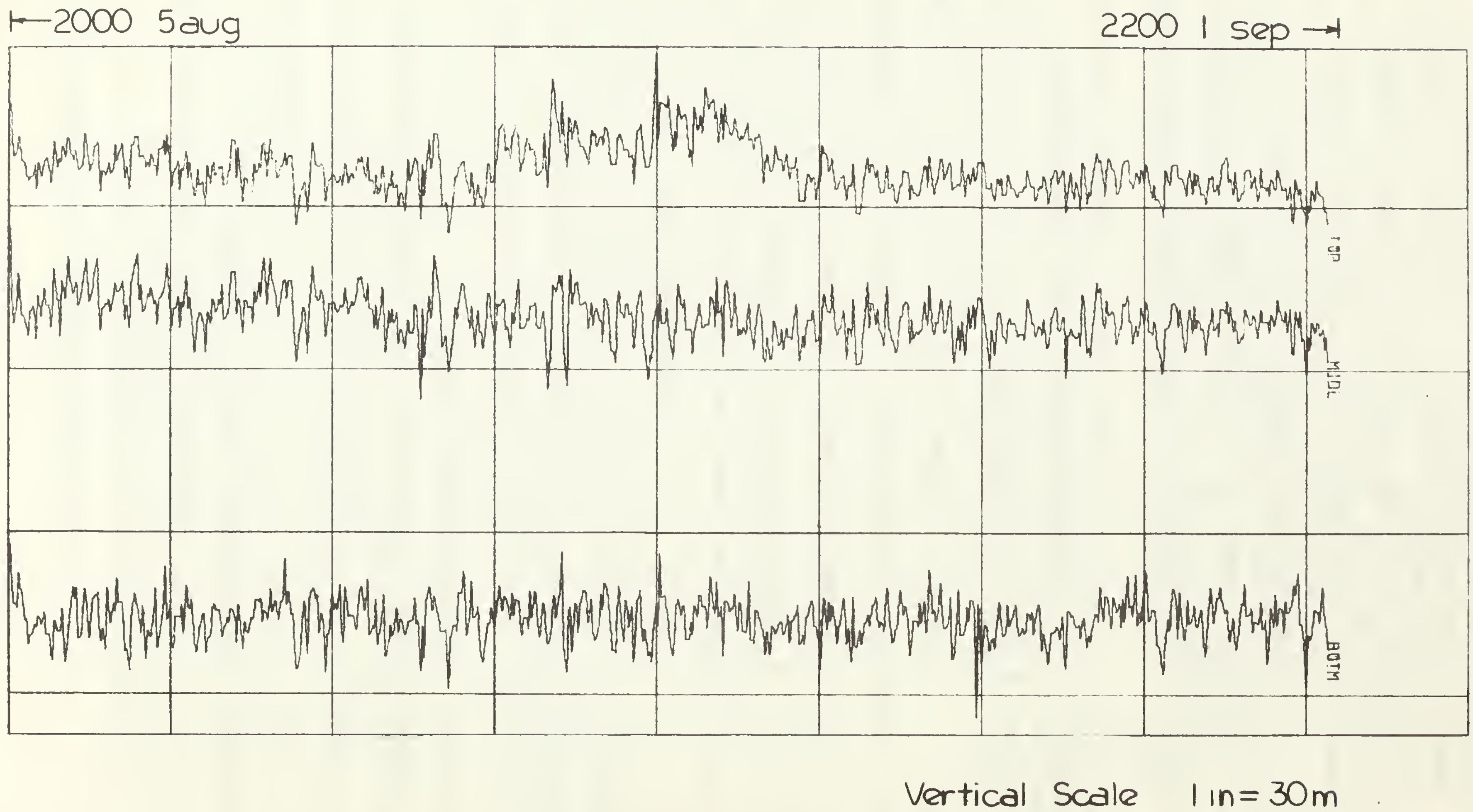

FIGURE 4
Thermocline Depths

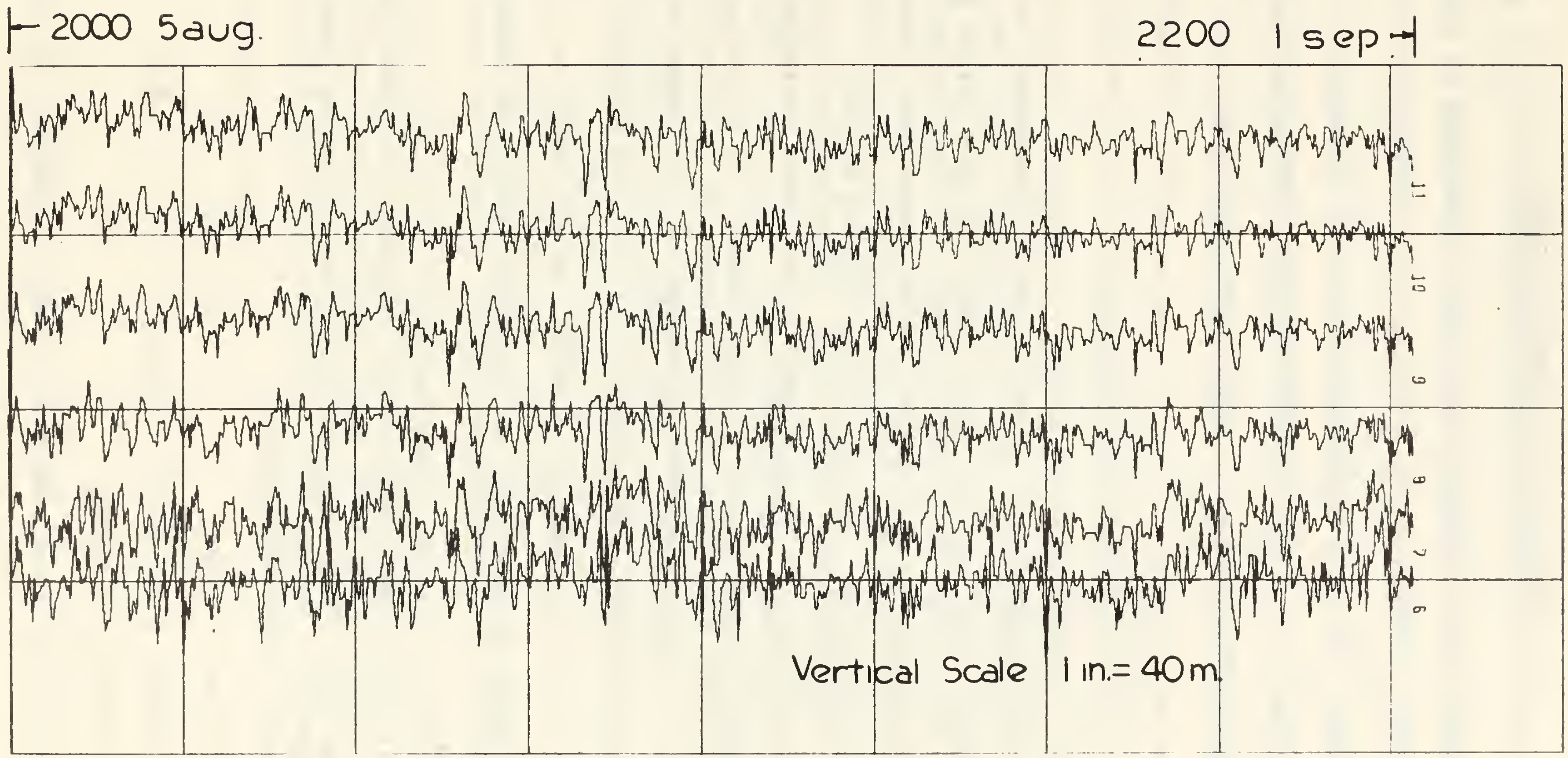

FIGURE 3
Isotherm Depths

V. COMPUTATION OF POWER SPECTRA

The reworked data obtained as described in Section IV were subjected to Fourier analysis to produce a power spectrum. Subroutine RHARM, which is contained in the IBM Scientific Subroutine Library, was used for this analysis. This subroutine uses an algorithm, developed by Tukey and Cooley [1965], to transform the depth fluctuations to a Fourier series of the form

$$\eta = \sum_{n=0}^{N} a_n \cos n\omega t + b_n \sin n\omega t$$

where $n\omega$ is the frequency of a given sinusoidal component in radians per hour, N is half the total number of data points plus one, t is time in hours, and η is the instantaneous wave elevation above its mean position. The sum of the squares of the coefficients, $a_n^2+b_n^2$, is proportional to the power contained in the nth component, whose period is N/(n) hours. a_0 is the mean value of the data, and to eliminate leakage of this large value into neighboring frequency components, all data was adjusted to a mean value of zero in subroutine AVERGE before RHARM was called. b_0 is always equal to zero, and is therefore ignored in subsequent manipulations.

Once the Fourier coefficients were obtained, they were hanned by subroutine HANN. This subroutine used the following recursion formula to refine the coefficients:

$$a_1' = 0.5a_1 - 0.25a_2$$

$$a_i' = -0.25a_{i-1} + 0.5a_i - 0.25a_{i+1} \quad i = 2,3,\ldots,N-1$$

$$a_N' = -0.25a_{N-1} + 0.5a_N$$

The sine coefficients were similarly treated. Hanning the data is a method of greatly reducing leakage of power from frequencies containing large amounts of power into neighboring frequencies. According to Bingham, et. al. [1967] if the Fourier transform is perfect for ω_1, ω_2, ω_3,... and ω is none of the ω_i, the leakage without hanning is proportional to $|\omega_i - \omega|^{-1}$ and hanning the coefficients reduces the leakage to a term proportional to $|\omega_i - \omega|^{-3}$.

The hanned data was then plotted on a graph of power (spectral density) versus frequency. Although this presentation gave indications of the frequencies of the major components of the internal waves present, it was objectionably noisy. To reduce this noise, a method of power spectrum smoothing was utilized to produce the final spectrum. This method, originated by Jones [1965] was used in subroutine SMOOTH. The method consists of first arranging the squares of the non-zero a and b coefficients in order and renaming them s, as follows:

$$\begin{array}{cccccc} a_0^2, & a_1^2, & b_1^2, & a_2^2, & b_2^2, & \ldots\ldots \\ s_1, & s_2, & s_3, & s_4, & s_5, & \ldots\ldots \end{array}$$

The smoothed spectrum is obtained by averaging these numbers in groups of 2m, except for the first group, which contains m, and the last group, which contains those numbers which do not fit into groups of (2p + 1)m, where p is the number of groups of 2m included in the data.

$$S_o = N/m \left[s_1 + 2\sum_{i=2}^{m} s_i \right]$$

$$S_1 = N/m \sum_{m+1}^{3m} s_i$$

$$\cdot$$

$$\cdot$$

$$S_p = N/m_p \sum_{(2p-1)m+1}^{N} s_i$$

where m_p is the number of pairs remaining in the last step and N is the total number of Fourier coefficients used in the smoothing. By this method of smoothing, S_o will be centered at $.25/N\,\delta t$, S_1 will be centered at $1/N\,\delta t$, S_2 at $2/N\,\delta t$, and so forth. Care must be taken to choose m sufficiently large that significant smoothing takes place without choosing it so large that it eliminates detail from the final spectrum.

Subroutine SMOOTH was run with m's of 2, 3, 4, 5, 10, 20, and 50. Of these, the best results were obtained with m = 4, and this grouping factor was used in the final analysis. After the data were smoothed, subroutine PERIOD was called to generate counters to be used for the final graphic output.

The data were broken into two groups, the first containing the first 512 data points (consecutive numbers 1 through 512) and the second containing the last 512 data points (consecutive numbers 139 through 651). More confidence may be placed in spectral estimates if the spectral estimates are the averages of several sets of independent data. The nearest approximation to this which was possible with this data was the treatment which was used. Had a sample size of 256 points been chosen, totally independent data sets could have been used, and confidence in the results would have been increased. The reduction in the number of data points by a factor of two, however, would have so significantly reduced the accuracy of the harmonic analysis that the increased confidence in the results would have been meaningless. For this reason, the use of two partially independent data sets of 512 points each was decided upon.

The two groups of hanned, smoothed data were then added together, term for term, by subroutine GROUP, and averaged. Plots of the results of this averaging for the top, center, and bottom of the thermocline are shown in Figures 5, 6, and 7.

Analysis of the depth fluctuations of selected isotherms was carried on in the same manner as analysis of the thermocline data. The smoothed spectra of fluctuations of the isotherms analyzed are shown in Figures 8, 9, 10, 11, 12, and 13. For comparison, Tabata's results for the 6°C and 10°C isotherms are shown in Figure 14.

The complete computer program used in data reduction is given in the Computer Program section of this thesis.

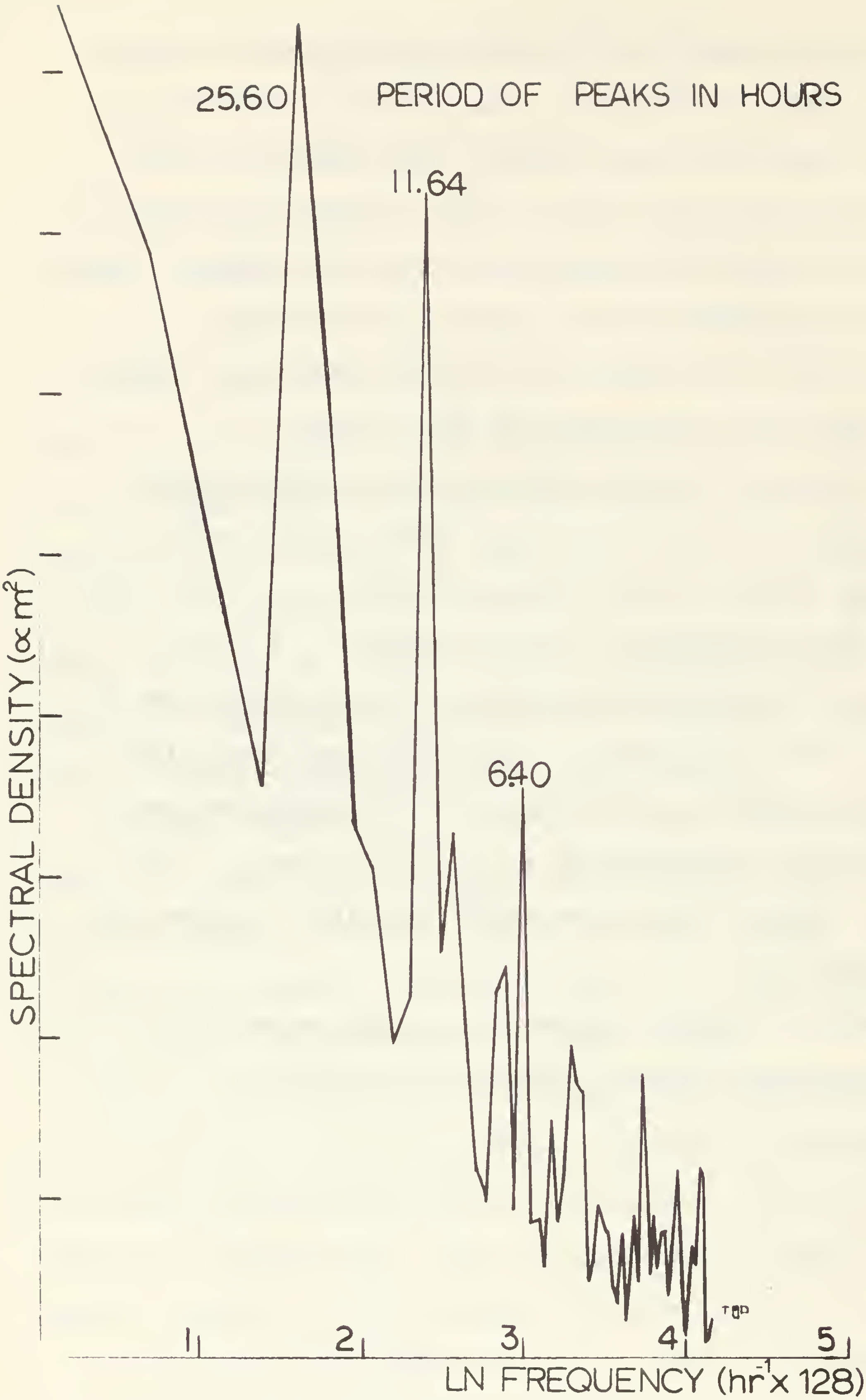

FIGURE 5

Z_t SPECTRUM

FIGURE 6

Z_C SPECTRUM

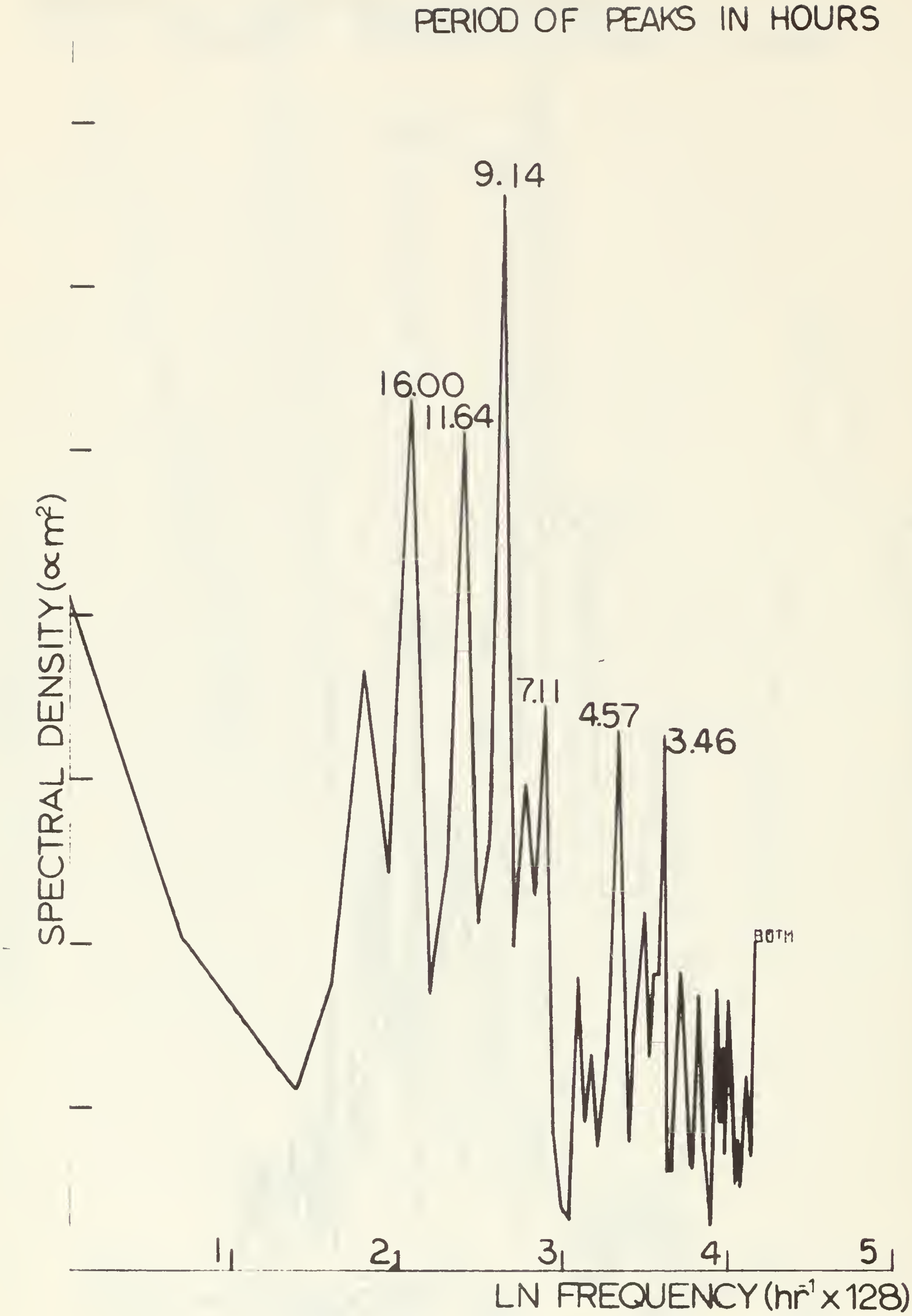

FIGURE 7

Z_{bt} SPECTRUM

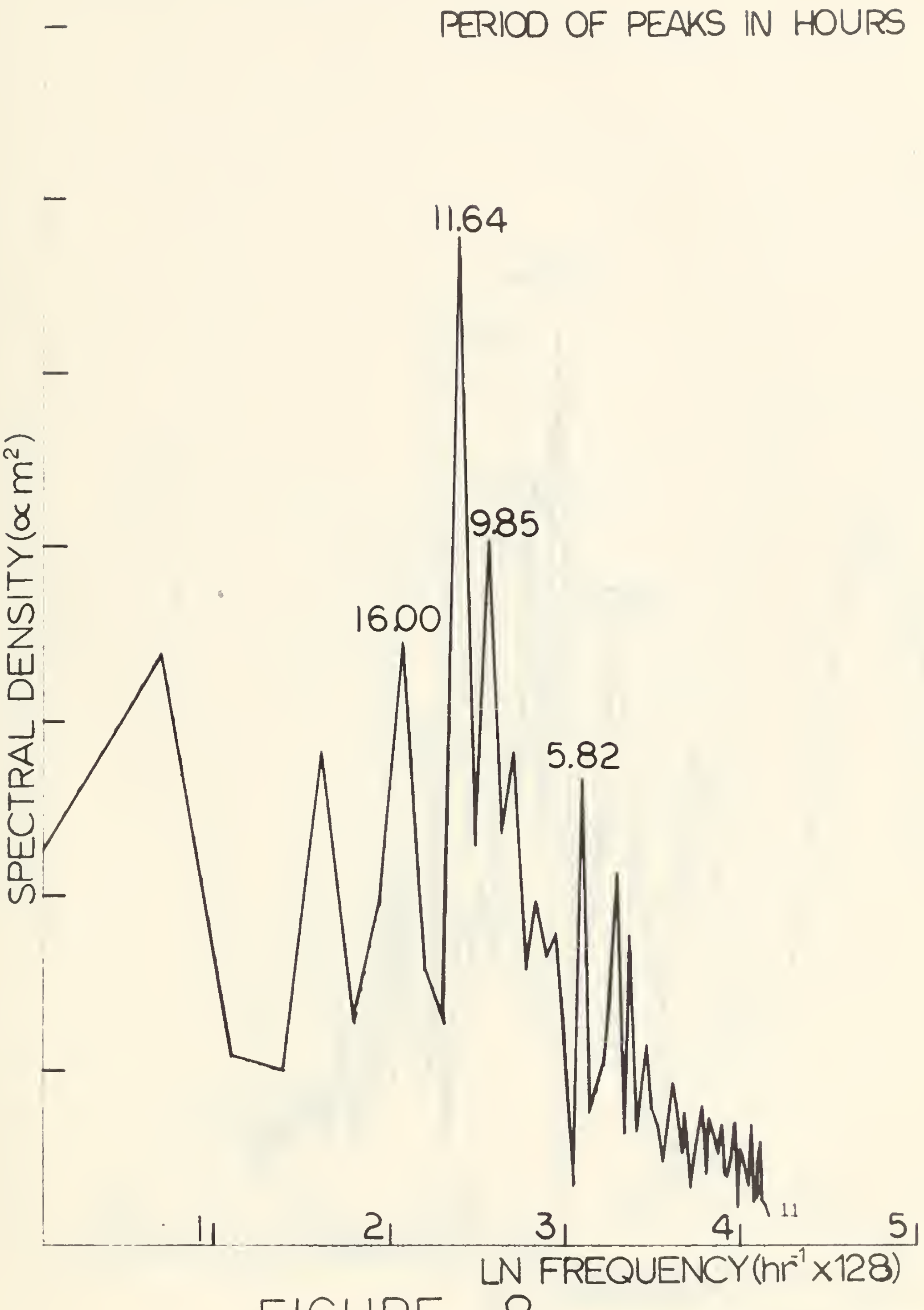

FIGURE 8

11°C ISOTHERM SPECTRUM

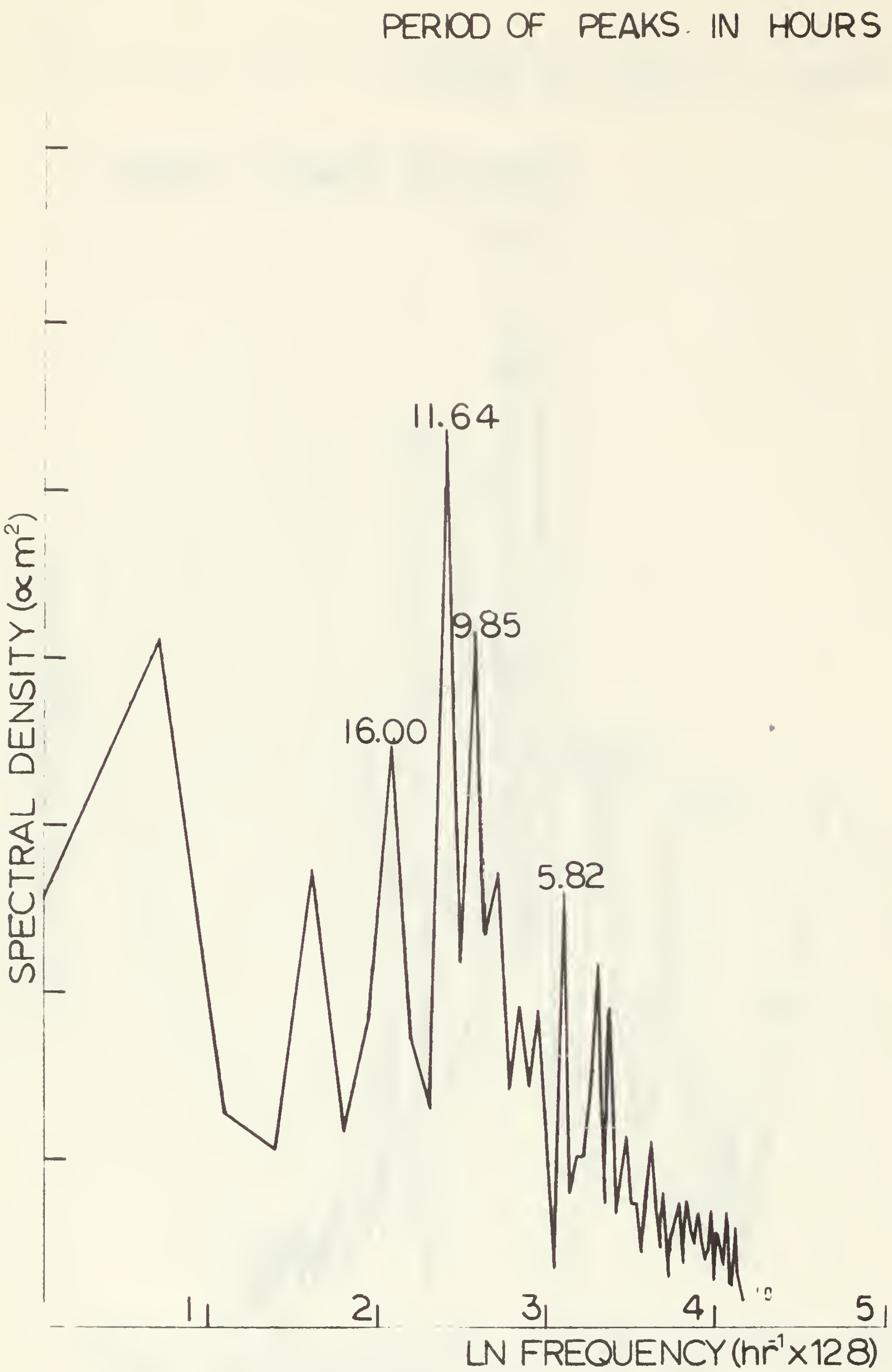

FIGURE 9

10°C ISOTHERM SPECTRUM

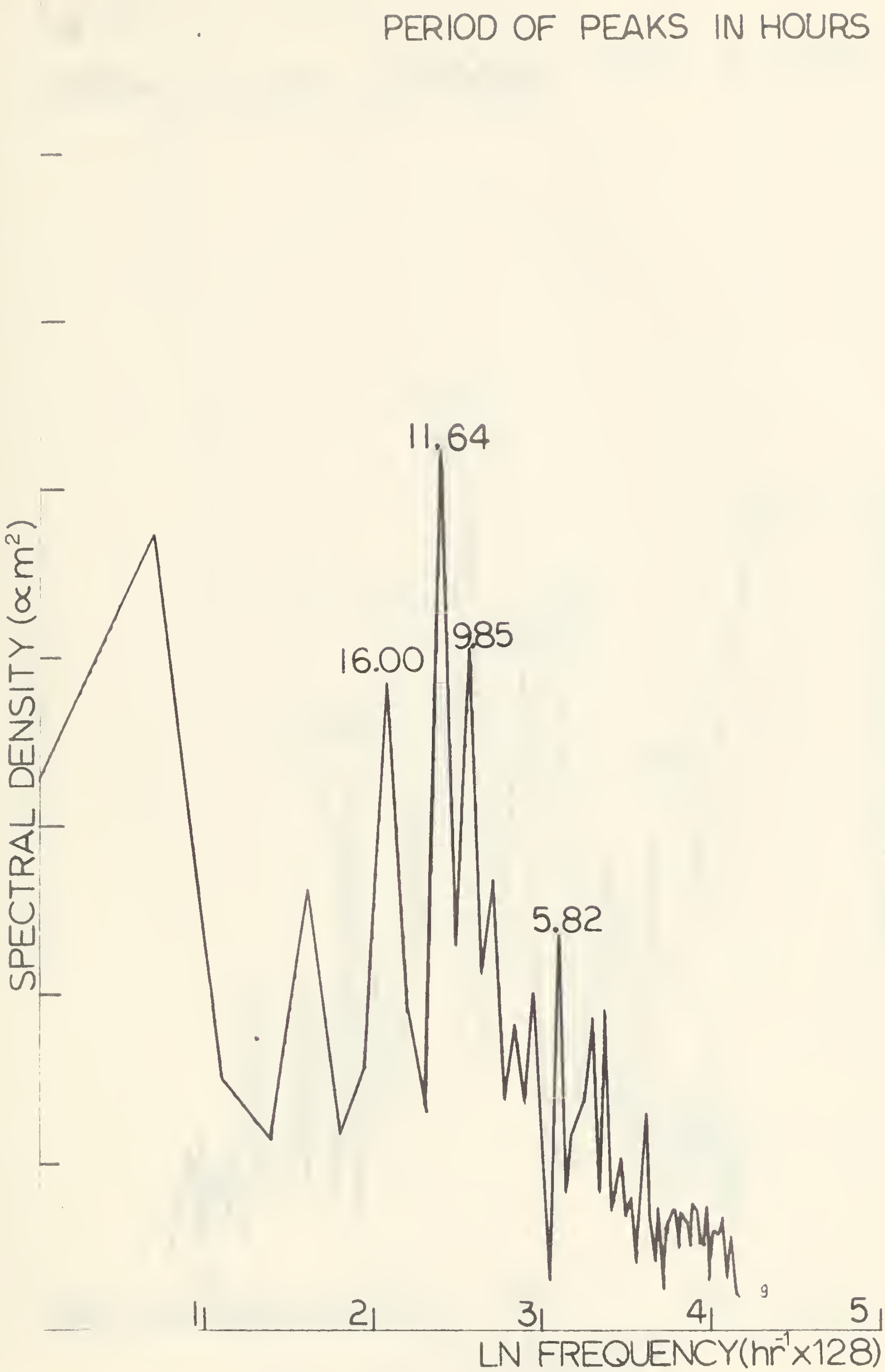

FIGURE 10

9°C ISOTHERM SPECTRUM

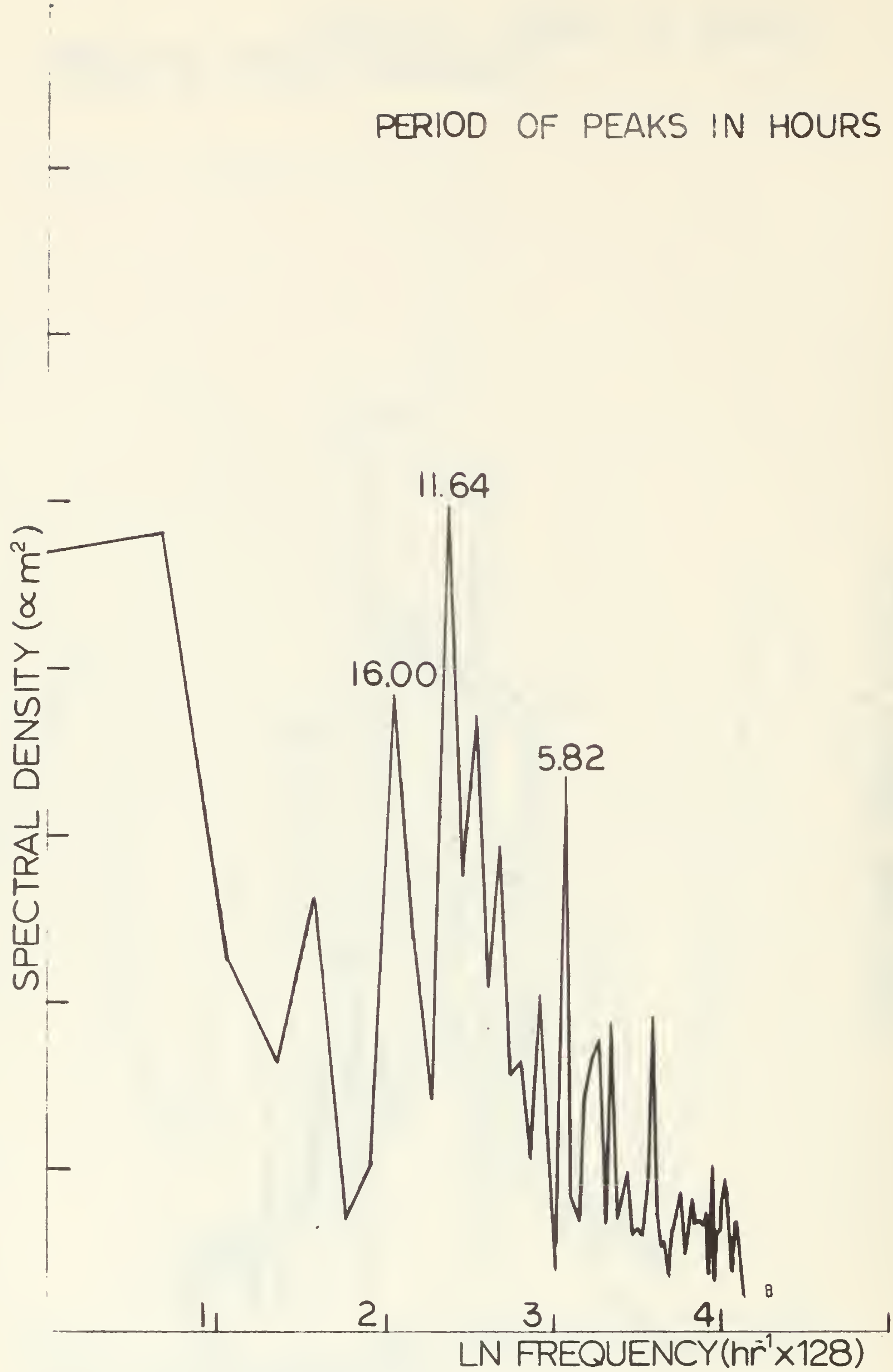

FIGURE 11

8°C ISOTHERM SPECTRUM

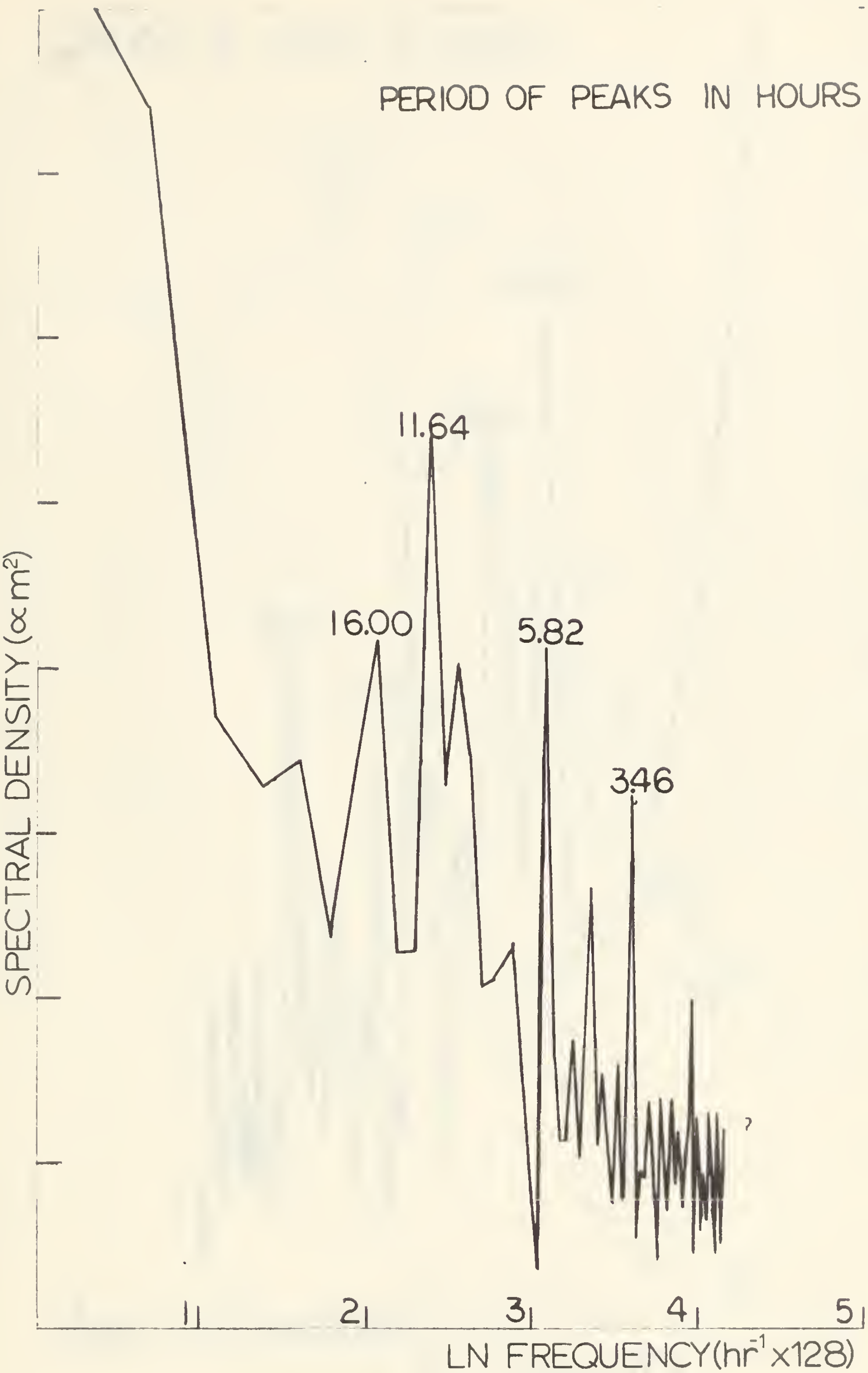

FIGURE 12

7°C ISOTHERM SPECTRUM

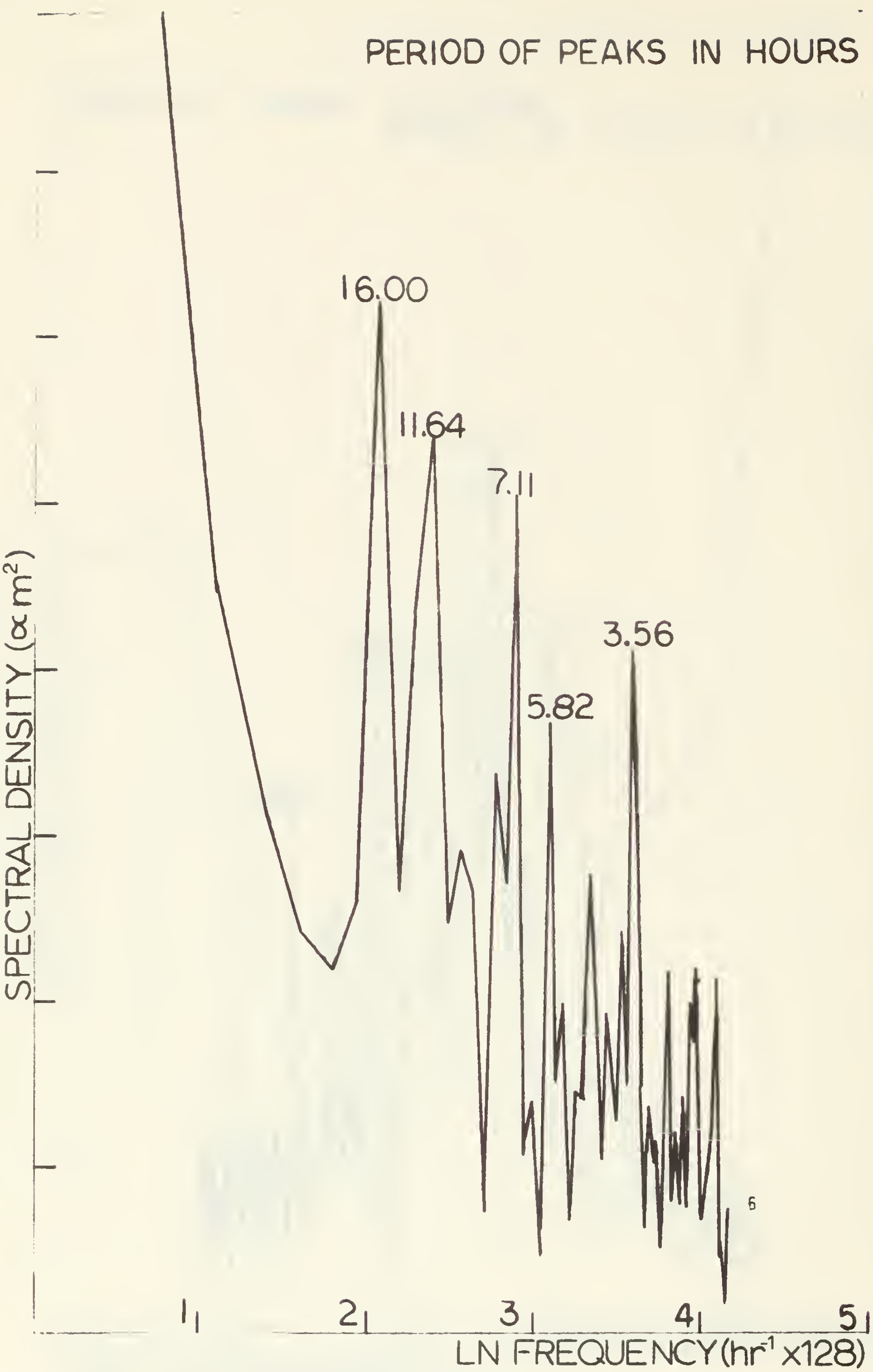

FIGURE 13

6°C ISOTHERM SPECTRUM

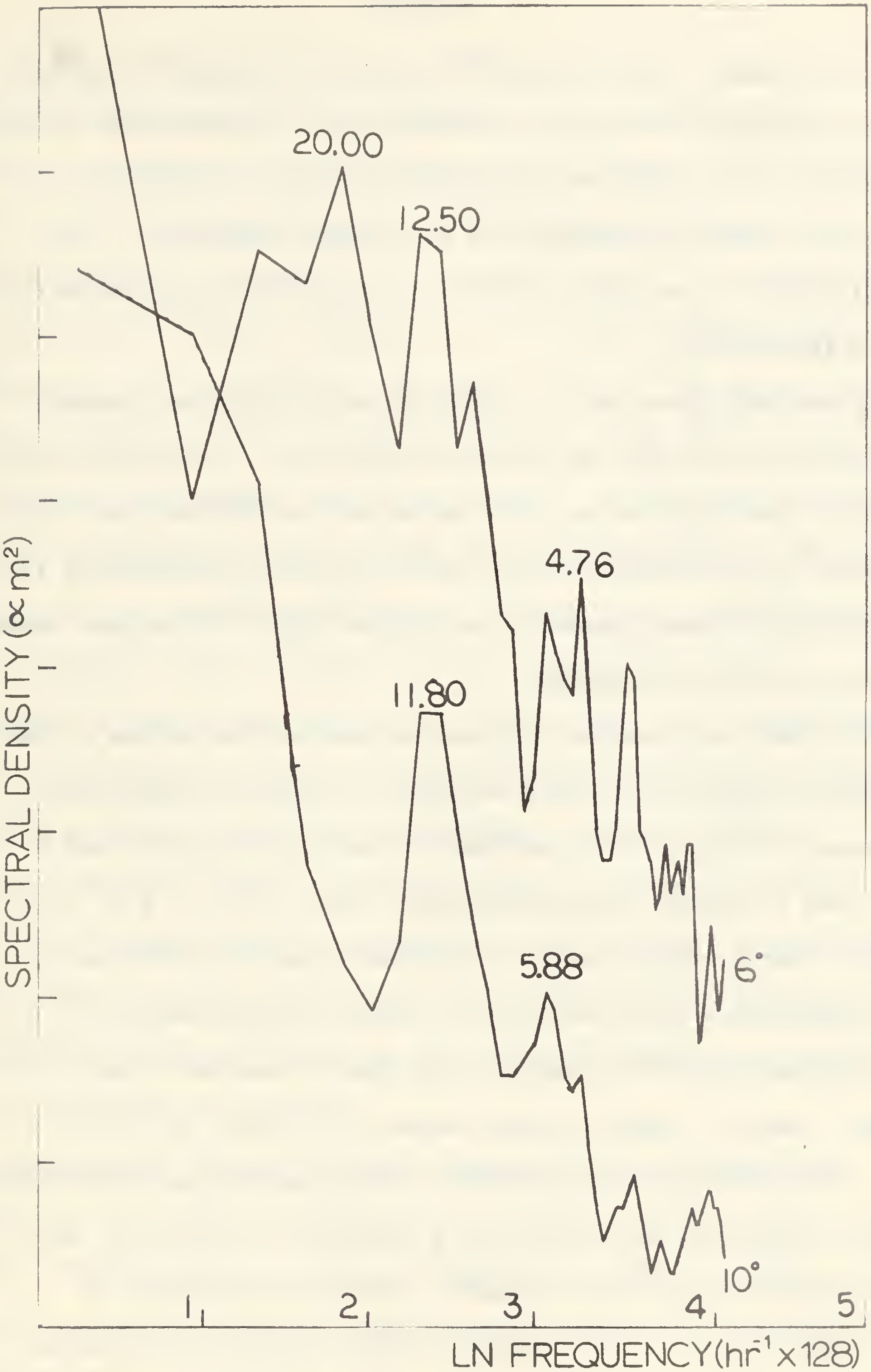

FIGURE 14

6°C & 10°C ISOTHERM SPECTRA (TABATA, 1965).

VI. RESULTS

In his paper, Tabata reports a strong semidiurnal component in the internal waves analyzed through depth fluctuations of the 6°C through 10°C isotherms, with greater power in higher frequencies noted in analysis of the deeper isotherms. This general behavior was also noted in the digitally analyzed data used in this thesis.

A power peak centered at an 11.64 hour period was present in the spectra for all the isotherms analyzed, as well as in the spectra of the top, center, and bottom of the thermocline--the shallowest feature studied--also shows a peak corresponding to a period of 25.6 hours, probably associated with a principal lunar diurnal (O_1) tidal component.

The power spectrum due to fluctuations of the bottom of the thermocline indicates a shift of power to higher frequencies, with peaks associated with periods of approximately 4.5 and 3.5 hours, and a maximum energy associated with waves of 9.14 hour period. Tabata reports waves of periods equal to five hours and slightly less than four hours present for the deepest of the isotherms which he studied, with which these data agree. In no case, however, does he report waves of periods near nine hours. Relative peaks in spectral density having periods between nine and ten hours (most often at a period of 9.85 hours) were found associated with all features. Waves with periods of 16.00 hours, which is very close to the inertial period of 15.7 hours at Ocean Station "P" were also noted in all spectra produced.

In the original plan for the analysis, the fluctuations of the thickness of the thermocline were also to be investigated, to determine whether the thermocline was oscillating in the fundamental mode--in which the entire thermocline oscillates as a unit--or mode two--in which the top and bottom of the thermocline are 180° out of phase, resulting in a periodic thickening and thinning of the thermocline's vertical extent. The spectra produced by analysis of the thickness showed nearly constant spectral density throughout the entire range of frequencies studied, except for large contributions in the low frequency end of the spectrum. This suggested that the thermocline was oscillating in the fundamental mode, and the spectral densities at high frequencies were due to noise. To verify this, the correlations of the top, center, and bottom of the thermocline were computed. It was found that the depth fluctuations of each of these features with the others was greater than +.90 which confirms the fact that the entire thermocline was oscillating as a unit. This conclusion agrees with the results published by Tabata, who found phase differences on the order of $\pm$ 20° between oscillations of the isotherms he analyzed.

The spectrum associated with the top of the thermocline showed a very large contribution due to waves of very long period. Examination of Figure 3 shows the reason for this. The top of the thermocline was slowly sinking with time, presumably due to mixing in the upper layer. This effect, although small in magnitude, would have been identified in the analysis as an internal wave with a period equal to the total length of the record, which in this case was 512 hours. In addition, during the period 16 August to 21 August, the water column, which was almost perfectly isothermal above and below the thermocline at all other times, developed near-surface transient thermoclines. As the non-Gaussian analysis used to obtain thermocline depths did not eliminate transient thermoclines before computing the depths associated with the main thermocline, these near-surface transients resulted in the calculation of an abnormally large variance by subroutine GAUSS1, which had the effect of moving the calculated top of the thermocline closer to the surface than it actually was. The fact that the time period during which this occurred was included in both sets of 512 hourly observations meant that an additional large power component associated with a period of 512 hours was found in the analysis. This is an apparent effect, rather than a real one, and even when combined with the long-term sinking of the thermocline, fails to obscure the results in the frequency range of interest.

Examination of Figure 3 also reveals that the center of the thermocline also showed a slight downward displacement with time, but of less magnitude than that of the top of the thermocline. This resulted in a relatively large amount of power associated with long-period internal waves at the center of the thermocline, but much smaller than the power contribution to the spectrum of the top of the thermocline. The bottom of the thermocline showed no change in mean depth with time, which indicates that surface layer mixing extended only to a depth slightly below the center of the thermocline. Although in most cases large amounts of power were associated with low frequencies, these frequencies were not considered of interest, and are not indicated on the graphs of the final spectra. A complete presentation of all data produced by the analysis is presented in the COMPUTER OUTPUT section.

Tabata's results for the 6°C and 10°C isotherms are presented in Figure 14 in a semilog plot for comparison with the results of the digital analysis. Although these curves differ in detail from those presented for the same isotherms analyzed digitally in this thesis, the overall form is strikingly similar. For both analyses of the 10°C isotherm, the greatest peak spectral density is associated with waves whose period is greater than the semi-diurnal tidal period. For both analyses of the 6°C isotherm, the spectral density is rather well-distributed throughout the higher frequency components, which also have a greater mean spectral density than the higher frequency components of the analyzed 10°C isotherm data. This indicates that the higher frequency components showed an increase in amplitude with increasing depth, a behavior which was also noted by Haurwitz, Stommel, and Munk [1959].

The Brunt-Vaisala frequency associated with the thermal structure of this time series is 3.79×10^{-2} sec^{-1}. This indicates that the thermal structure is capable of supporting internal waves of any period greater than 26.4 seconds. The Nyquist frequency for a digital time series sampled at hourly intervals is 0.5 hr^{-1}. Thus, there is a large range of frequencies which may be present but of which the data does not permit investigation.

The causes of the internal waves with periods near 24, 16, or 12 hours seem obvious. The 24 and 12 hour periods are probably associated with internal waves which are being driven by diurnal or semidiurnal tidal forces. As mentioned previously, the 16 hour period seems to be associated with inertial internal waves. The causes for the internal waves with periods between four and six hours--indicated both in this thesis and in Tabata's study--are not so easily theorized.

Another possible source of internal waves is baroclinic Rossby waves, but this type of wave is associated with extremely low frequencies, with periods on the order of days, so it would seem that these waves will not explain the higher frequency waves indicated by the analysis. Two other possible causes for these waves do exist, however. They might be long-period waves which have been reflected from a coast and have interacted with varying density gradients, which act as a weak filter, but this theory cannot be tested unless one knows the direction of propagation of the higher frequency internal waves.

The other possible source of these waves is a non-linear interaction between surface pressure disturbances, but in order to examine this as a possible cause, one must have records of both surface waves and weather. Although weather records exist for the time during which the bathythermographs were taken, no surface wave information is available. To be of value in investigating this process as a causative agent, the surface wave information would have to be very detailed, and wave records of the required detail can only be obtained in the deep ocean through highly sophisticated and careful measurements which are not compatible with the mission of a weather ship.

In conclusion, it seems that little can be said about these higher-frequency internal waves except that they exist. In order to say more, one would require a great deal more data than is available. Little has been done in the investigation of internal waves to date, and one can see that it represents a fertile subject for further research.

VII. RECOMMENDATIONS

During the execution of this thesis, a number of changes in the format and amount of data were wished. Had these changes been made, the analysis of the data would have been simplified and the accuracy of the results enhanced. For these reasons, the following recommendations are made.

In digitizing mechanical bathythermograph traces, it would greatly assist the user of the digitized data if the data through a thermocline were recorded as depths at equally spaced temperatures, instead of temperatures at equally spaced depths as is now done. This would provide an increased number of points in the region of maximum gradient, enabling the digitized data to be used to more accurately reconstruct the original trace. The method presently used to digitize expendable bathythermograph traces, which records a data point at the location where the trace departs from linearity by a specified amount, allows use of digitized data to almost perfectly reconstruct the trace.

If salinity data were also available for analysis, it would be a simple task to compute depths of isopycnals using a computer. If terms to second order involving salinity and pressure were introduced to the already-known temperature information, depth-density profiles of much greater accuracy would result, and this in turn could be analyzed to produce internal wave information of higher quality.

Closer spacing of observations would enable analysis of higher frequency components, since through aliasing the smallest wave period that can be detected using a sampling interval of one hour is two hours. Also, as was suggested by Tabata, utilizing data presently available over extremely long periods, with t as large as a year, would allow analysis of extremely low frequency internal waves.

As was indicated in the Results section of this thesis, there is a need for information regarding the direction of propagation of the internal waves indicated. This could be gotten using a cross array, consisting of two orthogonal arrays of the type described by Gilchrist [1966] for use in investigating the direction of propagation of surface waves. Several such arrays would be required to allow investigation of a large range of possible wavelengths, but it would seem that the information obtained as a result of such an expenditure of effort and equipment would be a highly significant contribution to the present knowledge of internal waves.

APPENDIX A

EXAMPLE OF COMPUTATION

The following four figures indicate graphically the operation of the subroutines used in computing the depths used in harmonica analysis. The data is from bathythermograph number 392 (Fig. 2).

Figure 15 indicates the operation of subroutine INTERP in fitting straight line segments between raw data points, and the choosing of a temperature corresponding to the center of thermocline (ITC).

Figure 16 shows the operation of subroutine PURGE in eliminating small inversions and duplications of temperatures, reducing the data to a set of temperature-depth pairs from the surface to the bottom of the trace at an interval of $0.1^{\circ}C$. This figure also indicates the operation of subroutines FINDZC and FINDZS in determining the depth of the center of the non-Gaussian thermocline and the depths of the selected isotherms.

Figures 17 and 18 show the thermal structures used in GAUSS1 and GAUSS2 to obtain the depth of the top and bottom of the thermocline, respectively.

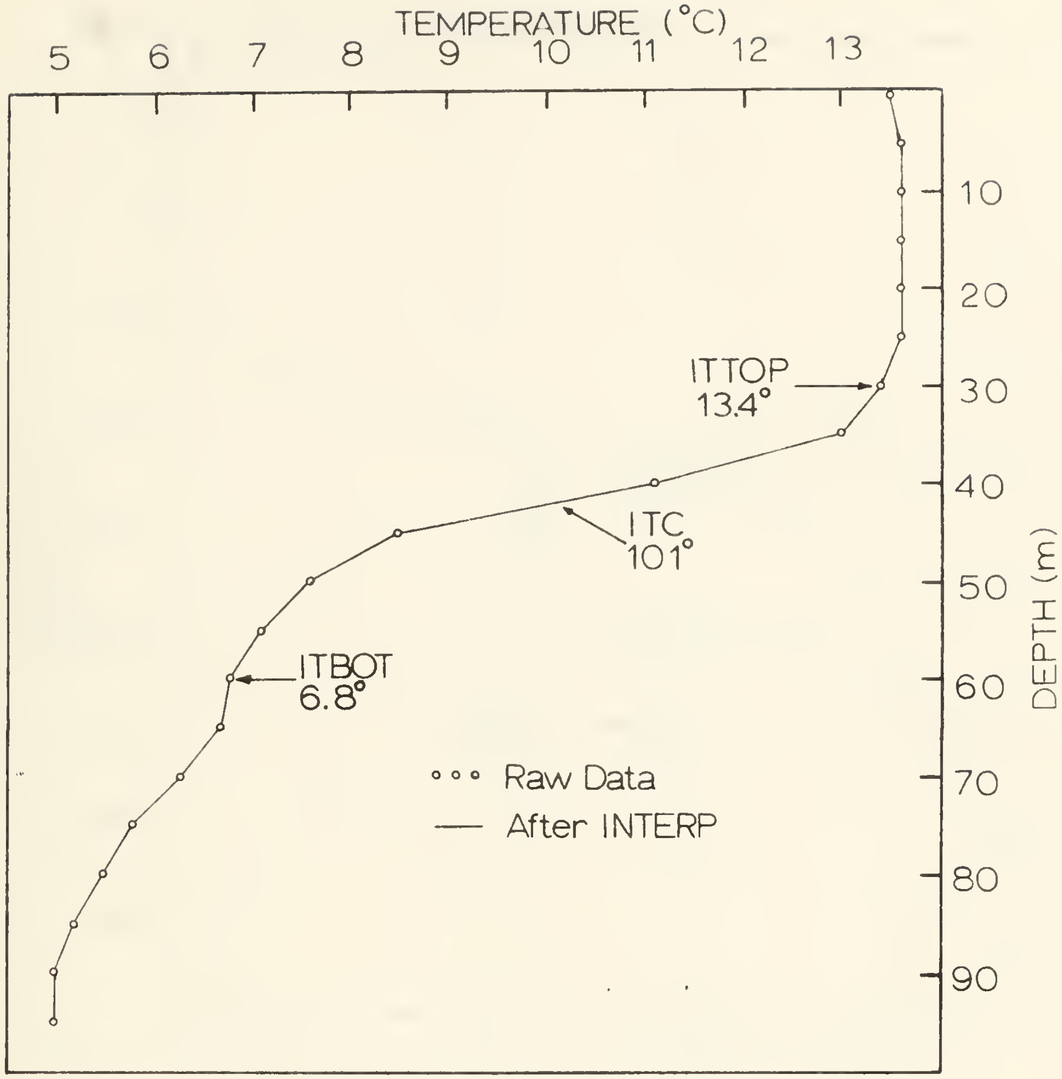

FIGURE 15
Subroutine INTERP

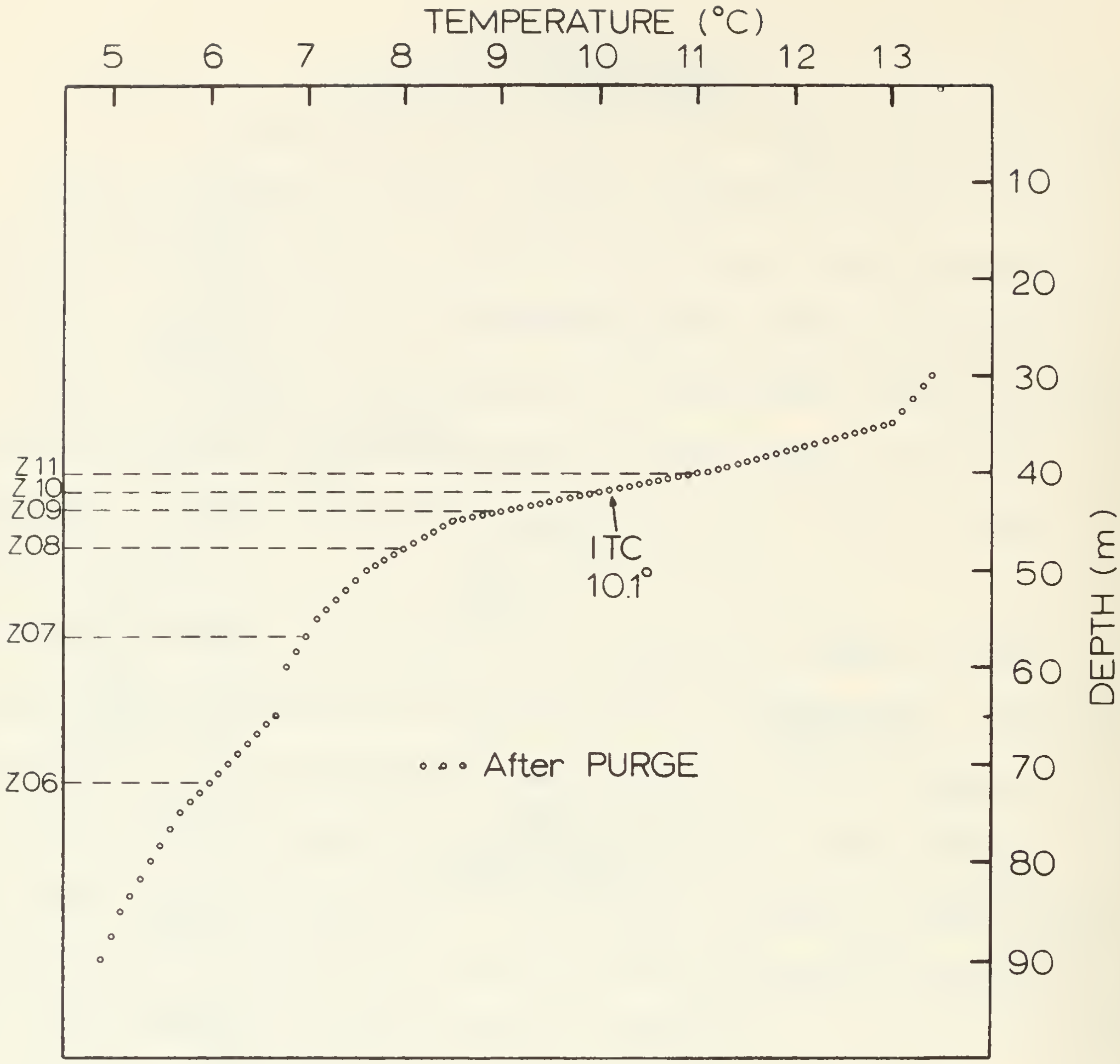

FIGURE 16
Subroutine PURGE,
Subroutine FINDZC,
and
Subroutine FINDZS

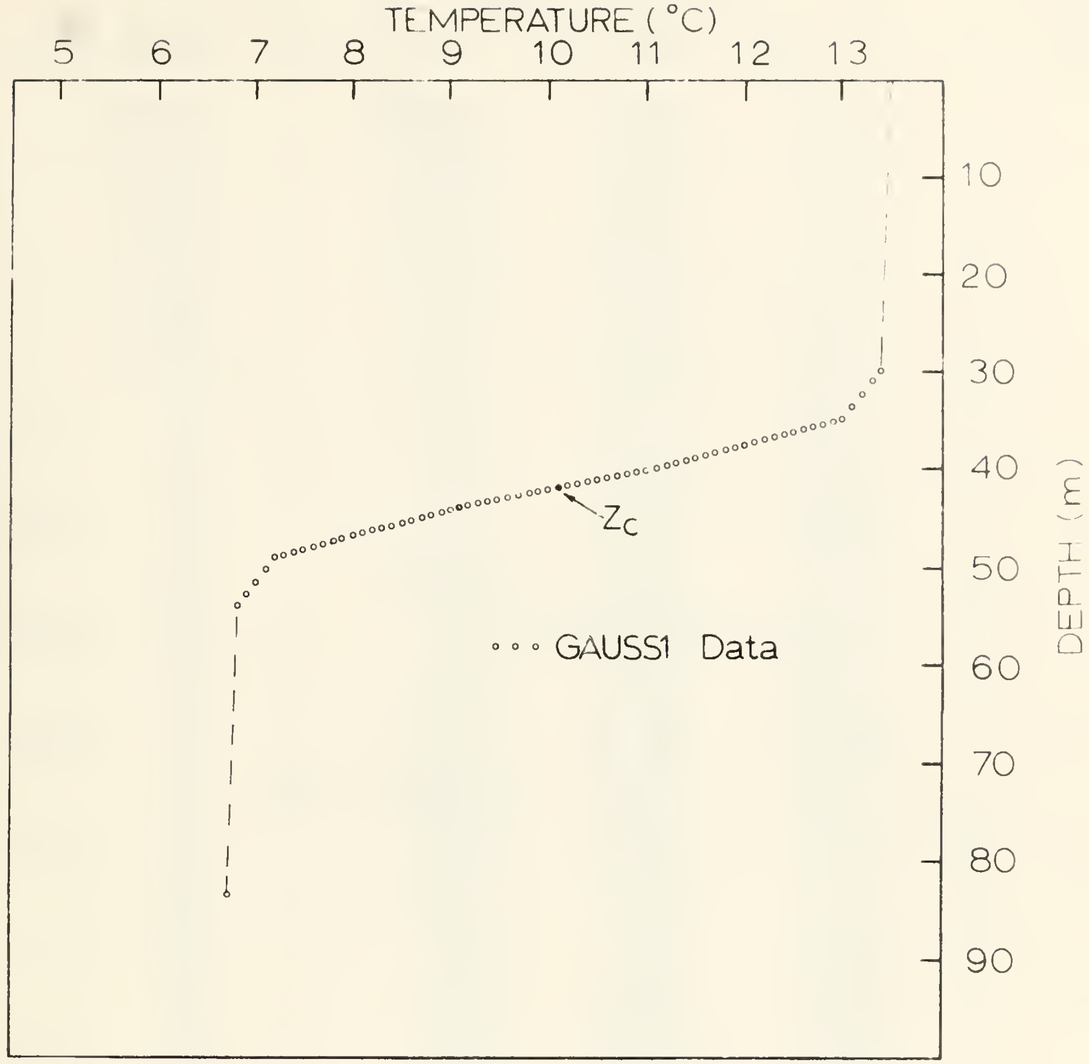

FIGURE 17
Subroutine GAUSS1

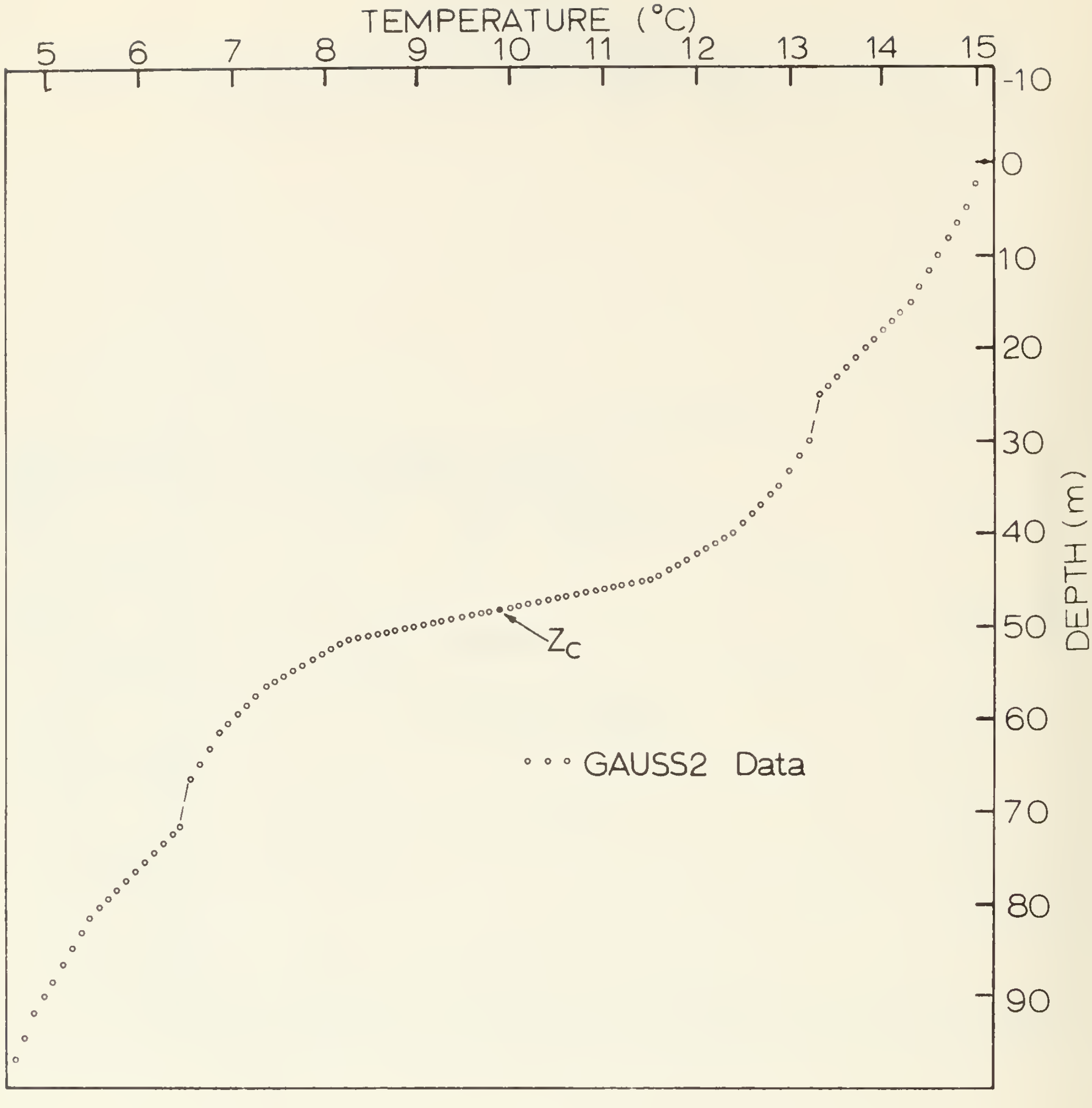

FIGURE 18
Subroutine GAUSS2

SPECTRAL ESTIMATES FOR GROUP NUMBER 1

PERIOD	TOP	CENTER	BOTTOM
256.00	1350.03	174.38	19.16
128.00	1103.64	77.31	44.45
64.00	120.10	95.60	15.27
42.67	94.05	25.38	20.09
32.00	87.53	31.76	9.43
25.60	130.19	56.68	38.14
21.33	102.32	29.97	57.98
18.29	45.52	34.23	49.46
16.00	64.68	87.54	100.22
14.22	48.13	51.23	34.16
12.80	35.91	38.87	45.06
11.64	148.41	161.63	97.09
10.67	46.51	65.00	42.13
9.85	41.05	103.19	40.72
9.14	16.10	32.54	80.42
8.53	20.24	71.41	22.10
8.00	7.53	23.48	36.36
7.53	35.92	44.72	38.08
7.11	33.53	38.47	43.42
6.74	13.78	45.54	4.96
6.40	59.62	24.03	5.42
6.10	13.55	8.35	3.14
5.82	5.89	49.11	28.08
5.57	9.94	23.59	15.05
5.33	19.22	34.63	10.57
5.12	14.99	18.16	4.11
4.92	10.94	21.24	16.40
4.74	34.88	45.15	21.99
4.57	23.18	26.11	53.53
4.41	31.61	59.61	40.11
4.27	8.10	14.50	7.97
4.13	11.68	21.83	28.66
4.00	17.20	27.56	12.42
3.88	12.50	17.87	24.02
3.76	12.42	10.35	29.74
3.66	7.75	7.99	17.65
3.56	5.35	16.90	24.79
3.46	11.12	18.88	46.63
3.37	4.46	18.30	7.61
3.28	5.54	10.48	7.70
3.20	16.05	24.40	21.84
3.12	2.29	8.35	31.32
3.05	28.51	7.34	25.94
2.98	21.62	17.88	3.45
2.91	8.61	20.58	11.73
2.84	17.57	6.42	15.31
2.78	6.80	18.75	26.55
2.72	14.41	15.26	4.88
2.67	8.17	14.66	7.67
2.61	7.28	12.53	2.65
2.56	11.15	18.91	6.31
2.51	10.31	8.33	8.67
2.46	14.05	7.03	11.61
2.42	7.60	14.77	13.75
2.37	3.15	1.97	11.27
2.33	2.40	7.55	8.55
2.29	5.89	16.59	20.14
2.25	13.70	14.67	9.19
2.21	7.39	13.26	8.39
2.17	9.85	11.59	12.24
2.13	16.17	10.16	8.70
2.10	11.41	17.28	19.91
2.06	1.65	7.11	11.97
2.03	2.14	6.99	12.82
1.99	2.64	1.63	25.07

SPECTRAL ESTIMATES FOR GROUP NUMBER 1

PERIOD	11 DEG	10 DEG	9 DEG
256.00	273.66	261.62	210.58
128.00	63.83	71.16	91.99
64.00	81.13	98.19	113.84
42.67	15.49	18.52	23.32
32.00	27.11	29.28	31.17
25.60	65.45	64.64	61.99
21.33	29.29	25.96	21.30
18.29	45.27	41.54	37.91
16.00	98.00	93.40	98.91
14.22	54.98	55.67	58.88
12.80	36.86	39.91	42.85
11.64	169.94	161.12	156.58
10.67	59.76	58.46	60.27
9.85	86.45	89.74	93.88
9.14	31.59	31.23	28.92
8.53	66.20	65.11	66.38
8.00	26.99	22.94	21.40
7.53	43.19	41.73	39.99
7.11	35.12	31.61	29.13
6.74	52.44	54.88	57.44
6.40	19.93	20.19	18.72
6.10	10.37	10.10	8.34
5.82	48.47	47.84	42.58
5.57	19.72	19.67	17.47
5.33	21.22	24.82	29.18
5.12	13.16	13.86	17.41
4.92	25.67	26.79	29.47
4.74	39.33	37.50	32.47
4.57	16.03	18.59	19.03
4.41	47.07	51.14	51.25
4.27	13.40	13.43	13.02
4.13	25.57	24.53	22.72
4.00	22.95	24.16	24.12
3.88	15.96	16.30	15.02
3.76	9.59	10.21	10.61
3.66	11.07	10.66	10.43
3.56	17.23	18.32	20.62
3.46	17.87	20.30	19.49
3.37	22.92	23.19	17.89
3.28	12.01	11.58	8.25
3.20	15.66	16.52	14.68
3.12	5.09	5.03	4.38
3.05	5.86	4.72	5.54
2.98	17.88	18.66	19.78
2.91	20.32	19.72	17.66
2.84	9.69	8.37	8.54
2.78	15.79	16.80	15.37
2.72	13.84	13.34	15.93
2.67	11.56	11.36	10.40
2.61	14.74	13.09	13.27
2.56	13.28	15.94	20.01
2.51	8.26	6.56	6.28
2.46	12.12	10.14	9.14
2.42	16.88	15.52	15.60
2.37	3.01	2.62	2.34
2.33	8.68	7.41	6.27
2.29	7.54	9.13	11.88
2.25	10.11	11.74	15.26
2.21	14.16	14.11	13.87
2.17	7.54	8.23	11.24
2.13	5.75	5.88	6.59
2.10	11.08	11.46	12.12
2.06	2.92	4.38	6.34
2.03	4.55	5.33	4.95
1.99	2.04	2.24	4.11

SPECTRAL ESTIMATES FOR GROUP NUMBER 1

PERIOD	8 DEG	7 DEG	6 DEG
256.00	164.78	179.46	235.34
128.00	129.29	224.85	270.58
64.00	115.01	162.44	172.24
42.67	34.11	47.31	104.09
32.00	43.68	54.21	63.55
25.60	62.56	81.88	79.20
21.33	6.17	58.53	47.90
18.29	29.03	106.05	78.58
16.00	107.04	99.85	113.75
14.22	77.82	71.47	64.55
12.80	51.32	85.02	131.76
11.64	150.26	162.63	168.32
10.67	70.64	81.96	60.41
9.85	84.23	92.23	60.90
9.14	27.83	58.51	49.41
8.53	77.20	50.88	21.92
8.00	28.06	41.53	75.20
7.53	35.01	47.83	80.99
7.11	24.00	39.33	86.15
6.74	57.43	42.73	28.65
6.40	21.65	12.21	19.81
6.10	5.00	7.59	14.34
5.82	61.39	103.22	94.61
5.57	17.39	35.84	37.59
5.33	17.96	38.76	61.74
5.12	22.29	9.60	6.62
4.92	35.10	41.82	30.37
4.74	30.51	18.39	24.97
4.57	13.55	29.50	46.12
4.41	53.24	40.96	49.66
4.27	12.45	17.95	31.90
4.13	25.96	42.56	63.11
4.00	23.16	28.28	41.16
3.88	10.73	14.97	28.30
3.76	10.97	32.88	45.66
3.66	17.16	14.16	41.44
3.56	20.07	31.28	94.63
3.46	42.61	86.33	101.53
3.37	20.63	10.14	13.16
3.28	6.05	22.31	6.96
3.20	12.41	15.48	34.15
3.12	4.19	28.65	26.31
3.05	9.05	22.92	27.07
2.98	19.21	10.70	11.19
2.91	22.03	26.70	20.16
2.84	11.30	23.64	68.37
2.78	13.07	18.31	12.14
2.72	20.97	26.19	24.85
2.67	8.59	18.77	18.18
2.61	15.35	15.45	8.85
2.56	18.28	18.59	8.82
2.51	11.36	20.89	23.45
2.46	3.21	19.45	24.60
2.42	18.38	31.24	47.12
2.37	3.90	11.29	40.13
2.33	5.74	15.77	8.63
2.29	12.89	13.49	15.45
2.25	20.05	27.29	27.04
2.21	13.89	13.72	23.94
2.17	14.80	33.53	42.12
2.13	6.94	18.84	49.99
2.10	17.86	14.31	6.35
2.06	14.26	37.02	13.11
2.03	6.93	11.23	3.57
1.99	5.42	28.84	15.93

SPECTRAL ESTIMATES FCR GROUP NUMBER 2

PERICD	TCP	CENTER	BOTTOM
256.0C	1873.16	52.36	29.24
128.00	1230.09	34.40	65.41
64.00	62.73	71.81	39.17
42.67	35.34	33.78	19.51
32.00	7.52	9.03	20.11
25.60	90.39	44.97	9.19
21.33	59.56	27.46	39.47
18.29	43.80	33.56	15.10
16.0C	15.79	41.30	41.12
14.22	4.08	9.56	10.88
12.8C	24.20	14.31	22.51
11.64	44.52	58.22	39.16
10.67	20.60	28.23	14.35
9.85	45.76	83.83	31.85
9.14	39.83	78.72	94.26
8.53	12.49	44.50	30.68
8.00	18.52	36.38	42.39
7.53	24.04	43.13	22.91
7.11	31.31	26.27	48.27
6.74	10.81	21.69	18.50
6.4C	34.62	26.15	4.79
6.10	8.79	5.32	4.89
5.82	17.31	61.58	19.61
5.57	5.10	13.72	8.98
5.33	20.25	27.41	24.39
5.12	7.43	28.91	16.09
4.92	18.44	24.20	13.47
4.74	17.09	43.48	26.44
4.57	22.07	22.05	34.15
4.41	12.50	25.19	12.74
4.27	4.51	16.64	12.88
4.13	4.02	6.62	12.C8
4.00	8.21	20.35	35.31
3.88	9.86	13.39	34.24
3.76	8.19	18.1C	4.96
3.66	3.97	5.08	30.45
3.56	3.49	8.71	23.32
3.46	9.54	39.34	40.22
3.37	2.46	8.65	8.75
3.28	8.32	6.48	8.51
3.20	7.52	13.95	11.41
3.12	10.27	7.18	16.97
3.05	18.60	16.51	13.C8
2.98	9.50	9.84	18.43
2.91	5.25	5.91	5.05
2.84	5.94	10.51	9.47
2.78	8.10	15.46	18.04
2.72	6.22	9.52	15.05
2.67	12.63	9.96	9.56
2.61	2.99	9.36	4.56
2.56	7.37	6.54	8.84
2.51	13.50	12.56	36.85
2.46	17.12	13.96	12.55
2.42	6.24	12.82	22.43
2.37	4.75	13.57	7.89
2.33	1.19	20.42	35.32
2.29	7.40	14.15	7.85
2.25	4.74	9.45	4.69
2.21	8.02	8.11	12.78
2.17	9.94	7.15	1.35
2.13	15.54	7.77	11.75
2.10	18.48	14.44	11.56
2.06	0.76	4.66	13.29
2.03	0.82	4.41	5.74
1.99	3.93	3.27	28.37

SPECTRAL ESTIMATES FOR GROUP NUMBER 2

PERIOD	11 DEG	10 DEG	9 DEG
256.00	52.16	69.90	108.77
128.00	26.01	31.23	38.58
64.00	54.40	66.58	75.30
42.67	27.93	32.99	36.67
32.00	12.73	13.47	14.18
25.60	47.71	44.67	42.49
21.33	21.50	21.45	25.32
18.29	35.28	32.62	24.70
16.00	40.05	45.20	54.94
14.22	8.69	13.78	18.46
12.80	13.88	12.51	8.94
11.64	60.50	53.13	52.29
10.67	31.83	29.07	30.83
9.85	74.73	75.01	68.06
9.14	62.45	62.82	55.57
8.53	46.58	43.35	40.45
8.00	36.11	33.86	33.10
7.53	35.29	35.05	32.63
7.11	31.00	26.05	24.60
6.74	18.77	20.93	22.63
6.40	29.29	28.22	22.97
6.10	3.03	4.18	3.45
5.82	57.95	55.82	51.14
5.57	10.39	12.71	14.96
5.33	13.85	16.17	18.22
5.12	28.18	27.08	34.45
4.92	28.18	28.90	28.48
4.74	45.51	49.04	41.54
4.57	9.48	11.09	13.60
4.41	24.08	25.39	24.76
4.27	12.55	14.16	15.29
4.13	10.37	11.26	10.01
4.00	22.87	21.64	17.14
3.88	15.23	13.69	11.97
3.76	18.28	19.52	20.96
3.66	8.13	7.64	5.53
3.56	9.43	10.50	13.65
3.46	19.05	24.26	32.10
3.37	9.19	8.84	9.03
3.28	8.85	7.80	8.40
3.20	14.66	15.48	14.16
3.12	8.10	7.28	5.28
3.05	14.92	17.01	16.74
2.98	9.45	7.67	8.27
2.91	11.37	9.91	11.06
2.84	6.44	7.14	10.95
2.78	13.31	13.44	12.51
2.72	9.73	9.70	9.37
2.67	9.18	8.74	9.78
2.61	12.80	14.21	16.39
2.56	2.42	4.57	8.24
2.51	8.99	9.65	14.19
2.46	8.45	9.08	11.27
2.42	11.11	12.28	13.77
2.37	5.68	8.81	9.51
2.33	13.06	15.17	16.70
2.29	9.79	10.11	11.41
2.25	3.36	3.40	7.66
2.21	13.41	13.17	13.00
2.17	2.21	2.81	6.88
2.13	4.79	4.42	5.49
2.10	12.13	12.21	9.89
2.06	7.11	8.91	9.91
2.03	4.67	4.57	4.76
1.99	4.76	4.45	3.54

SPECTRAL ESTIMATES FOR GROUP NUMBER 2

PERIOD	8 DEG	7 DEG	6 DEG
256.00	194.69	213.73	173.83
128.00	57.58	129.74	147.77
64.00	76.76	133.56	193.10
42.67	55.49	100.73	79.06
32.00	21.23	76.60	65.44
25.60	42.05	55.35	18.11
21.33	21.95	36.54	40.63
18.29	11.65	30.71	25.92
16.00	46.29	66.71	135.36
14.22	18.27	19.70	42.29
12.80	4.94	6.52	50.43
11.64	48.00	53.58	48.18
10.67	38.98	49.28	38.86
9.85	63.58	68.84	55.95
9.14	55.29	80.36	57.35
8.53	39.34	31.65	7.71
8.00	33.78	43.01	60.06
7.53	30.41	39.84	28.44
7.11	17.66	54.21	116.32
6.74	23.74	23.54	15.47
6.40	20.10	21.76	36.51
6.10	9.99	6.70	4.73
5.82	71.99	61.25	52.81
5.57	15.34	36.78	23.22
5.33	9.67	6.59	17.98
5.12	34.92	35.62	21.27
4.92	30.63	28.13	28.39
4.74	39.80	23.08	32.17
4.57	12.48	47.87	64.83
4.41	21.42	65.73	38.23
4.27	15.05	26.20	10.77
4.13	6.34	18.78	14.18
4.00	15.61	14.13	20.43
3.88	12.49	15.41	23.33
3.76	14.23	30.86	51.93
3.66	6.18	16.89	18.91
3.56	13.21	33.64	70.53
3.46	33.28	42.80	33.14
3.37	11.39	11.50	49.13
3.28	14.58	15.97	19.08
3.20	9.39	21.56	20.77
3.12	9.07	25.88	15.84
3.05	15.49	22.13	19.59
2.98	10.36	6.02	9.61
2.91	11.80	28.53	43.65
2.84	7.61	15.23	18.90
2.78	8.62	10.36	20.08
2.72	11.20	29.23	23.79
2.67	18.19	16.42	13.42
2.61	11.65	32.13	48.01
2.56	7.39	10.89	22.35
2.51	17.48	22.07	56.28
2.46	10.91	28.46	45.81
2.42	21.95	48.08	41.00
2.37	8.38	7.11	21.15
2.33	18.11	35.27	19.77
2.29	12.87	10.35	17.23
2.25	17.15	9.62	11.68
2.21	19.65	12.33	22.57
2.17	7.90	18.77	15.58
2.13	7.50	10.18	35.26
2.10	8.71	4.04	12.44
2.06	9.69	14.64	8.94
2.03	5.92	9.51	3.79
1.99	3.34	19.49	14.29

AVERAGED SPECTRAL ESTIMATES

NR	PERIOD	TOP	CENTER	BOTTOM
0.5	256.00	1611.60	113.37	24.20
1.0	128.00	1166.87	55.85	54.93
2.0	64.00	91.42	83.71	27.22
3.0	42.67	64.69	29.58	19.80
4.0	32.00	47.53	20.40	14.77
5.0	25.60	110.29	50.83	23.66
6.0	21.33	80.94	28.71	48.73
7.0	18.29	44.66	33.89	32.28
8.0	16.00	40.23	64.42	70.67
9.0	14.22	26.10	30.40	22.52
10.0	12.80	30.06	26.59	33.78
11.0	11.64	96.46	109.93	68.12
12.0	10.67	33.56	46.61	28.24
13.0	9.85	43.40	93.51	36.28
14.0	9.14	27.96	55.63	87.34
15.0	8.53	16.37	57.95	26.39
16.0	8.00	13.02	29.93	39.37
17.0	7.53	29.98	43.93	30.50
18.0	7.11	32.42	32.37	45.84
19.0	6.74	12.29	33.62	11.73
20.0	6.40	47.12	25.09	5.10
21.0	6.10	11.17	6.83	4.01
22.0	5.82	11.60	55.35	23.84
23.0	5.57	7.52	18.66	12.02
24.0	5.33	19.73	31.02	17.48
25.0	5.12	11.21	23.54	10.10
26.0	4.92	14.69	22.72	14.94
27.0	4.74	25.99	44.32	24.21
28.0	4.57	22.62	24.08	43.84
29.0	4.41	22.06	42.40	26.43
30.0	4.27	6.30	15.57	10.43
31.0	4.13	7.85	14.23	20.37
32.0	4.00	12.71	23.95	23.87
33.0	3.88	11.18	15.63	29.13
34.0	3.76	10.30	14.22	17.35
35.0	3.66	5.86	6.54	24.05
36.0	3.56	4.42	12.81	24.06
37.0	3.46	10.33	29.11	43.42
38.0	3.37	3.46	13.47	8.18
39.0	3.28	6.93	8.48	8.11
40.0	3.20	11.78	19.17	16.63
41.0	3.12	6.28	7.76	24.15
42.0	3.05	23.56	11.92	19.51
43.0	2.98	15.56	13.86	10.94
44.0	2.91	6.93	13.24	8.39
45.0	2.84	11.75	8.46	12.39
46.0	2.78	7.45	17.10	22.30
47.0	2.72	10.31	12.39	9.96
48.0	2.67	10.40	12.31	8.61
49.0	2.61	5.14	10.94	3.61
50.0	2.56	9.26	12.73	7.58
51.0	2.51	11.91	10.45	22.76
52.0	2.46	15.58	10.50	12.08
53.0	2.42	6.92	13.79	18.09
54.0	2.37	3.95	7.77	9.58
55.0	2.33	1.79	13.99	21.94
56.0	2.29	6.64	15.37	14.00
57.0	2.25	9.22	12.06	6.94
58.0	2.21	7.71	10.69	10.59
59.0	2.17	9.89	9.37	6.80
60.0	2.13	15.85	8.97	10.22
61.0	2.10	14.94	15.86	15.74
62.0	2.06	1.21	5.88	12.63
63.0	2.03	1.48	5.70	9.28
64.3	1.99	3.29	2.45	26.72

AVERAGED SPECTRAL ESTIMATES

NR	PERIOD	11 DEG	10 DEG	9 DEG
0.5	256.00	162.91	165.76	159.68
1.0	128.00	44.92	51.19	65.29
2.0	64.00	67.77	82.38	94.57
3.0	42.67	21.71	25.75	29.99
4.0	32.00	19.92	21.37	22.67
5.0	25.60	56.58	54.65	52.24
6.0	21.33	25.39	23.70	23.31
7.0	18.29	40.27	37.08	31.31
8.0	16.00	69.02	69.30	76.93
9.0	14.22	31.83	34.73	38.67
10.0	12.80	25.37	26.21	25.89
11.0	11.64	115.22	107.12	104.43
12.0	10.67	45.80	43.77	45.55
13.0	9.85	80.59	82.38	80.97
14.0	9.14	47.02	47.02	42.24
15.0	8.53	56.39	54.23	53.42
16.0	8.00	31.55	28.40	27.25
17.0	7.53	39.24	38.39	36.31
18.0	7.11	33.06	28.83	26.87
19.0	6.74	35.61	37.90	40.03
20.0	6.40	24.61	24.21	20.84
21.0	6.10	6.70	7.14	5.89
22.0	5.82	53.21	51.83	46.86
23.0	5.57	15.05	16.19	16.22
24.0	5.33	17.53	20.50	23.70
25.0	5.12	20.67	20.47	25.93
26.0	4.92	26.93	27.85	28.97
27.0	4.74	42.42	43.27	37.00
28.0	4.57	12.75	14.84	16.32
29.0	4.41	35.58	38.27	38.00
30.0	4.27	12.98	13.80	14.15
31.0	4.13	17.97	17.89	16.36
32.0	4.00	22.91	22.90	20.63
33.0	3.88	15.59	14.99	13.49
34.0	3.76	13.94	14.86	15.79
35.0	3.66	9.60	9.15	7.98
36.0	3.56	13.33	14.41	17.13
37.0	3.46	18.46	22.28	25.80
38.0	3.37	16.06	16.02	13.46
39.0	3.28	10.43	9.69	8.32
40.0	3.20	15.16	16.00	14.42
41.0	3.12	6.59	6.16	4.83
42.0	3.05	10.39	10.86	11.14
43.0	2.98	13.66	13.17	14.02
44.0	2.91	15.85	14.82	14.36
45.0	2.84	8.07	7.76	9.75
46.0	2.78	14.55	15.12	13.94
47.0	2.72	11.78	11.52	12.65
48.0	2.67	10.37	10.05	10.09
49.0	2.61	13.77	13.65	14.83
50.0	2.56	7.85	10.26	14.12
51.0	2.51	8.62	8.10	10.24
52.0	2.46	10.28	9.61	10.20
53.0	2.42	13.99	13.90	14.68
54.0	2.37	4.35	5.72	5.93
55.0	2.33	10.87	11.29	11.48
56.0	2.29	8.66	9.62	11.64
57.0	2.25	6.74	7.57	11.46
58.0	2.21	13.78	13.64	13.43
59.0	2.17	4.88	5.52	9.06
60.0	2.13	5.27	5.15	6.04
61.0	2.10	11.61	11.83	11.01
62.0	2.06	5.02	6.65	8.12
63.0	2.03	4.61	4.95	4.85
64.0	1.99	3.40	3.35	3.83

AVERAGED SPECTRAL ESTIMATES

NR	PERIOD	8 DEG	7 DEG	6 DEG
0.5	256.00	179.73	196.59	204.59
1.0	128.00	93.43	177.30	209.18
2.0	64.00	95.89	148.00	182.67
3.0	42.67	44.80	74.02	91.57
4.0	32.00	32.45	65.40	64.49
5.0	25.60	52.30	68.61	48.65
6.0	21.33	14.06	47.53	44.27
7.0	18.29	20.34	68.38	52.25
8.0	16.00	76.66	83.28	124.55
9.0	14.22	48.05	45.58	53.42
10.0	12.80	28.13	45.77	91.10
11.0	11.64	99.13	108.10	108.25
12.0	10.67	54.81	65.62	49.64
13.0	9.85	73.90	80.53	58.43
14.0	9.14	41.56	69.44	53.38
15.0	8.53	58.27	41.26	14.81
16.0	8.00	30.92	42.27	67.63
17.0	7.53	32.71	43.83	54.71
18.0	7.11	20.83	46.77	101.24
19.0	6.74	40.58	33.14	22.06
20.0	6.40	20.88	16.98	28.16
21.0	6.10	7.49	7.14	9.53
22.0	5.82	66.69	82.23	73.71
23.0	5.57	16.37	36.31	30.40
24.0	5.33	13.82	22.67	39.86
25.0	5.12	28.61	22.61	13.95
26.0	4.92	32.86	34.97	29.38
27.0	4.74	35.15	20.73	28.57
28.0	4.57	13.02	38.68	55.47
29.0	4.41	37.33	53.35	43.94
30.0	4.27	13.75	22.07	21.33
31.0	4.13	16.15	30.67	38.65
32.0	4.00	19.39	21.20	30.80
33.0	3.88	11.61	15.19	25.82
34.0	3.76	12.60	31.87	48.79
35.0	3.66	11.67	15.52	30.17
36.0	3.56	16.64	32.46	82.58
37.0	3.46	37.94	64.57	67.34
38.0	3.37	16.01	10.82	31.14
39.0	3.28	10.31	19.14	13.02
40.0	3.20	10.90	18.52	27.46
41.0	3.12	6.63	27.26	21.08
42.0	3.05	12.27	22.53	23.33
43.0	2.98	14.78	8.36	10.40
44.0	2.91	16.91	27.62	31.91
45.0	2.84	9.46	19.44	43.64
46.0	2.78	10.85	14.34	16.11
47.0	2.72	16.09	27.71	24.32
48.0	2.67	13.39	17.59	15.80
49.0	2.61	13.50	23.79	28.43
50.0	2.56	12.83	14.74	15.59
51.0	2.51	14.42	21.48	39.87
52.0	2.46	7.06	23.96	35.20
53.0	2.42	20.16	39.66	44.06
54.0	2.37	6.14	9.20	30.64
55.0	2.33	11.93	25.52	14.20
56.0	2.29	12.88	11.92	16.34
57.0	2.25	18.60	18.45	19.36
58.0	2.21	16.77	13.02	23.25
59.0	2.17	11.35	26.15	28.85
60.0	2.13	7.22	14.51	42.62
61.0	2.10	13.29	9.18	9.39
62.0	2.06	11.97	25.83	11.03
63.0	2.03	6.42	10.37	3.68
64.3	1.99	4.38	24.17	15.11

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C     MAIN PROGRAM
C     THIS PROGRAM READS TEMPERATURES AT EVEN DEPTH INCREMENTS
C     OF FIVE METERS, CALLS SUBROUTINES TO FIND DEPTHS OF THE
C     TOP, CENTER, AND BOTTOM OF THE THERMOCLINE, AND THE
C     DEPTH OF SIX SELECTED ISOTHERMS.  THESE DEPTHS ARE SUB-
C     JECTED TO A FAST FOURIER ANALYSIS TO DETERMINE THEIR
C     FOURIER COEFFICIENTS, WHICH ARE THEN HANNED, SMOOTHED IN
C     FREQUENCY BANDS, AND STORED FOR EACH GROUP OF 2**N DATA
C     CARDS.  THE PROGRAM MAY ALSO BE USED TO PRODUCE GRAPHIC
C     DISPLAYS OF FLUCTUATIONS OF DEPTHS WITH TIME, AND THE
C     FINAL AVERAGED POWER SPECTRA.
C
C     VARIABLES USED IN THIS PROGRAM ARE--
C       IGROUP--THE NUMBER OF GROUPS OF 2**N DATA CARDS USED
C       IN EACH ANALYSIS
C
C       IT--THE ARRAY OF TEMPERATURES IN EACH TRACE AS READ
C       FROM DATA CARDS
C
C       Z11,Z10,Z09,Z08,Z07,Z06--THE DEPTHS OF THE SIX
C       SELECTED ISOTHERMS, AS CHOSEN IN SUBROUTINE INTERP
C
C       KN--THE SUBSCRIPT OF THE DEPTH CHOSEN AS THE INITIAL
C       APPROXIMATION OF THE CENTER OF THE THERMOCLINE
C
C       ZT--THE DEPTH OF THE TOP OF THE THERMOCLINE
C
C       ZC--THE DEPTH OF THE CENTER OF THE THERMOCLINE
C
C       ZBT--THE DEPTH OF THE BOTTOM OF THE THERMOCLINE
C
C       J--USED AS THE SUBSCRIPT OF THE DEPTHS OBTAINED FOR
C       EACH DATA CARD
C
C       M--THE GROUPING NUMBER, A MEASURE OF THE NUMBER OF
C       INDIVIDUAL FOURIER COEFFICIENTS INCLUDED IN EACH
C       SPECTRAL BANDWIDTH.  NOTE THAT M MUST BE SPECIFIED IN
C       THE MAIN PROGRAM
C
C       AJ,BJ,ZJ--COUNTERS ASSOCIATED WITH THE CONSECUTIVE
C       NUMBER OF THE SPECTRAL DENSITIES, FOR USE IN GRAPHICAL
C       OUTPUT
C
C       END11,END10,ENDT,ENDC,ETC.--FINAL SPECTRAL ESTIMATES
C       USED TO OBTAIN AN AVERAGE POWER SPECTRUM
C
C     SUBROUTINES CALLED FROM MAIN PROGRAM ARE
C       INTERP
C       NONGA
C       AVERGE
C       HANN
C       SMOOTH
C       PERIOD
C       GROUP
C       RHARM--FOR DETAILS CONCERNING SUBROUTINE RHARM SEE THE
C       DOCUMENTATION FOR THE IBM SCIENTIFIC SUBROUTINE
C       LIBRARY
C
        DIMENSION IT(200),TX(200),X(200),APER(200),AJ(200),
     *ZJ(200),BJ(200),Z11(600),Z10(600),Z09(600),Z08(600),
     *Z07(600),Z06(600),B11(600),B10(600),B09(600),B08(600),
     *BC7(600),B06(600),ZT(600),ZC(600),ZBT(600),AZT(600),
     *AZC(600),AZBT(600),AZIN(600),AZOUT(600),ZIN(600),
     *BOUT(600),ARGIN(600),ARGOUT(600),INV(64),S(64),
     *ENDT(200),ENDC(200),ENDB(200),END11(200),END10(200),
     *END09(200),END08(200),END07(200),END06(200),T(200),
     *Z(200)
        REAL LABL1/' 11 '/,LABL2/' 10 '/,LABL3/'  9 '/,
     *LABL4/'  8 '/,LABL5/'  7 '/,LABL6/'  6 '/,LABL7/' TOP'
     */,LABL8/'MIDL'/,LABL9/'BOTM'/
        REAL*8 ITI1(12)/'BOX 80D9  AVERAGED POWER SPECTRUM OF
     *FOURIER    COEFFICIENTS DATA INPUT HAS MEAN OF ZERO H,
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     *S4,ZT'/
      REAL*8 ITI2(12)/'BOX 80D9  AVERAGED POWER SPECTRUM OF
     *FOURIER    COEFFICIENTS DATA INPUT HAS MEAN OF ZERO H,
     *S4,ZC'/
      REAL*8 ITI3(12)/'BOX 80D9  AVERAGED POWER SPECTRUM OF
     *FOURIER    COEFFICIENTS DATA INPUT HAS MEAN OF ZERO H,
     *S4,ZB'/
      REAL*8 ITITL1(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE 11 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      REAL*8 ITITL2(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE 10 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      REAL*8 ITITL3(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE  9 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      REAL*8 ITITL4(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE  8 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      REAL*8 ITITL5(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE  7 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      REAL*8 ITITL6(12)/'BOX 80  FOURIER ANALYSIS OF FLUCTUA
     *TIONS OF THE  6 DEGREE CENTIGRADE ISOTHERM     RHARM A
     *NALYSIS'/
      DO 3 IGROUP=1,2
      DO 1 J=1,512
      READ(5,2) (IT(I), I=1,20)
    2 FORMAT (13X,20I3)
      CALL INTERP(IT,20,T,Z,ICOUNT,KN,Z11,Z10,Z09,Z08,Z07,
     *Z06,J)
      CALL NONGA(Z,T,ICOUNT,ZT,ZC,ZBT,KN,J)
    1 CONTINUE
      CALL AVERGE (ZT,AZT)
      CALL AVERGE (ZC,AZC)
      CALL AVERGE (ZBT,AZBT)
      CALL AVERGE (Z11,B11)
      CALL AVERGE (Z10,B10)
      CALL AVERGE (Z09,B09)
      CALL AVERGE (Z08,B08)
      CALL AVERGE (Z07,B07)
      CALL AVERGE (Z06,B06)
      CALL RHARM(AZT,8,INV,S,IFERR)
      CALL RHARM(AZC,8,INV,S,IFERR)
      CALL RHARM(AZBT,8,INV,S,IFERR)
      CALL RHARM(B11,8,INV,S,IFERR)
      CALL RHARM(B10,8,INV,S,IFERR)
      CALL RHARM(B09,8,INV,S,IFERR)
      CALL RHARM(B08,8,INV,S,IFERR)
      CALL RHARM(B07,8,INV,S,IFERR)
      CALL RHARM(B06,8,INV,S,IFERR)
      CALL HANN (AZT,ZT)
      CALL HANN (AZC,ZC)
      CALL HANN(AZBT,ZBT)
      CALL HANN (B11,Z11)
      CALL HANN (B10,Z10)
      CALL HANN (B09,Z09)
      CALL HANN (B08,Z08)
      CALL HANN (B07,Z07)
      CALL HANN (B06,Z06)
      M=4
      CALL SMOOTH  (ZT,M,ITEST,AZT,AJ,IPTS)
      CALL SMOOTH  (ZC,M,ITEST,AZC,AJ,IPTS)
      CALL SMOOTH (ZBT,M,ITEST,AZBT,AJ,IPTS)
      CALL SMOOTH(Z11,M,ITEST,B11,AJ,IPTS)
      CALL SMOOTH(Z10,M,ITEST,B10,AJ,IPTS)
      CALL SMOOTH(Z09,M,ITEST,B09,AJ,IPTS)
      CALL SMOOTH(Z08,M,ITEST,B08,AJ,IPTS)
      CALL SMOOTH(Z07,M,ITEST,B07,AJ,IPTS)
      CALL SMOOTH(Z06,M,ITEST,B06,AJ,IPTS)
      CALL PERIOD(M,ITEST,APER,AJ,BJ)
      WRITE(6,30)
```

```
   30 FORMAT(1H1)
      WRITE(6,6) IGROUP
    6 FORMAT(//32X,'SPECTRAL ESTIMATES FOR GROUP NUMBER ',
     *I1,//)
      WRITE(6,7)
    7 FORMAT(30X,'PERIOD',10X,'TOP',4X,'CENTER',4X,
     *'BOTTOM',/)
      DO 8 I=1,IPTS
      WRITE(6,9) APER(I),AZT(I),AZC(I),AZBT(I)
    9 FORMAT(30X,F6.2,4X,3F10.2)
    8 CONTINUE
      WRITE(6,30)
      WRITE(6,6) IGROUP
      WRITE(6,10)
   10 FORMAT(30X,'PERIOD',7X,'11 DEG',4X,'10 DEG',5X,'9 DEG'
     *,/)
      DO 11 I=1,IPTS
      WRITE(6,9)  APER(I),B11(I),B10(I),B09(I)
   11 CONTINUE
      WRITE(6,30)
      WRITE(6,6) IGROUP
      WRITE(6,13)
   13 FORMAT(30X,'PERIOD',8X,'8 DEG',5X,'7 DEG',5X,'6 DEG',
     */)
      DC 14 I=1,IPTS
   14 WRITE(6,9)  APER(I),B08(I),B07(I),B06(I)
      DO 26 I=1,IPTS
      ENDT((IGROUP-1)*IPTS+I)=AZT(I)
      ENDC((IGROUP-1)*IPTS+I)=AZC(I)
      ENDB((IGROUP-1)*IPTS+I)=AZBT(I)
      END11((IGROUP-1)*IPTS+I)=B11(I)
      END10((IGROUP-1)*IPTS+I)=B10(I)
      END09((IGRCUP-1)*IPTS+I)=B09(I)
      END08((IGROUP-1)*IPTS+I)=B08(I)
      END07((IGROUP-1)*IPTS+I)=B07(I)
      END06((IGRCUP-1)*IPTS+I)=B06(I)
   26 CONTINUE
    3 CONTINUE
      CALL GROUP(AJ,BJ,APER,IPTS,ENDT,ENDC,ENDB,END11,END10,
     *END09,END08,END07,END06)
C     CALL DRAW FOR ISOTHERMS
C     CALL DRAW THERMOCLINES
C     INCREASE SIZE OF ELEMENTS OF ENDT,ENDC, AND ENDB
C     FOR GRAPHING
      DO 999 I=1,IPTS
      ENDT(I)=1.5*ENDT(I)
      ENDC(I)=1.5*ENDC(I)
  999 ENDB(I)=ENDB(I)*1.5
      WRITE(6,30)
      CALL DRAW(IPTS,BJ,END11,0,0,LABL1,ITITL1,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,END10,0,0,LABL2,ITITL2,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,END09,0,0,LABL3,ITITL3,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,END08,0,0,LABL4,ITITL4,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,END07,0,0,LABL5,ITITL5,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,END06,0,0,LABL6,ITITL6,1.,20.,0,0,2,
     *2,5,8,0,IL)
      CALL DRAW(IPTS,BJ,ENDT,0,0,LABL7,ITI1,1.,20.,0,0,2,2,
     *5,8,0,IL)
      CALL DRAW(IPTS,BJ,ENDC,0,0,LABL8,ITI2,1.,20.,0,0,2,2,
     *5,8,0,IL)
      CALL DRAW(IPTS,BJ,ENDB,0,0,LABL9,ITI3,1.,20.,0,0,2,2,
     *5,8,0,IL)
      STOP
      END
```

```
      SUBROUTINE INTERP(IT,IPTS,TX,X,ICOUNT,KN,Z11,Z10,Z09,
     *Z08,Z07,Z06,KK)
C   THIS SUBROUTINE, CALLED FROM THE MAIN PROGRAM, IS USED
C   TO LINEARLY INTERPOLATE BETWEEN DATA POINTS GIVEN AT
C   FIVE METER INCREMENTS TO DEPTHS CORRESPONDING TO TEMP.
C   INCREMENTS OF 0.1 DEGREE CENTIGRADE.  THIS PROGRAM DOES
C   NOT CONSIDER SMALL NEAR-SURFACE TRANSIENTS OR INVERSIONS
C
C   VARIABLES USED IN THIS SUBROUTINE
C     IT--INPUT TEMPERATURES, IN TENTHS OF A DEGREE, FROM
C     MAIN PROGRAM
C
C     TX--THE ARRAY OF INTERPOLATED TEMPERATURES
C
C     X--THE ARRAY OF INTERPOLATED DEPTHS ASSOCIATED WITH TX
C
C     ICOUNT--THE TOTAL NUMBER OF ELEMENTS IN X AND IN TX
C
C     IPTS--THE NUMBER OF TEMPERATURES PER TRACE AS READ
C     FROM DATA CARDS.     IN THE CASE OF THIS DATA IPTS=20
C
C     IT1,IT2,IT3,IT4,IT5,IT6--THE TEMPERATURES, IN TENTHS
C     OF A DEGREE, OF THE ISOTHERMS WHOSE DEPTH FLUCTUATIONS
C     ARE OF INTEREST.  THESE VALUES MUST BE SPECIFIED IN
C     THIS SUBROUTINE, AND ARE ASSOCIATED WITH Z11,Z10,Z09,
C     Z08,Z07,Z06 RESPECTIVELY IN LATER COMPUTATIONS.
C
C     T,TT,ITT--ARRAY NAMES ASSIGNED TO THE INTERPOLATED
C     TEMPERATURES AT VARIOUS POINTS IN THE COMPUTATIONS
C
C     Z,ZZ--ARRAY NAMES ASSIGNED TO THE INTERPOLATED DEPTHS
C
C     ITO--AN ARRAY USED TO FIND A TEMPORARY TOP, BOTTOM,
C     AND CENTER OF THE THERMOCLINE.
C
C     ITTOP--THE TEMPERATURE OF THE TEMPORARY TOP OF THE
C     THERMOCLINE.  DEFINED IN THIS SUBPROGRAM AS THE DEPTH
C     BELOW WHICH THE TEMPERATURE FIRST DECREASES MORE THAN
C     0.3 DEGREES IN FIVE METERS OF DEPTH.
C
C     ITBOT--THE TEMPERATURE OF THE TEMPORARY BOTTOM OF THE
C     THERMOCLINE.  DEFINED AS THE FIRST TEMPERATURE LESS
C     THAN ITTOP ABOVE WHICH THE TEMPERATURE DECREASES LESS
C     THAN 0.2 DEGREES IN FIVE METERS OF DEPTH
C
C     ITC--THE ARITHMETIC MEAN OF ITTOP AND ITBOT, USED IN
C     SUBROUTINE FINDZC TO DETERMINE THE SUBSCRIPT OF THE
C     DEPTH VALUE CORRESPONDING TO THE ESTIMATED CENTER OF
C     THE THERMOCLINE
C
C   SUBROUTINES CALLED FROM THIS SUBROUTINE ARE
C     PURGE
C     FINDZC
C     FINDZS
C
      DIMENSIONIT(1),TX(1),X(1),Z11(1),Z10(1),Z09(1),Z08(1),
     *Z07(1),Z06(1),T(200),TT(200),Z(200),ZZ(200),ITT(200),
     *ITO(200)
      ILESS=IPTS-1
C     GENERATE Z(I)
      DO 999 I=1,IPTS
      Z(I)=5.*(I-1)
  999 CONTINUE
      IEPS=0
      IT1=110
      IT2=100
      IT3=90
      IT4=80
```

```
      IT5=70
      IT6=60
      DO 90 II=1,ILESS
      ITO(II)=IT(II+1)-IT(II)
      IF(ITO(II).LT.-3) GO TO 91
   90 CONTINUE
   91 ITTOP=IT(II)
      DO 92 JJ=10,ILESS
      ITO(JJ)=IT(JJ+1)-IT(JJ)
      IF(ITO(JJ).GT.-2) GO TO 93
   92 CONTINUE
   93 ITBOT=IT(JJ)
      ITC=(ITTOP+ITBOT)/2
C     EXAMINE IT(I) VERSUS IT(I+1).  IF SHALLOWER TEMP.
C     IS GREATER THAN NEXT TEMPERATURE BELOW IT IN THE
C     WATER COLUMN, INTERPOLATE.  IF NOT, STORE.
      DO 1000 I=1,IPTS
      J=I+IEPS
      K=0
      IF(I.EQ.IPTS) GO TO 71
      IF(IT(I).GT.IT(I+1)) GO TO 5
      ITT(J)=IT(I)
      ZZ(J)=Z(I)
      GO TO 1000
    5 ITDIF=IT(I)-IT(I+1)
      ZDIF=5./ITDIF
      MAX=ITDIF+J-1
      DO 1001 JJ=J,MAX
      ZZ(JJ)=(JJ-J)*ZDIF+Z(I)
      ITT(JJ)=IT(I)-K
      K=K+1
 1001 CONTINUE
      IEPS=MAX-I
 1000 CONTINUE
      INT=ILESS+J
   71 ITT(J)=IT(I)
      ZZ(J)=Z(I)
  100 DO 8 I=1,J
      TT(I)=ITT(I)
    8 CONTINUE
      ICOUNT=J-1
      CALL PURGE (TT,ZZ,ICOUNT,TX,X)
      CALL FINDZC(TX,ITC,KN,ICOUNT)
      CALL FINDZS(TX,X,IT1,IT2,IT3,IT4,IT5,IT6,Z11,Z10,Z09,
     *Z08,Z07,Z06,KK)
   20 RETURN
      END
```

```
      SUBROUTINE PURGE (TT,ZZ,ICOUNT,TX,X)
C   SUBROUTINE PURGE TAKES AS INPUT THE ARRAY OF TEMPERATURE
C   AND DEPTH INFORMATION FROM SUBROUTINE INTERP AND
C   EXAMINES IT FOR DUPLICATION OF TEMPERATURE, CONSIDERING
C   A POSITIVE TEMPERATURE-DEPTH GRADIENT TO BE ISOTHERMAL.
C   (THIS WAS JUSTIFIED BY THE PARTICULAR DATA INVOLVED IN
C   THE WRITING OF THIS SUBPROGRAM.)  THE SUBROUTINE STORES
C   EACH TEMPERATURE IN THE TRACE AT THE SHALLOWEST DEPTH
C   AT WHICH IT APPEARS.
C
C   VARIABLES USED IN THIS SUBPROGRAM
C     TT,TX,ZZ,X--AS DEFINED IN SUBROUTINE INTERP
C
C     ICOUNT--AS OUTPUT, THE NUMBER OF TEMPERATURES OR DEPTH
C     PAIRS REMAINING AFTER DUPLICATIONS HAVE BEEN REMOVED.
C
      DIMENSION TT(1),TX(1),ZZ(1),X(1)
      TX(1)=TT(1)
      X(1)=ZZ(1)
      J=2
```

```
      DO 1 I=1,ICOUNT
      IF(TT(I+1).GE.TT(I)) GO TO 1
      TX(J)=TT(I+1)
      X(J)=ZZ(I+1)
      J=J+1
    1 CONTINUE
      ICOUNT =J-2
      RETURN
      END
```

```
      SUBROUTINE FINDZS(TX,X,IT1,IT2,IT3,IT4,IT5,IT6,Z11,
     *Z10,Z09,Z08,Z07,Z06,KK)
C     THIS SUBROUTINE TAKES THE PURGED ARRAYS OF TEMPERATURES
C     AND DEPTHS AND FINDS THE DEPTHS OF ISOTHERMS SELECTED
C     IN SUBROUTINE INTERP
C
C     VARIABLES USED IN THIS SUBROUTINE
C       TX,X--AS DEFINED IN SUBROUTINE INTERP
C
C       IT1,IT2,IT3,IT4,IT5,IT6--AS DEFINED IN SUBROUTINE
C       INTERP
C
C       Z11,Z10,Z09,Z08,Z07,Z06--AS DEFINED IN MAIN PROGRAM
C
C       KK--THE SUBSCRIPT OF THE DEPTH VALUE IN THE 2**N ARRAY
C
      DIMENSION TX(1),X(1),Z11(1),Z10(1),Z09(1),Z08(1),
     *Z07(1),Z06(1)
      DO 1 I=1,100
      IF(TX(I).LE.IT1) GO TO 2
    1 CONTINUE
    2 Z11(KK)=X(I)
      DO 3 J=I,100
      IF(TX(J).LE.IT2) GO TO 4
    3 CONTINUE
    4 Z10(KK)=X(J)
      DO 5 K=J,100
      IF(TX(K).LE.IT3) GO TO 6
    5 CONTINUE
    6 Z09(KK)=X(K)
      DO 7 L=K,100
      IF(TX(L).LE.IT4) GO TO 8
    7 CONTINUE
    8 Z08(KK)=X(L)
      DO 9 M=L,100
      IF(TX(M).LE.IT5) GO TO 10
    9 CONTINUE
   10 Z07(KK)=X(M)
      DO 11 MM=M,100
      IF(TX(MM).LE.IT6) GO TO 12
   11 CONTINUE
   12 Z06(KK)=X(MM)
      RETURN
      END
```

```
      SUBROUTINE FINDZC(TX,ITC,KN,ICOUNT)
C     THIS SUBROUTINE DETERMINES THE SUBSCRIPT OF THE DEPTH
C     CORRESPONDING TO ITC AS FOUND IN SUBROUTINE INTERP
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       TX,ITC--AS DEFINED IN SUBROUTINE INTERP
C
C       ICOUNT--AS DEFINED IN SUBROUTINE PURGE
C
```

```
C       KN--THE SUBSCRIPT OF THE DEPTH ASSOCIATED WITH ITC
C
        DIMENSION TX(1)
        DO 1 I=1,ICOUNT
        IF(TX(I).LE.ITC) GO TO 11
    1   CONTINUE
   11   KN=I
        RETURN
        END




        SUBROUTINE NONGA(Z,T,ICOUNT,ZT,ZC,ZBT,KN,JL)
C     THIS SUBROUTINE TAKES THE INPUT ARRAYS OF TEMPERATURE
C     AND DEPTH, DIVIDES THE DEPTHS AT THE DEPTH CORRESPONDING
C     TO ITC AND PRODUCES TWO SYMMETRICAL DEPTH ARRAYS, ONE A
C     'REFLECTION' OF THE DEPTHS FROM Z(KN) TO THE SURFACE AND
C     THE OTHER A 'REFLECTION' OF DEPTHS FROM Z(KN) TO THE
C     BOTTOM CF THE TRACE.
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       Z,T--AS DEFINED IN SUBROUTINE INTERP
C
C       ICOUNT--AS DEFINED IN SUBROUTINE PURGE
C
C       ZT,ZC,ZBT--AS DEFINED IN MAIN PROGRAM
C
C       JL--THE SUBSCRIPT OF THE DEPTHS ZT,ZC,ZBT IN THE ARRAY
C       OF 2**N TERMS USED IN THE FOURIER ANALYSIS
C
C       KN--THE SUBSCRIPT CORRESPONDING TO THE DEPTH OF THE
C       ESTIMATED CENTER OF THE THERMOCLINE
C
C       Z5--THE 'REFLECTED' ARRAY OF DEPTHS FROM Z(KN) TO THE
C       SURFACE
C
C       Z3--THE 'REFLECTED ARRAY OF DEPTHS FROM Z(KN) TO THE
C       BOTTOM OF THE TRACE
C
C       NUMBER,KDUMMY,NNN,NUMMER--COUNTERS USED IN THE
C       PROCESS OF PRODUCING GAUSSIAN THERMOCLINES FROM THE
C       NON-GAUSSIAN INPUT THROUGH 'REFLECTION'
C
C       SK--A WEIGHTING FACTOR, WHICH IS APPLIED TO THE
C       VARIANCE TO COMPENSATE FOR A LACK OF ACCURACY IN
C       COMPUTATION DUE TO USE OF FINITE DIFFERENCE TECHNIQUES
C       RATHER THAN INTEGRATION. ITS VALUE IS DEPENDENT UPON
C       THE TEMPERATURE INTERVAL (DELTA T) USED IN COMPUTATION
C
C     SUBROUTINES CALLED FROM THIS SUBROUTINE ARE
C       GAUSS1
C       GAUSS2
C
        DIMENSION Z(200),Z3(200),Z5(200),T(200),ZC(1),ZT(1),
       *ZBT(1)
        IF((T(1)-T(2)).GE.10.) SK=1.30000
        IF((T(1)-T(2)).LE.5.) SK=1.470000
        NUMBER=ICOUNT -KN+1
        KDUMMY=ICOUNT-KN
        NNN=(2*(ICOUNT-KN))+1
        ZC(JL)=Z(KN)
        DO 73 I1=1,KN
        Z5(I1) = Z(I1)
   73   CONTINUE
        IZ1=KN-1
        DO 7 I2=1,IZ1
        X = Z(KN) - Z(KN-I2)
        Z5(KN+I2) = Z(KN) + X
    7   CONTINUE
        NUMMER=2*KN-1
```

```
      CALL GAUSS1(Z5,NUMMER,ICOUNT,SK,ZT,ZC,JL)
    4 KKK=KN-1
      DO 74 I3=1,NUMBER
      Z3(KDUMMY + I3) = Z(KKK+I3)
   74 CONTINUE
      DO 59 I4=1,KDUMMY
      X = Z(KN+I4) - Z(KN)
      Z3(NUMBER - I4) = Z(KN) - X
   59 CONTINUE
      CALL GAUSS2(Z3,NNN,ICOUNT,SK,ZBT,JL)
      RETURN
      END



      SUBROUTINE GAUSS1(X,NT,ICOUNT,SK,ZT,ZC,JL)
C     THIS SUBROUTINE FINDS THE DEPTHS OF THE TOP AND CENTER
C     OF THE THERMOCLINE THROUGH GAUSSIAN METHODS
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C        X--CORRESPONDS TO THE Z5 ARRAY OF SUBROUTINE NONGA
C
C        NT--THE NUMBER OF ELEMENTS IN THE ARRAY OF X
C
C        Z1--THE DEPTHS USED IN COMPUTATION OF THE GAUSSIAN
C        THERMOCLINE.  BY DEFINITION, THE AVERAGE VALUE OF
C        CONSECUTIVE DEPTHS IN THE X ARRAY, THE MIDPOINT
C        OF 'DELTA T', AT WHICH THE DEPTHS MAKE THEIR
C        CONTRIBUTION
C
C        Z2--THE ARRAY OF VALUES CORRESPONDING TO THE SQUARES
C        OF THE VALUES OF Z1
C
C        JL--AS DEFINED IN NONGA
C
C        SK--AS DEFINED IN NONGA
C
C        SIGSQR--THE VARIANCE OF THE ARRAY OF Z1
C
C        SUM1--THE SUM OF THE ARRAY OF Z1
C
C        SUM2--THE SUM OF THE ARRAY OF Z2, OR THE SUM OF THE
C        SQUARES OF THE VALUES OF THE ARRAY OF Z1
C
      DIMENSION Z1(200),Z2(200),X(1),ZC(1),ZT(1)
      N=NT-1
      SUM1 =0.0
      SUM2=0.0
      DO 1 I=1,N
      Z1(I)=(X(I)+X(I+1))/2.
      Z2(I)=Z1(I)**2
      SUM1=SUM1+Z1(I)
    1 SUM2=SUM2+Z2(I)
      ZC(JL)=SUM1/N
      SIGSQR=(SUM2/N)-(ZC(JL)**2)
      ZT(JL)=ZC(JL)-SK*SQRT(SIGSQR)
      RETURN
      END



      SUBROUTINE GAUSS2(X,NT,ICOUNT,SK,ZBT,JL)
C     THIS SUBROUTINE FINDS THE DEPTH OF THE BOTTOM OF THE
C     THERMOCLINE THROUGH GAUSSIAN METHODS
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C        X--CORRESPONDS TO THE Z3 ARRAY OF SUBROUTINE NONGA
C
```

```
C       NT--THE NUMBER OF ELEMENTS IN THE ARRAY OF X
C
C       JL,SK--AS DEFINED IN SUBROUTINE NONGA
C
C       SUM1,SUM2,Z1,Z2--AS DEFINED IN SUBROUTINE GAUSS1
C
C       SIGSQR--AS DEFINED IN SUBROUTINE GAUSS1
C
      DIMENSION Z1(200),Z2(200),ZC(600),X(1),ZBT(1)
      N=NT-1
      SUM1 =0.0
      SUM2=0.0
      DO 1 I=1,N
      Z1(I)=(X(I)+X(I+1))/2.
      Z2(I)=Z1(I)**2
      SUM1=SUM1+Z1(I)
    1 SUM2=SUM2+Z2(I)
      ZC(JL)=SUM1/N
      SIGSQR=(SUM2/N)-(ZC(JL)**2)
      ZBT(JL)=ZC(JL)+SK*SQRT(SIGSQR)
      RETURN
      END


      SUBROUTINE AVERGE(AZIN,AZOUT)
C     THIS SUBROUTINE TAKES THE ARRAY OF DEPTHS OF A SELECTED
C     FEATURE (ISOTHERM OR THERMOCLINE), AND PRODUCES AN ARRAY
C     WITH A MEAN VALUE OF ZERO
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       AZIN--THE ARRAY OF INPUT VARIABLES, WITH AN ARBITRARY
C       MEAN VALUE
C
C       AZOUT--THE OUTPUT ARRAY OF VARIABLES, WITH A MEAN OF
C       ZERO
C
C       ZINBAR--THE SUMMATION OF VALUES OF AZIN
C
C       AZINB--THE MEAN OF VALUES OF AZIN
C
      DIMENSION AZIN(1),AZOUT(1)
      ZINBAR=0.0
      DO 1 I=1,512
    1 ZINBAR=ZINBAR+AZIN(I)
      AZINB=ZINBAR/512.
      DO 2 I=1,512
    2 AZOUT(I)=AZIN(I)-AZINB
      RETURN
      END


      SUBROUTINE HANN  (ZIN,BOUT)
C     THIS SUBROUTINE TAKES THE FOURIER COEFFICIENTS AS FOUND
C     IN SUBROUTINE RHARM, AND HANNS THEM, TO REDUCE LEAKAGE
C     BETWEEN NEIGHBORING FREQUENCY BANDS.
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       ZIN--THE ARRAY OF FOURIER COEFFICIENTS FOUND BY RHARM
C
C       BOUT--THE HANNED FOURIER COEFFICIENTS
C
      DIMENSION ZIN(1),BOUT(1)
      DO 1 I=1,2
      BOUT(I)=.5*ZIN(I)-.25*ZIN(I+2)
    1 CONTINUE
      DO 2 I=509,510
```

```
      BOUT(I+2)=-.25*ZIN(I)+.5*ZIN(I+2)
    2 CONTINUE
      DO 3 I=3,510
      BOUT(I)=-.25*ZIN(I-2)+.5*ZIN(I)-.25*ZIN(I+2)
    3 CONTINUE
      RETURN
      END



      SUBROUTINE SMOOTH(ARGIN,M,ITEST,ARGOUT,ZJ,ICOUNT)
C     THIS SUBROUTINE TAKES THE HANNED FOURIER COEFFICIENTS
C     AND AVERAGES THEM OVER FREQUENCY BANDS OF SPECIFIED
C     WIDTH.  THE NUMBER OF FREQUENCIES INCLUDED IN EACH BAND
C     IS EQUAL TO M, AS READ IN BY THE MAIN PROGRAM BEFORE
C     THIS SUBROUTINE IS CALLED.
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       ARGIN--THE ARRAY OF HANNED FOURIER COEFFICIENTS
C
C       M--A GROUPING FACTOR, AS DISCUSSED ABOVE
C
C       ITEST--THE NUMBER OF GROUPS OF 2M FOURIER COEFFICIENTS
C       WHICH IS ALSO ONE LESS THAN THE TOTAL NUMBER OF
C       SMOOTHED FREQUENCY BANDS
C
C       ARGOUT--THE ARRAY OF ITEST+1 FREQUENCY BANDS
C
C       ZJ--AS DEFINED IN MAIN PROGRAM
C
C       ICOUNT--THE TOTAL NUMBER OF FREQUENCY BANDS IN ARGOUT
C
C       FACTOR--A WEIGHTING FACTOR USED IN SMOOTHING
C
C       N--THE NUMBER OF VALUES IN THE ARRAY OF DEPTHS
C       SUBJECTED TO THE FOURIER ANALYSIS
C
      DIMENSION ARGIN(1),ARGOUT(1),ZJ(1)
      CM=M
      N=512
      FACTOR=N/CM
      ITEST=N/(2*M)
      JTEST=(N-1)/M
      IF(ITEST*2.GE.JTEST) GO TO 3
      ITEST=ITEST+1
    3 CONTINUE
      ICOUNT=ITEST+1
      DO1 KK=1,512
      ARGOUT(KK)=0.0
      ARGIN(KK)=ARGIN(KK)**2
    1 CONTINUE
      PART=0.0
      DO 2 JJ=3,M
    2 PART=PART+ARGIN(JJ)
      ARGOUT(1)=FACTOR*(ARGIN(1)+(2.*PART))
      ZJ(1)=1.
      DO 100 II=2,ITEST
      MM=(((2*II)-3)*M)+2
      MSTOP=((2*II)-1)*M+1
      DO 101 IJ=MM,MSTOP
  101 ARGOUT(II)=ARGOUT(II)+ARGIN(IJ)
      ARGOUT(II)=FACTOR*ARGOUT(II)
  100 ZJ(II)=II
      MN=MM+M
      ZJ(ICOUNT)=ICOUNT
      IDIFF=511-((ITEST*2)-1)*M
      IF(2*(IDIFF/2).EQ.IDIFF) GO TO 30
      IDIFF=IDIFF+1
   30 PRDIFF=IDIFF/2
      DIFF=512./PRDIFF
```

```
      DO 102 JK=MN,512
  102 ARGOUT(ICOUNT)=ARGOUT(ICOUNT)+ARGIN(JK)
      ARGOUT(ICOUNT)=DIFF*ARGOUT(ICOUNT)
      RETURN
      END



      SUBROUTINE PERIOD(M,ITEST,APER,AJ,BJ)
C     THIS SUBROUTINE IS USED TO GENERATE THE PERIODS
C     ASSOCIATED WITH THE SMOOTHED SPECTRAL BANDWIDTHS, AND TO
C     GENERATE THE FINAL SET OF COUNTERS FOR USE IN GRAPHICAL
C     DISPLAY
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       M--AS DEFINED IN SUBROUTINE SMOOTH
C
C       ITEST--AS DEFINED IN SUBROUTINE SMOOTH
C
C       APER--THE ARRAY OF PERIODS ASSOCIATED WITH THE ARRAY
C       OF SMOOTHEC BANDWIDTHS
C
C       AJ--AS DEFINED IN MAIN PROGRAM
C
C       BJ--DEFINED AS THE NATURAL LOGARITHM OF AJ FOR USE IN
C       PRODUCING A SEMILOG GRAPHICAL OUTPUT
C
      DIMENSION APER(1),AJ(1),BJ(1)
      IPTS=ITEST+1
      APER(1)=(2.*512.)/M
      DO 1 I=2,ITEST
    1 APER(I)=512./((I-1)*M)
      APER(IPTS)=2048./(1024.-(512.-((2*ITEST+1)*M)))
      DO 3 I=1,IPTS
    3 AJ(I)=512./(M*APER(I))
      DO 2 I=1,IPTS
    2 BJ(I)=ALOG(AJ(I))
      RETURN
      END



      SUBROUTINE GROUP(AJ,BJ,APER,IPTS,ENDT,ENDC,ENDB,END11,
     *END10,END09,END08,END07,END06)
C     THIS SUBROUTINE IS USED TO AVERAGE THE RAW SPECTRA
C     PRODUCED IN 'IGROUP' TIMES THROUGH THE MAIN PROGRAM ,
C     AND TO PRINT THE VALUES ASSOCIATED WITH THESE AVERAGED
C     SPECTRA
C
C     VARIABLES USED IN THIS SUBROUTINE ARE
C       AJ,BJ,APER--AS DEFINED IN SUBROUTINE PERIOD
C
C       IPTS--THE TOTAL NUMBER OF SPECTRAL BANDWIDTHS COMPUTED
C       IN SUBROUTINE SMOOTH
C
C       END11,END10,ENDT,ENDC,ETC.--AS DEFINED IN MAIN PROGRAM
C
      DIMENSION AJ(1),BJ(1),APER(1),ENDT(1),ENDC(1),ENDB(1),
     *END11(1),END10(1),END09(1),END08(1),END07(1),END06(1)
      DO 1 I=1,IPTS
      ENDT(I)=.5*(ENDT(I)+ENDT(I+IPTS))
      ENDC(I)=.5*(ENDC(I)+ENDC(I+IPTS))
      ENDB(I)=.5*(ENDB(I)+ENDB(I+IPTS))
      END11(I)=.5*(END11(I)+END11(I+IPTS))
      END10(I)=.5*(END10(I)+END10(I+IPTS))
      END09(I)=.5*(END09(I)+END09(I+IPTS))
      END08(I)=.5*(END08(I)+END08(I+IPTS))
      END07(I)=.5*(END07(I)+END07(I+IPTS))
```

```
      END06(I)=.5*(END06(I)+END06(I+IPTS))
    1 CONTINUE
      WRITE(6,30)
   30 FORMAT(1H1)
      WRITE(6,3)
    3 FORMAT(//40X,'AVERAGED SPECTRAL ESTIMATES',/)
      WRITE(6,4)
    4 FORMAT(30X,'NR',4X,'PERIOD',10X,'TOP',4X,'CENTER',4X,
     *'BOTTOM',/)
      DO 5 I=1,IPTS
    5 WRITE(6,6) AJ(I),APER(I),ENDT(I),ENDC(I),ENDB(I)
    6 FORMAT(28X,F4.1,4X,F6.2,4X,3F10.2)
      WRITE(6,30)
      WRITE(6,3)
      WRITE(6,7)
    7 FORMAT(30X,'NR',4X,'PERIOD',7X,'11 DEG',4X,'10 DEG',
     *5X,'9 DEG',/)
      DO 8 I=1,IPTS
    8 WRITE(6,6) AJ(I),APER(I),END11(I),END10(I),END09(I)
      WRITE(6,30)
      WRITE(6,3)
      WRITE(6,10)
   10 FORMAT(30X,'NR',4X,'PERIOD',8X,'8 DEG',5X,'7 DEG',5X,
     *'6 DEG',/)
      DO 9 I=1,IPTS
    9 WRITE(6,6) AJ(I),APER(I),END08(I),END07(I),END06(I)
      RETURN
      END
```

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13. ABSTRACT

Methods of investigating internal waves from bathythermograph information are discussed. A series of 651 hourly observations taken at Ocean Station "P" is analyzed by two methods to determine the characteristics of the internal waves present. One method involves a study of the fluctuations in depth of six selected isotherms; the other involves a study of the fluctuations in depth of the top, center, and bottom of the thermocline. An objective method is used to determine the depths of these features from the bathythermograph data.

Spectra of these features are calculated by Fast Fourier transform, hanned, and smoothed, and these spectra are compared with previously published results obtained from a difference method of analysis of the same time series.

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