

Some Considerations

Of Mr. Nic. Mercator, concerning the Geometrick and direct Method of Signior Cassini for finding the Apogees, Excentricities, and Anomalies of the Planets; as that was printed in the Journal des Scavans of Septemb. 2. 1669: which Considerations are here delivered in the Latine Tongue, wherein they were written by the Author, as chiefly regarding the Learn'd in Astronomy, viz.

Clarissimi Cassini Methodus

Investigandi Apogea, Excentricitates & Anomalias Planetarum, breviter Exposita & Demonstrata.

Supponit Cl. *Cassini*, ad Planetam in Ellipsi moventem extendi ab utroque foco duas rectas, quarum altera sit *medii*, altera autem *veri* motus linea. Constructio porro talis est;

<i>Fig. II.</i> L est Centrum Concentrici ABCDE.	R HG est linea recta.
B LD est Diameter.	B I est perpendicularis ad R HG.
B A, B C, B P, sunt intervalla apparetia.	I est Centrum Ellipseos.
D E, D F, D Q, sunt intervalla mediorum motuum.	L I est Excentricitas.
B E, B F, B Q; item DA, DC, DP, sunt lineæ rectæ.	I O = L I.
B E secat DA in H; BF secat DC in G; B Q secat DP in R.	O est focus, circa quem ordinatur medius motus; L, circa quem verus.
	I M = I N = LB.
	M est Apogeon; N, Perigeon; BLM Anomalia vera.

Demonstratio.

I. Illustrissimus ac Reverendiss. *Sethus Wardus*, quondam in Celeberr. Acad. Oxon. Professor Astronomiae Savilianus, nunc Episcopus Sarisburiensis, in *Examine Astronomiae Philolaicae*, edito Oxon. A. 1653. c. 6. docuit Methodum, ex data Anomalia media Planetarum, investigandi veram; quæ est hujusmodi:

Fig. III. C, est Centrum Ellipseos AEP: F, focus, circa quem ordinatur medius motus. S, focus, circa quem ordinatur verus motus. A, Apogeon. P, Perigeon, E, Erro sive Planeta. AIE, Anomalia media. ASE, Anomalia vera. FET, linea recta, ET = SE. ST est linea recta.

In \triangle SFT dantur, 1. SF distantia focorum: 2. FT = FE + ES = AP. 3. AFT, angulus externus, sive Anomalia media, æqualis summae angularum FST & T. Ergo inveniri potest FSE, sive Anomalia vera, æqualis differentiæ Angularum FST & T. Nimurum

Vt

Ut semi-summa laterum F T & F S, ad semi-differentiam eorundem;
Ita Tangens semi-summæ angulorum F S T & T, ad Tangentem semi-differentiæ eorundem.

Sed semi-summa laterum F T & F S invenitur, substituendo pro F T æqualem AP, cuius semis est AC, qui additus C S semissi ipsius F S, facit Semi-summam AS, distantiam Planetae maximam.

Tum, si ex semi-summa A S auferatur latus minus F S, restat semi-differentia laterum F A, æqualis PS, distantiae Planetae minimæ; ut sit

Regula ex Anomalia Media data inveniendi veram:

Ut A S, distantia Planetae maxima, ad PS, distantiam minimam;

Ita Tangens dimidiæ Anomalie mediæ, ad Tangentem dimidiæ Anomalie veræ.

Corollar. I. Si continuetur S E usque ad U, ita ut E U sit \equiv ipsi F E, & tota S U \equiv Axi AP; erit \triangle FSU angulus U semis Prostaphæreos F E S, ideoque æqualis semi-differentiæ angulorum Anomalie mediæ & veræ, h.e. ipsorum A F E & A S E; & externus A FU \equiv semi-summæ eorundem A F E & A S E angulorum, ablata scil. semi-differentiæ U F E ex majori AFE. Unde oriuntur duæ Analogiæ:

1. Ut Sinus semi-summæ Anomalie mediæ & veræ A FU, ad Sinum semi-differentiæ eorundem, U; Ita S U (= $\text{axi transverso } AP$) ad S F, distantiam focorum.

2. Ut Sinus semi-summæ Anomalie mediæ & veræ, A F V, ad Sinum Anomalie veræ F S U; Ita S U (vel axis A P) ad F U, subtensam Anomalie veræ: Ita quoque semi-axis A C, ad semi-subtensam U X, vel F X.

Corollar. II. Si in eodem Triangulo FSU, ex subtensiæ F U puncto medio X, erigatur perpendicularis X E; secabit illa S U in duas partes, quarum altera U E \equiv linea medii motus F E, altera vero SE est ipsa linea veri motus.

II. Fig. IV. Sit a Centrum Con- | cdb , est Angulus dimidiæ Anomalie
centrici cbf i. veræ, &

c ad, Diameter, eadémque linea | dci , Angulus dimidiæ Anomalie
Apsidum.

cb, Arcus Anomalie veræ, cui re- | ci & dh sunt lineæ rectæ, secantes
spondet se mutuo in g.

di, Arcus Anomalie mediæ. Itaque

Ab Intersectionis punto g demittatur ad cd perpendicularis g b. Erit igitur,

$db \cdot bg :: \text{Radius ad tang. } bdg \text{ vel } cdh$.

Et $cb \cdot bg :: \text{Rad. tang. } bcg \text{ vel } dc i$.

Ergo

Ergo $db \times \text{tang. } cdh = bg \times \text{Rad.} = cb \times \text{tang. } dc i.$

Quare $db \cdot cb :: \text{tang. } dc i \cdot \text{tang. } cdh$; hoc est, db erit ad cb , ut tangens dimidiæ Anomalie mediæ ad tangentem dimidiæ Anomalie veræ; adeoque (per Regulam supra expositam) ut distantia Planetæ maxima, ad distantiam minimam. Quare obrem db erit distantia Planetæ maxima, & cb , minimæ, & ab , excentricitatì.

Cumque idem eodem modo demonstretur de ceteris omnibus Intersectionum punctis, nimis. Perpendiculares ab ipsis ad cd lineam incidere in punctum b ; oportet, ut recta, jungens ipsas Intersectiones, congruat perpendiculari bgf .

III. Ductâ diametro hak , fiat arcus $k l = \text{arcui } id$, & ducantur kc & hl , secantes se mutuo in p . Ab h in bgf demittatur perpendicularis hr , eadēque parallela Apsidum lineæ ca ; erit angulus $r hs$ semi-differentia arcuum Anomalie veræ ch , & mediæ di . Tum ab eodem h puncto ducatur recta $h\beta$, faciens cum kh angulum $=$ angulo $r hs$, & occurrentis lineæ Apsidum in β . Erit $\Delta i a \beta h$ angulus $\beta a h$ mensura arcus ch , sive Anomalie veræ, & $\beta h a$ semi-differentia Anomalie veræ & mediæ (ex Constructione;) & externus $c \beta h$ (æqualis duobus internis & oppositis βah & βha , adeoque compositus ex Anomaliâ vera & semi-differentia ejus à media) erit semi-summa Anomalie veræ & mediæ. Ergo, per *Collarum I^{mi}* Analogiam priorem; Ut Sinus $c \beta h$, ad Sinum $\beta h a$; ita Radius $a h$, ad Excentricitatem a^2 . Sed supra demonstravimus quoque ab æqualem Excentricitatì. Ergo punctum β congruit puncto b .

Tum ex b excitetur ipsi hb perpendicularis bt ; Aio, hanc continuatam incidere in punctum Intersectionis p . Nam Triangula $r hs$ & bkt sunt similia, ex Constructione; quemadmodum & $\Delta m hpk$ simile est $\Delta o hgi$, cum eidem peripheriæ ch insistentes anguli $p kh$ & ghi sint æquales, nec non æqualibus peripheriis kl & id insistentes anguli $p hk$ & ghi æquales; quare & tertius hpk æqualis est tertio hgi . Et ex æqualibus $p hk$ & ghi ablatis æqualibus bht & rhs , restant æquales $p hb$ & ghr . Vnde sic arguo: $srb = t bh$, & $rhs = bht$, Ergo $hrs = btb$; ergo & Complementa horum ad semi-circulum sunt æqualia, nimis. $rsi = btk$; & $sig = tkp$, Ergo & $igs = kpt$, quibus ablatis ex æequalibus igh , & kph , restat $hgs = hpt$; & $ghr = phb$, Ergo & $hrg = bhp$. Sed hrg est rectus, Ergo & hbp rectus est. Cum vero & hbt rectus sit ex Constructione, erit $t b$ in directum ipsi bp . Cumque idem eodem modo demonstretur de quavis alia Intersectione linearum ab b & k ad congruentia Anomalie veræ & mediæ puncta ductarum; patet, non modò rectam, jungentem intersectiones, transituram per b punctum; sed & kb , lineam perpendicularem fore ad eandem Jungentem. q.e.dem.

Corollarium. Si à quovis punto Anomalie veræ, puta b , ad respondens punctum Anomalie mediae i ducatur recta hi ; excitata ē Centro Excentrici b , ipsi $c b d$ perpendicularis bf secabit ipsam bi in s eā ratione, qnam linea medii motū obtinet ad lineam veri motū.

Nam per *Corollarii I^{mi}* Analogiam posteriorem, hb est semi-subtensa; Ergò per *Corollarium II^{um}*, perpendicularis eretta ex b , nimir. bt , secat diametrum hk in t eā ratione, qnam linea medii motū obtinet ad lineam veri motū. Ergò & rs (sive bf) secat bi lineam eadem ratione in s ; propter demonstratam modò figurarum $t b h k p h b$ & $s r b i g h r$ similitudinem.

Cæterū ex laudata superius Reverendiss. *Wardi* Methodo inveniendi primam inæqualitatem, non est difficile, alium adhuc modum investigandi Apogea & Excentricitates, non minus directum & Geometricum, & Observationes quovis admittentem, producere; quem & paucis expōnam. Plures modos invenient Astrophili in Reverendiss. Viri *Astronomia Geometrica*, edita A. 1656, ad quam eos remitto. Interim

Fig. V. Sint l & d duo foci Ellipseos; t & u duo puncta veri motū Planetæ; arcus Ellipseos $t u$ ex l spectatus sub angulo $t lu$, & ex d , sub angulo $t du$; item distantia focorum ld ex t spectatus sub angulo dtl , & ex u , sub angulo dtu : Aio, differentiam angulorum $t lu$, $t du$, a qualem esse differentię angulorum dtl & dtu .

Cū enim trianguli lux tres anguli simul sumpti æquales sint trianguli dtx tribus angulis simul sumptis; si auferantur utrinque æquales lxu & dxt , reliq:orū duorum summam $ulx + lux$ erit = summæ reliquorum $tdx + dtx$, & ab his æqualibus summis si auferantur inæquales, v. g. ulx ex priori, & tdx ex posteriori, reliquorum, lux & dtx , differentia = est differentiæ ablatorum ulx & dtx ; quod erat propositum.

Centro l , intervallo axis transversi mn , describatur Circulus abc , cujus arcus ab rursus ex l spectatur sub angulo alb , & ex d , sub angulo adb ; item distantia focorum ld ex a spectatur sub angulo lad , & ex b , sub angulo lbd . Ergò rursus differentia angulorum alb & adb = est differentię angulorum lad & lbd . Sed per *Coroll. I.* angulus lad semis est anguli lad , & angulus lbd semis anguli $lt d$. Ergò horum angulorum lad & lbd differentia = est semi-differentiæ angulorum lad & $lt d$; ergò & angulorum alb & adb differentia = est semi-differentiæ angulorum ult & ndt , quorum prior est intervallum apparenſ duarum Observationum, posterior autem, intervallum motū mediū. Datā igitur horum intervallorum differentiā, datur quoque hujus (*differentia*) semis, nimir. differentia angulorum alb & adb . Sed alb idem est cum ul dato; Ergo datur quoque adb angulus, sub quo peripheria ab spectatur ex d .

Simili modo ostendetur, differentiam angulorum tly & tdy aequalē esse summæ angulorum $lt d$ & lyd ; nec non differentiam angulorum $b1c$ & bdc = esse summæ angulorum lbd & lcd . Cumque lbd semis sit ipsius $lt d$, & lcd semis ipsius lyd ; erit sanè summa ipsorum lbd & lcd = semi-summæ angulorum $lt d$ & lyd , hoc est, differentia angulorum $b1c$ & bdc = erit semi-differentia angulorum tly & tdy , quorum prior est intervallum apparen̄s duarum Observationum, posterior autem, intervallum motūs medii. Quare, data horum intervallorum differentiā, datur quoque hujus semis, nimir. differentia angulorum $b1c$ & bdc . Sed $b1c$ idem est cum tly dato; Ergo datur quoque bdc angulus, sub quo peripheria bc spectatur ex d .

Unde liquet, ex datis intervallis Observationum mediis & apparentibus, dari angulos, sub quibus ex d spectantur Circuli abc peripheriae quotvis, interceptæ à lineis veri motūs. Ergo, per Herigoni Theor. Plan. I. I. c. 3. Prop. 12. Schol. 1. totidem Circuli segmenta describi possunt, capacia angulorum, sub quibus isti arcus conspiciuntur ex d , quæ segmenta omnia se mutuo intersecabunt in d . Possunt igitur & hac Methodo inveniri Apogea & Excentricitates Planetarum, delineatione Geometricâ, adhibitis Observationibus quotvis; nec difficilis est, Circulos ducere, quam lineas rectas.

Sed ut demus id, quod verum est, Clarissimi Cassini delineationem Geometricam non-nihil expeditiorem esse; verendum est interim, ne, si $\alpha\pi\beta\gamma\delta\epsilon\eta\tau$ Astronomis expertam sectemur, Diagrammata requirat enormous magnitudinis, adeoque operosior evadat, quam ipse Calculus. Ad hunc autem accedentes, utramque Methodum aequipollere deprehendemus.

Adhibeamus enim ex Observationibus Tychonicis tres, quæ Dom. Cassini Diagrammati quodammodo consentiant; nimir. Observationem A, cum An. 1604, Mart. 28 d. 16 h. 23 m. Mars observatus fuit in $\approx 18 g. 37 m. 10 s.$ B, cum An. 1587, Mart. 6 d. 7 h. 23 m. idem Planeta visus fuit in $\approx 20 g. 43 m. 0 s.$ Denique C, cum An. 1600 Jan. 18 d. 14 h. 2 m. deprehenderetur in $\approx 8 g. 38 m. 0 s.$ Est igitur inter A & B intervallum apparen̄s $22 g. 54 m. 10 s.$ & huic respondens medium $25 g. 58 m. 40 s.$; at inter B & C intervallum apparen̄s $47 g. 5 m. 0 s.$ & medium $56 g. 21 m. 57 s.$ Itaque

(1173)

Methode Cassini, Fig. II.

1. In Triangulo DBH,
Dantur DB 10,00000
DBH 12 | 99
BDH 11 | 45
Quæritur BH 9,68 106

2. In Triangulo DBG.
Dantur DB 10,00000
DBG 28 | 18
BDG 23 | 54
Quær. BG 9,70653

3. In Triangulo HBG.
Dantur BH 9,68 106
BG 9,70653
HBG 41 | 17
Quær. BGH 64 | 95
Cujus Compl. GBI 25 | 05
Si auferas ex GBD 28 | 18
Restat IBD vel IBL 3 | 13

4. In Triangulo GB I.
Dantur BG 9,70653
GIB 90
GBI 25 | 05
Quær. BI 9,66363

5. In Triangulo IBL.
Dantur BI 9,66363
BL (semis τε BD) 9,69897
IBL 3 | 13
Quær. BLI 32 | 31 An. ve-
ra, & LI, 8,67284, Ex-
centricitas.

Methode Herigoni, Fig. V.

1. In Triangulo dbb,
Dantur db 10,00000
adb externus 24 | 44
bhd 11 | 45
Quær. bh 10,31894

2. In Triangulo dbg,
Dantur db 10,00000
cdb externus 51 | 72
bgd 23 | 54
Quær. bg 10,29347

3. In Triangulo hbg,
Dantur bh 10,31894
bg 10,29347
hbg 41 | 17
Quær. bbg (vel bbi) 64 | 95 = bsg
Et bbi = sgb = 90
Ergo bbi = gbs = 25 | 05
Ex gbi = gbs + sbi (= hbg - bbi) = 16 | 12
Aufer dbb = bbi - dbi = 12 | 99
Restat gbs + sbi - bbi + dbi = sbd (vel dbl) 3 | 13

4. In Triangulo gbs
Dantur bg 10,29347
bgs 90
gbs 25 | 05
Quær. bs 10,33637

5. In Triangulo dbl,
Dantur bd 10,00000
bl (semis τε bs) 10,03534
dbl 3 | 13
Quær. bl d 32 | 31 Anom. vera
Et ld 9,00926 Excentricitas.

Nimir. Ut Fig. II. BL 9,69897, ad LI, 8,67284;
Ita Fig. V. bl 10,03534, ad ld 9,00926.

Ex loco apparenti secundæ Observationis
auferatur angulus Anomalie veræ B LI
Restat locus Apogei

	s.	g.	m.	sec.
	5	25	43	0
	1	2	18	36
	4	23	24	24

Erat autem revera ævo *Tychonis Apogeon Martis* in Ω $28\frac{1}{2}$ d., à quo deficit iste locus, calculo inventus, solidis quinque gradibus. Porro, Ut B L 9, 69897, } Ita 5,18290 Log-us 152369 distantia med. ðtis, ad L I 8,67284; } ad 4,15677 Log-um 14347 Excentricitatis ðtis.

Est autem vera Excentric. ðtis 14179, quam ista, calculo inventa, excedit $16\frac{8}{9}$ particulis.

Cæterum in ratiocinio secundum utramque Methodum instituto notare licet non modò perpetuam Triangulorum similitudinem, sed & Epilogismi congruentiam; ne quis Apogei & Excentricitatis sic inventæ à vero discrepantiam censeat errori Calculi imputandam. Sed nec Observationum viatio contingit; quas in dubium vocare nil aliud foret, quām principia in Astronomia negare. Itaque restat, ut Hypothesin excutiamus.

Et *Ellipticæ* quidem Orbitæ Inventio sine controversia *Keplero* debetur; sed quibus Accelerationis & Retardationis gradibus incedant Planetæ, definire, non minus pertinet ad integrandam Hypothesin, quām ipsius Orbitæ determinatio. Quanquam autem ex Cl. *Cassini* (vel Interpretis ejus) sermone id nusquam apparet; attamen ex Constructione Problematis, & ejus Analysi, manifestum est, eum supponere, Planetam ex foco superiori videri prorsus æquabili motu incedere. Fuit sanè, cum idem ex illis *Kepleris*, quod ejus Scripta evolventibus liquere potest. Sed cum id Observationibus nequaquam congruere animadverteret, mutavit sententiam, & lineam veri motū Planetæ æqualibus temporibus æquales areas Ellipticas verrere professus est: Punctum autem, ex quo Planeta exactè æquabili motu procedere videtur, nullum omnino extare in hoc Universo, nisi id libratile statuere libeat. Nulli interim puncto propriū æquabilem videri incessum Planetæ, quām ipsi foco superiori Ellipseos. Neque inventus fuit hactenus, qui areas *Kepleri* phænomenis satisfacere posse negaret; sed, cum eas Calculo directo exhibere nec ipse nec post eum quisquam potuerit, causati sunt nonnulli, *Keplerum*, nimis indulgentem causis *Physicis*, à *Geometria* diversum abiisse; quasi causæ physicæ repugnant Geometriæ, aut minus Geometricum sit Problema, quod, nullā impacta physicarum causarum mentione, sic proponitur: *Data area Trilinei, inter lineas absidum, & veri motus, nec non peripheriam Ellipticam intercepti, invenire Angulum ad Solem.* Habent igitur à *Keplero* respondsum, qui illi *æquationes* objiciunt; nim. *Eant ipsi & Schema solvant.*

Quamvis autem religio fuerit *Keplero*, ab Hypothesi, quam *Naturam esse* planè persuasum habebat, recedere; quidni liberum foret aliis periculum facere, num via quævis alia detur, inæqualitatem Planetarum primam directo Calculo investigandi? Ideoque Vir Clariss. *Ism. Bullialdus* aggresus est ratiocinio Geometrico indagare, quā semitā, & quibus intentionis ac remissionis gradibus conveniret Planetas ferri, ut ab æquabili incessū norma, Astronomis ante *Keplerum* assumptā, ad eam, quam spectamus, inæqualitatem perduceremur. Perennant Illustrissimi viri monu-

monumenta, unde omnem hujus Inventi rationem haurire licet Astrophilis. Amplexus eandem Reverendiss. *Seth. Wardus*, primum ostendit, paria facere cum linea æquabilis motū circa alterum Ellipsoes umbilicum gyrata; deinde & Calculi directi methodo ornavit eā, quam paulò ante recitavimus: Ita ut nil amplius desiderari posset, quām ut *Urania* felicibus cæptis annueret. Cujus quidem nomine suscipere ausus fuit Illustris. Comes *Paganus*, edito, biennio post, ejusdem ferè tenoris Scripto, adeò veram esse Hypothesin, ut deprehensam circa Octantes discrepantiam, Astronomorum incitiae tributam mallet. At Cl. *Bullialdus*, audiendam potius ipsam Astronomiam ratus, Observatorum ore loquentem, secundis curis, adhibita prioribus Inventis limitatione quadam, discrepantiam illam exterminavit. Unde porrò intelligitur, Hypothesin illam, cui Cl. *Cassinius* investigationem Apogeorum & Excentricitatum superstruit, tantundem ferè deficere à vero quantum Cl. *Bullaldi* limitatio posset, atque ab illo defectu pullulare eum quem suprà notavimus, Calculi à Cœlo dissensum.

Tantum vero abest, ut de Eximii Viri Inventione vel minimum delibatum velim, ut quicquid hujus lucubratiunculae non hausi ex Reverendiss. *Wardo*, vel *Herigono*, id omne Ipsi libentissime acceptum referam, qui an-sun nobis præbuit hæc altius considerandi. Nec dubitamus, quin omnia ista multò uberiori ac luculentiori in promisso *Traictatu* exposita propediem reperturi simus, cujus Editionem maturam, pro eo qno flagramus divinissimæ Scientiæ amore, perquām avidè expectamus.

An Account of Three Books.

I. *Esercizi intorno alla Generatione Degli Infetti, fatte da Franciso Redi, Accademico della Crusca. In Firenze, A. 1668. in 4o.*

The Learned and Ingenious Author of this Book, lately come to the Publishers hands, though not yet (which is much disliked by the curious) into our Stationers Shops, doth with much industry undertake therein to evince, that there is no such thing as *Aequivocal Generation* but that every Animal is generated by the seed of another Animal, (its parent,) or, at least, from some Living and un-corrupted Plant, as out of Oak-Apples, and several Protuberances and Excrescencies of Vegetables.

First then, in the asserting of the *Universal* and true Generation of Insects by a peculiar and paternal Seed, the Author positively affirms, that he could never find, by all the Experiments and Observations, he ever made (of which he relateth a great number, by himself made upon all sorts of Animals) that ever any Insects were bred from Flesh, or Fish, or putrefied Plants, or any other Bodies, but such, as Flies had access unto, and scatter'd their seed upon; he having taken extraordinary care and pains to observe, that alwayes on the Flesh, before it did verminate, there late Flies of the self same kind with those, that were afterwards produc'd thence; and again, that no Worms would ever come from any Flesh in Vessels well cover'd, and defended from the access of Flies; so that to him there is no generation of Insects from any dead Animals, but such as have been fly-blown.

And least it should be objected, that the reason, why in vessels exactly clos'd, no Insect breeds, is the want of Air, necessary to all Generation, He hath carefully covered several vessels with very fine Naples-vaile, for the Air to enter, though Flyes could not; but that no worms at all were bred there, notwithstanding that many Flyes swarmed about them, invited by the smel of the Flesh inclosed therein.

Secondly, to make out the other part of his Position. viz. That those Animals that are not bred by the seed of other Animals, are produced from some live Plant, or its Excre-

Fig. II.

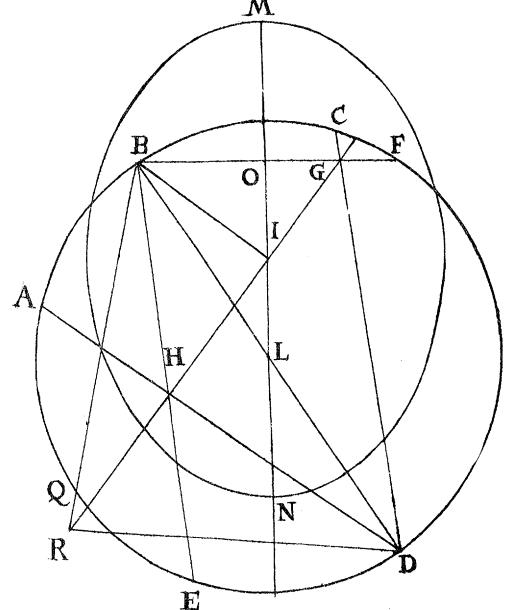


Fig. III.

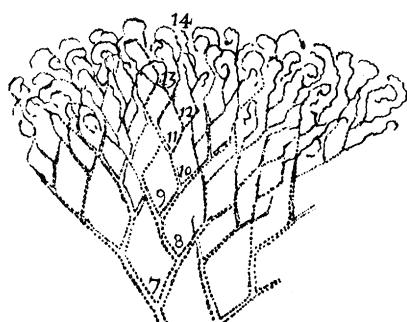
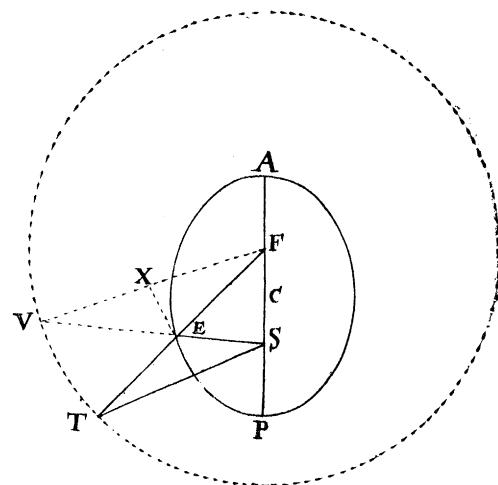


Fig. I.

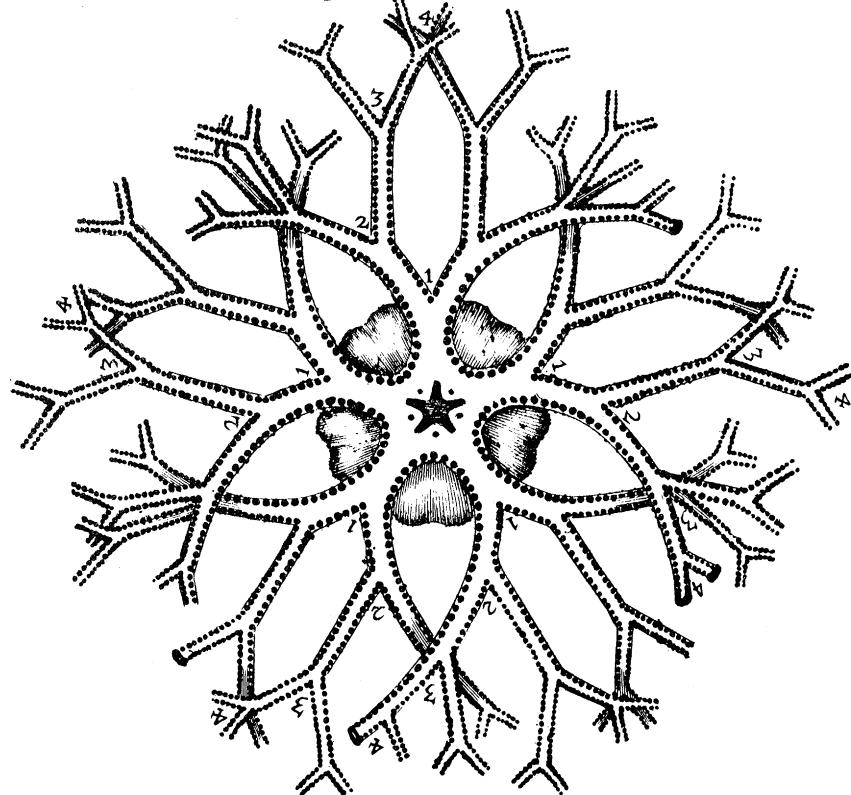


Fig. IV

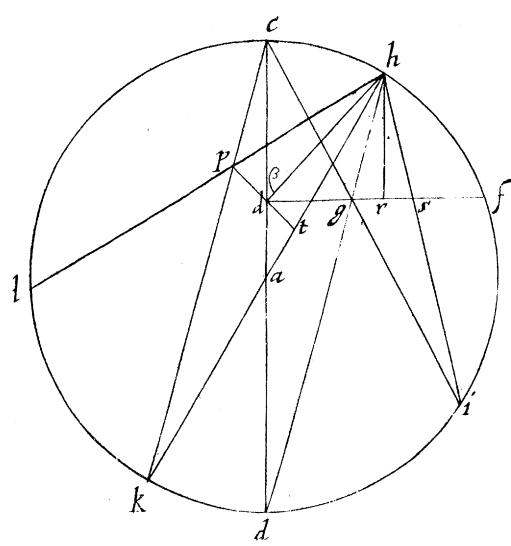


Fig. V.

