NAVIGATION

AND

NAUTICAL ASTRONOMY,

IN THEORY AND PRACTICE.

WITH

ATTEMPTS TO FACILITATE THE FINDING OF THE TIME AND THE LONGITUDE AT SEA.

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PREFACE.

THE following work is an attempt to exhibit, in a moderate compass, the theory and practice of Navigation and Nautical Astronomy;—to supply an elementary manual available for educational purposes at home, and which the young navigator may profitably consult in the exercise of his professional duties at sea.

To those already acquainted with the subject, even a cursory examination of the following pages will suffice to show, that I have ventured to depart from the plan adopted in existing treatises in several important particulars. I have, for instance, been much more sparing in the employment of logarithms, by the aid of which numbers it is usually recommended that every nautical calculation should be performed.

But having long entertained the conviction that the indiscriminate use of logarithms in the simpler operations of trigonometry is injudicious—since in such operations they save neither time nor trouble—I have resolved here to dispense with them in all those computations of navigation, in which the right-angled triangle only enters into consideration, and it is with the right-angled triangle almost exclusively that the practical business of navigation has to do.

In the introductory chapter, I have sufficiently prepared the learner for this innovation—if innovation it be considered; and I am not without hopes that persons whose practical experience in these matters qualifies them to form a correct judgment, will assent to the change thus introduced. Petty multiplications and divisions can be more expeditiously, and more satisfactorily, performed, without the aid of logarithms than with it; and that sort of assistance which rather retards than expedites the end in view, is in fact no assistance at all.

With the exception of this change in the mode of conducting the numerical operations, there will be found little of peculiarity or novelty in the treatment of the navigation proper, unless indeed it be in the uniform blending together of theory and practice. The custom of making a book on navigation to consist of only a collection of authoritative rules, either without any theory at all, or with the investigations thrown together in the form of a supplement or appendix, to be studied or not, as the learner pleases, is one which I think should now be abandoned. More attention is being paid to professional training, better provision for it is supplied, and a higher standard of qualification demanded, than was the case fifty or sixty years ago. And our elementary scientific text-books must harmonise with this improved state of our educational system: it is not enough now that a candidate for professional distinction knows what his book tells him; he must know what it *proves* to him—the *why* as well as the how. But it is more especially from an impartial examination of the *second* part of this treatise—the part devoted to Nautical Astronomy—that I indulge hopes of a favourable reception of my book. In this more advanced and more difficult portion of the subject, I have dispensed with formal "rules" in all cases where verbal precepts and directions would be long and tedious; and instead, have mapped out, as it were, a *blank form* of the route which the calculation is to take. Mathematical formulæ of any complexity are but ill adapted to verbal translation. By a person even but slightly acquainted with algebraical notation, the formula itself will, in general, be preferred for a working model, to the rule derived from it; but a blank form is preferable even to the symbolical expression, inasmuch as this, though indicating all the numerical operations, suggests nothing as to the most convenient way of ordering those operations. Blank forms have, in particular problems, been recommended, and even partially adopted before: but, I believe, not till now systematically given to replace rules; and I have no doubt that they will prove acceptable in actual practice at sea.* I would also invite attention to the manner in which the problem of finding the latitude from a single altitude of the sun off the meridian is discussed, more especially to the practical inferences at page 147. Less consideration than it deserves is given to this problem in former treatises, on account of an affirmed ambiguity in the calculated result. I think it is here shown that the ambiguity complained of is more imaginary than real.

The chapter "On Finding the Time at Sea," page 183, has also, I think, some claim to notice; as I believe I have introduced a practical improvement in the working of this important problem. I would more particularly refer to what is contained between page 192 and the end of the chapter. To the subject of the sixth chapter, "On Finding the Longitude at Sea," I have also—as it deserves—devoted much careful consideration: the part of this chapter to which every person critically disposed will turn, will of course be the article, on *Clearing the Lunar Distance.*

* Steps 1, 2, in the form at p. 143, should stand side-by-side: the narrowness of the page here renders this arrangement impracticable.

Besides the well-known logarithmic process of Borda—here a little modified—I have also given a method in which logarithms may be altogether dispensed with; in which subsidiary tables, and auxiliary arcs are not needed; and in which (besides a little common arithmetic) the whole *operation is performed with the aid of only a single small table—a table of natural cosines.*

Should, however, the computer prefer to use logarithms where I have employed common arithmetic, it is of course optional with him to do so; I have exhibited both modes of proceeding, and if any one takes the trouble to count the number of figures brought into operation in each method, he will find that, on the average, the arithmetical process will not require above half-a-dozen more than the logarithmic: and one advantage of the former is, that the work is more readily revised. This work, in all its details, it is better to preserve rather than to record a mere abstract; and even after the lapse of several hours, if a recurrence to it should lead to the detection of any numerical error, it will not be too late to put all to rights.

There is no merit in devising formulæ and rules for clearing the lunar distance; dozens of them may be easily educed from the same fundamental expression. I have carefully examined and compared all those which different authors have selected, and steadily resisting all bias of judgment in favor of that here proposed, I have been forced to the conclusion that it has a claim to adoption. The method most in esteem at present, is that first given by Krafft of St. Petersburg, which requires a table of versed and suversed sines, and another special table of "Auxiliary Arcs." This latter table is somewhat complicated, and will seldom furnish the arc required to within a second or two of the truth; but the method is nevertheless the most simple and convenient hitherto proposed: whether the dispensing with all such special tables, and thus securing accuracy to the nearest second, will entitle the process I have recommended to a favorable comparison with that just mentioned, others must of course determine.

The tables which are to accompany this work, in conjunction with the logarithmic tables already published in the present series of Rudimentary Treatises, will comprehend all those which are indispensably necessary in Navigation and Nautical Astronomy — and no more than are necessary. The table of natural cosines will give degrees, minutes, and seconds; and will be so arranged that the "Argument" will always appear at the top of the page, so that the extract to be made will always be found by running the eye down the page: there will never be any necessity to proceed upwards, a plan which will of course facilitate the references.

The logarithmic tables just adverted to may be bound up with those now in preparation, but it will be better to keep them distinct. A very little familiarity with them will enable the computer at once to put his hand on that one of the two collections which contains the particular table he wants, which table it will be more easy to find in a small volume than in a large one: the two volumes of tables will be distinguished one from the other by difference of colour in the covers.

I have only further to add, that most of the astronomical examples in this book are accommodated to the Nautical Almanac of the current year, 1858; they all refer to dates in advance of the time when they were framed, and are therefore of course all hypothetical.

J. R. YOUNG.

May, 1858.

*** The Index, or Table of Contents, will be found at the end of the Volume.

☞ The NAVIGATION TABLES, to accompany this Work, will shortly be published: Price, 1*s.* 6*d.*

NAVIGATION AND NAUTICAL ASTRONOMY.

INTRODUCTION.

As this rudimentary treatise is intended principally for the instruction of persons having only a very moderate acquaintance with mathematics, we shall devote a few introductory pages to the practical computations of the sides and angles of plane triangles, a portion of the general doctrine of Trigonometry that is indispensably necessary to the thorough understanding of the rules and operations of NAVIGATION.

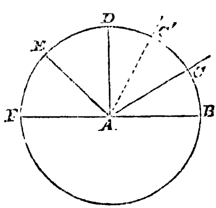
Although the path of a ship at sea is always traced upon a curved surface, and is usually a line of a complicated form, yet it fortunately happens that all the essential particulars respecting this curved line—essential, that is, to the purposes of Navigation—are derivable from the consideration of straight lines only, all drawn upon a plane surface; and the most complicated figure with which we have to deal, in Navigation proper, is merely the plane triangle.

And it may be as well here, at the outset, to apprise the beginner that he is not to suppose that the substitution of the straight line on a *plane* surface, for the spiral curve on a *spherical* surface, in the various computations of navigation, is a contrivance forced upon us on account of the difficulties attendant upon the discussion of the more intricate form of the latter, and that simplicity is attained at the expense of accuracy, by substituting a straight line on a plane, for a curve line on a sphere. It will be shown in the proper place (Chapter II.), that this substitution involves no error at all—that with the curvilinear form of a ship's path we have in reality nothing to do—that, in imagination, the curve may be straightened out, and that everything connected with the course, the distance sailed, and the difference of latitude made, may be accurately embodied in a plane triangle.

The learner will thus readily perceive that a familiarity with the rules for calculating the sides and angles of plane triangles is a preliminary indispensable to the attainment of a sound practical knowledge of navigation; and we therefore earnestly invite his attention to what is delivered in the following introductory chapter, written with an especial view to the declared object of the present rudimentary treatise.

PRELIMINARY CHAPTER.

ON THE COMPUTATIONS OF THE SIDES AND ANGLES OF PLANE TRIANGLES.—Every triangle consists of six *parts*, as they are called—the three sides, and the three angles. As lines and angles are magnitudes quite distinct in kind, we cannot directly combine a line and an angle in calculation, any more than we can combine a mile and a ton. To obviate this difficulty, and to convert all the magnitudes with which trigonometry deals into linear magnitudes only, employed in connection with abstract numbers, certain *trigonometrical lines*, or *numbers* having reference to the angles, are always used in the computations of trigonometry, instead of the angles themselves. It will be shown presently what these trigonometrical quantities are, and how completely they enable us to conduct investigations concerning the sides and angles of triangles, without the latter kind of magnitudes ever directly entering the inquiry: previously to this, however, it will be necessary to explain how angles themselves are measured.



About the vertex A, of any angle B A C, as a centre, let a circle B C D E F be described: the intercepted arc B C will vary as the angle B A C; that is to say, that if the angle change to B A C′, whether greater or smaller than the former, then will the intercepted arc change from B C to B C′, so as to give the proportion

angle B A C : angle B A C′ : : arc B C : arc B C′, as is obvious from prop. xxxiii. of Euclid's sixth book. And this is true whatever be the magnitude of the circle, or the length of the radius AB.

The circumference of the circle is conceived to be divided into 360 equal parts, called *degrees;* so that, from the above proportion, an angle at the centre, subtended by an arc of 40 degrees, is double the angle at the centre subtended by an arc of 20 degrees, three times the angle subtended by an arc of 10 degrees, and so on; and this is true whatever be the radius of the circle described about A.

The degrees of one circle differ of course in length from the degrees of another circle, when the two circles have different radii:—a degree being the 360th part of the circumference, whether the circle be small or great; yet it is plain, that if a circle, whether larger or smaller than that before us, were described about A, the arc of it, intercepted by the sides A B, A C of the angle, would be the same part of the whole circumference to which it belongs, that the arc B C of the circle above is of the whole circumference to which *it* belongs: in other words, the angle at the centre would subtend the same *number* of degrees, whatever be the length of the radius of the circle on which those degrees are measured:—the degrees themselves would be unequal in magnitude, but the *number* of them would be the same.

By viewing an angle in reference to the *number* of degrees in the circular arc which subtends it, as here explained, we arrive at a simple and effective method of estimating angular magnitude: the circular degree suggests the angular degree, which we may regard as the unit of angular measurement—the angular degree being that angle the sides of which intercept one degree of the circle. Angles are thus measured by *degrees*, and fractions of a degree—the measures applied being the same in kind as the quantities measured, just as in all other cases of measurement.

For the more convenient expression of fractional parts, a degree is conceived to be divided into sixty equal portions, called *minutes*, and each of these into sixty equal parts, called *seconds;* further subdivisions are usually regarded as unnecessary, so that whenever it is thought requisite to express an angle with such minute accuracy as to take note of the fraction of a second, that fraction is actually written as such.

The notation for degrees, minutes, and seconds, will be readily perceived from an instance or two of its use: thus, 24 degrees 16 minutes 28 seconds would be expressed, in the received notation, as follows: 24° 16′ 28″; and 4 degrees 9 minutes 12 seconds and three quarters of a second, would be written 4° 9′ 12″¾.

If we were required actually to construct an angle from having its measurement in this way given, and were precluded from the use of any peculiar mechanical contrivance for this purpose, we should first draw a straight line, as A B, in the preceding diagram; then, with the extremity A as centre, and with any radius that might be convenient, we should describe a circle B D F, &c.: the circumference of this circle we should divide into 360 equal parts, or the half of it, B E F, into 180 equal parts, or the fourth of it (the quadrant), B D, into 90 equal parts; we should then count from B as many of these parts, or *degrees*, as there are in the measure of the angle, adding to the arc, made up of these degrees, whatever fractional part of the next degree, in advance, was expressed by the minutes and seconds: the whole extent (B C) of arc, subtending the angle to be constructed, would thus be discovered; and by drawing A C, the required angle B A C would be exhibited. But the practical difficulties of all this would be very considerable, if not insurmountable; they need not, however, be encountered, as instruments for constructing angles, and for measuring those already constructed, are easily procurable: the common *protractor*, with which all cases of mathematical instruments is furnished, enables us speedily to effect the business with sufficient accuracy for all ordinary purposes. It is simply a semicircular arc divided into degrees, as above described, with the centre marked on the diameter connecting its extremities.

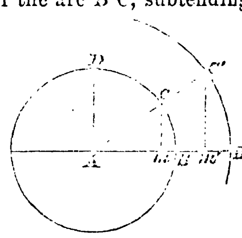
But the construction, or measurement, of angles upon paper, is a mechanical operation with which we have nothing to do in calculations respecting triangles; and we have adverted to it solely for the purpose of giving greater clearness and precision to the student's conception of angular measurement; to satisfy him, in fact, that the numerical expression for the value of any angle—using the notation explained above—does really convey an accurate idea of the amount of opening it refers to, and furnishes a sufficient datum for the actual construction of the angle, supposing no merely mechanical difficulties to stand in the way. Referring again to the diagram, at page 2, we have further to remark, that what must be added to any arc, or subtracted from it, to make it become a quadrant, or an arc of 90°, is called the *complement* of that arc: thus, C D is the complement of the arc B C; and D E, taken subtractively, is the complement of the arc B E. In like manner, what must be applied to an arc to make it a semicircumference, or 180°, is called the *supplement* of that arc: thus, C D F is the supplement of the arc B C, and E F the supplement of the arc B C E. The same terms apply to the angles subtended: thus, the angle C A D is the complement of the angle B A C; and the angle D A E, taken subtractively, is the complement of the angle B A E. In like manner, the angle C A F is the supplement of the angle B A C, and the angle E A F the supplement of the angle B A E.

For example, the complement of 24° 12′, whether we refer to arc or angle, is 65° 48′, and its supplement is 155° 48′.

The Trigonometrical Sines, Cosines, &c.

It has already been observed, that as an angle and a straight line cannot possibly be combined in any numerical calculation, it is necessary to employ either lines or abstract numbers instead of angles in all the rules and investigations of trigonometry, the quantities thus employed being, of course, such as to always suggest or indicate the angles themselves: we deduce them as follows:—

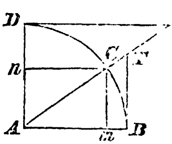
The SINE.—From the extremity C, of the arc B C, subtending the angle B A C at the centre of the circle, let a perpendicular C*m* be drawn to the radius A B: this perpendicular is the *sine* of the arc B C. The sine of an arc is, therefore, a line which may be thus defined :—It is the perpendicular, from the *end* of the arc, to the radius drawn to the *beginning* of that arc: all the arcs considered are supposed to have their origin at B.



Just as from the measure of an arc we derive the measure of the angle it subtends, so from the sine of an arc we deduce the sine of the angle. It would not do to regard, without any modification, the sine of an arc as the sine of the angle it subtends, because, though the angle remain unchanged, the subtending arc—and consequently the sine of it—may be of any length whatever, in the absence of all limitation as to the length of the radius. In order, therefore, that every angle may have a fixed and determinate sine, the radius is always regarded as the linear representation of the numerical unit or 1, upon which hypothesis it is plain that the sine of an angle will always be the same fraction, since $\frac{Cm}{AC}$ is always equal to $\frac{C'm'}{AC'}$ (Euc. 4. vi.), so that the fraction alluded to is no other than the ratio of the sine of the arc to the radius. It is this ratio or fraction that is called the *trigonometrical sine*, or sine of the *angle;* it is an *abstract number:* the sine of the *arc* is called the linear or *geometrical* sine:—it is *a straight line.*

The Cosine.—The cosine of the arc B C is the portion Am of the radius intercepted between the centre and the foot of the sine of the arc. The trigonometrical cosine, or the cosine of the angle B A C, which the arc subtends, is the numerical representation of Am conformably to the scale A B = 1. In other words, it is the ratio or fraction $\frac{Am}{AC}$ or $\frac{Am'}{AC'}$

The Tangent.—The tangent of the arc B C is the straight line B T, touching the arc at its commencement B, and terminating in T, where the prolonged radius through the end C of the arc meets it. The trigonometrical tangent, or tangent of the angle B A C, is the numerical value of the same line on the hypothesis that A B = 1. In other words,



it is the ratio $\frac{BT}{AB}$, for A B : B T : : 1 : $\frac{BT}{AB}$ the trig. tangent.

The Cotangent.—The cotangent of the arc B C is the line Dt, touching the complement of that arc at D, and terminating in A C prolonged. The trigonometrical cotangent, or cotangent of the angle B A C, is the numerical value of the same line, on the hypothesis that A B = 1. In other words, it is the ratio $\frac{Dt}{AD}$, for

$$AD : Dt : : 1 : \frac{Dt}{AD}, \text{ the trigonometrical cotangent.}$$

The Secant.—The secant of the arc B C is the line A T from the centre up to the tangent: its numerical value on the hypothesis of A B = 1, is the trigonometrical secant, or secant of the angle B A C. This numerical value is the ratio $\frac{AT}{AB}$, for

The COSECANT.—The Cosecant of the arc B C is the line At from the centre up to the cotangent: the trigonometrical cosecant, or cosecant of the angle B A C is the numerical value of At on the hypothesis that A D = 1; this value is the ratio $\frac{At}{AD}$, for AD: At : : 1 : $\frac{At}{AD}$. The learner will perceive that cosine, cotangent, and cosecant are nothing more than the sine, tangent, and secant of the *complement* of the arc or angle, the commencement of the complemental arc being considered as at D. It is also further obvious that any geometrical sine, cosine, &c., if divided by the radius of the arc with which it is connected will give the sine, cosine, &c., of the angle which that arc subtends at the centre: these trigonometrical quantities, though all pure numbers, may, as already explained, be represented by *lines*—the same lines as those employed in connection with the arc, provided only we agree to regard the radius of that arc as the linear representation of the unit 1. The advantage of thus regarding the radius as *unit* is that we can investigate the relations among the trigonometrical quantities defined above without introducing the radius as a divisor, since a unit-divisor may always be suppressed, and may avail ourselves of the aid of geometry for this purpose. Thus, referring to the right-angled triangles in the preceding diagram we have from Euclid, Prop. 47, Book I.

$$Cm^2 + Am^2 = AC^2,\ AT^2 = AB^2 + BT^2,\ At^2 = AD^2 + Dt^2$$

that is, the radius being regarded as = 1, and the angle being represented by A,

$$\sin^2 A + \cos^2 A = 1,\ \sec^2 A = 1 + \tan^2 A,\ \text{cosec}^2 A = 1 + \cot^2 A. \ldots (1)$$

Again, because the sides about the equal angles of equiangular triangles are proportional, the triangles ACm, ATB, AtD furnish the following proportions, namely:—

$$\begin{array}{l} \cos A : \sin A :: 1 : \tan A \\ \sin A : \cos A :: 1 : \cot A \\ \cos A : 1 :: 1 : \sec A \\ \tan A : 1 :: 1 : \cot A \\ \sin A : 1 :: 1 : \text{cosec } A \end{array}$$

$$\therefore \tan A = \frac{\sin A}{\cos A},\ \cot A = \frac{\cos A}{\sin A}, \quad . \ . \ . \ . \quad (2)$$

$$\sec A = \frac{1}{\cos A},\ \cot A = \frac{1}{\tan A},\ \text{cosec } A = \frac{1}{\sin A}, \quad . \ . \ . \quad (3)$$

From the relations (1) we see that

$\sin A = \sqrt{(1-\cos^2 A)}$, $\cos A = \sqrt{(1-\sin^2 A)}$, $\sec A = \sqrt{(1+\tan^2 A)}$
$\operatorname{cosec} A = \sqrt{(1+\cot^2 A)}$ . . . . . . (4)

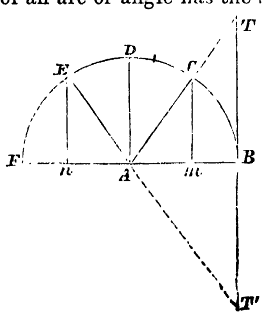
And from these, in conjunction with (2) and (3), we further see that when either the sine or the cosine of an angle is known, all the other trigonometrical values may be computed from it. Moreover, from (3) it appears that the following pairs of values are the reciprocals of each other, namely—

Sec A, cos A ; tan A, cot A ; sin A, cosec A.

But it would be out of place here to discuss the relations among the trigonometrical values at any greater length: for a more comprehensive view of the general theory of these quantities, the learner is referred to the rudimentary treatise on Trigonometry.

On the Trigonometrical Tables.

The numerical values of all the trigonometrical lines, conformably to the hypothesis, that radius = 1, are carefully computed, for all angles from A = 0° up to A = 90° and arranged in a *table.* Such a table is called a table of *natural* sines, cosines, &c., to distinguish it from a table of *logarithmic* sines, cosines, &c., to be hereafter adverted to. In the construction of such a table it is unnecessary to compute for angles above 90°, for, as a little reflection, on reference to the diagrams in which the trigonometrical lines are exhibited, is sufficient to show, the sine, cosine, &c. of an arc or angle above 90°, is a line of the same length as the sine, cosine, &c. of an arc or angle as much below 90°, so that the sine, cosine, &c., of an arc or angle has the same linear and numerical value as the sine, cosine, &c. of the *supplement* of that arc or angle, and this is a truth that the learner must always keep in remembrance: thus sin 120° = sin 60°, sin 135° = sin 45°, and so on. Similarly of the cosines, tangents, &c., only here it is to be noticed that the cosine of a supplemental arc lies in an *opposite direction* to the cosine of the arc itself, remembering that all arcs here considered are supposed to have one common origin or commencement:—the origin B in the preceding diagrams. For instance, if DE be made equal to DC the arc BE will be the



supplement of the arc BC, both arcs commencing at B, and it is this arc BE that we are to deal with as the supplement of BC, and not the equivalent arc FC. The cosine of the supplement of BC is therefore A*n*, a line which, though the same in length, is directly opposed in situation to the cosine A*m* of the arc BC. This opposition of direction we have means of indicating algebraically: the opposite signs + and — furnish these means, so that instead of writing

$$\cos 120° = \cos 60°, \text{ we should write } \cos 120° = -\cos 60°.$$

Similarly for the tangents: the tangent of BC is drawn from B upwards to meet the dotted line marking the prolongation of the radius through C; but the tangent of the supplemental arc B E is drawn downwards to meet the dotted line marking the prolongation of the radius E A—agreeably to the definition of the tangent. The two tangents though equal in length, being opposite in direction, we accordingly write $\tan 120° = -\tan 60°$. It is sufficient, however, that we know whether the *cosine* be plus or minus, in order to enable us to pronounce upon the algebraic sign of any other of the trigonometrical quantities belonging to an arc or angle between 0° and 180°: thus the equations (2), page 7, give us tangent and cotangent, and the others secant and cosecant. It is not our business to explain here how the natural sines, cosines, &c., are computed; as may be easily imagined, the work is of a very laborious character, but tables having been constructed once for all, there is no occasion for a repetition of the labour.

As to the use of such a table in facilitating calculations respecting the sides and angles of plane triangles, we offer the following explanations:

Referring to the diagram at page 6, we see that A*m* C, A B T are two similar right-angled triangles. In the first of these, 1 is the numerical value or representative of A C; in the second, 1 is the numerical representative of A B, the other sides being estimated according to this scale: the numerical values of these sides, for all values of the angle A, from A = 0° up to A = 90°, the values increasing minute by minute, are arranged in the table, the values of *m* C, A*m*, under the heads of sine and cosine of the angle A; and the values of B T, A T under the heads of tangent and secant of the same angle.

It follows, therefore, whatever right-angled triangle we may have to consider in actual practice, that the numerical values of the sides of *two* right-angled triangles, *similar* to it, will always be found already computed for us in the table. For instance, suppose we were dealing with a right-angled triangle of which the angle at the base is 34° 27′; we turn to the table for 34° 27′, this particular value of the angle A, and against it we find, under the head *Sine*, the number for mC, and under the head *Cosine*, the number for Am; and we know already that the number for A C is 1. Thus we know completely the numerical values of all the sides of a triangle AmC similar to that proposed for consideration: these values, as furnished by the table, are

$$mC = \cdot 56569,\ Am = \cdot 82462, \text{ and } AC = 1;$$

or, using the trigonometrical names by which these are called,

$$\sin 34° 27' = \cdot 56569,\ \cos 34° 27' = \cdot 82462, \text{ and rad} = 1.$$

Now although these tabular numbers are all abstract numbers, yet there is no hindrance to our regarding them as so many feet, or yards, or miles, provided only we take care to regard the radius as 1 foot, yard, or mile.

Again referring to the table for the particulars connected with the other similar triangle A B T, we find

$$BT = \cdot 68600,\ AT = 1\cdot 21268, \text{ and } AB = 1$$

or, using the language of the table,

$$\tan 34° 27' = \cdot 68600,\ \sec 34° 27' = 1\cdot 21268,\ \text{rad} = 1.$$

Suppose the hypotenuse of the triangle proposed to us is 56 feet, then comparing our triangle with the tabular triangle ACm, of which the hypotenuse is 1 foot, we know that, as the hypotenuse of the proposed triangle is 56 times the hypotenuse A C, the perpendicular of the former must be 56 times Cm, and the base 56 times Am: hence

$$56 \sin 34° 27' = \text{the required perpendicular},$$
$$\text{and } 56 \cos 34° 27' = \text{the required base}.$$

The work is as below—

sin 34° 27′=·56569	cos 34° 27′=·82462
56	56
339414	494772
282845	412310
The perpendicular=31·67864 feet	The base=46·17872 feet.

We see by this illustration that of the two tabular triangles

A*m*C, A B T, we do not take either, at random, to compare with the triangle under consideration:—we select that of the two in which the *radius* (1) corresponds to the side whose length is *given*. Such a selection is always to be made. If, for instance, the *base* of a right-angled triangle be given—say equal to 47 feet, and it be required, from this and the angle at the base—say 34° 27′, as before, to compute the perpendicular and hypotenuse, we then refer to the table for the triangle A B T; because in this it is the *base* that is 1: we thus have

47 tan 34° 27′ = the required perpendicular,
and 47 sec 34° 27′ = the required hypotenuse,

the work being as follows:

tan 34° 27′ = ·686	47 sec 34° 27′ = 47 ÷ cos 34° 27′
47	·8246)47 (57 ft. the hypotenuse.
4802	41230
2744	57700
The perpendicular 32·242 feet.	57722

These illustrations will, we think, suffice to convey to the learner a clear idea of the use of a table of natural sines, cosines, &c., in the solution of right-angled triangles; and we may, therefore, proceed at once to discuss the several cases that occur in practice. To oblique-angled triangles we shall devote a distinct article; but it may be well to apprise the learner, that nearly all the calculations concerning the course and distance sailed of a ship at sea, involve the consideration of right-angled triangles only.

Solution of Right-angled Triangles.

Of the six parts of which every triangle consists—the sides and the angles—any *three*, except the three angles, being given, the remaining three may be found by calculation. In a right-angled triangle *one* angle is always known, namely, the right-angle, so that it is sufficient for the solution that any two of the other five parts (except the two acute angles) be given. In a right-angled triangle, therefore, the given parts must be either

1. A side and one of the acute angles;
or 2. Two of the sides.

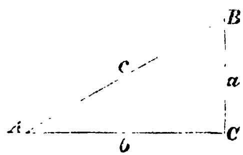
The reason why a knowledge of the three angles of a triangle will not enable us to find the sides, is that *all* triangles that are similar to one another, however their sides may differ, have the three angles in any one respectively equal to the three angles in any other; so that with the same three angles, an infinite variety of triangles may be constructed.

It follows, therefore, that in every practical example that can occur, the given quantities must be such as to place the example under one or other of the following four cases:

I. The hypotenuse and one of the acute angles given.
II. The base or perpendicular, and an acute angle given.
III. The hypotenuse and one of the other sides given.
IV. The base and perpendicular given.

We shall consider these four cases in order.

I. *The hypotenuse and one of the acute angles given.*——In the right-angled triangle in the margin, let the hypotenuse A B and the angle A at the base be given, and let the numerical values of the three sides be denoted by a, b, c, these small letters corresponding to the large letters denoting the opposite angles. We are to compare this triangle with the tabular triangle having the same base angle A, and of which the hypotenuse is 1: the perpendicular of this tabular triangle will be sin A, and its base cos A (see diagram, p. 5). And since our given hypotenuse is c times that of the tabular triangle, the perpendicular of our triangle must be c times that in the table, and the base, c times the tabular base; that is to say, for the required perpendicular and base we shall have



$$a = c \sin A, \text{ and } b = c \cos A.$$

If, however, the vertical angle B were given instead of the base angle A, then, since these angles are the complements of each other, we should have sin A = cos B, and cos A = sin B, so that

$$a = c \cos B, \text{ and } b = c \sin B.$$

Hence we deduce the following rule:

RULE 1. *For the Perpendicular.*—Multiply the given hypotenuse by the sine of the angle at the base, or by the cosine of the vertical angle.

2. *For the Base.*—Multiply the given hypotenuse by the cosine of the angle at the base, or by the sine of the vertical angle.

EXAMPLES.

1. In the right-angled triangle A B C are given the hypotenuse $c=48$ feet, and the angle $A=37° 28'$, to find the perpendicular a, and the base b, as also the angle B.

	sin 37° 28′=·6083	cos 37° 28′=·7937
A + B=90°	48	48
A=37° 28′	48664	63496
∴B=52° 32′	24332	31748
	a=29·1984 feet.	b=38·0976 feet.

2. Given $c=63$ yards, and $A=24° 19'$; to find a and b, as also B.

	sin 24° 19′=·4118	cos 24° 19′=·9113
A + B=90°	63	63
A=24° 19′	12354	27339
∴B=65° 41′	24708	54678
	a=25·9434 yards.	b=57·4119 yards.

3. Given the vertical angle $B=33° 12'$, and the hypotenuse $c=98$ feet, to find the remaining parts of the triangle.

cos 33° 12′=·8368	sin 33° 12′=·5476
98	93
66944	43808
75312	49284
a=82·0064 feet	b=53·6648 feet.

Also $A=90° - 33° 12'=56° 48'$.

In the foregoing operations only *four* decimal places have been taken from the table—a number of places amply sufficient for all the purposes of Navigation.

NOTE.—The learner will not forget that when one acute angle of a right-angled triangle is given, the other is virtually given, being the complement of the former; whenever therefore a side and an acute angle are given, we may always regard the angle *adjacent* to the given side as given. Now it will save the necessity of all reference to diagrams and formulæ or rules, if with the vertex of this adjacent angle as centre, and the given side as radius, we conceive an arc to be described, and notice whether the *required side* becomes a sine, a cosine, a tangent, or a secant; for, whichever of these it is, *that* is the name of the trigonometrical quantity to be taken from the table, for a multiplier of the given side, in order to produce the required side. The table is, of course, to be entered with that angle whose vertex is thus taken for centre.

4. The hypotenuse of a right-angled triangle is 38 feet, and the angle at the base 27° 42′: required the other sides, and the vertical angle.

Ans. $a=17{\cdot}662$ feet, $b=33{\cdot}645$ feet. $B=62°\ 18'$.

5. The hypotenuse is 76 feet, and the vertical angle 43° 18′: required the perpendicular and base.

Ans. $a=55{\cdot}313$ feet, $b=52{\cdot}121$ feet.

6. The hypotenuse is 521 feet, and the vertical angle 36° 6′: required the other sides.

Ans. $a=420{\cdot}97$, $b=306{\cdot}97$ feet.

II. *Base or perpendicular and one of the acute angles given.*—Let the base AC and the angle A be given: then we have to compare our triangle with that one of the two similar tabular triangles, whose base (the radius) is 1. The perpendicular of this tabular triangle will be tan A, and its hypotenuse sec A. (See diagram p. 6). And since our given base is b times that of the tabular triangle, our required perpendicular must be b times that of the tabular one, and our required hypotenuse also b times that of the tabular one, hence the required perpendicular and hypotenuse will be

$$a=b \tan A, \text{ and } c=b \sec A.$$

If it be the vertical angle B that is given instead of the base angle A, then since

$$\cot B=\tan A, \text{ and } \operatorname{cosec} B=\sec A,$$

we shall have

$$a=b \cot B, \text{ and } c=b \operatorname{cosec} B.$$

In the Tables, the secants and cosecants are frequently omitted, because from the fact that secant is 1 divided by cosine, and cosecant 1 divided by sine, they may be dispensed with. (See p. 7.) Making, therefore, these substitutions for secant and cosecant above, and remembering also that cotangent is 1 divided by tangent, the values of a and c may be expressed thus:

$$a = b \tan A, \text{ and } c = \frac{b}{\cos A}$$

$$a = \frac{b}{\tan B}, \text{ and } c = \frac{b}{\sin B}$$

and these expressions furnish the following rules:

Rule 1. *For the perpendicular.* Multiply the given base by the tangent of the base angle: or divide it by the tangent of the vertical angle.

2. *For the hypotenuse.* Divide the given base by the cosine of the base angle: or by the sine of the vertical angle.

NOTE.—As either of the two sides may be considered as *base*, if the perpendicular be given, namely, CB instead of AC, we have only to conceive the triangle to be turned about till base and perpendicular change positions, and then to apply the rule. (See also the NOTE at p. 13, the directions in which will enable the learner to dispense with formal rules).

1. At the distance of 85 feet from the bottom of a tower, the angle of elevation A of the top is found to be 52° 30′: required the height of the tower.

Here the base and the base angle are given to find the perpendicular, as in the margin. Hence the height of the tower may be concluded to be $110\frac{3}{4}$ feet.

$$\begin{array}{r} \tan 52° 30' = 1{\cdot}3032 \\ 85 \\ \hline 65160 \\ 104256 \\ \hline \text{Height} = 110{\cdot}7720 \text{ feet.} \end{array}$$

If the angle of elevation A, be taken not from the horizontal plane of the base of the tower but from the eye, by means of a quadrant or other instrument, then, of course, the height of the eye above that plane must be added. If in the present case the height of the eye be $5\frac{1}{4}$ feet, then the height of the tower will be 116 feet.

2. Required the length of a ladder that will reach from the point of observation to the top of the tower in the last example.

Here the base and base angle are given to find the hypotenuse, as in the margin. We conclude, therefore, that the length of the ladder must be 140 feet nearly.

$$\cos 52° 30' = \cdot 60{,}8{,}8)85 \ (140$$
$$\begin{array}{r} 609 \\ \hline 241 \\ 243 \\ \hline \end{array}$$

The division in the margin is what is called *contracted* division, which saves figures, and which may always be employed for this purpose whenever the divisor has several decimals. (See the Rudimentary Arithmetic.) The learner may easily prove the correctness of the two results in this and the former example by applying the principle of the 47th proposition of Euclid's fifth book, namely, that in a right-angled triangle the hypotenuse is equal to the square root of the sum of the squares of the other two sides: thus—

$$\begin{array}{r} 85 \\ 85 \\ \hline 425 \\ 680 \\ \hline 7225 \\ \hline \end{array} \qquad \begin{array}{r} 110 \cdot 8 \\ 110 \cdot 8 \\ \hline 8864 \\ 12188 \\ \hline 12276 \cdot 64 \\ \hline \end{array} \qquad \begin{array}{r} \text{Square of perp.} = 12277 \\ \text{Square of base} = 7225 \\ \hline 19502(140 \\ 1 \\ \hline 24)\ 95 \\ 96 \\ \hline \end{array}$$

∴ Hypotenuse = 140 ft. nearly.

Minute decimals are of course disregarded in all practical operations of this kind.

3. From the top of a ship's mast, 80 feet above the water, the angle of depression of another ship's hull was found to be 4°: required the distance between the ships.

The angle of depression is the angle between the horizontal line from the mast-head, and the slant line from the same point to the distant ship. The complement, therefore, of this angle is the angle B of the triangle ABC (p. 12), where A is the distant ship, and B the mast-head whence the angle of depression is taken. The angle B, therefore, is 90°—4°=86°, and the perpendicular or side, BC, adjacent to this being given, we have for AC the value $b = a \tan B = 80 \tan 86°$, and the work is as in the margin, from which it appears that the distance is 1144 feet.

$$\begin{array}{r} \tan 86° = 14 \cdot 3007 \\ 80 \\ \hline 1144 \cdot 056 \\ \hline \end{array}$$

4. From the edge of a ditch 18 feet wide, and which surrounded a fort, the angle of elevation of the top of the wall was found to

be 62° 40′; required the height of the wall, and the length of a ladder necessary to scale it.

Here $A = 62° 40'$ and $b = 18$: to find a and c.

tan 62° 40′ = 1·9347	cos 62° 40′ = ·4,5,9,2)18 (39·2 = c.
18	1378
154776	422
19347	413
a = 34·8246 feet	9
	9

Hence the height is 34·8 feet, and the length of the ladder 39·2 feet.

5. A flagstaff, known to be 24 feet in length, is observed to subtend an angle of 38′ at a ship at sea, and the angle of elevation of the cliff on the edge of which the staff is planted is also observed to be 14°. What is the distance of the ship from the cliff?

The distance or base of the triangle being b, it is plain that,—

$b \tan 14° 38'$ = the height of the top of the staff,
and $b \tan 14°$ = ,, ,, cliff,
$\therefore b(\tan 14° 38' - \tan 14°)$ = length of staff = 24 feet.

$$\therefore b = \frac{24}{\tan 14° 38' - \tan 14°} \text{ feet.}$$

tan 14° 38′ = ·26110
tan 14° = ·24933

·01,1,77)24 (2039 feet, the distance.
2354
46
35
11
11

6. Given the base 73 feet, and the angle at the base 52° 34′, to find the perpendicular and hypotenuse.

Ans. perp. = 95·365 ft., hyp. = 120·097 ft.

7. Given the base 327 feet, and the vertical angle = 35° 43′; required the perpendicular and hypotenuse.

Ans. perp. 454·8 ft., hyp. 560 feet.

8. From the top of a castle 60 feet high, standing on the edge

of a cliff, the angle of depression of a ship at anchor was observed to be 4° 52′. From the bottom of the castle, or top of the cliff, the angle of depression was 4° 2′. Required the horizontal distance of the ship, and the height of the cliff.*

Ans. dist. of ship 4100 feet, height of cliff, 289 ft.

9. The base of a right-angled triangle is $346\frac{1}{2}$ feet, and the opposite angle 54° 36′; required the perpendicular and hypotenuse. Perpendicular 246·2 ft., hyp. 425·1 ft.

III. *The hypotenuse and one of the other sides given.*—Representing the perpendicular, base, and hypotenuse by a, b, and c, as before, we have seen (p. 12) that:—

$$a = c \sin A, \text{ and } b = c \cos A.$$

$$\therefore \sin A = \frac{a}{c}, \text{ and } \cos A = \frac{b}{c}$$

and these expressions give the following rule:—

RULE.—Divide the perpendicular by the hypotenuse, the quotient will be the sine of the angle at the base. Divide the base by the hypotenuse, the quotient will be the cosine of the angle at the base. A reference to the table will, in either case, give the angle itself. An angle being thus found, the remaining side of the triangle becomes determinable by either of the foregoing rules. Or, without first finding an angle, the remaining side of the triangle may be computed from Euclid 47, I; for since by that proposition, $a^2+b^2=c^2$.

$$\therefore a = \sqrt{(c^2-b^2)} \text{ and } b = \sqrt{(c^2-a^2)}$$

or, which is the same, $a = \sqrt{\{(c+b)(c-b)\}}$ and $b = \sqrt{\{(c+a)(c-a)\}}$

EXAMPLES.

1. In a right-angled triangle are given the perpendicular $a=$ 192 feet, and the hypotenuse $c=240$: to find the angles A, B, and the base b.

The work is as follows:—

For the angle A.	*For the base b.*
240)192(·8=sin 53° 8′	cos 53° 8′=·6
192	and 240×·6=144=b.

$\therefore$ A=53° 8′. $\therefore$ B=90°−53° 8′=36° 52′, and b=144 feet.

* The learner will recollect, from Euclid, 29, I., that the angle of depression of a point A from an elevated point B, is equal to the angle of elevation of B from A.

To find the base b, without first computing the angle A, we have

$$b=\sqrt{\{(c+a)(c-a)\}}=\sqrt{\{432\times 48\}}=\sqrt{20736}$$

```
   20736(144
   1
24)107
    96
284)1136
    1136
```

The operation for this square root is in the margin.

2. Given the hypotenuse $c=54{\cdot}68$ feet, and the base $b=35{\cdot}5$, to find the angles A, B, and the perpendicular a.

```
For the angle A.                          For the perpendicular a.
54·68)35·5   ·(6492=cos 49° 31'           sin 49° 31'=·7606
      32808                                  54·68
       2692                                  ·7606
       2187                                  32808
        505                                 32808
        492                                38276
         13                             a=41·589608 feet.
         11
```

$\therefore$ A$=49°\ 31'$. $\therefore$ B$=90°-49°\ 31'=40°\ 29'$, and $a=41{\cdot}6$ feet.

In computing a, as above, it is plain that several more decimals are calculated than are at all necessary: the contracted method, as exhibited in the margin, dispenses with these superfluous figures (see the Rudimentary Arithmetic).

```
 54·68
  6067
 38276
  3281
    32
41·589
```

To find a independently of the angle A, we have

$$a=\sqrt{\{(c+b)(c-b)\}}=\sqrt{\{90{\cdot}18+19{\cdot}18\}}=\sqrt{1729{\cdot}6524}.$$

The extraction of this square root is exhibited in the margin. And the agreement of the two results is a sufficient confirmation of the accuracy of all the operations.

```
      1729·6524(41·589
      16
 81)  129
       81
  825)4865
      4125
  8308) 74024
        66464
         7560
```

3. Given the hypotenuse $c=200$ feet, and the base $b=118$ feet, to find the angles A, B, and the perpendicular a.

Ans. A=53° 51′, B=36° 9′, $a=161·5$ feet.

4. Given the hypotenuse $c=645$ feet, and the perpendicular $a=407·4$ feet, to find the angles A, B, and the base b.

Ans. A=39° 10′, B= 50° 50′, $b=500$ feet.

IV. *The base and perpendicular given.*—The letters denoting the sides and angles being as before, we have already seen (p. 14) that—

$$\left.\begin{array}{r} a = b \tan A \;\therefore\; \tan A = \dfrac{a}{b} \\ \text{Also } b = a \tan B \;\therefore\; \tan B = \dfrac{b}{a} \end{array}\right\} \text{and } c = \sqrt{(a^2+b^2)}$$

The rule therefore is as follows :—

Rule.—Divide the perpendicular by the base; the quotient will be the tangent of the angle at the base; or, divide the base by the perpendicular; the quotient will be the tangent of the angle at the vertex. An angle being found, the hypotenuse may be computed as already taught; or, from the general expression for c, above, without the aid of an angle.

EXAMPLES.

1. Given the base $b=35·5$, and the perpendicular $a=41·6$, to find A, B, and c.

For the angles A and B.		*For the hypotenuse.*
35·5)41·6(1·1718 = tan 49° 31′		$c=b\div\cos A$.
355	90°	cos. A = ·6,4,9,2)35·5 (54·68
61	∴B=40°29′	32460
355		3040
255		2597
2485		443
65	∴A=49° 31′, B=40° 29′, c=54·68 ft.	389
355		54
295		51
2840		

The same value of c will be given by the formula $c=\sqrt{(a^2+b^2)}$

2. Given $a=7564$ yards, and $b=3987$ yards, to find A, B, and c. Ans., A=62°12′, B=27°48′, $c=8550\frac{1}{2}$ yards.

3. Given the base of an isosceles triangle equal to 71 feet, and the altitude equal to 41·6 feet: required the other parts.

Ans., base angles each =49°31′, vertical angle =80° 58′, each of the equal sides 54·68 feet.

The preceding rules and practical illustrations exhibit, with all necessary fulness, the arithmetical operations which we would recommend always to be adopted by navigators and surveyors in the solution of right-angled triangles. Persons engaged in calculations of this kind, almost invariably use *logarithms;* the work is certainly thus made to appear, in general, somewhat shorter, but a little experience will prove that this greater brevity is attended with an increased consumption of time. The object of logarithms is not so much to save figures as to save time and trouble, and this latter object they signally effect in all the computations of trigonometry, except in those confined to right-angled triangles; and as before remarked, it is with right-angled triangles, almost exclusively, that seamen have to do in calculating the course, distance, &c., of a ship at sea.

Keeping, therefore, the special purposes of the present rudimentary treatise in view, we shall discuss the subject of oblique-angled triangles with less amplification. The following article on logarithms must, however, be previously studied, not only on account of the use of these numbers in the solution of oblique-angled triangles, but also because a familiarity with logarithms is indispensable in the operations of nautical astronomy.

On Logarithms.

Logarithms are a set of numbers contrived for the purpose of reducing the labour of the ordinary operations of multiplication, division, and the extraction of roots, and they are of especial service in most of the practical inquiries of trigonometry and astronomy.

In what has been delivered in the foregoing article, the arithmetical operations referred to, multiplication and division, have entered in so trifling a degree, that no irksomeness can have been experienced in the performance of them, and, therefore, the want of any facilitating principle cannot have been felt.

But the learner will readily perceive that if the work of any of the examples just given had involved the multiplication together of two or three sines or cosines, or successive divisions, by these, the calculations would have become tedious, and the risk of error, in dealing with so many figures, increased. Now as it is the main object of logarithms to convert multiplication into addition, and division into subtraction, the value of these numbers in computations such as those just mentioned is obvious. We shall in this article briefly show how the conversion alluded to is effected.

Two principles fully established in algebra will have to be admitted. (See Rudimentary Algebra).

1. That if N represent any number, and x and y any exponents placed over it, agreeably to the valuation for powers and roots, then,

$$N^x \times N^y = N^{x+y}, \text{ and } N^x \div N^y = N^{x-y}$$

2. That N being any positive number greater than unity, and n also any positive number chosen at pleasure, we can always determine the exponent x so as to satisfy the condition, $N^x = n$.

This last truth being admitted, it follows that every positive number (n) can be expressed by means of a single invariable number (N) with a certain suitable exponent (x) over it. For example, let 10 be chosen for the invariable number N, and let any number, say the number 5862, be chosen for n. Algebra teaches that the value of x that satisfies the condition $10^x = 5862$ is $x = 3{\cdot}768046$, so that

$$10^{3\cdot768046} = 5862, \text{ that is } 10^{\frac{3768046}{1000000}} = 5862$$

so that if the power of 10, denoted by the numerator of the exponent, were taken, and then the root of that power, denoted by the denominator, were extracted, the result would be the number 5862. It is the exponent of 10 just exhibited, namely, 3·768046, that is called the *logarithm* of the number 5862.

In like manner, if any other value be chosen for n, algebra always enables us to find the proper exponent to be placed over *the base* 10 to satisfy the condition $10^x = n$: thus—

$$10^{2\cdot514548} = 327,\ 10^{3\cdot677698} = 4761,\ 10^{4\cdot866524} = 73540$$

so that log 327 = 2·514548, log 4761 = 3·677698, log 73540 = 4·866524.

A table of the logarithms of numbers is nothing more than a table of the exponents of 10 placed against the several numbers themselves. Any number above unity, other than 10, might serve for the *base* of a system of logarithms, but there are peculiar advantages connected with the base 10 which have recommended it to general adoption.

The actual construction of a table of logarithms, notwithstanding the appliances of modern algebra, is a work of very considerable labour; but this labour once performed, arithmetical computations, that would otherwise be nearly impracticable, can be easily managed by the aid thus afforded, as we shall now see. Adverting to the first of the above algebraical propositions, we know that

$$10^x \times 10^y = 10^{x+y}, \text{ and } 10^x \div 10^y = 10^{x-y}.$$

The first equation shows that the logarithm of the *product* of two numbers is the *sum* of the logarithms of the factors or numbers themselves, and the second shows that the logarithm of the *quotient* of two numbers is the *difference* of the logarithms of the numbers themselves.

Hence if we have to multiply two numbers together, we look in the table for the logarithms of those numbers, take them out and *add* them; the sum we know must be the logarithm of the product sought, which product we find in the table against the logarithm. If we have to divide one number by another, we *subtract* the logarithm of the latter from that of the former, the remainder is the logarithm of the *quotient*, against which in the table we find inserted the quotient itself.

If several factors are to be multiplied together, then the logarithms of all are to be added together to obtain the logarithm of the product; and in the case of successive divisions, the logarithms of all the divisors are to be added together, and the sum subtracted from the logarithm of the proposed dividend; the remainder is the logarithm of the final quotient. The logarithmic operation for finding a product, at once suggests that for finding a power which is only a product raised from equal factors. If the power arises from p factors each equal to n, then it is plain that $p \log n$ must be the logarithm of that power, that is $\log n^p = p \log n$. If instead of a power of a number we have to compute a root, the pth root of n, then representing this root by r, that is putting,

$$n^{\frac{1}{p}} = r, \text{ we have } n = r^p \therefore \log n = p \log r \therefore \log r = \tfrac{1}{p} \log n.$$

Even if the root were still more complicated, as for instance, $n^{\frac{m}{p}}$, then, as before, representing it by r, we have

$$n^{\frac{m}{p}} = r \therefore n^m = r^p \therefore m \log n = p \log r \therefore \log r = \frac{m}{p} \log n$$

We thus derive the following practical rules for performing the more troublesome operations of arithmetic by logarithms.

Multiplication.—Take the log of each factor from the table and add them all together: the sum will be the log of the product. Refer to the table for this new log and against it will be found the number which is the product.

Division.—Subtract the log of the divisor from that of the dividend: the remainder is a log against which in the table will be found the quotient.

Powers and Roots. Multiply the log of the number whose power or root is to be found, by the exponent denoting the power or root, whether it be integral or fractional; the product will be a log against which in the table will be found the power or root sought. It may be proper to mention here that the *decimal part* only of the logarithm of a number is inserted in the table; there is no occasion to encumber the table with the preceding integer when the log has one, as this may always be prefixed without any such aid; and this is the principal advantage of making the number 10 the base of the table; for since

$$10^1 = 10, \ 10^2 = 100, \ 10^3 = 1000, \ 10^4 = 10000, \ \&c.,$$

we see at once, 1st, that the log of a number consisting of but a single integer, however many decimals may follow it, being *less* than 10, cannot have its log so great as 1; hence the integral part of the log of such a number must be 0. 2nd, that the log of a number consisting of two integral places, with decimals or not, that is, a number between 10 and 100, must lie between 1 and 2; hence the integral part of the log of such a number must be 1. 3rd, that the log of a number having three integral places, or lying between 100 and 1000, must have 2 for its integral part. Hence when any number is proposed, we have only to count how many integer places there are in it: the figure expressing the number of places, *minus* 1, will be the integral part or *characteristic* as it is sometimes called, of the log of the proposed number, and the proper decimals may then be united to it from the table.

Thus, as 235·6 consists of three integer places, the integral part of its log is 2; as 4368 consists of four places, the integer part of its log is 3, and so on. By referring to the table for the proper decimal parts we find

$$\log 235·6 = 2·372175, \text{ and } \log 4368 = 3·640283.$$

The tables here referred to are those published in the Rudimentary Series, under the title of "Mathematical Tables," to which is prefixed a much more comprehensive account of logarithms and their construction than is suitable for this place, and to which, therefore, the learner is referred for all additional information necessary.

Rules and Formulæ for the solution of oblique-angled Triangles.

The present article will be entirely practical. The space which this introductory chapter has already occupied forbids that further extension of it which a full discussion of the theory of oblique-angled triangles would demand, and which, in fact, is already accessible to the learner in the Rudimentary Trigonometry (chapter iii.). We might, indeed, have been justified in omitting the preliminaries on which we have been dwelling altogether, and have contented ourselves with a general reference, on these points, to the work just mentioned. But books on Trigonometry not having any special practical object in view—as we have in the present treatise—are generally deficient in that amount of mere arithmetical illustration which he who is in training for actual practice so much requires. And in such examples as are given in these books the writers usually consider each case as one in which the utmost attainable accuracy of result is to be secured, and they accordingly calculate their angles to *seconds.* Such refinements are worse than useless in Navigation; they tend to mislead the calculator, and to beget a false confidence in his conclusions: there are always errors in *the data*, practically unavoidable, which render the results of the computations founded upon them at best but approximations to the truth. It is the business of Nautical Astronomy to supply the short-comings of Navigation, and to rectify its inaccuracies.

Rules and Formulæ for Oblique-angled Triangles.

In the solution of oblique-angled triangles there are three cases, and only three cases, to be considered. The data or given parts must be either,—

1. Two angles and an opposite side, or two sides and an opposite angle.
2. Two sides and the included angle.
3. The three sides; to find the other parts.

I. *Given either two angles and an opposite side, or two sides and an opposite angle.*

RULE. If two sides are given, then the side opposite the given angle is to the other side as the sine of that given angle to the sine of the angle opposite the latter side. If two angles are given, then the sine of the angle opposite the given side is to the sine of the other given angle as the given side to the side opposite the latter angle.

This is more briefly expressed by the precept that the sides of triangles are to one another as the sines of the angles opposite to them: or by the formula

$$a : b :: \sin A : \sin B$$

where a, b stand for any two sides, and A, B for the angles opposite to them (Rudimentary Trig. p. 52). The first and third terms of this proportion are *always* the two given parts opposite to each other: when an angle is to be found, the first term of the proportion is a side; when a side is to be found, the first term is a sine.

NOTE. It is proper to apprise the learner before he proceeds to logarithmic operations, that the log sines, log cosines, &c., are all computed on the hypothesis that the numerical value of the trigonometrical radius is not 1 but 10^{10}; so that the log of this radius is 10. In consequence of this change, every log sine, log cosine, &c., is the logarithm of the *natural* sine, cosine, &c., increased by 10.

Example. Given two sides of an oblique-angled triangle, 336 feet and 355 feet, and the angle opposite the former 49° 26′: required the remaining angles?

Here are given $a=336$, $b=355$, and $A=49°\ 26'$, to find B.

As $a=336$,	of which the log is	2·52634	to be *subtracted.*
: $b=355$,	,, ,,	2·55023	
: : sin A, 49° 26′	,, ,,	9·88061	
		12·43084	
: sin B, 53° 23′	,, ,,	9·90450	*Remainder.*

In this operation the log at the top is subtracted from the sum of the two logs underneath, since, in logarithms, addition supplies the place of multiplication, and subtraction that of division. But, by a simple contrivance, the subtractive operation may be dispensed with, and the whole reduced to addition. The following is the plan adopted to bring this about; the subtractive log 2·52634, being before us in the table, instead of copying it out, figure by figure, we put down what each figure wants of 9, until we arrive at the last figure (in the present case 4), when we put down what *it* wants of 10. Thus, commencing at the 2, we write down 7; passing to the 5, we write down 4, to the 2, we write 7; to the 6, we write 3; to the 3, we write 6; and arriving at the 4, we write 6; so that, instead of the logarithm 2·52634, we write down 7·47366, which is evidently what the logarithm itself wants of 10; in fact, in proceeding as just directed, we have been merely subtracting, in a peculiar way, 2·52634 from 10, the remainder being 7·47366: this remainder is called the *arithmetical complement* of 2·52634; and a little practice will render it quite as easy, by looking at the successive figures of any log, to write down the arithmetical complement of that log as to copy out the log itself. Now, if, in the foregoing work, we had omitted to introduce the subtractive log 2·52634, our result 12·43084 would have been erroneous in *excess* by 2·52634; and if, in addition to suppressing this subtraction, we had actually *added* the complement 7·47366, the result would obviously have erred in excess by 10, an error very easily allowed for by the dismissal of 10 from the total amount. And this is the plan always adopted, so that instead of the above, the work would stand as below:

As $a=336$, arith. comp.	7·47366
: $b=355$ . . .	2·55023
: : sin A, 49° 26′ . . .	9·88061
: sin B, 53° 23′ . . .	9·90450

The result, 10 being suppressed, is the same as before, and a row of figures is dispensed with.

We shall give another example, worked out in this way.

2. Given one side of a plane triangle 117 yards, and the angles adjacent to it 22° 37′ and 114° 46′: required the other parts?

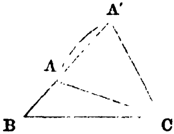
The sum of the two given angles being 137° 23′, the third angle is 180° — 137° 23′=42° 37′, the angle opposite the given side. Hence we have A=42° 37′, B=22° 37′, and a=117.

As sin A, 42° 37′	arith. comp.	0·16935
: sin B, 22° 37′	. . . .	9·58497
: : a=117	. . . .	2·06819
: b=66·45	. . . .	1·82251

It remains now to find the third side c, for which purpose we have given A=42° 37′, C=114° 46′, and a=117.

As sin A, 42° 37′ arith. comp.	. . .	0·16935
: sin C, 114° 46′ (supplement=65° 14′)		9·95810
: : a=117	. . . .	2·06819
: c=156·9	. . . .	2·19564

NOTE. The case in which the given parts are two sides and an angle opposite to one of them is, in certain circumstances, a case of *ambiguity:* in other words, there may be two *different* triangles having the same two sides and opposite angle in common, and the remaining three parts in each different, so that we may be in doubt as to *which* of the two triangles is that to which the given parts exclusively refer.



Thus let A B C be a triangle, and C A such that the arc A A′, described from C as centre, and with C A as radius, may cut B A prolonged in A′. It is plain that the two triangles, A·B C, A′ B C, will have the two sides C A, C B, and the angle B in the one, the same as the two sides C A′, C B, and the angle B in the other.

The angle B and the side C A or C A′ opposite to it being given, the rule would determine the *sine* of the angle A or A′ opposite the other given side. The angle connected with a sine in the table is *acute*, but we know that the obtuse angle which is its supplement has the same sine, so that in the ambiguous case the acute angle has no more claim to selection than the obtuse angle. It is plain that in the above diagram the angles B A C and A′ are supplements of one another, inasmuch as B A C and C A A′ are, and C A A′, C A′ A are equal.

If, however, the given angle B be *obtuse*, then there can be no ambiguity, since both the remaining angles must be acute.

Neither can there be any ambiguity if, B being acute, the side opposite to it is greater than the other given side; for the greater side being opposite to the greater angle, the angle whose sine is determined by the rule must be also acute, and less than the given one.

It thus appears that the ambiguity can have place only when the given angle is acute, and the side opposite to it *less* than the other given side. In these circumstances all we can say is, that the sought angle is either that furnished by the tables or its supplement. But in actual practice it can but seldom happen that we are so unacquainted with the form of our triangle, as to be in doubt as to whether the angle in question is acute or obtuse.

II. *Given two sides and the included angle.*

Rule.—As the sum of the two given sides
Is to their difference,
So is the tangent of half the sum of the opposite angles
To the tangent of half their difference.

Or, expressed in Algebraic symbols instead of words, the rule is,—

$$a + b : a \sim b :: \tan \tfrac{1}{2} (A + B) : \tan \tfrac{1}{2} (A \sim B)$$

the given parts being a, b, and C, and *consequently* A + B, since $A + B = 180° - C$, or $\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$. The last term of the proportion being found, a reference to the table gives us $\frac{1}{2}(A \sim B)$, which added to $\frac{1}{2}(A + B)$ gives the greater of the two angles A, B; and subtracted from $\frac{1}{2}(A + B)$ gives the less.

Example. Given two sides of a triangle equal to 47 and 85 respectively, and the angle between them 52° 40′, required the remaining parts—

Here $a = 85$, $b = 47$, $C = 52° 40'$

$$\therefore \tfrac{1}{2}(A + B) = 90° - 26° 20' = 63° 40';$$

Also $a + b = 132$, and $a - b = 38$.

As $a + b = 132$	arith. comp. .	7·87943
: $a - b = 38$	. . . . . .	1·57978
: : tan $\frac{1}{2}$ (A + B), 63° 40′	. . . . . .	10·30543
: tan $\frac{1}{2}$ (A — B), 30° 11′	. . . . . .	9·76464

The greater angle A = 93° 51′
The less angle B = 33° 29′

We have now to determine the side c, as follows:—

As sin B, 33° 29′ arith. comp.	. . . . .	·25830
: sin C, 52° 40′	. . . .	9·90043
: : $b = 47$	. . . .	1·67210
: $c = 67{\cdot}74$	. . . .	1·83083

III. *Given the three sides.*

For the solution of this case it is better to work from a formula than from any rule expressed in words. There are two formulæ adapted to logarithmic computation, and these very readily furnish a third. It is generally matter of indifference which of the three be employed, at least as respects accuracy of result; the second of them is, however, a little preferable on the score of brevity. In certain extreme and therefore unusual instances, however, one form is to be preferred to another to secure greater precision, as will be noticed presently. Let s stand for half the sum of the three sides, that is, let $s = \frac{1}{2}(a + b + c)$ then—

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)\,(s - c)}{b\,c}} \quad . \quad . \quad (1)$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s\,(s - a)}{b\,c}} \quad . \quad . \quad . \quad (2)$$

And dividing the first of these by the second, recollecting that sine divided by cosine gives tangent, we have for a third formula—

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s - b)\,(s - c)}{s\,(s - a)}} \quad . \quad . \quad (3)$$

If the angle A, to be determined, is foreseen to be so small as to amount to only a few minutes, then $\frac{1}{2}$ A had better be derived from the first formula or the third instead of from the second, because the *cosines* of angles differing but little from 0°, differ themselves by so small a quantity that the five or six leading decimals may equally belong to several consecutive cosines, so that if our table be limited to this extent of decimals, we shall find a succession of small angles with the same cosine against each, so that if we enter the table with this cosine, we shall be at a loss which of these small angles to select.

If the angle A be very near 180°, and therefore $\frac{1}{2}$ A very near 90°, then the second formula will be preferable to the first, because very near 90° the *sines* differ from one another only in their remote decimals.

These niceties, however, are only worth attention in cases where the minutest accuracy is desirable: in Navigation any one of the above formulæ is just as good as another.

EXAMPLE.—The three sides of a triangle are—

$$a = 195, \ b = 216, \ c = 291,$$

required the angle A?

By Formula (1).

a=195	
b=216 arith. comp.	7·66555
c=291 arith. comp.	7·53611
2)702	
s=351	
$s-b$=135	2·13033
$s-c$= 60	1·77815
	2)19·11014
sin $\frac{1}{2}$ A 21° 2′ . . .	9·55507
2	
∴ A=42° 4′	

By Formula (2.).

a= 195	
b= 216 arith. comp.	7·66555
c= 291	7·53611
2)702	
s= 351	2·54531
$s-a$= 156	2·19312
	2)19·94009
cos $\frac{1}{2}$ A 21° 2′ . . .	9·97005
2	
∴ A =42° 4′	

In each of the foregoing operations two arithmetical complements are introduced, consequently the result of the addition is in each case too great by 20; but after the division by 2 the final result is but 10 in excess, which additional 10 is necessary to complete the logarithmic sine, and cosine of the table (see p. 26).

THE PRINCIPLES OF NAVIGATION.

The business of navigation is to conduct a ship from any known place on the surface of the globe, to any other she may be intended to reach; as also to determine her position at any period of the voyage. The subject is divided into two distinct branches—Navigation proper, and Nautical Astronomy. It is with the former branch only that we are at present to be occupied: it comprehends all those operations, tributary to the ultimate object, which are independent of an appeal to the heavenly bodies, and which are matters of daily routine on shipboard.

If the direction in which a ship is sailing at any time, and the rate of her progress through the water, could always be measured with accuracy, there would be comparatively but little need for astronomical observations in navigating a vessel from one port to another; but impelled by the wind and the waves—forces proverbially fickle and inconstant—the practical difficulties in the way of such accuracy of measurement are insuperable; and therefore, as already observed, the mariner must content himself with approximations only to the truth. But, fortunately for him, the turbulence of the ocean can never disturb the tranquillity of the skies; and he knows that during all his own unavoidable aberrations from *his* proper path, the moon and the sun have never for an instant deviated from theirs. How the relative positions of these two bodies, or the position of the former in reference to the stars among which she moves, can enable the navigator to correct his own position, and thus, with renewed confidence, to start afresh, is an inquiry to be answered only by Nautical Astronomy.

CHAPTER I.

DEFINITIONS.—INSTRUMENTS.

In Geography and Navigation the earth is regarded as a sphere. It is known from actual measurements at various parts of its surface to slightly differ from this: it is a little flattened at the poles, as a body constantly rotating on an axis may be expected to be. But the departure from sphericity is so trifling, that no practical error of any moment can arise from our treating the earth as a globe, in laying down geographical positions, and in framing directions for sailing over its surface. With a view to these objects, certain lines are imagined to be traced on the surface of the earth; and on the artificial globes, on which the prominent features of this surface are depicted, the imaginary lines alluded to are actually drawn. Their definitions, with those of certain remarkable points, are as follows:—

Axis.—The axis of the earth is the diameter about which its daily rotation is performed; the direction of this rotation is from west to east; it is completed in twenty-four hours.

Poles.—The two extremities of the axis are called the poles of the earth: that to which we, in these countries, are nearest, is the North Pole, the other is the South Pole; as they are the extremities of a diameter, they are 180° apart.

Equator.—The equator is a great circle on the earth equally distant from the poles, dividing the globe into two equal parts, or hemispheres,—the northern hemisphere and the southern hemisphere. The poles of the earth are the poles of the equator, every point in this latter circle being 90° (of a great circle) distant from either pole. It must be observed, that by a *great* circle is meant a circle of the sphere, having for its centre the centre of the sphere: no greater circle can be traced upon its surface; all other circles are called small circles.

MERIDIANS.—Every semicircle drawn from one pole to the other is called the meridian of every place on the earth through which it passes. Of all the innumerable meridians that may be imagined on the globe of the earth, one is always selected by every civilised kingdom as a principal, or *first meridian*; it is usually that which passes through the national observatory: in this kingdom the first meridian is that of the Greenwich observatory, in France it is that of the Paris observatory.

LATITUDE.—The latitude of any place on the surface of the earth is the distance of that place from the equator, measured in degrees and minutes on the meridian of that place. The latitude is north when the place is situated in the northern hemisphere, and south when it is situated in the southern hemisphere. The latitude of each pole is 90°, that of any other spot must be less than 90°.

PARALLELS OF LATITUDE.—Every small circle on the globe, parallel to the equator, is called a parallel of latitude; every point on its circumference, being equally distant from the equator, has the same latitude.

DIFFERENCE OF LATITUDE.—The difference of latitude of any two places is the arc of a meridian contained between the two parallels of latitude passing through those places. If the places are both on the same side of the equator, their difference of latitude is found by subtracting the less latitude from the greater: if the places are one on each side of the equator, their difference of latitude is found by adding the two latitudes together.

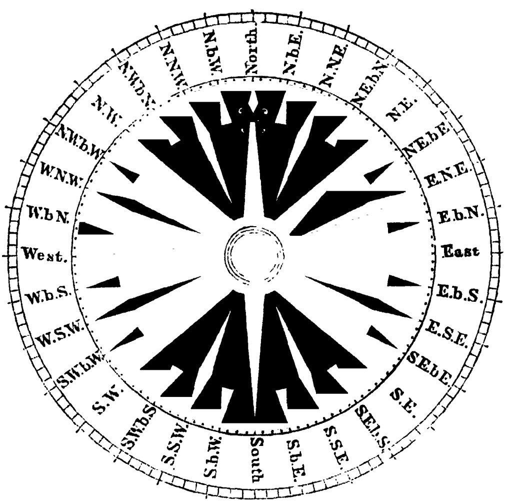
Longitude.—The longitude of any place on the earth is the arc of the *equator*, intercepted between the first meridian and the meridian of the place. If the place lie to the east of the first meridian it has east longitude, if it lie to the west it has west longitude. No place therefore can exceed 180° in longitude, whether east or west.

Difference of Longitude.—The difference of longitude of two places is the arc of the equator intercepted between the meridians of those places; if the places lie both east, or both west, of the first meridian, the difference of longitude is found by subtraction; but if one have east longitude and the other west, the difference is found by addition.

Horizon.—A plane conceived to touch the surface of the earth at any place, and to be extended to the heavens, is called the *sensible horizon* of that place. And a plane parallel to this, but passing through the centre of the earth, is called the *rational horizon* of that place. The horizon, whether sensible or rational, is thus a *plane;* but the remote bounding circle which, to an eye elevated above the surface of the ocean, appears to unite sea and sky, is that which mariners more commonly regard as the horizon, and call it the *sea-horizon*, or *offing*. The plane of this circle obviously *dips* below the planes of the sensible and rational horizons, and the amount of this depression is that which is called the *dip of the horizon*.

The Compass.—The straight line in which the plane of the meridian of any place cuts the sensible horizon of that place, is called the horizontal meridian, or north and south line; and the horizontal straight line, perpendicular to this, is the east and west line of the horizon. The sensible horizon is artificially represented by a circular card, on the under side of which is fixed a magnetised bar or Needle, in the direction of the north and south line, or horizontal meridian. The card being so suspended as to always remain horizontal, and to turn freely about its centre, the tendency of the needle to point north and south causes the meridian line, on the upper surface of the card, to settle in the proper direction; and the intervals between the four points E. W. N. S.—the four *cardinal points* as they are called—being subdivided, as in the annexed figure, the instrument is placed securely in a brass circular box or bowl with a glass cover, and hung upon brass hoops (*gimbals*), so that the horizontal position of the card may not be disturbed by the motion of the ship. This instrument is

THE MARINER'S COMPASS.



The four quadrants into which the meridian line N. S., and the east and west line E.W., divides the rim of the card, are each subdivided into eight equal parts called *points*, so that each point is an arc of 11° 15′, and this is further divided into quarter points. The outer rim of the card is divided into 360 degrees; the thirty-two points of the compass, and the angles at the centre which the corresponding lines make with the meridian (neglecting quarters of minutes) are exhibited in the following Table:—

NORTH.		POINTS.	ANGLES.	SOUTH.	
		¼	2° 49′		
		½	5° 37′½		
		¾	8° 26′		
N. b. E	N. b. W.	1	11° 15′	S. b. E.	S. b. W.
		1¼	14° 4′		
		1½	16° 52′½		
		1¾	19° 41′		
N. N.E.	N. N.W.	2	22° 30′	S. S.E.	S.S.W.
		2¼	25° 19′		
		2½	28° 7′½		
		2¾	30° 56′		
N.E. b. N.	N.W. b. N.	3	33° 45′	S.E. b. S.	S.W. b. S.
		3¼	36° 34′		
		3½	39° 22′½		
		3¾	42° 11′		
N.E.	N.W.	4	45° 0′	S.E.	S.W.
		4¼	47° 49′		
		4½	50° 37′½		
		4¾	53° 26′		
N.E. b. E.	N.W. b. W.	5	56° 15′	S.E. b. E.	S.W. b. W.
		5¼	59° 4′		
		5½	61° 52′½		
		5¾	64° 41′		
E. N.E.	W. N.W.	6	67° 30′	E. S.E.	W. S.W.
		6¼	70° 19′		
		6½	73° 7′½		
		6¾	75° 56′		
E. b. N.	W. b. N.	7	78° 45′	E. b. S.	W. b. S.
		7¼	81° 34′		
		7½	84° 22′½		
		7¾	87° 11′		
E.	W.	8	90° 0′	E.	W.

The compass is placed near the helm, and the line from the centre, in the direction of the ship's head, denotes the angle which its track is making with the meridian, or north and south line N. S. It is called a *rhumb* line. It must be noticed, however, that the N. S. line is not truly the horizontal meridian; the needle, which settles the position of this line, deviates from the true direction; it does not point accurately to the north, and the angle between the true meridian and that in which the needle settles, called the *magnetic* meridian, is the *variation* of the compass. The amount of this variation at any place may be discovered by Nautical Astronomy. Another correction is in general requisite: the iron in the ship necessarily influences the needle, the disturbance thus occasioned is called the *deviation* of the compass, the amount of which can be ascertained only by special experiments. Since the introduction of iron vessels, this local attraction has engaged a good deal of attention: we shall advert to the subject more fully in a future chapter.

COURSES.—So long as a ship sails on the same rhumb-line, her track makes the same angle with the successive meridians; this angle, indicated by the compass, is called the ship's course. If the course be not corrected for variation, it is the *compass-course;* when corrected, it is the *true course.* The compass course, be it observed, supposes the needle to be previously freed from the effects of the local attraction of the ship. The contrivances for this purpose will be noticed hereafter. The variation, whether to the right or left, when known, is easily allowed for: in what follows in the next chapter we shall suppose the allowance to be made, and the courses mentioned to be the true courses.

LEEWAY.—The course may also be affected by the *leeway,* or the oblique motion of the vessel occasioned by the action of the wind sideways, impelling the ship along a track oblique to the fore-and-aft line; this angle of deviation from the direction shown by the compass is the leeway; it is to be estimated and allowed for, according to circumstances, from the navigator's observation and experience of the behaviour of his ship.

RATE OF SAILING.—The rate at which a ship is sailing on any course is measured by an instrument called the *log,* or the *log-ship,* and a line attached to it called the *log-line,* which is about 120 fathoms in length.

The log itself is a wooden quadrant, of which the circular rim is loaded with lead, so that when it is *hove,* or thrown into the water, it settles in an upright position, with its centre just above the surface, and the log-line is so fastened to it that the face of the log is kept towards the ship, in order that it may offer the greatest resistance to being dragged along, as the line is being unwound from a reel by the advancing motion of the vessel. The length of line thus unwound in *half a minute* gives the distance run, or rate of sailing *per hour*, on the following principle:

The log-line is divided into equal parts, each part being the 120th of a nautical mile. Now a nautical mile—that is, the 60th part of a degree of the equator, or of a meridian, is about 6080 feet, so that each of the equal parts of the log-line is 50 feet 8 inches. The several divisions are marked by pieces of string passed through the strands of the log-line and *knotted*, the number of knots in any string indicating the number of parts between it and the end of the log-line; that is, how many parts have run off the reel. If, therefore, we take note of the number of knots reached in half a minute, we shall learn how many 120ths of a mile the ship has sailed in the 120th of an hour, which will, of course, be at the rate of so many miles per hour. As the knots thus give the miles per hour, sailors are in the habit of calling the miles sailed per hour so many knots.

The marks on the log-line do not commence at the log; a portion of line—about 10 or 12 fathoms, is suffered to run out before the marking begins. This portion is called the *stray-line*, which allows the log to settle in the water clear of the ship, before the half-minute commences; the termination of the stray-line is marked by a piece of red cloth, and at the instant this passes from the reel the half-minute sand glass is turned, and the reel stopped as soon as the sand is run out.

The above is the common log; but there is an improved instrument, called *Massey's* log, constructed on a different principle, and which is generally preferred.

The course and rate of sailing at any time being measured by the instruments now described, a record is kept of the progress and position of the ship from day to day. The actual distance run, and the difference of latitude and longitude made from noon to noon being deduced from this record, without the correcting aid of astronomical observations, we have the ship's account by *dead reckoning*.

CHAPTER II.

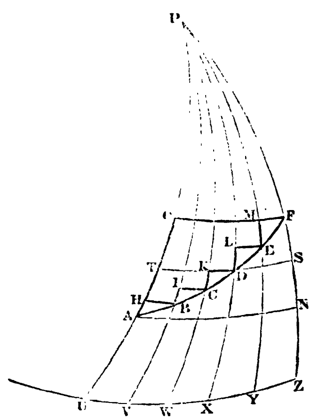
PLANE SAILING.—SINGLE COURSES.—COMPOUND COURSES.

PLANE sailing is usually defined to be the art of navigating a ship on the supposition that the earth is a *plane*. This definition is erroneous in the extreme: in all sailings the earth is regarded as what it really is—a sphere. Every case of sailing, from which the consideration of *longitude* is excluded, involves the principles of plane sailing; a name which merely implies, that although the path of the ship is on a *spherical* surface, yet we may represent the length of this path by a straight line on a *plane* surface, and may embody all the particulars necessary to be considered, longitude excepted, in a plane triangle.* This will sufficiently appear from the following investigation of the theoretical principles upon which plane sailing is founded. Let A, F, represent two places on the spherical surface of the ocean, the lines drawn from the pole P being meridians equidistant from one another, and so close together that the intercepted

* Even when longitude enters into consideration, it is still with the plane triangle only that we have to deal; and the reason that the sailings in which longitude is concerned—mid-latitude sailing and Mercator's sailing—are not comprehended under *plane* sailing, is that those sailings distinctly refer to, and are founded upon, the spherical figure of the surface sailed over; but, as the investigation here given in the text shows, the rules for plane sailing would equally hold good though the surface were a plane. Notwithstanding this truth, however, it is still incorrect to say that these rules are founded on the supposition that the earth *is* a plane;—no such supposition is made.

portions A B, B C, C D, &c., of the ship's track, in sailing from A to F, may each be regarded as a straight line.

The learner will at once see that we may conceive these portions of such trifling length, that it would be impossible to estimate them other than as straight lines :—they may be conceived, for instance, as only a yard or two long. Let also U Z be an arc of the equator, and draw the parallels of latitude as exhibited by the dark lines in the figure. A series of triangles, A H B, BIC, CKD, &c., will thus be formed on the surface of the sphere, so small, that each may be practically regarded as a plane triangle, without any sensible error. These plane triangles are all equi-angular; for the angles at H, I, K, &c., are all right angles, and the ship's track cuts every meridian which it crosses, while preserving the same course at the same angle. Consequently, by Euclid 4. VI., we have the continued proportion



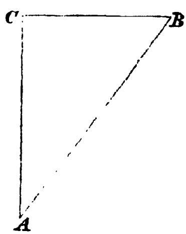
$$AB : AH :: BC : BI :: CD : CK, \&c.;$$

and since, in a continued proportion, one antecedent is to its consequent as the sum of the antecedents to the sum of the consequents (Euc. 5. V.), we have

$$AB : AH :: AB+BC+CD+\&c. : AH+BI+CK+\&c.$$

Now AB+BC+CD+&c., is the distance sailed from A to F on the course HAB; and AH+BI+CK+&c., is the difference of latitude AO between A, the place left, and F, the place arrived at.

Let now a right-angled plane triangle, similar to the little right-angled triangle AHB, be constructed; that is, a right-angled triangle in which A is the angle of the course, and



let the hypotenuse AB represent the distance sailed, that is, the length of AF on the globe; then it is obvious that the perpendicular AC will represent the difference of latitude AO; while the base CB — the side opposite to the course —will represent the sum of all the small departures, H B, I C, K D, &c., from the successive meridians which it crosses. For since

$$\text{AB} : \text{HB} :: \text{BC} : \text{IC} :: \text{CD} : \text{KD} \;\&\text{c.}$$
$$\therefore \text{AB} : \text{HB} :: \text{AB}+\text{BC}+\text{CD}+\&\text{c.} : \text{HB}+\text{IC}+\text{KD}+\&\text{c.}$$

But the plane triangle ABC is constructed so that

$$\text{AB} : \text{HB} :: AB : CB;$$

and, moreover, so that $AB = \text{AB}+\text{BC}+\text{CD}+\&\text{c.}$ on the globe, consequently

$$CB = \text{HB}+\text{IC}+\text{KD}+\&\text{c.}$$

This length CB is called the *Departure* made by the ship in sailing from A to F: there is no line corresponding to it on the globe; it merely expresses the sum of all the indefinitely small departures made by the ship in passing over the small intervals between the innumerable meridians conceived to be interposed between PU, the meridian left, and PZ, the meridian arrived at.

It is thus fully established that the distance sailed on any oblique course, the difference of latitude made, and the departure, may all be accurately represented by the sides of a right-angled plane triangle, the angle opposite to the departure being the angle of the course. Of the four things just mentioned; namely—Distance, Difference of Latitude, Departure, and Course, any two being given, the remaining two may, therefore, be determined by the solution of a right-angled plane triangle; and so far as these particulars are concerned, the results are obviously just the same as they would be if the ship were to sail on a plane surface instead of on a spherical surface; the curve meridians being replaced by parallel straight lines, and the perpendiculars to these regarded as the parallels of latitude. We do not make the supposition that the surface actually sailed upon is a plane, with the meridians parallel straight lines; but taking the surface as it really is—*spherical*, we find that, so far as the particulars mentioned above are concerned, we may replace it by such a plane surface. And this is the only justification of the name, Plane Sailing.

In the examples in plane sailing which follow, the learner is recommended to sketch the right-angled triangle in each case, regarding the top of the paper to be the north and the bottom the south, so that the east will be on the right hand and the west on the left. Having drawn a north and south line, representing the portion of meridian due to the difference of latitude; he should draw from the latitude arrived at, the base of the triangle for the departure,—towards the right if the departure be east, and towards the left if it be west; the hypotenuse will then represent the distance sailed, and the angle it makes with the difference of latitude, will be the course. The vertex of this angle is to be regarded as the centre of the compass, or of the sensible horizon at commencing the course; the angle will lie to the right or left, according as the sailing is towards the easterly or westerly side of the meridian started from. In all collections of tables for the use of navigators, there is inserted a Difference of Latitude and Departure Table, usually called a *Traverse Table;* by entering which, with the measured course and distance, we can get the corresponding difference of latitude and departure by *inspection.* The table usually extends up to distances of 300 miles, and may be used for greater distances by cutting up the greater distance into parts that will come within the limits of the table.*

* See the Navigation Tables which accompany this

Examples in Single Courses.

1. A ship from latitude 49° 30′ N. sails N.W. by N., a distance of 103 miles: required the latitude in, and the departure made?

The course being 3 points, is 33° 45′, the angle contained between the given hypotenuse 103 miles, and the required diff. lat.; hence by right-angled triangles, we have

For the diff. lat.

Diff. lat. = cos course × dist.

cos 3 points * = ·8315

dist. = 103

24945

8315

Diff. lat. N. = 85·6445 miles.

For the departure.

Dep. = sin course × dist.

sin 3 points = ·5556

dist. = 103

16668

5556

Dep. W. = 57·2268 miles.

By Inspection.—Referring to that page of the Traverse Table headed "3 Points," we find against the distance 103, in the column marked "Lat." the number 85·6, and in the column marked "Dep." the number 57·2; we infer, therefore, that the difference of latitude is 85·6 miles, and the departure 57·2 miles.

Since 60 miles is a degree, a nautical mile being a minute of the meridian, 85·6 miles = 1° 25′·6, which added to 49° 30′, the latitude left, gives 50° 55′·6 N. the lat. in.

2. A ship sails from lat. 37° 3′ N., S.W. by S. ½ S. a distance of 148 miles; required her latitude in and the departure made?

For the diff. lat.

Diff. lat. = cos course × dist.

cos 3½ points = ·773

dist. = 148

6184

3092

773

Diff. lat. S. = 114·404

= 1° 54′

Lat. left . . 37° 3′

Lat. in . . . 35° 9′N.

For the departure.

Dep. = sin course × dist.

sin 3½ points = ·6343

dist. = 148

50744

25372

6343

Dep. W. = 93·8764

Hence the departure is 93·9 miles W.

By Inspection.—With the course 3½ points, and distance 148, the Traverse Table gives diff. lat.=114·4, and dep. =93·9.

NOTE.—In the foregoing computations we see that several decimals in the results are superfluous. By using the contracted method of multiplication—as at page 19, and which is fully explained in the "Rudimentary Arithmetic," the unnecessary decimals may be dispensed with; thus, the four multiplications in the above examples, become contracted into the following by reversing the multipliers:

·8315	·5556	·773	·6343
301	301	841	841
8315	5556	773	6343
249	167	309	2537
85·64	57·23	62	507
		114·4	93·87

We have only to notice under what decimal place of the multiplicand the *units* figure of the multiplier stands, in order to determine the number of decimals in the product; thus, in the first operation the units figure is under the second decimal, therefore two decimals are to be marked off in the product. In like manner, two are to be pointed off in the second operation; one in the third; and two in the fourth.

3. A ship sails from lat. 15° 55′ S. on a S.E. ½E. course till she finds herself in lat. 18° 49′ S.: required the distance run and the departure made?

Latitude left 15° 55′ S.
Latitude in . 18° 49′ S.

Diff. lat. . 2° 54′=174 miles.

courses: it will be found at the commencement of the Navigation Tables to accompany this work, and will save the trouble of searching in the more extensive tables.

For the distance.

Dist. = diff. lat. ÷ cos course.

cos 4½ points = ·6,3,4,4)174 (274·3

```
   12688
   -----
    4712
    4441
   -----
     271
     254
   -----
      17
```

∴ Dist. = 274·3 miles.

For the departure.

Dep. = tan course × diff. lat.

tan 4½ points = 1·2185

dist. = 174, reversed 471

```
   12185
    8530
     487
   ------
```

Dep. E. = 212·02 miles.

By Inspection.—In that page of the traverse table devoted to the course 4½ points, and in the lat. column, is found 173·8, which is the nearest to the given diff. lat., 174. And against this number, in the proper columns, are found dist. = 274, and dep. = 211·8.

4. Yesterday at noon we were in lat. 38° 32′ N., and this day at noon we are in lat. 36° 56′ N. We have run on a single course between S. and E., at the rate of 5½ knots an hour: required the course steered and the departure made?

Lat. from 38° 32′ N.

Lat. in 36° 56′ N.

Diff. lat. 1° 36′ = 96 miles.

```
   24  = number of hours.
   5½
  ----
  120
   12
  ----
```

Dist. = 132 miles.

For the course.

Cos course = diff. lat. ÷ dist.

$$\frac{96}{132} = \frac{8}{11} = \cdot 7273 = \cos 43^\circ 20'$$

The departure may be found thus,

$$\sqrt{\{(132+96)(132-96)\}}$$

$$= \sqrt{\{228 \times 36\}} = \sqrt{8208}$$

$$= 90 \cdot 58.$$

For the departure.

Dep. = sin course × dist.

sin 43° 20′ = ·6862

dist. 132, reversed 231

```
   6862
   2059
    137
   -----
```

Dep. E. = 90·58 miles.

Hence the course steered is S. 43° 20′ E., or S.E. by S. ¾ E. nearly, and the departure is 90·58 miles Easterly.

This example may be solved by the traverse table, but not without some trouble; we should have to examine the several pages in which the distance 132 is inserted, till we came to that page in which, against this 132, stands 96, or a number near to this (viz. 96·5), in the lat. column. At the top of this page will be found the course nearly, namely, 43°, and in the dep. column the number 90.

5. A ship from lat 48° 40′ N. sails N.E. by N. 296 miles: required the lat. in, and the departure made?

Ans. lat. in, 52° 46′ N.; dep. E. 164·4 miles.

6. A ship from lat. 47° 30′ N. has sailed S.W. by S. a distance of 98 miles: required her lat. in, and the departure made? Ans. lat. in, 46° 9′ N.; dep. W. 54·45 miles.

7. A ship has sailed from lat. 37° 30′ N. to lat. 46° 8′ N., on a S.E. by S. course: required the distance run and the departure made?

Ans. dist. 98·6 miles; dep. E. 54·8 miles.

8. A ship from lat. 3° 16′ N. sails S.W. by W.¼W., until she has made 356 miles of departure: required her lat. in, and the distance sailed?

Ans. lat. in, 0° 17′ S.; dist. 415 miles.

9. A ship from lat. 36° 12′ N. sails in a direction between S. and W. till she arrives in lat. 35° 1′ N., having made 76 miles of departure: required her course, and distance sailed?

Ans. course S., 46° 57′ W.; dist. 104 miles.

10. A ship in lat. 3° 52′ S. is bound for a port bearing N.W. by W. ½W., in lat 4° 30′ N.: what distance on that course must the ship sail to reach the port, and what departure will she have made during the voyage?

Ans. dist. 1065 miles; dep. W. 939 miles.

11. A ship from lat. 50° 13′ sails between S. and E. 98 miles, till her departure is 82 miles: required her course, and the latitude arrived at?

Ans. course S., 56° 47′ E.; lat. in, 49° 19′ N.

12. If a ship take her departure at six o'clock in the evening from Cape Verde, in lat. 14° 45′ N., and sail W.S.W. ½W. at the rate of seven miles an hour until the

next day at noon: what will be her distance run, her departure, and the latitude in?

Ans. dist. 126 miles; dep. W., 120·6 miles; lat. in 14° 8′ N.

Compound Courses.

When a ship sails on different courses, as she usually does in a voyage of any length, the zig-zag track she describes is called a compound course or a *traverse*, and the determination of the single course and distance from the place left to that arrived at is called *resolving the traverse.*

In order to do this, the difference of latitude and departure for each distinct course must be found, and the aggregate of the several differences and departures taken for the single difference and departure which would be made by sailing from the place left to that reached on a single course. The determination of this course, and the corresponding distance, is then to be effected as in the preceding article.

In resolving a traverse it is usual to take the diff. lat. and dep. due to each of the component courses from the traverse table: and having prepared six columns, with the suitable headings, as in the annexed example, to insert each course, dist., diff. of lat., and departure, in its proper column. This done, we have only to add up all the differences of latitude marked N., and all marked S., and to take the difference of the two sums, and then to do the same with the departures marked E. and W., to obtain the diff. lat. and dep. due to the equivalent single course.

Examples in Compound Courses.

1. A ship from lat. 51° 24′ N. during the last twenty-four hours has run the following courses, namely:

1st. S.E., 40 miles.	4th. N.W. by W., 30 miles.
2nd. N.E., 28 miles.	5th. S.S.E., 36 miles.
3rd. S.W. by. W. 52 miles.	6th. S.E. by E., 58 miles.

Required the lat. in, and the direct course and distance to arrive at it?

TRAVERSE TABLE.*

COURSES.	DIST.	DIFF. LAT.		DEPARTURE.	
		N.	S.	E.	W.
S.E.	40		28·3	28·3	
N.E.	28	19·8		19·8	
S.W. by W.	52		28·9		43·2
N.W. by W.	30	16·7			24·9
S.S.E.	36		33·3	13·8	
S.E. by E.	58		32·2	48·2	
Direct course, S. 25° 59′ E.		36·5	122·7	110·1	68·1
			36·5	68·1	
Direct distance, 95·87 miles.			86·2	42	

The results of the above table show that the whole diff. lat. made is 86·2 miles S., and the departure 42 miles E., and from these we compute the direct course and distance as follows:

For the direct course.

Tan course = dep. ÷ diff. lat.

8,6·,2)42 (·4872 = tan 25° 59′
3448
752
6896
624
603
21

Lat. left . . .		51° 24′ N.
Diff. lat.	86·2 m.	1° 26′ S.
Lat. in		49° 58′ N.

For the distance.

Dist. = dep. ÷ sin course.

·4,3,8,1,1)42 (95·87 miles.
39430
2570
2191
379
350
29

* Before referring to the general traverse table, for the purpose of extracting the several particulars to be entered in this, it will be a security against putting any extract in the wrong column, if against each course and distance we put a small mark, as a cross, in each column where an entry connected with that course and distance is to be made, the mark being put sufficiently near the margin of the column to leave room for the entry to be placed against it. Thus: wherever N. occurs in the course, a mark is to be placed opposite to that course in the N. column: wherever S. occurs, a mark in the S. column. When E. occurs, mark in like

Hence the course is S.S.E. $\frac{1}{4}$ E. nearly, the distance is 95·87 miles, and the lat. in, 49° 58′ N.

NOTE. The learner should be here apprised that the balance of the departures, made in a succession of courses, is not in strictness the same as the single departure made in the single course from the place left to that ultimately reached by the traverse sailing. Suppose a ship in any latitude to sail due west or due east; then her entire distance will be also her departure. But if another ship were to sail from a lower latitude on the same meridian to the same place, it is obvious that her departure would exceed that of the former ship; and if she sailed from a higher latitude her departure would be less.

In a single day's run the inaccuracy of taking the balance of a set of departures as the departure due to the single equivalent course, is too small to lead to any practical error of consequence. We shall advert to this matter again at the close of the next chapter.

2. A ship from lat. 51° 25′ N. has sailed on the following courses, namely:

1st. S. S. E. $\frac{1}{4}$ E., 16 miles.
2nd. E. S. E., 23 miles.
3rd. S. W. by W. $\frac{1}{4}$ W., 36 miles.
4th. W. $\frac{3}{4}$ N., 12 miles.
5th. S. E. by E. $\frac{1}{4}$ E. 41 miles.

Required the latitude in, and the direct course and distance to reach it?

Ans. direct course S. 18° 12′ E.; dist. 62$\frac{3}{4}$ miles.

3. A ship from lat. 1° 12′ S. has sailed the following courses and distances, namely:

1st. E. by N. $\frac{1}{2}$ N., 56 miles.
2nd. N. $\frac{1}{4}$ E., 80 miles.
3rd. S. by E. $\frac{1}{2}$ E., 96 miles.
4th. N. $\frac{1}{4}$ E., 68 miles.
5th. E. S. E., 40 miles.
6th. N. N. W. $\frac{1}{2}$ W., 86 miles.
7th. E. by S., 65 miles.

manner the E. column, and when W. occurs, the W. column. This done, the traverse table may be referred to for the proper entries to be placed against the marks.

Required the lat. in, and the course and distance made good? Ans. lat. in, 0° 48′ N: course, N. 51° 47′ E.; dist. 193·8 miles.

4. Since last noon the following courses and distances have been run, namely:

1st. S. W. $\frac{3}{4}$ W., 62 miles.
2nd. S. by W., 16 miles.
3rd. W. $\frac{1}{4}$ S., 40 miles.
4th. S. W. $\frac{3}{4}$ W., 29 miles.
5th. S. by E., 30 miles.
6th. S. $\frac{3}{4}$ E., 14 miles.

Required the difference of latitude made, and the course and distance made good?

Ans. diff. lat. 1° 55′ S.; course, S. 43° 14′ W.; dist. 158 miles.

5. A ship from lat. 24° 32′ N. sails the following courses:

1st. S. W. by W., 45 miles.
2nd. E. S. E., 50 miles.
3rd. S. W., 30 miles.
4th. S. E. by E., 60 miles.
5th. S. W. by S. $\frac{1}{4}$ W., 63 miles.

Required her lat. in, her departure, and the direct course and distance?

Ans. lat. in, 22° 3′ N.; dep. 0; course, S. dist. 149 miles.

6. Yesterday noon we were in lat. 3° 18′ S., and since then we have run the following courses, namely:

1st. N. N. E., 22 miles.
2nd. N. by W., 30 miles.
3rd. N. E. by E., 40 miles.
4th. E. S. E., 25 miles.
5th. S. S. W., 18 miles.
6th. N. W. by N. $\frac{1}{2}$ W., 50 miles.
7th. N. E. $\frac{1}{2}$ E., 42 miles.
8th. W. by S. $\frac{1}{2}$ W., 45 miles.
9th. S. W. by S., 20 miles.
10th. E. by N. $\frac{1}{2}$ E. 62 miles.

Required our present lat. and dep., with the course and distance made good?

Ans. lat. in, 1° 39′ S.; dep. 58·4 miles E., course, N. 30° 32′ E. or N.N.E. $\frac{3}{4}$ E. nearly; dist. 115 miles.

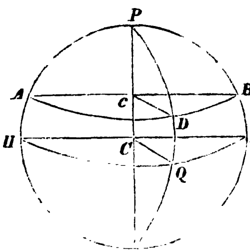
CHAPTER III.

PARALLEL SAILING.—MID-LATITUDE SAILING.

Parallel Sailing.

WHEN a ship sails due east or due west, her track is on a parallel of latitude, and the case is one of *parallel sailing*: her distance run is then the same as her departure, her difference of latitude is nothing, and her difference of longitude is determined upon the following principles.

In the annexed figure, let I Q H represent the equator,



and B D A any parallel of latitude; then C I will be the radius of the equator, and c B the radius of the parallel. Let B D be the distance sailed on this parallel, then the difference of longitude made will be measured by the arc I Q of the equator; and since similar arcs are to each other as the radii of the circles to which they belong, we have the proportion,

$$c\,\mathrm{B} : \mathrm{C\,I} :: \text{dist. B D} : \text{diff. long. I Q}.$$

But c B is the cosine of the latitude I B to the radius C I, that is, c B is C I times the trigonometrical cosine of the latitude, so that the above proportion is—

$$\mathrm{C\,I} \times \cos \text{lat.} : \mathrm{C\,I} :: \text{dist.} : \text{diff. long.}$$

$$\therefore \cos \text{lat.} : 1 :: \text{dist.} : \text{diff. long.} \quad . \; . \; . \; (1)$$

$$\therefore \text{diff. long.} = \frac{\text{dist. sailed}}{\cos \text{latitude}} \quad . \; . \; . \; . \; (2)$$

If the distance d between any two meridians be measured on a parallel whose latitude is l, and the distance d' between the same meridians be measured on another parallel whose latitude is l', then, calling the difference of longitude of the two meridians L, we have from (1), by alternation:

$$\cos l : d :: 1 : L$$
$$\cos l' : d' :: 1 : L$$
$$\therefore \cos l : \cos l' :: d : d' \quad \therefore d' = \frac{d \cos l'}{\cos l} \quad . \ . \ . \ (3);$$

that is, the intervals between any two meridians, measured on different parallels, are as the cosines of the latitudes of those parallels; so that if we know the length of a degree on the equator, or on any given parallel, we may thus readily find the length of a degree on any other given parallel. The proportion (1) or the equation (2) suffices for the solution of every example in parallel sailing; and, just as in plane sailing, we may embody the necessary particulars in a right-angled triangle. Thus, let the base represent the distance sailed, the hypotenuse the difference of longitude, in linear measure, and the angle between the two the latitude of the parallel; then, by right-angled triangles:

$$\text{hyp.} = \frac{\text{base}}{\cos \text{base angle}}, \text{ that is, diff. long.} = \frac{\text{dist. sailed}}{\cos \text{latitude}},$$

which is the equation (2).

We may, therefore, solve any problem in parallel sailing like a problem in plane sailing, by inspection of the traverse table: in order to this, we have only to regard the latitude of the parallel as *course*, and the distance sailed on it as *diff. lat.;* the corresponding *distance*, in the traverse table, will be diff. long. The perpendicular of our right-angled triangle has no significance; it serves merely to connect the other parts together.

NOTE.—If logarithms be used in working any example in parallel sailing, then, on account of the change in the radius of the table, the 1 in the proportion (1) must be changed into 10^{10}, this being the numerical value of the logarithmic radius. The proportion may be written thus:

cos lat. : radius : : dist. : diff. long.

where radius = 1 for the table of natural sines and cosines, and log radius = 10 for the table of log sines and cosines; so that, by logarithms, we should have:

log diff. long. = 10 + log dist. — log cos lat. . . . (4).

We think, however, that in general logarithms should be dispensed with, whenever the work by natural sines or cosines requires only one reference to the table.

Examples in Parallel Sailing.

1. A ship in lat. 49° 32′ N., and long. 10° 16′ W., sails due W. 118 miles; required the longitude arrived at?

diff. long. = dist. ÷ cos lat. cos 49° 32′ = ·6,4,90) 118 (182 miles.

		649
Long. left	10° 16′ W.	531
Diff. long. 182 miles	3° 2′ W.	519
Long. in	13° 18′ W.	12

By Inspection.—Taking the latitude (or rather 49°) as a course, and 118 as diff. lat., the corresponding distance in the traverse table is 180; but if we take 50° lat. as a course, and the same 118 as diff. lat. the corresponding distance in the table will be 184; half the sum of these, namely, 182, is therefore about the true diff. long.

2. A ship in lat. 36° 58′ N., and long. 20° 25′ W., is bound to St. Mary's, one of the Western Islands, in the same latitude, and in long. 25° 13′ W. What distance must she run to arrive at her destination?

dist = cos lat. × diff. long. cos 36° 58′ = ·799

diff. long. reversed 882

Long. of ship . .	20° 25′ W.	1598
Long. of St. Mary's	25° 13′ W.	639
Diff. long. .	4° 48′ = 288 miles.	64

Dist. = 230·1 miles.

By Inspection.—Taking 37° as a course, and 288 as a distance, the corresponding diff. lat. in the traverse table is 230, the distance required.

3. From two ports, both in lat. 32° 20′ N., and 256 miles apart, measured on the parallel, two ships sail due N., till they arrive at lat. 44° 30′ N. How many miles measured on the parallel reached are they apart?

This example is to be worked by the proportion or formula (3), and as there are two trigonometrical quantities concerned, namely, cos l, and cos l', we shall use logarithms.

As	cos l,	32° 20′	arith. comp.	·0732
:	cos l',	44° 30′	. . . .	9·8532
: :	d,	256	. . . .	2·4082
:	d',	216·1	. . . .	2·3346

Hence, measured on the parallel arrived at, the ships are 216 miles apart.

The work of this example, without logarithms, as indicated by the formula $d' = d \cos l' \div \cos l$, occupies more figures than that above, but probably not more time. The learner is recommended to solve it in this latter way, as an additional exercise.

4. A ship in lat. 53° 36′ N., and long. 10° 18′ E., sails due W. 236 miles: required the longitude arrived at?

Ans. long. in 3° 40′ E.

5. A ship in lat. 57° 29′ N., and long. 1° 47′ W., sails due E. 125 miles: required the longitude in?

Ans. long. in 2° 6′ E.

6. A ship in lat. 32° N. is bound to a port in the same latitude, but lying 6° 24′ of longitude to the E.: what distance has she to run? Ans. dist. 325·6 miles.

7. On a certain parallel 384 miles answers to 500 miles of diff. long.: required the latitude of the parallel?

Ans. lat. 39° 49′.

8. A ship from long. 81° 36′ W. sails W. 310 miles, and

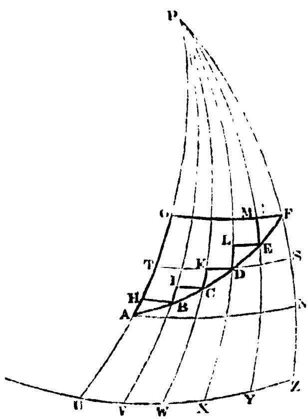
then finds by observation that her longitude is 91° 50′ W.: what is the latitude of the parallel on which she has sailed? Ans. lat. 59° 41′.

9. If a ship sail due E. 126 miles from the North Cape in Lapland, and then due N. till she arrives at lat. 73° 26′ N: how far must she sail due W. to reach the meridian of the North Cape? Ans. dist. 111·3 miles.

10. In what latitude will a ship's diff. long. be three times the distance she sails on the parallel having that latitude? Ans. lat. 70° 32′ *nearly*.

Mid-Latitude Sailing.

We have seen in the preceding article how the difference of longitude which a ship makes, may be determined when she sails on a parallel of latitude: we are now to consider the more general problem, namely, to find the difference of longitude made when the ship sails upon an oblique course. For the solution of this problem, without astronomical observations, Navigation offers two distinct methods: the one to be here explained, called *middle latitude* sailing; and the other, to be discussed in next chapter, called *Mercator's* sailing.

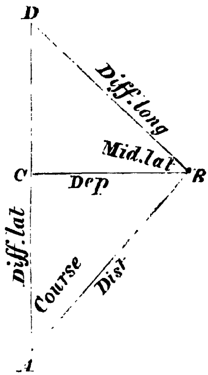


Mid-latitude sailing is a combination of plane sailing and parallel sailing; it proceeds on the supposition that what in plane sailing is called the departure, namely, H B + I C + K D + L E + M F, made by a ship, in sailing on the oblique rhumb A F, is equal to the distance T S, of the meridians of A and F, measured on the *middle parallel of latitude* between A and F, or between A and O.

Assuming then that T S is equal to the departure made by a ship in sailing from A to F, the rule for finding the difference of longitude between A and F, may be deduced as follows:

It has been seen in plane sailing that the difference of latitude A O, the distance run A F, and the angle A of the course may all be correctly represented in a right-angled triangle A B C, as in the margin.

Now the side of the triangle marked *departure* is, in the present hypothesis, the same as the mid-latitude distance between the meridians sailed from and arrived at, so that the difference of longitude made by the ship is the same as if it had sailed the *distance* C B on the mid-latitude parallel. We have now therefore a case of parallel sailing, the line C B representing the distance; so that, as in that sailing, if we make C B the base of a right-angled triangle, and the angle at the base the latitude of the parallel, that is the mid-latitude, it is plain that the hypotenuse B D, will be the difference of longitude.



We thus have two connected right-angled triangles; one, the lower in the above diagram, constructed conformably to the principles of plane sailing, the upper agreeably to the principles of parallel sailing; and what is *departure* in the lower triangle, is regarded as *distance* on the mid-latitude parallel in the upper. The perpendicular C D is superfluous except for the purpose of completing the triangle.

Now, by right-angled triangles, we have from the upper triangle,

$$\text{Diff. long.} = \frac{\text{departure}}{\text{cos mid-lat.}};$$

But from the principles of plane sailing, or from the lower triangle, we have

departure = dist. × sin course = diff. lat. × tan course.

Consequently,

$$\text{Diff. long.} = \frac{\text{departure}}{\text{cos mid-lat.}} = \frac{\text{dist.} \times \text{sin course}}{\text{cos mid-lat.}} \quad \frac{\text{diff. lat.} \times \text{tan course.}}{\text{cos mid-lat.}}$$

And these expressions embody the whole theory of parallel sailing. They may be stated as proportions thus:

1. cos mid. lat. : rad. (1) : : dep. : diff. long.
2. cos mid. lat. : sin course : : dist. : diff. long.
3. cos mid. lat. : tan course : : diff. lat. : diff. long.

If logarithms be used, then in the first proportion log rad. =10.

Examples in Mid-Latitude Sailing.—Single Courses.

1. A ship from latitude 52° 6′ N., and longitude 35° 6′ W. sails N.W. by W. 224 miles: required the lat. and long. arrived at?

For the diff. lat.

Diff. lat. = dist. × cos course.

cos 5 points =	·5556
224 reversed	422
	11112
	1111
	222
Diff. lat.	124·45 miles.

For the mid-lat.

60)124	
2° 4′ N	= diff. lat.
52° 6′ N	= lat. left.
54° 10′ N	= lat. in.
2)106° 16′	= sum of latitudes.
53° 8′	= ½ sum, or mid-lat.

For the diff. long. (By logarithms).

As	cos mid-lat.	53° 8′, arith. comp.		0·2219
:	sin course	5 points	. . . .	9·9198
: :	dist. .	. 224	. . . .	2·3502
:	diff. long.	. 310·4	. . . .	2·4919

Or, using proportion 3 instead of 2, the work will be,

As	cos mid-lat.	53° 8′, arith. comp.	0·2219
:	tan course	5 points . . .	10·1751
: :	diff. lat.	. 124·4 . . .	2·0948
:	diff. long.	. 310·4 . . .	2·4918

Hence the diff. long. is 5° 10′ W., which, added to 35° 6′ W., gives 40° 16′ W. for the long. in, the lat. in being 54° 10′ N.

By Inspection. For course 5 points, and distance 224, the traverse table gives dep. 186·2, and diff. lat. 124·4.

Again, for the mid.-lat. 53° as course, and the half of 186·2, namely 93·1, as diff. lat., the traverse table gives for dist. 155, the double of which is 310, the diff. long.

NOTE. The above method of determining the difference of longitude is not strictly accurate, since the departure is not exactly equal to the mid-latitude distance between the meridian left and the meridian reached. For a single day's run, however, the error is of no practical consequence, and in low latitudes, more especially if the angle of the course be large, that is, if the track of the ship be nearly due east or due west, the method may be depended upon, even for several days' run. But by applying to the mid-latitude the correction given in the table*, the method may always be employed with safety; the table is used thus:—Take out the correction under the given difference of latitude, and against the given mid-latitude. *Add* this correction to the mid-latitude; call the sum the *true* mid-latitude, and employ it, instead of the uncorrected mid-latitude in the calculation.

If the difference of latitude be not more than 1°, no correction will be necessary; when it is 2°, and under 3°, add 1′.

The principle on which the table is constructed will be explained at a future page. But it is easy to show here that some such table is necessary: thus—

Since diff. long. = dep. ÷ cos mid-lat. = dep. × sec mid-lat.

* See "Navigation Tables:" the table for correcting mid-latitude.

it follows that if there be any error in estimating the departure, that is, in regarding the mid-latitude distance between the meridians as equal to it, there will be an error still greater in the resulting diff. long. because *secant* always exceeds unity, so that in high latitudes the error in longitude may be seriously wide of the truth.

In the next example, where the diff. lat. is large, we shall work, for the diff. long. with the *true* mid-latitude.

2. A ship from lat. 51° 18′ N., long. 9° 50′ W., sails S. 33° 8′ W. a distance of 1024 miles: required the lat. and long. in?

For the diff. lat.

Diff. lat. = dist. × cos course.

cos 33° 8′ . .	·8374
1024 reversed .	4201
	8374
	167
	33
diff. lat.	857·4 miles.

For the true mid-lat.

6,0)85,7

14° 17′ S. = diff. lat.
51° 18′ N. = lat. left.

37° 1′ N. = lat. in.

2)88° 19′ = sum lats.

44° 9′½ = mid-lat.
Correction 27′

44° 36′½ = true mid-lat.

For the diff. long.

As cos true mid-lat. 44° 36′½, arith. comp.		0·1476
: sin course 33° 8′		9·7377
: : dist. . . 1024		3·0103
: diff. long. 786·3 = 13° 6′ . .		2·8956

If in this work the correction had been omitted, the diff. long. would have been 780·1, which is six miles in error.

9° 50′ + 13° 6′ = 22° 56′ W. long. in.

3. A ship from lat. 52° 6′ N., and long. 35° 6′ W., sails N.W. by W. 229 miles: required the lat. and long. arrived at?

Ans. lat. 54° 13′ N.; long. 40° 23′ W.

4. A ship from lat. 49° 57′ N., long. 5° 11′ W., sails between S. and W., till she arrives in lat. 38° 27′ N., when she finds she has made 440 miles of departure: what was the course steered, the distance run, and the long. arrived at?

Ans. course, S. 32° 32′ W.; dist. 818 miles; long. in, 15° 28′ W.

5. A ship from lat. 37° N. long., 22° 56′ W., steers N. 33° 19′ E., till she finds herself in lat. 51° 18′ N.: what longitude is she then in?

Ans. 9° 45′ W.

6. A ship from lat. 37° 48′ N., long. 25° 10′ W., is bound for a place in lat. 50° 13′ N., and long. 3° 38′ W.: required her course and distance?

Ans. course, N. 51° 7′ E.; dist., 1187 miles.

7. A ship from lat. 38° 42′½ N., long. 9° 8′½ W., sails on a W.S.W. course, a distance of 700 miles: required the lat. and long. arrived at?

Ans. lat. 37° 54′ N.; long. 12° 33 W.

8. A ship from lat. 40° 41′ N., long. 16° 37′ W., sails between N. and E. till she arrives at lat. 43° 57′ N., and finds that she has made 248 miles of departure: required the course, distance, and long. in?

Ans. course, 51° 41′ E.; dist. 316 miles; long. in, 11° W.

Note. From the principles discussed in the foregoing article, it is evident, as was observed at p. 50, that the determination of the direct course and distance from the balance of the departures on a compound course must involve some amount of error. If, at the end of a series of courses, it is found that the departures east just balance the departures west, the custom is to conclude that the ship has returned to the meridian left; but it is plain from the principles of mid-latitude sailing, that if a ship in N. lat. sail obliquely towards the N.E. quarter, and then, altering her course, sail towards the N.W. quarter, till she reach the same meridian:—it is plain that the mid-latitude distance between the meridians, on the first course, must exceed that on the second, and the correction taken from the table somewhat increases this excess. It follows, therefore, that the resultant of the departures in a traverse will not be the correct departure for the equivalent single course and distance, although in a day or two's run, the inaccuracy may be of no practical consequence.

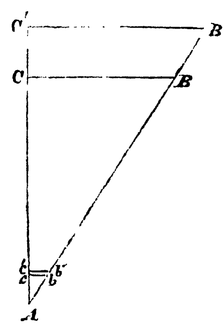
CHAPTER IV.

MERCATOR'S SAILING.—TRAVERSES BY BOTH SAILINGS.

THE principal object of mid-latitude sailing, as we have just seen, is to render the results of ordinary plane sailing available for the purpose of discovering difference of longitude. The determination of longitude may, indeed, be considered as the master problem of Navigation, and accordingly it has, more than any other, engaged the attention of scientific and nautical men. But of all the methods of solution hitherto proposed—excepting those dependent upon astronomical observations—that which we are about to explain is the most ingenious and satisfactory. It was first invented by Gerrard Mercator, a Fleming, who in 1556 published a chart, constructed upon peculiar principles, from which differences of longitude could be deduced.

The mathematical theory of this construction, however, and the tables necessary for bringing MERCATOR'S SAILING under the dominion of numerical computation, is due to an Englishman, Edward Wright, who formed his table of *Meridional Parts* after the manner now to be described.

Let A B, A C, and C B, in the annexed right-angled triangle, represent the distance run, the difference of latitude and the departure made on any single course, A. We know that the departure, C B, is not the representation of any line on the surface of the sphere, but the aggregate of all the minute departures shown in the diagram at p. 56, united in one continuous line. Let A *b c* be one of the elementary triangles in that diagram, *c b* being one of the elementary departures, and A *c* the elementary difference of latitude corresponding. Then since *c b* is a portion of a parallel of latitude, it will be to a similar portion of the equator or of the meridian, as the cosine of the latitude of the parallel is to radius (or 1), as was proved at p. 52; and this similar portion of the equator measures the difference of longitude between *c* and *b*.



If, therefore, the elementary distance, A *b*, be prolonged to *b'*, till the corresponding departure *c' b'* becomes *equal* to this difference of longitude, we shall have the following proportion, namely:

$$c\,b : c'\,b' :: \cos \text{lat. of } c\,b : 1$$

But (Euc. 4, VI.) $c\,b : c'\,b' :: \text{A}\,c : \text{A}\,c'$

$$\therefore \cos \text{lat. of } c\,b : 1 :: \text{A}\,c : \text{A}\,c'$$

$$\therefore \text{A}\,c' = \frac{\text{A}\,c}{\cos \text{lat. of } c\,b} = \text{A}\,c \times \sec \text{lat. of } c\,b \;.\;.\;. \quad (1)$$

It thus appears that if the proper difference of latitude A *c*, be increased to A *c'*, so that A $c' = \text{A}\,c \times$ sec lat. of *c*, the proper departure, *c b*, will become increased to *c' b'*, so that *c' b'* = diff. long. of *c* and *b*. In other words, a ship having made the small diff. lat. A *c*, and the corresponding departure *c b*, must continue her course till her diff. lat. A *c'* has increased to A $c \times$ sec lat. of *c*, in order that her increased departure *c' b'* may be equal to the diff. long. made in sailing from A to *b*. Now it is evident that if *all* the elementary differences of latitude are prolonged in this manner, the sum of all the corresponding increased elementary departures will be the whole diff. long.. made in sailing from A to B. Consequently, to represent the diff. long. between A and B, the diff. lat. A C must be prolonged till the length A C′ becomes equal to the sum of all the increased elementary differences of latitude, when the corresponding increased departure, C′ B′, will represent the diff. long. made in sailing from A to B. The business then, is to contrive means for finding, from A C, the proper enlargement of it, A C′. Wright proceeded as follows:

Taking the elementary differences of latitude each equal to a nautical mile, or one minute of the meridian, commencing at the equator, and calling the enlargements MERIDIONAL PARTS, he knew, from the relation (1) above, that—

Meridional Parts of 1′ = sec. 1′.
2′ = sec. 1′ + sec. 2′.
3′ = sec. 1′ + sec. 2′ + sec. 3′.
4′ = sec. 1′ + sec. 2′ + sec. 3′ + sec. 4′.
&c. &c.

And from these equalities he calculated the proper enlargement of the portions of the meridian, increasing minute by minute, from the equator, by help of the table of natural secants, thus:

	Lat.	Sum of nat. secants.	Mer. Parts.
Mer. Parts of	1′	= 1·0000000	= 1·0000000
	2′	= 1·0000000 + 1·0000002	= 2·0000002
	3′	= 2·0000002 + 1·0000004	= 3·0000006
	4′	= 3·0000006 + 1·0000007	= 4·0000013
	5′	= 4·0000013 + 1·0000011	= 5·0000024
	&c.	&c.	

It was by summing up the natural secants in this way that the first table of meridional parts was constructed. If we enter such a table with the latitude of A (preceding diagram), we shall find against that latitude the enlarged or *meridional latitude*; in like manner, entering with the latitude of C, we also find the corresponding meridional latitude: the difference of the two will be A C′ the *meridional difference of latitude*, or the sum of the two, if A and C are on opposite sides of the equator.

It is plain that a table of meridional parts, constructed after this method, will be the more strictly accurate the smaller the elementary portions of the meridian are taken; as, for instance, by taking them each half a minute in length, instead of a whole minute, as indeed was subsequently done. But Dr. Halley contrived means of constructing the table in another way, which way involved no inaccuracy at all; and the tables in existing use are all formed in this correct manner.* (See the Mathematical Tables.)

Referring now to the diagram at p. 63, we have the two following proportions for the solution of problems in Mercator's sailing, namely:

1. As rad. (1) : tan course : : mer. diff. lat. : diff. long.
2. As proper diff. lat. (A C) : dep. : : mer. diff. lat. : diff. long.

And, as in former cases, we think it will sometimes be more convenient to work examples in this sailing without logarithms than with them.

Examples in Mercator's Sailing. Single Courses.

1. A ship from lat. 52° 6′ N., and long. 35° 6′ W., sails N.W. by W. 229 miles: required her lat. and long. in?

By ex. 3, p. 60, the lat. in is found to be 54° 13′ N.; and

* For an account of Dr. Halley's method, and for further details on the progress of this part of Navigation, the inquiring student is referred to the Navigation and Nautical Astronomy, in "Orr's Circle of the Sciences."

to find the diff. long. we proceed by Mercator's sailing as follows:

Lat. in . 54° 13′ N. Mer. parts 3887	Diff. long. = tan course × mer. diff. lat.
Lat. left 52° 6′ N. ,, 3675	tan 5 points = 1·4966
Long. left 35° 6′ W. Mer. diff. lat. 212	212
Diff. long. 5° 17′ W. = 317 miles.	29932
Long. in. 40° 23′ W.	1497
	299
	Diff. long. . 317·28 miles.

The longitude is therefore the same as that previously found by mid-latitude sailing.

2. A ship from lat. 51° 18′ N., and long. 9° 50′ W., sails S. 33° 8′ W. 1024 miles: required the lat. and long. in?

The lat. is found in ex. 2, page 60, to be 37° 1′ N.

Lat. left 51° 18′ N. Mer. parts 3598	Diff. long. = tan. course × mer. diff. lat.
Lat. in 37° 1′ N. ,, 2394	tan 33° 8′ = ·6527
Mer. diff. lat. 1204	4021
Long. left 9° 50′ W.	6527
Diff. long. 13° 6′ W. = 786 miles.	1305
Long. in 22° 56′ W.	26
	Diff. long. 785·8 miles.

By Inspection. For the course 33°, and distance 256, being one-fourth of the given distance, the traverse table gives diff. lat. = 214·7, four times which is 858, therefore diff. lat. = 14° 18′ S., and hence the lat. in is 37° N. The meridional difference between the two latitudes is 1205. For one-fifth of this, namely 241, as diff. lat., and 33° as course, the traverse table gives, under *departure*, 156·9, five times which is 784, the miles of diff. long. This makes the long. in 22° 58′, two minutes too great. When the number with which we enter the traverse table is beyond the limits of the table, it may be a little more convenient to divide that number by 10: in this way 1205 will give 120·5. The nearest to this, under diff. lat. is 120·8, the corresponding departure being 78·4, ten times which is 784 for the diff. long. On account of small quantities being disregarded, the traverse table does not always give results with the same accuracy as computation.

3. Required, the course and distance between Ushant, in lat. 48° 28′ N., long. 5° 3′ W., and St. Michael's, in lat. 37° 44′ N., long. 25° 40′ W.?

For the course.

tan course = diff. long ÷ mer diff. lat.

Diff. long. = 20° 37′ = 1237 miles.

Ushant, lat.	48° 28′	Mer. pts.	3334
St. Michael	37° 44′	"	2448
Diff. lat. 644 m. =	10° 44′	Mer. D. lat. =	886

8,8,6)1237(1·3962 = tan 54° 23′
886
351
2658
852
7974
546
532
14

For the distance.

Dist. = diff. lat. ÷ cos course.

cos 54° 23′ = ·5,8,2,3)644 (1106 miles.
5823
617
582
35
35

Otherwise by logarithms.

Diff. long. 1237 . . .	3·0924
Mer. diff. lat. 886 . .	2·9474
tan 54° 24′	·1450

In strictness the angle, in both computations, is about 54° 23′½.

By Inspection. The diff. long. and the mer. diff. lat. being found as above, seek in the traverse table for the mer. diff. lat., in that diff. lat. column having the diff. long. in the corresponding dep. column. The page in which these are found will give the course: with this course and the true diff. lat. enter the table again for the distance. But the traverse table is not well adapted for the solution of examples of this kind; it usually gives but approximate results, and, as in the present case, the approximation may not be very close. In this example the distance by the table is 11 miles short of the truth as given by computation.

4. A ship from lat. 51° 9′ N. sails S.W. by W. 216 miles: required the lat. in, and the diff. long. made?

Ans. lat. 49° 9′ N.; diff. long. 4° 40′ W.

5. A ship sails from lat. 37° N. long., 22° 56′ W., on the course N. 33° 19′ E., till she arrives at lat. 51° 18′ N.: required the distance sailed and the long. arrived at?

Ans. dist. 1027 miles; long. 9° 45′ W.

6. A ship sails from lat. 42° 54′ N. on the course S.E.¼E., till her diff. long. is 134 miles: required the distance sailed, and the lat. in?

Ans. dist. 132½ miles; lat. 41° 25′ N.

7. A ship sails N.E. by E. from lat. 42° 25′ N., and long. 15° 6′ W., till she finds herself in lat. 46° 20′ N.: required the distance sailed and long. in?

Ans. dist. 423 miles; long. in 6° 54′ W.

8. A ship from lat. 51° 18′ N., long. 9° 50′ W., sails S. 33° 19′ W., till her departure is 564 miles: required her long. in? Ans. long. 23° 2′ W.

Compound Courses by Mid-Latitude and Mercator's Sailing.

In order to find the diff. lat. and the diff. long. made at the end of a series of courses, or a traverse, we must register the particulars of each course in a traverse table, as at page 49, and proceed in one or other of the two following ways:

1. Having found the diff. lat. and dep. made during the traverse as at page 49, determine from these the direct course and distance, and find the diff. long. due to this single course by either mid-latitude or Mercator's sailing, as in the foregoing articles.

2. Or: the several entries having been made in the traverse table as before, find the balance of the diff. lat. columns only; we shall thus discover the latitude in; and for the diff. long. we proceed thus:

From the latitudes at the beginning and end of each course find the corresponding mid-latitude, with which and the departure made during the course, deduce the diff. long. by mid-latitude sailing. The diff. long. being thus found for each distinct course, the whole diff. long. due to the traverse becomes known. But if Mercator's sailing be employed instead of the mid-latitude method, then there will be no occasion for the insertion of any departures in the table.

The following example, worked both by mid-latitude and Mercator's sailing, will sufficiently show how the tabulated quantities are to be arranged:

1. A ship from lat. 60° 9′ N., and long. 1° 7′ W., sailed the following courses and distances, namely:

1st. N.E. by N. 69 miles. 3rd. N. by W. ½ W. 78 miles.
2nd. N.N.E. 48 miles. 4th. N.E. 108 miles.
5th. S.E. by E. 50 miles.

Required the direct course and distance, and the lat. and long. in?

TRAVERSE TABLE.

Courses.	Dist.	Diff. lat.		Departure.	
		N.	S.	E.	W.
N. E. by N.	69	57·4		38·3	
N. N. E.	48	44·4		18·4	
N. by W. ½ W.	78	74·6			22·6
N. E.	108	76·4		76·4	
S. E. by E.	50		27·8	41·6	
∴ Direct course N. 34° 4′ E.		252·8		174·7	
Distance 272 miles.		27·8		22·6	
		225		152·1	

By the traverse table the diff. lat. 225, and dep. 152·1, gives for the course 34°, and for the distance, 272 miles. The computation is as follows:

For the course.

tan course = dep ÷ diff. lat.

22,5)152·1(·6760 = tan 34° 4′
1350
171
1575
135
135

For the distance.

dist. = diff. lat. ÷ cos course.

·8,2,8,4)225 (272 miles.
1657
593
580
13

For the diff. long. by mid-latitude sailing.

Latitude left . .	60° 9′ N.	
Diff. lat. 225 m. .	3° 45′ N.	
Latitude in . .	63° 54′ N.	
Sum of lats. . .	124° 3′	
½ sum, or mid-lat.	62° 1′½	
Correction . . .	2′½	
True mid-lat. .	62° 4′	

Diff. long. = dep. ÷ cos mid-lat.

cos 62° 4′ = ·46,8,4)152·1 (325 miles.
14052
1158
937
221
234

Hence the diff. long. by mid-lat. sailing is 325 miles E.

For the diff. long. by Mercator's sailing.

Latitude left	60° 9′ N.	mer. pts.	4545
Latitude in	63° 54′ N.	,,	5026
Meridional diff. lat.			481

Diff. long. = mer. diff. lat. × tan course.

tan course = ·676
184
2704
541
7
Diff. long. . 325·2 miles.

It thus appears that the diff. long. made during the traverse is 325 miles, on the supposition, however, that the traverse is correctly resolved into the single course and distance as given above; in other words, that the balance of the departures is the same as the departure that would be made by sailing on a single course from the place left to that arrived at. But, as already shown, such is not the case; and consequently the diff. long. just determined, must be affected with error. To avoid this error it is necessary to proceed according to the second of the two methods described above, that is, in one or other of the following ways:

WORK OF THE PRECEDING EXAMPLE ON MORE CORRECT PRINCIPLES.

1st. *By Mid-Latitude Sailing.*

2nd. *By Mercator's Sailing.*

NOTE. The object of each of the following solutions is to determine the *difference of longitude* correctly; the difference of *latitude* is always accurately ascertained as above; but to give a complete form to the work, the longitude table is annexed to the traverse table for finding the diff. lat. and the several departures.

1. *Solution by mid-latitude sailing.*

TRAVERSE TABLE.						LONGITUDE TABLE.				
Courses.	Dist.	Diff. lat.		Departure.		Lats.	Sums.	Mid. lats.	Diff. long.	
		N.	S.	E.	W.	60° 9′			E.	W.
N. E. by N.	69	57·4		38·3		61 6′	121° 15′	60° 37′	78	
N. N. E.	48	41·4		18·4		61° 50′	122° 56′	61° 28′	38	
N. by W. ½ W.	78	74·6			22·6	63° 5′	124° 55′	62° 27′		49
N. E.	108	76·4		76·4		64° 21′	127° 26′	63° 43′	174	
S. E. by E.	50		27·8	41·6		63° 53′	128° 14′	64° 7′	95	
		252·8							385	
		27·8							49	
	Diff. lat.	225						Diff. long.	336	

NOTE. As the diff. lat. made on any single course does not much exceed a degree, there is no need for any correction of the mid-lats. The two columns for diff. long. are filled up from the traverse table by this rule:—Take the mid-lat. as a course, and seek the corresponding departure in a *diff. lat.* column, against which, in the *dist.* column will be found the number of miles in the diff. long. In the table the course 60° and diff. lat. 38·3 gives dist. 77, and the course 61°, with same diff. lat., gives dist. 79; we therefore take 78 for the course 60°½, as 60° 37′ nearly is. In like manner

61° 28′, is regarded as the mean between 61° and 62°, and 62° 27′, as the mean between 62° and 63°, while 63° 43′, is regarded as 64°, so also is 64° 7′.

2. Solution by Mercator's sailing.

TRAVERSE TABLE.				LONGITUDE TABLE.				
Courses.	Dist.	Diff. lat.		Lats.	Mer. Parts.	Mer. D. L.	Diff. long.	
		N.	S.	60° 9′	4545		E.	W.
N. E. by N.	69	57·4		61° 6′	4662	117	78·3	
N. N. E.	48	44·4		61° 50′	4754	92	38·1	
N. by W. ½ W.	78	74·6		63° 5′	4916	162		49·2
N. E.	108	76·4		64° 21′	5088	172	172	
S. E. by E.	50		27·8	63° 53′	5023	65	97·3	
		252·8					385·7	
		27·8					49·2	
Diff. lat.		225				Diff. long.	336·5	

The two columns for diff. long. are, as before, supplied from the traverse table. By entering the table with the given course, we seek for the given mer. diff. lat. in a diff. lat. column, and against it, in the dep. column, we find the number of miles in the diff. long. To find the lat. and long. in, we have

Latitude left	60° 9′ N.	Longitude left	1° 7′ W.
Diff. lat. 225 m.	3° 45′ N.	Diff. long. 336 m.	5° 36′ E.
Latitude in	63° 54′ N.	Longitude in	4° 29′ E.

By the first mode of computation, in which the traverse is reduced to a single course and distance, the longitude in is 4° 18′, which is 11′ in error.

In order now to find the more correct single course and distance, we have

For the course.

tan course = diff. long ÷ mer. diff. lat.

Lat. left 60° 9′	Mer. parts	4545
Lat. in 63° 54′	,,	5026
Mer. diff. lat.		481

336 ÷ 481 = ·6985 = tan 34° 56′.

For the distance.

Dist. = diff. lat. ÷ cos course.

cos 34° 56′ = ·8,1,9,8)225 (275 mls.

1640

610

573

Hence the correct course is N. 34° 56′ E., and the distance 275 miles.

NOTE. It may be instructive to the learner to notice here that, agreeably to the general practice, in forming the column headed "lats.", we have disregarded every decimal below ·5 in the diff. lat. columns, and have replaced every decimal above ·5 by unit: and in consequence of this, the final latitude in the former column comes out 63° 53′ instead of 63° 54′, as it ought to do. Now, although a fastidious attention to minute accuracy is seldom absolutely necessary in operations of this kind, yet when precision can be attained with very little extra trouble, it is always better and safer to aim at it. By noticing the consecutive number in the table, the influence of the decimal may be much more accurately estimated than by the rough general principle of rejecting it altogether, or replacing it by unit. Thus, it is plain that each decimal in the column headed N. is very nearly ·5, or ½, and that in the column S. is very nearly equal to unit: the more correct "Longitude Table" above will therefore be as follows:—

LONGITUDE TABLE.

Lats.	Mer. Parts.	Mer. D. L.	Diff. Long. E.	Diff. Long. W.
60° 9′	4545			
61° 6′½	4663	118	78·9	
61° 51′	4756	93	38·6	
63° 5′½	4917	161		48·8
64° 22′	5090	173	173	
63° 54′	5026	64	96·8	

Longitude left . . .	1° 7′ W.	387·3
Diff. Long. 338·5 .	5° 38′½ E.	48·8
Longitude in . . .	4° 31′½ E.	338·5

We thus see that from not taking a more correct estimate of the decimals in the former work, the result was about 2 miles of longitude too little. If the several courses and distances in the above traverse be *correct*, the longitude now deduced cannot err from the truth by more than a small fraction of a minute, provided an accurate table of Mer. Parts has been used.

2. A ship from lat. 66° 14′ N., and long. 3° 12′ E., has sailed the following courses, namely:—

1st. N.N.E. ½ E.	46 miles.	3rd. N. ¾ W.	52 miles.
2nd. N.E. ½ E.	28 „	4th. N.E. by E. ¼ E.	57 „

5th. E.S.E. 24 miles:

required the latitude and longitude in?

Ans., lat. 68° 24′ N.; long. 7° 53′ E.

3. A ship from lat. 38° 14′ N., and long. 25° 56′ W., has sailed the following courses, namely:—

1st. N.E. by N. ¼ E.	56 miles.	4th. S.S.E.	30 miles.
2nd. N.N.W.	38 „	5th. S. by W.	20 „
3rd. N.W. by W.	46 „	6th. N.E. by N.	60 „

required the latitude and longitude in, and the correct single course and distance?

Ans., lat. in, 40° 2′½ N.; long. in, 25° 29′ W.; course, N. 10° 57′ E.; dist. 110½ miles.

Having now sufficiently discussed Mercator's sailing, we are in a condition to explain the principles on which the table for correcting the mid-latitude referred to at page 59 is constructed.

Let l represent the proper difference of latitude.

l' the meridional difference of latitude.

m the lat. in which the dist. between the two meridians = departure.

L the difference of longitude of those meridians.

Then for tan course, by plane, mid-lat., and Mercator's sailings, we have

$$\tan \text{course} = \frac{\text{departure}}{l} = \frac{\text{L} \times \cos m}{l} = \frac{\text{L}}{l'} \therefore \cos m = \frac{l}{l'}.$$

Hence, by dividing the proper diff. lat. l, by the meridional diff. lat. l', we get cos m, and thence m, the latitude of the parallel, the portion of which intercepted between the two meridians is exactly equal to the departure, the length of this intercepted portion being

$$L \times \cos m = \text{departure (see p. 57).}$$

It is the difference between m and the latitude of the middle parallel that is inserted in the table referred to at p. 59.

NOTE. Before concluding the present chapter, it may be as well to notice that, when in any example the diff. long. is given, and from knowing also two of the quantities, course, diff. lat., departure, or distance, it is required to find the lats. from and in, such example cannot be worked by Mercator's sailing. The proper diff. lat. and the mer. diff. lat. may be found, but not the lats. themselves: the problem must be solved by mid-latitude sailing, as in the following instance.

Ex. A ship sails in the N.W. quarter 248 miles, till her departure is 135 miles, and her diff. long. 310 miles: required the lat. from and in?

By plane sailing the diff. lat. is found to be 208 miles = 3° 28′, and from the equation last given above

$$\cos m = \frac{\text{dep.}}{L} = \frac{135}{310} = \cdot 4355 = \cos 64^\circ\ 11'.$$

Hence, the mid-latitude *corrected* is $m = 64^\circ\ 11'$; this is greater than the mid-latitude unmodified, by the correction in the table, namely by 3′; therefore the mid-lat. is 64° 8′, and proceeding as in the margin, we readily determine the lats. from and in. From neglecting the tabular correction the latitudes in other books are made too great.

mid. lat.	. 64° 8′
half diff. lat.	1° 44′
lat. from	. 62° 24′N.
lat. in	. 65° 52′N.

Again. If, with the diff. long., one lat. be given, and the course, dist., or dep., mid-latitude sailing is not applicable to the finding of the other lat. For example:

Ex. A ship from lat. 34° 29′ N., sails S. 41° W., till her diff. long. is 680 miles: required her lat. in?

Mer. diff. lat. = diff. long. ÷ tan course = 680 ÷ 8693 = 782

Lat. from, 31° 29′ N. . . Mer. Parts		2207
	Mer. diff. lat. . . .	782
Lat. in, 23° 6′ N. Mer. Parts . .		1425

Hence the latitude arrived at is 23° 6′ N.

CHAPTER V.

CURRENT SAILING—PLYING TO WINDWARD—TAKING DEPARTURES.

If a current act upon a ship, her rate of sailing is necessarily affected by it, and in general both her rate and the direction in which she would otherwise move through the water.

If the ship sail directly with or directly against the current, her rate only will be affected; but if she sail athwart the current, both her rate of sailing and her course become subjected to its influence.

The course, as determined from the compass (the usual corrections being made), marks the direction of the ship's head, and in this direction the ship moves a certain distance in a certain time; but the current carries her a certain other direction and distance in the same time, her actual motion being compounded of the two. It is thus the same—as far as position is concerned, disregarding the time of arriving at it—as if the ship had sailed the two distinct courses and distances in succession, so that current sailing resolves itself into a simple case of traverse sailing, as soon as the direction and velocity of the current are ascertained. The direction of the current, or the point of the compass towards which it flows, is called the *set* of the current; and its velocity, or rate, is called the *drift*.

The usual way of ascertaining the set and drift of a current unexpectedly met with at sea, is to take a boat a short distance from the ship; and, in order to keep it from being carried by the current, to let down, to the depth of about one hundred fathoms, a heavy iron pot, or some other sufficient weight, attached to a rope fastened to the stem of the boat, which by this means is kept steady. The log is then hove into the current, the *direction* in which it is carried, or the set of the current, is determined by aid of a boat compass; and the *rate* at which it is carried, or the hourly drift of the current, is given by the number of knots of the log-line run out in half a minute.

Examples in Current Sailing.

1. A ship sails N.W. a distance, by the log, of 60 miles, in a current that sets S.S.W., drifting 25 miles in the same time: required the course and distance made good?

This is the same as the following question, namely:—

A ship sails the following courses and distances—

1st. N.W. 60 miles. 2nd. S.S.W. 25 miles: what is the direct course and distance?

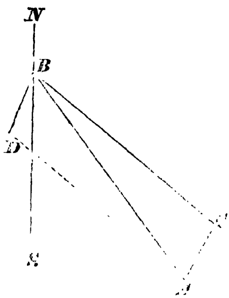
Courses.	Dist.	Diff. lat.		Departure.	
		N.	S.	E.	W.
N. W.	60	42·4			42·4
S. S. W.	25		23·1		9·6
Diff. lat. . .		19·3		Dep.	52

For this diff. lat. and dep. the course by plain sailing is N. 69° 38′ W., and the distance is 55½ miles.

2. A ship sailing at the rate of 7 knots an hour, is bound to a port bearing S. 52° W., but the passage is in a current which sets S.S.E., two miles an hour: it is required to shape the course?

Here one only of the two courses of the traverse is given, together with the resulting direct course; to find the other component course: we shall give two solutions, the second by the traverse table.

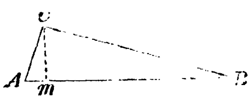
Let B A be in the direction of the port, and B the place of the ship, B D in the direction of the current = 2, and B C in the required direction = 7. Then in the triangle A B C, there are given the side B C = 7, the side C A = B D = 2, and the angle C A B = D B A = 22° 30′ + 52° = 74° 30′, to find the angle A B C. In order to this, we have (p. 26),



B C : C A : : sin 74° 30′ : sin A B C.
that is, 7 : 2 : : ·9636 : sin A B C.

$$\begin{array}{r} 2 \\ \hline 7)1{\cdot}9272 \\ \hline \sin 15^\circ\, 59' = {\cdot}2753 \end{array} \qquad \begin{array}{rl} \therefore \text{A B C} & = 15^\circ\, 59' \\ \text{A B S} & = 52^\circ \\ \hline \therefore \text{course C B S} & = 67^\circ\, 59' \end{array}$$

Otherwise.—In the triangle A B C, let A C, B C, and the angle C A B, measure the same as in the above diagram; and let C m be perpendicular to A B: then, by right-angled triangles,



A C sin A = C m, and B C sin B = C m . . . . (1).

Hence, entering the traverse table with A = 74° 30′ as a course, and A C = 2 as a distance, we get the dep. C m = 1·9.

Again, with the dep. C m = 1·9, and the distance B C = 7, we get the course B = 16°, which added to A B S = 52°, gives 68° for C B S, the required course.

The learner will observe that the solution previously given is at once derived from the equations (1): for from these

$$\sin B = \frac{C\,m}{B\,C} = \frac{A\,C \sin A}{B\,C}.$$

3. A ship runs N. E. by N. 18 miles in three hours, in a current setting W. by S. two miles an hour: required the course and distance made good?

Ans. course 1½ points, or N. by E. ½ E.: dist. 14 miles.

4. A ship in 24 hours sails the following courses in a current setting S.E. by S. 1½ miles an hour, namely:

1st. S.W. 40 miles.	3rd. S. by E. 47 miles.
2nd. W.S.W. 27 miles.	Current, S.E. by S. 36 miles.

required the direct course and distance made good?

Ans. course, S. 11° 50′ W., dist. 117 miles.

5. The port bears due E., the current sets S.W. by S. three knots an hour, the rate of sailing is 4 knots an hour: required the course to be steered?

Ans. course N. 51° E.

6. A ship sailing in a current has by her reckoning run S. by E. 42 miles, and by observations is found to have made 55 miles, of diff. lat. and 18 miles of dep.: required the set and drift of the current?

Ans. set S. 62° 12′ W., whole drift 30 miles.

Plying to Windward.

When a ship bound to a port has a foul wind, she can reach it only by *tacking*, that is, by crossing the wind on two or more courses, making a zigzag instead of a direct track. This is called plying to windward.

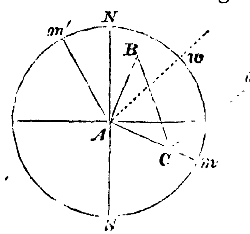
Having sailed a certain distance as near to the wind as she can, the ship tacks about, recrossing the current of air at the same angle; and thus she crosses and recrosses always at the same angle, till she arrives at her port.

Starboard signifies the righthand side, and larboard the lefthand side. When a ship plies with the wind on the right the *starboard tacks are aboard*, and when the wind is on the left the *larboard tacks are aboard.* When a ship sails as near as she can to the point from which the wind blows, she is said to be *close hauled.* The following example will sufficiently illustrate the calculations usually necessary in plying to windward, a subject in which the learner will perceive that some knowledge of oblique-angled triangles is requisite.

Examples in Plying to Windward.

1. Being within sight of my port bearing N. by E. ½ E. distant 18 miles, a fresh gale sprung up from the N.E.: with my larboard tacks aboard, and close hauled within six points of the wind, how far must I run before tacking about; and what will be my distance from the port on the second board?

In the annexed diagram A is the place of the ship, B that of the port, A C the distance on the first board, and C B that on the second. The direction of the wind is marked by *w* A, *w′* C.



As the ship sails within 6 points of the wind, the arc *w m* must be = 6 points, and if *w m′* be made also = 6 points, C B will be parallel to A *m′*. As *w* A C is 6 points, *w′* C A is 10, and since *w′* C B is also 6, B C A is 4. Again, B A N is 1½, and *w* A N is 4, ∴ *w* A B is 2½ ∴ C A B is 8½. Hence, in the triangle A B C, we have given the side A B = 18, and the angles A and C equal to 8½ and 4 points respectively, to find A C and C B.

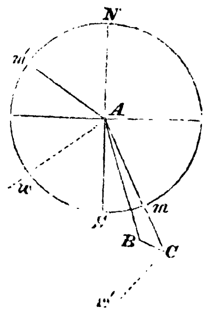
sin 4 points : sin 8½ points = sin 7½ points : : 18 : B C.
sin 4 points : sin 3½ points (B) : : 18 : A C.

sin 4 pts. (C) Arith. Comp.	·1505	sin 4 pts. (C) Arith. Comp.	·1505
: sin 8½ pts. (A)	9·9979	: sin 3½ pts. (B)	9·8024
: : A B = 18	1·2553	: : A B = 18	1·2553
: B C = 25·23	1·4037	: A C = 16·15	1·2082

Hence, the ship must sail 16 miles on the first tack, and then 25¼ miles on the second, to reach her port. The course on the second, or starboard tack, is 6 points — 4 points = 2 points, or N.N.W.

2. If a ship can lie within 6 points of the wind on the larboard tack, and within 5½ points on the starboard tack,

required her course and distance on each tack to reach a port lying S. by E. 22 miles, the wind being at S.W ?



Let A be the place of the ship, and B that of the port, and let the first course A C be on the starboard tack, the direction w A being that of the wind, and the arc $w\ m = 5\frac{1}{2}$ points. If the arc $w\ m'$ be made equal to 6 points, C B parallel to A m', will be the other course, or that on the larboard tack.

$$w\text{ A C} = 5\tfrac{1}{2} \therefore w'\text{ C A} = 10\tfrac{1}{2},\ w'\text{ C B} = 6 \therefore \text{B C A} = 4\tfrac{1}{2}.$$
$$\text{Also B A S} = 1, \text{ and } w\text{ A S} = 4 \therefore w\text{ A B} = 5 \therefore \text{B A C} = \tfrac{1}{2}.$$

Hence, in the triangle A B C, we have given the side A B = 22, and the angles A and C equal to ½ a point and 4½ points respectively, to find A C and C B.

sin 4½ pts. (C) Arith. Comp.	·1118	sin 4½ pts. (C) Arith. Comp.	·1118
: sin ½ pt. (A)	8·9913	: sin 11 (or 5) pts. (B) .	9·9198
: : A B = 22	1·3424	: : A B = 22	1·3424
: B C = 2·79	·4455	: A C = 23·66	1·3740

The course C A S on the starboard tack is 1½ points, or S. by E. ½ E., 23·66 miles: the course on the larboard tack, being equal to the angle m' A N, is 6 points or W.N.W. 279 miles. It is obvious, that when a ship close hauled is to reach her port on two tacks, she must steer on one tack till the bearing of the port is the same as the course on the other tack. And, as the foregoing illustrations sufficiently show, when the distance A B and the bearing of the port are known, we may always work by the following rule :—As the sine of the angle between the two courses is to the sine of the angle between the given distance and either course, so is that distance to the distance sailed on the other course.

3. A ship is bound to a port 80 miles distant, and directly to windward, which is N.E. by N. ½ E. and ...

reach her port at two boards, each within 6 points of the wind, and to lead with the starboard tack: required her course and distance on each tack?

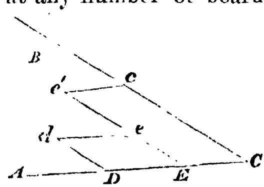
Ans. starboard tack, N.N.W. $\frac{1}{2}$ W., 104·5 miles;
larboard tack, E.S.E. $\frac{1}{2}$ E., 104·5 miles.

4. Wishing to reach a point bearing N.N.W., 15 miles, but the wind being at W. by N., I was obliged to ply to windward; the ship, close hauled, could make way within 6 points of the wind: required the course and distance on each tack?

Ans. larboard tack, N. by W., 17·65 miles;
starboard tack, S.W. by S., 4·138 miles.

5. The port bears N. by E. $\frac{1}{2}$ E., 18 miles; the wind blows from N.E., the ship after running 48 miles on the larboard tack within 6 points of the wind, tacks about: required her course and distance to the port on the second tack? Ans. course N. 57° 35′ W., dist. 49·58 miles.

Note.—Whether a ship, when close hauled, reaches a point at two boards or courses, or, by more frequent tacking, at any number of boards, the actual distance sailed is just the same. Thus, suppose, first, the ship A reaches the point B on two boards A C, C B; the whole distance sailed is A C + C B. Suppose, secondly, that she tacks at D, running D d parallel to C B, then tacking again, that she runs $d\ e$ parallel to A C, and so on till she arrives at some point c in C B, and then sails on her last course, c B. Then, because the opposite sides of a parallelogram are equal $d\ e$ = D E, and $e'\ c$ = E C ∴ A D + $d\ e$ + $e'\ c$ = A C. In like manner D d + $e\ e'$ = C c ∴ C B = D d + $e\ e'$ + c B. Hence, the distance A C + C B = A D + D d + $d\ e$ + $e\ e'$ + $e'\ c$ + c B.



When the port is directly to windward there may be some advantage in working up to it by a succession of short courses, as figured above; for the wind may change, and any change must be for the better,—and it is plain that at whatever intermediate point on the above zigzag path the ship may be, she is nearer her port B than she would be by running the same distance along A C, C B.

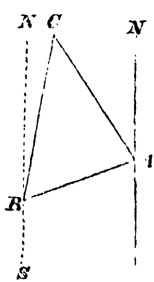
Taking Departures.

At the commencement of a voyage before the ship loses all sight of land, the distance and bearing of some known headland, lighthouse, or other object, the last familiar spot likely to be seen, is taken, and the ship is supposed to have taken her departure from that place, the direction opposite to the bearing and the distance being regarded as the first course and distance, and are entered as such on the log-board.

The bearing being taken by the compass, it is customary for experienced navigators to estimate the distance by the eye, but the more correct method of taking a departure is to observe two bearings of the object, measuring by the log the distance sailed in the interim between the observations, as in the following examples:—

Examples in Taking Departures.

1. Sailing down the Channel the Eddystone bore N.W. by N.; and after running W.S.W. 18 miles, it bore N. by E.: required the course and distance from the Eddystone to the place of the last observation?



In the annexed diagram, A represents the place of the ship at the first observation, B its place at the second, and C is the object observed. By the question the angle N A C is 3 points, and the angle N B C is 1 point, also the course of the ship S A B is 6 points. Consequently, for the number of points in the angles A, B, of the triangle A B C, we have

$A = 16 - 6 - 3 = 7$, $B = 16 - 10 - 1 = 5$, $\therefore C = 4$.

sin C, 4 points, Arith. Comp. . . .	·1505
: sin A, 7 points	9·9916
: : A B = 18	1·2553
: B C = 24·97	1·3974

As the course from B to the Eddystone C is N. by E., the course from Eddystone to B must be directly opposite, namely S. by W. Hence, the departure, or first course and distance is S. by W. 25 miles; the lat. and long. left being that of the Eddystone.

2. Sailing down the Channel the Eddystone bore N.W.; and after running W. by S. 8 miles, it bore N.N.E.: required the ship's course and distance from the Eddystone to the place of the last observation?

Ans. course S.S.W., distance 7·2 miles.

3. At three o'clock in the afternoon the Lizard bore N. by W. ½ W., and having sailed 7 knots an hour W. by N. ¼ N. till 6 o'clock, the Lizard bore N. E. ¾ E.: required the course and distance from the Lizard to the place of the last observation?

Ans. course S.W.¾W., distance 19·35 miles.

4. In order to get a departure I observed a headland of known latitude and longitude to bear N. E. by N.; and after running E. by N. 15 miles, the same headland bore W.N.W.: required my distance from the headland at each place of observation?

Ans. first dist. 8½ miles; second, 10·8 miles.

The ship having taken her departure, and her voyage being fairly commenced, she shapes her course according to her destination, by aid of a Mercator's Chart, in which are marked the obstacles and places of danger she must avoid. Her hourly progress, as measured by the log, and the courses she steers from noon till noon, together with other noteworthy particulars, are registered on the log-board, which is a large black board properly divided into columns for these several entries: the result of the 24 hours traverse —leeway, currents, &c., being allowed for—is determined every noon, as in the foregoing pages, and the latitude and longitude in, by dead reckoning, ascertained.

Whenever practicable, these are corrected by means of astronomical observations, and the true latitude and longitude found: the place of the ship may then be pricked off on the chart, and from this place as a fresh starting point the course is shaped for another stage in the journey. A specimen of a ship's journal will be given hereafter; but as the determination of the latitude and longitude of a ship, independently of the dead reckoning, or the latitude and longitude by account, requires a knowledge of nautical astronomy, we must now proceed to the second part of our subject; navigation proper terminating here.

END OF THE NAVIGATION.

NAUTICAL ASTRONOMY.

CHAPTER I.

DEFINITIONS—CORRECTIONS OF OBSERVED ALTITUDES.

NAUTICAL ASTRONOMY is that branch of the general science of astronomy which enables us to determine the situation of a ship at sea by means of celestial observations. It is, therefore, entirely occupied with the solution of one important problem, namely, the finding the latitude and longitude of any spot on the surface of the ocean:—of a place where the erection of a fixed observatory is impossible, and at which even the astronomical telescope cannot be used. It is because we are thus precluded from the advantages of an observatory, and of such instrumental aid as can be always supplied and employed on land, that observations at sea must be limited in their extent, and peculiar in their kind; and it is on these accounts that a special system of practical astronomy must be devised for sea purposes; and hence the propriety of the name *Nautical Astronomy.* The definitions which follow, however, have no exclusive application.

AXIS.—The axis of the heavens is merely the prolongation of the axis of the earth: the axis of the earth is the diameter about which that body really turns from west to east; the axis of the heavens is that about which the heavenly bodies *appear* to turn from east to west. In nautical astronomy, as well as in many parts of general astronomy, we may regard these heavenly bodies to be, as they seem to be, all equidistant from the centre of the earth, and situated in the apparent concavity surrounding us, called the heavens: the points where the axis pierces this concavity are the *poles of the heavens* or the *celestial poles.*

The learner need scarcely be informed that these are not determinate physical points fixed in space, like the poles of the earth; we regard only the direction of these points, not their linear distance: *linear* distances of points or objects in the heavens do not enter into consideration in nautical astronomy, which takes note of *angular* distances only. The angular distance of two objects is the angle at the eye between the visual rays, or straight lines, proceeding one from each object, and meeting at the eye; and it is plain that at whatever point in the straight line from the object that object be placed, the *angular distance* between the two will remain unaltered. In astronomy the eye of the observer is supposed to be at the centre of the earth, which is also the centre of our imaginary concavity; and the angular distance of any two celestial objects must be the same however small or however great the radius of that concavity is supposed to be. This angular distance is, in reality, observed from the surface of the earth, but it is, by a certain correction hereafter explained, always reduced to what it would be if the eye were at the centre: the radius of the *earth* is the only linear measure introduced.

EQUINOCTIAL.—The equinoctial, or the celestial equator, is that great circle of the celestial sphere of which the plane is perpendicular to the axis; it is therefore marked out by the plane of the terrestrial equator being extended to the heavens, the poles of which are the poles of the equinoctial.

MERIDIANS.—The celestial meridians too are, in like manner, traced by extending the planes of the terrestrial meridians to the heavens: they are semicircles perpendicular to the equinoctial, and terminating in the poles of that great circle.

and NADIR.—The zenith is that point of the celestial sphere which is directly over the head of the spectator: a straight line from the centre of the earth, through any place on its surface, if prolonged to the heavens, would mark the *zenith* of that place. And the point in the celestial sphere diametrically opposite to this, is the *nadir* of that place. The line joining the zenith and nadir is evidently the axis of the rational horizon of the place; and the points themselves the poles of the horizon.

VERTICAL CIRCLES.—The vertical circles of any place are the great circles perpendicular to the horizon of that place; they are also called circles of altitude, because the altitude of a celestial object is the height of it above the horizon measured in degrees of the vertical circle passing through it. It is plain that all vertical circles meet in the zenith and nadir; and that the complement of the altitude of any celestial body is the zenith distance of that body. Small circles parallel to the horizon are called parallels of altitude.

The most important of the vertical circles of any place is that which coincides with the meridian: when an object is upon this, its altitude is the greatest; it is the meridian altitude of the object: when the object is on the opposite meridian, or below the elevated pole, its altitude is the least.

The vertical circle at right angles to the celestial meridian, and which therefore passes through the east and west points of the horizon, is also distinguished from the others: it is called the *prime vertical.* When an object is on the meridian, it is either due south, or due north: when it is on the prime vertical, it is either due east or due west.

AZIMUTH.—The azimuth of a celestial body is the arc of the horizon comprehended between the meridian of the observer and the vertical on which the body is. The degrees in this intercepted arc obviously measure the angle at the zenith between the meridian and the vertical through the body. Vertical circles are also frequently called *azimuth circles.*

Amplitude.—This term is also applied to an arc of the horizon,—the arc, namely, comprised between the east point of the horizon, and the point of it where the body rises, or between the west point, and where it sets. Like the azimuth, the amplitude is measured by an angle at the zenith; the angle, namely, between the prime vertical and that which passes through the body at rising or setting; but, unlike the azimuth, the object must be in the horizon when we speak of its amplitude: whereas, whatever be its altitude, it always has azimuth.

Declination.—The declination of a celestial object is its distance from the equinoctial, measured on the celestial meridian which passes through it; so that what is latitude, as respects a point on the earth, is declination in reference to a point in the heavens; and as circles of latitude (terrestrial meridians) all meet at the poles of the earth, or of the equator, so circles of declination all meet at the poles of the heavens, or of the equinoctial. Also, parallels of latitude on the terrestrial, become parallels of declination on the celestial sphere.

Polar Distance.—By the polar distance of a celestial object is meant the arc of the declination circle, from the object, to that pole of the heavens which is elevated above the rational horizon. When the object is on the same side of the equinoctial as the elevated pole, the polar distance is evidently the complement of the declination, or, as it is called, the co-declination: when the object and the elevated pole are on contrary sides of the equinoctial, the polar distance is the declination increased by 90°.

The altitude of the pole, above the rational horizon of any place, is always equal to the latitude of that place. For the latitude is the distance of the zenith from the equinoctial, and therefore the distance between the zenith and the elevated pole is the complement of the latitude; and the same distance is equally the complement of the altitude of the pole above the rational horizon; this altitude is, therefore, equal to the latitude of the place. The depression of the equinoctial below the horizon, or its elevation above the horizon, in the opposite quarter, is the complement of the latitude, or the co-latitude, which is therefore measured by the angle the equinoctial makes with the horizon.

The circles and terms now defined comprehend all those in most frequent use in Nautical Astronomy, and it is always to be understood, whenever we have spoken of the distance between two points, as measured on an arc of one of these circles, that the *angular* distance, or the degrees and minutes of that arc is uniformly meant, and not the linear extent of the arc. The circles referred to having no definite radii, the arcs referred to can have no definite length, though they subtend determinate and calculable angles. We have now only to mention one or two other circles of the celestial sphere occasionally referred to in nautical observations.

THE ECLIPTIC.—This is the great circle described by the sun in its apparent annual motion about the earth; it is in reality the path actually described by the earth about the sun in the contrary direction. The ecliptic crosses the equinoctial at an angle of about 23° 27′½: this is called the obliquity of the ecliptic; it, as well as the points of intersection, is subject to a small change. The two points of intersection are called the equinoctial points; the sun, in its apparent annual path in the ecliptic, passes through one of these points on about the 21st of March, and through the other on about the 23rd of September. At these times the days and nights are equal at all places where the sun rises and sets, because any point in the *equinoctial*, in the apparent daily rotation of the heavens, is as long below the horizon as above, since the horizon of every place divides that and every other *great* circle into two equal portions. The poles are the only places on the earth at which the sun, when in either of the equinoctial points, neither rises nor sets: the equinoctial then coinciding with the horizon, the sun revolves with its centre describing that circle, one half of its disc above, and the other below it. The small advance of the sun in its annual path is too minute in 24 hours to sensibly affect this statement.

The two points of the ecliptic, 90° distant from the equinoctial points, are called the *solstitial* points, as the sun's apparent motion at these points is so slow that he seems almost stationary: he passes through them about the 21st of June and the 21st of December.

CELESTIAL LONGITUDE.—The ecliptic is the circle on which the longitude of every heavenly body is measured: the point from which longitude is measured is the vernal equinoctial point, which is called the first point of the constellation Aries; and, unlike terrestrial longitude, it is measured in one continued direction round the celestial sphere; so that while terrestrial longitude can never exceed 180°, celestial longitude may be of any extent short of 360°. The 360° of the ecliptic is conceived to be divided into twelve equal parts, called *signs;* each sign is therefore an arc of the ecliptic of 30°. The names of the constellations through which these signs pass, and the symbols by which they are denoted, are as follows:—

1. ♈ Aries (The Ram).
2. ♉ Taurus (The Bull).
3. ♊ Gemini (The Twins).
4. ♋ Cancer (The Crab).
5. ♌ Leo (The Lion).
6. ♍ Virgo (The Virgin).
7. ♎ Libra (The Balance).
8. ♏ Scorpio (The Scorpion).
9. ♐ Sagittarius (The Arrow).
10. ♑ Capricornus (The Goat).
11. ♒ Aquarius (The Water-bearer).
12. ♓ Pisces (The Fishes).

Of these, the first six signs are on the north of the equinoctial, and the others on the south. The belt of the heavens about 16° wide, 8° on each side of the ecliptic, and in which these constellations are situated, and within the limits of which the planets pursue their courses being called the *zodiac*, the 12 signs are frequently called the *signs of the zodiac.*

Celestial Latitude.—The latitude of a heavenly body is measured from the ecliptic, north or south, on a circle perpendicular to it; the circles of latitude all uniting in the poles of the ecliptic.

Right Ascension.—The right ascension of a celestial object is the arc of the equinoctial between the first point of Aries and the point where the declination circle through the object cuts the equinoctial. Thus, right ascension and declination in reference to an object in the heavens, correspond to latitude and longitude of a place on the earth. On the earth, longitude is measured from the first meridian (that of Greenwich in this kingdom); in the heavens, longitude and right ascension are both measured from the origin of the signs,—the first point of Aries, or where the ecliptic crosses the equinoctial, but always from W. to E.

We see from these definitions that, as in the terrestrial great circles, every great circle of the heavens is accompanied by another great circle at right-angles to it; thus, latitude and longitude, declination and right ascension, altitude and azimuth, are all pairs of arcs perpendicular to each other. Those great circles all of which are perpendicular to another great circle, in other words, those great circles that all unite in the poles of another, are frequently called *secondaries* to the latter: thus, the meridians are secondaries to the equinoctial; the circles of celestial latitude are secondaries to the ecliptic; and vertical circles, or circles of altitude, are secondaries to the horizon. No measures in the heavens are the degrees, minutes, &c., of a *small* circle; the distance between any two objects taken by an instrument is always the shortest distance; and on a spherical surface, the shortest distance between any two points is the arc of the *great circle* joining those points.

On Time:—Apparent, Mean, and Sidereal.

The interval of time between two successive appearances of the sun upon the same meridian, is the length of a *day;* not of a day according to civil reckoning, or as measured by the 24 hours of a clock, but of a *Solar day.* The interval spoken of, is not uniformly of the same length; for although the earth performs each of its diurnal rotations in exactly the same time, yet its annual motion of revolution round the sun is irregular. Solar days, therefore, vary slightly in length, and it is the mean of all these varying days that is taken for the common day, and divided into the hours, minutes, &c., as shown by clocks and chronometers, and referred to in the common business of life: the common day, therefore, is the *Mean Solar day,* being the mean of all the *Apparent Solar days.*

The *Day,* whether mean or apparent, is divided into 24 equal intervals, called hours; and each of these into minutes and seconds; an hour, minute, &c., of mean, or common time, is not precisely the same as an hour, minute, &c., of apparent time; but the 24th part of the day is always called an hour. We thus see that the apparent day, though not of invariable length, is a natural day: it is the actual interval between two consecutive passages of the sun over the meridian. But the mean day, though of invariable length, is an artificial day; it is not measured by the recurrence of any natural phenomenon. There is, however, a natural day, which, like the artificial mean day, is strictly invariable; it is called the *Sidereal Day,* and measures the interval between two successive appearances of the same fixed star on the meridian, and is the exact time occupied in one rotation of the earth on its axis. The distance of the fixed stars is so immense, that the earth's change of place from day to day produces not the slightest effect upon their apparent positions; whatever star be observed, and whatever part of its orbit the earth be in, it is always found that the interval between two consecutive passages of the star over the meridian is uniformly the same in length: the interval is 23h. 56m. 4·09s. of mean time. In the reckoning of astronomers, both the apparent and the mean day commences at noon, the former at *apparent* noon, or when the sun is actually on the meridian, the latter at *mean* noon, the instant when the sun *would be* on the meridian if his motion in right ascension were uniformly equal to his mean motion. But the sidereal day commences when the first point of Aries is on the meridian. In each kind of day the astronomical reckoning is carried on from 0h. to 24h. But the nautical day, in keeping a ship's account, is the same as the civil day, the reckoning beginning at midnight, counting 12 hours till noon, and then 12 more till the next midnight, when a new day begins. It will be observed, therefore, that the astronomical day does not commence till 12h. of the civil day have expired: thus, August 15, at 9 o'clock in the morning, or as it would be recorded in the ship's account, August 15 at 9h. A.M., in astronomical reckoning would be August 14, at 21h., that is, 3h. from the approaching noon, when a new astronomical day, namely, August 15 commences. It may be noticed here, that "A.M." signifies *in the morning* (*Ante Meridiem*); and "P.M." means *in the afternoon* (*Post Meridiem*).

HOUR-ANGLE.—The angle at the pole of the equinoctial which a meridian passing through the centre of the sun makes with the meridian of the place of observation is called the sun's *hour-angle* from apparent noon; this angle converted into time at the rate of 15° to an hour gives the apparent time at the place *after* noon, if the sun be westward of the meridian, and *before* noon if it be eastward. It is the time shown by a sundial. In observations for the time at sea, it is the sun's hour-angle that is usually the object sought: so that the time deduced is apparent time, which is readily converted into mean time by help of the table for the "Equation of Time," given at p. 1 of the Nautical Almanac, in which publication all the predicted phenomena concerned in Nautical Astronomy are recorded, like the common occurrences of life, in mean time.

The hour-angle for any other celestial object is, in like manner, the angle at the pole between the two meridians,—one through the zenith, and the other through the object; which angle is evidently always the difference between the right ascension of the meridian of the place, and that of the object expressed in degrees.

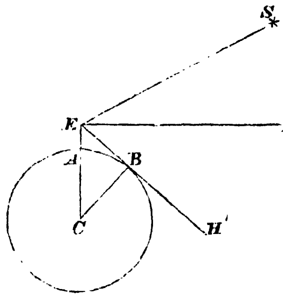
On the Corrections to be applied to Observed Altitudes to obtain the True Altitudes.

Altitudes of celestial objects are taken at sea by a quadrant or sextant, which measures the angular distance of the object above the *visible* horizon of the observer. This is the *observed* altitude: but if the eye, instead of being above, were level with the surface of the sea, the angular elevation of the object would be measured from the *sensible* horizon. This is called the *apparent* altitude, and is obviously less than the observed altitude. The higher the eye, the greater of course is the excess of the observed over the apparent altitude; a correction is therefore necessary to reduce the former to the latter, and this correction is always *subtractive*.

CORRECTION FOR DIP.—Let E be the place of the observer's eye, and S the situation of the object whose altitude is to be found in angular measure, that is, the angle S E H, E H being the horizontal line.* Then, the observer's visible horizon being the tangent to the earth, from E,

* The sensible horizontal line is in strictness drawn from A; but the nearest even of the heavenly bodies is so distant, that the length of A E may be considered as nothing in comparison; that is, the angle at S, subtended by A E, is immeasurably small.

the altitude given by the instrument will be the angle S E H′; the difference between these two, namely, the angle H E H′ is the Dip, or depression of the visible horizon, and is that which must be subtracted from the observed in order to obtain the apparent altitude of



Draw C B from the centre of the earth to the point of contact B; then H E H′, and the angle C, are each the complement of C E B, and are therefore equal; that is, C is equal to the angle of the dip.

Now (Euc. 36, III.), if r be put for the radius of the circle, and h for the height A E of the eye, we have $EB^2 = (2r+h)h = 2rh + h^2$. But since h^2 is very insignificant in comparison with $2rh$, it may without appreciable error be rejected, so that we shall have, $EB = \sqrt{2rh}$. Now, from the right-angled triangle E B C, we have $EB = EC \sin C = (r+h) \sin \text{dip}$; and, because the angle C is very small, never exceeding a few minutes, the arc may be taken for its sine; hence, equating the two expressions for E B, we have

$$(r+h)\,\text{dip} = \sqrt{2rh} \;\therefore\; \text{dip} = \frac{\sqrt{2rh}}{r+h} \text{ or } = \frac{\sqrt{2rh}}{r} \text{ very nearly,}$$

which is the length of the arc, to radius 1, that measures the angle of the dip due to the height h of the eye. This arc, for all values of h likely to occur in practice, is converted into minutes, and the table of "Corrections for Dip" formed.

As the *number* of minutes in the arc which measures C is the same, whatever be the radius of that arc, it follows that the number of minutes or nautical miles in the arc A B is the number of minutes in the dip; and since $\frac{\sqrt{2rh}}{r}$ is the *length* of the arc measuring C to radius 1, it follows that $\sqrt{2rh}$ is the length of the arc A B: this, therefore, being calculated in nautical miles for successive values of h, the table referred to may be constructed a little differently.

CORRECTION FOR SEMI-DIAMETER. — When the body whose altitude is to be taken is either the sun or the moon, the altitude furnished by the instrument is that of either the lowermost or uppermost point of the disc, called the lower or upper *limb* of the body; a correction, therefore, for semi-diameter must be applied, after that for dip, in order to get the apparent altitude of the centre. This correction is the angle subtended at the eye by the semi-diameter of the body observed; it is given for every day in the Nautical Almanac. The moon, however, being so much nearer to the earth than the sun, her diminution of distance, in ascending from the horizon towards the zenith, has a sensible effect upon her apparent magnitude; her semi-diameter measures more when she is in the zenith than when she is in the horizon, for she is nearer, by a semidiameter of the earth in the former case than in the latter, and there is a gradual augmentation of her diameter as she gradually ascends. The moon is only about sixty semi-diameters of the earth off when in the horizon, so that her semi-diameter when in the zenith, is about one-sixtieth part of the whole greater, and the amount of augmentation for any altitude is found by multiplying one-sixtieth of her horizontal semi-diameter by the sine of her altitude. In this way the table intitled, "Augmentation of the Moon's Semi-diameter," is constructed. The number of seconds placed against the altitude in this table must be added to the horizontal semi-diameter, given in the Nautical Almanac, to obtain the semi-diameter proper to that altitude.

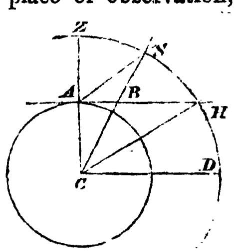
With respect to the sun, his distance from the earth is so great that the augmentation of his semi-diameter, as he increases his altitude, is practically insensible. Hence, *For the apparent alt. of the Sun's centre.*—To the observed alt. apply the corrections for dip and semi-diameter.

For the apparent alt. of the Moon's centre.—To the observed alt. apply the corrections for dip, semi-diameter, and augmentation.

CORRECTION FOR REFRACTION.—As the lower parts of the atmosphere surrounding the earth are compressed by the weight of the upper, the density of the air diminishes the higher we ascend. A ray of light, therefore, from any celestial object, upon entering our atmosphere, meets with an obstruction which becomes more and more sensible the deeper into it the ray penetrates. The ray is thus bent more and more out of its rectilinear course, and its path through the atmosphere, instead of being a straight line, is deflected into a curve concave to the earth. The direction of the object from which the ray proceeds, being judged of by the direction in which the ray arrives at the eye, is thus erroneously inferred: we see the object raised above its real place, and so, except when it is in the zenith, regard its altitude as greater than it actually is. The correction, therefore, for this *refraction* of the rays of light, is like that for dip, always *subtractive*. The more obliquely the rays enter the atmosphere, the greater is their refraction: when they enter perpendicularly, they are not refracted at all: hence when the object is in the horizon, the refraction is greatest; it diminishes as the object ascends, and becomes nothing at the zenith. In different states of the atmosphere the refraction for the same altitude, is of course different; the table gives the value of the correction for the *mean* state of the atmosphere, and to this is sometimes annexed a second table modifying the corrections of the former according to the actual condition of the atmosphere, as shown by the thermometer and barometer at the time and place of observation; but this additional table is but seldom made use of at sea. It is however given at p. 140, of the mathematical tables accompanying this work.

Correction for Parallax.—Before the altitude of any celestial object can be employed for any practical purpose, it must be reduced to what it would have been if taken not from the surface, but from the centre of the earth, and measured not from the sensible, but from the rational horizon of the place of observation. In the case of a fixed star, the distance is so immense, that the radius of the earth dwindles in comparison to a point, and there is no measurable difference between an altitude taken from the centre and an altitude taken, at the same time, from a point directly above the centre, on the surface. But as respects the sun and moon, especially the latter, the angle at the body subtended by the radius of the earth, and which is called the *Parallax in altitude*, is of appreciable magnitude.

Let S be the object observed, A the place of observation, A H the sensible, and C D the rational horizon: the observed altitude when corrected for dip, semi-diameter, and refraction, will be measured by the angle S A H, which is the true altitude of the centre above the sensible horizon, and S C D will be the true altitude of the centre above the rational horizon.



The difference between these two angles, since S C D = S B H, is

S B H—S A H=A S C, the parallax in altitude.

And the true altitude S C D, of the centre above the rational horizon C D, is

S C D=S B H=S A H+A S C, the true altitude.

Hence while the correction for refraction is subtractive, the correction for parallax is additive. The horizontal parallax is the angle A H C: this is given for every day in the year in the Nautical Almanac; that for the sun never varies much from 9″ but that for the moon changes considerably; it is given both for noon and midnight, Greenwich time.

From the horizontal parallax, the parallax in altitude is easily computed; for referring to the triangle S A C, we have the proportion

$$S C : A C :: \sin S A C : \sin A S C.$$

$$\text{or } H C : A C :: \sin S A Z : \sin A S C = \frac{A C}{H C} \cos S A H.$$

But $\frac{A C}{H C} = \sin A H C$, the hor. parallax, and $\cos S A H = \cos$ alt.

And since the parallax in altitude A S C is always a very small angle, we may substitute the seconds in the measuring arc for the time: we thus have,

Par. in alt. in seconds = Hor. Par. in seconds × cos alt.

And it is from this expression that the table for parallax in altitude is constructed. In the table headed "Correction of the Moon's Altitude," the joint correction for both refraction and parallax is given; it exhibits the value of parallax *minus* refraction.

The two corrections just explained (Refraction and Parallax) applied to the apparent altitude of any point in the heavens, reduces the apparent to the *true* altitude of that point, as if the observer's eye were at the centre of the earth, and the angular elevation taken from the rational horizon. Hence,

For the true altitude of the centre of Sun or Moon.—To the apparent altitude apply the corrections for refraction and parallax. As already observed the *stars* have no parallax.

In taking from the Nautical Almanac the measures there given for semi-diameter and horizontal parallax, it must not be forgotten that these measures are what they would be if observed from the centre of the earth *at the Greenwich time* recorded in the almanac. Now for the *moon*, they vary slightly but perceptibly from hour to hour, so that for any intermediate time at Greenwich the corresponding values must be found by proportion. The time at Greenwich, and the instant of any observation or event elsewhere, is the GREENWICH DATE of that observation or event; it is found by converting the longitude of the place of observation into time at the rate of 15° to an hour, as already noticed at page 94.

Having now explained all the corrections necessary to be applied to an altitude observed at sea, in order to deduce the true altitude, we shall proceed to a few examples: we must first remark, however, that even the observed altitude itself is affected with error; it is not that which an instrument entirely free from all imperfection would give. Such an instrument was never constructed by human hands. It is scarcely too much to say, that no chronometer, for instance, whatever the care and skill bestowed upon it, ever showed exact time; nor did any quadrant or sextant ever accurately measure an altitude. But this imperfection is of far less consequence than might at first be supposed: it is of but little moment whether a time-keeper lose or gain, provided only it lose or gain *uniformly*, because, from knowing its error at any one instant, we can easily, from the uniform increase of that error, compute its error at any other instant, and thence obtain the correct time. So with respect to the sextant or quadrant, the *index error*, as it is called, being known, and there are several ways of determining it as will be hereafter noticed, the proper allowance for it can always be made, and the correct observed altitude obtained, as in the examples following:

Examples of Correcting Altitudes taken at sea.

A STAR.—1. If the observed altitude of a star be 42° 36′, and the height of the eye 18 feet, what will its true altitude be, supposing the index error of the instrument to be

Observed Alt.		42° 36′ 0″
Index cor. .	—3′ 18″	. —0 7′ 29″
Dip	—4′ 11″	
Apparent alt.		42° 28′ 31″
Refraction		—1′ 4″
True altitude		42° 27′ 27″

2. The altitude of a star is 43° 12′, the height of the eye 18 feet, and the index error +2′ 24″: required the true altitude? Ans. true alt. 43° 9′ 11″.

3. The altitude of a star is 16° 33′, the height of the eye 17 feet, and the index error +3′: required the true altitude? Ans. true alt. 16° 28′ 42″.

The Sun.—4. On a certain day the observed altitude of the sun's lower limb was 28° 16′, the height of the eye was 20 feet, the index error was —2′ 38″, and the semi-diameter of the sun, as given for that day in the Nautical Almanac, was 16′ 4″: required the true altitude of the centre?

Note.—The sun's horizontal parallax may always be taken at 9″.

Observed alt. sun's L. L. . .		28° 16′ 0″
Index cor. . .	—2′ 38″	
Dip	—4′ 24″	+9′ 2″
Semi-diam. . .	+16′ 4″	
App. alt. centre		28° 25′ 2″
Refrac. and par.		—1′ 39″
True alt. centre		28° 23′ 23″

5. The observed altitude of the sun's lower limb on a certain day was 16° 33′, the height of the eye was 17 feet, the index error was + 3′, and the semi-diameter of the sun, as given in the Nautical Almanac for the day, was 16° 17′: required the true altitude of the centre?

Ans. true alt. 16° 45′ 1″.

6. The altitude of the sun's upper limb was 47° 26′, the height of the eye 20 feet, the index error —1′ 47″, and the

sun's semi-diameter 15′ 49″: required the true altitude of the centre? Ans. 47° 3′ 12″.

THE MOON.—7. The observed altitude of the moon's upper limb was 41° 23′, the index error was +2′, the height of the eye 15 feet, the horizontal semi-diameter at the time 15′ 10″, and the horizontal parallax 55′ 40″: required the true altitude of the moon's centre?

Observed alt. moon's U. L. . . .		41° 23′ 0″
Index cor.	+ 2′ 0″	
Dip	— 3′ 49″	—17′ 9″
Semi-diameter +10″ for Aug. . .	—15′ 20″	
App. alt. centre		41° 5′ 51″
Refraction and par., or correction of moon's apparent altitude .		+40′ 50″
True alt. moon's centre .		41° 46′ 41″

8. The observed altitude of the moon's upper limb was 46° 18′ 49″, the index error —6″, the height of the eye 20 feet, the moon's horizontal semi-diameter at the time 16′ 6″, and the horizontal parallax 59′ 7″: required the true altitude of the moon's centre?

Ans. true alt. 46° 38′ 11″.

9. The observed altitude of the moon's lower limb was 36° 39′ 46″, the index correction +2′ 17″, the height of the eye 22 feet, the moon's horizontal semi-diameter at the time 15′ 10″, and the horizontal parallax 55′ 33″: required the true altitude of the moon's centre?

Ans. true alt. 37° 35′ 52″.

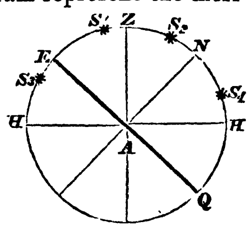
NOTE.—In the preceding examples the horizontal semi-diameter and the horizontal parallax of the moon, have been considered as those due to the body at the instant of observation. In the Nautical Almanac these quantities are given only for every noon and midnight at Greenwich, and they vary sufficiently, at least the latter, in the interval, to render it necessary, if strict accuracy be required, to make allowance for that variation whenever the Greenwich time at the instant of observation is intermediate between Greenwich noon and midnight. But in finding the *latitude* at sea, the omission of a single correction amounting only to a few seconds is not of much practical consequence, so that the allowance alluded to is usually disregarded. If the latitude can be determined to the nearest minute, it is as much as can be expected considering the difficulty of taking an altitude at sea with precision; and indeed it is as much as the safety of navigation requires. Still when the time of an observation of the moon is some hours distant from Greenwich noon or midnight, as we can easily allow for those hours, by a simple inspection of the noon and midnight horizontal parallax in the almanac, we may as well do so. When we come to treat of the problem of the *longitude*, we shall take more exact account of the small corrections of the moon's altitude.

CHAPTER II.

ON FINDING THE LATITUDE AT SEA FROM A MERIDIAN ALTITUDE.

THE best method of determining the latitude of a ship at sea, is that which is deduced from an observed altitude of a celestial object when on the *meridian* of the place. It is to be preferred on two accounts: first, because the observation can in general be made with greater accuracy; and secondly, because the necessary calculations are easier and fewer in number. The most desirable object to observe is the sun, which is on the meridian at the ship's apparent noon, and accordingly the opportunity of taking his altitude at that time should never be disregarded at sea. A star of known declination is also a very suitable object; but when the stars begin to appear the horizon generally becomes too obscure to be sufficiently well defined, a hindrance, however, which may be sometimes removed by employing an *Artificial Horizon* to be hereafter noticed. The moon is not so well calculated to give the latitude with accuracy as the sun or a star, because the moon's declination changes considerably even in an hour, and as the *declination* of the body observed, as well as its meridian altitude, must be known, if there be much error in the ship's longitude by account, and consequently in the Greenwich date of the observation, there will be a proportionate error in the declination, and hence in the latitude inferred. The declination of a star may be regarded as constant, so that there will in this case be no occasion for finding the Greenwich date of the observation: and the declination of the sun varies so slowly, that even a considerable error in the ship's longitude, and therefore in the Greenwich date of the observation, will occasion no error of consequence in the declination at the time of that observation. The way in which the latitude of the place of observation is deduced from the meridian altitude and declination of a celestial object is easily explained as follows:—

Let the circle in the annexed diagram represent the meridian of the observer at Z his zenith, and H H his rational horizon. Let also E Q be the equinoctial, and N the pole which is elevated above the horizon. Then in reference to an object S, on the meridian, S Z will always be the co-altitude, S Q or S E the declination, and E Z or H N the latitude. Now with respect to the elevated pole N, and the zenith Z, the object must be situated in one or other of the four positions marked S_1, S_2, S_3, S_4; and taking these in order we have for the latitude E Z,



$E\,Z = E\,S_1 + S_1\,Z$, that is lat. = dec. + zenith distance.
$E\,Z = E\,S_2 - S_2\,Z$, „ lat. = dec. — zenith distance.
$E\,Z = S_3\,Z - E\,S_3$, „ lat. = zenith dist. — declination.
$H\,N = H\,S_4 + S_4\,N$, „ lat. = altitude + co-declination.

In this last position, where the elevated pole is between the object and the zenith, the object is said to be *below* the pole: in the other positions it is situated above the pole. When the zenith is north of the object, the zenith distance of it is said to be north; and when the zenith is south of it, the zenith distance is said to be south: hence, we have the following rule for finding the latitude from the true altitude when the object is above the pole.

When the object is above the pole.—If the zenith distance and the declination have the same name, that is, if *both* be north or both south, their *sum* will be the latitude.

If the zenith distance and the declination have different names, that is, if one be north and the other south, their *difference* will be the latitude, of the same name as the greater.

When the object is below the pole, the latitude is equal to the sum of the true altitude and the co-declination, of the same name as the declination.

As it is necessary to know the declination of the object observed at the time of observation, or at the Greenwich date of it, we must know how to convert degrees, minutes, &c., of longitude into time, from the relations $15° = 1^h$, $15' = 1^m$, $15'' = 1^s$. These relations suggest the following rule:

Conversion of Longitude into Time.—Rule. Multiply the degrees, minutes, and seconds, each by 2. Divide each result by 30: the quotient, from the *degrees*, will be hours, and *twice* the remainder will be the minutes: we shall thus have the hours and minutes in the degrees. The quotient from the *minutes* will be minutes of time, and *twice* the remainder will be the seconds: we shall thus have the time in the minutes of longitude. And lastly, the quotient from the *seconds* will be seconds of time.

Example 1. Convert 34° 44′ 34″ into time.

The double of this is 68° 88′ 68″, and dividing each denomination separately by 3, cutting off the unit figure for the 0 suppressed in the divisor 30, and remembering to double each remainder, the operation will stand thus:

$$
\begin{array}{llll}
3)6{,}8^\circ & 8{,}8' & 6{,}8'' & \\
\hline
2 & 16 & & \\
 & 2 & 56 & \\
 & & 2 & \cdot 27 \\
2^h & 18^m & 58^s{\cdot}27 & \\
\hline
\end{array}
$$

The division of the seconds is carried on to decimals, these being always used instead of *thirds*.

2. Convert 108° 24′ 22″ into time.

$$
\begin{array}{llll}
3)21{,}6^\circ & 4{,}8' & 4{,}4'' & \\
7 & 12 & & \\
 & 1 & 36 & \\
 & & 1 & \cdot 47 \\
\hline
7^h & 13^m & 37^s{\cdot}47 & \\
\hline
\end{array}
$$

3. 84° 42′ 30″ in time is $5^h\ 38^m\ 50^s$.
4. 93° 37′ 41″ „ $6^h\ 14^m\ 30^s{\cdot}72$.
5. 230° 32′ 10″ „ $15^h\ 22^m\ 8^s{\cdot}7$.

The preceding method of converting degrees, &c. into time, will be found much more convenient than that in common use.

In order to convert time into angular measure, multiply the number of hours by 15′, the product is so many degrees. Divide the minutes and seconds by 4, and reckon every unit of remainder as 15′, if minutes be the dividend, and as 15″ if seconds be the dividend.

For example: let it be required to convert $3^h\ 14^m\ 23^s$ into angular measure, as also $5^h\ 19^m\ 37^s$.

$$
\begin{array}{ll|ll}
3^h = & 45^\circ & 5^h = & 75^\circ \\
14^m = & 3^\circ\ 30' & 19^m = & 4^\circ\ 45' \\
23^s = & 5'\ 45'' & 37^s = & 9'\ 15'' \\
\hline
 & 48^\circ\ 35'\ 45'' & & 79^\circ\ 54'\ 15''
\end{array}
$$

Latitude from Meridian Altitude of the Sun above the Pole.

RULE 1.—From the longitude by account, find the apparent time at Greenwich; that is, the Greenwich date of the observation in apparent time.

2. From p. I of the month in the Nautical Almanac, get the sun's declination at apparent noon at Greenwich, and from the hourly variation of the declination there given, and the Greenwich date, find the proper correction for that date: the declination at the time of observation will thus be obtained.

3. To the observed altitude apply the proper corrections for reducing it to the true altitude, which subtracted from 90° will give the zenith distance.

4. Mark the zenith distance N. or S. according as the zenith is north or south of the sun; then if the declination and zenith distance have the same marks, their sum will be the latitude: if they have different marks, their difference will be the latitude, of the same name as that of the greater of the two quantities.

NOTE.—After the preliminary reduction of the declination to the time of observation, the first step in the work, for obtaining the apparent altitude of the centre of the sun or moon from the observed altitude, comprehends the uniting of the three corrections for index error, dip, and semidiameter, into one: when the signs of these three items are not all alike, the finding of the balance of them is a little inconvenient. But both the index error and the dip being always known before the observation, their combined effect is also known, and may therefore be written down as one correction.

Examples. The Latitude from Meridian Altitude of the Sun above the Pole.

1. March 4, 1858, in longitude 86° 34′ W., the observed meridian altitude of the sun's lower limb was 46° 48′ 30″ (zenith N.), the correction for index and dip was —4′ 6″: required the latitude?

1. *For the app. time at Greenwich.*

Longitude by account . . .	86° 34′ W.
	2′
	3,0)17,2° 6,8′
	5 44
	2
Greenwich date (app. time) . . .	5^h 46^m

The variation for this time must be subtracted, as the declination is decreasing (See Nautical Almanac).

2. *For the decl., Greenwich date.*

Dec. app. noon, Nautical Alm.			
6° 25′ 46″ S.		Diff. for 1^h . .	—57″·81
—5′ 33″			5
Dec. . 6° 20′ 13″ S.		in 5^h . .	289″·05
		in 30^m . .	28″·91
		in 15^m . .	14″·45
		in 1^m . .	·96
	Variation in 5^h 46^m . . .	5′ 33″ =	—333″·37

3. *For the Latitude.*

Observed alt. sun's L. L. . . .		46° 48′ 30″
Index and dip. . . .	—4′ 6″	+12′ 3″
Semi-diam.	+16′ 9″	
App. alt. of centre		46° 0′ 33″
Refraction — parallax		—50″
True alt. of centre		45° 59′ 43″
		90°
True zenith distance		44° 0′ 17″ N.
Declination G. Date		6° 20′ 13″ S.
LATITUDE		37° 40′ 4″ N.

NOTE.—There is no absolute necessity to find the Greenwich date of the observation, in order to get the declination at that date. If we double the hourly variation, divide by 30, and then multiply by the number of degrees and fraction of a degree in the longitude, the proper correction of the declination will be obtained: thus,

Hourly variation	57″·81
	2
	3,0)11,5·62
	3·854
86 reversed	68
	30832
	2312
	193
Cor. of declin.	333″·37

The principle of this second method of correcting the declination for longitude is easily explained. The hourly variation is that due to 15° of longitude; hence, the double of it divided by 30 is the difference of declination due to 1° of longitude; and this difference multiplied by the degrees of longitude of the ship, must give the proper correction of declination. Any odd minutes in the longitude amounting to less than a quarter of a degree, may be disregarded, as they will not make 1″ difference in the result.

2. May 29, 1858, in longitude 31° 17′ W., the observed meridian altitude of the sun's lower limb was 65° 42′ 30″ (zenith N.), the index error was −1′ 9″, and the height of the eye 13 feet: required the latitude?

1. *For the app. time at Greenwich.*

Longitude by account	31°	17′ W.
		2
	3,0)6,2°	3,4
	2	4
		1
Greenwich date	2^h	5^m

2. *For the declin., Greenwich date.*

Noon Declin. . .	21° 37′ 24″ N.	Hourly diff. .	+22″·83
Cor. for long. . .	+48″		2
DECLINATION .	21° 38′ 12″ N.	in 2^h . .	45·66
		in 5^m . .	1·90
		Increase of declination in 2^h 5^m .	+47″·56

3. *For the Latitude.*

Observed alt. sun's L. L.		65° 42′ 30″
Index and dip	— 4′ 42″	+11′ 7″
Semi-diam.	+15′ 49″	
App. alt. of centre		65° 53′ 37″
Refraction — par.		— 22″
True alt. of centre		65° 53′ 15″
		90°
True zenith dist.		24° 6′ 45″ N.
Declin. Greenwich date		21° 38′ 12″ N.
LATITUDE		45° 44′ 57″ N.

The declination at the time of observation is found by the method in the NOTE as follows :—

Noon declination .	21° 37′ 24″	Diff. for 1^h	+22″·83
Cor. for longitude .	— 48″		2
DECLINATION .	21° 38′ 12″		3,0)4,5·66
			1·522
		31 reversed .	13¼
			4566
			152
			38
		Cor. of declin.	+47″·56

3. September 23, 1858, in longitude 94° E., the meridian altitude of the sun's upper limb was 75° 20′ (zenith S.), and the correction for index and dip was —4′ 42″ : required the latitude ?

1. *For the app. time at Greenwich.*

Longitude by account . . .	94° E.
	2
	3,0)18,8
Greenwich date, *before* noon . .	6^h 16^m

2. *For the declin., Greenwich date.*

Noon declin. . . .	0° 3′ 24″ S.	Diff. in 1^h	+ 58″·47
Cor. for E. longitude .	— 6′ 6″		6
DECLINATION .	0° 2′ 42″ N.	in 6^h	350″·82
		in 15^m	14″·62
		in 1^m	1″
Cor. of declination . .			+ 366″ = 6′ 6″

3. *For the Latitude.*

Obs. alt. U. L.		75° 20′ 0″
Ind. and dip . .	— 4′ 42″	— 20′ 41″
Semi-diam. . .	— 15′ 59″	
App. alt. centre		74° 59′ 19″
Ref. — par.		— 14″
True alt. centre		74° 59′ 5″
		90°
True zenith dist.		15° 0′ 55″ S.
Declination		0° 2′ 42″ N.
LATITUDE		14° 58′ 13° S.

In this example the Greenwich time of the observation was 6^h 16^m before the noon of the 23rd. The hourly difference is subtractive, because the *south* declination, in proceeding from the noon of the 23rd towards the noon of the 22nd, decreases. As the decrease of S. declination *exceeds* the S. declination at noon, the declination must have changed from N. to S. in the interval.

In finding the correction of declination for longitude, the learner will in general find the *second* method to be a little more easy and convenient than the first, and the work will be facilitated if he always prepare a blank form of the operation previously to commencing it. Nor should he neglect, when once the Nautical Almanac, or the book of Tables is in hand, to make all the use of it he can in anticipation of what he may want to extract: thus, at the time of taking out the declination, he should also take out the semi-diameter, putting it in its proper place in the blank form. The following is a specimen of such a form, when the second method of finding the declination at the Greenwich date of the observation is used.

Blank form for the Sun.

	° ′ ″		″
Noon Declin.	.. .. ..	Diff. in 1ʰ	...
Cor. for long.	.. ..		× 2
DECLINATION	.. .. ..	3,0)	....
			....
		Long.	× ....
		Cor. of decl. for long.	.....″

*** The longitude is to be taken only to the nearest *half degree*, or at most to the nearest quarter.

		° ′ ″
Observed altitude (L. L. or U. L.)		.. .. ..
Index and dip	..′ ..″ }	.. ..
Semi-diam.	.. ..	
App. alt. centre		.. .. ..
Ref. — par.		— .. ..
True alt. centre		.. .. ..
		90
True zenith dist.		.. .. ..
Declination		.. .. ..
LATITUDE		.. .. ..

The same form will serve equally for a planet, as in the following example:—

4. Jan. 29, 1858, in longitude 58° 37′ E., the observed

meridian altitude of Jupiter's lower limb was 49° 18′ 35″, (zenith N.), the index error was + 4′ 10″, and the height of the eye 22 feet: required the latitude?

Declin. on Merid. of G. . . .	13° 2′ 27″ N.	Diff. in 1h .	+ 5″·3
Correction for long. . . .	−21″		2
DECLINATION . .	13° 2′ 6″ N.		3,0)1,0·6
			·353
		Long. reversed . .	85½
			177
			28
			2
		Cor. for long. . .	20″·7

The correction of the declination is subtractive because the longitude is E

Obs. alt. L. L.		49° 18′ 35″
Ind. and dip. . . .	−27″	−8″
Semi-diam. . .	+19″	
App. alt. centre		49° 18′ 27″
Ref. − par.		−49″
True alt. centre		49° 17′ 38″
		90°
True zenith dist.		40° 42′ 22″ N.
Declination		13° 2′ 6″ N.
LATITUDE		53° 44′ 28″ N.

Latitude from Meridian Altitude of a fixed Star above the Pole.

As already remarked, a star changes its declination so slowly, that any correction for longitude is insensible. And as moreover a fixed star has no parallax in altitude, nor any diameter, the only corrections of the observed altitude will be those for index error, dip, and refraction; the rule, therefore, for deducing the latitude from a star is as follows:—

RULE 1. Correct the observed altitude for index, dip, and refraction; the result will be the true altitude, which subtracted from 90° will give the zenith distance.

2. Mark the zenith distance N. or S. according as the zenith is N. or S. of the star; then, if the declination, taken from the Nautical Almanac, and the zenith distance have the same marks, their sum will be the latitude; if they have different marks, their difference will be the latitude.

Examples. Meridian Altitude of a Star above the Pole.

1. April 11, 1858, the meridian altitude of Arcturus was observed to be 46° 15′ (zenith N.), the index correction was + 2′ 10″, and the height of the eye 20 feet: required the latitude?

Observed altitude .	46° 15′ 0″
Index . +2′ 10″ ; Dip . −4′ 24″	−2′ 14″
Apparent altitude .	46° 12′ 46″
Refraction . . .	−56″
True altitude . .	46° 11′ 50″
	90°
Zenith dist. . .	43° 48′ 10″ N.
Star's declin. Ap. 11	19° 55′ 5″ N.
LATITUDE . .	63° 43′ 15″ N.

2. May 1, 1858, the observed meridian altitude of Spica was 28° 45′ (zenith N.), the index error was — 2′ 20″, and the height of the eye 18 feet; required the latitude?

Observed altitude .	28° 45′ 0″
Index . — 2′ 20″ ; Dip . — 4′ 11″	— 6′ 31″
Apparent altitude .	28° 38′ 29″
Refraction . . .	— 1′ 46″
True altitude . .	28° 36′ 43″
	90°
Zenith distance. .	61° 23′ 17″ N.
Star's declin., May 1	10° 25′ 26″ S.
LATITUDE . .	50° 57′ 51″ N.

NOTE.—The time at Greenwich when a star or planet passes the meridian of that place is very nearly the same as the time at the ship when it passes the ship's meridian; so that, having the approximate time at the ship, we can ascertain by a reference to the Nautical Almanac, what stars or planets will be on the meridian of the ship about that time. The time of the meridian transit of each of the planets is actually given for every day of the year, and the time of a star's transit is found by subtracting the R. A. (right ascension) of the sun from the R. A. of the star, both of which are given in time in the Nautical Almanac: should the R. A. of the sun exceed that of the star, 24^h must be added to the latter. But several stars may be near the meridian of the ship at the same time: to prevent mistake as to the star actually selected from the almanac for observation, we may previously find, approximately, what altitude the star thus selected ought to have: in order to this, add the star's declination to the latitude by account if they are of different names, and subtract if they are of the same name: the result is the zenith distance or co-altitude of the star. By these aids—the time and the altitude—the star may be discovered some minutes before the time of transit, its altitude taken, and the index gradually moved as the star ascends, till it appears stationary, and is about to descend, at which instant it is on the meridian.

Referring to the first of the preceding examples for an illustration, we find from the Nautical Almanac, that on April 11, the R. A. of Arcturus was $14^h\ 9^m\ 14^s$, and that of the sun, $1^h\ 18^m\ 42^s$. The difference of these is $12^h\ 50^m\ 32^s$, which is the time of meridian transit of the star. It may be looked for by aid of the approximate altitude, at about 18 or 20 minutes to 1 o'clock in the morning, making ample allowance for error in the ship's time, and kept in contact with the horizon till it ceases to rise. In the second example it will be found that the observation was made at $10^h\ 43^m\ 19^s$.

But in the case of a *star* (not a planet), instead of making a particular selection from the stars in the Nautical Almanac and then finding its time of transit, it is better to fix upon the time, or rather upon the most convenient interval, and then seek in the almanac for the stars which pass the meridian in that interval, making our selection from among them. Thus, suppose it were required to find what stars will pass the meridian of the ship on April 11.

and 10 o'clock in the evening. Adding 8^h to the R. A. of the sun, we get the R. A. of the ship's meridian at 8^h P.M., and adding 2^h more, we get the R. A. of the meridian at 10^h P.M. The stars whose R. A. lie between these limits are those required. If the sum exceed 24^h, the excess is the R. A. of the meridian. On the day proposed, the sun's R. A. is $1^h\ 18^m\ 42^s$: hence, the R. A. of each of the required stars lies between $9^h\ 18^m\ 42^s$, and $11^h\ 18^m\ 42^s$. Within these limits the Nautical Almanac gives α Hydræ, θ Ursæ Majoris, ϵ Leonis, π Leonis, Regulus, &c.

The learner need scarcely be reminded that the sun's R. A. at Greenwich noon is not precisely the same as his R. A. at any other Greenwich date; but as the sun's mean motion in R. A. is only about 4^m a day, it would be needless to allow for change of R. A. in the present inquiry.

When the horizon is too obscure for the observation of an altitude, an *artificial horizon* is sometimes employed. This consists of a shallow trough of quicksilver, protected from wind and weather by a glass covering or roof. The observer placing himself at a convenient distance from this, so that the object and the reflected image of it may both be distinctly seen, the angular distance between the two is taken; and since the angular distance of the image below the horizontal plane is the same as that of the object itself above that plane, the instrument, corrected for index error, will give double the altitude, and there will be no correction for dip: hence, dividing by 2, the apparent altitude of the object will be obtained.

We shall now give a few examples for exercise in finding the latitude from a meridian altitude of the sun or a star.

Examples for Exercise.

1. April 27, 1858, in north latitude, and in longitude 87° 42′ W., the observed meridian altitude of the sun's

lower limb was 48° 42′ 30″ (zenith N.), the index error was + 1′ 42″, and the height of the eye 18 feet: required the latitude? Ans. latitude, 55° 36′ 56″ N.

2. August 14, 1858, in north latitude, and in longitude 51° W., the observed meridian altitude of the sun's upper limb was 47° 26′ (zenith N.), the index correction was — 1′ 47″, and the height of the eye 20 feet: required the latitude? Ans. latitude, 57° 15′ 59″ N.

3. Nov. 8, 1858, in south latitude, and in longitude 62° E., the meridian altitude of the sun's lower limb was 57° 12′ 30″ (zenith S.), the index correction was + 1′ 36″, and the height of the eye 30 feet: required the latitude? Ans. latitude, 49° 7′ 56″ S.

4. Nov. 21, 1858, in north latitude, and in longitude 165° E., the meridian altitude of the sun's lower limb was observed to be 47° 38′ (zenith N.), the index error was — 1′ 15″, and the height of the eye 17 feet: required the latitude? Ans. latitude, 22° 21′ 43″ N.

5. March 2, 1858, the meridian altitude of Arcturus was observed to be 47° 24′ 30″ (zenith N.), the index error was — 2′ 10″, and the height of the eye 17 feet: required the latitude? Ans. latitude, 62° 37′ 41″ N.

6. March 12, 1858, the meridian altitude of α Hydræ was observed to be 39° 24′ 30″ (zenith N.), the index error was — 2′ 10″, and the height of the eye 17 feet: required the latitude? Ans. latitude, 42° 40′ 4″ N.

7. July 10, 1858, the meridian altitude of Fomalhaut was observed to be 63° 38′ 30″ (zenith N.), the index correction was — 2′ 30″, and the height of the eye 24 feet: required the latitude? Ans. latitude, 3° 52′ 47″ S.

8. April 17, 1858, in longitude 15° W., the observed meridian altitude of the lower limb of the planet Mars was 57° 40′ 30″ (zenith N.), the index correction was + 2′, and the height of the eye 17 feet: required the latitude? Ans. latitude, 12° 24′ 57″ N.

9. June 13, 1858, in longitude 72° 30′ E., the observed

meridian altitude of the lower limb of Venus was 30° 40′ 10″ (zenith S.), the index correction was + 4′ 20″, and the height of the eye 24 feet: required the latitude?

Ans. latitude, 35° 38′ 0″ S.

Latitude from Meridian Altitude of the Moon above the Pole.

As the declination of the moon varies much more rapidly than that of any other heavenly body, it is given in the Nautical Almanac for every hour in the day, together with the average amount by which it varies in 10^{m} of the succeeding hour, that is, one sixth of the whole variation during that hour.

To find what the moon's declination is, when her altitude is taken, we must first determine the Greenwich date, in mean time, of the observation: if the mean time at the ship, as well as the longitude, be known, this of course is easily done; but if the ship's mean time cannot be depended upon, we must then refer to the Nautical Almanac for the Greenwich time of the moon's transit over the Greenwich meridian, and thence by means of the daily variation in the time of transit, and the longitude, find the ship's time of her transit over the ship's meridian; we shall thus get the time at the ship when the observation was made, and thence, by means of the longitude, the Greenwich date of that observation.*

The Greenwich date in hours and minutes being thus

* It may be as well to remark here that the Greenwich date of any observation at sea is at once shown by the chronometer, provided confidence can be placed in its regularity. Such is the perfection to which chronometers are now brought, that they may in general be depended upon for the determination of the mean time at Greenwich throughout a long interval. But an instrument of such delicate construction is very easily injured, and even variations of temperature will disturb in some degree its uniformity of action. It is therefore considered as necessary to the safety of navigation to be provided with methods of finding a ship's position on the ocean, independently of the chronometer.

found, we refer to the Nautical Almanac for the moon's declination at the *hour*, and correct it for the odd minutes by means of the "Diff. of Declin. for 10^m" before alluded to: the declination corresponding to the altitude will thus be obtained.

We shall evidently get the ship's time of transit over the ship's meridian by applying to the Greenwich time of transit over the Greenwich meridian the correction furnished by multiplying the daily difference of time in the Greenwich transit by the longitude, and dividing the product by 360: the following Table, however, enables us to dispense with this operation.

Table for finding the mean time of the Moon's transit over a given meridian from the time of the transit at Greenwich, and the daily variation.

DAILY VARIATION OF THE TIME OF GREENWICH TRANSIT.

Longitude	40m	42m	44m	46m	48m	50m	52m	54m	56m	58m	60m	62m	64m	66m
10°	1	1	1	1	1	1	1	1	1	2	2	2	2	2
20	2	2	2	2	3	3	3	3	3	3	3	3	3	4
30	3	3	4	4	4	4	4	4	4	5	5	5	5	5
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7
50	5	6	6	6	6	7	7	7	7	8	8	8	9	9
60	6	7	7	7	8	8	8	9	9	9	10	10	10	11
70	7	8	8	9	9	9	10	10	10	11	11	12	12	12
80	9	9	9	10	10	11	11	12	12	12	13	13	14	14
90	10	10	11	11	12	12	13	13	13	14	14	15	15	16
100	11	11	12	12	13	13	14	14	15	15	16	17	17	18
110	12	12	13	14	14	15	15	16	16	17	18	18	19	19
120	13	14	14	15	15	16	17	17	18	19	19	20	20	21
130	14	15	15	16	17	17	18	19	19	20	21	21	22	23
140	15	16	17	17	18	19	20	20	21	22	22	23	24	25
150	16	17	18	19	19	20	21	22	22	23	24	25	26	26
160	17	18	19	20	21	21	22	23	24	25	26	26	27	28
170	18	19	20	21	22	23	24	25	25	26	27	28	29	30
180	19	20	21	22	23	24	25	26	27	28	29	30	31	32

Although the above Table may be regarded as sufficiently accurate for the purpose intended, yet the learner is not to expect that the correction for longitude, which it gives by inspection, has the same precision as if it were deduced from direct computation; that is, by multiplying the daily variation by the longitude, and dividing by 360. Certain tables are absolutely indispensable in Nautical Astronomy, but we think it may be reasonably questioned whether the mariner is not sometimes encumbered with a greater abundance of this kind of aid than he really requires. As tables give, in general, only approximations to the truth, the more sparingly they are used, the greater will usually be the accuracy of the work. The computation adverted to above, is too trifling and easy to render a table to supply its place of much value; and we insert it, as it occupies but little room, more in compliance with custom than from necessity. From the foregoing remarks, the learner will be prepared for the following rule for finding the latitude of the ship from a meridian altitude of the moon when above the pole.

RULE 1. From the Nautical Almanac, take out the time of the moon's "Meridian Passage" at Greenwich on the given day, as also the daily variation.

2. From the longitude by account, and the foregoing table, or by independent calculation, reduce the time of the meridian passage at Greenwich to *the time at the ship* when the altitude was taken.

3. From the time thus deduced, and the longitude, find *the time at Greenwich* when the altitude was taken.

[NOTE.—These three precepts may be disregarded if the chronometer at the instant of observation be consulted.]

4. The time at Greenwich being ascertained, take the moon's declination for the *hour* from the Nautical Almanac, computing the correction for the odd minutes by aid of the difference in declination for 10^m.

5. From page III of the month take out the moon's semidiameter, increasing it by the "Augmentation" given in the tables. The correction for index-error, dip, and semi-diameter will reduce the observed altitude of the limb to the apparent altitude of the moon's centre.

6. To the apparent altitude of the centre, add the correction, parallax in alt. minus refraction (See Table XVII.), and the true altitude of the centre will be obtained. This subtracted from 90° will give the zenith distance, which is to be marked N. or S. according as the zenith is N. or S. of the moon. Then as in the case of the sun, take the sum or the difference of the zenith distance and the declination according as they have the same or different marks, and the result will be the latitude.

NOTE.—The moon's semi-diameter and horizontal parallax are given in the Nautical Almanac for every noon and midnight: the corrections for any intermediate Greenwich date may be easily estimated by taking the differences, and then the proportional part of each difference for the number of hours after noon or midnight.

Examples: Meridian Altitude of the Moon above the Pole.

1. May 17, 1858, in longitude 49° W., the meridian altitude of the moon's lower limb was observed to be 47° 18′ 30″ (zenith S.), the index correction was + 1′ 40″, and the height of the eye 20 feet: required the latitude.

1. *For the mean time at Greenwich when the altitude was taken.*

Moon's transit at G., May 17 .	$4^h\ 20^m{\cdot}5$	Daily diff. . .	$56^m{\cdot}4$
Cor. for long. 49° W. . . .	$+7{\cdot}7$	Long. (reversed)	94
Time at ship when alt. was taken	$4^h\ 28^m$		2256
Long. 49° W. in time . .	$+3\ \ 16$		508
Greenwich date of observation .	$7^h\ 44^m$		36,0)276,4($7^m{\cdot}7$ cor.
			252
			244

2. *For the Moon's Declination at $7^h\ 44^m$ at Greenwich.*

Declination May 17, at 7^h	23° 56′ 40″·7 N.	Diff. in 10^m, − 97″·33
Decrease in $7^h\ 44^m$.	− 7′ 8″·25	4·4
DECLINATION at $7^h\ 44^m$.	23° 49′ 32″ N.	38932
		3893
		428″·25 = 7′ 8″·25

3. *For the Moon's Hor. Semi-diameter and Hor. Parallax at 7ʰ 44ᵐ.*

Semi-dia. at noon.	16′ 19″·7	Diff. in 12ʰ	− 6″·2	Hor. par. . .	59′ 47″	Diff. − 22″·5
	− 4	in 6ʰ	3·1		− 15	11·25
SEMI-DIA. at 7ʰ 41ᵐ	16′ 16″	in 1½ʰ	·8	HOR. P. at 7ʰ 44ᵐ	59′ 32″	2·81
		in ¼ʰ	·1			·47
	Correction for 7ʰ 44ᵐ . .		− 4″ *	Cor. for 7ʰ 41ᵐ		− 15″ *

4. *For the Latitude of the Ship.*

Observed alt. of Moon's L. L. . . .			47° 18′ 30″
Index and dip . . .		− 2′ 44″	
Semi-diam.	16′ 16″	+ 16′ 28″	+ 13′ 44″
Augmentation	12″		
App. altitude of moon's centre . . .			47° 32′ 14″
Correction of moon's altitude . . .			+ 38′ 56″
True altitude of centre			48° 11′ 10″
			90°
Zenith distance			41° 48′ 50″ S.
Moon's declination			23° 49′ 32″ N.
LATITUDE			17° 59′ 18″ S.

2. Oct. 4, 1858, in longitude 60° 42′ W., the observed meridian altitude of the moon's lower limb was 30° 30′ 40″ (zenith N.), the index correction was + 5′ 42″, and the height of the eye 16 feet: required the latitude?

1. *For the mean time at Greenwich when the altitude was taken.*

Moon's transit at G. Oct. 3 .	21ʰ 45ᵐ	Diff. . .	46ᵐ·4
Cor. for longitude 60° 42′ W.	+ 7·8	Longitude.	60½
Time at ship . . .	21 52·8		2784
Long. 60° 42′ W. in time .	+ 4 2·8		23·2
Greenwich date of observ. .	25 55·6		36,0)280,7·2(7ᵐ·8 cor.
that is, Oct. 4 . .	1ʰ 55ᵐ·6		252
			287

* These small corrections for the horizontal semi-diameter and the horizontal parallax, need not be computed to the degree of nicety here observed. It will be quite sufficient if the Greenwich date be taken to the nearest half-hour, the "Diff." multiplied by it, and the product divided by 12; thus:—6″·2 × 8 ÷ 12 = 6″·2 × $\frac{2}{3}$ = 4″; and 22″·5 × $\frac{2}{3}$ = 15″,

2. *For the Moon's Declination Oct. 4, at $1^h\ 55^m{\cdot}6$ at Greenwich.*

Declin. Oct. 4, at 2^h .	7° 48′ 48″·3 N.	Diff. in 10^m	− 158″·5
Decrease in $4^m{\cdot}4$* . .	+ 1′ 9″·7		·44
DECLIN. at $1^h\ 55^m{\cdot}6$.	7° 49′ 58″ N.		6340
			634
			69″·74 = 1′ 9″·7

3. *For the Moon's Hor. Semi-diameter and Hor. Parallax at $1^h\ 56^m$.*

Semidiam. at noon	15′ 56″·1	Diff. in 12^h	− 3″·5	Hor. P.	58′ 20″·6	Diff.	12″·7
	·6	in 2^h	− ·6		− 2″·1		− 2″·1
SEMIDIAM. at 2^h	15′ 55″·5			H. P. at 2^h	58′ 18″·5		

4. *For the Latitude of the Ship.*

Observed alt. of Moon's L. L. . . .		30° 30′ 40″
Index and Dip .	+ 1′ 46″	
Semidiam. 15′ 56″ ; Augmentation 8″	+ 16′ 4″	+ 17′ 50″
Apparent alt. of moon's centre . . .		30° 48′ 30″
Correction of moon's app. altitude . .		+ 47′ 56″
True altitude of centre		31° 36′ 26″
		90°
Zenith distance		58° 23′ 34″ N.
Moon's declination		7° 49′ 58″ N.
LATITUDE		66° 13′ 32″ N.

NOTE.—From the foregoing examples the learner will perceive that the principal object of step 1 in the operation, that is, of finding the Greenwich date, is to enable us to get the declination with the necessary accuracy at the instant the altitude was taken. As the declination may

* The Greenwich date, $1^h\ 55^m{\cdot}6$, is $4^m{\cdot}4$ short of 2^h; and as the declination decreases as the time increases, it is less at 2^h than at the Greenwich date; so that the correction of the declination at 2^h, for the preceding $4^m{\cdot}4$, must be *added*.

increase or diminish by so much as nearly 3′ in 10^m of time, it is evident that the Greenwich date of the observation should not err by more than a minute or two minutes of the truth. This date, as already remarked, may in general be got more readily, and with greater precision, from the chronometer than from the longitude by account. Indeed, the longitude by *account*, should not be employed in this problem, unless it be known to differ by less than 30′ of the truth.

In step 3 of the operation there is no occasion for much precision in the Greenwich date: indeed, the correction for it may always be roughly allowed for by a glance at the 12^h differences furnished by the Nautical Almanac, without formally computing for it as above: it is often neglected altogether as being of but little moment. The following is the blank form of the necessary operations.

Blank Form for the Moon.

1. Transit at G.	. .^{h} . .^{m}	Daily diff.	. .^{m}
Cor. for long.	. .	Degrees of long.	× . .
Ship's date of obs.	. . . .		36,0) . . . . (. .^{m} Cor.
Longitude in time	. . . .		
G. date of obs.	. . . .*		

2. Declin. at above *hour*	. .° . .′ . .″	Diff. in 10^m	. .″(divided by 10†)
Cor. for *minutes*	. . . .	Minutes in G. date	× . .
DECLIN. at G. date	. . . . . .		. . .″ Cor. for minutes.

3. Moon's Semidiam. from Naut. Alm. To be corrected for the G. date by inspection.

Moon's Hor. Parallax from Naut. Alm. To be corrected for G. date by inspection.

* This may be got from chronometer.

† That is, remove the decimal point one place to the left.

4. Observed altitude (L. L. or U. L.)		..° ..′ ..″
Index and dip	..′ ..″ }	.. ..
Semidiam. + Augmen.	..′ ..″ }	
App. alt. of moon's centre		.. .. ..
Correction of app. alt. (Table XVII.)		+.. ..
True altitude of moon's centre		.. .. ..
		90
True zenith distance		.. .. ..
Moon's declination at G. date		.. .. ..
LATITUDE		.. .. ..

Examples for Exercise.

1. Aug. 30, 1858 *, in longitude 129° 30′ E., the observed meridian altitude of the moon's lower limb, was 41° 10′ (zenith N.), the index correction was −3′ 40″, and the height of the eye 18 feet: required the latitude?

Ans. latitude, 67° 7′ 6″ N.

2. Nov. 25, 1858, in longitude 22° 30′ W., the observed meridian altitude of the moon's upper limb was 72° 12′ 30″ (zenith N.), the index correction was − 2′ 10″, and the height of the eye 20 feet: required the latitude?

Ans. latitude, 40° 47′ 29″ N.

3. Nov. 29, 1858, at 8^h 46^m, A.M. Greenwich mean time, as shown by the chronometer, the observed altitude of the moon's lower limb when on the meridian of the ship was 38° 15′ (zenith N.), the index correction was − 2′ 10″, and the height of the eye 20 feet: required the latitude? †

Ans. latitude, 50° 9′ 24″ N.

4. Nov. 16, 1858, in longitude 82° 30′ E., the meridian

* The time of the moon's meridian-passage at Greenwich on August 29, is 16^h $10^m{\cdot}7$, which, according to civil reckoning, is Aug. 30, at 4^h $10^m{\cdot}7$ A.M. The learner will not forget that on shipboard the civil reckoning of time is employed; in the Nautical Almanac, the astronomical reckoning. See the work of ex. 2, p. 123.

† In this example the Greenwich date of the observation is given, namely, Nov. 28, 20^h 46^m.

altitude of the moon's lower limb was observed to be 64° 48′ (zenith S.), the index correction was + 6′ 40″, and the height of the eye 22 feet: required the latitude?

Ans. latitude, 24° 42′ 58″ S.

5. Dec. 13, 1858, in longitude 58° 45′ E., the observed meridian altitude of the moon's upper limb was 43° 25′ (zenith S.), the index correction was + 5′ 24″, and the height of the eye 24 feet: required the latitude?

Ans. latitude, 48° 24′ 5″ S.

6. Dec. 17, 1858, in longitude 18° 42′ W., the meridian altitude of the moon's lower limb was observed to be 52° 35′ (zenith N.), the index correction was – 3′ 40″, and the height of the eye 25 feet: required the latitude?

Ans. latitude, 59° 5′ 1″ N.

Latitude from a Meridian Altitude below the Pole.

The sun is on the meridian of any place below the pole at apparent midnight, that is, 12^h after apparent noon at that place; so that 12^h increased or diminished by the longitude in time, according as the place is W. or E. of Greenwich, will be the apparent time at Greenwich; that is, the Greenwich date of the observation: the declination at this time is found as in the examples already given, from the noon-declination in the Nautical Almanac.

For a fixed star the change of declination in 12^h is insensible, so that the declination will be the same as that given in the Nautical Almanac.

For a planet the declination varies sensibly in 12^h, so that, as in the case of the sun, the variation must be allowed for.

In the case of the moon the ship-time of transit over the mid-day portion of the meridian is to be found as in the foregoing examples: this time increased by 12^h and by half the daily difference of time will be the ship-time of her passage over the opposite portion of the meridian; that is, of her meridian passage below the pole. The proper correction of this time for longitude being then made, the Greenwich date of the observation, and thence, by aid of the Nautical Almanac, the declination at that date, is to be found as before. The rule, therefore, is as follows:

RULE 1. Find the declination of the object at the instant of observation, and thence its polar distance.

2. Apply to the observed altitude the proper corrections for obtaining the true altitude.

3. To the true altitude add the polar distance: the sum will be the latitude, of the same name as the declination.

NOTE.—When *above* the pole, the object rises till it arrives at the meridian, when, having attained its *greatest* altitude, it begins to descend: when *below* the pole, on the contrary, the object descends lower and lower till it arrives at the meridian, when having sunk to its *lowest* altitude it begins to ascend. It is only by seizing the instant at which the object appears stationary that its arrival at the meridian can be detected at sea; but it may be as well to notice that, rigorously speaking, this may not be the instant of the meridian transit after all; for it must be remembered that, besides the motion in altitude, there is also a motion in declination, so that it may happen, especially in the case of the moon, that this latter motion may cause the altitude to be the greatest or least a little before or a little after the meridian passage. With regard to the sun and planets, this circumstance is of no moment; but under particular circumstances the meridian altitude of the moon, as furnished by observation, on account of the rapid change in declination of that body, may differ from the altitude when actually on the meridian by 1′ or 2′. The moon, therefore, is the least eligible object from which to deduce the latitude.

Examples: Meridian altitude below the Pole.

1. July 2, 1858, in longitude 23° 10′ W., the observed meridian altitude of the sun's lower limb when below the

pole or at apparent midnight was 7° 40′, the index correction was + 3′ 20″, and the height of the eye 19 feet: required the latitude?

1. *For the Declination at the instant of observation.*

G. Noon declin. July 2 . .	23° 3′ 52″·2 N.	Diff. in 1ʰ .	— 11″·5
,, ,, July 3 . .	22° 59′ 16″·3 N.		2
2)	46° 3′ 8″·5	3,0)	2,3
G. Midnight declin. July 2	23° 1′ 34″ N.		·766
Cor. for longitude W. .	— 18″	Long. (reversed)	32
Declin. at instant of obs.	23° 1′ 16″ N.		1533
	90°		230
Polar distance . . .	66° 58′ 44″	Cor. of dec. for long.	17″·63

2. *For the Latitude of the Ship.*

Observed alt. sun's L. L.		7° 40′ 0″
Index and dip.	— 0′ 57″	+ 14′ 49″
Semi-diameter	+ 15′ 46″	
App. alt. of centre		7° 54′ 49″
Refraction—Parallax		— 6′ 30″
True alt. of centre		7° 48′ 19″
Polar distance		66° 58′ 44″
LATITUDE		74° 47′ 3″ N.

2. April 10, 1858, the observed altitude of the pole-star when on the meridian below the pole was 41° 36′, the index correction was — 4′ 10″, and the height of the eye 17 feet: required the latitude?

Obs. alt. of pole star . .		41° 36′ 0″		
Index cor. — 4′ 10″		16′ 14″		
Dip. . . — 4′ 4″				
App. altitude		41° 52′ 14″		
Refraction		— 1′ 5″	Declin. Ap. 10 .	88° 33′ 16″ N.
				90°
True altitude		41° 51′ 9″		
Polar distance		1° 26′ 44″	Polar distance .	1° 26′ 44″
LATITUDE . . .		43° 18′ 53″ N.		

3. May 15, 1858, in longitude 37° 42′ E., the observed altitude of the moon's lower limb when below the pole was 9° 25′; the index correction was + 2′ 8″, and the height of the eye 22 feet: required the latitude?

1. Moon's upper transit G. May 15	$2^h\ 12^m{\cdot}4$	Daily diff.	$+66^m{\cdot}1$
Half daily difference	33	Degrees of long.	40
Moon's lower transit . . .	14 45		36,0) 264,0 (7ᵐ cor.
Corrections for long. E. . .	— 7		
Ship's date of observation . .	$14^h\ 38^m$		
Longitude in time . . .	— 2 31		
Greenwich date of observation	$12^h\ 7^m$	(May be got from Chronom.)	

2. Moon's declin. at 12^h .	28° 14′ 6″· N.	Diff. in 10^m . .	+18″·85
Correction for 7^m . . .	+13″		·7
Declin. at G. date . .	28° 14′ 19″ N.	Cor. for 7^m . .	+13·195
	90°		
Polar distance . . .	61° 45′ 41″		

3. Moon's Hor. Semidiam. May 15 at 12^h 16′ 35″. Hor. Par. 60′ 42″.

4. Observed alt. moon's L. L.	9° 25′ 0″
Index and Dip. — 2′ 29″ } Semidiam. + Aug. +16′ 38″ }	14′ 9″
App. alt. of moon's centre	9° 39′ 9″
Correction of app. alt.	+54′ 21″
True alt. of moon's centre	10° 33′ 30″
Polar distance	61° 45′ 41″
LATITUDE	72° 19′ 11″ N.

NOTE.—In order that a celestial object may be above the horizon when it is below the pole, it appears that the latitude of the place must exceed the polar distance, the excess being the true altitude of the object. On account of the varying state of the atmosphere near the horizon, the refraction for altitudes below six or seven degrees, cannot be estimated with accuracy. And as the polar distance of the sun is never much less than 67°, and that of the moon never much less than 62°, it follows that for a meridian altitude of 6° or 7° the latitude, in the case of the sun below the pole, must not be less that 73° or 74°, and in the case of the moon, not less than 68° or 69°. Hence, such meridian observations on either of these two bodies are restricted to high latitudes, and are therefore not generally available at sea: but in the case of the fixed stars, opportunities occur in all latitudes of getting a meridian altitude below the pole sufficiently great to allow of the tables of refraction being used with safety. As the pole-star is always above the horizon in latitudes north of the equator, and on cloudless nights is always sufficiently visible, and easily recognised, it is the star more frequently selected for finding the latitude at sea, when north, than any other. Some short useful tables are given in the Nautical Almanac, (pp. 527-9) for finding the latitude from an altitude of the pole-star, whether it be on the meridian of the place of observation or not: the following is the rule there given, (p. 568) with an example of its application.

To find the latitude from an altitude of the Pole Star.—RULE 1. From the observed altitude, when corrected for the error of the instrument, refraction, and dip, subtract 2′: the result is the reduced altitude.

2. Reduce the mean time of observation at the place to the corresponding sidereal time, by the table at page 530, Nautical Almanac.

3. With the sidereal time found, take out the *first correction* (Naut. Alm., p. 527), with its proper sign. If the sign be +, the correction must be *added* to the reduced altitude; but if it be —, it must be subtracted; in either case the result will give an approximate latitude.

4. With the altitude and sidereal time of observation, take out the *second correction,* (p. 528): and with the day of the month, and the same sidereal time, take out the *third correction* (p. 529). These two corrections *added* to the approximate latitude, will give the latitude of the place.

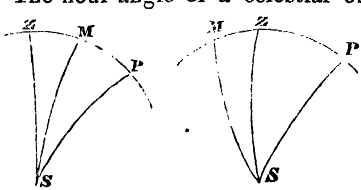
Example: March 6, 1858, in longitude 37° W., at 7ʰ

43m 35s, mean time, suppose the altitude of the pole-star, when corrected for the error of the instrument, refraction, and dip, to be 46° 17′ 28″: required the latitude?

Ship mean time	7h 43m 35s	
Long. 37° W. in time . .	2 28 0	
Greenwich mean time . .	10h 11m 35s	
Sidereal time at Greenwich mean noon		22h 55m 42s
Mean time at ship		7 43 35
Acceleration (page 530, Naut. Alm.) for 10h 12m .		1 41
Sidereal time of observation		6h 40m 58s
Reduced altitude		46° 15′ 28″
With *Argument* 6h 40m 58s, *First Correction*		— 10′ 7″
Approximate latitude		46° 5′ 21″
Arguments, 46° 17′, 6h 41m } *Second Correction*		+ 1′ 6″
Arguments, March 6, 6h 41m *Third Correction*		+ 2′ 31″
LATITUDE		46° 8′ 58″ N.

To find the Latitude when the Declination, Altitude, and Hour-angle are given.

The hour-angle of a celestial object at any place and at any instant is the angle at the pole included between the meridian of the place, and the meridian through the object at that instant. In the annexed diagram, let Z be the zenith of the place, P the elevated pole, and S the object observed, then Z S = co-altitude, P S = polar distance, P = the hour-angle, and P Z = the co-latitude of the place.



To find this last quantity the three former are supposed to be given, so that the solution may be effected by case 2 of oblique-angled spherical triangles (see Spherical Trigonometry, p. 19.) But the following method by right-angled triangles is the more easy.

Draw S M, perpendicular to the meridian of Z, dividing the oblique-angled triangle P Z S, into the two right-angled triangles P M S, Z M S: then by Napier's Rules (Sph. Trig. p. 11.) we have:—

From the triangle P M S, by taking P for middle part, and P S, P M, for adjacent parts,

$$\cos P = \cot PS \tan PM \quad \therefore \tan PM = \cos P \tan PS \quad . \; . \; . \; . \quad (1)$$

And by taking the hypotenuse P S for middle part, and P M, S M for opposite parts,

$$\cos PS = \cos PM \cos SM \quad . \; . \; . \; . \quad (2)$$

Again, from the triangle Z M S, by taking the hypotenuse Z S for middle part, we have

$$\cos ZS = \cos ZM \cos SM \quad . \; . \; . \; . \quad (3)$$

Consequently, dividing (2) by (3), we have,

$$\frac{\cos PS}{\cos ZS} = \frac{\cos PM}{\cos ZM} \therefore \cos ZM = \cos PM \cos ZS \sec PS \quad . \; . \quad (4)$$

Hence, P M being determined from equation (1), and then Z M from equation (4), the sum of these or their difference, according as M falls between P and Z, or not, will give P Z the co-latitude of the place. Bringing equations (1) and (4) together, the formulæ for computation are therefore,

$$\left. \begin{array}{l} \tan PM = \cos \text{hour-angle} \times \text{cotan declination} \\ \cos ZM = \cos PM \times \sin \text{alt.} \times \text{cosec declination} \end{array} \right\} \quad . \; . \; . \quad (A)$$

If the object observed off the meridian be the sun, the hour-angle is the apparent time from the ship's noon, that is, from the sun's passage over the meridian, converted into degrees. The chronometer gives the mean time at Greenwich of the observation, and we thence find, by help of the longitude by account, the ship's mean time of observation, affected only by the error of longitude. This mean time, by applying the correction for the equation of time, given in the Nautical Almanac, becomes converted into apparent time, and thus the time from the ship's apparent noon, or the hour-angle in time, becomes known.

But if the object be other than the sun, we must add the sun's right ascension at the instant of observation, to the apparent time after the ship's preceding noon, the sum, or its excess above 24^h, is the R. A. of the ship's meridian: the difference between this and the right ascension of the object is obviously the hour-angle in time.

It is proper to observe that in determining the latitude by computing the formulæ (A), there may be a doubt as to whether the point M would lie between the zenith and the pole or not, and consequently as to whether the sum or difference of P M, Z M is the co-latitude; but in general, the latitude by account must be near enough to the truth to remove all hesitation on this head.*

If, however, the object be near the meridian, this source of ambiguity may be always avoided, and there is *one* additional reason for preferring an observation near the meridian to one more distant from it:—the higher the object observed, the less likely is the refraction to be disturbed from its mean state. In preparing for a meridian altitude of the sun, it sometimes happens that although an observation can be well taken a few minutes before or after noon, yet that the sun becomes obscured by clouds when actually on the meridian. It is very useful, therefore, to know how the latitude may be obtained from an altitude *near* the meridian: a rule for this purpose may be investigated as follows:—

* When Z M is so small that whether added to, or subtracted from P M, the result, in either case, differs so little from the estimated co-latitude, that there seems no sufficient reason for preferring one to the other, the circumstance can occur only when the body is very near the prime vertical, that is, nearly due E. or due W. And when P M is so small as to make but little difference whether it be added to or subtracted from Z M, the circumstance can occur only when the body is very near the six o'clock hour circle. Except in one or other of these positions the observation may be made; and the latitude deduced with accuracy. It is necessary to notice, however, when the latitude and declination are of contrary names, that the co-declination is then measured from the *depressed* pole P'; so that P' M + Z M — 90° is the distance of Z above the equinoctial, that is the latitude of the ship; and in *this* case there can never be any ambiguity.

Referring to the triangle Z P S, the fundamental theorem of spherical trigonometry gives,

$$\cos ZS = \cos PZ \cos PS - \sin PZ \sin PS \cos P$$

$$\therefore \cos P = \frac{\cos ZS - \cos PZ \cos PS}{\sin PZ \sin PS}$$

Let z represent the zenith distance that S would have when actually upon the meridian, and let z' be the difference between this meridian zenith distance, and the zenith distance Z S, off the meridian found by observation: then since $ZS = z + z'$, the above equation is,

$$\cos P = \frac{\cos (z+z') - \cos PZ \cos PS}{\sin PZ \sin PS}$$

Hence, subtracting each side from 1, and remembering (Plane Trig. p. 27), that $1 - \cos P = 2 \sin^2 \frac{1}{2} P$, we have,

$$2 \sin^2 \tfrac{1}{2} P = \frac{\sin PZ \sin PS + \cos PZ \cos PS - \cos (z+z')}{\sin PZ \sin PS}$$

$$= \frac{\cos (PZ \sim PS) - \cos (z+z')}{\sin PZ \sin PS} \quad \text{(Plane Trig. p. 26).}$$

Now the difference $PZ \sim PS$, between the co-latitude and the polar distance, must be equal to the meridian zenith distance z, because that is also the difference between the co-latitude and the polar distance: therefore,

$$2 \sin^2 \tfrac{1}{2} P = \frac{\cos z - \cos (z+z')}{\sin PZ \sin PS} = \frac{\cos z - \cos z \cos z' + \sin z \sin z'}{\sin PZ \sin PS}$$

As the object S is near the meridian, and as objects near the meridian usually make comparatively but slow advance in altitude, the difference z', between the meridian zenith distance, and that actually observed, may in general be considered as sufficiently small to justify our regarding $\cos z'$ as equal to 1, and thus writing the above equation,

$$2 \sin \tfrac{1}{2} P = \frac{\sin z \sin z'}{\sin PZ \sin PS}$$

$$\therefore \sin z = 2 \sin PZ \sin PS \operatorname{cosec} z \sin^2 \tfrac{1}{2} P$$

Now the arc z' being very small, the number of seconds in it is very nearly equal to the number of times $\sin 1''$ is contained in $\sin z'$: consequently we have very nearly,

$$\text{No. of seconds in } z' = \frac{2}{\sin 1''} \sin PZ \sin PS \operatorname{cosec} z \sin^2 \tfrac{1}{2} P$$

$$= \frac{2}{\sin 1''} \cos \text{lat.} \cos \text{declin.} \operatorname{cosec} z \sin^2 \tfrac{1}{2} \text{hour-angle.}$$

In this way the correction z', or the number of seconds to be applied to the observed zenith distance off the meridian, to reduce it to the zenith distance on the meridian, may be obtained. But this zenith distance z must be known approximately before the formula can be used; it is supplied by the latitude by account; if this do not differ from the true latitude by more than about 15′, the true latitude itself will be deduced with tolerable accuracy. The method, however implied in the formulæ (A), is the more correct, though the work by those formulæ requires more references to tables. But if the corrected latitude, furnished by the method just discussed be used in place of the latitude by account, and the operation performed anew, the second result will have all the accuracy desirable; and from glancing at the formula, it will be seen that, as all the elements of the computation except cosec z, and cos lat. are the same in the two operations, the additional work will be very trifling, as the following example further shows:

We shall now give the result just obtained in the form of a practical rule.

Latitude from an Altitude near the Meridian.

Since the logarithm of $\frac{2}{\sin 1''}$ is 5·615455, the formula above expressed in words furnishes the following rule.

RULE 1. To the declination of the object, add the latitude by account when one is N and the other S; but when such is not the case, take the difference of the two: the result is the meridian zenith distance by account.

2. If the object be the sun, the apparent time from noon in degrees, &c., is the hour-angle. For any other object add the sun's R. A. at the instant to the apparent time since the preceding noon, the sum, or its excess above 24^h, is the

R. A. of the ship's meridian. The difference between this and the R. A. of the object is the hour-angle.

3. Add together the five following logarithms :—

   1. The constant logarithm, 5·615455.
   2. log cosine of the latitude by account.
   3. log cosine of the declination.
   4. log cosec of the mer. zenith dist., deduced from the latitude and declination.
   5. 2 log sin of half the hour-angle.

The sum of these logs, rejecting the tens from the index, is the log of a number of seconds called the "Reduction," which subtracted from the true zenith distance off the meridian, gives the true zenith distance on the meridian. When this and the declination are of the same name, their sum, when of different names, their difference is the latitude, of the same name as the greater.

NOTE.—As fractions of a second are disregarded in the reduction, the logs used in finding it need be taken from the tables only to the nearest minute.

Examples: Sun near the Meridian.

1. In latitude 56° 40′ N. by account, when the sun's declination was 14° 12′ N., at 0^h 16^m P.M., apparent time, the sun's true zenith distance was 42° 40′ N.: required the latitude?

Constant log			5·615455
Latitude by acct. . . .	56° 40′ N.	cos	9·739975
Declination	14° 12′ N.	cos	9·986523
∴ Mer. zen. dist. acct. .	42° 28′	cosec	10·170593
Half hour-angle . . .	2° 0′	2 sin	17·085638
	60) 396″	log	2·598184

Reduction	— 6′ 36″	5·615455
Zenith dist. from obs. .	42° 40′ 0″ N.	9·738820
Cor. mer. zenith dist. .	42° 33′ 24″ N.	9·986523
Declination	14° 12′ 0″ N.	10·169903
		17·085638
Corrected latitude . .	56° 45′ 24″ N.	2·596339 = log. 395″

The work on the right is a repetition of the operation above, substituting the computed latitude for the latitude by account; and as the former exceeds the latter by 5′, the mer. zenith dist. by account, in the first operation, becomes increased by 5′; that is, it is 42° 33′. As the reduction 395″ differs from the former by 1″, the more correct latitude is 56° 45′ 25″ N.

We shall now exhibit the work of the same example by the formulæ (A) at page 133.

1. Tan P M = cos hour-ang. × cot dec.

Hour-angle . . .	4° 0′	cos	9·998941
Declin.	14° 12′	cot	10·596813
P M	75° 46′	tan	10·595754

2. Cos Z M = cos P M cosec dec. sin. alt.

P M	75° 46′	cos	9·390708
Declin. . . .	14° 12′	cosec	10·610289
Altitude . . .	47° 20′	sin	9·866470
Z M	42° 31′ 25″	cos	9·867467
P M	75° 46′		
∴ Colat. =	33° 14′ 35″		
	90°		
LATITUDE .	56° 45′ 25″		

As the latitude here deduced is exactly the same as that above, we may infer that both results are strictly correct.

The student is strongly recommended to familiarise himself with both these methods of finding the latitude from an observation of the sun off the meridian, remembering that the first method is applicable only when the observation is made *near* the meridian; the second method is generally applicable, except under the circumstances pointed out in the foot-note, p. 134. The reason that the first method is somewhat preferable to the second, when the object is near the meridian, is that the *seconds* in the angles may be disregarded in taking out the logarithms, or rather that each angle may be taken to the nearest minute only. But the second method will always furnish a satisfactory test of the accuracy of the result deduced by the first: we shall now work out another example.

2. In latitude 48° 12′ N. by account, when the sun's declination was 16° 10′ S. at 0^h 20^m P.M., apparent time, the sun's true zenith distance was 64° 40′ N.: required the latitude?

Constant log			5·615455
Latitude by acct. . .	48° 12′ N.	cos	9·823821
Declination	16° 10′ S.	cos	9·982477
∴ Mer. Z.D. acct. . .	64° 22′	cosec	10·044995
½ hour-angle	2° 30′	2 sin	17·279360
	60) 557″	log.	2·746108
Reduction	— 9′ 17″		
Z.D. obs.	64° 40′ 0″ N.		
Mer. Z.D.	64° 30′ 43″ N.		
Declin.	16° 10′ 0″ S.		
LATITUDE	48° 20′ 43″ N.		

Constant log			5·615455
Corrected lat. . . .	48° 21′	cos	9·822546
Declination	16° 10′	cos	9·982477
M.Z.D.	64° 31′	cosec	10·044452
½ hour-angle	2° 30′	2 sin	17·279360
Reduction	555″	log.	2·744290
	557″		
Correction of lat. . .	+ 2″		

Hence the corrected latitude is 48° 20′ 45″ N.

The work of this example by the formulæ marked (A) is as follows:—

1. tan P M = cos hour-ang. × cot dec.

Hour-angle . . .	5° 0′ 0″	cos	9·998344
Declin.	16° 10′ 0″	cot	10·537758
P M . . .	73° 46′ 29″½	tan	10·536102
Z M . . .	64° 34′ 15″¾	This is found on next page.	
	138° 20′ 45″	(See foot-note, page 134.)	
	90°		
LATITUDE . .	48° 20′ 45″ N.	as determined by the former method.	

2. Cos Z M = cos P M cosec dec. sin alt.

P M	73° 46′ 29″½	cos	9·446249
Declin.	16° 10′ 0″	cosec	10·555280
Altitude	25° 20′ 0″	sin	9·631326
Z M	64° 34′ 15″¾	cos	9·632855

As in these two examples the corrections have been supposed to have been applied to the observed, to obtain the true zenith distance, and as also the hour-angle in time is considered to be known, we shall now work out a final example in which are given the latitude by account, the longitude, the observed altitude, and the Greenwich mean time, as shown by the chronometer.

3. August 21, 1858, A.M., in lat. 51° 40′ N. by account, and long. 2° 9′ W., the chronometer known to be $36^s{\cdot}2$ fast, showing $11^h\ 48^m\ 32^s$ Greenwich mean time, the observed altitude of the sun's lower limb was 50° 36′ (zenith N.), the index correction was + 1′ 20″, and the height of the eye 20 feet: required the true latitude?

1. *For the Hour-angle.*

G. mean time by Chron.	$11^h\ 48^m\ 32^s$
Error of Chron.	—36
G. date, Aug. 20	12^h + 11 47 56
Equa. of time G. date	—2 59
G. app. time, A.M.	11 44 57
Long. 2° 9′ W. in time	—8 36
App. time at ship, A.M. Aug. 21 . .	$11^h\ 36^m\ 21^s$

∴ Hour-angle = $23^m\ 39^s$, or 5° 55′

2. *For Declination and Equation of Time.*

Declin. noon Aug. 21 - . . .	12° 8′ 56″·9 N.
Diff. for 1^h, 49″·71 ∴ for 12^m .	+9 ·9
DECLIN. at G. date	12° 9′ 7″ N.

Eq. of time, noon Aug. 21 . . $2^m\ 58^s{\cdot}58$

NOTE.—The G. date being so near the noon of Aug. 21, the correction for Equa. of time is inappreciable. When the time from noon is considerable, the correction by means of the "Diff. for 1^h" need be made only for the nearest *hour*.

3. *For the true Zenith Distance.*

Observed alt. Sun's L. L.	. . .	50° 36′ 0″
Index cor. and dip. — 3′ 4″ }	. .	+ 12′ 47″
Semidiameter . . + 15′ 51″ }		
Apparent alt. of centre	. . .	50° 48′ 47″
Refraction—Parallax	. . . .	— 42″
True alt. of centre	. . . .	50° 48′ 5″
		90°
True zenith distance off meridian	. .	39° 11′ 55″ N.

4. *For the Latitude of the Ship.*

Constant log.			5·615455
Lat. by acct. . .	51° 40′ N.	cos	9·792557
Declination . .	12° 9′ N.	cos	9·990161
∴ M.Z.D. acct. .	39° 31′ N.	cosec	10·196336
½ Hour-angle . .	2° 57′½	2 sin	17·425458
	6,0) 104,7″ . . .	log.	3·019967
Reduction . . .	— 17′ 27″		
Z.D. off mer. . .	39° 11′ 55″ N.		
∴ Mer. Z.D. . .	38° 54′ 28″ N.		
Declin. . . .	12° 9′ 7″ N.		
LATITUDE . .	51° 3′ 35″ N.		

Constant log.			5·615455
Corrected lat. . . .	51° 4′	cos	9·798247
Declination . . .		cos	9·990161
Corrected Mer. Z.D. .	38° 54′	cosec	10·202066
½ Hour-angle . . .		2 sin	17·425458
Reduction . . .	1055″ . . .	log.	3·031387
	1047″		
	— 8″ Correction of lat.		
∴ CORRECTED LATITUDE . .	51° 3′ 27″ N.		

We shall test the degree of accuracy of this result by the formula (A).

Work of preceding Example by Formula (A), page 133.*

1. tan P M = cos hour-ang. × cot dec.

Hour-angle . .	5° 55′ 0″	cos	9·997680
Declination .	12° 9′ 7″	cot	10·666895
P M . . .	77° 47′ 5″½	tan	10·664575

2. Cos Z M = cos P M cosec dec. sin alt.

P M . . .	77° 47′ 5″½	cos	9·325481
Declination .	12° 9′ 7″	cosec	10·676738
Altitude . .	50° 48′ 5″	sin	9·889280
Z M . . .	38° 50′ 14″	cos	9·891499
P M . . .	77° 47′ 5″½		
	38° 56′ 51″½		
	90°		
TRUE LATITUDE	51° 3′ 8″½		

We see from this result, that the former method, even with the latitude by account, so much as nearly 37 miles in error, gives the latitude true to within less than half a mile, without computing the corrected reduction.

NOTE.—If the sun did not change his declination, *equal altitudes*, taken one before and the other after noon, would correspond to equal hour-angles, so that half the time elapsed between taking these equal altitudes would be the hour-angle at either observation. But on account of the

* The most troublesome part of a logarithmic operation is proportioning for the *seconds;* the computer will, in general, find the following the most convenient mode of proceeding, namely : Disregard the seconds, and enter the table with the degrees and minutes only, but against the log taken out write the tabular difference. When all the logs with their differences have been thus extracted, *then* compute in each case for the seconds, remembering that for every *co*-quantity the proportional part will be *subtractive.* The balance of these corrections for the seconds may then be incorporated with the sum of the logs from the table. As regards *arithmetical complements,* the corrections for *co*-quantities are to be added, in other cases they are to be subtracted. After the differences are all extracted from the table, it may be well to put the proper sign against each, to prevent mistake as to the additive and subtractive corrections.

change in declination, this method of deducing the hour-angle cannot be employed with safety, except under certain circumstances. When the latitude and declination are such that the sun passes the meridian near the zenith, half the elapsed time between equal altitudes, a few minutes before and a few minutes after noon, will give the hour-angle with sufficient accuracy, because in these circumstances, the sun's motion in altitude is so rapid, that the correction in altitude, due to the motion in declination, is passed over in a very short time; and the hour-angle, if the elapsed time between the two observations do not exceed about 30^m, may be safely inferred. In high latitudes, however, where the sun's motion in altitude is very slow, if the change in declination be rapid, the hour-angles on contrary sides of the meridian may be very unequal for equal altitudes.

Whatever minutes of latitude the ship may have moved from or towards the sun, in the interval of the observations, should be allowed for in taking the second altitude.

We shall now give the blank form for the foregoing operations.

Blank Form. Sun near the Meridian.

1. *For the Hour-angle.*

G. mean time by chron.	$..^h\ ..^m\ ..^s$
Error of chron.	.. ..
G. date, mean time	.. .. ..
Equa. of time G. date	.. ..
G. apparent time	.. .. ..
* Long. in time (— for W. + for E.)	.. .. ..
App. time at Ship	.. .. ..

∴ Hour-angle = $..^m\ ..^s$, or $..^\circ\ ..'\ ..''$

* It must be remembered that the longitude here is assumed to be correct; whatever error there is in it, there will be the same error in the hour-angle when converted into degrees and minutes: the *seconds*, however, in this angle may be disregarded.

2. *For the Declin. and Eq. of Time at G. date.*

Noon declin. (Naut. Alm.)		. .° . .′ . .
Diff. for 1^h	. .″ . .	
Time in minutes from G. noon		
	60). . . . .(. . .″ = . .′	
Declin. at G. date		. .° . .′
Equa. of time G. preceding noon		. .^{m}
Diff. in 1^h × No. of *hours* since that noon		
Equa. of time at G. date		. .

3. *For the True Zenith Distance off Meridian.*

Observed alt. (L. L. or U. L.)		. .° . .′ .
Index cor. and dip	. .′ . .″ }	. . .
Semidiameter	. . . . }	
Apparent alt. of centre		. . . . .
Refraction—Parallax		— .
True alt. of centre		. .
		90
True Zenith distance off Meridian		. .

4. *For the Latitude of the Ship.*

Constant log			5·615455
Lat. by acct.	. .° . .′ . .″	cos	. . . .
Declin.	. . . . . .	cos	. . . .
∴ Mer. Z.D.	. . . . . .	cosec	. . . .
½ Hour-angle	. . . . . .	2 sin	. . . .
	60) . . .″	log	. . . .
Reduction	— . .′ . .″		
Z.D. off merid.	. .° . .′ . .″		
Corrected M.Z.D.			
Declination			
Corrected Lat.			

It will be sufficient to take the above logs to the nearest minute; as also those following.

. . . .		.	5·615455
Corrected Lat.		cos	
Declination		cos	
Corrected M. Z. D.		cosec	
½ Hour angle		2 sin	
Corrected Reduction	. . . "	log	

The difference between this corrected reduction and the former, applied to the corrected latitude, will give the TRUE LATITUDE.

NOTE.—The term "near the meridian" must not be considered as always implying the same limit of distance. If in ex. 3 above, the latitude by account had been nearly equal to the declination, that is, about 12° N., the hour-angle employed would have been much too large for safety. For it is plain that in these circumstances, the sun's motion in altitude, even when very near the meridian, is rapid: his zenith distance when on the meridian is small, but when off it only a few minutes of time, the zenith distance is considerable. Now in the investigation of the rule, it is assumed that the difference between the two zenith distances is so trifling, that the cosine of that difference may be regarded as 1 without any error of consequence: we see, therefore, that in the circumstances here supposed, the method would be objectionable.

It may be further noticed, too, that as a small error in the hour-angle would correspond to a comparatively large arc of altitude, a comparatively large displacement of the pole would be necessary to make the erroneous hour-angle and the true altitude agree; and thus the latitude inferred would involve appreciable error. But, as already remarked, when the sun arrives at the meridian too near to the zenith for the present method to be trustworthy, on account of the reasons stated above, the hour-angle may be determined with sufficient accuracy by equal altitudes carefully taken before and after the meridian passage:—half the interval of time between the two observations being the hour-angle.

As the sun's motion in altitude when near the meridian is obviously greater and greater as his meridian zenith distance decreases, the hour-angle, must in the present problem, be less and less. The following short table will show within what time of the sun's passage over the meridian of the ship, the altitude "near the meridian" may always be taken.

Sun's Merid., Zenith Dist., or Difference of Lat. and Declination.												
5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
$0^h\ 3^m$	$0^h\ 5^m$	$0^h\ 9^m$	$0^h\ 12^m$	$0^h\ 15^m$	$0^h\ 20^m$	$0^h\ 25^m$	$0^h\ 30^m$	$0^h\ 35^m$	$0^h\ 40^m$	$0^h\ 50^m$	$0^h\ 55^m$	1^h

We shall now give a blank form, which may be followed whether the sun be near to, or remote from the meridian.

BLANK FORM. *Sun near to or remote from the Meridian.*

☞ The hour-angle*, declination, equation of time, and true alt. to be found as in the last Form.

1. Hour-angle	..° ..′ ..″	cos	....	2. PM	..° ..′ ..″	cos	....
Declination	.. .. ..	cot	....	Declin.	.. .. ..	cosec	....
PM =	.. .. ..	tan	....	Alt.	.. .. ..	sin	....
				ZM =	.. .. ..	cos	....

Then the sum or difference of P M, Z M is the CO-LATITUDE, when the lat. and declin. are of the same name. And the sum *minus* 90° is the LATITUDE when they are of different names.

As already noticed at page 134, the latitude by account will in general be guide sufficient as to whether the sum or difference of P M, Z M, is to be taken.

If either P M or Z M be so small that the error in the latitude by account may equal or exceed it, then, and then only, can there be any doubt as to whether the sum or difference of P M, Z M should be taken; and we shall be

* The hour-angle *here* is, of course, to be computed to *seconds* of the equinoctial. The declination and equation of time are taken from page II of the Nautical Almanac; and the "Diff. for 1^h" from page I.

apprised that our observation has been made when the sun is too near the six o'clock hour-circle, or too near the prime vertical. But without any reference to the latitude by account, whenever we know the position of the sun in reference to the six o'clock hour circle and prime vertical, all ambiguity may be removed by the following simple consideration, namely:—

No two perpendiculars to a great circle of the sphere can cross each other except at the distance of 90° * : hence,

1. *When the Latitude and Declination are of the same name,*

If the six o'clock hour circle, and the prime vertical, be both on the same side of the sun, the *difference* between P M, Z M must be taken: the result is the CO-LATITUDE. If the sun be between the six o'clock hour circle and the prime vertical, the sum of P M, Z M must be taken: the result is the CO-LATITUDE.

2. *When the Latitude and Declination are of different names,*

The sum of P M, Z M must be taken: the result, diminished by 90°, is the LATITUDE. [The sun, in this case, arrives at the prime vertical before it rises.]

Should the sun be actually *upon* the six o'clock hour circle, then P M will be 0, and Z M will be the co-latitude. And should it be actually *upon* the prime vertical, Z M will be 0, and P M will be the co-latitude. In the former case, the cosine of the hour-angle will be 0; in the latter,

$$\cos \text{hour-angle} = \tan \text{declination} \div \tan \text{latitude}.$$

We think, with these precepts and directions, the mariner can have no difficulty in determining his latitude from a

* Perpendiculars to the meridian all intersect in the E. and W. point of the horizon: these points are, therefore, the poles of the meridian.

Single Altitude off the meridian, when his *time* is pretty accurately known.

It will be observed that in the method just discussed, there is no restriction as to whether the sun be near the meridian or not; nor, being near the meridian, whether it be near the zenith or not. When near the zenith, it must, it is true, also be near the prime vertical; but on which side of the prime vertical, will be ascertained by noticing on which side the E. or W. point the altitude is taken; and also by noticing that the motion in altitude is quickest when the body is crossing the prime vertical.

Whether the sun be near to or remote from the zenith, the observation should always be made when there can be no doubt as to on which side of the six o'clock hour circle, or of the prime vertical, the body is. And what is here said in reference to the sun is equally applicable to any other celestial object.

In the following examples the learner is recommended, for the sake of practice, to work out the solutions, by both forms, of the cases in which the object observed is near the meridian.

Examples for Exercise.—Object off the Meridian.

1. At $18^m\ 45^s$ from apparent noon, in latitude 8° S. by account, the sun's true altitude was 74° 16′ (zenith N.), and his declination at that time 23° 27′ S: required the latitude true to the nearest minute?

Ans. latitude, 8° 23′ S.

2. In latitude 48° 12′ N. by account, when the sun's declination was 16° 10′ S., at $0^h\ 16^m$ P.M., apparent time, his true altitude was 25° 20′ (zenith N.): required the correct latitude? Ans. latitude, 48° 24′ 5″ N.

3. At $3^h\ 5^m\ 36^s$ P.M., apparent time, in north latitude, the sun's true altitude was 35° 4′ 7″, and his declination 10° 54′ 26″ N.: required the latitude?

Ans. lat. 50° 48′ 28″ N.

4. At 10^h 40^m A.M. apparent time, in north latitude, when the sun's declination was 16° 12′ 10″ N., his true altitude, S. of E., was 44° 56′: required the latitude?

Ans. lat. 58° 47′ 8″ N.

5. In latitude 50° 40′ N. by account, when the sun's declination was 11° 44′ 58″ N., his true altitude was 50° 52′ 29″ at 11^h 47^m 57^s, A.M.: required the correct latitude?

Ans. lat. 50° 47′ 49″ N.

6. At 9^h 30^m A.M., apparent time, in north latitude, when the sun's declination was 12° 28′ 40″ N., his true altitude S. of E., was 41° 30′: required the latitude?

Ans. lat. 50° 6′ 1″ N.

7. At 7^h 20^m A.M., apparent time, in north latitude, when the sun's declination was 18° 50′ 10″ N., his true altitude N. of E., was 24° 20′: required the latitude?

Ans. lat. 19° 14′ 53″ N.

8. What would have been the latitude in the last example if the sun had been S. of E. at the time of observation?

Ans. 70° 36′ 9″ N.

9. Oct. 29, 1858, P.M., in north latitude, and longitude 4° 40′ W., the Greenwich time, as shown by chronometer, which was 5^s slow, was 2^h 20^m 43^s, the observed altitude of the sun's lower limb was 46° 40′, the index correction was — 4′ 30″, and the height of the eye 17 feet: required the latitude?

Ans. lat. 12° 50′ 34″ N.

10. At 11^h 2^m 32^s P.M., apparent time, in longitude 0° 45′ W., the observed altitude of the Pole Star was 51° 22′, the index correction + 3′, the height of the eye 26 feet; and for apparent noon at Greenwich, on the day of observation, the Nautical Almanac gave the following particulars. (See the Rule, page 136.)

Sun's R. A. 6^h 51^m $11\frac{1}{2}^s$; star's R. A. 1^h 1^m 41^s; star's declin. 88° 26′ 56″: required the latitude to the nearest minute?

Ans. lat. 51° 47′ N.

11. Determine the latitude from the altitude of the Pole

Star, as given, with the other particulars, in the example at page 132.

Latitude from Two Altitudes of the Sun, and the Time between the observations.

It has been sufficiently shown in the foregoing article, that when the *time* is known, the latitude of the ship may always be found from a single altitude of the sun, provided we know his declination at that time. But if either the longitude by account, or the chronometer, be suspected of error too great to justify confidence in the time at ship, as deduced from them, then it will be necessary to enter upon the more complicated problem of DOUBLE ALTITUDES.

In this problem, the exact time, either at the ship or at Greenwich, is not necessary: it is the *interval* of time only, between the two observations, that it is requisite to know with accuracy; and this the chronometer, or even a good common watch, if the interval be not unreasonably long, will always measure with the desired precision.

In order to facilitate the solution of this problem of double altitudes, various tables have been constructed, and many rules and expedients devised; but we consider that the direct method, by Spherical Trigonometry, while it is more accurate, is fully as short, and much less burthensome to the memory.* Its investigation is as follows:

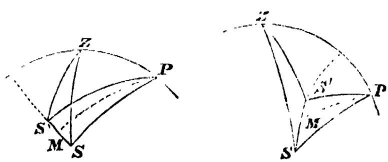
Let Z be the zenith of the ship, P the elevated pole, and S, S′ the two places of the sun when the altitudes are taken,

* The celebrated Delambre, after having carefully examined all the rules with which he was acquainted for the solution of this problem, came to the conclusion that the rigorous process, by spherical trigonometry, was to be preferred, as well for brevity as for accuracy of result. And another high practical authority, Captain Kater, entertains the same conviction as to the superiority of the direct over the indirect methods.—See "Encyclopædia Metropolitana," Art. Nautical Astronomy.

then in the annexed diagrams the following quantities will be given, namely—

The polar distances P S, P S′ }
co-altitudes . Z S, Z S′ } to find the co-latitude Z P.
hour-angle . . S P S′ }

There are thus three spherical triangles concerned, namely, the triangles P S S′, Z S S′, and Z S P, the first two, having the great circle arc S S′, joining the two positions of the sun for a common base, and the third having for base the co-latitude Z P.



1. In the triangle P S S′, we may regard the two sides P S, P S′ as equal, since the change of declination in passing from S to S′ is so small that S S′ may be safely considered as the base of an isosceles spherical triangle, of which each side is $\frac{1}{2}$ (P S + P S′), that is half the sum of the actual polar distances. Hence drawing the perpendicular P M, which will bisect the angle P and the base S S′, we shall have given in the right-angled triangle P M S, the side P S, and the angle S P M, to find S M = $\frac{1}{2}$ S S′.

2. In the triangle P S S′, there are now given the sides P S, S S′; or P S′, S S′, to find the angle P S S′ or P S′ S.

3. In the triangle Z S S′ we shall have given the three sides to find the angle Z S S′, and since the angle P S S′ is known from the solution of the first triangle, we shall thence have—

the angle P S Z = P S S′ − Z S S′ or P S S′ + Z S S′.

4. In the triangle P S Z we shall thus have given the sides S P, S Z, and the included angle, to find Z P.

Thus by the solution of three spherical triangles, we shall be able to determine the co-latitude of the ship without the aid of any but the common logarithmic tables, and the result will be rigorously correct, if the data are so, except in so far as it may be affected by the supposition that the magnitude of the arc S S′ would remain unaltered by our lengthening the shorter of the two polar distances by half their difference, and shortening the other as much. It is plain that the error of this supposition, always very small, becomes less and less as the interval between the observations becomes diminished. As the solution of the third triangle, in which two sides and the included angle are given, may be effected in various ways, we shall here give the investigation of what appears to us to be the preferable method.

From the fundamental formula of Spherical Trigonometry we have—

$$\cos Z P = \cos S P \cos S Z + \sin S P \sin S Z \cos S$$

$$\text{But } \sin S P = \cos S P \frac{\sin S P}{\cos S P} = \cos S P \tan S P$$

$$\therefore \cos Z P = \cos S P (\cos S Z + \sin S Z \tan S P \cos S)$$

Now tan S P cos S must be equal to the cotangent of some arc: call this arc α, so that

$$\tan S P \cos S = \cot \alpha = \frac{\cos \alpha}{\sin \alpha} \quad . \quad . \quad . \quad (1)$$

then the preceding equation becomes

$$\cos Z P = \cos S P \frac{\sin \alpha \cos S Z + \sin S Z \cos \alpha}{\sin \alpha}$$

in which we see that the numerator of the fraction is equal to sin (α + S Z). Consequently we have finally

$$\cos Z P = \frac{\cos S P \sin (S Z + \alpha)}{\sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If S P should be greater than 90°, tan S P will be negative; so that when cos S is positive, the right-hand member of (1) will be negative. In this case therefore (2) will be

$$\cos ZP = \frac{\cos SP \sin (SZ - \alpha)}{\sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

provided we still take cos S P positively, that is, use the supplement of S P. If S P and S each exceed 90°, (1) will be positive and (2) negative.

We shall now exhibit the work by the above method, in an example, regarding the necessary preliminary corrections for altitude, declination, and semidiameter, to have been made.

EXAMPLES.—*Latitude from two altitudes of the Sun, and the elapsed time.*

1. The two corrected zenith distances of the sun's centre are

$$ZS = 73° 54' 13'', \text{ and } ZS' = 47° 45' 51'',$$

the corresponding polar distances are

$$PS = 81° 42' \text{ N. and } PS' = 81° 45' \text{ N. } \therefore \tfrac{1}{2}(PS + PS') = 81° 43' 30'',$$

and the interval of time between the observations is 3^h: required the latitude?

1. *In the triangle P M S, to find the side S M.*

P S	81°	43′	30″	. . . . . .	sin 9·995455
S P M	22	30	0	. . . . . .	sin 9·582840
S M	22	15	11	. . . . . .	sin 9·578295
			2		
∴ S S′ =	44	30	22		

2. *In the triangle P S S′, to find the angle P S S′.*

S S′	44°	30′	22″	Arith. Comp.	sin 0·154291
P S′	81	45	0	. . . . . .	sin 9·995482
S P S′	45	0	0	. . . . . .	sin 9·849485
P S S′	86	39	0	. . . . . .	sin 9·999258

3. *In the triangle Z S S′, to find the angle Z S S′.*

Z S′	47°	45′	51″		
Z S	73	54	13	Arith. Comp.	sin 0·017369
S S′	44	30	22	Arith. Comp.	sin 0·154291
2)	166	10	26		
½ sum of sides =	83	5	13		
½ sum − Z S =	9	11	0	. . . . . .	sin 9·203017
½ sum − S S′ =	38	34	51	. . . . . .	sin 9·794919
					2)19·169596
½ Z S S′ =	22	36	26	. . . . . .	sin 9·584798
∴ Z S S′ =	45	12	52		
P S S′ =	86	39	0		
∴ P S Z =	41	26	8		

4. *In the triangle P S Z, to find first α, and thence the side Z P.*

P S	81°	42′	0″	. .	tan 10·835992
P S Z	41	26	8	. .	cos 9·874888
α =	11	0	42	. .	cot 10·710880
Z S =	73	54	13		
Z S + α =	84	54	55		

P S				. . . .	cos 9·159435
α	11°	0′	42″	Ar. Comp.	sin 0·718946
ZS+α	84	54	55	. . .	sin 9·998288
Lat. =	48	49	55*	. . .	sin 9·876669

Hence the Latitude is 48° 49′ 55″.

NOTE.—The first operation, namely, that for finding S S′, may be replaced by a process similar to that marked (4) just given; because, in both cases, two sides and the included angle of a triangle are given to find the third side; but, as already remarked, a trifling amount of accuracy, in the determination of S S′, has been sacrificed to the superior brevity of the work. It may not be amiss here to recompute S S′, in the manner in which Z P is computed above, for the sake of comparing the two results.

* The sum of the three logarithms is the *cosine* of Z P, consequently it is the *sine* of the latitude.

P S	81° 42′ 0″	tan 10·835992
S P S′	45 0 0	cos 9·849485
α =	11 39 26	cot 10·685477
P S′ =	81 45 0	
P S′ + α =	93 24 26 = Sup.* of 86° 35′ 34″	

P S	. . . .	cos 9·159435
α	11° 39′ 26″ Ar. C.	sin 0·694528
P S′ + α	86 35 34 . .	sin 9·999232
S S′ =	44 30 23 . . .	cos 9·853195

which differs from the former result only by 1″.

This example is well suited to test the general trustworthiness of the operation marked (1), as the interval between the observations, 3 hours, is tolerably large, and the change of declination, 1′ an hour, is an extreme supposition.

In reference to the method of solution here exemplified there are one or two remarks to be made which deserve the student's attention.

1. In the step marked (2) the angle P S S′ is inferred from its *sine*. Now to a sine belongs either of two angles—the supplements of each other, and it may be matter of doubt whether the angle taken from the table, in connection with this sine, should, in the case before us, be acute, as we have considered it to be in the above operation, or obtuse: we proceed to show how the ambiguity may be avoided.

The fundamental formula of Spherical Trigonometry gives

$$\cos PS' = \cos PS \cos SS' + \sin PS \sin SS' \cos PSS'$$

$$\therefore \cos PSS' = \frac{\cos PS' - \cos PS \cos SS'}{\sin PS \sin SS'}$$

Now, it is matter of indifference which of the two places of the celestial object we mark S or S′; so that in this formula we may always consider that cos P S′ is numerically greater than cos P S; and consequently numerically greater than cos P S cos S S′. And since the denominator is necessarily positive, the fraction necessarily takes the sign of cos P S′. Consequently, cos P S S′ always has the same sign as cos P S′, so that P S′ and P S S′ are always either both acute or both

* Instead of taking the supplement, for the purpose of getting its *sine* in the next column of the work, we may take merely the excess of P S′ + α above 90°, namely 3° 24′ 26″, and take out its *cosine*, which, of course, is

obtuse: hence, if we always take for P S′ that one of the two polar distances, whose sine is less than the sine of the other, the angle P S S′ will always be of the same species as the side P S′, that is, they will be either both acute or both obtuse.

As respects the *sun*, however, these considerations need not be attended to: whenever the two positions of that body are on the same side of the equinoctial, both angles will be either acute or obtuse,—acute when the declination is of the same name as the latitude, and obtuse when it is of contrary name. The sides of the polar triangle differ from equality in so trifling a degree, that the angles referred to may always be regarded as of the same species, except when the sun actually crosses the equinoctial in the interval of the observations; and even then, each angle will be so near to 90° that, whether they be regarded as acute or obtuse, can make no difference of importance.

2. In low latitudes it may happen that the arc S S′, if prolonged, would cut the meridian between P and Z, as in the second of the diagrams at page 151. In this case the angle P S Z will not be the difference of the angles P S S′, Z S S′, but their sum. It is plain that when the altitudes are both on the same side of the meridian, P S Z can be the *sum* only when the latitude is so low—the declination also being of the same name—that the sun would cross the meridian between P and Z; for if it crossed the meridian on the other side of Z, the great circle arc S S′, when prolonged, would necessarily cut the meridian still further from Z on that side: hence when the declination and latitude are of the same name, and we know that the latter is greater than the former, we may be sure that the difference, and not the sum, of the two angles in question must be taken, when both observations are on the same side of the meridian. When, however, under other circumstances, in these low latitudes, a doubt occurs, we may remove it by recomputing

vice versâ; and choosing that of the two results which differs the least from the latitude by account. But a more convenient way seems to be this; namely, to directly compute the angle P S Z, from the three sides of the triangle P S Z: the polar distance P S, the co-altitude Z S, and the co-latitude by account P Z, the operation being similar to that of step (3) above; the result will be an approximation to what the step referred to ought to give. And we may remark, that as an approximation only is to be expected, *seconds* in the several arcs need not be regarded; each may be taken to the nearest minute only.

When the true co-latitude P Z is thus ascertained, we may combine it with the polar distance P S, and the co-altitude Z S, in imitation of the operation (3), to determine the hour-angle Z P S, that is, the apparent time from noon, when the altitude nearest to the prime vertical was taken; and the correction for Equation of Time being applied, we shall get the mean time from noon when S was observed. We here suppose S to be that one of the two positions of the sun which is the nearer to the prime vertical, since the motion in altitude is quicker than when the sun is in the other position, and consequently a small error in the altitude has a less effect upon the hour-angle.

In the example worked at page 153, we have proceeded on the supposition that the altitudes of the sun have both been taken at the same place; but as, at sea, the ship usually sails on during the interval of the observations, it is necessary to allow for the change of place, and to reduce the first altitude to what it would have been if taken at the place of the second observation.

This is called the correction for the ship's run; it is obtained thus:—From the sun's bearing find the angle between the ship's direction, and the sun's direction at the first observation; then considering this angle as a course, and the distance sailed as the corresponding distance, find, either by the traverse table or by computation as in plane sailing, the diff. lat.: this will be the number of minutes by which the ship—or rather the ship's zenith—has advanced towards, or receded from, the sun in the interval, and will therefore be the number of minutes to be added to, or subtracted from, the first altitude, to reduce that altitude to what it would have been if taken by another person at the place of the second observation, and at the time of the first.

2. February 8, 1858, in latitude 35° N. by account, when the mean time at Greenwich, as shown by the chronometer, was $11^h\ 17^m\ 4^s$ A.M., the observed altitude of the sun's lower limb was 36° 10′, and his bearing S.¼E.: after running 27 miles, the observed altitude of the lower limb was 41° 20′, the time shown by the chronometer being $2^h\ 38^m\ 18^s$ P.M. The error of the instrument was +2′, and the height of the eye, 20 feet: required the latitude of the ship when the second observation was made.

1. *For the polar distances P S, P S′, and the angle S P S′ between them.*

First Observation.

G. Time, Feb. 7 . . .	$23^h\ 17^m\ 4^s$		
Noon Declin.	15° 18′ 4″ S.	Diff. 1^h	−47″·26
Cor. for $23^h\frac{1}{4}$ . .	−18 19		$23\frac{1}{4}$
DECLINATION . .	14 59 45 S.		14178
	90		9452
PS =	104 59 45		1181
			6,0) 109,8·79
			Cor. 18′ 19″

Second Observation.

G. Time, Feb. 8 . . .	$2^h\ 38^m\ 18^s$		
Noon Declin.	14° 59′ 10″ S.	Diff. 1^h	−47″·89
Cor. for $2^h\frac{1}{2}$ . .	− 2		$2\frac{1}{2}$
DECLINATION . .	14 57 10 S.		9578
	90		2395
P S′ =	104 57 10		6,0) 11,9·73
P S =	104 59 45		2′
	2) 209 56 55		

From Noon, Feb. 7	Time of 1st Observation	$23^h\ 17^m\ 4^s$	3^h =	45°
	. . 2nd . . .	26 38 18	21^m =	5° 15′
	Interval of Time .	3 21 14	14^s =	3′ 30″
	Hence the angle S P S′, in degrees, =			50 18 30

2. *For the true altitudes of the Sun's centre.*

First alt. sun's L. L.			36° 10′ 0″
Index and Dip	— 2′ 24″	} .	+ 13 51
Semi-diameter	16 15		
App. alt. of centre			36 23 51
Refraction — Parallax			— 1 12
True alt. of centre . . .			36 22 39

Second alt. sun's L. L.	41° 20′ 0″
Index, Dip, and Semi.	+ 13 51
App. alt. of centre	41 33 51
Refraction — Parallax	— 59
True alt. of centre . . .	41 32 52

Since the angle between the sun's bearing at the first observation, namely, S.$\frac{1}{4}$E., and the course of the ship afterwards, namely, N.E., is 11$\frac{3}{4}$ points, the ship has sailed, in the interval, within 4$\frac{1}{4}$ points of the direction opposite to the sun, a distance of 27 miles. With 27 miles dist. and 4$\frac{1}{4}$ points course, the Traverse Table gives 18′ for the corresponding diff. lat., so that the ship has receded 18′ from the sun during the interval of the observation. Consequently 18′ must be subtracted from the first true altitude to reduce it to what it would have been if a second observer had taken it at the place of the second observation at the time the first was made. The true altitudes at this latter place are therefore

36° 4′ 39″ and 41° 32′ 52″

∴ Z S = 53° 55′ 21″ and Z S′ = 48° 27′ 8″

3. *In the triangle P M S, to find S M = ½ S S′.*

Each of the equal sides of the triangle S P S′, regarded as isosceles, is ½ (P S + P S′) = 104° 58′ 28″, and S P M = ½ S P S′ = 25° 9′ 15″

P S	104°	59′	45″	. . .	sin	9·984952
S P M	25	9	15	. . .	sin	9·628445
S M	24	14	28½	. . .	sin	9·613397
			2			
∴ S S′ =	48	28	57			

4. *In the triangle P S S′, to find the angle P S S′.*

S S′	48°	28′	57″	Arith. Comp.	sin	0·125661
P S′	104	57	10	. . . .	sin	9·985039
S P S′	50	18	30	. . . .	sin	9·886204
P S S′	83	10	0	. . . .	sin	9·996904

5. *In the triangle Z S S′, to find the angle Z S S′.*

Z S′	48°	27′	8″			
Z S	53	55	21	Arith. Comp.	sin	0·092470
S S′	48	28	57	Arith. Comp.	sin	0·125661
	2)150	51	26			
½ sum of sides =	75	25	28			
½ sum — Z S =	21	30	7	. . . .	sin	9·564113
½ sum — S S′ =	26	56	31	. . . .	sin	9·656182
						2)19·438426
½ Z S S′ =	31	35	29	. . . .	sin	9·719213
∴ Z S S′ =	63	10	58			
P S S′ =	83	10	0			
∴ P S Z =	19	59	2			

6. *In the triangle P S Z, to find first α and thence the side Z P.*

P S	104° 59′ 45″	. . .	tan	10·572074
P S Z	19 59 2	. . .	cos	9·973030
α =	15 54 34	. . .	cot	10·545104
Z S =	53 55 21			
Z S — α =	38 0 47	[See formula (3), p. 153.]		

P S*		. . . .	cos	9·412878
α	15° 54′ 34″	Ar. Comp.	sin	0·562063
Z S — α	38° 0′ 47″	. . .	sin	9·789469
Lat. =	35° 32′ 35″	. .	sin	9·764410

Hence the latitude is 35° 32′ 35″ N.

* There are three references to the Tables with this arc, namely, one in

NOTE.—The illustrations now given will convey to the student a sufficient notion of the problem of Double Altitudes; and from the length of the computation involved in its solution, he will be prepared for the statement that it is a problem resorted to at sea only from necessity. This necessity, however, can but seldom occur; so long as the chronometer can be safely depended upon, and the longitude by account is not grossly in error, the *time* at the ship can always be obtained with sufficient precision to enable us to get the latitude from a single altitude of the sun, as fully explained at pages 136 and 146. And the latitude thus inferred from a single altitude, and the sun's hour-angle, is in general much more trustworthy than the latitude deduced from double altitudes, which should never be regarded as more than an approximation. Trifling errors in the data of a problem may accumulate to something considerable when they pervade a long course of operations. *One* of the two altitudes in the present problem we are pretty sure must be affected with error:—the altitude, namely, which is corrected for the run of the ship: there is, of course, some difficulty in getting the sun's bearing with precision, and there is a further liability to error in the estimated distance sailed.*

other for cosine. These may all be taken from the table at one opening; but it will be better to take out only two—as sine and tangent; then cosine is at once got by subtracting the tan from the sine, conceiving the latter to be increased by 10, for $\cos = \frac{\sin}{\tan}$ $\therefore \log \cos = 10 + \log \sin - \log \tan$, as in the margin.

$$10 + \sin PS = 19{\cdot}984952$$
$$\tan PS = 10{\cdot}572074$$
$$\cos PS = 9{\cdot}412878$$

* If the sun's true bearing or azimuth at either place of observation could be taken with precision, there would be no necessity for a second altitude; for we should then have a spherical triangle Z P S in which are given the polar distance P S, the co-altitude Z S, and the angle Z, to find the co-latitude Z P.

For the purposes of the problem in the text, however, the *true* bearings of the sun and of one place of observation from the other, are not necessary:—the *compass* bearings are sufficient, because the angle between the two directions—which is all that is wanted—is unaltered in magnitude

The sun's bearing can be taken with more accuracy when low than when high; so that in this problem the bearing is always taken with the less of the two altitudes; when therefore it is the *second* that is the less altitude, and the sun's bearing is taken, the point *opposite* to that of the ship's course, from the former position, must be used in reducing the second altitude to what it would have been if taken at the same place as the first; and the latitude will apply to that place. The following is the blank form of the operations:—

BLANK FORM. ***Latitude from Two Altitudes of the Sun.***

1. *For the polar distances P S, P S′, and the polar angle S P S′ between them.*

First Observation.

G. Time	. .^{h} . .^{m} . .^{s}		
Noon Declin.	. .° . .′ . .″	Diff. in 1^h	. .″
Cor. for hours fr. noon	. . . .	Hours from noon	× . .
DECLINATION	. . . . . .		
	90		6,0) . . . .″
P S =	. . . . . .	Correction	. .′ . .″

Second Observation.

G. Time	. .^{h} . .^{m} . .^{s}		
Noon Declin.	. .° . .′ . .″	Diff. in 1^h	. .″
Cor. for hours fr. noon	. . . .	Hours from noon	× . .
DECLINATION	. . . . . .		
	90		6,0) . . . .″
P S′ =	. . . . . .	Correction	. .′ . .″
P S =	. . . . . .		
2)	. . . . . .		
½ (P S + P S′) =	. . . . . . ☞ used for P S, in step 3.		

Interval of Time, converted into Degrees,

. .° . .′ . .″ = S P S′ ∴ ½ S P S′ = . .° . .′ . .″ = S P M.

---

the sun, and thence to deduce the latitude as indicated in this note, the

2. *For the true altitudes of the Sun's centre.*

First alt. (L. L. or U. L.)	. .° . .′ . .″	Second alt. (L. L. or U. L.)	. .° . .′ . .″
Index and Dip . .′ . .″ } Semi-diameter . . . . }	. . . .	Index and Dip . .′ . .″ } Semi-diameter . . . . }	. . . .
App. alt. of centre	. . . . . .	App. alt. of centre	. . . . . .
Refraction — Parallax	—	Refraction — Parallax	— . . . .
True alt. of centre	. . . . . .	True alt. of centre	. . . . . .

The sun's bearing when the less altitude was taken having been observed, and the course of the ship, or the bearing of the place of the greater altitude from that of the less being known, find by addition or subtraction, the angle between these two bearings. With this angle as a course, and the distance between the two places as a distance, find the corresponding diff. lat. from the Traverse Table; and this diff., taken as so many minutes, add to, or subtract from, the less altitude, according as the ship has advanced towards, or receded from the sun. The less altitude being thus corrected for run, subtract each altitude from 90°, and we shall have

ZS = . .° . .′ . .″, and ZS′ = . .° . .′ . .″

3. *In the triangle P M S, to find S M and thence 2 S M = S S′.*

PS	. .° . .′ . .″	sin	. . . .
SPM	. . . . .	sin	. . . .
SM	. . . . .	sin	. . . .
	2		
∴ S S′ =	. . . . .		

4. *In the triangle P S S′ to find the angle P S S′.*

SS′	. .° . .′ . .″	Ar. Comp. sin	. . . .
PS′	. . . . .	sin	. . . .
SPS′	. . . . .	sin	. . . .
PSS′	. . . . .	sin	. . . .

NOTE. As pointed out in the preceding page, the polar distance, used in this third step, is taken equal to the half sum of the actual polar distances.

5. *In the triangle Z S S′, to find the angle Z S S′ and thence P S Z.*

Z S′	..° ..′ ..″		
Z S	.. .. ..	Arith. Comp.	sin ....
S S′	.. .. ..	Arith. Comp.	sin ....
2)	.. .. ..		
½ sum of sides	.. .. ..		
½ sum — Z S	.. .. ..		sin ....
½ sum — S S′	.. .. ..		sin ....
			2) ....
½ Z S S′ =	.. .. ..		sin ....
∴ Z S S′ =	.. .. ..	The sum or difference. (See p. 156.)	
P S S′ =	.. .. ..		
∴ P S Z =	.. .. ..		

6. *In the triangle P S Z, to find first α and thence the latitude.*

PS	..° ..′ ..″	tan	P S	..° ..′ ..″	cos ....
P S Z =	.. .. ..	cos	α	.. .. .. Ar. Comp.	sin ....
α =	.. .. ..	cot	Z S ± α	.. .. ..	sin ....
Z S =	.. .. ..		LAT.	.. .. ..	sin ....
Z S ± α =	.. .. ..	(See formulæ, p. 152.)			

If instead of the sun the object observed be a star, step 1 is of course dispensed with, as the declination is got at once from the Nautical Almanac, and in step 2 there is no correction for semidiameter and parallax; the remainder of the operation is the same. But instead of taking two altitudes of the same star, a far more practicable and trustworthy method of finding the latitude is to take simultaneous altitudes of *two distinct stars.* This method has several advantages :—

1. No allowance is made for run of the ship, and thus all error involved in the course sailed and the bearing of the sun is avoided.

2. There is no risk incurred of losing a second observation from unfavourable weather.

3. The hour-angle, or the angle at the pole between the

two polar distances, is given at once by taking the difference of right ascensions of the two stars; so that neither the Greenwich date nor the time at ship requires to be known.

As, however, the polar distances of the two stars may differ considerably, the side S S′ cannot here be computed as in the case of the sun or of a single star: it must be found, in the triangle S P S′, in a way similar to that in which Z P was found in the triangle P S Z. But after what has been done, an example will suffice to make the operation intelligible.

It is proper to notice, however, that if there be but one observer, so that the altitudes, instead of being both taken at the same instant, must be taken in succession, the practical operation must be managed as follows:—The altitude of one star must be taken, and the time noted by a watch; the altitude of the other star must then be taken, and the time noted. After a short interval, the altitude of the second star must again be taken, and the time noted: we shall thus learn the second star's motion in altitude in a given time; and may thence, by proportion, find what its altitude was when the first star was observed; so that we shall have the altitudes of both at that instant.

Latitude from the Altitudes of two Stars taken at the same time.

Ex. In latitude 38° N. by account, the altitudes of α Pegasi and α Aquilæ, taken at the same instant, on the same side of the meridian, were respectively—

$$29° 49' 27'' \text{ and } 57° 29' 50'',$$

the index correction was —15″, and the height of the eye 41 feet: also, the Nautical Almanac gave the following particulars:—

α Pegasi.		α Aquilæ.	
Declination	14° 22′ 50″ N.	Declination	8° 28′ 2″ N.
Right Ascension	$22^h\ 57^m\ 6^s$	Right Ascension	$19^h\ 43^m\ 15^s$

Required the latitude?

1. *For the polar distances P S, P S′, and the polar angle S P S′ between them.*

	14°	22′	50″			8°	28′	2″
	90					90		
P S′ =	75	37	10		P S* =	81	31	58

Right Ascensions	22^h	57^m	6^s	3^h . . .	45°		
	19	43	15	13^m . . .	3	15′	
S P S′ in time =	3	13	51	51^s . . .		12	45″
				∴ S P S′ =	48°	27′	45″

2. *For the Stars' true zenith distances Z S′, Z S.*

Observed alt. of S′ . .	29°	49′	27″	. . . . of S	57°	29′	50″
Index and Dip . . .		—6	33			—6	33
Apparent alt. . . .	29	42	54		57	23	17
Refraction		—1	42				—37
True altitude . .	29	41	12		57	22	40
	90				90		
Z S′ =	60	18	48	Z S =	32	37	20

3. *In the triangle P S S′, to find S S′.*

P S′	75°	37′	10″	. .	tan	10·591091
S P S′	48	27	45	. .	cos	9·821586
α =	21	8	23	. .	cot	10·412677
P S =	81	31	58			
P S + α =	102	40	21			

P S′	75°	37′	10″	. . .	cos	9·395084
α	21	8	23	Ar. Comp.	sin	0·442922
P S + α	102	40	21	. . .	sin	9·989290
S S′ =	47	47	14	. . .	cos	9·827296

4. *In the triangle P S S′, to find the angle P S S′.*

S S′	47°	47′	14″	. Arith. Comp.	sin 0·130384
P S′	75	37	10	. . .	sin 9·986175
S P S′	48	27	45	. . .	sin 9·874205
P S S′	78	13	32	. . .	sin 9·990764

5. *In the triangle Z S S′, to find the angle Z S S′ and thence P S Z.*

Z S′	60°	18′	48″		
Z S	32	37	20	. Arith. Comp.	sin 0·268333
S S′	47	47	14	. Arith. Comp.	sin 0·130384
	2)140	43	22		
	70	21	41		
½ sum — Z S =	37	44	21	. . .	sin 9·786799
½ sum — S S′ =	22	34	27	. . .	sin 9·584194
					2)19·769710
½ Z S S′ =	50	5	41	. . .	sin 9·884855
∴ Z S S′ =	100	11	22		
P S S′ =	78	13	32		
∴ P S Z =	21	57	50		

6. *In the triangle P S Z, to find α and thence the latitude.*

P S	81°	31′	58″	. .	tan 10·827204
P S Z	21	57	50	. .	cos 9·967276
α =	9	7	9	. .	cot 10·794480
Z S =	32	37	20		
Z S + α =	41	44	29		

P S	81°	31′	58″	. . .	cos 9·168008
α	9	7	9	Ar. Comp.	sin 0·800021
Z S + α	41	44	29	. . .	sin 9·823324
Lat.	38	12	29	. . .	sin 9·791353

The blank form for the foregoing operation is as follows:—

BLANK FORM. *Latitude from Altitudes of two Stars taken at the same time.*

☞ That star is to be marked S′ of which the sine of the polar distance is the *less*.

1. *For the polar distances P S, P S′, and the polar angle S P S′.*

Declin. of S	..° ..′ ..″	of S′	..° ..′ ..″
	90		90
Polar dist. P S =	.. .. ..	P S′ =	.. .. ..
R. A. of S	..h ..m ..s		
,, of S′	.. .. ..		
S P S′ in time	.. .. ..	∴ S P S′ =	..° ..′ ..″

2. *For the true zenith distances Z S, Z S′.*

Observed alt. of S	..° ..′ ..″	of S′	..° ..′ ..″
Index and Dip	.. ..		.. ..
Apparent alt.	.. .. ..		.. .. ..
Refraction	—.. ..		—.. ..
True altitude	.. .. ..		.. .. ..
	90		90
Z S =	.. .. ..	Z S′ =	.. .. ..

3. *In the triangle P S S′, to find S S′.*

P S′	..° ..′ ..″	tan	P S′	..° ..′ ..″		cos ....
S P S′	.. .. ..	cos	α	.. .. ..	Ar. Comp.	sin ....
∴ α =	.. .. ..	cot	P S ± α	.. .. ..		sin ....
P S =	.. .. ..		S S′ =	.. .. ..		cos ....
S ± α =	.. .. ..	(See formulæ, p. 152.)				

The remaining steps, namely, 4, 5, and 6, are the same as those in the form at page 163.

Examples for Exercise in Double Altitudes.

1. In latitude 1° 34′ N. by account, two corrected zenith distances of the sun's centre were 54° 39′, and 19° 59′; the corresponding declinations were 5° 31′ 6″ S., and 5° 28′ 54″ S.; the interval of time between the observations was 2^h 20^m: required the latitude?

Ans. lat. 1° 29′ 28″ N.

2. In latitude 35° 27′ N. by account, when the mean time at Greenwich, as shown by the chronometer, was 11^h 17^m 4^s A.M., the observed altitude of the sun's lower limb was 36° 14′, and his bearing S. ¼ E. After running N.E. 27 miles, the observed altitude of the lower limb was 41° 24′, the time at Greenwich being 3^h 38^m 18^s P.M. The error of the instrument was —4′, and the height of the eye 20 feet. The Nautical Almanac gave—

Declin. at G. noon	preceding 1st observation,	15° 38′ 9″.	Diff. in 1^h,	—46″·5
,, G.,	following ,, ,,	15 19 32	,,	—47″·2

Sun's semidiameter 16° 14′.

Required the latitude of the place where the second observation was made? Ans., lat. 35° 20′ 2″ N.

3. In latitude 53° 30′ N. by account, the corrected zenith distances of Capella and Sirius, both observed at the same time, were—

Capella.	*Sirius.*
Zenith distance Z S′ = 29° 14′ 24″	Zenith distance Z S = 72° 5′ 48″
Polar distance P S′ = 44 11 39	Polar distance P S = 106 28 40.

Also the difference of their right ascensions was 1^h 33^m 45^s: required the latitude? Ans., lat. 53° 19′ 23″ N.

4. In latitude 28° 10′ S. by account, the sun being obscured at noon, its altitude was taken shortly afterwards, the chronometer at the instant showing 9^h 49^m 20^s; and when the same chronometer showed 10^h 44^m 45^s, the altitude was again taken. In the first observation the altitude

of the upper limb was 45° 33′; in the second the altitude of the lower limb was 42° 8′ 30″, the sun's bearing at the time being N. $\frac{1}{4}$ E. The ship's run in the interval was N.W. $\frac{3}{4}$ W. 6 miles; the allowance for index error and dip was —4′ 30″, and the Nautical Almanac gave for the Greenwich noon of the day—

Sun's declination 16° 34′ 4″ N. Diff. in 1^h, 42″·8
Sun's semidiameter 15′ 52″.

Required the latitude to the nearest minute at the place where the first observation was taken?

Ans., lat. 28° 0′ S.

NOTE.—In the foregoing examples a single altitude of the celestial object observed, has uniformly been regarded as *the* altitude at the time; but as it is not always easy to take an altitude with precision, it is customary, where much accuracy is required, to take several altitudes—usually three or five—in pretty rapid succession, that is, within a minute or two of each other, and to note the corresponding times: the intervals should be as nearly equal as practicable. The mean of the altitudes is then taken as the altitude corresponding to the mean of the times.

The learner is to understand, however, that in taking a set of altitudes, it is not the chronometer which is directly consulted for the corresponding times: the chronometer is never removed and carried about, but a good seconds watch is always employed. The mean of the times by watch, corresponding to the mean of the altitudes, being found, the watch is then carried to the chronometer, and its error on the chronometer ascertained; this error being allowed for, we have the time by chronometer corresponding to the mean of the altitudes; or the error is found immediately *before* the observations are taken.

CHAPTER III.

ON THE VARIATION OF THE COMPASS.

The angle by which the compass-needle—when uninfluenced by local circumstances—deviates from the true north and south line, is called the *variation of the compass* at the place through which that north and south line passes. The variation is different at different places, and is seldom long constant even at the same place. At London the variation was formerly easterly—in 1659 it was zero, the needle then pointing due north and south: it then slowly deviated from the plane of the geographical meridian towards the west, the deviation increasing till the year 1819, when the westerly limit, 24° 42′, appears to have been attained. Since then it has been slowly but irregularly returning, the variation at present being about 23° West. On shipboard the angular departure of the compass-needle from the plane of the geographical meridian, is the combined effect of the *variation* properly so called and the local attraction of the ship itself, which in iron vessels must of course be considerable. Contrivances have been introduced to neutralise this local attraction; an account of the most efficient of these will be found in the article on "The Compass," in Mr. Grantham's "Iron Shipbuilding," in Weale's Series of Rudimentary Treatises.

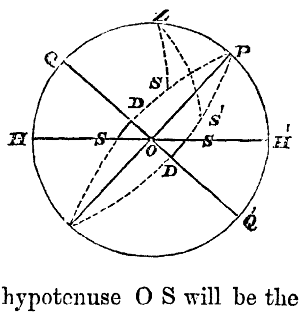
To ascertain at any place the amount by which the compass direction deviates from the direction of the true north and south line, is obviously a matter of much practical importance at sea: the following article will be devoted to the consideration of it.

Variation determined from the observed Amplitude of a celestial object.

In order to discover to what extent the compass is in error, it is plain that we must possess some means independent of that instrument of finding the *true bearing* of an object; the difference between this and the compass bearing will be the variation, or the error of the instrument.

At sea, the object must be one of the heavenly bodies: if it be in the horizon, that is, just rising or setting, the bearing is its *amplitude;* if it be above the horizon, the bearing is its *azimuth.* When the object is rising, the true amplitude is always measured from the E., and when it is setting from the W., and towards the north or south according as the declination is N. or S.

To compute the amplitude it is only necessary to know the declination of the object, and the latitude of the place.



For let P be the elevated pole, and Q Q′ the equinoctial; Z the zenith, and H H′ the horizon; then if S be the body at rising or setting, the perpendicular S D to Q Q′ will be its declination, the opposite angle O, at the east or west point of the horizon, will be the co-latitude of Z, and the hypotenuse O S will be the true amplitude; so that in the right-angled spherical triangle O D S we have sin S D = sin O S × sin O, that is sin declin. = sin amp. × cos lat.

$$\therefore \text{sin amplitude} = \frac{\text{sin declination}}{\text{cos latitude}}$$

Since refraction causes objects to appear in the horizon when they are on the average about 33′ below it, the compass bearing should be taken when the sun's centre, or the star selected, is about 33′ + dip *above* the sea horizon; so that allowing 16′ for the sun's semidiameter, the observed altitude of the sun's lower limb should be about 17′ + dip. It is to be observed that if the true amplitude found by calculation, and that taken by the compass, be both N. or both S., their difference will be the variation; but if one be N. and the other S. their sum will be the variation. The variation is E. when the true amplitude is to the right, and W. when it is to the left of the compass amplitude, that is to say, it is E. or W., according as the sun's true direction is to the right or left of the compass direction. It will be sufficient if the variation is found to the nearest minute.

EXAMPLES.—*Variation of the Compass from an Amplitude.*

1. February 20, 1858, the rising amplitude of Aldebaran, taken at sea with the azimuth compass in true latitude 27° 36′ N., was E. 23° 30′ N.: required the variation of the compass?

Declination of Aldebaran, Feb. 20, 1858, (Naut. Alm. p. 379.) 16° 13′ 21″ N.

log sin amplitude = log sin declin. — log cos latitude + 10

Declin.	16° 13′ 21″	....	sin 9·44618
Latitude	27 36 0	....	cos 9·94753
True amplitude E.	18 22 33 N.	...	sin 9·49865
Compass amplitude E.	23 30 0 N.		
Variation	5° 7′ 27″ E.		
or	5° 7′½ E.		

the true amplitude being to the right.

The magnetic or compass E. has receded 5° 7′ 27″ from the true E. towards the S., hence the magnetic N. must have deviated this amount E. from the true N.

2. July 10, 1858, the star Rigel was observed to set 9° 50′ to the N. of the W. point of the compass, in true latitude 48° 10′ N.: required the variation of the compass?

Declination of Rigel, July 10, 1858, 8° 21′ 54″ S.

Declin.	8° 21′ 54″	....	sin 9·16280
Latitude	48 10 0	....	cos 9·82410
True amplitude W.	12 35 56 S.	...	sin 9·33870
Compass amplitude W.	9 50 0 N.		
Variation	22° 25′ 56″ W., or 22° 26′ W., the true amplitude being to the left.		

3. February 15, 1858, in latitude 43° 36′ N. true, and longitude 20° W. by account, the setting amplitude of the sun's centre was observed to be W. 6° 45′ N. at $6^h\ 50^m$ P.M. apparent time: required the variation of the compass?

1. *For the declination at time of observation.*

App. Time at Ship	6^h	50^m
Long. in time W.	+1	20
App. Time at Greenwich . .	8	10

Sun's Noon Declin.	12° 39′ 57″·5 S.	Diff. 1^h, −51″·82
Cor. for $8^h\ 10^m$.	—7 3	$8\frac{1}{6}$
DECLINATION .	12 32 54 S.	41456
		864
		423·20

2. *For the True Amplitude.*

Declin.	12° 33′	. . . .	sin 9·33704
Latitude	43 36	. . . .	cos 9·85984
True amplitude W.	17 28 S.	. . . .	sin 9·47720
Compass amplitude W.	6 45 N.		
VARIATION .	24° 13′ W., the true amp. being to the left of the compass.		

The blank form for these operations is the following:

BLANK FORM.—*Variation of Compass from Sun's Amplitude.*

[The true amplitude is always measured from the E. when the object is rising, and from the W. when it is setting; and towards the N. or S. according as the declination is N. or S.]

1. *For Sun's declination at time of Observation.*

	Time at Ship	. .^{h} . .^{m} . .^{s}		
	Long. in Time	. . . . . .		
	Greenwich date	. . . . . .		
Sun's Noon Declin. at G.	. .° . .′ . .″		Diff. 1^h	. .″
Cor. for Time from Noon	. . . .		No. of hours	× . .
DECLINATION	. . . . . .		Cor.	. . .″

2. *For the true Amplitude, and thence the Variation.*

Declination	. .° . .′	sin . . . . . .
Latitude	. . . .	cos . . . . . .
True amplitude	. . . .	sin . . . . . .
Compass amplitude	. . . .	
VARIATION	. . . .	When the amplitudes are both N. or both S. this is the diff. of the two, otherwise it is their sum; E. if the true amp. is to the right, and W. if to the left, of the compass amp.

Examples for Exercise.

1. Jan. 1, 1858, the rising amplitude of Spica, in latitude 16° 21′ S. true, was observed by compass to be E. 16° 3′ N.: required the variation of the compass?

Ans. variation 26° 55′ E.

2. In latitude 21° 14′ N. true, when the declination of the sun, reduced to the time at the ship, was 19° 18′ 6″ S., its rising amplitude was observed to be E. 35° 20′ S.: required the variation of the compass?

Ans. variation 14° 34′ W.

3. March 11, 1858, at about 5^h 56^m A.M. apparent time, the sun's rising amplitude was observed to be E. 6° 36′ N.; the true latitude of the ship was 10° 2′ S., and her longitude by account 168° E.: required the variation of the compass?

Ans. variation 10° 38′ E.

4. Nov. 15, 1858, at about 6^h 45^m P.M. mean time, the sun's setting amplitude was W. 15° 40′ S.; the true latitude of the ship was 31° 56′ N., and her longitude by account 75° 30′ W.: required the variation of the compass?

Ans. variation 6° 27′ W.

5. Sept. 18, 1858, at about 5^h 50^m A.M. mean time, the sun's rising amplitude was E. 12° 10′ N.; the true latitude was 47° 25′ N., and the longitude by account 72° 15′ W.: required the variation?

Ans. variation 9° 9′$\frac{1}{2}$ E.

Variation determined from the observed Azimuth of a celestial object.

Azimuth like amplitude is an arc of the horizon: it is the measure of the angle at the zenith included between the meridian of the observer and the vertical through the object observed. In N. latitude the horizontal arc is here regarded as measured from the S. point of the horizon, and S. latitude from the N. point; towards the E. if the altitude be increasing, and towards the W. if it be decreasing.

In the diagram at page 172, let S′ be the object: the arc of the horizon which measures the angle H Z S′ is the true azimuth. To determine it, we have given the three sides of the oblique-angled spherical triangle Z S′ P; namely, the co-altitude P Z, the polar distance P S′, and the co-altitude Z S′: the angle P Z S′ may therefore be found by an operation similar to that marked (5) at page 160: the supplement of this angle is the true azimuth. As the operation referred to gives half the angle P Z S′, or $\frac{1}{2}$ Z, and that the supplement of Z is $2(90° - \frac{1}{2} Z)$, we have only to change *sin* in the final result to *cos*.

EXAMPLES. *Variation of the Compass from an Azimuth.*

1. April 20, 1858, at about 9^h A.M. apparent time, the altitude of the sun's lower limb was 36° 50′, and his bearing or azimuth, by compass, at the same time, S. 31° E. The true latitude of the ship was 50° 12′ N., and the longitude by account 13° W.: required the variation of the compass, the correction of the altitude for index and dip being −4′ 31″?

1. *For the Sun's polar distance at time of observation.*

Time at Ship, Ap. 19 . .	21^h 0^m
Long. in Time W. . . .	+52
App. Time at G.	21 52

Declin. Noon, Ap. 19,	11° 10′ 15″·2 N.	Diff.	+51″·64
Cor. for 22^h	+18 56		22
DECLINATION . .	11 29 11 N.		10328
	90		1033
Polar distance P S′ =	78 30 49		6,0)113,6·1
			+18′ 56″

2. *For the true co-altitude.*

Observed altitude L. L. .	36° 50′ 0″
Index and dip −4′ 31″ } Semidiameter +15 57 }	11 26
Apparent alt. of centre .	37 1 26
Refraction − Parallax .	−1 10
True altitude	37 0 16 ∴ CO-ALTITUDE Z S′ = 52° 59′ 44″

3. *For the true Azimuth, and thence the Variation.*

Polar distance	78°	30′	50″		
Co-altitude	52	59	40	Arith. Comp.	sin 0·09768
Co-latitude	39	48	0	Arith. Comp.	sin 0·19375
	2)171	18	30		
½ sum . . .	85	39	15		
½ sum – co-alt.	32	39	35	. . . . . .	sin 9·73211
½ sum – co-lat.	45	51	15	. . . . . .	sin 9·85586
					2)19·87940
½ Azimuth	29	30	0	. . . . . .	cos 9·93970
		2			
True Azimuth S.	59	0	E.		
Compass Azimuth S.	31	0	E.		
VARIATION . .	20°	0′	W.		

The variation is E. or W., according as the true azimuth is to the right or left of the observed azimuth, just as in an amplitude.

2. June 9, 1858, at about 5^h 50^m A.M. apparent time, in latitude 50° 47′ N. true, and longitude 99° 45′ W. by account, the bearing of the sun by compass was S. 92° 36′ E., when the altitude of his lower limb was 18° 35′ 20″; the index correction was + 3′ 10″, and the height of the eye 19 feet: required the variation of the compass?

1. *For the polar distance at time of observation.*

Time at Ship, June 8, . . .	17^h	50^m
Long. in Time W.	6	39
App. Time at G., June 9, . .	0	29

Declin. Noon, June 9,	22°	56′	38″	N.	Diff.	+ 12″·08
Cor. for 29^m . . .			+6			
DECLINATION .	22	56	44			
	90					
POLAR DISTANCE	67	3	16			

2. *For the true co-altitude.*

Observed alt. L. L.				18°	35′	20″
Index and dip	−1′	7″	} .		+14	40
Semidiameter	+15	47				
App. alt. of centre				18	50	0
Refraction − Parallax					−2	41
True altitude				18	47	19
				90		
CO-ALTITUDE				71°	12′	41″

3. *For the true azimuth and thence the variation.*

Polar distance . .	67°	3′	16″		
Co-altitude . .	71	12	40	Arith. Comp.	sin 0·02378
Co-latitude . . .	39	13	0	Arith. Comp.	sin 0·19911
	2) 177	28	56		
½ sum	88	44	28		
½ sum − co-alt. .	17	31	48	. . . . . .	sin 9·47886
½ sum − co-lat. .	49	31	28	. . . . . .	sin 9·88120
					2) 19·58295
½ Azimuth . .	51°	46′	50″	. . . . .	cos 9·79147
			2		
True Azimuth S. .	103	34	E.		
Compass Azimuth S.	92	36	E.		
VARIATION . . .	10	58	W., the true azimuth being to the left of the observed.		

The computation marked (3) in each of these two examples has been conducted in imitation of that at page 160. But there is another form for finding an angle A, from the three sides *a*, *b*, *c*, of a spherical triangle, which is somewhat shorter than that above, namely, the form—

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}}$$

s being half the sum of the sides. If we call the altitude *a*, the latitude *l*, and the co-declination or polar distance *p*; and put *s* for the ½ sum of these three, the formula, after an obvious transformation, will give—

$$\sin \tfrac{1}{2} \text{Azimuth} = \sqrt{\frac{\cos s \cos (s-p)}{\cos a \cos l}}$$

The work of step (3) above, by this formula, is as follows:—

Polar distance . .	67°	3′	16″		
Altitude	18	47	20	Arith. Comp.	cos 0·02378
Latitude	50	47	0	Arith. Comp.	cos 0·19911
2)	136	37	36		
½ sum	68	18	48	. . . . . .	cos 9·56765
½ sum − Polar dist.	1	15	32	. . . . . .	cos 9·99990
					2)19·79044
½ Azimuth . . .	51	46	50	. . . . . .	sin 9·89522

It is this method of working the step which we shall indicate in the following blank form:—

Blank Form. *Variation of the Compass from an Azimuth.*

[The azimuth is to be estimated from the S. in N. lat., and from the N. in S. lat.: towards the E. when the altitude is increasing, and towards the W. when it is decreasing.]

1. *For the declination.*

Time at Ship	. .ʰ . .m
Long. in time	. . . .
Greenwich date	. . . .*
G. Noon Declin.	. .° . .′ . .″
Diff. in 1ʰ . .″	
No. of hours × . .	
Correction . . .″ =	. . . .
Declination	. . . . . .
	90
Polar Distance	. . . . . .

2. *For the true altitude.*

Observed alt. (L. L. or U. L.)		. .° . .′ . .″
Index and dip	. .′ . .″	
Semi-diameter	. . . .	. . . .
App. alt. of centre		. . . . . .
Refraction —Parallax		— . . . .
True Altitude		. . . . . .

Note. When the object is a star, the declination is got at once from the Nautical Almanac; and the altitude requires no correction for semidiameter and parallax.

* The Greenwich date, mean time, may be obtained from the chronometer, properly corrected for gain or lo[illegible].

3. *For the true Azimuth, and thence the Variation.*

Polar distance	..° ..′ ..″		•
Altitude	.. .. ..	Arith. Comp.	cos ......
Latitude	.. .. ..	Arith. Comp.	cos ......
	2)		
½ sum	.. .. ..		cos ......
½ sum ∼ Polar dist.	.. .. ..		cos ......
			2) ....
½ Azimuth	.. .. ..		sin ......
	2		
True Azimuth	.. ..		
Compass Azimuth	.. ..		
VARIATION	.. ..		

☞ When the true azimuth is to the *left* of the compass azimuth, *subtract*; when to the *right*, *add*.

NOTE.—When the object observed is on the meridian, its bearing by compass will be the variation, which will be W. if the meridian be to the left of the compass bearing, and E. if it be to the right.

Examples for Exercise.

1. July 20, 1858, in latitude 21° 42′ N. true, and longitude 62° E. by account, the sun's observed azimuth was S. 100° 16′ E., at $7^h\ 4^m$ A.M. apparent time; the altitude of his lower limb was 23° 36′, allowing for index error, and the height of the eye 24 feet: required the variation of the compass? Ans. variation 3° 42′ W.

2. October 28, 1858, in latitude 36° 18′ S. true, and longitude 15° 30′ E. by account, the sun's observed azimuth was N. 86° 34′ W., at about $6^h\ 30^m$ P.M. mean time; the altitude of his lower limb was 12° 35′, allowing for index error, and the height of the eye was 30 feet: required the variation of the compass? Ans. variation 10° 36′ W.

3. November 3, 1858, in latitude 25° 32′ N. true, and longitude 85° W. by account, the sun's observed azimuth was

S. 58° 32′ W., at about $4^h\ 15^m$ P.M. mean time; the altitude of his lower limb was 15° 37′, the index correction was +1′ 20″, and the height of the eye 15 feet: required the variation of the compass? Ans. variation 5° 26′ E.

4. May 21, 1858, in latitude 52° 12′ N. true, the sun's azimuth by compass was S. 82° 58′ W., and the altitude of his lower limb was 23° 46′. The chronometer showed the Greenwich time of the observation to be $17^h\ 56^m\ 34^s$, May 20. The index correction was +2′ 30″, and the height of the eye 12 feet: required the variation of the compass?

Ans. variation 9° 24′ E.

NOTE.—The object of the preceding articles on the variation of the compass is to determine the angular departure of the N. point of the instrument from the true N. point of the horizon, at the time and under the circumstances in which the amplitude or azimuth is taken. If no provision have been made for neutralizing the influence of the ship itself on the needle, the variation thus determined will be compounded of variation proper and of the deviation from the position in which the needle would otherwise settle, caused by the local attraction. In iron ships this local attraction is of course considerable, and it is a great deal influenced by the position in reference to the meridian in which the ship is built. To determine the extent to which the deviation affects the variation proper, experiments must be made before the ship proceeds to sea, by turning her head in different directions, and comparing her compass with another compass on shore. To free the variation from the local disturbances thus ascertained, artificial magnets, and small boxes of iron chain, are recommended by the Astronomer Royal to be employed in the manner directed by him in a pamphlet to be had of the publisher of the present treatise. An account of the necessary operations for Compass Correction will also be found in Mr. Grantham's work on "Iron Ship-Building," before alluded to.

CHAPTER IV.

ON FINDING THE TIME AT SEA.

The determination of the time at sea is a problem of the first consequence. It is indispensably necessary to the discovery of the correct longitude, which indeed is nothing more than the interval between the ship's time and Greenwich time at the same instant, converted into degrees and minutes. As in most of the other problems of Nautical Astronomy, so here :—of the quantity sought we have generally some approximate value, more or less incorrect, and this is turned to account in the operation for finding the true value. At first sight the statement would appear contradictory, that erroneous data could aid in conducting to correct conclusions; but Nautical Astronomy abounds in instances where very material errors in the values with which we work have no practical influence upon the results arrived at. The reason is, that these erroneous values never enter directly into the mathematical portion of the inquiry: they merely serve the purpose of suggesting to us certain other quantities with which they are connected—which are actually employed in the computation—and which are such as to be incapable of error beyond a very limited extent. The ship's longitude *by account*, and her *estimated time*, never enter into the trigonometrical calculation of any nautical problem, claiming accuracy of result: but for the preparatory reductions for the sun's declination, or the equation of time, they may be used with every confidence; and it is only for such like purposes that they *are* used. These quantities vary so little in a considerable interval of time, that an error in time of one hour—equivalent to an error of 15° of longitude—will not affect the sun's declination to the extent of 1′; and as to the equation of time, the error will not average 1″.

Time deduced from an Altitude of the Sun.

Referring to the diagram at page 172, if S be the place of the sun at the time of observation, there will be given in the spherical triangle P Z S, the co-latitude P Z, the co-altitude Z S, and the polar distance P S, to find the hour angle Z P S, which measures the apparent time from noon.

As at page 179, putting a for the altitude of S, p for its polar distance, l for the latitude of Z, and s for the half sum of all three, we shall have for the hour angle P.

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\cos s \sin (s-a)}{\sin p \cos l}}$$

which is derived from the formula following (Spherical Trig. p. 18) :—

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}$$

by obvious substitutions.

The hour angle P being thus found, and converted into time, we shall have the apparent time at the ship; and by applying the equation of time, shall thence get the mean time at the ship, as in the examples following.

NOTE.—It will prevent confusion, and consequently all liability to mistake, if the time at a place at any instant be always measured from the noon at the place *preceding* that instant; that is, if it be always converted into time P.M. . Thus, $10^h\ 20^m$ A.M. in civil reckoning, means $10^h\ 20^m$ past the preceding midnight: it is better to regard it as $22^h\ 20^m$ past the preceding noon, pushing the date one day back, so that $10^h\ 20^m$ A.M. Jan. 4, is the same as $22^h\ 20^m$ Jan. 3.

EXAMPLES. *Time from an Altitude of the Sun.*

1. In latitude 50° 30′ N. true, and longitude 110° W. by account, the mean of a set of altitudes of the sun's lower limb was 11° 0′ 50″, the mean of the corresponding times

by the *watch* was 4^h 45^m P.M., the index correction was $-3' 20''$, and the height of the eye 20 feet: required the mean time at the ship, and the error of the watch?

From the Nautical Almanac at G. noon.

Sun's Declin. 0° 6′ 56″ S. Diff. in 1^h, $+58''\frac{1}{2}$. Equa. of Time, 7^m 42^s. Diff. $+0''\cdot858$. Semi-diameter, 15′ 58″.

1. *For the true altitude* a, *and polar distance* p.

Obs. alt.		11° 0′ 50″
Index and dip	− 7′ 44″ }	8 14
Semidiam.	+ 15 58 }	
App. alt. centre		11 9 4
Ref. − Parallax		−4 39
True alt. centre		11 4 25

Equa. of time *	7^m 42^s	Diff. in 1^h,	+ ·858″
Cor. for 12^h . .	+ 10		12
EQUA. OF TIME	7^m 52^s		10·296

Time per watch	4^h 45^m 0^s P. M.	
Long. 110 W.	+7 20	
Mean time at G.	12 5	nearly.

Noon Declin. at G.	0° 6′ 56″ S.	Diff. in 1^h,	+ 58″½
Cor. for 12^h 5^m . .	+11 47		$12\frac{1}{12}$
DECLINATION .	0 18 43 S		702
	90		5
POLAR DISTANCE	90 18 43		6,0)70,7″
			11′ 47″

* This is taken from page I of the month in the Nautical Almanac: there will be no sensible error in regarding the time as apparent instead of mean. The Almanac itself directs whether the reduced equation is to be added to or subtracted from the apparent time at the ship.

2. *For the mean time at the ship, and error of the watch.*

Altitude . .	11°	4′	25″	
Latitude . .	50	30	0	Arith. Comp. cos 0·196489
Polar distance	90	18	43*	Arith. Comp. sin 0·000006
	2)151	53	8	
½ sum . .	75	56	34	. . . cos 9·385411
½ sum − alt.	64	52	9	. . . sin 9·956812
				2)19·538718
½ Hour angle	36°	0′	48″	. . . sin 9·769359
			2	
∴ Hour angle =	72	1	36	
In Time =	4^h	48^m	6^s	Apparent time at ship
Equation of time		−7	52	
	4^h	40^m	14^s	Mean time at ship.
	4	45	0	Mean time per watch.
	0^h	4^m	46^s	Watch *fast* for mean time at ship.

The student will readily perceive the object of finding the error of the watch. The watch being assumed to be a sufficiently good one to be depended upon for regularity during the short time occupied in performing the foregoing calculation, when the operation is finished the watch—making the proper allowance for the error—will still show what the time is at the place where the observation was made: comparing it, therefore, now with the chronometer—which is never disturbed from its situation—we shall at once see by how much *it* differed from the time at the place of observation, at the instant that observation was made; that is, we shall get what is called the error of the chronometer on mean time at the place. A memorandum being made of this error, so that we may always be able to allow for it when consulting the

* The sine of this is cos 18′ 43″. And whenever the polar distance exceeds 90°, instead of the sine of it we may always take the cosine of the excess, and thus avoid the trouble of subtracting from 180°.

chronometer, we may at any future instant learn the time at that instant, at the place left,—provided, at least, the chronometer can be depended upon for regularity during the interval. Hence, by again finding the mean time at ship, and as before the error of the chronometer on that time, the difference of the errors will be the difference of longitude in time between the two situations of the ship. But we must defer further remarks on this subject till next article.

Since the determination of the time at sea requires that the altitude of the object observed should be taken with more than ordinary accuracy, a single observation for this purpose is seldom considered as sufficient; it is, therefore, usual to take a set of altitudes, and to employ the mean of the whole, taking the mean of the corresponding times by watch as the estimated time, as in the following example.

2. August 16, 1858, the following observations were taken in latitude 36° 30′ N. true, and longitude 153° E. by account: the index correction was —3′ 5″, and the height of the eye 27 feet: required the mean time at the ship, and the error of the watch?

Times per Watch.		*Alts. Sun's L. L.*
4^h 40^m 0^s P.M.		24° 18′
41 10		24 2
42 5		23 48½
43 0		23 36
44 17		23 19¼
5)210 32		5)119 4
4 42 6	. . Means . .	23 44 48

We are to proceed as if the observed altitude of the sun's lower limb were 23° 44′ 48″, and the corresponding time per watch 4^h 42^m 6^s P.M.

1. *For the true altitude and polar distance.*

Obs. alt. L. L.	. . . .		23°	44′	48″
Index and Dip.	− 8′ 12″	} .			
Semi-diameter	+ 15 50			+ 7	38
App. alt. centre	. . .		23	52	26
Ref. —Parallax	. . . .			−2	2
True alt. centre	. . .		23	50	24

Equa. of Time	4ᵐ 17ˢ		Diff. − ·49
Cor. for 18 hours	−9		81
			49
			39
EQUA. OF TIME	4 8	*Add.*	8·8

Time per Watch, Aug. 16	4^h	42^m	6^s P.M.
Long. 153° E.	−10	12	0
M. Time at G.. Aug. 15*	18	30 nearly	

Noon declin. at G.	14°	5′	13″ N.	Diff. −47″·13
				81½
				4713
Cor. for 18½ hours		−14	32	3770
DECLINATION	13	50	41	236
	90			6,0)87,1·9
POLAR DISTANCE	76	9	19	−14′ 32″

* Agreeably to what is recommended in the NOTE at p. 184, the time at Greenwich, at the instant of observation, is measured from the Greenwich noon *preceding* that instant, 24^h being tacitly added to the time per watch, to bring this about, and the date therefore put one day back. This is the same as if we had actually subtracted the longitude in time from the time per watch, getting for remainder (neglecting the 6^s) − 5^h 30^m; that is, 5^h 30^m *preceding* the noon of Aug. 16, which is the same as 18^h 30^m *after* the noon of Aug. 15.

2. *For mean time at ship and error of the watch.*

Altitude . .	23°	50′	24″	
Latitude . .	36	30	0	Arith. Comp. cos 0·094837
Polar distance	76	9	20	Arith. Comp. sin 0·012804
	2)136	29	44	
½ sum	68	14	52	. . . . cos 9·568911
½ sum − alt.	44	24	28	. . . . sin 9·844949
				2)19·521501
½ Hour angle	35°	12′	1″	. . . . sin 9·760750
		2		
∴ Hour angle =	70	24		
In Time =	4^h	41^m	36^s	Apparent time at ship.
Equation of time		+4	8	
	4	45	44	Mean time at ship.
	4	42	6	Mean time per watch.
	0^h	3^m	38^s	Watch *slow* for mean time at ship.

It may be remarked here that an error of a few seconds in the polar distance—which, of course, is to be expected—since the estimated time and estimated longitude are both to some extent incorrect, will make no appreciable difference in the value of the hour-angle deduced. The polar distance is always a large arc—never much less than 67°, and for large arcs the tabular differences of the *sines* are always small. Whatever error there may be in the polar distance, there will be half that error in the ½ sum, and in the ½ sum − alt.; but as the logs connected with these are log cos and log sin, their errors oppose one another. [See, however, the remarks at page 190.] It appears, from the Hourly Diff. in the declination, that if the combined errors of the time and longitude in the foregoing example amounted to so much as 1^h of time, or 15° of longitude, the error in the polar distance would be 47″. Suppose, for greater convenience of calculation, we assume the error to be 44″, and let us see what effect such an error would have upon the resulting time at the ship.

Suppose, now, that this error gives a correction of +22″ in the polar dist., then, applying this correction as recommended above, we have—

					0·199263
Polar distance	91°	36′	52″	Arith. Comp. sin	0·000172
½ sum . .	86	1	39	. . . cos	8·840593
½ sum — alt.	56	23	13	. . . sin	9·920538
					2)18·960566
Hour angle	17	35	21½	. . . sin	9·480283
			2		
∴ Hour angle =	35	10	43	= 2ʰ 20ᵐ 43ˢ in time.	

As this is 3ˢ less than the apparent time before deduced, it follows that the error of the watch is 31ᵐ 57ˢ.

We take this opportunity of showing the practical advantage of using the logarithmic tables as recommended in the foot-note at p. 142. Disregarding the seconds in the several arcs, we shall take out the several logarithmic values to the degrees and minutes only, writing against each the tabular difference to 100″: we shall then multiply each difference by the number of seconds which has been reserved, cutting off two figures from the right of the product for the division by 100, and shall then incorporate the aggregate of these quotients, previously marked + or — as directed in the foot note, with the sum of the logs extracted. In repeating the operation, we merely have to increase three arcs by 22″, 11″, and 11″ respectively: we shall therefore have only to multiply the differences against the arcs by these numbers in order, cutting off two figures as before. If the cutting off be postponed till the sum of the products is found, strict accuracy will be secured to the final figure of the result; and this is the plan we shall adopt in what follows.

		(*Tab. Diff. for 100″.*)	(*Parts for seconds.*)		(*Parts for seconds.*)	
Arith. comp. . .	cos 0·199263					
Arith. comp. . .	sin 0·000169	6 +	180	22″ +	132	
	cos 8·841774 ...	3030 —	84840	11″ —	33330	
	sin 9·920520 ...	140 +	280	11″ +	1540	
	18·961726	Cor. =	— 843,80	664)	— 31658(	— 48″
Correction for seconds	— 844					P = 35° 11′ 31″
	2)18·960882					P = 35° 10′ 43″
$\frac{1}{2}$ P = 17° 35′ 45$\frac{1}{2}$″ .... sin	9·480441 ...	664*				

* This difference stands against sin 17° 35′ in the table, and is to be taken out with the angle 17° 35′ itself, as well as the difference between the sine of this angle and the sine arrived at above; the latter difference, with two zeros annexed, divided by the former difference, gives the seconds, in the usual way.

In the extra work for correcting $\frac{1}{2}$ P, the parts for seconds amount to the half of — 316,58; so that the half of 31658 divided by 664 will give the seconds of correction for $\frac{1}{2}$ P, therefore the whole of 31658 divided by 664 must give the correction for P, as above. We have computed $\frac{1}{2}$ P to the nearest half-second mainly for the purpose of furnishing a better means of comparison between the two processes.

The time consumed in performing the necessary work in this way, is scarcely half that occupied by the operations before given, more rigid accuracy is in general also secured, the process can much more readily be revised if error be suspected, and after the first set of references to the table, no further references are necessary. There can be no hesitation about the proper *sign* to be written against the tabular difference: every *sine* is +, and every *cosine* is —; but *complements* always require a change of the sign that otherwise would be written. It will be remembered in the above that it was not the sine of 91° 36′ that was taken from the table, but the *cosine* of 1° 36′; so that the tabular difference 6 would have been marked —, only it is the *complement* of that cosine which is written down. It need scarcely be remarked that when seconds are to be *subtracted* from the arcs, the signs are opposite to those which would be annexed if the seconds were to be added. We shall now give the blank form for computing the hour-angle, and for correcting the first result in the manner here explained.

BLANK FORM. *Time at Ship from the Latitude and Sun's Altitude.*

1. *For the true altitude, polar distance, and equation of time.*

Obs. alt.		. .° . .′ . .″		
Index and Dip	. .′ . .″	. . . .		
Semi-diam.	. . . .			
App. alt. centre		. . . . . .		
Ref. — Parallax		—. . . .		
True alt. centre		. . . . . .		
Equa. of Time	. .ᵐ . .ˢ		Diff.	. . .″
Cor. for the hours	. .			× . . hours
EQUA. OF TIME	. . . .			
				. .″ Cor.*
Time per watch	. .ʰ . .ᵐ . .ˢ			
Long. in time	. . . . . .			
Mean Time at G.	. . . . . . nearly.			
Noon Declin. at G.	. .° . .′ . .″		Diff.	. . .″
Cor. for hours past noon	. . . .			× . . hours
DECLINATION	. . . . . .			
	90			60) . . . .″
POLAR DISTANCE	. . . . . .			. .′ . .″ Cor.

* The Nautical Alm. directs whether the equa. of time is additive or subtractive.

2. *For the mean time at ship, and error of the watch.*

Alt.	. .° . .′ . .″	*[Seconds reserved.]*	*Diff.*	*Pts. for secs.*
Lat.	. . . . . .	Comp. cos	. . . . +	. . . .
Polar dist.	. . . . . .	Comp. sin	. . . . —*	. . . .
2)	. . . . . .			
½ sum	. . . . . .	cos	. . . . —	. . . .
½ sum — alt.	. . . .	sin	. . . . +	. . . .
		. . . .	Cor. for secs.	. . . . .
Correction for seconds		. . .		
		2)		
½ hour angle	. . . . . .	sin	. . . . + † = D	
	2			
Hour angle =	. . . . . .	. .ʰ . .ᵐ . .ˢ Apparent time.		

Equa. of Time . . . .

. . . . . . Mean time at ship.

. . . . . . Mean time per watch.

. . . . . . Watch { fast / slow } for M. time at ship.

Cor. of this time . . (*See extra work below.*)

. . . . . . True error of the watch.

[Extra work for correcting the first result.]

Secs. of cor.	*Pts. for the secs.*
. .″	. . . .
. .	. . . .
. .	. . . .

D) . . . . . (. .″ = Cor. of former hour angle = . .ˢ in time.

* If the polar distance exceed 90°, the comp. *cos* of the *excess* is to be taken, in which case this sign will be *plus*.

† This difference is to be taken out of the Table at the same time as the ½ hour angle.

NOTE.—In taking an altitude, for the purposes of the present problem, it is desirable that the object should be as nearly due E. or due W. as possible, because in that situation, a small error in the altitude will have the least influence on the time: the nearer the object is to the meridian, when between it and the prime vertical, the less favourable is the observation to accuracy in the deduced hour-angle. When, however, the place is between the tropics, and the declination of the same name as the latitude, the proximity of the sun to the meridian will not be an objection: since under these circumstances, his motion in altitude is sufficiently rapid for a good observation at any point of his course.

Time deduced from an Altitude of a Star.

When instead of the sun, the object observed is a star, though the trigonometrical computation for finding the hour-angle remains the same, some of the preparatory work, in step (1) of the foregoing form, is different. The determination of the sun's hour-angle gave us at once the apparent time: but a star's hour-angle alone can give us no information as to the time of observation; yet if the star's *Right Ascension* be also known, then, combining this with the hour-angle, either by addition or subtraction, we shall know the R. A. of the meridian; and then again subtracting from this the R. A. of the sun, we shall finally obtain the *sun's* hour-angle, and thence the time at the ship when the star was observed. [See NOTE, p. 197.]

The star's R. A. at the time of observation, and the mean sun's R. A. at Greenwich noon, are both given in the Nautical Almanac; and therefore the latter being reduced to the Greenwich time of observation, we shall have the R. A. of each object at the same instant; and as just explained, the star's hour-angle at that instant being found, we shall have the R. A. of the mean sun and of the meridian of the ship at the same instant, and therefore the mean time. A single example will sufficiently illustrate what is here said.

NOTE.—Right ascension, be it remembered, is measured from W. to E., or from the first point of Aries *easterly* from 0° up to 360°, that is, in a direction contrary to the apparent diurnal rotation of the heavens: when therefore a star is to the W. of the meridian, its hour-angle must be added to its R. A. to get the R. A. of the meridian; and when it is to the E., its hour-angle must be subtracted. With regard to the sun, whether it be W. or E. of the meridian, its R. A., subtracted from the R. A. of the meridian, will give the sun's hour-angle from preceding noon.

The student must especially remember, that whenever we speak of one R. A. as being subtracted from another, with a view to obtaining a third R. A., it is always tacitly supposed that 24^h is added to the second when the first is greater than it. And whenever one R. A. is to be added to another to get a third, 24^h is always suppressed from the sum if it exceed that quantity. It is plain that there is no displacement of a celestial object by increasing its R. A. by 24^h, or by 360° if the R. A. be expressed in angular measure.

The hour-angle of a star, or planet, or of the moon, is its least angular distance from the meridian, whether the object be to the W. or E.; but the hour-angle of the sun is usually measured *westward*, that is, from the preceding noon.

Example. Time from an Altitude of a Star.

April 22, 1858, in true latitude 42° 12′ N. and longitude by account 44° 30′ E. when the mean time per watch was 8^h 2^m P. M. the observed altitude of the star Arcturus, eastward of the meridian was 73° 48′ in artificial horizon: the error of the instrument was + 7′ 34″: required the error of the watch?

1. *For the true altitude, the polar distance, and the R. A. of mean sun.**

Observed Alt.	73° 48′ 0″
Index cor.	+ 7 34
	2) 73 55 34
Apparent alt.	36° 57′ 47″
Refraction	— 1′ 17″
TRUE ALT.	36° 56′ 30″

NOTE.—When the artificial horizon is used, there is no correction for Dip. The index correction being applied, the result is twice the app. alt.

Time per watch . .	8^h 2^m 0^s	
Long. 44°½ E. in time .	2 58 0	
Mean Time at G.	6 4 0	nearly.

R. A. mean sun at G., noon . . .	2^h 1^m 0^s·51
Diff. for 1^h, + 9^s·86 ∴ for 6^h . . .	+ 59·16
R. A. MEAN SUN AT TIME OF OBS. . .	2 2 0

STAR'S R. A. 14^h 9^m 14^s, DECLIN. 19° 55′ 6″ N.

∴ POLAR DISTANCE = 70 4 54

2. *For the mean time at the place, and error of the watch.*

						Tab. Diff.	*Parts.*
Altitude	36°	56′	30″				
Latitude	42	12	0	Comp. cos	0·130296	191 +	0
Polar dist.	70	4	54	Comp. sin	0·026831	76 —	4104
	2)149	13	24				
½ sum	74	36	42	cos	9·424156	765 —	32130
½ sum — alt.	37	40	12	sin	0·786089	273 +	3276
					19·367372		— 329,58 Cor. for secs.
					— 330		
					2)19·367042		
½ Hour angle	28	51	2 2	sin	9·683521	382 +	

∴ Hour angle 57 42 4 = 3^h 50^m 48^s in Time E. of Meridian.

Star's R. A.	14	9	14
R. A. of Meridian	10	18	26
R. A. of mean sun	2	2	0
Mean time at place	8	16	26
Mean time per Watch	8	2	0
		14	26 Watch *slow* on mean time at place.

* The R. A. of the mean sun, at mean noon at Greenwich, is given at

In this additional time the R. A. of the mean sun would be increased by about 2^s, as appears by referring to the "Diff. for 1^h." Hence the error of the watch is $14^m\ 24^s$. *

In the following blank form we shall provide for the insertion in step 2 of the tabular differences, as in the above example. But since a star's declination is constant for a considerable interval of time, there will be no correction required for change of polar distance; a correction, however, for change in the mean sun's R. A. may be necessary, as in the preceding example.

BLANK FORM. *Time at Ship from the Latitude, and a Star's Altitude.*

1. *For the true altitude, the polar distance, and the R. A. of mean sun.*

Obs. Alt. ..° ..′ ..″	Time per watch ..^{h} ..^{m} ..^{s}
In. and Dip ..′ ..″ }	Long. in time .. .. ..
Refrac. —.. .. }	Mean time at G. .. .. .. nearly.
TRUE ALT.	R. A. of m. sun at G. noon ..^{h} ..^{m} ..^{s} Diff. $+9^s{\cdot}86$
NOTE.—When the altitude is taken in the artificial horizon, there is no correction for Dip. The index correction being made, the result is twice the *apparent* altitude.	Cor. for time after G. noon +.. .. Time × ..
	R. A. OF M. SUN AT T. OF OB. .. .. ..
	Cor. ...^{s}
	STAR'S R. A. ..^{h} ..^{m} ..^{s}, Declin. ..° ..′ ..″ (Naut. Alm.)
	90
	STAR'S POLAR DISTANCE .. .. ..

The mean sun's daily advance in R. A. is uniformly $3^m\ 56^s{\cdot}55$...; consequently his hourly increase in R. A. is $9^s{\cdot}86$.

In the next step, if the polar distance exceed 90°, the comp. *cos* of the *excess* is to be taken, when the sign annexed to the "Diff." will be +.

* The additional correction of the time here noticed is improperly omitted in most books on Nautical Astronomy. For the purpose of determining the longitude of the ship, it is of the first importance that the ship's time should be obtained with all possible accuracy. In the above example, the corrected mean time at the ship is $8^h\ 16^m\ 24^s$.

2. *For the mean time at the ship, and error of the watch.*

			Tab. Diff.	*Pts. for Secs.*
Altitude	.° ..′ ..″			
Latitude	.. .. ..	Comp. cos	.... +	....
Polar dist.	.. .. ..	Comp. sin	.... —	....
	2).			
½ sum	.. .. ..	cos	.... —	....
½ sum — alt.	.. .. ..	sin	.... +	....
		....		
	Cor for seconds	...		.....
		2).		
½ Hour angle	.. .. ..	sin	.... +	
	2			

∴ Hour angle .. .. .. = ..ʰ ..ᵐ ..ˢ in Time { E. / W. } of Meridian. If E. subtract: if W. add.

R. A. of Star	.. .. ..	
R. A. of Meridian	.. .. ..	
R. A. of Mean Sun	.. .. ..	(Subtract.)
Mean time at Ship	.. .. ..	
Mean time per Watch	.. .. ..	
	.. .. ..	Error of Watch

NOTE.—The error in the sun's R. A., due to this error in the time per watch, may now be allowed for, and the watch still further slightly corrected.

In this and the preceding problem, the latitude is assumed to be correct; but, in general, the error of a mile or so in this datum will have but very little influence on the time. We shall, by way of illustration, suppose the latitude in the example solved above to be 1′ below the truth, and examine into the effect of this error on the time. The tabular differences, already before us, will enable us to do this with but little trouble, as in the margin. We are to conceive 60″ added to the latitude, and therefore 30″ to the ½ sum, and also 30″ to the ½ sum — alt., and to find the "parts" for these additional seconds, just as the parts were found for the seconds before. It ap- 'om th- result th t 17 i- to b-

Tab. Diff.	*Parts.*
191 +	11460
765 —	22950
273 +	8190
	2) — 33,00
	Cor. = — 17

subtracted from 9·683521, thus reducing it to 9·683504 = sin 28° 50′ 58″ ∴ hour-angle = 57° 41′ 56″ = $3^h\ 50^m\ 48^s$ in time, within about a quarter of a second. The tabular difference 382, against sin 28° 51′ 2″, enables us to get the correction for seconds due to the difference —17, at once thus —1700 ÷ 382 = — 4, the number of seconds to be subtracted from 28° 51′ 2″. If the latitude had been 60″ too great, then the signs of the "parts" in the margin would all be changed ∴ 1700 ÷ 382 = 4 = the number of seconds to be added to 28° 51′ 2″; that is, the ½ hour-angle would have been 28° 51′ 6″, and the hour-angle, 57° 42′ 12″, which in time, is $3^h\ 50^m\ 49^s$.

If the latitude be assumed to be 2′ below the truth, then each of the parts should be doubled, and half the sum taken: but this half sum is obviously the same as the whole sum —3300 above, hence —3300 ÷ 382 = 9 = the number of seconds of correction of the ½ hour-angle, which angle is therefore 28° 50′ 53″, and consequently the hour angle is 57° 41′ 46″ = $3^h\ 50^m\ 47^s$ in time. Without correcting the time in this way, through the correction of the hour-angle, we may at once apply to the former the correction for the seconds of arc in time: thus 18″ = $1^s\frac{1}{5}$ of time; but as fractions of a second of time are disregarded, as well in the original result, as in the correction of it, the corrected time might on this account still err by nearly a second.

It is easy to see, from what has now been said, how readily the navigator may form an accurate estimate of his error in the time, arising from a small error in the latitude from which it has been deduced. But if the error of latitude amount to several minutes, the correction for the time found in this manner will be only approximative since the tabular differences are not constant from minute to minute. If instead of the latitude, it be the altitude that is supposed to involve a small error, the correction of the time due to that error may be found in a similar manner, but with even less trouble, because the only arcs affected by the error will be the ½ sum, and the ½ sum — alt. Thus, if the altitude be increased by 1′, the ½ sum will be increased by 30″, and the ½ sum — alt. will be diminished by 30″, so that the correction will be found as in the margin: and — 15600 ÷ 382 = — 41″ = 3ˢ in time nearly, so that a small error in the altitude will have a much greater effect upon the time than an equal error in the latitude: but in both cases it appears that an error in excess (not exceeding 2′) will produce about the same error on the time as an equal error in defect, the errors in the time having opposite signs. Beyond 2′ of error, whether in the latitude or in the altitude, the correction of the time must be regarded as only approximative.

Tab. Diff.	*Parts.*
765 —	22950
273 —	8190
	2)311,40
Cor. =	— 156

Examples for Exercise.

1. March 15, 1858, the mean of a set of altitudes of the sun's L. L., in lat. 16° 28′ 30″ N., and long. 99° 30′ W., by account, was 10° 36′ 20″, the mean of the corresponding times per watch was $6^h\ 45^m$ A.M., the index correction of the sextant was —2′ 30″, and the height of the eye 22 feet: required the mean time at the ship, and the error of the watch?

Ans. mean time at ship $6^h\ 56^m\ 14^s$;
error of watch $11^m\ 14^s$ slow.

2. April 26, 1858, in lat. 29° 47′ 45″ S., and long. by account 31° 7′ E., the mean of a set of altitudes of the star Altair was 25° 14′ 20″ to the E. of N., the mean of the times per watch was $2^h\ 12^m\ 30^s$ A.M., the index correction was + 10″, and the height of the eye 20 feet: required the mean time at the ship and the error of the watch?

Ans. mean time at ship $1^h\ 51^m$; error of watch $21^m\ 30^s$ fast.

NOTE.—In a similar manner may the time be deduced from an altitude of a planet; the only difference being that, as in the case of the sun, the observed altitude is to be corrected for parallax and semi-diameter as well as for refraction.

CHAPTER V.

ON FINDING THE ERROR AND RATE OF THE CHRONOMETER.

In the preceding chapter we have discussed at some length the interesting problem of determining the time at sea, and thence the error of the watch. If the time thus determined be compared with the time shown by the chronometer, we shall in like manner, by taking the difference of the two times, find the error of the chronometer on mean time at the place of observation. It is still more important, however, to know the error of the chronometer on mean time at Greenwich; and this may be easily ascertained provided the longitude of the place of observation be pretty accurately known; for, as already seen, if the mean time at the place and the longitude of that place be both known, the exact time at Greenwich is very readily obtained, and the difference between this time and that shown by the chronometer is the error on Greenwich mean time.

All chronometers *have* an error: this is always accurately determined, usually at an observatory—where a memorandum is kept of its performance as a time-keeper. The purchaser always receives a certificate stating how much the chronometer was too fast or too slow, for mean time at Greenwich, at the Greenwich mean noon at a specified date; and also how much it gains or loses, on the average, in 24 hours of mean time—that is to say, its *daily rate.*

The chronometer, accompanied with the proper certificate of its error and daily rate, is taken to sea, and after any interval of time, its daily rate being multiplied by the number of days elapsed and the product—called the accumulated rate—being combined with the original error, we are enabled to apply the proper correction to the time actually indicated by the chronometer, and thus to ascertain the mean time at Greenwich.

For example: Suppose on August 21, in longitude by account 38° 45′ W., when the mean time at the ship, as found by the method explained in last chapter, was $7^h\ 24^m$ P.M., that the chronometer showed $10^h\ 2^m\ 34^s$, and that the following certificate from the Greenwich Observatory stated—

Aug. 1, Mean Noon at G.	*Daily Rate.*
Error of Chron. $2^m\ 4^s{\cdot}75$ Fast	$2^s{\cdot}6$ Gaining.

Required the mean time at Greenwich corresponding to the mean time at ship?

Time at Ship Aug. 21	$7^h\ 24^m$ P. M.
Long. 38° 45′ W. in time . .	+ 2 35
Time at G. Aug. 21	9 59

From Aug. 1 to Aug. 21, at $9^h\ 59^m = 20^d\ 9^h\ 59^m$, or $20^d\ 10^h = 20^d\frac{5}{12}$

Correction for Daily rate . . .	$-\ 2^s{\cdot}6$
	20
For Accumulation in 20^d . . .	52
in $\frac{5}{12}$ . . .	1·08
For Accumulated rate . . .	−53·08
For original error	$-2^m\ 4^s{\cdot}75$
Whole correction . . .	−2 57·83
Chronometer showed . . .	$10^h\ 2^m\ 34^s$
MEAN TIME AT G. . . .	$9^h\ 59^m\ 36^s$

Hence, assuming the mean time at ship to have been correctly determined, and the chronometer to have maintained its rate, the longitude by account is 36^s of time in error—that is, it is 9′ too little; so that the corrected longitude is 38° 54′ W. The whole correction of the chronometer for the 20 days elapsed, that is, up to mean noon of Aug. 21 at Greenwich, being $2^m\ 4^s{\cdot}75 + 52^s$ subtractive, we may henceforth employ the following memorandum—

Aug. 21, *Mean Noon at G.*	*Daily Rate.*
Error of Chron. $2^m\ 56^s{\cdot}75$ Fast	$2^s{\cdot}6$ Gaining.

But, although implicit confidence may be placed in the original correction, yet we have no security that the daily rate may not have changed. It is of importance, therefore, from time to time to examine into this matter, and instead of taking the invariability of the original rate for granted, to ascertain the rate at subsequent periods anew. In order to do this efficiently, the navigator must wait till his ship arrives at some port or harbour, where it can remain for several days.* If the place have the advantage of an Observatory, the mean time there can always be obtained; if not, the mean time must be found by the methods explained in the last chapter, using the artificial horizon for taking the altitudes ashore, or else in the way hereafter directed. The mean time at the place, upon comparison with the mean time at the same instant as shown by the chronometer, will give the error of the chronometer on mean time at that place. A few days after this set of observations for the mean time let another set be taken, and the mean time again determined, and compared with that shown by the chronometer: the error of the chronometer on mean time at the place will be again ascertained; the difference between the two errors (or their sum, if of contrary names) will show how much the time-keeper has gained or lost in the interval between the two times of observation; from which we can readily find, by proportion, what has been its average gain or loss in 24 hours of that interval—that is, its *daily rate.*

Similar observations should be made at intervals as long

* The next best method to this, is to compare the Greenwich time, as shown by the chronometer, with the Greenwich time as determined by Lunar observations; to be discussed in next chapter. The difference of the times will show the error of the chronometer on Greenwich mean time; and subsequent observations being taken, and the difference of the times found in like manner, the daily rate of the chronometer, in the interval of time elapsed, may be inferred.

as the ship remains at the place; and it is probable that different daily rates will thus be deduced: it is the mean or average of all these which must be regarded as *the* daily rate of the chronometer; and on the day of the ship's departure a fresh memorandum is to be made of the error of the chronometer on Greenwich mean time, at the corresponding Greenwich date, and of the daily rate thus determined.

Whenever an astronomical clock can be referred to, the necessity for taking observations for the mean time at the place will of course be superseded: a daily comparison of the chronometer with the mean-time clock will show the daily rate of the former, which, if not uniform, will enable us to determine the mean daily rate; or the comparison may be made at equal intervals of two or three days.

The chronometer itself is not to be carried ashore for the purpose of comparison: a good seconds watch is to perform this office for it.

The following, from Woodhouse's Astronomy, p. 804, will serve as an illustration: the place is Cadiz:—

Days.	*Times of mean Noon.*	*Chron. too slow.*	*Differences.*
Sept. 8	$11^h\ 54^m\ 18^s{\cdot}18$	$5^m\ 41^s{\cdot}82$	
11	54 30·82	5 29·18	$-12^s{\cdot}64$
15	54 46·93	5 13·07	16·11
18	54 59·46	5 0·54	12·53
21	55 11·97	4 48·03	12·51
24	55 23·82	4 36·18	11·85
			— 65·64

Here the sum of the differences in 16 days is $65^s{\cdot}64$, and accordingly the mean daily rate, estimated by dividing the sum by the number of days, is $-4^s{\cdot}1025$.

But both the error of the chronometer on mean time at the place and its daily rate may be found without any reference to that mean time at particular instants, as the two following problems will show:—

1. *To find the Error of the Chronometer by equal Altitudes of a Star.*

The declination of a fixed star is constant,* so is the time during which the earth performs a rotation on its axis: hence, if equal altitudes of a fixed star be taken, one before and the other after its meridian passage, the meridian itself will bisect the angle at the pole between the two equal polar distances, and therefore half the time elapsed between the two observations—taken at the same place—will make known the exact time when the star was on the meridian. Now, the chronometer may surely be considered as sufficiently regular to measure the interval between the observations with the necessary accuracy, so that if the chronometer-times of the two observations be added together, and half the sum taken, the result will be the chronometer-time of the star's meridian passage.

But the R. A. of the star is the R. A. of the meridian on which it is; and if from this R. A., increased by 24^h if less than the mean sun's R. A., we subtract the latter for the preceding Greenwich noon, we shall have the mean time at the place at the instant of transit nearly, as at page 198. And applying to this the correction for longitude in time, we shall have the mean time at Greenwich nearly.

As in deducing this time the sun's R. A. for the preceding noon was employed, we can now, by means of the "Diff. for 1^h," find what correction of this R. A. is due to the time past that noon just determined, and apply it to the mean time of transit nearly, to get the more correct time, just as the like correction was applied at page 199. The difference between the time just found and the chronometer-time of transit will be the error of the chronometer on mean time at the place. The following is an example:—

* That is, it varies insensibly during the interval of time between the two observations here taken.

Observations on the Star Arcturus, Nov. 29, 1858, *in longitude* 98° 30′ *E.*

Altitudes E. and W. of Meridian.	Times shown by Chron.	Sum of Times.
43° 10′	11h 55m 47s 18 11 55	30h 7m 42s
43 30	11 57 57 18 9 45	30 7 42
43 50	12 0 7 18 7 35	30 7 42

Hence the Chronometer-time of the star's transit is 15h 3m 51s.

Arcturus R. A. Nov. 29 ...	14h 9m 13s	(to be increased by 24h, as R. A. of sun is greater.)
R. A. of mean sun at noon	16 20 48	Diff. for 1h ..... + 10s·76
		15¼
Mean Time of transit at place	21 48 25 nearly	
Long. 98° 30′ E. in time	6 34 0	5380
Mean time at Greenwich	15 14 25 nearly	1076
		269
		Cor. for 15¼h ... 164·09 = 2m 44s

Subtracting therefore this increase in the sun's R. A. for the 15h¼ past the noon, when the R. A. was as above, we have

Mean time of transit at place	21h 45m 41s	Mean time at G.	15h 11m 41s
Mean time as shown by chron.	15 3 31	....	15 3 31
Error of ch. on mean T. at place	6 42 10	Error on mean T. at G.	8 10

In taking the equal altitudes, the best mode of proceeding is this: having selected the star, which should be at a considerable distance from the meridian, that is, about three or four hours, take its altitude roughly with the sextant, then advance the index so that it may point to degrees and minutes without any fractions of a minute: suppose, as in the illustration just given, the index is advanced to 43° 10′, then waiting till the star has attained this altitude, let the time $11^h\ 55^m\ 47^s$ be noted. Now advance the index to—say 43° 30′, waiting till this altitude is reached and again note the time $11^h\ 57^m\ 57^s$. In like manner advance the index an additional 20′, and wait till the altitude 43° 50′ is attained, noting the time $12^h\ 0^m\ 7^s$, and so on till as many altitudes and times before the meridian passage have been taken as may be considered necessary.

Then without disturbing the index from its last position, wait till this last altitude is furnished by the star on the other side of the meridian, the time $18^h\ 7^m\ 35^s$ being noted and linked with the time when the equal altitude was before taken: and proceeding in this manner, moving the index 20′ the contrary way, after each observation, till we arrive at the altitude 43° 10′ at first taken, the series of observations will be completed, and the times corresponding to each pair of equal altitudes will have been noted. If the chronometer have gone uniformly during the interval between the first and last observation, the mean of the times corresponding to any pair of equal altitudes will be the same as the mean of the whole, that is, it will be the same as we should get by dividing the sum of all the times by the number of pairs, and taking half the quotient. But should there be a slight difference, the latter result is to be regarded as the chronometer-time of the star's transit.

The student will not fail to notice that this method of equal altitudes has the advantage of not requiring any corrections for the index error of the instrument, yet after the first of the altitudes, when the star has passed the meridian, is taken, the shifting the index of the sextant to its former place may not be accomplished with strict precision, it would therefore be better to take each of the altitudes, before the meridian transit, with a different sextant; to take the first altitude after the transit with the sextant last used, and the remaining altitudes with the other sextants used in reverse order. The indexes all remaining untouched, we have sufficient security that the altitudes on one side of the meridian are really equal to the corresponding altitudes on the other side, presuming the accuracy of the observations.

By the same method of equal altitudes may the time, by chronometer, of the sun's meridian passage be deduced, but on account of the sun's change of declination, in the interval of the observations, a separate computation for the influence of this change on the time becomes necessary: we think the determination of the time from a single altitude of the sun, as explained in last chapter, is to be preferred.

2. *To find the Rate of the Chronometer by equal Altitudes of the same Star, on the same side of the meridian, on different nights.*

It has already been stated (page 94) that the interval between two consecutive transits of the same fixed star over the same meridian is uniformly $23^h\ 56^m\ 4^s{\cdot}09$ of mean time: consequently the return of any fixed star to the same meridian is exactly $3^m\ 55^s{\cdot}91$ earlier at every reappearance. And on account of the strict uniformity in the diurnal motion, not only is the star thus accelerated in its return to the meridian, but equally in its return to any point in its diurnal path. It follows, therefore, that if an altitude of a star be taken, and the time by the chronometer be noted, and then after the lapse of any number of days the same altitude, on the same side of the meridian, be again taken, and the time noted—it follows that if we divide the difference of these chronometer times by the number of days, the amount by which the quotient differs from $3^m\ 55^s{\cdot}91$, will be the daily error of the chronometer. For example, June 6, 1858, at $10^h\ 30^m\ 12^s$ by chronometer, and on June 12, at $10^h\ 6^m\ 40^s$, a star on the same side of the meridian had equal altitudes: required the rate of the chronometer?

June 6	Time by Chronom.	10^h	30^m	12^s	
12	,, ,,	10	6	40	
6	Days elapsed 6)		23	32	
	Daily diff. by chron.		3	54·33	
	True daily diff.		3	55·91	
	Rate of chronom.		0	1·58	*Gaining*

It is plain that the chronometer must be gaining when the daily difference is less than it ought to be, and losing when it is greater. As in all cases of taking altitudes for the time, the nearer the object observed is to the prime vertical the better, and in the present case it is probable that a single altitude, if carefully taken, is preferable even to the mean of several altitudes. If several be taken, the altitudes must all be *read off*, and to do this without a second or two of error, is no easy matter; but in the case of a single altitude only, the reading off is unnecessary: the index should be clamped for that altitude, and the sextant left untouched till the second observation is taken, which, if practicable, should be on a night when the state of the atmosphere, as indicated by the barometer and thermometer, is nearly the same as it was on the night of the first observation. Of course here, as in the former problem, there is to be no correction for index error.

If different stars are observed, each with a different instrument, the mean of the rates, furnished by the several pairs of observations, is likely to be the more correct rate.

In the foregoing remarks and directions we have said nothing as to the choice of any particular star or stars, merely observing that, whatever star be selected, its position in the heavens should be as near to the prime vertical as possible; its altitude, however, should never be less than 10 or 12 degrees, because of the changes to which the refraction at low altitudes is subject; but it is not a matter of entire indifference which star is selected; for as the more rapid the motion of an object, the less does any small error in marking its exact position affect the time corresponding to that position, the nearer the star is to the equinoctial the better: so that when its position is in other respects favourable, that star which has the least declination should always be chosen in observations for time.

CHAPTER VI.

ON FINDING THE LONGITUDE AT SEA.

The longitude of any place on the surface of the globe is ascertained as soon as we can discover the time at that place and the time at Greenwich at the same instant, since we have only to convert the difference of the two times into degrees and minutes, reckoning 15° to the hour to effect the object. How to find the time at the place is a problem that has been sufficiently discussed in Chapter IV., and it is the office of the chronometer, when properly corrected for error and accumulated rate, to furnish the time at Greenwich. But the time at Greenwich as well as the time at the place may also be found by direct observations of the sun and moon, or of the moon and a star independently of the chronometer: that is, it can be found by what is called a *Lunar Observation.* This method of finding the Greenwich date of an observation and thence the longitude of the place where the observation was taken will be discussed in the next article: in the present we shall infer that date from the chronometer.

Longitude by Chronometer.

After what has been taught in the two preceding chapters, but little need be said here by way of explaining the principles of this method; an example will best convey the mode of proceeding, the learner bearing in mind that when the time at Greenwich is less than that at the place, the longitude is E.: when greater, the longitude is W.

August 16, 1858, in E. longitude, observations were taken of the sun, as recorded at p. 187. (Ex. 2), when the chronometer showed $6^h\ 36^m\ 40^s$ A. M. On July 14, the error of the chronometer on Greenwich mean time had been found to be $2^m\ 20^s$ fast, and its daily rate to be $3^s{\cdot}5$ gaining: required the longitude of the ship?

Aug. 15. Time by Chron.	18^h	36^m	40^s
Original error $-2^m\ 20^s$; Accumulation in 32^d, $-1\ \ 52$		-4	12
Time corrected to noon, Aug. 15	18	32	28
Correction for $18\frac{1}{2}^h$ . .			$-2{\cdot}7$
Mean time at G. Aug. 15 .	18	32	$25{\cdot}3$
Mean time at ship, Aug. 15 .	28	45	44
Longitude in Time . .	10	13	$18{\cdot}7$ E.

∴ LONGITUDE $= 153°\ 19'\ 41''$ E.

Daily rate . . .	$3^s{\cdot}5$ gaining
July 14 to Aug. 15	= 32 days
	70
	105
Accum. in 32^d . . .	$112^s = 1^m\ 52^s$

Gain in $18\frac{1}{2}^h = \dfrac{3{\cdot}5 \times 18\frac{1}{2}}{24} = 2^s{\cdot}7$

NOTE.—The mean time at the ship is found by the calculation at page 189, the sun's declination being corrected for the Greenwich time, here inferred from the *chronometer* to be $18^h\ 32^m\ 25^s{\cdot}3$. In the operation at page 188, the Greenwich time is estimated from the longitude and time *by account:* neither of which is necessary here.

In correcting the chronometer-time for error and rate, it will be observed that we have first applied the correction for the time up to the noon of Aug. 15, and have then corrected for the hours beyond this date. In strictness this is the way in which the corrections should be applied. If we had computed the gain upon $32^d\ 18^h\ 36^m\ 40^s$, we should have treated the time as if it had been accumulating at a sort of compound interest. It is true that in general this would not lead to any practical error, but if the original correction, the number of days elapsed, and the daily rate, be all considerable, there might be an error of a second or so in the Greenwich time.

BLANK FORM.—*Longitude by Chronometer.*

[Date]* Time by Chron.		$..^h\ ..^m\ ..^s$	Daily rate	$..^s$
Original error	$..^m\ ..^s$ }		Days elapsed	× . . .
Accum. in days elapsed	.. .. }	.. ..		
Time corrected to noon of Date		.. .. ..	Accum. rate	$...^s = ..^m\ ..^s$
Correction for time past noon		.. =	Daily rate × time past Noon ÷ 24^h.	
MEAN TIME AT GREENWICH		.. .. ..		

With the mean time at Greenwich thus determined, and the altitude, observed at the above chronometer-time, find now, by the proper form (pages 194 or 199), the corresponding mean time at ship: we shall then have

* The day is considered to commence at the preceding Greenwich noon, and the time shown by the chronometer is the approximate time after that noon.

Mean time at ship	..^{h} ..^{m} ..^{s}	Take the difference: if G. time is the less, the long. is E., otherwise it is W.
Mean time at Greenwich	.. .. ..	
Longitude in time	.. .. ..	∴ LONGITUDE = ..° ..′ ..″

which is E. or W. according as Greenwich time is less or greater than ship-time.

Examples for Exercise.

1. June 2 the true altitude of the sun's centre was 30° 2′, when the chronometer showed $5^h\ 1^m\ 0^s$; the latitude was 40° 5′ N. The chronometer on May 20 was 45^s slow for Greenwich time, and its rate $2^s{\cdot}1$ *losing*. The sun's declination at the time of observation was 22° 9′ 17″ N., and the corresponding equation of time was $2^m\ 31^s$, to be *subtracted* from apparent time: required the longitude of the ship?

Ans. longitude, 7° 29′ 49″ W.

2. May 19, in the afternoon, in latitude 42° 16′ N., the mean of a set of altitudes of the sun's lower limb was 43° 55′, the mean of the corresponding times by chronometer was $7^h\ 0^m\ 56^s$. On March 17, at noon, the chronometer was $1^m\ 18^s$ too fast for Greenwich mean time, and its rate was $7^s{\cdot}8$ *gaining:* the sextant had no index error, and the height of the eye was 25 feet:

Sun's Decl. G. mean noon.	*Equation of Time* (sub. from app. time).
19° 47′ 43″ N. Diff. for 1^h, + 31″·23	$3^m\ 49^s{\cdot}5$ Diff. for 1^h, — $0^s{\cdot}13$

required, the longitude of the ship?

Ans. longitude, 55° 44′ 45″ W.

3. August 20, 1858, in latitude 50° 20′ N., when the chronometer showed $2^h\ 41^m\ 12^s$, the observed altitude of the star Altair was 36° 59′ 50″ W. of the meridian; the index correction was +6′ 28″, and the height of the eye 20 feet. On Aug. 1, at noon, the chronometer was $17^m\ 45^s$ slow on Greenwich mean time, and its daily rate was $4^s{\cdot}3$ *losing;* required the longitude of the ship?

Ans. longitude, 141° 35′ 30″ E.

Longitude by Lunar Observations.

In the foregoing article we have explained how the longitude at sea may be determined by aid of the chronometer, an instrument of human contrivance, and consequently liable to those accidents and derangements to which all the constructions of man, are exposed. It is true, as we have previously shown, the errors and irregularities of the chronometer may from time to time, as suitable opportunities occur, be discovered and corrected; but such opportunities frequently offer themselves, only at wide intervals, and during these intervals the mariner has to assume that his time-keeper has uniformly maintained its rate, as last determined, and that through whatever changes of climate or fluctuations of weather he may have passed, and whatever hidden influences may have been in operation, nothing has disturbed this assumed regularity. And in truth, under ordinary circumstances he may make this assumption with safety; as far as skill and mechanical ingenuity are concerned, the chronometer may be regarded as a masterpiece of artistic construction, but of so delicate a character that the greatest care is necessary to preserve it in the condition in which it leaves the workman's hands. It is accordingly kept in an apartment by itself—the chronometer-room—out of which it is never taken during a voyage; it is imbedded in soft cushions, and, like the compass, suspended upon gimbals, so that the motion of the ship may not affect it by jerks and vibrations, and the atmosphere around it is, as far as possible, maintained, by means of lamps, at the same temperature, so that it may not suffer in its action from varying heat and cold. But notwithstanding all these precautions, it is evidently most desirable to be provided against accidental injury, and even against possible imperfections of construction; to have, in fact, some means to resort to beyond the reach of accident, and where all defect of workmanship is an absolute impossibility. Such means can be furnished only by the unerring mechanism of the skies.

The sun, moon, and stars supply to the mariner a celestial chronometer; and when all other resources fail him, he may read off his time from the dial-plate of heaven; but to decipher its indications requires some degree of scientific knowledge, and involves no inconsiderable amount of mathematical calculation: in the present article we shall investigate the theory, and exhibit the practical application, in as simple a manner as we can, of the problem of finding the time at Greenwich, and thence the longitude by the LUNAR OBSERVATIONS.

It may be well, however, in a few preliminary remarks, to convey to the learner some general notion of the leading features of this inquiry before entering upon the mathematical details.

And first we may observe that of all the heavenly bodies the moon is that whose apparent motion is the most rapid, and consequently that whose change of place in a small portion of time is most easily detected. The best way of estimating the change of place of a moving body in a given interval of time, is to measure its distance at the beginning and at the end of the interval from some object directly in the path it is describing: the further the object to which the motion is referred is situated out of this path, the less does the moving body advance towards it or recede from it in a given interval of time, and consequently the more difficult is it to estimate accurately the difference of distance when that interval is small.

Now, the immediate object of a Lunar Observation is to measure the angular distance at any instant between the moon and some known object, either directly in or very nearly in the path she is describing. The theory of the moon's motion is now so well understood, that what her distance will be from such known object at any future instant can always be predicted, and although her motion is not strictly uniform, yet it is sufficiently so, that if the distances from the object at two instants three hours apart be previously computed, her distance at any intermediate instant can be found by proportion, and conversely an intermediate distance being found by observation to have place, we can in like manner, by proportion, discover the intermediate time corresponding to that distance. Now, the distance of the moon from each of the several stars lying in or very near her path, as also her distance from the sun, are carefully computed for every three hours of every day in the year, and for several years in advance, and the results are all inserted in the Nautical Almanac; these "Lunar Distances" occupy from page XIII. to page XVIII. of every month.

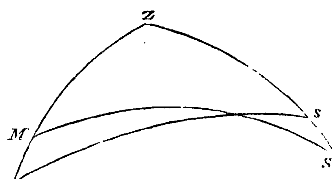
An observer at sea, wishing to know the time at Greenwich, measures with his sextant the distance of the moon either from the sun, or from one or the other of these selected stars, and after reducing the observed to the true distance, in a way hereafter to be explained, he refers to the Nautical Almanac for that distance, recorded there on the given day, which is the nearest distance preceding, in order of time, to that he has obtained, against which will be found the *hour*, Greenwich mean time, when that recorded distance had place, and he further knows that *his* distance occurred at a more advanced period of Greenwich time. To find how much more advanced, he takes the difference between the recorded distance at the hour just found, and the recorded distance at the third hour afterwards, as also the difference between *his* distance and that in the Almanac at first found: then as the former difference is to this, so is 3^h to the additional time required. But a shorter way of computing the proportional part of the time will be explained hereafter.

From this brief sketch of the course to be pursued, the learner will perceive that there is nothing laborious in finding the time at Greenwich by a Lunar Observation except the work that may be necessary for reducing the observed to the true distance; and this, it must be confessed, involves some amount of calculation. As the object is to clear the observed distance from the effects of parallax and refraction, the operation is called the problem of

Clearing the Lunar Distance.

In the annexed diagram let Z be the zenith of the place of observation, and let Z M, Z S be the two verticals on which the objects are situated at the time; *m*, *s*, the apparent places of the moon and sun, or of the moon and a star, and let M, S, be their true places.

As the moon is depressed by parallax, more than it is elevated by refraction, its true place, M, will be above its apparent place, *m*; but the sun or a star being, on the contrary, more elevated by refraction than depressed by parallax, its true place, S, will be below its apparent place, *s*.



The apparent zenith distances, Z *m*, Z *s*, are got at once, in the usual way, that is, by applying to the observed altitudes the corrections for dip, index error, and semidiameter, and subtracting each apparent altitude thus obtained from 90°: the apparent distance between the two objects, *m*, *s*, that is, the great-circle arc *m s*, is the observed distance itself: and the problem is to compute from these the *true* distance, that is the great-circle arc M S.

In the spherical triangle Z *m s* all the three sides will be given: hence the angle Z, or rather cos Z, may be found in terms of these known quantities. In the spherical triangle Z M S two of the sides, Z M, Z S, being the true co-altitudes, —obtained by applying the corrections for parallax and refraction to the apparent altitudes, and subtracting each result from 90°,—are known; so that the expression for cos Z, in the triangle Z M S, will involve the true distance M S as the only unknown quantity. Consequently, by equating the two expressions for cos Z, we shall have an equation in which M S is the only unknown, and this may therefore be determined by the ordinary operations of algebra. Let the apparent altitudes and the apparent distance be represented by the small letters a, a', and d; and the true altitudes and the true distance by the capital letters A, A′, and D: then (Spherical Trig. p. 5) we have first from the triangle Z m s, and then from the triangle Z M S,

$$\cos \mathrm{Z} = \frac{\cos d - \sin a \sin a'}{\cos a \cos a'}$$

$$\cos \mathrm{Z} = \frac{\cos \mathrm{D} - \sin \mathrm{A} \sin \mathrm{A}'}{\cos \mathrm{A} \cos \mathrm{A}'}$$

$$\therefore \frac{\cos \mathrm{D} - \sin \mathrm{A} \sin \mathrm{A}'}{\cos \mathrm{A} \cos \mathrm{A}'} = \frac{\cos d - \sin a \sin a'}{\cos a \cos a'}$$

$$\therefore \cos \mathrm{D} = (\cos d \sin a \sin a') \frac{\cos \mathrm{A} \cos \mathrm{A}'}{\cos a \cos a'} + \sin \mathrm{A} \sin \mathrm{A}'$$

$$= \left\{ \cos d + \cos (a + a') - \cos a \cos a' \right\} \frac{\cos \mathrm{A} \cos \mathrm{A}'}{\cos a \cos a'} + \sin \mathrm{A} \sin \mathrm{A}'$$

But (plane Trig. p. 30)

$$\cos d + \cos (a + a') = 2 \cos \tfrac{1}{2} \left\{ a + a' + d \right\} \cos \tfrac{1}{2} \left\{ (a + a') \sim d \right\}$$
$$= 2 \cos s \cos (s \sim d)$$

by putting s for $\frac{1}{2}(a + a' + d)$. Consequently

$$\cos \mathrm{D} = \frac{2 \cos s \cos (s \sim d) \cos \mathrm{A} \cos \mathrm{A}'}{\cos a \cos a'} - \cos \mathrm{A} \cos \mathrm{A}' + \sin \mathrm{A} \sin \mathrm{A}$$
$$= \frac{2 \cos s \cos (s \sim d) \cos \mathrm{A} \cos \mathrm{A}'}{\cos a \cos a'} - \cos (\mathrm{A} + \mathrm{A}')$$

Subtract each side of this equation from 1, then since

$$1 - \cos \mathrm{D} = 2 \sin^2 \tfrac{1}{2} \mathrm{D}, \text{ and } 1 + \cos (\mathrm{A} + \mathrm{A}') = 2 \cos^2 \tfrac{1}{2} (\mathrm{A} + \mathrm{A}')$$

we shall have, after dividing by 2,

$$\sin^2 \tfrac{1}{2} D = \cos^2 \tfrac{1}{2} (A + A') - \frac{\cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a'}$$
$$= \cos^2 \tfrac{1}{2}(A + A') \left\{ 1 - \frac{\cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a' \cos^2 \tfrac{1}{2}(A + A')} \right\}$$

Now, let the fraction within the brackets be represented by $\sin^2 C$, then the expression becomes

$$\sin^2 \tfrac{1}{2} D = \cos^2 \tfrac{1}{2} (A + A') \cos^2 C$$
$$\therefore \sin \tfrac{1}{2} D = \cos \tfrac{1}{2} (A + A') \cos C$$

Hence the formulæ for finding the true distance D are the following, namely,

$$\left. \begin{aligned} \sin C &= \sqrt{\frac{\cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a' \cos^2 \tfrac{1}{2} (A + A')}} \\ \sin \tfrac{1}{2} D &= \cos \tfrac{1}{2} (A + A') \cos C \end{aligned} \right\} \ldots . (I)$$

It may be satisfactory to the learner to mention that in assuming the fraction above to be equal to the square of some sine ($\sin^2 \tfrac{1}{2}C$), we do no more than assume that the fraction is some positive quantity less than unit. And we are justified in this assumption from the following considerations:

1. The fraction is *positive*. For every factor in the denominator is obviously positive, since neither of the altitudes can exceed 90°. Every factor in the numerator is also positive; the only one of these about which there could be any doubt is the factor $\cos s$; but to prove that $2s$ can never be so great as 180°, conceive the arc measuring the lunar distance to be extended both ways to the horizon: the arc thus completed would measure 180°, and the ends of it are cut off by the perpendiculars to the horizon—the altitudes—which are respectively *less* than those hypotenusal ends, because in a right-angled triangle, whether spherical or not, the perpendicular is less than the hypotenuse.

2. The fraction is not only positive, but it is less than unit. For if it were *equal* to unit, $\sin^2 \tfrac{1}{2} D$ would be nothing;

and if it were greater than unit, $\sin^2 \frac{1}{2} D$ would be negative, which no *square* can be.*

The preceding formulæ were first given by a French mathematician, *M. Borda.* The computation of the expression under the radical requires, we see, nothing but cosines; the result of this computation is, however, a sine, namely, sin C. In the right hand member of the other expression there occurs another cosine, cos C, and a cosine already employed; the final result being a sine. It would be as well, perhaps, to postpone the change from cosine to sine till the very last, so that in the arrangement of the work there should be no interruption to the vertical row of cosines, in which case the row of figures would terminate with two sines; that is, it might be as well to use the formulæ under the following slight change:—

$$\left.\begin{array}{l} \cos C = \sqrt{\dfrac{\cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a' \cos^2 \frac{1}{2} (A + A')}} \\ \sin \frac{1}{2} D = \cos \frac{1}{2} (A + A') \sin C \end{array}\right\} \ldots \text{(II)}$$

* These remarks should not be regarded as superfluous. In following the steps of a mathematical investigation, the learner should exercise that caution and circumspection which is often necessary to prevent too unqualified an interpretation of his symbols: for instance, in the inquiry above, he might hastily conclude, in the absence of such caution, that the formulæ arrived at conveyed a general truth in spherical trigonometry; the two spherical triangles Z M S, Z m s, being any whatever, partially superimposed, as in the diagram: the above remarks show that this would be too unqualified an inference. The author of this work himself committed a mistake of the like kind, many years ago, when writing on the present subject. Starting from the second expression for cos D above, namely from

$$\cos D = \left\{ \frac{2 \cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a' \cos (A + A')} - 1 \right\} \cos (A + A')$$

he replaced the quantity within the brackets by $2 \cos^2 C - 1$, that is by cos 2 C, and thus got the following formulæ for the true distance, namely,

$$\left.\begin{array}{l} \cos C = \sqrt{\dfrac{\cos s \cos (s \sim d) \cos A \cos A'}{\cos a \cos a' \cos (A + A')}} \\ \cos D = \cos 2 C \cos (A + A') \end{array}\right\}$$

which are true however only under the limitation that $A + A'$ is less than 90°. ("Young's Trigonometry," 1841, p. 194.)

Referring to the two expressions for cos Z at page 220, and subtracting each from 1, we have,

$$1-\cos Z=\frac{\cos a\cos a'+\sin a\sin a'-\cos d}{\cos a\cos a'}=\frac{\cos(a\sim a')-\cos d}{\cos a\cos a'}$$

$$1-\cos Z=\frac{\cos A\cos A'+\sin A\sin A'-\cos D}{\cos A\cos A'}=\frac{\cos(A\sim A')-\cos D}{\cos A\cos A'}$$

$$\therefore\ \frac{\cos(a\sim a')-\cos d}{\cos a\cos a'}=\frac{\cos(A\sim A')-\cos D}{\cos A\cos A'}$$

$$\therefore\ \cos D=\left\{\cos d-\cos(a\sim a')\right\}\frac{\cos A\cos A'}{\cos a\cos a'}+\cos(A\sim A')$$

Now we know (Plane Trig. p. 30), that,

$$\cos P+\cos Q=2\cos\tfrac{1}{2}(P+Q)\cos\tfrac{1}{2}(P\sim Q)$$

Let $\frac{1}{2}(P+Q)=A$, and $\frac{1}{2}(P\sim Q)=A'$, then we shall have,

$$P=A+A',\ \text{and}\ Q=A\sim A',\ \text{or else}\ Q=A+A',\ \text{and}\ P=A\sim A'$$

$$\therefore\ \cos(A+A')+\cos(A\sim A')=2\cos A\cos A'$$

$$\text{In like manner,}\ \cos(a+a')+\cos(a\sim a')=2\cos a\cos a'$$

Hence, substituting in the above value of cos D, we have,

$$\cos D=\left\{\cos d-\cos(a\sim a')\right\}\frac{\cos(A+A')+\cos(A\sim A')}{\cos(a+a')+\cos(a\sim a')}+\cos(A\sim A')\ldots\text{(III}$$

If, instead of subtracting, we add each side of the two expressions for cos Z to 1, we shall get, in like manner,

$$\cos D=\left\{\cos d+\cos(a+a')\right\}\frac{\cos(A+A')+\cos(A\sim A')}{\cos(a+a')+\cos(a\sim a')}-\cos(A+A')\ldots\text{(IV}$$

We shall now illustrate the use of these formulæ by an example.

Application of the Formulæ for Clearing the Distance.

1. Suppose the apparent distance of the moon's centre from a star to be 63° 35′ 14″, the apparent altitude of the moon's centre, 24° 29′ 44″, and the true altitude 25° 17′ 45″,

also the apparent altitude of the star, 45° 9′ 12″, and its true altitude 45° 8′ 15″: required the true distance?

Here $d = 63° 35' 14''$, $a = 24° 29' 44''$, $a' = 45° 9' 12''$.

$A = 25° 17' 45''$, $A' = 45° 8' 15''$

The work will be as follows:—

1. *By the Formulæ* (II).

	°	′	″		*Tab. Diff.*	*Parts.*
d	63	35	14			
a	24	29	44	Comp. cos 0·040919 . . .	96 +	4224
a'	45	9	12	Comp. cos 0·151655 . . .	212 +	2544
2)	133	14	10			+ 6768
s	66	37	5	. . . . cos 9·598660 . . .	487 —	2435
$s \sim d$	3	1	51	. . . . cos 9·999398 . . .	11 —	561
A	25	17	45	. . . . cos 9·956268 . . .	99 —	4455
A′	45	8	15	. . . . cos 9·848472 . . .	212 —	3180
A + A′	70	26	0	39·595372		—10631
				— 39 parts for secs.		— 38,63
				2)39·595333		
				19·797667		
½ (A + A′)	35	13	0	. . — cos 9·912210 +		
C	39	48	37	. . . . cos 9·885457 . . .	175)6500(37″	
					525	
C				. . . . sin 9·806254 . . .	253 1250	
				+ 94 pts. for 37″		
				9·806348 +		
∴ ½ D =	31	32	17½	. . . . sin 9·718558 . . .	343)6100(17″½	
∴ D =	63	4	35		343	
					2670	
					2401	
					269	

The minus sign is put before cos ½ (A + A′) to imply that it is to be subtracted from the quantity over it; and the plus sign is annexed to it to indicate its addition to the quantity similarly marked below.

2. *By the Formula* (IV).

d 63°	35′	14″	nat. cos 444835 +		
a 24	29	44 }			
a' 45	9	12 }			
$a+a'$ 69	38	56	nat. cos 347772 +		
			792607	. . . . .	log 5·899058
0	39	28	nat. cos 935704 +		
A 25	17	45 }	1283476		Comp. log 3·891612
A′ 45	8	15 }			
A + A′ 70	26	0	nat. cos 334903 —		
A ∾ A′ 19	50	30	nat. cos 940634		
			sum 1275537	. . . . .	log 6·105693
			787704 +	. . . .	log 5·896363
D 63	4	35	nat. cos 452801		

In the preceding operation we have not actually exhibited the parts for the seconds. As never more than two cosines are to be added together the parts for seconds should be incorporated into each at once: * but in comparing this method with the former, an estimate should be made of what is here suppressed, in reference to the extracts from the table of logarithms. (See p. 230.)

* In the Navigation Tables which are intended to accompany this work, will be found a very convenient table of natural cosines, by aid of which the trouble of correcting for seconds is scarcely worth mentioning. This table may also be found useful for other purposes. The author has before expressed his disapproval of the exuberant supply of tables with which most of the books on this subject abound. He is persuaded that a reference to a variety of tables, in one and the same operation, begets confusion and perplexity; more especially when any of these require to be modified, in every case of practice, by supplemental tables in the margin. He inclines to think that the navigator who has to work out an important problem, such as that in the text—where even a small inaccuracy is of consequence—would rather have a model to go by which should confine his attention to a single table, the use of which he is well acquainted with,

It is some advantage in this second way of finding the distance that the cosines of $a + a'$, $A + A'$ always occur in the same column, or in adjacent columns of the table: so do the cosines of $a \sim a'$, $A \sim A'$. Also the first and last logs occur in like manner at the same column or in adjacent columns, as do the two middle logs. The table of natural cosines, as given in the accompanying volume of tables, is moreover more easily employed than the table of log cosines. On these accounts some may possibly prefer the method now given. Both the methods might be abridged by the aid of special tables: but these are sometimes so perplexing, involving two or three small marginal tables of corrections, and requiring so much tact and judgment in the use, that we think the rigorous methods by the common tables are to be preferred. Indeed as a general principle the fewer the tables employed in the computations of nautical astronomy the better. Even the logarithmic portion of the foregoing work might we think be advisedly replaced by common arithmetic: the operation would then stand thus:—

even should he have to perform a few independent arithmetical operations, than have his mind perplexed by turning from table to table for the several items he is to put down ; more especially when these are not to be obtained, after all, without certain changes and very careful and vigilant scrutiny. Under this conviction, the author has here proposed a method which, besides a little arithmetic, requires reference only to one table, very easy to consult—a table of natural cosines.

It will, however, be understood that the preference here given to the arithmetical operation in next page, instead of to the logarithmic work in the last, is merely a matter of individual taste and opinion. The computor who uses the method in the text, will employ logarithms or not, as he thinks best.

d	63°	35′	14″	nat. cos 444835 +	
a	24	29	44		
a'	45	9	12		
$a+a'$	69	38	56	nat. cos 347772 +	
				792607	Multiplier (to be reversed)
$a \sim a'$	20	39	28	nat. cos 935704 +	
A	25	17	45	1283476	Divisor.
A′	45	8	15		
A + A′	70	26	0	nat. cos 334903	(*To be subtracted from quotient below*)
A ∾ A′	19	50	30	nat. cos 940634	
				1275537	Multiplicand
				706297	
				8928759	
				1147983	
				25511	
				7653	
				89	

12,8,3,4,7,6)	10109995	(787704
	8984332	334903 (*Subtract*)
	1125663	452801 nat. cos 63° 4′ 35″ = D
	1026781	
	98882	
	89843	
	9039	
	8984	
	55	

Should this nat. cos be negative, the *supplement* of the angle answering to it in the tables will be D.

An arithmetical operation like the preceding must not be judged of by the eye, in a comparison of it with a logarithmic process; in the latter the fingers are a good deal less exercised, but the mind a good deal more.

It may be proper to add here that the sign + or − annexed to any quantity, implies that that quantity is to be algebraically added to or subtracted from the next marked quantity below it, whatever the *prefixed* signs of the quantities may be. Whenever any of the cosines are negative, that is, when any of the angles exceed 90°, the negative sign is, of course, to be prefixed. The numbers whose logarithms are taken, are all regarded as positive: whether the final result belongs to a positive or negative number, is to be determined as in the common "rule of signs" in multiplication:—only an *odd* number of negative quantities can give a negative result. It may be further noticed that the cosines are all treated as whole numbers, and not as decimals.

The operation by this second method is easily expressed in a rule as follows:—

RULE *for Clearing the Apparent Distance.*

1. Write down, in order, the apparent distance, and the apparent altitudes; and take the sum and difference of the latter two.

2. Underneath, write the true altitudes; taking in like manner their sum and difference.

3. Referring now to the table of natural cosines, take out the cosine of the apparent distance, as also the cosine of each sum and difference.

4. Take the sum of the first and second cosines, then the sum of the second and third, and lastly the sum of the fourth and fifth.

5. These sums will give three *numbers.* Multiply the first and third of them together, and divide the product by the middle one, performing the operation either by logs or by common arithmetic; the result—the cosine of the sum of the true altitudes being *subtracted* from it—will be the cosine of the true distance. If this cosine be *negative* the supplement of the angle in the tables is to be taken.

NOTE.—In taking out the cosines, the best way of proceeding will be this: Having found the column headed with the degrees, take first the *seconds;* and having written the proper correction for these on a slip of paper, place this correction under the cosine answering to the minutes, and write down the result of the *subtraction.*

We shall give another example worked by this rule, and shall then sketch the blank form for each of the two methods of finding the true distance.

2. Given the apparent altitudes $a = 29° 27' 5''$, $a' = 25° 50' 51''$; the true altitudes $A = 29° 25' 30''$, $A' = 26° 41' 35''$, and the apparent distance $d = 99° 58' 58''$: required the true distance?

d	99°	58′	58″	nat. cos	— 173352 +	
a	29	27	5			
a'	25	50	51			
$a + a'$	55	17	56	nat. cos	569295 +	
					395943	Multiplier (to be reversed)
$a \sim a'$	3	36	14	nat. cos	998023 +	
A	29	25	30		1567318	Divisor
A′	26	41	35			
A + A′	56	7	5	nat. cos	557484	
A ∼ A′	2	43	55	nat. cos	998863	
					1556347	Multiplicand
					349593	
					4669041	
					1400712	
					77817	
					14007	
					622	
					47	

15,6,7,3,1,8)6162246(	393171		
4701954	557484	(*Subtract*)	
1460292	— 164313	= nat. cos	
1410586		99° 27′ 26″ = D	
49706			
47019			
2687			
1567			
1120			
1097			
23			

If instead of actually multiplying and dividing, we take the logarithms of the three numbers, the extracts from the table will be as follows:—

```
5·597586            6·195069          6·192010
      44                  83                83·7
       3                   2·77             11·16
--------                   2·21              1·95
5·597633             --------          --------
3·804843 = Comp. of 6·195157           6·192106
6·192106             --------          --------
--------
5·594582 = log   393171
     503         557484  (Subtract)
     ---         ------
 111) 79 (71  — 164313 = nat. cos 99° 27′ 26″
     777         ------
```

1ST BLANK FORM *for clearing the Lunar Distance.*

```
App. dist.        . .°  . .′  . .″ nat. cos . . . . . . +
App. alts.      { . .   . .   . . }
                { . .   . .   . . }
                 ----------------
Sum app. alts.    . .   . .   . .  nat. cos . . . . . . +
                                            -----------
                                            . . . . . . Multiplier
                                            -----------
Diff. app. alts.  . .   . .   . .  nat. cos . . . . . . +
                                            -----------
True alts.      { . .   . .   . . }         . . . . . . Divisor
                { . .   . .   . . }         -----------
                 ----------------
Sum true alts.    . .   . .   . .  nat. cos . . . . . .  (To be subtracted from
Diff. true alts.  . .   . .   . .  nat. cos . . . . . .     quotient below).
                                            -----------
                                        Sum . . . . . . Multiplicand
                                          × . . . . . .
                                            -----------
                                  Divisor) Product (Quotient
                                                  . . . . . . —
                                                  -----------
                                                  . . . . . . nat. cos of TRUE
                                                  -----------
                                                                 DISTANCE.
```

NOTE.—The "quotient" may be found as here indicated, by common multiplication and division, using the contracted methods, or by taking the logarithms of the three numbers, thus:—

Log Multiplier	. . . .
Comp. log Divisor	. . . .
Log Multiplicand	. . . .
Log Quotient	. . . . , 10 being rejected from the index.

2ND BLANK FORM *for clearing the Lunar Distance* (Borda's Method).

			Tab. Diff.	*Parts for secs.*
App. dist.	..° ..′ ..″			
App. alt.	.. .. ..	Comp. cos	. . . . +	. . . .
App. alt.	.. .. ..	Comp. cos	. . . . +	. . . .
½ sum	.. .. ..	cos	. . . . —	. . . .
½ sum ∼ app. dist.	.. .. ..	cos	. . . . —	. . . .
True alt.	.. .. ..	cos	. . . . —	. . . .
True alt.	.. .. ..	cos	. . . . —	. . . .
Sum true alts	.. .. ..	. . . .		— . . . . .
		. . . Parts for secs.		. . . . .
½ sum true alts.	.. .. ..	—cos . . . . +		
Angle C	.. .. ..	cos	. . .) . . . (. . ′	
Angle C	.. .. ..	sin	. . .	
		+ . . . Parts for secs.		
½ true distance				
TRUE DISTANCE				

By comparing the two forms the student will perceive that if in the first the multiplication and division be performed by logarithms, there will be the same number of references to tables in each: but in the first method the references are made with much greater facility, and consequently the work is completed in less time, and with less trouble: and, as both methods are equally accurate—giving the true distance to the nearest second—the first method, we think, claims the preference, on the ground of superior simplicity.

But it may be remarked, that whichever method be employed, an error of a few seconds—or of even so much as one or two minutes—in taking the altitudes, will have but very little influence on the resulting true distance, provided the observed distance be taken with accuracy. This is a valuable peculiarity; because, in preparing to take the distance, the sextant can be previously set to a division on the limb easily read off, the observer waiting till the anticipated distance has place, at the instant of which the altitudes may be taken by two other observers; and any small inaccuracy either in the readings off or in the observations themselves, will be of comparatively little consequence.

But, instead of a single observation, it is always best to take the mean of several. For this purpose, after the first anticipated distance is taken, with the corresponding altitudes, the index of the sextant can be moved a minute or two, according as the objects are approaching to or receding from each other, and another observation of the distance, with the corresponding altitudes, taken, and so on: the mean of the distances, and the means of the corresponding altitudes, are those from which the true distance is to be computed. It is of much more importance to deduce the *distance* from the mean of a set, than to so deduce the altitudes, since strict precision in the latter is not indispensable: indeed, as we have already remarked, the altitudes may be each in error to the extent of even one or two minutes, without materially affecting the result of the computation.*

* The reason of this may be explained as follows: The fraction in the formula (III), has for numerator and denominator numbers consisting of six or seven places of figures each. If the last two or three figures of each be *equally* increased or diminished, it is plain the value of the fraction cannot be materially altered; and it is equally plain that a small alteration [illegible] Again, the fraction [illegible]

It may happen, however, from the want of qualified assistants, that both distance and altitudes must be taken by the same observer. In this case, having set his sextant to the anticipated distance, shortly before this distance has place, let him take the altitude of each object, with another instrument, noting by the watch the corresponding times. Let him again observe the altitudes and times shortly after the distance is taken, having already noted the time of the distance itself. Then, by proportion, as the interval of time between the two altitudes of the same object is to the interval of time between one of those altitudes and the distance, so is the difference of the altitudes to the correction to be applied to the one altitude spoken of, to reduce it to what it would have been if taken at the instant of the distance. Of course, a mean distance, and a mean altitude of such object, can be inferred from several, as before. But the obscurity of the horizon may preclude the taking of the altitudes altogether: in this case they will have to be determined by computation. The method of computing altitudes will be explained hereafter.

Examples for Exercise.

1. The apparent distance d, the apparent altitudes a, a', and the true altitudes A, A′, are as follows, namely:—

$$d = 83^\circ\ 57'\ 33'',\ a = 27^\circ\ 34'\ 5'',\ a' = 48^\circ\ 27'\ 32''$$
$$A = 28^\circ\ 20'\ 48'',\ A' = 48^\circ\ 26'\ 49''$$

Required the true distance D? Ans. $D = 83^\circ\ 20'\ 54''$.

2. The apparent distance, the apparent altitudes, and the true altitudes are as follows, namely:—

differs from unity by a very small fraction; that is, it is equal to $1 + p$, p being very small. The formula is therefore

$$\cos d\ (1 + p) - \cos (a \backsim a') - p \cos (a \backsim a') + \cos (A \backsim A')$$

Now $\cos (a \backsim a')$ and $\cos (A \backsim A')$ have equal errors; these errors, therefore, here destroy each other, so that the only error remaining is that in $\cos (a \backsim a')$ multiplied by the very small fraction p. And similarly of form (IV).

$$d = 72^\circ\, 42'\, 20'',\ a = 20^\circ\, 13'\, 20'',\ a' = 31^\circ\, 17'\, 20''$$
$$A = 20^\circ\, 10'\, 43'',\ A' = 32^\circ\, 2'\, 14''$$

Required the true distance D? Ans. $D = 72^\circ\, 33'\, 8''$.

3. The apparent distance, the apparent altitudes, and the true altitudes, are as follows, namely:—

$$d = 56^\circ\, 56'\, 31'',\ a = 58^\circ\, 4'\, 35'',\ a' = 23^\circ\, 3'\, 4''$$
$$A = 58^\circ\, 3'\, 59'',\ A' = 23^\circ\, 51'\, 41''$$

Required the true distance D? Ans. $D = 56^\circ\, 16'\, 27''$.

4. The apparent distance, the apparent altitudes, and the true altitudes, are as follows, namely:—

$$d = 108^\circ\, 14'\, 34'',\ a = 24^\circ\, 50',\ a' = 36^\circ\, 25'$$
$$A = 25^\circ\, 41'\, 39'',\ A' = 36^\circ\, 23'\, 50''$$

Required the true distance D? Ans. $D = 107^\circ\, 32'\, 1''$.

5. The apparent distance, the apparent altitudes, and the true altitudes, are as follows, namely:—

$$d = 33^\circ\, 30'\, 21'',\ a = 28^\circ\, 24'\, 59'',\ a' = 61^\circ\, 36'\, 50''$$
$$A = 28^\circ\, 23'\, 14'',\ A' = 62^\circ\, 2'\, 0''$$

Required the true distance D? Ans. $D = 33^\circ\, 56'\, 48''$.

Determination of the Greenwich Time, and thence the Longitude, from a Lunar Distance.

As already stated (page 218), a variety of Lunar Distances are given in the Nautical Almanac for every day in the year, and for intervals of every three hours. During such an interval the motion of the moon in its path may be considered as sufficiently uniform to justify our inferring, without material error, what the distance would be on any intermediate instant, by proportion, or on the other hand, what the time would be corresponding to any intermediate distance. But it is evidently troublesome to work a proportion in which two of the terms are degrees, minutes, and seconds, and the third term hours. To save this trouble in all such proportions, Dr. Maskelyne, a former Astronomer Royal, calculated a table, called a table of *Proportional Logarithms:* it will be found in the Navigation Tables which accompany this volume:—we shall here explain the principles of its construction, and the use to be made of it.

PROPORTIONAL LOGARITHMS.—The number of seconds in 3^h is 10800, and if a be the number of seconds in any portion of time less than 3^h, then log 10800 − log a is what is to be understood by the proportional logarithm of a.

Hence, contrary to common logarithms, the greater the number a the less will be its proportional logarithm. In fact, these logarithms are analagous to what in common logarithms are called *arithmetical complements;*—the greater the log the less its arithmetical complement. As—

$$\text{Prop. log } a = \text{com. log } 10800 - \text{com. log } a = \text{com. log } \frac{10800}{a},$$

proportional logarithms are complements of the common logarithms—not to 10—but to com. log 10800. If a be actually equal to 10800, then prop. log. a = com. log $1 = 0$; just as in common logs, if a log be actually equal to 10, its complement is 0.

We thus see that a table of proportional logarithms of the numbers required is constructed by simply subtracting the common log of each number from the common log of 10800, that is, from 4·033424.

Let the difference between two consecutive lunar distances in the Nautical Almanac be D, and suppose the difference between an intermediate lunar distance determined at sea, and that of the two distances in the Almanac, which is the nearer to it, preceding, in order of time, to be d: then to find what portion (x^h) of time must be added to the time of this nearer distance to obtain the Greenwich time of the observed distance, we have the proportion,

$$D : d :: 3^h : x^h$$
$$\therefore \log x^h = \log 3^h + \log d - \log D$$
$$\therefore \log 3^h - \log x^h = \log D - \log d$$
$$= (\log 3^h - \log d) - (\log 3^h - \log D)$$

that is,

$$\text{P. } \log x^h = \text{P. } \log d - \text{P. } \log D,$$

where by x^h, d, and D, are meant the *number* of seconds in these several quantities.

The P. log D is inserted in the Nautical Almanac, between the distances there given at the beginning and end of every three hours, so that by subtracting this proportional log from P. log d, taken out of the table of proportional logarithms, the remainder will be a P. log, answering to which in the table will be found the portion of time to be *added* to the hour of the earliest distance, in order to get the Greenwich mean time of the observed distance. For example: Suppose it were required to find the Greenwich mean time at which the true distance between the moon and α Pegasi would be 41° 14′ 58″ on January 22, 1858. It appears, by inspecting the distances in the Nautical Almanac, that the time must be between *noon*, that is 0^h and 3^h; the *nearest* distance, *preceding* in order of time the given distance, is therefore the

Distance at noon	40°	29′	8″	. . . . P. log of diff.	2987 —
Given distance	41	14	58		
Difference .	0	45	50	. . . P. log	5941
Time after noon	1^h	31^m	10^s	. . . P. log	2954

But, although the moon's motion during the whole of the 3^h is sufficiently uniform to render the interval of time, thus determined by proportion, a close approximation to the true interval, yet to obtain the interval exactly, a correction for the moon's variable motion during that interval must be applied. The correction is found as follows:—

Take the difference between the P. logs against the two lies. Then with this difference, and the approximate interval, found as above, enter the short table given at p. 526 of the Almanac, and the proper correction will be found. Thus, in the example above, the P. log at noon is 2987, and the P. log at 3^h is 2936: the difference between these is 51. Turning to the table at p. 526 of the Almanac, we find opposite to $1^h\ 31^m$ (the nearest to $1^h\ 31^m\ 10^s$), and under 51, the correction 16^s; which, *added* to the approximate interval, $1^h\ 31^m\ 10^s$, because the P. logs here are *decreasing*, gives $1^h\ 31^m\ 26^s$ for the true interval from noon: hence the Greenwich mean time is $1^h\ 31^m\ 26^s$.

Proportional logarithms may be advantageously used in many other inquiries in which common proportion would else be necessary. And as in ordinary logarithms, we may always avoid subtraction by taking the complement of the P. log to 10·0000, and then rejecting this amount in the sum. For example,—

The observed altitude of a celestial object at $3^h\ 28^m\ 44^s$ was 20° 3′, and at $3^h\ 38^m\ 20^s$, the altitude was 20° 45′: what was its altitude at $3^h\ 33^m\ 47^s$?

First alt. .	. 20° 3′	Time	. $3^h\ 28^m\ 44^s$.	. . . .	. $3^h\ 28^m\ 44^s$
Second . .	. 20 45	,,	3 38 20	Time at req. alt.	3 33 47
Difference	. 0 42	Diff .	. 0 9 36	Difference .	. 0 5 3

	As $9^m\ 36^s$	Arith. comp.	P. log 8·7270
	: 5 3		P. log 1·5520
	:: 42′		P. log ·6320
	: 22′ 6″		P. log ·9110
First alt.	. 20° 3′ 0″		
	20° 25′ 6″	Altitude at $3^h\ 33^m\ 47^s$	

Having thus shown the use of proportional logarithms, we may now proceed to detail the operations necessary for obtaining the longitude by a Lunar Observation.

Longitude from a Sun-Lunar.

1. The first thing to be done is to get, either from the ship's account, or from the chronometer, the approximate

Greenwich date of the observations; by means of which the semi-diameters, horizontal parallax, declination, and equation of time, at the instant of observation, may be ascertained sufficiently near the truth for the purpose in view; for these quantities vary so little in even a long interval of time, that a considerable error in the Greenwich date can affect their value only in a very slight degree.

2. The next step in the work is, by applying the necessary corrections to the observed, to obtain the apparent, and true altitudes; and the apparent distance of the centres.

3. These preparatory operations having been performed, we shall then have data sufficient for finding both the mean time at the ship, and the mean time at Greenwich, at the instant the observations were made, as in the following examples:—

1. On February 12, 1848, at $4^h\ 16^m$ P.M. mean time, by estimation, in latitude 53° 30′ S., and longitude by account 39° 30′ E., the following lunar observation was taken:—

	Sun's L. L.	*Moon's L. L.*		*Nearest Limbs.*
Obs. alt.	29° 17′ 26″	25° 40′ 20″	Obs. dist.	99° 27′ 30″
Index cor.	—2 10	—1 10		—50
	29 15 16	25 39 10		99 26 40

The height of the eye was 20 feet: required the longitude?

Mean time at Ship, Feb. 12	$4^h\ 16^m$
Longitude E. in time	2 38
Estimated mean time at G.	1 38

Referring now to the Nautical Almanac, we take out the two semi-diameters, the sun's declination, the moon's horizontal parallax, and the equation of time—

Sun's noon declin. at G.	13° 52′ 18″ S.	Diff. for 1h,	—49″·87
Correction for 1h½	—1 15		1½ hours
Declination	13 51 3 S.	Eq. of Time.	·4987
	90	+14m 33s	2499
Polar Distance	76 8 57		—74·86

Moon's Hor. parallax at G. noon	58′ 36″	Diff. for 12h, —13″
Correction for 1h½ = ⅛ of 12h	—1″·6	Sun's semi-diam. 16′ 13″
Horizontal Parallax	58 34	Moon's semi-diam. 15 58

1. *For the Apparent and True Altitudes.*

Sun.		Moon.	
Obs. alt. L. L.	29° 15′ 16″	Obs. Alt. L. L.	25° 39′ 10″
Dip — 4′ 24″ / Semi + 16 13	+ 11 49	Dip. — 4′ 24″ / Semi + Aug. + 16 5	+ 11 41
App. alt. centre	29 27 5	App. alt. centre	25 50 51
Ref. — Parallax	— 1 35	Par. — Refraction	+ 50 44
True alt. centre	29 25 30	True Alt. centre	26 41 35

2. *For the Mean Time at Ship.*

					Tab. Diff.	*Pts. for secs.*
Alt. (sun)	29°	25′	30″			
Latitude	53	30	0	Comp. cos 0·225612		
Polar dist.	76	9	0	Comp. sin 0·012814	52 —	0
	2)159	4	30			
½ sum	79	32	15	cos 9·259268	1141 —	17115
½ sum — alt.	50	6	45	sin 9·884889	176 +	7920
				19·382583		— 91,95
				— 92		
				2)19·382491		
½ Hour angle	29	25	7	sin 9·691246		
			2			

Hour angle 58 50 14 =	3h 55m 21s	Apparent time at Ship
Equation of Time	+ 14 33	
Mean Time at Ship . .	4 9 54	

3. *For the True Distance, the Greenwich Time, and the Longitude.*

Obs. dist.	99° 26′ 40″	App. dist.	99° 58′ 58″	nat. cos	—173352 +
Sun's semi-diam.	+ 16 13	App. alts.	29 27 5		
Moon's + Aug.	+ 16 5		25 50 51		
		Sum app. alts.	55 17 56	nat. cos	569295 +
					395943 Multiplier
		Diff. app. alts.	3 36 14	nat. cos	998023 +
					1567318 Divisor
		True alts.	29° 25′ 30″		
			26 41 35		
		Sum true alts.	56 7 5	nat. cos	557484
		Diff. true alts.	2 43 55	nat. cos	998863
					1556347 Multiplicand
					×349593
				1567318)6162246(	393171
					—557484
		TRUE DISTANCE	99° 27′ 26″	nat. cos	—164313
		nce at noon (Naut. Alm.)	98 38 0	Prop. log (N. A.)	2725 —
		Difference	49 26	Prop. log	5612
		Mean time at Greenwich	1h 32m 35s	Prop. log	2887
		Mean time at ship	4 9 54		
		Longitude E. in time	2 37 19		

$2^h = 30°$
$37^m = 9°\ 15'$
$19^s = \quad 4'\ 45''$

LONGITUDE E. 39° 19′ 45″

If the *estimated* mean time at Greenwich, namely $1^h\ 38^m$, had been taken from the chronometer, we should now be able to infer from the correct Greenwich time, namely $1^h\ 32^m\ 35^s$, that the error of the chronometer on Greenwich mean time is $5^m\ 25^s$ *fast.*

NOTE.—When the estimated time at Greenwich, upon which the preparatory operations are founded, differs considerably from the true mean time at Greenwich, it will be prudent to glance at the results of those operations with a view to discovering whether this difference of time can cause any appreciable modification of them; that is to say, whether, 1st, the sun's declination requires any additional correction of consequence in reference to its influence on the time at the ship—as fully explained at page 190; and, 2nd, whether the additional correction of the moon's semi-diameter can have any sensible effect on the distance.

In the example above, the $2^m\ 35^s$, by which the estimated Greenwich time, namely, $2\frac{1}{2}$ hours, exceeds the true Greenwich time, authorises an additional correction of $-2''$ in the declination, and therefore of $+2''$ in the polar distance; this correction, however, is readily seen to have no sensible influence on the mean time at the ship. The change in the moon's semi-diameter, which diminishes only $4''$ in 12^h, is equally insensible.

It may be further remarked, that in determining the interval of time from the proportional logarithms, we have not here taken account of the correction of that interval for the moon's variable motion, which correction as noticed at page 237 is given in the Nautical Almanac. We think it right, however, to introduce it in the Blank Form to be hereafter given, as in certain cases, especially in low latitudes, it may considerably affect the longitude. The learner will, of course, remember that an error of, say $2'$, in the longitude, does not place the ship 2 *miles* out of its true position, except it be actually on the equator:—the error in *distance* would be 2 miles × cos lat., as shown at page 53; such an error in the present case would but little exceed a mile.

In the foregoing example the time at the ship has been deduced from the sun, but if this body be too near the meridian when the lunar is taken for its altitude to be safely employed for this purpose, the time must be inferred from that of the moon. Now in this case it is desirable to find the time at Greenwich before finding that at the ship, that is, to perform the operation marked (3) above before that marked (2), instead of after it, for the right ascension and declination of the moon change so rapidly that an error of but 9 or 10 minutes in the time may cause an error of so much as 4′ in the resulting longitude. It is desirable, therefore, that when time is to be computed from the moon, that the Greenwich date of the observation should be as accurate as possible: we shall give an example.

2. May 22, 1844, at 11^h^ 15^m^, estimated time, in latitude 50° 48′ N. and longitude 1° W. by account, the following lunar observation was taken, the moon being E. of the meridian:—

	Sun's L. L.		*Moon's L. L.*		*Nearest Limbs.*
Obs. alt.	57° 53′ 0″		22° 53′ 2″	Obs. dist.	56° 26′ 6″
Index cor.	+30		−20		−35
	57 53 30		22 52 42		56 25 31

height of the eye was 24 feet: required the longitude?

ı time at ship, May 21 . . .	23^h 15^m	
itude W. in time	4	
ıated mean time at Greenwich	23 19,	or 41^m before noon, May, 22.

ərring now to the Nautical Almanac, we take out the ›wing particulars, the sun's declination not being required, e from the proximity of that body to the meridian it is ɔosed to deduce the ship time from the moon.

ı's semi-diam., May 21, midnight	15′ 1″·3	Diff. for 12^h	+4″ ·4
	+4 ·15	for 41^m	·25
ı's semi-diam. at 23^h 19^m, May 21	15 5 ·4	Cor.	4 ·15
ı's Hor. par. May 21, midnight	55′ 7″·6	Diff. for 12^h	+16″·1
	+15·2	for 41^m	·9·
ı's Hor. par. at 23^h 19^m, May 21	55 22·8	Cor.	15 ·2

Sun's semi-diameter at noon, May 22, 15′ 49″.

1. *For the Apparent and True Altitudes.*

Sun.		Moon.	
Obs. alt. L. L.	57° 58′ 30″	Obs. alt. L. L.	22° 52′ 42″
Dip — 4′ 49″ } Semi. + 15 49 }	+ 11 0	Dip. — 4′ 49″ } Semi. + 15 5·4 } Augment. 5·5 }	+ 10 22
App. alt. centre	58 4 30	App. alt. centre	23 3 4
Ref. — Parallax	— 31	Parallax — Ref.	+ 48 41
True alt. centre	58 8 59	True alt. centre	23 51 45

2. *For the True Distance and Time at Greenwich.*

Obs. dist. 56° 25′ 31″ } App. dist. 56° 56′ 31″ nat. cos 545489 +
Sun's semi. + 15 49 } App. alts. { 58 4 30
Moon's + Aug. + 15 11 } { 23 3 4

Sum 81 7 34 nat. cos 154260 +

699749 Multiplier.

alts. { 58° 3′ 59″ Difference 35 1 26 nat. cos 818912 +
{ 23 51 45

973172 Divisor.

81 55 44 nat. cos 140402
·ence 34 12 14 nat. cos 827043

967445 Multiplicand
947996

5804670
870701
87070
6772
387
87

9,7,3 1,7,2)6769687(695631
5839032 — 140402

930655 555229 = nat cos 56° 16′ 25″
875855

54800
48659

6141
5839

302
292

10

True distance	56°	16′	25″		
Dist. at 21h (Naut. Alm.)	55	15	36	P. L. of Diff.	3221 —
	1	0	49	. . P. L.	4712
Interval after 21h	2h	7m	42s	. . P. L.	1491
∴ Mean time at Greenwich	23h	7m	42s	May 21	

Having thus got the correct mean time at Greenwich when the lunar distance was taken, we can now deduce the right ascension and declination of the moon with greater precision, as follows:—

Mean Sun's R. A.		*Moon's R. A.*		*Moon's Declin.*	
Noon, 21st	3h 56m 53s·8	23h	7h 55m 24s	23h	17° 5′ 12″ N.
Cor. for 23h 7m 42s	+ 3 48*	Cor. for 7m 42s	+ 16	Cor. for 7m 42s	— 1 2
Sun's R. A.	4 0 42	Moon's R. A.	7 55 40	Moon's Dec.	17 4 10 N.
					90
				POLAR DISTANCE	72 55 50

3. *For the Mean Time at Ship.*

					Tab. Diff.	*Parts for secs.*
Altitude	23°	51′	45″			
Latitude	50	48	0	Comp. cos 0·199263		
Polar dist.	72	55	50	Comp. sin 0·019597	65 —	3250
	2)147	35	35			
½ sum	73	47	47½	cos 9·446025	724 —	34390
½ sum — alt.	49	56	2½	sin 9·883829	177 +	443
				19·548714		— 371,97
				— 372		
				2)19·548342		
½ Hour angle	36	28	44	sin 9·774171		

∴ Hour angle 72 57 28, or	4h	51m	50s	E. of meridian
Moon's R. A.	7	55	40	
R. A. of meridian	3	3	50	
Mean Sun's R. A.	4	0	42	
Mean time at ship, May 22	0	56	52	before noon
or May 21	23	3	8	
Mean time at Greenwich	23	7	42	
Longitude W. in time	0	4	34	
∴ LONGITUDE W.,	1°	8′	30″	

If in this example the mean time at ship had been found before the mean time at Greenwich, the resulting longitude would have been about 4′ in error.

We shall conclude these illustrations with one more example.

3. September 2, 1858, at 4^h 50^m 11^s, as shown by the chronometer, in latitude 21° 30′ N., the following lunar observation was taken, the height of the eye being 24 feet:—

	Obs. alt. Sun's L. L.	*Obs. alt. Moon's L. L.*	*Obs. dist. N. L.*
	58° 40′ 30″	32° 52′ 20″	65° 32′ 10″
Index cor.	+2 10	+3 40	−1 10

Required the longitude?

Sun's Noon Declin. at G.	7° 56′ 46″·5 N.	Diff. for 1^h	−54″·96
Cor. for 4^h 50^m	−4 26		5
Declination	7 52 21	for 5^h	27480
	90	for 10^m	916
POLAR DISTANCE	82 7 39		6,0)26,5·64
			−4′ 26″

Sun's semi-diam.	15′ 53″·8	Moon's semi-diam.	16′ 17″
Equa. of time	25^s·35	Diff. for 1^h	+0″·796
Cor. for 4^h 50^m	3.85		5
EQUA. OF TIME corrected	29·2 *Sub.*	for 5^h	3980
		for 10^m	133
			+3·847

Moon's Hor. Parallax	59′ 35″·1	Diff. for 12^h	+5″·7
Cor. for 5^h	2″	for 5^h	+2″
HOR. PAR. corrected . .	59 37		

minutes and seconds may be easily obtained. But there is a table for furnishing this difference in the Nautical Almanac, page 530.

The difference between the Moon's R. A. at 23^h, and at the following noon, is (by Naut. Alm.) + 2^m 5^s, the proportional part of which, for 7^m 42^s is + 16^s. Also, the difference between the two declinations is — 8′ 1″, the proportional part of which for 7^m 42^s is — 1′ 2″.

1. *For the Apparent and True Altitudes.*

SUN.		MOON.	
Obs. alt. L. L.	58° 42′ 40″	Obs. alt. L. L.	32° 56′ 0″
Dip − 4′ 49″ } Semi. + 15 54 }	+ 11 5	Dip − 4′ 49″ } Semi. + 16 17 } Aug. + 9 }	+ 11 37
App. alt.	58 53 45	App. alt.	33 7 37
Ref. − Par.	− 30	Cor. of alt.	+ 48 26
True alt.	58 53 15	True alt.	33 56 3

2. *For the Mean Time at Ship.*

					Tab. Diff.	*Parts for secs.*
Sun's alt.	58°	53′	15″			
Latitude	21	30	0	Comp. cos 0·031322		
Polar dist.	82	7	39	Comp. sin 0·004124	29 −	1131
	2)162	30	54			
½ sum	81	15	27	cos 9·182196	1369 −	36963
½ sum − alt.	22	22	12	sin 9·580392	511 +	6132
				18·798034		− 319,62
				− 320		
				2)18·797714		
½ Hour angle	14	30	31½	sin 9·398857		
			2			
Hour angle	29	1	3	or 1^h 56^m 4^s Apparent time at ship		

Equation of time			− 29
MEAN TIME AT SHIP	1	55	35

[The hour-angle deduced above is rather small—too small for the ship time derived from it to be depended upon as accurate, except in particular circumstances. But, as noticed at p. 196, when, as in the present example, the place of observation is between the tropics, and the declination is of the same name as the latitude, the hour-angle may be much smaller than under other circumstances, without affecting the accuracy of the result. When the sun's hour-angle exceeds 2^h, as in general it should, it may be found by Table XVIII of the Mathematical Tables, from twice the

3. For the True Distance, the G. Time, and the Longitude.

Obs. dist.	65°	31′	0″	App. dist.	66°	3′	20″	nat. cos	405850		
Sun's semi.	+ 15	54		App. alts.	58	53	45				
Moon's + Aug.	+ 16	26			33	7	37				
				Sum	92	1	22	nat. cos —	035297 +		
									370553	Multiplier.	
True alts.	58°	53′	15″	Diff.	25	46	8	nat. cos	900556 +		
	33	56	3						865259	Divisor.	
Sum	92	49	18	nat. cos —	049228						
Diff.	24	57	12	nat. cos	906652						

857424 Multiplicand.
355073

2572272
600197
4287
429
26

8,6,5,2,5,9)3177211(367198
2595777 + 049228

581434   416426 nat. cos 65° 23′ 27″
519155

62279
60568

1711
865

846
779

67

True distance	65°	23′	27″		
Dist. at 3^h (N. A.)	66	24	23	P. L. of Diff.	2537 —
	1	0	56	. . . P. L.	4704
Interval of time	1^h	49^m	18^s	. . . P. L.	2167
Correction p. 526 (N.A.)			+ 1		
Mean time at Green.	3^h + 1	49	19		
Mean time at ship	1	55	35		
Longitude W. in time	2	53	44	∴ Longitude 43° 26′ W.	

And the error of the chronometer is 52^s *fast* on Greenwich mean time.

It now merely remains for us to give the blank form for a sun-lunar.

BLANK FORM.—*Longitude by Sun-Lunar.*

Estimated mean time at ship	..h ..m	
Estimated longitude in time	.. ..	(— for E. and + for W.)
Greenwich date	.. ..	(May be had from Chron.)

Sun's noon declin. at G.	..° ..′ ..″	Diff. for 1h	..″
Cor. for time past G. noon	.. ..	× ..	Hours past noon
Declin at G. date	.. .. ..		
	90	6,0).....	
POLAR DISTANCE	.. .. ..	..′ ..″	Cor of declin.
Sun's semi-diameter	..′ ..″	Moon's semi-diameter	..′ ..″
Equa. of time (p. I., N.A.)	..m ..s	Diff. for 1h	..″
Cor. for time past G. noon	..	× ..	Hours past noon
EQUA. OF TIME AT G. DATE	.. ..		
		..″	Cor. of Eq. of time
Moon's Hor. Parallax	..′ ..″	Diff. for 12h	..″
Cor. for time past G. noon	..	× ..	Hours past noon
HOR. PAR. AT G. DATE	.. ..		
		12)...	
		..″	Cor. of Hor. Par.

1. *For the Apparent and True Altitudes.**

SUN.	MOON.
See Blank Form, p. 113.	See Blank Form, p. 126.

2. *For the Mean Time at Ship, from the Sun's Alt.*

See Blank Form, p. 195.

* The blank forms for these it is scarcely necessary, at this stage of the learner's progress, even to refer to: the operations for deriving the apparent and true altitudes, whether of the sun or of the moon, from the observed altitude, are of such frequent recurrence, and, moreover, are so simple and obvious, that there can be no necessity to consult a form for them in working out the present problem.

3. *For the True Distance.*

Correct the observed distance for the two semi-diameters, taking account of the augmentation of the moon's semi-diameter, the same as in step 1; the result will be the apparent distance, with which and the apparent altitudes proceed as in the Form at p. 230 to find the true distance.

4. *For the Longitude.*

True distance	..° ..′ ..″		
Next earlier dist. (Naut. Alm.)	.. .. ..	P. L. of diff.	Diff. from next P. L. ..
	. .. ..	P. L.	With this diff. and t interval of time, find t
Interval of time	.h ..m ..s	P. L.	correction of that inter. in the Table at p. 526 N.
Cor. p. 526 Naut. Alm.	..		
True interval of time	.h ..m ..s *after* time of earlier dist. in Naut. Alm.		
Time of earlier dist.	+..		
Mean time at G.	.. .. ..		
Mean time at ship	.. .. ..		
Longitude in time	.. .. .. ∴ LONGITUDE ..° ..′ ..″		

NOTE.—When from the sun being too near the meridian, or from any other cause, the time at the ship must be deduced from the altitude of the moon instead of from that of the sun, then the true distance, and thence the mean time at Greenwich, should be obtained *before* the mean time at ship is computed, as in Example 2. The Blank Form for determining the time from the moon's altitude is the following:—

Time at Ship from the Moon's Altitude, and Time at Greenwich.

Mean Sun's R. A.		Moon's R. A.		Moon's Declin.	
R. A. at G. noon	..h ..m ..s	R. A. at the hour	..h ..m ..s	At the hour	..° ..′ .
Cor. for time past noon	.. ..	Cor. for min. and sec.	..	Cor.	.. .
R. A. at G. date	.. .. ..	R. A. at G. date	.. .. ..	Dec. G. date	.. .. .
					90
		POLAR DISTANCE			.. .. .

3. *For the Mean Time at Ship.*

				Tab. Diff.	*Pts. for secs.*
True alt.	. .° . .′ . .″				
Latitude	. . . . . .	Comp. cos	. . . .	. . . +	. . . .
Polar dist.	. . . . . .	Comp. sin	. . . .	. . . —	. . . .
	2).				
½ sum	. . . . . .	cos	. . . .	. . . —	. . . .
½ sum — alt.	. . . . . .	sin	. . . .	. . . +	. . . .
			. . . .	Cor. for secs.	. . . . .
		Cor. for secs.	. . .		
			2).		
½ Hour angle	. .° . .′ . .″	sin	. . . . . .		
∴ Hour angle	. . . . . ., or in time	. .h . .m . .s			

Moon's R. A.	. . . . . .	
R. A. of meridian	. . . . . .	(Sum if W., Diff. if E. of Merid.)
Mean sun's R. A.	. . . . . .	
Mean time at ship	. . . . . .	
Mean time at Green.	. . . . . .	
Longitude in time	. . . . . .	∴ Long. . .° . .′ . .″

NOTE.—In the work for clearing the observed distance from the effects of parallax and refraction, the cosines, although all decimals, may always be treated as whole numbers, as in the examples already exhibited. It may sometimes happen that when the cosines of the apparent distance, and the sum of the apparent altitudes, have contrary signs, they may be so nearly equal that their algebraic sum (in this case their numerical difference) may have a 0 in the place of the leading figure. It is best always to actually insert this 0 in the resulting multiplier; and in employing the multiplier, as such, to put the 0 in the unit's place, just as we should do if it were a significant figure, commencing the work of multiplication, however, with the *next* figure, rejecting, as in all other cases, the unit's figure of the multiplicand. It will generally be found that in the subsequent division the first place of the quotient will also be a 0, and that a significant figure can be given only after cutting off the unit's figure of the divisor. Attention to these particulars is necessary in order to avoid writing the figures of the quotient each a place to the left before its true place.

These remarks are not to be regarded as pointing to any distinction of cases, because there is no such distinction: all that the computer has to bear in mind is, that each of the three numbers, multiplier, divisor, and multiplicand, is to consist of at least *six* places; which number of places is not to be diminished by suppressing leading zeros, unless, indeed, the operations with these numbers be performed by logarithms, when the leading zeros are, of course, to be rejected.

Of the two factors marked multiplier and multiplicand, either may, of course, be placed under the other. It sometimes happens that the right-hand places of the latter are occupied by zeros: when such is the case, it will be better to make *it* the multiplier, and the other factor the multiplicand; for in reversing this multiplier the zeros have no influence. We shall now give an example or two for exercise.

Examples for Exercise: Longitude from Sun-Lunar.

1. January 21, 1858, at about 11ʰ A.M. estimated mean time, in latitude 40° 16′ S., and longitude by account 106° 30′ E., the following lunar was taken:—

Obs. alt. Sun's L. L.		*Obs. alt. Moon's U. L.*		*Obs. dist. N. L.*	
	68° 17′		16° 9′ 36″		70° 27′ 20″
Index cor.	+ 2′	Index cor.	+ 4″	Index cor.	— 2′ 15″

The height of the eye was 17 feet: required the longitude to the nearest minute? Ans. longitude 105° 44′ E.

2. May 18, 1858, at 4ʰ 30ᵐ P.M. mean time by estimation, in

latitude 14° 20′ N., and longitude by account 58° 30′ E., the following lunar was taken:—

Obs. alt. Sun's L. L.		*Obs. alt. Moon's L. L.*		*Obs. dist. N. L.*	
	83° 28′ 20″		69° 18′ 10″		71° 36′ 40″
Index cor.	—4′ 10″	Index cor.	+2′ 20″	Index cor.	—1′ 30″

The height of the eye was 24 feet: required the longitude to the nearest minute? Ans. longitude 56° 50′ E.

3. September 11, 1858, when the chronometer showed $4^h\ 30^m$ Greenwich mean time, in latitude 48° 38′ 7″ N., the following lunar was taken:—

Obs. alt. Sun's L. L.		*Obs. alt. Moon's L. L.*		*Obs. dist. N. L.*
	52° 8′		28° 14′ 10″	50° 33′ 28″
Index cor.	+2′	Index cor.	—2′ 10″	—1′ 5″

The height of the eye was 24 feet: required the longitude, and the error of the chronometer on Greenwich mean time?

Ans. longitude 39° 15′ W.: error of chron. $6^m\ 56^s$ *fast.*

Longitude by a Star-Lunar.

When the observed distance is that between the moon and a fixed star, instead of between the moon and the sun, the computations for the ship's time become a little modified. In the case of a fixed star, we have nothing to do with either parallax or semi-diameter, nor does the declination, as given in the Nautical Almanac for the day of observation, require any correction to adapt it to the instant when that observation is made. But whenever *time* is to be deduced from any celestial object other than the sun, Right Ascensions must always enter into the work. The star's hour-angle at the instant of observation is obtained exactly as the sun's hour-angle is obtained, but the former, in itself, can give us no information as to the time, which is necessarily the hour-angle of the mean sun at that instant, and which in astronomical reckoning is always the time past the preceding noon.

Now, without any direct observations on the sun, this hour-angle at once becomes known, provided we know the R. A. of the sun and the R. A. of the meridian at the instant referred to. The R. A. of the meridian is obtained from that of the star and the star's hour-angle at the instant: If the star be to the E. of the meridian, its R. A. (or this + 24^h) diminished by its hour-angle is the R. A. of the meridian; and if it be to the W. of the meridian, its R. A. increased by its hour-angle—or the excess of the sum above 24^h—is the R. A. of the meridian.

The R. A. of the meridian being thus obtained, we have only to subtract from it (increased by 24^h, if necessary for this purpose) the R. A. of the mean sun in order to get the mean sun's hour-angle from preceding noon,—that is, the mean time after that noon. These matters, however, have been sufficiently dwelt upon in Chapter IV, and after what has been done in the preceding article the student can require no additional instructions to render the following work of a star-lunar intelligible.

Examples: Star-Lunar.

1. August 7, 1858, at about half-past 3 o'clock in the morning, in latitude 49° 40′ N., and longitude by account 61° 30′ W., the following star-lunar was taken:—

Obs. alt. Aldebaran E. of Meridian.		*Obs. alt. Moon's L. L.*		*Obs. dist. N. L.*	
	32° 17′ 10″		32° 24′ 40″		41° 27′ 50″
Index cor.	— 2 18	Index cor.	+ 2 10	Index cor.	— 3 20

The height of the eye was 20 feet: required the longitude?

Time at ship, Aug. 6	15h	30m
Longitude W. in time	4	6
Estimated mean time at G.	19	36

Moon's semi-diameter 16′ 33″ Horizontal parallax 60′ 35″

Mean sun's R. A., noon, Aug. 6 .	8h	58m	55s·62	Hourly diff.		+ 9s·856
Correction for 19h 36m	+ 3		13			19
Mean sun's R. A. at est. time .	9	2	9			88704
						9856
Star's R. A.	4h	27m	48s	Cor. for 19h		187·264
Star's declination	16°	13′	26″	for 30m		4·928
	90			for 6m		·986
Polar distance .	73	46	34			193s = 3m 13s*

1. *For the Apparent and True Altitudes.*

STAR.

Obs. alt.	32°	14′	52″
Dip		—4	24
App. alt.	32	10	28
Refraction		—1	32
True alt.	32	8	56

MOON.

Obs. alt.			32°	26′	50″
Dip	—4′ 24″	}			
Semi.	+16 33	}		+12	19
Aug.	+10	}			
App. alt.			32	39	9
Cor. of alt.				+ 49	29
True alt.			33	28	38

* This correction may be obtained from the Table given at page 530 of the Nautical Almanac.

The Table here referred to shows by how much the mean sun's right ascension is increased in a given interval of mean time. In the above example the quantities taken out of the Table would be the following:—

Increase of R. A. in 19h	3m 7s·273
,, ,, in 36m	5 ·914
,, ,, in 19h 36m . . .	3m 13s

In general the correction for the increase of the mean sun's R. A., due to the time past Greenwich noon, may be more expeditiously found by help of this Table than by working for it as above.

The hourly difference of the R. A., to *four* places of decimals, as given in the Nautical Almanac, is 9s·8565: if the additional decimal be annexed to those in the text, the decimals in the correction for 19h will agree with

2. *For the Mean Time at Ship.*

Alt. (Star)	32°	8′	56″			
Latitude	49	40	0	Comp. cos 0·188939		
Polar dist.	73	46	34	Comp. sin 0·017669	61 —	2074
	2)155	35	30			
½ sum	77	47	45	cos 9·325534	973 —	43785
½ sum — alt.	45	38	49	sin 9·854233	206 +	10094
				19·386375		— 357,65
				— 358		
				2)19·386017		
	29°	33′	0″	sin 9·693009		
			2			
Star's hour-angle	59	6	0, or 3^h 56^m 24^s E. of meridian			

Star's R. A.	4	27	48	
R. A. of meridian	0	31	24	to be increased by 24^h
R. A. of mean sun	9	2	9	
Mean time at ship	15	29	15	

[As noticed at page 246, the hour-angle determined above may be otherwise expeditiously found, so soon as the result 19·386017 is obtained, by entering Table XVIII (Mathematical Tables) with 9·386017: thus—

Given number	9·386017	
Tab. numb. next less	9·384678	3^h 56^m
Tab. difference 5568)	133900*(24ˢ	
	11136	
	22540	

Hence, the hour-angle in time is 3^h 56^m 24^s, as above. And in this manner may the hour-angle in time be always determined.]

* Two zeros are always to be annexed to the remainder.

3. *For the True Distance, the G. Time and the Longitude.*

Obs. dist.	41°	24′	30″	App. dist.	41°	41′	13″	nat. cos 746791	
Moon's semi. + Aug.	+ 16	43		App. alts.	32	10	28		
					32	39	9		
				Sum	64	49	37	nat. cos 425354 +	
								1172145	Mult.
True alts.	32°	8′	56″	Difference	0	28	41	nat. cos 999962 +	
	33	28	38					1425316	Div.
Sum	65	37	34	nat. cos 412690					
Difference	1	19	42	nat. cos 999730					

1412420 Multiplicand.
5412711

1412420
141242
98869
2825
141
56
7

1,4,2,5,3,1,6)1655560(1161539
1425316 — 412690

230244 748849 nat. cos. 41° 30′ 33″
142532

True dist.	41°	30′	33″		87712
Dist. at 18^h	40	36	47	P. L. diff. 2254	85519
	0	53	46	. . . P. L. 5248	2193
Interval	1^h	30^m	20^s	. . . P. L. 2994	1425
Correction (N. A.)*		+4			768
True interval	1^h	30^m	24^s	after 18^h	712
	18				56
					43
Mean time at G.	19	30	24		
Mean time at ship	15	29	15		13
Long. W. in time	4	1	9	∴ LONGITUDE 60° 17′ 15″ W.	

* As in former cases, this correction is got from the Table at page 526 of the Nautical Almanac. The difference between the P. L. for 18^h and that for 21^h is 13, the P. logs being *decreasing;* and under this difference in the Table, and against the interval 1^h 30^m, we find the correction + 4.

2. September 18, in latitude 28° 45′ 11″ S., when the chronometer showed 5h 12m 30s Greenwich mean time, the following star-lunar was taken :—

Obs. alt. Antares W. of Meridian.		*Obs. alt. Moon's L. L.*		*Obs. dist. N. L.*	
	52° 18′ 40″		41° 36′ 10″		56° 7′ 40″
Index cor.	— 3 10	Index cor.	+4 20	Index cor.	+4 50

Required the longitude, and the error of the chronometer, the height of the eye being 20 feet?

Mean time at G. by chronometer 5h 12m 30s
Moon's semi-diameter 14′ 58″ Horizontal parallax 54′ 44″

Mean sun's R. A., noon, Sept. 18 . . .	11h 48m 27s·47
Correction for 5h 12m·5 (N. A., p. 530)* .	+ 51·34
Mean sun's R. A. at G. time by chron.	11 49 19
Star's R. A.	16h 20m 45s
Star's Declination	26° 7′ 2″ S.
	90
Polar distance	63 52 58

1. *For the Apparent and True Altitudes.*

Star.		Moon.		
Obs. alt. . . .	52° 15′ 30″	Obs. alt. . . .		41° 40′ 30″
Dip	—4 24	Dip.	— 4′ 24″	
App. alt. . . .	52 11 6	Semi.	+14 58	+10 44
Refraction . .	—45	Augm.	+10	
True alt. . . .	52 10 21	App. alt. . . .		41 51 14
		Cor. of alt. . .		+39 41
		True alt. . . .		42 30 55

* At the page of the Nautical Almanac here referred to, we have—

Correction for 5h	49s·2824
12m	1 ·9713
30s	·0821
Cor. for 5h 12m 30s	51 ·3358

2. *For the Mean Time at Ship.*

Star's alt.	52°	10′	21″			
Latitude	28	45	11	Comp. cos 0·057136	115 +	1265
Polar dist.	63	52	58	Comp. sin 0·046834	103 −	5974
	2)144	48	30			
½ sum	72	24	15	cos 9·480539	664 −	9960
½ sum — alt.	20	13	54	sin 9·538538	571 +	30834
				19·123047		+ 161,65
				+ 162		
				2)19·123209*		
	21	22	19	sin 9·561604		
			2			
Star's hour-angle	42	44	38	or 2^h 50^m 59^s W. of meridian.		

Star's R. A.	16	20	45
R. A. of meridian	19	11	44
R. A. of mean sun	11	49	19
Mean time at ship	7	22	25

If, stopping here, we enter Table XVIII of the Mathematical Tables with the number 9·123209, we shall get the hour-angle as at page 255, thus—

Given number . . .	9·123209	
Tab. numb. next less . .	9·118468 . . .	2^h 50^m
Tab. difference 8095)	474100(59^s	
	40475	
	69350	

Hence the hour-angle is 2^h 50^m 59^s, as above, fractions of a second being disregarded.

3. *For the True Distance, the G. Time, and the Longitude.*

Obs. dist.	56h 12m 30″	App. dist. 56° 27′ 38″ nat. cos				552510
Moon's semi. + Aug.	+ 15 8	App. alts. { 52 11 6 / 41 51 14				
		Sum 94 2 20 nat. cos				— 070434 +
						482076 Mult.
True alts. { 52° 10′ 21″ / 42 30 55		Difference 10 19 52 nat. cos				983786 +
						913352 Div.
Sum	94 41 16		nat. cos — 081726			
Difference	9 39 26		nat. cos 985830			
			904104 Multiplicand			
			670284			
			3616416			
			723283			
			18082			
			633			
			54			
			9,1,3,3,5,2)4358468(	477195		
			3653408 +	081726		
			705060	558921 nat. cos 56° 1′ 8		
			639346			
True dist.	56° 1′ 8″		65714			
Dist. at 3h	54 46 8	P. L. of diff. 2965	63935			
	1 15 0	. . . P. L. 3802	1779			
			913			
Interval of time	2h 27m 26s	. . . P. L. 0837	866			
Corr. (N. A. p. 526)	2		822			
True interval	2h 27m 28s after 3h		44			
	3					
Mean time at G.	5h 27m 28s					
Mean time at ship	7 22 25					
Long. E. in time	1 54 57	∴ LONGITUDE 28° 44′ 15″ E.				
Mean t. at G. by ch.	5 12 30					
Error of chron.	0 14 58	Slow on Greenwich mean time.				

As this error is considerable, it will be proper to ascertain, and allow for, its influence on the Sidereal Time, or the R. A. of the mean sun. By turning to page 530 of the Nautical Almanac, we find that the correction—or the acceleration of R. A.—for 14m 58s is + 2s: hence the true mean time at ship is $7^h\ 22^m\ 23^s$, and consequently the true longitude is 28° 43′ 45″ E.

If the work of the multiplication and division be performed by logarithms, instead of by common arithmetic, as above, the operation will be as at p. 230, or as follows:—

			Prop. Parts.	
482076	.... log	5·683047	63	
904104	.... log	5·956216	5·4	= 70
			1·88	
913352	Arith. comp. log	4·039387	23·5	= — 24
		46	·94	
477195	.... log	5·678696		+ 46

Blank Form.—*Longitude by Star-Lunar.*

Estimated mean time at ship	. .^{h} . .^{m}	
Estimated longitude in time	. . . .	(— for E. and + for W.)
Estimated Greenwich date	. . . .	(May be had from Chron.)

At G. date : moon's semi-diameter . .′ . .″ Hor. parallax . .′ . .″

Mean sun's R. A., or sidereal time at G. noon	. .^{h} . .^{m} . .^{s}
Cor. for G. time past noon (Naut. Alm. p. 530)	+ . . . .
Mean sun's R. A. at Greenwich date	. . . . . .

Star's R. A.	. .^{h} . .^{m} . .^{s}
Star's declination	. .° . .′ . .″
	90
Polar distance	. . . . . .

1. *For the Apparent and True Altitudes.*

Star.		Moon.		
Obs. alt.	. .° . .′ . .″	Obs. alt.		. .° . .′ . .″
Dip	—	Dip	— . .′ . .″	
		Semi.	. . . .	. . . .
App. alt.	. . . . . .	Augm.	. .	
Refraction	—	App. alt.		. . . . . .
		Cor. of alt.		+ . . . .
True alt.	. . . . . .	True alt.		. . . . . .

2. *For the Mean Time at Ship from Star's Altitude.*

			Tab. Diff.	Parts for secs.
Star's alt.	..° ..′ ..″			
Latitude	.. .. ..	Comp. cos	... +	....
Polar dist.	.. .. ..	Comp. sin	... —	....
	2).			
½ sum	.. .. ..	cos	... —	....
½ sum — alt.	.. .. ..	sin	... +	....
		....		......
		... Cor. for secs.		
		2).......		
	.. .. .. / 2	sin		

Star's Hour angle .. .. .. or ..ʰ ..ᵐ ..ˢ (— if E. or + if W. of meridian)

Star's R. A. .. .. ..

R. A. of meridian .. .. .. (to be increased by 24ʰ, if less than R. A. of sun)

R. A. of mean sun .. .. .. (to be subtracted from R. A. of mer.)

Mean time at ship .. .. ..

3. *For the True Distance.*

Obs. dist. ..° ..′ ..″ } App. dist. ..° ..′ ..″ nat. cos ......

Moon's semi. + Aug. .. .. } App. alts. { .. .. .. / .. .. ..

Sum .. .. .. nat. cos ...... +

...... Mult.

Difference .. .. .. nat. cos ...... +

...... Div.

True alts. { ..° ..′ ..″ / .. .. ..

Sum .. .. .. nat. cos ......

Difference .. .. .. nat. cos ......

...... Multiplicand

...... Multiplier reversed

Divisor)Product(Quotient.

...... = nat. cos sum of true alts. (Sub.)

Remainder = nat. cos true distance.

4. *For the Greenwich Mean Time and Longitude.*

True distance	..° ..′ ..″		*Diff.**
Next preceding dist. (Naut. Alm.)	.. .. ..	P. L. of diff.	..
	. .. ..	P. L.	
Interval of time	.^{h} ..^{m} ..^{s}	P. L.	
Correction (Naut. Alm. p. 526)	..		
True interval	. .. ..		
Time of preceding dist.	..		
Mean time at Greenwich	.. .. ..		
Mean time at ship	.. .. ..		
Longitude in time	.. .. ..	∴ LONGITUDE ..° ..′ ..″	

The difference between the above mean time at Greenwich, and the time shown by the chronometer, will be the error of the chronometer on Greenwich mean time at the instant of observation. If the error have been found at any previous instant, the difference of the errors will be the accumulated rate during the interval; and this divided by the number of days in that interval will be the daily rate.

Examples for Exercise: Longitude by Star-Lunar.

1. August 5, 1858, in latitude 24° 18′ N., and longitude by account 11° 15′ E., the mean time at ship per watch being 11^h 15^m P.M., the following star-lunar was taken:—

a Pegasi E. of Meridian.		*Moon's L. L.*		*Dist. N. L.*	
Obs. alt.	46° 35′ 0″	Obs. alt.	56° 26′ 10″	Obs. dist.	94° 32′ 10″
Index cor.	+ 1 30	Index cor.	— 2 0	Index cor.	+ 4 10

The height of the eye was 24 feet: required the error of the watch on ship mean time, and the longitude?

Ans. error of watch 18^m 22^s *fast*;
longitude 11° 9′ 15″ E.

* This is the difference between the P. L. taken from the Nautical Almanac, and the P. L. next following; it is required, in conjunction with

2. September 5, 1858, in latitude 8° 24′ S., when the chronometer, known to be $7^m\ 2^s$ slow on Greenwich mean time, showed $18^h\ 40^m\ 8^s$, September 4, the following star-lunar was taken early in the morning:—

Aldebaran E. of Meridian.		*Moon's L. L.*		*Dist. remote Limb.*	
Obs. alt.	36° 30′ 0″	Obs. alt.	57° 28′ 20″	Obs. dist.	65° 4′ 42″
Index cor.	+ 4 0	Index cor.	+ 2 20	Index cor.	— 2 10

The height of the eye was 20 feet: required the additional error of the chronometer, and the longitude of the ship?

Ans. additional error of the chronometer 7^s *slow*; longitude 66° 57′ 30″ W.

In all the foregoing examples the mean time at the ship has been deduced from the altitudes employed in clearing the lunar distances; but, as already remarked (pages 104, 242), neither the moon nor a star is so eligible for the determination of *time* as the sun; and even the sun, either from proximity to the meridian, or to the horizon, may not be in a favourable position for the purpose, when the distance between it and the moon is taken. Now, as in determining the longitude, it is just as important to know accurately the time at the place of observation, as the time at Greenwich, it is often necessary to observe for *ship-time* either before or after the lunar distance is taken, and thence to deduce the time at the place where, and at the instant when, that distance was observed. And here, again, the chronometer performs an important office: it furnishes us—with all needful accuracy—with the interval of time between observations for ship-time and those for the distance, which interval is of course not affected by the error of the chronometer, and only in a very minute degree by its daily rate; which, however, if known, may be allowed for.

If the time at ship be determined at a place A, and the

the approximate interval of time, for finding the correction of that interval given at p. 526 of the Almanac.

lunar distance be taken at another place B, the interval of time between the two sets of observations—corrected for the difference of longitude between A and B—being added to the time at A, if the ship was at A before it was at B, or subtracted in the contrary case, will give the time at B when the ship was there; that is, when the distance was taken.

The following skeleton form will sufficiently indicate what steps are necessary to find the time at B when the distance was observed there, from knowing the time when the ship was at A, the interval between the chronometer times when at A and at B, and the difference of longitude between A and B.

Mean time by chronometer when at A	..^{h} ..^{m} ..^{s}
. . . . B	.. .. ..
Interval of time by chronometer	.. .. ..
Correction for gain or loss in that interval	..
Interval of time corrected for rate	.. .. ..
Diff. long. of A and B in time	.. ..*
Interval of time corrected for diff. long.	.. .. ..
Mean time at ship when at A	.. .. ..
. . . . . B	.. .. ..
. . at Greenwich when at B	.. .. ..
Longitude of ship in time when at B	.. .. ..

It has already been remarked, that although an altitude from which the time at the place where it is taken is to be deduced, should be measured with all practicable accuracy, yet for the purpose of clearing the lunar distance merely, a like precision in the altitudes is not indispensably necessary. But circumstances may arise, from an obscure horizon or other causes, which may preclude the observations for altitudes altogether, though the distance may be readily taken. In

* If B is to the east of A, this must be added: if to the west, it must be subtracted.

such circumstances, the altitudes for clearing the distance must be determined by computation.

In order to compute the altitude of a celestial object at any instant, we must know the object's hour-angle with the meridian at that instant, and this requires that we know the *time*.

If the object be the sun, the time itself—corrected for the equation of time—is the hour-angle; but if the object be the moon or a star, the hour-angle will be the difference between the R. A. of the object and the R. A. of the meridian at the proposed instant; and to get these right ascensions, the time at the place for which the altitude is required must be known.

To find the time at a place B, where a lunar distance is taken, by means of the time at a place A, where altitudes are taken, the foregoing blank form suffices. And for determining the time at A, ample directions have already been given in Chapter IV.

The time at B when the lunar distance was observed, and thence the hour-angle of each object with the meridian being found, the declinations at the time, and the latitude of B being also known, it will be easy to compute the corresponding true altitudes; and thence, by applying the usual corrections for altitude the contrary way, to get the apparent altitudes when the distance was observed; so that we shall have all that is necessary for the determination of the true distance, and thence the longitude of the ship when at the place where the distance was observed.

How the true and apparent latitudes of an object are to be computed when the object's hour-angle, its declination, and the latitude of the place are given, may be explained as follows:—

Computation of Altitudes.

Referring to the diagram at page 151, or to that at page 172, we have, in the spherical triangle P Z S, the following quantities given, namely:—The co-latitude P Z, the polar distance P S, and the hour-angle P, given, to determine the co-altitude Z S; that is, there are given two sides and the included angle of a spherical triangle to determine the third side.

Formulæ for the solution of this case have already been investigated at page 152. If in imitation of what is there done, we put—

$$\tan Z P \cos P = \cot a = \frac{\cos a}{\sin a} \quad . \; . \; . \; . \quad (1)$$

we shall have—

$$\cos Z S = \frac{\cos Z P \sin (a + S P)}{\sin a}$$

$$\therefore \sin \text{alt.} = \frac{\cos Z P \sin (a + S P)}{\sin a} \quad . \; . \; . \; . \quad (2)$$

It will be observed here that of the trigonometrical quantities tan Z P, cos Z P, cos P, the only one that can ever become negative is cos P. When such happens to be the case, that is, when the hour-angle exceeds 90°, (1) is negative, and therefore if in this case we take cos P positive, the formula (2) will become

$$\sin \text{alt.} = \frac{\cos Z P \sin (a - S P)}{\sin a} \quad . \; . \; . \; . \quad (3)$$

which may always be employed when the hour-angle exceeds 90°. We do not say that it *must* be employed, because the form (2), as well here as at page 152, is applicable to all cases; but then, in using it, where it may be replaced by (3), the influence of the *signs* of the trigonometrical quantities must not be overlooked. When cos P is negative, cot a will be negative, so that the angle a' will be the supplement of that furnished by the Tables: this supplement, *added* to S P, will always give an angle $a' + $ S P, such that sin ($a' +$ S P) will be the same as sin ($a -$S P); but by using the latter the trouble of taking supplements is avoided. When P exceeds 90°, a will necessarily exceed S P, otherwise the sine of the altitude would be negative, which is impossible.* And it may, therefore, be further observed that S P can never be subtractive when the latitude and declination are of contrary names: in fact, for the hour-angle in this case to exceed 90°, the object must be below the horizon.

NOTE.—Since the altitudes employed in clearing the lunar distance are not required to the same degree of precision as those used in finding the time, it will be sufficient if they are computed to within 20″ or 30″ of the truth.

Examples of Computing Altitudes.

1. Given the co-latitude Z P=51° 56′, the polar distance S P=64° 13′, and the hour-angle P= 33° 30′, to find the altitude of the star.

Z P	51°	46′	0″	tan 10·103548	. . . . . .			cos 9·791596
P	33	30	0	cos 9·921107				α Ar. comp. sin 0·163188
α	43	22	30	cot 10·024655				α + S P sin 9·979200
S P	64	13	0	TRUE ALT.	59°	12′	9″	sin 9·933984
α + S P	107	35	30	Refraction			+ 35	
				APP. ALT.	59	12	44	

If the object had been the sun instead of a star, we should have had to have subtracted 5″ from this result for parallax, so that the apparent altitude would have been 59° 12′ 39″.

Although, as stated above, the true altitude need not be computed to extreme nicety as regards the seconds, yet small corrections such as this, to reduce the time to the apparent altitude, must not be neglected: the relative measures of the true and apparent altitudes must be scrupulously preserved, as the formula for clearing the observed distance sufficiently implies. On this account, when the object whose altitude is to be computed is the moon, the correction

* It may be remarked in reference to the formula (2) page 152, that since, as there noticed, (2) would be negative if S P and S were each to exceed 90°, such a case cannot exist; for cos Z P is always positive.

of altitude, applied to the true altitude as above, gives a result which should be regarded as only the approximate apparent altitude; because, in the Tables, this correction is adapted to the *apparent* and not to the *true* altitude; so that, when the approximate apparent altitude is obtained from the true, as above, we should again refer to the table, entering it now with this close approach to the apparent altitude, and take out the true correction of it: the correction previously applied belonging to an altitude somewhat too great. For example: Suppose that in the instance above, the object had been the moon, and that its horizontal parallax at the time had been 54′ 50″; then, referring to the table of "Correction of the Moon's Altitude," entering it with this horizontal parallax, and with the true altitude, 59° 12′ 9″, as if it were the apparent altitude, we find the corresponding correction to be 27′ 30″, which must be regarded as an approximate correction only, thus—

Moon's true altitude	59°	12′	9″
Approximate correction		—27	30
Approximate app. alt.	58	44	39
Cor. due to this app. alt.		—27	51
∴ Apparent Altitude	58	44	18

And even this is a second too great, as the Table shows; so that the correct apparent altitude is 58° 44′ 17″.

In the case of the sun or a star, the approximate correction will seldom differ by so much as a second from the true correction; and therefore need not in general be modified.

2. September 2nd, 1858, in latitude 21° 30′ N., and longitude, 43° 18′ W., by account, the distance between the sun and moon was taken, but the moon being near the horizon it was resolved to find its altitude by computation. The mean time at the ship, as determined from altitudes of the sun, was found to be $1^h\ 55^m\ 35^s$: required the altitude of the moon?

Mean time at ship	1^h	55^m	35^s
Longitude W. in time	2	53	12
Greenwich date of obs.	4	48	47

Mean Sun's R. A., and R. A. of Meridian.

R. A. at Greenwich, noon	10^h	45^m	$22^s{\cdot}61$
Correction for 4^h 48^m 47^s			47 ·44
Mean Sun's R. A.	10	46	10
Mean time at ship	1	55	35
R. A. of meridian	12	41	45

Moon's R. A., Declination, Hor. Parallax, and Hour-angle.

R. A. at 4^h	6^h	18^m	28^s	Declin. at 4^h	28°	10′	1″	N
Cor. for 48^m 47^s		2	12	Cor. for 48^m 47^s		−2	10	
Moon's R. A. G. date	6	20	40	Declination	28	7	51	
R. A. of meridian	12	41	45		90			
MOON'S HOUR-ANGLE	6	21	5	POLAR DIST.	61	52	9	
or	95°	16′	15″					
Moon's Hor. Parallax		59′	35″·1	Diff. for 12^h	+5″·7			
Correction			+2	for 5^h	+2			
HOR. PAR. G. date		59	37					

Computation of the Moon's Altitude.

Latitude	21°	30′	0″	cot 10·404602		. . . . 9·564075
Hour angle	95	16	15	cos 8·963134		α Ar. comp. sin 0·011501
α	76	52	20	cot 9·367736		α − S P sin 9·413083
Polar dist.	61	52	9	TRUE ALT.	5° 35′ 30″	sin 8·988659
α − S P	15	0	11	1st correction	− 50 22	
				Approx. app. alt.	4 44 8	
				2nd correction	− 49 4	
					4 46 26	
				3rd correction	− 4	
				APP. ALT.	4 46 22	

The following is the blank form for these operations :—

Blank Form: *Computation of the Moon's Altitude.*

1. *For the Greenwich Date.*

Mean time at ship	..^{h} ..^{m} ..^{s}
Longitude in time	.. .. ..
Greenwich date of obs.	.. .. ..

2. *For the Mean Sun's R. A., and R. A. of Meridian.*

R. A. at Greenwich, noon	..^{h} ..^{m} ..^{s}	
Cor. for G. date (Naut. Alm. p. 530)	.. .. ..	
Mean sun's R. A. at G. date	.. .. ..	(*Add*)
Mean time at ship	.. .. ..	
R. A. of meridian	.. .. ..	

3. *For Moon's R. A., Declination, Hor. Par., and Hour-angle.*

R. A. at hour of G. date	..^{h} ..^{m} ..^{s}		Declin. at hour	..° ..′ ..
Cor. for minutes and secs.	.. ..		Cor. for mins. and secs.	. ..
Moon's R. A. at G. date	.. .. ..	(*Sub. less from greater*)	Declin. G. date	.. .. ..
R. A. of meridian	.. .. ..			90
Moon's hour angle	.. .. .*		Polar dist.	.. .. ..
Hour-angle in degrees	..° ..′ ..″			
Moon's hor. parallax	..′ ..″	Diff. for 12^h	..″	
Cor. for time past noon	..	for time past noon	..	
Hor. Par. at G. date	.. ..			

4. *For the Moon's True and App. Altitude.*

Latitude	..° ..′ ..″	cot			sin ....
Hour angle	.. .. ..	cos		α Ar. comp.	sin ....
α	.. .. ..	cot		α ± SP	sin ....
Polar dist.	.. .. ..		True altitude	..° ..′ ..″	sin ....
α ± SP†	.. .. ..		1st correction	—	
			Approx. app. alt.	.. .. ..	
			2nd correction	— .. .. to be applied to true alt.	
			App. altitude	.. .. ..	
			3rd correction	— .. to be applied to app. alt.	
			App. altitude	.. .. ..	

☞ The corrections on the right are taken from the table of "Corrections of the Moon's Altitude," which is entered first with the true alt. and then with the corrected app. alt.

* If this remainder exceed 12^h, subtract it from 24^h.

† The lower sign to be used only when the hour-angle exceeds 90°, which can never happen when the latitude and declination have contrary names.

Blank Form: *Computation of a Star's Altitude.*

1. *For the Greenwich Date.*

Mean time at ship	..^{h} ..^{m} ..^{s}
Longitude in time	.. .. ..
Greenwich date of obs.	.. .. ..

2. *For the Mean Sun's R. A., R. A. of Meridian, Star's Hour-angle, and Polar Distance.*

R. A. at Greenwich, noon	..^{h} ..^{m} ..^{s}	
Cor. for G. date (Naut. Alm., p. 530)	.. .. ..	
Mean sun's R. A. at G. date	.. .. ..	(*Add*)
Mean time at ship	.. .. ..	
R. A. of meridian	.. .. ..	(*Sub. less from greater*)
R. A. of the star (Naut. Alm.)	.. .. ..	
Star's Hour-angle in Time	.. .. ..* or ..° ..′ ..″	
Star's declin. (Naut. Alm.)	..° ..′ ..″	
	90	
Polar distance	.. .. ..	

3. *For the Star's True and Apparent Altitude.*

Latitude	..° .. ..″	cot			sin ....
Hour-angle	.. .. ..	cos			α Ar. comp. sin ....
α	.. .. ..	cot			$\alpha \pm$ SP sin ....
Polar dist.	.. .. ..	True altitude	..° .. ..″		sin ....
$\alpha \pm$ SP†	.. .. ..	Refraction	+		
		App. altitude	.. .. ..		

If the object be the Sun, the mean time at ship, when the observation for the lunar distance was taken, corrected for the equation of time at that instant, will be the apparent

* If this remainder exceed 12^h, subtract it from 24^h.

† The lower sign has place only when the hour-angle exceeds 90°.

time at ship; that is, the sun's hour-angle: this being found the computation for the true altitude will be the same as (3) above, from which the apparent altitude is obtained by adding the refraction diminished by the sun's parallax in altitude. The preparation for the step (3), in the case of the sun, is therefore as follows:—

For the Sun's Hour-angle.

Mean time at ship	..^{h} ..^{m} ..^{s}			
Longitude in time	.. .. ..			
GREENWICH DATE OF OBS.	.. .. ..			
Sun's noon declin.	..° ..′ ..″		Diff. for 1^h	..″
Cor. for Greenwich date	.. ..		Cor. for G. date	..′ ..″
DECLINATION AT G. DATE	.. .. ..			
	90			
POLAR DISTANCE	.. .. ..			
Equation of time at G., noon		..^{m} ..^{s}	Diff. for 1^h	..^{s}
		..	Cor. for G. date	..^{s}
Equa. of time at G. date		..^{m} ..^{s}		
Mean time at ship		..^{h} ..^{m} ..^{s}		
SUN'S HOUR-ANGLE		.. .. ..* or ..° ..′ ..″		

Then proceed to calculate the true altitude as in step 3 for a star, adding refraction minus the parallax to the true, to obtain the apparent altitude.

Examples for Exercise: Computation of Altitudes.

1. In example 3, page 245, it is required to compute the true and apparent altitudes of the sun when the lunar distance was taken.

Ans. True altitude 58° 53′ 10″; apparent altitude 58° 53′ 40″.

2. August 16, 1858, in latitude 36° 30′ N., and longitude

* If this exceed 12^h, subtract it from 24^h.

153° E.. by account, when the mean time at ship was 4^h 45^m 44^s, required the true and apparent altitudes of the sun?

Ans. True altitude 23° 50′ 24″; apparent altitude 23° 52′ 26″.

3. April 26, 1858, in latitude 29° 47′ 45″ S., and longitude by account 31° 7′ E., the distance between the moon and the star Altair was taken, when the mean time at ship was 1^h 51^m A.M., it is required to compute the true altitude of the star to the nearest minute, and thence to deduce the apparent altitude?

Ans. True altitude 25° 8′; apparent altitude 25° 10′ 3″.

4. October 2, 1858, in latitude 46° 15′ N., and longitude by account 56° 24′ E., a star-lunar was taken, when the mean time at ship was 5^h 32^m 12^s A.M., it is required to compute the moon's true altitude, and thence to deduce the apparent altitude?

Ans. True altitude 49° 22′ 17″; apparent altitude 48° 44′ 14″.

NOTE.—In computing altitudes as above for the purpose of clearing the lunar distance, it will suffice if the true altitude is obtained to the nearest minute; but the corrections for deducing from this the apparent altitude should be applied with care, the seconds being always retained. Indeed, if the true and apparent altitudes are obtained with strict precision, and we equally increase or diminish these by even so much as a minute or two, the resulting true lunar distance will be affected in but a very trifling degree by the change, inasmuch as the relative values of the altitudes will be disturbed but in a very trifling degree. Also a few seconds,—any number, for instance, not exceeding 10″,—may be added to or taken from the apparent lunar distance, provided at the end of the work the resulting true distance be corrected for the overplus or deficient seconds in the apparent distance.

By so modifying the apparent quantities as to cause the seconds in each to be a multiple of 10″, we may save a little trouble in taking the parts for seconds, when the logarithmic method of clearing the lunar distance is employed: but in the mode of operation more specially dwelt upon in this work, such changes would produce no advantage.

Having now discussed all the more important problems of Nautical Astronomy, with as much fulness of detail as the limits of the present rudimentary treatise permit, it merely remains for us, in conclusion, to give a short account of what at sea is called a "Day's Work;" and to exhibit a brief specimen of a Ship's Journal, as promised at page 85.

CHAPTER VII.

DAY'S WORK AT SEA: THE SHIP'S JOURNAL.

As already noticed at page 84, as soon as a ship has taken her departure and her voyage fairly begun, the several courses on which she sails, as indicated by the compass, her hourly rate of sailing as determined by the log, together with the other particulars, leeway, currents, &c., affecting her progress, are all recorded in chalk on a large black board, called the *log-board.* These are afterwards copied into the *log-book,* and the courses being all corrected for leeway and variation of the compass, each corrected course, with the entire distance sailed on it, being known, a reference to the Traverse Table gives the corresponding difference of latitude and departure. The difference of latitude and departure due to the whole traverse is then found, and thence the direct course and distance sailed, as explained at page 48. Lastly, with this direct course and distance, the difference of longitude made is found either by parallel, mid-latitude, or Mercator's sailing, and thus the position of the ship at the end of the traverse is ascertained. These operations are regularly brought up to noon of each day; they comprise what is called a *Day's Work,* the result of which is the position of the ship at noon by *dead reckoning.*

Whenever astronomical observations for latitude or longitude are made, a distinct record of the result of these is inserted in the log-book; but since the ship's daily account is always closed at noon, and a fresh account opened, a latitude or longitude, determined by observations in the interim, is brought up to the following noon, by applying to it the latitude or longitude, by dead reckoning, made in the interval between the observations and that noon; so that in strictness, what is recorded as the result of observation at *noon*, is often made up, in small part, of the dead reckoning. The ship's position at noon being determined in this manner, the chart is referred to, and the place where she is being pricked off, she takes as it were a fresh departure from a known spot, and her course from it is then shaped, as at first, in accordance with her ultimate destination. When this is reached the log-book, thus completed, furnishes a Journal of the voyage.

As a specimen, we shall here exhibit a page of such a journal, subjoining the necessary day's work.

NOTE.—The initial letters, H, K, and F, stand for Hours, Knots (or miles), and Fathoms respectively. The *fathom* is not a fixed length of the log-line, like the *knot;* sometimes it is the eighth part of the knot, or something beyond six feet, but it is more convenient to take the tenth part of the knot, which is a little less than six feet, for the fathom; and this is supposed to be its length in the following specimen*; so that, as the knot represents a mile, the fathom will represent one-tenth of a mile.

The result of the day's work preceding the day to which the following page of the journal applies, is supposed to stand thus:—

Course.	Dist.	Lat. acct.	Lat. obs.	Long. acct.	Long. Obs.	Bearing and dist. of Lizard at noon
N. 58° E.	57m.	38° 19′ N.	38° 20′ N.	24° 11′ W.		N. 49°½ E. Dist. 1074 miles.

In strictness a fathom is 6 feet.

EXTRACT FROM A JOURNAL OF A VOYAGE FROM ST. MICHAEL'S TOWARDS ENGLAND.

H	K	F	Courses.	Winds.	Lee-way.	Remarks, Monday, Sept, 12, 18—, P.M.
1	4	6	N. N. E.	E.	1½	Moderate and clear weather.
2	5	0				Out first reef topsails, set royals, and flying jib.
3	5	3	N. by E. ¾ E.	E. ¼ N.	½	Light breezes and clear weather.
4	5	6				Ditto weather. Swell from E. from 4 P.M., till 8, for which allow a drift of 21 miles.
5	6	0				In royals and flying jib.
6	6	1				
7	5	8	E. S. E.	N. E.	1	Tacked.
8	5	7				
9	5	0			¼	Ditto weather
10	5	3			0	
11	5	8				Ditto weather.
Midnt.						
						Tuesday, Sept. 13, A.M.
1	5	9	E. S. E.	N. E.	1¼	Moderate and clear weather.
2	5	7				
3	5	3			1	Fresh breezes. In top-gallant sail.
4	5	4				
5	5	0	E. N. E.	N.	1½	In first reef topsails.
6	5	0				Strong breezes and cloudy.
7	4	8				In second reef topsails.
8	4	3				
9	3	9			2	Long. by chron. at 9, A.M., 23° 2′ W.
10	3	4				Flying clouds, with light showers.
11	3	3			2¼	Fresh gales and squally. Down jib and in spanker.
Noon	3	5				Lat. at noon by mer. alt. 38° 46′ N. Variation by azimuth, 20° W.

In order to complete this page of the journal, the day's work must now be computed: the compass courses recorded above being corrected for leeway—or the angle of deviation which the action of the wind sideways causes the ship to make with the fore-and-aft line—the distance, diff. lat. and departure due to each course, are to be taken from the traverse table, and thence the whole distance, diff. lat., and departure found, as also the compass course from the commencement of the traverse to the end, as at page 49.

This compass course being corrected for variation, gives the true course, with which and the distance we are to find from the traverse table, or by computation, the true diff.

lat., and thence by mid-latitude or Mercator's sailing, the diff. long. These differences, applied to the latitude and longitude determined by yesterday's work, make known the place of the ship; and the latitude and longitude of the place next to be worked for, being also known, it will merely remain, from these data, to find the bearing and distance of the spot to be reached, and to shape the course accordingly. The day's work is, therefore, as follows:—

Traverse Table.

Courses corrected for Leeway.	Dist.	Diff. lat. N.	Diff. lat. S.	Departure. E.	Departure. W.
N. ¾ E.	9·6	9·5		1·4	
N. by E. ¾ E.	23·0	22·3		5·6	
S. E. by E.	11·5		6·4	9·6	
S. E. by E. ½ E.	10·3		4·8	9·1	
E. S. E.	12·0		4·6	11·1	
S. E. ¾ E.	11·6		6·9	9·3	
S. E. by E.	15·7		8·7	13·0	
E. ½ N.	14·1	1·4		14·0	
E.	7·3			7·3	
E. ½ S.	6·8		0·7	6·8	
W. (Swell)	24·0				24
		33·2	32·1	87·2	24
Compass course N. 89° E.		32·1		24·0	
Distance 63 miles.		1·1		63·2	

Compass course	N. 89°	E
Variation	20	W.
TRUE COURSE	N. 69	E.

With the course 69°, and distance 63 miles, the traverse table gives 22·6 for the difference of latitude: hence—

Lat. left	38° 20′ N.	Meridional parts 2494	tan 69° = 2·6051
Diff. lat.	23 N.		92
LAT. BY ACCT.	38° 43′	 2523	52102
			23446
		Mer. diff. lat. 29	Diff. long. 75·548

Longitude left . .	24° 11′ W.
Diff. long. . .	1 16 E.
LONG. BY ACCT. . .	22 55 W.

The departure made from 9^h A.M. till noon is nearly 14 miles: with this and the mid-latitude about 38°½, the difference of longitude is found to be about 17′ E.: hence—

Longitude by chronom. at 9^h A.M.	23° 2′ W.
Diff. long. up to noon . .	17 E.
LONG. BY OBSERVATION AT NOON .	22 45 W.

Having thus got the latitude and longitude of the ship at noon, we may from these determine the course and distance to the port or place to be worked for—in the present case the Lizard, in lat. 49° 58′ N., and long. 5° 11′ W., as in ex. 3, p. 67.

Lat. ship (by obs.) 38° 46′	Mer. parts 2527	Long. ship.	22° 45′ W.
Lat. Lizard 49 58	Mer. parts 3471	,, Lizard	5 11 W.
∴ Diff. lat. = 672 miles :	Mer. diff. lat. 944	Diff. long.	17 34 = 1054 m.

944)1054(1·1165 = tan 48° 9′, and cos = ·6672)672(1007

Therefore, the course is N. 48° 9′ E., and the distance 1007 miles. Consequently, the work for the day being thus completed, we write the following results at the bottom of the page :—

Course.	Dist.	Lat. acct.	Lat. obs.	Long. acct.	Long. obs.	Bearing and dist. of
N. 69° E.	63 m.	38° 43′	38° 46′	22° 55′ W.	22° 45′ W.	Lizard N. 48° 9′ E. Dist. 1007 miles.

NOTE.—In the foregoing day's work the correction for the variation of the compass, as it is given in degrees and not in points, is applied to the direct course resulting from resolving the traverse ; but when the variation is expressed in points, like the leeway, each course may be corrected for both before casting up the log. The direction of the wind suggests the direction in which the leeway is to be estimated. When the ship is on the starboard tack, the allowance for leeway is to the left, and when on the larboard tack it is to the right. As regards variation, when it is westerly it must be allowed to the left of the compass course, and when easterly to the right. We shall only further add, that when the day's run is very considerable, and no observation for longitude has been obtained, the difference of longitude made will be more correctly determined by working for this difference agreeably to the principles explained at pages 71, 72.

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10.	HOMER. Odyssey, vii. to xii.	1*s.* 6*d.*
11.	HOMER. Odyssey, xiii. to xviii.	1*s.* 6*d.*
12.	HOMER. Odyssey, xix. to xxiv.; and Hymns	2*s.*
13.	PLATO. Apology, Crito, and Phædo	2*s.*
14.	HERODOTUS, i. ii.	1*s.* 6*d.*
15.	HERODOTUS, iii. iv.	1*s.* 6*d.*
16.	HERODOTUS, v. vi. and part of vii.	1*s.* 6*d.*
17.	HERODOTUS. Remainder of vii. viii. and ix.	1*s.* 6*d.*
18.	SOPHOCLES; Œdipus Rex	1*s.*
20.	SOPHOCLES; Antigone	2*s.*
23, 24.	EURIPIDES; Hecuba and Medea	1*s.* 6*d.*
26.	EURIPIDES; Alcestis	1*s.*
30.	ÆSCHYLUS; Prometheus Vinctus	1*s.*
40.	ARISTOPHANES; Acharnians	1*s.* 6*d.*
41.	THUCYDIDES, i.	1*s.*

Preparing for Press.

19. SOPHOCLES; Œdipus Colonæus.
21. SOPHOCLES; Ajax.
22. SOPHOCLES; Philoctetes.
25. EURIPIDES; Hippolytus.
27. EURIPIDES; Orestes.
28. EURIPIDES. Extracts from the remaining plays.
29. SOPHOCLES. Extracts from the remaining plays.
31. ÆSCHYLUS; Persæ.
32. ÆSCHYLUS; Septem contra Thebes.
33. ÆSCHYLUS; Choëphoræ.
34. ÆSCHYLUS; Eumenides.
35. ÆSCHYLUS; Agamemnon.
36. ÆSCHYLUS; Supplices.
37. PLUTARCH; Select Lives.
38. ARISTOPHANES; Clouds.
39. ARISTOPHANES; Frogs.
42. THUCYDIDES, ii.
43. THEOCRITUS; Select Idyls.
44. PINDAR.
45. ISOCRATES.
46. HESIOD.