

*The Solutions of three Chorographic Problems, by
a Member of the Philosophical Society of Oxford.*

THE three following Problems may occur at Sea, in finding the distance and position of *Rocks, Sands, &c.* from the shore; or in surveying the Sea Coast; when only two objects whose distance from each other is known, can be seen at one station: but especially they may be useful to one, that would make a *Map* of a Countrey by a Series of Triangles derived from one or more measured Bases; which is the most exact way of finding the bearing and distance of places from each other, and thence their true Longitude and Latitude; and may consequently occur to one that would in that manner measure a Degree on the Earth.

The First Problem (Fig. 1. and 2.)

There are two objects *B* and *C*, whose distance *BC* is known, and there are two stations at *A* & *E*, where the objects *B, C* being visible, & the stations one from another, the Angles *BAC, BAE, AEB, AEC*, are known by Observation, (which may be made with an ordinary Surveying *Semicircle*, or *Crossstaff*, or if the objects be beyond the view of the naked Ey, with a *Telescopic Quadrant*) to find the distances or lines *AB, AC, AE, EC*.

Construction.

In each of the triangles *BAE, CAE*, two angles at *A, E*, being known, the third is also known: then take any line $\alpha\epsilon$ at pleasure, on which constitute the triangles $\beta\alpha\epsilon, \alpha\epsilon\gamma$ respectively equiangular to the triangles *BAE, AEC*; join $\beta\gamma$. Then upon *BC* constitute the triangles *BCA, BCE* equiangular to the correspond-

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dent triangles $\beta\gamma\alpha$, $\beta\gamma\epsilon$, join AE , and the thing is manifestly done.

The Calculation.

Assuming $\alpha\epsilon$ of any number of parts, in the triangles $\alpha\beta\epsilon$, $\alpha\gamma\epsilon$, the angles being given, the sides $\alpha\beta$, $\alpha\gamma$, $\epsilon\beta$, $\epsilon\gamma$ may be found by Trigonometry: then in the Triangle $\beta\alpha\gamma$, having the angle $\beta\alpha\gamma$, and the legs $\alpha\beta$, $\alpha\gamma$, we may find $\beta\gamma$. Then $\beta\gamma. BC::\beta\alpha. BA::\beta\epsilon. BE::\gamma\alpha. CA::\gamma\epsilon. CE$.

The second Problem (Fig. 3 & 4.)

Three objects B, C, D , are given, or (which is the same) the sides, and consequently angles of the triangle BCD are given; also there are 2 points or stations A, E , such, that at A may be seen the three points B, C, E , but not D ; and at the station, E , may be seen A, C, D , but not B , that is the angles BAC , BAE , AEC , AED , (and consequently EAC , AEC), are known by observation: to find the lines AB, AC, AE, EC, ED .

Construction.

Take any line $\alpha\epsilon$ at pleasure, and at its extremitys make the angles $\epsilon\alpha\gamma$, $\epsilon\alpha\beta$, $\alpha\epsilon\gamma$, $\alpha\epsilon\delta$, equal to the correspondent observed angles EAC , EAB , AEC , AED . Produce $\beta\alpha$, $\delta\epsilon$, til they meet in ϕ , join $\phi\gamma$; then upon CB describe (according to 33. 3. Eucl) a Segment of a circle, that may contain an angle $=\gamma\phi\beta$; and upon CD describe a Segment of a circle capable of an angle $=\gamma\phi\delta$; suppose F the common section of these 2 circles; join FB, FC, FD ; then from the point C , draw forth the lines CA, CE , so that the angle FCA may be $=\phi\gamma\alpha$, and $FCE = \phi\gamma\epsilon$; so A, E , the common sections of CA, CE , with FB, FD , will be the points required, from whence the rest is easily deduced.

The Calculation.

Assuming $\alpha\epsilon$ of any number, in the triangles $\alpha\gamma\epsilon$, $\alpha\phi\epsilon$, all the angles being given, with the side $\alpha\epsilon$ assum'd,

sum'd, the sides $\alpha\gamma$, $\varepsilon\gamma$, $\alpha\phi$, $\varepsilon\phi$, will be known; then in the triangle $\gamma\alpha\phi$, the angle $\gamma\alpha\phi$ with the legs $\alpha\gamma$, $\alpha\phi$ being known, the angles $\alpha\phi\gamma$, $\alpha\gamma\phi$ with the side $\phi\gamma$ will be known: then as for the rest of the work in the other figure, the triangle BCD having all its sides and angles known, and the angles BFC , BFD , being equal to the found $\beta\phi\gamma$, $\beta\phi\delta$; how to find FB , FC , FD by *Calculation* (and also *Protraction*) has been shewn by Mr *Collins* (in *Phil. Transact* n. 69 p. 2093.) as to all its cases, which may theretore supersede my shewing any other way.

But here it must be noted, that if the sum of the observed angles, BAE , AED , is 180 degrees: then AB and ED cannot meet, because they are parallel, and consequently the given solution cannot take place; for which reason I here subjoin another.

Another Solution.

Upon BC (Fig. 5.) describe a segment BAC of a circle, so that the angle of the segment, may be equal to the observed $\angle\beta\alpha\gamma$, (which as above quoted is shewn 33. 3. Euclid) and upon CD describe a segment CED of a circle, capable of an angle equal to the observed $\angle CED$; from C draw the diameters of these circles CG , CH ; then upon CG describe a segment of a circle GFC , capable of an angle equal to the observed $\angle AEC$; likewise upon CH describe a circles segment CFH , capable of an angle equal to the observed $\angle CAE$: suppose F the common section of the two last circles HFC , GFC , join FH , cutting the circle HEC in E , join also FG , cutting the circle GAC in A : I say that A , E , are the points required.

Dem :

For the $\angle BAC$ is $=\beta\alpha\gamma$ by construction of the segment, also the angles CEH , CAG , are right, because each exists in a semicircle: therefore a circle being described upon CF as a diameter, will pass thro E , A ; There-

fore the angle $\angle CAE = \angle CFE = CFH =$ (by construction) to the observed angle $\gamma \alpha \epsilon$. In like manner the $\angle CEA = \angle CFA = CFG =$ observ'd angle $\gamma \epsilon \alpha$.

If the stations A, E fall in a right line with the point C , the lines GA, HE being parallel, cannot meet: but in this case the problem is indeterminate and capable of infinite Solutions. For as before upon CG describe a Segment of a circle capable of the observed $\angle \gamma \epsilon \alpha$, and upon CH , describe a Segment capable of the observed $\gamma \alpha \epsilon$: then thro C , draw a line any way cutting the circles in A, E , these points wil answer the question.

The Third Problem.

Four points B, C, D, F , (Fig. 6.) or the 4 sides of a quadrilateral, with the angles comprehended are given; also there are 2 stations A and E such, that at A , only B, C, E are visible, and at E only A, D, F , that is, the angles BAC, BAE, AED, DEF are given: to find the places of the two points A, E ; and consequently, the lengths of the lines AB, AC, AE, ED, EF .

Construction.

Upon BC (by 33. 3. Eucl.) describe a segment of a circle, that may contain an angle equal to the observed angle BAC , then from C draw the chord CM , or a line cutting the circle in M , so that the angle BCM may be equal to the supplement of the observed angle BAE , i. e. its residue to 180 degrees. In like manner on DF describe a segment of a circle, capable of an angle equal to the observ'd DEF , and from D draw the chord DN , so that the angle FDN may be equal to the supplement of the observ'd angle AEF , join MN , cutting the 2 circles in A, E : I say A, E , are the two points requir'd.

Dem:

Join AB, AC, ED, EF , then is the $\angle MAB = \angle BCM$ (by 21. 3. Eucl. :) = supplement of the observ'd $\angle BAE$ by construction, therefore the constructed $\angle BAE$

$\angle BAE$ is equal to that which was observed. Also the $\angle BAC$ of the segment, is by construction of the Segment, equal to the observ'd $\angle BAC$. In like manner the constructed angles AEF , and DEF , are equal to the correspondent observed angles AEF , DEF , therefore A, E are the points required.

The Calculation.

In the Triangle BCM , the $\angle BCM$ ($=$ supplement of BAE) and $\angle BMC$ ($= BAC$) are given, with the side BC ; thence MC may be found; in like manner DN in the $\triangle DNF$ may be found. But the $\angle MCD$ ($= BCD$ -- BCM) is known with its legs MC, CD , therefore its base MD , and $\angle MDC$ may be known. Therefore the $\angle MDN$ ($= CDF$ -- CDM -- FDN) is known, with its legs MD, DN ; thence MN with the angles DMN, DNM , will be known. Then the $\angle CMA$ ($= \angle DMC + DMN$) is known, with the $\angle MAC$ ($= MAB + BAC$) and MC before found; therefore MA and AC will be known. In like manner in the triangle EDN . the angles E, N , with the side DN being known, the sides EN, ED will be known; therefore AE ($= MN$ -- MA -- EN) is known. Also in the triangle ABC , the $\angle A$ with its sides BC, CA being known, the side AB will be known, with the $\angle BCA$; so in the triangle EFD , the $\angle E$ with the sides, ED, DF being known, EF will be found, with the $\angle EDF$. Lastly in the triangle ACD , the $\angle ACD$ ($= BCD$ -- BCA) with its legs AC, CD being known, the side AD will be known, and in like manner EC in the triangle EDC .

Note that, in this problem, as also in the first and second, if the two stations fall in a right line with either of the given objects: the *locus* of A , or E , being a circle, the particular point of A , or E , cannot be determined from the things given.

As to the other cases of this third problem, wherein A , and E , may shift places, i. e. only D, F, E may be vi-

visible at *A*, and only *A, B, C* at *E*; or wherein *B, D, E*, may be visible at *A*, and only *C, F, A*, at *E*; or wherein *A* may be of one side of the quadrilateral, and *E* on the other; or one of the stations within the quadrilateral, and the other without it: I shall for brevity sake omit the figures, and diversity of the Signs \dagger and $--$ in the calculation, and presume that the Surveyour will easily direct himself in those cases, by what has been said.

The Solution of this third problem is general, and serves also for both the precedent. For suppose *C, D* the same point in the last figure, and it gives the solution of the second problem: but if *B, C*, be supposed the same points with *D, F*, by proceeding as in the last, you may directly solve the first problem.

A Letter from William Molyneux Esq; to one of the Secretarys of the R. S. concerning the Circulation of the blood as seen, by the help of a Microscope, in the Lacerta Aquatica.

Dublin Octob. 27. 1685.

Sir,

OUR Society lately received transcripts of two of D^r *Gardens* Letters, the first dated from *Aberdeen* July 17. 1685. to D^r *Middleton*; the other Sept. 4. 1685. to D^r *Plot*. To both these Letters I have something to say.

In the first he gives an Account of the Visible Circulation of the blood in the *Water-Newt* or *Lacerta Aquatica*; truly I am heartily glad, that this Learned and Ingenious D^r has hit upon this Experiment; tis now above two years and an half, since I first Discovered this surprising

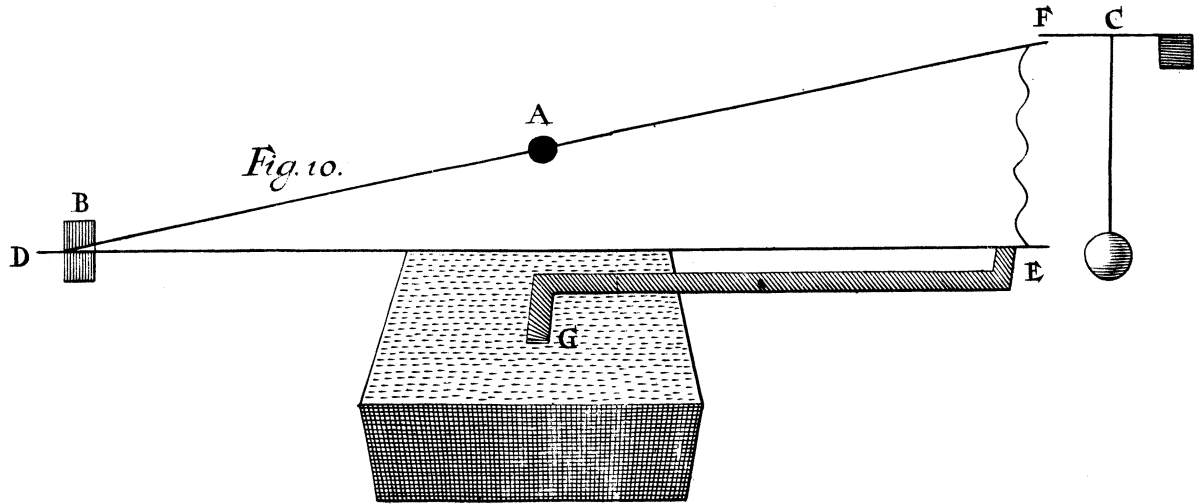
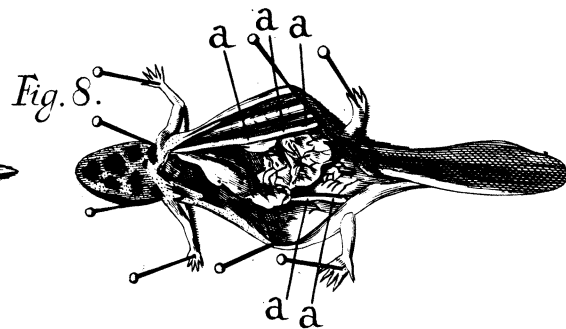
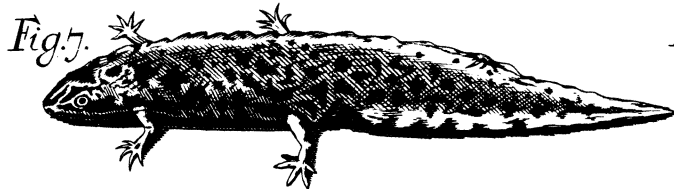
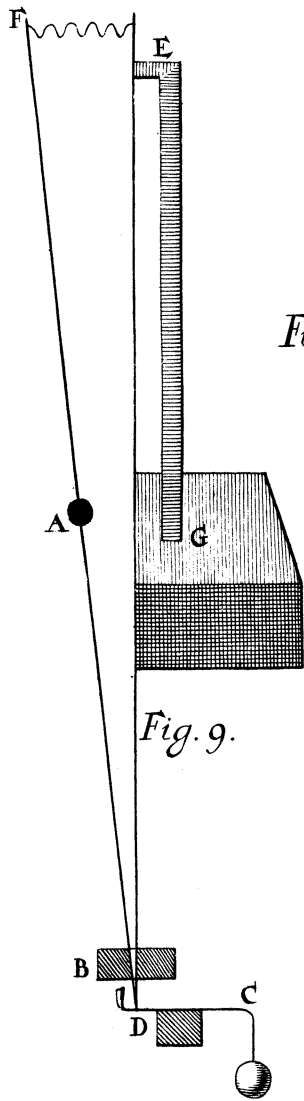
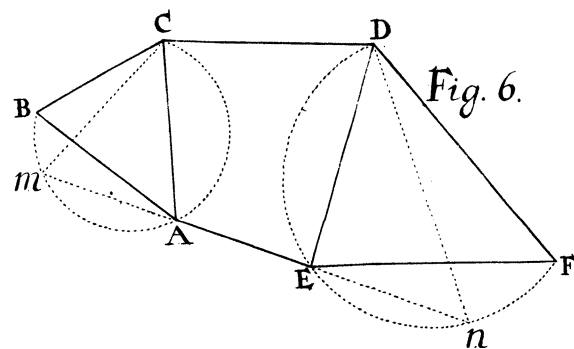
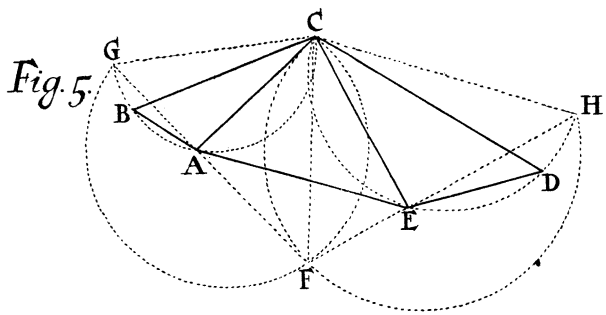
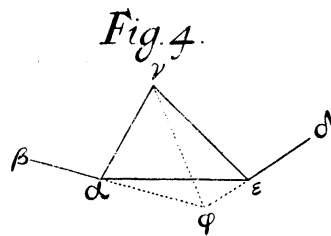
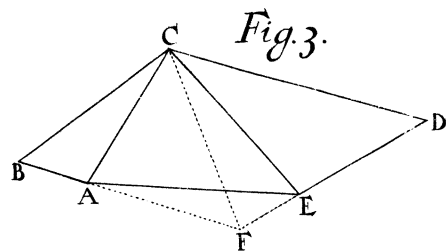
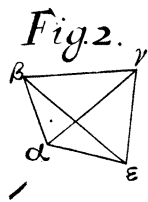
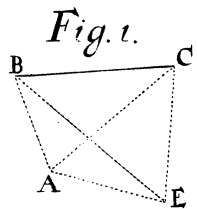


Fig. 11.

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SEMPIDES

- a. Anglicus & Græcus.
- b. Rom. Bernardi.
- g. Rom. Grecii.
- q. Rom. Villalpandi.
- r. Rhinlandicus.
- p. Parisinus Auzotii.
- c. Catholicus Mori.
- l. Venetus Auzotii.
- m. Bononiensis Picardi.
- n. Bononiensis Riccioli.