TREASURY DEPARTMENT

U. S. COAST AND GEODETIC SURVEY

W. W. DUFFIELD

SUPERINTENDENT

TOPOGRAPHY

PHOTO-TOPOGRAPHIC METHODS AND INSTRUMENTS

By J. A. FLEMER, Assistant

APPENDIX No. 10—REPORT FOR 1897

WASHINGTON
GOVERNMENT PRINTING OFFICE
1898

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PREFACE.

In the annual report of the Superintendent of the United States Coast and Geodetic Survey for 1893, Appendix No. 3, a description is given of phototopography as practiced in Italy and in the Dominion of Canada.

The Canadian surveying camera and the Italian phototheodolite, which have been described in said appendix (No. 3, 1893), have both been replaced by improved and more effective instruments, which will be described in the paper herewith presented, together with other photographic surveying instruments that may be regarded as typical representations of the different forms now in use.

Notwithstanding the rapid rise in the popularity of photographic surveying in general, we still meet with many who express doubt as to the practical value and accuracy of photographic surveying methods, either from ignorance of those methods, from defective results obtained from the application of photography to the survey of areas not adapted for a phototopographic development, or, more frequently, from that extreme conservatism which meets all innovations with more or less doubt and distrust. Others, again, may have failed to take kindly to photographic surveying, supposing a thorough familiarity with the theories and laws of optics, descriptive geometry, perspective drawing, and general cartography to be essentials, without which no practical knowledge and understanding of photogrammetry may be obtained.

Although it should be admitted that such knowledge will enable the student to master phototopography in a rapid and easy manner, giving him a great advantage in and an enlarged field for the practical application of the same, or in teaching its methods to others, yet the fundamental principles underlying this art are so simple that it is believed any topographer or land surveyor, with the knowledge that he should possess as such, can readily acquire enough of the theoretical fundamental principles to become fully able to apply photography successfully to practical surveys.

Although it will not fall within the scope of this paper to enter into the study of either optics, descriptive geometry, perspective, photo-chemical analysis, or cartography, it will show in a general manner how photography has been applied to topographic surveys by describing the simple processes and methods that will suffice to direct beginners in their practical applications, leaving it to experience and subsequent special study to determine the measure of success, the more so as several excellent works and text-books on photographic surveying have recently been published in the English, French, Italian, and German languages.

The compiler of this paper having consulted all available publications describing phototopographic methods, both foreign and domestic, gladly expresses his indebtedness for information on this subject to Capt. E. Deville, surveyor-general of Dominion lands; to Mr. W. F. King, Alaskan boundary commissioner to Her Majesty, Ottawa, Canada; to Col. A. Laussedat, director of the Conservatoire des Arts et Métiers, Paris; and particularly to the following publications:

La Fototopografia in Italia, Rivista Marittima, L. P. Paganini, 1889, Fasc. VI and VII.

Nuovi Appunti di Fototopografia, Rivista Marittima, L. P. Paganini, 1894, E. C. Forzani.

Zeitschrift für Vermessungswesen.

Die photographische Messkunst, Prof. Franz Schiffner, Halle a. S., Wilhelm Knapp, 1892.

Photographic Surveying, E. Deville, Ottawa, 1895.

Zeitschrift für Instrumentenkunde.

Comptes Rendus de l'Académie des Sciences, Paris, Revue Scientifique, No. 26, I; No. 3, II; 1894.

Die photographische Messkunst, Franz Schiffner, Halle a. S., 1892.

APPENDIX NO. 10—1897.

PHOTOTOPOGRAPHIC METHODS AND INSTRUMENTS.

By J. A. Flemer.

INTRODUCTION.

Topography is that branch of surveying which pictures the shape of the outer visible surface of the earth, in reduced scale, as a horizontal projection, yet showing the relative positions of points of the terrene also in the vertical sense. It is, therefore, supplementary to geodesy in representing areas of the earth's surface, including all the necessary details and changes in the terrene, by means of instrumental measurements made in the field.

The work of filling in the details—topographic surveying in the closer sense—may be accomplished by various methods, differing in the matter of costs, time, and attainable accuracy; one may be advantageously employed for one class of work, while another may be preferable for another class or locality, under different conditions, and the method best adapted for any particular region should be employed to obtain the best results. Minute and detailed methods, with ensuing accurate results, should be applied to cities and all closely settled regions, to the coastal belts, larger river valleys and lakes, particularly when navigable, and this work should be platted on a large scale.

Arid, barren, and mountainous regions, as well as prairies and swamp lands, when sparsely settled, should be more generalized in their cartographic representation and platted on a small scale.

Topographic surveys may be accomplished in various ways, of which the following are the methods and instrumental outfits more frequently in use:

I. The direct platting to scale in the field of all features to be represented on the finished chart:

(*a*) With a plane-table and steel tape measure.

(*b*) With a plane-table and telemeter or stadia rods.

(*c*) With a tachygraphometer and telemeter or stadia rods.

(*d*) With either outfit mentioned under *a* and *b*, but with a leveling instrument in addition for a more precise location of the horizontal contours.

(*e*) Using a barometer instead of a level for less accurate work.

II. The compilation of all available data—cadastral surveys, public land and county surveys, railroad and canal surveys—giving principally the horizontal distances and making a supplementary survey to supply the missing data, which in this case are principally elevations that may be supplied by leveling profiles, by trigonometric leveling, by interpolation and sketching.

III. The records of the survey are in the shape of field notes and sketches (tachymetry), the map being produced by platting the recorded data in the office:

(*a*) With a surveyor's compass and steel tape, locating the relative positions of characteristic points in the horizontal sense, while their relative elevations are ascertained by means of a level and minor details are sketched.

(*b*) By means of a transit and steel tape points are determined both geographically and hypsometrically (using vertical angles), and minor details by sketching.

(*c*) By means of a transit and telemeter or stadia rods.

(*d*) By means of a tachymeter and stadia rods (elevations being obtained mechanically with the instrument).

(*e*) By means of a transit with steel tape or telemeters, combined with a leveling instrument (fcr locating horizontal contours).

(*f*) By using a specially constructed aneroid barometer (Goldschmidt's) in place of the level for locating and tracing the horizontal contours in the field.

IV. The field records for developing the terrene are represented by photographic negatives, taken under special conditions (for phototopographic purposes) from known stations:

(*a*) With a camera or phototheodolite, telemeters, or other distance measures (and often a barometer for obtaining elevations).

(*b*) With a surveying camera, a separate theodolite, telemeters and aneroid barometer.

(*c*) With a photographic plane table, a distance measure, and aneroid barometer.

(*d*) With a surveying camera, a separate plane table, and distance measure, frequently using an aneroid barometer for camera stations occupied without the plane table.

V. The topographic survey may be accomplished by means of a specially constructed surveying camera attached to a free or captive balloon.

After the area which is to be surveyed has been covered with a net of triangles and polygons it will have been provided with a framework of lines of known lengths and direction (triangulation), forming a skeleton survey of the country, and after the natural and artificial features have been filled in by one of the numerous topographic methods (just mentioned) with more or less detail and accuracy we will have a topographic survey of the area of more or less precision.

A good example of changing the method with the locality may be cited in the new survey of Italy, where Paganini's results fully proved the efficiency of phototopography for alpine work (platted on a scale of 1:25000 and 1:50000) and led to the adoption of the phototheodolite as an auxiliary instrument to the plane table, the latter being used for mapping the areas below 2,000 meters, while the phototheodolite was exclusively used for the delineation of the terrene situated above that altitude.

Photogrammetry proper (or *metrophotography*) should be applied to the art of taking perspective views of buildings with a photographic camera for the purpose of constructing therefrom their elevations and ground plans, and it is used principally for architectural, archæological, and engineering purposes.

The term *phototopography* (or *topophotography*) should be generally adopted for all topographic surveys based on perspective views of the terrene obtained by means of the camera.

Under *photographic survey* we could then class all surveys based on photographic data which do not include the orographic delineation of the terrene.

Iconometry means the measuring of dimensions of objects from their perspectives ("Bildmesskunst"), and this term could well be applied to those graphic constructions which serve to convert perspectives into horizontal projections; iconometry is the reverse of perspective drawing.

Photography has been very successfully employed for topographic surveys in Italy, Austria, and Canada, and for the production of the extensive topographic reconnaissance maps of southeastern Alaska.

Although this method, invented and elaborated by Colonel Laussedat, found its first application in France, still, both in France and in Germany, it was originally preempted by the military authorities, under whose auspices it was developed and chiefly used for so-called secret or military surveys; lately, however, photography has found a wider and more general application to surveying in those two countries, and we find this method now in use also in Greece, Spain, Portugal, Norway, Belgium, Mexico, Chile, Peru, Tonquin, Brazil, Argentine Republic, Switzerland, and England.

Although Lieut. Henry A. Reed has, for several years past, taught phototopography, theoretically and practically, at the United States Military Academy at West Point, there seems to be no record of any further work of this kind undertaken by others in the United States.

In the following paper we will treat principally of those photogrammetric methods which are applicable to topographic surveys, although the same principles underlie also the methods in use when applying photography to—

Geological surveys.—For the study of changes in glaciers (glacial motion or variation) based upon the comparison of glacier maps, obtained at stated time intervals from identical and known camera stations; for volcanic eruptions and their effects; for the study of periodical changes in sand dunes due to recurrent winds blowing from one direction at regular intervals, etc.

Meteorologic observations.—For the study of the higher aerial currents and cloud altitudes, based upon iconometric cloud charts, obtained by simultaneous photographic records on plates exposed at different stations at stated time intervals; for the study of the paths of lightning, their lengths, etc.

Hydrographic surveys.—For the location of rocks, buoys, etc.; for the study of fluvial currents, riparian changes due to corrosion, erosion, etc.; for obtaining coast views from points marked on the sailing charts to facilitate the locating of the position of vessels when approaching land, etc.

Engineering.—To estimate the amount of work done at any date by means of photographic surveys that show the status of the work (excavations, fills, structural buildings, etc.) at stated time intervals, etc.

Architectural purposes.—For constructing the ground plans and elevations of old buildings from their perspective views (photographs), for purposes of remodeling, renovation, or preservation.

Military and secret surveys, and so on.

CHAPTER I.

FUNDAMENTAL PRINCIPLES OF ICONOMETRY.

If only one perspective of an object, including its distance line, the principal point, and the horizon line is given—in other words, if the point of view and the central projection upon a vertical plane of an object are given—the object itself can not yet be determined regarding its position and dimensions. In the same way the geographic position of a point can not be located

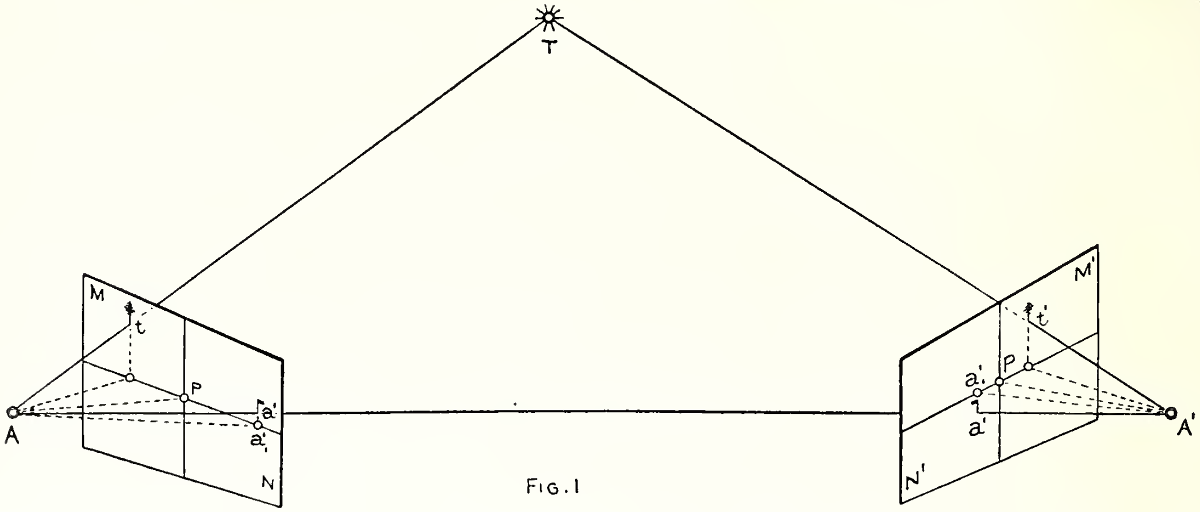


FIG. 1

by means of the plane table from a known station, unless direct measurements to ascertain the distance of such point from the station are resorted to.

If, however, two different perspectives (including their elements) of the same object, obtained from two different known stations, are given, the dimensions and the position of the object with reference to the two stations may be determined iconometrically in a manner analogous to that in

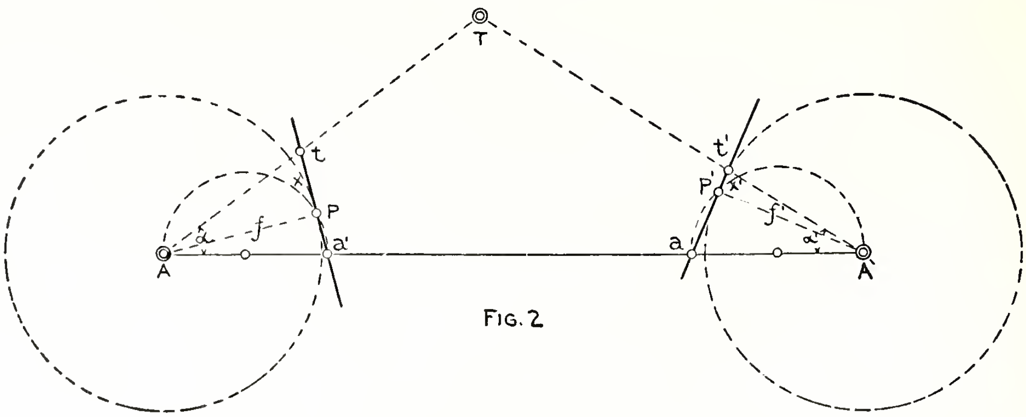


FIG. 2

which a point is located (by intersection) on the plane-table sheet by being observed upon from two different plane-table stations.

Referring to fig. 1, the positions of the camera stations A and A', also the distance $A\ A'$, may be given, and two photographs containing the image t of an object T, including the image a' of the other camera station, may have been obtained from the two stations.

If the base line $A A'$, fig. 2, be laid down on paper, in reduced scale, and if the pictures MN and $M' N'$, fig. 3, be brought into the same relative positions with reference to the platted line which they had at the time of their exposure in the field, the position T of the pictured point (with reference to the platted points A and A') may be located by drawing the rays $A\ t$ and $A'\ t'$ to their intersection. To locate the platted position of T the horizontal projections of the rays $A\ t$ and $A'\ t'$ are brought to their intersection on the platting sheet, fig. 4, which may be done by ascertaining

FIG. 3

the proper positions of the lines of intersection of the picture planes with the horizontal platting plane with reference to A and A' (by "orienting" the picture traces).

The map being the orthogonal projection of the terrene in horizontal plan, the horizontal projections of the perspectives (or picture planes exposed in the vertical plane) will appear as straight lines, termed "picture traces," fig. 4.

The correct orientation of the picture traces forms the most important part of iconometric

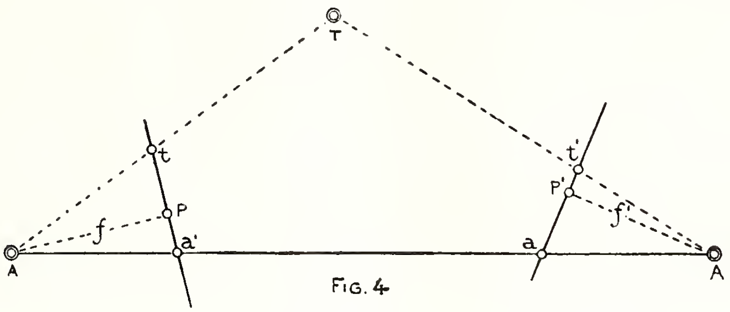


FIG. 4

platting, the subsequent location of picture points being accomplished by bringing the horizontal projections of the visual rays—lines of direction—drawn to identical points to their corresponding intersections.

I. ORIENTING THE PICTURE TRACES ON THE WORKING SHEET.

(1) A base line AA', measured in the field, has been platted to scale, fig. 2, and two pictures, MN and $M'N'$, fig. 3, had been obtained from the camera stations A and A^1 respectively by means of a surveying camera. The focal lengths of the pictures ($=f$ and f' respectively), the positions of the principal points (P and P'), and the horizon lines may also be given.

It is desired to locate T with reference to AA' upon the working sheet.

The distances: $AP=f$; $A'P'=f'$ (fig. 4); tP, $t'P'$, Pa' and $P'a$ (to be measured on the pictures MN and $M'N'$ respectively) and the line AA' are given.

The distances Aa' and $A'a$ may be found graphically (by constructing the right-angle triangles APa' and $A'P'a$), or they may be computed from the equations:

$$Aa' = \sqrt{(AP)^2 + (Pa')^2}$$

$$A'a = \sqrt{(A'P')^2 + (P'a)^2}$$

These distances are now laid off upon AA' from A and A' respectively, semicircles are described over Aa' and $A'a$, and two circles are drawn about A and A' with f and f' respectively, as radii.

The intersections P and P' of these two pairs of circles locate the horizontal projections of the principal points on the two picture-traces, the latter being represented by the tangents Pa' and $P'a$. The distances x $(= Pt)$ and x' $(= P't')$ are now measured on the pictures and laid off on the tangents as indicated in fig. 2, when the intersection of the lines drawn from A and A' through the points (just found) t and t' will locate the horizontal projection of T with reference to A and A'.

(2) The instrument used was a camera or phototheodolite:

In this case the angles α and α' (fig. 2) may be measured directly in the field.

We now plat the angles α and α' upon the base line AA' and make $AP = f$ and $A'P' = f'$.

The perpendiculars to AP and $A'P'$ in P and P', respectively, will represent the picture-traces (ta' and $t'a$) in correct orientation.

(3) When several pictured points (triangulation points) and the base line are given on the working sheet, the orientation of the picture-traces upon the map-projection may be accomplished as follows (fig. 5):

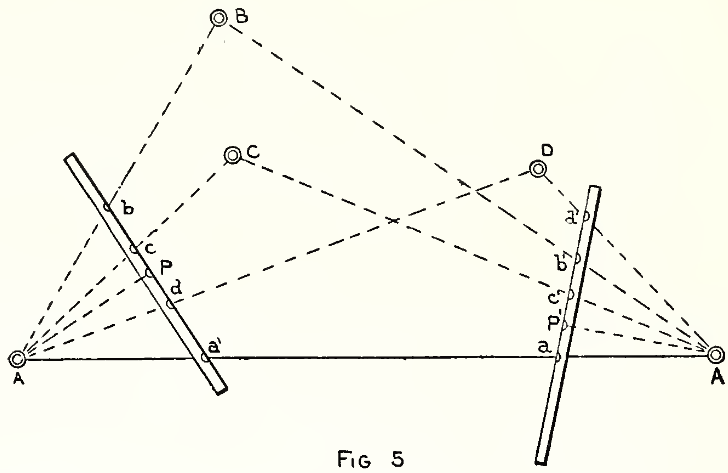


FIG 5

The rays AB, AC, AD, and $A'B$, $A'C$, $A'D$ are drawn upon the iconometric platting sheet, the points B, C, and D being already platted on the same.

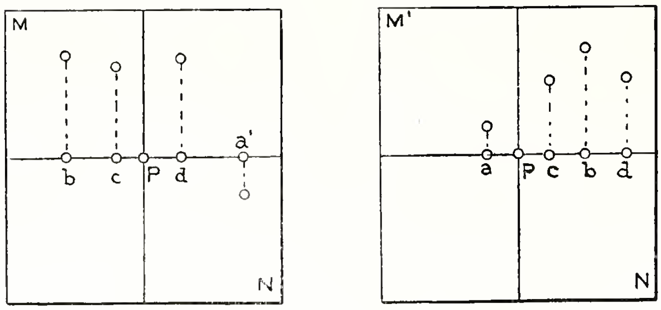


FIG 6

The points b, c, P, d, and a are transferred from the horizon line $00'$ of the negative MN (fig. 6) upon the perfectly straight edge of a strip of paper, which is placed upon the radials drawn from A (as center) to the points B, C, D. The strip is now moved about until

b falls upon the ray AB
c falls upon the ray AC
d falls upon the ray AD
a' falls upon the line AA'

The line AP should now be perpendicular to the straight edge of the paper strip, and the line *bcda'* drawn upon the working sheet (along the straight edge of the paper strip) will represent the oriented picture-trace of MN—

AP will be the distance line, and

P will be the horizontal projection of the principal point.

The same having been done regarding the point A' and its picture $M'N'$, both picture-traces will be oriented and the positions of any additional points, that may be identified on both pictures, may be located by platting their abscissæ (measured on the horizon lines of the pictures, regarding P as the origin of the coordinates) upon the picture-traces on the proper sides of the principal points. Lines drawn from the station points, A and A', through such corresponding points on the picture-traces will locate the relative positions of such points on the platting sheet by their points of intersection.

II. ARITHMETICAL DETERMINATION OF THE PRINCIPAL AND HORIZON LINES ON THE PICTURES.

In the preceding it had been assumed that each perspective was provided with the principal and horizon lines, which would be the case when an *adjusted* surveying camera or phototheodolite

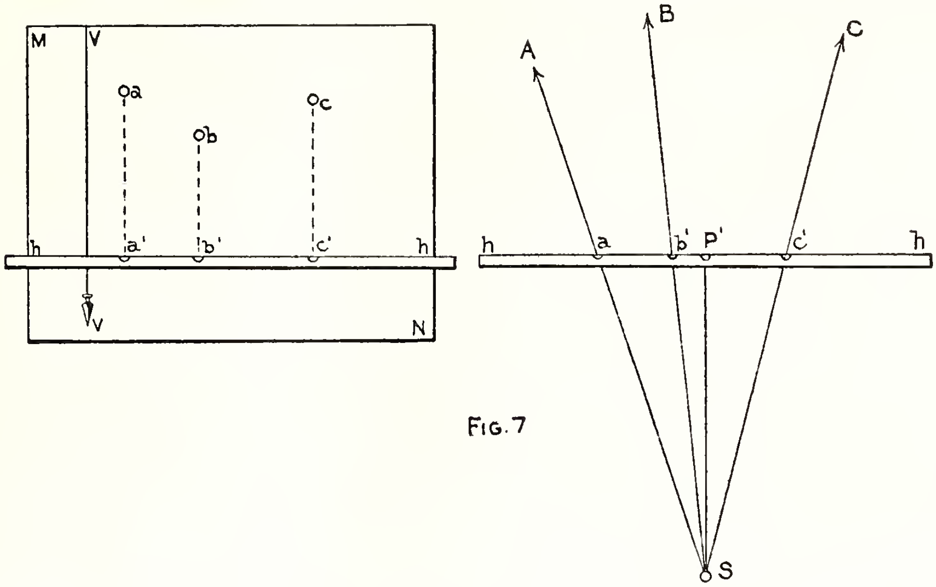


FIG. 7

had been used for obtaining the pictures. When an ordinary camera (with provisions to maintain the picture plane in a vertical position) or an unadjusted surveying camera is used, the correct position of the principal and horizon lines as well as the length of the distance line (focal length) must be ascertained, which may be accomplished in various ways:

(1) *Determination of the principal point and distance line of the perspective.*—A plumb-bob, suspended in such a way that the plumbline will be photographed upon the negative, may serve to establish the direction of the principal line VV (fig. 7) upon the trial plate.

The negative may also contain the images a, b, c, ... of three or more points A, B, C, ... of known positions. A line hh is drawn upon the negative perpendicular to VV, and the straight edge of a paper strip is placed upon this line. The pictured points a, b, c, ... are now projected upon the straight edge of the paper by drawing parallels to VV through the points a, b, c, ... (fig. 7).

After having drawn radials from the platted station S to the points A', B', C', ... the paper strip is adjusted over the former in such a way that the image projections a', b', c' will fall upon their corresponding radials, when the position (as indicated by the line hh) of the paper strip's edge will be the oriented picture trace. If we now draw a line (SP') from the platted station S perpendicular to hh, the point P' will be the horizontal projection of the principal point P, and SP' will be the distance line ($=f$) for the picture MN.

Whenever the positions of the points A, B, C, ... with reference to the station S are not known, it will become necessary to observe the horizontal angles ASB, BSC, CSD, ... instrumentally from the station S, and plat the same upon a sheet of paper in order to adjust the paper strip upon the radials, in the manner just described, to find the principal point and distance line (focal length).

(2) *Determination of the position of the horizon line on the perspective.*—When the elevations AA', BB', CC', ... of the points A, B, C, ... above the horizon of the station (S) are known, the position of the horizon line (oo') (fig. 8) may be found by constructing or by computing the lengths of the ordinates aa', bb', cc', ... from the relations:

$$aa' : AA' = Sa' : SA'$$
$$bb' : BB' = Sb' : SB'$$
$$cc' : CC' = Sc' : SC'$$

. . . .
. . . .
. . . .

whence

$$aa' = \frac{Sa' . AA'}{SA'} = y'$$

$$bb' = \frac{Sb' . BB'}{SB'} = y', \text{ etc.}$$

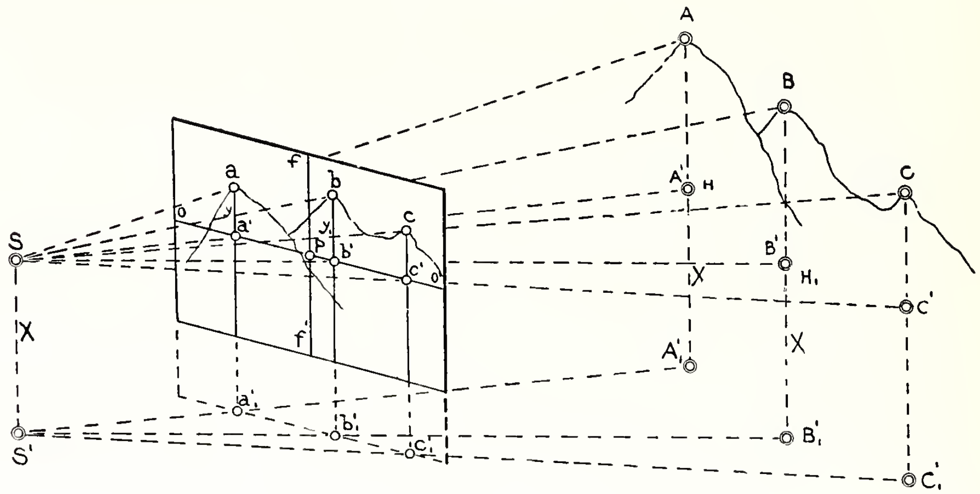


FIG. 8

The distances Sa', Sb', Sc', . . . are taken from the platting sheet (fig. 8) and the distances SA', SB', SC', ... as well as the differences in elevation AA', BB', CC', ... are known (if the points A, B, C, . . . had been located in the horizontal and vertical sense with reference to the station S).

For example:

Difference in elevation between A and $A' = 100^m$.
Distance of A' from the station $S = 1000^m$.
Distance Sa', measured on the platting sheet, $= 0{\cdot}5^m$.

The ordinate $aa' = \frac{0{\cdot}5 \times 100}{1000} = 0{\cdot}05^m$.

The horizon line (oo') on the negative will be 50 mm. vertically below (parallel with VV) the pictured point a.

The direction of VV (the principal line) being parallel to the pictured plumb line, this distance aa' is laid off in the same direction below a, and a line oo', drawn at right angles to VV through a', will locate the horizon line. The ordinates bb', cc', ... of the other pictured points may well serve to check this position of oo'. The horizon line will be the tangent to the arcs described with aa', bb', cc', ... about a, b, c, ... respectively, as centers.

The principal point P, may now be transferred to the negative by using the paper strip, and the line drawn through P perpendicular to oo' will be the principal line for the picture MN.

III. GRAPHIC METHOD FOR DETERMINING THE POSITIONS OF THE PRINCIPAL AND HORIZON LINES ON THE PERSPECTIVES.

The following graphic method for orienting the picture trace and locating the principal and the horizon lines was published by Prof. F. Schiffner in 1887; it is also mentioned by Prof. F. Steiner.

Three points, A, B, and C (fig. 9), may be given with reference to the station S upon the platting sheet.

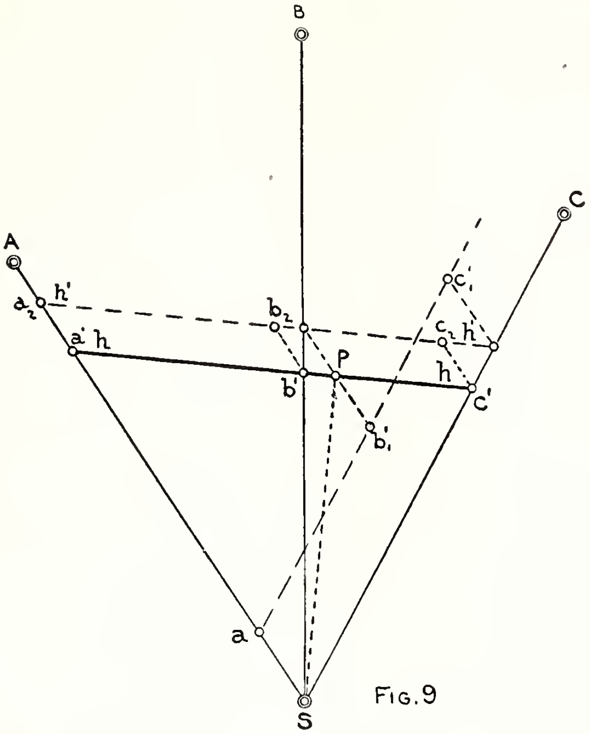


FIG. 9

From S radials are drawn through A, B, and C. Through a point a on the ray SA a parallel to SC is drawn, and the distance $a'b'$ (taken from the negative MN) is laid off from a ($= ab'_1$) upon this parallel, while the distance $b'c'$ is laid off upon the same line from b'_1 ($= b'_1c'_1$). Parallels to the radial SA are now drawn through the points b'_1 and c'_1 and prolonged to intersect the radials SB and SC. The line ($h'h'$) connecting these two points of intersection will be parallel with the direction of the picture trace.

The same distances $a'b'$ and $b'c'$ (taken from the negative) are laid off upon this line $h'h'$ from a_2 ($= a_2b_2$) and from b_2 ($= b_2c_2$). The lines drawn through these points b_2 and c_2, and parallel with the radial SA, are brought to intersections with the radials SB and SC, when the line (hh) passing through these intersections will represent the picture trace correctly placed (oriented) with reference to S, A, B, and C.

The distance SP of S from hh represents the distance line (focal length) of the picture MN, while the point P'' will be the horizontal projection of the principal point P.

After having transferred P'' (with reference to a', b', and c'), by means of a paper strip, to the negative MN, a parallel to VV, drawn through the transferred point P, will locate the principal line upon the negative.

The horizon line may now be located in the same manner as shown under II, 2, adopting the graphic solution.

IV. THE FIVE-POINT PROBLEM (BY PROFESSOR STEINER).

In the methods just described it had been assumed that the position of the camera station was known with reference to the surrounding points *A*, *B*, *C*. . . . .

In case the panorama pictures were taken from a camera station of unknown position and a series of known points are pictured upon the panorama views, the position of the camera station may be found (with reference to the surrounding points of known positions), and the orientation of the picture trace may be accomplished by means of Prof. F. Steiner's so-called "five-point problem" (fig. 10), if one of the views contains the pictures of *five* or more points of known positions.

The panorama view *MN* may contain the images *a*, *b*, *c*, *d*, and *e* of the points *A*, *B*, *C*, *D*, and *E* (already plotted upon the working sheet), and also the picture of a suspended plumb line or other vertical (or horizontal) line.

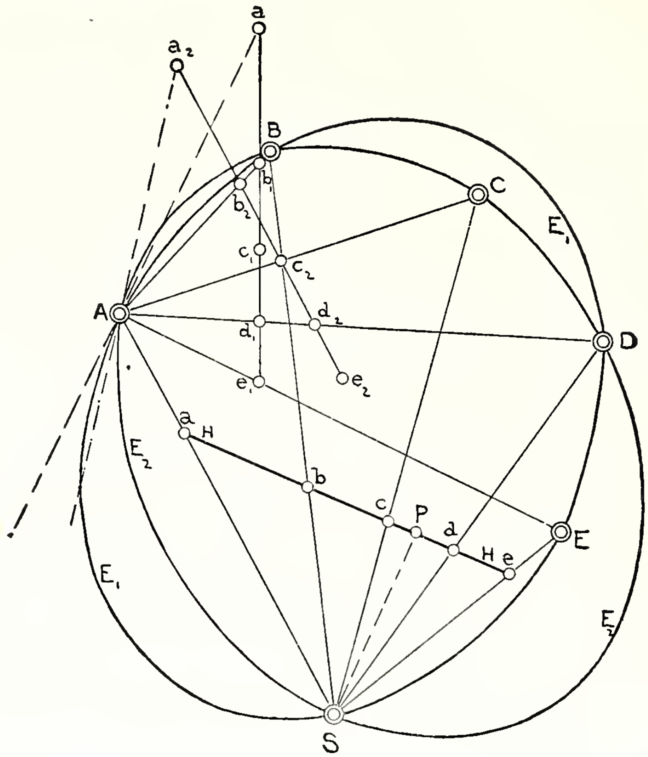


FIG. 10

The points *a*, *b*, *c*, *d*, and *e* of the negative are again projected upon the straight edge of a paper strip = *a'*, *b'*, *c'*, *d'*, and *e'*.

Radials are now drawn from one (*A*) of the five plotted points, as a center, to the other four, *B*, *C*, *D*, and *E*. The marked paper strip is then placed over the radials in such a way that

b' falls upon *AB*, *d'* falls upon *AD*, *e'* falls upon *AE*,

when the strip will have the position a_1, b_1, c_1, d_1, e_1. The line drawn through *A* and a_1 (the latter transferred by means of the strip) will be the tangent in *A* to the ellipse E_1 (passing through *A*, *B*, *D*, *E* and through the station point *S*).

The paper strip is now placed over the radials *AB*, *AC*, and *AD*, so that

b' falls upon *AB*, *c'* falls upon *AC*, *d'* falls upon *AD*,

when the strip will have the position a_2 b_2 c_2 d_2 e_2, and the line Aa_2 will be the tangent in *A* to the ellipse E_2 (passing through the points *A*, *B*, *C*, *D* and the station point *S*).

The position of the station point S on the working sheet (with reference to the five points A, B, C, D, and E) will be identical with the point of intersection of the two ellipses E_1 and E_2.

(1) *Determination of the principal point and distance line in the perspective.*—The distance line and the principal point are now found by drawing the radials SA, SB, SC, SD, and SE, and placing the paper strip over these in such a way that

a' falls upon SA, b' falls upon SB, c' falls upon SC, d' falls upon SD, e' falls upon SE,

which position is indicated by the line HH. The perpendicular upon HH passing through S (= SP) is the distance line and P is the principal point projected into horizontal plan, which

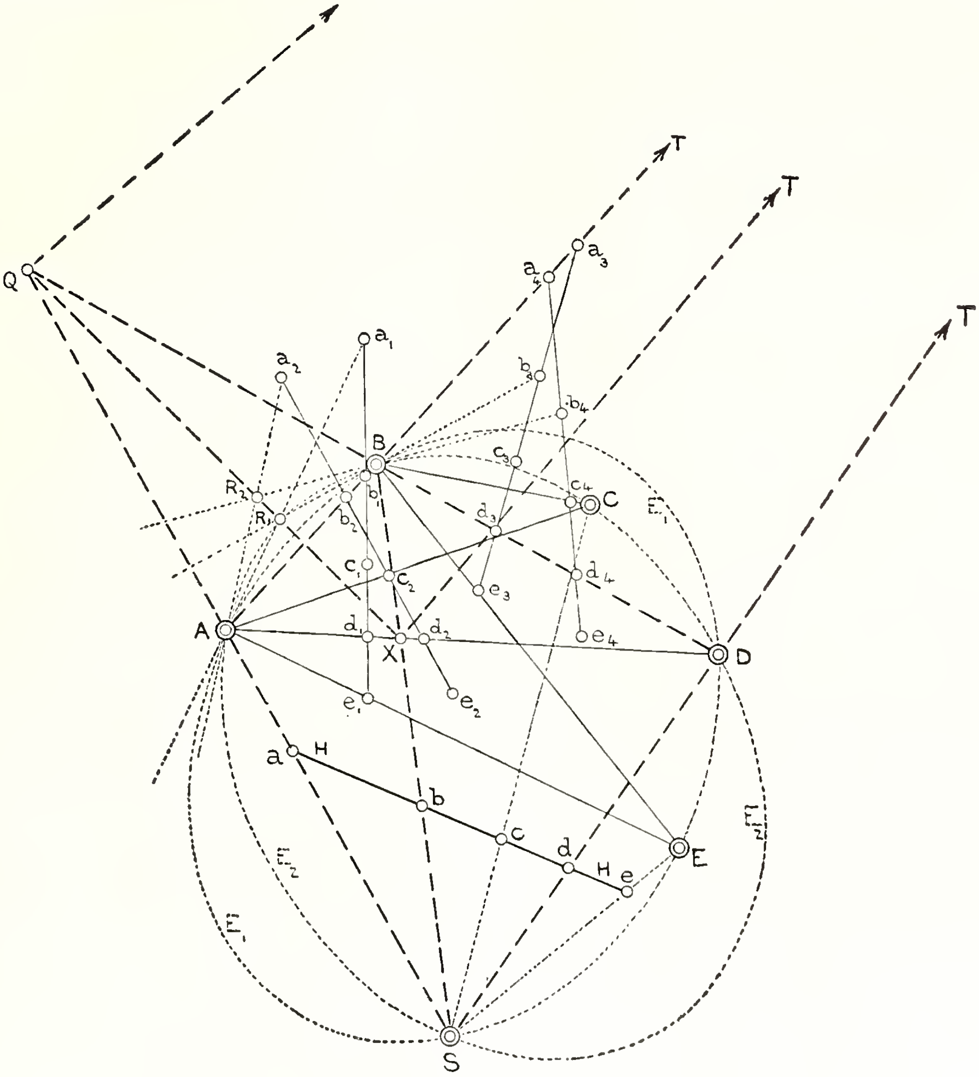


FIG. 11

may now be transferred to the picture by means of the paper strip in order to locate the principal line in a similar manner to that mentioned in the preceding pages.

(2) *Simplified construction for locating the camera station by means of the five-point problem.*—The preceding method is rather complicated, but Professor Schiffner devised the following construction (fig. 11), in which the drawing of the ellipses E_1 and E_2 is avoided:

The same five points, A, B, C, D, and E, with their images a, b, c, d, and e, on one plate MN, may be given.

The two lines, b_3B and b_4B, tangent in B to the two ellipses E_1 and E_2, respectively, are located precisely in the same manner as the two tangents a_1A and a_2A were found for the point A.

The intersections R_1 and R_2 of the tangent pairs a_1 A, b_3 B and a_2 A, b_4 B (belonging to the ellipses E_1 and E_2, respectively) are situated upon a line Qx, forming one side of the polar triangle QxT, common to both ellipses. This line Qx intersects the diagonal AD in x and the quadrilateral side BD in Q, and the lines drawn through Q from A and through x from B will intersect each other in the fourth point of intersection (S) of the two ellipses.

The quadrilateral $ABDS$, obtained by connecting the four points of intersection of the two ellipses, has the point x as the intersection of its diagonals. By prolonging the sides BD and AS to their point of intersection Q and the sides AB and SD to their point of intersection T, the three diagonal points QxT will form the polar triangle common to the two ellipses.

Also this method remains complicated and requires many lines to be drawn before the picture trace and the camera station (S) may be plotted.

(3) *Special application of the five-point problem for the case when the five points range themselves into a triangle.*—The application of the "five-point problem" becomes very much simplified, however, for the special case when the five points range themselves into a triangle, of which two sides (AC and CE) contain three points each (fig. 12).

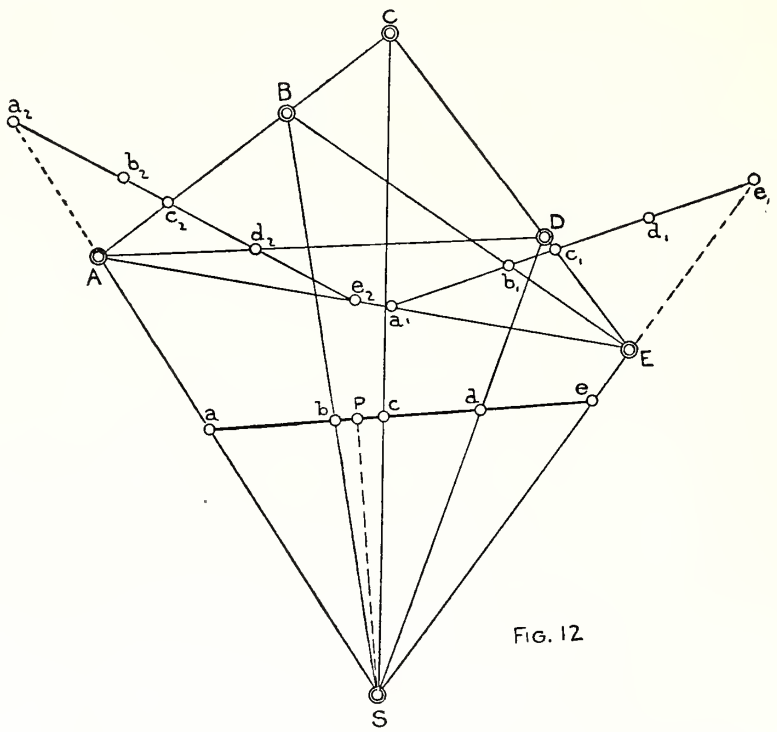


FIG. 12

If we now place the strip of paper upon the radials drawn from A, so that

e' falls upon AE, d' falls upon AD, c' falls upon AC,

it will have the position a_2 b_2 c_2 d_2 e_2, and the first ellipse (E_1) will resolve itself into the lines CE and Aa_2.

If we now place the paper strip a' b' c' d' e' upon the radials drawn from E to A, B and C, so that a' falls upon EA, b' upon EB, and c' upon EC, it will assume the position a_1 b_1 c_1 d_1 e_1, and the second ellipse (E_2) will have resolved itself into the lines AC and Ee_1.

The intersection S of the two lines Aa_2 and Ee_1 will locate the station point with reference to the five given points, and by placing the paper strip upon the radials SA, SB, SC, SD, and SE in such a way that a' falls upon SA, b' upon SB, etc., its edge will locate the picture trace.

(4) *To find the elevation* (x) *of a camera station* (S) *that has been located by means of the "five-point problem."*—In order to ascertain the elevation of the unknown station S, platted after one of the preceding methods, it will become necessary to know the elevations of at least two of the five points.

Let the elevation of the station S fig. 8 be designated by x.

The elevation of $A = H$ and of $B = H_1$. The ordinates $aa' = y$ and $bb' = y_1$.

From the relation $S'a_1' : S'A_1' = aa' : AA'$;

or

$$Sa' : SA' = y : (H - x)$$

we find

$$y = \frac{Sa'}{SA'} (H - x) \text{ and}$$

$$y_1 = \frac{Sb'}{SB'} (H_1 - x).$$

The difference between y and y_1 may be measured on the negative, hence

$$y - y_1 = m$$

is known, and the value for x may be found from the equation

$$y - y_1 = (H - x) \frac{Sa'}{SA'} - (H_1 - x) \frac{Sb'}{SB'} = m.$$

The values for Sa', SA', Sb', and SB' may be taken directly from the platting sheet, while those for H and H_1 are found in the triangulation records.

If we write the above equation in the general form—

$$\frac{H - x}{n} - \frac{H_1 - x}{o} = m,$$

the elevation x of the camera station S may be computed from—

$$x = \frac{mno - Ho + H_1 n}{n - o}.$$

The numerical values for the ordinates y and y_1 (locating the position of the horizon line on the perspective) may now be computed from the equations—

$$y = \frac{H - x}{n} \text{ and}$$

$$y_1 = \frac{H_1 - x}{o}$$

V. THE THREE-POINT PROBLEM.

If the triangulation points are not sufficiently close together that five or more points may be pictured on one perspective, and if stations are occupied with the camera that are not connected with the trigonometric survey, it will become necessary to employ other means to determine the position of the camera station with reference to the surrounding triangulation points.

In order to connect the camera station with the triangulation system by direct measurements and observations, made at the camera station, it will be requisite that at least three triangulation points be visible from such station, unless the location of the camera station is to be made by observations made from other stations. In the latter case the occupation of two (better three) triangulation points, if favorably located, would suffice to establish the ("concluded") position of the camera station.

The determination of the position of an occupied point by observing upon three fixed and known points is generally known as the "three-point problem," "station platting," "station pointing," or "*Pothenot's* method," although Snellius had used the same method in his trigonometric work in the Netherlands in the second decade of the seventeenth century. Let A, B, and C, fig. 13, be the three points, the positions of which are known. A fourth undetermined point S

may have been occupied from which the horizontal angles $A\ S\ B = M$ and $B\ S\ C = N$ may have been observed instrumentally. The position of S with reference to A, B, and C may then be ascertained in various ways.

(1) *Using the three-arm protractor (mechanical application of the three-point problem).*—The simplest (and crudest) method is purely mechanical in its application. The two horizontal angles M and N are laid off upon a three-arm protractor ("station pointer"), or upon a piece of tracing paper, moving the three radials SA, SB, and SC over the three fixed and platted points A, B, and C until the three radials SA, SB, and SC bisect their corresponding points A, B, and C. Holding the two angles M and N unchanged in this position, the point S is transferred to the working sheet.

(2) *Graphic solution of the three-point problem*—

(*a*) *Using the so-called "two-circle problem."*—Theoretically the best graphic method is that which locates the position of the fourth point S, fig. 13, as the intersection of two circles, one passing through A and B and having all angles of circumference $= ASB = M$ over AB that may be drawn over the line AB as chord, the other circle passing through B and C and having over BC as chord all angles of circumference equal to $BSC = N$.

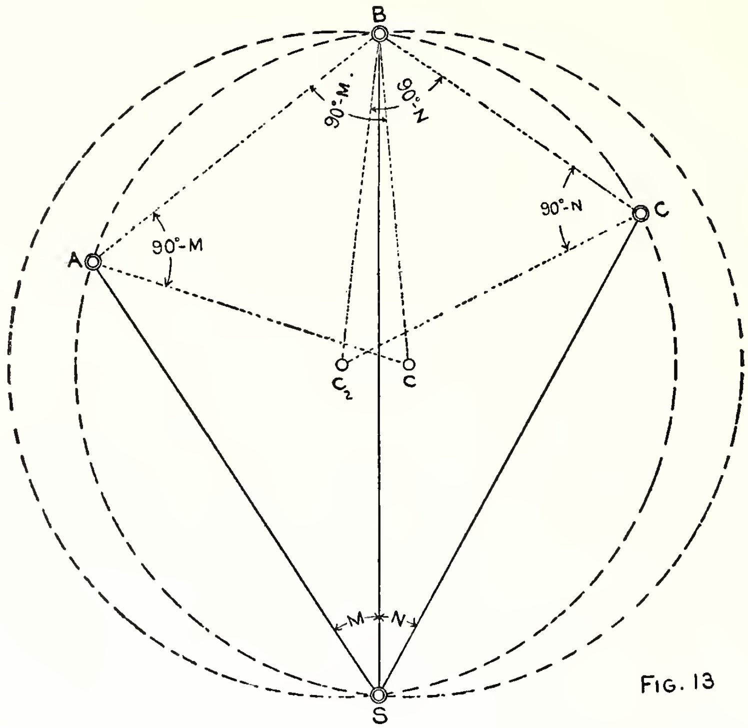


FIG. 13

From the platted triangle side AB we lay off at A and B the angles BAC and ABC_1 each equal to:

$$\frac{180 - 2\,(ASB)}{2} = 90° - ASB = 90° - M$$

and about the point c_1, thus obtained, a circle ABS is described with the radius $= c_1A = c_1B$. The observed angle $ASB = M$ will then be an angle of circumference over AB, and the point S will be located somewhere on the arc over the chord AB.

By means of the angle $BSC = N$ a second circle BCS is described over the triangle side BC, in a similar manner, about c_2 as center with the radius $c_1B = c_1C$. The observed second angle $BSC = N$ will be an angle of circumference over the chord BC, hence the point S will be situated also upon the arc over the chord BC and the true position of S is at the point of intersection S of the two circles.

(*b*) *Using the method of Bohnenberger and Bessel.*—The following constructive method (devised by Bohnenberger and Bessel) is readily applied and of a very simple character, fig. 14. If we describe a circle through two of the three given points A and B and the station S as the third point the angles

$$ASB = ACB = M \text{ and}$$
$$BSC = BAC = N$$ (being angles of circumference upon the same arcs AD and DC respectively).

Hence, if we lay off the observed horizontal angle M on the base line AC at C and the other horizontal angle N on AC at A, the point of intersection D of their convergent sides CD and AD will be on the line connecting the third point B and the platted station S. After having thus found the direction of the line DB the position of S on the line DB may be found as follows:

At any point x of the line DB the observed angles M and N are laid off to either side of DB, in the sense in which they were observed. Lines AS and CS drawn through A and C parallel to xy and xz, respectively, will locate the position of the station S (upon DB) with reference to the three points A, B, and C.

This construction is only recommended when BD is sufficiently long (in fig. 14 it is evidently too short) to admit of a correct prolongation of its direction toward S.

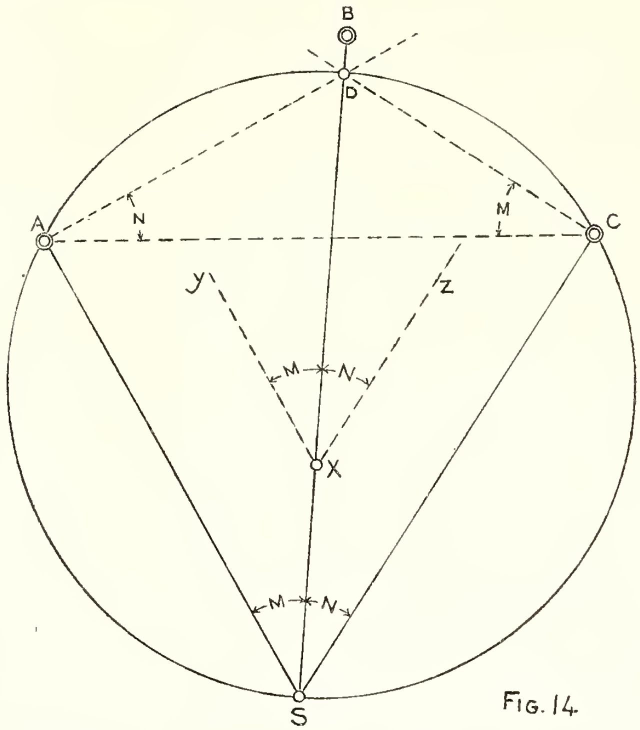


Fig. 14.

The picture trace containing the horizontal projections of the pictured points a, b, and c may now be oriented in the known manner by adjusting the paper strip over the radials SA, SB, and SC.

VI. ORIENTATION OF THE PICTURE TRACES, BASED UPON INSTRUMENTAL MEASUREMENTS MADE IN THE FIELD.

When no points are known of the area to be mapped phototopographically the elements of the perspective (horizon line, principal point, and distance line) can no longer be ascertained from the photograph alone, but instrumental observations will have to be resorted to. This method, having been adopted by Capt. E. Deville, will be described in Chapter III, II, 3, in connection with the Canadian method.

VII. RELATIONS BETWEEN TWO PERSPECTIVES OF THE SAME OBJECT VIEWED FROM DIFFERENT STATIONS ("KERNELPOINTS" AND "KERNELPLANES").

A more generalized application of photogrammetric methods has been inaugurated since Prof. G. Hauck published his investigations and results regarding the relationship existing between systems of three lines, each of the latter being in a different plane. ("Theorie der trilinearen Verwandtschaft ebener Systeme," Journal für reine und angewandte Mathematik, herausgegeben von L. Kronecker und A. Weierstrass, 1883, Bd. 95.)

The practical value of Professor Hauck's deductions had been tested by the students attending his lectures in 1882 during the exercises which are connected with the course in descriptive geometry at the Technical High School in Berlin (Charlottenburg).

(1) *Kernelpoints and kernelplanes.*—In his discussion of the relationship existing between two perspectives of the same object taken from different stations, Professor Hauck has evolved some

properties which may be very useful and of value in iconometric platting. The principal law involved in the application of photogrammetry may be stated as follows:

If two projections (perspectives or photographs) of the same object are projected by perspective rays emanating from the "kernelpoints" ("kernpunkte") as centers the line of intersection of the two planes of projection (picture planes) will be their perspective axis.

With the aid of this law the projection on a third plane of an object may be deduced from the given projections on two planes of the same object. Or, for our case:

If two photographs, MN and $M'N'$, taken from two stations S and S' (and representing the same object), are given, the orthogonal horizontal projection (ground plan) of the same object may be constructed therefrom.

Professor Hauck's methods are also applicable to photographs obtained when the plate was exposed in an inclined position. In order to illustrate the connection existing between two different perspectives of the same object, we will refer to fig. 15, representing the simple case where the two perspective planes (MN and $M'N'$) are vertical.

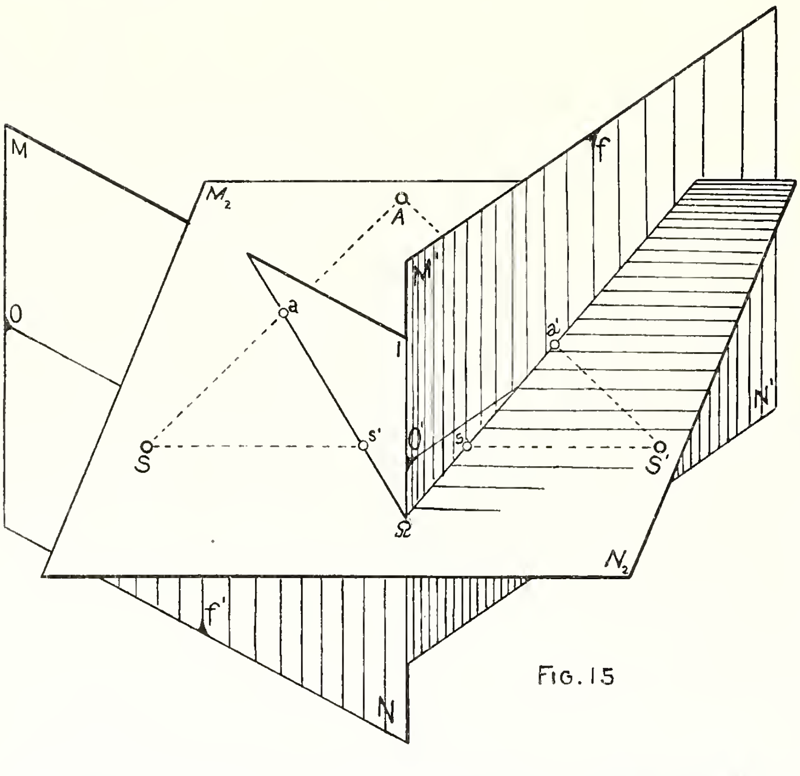


Fig. 15

Let S and S' represent the two camera stations (centers of projection or points of view for the vertical picture planes MN and $M'N'$), s' the picture of S' in MN, s the picture of S in $M'N'$, $I\Omega$ the line of intersection of the two picture planes MN and $M'N'$, a image of the point A in MN, a' image of the point A in $M'N'$.

The two pictured points s and s' are the so-called "kernelpoints" (kernpunkte), and any plane ("kernelplane") passing through the line (base line) SS' will contain the "kernelpoints" s and s'.

The position of the "kernelpoints" may be found graphically by passing a plane ("kernelplane") through the two stations and a third point A (pictured in both planes MN and $M'N'$), which will intersect the first picture plane MN in the line as' and the picture plane $M'N'$ in sa'. Then the following conditions will prevail:

1. The lines of intersection as' and $a's$ will intersect the line $I\Omega$ in one point (Ω).
2. The pictures a and a' of the point A will be on the lines as' and $a's$.
3. The lines as' and $a's$ will pass through the pictures (s' and s) of the two camera stations (S' and S).

The lines $S'A$, SA, SS', as', and $a's$ being situated in the "kernelplane" M_2N_2, all lines as' (for all points of the object pictured in MN) will pass through the picture s' ("kernelpoints") of the second camera station S', and all lines $a's$ (for all points of the object pictured in $M'N'$) will pass through the picture s of the first camera station S. Furthermore, all lines (as' and $a's$) joining the two perspectives (pictures) of identical points (A) with the corresponding "kernelpoints" (s' and s) will intersect the line of intersection ($I\Omega$) of the two picture planes (MN and $M'N'$) in the same point (Ω).

Therefore, if two photographs (MN and $M'N'$) of the same object (A) contain the pictures (s' and s) of their reciprocal stations (S' and S), conditions peculiarly adapted for the facilitation of the iconometric constructions will arise, inasmuch as such pictured stations (s and s') will be "kernelpoints."

The line of intersection ($I\Omega$) of the two picture planes (MN and $M'N'$) may also play an important part in the iconometric platting, not only for pictures exposed in vertical planes, but even more so when they are exposed in inclined planes.

If two pictures MN and $M'N'$ are given (in fig. 16 their traces are represented as HH and $H'H'$, respectively) representing the same object (viewed from two stations S and S'), then the pictures s and s' ("kernelpoints") of the reciprocal camera stations may be located upon the picture planes by construction (if they are not shown in the field of the pictures), as shown in fig. 16.

The horizontal projections (s_1 and s_1') of the "kernelpoints" (s and s') are identical with the points of intersection of the base line (SS') and the pictures traces (HH and $H'H'$). The horizontal projections of the line of intersection ($I\Omega$, fig. 15) of the two picture planes (MN and $M'N'$) will be represented by the point of intersection (i) of the two picture traces (HH and $H'H'$).

Hence, if we revolve the picture planes about their ground lines until they fall within the horizontal plane of the ground plan, the line $I\Omega$, fig. 15 (common to both picture planes), will be represented by the lines $i(I)$, fig. 16, and the "kernelpoints" s and s' of the revolved planes will fall upon the lines $s_1(s)$ and $s'_1(s')$, respectively. (These lines are perpendiculars upon the picture traces (HH and $H'H'$) in the horizontal projections of the "kernelpoints".)

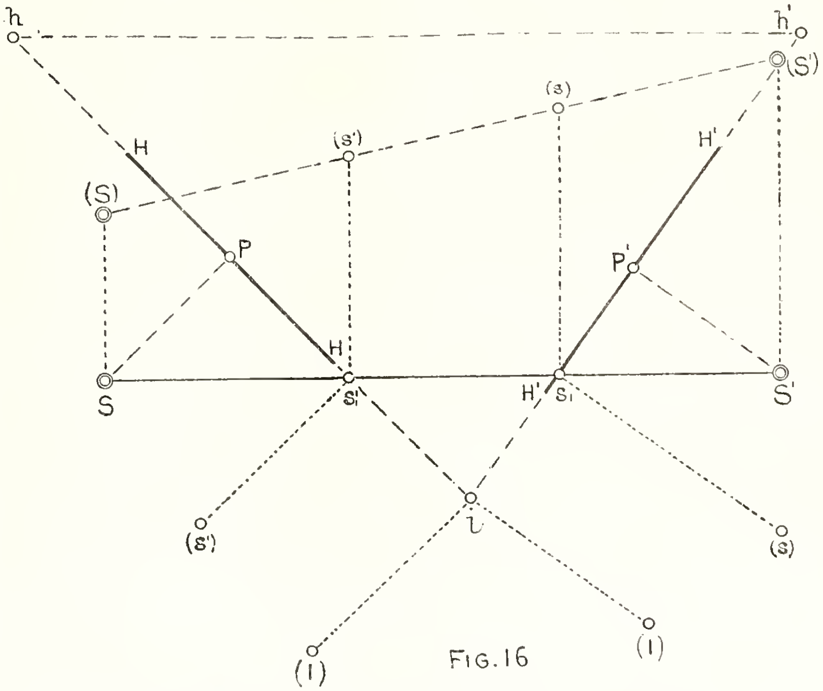


FIG. 16

To find the lengths $s_1(s)$ and $s'_1(s')$ (ordinates of the "kernelpoints" in the picture planes), perpendiculars are erected in S and S', fig. 16, and their lengths are made equal to the elevations of the respective camera horizons above the ground plane $= S(S)$ and $S'(S')$, respectively. The line $(S)(S')$, connecting the camera stations S and S' (in fig. 16 the vertical plane passing through the camera stations S and S' has been revolved about the horizontal projection of the base line SS' until it coincides with the horizontal ground plane) will intersect the lines $s_1(s)$ and $s'_1(s')$ (which are perpendicular to the horizontal projection of the base line in the "kernelpoints" s_1 and s'_1), and the lengths $s_1(s)$ and $s'_1(s')$ will equal the ordinates of the kernelpoints. In this manner the "kernelpoints" may be located in the picture plane of any photograph.

(2) *Use of the line of intersection ($I\Omega$) of two picture planes (MN and $M'N'$) which show identical objects viewed from two different stations (s and s').*—If a series of characteristic points of the terrene, pictured in a vertical picture plane MN, fig. 17, are connected with the "kernelpoint" s by straight lines, these will (when prolonged) intersect the line $I\Omega$, and if the pictures of the identical points in the vertical picture plane $M'N'$ are joined with the "kernelpoint" s', and if these lines are likewise prolonged to intersect the line ($I\Omega$), forming the intersection of the two picture planes (MN and $M'N'$), the series of intersections of $I\Omega$ with the first group, belonging to MN, will be identical with the intersections of $I\Omega$ with the second group of lines, belonging to $M'N'$.

If we now imagine the line $I\Omega$ provided with a scale of equal parts, with zero in the ground plane GG, fig. 17, lines drawn through the "kernelpoints" and identical points of objects pictured in both picture planes (MN and $M'N'$) will intersect identical points of the scale. The space (OO') intercepted on the scale by the horizon lines of the two picture planes will represent the difference in elevation between the two camera stations (S and S'). This scale may be drawn to show on both lines $I\Omega$ of the pictures when separated.

The picture (photograph) itself frequently may not be sufficiently extended to contain the line $I\Omega$, in which case the scale may still be utilized by laying it off on a line xx'' on picture MN and on a line zz'' on picture $M'N'$, where xx'' and zz'' are parallel with the line of intersection ($I\Omega$) of the two picture planes MN and $M'N'$ and as long as the following relation remains fulfilled:

$$s\Omega \; : \; sx' = s'\Omega \; : \; s'z'$$

For a second point B pictured as b and b' on the two picture planes MN and $M'N'$, respectively, the following proportions must stand:

$$s\beta \; : \; sx_0 = s'\beta \; : \; s'z_0$$

From the similarity of the triangles sx_0x', $s\beta\Omega$, $s'z_0z'$, and $s'\beta\Omega$, we find:

$$x_0x' = z_0z'$$

($\beta\Omega$ being common to both triangles $s\beta\Omega$ and $s'\beta\Omega$), which means the spaces on the scales xx'' and

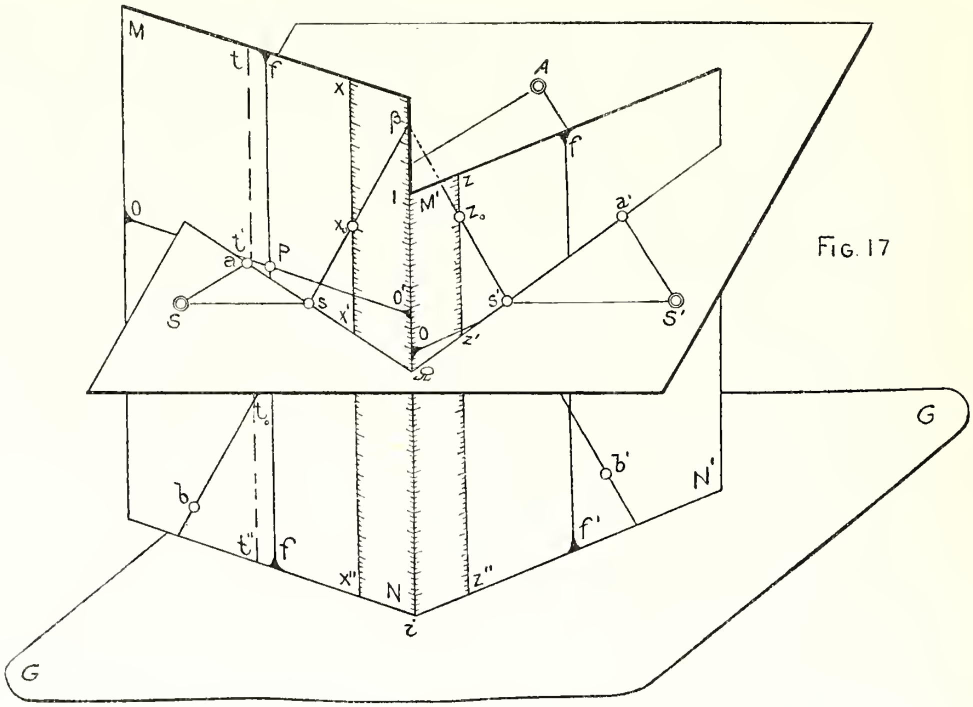


FIG. 17

zz'' are the same in numerical value. The two scales (or one of them) may also be placed beyond s and s'—for example, at tt''—in which case:

$$s\beta \; : \; st_0 = s\beta \; : \; sx_0$$
$$= s'\beta \; : \; s'z_0$$

when the scale tt'' should be read in the directions from t' toward t_0. It may generally be stated that the scales should be placed parallel to $I\Omega$ and at distances from the "kernelpoints" in proportion to the distances from the latter to the line of intersection of the picture planes, their correct positions being best found graphically from the horizontal projection or from the ground plan. To avoid the obscuring of details on the photographs it is recommended to draw these scales outside of the picture proper.

To find the best position of the second scale on the second picture, graphically, after the position for the first scale on the first picture has been decided upon, we will again refer to fig. 16, where HH and $H'H'$ = picture traces, S and S' = horizontal projections of the camera stations, P and P' = traces of the principal lines ff and $f'f'$, and h = selected positions for the first scale.

To find the corresponding position of the second scale, draw a line hh' parallel to SS' through h,

$$s'_1 i \; : \; s'_1 h = s_1 i \; : \; s_1 h'$$

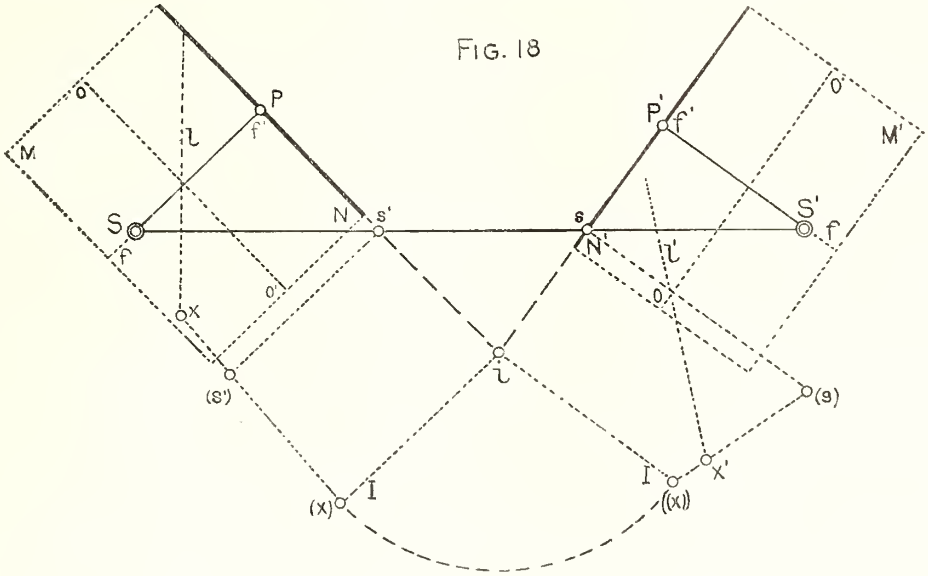
whence $s_1 h'$ = distance of the second scale from the "kernelpoint" s_1 in the second picture plane.

The conditions and relations just described, and first discussed by Prof. G. Hauck, may often serve with advantage in iconometric platting (in the following we will refer to them again).

For example: If we consider the case of a straight line L, fig. 18, shown on MN as l, of which, however, only the short piece l' is pictured on $M'N'$, and it is desired to locate a point x, identified on l in MN, but falling on the prolongation of l' outside of the picture limit of $M'N'$, we may proceed as follows:

The pictured point x on l in MN is connected with the "kernelpoint" (s') and this line $(s')\,x$ is prolonged to intersect Ii in (x). After transferring this point (x) to the line $i\,I$ of the second

FIG. 18



picture plane $M'\,N'$ to $((x))$, the latter point is connected with the "kernelpoint" (s), and the intersection of $((x))\,(s)$ and line l' will be the point sought, x', of the prolonged line l'.

VIII. TO PLAT A FIGURE, SITUATED IN A HORIZONTAL PLANE, ON THE GROUND PLAN BY MEANS OF ITS PERSPECTIVE.

In topographic surveys, figures in level planes are not frequently dealt with, except when locating the outlines of lakes and marshes, including coast lines, and the simplest way to plat these would be to expose photographic plates (held in a horizontal position) from a balloon at points of known positions and at identical or known elevations.

The platting of such figures, when photographed on vertically exposed (also inclined) plates from stations higher than the figure's plane, will also be an easy matter. It may even be accomplished if but one view of such figure had been obtained from only one station (of a known position), provided the difference in elevation between the camera station and the figure's (horizontal) plane, the principal point, and the focal length of the view are known.

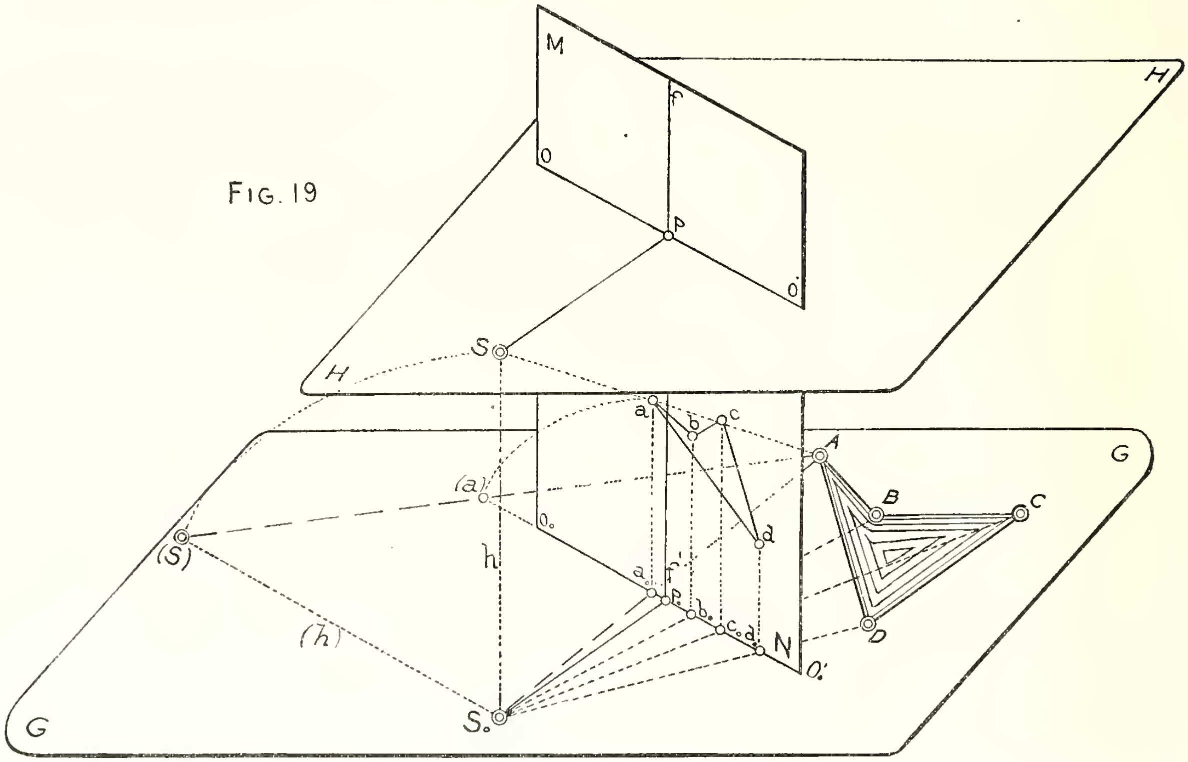
With reference to fig. 19: $H\,H$ = Horizon plane of the camera station S, $M\,N$ = Picture plane (vertical), $G\,G$ = Ground plane or horizontal plane coinciding with the surface of the lake $A\,B\,C\,D$, $S\,S_0 = h$ = difference in elevation between the camera station S and the surface of the water in the lake $A\,B\,C\,D$.

From the picture $a\ b\ c\ d$ (of the lake $A\ B\ C\ D$) with focal length $= SP$ and known difference in elevation $= h$ the horizontal projection of the lake $A\ B\ C\ D$ is to be plotted.

The ground line $O_o\ O_o'$ (intersection of ground plane $G\ G$ and picture plane $M\ N$) is drawn through P_o parallel to the horizon line $O\ O'$ ($P\ P_o = h$, measured in the platting scale). If we now project the pictured points a, b, c, and d upon $O_o\ O_o' = a_o$, b_o, c_o, and d_o, and draw radials from the platted station S_o through the points a_o, b_o, c_o, and d_o, they will pass through the points A, B, C, and D (which are to be platted), and the latter could be located if their distances from S_o were known.

We now regard the vertical plane, passing through the camera station S and pictured point a, which intersects the ground plane in the line $S_o\ a_o$ or in $S_o\ A$. From the similar triangles $S\ S_o\ A$

FIG. 19



and $a\ a_o\ A$ we can find the distance $S_o\ A$ (the horizontal distance from the camera station to the point sought, A) either graphically or arithmetically.

When the vertical plane SS_oA is revolved about S_oA until it coincides with the ground plane GG, the points S and a will assume the positions (S) and (a) respectively, in the ground plane, and the line connecting (S) and (a) will pass through the point A of the lake. Hence, A may be located in the ground plan as the intersection of $(S)(a)$ with S_oa_o.

The same may be done for the points B, C, and D by revolving the vertical planes SS_oB, SS_oC, and SS_oD about S_ob_o, S_oc_o, and S_od_o, respectively, into the ground plane to locate the positions of B, C, and D.

To avoid a multiplicity of lines on the working or platting sheet, these constructions are preferably made on a separate sheet of paper, and the following construction may be adopted:

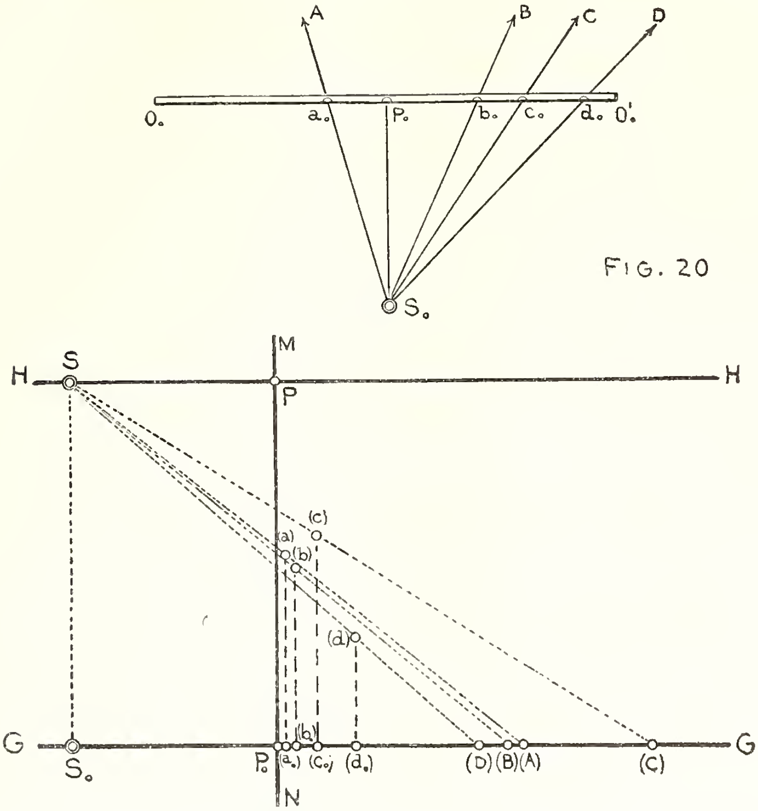


FIG. 20

The vertical planes S_0a_0, S_0b_0, S_0c_0, and S_0d_0 may be revolved about SS_0 as axis until they all coincide with the principal plane $SS_0\,PP_0$ (fig. 20), where the paper surface may represent the principal plane.

HH = trace of the horizon plane in the principal plane.
MN = trace of the picture plane in the principal plane.
GG = trace of the ground plane in the principal plane.
SS_0 = difference in elevation between the station S and the ground plane (surface plane of the lake $ABCD$), measured in the platting scale.
SP = S_0P_0 = true length of the focal distance for the photograph MN.

The radials S_0a_0, S_0b_0, S_0c_0, and S_0d_0 are laid off upon the line GG from S_0. The verticals $(a_0)(a)$, $(b_0)(b)$, $(c_0)(c)$, and $(d_0)(d)$ are made equal to the ordinates aa_0, bb_0, cc_0, and dd_0 (measured on the picture). Radials drawn from S through (a), (b), (c), and (d) will cut off on the line GG the horizontal distances $S_0(A)$, $S_0(B)$, $S_0(C)$, and $S_0(D)$, equal to the horizontal distances S_0A, S_0B, S_0C, and S_0D, measured in the platting scale. If these distances are laid off upon the radials S_0a_0, S_0b_0, S_0c_0, and S_0d_0 the positions of the characteristic points A, B, C, and D of the lake will be platted in the scale of the map with reference to the ground line $O_0O'_0$ (which on the platting sheet is identical with the picture trace) and the platted station S_0.

The same result may be arrived at by utilizing the orthogonal projection of the points a, b, c, and d and A, B, C, and D in the principal plane, instead of revolving the vertical planes into the principal plane.

With reference to fig. 21: PP = Principal plane, MN = Picture plane, HH = Horizon plane (containing camera station S), GG = Ground plane or surface plane of the lake.

If we draw the radials S_0a_0, S_0b_0, S_0c_0, and S_0d_0 from S_0 (orthogonal projection of S in GG) through the orthogonal projections a_0, b_0, c_0, and d_0 of the pictured points a, b, c, and d on the ground line O_0O_0', the points sought will be situated upon those radials.

If we now project the points a, b, c, and d (in the picture plane) upon the principal line $= \alpha$, β, γ, and δ, the radials $S\alpha$, $S\beta$, $S\gamma$, and $S\delta$, drawn in the principal plane PP, will locate the

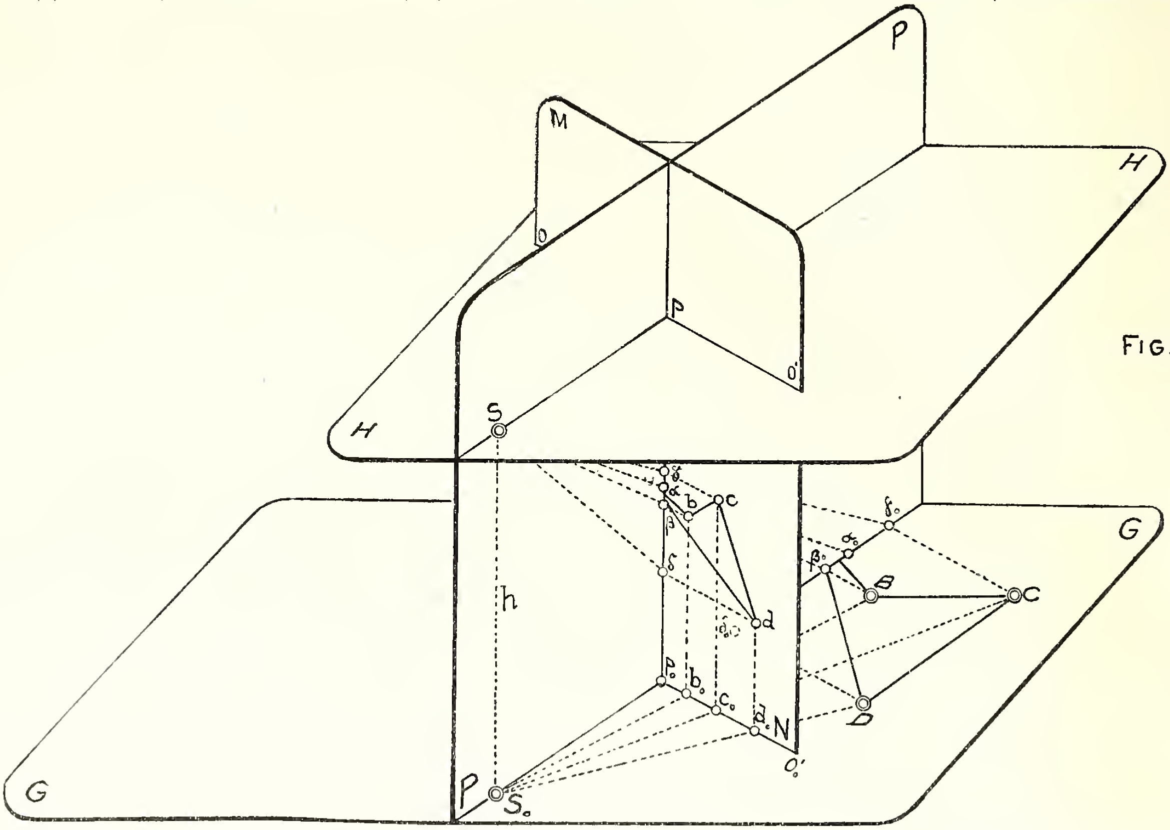


FIG. 21

points α_0, β_0, γ_0, δ_0 upon the line S_0P_0 (in the ground plane), they are the orthogonal projections of the points A, B, C, and D in GG upon S_0P_0. The points A, B, C, and D in the ground plane may therefore be found by erecting perpendiculars upon S_0P_0 in α_0, β_0, γ_0, and δ_0. The intersec-

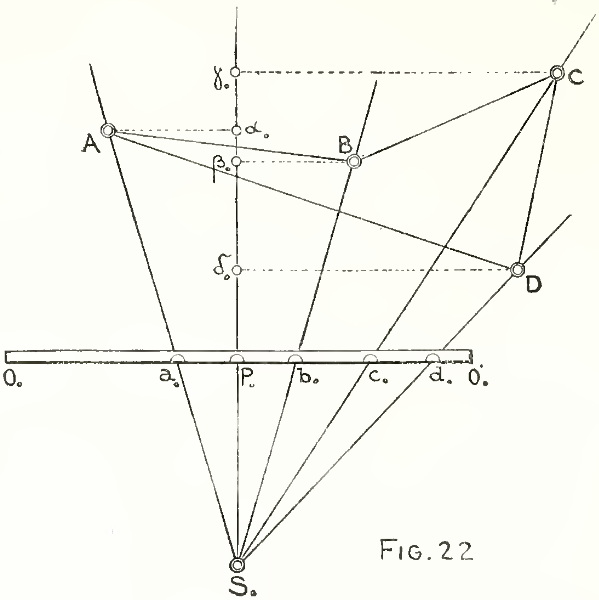


FIG. 22

tions of these with the radials S_0a_0, S_0b_0, S_0c_0, and S_0d_0 will locate the positions of the points A, B, C, and D on the platting sheet.

This construction is also preferably made on a separate sheet of paper. The radials S_0a_0, S_0b_0, S_0c_0, and S_0d_0, fig. 22, are drawn through their corresponding points on the platted picture trace (or ground line) O_0O_0', and the rest of the construction (fig. 23) is made by regarding the paper surface as the principal plane. The designations are the same as in fig. 20. The points δ, β, α,

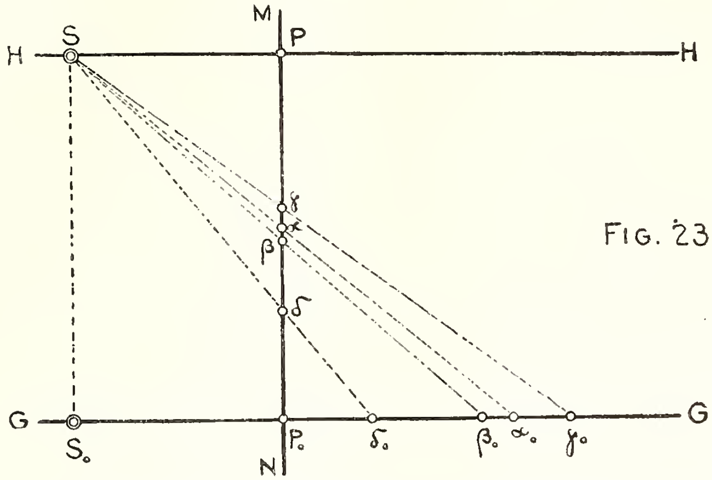


FIG. 23

and γ, fig. 23, on the line PP_0 (principal line) represent the projections of the pictured points a, b, c, and d in the principal plane; hence, their positions are found by transferring the ordinates of the pictured points to PP_0 from P_0:

$$P_0\ \delta = dd_0;\ \ P_0\ \beta = bb_0;\ \ P_0\ \alpha = aa_0,\ \text{and}\ P_0\ \gamma = cc_0$$

The radials from S through δ, β, α, and γ locate the points δ_0, β_0, α_0, and γ_0 on the line GG (or S_0P_0), fig. 23.

By transferring the distances $S_0\delta_0$, $S_0\beta_0$, $S_0\alpha_0$, and $S_0\gamma_0$, fig. 23, to the line S_0P_0 from S_0, fig. 22, and drawing lines through δ_0, β_0, α_0, and γ_0 parallel with O_0O_0', their intersections with the corresponding radials S_0d_0, S_0b_0, S_0a_0, S_0c_0 will locate the platted positions of the points D, B, A, and C of the lake.

IX. TO DRAW A PLANE FIGURE ON THE GROUND PLAN, BY MEANS OF THE SO-CALLED "METHOD OF SQUARES," IF ITS PERSPECTIVE AND THE ELEMENTS (POINT OF VIEW AND DISTANCE LINE) OF THE VERTICAL PICTURE PLANE ARE GIVEN.

If we imagine the figure covered with a net of squares, one set of its sides being parallel with

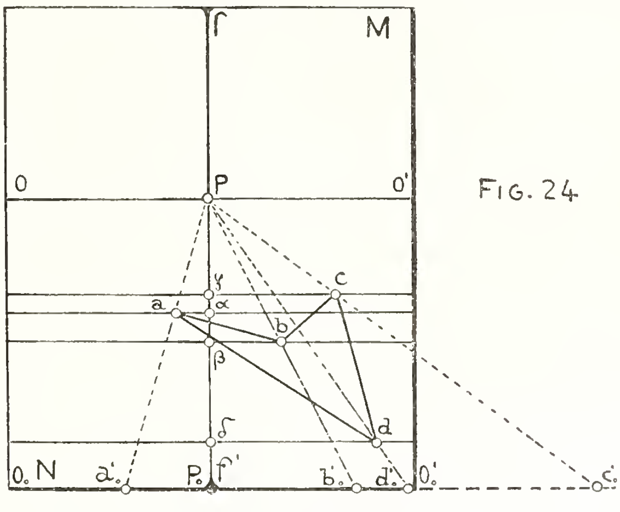


FIG. 24

and the other set being perpendicular to the ground line, such net may be utilized to draw the outline of the figure upon the ground plan, it being only necessary to cover the pictured figure *a b c d* with the perspective of the selected net in the ground plane, i. e., the lines forming the squares of the perspective must have the proper relation to the principal ray and horizon line.

The simplest disposition of the lines (forming the auxiliary network) for locating the figure is the one mentioned above (parallel with and perpendicular to O_oO_o'), but any other selection may be made. The squares may be of equal size or not, and the directions of the lines composing the network may be given any direction.

In fig. 24, in illustration of this method, the lines in the perspective which correspond to the

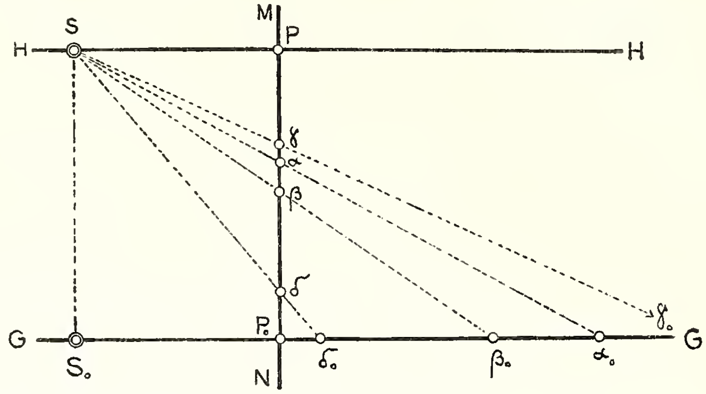


FIG. 25

sides of the rectangles that are perpendicular to the ground line O_oO_o' will vanish in the principal point P, and those parallel with the ground line O_oO_o' will be parallel with the horizon line OO'. Selecting the lines of this network so that two lines of each system will pass through one of the

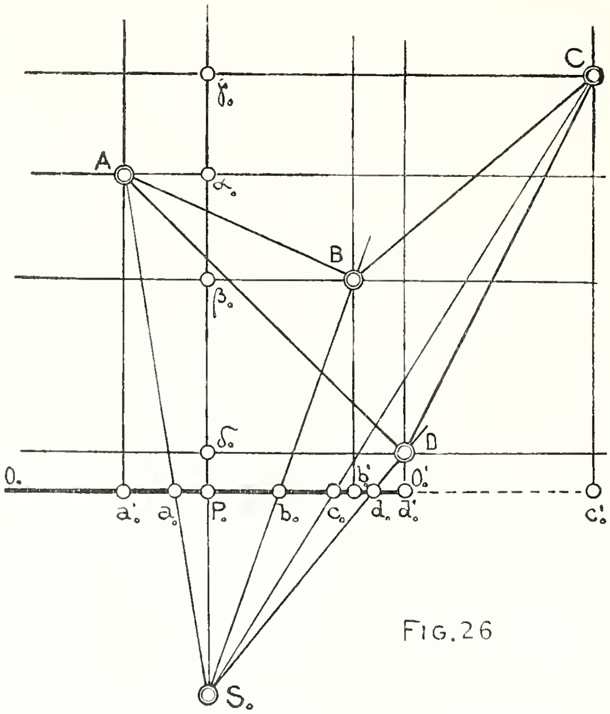


FIG. 26

characteristic points of the figure *abcd*, the perspective of this net will appear as shown in fig. 24, where O_oO_o' represents the ground line of the picture plane MN.

If we now plat the principal plane SS_oP_oP, fig. 25, retaining the same designations as for fig. 20, the points δ_o, β_o, α_o, and γ_o will represent (in the ground plane) the intersections, with the horizontal projection of the principal ray SP ($= S_oP_o$), of those net lines that had been drawn through D, B, A, and C parallel with the ground line.

After platting the picture trace O_oO_o' (of the perspective MN, fig. 24) in the ground plane by means of the radials S_oa_o, S_ob_o . . . . . . , the distances $S_o\delta_o$, $S_o\beta_o$ . . . . . . (fig. 25) laid off upon S_oP_o, fig. 26, will locate those net lines (parallel with O_oO_o') in the ground plane which correspond to the lines $d\delta$, $b\beta$ . . . . shown in the perspective MN, fig. 24.

If we now transfer the points a_o', P_o, b_o', d_o', and c_o' from fig. 24 to a strip of paper, and place this upon the picture trace O_oO_o', fig. 26, that the points P_o will coincide, the lines $a_o'A$, $b_o'B$, . . . drawn parallel with S_oP_o will represent the net lines which are perpendicular to the ground line $O_oO'_o$.

Thus the platted positions of the points A, B, C, and D are located on the ground plane by the intersections of the corresponding net lines of both systems, as shown in fig. 26.

The points A, B, C, and D will, of course, also be bisected by the radials S_oa_o, S_ob_o . . . . . , which fact may make it more advantageous to select some other disposition of the net lines for a figure of a different shape.

When the figure has a sinuous perimeter the squares of the network should be selected of a size sufficiently small to enable the draughtsman to draw the perimeter sections falling within the squares sufficiently accurate to obtain a correct representation of the general outline.

X. THE VANISHING SCALE.

We had seen, fig. 26, that the radials drawn from the so-called "foot of the station" (S_o) represent the directions to the points A, B, C . . . . . in the ground plane, and if we could determine the distances S_oA, S_oB . . . . (from the foot of the station S_o to the points to be platted A, B, ...) from the perspective in some manner the location of the platted positions in the ground plane would become an easy matter.

The distances S_oA, S_oB . . . . . . , fig. 26, may be determined from the perspective by means of the so-called vanishing scale, which may be constructed as follows, fig. 27:

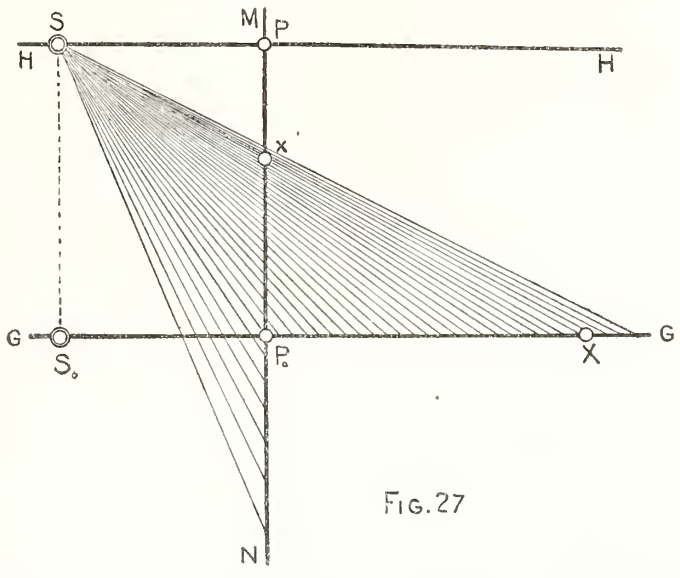


FIG. 27

MN = trace of picture plane in the principal plane, HH = trace of horizon plane in the principal plane, GG = trace of ground plane in the principal plane, SS_o = elevation of the station S above the ground plane GG, or above the foot of the station S_o.

A scale of equal parts is laid off upon GG, to either side of P_o, and radials are drawn from S through the graduation points of the scale; their intersections with MN form the vanishing scale, which may serve to locate distances from the foot of the station to points to be platted in the ground plane.

The picture trace O_0O_0', fig. 28, may have been platted and the radials S_0a_0, S_0b_0, ... may have been drawn on the working sheet.

It is desired to locate the position of a point, A, in the ground plane by means of the vanishing scale and the picture a, fig. 29, of the point A.

Take the ordinate aa_0 from the perspective MN, fig. 29 (vertical distance of a above the ground line O_0O_0'), and lay it off upon the vanishing scale (fig. 27), PP_0 from $P_0 = P_0x$.

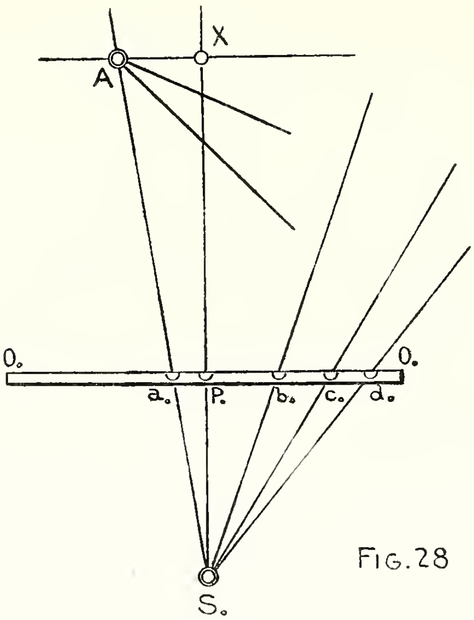


FIG. 28

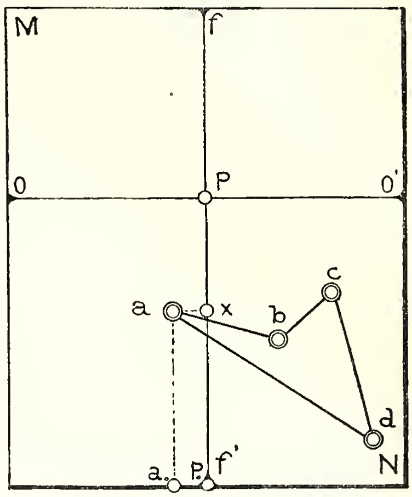


FIG. 29

The line ax, fig. 29, parallel with the horizon line OO' and passing through a in the perspective, corresponds with the line AX, fig. 28, parallel with the ground line and passing through A in the ground plane.

Hence, if we lay off S_0X, fig. 27, upon S_0P_0 from S_0, fig. 28, the point A (in the ground plane) will be situated upon the line XA, fig. 28, drawn through X and parallel with the ground line O_0O_0'. The intersection of the radial S_0a_0 with this line XA, fig. 28, will be the point A.

CHAPTER II.

PHOTOGRAPHS ON INCLINED PLATES.

In the preceding we have regarded photographic plates (perspectives) only that had been exposed in a vertical plane, and although the use of inclined plates for phototopographic purposes is not to be generally recommended (on account of the complications that will arise in the ordinarily simple constructions in iconometric platting from vertically exposed plates, and because the relations which exist between the elements of the perspective and the orthogonal projection in horizontal plan of the pictured objects will not be so readily recognized), still, occasions may arise where the selection of the available or accessible stations will be so circumscribed that the exposure of inclined plates will become necessary in order to control the inaccessible terrene (above or below the camera station).

Photographs may also have been obtained with an ordinary camera, without any device for adjusting the plate in vertical plane, or the use for iconometric platting of the photographs (perhaps taken only for illustrative purposes) may have been an afterthought.

With reference to fig. 30 we have:

PP = principal plane.
HH = horizontal plane passing through the second nodal point of the camera lens (at the station S).
GG = ground plane.
MN = picture plane.
$O'P$ = trace of picture plane MN, in the horizon plane HH.
O'_oP_o = ground line of picture plane MN.
S_o = foot of the station S.
$P'P_o$ = principal line of the picture plane.
P' = principal point of the perspective MN.
SS_o = vertical of the station; it will penetrate the picture plane MN above (or below) the horizon line at s. The trace s of this vertical sS_0 in the picture plane is the vanishing point for the perspectives of all vertical lines that may be pictured on MN.
$P'SP = PsS = \alpha$ = angle of inclination of the plate MN.
SP = (horizontal) line from S perpendicular to horizon line $O'P$.
SA = line of direction from S to a point A, pictured in MN as a.

If we revolve SP, in the vertical plane PP, about P until SP falls within the picture plane, then the point S will fall into (S) and the line Sa will fall into $(S)a$.

The vertical plane containing the line SA and passing through SS_o will intersect the ground plane in S_oa_o. If we now revolve the line S_oP_o, in the vertical plane PP, about P_o until S_oP_o falls within the picture plane MN, then the point S_o will fall into (S_o) and the trace S_oa_o will have assumed the position $(S_o)a_o$, and the intersection A of the trace S_oa_o with the line of direction Sa will locate the platted position of the pictured point a in the ground plane GG.

The line sa intersects the ground line in a_o, and S_oa_o will be the radial in the ground plane to the platted position of A and passing through the foot S_o of the station S.

To find A on S_oa_o we first locate in the picture plane the intersection (A) of the revolved lines $(S)a$ and $(S_o)a_o$. This point (A) revolved in the vertical plane a_oS_oS about a_o will locate A upon S_oa_o.

To locate the position of A in GG, in the manner just shown, we should know the position of the line $O'P$, as well as the points S and P. These are known or may readily be found if the position of the principal point P', the length of the distance line SP', and the value of the angle of inclination (α) for the plate are known.

When a photographic plate in a surveying camera is intentionally exposed in an inclined position, it will generally be exposed in such a way that the principal line ff' still coincides with the intersection of the picture plane MN and the principal plane PP, fig. 30.

When the angle of inclination α is an angle of elevation (depression) the horizon line (intersection of horizon plane and inclined picture plane) will fall below (above) the line representing the horizon line on the plate when exposed vertically. In order to use the inclined plate for

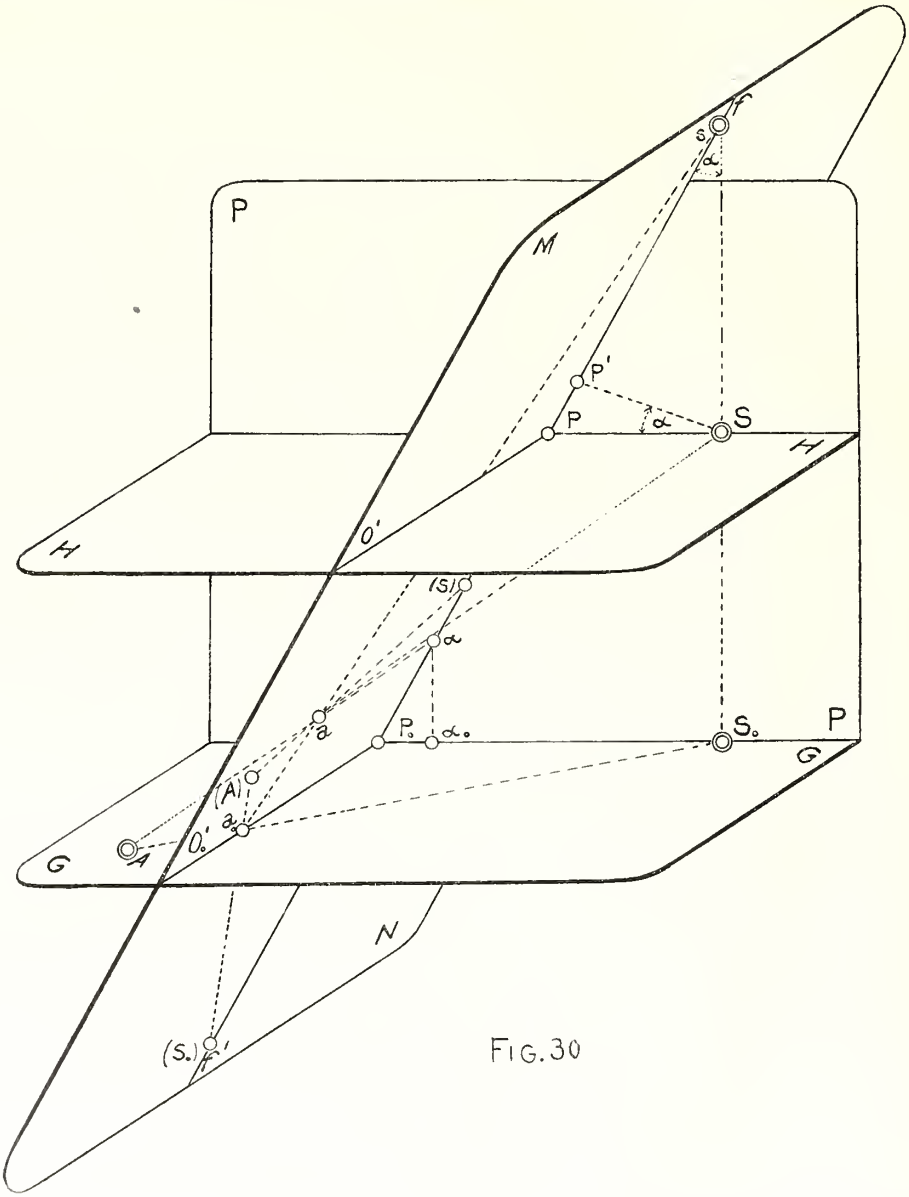


FIG. 30

iconometric purposes the angle of inclination should be observed directly in the field, and, if the constant focal length of the camera ($=f$) is known, the line SP, fig. 30, may be found as the hypothenuse of the right-angle triangle with angle $=\alpha$ and adjoining side $=f$.

I. TO PLAT THE PICTURE TRACE OF AN INCLINED PLATE.

In order to plat the picture trace the horizontal angle, included between the optical axis of the inclined camera and the horizontal direction to some known point, should be measured. Should the length SS' (elevation of station S above the foot of the station, fig. 32), the position of the line connecting two camera stations, and also the position of a third point A (visible from both stations) be known, no horizontal angle α needs to be measured instrumentally, provided the plates containing the picture a of the third point A are oriented in such a way that the picture a be bisected by the vertical thread or principal line ff' of the perspective.

With reference to fig. 31 we have

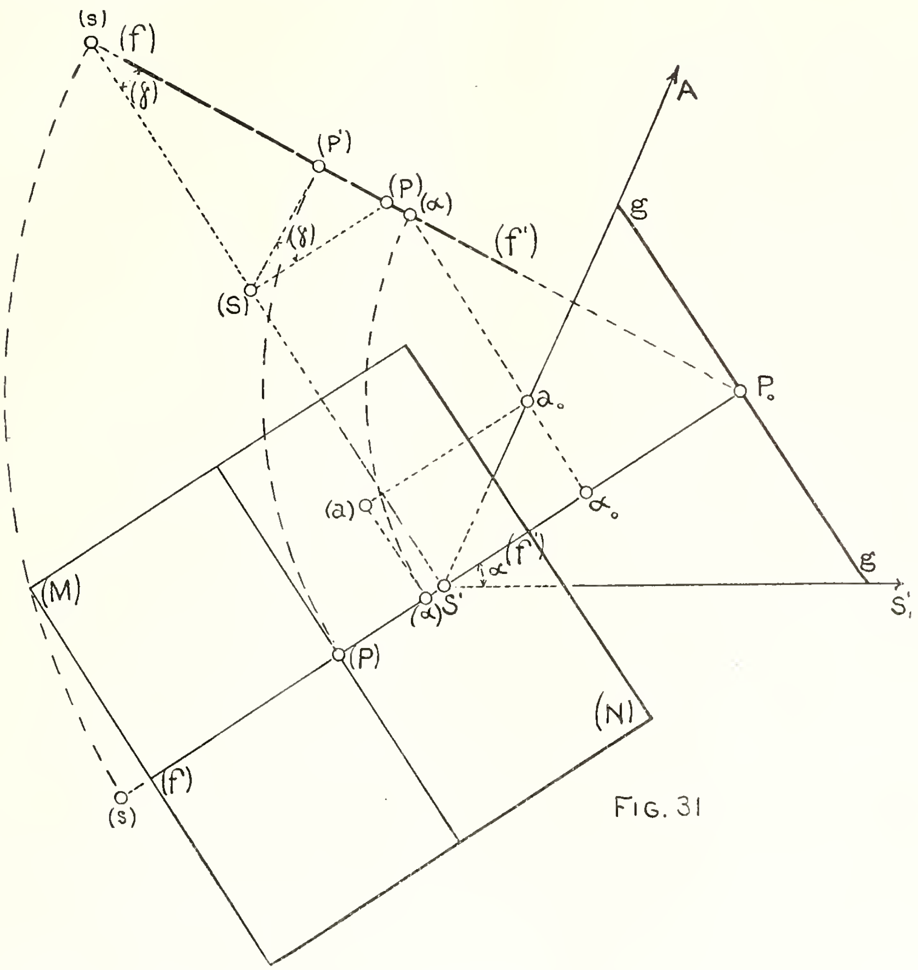


FIG. 31

S' = platted position of the station S

$S'S_1'$ = platted length and direction of the base line.

The horizontal angle α (at S') included between this line of direction $S'S_1'$ and the principal plane (or horizontal projection of optical axis $S'P_0$) may have been observed in the field. The line $S'S$ in fig. 32 represents the elevation of the station S (laid off in the platting scale). If we revolve this line $S'S$ about $S'P_0$ into the platting plane it will assume the position shown as $S'(S)$ in fig. 31. After erecting at (S) a line $(S)(P)$ perpendicular to $S'(S)$ the angle of inclination γ of the plate MN is laid off upon $(S)(P)$ from (S).

$(S)(P')$ is made equal to the constant focal length $(=f)$ of the camera, and the line drawn perpendicular to $(S)(P')$ through (P') will represent the principal line $(f)(f')$ of the perspective MN, revolved about $S'P_0$ into the platting plane. The point of intersection (s) of $(S)S'$ with $(f)(f')$ represents the vanishing point for all vertical lines shown on the picture.

The point of intersection P_0 of the line $(f)(f')$ and the horizontal projection of the optical axis $S'P_0$ will be the trace in the ground plane of the inclined principal line ff'.

The line P_0g, perpendicular to $S'P_0$ in P_0, is the ground line or the trace of the inclined picture plane MN in the platting plane GG.

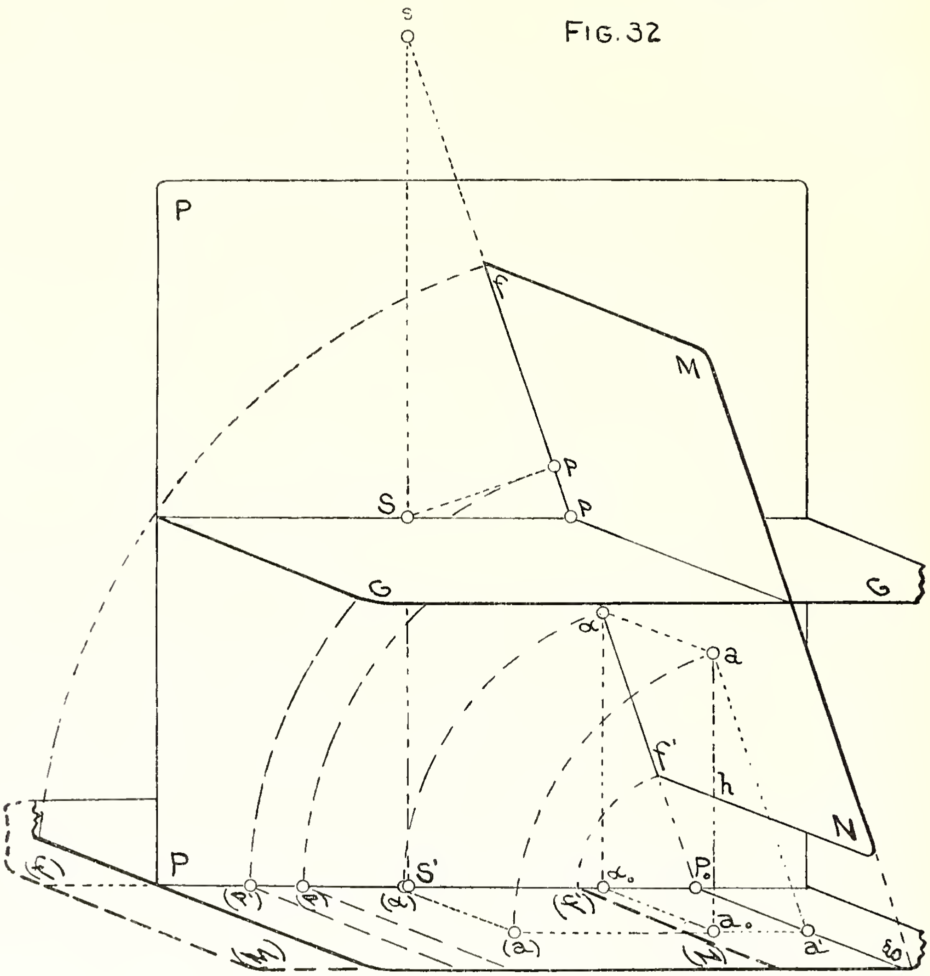
II. PLATTING THE LINES OF DIRECTION TO POINTS PICTURED ON AN INCLINED PHOTOGRAPHIC PLATE.

The inclined picture plane MN, fig. 32, is revolved about P_0g into the drawing or ground plane, when the picture will appear as $(M)(N)$, the principal point P falling upon $S'P_0 = (f)(f')$ in (P) and $(P)P_0 = PP_0$.

To plat the direction to a point A from S', we first locate the orthogonal projection a_0 (in the ground plane) of the pictured point a, fig. 31.

The image point a, fig. 32, projected upon ff' or upon $PP_0 = \alpha$ and a circle described about

FIG. 32



P_0 with $P_0(\alpha)$ will locate the position (α) of the projected point on the principal line $(f)(f')$, revolved into the platting or ground plane.

The perpendicular to $S'P_0$ in α_0 and the vertical to the ground plane GG from a, fig. 32 intersect each other in a_0, and $S'a_0$, fig. 31, is the horizontal projection (in the ground or platting plane) of the line of direction or radial from S' to the point A.

III. DETERMINATION OF THE ALTITUDES OF POINTS PICTURED ON INCLINED PLATES

We refer again to fig. 32. It is desired to find the elevation H of the point A (pictured in a) above the ground plane GG.

Projecting a upon the principal plane PP we find α on ff'; the vertical through α intersects the horizontal projection of the principal ray $S'P_0$ in α_0, fig. 32; hence, $\alpha\alpha_0$ represents the elevation of the point A above GG, measured in the platting scale.

With reference to fig. 31, this elevation aa_0 (fig. 32) of a above the ground plane is found by projecting a upon $P'P_0$ ($= \alpha$ in fig. 32); the corresponding point on the principal line revolved about P_0 into the platting plane is (α) and its orthogonal projection upon the principal plane, the latter revolved into the platting plane about $S'P_0$, fig. 31, is (a); hence, the elevation of A above the ground plane is $= (\alpha)\ \alpha_0 = h$, to be measured in the platting scale.

If $D =$ distance of the platted point from S', taken from the platting sheet, $H =$ elevation of

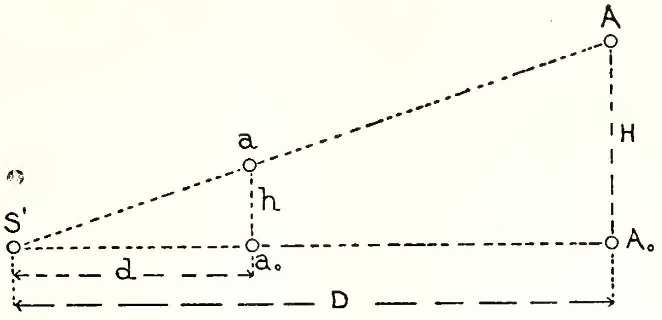


FIG. 33.

the point A above the ground plane GG, $h = (\alpha)\ \alpha_0 = aa_0$, fig. 32, $= (\alpha)\ \alpha_0$, fig. 31, $= aa_0$, fig. 33 $S'\alpha_0 = d$ (fig. 31), taken from the platting sheet, the elevation H of the point A may be found either graphically from a diagram, fig. 33, or it may be computed from the relation:

$$H = \frac{D\,h}{d}$$

IV. APPLICATION OF PROFESSOR HAUCK'S METHOD.

The constructions just described for locating the horizontal directions to points photographed on inclined plates may be greatly simplified by applying Professor Hauck's method, by utilizing the properties of the "kernel points" of two photographs obtained from different stations but comprising the same ground.

With reference to fig. 34: S and $S' =$ the two camera stations.

S_0 and $S_0' =$ the foot points of S and S' respectively.
MN and $M'N' =$ inclined picture planes; both contain the image a and a' of a point A and the pictures s' and s ("kernel points") of the stations S' and S.
α_0 and $\alpha_0' =$ orthogonal projections (in the ground plane GG) of a and a' respectively.
$A_0 =$ orthogonal projection of A in the ground plane.
Σ, s' and $\pi =$ kernel points for picture plane MN.
Σ', s and $\pi' =$ kernel points for picture plane $M'N'$.

These "kernel points" are of importance, inasmuch as—

The horizontal direction S_0A_0 (or $S_0'A_0$) intersects the ground line gg' of MN (or $M'N'$) in α_0 (or α'_0). The line connecting a and s' ("kernel point") in MN and the connection of a' and s in $M'N'$ intersect each other in the same point Ω of the line of intersection of the two picture planes, and also intersect the ground lines gg' of the picture planes in the "kernel points" π and π', respectively. All lines in MN, connecting s' with pictured points, and those in $M'N'$, connecting s with the pictures in $M'N'$ of the same points, intersect each other in points Ω of the line of intersection of the two inclined picture planes. The kernel points Σ and Σ' are the intersections of the verticals passing through the camera stations (S and S'), with the inclined picture planes. They are the "vanishing points" for the pictures of all vertical lines shown on the negatives, and whenever the pictures contain images of vertical lines the intersections of these would locate Σ and

Σ' on MN and $M'N'$, respectively. Still, when the picture plane is inclined in such a way that the principal line of the same would coincide with that of the vertically exposed plate (when the former were revolved about a line as axis passing through the second nodal point and parallel

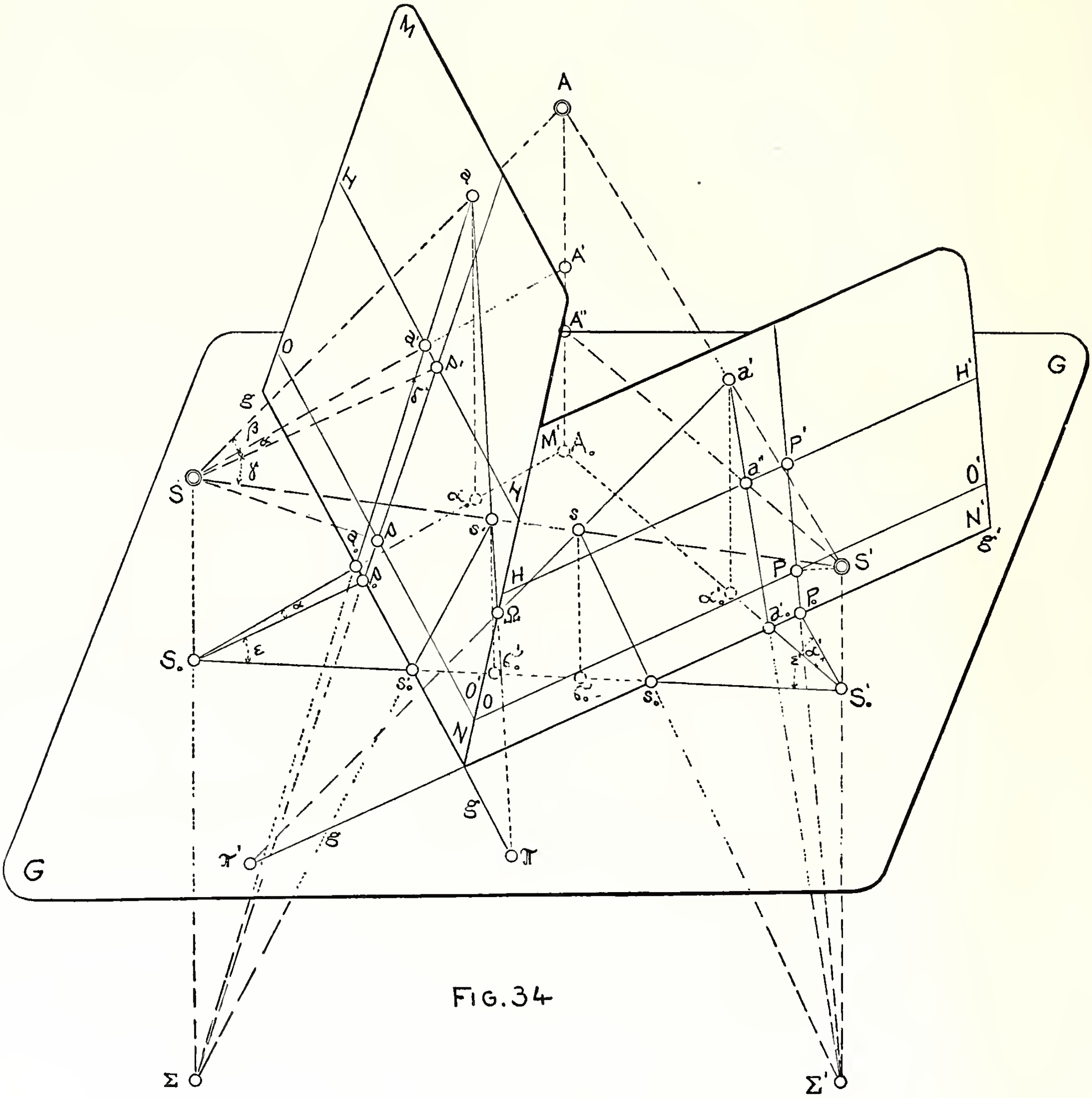


FIG. 34

with the horizon line OO' or HH', fig. 34) the kernel point Σ may more readily be located upon ff', as previously shown for s in fig. 32.

In order to locate the position of A_0, fig. 34, with reference to a on MN and a' on $M'N'$ we connect a and Σ and also a' with Σ', which lines locate a_0 and a_0' upon the ground lines of the picture planes MN and $M'N'$. The intersection of the lines S_0a_0 and $S_0'a_0'$ will give the position of A_0 in the ground plane GG.

CHAPTER III.

PHOTOTOPOGRAPHIC METHODS.

I. ANALYTICAL OR ARITHMETICAL METHODS.

(1) *Method of Prof. W. Jordan.*—In 1874 Professor Jordan made a map of the oasis "Dachel,' including the village "Gassr-Dachel," based on photographs taken with an ordinary camera by Remelé, obtained on Gerhard Rohlf's African expedition during the winter of 1873–74. Care was exercised to expose the plates in vertical plane, and horizontal directions to at least three points for each photograph were instrumentally measured to obtain the data needed for the proper orientation of the pictures. Vertical angles to at least two such points (for every picture) were also observed to give the means for locating the horizon lines of the pictures and thus enabling the draftsman to deduce the elevations of other points pictured on the photographs.

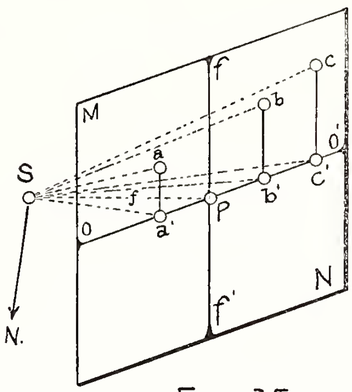


FIG. 35

With reference to fig. 35 we have: OO' = horizon line, ff' = principal line, P = principal point, SP = focal length $= f$, variable for different pictures.

The ordinates aa', bb', and $cc' = y_1$, y_2, and y_3, respectively.

The abscissæ of the three points a, b, and c be x_1, x_2, and x_3 respectively.

The horizontal angles included between the principal ray and the horizontal directions Sa', Sb', and $Sc' = \alpha_1$, α_2, and α_3 respectively.

The azimuthal angles (between the meridian SN and the horizontal directions Sa', Sb', and Sc') $= \varphi_1$, φ_2, and φ_3

Then $\alpha_2 - \alpha_1 = \varphi_2 - \varphi_1 = \varepsilon_1$ and $\alpha_3 - \alpha_2 = \varphi_3 - \varphi_2 = \varepsilon_2$

The elevations of the points A, B, and C above the plane of reference or above the ground plane $= H_1$, H_2, and H_3

As the photographic plate MN had been exposed in vertical plane, it will be evident that for the three points a, b, and c pictured on the perspective MN, fig. 35—

$$x_1 = f \tan \alpha \qquad x_2 = f \tan \alpha_2 \qquad x_3 = f \tan \alpha_3$$

or,

$$x_2 - x_1 = f(\tan \alpha_2 - \tan \alpha_1) = f \frac{\sin(\alpha_2 - \alpha_1)}{\cos \alpha_1 \cos \alpha_2}$$

and

$$x_3 - x_2 = f(\tan \alpha_3 - \tan \alpha_2) = f \frac{\sin(\alpha_3 - \alpha_2)}{\cos \alpha_3 \cos \alpha_2}$$

The values $x_2 - x_1$ and $x_3 - x_2$ may be scaled off directly on the photograph, and the values for $\alpha_2 - \alpha_1$ and $\alpha_3 - \alpha_2$ may be taken from the field records of the observed angles.

Hence $\frac{\cos \alpha_3}{\cos \alpha_1}$ may be computed from the equation

$$\frac{x_2 - x_1}{x_3 - x_2} = \frac{\cos \alpha_3}{\cos \alpha_1} \frac{\sin(\alpha_2 - \alpha_1)}{\sin(\alpha_3 - \alpha_2)}$$

If we substitute $\tan \gamma$ for $\frac{\cos \alpha_3}{\cos \alpha_1}$

and as

$$\frac{1 + \tan \gamma}{1 - \tan \gamma} = \tan(45^\circ + \gamma)$$

we may now write

$$\tan(45^\circ + \gamma) = \frac{1 + \frac{\cos\alpha_3}{\cos\alpha_1}}{1 - \frac{\cos\alpha_3}{\cos\alpha_1}} = \frac{\cos\alpha_1 + \cos\alpha_3}{\cos\alpha_1 - \cos\alpha_3}$$

$$= \frac{\cos\frac{\alpha_1+\alpha_3}{2}\cos\frac{\alpha_1-\alpha_3}{2}}{\sin\frac{\alpha_1+\alpha_3}{2}\sin\frac{\alpha_1-\alpha_3}{2}} = \cot\frac{\alpha_1+\alpha_3}{2}\cot\frac{\alpha_1-\alpha_3}{2}$$

and $\tan\frac{\alpha_1+\alpha_3}{2} = \cot(45^0+\gamma)\cot\frac{\alpha_1-\alpha_3}{2}$

From this equation we compute $\alpha_1 + \alpha_3$, and after subtracting

$$\alpha_3 - \alpha_2 = \varphi_3 - \varphi_2 = \varepsilon_2$$

from

$$\alpha_2 - \alpha_1 = \varphi_2 - \varphi_1 = \varepsilon_1$$

we find

$$\alpha_1 - \alpha_3 = \varphi_1 - \varphi_3$$

knowing $\alpha_1 + \alpha_3$ and $\alpha_1 - \alpha_3$ we can readily find α_1 and α_3, also,

$$\alpha_2 = \alpha_1 + \varepsilon_1 \text{ or}$$

$$= \alpha_3 - \varepsilon_2$$

We had found:

$$x_2 - x_1 = f\frac{\sin(\alpha_2-\alpha_1)}{\cos\alpha_1\cos\alpha_2} = f\frac{\sin\varepsilon_1}{\cos\alpha_1\cos\alpha_2}; \text{ hence } f = \frac{(x_2-x_1)\cos\alpha_1\cos\alpha_2}{\sin\varepsilon_1}$$

$$x_3 - x_2 = f\frac{\sin(\alpha_3-\alpha_2)}{\cos\alpha_3\cos\alpha_2} = f\frac{\sin\varepsilon_2}{\cos\alpha_3\cos\alpha_2}; \text{ whence } f = \frac{(x_3-x_2)\cos\alpha_3\cos\alpha_2}{\sin\varepsilon_2}$$

Thus the abscissæ x_1, x_2, and x_3, (the principal line ff') and the focal length f may be found.

With the aid of the observed vertical angles β the horizon line OO' may be located on the photograph. For example, if the vertical angle $\beta_3 = c\,S\,c'$ had been observed to the point C, we find:

$$y_3 = Sc' \tan\beta_3$$
$$= \frac{f}{\cos\alpha_3}\tan\beta_3$$

The horizon line OO' will fall below the pictured point c by the vertical distance $\frac{f}{\cos\alpha_3}\tan\beta_3$, and for the point a the vertical distance to the horizon line would be

$$y_1 = \frac{f}{\cos\alpha_1}\tan\beta_1$$

At least two vertical angles having been observed for each plate, the horizon line OO' may thus be located and marked upon the negative, when the principal point P may also be marked on OO' by means of the abscissæ x_1, x_2, and $x_3 = a'P$, $b'P$, and Pc', respectively.

(2) *Method of Dr. G. Le Bon.*—Dr. Le Bon, who used his instrument chiefly for the draughting of ancient buildings and monuments in India, provided the ground-glass plate of his camera with a net of squares, each square having sides 1 centimeter long, the latter being drawn parallel with the horizon—and principal lines, which latter two were subdivided into millimeters.

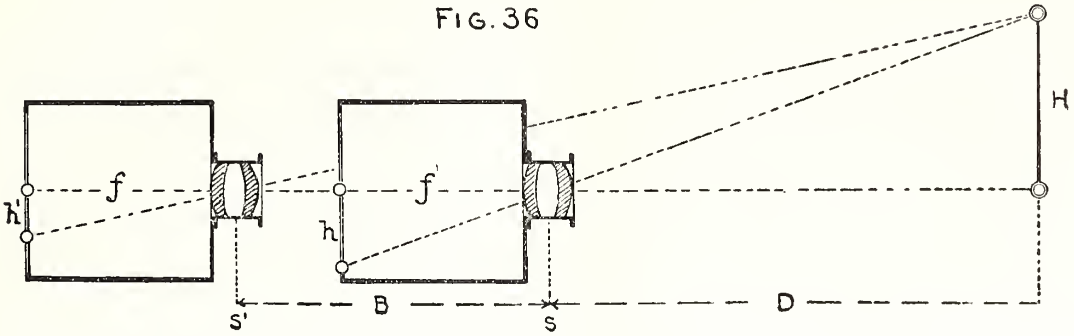
This arrangement enabled the operator to obtain the measurements of objects directly by inspection of the image on the (graduated) ground-glass plate. To determine the dimensions of the front of a building, a certain distanee is measured direetly upon the same and a picture is then taken by exposing a photographie plate in vertieal plane and parallel to the base of the front of the building.

For example:

(*a*) To find the distanee D of an objeet of unknown height H.

Two stations, S and S', are oecupied on a base line (whieh is measured directly in the field) laid off in a direction at right angles to the base of the objeet, fig. 36.

FIG. 36



If the height of the image, measured on the graduated ground glass, at the first station S is h and the focal length for both exposures be the same $=f$, then

$$D : H = f : h$$

and for the seeond station S'

$$D + B : H = f : h'$$

By dividing the second equation by the first, we find:

$$\frac{D+B}{D} = \frac{h}{h'}\ ;\ \frac{B}{D} = \frac{h}{h'} - 1 = \frac{h-h'}{h'}$$

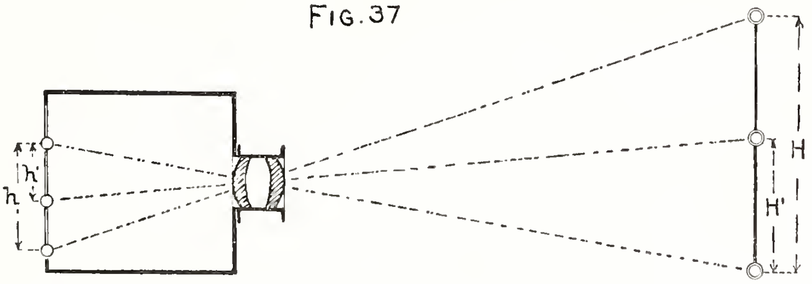
whence:

$$D = \frac{B.h'}{h-h'}$$

B is given and h and h' are measured direetly on the ground glass.

(*b*) It is desired to find the height H of an objeet of whieh the fractional length H' had been measured directly, fig. 37.

FIG. 37



On the image of the objeet on the graduated ground-glass plate the heights h and h' may be read off direetly, and H' being known we find H from the equation

$$H = H' \frac{h}{h'}$$

(3) *Method of L. P. Paganini* (*Italian method*).—This method was developed for the topographic survey of Italy, made under the auspices of the Royal Italian Military Geographical Institute, and a detailed description of the same, with numerical examples, has been published in Appendix

No. 3, Report for 1893 of the Superintendent of the United States Coast and Geodetic Survey. Also, Dr. C. Koppe and Prof. F. Steiner give preference to the arithmetical method for photogrammetric surveys in general.

GENERAL ARITHMETIC DETERMINATION OF THE ELEMENTS OF THE ITALIAN PHOTOGRAPHIC PERSPECTIVES.

The panoramic views which subserved the map making were obtained by ten successive exposures. After each exposure the camera was moved in azimuth by a horizontal angle of 36°, and as each plate subtends a horizontal angle of 42°, the two ends of adjoining plates have a common margin of a width of 3° in arc, corresponding to a width of 15 millimetres. These common margins of two adjoining plates serve principally to ascertain whether the adjustments of the phototheodolite have been changed during the occupancy of a station.

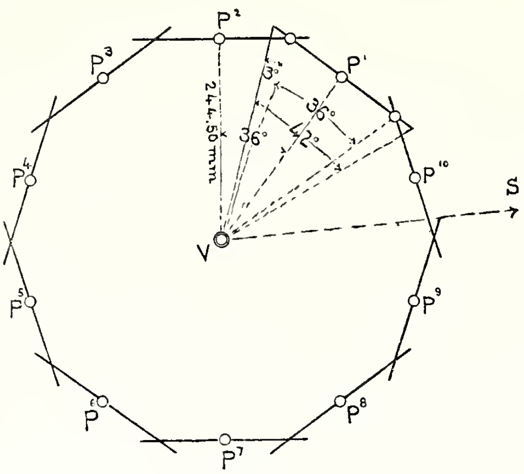


FIG. 38

(*a*) *Orientation of the picture trace.*—The horizontal projection of one complete panorama composed of ten plates will be a regular decagon, fig. 38, with a radius of the inscribed circle equal to the principal focal length (constant) of the camera.

P', P^2, ... P^{10} = (horizontal) picture traces,
V = panorama station,
$VP' = VP^2 =$ . . . . $VP^{10} = f =$ principal focal length of camera.

After the position of one panoramic view has been platted on the map, its picture trace will be oriented, and with it the remaining nine views of the panorama.

After the horizontal angle ω, fig. 39, included between the principal ray VP' of view P' and the horizontal direction to the triangulation point S, fig. 39, has been platted the orientation of each succeeding view P^2, P^3 . . . . . P^{10} is accomplished by adding successively 36°, 72°, 108° . . . . . $(360° - \omega°)$ to the angle ω.

(*b*) *Platting the lines of direction to pictured points of the terrene.*—The orientation of the panorama having been made, the lines of direction to points photographed on the panorama plates may readily be platted.

The plate MN, fig. 39, may represent a vertical photographic plate oriented with reference to the known point S, pictured on MN as s.

OO' = horizon line,
V = point of view of the perspective MN,
ω = angle of orientation for this plate with reference to S,
$VP = f =$ (principal) focal length,
ss', perpendicular to $OO' = y =$ ordinate of the image s,
sx perpendicular to $ff' = x =$ abscissa of the point s.

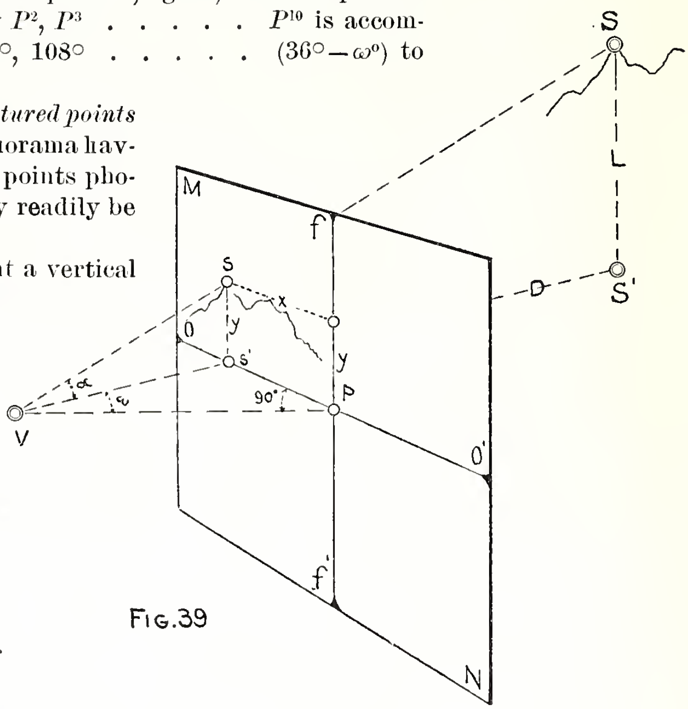


FIG. 39

From the rectangular triangle $VP's'$, fig. 39, we find:

$$x = f \tan \omega.$$

If the camera station V and the known point A have been platted and the picture trace OO', fig. 40, has been oriented, the horizontal projection of the ray from V to S may be found as follows:

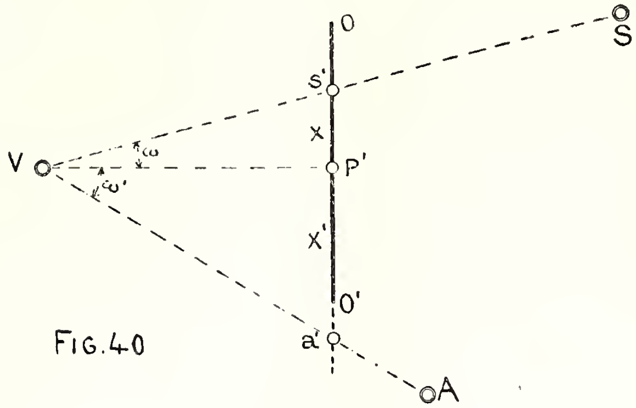


FIG. 40

The abscissa $P's' = x$, fig. 39, is laid off on OO', fig. 40, from P' in the sense of the direction to S (whether S is to the right or to the left of the principal line ff', fig. 39) with reference to the principal point P', locating s' (the orthogonal projection of the pictured point in the ground plane) and a line drawn from V through $s' = Vs'$, which will be the ray VS, fig. 39, projected in the platting plane.

The position of S on the platting sheet is obtained by finding the point of intersection of two or more lines of direction, obtained in a similar manner, from other pictures containing images of S and taken from different stations, as all rays to the same object, seen from different stations, must intersect each other in the same point on the platting sheet.

The elevations of pictured terrene points are readily determined after the selected points (identified on several pictures) have been determined and platted in horizontal plane, in the manner just described.

If the elevation of the station V is known, the elevation of the line of horizon OO' on the plate, fig. 39, may easily be obtained by adding the height of instrument to the elevation of V.

(c) *Determination of the elevations of pictured points.*—Disregarding the effects of curvature and refraction, the elevations of all the points on the plate which are bisected by the horizon line OO' have the same elevation as the optical axis of the instrument at V.

The elevations of pictured points, above or below the horizon line, are obtained by determining their elevation above or their depressions below the line OO'.

If D = horizontal distance from station V to a point S, fig. 39,
= VS', fig. 39, to be measured in the platting scale.
L = difference in elevation between point S and station V.
= SS' (S' being the orthogonal projection of S upon the platting plan).
d = horizontal distance of the picture s of S from V.

We find from the similar triangles $Vs's$ and VSS':

$$L : D = y : d \qquad 1$$

$$L = \frac{Dy}{d}. \qquad 2$$

From the rectangular triangle $VP's'$ follows:

$$d = \frac{f}{\cos \omega} = f \sec \omega, \qquad 2a$$

whence

$$L = \frac{Dy}{f \sec \omega}. \qquad 3$$

Should the point S be bisected by the vertical thread (principal plane) then

$$\omega = 0 \text{ and } \sec \omega = 1, \text{ or,}$$

$$L = \frac{Dy}{f}. \qquad 3a$$

This formula would answer for all points of the perspective if the image plate were a cylindrical surface of radius $= f$ (instead of being a tangential plane to such cylinder), if the decagon were a circle (as it is the case for the sensitive film of the panoramic cameras, and Colonel Moessard's cylindrograph, which will be described later).

Differences of elevation, taken from the perspectives, are positive or negative according to the relative positions of the pictured points with reference to the horizon line OO', fig. 39, whether above OO' or below the same, and the apparent elevations of such points (above mean sea level) are obtained by adding their ordinates (L, fig. 39) to or subtracting them from the elevation of the camera station (V, fig. 39).

By comparing the elevations thus obtained for identical points from photographs exposed from different stations the hypsometric determinations of secondary points of the terrene may be checked.

(*d*) *Checking the position of the horizon line of a photograph.*—To check the position of the horizon line OO', fig. 39, photographs are selected which show the images of two or more triangulation points, the elevations of the latter, determined from the photographs, are compared with those given in the triangulation records and discrepancies are adjusted by shifting the line OO'. Should the elevations of the triangulation points be unknown, or should the pictures from any station not contain the pictures of such points, this check may still be made by measuring the vertical angles (α, fig. 39, with the vertical circle of the phototheodolite from such camera station) to a series of prominent points (S, fig. 39) and comparing their computed ordinates (L, fig. 39) with those obtained from the pictures.

We find from the similar triangles VSS' and Vss', fig. 39:

$$\tan SVS' = \tan \alpha = \frac{L}{D} = \frac{y}{d}$$

and we had according to formula $2a$:

$$d = \frac{f}{\cos \omega}$$

hence

$$\tan \alpha = \frac{y}{f} \cos \omega, \text{ or} \qquad 4$$

$$y = \frac{f. \tan \alpha}{\cos \omega} \qquad 5$$

where ω is the horizontal angle included between the vertical plane (VSS') passing through the camera station V and the point S, fig. 39, and the principal plane (Vff'). This angle ω should be measured (with the horizontal circle of the phototheodolite) for several points S at every station, whence a limited or insufficient number of triangulation points may be seen.

If the computed values for y, formula 5, are not in accord with those obtained by direct measurement on the photograph, the horizon line OO', fig. 39, must be adjusted until the values for the ordinates measured on the picture are the same as those computed by aid of formula 5.

The necessity of the precise determination of the value f (focal length) is evident from the preceding, and if the panorama pictures contain a sufficient number of well-defined pictures of surrounding triangulation points, the determination of f may readily be made by means of the adjusted horizon line OO', fig. 39.

(*e*) *Determination of the focal length f.*—The phototheodolite is set up over a well-determined point and adjusted. A plate is exposed in vertical plane in such a way that the vertical thread ff' bisects a known geodetic point S, fig. 41, which can readily be identified upon the ground-glass plate of the camera. (It is also desirable that the ordinate y, fig. 41, be sufficiently long to assure a correct measurement of its length to be made on the picture.) There will be given, fig. 41:

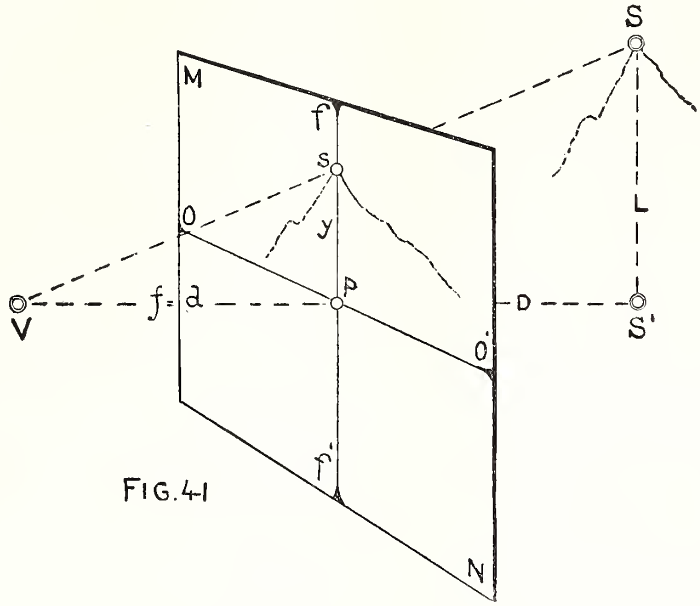


FIG. 41

L = difference of elevation of bisected point S and panorama station V, D = horizontal distance between S and V, y = ordinate of pictured point s.

From equation 3*a* we find

$$f = \frac{D.y}{L}$$

which will be a fairly accurate value if the horizontal position of the camera was assured and if the ordinate y was correctly measured on the negative.

Another value for f may be found from equation 5:

$$f = \frac{y.\cos\omega}{\tan\alpha}$$

if the picture contained triangulation points enough to adjust the horizon line by computing their ordinates:

$$y = \frac{L.d}{D}$$

By using the mean of these determinations for f the computations (based upon the new values for x and y) may be repeated until perfect agreement is reached.

(*f*) *Determination of the principal point of the perspective.*—The great number of triangulation points established in Italy, with special reference to the phototopographic survey, facilitates the application of the photogrammetric method and assures the accurate determination of the perspective elements. Although the Italian pictures command a horizontal angle of but 42°, the greater number of them contain the pictures of several triangulation points, and it can be ascertained simultaneously with the determination of the value of f whether the picture P, of the intersection of the cross wires (OO' and ff') coincides with the principal point of view, P, upon the perspective, fig. 42.

s and s' = pictures of two triangulation points S and S' on the photograph MN, V = station point or point of view; ss and $s's'$ = y and y' respectively = verticals upon the horizon line OO' through the picture points s and s', = ordinates of the triangulation points; SS_1 and $S'S_1'$ = L and L' respectively, = differences in elevation between the triangulation points and camera station; D and D' = horizontal distances from V to S and S' respectively; x and x' = abscissæ of pictured

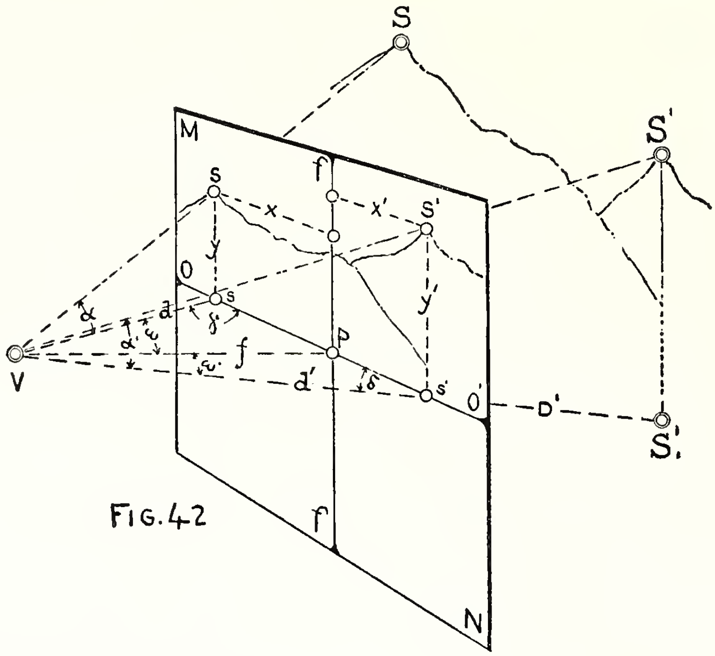


FIG. 42

points s and s'; d and d' = horizontal distances of the pictured triangulation points from the point of view V.

It is desired to find VP and the position of P with reference to s and s', or the abscissæ x and x'.

L, L', D, D', y, and y' are known, or they may be found by direct measurements on the chart projection and upon the photograph. Hence:

$$d = \frac{D.y}{L}$$

$$d' = \frac{D^1.y^1}{L'}$$

The horizontal angle sVs' ($= \omega + \omega'$) being observed in the field the other two angles, γ and δ, of the horizontal triangle sVs', may be computed as follows:

$$\tan \frac{\gamma - \delta}{2} = \frac{d' - d}{d + d'} \cot \frac{sVs'}{2}$$

By substituting H for $\frac{\gamma - \delta}{2}$ and M for $\frac{\gamma + \delta}{2} = 90^0 - \frac{sVs'}{2}$

we will find

$$\gamma = M + N$$
$$\delta = M - N$$

From the two triangles sVP and PVs' (both are rectangular at P) we find

$$f = d.\ \sin\gamma = d'\ \sin\delta$$
$$x = f.\ \cot\gamma$$
$$x' = f.\ \cot\delta$$

also, as a check, the angles of orientation:

$$\omega = 90^\circ - \gamma$$
$$\omega' = 90^\circ - \delta$$

To check the abscissæ the length ss' is carefully measured upon the negative, which length should equal the computed value of

$$x + x' \text{ and also } = \frac{(d + d')\sin\dfrac{sVs'}{2}}{\cos\dfrac{\delta - \gamma}{2}}$$

Should the horizontal angle sVs' not have been measured in the field for some reason, then the angles γ and δ may be found by computation, after carefully measuring ss' on the negative and using the formulas

$$\tan\frac{\delta}{2} = \sqrt{\frac{(p - d')(p - ss')}{p(p - d)}} \text{ and}$$

$$\tan\frac{\gamma}{2} = \sqrt{\frac{(p - d)(p - ss')}{p(p - d')}}$$

where:

$$p = \frac{d + d' + ss'}{2}$$

the angles of elevation α and α', which are obtained either by direct measurement in the field or computed from the formulas

$$\tan\alpha = \frac{L}{D}$$

$$\tan\alpha' = \frac{L'}{D'}$$

serve to obtain checks on the values, measured on the negatives, for y and y' by using the formulas

$$y = \frac{f}{\cos\omega}\tan\alpha \text{ and}$$

$$y' = \frac{f}{\cos\omega'}\tan\alpha'$$

the value for f in above formulas being the same as found from the equation

$$f = d.\ \sin\gamma = d'.\ \sin\delta$$

By repeating the computation with these values for y and y' (if any discrepancy is noted between these new and the former values for y and y') the true value for f may be obtained very closely.

For all practical purposes, however, it suffices to take several pictures with a constant focal length, and to take the mean value of the different f determined from those pictures.

(*g*) *Franz Hafferl's method for finding the focal length of a photograph from the abscissæ of two pictured triangulation points.*—When the horizontal distances D and D' are great, compared with the differences in elevation (L and L') between the points in question (S and S') and the camera station V, fig. 42, the ordinates y and y' will be short, their lengths will be difficult to be measured, and it may be better in that case to determine the value for f by means of the abscissæ of the pictured points, fig. 43.

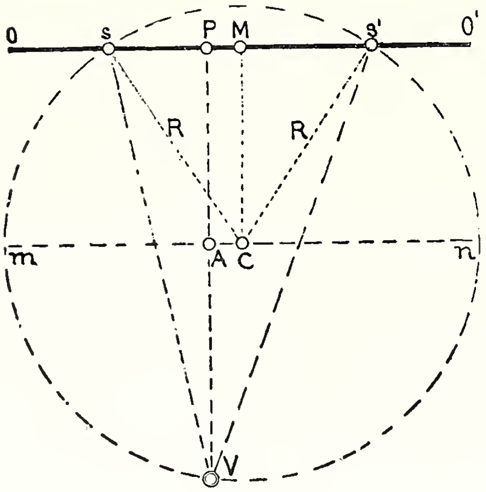


FIG. 43

OO' = platted (and oriented) picture trace, Vs and Vs' = platted horizontal directions from the camera station V to the triangulation points S and S' (pictured as s and s'), VP = perpendicular to the picture trace through V.

It is desired to find f.

Describe a circle through the three points V, s and s', the center of which may be at C.

The angle $sCs' = 2\,(sVs')$. The perpendicular through C to ss' ($= CM$) will bisect this line and the center angle sCs' into two equal parts; hence, sCM and $s'CM$ each $= sVs'$, and if the radius of the circle passing through s, s' and $V = R$ we will have the following relation (from the triangle sMC):

$$sC = R = \frac{sM}{\sin sCM} = \frac{x + x'}{2} \cdot \frac{1}{\sin sCM}; \quad sM = \frac{x + x'}{2}$$

Having drawn the diameter mn parallel with OO', we will have

$$f = VP = VA + AP$$

VA being vertical to mn it will be the middle proportional to mA and An:

$$mA : AV = AV : An \text{ or}$$

$$mA \cdot An = AV^2$$

We can now replace mA by $(mC - AC) = R - \frac{x' - x}{2}$

and as

$$An = nC + AC = R + \frac{x' - x}{2}$$

we find:

$$AV = \sqrt{\left(R - \frac{x' - x}{2}\right)\left(R + \frac{x' - x}{2}\right)}$$

and finally:

$$AP = CM = SM \cot MCs$$

$$= \frac{x' + x}{2} \cot sCM$$

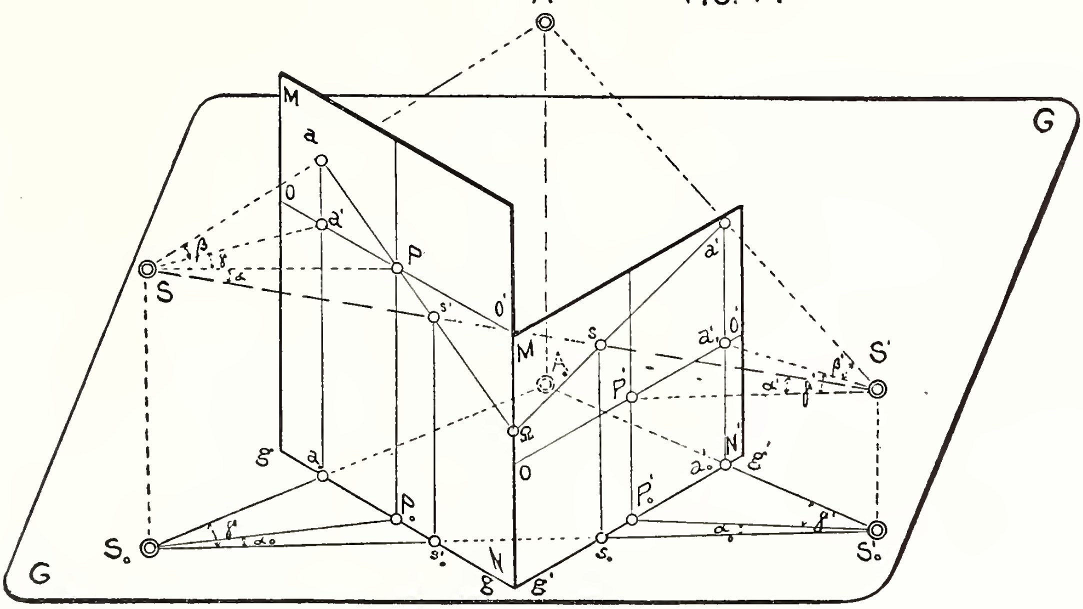
(4) *General arithmetical method for finding the platted positions of points pictured on photographic perspectives (exposed in vertical plane).*—If we refer the pictured points to the principal point P by

means of the rectangular system of coordinates formed by the principal line ff' and the horizon line OO' we will have with reference to fig. 44:

S and S' = two camera stations; MN and $M'N'$ = two picture planes exposed in vertical plane, one from station S, the other from station S'; aa' $(= y)$ and $a'P$ $(= x)$ = coordinates of pictured point a on MN; $a'a'_1$ $(= y')$ and a'_1P' $(= x')$ = coordinates of a' pictured on $M'N'$; f = focal length (the same for both pictures MN and $M'N'$); $D = S_0A_0$ = horizontal distance of A from station S; $D' = S'_0A_0$ = horizontal distance of A from station S'; $d = Sa' = S_0a_0$ = horizontal distance of pictured point a from point of view S; $d' = S'a_1' = S_0'a_0'$ = horizontal distance of pictured point a' from point of view S'; H = elevation of A above horizon plane of station S. H' = elevation of A above horizon plane of station S'; $B = S_0S_0'$ = horizontal distance between the stations S and S'; α and α' = horizontal angles included between B and the principal planes passing through S and S', respectively.

If the camera (theodolite) was in perfect adjustment, if the base line B is known, and if the

FIG. 44.



angles α and α' had been observed, we will know the values of B, α, α', f, and the coordinates x, y, x', and y', the latter being obtained by direct measurement on the negatives.

We can now compute—

(1) The horizontal angle γ, included between the principal plane and any horizontal direction, Sa', fig. 44, from the equation:

$$\tan \gamma = \frac{x}{f} \text{ or } \tan \gamma' = \frac{x'}{f}$$

(2) The angle of elevation β of the line of direction Sa to any point, A, pictured as a on the photograph MN, from the equation:

$$\tan \beta = \frac{y}{d} \text{ or } \tan \beta' = \frac{y'}{d'}$$

As

$$d = \sqrt{f^2 + x^2} \text{ or } d' = \sqrt{f^2 + (x')^2}$$

we may also write:

$$\tan \beta = \frac{y}{\sqrt{f^2 + x^2}}$$

or

$$\tan \beta' = \frac{y'}{\sqrt{f^2 + (x')^2}}$$

We know the length S_0S_0' $(= B)$ of the triangle $S_0A_0S_0'$, and the angles γ, α, γ' and α' also being given, we have

$$D:B = \sin(\gamma' + \alpha') : \sin[180° - (\gamma + \alpha + \gamma' + \alpha')]$$
$$= \frac{\sin(\gamma' + \alpha')}{\sin(\gamma + \alpha + \gamma' + \alpha')}$$

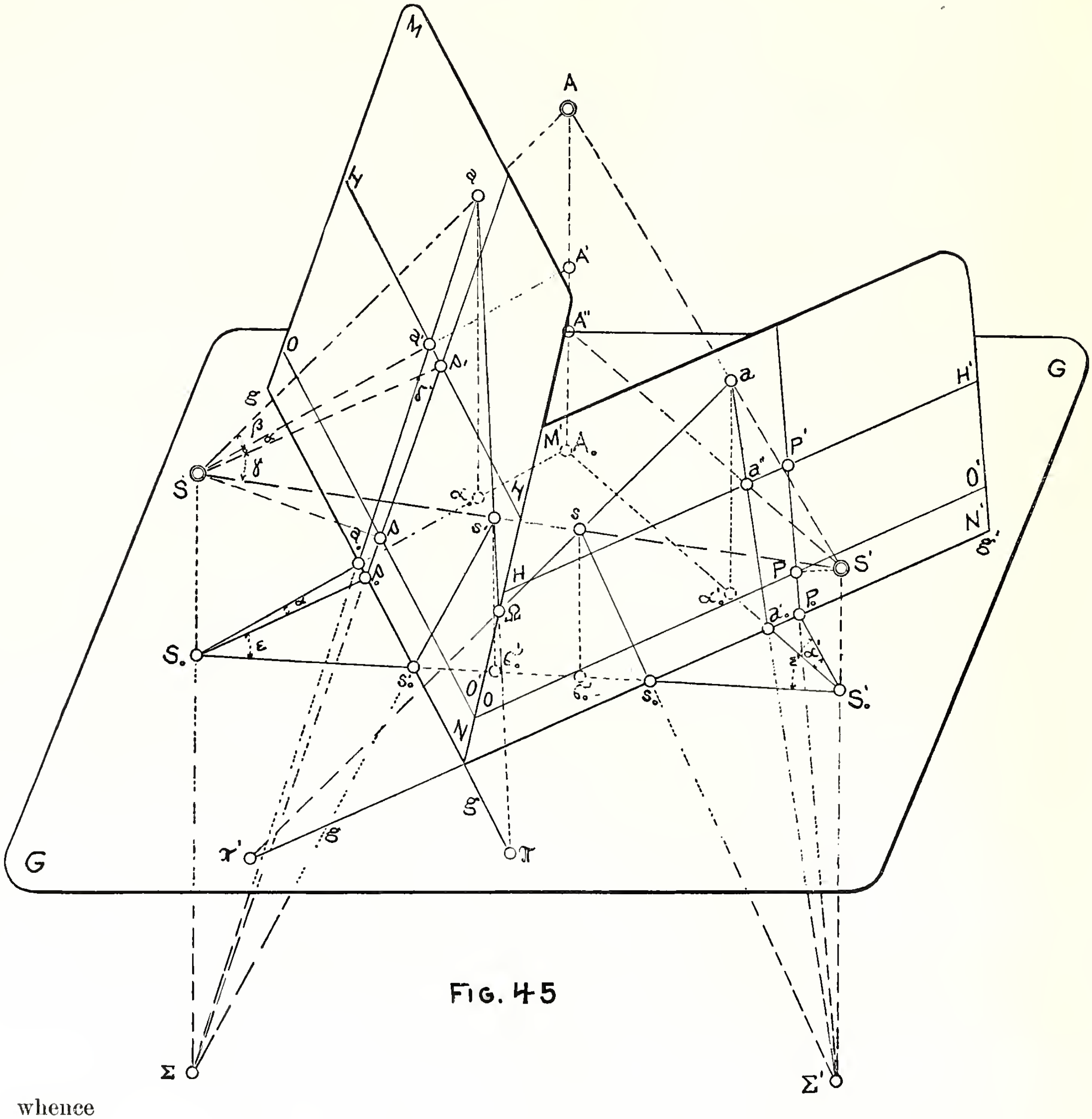


FIG. 45

whence

$$S_0A_0 = D = \frac{B \sin(\gamma' + \alpha')}{\sin(\gamma + \alpha + \gamma' + \alpha')}$$

or

$$S_0'A_0 = D' = \frac{B \sin(\gamma + \alpha)}{\sin(\gamma + \alpha + \gamma' + \alpha')}$$

The difference in elevation, H, between the point A and camera station S may be found from

$$\frac{H}{D} = \tan \beta$$

whence

$$H = D \tan \beta$$

or

$$H' = D' \tan \beta.$$

(5) *General arithmetical method for finding the platted positions of points pictured on photographic perspectives for inclined picture planes.*—For inclined picture planes we will have to take into consideration the angle of inclination of the plate—the angle which is included between the optical axis of the inclined camera and the horizon plane of the camera station.

We have, with reference to figs. 45 and 46:

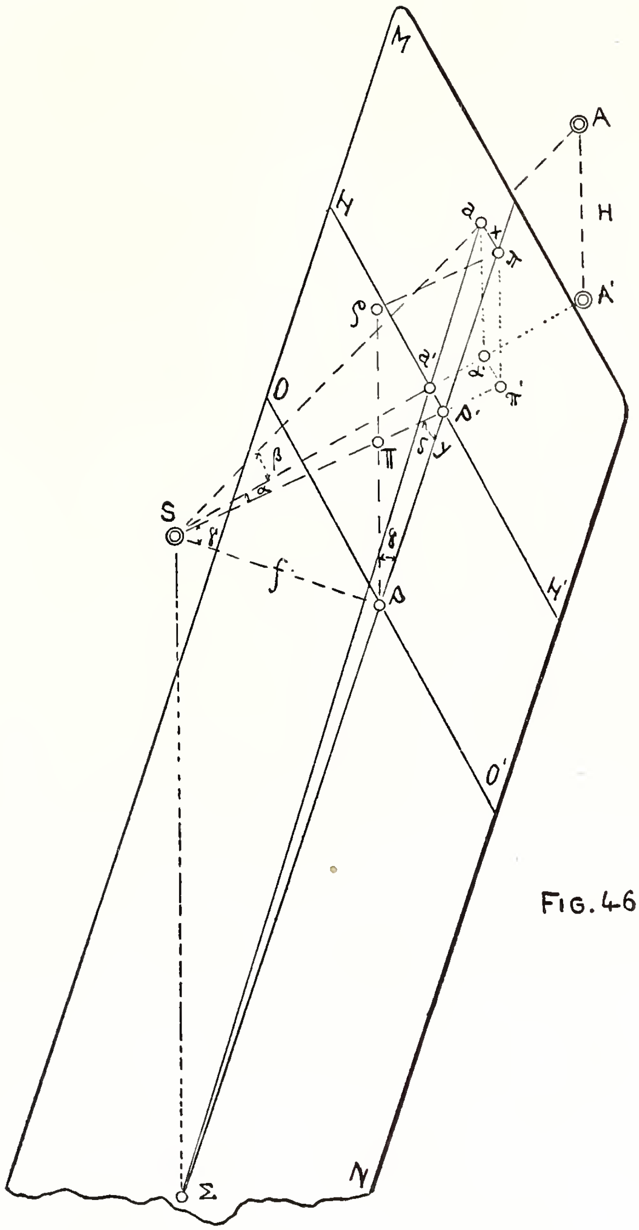


FIG. 46

α = horizontal angle between the principal plane of station S and the vertical plane passing through station S and the point A, pictured as a on inclined picture MN; β = angle of elevation of the point A observed from S; γ = angle of inclination of the photographic plate MN; $\delta = 180° - \gamma$; OO' = horizon line on MN when vertical, permanently marked on the camera; P = principal point for the vertical plate, also permanently marked as the intersection of the principal and horizon lines when the plate is vertical; $P\pi = y =$ ordinate of a on MN (fig. 46); $a\pi = x =$ abscissa of a on MN, very nearly $= a'P'$; $\Sigma =$ vanishing point ("kernel point") for all vertical lines pictured on MN.

From inspection of fig. 46 it will follow directly:

$$\begin{aligned}\tan\beta &= \frac{a\alpha'}{S\alpha'} = \frac{\pi\pi'}{S\alpha'} = \frac{S\Pi}{S\alpha'} \\ &= \frac{P\rho - P\Pi}{\sqrt{x^2 + (S\pi')^2}} = \frac{y\cos\gamma - f\sin\gamma}{\sqrt{x^2 + (S\Pi + \Pi\pi')^2}} \\ &= \frac{y\cos\gamma - f\sin\gamma}{\sqrt{x^2 + (S\Pi + \rho\pi)^2}} \\ &= \frac{y\cos\gamma - f\sin\gamma}{\sqrt{x^2 + (f\cos\gamma + y\sin\gamma)^2}}\end{aligned}$$

and

$$\tan\alpha = \frac{\alpha'\pi'}{S\pi'} = \frac{x}{S\Pi + \rho\pi} = \frac{x}{f\cos\gamma + y\sin\gamma}$$

(We had found for the vertically exposed plate

$$\tan\beta = \frac{y}{\sqrt{x^2 + f^2}} \text{ and}$$

$$\tan\alpha = \frac{x}{f}$$

The preceding formulas for tan α and tan β will assume the form of the latter if the angle of inclination γ is reduced to zero, as $\sin\gamma = \sin 0 = 0$ and $\cos\gamma = \cos 0 = 1$.)

After having thus found α and β (also α' and β') we can now compute the value for $D = S_0A_0$ and for $H = AA'$

With reference to fig. 45 we have

$$\frac{D}{B} = \frac{\sin(\varepsilon' - \alpha')}{\sin[180° - (\alpha + \varepsilon + \varepsilon' - \alpha')]}$$

hence

$$D = \frac{B\sin(\varepsilon' - \alpha')}{\sin(\alpha + \varepsilon + \varepsilon' - \alpha')}$$

and from

$$\tan\beta = \frac{H}{D}$$

we find

$$\begin{aligned}H &= D\tan\beta \\ &= \frac{D\,(y\cos\gamma - f\sin\gamma)}{\sqrt{x^2 + (f\cos\gamma + y\sin\gamma)^2}}\end{aligned}$$

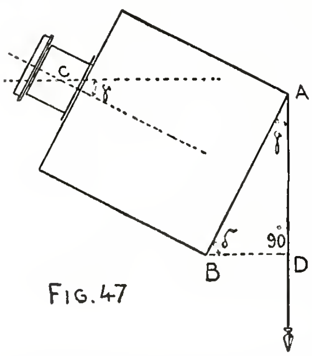


FIG. 47

If an ordinary surveying camera, with a constant focal length, is used, and when it should become desirable to expose a photographic plate in an inclined plane, the complement δ of the angle of inclination of the optical axis ($= \gamma$) may be determined more readily (but only approximately) than the latter by carefully measuring the distances AD, fig. 47 (in the direction of the line of a suspended plumb bob), and DB, supposing AB to be parallel with the photographic plate.

(6) *General analytical determination of the elements of a photographic perspective.*—If, in addition to the photographs, data obtained by instrumental observations are given for a graphical determination of the focal lengths of the pictures, their horizon lines and principal points, then these elements may also be determined by computation.

A picture, MN, may contain the images a, b, and e of three known points, A, B, and C, the position of the camera station (whence this picture was obtained) being likewise known with reference to the three platted points A', B', and C', fig. 48.

To orient the picture trace (or ground line) gg' with reference to the platted station S', and the platted points A', B', and C', the latter are preferably referred to a system of coordinates having the platted station S' as origin.

In fig. 48, for example, a rectangular system of coordinates, $S'Y$ and $S'X$, has been adopted, with the origin in S', and axis of abscissa passing through one of the three triangulation points.

The coordinates of the three triangulation points A', B', and C', platted on the chart projection, are found by measurement $= X_1 Y_1$, $X_2 Y_2$, and X_3, respectively.

The coordinates of the orthogonal projections (on the picture trace gg') of the corresponding points, pictured on the photograph MN, may be designated by $x_{\text{I}} y_{\text{I}}$, $x_{\text{II}} y_{\text{II}}$, and x_{III}, respectively.

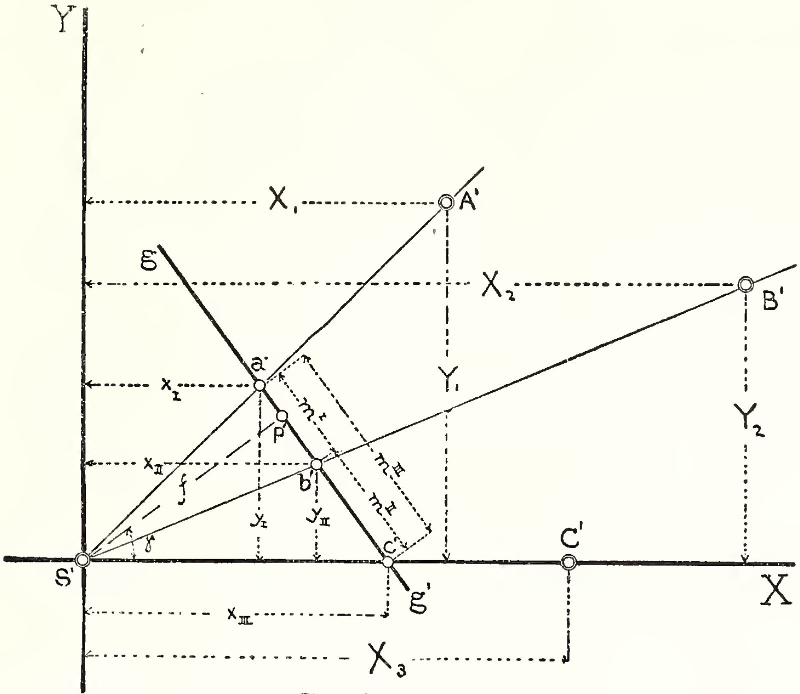


FIG. 48

The horizontal distances between a and b, b and c, a and c (which are the same as those between a' and b', b' and c', a' and c' on the picture trace) may be m^{I}, m^{II}, and m^{III}, respectively.

We will find directly, from an inspection of fig. 48:

$$(1)\quad y_{\text{I}} : x_{\text{I}} = Y_1 : X_1$$
$$(2)\quad y_{\text{II}} : x_{\text{II}} = Y_2 : X_2$$
$$(3)\quad y_{\text{I}} : y_{\text{II}} = m^{\text{III}} : m^{\text{II}}$$
$$(4)\quad (x_{\text{III}} - x_{\text{I}}) : (x_{\text{II}} - x_{\text{I}}) = m^{\text{III}} : m^{\text{I}}$$
$$(5)\quad (x_{\text{III}} - x_{\text{I}})^2 + y_{\text{I}}^2 = (m^{\text{III}})^2$$

From these five equations the five unknown quantities x_{I}, y_{I}, x_{II}, y_{II}, and x_{III}—the coordinates of the points a', b', and c', which are to be located—may be computed.

From the area of the triangle $S'a'c'$

$$\frac{y_{\text{I}} \cdot x_{\text{III}}}{2} = \frac{f \cdot m^{\text{III}}}{2}$$

we find the focal length

$$f = \frac{y_{\text{I}} \cdot x_{\text{III}}}{m^{\text{III}}}$$

The horizontal angle of orientation γ—included between the principal ray $S'P'$ and the horizontal direction to C $(= S'C')$—may be found from the equation:

$$\cos \gamma = \frac{f}{x_{III}} \text{ or } = \frac{y_I}{m^{III}}$$

The principal point P' may now be located upon gg' from c' by making

$$P'c' = x_{III} \sin \gamma.$$

The differences in elevation between the camera station S and the three triangulation points A, B, and C being known, it will now be an easy matter to draw the horizon line upon the photograph and mark the position of the principal point P on the same.

II. GRAPHICAL ICONOMETRIC METHODS.

(1) *Method of Col. A. Laussedat.*—Colonel Laussedat's methods of constructing topographic maps from perspective views of the terrene, having been widely published, form the groundwork for all subsequent work in this direction; they are chiefly of a graphical character and they are in harmony with the laws of perspective.

Laussedat considers two cases in reconnaissance surveys for geographic expeditions to which photo-topographic methods may be applied with advantage:

(1) The explorer may remain sufficiently long in one locality to make a survey on a large scale, say 1:20 000, and even larger for special purposes.

(2) The explorer moves rapidly from place to place, gathering only the most necessary data on his itinerary to enable him to plat the topography of the traversed country as a "running survey" on a small scale—say 1:50 000 and even smaller—preserving and representing only the principal topographic features met with on the track survey.

In the first-mentioned case the explorer will measure one or more base lines, with as great an accuracy as the means at hand and the time at his disposal will admit. He will then cover the area to be mapped with a system of triangles, connected with (or founded upon) the base line, and, inasmuch as the triangulation stations will be occupied with the surveying camera, the scheme should be laid out with due reference to the subsequent iconometric platting of the topographic features.

When applying the ordinary surveying methods the triangulation scheme would probably be laid out with a view toward covering as large a territory as possible, occupying the least number of intervisible points. With the use of photography, however, the conditions are changed; every topographic feature that is to be platted iconometrically should be seen from two or more camera stations. The latter are to be triangulation stations, or they will have to be tied on to the general scheme by special supplementary instrumental observations. Still it is not always essential that the highest peaks, which may be included in the trigonometric survey (as concluded points), should also be occupied with the camera, as frequently other camera stations will answer the requirements just as well.

Regarding the second case, where the explorer follows a certain route, making only the most necessary (and at best but short) side excursions, the photo-topographic method is even of greater value than in the first case, particularly when traversing open and broken country. For this kind of reconnoissance it may be well claimed that the photographic method surpasses all other surveying methods regarding the amount of data which may be collected in a limited time period.

All topographic operations and instruments serve to measure vertical and horizontal angles, and a photographic perspective (of which the focal length and the positions of the horizon line and principal point are known) will give all the data needed to determine the vertical and horizontal angles of lines of direction drawn from the point of view to all points pictured on the photograph.

The points A and B, pictured on the vertical plate MN, fig. 49, may represent the images of two distant mountain peaks; a and b will be their orthogonal projections upon the horizon line HH' (picture trace in horizontal plane HH).

$a\,S\,b = \alpha =$ horizontal angle between lines of direction from the station to the two peaks, A and B. SP (perpendicular to HH') = distance line or focal length of the picture MN.

The vertical angles β and γ may be shown, in horizontal plan, by revolving the vertical

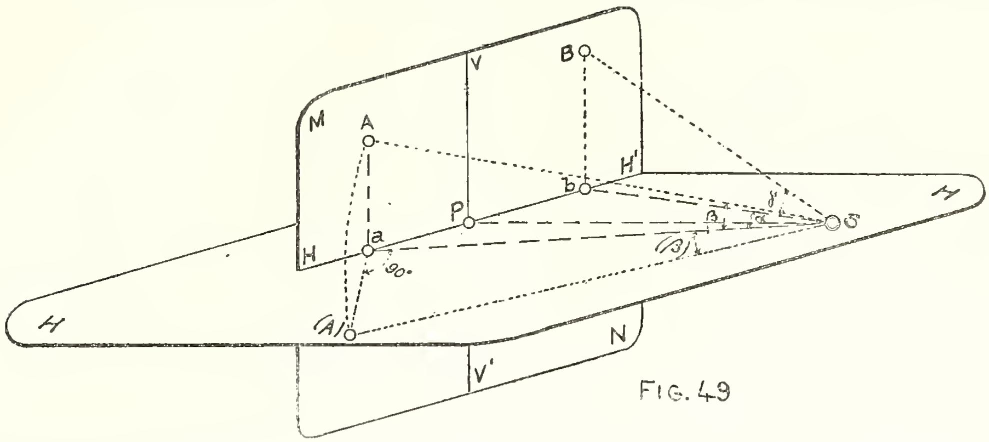


FIG. 49

planes passing through SA and SB about the lines Sa and Sb, respectively, until they coincide with the horizon plane HH. This has been done in fig. 49 for the vertical plane $S\,a\,A$:

$$a\,A = a\,(A) \text{ and } (A)\,a\,S = A\,a\,S = 90^\circ$$
$$A\,S\,a = (A)\,S\,a = \beta.$$

The vertical angles β and γ may now be measured in horizontal plan as (β) and (γ).

To indicate the general method of iconometric platting, and to show how the platted features of the terrene may be obtained from the photographs, we will refer to figs. 50 and 51.

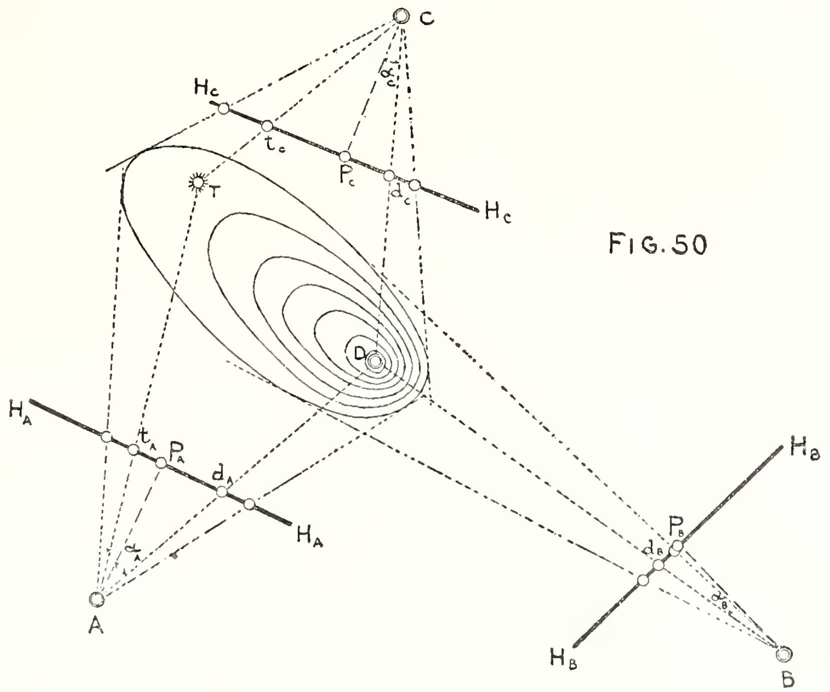


FIG. 50

A, B, and C are three camera stations, platted in horizontal plan, whence three perspectives, I, II, and III, fig. 51, of the same knoll D were obtained. The traces of these three pictures on the platting sheet, fig. 50, may be $H_A\,H_A$, $H_B\,H_B$, $H_C\,H_C$. All three photographs may have been obtained with the same camera of constant focal length—the distance lines $P_A\,A$, $P_B\,B$, and $P_C\,C$ are of equal length.

(*a*) *Locating points identified on several photographs on the platting sheet.*—The three stations *A*, *B*, and *C* are platted, either as parts of the triangulation system, or by measuring the base line *AB* on the ground and measuring the horizontal angles *CAB*, *CBA*, and *ACB*, after which the sides *AC* and *BC* may be found graphically (or by computation) and the triangle *ABC* may now be

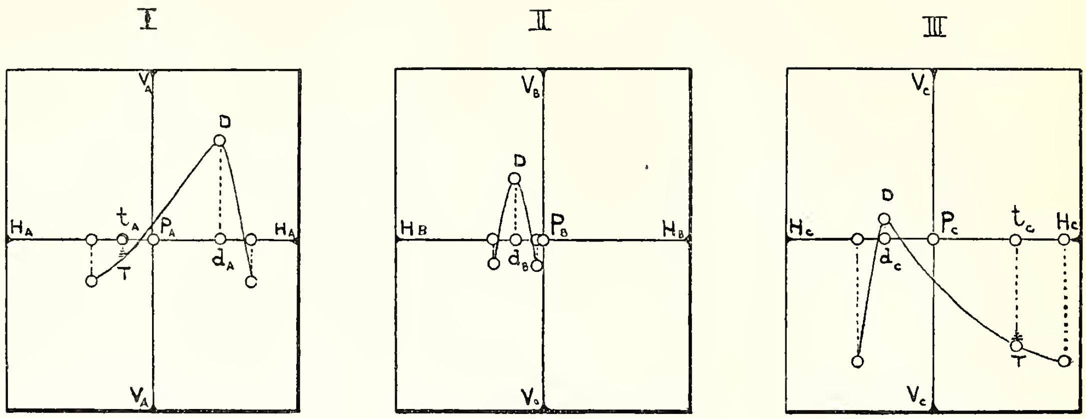


FIG. 51

platted upon the working plan. Horizontal angles or directions to *D* having also been observed from *A*, *B*, and *C*, its position with reference to those three points may also be platted. To plat the three picture traces *HH* we must know the horizontal angles *PAd* (= α), which are observed in the field for each picture by means of the horizontal circle attached to the phototheodolite.

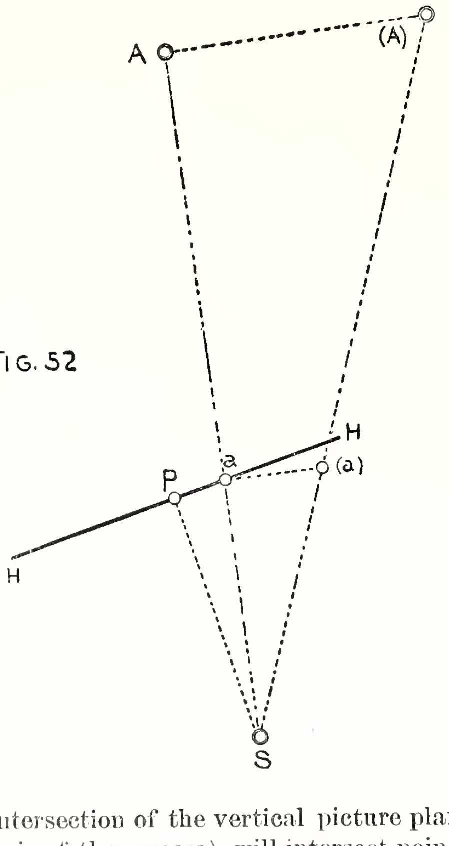


FIG. 52

The angles α are platted as α_A, α_B, and α_C, fig. 50, and the constant focal length (=*f*) of the three negatives I, II, and III, fig. 51, is laid off on the radials AP_A, BP_B, and CP_C. Perpendiculars erected to these lines in P_A, P_B, and P_C, respectively, will represent the oriented picture traces H_AH_A, H_BH_B, and H_CH_C, when the abscissæ P_Ad_A, P_Bd_B, and P_Cd_C, measured on the negatives I, II, and III, should equal the lengths P_Ad_A, P_Bd_B, and P_Cd_C on the picture traces.

The point *D* is termed a "reference point." Every picture that is to be used in iconometric platting should contain the image of at least one such reference point of known position in both the horizontal and vertical sense.

After the picture traces *HH* have once been platted, any other point, *T*, of the terrene, shown on two or more photographs, may readily be platted from the photographs without requiring instrumental measurements in the field.

To locate the platted position of the point *T*, shown on two pictures, I and III, as *t*, the abscissæ, P_At_A and P_Ct_C, are laid off on the picture traces H_AH_A and H_CH_C, respectively, from P_A and P_C and on the proper side of *P* to correspond with the position of the image *t* with reference to the principal point, *P*, of the perspectives. Lines drawn from *A* and *C* through t_A and t_C, fig. 50, represent the lines of horizontal directions to *T*, and their point of intersection locates the position of *T* on the plat with reference to *A*, *B*, and *C*.

(*b*) *Determination of the elevations of pictured points.*—The horizon line *HH'* of a perspective, fig. 49, being the intersection of the vertical picture plane *MN* with the horizon plane (passing through the optical axis of the camera), will intersect points in the picture which in nature have the same elevation as the optical axis of the camera or as the point of view *S*.

The distances Sa and SA, fig. 52, are measured on the platting sheet and the ordinate aA, fig. 49, of the pictured point a, on the negative. Perpendiculars are then erected to SA in A and a and the latter is made equal to the ordinate of a taken from the picture $= Aa = a\,(a)$, fig. 52. If we now draw the line $S\,(a)$ (to its intersection with the perpendicular in A), then the triangles $Sa\,(a)$ and $SA\,(A)$ will be similar and the angle $AS\,(A)$ will represent the vertical angle (of elevation) of the visual ray from S to A revolved about SA into the plane of the horizon or into the platting plan. From the similar triangles $Sa\,(a)$ and $SA\,(A)$ we derive the proportional equation:

$$A\,(A) : SA = a\,(a) : Sa$$

whence

$$A\,(A) = \frac{a\,(a)\,.\,SA}{sa}$$

The value found for $A\,(A)$ measured on the platting scale will give the difference in elevation between camera station horizon and the point A.

In practical work the elevations of the camera stations are known, and by adding the height of the instrument including the value $A\,(A)$ to the elevation of S, fig. 49, the absolute height of A will be found, which, however, is still to be corrected for curvature and refraction.

A second value for the elevation of A may be found in the same manner for another negative containing the image a (taken from another station), and the mean of several such determinations is adopted for the final value for the height of A.

(*c*) *Drawing the plan, including horizontal contours.*—After some little practice points, pictured on different negatives but representing identical points in nature, will readily be identified by the observer and he will soon be able to pick out the characteristic points to reproduce the water courses, watersheds, roads, canals, etc., on the platting sheet. After these principal guide lines have been well located on the chart, buildings, outlines of woods, marshes, etc., are platted, including everything that is to be shown on the finished map.

Enough points should be platted iconometrically to give a good control for a correct delineation of the relief. When the number of points determined on the plan is sufficient, or if they are favorably located to give an adequate control only for the delineation in the horizontal sense, additional points should be platted in order to obtain an equally good control of the terrene in the vertical sense.

The planimetric work completed, elevations of as many of the platted points as seem necessary (or additional ones) are determined and inscribed on the chart. Horizontal and equidistant contours may now be drawn, by interpolation, to harmonize with the elevations suffixed on the chart to the points of control, conforming their courses (between the located points) to the configuration of the terrene, as it is shown on the photographs.

It can not be denied that a certain amount of study and practical application are required to enable the draftsman to correctly interpret forms of the terrene, shown in perspective. Yet, it should also be admitted that such translation or conversion of the relief of the terrene into the horizontal map projections may be far more accurately accomplished (at one's leisure) by means of geometrically correct perspectives, than could be accomplished by sketching in the field. When topographic features are sketched, as seen from one direction, they will frequently be found to have been misconceived when they are seen again from another (not anticipated) point of view. Of course, the platted forms may then be corrected in a measure, at least, still, many details are sketched which will not be seen again from other stations, and, even those that are seen again under other conditions may not be modified to conform to their true shapes, unless the original station, whence they were first seen and sketched, could be reoccupied to verify the suggested changes and corrections. Generally speaking, topographers regard a second occupation of a station with little favor, it being considered too great a waste of time, retarding progress, and considerably increasing the cost of the work.

In iconometric platting, however, it is always an easy matter to refer back again to panoramic views obtained from some other station, and the platting of topographic details should not be attempted without having first made a careful study of and a close comparison between the various pictures representing the same features but seen from different points of view.

(2) *Method of Dr. A. Meydenbaur.*—The pantoscopic lens (made by E. Bush, Rathenow,

Prussia) of Dr. Meydenbaur's surveying camera commands an angle of about 100°. By excluding an external ring of the effective disk of these lenses by means of a diaphragm, pictures are obtained subtending an angle of but 66°, requiring six plates for a complete panorama.

This camera has neither telescope nor vertical circle but it is provided with a horizontal circle, thus enabling the operator to control the revolutions of the camera in azimuth.

After this camera has been set up and adjusted over a station the panorama is photographed by exposing six plates in succession, each successive turn in azimuth of the camera covering an angle of 60°, fig. 53, and two adjoining plates lapping over each other by 3° in arc. These common margins (like Paganini's plates) contain identical sections of the panorama view. They may serve to find the value for the focal length of the pictures, and they control the permanency of the camera's adjustments during one complete revolution in azimuth.

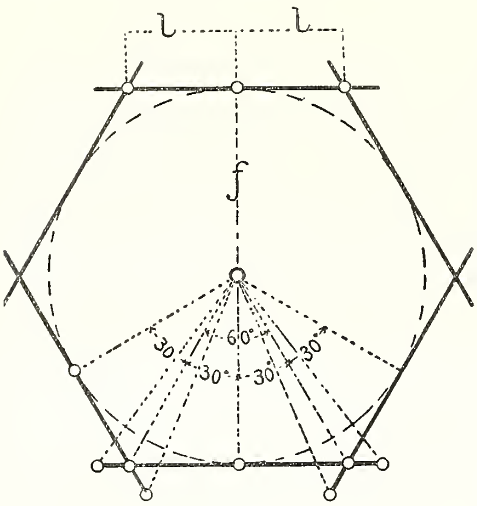


FIG. 53

(*a*) *Determination of the focal length for the panorama views.*—From the six plates, covering the entire horizon from one station, objects may be selected on the center lines of the common margins of adjoining plates which should be equidistant from the principal lines of the two plates.

After having selected a series of such reciprocal points (using a magnifying glass if necessary) on all six plates, we will have obtained twelve determinations, represented by the length l, for the position of the principal line. The greatest discrepancy between any two values should not exceed 0·2 mm, if the instrument was well adjusted. The sum $=2l$ of two such distances (between two of the corrected principal lines) will represent the effective lengths of one picture, or the length of one side of a regular hexagon, with an inscribed circle of the radius equal to the constant focal lengths ($=f$) of the negatives.

This length $=f$ may be found graphically or it may be computed from the formula:

$$f=\frac{l}{\tan 30^\circ}$$

When positive prints are to be used in the iconometric map construction it will become necessary to correct this focal length f to correspond with any changes that may have taken place in the dimensions of the prints when compared with their negatives. By comparing the distances between the "teeth" (marking the principal and horizon lines) on the negative with those included between their contact prints on the positive the total linear changes of the print in the directions of the principal and horizon lines are readily found.

We have with reference to fig. 54: $ab=$ original length included between the teeth marking the horizon line on the negative. $a'b'=$ length of horizon line (included between the pictured teeth) on the positive print. $co=f=$ constant focal length of camera or negative.

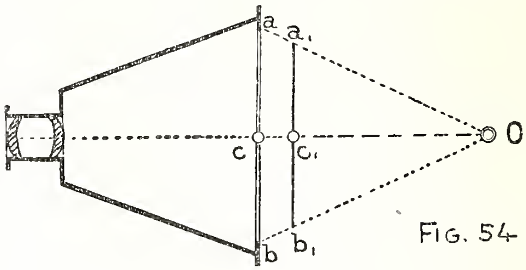


FIG. 54

If we draw the triangle abO, place the line $a'b'$ (measured on the print) parallel with ab and move the same (maintaining its direction parallel with ab) toward (or from) O until a' falls upon ao and b' upon bo, then $c'O$ will be the focal length of the photograph ("contracted," in our case). This determination of f should be made for every print that is to be used in the iconometric map construction.

The topographic map is graphically constructed from the negatives and prints in a manner very similar to that described for Colonel Laussedat's method.

(*b*) *General method of iconometric plotting.*—With reference to fig. 55 we have:

I and II = two negatives of plates exposed from camera stations I and II, respectively.

I II = baseline, measured in the field.

The elevations of camera stations I and II may be known and negative I may contain the image of station II, negative II that of station I. After the baseline I II has been platted in reduced scale (in the scale of the proposed map), circles are described about I and II as centers

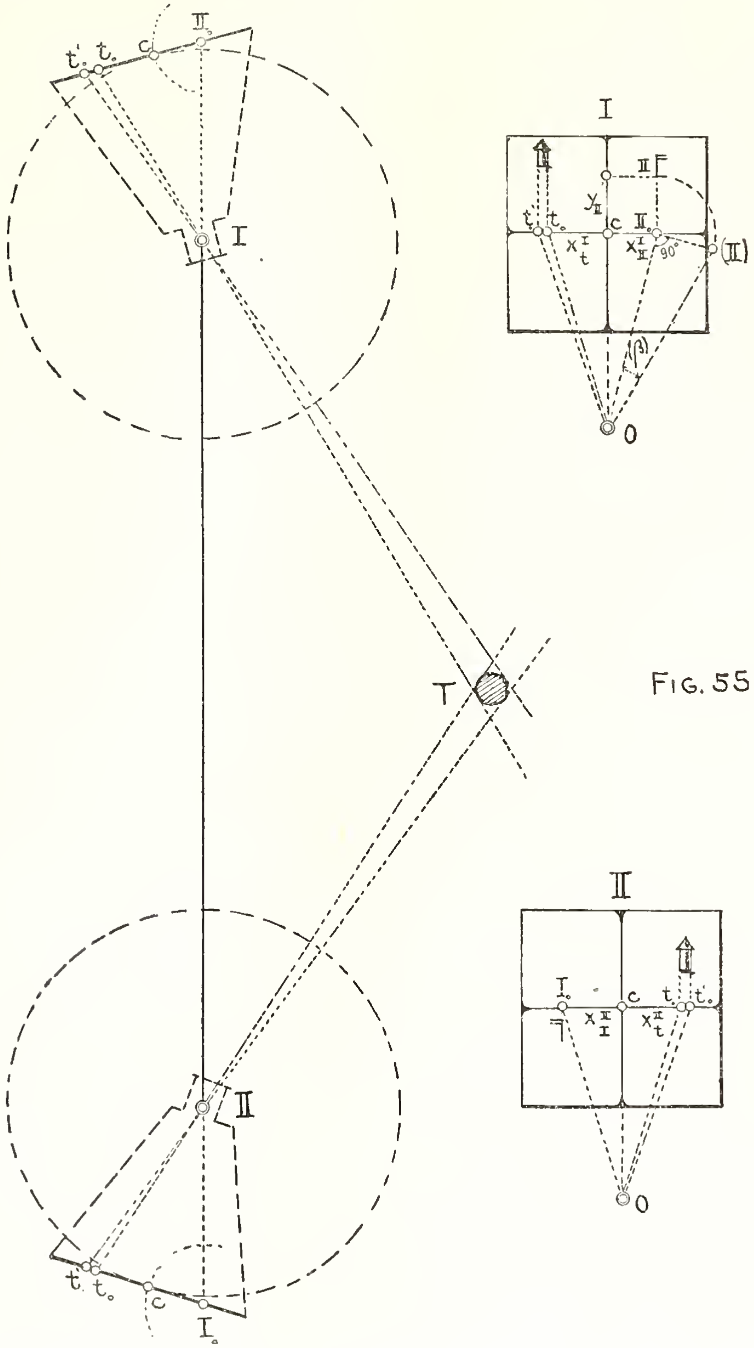


FIG. 55

with radii $= cO = f =$ constant focal length of the negatives. Then make

$$I\,II_0 = O\,II_0 \text{ (Pl. I) and}$$
$$II\,I_0 = O\,I_0 \text{ (Pl. II).}$$

Describe arcs from II_0 with $II_0c = x^{I}_{II}$ (plate I) and from I_0 with $I_0c = x^{II}_{I}$ (Pl. II) as radii, transpose $ct_0 = x^{I}_{t}$ (Pl. I) on the tangent II_0c and $ct_0 = x^{II}_{t}$ (Pl. II) on the tangent I_0c.

The prolongations of $t_0 I$ and $t'_0 I$ will be tangential directions to the sides of the tower T (pictured on Pls. I and II) from camera station I, and t_0II and t'_0II will be the tangential directions to the sides of the same tower T from station II. These four tangents intersect each other at T in a quadrangle, the inscribed circle of which will represent the position of the tower (in horizontal plan) with reference to the baseline I II.

Any other points, common to both Pls. I and II, may be located in horizontal plan in pre-

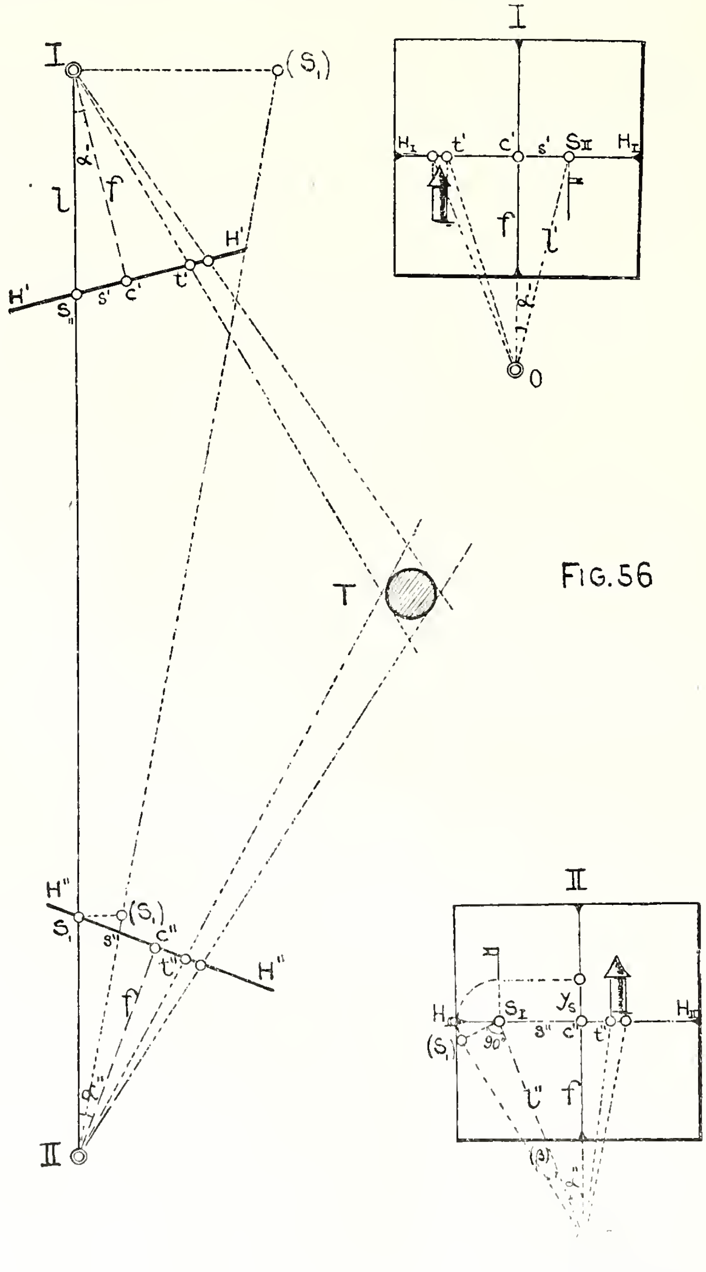


FIG. 56

cisely the same manner. The method just described is general in character, but when the camera is provided with a horizontal circle, enabling the observer to cover the horizon with six plates by revolving the camera exactly 60° in azimuth after each exposure, the following method is generally applied:

The constant focal length $= f$ of the negatives is laid off on the principal line below the principal point $= c'O$ for negative I and $= c''O$ for negative II. The images of the stations are projected upon the horizon lines, S_{II} upon $H_I H_I$ (Plate I) and S_I upon $H_{II} H_{II}$ (Pl. II) when

$c'OS_{II} = \alpha'$ represents the horizontal angle included between the principal plane and base line I II, and $c''OS_I = \alpha''$ represents the corresponding horizontal angle for station II. These angles α' and α'' are transferred from the negatives I and II to the corresponding ends of the base line I II, as indicated in fig. 56.

After laying off the focal length f from the base stations I and II upon the sides of the angles α' and α'' ($= Ic'$ and IIc'' respectively) and erecting perpendiculars ($H'H'$ and $H''H''$) in c' and c'' to Ic' and IIc'' respectively, they will represent the oriented picture traces of negatives I and II.

The remaining two sets, of five plates each, of the panorama views at the stations I and II, are easily oriented and platted, the next plate in order at station I, for instance, will have the optical axis in the direction $\alpha' + 60°$, the third: $\alpha' + 120°$, etc.

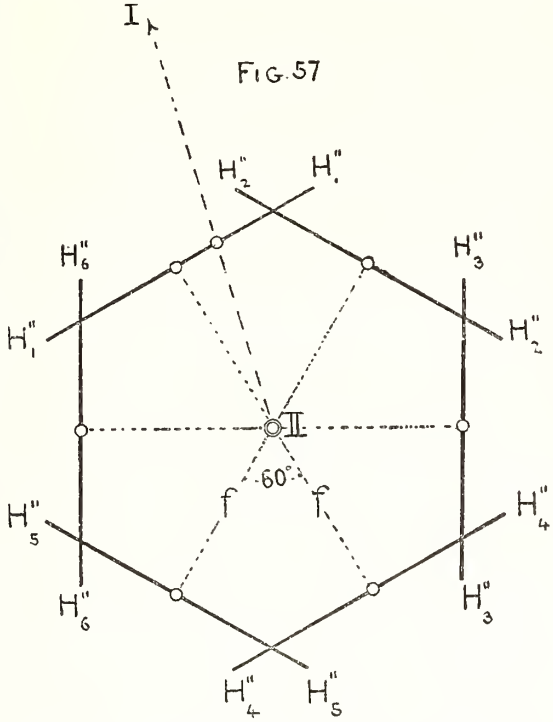


FIG. 57

After all the horizontal projections of the vertical plates (picture traces) H_1H_1, H_2H_2, ... H_6H_6, fig. 57, have been platted at both stations I and II, the horizontal projections of all points that may be identified on two plates are marked and platted by locating the intersections of the lines of direction drawn through the projections on the picture traces of the pictured points in the same manner as shown in fig. 56 for the tower T. Every platted camera station will be surrounded by a regular hexagon formed by the picture traces of the six plates comprising the panorama set.

(*c*) *Determination of the elevations of pictured points of the terrene.*—The projection in horizontal plan of an object having been platted, the elevation $I(S_I)$ of that object S_I above (or the depression of it below) the horizon, HH, of the camera station II may be found as follows:

The lengths IIS_I ($= OS_I$ on Pl. II) and I II, fig. 56, may be measured on the platting sheet, and the ordinate y_s may be taken from the negative II.

We erect perpendiculars to I II in $S_I = y_s = S_I$ (S_1) and in I, then draw the line $II(S_I)$ to its intersection (S_I) with the perpendicular to I II in I, when the length $I(S_I)$, measured in the platting scale, will represent the difference in elevation between the points I and II.

By computation we would find from:

$$I(S_I) : y_s = I\,II : S_I\,II$$

$$I(S_I) = y_s \frac{I\,II}{S_I\,II}$$

If the scale of the map is $\frac{1}{M}$, we will have:

$$I(S_I) = M.y_s.\frac{I\,II}{S_I\,II}$$

The values of y_s, $I\,II$, and $S_I II$ are found by direct measurements with a small ivory scale divided into 0·5 mm., of which 0·1 mm. may be estimated after a little practice.

(3) *Method of Capt. E. Deville* (*Canadian method*).—This so-called Canadian method has been in use under the auspices of the department of the interior of the Dominion of Canada since 1888. Deville has given a full account of these methods in Photographic Surveying, published at the government printing bureau, at Ottawa, in 1895, and the following paragraphs have been largely taken from Deville's book:

(*a*) *General remarks on field work.*—The area to be surveyed is covered with a triangulation net, preferably before the phototopographic survey is commenced, and a secondary triangulation is carried along with the phototopographic work to locate the camera stations in both the horizontal and vertical sense, with reference to the primary triangulation stations already established.

The surveyor makes a rough plat of the entire triangulation (in the field), on which he locates all the stations occupied to enable him to recognize the weak points of his work and to plan his operations with a thorough understanding and good assurance of success. The instrumental work in the field is done merely to locate the camera stations and certain reference points (if the triangulation points are not sufficiently close together), all topographic features being deduced from the pictures.

The camera stations are located either by angles taken from the station to surrounding triangulation points, by resecting, or by angles observed from the latter to the signal left over the camera station, by intersecting, or by both methods combined.

The final value of the work depends in a great measure upon a judicious selection of the camera stations in order to bring the relief of the entire terrene under proper control and to be enabled to plot all points needed for a full development of the terrene by the method of intersections of horizontal lines of direction.

Other methods for platting the topographic features and details are employed only when the method of intersections fails on account of the camera stations not being well situated, or on account of an insufficiency of data to give the requisite number of horizontal lines of direction for a good location of points by "intersecting."

Each camera station should be marked by a signal of some kind before leaving it, not to be shown on the pictures, but to be observed upon with the transit or altazimuth from other stations in order to locate the correct position of the camera station on the platting or working plan.

Frequently it will be of advantage to set the camera up excentrically over a triangulation station in order to include certain additional parts of the landscape in the views. The position of the excentric camera station, with reference to the triangulation point, can readily be ascertained, and should always be carefully recorded.

Complete panorama sets are not taken at every camera station, it being preferred, rather, to increase the number of stations, often occupying a station to obtain a single view only, if by doing so better intersections for the iconometric location on the platting sheet of some special feature are obtained. Multiplicity of stations demands but a small increase in labor, either in the field, in the extra observations of directions to reference points for their location, or in the iconometric platting in the office, and enough stations should always be occupied to give a full control of the relief of the area to be surveyed.

A certain section of the terrene may be so located that it will be a difficult matter to select more than one station whence it may be seen. In such a case the method of "vertical intersections" may often become useful: Two or more views of such area are taken from stations at different *elevations*, the greater the difference in altitude between such stations the longer will the base line be, and the better are the intersections which locate the features in question, if the latter are not too far away.

As enough plates should be exposed to cover the ground completely, the camera stations will have to be distributed in such a way that all valleys, sinks, and depressions, that may be represented in the scale of the map, are well controlled (i. e., seen from different camera stations). It is evident, therefore, that the number of stations to be occupied for the phototopographic development of a certain area will depend in a great measure upon the character of the terrene and upon the scale of the chart.

Two or three well-defined points (so-called reference points) in each panorama view (covered by one plate) are observed with the transit or altazimuth noting and recording the vertical and horizontal angles upon the outline sketch made for every plate exposed. Such sketches serve to identify points with far more certainty than a mere designation or description of the points observed upon. The general triangulation notes are kept in the usual manner.

Vertical angles are observed to check the position of the horizon line on every photograph and to correct errors due to small changes in the level adjustments of the camera that may arise during the transportation of the instrument over a rough trail. The horizontal angles are needed for the location of the camera stations and for the orientation of the pictures (picture traces) on the platting sheet for the subsequent map construction.

(*b*) *General remarks on the iconometric platting of the survey.*—The field notes of the phototopographic surveys made in the Northwest Territory of the Dominion of Canada by the topographical surveys branch of the department of the interior (under Capt. E. Deville, surveyor of Dominion lands), are platted on a scale of 1:20000, but the maps are published on a scale of 1:40000, with (equidistant) contours of 100 feet vertical intervals.

The phototopographic reconnaissance in southeastern Alaska, executed by Dominion land surveyors under Dr. W. F. King, Alaskan boundary commissioner to Her Majesty, was platted on a scale of 1:80000 and published on a scale of 1:160000, with horizontal contours of 250 feet vertical intervals.

It is assumed that the triangulation computations have been made and that the triangulation points have all been platted, and that their elevations above the adopted reference plane have been affixed to the marked points on the platting sheet.

The triangle sides of the secondary triangulation scheme (executed during the phototopographic survey) are now computed (the corrections to the horizontal angles, indicated by the closing errors, having been applied), the latitudes and departures (from every secondary point to the nearest primary station) are computed, and the secondary stations are then platted by their latitudes and departures (unless the primary triangulation sides are too long).

The camera stations (not included in the secondary triangulation scheme) are now platted with reference to the triangulation points, using a table of chords or a station pointer (three-arm vernier protractor). If many points had been observed upon from the camera station, the horizontal angles are preferably laid off on a piece of tracing paper, and this improvised multi-arm protractor is used like a station pointer to locate the station.

The surveyor should endeavor to obtain at least one direction from a triangulation station to every camera station; the (iconometric) platting will then be less troublesome and more accurate.

Photographs should not be used for platting the positions of camera stations, as this would not locate them with sufficient precision, and enough angles should always be observed in the field to locate every occupied station in the manner just mentioned.

From the original negatives copies are made, enlarged to 9½ by 13 inches on heavy bromide paper, more recently, however, a special brand of bromide paper, known as "platino-bromide," has been used by Captain Deville. The enlargement adopted in Canada for these bromide prints is about 2.1 times larger than the negatives, which ratio was selected to utilize the full width of the paper found in market.

These bromide enlargements are used extensively in the map construction, and they should be made with great care to reduce distortion to a minimum. Before using the prints for the map construction any distortion due to the enlarging process should be ascertained, which is done in the following manner:

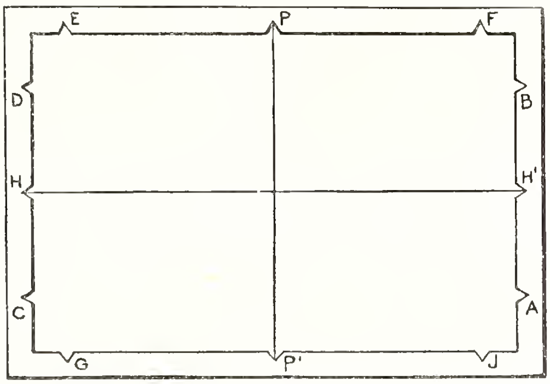


FIG. 58

Join the middle notches H and H', indicating the position of the horizon line, and P and P', representing the position of the principal line, fig. 58, and with a set square test these two lines for perpendicularity. Take with a pair of dividers the distance between the two notches A and B (which on the negative is equal to one-half of the constant focal length) and see whether it is equal to the distance of the corresponding two notches C and D. Now apply one of the points of the dividers in P; the other should come in E and F. Transfer the point to P' and check the length $P'G$ and $P'J$ in the same way. If the print satisfies all these tests, it may be used for the iconometric platting; if it does not, it is returned to the photographer with a request for a better one.

(c) *Platting the picture traces.*—Every photograph contains at least one, generally several, of the triangulation points platted on the working sheet, and the traces of both the picture plane and principal plane are found and platted on the plan as follows:

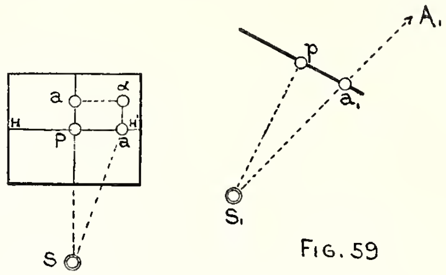


FIG. 59

The distance, or principal line PS, fig. 59, is made equal to the focal length of the picture, and the image point α of the point A is projected upon the principal line $(=a)$ and upon the horizon line $(=a')$. If S_1 represents the platted position of the camera station S on the plan, and if S_1A_1 represents the horizontal direction on the plan from S to A, we make $S_1a_1 = Sa'$ (taken from the photograph) and from a_1, as center, with $\alpha\,a$ $(= Pa')$ as radius, describe a circle to which S_1p is drawn tangent, then $S_1p =$ trace of principal plane and the perpendicular to S_1p through $a_1 = pa_1 =$ trace of picture plane.

Instead of making this construction on the "photograph board," which will be described further on, it can be made on the plan itself, as follows:

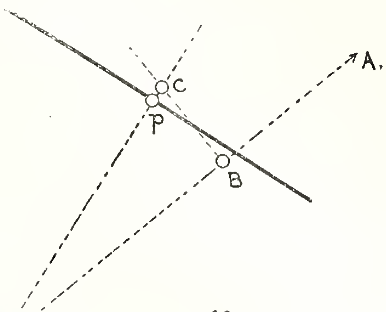


FIG 60

On S_1A_1 take S_1B, fig. 60, equal to the focal length of the print; erect BC perpendicular to S_1A_1 in B and equal to $\alpha\,a$, fig. 59. Join S_1C and take S_1p equal to the focal length; at p erect a perpendicular to S_1C and it will represent the trace of the picture plane, while S_1C is the trace of the principal plane.

Another simple method, avoiding the drawing of constructive lines on the plan, is as follows: Take a triangle of hard rubber or wood and mark off along one side the focal distance SP, fig. 59, of the print, $= ab$, fig. 61, and carefully notch the triangle side at b so that the center of a fine needle, marking the platted station point, will just fit into the notch. From the print, fig. 59, take the abscissa of the pictured point $\alpha = a\alpha = Pa'$ between the points of a pair of dividers, move the triangle, fig. 61, about the needle (which marks the platted station) with the left hand until ac, fig. 61, is equal to the distance $a\alpha$ held between the points of the dividers. The triangle is held securely in this position and lines are drawn along the triangle sides ab and ac. Prolong ac beyond a and check the distance ac again to be $= a\alpha$. The line bc represents the horizontal direction from the platted station b to the platted reference point c (on the negative, fig. 59, the picture of the corresponding reference point is α). We will now have: $ba =$ trace of the principal plane, $ac =$ trace of the picture plane, $a =$ projection of the principal point on the platting sheet.

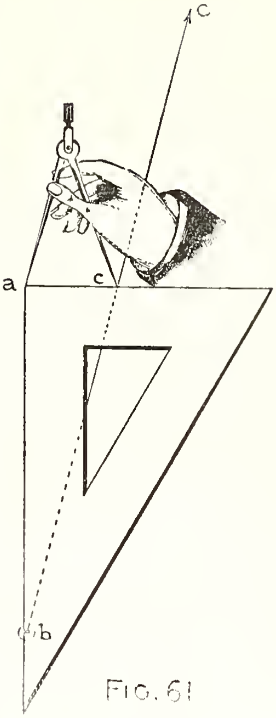


FIG. 61

The trace of the principal plane ($= ab$) is preferably marked on the platting sheet by a short line only, bearing an arrow pointing toward the platted station (b) whence the picture was taken, and the principal point a is marked to correspond with the designation of the print. It may be remarked here that as few lines as possible are drawn on the platting sheet to avoid confusion and mistakes. (See photograph board.)

(*d*) *The identification of pictured points on several photographs representing identical points of the terrene.*—The topographic survey being platted mainly by the intersections of horizontal directions, points controlling the relief of the same area must be identified on sets of pictures taken from different stations. When selecting such points on a photograph preference should be given to those which best define the surface relief or terrene, like characteristic points of ridges, peaks, saddles, changes of slope, changes in the river courses, etc., each point being marked by a dot in red ink on the photograph and having a number or symbol affixed to it. It will now be necessary to identify as many of these points as possible on other photographs, covering the same area, and these are similarly marked by red dots, and identical points are given the same designation by number or symbol in red ink.

The identification itself of points on several pictures offers no serious difficulties, and, with some practice, as many points as may be needed for a full development of the terrene, even under different illumination of the pictured areas, may be picked out with rapidity and certainty.

(*e*) *Application of Professor Hauck's method for the identification of pictured points.*—In cases of doubt, when attempting to identify the same point on two different photographs, beginners may take advantage of Professor Hauck's method, which has been described in Chapter I, Paragraph VII.

The two photographs are pinned side by side on a drawing board. The images of the camera stations whence the pictures were obtained are "kernelpoints," and if they fall outside of the picture limits they are determined from the ground plan and platted on the drawing board. The parallels to the principal lines of the photographs on which the scales are to be laid off are drawn in the manner explained in Chapter I and the scales are fixed in position. A fine needle is now inserted into each of the "kernelpoints" and the loop at one end of a fine silk thread is dropped over each needle, the other end of the thread being secured by a slender rubber band to a small paper weight (fig. 62).

A well-defined point is now identified on *both* photographs, sufficiently far from the "kernel points," and one thread is moved by taking the paper weight up and passing its thread, under gentle tension of the rubber band, through the point just identified on the photograph, when the weight is deposited upon the drawing board, holding the thread in the given position. The same operation is repeated with the other silk thread and the other photograph, when the two threads should intersect the scales at identical division marks. If they do not, one of the scales is to be moved until both threads bisect the same division marks of the scales.

Now the identification of the doubtful points may be proceeded with. Having selected a point on one of the photographs, the corresponding silk thread is moved to bisect that point, noting the position of the thread with reference to the scale in this position. The other thread is moved to bisect the corresponding graduation mark on the second scale, when this thread will also bisect the corresponding point on the second photograph.

(*f*) *Platting the intersections of horizontal directions to pictured points.*—After enough pictures have been selected to control the cartographic development of a certain area and the identification and marking of corresponding points have been completed, projections of all these points on the horizon lines of the pictures are marked (their abscissæ are measured) and transferred to the straight edge of a strip of paper, including the marking on the paper strip of the principal point of every photograph. Each paper strip bears the same designation (in red ink) as the picture to which it belongs.

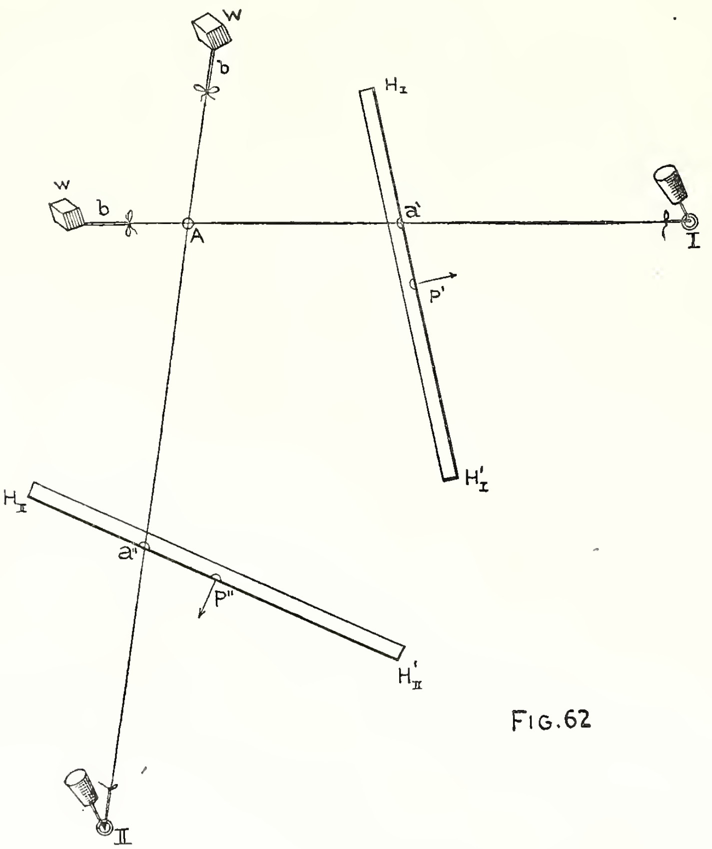


FIG. 62

These strips are now placed upon the platting sheet on the picture traces to which they belong in such a manner that the principal points of paper strips and picture traces coincide, and in this position they are securely held to the working sheet by means of small thumb tacks or paper weights.

To plat the horizontal projections of a point, shown and marked on two prints, two fine needles are inserted into the platted station points *I* and *II*, fig. 62 (of the two prints) and a fine silk thread attached to a small paper weight *w* (by fine rubber band *b*) is secured to each needle by a loop.

The thread attached to station needle *I* is now moved over the weighted paper strip (indicating the picture trace on the plan) until it bisects the horizontal projection a' of the picture point *a*. The weight is now placed upon the working plan, holding this thread (under slight tension) in this position. The second thread, attached to the needle in station *II*, is placed over the projection a'' of the image of the point *A*, also under tension of the rubber band. The point of intersection of the two threads will be the position on the plan of the point to be platted (= *A*). After this position of *A* upon the plan has been checked, in the same manner, by means of another photograph (thread and paper strip) taken from a third station, *III*, and containing the image a''' of the point *A*, its platted position is marked by a dot in red ink and its designation corresponding with that given on the prints is also affixed.

After a sufficient number of points have been platted in this manner by intersections, and after they have all been supplied with the letters or numerals given them on the prints, their elevations are determined and also added in red ink. Frequently the designation of the points by letters or symbols are only added in pencil on the working sheet, to be erased after the elevations of the points have been affixed to them in red ink.

When the strips of paper should overlap each other, as shown in fig. 63, the part *CD* of the picture trace *PQ* is marked off on the strip *MN* lying under *PQ*, the paper strip *PQ* is placed in proper position, and the marks on its edge are transferred to the line *CD*. The strip *PQ* is now placed under *MN*, the marks on the latter along *CD* serving the same purpose as those of *PQ*.

When a station, A, fig. 64, falls so close to the edge of the working board that the trace QR (of the picture plane) falls outside of the limits of the plan, then the trace AC of the principal plane is produced to B, making $AB = AC =$ focal length of the picture, and MN is drawn perpendicular to BC or parallel to QR. The line MN will, with reference to QR, occupy the same position as the focal plane of the camera does to the picture plane of the perspective. The direction of a point of the photograph projected in Q on the picture trace is found by joining NA and producing to the opposite side of A.

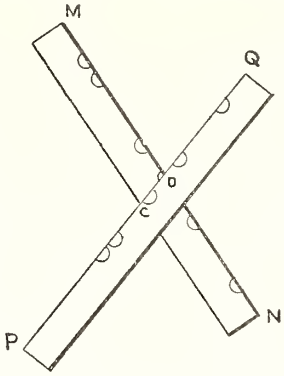


FIG. 63

As mentioned before, the intersection of the first two lines of direction should be checked either by a third line or otherwise before the position on the plan of a pictured point should be accepted as correct. Such intersections may, for instance, be checked by determining the height of the point from both photographs. Unless correctly platted and correctly identified, the two values for its height will not agree. This check, however, does not guard against slight errors in platting. A check may also be obtained by drawing a line, on which the point is situated, with the perspectograph or perspectometer, but the best check will always be a third intersecting line of direction from a third station.

(*g*) *Platting pictured points iconometrically by vertical intersections.*—We had seen how the base line between two stations is projected into horizontal plan for the method of horizontal intersections hitherto considered, but when two camera stations are occupied at different elevations (and close together horizontally) to locate features of the terrene by intersections, the so-called "method of vertical intersections" is employed. With this method the base line (its horizontal projection being either too short or more frequently falling into the direction in which the points to be located iconometrically are situated) is projected upon a vertical plane. The greater the difference in elevation between the two stations, the greater the length of this base-line projection in vertical plane, and also the better the location of the points by vertical intersections will be.

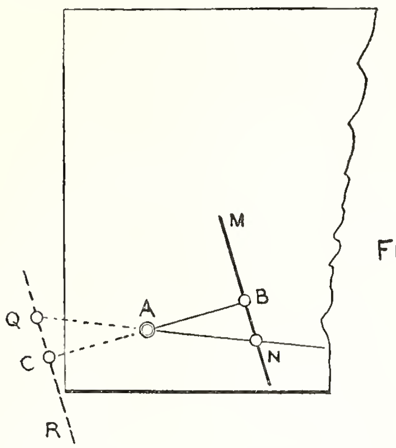


FIG. 64

We will have with reference to fig. 65:

A and B = positions of the two camera stations, platted upon the working sheet. (A is more elevated than B). aB = horizontal projection of the base line AB. A_N and B_N = two negatives (showing the images d_A and d_B of the same point D) exposed at the stations A and B respectively. $H_{AB}H_{AB}'$ and H_BH_B' = picture traces of the two negatives on the ground plane or working sheet. $aP_A' = BP_B'$ = focal length of the negatives A_N and B_N.

We will assume that the horizontal plane passing through the lower station (B) is the ground or platting plane, and the principal plane of the negative A may be taken as the vertical plane of projection. $H_{AB}H_{AB}'$ will then be the trace of the picture plane A_N on the ground plane.

Furthermore, the principal plane, of which aP_A' is the trace in the ground plane, is supposed to be revolved about aP_A' into the ground or platting plane in order to simplify the construction.

To plat the position in the ground plane of a point D, pictured on A_N and B_N as d_A and d_B respectively, the rays Ad_A and Bd_B are projected upon the vertical plane (revolved about aP_A' into the ground plane) when (d_1), in fig. 65, will represent their point of intersection d, projected into the vertical plane $= d_1$, and revolved about aP_A' into the platting plane $= (d_1)$.

The ray $Ad_A = AD$ intersects or penetrates the picture plane A_N at a distance $= d_Ad_{AB}'$ vertically above d_A', on its picture trace $H_{AB}H_{AB}'$ (ground line of picture A_N). This ordinate is laid off upon $P_A'H_{AB} = P_A'$ (d_A), when (d_A) will be the projection on the vertical plane of the pictured point d_A.

The vertical through a projected upon the vertical plane is represented as a (A), and if we make a $(A) = P_AP_{AB}'$ (of picture A_N) = difference in elevation between the two stations A and B, then (A) will be the upper camera station A projected into the vertical plane, and the line connecting A with the point (d_A), just found, will be the projection into vertical plane of the ray Ad_A or AD (revolved about aP_A' with the vertical plane into the platting plane).

The ray $Bd_B = BD$ intersects the second picture plane B_N in d_B. If we draw through d_B' (projection of d_B in ground line H_BH_B') a perpendicular to $aP_A' = d_B'd_{1B}$, then d_{1B} will be the projection into the vertical plane of the horizontal projection in the picture trace of the picture

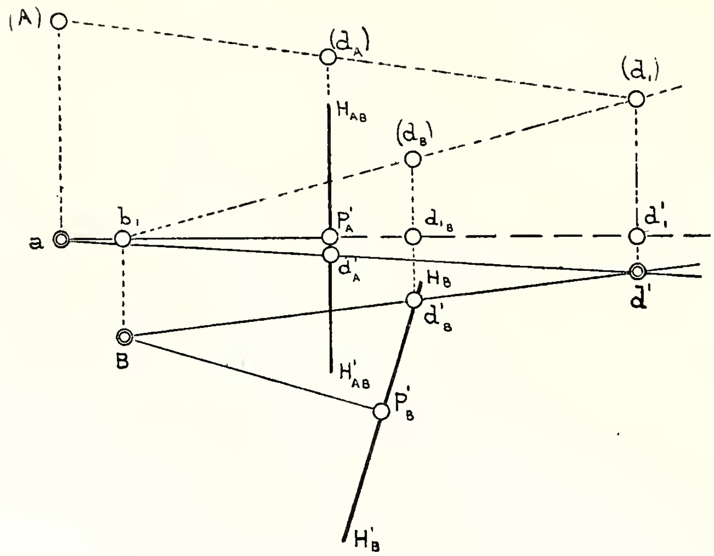
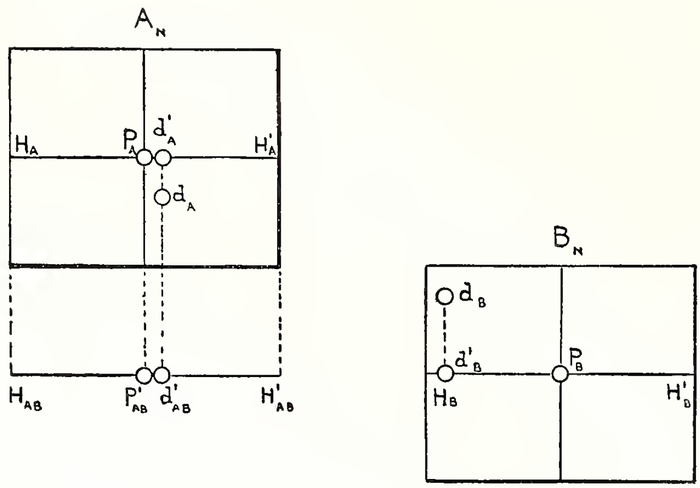


FIG. 65



point d_B. Producing $d_B'd_{1B}$ and making d_{1B} $(d_B) = d_Bd_B'$ (measured on the negative B_N) will locate at (d_B) the projection of the pictured point d_B upon the vertical plane.

The perpendicular to aP_A' through B will locate the projection into the vertical plane $=b_1$ of the platted station B, hence the line connecting b_1 with (d_B) will be the projection into the vertical plane of the ray $Bd_B = BD$.

The intersection (d_1) of b_1 (d_B) with (A) (d_A) locates the projection into vertical plane of the point sought, d, and the horizontal projection of this point d (the platted position of the original point, D) will be on the line (d_1) d_1' (which is the vertical through d, or in our case the perpendicular to aP_A' from (d_1)), and either horizontal directions ad_A' or Bd_B', produced to intersect this perpendicular (d_1) d_1' will locate the horizontal projection d' of the point d, representing the position on the platting sheet of the point D with reference to the platted stations a and B. (The location of d' as the intersection of the horizontal directions ad_A' and Bd_B' would not be very accurate for our case, and far less so for pictured points on the other side of the principal point P_B', where the angles of intersection of the horizontal directions would be even smaller than at d'.)

The point d_1' being the projection into the vertical plane of the point d' (= horizontal projection into the ground plane of the point d) the length $(d_1)\ d_1'$, measured on the platting scale, will represent the elevation of the point D above station B.

(h) *Iconometric determination of elevations.*—Generally speaking, one perspective will not suffice to determine the height of a point, although there are exceptions, like the points on the horizon line, which have the same elevation as the camera station.

With reference to fig. 66 we have: d_1 = horizontal projection of the point D. Bd_1 = horizontal distance between platted station B and platted position d of point D (measured in platting scale on working sheet). Bd_{1B} = horizontal distance between station B and projection of pictured point d_B in ground line $H_B\ H_B'$, measured on platting sheet. $d_{1B}\ (d_B) = h$ = ordinate of pictured point d_B above ground line (revolved with vertical plane about Bd_{1B} into platting plane), measured

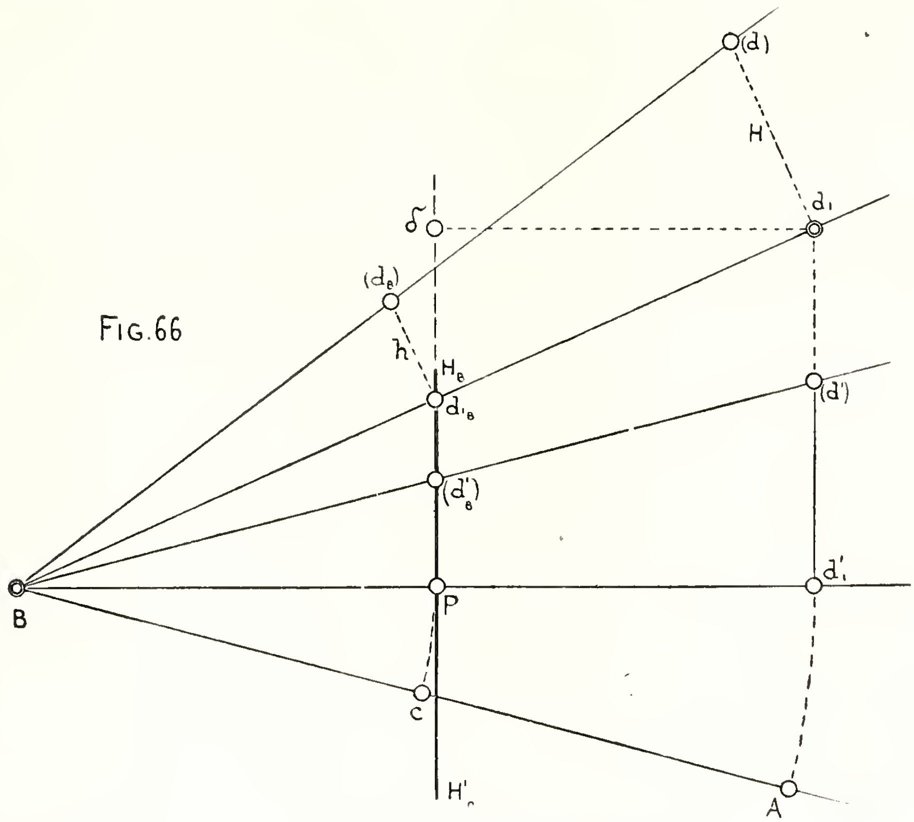


FIG. 66

on picture. $d_1\ (d) = H$ = height of point d above the ground plane (revolved into the ground plane with the vertical plane about Bd_1). Measured on the platting scale, it will give the height of D above the camera horizon (ground plane = horizon plane).

The height H is a fourth proportional to the three known lengths Bd_1, Bd_{1B}, and $d_{1B}\ (d_B)$.

After projecting the platted point d and the pictured point d_B into the principal plane, and after revolving the latter about the principal line BP into the platting, or ground plane, we will have:

$P\ (d_B') = h$ = height of pictured point d_B above the platting plane. (d'_B) = pictured point d_B, projected into the principal plane and revolved with the latter about the principal line into the platting plane. $(d')\ d_1'$ = vertical height of the point d, projected into the vertical plane and revolved with the latter, about the principal line, into the platting or horizon plane; hence $(d')\ d_1' = H$ = elevation of d above the horizon plane.

This height = H being the fourth proportional to the three known lengths:

$BP = f$ = focal length of the print. $P\ (d_B')$ = ordinate of pictured point d_B, measured on photograph, and $Bd_1' = f + Pd_1'$; where: Pd_1' = vertical distance between the platted point d_1 and the picture trace H_BH_B', to be measured on the platting sheet, its value ($=H$) may be found mechanically with aid of an ordinary sector, fig. 67, as follows:

Take with a pair of dividers the (ordinate) distance from the pictured point d_B to the horizon line (on the photograph), place one point of the dividers on the division C of the sector, fig. 67,

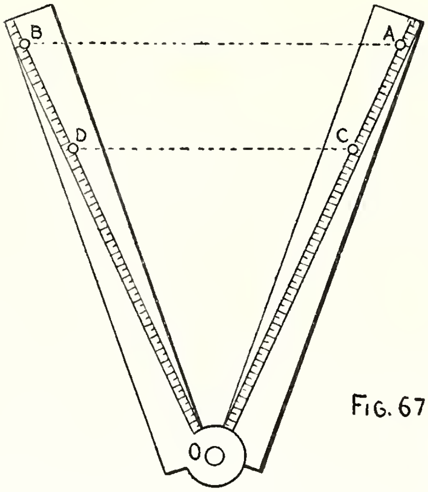


FIG. 67

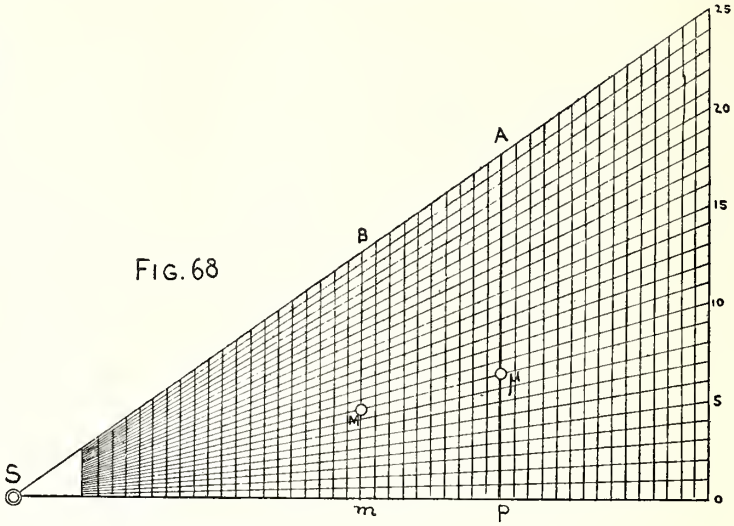


FIG. 68

where OC = focal length of the photograph, and open the arms of the sector until the second point of the dividers coincides with the corresponding division D of the other arm of the sector (OD being equal to OC = focal length), now add the length $d_1'P$, fig. 66 (horizontal distance of the platted point d_1 to the picture trace H_BH_B' projected into vertical plane), to the focal length $=f$ $= OC$, fig. 67, by placing one point of the dividers in C, when the other point may coincide with the division A of the scale OC. Hold the sector unchanged, turn the dividers about the point A, and bring the second point to the graduation mark B of scale OD, B corresponding to A, or $OB = OA$; when AB will represent the height H of the point d above the horizon plane of the station B, to be measured on the platting scale.

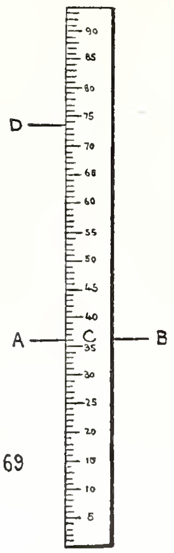


FIG. 69

(*i*) *Iconometric determination of elevations by means of the so-called "scale of heights."*—Another method consists in making use of the "scale of heights," fig. 68. Make $SP = f$ = focal length of the perspective, erect PA perpendicular to SP in P, and divide both lines into equal parts. Draw radials from S through the points of division on PA, and through those of SP draw parallels to PA. Now, with a pair of dividers take from the photograph the distance from the pictured point to the horizon line (the ordinate of the pictured point corresponding to $P(d'_b) = h$, in fig. 66) and transfer it to PA from $P = P\mu$. The position of μ may be found to correspond to the line $S\mu$, passing through the point 9 of the graduation on PA.

With a pair of dividers take now (from the platting sheet) the vertical distance from the horizontal projection of the point to the picture trace ($= \delta d_1$ in fig. 66) and transfer it to the right or left of P according as the point of the plan falls beyond the picture trace or between the platted station and the picture trace. In fig. 68 it is shown as falling between the station and the picture trace into m. The line mB, drawn parallel with PA, is intersected by the radial $S\mu$ (corresponding to scale division mark 9) in M. The distance mM, measured on the platting scale, will be the height of the point above (or below) the station.

A scale, fig. 69, is conveniently pinned somewhere, perpendicularly to a line AB, the division C of the scale, corresponding to AB, being the height of the camera station. One point of a pair of dividers with which the length AB was taken off the "sector," or with which the length mM

was taken off the "scale of heights," is set in C, fig. 69, and the division mark D of the scale, coinciding with the other point of the dividers, will indicate the height of the point above the plane of reference or datum plane.

This height is entered in pencil on the plan, inclosed in a small circle, to distinguish it from the number of the station. It is checked by means of a second photograph, and when the discrepancy between the two values for the elevation of the point is within the permissible limits of error, their mean value is entered in red ink on the plan and all pencil figures are erased.

Any marked difference in the value for the height of a point obtained from two photographs would indicate either that the two points selected on the photographs do not represent the same point of the terrene, or that an error in platting or in finding the height had been made. A third intersecting line from a third station would dispose of the first two alternatives, and a new measurement of the height will show whether an error had been made, or whether the discrepancy is due to unavoidable errors.

(*j*) *The use of the so-called "photograph board."*—The various constructions described in the preceding pages, if made directly on the platting sheet and on the photographs, would produce confusion in the iconometric platting, owing to the intricacy of the lines, and would obscure many details in the pictures. Captain Deville, therefore, has had a special drawing board prepared on which as many of the construction lines are drawn, once for all, as would have to be repeated for the different prints of uniform size and which had been obtained with the same camera.

This so-called "photograph board" is an ordinary drawing board, covered with tough drawing paper, the surface of which is to represent alternatively either the picture plane or the principal plane (both revolved into the horizon plane). It is used in conjunction with the photographs or negatives.

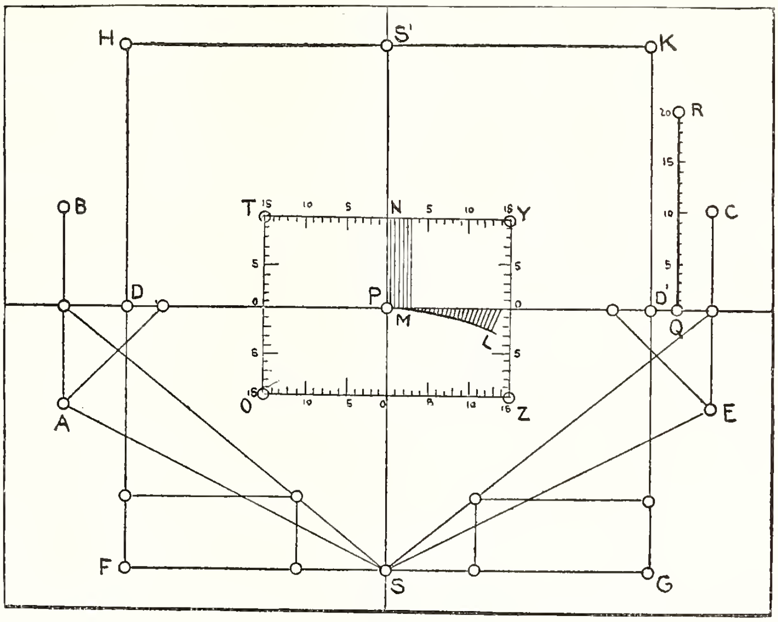


FIG. 70

Two lines, DD' and SS', fig. 70, are drawn at right angles to each other. They represent the horizon and principal lines, while $PD = PD' = PS' = f =$ focal length, so that D, D', S, and S' are the left, right, lower, and upper distance points, respectively.

The photograph is placed in the center of the board, the principal line coinciding with SS' and the horizon line with DD', in which position ($TYZO$) it is secured to the board by means of small thumb tacks or pins. The four scales, forming the sides of the square $OTYZ$, serve, among other purposes, to locate lines parallel either with SS' or DD' (without actually drawing the parallels) on the photograph, the latter falling within the limits of the square $OTYZ$. At a suitable distance from the distance point D' a perpendicular QR is drawn, on which are marked by means of a table of tangents the angles formed with DQ by lines drawn from D. This scale may be used for measuring the altitudes or azimuthal angles of points of the photograph, as will be explained in a separate paragraph later.

From S as a center with $SP = f =$ focal length an arc of a circle PL is described and divided into equal parts. Through these points of division, and between PL and PD', lines are drawn converging to S. Parallels MN to the principal line are also drawn, as shown in fig. 70. All these lines are used in connection with the scale of degrees and minutes QR.

The studs of the "centro lineads" (to be mentioned later) are fixed in A, B, C, and E, the lines AB and CE joining their centers, and those required for adjusting the centro lineads are drawn on the photograph board to be used as will be explained in a later paragraph. The square $FGKH$ is constructed on the four distance points S, S', D', and D.

(k) *Construction of the traces of a figure's plane.*—If one wishes to use a perspective instrument for converting a figure—situated in an inclined plane of which the perspective (photograph) is given—into the projection of the figure, into horizontal plan, it will be necessary to locate the traces of the figure's plane on the principal and picture planes.

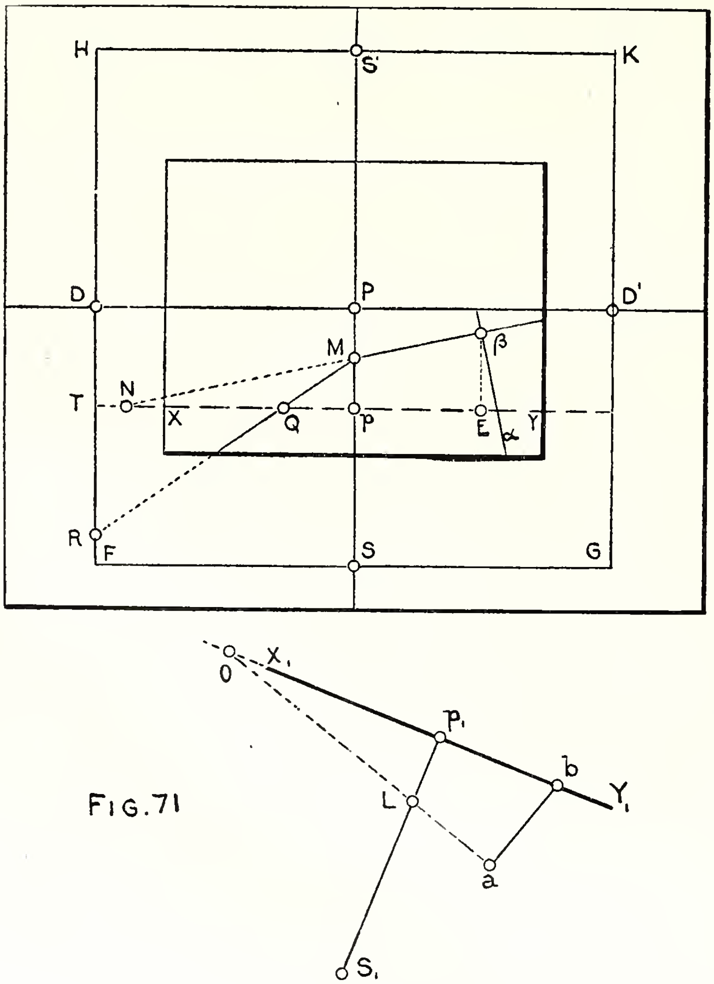


FIG. 71

We may distinguish between two cases that frequently arise in practical work.

(1) The inclined plane, containing the figure, may be given by the line of greatest slope, or,

(2) The inclined plane may be given by three or more points.

First case: The inclined plane containing the figure is given by its line of greatest slope; this may be an inclined road, the drainage line of a straight valley, the trend of a torrent, the surface of a live glacier, etc.

This line of greatest slope may be represented on the plan by a line ab, fig. 71, the altitude of a being known.

After the photograph has been pinned to the photograph board, the ground line XY is drawn, taking the horizontal plane through a as the ground plane.

On the platting board aO is drawn through a perpendicular to the horizontal projection ab of the line of greatest slope, and it is produced to its intersections L and O with the principal line S_1p_1 and with the picture trace X_1Y_1.

On the photograph pE is made equal to p_1b; at E a perpendicular to XY is erected and produced to the intersection β with the pictured line of greatest slope.

If we now make pN (on the photograph) $= p_1o$ (of the plan) and join $N\beta$ on the picture, the line $N\beta$ will represent the trace of the required plane (the figure's plane) on the picture plane.

If pQ (on the photograph) is made equal to p_1L (of the plan) and Q joined with M, MQ will represent the trace of the required plane in the principal plane, revolved about SS' (on the photograph board) into the picture plane, the station S falling in D.

Producing MQ to R, DR will represent the vertical distance of the station S above the plane $RM\beta$.

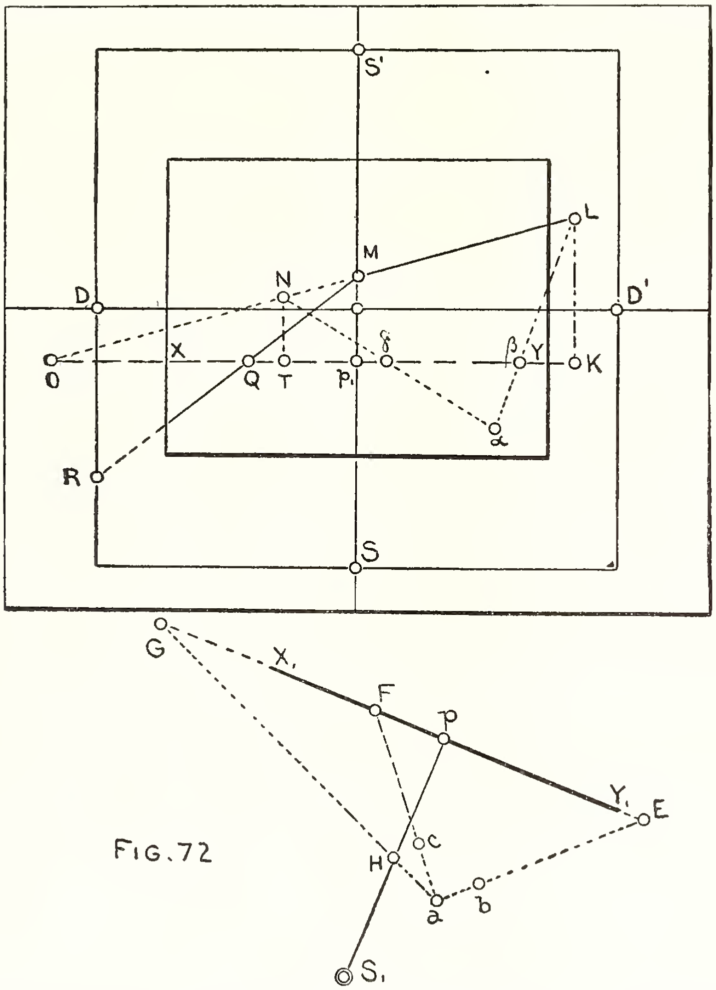


FIG. 72

Second case: The inclined plane containing the figure is given by three points.

Take for ground plane the horizontal plane containing one of the points a, fig. 72, and draw the ground line XY on the photograph. On the platting sheet join a to the two remaining points, b and c, and produce these lines ab and ac to the intersections E and F with the picture trace.

On the photograph make p_1K equal to pE and draw KL perpendicular to XY. Join the perspectives α and β of the points shown in a and b on the plan and produce to the intersection with KL. Make p_1T equal to pF, draw TN perpendicular to XY, and produce to the intersection N with the line joining the perspectives α and γ (of a and c). Join N and L, when NL will represent the trace of the required plane on the picture plane.

Produce LN to O and make $pG = p_1O$; join a and G and make $p_1Q = pH$. The line MQ will represent the trace of the required plane on the principal plane, revolved about SS' into the picture plane, the station S being now in D. Here again DR is the vertical distance of station S above the plane containing the three given points a, b, and c.

(*l*) *Contouring.*—After the heights of a sufficient number of points have been determined to give a good development of the terrene that is to be mapped, the contour lines are drawn in by interpolation between the points of which the elevations had been established.

In a moderately rolling country a limited number of points of known elevations will suffice to draw the contour lines with precision; but in a rocky region, where abrupt changes and irregular forms predominate, it is almost impossible to plat enough control points to enable the iconometric draftsman to render a faithful representation of the relief of the broken terrene, and it is here that a close study of the photographs will give the greatest assistance in modifying the courses of the contours to represent the characteristic features of the terrene.

The value of photographic views for a correct or naturalistic delineation of the topography of a given area is generally acknowledged by experienced topographers, even when using

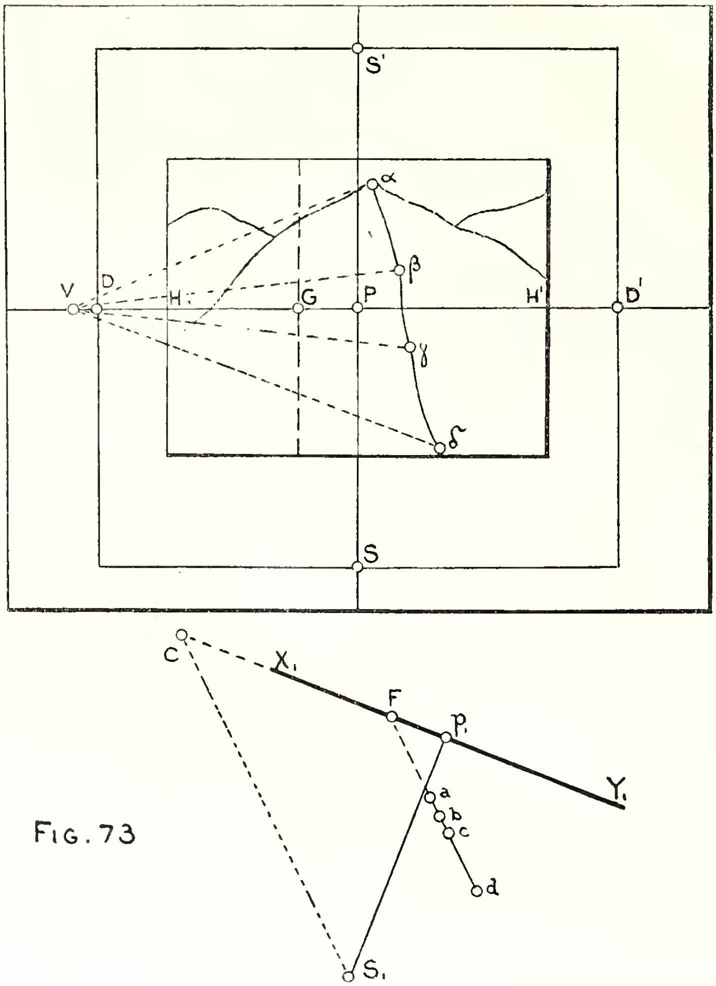


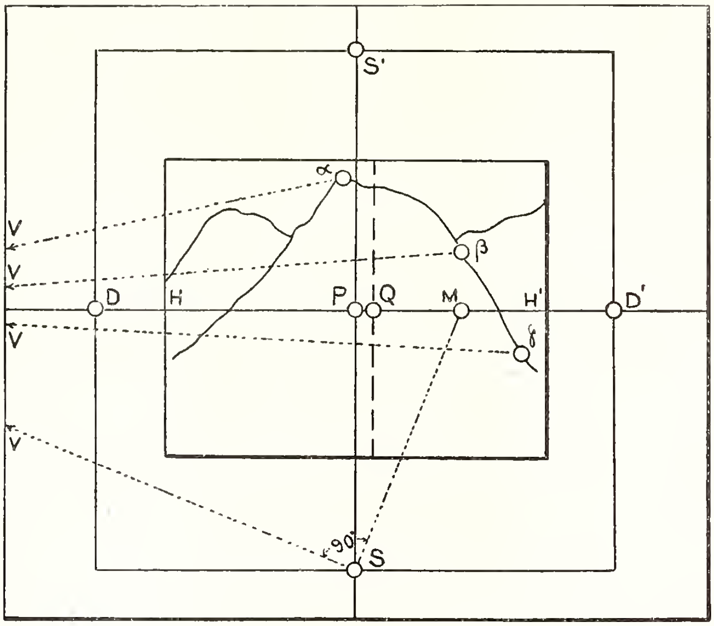
FIG. 73

instrumental methods alone for the control work, as a minute study of the pictured terrene (the photographs) will always aid the draftsman (when inking the topographic sheet) to draw the contours (of which the main deflections had been located instrumentally) with a more natural and artistic reproduction of nature's forms than could be attained by mechanically inking the penciled lines as obtained solely by instrumental measurements.

Instead of drawing the contour lines at once on the plan, the draftsman may begin by sketching them on the photographs, following the same rules for their location as if he were drawing them on the plan, for the image of every platted point is already marked on the photograph and its elevation may be taken from the working plan. By following this course he will be enabled to follow the inequalities of the surface very closely. Those perspectives of the contours on the pictures will greatly facilitate their horizontal projections to be drawn upon the plan. They may also be transferred to the plan by means of the perspectograph or perspectometer if accuracy is to give place to rapidity.

A sufficient number of tertiary points having been platted by intersections, there will be no difficulty in drawing the contour lines (by interpolation) between such points. It may happen, however, that the number of the control points is too small and that the latter are too far apart to give a good definition of the terrene (as in a topographic reconnaissance), and then it will become necessary to resort to other (frequently less accurate) methods for locating the contours on the plan.

For example, the ridge $a\ b\ c\ d$ of a mountain range appearing as $\alpha\ \beta\ \gamma\ \delta$ on a photograph, fig. 73, may be divided by the contour planes by assuming it to be contained in a vertical plane.



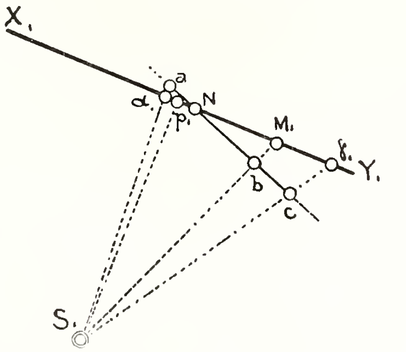


FIG. 74.

On the plan, fig. 73, we produce the projection ad of the ridge to the intersection F with the picture trace X_1Y_1 and draw through the projection S_1 of the station S_1C parallel to ad.

The photograph having been pinned to the photograph board, take from the principal point P on the horizon line $PV = p_1C$ and $PG = p_1F$. At G place the scale of equidistances perpendicular to the horizon line, the division at G corresponding to the height of the station, and join the marks of the scale to the vanishing point V.

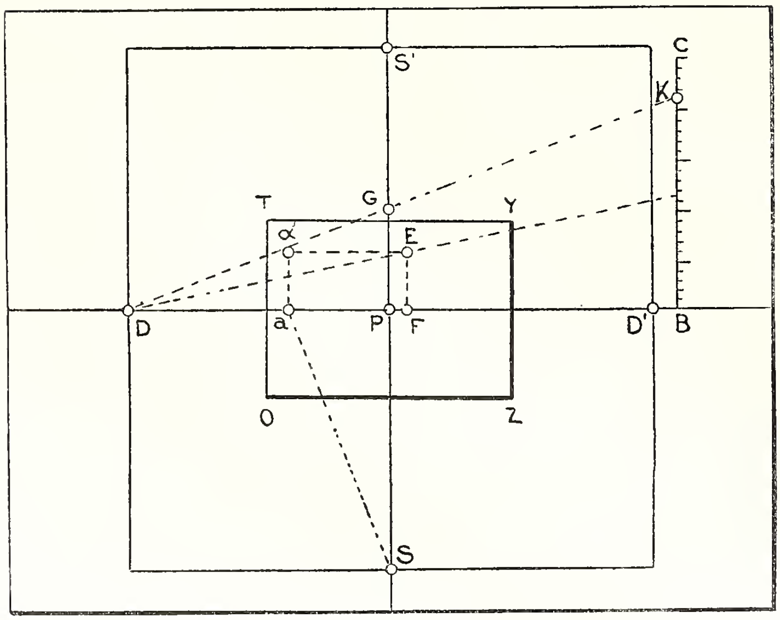
Having now the successive points of intersection of the ridge by the successive contour planes, their distances from the principal line SS'—their abscissæ—are marked upon the edge of a strip of paper in the usual manner. The intersection of the radials from S_1 through the points marked on the paper strip with the projection $a\ .\ .\ d$ of the ridge $\alpha\ .\ .\ \gamma$ will give the intersections of the contour lines.

Should the mountain have rounded forms and no well-defined ridge, the visible outline on the photograph may be assumed to be contained in a vertical plane perpendicular to the line of direction drawn to the middle of the ridge outline.

The construction, fig. 74, is made by drawing the line of direction *SM* to the middle of the ridge outline and *SV* perpendicular to *SM*. On the plan $p_1 M_1$ is made $= PM$, and from the projection a of the summit α of the mountain a perpendicular ac to $S_1 M_1$ is drawn, which will represent the horizontal projection (ac) of the pictured outline ($\alpha\gamma$); it is produced to the intersection N with the picture trace $X_1 Y_1$. PQ is taken (on the photograph) equal to $p_1 N$ (on the plan), and the scale of equidistances is placed at Q perpendicular to the horizon line DD'. The division mark at Q corresponds with the elevation of the station S, and the points of division corresponding with the contours are joined to V and produced to their intersections with the outline α . . γ. The platting is done as in the preceding case, or the lines of direction drawn to the points of intersection of the outline $\alpha\gamma$ by the contour planes may simply be platted and the contour lines on the plan may be drawn tangent to these lines of direction.

The horizon line, containing the perspectives of all points having the same elevation as the station, represents the perspective of a contour line when the camera horizon is identical with a contour plane. The iconometric draftsman should pay particular attention to geologic forms and to the origin of topographic features, as without such applied knowledge a correct interpretation of such forms and their cartographic representation would require the cartographic location of a vast number of control points to obtain a faithful representation of the terrene forms. Although the latter may often result from the successive or combined actions of many agencies, they will yet have similar recurrent characteristic shapes when produced by the same causes, and the contours, being the means for delineating the cartographic representation of the terrene shaped by identical agencies, should also show a corresponding characteristic similarity.

Fig. 75



(*m*) *The photograph protractor.*—The angle included between the line of direction (to a point of a photograph) and the horizon, or the principal plane—the vertical or altitude and the horizontal or azimuth angle—is sometimes wanted.

The horizontal angle may be obtained directly on the photograph board by joining the station S, fig. 75, and the projection a (on the horizon line) of the pictured point α. If required in arc measure, the distance Pa may be transferred to the principal line SS' from $P = PG$; D is joined to G and produced to the scale of degrees and minutes BC, where the graduation mark k indicates the value of the horizontal angle in arc measure.

When many such angles are to be measured, the horizontal scales TY and OZ, fig. 75, may be divided into degrees and minutes by means of a table of tangents, using as radius the focal length SP.

The altitude is the vertical angle at S of the right-angle triangle, having for sides Sa and $a\alpha$. To construct it, take $DF = Sa$, draw FE parallel with and equal to $a\alpha$, join D and E, and produce DE to the scale BC of degrees and minutes.

This construction will be facilitated by the lines already drawn on the photograph board, fig. 70. With a pair of dividers take the distance (abscissa) from the pictured point α to the principal line SS', fig. 75, and carry it from P, fig. 70, in the direction PD', and from the point so obtained take the distance to the arc ML, fig. 70, measuring in the direction of the radials marked on the board, which will represent the distance PF, fig. 75. Then, with the dividers, carry $a\alpha$ to FE, fig. 75, which is that one of the parallel lines MN of fig. 70 that corresponds to the point F. The construction may now be completed in the manner already explained.

A protractor may be constructed to measure these angles directly by drawing lines on a transparent plate parallel with the principal line—they contain points having identical azimuths—and curves containing points of identical altitudes.

The azimuthal lines may be found by platting the horizontal angles in S, fig. 70, and drawing lines parallel to the principal line SS' through the points of intersection of the radials with the horizon line DD'.

If we regard the horizon and principal lines as axes of coordinates and denote the altitude αa of a point a pictured as α, fig. 75, by h, the equation of the curve of altitude h may be written—

$$y^2 = (x^2 + f^2) \tan^2 h.$$

This also is the equation of an hyperbola of which the principal and horizon lines are the transverse and conjugate axes, and of which the principal point is the center.

One of the hyperbola's branches represents the points above the horizon, and the other branch the points of equal altitude below the horizon. The asymptotes are lines intersecting each other at the principal point, and including angles with the horizon line equal to h. This hyperbola represents the trace on the picture plane of the cone of visual rays which include the angle h with the horizon plane.

These hyperbolic curves of equal altitude may be obtained by computation, using the preceding formula and substituting different values for h, or they may be obtained graphically by platting a series of points for each curve by reversing the construction given above for finding the altitude of the pictured point α, fig. 75. The angular distance between the lines representing points of equal azimuths (or those of equal altitudes) will depend upon the degree of precision required.

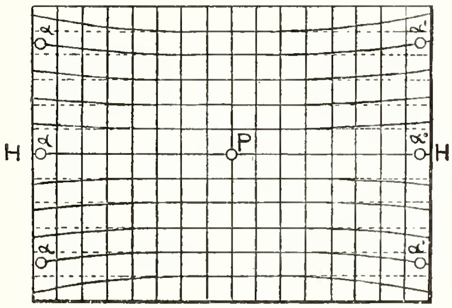


FIG. 76

The complete protractor is shown in fig. 76. It may be made in the same manner as mentioned for the perspectometer by drawing it on paper on a large scale, reducing it by photography, and making a transparency by bleaching in bichloride of mercury.

(4) *Method of V. Legros for determining the position of the horizon line.*—Commandant V. Legros recommends the use of these hyperbolas for locating the horizon line of a vertically exposed photographic plate:

When a camera with the photographic plate adjusted in vertical plane is rotated horizontally in azimuth, the plate remaining vertical, any point α, fig. 76, will describe a hyperbola $\alpha\alpha'$ on the ground glass plate. The nearer the observed point α approaches the horizon line the smaller the curvature of its hyperbolic trace on the ground glass will become, and a point a^0 which traverses the ground glass plate in a straight line HH' will have the same elevation as the second nodal point of the camera lens. Its angle of elevation will be ± 0 or HH' will be the horizon line of the plate. To locate the horizon line experimentally in this way the ground glass plate is best provided with a series of equidistant horizontal and vertical lines, after the manner of Dr. Le Bon's ground glass plates.

(5) *Method of Prof. S. Finsterwalder for locating contours on the plan.*—Prof. S. Finsterwalder's method for the iconometric location of horizontal contours is based upon the following consideration:

The pictured outline of a terrene form is regarded as the trace of the terrene surface in a plane vertical to the platting or ground plane and containing the pictured outline. This method is well adapted for the development of the terrene forms of a moderately rolling country.

The camera stations are specially selected with reference to the use of this method, with a view toward obtaining pictures with a sufficient number of such outlines of the terrene forms to enable the iconometric draftsman to give a good definition of the relief of the region to be platted.

The pictured outlines of terrene forms may be regarded as falling within vertical planes, and the rays from the point of view—second nodal point of camera lens—to the pictured points of such outline will form a cone with apex in the point of view, its base being formed by the pictured outline.

Any horizontal plane containing a contour A will intersect such a cone of rays in a curve B, the latter touching A in one point. This curve B may be platted on the working sheet by laying off, upon a few rays from the platted station to points of the pictured outline, the distance:

$$h \cot \beta$$

and the points thus located on the radials from the station point, if connected by a continuous line, will represent the curve B platted in horizontal plan.

> h=difference in elevation between the station (whence the picture was taken) and the horizontal contour A.
>
> β=vertical angle to each point of the outline bisected by the vertical plane passing through its radial or visual ray.

The direction of the pictured outline is now platted on the plan, and where it bisects the curve B will be a point of the contour A. As we naturally would draw not only one curve B, but rather a series of them corresponding to several horizontal planes, passing through a series of contours A of various elevations, the construction may be simplified, inasmuch as the curves B—being lines of intersection of the same cone of rays with a series of parallel (horizontal contour planes) planes—will all be similar in shape, their corresponding points (points on the same radials) having the same relative positions with reference to the platted station, the value $h \cot \beta$ need only be determined for one point of the other curves B if one curve B had been drawn, the others being parallel with the first.

CHAPTER IV.

PHOTOGRAMMETERS.

The practical *value of a photogrammeter* (photographic surveying instrument) depends greatly upon the quality and general uniformity of its lens or lenses, upon the rigidity of the component parts of the apparatus, its easy transportability, and on the rapidity with which it may be put into adjustment.

A good *phototopographic lens* should be free from spherical aberration (or diffusion of the light rays); it should possess no chromatic aberration, nor should the image show distortion of any kind, and the field of view (the range of lens) should be large, rapidity of the lens being desirable, but less important than the other requirements just mentioned.

The *principal lenses in use* for phototopographic purposes are: *Dallmeyer's* rapid rectilinear, *Steinheil's* aplanat, *Bush's* pantoscopic, *Görz's* double anastigmat, and, more recently, *Zeiss's* anastigmat lens.

The nodal points, the focal length, arc of visibility, and the arc which is perfectly free from distortion of every kind should be known for every lens used for phototopographic purposes, and the manufacturers of all good lenses are best fitted to determine those values with great precision for every lens.

I. REQUIREMENTS TO BE FULFILLED BY A TOPOGRAPHIC SURVEYING CAMERA.

A good surveying camera or photogrammeter for topographic work should produce negatives which are geometrically true perspectives the elements of which should be known, and the following desiderata should be fulfilled:

First. The plates to be exposed should be adjustable into vertical plane.

Second. The distance between image point and sensitive plate should be maintained unchanged for all plates.

Third. This distance—the constant focal length—should be known or will have to be determined for every instrument.

Fourth. Means should be provided to trace or locate the horizon line upon every negative or print.

Fifth. Means should be provided for locating the principal point upon every negative.

Sixth. A ready orientation of the photographs (the picture traces) for iconometric platting should be provided for; and we may add as

Seventh. Enough characteristic stations (besides the triangulation points needed for the instrumental control) are to be occupied with the surveying camera to give a full development of the terrene, which is to be mapped.

Until recently photographic surveying instruments were not procurable in open market. Nearly every observer who made practical application of the photographic methods for topographic surveys had an apparatus constructed for his particular need and according to his individual ideas.

In the following we will describe such photogrammeters as may be regarded as special types, constructed to fulfill different requirements.

II. ORDINARY CAMERAS ADAPTED FOR SURVEYING PURPOSES.

These cameras are generally supported by three leveling screws, and they are provided with a circular level, or with two cross levels, for adjusting the sensitive plate into vertical plane. The distance between lens and sensitive plate (focal distance) may be made invariable by means of two rods *Sp*, fig. 77 (*Werner's* apparatus, made by R. Lechner, of Vienna, in Austria), or by means of two arms *H* and clamp screw *M*, after the bellows had been extended by aid of the pinion *K* and rack movement to that point indicated by the vernier *n*, fig. 78, as the proper focal length for

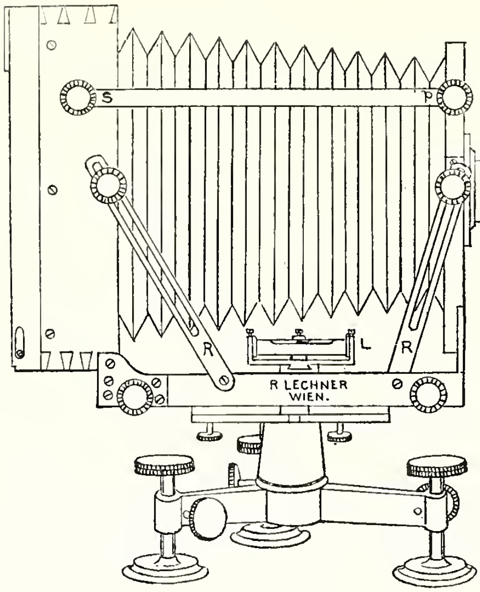


FIG. 77

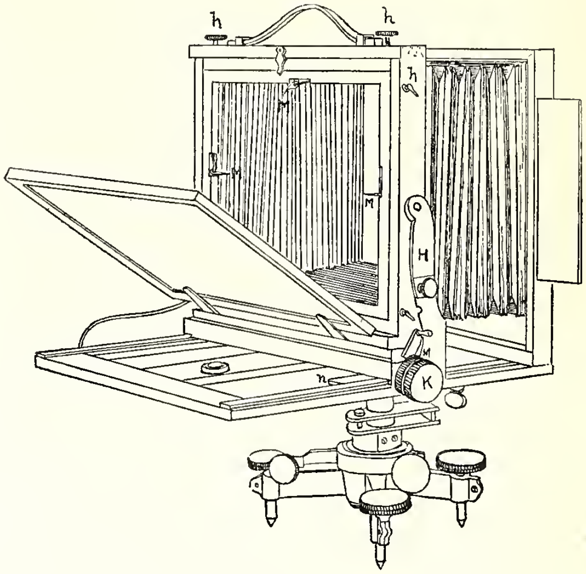


FIG. 78

infinite distance. The arrangement shown in fig. 78, represents the apparatus of Dr. *Vogel* and Professor *Doergens*, made by Stegemann, of Berlin, in Prussia.

Dr. *G. Le Bon* also used a similarly modified camera for his archæological researches in India (undertaken under the auspices of the French ministry of culture).

Short brass points *M*, fig. 79, serve to locate the horizon and principal lines on the negatives by protecting the sensitive plates against the action of those light rays which they intercept. In some instances those points *M* may be brought into direct contact with the sensitive film surface of the plate by turning a button, thus producing a sharp, well-defined image of the outlines of the teeth on the negative.

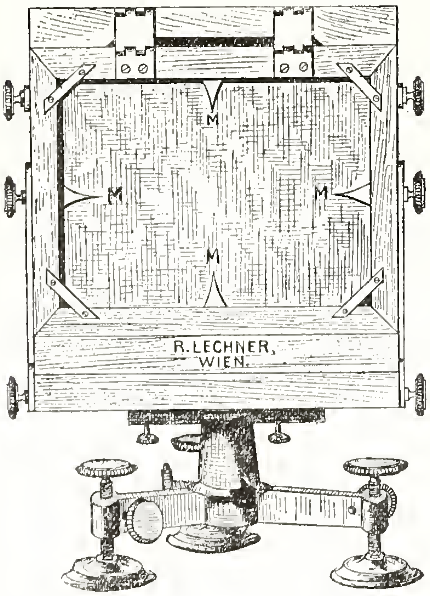


FIG. 79

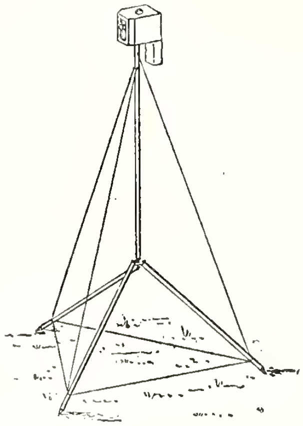


FIG. 80

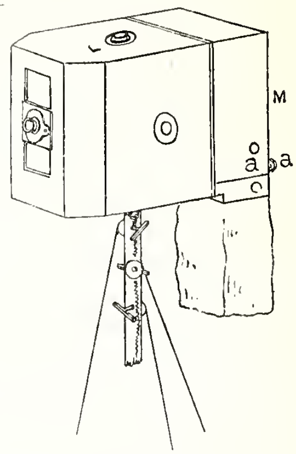
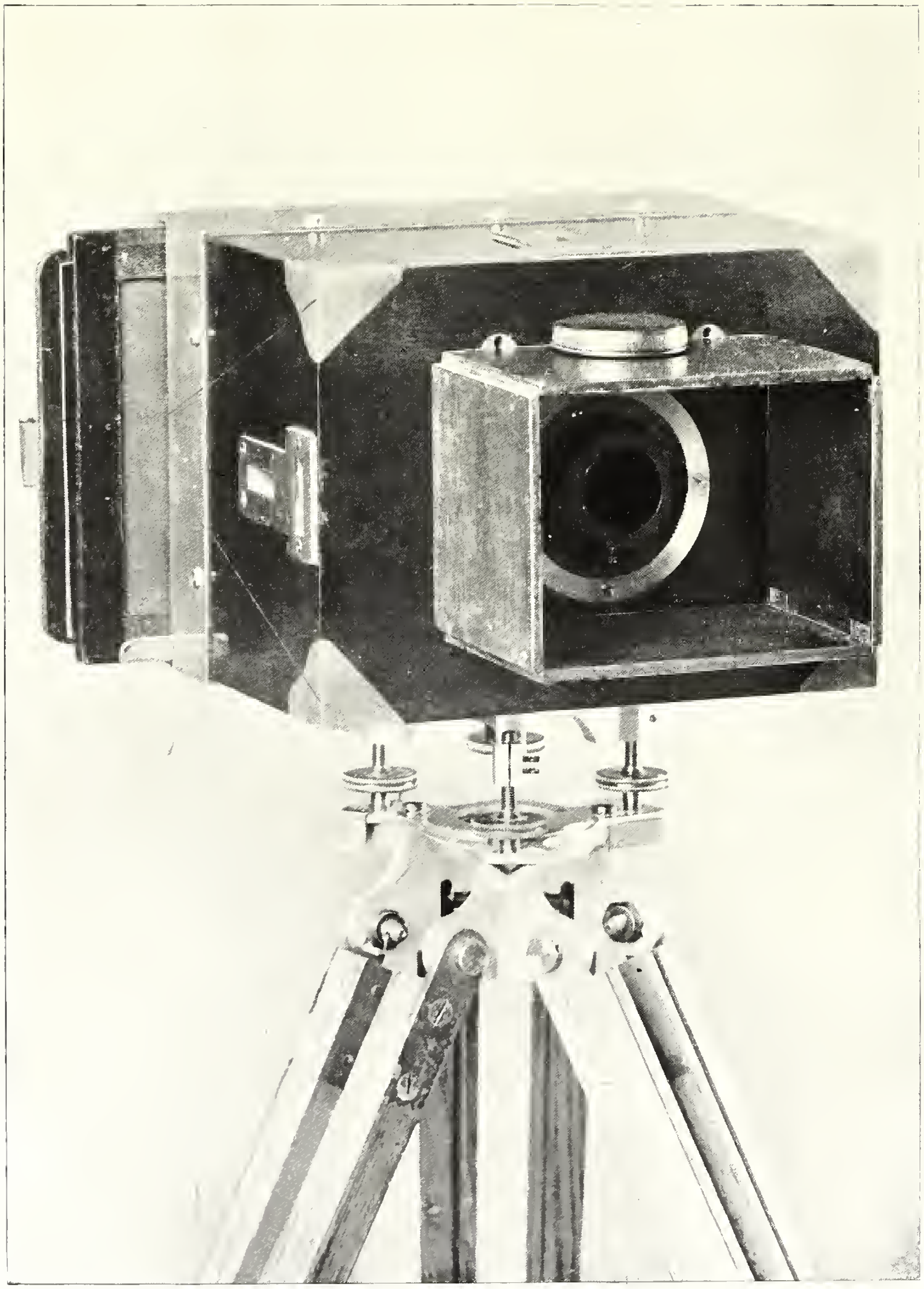
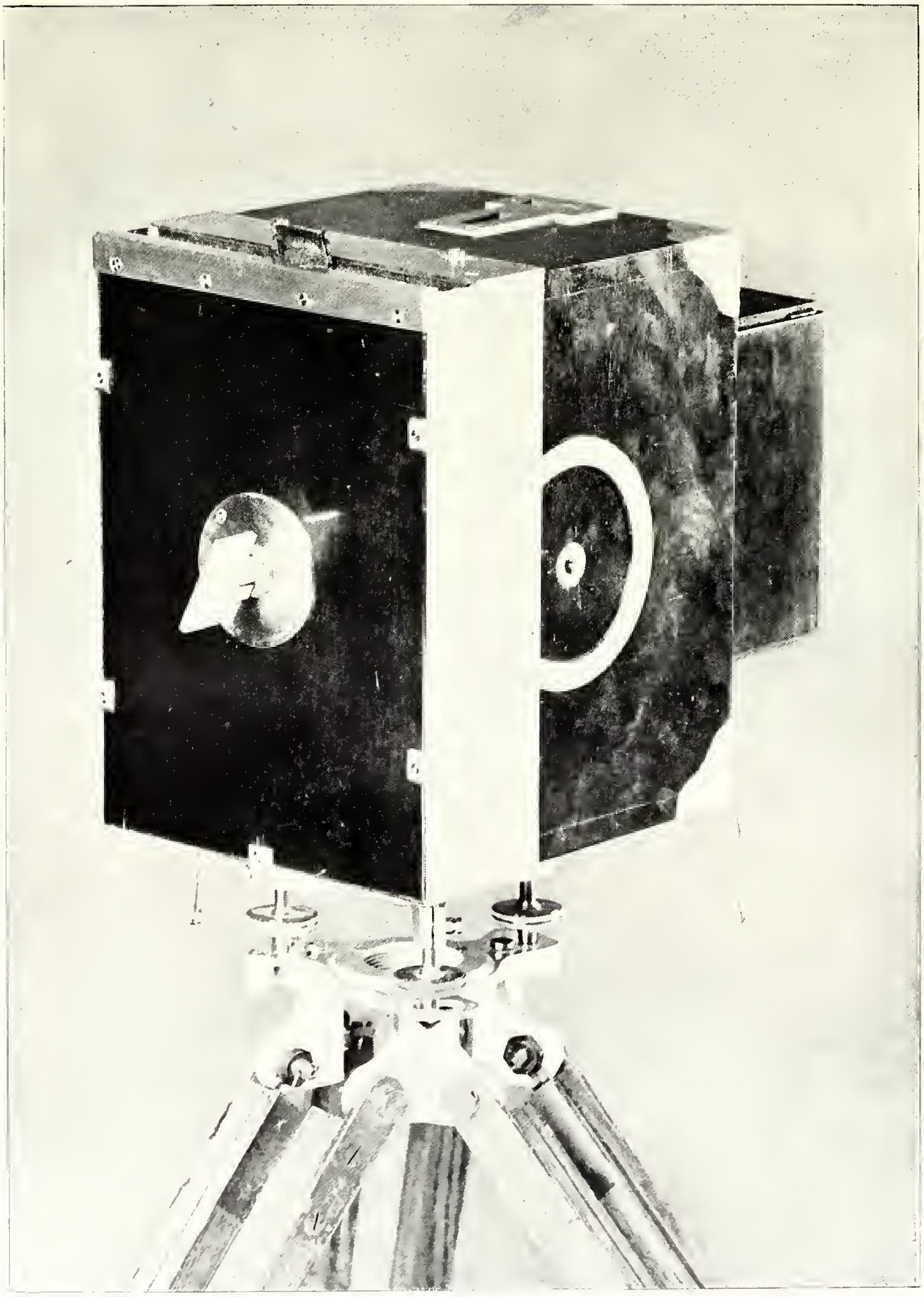


FIG. 81

The use of such modified cameras should not be extended beyond preliminary work; for extensive use the results will not be sufficiently uniform and accurate.



CANADIAN (E. DEVILLE'S) SURVEYING CAMERA.



CANADIAN (E. DEVILLE'S) SURVEYING CAMERA—VERTICAL POSITION.

III. SPECIAL SURVEYING CAMERAS WITH CONSTANT FOCAL LENGTHS.

(1) *Dr. A. Meydenbaur's surveying camera.*—Among the numerous patterns of this class of instruments Dr. Meydenbaur's is probably the earliest form. Fig. 80 shows *Meydenbaur's new, small-sized magazine camera.* The plates are successively pressed against a metal frame secured at a constant distance from the lens. After an exposure the plate is dropped into a leather sack *b*, fig. 81, attached to the camera. The dimensions of the camera box are 9 by 12 centimetres, it weighs 750 grammes, and it is mounted on a rod which is joined at its lower end to three short legs in such a way that the four pieces may be folded together to form a stout cane 0·85 metre long. The lower ends of the three legs of this tripod, and the upper end of the supporting rod are connected by twisted violin strings to which tension may be given by turning the ratchet wheels indicated in fig. 81. The leather pouch, together with twelve plates, weigh about 500 grammes.

The sensitive plate may be adjusted into vertical plane by means of a ball and socket connec-

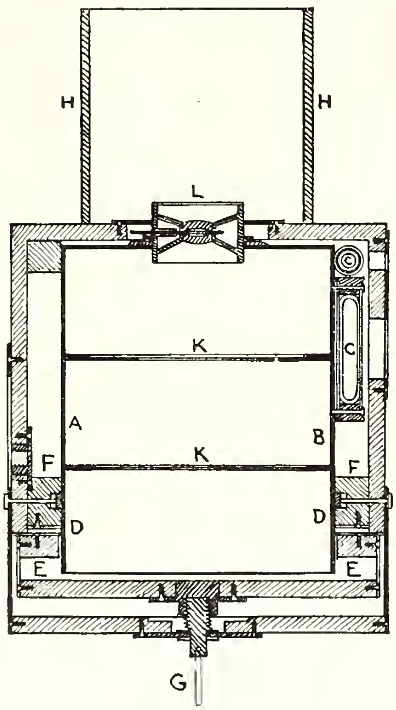


FIG. 84.

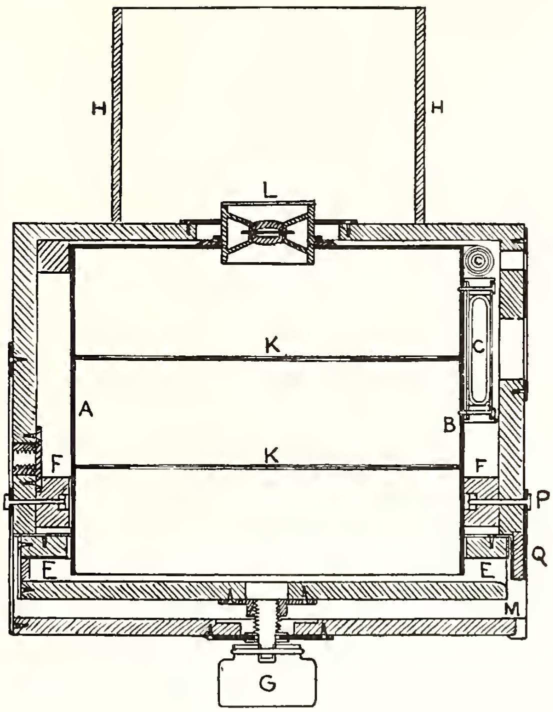


FIG. 85

tion between the camera and upper end of the tripod rod, together with the circular level *L*, shown on the upper face of the camera box in fig. 81.

(2) *E. Deville's new surveying camera.*—The following description of the new Canadian surveying camera is taken from *Deville's Photographic Surveying*, Ottawa, 1895. This camera is shown in figs. 82 and 83. Figs. 84 and 85 represent sections of the instrument.

The camera proper is a rectangular metal box *AB* (figs. 84 and 85) open at one end. It carries the lens *L* and two sets of cross-levels *CC*, which may be observed through openings in the outer mahogany box. The metal box is supported by wooden blocks and a frame *FF*, held in position by two bolts *DD*.

The plate holder is made for single plates; it is inserted into the carrier *EE*, which may be moved forward and backward by turning the screw *G*.

A *folding shade HH*, hooked to the front of the camera, and *diaphragms KK*, inside of the metal box, intercept all light that does not contribute to the formation of the image on the photographic plate.

The camera rests on a *metal triangular base*, fig. 86, with three-foot screws, exactly like the base of the transit which is used in conjunction with Deville's camera, so that either camera or transit may be placed on the same tripod at any time. The camera may be set up with the longer side either horizontal or vertical, figs. 82 and 83. Both transit and tripod are carried by the surveyor, while one camera with one dozen plates (in the single plate holders), without a tripod, are taken by one of the men who always accompany the surveyor. The assistant surveyor has a second camera, with 12 plates and a separate tripod.

The legs of these *tripods*, when folded together, are 20 inches long and are placed under the box of the transit, in a separate sole-leather case, to be carried on the back of the surveyor. The tripod of the assistant surveyor's camera is similarly attached to the sole-leather case of his camera.

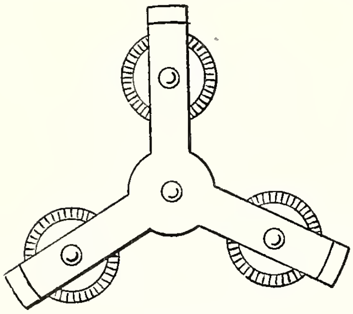


FIG. 86

The lens of this camera is a Zeiss anastigmat, No. 3 of series V, focal length = 141 millimetres with a deep-orange color screen in front.

Having set the camera up on the tripod, the plate-holder carrier *E* is moved back as far as it will go by turning the screw *G*, the plate holder is inserted through the opening *ME*, the slide is withdrawn, and the carrier is moved forward by revolving the screw *G*, until the plate is in contact with the back of the metal box *AB*. In order to secure a perfect contact, the carrier has a certain amount of free motion. The camera should now be turned in the proper direction; the field embraced by the plate is indicated by lines drawn on the outside of the mahogany box. The camera is now carefully leveled, the exposure made, and the plate holder withdrawn (after the slide had been inserted) by repeating the same operations, however, in the inverse order.

The *levels CC* are rigidly attached to the metal camera box without any means of adjustment. They are, however, very nearly adjusted by the maker. For this purpose he takes the metal box out of the mahogany casing and places it on a piece of plate glass which had been leveled like an artificial horizon. By filing down one end or the other of the level's outer case he brings each bubble very nearly into the middle of its tube. These tubes have continuous numbers on the graduation marks, as illustrated in fig. 87.

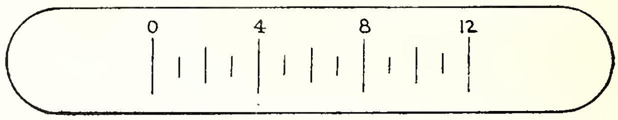


FIG. 87

Accompanying each camera is a *piece of plate glass*, ¼ inch thick and 11 inches long, which can be inserted into the carrier in place of the plate holder. That end of the plate glass which projects outside of the camera when it is thus inserted is coated on the back with a varnish of gum guaiacum (dissolved in alcohol) to which some lampblack has been added. This coating has very nearly the same refractive index as glass, precluding all reflections from the back of the plate glass.

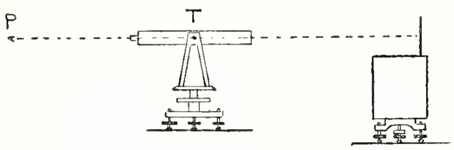


FIG. 88

When the camera is received from the maker *the exact readings of the levels, CC*, when the back of the metal box (against which the photographic plate is pressed) is vertical, *should be ascertained*. To do this the bolts *P*, fig. 85, next to the opening *M*, are unscrewed and removed. *Q* may then slide backwards and be taken out. The piece of coated plate glass is now inserted into the carrier *E*, figs. 84 and 85, and pressed into contact with the metal box by revolving the screw *G*. The camera is placed on its tripod and leveled. Immediately in front and at the same height as the camera a transit (or a leveling instrument) *T*, fig. 88, is set up, and, after carefully adjusting it, a distant but well-defined point *P* is selected on the same level with the transit and camera. The intersection of the threads of the telescope is brought to coincide with *P*, and the telescope is clamped in this position to the vertical circle. Turning it in azimuth the image of *P*, reflected by the plate glass, should appear at the intersection of the telescope's threads. If it does, the face of the plate glass is vertical and the position of the bubble in the tube of the level, directed at right angles to the plate glass, is the correct one for adjusting the instrument in the future. If it does not, the camera must be tilted forward or backward by means of the foot screws until coincidence is established. The bubble of the level may or may not now be in the middle of the tube, but its position, whatever it is, will be the correct one for the future when adjusting the camera at any station. This level reading should therefore be recorded, and whenever the camera is to be leveled in its subsequent use it must be remembered that the bubble is to be given the same position.

This level reading determination is to be made for the two positions of the camera in which it is used, figs. 82 and 83, horizontal and vertical.

The next step is to *locate the position of the principal point* on the vertical photographic plate, and *to determine the length of the distance line or the constant focal length.*

Select a station so that a number of distant and well-defined points may be found on the horizon line, as laid down by the maker of the camera. The view selected may be the distant shore of a lake, a large building, or a row of buildings. Set up the tripod and adjust the transit. Find two points E and F, fig. 89, on the horizon line (with a zenith distance of 90°) that both come within the field of the camera, when set horizontal, both points being near the edges of the plate. Measure the angle ω between them.

Find two other points G and H, also on the horizon line, and such a distance apart that they both come within the field of the camera when the same is vertical, fig. 83. Now replace the transit by the camera in the horizontal position, fig. 82, turn it in azimuth to take in E and F, level carefully and expose a plate.

Set the camera in the vertical position, fig. 83, turn it so that it takes in the observed points G and H, level carefully and expose another plate. The first plate, after development, will show the two points E and F on a line very nearly parallel to the edges AB and CD, fig. 89, of the metal box. The principal point, of course, will be on this line. Cut this line into the film with a fine needle point and straightedge.

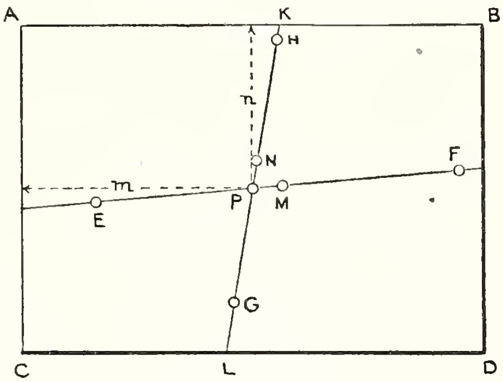


FIG. 89

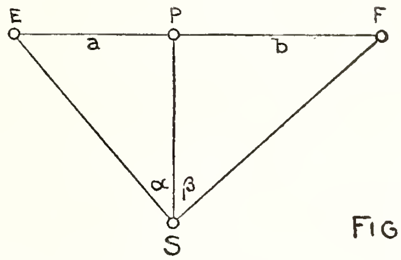


FIG. 90

The second plate, exposed in the vertical position, after development gives another horizon line GH, fig. 89, which may be transferred to the first plate by means of the distances AK, and CL to the corners of the metal box. This (principal) line is likewise cut through the film with a fine needle point and straightedge, the principal point P is at the intersection of both lines EF and GH.

The length of the distance line, $SP = f$, fig. 90, may be computed from the observed horizontal angle ω, included between SE and SF, and from the distances $EP = a$ and $PF = b$, measured on the negative.

Let S, fig. 90, be the second nodal point of the camera lens, α and β the angles ESP and PSF.

$$\alpha + \beta = \omega.$$

The lengths of a and b are known and if we designate the focal length SP by f, we will have:

$$\tan \alpha = \frac{a}{f}$$

$$\tan \beta = \frac{b}{f}$$

$$\tan \alpha \times \tan \beta = \frac{a\,b}{f^2}$$

Hence:

$$\tan(\alpha+\beta)=\tan\omega=\frac{\frac{a}{f}+\frac{b}{f}}{1-\frac{ab}{f^2}},\text{ or:}$$

$$f^2=\frac{a+b}{\tan\omega}f-ab=0$$

after resolving this quadratic equation we find:

$$f=\frac{a+b}{2\tan\omega}+\sqrt{\frac{(a+b)^2}{4\tan^2\omega}+ab}$$

Having found the focal length and the principal point, reference marks are to be made on the edges of the metal box to indicate the horizon line, the principal line, and the focal length on the negatives, or on the enlargements made from the latter.

Measure the distance m, fig. 89, from P to AC. From the corresponding corners A and C, fig. 91, of the metal box, lay off m on AR and CT. With a very fine and sharp file held in the direction of the lens, cut into the edge of the metal a clean and sharp notch at T and another at R.

Repeat the same operation at the corners A and B, fig. 91, with the distance n from P to AB, fig. 89.

The lines OQ and RT will be the horizon and principal lines of the negatives when the camera is leveled to bring the bubble into its proper position, as has been mentioned in the foregoing.

From R and T, fig. 91, lay off the distances Rr, Rr', Tt, $Tt'=\frac{f}{2}=$ one-half of the constant focal length.

From O and Q measure Oo, Oo', Qq, $Qq'=\frac{f}{4}=$ one-fourth of the focal length, and at each one of these points make a notch with the file held in the direction of the lens.

Every photograph will now show twelve triangular projections into the dark border of the

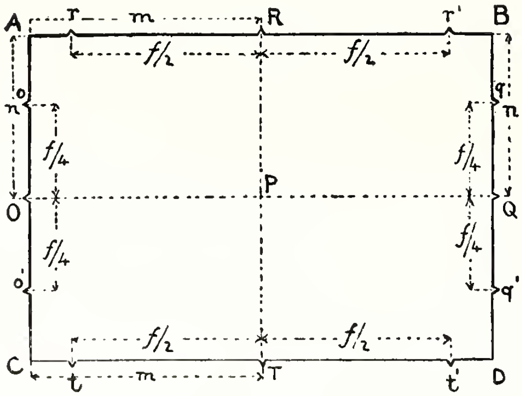


FIG. 91

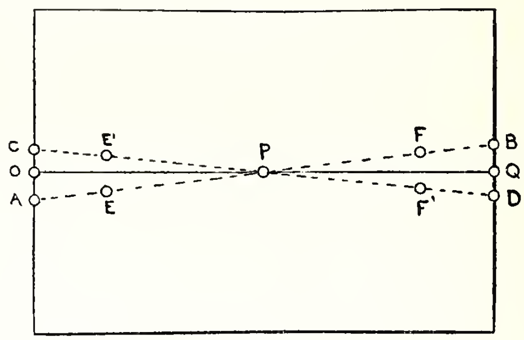


FIG. 92

photograph. Four of these projections serve to fix the horizon and principal lines; the remaining eight give the focal length value.

It now remains necessary *to find the correct readings of the transverse levels* (those placed parallel with the sensitive plate), when the horizon and principal lines pass exactly through their notches of the metal box.

Again set up the camera facing the same distant view as before, but in adjusting it bring the bubble of the transverse level near one end of the tube, note the level reading and expose a plate. After development it will give an horizon line EF, fig. 92, cutting the border of the negative in A and B at some distance from the pictured notches O and Q. Now change the adjustment of the camera by bringing the bubble of the transverse level to the other end of the tube, note the level reading and expose another plate. This will give another horizon line $E'F'$, cutting the border of the negative in C and D.

Great care should be exercised in both cases to maintain the other level (the one at right angles to the sensitive plate) at its proper reading in order to expose both plates in vertical plan.

After measuring *CO* and *OA* or *BQ* and *QD*, a simple proportion will give the proper reading of the transverse level, which will bring the horizon line of the vertically exposed plate through the two notches *O* and *Q* of the metal box.

The correct reading of the other transverse level is found by the same method, with the camera in the vertical position, fig. 83.

All these operations must be executed with great care and precision, and with the help of a microscope of moderate power, as the subsequent iconometric platting of pictured points is based upon the determination of the ordinates and abscissæ of such points on the photographs, with reference to the principal and horizon lines, as a system of rectangular coordinates.

It had been assumed that the levels were placed very nearly in correct adjustment by the maker, as previously mentioned. If found too much out, they should of course be first approximately adjusted by setting the metal box on a leveled plate. For this purpose the plate glass sent out with every instrument is set on the camera base and leveled like an artificial horizon.

(3) *Use of the instruments comprised in the Canadian phototopographic outfit.*—The instruments and tripod being made as light as possible, steadiness is secured by a net suspended between the tripod legs in which a heavy stone is placed. With this device better photographs and more precise observations are obtained, and there is no risk of the instruments (resting upon the tripod) being blown over during one of the sudden and strong gusts of wind so frequently encountered on elevated peaks in the mountains.

After having arrived at a triangulation station, the surveyor adjusts the transit and observes the azimuth and zenith distances of all signals marking the triangulation and camera stations that may be visible from his position. If accompanied by his assistant, each reads one vernier and both enter the readings in record books. After completion of the observations they compare notes. Any discrepancy that may be discovered in the recorded data is corrected on the spot.

The camera is carried in a sole-leather case containing also twelve filled plate holders. When more plates are needed they (with the necessary holders) must be carried in a separate receptacle. Taking the camera out of the case, the leveling base, fig. 86, is screwed to it, and the camera is then placed upon the tripod, from which the transit had been removed, without disturbing the position of the tripod; the shade or hood is now unfolded and attached to the hooks at the front of the camera, fig. 82. A plate holder is inserted into the carrier, and its number is recorded upon a rough outline sketch of the view commanded by the field of the camera image, entering also such notes as may be of value for the development of the plate and for the iconometric platting of the topography recorded upon it (by the action of the light). Having made sure that the cap is on the lens, the slide is withdrawn from the plate holder and the plate is brought into contact with the frame of the metal box by turning the screw *G*, figs. 84 and 85, devised for this purpose. The surveyor now turns the camera in azimuth until the lines on the upper face of the wooden casing show that it is properly directed or oriented to include the panorama section to be photographed between the lines, the field of view coinciding with the outline sketch bearing the number of the plate holder in the camera. Sighting along the converging lines, shown on the side-face of the wooden camera casing, he can assure himself whether the view on the image plate reaches high or low enough. If it does not, he will put the longer dimension of the camera upright, unless the camera was already in that position. He levels carefully, in the manner previously described, and exposes the plate. Whenever the sun shines inside of the front hood it should be shaded off during the exposure of the plate by holding something above the hood. Under no circumstances should the sun be permitted to shine upon the lens.

Every evening, after returning to the survey camp, the surveyor replaces the exposed plates in his dark tent by new ones, using a ruby-colored light. He also marks the exposed plates in one corner, before removal from the holder, with his initials, the number of the dozen and of the plate (the same number as given to the corresponding outline sketch), using a soft lead pencil for this purpose; e. g., IV, 5, means plate No. 5 of the fourth dozen, or the forty-first plate. The exposed plates are now placed into a double tin box, fig. 93, which can be closed hermetically, and which will float when filled with two dozen plates, should the same be accidentally thrown into water. These boxes are shipped to the head office in Ottawa, where the plates are developed by a specialist.

The data obtained with the aid of the transit for triangulation purposes are recorded in the field book in the usual manner, as customary for such work.

The horizontal angles observed with the transit (or altazimuth instrument) to the points of the terrene marked on the outline sketch which accompanies each negative, serve not only for the orientation of the horizontal projection of the plate on the plan (the so-called "picture trace"), but they also serve to counteract in a measure and to ascertain the distortion of the paper prints (or photographic enlargements). The vertical angles, together with the platted distances, are used to check and verify the position of the horizon line on the different photographs.

The most important camera stations are occupied by the surveyor; the secondary stations by the assistant surveyor, with his own camera. No trigonometric observations are made by the assistant while occupying the secondary stations.

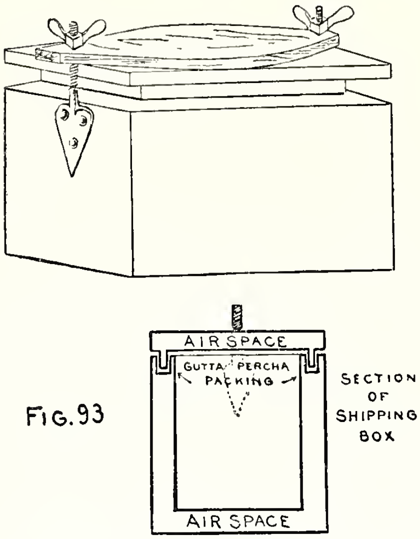


FIG. 93

All views are taken with the same stop: *f*/36.

(4) *The United States Coast and Geodetic Survey camera.*—The original type of the Coast and Geodetic Survey camera, used in connection with the Alaskan boundary survey, was similar in form to Deville's original camera, except that it had a special tripod with ball and socket adjustment and that the teeth which serve to mark the principal and horizon lines on the negative could be turned by revolving *one* button to be pressed into contact with the photographic plate.

This camera was also provided with a ground glass, enabling the surveyor to inspect the entire field controlled by each plate before exposure, and giving ready means for testing the positions of the teeth which mark the horizon line.

The camera itself was a plain rectangular box made of well-seasoned mahogany $6\frac{3}{4}$ by $5\frac{5}{8}$ by $9\frac{1}{4}$ inches in size, and it was used always in the same position, with the short faces vertical. The bamboo tripod legs were composed of three pieces, each 16 inches long, and screwed together at the joints. When dismembered the tripod was carried in a sole-leather packing case together with the camera, twelve plates (in six double plate holders), notebook, barometer, thermometer, yellow color screen, etc.

The new phototopographic camera of the Coast and Geodetic Survey is a phototheodolite, resembling Colonel Laussedat's latest pattern which will be described in the following pages.

IV. SURVEYING CAMERAS COMBINED WITH GEODETIC INSTRUMENTS.

(Phototheodolites, photographic plane tables, etc.)

The data acquired in the field with photogrammeters of the class just described had to be supplemented with observations made in the field with some geodetic instrument (transit, plane table, etc.) in order to obtain complete topographic surveys of the regions traversed by the phototopographic surveying party.

The idea of combining surveying instruments with a photographic camera into single compact and serviceable instruments originated very early with phototopographic workers, and refined phototheodolites and photographic plane tables are to this day the favorite phototopographic instruments in Europe, whence they are also exported to other countries.

These more or less complicated instruments have been devised to secure great precision in the work undertaken with them, and refined methods are employed for the field observations, for the culling of data from the photographic perspectives, and for the computations made in the office to increase the general precision of data derived from the operations executed in the field.

Generally speaking, the best results for topographic purposes are obtained by means of photography, if we bear in mind that phototopography essentially and primarily is a constructive and graphic art, based upon graphic or pictorial records (which are nothing more than central projections in vertical plan of objects and their dimensions, that are to be transposed graphically into orthogonal projections into horizontal plan). Instrumental observations being required only to furnish such elements as may be needed to make the graphic transpositions (iconometric platting in a reduced scale) of the lines of directions and distances, and also to obtain checks or a proper control for the work in its entirety.

Photographic surveys have been conducted principally in regions where other surveying

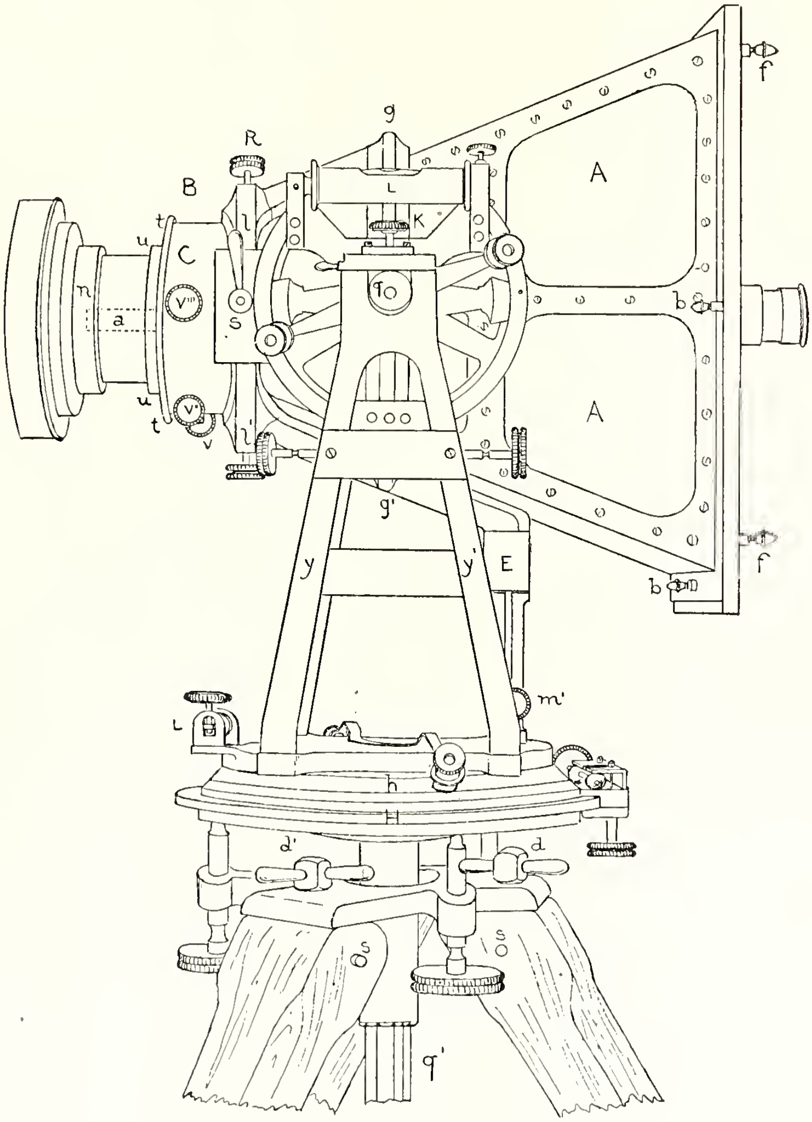


FIG 94

methods are either precluded or where their application would entail great cost and consume too much time, and such regions are characterized chiefly by a rugged and broken topography.

The necessity, therefore, lies close at hand to devise instruments that will not readily get out of adjustment or drop to pieces when transported over rugged mountain trails, and the more simplified their structural composition the more available will they become for the production of rapid and accurate work.

It is at once evident that the combination of a camera and a surveying instrument into a well-united, well-balanced, easily manipulated, and essentially light and withal rigid instrument is not easily accomplished. It is not surprising therefore, when searching the published descriptions of phototheodolites and other photogrammeters, to come upon a great number of types in which the many difficulties have been overcome, more or less successfully, by various devices.

We may find: A large-sized theodolite with a small camera, placed centrally between the Y supports, after removal of the telescope from the latter, both being interchangeable;

A large camera mounted upon the horizontal circle with a telescope and vertical circle attached eccentrically (at either side of the camera);

A large centrically located camera, the lens of which serves at the same time as objective of the telescope, the corresponding eyepiece being at the center of the frame that ordinarily supports the ground glass plate (in this form the camera itself is the telescope);

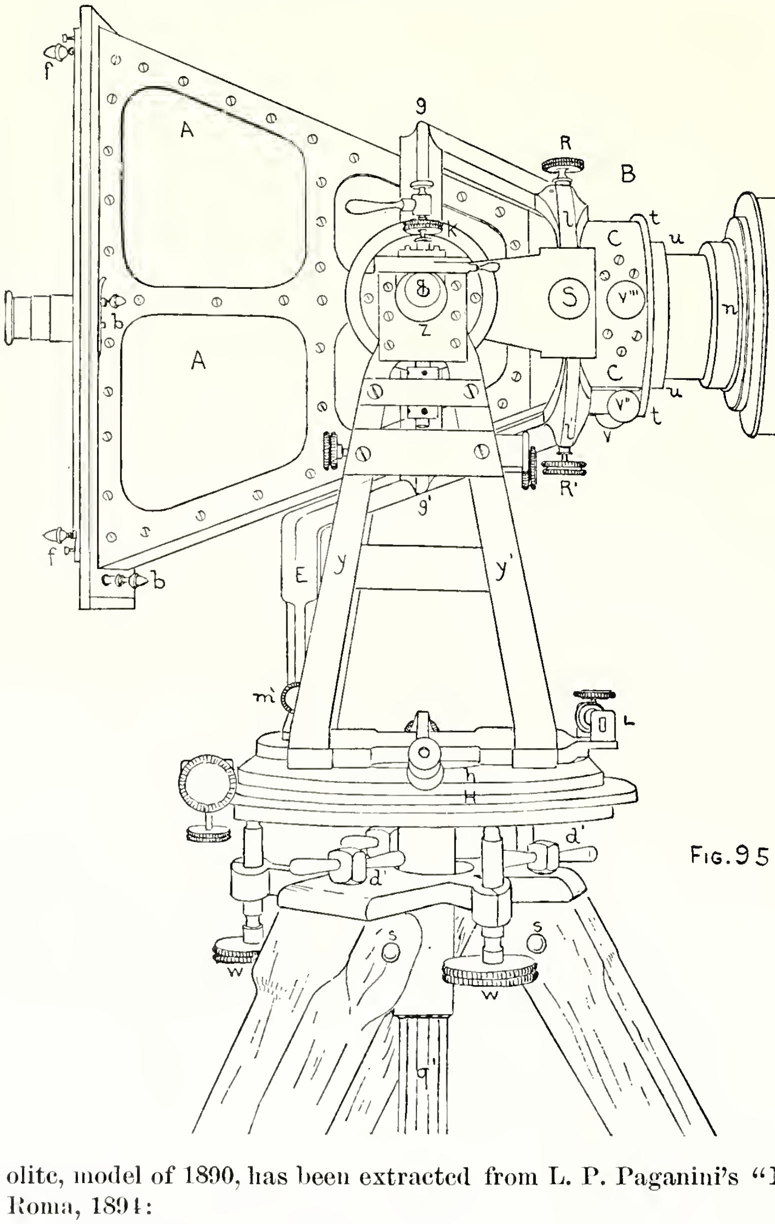


Fig. 95

Instruments where the board of the plane table has been replaced by a surveying camera, the upper face of which receives and supports the plane-table sheet and plane-table alidade; also various other combinations (some with compass attachments).

This class of instruments has been in use for large scale surveys and where the instrumental outfit could readily be brought very near the stations to be occupied by convenient means of transportation, the instruments rarely being subjected to such primitive and rough methods of transportation over long distances, as it generally has been the case on our continent when surveying cameras have been used.

(1) *The new Italian phototheodolite, devised by L. P. Paganini.*—Paganini's model of 1884 has been described in Appendix No. 3, United States Coast and Geodetic Survey Report for 1893.

The following description of Paganini's new phototheodolite, model of 1890, has been extracted from L. P. Paganini's "Nuovi appunti di fototopografia," Roma, 1894:

The general form and the dimensions of the camera box of Paganini's new phototheodolite remain about the same as with the older model, the principal change resting in the omission of the eccentric telescope which has been replaced by the centrally mounted camera, which may, at will of the observer, be converted into a telescope.

The telescopes which we generally find attached to surveying instruments consist of a tube, slightly conical in shape, having a positive lens or a system of convergent lenses at one end (the "objective") which produce within the telescope a real and inverted image—the same as the camera lens—of any object toward which the lens may be directed. The other, smaller end of the telescope tube, has a still smaller tube inserted into it which may be moved in the direction of the axis of the tube. This second tube also contains a system of convergent lenses—so-called "ocular lens" or "eyepiece" of the telescope—which serve to project an enlargement of the image in the telescope upon the retina of the observer's eye. In the image plane of the objective (within the telescope), is the so-called diaphragm—a ring-shaped metal disk—to one side of which a pair of cross hairs—spider webs, cocoon threads, or lines cut into a thin piece of plate glass—is attached in such a way that the hairs fall within the image plane. One hair is vertical and the other horizontal, their point of intersection coinciding with the optical axis of the telescope.

The old camera was provided with the objective, and a corresponding eyepiece had only to be added to convert the camera into a surveying telescope. In the instrument under consideration the eyepiece consists of a positive lens set, known in optics as "Ramsden's ocular lens." The inner wall surfaces of the camera box should be well blackened to avoid any side reflection and a consequent dimness in the appearance of the cross wires.

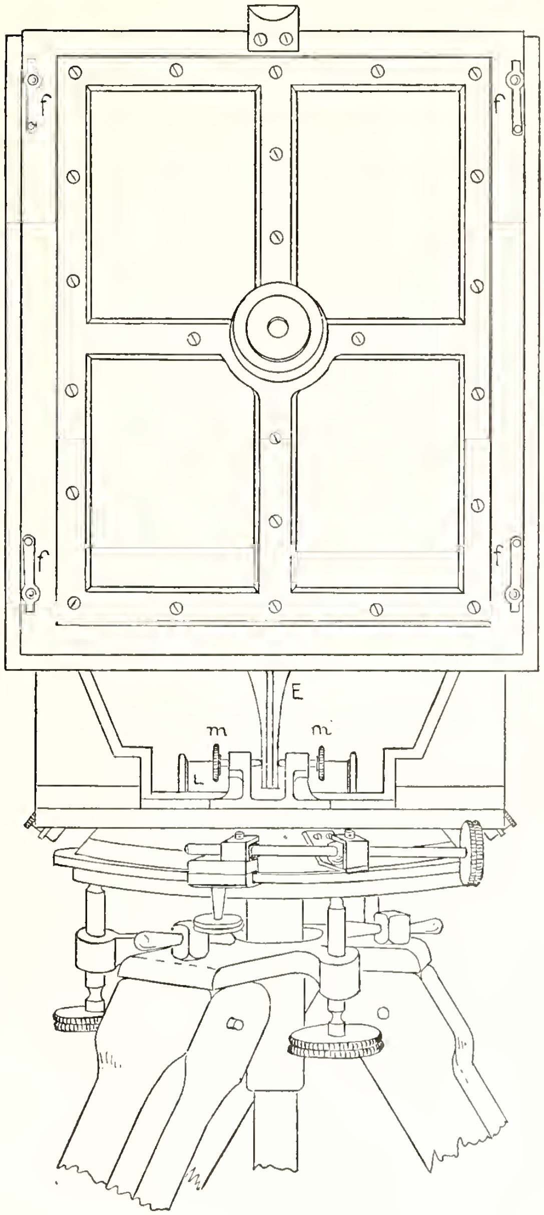


FIG. 96

The camera proper consists of two parts, a truncated pyramid *A*, figs. 94 to 98, and a cylindrical attachment *B*, into which the tube *t* is inserted. A second tube within the cylinder *t* may be moved in the direction of the optical axis by means of a screw, the threads of which have a rise of one millimetre. By revolving the inner tube the lens is brought nearer to or farther from the image plane, the lens remaining parallel with the image plane at any position that may thus be given to the lens.

A scale *a*, figs. 94 and 98, graduated to millimetres, is permanently attached to the tube *t* and it lies very close to the ring *n*, the circumference of which is divided into ten equal parts. (This graduated ring *n* is soldered upon the cylinder *u* containing the camera lens.) This scale *a* (extending in a direction parallel to the optical axis of the lens) has a mark, coinciding with the index rim of the ring *n*, thus indicating the focal length of the camera lens when focused upon objects at infinite distance. The millimetre graduation of the scale *a*, extending from the zero mark in the direction toward the ground glass serves to ascertain the focal lengths for objects nearer the camera station. The circumferential graduation on the ring *n* serves to read one-tenth of one revolution of the tube *u*, which is equal to an axial motion of the lens of 0·1 millimetre, hence the focal length for any object focused upon may be read to single millimetres on the scale *a* and to tenths of a millimetre on the graduated ring *n*.

The construction of this phototheodolite is such that the optical axis of the camera lens is always at right angles to the picture plane—the ground-glass surface or the sensitive film of the photographic plate. The intersection of the optical axis and the picture plane, the principal point, is marked by the intersection *P*, fig. 97, of the two very fine platinum wires *O O'* and *f f'*, one horizontal and the other vertical when the instrument is in adjustment. These wires are stretched across the back of the camera box as close as possible to the picture plane. The buttons *b*, figs. 94 and 95, serve to give tension to the wires. The wire *O O'* corresponds to the horizon line and the vertical wire *f f'* corresponds to the principal line of the perspective represented by the image on the ground-glass plate.

Fig. 96 shows the rear view of this instrument, the ground glass having been replaced by an opaque plate, strengthened by a metal frame and ribs, which supports the Ramsden eyepiece in the center, its optical axis coinciding with that of the camera lens. The cross wires $O\ O'$, $f\ f'$, at the rear of the camera, serve also for the astronomical telescope into which the camera may be converted by attaching the opaque plate with central eyepiece as shown in fig. 96. The fitting of this eyepiece allows for axial motion to adjust its position to avoid parallax.

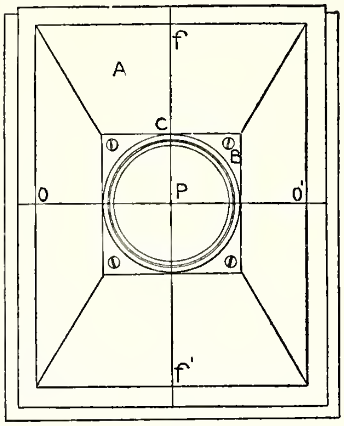


FIG. 97

The rear opaque plate and the other sides of the camera box are made of cardboard (impregnated with chemicals to render it impervious to moisture), and they are stiffened by frames and ribs of metal as illustrated in figs. 94 and 96.

The cylindrical part B, figs. 94, 95, and 97, is inclosed by a solid metal collar C, which is held in position within the metal ring $l\ l'$ by four screws R, R', S, S'. This ring $l\ l'$ is connected with the frame $g\ g'$ by means of two arms $l\ g$ and $l'\ g'$, all being cast into one piece. The frame $g\ g'$ has pivots q attached to it which form the horizontal axis of rotation for the camera.

This instrument is provided with a vertical circle, fig. 94, horizontal circle H, figs. 94, 95, and 98, verniers, reading microscopes, levels L, figs. 94 and 96, clamps, and slow-motion screws, forming a complete transit with centrally mounted "camera-telescope."

Fig. 98 represents a vertical section of this instrument. The scale a, already described, is here placed on top of the tube u to illustrate its function better. yy = uprights, supporting the horizontal axis of rotation of the "camera-telescope." h = alidade supplied with verniers. H = lower limb or horizontal circle bearing the graduation; it is supported on the tripod head T by three leveling screws W. a = casing for conical center. q' = central clamp-screw, firmly uniting T and H (it guards against an accidental falling off of the instrument from the tripod); it screws into a ball which is supported by the hemispherical socket w of the lower part of a.

The horizontal circle has a diameter of 10·5 centimetres. It is graduated into thirty minutes, and its verniers read to single minutes.

The photographic plates are 18 by 24 centimetres, the same size as for the 1884 model camera.

The objective lens was an aplanat of Steinheil of 237·7 millimetres focal length. More recently, however, the Italian phototheodolites have been provided with anastigmats of Zeiss.

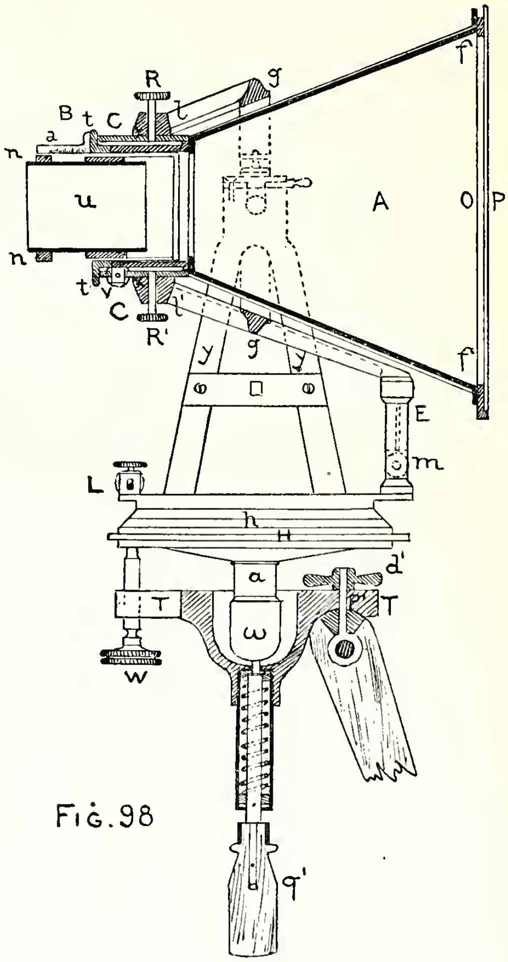


FIG. 98

The column E, figs. 94, 95, 96, and 98, forming a prolongation of the lower arm $l'\ g'$, is held in place by two counter screws m and m', fig. 96, which serve to hold the horizontal axis of rotation of the camera in a fixed position, avoiding accidental changes during the execution of a set of panorama pictures.

After unscrewing the nuts d', fig. 98, the tripod legs may be removed. They serve as "alpenstocks" when the instrument is being transported from station to station. The camera-telescope is lifted out of the wyes and packed in a separate case; the lower part of the instrument is packed in another case, and the plate holders and plates are transported in a third case.

(2) *Photogrammetric theodolite of Prof. S. Finsterwalder.*—This phototheodolite (manufactured by Max Ott (A. Ott), of Kempten, in Bavaria) was devised by Dr. Finsterwalder after many years of practical work and experience incidental to his Alpine surveys and studies of glacial motion. This experience taught him the desirability of producing a camera compactly built, rigidly constructed in all its parts, and yet having a minimum of weight. To avoid the extra weight when transporting a separate theodolite (with the surveying camera) for the trigonometric location of the stations occupied with the camera, he provided the surveying camera with the means for observing horizontal and vertical angles.

Professor Finsterwalder's phototheodolite is illustrated in fig. 99. The entire outfit weighs 10 kilogrammes, which weight is distributed as follows:

	Kilogrammes.
The instrument per se	2·7
Carrying case for same	2·4
The tripod	1·7
One dozen leather plate holders, including the twelve plates	2·5
Packing case for the latter	0·7

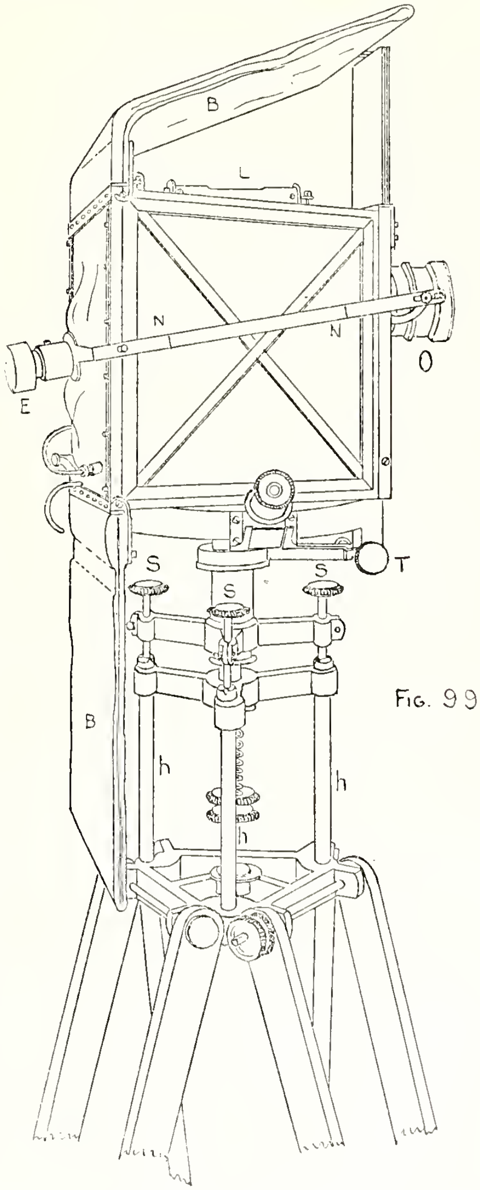


Fig. 99

Professor Finsterwalder has used a double anastigmat of Görz and later an anastigmat of Zeiss, with a constant focal length of 150 millimetres. With this focus the lens will photograph perspectively correct a plate of 160 by 200 millimetres. The plates have a size of 120 by 160 millimetres and they command an effective horizontal field of 53°, enabling the observer to cover the complete panorama with seven plates.

For the central or normal position of the objective the camera commands an effective vertical field of ±20°. This range would often be insufficient, particularly when photographing mountainous terrene of an alpine character, therefore it was deemed advisable to mount the objective on a slide, which will permit a considerable change in the vertical sense. Owing to this device, objects subtending an angle of depression of 35°, together with others subtending an angle of elevation of 5°, may still be photographed on the same plate, giving a vertical control of 40° in all.

In extreme cases, when it should become desirable to photograph objects subtending angles of +35° and of −35°, or 70° in all, Professor Finsterwalder recommends the exposure of two plates in succession, commanding the same (identical) horizontal angle, exposing one with the maximum elevation of the objective slide and the other with the maximum depression of the lens. Thus, inclined pictures are not only avoided, but the effective surface of the plate is utilized to the best advantage, and the weight of glass to be carried is reduced to the minimum.

In order to obtain uniformly accurate results with the relatively short focal length (maintaining a constant distance between the lens and the sensitive surface of the plates), the plates are not inserted into plate holders (where the variable thickness of the glass would affect the so-called "constant focal length"), but they are pressed directly against a metal frame, which forms the back of the camera box, very similar to the arrangement described for Captain Deville's (Canadian) camera. To do this, use has been made of Dr. Neuhauss's leather plate holders, formed like a sack *B*, fig. 99. The inner edges of the metal frame are graduated in order to locate the principal and horizon lines upon the negatives.

These leather sacks have metal slat arrangements, and the transfer of the plate from the sack to the camera is made by hooking the sack with its mouth to the upper edge of the rear camera side. While holding the bag in a vertical position the slats are opened and the plate is allowed to slide from the sack into the carrier to be exposed.

Springs are provided at the back of the camera box to check against a sudden dropping of the plate into the metal carrier, to avoid a breaking or cracking of the plate by striking the closed lower metal slide of the plate carrier too hard. These springs also serve to press the plate, when in position for exposure in the carrier, into perfect contact with the graduated metal frame at the back of the camera box.

By withdrawing the upper curved handle, fig. 99, at the back of the camera, the tension of the springs may be reduced and the plate glides into position to be exposed. After exposure the lower slide is withdrawn and the plate will slip into the empty sack *B*, which had been hooked to the lower edge of the camera back for this purpose, as illustrated in fig. 99.

The eccentricity of the center of gravity, by applying the weight of the sack and plate to one side of the camera, does not affect the adjustments of the instrument sufficiently to throw the photographic plate out of the vertical plane in which the exposure should be made. This camera theodolite is accurately balanced when no sack is attached, in which form it is used to measure the angles that may be needed to locate the camera station (geographically, and also in the vertical sense) with reference to surrounding trigonometric signals.

In order to convert this camera into a theodolite (with centrally located telescope), the back of the camera is provided with a telescopic eyepiece *E*, of a magnifying power of from 7 to 8. This eyepiece is adjusted to form a surveying telescope with the camera lens *O* as objective. It is provided with cross wires or webs, and a shutter affords the means to shut out the light when the instrument is used for photographing.

The camera lens (objective of "camera telescope") being movable in the vertical sense within a range of 100 millimetres, all objects falling within a range of $\pm 17^\circ$ may be bisected with this telescope. The definition of points to be bisected, when above or below the camera horizon, would be very poor if the eyepiece *E* were rigidly fixed in the horizontal position, but by means of the metal arms *NN* the eyepiece may be revolved about a horizontal axis in such a way that it will always be directed to the center of the camera lens.

With the double anastigmat of Görz, which produces a perfectly flat picture (with neither spherical, chromatic, nor astigmatic aberration or distortion), a change in the focus of the eyepiece will rarely be required.

Horizontal angles may be observed directly by means of a horizontal circle of 120 millimetres diameter, which is provided with two verniers reading to single minutes. A series of experimental tests has proven that horizontal angles observed between points of considerable difference in altitude may be obtained within a limit of error of 0·4′. This instrument, therefore, gives results sufficiently accurate to locate the camera station trigonometrically with reference to surrounding fixed points of known positions, if they are not too far distant to be defined with this low-power telescope.

Vertical angles, however, can not be obtained directly. Still, by means of a scale and vernier attached to the camera-lens slide (or front board) the change of the camera lens from its central or normal position (that is, a value directly proportional to the tangent of the vertical angle) may be read to 0·05 millimetre. The slide motion of the front board is accomplished with a rack and pinion, and experience has proven that the observations may be obtained within a limit of error (converted into arc measure) of 1 minute.

The three rods, designated by *h* in fig. 99, are each 100 millimetres long. They serve to elevate the instrument support and the three leveling screws *S* sufficiently high above the tripod to allow full play for the leather plate holders *B*, when they are placed in position to receive the exposed plate. The tripod legs may be folded together to one-half their length.

No ground-glass plate being provided, a special finder has been devised correctly showing the field controlled by the plate for any position of the camera lens. (See Zeitschrift für Instrumentenkunde, October, 1895.)

(3) *Photo-theodolite for precise work, by O. Ney.*—This instrument has been patented in the

German Empire, and the following description has been taken from Zeitschrift für Instrumentenkunde, page 55, 1895:

In the construction of this instrument, figs. 100 and 101, it has been sought to satisfy the following requirements:

First. The camera should be sufficiently large to produce clear and well-defined perspectives.

Second. The general disposition of weight and mass should be symmetrical (the camera and the telescope of the theodolite were to be mounted centrally).

Third. The weight of the instrument should be reduced to the minimum consistent with rigidity and sufficient strength to assure a free and easy manipulation, as well as durability or permanency of its adjustments when used in the field.

This instrument is composed of two distinct parts, the camera proper (with horizontal circle) and a complete transit. They may be used separately and independently, but always upon the same support and with the same tripod. The interchange

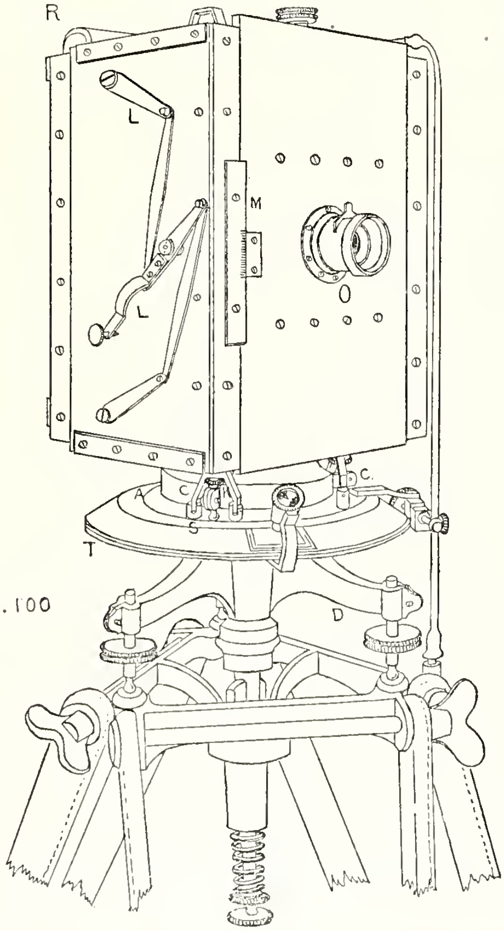


Fig. 100

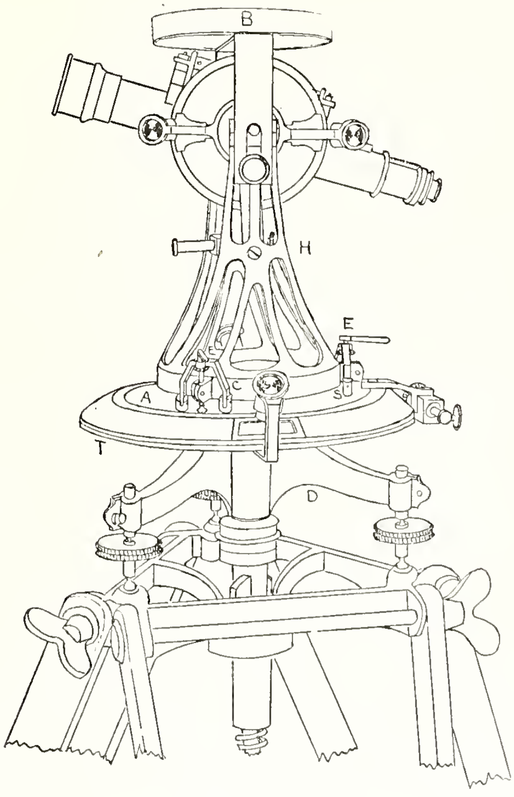


Fig 101

between camera and transit is readily accomplished (both being centered over the same instrument support) with accuracy and expediency.

The principal advantages attached to this disposition of the component parts of the phototheodolite may be cited as follows:

First. The symmetrical and central mounting of the camera and transit telescope insures accuracy in the results.

Second. The weight of each separate instrument—camera and transit—has been reduced to a minimum.

Third. A disturbance of the adjustments of the instrument support (including tripod) may be completely avoided by having the plate inserted and the slide withdrawn before placing the camera box into position upon the upper alidade limb.

The carrying into effect of the ideas just mentioned has been greatly aided by supplying all leveling and clamp screws with spherical ends resting upon plates in such a manner that a free play of motion will take place. These spherical terminations of the screws were originally devised by Reichel.

The two forms in which this instrument may be used are shown in figs. 100 and 101. The former shows the photo-topographic camera (similar to Professor Finsterwalder's instrument), and the latter shows the transit with compass *B*.

D is the very rigid, yet essentially light, instrument support, the three arms being cast into one piece with the bearing for the conical pivot attached to the horizontal limb *T*.

T is the lower graduated limb and *A* is the upper limb of the alidade bearing the verniers.

A large circular level is attached to the center of the upper limb of the alidade. The latter has three hardened plates inserted into its upper surface (at *S*, figs. 100 and 101), one with a plane surface, the second with a conical cavity, and the third with a V-shaped groove or slot. They form the supports for the spherical terminations of the three screws *K*, fig. 100, attached to either the transit or the surveying camera. These screws are received between the flanges *C* that form a part of the base ring supporting either the camera or the telescope wyes *H*, fig. 101.

The two sets of three screws *K* (one for camera and one set for the transit) serve to adjust the horizon lines of both instruments and to bring them into the same horizontal plane.

The transit telescope is arranged for stadia reading (after Porro's method), with 100 as the constant factor. The telescope level reads to 20″, and the final adjustment of the transit is accomplished by means of this level. The striding compass *B*, fig. 101, is graduated to read to 30′, whereas the horizontal circle reads either to 10″ or 20″, according to the size of the instrument.

In order to secure the transit and the camera to the horizontal circle (which both have in common) three horseshoe-shaped clasps (shown near *C*, figs. 100 and 101) are hinged to the upper limb *A* of the alidade in such a way that they straddle either set of the three screws *K* of the projecting flanges *C* (when they are turned up as shown in the figs. 100 and 101).

Each of these clasps has a clamp screw with lever handle *E*, fig. 101, and by tightening these three clamp screws they are brought to bear upon the hardened heads of the screws *K*, making a firm connection between the upper limb of the alidade and the superimposed transit or camera. This connection is easily made, and it does not disturb the adjustments of the instrument.

This instrument is made in two sizes; one has plates 13 by 18 centimetres, and the other 18 by 24 centimetres. To avoid changes in the dimensions of the camera box, due to hygroscopic influences of the atmosphere, the box is constructed entirely of aluminium. The plate holders and the movable plate carrier, however, are made of mahogany, impregnated with chemicals to make the wood impervious against moisture.

To avoid any possible change in the constant focal length, due to an uneven thickness of photographic plates or of the plate holders, the movable carrier may be moved toward the camera lens by means of the levers *L*, fig. 100, until the sensitive surface of the photographic plate is brought into contact with a metal frame, securely fastened to the sides of the camera box, and which has a centimetre graduation filed into its inner edges. The distance of the rear surface of the graduated frame from the second nodal point of the camera lens constitutes the constant focal length of the camera.

The centimetre graduation on the inner edges of the metal frame, reproduced on the margin of the negatives, serves to ascertain whether the sensitive films (or the contact prints) have undergone any change during the process of development and also to ascertain the amount of correction to be applied to the perspective, if found to be distorted, before using it for the iconometric platting.

The camera is provided with a pair of cross levels to enable the observer to detect any change in its adjustments prior to exposing a plate. These levels are graduated to read to 20″ of arc. When the instrument is in perfect adjustment, the picture plane will be in a vertical plane and the principal ray will be in the same horizontal plane as the optical axis of the telescope (when level), if the camera were replaced by the transit without disturbing the tripod's position.

When this camera theodolite is adjusted, the vernier *M*, fig. 100, will read zero for the normal position of the lens. Still, the objective may be elevated or depressed by 35 millimetres, which change from the normal or central position of the lens may be read correctly within 0.1 millimetre on the scale and vernier M, fig. 100. The pneumatic camera shutter is arranged both for time and for instantaneous exposures, a special device guarding against the possibility of exposing a plate before it is brought into perfect contact with the graduated metal frame, previously mentioned.

The plate holder can not be withdrawn from the camera before the slide has been replaced, nor as long as the plate is in contact with the graduated frame.

(4) *The phototheodolite of Dr. C. Koppe.*—Dr. C. Koppe, professor at the Technical High School in Braunschweig, Germany, is an ardent advocate of photogrammetry and he has done much toward popularizing photographic surveying in Germany. His work on photogrammetry, published in 1889, is an excellent manual both in respect to theory and practice. In 1896 he published a treatise on photogrammetry applied to cloud photography for meteorological research.

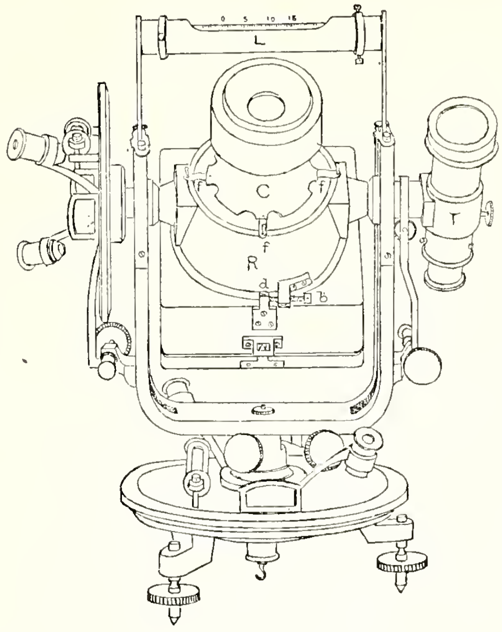


FIG. 102

This phototheodolite, fig. 102, has a centrally mounted camera with the telescope on one side and the vertical circle on the other. The horizontal axis between the two wyes has been widened into a conical ring R, into which the camera C may be inserted. Four stout springs f press the camera C tightly against the ring surface forming the base of the conical ring R. After insertion into the ring, the camera C is revolved within the former until the end of the screw b abuts against the stop d, when the horizon line of the perspective (negative) should be horizontal.

The camera axis is parallel with the optical axis of the telescope T, both axes being in the same horizontal plane when the vernier of the vertical circle reads zero. When elevating or depressing the telescope T the camera axis will follow the same motion, both remaining parallel. The instrument will be in equilibrium with the camera de- or attached. The horizontal axis of this instrument may be adjusted by means of the striding level L, which, when necessary, may be replaced by a striding compass in a manner similar to that illustrated in fig. 101.

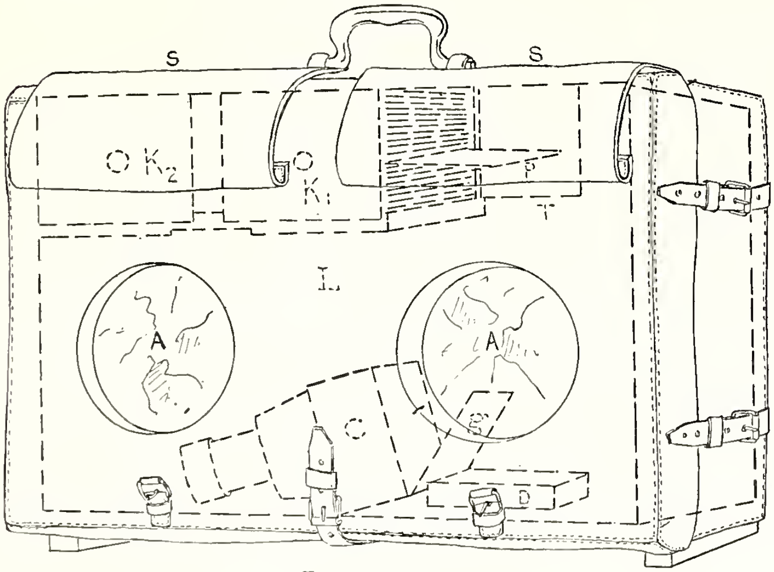


FIG. 103

Since the telescope may be reversed in the wyes, an error of collimation and any index error of the vertical circle may be found or eliminated.

There are neither slides nor plate holders provided with this instrument, the plates being inse ted directly into the camera. This may be done in the field by aid of the packing case specially constructed to serve as a dark chamber, fig. 103.

This case is made of wood with double doors, each door having a circular hole A, which is filled in with a flexible, light-tight, and dark-colored material, forming sleeves in such a way that the hands of the operator may be thrust through an elastic opening in the center (of the circular openings). The fabric will close tightly around the wrists—when the interior of the case will be perfectly dark—and the sleeves *A* will permit free play to the hands for manipulating the camera and plates within the space *L* of the case.

This case is inclosed with a tight-fitting sole-leather covering, having two flaps *S* to protect the openings *A* against the admission of dust when the packing case is transported on the back of the instrument bearer.

The entire instrument, except the tripod, may be packed into this case for transportation. It also contains two boxes, K_1 and K_2; the former receives the exposed plates (negatives) while the latter contains the supply of unexposed plates.

When a plate is to be exchanged the camera *C* is placed into the packing case and both doors as well as the leather main flap or cover are securely closed; both hands are now inserted through *A*, and after the sleeves are tightly closed about the wrists the camera is opened, the exposed plate removed and placed into the box K_1 (as shown at *P*, fig. 103). The door *T* is closed and a new plate, taken from the box K_2, is placed into the camera (as shown by *g*, fig. 103) and the camera back is closed, when the camera will be ready for another exposure.

The constant focal length of this camera is represented by the distance between the second nodal point of the lens and the rear surface of a metal frame (similar to that of Ney's phototheodolite) permanently attached to the rear of the camera box.

The inner edges of this metal diaphragm or frame are graduated into centimetres; the middle graduation marks of the horizontal sides of the frame locate the principal line, while the middle graduation marks of the vertical sides represent the termini of the horizontal line on the perspectives. The focal length, once determined, will remain unchanged for all plates.

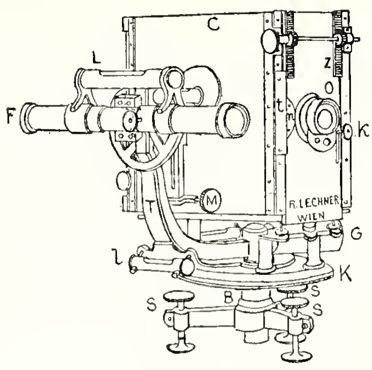


FIG 104.

This instrument has been manufactured for Professor Koppe by F. Randhagen, in Hanover, Germany.

The "Topographic Bureau" of the Swiss Republic has used a phototheodolite constructed after the model of Dr. Koppe's instrument. The experience in Switzerland, however, seems to have decided the topographic bureau *not* to replace the plane table by the phototheodolite for general topographic surveys executed by that bureau.

(5) *Phototheodolite devised by V. Pollack, manufactured by R. Lechner in Vienna, Austria.*—With this instrument (fig. 104) the camera *C* is centrally located, and it rests upon a horizontal circle. The telescope *F* and the vertical circle are mounted at one side of the camera, a weight *G* counterbalancing both on the other side of the camera.

Aluminum has been used very freely in the construction of this phototheodolite in order to reduce the weight as low as possible. This instrument has been manufactured in two sizes; the horizontal circle of the small-sized one is graduated to 30′, the verniers reading 1′, while the larger one has a circle graduated to 20′, and its verniers read 20″. The telescope *F* is mounted similarly to that of the so-called Danish plane table alidade.

The adjustment of the horizontal axis of revolution of the telescope *F* is accomplished by means of a special level. Clamps and slow-motion screws are provided for both the horizontal and vertical circles. The telescope has a focal length of 27 centimetres and an opening of 31 millimetres, with a magnifying power of 9 to 18 diameters. The telescope is arranged for stadia reading, and it has 100 as the constant multiplier. The telescope level *L* is graduated to 10″ or 20″. The vertical circle is graduated to 20′ and its two verniers read to 20″.

The camera box is made of aluminum and it is provided with a Zeiss anastigmat. By means of the rack and pinion *z* the lens may be elevated or depressed by either 30 or 50 millimetres, according to the size of the instrument. The scale *t*, with vernier *n*, serves to measure the vertical deviation of the lens from its normal position. Also this camera is provided with a graduated metal frame, the inner edges of which have either a centimetre or five-millimetre graduation, which is reproduced upon the margins of the negatives. They serve not only to locate the horizon and the principal lines upon the perspectives, but they also give the means to discover any distortion that may arise in the pictures due to the wet process of development.

This metal graduated frame is brought into contact with the sensitive surface of the film by a simple mechanical contrivance in such a way that the focal length for all negatives is constant, even if the plate holders or glass plates should not be equally thick.

(6) *Col. A. Laussedat's latest phototheodolite.*—This instrument (figs. 105 and 106) has been manufactured by E. Ducretet and L. Lejeune, in Paris, France.

Both transit telescope *L* and camera *C* are centrally mounted, the latter above the former. The camera may also be used alone, independently of the transit, and it may then be mounted upon the tripod (fig. 106) by means of a special pivot or spindle *S'*. The transit may likewise be used alone, without the camera, for trigonometric observations.

S = leveling screws. c_1 = central clamp screw. *C* = camera, and *B* = magazine for fifteen plates. *O* = objective of the camera; it is a rectilinear wide-angle lens of 75 millimetres focal length. *H* = sliding front plate of camera, provided with pinion and rack movement, *R*, to elevate or depress the lens. *V* = finder to show the extent of the field covered by the photographic plate, although a focusing glass, *G*, fig. 106, is also provided. *L* = transit telescope provided with stadia wires. *Ce* = vertical circle, graduated to 30'. *M* = Wye supports of the telescope axis of revolution, their prolongation forming the camera support. *A* = horizontal circle graduated to 30'; its clamp and slow-motion screw are indicated at *P'*. *N* = adjustable level. *D* = declination or box compass.

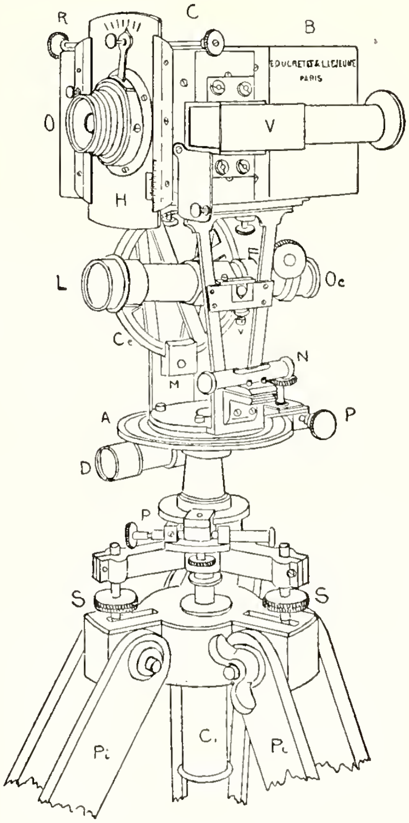


Fig. 105

Several loaded magazines, each containing 15 plates, may be carried with this instrument and the plates may be exchanged in full daylight without having to remove the camera. The photographic plates are $6\frac{1}{2}$ by 9 centimetres, but enlarged prints are used for the iconometric platting. Six plates cover the entire horizon and will form a complete panorama.

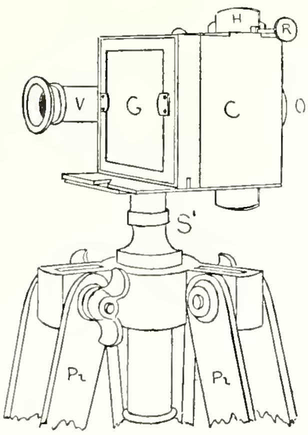


Fig. 106

The lens is provided with an iris shutter. It may be focused for short distances or infinity by turning a lever over a scale showing the distances in metres attached to the front board, *H*, of the camera.

In fig. 106 the camera is represented with the magazine, *B*, removed and replaced by the ground glass plate, *G*.

The entire outfit, excepting the tripod, may be transported in one carrying case (with shoulder or pack straps) of 39 by 28 by 17 centimetres size and 8 kilogrammes weight if but one magazine filled with 15 plates is included.

(7) *The phototheodolite of Starke and Kammerer.*—This instrument, fig. 107, is somewhat similar in construction to Professor Finsterwalder's phototheodolite; both have neither telescope nor vertical circle, being provided with camera telescopes instead.

An ordinary skeleton tripod supports the three leveling screws, *S*, and a central clamp screw with spiral spring, *P*, securely connects the tripod head with the instrument proper. *H* represents the horizontal circle, graduated to 20', but by means of two verniers and microscopes, *L*, horizontal angles may be read to 1'.

The vertical axis of revolution, ending in three horizontal arms, B_1, B_2, B_3, may be adjusted with the aid of the leveling screws *S* and the cross levels l_1, l_2. The plate *D*, forming the support of the cross levels, is firmly united with the arm B_2.

E = upper clamp screw. M = upper tangent screw for slow motion. F_1, F_2, F_3 = three leveling screws supporting the camera telescope; they rest in grooves on the arms B_1, B_2, B_3. l_3, l_4 = cross levels, attached to the camera telescope, figs. 107 and 108; they serve to adjust the photographic plate into vertical plane, using the three leveling screws F_1, F_2, F_3 for this purpose. S = movable front board or lens slide, figs. 107 and 108. Q = handle to facilitate the mounting of the camera,

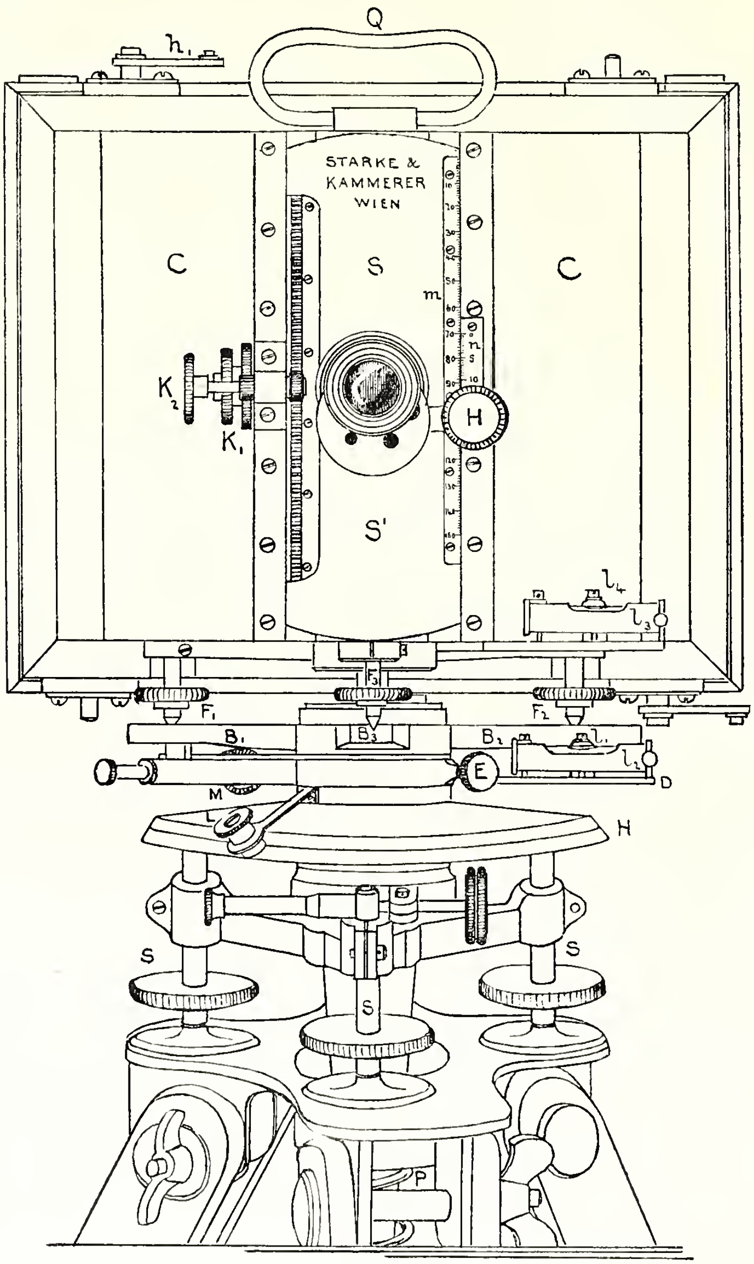


FIG. 107

C, upon the three arms B_1, B_2, B_3. K_1 = pinion for elevating or depressing the front board S, which has a corresponding rack, as illustrated in fig. 107. K_2 = differential pinion for slow motion of the front board. H = clamp screw for fixing the lens in any position above or below its central or normal position. m = millimetre scale for measuring any vertical change of the lens from its normal position, the vernier n permitting such change to be read to 0·05 millimetre.

The camera may be securely united with the vertical axis of the horizontal circle by a clamp screw manipulated from within the camera box.

When the zero mark of the vernier *n* coincides with the 70 mark of the scale *m*, the lens should be in its central or normal position. The slide *S* may be moved 70 millimetres up or down; from 70 to 140 it falls above the normal position.

The lens is a Zeiss anastigmat, $^{f}/_{18}$, with a focal length of about 212 millimetres.

When the camera lens is suitable for photo-topographic purposes, the horizontal change in the distance between its second nodal point and the image plane should only be:

	0·09	0·11	0·15	0·22	0·45 millimetre for:
distances of	500	400	300	200	100 metres.

Hence focusing may be dispensed with for general photo-topographic purposes; still, in order that this instrument—for special purposes—may also produce sharp and well-defined pictures of objects close to the camera, the lens mount is such to allow a motion in the direction of the optical

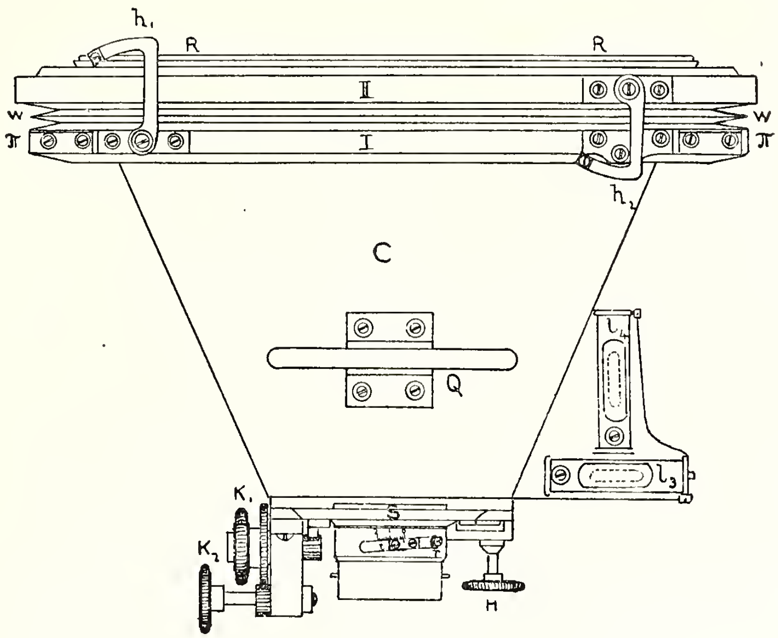


FIG. 108

axis within a range of 2 millimetres, whereby objects but 23 metres away from the camera may still be brought into focus.

The external tube of the lens mount has a helical groove or slot, fig. 108, in which a small metal block *t*, provided with an index mark, may glide freely. This block is attached to the inner tube of the lens mount, and a screw *r* at one end of the slot serves to clamp the two tubes together, when the focal length will be maintained constant for any length of time. When the screw *r* is loosened and the outer tube revolved from left to right, the focal length will be shortened. When the block *t* has passed from one end of the slot to the other, the focal length will have suffered a change of 2 millimetres. The two positions of the index mark on the block *t*, for these extreme limits, are marked on the edge of the slot on the outer tube, 0 and 2, fig. 108; the interval being divided into twenty equal parts, one part will correspond with an axial motion of the camera lens of 0·1 millimetre.

A metal frame is attached to the back of the camera box, its rear surface coinciding with the picture plane. The inner edges of this frame are provided with a centimetre graduation; the middle marks (triangular file cuts) of the vertical sides of this frame designate the termini of the horizon line on the negative, while the middle notches of the two horizontal sides indicate the position of the principal line. When the instrument is in adjustment, the principal line will be vertical, the horizon line will be horizontal, and their point of intersection will be the principal point of the photographic perspective. The opening of this metal frame is 17·8 by 22·8 centimetres, which is also the effective size of the pictures.

The two frames *I* and *II* in figs. 109 and 110 give the means at hand to make a light-tight connection between the single plate holders (or ground-glass plate) and the camera telescope. The short bellows *w*, connecting frame *I* with *II*, will admit the frame *II* to be moved a little while *I* remains fixed to the camera box. Each of these two frames is provided with two hooks, frame *I* having one upper hook h_1, figs. 107 and 108, and a similar hook near the lower corner diagonally opposite h_1. The hook h_2, fig. 108, is attached to the upper corner (opposite hook h_1) of frame *II*, which also has a similar lower hook diagonally opposite h_2 and directly under h_1.

Fig. 110 represents a partial section of the rear end of the camera, showing the ground-glass attachment *V*. Frame *II* is fastened to frame *I* by means of the upper left hook h_2 and the lower right hook.

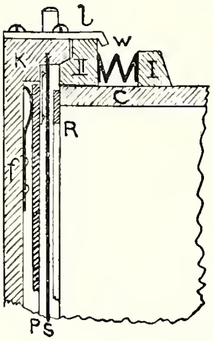


FIG. 109

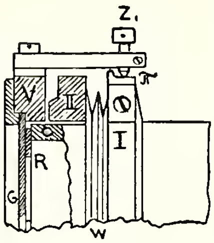


FIG. 110

The ground-glass frame *V* is supported by the screws z_1 and z_2, figs. 110 and 111, the points of which rest upon the metal plates π, figs. 108 and 110, attached to the permanently fixed frame *I*. The face of the ground glass *G*, fig. 110, is brought into contact with the rear surface of the graduated metal frame *R*, fig. 110, by means of the upper right and lower left hooks.

The ground-glass attachment *V* also has the eyepiece, which forms a telescope with the camera lens, converting the camera into a camera telescope. The position of the optical axis of the eyepiece may be adjusted vertically by turning the screws z_1 and z_2 until the line of collimation of eyepiece and camera lens fall together into the plane of the camera horizon (the camera lens being in its normal position, or the zero mark of the vernier *n* coinciding with the 70 mark of the scale *m*, fig. 107). In this position points on the horizon may be sighted through the eyepiece of the ground-glass attachment; but when the camera lens had been moved up or down some distance away from its normal position the eyepiece can no longer be used with its optical axis horizontal, and the stops p_1 and p_2, fig. 111, are now unfastened and the eyepiece is tilted up or down (rotating it about the horizontal axis x_1 x_2, fig. 111) until its optical axis is directed to the center of the object glass, when the image of the point to be bisected will appear well defined.

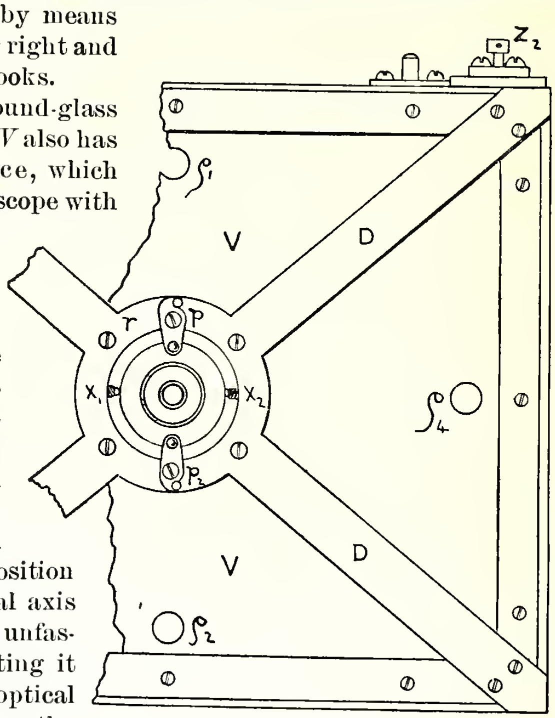


FIG. 111

The circular openings ρ, shown in the ground-glass attachment, fig. 111, serve to examine the middle notches of the inner edges of the sides of the graduated metal frame *R*, which define the horizon, and the principal lines of the perspective, thus giving the means to test the positions of those lines and to adjust the same, if necessary.

The outer wooden frame *V*, fig. 110, of the ground-glass attachment is strengthened with two metal diagonal ribs D_1, fig. 111, which are joined at their intersection by a ring *r*, the latter forming the support for the eyepiece, which may be revolved about the horizontal axis x_1 x_2, as has been already mentioned.

Each holder contains a single plate, and fig. 109 illustrates a section through the upper rear part of the camera box with a plate holder *K* in position:

P = dry plate; it rests at its four corners upon the springs *f*. *S* = hard rubber slide, which is completely withdrawn when making an exposure of the plate. R = graduated metal frame permanently fixed to the rear end of the camera box C.

We will now describe how the plate holder is attached to the camera for exposing a plate:

Frame II is set free from frame I, and K is hung to the frame II by means of the bent plate l, fig. 109, when the beveled projecting edge of K closes into the rebate of frame II, producing a light-tight connection. K is now secured to frame II by the upper left and lower right hooks (which is the position shown in fig. 109). The hard-rubber slide S is now withdrawn, and the pair of hooks—upper right and lower left—are tightened to draw the holder K forward until the sensitive film surface is brought into contact with the graduated metal frame R at the back of the camera C, the springs f taking up any lost motion and insuring a perfect contact.

The lens is now uncapped, the exposure made, and the plate holder is withdrawn by repeating the same operations in the inverse order: unfastening the pair of hooks—upper right and lower left—inserting the slide S, and drawing back the last two hooks—lower right and upper left.

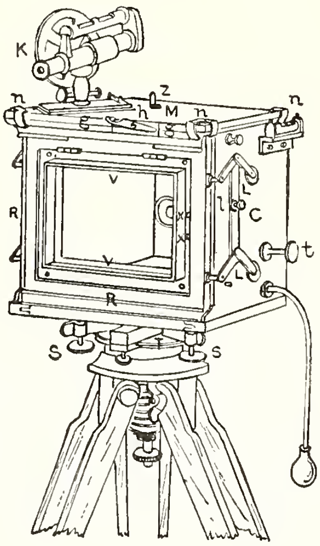


FIG. 112

(8) *Captain Hübl's plane-table photogrammeter.*—This instrument is made by R. Lechner in Vienna, Austria, and it has been described in "Lechner's Mittheilungen aus dem Gebiete der Photographie und Kartographie," Verlag von R. Lechner (Wilhelm Müller) Graben 31, Wien.

The result aimed at in topography generally being the graphic representation of the terrene, Captain Hübl replaced the theodolite of the ordinary photogrammeter by a plane table with alidade, thus being enabled to plat the directions required for the orientation of the picture traces, as well as those needed for the location of the camera stations, directly in the field upon the plane table.

For this purpose the top M, fig. 112, of the camera C, (21 by 21 centimetres) is disposed for use as a plane table. It receives the paper sheet, which is held in position by four metal corner clamps n.

Fig. 113 shows the plane table (or upper surface of the camera) $a\ b\ c\ d$, which has two pivots, z and z^1, about which the ruler LL of the alidade K may be revolved in azimuth. If zf, fig. 113, represents the constant focal length, eg will be the horizontal projection of the picture trace. By placing the ruler LL of the alidade upon the pivot z the horizontal projections of horizontal directions emanating from z (representing the platted station point) as a center to those points of the perspective which serve to orient the picture may be drawn upon the paper between the sector $e\ z\ g$.

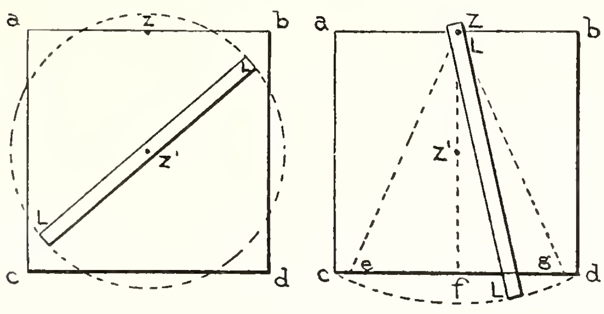


FIG. 113

The central pivot z', fig. 113, serves as the vertical axis of rotation for the alidade ruler LL when drawing the horizontal directions to known points (signals over trigonometric stations, visible from the camera station) to locate the position of the station with reference to surrounding triangulation points. The line zf or $z'f$ represents the horizontal projection of the principal ray (or of the optical axis of the camera). It is the trace of the principal plane upon the horizontal-projection plane.

With reference to fig. 112:

c = camera box made of aluminum, with constant focal length. k = plane-table alidade, arranged for stadia reading, with vertical circle. z = pivot over second nodal point of the camera lens. z^1 = pivot vertically above center of instrument (in prolongation of the vertical axis of rotation for the camera or plane-table).

At *e* and *g*, fig. 113, are two stops representing the ends of the photographic field *ezg*, which is identical with the horizontal angle commanded by each plate.

The lever *h*, fig. 112, serves to locate the principal point *f*, fig. 113; when the edge of the ruler *LL* abuts against the upturned lever *h*, and the principal ray *zf* (bisecting the angle *ezg*) may be drawn upon the plane-table sheet.

With reference to fig. 112: *b* = rubber bulb for operating the pneumatic shutter of the camera. *t* = head of pinion which serves to elevate or depress the camera lens, the change from the normal position of the lens being read on a scale with vernier. *n* = spirit level, two being provided (at right angles) for adjusting the instrument. *R* = movable plate carrier. *LL* = lever for moving the plate carrier *R* forward (toward the lens) until the sensitive surface of the plate is brought into contact with the graduated metal frame *vv*.

The horizon and the principal line may be located upon the perspectives by means of the centimetre graduations on the inner edges of the metal frame *vv*, or two fine wires may be attached to the corresponding points of the graduation.

The camera is supported by the three leading screws *s*, their upper ends resting in three slots of the lower face of the camera box. The latter is firmly united with the tripod head by means of a central clamp screw with spiral spring. *T*=graduated horizontal circle with clamp screw. It serves to enable the observer to turn the camera by an equal amount in azimuth after each exposure. *xx*=correction screws to adjust the graduated metal frame *vv* to bring the principal point into the optical axis of the camera lens.

The plane-table *M*, with alidade *K*, serves to locate the camera station in both the vertical and horizontal sense. If the camera stations are not very close together, the plane-table may also serve for the location of tertiary points and for the sketching of details.

This photographic plane-table is well suited for topographic reconnaissance surveys. The results obtained by means of the same may not be as precise as those obtained with the more complicated and refined phototheodolites, but it is more easily transported, is very simple in manipulation, and the adjustments are not liable to be easily disturbed. The instrument is compact, well conceived, and excellently executed.

The size of the photographic plate is 12 by 16 centimetres, giving an effective picture within the graduated margin of 10 by 14 centimetres.

The cube-shaped camera has sides of 21 centimetres length, and weighs 3·5 kilograms. The packing case, including the entire outfit and stout tripod (three folding legs), weighs only 11·5 kilograms. The cost in Vienna of the complete instrument is 400 florins.

V. PANORAMIC CAMERAS.

The lenses of the older surveying cameras gave correct perspectives only for small angles, rarely exceeding 30°, and Martens, in Paris, was probably the first to devise a so-called panoramic camera to photograph larger sections of the horizon on one plate, even with lenses that ordinarily would cover but a small angular field.

If the objects to be photographed are far enough distant to permit the use of a constant focal length of lens for the picture, and if the lens may be rotated about a vertical axis passing through the second nodal point of the lens system, such panoramic views may be obtained upon a sensitized surface bent into a half cylinder whose radius equals the constant focal length of the lens and whose axis coincides with the vertical axis of rotation of the camera lens.

The topographic cylindrograph of R. Moessard.—The following-described apparatus has been devised by R. Moessard (commandant du Génie, attaché au service géographique de l'armée), of St. Cyr, France.

The hemicylindrical camera box, fig. 114, rests upon a tripod, with three leveling screws to adjust the verticality of the axis of revolution *aa* of the camera lens *O*, which axis coincides with that of the half cylinder formed by the sensitized surface of the film. The latter may be replaced by a half-cylindrical ground-glass plate.

The camera lens *O* may be rotated by hand about *aa*, using the sight ruler *S* as lever. By viewing the landscape through the sights *PP'* of the lever *S*, the proper timing for the exposure of the different panorama sections may be estimated. The space between the lens O and the frame RR is filled in with a light-tight fabric, allowing full play for the rotating objective O.

The upper surface of the topographic cylindrograph is provided with an azimuth compass C and a pair of cross levels A and B. The bent frame forming the guide for the sensitive film has graduations on the inner edges, which form the margins of each panoramic view.

The divisions of the upper and lower (horizontal) scales correspond to degrees in arc, while the divisions of the vertical sides are graduated to read $\frac{f}{100}$, where $f =$ constant focal length of the lens $O =$ radius of the cylindrical sensitive surface of the film.

Four movable indices are provided, two, H and H', fig. 115, serve to mark the horizon line of the half panorama, and the other two, N and E, indicate the magnetic north-and-south line and the magnetic east-and-west line for each half panorama, the compass C, with the sight ruler S, giving the means for properly setting the index marks N and E for each view. Thus the magnetic azimuths of horizontal directions may be taken directly from the pictures.

The vertical angles are readily found by means of the ordinates of pictured points (above or below the horizon line HH') measured in one-hundredths of the focal length f, using the photographed scales on the vertical margins of the pictures for this purpose.

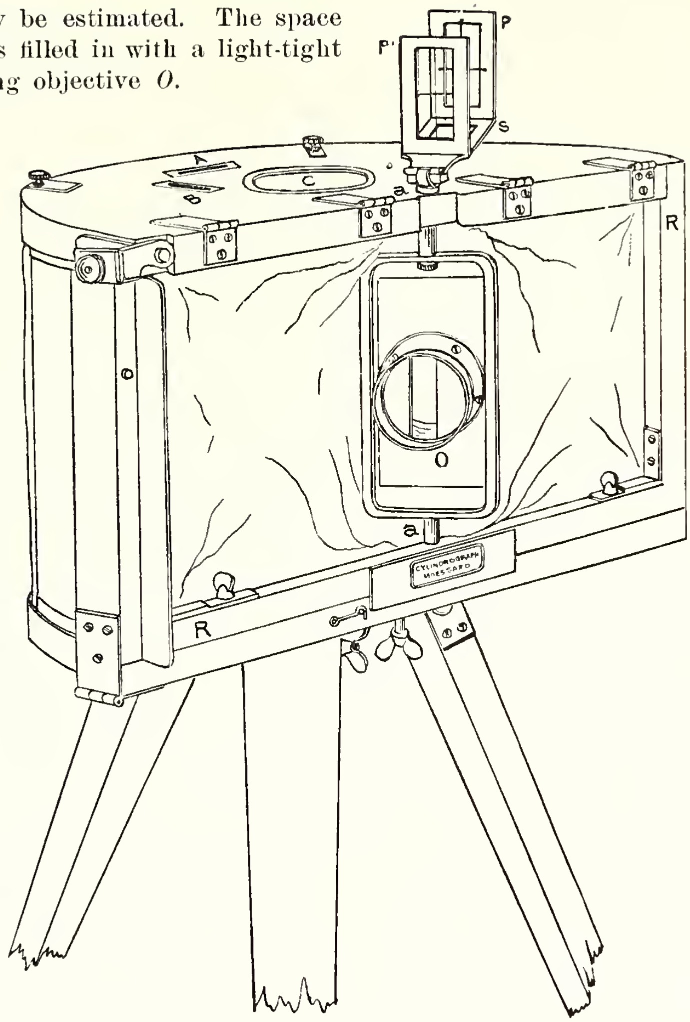


FIG 114.

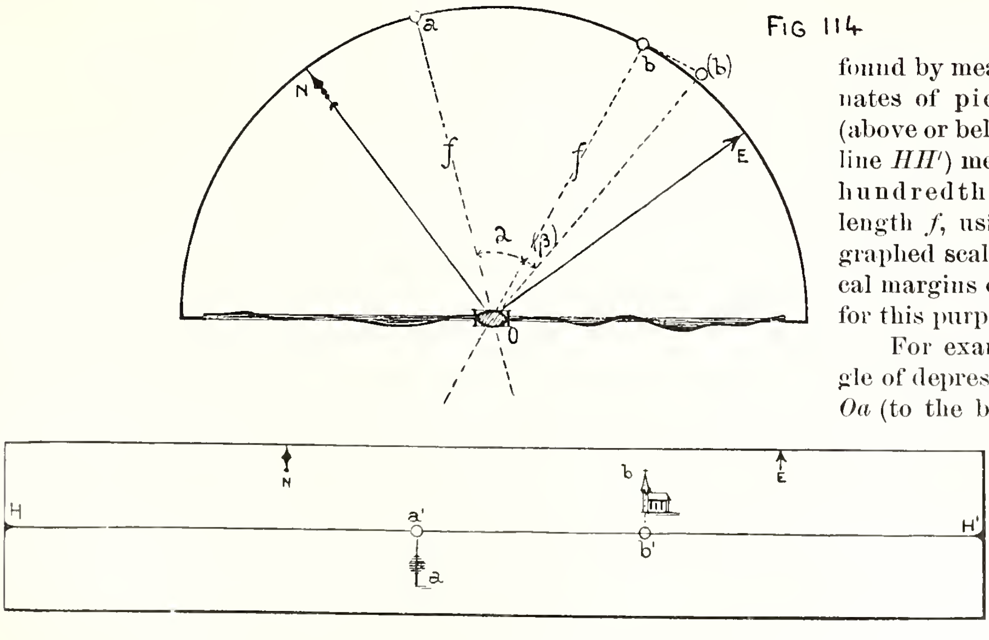


FIG. 115

For example: The angle of depression of the ray Oa (to the base of the pictured tree a), fig. 115, may be found from

$$\tan \beta = \frac{aa'}{f}$$

or when aa', measured on the side scale, is found to be equal to 25 divisional parts:

$$\tan \beta = \frac{aa'}{100} = 0.25$$

To determine whether the levels A and B, fig. 114, read zero when the cylindrical film is vertical, and also to ascertain whether the index marks H and H', fig. 115, representing the horizon line, are correctly placed, we may proceed as follows:

A theodolite, fig. 116, is set up about 10 or 15 metres behind the cylindrograph (after the back of the camera had been removed to bring the index marks H and H' into view), and both instruments are leveled. After bisecting the upper edge of the cylindrograph the telescope of the theodolite is moved in azimuth, when the bisection should continue. The same should be the case for the lower surface edge of the cylindrograph after depressing the telescope of the theodolite to bisect that edge. Does this not take place, the cylindrograph will have to be adjusted by means of the leveling screws until the bisection takes place, when the level A is to be changed to read zero for this position of the cylindrograph.

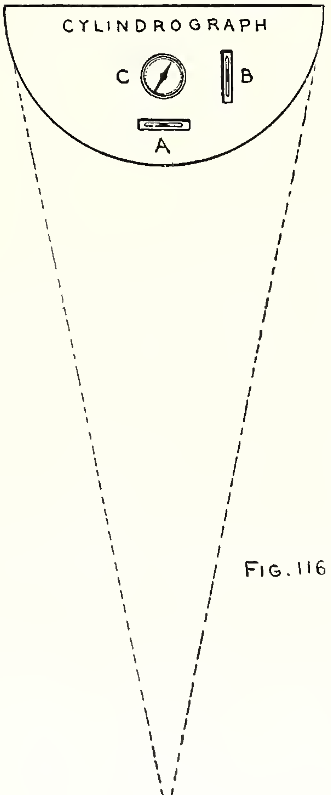


FIG. 116

The theodolite is now set up in the direction of the level A, at one side of the cylindrograph, and the level B is adjusted in the same manner as just indicated for A.

To adjust the indices H and H' into the horizontal plane (containing the optical axis of the adjusted cylindrograph) a comparison may be made on a cylindrograph picture, showing several points of known elevations, the elevation of the cylindrograph being also known, or the theodolite may be set up with the horizontal telescope at the same elevation with the optical axis of the adjusted cylindrograph. The horizontal telescope of the theodolite is now moved in azimuth until a well-defined point is bisected, which point may be identified on the ground glass of the cylindrograph. The image of this point on the ground glass is marked and the cylindrograph is moved in azimuth, marking the image on the ground glass in two more places. A (horizontal) line passing through these marked points should pass through H and H'.

The objective O is attached to a funnel-shaped box within the camera, permitting the simultaneous exposure of a vertical strip of film having a width of but 62 millimetres. Points of the film that would be pictured outside of this strip can not be acted upon by the light unless the objective is revolved about the axis aa.

After the time needed for the correct exposure of this strip (of 62 millimetres width) has been ascertained, the correct exposure may be given the entire semicylinder by moving the sight ruler S with a quick and uniform motion about aa from one extreme end of the film to the other.

The semicylindrical film being 860 millimetres long, each strip of the film would then have been exposed the sixty-two eight hundred and sixtieth part of the time required to make one full revolution of the objective. If one complete revolution required ten seconds, and if the correct exposure for the strip was found to be five seconds, each strip would have received an exposure of $\frac{10 \times 62}{860}$ seconds $= 0.72$ second. To give each strip the required exposure of five seconds the entire revolution of the lens should be repeated $\frac{5}{0{\cdot}72}$ times in succession, or about seven times, each complete revolution taking ten seconds.

As yet these instruments are not made sufficiently precise to be recommended for phototopographic surveys. The conception of this instrument, however, is ingenious, and where the question of transportation need not be considered the topographic cylindrograph in a more perfected form may give good results for surveying purposes.

CHAPTER V.

ICONOMETERS AND PERSPECTOGRAPHS.

We understand under iconometers a series of instruments that have been devised to simplify the constructions of phototopographic platting (iconometry).

After two drawing boards have been covered with paper (gummed down on the edges) both sheets are provided with a chart projection upon which all trigonometric (triangulation) points are platted and their elevations inscribed.

The constructions incidental to the iconometric platting of the phototopographic survey may be divided into three classes:

First. The platting of all horizontal directions, that had been observed instrumentally, for the location of the camera stations and for the orientation of the panorama views.

Second. The determination of the horizontal projection of points pictured on three or more photographs taken from different stations.

Third. The determination of the elevations of the various camera stations and tertiary points (that are located iconometrically) to facilitate the platting of the horizontal contours of the terrene.

The principal instruments used for the iconometric platting of the phototopographic survey in Italy have been described in Appendix No. 3 of the United States Coast and Geodetic Survey Report for 1893. They are:

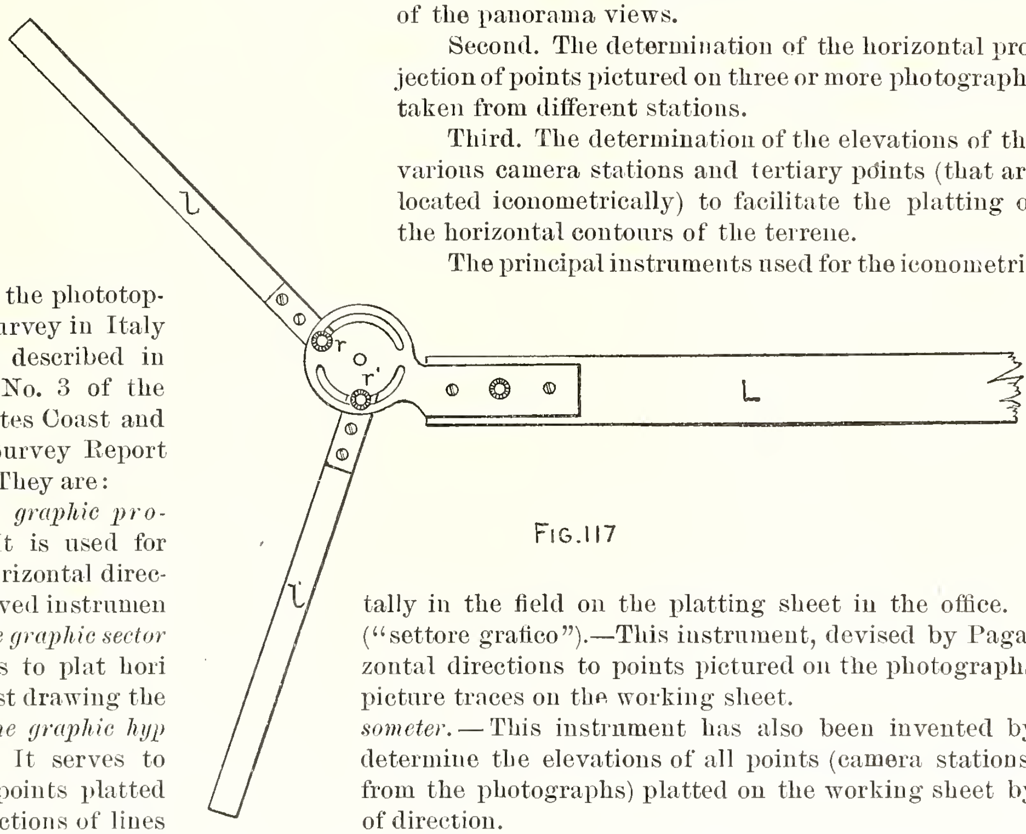


FIG. 117

I. *The graphic protractor.*—It is used for platting horizontal directions observed instrumen tally in the field on the platting sheet in the office.

II. *The graphic sector* ("settore grafico").—This instrument, devised by Paganini, serves to plat hori zontal directions to points pictured on the photographs without first drawing the picture traces on the working sheet.

III. *The graphic hyp someter.*—This instrument has also been invented by Paganini. It serves to determine the elevations of all points (camera stations, as well as points platted from the photographs) platted on the working sheet by the intersections of lines of direction.

IV. *The centrolinead.*—Reference has been made to this instrument under the description of the Canadian photograph board. Captain Deville uses this instrument for drawing lines to a vanishing point falling outside of the limits of the platting sheet.

The distance between the principal point and the vanishing points of lines increases the nearer parallel to the picture plane such lines are. Lines parallel with the picture plane have their vanishing point at infinite distance from the principal point; practically they have no vanishing point. Their perspectives are parallel with the original lines.

It often occurs in iconometric platting that the vanishing points of some lines fall outside of the limits of the drawing board, and, in order to draw a line which, if produced, would pass through the distant vanishing point, special constructions would have to be made to locate the direction of such a line.

This instrument, fig. 117, is used instead of making such auxiliary constructions on the photograph board. It is composed of a wooden straightedge, *L*, and two wooden movable arms,

l and l', which may be given any inclination against the straightedge L. The clamp screws, r and r', serve to fix the arms l and l' permanently in any position.

The photograph board, fig. 70, is provided with four points, A, B, C, and E, indicating the centers of the studs against which the arms l and l' play or rest when the centrolinead is used on the photograph board. The distance between the studs may vary, but each two forming a pair are generally placed from 6 to 8 inches apart, and, the arms of the centrolinead being held in contact with the studs, the various directions of the ruler L will intersect each other in one common point.

With reference to fig. 118 we have:

A and B = one pair of studs permanently fixed upon the photograph board. OA and OB = movable arms of the centrolinead, now clamped in the position given them in the figure. OC = ruler of the centrolinead (= L in fig. 117).

If we describe a circle through the three points A, O, and B—the angle AOB remaining constant—the angle AOB will be an angle of the periphery AB for any position given the ruler OC (= L, fig. 117) as long as OA and OB (l and l', fig. 117) remain in contact with the studs A and B. When OC is changed to assume the position $O'C'$ the intersection, V, of the two lines OC and $O'C'$ will also be on the periphery of the circle because the angle AOV ($AO'V$) remains the same and must subtend the same arc AV as long as the studs A and B remain unchanged. Hence, for the assumed position of the studs the directions of all lines drawn along the fiducial edge of the ruler OC (giving O all positions on the arc AOB) will pass through the point V—they will vanish at V.

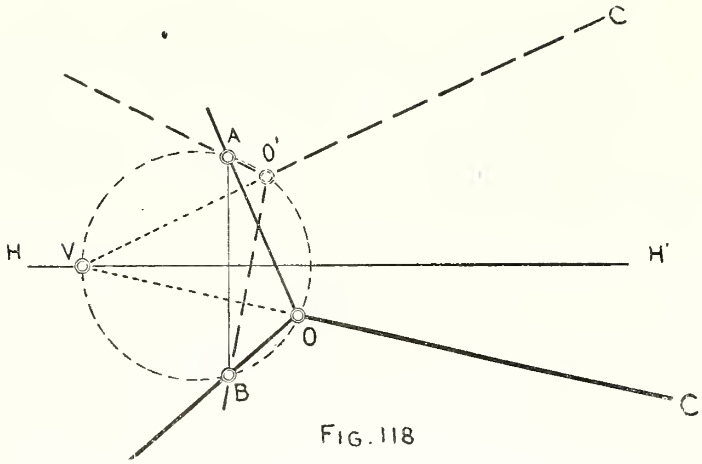


FIG. 118

In the iconometric work of the Canadian surveys the centrolinead is used only for drawing the perspectives of horizontal lines, their vanishing points being on the horizon line. The studs A and B are placed on the photograph board on a line AB, perpendicular to the horizon line and at equal distances from the latter. The horizon line HH' (DD' in fig. 70) becomes a diameter of the circle $AOBV$, and $VA = VB$. If the movable arms of the centrolinead include the same angles with the direction of the fiducial edge of the straightedge, the line OC, bisecting the angle AOB, must pass through V midway between A and B.

The distance of the vanishing point, V, from the principal point, P, may be varied at pleasure by changing the inclination of the arms, l and l', against the direction of the fiducial edge of the ruler L. When the direction of the arms l and l' falls together and is perpendicular to L, the vanishing point will fall at infinite distance from the principal point P and the lines drawn along the fiducial edge of the straightedge L will become parallel with the horizon line HH'.

The distance of the vanishing point V from P may also be varied by changing the distance between the studs A and B or C and E, fig 70—increasing this distance will enlarge the circle $AOBV$ and V moves farther off from P, reducing that distance will decrease the diameter of the circle $AOBV$ and V will approach the principal point P. The practice in Canada, however, is to retain the position of the studs unchanged on the photograph board and to change the inclination of the arms l and l' of the centrolinead instead.

If we gradually close the arms l and l', V will approach the line AB and when the angle AOB becomes equal to 90° the arc AOB will have become a semicircle, and the intersection of AB with HH' will be the center of the circle $AOBV$, the distance of both O and V from AB will be equal to $\frac{AB}{2}$, continuing to close the arms l and l', V will approach closer to AB without ever reaching it.

(1) *To set the arms (l and l') of the centrolinead if the direction to the vanishing point (V) is given by a line in the ground plan.*

With reference to fig. 119 we have:

P = principal point on the photograph board. A and B = positions of the studs. Sv = given direction of the line on the ground plan, when V will be the vanishing point for that line.

We revolve the picture plane about the horizon line, as axis, into the horizontal platting plane when the station may fall in S, fig. 119, SP being then the distance line or focal length projected into horizontal plan. Should the point V fall upon the drawing board we could describe a circle through AB and V and place the fiducial edge of the centrolinead's straight-edge upon DP (upon the horizon line) with the axis of rotation O of the arms l and l' in D upon the circle, then bring the arms l and l' into contact with the studs A and B and clamp them in this position. Still, in this case there would be no use for the centrolinead, the point V being accessible.

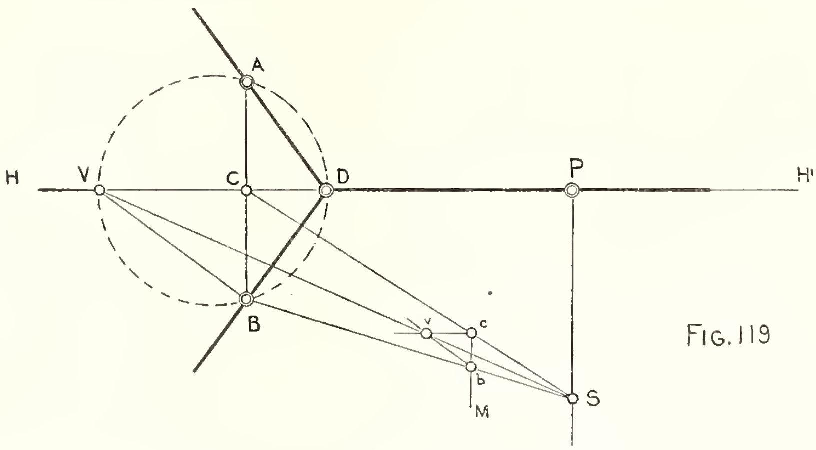


FIG. 119

To set the arms for an inaccessible point V we again refer to fig. 119. Join the points V and B, the angle VDB—the inclination of the lower arm l' against the ruler L—is equal to VBA, both angles subtending equal arcs of the same circle. Draw the lines CS and BS. At any point c on CS draw cM and cv parallel to AB and DP and join b and v. By reason of similarity of triangles, vb must be parallel to VB and the angle

$$vbc = VBC = BDV.$$

Hence, the arms of the centrolinead may be set in the case under consideration by placing the ruler L on Mb, the axis of rotation, O, coinciding with b, and adjusting the lower arm l' of the centrolinead to coincide with bv. The other arm l, having the same inclination against the ruler L as the arm l', may be set by placing the ruler L upon the horizon line DP and moving it along this line until the lower adjusted arm l' comes into contact with the stud B, then moving the other arm l about O until it comes into contact with the stud A and clamping it also.

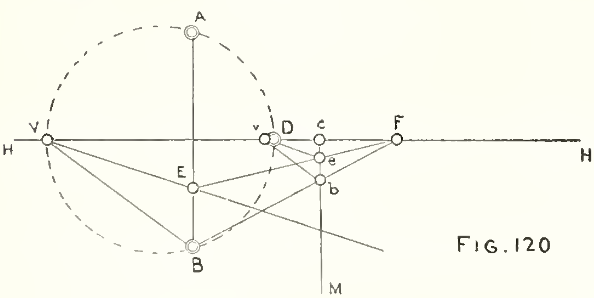


FIG. 120

The lines BS, CS, Mc, and cv are drawn once for all upon the photograph board, fig. 70. The only line to be drawn for setting the arms of the centrolinead is Sv, which is the direction of the given line on the ground plan. The line bv need not be drawn, the points b and v being located by drawing cv parallel with the horizon line and cM or cb parallel with the distance line SP.

(2) *To set the arms of the centrolinead, if the given line VE belongs to the perspective:*

Take any point F, fig. 120, on the horizon line, join F with E and F with B, then draw eM parallel to AB. Through e draw ev parallel to EV and join vb. Owing to the similarity of triangles vb will be parallel with VB and the angle $vbe = VBA$, which is the inclination of the arm against the ruler L of the centrolinead.

FB and eM are permanently drawn on the photograph board, but FE and ve will have to be drawn for every given line. In this case two lines will have to be drawn instead of one, as in the preceding case.

Centrolineads are usually sold in pairs; one serves to work on the left side of the principal point and the other on the right side.

V. The perspectometer (as used by Capt. E. Deville).—The perspectometer is used to dispense with the construction of the squares on the perspective when applying the "method of squares" (Chapter I, Paragraph IX) to draw a figure in the ground plan by means of its perspective.

On a thin, transparent film (glass, xylonite, isinglass, horn, etc.,) two parallel lines AB and DD', fig. 121, are drawn intersecting the common perpendicular pP. Make $DP=PD'=pA=pB=$ distance line (focal length) and from p lay off on AB (to both sides of p) equal distances:

$$pm=mn=no \;\ldots\ldots\; =pm'=m'n'=n'o'= \;\ldots\ldots$$

Join these points of division to P and draw lines through the corresponding intersections of the radials from P with the perpendiculars AD and BD', rr', tt' . . . . . . which lines will be parallel with AB and DD'.

The use of the perspectometer.—The perspectometer is placed upon a perspective with P on the principal point and DD' coinciding with the horizon line. The ground line of the perspective may fall in XY, fig. 121, it will be divided into equal parts by the radials from P, and the trapezoids of the perspectometer represent the perspectives of the squares in the ground plane having the equal parts on XY as sides.

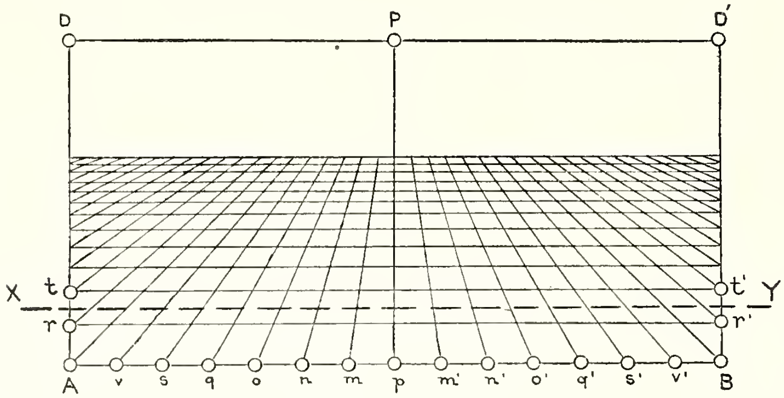


FIG. 121

By placing the perspectometer on the perspective in the manner indicated above the squares covering the perspective of the figure that is to be platted iconometrically on the ground plan are at once apparent, and only those required for the drawing of the figure in question are drawn on the ground plan.

The sides of the squares to be drawn on the ground plan (their side lengths are equal to the divisions on the ground line between the radials drawn from P) are laid off from the trace of the principal plane on the ground line, and the position of the front line nearest the picture trace (or ground line) is laid off on the ground plan either by estimation or construction. The estimation of the position of this line (corresponding to tt') on the ground plan is made by noting the fraction of a square's side which represents the distance (between tt' and XY, fig. 121) from the ground line on the perspective.

The same perspectometer serves only for perspectives which have the same distance line (like photographs of distant objects taken with the same lens), different distance lines requiring different perspectometers.

The width $p\ P$ should be equal to the height of the horizon line above the foot of the picture; the radials from P need not extend beyond the width of the picture, the distance points D and D' having been taken as the limit of the perspectometer in the figure (121) merely to show more fully the principles involved in its construction.

The length of a single division on the line AB should be selected with reference to the resulting equal division lengths of the lowest ground line used for the pictures, as the dimensions of the division lengths on the latter give the measure for the sides of the squares to be drawn on the ground plan.

These division lengths on the ground line should be in harmony with the scale of the plan and with the degree of accuracy that may be required for the delineation of the topographic features. The smaller the size of the squares is selected on the ground plan the more accurately the transfer of the figure from its perspective to the ground plan may be made, the same principles being involved in this method of iconometric platting as in the well-known method of reducing drawings by means of two sets of (hair) squares, the ratio of their sides corresponding to the scale of the required reduction.

Captain Deville recommends the perspectometer to be made by first drawing it on paper in a fairly large scale, and then making a negative of it, reduced photographically to the desired size of the finished perspectometer. A positive copy may now be made on a transparency plate, which, if bleached in a solution of bichloride of mercury, will show white lines on clear glass. For the sake of better preservation such perspectometer, when completely dry and hard, should be varnished.

When using the perspectometer for transferring figures from their perspectives to the ground plan, when such figures are situated in planes perpendicular to the picture plane but inclined against the horizon plane, the center of the perspectometer is placed upon the principal point P of the picture plane, the same as before, but the perspectometer is now revolved about P until the parallel lines of the same are parallel with the trace of the inclined (figure's) plane on the picture plane. In this case the trapezoids of the perspectometer represent the perspective of a net of squares situated in the inclined plane, the squares of which are now to be projected into the ground plane.

This net of squares in the inclined plane, when projected into the ground plane, will be composed of rectangular figures of equal size, their long sides being in a direction at right angles to the picture trace (or ground line) and of a length equal to that which is intercepted between two adjoining radials of the perspectometer on the trace of the inclined plane (on the picture plane), while the short sides of those rectangles (forming the projection in the ground plan of the squares in the inclined figure's plane) will be equal to the lengths obtained on the ground line by projecting the points of intersection of the radials of the perspectometer with the inclined plane's trace on the picture plane upon the ground line of the picture plane.

The construction of the rectangular net on the ground plan may now be made in an analogous manner to that mentioned for the squares, and the drawing in of the figure on the ground plan with reference to its position within the trapezoids of the perspectometer is accomplished in the usual manner.

Should the figures be situated in planes that are inclined to both the picture and the ground planes, then the figure is first projected upon a plane perpendicular to the picture plane, and having the same trace in the latter as the inclined plane.

VI. The perspectograph.—Numerous instruments have been devised for drawing perspectives from plans or from nature, mechanically, or by means of optical devices, some of which may inversely become of use for transcribing perspectives of figures into orthogonal projections.

The perspectograph, invented by H. Ritter, serves to construct the orthogonal projection of a plane figure from its perspective, or to draw the perspective from the plans of the object without referring to the object itself.

Ritter's instrument, manufactured by C. Schrœder & Co., in Frankfort-on-the-Main, has been patented in Germany, October 13, 1883, under No. 29002. It was devised primarily for architectural purposes.

This instrument in its present form, composed largely of wood, is not well suited for surveying purposes, as it contains too many sources of error due to lost motion in its bearings, still, its theory being sound, there is no reason to question its ultimate value, even for precise work, if it were carefully made by an expert mechanician (excluding the use of wood and using metal throughout), being guided in its construction by the demands of the greatest precision attainable. As a carefully constructed instrument based on the present pattern may become useful in platting the data of a topographic reconnaissance where, in the nature of the work, rapidity in making the results practically available is of greater importance than a high degree of accuracy, the following description of this instrument may not be out of place here. For its methods of use in phototopographic surveying we respectfully refer to Capt. E. Deville's work on "Photographic Surveying" already mentioned.

We have seen (Chapter I) that the platted position of a point in the ground plan may be found from its perspective (in vertical plane) by locating the point of intersection of the horizontal projection of the ray: "station—pictured point" with the line of direction itself. (The latter with its vertical plane is revolved about the trace of the vertical plane in the ground plane (as axis of rotation) into the ground plane in which plane the point of intersection is located.)

With reference to fig. 122 we have:

S = camera station or point of view. μ = perspective (image) of a point M, to be platted in the ground plan. s = foot of the station S. XY = ground line of the picture plane (vertical) MN. M = platted position of the point M in the ground plane GG.

If we draw through the foot of the station s a line parallel to the ground line XY, and make its length $s\ (S)$, equal to sS, join (S) and the platted point M, then it will follow from the similarity of the triangles $O\mu M$ and sSM that:

$$sS : O\mu = Ms : MO$$

From the similar triangles $s(S)M$ and $O(\mu)M$ we find

$$s(S) : O(\mu) = Ms : MO$$

hence

$$s(S) : O(\mu) = sS : O\mu$$

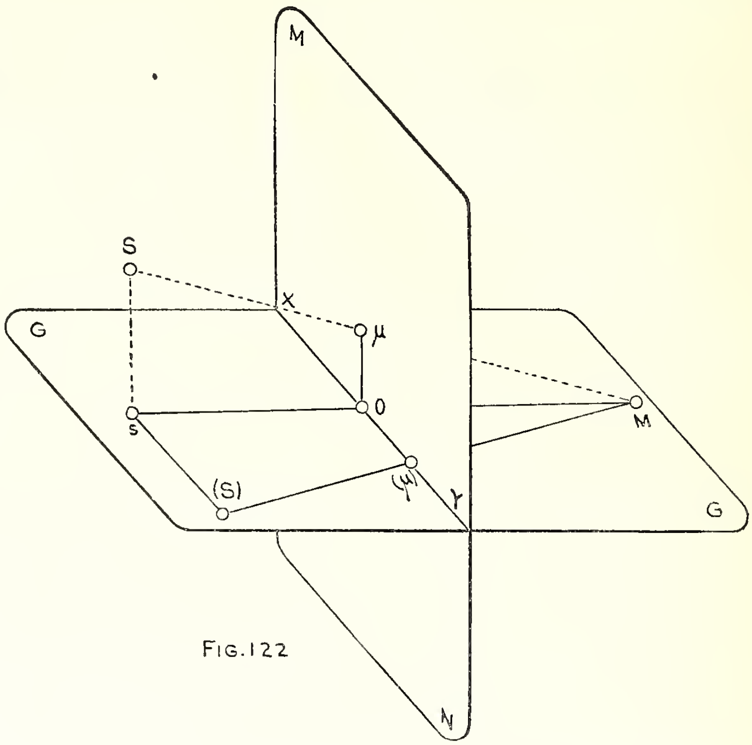
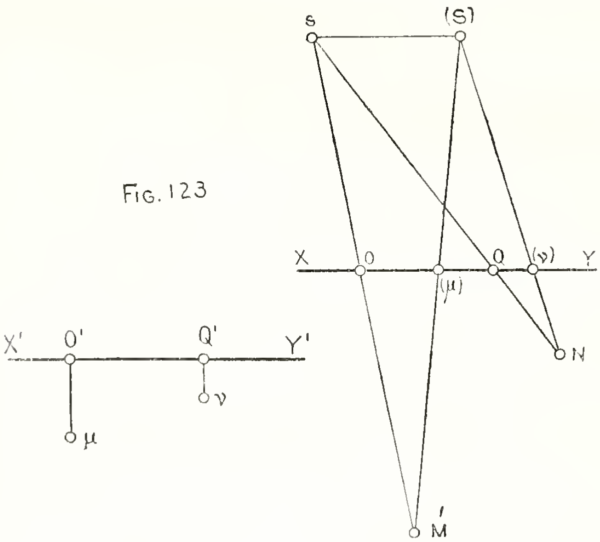


FIG. 122

Having made $sS = s(S)$, the last equation can only prevail if $O\mu = O(\mu)$.

To find, therefore, the perspective μ of a point M, given on the ground plan, we first draw a line $s\ (S)$ through the platted station in the ground plane parallel with the ground line XY, making $s(S)$ = height of the station S above the ground plane. Draw the lines sM and $(S)M$, which will intersect the ground line XY, in O and (μ), fig. 123. On the ground line $X'Y'$, drawn in another place of the working sheet, we assume a point O', representing O of the ground plan, and erect $o\mu$ perpendicular to $X'\ Y'$ in O' and make $O'\mu = O(\mu)$, when μ will be the perspective of M in the reverse position of the perspective. The perspective of any other point, N, given on the ground plan may be found in the same way, making $O'Q' = OQ$ and $Q'v = Q(v)$.

FIG. 123



Ritter devised the perspectograph to perform this construction, illustrated in fig. 123, mechanically.

Fig. 124 illustrates the general arrangement of Ritter's perspectograph. sM and $(S)M$ = two slotted wooden arms carrying the tracer, M, at their point of intersection. The connections at s, O, (S), and (μ) are such that the rulers sM and $(S)M$ may slide through these points. The slide connections, s and (S), may also be moved along the groove or slot of the wooden ruler RT. The sliding piece O is secured to a rod which in turn may slide in the groove of the wooden ruler XY, being connected at its other end D with a system of arms or levers joined together after the manner of a pantograph. The distance OD is maintained unchanged while the instrument is in use.

The center of *s* is placed directly upon the point that marks the platted camera station on the ground plan. The ruler *RT* is placed parallel to the ground line of the picture plane, and *s* and *RT* are now secured in this position on the ground plan.

When the arm *sM* is moved, *s* being held in a fixed position (to coincide with the platted station point), the point *O* will follow the motions of the arm *sM*, also applying its motion directly to the arm *OD* (which slides in the groove of *XY*) and indirectly to the arms of the pantograph system.

The fourth sliding piece (μ), being connected with the joint *A* of the pantograph system by means of a separate piece, insures a permanently fixed distance between (μ) and *A* while the instrument is in use.

The pantograph system is composed of six pieces: Four straight arms, *AB*, *AC*, $F\mu$, and $F\mu'$ and two double arms or levers, *CDE* and *BDG*, which are bent at right angles at their points of junction *D*. The sides of the two parallelograms *ABCD* and *DGFE* are all of equal lengths, and the six arms are joined in *A*, *B*, *C*, *D*, *E*, *F*, and *G*. The arms $F\mu$ and $F\mu'$ are twice as long as the length of the side of the parallelograms.

The pencil which describes the perspective may be attached to the free end of either arm $F\mu$ or $F\mu'$.

The angles *GDB* and *EDC* being each equal to 90°, the sum of the two other angles *CDB*

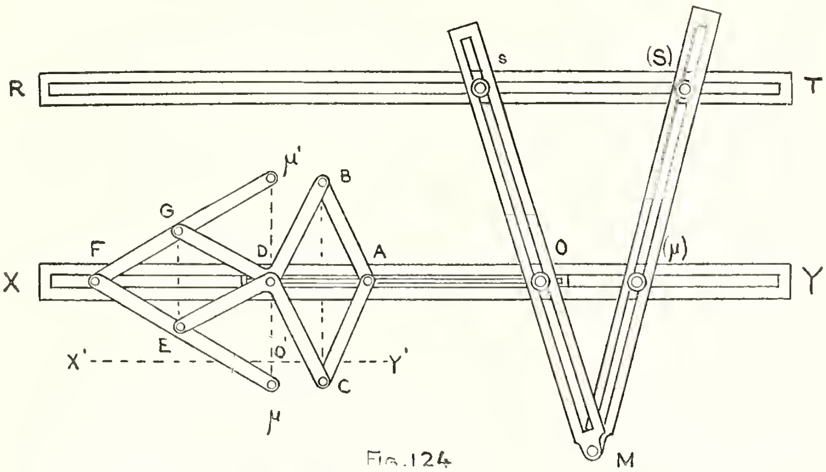


Fig. 124

and *GDE* must be equal to 180°, and as the sum of two adjacent angles in a parallelogram is equal to 180°, it follows that

$$CDB + GDE = CDB + DCA$$

or:

$$GDE = DCA$$

This shows that the two parallelograms *FGDE* and *CDBA* must be equiangular, and as their sides are equal in length, the parallelograms themselves must be equal, and the diagonals *FD* and *GE* of the one are equal to *BC* and *AD* of the other, respectively.

The two long arms $F\mu'$ and $F\mu$ being of equal lengths, $\mu\mu'$ will be parallel to *GE*, both will be perpendicular to the direction of *XY*, and $\mu\mu'$ will pass through *D*. We have, therefore, $D\mu' = D\mu = GE = DA$.

Use of the perspectograph.—The sliding piece *s* is secured to the working board, over the platted position of the camera station on the ground plan, still permitting a gliding movement of the arm *sM* in the direction *sM* (fig. 124). The center line of *RT* is brought into a position parallel to the platted ground line, and its position is also secured to the board. The sliding piece (*S*), finally, is moved from *s* (in the groove of *RT*) until *s* (*S*) is equal to the elevation of the station *S* above the ground plane, also securing (*S*) in this position, when it will still permit a gliding movement of the arm (*S*) *M* in the direction of (*S*) *M*. The center line of the wooden ruler *XY* is placed upon the ground line (picture trace) on the ground plan.

The manipulation of the instrument and its general working will now readily be understood.

For instance, when the tracer M is moved in a direction parallel to RT or to XY, the arm sM will move the slide OD in the same direction. The distance $O\,(\mu)$ remaining unchanged—as long as $s\,(S)$ undergoes no change—$(\mu)\,A$ will also remain of a constant length. Hence, AD and also GE, as well as $D\mu$, undergo no changes, and the pencil in μ or in μ' will trace a line parallel to XY, representing the perspective of a line of the ground plan (the one traced by M) parallel to the picture plane.

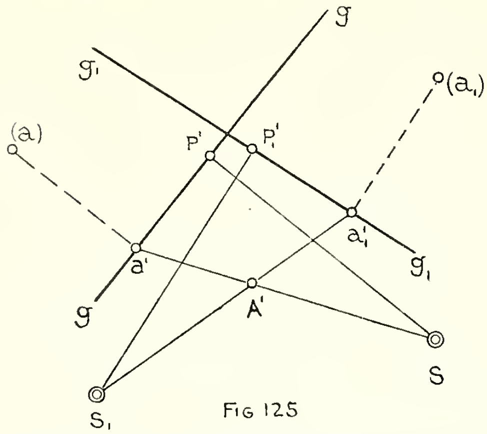


Fig 125

When M is moved in the direction of sM, away from XY, the positions of O and D remain the same, but $O\mu$ will be lengthened, (μ) moves to the right, or away from O, carrying the point A with it ($A\,(\mu)$ being a constant length) and increasing the length of the diagonal DA in proportion to the increase of the length of $O\,(\mu)$. DA being equal to $GE = D\mu$ $(D\mu')$, the latter will also be lengthened, and μ will be moved down, or away from XY, by the same amount as (μ) is moved to the right. The relation between the construction made in fig. 123 and the mechanical platting by means of the perspectograph will now be evident.

VII. Professor Hauck's trikolograph.—This instrument has been described by Dr. G. Hauck in a memorial commemorating the opening of the new building of the Royal Technical High School at Charlottenburg, near Berlin, November 2, 1884. It serves to reconstruct an object from two perspectives of the same that had been obtained from two different points of view.

The principles which underlie the construction of this instrument hold equally good for the construction of an instrument to be used for the mechanical platting of the ground plan of any object represented on two photographs obtained from different stations.

In 1887, Prof. F. Schiffner already suggested the changes to be made to Dr. Hauck's instrument, in order to render it available as an instrument of precision for the use of the phototopographer; still it seems that mechanical difficulties in its manufacture are yet to be overcome, as the writer has not met with any record of such a perfected instrument having been either in use or even been constructed.

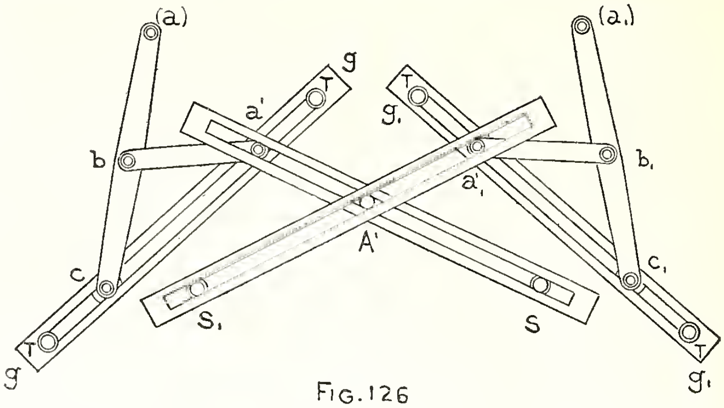


Fig.126

In Chapter I it had been shown that a point, A, photographed from two stations S and S_1, may be platted in horizontal plan, if the two picture traces, gg and g_1g_1, and the two camera stations, S and S_1, are given on the horizontal plan, fig. 125.

The two picture planes may be revolved about their ground lines, gg and g_1g_1, into the ground or platting plane, when (a) and (a_1) will be the two images of the point, A, revolved into the ground plane. If we draw lines through (a) and (a_1) perpendicular to the corresponding ground lines gg and g_1g_1, then a' and a_1' will be the (horizontal) projections of the picture points, a and a_1, into the platting plane, and the intersection, A', of the radials Sa' and S_1a_1' will locate the positions on the platting sheet of the point A, pictured on the two plates as a and a_1, respectively.

This graphic determination of the platted position A' of the point A may be accomplished mechanically by placing slotted rulers with their center lines upon gg and g_1g_1, fig. 126, and indicating the directions of the perpendiculars, dropped from the pictured points (revolved into the horizontal plan) upon the ground lines, by two arms $(a)\,bc$ and $a'b$ of a pantograph combination, where

$$(a)b = bc = a'b$$

or

$$(a_1)b_1 = b_1c_1 = a'_1b_1$$

The points $(a)a'$ and c will always be situated on the periphery of a semicircle described about b as the center, and, as the points c and a' are permanently held on the line gg, the angle $(a)\,a'c$ (angle of the periphery subtending the semicircle) will be equal to 90° for all inclinations that may be given $(a)c$ against gg. The directions of the radials Sa' are laid down mechanically by means of two slotted rulers Sa' and $S_1a'_1$, held in position by the studs in S and a' (and S_1 and a'_1, respectively), both rulers being revolvable about the fixed points S and S.

This instrument, of which the characteristic features are illustrated in fig. 126, performs the constructions mechanically that were made graphically or geometrically in fig. 125.

The slotted rulers gg and g_1g_1 are secured to the platting board (their center lines on the picture traces) by means of thumb tacks T. The pantograph arms $(a)\,c - (a_1)c_1 -$ and $a'b - (a'_1)\,b_1 -$ are connected with these rulers by means of sliding joints c (and c_1) and a' (and a'_1), while the studs which mark the stations S and S_1 end in cylindrical projections that fit into the slots of the rulers Sa' and $S_1a'_1$, the latter fitting also over similar cylindrical attachments to a' and a'_1, in such a way that the rulers Sa' and $S_1a'_1$ may freely glide over the points S and a' (or S_1 and a'_1) and at the same time may revolve about the fixed points S and S_1, respectively.

The points (a) and (a_1) are provided with tracers, and a pencil slide is attached to the intersection of the rulers Sa' and $S_1a'_1$ (in A') in such a way that the pencil point may freely slide either way in the grooves of Sa' and $S_1a'_1$.

A comparison between the figures Nos. 126 and 127 will plainly show that A' will always represent the platted position of the point A, derived from its two images a and (a_1) (revolved into horizontal plan). Still, it may not always be possible to identify both images of the same point on the two pictures, and, in order to apply Professor Hauck's method to identify the second image (on the second photograph) by means of the so-called "kernelpoints," the instrument shown in fig. 126 should be modified in such a way that the point of the second tracer may always be upon the image (on the second picture) which corresponds to the point designated by the first tracer on the first picture (revolved into the ground plane).

We had seen (Chapter I) that the line connecting the image of any point A on the first picture with the image of the second camera station (with the kernelpoint (s_1), fig. 127)—and the line —connecting the image of the same point A on the second picture with the image of the first camera station (with the kernelpoint (s), fig. 127)—will bisect the same point σ of the line of intersection of the two picture planes.

The picture planes being vertical, this line of intersection will be represented by the vertical line through the point Ω of the ground plane (through the point of intersection of the two picture traces or ground lines gg and g_1g_1). The picture planes having been revolved about their ground lines as axes into the horizontal plane, this line of intersection, $\sigma\Omega$, also revolved into the ground plane (once about gg and once about g_1g_1) will appear twice in the platting plane, once as $\Omega(\sigma)$, perpendicular to gg in Ω, and again as $\Omega(\sigma_1)$, perpendicular to g_1g_1 in Ω.

As the points (σ) and (σ_1) represent the same point σ revolved into the horizontal plane, once about gg and again about g_1g_1 as axes, the lengths $(\sigma)\,\Omega$ and $(\sigma_1)\,\Omega$ must be equal.

In order, therefore, that this instrument, fig. 126, may work in harmony with the principles that underlie Professor Hauck's method, it will have to be modified to fulfill the following conditions:

A line drawn through the kernelpoint s_1, fig. 127, and any point pictured on the first photograph and a line drawn through the kernelpoint s and the image of the same point on the second photograph are to intersect the line of intersection of both picture planes in the same point σ, or the two lines revolved (with the picture planes) into the horizontal plane must bisect the revolved lines $(\sigma)\,\Omega$ and $(\sigma_1)\,\Omega$ (of the line of intersection of the picture planes) in points (σ) and (σ_1), both to be equidistant from Ω.

The complete instrument, in a general way, is represented in fig. 127. The two slotted rulers gg and g_1g_1 of fig. 126 have been supplied with additional arms $\Omega(\sigma)$ and $\Omega(\sigma_1)$, each arm including an angle of 90° with its ruler. These rectangular elbow pieces are secured to the platting board by four thumb tacks T after the rulers $g\Omega$ and $g_1\Omega$ had been placed with their center lines upon the picture traces gg and g_1g_1, respectively, in such a way that the intersections of the center lines of the elbow rulers (at the rectangular elbow ends of the rulers) coincide with the intersection Ω of the ground lines or picture traces gg and g_1g_1.

The pantograph arms, representing the ground lines of the pictures, are attached to the rulers the same as shown in fig. 126. Studs are inserted into the kernelpoints (s_1) and (s), and the arms $\Omega(\sigma)$ and $\Omega(\sigma_1)$ support a ruler $(\sigma)(\sigma_1)$, which may glide freely over these arms of the elbow pieces. To cut off equal lengths by this ruler $(\sigma)(\sigma_1)$ on the elbow arms $\Omega(\sigma)$ and $\Omega(\sigma_1)$, the angle $d(\sigma)e$ is adjustable, and it should be regulated for each set of two picture traces to make:

$$(\sigma)\Omega = (\sigma_1)\Omega$$

When $(\sigma)d$ is moved along the slot of $(\sigma)\Omega$ the slide point (σ_1) will move along $(\sigma_1)\Omega$, $\Omega(\sigma)$ always being equal to $\Omega\ (\sigma_1)$.

The screw d serves to clamp the angle $d(\sigma)e$ for any opening corresponding to the angle

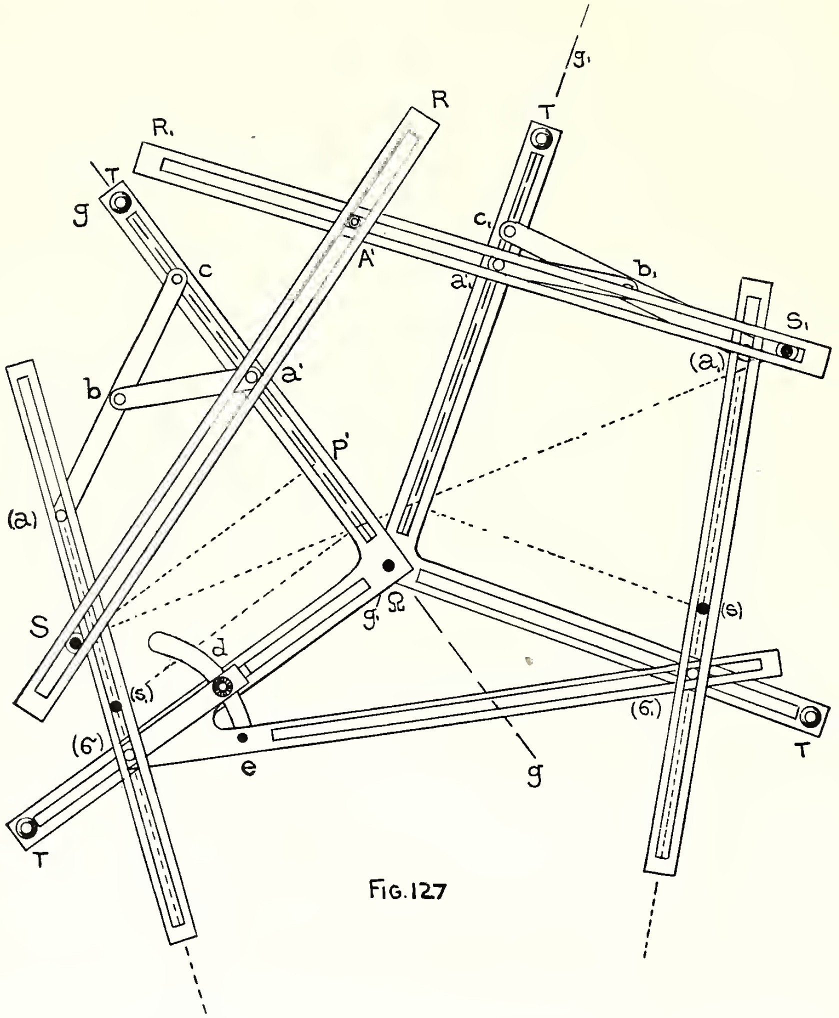


FIG. 127

$g\Omega g_1$ included between the picture traces. Slotted rulers are now placed over the studs that mark the kernelpoints (s_1) and (s), their slots also receiving the cylindrical prolongations of the tracers (a) and (a_1) and those of the slide points (σ) and (σ_1), respectively. To complete the instrument, two slotted rulers RS and R_1S_1 are finally placed over the studs S and S_1 (marking the platted positions of the two stations) and over the sliding joints a' and a' (which are the same as those in fig. 126). At their point of intersection A' the sliding pencil point is inserted (into the slots of these two rulers), which finally completes this instrument as illustrated in fig. 127.

If we now move the tracer (a) on the first photograph, the pantograph arms $(a)\ c$ and ba' will change the position of the ruler SR (into the direction of the radial from S, to the horizontal projection, on the picture trace, of the pictured point designated by the tracer point (a) on the first photograph), and the ruler $(a)\ (s_1)$ is moved, locating the point (σ).

This change in the position of (σ) produces a corresponding change in the sliding point (σ_1), which in turn changes the position of the tracer (a_1), causing the pantograph arms (a_1) c_1 and $b_1a'_1$ to move, and a change in the position of a'_1 will cause the radial ruler R_1S_1 to assume a new position also. The intersection of RS with the new position of R_1S_1 will locate the platted position in horizontal plan of the point under the tracer (a) on the first photograph, without having actually identified the corresponding image of the (same) point under the tracer (a_1) on the second picture.

If a line on either photograph is followed out by one of the tracers (a) or (a_1), the pencil point A' will draw the horizontal projection of the line given in perspective (the second tracer being observed chiefly as a check or to aid the general working of the instrument by a gentle tapping when the movements of the various parts of the instrument are retarded by too much friction or lost motion).

Until now no perfect perspectograph has been constructed, and, no matter how accurately such instruments, like the one just described, may be made by the mechanician, there will always remain some unavoidable imperfections in the material or in the workmanship of the instrument that will produce more or less error in the results.

For accurate and precise work, therefore, the iconometric platting should be accomplished with the aid of graphic or geometrical constructions for all the control work of the survey, using perspective instruments only for filling in such details which, in an instrumental survey of a similar character, would be sketched in by the topographer.