U. S. TREASURY DEPARTMENT

PUBLIC HEALTH SERVICE

Public Health Bulletin No. 218

April 1935

STUDIES IN ILLUMINATION

IV. Daylight in Buildings.—A Study of the Effect of the Height and Width of Windows and of the Reflecting Power of the Walls and Ceiling Upon the Natural Illumination Within a Building

By

JAMES E. IVES, Senior Physicist

FREDERICK L. KNOWLES, Assistant Physicist

and

LEWIS R. THOMPSON, Assistant Surgeon General

U. S. Public Health Service

PREPARED BY DIRECTION OF THE SURGEON GENERAL

UNITED STATES

GOVERNMENT PRINTING OFFICE

WASHINGTON : 1935

For sale by the Superintendent of Documents, Washington, D. C. - - - - - - - - Price 15 cents

CONTENTS

ILLUSTRATIONS

PLATES

FIGURES

TABLES

APPENDIX TABLES

STUDIES IN ILLUMINATION

IV. Daylight in Buildings [1]

1. INTRODUCTION

The fenestration of buildings is an important matter because upon the area and position of windows depends the amount and character of the light under which we live and work. The height, width, location, and orientation of the windows of a room will determine the amount and distribution of the light in the room.

Insufficient or improperly directed light is undoubtedly injurious to our eyes, produces discomfort, and lowers productive efficiency. Abundance of light also promotes cleanliness and a cheerful attitude of mind.

The design of windows in factories, offices, and schools is especially important. In a factory or office, insufficient illumination will, in addition to causing eye strain, materially lower the productive efficiency of the workers. In a school, insufficient or improper fenestration may be a reason for the backwardness of pupils, will cause eye strain, and possibly produce myopia in the younger pupils.

It must be remembered that the fenestration of a building not only affects those occupying the building immediately after its erection, but also those who will occupy it during its whole life.

Until recently little information has been available on the effect of the height, width, location, and orientation of the windows upon the lighting of a room. Also, much of the information that has been published thus far has been based upon measurements made with small models. The present study was made with a full-sized building erected especially for the purpose, actual daylight being the source of the illumination. In this building measurements were made with the ceiling and walls painted a mat white, and also a mat black, so that the effect of the light reflected from the ceiling and walls could be determined. Furthermore, at the same time that measurements of illumination were made within the building, measurements were made of the brightness of the particular portion of the sky producing the illumination within the building. Thus it has been possible for the first time to correlate illumination within an actual building with the brightness of the sky, measured out of doors, producing it, and to

[1] Received for publication Nov. 8, 1934.

reduce the results to the basis of foot-candles per unit of sky brightness. Allowance was also made for the light reflected and absorbed by the glass of the window and, in the final analysis, allowance was made for the light lost by the obstruction of sash bars and casings, and by dirt on the glass.

2. ACKNOWLEDGMENTS

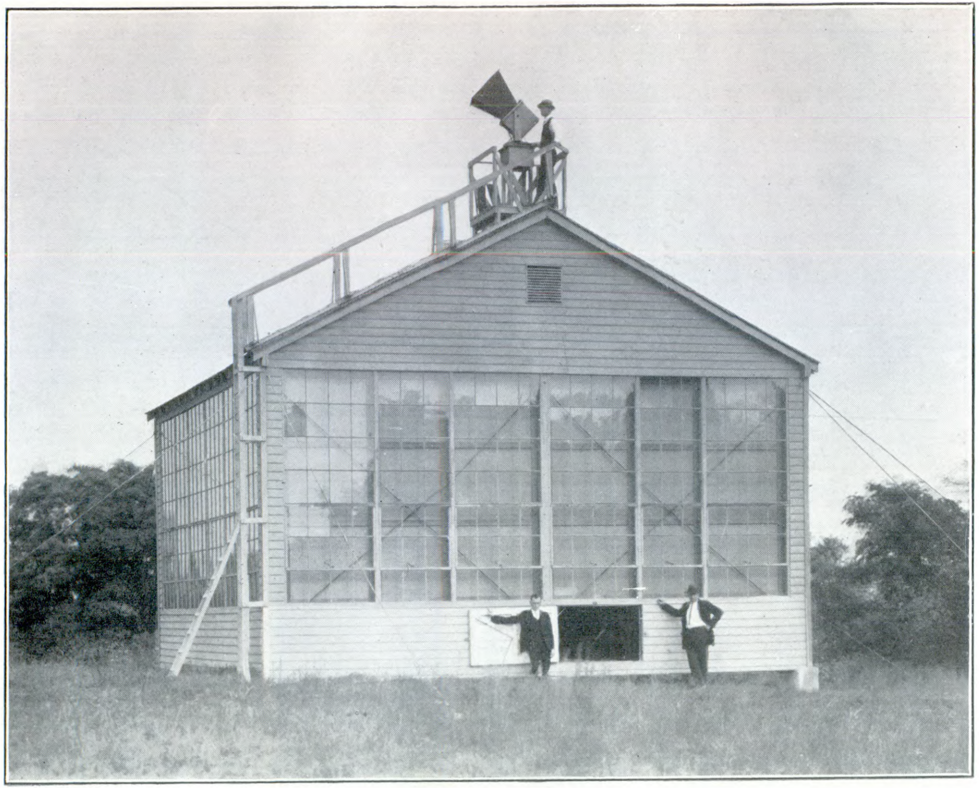
The authors wish to acknowledge their indebtedness to Dr. W. A. Taylor, Chief of the Bureau of Plant Industry of the Department of Agriculture, for permission to erect the experimental building on the Arlington Experiment Farm, to Prof. H. H. Higbie of the University of Michigan, Ann Arbor, Mich., who acted as a consultant during the course of the research, and to Senior Statistician Rollo H. Britten of the United States Public Health Service for assistance rendered in the analysis of the results. The assistance rendered by Mr. J. Harold Link in making the measurements is also acknowledged.

3. DESCRIPTION OF THE EXPERIMENTAL BUILDING

The experimental building in which the studies were made is situated on a knoll on the grounds of the Department of Agriculture Experiment Farm in Arlington, Va., across the Potomac from Washington. The latitude and longitude of the experimental building are 38°53′ N. and 77°04′ W. It is about 3 miles west of the Capitol, and the meterological conditions are essentially the same as in Washington, except that the air in its neighborhood may be somewhat clearer. The site was selected so that the outlook from the buildings would be unobstructed by adjacent buildings and not seriously obstructed by trees. The building is shown in plate I. The trees shown in the picture are much farther from the building than they appear to be, at least 300 feet away, so that they do not have any sensible effect upon the light entering the windows.

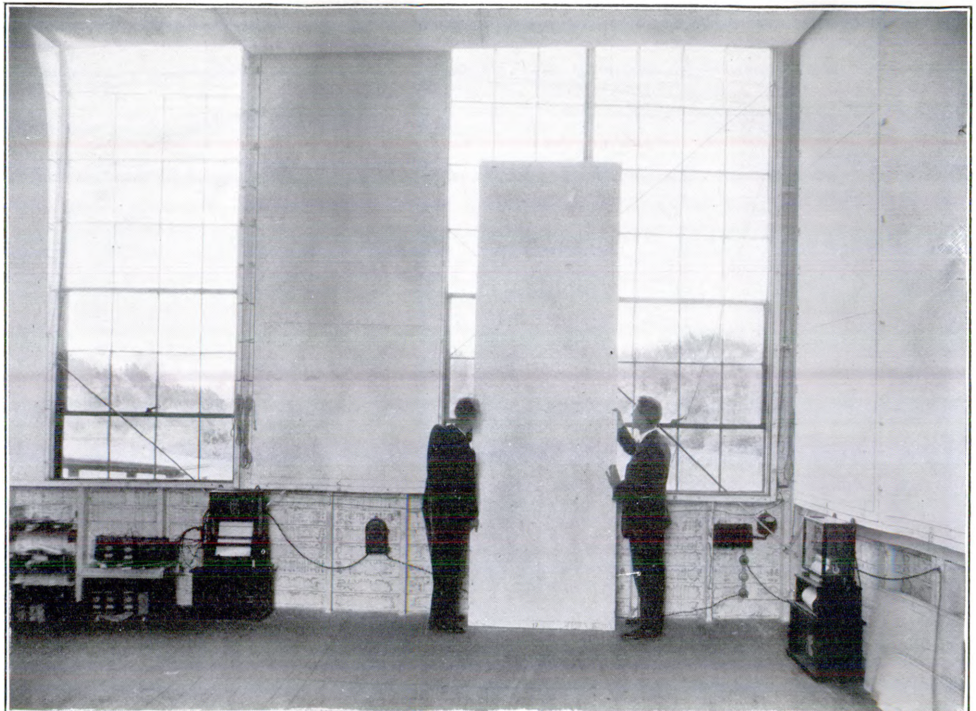
The building is orientated accurately north and south so that the sides face north, west, south, and east, respectively. It is built of wood, except the window sash, which is of steel. The building is 30 feet square and approximately 15 feet from the floor to the eaves. It is braced internally with sixteen ⅝-inch steel rods running diagonally across the windows and externally with eight similar steel rods fastened to the upper corners of the building and anchored to concrete bases at some distance from the building. The steel rods in both cases are equipped with turnbuckles.

The side walls are made up mostly of sections of windows. As will be seen from plate I, each wall contains six window sections placed symmetrically. The window sections are approximately 12 feet high, measured from the sill to the top of the window. The sill is 3 feet 4 inches above the floor. Of the window sections in each wall,



EXPERIMENTAL DAYLIGHT BUILDING.

PLATE II



INTERIOR OF BUILDING SHOWING MOVABLE SHUTTERS AND RECORDING POTENTIOMETERS.

4 are large and 2 are small. The large sections are 4.86 feet wide and the small sections 3.75 feet wide. The large sections are 4 panes wide and the small 3 panes wide. Each wall contains 5 mullions.

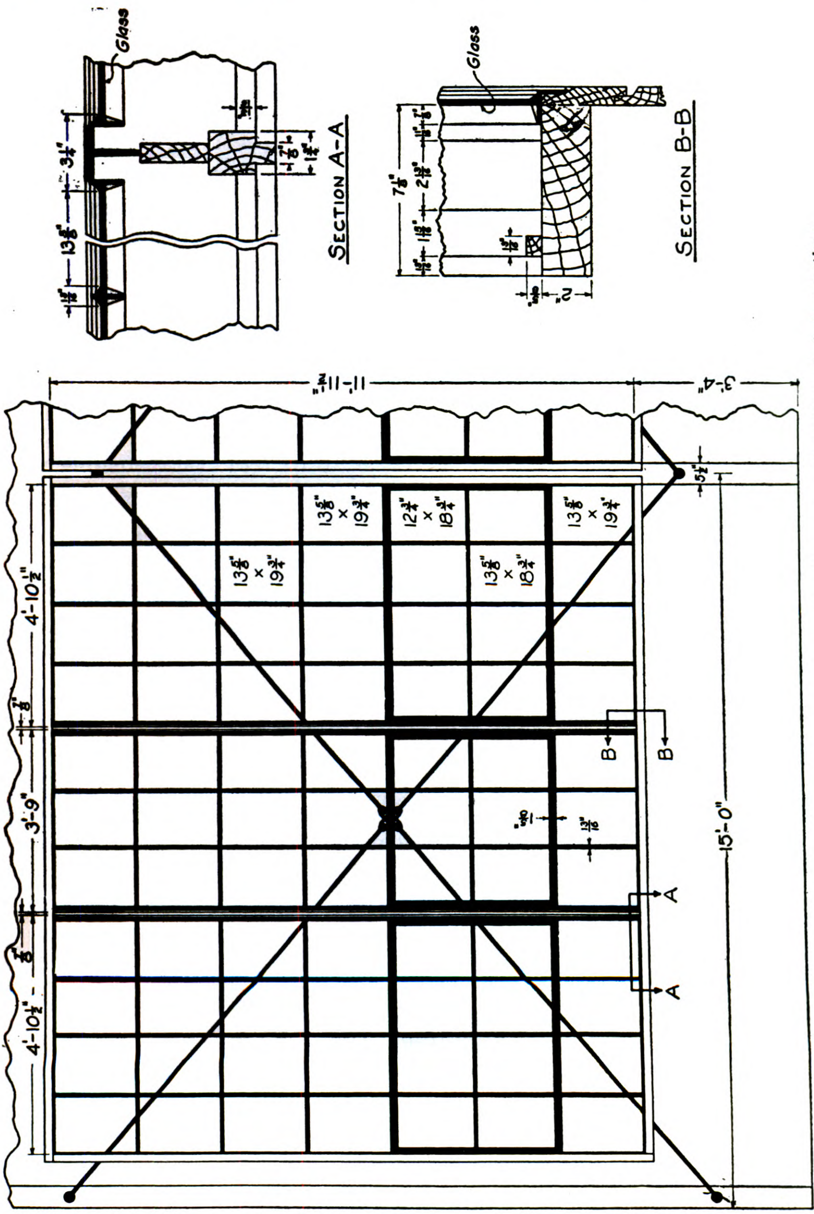


FIGURE 1.—Detailed diagram of one-half of the north wall, showing dimensions of window sections.

The center mullion is 6.7 inches wide and the other mullions 3.25 inches wide. The steel sash of the windows is glazed with "A" quality window glass about 3.2 mm thick. The individual panes measure about 14 by 20 inches. About 14 percent of the area of the window is taken up by the steel sash, about 2 percent by wooden casings, and about 1 percent by the tie rods.

A detailed diagram of one-half of the north wall is shown in figure 1. This diagram shows the glass area of the window and the dimensions of the window sections.

As shown in plate II, the width of the windows was varied by means of shutters which fit into the window sections. The shutters were made of wallboard supported by wooden frames. They were made in different sizes so that any or all of the window sections could be covered. The use of shutters made possible the variation of the window openings, both as to position and size. Boards about 4 inches wide were fitted between the mullions and the shutters to prevent any light entering from the sides.

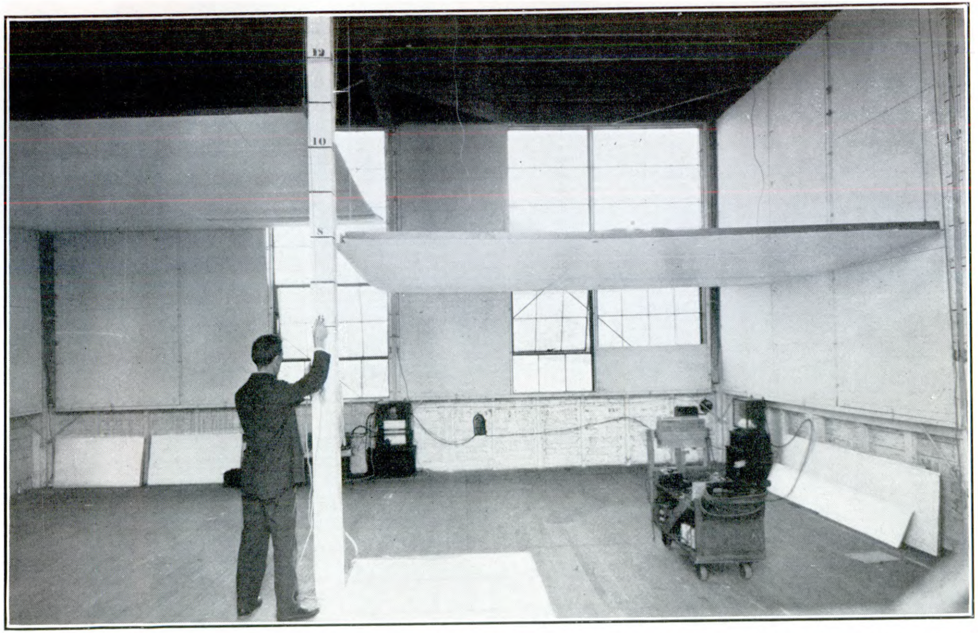
The ceiling was movable and made in four sections, as shown in plate III, each section being made of wallboard supported by a wooden frame. By means of ropes and pulleys, these sections were adjusted for ceiling heights of 9, 12, and 15 feet. Extensions were provided from the edge of the ceiling to the glass of the windows, so that when the height of the ceiling was less than 15 feet, light could not enter the room from the portion of the window above the ceiling.

The ceiling and walls, including the inside of the shutters, of the building were first painted white with a mat water-color paint, having a reflecting power of about 78 percent. After a complete set of measurements of illumination had been made under these conditions, the walls were painted a mat black, having a reflecting power of about 4 percent. The series of measurements were then repeated. After their completion the ceiling was painted black with the same paint as the walls, and the series of measurements were again repeated. In this way it was possible to determine the effect of the reflection of light from the walls and ceiling. The coefficient of reflection of the floor, when clean, varied from 12 to 17 percent, the average being about 13 percent.

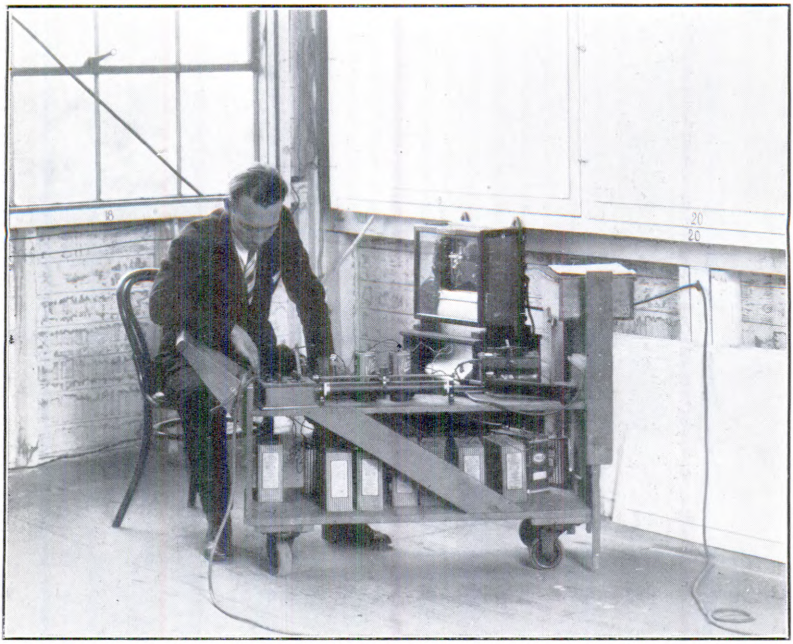
4. MEASUREMENTS OF ILLUMINATION

Measurements of the illumination within the building were made on a horizontal plane 36 inches above the floor at 36 stations. The positions of these stations were marked on the floor and are shown in the floor diagram given in figure 2. The stations are 6 feet apart, except near the wall where there is a row 2 feet from the wall, and 4 feet from the adjacent row.

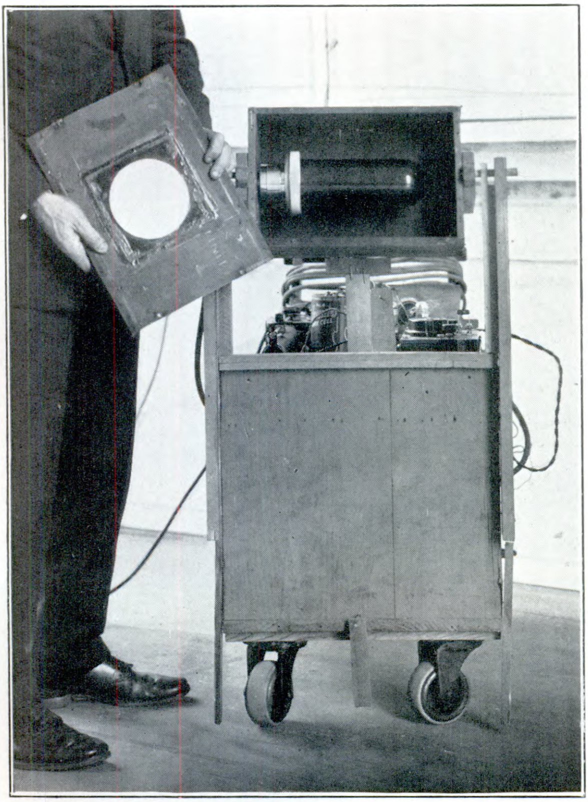
In order to eliminate as far as possible the personal equation in the measurements of illumination, and to provide a rapid method of making the measurements from station to station, the measurements were made with a photoelectric cell connected through an amplifying



INTERIOR OF BUILDING, SHOWING MOVABLE CEILING, RECORDING POTENTIOMETERS, AND WAGON CARRYING PHOTOELECTRIC CELL WITH AMPLIFYING CIRCUIT.



WAGON CARRYING PHOTOELECTRIC CELL AND AMPLIFYING CIRCUIT, WITH THE RECORDING POTENTIOMETER IN THE BACKGROUND.



CASE PHOTOELECTRIC CELL AND OPAL GLASS PLATE COVER, MOUNTED ON THE WAGON.

circuit to a recording potentiometer, the photoelectric cell and the amplifying circuit being mounted on a movable carriage, like a tea-wagon, which could be moved easily from station to station.

The photoelectric cell was of the type developed by Case (1, 2), a highly exhausted vacuum tube about 5½ inches long and 2 inches in diameter. The photoelectric substance in this cell is barium or some compound of barium. The response of the cell to light is made the same as that of the average human eye by covering it with a specially designed amber-colored glass filter. The Case cell, covered by the amber-colored glass filter, was investigated by W. W. Coblentz of the Bureau of Standards, and its response was found to agree very closely with the visibility curve of the eye (3). The cell was enclosed in a wooden box, painted a mat black on the inside, and having a circular opening, 5 inches in diameter, at the top. To eliminate as far as possible any directional effect of the light falling on it, the circular opening above the cell was covered with a thin plate of opal glass, about 2 millimeters thick, having its upper surface ground to a mat finish.

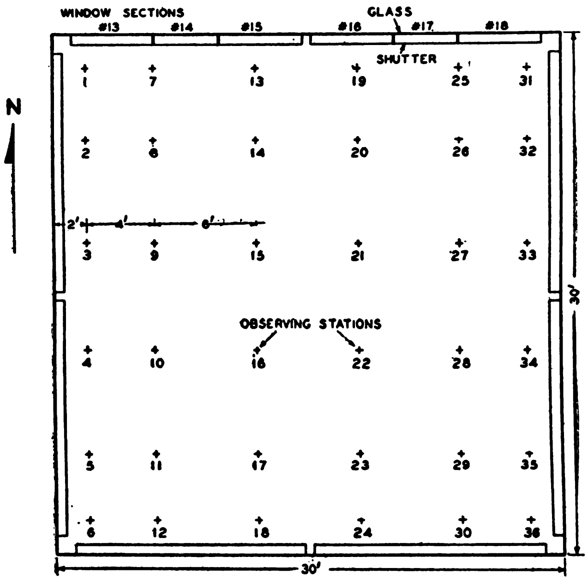


FIGURE 2.—Floor diagram, showing positions of observing stations in relation to the window sections.

The photoelectric cell and the amplifying circuit were connected to a recording potentiometer by means of a long flexible cable. The carriage, with the box containing the photoelectric cell and the amplifying circuit mounted upon it, is shown in plate IV. Behind the carriage can be seen the recording potentiometer. The photoelectric cell in the box, with the opal glass cover removed, is shown in greater detail in plate V.

A diagram of the amplifying circuit is shown in figure 3. It is a Wheatstone-bridge arrangement with vacuum tubes forming two arms of the bridge and 50,000-ohm resistances forming the other two arms. The grid of one tube was connected to the barium electrode of the photoelectric cell, and the grids of both tubes were connected to a closed circuit containing a battery so that their potentials could be varied. The amplifying circuit was balanced by covering the photoelectric cell and varying the potential of the grid of the second tube until no current went to the recording potentiometer. The amplifying circuit was shielded by enclosing it in a metal box.

Readings recorded on the potentiometer were compared with readings of illumination obtained with a Macbeth illuminometer at least

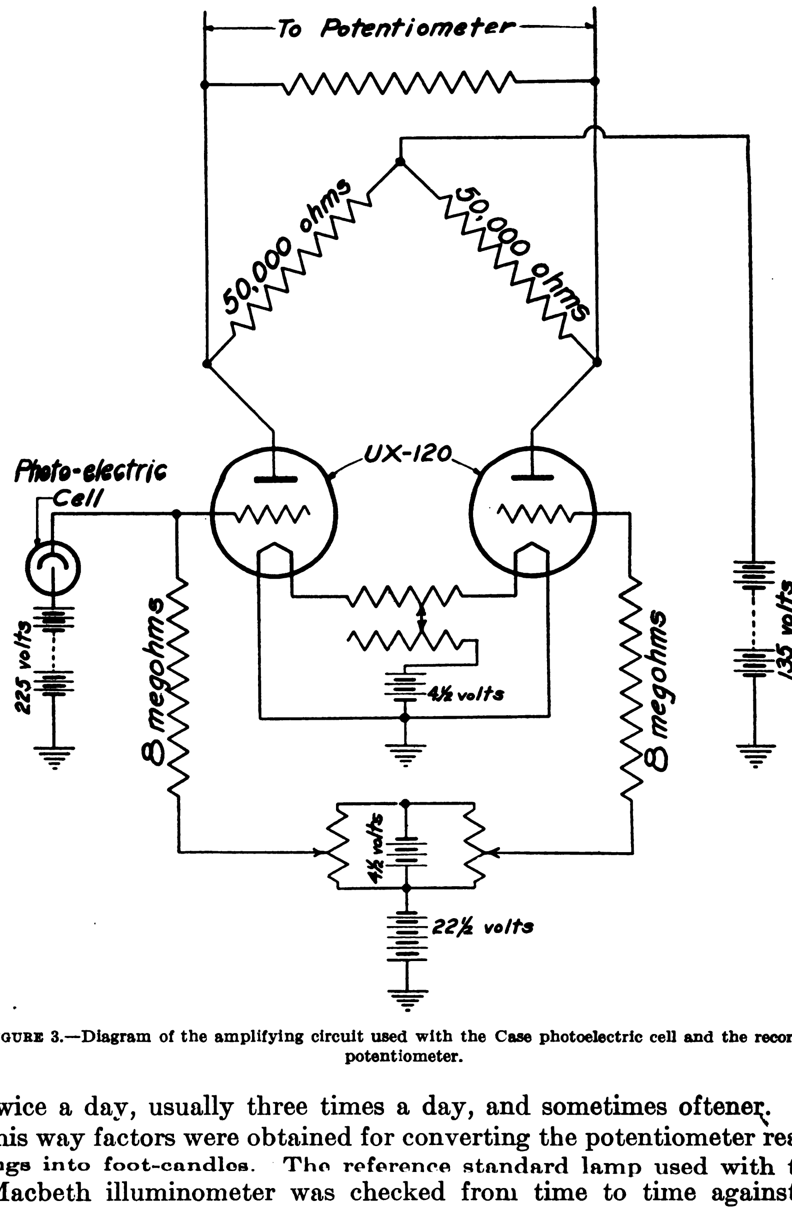


FIGURE 3.—Diagram of the amplifying circuit used with the Case photoelectric cell and the recording potentiometer.

twice a day, usually three times a day, and sometimes oftener. In this way factors were obtained for converting the potentiometer readings into foot-candles. The reference standard lamp used with the Macbeth illuminometer was checked from time to time against a standard lamp supplied by the Bureau of Standards.

PLATE VI

SKY BRIGHTNESS METER MOUNTED ON PLATFORM ON THE ROOF OF THE BUILDING.

Since the transmission of the daylight filter used with the Macbeth illuminometer was determined for light of a color temperature of 2,750° K, and since the color temperature of the light from a clear north sky varies from 12,000° to 26,000° K (4, 5) it was necessary to apply a correction factor to the readings obtained with the illuminometer. For this purpose it was assumed that the average color temperature of the light from a clear sky was 15,000° K, and the values obtained with the Macbeth illuminometer were multiplied by the factor 1.25, given in the calibration certificate for this filter, as the factor for this color temperature.

The measurements recorded in the present study were made only with window sections facing north and with no direct sunlight entering the building. Several preliminary series of measurements were made, but the measurements recorded in this Bulletin were made during the year 1933 between the hours of 8 a. m. and 5 p. m., from May to July, and from September to November. The measurements were made only on clear days, the days being distributed as follows: 3 in May, 5 in June, 9 in July, 12 in September, 8 in October, and 3 in November. From 1 to 12 sets of readings of the illumination were taken in a day; the average number taken on a day being 4.

5. MEASUREMENTS OF SKY BRIGHTNESS

A Case photoelectric cell, connected to a recording potentiometer, was originally used to make measurements of the sky brightness, but the readings obtained in this way, on account of the lack of sensitivity of the method, were found to be very inaccurate. These instruments were therefore replaced with a Weston illuminometer (6), which consists of two photronic cells covered with a compensating light filter, connected to a sensitive current indicator calibrated directly in foot-candles. Readings were made directly on the current indicator.

The photronic cells of the Weston illuminometer were mounted on the bottom of a wooden box to which was attached a rectangular horn made of galvanized iron. The horn and the box were painted a mat black on the inside. The horn was designed so that, when suitable diaphragms were used, the light from a segment of the sky, approximately 30° in altitude and 30° in azimuth, would fall on the photronic cells. The horn was fastened to the wooden box by four bolts so that it could be removed at will. The box and the horn, mounted on horizontal bearings, were supported on a turntable on ball bearings so that the horn could be pointed in any direction.

The whole apparatus, which may be called a sky-brightness meter, mounted on a table on a platform on the roof of the building, is shown in plate VI. Flexible wires connected the photronic cells to the current indicator. Figure 4 gives the dimensions of the brightness meter. In the figure, *A* is a side view of the brightness meter, showing the horn and box containing the photronic cells mounted on bearings so that the horn can be pointed in any direction; *B* is a vertical section, and *C* and *D* horizontal and vertical sections respectively, looking from above and from the rear of the brightness meter respectively.

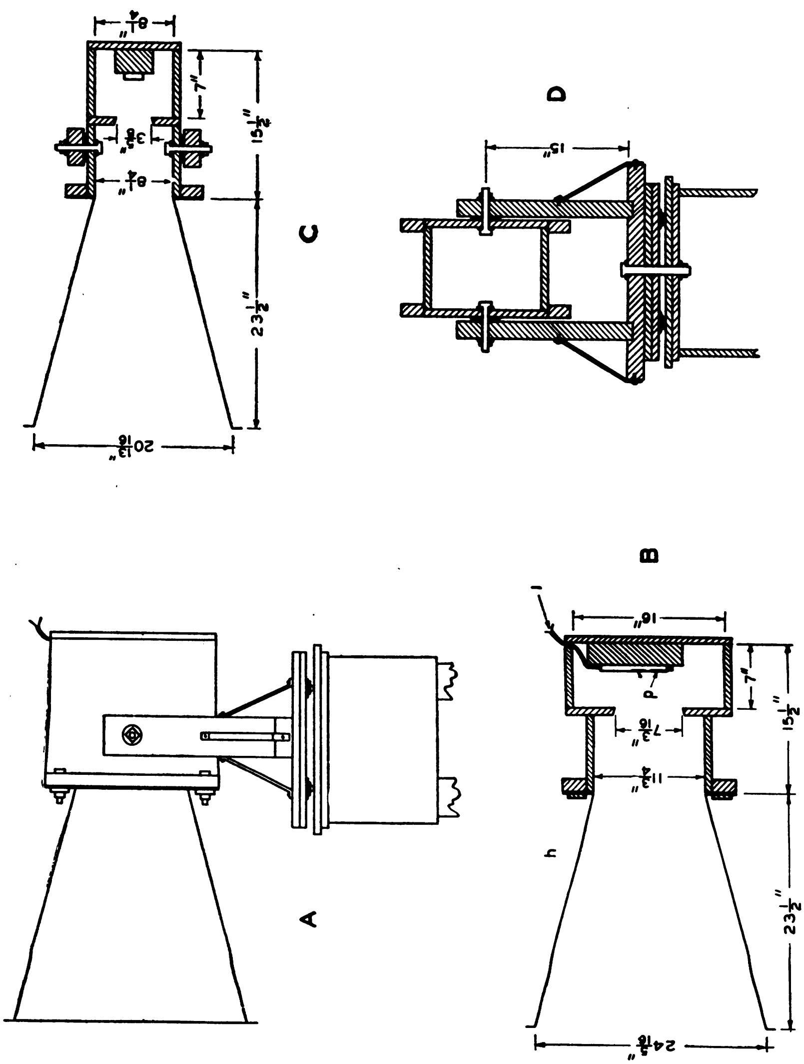


FIGURE 4.—Details of the sky-brightness meter. A is the side view; B, a vertical section; C and D, horizontal and vertical sections, looking from above and from the rear, respectively; h is the horn; p, the photronic cells; and l, the wires to the current indicator.

In the drawings, *h* is the horn, *p* the photronic cells, and *l* the wires going to the current indicator.

For purposes of measurement, the entire northern sky, extending from 90° west of north to 90° east of north, and from the horizon to the zenith, was divided into 18 segments, as shown in figure 5, each segment being 30° in azimuth and 30° in altitude. For convenience, the six segments in the lowest zone, the zone from 0° to 30° in altitude, were lettered from *A* to *F;* those in the middle zone, 30° to 60° in altitude, from *G* to *L;* and those in the highest zone, 60° to 90° in altitude, from *M* to *R*. Since the segments change their shape and area as their altitude changes, diaphragms, having openings of the proper shape and area, were used in front of the horn, so that light from a single segment only fell on the photronic cells.

The widths of the openings in the diaphragms which would approximately fulfill this condition were determined from the ratio of the arcs S_1, S_2, and S_3 in figure 6. From figure 6 it is evident that the lengths of these arcs are proportional to the cosines of their altitudes. For arcs having altitudes of 0°, 30°, and 60°, they would therefore be proportional to the numbers 1.000, 0.866, and 0.500, respectively.

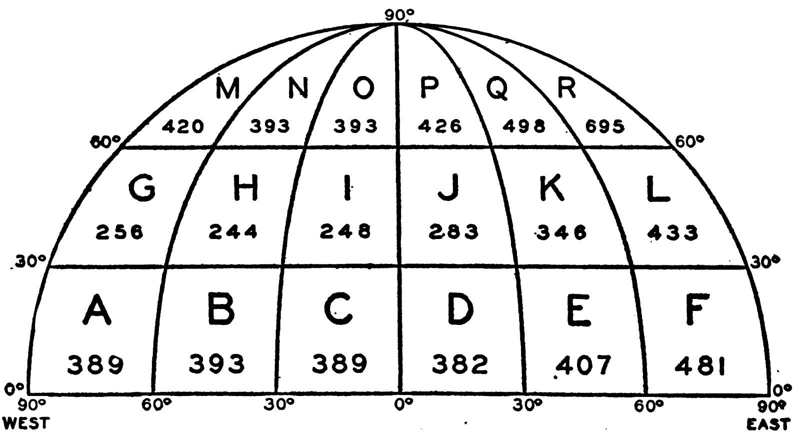


FIGURE 5.—Diagrammatic representation of the entire north sky divided into 18 segments, each being 30 degrees in altitude and azimuth, and identified by a letter. The figures in the segments give the brightness of the clear sky in candles per square foot in Washington on June 2, 1933, from 11:04 to 11:11 a.m.

Since the hood at its lower edge had a width of 20.8 inches, the widths of the openings of the diaphragms at their lower edges should then be 20.8, 18.0, and 10.4 inches, respectively, and 18.0, 10.4, and 0.0 at their upper edges. The diaphragms actually used had the dimensions shown in figure 7. One of the dimensions it will be noted was only 17.1 inches instead of 18.0 inches. This slight narrowing of the opening, however, could have had very little effect upon the results obtained for the brightness of the individual sky segments.

The readings on the current indicator were calibrated for brightness with a Macbeth illuminometer. The photometer head of the Macbeth illuminometer was rigidly attached to the wooden box of the brightness meter, so that it pointed towards the same segment of the sky as the brightness meter and simultaneous readings were made with both instruments. The sighting aperture of the Macbeth illuminometer was pointed towards the center of a segment, and the value for sky brightness thus obtained was taken as the average brightness of the segment. Calibrations were made for each of the three zones separately, giving three separate constants.

As in the case of measurements of illumination with the Macbeth illuminometer, the color temperature of the sky was taken at 15,000° K and a correction factor of 1.25 was applied to the readings obtained with the daylight filter. No attempt was made to correct for the color temperature of different portions of the sky, although different portions of the sky differ materially in their color. These differences in color, however, would probably not make any great error in the values of the brightness, not more than 8 percent at the most, and in general very much less.

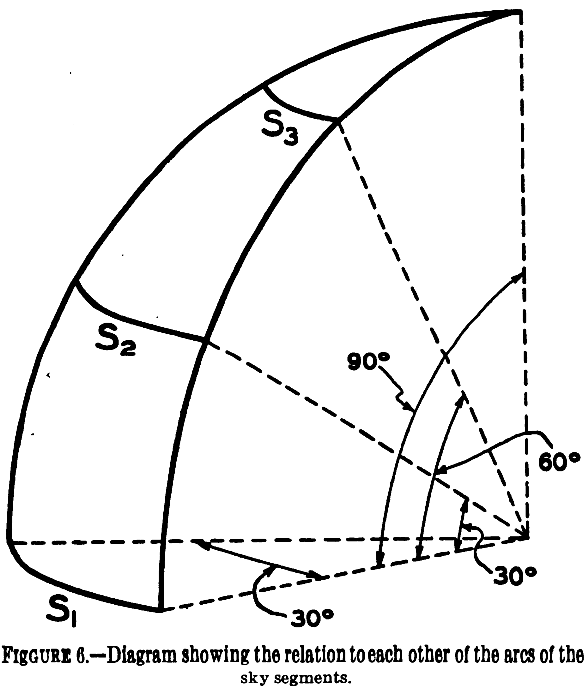


FIGURE 6.—Diagram showing the relation to each other of the arcs of the sky segments.

About 12,000 measurements of brightness were made, although only about 8,000 were used in connection with the measurements of illumination within the building. About 8,000 measurements of brightness were made in the lower zone, 2,900 in the middle zone, and 1,100 in the upper zone. About 5 minutes were required to make a complete set of 18 readings.

To give an idea of the magnitude and distribution of the sky brightness in Washington on a clear day in summertime, the values of the brightness obtained for each of the segments on June 2, from 11:04 to 11:10, expressed in candles per square foot,[2] are given in figure 5.

The averages of all the measurements for a given month, hour, and segment of the sky, are given in tables A and B in the appendix. From the tables it will be noted that in some instances the average brightness of a given segment varied greatly with the time of day. Thus the average brightness of segment *L* in June decreased from 1892 candles per square foot at 8:30 a. m. to 175 at 3:30 p. m., and the

[2] 1 candle per square foot is equal to 3.381 millilamberts.

average brightness of segment *G* increased from 236 at 8:30 a. m. to 1,054 at 3:30 p. m. The variations in the brightness of the other segments are, however, less than these, and the variations in the fall are less than those in the summer. Also, the changes in the northern segments are less than those in the western or eastern segments. The average brightness of all of the 18 segments of the north sky from May to July and September to November, from 8 a. m. to 4 p. m., was found to be 372 candles per square foot. The greatest single value of brightness observed was 2,052 candles per square foot for segment *L* at 8:16 a. m. on June 3. The smallest value observed was 102 candles per square foot for segment *P* at 3:39 p. m. on October 26.

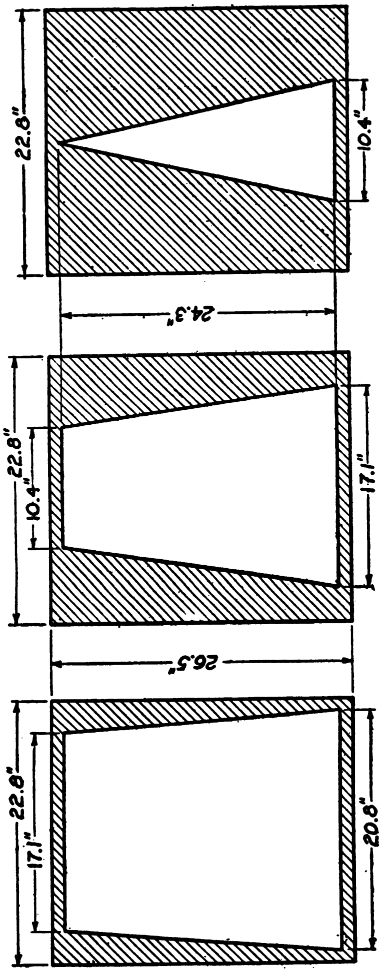


FIGURE 7.—Diaphragms used with sky-brightness meter.

These very high and very low values of sky brightness were only obtained for segments of the sky visible from stations in the corners of the room near the windows, or early in the morning, or late in the afternoon.

The measurements of sky brightness, for the months during which the final observations were made, gave average brightnesses from 8 a. m. to 4 p. m. for the segments of the clear north sky illuminating the windows of the building, of 482, 535, 444, 285, 244, and 242 candles per square foot, for the months of May, June, July, September, October, and November, respectively. The average brightness of the clear north sky for different hours of the day for June and July, and for September and October, together with the corresponding values for November, are shown in figure 8, the values being taken from tables A and B in the appendix. It will be noted that the average brightness of the clear north sky in Washington is fairly constant from 8 a. m. to 4 p. m. in the fall months, but shows a marked decrease from 8 a. m. to 4 p. m. in June and July. The

104087—35——3

curves also show that there is a marked decrease in the average brightness of the clear north sky from summer to fall.

Measurements of the brightness of the clear sky made by Kimball and Hand (7) in Washington and Chicago in 1921, and by Basquin (8) in Chicago in 1897, 1898, and 1899, are of the same order as those obtained in this study.

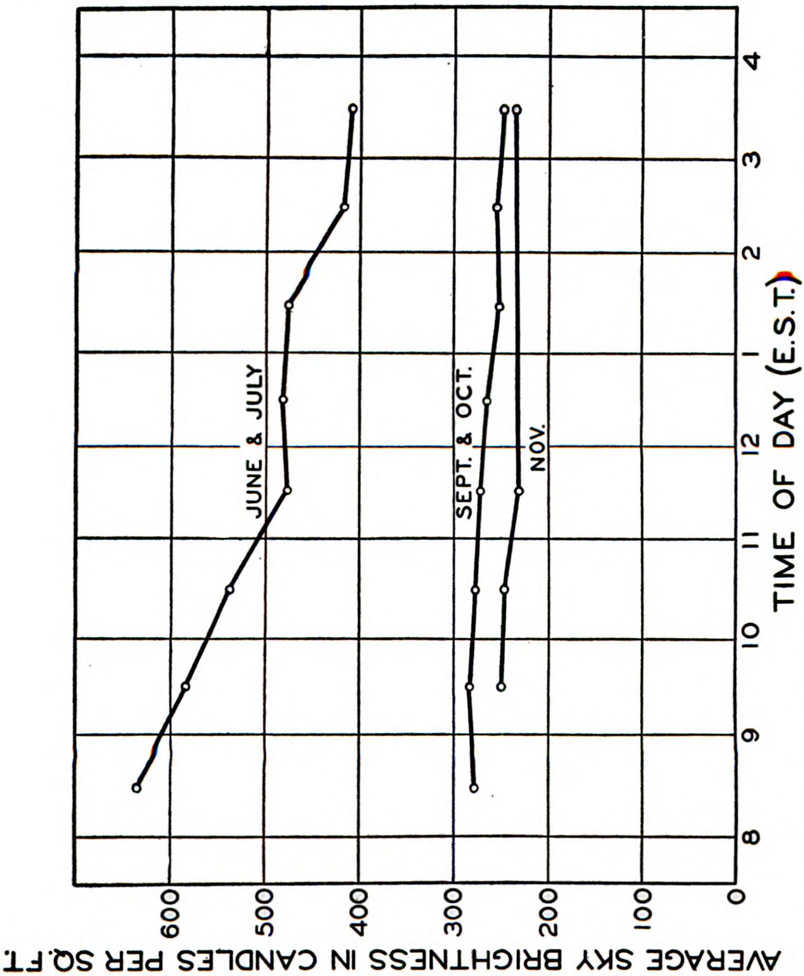


FIGURE 8.—Variation of the average brightness of the clear north sky in Washington, D. C. with the time of day for June and July, September and October, and November.

A more complete analysis of all the brightness measurements made in connection with this study will be found in another paper by two of the authors (17).

6. METHOD OF CALCULATING EFFECTIVE BRIGHTNESS OF SKY VISIBLE THROUGH A WINDOW

Since the amount of illumination received at any point in a room from a window depends, among other things, upon the area of the window projected upon the sky vault, it was necessary to determine these projected areas for each of the 6 window sections for each of the 36 stations in the building.

For this purpose the horizontal and vertical angles made by the lines joining each of the 18 stations, shown as 1 to 18 on figure 2, with the 4 corners of each of the 6 window sections, were measured with a sextant for window heights of 6, 9, and 12 feet, respectively.

These angles were then laid off for each window section on a sky-vault diagram (fig. 9), somewhat similar to that described by P. J. and J. M. Waldram (9), and the four points so obtained for each window section joined together by straight lines. The resulting figures are the projections of the window sections on the sky vault diagram for each of the observing stations.

The simplest representation of the 18 segments, each 30° in azimuth and 30° in altitude, shown in figure 5, into which the quarter sphere

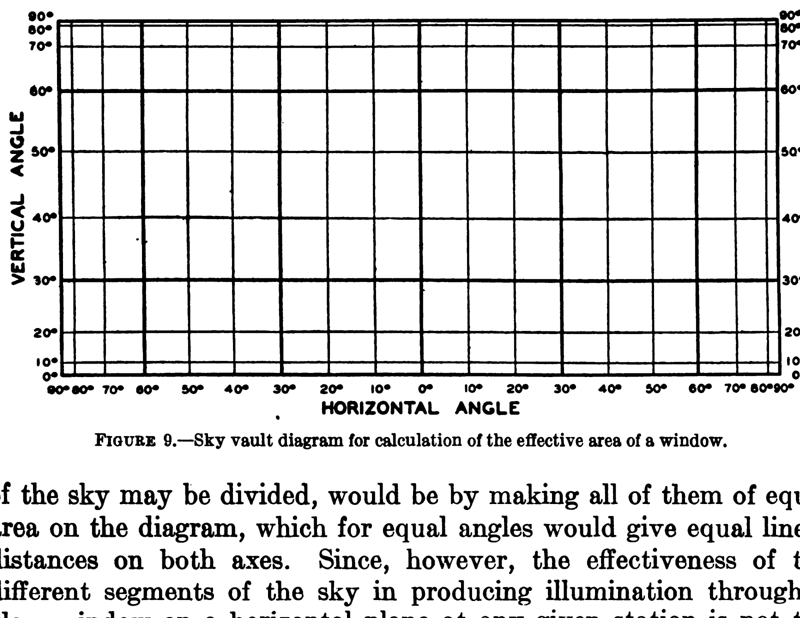


FIGURE 9.—Sky vault diagram for calculation of the effective area of a window.

of the sky may be divided, would be by making all of them of equal area on the diagram, which for equal angles would give equal linear distances on both axes. Since, however, the effectiveness of the different segments of the sky in producing illumination through a glass window on a horizontal plane at any given station is not the same, the areas have not been made equal but have been made approximately proportional to their contribution to the illumination. This proportionality has been achieved by contracting the vertical scale of the diagram at both its upper and lower limits. The vertical scale has been contracted at its lower limit because the lower segments of the sky contribute less and less to the illumination on the horizontal working plane as their altitude decreases, and at its upper limit because the areas of the segments of the sky decrease in size and therefore in their ability to produce illumination as their altitude increases.

In addition, however, to these factors, which were allowed for by the Waldrams in their diagram, a correction has been applied to the diagram used in this study for the light lost by the reflection and absorption of the glass of the windows. This correction has been

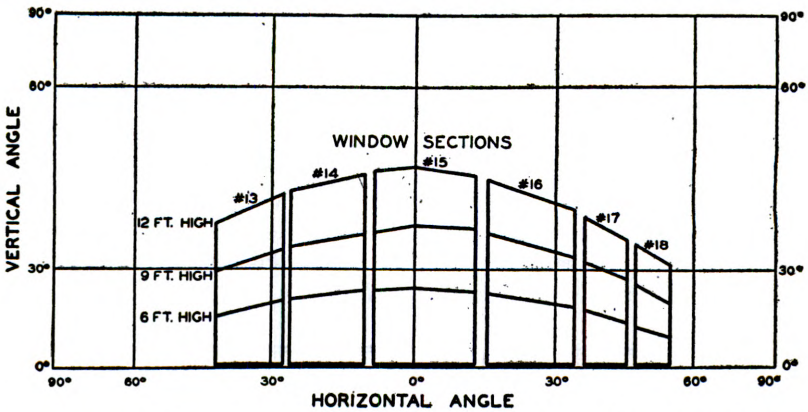


FIGURE 10.—Projections of areas of window sections on the sky vault diagram for observing station number 15 in Figure 2.

applied both to the horizontal and vertical coordinates of the diagram. Its effect is clearly shown in the spacing of the horizontal coordinates. In the original Waldram diagram the angles are equispaced on the horizontal axis, but it will be noted that in figure 9, the spacings

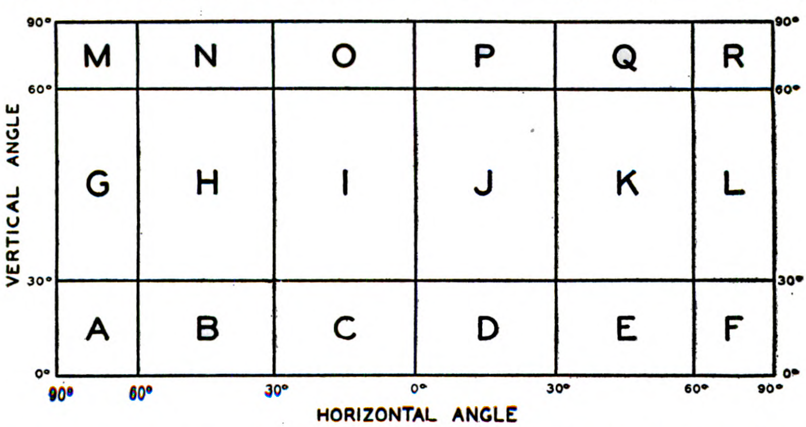


FIGURE 11.—Sky vault diagram showing lettered segments corresponding to the segments shown in Figure 5.

decrease on either side of 0°. This is due to the fact that more and more of the light incident upon the glass of the windows is lost by reflection and absorption as the angle of incidence of the light increases. The method of calculating the value of this correction is given in the section on theoretical considerations, pages 42–44.

The results obtained in this manner for station 15 (fig. 2) are shown in figure 10. It will be noted that for station 15 (fig. 2), which is 12 feet from the window and near the middle of the room, the projections for all three window heights are in the lower and middle sky zones. For a window 6 feet high, they are all in the lower zone, which means that for a window 6 feet high only portions of the lower zone of the sky are visible from station 15. It will also be noted that from station 15 no portion of the sky having an azimuth greater than 60° is visible. Eighteen diagrams similar to figure 10 were prepared, one for each of the 18 stations. In figure 10, the diagram is divided into 18 segments, 30° in azimuth and 30° in

U. S. Public Health Service - Office of Industrial Hygiene & Sanitation
Sky Brightness Calculations, Expr. Illum. Cabinet, Arlington, Va.

Date Time Ceiling Height ---- feet. Survey No.
Sheet No.

Sky Seg.	Sky Bright-ness	WINDOW NO.											
		Sta. No.		Sta. No.		Sta. No.		Sta. No.		Sta. No.		Sta. No.	
A													
B													
C													
D													
E													
F													
G													
H													
I													
J													
K													
L													
M													
N													
O													
P													
Q													
R													
	TOTAL												

Industrial Hygiene, Illum. Form No. 7

FIGURE 12.—Form used for calculating average brightness of the effective areas of the window sections.

altitude, each segment representing a segment of the north sky, as shown in figure 5, these segments being identified in the measurements of brightness as segments *A*, *B*, *C*, etc., segments *A* to *F* being in the lower zone, *G* to *L* in the middle zone, and *M* to *R* in the upper zone, as shown in figure 11.

The average brightness of the portion of the sky vault producing the illumination at a station through a window was taken as the sum of the weighted brightnesses of the sky segments, the brightness of each segment being weighted by the portion of the area of the window projected on that segment..

The form used for calculating the average brightness of the portions of the sky vault subtended at the stations by the window is shown in figure 12. In the first column are given the capital letters indicating the sky segments subtended in whole or in part by the window; in the second column are recorded the brightnesses of each of these segments as measured with the sky brightness meter; in the third column, the ratios in percent of the portions of the sky segments subtended by the window sections to the total area of these portions, and in the fourth column the products of the values in columns 2 and 3, these products giving the weighted brightnesses of each sky segment as seen through the window. The total of these weighted brightnesses was taken as the average brightness in candles per square foot of the portion of the sky vault subtended at the station by the window.

7. CONCURRENT MEASUREMENTS OF ILLUMINATION AND SKY BRIGHTNESS

Measurements of illumination and sky brightness were made only on clear days, since on cloudy days the illumination varied too rapidly to get satisfactory measurements, either of illumination or sky brightness. Measurements were made between the hours of 8 a. m. and 5 p. m. While measurements of the illumination were being made within the building, measurements of sky brightness were being made on the roof. Twenty to thirty minutes were required to make a set of readings of illumination at each of the 36 stations within the building and during that time several sets of readings were made of the brightness of the sky segments.

In the computation of the illumination-brightness ratio, each value of the illumination measured at a given station was divided by the corresponding average brightness of the portion of the sky illuminating that station, and the average taken of six separate determinations of this ratio. Three of the measurements, at a given station, were made in the forenoon and three in the afternoon, between the hours of 8 a. m. and 4 p. m. The values of the illumination obtained at a given station for different times of the day differed widely from each other, varying with the time of day, being in some cases several times as great. This was also true of the corresponding values of sky brightness. The ratio, however, of the illumination to sky brightness at a given station was much more constant, since in general when the sky brightness was high the illumination was high, and vice versa.

After a very large number of preliminary measurements, during the course of which the apparatus and methods were modified and improved, the final measurements were made during the summer and fall of 1933, from May to November, inclusive.

The measurements of illumination were made with windows of three different widths, using 2, 4, or 6 sections respectively, giving windows 9.75, 17.25, and 27.00 feet wide. The windows were all located centrally in the north wall of the building and in all cases the top of the window extended to the ceiling.

The illumination for each of the three window widths was determined for ceiling heights of 9, 12, and 15 feet, respectively, and for white walls and white ceiling, for black walls and white ceiling, and for black walls and black ceiling.

The illumination was therefore measured at each of the 36 stations under 27 different conditions, and for each of these conditions 6 sets of readings were usually taken, 3 in the forenoon and 3 in the afternoon. The results given in this report are therefore based upon some 6,000 measurements within the building.

From the values obtained for the illumination within the building, expressed in foot-candles, and from the corresponding values of the brightness of the sky visible through the window at a given station, calculated according to the method described in section 6, values were determined for the ratio of the illumination to the sky brightness. The results have been expressed in the tables in foot-candles per 100 candles per square foot. One hundred candles per square foot gives a convenient unit. The value of sky brightness proposed for adoption as an international standard by the International Commission on Illumination is $\frac{500}{\pi}$, or 159.1 candles per square foot (16). A uniform sky brightness of 159.1 candles per square foot would give an illumination of 500 foot-candles on a horizontal surface exposed to an unobstructed hemisphere of sky. Five hundred foot-candles is said to be about the average illumination under a completely unobstructed sky in moderately dull weather in Great Britain, from 9 a. m. to 3 p. m., from February to October. In the United States the average illumination on a cloudy day under a completely unobstructed sky is somewhat greater than in Great Britain, and a sky brightness of $\frac{682}{\pi}$ or 217.1 candles per square foot has been proposed as the standard (16). Six hundred and eighty-two foot-candles is said to be the minimum value of the illumination at Washington due to an unobstructed sky for 4 hours either way from the meridian on a cloudy day at either the vernal or autumnal equinox. The values of illumination given in tables I to III, it will be noted, are for a sky having a brightness of 100 candles per square foot, or for a sky brightness only about half of that proposed for the American standard.

8. RESULTS OF THE STUDY

From the averages of the concurrent measurements of illumination and sky brightness for different window heights, window widths, and for white or black ceilings or walls, values are given in tables I, II, and III, for the illumination within the building at stations 5 feet apart, and 5 feet from the walls, expressed as the illumination in foot-candles that would be produced by a sky having an average brightness of 100 candles per square foot. These values were obtained by interpolation on smoothed curves plotted from the values obtained at the original 36 stations within the building which are given in tables C, D, and E in the appendix. Locations 5 feet apart were chosen for the display of the final results as being more convenient for practical use in every day problems than the locations of the original stations. The locations of the stations 5 feet apart are shown in figure 13. The stations are identified by the numbers 11, 12, etc., the first figure in the identifying number indicating the position of the row from north to south, and the second figure, the position of the row from east to west, the windows being on the north side of the building.

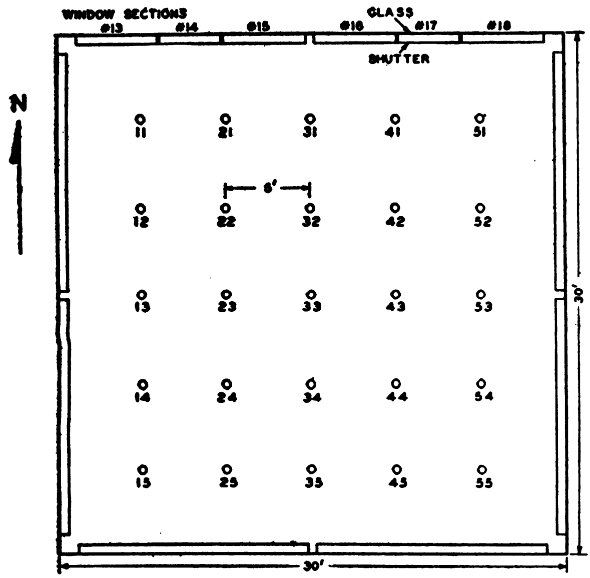


FIGURE 13.—Floor diagram showing position of stations, 5 feet apart, for which the illuminations are given tables I to VI.

TABLE I.—*Illumination in foot-candles for white ceiling and white walls, at stations 5 feet apart, for a sky brightness of 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	2.2	4.6	6.5	6.3	10.2	13.2	14.5	19.4	23.8	24.2	31.0	36.7	33.7	44.0	51.0	42.0	54.4	64.2
12 or 52	2.3	4.3	6.0	5.0	8.9	12.4	8.0	14.3	20.0	11.8	19.8	28.2	15.4	25.4	36.8	19.0	30.7	45.3
13 or 53	1.7	3.5	5.1	3.5	7.0	10.1	5.3	10.5	15.2	7.1	13.8	20.2	8.9	16.7	25.7	10.4	19.5	30.5
14 or 54	1.2	2.9	4.3	2.5	5.4	8.3	3.9	7.9	12.2	5.2	10.2	16.0	6.3	12.3	20.4	7.6	14.2	24.0
15 or 55	1.2	2.4	3.8	2.2	4.5	7.4	3.3	6.6	10.8	4.3	8.7	14.0	5.1	10.4	17.3	5.9	11.8	20.4
21 or 41	9.3	12.4	14.7	18.3	24.8	27.9	27.3	36.0	40.0	34.1	43.8	50.0	38.4	49.6	57.7	41.3	53.0	63.6
22 or 42	4.5	7.4	9.9	8.4	14.1	19.1	11.9	19.8	27.7	14.5	24.4	35.0	16.8	29.3	40.9	18.3	33.4	45.5
23 or 43	2.3	4.5	6.5	4.4	8.6	13.0	6.4	11.9	18.6	8.1	15.1	23.3	9.5	18.3	28.0	11.0	21.1	32.5
24 or 44	1.5	3.3	4.9	2.9	6.0	9.2	4.4	8.4	13.4	5.7	10.7	17.3	6.8	12.8	21.2	8.1	14.7	24.8
25 or 45	1.2	2.4	3.7	2.4	4.7	7.7	3.4	6.8	11.3	4.3	8.7	14.3	5.1	10.4	17.6	6.0	11.7	20.2
31	20.5	29.5	33.6	28.5	38.3	42.6	34.6	45.1	49.5	37.6	48.6	55.2	39.5	50.7	60.1	40.8	52.2	64.2
32	6.6	11.8	16.5	11.1	18.4	24.5	14.2	22.9	31.5	16.1	26.2	36.9	17.4	29.2	41.9	18.0	31.7	46.8
33	2.8	5.6	7.6	5.2	10.0	14.0	7.4	13.0	19.9	8.9	16.0	25.0	9.9	18.4	29.4	10.7	20.9	32.7
34	1.7	3.5	4.8	3.2	6.3	9.7	4.7	8.8	14.0	5.7	11.1	18.0	6.9	13.1	21.8	8.0	14.9	25.3
35	1.3	2.6	3.6	2.4	4.7	7.4	3.6	6.8	10.9	4.4	8.6	14.2	5.1	10.2	17.2	6.0	11.7	19.9

Table II.—*Illumination in foot-candles for white ceiling and black walls, at stations 5 feet apart, for a sky brightness of 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	0.9	2.5	3.7	3.8	6.7	8.8	10.4	14.2	17.7	18.3	23.7	28.3	25.7	34.6	40.0	31.6	42.8	50.7
12 or 52	1.5	3.0	4.3	3.6	6.3	9.0	5.9	9.9	14.8	8.8	13.8	20.7	11.6	17.9	26.6	14.3	21.9	32.5
13 or 53	1.0	2.2	3.2	2.3	4.4	6.5	3.6	6.5	9.7	4.8	8.6	12.7	5.8	10.2	15.7	6.6	11.7	18.3
14 or 54	.6	1.5	2.2	1.4	2.7	4.2	2.3	3.9	6.1	3.1	5.1	7.9	3.7	6.1	9.8	4.5	7.0	11.4
15 or 55	.5	.9	1.4	.9	1.7	2.8	1.4	2.4	4.1	1.8	3.3	5.4	2.1	4.0	6.5	2.4	4.3	7.6
21 or 41	8.1	10.4	12.2	16.0	21.8	24.1	23.6	31.6	34.8	28.7	37.3	43.3	31.2	41.8	49.4	32.7	44.1	53.6
22 or 42	3.8	6.1	8.3	7.1	11.8	16.0	10.1	16.4	23.1	12.2	19.9	29.0	14.0	23.5	33.5	15.0	25.8	36.7
23 or 43	1.7	3.2	4.8	3.3	6.1	9.5	4.8	8.4	13.4	6.0	10.6	16.3	7.0	12.6	18.7	7.8	14.2	21.1
24 or 44	1.0	2.0	3.1	1.9	3.5	5.4	3.0	4.8	7.6	3.8	6.1	9.5	4.5	7.2	11.3	5.0	8.2	13.0
25 or 45	.6	1.0	1.6	1.2	2.1	3.3	1.7	3.0	4.8	2.1	3.9	6.0	2.4	4.5	7.2	2.7	4.9	8.1
31	19.5	27.7	31.3	26.6	35.8	39.4	31.1	41.3	45.2	32.8	43.5	49.8	33.8	44.4	53.6	34.4	45.1	56.6
32	6.1	10.0	15.0	10.1	16.2	21.5	12.7	19.9	27.3	14.3	22.4	31.6	15.1	24.7	35.5	15.3	26.3	39.2
33	2.4	4.3	6.2	4.3	7.6	10.9	6.1	9.8	15.0	7.1	11.8	18.0	7.6	13.0	20.4	8.1	14.4	22.9
34	1.3	2.2	3.1	2.4	3.9	6.0	3.5	5.5	8.4	4.1	6.7	10.5	4.7	7.7	12.4	5.2	8.5	14.1
35	.8	1.2	1.6	1.4	2.2	3.4	2.2	3.2	4.9	2.5	4.0	6.3	2.7	4.6	7.6	2.9	5.0	8.7

Table III.—*Illumination in foot-candles for black ceiling and black walls, at stations 5 feet apart, for a sky brightness of 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	0.4	1.5	2.0	2.7	4.7	5.5	8.0	10.5	12.6	13.7	18.0	21.7	19.0	27.3	32.1	23.0	34.0	41.6
12 or 52	1.1	2.3	3.0	2.7	4.8	6.4	4.5	7.4	10.7	6.5	10.2	15.2	8.4	13.0	20.0	10.0	15.8	25.0
13 or 53	.7	1.6	2.2	1.6	3.2	4.6	2.4	4.7	7.0	3.2	6.3	9.2	3.9	7.2	11.4	4.4	8.0	13.0
14 or 54	.4	1.0	1.4	.8	1.9	2.8	1.3	2.7	4.0	1.8	3.5	5.2	2.1	4.1	6.6	2.6	4.6	7.6
15 or 55	.4	.6	.9	.7	1.2	1.9	1.0	1.7	2.7	1.2	2.2	3.5	1.3	2.6	4.2	1.4	2.8	4.9
21 or 41	7.0	9.2	10.0	13.5	18.7	20.0	18.7	26.4	29.0	22.2	31.0	36.0	23.5	33.8	41.0	24.0	35.2	44.3
22 or 42	3.2	5.3	7.0	6.1	10.0	13.0	8.3	13.5	18.5	9.5	16.0	23.0	10.2	18.3	26.5	10.5	19.5	28.7
23 or 43	1.2	2.5	3.8	2.5	4.9	7.4	3.5	6.5	10.5	4.2	8.1	12.6	4.8	9.5	14.2	5.2	10.4	15.7
24 or 44	.6	1.4	2.2	1.2	2.5	3.8	1.8	3.3	5.3	2.2	4.1	6.6	2.6	4.8	7.8	2.9	5.4	8.8
25 or 45	.4	.7	1.0	.8	1.4	2.2	1.0	2.0	8.2	1.2	2.6	4.0	1.3	3.1	4.7	1.4	3.4	5.3
31	17.0	24.5	27.5	21.8	30.2	33.1	24.3	33.9	37.5	25.2	35.2	41.0	25.5	35.6	44.3	25.6	36.0	47.0
32	5.2	8.5	13.0	8.5	13.8	17.8	10.2	16.6	22.3	10.6	18.2	25.3	10.7	19.3	28.1	10.7	19.8	30.5
33	1.6	3.4	5.0	3.1	5.9	8.5	4.2	7.4	11.8	4.7	8.8	14.0	5.0	9.8	15.6	5.3	10.6	16.8
34	.8	1.4	2.2	1.4	2.6	4.2	2.0	3.7	5.9	2.3	4.6	7.2	2.6	5.2	8.4	2.9	5.5	9.5
35	.4	.7	1.0	.8	1.4	2.2	1.2	2.0	3.2	1.4	2.6	4.0	1.5	3.1	4.7	1.6	3.4	5.3

It is evident that the quantities given in tables I to III are ratios, the ratios of the illumination, measured within the building, to the brightness of the sky producing that illumination, measured outside the building.

In tables I to III values of this illumination-brightness ratio are given for the three conditions of white ceiling and white walls, white

104087—35——4

ceiling and black walls, and black ceiling and black walls, respectively, for stations 5 feet apart, for window widths of 5, 10, 15, 20, 25, and 30 feet, and window heights of 6, 9, and 12 feet, the top of the window being at the level of the ceiling in each case and the bottom of the window 3 feet 4 inches above the floor.

The original observations of illumination and sky brightness are not given since the publication of these values would necessitate 18 tables, each table containing some 684 measured values, or 11,664 values in all.

By subtracting the values of illumination given in table III from those in table I, the illumination produced by the light reflected from the ceiling and walls is obtained. In the same way we get from tables III and II the illumination produced by the light reflected from the ceiling and from tables II and I, the light reflected from the walls. These reflected illuminations are given in tables IV, V, and VI, respectively. The sums of the corresponding values in tables V and VI give the values in table IV.

Since the illumination-brightness ratio was obtained in all cases by dividing the illumination observed at a given station by the effective brightness of the portion of the sky visible through the window at that station, the illumination-brightness ratio is given accurately when the ceiling and walls are black, as in the case of the results given in table III, since in this case none of the illumination on the horizontal working plane is due to light reflected from the ceiling and walls. When, however, either the ceiling or walls, or both, are white, part of the illumination on the horizontal working plane is due to light reflected from them, and since the light falling on ceiling and walls comes either directly or indirectly from the *whole* quarter sphere of the sky, the brightness of the sky producing the *reflected* illumination (given in tables IV to VI) may be taken as the average brightness of the quarter sphere of the sky. It is not easy to determine the part played by each of the 18 segments of the quarter sphere in producing the light falling on the ceiling and walls. Obviously the extreme eastern and western segments and the segments in the upper zone affect the ceiling and walls but little and it is these segments that have the highest sky brightness. The other segments of sky vary little from one another in brightness and the average brightness of all of them is of the same order as any one of them so that the illumination-brightness ratios given in tables IV and V for the light reflected from the ceiling and walls will at least approximate the values for an average brightness of the sky of 100 candles per square foot.

TABLE IV.—*Illumination in foot-candles reflected from white ceiling and white walls, at stations 5 feet apart, for a sky brightness of approximately 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	1.8	3.1	4.5	3.6	5.5	7.7	6.5	8.9	11.2	10.5	13.0	15.0	14.7	16.7	18.9	19.0	20.4	22.6
12 or 52	1.2	2.0	3.0	2.3	4.1	6.0	3.5	6.9	9.3	5.3	9.6	13.0	7.0	12.4	16.8	9.0	14.9	20.3
13 or 53	1.0	1.9	2.9	1.9	3.8	5.5	2.9	5.8	8.2	3.9	7.5	11.0	5.0	9.5	14.3	6.0	11.5	17.5
14 or 54	.8	1.9	2.9	1.7	3.5	5.5	2.6	5.2	8.2	3.4	6.7	10.8	4.2	8.2	13.8	5.0	9.6	16.4
15 or 55	.8	1.8	2.9	1.5	3.3	5.5	2.3	4.9	8.1	3.1	6.5	10.5	3.8	7.8	13.1	4.5	9.0	15.5
21 or 41	2.3	3.2	4.7	4.8	6.1	7.9	8.6	9.6	11.0	11.9	12.8	14.0	14.9	15.8	16.7	17.3	17.8	19.3
22 or 42	1.3	2.1	2.9	2.3	4.1	6.1	3.6	6.3	9.2	5.0	8.4	12.0	6.6	11.0	14.4	7.8	13.9	16.8
23 or 43	1.1	2.0	2.7	1.9	3.7	5.6	2.9	5.4	8.1	3.9	7.0	10.7	4.7	8.8	13.8	5.8	10.7	16.8
24 or 44	.9	1.9	2.7	1.7	3.5	5.5	2.6	5.1	8.1	3.5	6.6	10.7	4.2	8.0	13.4	5.2	9.3	16.0
25 or 45	.8	1.7	2.7	1.6	3.3	5.5	2.4	4.8	8.1	3.1	6.1	10.3	3.8	7.3	12.9	4.6	8.3	14.9
31	3.5	5.0	6.1	6.7	8.1	9.5	10.3	11.2	12.0	12.4	13.4	14.2	14.0	15.1	15.8	15.2	16.2	17.2
32	1.4	2.8	3.5	2.6	4.6	6.7	4.0	6.3	9.2	5.5	8.0	11.6	6.7	9.9	13.8	7.3	11.9	16.3
33	1.2	2.2	2.6	2.1	4.1	5.5	3.2	5.6	8.1	4.2	7.2	11.0	4.9	8.6	13.8	5.6	10.3	16.3
34	.9	2.1	2.6	1.8	3.7	5.5	2.7	5.1	8.1	3.4	6.5	10.8	4.3	7.9	13.4	5.1	9.4	15.8
35	.9	1.9	2.6	1.6	3.3	5.2	2.4	4.8	7.7	3.0	6.0	10.2	3.6	7.1	12.5	4.4	8.3	14.6

TABLE V.—*Illumination in foot-candles reflected from the white ceiling, at station, 5 feet apart, for a sky brightness of approximately 100 candles per square foots for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	0.5	1.0	1.7	1.1	2.0	3.3	2.4	3.7	5.1	4.6	5.7	6.6	6.7	7.3	7.9	8.6	8.8	9.1
12 or 52	.4	.7	1.3	.9	1.5	2.6	1.4	2.5	4.1	2.3	3.6	5.5	3.2	4.9	6.6	4.3	6.1	7.5
13 or 53	.3	.6	1.0	.7	1.2	1.9	1.2	1.8	2.7	1.6	2.3	3.5	1.9	3.0	4.3	2.2	3.7	5.3
14 or 54	.2	.5	.8	.6	.8	1.4	1.0	1.2	2.1	1.3	1.6	2.7	1.6	2.0	3.2	1.9	2.4	3.8
15 or 55	.1	.3	.5	.2	.5	.9	.4	.7	1.4	.6	1.1	1.9	.8	1.4	2.3	1.0	1.5	2.7
21 or 41	1.1	1.2	2.2	2.5	3.1	4.1	4.9	5.2	5.8	6.5	6.8	7.3	7.7	8.0	8.4	8.7	8.9	9.3
22 or 42	.6	.8	1.3	1.0	1.8	3.0	1.8	2.9	4.6	2.7	3.9	6.0	3.8	5.2	7.0	4.5	6.3	8.0
23 or 43	.5	.7	1.0	.8	1.2	2.1	1.3	1.9	2.9	1.8	2.5	3.7	2.2	3.1	4.5	2.6	3.8	5.4
24 or 44	.4	.6	.9	.7	1.0	1.6	1.2	1.5	2.3	1.6	2.0	2.9	1.9	2.4	3.5	2.1	2.8	4.2
25 or 45	.2	.3	.6	.4	.7	1.1	.7	1.0	1.6	.9	1.3	2.0	1.1	1.4	2.5	1.3	1.5	2.8
31	2.5	3.2	3.8	4.8	5.6	6.3	6.8	7.4	7.7	7.6	8.3	8.8	8.3	8.8	9.3	8.8	9.1	9.6
32	.9	1.5	2.0	1.6	2.4	3.7	2.5	3.3	5.0	3.7	4.2	6.3	4.4	5.4	7.4	4.6	6.5	8.7
33	.8	.9	1.2	1.2	1.7	2.4	1.9	2.4	3.2	2.4	3.0	4.0	2.6	3.2	4.8	2.8	3.8	5.5
34	.5	.8	.9	1.0	1.3	1.8	1.5	1.8	2.5	1.8	2.1	3.3	2.1	2.5	4.0	2.3	3.0	4.6
35	.4	.5	.6	.6	.8	1.2	1.0	1.2	1.7	1.1	1.4	2.3	1.2	1.5	2.9	1.3	1.6	3.4

TABLE VI.—*Illumination in foot-candles reflected from the white walls, at stations 5 feet apart, for a sky brightness of approximately 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	1.3	2.1	2.8	2.5	3.5	4.4	4.1	5.2	6.1	5.9	7.3	8.4	8.0	9.4	11.0	10.4	11.6	13.5
12 or 52	.8	1.3	1.7	1.4	2.6	3.4	2.1	4.4	5.2	3.0	6.0	7.5	3.8	7.5	10.2	4.7	8.8	12.8
13 or 53	.7	1.3	1.9	1.2	2.6	3.6	1.7	4.0	5.5	2.3	5.2	7.5	3.1	6.5	10.0	3.8	7.8	12.2
14 or 54	.6	1.4	2.1	1.1	2.7	4.1	1.6	4.0	6.1	2.1	5.1	8.1	2.6	6.2	10.6	3.1	7.2	12.6
15 or 55	.7	1.5	2.4	1.3	2.8	4.6	1.9	4.2	6.7	2.5	5.4	8.6	3.0	6.4	10.8	3.5	7.5	12.8
21 or 41	1.2	2.0	2.5	2.3	3.0	3.8	3.7	4.4	5.2	5.4	6.0	6.7	7.2	7.8	8.3	8.6	8.9	10.0
22 or 42	.7	1.3	1.6	1.3	2.3	3.1	1.8	3.4	4.6	2.3	4.5	6.0	2.8	5.8	7.4	3.3	7.6	8.8
23 or 43	.6	1.3	1.7	1.1	2.5	3.5	1.6	3.5	5.2	2.1	4.5	7.0	2.5	5.7	9.3	3.2	6.9	11.4
24 or 44	.5	1.3	1.8	1.0	2.5	3.9	1.4	3.6	5.8	1.9	4.6	7.8	2.3	5.6	9.9	3.1	6.5	11.8
25 or 45	.6	1.4	2.1	1.2	2.6	4.4	1.7	3.8	6.5	2.2	4.8	8.3	2.7	5.9	10.4	3.3	6.8	12.1
31	1.0	1.8	2.3	1.9	2.5	3.2	3.5	3.8	4.3	4.8	5.1	5.4	5.7	6.3	6.5	6.4	7.1	7.6
32	.5	1.3	1.5	1.0	2.2	3.0	1.5	3.0	4.2	1.8	3.8	5.3	2.3	4.5	6.4	2.7	5.4	7.6
33	.4	1.3	1.4	.9	2.4	3.1	1.3	3.2	4.9	1.8	4.2	7.0	2.3	5.4	9.0	2.8	6.5	10.8
34	.4	1.3	1.7	.8	2.4	3.7	1.2	3.3	5.6	1.6	4.4	7.5	2.2	5.4	9.4	2.8	6.4	11.2
35	.5	1.4	2.0	1.0	2.5	4.0	1.4	3.6	6.0	1.9	4.6	7.9	2.4	5.6	9.6	3.1	6.7	11.2

A correction for the amount of light reflected from the black surfaces has been made in tables II and III so that the results for black walls and ceiling have been reduced as nearly as possible to those for perfectly black surfaces. In these calculations the reflecting powers of the white walls or ceilings and of the black walls or ceilings have been taken as 78 and 4 percent, respectively.

The values given in tables III, V, and VI are displayed in figure 14 which gives the results in the form of a chart. In this chart the distribution of the illumination-brightness ratio, in the experimental daylight building, is given for each combination of window width and window height. The chart is divided into 18 squares, one square for each combination of window width and window height, the window widths varying from 5 to 30 feet, and the window heights from 6 to 12 feet. In each square are given the values of the illumination-brightness ratios on the working plane 36 inches above the floor at stations 5 feet apart. The middle number of each group of three gives the illumination-brightness ratio for black ceiling and black walls; that is, for the illumination coming directly from the window. The upper number gives the illumination reflected from the ceiling, and the lower number gives the illumination reflected from the walls. The illumination in a room having white ceiling and walls can be found by adding together all three values. It may be pointed out that the illumination for black ceiling and walls will approximate that for a very large room and very high ceiling, or for a very dirty ceiling and walls, or for ceiling and walls having very low reflecting powers.

An inspection of tables IV, V, and VI shows clearly the increase in illumination upon the horizontal working plane due to the light reflected from either the ceiling or the walls, or from both together. It will be noted from table IV that the reflected illumination from the ceiling and walls is greatest near the windows and, going from the windows to the back of the room, decreases rapidly at first and then gradually. Table V shows that the illumination due to the ceiling is greatest near the windows and decreases with the distance from the window; it increases with increase of window height and with increase of window width. Table VI shows that the illumination due to the walls varies with the location in the room and with the height and width of the window; in general, it is greatest near the window and the rear wall and has a minimum value near the middle of the room. The contributions to the illumination from the ceiling and the walls, respectively, are in general of the same order of magnitude, although their relative values differ for different positions in the room.

In table VII are given the ratios, expressed in percent, of the illumination reflected from the ceiling and walls to the total illumination. The figures were obtained by dividing the values given in table IV by the corresponding values in table I. It will be seen from table VII that the contribution of the illumination reflected from the ceiling and walls to the total illumination is very considerable, varying from 17 to 82 percent. The average contribution at all stations, for all widths and heights of windows, is 51 percent. In general the contribution from the ceiling and walls increases from the window to the rear of the room and decreases from the walls toward the middle of the room, the percentage increase or decrease being more marked the less the width of the window.

TABLE VII.—*Ratio in percent of the illumination reflected from the ceiling and walls to the total illumination for stations 5 feet apart, for different combinations of width and height of window*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	82	67	69	57	54	58	45	46	47	43	42	41	44	38	37	45	38	35
12 or 52	52	46	50	46	46	48	44	48	46	45	48	46	46	49	46	47	48	45
13 or 53	59	54	57	54	54	54	55	55	54	56	54	54	56	55	56	58	57	57
14 or 54	67	66	67	68	65	66	67	66	67	65	66	68	67	66	68	66	68	68
15 or 55	67	75	76	68	73	74	70	74	75	72	75	75	74	75	76	78	76	76
21 or 41	25	26	32	26	25	28	32	27	28	35	29	28	39	32	29	42	34	30
22 or 42	29	28	29	27	29	32	30	32	33	34	34	34	39	38	35	43	42	37
23 or 43	48	48	42	43	46	43	45	45	44	48	46	46	50	48	49	53	51	52
24 or 44	60	58	55	59	58	60	59	61	60	61	62	62	62	62	63	64	63	64
25 or 45	67	71	73	67	70	71	71	71	72	72	70	72	74	70	73	77	71	74
31	17	17	18	24	21	22	30	25	24	33	28	26	35	30	26	37	31	27
32	21	25	21	23	25	28	28	28	29	34	30	31	38	34	33	41	38	35
33	43	39	34	38	41	38	43	43	41	47	47	44	48	47	47	51	49	50
34	53	62	54	56	59	57	57	58	58	60	59	60	62	60	62	65	63	62
35	69	73	72	67	70	70	67	71	71	68	70	72	71	70	73	73	71	73

To display graphically some of the results given in table I for white ceiling and white walls, the illuminations along the center line of the room, going from the window to the back wall, from station 31 to station 35, and from one side of the room to the other, from station 13 to station 53 (see fig. 13), have been plotted in figures 15 and 16. The corresponding values for black ceiling and black walls given in table III, have been plotted in figures 17 and 18. Although these

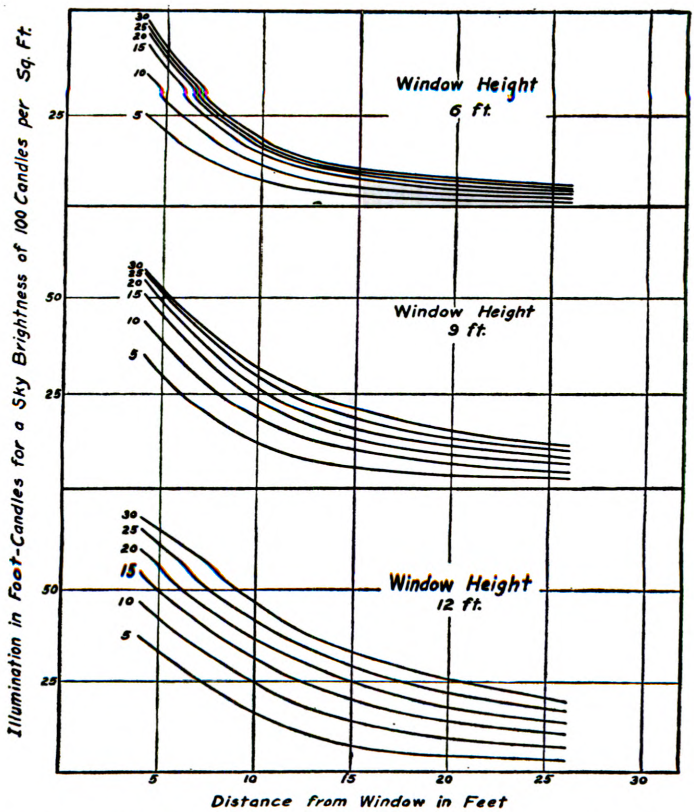


FIGURE 15.—Variation of illumination from window to back wall, along center line of room, for white ceiling and white walls, for windows of different heights and widths. The numbers adjacent to the curves give the widths of the windows in feet.

curves are only for the central lines of the room, they give a fairly good idea of the way the illumination varies over a large part of it.

The actual distribution of the illumination in any room will, in general, change somewhat with the dimensions of the room, being influenced by its width and its depth, because the illumination is affected by the reflections from the ceiling and walls. For the case of black ceiling and walls, since these surfaces no longer reflect any light, the results will be unaffected by the dimensions of the room and will be true for a room of any dimensions. For any room approximately 30 feet square, the value of the illumination will be somewhere between the values given in table I and in table III, since the reflecting powers of the ceiling and walls of the average room will lie between the values of the reflecting powers for these two cases, viz., between 78 and 0 percent. The reflecting power of the ceiling and walls in any room is seldom, if ever, above 78 percent, and never as low as 0 percent. The average reflecting power of the walls of an office or schoolroom is probably not more than 60 percent. For workshops the reflecting

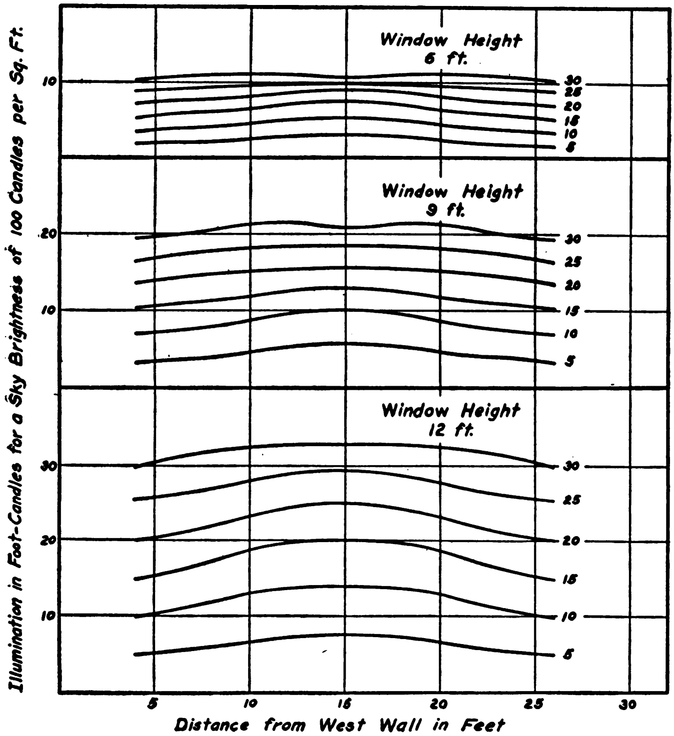


FIGURE 16.—Variation of illumination from one side of the room to the other, along center line of room, for white ceiling and white walls, for windows of different heights and widths. The numbers adjacent to the curves give the widths of the windows in feet.

power of the walls will usually be considerably less. The effect of the light reflected from the ceiling and walls is shown clearly by a comparison of figures 15 and 16 with figures 17 and 18. It is seen that when the ceiling and walls were white, the illumination was much higher than when they were black; that the increase in illumination due to the ceiling and walls was greatest near the walls, and that the effect of the reflected light was to make the distribution of the illumination throughout the room somewhat more uniform.

If the ceiling and walls are colored instead of being white, the amount of light reflected from them will depend upon the reflecting power of the colors. Mat white surfaces, when clean, will have a reflecting power of about 80 percent. Gray will have a reflecting power varying from 70 to 20 percent according to the degree of darkness; ivory, from 77 to 73 percent; buff, about 60 percent; tan, 28 percent; pink, 50; dark red, 16; light green, 63; dark green, 21, and light blue, 52 percent. These values have been taken from those given by Cady and Dates in "Illuminating Engineering" for light from Mazda lamps, but they will approximate the values for daylight.

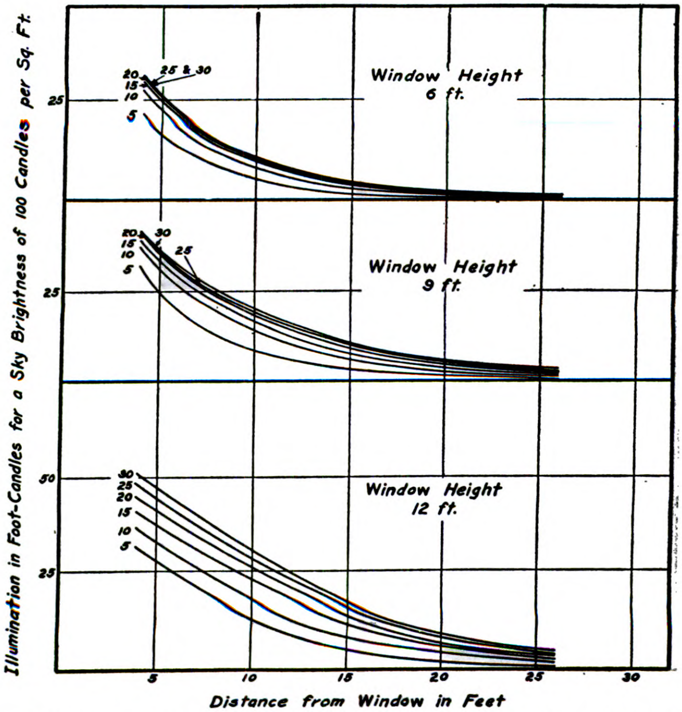


FIGURE 17.—Variation of illumination from window to back wall, along center line of room, for black ceiling and black walls, for windows of different heights and widths. The numbers adjacent to the curves give the widths of the windows in feet.

The effect of increasing either the height of the window or its width is shown in figures 19 and 20, both for white ceiling and white walls, and for black ceiling and black walls, the values plotted being those obtained in the middle of the room at station 33. Similar graphs are given in figures 21 and 22 for the values of the illumination at the rear of the room, 25 feet from the window (station 35). It will be noted that in figures 19 to 22 the curves have been extended so as to include values for window heights ranging from 4 to 12 feet, and for window widths from 5 to 30 feet, although the window heights actually used in the study were only varied from 6 to 12 feet, and the window widths from 9.75 to 27 feet. It is believed however that these extensions are justified.

The values of the illumination in other parts of the room vary with the width and height of the window, in a manner more or less similar to those at station 33, as will be evident from an inspection of tables I and III. It will be noted from the curves of figures 19 and 20, and 21 and 22, that for the same width and height of window the illumination

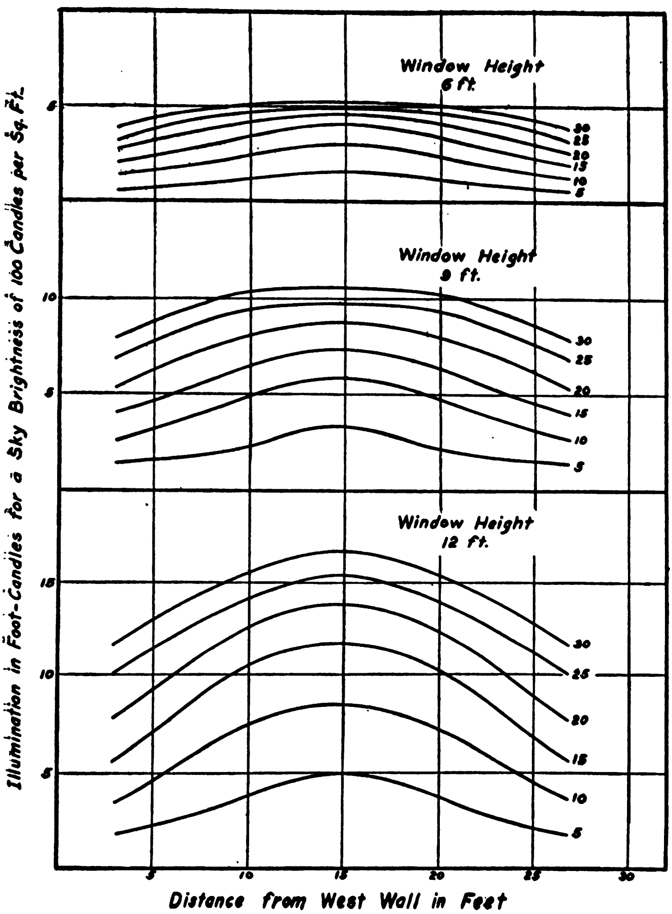


FIGURE 18.—Variation of illumination from one side of the room to the other, along center line of room, for black ceiling and black walls, for windows of different heights and widths. The numbers adjacent to the curves give the widths of the windows in feet.

is in some cases nearly twice as great for white walls and ceiling as for black walls and ceiling. Furthermore, an inspection of figures 19 and 20 brings out the very important fact that at the middle of the room an increase in the height of a window produces a much greater increase of illumination than a proportional increase of width. Doubling the height of a window approximately trebles the illumination at the middle or rear of the room, whereas doubling the width of the

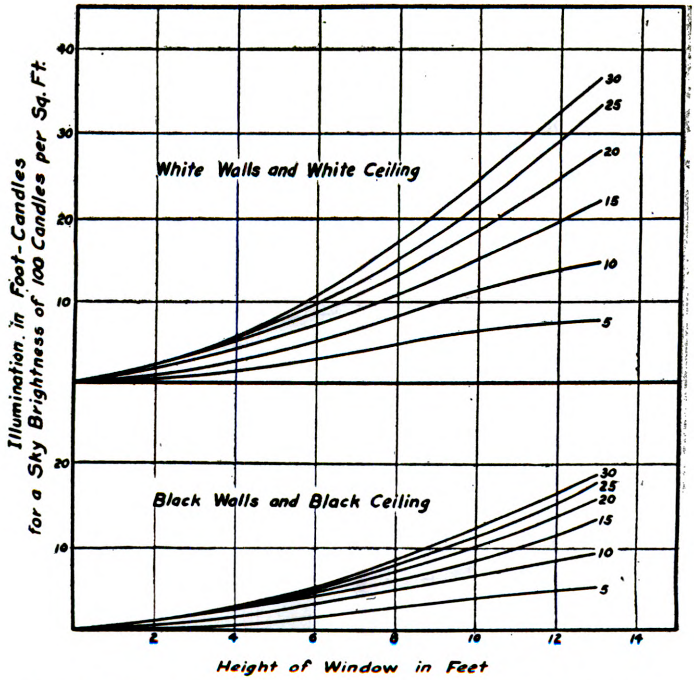


FIGURE 19.—Variation of illumination at middle of room with height of window, for windows of different widths. The numbers adjacent to the curve give widths of the windows in feet.

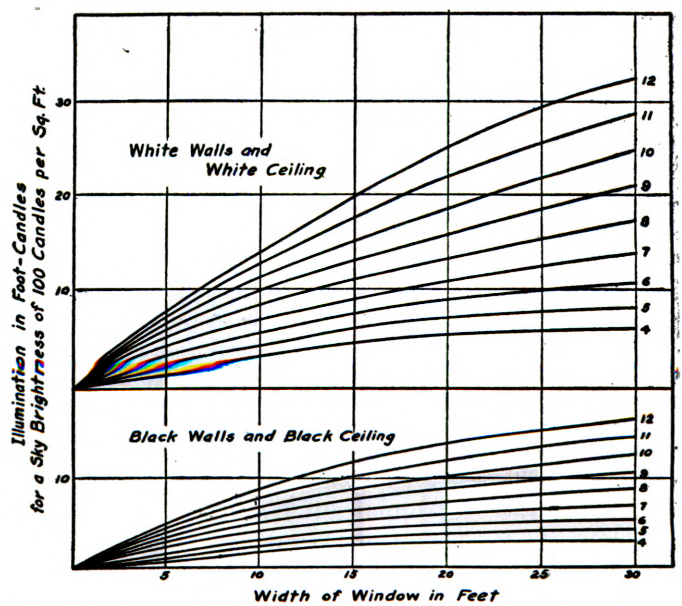


FIGURE 20.—Variation of illumination at middle of room with width of window, for windows of different heights. The numbers adjacent to the curves give the heights of the windows in feet.

window does not even double the illumination. Therefore, for the same amount of window area, a greater increase of illumination is produced by increasing the height than by increasing the width of a window.

At points very near the window the illumination will, except for the light reflected from the walls and ceiling, be everywhere approximately the same,[3] since such points subtend the whole northern sky.

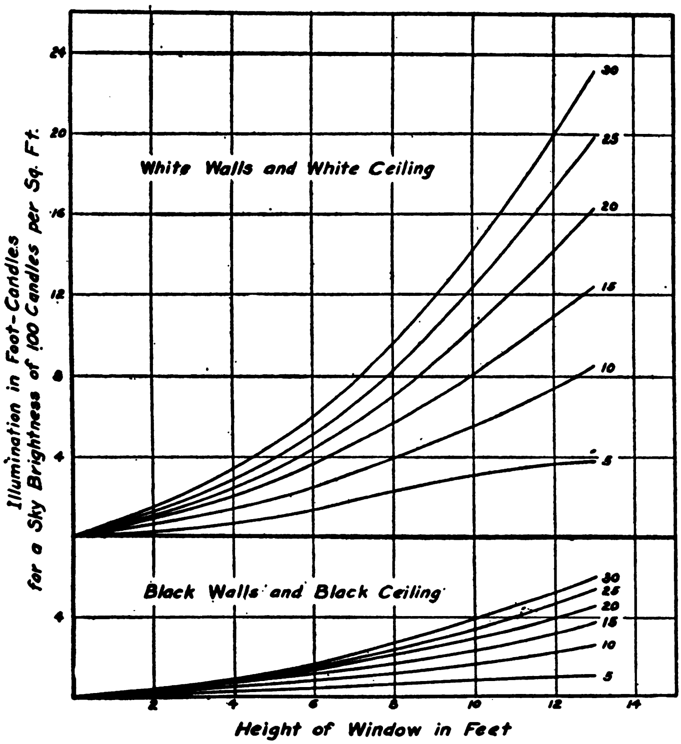


FIGURE 21.—Variation of illumination at rear of room with height of window, for windows of different widths. The numbers adjacent to the curves give the widths of the windows in feet.

For points very near the window, therefore, changing the height or width of the window would have very little effect upon the illumination. As we move away from the window, however, toward the rear of the room, the effect of increasing the height or width of the window increases rapidly, the illumination due to the increased height increasing more rapidly than the illumination due to the increased width. Outside the window on the other side of the glass, the illumination on a horizontal plane due to the quarter sphere of the sky, for a sky brightness of 100 candles per square foot, will be $\frac{100 \times \pi}{2}$ or 157 foot-candles.

[3] This was not quite true for the values obtained in the experimental building, since the plane on which the measurements were made was 4 inches more or less below the sill of the window.

Added to this will be a small amount of illumination coming from inside the room due to reflection from the ceiling and walls. On the other side of the glass, inside the room, the illumination on the horizontal working plane will be much less owing to the loss of light, by reflection and absorption of the glass.

In a consideration of the efficiency of the lighting of a room by means of windows, it is necessary to know two things: (a) The distribution of the light supplied by the windows upon the working

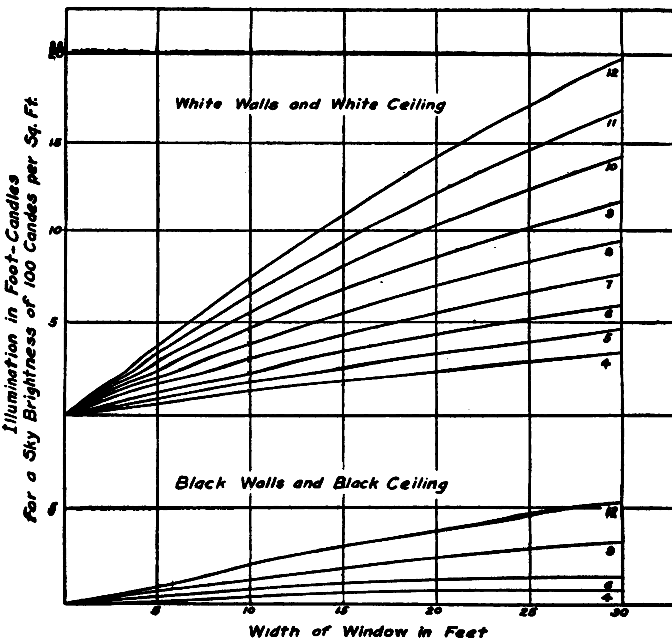


FIGURE 22.—Variation of illumination at rear of room with width of window, for windows of different heights. The numbers adjacent to the curves give the heights of the windows in feet.

plane, and (b) its intensity. The results of this study give us information upon both of these points.

For a room of moderate size with white ceiling and walls, our results give very accurately the distribution of the light for windows of varying width and height, and they show how, for a given glass area, the width and height of the window can be adjusted to each other so as to obtain the distribution and intensity of light which is desired. This can be seen very clearly from the distribution chart (fig. 14), where the values of illumination throughout the room are given for windows of various widths and heights. In this chart the illumination coming directly from the windows is given in large figures. Above and below the large figures are given the illumination reflected from the ceiling and walls, respectively. If it is remembered that the values of the reflected illumination are for a coefficient of reflection of 78 percent, the amount reflected from ceiling or walls having any other coefficients of reflection can be determined by taking proportional parts of the reflected illumination given in the chart for either walls and ceilings. Thus, for a window 10 feet wide and 6 feet high, the direct illumination at the middle of the room is 3.1 foot-candles; the reflected illumination from the ceiling, 1.2 foot-candles, and that from the walls, 0.9 foot-candles. The total illumination is therefore 3.1+1.2+0.9, or 5.2 foot-candles. If the coefficient of reflection of the ceiling and walls is 40 percent instead of 78 percent, the total illumination becomes 3.1+0.6+0.5, or 4.2 foot-candles, instead of 5.2.

TABLE VIII.—*Showing degree of uniformity of illumination for each combination of window width and window height for white ceiling and white walls*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	9.3	6.4	5.2	4.5	3.8	3.2	2.4	2.3	2.1	1.6	1.6	1.5	1.2	1.2	1.2	1.0	1.0	1.0
12 or 52	8.9	6.9	5.6	5.7	4.3	3.4	4.3	3.2	2.5	3.2	2.5	1.9	2.6	2.0	1.6	2.2	1.8	1.4
13 or 53	12.1	8.4	6.6	8.1	5.5	4.2	6.5	4.3	3.3	5.3	3.5	2.7	4.4	3.0	2.3	4.0	2.8	2.1
14 or 54	17.1	10.2	7.8	11.4	7.1	5.1	8.9	5.7	4.1	7.2	4.8	3.4	6.3	4.1	2.9	5.5	3.8	2.7
15 or 55	17.1	12.3	8.8	13.0	8.5	5.8	10.5	6.8	4.6	8.7	5.6	3.9	7.7	4.9	3.5	7.1	4.6	3.1
21 or 41	2.2	2.4	2.3	1.6	1.5	1.5	1.3	1.3	1.2	1.1	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0
22 or 42	4.6	4.0	3.4	3.4	2.7	2.2	2.9	2.3	1.8	2.6	2.0	1.6	2.4	1.7	1.5	2.3	1.6	1.4
23 or 43	8.9	6.6	5.2	6.5	4.5	3.3	5.4	2.8	2.7	4.6	3.2	2.4	4.2	2.8	2.1	3.8	2.6	2.0
24 or 44	13.7	8.9	6.9	9.8	6.4	4.6	7.9	5.4	3.7	6.6	4.5	3.2	5.8	4.0	2.8	5.2	3.7	2.6
25 or 45	17.1	12.3	9.1	11.9	8.1	5.5	10.2	6.6	4.4	8.7	5.6	3.9	7.7	4.9	3.4	7.0	4.6	3.2
31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
32	3.1	2.6	2.0	2.6	2.1	1.7	2.4	2.0	1.6	2.3	1.0	1.5	2.3	1.7	1.4	2.3	1.7	1.4
33	7.3	5.3	4.4	5.5	3.8	3.0	4.7	3.5	2.5	4.2	3.0	2.2	4.0	2.8	2.0	3.9	2.6	2.0
34	12.1	8.4	7.0	8.9	6.1	4.4	7.4	5.1	3.5	6.6	4.4	3.1	5.7	3.9	2.8	5.2	3.6	2.5
35	15.8	11.3	9.3	11.9	8.2	5.8	9.6	6.6	4.5	8.5	5.7	3.9	7.7	5.0	3.5	7.0	4.6	3.2

TABLE IX.—*Showing degree of uniformity of illumination for each combination of window width and window height for black ceiling and black walls*

Station	Width of window in feet																	
	5			10			15			20			25			30		
	Height in feet			Height in feet			Height in feet			Height in feet			Height in feet			Height in feet		
	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12	6	9	12
11 or 51	42.5	16.3	13.7	8.1	6.4	6.0	3.0	3.2	3.0	1.8	2.0	1.9	1.3	1.3	1.4	1.1	1.1	1.1
12 or 52	15.4	10.7	9.2	8.1	6.3	5.2	5.4	4.6	3.5	3.9	3.5	2.7	3.0	2.7	2.2	2.6	2.3	1.9
13 or 53	24.3	15.3	12.5	13.6	9.4	7.2	10.1	7.2	5.4	7.9	5.6	4.5	6.5	4.9	3.9	5.8	4.5	3.6
14 or 54	42.5	24.5	19.6	27.3	15.9	11.8	18.7	12.6	9.4	14.0	10.1	7.9	12.1	8.7	6.7	9.8	7.8	6.2
15 or 55	42.5	40.8	30.6	31.1	25.2	17.4	24.3	19.9	13.9	21.0	16.0	11.7	19.6	13.7	10.5	18.3	12.9	9.6
21 or 41	2.4	2.7	2.8	1.6	1.6	1.7	1.3	1.3	1.3	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.1
22 or 42	5.3	4.6	3.9	3.6	3.0	2.5	2.9	2.5	2.0	2.7	2.2	1.8	2.5	1.9	1.7	2.4	1.8	1.6
23 or 43	14.2	9.8	7.2	8.7	6.2	4.5	6.9	5.2	3.6	6.0	4.3	3.3	5.3	3.6	3.1	4.9	3.5	3.0
24 or 44	28.3	17.5	12.5	18.2	12.1	8.7	13.5	10.3	7.1	11.5	8.6	6.2	9.8	7.4	5.7	8.8	6.7	5.3
25 or 45	42.5	35.0	27.5	27.2	21.6	15.0	24.3	17.0	11.7	21.0	13.5	10.2	19.6	11.5	9.4	18.3	10.6	8.9
31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
32	3.3	2.9	2.1	2.6	2.2	1.9	2.4	2.0	1.7	2.4	1.9	1.6	2.4	1.8	1.6	2.4	1.8	1.5
33	10.6	7.2	5.5	7.0	5.1	3.9	5.8	4.6	3.2	5.4	4.0	2.9	5.1	3.6	2.8	4.8	3.4	2.8
34	21.2	17.5	12.5	15.6	11.6	7.9	12.1	9.2	6.4	11.0	7.7	5.7	9.8	6.8	5.3	8.8	6.5	4.9
35	42.5	35.0	27.5	27.2	21.6	15.0	20.2	17.0	11.7	18.0	13.5	10.2	17.0	11.5	9.4	16.0	10.6	8.9

An important element in the lighting of a room by windows is the *uniformity* of the illumination; that is, the variation of the illumination at any point of the working plane from its minimum value. A measure of uniformity has been called the "uniformity coefficient" and has been defined as the ratio of the maximum to the minimum illumination on the working plane. To indicate the uniformity of the illumination found for white ceiling and white walls, and for black ceiling and black walls, tables VIII and IX have been prepared from tables I and III, showing the *uniformity* of the illumination for each combination of window width and window height. This has been done for each combination by dividing each of the 15 values of illumination for that combination into the maximum illumination. The greatest of the 15 values so obtained will be approximately the uniformity coefficient for that combination. It must be remembered that the greater the value of the coefficient, the less the uniformity. An inspection of table VIII shows that the uniformity of the illumination at all stations is increased in all cases by increasing either the height or the width of the window, or by increasing both together. Thus the uniformity of the illumination in our building for a window 25 feet wide was greater than for one 5 feet wide for all heights of window; and for each window width was more uniform for a window 12 feet high than for one 6 feet high. By comparing the values for station 35 (fig. 13), at the back of the room, for windows 10 and 20 feet wide, respectively, it is seen that doubling the width of the window increases the uniformity of the illumination by about 50 percent, whereas doubling the height of the window increases the uniformity by about 100 percent. An inspection of table IX, for a black ceiling and black walls, leads to the same conclusions as the inspection of table VIII.

The method of calculating the effective area of the sky vault subtended at any station in the room was described in section 6 and the sky vault diagram used for this purpose was shown in figure 9. As has been stated, this diagram is a modification of the diagram used by the Waldrams (9), a correction having been applied to it by us for the reflection and absorption by the glass of the incident light. As an example of the method, the projections of the six window sections visible at station 15 (fig. 2) were shown in figure 10.

Since the area of the projection of a window on the sky vault diagram, for any given station in the room, is a measure of the effectiveness of that window in producing illumination on the horizontal working plane at that station, the illuminations found at each of the 36 stations when the ceiling and walls were black, for windows of different widths and heights, should be proportional to the areas of the projections of the windows on the sky vault diagram. The areas of these projections for each of the 36 stations, expressed as fractions of the area of the projection of the whole quarter sphere of the sky on the sky vault diagram, are given as table F in the appendix. From table F, and the corresponding values of illumination given in table E, table G has been obtained by dividing the values in table E by the corresponding values in table F. If the value of the effective area of the sky vault for each station has been calculated correctly, and if the value of the illumination at each corresponding station has been measured correctly, we might expect that the ratio of one to the other would be constant. It will be noted that the values in table G fluctuate around a mean value of 77. Most of the values lie between 65 and 85. Considering the many sources of error in making the calculations and the measurements, the proportionality between the values of the calculated effective areas and the measured values of the illumination is probably as close as could be expected. The values of the ratio given in table G may be said to confirm the distribution of the illumination throughout the room given in table III.

It may be noted that the fractional values given in table F are equal to the ratios of the illumination at the given stations to the illumination at a point very near the window, since at a point very near the window the value of the fraction becomes equal to unity. It may also be noted that it follows from this that the mean value of the ratios given in table G, viz, 77, indicates the number of foot-candles of illumination that we might expect on a horizontal plane, 36 inches above the floor, for a sky brightness of 100 candles per square foot, at a point near the window, when the ceiling and walls are black, and the windows are glazed in the manner described, and a small amount of the light falling on the window is absorbed by dirt on the glass.

9. APPLICATION OF RESULTS OF STUDY TO DESIGN OF WINDOWS

As has been already stated, the values of illumination given in tables I, II, and III, and on the distribution chart, are for a sky brightness of 100 candles per square foot. Since the illumination at any point in a room is directly proportional to the brightness of the portion of the sky illuminating that point, to determine the intensity of the illumination at that point from the values given in tables I to III, or from the distribution chart (fig. 14), for any time of day or year, it is necessary to know the average brightness of that portion of the sky for that time.

The problem of providing sufficient natural illumination in a building is somewhat simplified by the fact that the daylight illumination on a clear day during the working period, say from 8 a. m. to 4 p. m., is generally sufficient near the windows and, consequently, in a building of modern design it is only necessary to consider the adequacy of the illumination in the part of the room distant from the windows, say at points in the working plane at a distance of 25 feet from the window, in the late afternoon, at the darkest time of the year, say at 4 p. m. in December.

It is impossible at the present time to give the values of the brightness of different portions of a clear sky throughout the United States at 4 p. m. in December. Fortunately, however, it is possible to determine the average brightness of a clear sky for any hour of the day and for any time of the year from values of daylight illumination over the

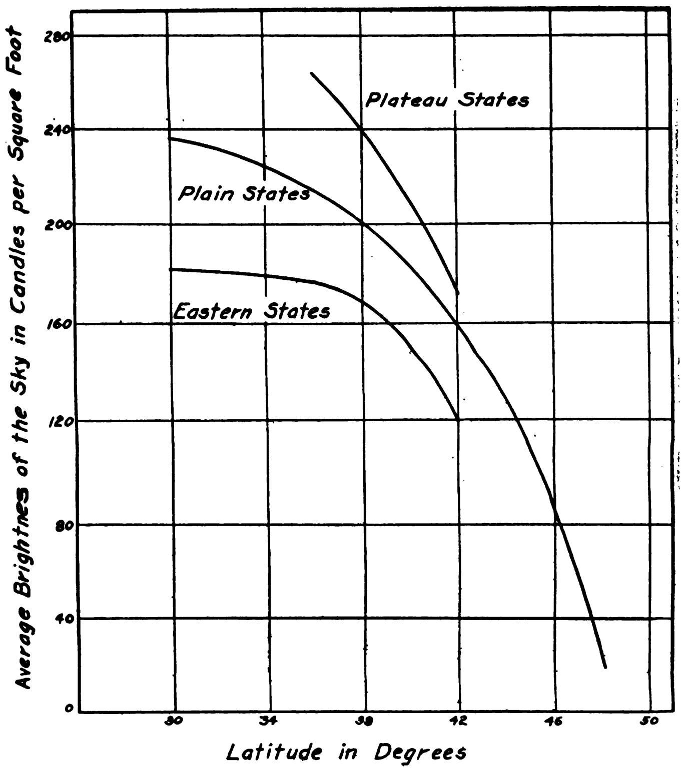


FIGURE 23.—Variation of the average brightness of the whole clear sky with latitude, at 4 p. m. on December 21, for different regions in the United States.

greater part of the United States, measured and computed by H. H. Kimball, since the average brightness of the sky can be easily calculated if the average illumination is known. (See section on theoretical considerations, p. 44). From values of brightness obtained in this way, table X has been prepared giving the hourly average brightness of the sky for different regions and latitudes in the United States on clear days on December 21, March 21, June 21, and September 21. The values for 4 p. m. on December 21 have been plotted in figure 23, showing the variation of the sky brightness with latitude for the Eastern States, Plain States, and Plateau States.

If, for example, we wish to know what the illumination on a horizontal working plane, 36 inches above the floor, in a room comparable in size to that used in the experimental building will be at 4 p. m. on December 21, at a point 25 feet from the window (station 35), with a window 10 feet wide and 6 feet high, with white ceiling and walls, at Jacksonville, Fla., we go to table I and see that the total illumination (from window, ceiling, and walls) from a window of these dimensions for a sky brightness of 100 candles per square foot, is 2.4 foot-candles. Turning to table X we see that for the latitude of 30 degrees in the Eastern States the average sky brightness at 4 p. m. is 181 candles per square foot. Since the ratio of 181 to 100 is 1.81, the average illumination at a point 25 feet from the window at 4 p. m. on December 21 in Jacksonville, Fla., will be 2.4×1.81, or 4.3 foot-candles. In Boston, in a latitude of about 42 degrees, the average brightness of the sky, as shown in table X, at 4 p. m. on December 21, is about 119 candles per square foot. The average illumination in that city, under the same conditions as in Jacksonville, would therefore be 2.4×1.19, or 2.9 foot-candles, at 25 feet from the window, as compared with 4.3 foot-candles in Jacksonville. A window of certain dimensions facing in a given direction will produce less illumination in Boston than in Jacksonville. In designing the lighting of a building by daylight in a given locality the average brightness of the sky in that locality may well be considered.

Table X.—*Average brightness of the whole clear sky in the United States in candles per square foot, by season, region, latitude, and hour of the day* [1]

[Explanatory note: The Eastern States may be taken as those bordering the Atlantic Ocean, and having an elevation of from 0 to 500 feet; the Plain States as those lying between the Mississippi River and the Rocky Mountains, elevation 1,500 to 2,000 feet; and the Plateau States as those lying between the Rocky Mountains and the Sierra Nevada and Cascade Mountains, elevation about 7,000 feet. States lying between these regions will have intermediate values]

Region	Latitude in degrees	Local apparent time							
		Noon	11 a. m. or 1 p. m.	10 a. m. or 2 p. m.	9 a. m. or 3 p. m.	8 a. m. or 4 p. m.	7 a. m. or 5 p. m.	6 a. m. or 6 p. m.	5 a. m. or 7 p. m.
December 21									
Eastern States	30	350	339	299	248	181	21	--------	--------
Do	36	330	320	294	241	176	--------	--------	--------
Do	42	311	293	267	215	119	--------	--------	--------
Plain States	30	393	388	350	311	241	21	--------	--------
Do	36	366	359	341	294	214	--------	--------	--------
Do	42	359	349	322	279	160	--------	--------	--------
Do	48	320	315	286	232	28	--------	--------	--------
Plateau States	36	404	398	387	350	265	--------	--------	--------
Do	42	395	384	360	314	172	--------	--------	--------

[1] The values in this table have been calculated from values given by H. H. Kimball (11) for the illumination on a horizontal plane.

Table X.—*Average brightness of the whole clear sky in the United States in candles per square foot, by season, region, latitude, and hour of the day*—Continued

Region	Latitude in degrees	Local apparent time							
		Noon	11 a. m. or 1 p. m.	10 a. m. or 2 p. m.	9 a. m. or 3 p. m.	8 a. m. or 4 p. m.	7 a. m. or 5 p. m.	6 a. m. or 6 p. m.	5 a. m or 7 p. m.
					March 21				
Eastern States	30	434	435	388	320	239	153	18	--------
Do	36	449	422	380	328	266	184	19	--------
Do	42	433	411	370	333	267	198	19	--------
Plain States	30	484	491	438	372	308	212	18	--------
Do	36	480	454	410	368	315	232	19	--------
Do	42	475	453	412	387	327	258	19	--------
Do	48	447	424	409	361	328	254	19	--------
Plateau States	36	523	499	455	411	340	289	19	--------
Do	42	495	477	430	408	360	294	19	--------
					June 21				
Eastern States	30	476	454	397	331	280	202	127	--------
Do	36	491	471	441	372	290	230	155	38
Do	42	496	477	451	386	313	260	194	105
Plain States	30	554	528	462	387	335	259	191	--------
Do	36	532	512	482	406	324	267	187	50
Do	42	526	505	481	414	338	286	217	147
Do	48	516	496	474	419	371	306	250	205
Plateau States	36	586	545	523	452	365	315	264	62
Do	42	565	541	515	444	370	315	250	177
					September 21				
Eastern States	30	413	413	374	305	236	150	21	--------
Do	36	425	403	365	320	260	178	22	--------
Do	42	420	398	365	327	256	194	22	--------
Plain States	30	466	472	421	350	282	192	21	--------
Do	36	462	439	395	348	291	202	22	--------
Do	42	448	425	384	352	277	212	22	--------
Do	48	425	401	387	336	288	217	24	--------
Plateau States	36	504	478	430	380	338	271	22	--------
Do	42	493	464	420	396	333	291	22	--------

It should be noted that in this discussion we have left out of consideration the effect of the orientation of the windows. The direction in which a window faces may make a great difference in the intensity of the illumination at any particular time of the day, and also in the amount of direct sunlight entering the room. Also, the information given is for the light from a clear sky. The brightness of a cloudy sky varies greatly (7).

Ultimately, the size of window required depends upon the amount of illumination which is set up as the standard requirement. If, for instance, in a building at Jacksonville, Fla., with white ceiling and white walls, we say that the illumination at 4 p. m. on December 21, when the sky is clear, at a distance of 25 feet from the window, shall not on an average be less than 6 foot-candles, we again turn to table X or figure 23 and see that for Jacksonville, in latitude 30 degrees, the average sky brightness is 181 candles per square foot. From table I we can find what the dimensions of the window must be to give any desired illumination when the brightness of the sky is 100 candles per square foot. In order to use table I, however, the illumination of 6 foot-candles at 181 candles per square foot must be reduced to what it would be if the illumination were 100 candles per square foot. This is done by multiplying the 6 foot-candles by the ratio of 100 to 181, or by 0.55. Doing this we get 3.3 foot-candles. Looking at table I, we see that at a distance of 25 feet from the window (station 35) to obtain an illumination of 3.3 foot-candles, if the window is 5 feet wide, the height of the window must be at least 11 feet; if 10

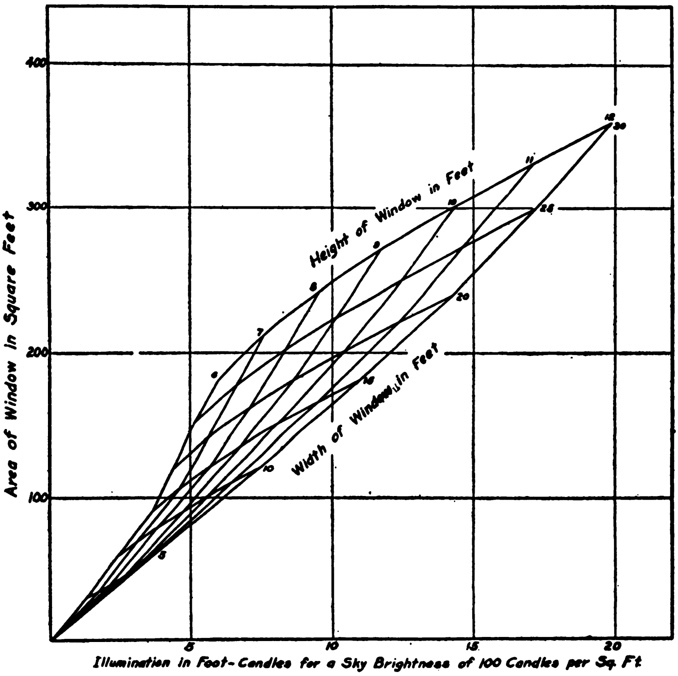


FIGURE 24.—Relation of the area of a window to the illumination which it produces at a distance of 25 feet, for windows of different widths and heights.

feet wide, it must be at least 7 feet high; and if 15 feet wide, it must be at least 6 feet high. It may be noted that the areas of the windows having these dimensions are 55, 70, and 90 square feet, respectively, so that we see that greater window areas are required to produce the 6 foot-candles of illumination as the height of the window is decreased.

The relation between illumination and width, height, and area of window, at a distance of 25 feet from the window (station 35), when the ceiling and walls are white, is shown diagrammatically in figure 24. A similar diagram could be constructed for any other point in the room. This diagram shows the relative importance of window width and height. For instance, windows 25 feet wide and

6 feet high, 15 feet wide and 10 feet high, and 12.5 feet wide and 12 feet high, respectively, have the same window area of 150 square feet but produce illuminations of 5.1, 8.1, and 9.2 foot-candles, respectively. The illumination for the third case is almost double that for the first. Another interesting use of the diagram is for determining window width and height when a given illumination is required. To produce an illumination of 5 foot-candles, with a sky brightness of 100 candles per square foot, a window area varying from 66 to 105 square feet is seen to be necessary, the area required depending upon the relation of the width to the height of the window. A window 7 feet wide and 12 feet high produces as much illumination as a window 30 feet wide and 6 feet high. Window area is an important element in the structural design of a building. The height of the window is limited by the height of the ceiling, and the area of the window is bound up with the problem of heating the building and with architectural considerations.

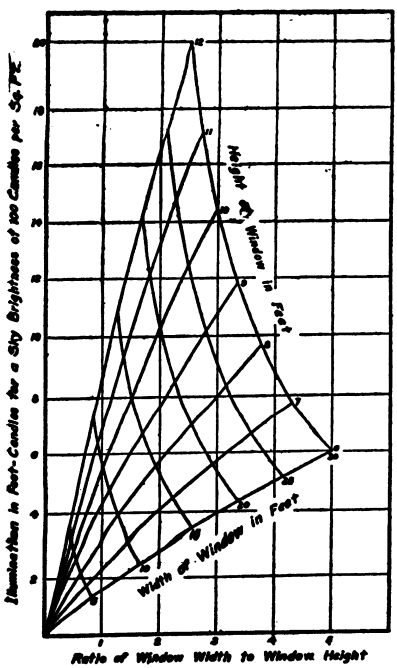


FIGURE 25.—Variation of the illumination produced by a window at a distance of 25 feet with the ratio of window width to window height, for windows of different widths and heights.

Commercial window sash, on account of structural and architectural considerations, comes in certain fixed sizes in which the ratio of window width to window height is fixed. In this connection the curves of figure 25 have been plotted, for the same station and conditions as in figure 24, giving the illumination for windows in which the ratio of window width to window height is varied.

10. COMPARISON OF EXPERIMENTAL RESULTS WITH VALUES CALCULATED FROM THE FORMULAS OF OTHER INVESTIGATORS

P. J. and J. M. Waldram (9) have derived a formula for calculating the illumination due to a vertical window. They find that the illumination, I, on a horizontal plane, may be expressed in foot-candles as follows:

(1) $$I=\frac{\phi B}{4}(\cos 2\theta_1-\cos 2\theta_2)$$

where ϕ is the horizontal angle subtended by the window

B, the brightness of the sky seen through the window, in candles per square foot, assumed to be uniform over the whole sky;

θ_1, the vertical angle between the horizontal plane and the bottom of the window, and

θ_2, the vertical angle between the horizontal plane with the top of the window.

If the horizontal plane is at the height of the bottom of the window $\theta_1 = o$ and formula (1) becomes

$$I = \frac{\phi B}{4}(1 - \cos 2\theta_2) \tag{2}$$

Higbie (10) has also developed a formula for the illumination due to a window which is in the following form:

$$I = -\frac{ab}{2}\left[\frac{1}{\sqrt{a^2+f^2}}\left(\tan^{-1}\frac{m}{\sqrt{a^2+f^2}} + \tan^{-1}\frac{n}{\sqrt{a^2+f^2}}\right) - \frac{1}{\sqrt{a^2+g^2}}\left(\tan^{-1}\frac{m}{\sqrt{a^2+g^2}} + \tan^{-1}\frac{n}{\sqrt{a^2+g^2}}\right)\right] \tag{3}$$

Where:

I is the illumination in foot-candles on a horizontal plane at any point *P* in the room

a, the perpendicular distance of the point *P* from the plane of the window

b, the brightness of the window in candles per square foot, assumed to be uniform

f, the height of the top of the window above the horizontal plane

m, the distance to one side of the window of the foot of the perpendicular from *P* to the plane of the window

n, the distance to the other side of the window of the foot of the perpendicular from *P* to the plane of the window

g, the height of the bottom of the window above the horizontal plane.

If, in this formula, the height of the horizontal plane, which in general represents the working plane, is equal to the height of the bottom of the window, *g*, becomes equal to zero. If further we calculate the illumination along a line perpendicular to one edge of the window, one of the constants *m* or *n* becomes equal to zero. Let us say $n = o$. The formula then becomes much simpler and reduces to

$$I = \frac{b}{2}\left[\tan^{-1}\left(\frac{m}{a}\right) - \frac{a}{\sqrt{a^2+f^2}}\tan^{-1}\left(\frac{m}{\sqrt{a^2+f^2}}\right)\right] \tag{4}$$

In the Waldram formulas, (1) and (2), the variables are angles, and in the Higbie formulas, (3) and (4), linear distances.

It will be noted that neither the Waldram nor the Higbie formulas take into account the illumination reflected from the ceiling and the walls. Tables I to VII show that in some cases the reflected illumination is quite large, and near the walls and in the rear of the room forms a large part of the whole. Their formulas are therefore for a black ceiling and black walls. It will also be noted that in the Waldram formula no account is taken of the loss of light by reflection and absorption by the glass of the window. In Higbie's formula the

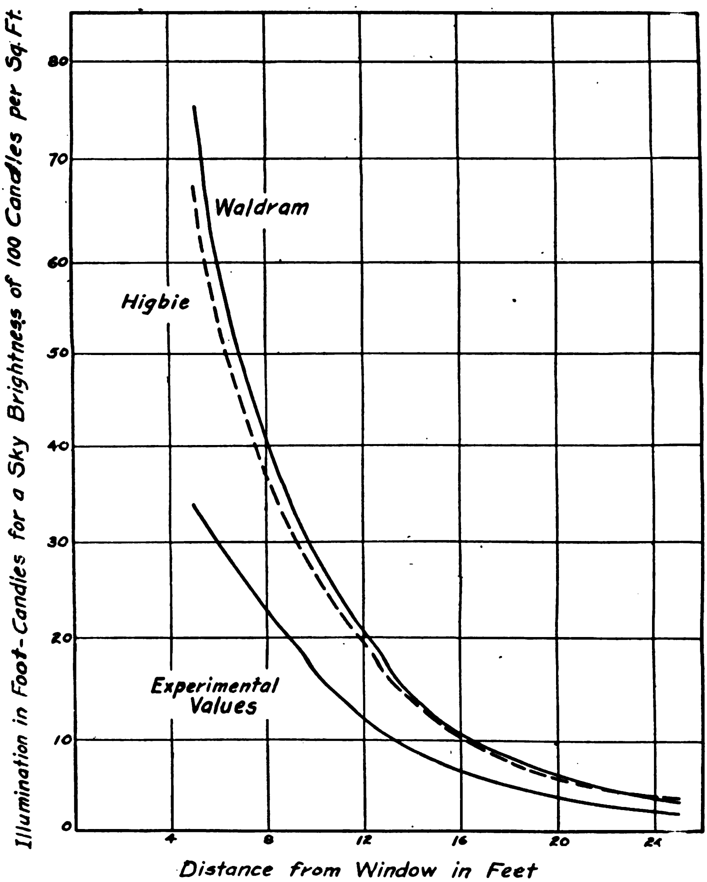


FIGURE 26.—Comparison of experimental values obtained for the illumination with black ceiling and black walls, along the center line from the window to the back wall, with values calculated from the formulas of Waldram and Higbie.

brightness, *b*, is the brightness of the window and the distribution of the brightness over the window is assumed to be uniform.

In figure 26 are given three curves, plotted from values given in table XI, showing the falling off of the illumination, for a sky brightness of 100 candles per square foot, as we go from the middle of a window 15 feet wide and 9 feet high to the rear of the room. Curve 1 gives our experimental values, for black ceiling and walls, for stations 31 to 35, taken from table III, and curves 2 and 3 have been calculated from the Waldram and Higbie formulas (2) and (4), respectively, for the same stations.

It will be noted that the values calculated from the formulas of Waldram and Higbie agree fairly well together but are much higher than the values obtained by us experimentally. These differences can be explained, however, by the fact that the formulas of Waldram and Higbie do not make any allowance for the loss of light due to the reflection and absorption of the glass, or for the loss of light due to the obstruction of light by dirt on the glass and by the framework holding the glass. Turner-Szymanowski (18) found that the loss due to the reflection and absorption of the clean glass varied from 12 to 22 percent according to the position of the observer in the room. An accumulation of dirt during 5 weeks added from 5 to 10 percent more to the loss. Our windows were washed whenever they appeared to be dirty, about once a month. Assuming that the loss of light in our study from reflection and absorption and from dirt was equal to the average of the losses found by Turner-Szymanowski, we would have a loss of light from these two causes for our window of about 23 percent. In our window the steel sash holding the glass, the wooden rims about the window, and the tie-rods, took up about 17 percent of the area of the window, producing an additional loss of light of about 17 percent. The total loss of light from all three causes would then be about 40 percent. Deducting 40 percent from the values obtained from the Waldram and Higbie formulas brings their theoretical values very close to our experimental findings, as is evident from an inspection of table XI. It will be noted from table XI that, comparing the mean of the adjusted calculated values in the seventh column with our experimental values in the second column, the greatest difference occurs in the value of the illumination near the window. This is probably due to three causes: 1, that the loss of light by reflection from the glass is much greater near the window than far from it, whereas an average value for the loss was used in the calculated values; 2, that the height of the window sill in our building was 4 inches above the horizontal plane on which the illumination was measured, which would tend to make our experimental values too small near the window; and 3, the effect of the center mullion, 5½ inches wide, which has its greatest effect near the window.

TABLE XI.—*Comparison of experimental values of illumination for black ceiliug and black walls with values calculated from Waldram's and Higbie's formulas for a window 15 feet wide and 9 feet high. Illuminations are expressed in foot-candles for a sky brightness of 100 candles per square foot*

Distance from window in feet	Experimental values	Calculated values		Calculated values adjusted for light losses		Mean of adjusted calculated values
		Waldram	Higbie	Waldram ×.60	Higbie ×.60	
5	33.9	75.2	67.8	45.1	40.7	42.9
10	16.6	28.8	26.9	17.3	16.1	16.7
15	7.4	12.3	12.1	7.4	7.3	7.4
20	3.7	6.1	5.6	3.7	3.4	3.6
25	2.0	3.3	2.6	2.0	1.6	1.8

11. THEORETICAL CONSIDERATIONS

METHOD OF CALCULATION OF THE LIGHT LOST BY THE REFLECTION AND ABSORPTION OF THE GLASS OF THE WINDOWS

In discussing the sky vault diagram shown in figure 9, we have referred to the correction applied to this diagram for the loss of light in passing through the glass of the window and the dependence of this loss upon the angle at which the light falls upon it.

Glass reflects, refracts, and absorbs light falling upon it, the amount of light reflected, refracted, and absorbed depending upon the angle at which the light falls upon it. Ordinary window glass transmits only about 92 percent of the light falling upon it at right angles, and this amount decreases as the angle of incidence of the light increases.

Fresnel developed an equation from which the fraction R, of the light reflected from the surface of the glass, may be calculated, namely

$$R = \frac{1}{2}\left[\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)}\right] \qquad (1)$$

where i=angle of incidence
r=angle of refraction
R=ratio of intensity of reflected light to incident light.

Since $\frac{\sin i}{\sin r}=n$, where n is the index of refraction of the glass, when $i=0$, that is, when the light is incident normally on the glass, equation (1) becomes $R=\left(\frac{n-1}{n+1}\right)^2$

Since R is the fraction of the incident light reflected from the front surface of the glass, $1-R$ is the fraction of the light transmitted through the front surface.

TABLE XII.—*Fractional part of the incident light transmitted through window glass 3 millimeters thick, having an index of refraction of 1.5, and an absorption factor of 0.014, for angles of incidence from 0 to 90 degrees*

Angle of incident light	0°	10°	20°	30°	40°	50°	60°	70°	80°	85°	90°
Fraction reflected from front surface	0.040	0.040	0.040	0.042	0.046	0.058	0.089	0.171	0.338	0.613	1.000
Fraction transmitted through front surface	.960	.960	.960	.958	.954	.942	.911	.829	.612	.387	.000
Fraction transmitted through both surfaces	.919	.919	.918	.916	.908	.887	.832	.703	.438	.237	.000
Ratio of light transmitted for any angle of incidence to that transmitted for normal incidence	1.000	1.000	.999	.997	.988	.965	.905	.765	.477	.258	.000

Within the glass the light which has entered it suffers multiple reflections from both the front and back surfaces of the glass, and the amount finally transmitted by it through both surfaces is the amount passing through the front surface and the amount reflected within the glass from the front surface, less a small amount of the light absorbed within the glass. Stokes (12, 13, 14, 15) has shown that the proportion, F, of the light passing through the glass to that falling upon it, is given by

$$F=\frac{(1-R)^2k^t}{1-R^2k^{2t}} \tag{2}$$

where R is the fraction of the light reflected from the front surface of the glass, given by equation (1), k is equal to $1-\alpha$, where α is the fraction of the light absorbed in passing through a distance of 1 centimeter of the glass and t is the distance traversed by the light in the glass which is equal to $t_0 \sec r$ where t_0 is the thickness of the glass and r is the angle of refraction. It will be noted that the fraction F, transmitted through the glass varies with the angle of incidence of the light; the index of refraction, n, of the glass; its absorption factor, α, and its thickness, t_0.

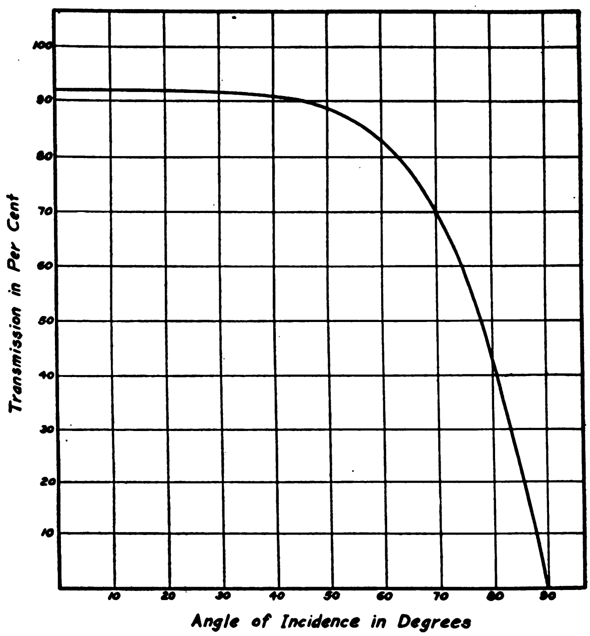


FIGURE 27.—Calculated values or the transmission of light by window glass for angles of incidence from 0 to 90 degrees. Thickness of glass assumed to be 3.0 mm.; index of refraction, 1.5; and absorption factor, 0.014.

The indices of refraction of two samples of the glass used in the windows of the experimental daylight building were determined with a microscope and were found to be 1.53 for one sample and 1.50 for the other. The International Critical Tables, vol. 5, p. 264, gives 0.0140 as the absorption factor for glass.

The proportion of the total light transmitted by the glass of the windows, calculated from the formula given in equation 2, for a thickness of 3 millimeters, an index of refraction of 1.5 and an absorption factor of 0.0140, is given in table XII and plotted in figure 27. It will be noted that the greatest transmission, about 92 percent, occurs at normal incidence. From normal incidence to 40° the amount of light transmitted does not vary much; beyond 40° it becomes less, and at 60° begins to decrease rapidly.

The absorption of light within the glass, 0.0140 for one centimeter, is small compared with the loss due to reflection at the front and back surfaces. If the loss due to absorption is neglected the formula reduces to

$$F=\frac{1-R}{1+R} \tag{3}$$

If both the multiple reflections and the absorption within the glass are neglected, the fraction of the incident light transmitted through both surfaces will be given by

$$F=(1-R)^2 \tag{4}$$

The values obtained from formulas (1) and (2) are given in the second and fourth rows of table XII.

In applying the correction for the amount of light lost by reflection and absorption of the glass of the window, the spacings of the vertical and horizontal scales of the sky-vault diagram shown in figure 9 were corrected in accordance with the ratios given in the bottom row of figures in table XII.

CALCULATION OF THE AVERAGE BRIGHTNESS OF THE SKY FOR DIFFERENT REGIONS AND LATITUDES IN THE UNITED STATES FROM THE VALUES GIVEN BY KIMBALL FOR THE ILLUMINATION ON A HORIZONTAL PLANE

The sky brightness calculated from the illumination on a horizontal plane due to the hemisphere of the sky is the average brightness of the whole sky. It does not tell us anything about the distribution of the brightness over the hemisphere of the sky, although the brightness will in general vary greatly from point to point, being, for a clear sky, greatest near the sun and least at a point opposite the sun and a little less than 90° from it. It is useful, however, as giving the average value.

Kimball (11) in his paper on "Variations in Total and Luminous Solar Radiation with Geographical Position in the United States" gives the total illumination on a horizontal plane from the sun and the sky, by hours of the day and months of the year, for different regions and latitudes in the United States. He also gives the ratio of the amount of light coming from the sun to the amount coming from the sky for different solar altitudes. He also gives the solar altitude for different latitudes, months of the year, and hours of the day. It is therefore possible to calculate the illumination coming from the sky alone from the values given by him for the total illumination on a horizontal plane.

But the illumination I, in foot-candles, coming from a hemisphere of sky of uniform brightness B is equal to the brightness B, expressed in candles per square foot, multiplied by π, where $\pi=3.1416$,

$$\text{or } I=\pi B$$

$$\text{and } B=\frac{I}{\pi}$$

Therefore, if from the values of the total illumination from the sun and the sky we determine the illumination from the sky alone, we can determine an *average* brightness of the sky by dividing these values by 3.1416. Values obtained in this way for the winter and summer solstices and the spring and fall equinoxes, for the Eastern, Plain, and Plateau States, are given in table X. In figure 23 are plotted the values for December 21 at 4 p. m.

12. SUMMARY AND CONCLUSIONS

1. A study has been made of the intensity and distribution of daylight within an experimental building by changing the height and width of the windows and the reflecting power of the ceiling and walls. The study was made with light from the sky alone and no investigation was made of the effect of direct sunlight entering the windows. The illumination was measured on a horizontal plane 36 inches above the floor at 36 stations within the building, for window widths of 9.75, 17.25, and 27 feet, and for window heights of 6, 9, and 12 feet for each of these widths. In each case the top of the window extended to the ceiling. Complete series of measurements were made in the building (1) with the ceiling and walls painted a mat white, (2) with the ceiling white and the walls a mat black, and (3) with both ceiling and walls a mat black.

2. While measurements of illumination were being made within the building, measurements of the brightness of the portion of the sky producing the illumination were made outside the building, so that the illumination within the building and the brightness of the portion of the sky producing it could be correlated. The results of the study are expressed as the illumination that would be produced by a sky having a brightness of 100 candles per square foot (338 millilamberts). The results are therefore independent of variations in the brightness of the sky and depend only on the dimensions and location of the windows, the reflecting power of the walls and ceiling, and the physical properties of the glass used in the windows, its thickness, index of refraction, coefficient of absorption, and its degree of cleanliness.

3. From the results obtained for the three combinations used, viz, white ceiling and white walls, white ceiling and black walls, and black ceiling and black walls, the illumination has been separated into its three components, viz, that coming directly from the window, that reflected from the ceiling, and that reflected from the walls. These three components of the illumination are shown on a distribution chart for 25 points in the room, spaced 5 feet apart, for 15 different combinations of window width and window height. The distribution chart gives the distribution of the illumination on a horizontal plane 36 inches above the floor, for any combination of window width, window height, and for reflecting powers of ceiling and walls of 0

and 78 percent. If the average brightness of the sky seen through the window is known, the intensity of the illumination at any point in a comparable room can be found by multiplying the figures on the chart by the average brightness of the sky expressed in hundreds of candles per square foot.

4. The results show that at the middle and rear of the room an increase in the height of a window produces a much greater increase of illumination than a proportional increase of width. Doubling the height of a window approximately trebles the illumination at the middle or rear of the room, whereas doubling the width of the window does not even double the illumination. Therefore, for the same amount of window area a greater increase of illumination is produced by increasing the height than by increasing the width of the window. The results also show that the uniformity of the illumination is increased by increasing either the height or the width of the window, or by increasing both together. A greater increase of uniformity is produced by increasing the height of the window than by a proportionate increase of width.

5. It is shown that the contribution of the illumination reflected from a white ceiling and white walls to the total illumination is very considerable, varying from 17 to 82 percent of the total. The average contribution of the reflected illumination, for all widths and heights of windows, is 51 percent. In general the percentage contribution of the reflected illumination increases as we go from the window to the rear of the room, and decreases as we go from the side walls toward the middle of the room. By comparing the values of illumination given for white ceiling and walls with those for black ceiling and walls, it is seen that the effect of the light reflected from the ceiling and walls is to make the distribution of the illumination throughout the room more uniform.

6. Figures are given showing the brightness of different portions of the clear north sky at Washington, D. C., for different times of the day and for different seasons of the year. Information is also given as to the average brightness of the sky for different regions and latitudes in the United States, for different seasons of the year and different times of the day.

7. A method has been developed for determining the illumination produced by light from the sky within a building having vertical windows of given dimensions, for any time of day or season of the year in the United States.

8. The experimental results of the study have been compared with values for the illumination calculated from the theoretical formulas of Waldram and Higbie. A fair agreement was found when allowance was made for loss of light by reflection and absorption of the incident light by the glass of the windows, and for the obstruction of the light

by dirt on the glass and by sash bars, casings, and tie rods. The total loss of light due to these causes was estimated to be about 40 percent.

9. The loss of light in ordinary window glass, 3 millimeters thick, due to reflection and absorption of the incident light, has been calculated, and a table given showing the amount of light transmitted for angles of incidence varying from 0 to 90 degrees.

REFERENCES

1. Ives, James E.: Records of Daylight by the Photo-Electric Cell. Trans Illuminating Engineering Soc., 20, 498–510, 1925.
2. Ives, James E.: Studies in Illumination III. A Study of the Loss of Light Due to Smoke on Manhattan Island, New York City, during the year 1927, Especially in its Relation to the Nature of the Weather, the Relative Humidity of the Air, and the Velocity and Direction of the Wind. Public Health Bulletin No. 197, Government Printing Office, Washington, D. C., 1930.
3. Coblentz, W. W.: Various Photo-Electrical Investigations. Scientific Papers of the Bureau of Standards, No. 462, 602–603, 1922.
4. Priest, I. G.: Preliminary Data on the Color of Daylight at Washington. Jour. Optical Soc. of America, 7, 78–79, 1923.
5. Taylor, A. H.: The Color of Daylight. Trans. Illuminating Engineering Soc., 25, 154–160, 1930.
6. Goodwin, Jr., W. N.: The Photronic Illumination Meter. Trans. Illuminating Engineering Soc., 27, 828–835, 1932.
7. Kimball and Hand: Sky-Brightness and Daylight Illumination Measurements. Trans. Illuminating Engineering Soc., 16, 255–275, 1921. Monthly Weather Review, 49, 481–488, 1921.
8. Basquin, O. H.: Daylight Illumination II. Brightness of the Sky. The Illuminating Engineer (New York), 1, 823–829, 930–936, 1906–07.
9. Waldram, P. J. and J. M.: Window Design and the Predetermination of Daylight Illumination. The Illuminating Engineer (London) April–May 1923, pp. 90–117.
10. Higbie, H. H.: Prediction of Daylight from Vertical Windows. Trans. Illuminating Engineering Soc., 20, 433–473, 1925.
11. Kimball, H. H.: Variations in the Total and Luminous Solar Radiation with Geographical Position in the United States. Monthly Weather Review, 47, 769–793, 1919.
12. Stokes, George G.: On the Intensity of the Light reflected from or transmitted through a pile of plates. Proc. Royal Society, 11, p. 547, 1862.
13. Walsh, J. W. T.: Photometry. Constable & Co., Ltd., London, 1926, p. 117.
14. Benford, Frank: Reflection and Transmission by Parallel Plates. Jour. Optical Soc. of America, 7, 1017–1025, 1923.
15. Walsh, J. W. T.: Appendix to Transmission Factor of Commercial Window Glasses. Illumination Research, Technical Paper No. 2, Department of Industrial and Scientific Research, His Majesty's Stationery Office, 1926.
16. Report of the Committee on Daylight Illumination. Proceedings, International Congress on Illumination, 1928, p. 473.
17. Ives, James E., and Knowles, F. L.: Recent Measurements of the Brightness of the Clear North Sky in Washington, D. C. Trans. Illuminating Engineering Soc., 30, 281–291, 1935.
18. Turner-Szymanowski, Waclaw: A Rapid Method for Predicting the Distribution of Daylight in Buildings. Engineering Research Bulletin No. 17, January 1931. Department of Engineering Research, University of Michigan.

APPENDIX

TABLE A (Appendix).—*Average brightness, in candles per square foot, of the clear north sky in Washington, D. C., from 8 a. m. to 4 p. m., for the months of May, June, and July, by zones and segments*

MAY

Hour ending	Sky segments																		Average for all segments
	Lower zone						Middle zone						Upper zone						
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	
9																			
10																			
11	451	436	414	441	525	664	359	342	349	467	507	709	376	547	567	626	778	755	517
12	458	508	483	504	601	711													
1	562	501	460	464	471	483	468	382	335	330	339	366	936	644	528	507	518	574	493
2	691	561	473	444	441	453	658	436	342	299	282	310	1,005	629	481	427	399	406	486
3	735	555	437	387	380	389	739	450	316	251	226	221	893	538	402	337	282	280	434
4																			
Average for all hours	579	512	453	448	484	540	556	402	336	337	338	402	802	590	494	474	494	504	

JUNE

Hour ending	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Average for all segments
9	407	413	424	550	931	1,575	236	262	308	453	806	1,892	279	296	367	486	702	1,042	635
10	410	436	446	504	742	1,126	268	309	337	424	638	1,293	336	362	407	511	715	1,246	584
11	464	440	429	450	547	732	392	383	404	468	617	917	601	584	618	719	908	1,371	614
12	441	484	427	419	450	533	346	316	313	339	394	503	590	568	578	600	645	804	483
1	483	467	444	426	430	456	456	389	350	310	318	356	827	669	609	607	609	629	491
2	672	544	463	417	424	437	758	496	386	342	323	390	1,145	705	572	508	461	454	528
3	746	582	417	370	373	389	907	396	268	211	191	186	891	430	315	251	224	216	407
4	853	615	429	367	376	399	1,054	403	265	208	182	175							
Average for all hours	560	488	435	438	534	706	552	369	329	344	434	714	667	516	495	526	609	823	

JULY

Hour ending	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Average for all segments
9																			
10																			
11	402	396	392	410	511	672	270	266	290	356	477	708	420	419	444	523	661	686	461
12	419	403	394	404	447	575	313	270	306	343	419	565	491	514	532	568	662	860	471
1	487	429	396	383	406	448	510	404	363	353	382	437	701	568	538	541	558	607	473
2	628	493	410	369	379	384	588	392	299	253	232	229	936	580	427	366	340	339	424
3	752	555	422	360	367	383	748	431	280	222	202	202	1,145	507	356	275	245	241	427
4	974	634	434	377	356	377	1,175	444	350	211	188	184	531	337	242	194	171	162	408
Average for all hours	610	485	408	384	411	473	601	368	315	290	317	388	704	488	423	411	440	482	

Table B (Appendix).—*Average brightness, in candles per square foot, of the clear north sky in Washington, D. C., from 8 a. m. to 4 p. m. for the months of September, October, and November, by zones and segments*

SEPTEMBER

Hour ending	Sky segments: Lower zone: A	B	C	D	E	F	Middle zone: G	H	I	J	K	L	Upper zone: M	N	O	P	Q	R	Average for all segments
9	345	345	315	296	391	692	166	166	164	184	261	442	161	161	169	193	246	323	279
10	324	335	324	306	353	556	175	174	178	198	261	420	212	205	214	237	302	399	287
11	324	339	338	323	361	492	192	184	188	206	260	396	254	244	246	274	336	459	301
12	343	362	366	347	347	379	210	184	183	191	213	279	315	276	271	283	328	414	294
1	556	523	510	454	406	376	356	296	232	204	205	228	420	311	281	295	311	365	292
2	433	363	324	317	331	308	299	211	172	161	161	168	334	237	189	170	166	173	256
3	556	390	325	312	337	329	401	248	190	166	165	167	377	281	215	190	182	186	279
4	742	432	324	306	338	336	510	276	190	163	163	164	383	269	204	174	163	161	204
Average for all hours	453	386	353	333	358	434	289	217	187	184	211	283	307	248	224	227	254	310	--------

OCTOBER

Hour ending	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Average for all segments
9																			
10	308	323	305	280	323	494	178	180	180	186	228	364	196	188	191	208	251	338	282
11	302	309	302	279	297	367	190	180	181	187	218	314	229	210	206	221	258	336	255
12	312	305	308	295	298	331	204	182	182	183	199	253	246	220	210	215	242	303	249
1	348	307	309	308	308	303	247	190	173	171	173	158	284	223	199	192	198	223	240
2	402	321	309	311	323	306	296	202	172	167	167	172	312	231	194	181	182	194	247
3	476	311	278	289	318	304	309	198	161	157	160	160	254	195	162	149	145	147	231
4	482	286	234	247	289	291	270	171	136	135	142	142	193	156	130	118	118	118	203
Average for all hours	376	309	292	287	308	342	242	186	169	169	184	223	245	203	185	183	199	237	--------

NOVEMBER

Hour ending	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Average for all segments
9																			
10	313	322	312	288	311	431	187	187	187	185	217	335	188	183	183	189	219	287	251
11	326	338	341	320	324	378	190	184	187	182	196	284	196	182	180	186	209	260	248
12	331	336	345	339	318	327	189	169	168	169	171	201	196	175	166	168	182	218	232
1	333	338	344	348	334	315													
2																			
3																			
4	490	258	190	173	174	173	436	260	188	167	164	164	358	267	216	195	187	187	236
Average for all hours	359	318	306	294	292	325	250	200	182	176	187	244	234	202	186	184	199	238	--------

TABLE C (Appendix).—*Illumination in foot-candles for white ceiling and white walls, at stations 6 feet apart, for a sky-brightness of 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet								
	9.75			17.25			27		
	Height of window in feet			Height of window in feet			Height of window in feet		
	6	9	12	6	9	12	6	9	12
1 or 31	1.52	2.98	4.19	5.32	7.14	8.47	53.8	54.0	52.9
2 or 32	3.20	5.89	8.13	10.1	15.4	19.1	29.1	40.4	46.5
3 or 33	3.29	6.51	9.63	7.24	12.3	18.0	12.6	21.4	33.0
4 or 34	3.07	5.75	8.54	5.42	9.64	14.6	8.04	15.4	23.9
5 or 35	2.23	4.59	6.91	3.68	7.38	11.2	5.58	13.0	18.4
6 or 36	2.53	4.52	6.78	4.20	7.62	11.0	6.00	11.3	17.7
7 or 25	3.27	5.06	6.70	32.1	34.6	34.0	61.0	63.7	66.5
8 or 26	6.82	11.1	14.0	18.5	26.5	31.8	34.1	45.3	55.0
9 or 27	4.31	8.61	12.3	8.00	15.1	21.2	12.4	23.2	35.1
10 or 28	2.79	6.12	9.32	5.38	10.2	15.5	8.10	15.4	24.9
11 or 29	2.06	4.85	7.68	3.65	7.60	12.5	5.33	11.4	19.7
12 or 30	2.56	5.24	7.81	4.53	8.39	13.0	6.32	12.6	19.9
13 or 19	47.3	50.2	50.1	54.8	57.8	56.4	54.1	60.0	62.0
14 or 20	22.0	29.9	34.4	30.5	41.0	47.3	33.5	47.6	57.1
15 or 21	6.58	12.7	17.0	9.97	18.2	25.8	12.7	23.7	36.8
16 or 22	3.70	6.95	10.9	6.06	11.1	17.2	8.38	16.0	25.9
17 or 23	2.18	4.81	7.39	3.72	6.93	12.2	5.38	11.2	19.6
18 or 24	2.59	5.16	7.50	4.50	8.56	12.7	6.70	13.2	20.2

TABLE D (Appendix).—*Illumination in foot-candles for white ceiling and black walls, at stations 6 feet apart, for a sky-brightness of 100 candles per square foot, for different combinations of width and height of window.*

Station	Width of window in feet								
	9.75			17.25			27		
	Height of window in feet			Height of window in feet			Height of window in feet		
	6	9	12	6	9	12	6	9	12
1 or 31	0.10	0.54	0.62	1.89	2.64	3.12	39.2	42.3	39.8
2 or 32	.96	2.52	3.47	5.16	7.96	10.7	18.0	26.5	30.0
3 or 33	1.42	2.77	4.64	3.71	5.55	9.39	6.83	11.3	18.5
4 or 34	1.08	2.26	3.68	2.26	3.68	5.96	3.42	5.96	10.4
5 or 35	.64	1.34	2.00	1.35	2.32	3.85	1.83	3.53	6.06
6 or 36	.43	1.08	1.53	1.04	1.69	2.92	1.07	2.48	4.31
7 or 25	1.84	2.82	2.99	25.9	28.4	28.1	52.2	56.8	57.2
8 or 26	5.15	8.49	9.57	14.5	19.7	25.6	26.6	35.5	42.7
9 or 27	3.10	5.61	8.38	6.32	9.85	14.8	9.20	15.6	24.3
10 or 28	1.60	3.26	4.78	3.47	5.03	8.31	4.65	7.91	11.9
11 or 29	.99	1.80	2.79	1.66	2.79	4.86	2.55	4.64	7.42
12 or 30	.75	1.39	1.92	1.14	2.10	3.54	1.28	3.15	4.81
13 or 19	43.3	47.4	47.9	48.7	55.4	53.5	51.2	58.4	57.5
14 or 20	18.7	28.4	31.7	26.6	34.9	41.6	27.4	39.0	49.6
15 or 21	5.71	9.49	14.0	8.90	14.3	21.6	10.6	18.1	27.8
16 or 22	2.72	4.67	6.66	4.55	6.69	10.6	5.67	9.38	14.6
17 or 23	1.27	2.04	2.95	2.00	3.25	5.40	2.69	5.16	8.18
18 or 24	.96	1.62	2.03	1.51	2.45	3.97	1.69	3.68	5.78

TABLE E (Appendix).—*Illumination in foot-candles for black ceiling and black walls, at stations 6 feet apart, for a sky-brightness of 100 candles per square foot, for different combinations of width and height of window*

Station	Width of window in feet								
	9.75			17.25			27		
	Height of window in feet			Height of window in feet			Height of window in feet		
	6	9	12	6	9	12	6	9	12
1 or 31	----------	----------	----------	0.58	0.69	0.65	31.3	38.2	36.6
2 or 32	0.79	1.71	2.19	3.73	5.64	7.20	13.2	23.0	25.9
3 or 33	1.16	2.39	3.62	2.61	4.68	6.92	5.1	10.1	15.5
4 or 34	.65	1.66	2.66	1.59	2.56	4.15	2.08	4.82	7.05
5 or 35	.36	1.04	1.63	.86	1.66	2.48	1.23	2.55	4.72
6 or 36	.21	.64	1.14	.67	1.35	2.11	.89	1.81	3.95
7 or 25	.91	1.21	1.40	22.5	22.6	21.6	41.7	52.1	52.9
8 or 26	3.88	6.80	7.66	11.0	15.1	18.9	18.7	30.6	36.1
9 or 27	2.41	4.98	6.76	4.63	7.81	11.2	6.71	13.4	19.9
10 or 28	.99	2.44	4.01	2.03	3.56	6.03	2.85	5.95	9.21
11 or 29	.55	1.38	2.10	1.31	1.83	3.24	1.48	3.20	5.57
12 or 30	.34	.77	1.60	.72	1.28	2.53	.91	2.15	3.88
13 or 19	41.6	42.0	41.4	41.6	45.5	44.4	40.1	50.6	51.3
14 or 20	16.6	23.7	25.9	19.5	27.2	31.8	19.6	33.7	42.8
15 or 21	4.94	9.07	12.1	6.80	11.6	17.0	7.76	15.8	23.1
16 or 22	1.50	3.05	5.17	2.67	4.81	7.68	3.46	6.69	10.88
17 or 23	.68	1.65	2.38	1.23	2.14	3.70	1.47	3.39	5.75
18 or 24	.40	.85	1.48	.65	1.48	2.56	1.08	2.65	4.30

TABLE F (Appendix).—*Effective area of the sky vault visible at each station in the experimental building expressed as a fraction of the total effective area of the quarter sphere of the sky*

Station	Width of window in feet								
	9.75			17.25			27		
	Height of window in feet			Height of window in feet			Height of window in feet		
	6	9	12	6	9	12	6	9	12
1 or 31	0.001	0.001	0.001	0.009	0.012	0.014	0.455	0.512	0.513
2 or 32	.014	.024	.034	.051	.084	.106	.173	.265	.311
3 or 33	.014	.030	.046	.032	.047	.095	.061	.104	.173
4 or 34	.009	.019	.032	.017	.033	.056	.030	.056	.095
5 or 25	.006	.013	.020	.010	.022	.036	.016	.035	.056
6 or 36	.004	.008	.014	.006	.014	.024	.010	.022	.039
7 or 25	.015	.023	.025	.309	.351	.354	.584	.672	.680
8 or 26	.053	.090	.100	.151	.236	.262	.259	.394	.447
9 or 27	.028	.052	.080	.052	.097	.147	.080	.148	.224
10 or 28	.015	.029	.046	.027	.050	.080	.041	.073	.120
11 or 29	.007	.017	.026	.012	.029	.045	.020	.043	.067
12 or 30	.005	.011	.018	.009	.018	.030	.013	.027	.044
13 or 19	.547	.622	.633	.592	.678	.692	.596	.686	.700
14 or 20	.192	.282	.339	.282	.418	.505	.304	.459	.559
15 or 21	.050	.096	.137	.078	.147	.214	.097	.184	.271
16 or 22	.021	.038	.062	.034	.061	.101	.045	.080	.133
17 or 23	.009	.019	.029	.015	.032	.049	.022	.046	.072
18 or 24	.006	.012	.019	.010	.021	.033	.015	.032	.050

Table G (Appendix).—*Ratio of the illumination, expressed in foot-candles for a sky-brightness of 100 candles per square foot, at each station in the experimental building, to the ratio of the effective area of the sky vault visible at that station to the total effective area of the quarter sphere of the sky*

[Explanatory note: The values given in this table were obtained by dividing the values in table E by the corresponding values in table F]

Station	Width of window in feet: 9.75 — Height of window in feet: 6	9.75 — 9	9.75 — 12	17.25 — 6	17.25 — 9	17.25 — 12	27 — 6	27 — 9	27 — 12
1 or 31	--------	--------	--------	64	58	46	69	75	71
2 or 32	56	71	64	73	67	68	76	87	83
3 or 33	83	80	79	82	100	73	84	97	90
4 or 34	72	87	83	94	78	74	69	86	74
5 or 35	60	80	82	86	75	69	77	73	84
6 or 36	52	80	81	112	96	88	89	82	101
7 or 25	61	53	56	73	64	61	71	78	78
8 or 26	73	76	77	73	64	72	72	78	81
9 or 27	86	96	84	89	80	76	84	90	89
10 or 28	66	84	87	75	71	75	70	82	77
11 or 29	79	81	81	109	63	72	74	74	83
12 or 30	68	70	89	80	71	84	70	80	88
13 or 19	76	68	65	70	67	64	67	74	73
14 or 20	86	84	76	69	65	63	64	73	77
15 or 21	99	94	88	87	79	79	80	86	85
16 or 22	71	80	83	78	79	76	77	84	82
17 or 23	76	87	82	82	67	76	67	74	80
18 or 24	67	71	78	65	70	78	72	83	86

O